CONTROLLER DESIGN ON MATLAB PLATFORM BY DISCRETE FREQUENCY RESPONSE

OBJECTIVE

To examine the consequences of negative feedback of different gains on Gain and Phase margins in a digital OLTF. As well as looking at the sensitivity of choice of sampling time on the same.

WHAT IS GAIN AND PHASE MARGIN

Consider a system with open loop gain AOL closed with a negative feedback β.

$$\frac{AOL}{1+\ \beta AOL}$$

We can easily observe that this relation can approach ∞ when the product $\beta A_{OL} = -1$. Bode plots are used to determine just how close a system is to thereby become unstable.

One measure of proximity to instability is the **gain margin**. It is defined as the distance separation of |βAOL| in dB from 0dB at the frequency where phase of βAOL is -180° (**phase crossover frequency**).

Another equivalent measure of proximity to instability is the **phase margin**. It is defined as the distance of the phase in degrees above −180° at the frequency where its magnitude reaches unity (**gain crossover frequency**).

For discrete-time systems, bode evaluates the frequency response on the unit circle

$$\mathrm{z} = \mathrm{e}^{\mathrm{j}\omega\mathrm{Ts}}\ and\ 0 \le \omega \le \omega_{\mathrm{N}} = \frac{\pi}{\mathrm{Ts}}$$

where Ts is the sampling time and ω_N is the Nyquist frequency

THE SYSTEM'S OPEN LOOP TRANSFER FUNCTION

$$10^{-5} * \frac{4.711z + 4.644}{z^3 - 3.5z^2 + 2.753z - 0.8781}$$

With nominal sampling time as 0.01sec and is closed with a negative feedback gain which varies from 0.5 to 10.

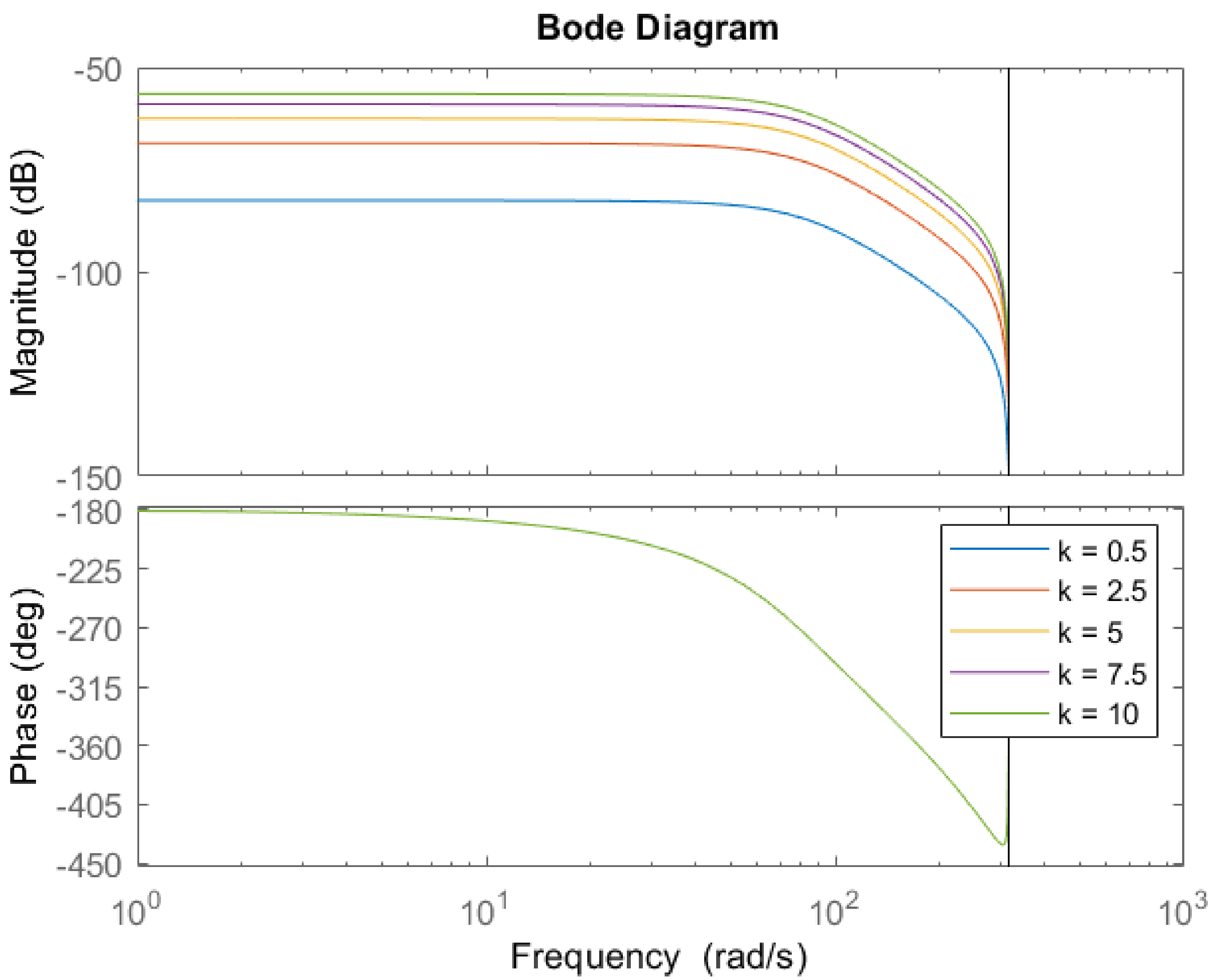
EFFECT OF VARIABLE GAIN ON GAIN AND PHASE MARGIN

When we multiply with a gain, it doesn't affect the phase crossover frequency therefore its always calculated at the same frequency and that is 0 rad/sec.

However, gain margin decreases due to change in the magnitude of the transfer function which decreases on increasing β as its given by:

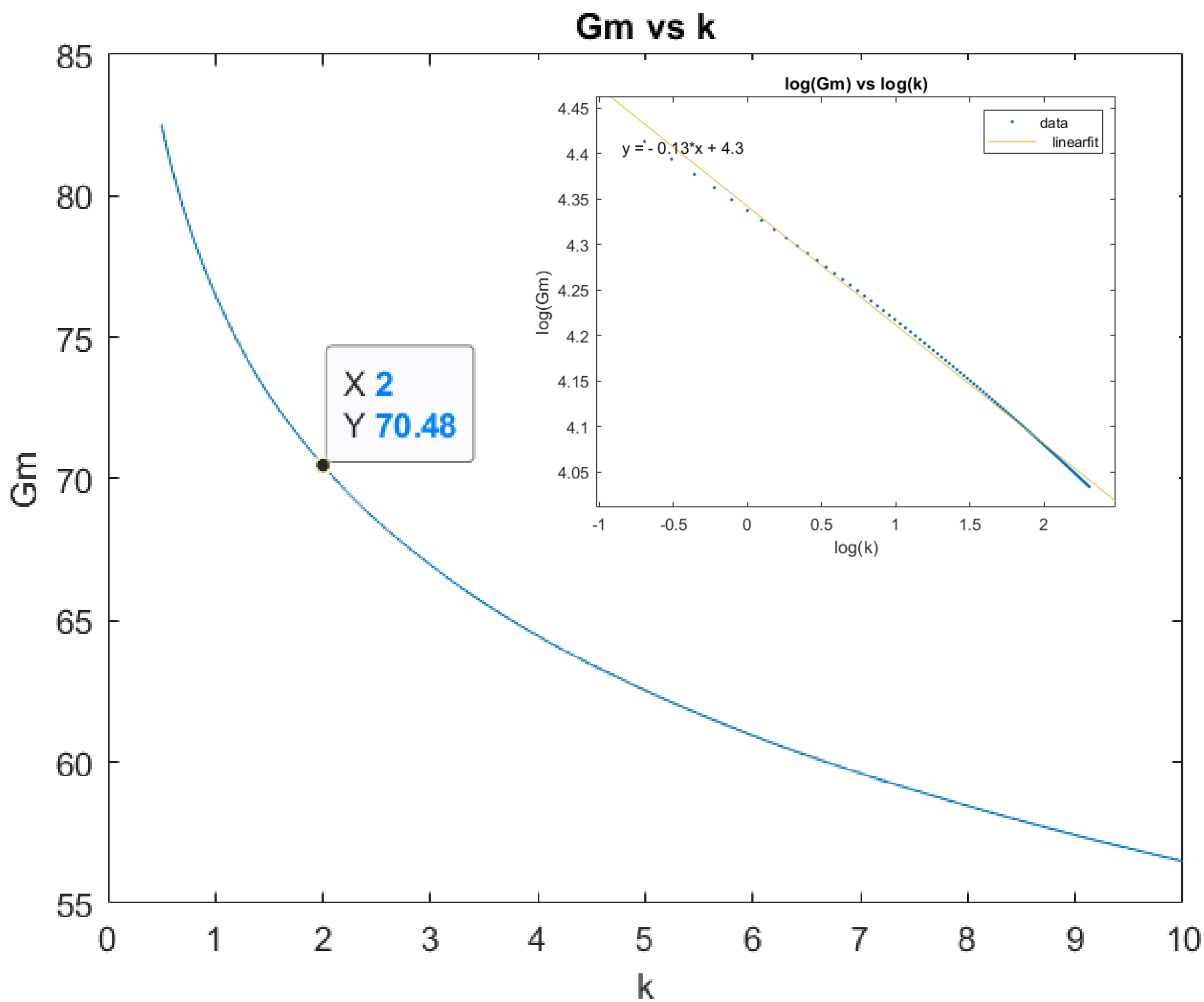
$$Gm = -log\beta - logA_{OL}$$

The very same thing can be observed with the bode plot.



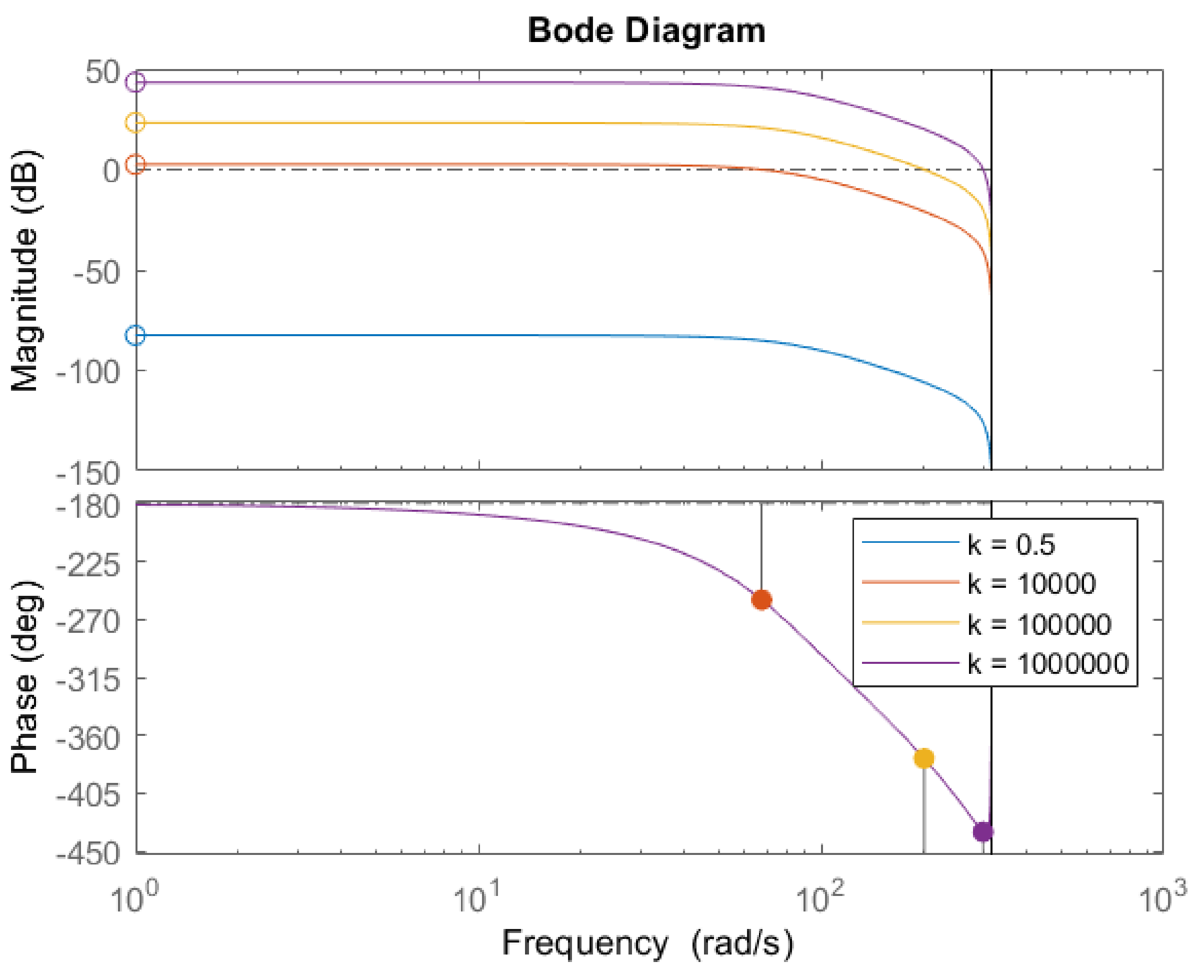
β	0.5	2.5	5	7.5	10
Gm	82.5	68.5	62.5	59	56.4

A negative gain margin indicates that stability is lost by decreasing the gain, while positive gain margins indicate that stability is lost by increasing the gain. This is very well visible from the plot shown below. As the gain increases from 0.5 to 10, the Gm which was initially 82.5 fell down to 56.4. And as we know, lower absolute value of Gain margin implies lower proximity to instability.



Also, it was found that the log Gm vs log k assumed an almost linear relationship for the region where k varies from 0.5 to 10.

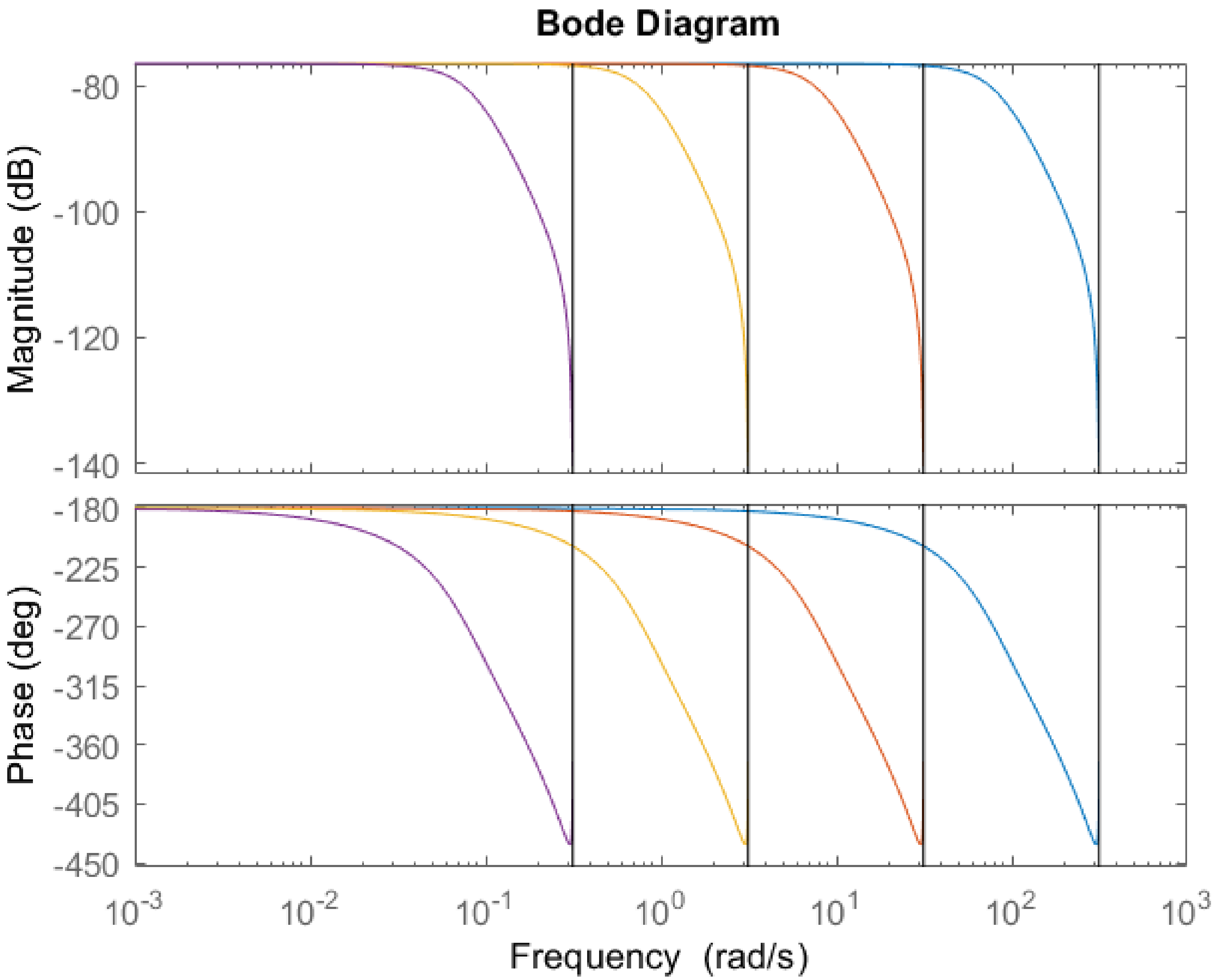
It is important to know that the phase margin is undefined as there is no point where its magnitude reaches 0dB. Hence, we cannot observe the effect of change of k over phase margin. But we can say that since gain cross over frequency changes due to change of k. Therefore, in general the phase margin would change as it would be calculated at some different frequency for every k.



It's interesting to note that for very high k values we do get 0dB and with increase of k, we actually get an increasing value of gain crossover frequency.

Effect of variable sampling time on gain and phase margin

You may assume that the change of sampling time changes the Gain and Phase margin but as we mentioned earlier how z and Ts are related, it can be easily concluded that for a given k as we increase the sampling time, the magnitude and phase plots get squeezed as ωT should remain constant for a phase. It can also be visualized from the plots below…



Therefore, margins are not affected at all but gain crossover frequency and phase crossover frequency would change.

It is important to mention that this system is inherently unstable as well as the Barkhausen criterion is correct only for minimum phase functions to predict the stability.

Submitted by:

Harsh Kadian (2017eeb1142)

Mrityunjay (2017eeb1153)

Pradeep (2017eeb1159)

Group 15