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CONTROLLER DESIGN ON MATLAB PLATFORM USING ANALOG ROOT LOCI

OBJECTIVE

- To design a cascade feedback PID controller for a third-order OLTF given by:

$$G(s) = \frac{1}{(s+15)(s^2+2s+10)}$$

  to realise a second-order CLTF such that ζ = 0.5 and ω ≥ 4 rad/s

- To perform sensitivity analysis for the variation of key parameters in the denominator.

CONTROLLER DESIGN

A general PID controller is given by:

$$C(s) = k(1 + \frac{1}{sTi} + sTd)$$

and, the closed-loop expression for the required system would, therefore, result in

$$Y(s) = G(s)\frac{H(s)}{1+G(s)H(s)}$$

$$Y(s) = \frac{1}{1 + \frac{(s+15)(s^2+2s+10)}{\frac{k(1+sTi+s^2TdTi)}{sTi}}}$$

$$Y(s) = \frac{1}{1 + \frac{s(s+15)(s^2+2s+10)}{kTd(s^2+\frac{s}{Td}+\frac{1}{TiTd})}}$$

Comparing the last equation, It can be quickly concluded that for being reduced to a second-order CLTF $Td = 0.5 \; and \; Ti = 0.2$. Moreover, the system's transfer function becomes:

$$Y(s) = \frac{kTd}{kTd + s(s+15)}$$

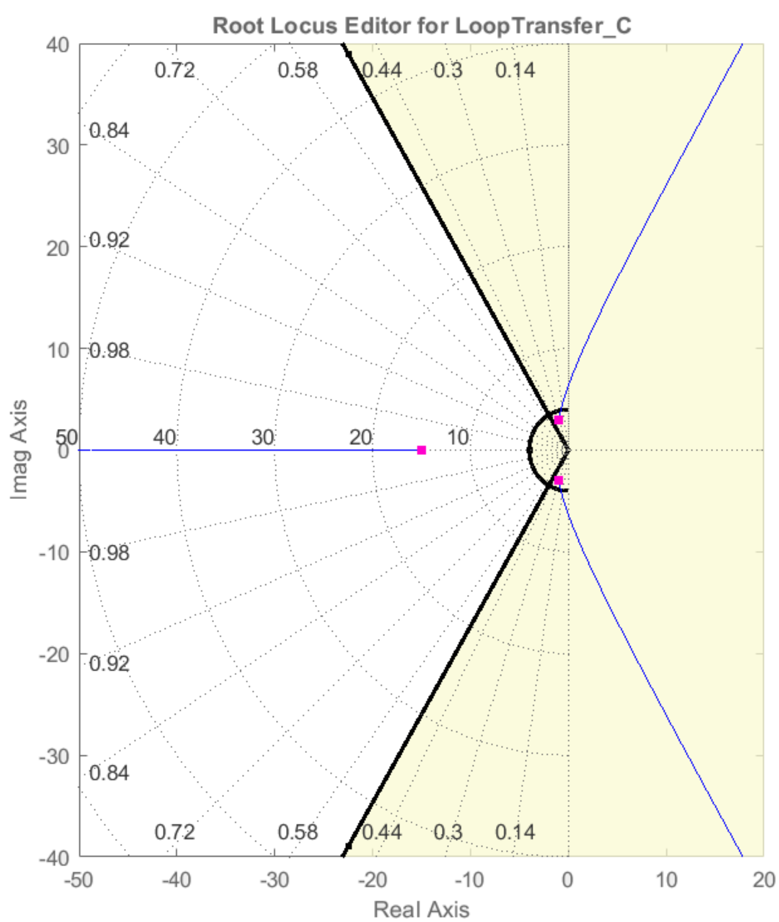
THE CLTF

To comply according to the given constraints, comparing with the standard second-order equation results in ω = 15rad/sec and $kTd = \omega^2 \ i.e. 225$

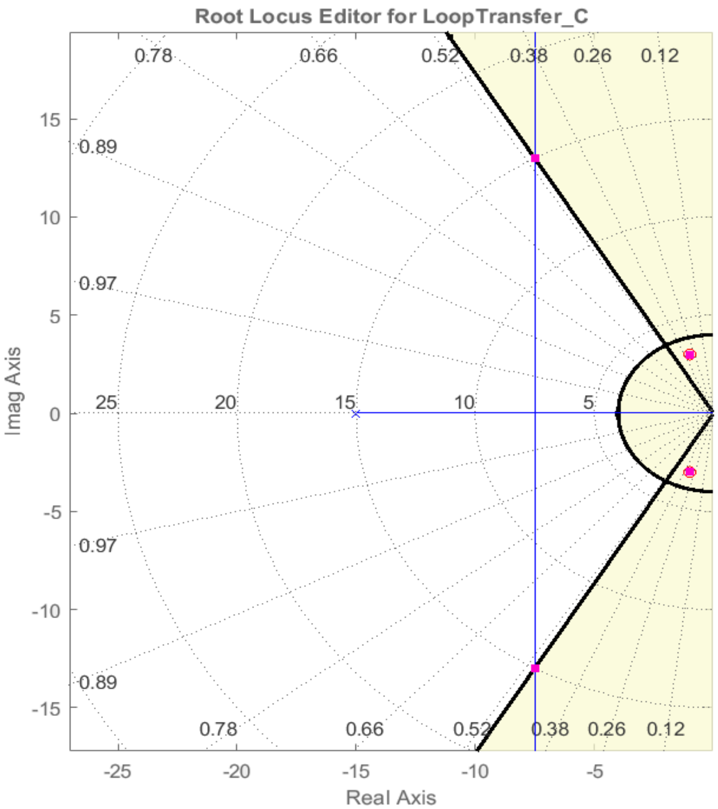
Hence,

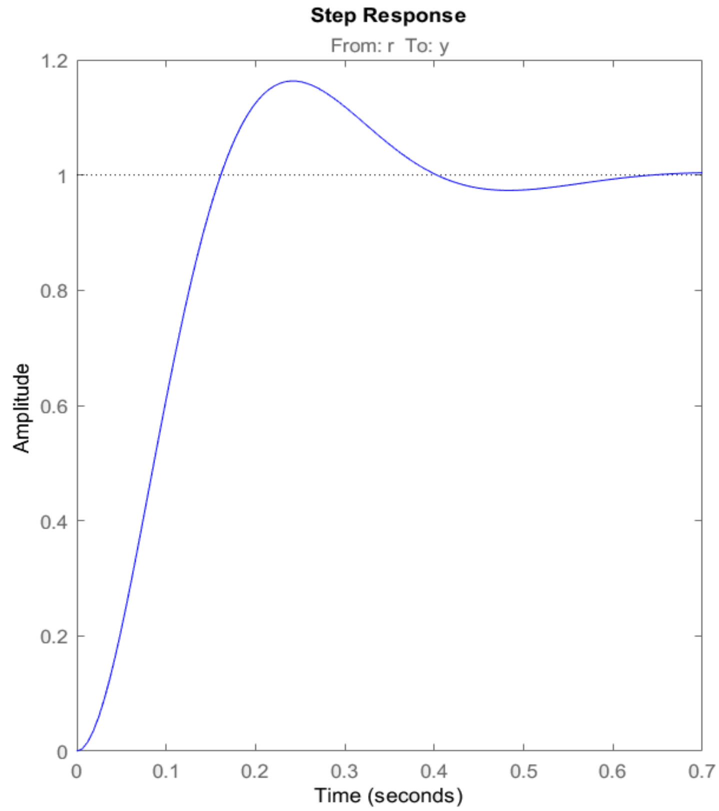
$$Y(s) = \frac{225}{s^2 + 15s + 225}$$

The similar expression could be achieved conveniently from the root loci of G(s) as well.



By the constraints, first and foremost no pole should lie within the restricted area, which makes us employ complex zeros in C(s) to cancel out the poles of G(s). For a PID controller, one pole should be placed at zero. The following result in the root loci shown as below:





Since ζ = 0.5, the poles should lie on the straight lines representing the constant ζ and finally choosing the gain along the root loci results in ω = 15rad/sec. The step response for the CLTF thereby obtained is shown on the right.

SENSITIVITY ANALYSIS

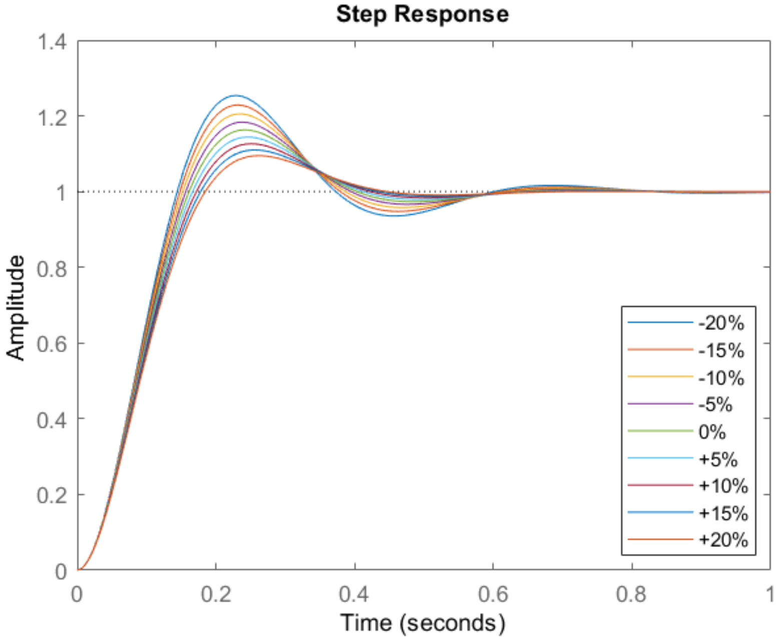
With the variation in ζ and ω, the poles of the transfer function Y(s) would shift along the straight line and the circle respectively. Therefore, locus of change of poles with variation in ζ would result in an arc and ω would result in movement along the straight line.

DYNAMIC RESPONSE WITH VARIATION IN ζ and ω

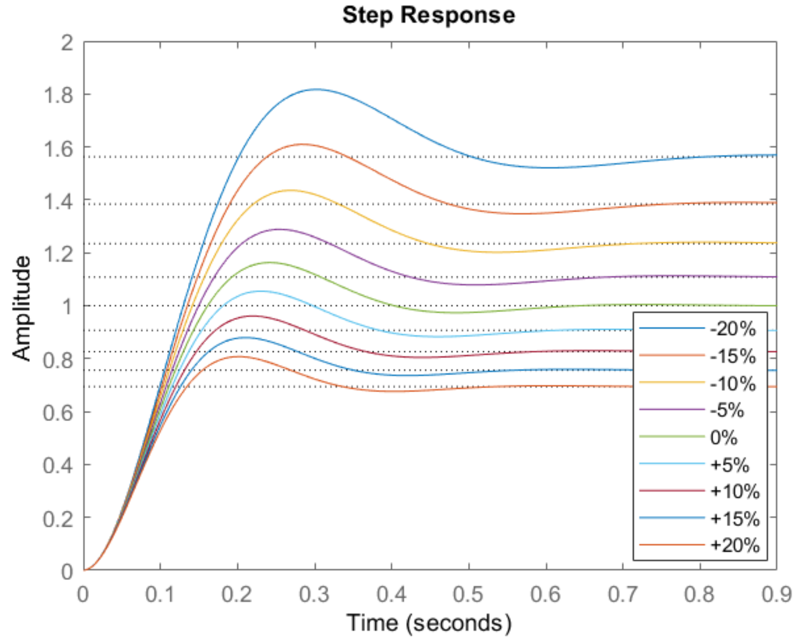
The performance characteristics of dynamic response with variation in ζ and ω are tabulated below:

Qualitative analysis of variation in ζ reveals that the system response should damp faster for a higher value of ζ which indeed is true. It would result in lower overshoots as well as settling time and rise time would be reduced as well.

For the variation in ω, the increase in the corresponding value should result in much more sustained oscillations. Apart from that, the DC gain would be affected due to change in denominator parameters only and changes accordingly.



Variation in ζ



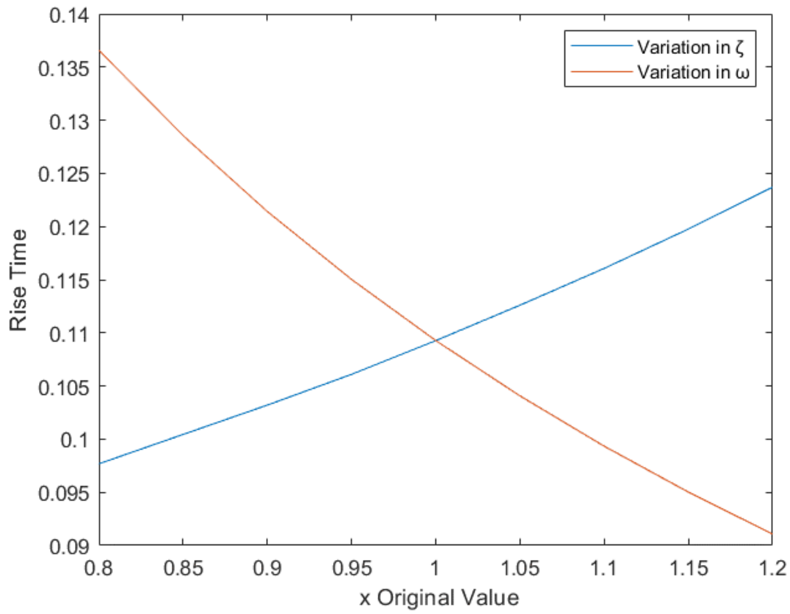
Variation in ω

Variation	Rise Time	Settling Time	Overshoot	Peak
-20%	0.0976	0.5606	25.3740	1.2537
-15%	0.1004	0.5595	22.8773	1.2287
-10%	0.1032	0.5563	20.5192	1.2051
-5%	0.1060	0.5500	18.3430	1.1834
0%	0.1092	0.5383	16.293	1.1629
5%	0.1125	0.5107	14.4005	1.1440
10%	0.1160	0.3887	12.6321	1.1263
15%	0.1197	0.3927	10.9928	1.1099
20%	0.1237	0.3962	9.47726	1.0947

Variation	Rise Time	Settling Time	Overshoot	Peak
-20%	0.1365	0.6729	16.293	1.817
-15%	0.1285	0.6334	16.293	1.609
-10%	0.1214	0.5982	16.293	1.435
-5%	0.1150	0.5667	16.293	1.288
0%	0.1092	0.5383	16.293	1.629
5%	0.1040	0.5127	16.293	1.054
10%	0.0993	0.4894	16.293	0.961
15%	0.0950	0.4681	16.293	0.879
20%	0.0910	0.4486	16.293	0.807

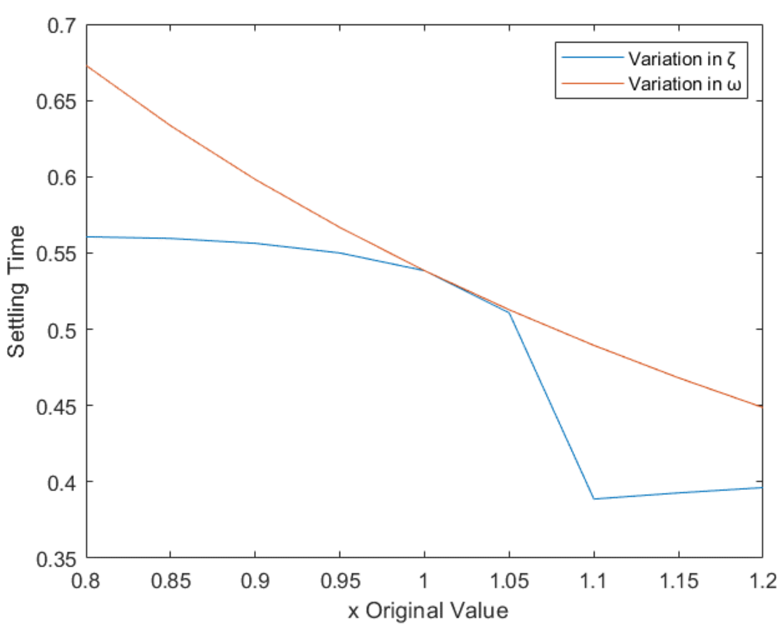
RISE TIME

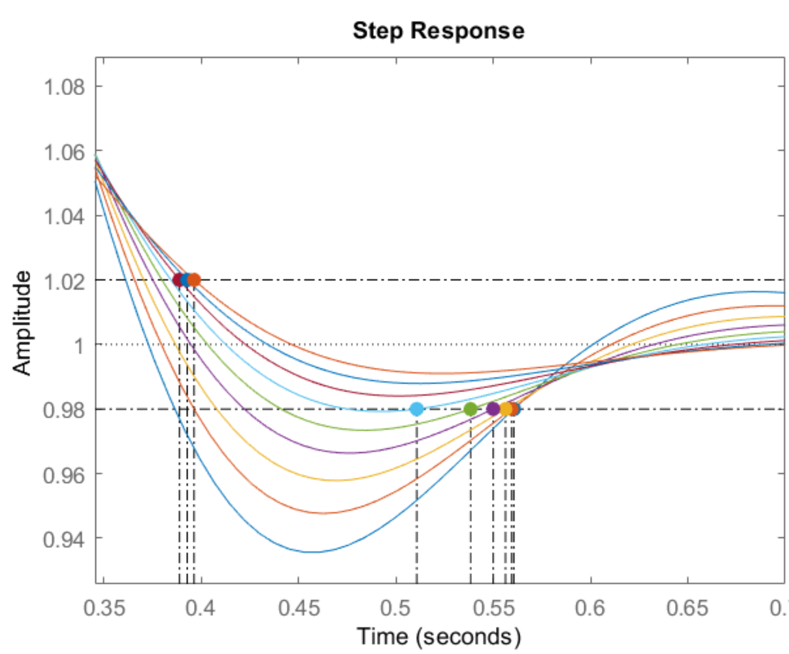
With increase in damping coefficient, the system would behave slowly, therefore, would result in the increase of rise time. On the other hand, with increase in natural frequency, the system would oscillate faster, therefore, making the system more responsive. It directly leads to decrease in rise time.



SETTLING TIME

Settling time decreases for both the variations. As we already know settling time is a subject to mathematical formulation (to be less than 2%) and therefore explains the massive discontinuity of variation in ζ. Apart from that, with the increase in ω settling time decreases due to improved responsiveness of the system, however, due to variation in ζ, damping increases which reduce the variations away from the steady-state as well and therefore reaches within the 2% range faster.





OVERSHOOT AND PEAK

Increase in damping coefficient reduces the overshoot heavily and is readily understood by intuition as well. It affects the peak value in a similar sense. On the other hand, with change in ω there is negligible change in overshoot. This explains the high concurrency in the behaviour among the variations in ω, but it is also readily visible that the peak value decreases due to the change in steady-state gain (as the variations are only visible in the denominator parameters).