# Algorithm for Constrained Path Optimization Problem on Road Networks

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### 1 Introduction

The problem of finding a path in road networks has been of great importance. Given the significance of this problem area, several researchers (e.g., [1, 2]) have explored it from different aspects. Among these, the most fundamental is computing a path between a source and destination under a given preference metric. The preference metric of choice has typically been the minimization of distance (e.g., [6]), time (e.g., [2]), or fuel (e.g., [5]). This paper makes the following contributions:

- 1. We propose a novel parallel approach, called *Parallel-Spatial-RG*, for the CPO problem on road networks.
- 2. Parallel-Spatial-RG uses intelligent task assignment policy and demonstrates an almost linear speed-up with an increase in the number of cores.
- 3. Experimentally evaluate Parallel-Spatial-RG on real road networks.

### 2 Problem Definition

*Input:* consists of the following:

- (1) A road network, G(V, E), where each node  $v \in V$  is associated with certain spatial coordinates and each edge  $e \in E$  associated with a cost and a score value.
- (2) A source  $s \in V$  and a destination  $d \in V$ .
- (3) A positive value *overhead*. We define the term budget as the sum of overhead and the cost of the shortest path between s and d.

Output: A directed path P\* between s and d.

Objective function: Maximize  $\Gamma(P*)$ 

Constraint:  $\Phi(P*) \leq budget$ 

## 3 Proposed Approach

A pseudocode of the algorithm is presented in Algorithm 1. Each call to the algorithm primarily takes the following input: (i) "source node" u, (ii) "destination node" v and (iii) remaining budget  $\beta$ . In the first call to the Spatial-RG algorithm, u, v, and  $\beta$  would be set according to the input values given while defining the CPO query. Thereafter, u, v, and  $\beta$  would change during the course of the recursion calls.

### 3.1 Time Complexity Analysis of Parallel-Spatial-RG:

In the worst case, an instance of Parallel-Spatial-RG algorithm would iterate over m feasible edges, and for each iteration, it would again iterate for  $\beta$  times. Following this, it would have two recursion calls inside the inner loop. Thus, the time complexity for one recursion depth is  $O(2m\beta)$ . For a maximum recursion depth of  $\theta$ , the total time complexity of Parallel-Spatial-RG would be  $O((2m\beta)^{\theta})$ .

#### Algorithm 1 Spatial-RG Algorithm

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Input: (a) Input graph G(V, E); (b) source node u; (c) destination node v; (d) Remaining budget \beta; (e)
current level; (f) maximum recursion depth \theta.
Output: (a) A directed path P between u and v
 1: P \leftarrow \text{minimum cost path between } u \text{ and } v
    if \Phi(P) > \beta then
         Return Null
 4: end if
    if level = \theta then /*Maximum recursion depth reached*/
         Return P
 7: end if
 8: s_p \leftarrow \Gamma(P) /*stores value of optimizing metric of P */
    for all edge e = (x, y) \in E with \Gamma(e) > 0 and e inside ellipse(u, v, \beta) do
         b \leftarrow \text{Euclidean\_Distance}(\mathbf{u}, \mathbf{x})
10:
11:
         while b < \beta - \Phi(e)-Euclidean_Distance(y, v) do
              P_1 \leftarrow \text{Spatial-RG}(u, x, b, level + 1)
12:
              P_2 \leftarrow \text{Spatial-RG} (y, v, \beta - b - \Phi(e), level + 1)
13:
              P_{\text{new}} \leftarrow P_1 \cup e \cup P_2
14:
              if (P_1 \cap P_2) = null \& \Gamma(P_{\text{new}}) > s_p then
15:
                   P \leftarrow P_{\text{new}} \text{ and } s_p \leftarrow \Gamma(P_{\text{new}})
16:
              end if
17:
              b \leftarrow b + 1
18:
         end while
19:
    end for
```

### 4 Experimental Analysis

We conducted experimental analysis on the real road networks of London, Delhi, and Buenos Aires (obtained from [4]). Due to lack of space, we are presenting only a summary of our results in this paper. Please refer to [3] for a detailed experimental analysis. In this paper, we present our results on the London dataset. This dataset has 285050 nodes and 749382 edges. Edges are selected uniformly at random from across the network and are assigned (randomly) a score value between 1 and 15. Other edges has 0 score value. Our experiments indicate that both the runtime and the score gain of Parallel-Spatial-RG increases as we increase the overhead and the density of the edges with non-zero score values. Please refer to [3] for more details on this experiment. Figure 1 illustrates the results.



Figure 1: Comparison of Parallel-Spatial-RG, MSWBS, and ILS\*(CEI).

### References

21: Return P

- [1] V. T. Chakaravarthy and et al. Scalable single source shortest path algorithms for massively parallel systems. *IEEE Trans on PDS*, 28(7):2031–2045, 2017.
- [2] Ugur Demiryurek and et al. Online computation of fastest path in time-dependent spatial networks. In *Proc. SSTD, LNCS Vol. no 6849*, pages 92–111. Springer, 2011.
- [3] Kousik Kumar Dutta, Ankita Dewan, and Venkata M. V. Gunturi. A multi-threading algorithm for constrained path optimization problem on road networks. *CoRR*, abs/2208.02296, 2022.

- [4] Alireza Karduni and et al. A protocol to convert spatial polyline data to network formats and applications to world urban road networks. *Scientific Data*, 3(160046), 2016.
- [5] Yan Li and et. al. Physics-guided energy-efficient path selection: a summary of results. In *Proc. of the* 26th ACM SIGSPATIAL, pages 99–108, 2018.
- [6] Sibo Wang and et al. Effective indexing for approximate constrained shortest path queries on large road networks. *Proc. VLDB Endow.*, 10(2):61–72, October 2016.