

# Tree Traversals & Expression Tree

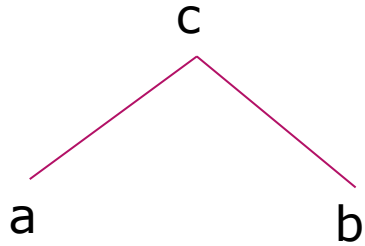
# Binary Trees - Traversals

- ▶ **Traversals**
  - ▶ **Preorder**
  - ▶ **Inorder**
  - ▶ **Postorder**
- ▶ **Expression Tree**

# Trees Traversals

- ▶ **Traversals**
  - ▶ **Visiting the nodes in a tree**
  - ▶ **Order of visit ?**

# Tree Traversals

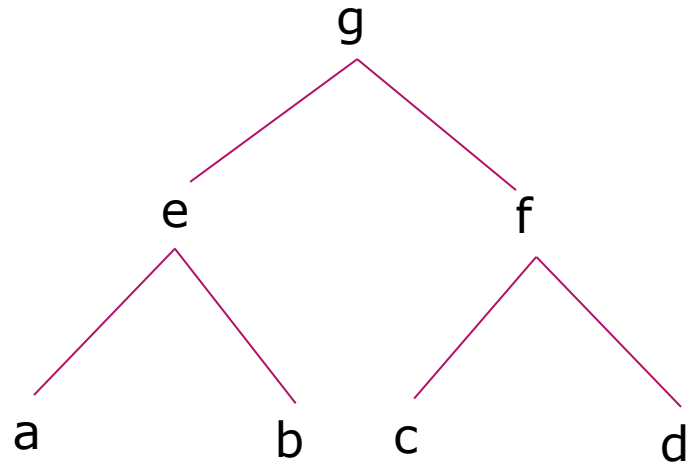


List the nodes in the tree.

a, c, b  
a, b, c  
c, a, b  
c, b, a  
.....

In which order to list the nodes?

# Tree Traversals



List the nodes.

a, e, b, c, f, d, g

a, b, e, c, d, f, g

.....

In which order ?

Is there any standard order?

# Example - Expression Tree

- ▶ **Language Translator (Compiler / Interpreter)**
  - ▶ Steps in translating  $x = a+b$
  - ▶ Semantics of  $x = a+b$  ?
  - ▶ In general, statements of the form *identifier* = <expression>

# Translation – Assignment Statement

- ▶ Language Translator - Interpreter
  - ▶ Steps in translating  $x = a+b$
- ▶ Semantics of  $x = a+b$  ?
  - ▶ Evaluate  $a+b$ , store the value in variable  $x$
- ▶ In general for  $variable = \langle expression \rangle$ 
  - ▶ Evaluate r.h.s *expression*, store value in the l.h.s *variable*

# Expression Evaluation

- ▶ **Expressions**

- $a + b * c$
- $(a + b * c + d) + (e / f + b * c + d \% g - h)$
- $x \ \&\& \ y \ || \ p \ \&\& \ q$

- ▶ **Order of evaluation of subexpressions**

- ▶ Based on **operator precedence** and **associativity**



# Expression Evaluation- precedence

- ▶  **$a+b*c$** 
  - **$*$**  has higher precedence than  **$+$**
  - **$b*c$**  to be evaluated first
- ▶ **Parenthesizing**
  - **$a+(b*c)$**

# Expression Evaluation-associativity

- ▶  **$a+b+c$**

- **$+$**  is left associative

- **$a+b$**  to be evaluated first

- ▶ **Parenthesizing**

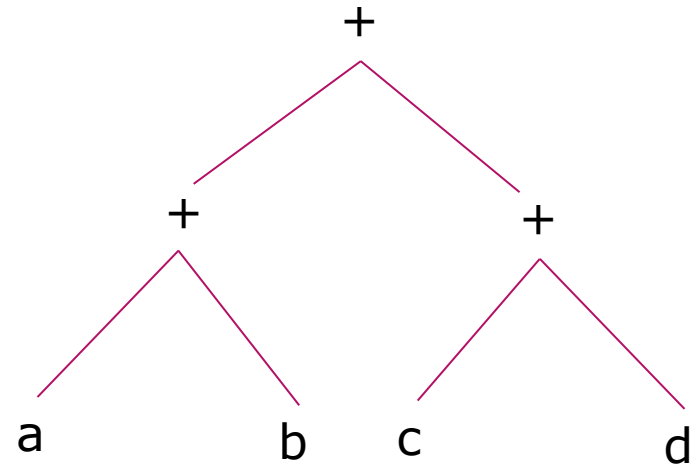
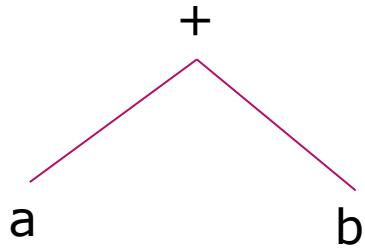
- $(a+b) + c$**

-

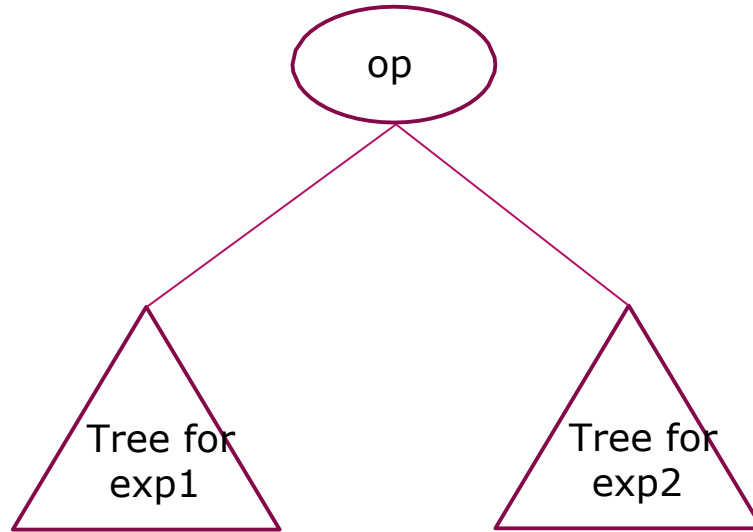
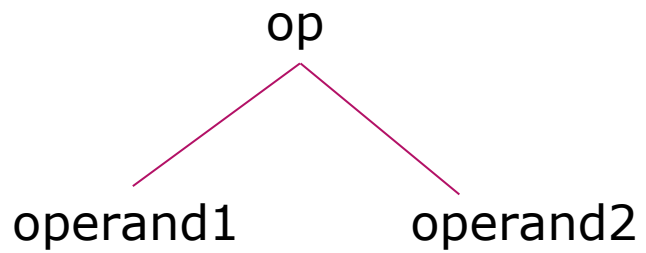
# Expression Evaluation – order of evaluation of subexpressions

- ▶ **Order of evaluation of subexpressions**
  - ▶ **Based on operator precedence and associativity**
- **$(a+b)*(c+d)$**
- **$a+b*c+d$**
- **$(a+b*c+d) + (e/f+b*c+d\%g-h)$**
- **$x \ \&\& \ y \ || \ p \ \&\& \ q$**

# Expression Tree

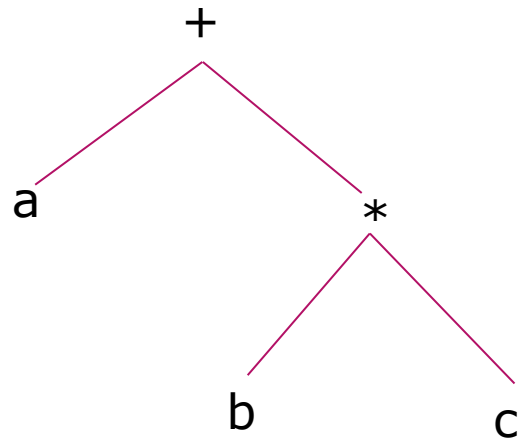


# Expression Tree

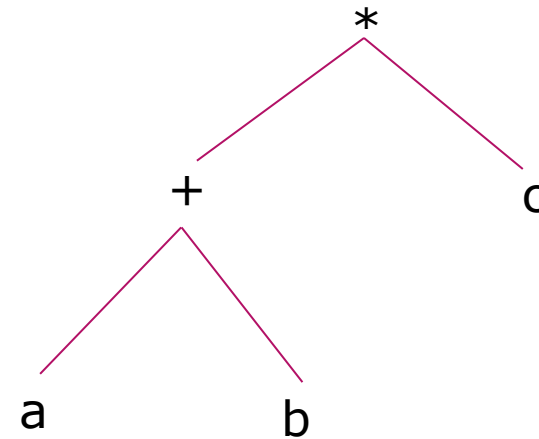


Tree for **exp1 op exp2**

# Expression Tree



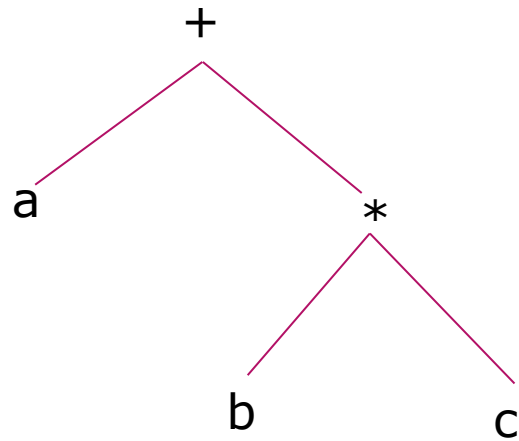
$a + (b * c)$



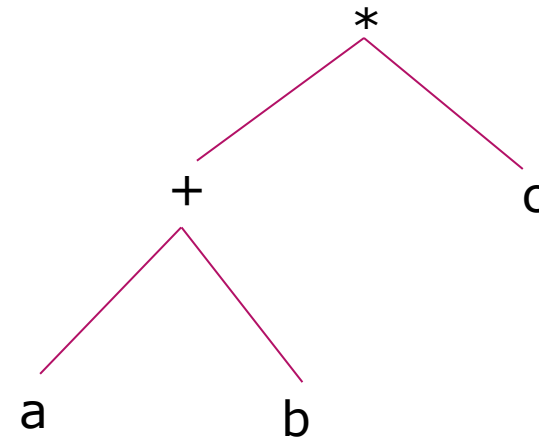
$(a + b) * c$

Order of evaluation ?

# Expression Tree



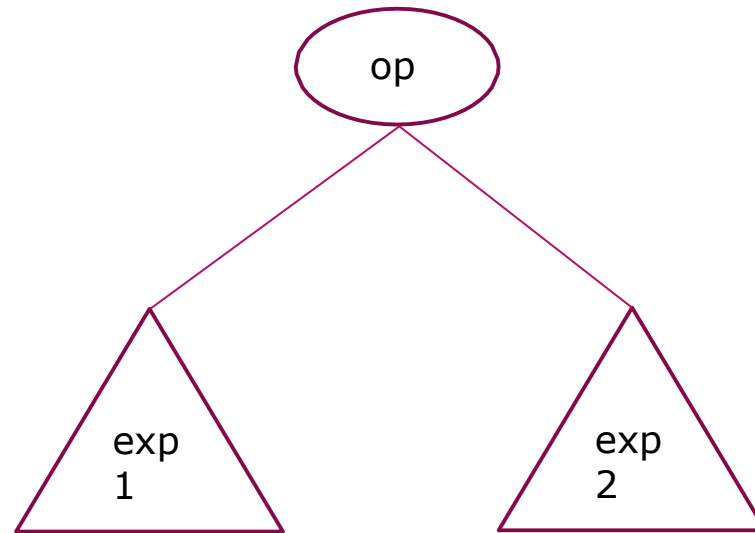
$a + (b * c)$



$(a + b) * c$

Order of evaluation : Evaluate the subtrees first

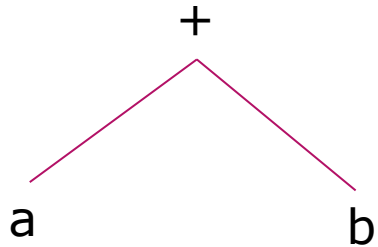
# Expression Tree - evaluation



**Evaluate exp1 → v1**  
**Evaluate exp2 → v2**  
**Evaluate v1 op v2**



# Expression Tree – listing nodes

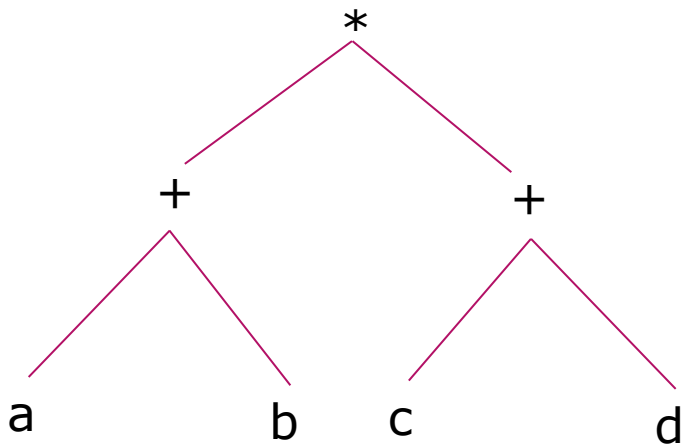


**+ab** root, lchild, rchild

**a+b** lchild, root, rchild

**ab+** lchild, rchild, root

# Expression Tree – listing nodes

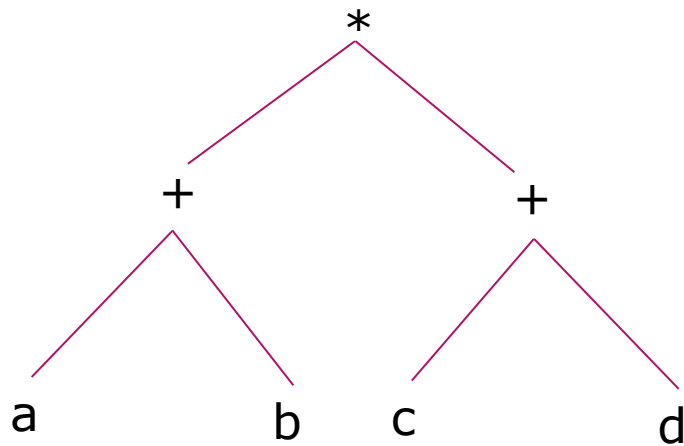


**\*+ab+cd** root, left subtree, right subtree

**a+b\*c+d** left subtree, root, right subtree

**ab+cd+\*** left subtree, right subtree, root

# Tree Traversals



**\*+ab+cd**    preorder    **D L R**

**a+b\*c+d**    inorder    **L D R**

**ab+cd+\***    postorder    **L R D**

# Tree Traversals

- ▶ **Preorder**
  - ▶ Root, Left subtree in preorder, Right subtree in preorder
- ▶ **Inorder**
  - ▶ Left subtree in inorder, Root, Right subtree in inorder
- ▶ **Postorder**
  - ▶ Left subtree in postorder, Right subtree in postorder, root

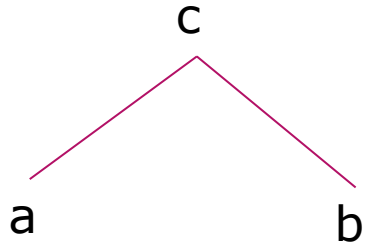
# Tree Traversals

- ▶ **Algorithms**
  - ▶ **Recursive – direct and easy**
  - ▶ **Iterative – using Stack**

# Tree Traversals

- ▶ Tree Traversal (**Tree Walk** in CLRS)
- ▶ Preorder
  - ▶ Visit **Root**, Left subtree in preorder, Right subtree in preorder
- ▶ Inorder
  - ▶ Left subtree in inorder, Visit **Root**, Right subtree in inorder
- ▶ Postorder
  - ▶ Left subtree in postorder, Right subtree in postorder, visit **Root**

# Tree Traversals

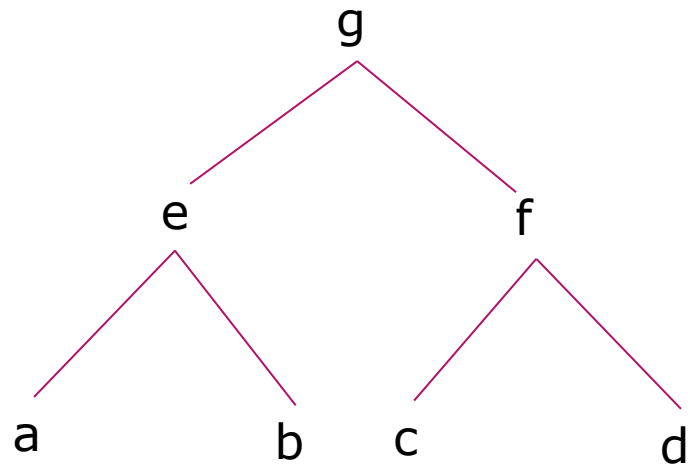


Preorder:     c   a   b

Inorder:     a   c   b

Postorder:   a   b   c

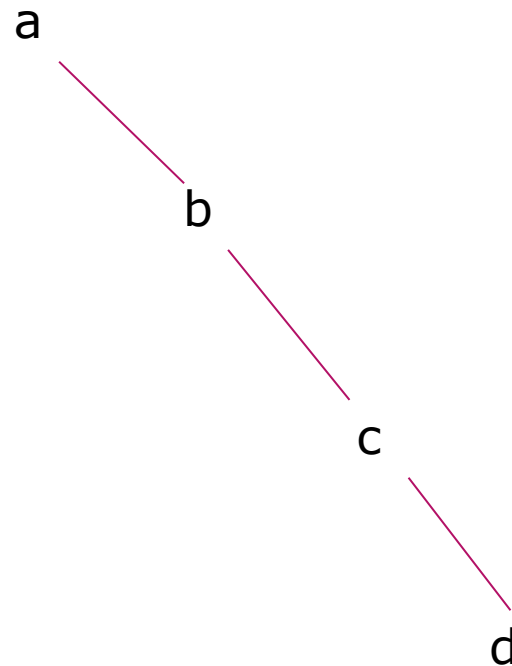
# Tree Traversals



List the nodes in preorder, inorder, postorder



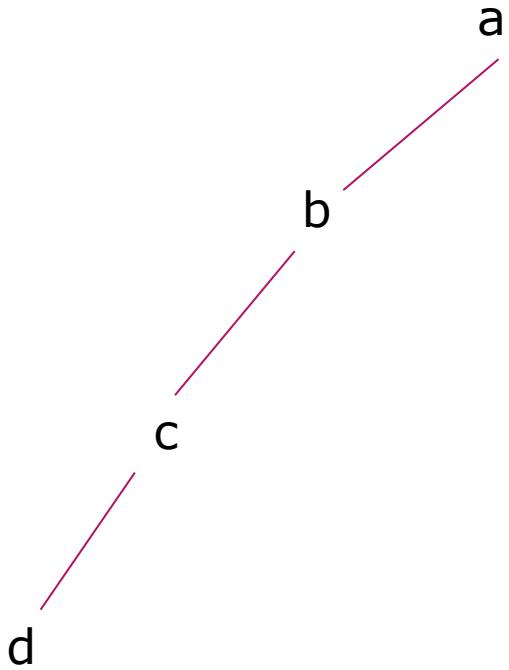
# Tree Traversals



List the nodes in preorder, inorder, postorder

Skewed Tree (Right skewed)

# Tree Traversals



List the nodes in preorder, inorder, postorder

# Inorder Tree Walk - Algorithm

```
INORDER-TREE-WALK (x)
    if x ≠ NIL
        INORDER-TREE-WALK (x.left)
        print x.data
        INORDER-TREE-WALK (x.right)
```

# Inorder Tree Walk - Algorithm

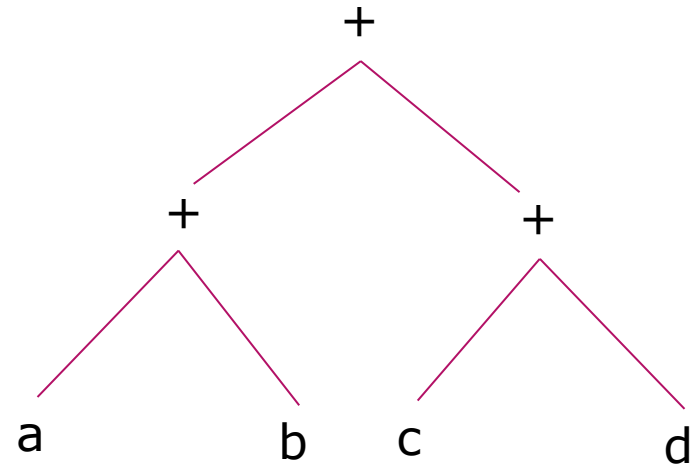
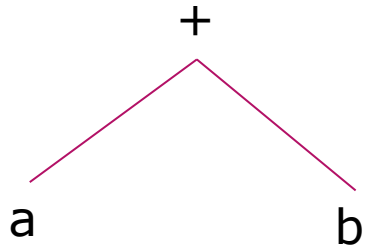
```
INORDER-TREE-WALK (x)
    if x ≠ NIL
        INORDER-TREE-WALK (x.left)
        print x.data
        INORDER-TREE-WALK (x.right)
```

- x is a node in the tree
- To traverse the entire tree T, invoke as INORDER-TREE-WALK (T.root)

# Tree Walk - Algorithms

- ▶ **Write recursive algorithms for**
  - `PREORDER-TREE-WALK()`
  - `POSTORDER-TREE-WALK()`

# Exercise: Expression Tree - Traversals



# Exercise: Expression Tree - Traversal

