VIVEKANANDHA COLLEGE OF TECHNOLOGY FOR WOMEN
ELAYAMPALAYAM, TIRUCHENGODE – 637205.

MATRICES AND CALCULUS

UNIT-1 (MATRICES)

(2021-2022)

I. Answer all the questions $5 \times 2 = 10$

1. Find the Sum and the product of the matrix A = $\begin{pmatrix} 1 & 1 & 5 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$
2. State Cayley Hamilton theorem.
3. Find the eigenvalues of the matrix $\begin{pmatrix} 4 & 1 \\ 3 & 2 \end{pmatrix}$
4. Find the eigenvalues of the inverse of the matrix A = $\begin{pmatrix} 2 & 1 & 5 \\ 0 & 4 & 4 \\ 0 & 0 & 5 \end{pmatrix}$
5. If 1 and 2 are the eigenvalues of a 2×2 matrix A, what are the eigenvalues of A^2 and A^{-1}.

II. Answer all the questions $1 \times 8 = 8$ & $1 \times 16 = 32$

6. Find the eigenvalues and eigenvectors of $\begin{pmatrix} 4 & -20 & -10 \\ -2 & 10 & 4 \\ 6 & -30 & -13 \end{pmatrix}$ (8)
7. Verify Cayley-Hamilton Theorem for the matrix $\begin{pmatrix} 8 & -8 & 2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{pmatrix}$. Hence find its inverse and A^4 (16)
8. Reduce the Q.F. $6x^2+3y^2+3z^2-4xy-2yz+4zx$ into a canonical form and find the nature of the Q.F. (16)

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MATRICES AND CALCULUS

UNIT-2 (DIFFERENTIAL CALCULUS)

(2021-2022)

I. Answer all the questions $5\times 2=10$

1. Evaluate $\lim\limits_{x\to 2^+}\frac{x^2+x-6}{|x-2|}$
2. Find $\frac{d}{dx}\left[(\sin x)^{\cos x}\right]$
3. Find the domain of the function $f(x)=\frac{x+4}{x^2-9}$
4. Explain why the function is discontinuous at the given number a $f(x)=\frac{1}{x+2}, a=-2$
5. Find the critical values of the function $f(x) = 2x^3-3x^2-36x$

II. Answer all the questions $5\times 8=40$

6. Find an equation of the tangent and normal lines to the given curve at specified point $f(x)=\frac{x^2-1}{x^2+x+1}$, (1,0)
7. (i) Find y' if $y=x^{x^{x^{\cdot^{\cdot^{\infty}}}}}$ (ii) If $\sin(x+y) = y^2\cos x$, then find $\frac{dy}{dx}$.
8. (i) Prove that equation $x^3-15x+c = 0$ has atmost one real root in the interval [-2, 2].
   (ii)If $f(1) = 10$ and $f'(x)\geq 2$ for $1\leq x\leq 4$ how small can f(4) possibly be?
9. Find the local maximum and minimum values of function $f(x) = x^5-5x+3$ using both the first and second derivatives tests.
10. (i) Suppose f and g are continuous functions such that g(2) = 6 and $\lim\limits_{x\to 2}\left[3f(x)+f(x)g(x)\right]=36$. Find f(2)
    (ii) Show that $f(x) = 3x^2+2x-1$ is continuous at x=2.

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MATRICES AND CALCULUS

UNIT-3 (FUNCTIOS OF SEVERAL VARIABLES)

(2021-2022)

I. Answer all the questions $5\times 2=10$

1. Evaluate $\lim\limits_{\substack{x\to\infty \\ y\to 2}} \dfrac{xy+5}{x^2+2y^2}$
2. Find $\dfrac{dy}{dx}$ when $x^3+y^3 = 3axy$
3. Find the domain of the function $u = \dfrac{y^2}{x}$, $v = \dfrac{x^2}{y}$ find $\dfrac{\partial(u,v)}{\partial(x,y)}$
4. If $u = f\left(\dfrac{x}{y},\dfrac{y}{x},\dfrac{z}{x}\right)$, then prove that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} + z\dfrac{\partial u}{\partial z} = 0$
5. Write two properties of jacobians.

II. Answer all the questions $5\times 8=40$

6. If $u=\log(x^3+y^3+z^3-3xyz)$, Show that $\left(\dfrac{\partial}{\partial x}+\dfrac{\partial}{\partial y}+\dfrac{\partial}{\partial z}\right)^2 u=\dfrac{-9}{(x+y+z)^2}$
7. Show that the functions $u = x+y-z$, $v = x-y+z$, $w = x^2+y^2+z^2-2yz$ are dependent. Find the relation between them.
8. Expand the function $\sin(xy)$ in powers of $x-1$ and $y-\dfrac{\pi}{2}$ upto second degree terms.
9. Find the maxima and minima of $x^4+y^4-2x^2+4xy-2y^2$
10. Find the maximum volume of the largest rectangular parallelepiped that can be inscribed in an ellipsoid $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}+\dfrac{z^2}{c^2}=1$

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MATRICES AND CALCULUS

UNIT-4 (INTEGRAL CALCULUS)

(2021-2022)

I. Answer all the questions $5\times 2=10$

1. Evaluate $\int \frac{\log x}{x}dx$
2. Evaluate $\int \frac{\sin 2x}{1+\cos^2 x}dx$
3. $\int_0^1 \tan^{-1}x\, dx$
4. Evaluate $\int \frac{1}{\sqrt{a^2-x^2}}dx$ by using trigonometric substitution.
5. For what values of p in the integral $\int_1^\infty \frac{1}{x^p}dx$ convergent?

II. Answer all the questions $5\times 8=40$

6. Evaluate $\int_0^3 \left(x^2-2x\right)dx$ by using Riemann sum by taking right end points as the sample points.
7. Find the reduction formula for $\int \sin^n x\, dx, n\geq 2$ is an integer and $\int_0^{\pi/2} \sin^n x\ dx$
8. Evaluate $\int \frac{x^4-2x^2+4x+1}{x^3-x^2-x+1}\, dx$
9. Evaluate $\int_0^1 \frac{\log(1+x)}{1+x^2}\, dx$
10. Evaluate $\int (3x-2)\sqrt{\left(x^2+x+1\right)}\, dx$

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MATRICES AND CALCULUS

UNIT-5 (MULTIPLE INTEGRALS)

(2021-2022)

I. Answer all the questions $5\times 2=10$

1. Evaluate $\int_0^3\int_0^2 e^{x+y}dydx$
2. Sketch roughly the region of integration for $\int_0^1\int_0^x f(x,y)dydx$
3. Find the domain of the function $\int_0^{\pi}\int_0^{\sin\theta} rdrd\theta$
4. Change the order of integration of $\int_0^a\int_y^a f(x,y)dxdy$
5. Express the region $x\geq 0,\ y\geq 0,\ z\geq 0,\ x^2+y^2+z^2\leq 1$ by triple integration.

II. Answer all the questions $5\times 8=40$

6. Change the order of integration in $\int_0^a\int_{\frac{x^2}{a}}^{2a-x} xydxdy$ and hence evaluate the same.
7. (i) Find the area bounded by the parabolas $y^2 = 4 - x$ and $y^2 = 4 - 4x$ as a double integral and evaluate it.
   (ii) Find the area of the cardioid $r=a\left(1+\cos\theta\right)$, using a double integral.
8. (i) Evaluate $\int_0^1\int_0^{b\left(1-\frac{x}{a}\right)}\int_0^{c\left(1-\frac{x}{a}-\frac{y}{b}\right)} x^2z\,dz\,dy\,dz$
   (ii) Evaluate $\int_0^a\int_0^b\int_0^c\left(x^2+y^2+z^2\right)dx\,dy\,dz$
9. Find the volume of that portion of $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ which lies in the first octant using triple integration.
10. Evaluate by changing to polar co-ordinates, the integral $\int_0^a\int_y^a\frac{x^2}{\sqrt{x^2+y^2}}dxdy$

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