

#### **LESSON PLAN**

Sub Code & Name: MA 3251

#### STATISTICS AND NUMERICAL METHODS

Unit: I Branch:

Semester: II

#### UNIT I TESTING OF HYPOTHESIS

Large sample test based on Normal distribution for single mean and difference of means - Tests based on t,  $Chi^2$  and F distributions for testing means and variances – Contingency table (Test for Independency) – Goodness of fit.

**Reference:** Grewal. B.S., and Grewal. J.S., "Numerical Methods in Engineering and Science", 9th Edition, Khanna Publishers, New Delhi, 2007.

	PART – A			
Q.No.		BT Level	Competence	PO
1.	What is statistical hypothesis?(Nov/Dec-2017)	BTL -1	Remembering	PO1
2.	Define chi-square. ?(Nov/Dec-2017)	BTL -1	Remembering	PO1
3.	Write type I and type II errors .(Apr/May-ov/Dec-2013)(May/Jun-2016)	BTL -1	Remembering	PO1
4.	What are the assumptions in 't' distribution?(Nov/Dec-2016)(Apr/May-2015)	BTL -1	Remembering	PO1
5.	State the important properties of the t-distribution. (Apr/May-2015)	BTL -1	Remembering	PO1
6.	Write any three applications of Chi-Square distribution.(May/Jun-2014)	BTL -1	Remembering	PO1
7.	Define null and alternative hypothesis.	BTL -2	Understanding	PO2
8.	When do we use the t-distribution? (Nov/Dec-2016)	BTL -2	Understanding	PO2
9.	What is meant by level of significance? (Apr/May-2016)	BTL -2	Understanding	PO2
10.	Define Standard error and Critical region. (Nov/Dec-2016)	BTL -2	Understanding	PO2
11.	Write any two applications of 't'-distribution. (Nov/Dec-	BTL -3	Applying	PO3
12.	Write the condition for the application of $\chi^2$ test.	BTL -3	Applying	PO3
13.	Write any three applications of 'F' distribution. (Nov/Dec-2015)	BTL -6	Creating	PO1,PO2, PO5
14.	State the important properties of F-distribution. (Nov/Dec-2011)	BTL -4	Analyzing	PO1,PO2, PO5
15.	Define sampling distribution. (Apr/May-2013)	BTL -4	Analyzing	PO1,PO2, PO5
16.	Define Chi-square test of goodness of fit. (Apr/May-2014)	BTL -3	Applying	PO5

17	Write down the form of the 95% confidence interval for the population mean in terms of population S.D.	BTL -5	Evaluating	PO1,PO2
18.	What is the Standard error of the difference between the means of two large samples drawn from different populations with known SD's	BTL -5	Evaluating	PO1,PO2
19.	What is the test statistic used to test the significance of the difference between small sample, mean and population?	BTL -6	Creating	PO1,PO2
20.	What is the test statistic used to test the significance of the ce between the means of two small samples?	BTL -4	Analyzing	PO1,PO2
21	Write down the formula of test stastistic 'Z' to test the significance of difference between the means (large s)	BTL -3	Applying	PO5
22	Write down the formula of test statistic 'Z' to test the significance of difference between the proportions(large samples).	BTL -3	Applying	PO5
23	What is the test statistic used to test the signifiance of the difference between the means of two small samples of same size, when the sample items are correlated?	BTL -6	Creating	PO1,PO3
24	What are the expected frequency of 2x2 contigency table given below.    a b   C d	BTL -4	Analyzing	PO1,PO. PO12
25	Write down the 1% and 5% critical values for right tailed two tailed Tests.	BTL -4	Analyzing	PO1,PO2
26	What is the difference between confidence limits and tolerance limits?	BTL -3	Applying	PO12
27	What are the assumptions of large sample?	BTL -5	Evaluating	PO1,PO2
28	What is test of goodness of fit?	BTL -3	Applying	PO12
29	Define hypothesis	BTL -6	Creating	PO1,PO2
30	What is meant by population?	BTL -4	Analyzing	PO1,PO2

Test of fidelity and selectivity of 190 radio receivers	1.(a)	The mean life by a compan deviation of s mean life tim 1600 hours.	y is four 90 hour ne of the	nd to b s. Exam e tubes	e 1580 ine th produ	0 hours ne hypo		BTL -1	Remembering	PO1			
Fidelity  Selectivit   Average   Hig   Y   Low   e   h    Low   6   12   32    Average   33   61   18    High   13   15   0    Use 0.01 level of significance to test whether there is a relationship between fidelity and selectivity. (Nov/Dec-2017)  A sample of 100 students is taken from a large population.  The mean height of the students in this sample is 160cms.  Can it be reasonably regarded that this sample is 160cms.  Can it be reasonably regarded that this sample is 160cms.  Can it be reasonably regarded that this sample is 160cms.  Can it be reasonably regarded that this sample is 160cms.  Can it be reasonably regarded that this sample is 160cms.  Can it be reasonably regarded that this sample is 160cms.  Can it be reasonably regarded that this sample is 160cms.  Can it be reasonably regarded that this sample is 160cms.  Can it be reasonably regarded that this sample is 160cms.  Can it be reasonably regarded that this sample is 160cms.  Can it be reasonably regarded that this sample is 160cms.  BTL -1  Remembering PO1  2.(b)  Fair Br\W Bl\R Total  Fair Br\	1. (b)												
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Sample II 15 14 15 19 15 18 16  (Apr/May-2016)  The following data gives the number of aircraft accidents that occurred during the various days of a week. Find whether the accidents are uniformly distributed over the week.	- (-)												
3.(b) The following data gives the number of aircraft accidents that occurred during the various days of a week. Find whether the accidents are uniformly distributed over the week.		<u> </u>	$\vdash$					+	1	.4	BTL -2	Understanding	PO2
3.(b)  The following data gives the number of aircraft accidents that occurred during the various days of a week. Find whether the accidents are uniformly distributed over the week.		l		14   1	5   1	.9 1!	5 18	16					
occurred during the various days of a week. Find whether the accidents are uniformly distributed over the week.	3.(b)	, , , ,	,	ives the	num	her of	aircraft :	accider	nts th	at			
		occurred dur	ing the	various	days	of a w	eek. Fin	d whet					
DTI O		accidents are	unifor	nly dist	tribute	ed over	the we	ek.			DT: 0		200
Su Mo Tue We Th Fr Sa BTL-2 Understanding PO2				Su	Мо	Tue	We	Th	Fr	Sa	RIT-7	Understanding	PO2
Days n n s d u i t				n	n	S	d	u	i	t			
No. of accidents		No. of accid	ents	14	1.0	0	13	11	0	44			
										14			

4. (a)	Two independent samples of 8 and 7 items respect the following Values of the variable(weight in kgs) Use 0.05 LOS to test	tively had			
	Sample I 9 11 13 11 15 9 12 1	4	BTL -2	Understanding	PO2
	Sample II   10   12   10   14   9   8   10   whether the variances of the two population's sam	unlo aro			
	equal. (Apr/May-2014)	ipie are			
5. (a)	A group of 10 rats fed on diet A and another group fed on diet B. Recorded the following increase the following increase in weight.(gm)				
	Diet A 5 6 8 1 12 4 3 9 6 10		BTL -2	Understanding	PO2
	Diet B 2 3 6 8 10 1 2 8 Find the variances are significantly different. (Use F (Nov/Dec-2014)	F—test)			
5 (b)	The marks obtained by a group of 9 regular course students and another group of 11 part time course a test are given below:				
	Sample     5     6     6     5     6     5     6     5       I     6     2     3     4     0     1     7     9     8		BTL -4	Applying	PO12
	Sample   6   7   7   6   6   5   7   6   7     II   2   0   1   2   0   6   5   4   2     Examine whether the marks obtained by regular st	6 6 6 8 6 udents and			
	part — time students differ significantly at 5% and 1 levels of significance. (Apr/May-2012)	1%			
6. (a)	Two independent samples of sizes 8 and 7 contains following values.	ed the			
	Sample         19         17         15         21         16         18         16	14	BTL -2	Understanding	PO2
	Sample II 15 14 15 19 15 18 16  Test if the two populations have the same variance				
6.(b)	In a certain factory there are two independent pro				
	manufacturing the same item. The average weight of 250 items produced from one process is found to Ozs, with a standard deviation of 12 Ozs, while the corresponding figures in a sample of 400 items fro process are 124 and 14. Is the difference between sample means significant? (Nov/Dec-2014)	to be 120 e m the other	BTL -4	Applying	PO12
	Records taken of the number of male and female be 800 families having four Children are as follows: Number of male births: 0 1 2 3	oirths in			
7. (a)	Number of female births :4 3 2	1 0 36 64 hypothesis ale birth is	BTL -4	Analyzing	PO1,PO2, PO5
	equal to female birth, namely $p = \frac{1}{2} = q$ . (Apr/May-	-2014)			

7. (b)				ctric bulbs w ere obtained	ere tested for	length			
				Type - I	Type - II				
		Samp	le size	8	7				
		Samp	le mean	1234 hrs	1036 hrs		BTL -3	Applying	PO12
			le S.D	36 hrs	40 hrs				
	Analyze tl	hat, is t	he differen	ce in the me	ans sufficient t	0			
			•	ior to type II	regarding the	length			
	of life? (N		<u> </u>						
8. (a)	A survey of following			h 5 children	each revealed	the			
	Boys	5	4 3	2 1	0				
	Girls	0	1 2	3 4	5		BTL -6	Croating	PO1,PO2,
	Famili						BIL-0	Creating	PO12
	es	14	56 110	88 40	12				
					esis that male a	nd			
	female bi	rths are	e equally p	obable?					
8.(b)					ple of 100 fiel				
					mple of 150 fie	elds			
				re. Assuming					
				kg for the ur	BTL -2	Understanding	PO2		
	(Apr/May		erence bety	veen the me					
	(/ (p1/ (v1a)	2013)							
9. (a)		•	•		7 from a norn				
	populatio				he variables Sa 14 16 14 1				
	Cample 2	18 16			BTL -1	Remembering	PO1		
	Sample 2			ce between					
				//Dec-2016)	tile illeans of s	ampies			
	,			<u> </u>					
9.(b)					ishmen has a r				
				iation of 6.4 mericans ha					
				6.3cms. Do					
				ige, taller tha	BTL -1	Remembering	PO1		
	(Apr/May		in the aver	ige, tailer till					
	(, .p., ividy	_010)							
40 / 1	 		1 .	ub - C !! .					
10.(a)	I wo rand	om san		the following		7			
	Sampl	Ci	Sampl		squares of n from the				
	e	Size	e Mean		ean				
		4.5				1	BTL -1	Remembering	PO1
	1	10	15		90	4			
	2	12	14		.08	]			
			•		e from the sam	е			
			on. (Nov/D			_			
10.(b)					of 10 patient				
					e B.P. 8, 8, 7, 5, ne medicine wa		DT: 1		DO4
							BTL -1	Remembering	PO1
	responsib	ופוטו נ	ne morease	: III D.Y. 3% I	o.s (Apr/May-	2012)			

11.(a)	200 digits were chosen at random from a set frequencies of the digits were given below: Use $\chi^2$ test to access the correctness of hypodigits were distributed in equal nos. in the tathe values of $\chi^2$ are16.9, 18.3, 19.7 for 9, 10 freedom at 5% level of significance.	othesis that the able, given that	BTL -3	Applying	PO12
11.(b)	4 coins were tossed 160 times and the follow obtained  No. of heads  Cobserved frequencies: 17 52 54 32  Under the assumption that the coins are bal expected frequencies of getting 0, 1, 2, 3, 4 h the goodness of fit. (Nov/Dec-2014)	4 L 6 anced, find the	BTL -5	Evaluating	PO1,PO2, PO5
12.(a)	A sample of 200 persons with a particular disselected. Out of these, 100 were given a dru were not given any drug. The result are as for the persons of the	BTL -1	Remembering	PO1	
12.(b)	A certain stimulus administered to each of 1 resulted in the following increase of blood p 1,3,0,-2,1,5,0, 4 & 6 can it be concluded tha will, in general, be accompanied by an increa pressure?	ressure 5,2,8,- t the stimulus	BTL -6	Creating	PO1,PO2, PO12
13.(a)	In a referendum submitted by the students to university, 850menand 560 women voted. 50 women voted favorably. Does this indicate a difference of opinion between men and women on this matter at 1% level of signification (Nov/Dec-2014)	BTL -1	Remembering	PO1	
13.(b)	Random samples drawn from two places gave data relating to the heights of male adults:  Place A Place B Mean height 8 7 S.D 1234 hrs 1036 hrs No. of sample 36 hrs 40 hrs Test at 5 % level, that the mean height is the the two places. (Apr/May-2012)		BTL -2	Understanding	PO2
14.(a)	In a random sample of 1000 people from citto be consumers of rice. In a sample of 800 fare found to be consumers of rice. Does this significant difference between the two cities proportion of rice consumers is concerned?	rom city B, 400 data give a	BTL -4	Analyzing	PO1,PO2, PO5

	(Nov/Dec-2012)			
14.(b)	In a year there are 956 births in a town A of which 52.5% were male while in towns A and B combined, this proportion in a total of 1406 births was 0.496.Is there any significant difference in the proportion of male births in the two towns ? (Apr/May-2011)	BTL -2	Understanding	PO2

	UNIT - II DESIGN OF EXPE	ERIMENTS		
	One way and two way classifications - Completely randomiz	•	andomized block des	ign — Latin
	square design - 2 <sup>2</sup> factor PART - A	ial design.		
Q.No	Question	BT Level	Competence	РО
1.	Write the advantages of Latin Square (Nov/Dec-2017)	BTL -1	Remembering	PO1
2.	What are the conditions to be followed in one way classification?(Nov/Dec-2017)	BTL -1	Remembering	PO1
3.	What is meant by analysis of variance?(May/Jun-2016)	BTL -1	Remembering	PO1
4.	Why a 2x2 Latin square is not possible?Explain.(May/Jun-2016)(May/Jun-2014).	BTL -1	Remembering	PO1
5.	Define Replication and Randomization.(Nov/Dec-	BTL -1	Remembering	PO1
6.	What is the advantage of factorial experiment? (Nov/Dec-2016)	BTL -1	Remembering	PO1
7.	What is the aim of design of experiment?(Apr/May-2015)	BTL -2	Understanding	PO2
8.	What are the basic principles of experimental design? (Apr/May-2015)	BTL -2	Understanding	PO2
9.	Write the advantages and disadvantages of RBD?(Apr/May-2015)	BTL -2	Understanding	PO2
10.	What is Latin Square design ?	BTL -2	Understanding	PO2
11.	Define Raw Sum of Squares and Correction factor	BTL -3	Applying	PO1,PO2,PO12
12.	Write any 3 applications of LSD. (Nov/Dec-2014)	BTL -3	Applying	PO1,PO2,PO12
13.	How do you calculate the Correction factor in LSD? (Nov/Dec-2012)	BTL -3	Applying	PO1,PO2,PO12
14.	What do you mean by design of nts?(Nov/Dec-2014)	BTL -4	Analyzing	PO5

15.	What are the subject matters included in the design of experiment?	BTL -4	Analyzing	PO5
16.	What are the assumptions in ANOVA? ?(Apr/May-	BTL -4	Analyzing	PO5
17.	are the three essential steps to plan an experiment?	BTL -5	Evaluating	PO1,PO2,PO5
18.	What are the basic steps in ANOVA? ?(Apr/May-2014)	BTL -5	Evaluating	PO1,PO2,PO5
19.	Write the steps to find F-ratio. (Nov/Dec-2016)	BTL -6	Creating	PO1,PO2,PO5
20.	Discuss the advantages of Completely Randomized block design.	BTL -6	Creating	PO1,PO2,PO5
21	State the uses of ANOVA. ? (Apr/May-2015)	BTL -4	Analyzing	PO12
22	Explain the word treatment in ANOVA. ?(Apr/May-2015)	BTL -4	Analyzing	PO12
23	What do you mean by 2-way classification?	BTL -4	Analyzing	PO12
24	Indicate the characteristics of a good experimental Design (Nov/Dec-2011)	BTL -5	Evaluating	PO1,PO2,PO5
25	What are the important designs of experiments?	BTL -5	Evaluating	PO1,PO2,PO5
26	What is an experimental error ? (Nov/Dec-2011)	BTL -6	Creating	PO1,PO2,PO5
27	What is meant by CRD? ?(Apr/May-2012)	BTL -6	Creating	PO1,PO2,PO5
28	Compare RBD and LSD.	BTL -3	Applying	PO1,PO2,PO5
29	Compare LSD and RBD. ?(Apr/May-2015)	BTL -3	Applying	PO1,PO2,PO5
30	What are the uses of Chi-Square test?	BTL -4	Analyzing	PO5
	PART - B	1		
1.(a)	The accompanying data resulted from an experiment comparing the degree of soiling for fabric copolymerized with the 3 different mixtures of met acrylic acid. Analyse the classification.  Mixture 1: 0.56	BTL -1	Remembering	PO1
1. (b)	A set of data involving 4 tropical food stuffs A, B, C, D tried on 20 chicks is given below. All the 20 chicks are treated alike in all respects except the feeding treatments and each feeding treatment is given to 5 chicks. Analyze the data:  A 55 49 42 21 52  B 61 112 30 89 63  C 42 97 81 95 92  D 169 137 169 85 154  (Apr/May-2016)	BTL -2	Understanding	PO2

2. (a)	The following table brands of electric			es in ho	urs of f	four				
	A:1610, 1610, 1	.650, 16	580, 1	700, 1	720,	1800				
	B: 1580, 1640, 1	•	•					BTL -1	Romambaring	PO1
	C:1460, 1550, 1							DIL-1	Remembering	POI
	ldentify an analyst of the mean lives of 2014)									
2.(b)	A company appoir	its 4 sale	smen	А, В, С а	and D a	and				
	observes their sale									
	monsoon. The figu	res are g	given ir	SALESN						
	SEASON		1	2	3	4		BTL -2	Understanding	PO2
	SUMMER		45	40	28	37				
	WINTER		43	41	45	38				
	MONSOON		39	39	43	41				
3.	Carry out an Analy						<u> </u>			
	order to determine									
	in the durability of 5 are selected fror						ıze			
	repair during the f						2			
	results are as follo		ew of	the abo	ve data	a, what				
	conclusion can you		441/50					BTL -1	Remembering	PO1
			MAKES					DIL-1	Remembering	101
		A 5	8 8	7						
		6	10	3						
		8	11	5						
		9	12	4						
		7	4	1						
4.	(Apr/May-2012)	<u> </u>								
	Five doctors each									
	disease and observ			•						
	store cover. The red	esuits are	e as to	<del>llows (r</del>	ecover	<del>y time ii</del>	1			
	daysy		TRE	ATMEN <sup>1</sup>	T					
	DOCTOR	Α	В	С	D	Е		BTL -2	Understanding	PO2
	А	10	14	23	18	20				
	В	11	15	24	17	21				
	С	9	12	20	16	19				
	D	8	13	17	17	20				
	E E	12	15	(a) da a	15	22				
5.	Estimate the differ treatments for the									
	2017)	320100		3,01010	(,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,		BTL -3	Applying	PO1,PO2,PO5	
	Perform a 2-way A	NOVA o	n the c	data giv	en belo					
			Т	reatme	nt 1					
	_	<b>~</b> !	1	2	3					
	en t	1	30	26	38	7				
						-				
						1				
					1				1	

		2	24	29	28					
		3	33	24	35					
		4	36	31	30					
		5	27	35	33					
	Use the coding met	thod sub	tractin	g 30 fro	m the gi	iven no.				
	(Nov/Dec-2016)			_						
6.	The following data					work				
	from Monday to Fr	riday by 2	4 differ		tes.					
				DAYS	T					
		MON	TUE	WED	THU	FRI				
	1	22	26	25	25	31				
	2 3	25	27	28	26	29		BTL -2	Understanding	PO2
	3 3	26	29	33	30	33			_	
	Test at <del>5</del> % leve#of									
	among the means									
	significant and also means obtained fo									
	significant. (Nov/De			uays of	tile wee	K ale				
7.	Analyze the variar	-		lowing	Latin sc	nuare of				
	yields of paddy w			_		•				
	methods of cultivat	tion								
	D12	22 A12	21 C1	.23 B1	122					
	B12	24 C12	.3 A1	22 D	125			BTL -4	Analyzing	PO5
	A12	20 B11	.9 D1	L20 C1	l21			DIL-4	Analyzing	105
	C12	22 D12	3 B1	21 A	122					
	Examine whether t					tion have	e			
	given significantly of									
8. (a)	The following data									
0. (a)	three burners A, B,				•					
	tests were made or				oread ov	ver 3 day	S.			
	-	A16	B17	C20				DTI 4	B l	BO4
	_	B16	C21	A15				BTL -1	Remembering	PO1
		C15	A12	B13						
	(Nov/Dec-2013)									
6 (1)	A variable tr									
8.(b)	varieties in a									
	experiment ar	na tne pe C25 B23				below.				
		D19 C21								
		<del>D17 C20</del>		_			$\dashv$	BTL-5	Evaluating	PO5
	E	B21 A1								
		(Nov	/Dec-2	012)						

9.	A farmer wishes to test the effects of four different fertilizers A, B, C, D on the yield of Wheat. In order to eliminate sources of error due to variability in soil fertility, he uses the fertilizers, in a Latin square arrangement as indicated in the following table, where the numbers indicate yields per unit area.           A18       C21       D25       B11         D22       B12       A15       C19         B15       A20       C23       D24         Design an analysis power and controlled a significant difference between the fertilizers at $\alpha$ =0.05 and $\alpha$ =0.01 levels of significance.	BTL -1	Remembering	PO1
10.	Set up the analysis of variance for the following results of a Latin Square Design(use $\alpha = 0.01$ ) level of			
	significance	BTL -4	Analyzing	PO5
	A12 C19 B10 D8 C18 B12 D6 A7	DIL-4	Analyzing	F03
	B22 D10 A5 C21			
	D12 A7 C27 B17			
11.	In a 5x5 Latin square experiment, the data collected is given in the matrix below Yield per plot is given in quintals for the five different cultivation treatments A, B, C,D and E. Perform the analysis of variance.  A48 E66 D56 C52 B61 D64 B62 A50 E64 C63 B69 A53 C60 D61 E67 C57 D58 E67 B65 A55 E67 C57 B66 A60 D57	BTL -6	Creating	PO1,PO2,PO5
12	In a Latin square experiment given below are the yields in quintals per acre on the paddy crop carried out for testing the effect of five fertilizers A, B, C, D, E. Analyze the data for variations.  B25 A18 E27 D30 C27 A19 D31 C29 E26 B23 C28 B22 D33 A18 E27 E28 C26 A20 B25 D33 D32 E25 B23 C28 A20	BTL -3	Applying	PO1,PO2,PO5
13.	Find out the main effects and interaction effects in the following 2 <sup>2</sup> factorial experiment and write down the analysis of variance table			
	BLOCKS 1 A B AB 0 10 1 11	BTL -3	Applying	PO1,PO2,PO5
	1 64 25 30 60			
	II			
	III 76 12 41 17 IV 75 33 25 10			

14.	of potash ar All the coml two levels o with 4 replic the followin The yields o	nd supe bination of sulph cation f og table obtained	er phosp ns of 2 l ate (k) o or each d are giv	ohate levels of pota . The y	on the of supe ash wei yields o the fol	we effect of sulphate yields of potatoes. In phosphate (p) and the studied in a RBD btained are given in lowing table. Usion (with $\alpha =$	BTL -3	Applying	PO1,PO2,PO1 2	
	BLOCKS	Yi	elds (Pe	er Plot	)					
		(1)	a	b	ab					
		23	25	22	38					
		Р	(1)	K	KP					
		40	26	36	38					
		(1)	K	KP	Р					l
		29	20	30	20					l
	IV	KP	K	Р	(1)					l
		34	31	24	28					l

## UNIT III SOLUTION OF EQUATIONS AND EIGENVALUE PROBLEMS

Newton Raphson method — Gauss elimination method — pivoting — Gauss Jordan methods — Iterative methods of Gauss Jacobi and Gauss Seidel — Matrix inversion by Gauss Jordan method — Eigen values of a matrix by power method.

Textbook : Grewal. B.S., and Grewal. J.S., "Numerical Methods in Engineering and Science", 9th Edition, Khanna Publishers, New Delhi, 2007.

PART - A

Q.No	Questions	BT Level	Competence	РО
1	State the order (rate) of convergence and convergence condition for Newton Raphson method. (A.U.N/D 2017, N/D 2011,2012, M/J 2013)	BTL-4	Analyzing	PO1
2	Give Newton Raphson iterative formula. (A.U N/D 2009,M/J 2012,2014)	BTL-2	Understanding	PO1,PO2 ,PO3
3	Establish an iteration formula to find the reciprocal of a positive number N by Newton Raphson method. (A.U.N/D 2010, M/J 2012)	BTL-1	Remembering	PO1,PO2
4	State the principle used in Gauss-Jordan method. (A.U M/J 2011)	BTL-1	Remembering	PO1
5	Give the sufficient condition of convergence of Gauss Seidel method. (A.U M/J 2011)	BTL-1	Remembering	PO1
6	Write the conditions for convergence in Gauss Seidel iterative technique. (or) When the method of iteration will be useful ? (A.U M/J 2009)	BTL-3	Applying	PO1
7	State Gauss Seidel method. (A.U M/J 2011,N/D 2012)	BTL-1	Remembering	PO1,PO2
8	Gauss Seidel method always converges — True or False (A.U	BTL-1	Remembering	PO1,PO2

	M/J 2016)			
9	Write the first iteration values of x,y,z when the equations $27x+6y-z=85$ , $6x+15y+2z=72$ , $x+y+5z=110$ are solved by Gauss Seidel method. (A.U N/D 2009,M/J 2012,2016)	BTL-3	Applying	PO1
10	Compare Gauss Elimination and Gauss Jordan methods for solving linear systems of the form AX=B. (A.U M/J 2016)	BTL-1	Remembering	PO1
11	What type of Eigen value can be obtained using power method? (A.U.N/D 2017, N/D 2011,2012, M/J 2014)		Remembering	PO1
12	Find the dominant eigen value of $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ by power method. <b>(A.U M/J 2012)</b>	BTL-1	Remembering	PO1
13	On what type of equations Newton's method can be applicable ? (A.U A/M 2016)	BTL-1	Remembering	PO1,PO2 ,PO5
14	By Gauss elimination method solve $x+y=2$ and $2x+3y=5$ . (A.U M/J 2014)	BTL-1	Remembering	PO1
15	Why Gauss Seidel iteration is a method of successive corrections? (A.U M/J 2016)	BTL-4	Analyzing	PO1
16	What are the merits of Newton's method of iteration?	BTL-1	Remembering	PO1
17	Give two direct methods to solve a system of linear equations (A.U A/M 2013)	BTL-2	Understanding	PO2
18	Compare Gauss Elimination with Gauss Seidel method.( A.U M/J 2017)	BTL-1	Remembering	PO1
19	Find inverse of A = $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ by Gauss Jordan method.	BTL-1	Remembering	PO1,PO2
	(A.U M/J 2013)			
	PART-B	T	T	
1	Solve $x \log_{10} x = 12.34$ with $x_0 = 10$ using Newton's method. (A.U.N/D 2017, N/D 2011,2012, M/J 2013)	BTL-4	Analyzing	PO1,PO2 ,PO5
2	Find the negative root of the equation $\sin x = 1 + x^3$ by using Newton Raphson method. (A.U M/J 2015)	BTL-4	Analyzing	PO1,PO2 ,PO5
3	Solve the following equation by Gauss Elimination method $10x-2y+3z=23$ $2x+10y-5z=-33 \text{ (A.U.N/D 2017, N/D }$ $3x-4y+10z=41$ <b>2011,2012, M/J 2014)</b>		Evaluating	PO1,PO2 ,PO5
4	Solve the equation by Gauss Jordan method : $2x_1+x_2+4x_3=4$ $x_1-3x_2-x_3=-5$ $3x_1-2x_2+2x_3=-1$	BTL-5	Evaluating	PO1,PO2 ,PO5

	[2 2 2]	BTL-2	Understanding	
5	Find the inverse of $\begin{bmatrix} 2 & 2 & 3 \\ 2 & 1 & 1 \\ 1 & 3 & 5 \end{bmatrix}$ using Gauss Jordan method.	DIL-2	Understanding	PO1,PO2
6	Solve by Gauss Siedel method $x+y+54z=110$ $27x+6y-z=85$ $6x+15y+2z=72$ (A.U.N/D 2017, N/D 2011,2013, M/J 2014)	BTL-2	Understanding	PO1,PO2
7	Find the dominant (largest) eigen value and the corresponding eigen vector of A = $\begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix}$ by power method. (A.U M/J 2015)	BTL-5	Evaluating	PO1,PO2 ,PO5
8	Find the numerically largest eigen value of A = $\begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \end{bmatrix}$ by power method and the corresponding eigen vector. (A.U M/J 2011,N/D 2012)	BTL-5	Evaluating	PO1,PO2 ,PO5
9	Find the numerically largest eigen value of A = $\begin{bmatrix} 5 & 4 & 3 \\ 10 & 8 & 6 \end{bmatrix}$ by power method with the initial eigen $\begin{bmatrix} 20 & -4 & 22 \end{bmatrix}$ vector $X_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ (A.U M/J 2016)	BTL-5	Evaluating	PO1,PO2 ,PO5

## UNIT IV INTERPOLATION, NUMERICAL DIFFERENTIATION AND NUMERICAL INTEGRATION

Lagrange's and Newton's divided difference interpolations — Newton's forward and backward difference interpolation — Approximation of derivates using interpolation polynomials — Numerical

single and double integrations using Trapezoidal and Simpson's 1/3 rules.

Textbook : Grewal. B.S., and Grewal. J.S., "Numerical Methods in Engineering and Science", 9th Edition, Khanna Publishers, New Delhi, 2007.

PART	- A
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Q. No	Opping: C214.2  Questions	BT Level	Competence	РО
1	Define interpolation and extrapolation? (A.U.N/D 2017, N/D 2011,2012, M/J 2013)	BTL-4	Analyzing	PO1
2	State Newton's formula on interpolation. When it is used? (A.U.N/D 2017, N/D 2011,2012, M/J 2014)	BTL-1	Remembering	PO1,PO2
3	Say True or False. — Newton's divided difference formula is applicable only for equally spaced intervals.  (A.U M/J 2011)	BTL-2	Understanding	PO1,PO2
4	State Newton's divided difference formula.	BTL-4	Analyzing	PO2
5	State Lagrange's interpolation formula	BTL-1	Remembering	PO1
6	Use Lagrange's formula to find the quadratic polynomial that takes these values $ \begin{array}{ccccccccccccccccccccccccccccccccccc$	BTL-2	Understanding	PO1
7	By differentiating Newton forward and backward difference formula, find the first derivative of the function f(x) (A.U M/J 2013)	BTL-2	Understanding	PO1,PO2
8	Write down the Newton – cotes quadrature formula.	BTL-1	Remembering	PO1
9	What is the geometrical interpretation of Trapezoidal rule? (A.U M/J 2016,N/D 2012)	BTL-1	Remembering	PO1
10	Using Trapezoidal rule evaluate $\int_{0}^{\pi} \sin x dx$ by dividing the range into 6 equal parts.	BTL-1	Remembering	PO1
11	Why is Trapezoidal rule so called? (A.U N/D 2011,N/D 2014)	BTL-2	Understanding	PO1,PO2
12	What are the truncation errors in Trapezoidal and Simpson's rules of numerical integration?	BTL-4	Analyzing	PO1
13	What is the condition for Simpson's 3/8 rule and state the formula.	BTL-4	Analyzing	PO1,PO2
14	Using Simpson's rule find $\int e^{\frac{4}{3}} dx$ given $e = \frac{1}{0}$ , $e = \frac{2.72}{1}$ ,	BTL-4	Analyzing	PO1
15	Compare Trapezoidal rule and Simpson's 1/3 <sup>rd</sup> rule for evaluating numerical integration. (A.U M/J 2015,N/D 2017)	BTL-1	Remembering	PO1

PART - B

1	Construct Newton's forward interpolation polynomial for the following data.  x: 4 6 8 10  y: 1 3 8 16  Use it to find the value of y for x = 5. (A.U M/J 2011,A/M 2012)	BTL-5	Evaluating	PO1,PO2 , PO3,PO5
2	The following data are taken from the steam table Temp $^{\circ}$ c : 140 150 160 170 180 Pressure kg f/cm $^{2}$ : 3.685 4.854 6.302 8.076 10.225 Find the pressure at temperature t = 175 $^{\circ}$ .	BTL-4	Analyzing	PO1,PO2
3	Using Lagrange's interpolation formula calculate the profit in the year 2000 from the following data  Year: 1997 1999 2001 2002  Profit in lakhs  of Rs.  (A.U.N/D 2017, N/D 2011,2012, M/J 2013)	BTL-5	Evaluating	PO1,PO2, PO5,PO12
4	Find the polynomial f(x) by using Lagrange's formula and hence find f(3) for  x: 0	BTL-4	Analyzing	PO1,PO2, PO5,PO12
5	Using Newton divided difference formula find $u(3)$ given $u(1) = -26$ , $u(2) = 12$ , $u(4) = 256$ , $u(6) = 844$ .	BTL-5	Evaluating	PO1,PO2, PO5,PO12
6	From the given table, the values of y are consecutive terms of a series of which 23.6 is the sixth term. Find the first and tenth terms of the series.  x: 3     4     5     6     7     8     9  y: 4.8     8.4     14.5     23.6     36.2     52.8     73.9  (A.U M/J 2016)	BTL-4	Analyzing	PO1,PO2, PO5,PO12
7	The following data gives the velocity of a particle for 20 seconds at an interval of 5 seconds. Find the initial acceleration using the entire data  time (sec.): 0 5 10 15 20  velocity (m/sec.): 0 3 14 69 228  (A.U N/D 2015)	BTL-5	Evaluating	PO1,PO2, PO5,PO12
8	Using Trapezoidal rule, evaluate $\int_{-1}^{1} \frac{dx}{1+x^2}$ taking 8 intervals.	BTL-5	Evaluating	PO1,PO2, PO5,PO12

9	Find an approximate value of log $_{\rm e}$ 5 by calculating to four decimal places by Simpson's rule the integral $\int_{4x}^{5} \frac{dx}{+5}$ dividing the range into 10 equal parts. (A.U A/M 2016)	BTL-3	Applying	PO1,PO2, PO5,PO12
10	Evaluate $\int_{0}^{6} \frac{dx}{1+x^2}$ by dividing the range into 6 equal parts using Simpson's rule.	BTL-3	Applying	PO1,PO2, PO5,PO12
11.	Evaluate $\int_{0}^{1} \frac{dx}{1+x^2}$ take h = 0.125. Hence find $\pi$ using Simpson's rule. (A.U.N/D 2017, N/D 2011,2012, M/J 2014)	BTL-5	Evaluating	PO1,PO2, PO5,PO12
12.	Compute $\int_0^1 \frac{xdx}{x^3 + 10}$ using Trapezoidal rule and Simpson's rule with the number of points 3,5,9. <b>(A.U M/J 2017)</b>	BTL-3	Applying	PO1,PO2, PO5,PO12

## UNIT V NUMERICAL SOLUTION OF ORDINARY DIFFERENTIAL EQUATIONS

Taylor's series method - Euler's method - Modified Euler's method - Fourth order Runge-Kutta method for solving first and second order equations - Milne's predictor-corrector methods for solving first order equations - Finite difference methods for solving second order equation.

	PART - A			
CO Map	pping:			
Q.No	Questions	BT Level	Competence	PO
1.	State Modified Euler algorithm to solve $y' = f(x, y), y(x) = y$ at x=x0+h. ( <b>A.U.N/D 2017, N/D 2011,2012, M/J 2013</b> )	BTL -1	Remembering	PO1
2.	State the disadvantage of Taylor series method. (A.U N/D 2009,M/J 2012,2014)	BTL -1	Understanding	PO1
3.	Write the merits and demerits of the Taylor method of solution. (A.U.N/D 2010, M/J 2012)	BTL -5	Understanding	PO1
4.	Which is better Taylor"s method or R. K. Method?(or)	BTL -1	Remembering	PO1

	State the special advantage of Runge-Kutta method over taylor series method. (A.U M/J 2011)			
5.	Compare Runge-Kutta methods and predictor – corrector methods for solution of initial value problem. (A.U M/J 2011)	BTL -1	Remembering	PO1
6.	What is a Predictor-corrector method of solving a differential equation?( <b>A.U M/J 2009</b> )	BTL -1	Understanding	PO2,PO 5
7.	State the third order R.K method algorithm to find the numerical solution of the first order differential equation. (A.U M/J 2011,N/D 2012)	BTL -1	Remembering	PO1
8.	Write Milne"s predictor formula and Milne"s corrector formula. (A.U M/J 2012,N/D 2014)	BTL -1	Understanding	PO1
9.	Write down Adams-Bashforth Predictor and Adams-Bashforth corrector formula. (A.U N/D 2011)	BTL -1	Understanding	PO1
10.	State Euler formula. (A.U M/J 2013)	BTL -1	Understanding	PO1
11.	Write down finite difference formula for y'(x) and y"(x) (A.U M/J 2012,N/D 2014)	BTL -1	Understanding	PO1
12.	Write down the Taylor series formula for solving first order ODE.	BTL -1	Understanding	PO1
13.	Using Taylor series method, find the value of y (0.1), from $f(x,y)=x^2+y^2$ and $y(0)=1$ correct to 4 decimal places	BTL -4	Analyzing	PO2
14.	Compare Taylor series method and RungeKutta method.	BTL -2	Remembering	PO5
15.	What are the advantages of R-K method over Taylor series method? (A.U N/D 2017)	BTL -2	Remembering	PO5
16.	Compare Single-step method Multi-step methods	BTL -1	Remembering	PO1
17.	Write down the error in Adam's predictor and corrector formulas	BTL -1	Understanding	PO1
18.	Write down the error in Milne's predictor and corrector formulas	BTL -1	Understanding	PO1
19.	Compare Adam's Bashforth method with RungeKutta method	BTL -1	Understanding	PO1
	PART-B			
1.	Using Taylor"s series method find y at $x = 0.1$ if $f(x,y) = x^2y - 1$ , $y(0) = 1$	BTL -1	Remembering	PO1,PO2
2	Solve: $y'' = x + y$ ; $y(0) = 1$ , by Taylor's series method. Find the values y at $x = 0.1$ and $x = 0.2$	BTL -3	Applying	PO1,PO2 ,PO5
3	Using Taylor's series method find y(1.1) given $y''=x + y$ , $y(1) = 0$	BTL -1	Remembering	PO1,PO2

4	Using Euler's method find y(0.2) and y(0.4) from $y'' = x + y, y(0) = 1 \text{ with } h = 0.2$	BTL -1	Remembering	PO1,PO2 ,PO5
5	Consider the initial value problem $y'' = y - x^2 + 1$ , $y(0) = 0.5$ using the modified Euler's method, find $y(0.2)$	BTL -2	Understanding	PO1,PO2 ,PO5
6	Using R.K method of fourth order, Solve $\frac{dy}{dx} = \frac{y_2 - x_2}{\frac{y_1 + x_2}{2}}$ with y(0) = 1 at x = 0.2.	B <u>TL -1</u>	Remembering	PO1,PO2 ,PO5
7	Using Milne"s method find y(4.4) givev $5xy$ " + $y^2 - 2$ = 0 given y(4) = 1, y(4.1) = 1.0049, y(4.2) = 1.0097 and y(4.3) = 1.0143.	BTL -1	Remembering	PO1,PO2 ,PO5
8	Obtain the approximate value of y at $x = 0.1 \& 0.2$ for the differential equation $\frac{dy}{dx} = 2y + 3e^x$ $y(0) = 0$ by Taylor's Series method. Compare the numerical solution obtained with the exact solution	BTL -3	Applying	PO1,PO2 ,PO12
9	Solve $\frac{dy}{dx} = \sin x + \cos y$ , $y(2.5) = 0$ by Modified Euler's method by choosing $h = 0.5$ , find $y(3.5)$	BTL -3	Applying	PO1,PO2 ,PO12
10	Solve $(1+x)\frac{dy}{dx} = -y^2$ , $y(0) = 1$ by Modified Euler's method by choosing $h = 0.1$ , find $y(0.1)$ and $y(0.2)$	BTL -3	Applying	PO1,PO2 ,PO12
11	Apply Runge – Kutta method, to find an approximate value of y when $x = 0.2$ given that $\frac{dy}{dx} = x + y$ , $y(0) = 1$ .	BTL -5	Evaluating	PO1,PO2 ,PO5
12	Given $\frac{dy}{dx} = x - y^2$ $y(0) = 0$ , $y(0.2) = 0.02$ , $y(0.4) = 0.0795$ and $y(0.6) = 0.1762$ . Compute $y(1)$ using Milne's Method.	BTL -3	Applying	PO1,PO2 ,PO12
13	Using Milne's method to find y(4.4) given that $5xy' + y^2 - 2 = 0$ given that $y(4) = 1$ , $y(4.1) = 1.0049$ , $y(4.2) = 1.0097$ , $y(4.3) = 1.0143$	BTL -1	Remembering	PO1,PO2 ,PO5

## **ANSWERS FOR TWO MARK QUESTIONS**

**UNIT —I-TESTING OF HYPOTHESIS** 

## PART-A(2 MARKS)

### (1). What is statistical hypothesis? (Nov/Dec-2017)

A **statistical hypothesis** is a **hypothesis** concerning the parameters or from of the probability distribution for a designated population or populations, or, more generally, of a probabilistic mechanism which is supposed to generate the observations

### (2). Define chi-square. ?(Nov/Dec-2017)

$$\chi^2 = \sum_{i=1}^n (O_i - E_i)^2 / E_i$$

## (3) Write type I and type II errors.(Apr/May-2015)(Nov/Dec-2013)(May/Jun-2016)

Type I error : Rejecting  $H_0$  when is true.

Type II error : Accepting  $H_0$  when it is false.

### (4) What are the assumptions in 't' distribution?(Nov/Dec-2016)(Apr/May-2015)

- (i) The parent population from which the sample is drawn is normal.
- (ii) The sample is random.

#### (5) State the important properties of the t-distribution.(Apr/May-2015)

- (i) For sufficiently large value of n, the t-distribution tends to the standard normal distribution.
  - (ii) The mean of the t-distribution is zero
  - (iii). The probability curve of the t-distribution is similar to the std.normal curve and is symmetric about t=0,bell-shaped.

## 6). Write any three applications of Chi-Square distribution. (May/Jun-2014)

- (i) To test the goodness of fit.
- (ii) to test the independence of attributes.
- (iii) To test the homogeneity of independent estimates of population.

#### (7) Define null and alternative hypothesis.

For applying the tests of significance ,we first set up a hypothesis which is a definite statement about the population parameter called Null hypothesis. Any hypothesis which is complementary to null hypothesis is called an alternative hypothesis.

#### (8) When do we use the t-distribution?

When the sample size is 30 or less and the population standard deviation is unknown, we use the t-distribution.

## (9) What is meant by level of significance?

The probability  $\alpha$  that a random value of the statistic 't' belongs to the critical region is known as level of significance.

## (10) Define Standard error and Critical region.

The standard deviation of the sampling distribution of a statistic is known as the standard error. A region corresponding to a statistic 't' in the sample S amounts to rejection of the null hypothesis is called critical region.

#### (11) Write any two applications of 't'-distribution.

The t-distribution is used to test the significance of the difference between

- (i) the mean of the small sample and mean of the population.
- (ii) The coefficient of correlation in the small sample and that in the population

assumed zero.

- (12) Write the condition for the application of  $\chi^2$  test.
  - (i) The sample observations should be independent.
  - (ii) N, the total frequency should be at least 50.
    - (iii) Theoritical cell frequency should be less 5.
- (13) Write any three applications of 'F' distribution.

F-test is used to test whether

- (i) Two independent samples have been drawn from the normal populations with the same variance  $\sigma^2$  .
- (ii) Two independent estimate of the population variance are homogeneous are not.
- (14) State the important properties of F-distribution.
- (i) The square of the t-variate with n degrees of freedom follows a F-distribution with 1 and n of freedom.
- 15) Define sampling distribution.

Different samples from the same population will result in general in distinct estimates, will form a statistical distribution called sampling distribution.

(16) Define Chi-square test of goodness of fit.

Chi-square test of goodness of fit is a test to find if the deviation of the experiment from theory is just by chance or it is due to the inadequacy of the theory to fit the observed data.

(17) Write down the form of the 95% confidence interval for the population mean in terms of population S.D.

$$\left(\overline{X} - 1.96 \frac{\sigma}{\sqrt{n}}, \overline{X} + 1.96 \frac{\sigma}{\sqrt{n}}\right)$$

(18) What is the Standard error of the difference between the means of two large samples drawn from different populations with known SD's.

$$\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

(19) What is the test statistic used to test the significance of the difference between small sample, mean and population?

$$t = \frac{\overline{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

(20) What is the test statistic used to test the significance of the difference between the means of two small samples?

$$t = \frac{\overline{x_1 - x_2}}{\sigma \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

(21) Write down the formula of test stastistic 'Z' to test the significance of difference between the means (large samples).

$$Z = \frac{\overline{x_1 - x_2}}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

(22) Write down the formula of test statistic 'Z' to test the significance of difference between the proportions(large samples).

$$Z = \frac{p_1 - p_2}{\sqrt{\frac{P_1 Q_1}{n_1} + \frac{P_2 Q_2}{n_2}}}$$

(23) What is the test statistic used to test the signifiance of the difference between the means of two small samples of the same size, when the sample items are correlated?

$$t = \frac{\vec{d}}{s\sqrt{n-1}}$$
, where  $d_i = x_i - y_i$ 

(24) What are the expected frequency of 2x2 contigency table given below.



$\frac{(a+b)(a+c)}{N}$	$\frac{(a+b)(b+d)}{N}$
(a+c)(d+c)	(d+b)(d+c)
N	N

(25) Write down the 1% and 5% critical values for right tailed and teo tailed Tests.

1% 5%

Two tailed test : 2.58 1.96
Right tailed test : 2.33 1.645

(26) What is the difference between confidence limits and tolerance limits

Confidence limits: To estimate a parameter of a population

Tolerance limits: To indicate between what limits one can find a certain proportion of a population.

- (27) What are the assumptions of large sample?
  - (i) it should be normal
  - (ii) values given by the samples are suffienctly close to the populatio parameters.
- (28) What is test of goodness of fit?

To determine whether the actual sample distribution matches a known theoretical distribution.

(29) Define hypothesis

Hypothesis is a statement about the population parameter.it is tested on the basis of the outcome of the random sample.

There are 2 types (i) null hypothesis and (ii) alternate hypothesis

(30) What is meant by population?

A population in statistics means a set of objects which are measurement or observations pertaining to the objects.

# UNIT —II-DESIGN OF EXPERIMENTS PART-A(2 MARKS)

1). Write the advantages of Latin Square design. (Nov/Dec-2017)

**Advantages of latin square designs**. Controls more variation than CR or RCB **designs** because of 2- way stratification.

(2). What are the conditions to be followed in one way classification? (Nov/Dec-2017)

In <u>statistics</u>, **one-way** <u>analysis of variance</u> (abbreviated **one-way ANOVA**) is a technique that can be used to compare means of two or more samples (using the <u>F distribution</u>). This technique can be used only for numerical response data, the "Y", usually one variable, and numerical or (usually) categorical input data, the "X", always one variable, hence "one-way"

(3). What is meant by analysis of variance? (May/Jun-2016)

Analysis of Variance is a technique that will enable us to test for the significance of the difference among more than two sample means.

4). Why a 2x2 Latin square is not possible? Explain. (May/Jun-2016) (May/Jun-2014).

Consider a nxn latin Square design ,then the degrees of freedom for SSE

$$= (n^2 - 1) - (n - 1) - (n - 1) - (n - 1)$$
$$= (n - 1)(n - 2)$$

For n=2, degrees of freedom of SSE=0 and hence MSE id not defined. Comparision is not possible. Hence 2x2 Latin Square is not possible.

(5) Define Replication and Randomization. (Nov/Dec-2016)

**Replication** is the repetition of an <u>experimental</u> condition so that the variability associated with the phenomenon can be estimated. In other words replication as "the repetition of the set of all the treatment combinations to be compared in an experiment. Each of the repetitions is called a **replicate**."

A method based on chance alone by which study participants are assigned to a treatment group. **Randomization** minimizes the differences among groups by equally distributing people with particular characteristics among all the trial arms.

- (6) What is the advantage of factorial experiment?(Nov/Dec-2016)
- (i) Factorial designs allow additional factors to be examined at no additional cost
- (ii) Factorial designs allow the effects of a factor to be estimated at several levels of the other factors, yielding conclusions that are valid over a range of experimental conditions.

(7) What is the aim of design of experiment? (Apr/May-2015) (May/Jun-2014)

The design of experiments (DOE, DOX, or experimental design) is the design of any task that aims to describe or explain the variation of information under conditions that are hypothesized to reflect the variation.

- (8) What are the basic principles of experimental design?(Apr/May-2015)
  - (i) Replication

### (ii) Randomization and Local control.

(9) Write the advantages and disadvantages of RBD?(Apr/May-2015) Advantages:

(i). Accuracy (ii) Flexibility (iii) Easy to analyze Disadvantage: It is not suitable for large number of treatment

### (10) What is Latin Square design?

A useful method of eliminating fertility variations consist in an experimental layout which will control in 2 perpendicular directions such a layout is a LSD.

(11)Define Raw Sum of Squares and Correction factor.

The expression  $\sum \sum x^2_{ij}$  is known as RSS and the expression  $\frac{G^2}{N}$ , where  $G^2 = \sum \sum x_{ij}^2$  is called

$$\frac{G^2}{N}$$
, where  $G^2 = \sum \sum x_{ij}^2$  is called

the correction factor.

(12) Write any 3 applications of LSD.

- (i) The statistical analysis is simple.
- (ii) Even with the missing data analysis remains relatively simple.
- (iii) More than one factor can be investigated simultaneously.

## (13) How do you calculate the Correction factor in LSD?

By squaring the grand total and dividing it by the number of observations, we calculate the correction factor.

### (14) What do you mean by design of experiments?(Nov/Dec-2014)

It is defined as the logical construction of the experiment in which the degree of uncertainty with which the inference is drawn ,may be well defined.

- (15) What are the subject matters included in the design of experiment?
  - (i) Planning of the experiment.
  - (ii) Obtaining relevant information from it regarding the statiscal hypothesis

under study.

### (16) What are the assumptions in ANOVA?

Each of samples is drawn from a normal population and the variances for the population from which samples have been drawn are equal.

- (17) What are the three essential steps to plan an experiment?
- (i) A statement of the objective.
- (ii) Statement should clearly mention the hypothesis to be tested.
- (iii) Description should include the type of experimental material, size of the experiment and the number of replications.

#### (18) What are the basic steps in ANOVA?

- (i) Estimate the population variance among the sample means.
- (ii) Estimate the population variance from the variance within the sample means.

## (19) Write the steps to find F-ratio.

$$F = \frac{S_1^2}{S_2^2} = \frac{Variance\ between samples}{Variance\ within\ samples}$$

- (20) Discuss the advantages of Completely Randomized block design.
- (i) easy to lay out
- (ii) allows flexibility (iii)simple

statiscal information

- (iv). The lot of information due to missing data is smaller than with any other design
- (21) State the uses of ANOVA.
  - (i) The effects of some fertilizer on the yields are significantly different.
  - (ii) The mean qualities of outputs of various machines differ significantly.
- (22) Explain the word treatment in ANOVA.

The word treatment in ANOVA is used to refer to any factor in experiment is controlled at different levels or values.

(23) What do you mean by 2-way classification?

In two way classification, the datas are classified according to different criteria or factors.

- (24) Indicate the characteristics of a good experimental design.
  - (i) Absolute (ii) Comparative.
- (25) What are the important designs of experiments?
  - (i) Completely Randomized design(or) One-Way classification
  - (ii) Randomized Block Design (or) Two-Way classification
  - (iii) Latin Square Design (or) Threee-Way classification.
- (26) What is an experimental error?

The variation from plot to plot caused by uncontrolled factors is known as experimental error.

(27) What is meant by CRD?

It is defined as a type of experimental design where the experimental units are allocated to the treatments in a completely random fashion. This is used to study the effects of one primary factor without the need to take other nuisance variables into account.

(28)Compare RBD and LSD.

RBD is more efficient than CRD for most types of experiment work.

In CRD, grouping of the experiments sixe so as to allocate the treatments at random to the experimental units is not done. But in RBD , treatments are allocated at random within the units of each stratum.

RBD is more flexible than CRD, since no restrictions are placed on the number or treatments or the number if replicatins.

(29) Compare LSD and RBD.

In LSD, the number of treatments is equal to the number of replications , whereas there are no such restrictions on treatments and replications in RBD.

- (30) What are the uses of Chi-Square test?
  - (i) To test significance difference between experimental values and theoretical values.
  - (ii) To find whether two or more attributes are associated or not.

## UNIT III SOLUTION OF EQUATIONS AND EIGENVALUE PROBLEMS

Newton Raphson method – Gauss elimination method – pivoting – Gauss Jordan methods – Iterative

methods of Gauss Jacobi and Gauss Seidel – Matrix inversion by Gauss Jordan method – Eigen values of a matrix by power method.

Textbook: Grewal. B.S., and Grewal. J.S., "Numerical Methods in Engineering and Science", 9th Edition, Khanna Publishers, New Delhi, 2007.

1.State the order (rate) of convergence and convergence condition for Newton Raphson method.

**Sol.** The order of convergence of Newton Raphson method is 2

(quadratic) and convergence condition is  $|f(x)f''(x)| < |[f'(x)]^2$ .

2. Give Newton Raphson iterative formula.

Sol. 
$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x)_n}, n = 0, 1, 2, \dots$$

3. Establish an iteration formula to find the reciprocal of a positive number N by Newton Raphson method.

**Sol.** Let 
$$x = 1/N$$

$$\Rightarrow N = \frac{1}{x} \Rightarrow \frac{1}{x} - N = 0$$
(i.e.)  $f(x) = \frac{1}{x} - N \Rightarrow f(x) = \frac{1}{x_n} - N, f'(x) = -\frac{1}{x_n^2}$ 

By Newton Raphson method,

$$x = x - \frac{f(x_n)}{f'(x_n)} = x - \frac{\frac{1}{x_n} - N}{-\frac{1}{x_n^2}} = x + x \frac{2}{x_n} \left(\frac{1}{x_n} - N\right)$$

$$= x \left(2 - Nx\right).$$
the principle used in Gauss-Jordan method.

4. State the principle used in Gauss-Jordan met

**Sol.** In the equation AX = B, the matrix A is transformed into an identity matrix.

5. Give the sufficient condition of convergence of Gauss Seidel method.

The absolute value of the leading diagonal element is greater than the sum of the absolute values of the other elements in that row, which is called diagonally dominant.

6. Write the conditions for convergence in Gauss Seidel iterative technique. (or) When the method of iteration will be useful?

The coefficient matrix should be diagonally dominant.

7. State Gauss Seidel method.

As soon as a new value for a variable is found by iteration it is used immediately in the Sol. following equations. This method is called Gauss Seidel method.

8. Gauss Seidel method always converges – True or False.

Sol. False.

9. Write the first iteration values of x,y,z when the equations 27x+6y-z=85, 6x+15y+2z=72, x+y+5z = 110 are solved by Gauss Seidel method.

Here the coefficient matrix is diagonally dominant. Then

$$x = \frac{1}{27} (85 - 6y + z) \dots (1)$$

$$y = \frac{1}{15} (72 - 6x - 2z) \dots (2)$$

$$z = \frac{1}{5} (110 - x - y) \dots (3)$$

**First Iteration** 

Put 
$$y = 0$$
,  $z = 0$  in (1), we get  $x = 3.148$   
Put  $x = 3.148$ ,  $z = 0$  in (2), we get  $y = 3.451$   
Put  $x = 3.148$ ,  $y = 3.451$  in (3), we get  $z = 20.662$ 

# 10.Compare Gauss Elimination and Gauss Jordan methods for solving linear systems of the form AX=B.

**Sol.** In Gauss Elimination method, the coefficient matrix reduced to upper triangular matrix and we get the solution by back substitution whereas in Gauss Jordan method, the coefficient matrix reduces to an unit or identity matrix and we get the solution without using back substitution.

## 11. What type of Eigen value can be obtained using power method?

Sol. Dominant eigen value.

12. Find the dominant eigen value of 
$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$
 by power method.

**Sol.** Dominant eigen value = 5.3722

## 13. On what type of equations Newton's method can be applicable?

**Sol.** Newton's method can be applicable to the solution of both algebraic and transcendental equation and can be also used when the roots are complex.

## 14. By Gauss elimination method solve x + y = 2 and 2x + 3y = 5.

**Sol.** The augmented matrix is

$$[A,B] = \begin{vmatrix} 1 & 1 & 2 \\ 2 & 3 & 5 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 & 2 \\ 2 & 3 & 5 \end{vmatrix}$$

$$= \begin{vmatrix} 0 & 1 & 1 \end{vmatrix} R_2 = R_2 - 2R_1$$

By back substitution, x + y = 2 - (1)

$$y = 1$$

(1) becomes, 
$$x+1=2$$

$$x=1$$

Hence x = 1, y = 1.

## 15. Why Gauss Seidel iteration is a method of successive corrections?

**Sol.** Because we replace approximations by corresponding new ones as soon the latter have been computed.

## 16. What are the merits of Newton's method of iteration?

**Sol.** Newton's method is successfully used to improve the result obtained by other methods. It is applicable to the solution of equations involving algebraical functions as well as transcendental functions.

## 17. Give two direct methods to solve a system of linear equations.

## **Sol.** Gauss Elimination method and Gauss Jordan method.

## 18. Compare Gauss Elimination with Gauss Seidel method.

**Gauss Elimination** 

Gauss Seidel

i.Direct method

i. Indirect method

ii. Used to find inverse of

ii. Used to solve system of

the matrix also.

equations only

iii. Diagonally dominant condition iii. Diagonally dominant condition

is not insisted.

is insisted.

## UNIT IV INTERPOLATION, NUMERICAL DIFFERENTIATION AND NUMERICAL INTEGRATION

Lagrange's and Newton's divided difference interpolations – Newton's forward and backward difference interpolation – Approximation of derivates using interpolation polynomials – Numerical single and double integrations using Trapezoidal and Simpson's 1/3 rules.

Textbook: Grewal. B.S., and Grewal. J.S., "Numerical Methods in Engineering and Science", 9th Edition, Khanna Publishers, New Delhi, 2007.

## 1. Define interpolation and extrapolation?

The process of computing the value of a function inside the given range is called interpolation. The process of computing the value of a function outside the given range is called extrapolation.

## 2. State Newton's formula on interpolation. When it is used?

Newton's forward interpolation formula is

$$y = y_{0} + u\Delta y_{0} + \frac{u(u-1)}{2!} \Delta^{2} y_{0} + \frac{u(u-1)(u-2)}{3!} \Delta^{3} y_{0} + \dots$$
where  $u = \frac{x - x_{0}}{h}$ 

This formula is used mainly for interpolating the values of y near the beginning of a set of tabular values.

Newton's backward interpolation formula is
$$y = y_n + u\nabla y_n + \frac{u(u+1)}{2!}\nabla^2 y_n + \frac{u(u+1)(u+2)}{3!}\nabla^3 y_n + \dots$$
where  $u = \frac{x - x_n}{h}$ 

This formula is used mainly for interpolating the values of y near the end of a set of tabular values.

## 3. Say True or False. – Newton's divided difference formula is applicable only for equally spaced intervals.

**Sol**. False.

4. State Newton's divided difference formula.

Sol. 
$$y = y_0 + (x - x_0)\Delta y_0 + (x - x_0)(x - x_0)\Delta^2 y_0 + (x - x_0)(x - x_0)(x - x_0)\Delta^3 y_0 +$$

5. State Lagrange's interpolation formula

Sol.

$$y = f(x) = \frac{(x - x_1)(x - x_2)(x - x_3).....(x - x_n)}{(x_0 - x_1)(x_0 - x_2)(x_0 - x_3).....(x_0 - x_n)} y_0$$

$$+ \frac{(x - x_0)(x - x_2)(x - x_3).....(x - x_n)}{(x_1 - x_0)(x_1 - x_2)(x_1 - x_3)......(x_1 - x_n)} y_1$$

$$+ \frac{(x - x_0)(x - x_1)(x - x_3)......(x - x_n)}{(x_2 - x_0)(x_2 - x_1)(x_2 - x_3)......(x_2 - x_n)} y_2$$

$$+ \frac{(x - x_0)(x - x_1)(x - x_2)(x - x_3)......(x - x_{n-1})}{(x_n - x_0)(x_n - x_1)(x_n - x_2)(x_n - x_3)......(x_n - x_{n-1})} y_n$$

6. Use Lagrange's formula to find the quadratic polynomial that takes these values

$$x : 0 \quad 1 \quad 3 \\ y : 0 \quad 1 \quad 0$$

Then find y(2).

**Sol.** By Lagrange's formula

$$y = f(x) = \frac{(x - x_1)(x - x_2)}{(x_0 - x_1)(x_0 - x_2)} y_0 + \frac{(x - x_0)(x - x_2)}{(x_1 - x_0)(x_1 - x_2)} y_1$$

$$+ \frac{(x - x_0)(x - x_1)}{(x_2 - x_0)(x_2 - x_1)} y_2$$

$$y = f(x) = \frac{(x - 1)(x - 3)}{(0 - 1)(0 - 3)} \cdot 0 + \frac{(x - 0)(x - 3)}{(1 - 0)(1 - 3)} \cdot 1 + \frac{(x - 0)(x - 1)}{(3 - 0)(3 - 1)} \cdot 0$$

$$y(x) = \frac{x^2 - 3x}{-2}$$
Hence  $y(2) = 1$ .

7. By differentiating Newton forward and backward difference formula, find the first derivative of the function f(x).

Sol. Newton forward interpolation formula is 
$$y = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \dots$$
where  $u = \frac{x - x_0}{h}$ 

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= \frac{1}{\Delta} \left[ \frac{2u - 1}{\Delta} \Delta^2 + \frac{3u_2 - 6u + 2}{6} \Delta_3 \right]$$

$$h \left[ \begin{array}{c} y_0 \\ 2 \end{array} \right]$$

$$+ \frac{2u_{3} - 9u_{2} + 11u - 3}{12} \Delta_{4} y_{0} + \dots$$
Newton backward interpolation formula is
$$y = y_{n} + u \nabla y_{n} + \frac{u(u+1)}{2!} \nabla^{2} y_{n} + \frac{u(u+1)(u+2)}{3!} \nabla^{3} y_{n} + \dots$$
where  $u = \frac{x - x_{n}}{h}$ 

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= \frac{1}{12} \begin{bmatrix} \nabla y_{n} + & y_{n} + & y_{n} + & y_{n} \\ & \nabla y_{n} + & \nabla y_{n} + & \nabla y_{n} \\ & & 6 \end{bmatrix}$$

$$+ \frac{2u_{3} + 9u_{2}}{12} + \frac{1}{12} \begin{bmatrix} \nabla y_{n} + & y_{n} + & y_{n} \\ & & 1 \end{bmatrix}$$

8. Write down the Newton – cotes quadrature formula..

$$\int_{x_0}^{x_n} f(x)dx = h\left\{ ny_0 + \frac{n^2}{2}\Delta y_0 + \frac{1}{2}\left(\frac{n^3}{3} - \frac{n^2}{2}\right)\Delta^2 y_0 + \frac{1}{6}\left(\frac{n^4}{4} - n^3 + n^2\right)\Delta^3 y_0 + \dots \right\}$$

## What is the geometrical interpretation of Trapezoidal rule?

**Sol.** We are finding the area of the curve enclosed by y = f(x), the X-axis, the ordinates x = a and x = ab by using the area of trapezium.

10. Using Trapezoidal rule evaluate  $\int \sin x dx$  by dividing the range into 6 equal parts.

**Sol.** 
$$h = \frac{\pi - 0}{6} = \frac{\pi}{6}$$

When  $h = \frac{\pi}{6}$ , the values of  $y = \sin x$  are

$$x: 0$$
  $\frac{\pi}{6}$   $\frac{2\pi}{6}$   $\frac{3\pi}{6}$   $\frac{4\pi}{6}$   $\frac{5\pi}{6}$   $\pi$ 

Trapezoidal rule is

$$\int_{0}^{\pi} \sin x dx = \frac{h}{2} \left[ (y_0 + y_n) + 2(y_1 + y_2 + y_3 + \dots y_{n-1}) \right]$$

$$= \frac{\pi}{6(2)} \left[ (0+0) + 2(0.5+0.8660+1+0.8660+0.5) \right]$$

$$= 0.9770$$

## 11. Why is Trapezoidal rule so called?

The Trapezoidal rule is so called, because it approximates the integral by the sum of n

### trapezoids.

## 12. What are the truncation errors in Trapezoidal and Simpson's rules of numerical integration?

Sol. Error in the Trapezoidal rule is  $-\frac{n}{12}f'(\theta)$ . Error in the Trapezoidal rule is of the order  $h^2$ .

Error in the Simpson's one-third rule is  $-\frac{h^5}{90}f^{IV}(\theta)$ . Error in Simpson's one-third rule is of the order  $h^4$ .

Error in the Simpson's three eighth rule is  $-\frac{3h^5}{80}f^{IV}(\theta)$ . Error in the Simpson's three eighth rule is of the order  $h^4$ .

## 13. What is the condition for Simpson's 3/8 rule and state the formula.

**Sol.** The condition for Simpson's 3/8 rule is the number of sub-intervals should be a multiple of 3. Simpson's 3/8 rule is

$$\int_{x_0}^{x_n} f(x)dx = \frac{3h}{8} \left[ (y_0 + y_1) + 3(y_1 + y_2 + y_4 + y_5 + y_7 + \dots) + 2(y_3 + y_6 + y_9 + \dots) \right]$$

14. Using Simpson's rule find 
$$\int_{-x}^{4} e^{x} dx$$
 given  $e^{0} = 1$ ,  $e^{1} = 2.72$ ,  $e^{2} = 7.39$ ,  $e^{3} = 20.09$ ,  $e^{4} = 54.6$ 

**Soln** The following data is

Simpson's 1/3<sup>rd</sup> rule is

$$\int_{x_0}^{x_n} f(x)dx = \frac{h}{3} \Big[ (y_0 + y_n) + 4(y_1 + y_3 + y_5 + \dots) + 2(y_2 + y_4 + y_6 + \dots) \Big]$$

$$\int_{0}^{4} e^x dx = \frac{1}{3} \Big[ (1 + 54.6) + 4(2.72 + 20.09) + 2(7.39) \Big]$$

$$= 53.8733$$

## 15. Compare Trapezoidal rule and Simpson's 1/3<sup>rd</sup> rule for evaluating numerical integration.

- **Sol.** i) In Newton Cotes Quadrature formula, if we put n = 1 we get Trapezoidal rule whereas if we put n = 2, we get Simpson's  $1/3^{rd}$  rule.
  - ii) In Trapezoidal rule, the interpolating polynomial is linear whereas in Simpson's  $1/3^{rd}$  rule, the interpolating polynomial is of degree 2.
  - iii) In Trapezoidal rule, there is no restriction on the number of intervals whereas in Simpson's  $1/3^{rd}$  rule, the number of intervals should be even.

## UNIT V NUMERICAL SOLUTION OF ORDINARY DIFFERENTIAL EQUATIONS

Taylor's series method - Euler's method - Modified Euler's method - Fourth order Runge-Kutta method for solving first and second order equations - Milne's predictor-corrector methods for solving first order equations - Finite difference methods for solving second order equation.

1. State Modified Euler algorithm to solve y' = f(x, y), y(x) = y at x = x0 + h. (A.U.N/D 2017, N/D

2011,2012, M/J 2013)

$$y_{n+1} = y_n + hf \left[ x_n + \frac{h}{2} y_n + \frac{h}{2} f(x_n, y_n) \right]$$

$$y_1 = y_0 + hf \left[ x_0 + \frac{h}{2} y_0 + \frac{h}{2} f(x_0, y_0) \right]$$

2. State the disadvantage of Taylor series method.

(A.U N/D 2009,M/J 2012,2014)

#### **Solution:**

In the differential equation f(x, y),  $\frac{dy}{dx} = f(x, y)$  the function f(x, y), may have a complicated

algebraical structure. Then the evaluation of higher order derivatives may become tedious. This is the demerit of this method.

# 3. Write the merits and demerits of the Taylor method of solution. (A.U.N/D 2010, M/J 2012) Solution:

The method gives a straight forward adaptation of classic to develop the solution as an infinite series. It is a powerful single step method if we are able to find the successive derivatives easily.

If f(x,y) involves some complicated algebraic structures then the calculation of higher derivatives becomes tedious and the method fails. This is the major drawback of this method.

However the method will be very useful for finding the starting values for powerful methods like Runge - Kutta method, Milne"s method etc.,

4. Which is better Taylor"s method or R. K. Method?(or) State the special advantage of Runge-Kutta method over taylor series method (A.U M/J 2011)

## **Solution:**

- R.K Methods do not require prior calculation of higher derivatives of y(x), as the Taylor method does. Since the differential equations using in applications are often complicated, the calculation of derivatives may be difficult.
- $\triangleright$  Also the R.K formulas involve the computation of f (x, y) at various positions, instead of derivatives and this function occurs in the given equation.
- 5. Compare Runge-Kutta methods and predictor corrector methods for solution of initial value problem. (A.U M/J 2011)

#### **Solution:**

Runge-Kutta methods

- Runge-methods are self starting, since they do not use information from previously calculated points.
- As mesne are self starting, an easy change in the step size can be made at any stage.

  3. Since these methods require several evaluations of the function f (x, y), they are time consuming.
- ➤ In these methods, it is not possible to get any information about truncation error.

Predictor Corrector methods

- > These methods require information about prior points and so they are not self starting.
- > In these methods it is not possible to get easily a good estimate of the truncation error.
- 6. What is a Predictor-corrector method of solving a differential equation? (A.U M/J 2009) Solution:
  - $\triangleright$  Predictor-corrector methods are methods which require the values of y at  $x_n, x_{n-1}, x_{n-2}, \ldots$  for computing the value of y at . x  $_{n+1}$
  - We first use a formula to find the value of y at x <sub>n+1</sub> and this is known as a predictor formula.

The value of y so got is improved or corrected by another formula known as corrector formula

7. State the third order R.K method algorithm to find the numerical solution of the first order differential equation. (A.U M/J 2011,N/D 2012)

**Solution:** To solve the differential equation y' = f(x, y) by the third order R.K method, we use the following algorithm.

$$\begin{aligned} k_1 &= hf(x,y) \\ k_2 &= hf\left(x + \frac{h}{2}, y + \frac{k_1}{2}\right) \\ k_3 &= hf(x + h, y + 2k_2 - k_1) \\ \text{and } \Delta y &= \frac{1}{6}(k_1 + 4k_2 + k_3) \end{aligned}$$

8. Write Milne's predictor formula and Milne's corrector formula.

(A.U M/J 2012,N/D 2014) Solution:

> Milne"s predictor formula is

$$y_{4,p} = y_0 + \frac{4h}{3} [2y_1' - y_2' + 2y_3']$$

➤ Milne"s corrector formula is

$$y_{4,c} = y_2 + \frac{h}{3} [y_2' + 4y_3' + y_4']$$

9. Write down Adams-Bashforth Predictor and Adams-Bashforth corrector formula. (A.U N/D 2011)

#### **Solution**

Adams-Bashforthpredictor formula is

$$y_{4,p} = y_3 + \frac{h}{24} [55y_3' - 59y_2' + 37y_1' - 9y_0']$$

Adams-Bashforthcorrector formula is

$$y_{4,c} = y_3 + \frac{h}{24} [9y_4' + 19y_3' - 5y_2' + y_1']$$

10.State Euler formula

(A.U M/J 2013)

## **Solution:**

$$y_{n+1} = y_n + hf[x_n, y_n]$$
 when  $n = 0, 1, 2, \dots$ 

11. Write down finite difference formula for y'(x) and y"(x) (A.U M/J 2012,N/D 2014)

## **Solution:**

$$y'(x) = \frac{y_{i+1} - y_i}{h}, \quad y''(x) = \frac{y_{i-1} - 2y_i + y_{i+1}}{h^2}$$

$$y_n = \frac{1}{1!} y_n = \frac{1}{2!} y_n = \frac{1}{3!} y_n = \dots$$

4decimal places

## **Solution:**

$$y'=2+2yy'+2(y')^2$$
  $y'=1$   
 $y^{iv}=2yy'+6y'y'$ 

$$y'=2;y'=8;y^{iv}=28$$

By using Taylor series formula,  $y_1 = 1.11145$ 

14. Compare Taylor series method and Runge Kutta method.

#### **Solution:**

- The use of R-K method gives quick convergence to the solutions of the differential equations than Taylor's series method.
- The labour involved in R-K method is comparatively lesser.
- In R-K method, the derivatives of higher order are not required for calculation as in Taylor series method.

15. What are the advantages of R-K method over Taylor series method?

### **Solution:**

The Rungekutta methods are designed to give greater accuracy and they possess the advantage of requiring only the function values at some selected points on the sub interval.

16. Compare Single-step method Multi-step methods. (A.U N/D 2017)

#### **Solution:**

S.No Single-step method Multi-step method

	1	It requires only the numerical value $y_{i+1}$ order to compute the next value $y_{i+1}$	It requires not only the numerical valueyibut also atleast four of the past valuesy <sub>i-1</sub> ,y <sub>i-2,,</sub>
	2	Taylor series, Euler's and R-K methodsare single step methods	Milne's, Adam's methods are multi stepmethods

17. Write down the error in Adam's predictor and corrector formulas.

## **Solution:**

Order of error is  $h^5$ 

Error in predictor 
$$h^5 \frac{251}{720} y^{\nu}$$

Error in corrector 
$$h^5 \frac{19}{120} y^{\nu} \xi$$

18. Write down the error in Milne's predictor and corrector formulas.

## **Solution:**

Order of error is  $h^5$ 

Error in predictor 
$$h^5 \frac{14}{45} y^{\nu} \xi$$

Error in corrector 
$$h^5 \frac{1}{90} y^{\nu} \xi$$

19.Compare Adam's Bashforth method with RungeKutta method

## **Solution:**

S. No	Adam's Bashforth Method	Runge-Kutta Method
1	Multi step method	Single step method
2	Need four prior values of y <sub>i</sub> 's	Need only the last prior value
3	Does not permits changes in the step size	Permits changes in the step size
		·