Reg. No. : E N G G T R E E . C O M

Question Paper Code : 51315

B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2024.

First Semester

Civil Engineering

MA 3151 – MATRICES AND CALCULUS

(Common to : All Branches (Except B.E. Marine Engineering))

(Also Common to PTMA 3151-Matrices and calculus for B.E. (Part-Time)
First Semester-All Branches-Regulations 2023)

(Regulations 2021)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. If λ is an eigenvalue of a matrix A, then prove that λ^2 is an eigenvalue of A^2.

2. If $x = [-1, 0, 1]^T$ is the eigenvector of the matrix $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$, then find the corresponding eigen value.

3. Sketch the graph of the function $f(x) = 2.0 - 0.4x$ and find the domain of the function.

4. Differentiate $y = x\tan(\sqrt{x})$ with respect to x.

5. Verify Euler's theorem for the function $u = x^2 + y^2 + 2xy$.

6. If $u = x - y, v = y - z, w = z - x$, then find the Jacobian $\frac{\partial(u,v,w)}{\partial(x,y,z)}$.

7. What is wrong with the equation $\int_{-2}^{1}\left[\frac{1}{x^4}\right]dx = \int_{-2}^{1}\left[x^{-4}\right]dx = \left[\frac{x^{-3}}{-3}\right]_{-2}^{1} = -\frac{3}{8}$.

8. Evaluate $\int_{-1}^{1}\left[\frac{\tan x}{1+x^2+x^4}\right]dx$ by using the concept of odd and even functions.

9. Evaluate $\int_1^2 \int_0^{x^2} [x] dy\,dx$.

10. Write the integral equation for the regions $x \geq 0$, $y \geq 0$, $z \geq 0$, $x^2 + y^2 + z^2 \leq 1$ by triple integration.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the eigenvalues and eigenvectors of the given matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{bmatrix}$. (8)

(ii) Using Cayley-Hamilton theorem, find the inverse of the given matrix $A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 1 \\ 1 & 0 & -2 \end{bmatrix}$. (8)

Or

(b) Reduce the quadratic form $3x_1^2 + 5x_2^2 + 3x_3^2 - 2x_2x_3 + 2x_3x_1 - 2x_1x_2$ to a canonical form by orthogonal reduction. (16)

12. (a) (i) Find the value of $\lim_{x \to 2} \left[\frac{x^2 - 2}{x^3 - 3x + 5} \right]^2$. (6)

(ii) Find the local maximum and minimum values of the function $f(x) = x + 2\sin x$ in the interval $0 \leq x \leq 2\pi$. (10)

Or

(b) (i) Find an equation of the tangent line to the curve $y = \frac{e^x}{(1 + x^2)}$ at the point $(1, e/2)$. (8)

(ii) Find the absolute maximum and absolute minimum values of the function $f(x) = \log\left[x^2 + x + 1\right]$ in the interval $[-1, 1]$. (8)

13. (a) (i) If $u = \log\left[x^2 + y^2 + z^2\right]$ then find the value of $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$? (8)

(ii) The temperature at any point (x, y, z) in space is given by $T = 400xyz^2$. Find the maximum temperature on the surface of the unit sphere $x^2 + y^2 + z^2 = 1$. (8)

Or

(b) (i) Expand $f(x, y) = e^{x+y}$ about the point $(0,0)$ in powers of x and y upto third degree terms by using Taylor's series. (8)

(ii) Find the maxima and minima for the given function $f(x, y) = x^3 y^2 [1 - x - y]$. (8)

14. (a) (i) Evaluate $\int x^2 e^x \, dx$ by using integration by parts. (8)

(ii) Evaluate the integral $\int \sin^4 x \, dx$. (8)

Or

(b) (i) Evaluate $\int \sqrt{a^2 - x^2} \, dx$. (8)

(ii) Evaluate $\int \frac{1}{(x^2 - a^2)} dx$ by using partial fraction. (8)

15. (a) (i) Evaluate $\int_0^{\pi/2} \int_0^{\sin\theta} [r] \, d\theta \, dr$. (8)

(ii) Change the order of integration in

$\int_0^a \int_x^a \left[x^2 + y^2\right] dy \, dx$ and hence evaluate it. (8)

Or

(b) (i) Evaluate $\iint [xy] \, dx \, dy$ over the positive quadrant of the circle $x^2 + y^2 = a^2$. (8)

(ii) Find the volume of the sphere $x^2 + y^2 + z^2 = 3^2$ by using triple integration. (8)

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Reg. No. : E N G G T R E E . C O M

Question Paper Code : 21272

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2023.

First Semester

Civil Engineering

MA 3151 — MATRICES AND CALCULUS

(Common to : All Branches (Except Marine Engineering))

(Regulations 2021)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. Find the eigenvalues of A^{-1} and A^2 if $A = \begin{pmatrix} 3 & 0 & 0 \\ 8 & 4 & 0 \\ 6 & 2 & 5 \end{pmatrix}$.

2. State Cayley-Hamilton theorem.

3. Sketch the graph of the function $f(x) = \begin{cases} x^2 & if\ -2 \le x \le 0 \\ 2-x & if\ 0 < x \le 2 \end{cases}$.

4. The equation of motion of a particle is given by $s = 2t^3 - 5t^2 + 3t + 4$ where s is measured in meters and t in seconds. Find the velocity and acceleration as functions of time.

5. If $u = \frac{x}{y} + \frac{y}{z} + \frac{z}{x}$, find $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}$.

6. Write any two properties of Jacobians.

7. Evaluate $\int_0^{\frac{\pi}{2}} \sin^9 x\, dx$.

8. Prove that the integral $\int_1^{\infty} \frac{1}{x} dx$ is divergent.

9. Evaluate $\int_1^2\int_1^3 xy^2\,dx\,dy$.

10. Find the area of a circle $x^2+y^2=a^2$ using polar coordinates in double integrals.

PART B — ($5 \times 16 = 80$ marks)

11. (a) (i) Find the eigenvalues and eigenvectors of $A=\begin{pmatrix}1 & 1 & 1\\ 0 & 2 & 1\\ -4 & 4 & 3\end{pmatrix}$. (8)

(ii) Using Cayley-Hamilton theorem, find A^{-1} if $A=\begin{pmatrix}1 & 3 & 7\\ 4 & 2 & 3\\ 1 & 2 & 1\end{pmatrix}$. (8)

Or

(b) Reduce the quadratic form $8x^2+7y^2+3z^2-12xy-8yz+4zx$ into the canonical form and hence find its rank, index, signature and nature. (16)

12. (a) (i) Let $f(x)=\begin{cases}\sqrt{-x} & \text{if } x<0\\ 3-x & \text{if } 0\le x\le 3\\ (x-3)^2 & \text{if } x>3\end{cases}$. Evaluate each of the following limits, if they exist.

(1) $\lim_{x\to 0^-} f(x)$

(2) $\lim_{x\to 0^+} f(x)$

(3) $\lim_{x\to 3^-} f(x)$

(4) $\lim_{x\to 3^+} f(x)$

(5) $\lim_{x\to 0} f(x)$

(6) $\lim_{x\to 3} f(x)$

Also, find where $f(x)$ is continuous. (8)

(ii) Find the n^{th} derivative of $f(x)=xe^x$. (4)

(iii) Differentiate $F(t)=\dfrac{t^2}{\sqrt{t^3+1}}$. (4)

Or

(b) (i) Use logarithmic differentiation to differentiate $y = \frac{x^{3/2}\sqrt{x^2+1}}{(3x+2)^5}$. (8)

(ii) Discuss the curve $f(x) = x^4 - 4x^3$ for points of inflection, and local maxima and minima. (8)

13. (a) (i) Given the transformations $u = e^x \cos y$ and $v = e^x \sin y$ and that f is a function of u and v and also of x and y, prove that

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = (u^2 + v^2)\left(\frac{\partial^2 f}{\partial u^2} + \frac{\partial^2 f}{\partial v^2}\right) \quad (8)$$

(ii) Expand $e^x \cos y$ in a series of powers of x and y as far as the terms of the third degree. (8)

Or

(b) (i) Examine for extreme values of $f(x, y) = x^3 + y^3 - 12x - 3y + 20$. (8)

(ii) A rectangular box, open at the top is constructed so as to have a volume of 108 cubic meters. Find the dimensions of the box that requires the least material for its construction. (8)

14. (a) (i) Find a reduction formula for $\int e^{ax} \sin^n x\, dx$. (8)

(ii) Integrate the following : $\int \frac{x^4 - 2x^2 + 4x + 1}{x^3 - x^2 - x + 1} dx$. (8)

Or

(b) (i) Evaluate $\int \sqrt{\frac{1-x}{1+x}}\, dx$. (8)

(ii) Find the centre of mass of a semicircular plate of radius r. (8)

15. (a) (i) Change the order of integration in $\int_0^4 \int_{x^2/4}^{2\sqrt{x}} xy\, dy\, dx$ and then evaluate it. (8)

(ii) Find the area enclosed by the curves $y = 2x - x^2$ and $x - y = 0$. (8)

Or

(b) (i) Find the volume of the tetrahedron bounded by the planes $x = 0$, $y = 0$, $z = 0$ and $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$. (8)

(ii) Find the moment of inertia of a hollow sphere about a diameter, given that its internal and external radii are 4 meters and 5 meters respectively. (8)
