Topics: Normal distribution, Functions of Random Variables

1. The time required for servicing transmissions is normally distributed with μ = 45 minutes and σ = 8 minutes. The service manager plans to have work begin on the transmission of a customer's car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?

   A. 0.3875
   B. 0.2676
   C. 0.5
   D. 0.6987

   ANS:- Mean=45 & St. Deviation=8
   Work will commence after 10mins so mean would be 45+10=55
   Car will be ready within 1hr=60
   So,60-55/8=0.62 , from the z value table=0.723
   Probability of service manager can't meet his commitment would be 1.
   1-0.723= 0.267

2. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean μ = 38 and Standard deviation σ=6. For each statement below, please specify True/False. If false, briefly explain why.
   A. More employees at the processing center are older than 44 than between 38 and 44.
      ANS:- Mean=38 & St. Deviation=6
      Probability of employees older then 44
      (44,38,6)=1-0.841 = 0.158
      Probability of employees b/w 38&44
      (38,38,6)=0.5
      0.841-0.5 = 0.341
      As we can see probability of employees b/w 38&44 is more, so therefore this statement is false.
   B. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.
      ANS:-Mean=38 St. Deviation=6,x=30
      Probability of employees under the age 30
      (30,38,6)= 0.091
      Total no of employees 400*0.091 = 36.4,so therefore this statement is true.

3. If $X_1 \sim N(\mu, \sigma^2)$ and $X_2 \sim N(\mu, \sigma^2)$ are *iid*normal random variables, then what is the difference between 2 X_1 and $X_1 + X_2$? Discuss both their distributions and parameters.

4. Let X ~ N(100, 20 ^{2}). Find two values, *a* and*b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

   A. 90.5, 105.9
   B. 80.2, 119.8
   C. 22, 78
   D. 48.5, 151.5
   E. 90.1, 109.9

   ANS:- Mean=100 & std.deviation=20
   Probability of random variable taking a value b/w them is 0.99
   (0.99)= 2.32
   2.32*20+100=146.4
   -2.32*20+100=53.48
   Hence D option would be the nearest value.

5. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions $Profit_1 \sim N(5, 3^2)$ and $Profit_2 \sim N(7, 4^2)$ respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45

   ANS:- annual profit=proft1 + profit2
   Mean=profit1 (mean) +profit2(mean) = (5+7)=12
   St. Deviation=sqrt of 3&4
   =(9+16) = 25 = 5.
   12*45= 540- Mean
   5*45=225-Std.deviation

   A. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
      ANS:- Range= 95%

      (0.95,540,225) = 99.00 , 980.99.

   B. Specify the 5th percentile of profit (in Rupees) for the company.
      ANS:- 5th percentile, for that we need to find the z score value
      0.5000-0.050= 0.4500
      Z score value of 90% = 1.645
      Mean+(-1.645)*St. Deviation
      540-1.645*225 = 169.87

   C. Which of the two divisions has a larger probability of making a loss in a given year?
      ANS:- Probability of making loss 1st =(0,5,3) = 0.0477
      Probability of making loss 2nd =(0,7,4) = 0.040
      1st division made more loss compare to 2nd division.