Project: Part 2

P2: Problems

Exercise 1.1

Given equation:

$$\frac{d}{dt}\begin{bmatrix} e_1 \\ \dot{e}_1 \\ e_2 \\ \dot{e}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{4C_\alpha}{m\dot{x}} & \frac{4C_\alpha}{m} & -\frac{2C_\alpha(l_f-l_r)}{m\dot{x}} \\ 0 & 0 & 0 & 1 \\ 0 & -\frac{2C_\alpha(l_f-l_r)}{I_z\dot{x}} & \frac{2C_\alpha(l_f-l_r)}{I_z} & -\frac{2C_\alpha(l_f^2+l_r^2)}{I_z\dot{x}} \end{bmatrix}\begin{bmatrix} e_1 \\ \dot{e}_1 \\ e_2 \\ \dot{e}_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \frac{2C_\alpha}{m} & 0 \\ 0 & 0 \\ \frac{2C_\alpha l_f}{I_z} & 0 \end{bmatrix}\begin{bmatrix} \delta \\ F \end{bmatrix}$$

Equations for evaluation:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{8000}{1888.6\dot{x}} & 0 & -\dot{x} - \frac{6400}{1888.6\dot{x}} \\ 0 & 0 & 0 & 1 \\ 0 & -\frac{6400}{25854\dot{x}} & 0 & -\frac{173384}{25854\dot{x}} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ \frac{40000}{1888.6} \\ 0 \\ \frac{62000}{25854} \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Controllability matrix $P = [B \quad AB \quad A^2B \quad A^3B]$

Observability matrix $Q = \begin{bmatrix} C \\ CA \\ CA^2 \\ CA^3 \end{bmatrix}$

We use rank(P), rank(Q) and the size of matrix A (n=4) to determine the controllability and observability of the given system at the following longitudinal velocities: 2 m/s, 5 m/s and 8 m/s and given variable values. The process was done with Python and the terminal outputs are shown below.

```
Run    Q1 ×
/Users/ryanwu/Project2/bin/python /Users/ryanwu/Documents/CMU/24-677 Modern Control Theory/Project/Project2/Q1.py
At 2 m/s:
Rank of controllability matrix P: 4, Controllable: Yes
Rank of observability matrix Q: 4, Observable: Yes
=======================================================
At 5 m/s:
Rank of controllability matrix P: 4, Controllable: Yes
Rank of observability matrix Q: 4, Observable: Yes
=======================================================
At 8 m/s:
Rank of controllability matrix P: 4, Controllable: Yes
Rank of observability matrix Q: 4, Observable: Yes
=======================================================
```

Figure 1. The controllability and observability at each given velocity values.

Exercise 1.2

(a)

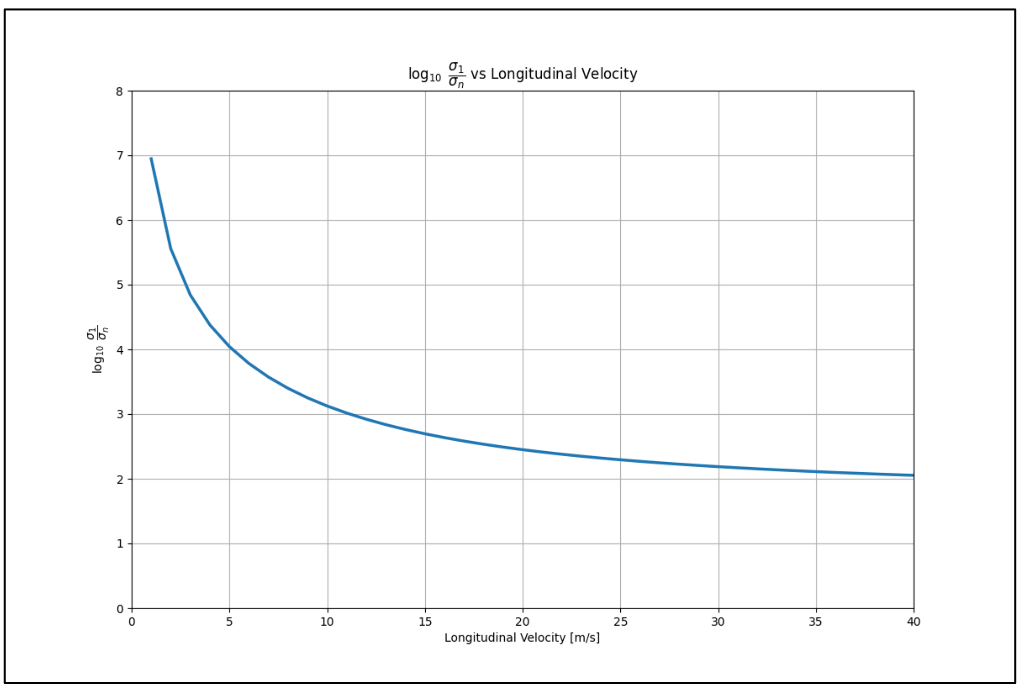


Figure 2. The plot for $log_{10}\frac{\sigma_1}{\sigma_n}$ versus the desired longitudinal velocity range.

(b)

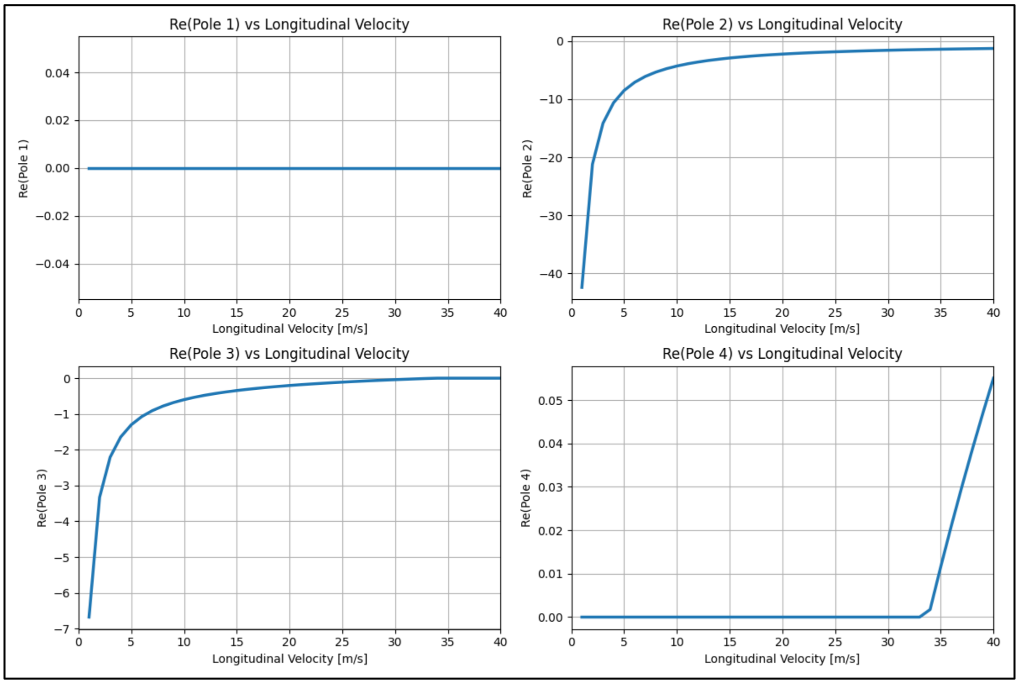


Figure 3. The plots for $Re(p_i)$ versus the desired longitudinal velocity range.

From Figure 2, we can observe that the value of the logarithm of the greatest singular value divided by the smallest converges to a smaller value, which means the system is less likely to be defected and becoming more and more controllable. However, from Figure 3, we can observe as the velocity increased, the poles became less and less negative, which means the overall system is becoming less and less stable.

Exercise 2

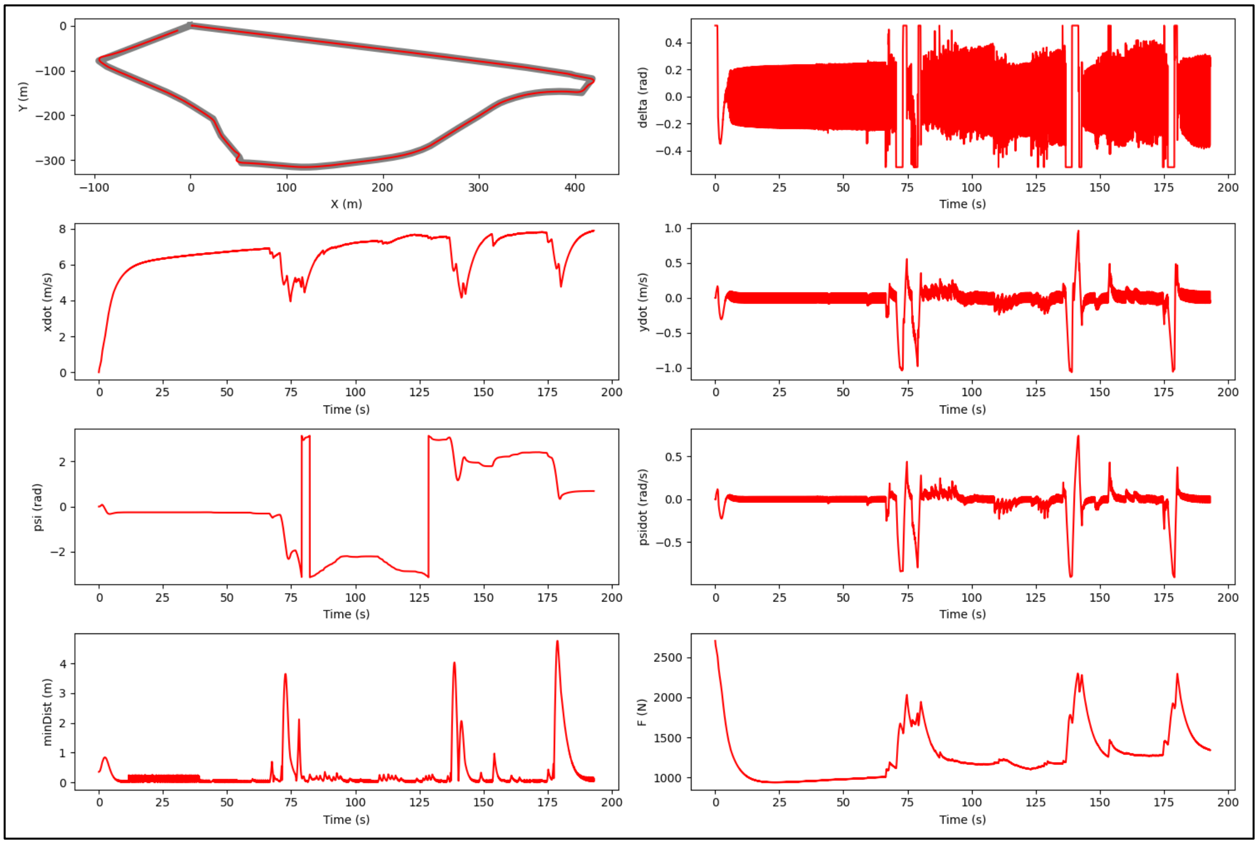
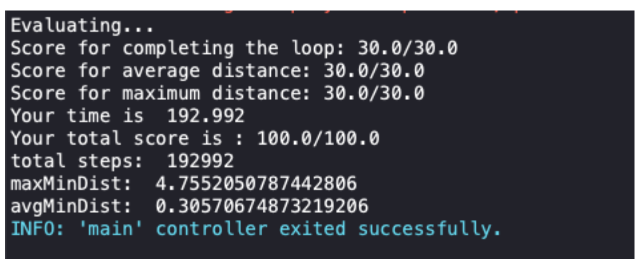


Figure 4. The final completion plot using poles: [-4, -1, -3, -2].



```
Evaluating...
Score for completing the loop: 30.0/30.0
Score for average distance: 30.0/30.0
Score for maximum distance: 30.0/30.0
Your time is  192.992
Your total score is : 100.0/100.0
total steps:  192992
maxMinDist:  4.7552050787442806
avgMinDist:  0.30570674873219206
INFO: 'main' controller exited successfully.
```

Figure 5. The final completion score using poles: [-4, -1, -3, -2].