SEE609: Mathematical and Computational Methods in Engineering

Semester I: 2024-25

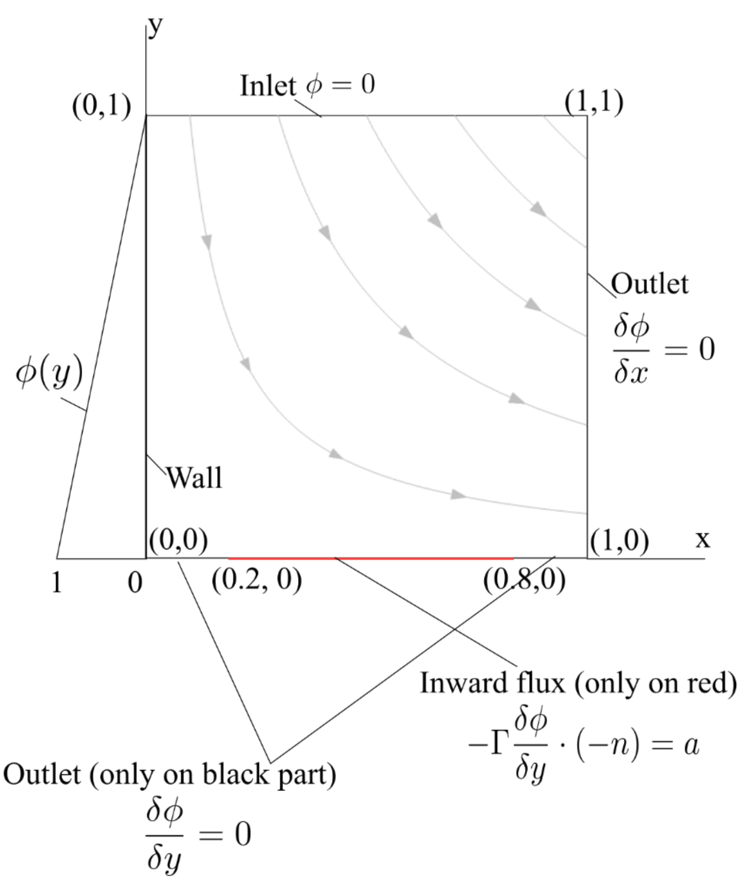
Final Project

Due date: 15 Nov 2024. Mid night (on Hello IITK)

NOTE: Submit a pdf file describing the derivations, problem formulations, discretization, equation system, boundary condition etc. Also, submit the results (i.e., convergence plots, contours, heatmaps, etc. from the code) in the main pdf file as well with associated discussions. Submit the standalone fully running code in a separate zipped file

Problem: Solve the 2D convection-diffusion transport of a scalar quantity in the known velocity field shown in the figure below. The transport equation to be solved is expressed in integral form as:

$$\int_S \rho\phi\mathbf{v}\cdot\mathbf{n}\,\mathrm{d}S = \int_S \Gamma\nabla\phi\cdot\mathbf{n}\,\mathrm{d}S + \int_\Omega \mathrm{q}_\phi\,\mathrm{d}\Omega$$



The known velocity field is u = x and v = −y, the diffusivity coefficient is Γ = 0.01, ρ = 1, the source term is $q_\phi = xy$, and the boundary conditions are:

- Inlet (Top wall: y=1): φ = 0
- Outlet (Right wall: x=1): $\frac{\partial \phi}{\partial x} = 0$
- Outlet(Bottom wall: y=0. For the two black sections between (x=0-0.2, and x=0.8-1)
    - $\frac{\partial \phi}{\partial y} = 0$
- Flux(Bottom wall: y=0. For the red section between x=0.2-0.8)
    - $\left(-\Gamma \frac{\partial \phi}{\partial y}\right) \cdot (-n) = a$
    - The values of a are provided below
- Dirichlet (Left wall: x=0): $\phi(y) = 1 - y$

Write your code in a modular structure, asking the user for (or receiving as an argument): i) the number of CVs in each direction and the type of convective discretization (UDS/CDS). You can use any inbuilt solver for the matrix solution. Clearly describe your meshing, discretization, UDS and CDS methods, boundary condition implementation, and other details. After developing the code do the following:

1) Use a base value of $a = 0.1$. Refine the mesh till grid independence is achieved. Estimate the total/average flux on the left boundary as the variable of interest. Plot the value of the flux as the mesh size reduces, then choose the final mesh size after which the variable changes are negligible. Do this analysis for both UDS and CDS
2) Use $a = 0.1$. Using Richardson extrapolation, get the exact value of flux. Describe the procedure in the pdf file.
3) Get the distribution of ϕ over the domain in a heatmap. Get these heatmaps for the following values of a. $a = [-0.1, -0.05, 0, 0.05, 0.1]$. Comment on the change in solution