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Optimization – Marketing Budget Allocation Report

Team

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Objective

We use Linear Programming in this project to build a simple marketing budget allocation strategy to maximize returns. We come up with budget allocations across various marketing mediums based on the insights provided by two different consulting firms about the ROI on each medium.

Approach

Our primary objective is to maximize the return on investment (ROI) across multiple platforms. With ten distinct platforms to consider and specific organizational constraints to adhere to, we modelled the Linear Programming Problem as follows:

Platforms Under Consideration:

- **Print**, **TV, SEO, AdWords, Facebook, LinkedIn, Instagram, Snapchat, Twitter, and Email**

The Constraints:

1. The combined investment in **Print** and **TV** should not surpass the collective amount spent on **Facebook** and **Email**.
2. The total investment in social media platforms (encompassing **Facebook**, **LinkedIn**, **Instagram**, **Snapchat**, and **Twitter**) should be at least double that of **SEO** and **AdWords**.
3. Each platform has an individual cap, ensuring no more than $3M is invested in any single one.

Note: We interpreted these constraints as if every column listed in the constraints will exist in the input data. We interpreted social media platforms as if it were an umbrella term, and added functionality to still compute the optimal solution if one of these platforms is deleted. However, since there is no way in the input file to identify if something is a social media, we did not support adding in a new social media platform to fall in this umbrella.

Modeling the Allocation:

To navigate this multifaceted scenario, we employed the **Linear Programming Model** using Gurobi optimizer. Here's a brief overview:

Decision Variables:

Given the ten platforms, we introduced ten decision variables, each representing the investment amount for its respective platform.

```
adver_mod_x = adver_mod.addMVar(len(var_names), ub=3, name=var_names)
```

Code Block 1: Decision variables defined using Gurobi in python

Objective Function:

The primary goal is to maximize the ROI. Using the ROI data from our consulting firm, the objective function was set up to sum the product of the ROI and the respective decision variable for each platform, aiming to maximize this sum.

```
adver_mod.setObjective(
    gp.quicksum(roi_dict[name] * var_dict[name] for name in var_names), sense=gp.GRB.MAXIMIZE)
```

Code Block 2: Objective function defined using Gurobi in python

Constraints:

The constraints, as detailed above, were incorporated to ensure our investments align with organizational guidelines. Specifically:

```
# Marketing Budget
adver_mod.addConstr(
    gp.quicksum(var_dict[name] for name in var_names) <= budget)

# Print & TV Budget Constraint
adver_mod.addConstr(var_dict['print'] + var_dict['tv'] - var_dict['facebook'] - var_dict['email'] <= 0
    )

# Social Media Budget Constraint
adver_mod.addConstr(
    gp.quicksum(
    var_dict[social] for social in ['facebook', 'linkedin', 'instagram', 'snapchat', 'twitter']
    )
    >= 2 * var_dict['seo'] + 2 * var_dict['adwords']
)
```

Code Block 3: Constraints defined using Gurobi in python

With the model set, the next steps involve solving it and analyzing the results. This will provide a clear roadmap for our marketing budget allocation, ensuring we get the most bang for our buck.

Analysis

In the quest to maximize our return on investment (ROI), the Gurobi optimization model was employed, leveraging the ROI data provided by our consulting firm. Let's delve into the findings.

ROI Data from First Consulting Firm:

Platform	Print	TV	SEO	AdWords	Facebook	LinkedIn	Instagram	Snapchat	Twitter	Email
ROI	0.031	0.049	0.024	0.039	0.016	0.024	0.046	0.026	0.033	0.044

Table 1: ROI estimates from first firm

Optimal Investment Strategy:

Based on the Gurobi model, the following investment strategy was suggested:

Platform	Print	TV	SEO	AdWords	Facebook	LinkedIn	Instagram	Snapchat	Twitter	Email	Objective Value
Investment ($M)	0.0	3.0	0.0	1.0	0.0	0.0	3.0	0.0	0.0	3.0	0.456

Table 2: Optimal investments based on ROI estimates from first firm

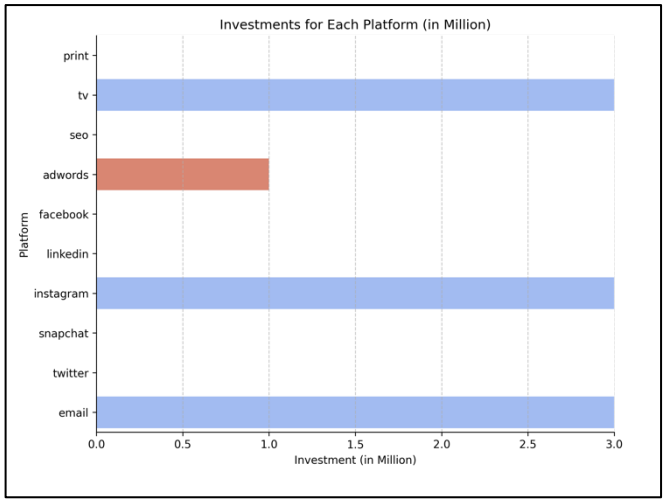

Figure 1: Optimal investments based on ROI estimates from first firm

Key Findings:

From the optimal allocation, a few key insights emerge:

1. The strategy leans heavily towards investing in platforms with the highest ROI. Specifically, the top 4 platforms in terms of ROI are **TV**, **Instagram**, **Email**, and **AdWords**.
2. The projected ROI, based on this allocation, stands at a promising $0.456 million.
3. Platforms like **Print**, **SEO**, **Facebook**, **LinkedIn**, **Snapchat**, and **Twitter** didn't receive any allocation, indicating they might not be the most efficient avenues for our current marketing goals.

In essence, the model suggests a focused approach, channeling funds into platforms that promise the highest returns. This strategy not only maximizes our ROI but also ensures that our marketing budget is utilized efficiently.

Comparative Analysis

While the initial results from our Gurobi optimization model, based on the first consulting firm's ROI data, were promising, our boss (fictional, ofcourse!) sought a second opinion on the ROI data to ensure comprehensive decision-making. Let's get into what we found.

ROI Data Comparison:

Platform	Print	TV	SEO	AdWords	Facebook	LinkedIn	Instagram	Snapchat	Twitter	Email
First Firm's ROI	0.031	0.049	0.024	0.039	0.016	0.024	0.046	0.026	0.033	0.044
Second Firm's ROI Estimate	0.049	0.023	0.024	0.039	0.044	0.046	0.026	0.019	0.037	0.026

Table 3: ROI estimates from first and second firm

Optimal Investment Strategy Comparison:

Based on the Gurobi model, the following investment strategies were suggested:

Platform	Print	TV	SEO	AdWords	Facebook	LinkedIn	Instagram	Snapchat	Twitter	Email	Objective Value
First Firm's Allocation	0.0	3.0	0.0	1.0	0.0	0.0	3.0	0.0	0.0	3.0	0.456
Second Firm's Allocation	3.0	0.0	0.0	1.0	3.0	3.0	0.0	0.0	0.0	0.0	0.456

Table 4: Optimal investments based on ROI estimates from first and second firm

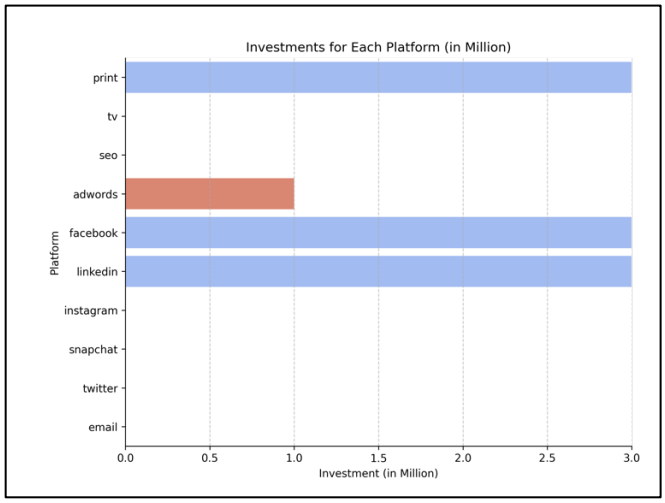

Figure 2: Optimal investments based on ROI estimates from second firm

Key Insights:

1. Interestingly, despite differences in ROI estimates from the two consulting firms, the projected ROI remains consistent at $0.456 million.
2. The second firm's data shifted our investment strategy. While the initial recommendation was to invest in **TV**, **Instagram**, **Email**, and **AdWords**, the new data suggests **Print**, **Facebook**, **LinkedIn**, and **AdWords** as the top platforms.

3. The consistency in the objective value underscores the importance of the top ROI platforms. Even though the specific platforms changed, the ROI for the top 4 platforms remained consistent between the two sets of data.

In conclusion, while the specific platforms of investment shifted based on the second firm's data, the overarching strategy remains consistent: invest in platforms with the highest ROI. This dual analysis reinforces our confidence in the proposed strategy, ensuring we're making a well-informed decision.

Deeper Analysis

In light of the two sets of ROI data and their respective optimal allocations, we embarked on a more granular analysis to understand the implications of each strategy. Let's dissect the findings.

Allocation Comparison:

Firstly, it's evident from our comparative analysis that the allocations derived from the first and second consulting firms' ROI data are **not the same**. This divergence underscores the sensitivity of our optimization model to the input ROI data.

Impact of Using Different Allocations:

1. **Using the Second Allocation with the First ROI Data**:
   - If we were to adopt the allocation strategy derived from the second ROI data, assuming the first ROI data is accurate, our objective would be $0.252 million, a decrease from the optimal objective of $0.456 million. This represents a potential loss in ROI.
2. **Using the First Allocation with the Second ROI Data**:
   - Conversely, if we were to use the allocation strategy based on the first ROI data, assuming the second ROI data is accurate, our objective would be $0.264 million, again a decrease from the optimal objective of $0.456 million. This too indicates a potential loss in ROI.

Reflecting on the Third Constraint:

The allocations for both first and second firm definitely change and the ROI increases by 9 thousand dollars. However, a 9 thousand dollar increase is just 0.09% of the overall budget, which is insignificant. Hence, we don't think the third condition set by the boss makes as much of a difference in this particular condition.

Given the discrepancies in objectives when using non-optimal allocations, the third constraint, which caps the investment in any single platform to $3M, appears to be a valuable safeguard. It ensures that we don't overcommit to a single platform, thereby mitigating risks associated with potential inaccuracies in ROI data.

Key Takeaways:

- The difference in allocations and the subsequent objectives, when using non-optimal strategies, highlight the critical importance of accurate ROI data. Even minor deviations can lead to significant financial implications.

- The third constraint, which promotes diversification, acts as a buffer against potential inaccuracies in ROI data. It's a strategic move that aligns with past experiences and lessons learned, ensuring a more balanced and risk-averse approach.

Sensitivity Analysis

In the dynamic landscape of marketing, ROI values for various platforms can fluctuate. To ensure our marketing strategy remains robust and adaptable, we conducted a sensitivity analysis on the ROI coefficients, using the first consulting firm's ROI data as our baseline.

The sensitivity analysis provides the range (lower and upper bounds) within which the ROI of each platform can vary without changing the optimal allocation:

Platform	Lower Bound	Upper Bound
Print	-∞	0.049
TV	0.039	0.062
SEO	-∞	0.039
AdWords	0.033	0.046
Facebook	-∞	0.029
LinkedIn	-∞	0.039
Instagram	0.039	∞
Snapchat	-∞	0.039
Twitter	-∞	0.039
Email	0.029	∞

- Platforms like **Print**, **SEO**, **Facebook**, **LinkedIn**, **Instagram**, **Snapchat**, **Twitter,** and **Email** have unbounded sensitivity in one or both directions. This indicates that these platforms have a wider flexibility in ROI changes without affecting the optimal allocation, given the constraints in place.

- Platforms like **TV** and **AdWords** have tighter bounds, indicating that even slight changes in their ROI values could potentially alter the optimal allocation.

- The third constraint, which caps the investment in any single platform to **$3M**, plays a pivotal role in these bounds. It ensures that we don't overcommit to a single platform, thereby promoting diversification and mitigating risks associated with potential inaccuracies in ROI data.

Reinvestment Analysis

With the new directive to reinvest half of our returns, we set out to determine the optimal allocation for each month, taking into account the monthly ROI and the constraints set by our boss. To understand the implications of reinvesting half of our returns, we performed a sensitivity analysis by updating the objective function and budget constraint based on the investment strategy employed.

```
for i in range(len(monthly_roi_df)):
    # Create a dictionary mapping column names to their respective ROI values
    roi_dict = dict(zip(var_names, monthly_roi_df.iloc[i][1:].values * 0.01))

    # Initiate model
    adver_mod = gp.Model()
    # Specify number of decision variables and assign names
    adver_mod_x = adver_mod.addMVar(len(var_names), ub=3, name=var_names)
    # Create a dictionary to map variable names to the decision variable objects
    var_dict = {name: adver_mod_x[j] for j, name in enumerate(var_names)}

    # Set objective using the roi_dict
    adver_mod.setObjective(
    gp.quicksum(roi_dict[name] * var_dict[name] for name in var_names), sense=gp.GRB.MAXIMIZE
    )

    # Add constraints
    adver_mod.addConstr(
     gp.quicksum(var_dict[name] for name in var_names) <= budget
    )  # Marketing Budget
    adver_mod.addConstr(
     var_dict['print'] + var_dict['tv'] - var_dict['facebook'] - var_dict['email'] <= 0
    )  # Print & TV Budget Constraint
    adver_mod.addConstr(
     gp.quicksum(
     var_dict[social] for social in ['facebook', 'linkedin', 'instagram', 'snapchat', 'twitter']
     if social in var_names
     )
     >= 2 * var_dict['seo'] + 2 * var_dict['adwords']
    )  # Social Media Budget Constraint

    adver_mod.Params.OutputFlag = 0
    # Optimize the model
    adver_mod.optimize()
    optimal_allocations = list(adver_mod_x.x)
    optimal_allocations.extend([budget, adver_mod.objVal])
    # Update marketing budget
    budget = base_budget + 0.5 * adver_mod.objVal
    monthly_allocation[monthly_roi_df.iloc[i][0]] = optimal_allocations
allocation_df = pd.DataFrame.from_dict(monthly_allocation, orient='index')
allocation_df.columns = var_names + ['Budget', 'ROI']
```

Code Block 4: Core logic for re-investment strategy using Gurobi in python

Results:

The table below showcases the optimal allocation for each month:

Month	Print	TV	SEO	AdWords	Facebook	LinkedIn	Instagram	Snapchat	Twitter	Email	Budget	ROI
January	3.0	0.0	0.0	1.333	0.0	0.0	2.667	0.0	0.0	3.0	10.0	0.373
February	3.0	0.0	0.0	2.396	3.0	0.0	0.0	0.0	1.791	0.0	10.187	0.406
March	0.0	0.0	0.0	3.0	0.0	3.0	1.203	0.0	3.0	0.0	10.203	0.408
April	0.0	0.0	0.0	3.0	0.0	3.0	3.0	0.0	1.204	0.0	10.204	0.400
May	1.2	0.0	0.0	0.0	0.0	0.0	3.0	0.0	3.0	3.0	10.2	0.411
June	3.0	0.0	0.0	0.0	0.0	0.0	3.0	0.0	1.206	3.0	10.206	0.424
July	0.0	0.0	0.0	3.0	1.212	0.0	3.0	0.0	3.0	0.0	10.212	0.428
August	2.714	0.0	0.0	1.5	0.0	0.0	0.0	0.0	3.0	3.0	10.214	0.438
September	0.609	0.0	0.0	3.0	0.0	3.0	0.0	0.0	3.0	0.609	10.219	0.403
October	0.0	0.0	0.0	3.0	0.0	3.0	3.0	0.0	0.0	1.201	10.201	0.371
November	3.0	0.0	0.0	1.186	0.0	0.0	3.0	0.0	0.0	3.0	10.186	0.442
December	3.0	2.11	0.0	0.0	3.0	0.0	0.0	0.0	0.0	2.11	10.221	0.433

Table 5: Optimal investments over months based on reinvestment strategy

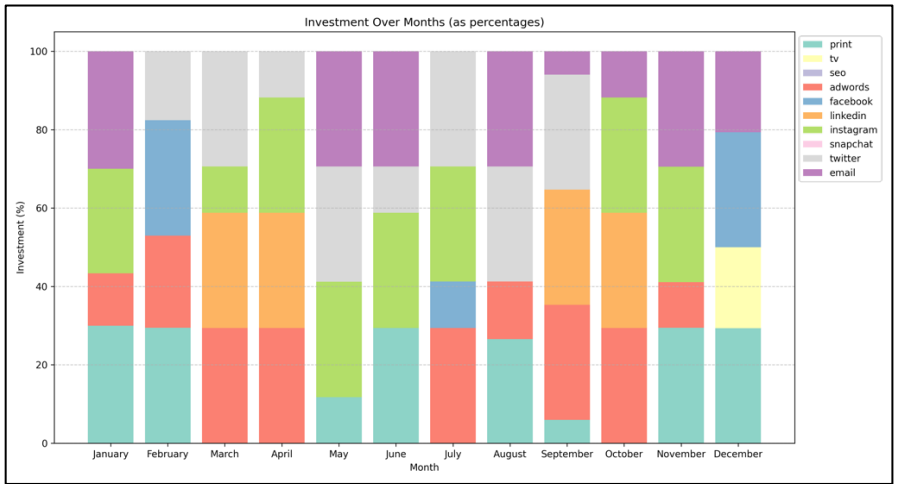

Figure 3: Optimal Investments over Months

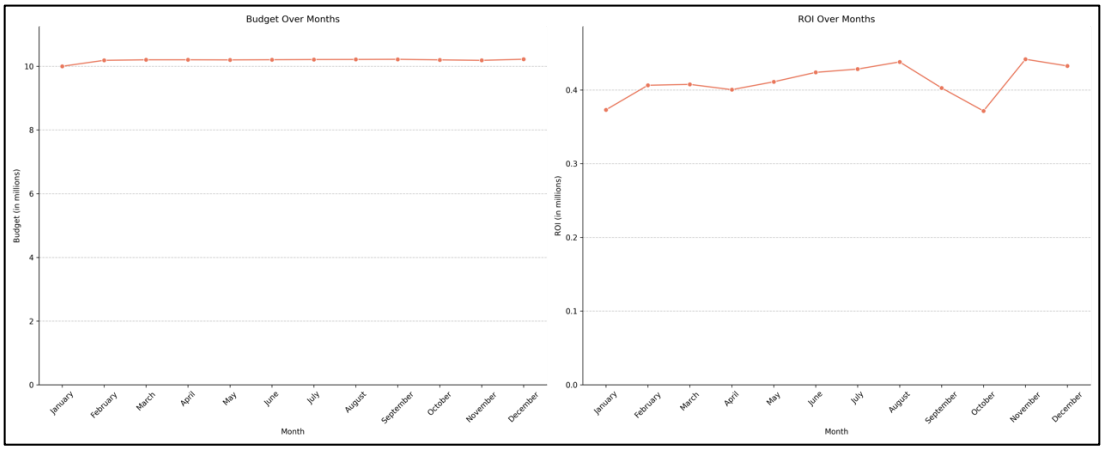

Figure 4: Budget and ROI Trend over Months

Key Insights from Reinvestment Analysis

- The optimal allocation varies each month, reflecting the dynamic nature of ROI values and the impact of reinvesting half of our returns.
- The constraints set by our boss play a pivotal role in shaping the monthly allocations. For instance, the cap on individual platform investments ensures we don't overcommit to a single platform, promoting diversification and mitigating risks.
- The reinvestment strategy, while amplifying our budget over time, requires a meticulous approach to ensure we're optimizing our allocations in line with the changing ROI values.

Stability Analysis of the Monthly Budget Allocation

A stable budget allocation, where monthly fluctuations are minimized, ensures predictability and can help in long-term planning. Let's assess the stability of our current allocation strategy.

By examining the month-to-month changes in the allocation for each platform, we observe that there are instances where the monthly change in spend for certain platforms exceeds the $1M threshold, indicating that our current allocation strategy is not stable.

Modeling Stability:

If we were to model for a stable budget allocation, we would **introduce additional constraints** to our optimization model. Specifically, for each platform and for each consecutive month, the difference in allocation should be constrained to be less than or equal to $1M.

Mathematically, this can be represented as:

$$\left|x_{i,t} - x_{i,t-1}\right| \leq 1$$

Where:

- $x_{i,t}$ is the allocation for platform i in month t.
- $x_{i,t-1}$ is the allocation for platform i in the previous month $t-1$.

Incorporating these constraints would ensure that our allocation remains stable month over month, while still aiming to maximize our ROI.

Therefore, while our current allocation strategy promises a high ROI, it lacks stability in monthly spend across platforms. To achieve a balance between maximizing returns and ensuring budgetary stability, we might consider revising our optimization model to include constraints that limit month-to-month fluctuations. This approach would provide a more predictable and consistent marketing strategy, aiding in long-term planning and stakeholder assurance.