

Dimensionality Reduction

Question 1: Note: In this question, all columns will be written in their transposed form, as rows, to make the typography simpler. Matrix M has three rows and three columns, and the columns form an orthonormal basis. One of the columns is $[2/7, 3/7, 6/7]$, and another is $[6/7, 2/7, -3/7]$. Let the third column be $[x, y, z]$. Since the length of the vector $[x, y, z]$ must be 1, there is a constraint that $x^2 + y^2 + z^2 = 1$. However, there are other constraints, and these other constraints can be used to deduce facts about the ratios among x , y , and z . Compute these ratios.

Q1

Let C_1 be $[2/3, 3/3, 6/3]$, C_2 be $[4/3, 2/3, -3/3]$ and C_3 be $[x, y, z]$

The dot Product of any two columns must be zero

$$C_1 \cdot C_2 = (2/3 * 4/3) + (3/3 * 2/3) + (6/3 * -3/3) = 0$$

$$C_2 \cdot C_3 = (4/3 * x) + (2/3 * y) + (-3/3 * z) = 0$$

$$6x + 2y - 3z = 0 \rightarrow \text{Eq 1}$$

$$C_3 \cdot C_1 = (x * 2/3) + (y * 3/3) + (z * 6/3) = 0$$

$$2x + 3y + 6z = 0 \rightarrow \text{Eq 2}$$

$$2 * \text{Eq 1} + \text{Eq 2} \rightarrow 12x + 4y - 6z + 2x + 3y + 6z = 0$$

$$14x + 7y = 0 \rightarrow y = -2x$$

$$3 * \text{Eq 2} - \text{Eq 1} \rightarrow 6x + 9y + 18z - 6x - 2y + 3z = 0$$

$$7y + 21z = 0 \Rightarrow y = -3z$$

$$x : y : z = -2 : 1 : 3$$

Question 2: Find the eigenvalues and eigenvectors of the following matrix:

2	3
3	10

You should assume the first component of an eigenvector is 1. Then, find out One eigenvalue and One eigenvector.

(2)
ans

Let the given matrix be $A = \begin{bmatrix} 2 & 3 \\ 3 & 10 \end{bmatrix}$ and the eigen vector be of the form $\begin{bmatrix} x \\ y \end{bmatrix}$

$$Ax = \lambda x \rightarrow \begin{bmatrix} 2 & 3 \\ 3 & 10 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \lambda \begin{bmatrix} x \\ y \end{bmatrix} \rightarrow 2x + 3y = \lambda x \text{ and } 3x + 10y = \lambda y$$

$$3 + 10y = (2 + 3y)y$$

$$3y^2 - 8y + 3 = 0 \rightarrow y = 3, -\frac{1}{3}$$

The eigen vectors are $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ -1/3 \end{bmatrix}$.

The eigen values are $2 + 3y = \lambda \rightarrow \lambda = 2 + 3 \times 3$

$$\lambda = 2 + 3(-\frac{1}{3}) = 1$$

Question 3: Suppose $[1,3,4,5,7]$ is an eigenvector of some matrix. What is the unit eigenvector in the same direction? Find out the components of the unit eigenvector.

(3)
ans

Given eigen vector of some matrix be $M = [1, 3, 4, 5, 7]$

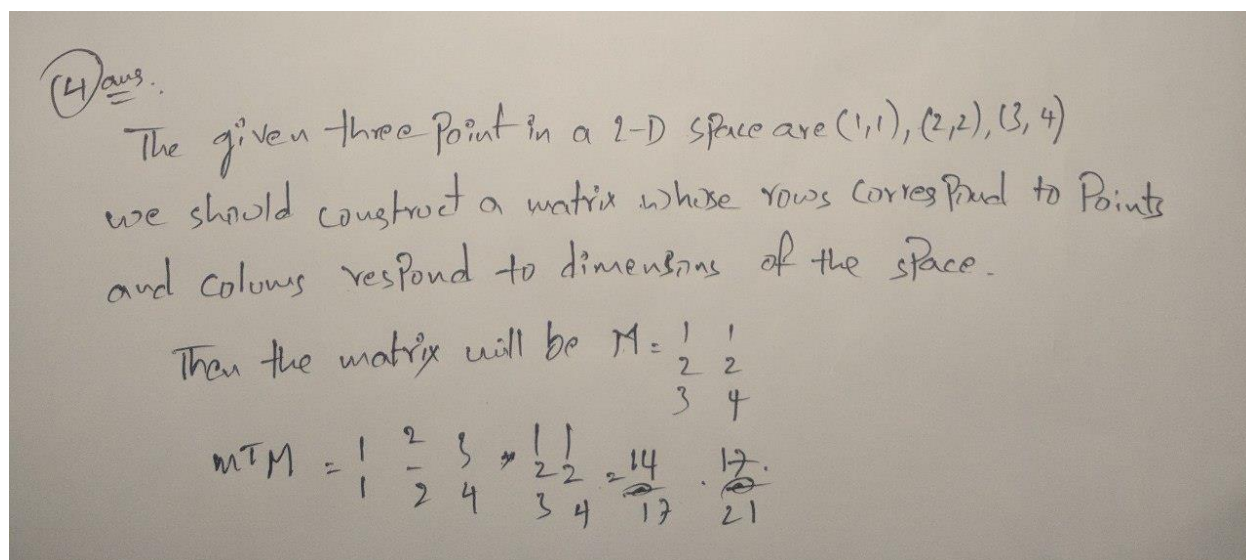
To get the unit eigen vector of given matrix.

Sum of the squares $= 1^2 + 3^2 + 4^2 + 5^2 + 7^2 = 100$ and its square root is 10.

Unit Eigen vector $= [\frac{1}{10}, \frac{3}{10}, \frac{4}{10}, \frac{5}{10}, \frac{7}{10}]$

Question 4: Suppose we have three points in a two dimensional space: $(1,1)$, $(2,2)$, and $(3,4)$. We want to perform PCA on these points, so we construct a 2-by-2 matrix, call it N , whose eigenvectors are the directions that best represent these three points. Construct the matrix N

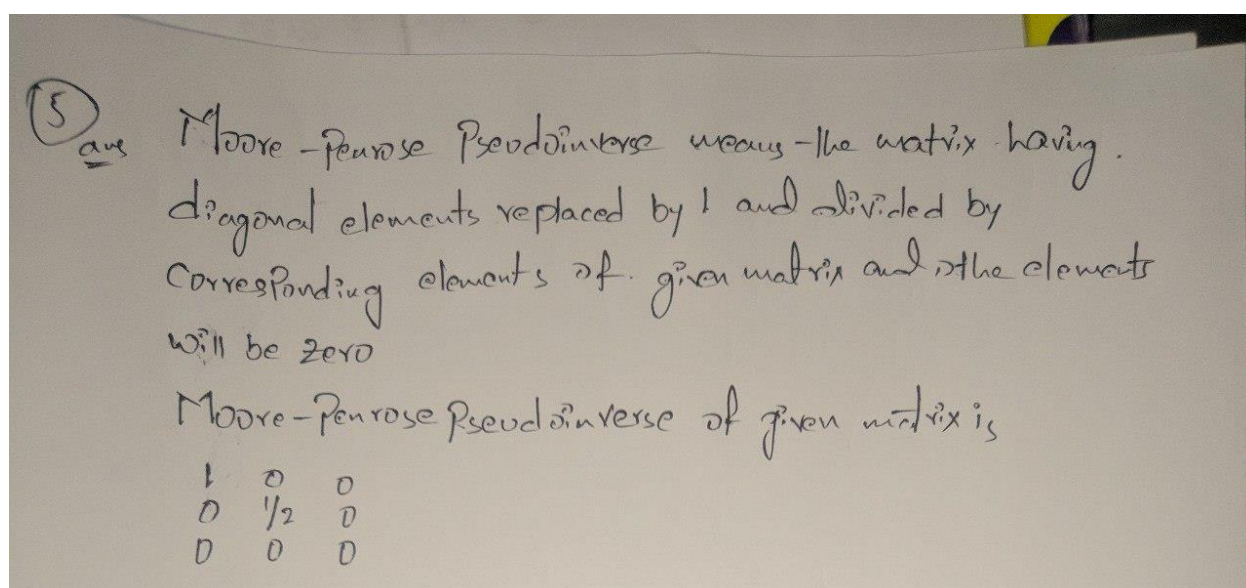
and identify, its elements.



Question 5: Consider the diagonal matrix $M =$

1	0	0
0	2	0
0	0	0

Compute its Moore-Penrose pseudoinverse.



Question 6: When we perform a CUR decomposition of a matrix, we select rows and columns by using a particular probability distribution for the rows and another for the columns. Here is a matrix that we wish to decompose:

1	2	3
4	5	6
7	8	9
10	11	12

Calculate the probability distribution for the rows.

Ans

Probability with which we choose row

\therefore Sum of squares of element in the rows.

Sum of squares of elements in the matrix

Sum of squares of elements in matrix

$$= 12 \times 13 \times 25/6 = 3900/6 = 650$$

$$P(R_1) = \frac{1^2 + 2^2 + 3^2}{650} = 14/650 = 0.02$$

$$P(R_2) = \frac{4^2 + 5^2 + 6^2}{650} = 77/650 = 0.12$$

$$P(R_3) = \frac{7^2 + 8^2 + 9^2}{650} = 194/650 = 0.298$$

$$P(R_4) = \frac{10^2 + 11^2 + 12^2}{650} = \frac{365}{650} = 0.56$$