

MapReduce and PageRank

Question 1:

Suppose our input data to a map-reduce operation consists of integer values (the keys are not important). The map function takes an integer i and produces the list of pairs (p,i) such that p is a prime divisor of i . For example, $\text{map}(12) = [(2,12),(3,12)]$.

The reduce function is addition. That is, $\text{reduce}(p,[i_1,i_2,\dots,i_k])$ is $(p,i_1+i_2+\dots+i_k)$.

Compute the output, if the input is the set of integers 15, 21, 24, 30, 49.

Answer 1: The output of map function is

$\text{map}(15) = [(3, 15), (5, 15)]$

$\text{map}(21) = [(3, 21), (7, 21)]$

$\text{map}(24) = [(2, 24), (3, 24)]$

$\text{map}(30) = [(2, 30), (3, 30), (5, 30)]$

$\text{map}(49) = [(7, 49)]$

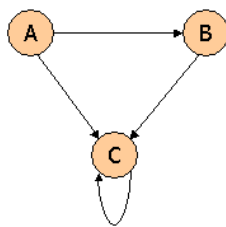
These are the respective prime divisors of inputs

The output of reduce function is

$\text{reduce}(2, 54), \text{reduce}(3,90), \text{reduce}(5,45), \text{reduce}(7, 70)$.

Question 2:

Consider three Web pages with the following links:



Suppose we compute PageRank with a β of 0.7, and we introduce the additional constraint that the sum of the PageRanks of the three pages must be 3, to handle the problem that otherwise any multiple of a solution will also be a solution. Compute the PageRanks a , b , and c of the three pages A, B, and C, respectively.

$$2)_{\text{ans}} \text{ for } a = \beta(a) + (1-\beta)$$

$$b = \beta(a/2) + (1-\beta)$$

$$c = \beta(a/2 + b + c) + (1-\beta)$$

$$\therefore \text{Here } \beta = 0.7, a+b+c = 3$$

$$\therefore a = 0.7(0) + (1-0.7) = 0.3$$

$$b = 0.7(0.3/2) + (1-0.7) = 0.405$$

$$c = 0.7(0.3/2 + 0.405 + c) + (1-0.7)$$

$$= 0.405 + 0.7(0.405) + (0.7)(c).$$

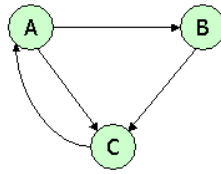
$$C = 1.7(0.405) + 0.7(c) \Rightarrow (1-0.7)C = 1.7(0.405).$$

$$= 0.3C = 0.6885$$

$$C = \frac{0.6885}{0.3}$$

$$= 2.295.$$

Question 3:



Suppose we compute PageRank with $\beta=0.85$. Write the equations for the PageRanks a , b , and c of the three pages A, B, and C, respectively.

Q3 Here

$$a = 0.85 \times c + (1-0.85) \frac{1}{3}$$

$$a = 0.85c + 0.05$$

$$b = 0.85 \times 0.5 \times a + 0.05$$

$$b = 0.425a + 0.05$$

$$c = 0.85 \times [0.5 \times a + b] + 0.05$$

$$= 0.425a + 0.85b + 0.05$$

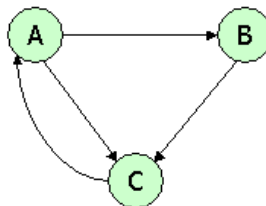
Formulae:

$$a = \beta \times c + (1-\beta) \frac{1}{3}$$

$$b = \beta \times a/2 + (1-\beta) \frac{1}{3}$$

$$c = \beta \times (a/2 + b) + (1-\beta) \frac{1}{3}$$

Question 4:



Assuming no "taxation," compute the PageRanks a , b , and c of the three pages A, B, and C, using iteration, starting with the "0th" iteration where all three pages have rank $a = b = c = 1$. Compute as far as the 5th iteration, and also determine what the PageRanks are in the limit.

(4) ~~18~~

formulae

$$a = c \quad b = a/2 \quad c = a/2 + b$$

0th iteration:

$$a = 1 \quad b = 1 \quad c = 1$$

1st iteration:

$$a = 1 \quad b = 1/2 \quad c = 1/2 + 1 = 3/2$$

2nd iteration

$$a = c = 3/2; \quad b = a/2 = 3/4; \quad c = 3/4 + 3/2 = 9/4$$

3rd iteration:

$$a = c = 9/4; \quad b = a/2 = 9/8; \quad c = 9/8 + 9/4 = 27/8$$

4th iteration:

$$a = c = 27/8; \quad b = a/2 = 27/16; \quad c = 27/16 + 27/8 = 81/16$$