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Data Science Interview Prep

Question 1

This problem was asked by Facebook.

There is a fair coin (one side heads, one side tails) and an unfair coin (both sides tails). You pick one at random, flip it 5 times, and observe that it comes up as tails all five times. What is the chance that you are flipping the unfair coin?

Solution:

This problem can be solved using Bayes Rule. We are asked to calculate the following quantity: $P(\text{Unfair} \mid \text{TTTTT})$.

We have that

$$\begin{aligned}
P(\text{Unfair} \mid \text{TTTTT}) &= \frac{P(\text{TTTTT} \mid \text{Unfair}) \cdot P(\text{Unfair})}{P(\text{TTTTT})} \\
&= \frac{P(\text{TTTTT} \mid \text{Unfair}) \cdot P(\text{Unfair})}{P(\text{TTTTT} \mid \text{Unfair}) \cdot P(\text{Unfair}) + P(\text{TTTTT} \mid \text{Fair}) \cdot P(\text{Fair})} \\
&= \frac{1^5 \cdot 0.5}{1^5 \cdot 0.5 + 0.5^5 \cdot 0.5} \\
&= \frac{0.5}{0.5 + 0.015625} \\
&= \boxed{0.9697}
\end{aligned}$$

Question 2

This problem was asked by Lyft.

What is the expected number of coin flips needed to get two consecutive heads?

Solution:

This is a slighly more complicated version of the classic problem – expected number of coin flips needed to get heads (which is simply the expected value of the geometric random variable with $p = 0.5$)

We can represent this sytem as a Markov chain as follows:

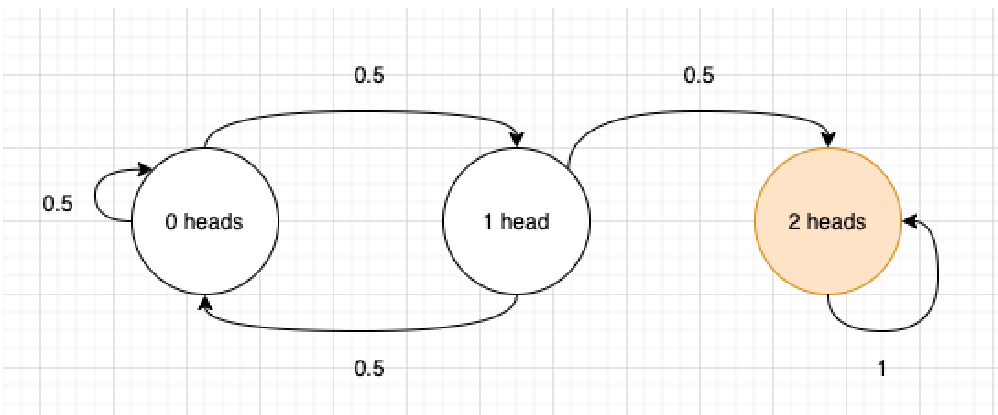

Figure 1: We start with 0 heads. With 0.5 probability, we see 1 head. After we have seen 1 head, if we see another head (which happens with $p = 0.5$), we are in the absorbing state 2. If we see a tails ($p = 0.5$), we go back to state 0.

If we are in state 2, the expected time to see two heads is $E[2] = 0$ as we have already seen two heads.

If we are in state 1, the expected time to see two heads is given by (using the law of total expectation):

$$
\begin{aligned}
E[1] &= 1 + \frac{1}{2} \cdot E[0] + 12 \cdot E[2] && (1) \\
&= 1 + \frac{1}{2} \cdot E[0] && (2)
\end{aligned}
$$

If we are in state 0, the expected time to see two heads is given by:

$$E[0] = 1 + \frac{1}{2} \cdot E[0] + \frac{1}{2} \cdot E[1] \qquad (3)$$

Substituting (2) in (3), we have:

$$E[0] = 1 + \frac{1}{2} \cdot E[0] + \frac{1}{2} \cdot \left(1 + \frac{1}{2} \cdot E[0]\right) \tag{4}$$

$$E[0] = \boxed{6} \tag{5}$$

Thus, if we are in state 0, that is when we start the experiment, the expected number of flips to see 2 heads is 6.