UNIVERSITÉ CÔTE D'AZUR

MAGS MASTER IN ASTROPHYSICS AND SPACE SCIENCE

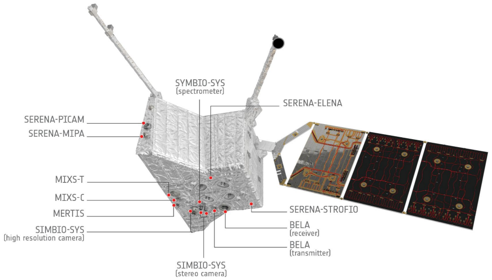
Testing General Relativity with BEPI

(Earth-Mercury Distance Measurements)

Presented by -Manidipa Banerjee

Introduction:

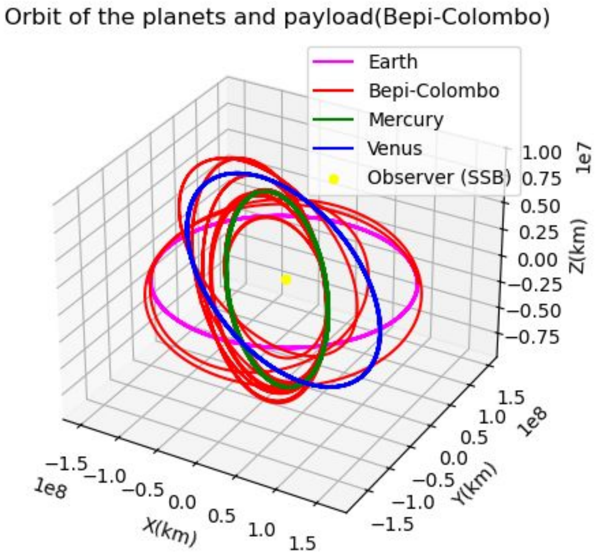
- Joint mission by ESA and JAXA to Mercury
- Two orbiters: **Mercury Planetary Orbiter(MPO), Mercury Magnetospheric Orbiter(MMO)** & Mercury Transfer Module(MTM)
- Aim is to study Mercury's composition, atmosphere, **Mercury Orbiter Radio Science Experiment(MORE)** to test GR
- Launched in October 2018, Arrived at Mercury on November 2026(1 year scientific operation of MPO)



http://sci.esa.int/bepicolombo/59287-bepicolombo-mpo-science-instruments/

Different Phases of BEPI:

- Used SPICE data to visualise different phases of BEPI
- **Launch date:** 20 October 2018, 01:45 UTC
- **Flyby of Earth:** April 2020
- **Flyby of Venus:** October 2020, August 2021
- **Flyby of Mercury:** October 2021, June 2022, June 2023, September 2024, December 2024, January 2025
- **Mercury Orbiter(MPO):** November 2026(Planned)



PPN formalism:

- Set of parameters used to estimate deviations from Newtonian Gravity
- Parameterized by Eddington-Robertson-Schiff parameters, β and γ
- When $\beta=\gamma=1$, refers to GR
- Alternative theories described by changing β and γ values or by adding additional parameter(Graviton with a non-zero mass leads to introducing additional parameter called Compton Wavelength of Graviton,λg.
- For N-body point mass system, the acceleration in PPN given by:

$$
\begin{aligned}
a_T^{PPN} = & -\sum_{A\neq T}\frac{\mu_A}{r_{AT}^3}\mathbf{r}_{AT} \\
& -\sum_{A\neq T}\frac{\mu_A}{r_{AT}^3 c^2}\mathbf{r}_{AT}\Bigg\{\gamma v_T^2 + (\gamma+1)v_A^2 - 2(1+\gamma)\mathbf{v}_A\cdot\mathbf{v}_T - \frac{3}{2}\left(\frac{\mathbf{r}_{AT}\cdot\mathbf{v}_A}{r_{AT}}\right)^2 \\
& \qquad -\frac{1}{2}\mathbf{r}_{AT}\cdot\mathbf{a}_A - 2(\gamma+\beta)\sum_{B\neq T}\frac{\mu_B}{r_{TB}} - (2\beta-1)\sum_{B\neq A}\frac{\mu_B}{r_{AB}}\Bigg\} \\
& +\sum_{A\neq T}\frac{\mu_A}{c^2 r_{AT}^3}[2(1+\gamma)\mathbf{r}_{AT}\cdot\mathbf{v}_T - (1+2\gamma)\mathbf{r}_{AT}\cdot\mathbf{v}_A](\mathbf{v}_T-\mathbf{v}_A) \\
& +\frac{3+4\gamma}{2}\sum_{A\neq T}\frac{\mu_A}{c^2 r_{AT}}\mathbf{a}_A
\end{aligned}
$$

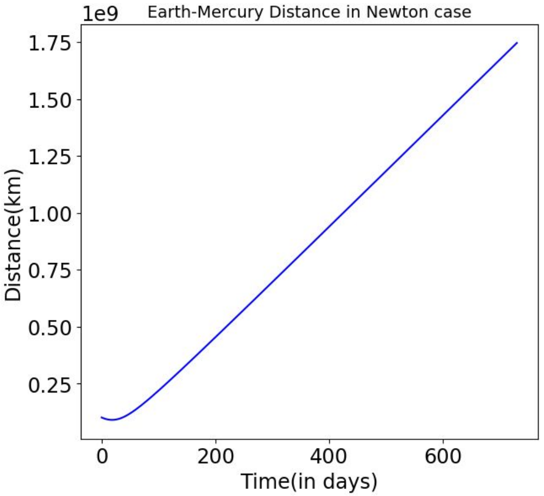
Simulation :

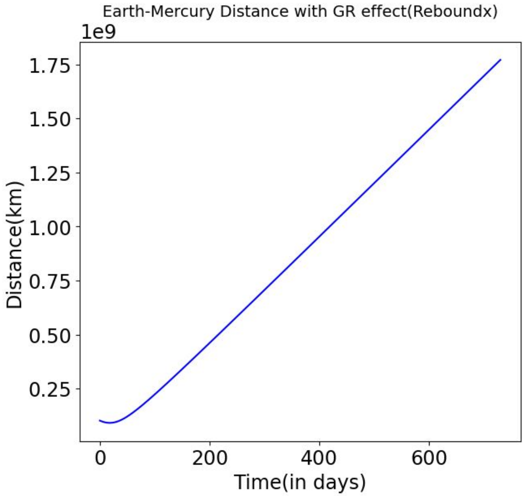
To see the impact on Earth-Mercury Distance:

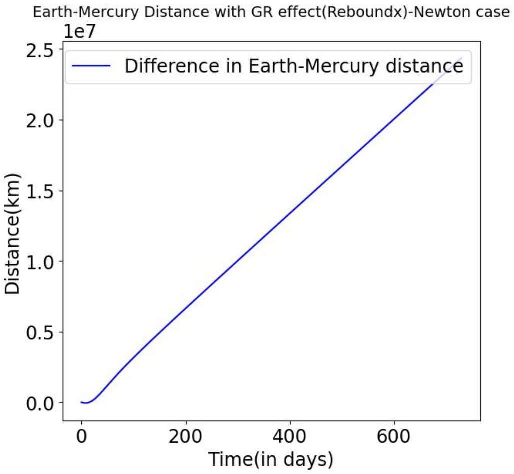
- Newtonian Gravity using REBOUND, integrating orbits of all 8 planets, 3 big asteroids (Ceres, Pallas, Vesta), the MPO, the Sun
- Integrator = WHfast
- Start Date = 2026-11-01 00:00 UTC(when MPO orbit is stable around Mercury)
- Simulation’s unit = ('days', 'km', 'kg')
- Observer at Solar System Barycenter
- Initial state vectors obtained from bc_plan_v430_20241120_001.tm meta-kernel from SPICE.
- REBOUNDx to add GR, Sun’s J2 effect
- Shapiro delay defined
- Additional accelerations added to the simulation using REBOUNDx
- This simulation will compute theoretical observable(Earth-Mercury distance) and Doppler measurement
- Covariance matrix analysis to estimate expected accuracy on the constraint parameters
  - Partial derivative matrix, J = δO/δP (O is the observable and P are the parameters: cartesian coordinates, velocities, β, γ, *λg.*
  - Covariance matrix, Cov(P) = (*J^T* W *J*) *^(-1)*
  - Diagonal of the Cov(P) represents the covariance vector of the fitted parameters, used to estimate σ(standard deviation)

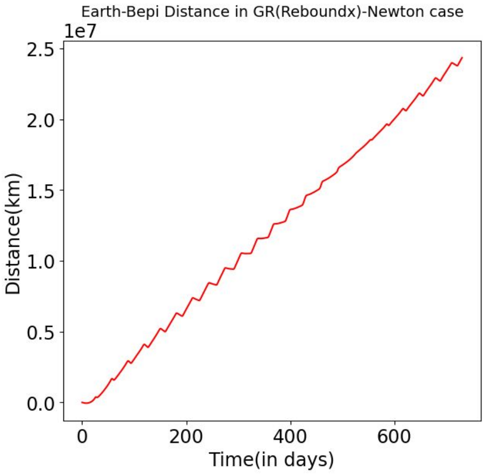
Phase 1:

- Newtonian Gravity & GR from REBOUNDx:
- Integrating orbits of all 8 planets, 3 big asteroids (Ceres, Pallas, Vesta), the MPO, the Sun
- Simulation time: 2 years
- With GR effect, noticeable difference over time in the earth-mercury distance measurements.
- Oscillations due to periodic gravitational influence by the perturbers
- Linear trend



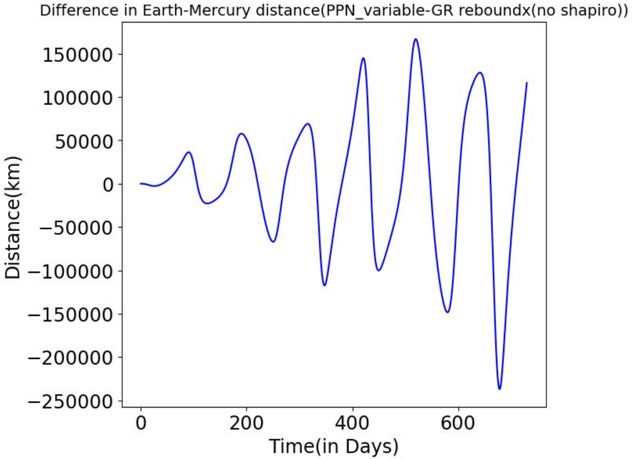


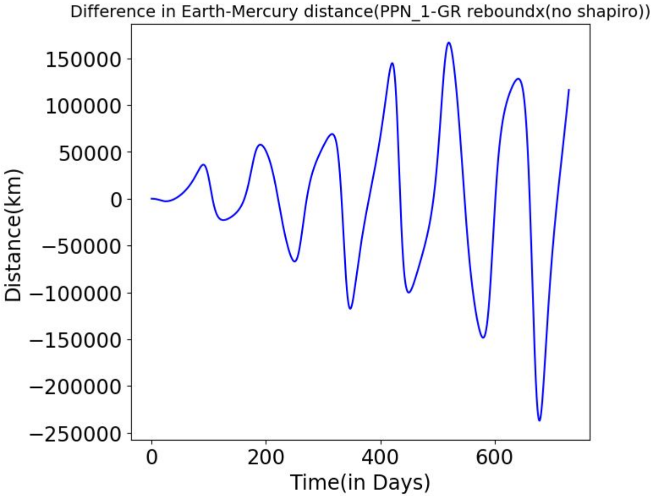


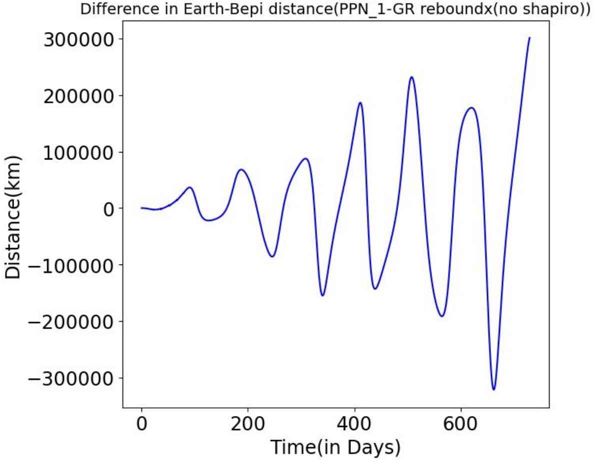


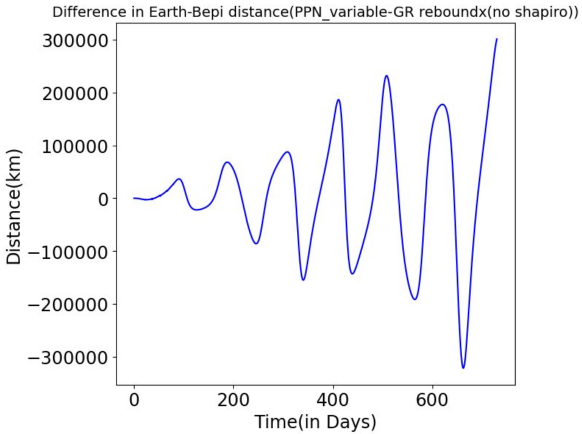
Phase 1:

- Due to heavy computation, 5 objects now considered over 2 year period:(Sun, Mercury, Bepi, Earth, Jupiter)
- PPN(gamma=beta=1), PPN with gamma=beta=variable
- Shapiro delay introduced(for Sun, Jupiter)
- Sun’s J2 effect introduced
- Increasing oscillatory amplitude in the difference in distance due to relativistic effects, influenced significantly by Jupiter and Sun.



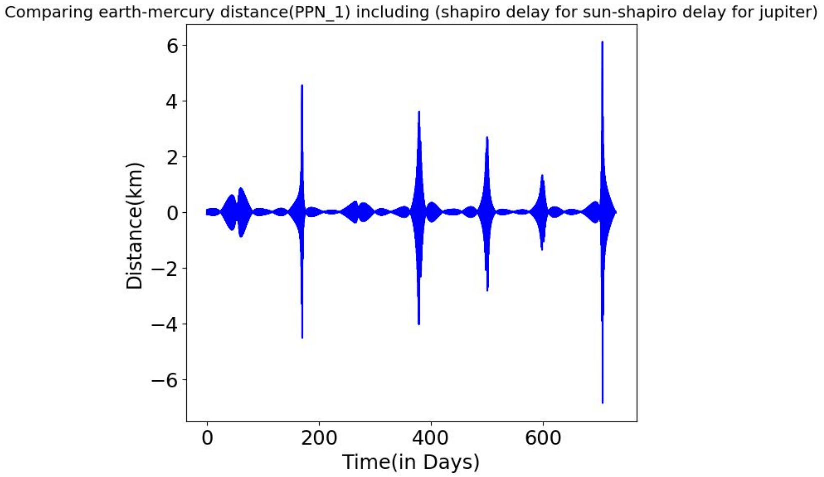


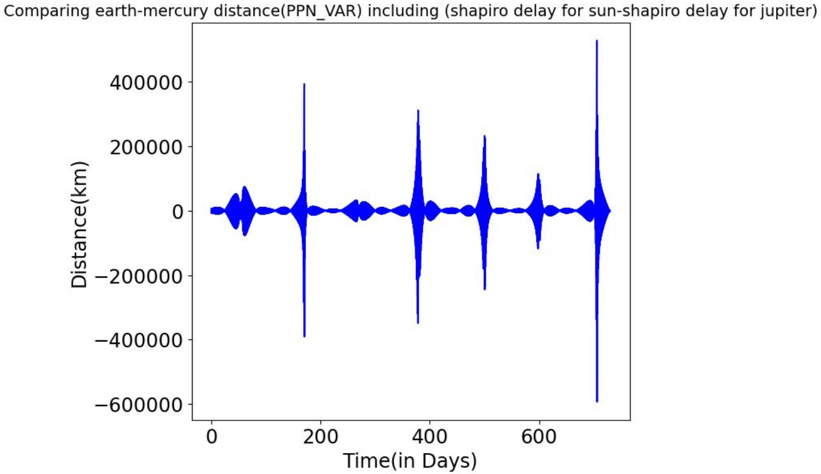


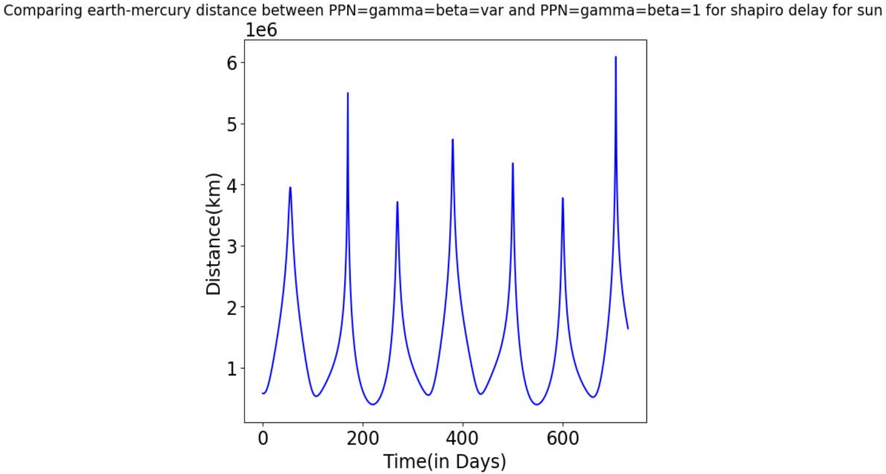


Phase 1:

- Shapiro delay: delay in the travel time of light when it passes by a massive object
- Shapiro delay computation depends on variable values of beta and gamma, increasing oscillatory behaviour than PPN with beta=gamma=1(consistent shift in the distance)
- Periodic variation in earth-mercury distance
- Sun's shapiro delay dominates the jupiter's due to its large mass and close proximity

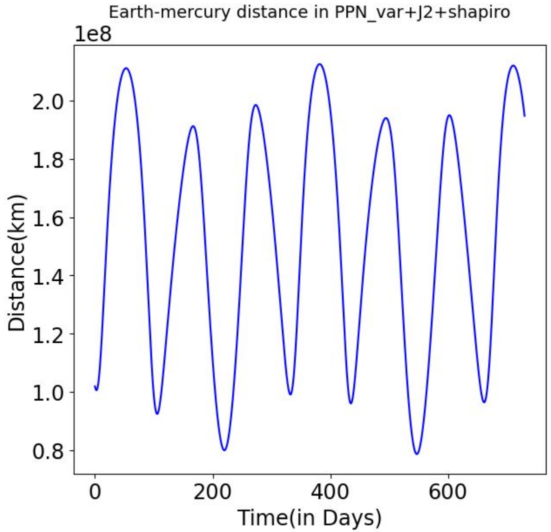


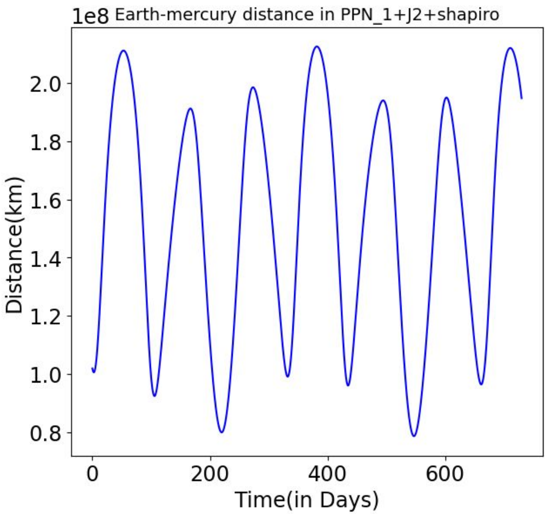


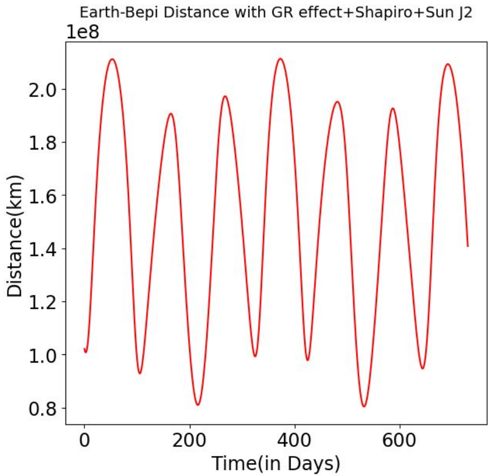


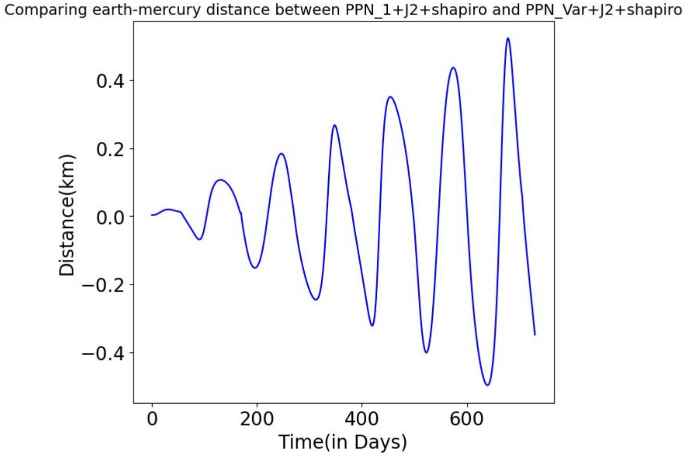
Phase 1:

- Variations due to relativistic effects, time delay induces an increase on the earth-mercury distance





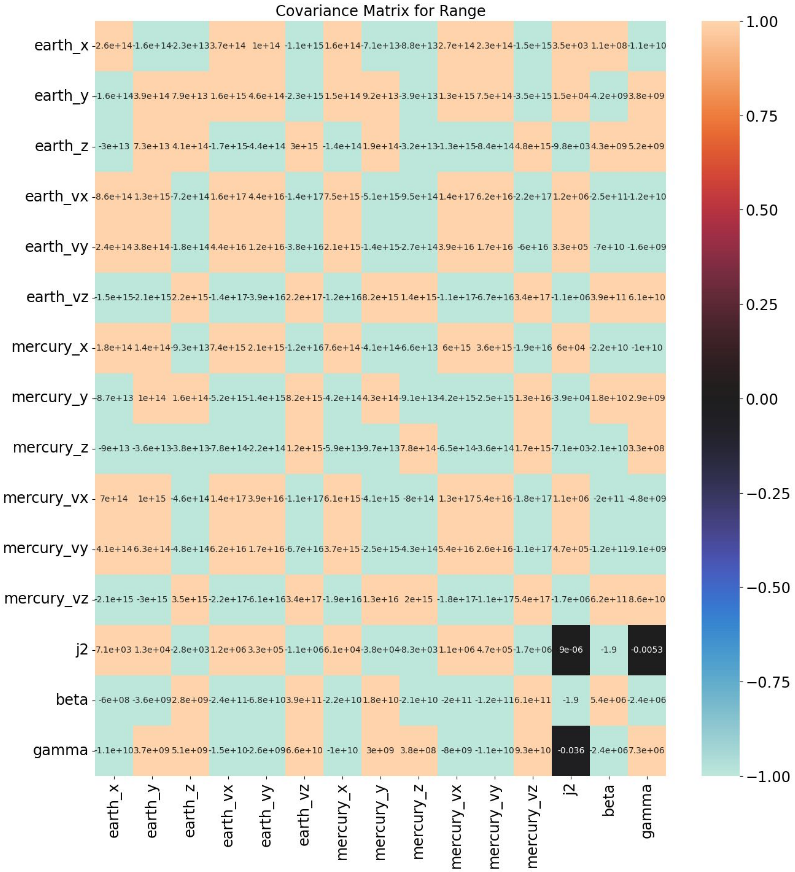


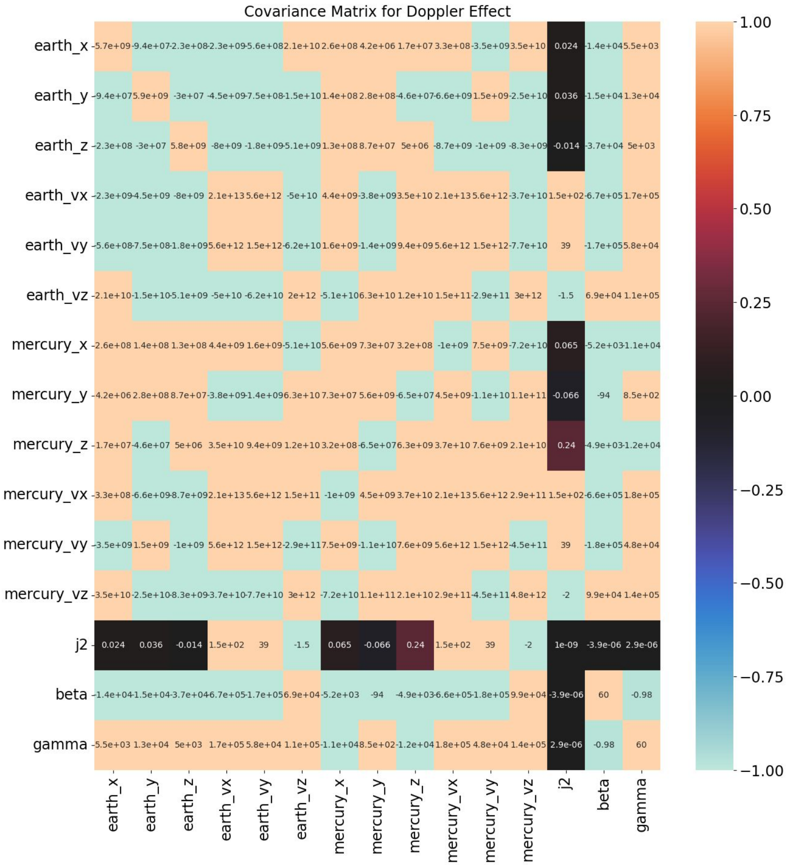


Phase 2:

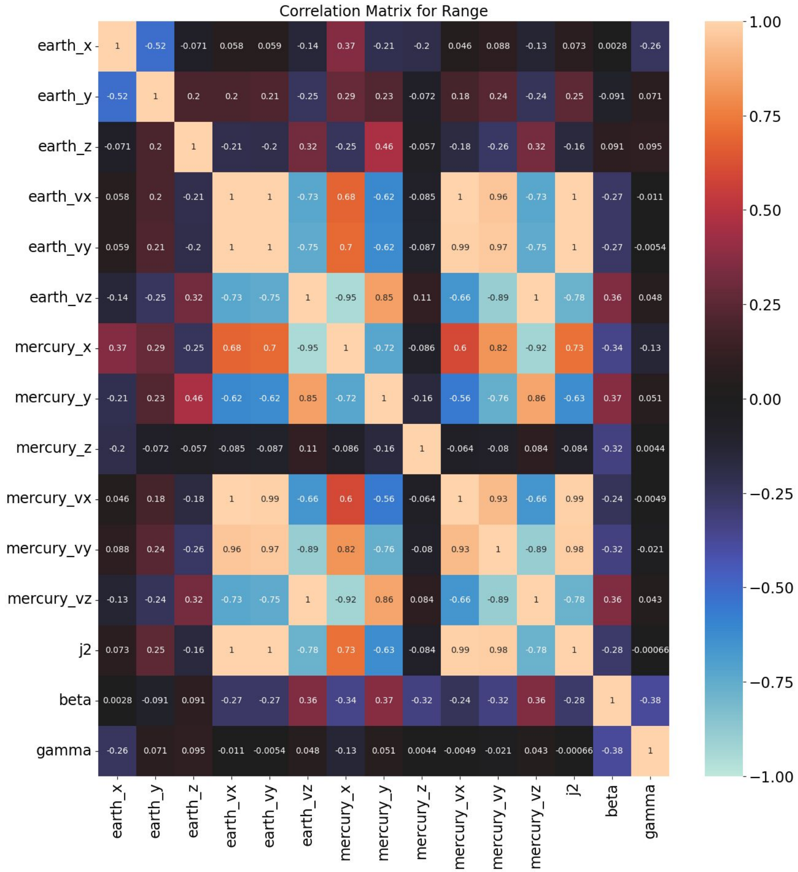
- Accounting for the 3 asteroids, Jupiter, initial conditions of the sun J2, Earth, Mercury orbits, and GRT as well as the Shapiro delay for determination of Earth-mercury distance and Doppler effect
- 10 days simulation with 8 hours integration per day,(time step=60 sec)
- Uncertainties of the parameters:
  - dx0 = .1 #(km)
  - dy0 = .1
  - dz0 = .1
  - dvx0 = 0.0001 #(km/s)
  - dvy0 = 0.0001
  - dvz0 = 0.0001
  - J2 = 1e-7
  - dJ2 = J2*.1
  - beta = 1
  - dbeta = 1e-5
  - gamma = 1
  - dgamma= 1e-5
  - # sigma_range=10 cm
  - # Sigma_doppler @ 60 s = 12 micron/s
- As discussed in ‘Simulation’ section, Covariance matrix(will have M rows: no. of observations and N columns: no. of parameters) will be computed through Jacobian and Weight matrix(W).
- The square root of the covariance vector from the covariance matrix represents the accuracy of the fitted parameters.

Phase 2:

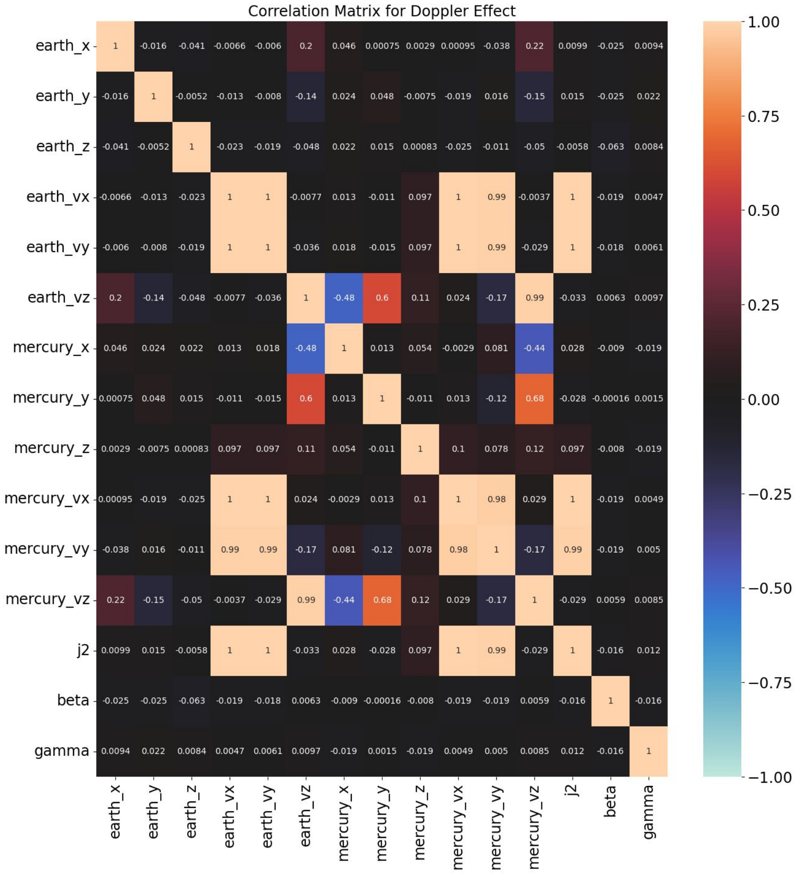

Covariance Matrix for Range
earth_x
earth_y
earth_z
earth_vx
earth_vy
earth_vz
mercury_x
mercury_y
mercury_z
mercury_vx
mercury_vy
mercury_vz
j2
beta
gamma
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00


Covariance Matrix for Doppler Effect
earth_x
earth_y
earth_z
earth_vx
earth_vy
earth_vz
mercury_x
mercury_y
mercury_z
mercury_vx
mercury_vy
mercury_vz
j2
beta
gamma
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00

Phase 2:



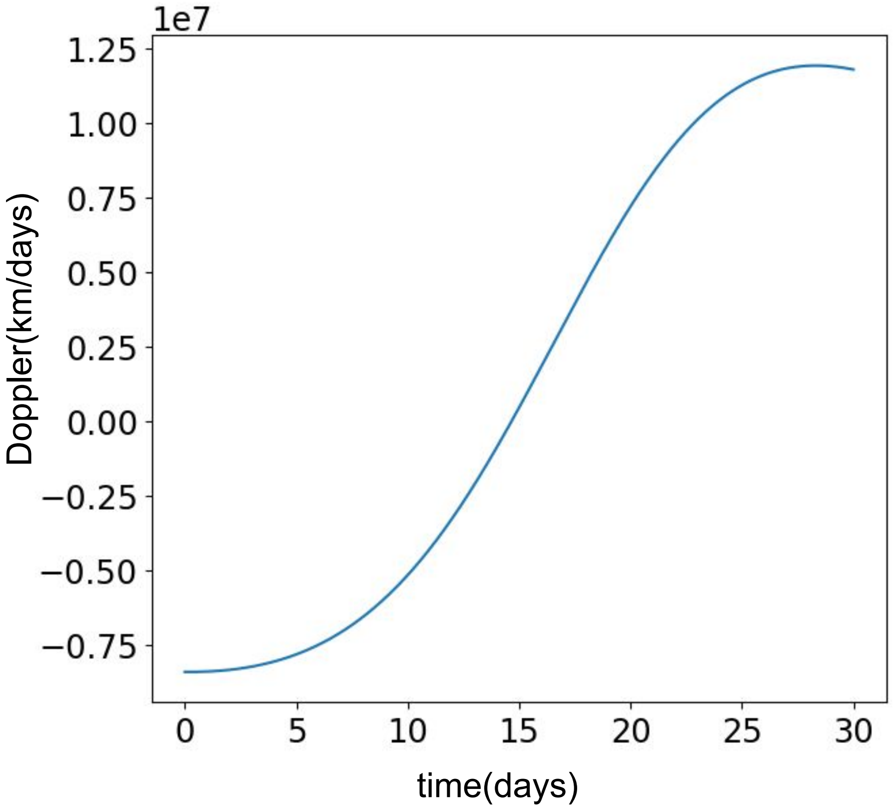
Correlation Matrix for Range
earth_x
earth_y
earth_z
earth_vx
earth_vy
earth_vz
mercury_x
mercury_y
mercury_z
mercury_vx
mercury_vy
mercury_vz
j2
beta
gamma
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00



Correlation Matrix for Doppler Effect
earth_x
earth_y
earth_z
earth_vx
earth_vy
earth_vz
mercury_x
mercury_y
mercury_z
mercury_vx
mercury_vy
mercury_vz
j2
beta
gamma
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00

Phase 2:

- The estimated σ obtained on the parameters are:
  - ['earth_x', 'earth_y', 'earth_z', 'earth_vx', 'earth_vy', 'earth_vz', 'mercury_x', 'mercury_y', 'mercury_z', 'mercury_vx', 'mercury_vy', 'mercury_vz', 'j2'', 'beta', 'gamma'] = [1.60319693e+07, 1.96933327e+07, 2.01303978e+07, 4.00880653e+08, 1.09336276e+08, 4.68607428e+08, 2.75054805e+07, 2.06769082e+07, 2.79452879e+07, 3.62587968e+08, 1.60753869e+08, 7.36257439e+08, 3.00247387e-03, 2.33254377e+03, 2.70136343e+03] #for range
  - ['earth_x', 'earth_y', 'earth_z', 'earth_vx', 'earth_vy', 'earth_vz', 'mercury_x', 'mercury_y', 'mercury_z', 'mercury_vx', 'mercury_vy', 'mercury_vz', 'j2'', 'beta', 'gamma'] = [7.55513453e+04, 7.66262123e+04, 7.63818185e+04, 4.59512669e+06, 1.21921136e+06, 1.40874912e+06, 7.50028871e+04, 7.48517272e+04, 7.93033883e+04, 4.59752502e+06, 1.23455684e+06, 2.18327226e+06, 3.16392533e-05, 7.72898166e+00, 7.75146218e+00] #for doppler



Phase 3:(Graviton)

- The Graviton is a hypothetical particle proposed in quantum field theory to mediate the force of gravity.
- Massive Gravity theory proposes graviton with a non-zero mass.
- One prediction of the MG theory is Yukawa suppression(1/r falloff of the Newtonian potential)
- Yukawa suppression depends on compton wavelength, *λg*
- It relates to the graviton mass by,

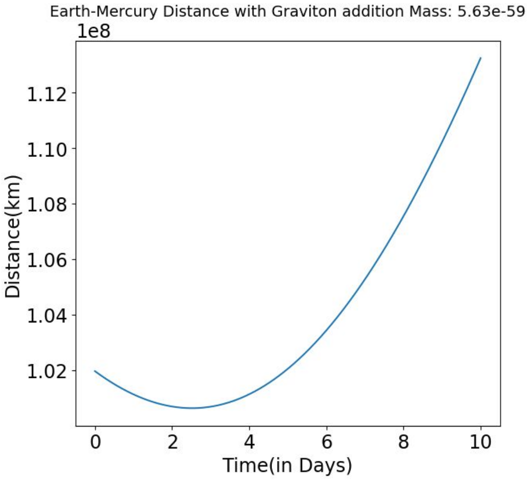
$$m_g = \frac{h}{c\lambda_g}$$

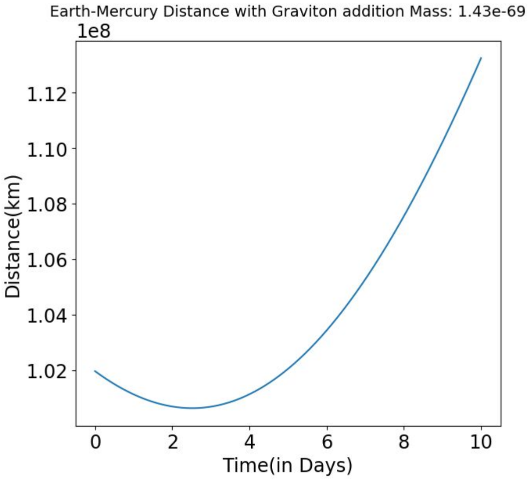
- Smaller *λg* leads to stronger suppression.
- In the PPN, the graviton introduces an additional radial acceleration added to the PPN acceleration. The additional acceleration therefore defined by,

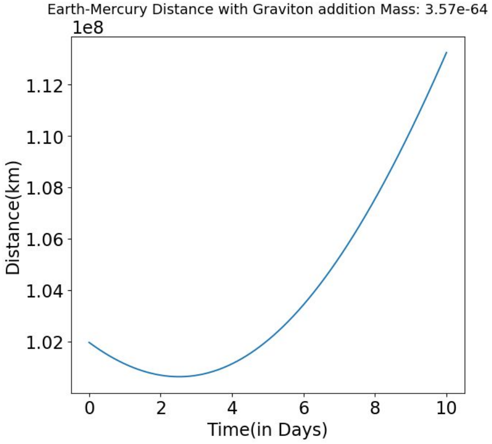
$$\delta \mathbf{a}_A^{\lambda_g} = \frac{1}{2\lambda_g^2} \sum_{A \neq T} \frac{\mu_T}{r_{AT}} \mathbf{r}_{AT} + \mathcal{O}(\lambda_g^{-3})$$

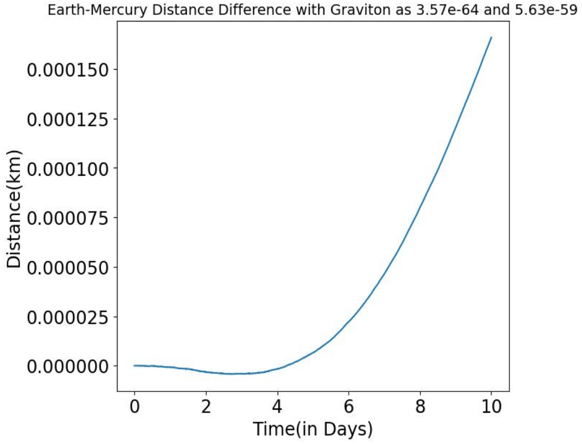
Phase 3:

- 10 days simulation with an addition of graviton mass
- Integrating SUN, BEPI, Mercury, Jupiter and Earth orbits
- Variations due to relativistic effects, addition of acceleration due to graviton as well as time delay induces an increase on the earth-mercury distance over time, with the graviton mass corresponds to 5.63e-59 kg shows max effect.
- To construct Jacobian, an uncertainty of 10^12 km was adopted for *δλg.*

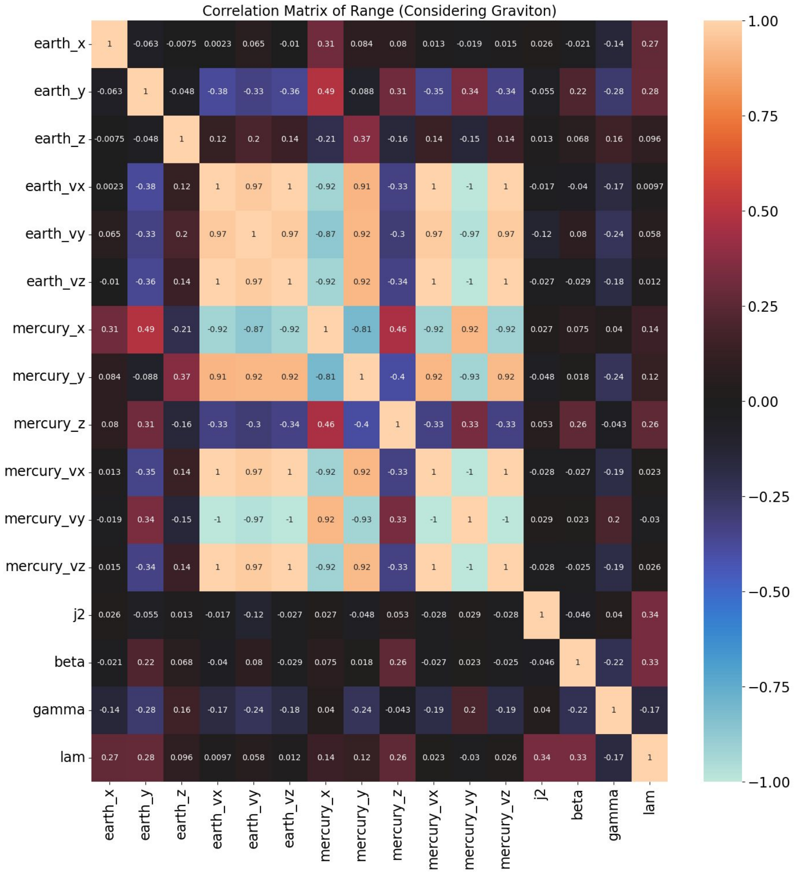


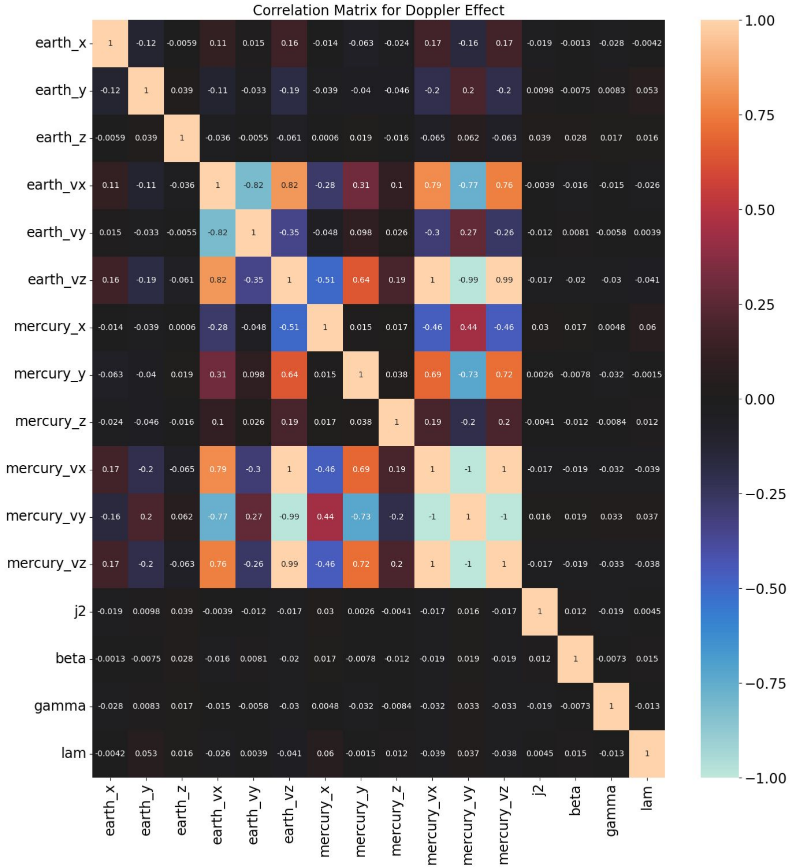




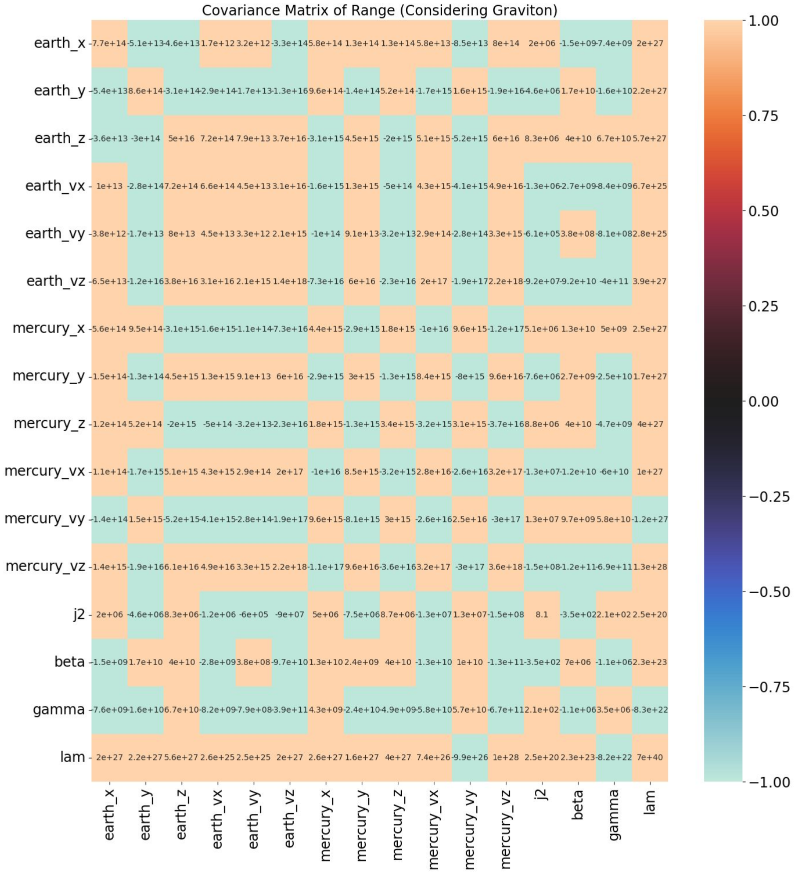


Phase 3:

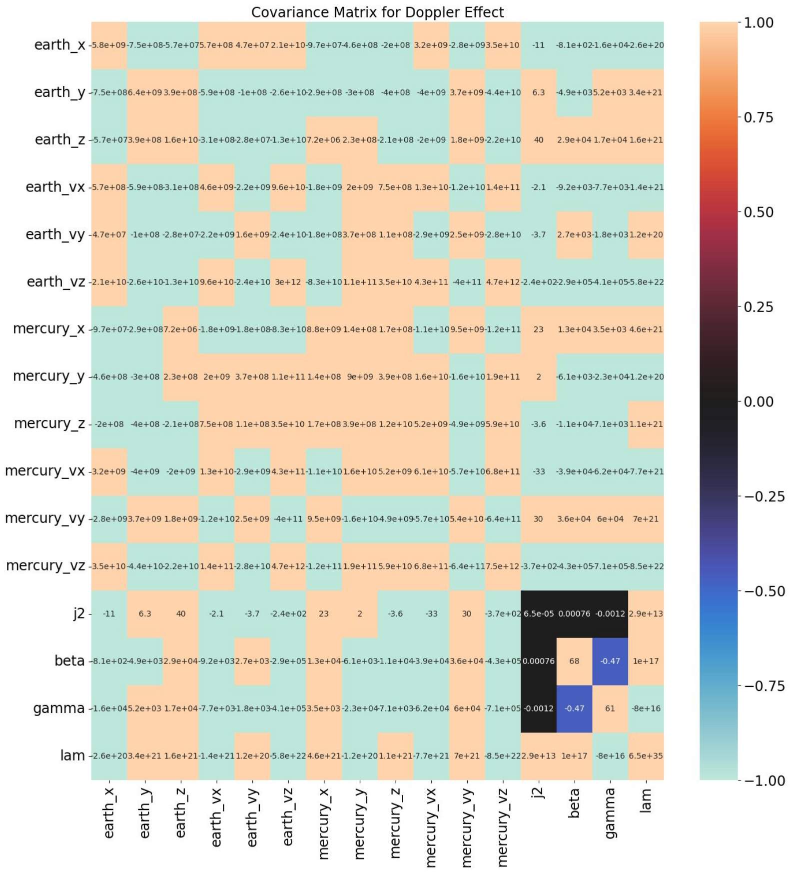

Correlation Matrix of Range (Considering Graviton)
earth_x
earth_y
earth_z
earth_vx
earth_vy
earth_vz
mercury_x
mercury_y
mercury_z
mercury_vx
mercury_vy
mercury_vz
j2
beta
gamma
lam
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00


Correlation Matrix for Doppler Effect
earth_x
earth_y
earth_z
earth_vx
earth_vy
earth_vz
mercury_x
mercury_y
mercury_z
mercury_vx
mercury_vy
mercury_vz
j2
beta
gamma
lam
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00

Phase 3:



Covariance Matrix of Range (Considering Graviton)
earth_x
earth_y
earth_z
earth_vx
earth_vy
earth_vz
mercury_x
mercury_y
mercury_z
mercury_vx
mercury_vy
mercury_vz
j2
beta
gamma
lam
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00



Covariance Matrix for Doppler Effect
earth_x
earth_y
earth_z
earth_vx
earth_vy
earth_vz
mercury_x
mercury_y
mercury_z
mercury_vx
mercury_vy
mercury_vz
j2
beta
gamma
lam
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00

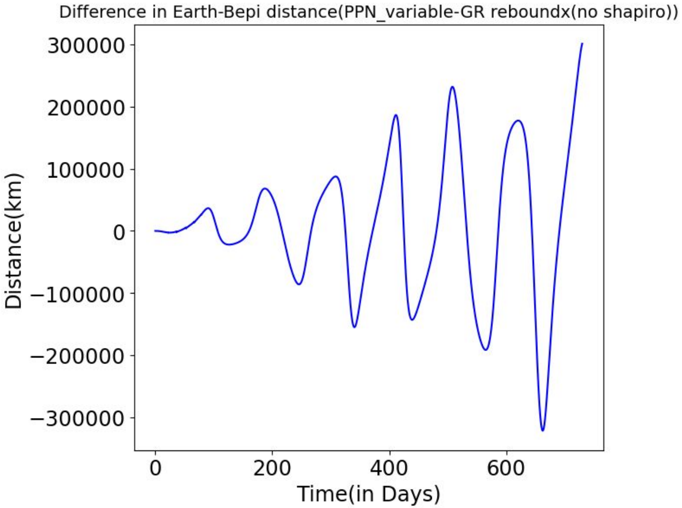
Phase 3:

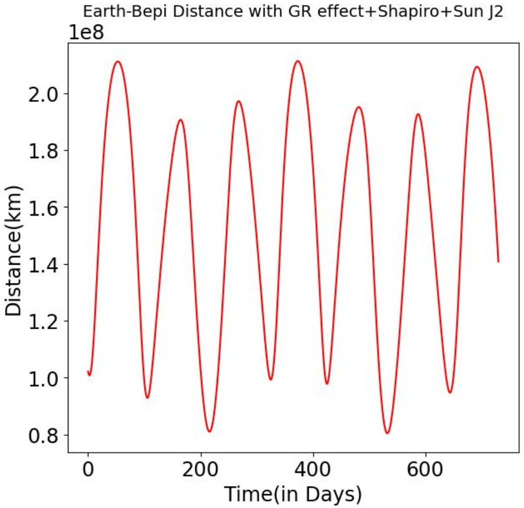
- The estimated σ obtained on the parameters are:
    - ['earth_x', 'earth_y', 'earth_z', 'earth_vx', 'earth_vy', 'earth_vz', 'mercury_x', 'mercury_y', 'mercury_z', 'mercury_vx', 'mercury_vy', 'mercury_vz', 'j2'', 'beta', 'gamma', 'λg'] = [2.77863525e+07, 2.92784092e+07, 2.22759642e+08, 2.57807181e+07, 1.80458395e+06, 1.18534261e+09, 6.65727619e+07, 5.48581437e+07, 5.78896700e+07, 1.67322707e+08, 1.58066375e+08, 1.89150544e+09, 2.85161172e+00, 2.63984052e+03, 1.87649120e+03, 2.65113498e+20]#for range
    - ['earth_x', 'earth_y', 'earth_z', 'earth_vx', 'earth_vy', 'earth_vz', 'mercury_x', 'mercury_y', 'mercury_z', 'mercury_vx', 'mercury_vy', 'mercury_vz', 'j2'', 'beta', 'gamma', 'λg'] =[7.62089989e+04, 8.00114983e+04, 1.26971979e+05, 6.76557514e+04, 3.96185439e+04, 1.73569946e+06, 9.39368707e+04, 9.47008882e+04, 1.08217392e+05, 2.46065478e+05, 2.31414900e+05, 2.74652853e+06, 8.03407389e-03, 8.23398815e+00, 7.82140074e+00, 8.08014330e+17] #for doppler

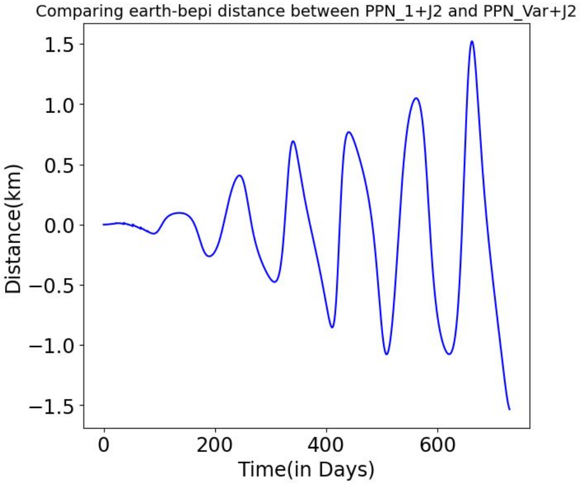
Reference:

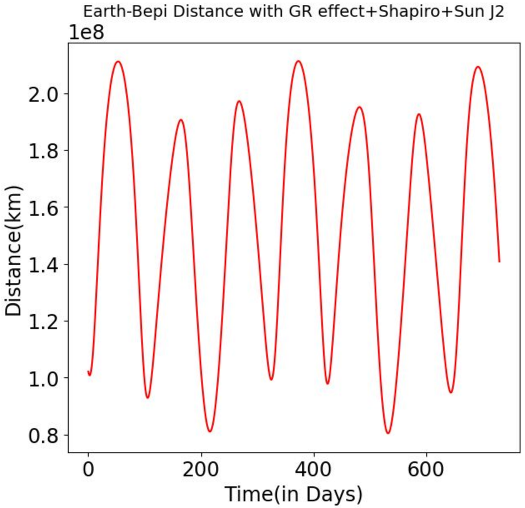
- 2303.01821 (Testing Theories of Gravity with Planetary Ephemerides)
- INPOP19a planetary ephemerides (INPOP19a planetary ephemerides)
- [1901.04307] Constraining the mass of the graviton with the planetary ephemeris INPOP (Constraining the mass of the graviton with the planetary ephemeris INPOP)
- [2303.05298] Testing the mass of the graviton with Bayesian planetary numerical ephemerides B-INPOP
- ESA - BepiColombo
- http://spiftp.esac.esa.int/data/SPICE/BEPICOLOMBO/misc/skd/BEPICOLOMBO.zip

Extras:

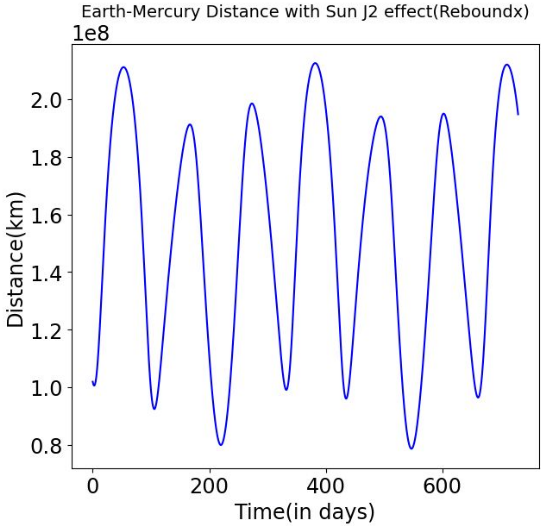

Difference in Earth-Bepi distance(PPN_variable-GR reboundx(no shapiro))
Distance(km)
Time(in Days)
300000
200000
100000
0
−100000
−200000
−300000
0
200
400
600


Earth-Bepi Distance with GR effect+Shapiro+Sun J2
1e8
Distance(km)
Time(in Days)
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0
200
400
600

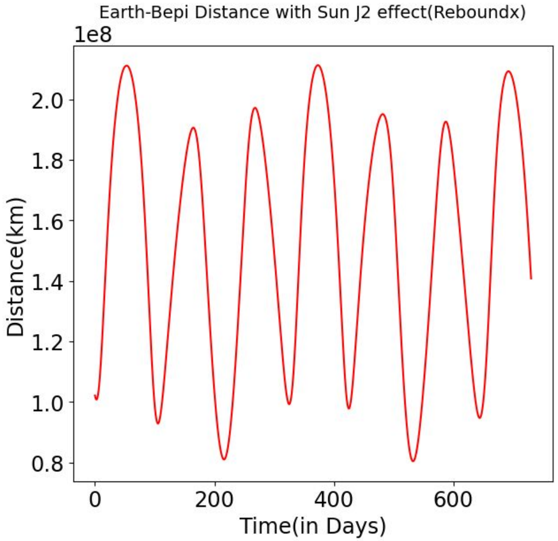

Comparing earth-bepi distance between PPN_1+J2 and PPN_Var+J2
Distance(km)
Time(in Days)
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
0
200
400
600


Earth-Bepi Distance with GR effect+Shapiro+Sun J2
1e8
Distance(km)
Time(in Days)
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0
200
400
600

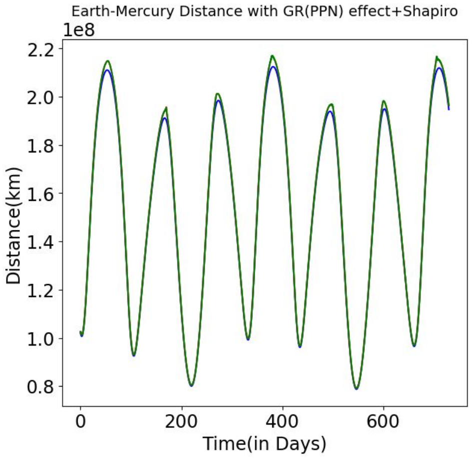
Extras:



Earth-Mercury Distance with Sun J2 effect(Reboundx)
1e8
2.0
1.8
1.6
1.4
1.2
1.0
0.8
Distance(km)
0
200
400
600
Time(in days)



Earth-Bepi Distance with Sun J2 effect(Reboundx)
1e8
2.0
1.8
1.6
1.4
1.2
1.0
0.8
Distance(km)
0
200
400
600
Time(in Days)



Earth-Mercury Distance with GR(PPN) effect+Shapiro
1e8
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0.8
Distance(km)
0
200
400
600
Time(in Days)