```
In [1] : %matplotlib inline
```

```
In [2] : import numpy as np
         import sympy as sp
         import matplotlib.pyplot as plt
         from matplotlib import colors, rc, animation
         from IPython.display import set_matplotlib_formats, HTML, Image

         set_matplotlib_formats('png', 'pdf')
         rc('animation', html='html5')
```

```
In [3] : sp.init_printing()
```

Ce module sert à simuler le comportement d'une corde, fixée à ses deux extrémités, homogène et inextensible que l'on fait vibrer, comme une corde d'un violon.

1 Code : harmoniques (conditions aux bords homogènes)

```
In [31] : class CordeHarmo:
              omega, x, t = sp.symbols("omega0 x t", real=True)
              cel = sp.symbols("c", real = True)
              N = sp.symbols("N", integer=True)
              m = sp.Idx("m", N)
              exprHarmo = sp.sin(m*omega*x/cel)*sp.cos(m*omega*t)

              def __init__(self, L, c):
                  """L : longueur de la corde
                  c : célérité de l'onde"""
                  self.L = L
                  self.cval = c
                  self.dom = np.linspace(0, L, 100)
                  puls = c*sp.pi/L
                  x,t = self.x,self.t
                  omega = self.omega
                  cel,m = self.cel,self.m
                  har = self.exprHarmo.subs({cel:c,omega:puls})
                  self.harm  = har
                  self.funcH = sp.lambdify((x,t,m), har, "numpy")

              @staticmethod
              def _legendeHarm(h):
                  return r"$m = %d$" % h

              def plotHarmo(self, t, li):
                  fig = plt.figure(1, figsize=(8,5))
                  ax = fig.add_subplot(111)
                  ax.grid(True)
                  dom = self.dom
```

```
        if hasattr(li, '__iter__'):
            for m in li:
                lab = self._legendeHarm(m)
                valeurs = self.funcH(dom, t, m)
                ax.plot(dom,valeurs,label=lab)
        else:
            lab = self._legendeHarm(li)
            valeurs = self.funcH(dom, t, li)
            ax.plot(dom,valeurs, label=lab)
        ax.set_title(r"Harmoniques $m$")
        ax.legend()
        fig.tight_layout()

    @staticmethod
    def _limMargin(a,b):
        wind = np.absolute(b-a)
        return (a-.1*wind,b+.1*wind)

    def animateHarm(self, t0, t1, h):
        """Animation du h-ème harmonique"""
        dom = self.dom
        fig = plt.figure(2,figsize=(8,5))
        ax = fig.add_subplot(111)
        ax.grid(True)
        ax.set_xlabel("Position $x$ (m)")
        ax.set_ylabel("Altitude $y(x,t)$")
        ax.set_title("%d-ème harmonique" % h)

        fps=30
        animtime=10
        N = int(np.ceil(animtime*fps))
        timestep = (t1-t0)/N
        interv = 1000/fps

        # Valeurs
        func = self.funcH
        states=[func(dom,t0+i*timestep,h) for i in range(N)]
        self.record = states

        line, = ax.plot(dom,states[0])
        timetext = ax.text(0.1,0.95,
            r"$t=%f$" % t0,
            transform=ax.transAxes)
        # Fenêtre
        lims = (np.min(np.asarray(states)),
               np.max(np.asarray(states)))
        ax.set_ylim(self._limMargin(*lims))
```

```
            def update(i):
                ti = t0+i*timestep
                timetext.set_text(r"$t=%f$" % ti)
                line.set_data(dom,states[i])
                return line, timetext

            anim = animation.FuncAnimation(fig,
                    update, frames=N,interval=interv,
                    blit=True)
            self.anim_harm = anim
```

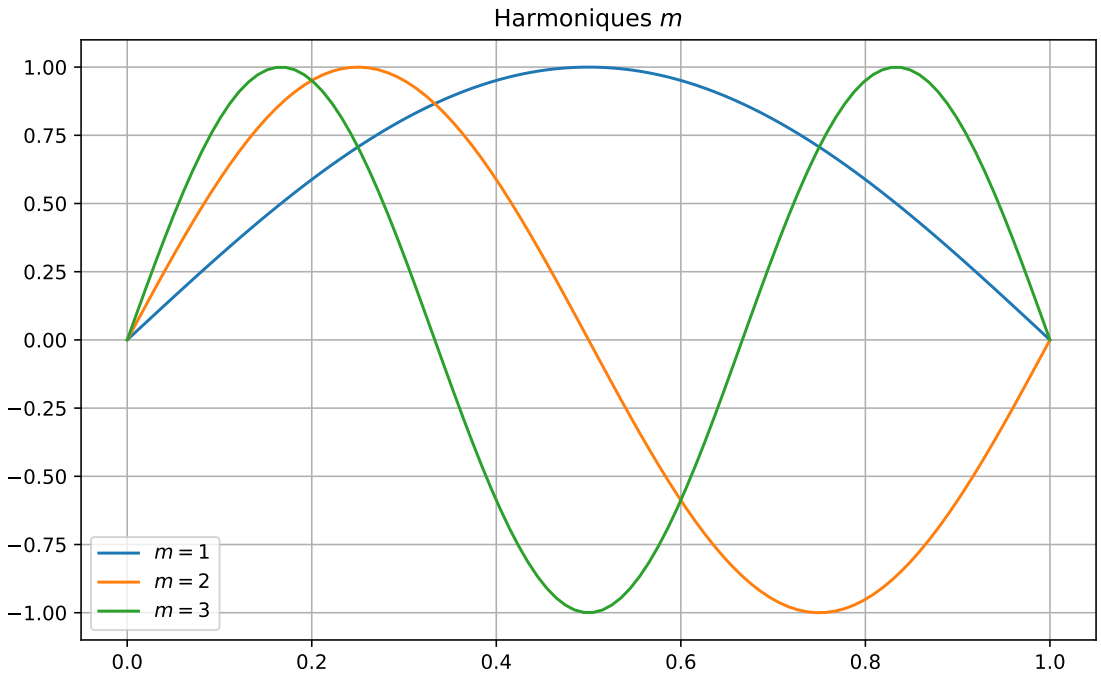
In [27] : `cor = CordeHarmo(1, 10)`

In [23] : `cor.harmo`

Out[23] :

$$\sin\left(\frac{\omega_0 x}{c} m\right) \cos\left(\omega_0 t m\right)$$

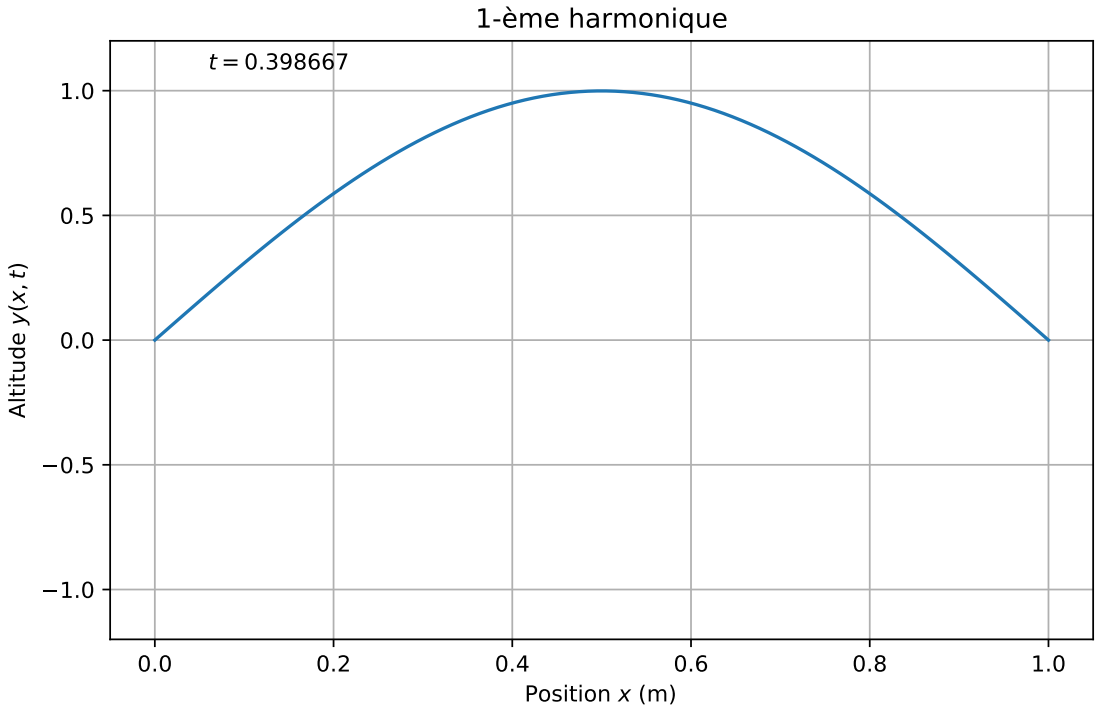
In [24] : `cor.plotHarmo(0, [1,2,3])`



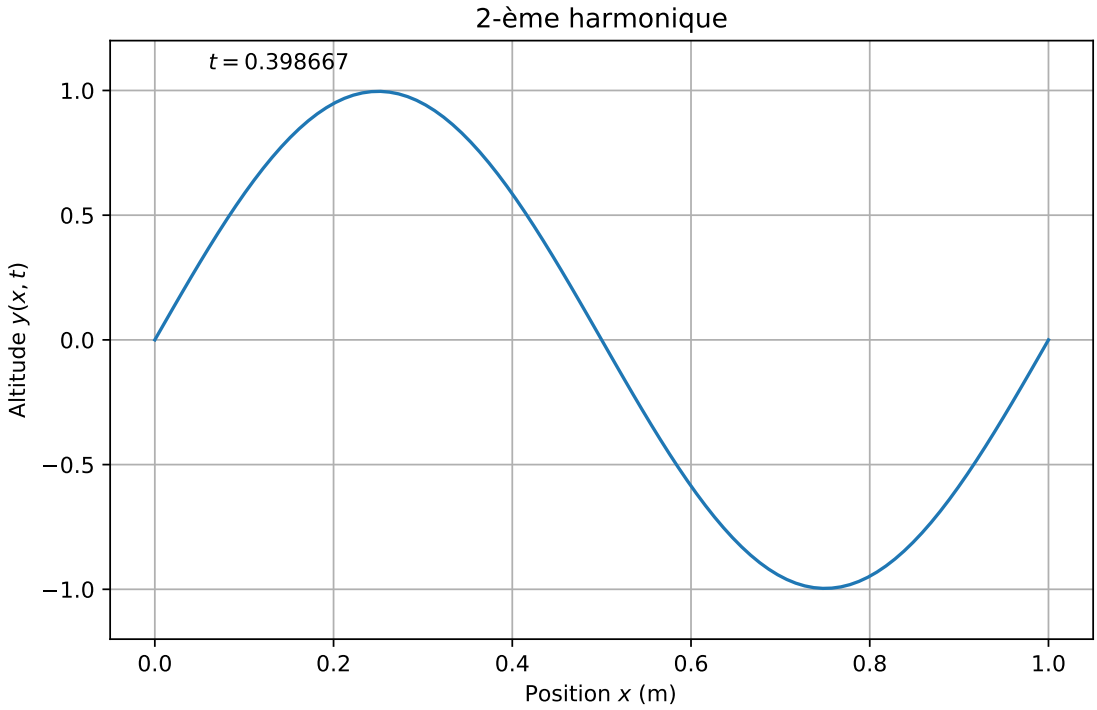
In [28] : `cor.animateHarm(0,0.4,1)`

```
cor.anim_harm
```

```
Out[28] : <matplotlib.animation.FuncAnimation at 0x1fdb5125ac8>
```

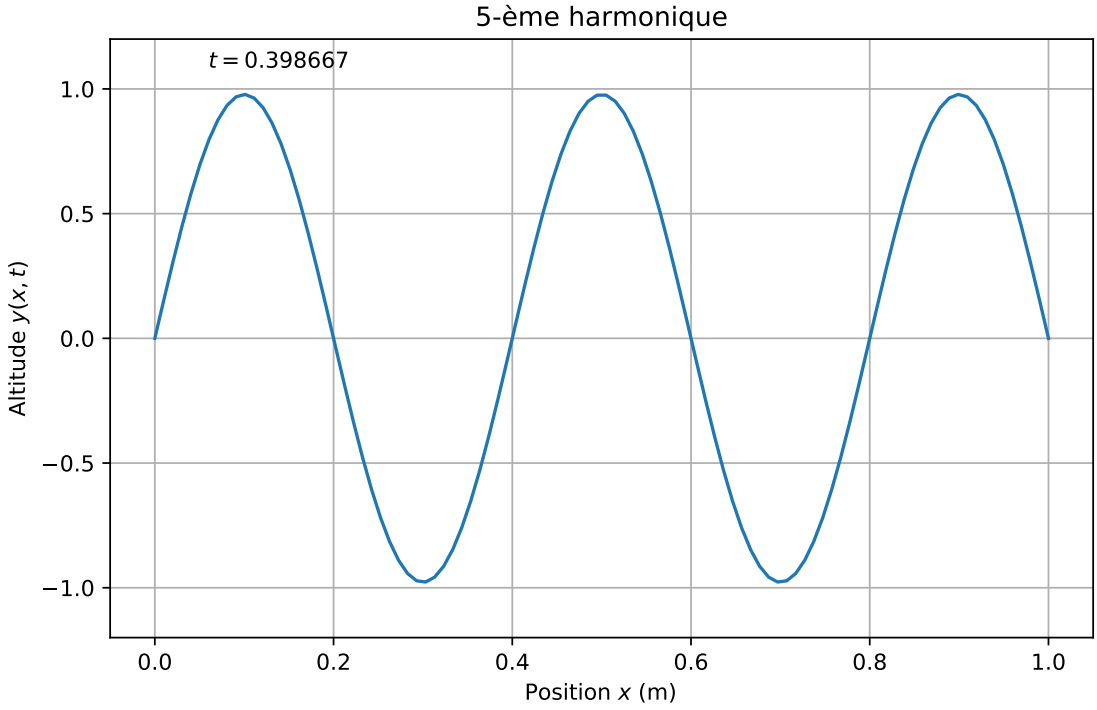



```
In [29] : cor.animateHarm(0,0.4,2)

          cor.anim_harm

Out[29] : <matplotlib.animation.FuncAnimation at 0x1fdb4006e10>
```




```
In [30] : cor.animateHarm(0,0.4,5)
          cor.anim_harm
```

```
Out[30] : <matplotlib.animation.FuncAnimation at 0x1fdb3fc0470>
```



```
In [76] : class Corde(CordeHarmo):
              def __init__(self, L, c, coeffs):
                  CordeHarmo.__init__(self, L, c)
                  expr = sum(el*self.harm.subs({self.m:i}) for i,el in enumerate(coeffs))
                  self.expr = expr
                  func = sp.lambdify((self.x,self.t), expr, "numpy")
                  self.func = func

              def plot(self, t):
                  fig = plt.figure(3, figsize=(8,5))
                  ax = fig.add_subplot(111)
                  ax.grid(True)
                  dom = self.dom
                  valeurs = self.func(dom, t)
                  ax.plot(dom, valeurs)

              @staticmethod
              def _limMargin(a,b):
                  wind = np.absolute(b-a)
                  return (a-.1*wind,b+.1*wind)

              def animate(self, t0, t1):
                  dom = self.dom
                  fig = plt.figure(2,figsize=(8,5))
                  ax = fig.add_subplot(111)
                  ax.grid(True)
                  ax.set_xlabel("Position $x$ (m)")
                  ax.set_ylabel("Altitude $y(x,t)$")
                  ax.set_title("Corde")

                  fps=30
                  animtime=10
                  N = int(np.ceil(animtime*fps))
                  timestep = (t1-t0)/N
                  interv = 1000/fps

                  # Valeurs
                  func = self.func
                  states=[func(dom,t0+i*timestep) for i in range(N)]
                  self.record = states

                  line, = ax.plot(dom,states[0])
                  timetext = ax.text(0.1,0.95,
                      r"$t=%f$" % t0,
                      transform=ax.transAxes)
```

```
        # Fenêtre
        lims = (np.min(np.asarray(states)),
                np.max(np.asarray(states)))
        ax.set_ylim(self._limMargin(*lims))

        def update(i):
            ti = t0+i*timestep
            timetext.set_text(r"$t=%f$" % ti)
            line.set_data(dom,states[i])
            return line, timetext

        anim = animation.FuncAnimation(fig,
                update, frames=N,interval=interv,
                blit=True)
        self.anim = anim
```

In [164] :
```
omega0 = 10*np.pi
def coeff(n):
    npi = n*np.pi
    return -2*np.sin(npi/2)/npi**2
coeffs = [coeff(n) for n in range(1,100)]
```

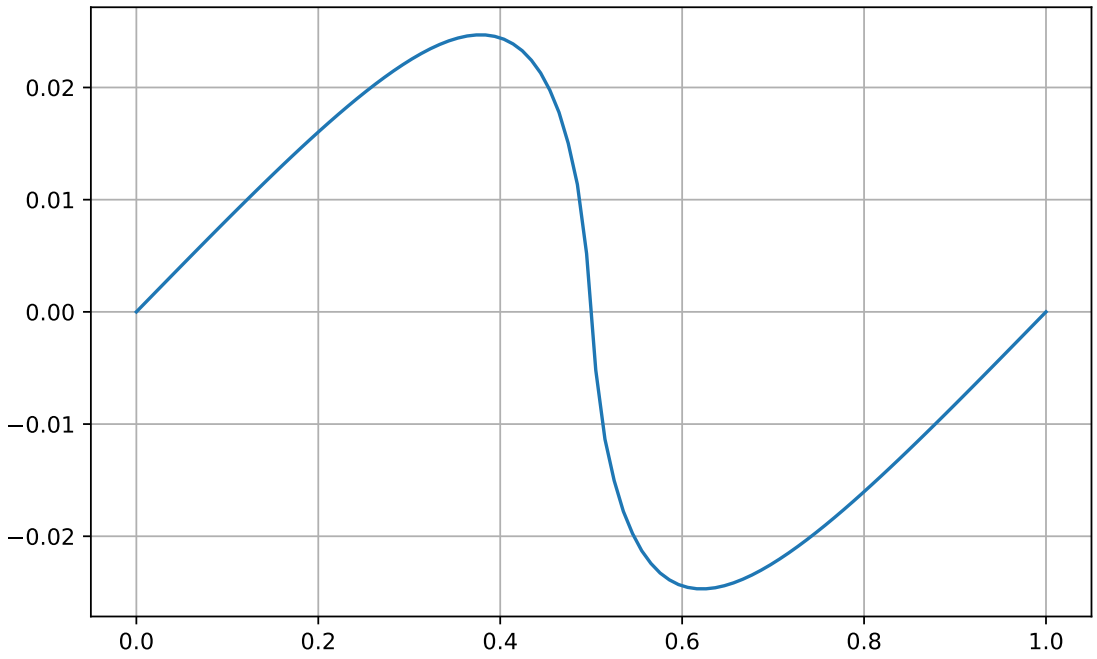
In [165] :
```
vibr = Corde(1,10, coeffs)
```

In [166] :
```
vibr.func(0.2,0.02)
```

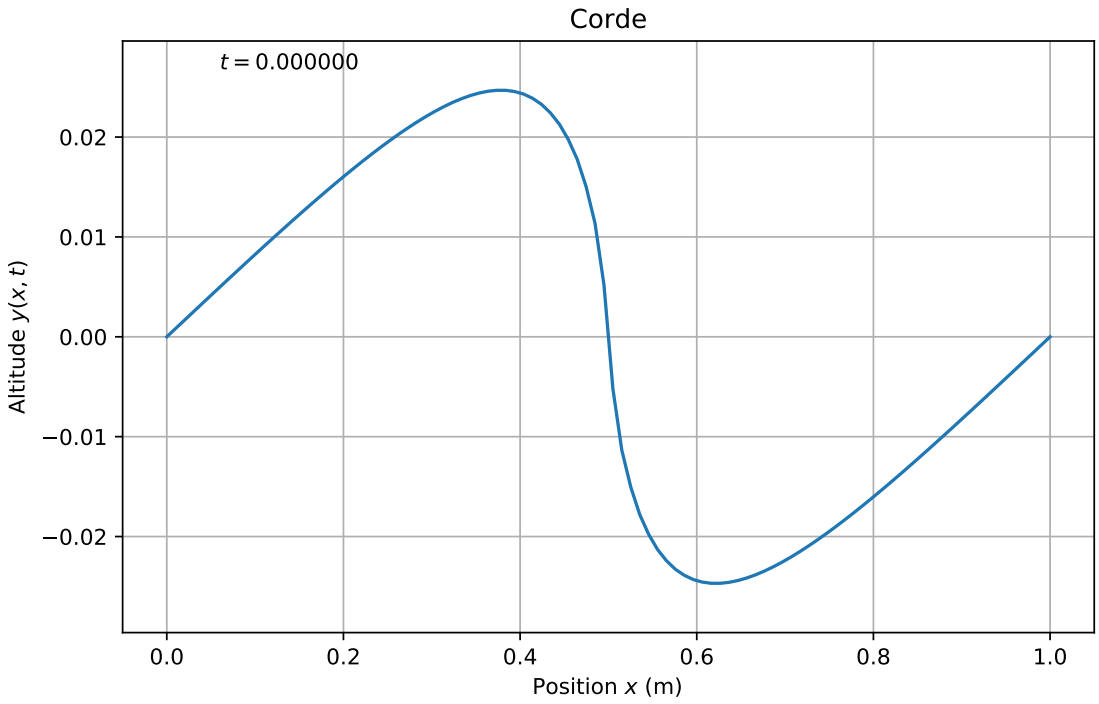
Out[166] :

$$0.0121957628775$$

In [167] :
```
vibr.plot(0.2)
```



In [168] : vibr.animate(0,5)



In [169] : vibr.anim

Out[169] : <matplotlib.animation.FuncAnimation at 0x1fdb52ec358>