```
In [35]: %matplotlib inline
         import matplotlib as mpl
         import matplotlib.pyplot as plt

         import numpy as np
         import sympy as sp
         import scipy.special as spec
         import scipy.integrate as inte

         r = sp.symbols('r')
         t = sp.S('t')

         sp.init_printing()
```

Le potentiel vecteur $\mathbf{A} = A(r,t)\mathbf{e}_\theta$ est solution de l'équation des ondes

$$\left(\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\mathbf{A} = -\mu_0\mathbf{J}(r,t),$$

avec $\mathbf{J}(r,t) = \dfrac{i(t)\delta(r)}{2\pi r}\mathbf{e}_\theta$.
Pour un courant sinusoïdal $i(t) = I\exp(i\omega t)$, le potentiel s'écrit $A(r,t) = f(r)\exp(i\omega t)$ et l'EDP se réduit à

$$\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}f}{\mathrm{d}r}\right) + k^2 f(r) = -\frac{\mu_0 I\,\delta(r)}{2\pi r}, \tag{1}$$

avec $k = \dfrac{\omega}{c}$.
La solution générale prend la forme

$$f(r) = AJ_0(kr) + BY_0(kr)$$

où J_0, Y_0 sont les 0-ièmes fonctions de Bessel de la première et seconde espèce, solutions de

$$\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}f}{\mathrm{d}r}\right) + k^2 f(r) = 0,$$

et A et B dépendent de la pulsation ω du courant.

```
In [112]: c = 3e8
          omega = sp.S('omega')
          k = omega/c

          puls = [2*np.pi*n*1e7 for n in range(8,11)] + [2*np.pi*n*1e6 for n in range(1,4)]

          B_field_omega = (sp.besselj(0,k*r) - sp.bessely(0,k*r))*sp.cos(omega*t)
          B_field = sum([B_field_omega.subs(omega, om) for om in puls])
          B_function = sp.lambdify((r, t), B_field, modules=['numpy',{"besselj":spec.jn, "bessely":s

          B_field
```

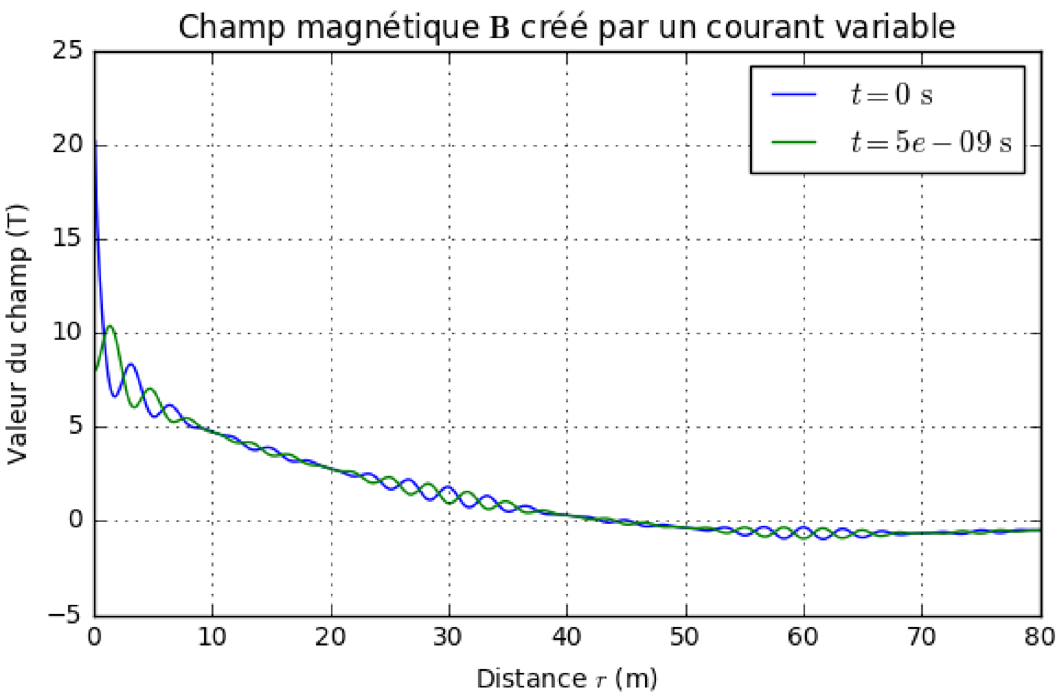
Out[112]:

$$\left(J_0\left(0.020943951023932r\right) - Y_0\left(0.020943951023932r\right)\right)\cos\left(6283185.30717959t\right)+\left(J_0\left(0.0418879020478639r\right) - Y_0\left(0.04188790\right.\right.$$

```
In [142]: rmin = 0.1
          rmax = 80

In [157]: def graphe_B(times):
              '''
              Construit les graphes du champ magnétique B aux temps donnés dans la liste
              "times"
```

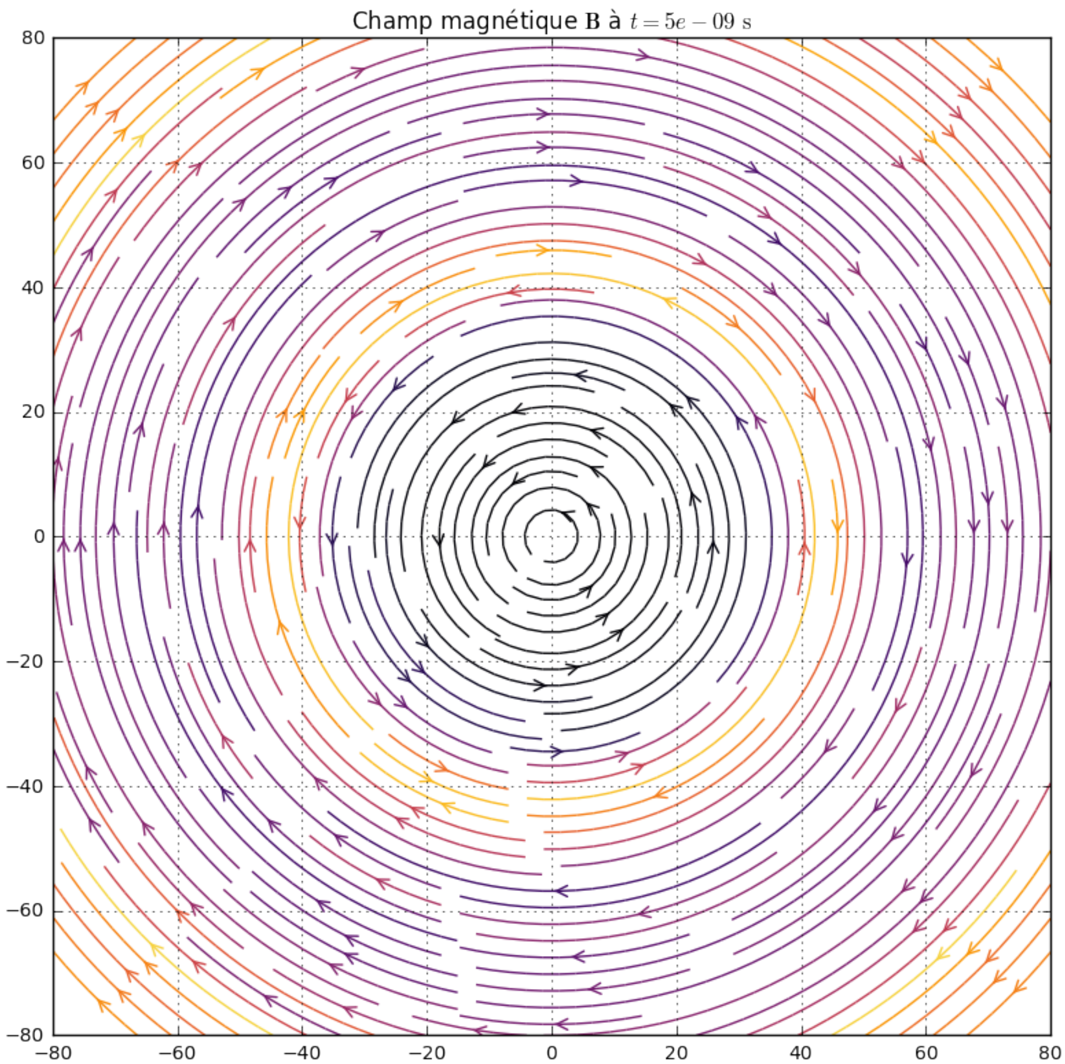
```
    '''
    radii = np.linspace(rmin, rmax, 10*rmax)

    fig, ax = plt.subplots(1,1, dpi=150)

    if hasattr(times, '__iter__'):
        for ti in times:
            leg = r'$t= {:g}$'.format(ti)
            leg = leg + r"$\ \mathrm{s}$"
            champ = B_function(radii, ti)
            ax.plot(radii, champ, label=leg)
    else:
        champ = B_function(radii, ti)
        leg = r'$t = {:g}$'.format(ti)
        leg = leg + r"$\ \mathrm{s}$"
        ax.plot(radii, champ, label=leg)

    ax.grid()
    ax.legend()
    ax.set_xlabel("Distance $r$ (m)")
    ax.set_ylabel("Valeur du champ (T)")
    ax.set_title(r'Champ magnétique $\mathbf{B}$ créé par un courant variable')

    fig.tight_layout()

    return fig, ax
```

```
In [158]: times = [0.000000001*k for k in [0, 5]]

          fig_Btheta, ax_Btheta = graphe_B(times)
          fig_Btheta.savefig('profil_champ.png')
```



```
In [132]: def build_field(t):
              wind = rmax

              Y, X = np.ogrid[-wind:wind:wind*10j, -wind:wind:wind*10j]

              def field_func(x,y):
                  r = np.sqrt(x*x+y*y)
                  Btheta = B_function(r, t)
                  direct = np.array([-y/r, x/r])
                  return Btheta*direct

              field_X, field_Y = field_func(X, Y)


              color = np.exp(-np.sqrt(field_X**2 + field_Y**2))

              fig, ax = plt.subplots(1, 1, figsize=(8,8))
              ax.grid()
              ax.set_aspect('equal')

              ax.set_xlim((-wind,wind))
              ax.set_ylim((-wind,wind))

              title_text = r'Champ magnétique $\mathbf{B}$ à '
              title_text += r"$t={:g}$".format(t)
              title_text += r" $\mathrm{s}$"
              ax.set_title(title_text)

              Z = np.exp(-2*np.sqrt(field_X**2+field_Y**2))
              Z = np.nan_to_num(Z)

              strm = ax.streamplot(X,Y, field_X, field_Y,
                  arrowstyle='->',
                  arrowsize=2,
                  color=Z,
                  cmap=plt.cm.inferno,density=2
                  )

              #fig.colorbar(strm.lines)

              fig.tight_layout()

              return fig
```

```
In [144]: t = 5e-9

          output_field = build_field(t)
          output_field.savefig("champmag_courant_variable.png")
```


Champ magnétique $\mathbf{B}$ à $t = 5e-09$ s
80
60
40
20
0
−20
−40
−60
−80
−80
−60
−40
−20
0
20
40
60
80