This short document aims to detail the mathematical structure and background of the processes simulated in this library.

1 Poisson processes

A D-dimensional Poisson process is a stochastic process yielding random events $(s_i)_i$ in a subset $A \subseteq \mathbb{R}^d$. It is governed by a deterministic event rate $\lambda\colon A \longrightarrow \mathbb{R}_+$. The total number of events $N(B)$ in a subset $B \subset A$ follows a Poisson distribution of parameter $\int_A \lambda(s)\ ds$, so that areas wherein λ takes larger values have more events. We denote $(s_i)_i \sim \mathcal{PP}(\lambda)$.

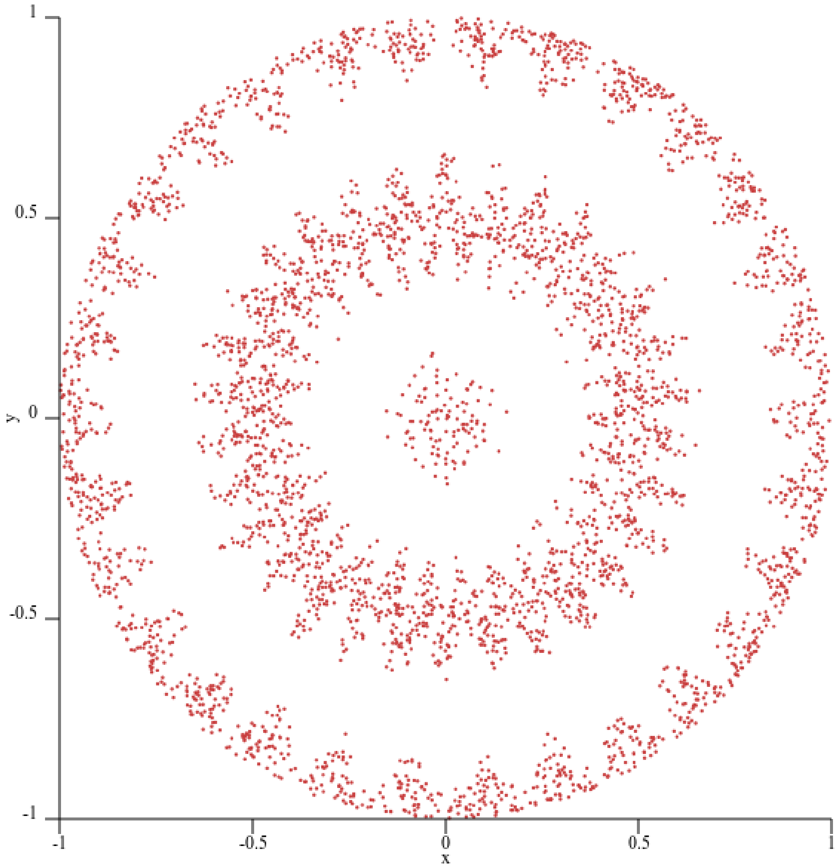

Figure 1: A Poisson process on the unit circle. 4003 events were simulated, the intensity function is $\lambda(r, \theta) = 4000\cos(4\pi(r + 0.06\cos(30\theta)))$.

Extension to vector-valued processes is evident, but not discussed here nor implemented in the library.

2 Hawkes processes

Hawkes processes are a set of time-dependent point processes where the event rate depends on past events. A multivariate Hawkes process has D point process components which are Poisson process conditionally to the process history $\mathcal{H}_t = \{(s_n, c_n)\}$, with rate

$$\lambda_k(t|\mathcal{H}_t) = \lambda_{0,k}(t) + \sum_{j=1}^{D} \sum_{s \in \mathcal{H}_t} g_{j,k}(t-s) \quad (1)$$

where $g_{j,k} : \mathbb{R}_+ \longrightarrow \mathbb{R}_+$. Each event is marked with an index $c_n \in \{1, \ldots, D\}$ indicating the process on which it occured.

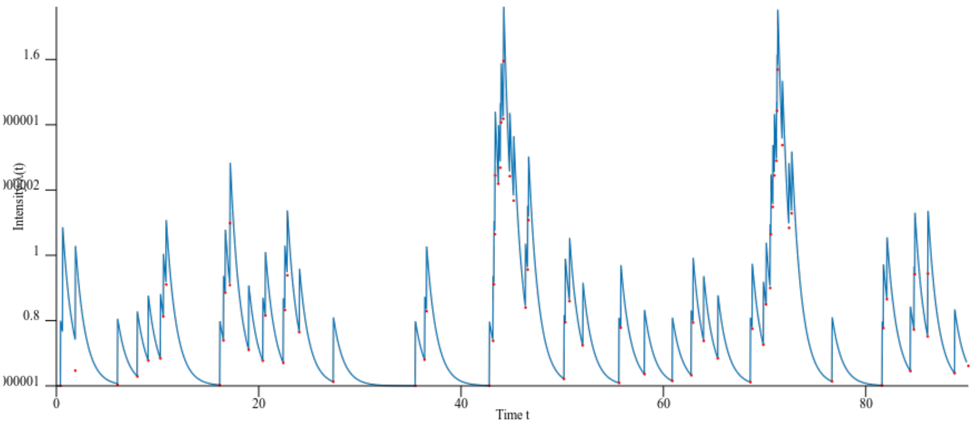

Figure 2: The intensity process $\lambda(t)$ of a one-dimensional Hawkes process.