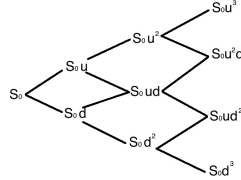


Computer Problem Set 1 The Cox-Ross-Rubinstein model

The present problem set is attached to Chapter 2 of the lectures notes. Please read its content in order to proceed to the requested implementation.



1. Consider the n -periods binomial model (see the example above with $n = 3$) defined by the coefficients $S_0 > 0, T > 0$,

$$u = u_n = e^{bh_n + \sigma\sqrt{h_n}}, d = d_n = e^{bh_n - \sigma\sqrt{h_n}}, h_n := \frac{T}{n},$$

for some given parameters $\sigma > 0, b \geq 0$. We denote by S_j^n the vector in \mathbb{R}^{j+1} of possible prices at time j defined by $S_j^n(i) := S_0 u^{j-i} d^i, i = 0, \dots, j$.

- (a) Build a function **Sn**(T, n, b, σ, j) which returns the vector S_j^n .
- (b) Consider a European call option with maturity T and strike $K \in \mathbb{R}_+$. Build a function **Payoffn**(T, n, b, σ, K) which returns the payoff vector of the option at maturity.
- (c) Let $r \geq 0$ be a constant interest rate. Build a function **Calln**(T, n, r, b, σ, K) which returns the price at time zero of the European call option.
- (d) At each time $t_j^n := jh_n$, denote by θ_j^n the hedging strategy corresponding to the above European option. Build a function **Deltan**(T, n, r, b, σ, K, j) which returns the vector of all values of θ_j^n .
- (e) Examine the dependence effect of the strike K on the functions **Calln**, **Deltan**, and comment. You may use various values of $K_i = 80 + i, i = 0, 1, 2, \dots, 40$, a number of periods $n = 50$, and the parameters:

$$\sigma = 0.3, \quad r = b = 5 \%, \quad S_0 = 100, \quad T = 2 \text{ (years)}. \quad (1)$$

2. We now consider the limiting Black-Scholes formula.

- (a) Build a function **Call**(T, r, σ, K) which returns the Black-Scholes price at time zero of the European call option. *The cumulative distribution function of the centered standard gaussian distribution is available in most softwares (Python : `scipy.stats.norm.cdf`, VBA-excel: `NormDist`, Scilab: `cdfnor`, Matlab: `cdf`).*
- (b) Define $\mathbf{err}(T, n, r, b, \sigma, K) := \frac{\mathbf{Calln}(T, n, r, b, \sigma, K)}{\mathbf{Call}(T, r, \sigma, K)} - 1$, plot the graph of this relative error in terms of the number of periods n , and comment. You may use the parameters (1) together with a strike $K = 105$.