



৯ম শ্রেণি একাডেমিক প্রোগ্রাম ২০২০

গণিত

লেকচার : M-25

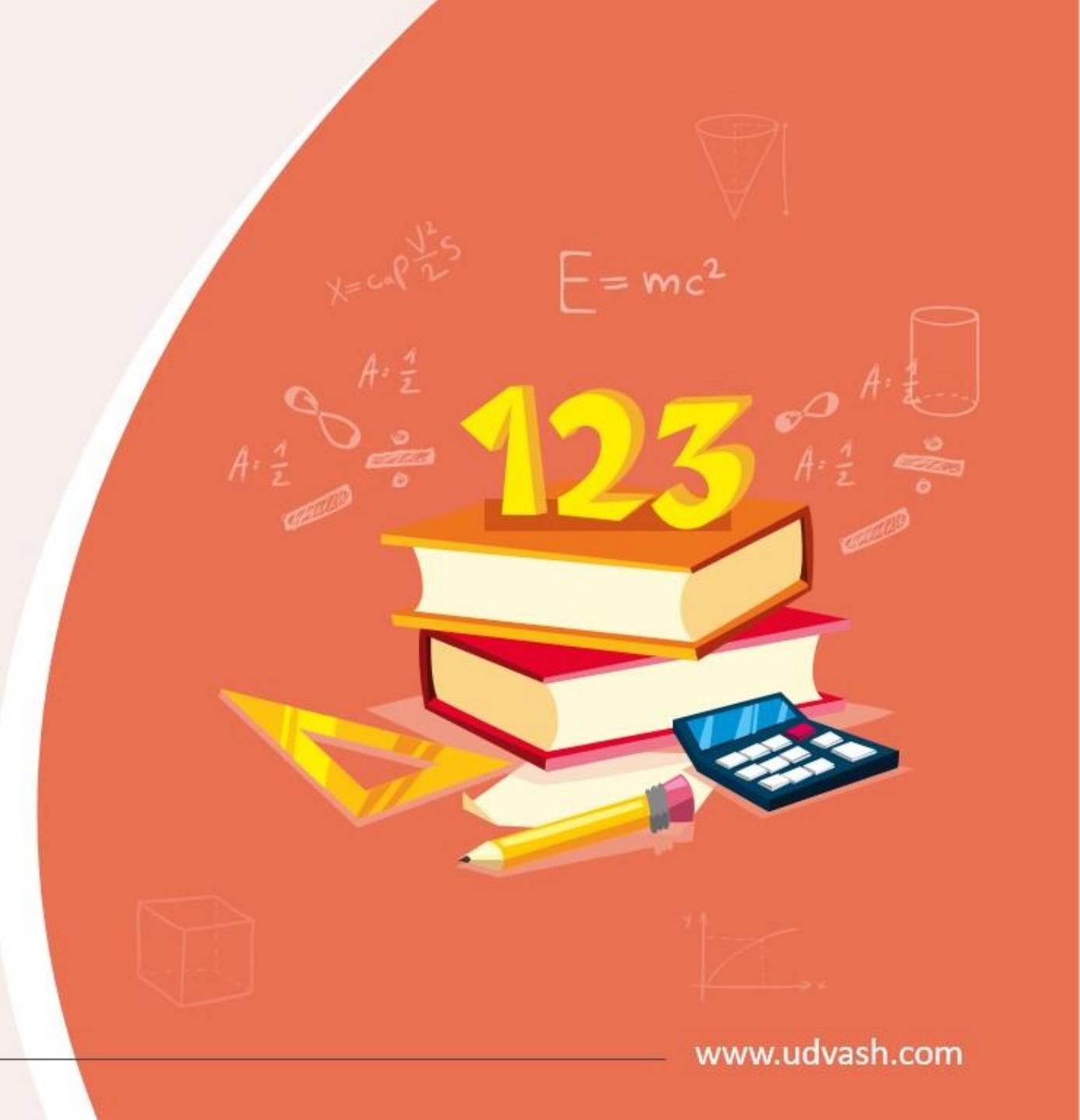
অধ্যায় ০৯ : ত্রিকোণমিতিক অনুপাত



উদ্বাশ

একাডেমিক এবং প্রতিশিল্প কেন্দ্র

$$x = \sqrt{\frac{b^2}{c} + c - \frac{b}{2}}$$



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অনুশীলনী ৯.১

□ $\sin A = \frac{3}{4}$ হলে, A কোণের অন্যান্য ত্রিকোণমিতিক অনুপাত নির্ণয় করো।

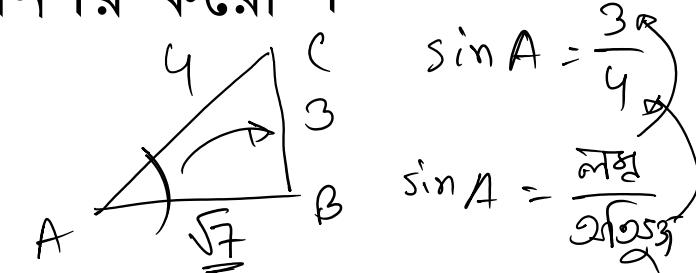
$$\cos A = \frac{\text{গুরুত্ব}}{\text{অঙ্গুহি}} = \frac{\sqrt{7}}{4}$$

$$\tan A = \frac{\text{লম্ব}}{\text{ভূজি}} = \frac{3}{\sqrt{7}}$$

$$\sec A = \frac{4}{\sqrt{7}}$$

$$\csc A = \frac{4}{3}$$

$$\cot A = \frac{\sqrt{7}}{3}$$



$$AB^2 + CB^2 = AC^2$$

$$\text{ব. } AB^2 = AC^2 - CB^2 \\ = 4^2 - 3^2$$

$$\text{ব. } AB^2 = 7$$

$$\therefore AB = \sqrt{7}$$

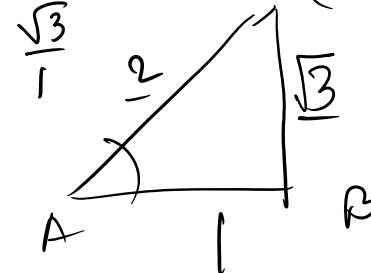
অনুশীলনী ৯.১

- ABC সমকোণী ত্রিভুজের $\angle B$ কোণটি সমকোণ। $\tan A = \sqrt{3}$ হলে, $\sqrt{3} \sin A \cos A = \frac{3}{4}$ এর সত্যতা যাচাই করো।

$$\sin A = \frac{\sqrt{3}}{2}$$

$$\cos A = \frac{1}{2}$$

$$\begin{aligned}\tan A &= \sqrt{3} = \frac{\sqrt{3}}{1} \\ &= \frac{\text{পৃষ্ঠা}}{\text{ধূঘূলি}}\end{aligned}$$



$$\begin{aligned}LHS &= \sqrt{3} \cdot \underline{\sin A} - \underline{\cos A} \\ &= \sqrt{3} \cdot \times \frac{\sqrt{3}}{2} \times \frac{1}{2} \\ &= \frac{3}{4} \\ &= RHS\end{aligned}$$

$$\begin{aligned}AC &= \sqrt{(\sqrt{3})^2 + 1^2} \\ &= \sqrt{4} \\ &= 2\end{aligned}$$

Poll Question- 01

□ $\sec^2 A + \tan^2 A = \frac{7}{12}$ হলে, $\sec^4 A - \tan^4 A$ এর মান কত?

- (a) 1
- (b) 84
- (c) $\frac{12}{7}$
- (d) $\frac{7}{12}$

$$\sec^2 A + \tan^2 A = \frac{7}{12}$$

$$\text{or, } 1 + \tan^2 A + \tan^2 A = \frac{7}{12}$$

$$\text{or, } \frac{2\tan^2 A + 1}{1} = \frac{7}{12} \quad \text{--- (1)}$$

$$\sec^4 A - \tan^4 A$$

$$= (\sec^2 A)^2 - \tan^4 A$$

$$= (1 + \tan^2 A)^2 - \tan^4 A$$

$$= 1 + 2\tan^2 A + \tan^4 A - \tan^4 A = 1 + 2\tan^2 A = \frac{7}{12}$$

অনুশীলনী ৯.১

□ প্রমাণ করো, $\frac{\sin A}{\cosec A} + \frac{\cos A}{\sec A} = 1$.

$$\text{LHS} = \frac{\sin A}{\cosec A} + \frac{\cos A}{\sec A}$$

$$= \sin A \cdot \sin A + \cos A \cdot \cos A$$

$$= \sin^2 A + \cos^2 A$$

$$= 1$$

$$= \text{RHS}$$

অনুশীলনী ৯.১

□ প্রমাণ করো, $\frac{1}{1 + \sin^2 A} + \frac{1}{1 + \csc^2 A} = 1.$

$$\text{LHS} = \frac{1}{1 + \sin^2 A} + \frac{1}{1 + \csc^2 A}$$

$$= \frac{1}{1 + \sin^2 A} + \frac{1}{1 + \frac{1}{\sin^2 A}}$$

$$= \frac{1}{1 + \sin^2 A} + \frac{1}{\frac{\sin^2 A + 1}{\sin^2 A}}$$

$$= \frac{1}{1 + \sin^2 A} + \frac{\sin^2 A}{1 + \sin^2 A}$$

$$= \frac{1 + \cancel{\sin^2 A}}{1 + \cancel{\sin^2 A}} = 1 = \text{RHS}$$

ব্যাখ্যা

অনুশীলনী ১.১

$$a = \sqrt{a^2}$$

(১) $\tan A + \sin A = a$ এবং $\tan A - \sin A = b$ হলে প্রমাণ করো যে, $a^2 - b^2 = 4\sqrt{ab}$

$$\text{LHS} = a^2 - b^2$$

$$\begin{aligned}
 &= (\underbrace{\tan A + \sin A}_{a})^2 - (\underbrace{\tan A - \sin A}_{b})^2 ; \quad \left[\because (\underbrace{a+b}_{\tan A + \sin A})^2 - (\underbrace{a-b}_{\tan A - \sin A})^2 = 4ab \right] \\
 &= 4 \sqrt{\tan^2 A - \sin^2 A} \\
 &= 4 \sqrt{(\tan A + \sin A)(\tan A - \sin A)} \\
 &= 4 \sqrt{ab} \\
 &= \text{RHS}
 \end{aligned}$$

$\sin^2 A + \cos^2 A = 1$
 $\sin^2 A = 1 - \cos^2 A$
 $\tan^2 A \cdot \cos^2 A$
 $= \frac{\sin^2 A}{\cos^2 A} \cdot \cos^2 A$
 $= \underline{\sin^2 A}$



উক্তাল

একাডেমিক এবং প্রশিক্ষণ কেন্দ্র

গণিত

অধ্যায় ০১ : ত্রিকোণমিতিক অনুপাত

অনুশীলনী ৯.১

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

□ প্রমাণ করো, $\frac{\tan A}{1-\cot A} + \frac{\cot A}{1-\tan A} = \sec A \cosec A + 1.$

$$\begin{aligned}
 LHS &= \frac{\frac{\sin A}{\cos A}}{1 - \frac{\cos A}{\sin A}} + \frac{\frac{\cos A}{\sin A}}{1 - \frac{\sin A}{\cos A}} = \frac{\sin^2 A \cdot \sin A - \cos^2 A \cdot \cos A}{\cos A \cdot \sin A (\sin A - \cos A)} \\
 &= \frac{\sin^3 A - \cos^3 A}{\cos A \cdot \sin A (\sin A - \cos A)} \\
 &\quad \cancel{(\sin A - \cos A)} \quad \cancel{(\sin^2 A + \sin A \cdot \cos A + \cos^2 A)} \\
 * &= \frac{\sin^2 A}{\cos A (\sin A - \cos A)} + \frac{\cos^2 A}{\sin A (\cos A - \sin A)} = \frac{\cos A \cdot \sin A \cdot (\sin A - \cos A)}{\cos A \cdot \sin A} \\
 &= \frac{1 + \sin A \cdot \cos A}{\cos A \cdot \sin A} \\
 &= \frac{1}{\cos A \cdot \sin A} + \frac{\sin A \cdot \cos A}{\sin A \cdot \cos A} = \sec A \cdot \cosec A + 1 = R.H.S
 \end{aligned}$$

গণিত



উদ্বান

একাডেমিক এবং প্রাথমিক বেতার

গণিত

অধ্যায় ০৯: ত্রিকোণমিতিক অনুপাত

অনুশীলনী ৯.১

□ $\sin^4 A + \sin^2 A = 1$ হলে, প্রমাণ করো যে, $\tan^4 A - \tan^2 A = 1$

$$\sin^4 A + \sin^2 A = 1$$

কৃ, $\sin^4 A = 1 - \sin^2 A$

বা, $\sin^4 A = \cos^2 A$

$$\frac{\sin^4 A}{\cos^4 A} = \frac{\cos^2 A}{\cos^4 A}$$

$\star \star$

বা, $\tan^4 A = \frac{1}{\cos^2 A}$

বা, $\tan^4 A = \sec^2 A$

বা, $\tan^4 A = 1 + \tan^2 A$

সু, $\left\{ \begin{array}{l} \tan^4 A - \tan^2 A = 1 \\ \end{array} \right.$

(Proved)

Poll Question- 02

$\cot^4 A - \cot^2 A = 1$ হলে, $\cos^4 A$ এর মান কত?

$$\underline{\cos^4 A}$$

(a) 1

(b) $1 - \cos^2 A$

(c) $1 + \cos^2 A$

(d) কোনোটিই নয়

$$\cot^4 A - \cot^2 A = 1$$

বা, $\cot^4 A = 1 + \cot^2 A$

বা, $\cot^4 A = \operatorname{cosec}^2 A$

বা, $\frac{\cos^4 A}{\sin^4 A} = \frac{1}{\sin^2 A}$

বা, $\cos^4 A = \frac{\sin^4 A}{\sin^2 A}$
 $= \sin^2 A$

$\therefore \cos^4 A = 1 - \cos^2 A$

অনুশীলনী ৯.১

□ প্রমাণ করো, $\frac{\cos A}{1-\tan A} + \frac{\sin A}{1-\cot A} = \sin A + \cos A.$

$$\begin{aligned}
 \text{LHS} &= \frac{\cos A}{1 - \frac{\sin A}{\cos A}} + \frac{\sin A}{1 - \frac{\cos A}{\sin A}} \\
 &= \frac{\cos A}{\cos A - \sin A} + \frac{\sin A}{\sin A - \cos A} \\
 &= \frac{\cos^2 A}{\cos A - \sin A} + \frac{\sin^2 A}{\sin A - \cos A} \\
 &= \frac{\cos^2 A}{\cos A - \sin A} - \frac{\sin^2 A}{\cos A - \sin A}
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{\cos^2 A - \sin^2 A}{\cos A - \sin A} \\
 &= \frac{(\cos A + \sin A) \cdot (\cos A - \sin A)}{(\cos A - \sin A)} \\
 &= \cos A + \sin A \\
 &= \text{RHS}
 \end{aligned}$$

অনুশীলনী ৯.১

□ প্রমাণ করো, $\frac{\sec A + \tan A}{\cosec A + \cot A} = \frac{\cosec A - \cot A}{\sec A - \tan A}$

$$\sec^2 A = 1 + \tan^2 A \quad \text{or, } \sec^2 A - \tan^2 A = 1$$

$$\therefore \cosec^2 A - \cot^2 A = 1$$

$$\text{LHS} = \frac{\sec A + \tan A}{\cosec A + \cot A}$$

$$= \left[\frac{\sec A + \tan A}{\cosec A + \cot A} \times \frac{\sec A - \tan A}{\cosec A - \cot A} \right] \times \left(\frac{\cosec A - \cot A}{\sec A - \tan A} \right)$$

$$= \left(\frac{\sec^2 A - \tan^2 A}{\cosec^2 A - \cot^2 A} \right) + \frac{\cosec A - \cot A}{\sec A - \tan A}$$

$$= 1 \times \frac{\cosec A - \cot A}{\sec A - \tan A} = \text{RHS}$$

অনুশীলনী ৯.১

□ প্রমাণ করো, $\frac{\cosec A}{\cosec A - 1} + \frac{\cosec A}{\cosec A + 1} = 2 \sec^2 A.$

$$\cot^2 A + 1 = \cosec^2 A$$

$$\text{LHS} = \frac{\cosec A (\cosec A + 1) + \cosec A (\cosec A - 1)}{(\cosec A - 1) (\cosec A + 1)}$$

$$= \frac{\cosec^2 A + \cosec A + \cosec^2 A - \cosec A}{\cosec^2 A - 1}$$

$$= \frac{2 \cosec^2 A}{\cot^2 A} = 2 \cdot \frac{1}{\frac{\cos^2 A}{\sin^2 A}} = 2 \cdot \frac{1}{\cos^2 A} = 2 \sec^2 A = \text{RHS}$$

Poll Question- 03

$\frac{5}{\sec^2 B} + \frac{2}{1+\cot^2 B} + 3 \sin^2 B$ এর মান কত?

(a) 5

(b) 2

(c) 3

(d) 4

$$= \frac{5}{\sec^2 B} + \frac{2}{\frac{1}{\sin^2 B}} + 3 \sin^2 B$$

$$= 5 \cdot \frac{1}{\sec^2 B} + 2 \sin^2 B + 3 \sin^2 B$$

$$= 5 \cos^2 B + 5 \sin^2 B$$

$$= 5 (\cos^2 B + \sin^2 B)$$

$$= 5 \cdot 1 = 5$$

$$\begin{aligned} & 1 + \cot^2 B \\ &= 1 + \frac{\cos^2 B}{\sin^2 B} \\ &= \frac{\sin^2 B + \cos^2 B}{\sin^2 B} \end{aligned}$$

$$= \frac{1}{\sin^2 B}$$

অনুশীলনী ৯.১

□ প্রমাণ করো, $\frac{\sin A}{1-\cos A} + \frac{1-\cos A}{\sin A} = 2\cosec A.$

$$\begin{aligned}
 \text{LHS} &= \frac{\sin A}{1-\cos A} + \frac{1-\cos A}{\sin A} \\
 &= \frac{\sin^2 A + (1-\cos A)^2}{(1-\cos A) \cdot \sin A} \\
 &= \frac{\sin^2 A + 1 - 2\cos A + \cos^2 A}{(1-\cos A) \cdot \sin A} \\
 &= \frac{1 + 1 - 2\cos A}{(1-\cos A) \cdot \sin A} \\
 &= \frac{2 - 2\cos A}{(1-\cos A) \cdot \sin A} \\
 &= \frac{2(1-\cos A)}{(1-\cos A) \cdot \sin A} \\
 &= 2 \cdot \frac{1}{\sin A} \\
 &= 2\cosec A \\
 &= \text{RHS}
 \end{aligned}$$

ଅନୁଶୀଳନୀ ୯.୧

 প্রমাণ করো, $(\tan A + \sec A)^2 = \frac{1+\sin A}{1-\sin A}$.

$$\begin{aligned}
 \text{RHS} &= \frac{1 + \sin A}{1 - \sin A} \\
 &\stackrel{=} {\frac{(1 + \sin A)(1 + \sin A)}{(1 - \sin A)(1 + \sin A)}} \\
 &= \frac{(1 + \sin A)^2}{1 - \sin^2 A} \\
 &\stackrel{=} {\frac{(1 + \sin A)^2}{\cos^2 A}}
 \end{aligned}$$

$$\begin{aligned}
 &= \left(\frac{1 + \sin A}{\cos A} \right)^2 \\
 &= \left(\frac{1}{\cos A} + \frac{\sin A}{\cos A} \right)^2 \\
 &= (\sec A + \tan A)^2 \\
 &= \text{LHS}
 \end{aligned}$$



অনুশীলনী ৯.১

□ প্রমাণ করো, $\frac{\cot A + \tan B}{\cot B + \tan A} = \cot A \cdot \tan B$

$$\begin{aligned}
 \text{LHS} &= \frac{\frac{\cos A}{\sin A} + \frac{\sin B}{\cos B}}{\frac{\cos B}{\sin B} + \frac{\sin A}{\cos A}} & = & \frac{\sin B \cdot \cos A}{\sin A \cdot \cos B} \\
 &= \frac{\cancel{\cos A \cdot \cos B + \sin A \cdot \sin B}}{\sin A \cdot \cos B} & = & \frac{\cancel{\cos A}}{\sin A} \cdot \frac{\cancel{\sin B}}{\cos B} \\
 &= \frac{\cancel{\cos B \cdot \cos A + \sin A \cdot \sin B}}{\sin B \cdot \cos A} & = & \cot A \cdot \tan B \\
 &= \frac{1}{\frac{\sin A \cdot \cos B}{\sin B \cdot \cos A}} & = & \text{RHS}
 \end{aligned}$$

অনুশীলনী ৯.১

□ প্রমাণ করো,

$$\sqrt{\frac{1-\sin A}{1+\sin A}} = \sec A - \tan A.$$

LHS =

$$\sqrt{\frac{1-\sin A}{1+\sin A}}$$

$$= \sqrt{\frac{(1-\sin A)(1-\sin A)}{(1+\sin A)(1-\sin A)}}$$

$$= \sqrt{\frac{(1-\sin A)^2}{1-\sin^2 A}}$$

$$= \sqrt{\frac{(1-\sin A)^2}{\cos^2 A}}$$

$$= \frac{1-\sin A}{\cos A}$$

$$= \frac{1}{\cos A} - \frac{\sin A}{\cos A}$$

$$= \sec A - \tan A$$

= RHS

অনুশীলনী ৯.১



$\cos A + \sin A = \sqrt{2} \cos A$ হলে, প্রমাণ করো, $\underline{\cos A} - \underline{\sin A} = \sqrt{2}(\underline{\sin A})$.

$$\cos A + \underline{\sin A} = \sqrt{2} \cos A$$

$$\text{সু, } \sin A = \sqrt{2} \cos A - \cos A$$

$$\sin A = \cos A (\sqrt{2}-1)$$

$$\text{বা, } (\sqrt{2}+1) \sin A = (\sqrt{2}+1)(\sqrt{2}-1) \cos A$$

$$\text{সু, } \sqrt{2} \sin A + \sin A = (\sqrt{2}^2 - 1^2) \cdot \cos A$$

$$\text{সু, } \sqrt{2} \sin A = \cos A - \sin A$$

$$\boxed{\text{LHS} = \text{RHS}}$$

Poll Question-04

□ $x \cos A - y \sin A = 2$ ও $x \sin A + y \cos A = 4$ হলে,

নিচের কোনটি সঠিক?

(a) $x^2 + y^2 = 10$

(b) $x^2 - y^2 = 12$

~~(c)~~ $x^2 + y^2 = 20$

(d) $x^2 - y^2 = 6$

$$\begin{aligned}
 & x \cos A - y \sin A = 2 \\
 & x^2 \cos^2 A - 2x \cos A \cdot y \sin A + y^2 \sin^2 A = 4 \quad (1) \\
 & x \sin A + y \cos A = 4 \\
 & x^2 \sin^2 A + 2x \sin A \cdot y \cos A + y^2 \cos^2 A = 16 \quad (2) \\
 & (1) + (2) \\
 & x^2 (\sin^2 A + \cos^2 A) + y^2 (\sin^2 A + \cos^2 A) = 20 \\
 & x^2 + y^2 = 20
 \end{aligned}$$

অনুশীলনী ৯.১

□ $\cosec A - \cot A = \frac{4}{3}$ হলে, $\cosec A + \cot A$ এর মান কত?

$$\cosec^2 A - \cot^2 A = 1$$

বা, $(\cosec A + \cot A)(\cosec A - \cot A) = 1$

বা, $(\cosec A + \cot A) \cdot \frac{4}{3} = 1$

$$\cosec A + \cot A = \frac{3}{4}$$

অনুশীলনী ৯.১

□ $\cot A = \frac{b}{a}$ হলে, $\frac{a\sin A - b\cos A}{a\sin A + b\cos A}$ এর মান কত?

$$\cot A = \frac{b}{a}$$

$$\text{সু, } \frac{\cos A}{\sin A} = \frac{b}{a}$$

$$\text{সু, } \frac{\cos A}{\sin A} \cdot \frac{\cos A}{\sin A} = \frac{b \cos A}{a \sin A}$$

$$\text{সু, } \frac{\cos^2 A}{\sin^2 A} = \frac{b \cos A}{a \sin A}$$

$$\text{সু, } \frac{\sin^2 A - \cos^2 A}{\sin^2 A + \cos^2 A} = \frac{a \sin A - b \cos A}{a \sin A + b \cos A}$$

$$\frac{a \sin A - b \cos A}{a \sin A + b \cos A}$$

$$a \sin A + b \cos A$$

$$= \sin^2 A - \cos^2 A$$

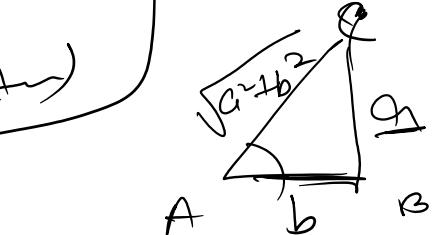
$$= \left(\frac{a}{\sqrt{a^2+b^2}} \right)^2 - \left(\frac{b}{\sqrt{a^2+b^2}} \right)^2$$

$$= \frac{a^2}{a^2+b^2} - \frac{b^2}{a^2+b^2}$$

$$\frac{a \sin A - b \cos A}{a \sin A + b \cos A} = \frac{a^2 - b^2}{a^2 + b^2}$$

$$(A)$$

$$\cot A = \frac{b}{a}$$



$$\sin A = \frac{a}{\sqrt{a^2+b^2}}$$

$$\cos A = \frac{b}{\sqrt{a^2+b^2}}$$

অধ্যায় ০৯ : ত্রিকোণমিতিক অনুপাত

গণিত



উক্তার্থ

একাডেমিক এবং প্রাইভেট কেয়ার

না বুঝে
মুখস্ত করার
অভ্যাস প্রতিভাকে
ধ্বংস করে



উদ্বাশ

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