Modeling echo chambers and polarization dynamics in social networks – review and model extension

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Assumptions [1]

- ⋆ more active users tend to show more extreme opinions,
- ⋆ agents sharing similar opinions can mutually reinforce each other and move towards more extreme views.

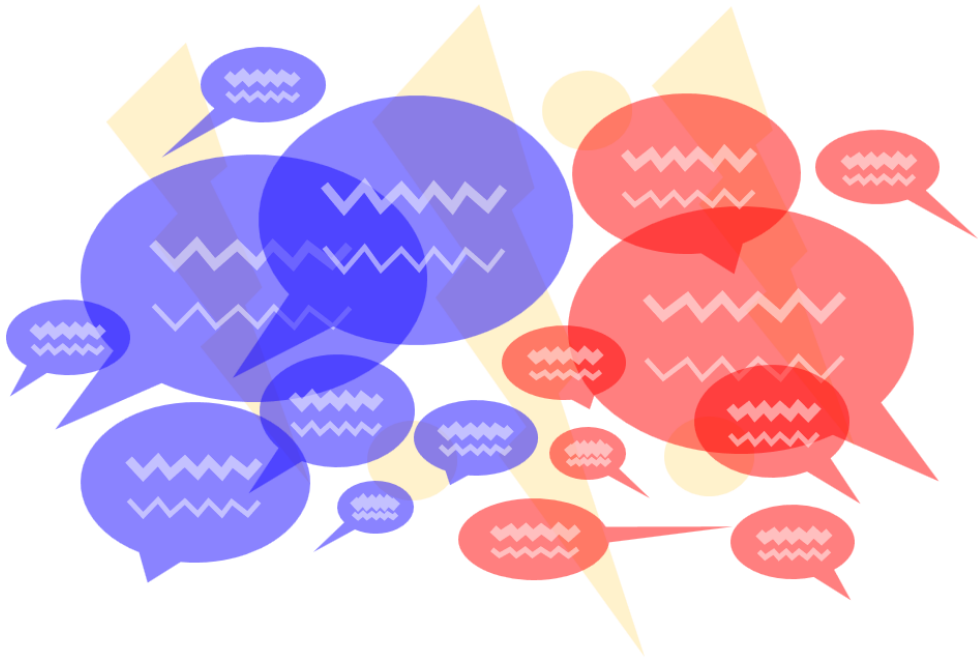


Figure source: Own elaboration.

Considered model [1]

We consider N agents, each with random opinion variable $x_i(t_0) \in [-1, 1]$, and the *activity* parameter a_i drawn from the power-law distribution.

At time t:

1. We initialize the temporary adjacency matrix A_{ij} with zeros,
2. Agent i is activated with probability a_i,
3. Active agent influences m distinct agents, so that $A_{ji}(t) = 1$, choosing j-th agent with probability

$$p_{ij} = \frac{|x_i - x_j|^{-\beta}}{\sum_j |x_i - x_j|^{-\beta}},$$

4. Agent j influenced by agent i may reciprocate the influence with probability r, and then $A_{ij}(t) = 1$,
5. Opinions change

$$\dot{x}_i = -x_i + K \sum_{j=1}^{N} A_{ij}(t) \tanh(\alpha x_j).$$

We discretize the above model and integrate the system of N coupled ODEs using fourth-order Runge-Kutta method with a time step $dt = 0.01$.

Interpretation of the parameters:

- $\alpha > 0$ – topic *controversialness*,
- $K > 0$ – social interaction strength,
- $\beta \geqslant 0$ – homophily strength.

Example: network with $m = 2$.

$$t = t_1$$

$$A(t) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

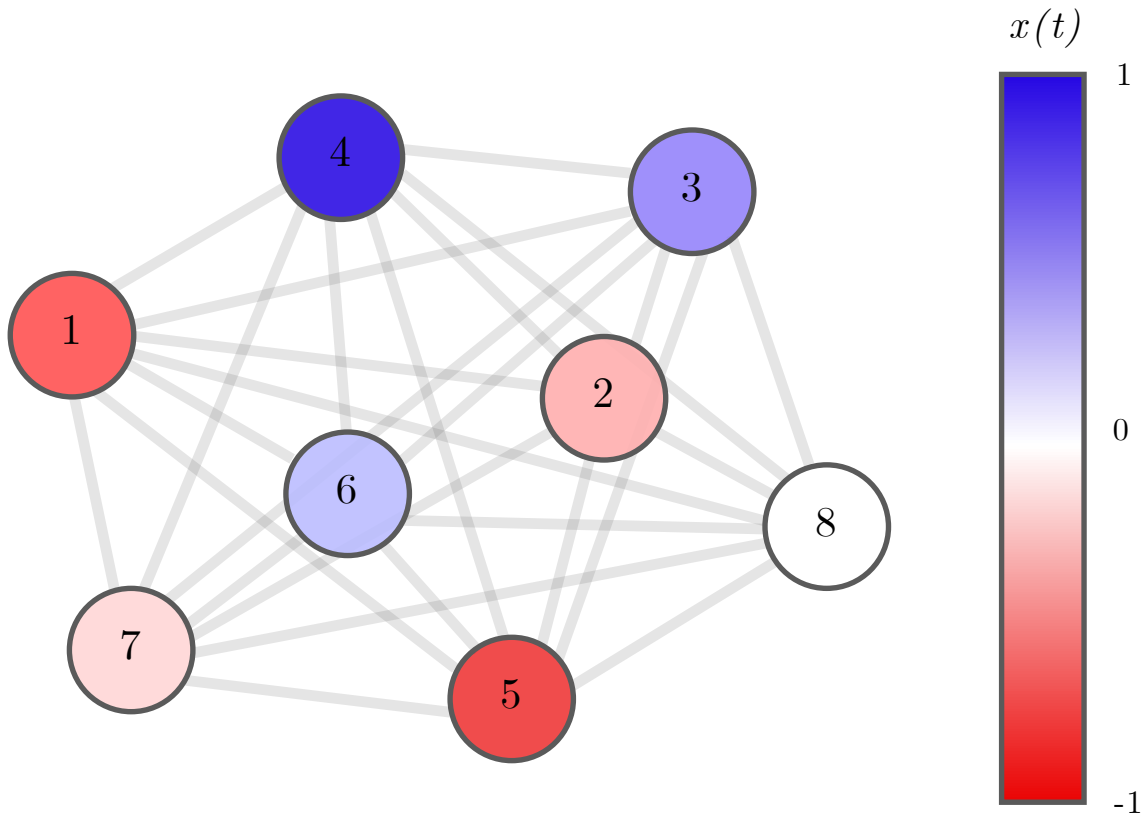
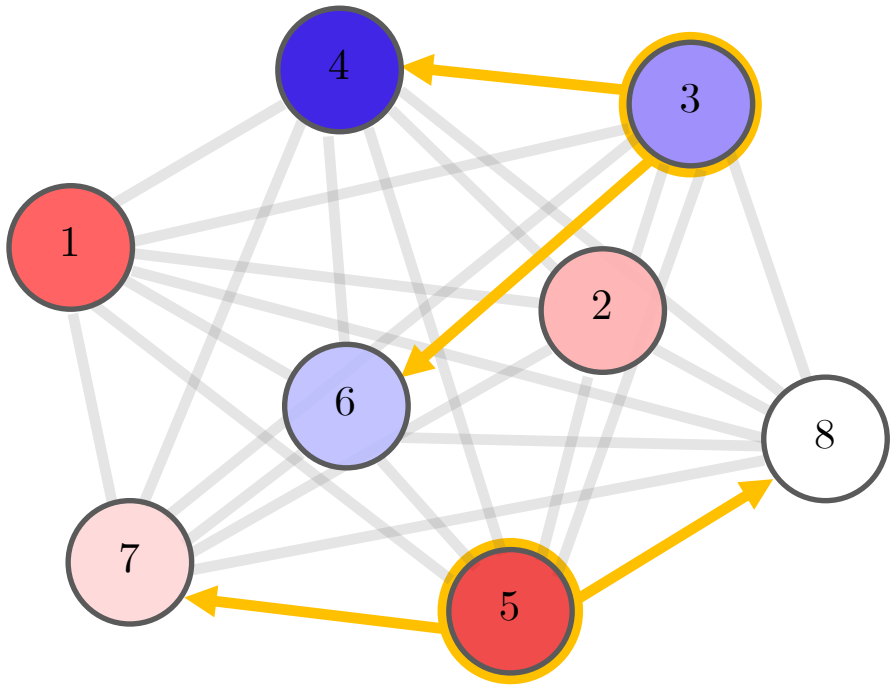


Figure source: Own elaboration.

$$P\left(\textcircled{i} = \textcircled{i}\right) = a_i, \qquad P(\underbrace{A_{ji}(t) = 1}_{\textcircled{i} \to \textcircled{j}}) = \frac{|x_i - x_j|^{-\beta}}{\sum_j |x_i - x_j|^{-\beta}}$$

$t = t_1$

$$A(t) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$



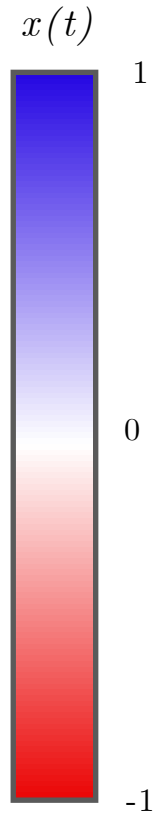
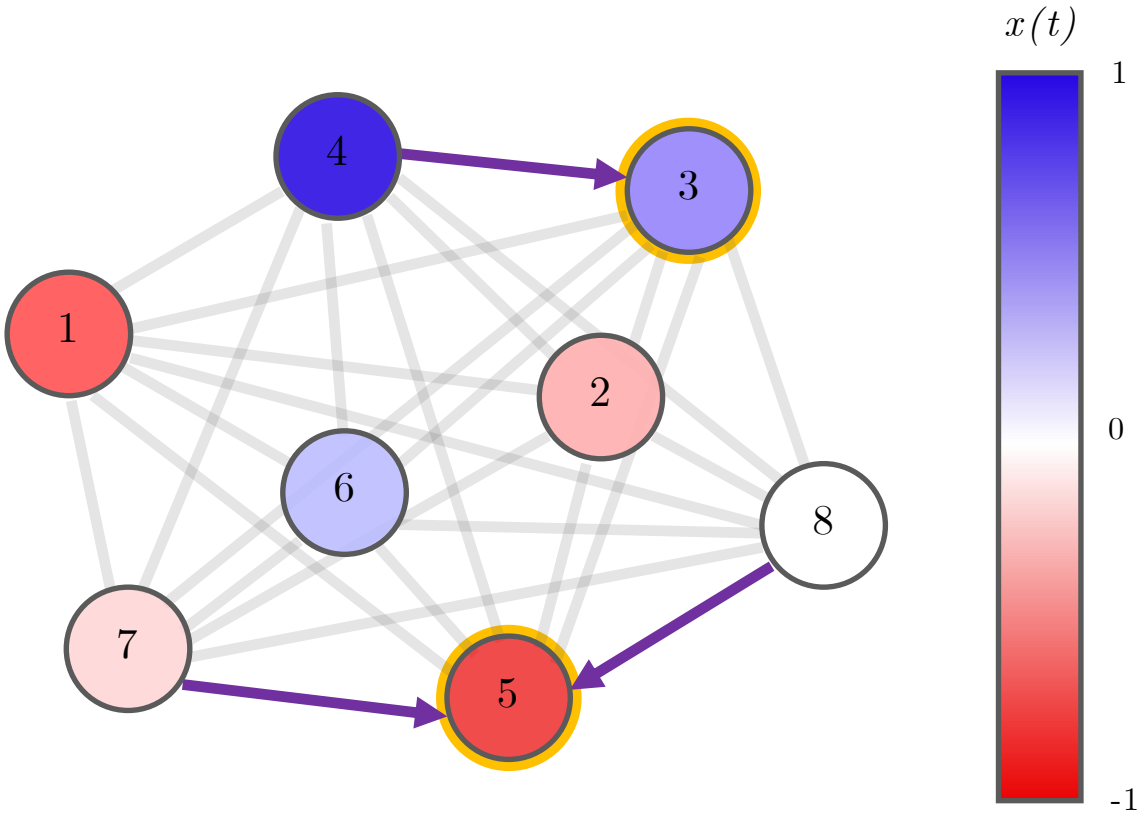


Figure source: Own elaboration.

$$P(\underbrace{A_{ij}(t) = 1 \mid A_{ji}(t) = 1}_{j \to i \mid i \to j}) = r$$

$t = t_1$

$$A(t) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$



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$$\dot{x}_i = -x_i + K \sum_{j=1}^{N} A_{ij}(t) \tanh(\alpha x_j)$$

$t = t_2$

$$A(t) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

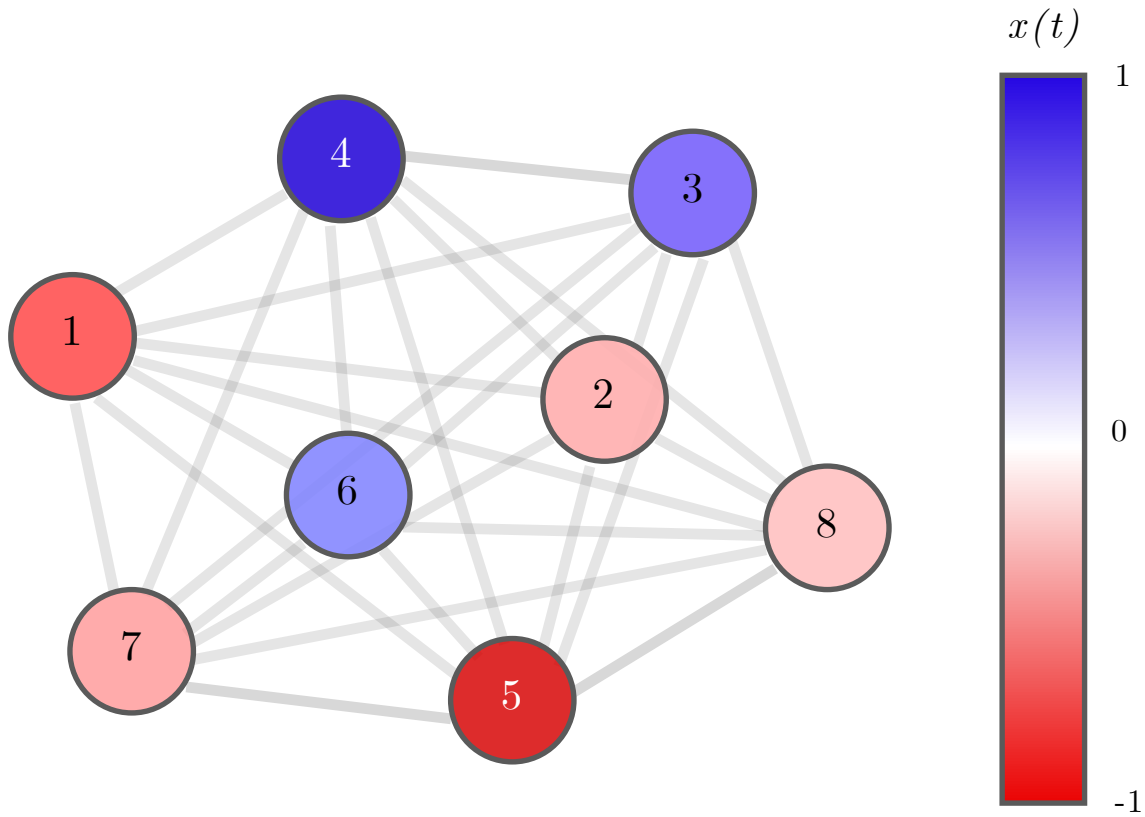


Figure source: Own elaboration.

From now we assume* $N = 1000$ and $m = 10$.

*Unless stated differently.

Results from [1]

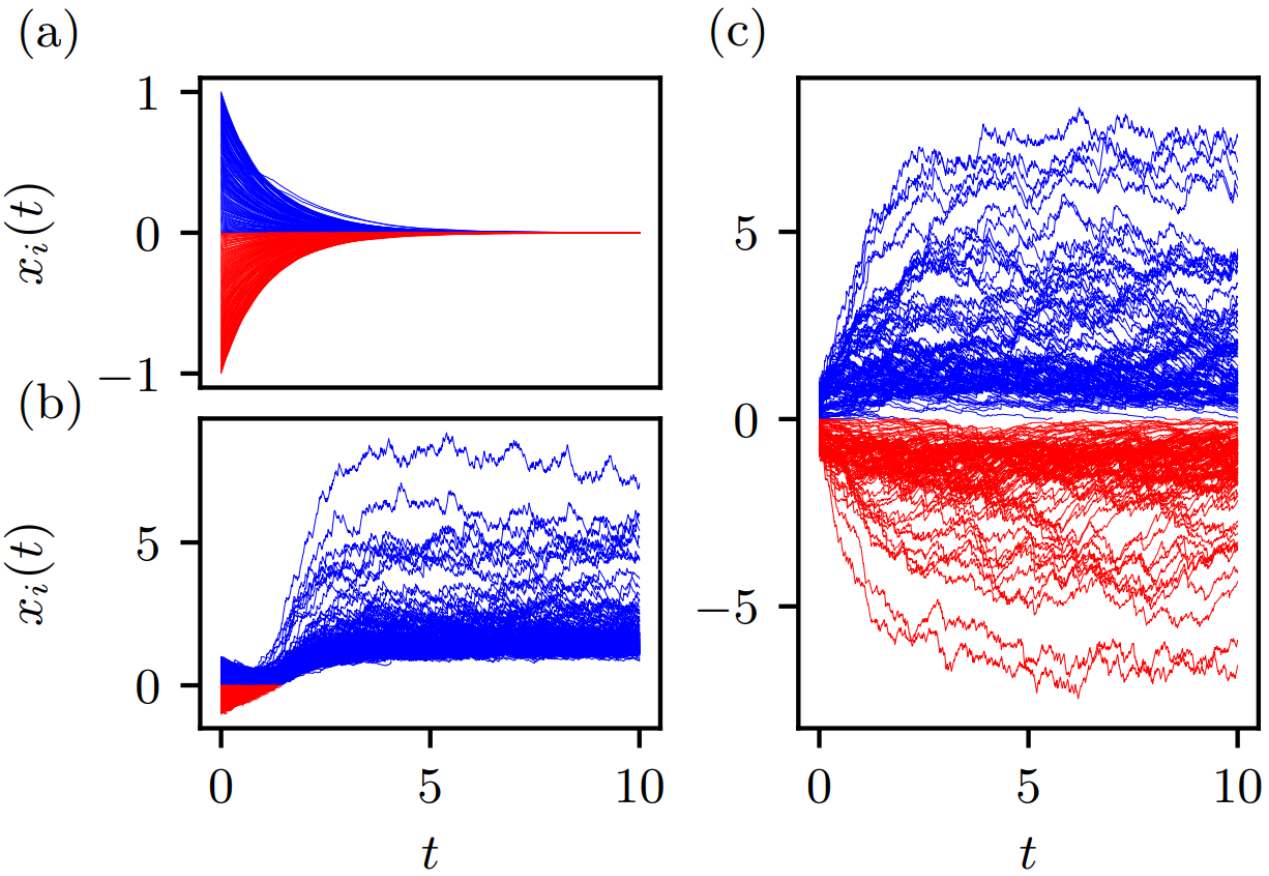
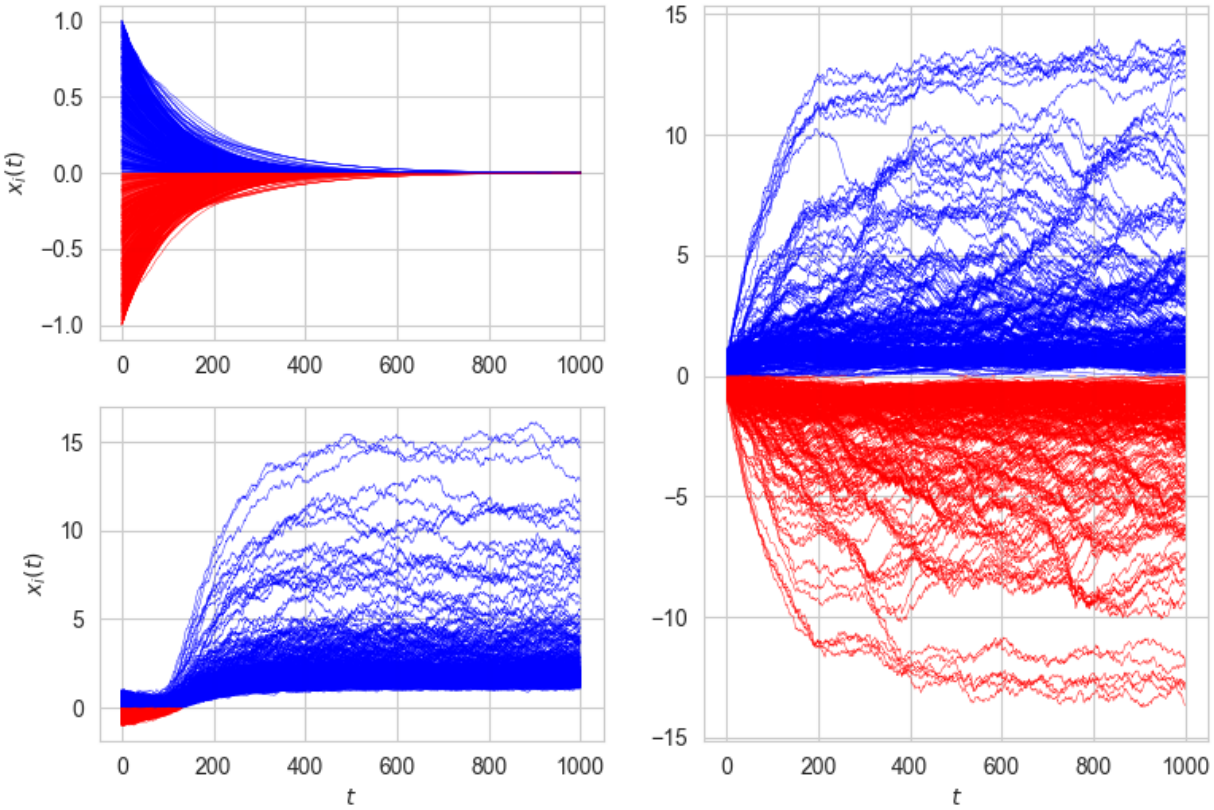


Figure: Temporal evolution of the agents' opinions for $K = 3$ and $r = 0.5$. (a) Neutral consensus for which all opinions converge to zero ($\alpha = 0.05, \beta = 2$). (b) One-sided radicalization ($\alpha = 3, \beta = 0$). (c) Opinion polarization, in which opinions split into two opposite sides ($\alpha = 3, \beta = 3$).

Our results



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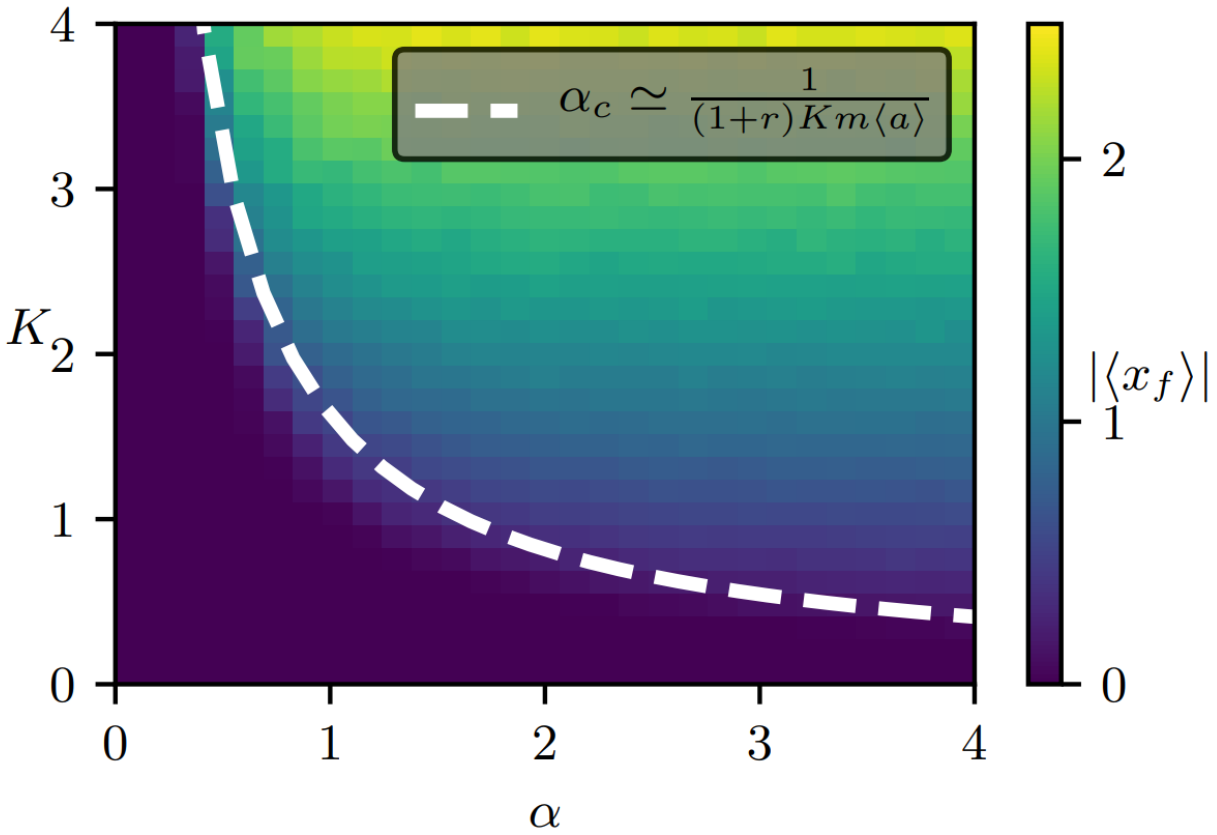

Figure: Transition from consensus to radicalization dynamics. Absolute values of the average final opinions $|\langle x_f \rangle|$ in K-α phase space for $N = 1000$, $\beta = 0.5$ and $r = 0.5$.

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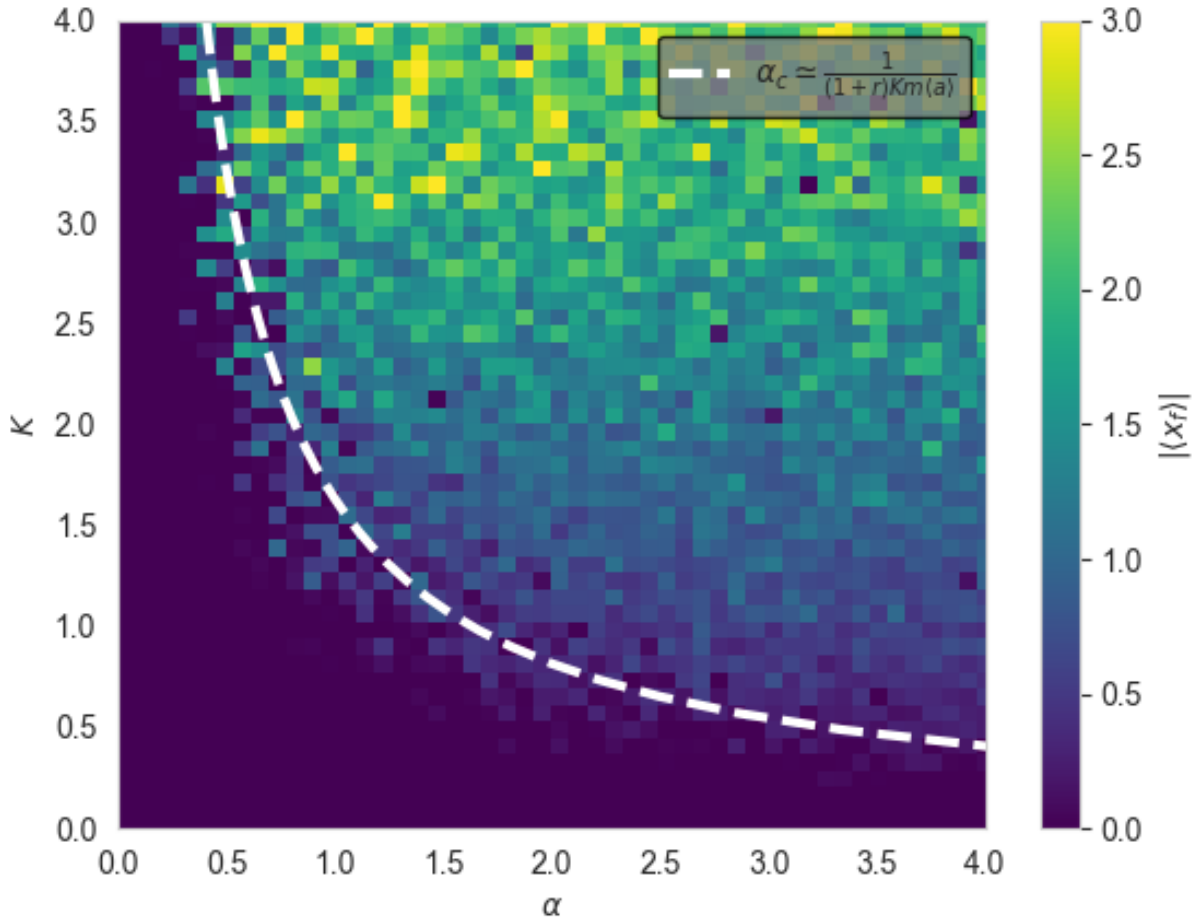

Figure: Transition from consensus to radicalization dynamics. Absolute values of the average final opinions $|\langle x_f \rangle|$ in K-α phase space for $N = 100$, $\beta = 0.5$ and $r = 0.5$.

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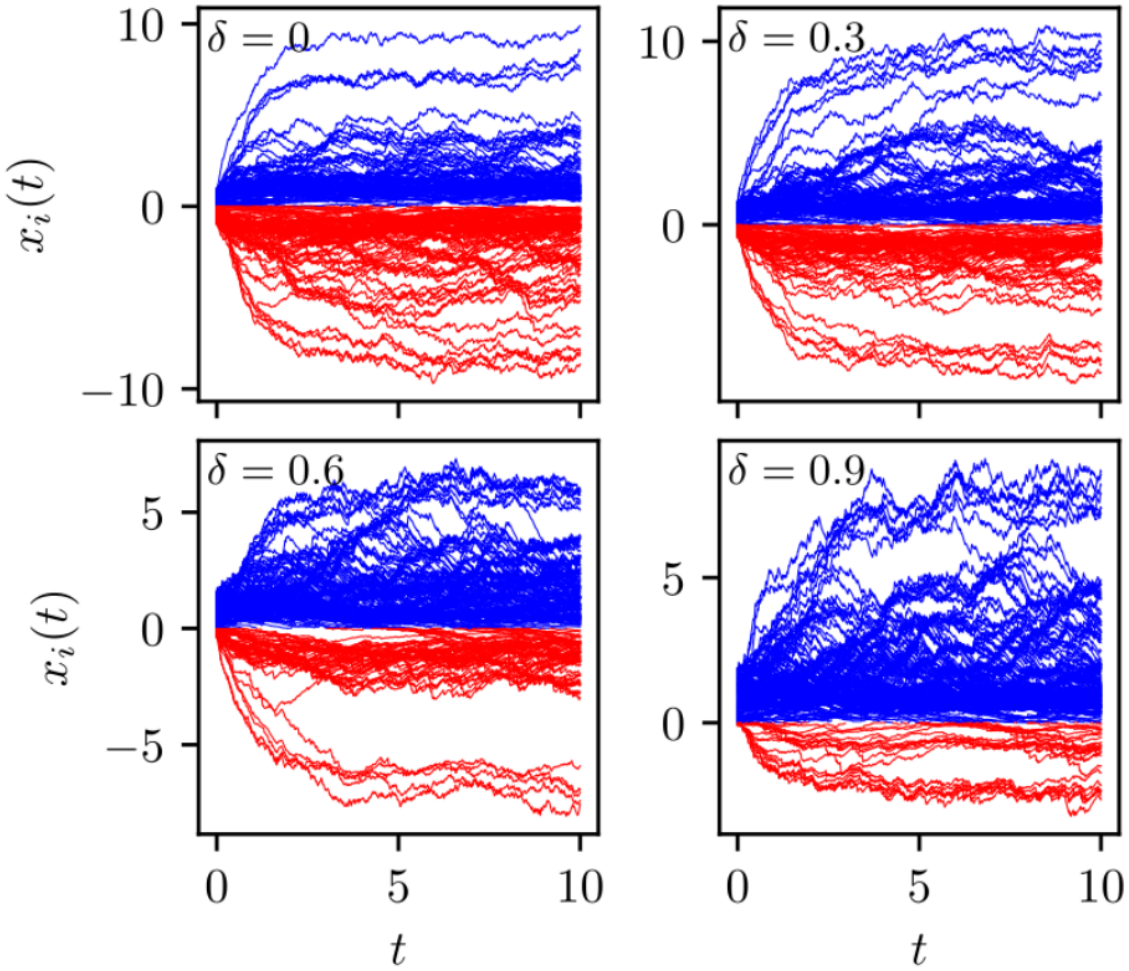
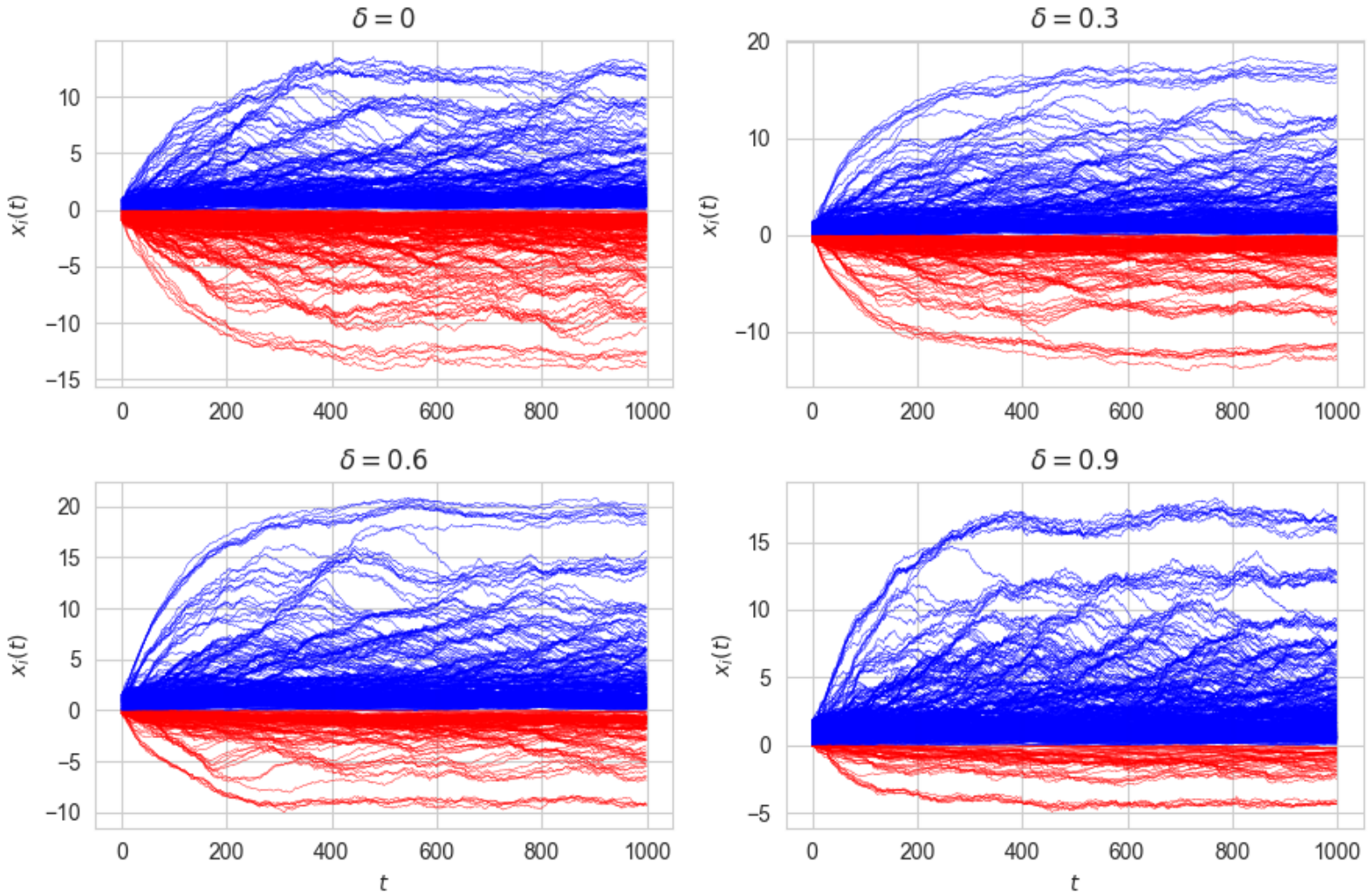


Figure: Opinion evolution for asymmetric initial conditions $x_i(0) \in [-1, 1] + \delta$. Model parameters are the same as in figure 1 subpanel (c).

Our results



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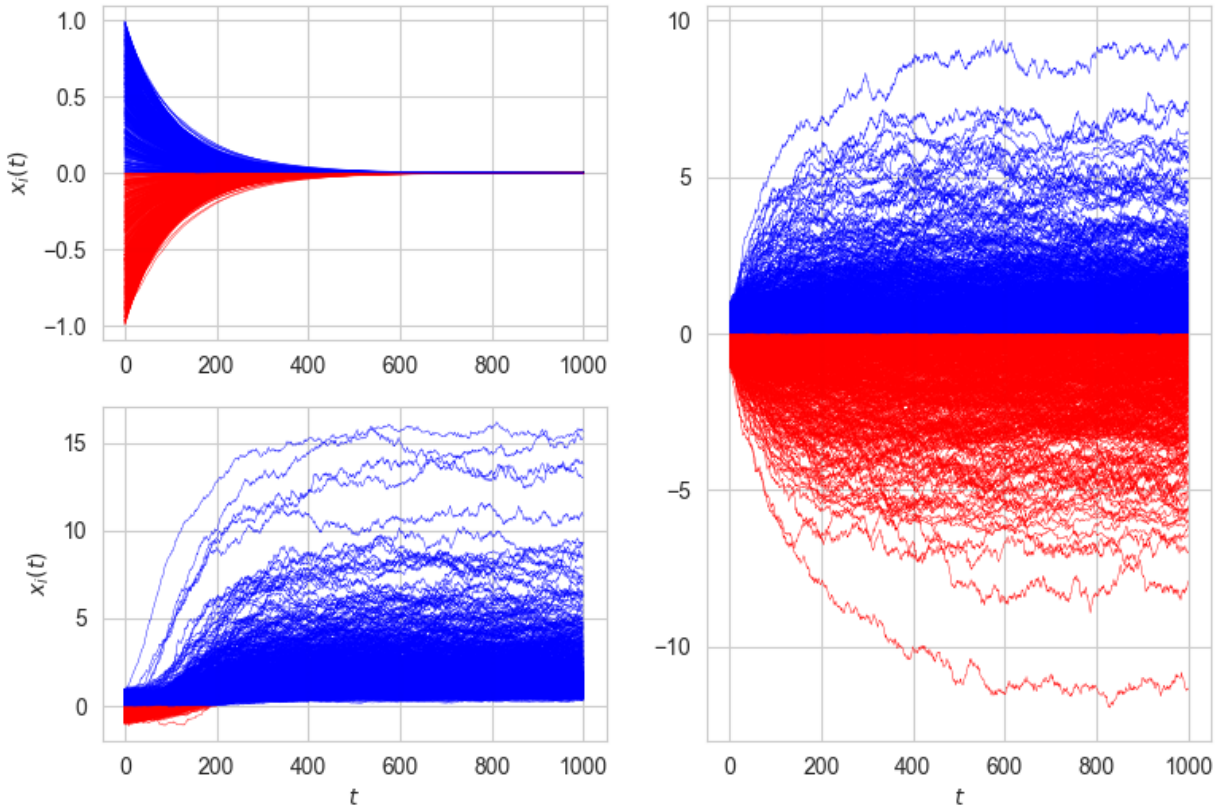
Model extension suggestion

From now, an active agent interacts with m of **his neighbours** from the **predefined** social network.

Our results for a BA(1000, 20) network



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Our results for a WS(1000, 20, 0.2) network

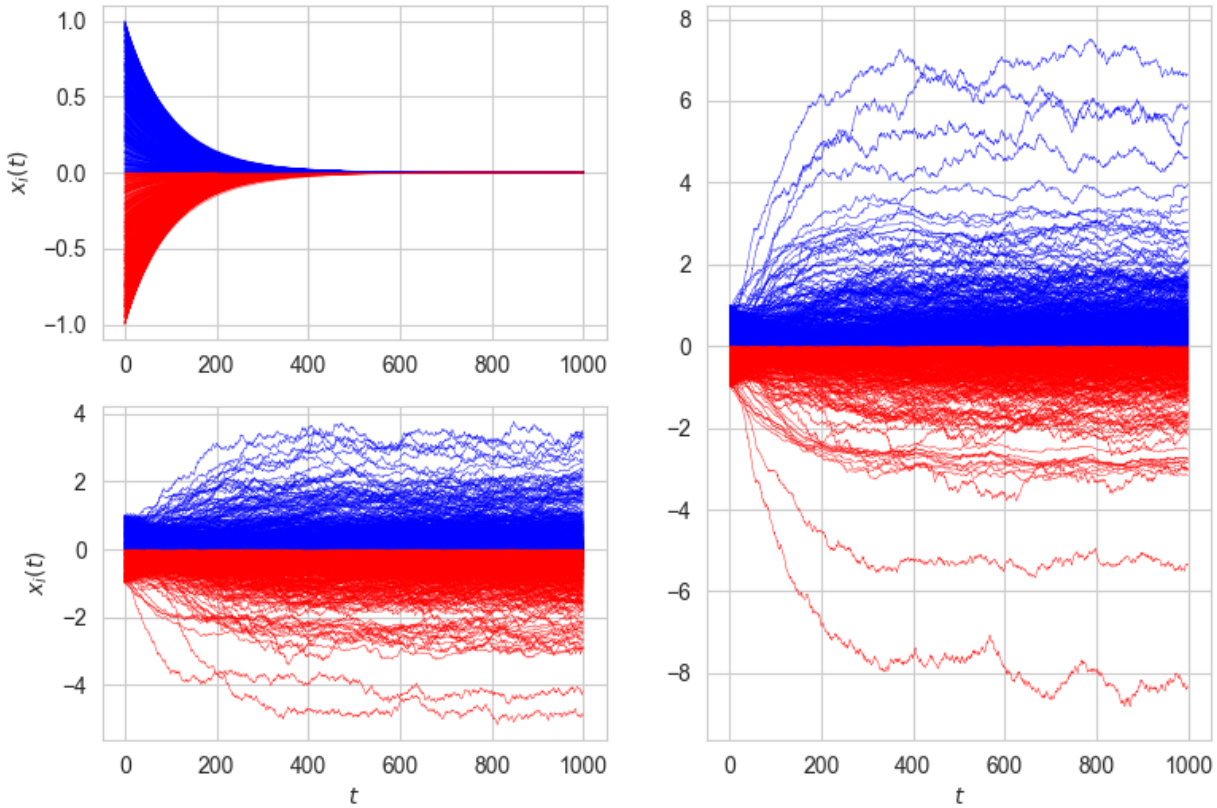
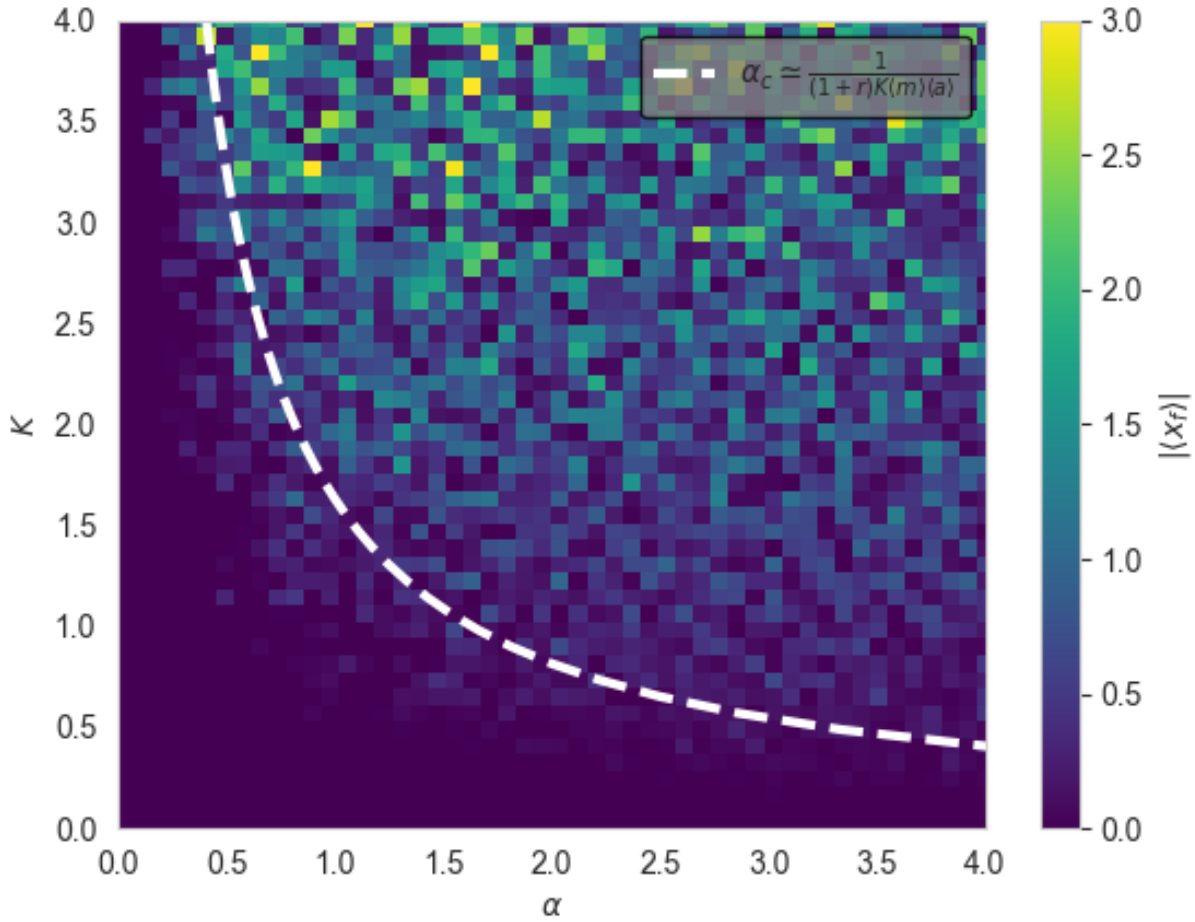


Figure: Temporal evolution of the agents' opinions for $K = 3$ and $r = 0.5$. (a) Neutral consensus for which all opinions converge to zero ($\alpha = 0.05, \beta = 2$). (b-c) Opinion polarization, in which opinions split into two opposite sides $\alpha = 3, \beta = 0$ for (b), $\alpha = 3, \beta = 3$ for (c).

Our results for a WS(1000, 20, 0.2) network



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Project contribution

- ⋆ Marcin Kostrzewa – *key research, extensions ideas, initial model implementation,*
- ⋆ Jakub Koral – *extensions ideas and implementation, plots, extensive simulations,*
- ⋆ Marcin Miśkiewicz – *extensions ideas and implementation, slides, explanatory diagrams, speaking*

References

[1] Fabian Baumann et al. “Modeling Echo Chambers and Polarization Dynamics in Social Networks”. In: *Phys. Rev. Lett.* 124 (4 2020), p. 048301.