Bayesian variational inference for a mixture of factor analyzer

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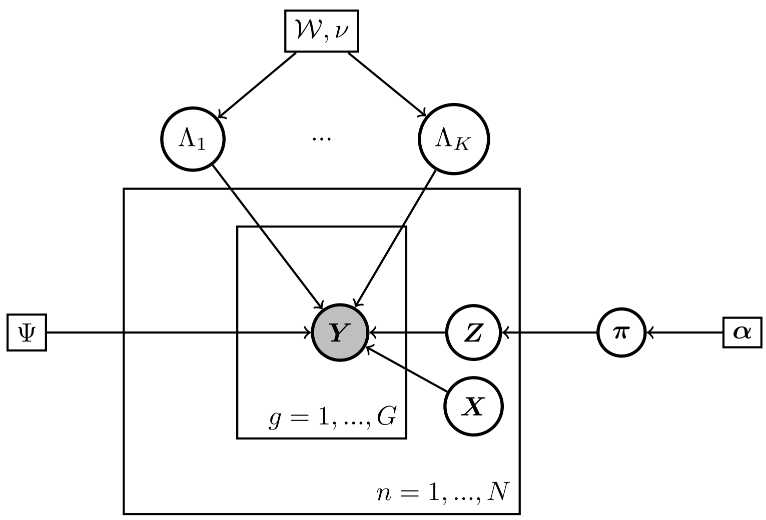
10 January 2014

1 Model

1.1 Notations

$n = 1, ..., N$	Samples
$g = 1, ..., G$	Genes
$k = 1, ..., K$	Factor analyzers
D	Number of observed variables
E	Number of latent variables
$\boldsymbol{y}^{n,g} \in \mathbb{R}^D$	Counts sample n gene g
$\boldsymbol{x}^n \in \mathbb{R}^E$	Factors sample n
$\boldsymbol{z}^n \in \{0,1\}^K$	Indicator factor analyzers sample n
$\boldsymbol{\pi} \in \mathbb{R}^K$	Mixture coefficients
$\Lambda^k \in \mathbb{R}^{D \times E}$	Precision matrix factor analyser k



1.2 Factors

$$P(\boldsymbol{y}^{n,g}|x^n, z^n, \Lambda, \Psi) = \prod_k N(\boldsymbol{y}^{n,g}|(\Lambda^k)^{-1}\boldsymbol{x}^n, \Lambda^k {\Lambda^k}^{-1} + \Psi) \tag{1}$$

$$P(\boldsymbol{x}^n) = N(\boldsymbol{x}^n|0, I_E) \tag{2}$$

$$P(\boldsymbol{z}^n|\pi) = Mult(\boldsymbol{z}^n|\pi) \tag{3}$$

$$P(\Lambda^k) = \mathcal{W}(\Lambda^k|W,\nu) \tag{4}$$

$$P(\boldsymbol{\pi}|\alpha) = Dir(\boldsymbol{\pi}|\boldsymbol{\alpha}) \tag{5}$$

1.3 Evidence

$$\prod_{n,g} P(\boldsymbol{y}^{n,g}) = \int_{\boldsymbol{\theta}} P(\boldsymbol{\theta}) \prod_{n,g} P(\boldsymbol{y}^{n,g}|\boldsymbol{\theta}) \tag{6}$$

$$= \int_{\boldsymbol{\pi}} Dir(\boldsymbol{\pi}|\boldsymbol{\alpha}) \int_{\Lambda} \prod_k \mathcal{W}(\Lambda^k|W,\nu) \tag{7}$$

$$\prod_n \left[\sum_k P(z_k^n|\boldsymbol{\pi}) \int_{\boldsymbol{x}^n} N(\boldsymbol{y}^{n,g}|(\Lambda^k)^{-1}\boldsymbol{x}^n, \Lambda^k {\Lambda^k}^{-1} + \Psi) \right] \tag{8}$$