# Variational Inference for Bayesian Mixture of Factor Analysers

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## 1 Model

### 1.1 Notations

| | |
|---|---|
| $P$ | Number of variables |
| $Q$ | Number of components |
| $S$ | Number of factor analysers |
| $N$ | Number of samples |
| $G$ | Number of genes |

Table 1: Indices

| | | |
|---|---|---|
| $\boldsymbol{y}^{n,g}$ | $P$ | Counts sample $n$ gene $g$ |
| $\boldsymbol{x}^{n,g}$ | $Q$ | Components sample $n$ gene $g$ |
| $\boldsymbol{z}^{n,g}$ | $Q$ | Indicators factor analysers sample $n$ gene $g$ |
| $\boldsymbol{\pi}$ | $S$ | Mixture coefficients |
| $\Lambda^s$ | $P \times Q$ | Loading matrix factor analyser $s$ |
| $\boldsymbol{\nu}^s$ | $Q$ | Precisions components factor analyser $s$ |

Table 2: Random vectors

| | | |
|---|---|---|
| $a$ | 1 | Shape gamma distribution |
| $b$ | 1 | Rate gamma distribution |
| $\alpha$ | 1 | Dirichlet parameter |
| $\Psi$ | $Q \times Q$ | Noise matrix |

Table 3: Hyperparameters

## 1.2 Conditional probability distributions

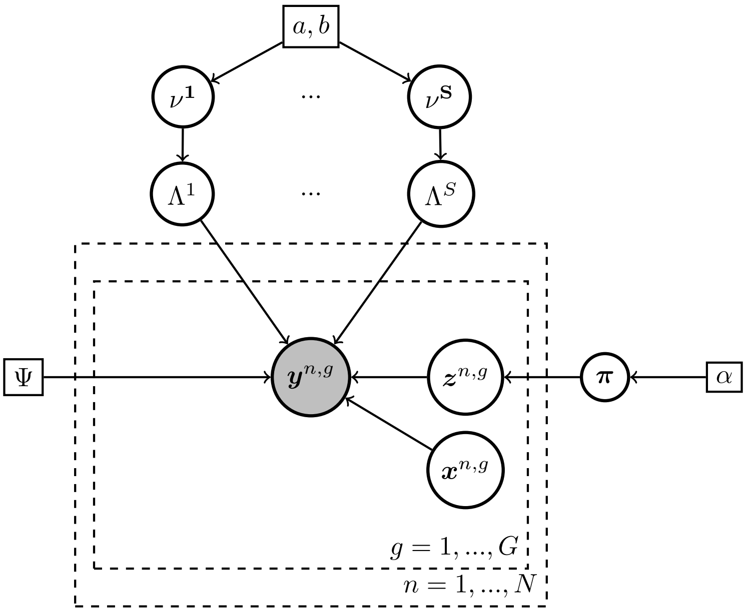


Figure 1: Graphical model

$$P(\boldsymbol{y}^{n,g}|\boldsymbol{x}^{n,g}, \boldsymbol{z}^{n,g}, \boldsymbol{\pi}, \Lambda) = \sum_{s=1}^{S} \mathcal{N}(\boldsymbol{y}^{n,g}|\Lambda^s \boldsymbol{x}^{n,g}, \Lambda^s \Lambda^{sT} + \Psi) P(z_s^{n,g}|\boldsymbol{\pi}) \tag{1}$$

$$P(\boldsymbol{x}^{n,g}) = N(\boldsymbol{x}^{n,g}|0, I_Q) \tag{2}$$

$$P(\boldsymbol{z}^{n,g}|\boldsymbol{\pi}) = Mult(\boldsymbol{z}^{n,g}|\boldsymbol{\pi}) \tag{3}$$

$$P(\boldsymbol{\pi}|\alpha) = Dir(\boldsymbol{\pi}|\alpha) \tag{4}$$

$$P(\Lambda^k) = \prod_{q=1}^{Q} N(\Lambda^s{}_{:q}|0, I_P \frac{1}{\nu_q^s}) \tag{5}$$

$$P(\boldsymbol{\nu}^s) = \prod_{q=1}^{Q} \Gamma(\nu_q^s|a, b) \tag{6}$$