ASSIGNMENT

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CONTENTS

1 UNIFORM RANDOM NUMBERS

Let U be a uniform random variable between 0 and 1

1.1. Generate 10^6 samples of U using a C program and save into a file called uni.dat .
Solution: Download the file:

```
$ wget https://github.com/Manikanta0705/
    Assignment/blob/main/code/1−1.c
$ wget https://github.com/Manikanta0705/
    Assignment/blob/main/code/source.h
```

and compile and execute the C program using

```
$ gcc 1−1.c −lm −wall −g
$ ./a.out
```

1.2. Load the uni.dat file into python and plot the empirical CDF of U using the samples in uni.dat. The CDF is defined as
Solution: The following code plots Fig. 1.2

```
$ wget https://github.com/Manikanta0705/
    Assignment/blob/main/codes/1−2.py
```

It is executed with

```
$ python3 1−2.py
```

$$F_U(x) = Pr(U \leq x) \tag{1.2.1}$$

Graph of CDF is as follow:

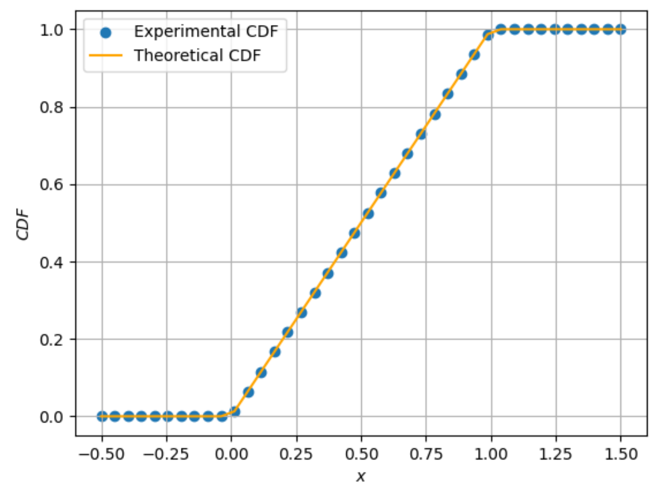

Fig. 1.2.1. CDF of U

1.3. Find a theoretical expression for $F_U(x)$.
Solution: Since We have,

$$P_U(x) = \begin{cases} 1 & x \in (0,1) \\ 0 & otherwise \end{cases} \tag{1.3.1}$$

on integrating for CDF we get,

$$F_U(x) = \int_{-\infty}^{x} P_U(t)dt \tag{1.3.2}$$

$$F_U(x) = \begin{cases} \int_{-\infty}^{x} 0dx & x \in (-\infty,0) \\ \int_{0}^{x} 1dx & x \in (0,1) \\ \int_{0}^{1} 1dx & x \in (1,\infty) \end{cases} \tag{1.3.3}$$

$$F_U(x) = \begin{cases} 0 & x \in (-\infty,0) \\ x & x \in (0,1) \\ 1 & x \in (1,\infty) \end{cases} \tag{1.3.4}$$

1.4. Write a C program to find the mean and variance of U.
Solution: download C program

```
$ wget https://github.com/Manikanta0705/
    Assignment/blob/main/code/1−4.c
```

and compiled and executed with

```
$ gcc 1−4.c −lm −Wall −g
$ ./a.out
```

$$E[U] = 0.500007 \tag{1.4.1}$$
$$\text{Var}[U] = 0.083301 \tag{1.4.2}$$

1.5. Verify your result theoretically given that

$$E[U^k] = \int_{-\infty}^{\infty} x^k dF_U(x) \tag{1.5.1}$$

Solution: we have

$$E[U] = \int_{-\infty}^{\infty} x dF_U(x) \tag{1.5.2}$$
$$= \int_0^1 x dx \tag{1.5.3}$$
$$= 0.5 \tag{1.5.4}$$

From (1.4.1), we have,

$$E[U] = 0.500007 \approx 0.5 \tag{1.5.5}$$

Similarly,

$$\text{Var}[U] = E[U^2] - (E[U])^2 \tag{1.5.6}$$
$$= \int_{-\infty}^{\infty} x^2 dF_U(x) - 0.25 \tag{1.5.7}$$
$$= \int_0^1 x^2 dx - 0.25 \tag{1.5.8}$$
$$= 0.3333... - 0.25 = 0.083333... \tag{1.5.9}$$

From (1.4.2),we get

$$\text{Var}[U] = 0.083301 \approx 0.08333.. \tag{1.5.10}$$

Hence Verified.

2 Central Limit Theorem

2.1. Generate 10^6 samples of the random variable

$$X = \sum_{i=1}^{12} U_i - 6 \tag{2.1.1}$$

using a C program, where $U_i, i = 1, 2, \ldots, 12$ are a set of independent uniform random variables between 0 and 1 and save in a file called gau.dat.

Solution: Download the file

```
$ wget https://github.com/Manikanta0705/
    Assignment/blob/main/code/2−1.c
```

Use source.h from the prob1.1
And run the code as:

```
$ gcc 2−1.c −lm −Wall −g
$ ./a.out
```

2.2. Load gau.dat in python and plot the empirical CDF of X using the samples in gau.dat.

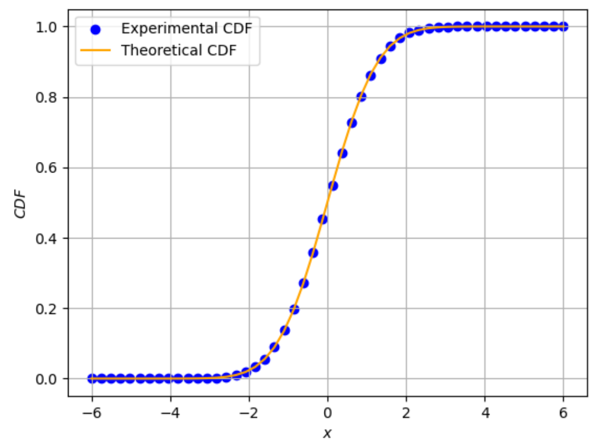

Fig. 2.2.1. The CDF of X

Solution: The following gitlink plots Figure 2.2.1

```
wget https://github.com/Manikanta0705/
    Assignment/blob/main/code/2−2.py
```

2.3. Load gau.dat in python and plot the empirical PDF of X using the samples in gau.dat. The PDF of X is defined as:

$$p_X(x) = \frac{d}{dx} F_X(x) \tag{2.3.1}$$

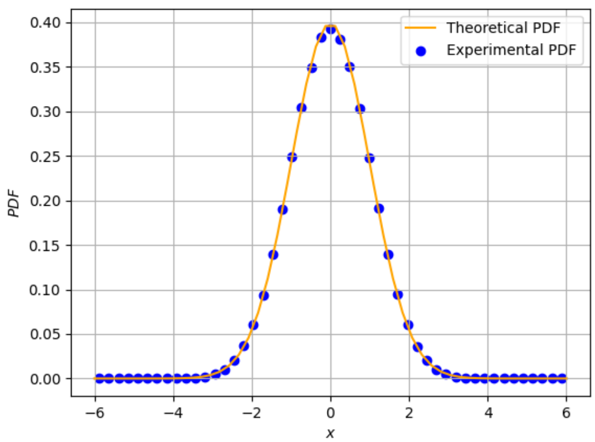

Fig. 2.3.1. The PDF of X

Solution: The following git link plots Figure 2.3.1

```
wget https://github.com/Manikanta0705/
    Assignment/blob/main/code/2−3.py
```

2.4. Find the mean and variance of X by writing a C program.
Solution: Download the following file:

```
wget https://github.com/Manikanta0705/
    Assignment/blob/main/code/2−4.c
```

and compile and execute the C program using

```
$ gcc 2−4.c −lm −wall −g
$ ./a.out
```

Values Obtained:

$$\boxed{\text{Mean = -0.000241}} \quad \boxed{\text{Variance = 1.000726}} \tag{2.4.1}$$

2.5. Given that:

$$p_X(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right), -\infty < x < \infty, \tag{2.5.1}$$

repeat the above exercise theoretically
Solution:

$$E[X] = \int_{-\infty}^{\infty} \frac{x}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right) \tag{2.5.2}$$

$$= -\frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right)\Bigg|_{-\infty}^{\infty} \tag{2.5.3}$$

$$= 0 \tag{2.5.4}$$

Also,

$$E[X^2] = \int_{-\infty}^{\infty} \frac{x^2}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right) \tag{2.5.5}$$

$$= -\frac{x}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}\Bigg|_{-\infty}^{\infty} + \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \tag{2.5.6}$$

$$= 0 + \frac{1}{\sqrt{2\pi}} \times \sqrt{2\pi} \tag{2.5.7}$$

$$= 1 \tag{2.5.8}$$

Thus,

$$\text{var}(X) = E[X^2] - E[X]^2 \tag{2.5.9}$$

$$= 1 \tag{2.5.10}$$

Therefore, the mean is 0 and the variance is 1.

$$\Pr(X > x) = Q(Z > x) \tag{2.5.11}$$

$$= Q(z) \tag{2.5.12}$$

$$CDF = \Pr(X < x) \tag{2.5.13}$$

$$= 1 - Q(z) \tag{2.5.14}$$

3 From Uniform to Other

3.1 Generate samples of:

$$V = -2\ln(1 - U) \tag{3.1.1}$$

and plot its CDF.
Solution:

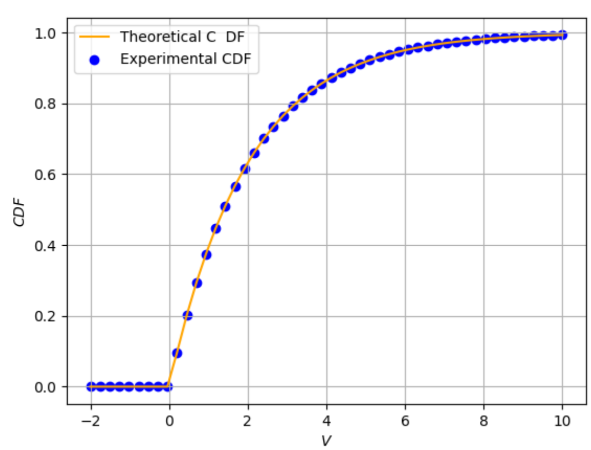

Fig. 3.1.1. The CDF of V

The following plots Figure 3.1.1

```
wget https://github.com/Manikanta0705/
    Assignment/blob/main/codes/3−1.py
```

3.2 Find a theoretical expression for $F_V(x)$.
Solution:

$$F_V(x) = \Pr\left(V \leq x\right) \tag{3.2.1}$$

$$= \Pr\left(-2\ln(1-U) \leq x\right) \tag{3.2.2}$$

$$= \Pr\left(1-U \geq \exp\left(-\frac{x}{2}\right)\right) \tag{3.2.3}$$

$$= \Pr\left(U \leq 1-\exp\left(-\frac{x}{2}\right)\right) \tag{3.2.4}$$

$$= F_U\left(1-\exp\left(-\frac{x}{2}\right)\right) \tag{3.2.5}$$

Therefore,

$$F_V(x) = \begin{cases} 0, & 1-\exp\left(-\frac{x}{2}\right) \in (-\infty, 0) \\ 1-\exp\left(-\frac{x}{2}\right), & 1-\exp\left(-\frac{x}{2}\right) \in (0, 1) \\ 1, & 1-\exp\left(-\frac{x}{2}\right) \in (1, \infty) \end{cases} \tag{3.2.6}$$

$$\implies F_V(x) = \begin{cases} 0, & x \in (-\infty, 0) \\ 1-\exp\left(-\frac{x}{2}\right), & x \in (0, \infty) \end{cases} \tag{3.2.7}$$