AI1110
Assignment 11

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Outline

Problem Statement

Question:

Given a discrete type random variable n taking the values 1,2,... and a sequence of random variables X_k independent of n, we form the sum

$$S = \sum_{k=1}^{n} x_k$$

This sum is a random variable specified as follows: For a specific ζ,n(ζ) is an integer and s(ζ) equals the sum of the numbers $X_k(\zeta)$ for k from 1 to n(ζ). We maintain that if the random variables X_k have the same mean, then

$$E\{s\} = \eta E\{n\}, \text{where} E\{x_k\} = \eta$$

Solution

Clearly,$E\{x_k|n = n\} = E\{X_k\}$ beacuse x_k is independent of n.
Hence

$$E\{s|n = n\} = E\left\{\sum_{k=1}^{n} x_k|n = n\right\} = \sum_{k=1}^{n} E\{x_k\} = \eta n \tag{1}$$

from (1) and $E\{g(x,y)|x\} = \int_{-\infty}^{\infty} g(x,y)f(y|x)dy$

$$E\{s\} = E\{E\{s|n\}\} = E\{\eta n\} \tag{2}$$

Solution

if the random variables x_k are uncorrelated with the same variance σ^2, *then*

$$E\{s^2\} = \eta^2 E\{n^2\} + \sigma^2 E\{n\} \tag{3}$$

$$\implies E\{s^2|n = n\} = \sum_{i=1}^{n}\sum_{k=1}^{n} E\{x_i x_k\} \tag{4}$$

$$\text{where} E\{x_i x_k\} = \begin{cases} \sigma^2 + \eta^2 & i = k \\ \eta^2 & i \neq k \end{cases} \tag{5}$$

Solution

with i = k and $n^2 - n$ terms with i $\neq k$

$$\implies (\sigma^2 + \eta^2)n + \eta^2(n^2 - n) = \eta^2 n^2 + \sigma^2 n \tag{6}$$

this yields because

$$E\{s^2\} = E\{E\{s^2|n\}\} = E\{\eta^2 n^2 + \sigma^2 n\} \tag{7}$$