Assignment 8

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Outline

Problem Statement

QUESTION

A player wins \$1 if he throws two heads in succession, otherwise he loses two quarters. If the game is repeated 50 times, what is the probability that the net gain or loss exceeds
(a) \$1 (b) \$5

Given

Let's denote the situation of the problem by random variable's X and Y such that $X \in \{0, 1\}, Y \in \{0, 1\}$
where,

Variable	Event
$X = 0$	Net gain doesnot exceed \$1
$X = 1$	Net gain or loss exceeds \$1
$Y = 0$	Net gain doesnot exceed \$5
$Y = 1$	Net gain or loss exceeds \$5

Table 1

Consider an experiment consisting of 50 Bernoulli trials and denote the number of two heads in succession obtained by a binomial random variable $Y \in \{0, 1, \ldots, 50\}$. This can be expressed as a binomial distribution with probability mass function given by:

$$p_Y(k) = \binom{n}{k}(1-p)^{n-k}p^k,\ 0 \leq k \leq n \tag{1}$$

where $n = 50$ and $p = 0.25$

Solution

(a) Let k represent the number of wins required in 50 games so that the net gain or loss does not exceed \$1. This gives the net gain to be

$$-1 < k - \frac{50-k}{2} < 1 \tag{2}$$

$$16 < k < 17.3 \tag{3}$$

$$k = 17 \tag{4}$$

The desired probability is given by:

$$p_{X=0}(17) = \binom{50}{17}(1-0.25)^{33}(0.25)^{17} \tag{5}$$

$$= 0.432 \tag{6}$$

$\therefore \Pr\left(X = 1\right) = 1 - 0.432 = 0.568$

Solution

(a) Let k represent the number of wins required in 50 games so that the net gain or loss does not exceed \$5. This gives

$$-5 < k - \frac{50-k}{2} < 5 \tag{7}$$

$$13.3 < k < 20 \tag{8}$$

The desired probability is given by:

$$p_{Y=0}(17) = \sum_{k=14}^{19} \binom{50}{n} (1-0.25)^{n-k} (0.25)^k \tag{9}$$

$$= 0.349 \tag{10}$$

$\therefore \Pr(Y = 1) = 1 - 0.349 = 0.651$