

# Assignment 8

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# Problem Statement

## QUESTION

A player wins \$1 if he throws two heads in succession, otherwise he loses two quarters. If the game is repeated 50 times, what is the probability that the net gain or loss exceeds

(a) \$1 (b) \$5

# Given

Let's denote the situation of the problem by random variable's  $X$  and  $Y$  such that  $X \in \{0, 1\}, Y \in \{0, 1\}$

where,

Variable	Event
$X = 0$	Net gain doesnot exceed \$1
$X = 1$	Net gain or loss exceeds \$1
$Y = 0$	Net gain doesnot exceed \$5
$Y = 1$	Net gain or loss exceeds \$5

Table 1

Consider an experiment consisting of 50 Bernoulli trials and denote the number of two heads in succession obtained by a binomial random variable  $Y \in \{0, 1, \dots, 50\}$ . This can be expressed as a binomial distribution with probability mass function given by:

$$p_Y(k) = \binom{n}{k} (1-p)^{n-k} p^k, \quad 0 \leq k \leq n \quad (1)$$

where  $n = 50$  and  $p = 0.25$

# Solution

(a) Let  $k$  represent the number of wins required in 50 games so that the net gain or loss does not exceed \$1. This gives the net gain to be

$$-1 < k - \frac{50 - k}{2} < 1 \quad (2)$$

$$16 < k < 17.3 \quad (3)$$

$$k = 17 \quad (4)$$

The desired probability is given by:

$$p_{X=0}(17) = \binom{50}{17} (1 - 0.25)^{33} (0.25)^{17} \quad (5)$$

$$= 0.432 \quad (6)$$

$$\therefore \Pr(X = 1) = 1 - 0.432 = 0.568$$

# Solution

(a) Let  $k$  represent the number of wins required in 50 games so that the net gain or loss does not exceed \$5. This gives

$$-5 < k - \frac{50 - k}{2} < 5 \quad (7)$$

$$13.3 < k < 20 \quad (8)$$

The desired probability is given by:

$$p_{Y=0}(k) = \sum_{k=14}^{19} \binom{50}{n} (1 - 0.25)^{n-k} (0.25)^k \quad (9)$$

$$= 0.349 \quad (10)$$

$$\therefore \Pr(Y = 1) = 1 - 0.349 = 0.651$$