Assignment 9

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Outline

Problem Statement

Question

Let x and y be jointly normal random variables with parameters $\mu_x, \mu_y, \sigma_x^2, \sigma_y^2$ and r.Find a necessary and sufficient condition for x+y and x-y to be independent.

Solution

We have,
z = x + y
w = x - y
are jointly normal random variables.
Hence if they are uncorrelated,then they are also independent.

$$
\begin{aligned}
Cov(z, w) &= E[(z - \mu_z)(w - \mu_w)] && (1)\\
&= E[(x - \mu_x) + (y - \mu_y)(x - \mu_x) - (y - \mu_y)] && (2)\\
&= Var(x) - Var(y) && (3)\\
&= \sigma_x^2 - \sigma_y^2 && (4)
\end{aligned}
$$

Solution

The random variables z and w are uncorrelated.

$$Cov(z, w) = 0 \tag{5}$$

$$\implies \sigma_x^2 = \sigma_y^2 \tag{6}$$

Hence $\sigma_x^2 = \sigma_y^2$ is the necessary and sufficient condition for the independence of x+y and x-y.