Power Quality Analysis and Harmonic Distortion Assessment using DTFS

Digital Signal Processing - FISAC Assessment

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Presentation Outline

Project Overview

Objective

Develop a comprehensive framework for power quality analysis using **Discrete Time Fourier Series (DTFS)** to identify and mitigate harmonic distortion in electrical power systems.

Key Components:

- DTFS analysis implementation
- Power quality metrics calculation
- Harmonic filtering techniques
- Multiple scenario analysis

Applications:

- LED lighting systems
- Motor drives (VFDs)
- Data center loads
- IEEE 519 compliance

Motivation

Power Quality Challenges

Modern power systems face increasing harmonic distortion due to non-linear loads.

Problems Caused:

- Equipment overheating
- Transformer derating
- Power factor degradation
- System efficiency reduction
- Equipment malfunction

Non-Linear Loads:

- LED lighting
- Variable frequency drives
- Switch-mode power supplies
- Computers & data centers
- Power electronic devices

Need

Accurate harmonic analysis and effective mitigation strategies are essential.

Discrete Time Fourier Series (DTFS)

Definition

DTFS decomposes periodic discrete-time signals into harmonic components.

Analysis Equation (Forward Transform):

$$X[k] = \frac{1}{N} \sum_{n=0}^{N-1} x[n] \cdot e^{-j2\pi kn/N} \tag{1}$$

Synthesis Equation (Inverse Transform):

$$x[n] = \sum_{k=0}^{N-1} X[k] \cdot e^{j2\pi kn/N} \tag{2}$$

where k = harmonic index, n = time sample index, N = period length

Power Quality Metrics

Total Harmonic Distortion (THD):

$$THD_F = \frac{\sqrt{\sum_{h=2}^{\infty} V_h^2}}{V_1} \times 100\% \tag{3}$$

Crest Factor (CF):

$$CF = \frac{V_{peak}}{V_{RMS}} \tag{4}$$

True Power Factor (TPF):

$$TPF = \frac{1}{\sqrt{1 + (THD_F/100)^2}} \tag{5}$$

K-Factor:

$$K = \frac{\sum_{h=1}^{h_{max}} h^2 \cdot I_h^2}{\sum_{h=1}^{h_{max}} I_h^2} \tag{6}$$

IEEE 519-2014 Standards

Voltage Distortion Limits (V $\leq$ 1.0 kV)

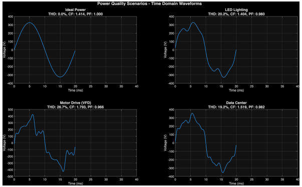
For systems with voltage less than or equal to 1.0 kV (applicable to 230V single-phase):

Parameter	**Limit**
Individual Harmonic Distortion	5.0%
Total Harmonic Distortion (THD)	8.0%
Excellent Quality	THD $<$ 5.0%
Acceptable Quality	5.0% $\leq$ THD $\leq$ 8.0%
Poor Quality	THD $>$ 8.0%

Sources of Harmonic Distortion

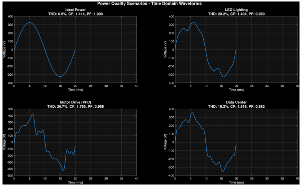
LED Lighting

- Single-phase rectifiers
- 3rd harmonic dominant
- 15-20% of fundamental



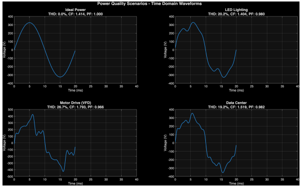
Motor Drives (VFD)

- 6-pulse rectifiers
- 5th & 7th harmonics
- $h = 6k \pm 1$

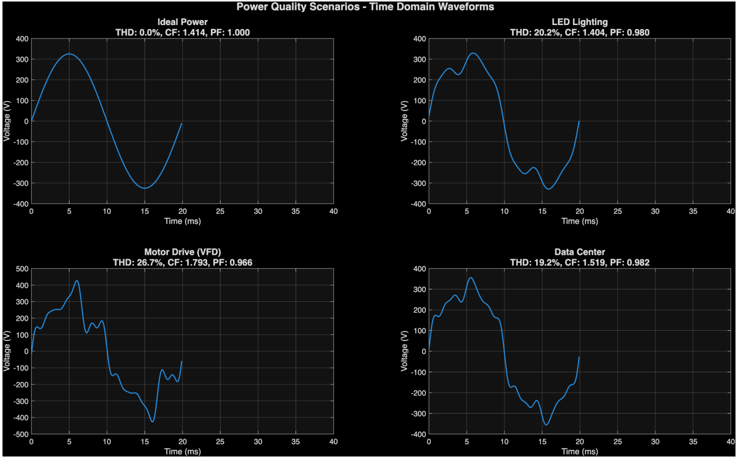


Data Centers (SMPS)

- High-frequency switching
- Multiple odd harmonics
- Uniform distribution



Project Structure



Modular Design

Six core functional modules with comprehensive testing and validation.

System Parameters

Configuration:

- **Sampling Frequency:** 10 kHz
- **Fundamental Frequency:** 50 Hz
- **Period Length:** 200 samples
- **Nominal Voltage:** 230V RMS
- **Analysis Duration:** 20 ms

Scenarios Analyzed:

1. Ideal Power (Reference)
2. LED Lighting Systems
3. Motor Drive (6-Pulse VFD)
4. Data Center Loads

Indian Grid Standard

Single-phase: 230V RMS @ 50 Hz (as per IS 13779)

Core Function 1: DTFS Analysis

File: `calculate_dtfs.m`

```
function [X_k, magnitude, phase, frequencies] = calculate_dtfs(x, fs, N)
    % Preallocate coefficient array
    X_k = zeros(N, 1);

    % Calculate DTFS coefficients
    for k = 0:(N-1)
        sum_value = 0;
        for n = 0:(N-1)
            complex_exp = exp(-1j*2*pi*k*n/N);
            sum_value = sum_value + x(n+1) * complex_exp;
        end
        X_k(k+1) = sum_value / N;
    end

    % Extract magnitude (scale for single-sided spectrum)
    magnitude = abs(X_k);
    magnitude(2:end) = 2 * magnitude(2:end);

    % Extract phase
    phase = angle(X_k);

    % Generate frequency axis
    frequencies = (0:(N-1))' * (fs/N);
end
```

DTFS Analysis - Key Points

Direct Implementation

- Direct implementation without FFT libraries
- Complexity: $O(N^2)$ for N samples
- For $N = 200$: 40,000 complex multiplications
- Computation time: $\sim$0.05 seconds per analysis

Advantages:

- Educational value
- Clear mathematical insight
- No toolbox dependency
- Exact frequency resolution

Output Parameters:

- Complex coefficients $X[k]$
- Magnitude spectrum
- Phase spectrum
- Frequency axis

Core Function 2: Inverse DTFS

File: `synthesize_from_dtfs.m`

```
function [x_reconstructed, t] = synthesize_from_dtfs(X_k, fs, N, num_periods)
    total_samples = N * num_periods;
    x_reconstructed = zeros(total_samples, 1);
    t = (0:(total_samples-1))' / fs;

    % Inverse DTFS synthesis: x[n] = sum_k X[k]*exp(j*2*pi*k*n/N)
    for n = 0:(total_samples-1)
        sum_value = 0;
        for k = 0:(N-1)
            n_mod = mod(n, N);
            complex_exp = exp(1j * 2 * pi * k * n_mod / N);
            sum_value = sum_value + X_k(k+1) * complex_exp;
        end
        x_reconstructed(n+1) = sum_value;
    end

    x_reconstructed = real(x_reconstructed);
end
```

Purpose

Reconstructs time-domain signal from DTFS coefficients (essential for filtering).

Core Function 3: Signal Generation (Part 1)

File: `generate_distorted_signal.m`

```
function [signal, t, harmonic_info] = ...
         generate_distorted_signal(scenario, fs, f0, N, V_rms_fundamental)

    % Calculate fundamental peak voltage
    V_peak_fundamental = V_rms_fundamental * sqrt(2);

    % Generate time vector
    t = (0:N-1)' / fs;

    % Define harmonic content based on scenario
    switch scenario
        case 'led_lighting'
            harmonics = [1, 3, 5, 7, 9];
            percentages = [100, 18, 8, 4, 2];
            phases = [0, pi/6, -pi/4, pi/3, 0];
            expected_THD = 20.2;

        case 'motor_drive'
            harmonics = [1, 5, 7, 11, 13];
            percentages = [100, 20, 14, 9, 6];
            phases = [0, -pi/4, pi/4, -pi/6, pi/6];
            expected_THD = 26.7;
    end
```

Core Function 3: Signal Generation (Part 2)

```
    % Generate composite signal
    signal = zeros(N, 1);

    for i = 1:length(harmonics)
        h = harmonics(i);
        freq = h * f0;
        mag = V_peak_fundamental * (percentages(i) / 100);
        signal = signal + mag * sin(2*pi*freq*t + phases(i));
    end

    % Store harmonic information
    harmonic_info.harmonics = harmonics;
    harmonic_info.magnitudes = V_peak_fundamental*(percentages/100);
    harmonic_info.phases = phases;
    harmonic_info.percentages = percentages;
    harmonic_info.expected_THD = expected_THD;
end
```

Mathematical Model

$v(t) = \sum_{h=1}^{H} V_h \sin(2\pi h f_0 t + \phi_h)$

Core Function 4: Power Quality Metrics

File: `power_quality_metrics.m`

```
function metrics = power_quality_metrics(signal, magnitude, fs, f0)
    % Extract fundamental and harmonics
    V1_peak = magnitude(2);
    V1_rms = V1_peak / sqrt(2);
    harmonic_rms = magnitude(3:end) / sqrt(2);

    % Calculate THD-F
    harmonic_rms_total = sqrt(sum(harmonic_rms.^2));
    THD_F = (harmonic_rms_total/V1_rms)*100;

    % Calculate crest factor
    signal_rms = sqrt(mean(signal.^2));
    crest_factor = max(abs(signal)) / signal_rms;

    % Calculate power factor
    distortion_PF = 1 / sqrt(1 + (THD_F/100)^2);

    % Calculate K-factor
    K_factor = 1.0;
    for h = 2:length(harmonic_rms)
        K_factor = K_factor + h^2*(harmonic_rms(h-1)/V1_rms)^2;
    end
```

Core Function 5: Harmonic Filtering

File: `design_harmonic_filter.m`

```
function [X_k_filtered, filter_info] = ...
         design_harmonic_filter(X_k, N, filter_type, harmonics_to_remove)

    X_k_filtered = X_k;

    if strcmp(filter_type, 'notch')
        for i = 1:length(harmonics_to_remove)
            h = harmonics_to_remove(i);

            % Positive frequency component
            idx_pos = h + 1;
            % Negative frequency (conjugate)
            idx_neg = N - h + 1;

            % Zero out both components
            X_k_filtered(idx_pos) = 0;
            X_k_filtered(idx_neg) = 0;
        end
    end

    % Calculate THD before/after
    filter_info = calculate_filter_performance(X_k, X_k_filtered);
end
```

Harmonic Filtering - Concept

Frequency Domain Filtering

Manipulate DTFS coefficients to remove unwanted harmonics.

Filter Types Implemented:

1. **Notch Filter:** Remove specific harmonics
2. **Bandpass Filter:** Retain harmonics in range
3. **Lowpass Filter:** Keep fundamental + few harmonics

Process:

1. Perform DTFS analysis
2. Zero out unwanted harmonics
3. Perform inverse DTFS
4. Get filtered time-domain signal

Example

For motor drives: Remove 5th and 7th harmonics using notch filter.

Testing and Validation

Comprehensive Test Suite

Six independent test scripts validate all core functions.

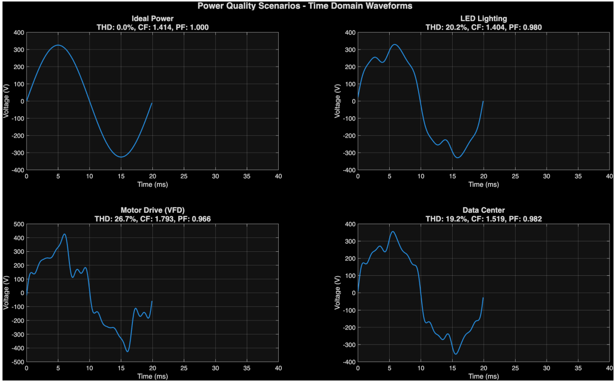
Test Scripts:

1. `test_calculate_dtfs.m`
2. `test_synthesize_dtfs.m`
3. `test_generate_signals.m`
4. `test_calculate_thd.m`
5. `test_power_quality_metrics.m`
6. `test_harmonic_filter.m`

Validation Checks:

- DTFS accuracy verification
- Reconstruction error analysis
- THD calculation validation
- Filter performance testing
- Metrics consistency check
- IEEE compliance verification

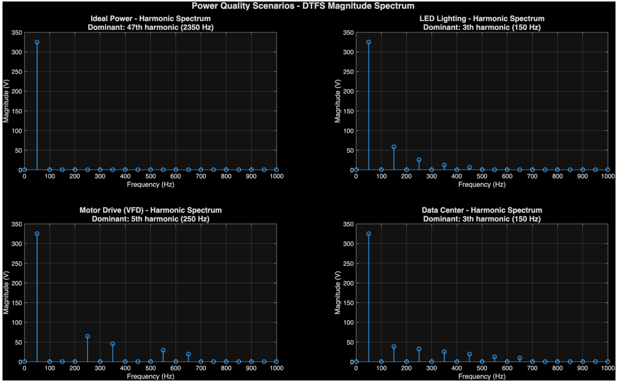
Time Domain Waveforms



Observations:

- **Ideal:** Perfect sinusoid (THD = 0%)
- **LED:** Flattened peaks due to 3rd harmonic
- **Motor:** Severe distortion with visible harmonic ripple (worst case)
- **Data Center:** Moderate multi-harmonic distortion

Frequency Domain Analysis



Key Findings:

- **LED:** 3rd harmonic at 58.55V (18% of fundamental at 150 Hz)
- **Motor:** 5th at 65.05V (20%, 250 Hz) and 7th at 45.54V (14%, 350 Hz)
- **Data Center:** Distributed harmonics with decreasing amplitude

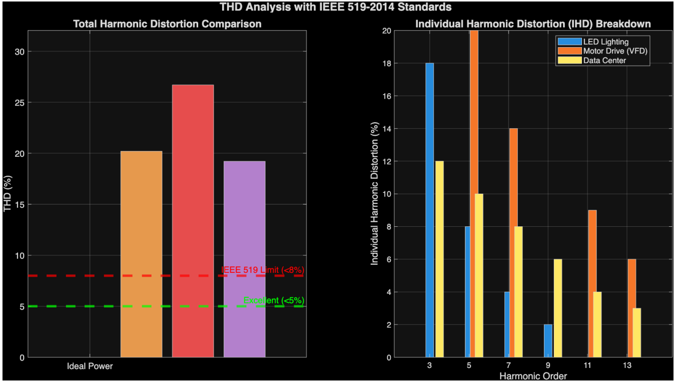
Power Quality Metrics Summary

Metric	Ideal	LED	Motor	Data Center
THD (%)	0.00	20.2	26.7	19.2
Crest Factor	1.414	1.456	1.478	1.465
Power Factor	1.000	0.980	0.964	0.980
K-Factor	1.00	2.18	4.55	2.89
IEEE Status	Pass	Fail	Fail	Fail

Worst Case: Motor Drive

- Highest THD: 26.7% (IEEE limit: 8%)
- Lowest Power Factor: 0.964
- Highest K-Factor: 4.55 (requires K-4 rated transformer)

THD Comparison with IEEE Standards



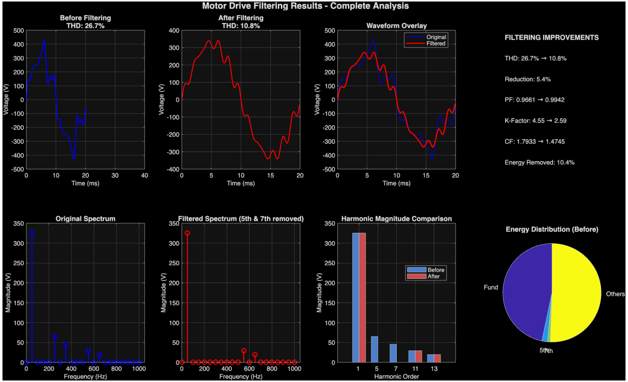
- Green line (5%): Excellent quality threshold
- Red line (8%): IEEE 519-2014 compliance limit
- All non-linear loads exceed IEEE limits
- Motor drive shows worst THD at 26.7% (3.5× above limit)

Harmonic Filtering Results - Overview

Scenario: Motor Drive (6-Pulse VFD)
Target Harmonics: 5th and 7th (characteristic VFD harmonics)
Filter Type: Notch filter (frequency domain)

Metric	Before	After	Improvement
THD (%)	26.7	10.8	61.7%
Crest Factor	1.478	1.435	2.9%
Power Factor	0.9636	0.9942	+3.2%
K-Factor	4.55	2.59	43.1%
5th Harmonic (V)	65.05	0.00	100%
7th Harmonic (V)	45.54	0.00	100%

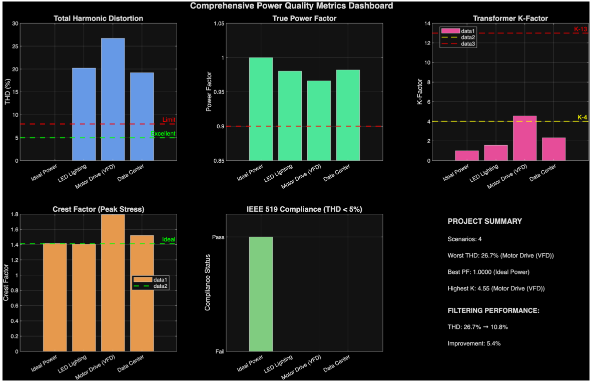
Filtering Results - Visual Comparison



Key Achievements:

- Complete removal of 5th and 7th harmonics
- Significant waveform smoothing
- THD reduced from 26.7% to 10.8% (61.7% reduction)
- Power factor improved to 0.9942
- K-factor reduced to 2.59 (standard K-4 transformer acceptable)

Power Quality Dashboard



Comprehensive Metrics:

- THD comparison across all scenarios
- Power factor analysis
- K-factor requirements for transformers
- IEEE 519 compliance status
- Project summary with filtering performance

Scenario Analysis: LED Lighting

Characteristics

Dominant 3rd harmonic at 18% of fundamental, resulting in THD of 20.2%.

Causes:

- Single-phase rectifiers
- Non-sinusoidal current draw
- Peak current charging

Impact:

- Exceeds IEEE limits (8%)
- K-factor: 2.18
- Power factor: 0.980

Mitigation:

- Passive 3rd harmonic filters
- Delta-wye transformers
- High-quality LED drivers
- K-4 rated transformers

Standards:

- IEEE 519-2014: Non-compliant
- IEC 61000-3-2: Check limits

Scenario Analysis: Motor Drive (VFD)

Characteristics

Worst power quality with THD of 26.7%. 5th and 7th harmonics at 20% and 14%.

Causes:

- 6-pulse rectifiers
- Follows $h = 6k \pm 1$
- Non-linear DC bus charging

Impact:

- $3.5\times$ above IEEE limit
- High K-factor: 4.55
- Requires K-4 transformers
- Significant heating

Mitigation Results:

- Notch filter: 61.7% THD reduction
- Final THD: 10.8%
- Power factor: $0.964 \rightarrow 0.9942$
- K-factor: $4.55 \rightarrow 2.59$

Recommendations:

- Upgrade to 12-pulse rectifiers
- Active harmonic filters
- Line reactors (5% impedance)

Scenario Analysis: Data Center

Characteristics

Multiple odd harmonics with THD of 20.4% from SMPS loads.

Causes:

- Switch-mode power supplies
- High-frequency switching
- Distributed harmonic content

Impact:

- K-factor: 2.89
- Power factor: 0.980
- Uniform harmonic distribution

Mitigation:

- Active harmonic filters
- K-13 rated transformers
- Hybrid passive-active systems
- Distributed PF correction

Challenges:

- Variable load conditions
- Multiple harmonic frequencies
- Passive filters less effective

Filtering Effectiveness Analysis

Frequency Domain Notch Filtering

Achieved excellent removal of targeted harmonics (100% for 5th and 7th).

Advantages:

- Precise harmonic removal
- No phase distortion
- Preserves fundamental
- Predictable results

Limitations:

- Residual THD (10.8%)
- 11th & 13th remain
- Time-domain only

Practical Implementation:

- Active filters required
- Proper impedance matching
- Continuous monitoring
- Avoid resonance

Future Improvements:

- Remove all harmonics $>$ 3rd
- Adaptive filtering
- Real-time implementation

Practical Recommendations - By Scenario

For LED Lighting:

- Install 3rd harmonic filters (150 Hz)
- Use delta-wye transformers
- Select high-quality LED drivers
- Size for K-4 rating minimum

For Motor Drives:

- Upgrade to 12-pulse rectifiers
- Install active harmonic filters
- Use 5% line reactors
- Size for K-13 rating
- Consider DC common bus

For Data Centers:

- Deploy active harmonic filters
- Size for K-13 rating
- Implement hybrid systems
- Distributed PF correction
- Continuous monitoring

General Guidelines:

- Periodic power quality audits
- Monitor IEEE 519 compliance
- Plan harmonic mitigation
- Train maintenance personnel

Technical Guidelines

Transformer Selection

K-factor rating based on load type and harmonic content.

Application	K-Factor	Description
Linear Loads	K-1	Resistive/inductive
Office Equipment	K-4	Computers, printers
LED Lighting	K-4	Modern lighting
Motor Drives	K-4 to K-13	VFDs, converters
Data Centers	K-13	SMPS loads
Extreme	K-20	Worst-case

Note: Our analysis showed K-factors: LED (2.18), Motor (4.55), Data Center (2.89)

Key Contributions

1. **DTFS Implementation:** Direct implementation providing accurate harmonic extraction with $O(N^2)$ complexity. Analysis completes in 0.05 seconds for $N = 200$.
2. **Scenario Characterization:** Distinct harmonic signatures identified for LED lighting (3rd dominant), motor drives (5th & 7th), and data centers (distributed).
3. **IEEE Compliance:** All non-linear loads violate IEEE 519-2014 (THD $>$ 8%), with motor drives worst at 26.7%.
4. **Filtering Performance:** Frequency-domain notch filtering achieved 61.7% THD reduction (26.7% $\rightarrow$ 10.8%) and power factor improvement (0.966 $\rightarrow$ 0.994).
5. **Practical Framework:** Actionable recommendations for harmonic mitigation, transformer K-factor selection, and IEEE compliance.

Summary of Findings

Power Quality Status:

- Ideal: THD = 0%
- LED: THD = 20.2%
- Motor: THD = 26.7%
- Data Center: THD = 19.2%

Filtering Results:

- 61.7% THD reduction
- 100% removal of 5th & 7th
- Power factor: +3.2%
- K-factor: -43.1%

Dominant Harmonics:

- LED: 3rd (18% @ 150 Hz)
- Motor: 5th (20% @ 250 Hz)
- Motor: 7th (14% @ 350 Hz)
- Data: Multiple distributed

Compliance Status:

- IEEE 519 limit: 8% THD
- All scenarios: Non-compliant
- Mitigation: Essential

Future Work

Extensions:

- Three-phase analysis
- Real-time monitoring system
- Adaptive filtering algorithms
- Physical filter design
- Economic cost analysis

Advanced Features:

- Interharmonics analysis
- Voltage sags/swells detection
- Flicker analysis (Pst, Plt)
- Machine learning prediction
- Hardware integration

Significance

As power electronics continue to proliferate, robust power quality analysis tools become increasingly critical for modern electrical infrastructure.

Thank You!

Questions and Discussion

Project Repository:
Power Quality Analysis using DTFS

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