Solved questions from reference book

1. **A gas undergoes a reversible non-flow process according to the relation $p = (-3V + 15)$ where V is the volume in m^3 and p is the pressure in bar. Determine the work done when the volume changes from 3 to 6 m^3.**

Solution:

$$p = (-3V + 15) \text{ bar } = (-3V + 15) \times 10^5 \text{ N/m}^2$$

Work done

$$\begin{aligned} W_{1-2} &= \int pdV = \int_{V_1}^{V_2} (-3V + 15) \times 10^5 dV \\ &= \left| -3\frac{V^2}{2} + 15V \right|_3^6 \times 10^5 \\ &= \left[\frac{-3}{2}(6^2 - 3^2) + 15(6 - 3) \right] \times 10^5 \end{aligned}$$

$$= (-40.5 + 45) \times 10^5 = 40.5 \times 10^5 \text{Nm (or J)} = 4.5 \times 10^5 \text{Nm}$$

2. **A non- flow reversible process occurs for which pressure and volume are correlated by the expression $p = (V^2 + 6/V)$ where p is in bar and V is in m^3. What amount of work will be done when volume changes from 2 to 4 m^3 ?**

Solution:

$$p = \left(V^2 + \frac{6}{V}\right) \text{ bar } = \left(V^2 + \frac{6}{V}\right) \times 10^5 \text{ N/m}^2$$

Work done, $W_{1-2} = \int pdV = \int_{V_1}^{V_2} \left(V^2 + \frac{6}{V}\right) \times 10^5 dV$

$$= 10^5 \left[\frac{V^3}{3} + 6\log_e V\right]_2^4 = 10^5 \left[\frac{4^3 - 2^3}{3} + 6\log_e \frac{4}{2}\right]$$

$$= (18.67 + 4.16) \times 10^5 = 22.83 \times 10^5 \text{Nm (or J)}$$

3. **Consider a gas enclosed in a piston- cylinder arrangement. The gas is initially at 150kPa and occupies a volume of 0.03 m^3. The gas is now heated until the volume of the gas increases to 0.1 m^3. Calculate the work done by the gas if volume of the gas is inversely proportional to the pressure.**

Solution: The path followed by the system is $V \propto \frac{1}{p}$. That is

$$\begin{aligned} pV &= \text{constant } C = p_1V_1 = p_2V_2 \\ \text{or } p &= \frac{C}{V} \end{aligned}$$

Work done, $W_{1-2} = \int pdV = \int_{V1}^{V2} \frac{C}{V} dV$

$$\begin{aligned} &= C\log_e \frac{V_2}{V_1} = p_1V_1\log_e \frac{V_2}{V_1} \\ &= 150 \times 10^3 \times 0.03\log_e \frac{0.1}{0.03} \\ &= 5.41 \times 10^3 \text{Nm (or J)} = 5.41 \text{ kJ} \end{aligned}$$

4. **0.75 kg of an ideal gas at 15°C temperature is contained in a cylinder of 0.5 m^3 capacity. The gas is heated at constant pressure until it attains a temperature of 150°C. Determine the density of gas when it has been heated.**

Solution:

$$T_1 = 15^\circ\text{C} = 273 + 15 = 288 \text{ K}; T_2 = 273 + 150 = 423 \text{ K}$$

$$v_1 = \frac{V}{m} = \frac{0.5}{0.75} = 0.667 \text{ m}^3/\text{kg}.$$

Since pressure remains constant, the Charles' law applies and, therefore,

$$\frac{v_1}{T_1} = \frac{v_2}{T_2}$$

$$v_2 = v_1 \frac{T_2}{T_1} = \frac{0.667 \times 423}{288} = 0.98 \text{ m}^3/\text{kg}$$

Density is reciprocal of specific volume and, therefore,

$$\rho_2 = \frac{1}{v_2} = \frac{1}{0.98} = 1.02 \text{ kg/m}^3$$

5. **Determine the mass of nitrogen present in a vessel of 2.5 m^3 capacity. The pressure and temperature gauges mounted on the vessel indicate gas to be at 80 bar pressure and 25°C temperature. For nitrogen gas constant $R = 297$ J/kg deg.**

Solution: $p = 80$ bar $= 80 \times 10^5$ N/m^2; $T = (25 + 273) = 298$ K

Invoking characteristic gas equation, $pV = mRT$

$$80 \times 10^5 \times 2.5 = m \times 297 \times 298$$

∴ Mass of nitrogen, $m = \frac{80\times10^5\times2.5}{297\times298} = 225.97$ kg

6. **A high-altitude chamber, the volume of which is 30 m^3, is put into operation by reducing the pressure from 1.013 bar to 0.35 bar and temperature from 27°C to 5°C. How many kg of air must be removed from the chamber during the process? Express this mass as volume measured at 1.013 bar and 27°C. Take $R = 287$ J/kgK for air.**

Solution : Invoking characteristic gas equation, $PV = mRT$

Initial mass of air, $m_1 = \frac{p_1V_1}{RT_1} = \frac{(1.013\times10^5)\times30}{287(273+27)} = 35.296$ kg

Final mass of air, $m_2 = \frac{p_2V_2}{RT_2} = \frac{0.35\times10^5\times30}{287(273+5)} = 13.160$ kg

∴ Mass of air removed during the process,

$$m_1 - m_2 = 35.296 - 13.160 = 22.136 \text{ kg}$$

Volume of this mass at 1.013 bar and 27°C is given by,

$$V = \frac{mRT}{p} = \frac{22.136 \times 287 \times (273 + 27)}{1.013 \times 10^5} = 18.815 \text{ m}^3$$

7. **A closed vessel contains 3 kg of CO_2 at pressure 70kPa and temperature 300 K. Heat is supplied to the vessel till the gas attains 140kPa of pressure. Calculate: (a) final temperature, (b) work done on or by the gas, (c) heat added, and (d) change in internal energy. For CO_2 : take $c_v = 0.65$ kJ/kgK.**

Solution: The closed vessel implies that the change in the state of the system is at constant volume and that the process is isochoric. For such a constant volume process, $\frac{p_1}{T_1} = \frac{p_2}{T_2}$

∴ Final temperature, $T_2 = \frac{p_2}{p_1} T_1 = \frac{140}{70} \times 300 = 600$ K

(b) No volume change takes place and consequently the work done on or by the gas is zero

(c) Heat added during a constant volume process is given by

$$Q = mc_v(T_2 - T_1) = 3 \times 0.65 \times (600 - 300) = 585 \text{ kJ}$$

(d) From the non-flow energy equation,

$$dU = \delta Q - \delta W = 585 - 0 = 585 \text{ kJ (increase)}$$

$$dU = mc_v(T_2 - T_1) = 3 \times 0.65 \times (600 - 300) = 585 \text{ kJ}$$

8. **An oxygen cylinder of 0.25 m^3 capacity contains oxygen at a pressure of 30bar and 293 K temperature. The stop valve is opened, and a certain mass of oxygen is used. Measurements show that the oxygen left in the cylinder is at 15bar pressure and 288 K temperature. Determine the mass of oxygen used.**

After closure of the valve, the oxygen remaining in the cylinder gradually attains its initial temperature of 293 K. Make calculations for the amount of heat transferred through the

cylinder walls to the atmosphere. It may be presumed that oxygen has a density of $\mathbf{1.43\ kg/m^3}$ at $\mathbf{0°C}$ and $\mathbf{1.01325 bar}$, and $\boldsymbol{\gamma = 1.4}$.

Solution: Applying characteristics gas equation, $pv = mRT$, we have

$$R = \frac{V}{m} \times \frac{p}{T} = \frac{p}{\rho T} = \frac{1.01325 \times 10^5}{1.43 \times 273} = 259.55\ \text{J/kgK}$$

Initial mass of oxygen in the cylinder,

$$m_1 = \frac{p_1 V_1}{R T_1} = \frac{30 \times 10^5 \times 0.25}{259.55 \times 293} = 9.862\ \text{kg}$$

After the valve is opened and the gas is used, oxygen left in the cylinder.

$$m_2 = \frac{p_2 V_2}{R T_2} = \frac{15 \times 10^5 \times 0.25}{259.55 \times 288} = 5.017\ \text{kg}$$

$\therefore$ mass of oxygen used $= m_1 - m_2 = 9.862 - 5.017 = 4.845$ kg

(b) From the non-flow energy equation; $\delta Q = \delta W + dU$

The cylinder is rigid (no change in volume takes place) and consequently the work done on or by the gas is zero.

$$\therefore \quad \text{Heat exchange } \delta Q \quad = dU = m c_v dT = m\left(\frac{R}{\gamma - 1}\right) dT$$

$$= 5.017 \times \frac{259.55}{1.4 - 1} \times (293 - 288) = 16277\ \text{J} \approx 16.28\ \text{kJ}$$

9. **An ideal gas requires $\mathbf{1150\ kJ/kg}$ of heat to raise its temperature from $\mathbf{20\ °C}$ to $\mathbf{100°C}$ when heated at constant pressure. When heat is supplied to the same gas at constant volume, the heat requirement is $\mathbf{825\ kJ}$ for the same temperature range. Determine the specific heat at constant pressure, specific heat at constant volume, diabatic exponent, characteristic gas constant and the molecular mass of the gas.**

Solution: At constant pressure process, $Q = m c_p dT$

$$1150 = 1 \times c_p(100 - 20);\ c_p = 14.375\ \text{kJ/kgK}$$

At constant volume process, $Q = m c_v dT$

$$825 = 1 \times c_v(100 - 20);\ c_v = 10.312\ \text{kJ/kgK}$$

Adiabatic exponent, $\gamma = \frac{c_p}{c_v} = \frac{14.375}{10.312} = 1.394$

Characteristic gas constant, $R = c_p - c_v$

$$= 14.375 - 10.312 = 4.063\ \text{kJ/kgK}$$

Molecular mass, $M \equiv \frac{R_{\text{mol}}}{R} \equiv \frac{8.314}{4.063} \equiv 2.046$

10. **A closed rigid vessel containing $\mathbf{10\ kg}$ of oxygen at $\mathbf{290\ K}$ is supplied heat until its pressure becomes two-fold that of initial value. Identify the process and calculate the final temperature, change in internal energy and enthalpy, and heat ineraction across the system boundary.**
Take $\boldsymbol{c_v = 0.65}$ kJ/kgK.

Solution: Characteristic gas constant, $R = \frac{R_{\text{mol}}}{\text{molecular mass}} = \frac{8314}{32} = 259.8\ \text{J/kgK}$

Since the vessel is closed and rigid, there will be no change in the volume of gas enclosed in it. As such the situation corresponds to that of an isochoric (constant volume) process. Since volume remains constant, Charles law applies and therefore,

$$\frac{T_2}{T_1} = \frac{p_2}{p_1};\ T_2 = \frac{p_2}{p_1} \times T_1 = 2 \times 290 = 580\ \text{K}$$

$$dU = m c_v (T_2 - T_1) = 10 \times 0.65 \times (580 - 290) = 1885\ \text{kJ}$$

Now, $c_p = c_v + R = 0.65 + 0.2598 = 0.9098$ kJ/kgK

$$dH = m c_p (T_2 - T_1) = 10 \times 0.9098 \times (580 - 290) = 2638.4\ \text{kJ}$$

From non-flow energy equation,

$$Q_{1-2} = W_{1-2} + dU$$

But $W_{1-2}=0$ as volume remains constant.

$$\therefore Q_{1-2}=0+1885=1885 \text{ kJ}$$

11. $\mathbf{1.5}$ kg of Nitrogen contained in a cylinder at pressure $\mathbf{6bar}$ and temperature $\mathbf{300\ K}$ expands three times its original volume. Assuming the expansion process to be isobaric, make calculations for: (i) initial volume, (ii) final temperature, (iii) work done by gas, (iv) heat added and (v) change in internal energy.

For Nitrogen: $c_p=1.05$ kJ/kgK and $R=295$ J/kgK.

Solution: The initial volume of the gas can be worked out by invoking characteristic gas equation, i.e.,

$$V_1=\frac{mRT_1}{p_1}=\frac{1.5\times 295\times 300}{6\times 10^5}=0.221 \text{ m}^3$$

(ii) Final volume, $V_2=3V_1=3\times 0.221=0.663 \text{ m}^3$

For a constant pressure process, Charle's law applies, i.e, $\frac{V_1}{T_1}=\frac{V_2}{T_2}$

$\therefore$ Final temperature, $T_2=\frac{V_2}{V_1}T_1=3\times 300=900$ K

(iii)

$$\text{Work done, } W_{1-2} \begin{aligned}[t] &=p(V_2-V_1)=6\times 10^5[3\times 0.663-0.221] \\ &=2.652\times 10^5 \text{Nm(or J)}=265.2 \text{ kJ}\end{aligned}$$

(iv) Heat added, $Q_{1-2}=mc_p(T_2-T_1)=1.5\times 1.05(900-300)=945$ kJ

(v) From non-flow energy equation

$$dU=Q_{1-2}-W_{1-2}=945-265.2=679.8 \text{ kJ (increase)}$$

12. It is desired to compress 10 kg of gas from $\mathbf{1.5\ m^3}$ to $\mathbf{0.3\ m^3}$ at a constant pressure of 15 bar. During this compression process, the temperature rises from $\mathbf{20^\circ C}$ to $\mathbf{150^\circ C}$ and the increase in internal energy is $\mathbf{3250\ kJ}$. Calculate the work done, heat interaction and change in enthalpy during the process. Also, workout the average value of specific heat at constant pressure

Solution:

Work done

$$\begin{aligned} W_{1-2} &=p(V_2-V_1) \\ &=15\times 10^5(0.3-1.5)=-18\times 10^5 \text{Nm(or J)}=-1800 \text{ kJ}\end{aligned}$$

The negative sign indicates that the work must be done on the system.

From non-flow energy equation, $\delta Q=\delta W+dU$

$$\begin{aligned} Q_{1-2} &=W_{1-2}+(U_2-U_1) \\ &=-1800+3250=1450 \text{ kJ}\end{aligned}$$

Since it is positive, heat has been supplied to the system.

For a constant pressure process, the change in enthalpy equals the heat supplied.

$$\therefore dH=Q_{1-2}=1450 \text{ kJ}$$

For unit mass of gas, $dh=\frac{1450}{10}=145$ kJ/kg

Average value of specific heat at constant pressure,

$$c_r=\frac{dht}{dT}=\frac{145}{(150-20)}=1.12 \text{ kJ/kgK}$$

13. At $\mathbf{100kN/m^2}$ pressure and $\mathbf{288\ K}$ temperature, the mass density of certain gas is $\mathbf{1.875\ kg/m^2}$. Under constant pressure conditions, $\mathbf{1.2}$ kg of this gas requires $\mathbf{300}$ kJ of heat to raise its temperature from $\mathbf{15^\circ C}$ to $\mathbf{300^\circ C}$. Make calculations for: (i) characteristic gas constant, (ii) specific heat capacity at constant pressure and at constant volume, (iii) change of internal energy and work interaction.

Solution: Applying characteristic gas equation, $pV=mRT$, we have.

$$R=\frac{V}{m}\times\frac{p}{T}=\frac{p}{\rho T}$$

$$R=\frac{100\times 10^3}{1.875\times 288}=185.2\ \mathrm{J/kgK}$$

(ii) At constant pressure process: $Q=mc_p dT$

$$300=1.2\times c_p\times(300-15);\ c_p=0.8772\ \mathrm{kJ/kgK}$$

Using the relation $c_p-c_v=R$, we get

$$c_v=c_p-R=0.8772-0.1852=0.6920\ \mathrm{kJ/kgK}$$

(iii) Change in internal energy, $dU=mc_v dT$

$$=1.2\times 0.692(300-15)=236.66\ \mathrm{kJ}$$

(iv) From the non-flow energy equation: $\delta \mathrm{Q}=\delta W+dU$

∴ Work interation $W_{12}=Q_{12}-dU=300-236.66=63.34$ kJ

The positive sign implies that work is done by the gas on the surroundings.

14. A vertical piston cylinder assembly that initially has a volume of $0.1\ \mathrm{m}^3$ is filled with 0.1 kg of nitrogen gas. This is weighted so that the pressure of nitrogen is always maintained at 1.15 bar. Heat transfer is allowed to take place until the volume is reduced to 75 per cent of the initial volume. Determine:

(a) initial and final temperatures of nitrogen

(b) magnitude and direction of heat transfer.

Assume the process to be quasi static and take $c_v=0.745$ kJ/kgK for nitrogen.

Solution: For nitrogen (molecular mass = 28), the gas constant is

$$R=\frac{R_{\text{mol}}}{M}=\frac{8314}{28}=296.93\ \mathrm{J/hgK}$$

From ideal gas equation $pV=mRT$

$$T_1=\frac{p_1V_1}{mR}=\frac{1.15\times 10^5\times 0.1}{0.1\times 296.93}=387.3\ \mathrm{K}$$

For a constant pressure process, Charle's law applies, i.e., $\frac{V_1}{T_1}=\frac{V_2}{T_2}$

∴ Final temperature $T_2=\frac{V_2}{V_1}\times T_1=0.75\times 387.3=290.47$ K

(b)

$$\text{Work done } W_{1-2}\begin{aligned}[t]&=p(V_2-V_1)=1.15\times 10^5(0.75\times 0.1-0.1)\\&=-2875\mathrm{Nm}(\text{ or J})=-2.875\ \mathrm{kJ}\end{aligned}$$

Change in internal energy, $dU=mc_v(T_2-T_1)$

$$=0.1\times 0.745(290.47-387.3)=-7.214\ \mathrm{kJ}$$

From the non-flow energy equation, $\delta Q=\delta W+dU$, we have

Heat interaction $Q_{1-2}=-2.875-7.214=-10.089$ kJ

The negative sign shows that the heat is rejected by the system.

15. **A closed system consists of a fluid inside a cylinder fitted with a frictionless piston. When the fluid was stirred by a paddle wheel, 120 kJ of mechanical work was supplied along with 40 kJ of energy in the form of heat. At the same time, the piston moved in such a way that pressure remained constant at 200kPa and the volume changed from 2 m^3 to 4 m^3. Make calculations for the change in internal energy and enthalpy of the fluid system**.

Solution: Work done by the fluid on the piston,

$$W_{1-2}\begin{aligned}[t]&=p(V_2-V_1)=200\times 10^3(4-2)\\&=4\times 10^5\mathrm{Nm}(\text{ or J})=400\ \mathrm{kJ}\end{aligned}$$

Work done on fluid by paddle wheel $=-120$ kJ

∴ Net work done by the fluid $=400-120=280$ kJ

From non-flow energy equation, $\delta Q=\delta W+dU$.

Change in internal energy,

$$dU = Q_{1-2} - W_{1-2}$$
$$= 40 - 280 = -240 \text{ kJ}$$

Change in enthalpy equals the sum of change in internal energy and the change in pressure volume product. That is.

$$dH = (U_2 - U_1) + (p_2V_2 - p_1V_1)$$
$$= (U_2 - U_1) + p(V_2 - V_1) \; (\because p_1 = p_2 = p)$$
$$= -200 + 200 \times 10^3(4 - 2) = -200 \text{ kJ}$$

16. 100 litres of hydrogen gas at 300 K temperature and 5 bar pressure is contained in a cylinder fitted with a frictionless piston. The piston carries some dead weight, can move freely and its upper part is exposed to atmospheric pressure. Identify the process.
There is input of paddle work to the gas and its temperature rises to 350 K. Evaluate (i) work done, (ii) change in internal energy and enthalpy, and (iii) paddlé work input. Take $c_p = 14.25$ kJ/kgK and $c_v = 10.1$ kJ/kgK.

Solution: The dead weight exerts a constant force on the piston, and as such the situation corresponds to displacement work under isobaric conditions.

Gas constant,

$$R = c_p - c_v = 14.25 - 10.10$$
$$= 4.15 \text{ kJ/kgK} = 4150 \text{ J/kgK}$$

As the pressure remains constant, Charles law applies and therefore,

$$V_2 = \frac{T_2}{T_1}V_1 = \frac{350}{300} \times (100 \times 10^{-3}) = 0.1167 \text{ m}^3$$

Mass of hydrogen, $m = \frac{p_1V_1}{RT_1} = \frac{5\times10^5\times0.1}{4150\times300} = 0.0402$ kg

(i)

Displacement work, $W_{1-2} = \int pdV = p(V_2 - V_1)$

$$= 5 \times 10^5(0.1167 - 0.1) = 8350\text{Nm(or J)} = 8.35 \text{ kJ}$$

(ii) Change in internal energy, $dU = mc_v(T_2 - T_1)$

$$= 0.0402 \times 10.1(350 - 300) = 20.30 \text{ kJ}$$

Change in enthalpy, $dH = mc_p(T_2 - T_1)$

$$= 0.0402 \times 14.25(350 - 300) = 28.64 \text{ kJ}$$

(iii)

Paddle work input $= W_{1-2} + dU$

$$= 8.35 + 20.30 = 28.65 \text{ kJ}$$

17. A fluid expands reversibly behind a piston from initial conditions of pressure 600kPa and volume 0.03 m³ to a final volume of 0.09 m³. Presuming isothermal conditions, estimate the work done, change in internal energy, and heat supplied or rejected by the fluid system.

Solution: For an isothermal process, the work done during expansion follows from the relation:

$$W_{1-2} = p_1V_1\log_e\frac{V_2}{V_1}$$
$$= (600 \times 10^3) \times 0.03 \times \log_e\frac{0.09}{0.03} = 19775\text{Nm (or J)}$$

(b) Since there is no change in temperature during an isothermal process, change in internal energy of the system equals zero.

(c) From non-flow energy equation.

$$\delta Q = \delta W + dU$$
$$\delta Q = 19775 + 0; \delta Q = 19775 \text{ J} = \mathbf{19.775}\text{kJ}$$

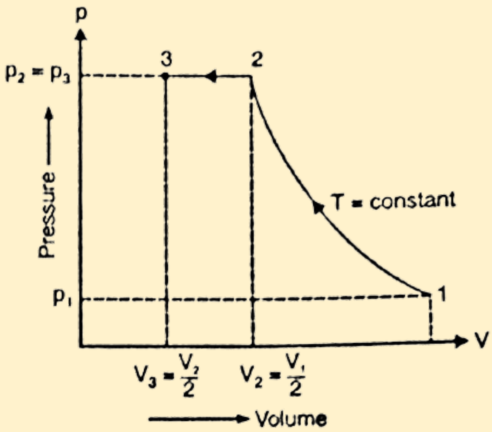
The positive sign indicates that heat is given to the system.

18. Air initially at 60kPa pressure, 800 K temperature and occupying a volume of $0.1\ \mathrm{m}^3$ is compressed isothermally until the volume is halved and subsequently it goes further compression at constant pressure till the volume is halved again. Sketch the process on $p - V$ plot and make calculations for the total work done and total heat interaction for the two processes. Assume ideal gas behaviour for air and take $c_p = 1.005$ kJ/kgK.

Solution: $V_1 = 0.1\ \mathrm{m}^3; V_2 = 0.05\ \mathrm{m}^3$ and $V_3 = 0.025\ \mathrm{m}^3$

Isothermal process 1-2

$$p_2 = p_1 \frac{V_1}{V_2} = 0.6 \times \frac{0.1}{0.05} = 1.2\text{bar}$$



Constant pressure process

$$T_3 = T_2 \frac{V_3}{V_2} = 800 \times \frac{1}{2} = \quad 400\ \mathrm{K}$$

$$(T_2 = T_1 = 800\ \mathrm{K})$$

From the characteristic gas equation,

$$m = \frac{p_1 V_1}{R T_1} = \frac{0.6 \times 10^5 \times 0.1}{287 \times 800} = 0.0261\ \mathrm{kg}$$

Work interactions.

$$W_{1-2} \quad = p_1 V_1 \log_e \frac{V_2}{V_1} = 0.6 \times 10^5 \times 0.1 \times \log_e \left(\frac{1}{2}\right)$$

$$= -4158.9\mathrm{Nm}(\text{ or J}) \ = -4.159\ \mathrm{kJ}$$

$$W_{2-3} \quad = p_2 (V_3 - V_2) = 1.2 \times 10^6 (0.025 - 0.05)$$

$$= -3000\mathrm{Nm}(\text{ or J}) \ = -3\ \mathrm{kJ}$$

Total work done, $W_{1-3} = W_{1-2} + W_{2-3} = -4.159 - 3 = -7.159$ kJ

Heat interactions.

$$Q_{1-2} = W_{1-2} + dU = W_{1-2} + (U_2 - U_1)$$
$$= W_{1-2} + m c_v (T_2 - T_1)$$
$$= -4.159\ \mathrm{kJ}$$
$$(dU = 0 \text{ as } T_2 = T_1)$$
$$Q_{2-3} = W_{2-3} + (U_3 - U_2) = W_{2-3} + m c_\mathrm{v} (T_3 - T_2)$$
$$= W_{2-3} + m c_\mathrm{v} (T_3 - T_1)$$

From the relation $c_p - c_p = R$, we have

Then:

$$c_v = c_p - R = 1.005 - 0.287 = 0.718\ \mathrm{kJ/KgK}$$
$$Q_{2-3} \quad = -3 + 0.0261 \times 0.718(400 - 800) = -10.496\ \mathrm{kJ}$$
$$\therefore\ Q_{1-3} \quad = Q_{1-2} + Q_{2-3} = -4.159 + (-10.496) = -14.655\ \mathrm{kJ}$$

19. 2 kg of an ideal gas is compressed adiabatically from pressure 100kPa and temperature 220 K to a final pressure of 400kPa. Make calculations for: (a) initial volume, (b) final volume and temperature (c) work performed, (d) heat added to or subtracted from the system, and (e) change in internal energy. It may be presumed that for the given ideal gas $c_p = 1$ kJ/kgK and $c_p = 0.707$ kJ/kgK.

Solution:

$$c_p - c_v \quad = R$$
$$\therefore R \quad = (1 - 0.707) = 0.293\ \mathrm{kJ/kgK} = 293\ \mathrm{J/kgK}$$

(a) Invoking characteristic gas equation,

$$V_1 = \frac{m R T_1}{p_1} = \frac{2 \times 293 \times 220}{100 \times 10^3} = 1.29\ \mathrm{m}^3$$

(b) Adiabatic index, $y = \frac{c_p}{c_v} = \frac{1}{0.707} = 1.414$

For an adiabatic process: $p_1V_1^\gamma = p_2V_2^\gamma$

$\therefore$ Final yolume, $V_2 = \left(\frac{p_1}{p_2}\right)^{1/\gamma} \times V_1 = \left(\frac{100}{400}\right)^{1/2.44} \times 1.29 = 0.484 \text{ m}^3$

Again, from characteristic gas equation,

$$\text{Final temperature, } T_2 = \frac{p_2V_2}{mR} = \frac{400 \times 10^3 \times 0.484}{2 \times 293} = 330.4 \text{ K}$$

(c)

$$\begin{aligned}\text{Work performed} &= \frac{p_1V_1 - p_2V_2}{\gamma - 1} \\ &= \frac{100 \times 10^3 \times 1.29 - 400 \times 10^3 \times 0.484}{1.414 - 1} \\ &= -156.04 \times 10^3 \text{Nm(or J)} = -156.04 \text{ kJ}\end{aligned}$$

The negative sign indicates that work was done on the gas system.

(d) Adiabatic compression implies that there is no heat transfer to or from the system.

(e) From non-flow energy equation,

$$dU = \delta Q - \delta W = 0 - (-156.04) = 156.04 \text{ kJ}$$

The positive value of dU implies that internal energy of the gas has increased.

20. A certain perfect gas is contained in a perfectly insulated piston cylinder arrangement at 100 kPa pressure and 290 K temperature. When the gas undergoes a non-flow reversible compression process to 500kPa, its temperature rises to 350 K. If the work done on the gas during the compression process is 50 kJ/kg, evaluate the adiabatic exponent, specific heat at constant volume, the characteristic gas constant and molecular mass of the gas.

Solution: Perfect insulation implies no transfer of heat and hence the process is adiabatic.

From the relation, $\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$

$$\frac{\gamma - 1}{\gamma} = \frac{\log_e\left(\frac{T_2}{T_1}\right)}{\log_e\left(\frac{p_2}{p_1}\right)} = \frac{\log_e\left(\frac{350}{290}\right)}{\log_e\left(\frac{500}{100}\right)} = \frac{0.1880}{1.6094} = 0.1168$$

$$\gamma - 1 = 0.1168\gamma; \quad \therefore = \frac{1}{1 - 0.1168} = 1.132$$

From non-flow energy equation,

$$\begin{aligned}Q_{1-2} &= W_{1-2} + dU \\ 0 &= -50 + dU \quad \therefore dU = 50 \text{ kJ/kg}\end{aligned}$$

Now,

$$dU = mc_v(T_2 - T_1)$$

$$50 = 1 \times c_v \times (350 - 290); \; c_v = 0.833 \text{ kJ/kgK}$$

Characteristic gas constant, $R = c_p(\gamma - 1)$

$$= 0.833 \times (1.132 - 1) = 0.1099 \text{ kJ/kgK}$$

$$\text{Molecular mass, } M = \frac{8.314}{0.1099} = 75.65$$

21. 0.5 kg of an ideal gas expands adiabatically until its pressure is halved. During expansion, the gas does 30 kJ of external work and its temperature falls from 500 K to 410 K. Make calculations for the adiabatic exponent and the characteristic gas constant.

Solution: From the relation.

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \text{ and noting that } p_2 = \frac{p_1}{2}$$

$$\frac{\gamma - 1}{\gamma} = \frac{\log_e \frac{T_2}{T_1}}{\log_e \frac{p_2}{p_1}} = \frac{\log_e \frac{410}{500}}{\log_e \frac{1}{2}}$$

$$= \frac{-0.1985}{-0.693} = 0.2864$$

$$\text{or } \gamma = 1 = 0.2864\gamma; \ \therefore \gamma = \frac{1}{1 - 0.2864} = 1.401$$

For adiabatic expansion process $\delta Q - 0$ and therefore, from non-flow energy equation,

$$\delta Q = 8W + dU$$

$$W_{1-2} = -dU = -mc_v(T_2 - T_1)$$

$$30 = -0.5 \times c_v(410 - 500); \ \therefore c_v = 0.667 \text{ kJ/kgK}$$

$$\text{Gas constant, } R = c_v(\gamma - 1) = 0.667 \times (1.4 - 1)$$

$$= 0.267 \text{ kJ/kgK} = 267 \text{ J/kgK}$$

22. An insulated cylinder of 0.4 m diameter and 0.8 m length contains 10 kg of oxygen. paddle work is done on the gas to increase its pressure from 3 bar to 6 bar. Determine the change in internal energy, work done on the gas and the change in enthalpy.

$c_p = 0.91$ kJ/kgK and $c_v = 0.64$ kJ/kgK.

Solution : Volume of cylinder, $V = \frac{\pi}{4} d^2 l = \frac{\pi}{4}(0.4)^2 \times 0.8 = 0.10 \text{ m}^3$

Gas constant, $R = c_p - c_v = 0.91 - 0.64 = 0.27$ kJ/kgK $= 270$ J/kgK

Initial State: $p_1 = 3$ bar; $V_1 = 0.1 \text{ m}^3$ and $m = 10$ kg

Then from characteristic gas equation,

$$T_1 = \frac{p_1 V_1}{mR} = \frac{3 \times 10^5 \times 0.1}{10 \times 270} = 11.11 \text{ K}$$

Final state: $p_2 = 6$ bar; $V_2 = 0.1 \text{ m}^3$ and $m = 10$Kg

$$T_2 = \frac{p_2 V_2}{mR} = \frac{6 \times 10^5 \times 0.1}{10 \times 270} = 22.22 \text{ K}$$

(i) Change in internal energy, $dU = mc_v(T_2 - T_1)$

$$= 10 \times 0.64(22.22 - 11.11) = 71.10 \text{ kJ} = 71100 \text{ J}$$

(ii) Using non-flow energy equation, $\delta Q = \delta W + dU$

Here $\delta Q = 0$ as the cylinder is insulated.

$$\therefore \ \delta W = -dU = -71.10 \text{ kJ}$$

The negative sign indicates that work is done on the gas.

(iii) Enthalpy represents the sum of internal energy and pressure-volume product. That is

$$H = U + pV$$

Change in enthalpy,

$$dH = dU + d(pV)$$

$$= dU + (p_2V_2 - p_1V_1)$$

$$= 71100 + (6 \times 10^5 \times 0.1 - 3 \times 10^5 \times 0.1)$$

$$= 71100 + 30000 = 101100 \text{ J}$$

23. 3 kg of air kept at an absolute pressure of 100 kPa and temperature of 300 K is compressed polytropically until the pressure and temperature become 1500 kPa and 500 K respectively. Evaluate the polytropic exponent, the final volume, the work of compression and the heat interaction. Take gas constant $R = 287$ J/kgK.

Solution: From the relation $\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$

$$\frac{n-1}{n}=\frac{\log_e \frac{T_2}{T_1}}{\log_e \frac{p_2}{p_1}}=\frac{\log_e \frac{500}{300}}{\log_e \frac{1500}{100}}=\frac{0.5108}{2.708}=0.1886$$

or $n-1=0.1886n; n=\frac{1}{1-0.1886}=1.23$

Initial volume, $V_1=\frac{mRT_1}{p_1}=\frac{3\times287\times300}{100\times10^3}=2.583\ \mathrm{m}^3$.

Final volume, $V_2=\frac{mRT_2}{p_2}=\frac{3\times287\times500}{1500\times10^3}=0.287\ \mathrm{m}^3$

Work of compression, $W_{1-2}=\frac{p_1V_1-p_2V_2}{n-1}=\frac{mR(T_1-T_2)}{n-1}$

$$=\frac{3\times287\times(300-500)}{1.23-1}=-748696\mathrm{Nm}(\text{ or J})\simeq-748.7\ \mathrm{kJ}$$

The negative sign indicates that work must be done on the gas.

Now, $c_v=\frac{R}{\gamma-1}=\frac{287}{1.4-1}=7175\ \mathrm{J/kgK}=0.7175\ \mathrm{kJ/kgK}$

$$dU=mc_v(T_2-T_1)=3\times0.7175\times(500-300)=430.5\ \mathrm{kJ}$$

From non-flow energy equation,

$$Q_{1-2}=W_{1-2}+dU=-748.7+430.5=-318.2\ \mathrm{kJ}$$

The negative sign indicates that heat is rejected by the gas.

24. 2 kg of an ideal gas occupies a volume of $0.3\ \mathrm{m}^3$ at 10bar pressure and 500 K temperature. When this gas expands polytropically $\left(pV^{1.2}=C\right)$ the internal energy decreases by 300 kJ. Presuming adiabatic exponent $\gamma=1.4$, determine:

(a) specific gas constant,

(b) final temperature, pressure, and volume of gas, and

(c) heat and work interactions across the system boundary.

Solution : From characteristic gas equation, $pV=mRT$

Gas constant, $R=\frac{pV}{mT}=\frac{10\times10^5\times0.3}{2\times500}=300\ \mathrm{J/kgK}$

(b) From the relation, $\left(c_p-c_v\right)=R$ and $\frac{c_p}{c_v}=\gamma$

$$c_v=\frac{R}{\gamma-1}=\frac{300}{1.4-1}=750\ \mathrm{J/kgK}=0.75\ \mathrm{kJ/kgK}$$

Change in internal energy, $dU=U_2-U_1=mc_v(T_2-T_1)$

The decrease in internal energy implies that dU is negative.

$$-300=2\times0.75(T_2-500);\ \therefore T_2=300\ \mathrm{K}$$

From the relations:

$$\frac{T_2}{T_1}=\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}=\left(\frac{V_1}{V_2}\right)^{n-1}$$

$$p_2=p_1\left(\frac{T_2}{T_1}\right)^{n-1}=10\times\left(\frac{300}{500}\right)^{\frac{1.2}{1.2-1}}=0.467\mathrm{bar}$$

$$V_2=V_1\left(\frac{p_1}{p_2}\right)^{1/n}=\left(\frac{10}{0.467}\right)^{1/1.2}\times0.3=3.85\ \mathrm{m}^3$$

(c)

Work done,

$$W_{1-2}=\frac{p_1V_1-p_2V_2}{n-1}=\frac{mR(T_1-T_2)}{n-1}$$

$$=\frac{2\times300\times(500-300)}{1.2-1}=600\times10^3\ \mathrm{J}=600\ \mathrm{kJ}$$

From non-flow energy equation,

$$Q_{1-2}=W_{1-2}+(U_2-U_1)=600+(-300)=300\ \mathrm{kJ}$$

25. One kg of air at a pressure of 7bar and a temperature of 360 K undergoes a reversible polytropic process which may be represented by $pV^{1.1}$ = constant. If the final pressure is 1.4bar, evaluate (a) final specific volume and temperature, (b) work done and heat transfer. How the work and heat interactions would be affected if the process is irreversible and 15 kJ of work is lost due to internal friction?

Take $R = 287$ J/kgK and $\gamma = 1.4$

Solution: Assuming air to be perfect gas, $p_1V_1 = mRT_1$

$$\therefore V_1 = \frac{1 \times 287 \times 360}{7 \times 10^5} = 0.1476\ \mathrm{m^3/kg}$$

From the given polytropic law,

$$V_2 = V_1\left(\frac{p_1}{p_2}\right)^{1/n} = 0.1476 \times \left(\frac{7}{1.4}\right)^{\frac{1}{1.1}} = 0.637\ \mathrm{m^3/kg}$$

$$T_2 = \frac{p_2V_2}{mR} = \frac{1.4 \times 10^5 \times 0.637}{1 \times 287} = 310.73\ \mathrm{K}$$

(b) Work done, $W_{1-2} = \frac{p_1V_1 - p_2V_2}{n-1} = \frac{mR(T_1 - T_2)}{n-1}$

$$= \frac{1 \times 287 \times (360 - 310.73)}{1.1 - 1} = 141405\mathrm{Nm(\ or\ J)} = 141.405\ \mathrm{kJ}$$

Now, $c_v = \frac{R}{\gamma-1} = \frac{287}{1.4-1} = 717.5$ J/kgK $= 0.7175$ kJ/kgK

Change in internal energy, $dU = mc_v(T_2 - T_1)$

$$= 1 \times 0.7175(310.73 - 360) = -35.35\ \mathrm{kJ/kg}$$

From non-flow energy equation, $\delta Q = \delta W + dU$

$$Q_{1-2} = W_{1-2} + dU = 141.405 + (-35.35) = 106.055\ \mathrm{kJ/kg}$$

(c) The end states are the same, but the process is irreversible with work dissipation in friction.
$\therefore dU = -35.35$ kJ/kg since the end states are the same and internal energy is the property or point function.

Net work done = displacement work - work dissipated in friction.

$$= 141.405 - 15 = 126.405\ \mathrm{kJ}$$

and net heat transfer is given by.

$$Q_{net} = dU + (\text{ displacement work } - \text{ work dissipated in friction })$$
$$= -35.35 + (141.405 - 15) = 91.055\ \mathrm{kJ/kg}$$

26. Air flows through a duct. The pressure and temperature at station 1 are $p_1 = 0.7$ atmosphere and $T_1 = 30°$C, respectively. At a second station, the pressure is 0.5 atm. Calculate the temperature and density at the second station. Assume the flow to be isentropic.

Solution.

$$p_1 = 0.7\ \mathrm{atm} = 0.7 \times 1.0133 \times 10^5\ \mathrm{N/m^2}$$

Because 1 atm $= 1.0133 \times 10^5$ N/m^2 and $T_1 = 30°$C, i.e. $T_1 = 30 + 273 = 303$ K.

Using the state equation, we get.

$$\boxed{\rho_1 = \frac{p_1}{RT_1}} = \frac{0.7 \times 1.0133 \times 10^5}{287 \times 303} = 0.8157\ \mathrm{kg/m^3}$$

$$\boxed{\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\gamma/(\gamma-1)}}$$

Therefore,

$$\frac{T_2}{T_1} = \left(\frac{0.5}{0.7}\right)^{(\gamma-1)/\gamma} = \left(\frac{0.5}{0.7}\right)^{0.4/1.4} = 0.908$$

$$T_2 = (0.908)(303) = 275.12\ \mathrm{K}$$

$$\boxed{\frac{\rho_2}{\rho_1} = \left(\frac{T_2}{T_1}\right)^{1/(\gamma-1)}} = (0.908)^{1/0.4} = 0.786$$

Thus,

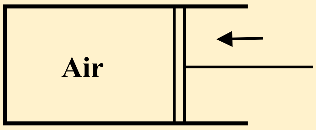
$$\rho_2 = (0.786)(0.8157) = 0.641 \text{ kg/m}^3$$

Hence, the temperature and density at the second station are.

$$T_2 = 2.12°\text{C}, \ \rho_2 = 0.641 \text{ kg/m}^3$$

Q 27 Statement for Linked Answer Questions 27 and 28:

A piston compresses 1 kg of air inside a cylinder as shown.



The rate at which the piston does work on the air is $3000\ W$. At the same time, heat is being lost through the walls of the cylinder at a rate of $847.5\ W$.

Q27. After 10 seconds, the change in specific internal energy of the air is

(A) 21,525 J/kg (B) —21,525 J/kg

(C) 30,000 J/kg (D) — 8,475 J/kg

Q 28 Given that the specific heats of air at constant pressure and volume are $C_p = 1004.5\ J/kg\text{-}K$ and $C_v = 717.5\ J/kg\text{-}K$ respectively, the corresponding change in the temperature of the air is

(A) 21.4 K (B) —21.4 K

(C) 30 K (D) —30 K

Solution q27:

Rate of work$= -3000\ W$

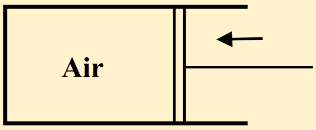
Work done for 10 seconds

$$= -3000 \times 10 = -30000\ J$$

Heat lost $= -847.5\ W$

Total heat lost in 10 seconds

$$= -847.5 \times 10 = -8475\ J$$



Applying first law of thermodynamics

$$dq = du + dw$$

For unit mass

$$-8475 = du - 3000$$

$$du = 21525\, J/kg$$

Solution q28:

Change in internal energy is given by-

$$du = c_v dT$$

$$21525 = 717.5\, dT$$

$$dT = 30\, K$$