Kinematic Synthesis and Optimization of Continuous Passive Motion Mechanisms for Knee

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Abstract*—Continuous Passive Motion (CPM) has been totally accepted as a physical therapy in the recent decades. There have been a number of CPM systems from the most complicated mechatronic systems to the least, developed for this purpose. Despite all improvements in manufacturing such machines, an optimum design needing an acceptably low power for efficient functioning has not yet been targeted. The aim of this paper is to kinematically synthesize a linkage-based mechanism for knee CPM with three-accuracy-point approach. An optimization problem is then defined to find the mechanism with the least amount of power generated by the driver joint and force generated at knee joint.

***Keywords*—Knee CPM; Optimization; Modeling and simulation; Rehabilitation**

I. Introduction

In order to prevent the development of stiffness of a joint following a surgery or injury, the concept of applying Continuous Passive Motion (CPM) arises. Salter, who had invented this concept, believed that joint degeneration caused by immobilization of a joint can be avoided by applying continuous motion to the joint and the motion should also be passive since the patient may not be able to move his\her joint persistently [1].

According to a study done by Montgomery, patients who had been subjected to CPM after a knee arthroplasty appeared to have better initial improvements and less knee swelling compared to those subjected to active physical therapy (APT) [2].

In order to fulfil the purpose of a more efficient and convenient rehabilitation, CPM machines have been introduced to provide such motions for joints, or in some cases, to strengthen the muscles which have been weakened due to different causes. Multiple Degrees of Freedom (DoF) systems have been proposed and developed for physical therapy of joints.

For instance, a multi-DoF wire-driven mechanism has been suggested by Homma et al. for hip, knee and ankle rehabilitation [3]. This mechanism is, however, mentioned to be a human-friendly system, it appears to have its complexity when it comes experiment and analysis.

A prototype robot-aided rehabilitation system for clinical neurological applications have been tested by Krebs. This robot, named MIT-MANUS, assists the upper extremities by providing multi-DoF rehabilitation motion [4].

The Therapeutic Exercise Machine (TEM), a multi-DoF system, has been developed for rehabilitation purposes of lower extremities, the hip and knee joints [5].

Other CPM machines to be mentioned are GPI controlled CPM machine able to follow special trajectory have been proposed by [6], an intelligently controlled 1-DoF rehabilitation robotic system proposed by two stages of teaching and therapy [7].

In [8] a fully mechanical knee rehabilitation system with crank rocker mechanism have been proposed which the therapist can control the range of the motion by setting the position of the actuator.

The use of links in the aforementioned mechanisms allows a more accurate modeling and kinematic analysis of the system. In addition, rigid links are more stable during motion; therefore, possible inconvenience or damage to the patient's joint are avoided.

Specifically focusing on CPM for knee joint, there are several mechanisms which can produce partial flexion or extension of knee. These mechanisms may consist of two four-bar linkages, two slider-crank mechanisms or a combination of both. In this essay, two of the mechanisms which can produce the desired motion for knee joint are analyzed kinematically and simulated by MATLAB SimMechanics. An optimization problem is defined based on the force at knee joint and power generated by the actuator. A comparison is then implemented between the designed mechanisms with regard to force and power.

II. Design Requirements for Kinematic Synthesis

Two mechanisms are proposed for knee CPM machines: 1. Combination of two four-bar mechanisms, 2. Combination of a four-bar mechanism and a slider-rocker.

The first mechanism consists of two four-bar linkages, one of which is the driver and the other one includes links on which the thigh and lower leg of the subject are going to be embedded. As it is shown in Fig. 1, the first four-bar linkage consists of link 1, 2 and a part of link 3.

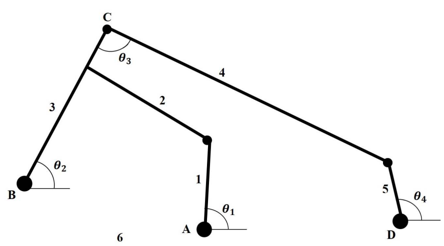


Fig. 1. Schematic of the 1st mechanism

In this mechanism, joint A is the driver; therefore, the range of motion of link 1 is known to the designer. This joint may be the joint where a motor should be located. Range of motion of link 3, the thigh, should also be known to the designer based on how the rehabilitation process is going to be done for any special case. With these parameters known, designing the first four-bar linkage is possible.

In the second four-bar linkage, which includes link 3, 4 and 5, range of θ_2 is known as explained before and range of θ_3, range of motion of knee joint, should also be known according to the status of the patient. Link 3 is where the thigh should be positioned; therefore, its length is also known. Having these parameters known, designing the second mechanism is also feasible. Link 6 is the base link of the mechanism. The known and unknown parameters of the problem are specifically shown in Table 1.

The second mechanism which is able to follow the proper motion, is a combination of one four-bar linkage and one slider-rocker mechanism. The slider-rocker mechanism is the driver and the four-bar linkage is where the thigh and the lower leg of the case is positioned. As it is indicated in Fig. 2, the slider-rocker mechanism, the driver mechanism, consists of Link 2 and 3.

Range of motion of the slider, δS, and range of θ_1, range of motion of thigh, should be known to the designer. Therefore, by knowing the initial position of the slider, S, the mechanism can be designed.

The four-bar linkage mechanism includes link 3, 4 and 5. This mechanism is designed just the same as the 2nd crank-rocker explained in the previous mechanism. Link 6 is the base. The known and unknown parameters related to this mechanism are specifically shown in Table 2.

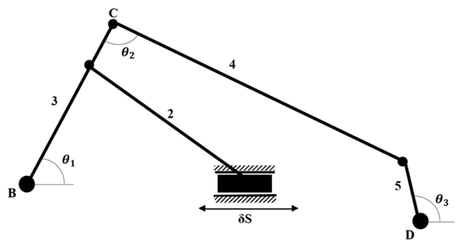


Fig. 2. Schematic of the 2nd mechanism

TABLE I. KNOWN AND UNKNOWN PARAMETERS OF TWO FOUR-BAR LINKAGES

Known	Unknown
1st Crank-Rocker Mechanism	
Range of motion of joint A (θ_1, driver)	Crank Length (Link 1)
Range of motion of thigh (θ_2)	Rocker Length (Link 3)
Angle of AB relative to horizontal axis	Coupler Length (Link 2)
Distance from A to B	Range of motion of other joints
2nd Crank-Rocker Mechanism	
Range of motion of thigh (θ_2)	Distance from B to D
Range of motion of knee joint (θ_3, Joint C)	Rocker Length (Link 5)
Angle of BD relative to horizontal axis	Coupler Length (Link 4)
Crank length (Link 3)	Range of motion of joint D

Thigh and the lower leg lengths are obtained from [9]. The essential data are provided in Table 3.

In order to design the mechanisms based on three-accuracy-point method, the accuracy points should be defined. To fulfil this purpose, the Chebyshev method is applied. [10]

The input range of motions of mechanisms is used as the interval 2h in this method. Using the schematic shown in Fig.3, the accuracy points are then defined as follows

$$a_1 = h\,(1 - \cos(\pi/6))$$
$$a_2 = h$$
$$a_3 = h\,(1 + \cos(\pi/6)) \quad (1)$$

wherein a_i (i = 1,2,3) is the accuracy point obtained according to the input range of motion. The accuracy points used in each mechanism are shown in Table 4.

TABLE II. KNOWN AND UNKNOWN PARAMETERS OF TWO FOUR-BAR LINKAGES

Known	**Unknown**
Slider-Rocker Mechanism	
Slider Course (δS, driver)	Slider offset from ground
Slider horizontal position with respect to joint B	Rocker Length (Link 2)
Range of motion of thigh (θ_2)	Coupler Length (Link 1)
Crank-Rocker Mechanism	
Range of motion of thigh (θ_2)	Distance from B to D
Range of motion of knee joint (θ_2, Joint C)	Rocker Length (Link 4)
Angle of BD relative to horizontal axis	Coupler Length (Link 3)
Crank length (Link 2)	Range of motion of joint D

TABLE III. SEGMENT WEIGHTS, LENGTHS, RADII OF GYRATION

Segment	**Males**	**Females**
Segment Weights (Kg)[1]		
Thigh	10.5	11.75
Lower Leg	4.75	5.35
Segment Lengths (cm)[2]		
Thigh	23.2	24.9
Lower Leg	24.7	25.7
Segment Radii of gyration[3] (Proximal)		
Thigh	54	53.5
Lower Leg	52.9	51.4

[1] Segment weights are expressed in total body weight.
[2] Segment lengths are expressed in total body height.
[3] Segmental radii of gyrations are expressed in segment length.

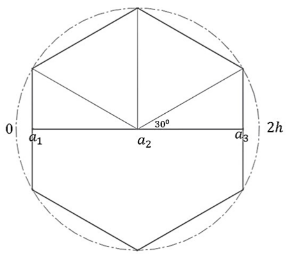


Fig. 3. Determination of three accuracy points with Chebyshev spacing

TABLE IV. ACCURACY POINTS CALCULATION.

1st Mechanism (Combination of two four-bar linkages)			
$\delta\theta_1 = +90°;\theta_{1,0} = 0°$	$\theta_{1,1} = 6°$	$\theta_{1,2} = 45°$	$\theta_{1,3} = 84°$
$\delta\theta_2 = +40°;\theta_{2,0} = 0°$	$\theta_{2,1} = 2.68°$	$\theta_{2,2} = 20°$	$\theta_{2,3} = 37.3°$
$\delta\theta_3 = -70°;\theta_{3,0} = 180°$	$\theta_{3,1} = 175.3°$	$\theta_{3,2} = 145°$	$\theta_{3,3} = 114.7°$
2nd Mechanism (Combination of a four-bar linkage and a Slider-rocker)			
$\delta S = 10cm; S_0 = 40cm$	$S_1 = 49.3cm$	$S_2 = 45cm$	$S_3 = 40.7cm$
$\delta\theta_1 = +90°;\theta_{1,0} = 0°$	$\theta_{1,1} = 6°$	$\theta_{1,2} = 45°$	$\theta_{1,3} = 84°$
$\delta\theta_2 = -70°;\theta_{2,0} = 180°$	$\theta_{2,1} = 2.68°$	$\theta_{2,2} = 20°$	$\theta_{2,3} = 37.3°$

A. Maintaining the Integrity of the Specifications

Both mechanisms are 1-DOF mechanisms each of which consists of two mechanisms. The kinematic synthesis of the mechanisms used in the suggested mechanisms for CPM machines can be achieved by three-accuracy-point method. [9]

In this method, Eq. (2) is formed leading to a system of equations with three unknowns based on the three-accuracy points. To solve this system, six constants, ω_i, are defined as in Eq. (3).

Solving the system of equation with the help of these constants, the parameters K_i will be obtained. Defining the K_i (i=1,2,3) as in Eq. (4), the unknown lengths of the four-bar linkages will be obtained.

$$K_1 \cos\varphi_i - K_2 \sin\psi_i + K_3 = \cos(\varphi_i - \psi_i) \qquad (2)$$

$$\begin{aligned}
\omega_1 &= \cos\varphi_1 - \cos\varphi_2 \\
\omega_2 &= \cos\psi_1 - \cos\psi_2 \\
\omega_3 &= \cos(\varphi_1 - \psi_1) - \cos(\varphi_2 - \psi_2) \\
\omega_4 &= \cos\varphi_1 - \cos\varphi_3 \\
\omega_5 &= \cos\psi_1 - \cos\psi_3 \\
\omega_6 &= \cos(\varphi_1 - \psi_1) - \cos(\varphi_3 - \psi_3)
\end{aligned} \qquad (3)$$

$$\begin{aligned}
K_1 &= \frac{l_4}{l_3} \\
K_2 &= \frac{l_4}{l_1} \\
K_3 &= \frac{{l_1}^2 - {l_2}^2 + {l_3}^2 + {l_4}^2}{2l_1 l_3}
\end{aligned} \qquad (4)$$

wherein l_1, l_2, l_3 and l_4 are the crank, coupler, rocker and base lengths, respectively.

This method is also applicable to the slider-rocker mechanism. The procedure is just the same as what has been explained. The constants are defined as follows

$$\begin{aligned}
\omega_1 &= s_1 \cos\varphi_1 - s_2 \cos\varphi_2 \\
\omega_2 &= \sin\varphi_1 - \sin\varphi_2 \\
\omega_3 &= {s_1}^2 - {s_2}^2 \\
\omega_4 &= s_1 \cos\varphi_1 - s_3 \cos\varphi_3 \\
\omega_5 &= \sin\varphi_1 - \sin\varphi_3 \\
\omega_6 &= {s_1}^2 - {s_3}^2
\end{aligned} \qquad (5)$$

For the second mechanism, the unknown lengths of the four-bar linkages will be obtained using the system of equations with three unknowns as follows

$$K_1 s_i \cos\varphi_i + K_2 \sin\varphi_i - K_3 = {s_i}^2 \quad (i = 1,2,3) \qquad (2)$$

Solving the system of equation, K_i is obtained. Then, the unknowns of the mechanisms can be calculated using the following equations

$$\begin{aligned}
K_1 &= 2l_1 \\
K_2 &= 2l_1 l_3 \\
K_3 &= {l_1}^2 - {l_2}^2 + {l_3}^2
\end{aligned} \qquad (7)$$

wherein l_1, l_2 and l_3 are rocker and coupler lengths and the slider offset from base link, respectively.

According to the initial conditions provided by the designer, the equations obtain a number of sets of lengths and angles for each mechanism.

III. INITIAL CONDITIONS, CONSTRAINTS AND OPTIMIZATION

In order to provide a number of different mechanisms, some initial conditions have been defined. For the 1st mechanism which is a combination of two four-bar linkages, the initial condition is considered to be position of joint A with respect to B for the first four-bar linkage. These angles vary in ranges shown in Table 5.

For the second mechanism which includes one four-bar linkage and one slider-rocker mechanism, the initial condition contains the angle of BD with respect to horizontal axis and the initial position of the slider. It should also be mentioned that in this mechanism link 2 is not a straight link as the other links are. It includes two links which make an angle with each other and form a solid link. Fig 4. These parameters vary in the range shown in Table 6.

While checking different initial conditions, each solution should be checked based on the constraints defined by the designer. Link 3 of the 1st mechanism should be equal to the thigh length; therefore, while designing the 1st four-bar linkage mechanism this constraint should be taken into account. For the 2nd mechanism which includes a slider, it should be mentioned that the links do not be of too large lengths.

After checking all the initial conditions, the driver power and the force on knee joint should be optimized in order to obtain the most proper mechanism for the case.

III. Simulation Results

Both mechanisms are simulated in MATLAB SimMechanics. The simulation yields the mechanisms which suggest a minimum of driver power and knee-joint force. The link lengths of the desired mechanisms are shown in Table 7. The motion of the optimized mechanisms is also available in Fig. 5-6. The simulated models are planar.

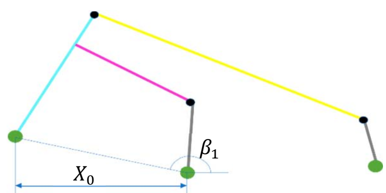
As it is obvious from the diagrams obtained from the simulation, in the 1st mechanism the minimum power and force on knee joint are 20.68 N.m/s and 235.46 N, and in the second mechanism 52.6 N.m/s and 1046 N, respectively. Fig. 6 and Fig. 7.

TABLE V. Initial Condition for the 1st Mechanism (Four-bar + Four-bar Linkage)

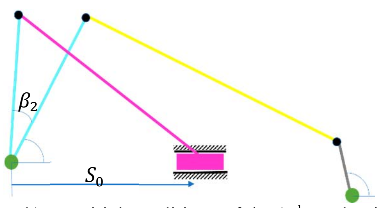
Angle	Range of variation
Angle of BA with respect to horizontal axis (β_1)	140-180°
Horizontal position of BA (X_0)	30-80 cm

TABLE VI. Initial Condition for the 2nd Mechanism (Slider-Rocker + Four-bar Linkage)

Angle	Range of variation
Angle of Link 2 (β_2)	0-40°
Initial position of the slider (S_0)	40-140 cm



a) Initial conditions of the 1st mechanism

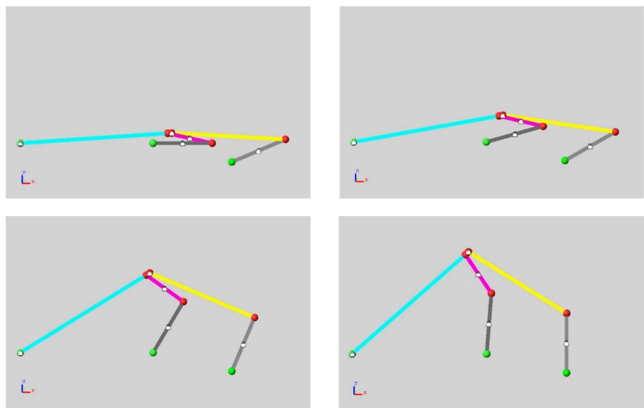


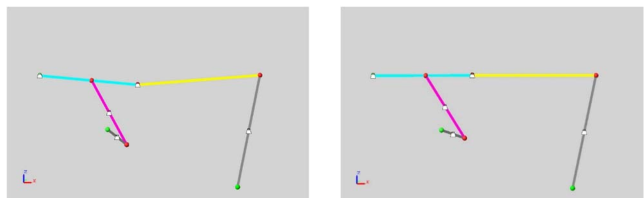
b) Initial conditions of the 2nd mechanism

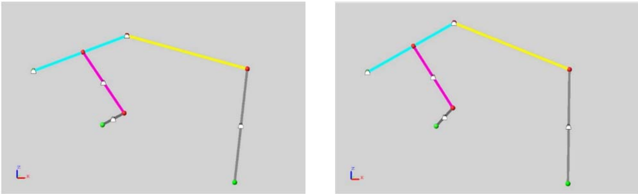
Fig. 4. Initial conditions

TABLE VII. Link Lengths

1st Mechanism (Combination of two four-bar linkages)			
Based on minimum knee force		***Based on minimum power***	
Link Number	Length (cm)	Link Number	Length (cm)
1	15	1	10
2	12	2	29
3	40 = 39 + 1	3	40 = 21 + 19
4	30	4	30
5	15	5	46
6	56	6	92
X_0	35	X_0	35
β_1	0°	β_1	-59°
2nd Mechanism (Combination of a four-bar linkage and a Slider-rocker)			
Based on minimum knee force		***Based on minimum power***	
Link Number	Length (cm)	Link Number	Length (cm)
1	169	1	67
2	40	2	40
3	50	3	39
4	46	4	16
5	91.9	5	68.3
S_0	-59	S_0	-22.3
β_2	40°	β_2	40°

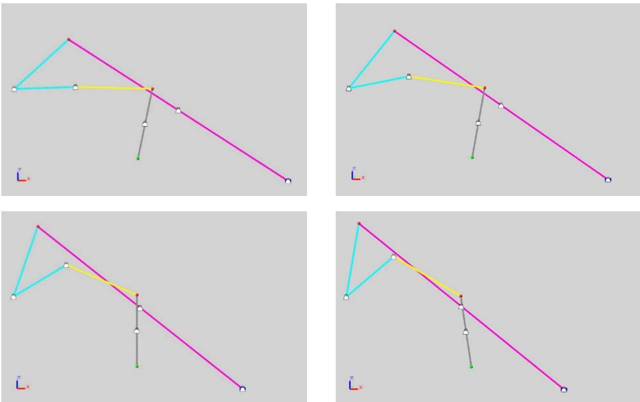

a) Based on minimum force of knee joint



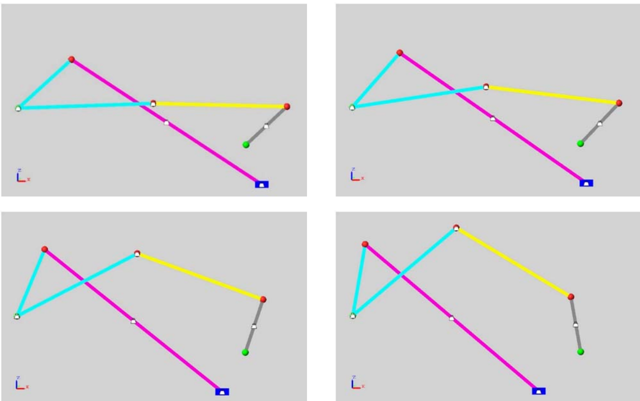


b) Based on minimum power of the driver

Fig. 5. Motion of the 1st mechanism

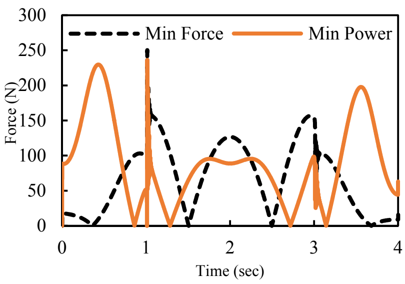


a) Based on minimum force of knee joint

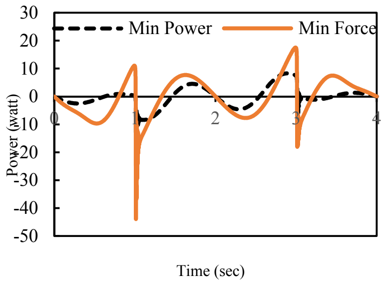


b) Based on minimum power of the driver

Fig. 6. Motion of the 2nd mechanism

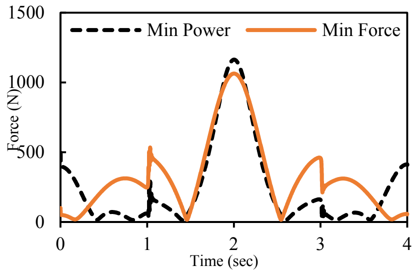


a) Force of knee joint in the 1st mechanism

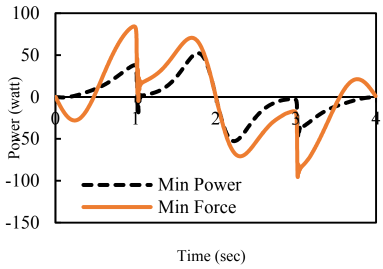


b) Driver power in the 1st mechanism

Fig. 7. Related diagrams of the 1st mechanism



a) Force of knee joint in the 2nd mechanism



b) Driver power in the 2nd mechanism

Fig. 8. Related diagrams of the 2nd mechanism

IV. CONCLUSION

Two different mechanisms, one consisting of two four-bar linkages and the other consisting of one four-bar linkage and one slider-crank mechanism, offering CPM for knee have been designed using three-accuracy-point approach. The mechanisms are designed considering specific design requirements and their motion has been studied through simulation. An optimization problem is then defined in order to find the mechanism which provides the least amount of actuation power and force generated at knee joint. The corresponding results are provided in the previous section.

V. REFERENCES

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