



# University of Victoria

**DEPARTMENT OF ELECTRICAL AND COMPUTER  
ENGINEERING**

## **Elec 572 Project** **Pre-Image**

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# 1. Θ FUNCTION:

The pre-image mapping for data set is as follows:

**Step 1:** Convert the input Hexadecimal string to Binary and reverse the binary as shown.

For z=1					
2	0	0	0	1	1
1	0	0	0	1	0
0	0	0	0	0	0
4	1	0	0	0	0
3	0	1	1	1	0
	3	4	0	1	2

for z = 2					
2	0	1	1	1	1
1	0	0	0	0	1
0	0	0	1	0	1
4	0	1	1	1	0
3	1	1	0	0	0
	3	4	0	1	2

for z = 3					
2	0	0	0	1	1
1	0	1	1	1	0
0	0	0	1	1	0
4	1	0	1	0	0
3	0	0	0	0	0
	3	4	0	1	2

for z = 4					
2	0	1	1	1	1
1	0	0	0	1	0
0	0	1	1	0	1
4	0	0	0	0	0
3	0	0	1	1	0
	3	4	0	1	2

for z = 5					
2	0	0	1	1	1
1	0	0	1	1	1
0	0	0	1	1	0
4	1	0	0	1	0
3	0	1	0	1	1
	3	4	0	1	2

for z = 6					
2	0	0	1	1	1
1	1	1	0	0	1
0	1	1	1	1	0
4	0	1	1	1	0
3	1	0	0	1	1
	3	4	0	1	2

for z = 7					
2	1	1	0	0	0
1	0	0	0	0	0
0	0	0	0	1	1
4	0	1	1	1	1
3	0	0	1	1	1
	3	4	0	1	2

for z = 8					
2	1	0	1	0	0
1	0	1	0	1	1
0	1	0	0	1	1
4	0	1	1	1	1
3	1	1	1	1	1
	3	4	0	1	2

**Step 2: Expand inverted data into plane**

y = 0					
7	1	0	0	1	1
6	0	0	0	1	1
5	1	1	1	1	0
4	0	0	1	1	0
3	0	1	1	0	1
2	0	0	1	1	0
1	0	0	1	0	1
0	0	0	0	0	0
	4	3	0	1	2

y = 1					
7	0	1	0	1	1
6	0	0	0	0	0
5	1	1	0	0	1
4	0	0	1	1	1
3	0	0	0	1	0
2	0	1	1	1	0
1	0	0	0	0	1
0	0	0	0	1	0
	4	3	0	1	2

y = 2					
7	1	0	1	0	0
6	1	1	0	0	0
5	0	0	1	1	1
4	0	0	1	1	1
3	0	1	1	1	1
2	0	0	0	1	1
1	0	1	1	1	1
0	0	0	0	1	1
	4	3	0	1	2

y = 3					
7	1	1	1	1	1
6	0	0	1	1	1
5	1	0	0	1	1
4	0	1	0	1	1
3	0	0	1	1	0
2	0	0	0	0	0
1	1	1	0	0	0
0	0	1	1	1	0
	4	3	0	1	2

y = 4					
7	0	1	1	1	1
6	0	1	1	1	1
5	0	1	1	1	0
4	1	0	0	1	0
3	0	0	0	0	0
2	1	0	1	0	0
1	0	1	1	1	0
0	1	0	0	0	0
	4	3	0	1	2

**Step 3:** after equation 1 of theta function

7	1	1	1	0	0
6	1	0	0	1	1
5	1	1	1	0	1
4	1	1	1	1	1
3	0	0	1	1	0
2	1	1	1	1	1
1	1	1	1	0	0

**Step 3:** after equation 2 of theta function

7	1	0	0	0	1
6	0	0	0	0	0
5	0	0	0	0	1
4	0	1	0	0	0
3	1	1	1	1	1
2	0	1	1	1	0
1	0	0	1	1	0
0	0	0	1	1	0
	4	3	0	1	2

**Step 4:** after equation 3 of theta function

7	0	1	1	1	0
6	0	0	1	1	1
5	1	0	0	1	0
4	0	0	0	1	1
3	1	1	0	0	1
2	0	1	1	1	0
1	1	1	1	1	0
0	0	1	0	0	0

7	1	1	1	1	0
6	0	1	1	1	1
5	0	1	1	1	1
4	1	1	0	1	0
3	1	1	1	1	1
2	1	1	0	0	0
1	0	1	0	0	0
0	1	0	1	0	0

7	0	0	0	1	0
6	0	0	0	1	1
5	1	1	1	1	1
4	0	1	1	1	0
3	1	0	0	1	0
2	0	1	0	0	0
1	0	0	0	1	1
0	0	0	1	1	0

7	1	1	0	1	1
6	0	0	0	0	0
5	1	1	0	0	0
4	0	1	1	1	1
3	1	1	1	0	1
2	0	0	0	0	0
1	0	0	1	1	1
0	0	0	1	0	0

7	0	0	1	0	1
6	1	1	0	0	0
5	0	0	1	1	0
4	0	1	1	1	1
3	1	0	0	0	0
2	0	1	1	0	1
1	1	1	0	0	1
0	0	0	1	0	0

**Step 5:** converted into slices

for Z=0					
2	0	0	1	0	1
1	0	0	1	0	0
0	0	0	1	1	0
4	1	0	1	0	0
3	0	1	0	0	0
	3	4	0	1	2

for Z=1					
2	1	1	0	0	1
1	0	0	1	1	1
0	0	0	0	1	1
4	0	1	0	0	0
3	1	1	1	1	0
	3	4	0	1	2

for Z=2					
2	0	1	1	0	1
1	0	0	0	0	0
0	0	1	0	0	0
4	1	1	0	0	0
3	0	1	1	1	0
	3	4	0	1	2

for Z=3					
2	1	0	0	0	0
1	1	1	1	0	1
0	1	0	0	1	0
4	1	1	1	1	1
3	1	1	0	0	1
	3	4	0	1	2

for Z=4					
2	0	1	1	1	1
1	0	1	1	1	1
0	0	1	1	1	0
4	1	1	0	1	0
3	0	0	0	1	1
	3	4	0	1	2

for Z=5					
2	0	0	1	1	0
1	1	1	0	0	0
0	1	1	1	1	1
4	0	1	1	1	1
3	1	0	0	1	0
	3	4	0	1	2

for Z=6					
2	1	1	0	0	0
1	0	0	0	0	0
0	0	0	0	1	1
4	0	1	1	1	1
3	0	0	1	1	1
	3	4	0	1	2

for Z=7					
2	0	0	1	0	1
1	1	1	0	1	0
0	0	0	0	1	0
4	1	1	1	1	0
3	0	1	1	1	0
	3	4	0	1	2

## 2.Rho ( $\rho$ ) function:

*As an example, we took the following dataset and performed preimage process for the verification of the Pre-image function by mapping technique.*

**Input Bit:** 62509a9dc54838ebc9f5887fea809585eba566f47e17baf474

**Step 1:** Convert the input Hexadecimal string to Binary and reverse the binary as shown.

Input (Hex)	Converted to Binary	Inverted Binary
dc	11011100	00111011
72	01110010	01001110
62	01100010	01000110
e5	11100101	10100111
fc	11111100	00111111
01	00000001	10000000
57	01010111	11101010
2d	00101101	10110100
e5	11100101	10100111
7a	01111010	01011110
1f	00011111	11111000
ff	11111111	11111111
9c	10011100	00111001
8a	10001010	01010001
37	00110111	11101100
df	11011111	11111011
56	01010110	01101010
3b	00111011	11011100
4d	01001101	10110010
89	10001001	10010001
41	01000001	10000010
b9	10111001	10011101
62	01100010	01000110
4f	01001111	11110010
bc	10111100	00111101

**Step 2:** Mapped the inverted Hex strings into slices

for Z=0					
2	0	1	1	1	0
1	1	0	1	1	1
0	1	0	0	0	0
4	1	0	1	1	0
3	1	1	1	0	1
	3	4	0	1	2

for Z=1					
2	1	1	1	1	0
1	0	1	0	1	0
0	0	0	0	1	1
4	1	0	0	0	1
3	0	0	1	1	1
	3	4	0	1	2

for Z=2					
2	0	1	1	1	1
1	1	0	0	1	1
0	1	1	1	0	0
4	1	1	0	0	0
3	1	0	1	1	0
	3	4	0	1	2

for Z=3					
2	1	0	1	1	1
1	0	1	0	0	1
0	0	1	1	0	0
4	1	1	0	1	0
3	1	1	1	0	1
	3	4	0	1	2

for Z=4					
2	0	1	1	1	1
1	0	1	0	1	0
0	0	1	1	1	0
4	0	1	0	1	0
3	0	0	1	1	1
	3	4	0	1	2

for Z=5					
2	0	1	0	1	0
1	1	1	0	0	1
0	1	1	0	1	1
4	0	1	0	1	1
3	0	0	0	0	1
	3	4	0	1	2

for Z=6					
2	0	0	0	1	0
1	1	1	0	1	0
0	1	1	1	1	1
4	1	0	1	0	1
3	1	0	1	1	0
	3	4	0	1	2

for Z=7					
2	1	0	0	1	1
1	1	0	0	0	0
0	1	1	1	0	0
4	0	1	0	1	0
3	0	1	1	0	0
	3	4	0	1	2

**Step 3:** Expand each slice into sheet



for X=3								
2	0	1	0	1	0	0	0	1
1	1	0	1	0	0	1	1	1
0	1	0	1	0	0	1	1	1
4	1	1	1	1	0	0	0	0
3	1	0	1	1	0	0	0	0
	0	1	2	3	4	5	6	7

for X=4								
2	1	1	1	0	1	1	0	0
1	0	1	0	1	1	1	1	0
0	0	0	1	1	1	1	1	1
4	0	0	1	1	1	1	0	1
3	1	0	0	1	0	0	0	1
	0	1	2	3	4	5	6	7

for X=0								
2	1	1	1	1	1	0	0	0
1	1	0	0	0	0	0	0	0
0	0	0	1	1	1	0	1	1
4	1	0	0	0	0	0	1	0
3	1	1	1	1	1	0	1	1
	0	1	2	3	4	5	6	7

for X=1								
2	1	1	1	1	1	1	1	1
1	1	1	1	0	1	0	1	0
0	0	1	0	0	1	1	1	0
4	1	0	0	1	1	1	0	1
3	0	1	1	0	1	0	1	0
	0	1	2	3	4	5	6	7

for X=2								
2	0	0	1	1	1	0	0	1
1	1	0	1	1	0	1	0	0
0	0	1	0	0	0	1	1	0
4	0	1	0	0	0	1	1	0
3	1	1	0	1	1	1	1	0
	0	1	2	3	4	5	6	7

*Perform Rho operation by shifting the bits into the sheet*

for X=3								
2	1	0	1	0	0	0	1	0
1	1	1	0	1	0	0	1	1
0	0	1	1	1	1	0	1	0
4	1	1	1	1	0	0	1	0
3	0	1	0	1	0	1	1	0
	0	1	2	3	4	5	6	7

for X=4								
2	0	1	1	1	0	1	1	0
1	1	1	1	0	0	1	0	1
0	1	1	1	1	1	0	0	1
4	0	1	0	0	1	1	1	1
3	1	0	0	1	0	0	0	1
	0	1	2	3	4	5	6	7

for X=0								
2	1	1	0	0	0	1	1	1
1	0	0	0	0	1	0	0	0
0	0	0	1	1	1	0	1	1
4	0	0	0	0	1	0	1	0
3	1	1	1	1	0	1	1	1
	0	1	2	3	4	5	6	7

for X=1								
2	1	1	1	1	1	1	1	1
1	1	0	1	0	1	1	1	0
0	1	0	0	1	1	1	0	0
4	0	1	1	1	0	1	1	0
3	0	1	0	0	1	1	0	1
	0	1	2	3	4	5	6	7

for X=2								
2	1	1	0	0	1	0	0	1
1	0	0	1	0	1	1	0	1
0	1	0	0	1	0	0	0	1
4	1	1	0	0	1	0	0	0
3	0	1	1	0	1	1	1	0
	0	1	2	3	4	5	6	7

**Step 4:** After performing the Rho operation, we again take the bits and arrange them into their respective slices:

for Z=0					
2	1	0	1	1	1
1	1	1	0	1	0
0	0	1	0	1	1
4	1	0	0	0	1
3	0	1	1	0	0
	3	4	0	1	2

for Z= 1					
2	0	1	1	1	1
1	1	1	0	0	0
0	1	1	0	0	0
4	1	1	0	1	1
3	1	0	1	1	1
	3	4	0	1	2

for Z=2					
2	1	1	0	1	0
1	0	1	0	1	1
0	1	1	1	0	0
4	1	0	0	1	0
3	0	0	1	0	1
	3	4	0	1	2

for Z=3					
2	0	1	0	1	0
1	1	0	0	0	0
0	1	1	1	1	1
4	1	0	0	1	0
3	1	1	1	0	0
	3	4	0	1	2

for Z=4					
2	0	0	0	1	1
1	0	0	1	1	1
0	1	1	1	1	0
4	0	1	1	0	1
3	0	0	0	1	1
	3	4	0	1	2

for Z = 5					
2	0	1	1	1	0
1	0	1	0	1	1
0	0	0	0	1	0
4	0	1	0	1	0
3	1	0	1	1	1
	3	4	0	1	2

for Z=6					
2	1	1	1	1	0
1	1	0	0	1	0
0	1	0	1	0	0
4	1	1	1	1	0
3	1	0	1	0	1
	3	4	0	1	2

for Z=7					
2	0	0	1	1	1
1	1	1	0	0	1
0	0	1	1	0	1
4	0	1	0	0	0
3	0	1	1	1	0
	3	4	0	1	2

#### **Step 4:**

Binary Data	Inverted Binary	Output (Hex)
00111011	11011100	dc
10011100	00111001	39
10010001	10001001	89
01111010	01011110	5e
11111001	10011111	9f
00001000	00010000	10
10101110	01110101	75
00101101	10110100	b4
11010011	11001011	cb
11100101	10100111	a7
11000111	11100011	e3
11111111	11111111	ff
11001001	10010011	93
10100010	01000101	45

01110110	01101110	6e
11111111	11101111	ef
01001101	10110010	b2
01101110	01110110	76
01010110	01101010	6a
10010001	10001001	89
00001010	01010000	50
01110110	01101110	6e
11001000	00010011	13
11110010	01001111	4f
01001111	11110010	f2

### 3. Pi ( $\pi$ ) FUNCTION:

Let the Input state array be **A** and Output state array be **A'**.

Following are the steps for calculation of pi function:

1. For all triples  $(x, y, z)$  such that  $0 \leq x < 5$ ,  $0 \leq y < 5$ , and  $0 \leq z < w$ ,  
let  $\mathbf{A'} [x, y, z] = \mathbf{A}[(x + 3y) \bmod 5, x, z]$ . (1)
2. Return **A'**.

The movement of bits by substituting the X and Y values from the above equation is shown as follows:

S:No	Input	Output
	A	A'
1	00	00
2	10	11
3	20	22
4	30	33
5	40	44
6	01	30
7	11	41
8	21	02
9	31	13
10	41	24
11	02	10
12	12	21
13	22	32
14	32	43
15	42	04
16	03	40
17	13	01
18	23	12
19	33	23
20	43	34
21	04	21
22	14	31
23	24	42
24	34	03

25	44	14
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Consider the equation (1), to find the pre-image function

Compare the similarities,

$$X' = (X+3Y) \bmod 5 \quad (2)$$

$$Y' = X \quad (3)$$

$$Z' = Z \quad (4)$$

Now, take equation (2)

$$X' = X + 3Y$$

$$Y = \frac{X' - X}{3}$$

$$Y = (X' - X) 3 \bmod^{-1} 5$$

Value of  $3 \bmod^{-1} 5$  yields 2

$$Y = 2(X' - X) \bmod 5$$

$$\text{And } X = Y'$$

$$Y = 2(X' - Y') \bmod 5$$

To make the mod 5 value return +ve , add +5

$$Y = 2(X' - Y' + 5) \bmod 5$$

Convert it into equation, to get the final pre-image equation as

$$\underline{A'[x, y, z] = A[Y, 2(X - Y + 5) \bmod 5, z]}$$

## 4. chi FUNCTION:

*Function : XoR ing the bits in the entire row*

$$\mathbf{A'} = \mathbf{A} \oplus (\overline{\mathbf{B}} \cdot \mathbf{C})$$

The forward function for chi is given by the below equation

*Input be state array  $\mathbf{A}$  and Output state be array  $\mathbf{A'}$ .*

*Now,*

1. For all triples  $(x, y, z)$  such that  $0 \leq x < 5$ ,  $0 \leq y < 5$ , and  $0 \leq z < w$ , let

$$\mathbf{A'}[x, y, z] = \mathbf{A}[x, y, z] \oplus ((\mathbf{A}[(x+1) \bmod 5, y, z] \oplus 1) \cdot \mathbf{A}[(x+2) \bmod 5, y, z]). \quad (1)$$

2. Return  $\mathbf{A'}$ .

*Using equation 1, we will convert the output bits into the input for the purpose of pre-image.*

Input	Output
ABCDE	A'B'C'D'E'
00000	0 0 0 0 0
10101	0 0 0 0 1
01011	0 0 0 1 0
01010	0 0 0 1 1
10110	0 0 1 0 0
00001	0 0 1 0 1
10100	0 0 1 1 0
10111	0 0 1 1 1
01101	0 1 0 0 0
01000	0 1 0 0 1
00010	0 1 0 1 0
00011	0 1 0 1 1
01001	0 1 1 0 0
01100	0 1 1 0 1
01111	0 1 1 1 0
01110	0 1 1 1 1
11010	1 0 0 0 0
00101	1 0 0 0 1
10000	1 0 0 1 0
11011	1 0 0 1 1

00100	1 0 1 0 0
10001	1 0 1 0 1
00110	1 0 1 1 0
00111	1 0 1 1 1
10010	1 1 0 0 0
11101	1 1 0 0 1
11000	1 1 0 1 0
10011	1 1 0 1 1
11110	1 1 1 0 0
11001	1 1 1 0 1
11100	1 1 1 1 0
11111	1 1 1 1 1

*Present the output to input for the purpose of deriving the equation,*

ABC/DE	00	01	10	11
000	0	1	0	0
001	1	0	1	1
010	0	0	0	0
011	0	0	0	0
100	1	0	1	1
101	0	1	0	0
110	1	1	1	1
111	1	1	1	1

*Derive the k map from above.*

K map when A=0

DE/ABC	000	001	011	010
00	0	1	0	0
01	1	0	0	0
11	0	1	0	0
10	0	1	0	0

K map when A=1

K map when A=0

DE/ABC	000	001	011	010
00	1	0	1	1
01	0	1	1	1
11	1	0	1	1



10	1	0	1	1
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Group 1's together to form equation from the kmap

$$AB + \overline{A}\overline{B}\overline{C}D + \overline{A}\overline{B}C\overline{E} + \overline{A}\overline{B}C\overline{D}E + \overline{A}\overline{B}CD + \overline{A}\overline{B}C\overline{E} + \overline{A}\overline{B}C\overline{D}E$$

Simplifying above equation we get,

$$AB + \overline{B}(A \oplus C)(D + \overline{E}) + \overline{B}\overline{D}E(A \otimes C) \quad (2)$$

Following is the equation for pre-image function,

$$\underline{Y = AB + \overline{B}(A \oplus C)(D + \overline{E}) + \overline{B}\overline{D}E(A \otimes C)}$$

## 5. IOTA FUNCTION:

The forward mapping function of iota is based on the *two algorithms*. These algorithms are as follows:

### Algorithm 1: find $rc(t)$

*Input* be integer  $t$  and *Output* be bit  $rc(t)$ .

Following are the steps in algorithm.

1. If  $t \bmod 255 = 0$ , return 1.
2. Let  $R = 10000000$ .
3. For  $i$  from 1 to  $t \bmod 255$ , let:
  - a.  $R = 0 \parallel R$ ;
  - b.  $R[0] = R[0] \oplus R[8]$ ;
  - c.  $R[4] = R[4] \oplus R[8]$ ;
  - d.  $R[5] = R[5] \oplus R[8]$ ;
  - e.  $R[6] = R[6] \oplus R[8]$ ;
  - f.  $R = \text{Trunc8}[R]$ .
4. Return  $R[0]$

### Algorithm 2: $\iota(A, ir)$

Input is state array  $A$ ; round index  $ir$ . And output is state array  $A'$ .

1. For all triples  $(x, y, z)$  such that  $0 \leq x < 5$ ,  $0 \leq y < 5$ , and  $0 \leq z < w$ , let  $A'[x, y, z] = A[x, y, z]$ .
2. Let  $RC = 0w$ .
3. For  $j$  from 0 to  $l$ , let  $RC[2j - 1] = rc(j + 7i_r)$ . (1)
4. For all  $z$  such that  $0 \leq z < w$ , let  $A'[0, 0, z] = A'[0, 0, z] \oplus RC[z]$ . (2)
5. Return  $A'$ .

*The effect of  $\iota$  is to modify bits of Lane  $(0, 0)$  such that it depends on the round index  $ir$  with other 24 lanes not affected by  $\iota$ .*

We know that,  $2^l = 8$ , So  $l = 3$  Whereas,  $l$  in the equation 1 Varies from 0 to 3  
the number of rounds here,  $i_r = 2$

*calculate values for RC (based on equation 1, for Round 1)*

When  $j = 0$ ,  $RC[0] = rc[7]$   
When  $j = 1$ ,  $RC[1] = rc[8]$   
When  $j = 2$ ,  $RC[3] = rc[9]$   
When  $j = 3$ ,  $RC[7] = rc[10]$

*calculate the values for RC (based on equation 1, for Round 2)*

When  $j=0$ ,  $RC[0] = rc[14]$

When  $j=1$ ,  $RC[1] = rc[15]$

When  $j=2$ ,  $RC[3] = rc[16]$

When  $j=3$ ,  $RC[7] = rc[17]$

*By performing the operations based on the algorithm 1 and Substituting the values of  $t$ , we get value of  $rc$ .*

As we need value for  $rc[17]$ , we need  $t$  only till 17 where,  $t$  ranges from 0 to 255.

Now	$t=0$ , $R= 10000000$	$rc[0] = 1$
	$t=1$ , $R= 01000000$	$rc[1] = 0$
	$t=2$ , $R= 00100000$	$rc[2] = 0$
	$t=3$ , $R= 00010000$	$rc[3] = 0$
	$t=4$ , $R= 00001000$	$rc[4] = 0$
	$t=5$ , $R= 00000100$	$rc[5] = 0$
	$t=6$ , $R= 00000010$	$rc[6] = 0$
	$t=7$ , $R= 00000001$	$rc[7] = 0$
	$t=8$ , $R= 10001110$	$rc[8] = 1$
	$t=9$ , $R= 01000111$	$rc[9] = 0$
	$t=10$ , $R= 10101101$	$rc[10] = 1$
	$t=11$ , $R= 11011000$	$rc[11] = 1$
	$t=12$ , $R= 01101100$	$rc[12] = 0$
	$t=13$ , $R= 00110110$	$rc[13] = 0$
	$t=14$ , $R= 00011011$	$rc[14] = 0$
	$t=15$ , $R= 10000011$	$rc[15] = 1$
	$t=16$ , $R= 11001111$	$rc[16] = 1$
	$t=17$ , $R= 11101001$	$rc[17] = 1$

*For Round 1 RC values are as follows:*

When  $j=0$ ,  $RC[0] = rc[7] = 0$

When  $j=1$ ,  $RC[1] = rc[8] = 1$

When  $j=2$ ,  $RC[3] = rc[9] = 0$

When  $j=3$ ,  $RC[7] = rc[10]=1$

*we can get the equation for Round 1 based on these observations:*

For  $j = 0$  to  $3$ ,

$$RC[2^j - 1] \oplus [j \bmod 2] \quad (3)$$

*For Round 2 RC values are as follows:*

When  $j=0$ ,  $RC[0] = rc[14] = 0$

When  $j = 1$ ,  $RC[1] = rc[15] = 1$

When  $j = 2$ ,  $RC[3] = rc[16] = 1$

When  $j = 3$ ,  $RC[7] = rc[17] = 1$

*we can get the equation for Round 2 based on these observations:*

For  $j = 0$  to  $3$ ,

$$RC[2^j - 1] \oplus [2^j - 1] \bmod 2 \quad (4)$$

*As seen that the Round 2 is derived from Round 1 i.e. the equation 4 is derived from equation 3, it is sufficient for proposed evaluation.*