Due date: Friday May 1, by 2pm

You must submit your assignment electronically and as a single file via the LMS page for this subject. Your solutions must include your workings. **Note that this assignment is worth 10% of your overall mark for this subject.** Advice for preparing an electronic version of your assignment is provided on **this LMS page (click here)** under the heading: 'Some ideas for submitting your assignments online'. Do not submit your file as a ZIP file.

In submitting your work, you are consenting that it may be copied and transmitted by the University for the detection of plagiarism. Please start with the following statement of originality, which must be included near the top of your submitted assignment:

"This is my own work. I have not copied any of it from anyone else."

Please round your answers to three decimal places if rounding is necessary. Alternatively, if you choose to express your answers as fractions, please ensure that the fraction is reduced to its simplest form. E.g. 10/20 should be expressed as 1/2.

1. Suppose that in a food storage facility, 0.15% of the food can be rotten at any given time, due to imperfect temperature controls. To maintain and monitor the quality of the stored food, a sample of 960 food containers is selected at regular intervals for inspection. Let X denote the number of containers with rotten food found in a given inspection sample. It follows that for this scenario, we can write: $X \sim \text{BIN}(960, 0.0015)$.

   Using this information, answer the following questions. Your calculations must be carried out by hand. Do not use software (e.g. R, Excel) for these questions. The use of a scientific calculator for basic calculations is permitted. Remember to show your workings.

   (a) What are $E(X)$ and $\text{Var}(X)$?

   (b) Using the probability mass function (pmf) for a Binomial random variable, find the probability that there are no containers with rotten food in the sample. I.e., find $P(X = 0)$. *Hint: When computing* $\binom{960}{0}$*, look for simplifications. Do not, for example, try and compute the factorial of* 960.

   (c) What is the probability that there is exactly one container with rotten food in the sample?

   (d) Suppose that to justify introducing a new temperature control system, the manager requires at least 3 containers to have rotten food, out of an inspection sample of 960 containers. What is the probability of this event occurring?

   (e) It is well known that the Poisson distribution ($\text{POIS}(np)$) provides an excellent approximation to the Binomial distribution ($\text{BIN}(n, p)$) when p is small and n is large[1]. Given this fact, use the Poisson distribution to approximate the probabilities you calculated in (b), (c) and (d). Comment on your results. Are the approximations close?

2. Suppose that the arrival schedule of delivery trucks at a certain distribution centre can be modelled using a Poisson process, with an arrival rate of 5 trucks per hour. The contents of 7 delivery trucks can be processed each hour in the delivery bay of the distribution centre. If the delivery bay staff are busy processing a truck's contents, any newly-arriving trucks will wait in a queue and join the end of the queue behind any other trucks already waiting. You may assume that the system can be modelled as an $M|M|1$ queue.

   Answer the following questions.

   (a) What is the traffic intensity, ρ, for this delivery bay?

   (b) What is the average amount of time that a truck will spend being unloaded in the delivery bay?

   (c) On average, how long will a driver spend waiting in the queue before they can unload the contents of their truck in the delivery bay?

   (d) What is the long term average number of trucks in the system at any one time (this includes those in the queue and those unloading in the delivery bay)?

   (e) On average, how long will a driver spend in the system (this includes time waiting in the queue and time unloading in the delivery bay)?

[1] To understand why this may useful, think about computation issues that can arise when dealing with factorials of very large numbers in the Binomial pmf.

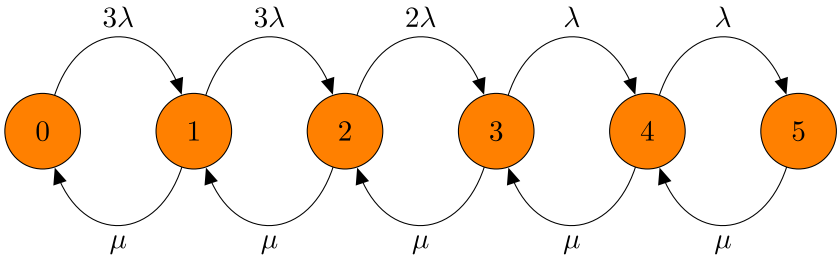
(f) This question should be carried out in R. You must include your R code in your answers.

Suppose that the staff of the distribution centre fall ill, and are replaced with temp workers, who operate with an as yet unknown delivery truck processing rate.

i. Using R and restricting $5 < \mu < 15$, plot the average amount of time spent waiting in the queue, as a function of μ ($\lambda = 5$ is fixed). Note, you may not wish to allow μ to get too close to 5 for the purpose of this plot. Describe the plot in the context of the problem.

ii. If new performance standards dictate that the average time a truck remains in the queue must not exceed 15 minutes, then what is the minimum rate at which the new staff need to process the contents of a delivery truck, so that they meet these performance standards?

3. Recall the server repair question in Lab 6. Suppose that now there are five servers, and only one repairer. Let the states, depicted below, represent the number of broken down servers.



Assume that the system is in a steady state. Answer the following questions.

(a) By equating the rate out of state 0 with the rate into state 0, find an expression for P_1 in terms of $\rho = \lambda/\mu$ and P_0.

(b) Similarly, find an expression for P_2 in terms of ρ and P_1 and then use you answer to (a) to find the expression for P_2 in terms of ρ and P_0.

(c) Now find expressions for P_3, P_4 and P_5 in terms of ρ and P_0.

(d) Using the fact that $\sum_{n=0}^{5} P_n = 1$, derive an expression for P_0 that depends only on ρ.

(e) Suppose that the breakdown rate of each server is $\lambda = 1$ per week and that the repairer can repair servers at a rate of 3 per week. At any given time, what is:

i. The probability that there are no servers broken?

ii. The probability that all servers are broken?

iii. The probability that exactly one server is broken?

(f) Suppose that it is important that at least two servers are working to satisfy demand. Using your answers above, what is the probability that at least two servers will be working at any given time?