Assignment_2_Maninderpreet Singh Puri_20494381

by Maninderpreet Singh Puri

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Great work

STM4PSD **ASSIGNMENT 2**

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STUDENT NUMBER: **20494381**

1.
(a)

Given X ~ BIN (960, 0.0015)

E (X) = *np*= 960 * 0.0015= 1.44 *Using Result 5.3.1*

Var (X) = *np (1-p)* = 960 * 0.0015 * (1- 0.0015) = 1.438 *Using Result 5.3.1*

(b)

Given P (X=0), n=960

Using Binomial distribution X ~ BIN (*n*,*p*) and $pX(x) = \binom{n}{x} p^x (1-p)^{n-x}$

- ⇨ P (X=0) = $\binom{960}{0} 0.0015^0 (1-0.0015)^{960-0}$
- ⇨ $\binom{960}{0}$ can be written as $\frac{960}{0!(960-0)!} = 1$ *Using 5.2.1*
- ⇨ P (X=0) = $1 * 1\ (0.9985)^{960} = 0.237$

So, the probability that there are no containers with rotten food in the sample is 0.237

(c)

- ⇨ P (X=1) = $\binom{960}{1} 0.0015^1 (1-0.0015)^{960-1}$
- ⇨ 960 * 0.0015 * $(0.9986)^{959}$= 0.341

So, the probability that there is exactly one container with rotten food in the sample is 0.341

(d)

Probability of at least 3 containers to have rotten food P (At least 3) can be described as

P (At least 3) = 1- (P (X=0) + P (X=1))+p(x=2)

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So using the results from (b) and (c)

P (At least 3) = 1- 0.237 + 0.341) = 1- 0.578 = 0.4217

(e)

Given E (X) = 1.44

For Poisson Distribution E (X) can be described as λ. So λ = 1.44.

Using Poisson distribution X ~ POIS (λ) and $pX(x) = \frac{\lambda^x e^{-\lambda}}{x!}$

- ⇨ P (X=0) = $\frac{1.44^0 e^{-1.44}}{0!}$ = 0.237
- ⇨ P (X=1) = $\frac{1.44^1 e^{-1.44}}{1!}$ = 0.341
- ⇨ P (At least 3) = 1- (P (X=0) + P (X=1)) = 1 – (0.237 + 0.341) = 0.422

Both approximations of Poisson and Binomial distributions are close.

2.

(a)

According to the information given in the question, λ = 5 and μ = 7.

So the Traffic intensity for the delivery bay is ρ = $\frac{\lambda}{\mu}$ *Using reading 6.4.1*

ρ = $\frac{5}{7}$ = 0.714

(b)

Using reading 6.4.2

Average amount of time that a truck will spend being unloaded in the delivery bay can be given by

Expected (mean) service time = $\frac{1}{\mu} = \frac{1}{7}$ = 0.14 * 60 (into minutes) = 8.57 minutes

(c)

Using Result 6.4.2

Average waiting time of a truck before unloading the contents of their truck in the delivery bay can be given by

$W_Q = \frac{\lambda}{\mu(\mu-\lambda)}$ = where λ = 5 and μ = 7.

So by substituting value in W_Q we get,

⇨ $\frac{5}{7(7-5)} = \frac{5}{14}$ = 0.357 * 60 (into minutes) = 21.42 minutes

(d)

Using Result 6.4.1

Long term average number of trucks in the system at any one time can be given by

$L = \frac{\lambda}{\mu-\lambda} = \frac{5}{7-5} = \frac{5}{2} = 2.5$

(e)

Average time a driver will spend in the system (this includes time waiting in the queue and time unloading in the delivery bay) can be given by,

$W = \frac{L}{\lambda} = \frac{2,5}{5}$ = 0.5 * 60 (into minutes) = 30 minutes

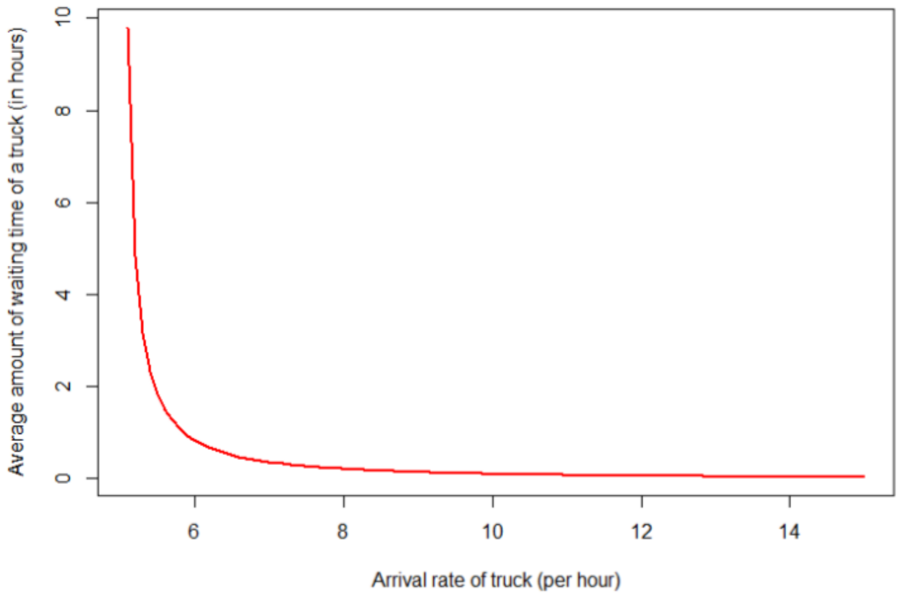
(f)

i.

R code:

```
windows(height = 6, width= 8)

WQ <- function(lambda , mu){

out <- lambda/(mu*(mu - lambda))

}

curve(WQ(5 , x), from = 5.1, to = 15 ,ylab = "Average amount of waiting time of a truck (in hours)", xlab ="Arrival rate of trucks (per hour)", lwd= 2, col ="red")
```

Output:



1

So according to the graph the average time spent in the queue decreases as lambda increases.

ii.

So from the given information in the question we can say that,

W_Q = 15, and taking $\lambda = 5$ we can solve for μ.

Note that 15 minutes is equal to 1 / 4 of an hour. We therefore must solve WQ = λ / μ(μ−λ) = 1 / 4

$W_Q = \frac{\lambda}{\mu(\mu-\lambda)}$

- ⇨ $15 = \frac{5}{\mu(\mu-5)}$ -2
- ⇨ $3\mu^2 - 15\mu - 1 = 0$ *(Quadratic equation)*
- ⇨ Now solving Quadratic equation by Quadratic formula we get,
- ⇨ $\mu \cong -0.065$ and 5.065

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So ignoring the negative value of μ we can say that the minimum rate at which the new staff need to process the contents of a delivery truck, so that they meet these performance standards is 5.

3.

(a)

We have μ P1 = 3 λ P0

So that $P1 = 3\,\frac{\lambda}{\mu}\,P0$

⇨ $P1 = 3\,\rho\,P0$ *As given $\rho = \lambda/\mu$*

(b)

The rate out of state 1 is $(\mu + 3\lambda)\,P1$ and the rate into state 1 is $3\lambda\,P0 + \mu\,P2$

Equating both we get.

⇨ $(\mu + 3\lambda)\,P1 = 3\lambda\,P0 + \mu\,P2$
⇨ $P2\,\mu = (\mu + 3\lambda)\,P1 - 3\lambda\,P0$
⇨ $P2 = (1 + 3\,\rho)\,P1 - 3\,\rho\,P0$
⇨ $P2 = (1 + 3\,\rho)\,(3\,\rho\,P0) - 3\,\rho\,P0$ *Using $P1 = 3\,\rho\,P0$*
⇨ $P2 = 9\,\rho^2 P0$

(c)

The rate out of state 2 is $(\mu + 2\lambda)\,P2$ and the rate into state 2 is $3\lambda\,P1 + \mu\,P3$

Equating both we get.

⇨ $(\mu + 2\lambda)\,P2 = 3\lambda\,P1 + \mu\,P3$
⇨ $P3\,\mu = (\mu + 2\lambda)\,P2 - 3\lambda\,P1$
⇨ $P3 = (1 + 2\,\rho)\,P2 - 3\,\rho\,P1$
⇨ $P3 = (1 + 3\,\rho)\,(9\,\rho^2 P0) - 3\,\rho\,(3\,\rho\,P0)$ *Using $P1 = 3\,\rho\,P0$ and $P2 = 9\,\rho^2 P0$*

⇨ $P3 = 18\,\rho^3 P0$

The rate out of state 3 is $(\mu + \lambda)\,P3$ and the rate into state 3 is $2\lambda\,P2 + \mu\,P4$

Equating both we get.

⇨ $(\mu + \lambda)\,P3 = 2\lambda\,P2 + \mu\,P4$
⇨ $P4\,\mu = (\mu + \lambda)\,P3 - 2\lambda\,P2$
⇨ $P4 = (1 + \rho)\,P3 - 2\,\rho\,P2$
⇨ $P4 = (1 + \rho)\,(18\,\rho^3 P0) - 2\,\rho\,(9\,\rho^2 P0)$ *Using $P3 = 18\,\rho^3 P0$ and $P2 = 9\,\rho^2 P0$*

⇨ $P4 = 18\,\rho^4 P0$

The rate out of state 4 is $(\mu + \lambda)\,P4$ and the rate into state 4 is $\lambda\,P3 + \mu\,P5$

Equating both we get.

⇨ $(\mu + \lambda)\,P4 = \lambda\,P3 + \mu\,P5$
⇨ $P5\,\mu = (\mu + \lambda)\,P4 - \lambda\,P3$
⇨ $P5 = (1 + \rho)\,P4 - \rho\,P3$
⇨ $P5 = (1 + \rho)\,(18\,\rho^4 P0) - \rho\,(18\,\rho^3 P0)$ *Using $P3 = 18\,\rho^3 P0$ and $P4 = 18\,\rho^4 P0$*

$\Rightarrow$ $P5 = 18\,\rho^5 P0$

(d)

According to the given information in the question, sum of all states is 1.

So,

$P0 + P1 + P2 + P3 + P4 + P5 = 1$

$P0 + 3\,\rho\, P0 + 9\,\rho^2 P0 + 18\,\rho^3 P0 + 18\,\rho^4 P0 + 18\,\rho^5 P0 = 1$

$(1 + 3\,\rho + 9\,\rho^2 + 18\,\rho^3 + 18\,\rho^4 + 18\,\rho^5)P0 = 1$

$P0 = (1 + 3\rho + 9\,\rho^2 + 18\,\rho^3 + 18\,\rho^4 + 18\,\rho^5)^{-1}$

(e)

Given $\lambda = 1,\ \mu = 3$

$\rho = \frac{\lambda}{\mu} = \frac{1}{3}$

i.

Probability that no server broken is

$P0 = (1 + 3\,(\frac{1}{3}) + 9((\frac{1}{3})^2) + 18((\frac{1}{3})^3) + 18((\frac{1}{3})^4) + 18((\frac{1}{3})^5)^{-1}$ *Using (d)*

- $\Rightarrow$ $P0 = (1 + 1 + 1 + \frac{2}{3} + \frac{2}{9} + \frac{2}{27})^{-1}$
- $\Rightarrow$ $(\frac{107}{27})^{-1} = \frac{27}{107} = 0.252$

ii.

Probability that all servers are broken is

- $\Rightarrow$ $P5 = 18\,\rho^5 P0$
- $\Rightarrow$ $P5 = 18 * (\frac{1}{3})^5 * \frac{27}{107} = 0.019$

iii.

Probability that exactly one server broken is

$P1 = 3\,\rho\, P0$

$= 3 * \frac{1}{3} * 0.252 = 0.252$

(e)

Probability of at least two servers working at any given time can be given as,

P0 + P1 + P2 +P3 = 1- (P4 + P5)

- ⇨ P4 = 18 ρ^4P0 = 18 $*$ $(\frac{1}{3})^4 * \frac{27}{107}$ = 0.056
- ⇨ P0 + P1 +P2 + P3 = 1- (0.056 + 0.019) = 0.925 *Using (e) ii.*

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FINAL GRADE

95/100

GENERAL COMMENTS

Instructor

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QM **Great work**

Text Comment. +p(x=2)

QM **-2**

PAGE 2

QM **-1**

PAGE 3

PAGE 4

Text Comment. Note that 15 minutes is equal to 1 / 4 of an hour. We therefore must solve WQ = λ / μ(μ−λ) = 1 / 4

QM **-2**

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