Homework #2: Euler Method

Due Date: Friday, Sept. 26 @ 12:00pm

Carbon dating. The carbon isotope ${}^{14}_{2}\mathrm{C}$ is widely used for dating of ancient artifacts containing 6 biological material. It undergoes β^- decay with a half-life time of $T_{1/2} = 5700$ years. Suppose an ancient artifact originally contained 10^{-12} kg of ${}^{14}_{2}\mathrm{C}$ (recall that 1 mol of a substance, which has a mass corresponding to the atomic mass-number in grams, contains $N_\mathrm{A} = 6.022 \cdot 10^{23}$ particles).

1. Derive analytically the relation between half-life time $T_{1/2}$ and decay constant τ as defined in class.
2. Write a computer code to numerically calculate the activity of the sample, defined as $R(t) = -dN/dt$, over a duration of 20,000 years. Use numerical time-step widths of 10 and 100 years. Plot the results in appropriate units with the exact (analytical) solution in the same graph.
3. Increase the time-step width to 1,000 years and re-plot. Is the accuracy of the numerical solution still acceptable? (e.g., what is the percentage deviation from the exact result after 2 half-lives?) Is the deviation from the exact result as large as you would expect from the neglected second-order term?

Golf. Write a program to calculate the trajectory of a golf ball of mass 46 grams and calculate the trajectories as a function of angle (use $\theta = 45°$, $30°$, $15°$ and $9°$). Choose the initial velocity of the golf ball to be 70 m/s. For the drag, assume a general form of $F_\mathrm{drag} = -C\rho A v^2$, where ρ is the density of air (at sea level), 1.29 kg/m^3, A is the frontal area of the golf ball, 0.0014 m^2, and C is a coefficient to be discussed below. For each angle, calculate and compare the trajectories for the following cases:

1. ideal trajectory: no drag and no spin.
2. smooth golf ball with drag: choose $C = 1/2$.
3. dimpled golf ball with drag: choose $C = 1/2$ for speeds up to $v = 14$ m/s and $C = 7.0/v$ at higher velocities. This transition to a reduced drag coefficient is due to turbulent flow, caused by the dimples.
4. dimpled golf ball with drag and spin: use a Magnus force $\mathbf{F}_\mathrm{Magnus} = S_0 \boldsymbol{\omega} \times \mathbf{v}$ with a backspin of $S_0\omega/m = 0.25\ \mathrm{s}^{-1}$ for a typical case.

Pointers. Start with the ideal trajectory first and validate it. Does the total distance traveled make sense? What are the final velocities? For each subsequent trajectory, you should be able to copy the Euler algorithm and add additional terms. Solve for x, y, v_mag, C, v_x, v_y; in that order. Be careful with your units. Use `DEBUG=True` and keep your print statements.

Submission. Write two Python scripts, `carbon.py` and `golf.py`, that accept the following arguments: `carbon.py --plot=`*`step_width`*, where *step_width* is the time-step width, `golf.py --plot=`θ, where θ is the angle. Write a `Makefile` that runs the following: `make carbon WIDTH=`*`width`* and `make golf THETA=`*`theta`*, where `WIDTH` and `THETA` are optional arguments. Write a LaTeX document outlining the problem, detailing your solution and discussion of your results. The document should include the requested figures. The document should be in pdf format and you should use colors and different marker symbols to enhance the readability of your figures. Report any "Contributions" to your work (see syllabus). Submit the python scripts, `Makefile`, and LaTeX document to both GitHub and Canvas.

GitHub organization. To better organize the homework submissions for grading, create a single directory `EN566_MatMod_`*`lastname_firstname`*. Inside this directory, create a directory for each homework assignment, i.e., `hw1`, `hw2`, etc.