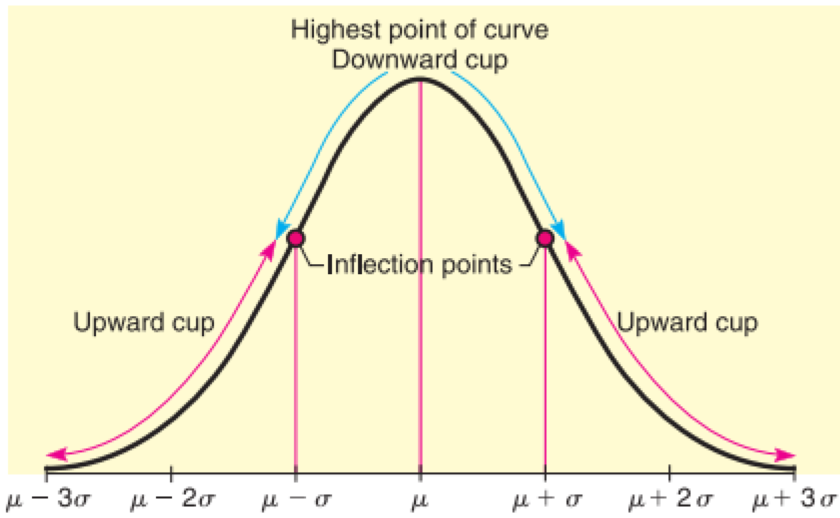
Normal Probability Distributions

Normal Curve: The graph of normal distribution is called a normal curve. It possesses a shape very much like the cross section of a pile of dry sand. The normal curve is also called a bell-shaped curve.

A general normal curve is smooth and symmetric about the vertical line extending upward from the mean μ. Notice that te highest point of the cure occurs over μ. The normal curve never touches the horizontal axis. The parameter σ controls the spread of the curve. The curve is quite close to the horizontal axis at $\mu + 3\sigma$ and $\mu - 3\sigma$.Thus, if the standard deviation μ is large, the curve will be more spread out: if it is small, the curve will be more peaked.



In above graph it shows the normal curve cupped downward for an ineval on either side of the mean μ. Then it betins to cup upward as we go to the lower part of the bell. The exacet place where the transition b/w the upward and downward cupping occur are above the points $\mu + \sigma$ and $\mu - \sigma$. In terms of calculus, transition points such as these are called inflection points.

Important Properties of a Normal Curve

1. The curve is bell-shaped, with the highest point over the mean μ.
2. The curve is symmetric about the vertical line through μ.
3. The curve approaches the horizontal axis but never touches or crosses it.

> 4. The inflection (transition) point b/w cupping upward and downward occur above $\mu + \sigma$ and $\mu - \sigma$.
> 5. The area under the entire curve is 1.

`Note:` The parameters that controls the shape of a normal curve are the mean μ and the standard deviation μ. when both μ and σ, are specified, a specific normal curve is determined. In brief, μ locates the balance point and σ determines the extent of the spread.

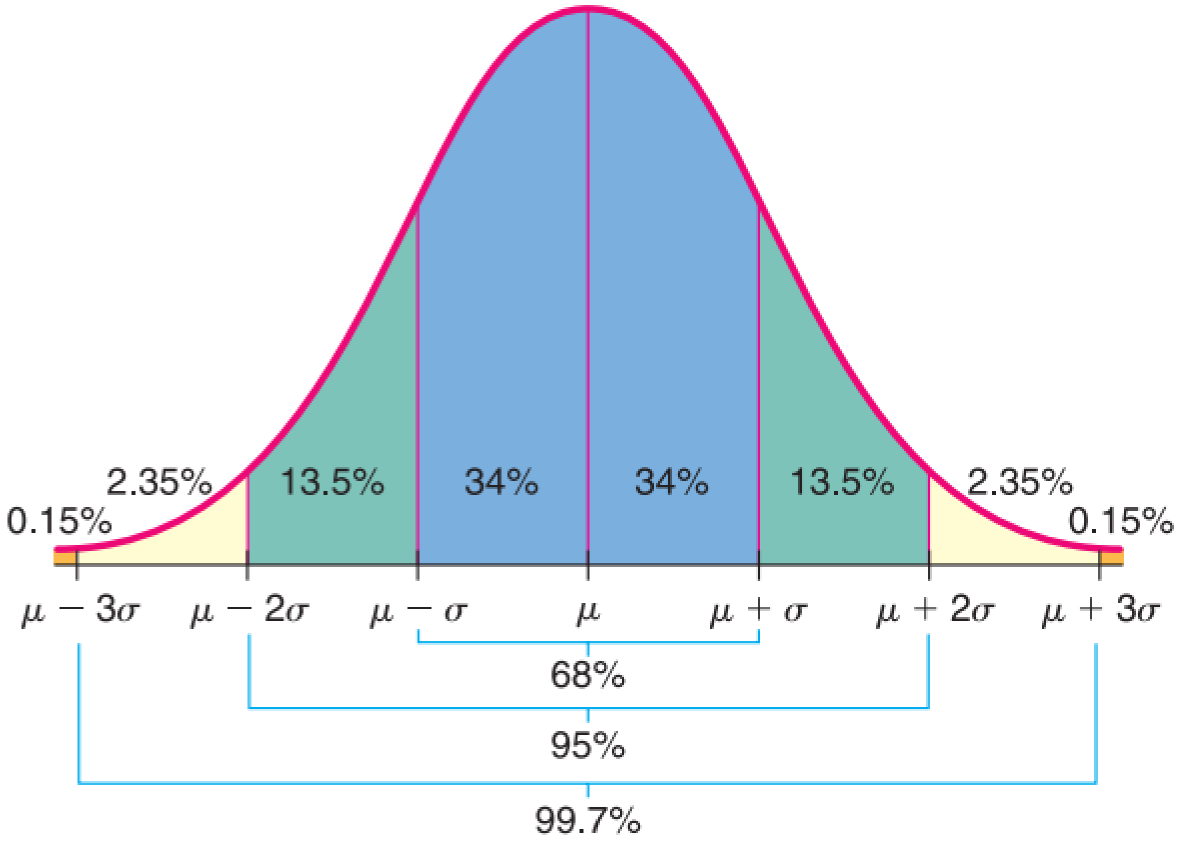
> `COMMENT` The normal distribution curve is always above th horizontal axis. the area beneath the curve and above the axis is exactly 1. such, the normal distribution curve is an example of density curve. The formula used to generate the shape of the normal distribution curve is called the **normal density function**. if x is a normal random variable with mean μ and standard deviation σ, the romula for the normal density function is
>
> $$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$
>
> The graph of the normal distribution is important because the portion of the area under the curve above a given interval represents the probability that measurement will lie in that interval.

Empirical Rule

For Normal distribution,

1. Approximately 68% of the data values will lie within 1 standard deviation on each side of the mean.
2. Approximately 95% of the data values will lie within 2 standard deviations on each side of the.
3. Approximately 99.7% of the data values will lie within 2 standard deviations on each side of the.

Area Under a Normal Curve



The above statementis called the *empirical rule* because, for symmetric, bell-shaped distributions, the given percentages are observed in paractice.

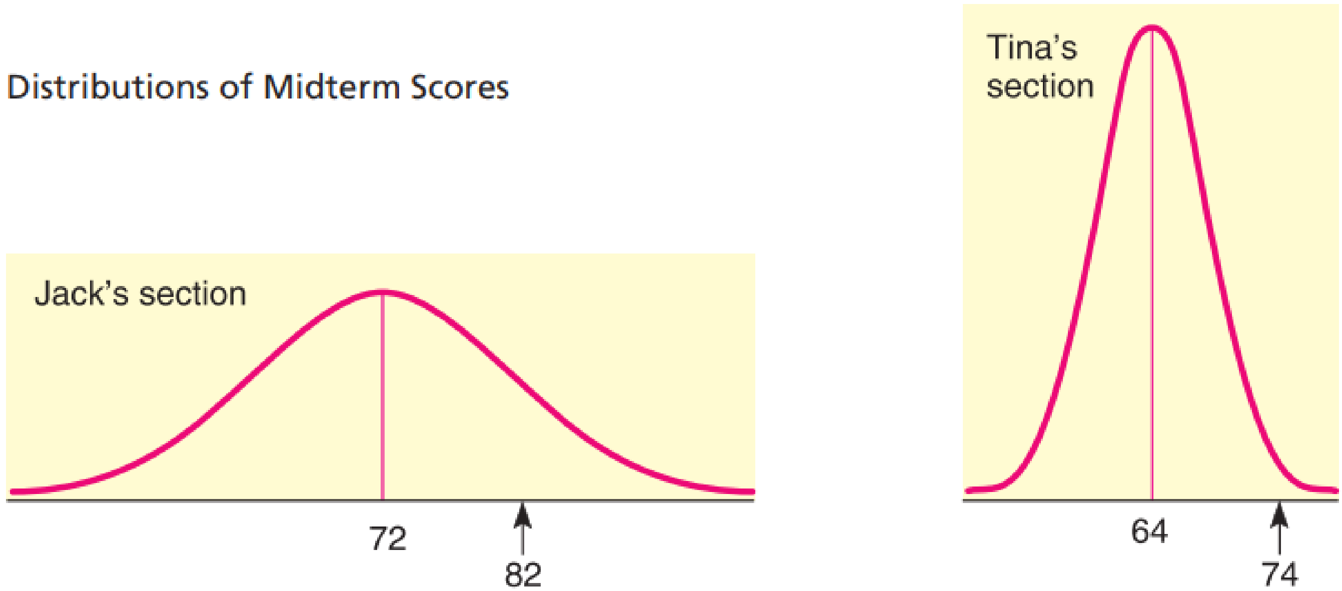
Standard Normal Distribution

Standard Normal Distribution: it is a way to compare and compute for all kind of normal distribution with different pair of values (μ, σ) because it will be very futile task to try to set up a table of areas under the normal curve for each different pair of (μ, σ) combination.

We achieve this standardization by considering how many standard deviations are measurement lies from the mean. In this way we can compare a value in one normal distribution with a value in another, different normal distribution.

> ## For example
>
> Suppose Tina and Jack are in two different sections of the same course. Each section is quite large, and the scores on the midterm exams of each section follow a normal distribution.

> In Tina's section the mean was 64 and her score was 74. In Jack's section the was 72 and his score was 82. Both Tina and Jack pleased that their scores were each 10 points above the average of each respective section.
>
> However, the fact that each was 10 points above average does no really tell us how each did with respect to the other students in the section.
>
> Tina's 74 was higher than most of th eother score in her section, while Jack's 82 is only an upper-middle score in his section. Tina's score is far better with respect to her class than Jack's score is with respect to his class.



The above example demostrates that it is not sufficient to know the differeence b/w a measurement and the mean of a distribution. we need also to consider the spread of the curve, or the standard deviation. What we really want to know is the number of standard deviation b/w a measurement and the mean.

z-number: The number z is the number of standard deviations b/w a measurement x and the mean μ of a normal distribution with standard deviation σ:

$$\begin{pmatrix}\text{Number of standard deviations} \\ \text{between the measurement and} \\ \text{the mean}\end{pmatrix} = \left(\frac{\text{Difference between the measurement and the mean}}{\text{Standard deviation}}\right)$$

Standard Score

The `z-value` or `z score` (also known as standard score) gives the number of standard deviations between the oritinal measurement x and the mean μ of the x distribution.

$$z = \frac{x - \mu}{\sigma}$$

Note: The mean is a special value of a distribution. if x becomes the μ (equal to the mean of the original distribution) then z score will become 0.

$$z = \frac{x - \mu}{\sigma} = \frac{\mu - mu}{\sigma} = 0$$

Standard Score Interpretation

A standard score or z score of a measurement tells us the number of standard deviations the measurement is from the mean.

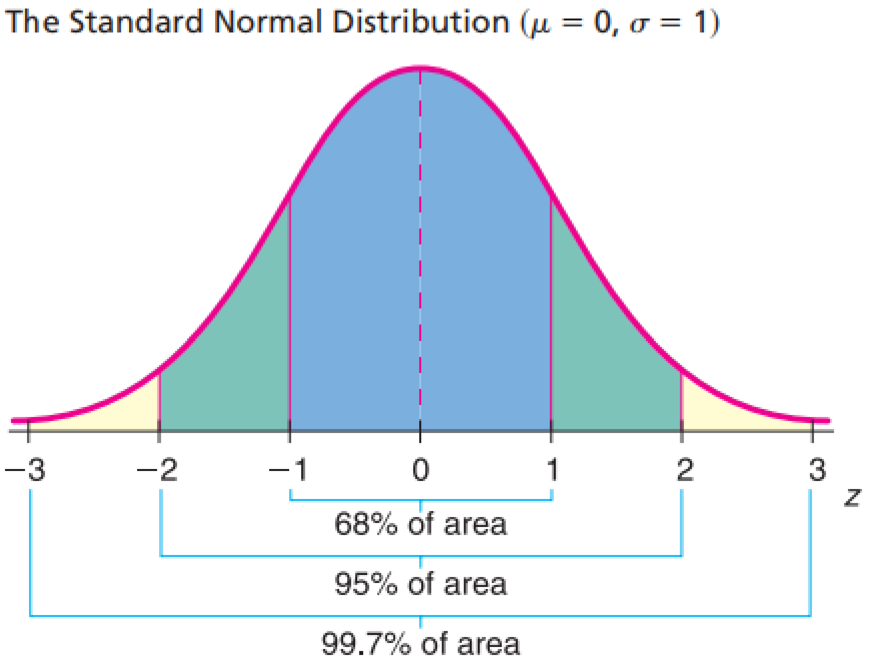
- A Standard score close to zero tells us the measurement is near the mean of the distribution.
- A positive standard score tells us the measurement is above the mean.
- A negative standard score tells us the measurement is below the mean.

Given an X distribution with mean μ and standard deviation σ, the **raw score** corresponding to a z score is $x = z\sigma + \mu$.

`Standard Normal Distribution:` The **Standard Normal Distribution** is a normal distribution with mean $\mu = 0$ and standard deviation $\sigma = 1$.

If the original distribution of **x values is normal**, then the corresponding z values have a normal distribution as well. The z distribution has a mean of 0 and a standard deviation of 1.

Note: Any normal distribution of x values can be converted to the standard normal distribution by converting all x values to their corresponding z values. The resulting standard distribution will always have mean $\mu = 0$ and standard deviation $\sigma = 1$.



> **Standard Normal Distribution Interpretation**
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> When we have the standard normal distribution,
>
> - The standard deviation is 1.
> - the mean is 0.
> - any normal distribution can be converted to a standard normal distribution by converting all the measurement to standard z scores.

Note: The advantage of standard normal distribution is that we need to only calcuate the area under single cure, as in result we have extensive tables that shows the area under the standard normal curve for almost any interval along the z-axis. The

The areas are important because each area is equal to the probability that the measurement of an item selected at random falls in this interval.

- `Note:` Because the normal distribution is continuous, there is no area under the curve exactly over a specific z. therefore
- Probabilities such as $P(z \geq z_1)$ are the same as $P(z > z_1)$.

Area Under any Normal Curve

To find area and probability for a random variable x values to z values using the formula $$z = \frac{x-\mu}{\sigma}$$ after converting the normal curve to standard normal curve we can use the standard normal distribution table.