# Pseudo-Inverse Matrix for Data Scientists

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# 1 Intuition behind Pseudo-inverse

The inverse of a matrix depends on it's adjunct (adj(A)) and it's determinant (det(A)), and is given by $A^{-1} = \frac{adj(A)}{|A|}$. This formula shows that for the matrix A had to be a non-singular square matrix for it's inverse to exist. But we don't always encounter square matrix in our lives and $Moore - Penrose - Pseudoinverse$ of a matrix gives a fair appromation to the inverse of a matrix.

# 2 Applications for a Data Scientists

As a data scientists we come across problems which require computing the inverse of matrices. We have to deal with problem $Y = XW$, where Y is a column matrix of the response/target variables and X is the N x D marrix with N-datasets and D-dimensions.

Each regression problem tries to solve for the weights, W, that minimizes the expected error. And the elegent way tosolve for W is through linear algebra like

$$W_{ML} = X^{-1}Y \tag{1}$$

where X inverse is the input matrix and depends on the size of the input dataset. This matrix is not necessarily a square matrix and hence to compute the weights we do need a Pseudoinverse matrix for X which is given as

$$X^{+} = (X^{T}X)^{-1}X^{T} \tag{2}$$

This was the simplest form of the regression equation. But, a majority of times we preprocess the imnput data (X) or do some feature extraction from it, before feeding it into the model. This extracted feature can be expressed in terms of the basis function $\phi(x)$.

In this case also, we need to calculate the inverse of basis $\phi(x)$ matrix. This is done easily through the pseudoinverse calculation.

$$\phi^{+} = (\phi^{T}\phi)^{-1}\phi^{T} \tag{3}$$

$$W_{ML} = (\phi^{T}\phi)^{-1}\phi^{T}Y \tag{4}$$