CHUA'S CIRCUIT

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1 Abstract

The Chua circuit, known to exhibit chaotic behaviour, has been constructed and studied. The goal was to investigate the experimental underpinning of the theory. The component relationship of the resistors, capacitors, inductors and operational amplifiers have been measured and established within their domain of validity.

2 Introduction

The Chua Circuit is the simplest electronic circuit exhibiting chaos, and many well-known bifurcation phenomena, as verified from numerous laboratory experiments, computer simulations, and rigorous mathematical analysis.

The Chua Circuit was invented in the fall of 1983 (Chua, 1992) in response to two unfulfilled quests among many researchers on chaos concerning two wanting aspects of the Lorenz Equations (Lorenz, 1963). The first quest was to devise a laboratory system which can be realistically modeled by the Lorenz Equations in order to demonstrate chaos is a robust physical phenomenon, and not merely an artifact of computer round-off errors. The second quest was to prove that the Lorenz attractor, which was obtained by computer simulation, is indeed chaotic in a rigorous mathematical sense. The existence of chaotic attractors from the Chua circuit had been confirmed numerically by Matsumoto (1984), observed experimentally by Zhong and Ayrom (1985), and proved rigorously in (Chua, et al, 1986). The basic approach of the proof is illustrated in a guided exercise on Chua's circuit in the well-known textbook by Hirsch, Smale and Devaney (2003).

2.0.1 Chua's Circuit and Realization

The circuit diagram of the Chua Circuit is shown in Figure 1. It contains 5 circuit elements. The first four elements on the left are standard off-the-shelf linear passive electrical components; namely, inductance L ¿ 0, resistance R ¿ 0, and two capacitances C1 ¿ 0 and C2 ¿ 0. They are called passive elements because they do not need a power supply (e.g., battery). Interconnection of passive elements always leads to trivial dynamics, with all element voltages and currents tending to zero (Chua, 1969).

- Local Activity is Necessary for Chaos
- The Chua Diode is Locally Active
- Oscilloscope Displays of Chaos

The Chua Circuit has been built and used in many laboratories as a physical source of pseudo random signals, and in numerous experiments on synchronization studies, such as secure communication systems and simulations of brain dynamics. It has also been used extensively in many numerical simulations, and exploited in avant-garde music compositions (Bilotta et al, 2005), and in the evolution of natural languages (Bilotta and Pantano, 2006).

Arrays of Chua Circuits have been used to generate 2-dimensional spiral waves, 3-dimensional scroll waves, (Munuzuri et al, 1993) and stationary patterns, such as Turing and other exotic patterns, (Munuzuri and Chua, 1997), (Madan, 1993), as illustrated in Figures 11(a), (b), and (c), respectively. Such high-dimensional attractors have been exploited for applications in image processing, neural networks, dynamic associative memories (Itoh and Chua, 2004), complexity (Chua, 1998), emergence (Arena et al, 2005), etc.

To include the effect of the inductor's resistance into the mathematical description, we analyze it as a resistor in series with an ideal inductor. Consequently, the differential equations describing Chua's Circuit read:

$$C_1 \frac{\mathrm{d}V_1}{\mathrm{d}t} = \frac{V_2 - V_1}{R_\mathrm{c}} - I_\mathrm{nl}(V_1)$$

$$C_2 \frac{\mathrm{d}V_2}{\mathrm{d}t} = \frac{V_1 - V_2}{R_\mathrm{c}} + I_L\,,$$

$$L \frac{\mathrm{d}I_L}{\mathrm{d}t} = -V_2 - R_L\, I_L\,.$$

The resistance Rc is the coupling resistor, labeled 1/G in Hobson and Lansbury's article. A short derivation of these equations should be contained in your lab report.

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3 Methods

In the laboratory, the following tasks were done:

1. The inductor we have readily available is one with 10 mH. Take a multimeter and measure its resistance RL.

2. Assemble Chua's circuit. Since the inductor is different from the one used in [1], some resistances should be chosen differently from the circuit diagram: take R1 = 2.2 k and R4 = 1 k, the other values as in the diagram. For the variable resistance Rc we use a multiturn potentiometer which allows fine control of its resistance.

3. Determine experimentally the value of Rc which corresponds to the onset of chaos. Do you see a sharp transition, or a period doubling cascade as for the logistic map?

4. Tune the circuit back into a regime where it is oscillating stably with larger amplitude. Switch the oscilloscope into Y -T-mode, attach the two channel probes to the outputs of the two operational amplifiers, and use the oscilloscope "save" function to store a time series sample of both channel inputs. This data is used later to verify the theoretical response curve of the nonlinear resistor.

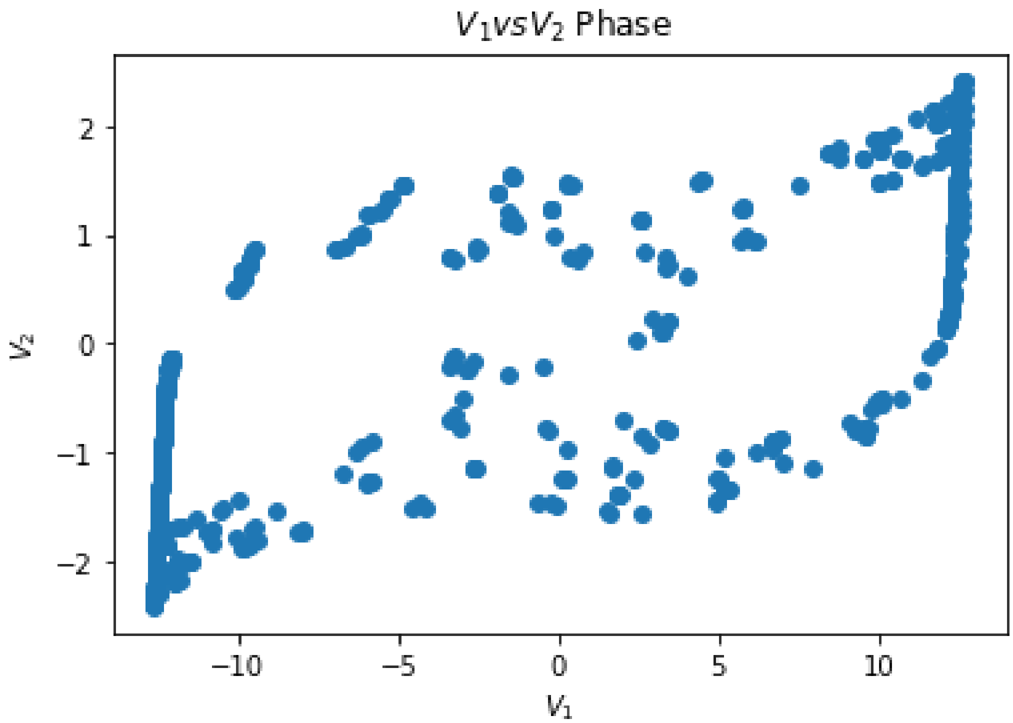
5. If you have time: Use an inductor with 100 mH, also available in the lab, and try if you can—potentially modifying the values of the capacitors or resistors as well— find a regime as well. If this is successful, you will likely obtain less hysteresis in the response curve of the nonlinear resistors, and consequently better agreement of theory 3 and experiment. (Note: the 100 mH inductor might have too big a resistance to excite nonlinear oscillations, so this is not guaranteed to work!)

4 Results

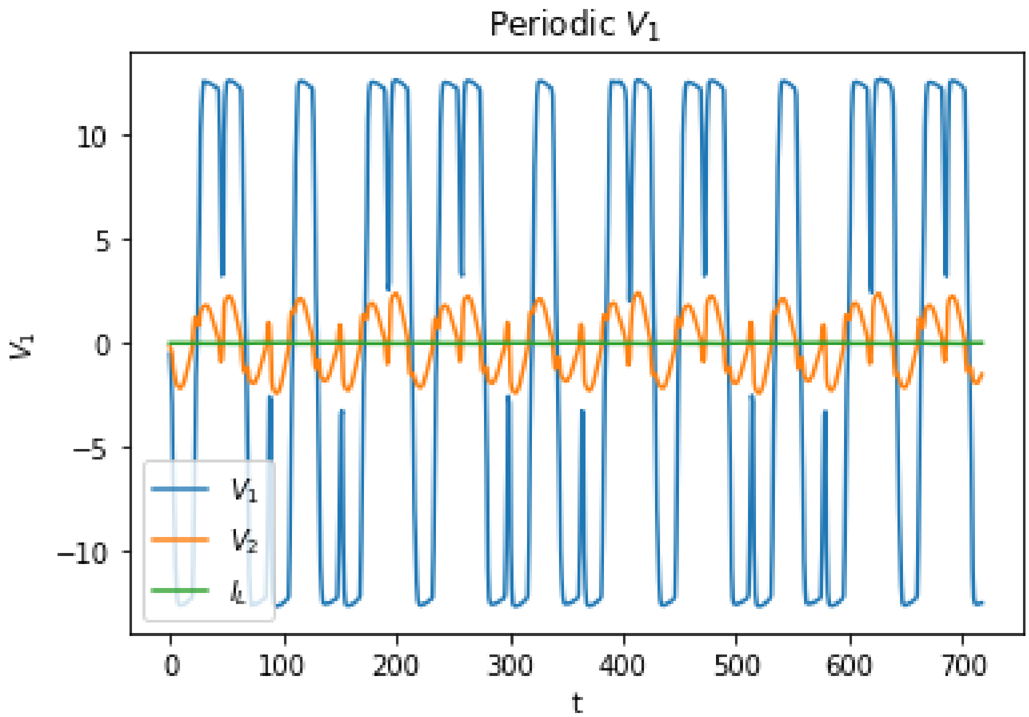
- FIGURE 1, FIGURE 2 AND FIGURE 3 ARE OBTAINED BY CODING UP THE DIFFERENTIAL EQUATIONS CHUA'S CIRCUIT IN PYTHON. THE CODES ARE ALSO SUBMITTED ALONG WITH THE REPORT

- FIGURE 4 AND FIGURE 5 DEPICTS THE RESPONSE CURVE OF INVERSE RESISTORS. IT IS OBTAINED BY USING DATA OBTAINED FROM OSCILLOSCOPE

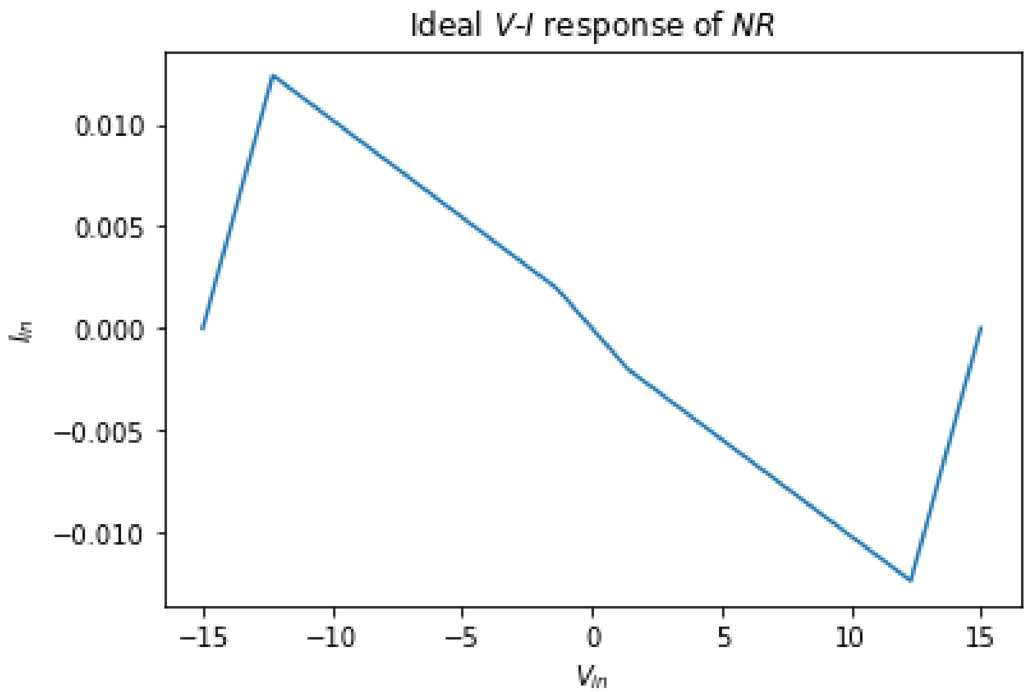
- FIGURE 6 IS THE PICTURE THAT WE OBTAINED IN THE OSCILLOSCOPE WHILE OBSERVING CHAOS PHENOMENA



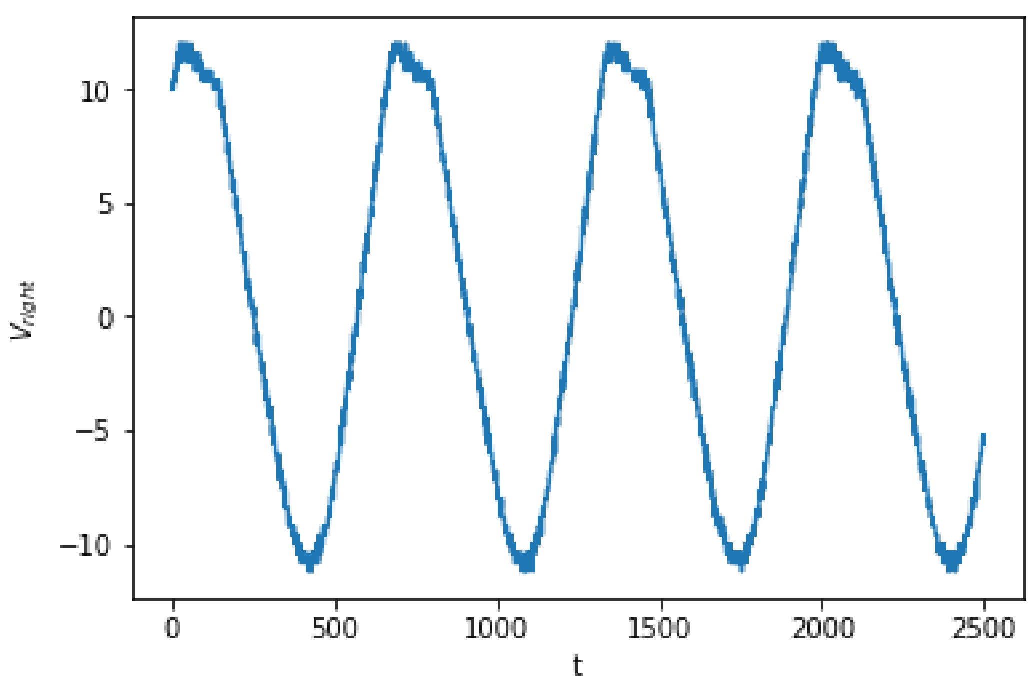
- FIGURE 1



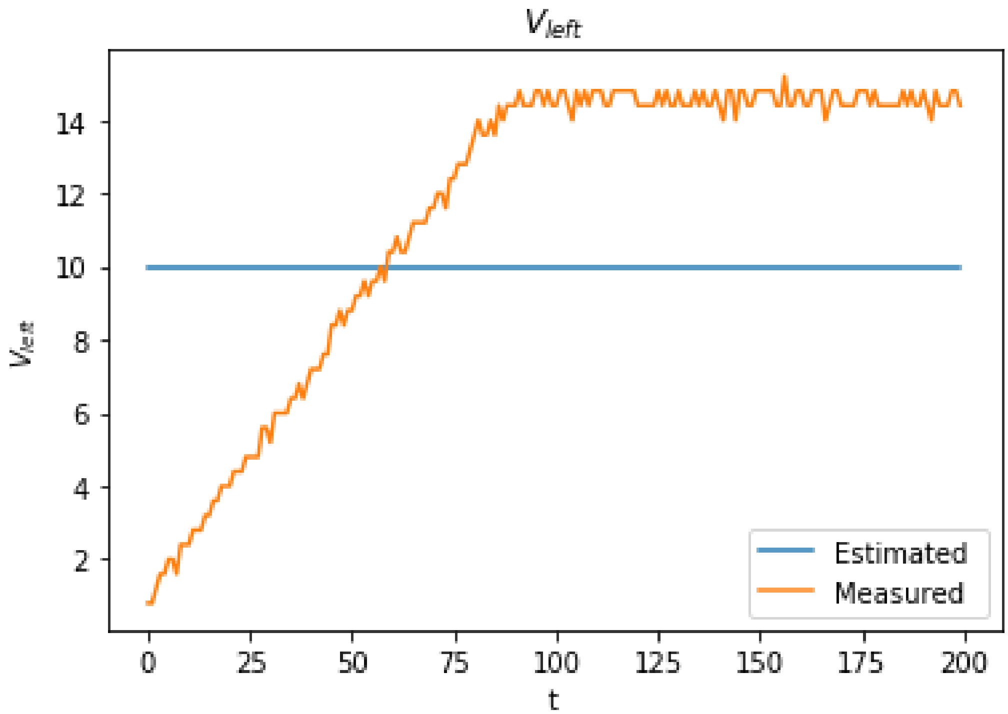
- FIGURE 2



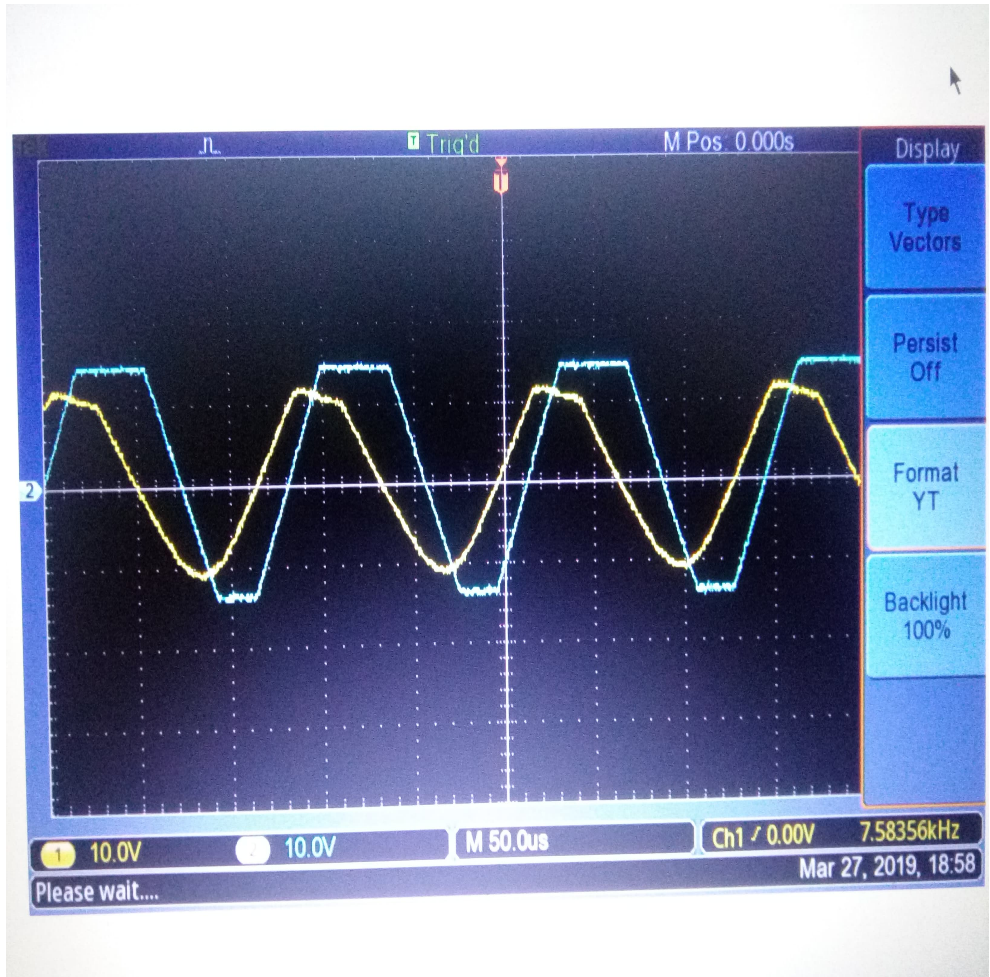
- FIGURE 3



- FIGURE 4


V_{left}
14
12
10
8
6
4
2
V_{left}
Estimated
Measured
0
25
50
75
100
125
150
175
200
t

- FIGURE 5

- FIGURE 6 

5 Conclusions

Hence, In this way Classic Chaotic Phenomenon i.e. CHUA'S CIRCUIT was observed in the lab and simulated.

6 References

1 P.R. Hobson and A.N. Lansbury, A simple electronic circuit to demonstrate bifurcation and chaos, Phys. Educ. 31 (1996), 39–43.

2 Dr. Leon O. Chua, Dept. of EECS, University of California, Berkeley(http://www.scholarpedia.org/article/ Chua$_c$*ircuit*)