

**Q.N.3:** Show that it is never optimal to exercise an American call option on a non-dividend-paying stock before expiration

Given points: 4

**Ans:**

The decision to exercise or hold American call option depends on time value  $t$  and underlying stock value  $L(t)$ . Exercise time  $p$  is chosen to maximize the value of the option.

It is never optimal to exercise an American call option on a non-dividend-paying stock before expiration because that exercise requires payment of the strike price  $K$  (assume). Option holder saves the interest on  $K$ , by holding onto  $K$  until the expiration time.

Let, 2 portfolios: ( $p$  = exercise time)

A1: an American call  $c$ ,  $Ke^{-r(T-t)}$  cash

A2: a share  $L$

Case 1: if Exercise time  $p < T$ , then,  $A1 = (L-K) + Ke^{-r(T-p)} < L = A2$

Case 2: if Exercise time  $p = T$ , then,  $A1 = \max(L-K, 0) + K = \max(L, K) \geq L = A2$

Which follows,  $A1 \geq A2$  all the times, therefore, one should never take  $p < T$ .