Dhirubhai Ambani Institute of Information and Communication Technology

ADSP

LAB – 8 : Square Wave

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EXERCISES

Question) Square wave experimentation.

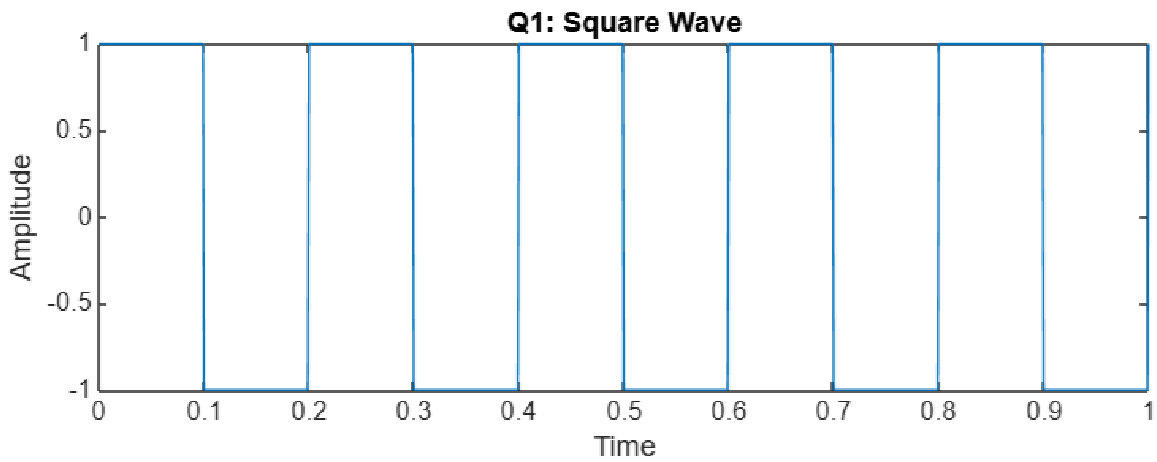
1. **Generate a square wave signal**
2. **Define an impulse response of the ideal low pass filter**
3. **Convolve square wave signal with impulse response**
4. **Plot 1 and 3, and note the observations of gibbs formula**
5. **Write derivation and theorem of gibbs formula and explain**

CODE:

1)

```
% 202411012

% 1) Generating a square wave

% Parameters
Fs = 1000;                  % Sampling frequency (Hz)
T = 1;                      % Duration (sec)
t = 0:1/Fs:T;               % Time vector
```

```
f0 = 5;                       % Square wave frequency

% Generate square wave
x = square(2*pi*f0*t);

% Plot the square wave
figure;
subplot(2,1,1);
plot(t, x); title('Q1: Square Wave'); xlabel('Time'); ylabel('Amplitude');
axis tight;
```

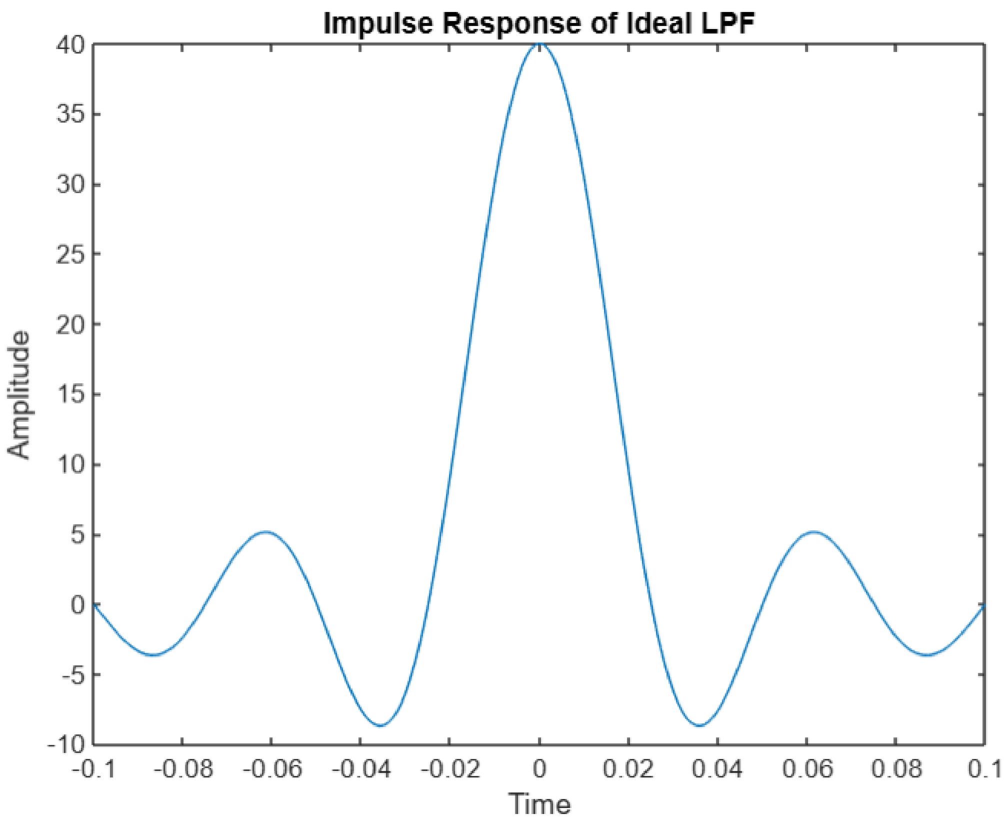


2)

An **ideal LPF** in time domain is:

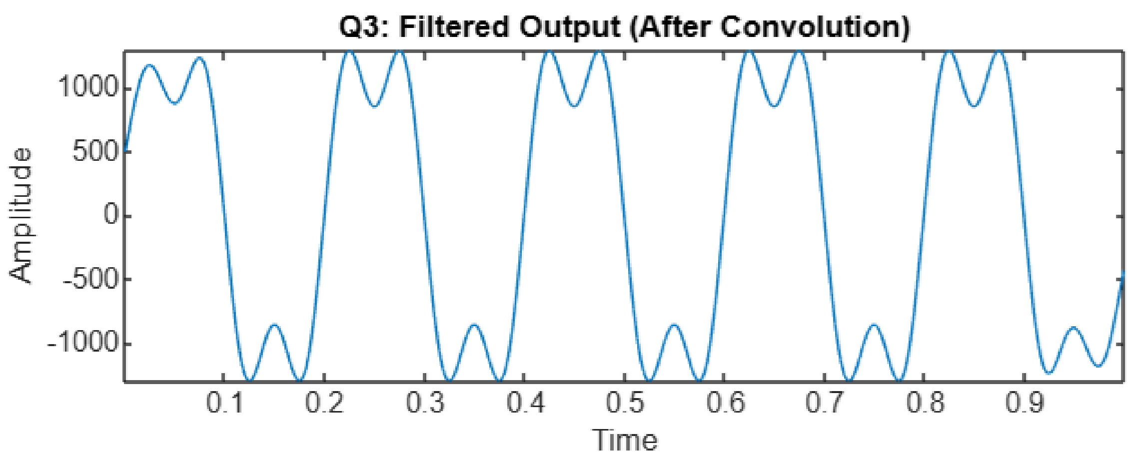
$$h(t) = 2f_c \cdot \mathrm{sinc}(2f_c t)$$

```
w% Ideal low-pass filter
fc = 20;                          % Cutoff frequency
t_h = -0.1:1/Fs:0.1;              % Time vector for h(t)
h = 2 * fc * sinc(2 * fc * t_h);  % Impulse response

% Optional: plot impulse response
figure;
plot(t_h, h); title('Impulse Response of Ideal LPF'); xlabel('Time');
ylabel('Amplitude');
```



3)

```
% Convolve square wave with LPF
y = conv(x, h, 'same');  % 'same' keeps output length same as x

% Plot filtered output
subplot(2,1,2);
plot(t, y); title('Q3: Filtered Output (After Convolution)'); xlabel('Time');
ylabel('Amplitude');
axis tight;
```



4)

After convolution, observe:

- Oscillations near the sharp edges of the square wave (especially where signal jumps from -1 to 1).
- These **ringing artifacts** are due to **Gibbs phenomenon** — overshoots near discontinuities.

Gibbs phenomenon:

- When approximating a **discontinuous signal** using a finite number of Fourier terms (or filtering using ideal filters), **overshoots** occur near jumps.
- These overshoots **do not vanish**, even if you add more terms — they settle to a constant height (~9% overshoot for square waves).

5)

STEP 5: Gibbs Phenomenon — Derivation & Theorem

◆ Theorem (Gibbs Phenomenon):

Let $f(t)$ be a piecewise smooth, periodic function with a jump discontinuity at $t = t_0$. The partial sum $S_N(f)$ of the Fourier series **overshoots** at that point and does not vanish even as $N \to \infty$:

$$\lim_{N\to\infty} \sup_{t\to t_0} |S_N(f)(t) - f(t)| \approx 0.08949 \cdot \Delta f$$

Where:

- $\Delta f = f(t_0^+) - f(t_0^-)$ is the jump size.

This ~**9% overshoot** is persistent and forms the **Gibbs ringing** observed in reconstructed or filtered signals.

◆ Sketch of Derivation:

- Fourier partial sums reconstruct a signal using sinusoids:

$$S_N(t) = \sum_{n=-N}^{N} c_n e^{j2\pi nt/T}$$

- Around a jump (e.g., square wave's rising edge), the sinc-like kernel causes overshooting due to **sidelobe accumulation**.
- The oscillations converge, but the **maximum overshoot amplitude remains**.

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