1.2.13 Expectation of Normal Distribution:-

→ Expected value $E[x]$ can be found by simply multiply the probability distribution function with x and integrate over all possible values.

$$\rightarrow E[x] = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{+\infty} x \cdot e^{-1/2\left(\frac{x-\mu}{\sigma}\right)^2} \cdot dx$$

$(x + \mu - \mu)$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} (x-\mu) \cdot e^{-1/2((x-\mu)/\sigma)^2} \cdot dx + \mu \cdot \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-1/2\left(\frac{x-\mu}{\sigma}\right)^2} \cdot dx$$

→ Let $y = x - \mu$;

$$\therefore E[x] = \underbrace{\frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} y \cdot e^{-1/2(y/\sigma)^2} \cdot dx}_{\text{odd function}} + \mu \cdot \underbrace{\int_{-\infty}^{\infty} f_x(x) \cdot dx}_{=1}$$

odd function.
i.e: $f(-y) = -f(y)$

→ First one is symmetric about y-axis, hence value of that integral is 0.

$$\therefore E[x] = 0 + \mu \cdot 1 = \mu$$

$$\therefore \boxed{E[x] = \mu}$$

⇒ Variance :- σ^2

⇒ Standard deviation :- $\sigma = \sqrt{\sigma^2} = \sigma$

Standard Normal Distribution :-

→ The expected value of a standard normal random variable X is:
+ expected value = 0
+ Variance = 1
+ standard deviation = 1

GATE-2008 :- Let X be a random variable following normal distribution with mean +1 and variance +4. Let Y be another normal variable with mean −1 and variance unknown. If ($P(X <= -1)$) $= P(Y >= 2)$, The standard deviation of Y is:
(A.) 3 (B.) 2 (C.) $\sqrt{2}$ (d.) 1

⇒ For X; $\mu_x = 1$ & $\sigma_x^2 = 4$;
then $\sigma_x = 2$.

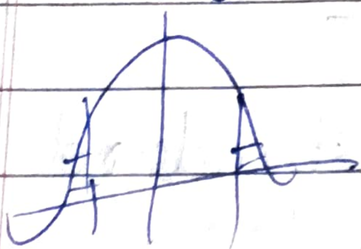
→ For Y; $\mu_y = -1$ and $\sigma_y^2 = ?$, to find σ_y.

$$P(X \le -1) = P(Y \ge 2)$$

$$\therefore Z_x = \frac{X - \mu_x}{\sigma_x} = \frac{-1-1}{2} = -1$$

$$z_Y = \frac{Y - \mu_Y}{\sigma_Y} = \frac{2-(-1)}{\sigma_Y} = \underline{\underline{\frac{3}{\sigma_Y}}}$$

$$\therefore \underbrace{P(z \le -1)} = P\left(Z \ge \frac{3}{\sigma_Y}\right)$$



$$\Downarrow$$

$$P(Z \ge 1) = P\left(Z \ge \frac{3}{\sigma_Y}\right)$$

$$\therefore \frac{3}{\sigma_Y} = 1 \Rightarrow \underline{\underline{\sigma_Y = 3}}$$