Sunday

5. Permutation & Combinations

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5.1 Combinatorics Basics

⇒ Permutation: It is different arrangements of a given number of elements taken one by one, or some, or all at a time.

→ For example, if we have two elements A and B, then there are two possible ways of arrangements, AB and BA.

→ Number of permutations when 'r' elements are arranged out of a total of 'n' elements is: $^nP_r = \frac{n!}{(n-r)!}$

e.g.: Let n=4 (A,B,C,D) and r=2.
Total permutations $= \frac{n!}{(n-r)!}$

$= \frac{4!}{(4-2)!}$

$= \frac{4!}{2!} = \frac{4\times3\times2!}{2!}$

$= 12$

→ These twelve permutations are:
AB, AC, AD, BA, BC, BD, CA, CB, CD, DA, DB, DC.

⇒ **Combination:-**

→ It is the different selections of a given number of elements taken one by one, or some, or all at a time.

→ For example, if we have two elements A and B, then there is only one way select two items, we select both.

→ Number of combinations when 'r' elements are selected out of a total of 'n' elements is:-

$$^nC_r = \frac{n!}{r!(n-r)!}$$

e.g.:- Let n=4 (A, B, C, D) and r=2 (All combinations of size 2).

→ Total combinations will be $= \frac{4!}{2!(4-2)!}$

$= \frac{4 \times 3 \times 2!}{2! \times 2 \times 1}$

$= 6$

→ Can also be solved like:

$= \frac{4 \times 3}{2 \times 1} = 6$

⇒ Counting Principles:-

⇒ There are two basic counting principles: sum rule and product rule.

1.) Sum Rule:- If a task can be done in one of n_1 ways or one of n_2 ways, where none of the set of n_1 ways is the same as any of the set of n_2 ways, then there are $n_1 + n_2$ ways to do the task.

e.g.:- No. of ways to order a dish from 5 veg dishes and 6 non-veg dishes is $5+6 = 11$ ways

2.) Product Rule:- If a task can be broken down in a sequence of K-subtasks, where each subtask can be performed in $n_1, n_2, n_3, \dots n_k$ respectively, then the total no. of ways the task can be performed is $n_1 * n_2 * \dots * n_k$.

e.g.:- In how many ways can 3 winning prizes be given to the top 3 players in a game played by 12 players?

→ We have to distribute 3 prizes among 12 players. This task can be divided into 3 subtasks of assigning a single prize to a certain player.

→ Giving out the first prize can be done in 12 different ways. After giving out the first prize, two prizes remain and 11 players remain.

→ Similarly, the second prize and third prize can be given in 11 ways and 10 ways.

→ The total number of ways by the product rule = $12 \times 11 \times 10 = 1320$

Question:1 In how many ways can a person choose a project from three lists of projects of sizes 10, 15 and 19 respectively?

→ List 1: 10 ways
List 2: 15 ways
List 3: 19 ways

∴ Total no. of ways to select a project
$= 19 + 15 + 10$
$= 34 + 10 = 44$ ways

→ If, it was to select 2 projects, then, $= \frac{\cancel{44}^{22} \times 43}{2 \times 1} = 946$ ways

Question:-2 How many distinct license plates are possible in the given format- Two alphabets in uppercase, followed by two digits then a hyphen and finally four digits. sample: AB12-3456.

⇒ Two alphabets in uppercase = 26×26 followed by two digits.

→ No. of ways to select a digit = 10
→ No. of ways to select an alphabet = 26

∴ Total no. of distinct licence plates can be formed = $26 \times 26 \times 10 \times 10 \times 10^4$
$= \underline{676 \times 10^6}$ ways

Question:-3 How many variable names of length upto 3 exist if the variable names are alphanumeric and case sensitive with the restriction that the first character has to be an alphabet?

⇒ $52 + 10 = 62$ ways

compulsory alphabet = 52 ways (A-Z, a-z)

→ For variable length of 1 : 52 ways
→ for variable of length 2 : 52×62 ways
→ For variable of length 3 : $52 \times 62 \times 62$ ways

∴ Total ways = $52(1 + 62 + 62^2) = 52(3906 + 1)$
$= 52(1 + 62(63)) = 52(3907)$
$= \underline{203164}$ ways