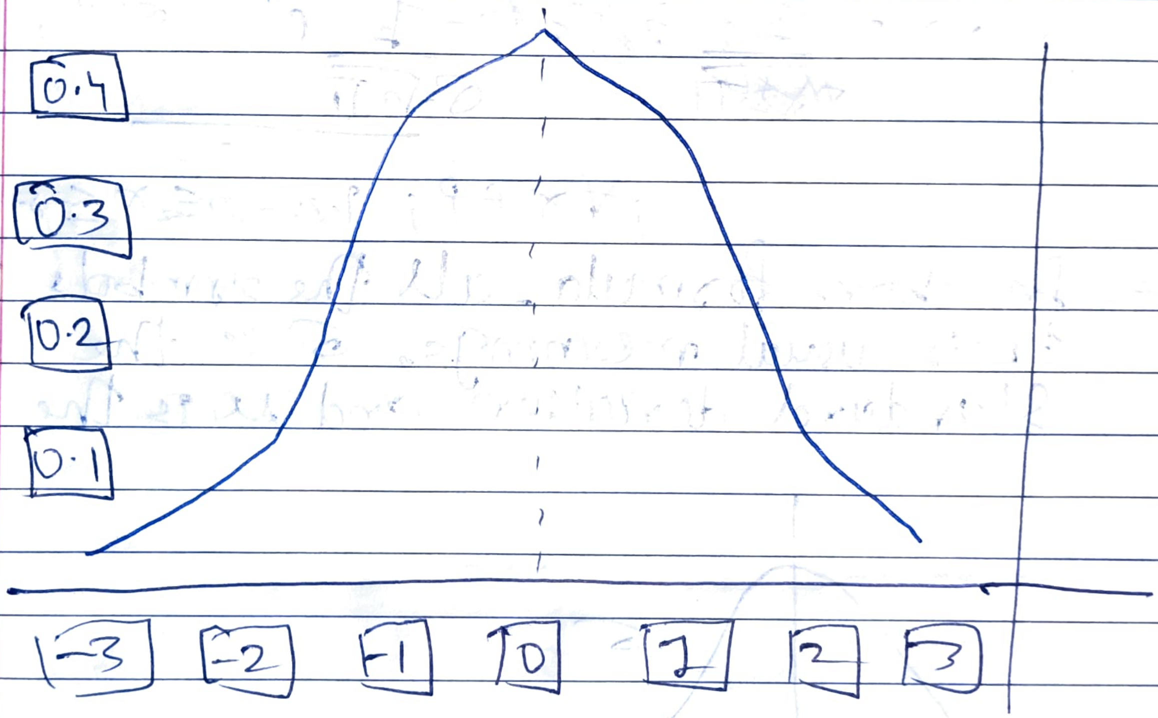
1.2.12 Normal Distribution:-

→ It is also known by other names such as –
Gaussian Distribution, Bell shaped Distribution.



→ It can be observed ~~that~~ from the above graph that the ~~gaussian~~ distribution is symmetric about its center, which is also the mean (0 in this case).

→ This makes the probability of events at equal deviations from the mean, equally probable.

→ The density is highly centered around the mean, which translates the lower probabilities for values away from the mean.

Date 4/10/23
Page 6

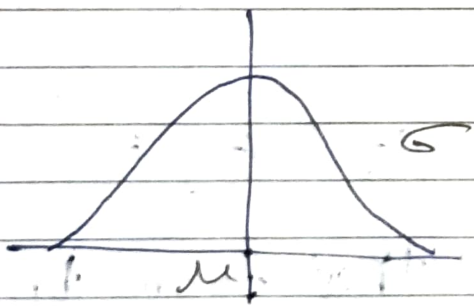
⇒ Probability Density Function :-

→ The probability density function of the general normal distribution is given as :-

$$f_X(x) = \cancel{\frac{1}{\sigma\sqrt{2\pi}}\, e^{-1/2}} \text{ i.e. } = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-1/2\left(\frac{x-\mu}{\sigma}\right)^2}$$

$; \forall x \in R;$ i.e. $-\infty \leq x \leq \infty$

⇒ In above formula, all the symbols have their usual meanings, σ is the standard deviation and μ is the Mean.



⇒ The z-score is a measure of how many standard deviations away from a data point is from the mean. Mathematically;

$$\boxed{z\text{-score} = \frac{x-\mu}{\sigma}}$$

⇒ Standard Normal Distribution:-

⇒ In the General Normal Distribution, if the Mean is set to 0 and the Standard deviation is set to 1, then the corresponding distribution obtained is called the Standard Normal Distribution.

⇒ The probability Density function now becomes:-

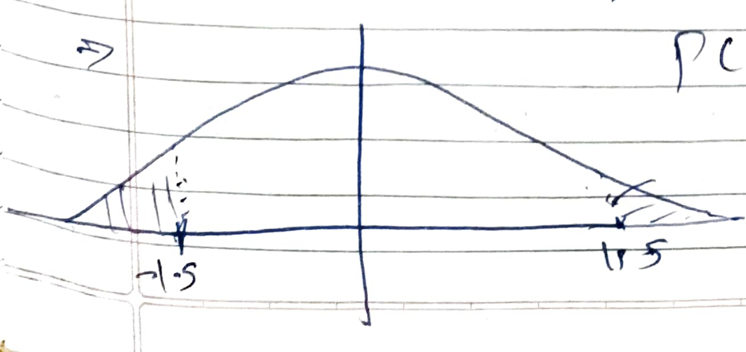
$$f_X(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \quad , \sigma = 1 \;\&\; \mu = 0 \quad , -\infty < x < \infty$$

⇒ If X is a normal random variable, with $E(X) = \mu$ and $V(X) = \sigma$, the random variable $Z = \frac{X-\mu}{\sigma}$, is a normal random variable with $E[Z] = 0$ and $V[Z] = 1$. That is, Z is a standard normal variable.

⇒ $P(1.4 < Z < 1.8) = P(Z \leq 1.8) - P(Z \leq 1.4)$

⇒ $P(Z \geq 1.3) = 1 - P(Z \leq 1.3)$

⇒



$P(Z \leq -1.5) = P(Z \geq 1.5)$
$= 1 - P(Z \leq 1.5)$

Date 4/10/23
Page 62

EXAMPLE:- Suppose that the current measurements in a strip of wire are assumed to follow a normal distribution with a mean of 10 milliamperes and a variance of 4 (milliamperes)^2. What is the probability that a measurement exceed 13 mA?

$\Rightarrow$ $x = 13$;
$\mu = 10$
$\sigma^2 = 4$
$\Rightarrow \sigma = 2$

To find $P(x \geq 13)$

$$z = \frac{x-\mu}{\sigma} = \frac{13-10}{2} = \frac{3}{2} = 1.5$$

$$\therefore P(z \geq 1.5) = 1 - P(z \leq 1.5)$$
$$= 1 - 0.93319 \text{ [from table]}$$
$$= 0.06681$$

$\Rightarrow$ The z-score is a measure of how many standard deviations away a data point is from the mean.

$\Rightarrow$ Mathematically, z-score $= \dfrac{x-\mu}{\sigma}$.

$\Rightarrow$ The exponent 'e' in the above formula is the square of the z-score times $-1/2$.

→ This is actually in accordance to the observations that we made above.

→ Values away from the mean have a lower probability compared to the values near the mean.

→ Values away from the mean will have a higher z-score and consequently a lower probability since the exponent is negative. The opposite is true for values closer to the mean.

→ This gives way for the 68-95-99.7 rule, which states that the percentage of values that lie within a band around the mean in a normal distribution with a width of two, four and six standard deviations, comprise 68%, 95% and 99.7% of all the values. The figure given below shows this rule: