1.2.8. Exponential Distribution:

→ Suppose we are posed with the question- How much time do we need to wait before a given event occurs?

→ Since, the time we need to wait is unknown, we can think of it as a Random variable.

→ For what time? - Comes exponential distribution.

→ If the probability of the event happening in a given interval is proportional to the length of the interval, then the Random variable has an exponential distribution.

→ The support (set of values the Random variable can take) of an Exponential Random Variable is the set of all Positive real numbers.

⇒ **Probability Density Function:**

→ For a positive real number λ the probability density function of a Exponentially distributed Random variable is given by:-

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x} & ; \text{if } x \in R_X \quad [R_X = [0, \infty)] \\ 0 & ; \text{if } x \notin R_X \end{cases}$$

→ To check if the above function is a legitimate probability density function, we need to check if it's integral over its support is 1.

$$\int_0^\infty \lambda \cdot e^{-\lambda x} \cdot dx = \lambda \cdot \left[\frac{e^{-\lambda x}}{-\lambda}\right]_0^\infty \quad [\text{for } x \text{ tends to } \infty, \text{ it's } 0]$$

$$= -1 \cdot [0 - 1] = \underline{1}$$

Cumulative Density function [CDF]:-

→ As we know, the cumulative density function is nothing but the sum of probability of all events upto a certain value of $x = t$.

→ In the Exponential distribution, the cumulative density function $F(x)$ is given by-

$P(X \leq x)$ $\quad R_x = [0, \infty)$

for $P(x \leq t) = F(x) = \int_0^t \lambda \cdot e^{-\lambda x} \cdot dx$

$$= \lambda \cdot \left[\frac{e^{-\lambda x}}{-\lambda}\right]_0^t$$

$$= -1 \cdot [e^{-\lambda t} - 1] = 1 - e^{-\lambda t}$$

$$\therefore \boxed{F(x) = CDF = 1 - e^{-\lambda t}}$$

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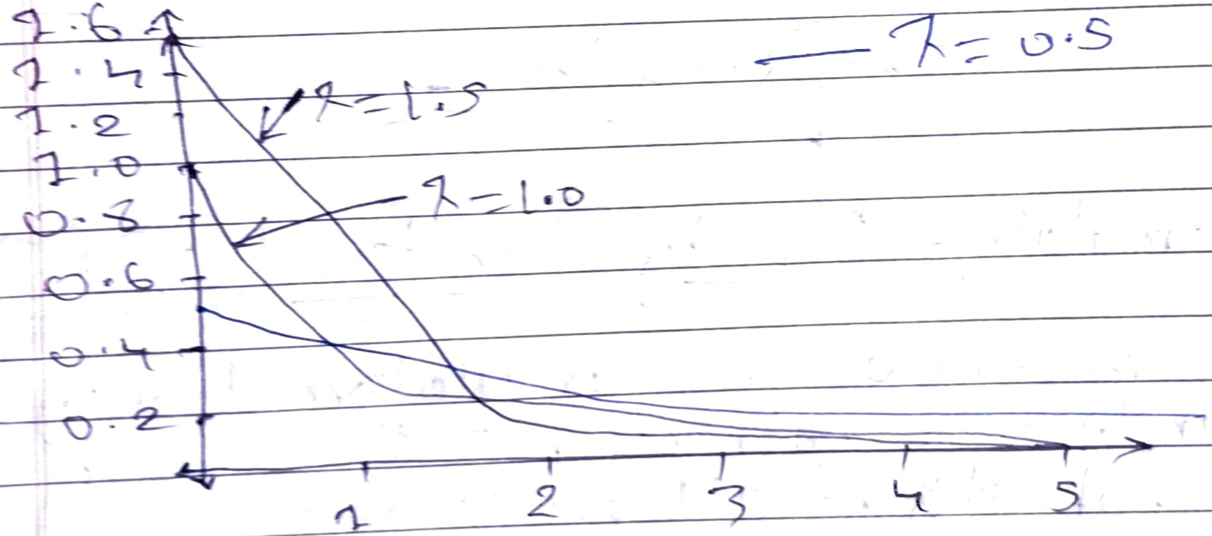
e.g.:- $P(x>5) = 1 - P(x \le 5)$

$P(2 \le x \le 4) = P(x \le 4) - P(x \le 2)$

→ Here, λ is the rate parameter and it affects on the density function are illustrated below:-



Properties of Exponential Distribution: