1.2.7 Expectation of Poisson distribution:

→ The expected value of the Poisson distribution can be found by summing up products of values with their respective probabilities.

$$E[X] = \sum_{x=0}^{\infty} x \cdot p_X(x) \quad [p_X(x) = P(X=x)]$$

⇒ We know that $p_X(x) = \frac{e^{-\lambda} \cdot \lambda^x}{x!}$

$$= \sum_{x=0}^{\infty} x \cdot \frac{e^{-\lambda} \cdot \lambda^x}{x!}$$

$$= 0 + \sum_{x=1}^{\infty} x \cdot \frac{e^{-\lambda} \cdot \lambda^x}{x!} \quad [x=0,\ 0 \cdot x = 0]$$

→ Let $x = y+1$;

$$= \sum_{y=0}^{\infty} (y+1) \frac{e^{-\lambda} \cdot \lambda^{y+1}}{(y+1)!}$$

$$= \sum_{y=0}^{\infty} \cancel{(y+1)} \cdot \frac{e^{-\lambda} \lambda^y \cdot \lambda}{\cancel{(y+1)} \cdot y!}$$

$$= \lambda \cdot \sum_{y=0}^{\infty} \frac{e^{-\lambda} \cdot \lambda^y}{y!}$$

$$= \lambda \cdot \sum_{y=0}^{\infty} p_Y(y) \quad [\text{sum of all probability is } 1]$$

$$= \lambda \qquad \boxed{\therefore E[X] = \lambda} \quad \boxed{\lambda = np}$$

⇒ Variance & standard deviation :-

→ The Variance of the Poisson distribution can be found using the Variance formula -

→ $Var[X] = \lambda$

→ Standard deviation $[X] = \sqrt{\lambda}$

GATE-2017 :- If a random variable X has a Poisson distribution with mean 5, then the expression $E[(x+2)^2]$ equals ____

NOTE :- This question appeared as NAT.
(A) 54 (B) 55 (C) 56 (D) 57

⇒ $E[x] = 5$

$V[X] = E[x] = 5$

$V[x] = E[x^2] - (E[x])^2$

$\therefore 5 = E[x^2] - (5)^2$
$\therefore E[x^2] = 5+25 = 30$
$\therefore E[x^2] = 30$

$E[(x+2)^2] = E[(x^2+4x+4)]$

$= E[x^2] + 4 \cdot E[x] + 4$
$= 30 + 4 \cdot 5 + 4 = 30 + 20 + 4$
$= 54$

ISRO CS-2009: If the pdf of a Poisson distribution is given by $f(x) = \frac{e^{-2} \cdot 2^x}{x!}$, then the mean is:-

(A.) 2^x . (B.) 2 (C.) -2 (D.) 1

→ Comparing it with $f(x) = \frac{e^{-\lambda} \cdot \lambda^x}{x!}$

$\lambda = 2$

GATE-2013:- Suppose 'p' is the number of cars per minute passing through a certain road junction between 5PM and 6PM, and 'p' has a Poisson distribution with mean 3. What is the probability of observing fewer than 3 cars during any given minute in this interval?

(A.) $8/(2e^3)$ (B.) $9/(2e^3)$ (C.) $17/(2e^3)$
(D.) $26/(2e^3)$

→ From question, mean $= E[x] = \lambda = 3$

To find $P(x<3) = P(x=0) + P(x=1) + P(x=2)$

$$= \frac{e^{-3} \cdot 3^0}{0!} + \frac{e^{-3} \cdot 3^1}{1!} + \frac{e^{-3} \cdot 3^2}{2!}$$

$$= \frac{1}{e^3}\left[1 + 3 + \frac{9}{2}\right] = \frac{1}{e^3}\left(\frac{8+9}{2}\right)$$

$$= \frac{17}{2e^3}$$