1.2.6 Poisson distribution:-

→ Suppose an event can occur several times within a given unit of time. When the total number of occurrences of the event is unknown, we can think of it as a random variable. This random variable follows the Poisson Distribution.

→ The Poisson distribution is a limiting case of the Binomial distribution when number of trials becomes very large and the probability of success is small.

→ The probability of "x" successes in "n" trials in a Binomial Experiment with success probability "p", is:-

$$P(x) = \binom{n}{x} \cdot p^x \cdot (1-p)^{n-x}$$

→ Let us denote the Expected value of the Random variable by $\lambda = np$

→ Rewriting $P(x)$ in terms of λ, $p = \frac{\lambda}{n}$

$$\therefore P(x) = \binom{n}{x} \left(\frac{\lambda}{n}\right)^x \cdot \left(1 - \frac{\lambda}{n}\right)^{n-x}$$

Date 4/10/23
Page 39

→ Expanding Binomial coefficient;

$$P(x) = \frac{n(n-1)\cdot(n-2)\cdots(n-x+1)}{x!}\cdot\left(\frac{\lambda}{n}\right)^x\left(1-\frac{\lambda}{n}\right)^{n-x}$$

$$= \frac{\cancel{n}}{\cancel{n}}\left(\frac{n-1}{n}\right)\cdot\left(\frac{n-2}{n}\right)\cdots\left(\frac{n-x+1}{n}\right)\cdots\frac{\lambda^x}{x!}\left(1-\frac{\lambda}{n}\right)^{n}\cdot\left(1-\frac{\lambda}{n}\right)^{-x}$$

$n \to \infty$

$$\lim_{n\to\infty}\left(1-\frac{\lambda}{n}\right)^n = e^{\lim_{n\to\infty} \frac{-\lambda}{\cancel{n}} \times \cancel{n}} = e^{-\lambda}$$

$$= 1\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\cdots\left(1-\frac{x+1}{n}\right)\cdot\frac{\lambda^x}{x!}\cdot e^{-\lambda}$$

as $n \to \infty$; $c/n = 0$

$$= 1\cdot 1\cdot 1\cdots 1\cdot\frac{\lambda^x}{x!}\cdot e^{-\lambda}$$

$$\therefore \boxed{P(x) = e^{-\lambda}\cdot\frac{\lambda^x}{x!}}$$

Probability Mass function for Poisson distribution;

$$PMF = P(x) = \frac{e^{-\lambda}\cdot\lambda^x}{x!}$$

where $\lambda = np$

EXAMPLE:- For the case of the thin copper wire, suppose that the number of flaws follows a Poisson distribution with a mean of 2.3 flaws per millimeter. Determine the probability of exactly 2 flaws in 2 millimeter of wire.

⇒ for 1 mm; $\lambda = 2.3$

for 2 mm; $\lambda = 2 \times 2.3 = \underline{4.6}$

$$P(x=2) = \frac{e^{-\lambda} \cdot \lambda^{x}}{x!}$$

$$= \frac{e^{-4.6} \cdot (4.6)^2}{2!}$$

$$= e^{4.6} \cdot \left(\frac{21.16}{2}\right)$$

$$= e^{-4.6} \times 10.58$$

$$\Rightarrow P(x) = \underline{1.117}$$ 0.10635