1.3.3 Properties of Conditional Probability:-

⇒ Conditional probability $= P(A|B)$

$$= \frac{P(A \cap B)}{P(B)}$$

Conditional Probability of Independent Events:-

→ If A & B are independent events,
then, $P(A \cap B) = P(A) \cdot P(B)$.

→ If A and B are independent events the conditional probability of event B given event A, P(B|A) is the essentially the probability of event B, P(B).

→ The formula is given by $P(B|A) = P(B)$.

→ Or, the conditional probability of two independent events are:

- When given event A, the probability of event B occuring is given by
$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{P(A).P(B)}{P(A)} = P(B)$$

- And, the given event B, probability of event A occuring is given by
$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A).P(B)}{P(B)} = P(A)$$

Conditional Probability of Mutually Exclusive Events:-

→ Under the probability theory, mutually exclusive events are events that cannot occur simultaneously.

→ Therefore, the probability of mutually exclusive events is always 0.

i.e., $P(B|A) = 0$, and $P(A|B) = 0$

Date 9/10/23
Page 73

Chain Rule or Multiplication Rule:-

→ Let E be an event happening given F be another event that has occurred. In that condition, the formula of conditional probability can be written as :-

$$P(E \cap F) = P(E|F) \cdot P(F)$$

→ This is known as a chain rule or multiplication rule.

→ Typically, it states that the probability of observing events, E and F, is the product of the probability of observing the F event and the probability of observing E given that event F has been observed.

→ The generalized form of multiplication rule is;

$$P(E_1 \cap E_2 \cap \ldots \cap E_n) = P(E_1) \cdot P(E_2|E_1) \cdot P(E_3|E_2, E_1) \ldots P(E_n|E_1 \ldots E_{n-1})$$

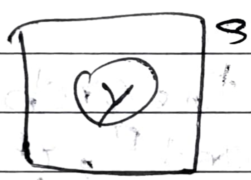
$$= P(E_1) \cdot P(E_2|E_1) \ldots P(E_n|E_1 \ldots E_{n-1})$$

⇒ Following are some fundamental properties of conditional probability:-

1.) **Property-1:-**

→ Suppose, X and Y be the 2 events of a sample space S of an experiment, then it can be said that;

$$P(S|Y) = P(Y|Y) = 1$$



$$P(S\cap Y) = P(Y)$$

2.) **Property-2:-**

⇒ Let X and Y are 2 events of a sample space S, and F is the event such that $P(F) \neq 0$;

$$P((X\cup Y)|F) = P(X|F) + P(Y|F) - P((X\cap Y)|F)$$

From; $P(X\cup Y) = P(X) + P(Y) - P(X\cap Y)$

3.) Property-3 :-

→ In conditional probability, the order of the sets or events matters, so;

$$P(A|B) \neq P(B|A)$$

→ The compliment formula holds only in the context of the first ~~argument~~ argument, there is not any corresponding formula for $P(A|B')$.

- Hence, $P(A|B') \neq 1 - P(A|B)$

→ But, $P(A'|B) = 1 - P(A|B)$