1.2.11 Expectation of Uniform Distribution:-

→ The PDF is as follows;

$$F(x) = \begin{cases} 1/b-a; & a \le x \le b; \\ 0; & \text{otherwise} \end{cases}$$

~~E(x)~~

⇒ **Expected or Mean-Value:-**

→ Using the basic definition of Expectation we get -

$$E(x) = \int_{-\infty}^{\infty} x \cdot f(x)\, dx.$$

$$= \int_a^b \frac{x}{b-a} \cdot dx$$

$$= \frac{1}{b-a} \left[\frac{x^2}{2} \right]_a^b$$

$$= \frac{(b^2 - a^2)}{2 \cdot b - a} = \frac{(b+a)\cancel{(b-a)}}{2 \cdot \cancel{(b-a)}} = \frac{b+a}{2}$$

$$\therefore \boxed{E(x) = \frac{b+a}{2}}$$

⇒ <u>Variance:</u>

⇒ Using the formula for variance;

$$V[x] = E[x^2] - (E[x])^2$$

$$E[x^2] = \int_a^b \frac{x^2}{b-a}\, dx$$

$$= \frac{1}{b-a}\left[\frac{x^3}{3}\right]_a^b$$

$$= \frac{1}{(b-a)} \cdot \frac{(b^3-a^3)}{3}$$

$$= \frac{(b-a)(b^2+a^2+ab)}{(b-a)3}$$

$$= \frac{b^2+a^2+ab}{3}$$

$$V[x] = \frac{(a+b)^2 - ab}{3}$$

$$E[x] = \frac{b+a}{2}$$

$$(E[x])^2 = \frac{(b+a)^2}{4}$$

$\therefore V[x] = E[x^2] - (E[x])^2$

$$= \frac{(b+a)^2 - ab}{3} - \frac{(b+a)^2}{4}$$

$$= \frac{4(b+a)^2 - 4ab - 3(b+a)^2}{12}$$

$$= \frac{(b+a)^2 - 4ab}{12}$$ (~~very small; so negligible~~)

$$\therefore V[x] = \frac{b^2 + 2ab - 4ab + a^2}{12} = \frac{(b-a)^2}{12}$$

⇒ Standard deviation:-

→ By the basic definition of standard deviation,

$$\sigma = \sqrt{V(x)} = \frac{b-a}{2\sqrt{3}}$$

EXAMPLE:- The current (in mA) measured in a piece of copper wire is known to follow a uniform distribution over the interval [0, 25]. Find the formula for the probability density function f(x) of the random variable x representing the current. Calculate the mean, variance, and standard deviation of the distribution and find the cumulative distribution function F(x).

Date 4/10/23
Page 58

$\Rightarrow f(x) = \begin{cases} 1/25; & 0 \le x \le 25 \\ 0; & \text{otherwise} \end{cases}$

$F(x) = \begin{cases} 0 ; & x < 0 \\ x-0/25-0 = x/25; & 0 \le x \le 25 \\ 1; & x > 25 \end{cases}$

$\text{Mean} = \frac{b+a}{2} = \frac{25+0}{2} = \underline{12.5}$

$\text{Variance} = \frac{(b-a)^2}{12} = \frac{(25)^2}{12} = \frac{625}{12}$

$\text{Standard deviation} = \sigma = \frac{(b-a)}{2\sqrt{3}}$

$\therefore \sigma = \frac{25}{2\sqrt{3}}$

Question:- If x is uniformly distributed in $(-1, 4)$ then,
(i.) its mean (ii.) variance (iii.) standard deviation (iv.) median

$\rightarrow$ (i.) $E[x] = \frac{b+a}{2} = \frac{4-1}{2} = \underline{1.5}$

(ii.) $V[x] = \frac{(b-a)^2}{12} = \underline{\frac{25}{12}}$

(iii.) $\sigma = \sqrt{V[x]} = \frac{5}{2\sqrt{3}}$

(iv.) $\text{Median} = \frac{b+a}{2} = \frac{4-1}{2} = \underline{1.5}$