1.2.5 Expectation of Binomial distribution:-

⇒ Probability Mass function for binomial distribution is:-

$$PMF = P(x=i) = \binom{n}{i} p^i (1-p)^{n-i}$$

⇒ Expectation:-

$$E[x] = \sum_{i=0}^{n} i \cdot P(x=i)$$

$$= \sum_{i=0}^{n} i \cdot \binom{n}{i} p^i (1-p)^{n-i}$$

$$= \sum_{i=1}^{n} i \cdot \frac{n!}{(n-i)!\, i!} p^i (1-p)^{n-i}$$

$$= \sum_{i=1}^{n} \frac{n!}{(n-i)!\,(i-1)!} \cdot p^i \cdot (1-p)^{n-i}$$

$$E[x] = np \cdot \sum_{i=1}^{n} \frac{(n-1)!}{(n-i)!\,(i-1)!} \cdot p^{i-1} (1-p)^{n-i}$$

⇒ Let $i-1=k$;

$$E[x] = np \cdot \sum_{k=0}^{n-1} \frac{(n-1)!}{(n-1-k)!\, k!} \cdot p^k \cdot (1-p)^{n-1-k}$$

$$= np\,[p+(1-p)]^{n-1} = np$$

$$\therefore E[x] = np$$

⇒ **Variance:-**

$V[x] = E[x^2] - (E[x])^2$

$\therefore V[x] = npq \qquad (q = 1-p)$

⇒ standard deviation $= S[x] = \sqrt{V[x]}$

$\therefore S[x] = \sqrt{npq}$

EXAMPLE:- A die is thrown repeatedly 36 times in all. Find $E(x)$ and $V(x)$ where x is the number of sixes obtained.

⇒ $n = 36$,
probability of getting six $= p = \frac{1}{6}$

$q = 1 - p = \frac{1}{1} - \frac{1}{6} = \frac{5}{6}$

$\therefore E[x] = np = 36 \times \frac{1}{6} = 6$

$\therefore V[x] = npq = 36 \times \frac{1}{6} \times \frac{5}{6} = 5$

The expectation of a binomial distribution represents the average or mean number of successes in a fixed number of independent Bernoulli trials, where each trial has the same probability of success.