## 1.2.4 Binomial Distribution:-

⇒ A binomial random experiment consists of 'n' Bernoulli trials such that:-

1. The trials are independent
2. Each trial results in only two possible outcomes, labeled as "success" and "failure".
3. The probability of a success in each trial, denoted as 'p', remains constant.

⇒ The random variable X that equals the number of trials that result in a success is a binomial random variable with parameters $0<p<1$ and $n=1,2,\ldots$

⇒ The probability mass function:-

$$P(X=x)=f(x)={}^{n}C_{x}\,p^{x}(1-p)^{n-x}$$

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$\rightarrow P(X=x) = P(x=1) + P(x=2) + \dots + P(x=x)$

EXAMPLE :- Suppose a biased coin comes up heads with probability 0.3 when tossed. The probability of seeing exactly 4 heads in 6 tosses is :-

$\rightarrow$ $n=6$ ; $p=0.3$

$x$: getting 4 heads

$P(x=4) = {}^6C_4 \, (0.3)^4 (1-0.3)^{6-4}$

$= 15 \times (0.3)^4 \times (0.7)^2$

$= 0.059535$

EXAMPLE :- An airline sells 65 tickets for a plane with capacity of 60 passengers. This is done because it is possible for some people to not show up. The probability of a person not showing up for the flight is 0.1. All passengers behave independently. Find the probability of the event that the airline does not have to arrange seperate tickets for excess people.

$\rightarrow$ $x$: No. of people showed up for flight

~~Pbxegdbs~~ If people appears are less than or equal to 60, then airline does not have to arrange seperate tickets for excess people.

$\therefore P(X \le 60) = 1 - P(X \ge 61)$

$$= 1 - \Big( {}^{65}C_{61} (0.9)^{61} (0.1)^{4} + {}^{65}C_{62} (0.9)^{62} (0.1)^{3} + {}^{65}C_{63} (0.9)^{63} (0.1)^{2} + {}^{65}C_{64} (0.9)^{64} (0.1)^{1} + {}^{65}C_{65} (0.9)^{65} (0.1)^{0} \Big)$$

$$= 1 - \Big( (0.9)^{61} [67.704 + 39.312 + 16.848 + 4.7385 + 0.6561] \Big)$$

$$= 1 - \left[ (0.9)^{61} (114.2586) \right]$$

$$= 1 - [0.1848]$$

$$= \underline{0.8152}$$

$\Rightarrow$ <u>Theorem</u>: Let $A$ be some event ~~that~~ associated with a random Experiment $E$, such that $P(A) = p$ and $P(A') = q = 1 - p$. Assuming that $p$ remains ~~the~~ the same for all repetitions, if we consider '$n$' independent repetitions (or trials) of $E$, and if the random variable (RV) $X$ denotes the number of times the event $A$ has occurred then $X$ is called a binomial random variable with parameters ~~p~~ '$n$' and '$p$' or we can say that $X$ ~~follows~~ follows a <u>binomial distribution</u> with parameters '$n$' and '$p$', or ~~symbolical~~ symbolically $B(n, p)$.

→ Obviously the possible values that $x$ can take, are, $0, 1, 2, \ldots, n$.

→ By the theorem under Bernoulli's trials, the probability mass function of a binomial RV is given by:-

$$P(X=r) = {}^nC_r \cdot q^{n-r} \cdot p^r \; ; \; r = 0, 1, 2, \ldots, n$$

where, $p+q=1$

→ Note:-

1. Binomial distribution is a legitimate probability distribution since,

$$\sum_{r=0}^{n} P(X=r) = \sum_{r=0}^{n} {}^nC_r \cdot q^{n-r} p^r = (q+p)^n = 1$$

2. The mean of the Binomial distribution is given by:-

$$E(X) = \sum_r x_r p_r = np \; ;$$

also, $E(x^2) = \sum_r x_r^2 \cdot p_r$

3. The variance of the Binomial Distribution is given by:-

$$Var(X) = E(X^2) - (E(X))^2 = npq$$