1.4 PYQs :-

GATE-2012: Consider a random variable x that takes values +1 and -1 with probability 0.5 each. The value of the cumulative distribution function $F(x)$ at $x=-1$ and +1 are:-

(A) 0 and 0.5 (C) 0.5 and 1
(B) 0 and 1 (D) 0.25 and 0.75

$\Rightarrow$ x : -1 1
$P(x)$: 0.5 0.5

$F(t=-1) = P(x \le -1) = 0 + 0.5$

$F(t=1) = P(x \le 1) = P(x=-1)$
$+ P(x<-1)$
$+ P(x=1) + P(x<1)$
$= 0.5 + 0 + 0.5 + 0$
$= 1$

GATE-2004:- An examination paper has 150 multiple-choice questions of 1-mark each, with each question having 4 choices. Each incorrect answer fetches -0.25 mark. Suppose 1000 students choose all their answers randomly with uniform probability. The sum total of the expected marks obtained by all these students is;

(A) 0 (B) 2550 (C) 7525 (D) 9375

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⇒ 1-question

1 correct → 1/4 (1)

3 wrong → 3/4 (-0.25)

→ Expected marks for 1 question:

$= (-0.25)\left(\frac{3}{4}\right) + 1\left(\frac{1}{4}\right)$

$= 1/16$

→ Hence, expected marks $= \frac{1}{16} \times 150 \times 1000$

$= 9375$

GATE-2019:- Given set $A = \{2,3,4,5\}$ and set $B = \{11,12,13,14,15\}$, 2 numbers are randomly selected, one from each set. What is the probability that the sum of the 2 numbers equal 16?

(A.) 0.20 (B.) 0.25 (C.) 0.30 (D.) 0.33

⇒ Total possible choice $= 4 \times 5 = 20$

Possible combinations = {2,14}
{3,13}
{4,12}
{1,15}

$= 4/20 = 1/5 = 0.2$

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GATE-2018: Two people, P and Q decide to independently roll 2 identical dice, each with 6 faces, numbered 1 to 6. The person with the lower number wins. In case of a tie, they roll the dice repeatedly until there is no tie. Define a trial as a throw of the dice by P and Q. Assume that all 6 numbers on each dice are equi-probable and that all trials are independent. The probability (rounded to 3 decimal places) that one of them wins on the third trial is

⇒ To have a win on third trial, first 2 trials must be a tie.

$S = \{1, 2, 3, 4, 5, 6\}$

Tie, if P & Q gets same on a roll, e.g.; (1,1) (2,2) (3,3) (4,4) (5,5)

$P(\text{Tie}) = \frac{6}{36} = \frac{1}{6}$

$P(\text{NOT tie}) = 1 - \frac{1}{6} = \frac{5}{6}$

$\therefore P(\text{Tie}, \text{Tie}, \text{wins}) = \frac{1}{6} \times \frac{1}{6} \times \frac{5}{6} = \frac{5}{216}$

$= 0.023$

GATE-2021: For a given biased coin the probability that the outcome of a toss is a head is 0.4. This coin is tossed 1000 times. Let x denote the random variable whose value is the number of times that head appeared in these 1000 tosses. The standard deviation of x (rounded to 2 decimal places) is ______

$\Rightarrow$ $n = 1000$

$p = 0.4$

$q = 1 - p = 0.6$

Variance $= npq = 1000 \times 0.4 \times 0.6$

$= 1000 \times 0.24$

$= \underline{240}$

Standard deviation $= \sqrt{240} = \underline{15.49}$

Question-17: The mean and standard deviation of a sample were found to be 9.5 and 2.5, respectively. Later, an additional observation 15 was added to the original data. Find the SD of the 11 observation.

$\Rightarrow$ Consider there are 10 such observations.

$\bar{x} = 9.5$

$\frac{\Sigma x_i}{n} = 9.5 \Rightarrow \Sigma x_i = \underline{95}$

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$\sigma = 2.5 \quad V[x] = 6.25$

$\therefore E[x^2] - (E(x))^2 = 6.25$

$\therefore E(x^2) = 6.25 + 9.5 = 96.5$

Corrected mean $= \frac{95+15}{11} = 10$

Corrected variance $= \frac{1}{11} \times (965 + 225) - 100$

$= \frac{1}{11}(965+225) - 100$

$= \frac{1190 - 1100}{11} = \frac{90}{11}$

$\sigma = \sqrt{90/11} = 2.86$

Question: A man is known to speak truth 3 out of 4 times. He throws a die and report that it is 6. Find the probability that it is actually 6.

$\Rightarrow$ E: Event when the man reports 6 in throwing a die.
S1: six occurs
S2: event six does not occur.

$P(S1) = \frac{1}{6}$ and $P(S2) = \frac{5}{6}$

$P(E|S_1)$: probability that man reports six and six has occurred actually
= probability of speaking truth $= \frac{3}{4}$

$P(E|S_2)$: probability that man reports six and six has not occurred actually
= probability of speaking lie $= \frac{1}{4}$

$P(S_1|E)$ = Probability that six occurs and person is telling the truth

$$= \frac{P(S_1) \times P(E|S_1)}{P(E)}$$

$$= \frac{P(S_1) \times P(E|S_1)}{P(S_1) \times P(E|S_1) + P(S_2) \times P(E|S_2)}$$

$$= \frac{\frac{1}{6_2} \times \frac{3}{4}}{\frac{1}{6} \times \frac{3}{4} + \frac{5}{6} \times \frac{1}{4}} = \frac{\frac{1}{8}}{\frac{1}{8} + \frac{5}{24}}$$

$$= \frac{1}{8} \times 3 = \frac{3}{8}$$