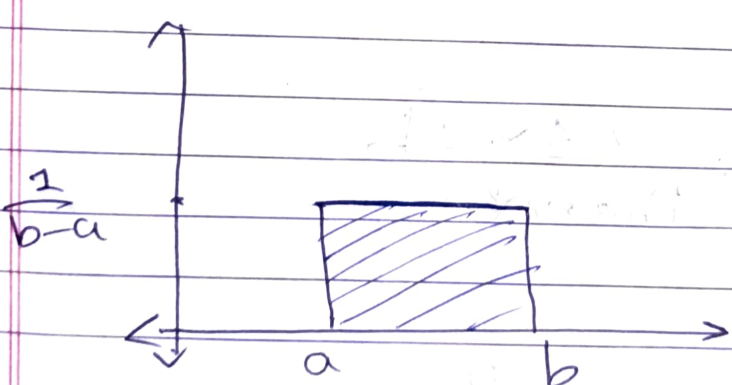
1.2.10 Uniform Distribution:-

→ The uniform distribution, also known as the Rectangular Distribution, is a type of Continuous Probability Distribution.

→ It has a continuous Random variable x restricted to a finite interval $[a,b]$ and its probability function $f(x)$ has a constant density over this interval.

→ The uniform probability distribution function is defined as:-

$$f(x) = \begin{cases} \frac{1}{b-a} ; & a \le x \le b \\ 0 ; & \text{otherwise} \end{cases}$$



→ To check if the above function is a legitimate probability density function, we need to check if it's integral over its support is 1.

$$\int_a^b \frac{1}{b-a} \cdot dx = \frac{1}{b-a} [x]_a^b$$

$$= \frac{1}{(b-a)}[b-a] = 1$$

→ Hence; its a legitimate probability density function.

⇒ Cumulative Density Function:-

→ As we know, the cumulative density function is nothing but the sum of probability of all events upto a certain value of $x = t$.

→ In the uniform distribution, the cumulative density function $F(x)$ is given by:-

$$f(x) = \begin{cases} \frac{1}{b-a}; & a \le x \le b \\ 0; & \text{otherwise} \end{cases}$$

→ For $t \le a$.

$$F(t) = P(x \le t) = \int_{-\infty}^{a} 0 \cdot dx = 0$$

→ for $a \le t \le b$

$$F(t) = P(x \le t) = \int_{a}^{t} \frac{1}{b-a} \cdot dx = \frac{1}{b-a}[x]_a^t$$

$$\therefore F(t) = \frac{t-a}{b-a}$$

→ For $t \geq b$,

$$P(X \leq t) = \int_{-\infty}^{t} \frac{1}{b-a} \, dx$$

$$= \int_{-\infty}^{a} \frac{1}{b-a} \, dx + \int_{a}^{b} \frac{1}{b-a} \, dx$$

$$+ \int_{b}^{t} \frac{1}{b-a} \, dx$$

[For all other intervals except betn a & b; ~~F(x)~~ $F(x) = 0$]

$$= 0 + \int_{a}^{b} \frac{1}{b-a} \, dx + 0$$

$$= 1$$

∴ For; $t \geq b$; $P(X \leq t) = 1$

$$\therefore F(t) = \begin{cases} 0 ; & t \leq a \\ t-a/b-a ; & a \leq t < b \\ 1 ; & t \geq b \end{cases}$$

EXAMPLE:- Let x be a uniform random variable with support $[5, 10]$. Compute the probability $P(7 < x < 9)$.

$\Rightarrow$ $f(x) = \begin{cases} 1/10-5 = 1/5, & 5 \leq x \leq 10 \\ 0, & \text{otherwise.} \end{cases}$

$\therefore P(7<x<9) = \int_7^9 f(x).dx$

$= \int_7^9 \frac{1}{5} dx = \frac{1}{5}[9-7] = \frac{2}{5}$

$\Rightarrow$ for $P(7<x<12) = \int_7^{10} f(x).dx + \int_{10}^{12} 0\, dx$

$= \frac{1}{5}[10-7] = \frac{3}{5}$

$\Rightarrow$ Using cumulative density function;

i.e.; $\frac{t-5}{5}$, $5 \leq x \leq t$

$\therefore P(7 < x \leq 9) = P(x \leq 9) - P(x \leq 7)$

$= \frac{9-5}{5} - \frac{7-5}{5}$

$= \frac{4}{5} - \frac{2}{5}$

$\therefore P(7<x<9) = \frac{2}{5}$