1.2.2 Bernoulli Distribution.

→ Bernoulli Distribution depends on Bernoulli Trial. [Also Known as Binomial Trial]

⇒ Bernoulli Trial :- It is a random experiment that has only 2 outcomes (usually called a "success" or a "Failure").
- Getting a heads
- Getting a six on dice
- Getting two heads when we toss two coin simultaneously.

→ The distribution formed by Bernoulli Trial is called Bernoulli Distribution

→ In an experiment whose outcome can be classified as either a success or failure.

⇒ Let $(x=1)$ if the outcome is success and $(x=0)$ if it is a failure,

The PMF is given by,

$P(0) = P(x=0) = 1-p$

$P(1) = P(x=1) = p$, where p is the probability of success

x is Bernoulli random variable.

$\therefore P(x=x) = p^x (1-p)^{1-x}$; $x = \{0,1\}$

e.g.:- Tossing 1 coin. Getting a head.

$P(x=1) = P(H) = 1/2$
$P(x=0) = P(T) = 1/2$

e.g.:- x: Getting a six

$\therefore P(x=1) = 1/6$
$\therefore P(x=0) = P(2/3/1/4/5) = 5/6$

e.g.:- Tossing 2 coins simultaneously. And,
x: Getting 2 heads

$\therefore P(x=1) = P(HH) = 1/4$
$\therefore P(x=0) = P(TT, HT, TH) = 3/4$

$PMF = \left(\frac{1}{4}\right)^x \left(\frac{3}{4}\right)^{1-x}$ (All trials are 1)

Example:- A basketball player can shoot a ball into the basket with a probability of 0.6. What is the probability that he misses the shot?

$\Rightarrow P(x=1) = 0.6, \quad P(x=0) = 1 - P(x=1)$
$= 1 - 0.6 = 0.4$

→ In case of the Bernoulli trial, there are only the two possible outcomes, but in the case of the binomial distribution, we get the number of successes in a sequence of independent experiments.

⇒ **Theorem:-** If the probability of occurrence of an event (probability of success) in a single trial of a Bernoulli's experiment is p, then the probability that the event occurs exactly r times out of n-independent trials is equal to $^nC_r q^{n-r} p^r$,

where $q = 1 - p$, the probability of failure of the event.

→ Required probability $= {}^nC_r\, q^{n-r} \cdot p^r$

where,
p = Probability of Success
$q = 1 - p$ = Probability of Failure
n = Number of Independent Trials
r = The number of times an event occurred

⇒ **Proof:-**

→ Getting exactly r successes means getting r successes and $(n-r)$ failures simultaneously.

$\therefore$ P(getting r successes and $n-r$ failures) $= q^{n-r} \cdot p^r$ (since, the n trials are independent).
[By product theorem]

Date 3/10/23
Page 27

→ The trials, from which the successes are obtained, are not specified. There are ^nC_r ways of choosing r trials for successes.

→ Once the r trials are chosen for successes, the remaining $(n-r)$ trials should result in failures.

→ These ^nC_r ways are mutually exclusive.

→ In each of these ^nC_r ways,
P(getting exactly r successes) $= q^{n-r} \cdot p^r$

→ Therefore, by addition Theorem, the required probability $= {^nC_r} \cdot q^{n-r} \cdot p^r$