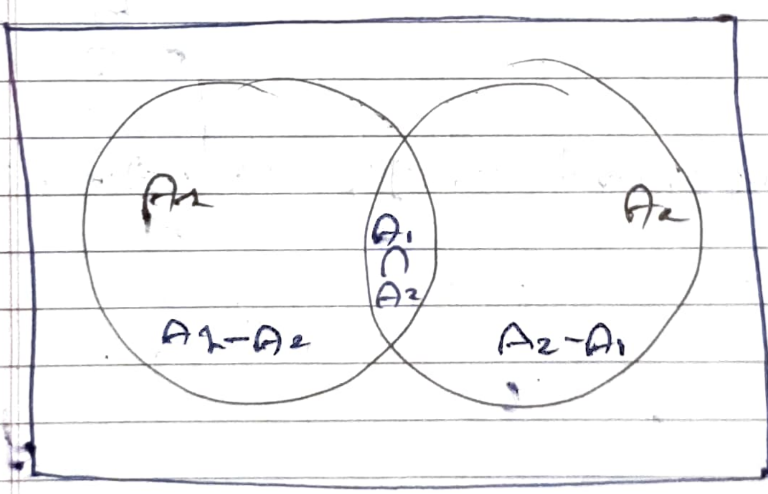
Wednesday

Date 24/4/24

55 Inclusion Exclusion Principle :-

→ In field of combinatorics, it is a counting method used to compute the cardinality of the union set.

→ For 2 finite sets A_1 and A_2, which are subsets of Universal set, then $(A_1 - A_2)$, $(A_2 - A_1)$ and $A_1 \cap A_2$ are disjoint sets.



→ Hence, it can be said that

$$|(A_1 - A_2) \cup (A_2 - A_1) \cup (A_1 \cap A_2)|$$
$$= |A_1| - |A_1 \cap A_2| + |A_2| - |A_1 \cap A_2| + |A_1 \cap A_2|$$

$$\therefore |A_1 \cup A_2| = |A_1| + |A_2| - |A_1 \cap A_2|$$

EXAMPLE: How many numbers between 1 and 1000, including both, are divisible by 3 or 4?

⇒ Numbers divisible by 3: $\left\lfloor \frac{1000}{3} \right\rfloor = 333$

Number divisible by 4: $\left\lfloor \frac{1000}{4} \right\rfloor = 250$

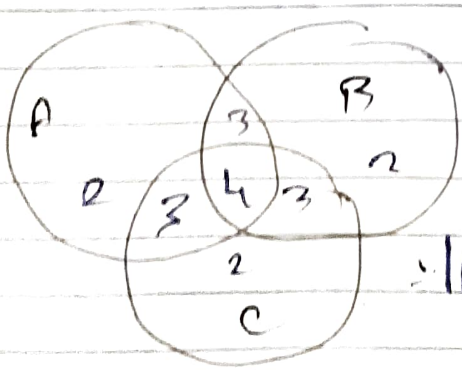
Numbers divisible by 12: $\left\lfloor \frac{1000}{12} \right\rfloor = 83$

∴ Numbers divisible by 3 or 4 $= 333 + 250 - 83$
$= 250 + 250$
$= \underline{500}$

⇒ similarly for 3 finite sets A_1, A_2 & A_3

$$|A_1 \cup A_2 \cup A_3| = |A_1| + |A_2| + |A_3| - |A_1 \cap A_2| - |A_2 \cap A_3| - |A_1 \cap A_3| + |A_1 \cap A_2 \cap A_3|$$

EXAMPLE: As shown in the diagram, 3 finite sets A, B and C with their corresponding values are given. Compute $|A \cup B \cup C|$



$|A| = 12$, $|B| = 12$, $|C| = 12$
$|A \cap B| = 7$, $|A \cap C| = 7$
$|B \cap C| = 7$; $|A \cap B \cap C| = 4$

∴ $|A \cup B \cup C| = 12 + 12 + 12 - 21 + 4$
$= 36 - 21 + 4$
$= 40 - 21 = 19$

⇒ <u>Principle</u>:- Inclusion-Exclusion principle says that for any number of finite sets $A_1, A_2, A_3, \ldots A_n$.

→ Union of the sets is given by =
sum of sizes of all single sets
− Sum of all 2-set intersections
+ Sum of all 3-set intersections
− Sum of all 4-set intersections
+ ----
+ $(-1)^{i+1}$ sum of all i^{th} set intersections

→ In general it can be said that:-

$$\left|\bigcup_{i=1}^{n} A_i\right| = \sum_{i=1}^{n} |A_i| - \sum_{1 \le i \le j \le n} |A_i \cap A_j|$$

$$+ \sum_{1 \le i \le j < k \le n} |A_i \cap A_j \cap A_k| - \cdots + (-1)^{n-1} |A_1 \cap \ldots \cap A_n|$$