Monday

1.2 Random Variable:-

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1.2.1 Random variables:-

⇒ Random variable is basically a function which maps from the set of sample space to set of real numbers.

⇒ Example:- Suppose that two coins (unbiased) are tossed.

x = number of heads
[x is a random variable or function]
Here, the sample space
$S = \{HH, HT, TH, TT\}$.
The output of the function will be;
$X(HH) = 2$
$x(HT) = 1$
$x(TH) = 1$
$x(TT) = 0$

⇒ formal definition:-

$x : S \to R$

x = random variable (It is usually denoted using capital letter).
S = set of sample space
R = set of real numbers

⇒ Suppose a random variable x takes "m" different values;
i.e., sample space $x = \{x_1, x_2, \dots, x_m\}$
with probabilities $P(x = x_i) = P_i$;
where $1 \leq i \leq m$

⇒ The probabilities must satisfy the following conditions:-

$0 \leq p_i \leq 1$; where $1 \leq i \leq m$

$p_1 + p_2 + p_3 + \cdots + p_m = 1$

⇒ Or, we can say $0 \leq p_i \leq 1$ and $\Sigma p_i = 1$.

⇒ Example:- Suppose that two coins (unbiased) are tossed.

X = number of heads.

Hence, possible values for random variable X are 0, 1, 2.

$X = \{0, 1, 2\}$

$P(X=0)$ = probability that number of heads is 0.

$\therefore P(X=0) = P(TT) = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{4}$

$\therefore P(X=1) = P(HT \mid TH)$

= Probability that no. of head is 1

$= \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

$P(X=2) = P(HH)$

$= \frac{1}{4}$

$P(X=2) + P(X=1) + P(X=0)$

$= \frac{1}{4} + \frac{1}{2} + \frac{1}{4} = 1$

Random Variable

- Discrete Random variable
- Continous Random variable

$K_1, K_2, \ldots K_n$

$[1,2)$ $[3,4]$

0 —— 1

Discrete Random variable:-

→ A random variable x is said to be discrete if it takes on finite number of values.

→ The probability function associated with it is said to be PMF = Probability mass function.

→ $P(x_i)$ = Probability that $x = x_i$ = PMF of $x = p_i$.

1. $0 \leq p_i \leq 1$
2. $\Sigma p_i = 1$, where sum is taken over all possible values of x.

⇒ Example:- Let $S = \{0, 1, 2\}$.

x_i	0	1	2
$P(x = x_i)$	?	0.3	0.5

Find the value of $P(x=0)$.

⇒ We know sum of $P(x_i) = 1$

$\therefore P(0) + P(1) + P(2) = 1$

$\therefore P(0) + 0.3 + 0.5 = 1$

$\therefore \underline{P(0) = 0.2}$

Continuous Random Variable:

→ A random variable x is said to be continous if it takes on infinite number of values.

→ The probability function associated with it is said to be PDF = Probability Density Function.

→ PDF:- If x is continuous random variable.

$$\to P(x < x < x + dx) = \int_{x}^{x+dx} f(x) \, dx$$

1. $0 \leq f(x) \leq 1, \; \forall x$
2. $\int f(x) dx = 1, \; \forall x$

→ Then $P(x)$ is said to be PDF of the distribution.

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⇒ **Example:** Compute the value of $P(1<x<2)$.

Such that

$$f(x) = \begin{cases} kx^3; & 0 \le x \le 3 \\ 0, & \text{otherwise} \end{cases}$$

- Where, $f(x)$ is a density function.

⇒ If a function f is said to be density function, then sum of all probabilities is equals to 1.

⇒ Since it is a continous random variable, Integral value is 1 overall sample space.

$$\therefore \int_0^3 kx^3\,dx = 1$$

$$\therefore k\left[\frac{x^4}{4}\right]_0^3 = 1$$

$$\therefore k\left[\frac{81}{4}\right] = 1 \Rightarrow k = \frac{4}{81}$$

$$\therefore P(1<x<2) = \int_1^2 kx^3 . dx$$

$$= k\left[\frac{x^4}{4}\right]_1^2$$

$$= \frac{4}{81}\left[\frac{16-1}{4}\right] = \frac{\cancel{15}^{5}}{\cancel{81}_{27}}$$

$$= \frac{5}{27}$$