SSY281 Model Predictive Control

Assignment 3

MPC practice and Kalman Filter

Due February 12 at 23:59

Systems & Control

Department of Electrical Engineering

Chalmers University of Technology

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Instructions

This assignment is **individual** and must be solved according to the following rules and instructions:

- Written report:
  - For those questions where a text (motivations, explanations, observations from simulations) rather than a numerical value is asked, the answers should be concisely written in the report.
  - Figures included in the report should have legends, and axes should be labelled.
  - The report should be uploaded *before the deadline* in Canvas.
  - Name the report as A3_X.pdf, where X is your *group* number (see Canvas page).
- Code:
  - Your code should be written in the Matlab template provided with this assignment following the instructions therein.
  - Name the Matlab scripts as A3_X.m where X is your *group* number.
- Grading:
  - This assignment if worth **15 points** in total.

1 Set-point tracking and disturbance modeling

Consider the following system with two inputs and two outputs:

$$A = \begin{bmatrix} 0.5 & 0 & 0 & 0 \\ 0 & 0.6 & 0 & 0 \\ 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0.6 \end{bmatrix} \quad B = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.4 \\ 0.25 & 0 \\ 0 & 0.6 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

Question 1 (**Points: 1**). *Calculate the state and inputs set-points* (x_s, u_s) *corresponding to the output set-point* $y_s = \begin{bmatrix} 1 & -1 \end{bmatrix}^T$. *Is regulation of* y *to* y_s *achievable? Answer this question and motivate your answer in the report.*

Question 2 (**Points: 1**). *Assume that only the first control input is available for control, i.e.,*

$$B = \begin{bmatrix} 0.5 \\ 0 \\ 0.25 \\ 0 \end{bmatrix}.$$

Can you still regulate y *to* $y_s = \begin{bmatrix} 1 & -1 \end{bmatrix}^T$*? Find the steady state that minimizes two-norm of the output error and motivate the procedure in the report.*
Hint: *The Matlab command* `quadprog` *can be used.*

Question 3 (**Points: 1**). *Consider the case that both control inputs are available and*

$$C = \begin{bmatrix} 1 & 1 & 0 & 0 \end{bmatrix}.$$

Calculate the state and inputs set-points (x_s, u_s) *corresponding to the output set-point* $y_s = 1$. *Can* $y = y_s$ *be achieved? Find the steady state that minimizes two-norm of the input while the output error is zero. Motivate the procedure in the report.*
Hint: *The Matlab command* `quadprog` *can be used.*

2 Control of a chemical reactor

In this section, you will apply MPC to a linearized model of a chemical tank reactor, in which the species A undergoes a reaction and the species B is produced. The model states are

$$x = \begin{bmatrix} c \\ T \\ h \end{bmatrix},$$

where c is concentration of A, T is reactor temperature, and h is the reactor tank level. The control variables u are the coolant temperature (the reaction is exothermic, i.e. it generates heat) and the outlet flow rate. The inlet flow rate acts as an unmeasured disturbance p. The linearized and discretized model (the sampling period is 1 min) at the desired operating point is given by

$$x(k+1) = Ax(k) + Bu(k) + B_p p(k), \tag{1}$$

with

$$A = \begin{bmatrix} 0.2681 & -0.00338 & -0.00728 \\ 9.703 & 0.3279 & -25.44 \\ 0 & 0 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$B = \begin{bmatrix} -0.00537 & 0.1655 \\ 1.297 & 97.91 \\ 0 & -6.637 \end{bmatrix}, \quad B_p = \begin{bmatrix} -0.1175 \\ 69.74 \\ 6.637 \end{bmatrix}.$$

An offset-free RH controller can be designed with an augmented model to account for the unknown disturbance. Consider the three following disturbance models

1. $n_d = 2,\ B_d = 0_{3\times 2},\ C_d = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix},$

2. $n_d = 3,\ B_d = 0_{3\times 3},\ C_d = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix},$

3. $n_d = 3,\ B_d = \begin{bmatrix} 0_{3\times 2} & B_p \end{bmatrix},\ C_d = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}.$

Question 4 (**Points: 3**). *Construct an augmented model with the matrices B_d and C_d for each disturbance model given above. In which case the augmented system is detectable? Answer and motivate your answer in the report.*

Question 5 (**Points: 3**). *Design a Kalman filter for each detectable augmented model to estimate its states.*

Question 6 (**Points: 1**). *Assume that we are only interested to control the first and the third outputs. Therefore, the new output would be $y(k) = C_s x(k)$ with $C_s = HC$ in which*

$$H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

*In case the output set point, i.e. y_{sp}, is zero, one can find the steady state target using the estimation of the disturbance, look at Section **5.2** of the lecture notes. Find matrix M_{ss}, defined as follows, for each detectable augmented system.*

$$\begin{bmatrix} x_s \\ u_s \end{bmatrix} = M_{ss}\hat{d}(k)$$

Question 7 (**Points: 5**). *Design a RHC for each detectable augmented system and simulate it with the following parameters where N and M are the prediction and control horizons, respectively. Consider zero as the initial condition for your observer.*

$$N = 10,\ M = 3,\ R = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix},$$

$$Q = P_f = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.001 & 0 \\ 0 & 0 & 1 \end{bmatrix},\ x(0) = \begin{bmatrix} 0.01 \\ 1 \\ 0.1 \end{bmatrix},$$

To simulate the controlled system, use the disturbance $p(k)$ defined as:

$$p(k) = \begin{cases} 0 & \text{if } k \leq 10 \\ 0.01 & \text{if } k > 10 \end{cases} \tag{2}$$

In which case the controller is able to remove the off-set? Motivate your answer in the report.