SSY281 Model Predictive Control

Assignment 4

Optimization basics and QP problems

$\boxed{\text{Due February 19 at 23:59}}$

Systems & Control

Department of Electrical Engineering

Chalmers University of Technology

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Instructions

This assignment is **individual** and must be solved according to the following rules and instructions:

- Written report:
  - For those questions where a text (motivations, explanations, observations from simulations) rather than a numerical value is asked, the answers should be concisely written in the report.
  - Figures included in the report should have legends, and axes should be labeled.
  - The report should be uploaded *before the deadline* in Canvas.
  - Name the report as A4_X.pdf, where X is your *group* number.
- Code:
  - Your code should be written in the Matlab template provided with this assignment following the instructions therein.
  - Name the Matlab scripts as A4_X.m, N1_X.m, and Ninf_X.m where X is your *group* number.
  - Strictly follow the instructions in the Matlab template.
- Grading:
  - This assignment if worth **15 points** in total.

1 Optimization Basics

Consider the optimization problem

$$\begin{aligned} & \min_x f(x) \\ \text{s.t.} \quad & g(x) \leq 0 \\ & h(x) = 0 \end{aligned} \tag{1}$$

where

$$\begin{aligned} f &: \mathbb{R}^n \to \mathbb{R} \quad g : \mathbb{R}^n \to \mathbb{R}^m \\ h &: \mathbb{R}^n \to \mathbb{R}^q \quad x \in \mathbb{R}^n \end{aligned}$$

Unless otherwise stated, assume that f,g and h are general nonlinear functions.

Question 1 (Points: 3). *Consider the optimization problem (1). Let x_1 and x_2 be feasible points.*

(a) Can you find simple conditions such that $z = \frac{x_1+x_2}{2}$ is a feasible solution?

(b) Can you find conditions such $z = \lambda x_1 + (1-\lambda)x_2$ with $0 \leq \lambda \leq 1$ is never worse than both x_1 and x_2? If this conditions holds, can it happen that z is better than both of them?

(c) Can you conclude the same for $\lambda = 0.5$?

Question 2 (Points: 3). *Which of the following sets are convex? Motivate your answers.*

(a) A slab, i.e., $\{x \in \mathbb{R}^n | \alpha \leq a^\top x \leq \beta\}$

(b) The set

$$M = \{x| \; ||x-y|| \leq f(y) \text{ for all } y \in S\},$$

where $S \subseteq \mathbb{R}^n$.

(c) A set of points closer to a given point than to a given set, i.e.,

$$\{x| \; ||x-x_0||_2 \leq ||x-y||_2 \text{ for all } y \in S\},$$

where $S \subseteq \mathbb{R}^n$.

Question 3 (Points: 3). *Linear programs have the general form:*

$$\min_x b^\top x$$
$$s.t. \quad Fx \leq g$$

Show that the following problems can be rewritten as linear programs:

(a) The 1-norm objective is given by:

$$\min_x ||Ax||_1$$
$$s.t. \quad Fx \leq g$$

(b) The ∞-norm objective is given by:

$$\min_x ||Ax||_\infty$$
$$s.t. \quad Fx \leq g$$

Question 4 (Points: 3). *Solve the following linear regression problems:*

(a) Fill in the function `N1.m` which takes A and b as inputs and returns the x^* that minimizes $||Ax^* - b||_1$.

(b) Fill in the function `Ninf.m` which takes A and b as inputs and returns the x^* that minimizes $||Ax^* - b||_\infty$.

***Note**: Use `linprog` to solve the above problems!*

2 QP Problems

Question 5 (Points: 3). *Consider the optimization problem:*

$$\begin{aligned} \min_{x,u} f(x,u) = & \frac{1}{2}(x_1^2 + x_2^2 + u_0^2 + u_1^2) \\ s.t. \quad & 2.5 \leq x_1 \leq 5 \\ & -1 \leq x_2 \leq 1 \\ & -2 \leq u_0 \leq 2 \\ & -2 \leq u_1 \leq 2 \end{aligned} \tag{2}$$

resulting from a finite-time constrained optimal control problem for the SISO process: $x_{k+1} = 0.5x_k + u_k$, with initial state $x_0 = 2$.

(a) *Solve the QP using MATLAB.*

(b) *Do the KKT conditions hold at the solution found at point (a)? Which constraints are active? (Hint: The Lagrangian multipliers are calculated by the MATLAB command* ***`quadprog`****).*

(c) *What would happen in the optimization problem if we remove lower bound on x_1, and what if we remove the upper bound on x_1? Why?*