SSY281
Assignment 6
Feasibility, alternative formulations of MPC

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Linear MPC Design

1 Question 1

As the states of the system gets close to the origin, the constraints become inactive. Therefore the contrained LQ controller can now be looked upon as an unconstrained LQ controller. This makes the terminal cost $\mathbb{V}_f$ as that of unconstrained LQ value function as

$$V_f(x) = V_{\infty}^{uc}(x) = x^T P x \tag{1}$$

where P is the solution of the algebraic Riccati equation, which when solved, results in the value P to be as

$$\begin{bmatrix} 16.09 & 32.98 \\ 32.98 & 93.93 \end{bmatrix} \tag{2}$$

For a linear quadratic MPC with linear constraints on the system $x^+ = Ax + Bu$ and with positive definite matrices Q and R and choosing the terminal cost V_f as the value function of the corresponding unconstrained infinite LQ controller and by choosing the feasibility set $\mathcal{X}_N$ as 3 step backward reach-ability of the terminal constraints on state set i.e $Pre_3(\mathbb{X}_f)$, the plot of the feasibility set looks as follows in Figure 1. (($\mathbb{X}_f$) is assumed to be control invariant in this case).

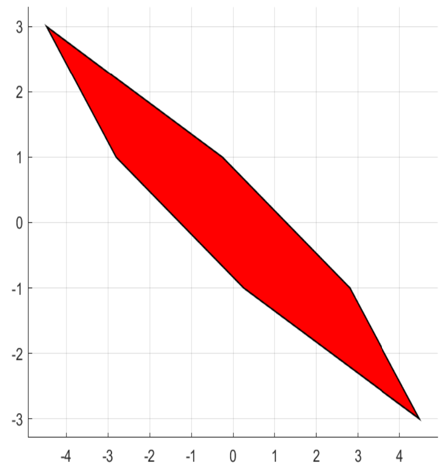

Figure 1: Feasibility set of the MPC controller for the given conditions

2 Question 2

The MPC controllers for the three given horizon length at 10, 15 and 20 were designed whose simulated results are as shown in the Figures 2, 3, 4 respectively.

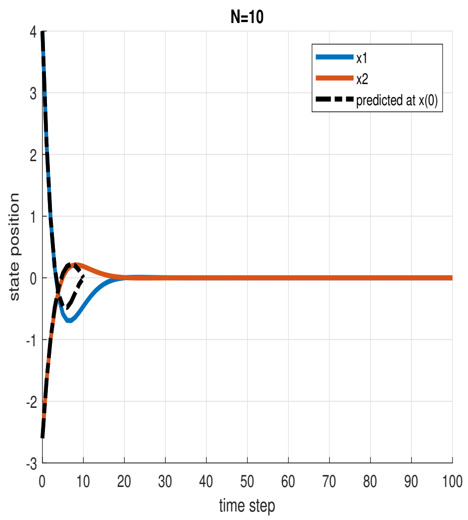

Figure 2: MPC cotroller for N =10

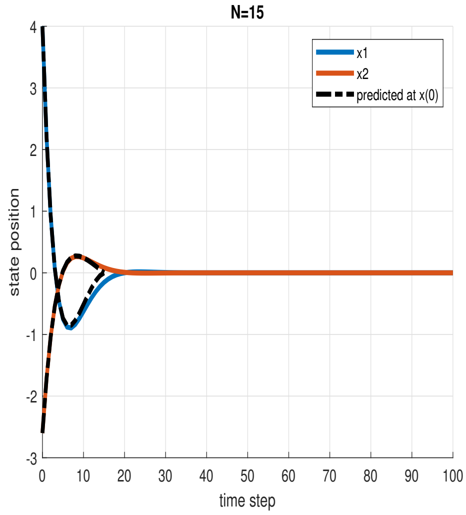

Figure 3: MPC cotroller for N =15

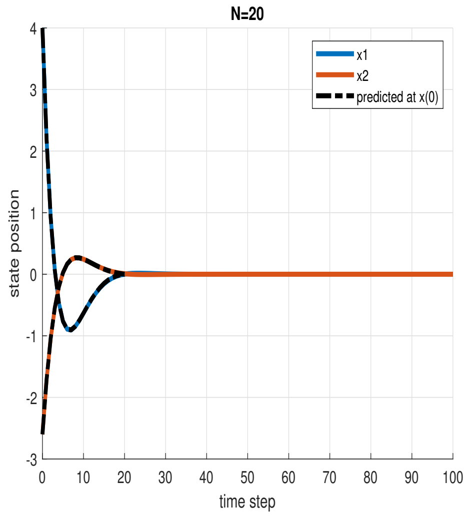

Figure 4: MPC cotroller for N =20

In these plots, one can observe that as the length of the horizon is increased, the precision of the prediction of designed MPC is improved. This can be accounted for the fact that at relatively small horizon lengths, the future state prediction outside the length of the horizon tends to change the final cost function which eventually affects the optimal control law which is applied in the next prediction stage and ends up misguiding the prediction trajectory more. As the length of the prediction control horizon is increased, this effect seems to reduce, giving higher accuracy for prediction.

3 Question 3

Clearly, for persistent feasibility $\mathcal{X}_f$ needs to be a subset of the domain C_∞. Since it is asked to assume N=1, which means the set $\mathcal{X}_f$ must be reached in one step. This can be achieved using the forward reachability starting from the origin such that the achieved set of reachability in one step which, when in intersection with the domain of C_∞ must give us the desired feasibility set $\mathcal{X}_f$, which can also be formulated as

$$\mathcal{X}_f = Reach_1(\mathcal{X}_N) \cap C_\infty \tag{3}$$

This makes the set $\mathcal{X}_f$ as the subset of the domain C_∞ which is reached with N=1, which ensures the persistent feasibility.