SSY281

Assignment 6

MPT and Persistent Feasibility

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MPT

1 Question 1

The H matrix is represented such that each row of matrix A (A_i) times the vector b can be formulated as a halfspace equation $A_i x \leq b$. Since a polyhedron is n intersection of a finite number of halfspaces and hyperplane, the polyhedron corresponding to the H representation is therefore defined as

$$\mathcal{P} = \{x \in \mathbb{R}^n | Ax \leq b, A_e x = b_e\} \tag{1}$$

accordingly matrix H is as shown in equation 2

$$H = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ -1 & -1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \tag{2}$$

While the polyhedron corresponding to the V-representation is a convex set of vertices and rays (V and R), represented as

$$V = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & -1 \end{bmatrix} \qquad R = [1 - 1] \tag{3}$$

2 Question 2

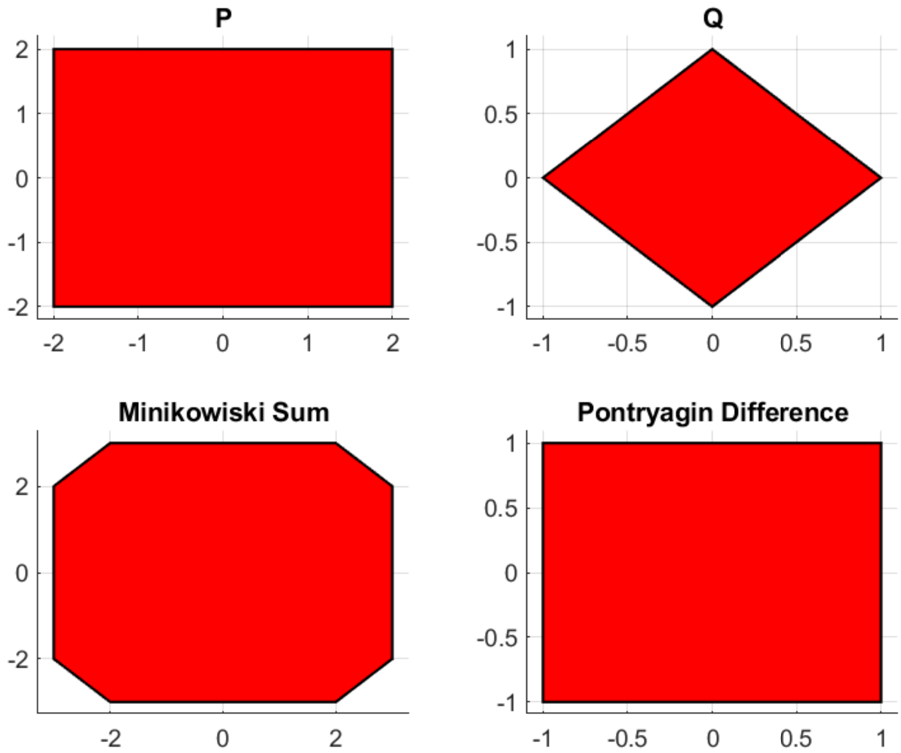

Figure 1: Minikowiski sum and Pontryagin difference of P and Q

3 Question 3

The set $\mathcal{S}$ is positively invariant for the given system $x^+ = Ax$ if $x(0) \in \mathcal{S} \implies x(k) \in \mathcal{S} \quad \forall \quad k \in N_+$. This can be graphically verified by defining the halfspace of the the systems $x^+ = Ax$ as

$$A_{in}x \leq b_{in} \implies A_{in}A^{-1}x^+ \leq b_{in} \tag{4}$$

And check if this is a subset of S. This is proved by the figure 2 obtained using MATLAB and therefore S is positively invariant.

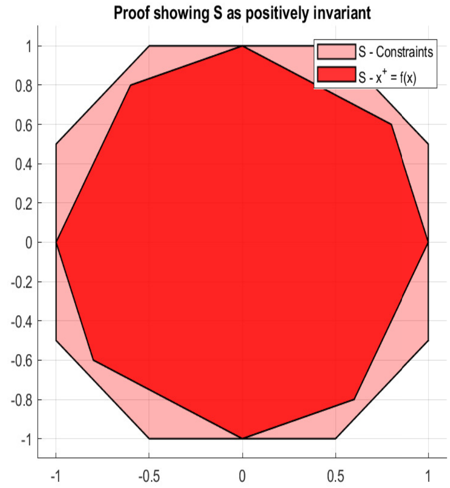

Figure 2: Graphical proof to show S as positive invariant

4 Question 4

Using the *Reach_7.m* function, the derived set $Reach(\mathcal{S})$ gives a set of future state i.e x^+ such that there exists an admissible input signal u that projects $x \in \mathcal{S}$ to x^+ which is again within the domain of S as shown in the Figure 3

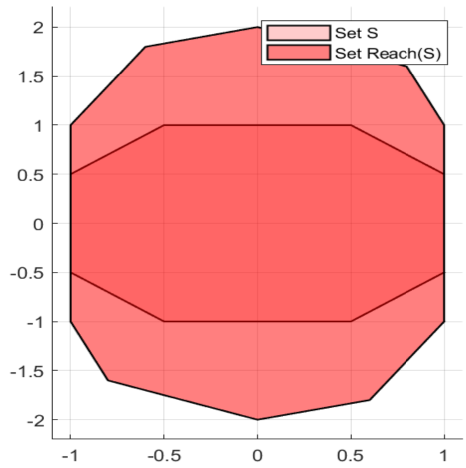

Figure 3: Forward Reachable set of set S

5 Question 5

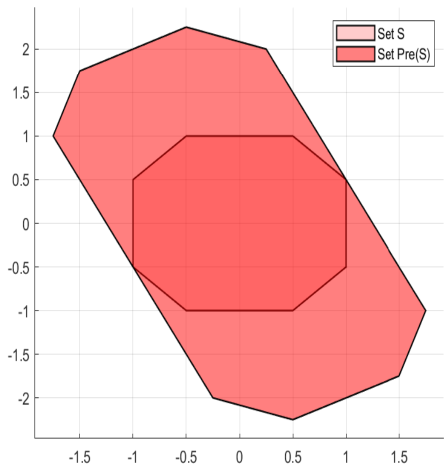

Figure 4: Backward reachability set Pre(S) of set S

6 Question 6.1

N = 26

7 Question 6.2

At N = 2 and for the given initial state conditions x_0 and by using invariantset command of MPT to achieve maximal invariant set $\mathcal{X}_{\{}$, the following plot is as shown in figure 5 which shows that x_0 belongs to the feasibility set.

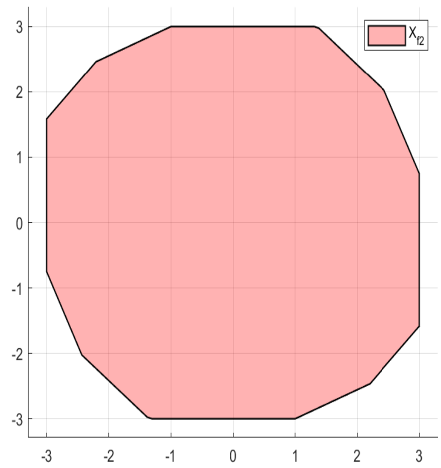

Figure 5: Invariant set (maximal) at N = 2

To check if this RHC will converge to the origin, it's closed loop poles must be within the unit circle of the stability region, as shown in Equation 5, proving the convergence of the RHC solution.

$$\lambda_i(A + BK_0) = \begin{bmatrix} 0.6964 \\ 0.6964 \end{bmatrix} \tag{5}$$

8 Question 6.3

The size of the feasibility set $\mathcal{X}_\mathcal{N}$ depends and is proportional to the terminal state constraint set. Since the terminal constraints set at N = 2 was set at maximal compared to that at N=26 which only had origin as the constraint. This explains why the feasibility set at N = 2 is larger than that of set at N = 26 as shown in Figure 6

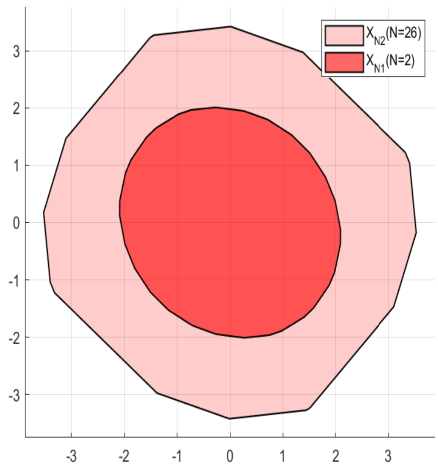

Figure 6: Feasibility sets for N=26 and N=2