SSY281
Assignment 3
MPC practice and Kalman Filter

Manish Suvarna

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Set-point tracking and disturbance modeling

1 Question 1

At steady state, i.e $C \cdot x_s = y_s$ and the condition for setpoint tracking becomes as shown in Equation1 and which gives the states and inputs set-points (x_s, u_s) as shown in 2

$$\begin{bmatrix} I - A & -B \\ C & 0 \end{bmatrix} \begin{bmatrix} x_s \\ u_s \end{bmatrix} = \begin{bmatrix} 0 \\ y_s \end{bmatrix} \tag{1}$$

$$x_s = \begin{bmatrix} 2.5 \\ -1.5 \\ 1.25 \\ -2.25 \end{bmatrix} \quad u_s = \begin{bmatrix} 2.5 \\ -1.5 \end{bmatrix} \tag{2}$$

On using the obtained values of x_s and substituting them in the equation $y = C \cdot x_s$ we reach the point y_s. Hence regulation of y to y_s is achieved.

2 Question 2

This is a steady state target problem with more outputs than inputs $(p > m)$ situation. Therefore the optimization problem to find the best steady-state targets now becomes as shown in Equation 3 and the steady state values that minimizes this two-norm error function was found by using quadprog in MATLAB, and are as shown in 4

$$\begin{aligned} min_{x_s,u_s}(|Cx_s - y_s|^2{}_Q), \quad Q \succeq 0 \\ subject \quad to, \\ \begin{bmatrix} I-A & B \end{bmatrix} \begin{bmatrix} x_s \\ u_s \end{bmatrix} = 0 \end{aligned} \tag{3}$$

$$x_s = \begin{bmatrix} 0.4 \\ 0 \\ 0.2 \\ 0 \end{bmatrix} \tag{4}$$

The realized system output using x_s in 4 is not equalt to y_s because if there are more outputs than inputs, all the set-points cannot be reached in steady state and we have to be satisfied with the output coming to as close as possible to the set-point values.

3 Question 3

This is a Steady-state target problem with more inputs than outputs ($p_z < m$) where p_z is the number of the selected controlled outputs. The optimization problem in this case involves the objective function as shown in 5 whose minimization gives the values of states x_s as shown in 6

$$\begin{aligned} min_{x_s,u_s}(|u_s - u_{sp}|^2{}_{R^s} + |Cx_s - y_s|^2{}_{Q^s}), \quad R_s \succeq 0 \\ subject \quad to \\ \begin{bmatrix} I-A & -B \\ C_z & 0 \end{bmatrix} \begin{bmatrix} x_s \\ u_s \end{bmatrix} = \begin{bmatrix} 0 \\ y_s \end{bmatrix} \end{aligned} \tag{5}$$

$$x_s = \begin{bmatrix} 0.5 \\ 0.5 \\ 0.25 \\ 0.75 \end{bmatrix} \tag{6}$$

By having hard constraints on the controlled outputs in steady-state, $y = y_s$ is achieved for that particular controlled output while the non-controlled outputs are bought as close as possible to the set-points using remaining flexibility of control.

Control of Chemical Reactor

4 Question 4

The augmented model for the given plant with a load disturbance model is as shown in Equation 7. The condition for this model to be detectable is as shown in Equation 8 where n and n_d are the number of states and disturbances in the model respectively.

$$\begin{gathered}\begin{bmatrix} x \\ d \end{bmatrix}^+ = \begin{bmatrix} A & B \\ 0 & I \end{bmatrix} \begin{bmatrix} x \\ d \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u \\ y = \begin{bmatrix} C & C_d \end{bmatrix} \begin{bmatrix} x \\ d \end{bmatrix}\end{gathered} \tag{7}$$

$$rank \begin{bmatrix} I - A & -B_d \\ C & C_d \end{bmatrix} = n + n_d \tag{8}$$

According to this condition mentioned in 8, only system (a) and system(c) were found to be detectable with system (b) being non-detectable.

5 Question 5

Using the dare command in MATLAB which solves for the unique stabilizing solution of the Ricatti equation and on comparing the dualities between the LQR and a kalman filter, the inputs to the dare command were tweaked according to this duality and the resulting Kalman gains for the two detectable augmented systems (a) and (c) were found to be as shwon in Equation 9

$$K_a = \begin{bmatrix} 0.233 & 0.003 & 0.006 \\ -0.0003 & 0.998 & -0.003 \\ -0.036 & -0.031 & 0.283 \\ 0.506 & -0.003 & -0.007 \\ 0.035 & 0.028 & 0.44 \end{bmatrix} \quad K_c = \begin{bmatrix} 0.232 & 0.0007 & -0.019 \\ -0.0002 & 0.999 & 0.002 \\ -0.0417 & -0.015 & 1.11 \\ 0.507 & -0.0009 & 0.0168 \\ 0.0393 & 0.0176 & -0.155 \\ -0.0006 & 0.0027 & 0.116 \end{bmatrix} \tag{9}$$

6 Question 6

Referring to the equality constraints in the Steady-state problem with disturbance as shown in 10 and M_{ss} to be a matrix co-efficient of disturbance signal $\hat{d}$ which is used to determine the state and input set-points (x_s, u_s) as defined in the question, $M_s s$ then becomes, as shown in Equation 11

$$\begin{bmatrix} I-A & -B \\ HC & 0 \end{bmatrix} \begin{bmatrix} x_s \\ u_s \end{bmatrix} = \begin{bmatrix} B_d\hat{d} \\ y_s - HC_d\hat{d} \end{bmatrix} \tag{10}$$

$$M_{ss} = \begin{bmatrix} I-A & -B \\ HC & 0 \end{bmatrix}^{-1} \begin{bmatrix} B_d \\ -HC_d \end{bmatrix} \tag{11}$$

The corresponding M_ss matrices for the two detectable augmented model (a) and (c) are as shown in Equation 12

$$Mss_a = \begin{bmatrix} -1.0 & 0 \\ 112.24 & 18.27 \\ 0 & -1.0 \\ 65.64 & -10.145 \\ 0 & 0 \end{bmatrix} \quad Mss_c = \begin{bmatrix} -1.0 & 0 & 0 \\ 112.24 & 18.27 & 120.42 \\ 0 & -1.0 & 0 \\ 65.64 & -10.14 & -66.85 \\ 0 & 0 & 1.0 \end{bmatrix} \tag{12}$$

7 Question 7

The system states, their estimation and the series of inputs computed using RHC method for System (a) and System (c) are as shown in the Figure 1 and 2 respectively.

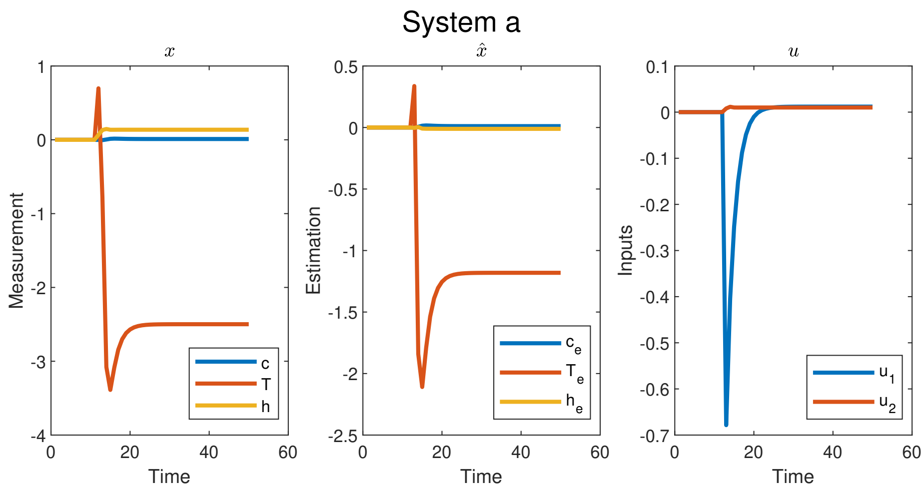

Figure 1: States, Estimation and Inputs of system a using RHC

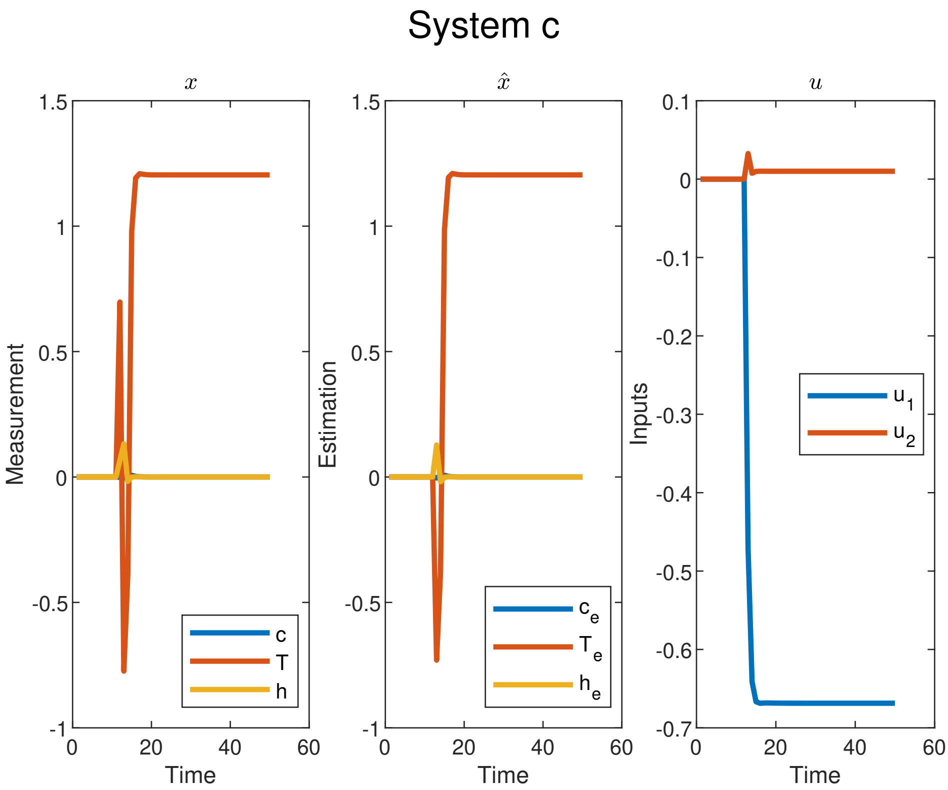

Figure 2: States, Estimation and Inputs of system c using RHC

It can clearly be seen that the off-set exists in the estimation of system (a) when compared to that of system (c) whose estimation are undisturbed even after the disturbance appears. This difference in behaviour can be accounted to considering all the existing disturbance for all the states in system c's augmented model whereas the distrbance was considered only for two states in the system a's augmented model.