SSY281 Model Predictive Control

Assignment 6

MPT and Persistent Feasibility

Due March 4 at 23:59

Systems & Control
Department of Electrical Engineering
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Instructions

This assignment is **individual** and must be solved according to the following rules and instructions:

- Written report:
  - For those questions where a text (motivations, explanations, observations from simulations) rather than a numerical value is asked, the answers should be concisely written in the report.
  - Figures included in the report should have legends, and axes should be labeled.
  - The report should be uploaded *before the deadline* in Canvas.
  - Name the report as A6_X.pdf, where X is your *group* number.
- Code:
  - Your code should be written in the Matlab template provided with this assignment following the instructions therein.
  - Name the Matlab script as A6_Xm, Pre_X.m, Reach_X.m, ShortestN_X.m, and RHCXf_X.m where X is your *group* number.
  - Strictly follow the instructions in the Matlab template.
- Grading:
  - This assignment if worth **15 points** in total.

1 MPT

After installing the Multi-Parametric Toolbox 3 (MPT) in Matlab, run `mpt_demo_sets1` and `mpt_demo2` in MATLAB to study the two demos of the MPT package.

Question 1 (**Points: 1**). *Use the commands in MPT to find the V- and the H-representation of the following polyhedron, plot them and explain the differences in the report.*

$$A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \\ -1 & -1 \\ 1 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

Note: *do not consider the tail in the V representation case, i.e., only consider the corner points to define the polyhedron.*

Question 2 (**Points: 1**). *Use the commands in MPT to compute the Minkowiski sum and the Pontryagin difference of the following polytopes (i.e. $P+Q$ and $P-Q$) and plot them.*

$$P = \{x : A_1 x \le b_1,\ x \in \mathbb{R}^2\}$$

$$Q = \{x : A_2 x \le b_2,\ x \in \mathbb{R}^2\}$$

$$A_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 0 & -1 \\ -1 & 0 \end{bmatrix}, \quad b_1 = \begin{bmatrix} 2 \\ 2 \\ 2 \\ 2 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -1 & -1 \\ 1 & 1 \\ 1 & -1 \\ -1 & 1 \end{bmatrix}, \quad b_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

2 Persistent Feasibility

Question 3 (**Points: 3**). *Consider*

$$x^+ = Ax, \quad A = \begin{bmatrix} 0.8 & 0.4 \\ -0.4 & 0.8 \end{bmatrix}, \tag{1}$$

and show that set S, defined as follows, is positively invariant for this system.

$$S := \{x :\ A_{in} x \le b_{in},\ x \in \mathbb{R}^2\},$$

$$A_{in} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 0 \\ 0 & -1 \\ 1 & 1 \\ 1 & -1 \\ -1 & 1 \\ -1 & -1 \end{bmatrix}, \quad b_{in} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1.5 \\ 1.5 \\ 1.5 \\ 1.5 \end{bmatrix}.$$

Question 4 (**Points: 3**). *Consider*

$$x^+ = Ax + Bu, \quad A = \begin{bmatrix} 0.8 & 0.4 \\ -0.4 & 0.8 \end{bmatrix}, \; B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \tag{2}$$

with $-1 \leq u \leq 1$.

Fill in function `Reach_X.m` *which calculates on step reachable set from* S*, which is defined in Question 3. Depict* S *and the reachable set you have calculated in the same plot.*

Hint. *You can use* `alpha` *in your plot option to adjust transparency of the polyhedrons in the figure.*

Note. *In this question you cannot use the* `reachableSet` *command.*

Question 5 (**Points: 3**). *Fill in function* `Pre_X.m` *which calculates one step "Pre" of* S *for system* (2)*. Depict* S *and the "Pre" set you have calculated in the same plot.*

Hint. *You can use* `projection` *command in MPT.*

Question 6 (**Points: 4**). *Consider*

$$x^+ = Ax + Bu, \quad A = \begin{bmatrix} 0.9 & 0.4 \\ -0.4 & 0.9 \end{bmatrix}, \; B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \tag{3}$$

and design a RH controller with $P_f = \mathbf{0}, Q = I_{2\times 2}, R = 1,$ *and* $\mathcal{X}_f = \mathbf{0}_{2\times 1}$*. Let* $x(0) = \begin{bmatrix} 2 & 0 \end{bmatrix}^\top$ *be the initial condition and consider the following constraints.*

$$|x_i(k)| \leq 3, \; |u(k)| \leq 0.1 \quad \forall k \in \{0, 1, 2, \ldots\}, \; \forall i \in \{1, 2\}$$

1. *Find the shortest prediction horizon N such that the RH controller is feasible.*

2. *Set $N = 2$ and choose $\mathcal{X}_f$ as the maximal control invariant set for the system. Is the RH controller still feasible until convergence to the origin?*

   Hint. *You can use* `invariantSet` *command in MPT.*

3. *Plot the set of feasible initial states for the controllers designed at the previous two points. How do you explain the difference? What is the size of the optimization problems (number of optimization variables and constraints) underlying the two RH controllers?*

   Hint. *You can use* `reachableSet` *command in MPT or* `Pre_X.m` *to calculate feasible initial states.*