Stochastic optimization algorithms
Lecture 12, 20200929

Ant colony optimization (I)

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Today’s learning goals

- After this lecture you should be able to
  - give examples of complex cooperative ant behavior,
  - describe the (main) method by which ants communicate,
  - describe and explain a model of cooperative foraging,
  - define the travelling salesman problem (TSP),
  - describe ant colony optimization (ACO) in general, and ant system (AS), in particular.

Ants

- Ants are one of the most widespread species on the planets.
- One of their most amazing characteristics (shared with bees and termites) is their remarkably complex cooperative behavior.

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Cooperative ant behavior

- Examples
  - Cooperative food transportation

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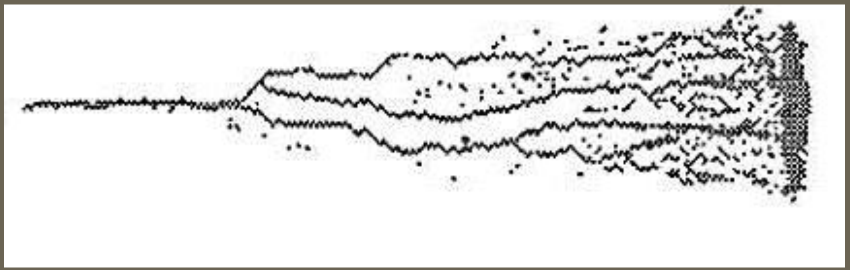
Cooperative ant behavior

- Examples
  - Dynamic bridge-building

Cooperative ant behavior

- Examples
  - Cooperative foraging (food search and retrieval)



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Cooperative ant behavior

- Video examples
  - Dynamic bridge-building
  - Cooperative transport (of an entire ant colony)

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Cooperative ant behavior

- Based on these examples, one might think that
  - … ants have very skilled leaders
  - … ants are capable of sophisticated long-range communication

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Cooperative ant behavior

- Based on these examples, one might think that
  - … ants have very skilled leaders
  - … ants are capable of sophisticated long-range communication
- NO! Neither of those assertions is true!
- So, *how* do ants achieve their complex cooperative behaviors?

Pheromones

- Ants communicate indirectly by means of secreted substances known as **pheromones**.
- As they move, they deposit a pheromone trail that other ants can follow.
- Note that pheromones are volatile. That is, they evaporate after a while.

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Stigmergy

- This mechanism, i.e. indirect communication via local modification of the environment is known as **stigmergy**.
- Video link

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Cooperative foraging: Model[1]

- Phenomenological model: At a two-way junction, ants select a direction **probabilistically** according to

$$p_{\text{left}} = \frac{(C+L_1)^m}{(C+L_1)^m + (C+R_1)^m}$$

$$p_{\text{right}} = 1 - p_{\text{left}}$$

- … where *C* and *m* are constants, and L_1 and R_1 are the number of ants that have previously selected the left and right direction, respectively.

Deneubourg *et al.* The self-organizing exploratory pattern of the argentine ant, Journal of Insect Behavior, **3**, pp. 159-168, 1990.

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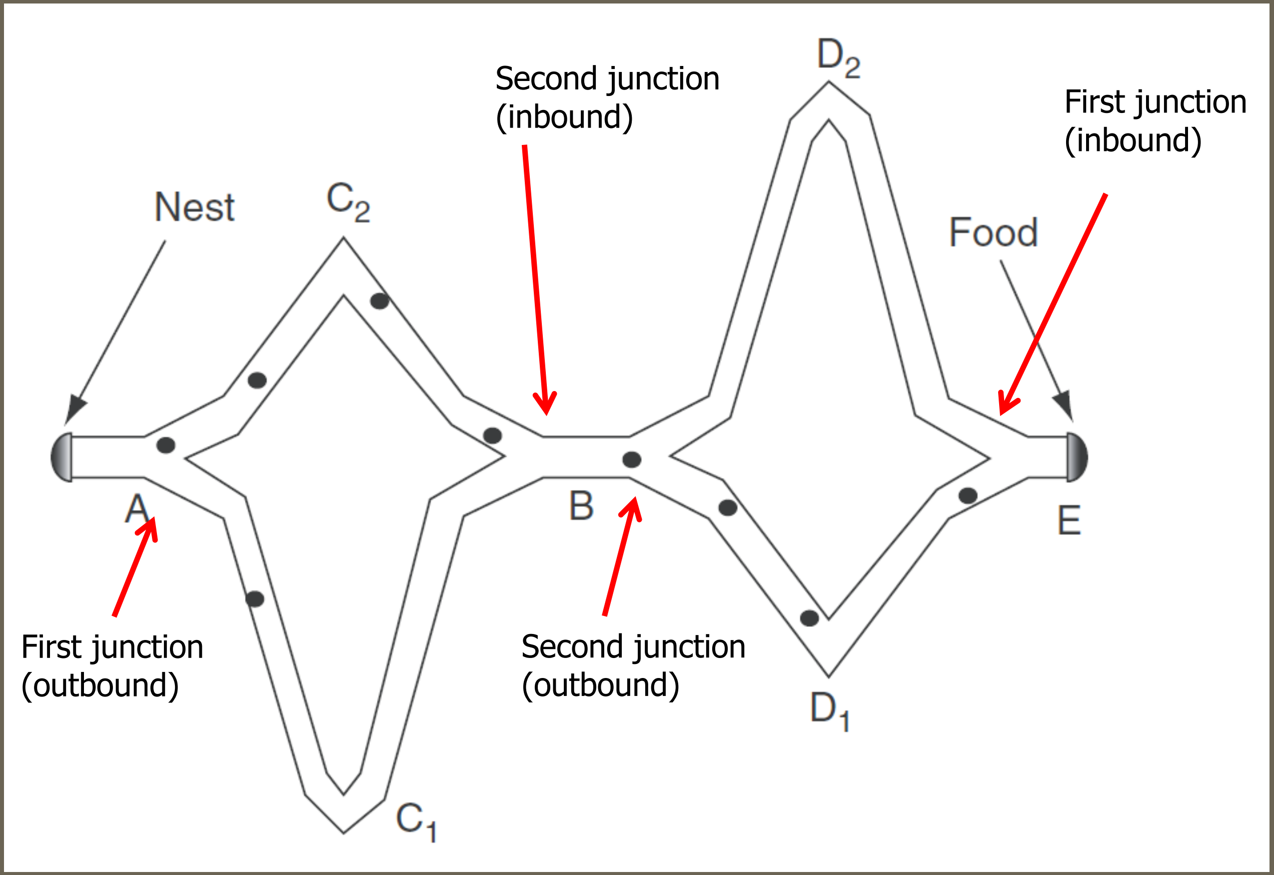
Cooperative foraging: Experiment



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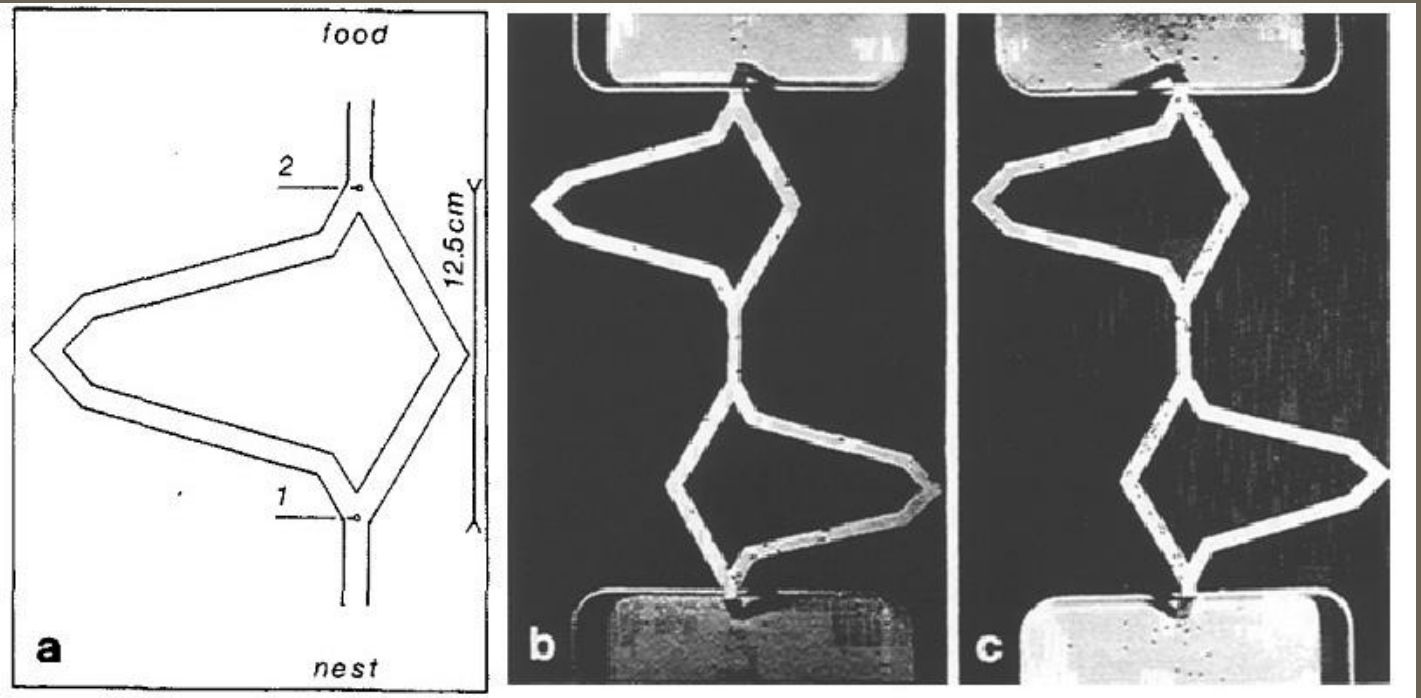

Fig. 1. A colony of *I. humilis* selecting the short branches on both modules of the bridge; a) one module of the bridge, b) and c): photos taken 4 and 8 min after placement of the bridge

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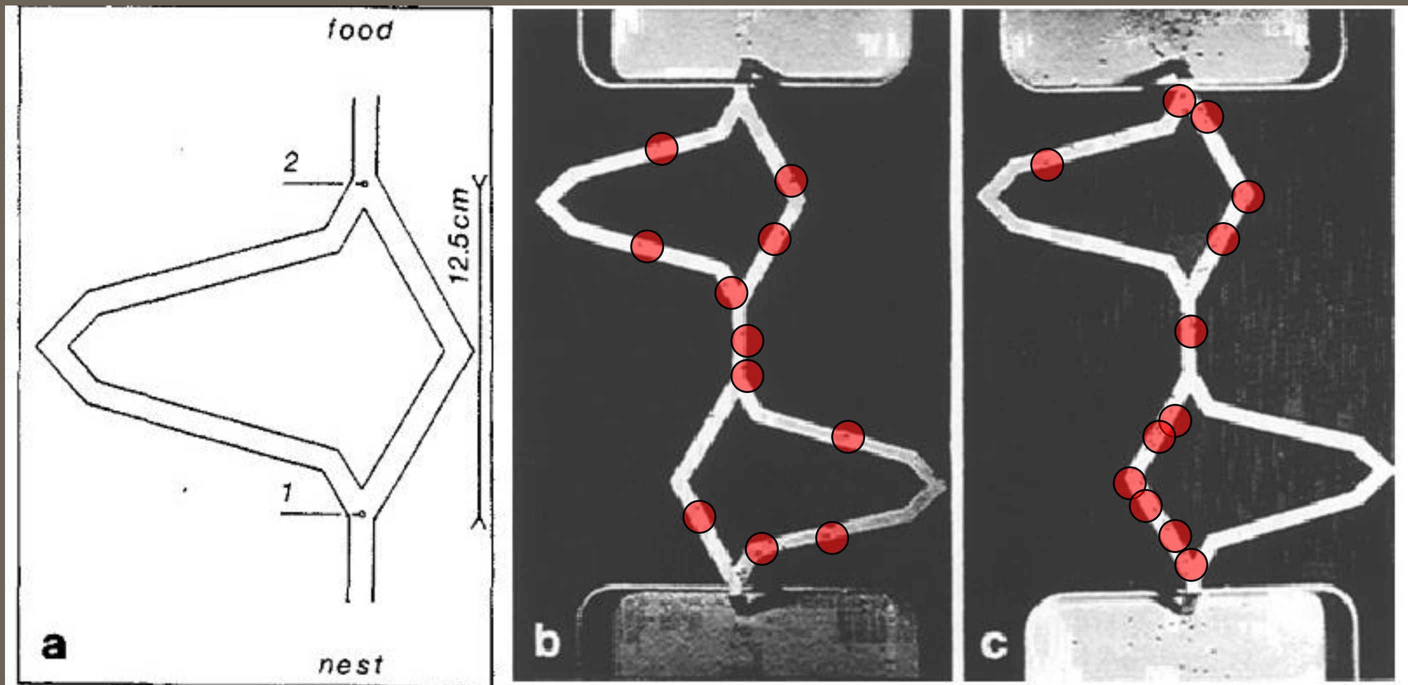

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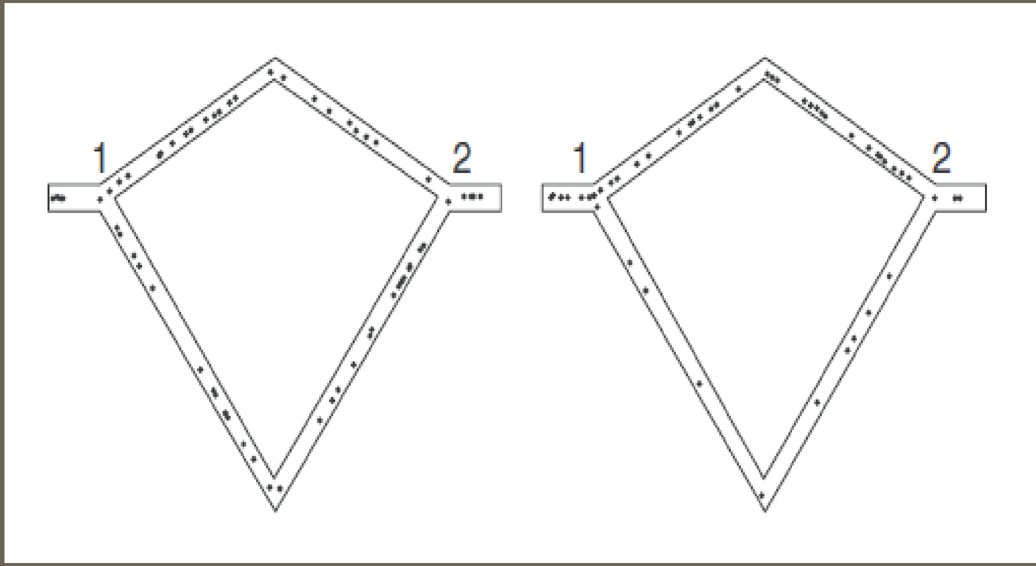
Cooperative foraging: Experiment

- Deneubourg et al. found that one obtains a very good fit to the observed ant behavior by setting C = 20, m = 2.

Cooperative foraging: Simulation

- Numerical simulation of this model, using a simplified arena



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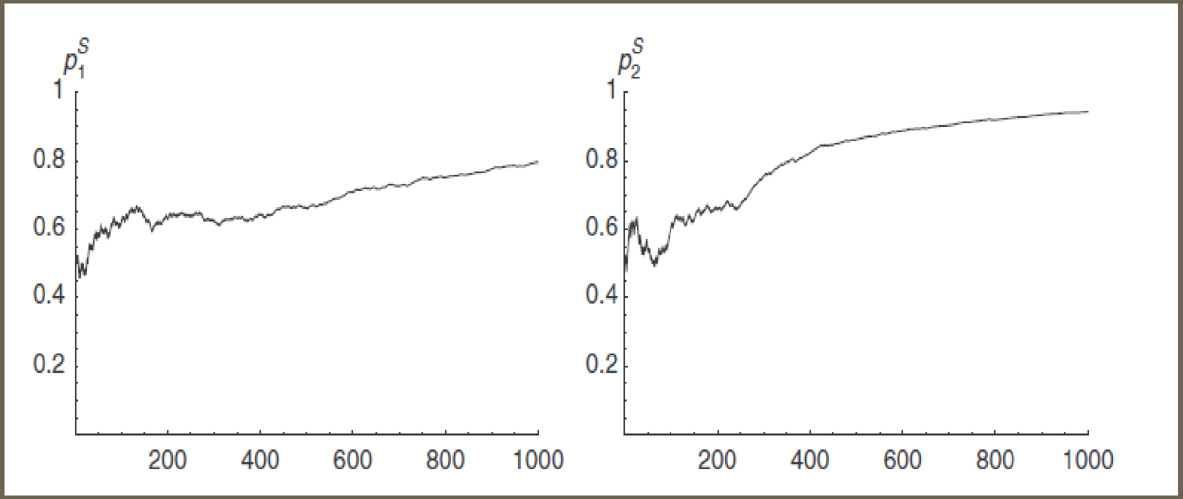
Cooperative foraging: Simulation

- Results: Most ants will take the shorter path after a while:



Positive reinforcement

- The cooperative behavior of the ants is based on **positive reinforcement** – ants generate a scent trail, other ants follow, thereby reinforcing the trail etc.
- Mostly, this works well in ant foraging.
- However, positive reinforcement can be dangerous!

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Positive reinforcement

- The *circle of death*: Ants follow an ever-strengthening pheromone trail, until they drop dead from exhaustion.
- [Video link]

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Ant colony optimization

- After some time, other researchers (notably Marco Dorigo) realized that the cooperative behavior of ants can be used as inspiration for an optimization algorithm for graph search, i.e. minimizing the length (or some other cost measure) of paths defined on a graph.
- A typical example is the travelling salesman problem:
  – Find the shortest path between n nodes (cities) such that
    - …each city is visited exactly once…
    - …except that, after visiting the final city, one returns to the city of origin.

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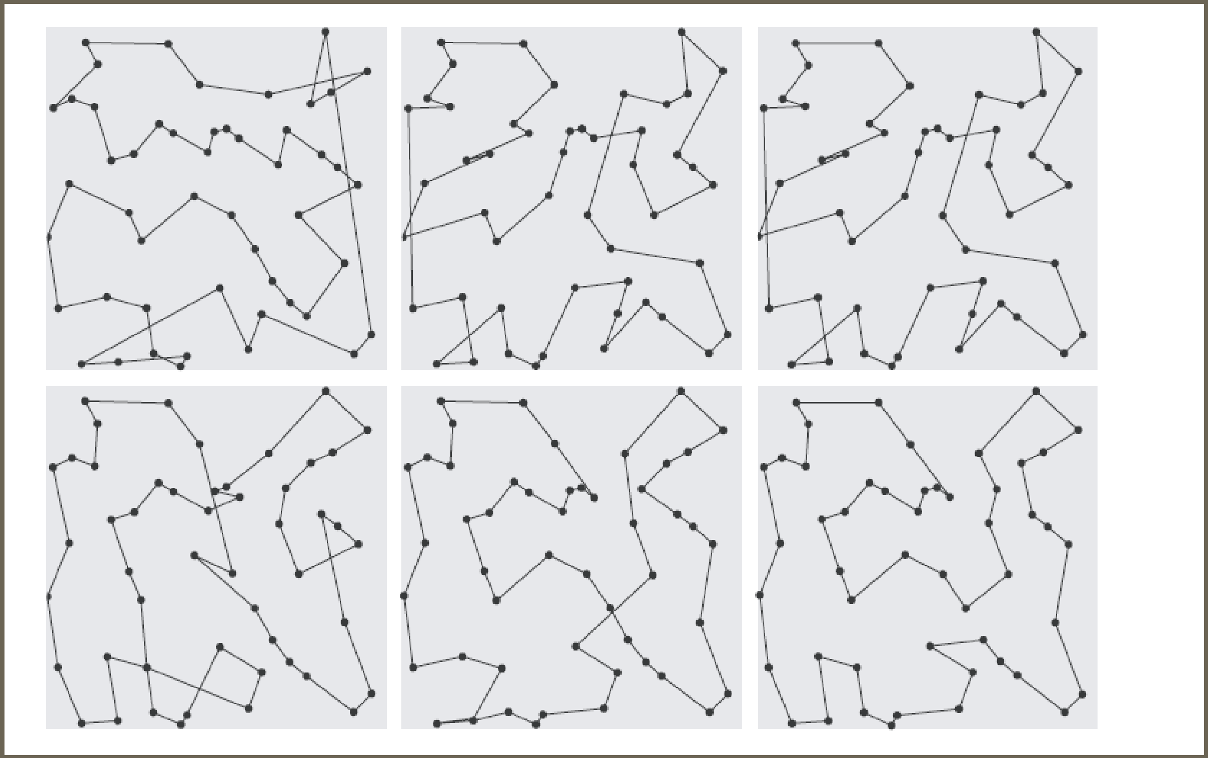
Travelling salesman problem (TSP)



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Ant colony optimization

- The are many different versions of ant colony optimization (ACO).
- One of the first versions is ant system (AS), which we will study now.
- ACO algorithms are applied to a graph known as the **construction graph**.
- Examples, see next slide (and pp. 104-105 in the book).

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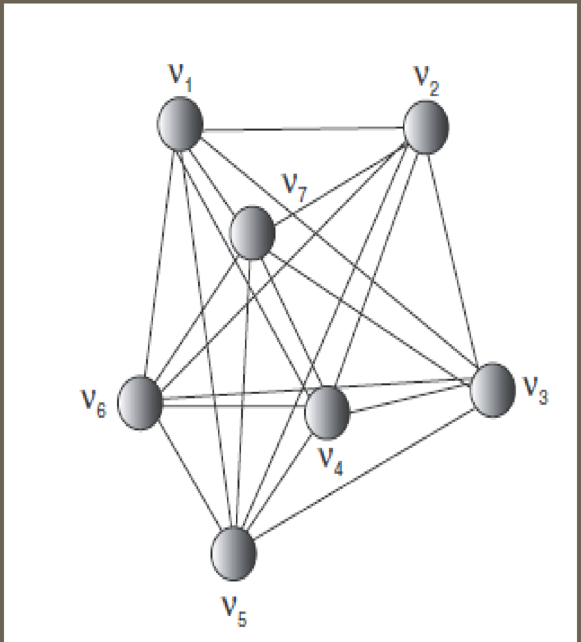
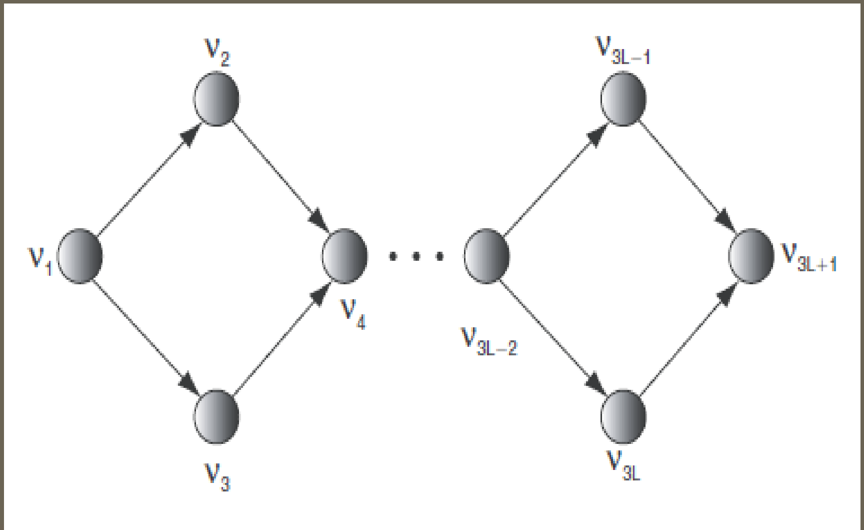
Construction graphs

- Construction graph for TSP
- Straightforward interpretation



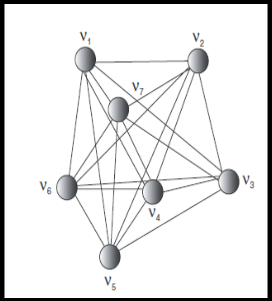
Construction graphs

- Chain construction graph
- Used for generating binary numbers of length L (that can then be used when forming variables)
- Graph with 3L+1 nodes and 4L edges:
  - At ν_1 if an up-move is chosen (move to ν_2), output 1.
  - If instead a down-move is chosen (move to ν_3), output 0.
  - Then move (deterministically) to ν_4 and repeat ...



Ant system

- Use TSP as an example!



- Algorithm 4.1 in the book.
- Important concepts
  - Tabu list
  - Visibility
  - Pheromone update rule

1. Initialize pheromone levels:

   $$\tau_{ij} = \tau_0, \quad \forall i, j \in [1, n].$$

2. For each ant k, select a random starting node, and add it to the (initially empty) tabu list L_T. Next, build the tour S. In each step of the tour, select the move from node j to node i with probability $p(e_{ij}|S)$, given by:

   $$p(e_{ij}|S) = \frac{\tau_{ij}^{\alpha} \eta_{ij}^{\beta}}{\sum_{v_l \notin L_T(S)} \tau_{lj}^{\alpha} \eta_{lj}^{\beta}}.$$

   In the final step, return to the node of origin, i.e. the first element in L_T. Finally, compute and store the length D_k of the tour.
3. Update the pheromone levels:
   3.1. For each ant k, determine $\Delta\tau_{ij}^{[k]}$ as:

   $$\Delta\tau_{ij}^{[k]} = \begin{cases} \frac{1}{D_k} & \text{if ant } k \text{ traversed the edge } e_{ij}, \\ 0 & \text{otherwise.} \end{cases}$$

   3.2. Sum the $\Delta\tau_{ij}^{[k]}$ to generate $\Delta\tau_{ij}$:

   $$\Delta\tau_{ij} = \sum_{k=1}^{N} \Delta\tau_{ij}^{[k]}.$$

   3.3. Modify τ_{ij}:

   $$\tau_{ij} \leftarrow (1-\rho)\tau_{ij} + \Delta\tau_{ij}.$$

4. Repeat steps 2 and 3 until a satisfactory solution has been found.

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Ant system: Step 1

- Typical initialization:
- $\tau_0 = N/D^{nn}$, where N is the number of ants, and D^{nn} is the *nearest-neighbour path* (obtained by starting at a random node and then moving to the nearest neighbour etc).
- Alternatively, just set τ_0 to any small value.

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Ant system: Step 2

- Building the tour.
  - Select a random start node.
  - Generate an (empty) tabu list (= list of visited nodes).
  - At the current node (j), select the move to node *i* probabilistically, using

$$p(e_{ij}|S) = \frac{\tau_{ij}^{\alpha}\eta_{ij}^{\beta}}{\sum_{\nu_l \notin L_T(S)} \tau_{lj}^{\alpha}\eta_{lj}^{\beta}}.$$

  - Then, after completing the tour, store its length D_k.

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Ant system: Step 2

probability of going to node *i,* i.e. selecting edge e_{ij} for the move

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The tour *S* generated, *so far*
Initially empty, then add the first node selected (after the first move) etc.

Ant system: Step 2

probability of going to node *i*, i.e. selecting edge e_{ij} for the move

Pheromone level on edge e_{ij}, raised to the power α.

Visibility of node *i* from node *j*. (taken as $1/d_{ij}$ for standard TSP), raised to the power β.

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Normalization factor (so that p can be treated as a probability.

The tour *S* generated, *so far* Initially empty, then add the first node selected (after the first move) etc.

Sum over all unvisited nodes, i.e. nodes that are *not* yet in the tabu list.

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Ant system: Step 3

- Update pheromones
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evaporation rate

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   In the final step, return to the node of origin, i.e. the first element in L_T. Finally, compute and store the length D_k of the tour.

3. Update the pheromone levels:

   3.1. For each ant k, determine $\Delta\tau_{ij}^{[k]}$ as:

$$\Delta\tau_{ij}^{[k]} = \begin{cases} \frac{1}{D_k} & \text{if ant } k \text{ traversed the edge } e_{ij}, \\ 0 & \text{otherwise.} \end{cases}$$

   3.2. Sum the $\Delta\tau_{ij}^{[k]}$ to generate $\Delta\tau_{ij}$:

$$\Delta\tau_{ij} = \sum_{k=1}^{N} \Delta\tau_{ij}^{[k]}.$$

   3.3. Modify τ_{ij}:

$$\tau_{ij} \leftarrow (1-\rho)\tau_{ij} + \Delta\tau_{ij}.$$

4. Repeat steps 2 and 3 until a satisfactory solution has been found.

Ant system

- Probabilistic selection of the next node (similar to RWS in GAs).
- It is ***not*** so that the artificial ants always go to the node for which $p(e_{ij}|S)$ is maximal!

1. Initialize pheromone levels:

$$\tau_{ij} = \tau_0, \quad \forall i, j \in [1, n].$$

2. For each ant k, select a random starting node, and add it to the (initially empty) tabu list L_T. Next, build the tour S. In each step of the tour, select the move from node j to node i with probability $p(e_{ij}|S)$, given by:

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Ant system

- Note that, in general,

$$\tau_{ij} \neq \tau_{ji}$$

- However, at least for TSP,

$$\eta_{ij} = \eta_{ji}$$

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$\tau_{ij} = \tau_0, \quad \forall i, j \in [1, n].$

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4. Repeat steps 2 and 3 until a satisfactory solution has been found.

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Ant system

- Use TSP as an example!
- Algorithm 4.1 in the book.
- Important concepts
  - Tabu list
  - Visibility
  - Pheromone update rule
- Typical parameters:
  - $\alpha = 1$
  - $\beta = 2$ to 5
  - $\rho = 0.5$

1. Initialize pheromone levels:

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   $$\tau_{ij} \leftarrow (1-\rho)\tau_{ij} + \Delta\tau_{ij}.$$

4. Repeat steps 2 and 3 until a satisfactory solution has been found.

Today's learning goals

- After this lecture you should be able to
  - give examples of complex cooperative ant behavior,
  - describe the (main) method by which ants communicate,
  - describe and explain a model of cooperative foraging,
  - define the travelling salesman problem (TSP),
  - describe ant colony optimization (ACO) in general, and ant system (AS), in particular.