Stochastic optimization algorithms
Lecture 11, 20200925

Evolutionary algorithms: Applications (II)

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NOTE:

- Make sure to attend the lecture on Tuesday!
- I will then introduce the next algorithm (Ant colony optimization).

- Check the FAQ frequently! I have added a lot of information regarding HP2 there, for example!

Today’s learning goals

- After this lecture you should be able to
  - Describe orbit determination in astrophysics, using GAs
  - Describe financial time series prediction, using EA (LGP)
- Note that there will be no exam questions regarding the details of any of the applications – they are meant as inspiring illustrations.

Orbit determination of interacting galaxies

- Many galaxies are parts of pairs (or larger groups)
- Due to gravitational forces, the galaxies sometimes collide (a process that takes millions of years), forming arms, tails etc.

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Orbit determination of interacting galaxies

- For astrophysicists, determining the orbits of such pairs is of importance in the contexts of …
  - … star formation
  - … dark matter distributions
  - … spiral structure formation etc.

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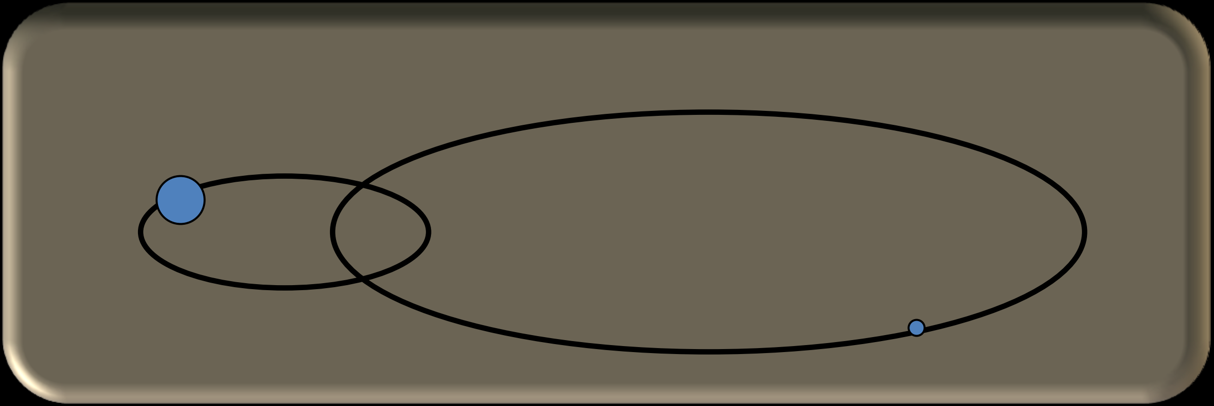
Orbit determination of interacting galaxies

- Given the positions and velocities of two (point) particles at any given time, the orbit can be determined.

$$\ddot{x}_1 = -G\frac{m_2(x_1 - x_2)}{\|x_1 - x_2\|^3}, \ddot{x}_2 = -G\frac{m_1(x_2 - x_1)}{\|x_1 - x_2\|^3}$$



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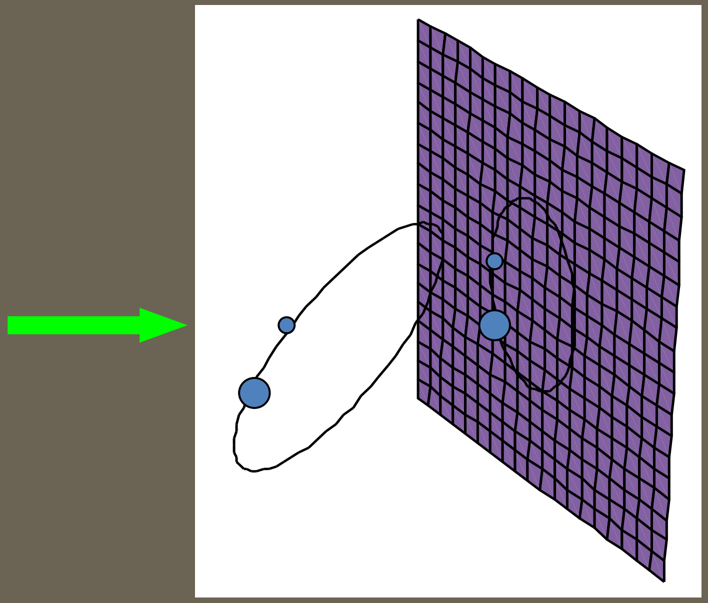
Orbit determination of interacting galaxies

- Problems:
  - One can only observe the projection of an orbit on the plane of the sky ($\Delta X, \Delta Y$).
  - One can only determine velocity components along the line-of-sight (ΔV_z)



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Orbit determination of interacting galaxies

- Moreover, galaxies are not point particles!
  - For each galaxy, two angles (inclination and position angle) are needed to determine the orientation of the galactic disc.
- Also, the masses of the two galaxies must be estimated.
- Thus, there are (at least) 9 unknown variables: $\Delta Z, \Delta V_x, \Delta V_y, I_1, PA_1, I_2, PA_2, M_1, M_2$.
- In our case, we used a GA to determine the value of these 9 parameters.

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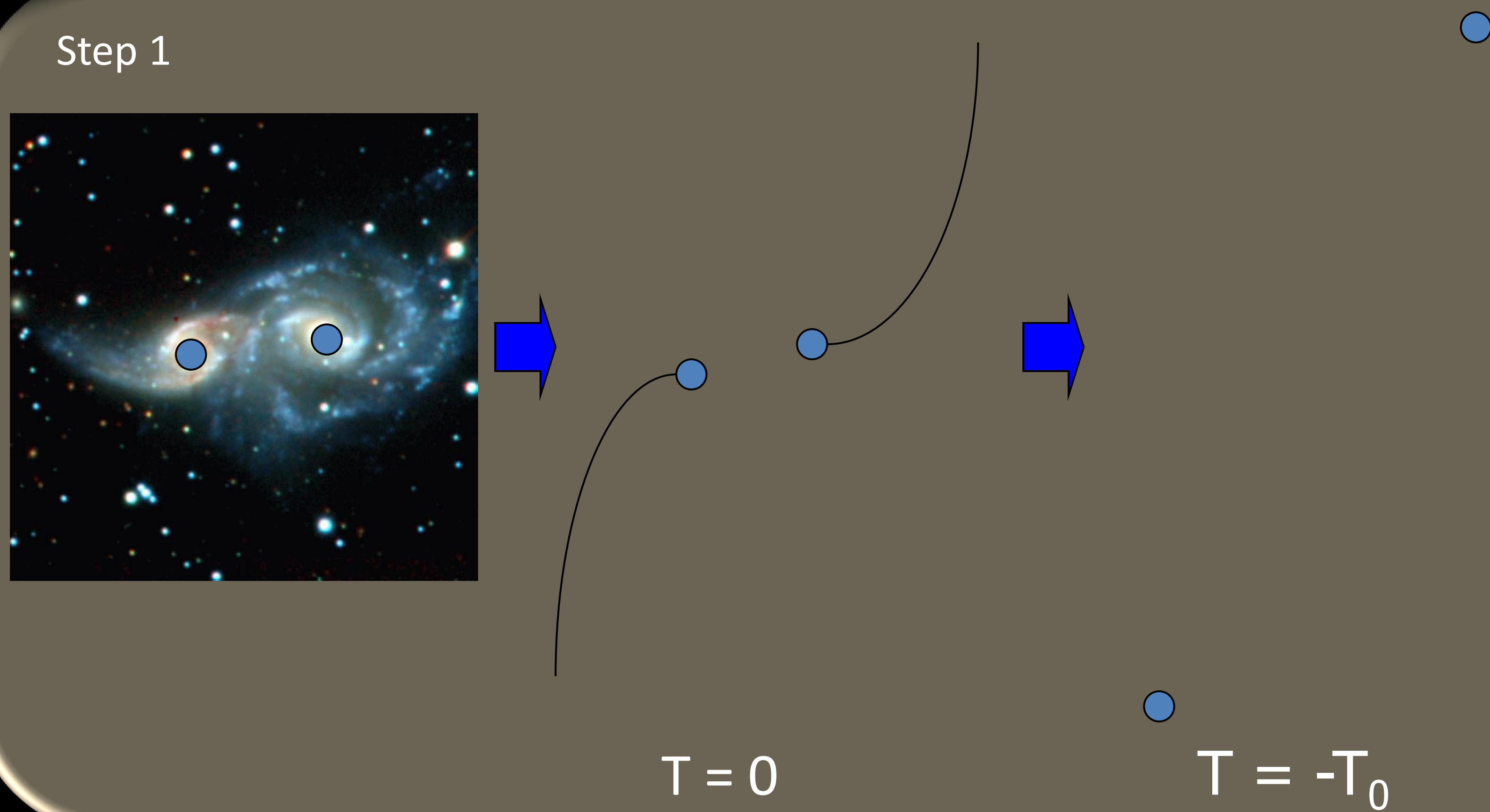
Orbit determination of interacting galaxies

- Assuming that one has a set of values for the unknown parameters, how does one test these values?
- Basically, (in simulation) one…
  - …integrates backwards in time (two point particles, one for each galaxy),
  - …adds a circularly symmetric disc (consisting of many particles) to each galaxy,
  - …integrates forward in time until the present time,
  - …compares the mass (light) distribution with the observed distribution.

Orbit determination of interacting galaxies

Step 1



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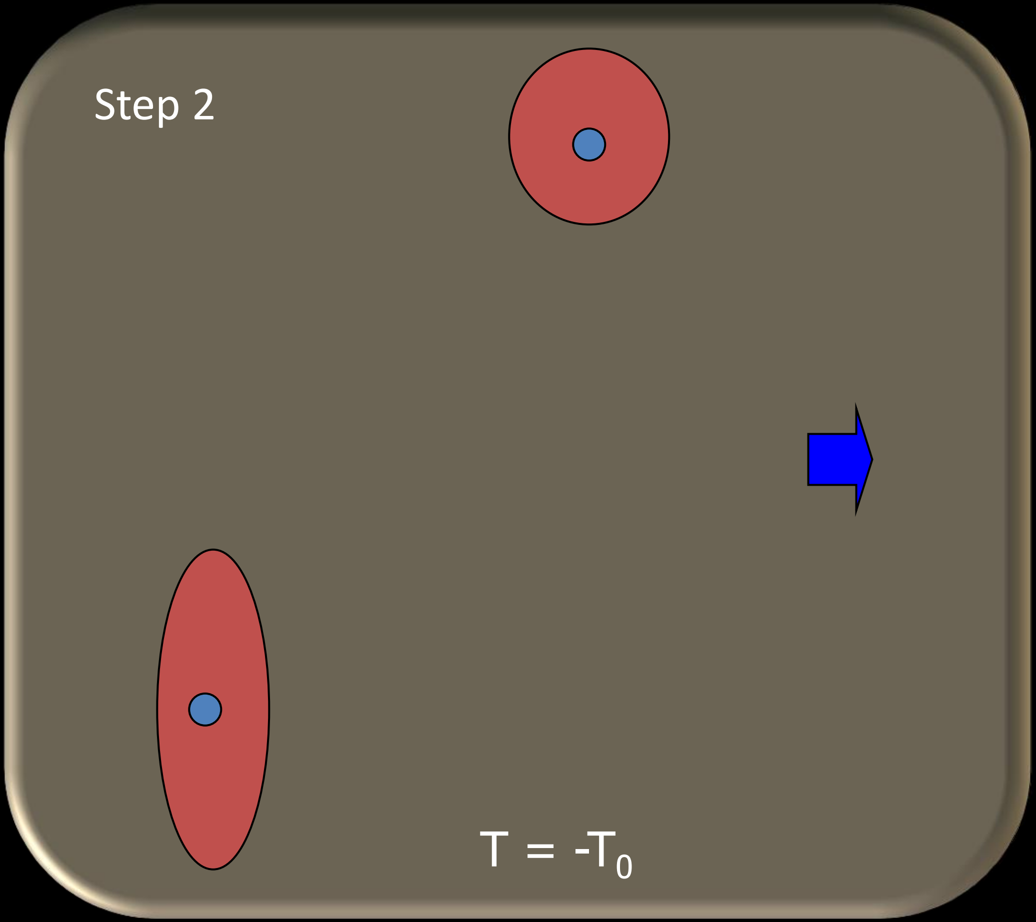
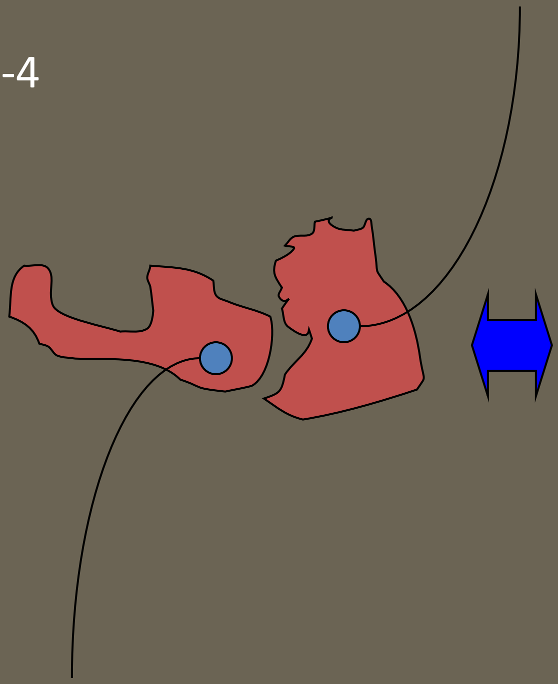
Orbit determination of interacting galaxies



Application 4: Orbit determination

Orbit determination of interacting galaxies

Steps 3-4



T = 0

Orbit determination of interacting galaxies

- Note:
  - Since one must carry out many evaluations (for the GA) the integration must be fast.
  - Thus, the circularly symmetric disc (used in the forward integration) consists of a set of massless particles, neglecting interparticle forces.
  - In violent galactic interactions (such as the one considered here), the forces acting on any star is dominated by the overall gravity field of the two galaxies, not the gravitational pull from nearby stars.

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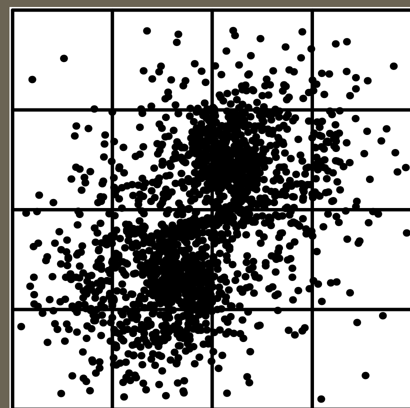
Orbit determination of interacting galaxies

- Evaluating the results:
  - Ideally, one should compare the mass distributions.
  - For the observed galaxies, one only has the light distribution.
  - In general (in galactic astrophysics) one assumes a constant mass-to-light ratio, so that light can be transformed to mass.
  - Here, we overlaid a grid on the image of the galaxies (simulated and observed) and the computed the difference in the mass distribution.

Orbit determination of interacting galaxies



0.003	0.023	0.050	0.014
0.023	0.216	0.465	0.066
0.100	0.580	0.157	0.020
0.036	0.174	0.026	0.004

- Fitness $f = 1/(1+\delta)$, where

$$\delta = \sum_{i,j}(m_{ij}{}^{\mathrm{obs}} + m_\varepsilon)\left|\ln \frac{m_{ij}+m_\varepsilon}{m_{ij}{}^{\mathrm{obs}}+m_\varepsilon}\right|$$

(m_ε is a small constant)

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Orbit determination of interacting galaxies

- Many candidate solutions (~10,000 – 100,000) were tested by the genetic algorithm until a satisfactory solution is found.
- The method was found to work well on a set of test cases (where the images were artificial, and the orbits were known).
- Next, it was applied to a real pair of galaxies, namely M51, the so-called whirlpool galaxy.

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The case of M51

- The M51 system, consisting of the main galaxy (NGC 5194) and its interaction partner (NGC 5195), is one of the most thoroughly studied galaxy pairs.

The case of M51

- A reasonable guess would be that NGC 5195 is located in the same plane as the disc of NGC 5194, and that it has recently generated magnificent spiral arms.
- Such a model (which gave rise to the term ”M51-like systems”) was made by Toomre and Toomre 1972.

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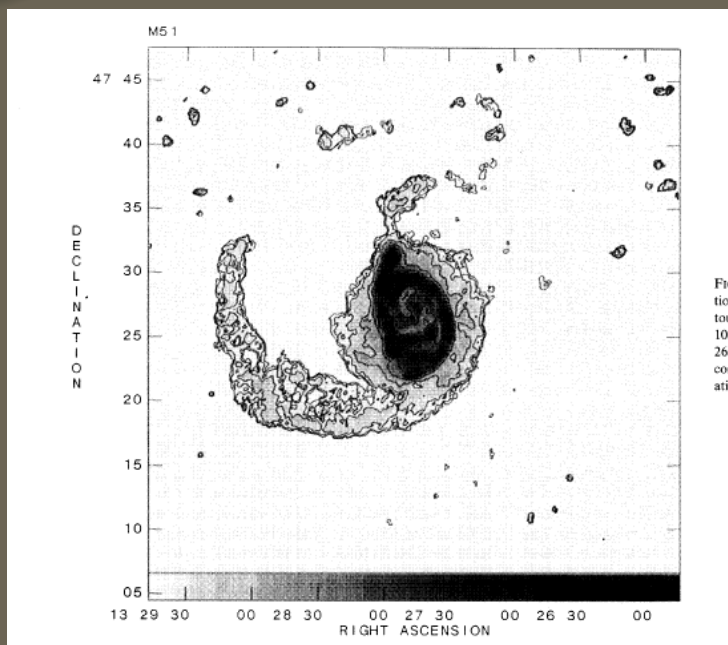
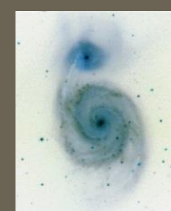
The case of M51

- However, in 1991, a gigantic arm of atomic hydrogen (HI) was discovered by Rots et al.





Application 4: Orbit determination

The case of M51

- The HI-arm is truly enormous, and cannot have been formed in a recent passage by NGC 5195.
- Thus, M51 is not an M51-type system!
- Engström and Athanassoula attempted to model the NGC5194/5195 interaction, but did not manage to fit the velocity field.

The case of M51

- Karl Johan Donner and I applied the method to the NGC 5194/5195 system. *
- Our aim was to fit the positions and velocities within the HI-arm.
- Thus, we blocked out the central region of the NGC 5194 galaxy.

* Wahde, M. & Donner, K.J. *Determination of the orbital parameters of the M51 system using a genetic algorithm,* Astronomy & Astrophysics, **379**, pp. 115-124, 2001.

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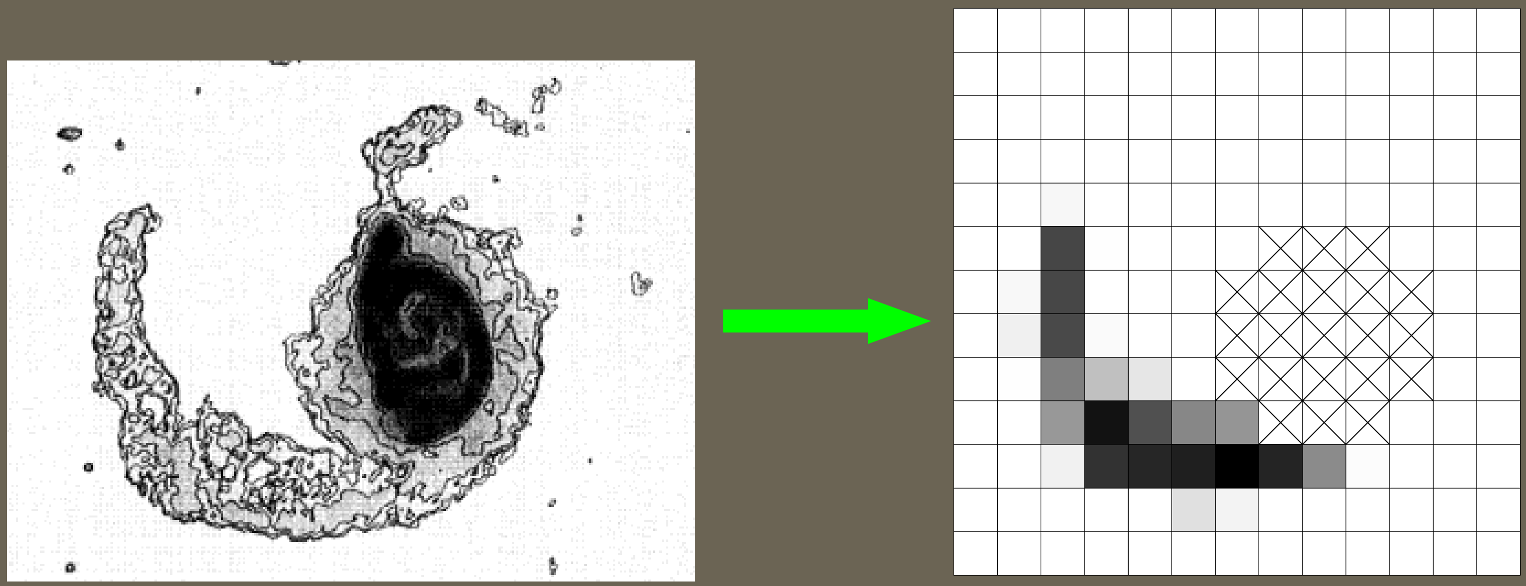
The case of M51



Application 4: Orbit determination

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The case of M51

- We found that, rather than being in the same plane as NGC 5194, the orbit of NGC 5195 is almost perpendicular to the plane of the sky.

- On our best-fit orbit, NGC 5195 is presently located 147 kpc behind NGC 5194, and moving on a hyperbolic orbit. The pericentre passage occurred 908 Myr ago, and the disc (of NGC 5194) was passed at a distance of 17 kpc from the centre, 849 Myr ago.

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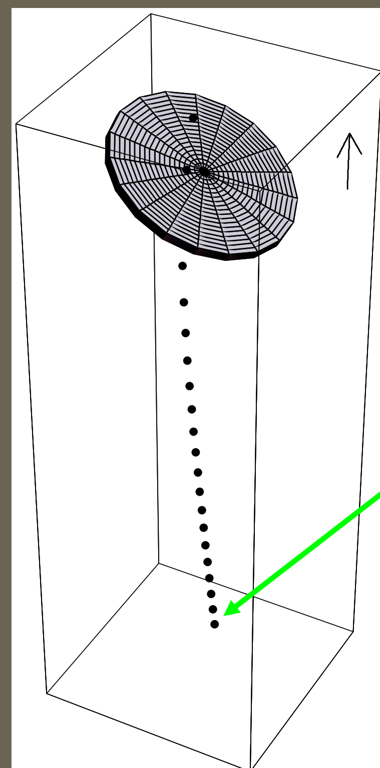
The case of M51

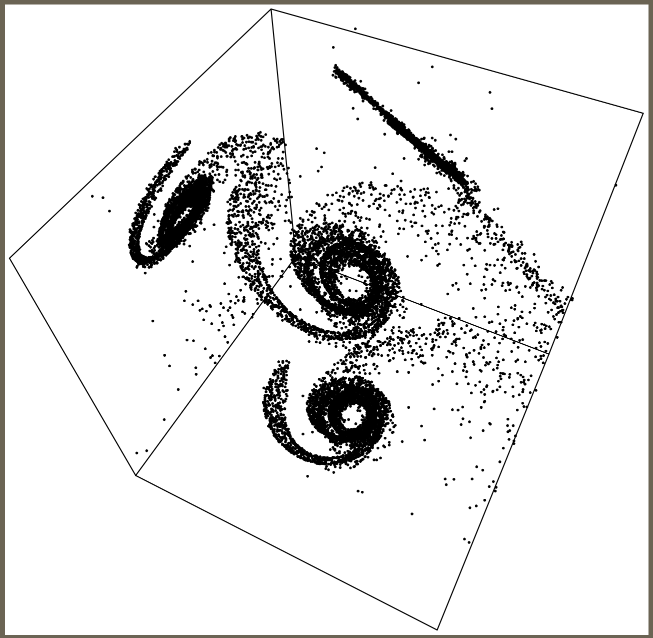


Current position of NGC5195

Application 4: Orbit determination

The case of M51

- A view of the results obtained using the best-fit parameters (three different projections shown)



Application 4: Orbit determination

The case of M51

- As a final test, we also carried out a simulation using full self-gravity, i.e. including also all interparticle forces.
- Nominally scales as N^2 (where N is the number of particles), rather than N as for the massless discs.
- However, using various techniques one can reduce the computation time to $N \log N$.

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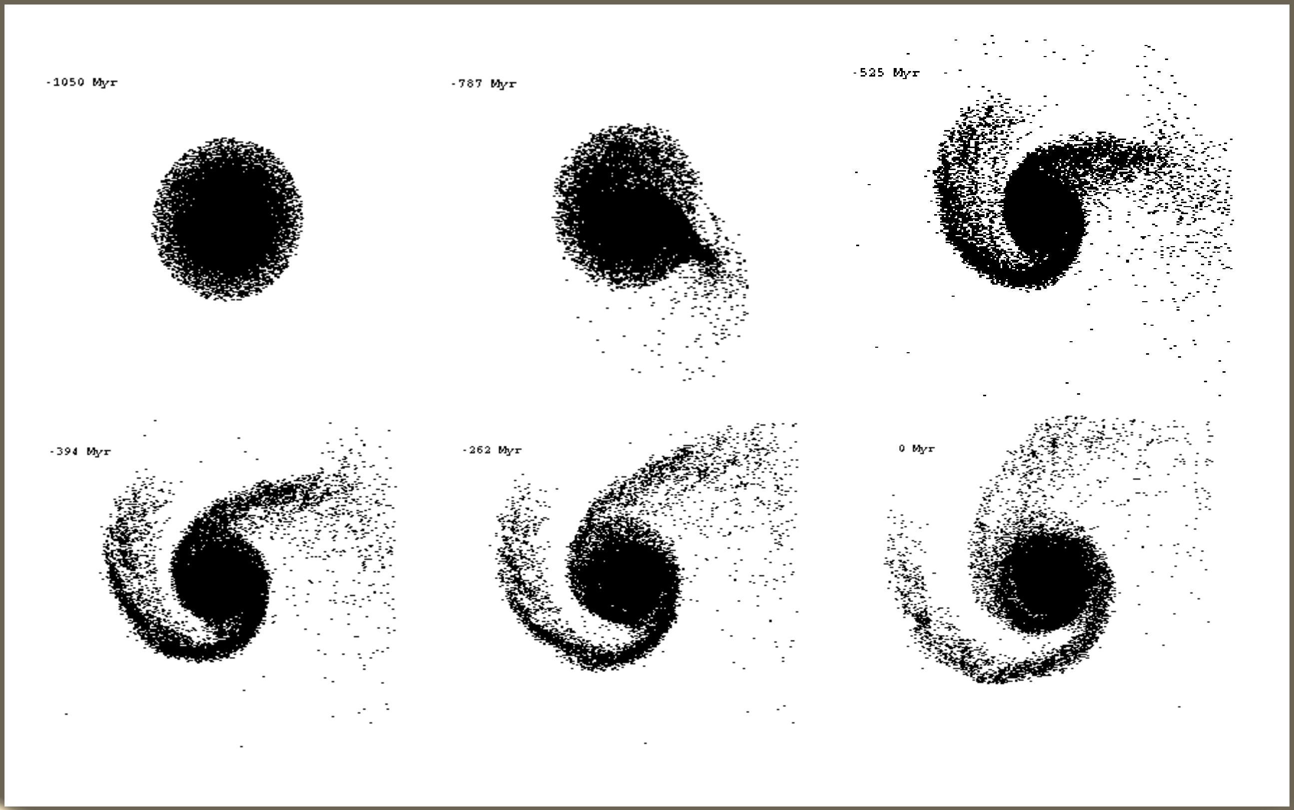
The case of M51



Application 4: Orbit determination

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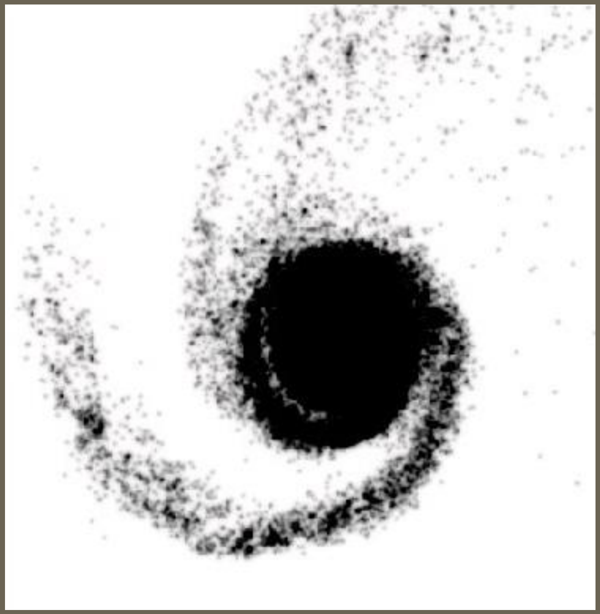
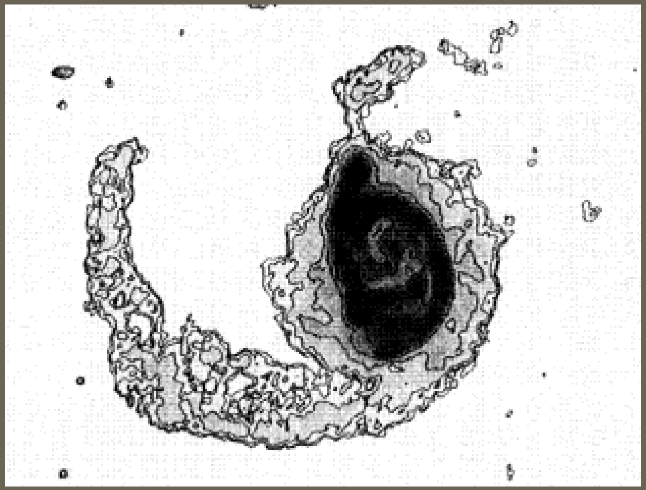
The case of M51

- Best-fit model compared to the observed galaxy:





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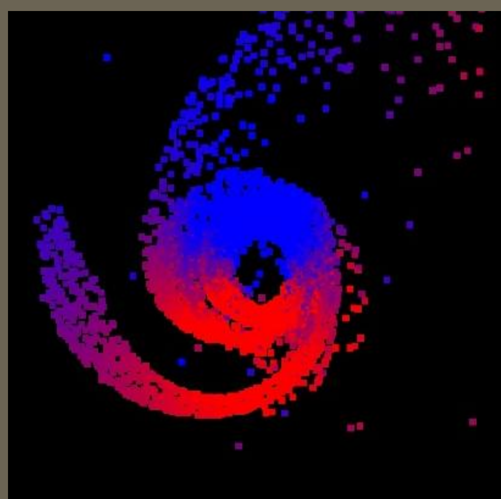
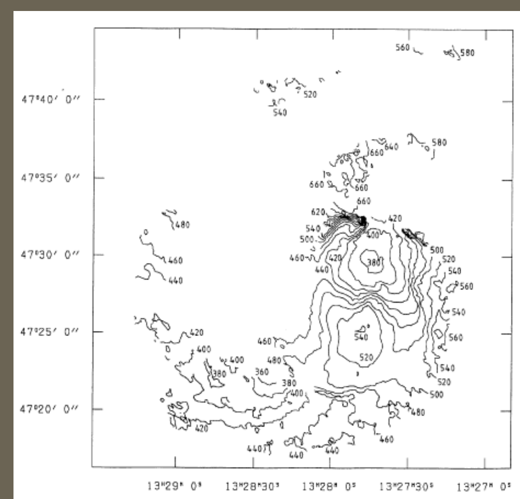
The case of M51

- Even though the position of the arm in our best-fit simulation corresponds well to observations, the velocity field is not reproduced perfectly:





The case of M51

- Summary:
  - A method for determining the orbits of interacting galaxies has been developed.
  - The method uses genetic algorithms to find the orbits of the galaxies.
  - The method has been applied to the NGC 5194/5195 system.
  - On the best-fit orbit, NGC 5195 passed the disc of NGC 5194 almost a billion years ago, and is now located far behind the disc of that galaxy.
  - The main discrepancy between our model and the observed galaxy lies in the velocity field.

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Today's learning goals

- After this lecture you should be able to
  - Describe orbit determination in astrophysics, using GAs.
  - Describe financial time series prediction, using EA (LGP)

Financial time series prediction

- The goal of this project has been *to investigate whether one can find optimized (combinations of) patterns (explained below) in financial time series that are human-interpretable* and...
- ...*which generate results significantly above market returns*.
- A general reference for this topic can be found here.

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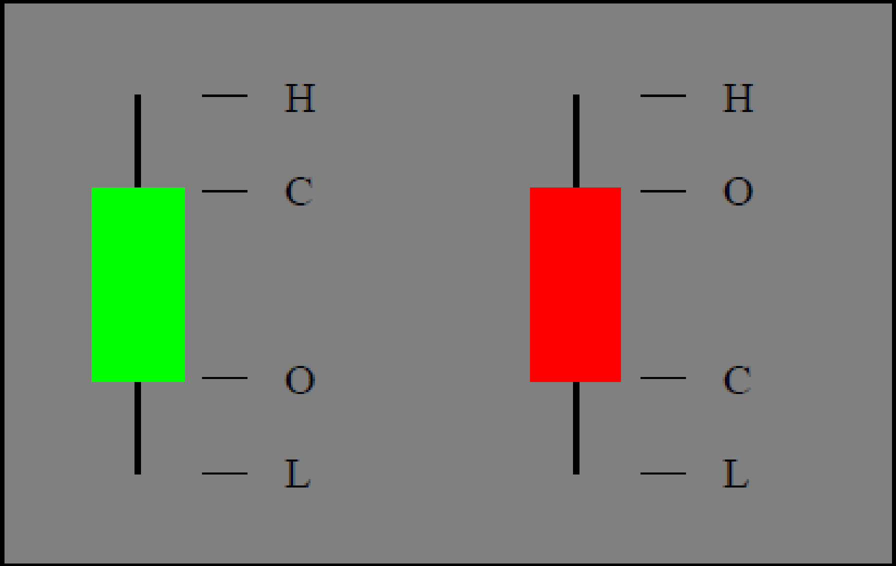
Financial time series

Application 5: Financial time series prediction

Candlestick representation

- H = high, C = close,
  O = open, L = low
- Body top (B_t)= max{C,O}
- Body bottom (B_b)= min{C,0}
- Body = C – O
- Upper shadow = H – B_t
- Lower shadow = B_b – L
- Can be defined for any duration.
- Here, each candlestick represents one trading day.



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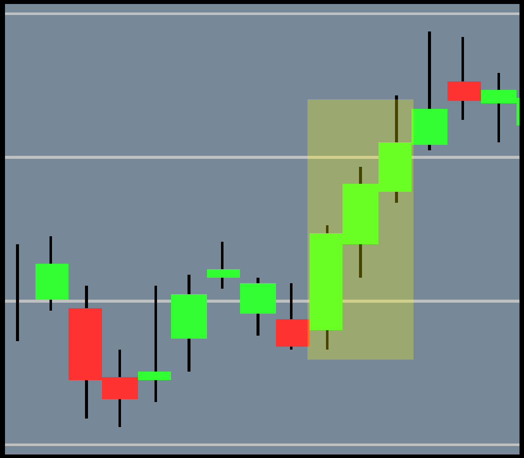
Candlestick patterns

- A (short) sequence (1 – 3, typically) of candlesticks fulfilling some condition.
- Examples:



"Three white soldiers"

"Shooting star"

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Candlestick patterns

- Should be approached with a healthy dose of scepticism!
- It is easy to show (empirically) that, by themselves and in isolation, candlestick patterns have ***zero predictive value***!
- On the other hand, there *are* successful traders who claim (at least) to make use of such patterns.
- However, no skilled trader would use *only* a single candlestick pattern as a basis for decision-making.

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Candlestick patterns

- A trader might use, say, a certain pattern occurring at the end of a downtrend:

- ...however, the traders' descriptions of such situations are often *too vague to be testable*!

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Method (brief summary)

- Define a generic (parameterized) candlestick pattern.
- Combine a sequence of such patterns to form a *pattern set.*
- Encode the patterns sets in chromosomes (strings of digits) The encoding scheme is described in detail in the paper.
- Apply *linear genetic programming* (non-length preserving) to evolve pattern sets, based on their performance over a large training data set.
- Check the results (one-day returns) over validation data, and then apply to previously unseen test data.

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Data

- Data set 1: 60 large-cap stocks from the US market
- Data set 2: 60 small-cap stocks from the US market
- Data collected from January 2005 to February 2016.
- 2789 trading days in total.
- Each data set divided into two disjoint sets, a training set (30 stocks) and a validation set (30 stocks).
- Thus, each data set contains 2789 x 30 = 83670 data points, from which 2788 x 30 = 83640 one-day returns can be computed.

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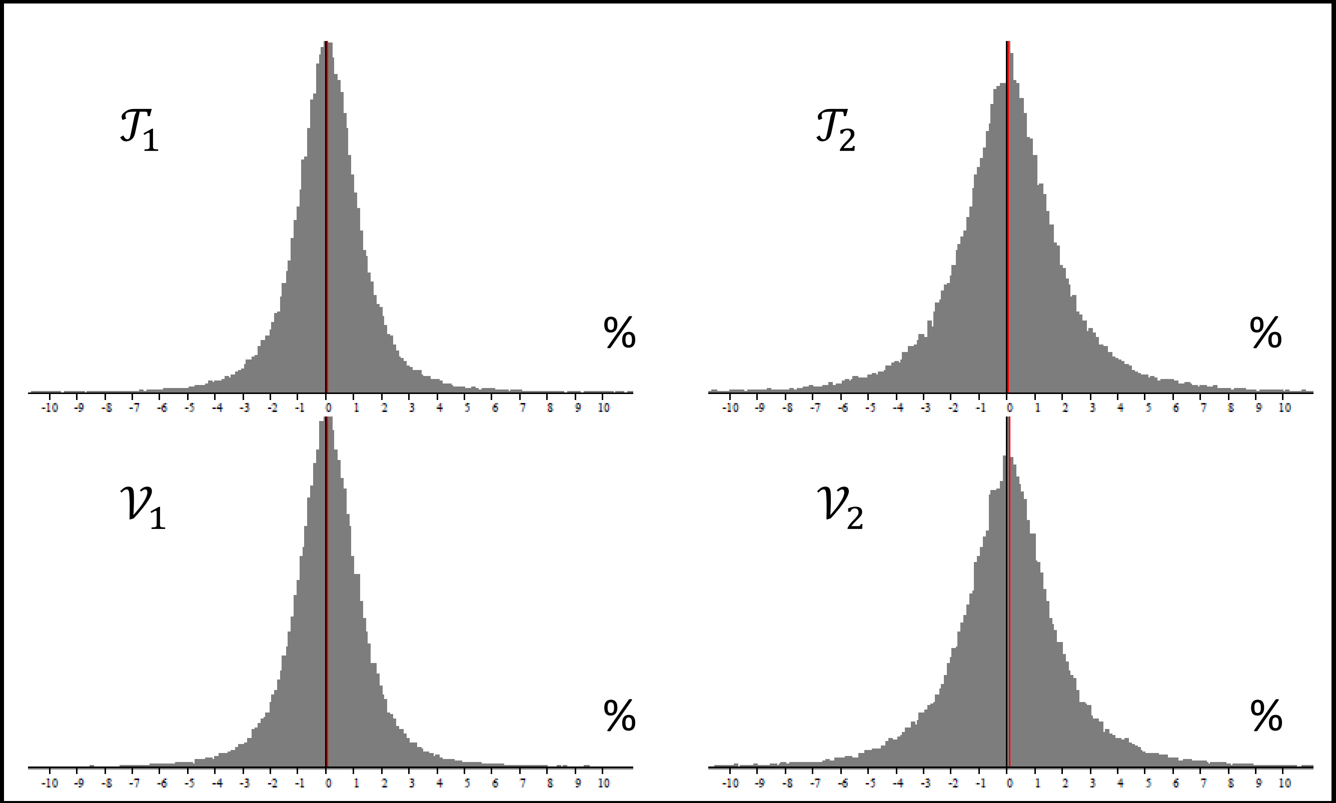
Data: One-day return distributions



- Average near 0 (0.05-0.07%)

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Evolutionary procedure

- Objective function: The Sharpe ratio (one-day returns)

$$S = \frac{\overline{r}_1 - \rho_1}{\sigma},$$

- Two running modes
  - *Tuning*: Start from a loosely defined pattern set (specified by the user) and tune it.
  - *Discovery*: Start from a population of completely random patterns sets.

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Results

- The best evolved pattern sets achieved highly significant positive results over the respective *validation* sets.

		Training			Validation				
ID	Tr., Val.	#	$\overline{r}_1$	S	#	$\overline{r}_1$	S	z	p
PS1	$\mathcal{T}_1, \mathcal{V}_1$	641	0.758	0.243	609	0.775	0.258	6.93	< 0.0001
PS2	$\mathcal{T}_1, \mathcal{V}_1$	518	0.626	0.201	456	0.518	0.174	3.74	< 0.0001
PS3	$\mathcal{T}_1, \mathcal{V}_1$	491	0.667	0.207	484	0.781	0.201	6.86	< 0.0001
PS4	$\mathcal{T}_2, \mathcal{V}_2$	657	1.038	0.187	599	1.147	0.184	4.21	< 0.0001
PS5	$\mathcal{T}_2, \mathcal{V}_2$	602	0.916	0.186	547	0.550	0.138	3.21	< 0.0007

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Number of trades.

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One-day return (%) over previously unseen data.
Note! - To be compared with market returns of around 0.05-0.07 (%).

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p-values for *rejection* of the null hypothesis, namely that the distribution of one-day returns (from the pattern set) is equal to the market return distribution.

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Results: Detailed example (PS1)

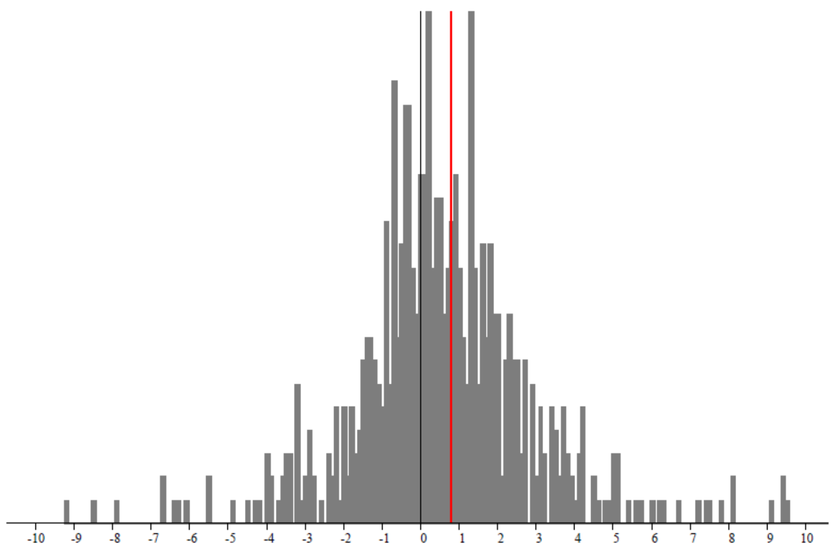

Fig. 5: The distribution of one-day returns obtained over the validation set for PS1. The average one-day return, indicated by a red vertical line, is equal to 0.775%. Note that the 11 most positive instances (returns above 10%) are not shown in the figure. The most negative instance (with a return of −9.16%) does appear in the figure, however.

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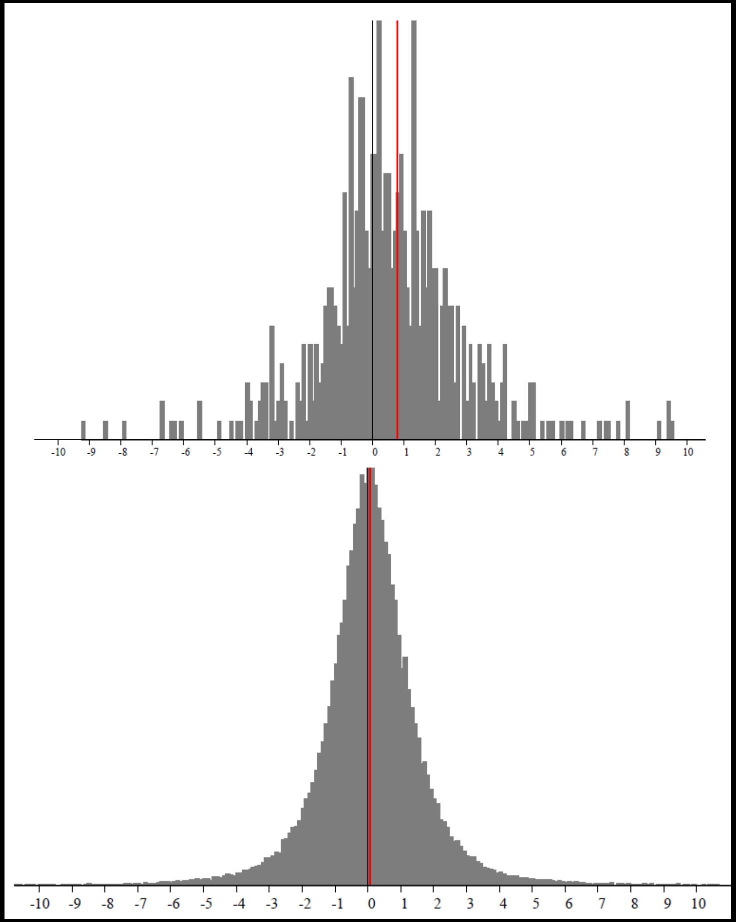
Results: Detailed example (PS1)

Pattern set 1 (PS1)

Market return (validation set)



Application 5: Financial time series prediction

Results: Additional validation

- The PS1 pattern set was then applied to the *other* validation set (V_2).
- Surprisingly, PS1 (trained over large-cap stocks) did very well also on V_2 (data from small-cap stocks):
  - Average one-day return 0.647%
  - p-value < 0.0001.

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Results: Forward tests

- Carried out *after* the paper was written: Based on data collected from mid-February 2016 until Aug. 2017, over a large set (200+) instruments (mid-cap and large-cap, US market, mostly different from the training set, but some overlap).
- Again applying the best pattern set (PS1):
  - One-day return of 0.250%,
  - 274 trades in total.
- Perhaps too few (so far) to draw firm conclusions, but the results are promising nonetheless.

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Results: What about trading?

- Of course, no trader would exit trades blindly after a single day.
- Instead, the pattern set would be used to provide *entry points* for trades.
- What if one does this, and then adds a very simple rule for exiting the trade (in this case, a rule with a single parameter, hard set before collecting data)?
- With this exit rule, the average trade duration has been around 6 days (trading since mid-February 2016)
- Result of a 1.5-year test:

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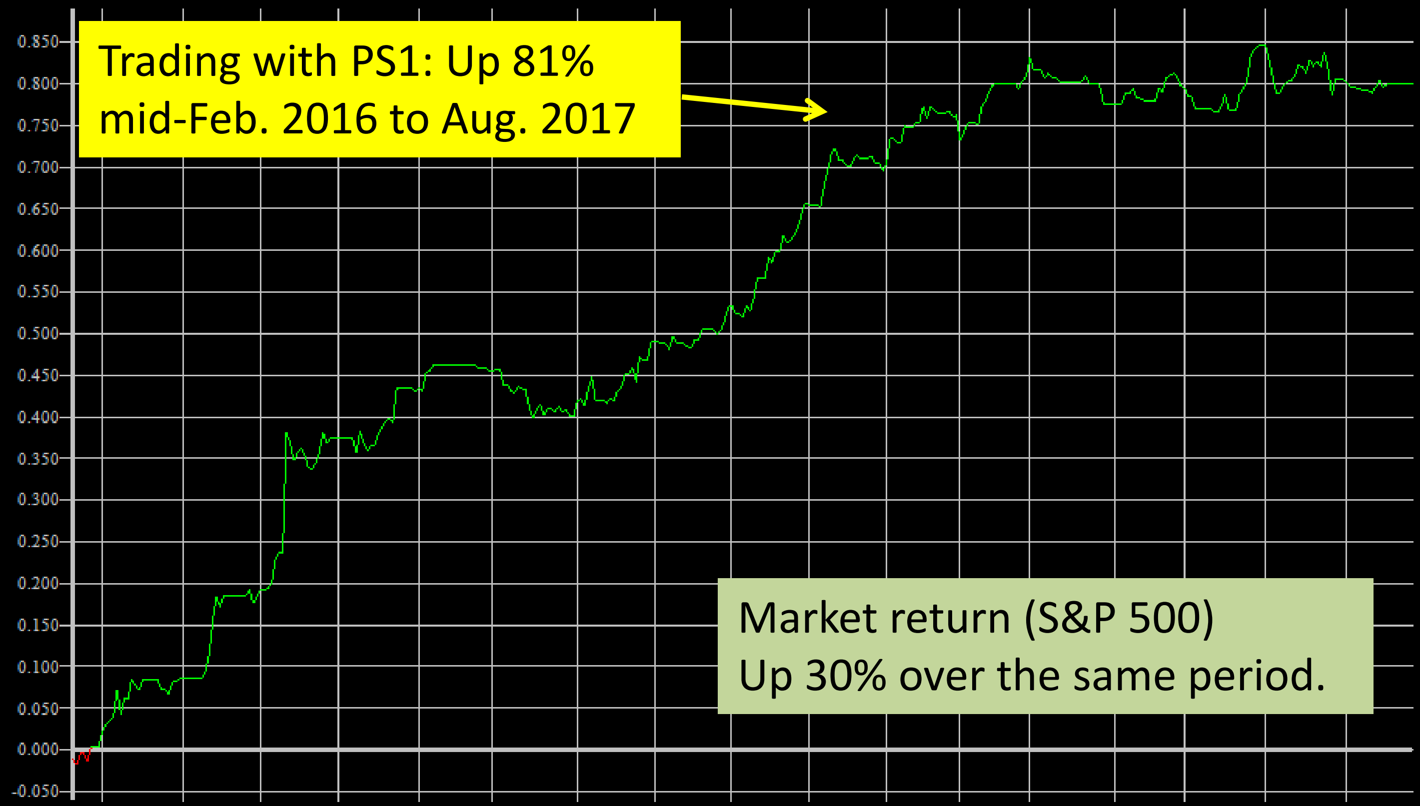
Results: What about trading?



Application 5: Financial time series prediction

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Today's learning goals

- After this lecture you should be able to
  - Describe orbit determination in astrophysics, using GAs.
  - Describe financial time series prediction, using EA (LGP)

Home problem 2

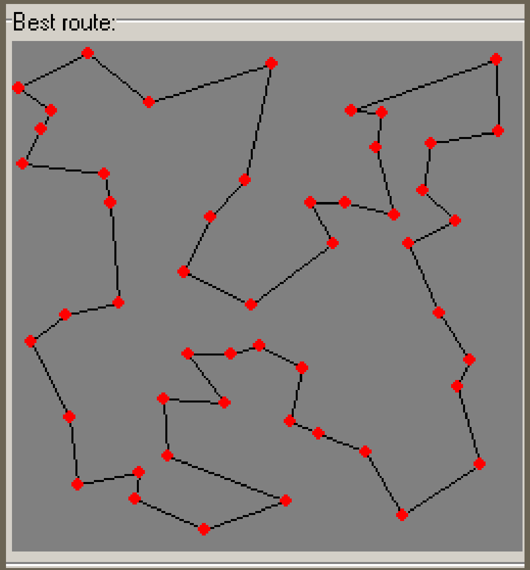
- Available on the web page. Strict deadline: 20201014
- You should start working on the problems as soon as possible – HP2 is more challenging than HP1.
- The theory for Problem 2.1c will be considered on Tuesday (Sept. 29th) and the theory for Problem 2.2 will be presented in the lecture on Friday (Oct. 2nd).
- In the mean time, you can start with 2.1a-b,d-e as well as 2.3 and 2.4 if you wish.

Home problem 2

- Read the instructions carefully before starting to solve the problems! Important to consider:
  - *What, exactly, is being asked in each (sub-)problem?*
  - *What information should be given in the report?*
  - *Which programs should be submitted?*

Problem 2.1

- Travelling salesman problem
- 50 cities
- Solve using (i) ACO and (ii) GA.
- Note: For ACO you <u>must</u> use the provided skeleton file AntSystem.m



Problem 2.2

- Particle swarm optimization (PSO): Make a contour plot to identify (all) the minima, and then find their exact locations using PSO, for the function:

$$f(x,y) = (x^2 + y - 11)^2 + (x + y^2 - 7)^2,$$

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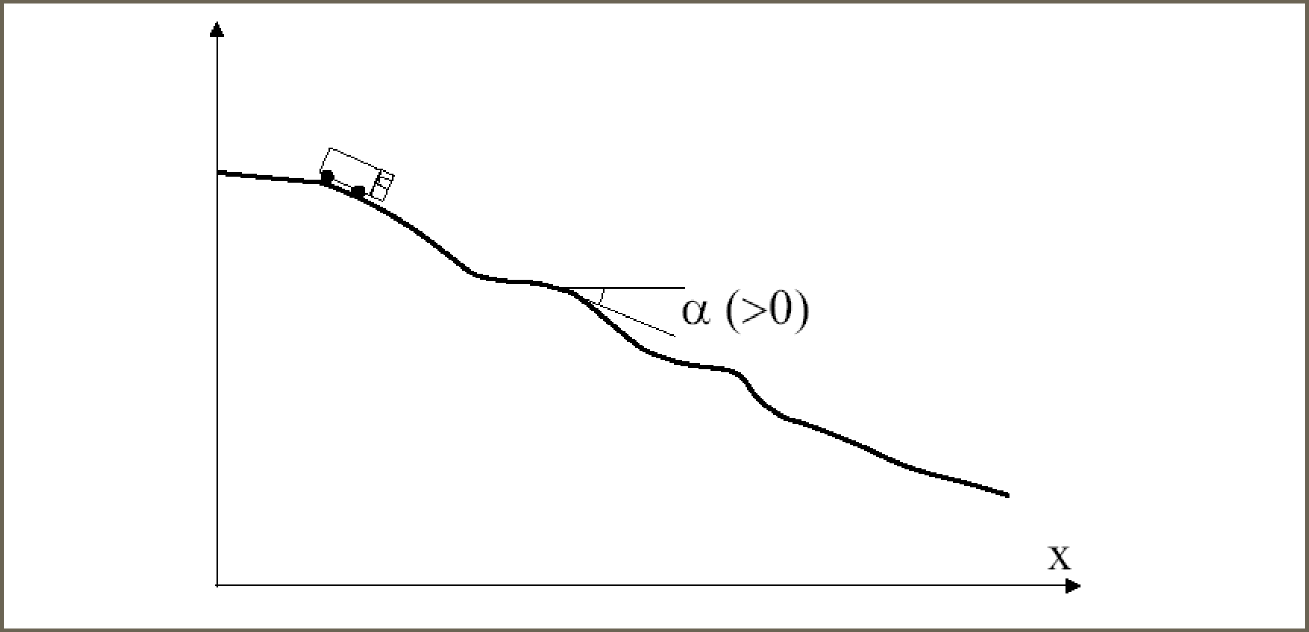
Problem 2.3

- Optimization of truck braking systems using a GA:



Problem 2.4

- (Noise-free) function fitting using LGP