

Uncertainty in the measurement of Time difference in Time of flight Method:

Since difference in time of flight of two particles of mass m_1 and m_2 is given by:

$$\Delta t = \frac{Lc}{2p^2} (m_1^2 - m_2^2) = \frac{Lc}{2p^2} K$$

$$\text{Where } K = (m_1^2 - m_2^2)$$

If

$$u \equiv u(x, y, z)$$

Then uncertainty in the measurement of u is given by formula:

$$\sigma_u^2 = \left(\frac{\partial u}{\partial x}\right)^2 \sigma_x^2 + \left(\frac{\partial u}{\partial y}\right)^2 \sigma_y^2 + \left(\frac{\partial u}{\partial z}\right)^2 \sigma_z^2$$

$$\sigma_{\Delta t}^2 = \left(\frac{\partial \Delta t}{\partial L}\right)^2 \sigma_L^2 + \left(\frac{\partial \Delta t}{\partial p}\right)^2 \sigma_p^2 \quad (1)$$

$$\frac{\partial \Delta t}{\partial L} = \frac{cK}{2p^2} \quad \text{and} \quad \frac{\partial \Delta t}{\partial p} = \frac{-cLK}{p^3}$$

$$\sigma_{\Delta t}^2 = \frac{c^2 K^2}{4p^4} \sigma_L^2 + \frac{c^2 L^2 K^2}{p^6} \sigma_p^2$$

$$\frac{\sigma_{\Delta t}^2}{\Delta t^2} = \frac{c^2 K^2 4p^4}{4p^4 L^2 c^2 K^2} \sigma_L^2 + \frac{c^2 L^2 K^2 4p^4}{p^6 L^2 c^2 K^2} \sigma_p^2$$

$$\frac{\sigma_{\Delta t}^2}{\Delta t^2} = \frac{\sigma_L^2}{L^2} + \frac{4\sigma_p^2}{p^2}$$

$$\text{For } \frac{\sigma_L}{L} \ll \frac{\sigma_p}{p}$$

$$\frac{\sigma_{\Delta t}}{\Delta t} = \frac{2\sigma_p}{p} \Rightarrow \sigma_{\Delta t} = \frac{2\sigma_p}{p} * \Delta t$$

Minimum Time difference due to error in momentum measurement will be:

$$\Delta t' = \Delta t - \sigma_{\Delta t}$$