

Assignment 7 : Manual Calculations

Batch Gradient Descent

Step 1 : Read Dataset (x, y) , epochs = 2,

$$m=1, c=1, \eta=0.1, n_s=2$$

x	y
0.2	3.4
0.4	3.8

Step 2 : iter = 1

$$\text{Step 3 : } E = \frac{1}{2n_s} \sum (y_i - mx_i - c)^2$$

$$\frac{\partial E}{\partial m} = \frac{1}{n_s} \left[\sum_{i=1}^{n_s} (y_i - mx_i - c)(x_i) \right]$$

$$= \frac{1}{2} [(3.4 - 1)(0.2) + 1)(0.2) + (3.8 - 1)(0.4) + 1)(0.4)]$$

$$= \frac{1}{2} [(4.2)(0.2) + (4.4)(0.4)] = \frac{1}{2} [0.84 + 1.76] \\ = -1.3$$

$$\frac{\partial E}{\partial c} = \frac{1}{n_s} \left[\sum_{i=1}^{n_s} (y_i - mx_i - c) \right] = \frac{1}{2} [4.2 + 4.4] = \frac{1}{2} [8.6] \\ = -4.3$$

$$\text{Step 4 : } \Delta m = -\eta \frac{\partial E}{\partial m} = -(0.1) \times (-1.3) = 0.13$$

$$\Delta c = -\eta \frac{\partial E}{\partial c} = -(0.1) \times (-4.3) = 0.43$$

$$\text{Step 5 : } m = 1 + 0.13 = 1.13$$

$$c = -1 + 0.43 = -0.57$$

$$\text{Step 6 : } \text{iter} = \text{iter} + 1 = 1 + 1 = 2$$

Step 7 : If 2 > 2 \Rightarrow false

Go to Step 3

$$\text{Step 3: } \frac{\partial \epsilon}{\partial m} = -\frac{1}{n_s} \left[\sum_{i=1}^{n_s} (y_i - mx_i - c)x_i \right]$$

$$= -\frac{1}{2} [(3.4 - (1.13 \times 0.2) + 0.57) \times 0.2 + (3.8 - (1.13 \times 0.4) + 0.57) \times 0.4]$$

$$= -\frac{1}{2} [3.2994 + 1.5672] = -2.4333$$

$$\frac{\partial \epsilon}{\partial c} = -\frac{1}{2} [3.744 + 3.918] = -3.831$$

$$\text{Step 4: } \Delta m = -\eta \frac{\partial \epsilon}{\partial m} = -(0.1) \times (-2.4333) = 0.24333$$

$$\Delta c = -\eta \frac{\partial \epsilon}{\partial c} = -(0.1) \times (-3.831) = 0.3831$$

$$\text{Step 5: } m = m + \Delta m = 1.13 + 0.24333 = 1.37333$$

$$c = c + \Delta c = -0.57 + 0.3831 = -0.1869$$

$$\text{Step 6: } \text{iter} = \text{iter} + 1 = 2 + 1 = 3$$

$$\text{Step 7: } \text{if } \text{iter} > \text{epoch} \Rightarrow 3 > 2 \Rightarrow \text{go to step 8.}$$

$$\text{Step 8: } \text{print}(m, c) = (1.37333, -0.1869)$$

$$\text{Step 9: } \text{mse of data}$$

$$\begin{aligned} \text{mse} &= \frac{[3.4 - (1.37333 \times 0.2) + 0.1869]^2 + [3.8 - (1.373 \times 0.4) + 0.1869]^2}{2} \\ &= \frac{[10.97089] + [1.81687]}{2} \\ &= 11.39388 \end{aligned}$$