

Assignment 9: Momentum Gradient Descent

Manual Calculation

Step 1: Read $[x, y], m = -1, \eta = 0.1, \gamma = 0.9, \text{epochs} = 2$

$$v_m = 0, v_c = 0$$

Step 2: iter = 1

Step 3: sample = 1

$$\text{Step 4: } \mathcal{E} = \frac{1}{2}(y_i - mx_i - c)^2$$

$$\frac{\partial \mathcal{E}}{\partial m} = -(3.4 - (-1)(0.2))(0.2) = -(4.2)(0.2) = -0.84$$

$$\frac{\partial \mathcal{E}}{\partial c} = -(4.2) = -4.2$$

$$\text{Step 5: } v_m = \gamma v_m + \eta \frac{\partial \mathcal{E}}{\partial m} = (0.9)(0) - (0.1)(-0.84) = 0.084$$

$$v_c = (0.9)(0) - (0.1)(-4.2) = 0.42$$

$$\text{Step 6: } m = -1 + 0.084 = -0.916 / c = -1 + 0.42 = -0.58$$

Step 7: sample = 1 + 1 = 2

Step 8: if sample > n: 2 > 2 \Rightarrow false

go to step 4

$$\begin{aligned} \text{Step 9: } \frac{\partial \mathcal{E}}{\partial m} &= -(3.8 - (-0.916 \times 0.4) + 0.58) \times 0.4 \\ &= -(3.9464) \times 0.4 = -1.57856 \end{aligned}$$

$$\text{Step 10: } v_m = (0.9)(0.084) - (0.1)(-1.57856) = 0.082256$$

$$v_c = (0.9)(0.42) - (0.1)(-3.9464) = 0.77264$$

$$\text{Step 11: } m = -0.916 + 0.082256 = -0.833744$$

$$c = -0.58 + 0.77264 = 0.19264$$

Step 12: Sample = 2+1 = 3

Step 13: If sample > nS = 3 > 2 = true

go to step 14

Step 14: iter = 1+1 = 2

Step 15: If iter > epoch = 1 > 2 = false

go to step 3

Step 3: Sample = 1

Step 4: $E = \frac{1}{2} (y - mx - c)^2$

$$\frac{\partial E}{\partial m} = -(3.4 - (1.6625 \times 0.2) - 0.19264) \times 0.2$$

$$= -(2.97411) \times 0.2 = -0.59482$$

$$\frac{\partial E}{\partial c} = -2.9741$$

$$\text{Step 5: } \Delta m = (0.9) \times (0.08225) - (0.1) \times (-0.59482) = 0.133507$$

$$\Delta c = (0.9) \times (0.772641) - (0.1) \times (-2.97411) = 0.992787$$

$$\text{Step 6: } m = 1.6625 + 0.133507 = 1.799757$$

$$c = 0.19264 + 0.992787 = 1.185427$$

Step 7: Sample = 1+1 = 2

Step 8: If sample > nS = 2 > 2 = false

go to step 4

$$\text{Step 4: } \frac{\partial E}{\partial m} = -(3.8 - (1.799757) \times (0.4) - 1.185427) \times 0.4$$

$$= 0.83786$$

$$\frac{\partial E}{\partial c} = -2.09467$$

$$\text{Step 5: } V_m = (0.9)(0.133507) - (0.1)(0.83786) = 0.20394$$

$$V_c = (0.9)(0.992787) - (0.1)(-2.09467) = 1.10297$$

$$\text{Step 6:- } m = 1.299754 + 0.20394 = 1.503697$$

$$c = 1.10297 + 1.185427 = 2.288397$$

$$\text{Step 7:- } \text{iter} = \text{iter} + 1 = 3$$

$$\text{Step 8:- } \text{if } \text{iter} > \text{epochs} = 3 \Rightarrow \text{false}$$

go to step 9

$$\text{Step 9:- } \text{print}(m, c)$$

$$1.503697, 2.288397$$

$$\text{Step 10:- } \text{mse} = \frac{(2.891364) + (2.889875)}{2}$$

$$= \frac{5.781239}{2}$$

$$\text{mse} = 2.890619$$