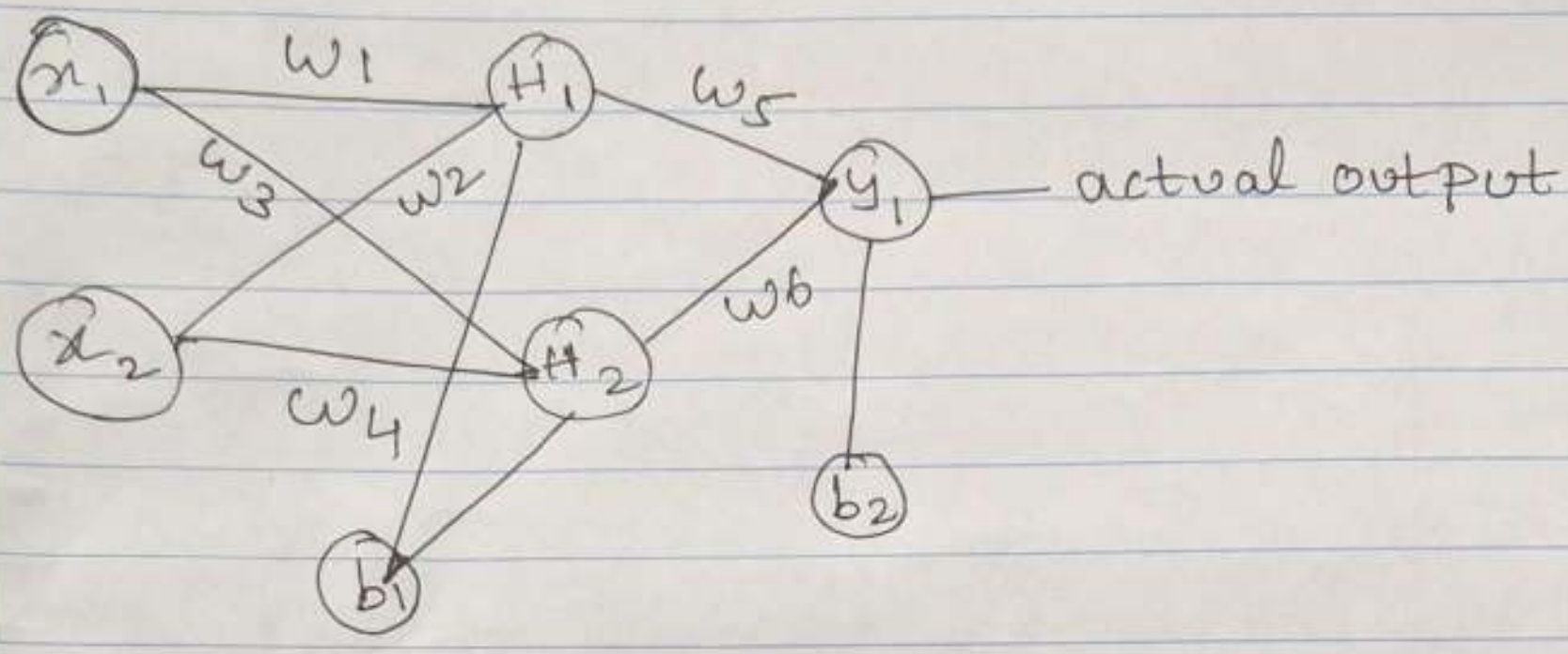

x_1, x_2 → Input Layer
H_1, H_2 → Hidden Layer
y_1 → output Layer
b_1, b_2 → bias

Considering activation function as Sigmoid

$$\text{Sigmoid} = \frac{1}{1+e^{-x}}$$

Considering input values and bias's as,

$x_1 = 0.25$ $b_1 = 0.45$
$x_2 = 0.75$ $b_2 = 0.15$

Initial Weights of the network,

$w_1 = 0.10$ $w_4 = 0.55$
$w_2 = 0.30$ $w_5 = 0.65$
$w_3 = 0.40$ $w_6 = 0.85$

Target value as,

$T = 0.03$

So, first we will do Forward step:

For $H_1 = x_1 * w_1 + x_2 * w_2 + b_1$

$= 0.25 * 0.10 + 0.75 * 0.30 + 0.45$

$= 0.7$

output of $H_1 = \frac{1}{1+e^{-H_1}}$

$= \frac{1}{1+e^{-0.7}}$

$= 0.6681879$

For $H_2 = x_1 * w_3 + x_2 * w_4 + b_1$

$= 0.25 * 0.40 + 0.75 * 0.55 + 0.45$

$= 0.9625$

output of $H_2 = \frac{1}{1+e^{-0.9625}}$

$= 0.72362$

For $y_1 =$ output $H_1 * w_5 +$ output $H_2 * w_6 + b_2$

$= 0.66817 * 0.65 + 0.72362 * 0.85 + 0.15$

$= 0.4343 + 0.6151 + 0.15$

$$= 1.1994$$

$$\text{output of } y_1 = \frac{1}{1+e^{-y_1}}$$

$$= \frac{1}{1+e^{-1.1994}}$$

$$= 0.76842$$

Since here, y_1 value is not matching with the target value, we will now calculate the total Error.

Calculating Total Error,

$$E_{Total} = \Sigma \frac{1}{2}(Target - Output)^2$$

$$= \frac{1}{2}(0.03 - 0.76842)^2$$

$$= 0.27263$$

As we got the total Error, now we will back propagate this Error to update the weights ($w_1, w_2, w_3, w_4, w_5, w_6$).

So, We will now do Backward step:

Calculating Error at w_5

$$\text{Error at } w_5 = \frac{\partial E_{Total}}{\partial w_5}$$

$$\frac{\partial E_{Total}}{\partial w_5} = \frac{\partial E_{Total}}{\partial out_{y_1}} * \frac{\partial out_{y_1}}{\partial y_1} * \frac{\partial y_1}{\partial w_5}$$

$$E_{Total} = \frac{1}{2}(T_1 - out_{y_1})^2$$

$$= \frac{1}{2}(T_1^2 - 2 \times T_1 \times out_{y_1} + out_{y_1})^2$$

$$\frac{\partial E_{Total}}{\partial out_{y_1}} = -T_1 + out_{y_1}$$

$$= -0.03 + 0.76842$$

$$= 0.73842$$

$$out_{y_1} = \frac{1}{1+e^{-y_1}} \Rightarrow e^{-y_1} = \frac{1}{out_{y_1}} - 1$$

$$\frac{\partial out_{y_1}}{\partial y_1} = \left(\frac{1}{1+e^{-y_1}}\right)^2 \times e^{-y_1}$$

$$= (out\ y_1)^2 \times \left(\frac{1}{out_{y_1}} - 1\right)$$

$$= out_{y_1}(1 - out_{y_1})$$

$$= 0.76842(1 - 0.76842)$$

$$= 0.17795$$

$$y_1 = out_{H_1} * w_5 + out_{H_2} * w_6 + b_2$$

$$\frac{\partial y_1}{\partial w_5} = out_{H_1} * 1 + 0 + 0$$

$$= out_{H1}$$

$$= 0.6682$$

$$\text{so, } \frac{\partial E_{Total}}{\partial w_5} = \frac{\partial E_{Total}}{\partial out_{y_1}} * \frac{\partial out_{y_1}}{\partial y_1} * \frac{\partial y_1}{\partial w_5}$$

$$= 0.73842 * 0.17795 * 0.6682$$

$$= 0.0878$$

Assuming Learning rate $\eta = 0.5$

$$\text{Updated } w_5 = w_5 - \eta * \frac{\partial E_{Total}}{\partial w_5}$$

$$= 0.65 - 0.5 * 0.0878$$

$$= 0.6061$$

Error at $w_6 = \frac{\partial E_{Total}}{\partial w_6}$

$$\frac{\partial E_{Total}}{\partial w_6} = \frac{\partial E_{Total}}{\partial out_{y_1}} * \frac{\partial out_{y_1}}{\partial y_1} * \frac{\partial y_1}{\partial w_6}$$

$$y_1 = outH_1 * w_5 + outH_2 * w_6 + b_2$$

$$\frac{\partial y_1}{\partial w_6} = 0 + outH_2 + 0$$
$$= 0.72362$$

So, $\frac{\partial E_{Total}}{\partial w_6} = 0.73842 * 0.17795 * 0.72362$
$= 0.0951$

updated $w_6 = w_6 - \eta * \frac{\partial E_{Total}}{\partial w_6}$

$= 0.85 - 0.5 * 0.72362$
$= 0.85 - 0.5 * 0.0951$
$= 0.80246$

Now, we will update w_1, w_2, w_3 & w_4

Error at w_1

$$\frac{\partial E_{Total}}{\partial w_1} = \frac{\partial E_{Total}}{\partial outH_1} * \frac{\partial outH_1}{\partial H_1} * \frac{\partial H_1}{\partial w_1}$$

$$\frac{\partial E_{Total}}{\partial out_{H_1}} = \frac{\partial E_{Total}}{\partial y_1} * \frac{\partial y_1}{\partial out_{H_1}}$$

$$\frac{\partial E_{Total}}{\partial y_1} = \frac{\partial E_{Total}}{\partial out_{y_1}} * \frac{\partial out_{y_1}}{\partial y_1}$$

$$= 0.73842 * 0.17795$$

$$= 0.1314$$

$$y_1 = out_{H_1} * w_5 + out_{H_2} * w_6 + b_2$$

$$\frac{\partial y_1}{\partial out_{H_2}} = w_5 + 0 + 0$$

$$= 0.65$$

$$\frac{\partial E_{Total}}{\partial out_{H_1}} = 0.1314 * 0.65$$

$$= 0.0854$$

$$out_{H_1} = \frac{1}{1 + e^{-H_1}}$$

$$\frac{\partial out_{H_1}}{\partial H_1} = out_{H_1}(1 - out_{H_1})$$

$$= 0.66818(1 - 0.66818)$$

$$= 0.2217$$

$$H_1 = w_1 x_1 + w_2 x_2 + b_1$$

$$\frac{\partial H_1}{\partial w_1} = x_1 + 0 + 0$$
$$= 0.25$$

so, $\frac{\partial E_{Total}}{\partial w_1} = 0.0854 * 0.2217 * 0.25$

$$= 0.00473$$

updated $w_1 = w_1 - \eta \frac{\partial E_{Total}}{\partial w_1}$

$$= 0.10 - 0.05 * 0.00473$$
$$= 0.0976$$

Error at w_2

$$\frac{\partial E_{Total}}{\partial w_2} = \frac{\partial E_{Total}}{\partial out_{H1}} * \frac{\partial out_{H1}}{\partial H_1} * \frac{\partial H_1}{\partial w_2}$$

$$H_1 = w_1 x_1 + w_2 x_2 + b_1$$

$$\frac{\partial H_1}{\partial w_1} = 0 + x_2 + 0$$
$$= 0.75$$

so, $\frac{\partial E_{Total}}{\partial w_1} = 0.0854 * 0.2217 * 0.75$

$$= 0.0142$$

$$\text{updated } w_2 = w_2 - \eta * \frac{\partial E_{Total}}{\partial w_2}$$

$$= 0.30 - 0.5 * 0.0142$$

$$= 0.2929$$

Error at w_3

$$\frac{\partial E_{Total}}{\partial w_3} = \frac{\partial E_{Total}}{\partial out_{H_2}} * \frac{\partial out_{H_2}}{\partial H_2} * \frac{\partial H_2}{\partial w_3}$$

$$\frac{\partial E_{Total}}{\partial out_{H_2}} = \frac{\partial E_{Total}}{\partial y_1} * \frac{dy_1}{\partial out_{H_2}}$$

$$y_1 = out_{H_1} * w_5 + out_{H_2} * w_6 + b_2$$

$$\frac{\partial y_1}{\partial out_{H_2}} = 0 + w_6 + 0$$

$$= 0.85$$

$$\frac{\partial E_{Total}}{\partial out_{H_2}} = 0.1314 * 0.85$$

$$= 0.11169$$

$$out_{H_2} = \frac{1}{1 + e^{-H_2}}$$

$$\frac{\partial out_{H_2}}{\partial H_2} = out_{H_2}(1 - out_{H_2})$$

$$= 0.72362\,(1 - 0.72362)$$

$$= 0.199994$$

$$H_2 = x_1 * w_3 + x_2 * w_4 + b_1$$

$$\frac{\partial H_2}{\partial w_3} = x_1 + 0 + 0$$
$$= 0.25$$

$$\text{So, } \frac{\partial E_{Total}}{\partial w_3} = 0.11169 * 0.19999 * 0.25$$
$$= 0.00558$$

$$\text{updated } w_3 = w_3 - \eta * \frac{\partial E_{Total}}{\partial w_3}$$
$$= 0.40 - 0.5 * 0.00558$$
$$= 0.3972$$

Error at w_4

$$\frac{\partial E_{Total}}{\partial w_4} = \frac{\partial E_{Total}}{\partial out_{H2}} * \frac{\partial out_{H2}}{\partial H_2} * \frac{\partial H_2}{\partial w_4}$$

$$H_2 = x_1 * w_3 + x_2 * w_4 + b_1$$

$$\frac{\partial H_2}{\partial w_4} = 0 + x_2 + 0$$
$$= 0.75$$

$$\text{So, } \frac{\partial E_{Total}}{\partial w_4} = 0.11169 * 0.19999 * 0.75$$
$$= 0.01675$$

$$\text{updated } w_4 = w_4 - \eta * \frac{\partial E_{total}}{\partial w_4}$$

$$= 0.55 - 0.5 * 0.01675$$

$$= 0.5416$$

So, all the new updated weights are:

$w_1 = 0.0976$

$w_2 = 0.2929$

$w_3 = 0.3972$

$w_4 = 0.5416$

$w_5 = 0.6061$

$w_6 = 0.8025$