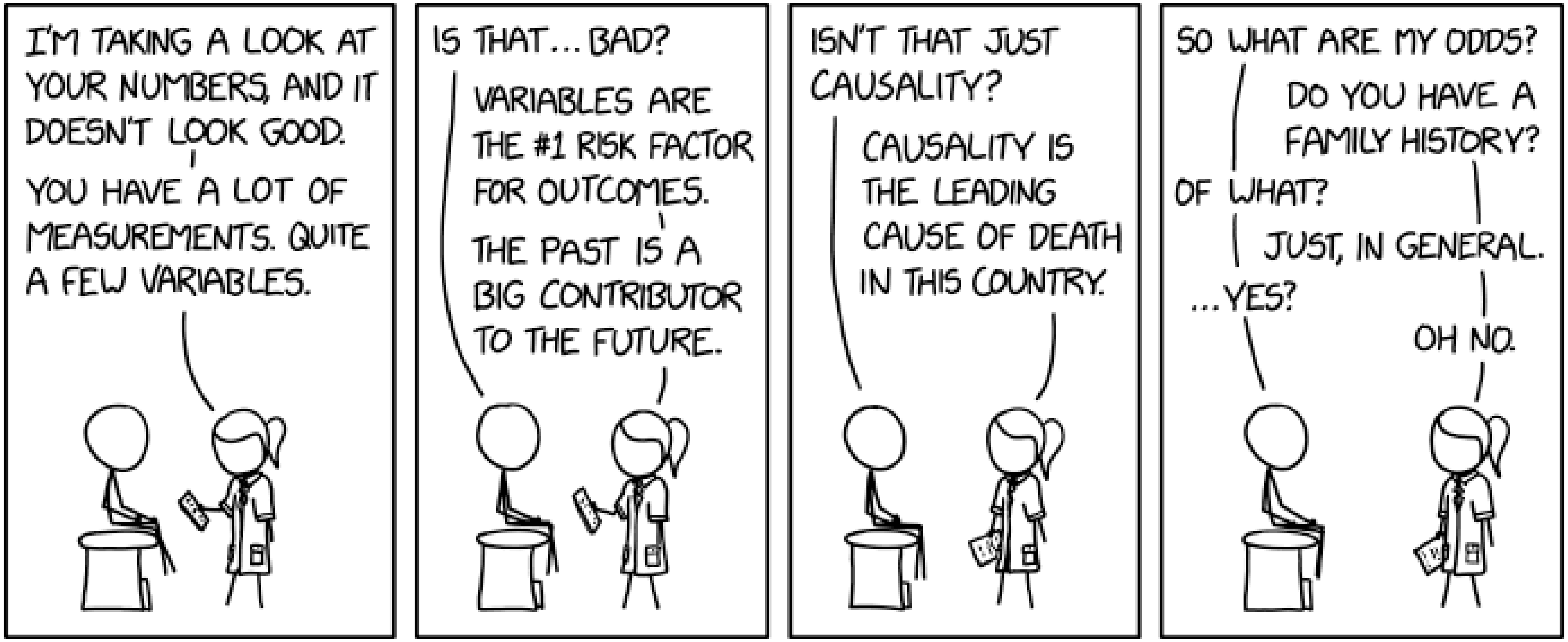

Source: xkcd.com/2620

INTRODUCTION TO FORECASTING & TIME SERIES STRUCTURE

Dr. Susan Simmons

Institute for Advanced Analytics

TIME SERIES DATA

Time Series Data

- A time series is an ordered sequence of observations.
  - Ordering is typically through ***equally spaced*** time intervals.
  - Possibly through space as well.
- Used in a variety of fields:
  - Agriculture: Crop Production
  - Economics: Stock Prices
  - Engineering: Electric Signals
  - Meteorology: Wind Speeds
  - Social Sciences: Crime Rates

Time Series Data

- We will begin our time series discussions with univariate time series (only one time series…one variable, we will call it Y).
- Multivariate time series will be in Fall 2.

Time Series Data

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Date	Y
January 2000	23
February 2000	18
March 2000	20
April 2000	25
May 2000	21

Time Series Data

- We will begin our time series discussions with univariate time series (only one time series…one variable, we will call it Y).
- Multivariate time series will be in Fall 2.

Date	Y
January 2000	23
February 2000	18
March 2000	20
April 2000	25
May 2000	21

Y_1

Time Series Data

- We will begin our time series discussions with univariate time series (only one time series…one variable, we will call it Y).
- Multivariate time series will be in Fall 2.

Date	Y
January 2000	23
February 2000	18
March 2000	20
April 2000	25
May 2000	21

Y_2

Time Series Data

- We will begin our time series discussions with univariate time series (only one time series…one variable, we will call it Y).
- Multivariate time series will be in Fall 2.

Date	Y
January 2000	23
February 2000	18
March 2000	20
April 2000	25
May 2000	21

Y_3

Time Series Data

- We will begin our time series discussions with univariate time series (only one time series…one variable, we will call it Y).
- Multivariate time series will be in Fall 2.

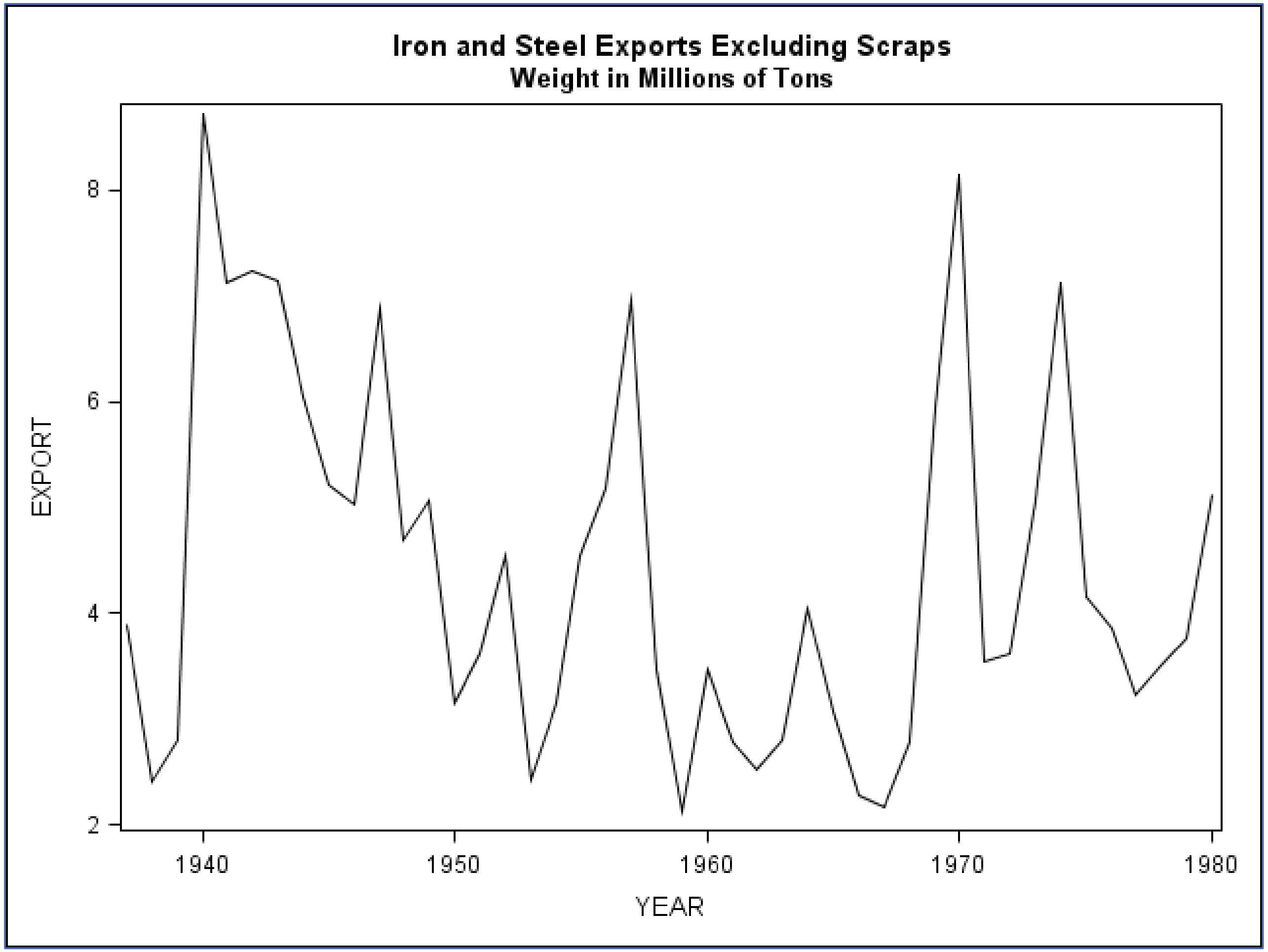
Date	Y
January 2000	23
February 2000	18
March 2000	20
April 2000	25
May 2000	21

Y_3

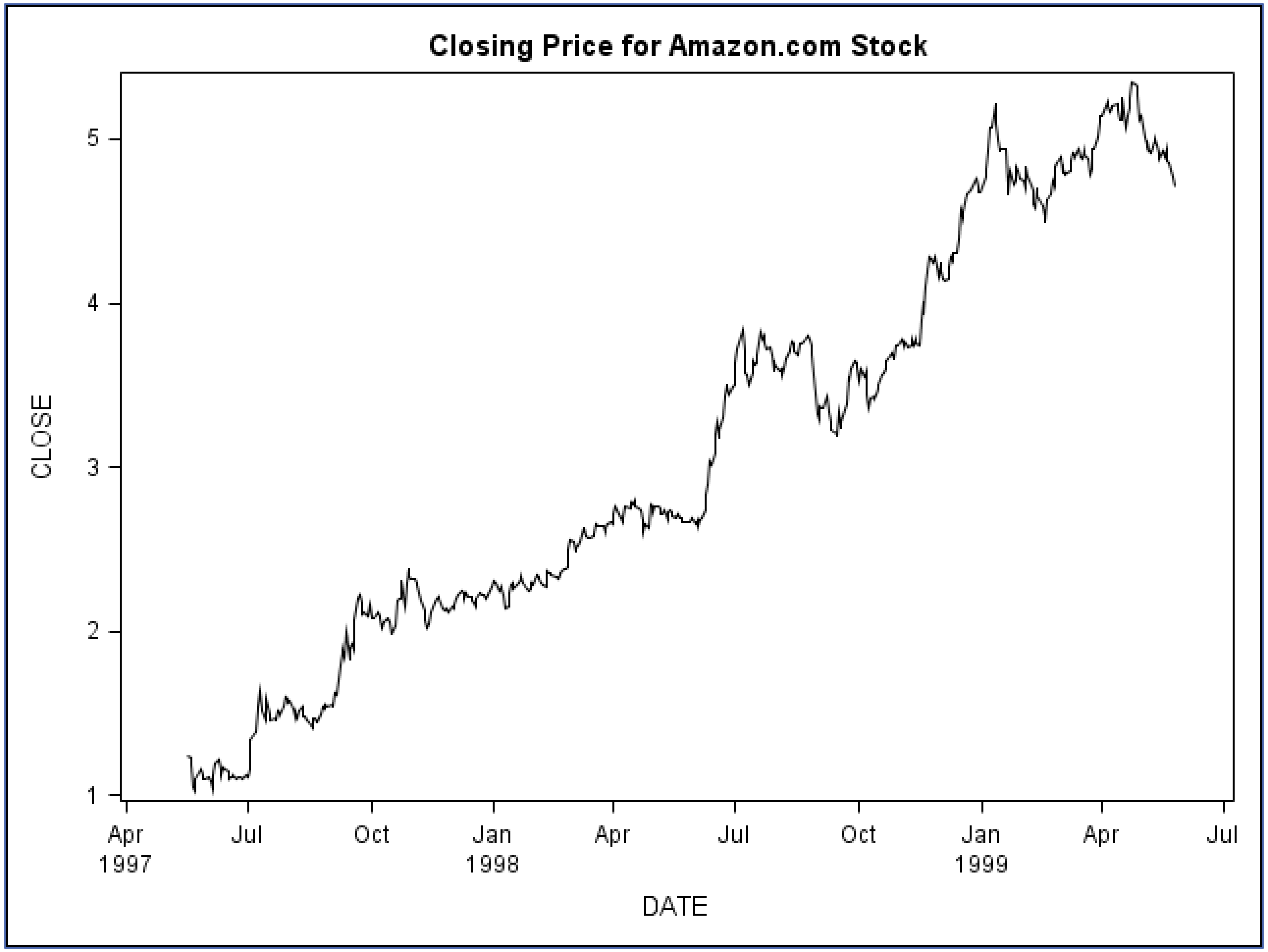
Y_t

CAREFUL: Since we are assuming equally spaced, you will need to take care of missing values !!

Example 1: Iron and Steel Exports

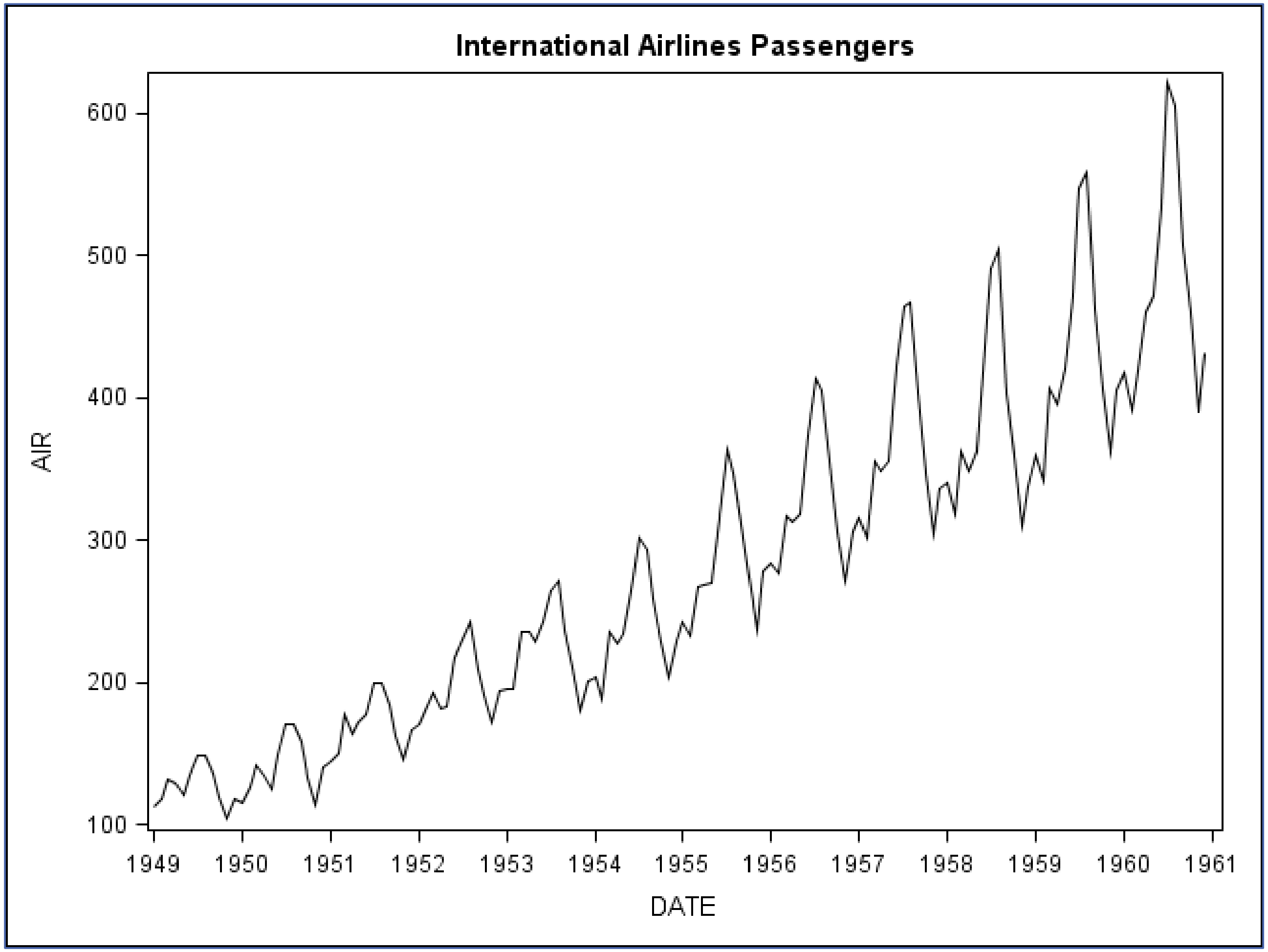


Example 2: Amazon.com Stock

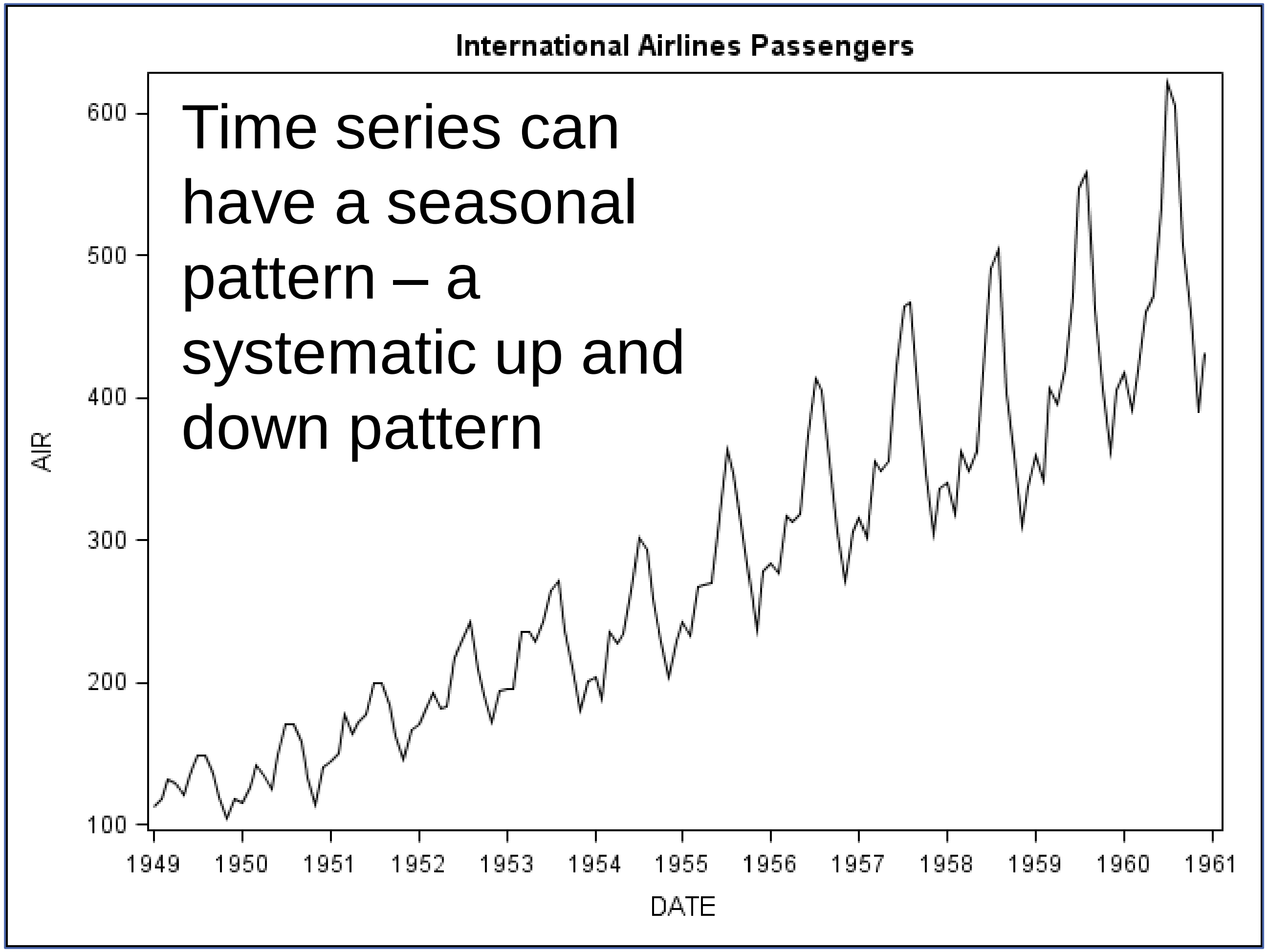


Example 2: Amazon.com Stock

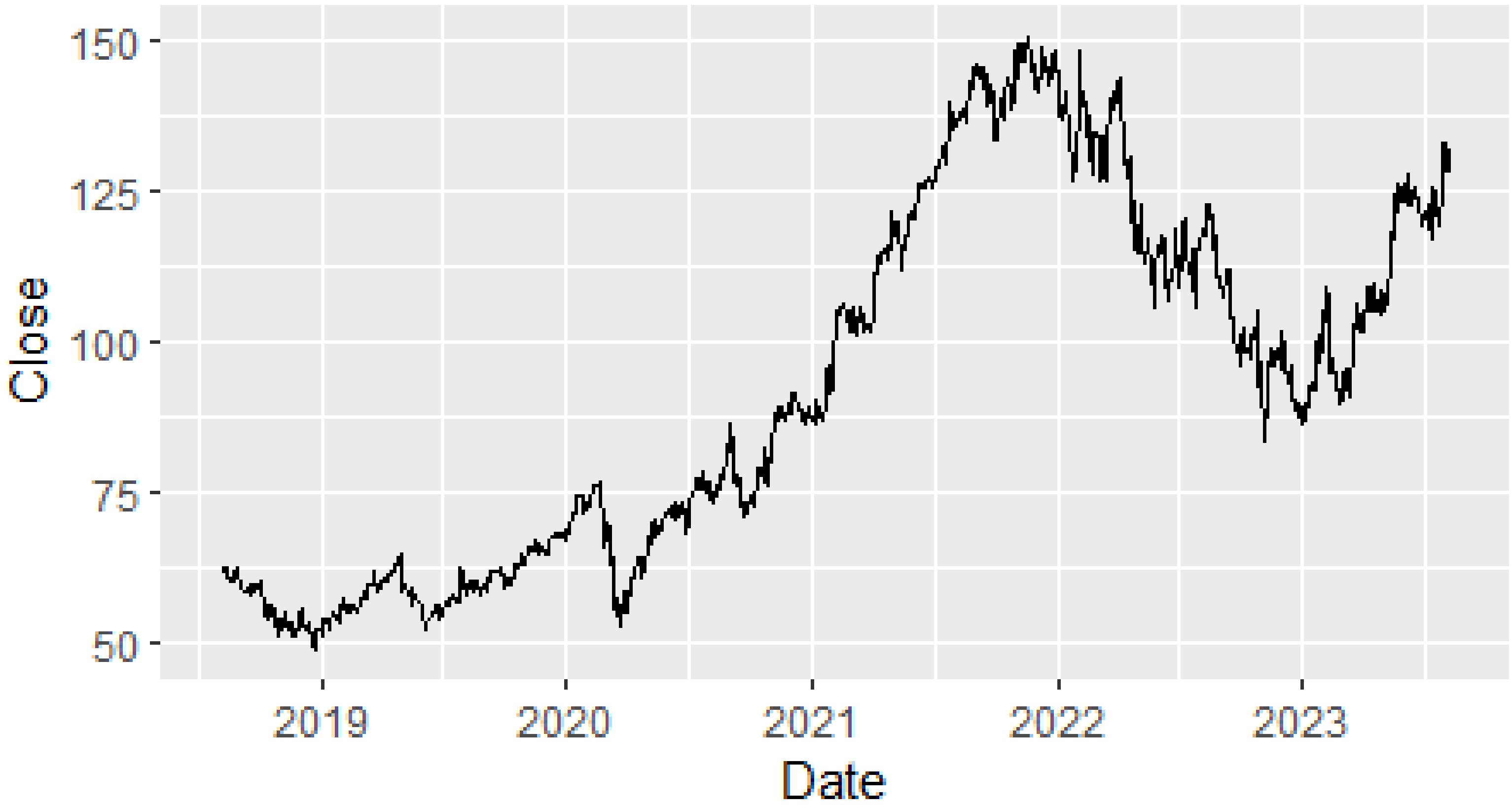
Example 3: Airlines Passengers



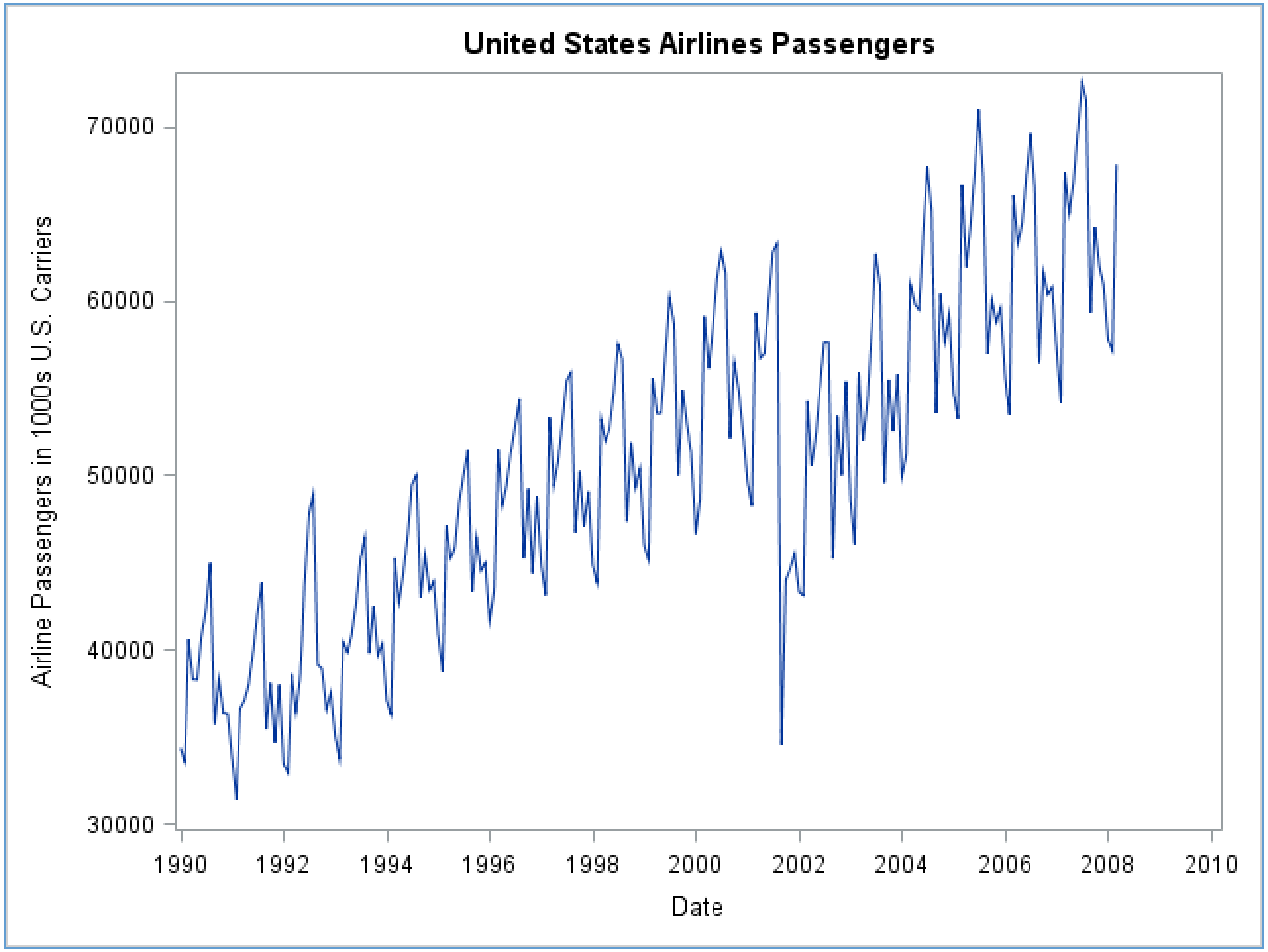
Example 3: Airlines Passengers



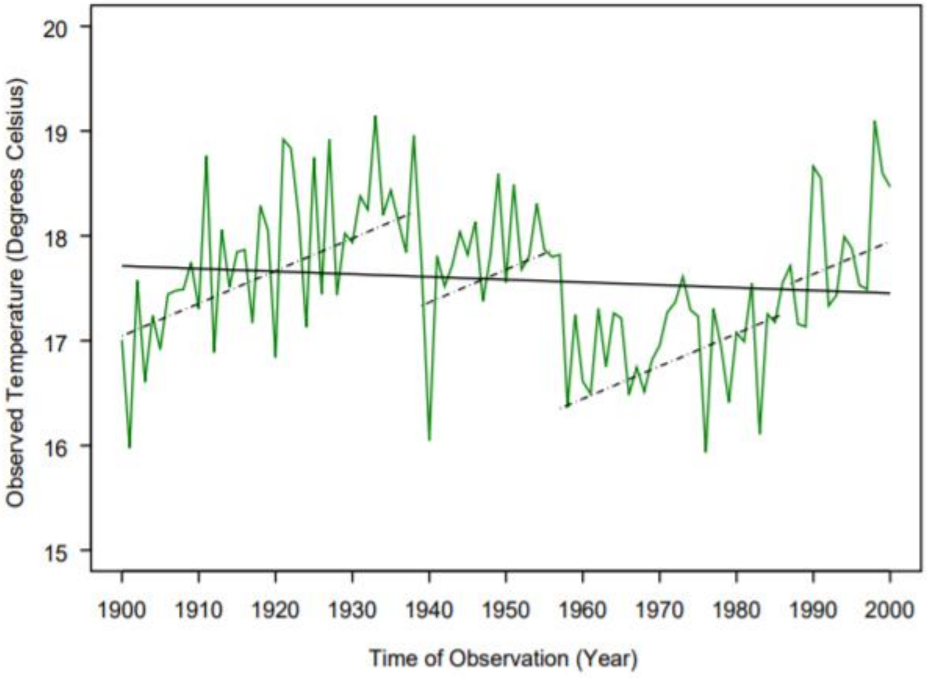
Google stock from 2018-2023



Example 5: Airline Passengers Again



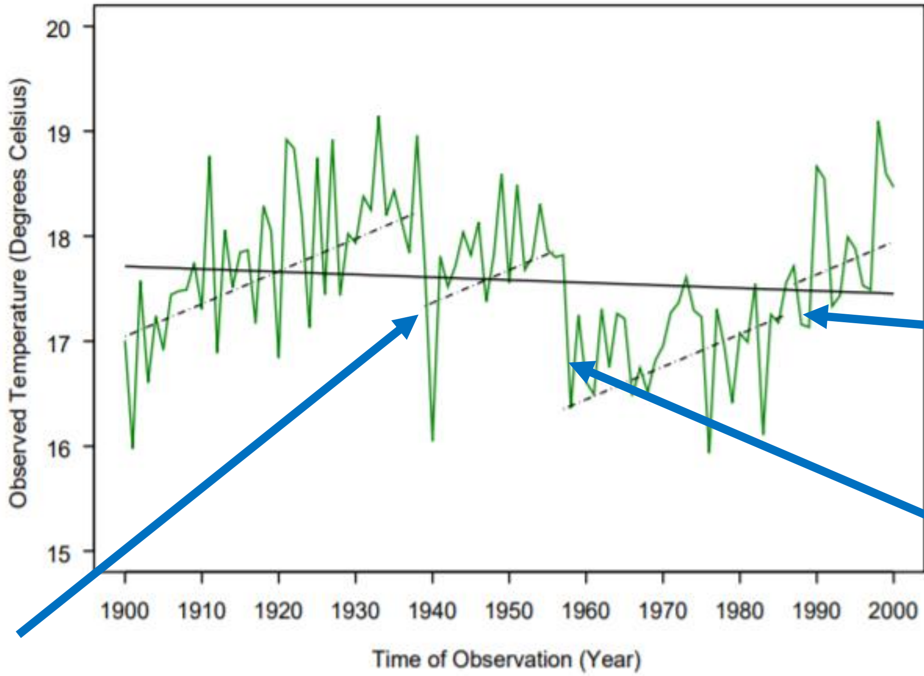
Temperature over the past century for Tuscaloosa, Alabama



Yearly Temperatures at Tuscaloosa AL With Least Squares Trends

Source: Dr. Robert Lund

Temperature over the past century for Tuscaloosa, Alabama



Yearly Temperatures at Tuscaloosa AL With Least Squares Trends

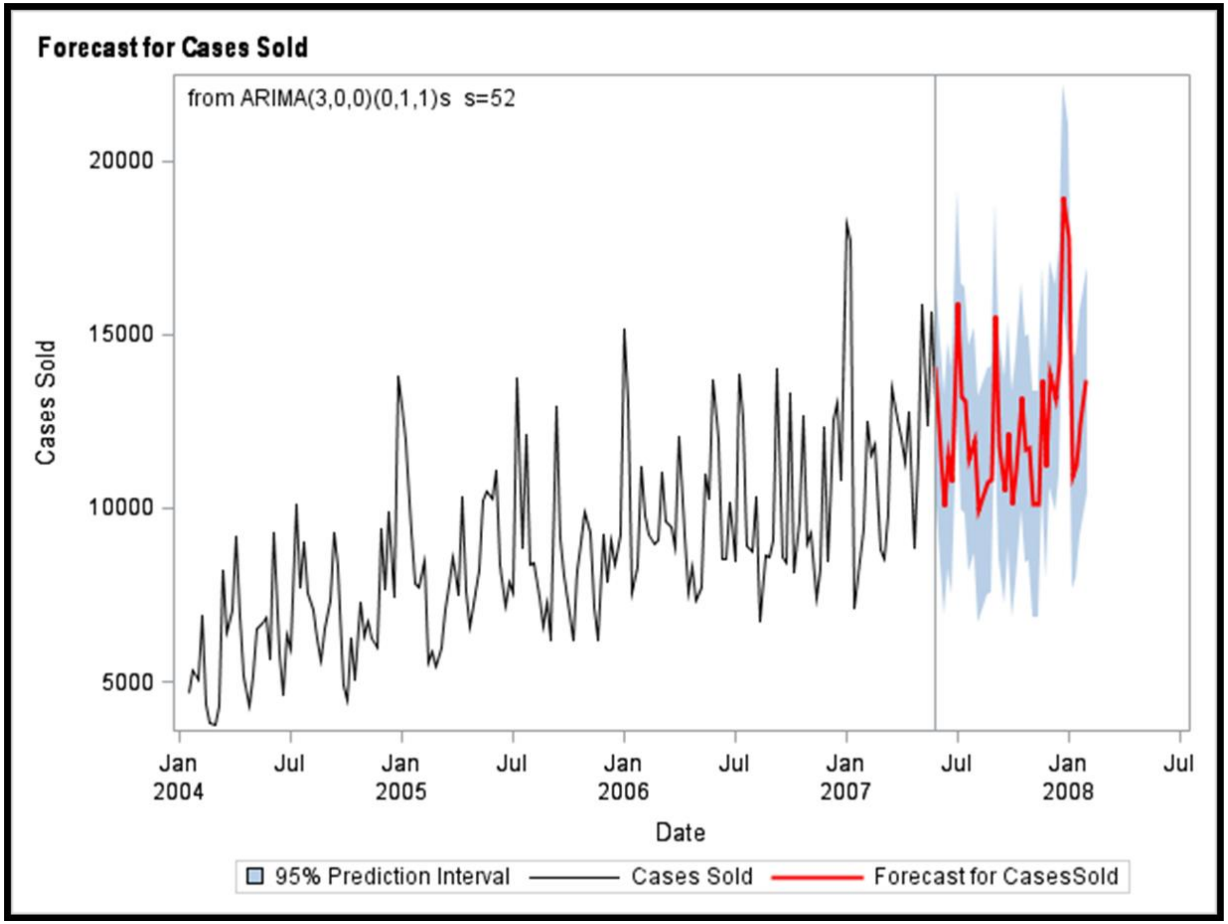
Station was relocated

Station relocated and instrumentation was changed

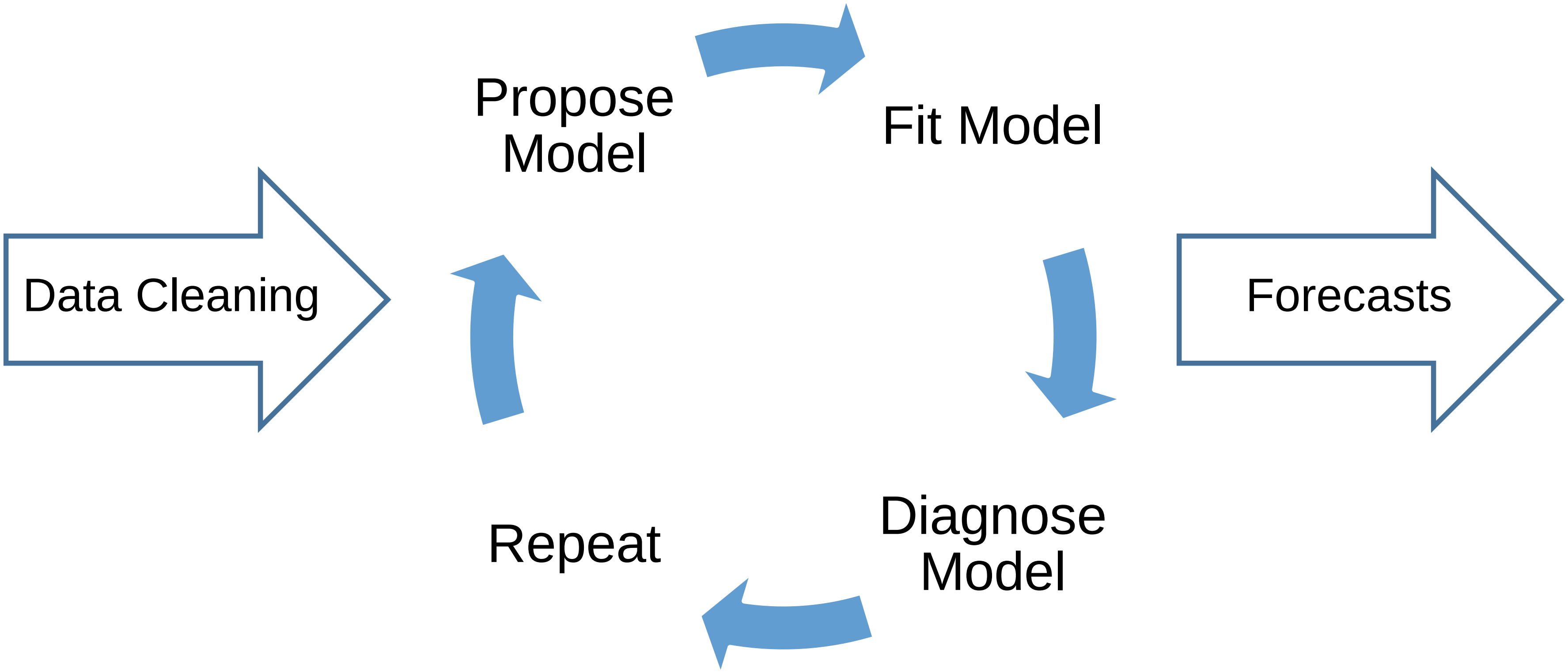
Thermometer was changed

Source: Dr. Robert Lund

Time Series to Forecast



Forecasting Process

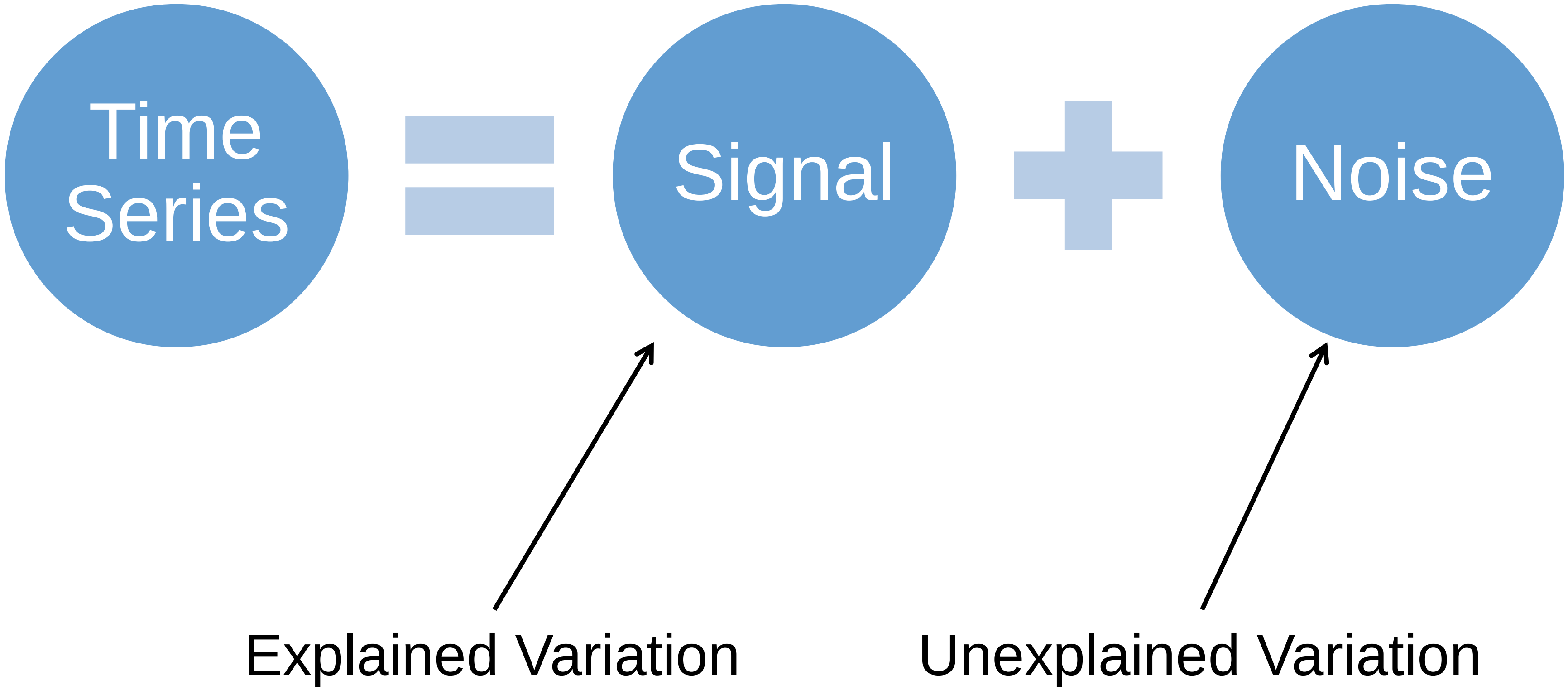


SIGNAL AND NOISE

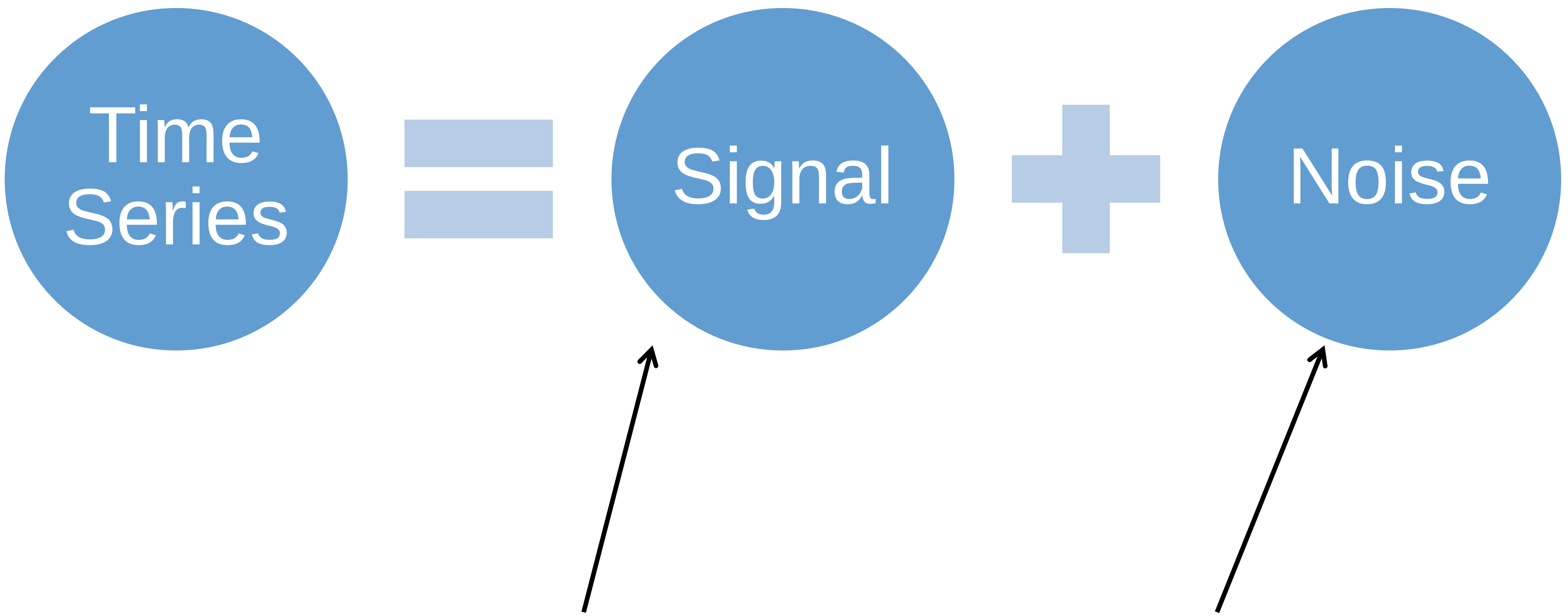
Statistical Forecasting

Time Series = Signal + Noise

Statistical Forecasting



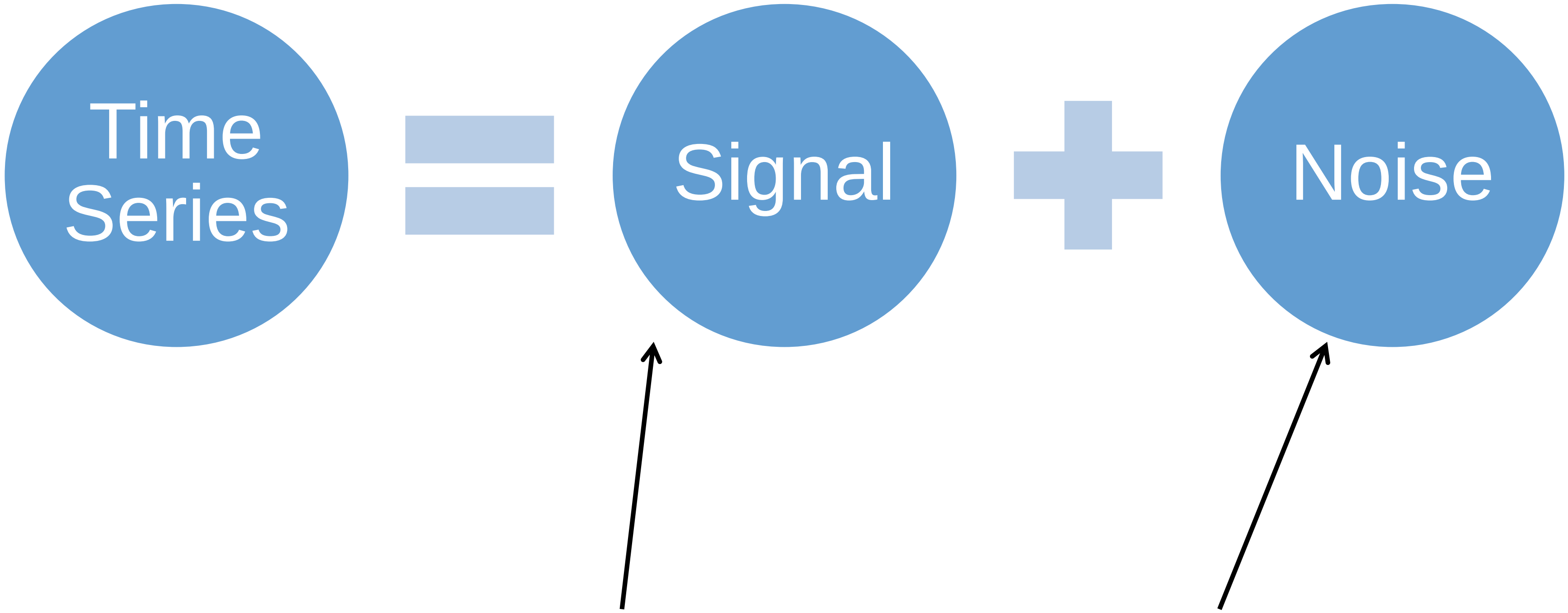
Statistical Forecasting



Explained variation:
Trend/Cycle
Seasonality

Error

Statistical Forecasting

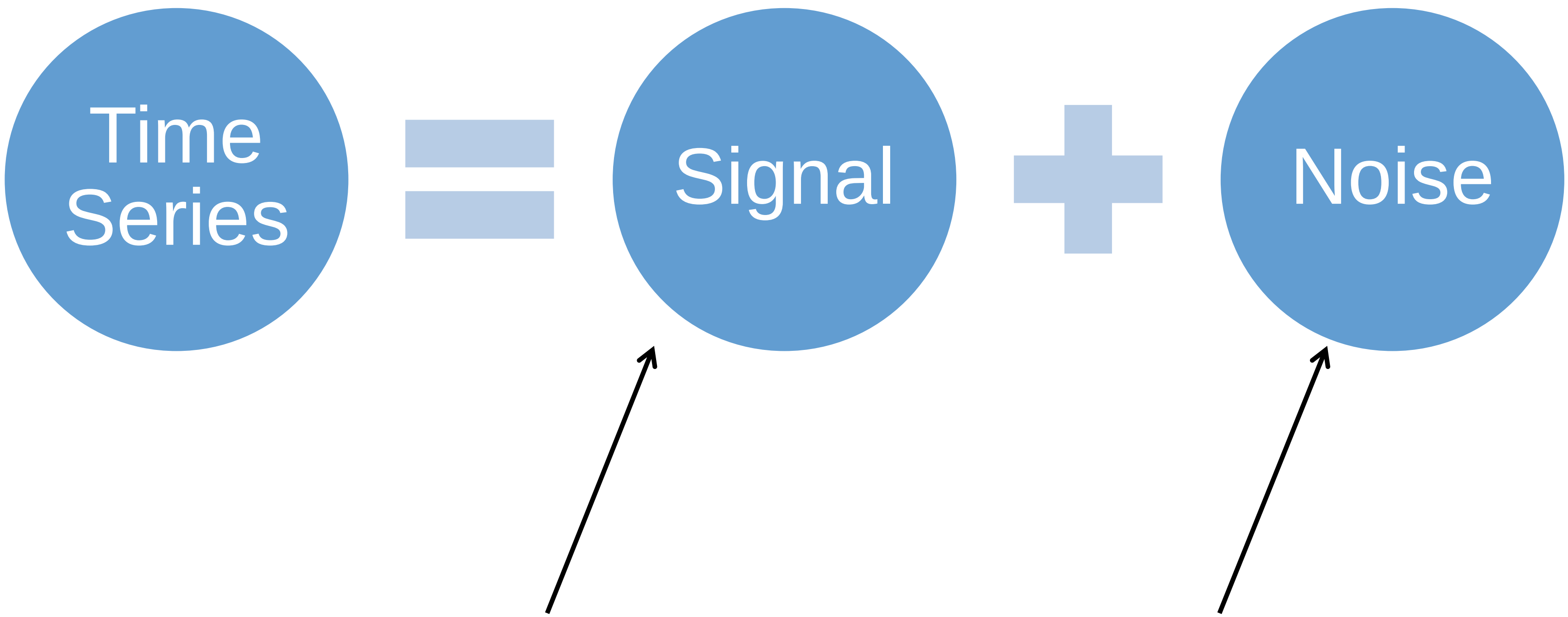


Explained variation:
Trend/Cycle
Seasonality
Dependence Structure

Error

This will be discussed in ARIMA

Statistical Forecasting



Forecasts extrapolate signal portion of model.

Confidence intervals account for uncertainty.

DECOMPOSITION

Time Series Decomposition

- If a time series only has trend/cycle patterns, there is no need to decompose
- If a time series has both trend/cycle patterns AND seasonal variation, we can decompose series into these individual parts:
  - Trend/Cycle patterns
  - Seasonal variation
  - Error

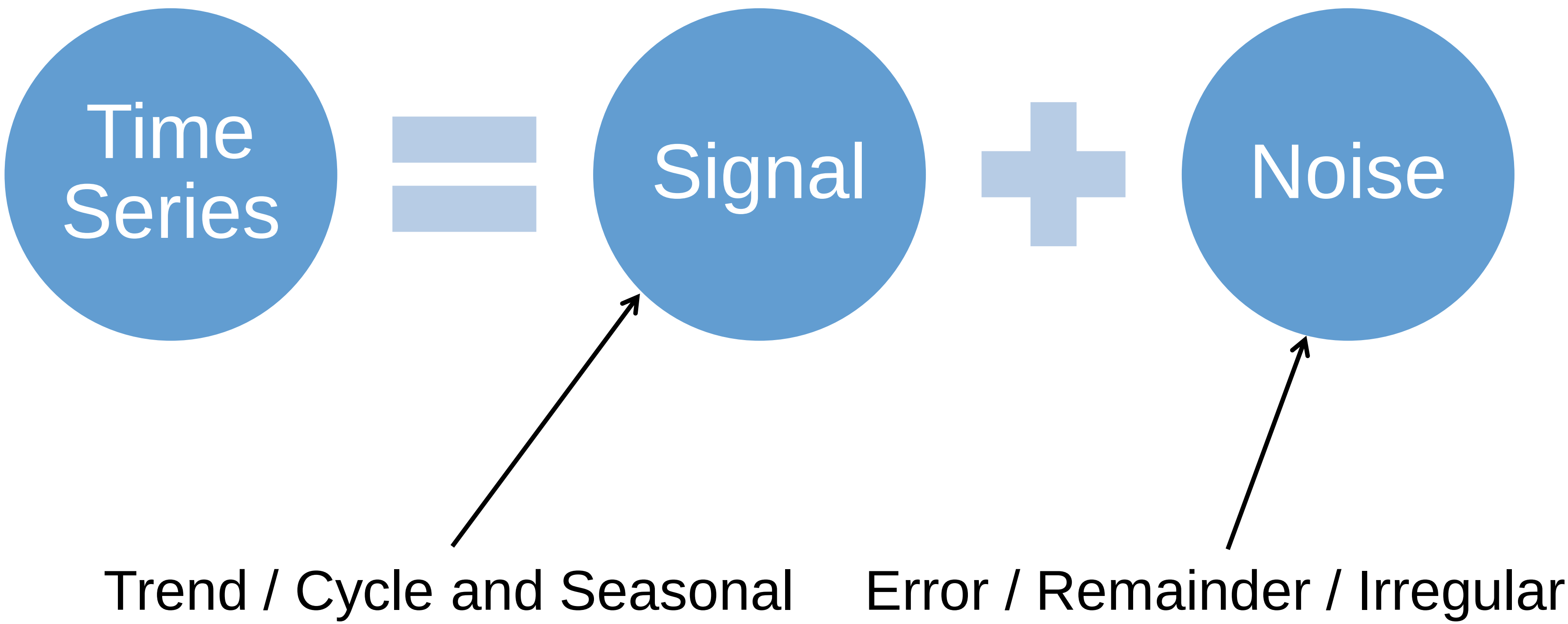
Time Series Decomposition

- If a time series only has trend/cycle patterns, there is no need to decompose
- If a time series has both trend/cycle patterns AND seasonal variation, we can decompose series into these individual parts:
  - Trend/Cycle patterns
  - Seasonal variation
  - Error

In order to decompose a time series, you must specify a seasonal component

Time Series Decomposition

- The signal part of the time series can typically be broken down into two components:



Time Series Decomposition

- The whole time series can now be thought of like the equations below.
  - Additive:

$$Y_t = T_t + S_t + R_t$$

  - Multiplicative:

$$Y_t = T_t \times S_t \times R_t$$

Time Series Decomposition

- The whole time series can now be thought of like the equations below.
  - Additive:

$$Y_t = T_t + S_t + R_t$$

Trend / Cycle

  - Multiplicative:

$$Y_t = T_t \times S_t \times R$$

Time Series Decomposition

- The whole time series can now be thought of like the equations below.
  - Additive:

$$Y_t = T_t + S_t + R_t$$

Seasonal

  - Multiplicative:

$$Y_t = T_t \times S_t \times R_t$$

Time Series Decomposition

- The whole time series can now be thought of like the equations below.
  - Additive:

$$Y_t = T_t + S_t + R_t$$

Error

  - Multiplicative:

$$Y_t = T_t \times S_t \times R_t$$

Time Series Decomposition

- The whole time series can now be thought of like the equations below.
  - Additive:

$$Y_t = T_t + S_t + R_t$$

  - Multiplicative:

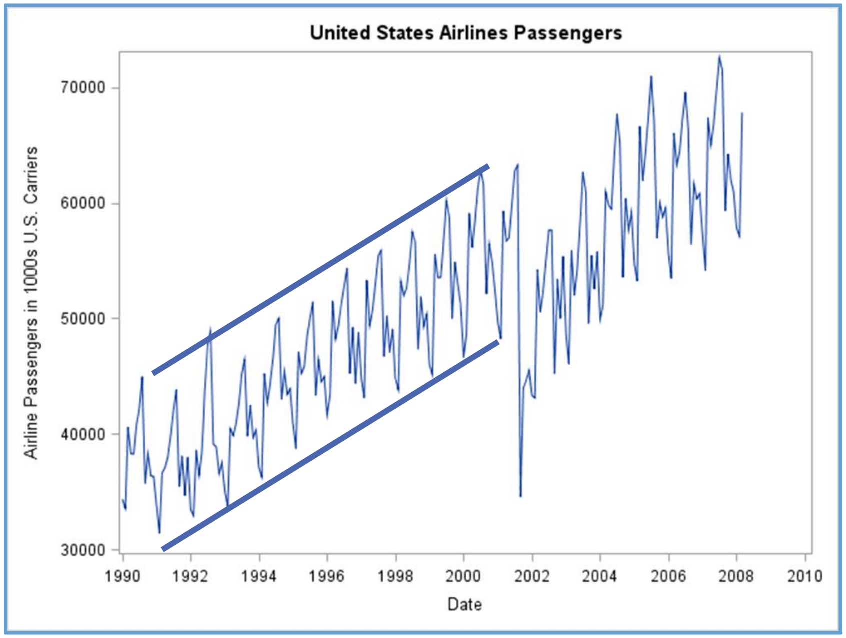
$$Y_t = T_t \times S_t \times R_t$$

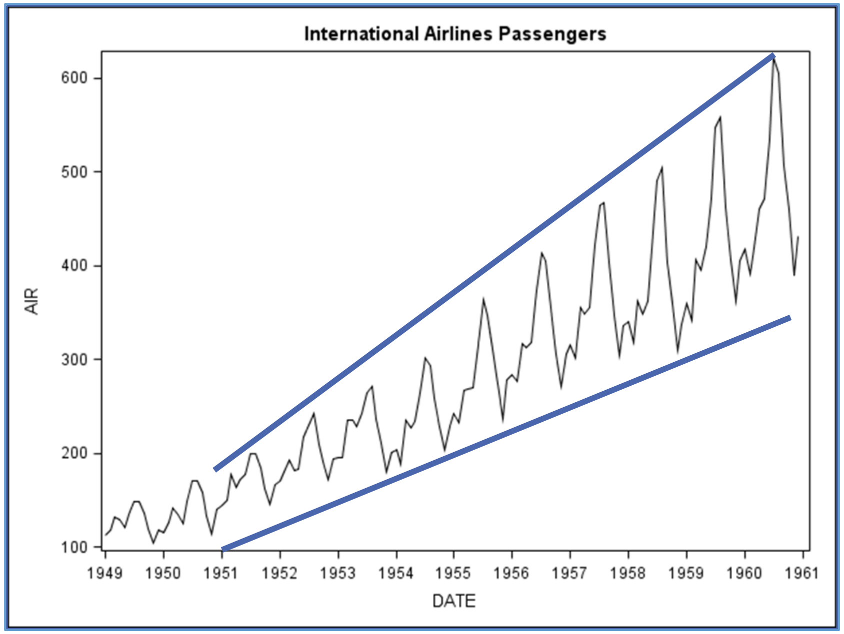
OR

$$\log(Y_t) = \log(T_t) + \log(S_t) + \log(R_t)$$

Additive vs. Multiplicative

- Additive – magnitude of variation around trend / cycle remains constant.
- Multiplicative – magnitude of the variation around trend / cycle proportionally changes.





Seasonally Adjusted Data

One advantage of time series decomposition is that we are able to create seasonally adjusted data (i.e. remove the "effect of Seasonality")

This allows analysts to understand the trend of the series

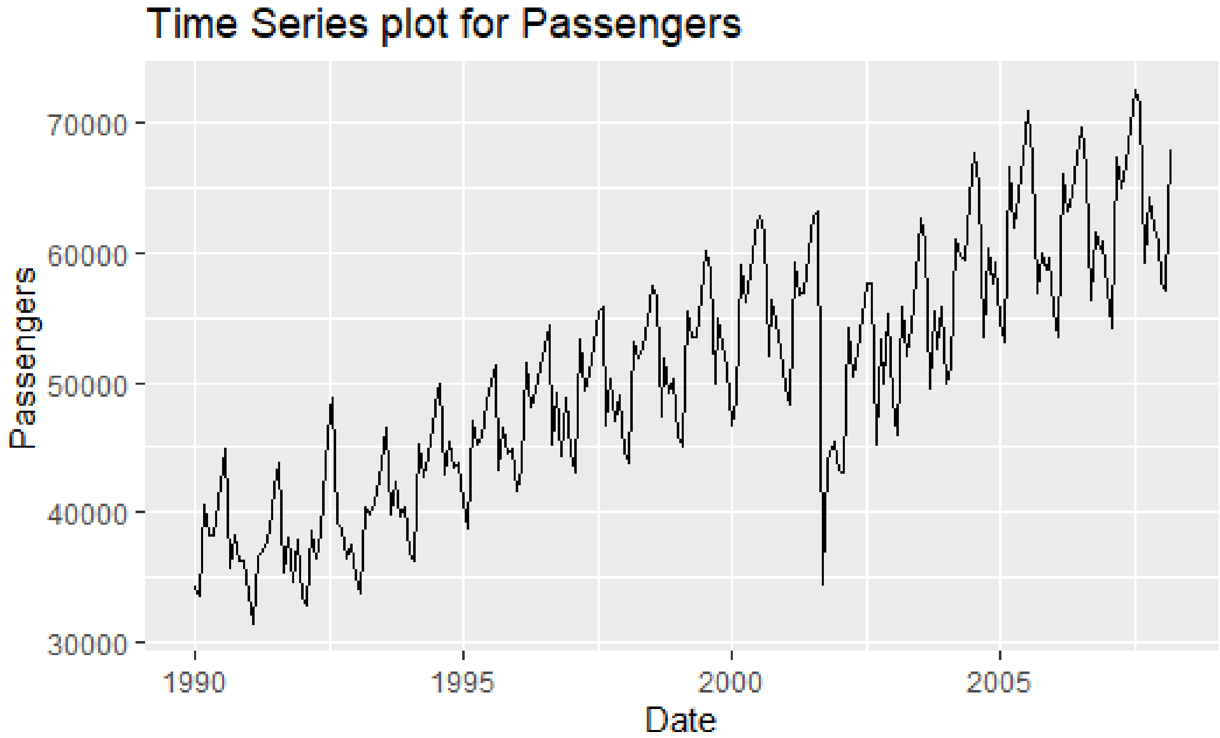
$$Y_t = T_t + S_t + R_t$$

$$Y_t - S_t \qquad\qquad (T_t + R_t)$$

The seasonal length of the time series is the length of one season (how long til the series repeats the "pattern")

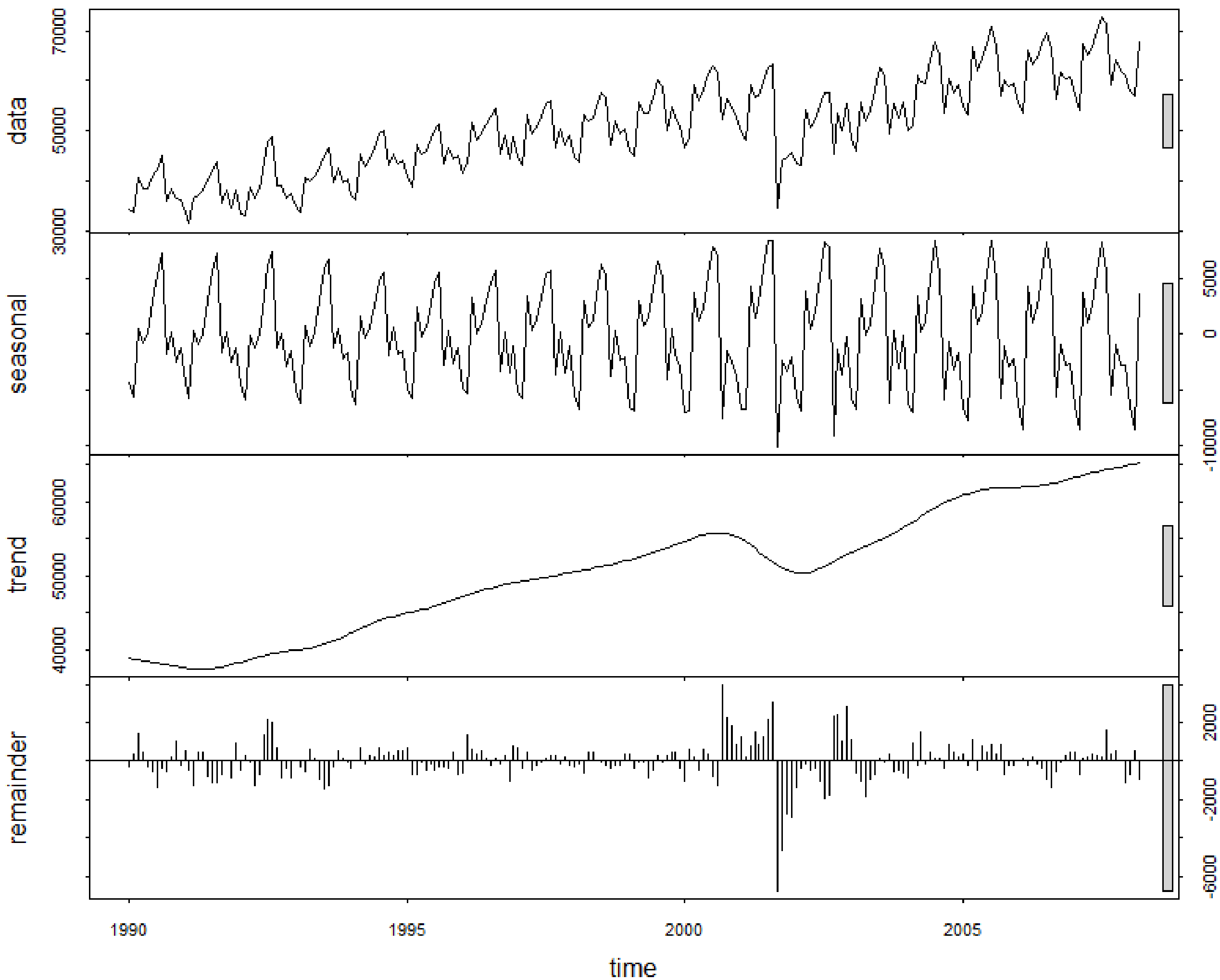
Airline data set

- Data contains number of US airline passengers from January 1990 – March 2008
- Data is monthly (length of season is 12…repeats pattern every 12 observations)



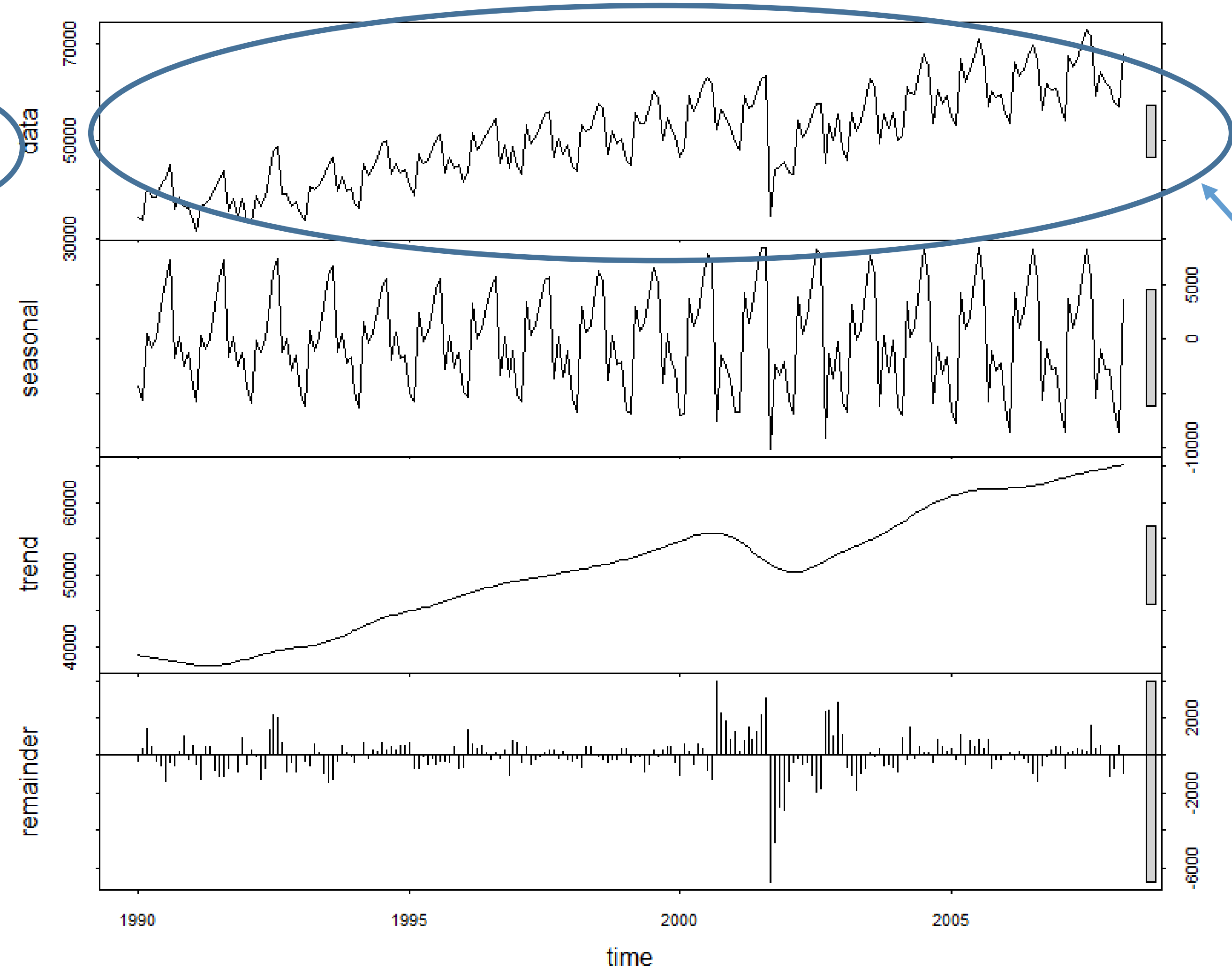
Time Series Decomposition

$Y_t = S_t + T_t + R_t$



Time Series Decomposition

$$Y_t = S_t + T_t + R_t$$



Original data

Time Series Decomposition

Y_t

$=$

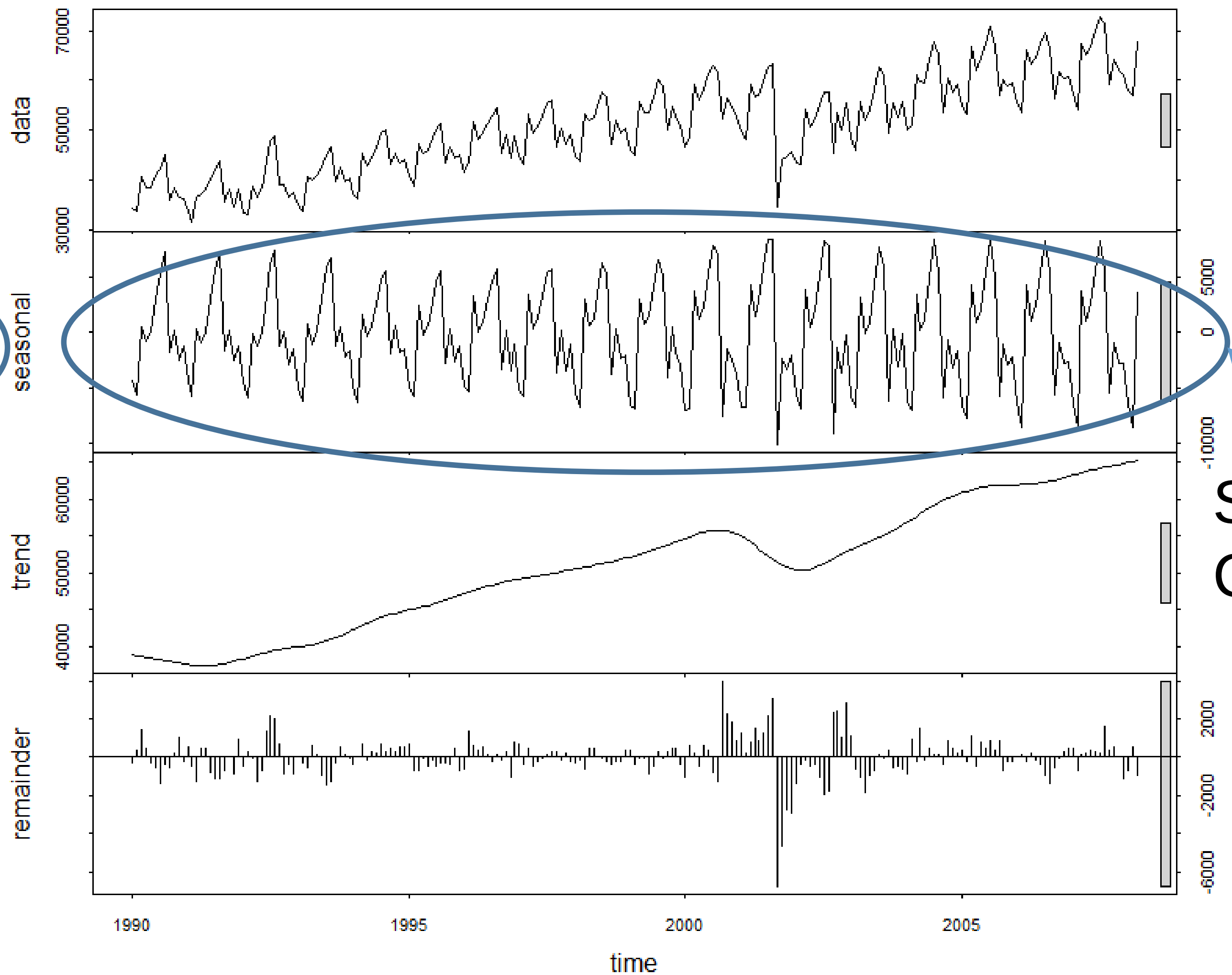
S_t

$+$

T_t

$+$

R_t



Seasonal Component

Time Series Decomposition

Y_t

$=$

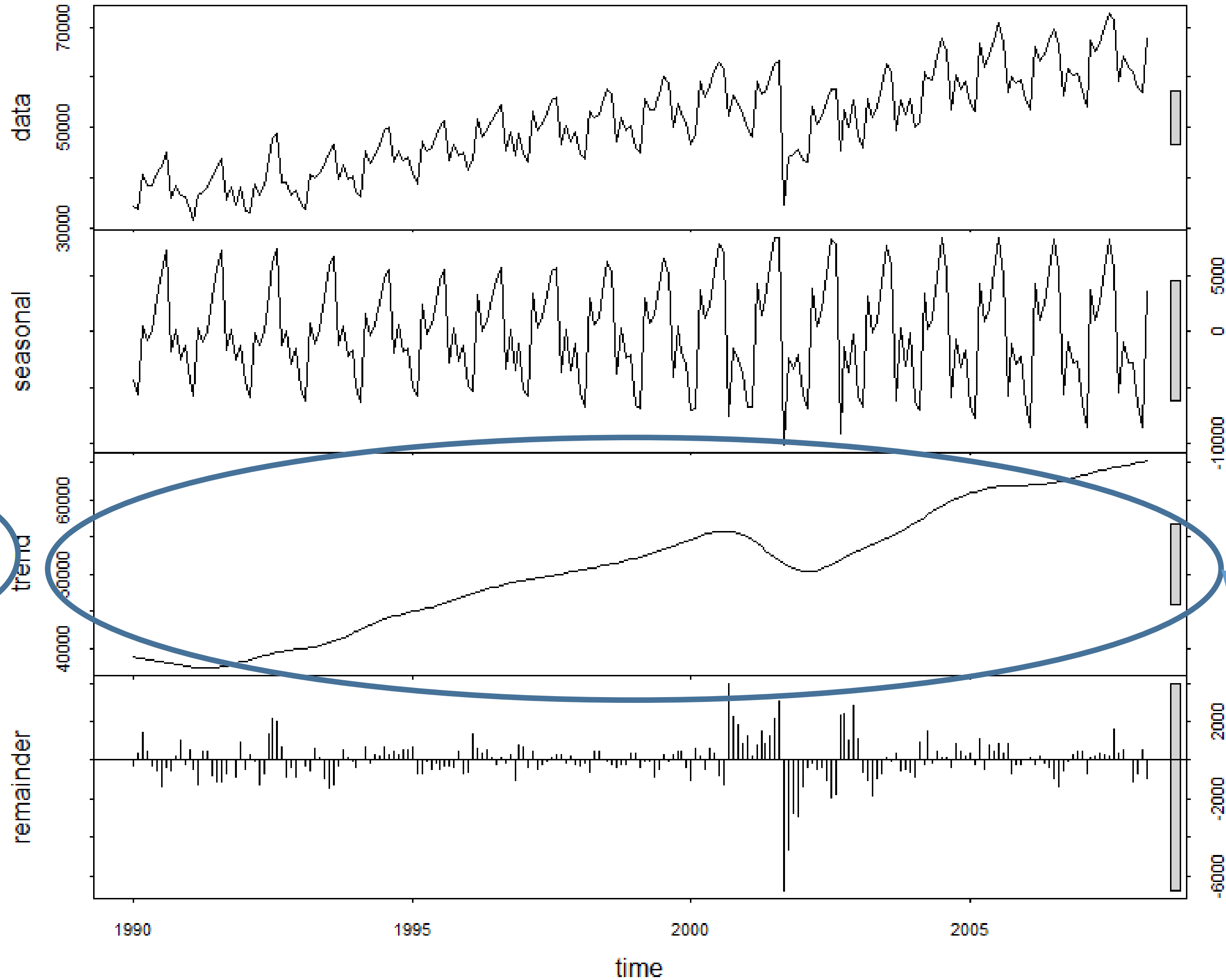
S_t

$+$

T_t

$+$

R_t



Trend Component

Time Series Decomposition

Y_t

$=$

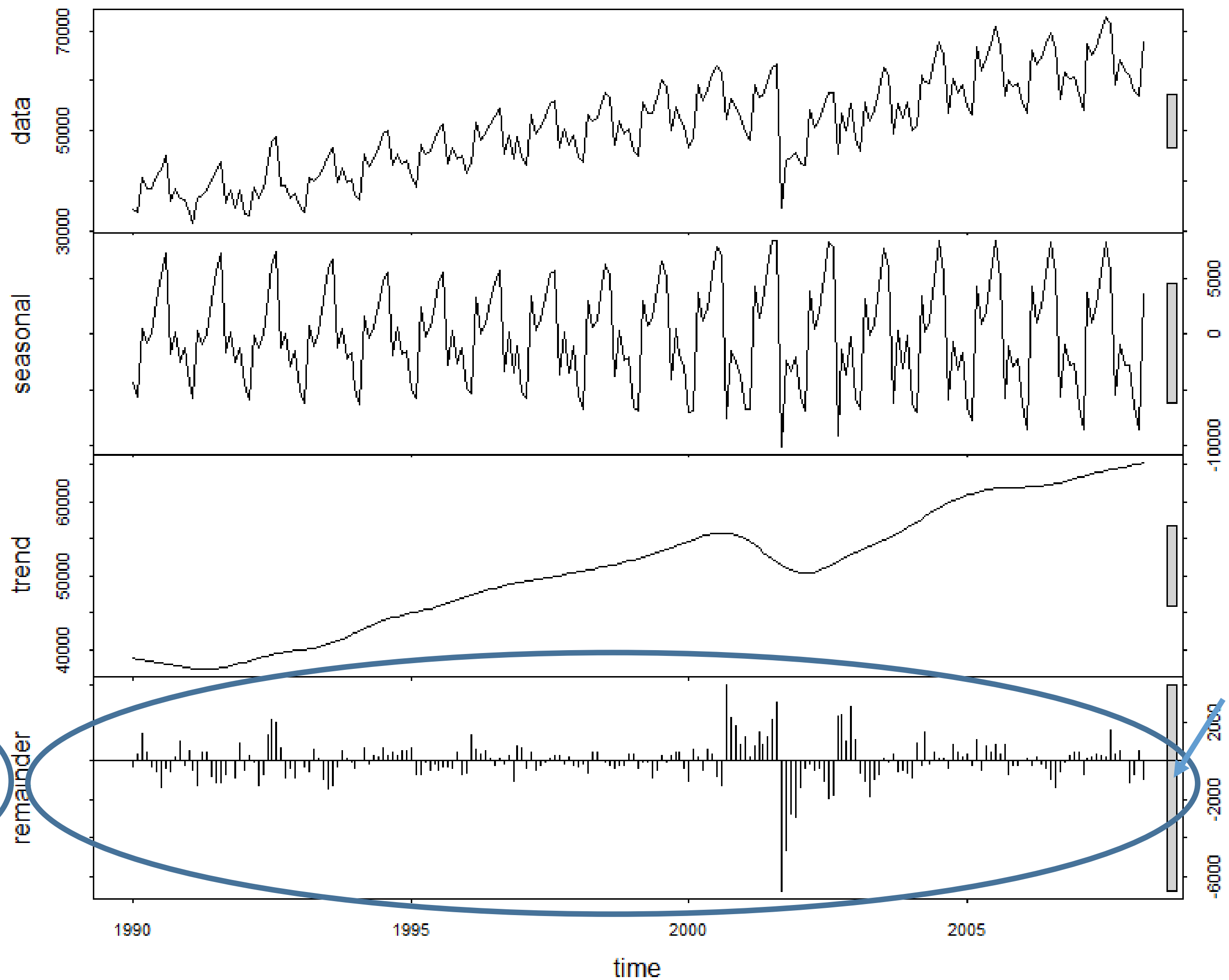
S_t

$+$

T_t

$+$

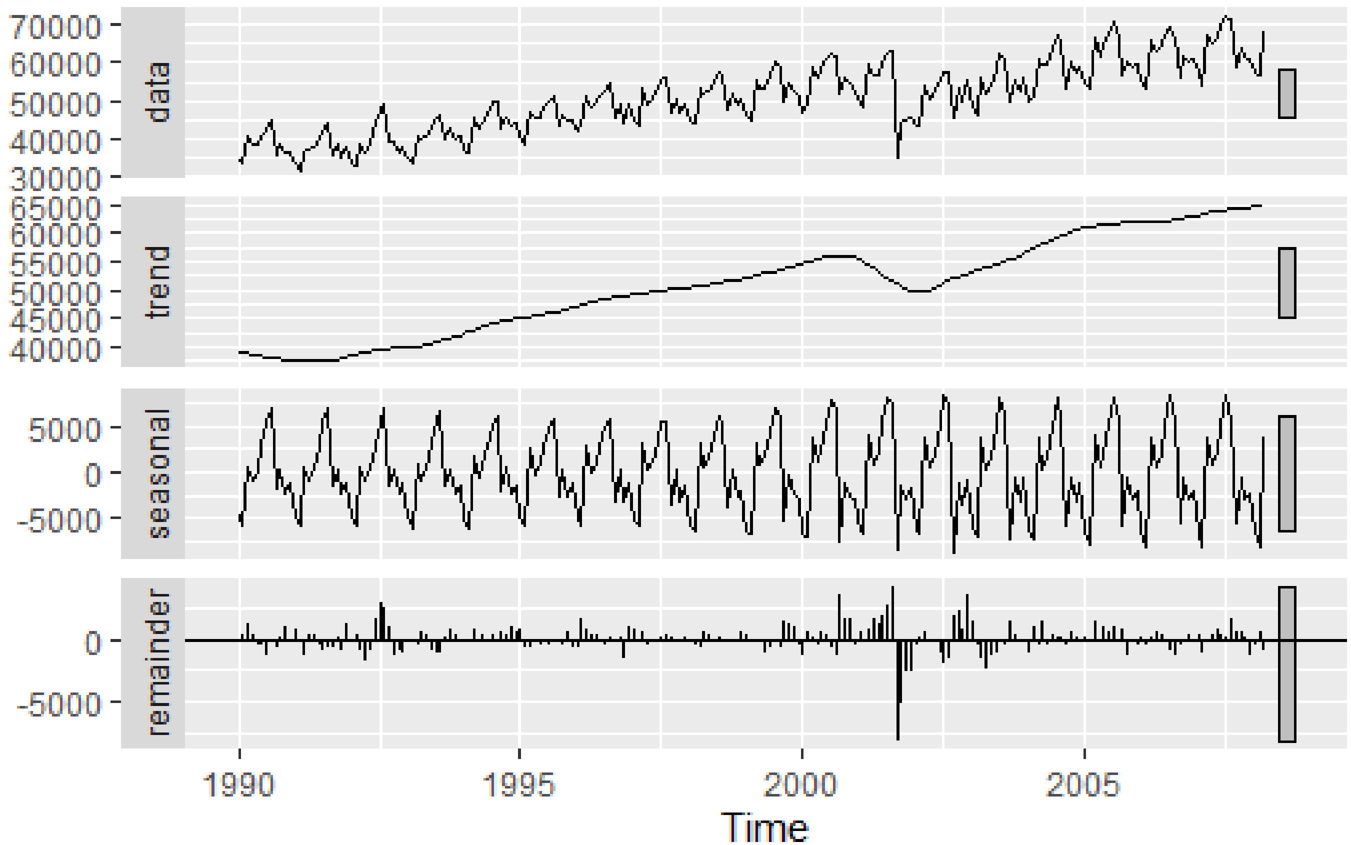
R_t



Remainder (What's left over)

Time Series Decomposition

```
# Time Series Decomposition ...STL#
Passenger <- ts(USAirlines$Passengers, start = 1990, frequency =12)
decomp_stl <- stl(Passenger, s.window = 7)

# Plot the individual components of the time series
plot(decomp_stl)

autoplot(decomp_stl)
```


70000
60000
50000
40000
30000
data
65000
60000
55000
50000
45000
40000
trend
5000
0
-5000
seasonal
0
-5000
remainder
1990
1995
2000
2005
Time

Pull off the different components

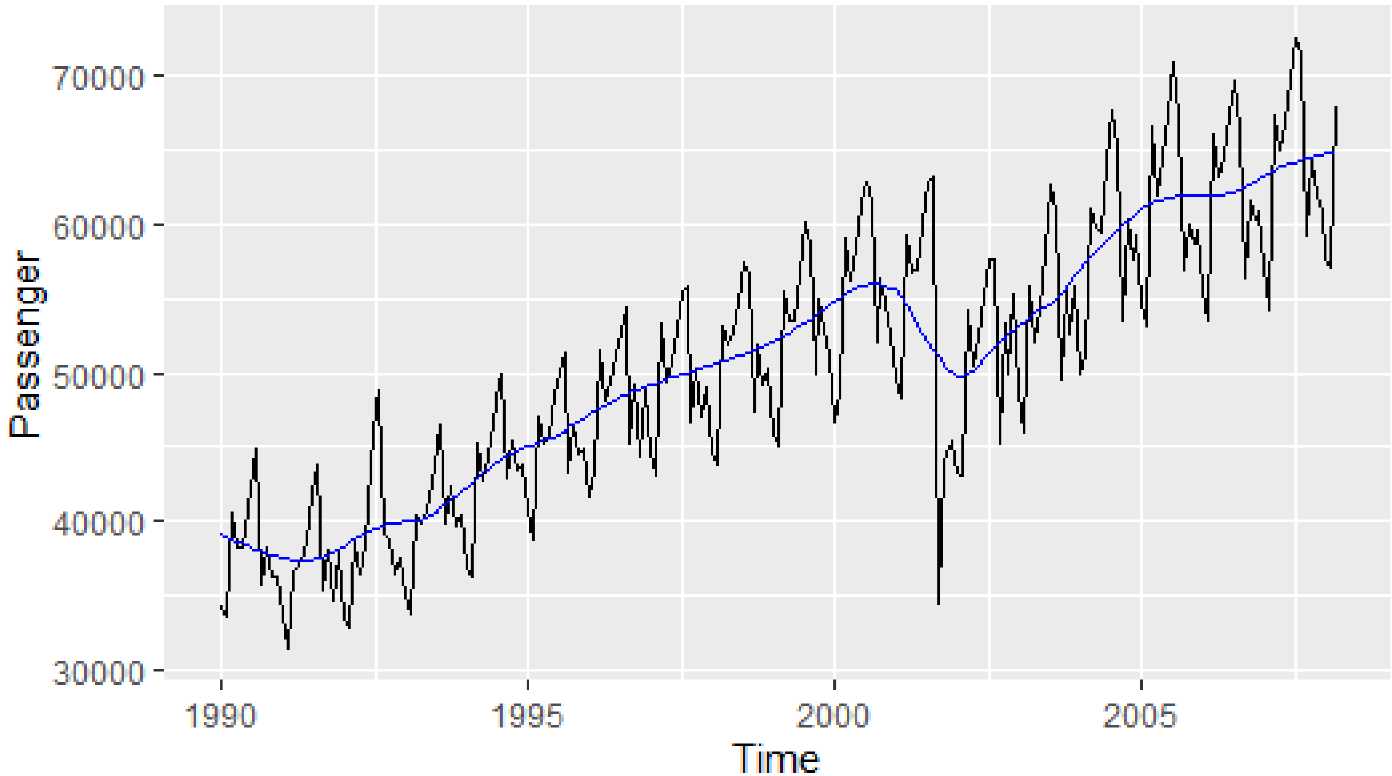
```
> head(decomp_stl$time.series)
          seasonal    trend remainder
Jan 1990 -4526.7610 39081.77 -207.0131
Feb 1990 -5827.5592 38942.75  420.8128
Mar 1990   560.4986 38803.72 1213.7829
Apr 1990  -802.2312 38664.69  404.5406
May 1990   139.2095 38533.15 -423.3574
Jun 1990  2953.8857 38401.61 -563.4910
```

Passengers
34348
33536
40578
38267
38249
40792

Time Series Decomposition

```
autoplot(Passenger)+
geom_line(aes(y=decomp_stl$time.series[,2]),
color="blue")
```

Overlay the trend component


70000
60000
50000
40000
30000
Passenger
1990
1995
2000
2005
Time

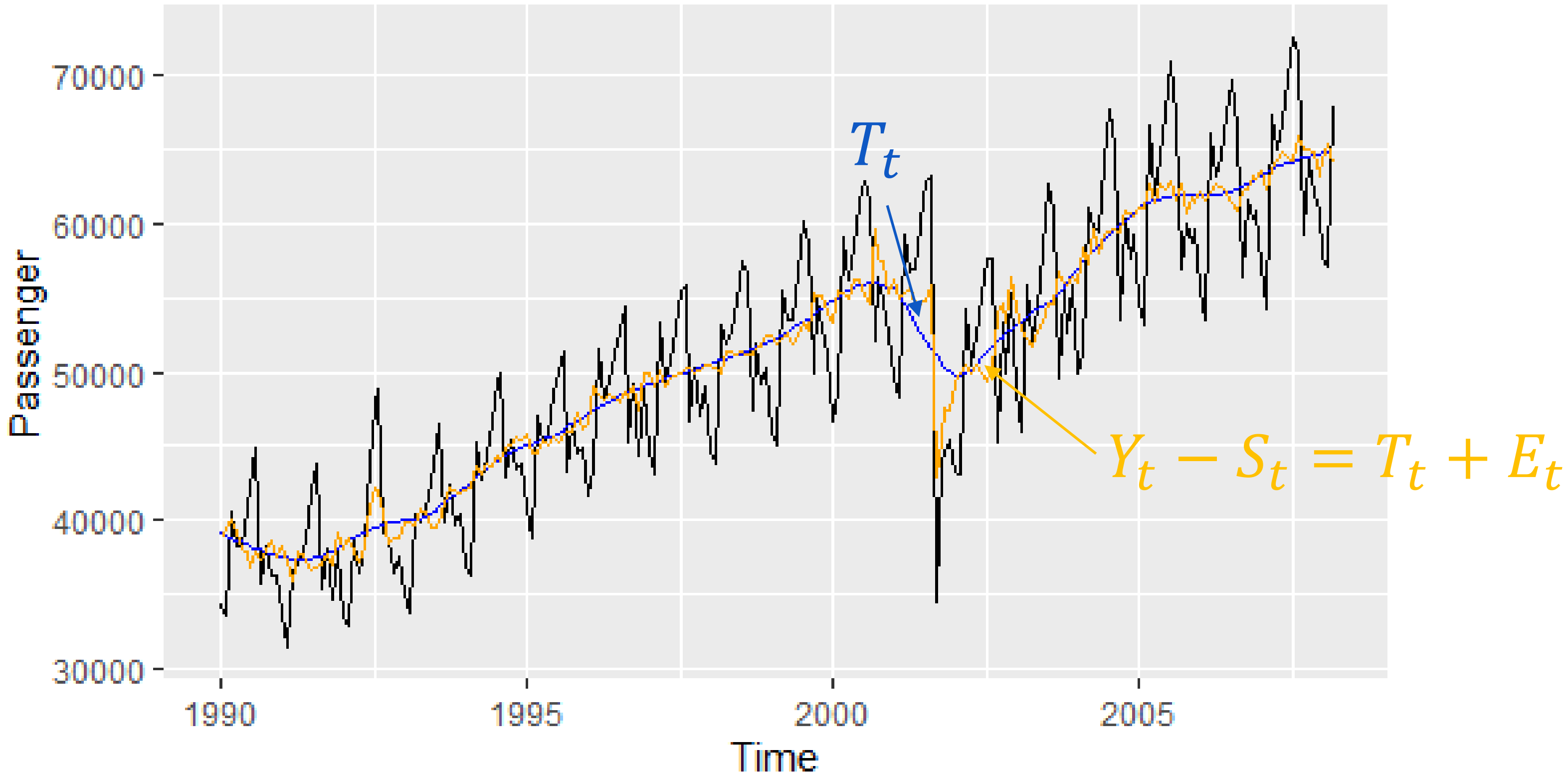
Time Series Decomposition

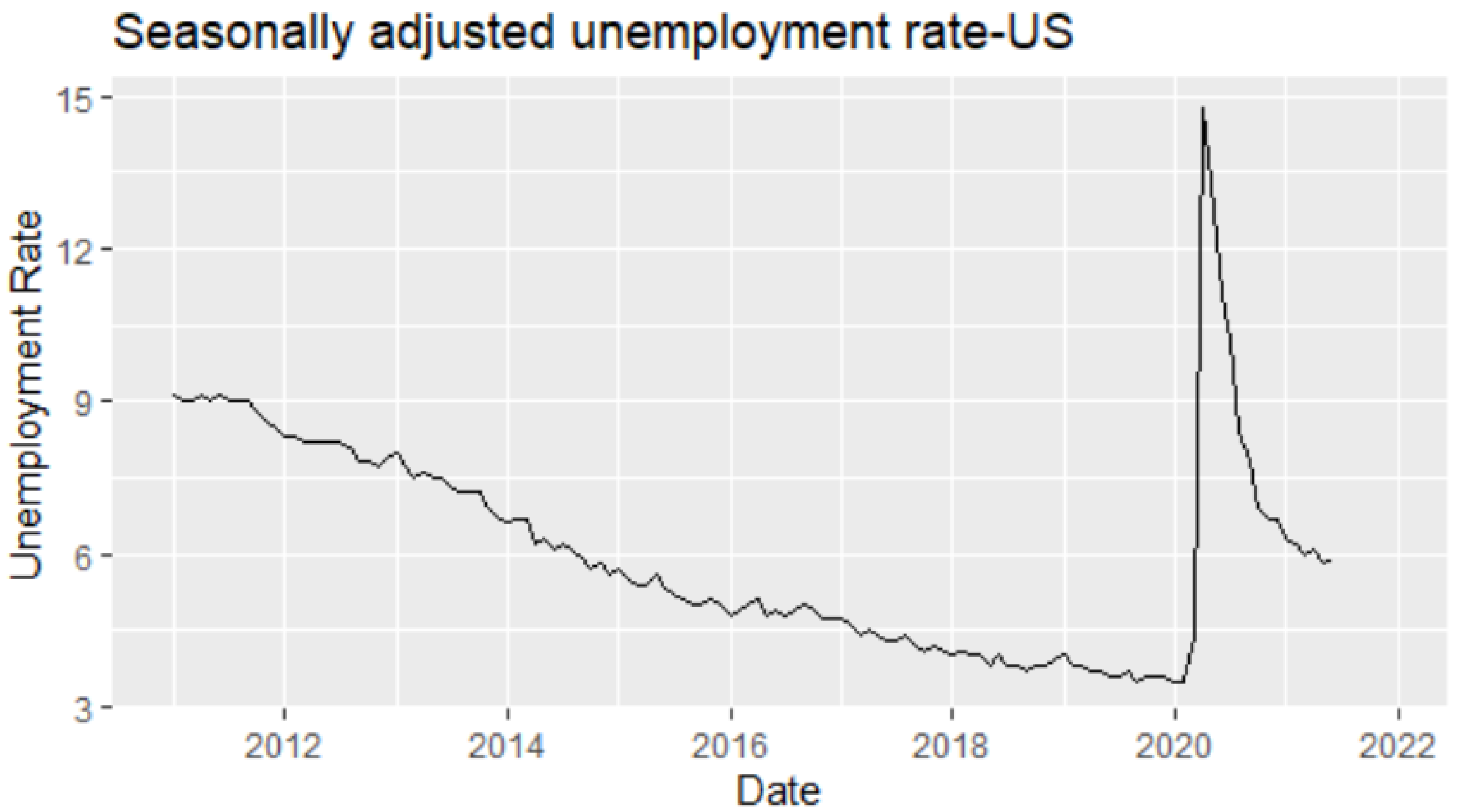
```
seas_adj=Passenger-decomp_stl$time.series[,1]

autoplot(Passenger) +
  geom_line(aes(y=decomp_stl$time.series[,2]),color="blue")
+ geom_line(aes(y=seas_adj),color="orange")
```

Overlay the trend component

Overlay seasonally adjusted

Time Series Decomposition

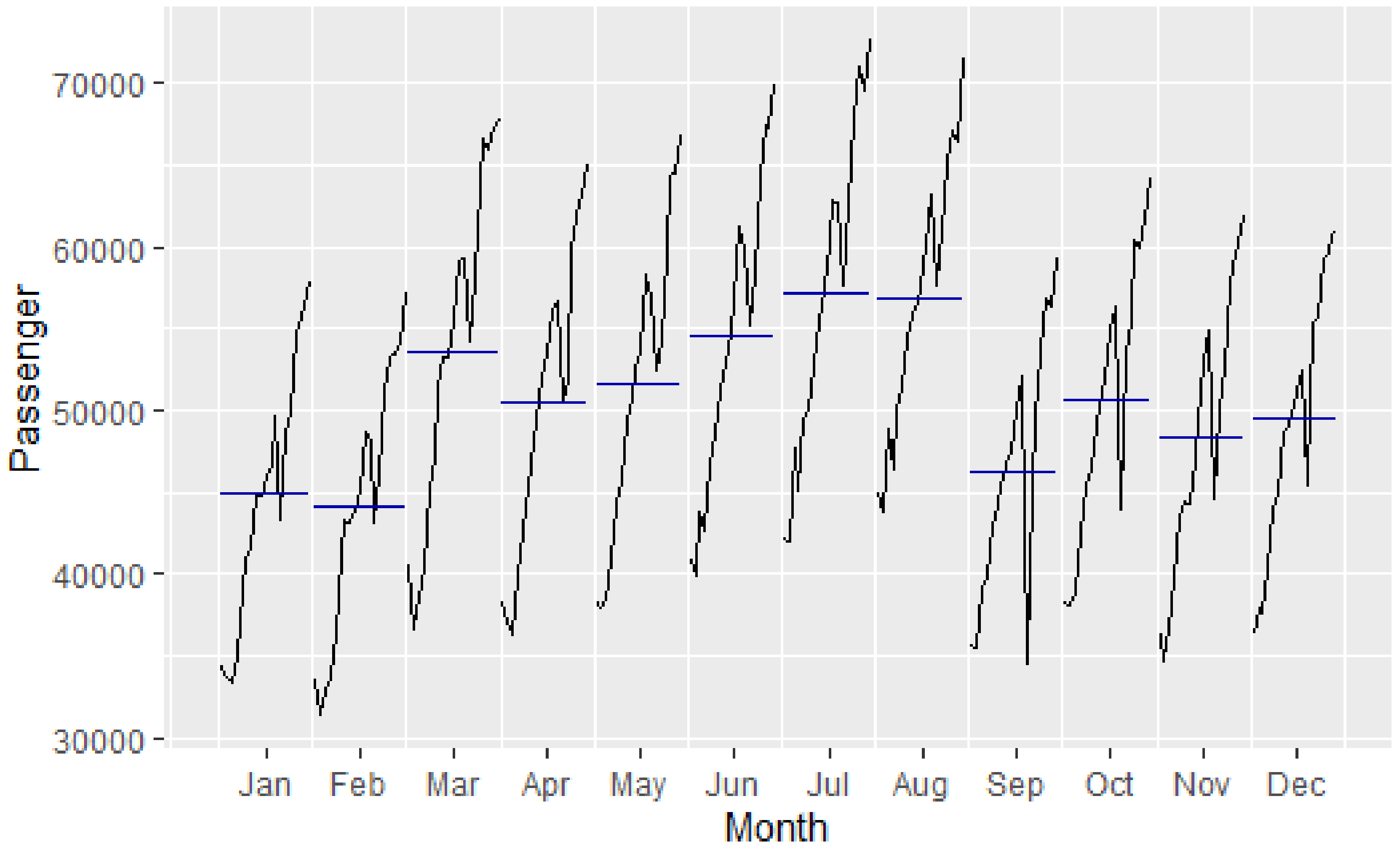



Seasonally adjusted unemployment rate-US
Unemployment Rate
15
12
9
6
3
2012
2014
2016
2018
2020
2022
Date

Time Series Decomposition – R

```
ggsubseriesplot(Passenger)
```

Time Series Decomposition



Decomposition Techniques

- There are many different ways to calculate the trend/cycle and seasonal effects inside time series data.
- Here are 3 common techniques:
  1. Classical Decomposition

Decomposition Techniques

- There are many different ways to calculate the trend/cycle and seasonal effects inside time series data.
- Here are 3 common techniques:
  1. Classical Decomposition
     a. Default in SAS (Can be done in R)
     b. Trend – Uses Moving / Rolling Average Smoothing
     c. Seasonal – Average De-trended Values Across Seasons

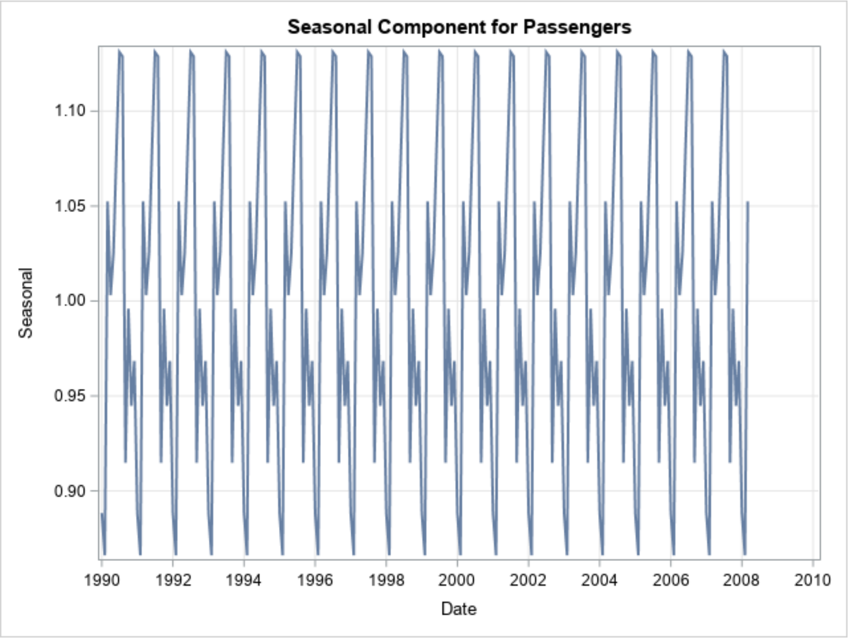
Decomposition Techniques

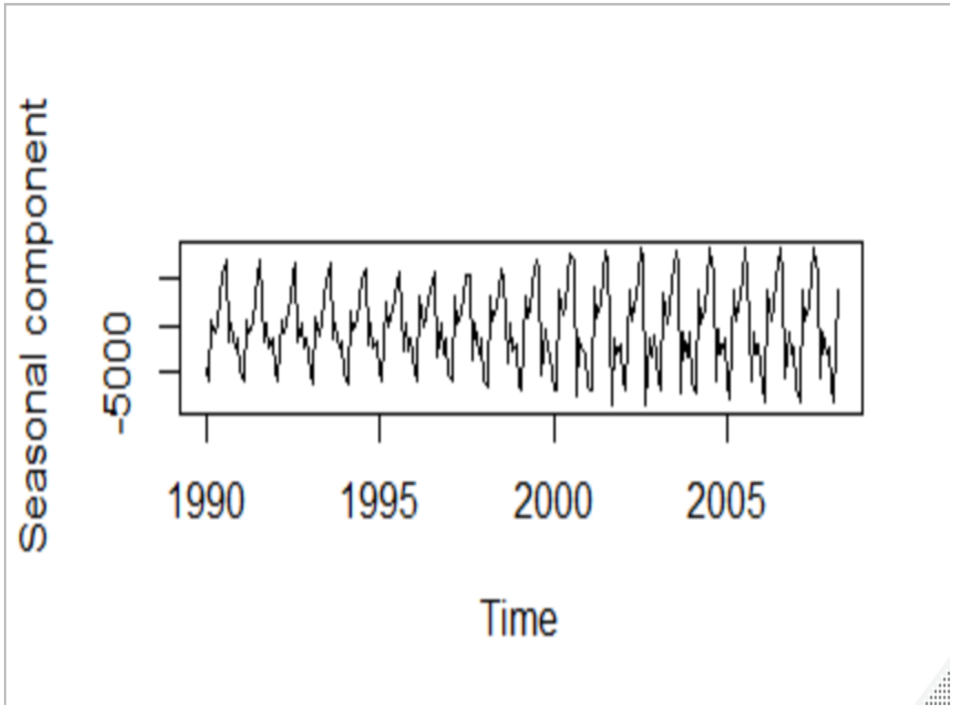
- There are many different ways to calculate the trend/cycle and seasonal effects inside time series data.
- Here are 3 common techniques:
  1. Classical Decomposition
  2. X-13 ARIMA Decomposition (self study)
     a. Trend – Uses Moving / Rolling Average Smoothing
     b. Seasonal – Uses Moving / Rolling Average Smoothing
     c. Iteratively Repeats Above Methods and ARIMA Modeling
     d. Can handle outliers

Decomposition Techniques

- There are many different ways to calculate the trend/cycle, and seasonal effects inside time series data.
- Here are 3 common techniques:
  1. Classical Decomposition
  2. X-13 ARIMA Decomposition
  3. STL (Seasonal and Trend using LOESS estimation) Decomposition
     a. Default of stl Function in R (Not available in SAS)
     b. Uses **LO**cal regr**ESS**ion Techniques to Estimate Trend and Seasonality
     c. Allows Changing Effects for Trend and Season
     d. Adapted to Handle Outliers

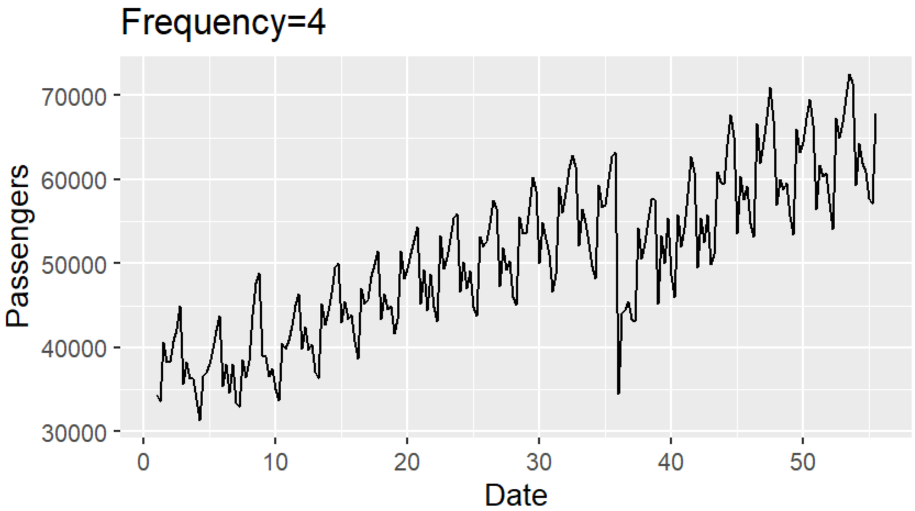
Comparison of Classical versus STL seasonal decomposition

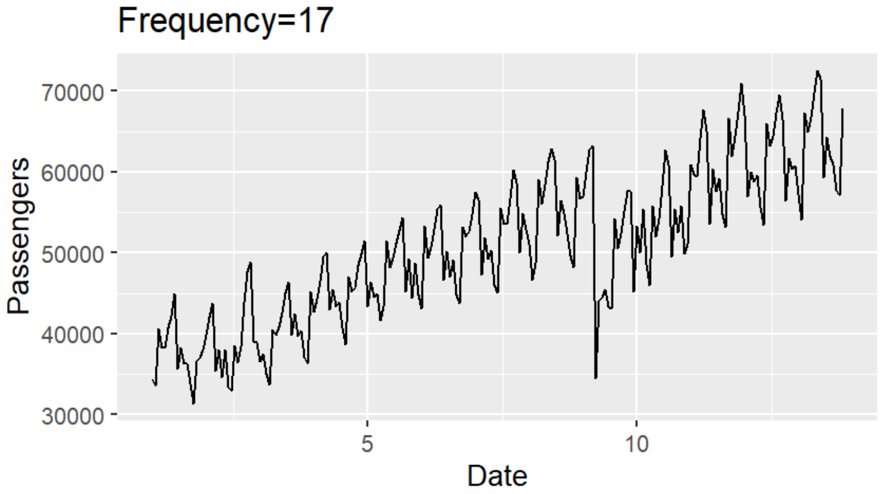




Cautions on decomposition

- Decomposition will NOT tell you if you have seasonal data (nor the length of seasonality)
- Not a really good test, but…





Measures for “strength” of trend and/or seasonality

- Measures provided by Hyndman and Athanasopoulos
- Values of F close to 0 indicate little strength and values close to 1 indicate high strength

$$F_T = \max\left(0, 1 - \frac{\mathrm{Var}(R_t)}{\mathrm{Var}(T_t + R_t)}\right).$$

$$F_S = \max\left(0, 1 - \frac{\mathrm{Var}(R_t)}{\mathrm{Var}(S_t + R_t)}\right).$$

```
> Ft=max(0,1-var(decomp_stl$time.series[,3])/(var(decomp_stl$time.series[,3])+var(decomp_stl$time.series[,2])+2*cov(decomp_stl$time.series[,3],decomp_stl$time.series[,2])))

> Ft
[1] 0.980

> Fs=max(0,1-var(decomp_stl$time.series[,3])/(var(decomp_stl$time.series[,3])+var(decomp_stl$time.series[,1])+2*cov(decomp_stl$time.series[,3],decomp_stl$time.series[,1])))

> Fs
[1] 0.934
```