SEASONALITY MODELS

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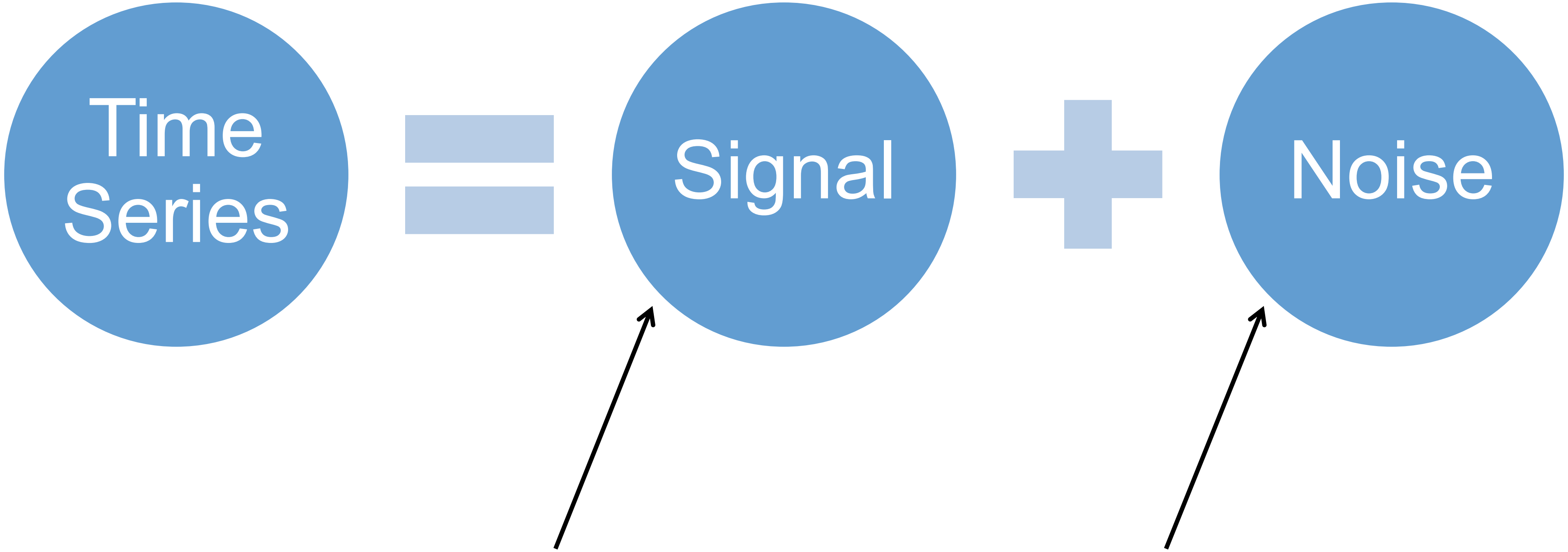
Institute for Advanced Analytics

QUICK REVIEW

Time Series Data



Forecasts extrapolate signal portion of model.

Confidence intervals account for uncertainty.

Time Series Data

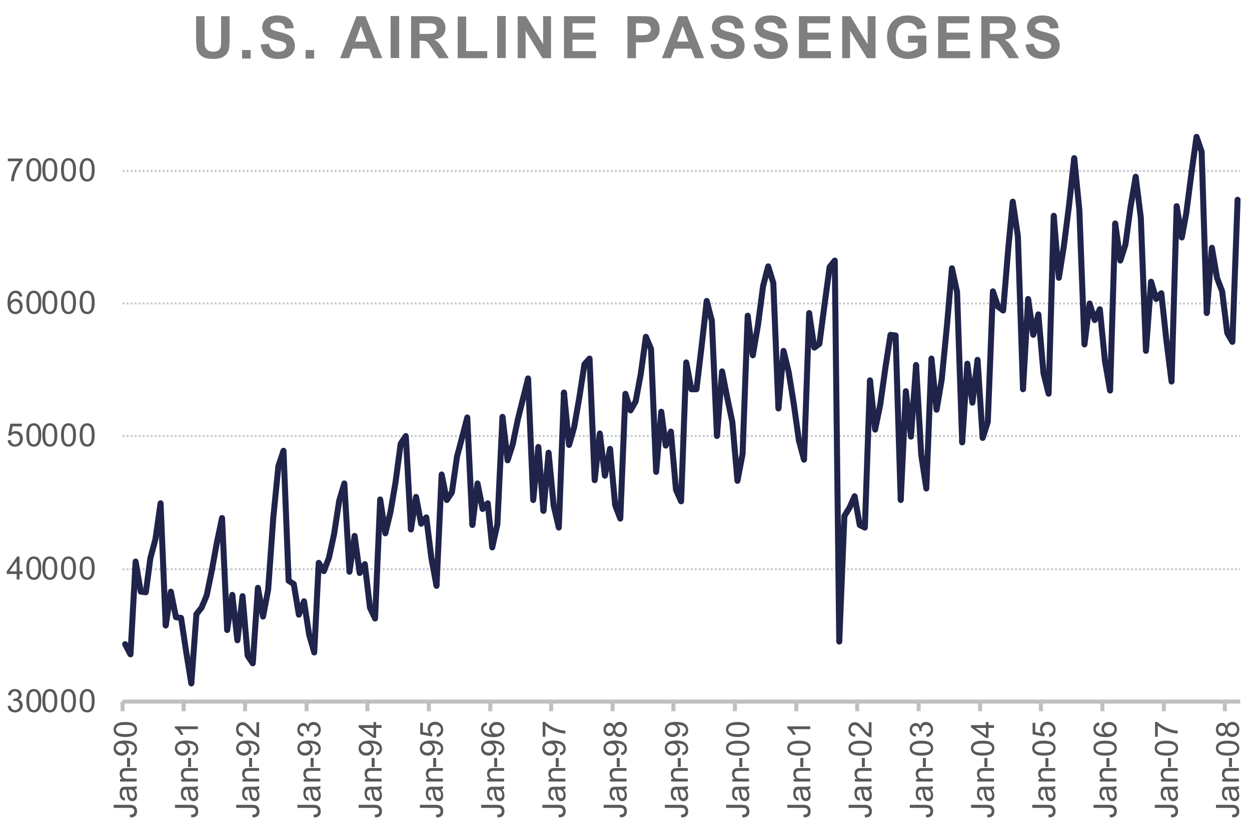
Original Series

=

Trend / Cycle

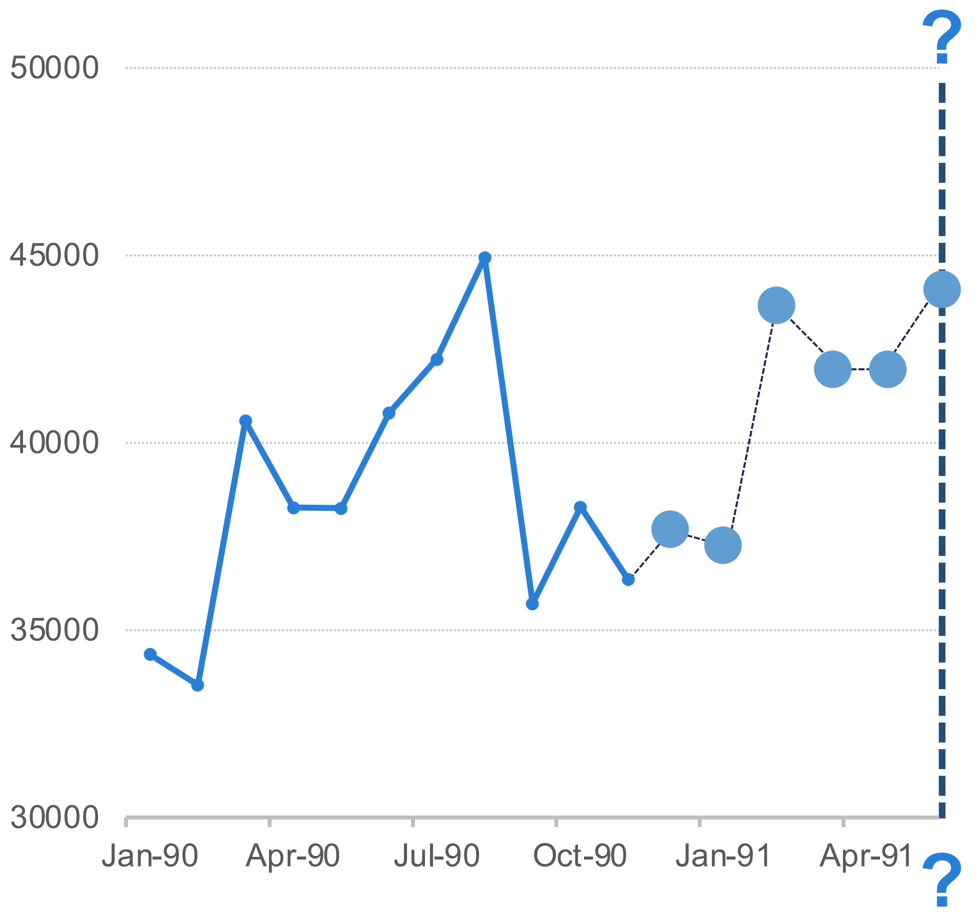
Season

Error



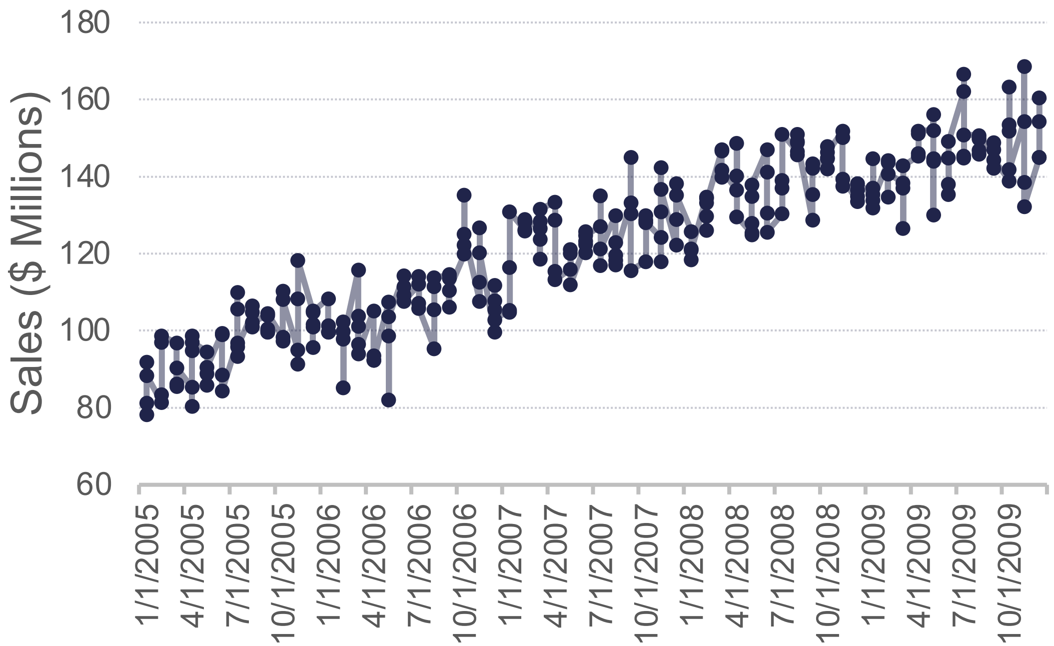
Exponential Smoothing Models

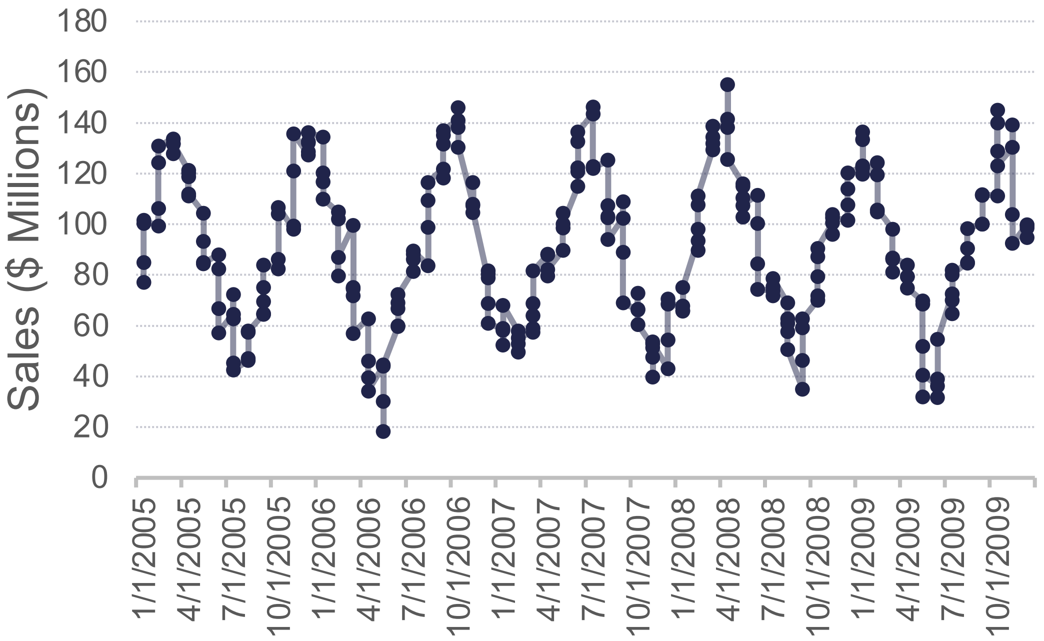
Original Series	$\hat{Y}_{t+k} = L_t + kT_t + S_{t-p+k}$
=	=
Level	$L_t = \theta Y_t + (1-\theta)L_{t-1}$
Trend	$T_t = \gamma(L_t - L_{t-1}) + (1-\gamma)T_{t-1}$
Season	$S_t = \delta(Y_t - L_t) + (1-\delta)S_{t-p}$



Stationarity

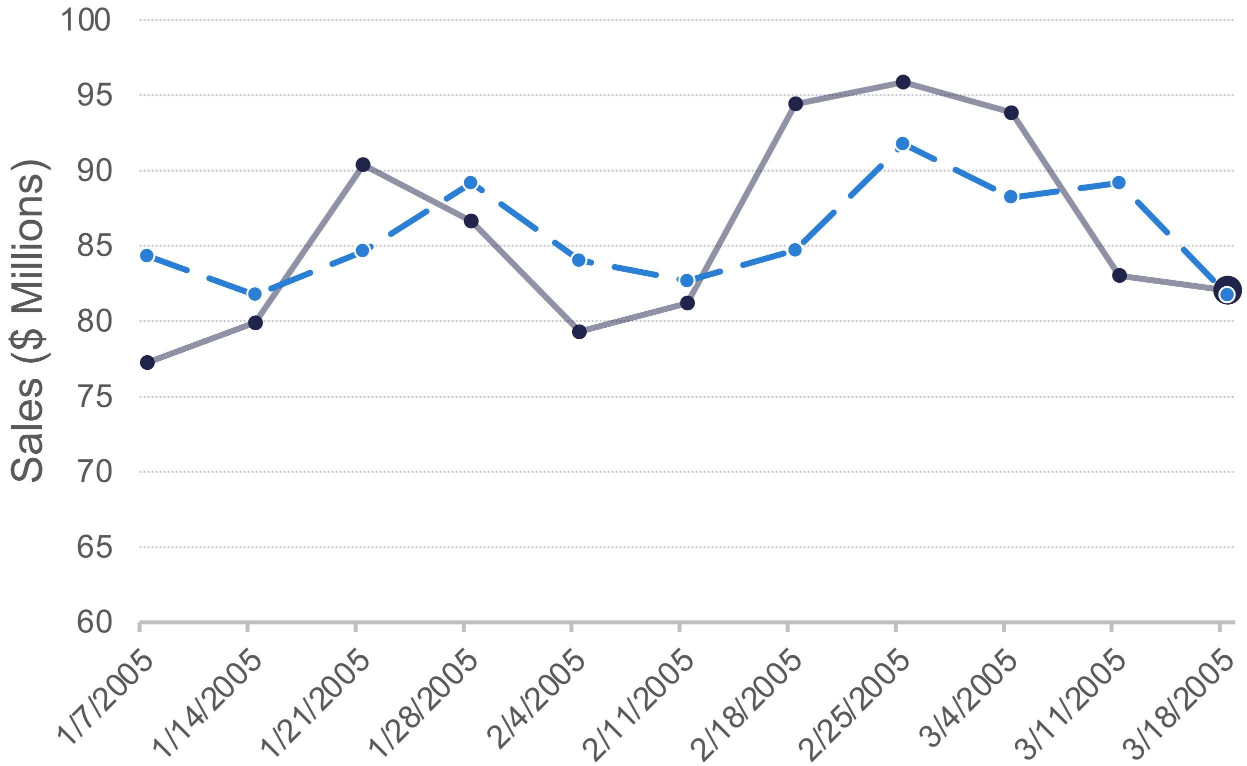
- Need consistency of mean and variance.
- What about changes in mean – trending, seasonality? **NOT** stationary.





ARIMA Models

- AR – forecast a series based solely on the past values in the series – called **lags.**
- MA – forecast a series based solely on the past errors in the series – called **error lags**.



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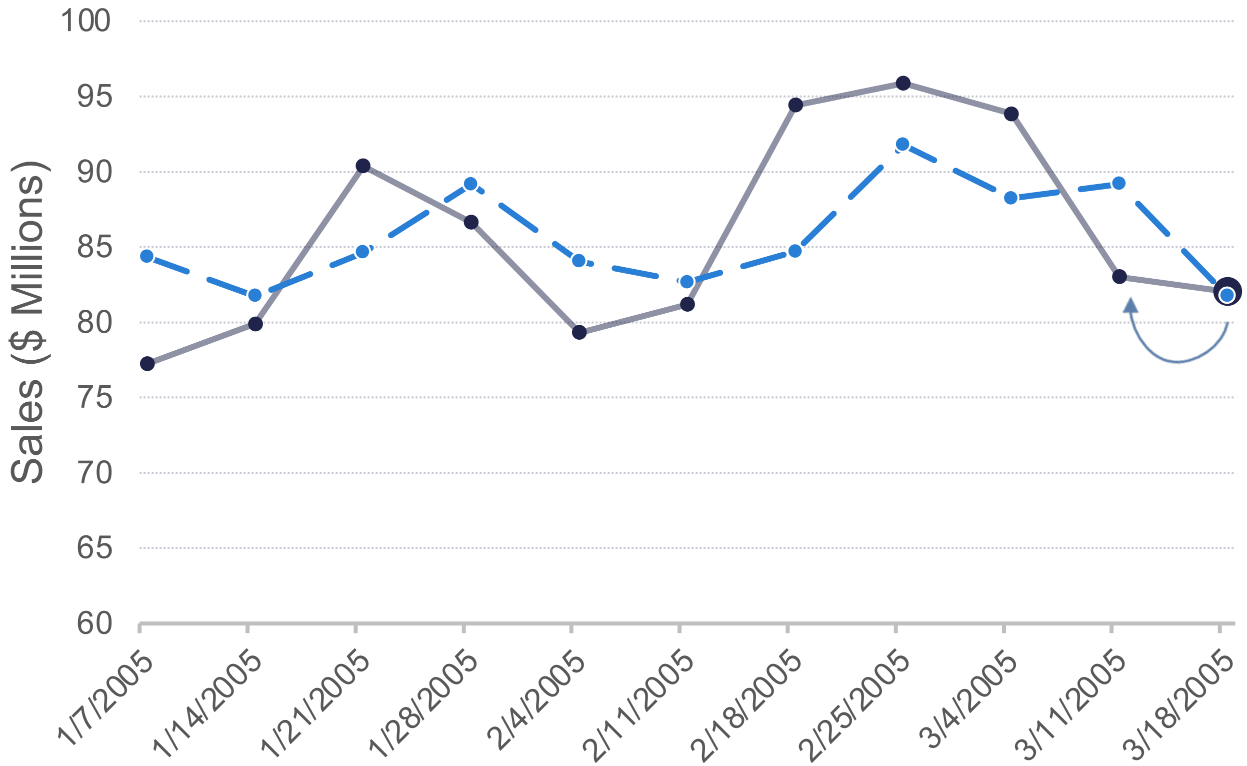

ARIMA Models

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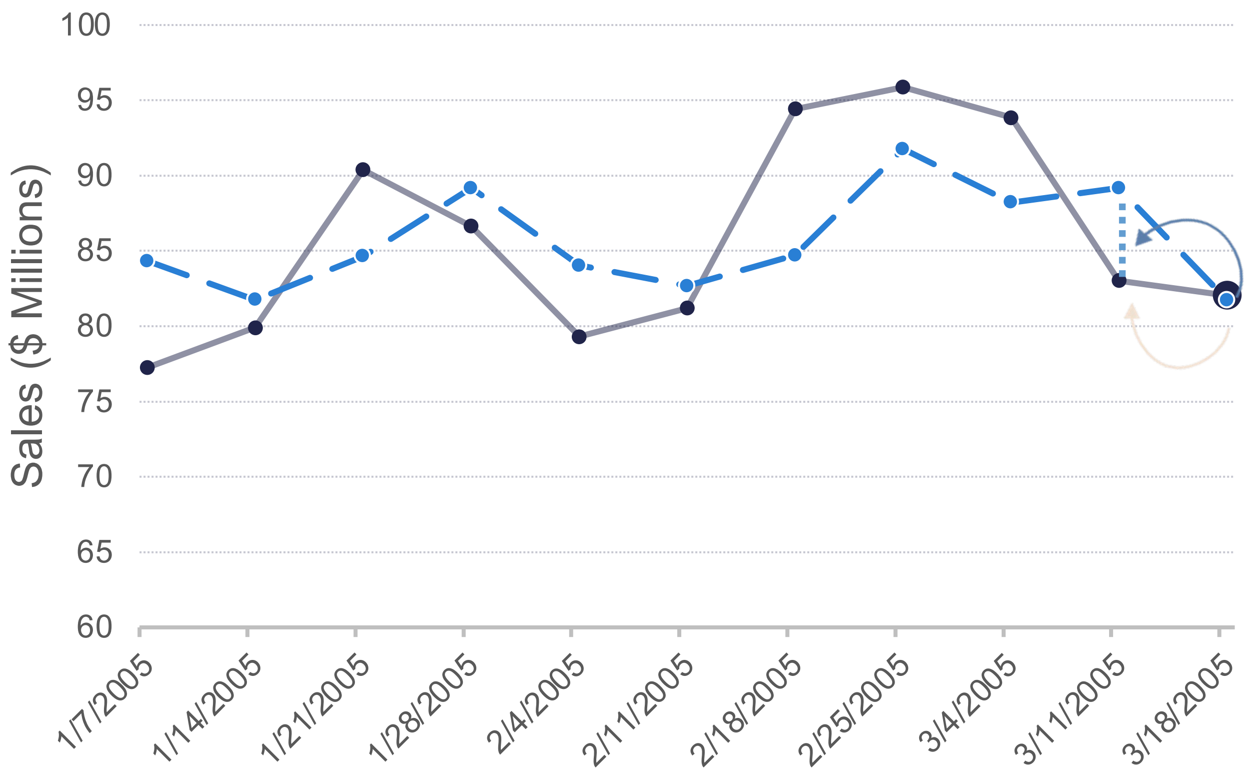


ARIMA Models

- ARIMA Models are typically written as the following:

$$\text{ARIMA}(p, d, q)$$

# of AR Terms

# of MA Terms

# of First Differences

U.S. Airlines Passengers 1990 – 2007

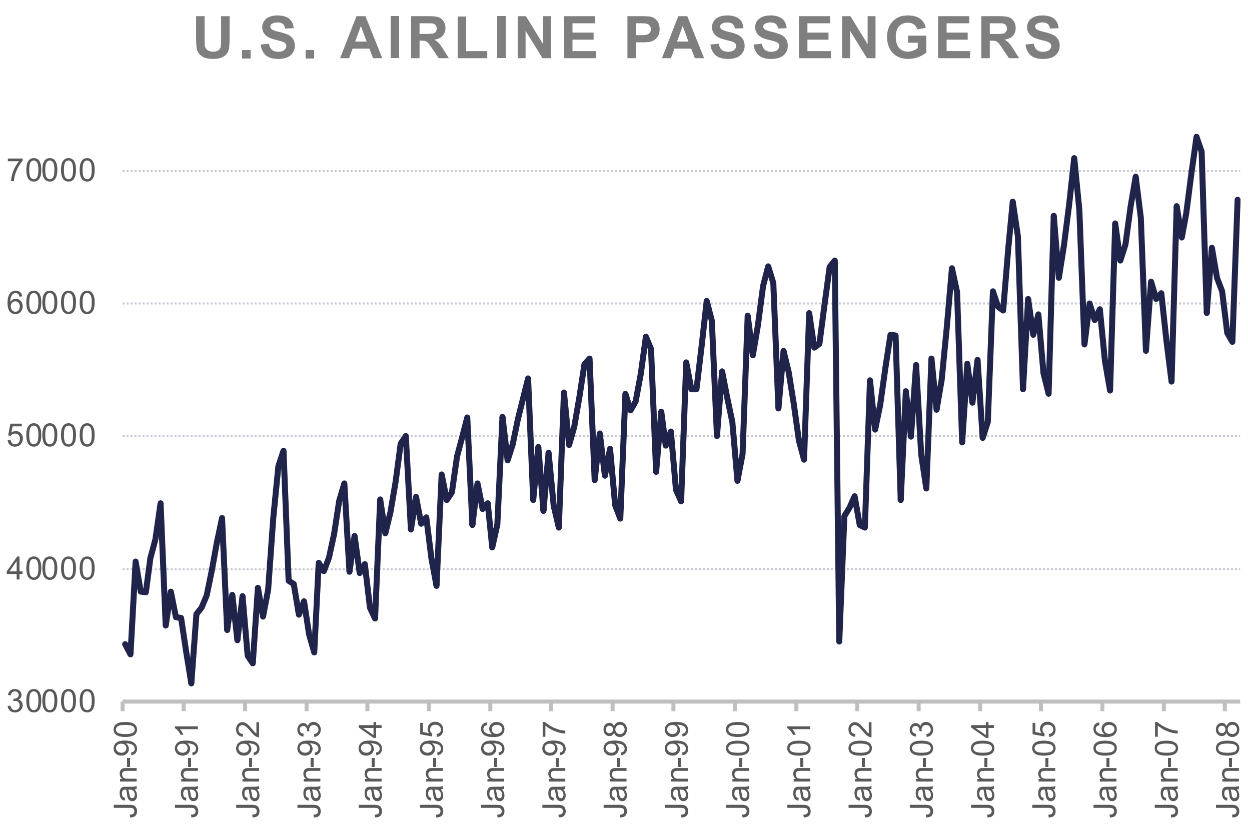
Original Series

=

Trend / Cycle

Season

Error



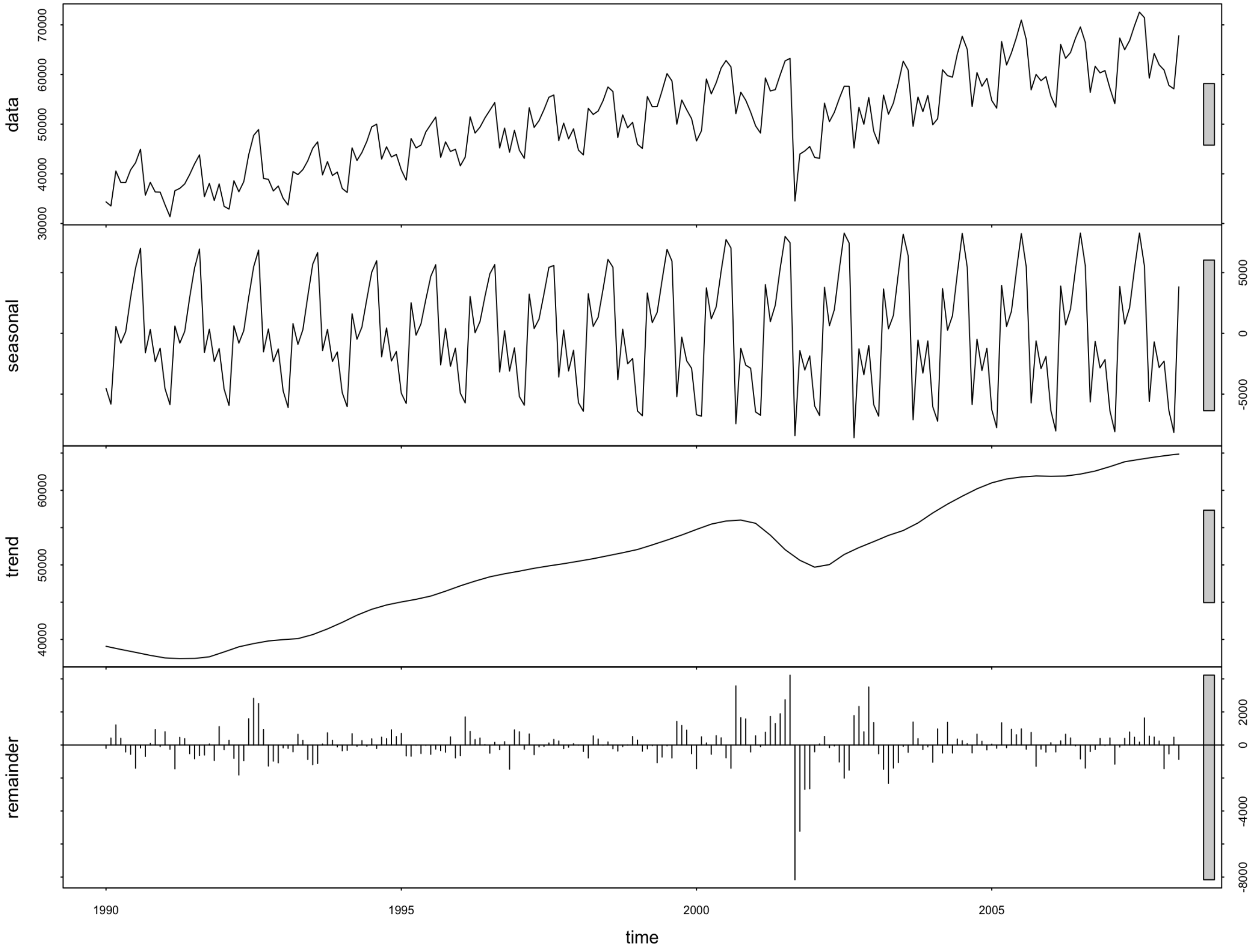
Split into Training and Validation

```
training <- subset(Passenger, end =
length(Passenger)-12)

test <- subset(Passenger, start =
length(Passenger)-11)


decomp_stl <- stl(training,
                  s.window = 7)

plot(decomp_stl)
```



Exponential Smoothing Models

Original Series = $\widehat{Y}_{t+k} = L_t + kT_t + S_{t-p+k}$

Level: $L_t = \theta Y_t + (1-\theta)L_{t-1}$

Trend: $T_t = \gamma(L_t - L_{t-1}) + (1-\gamma)T_{t-1}$

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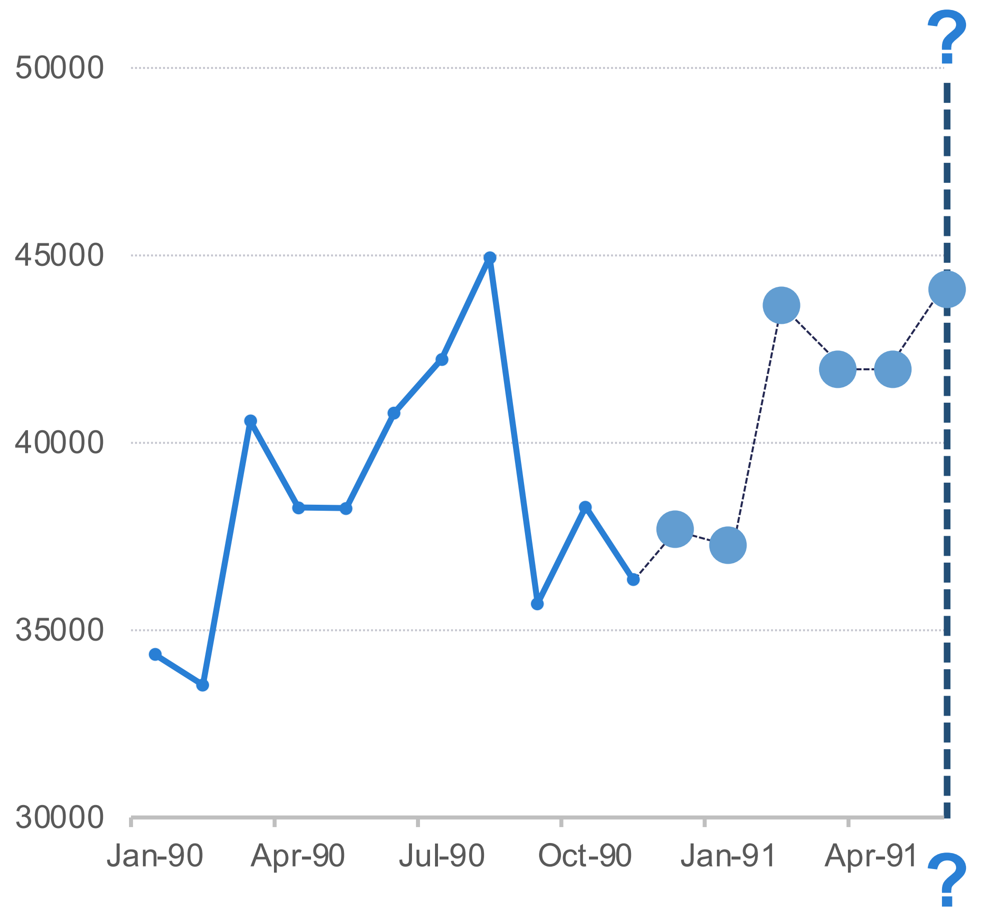


Exponential Smoothing Models

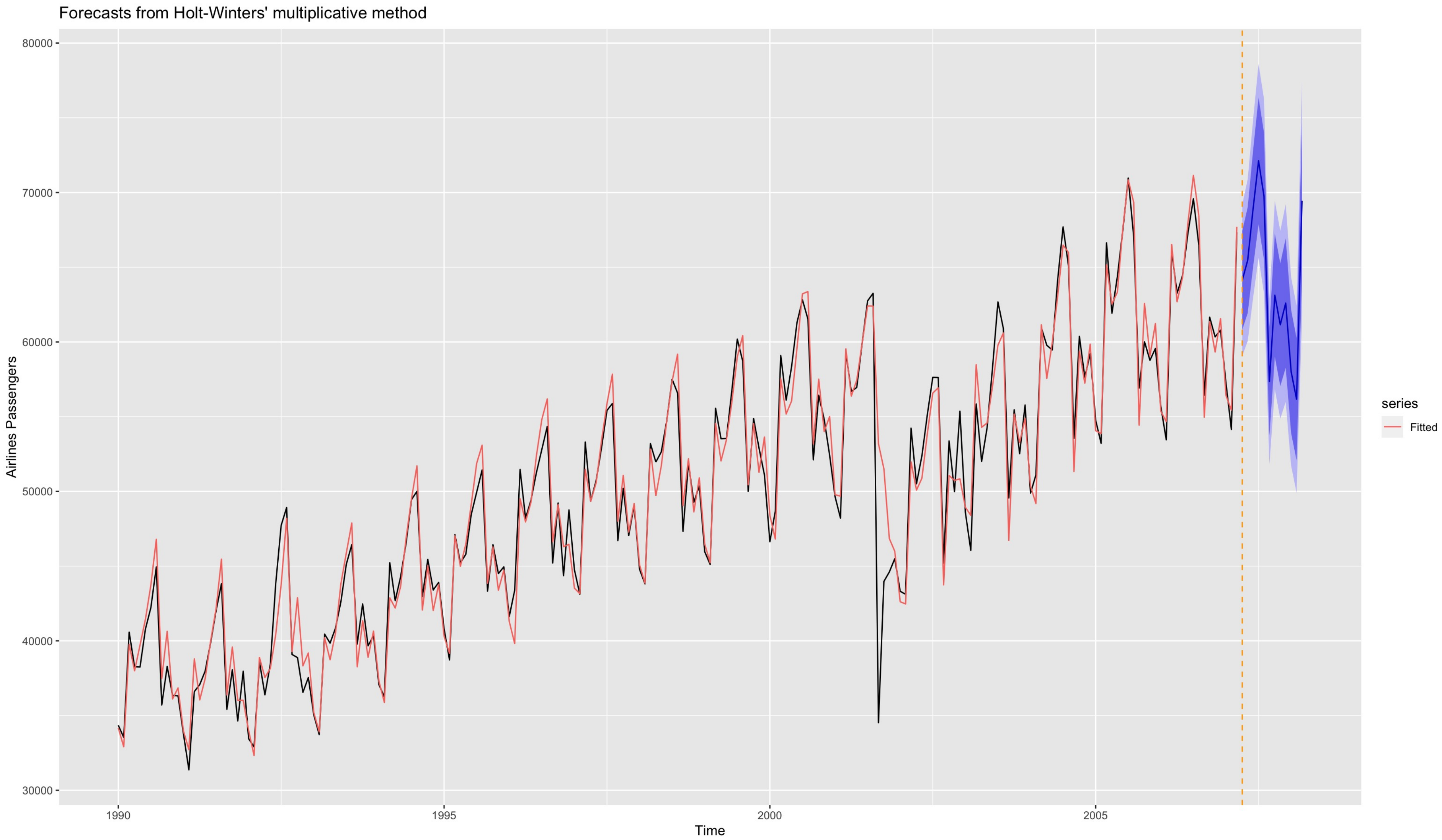
```
HWES.USAir.train <- hw(training, seasonal = "multiplicative", initial='optimal',h=12)

autoplot(HWES.USAir.train) +
  autolayer(fitted(HWES.USAir.train), series="Fitted") +
  ylab("Airlines Passengers") +
  geom_vline(xintercept = 2007.25,color="orange",linetype="dashed")

HW.error <- test - HWES.USAir.train$mean

HW.MAE <- mean(abs(HW.error))
HW.MAPE <- mean(abs(HW.error)/abs(test))*100
```

Exponential Smoothing Models



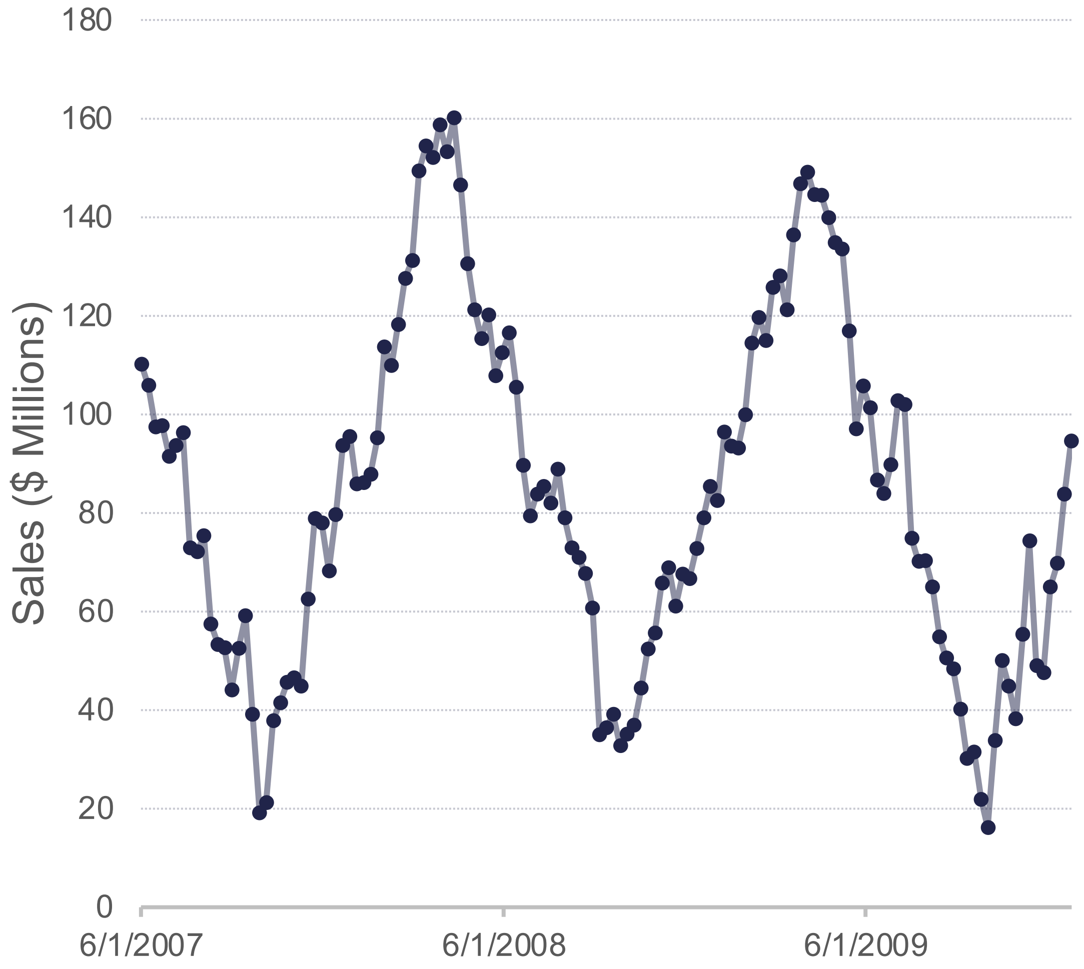
Model Evaluation on Test Data

Model	MAE	MAPE
HW Exponential Smoothing	1134.58	1.76%

SEASONALITY

Seasonality

- Seasonality is the component of time series that represents the effects of seasonal variation.
- Component that describes repetitive behavior known as seasonal periods.
  - Seasonal period = S
  - Seasonal factors repeat every S units of time.



Seasonality and Stationarity

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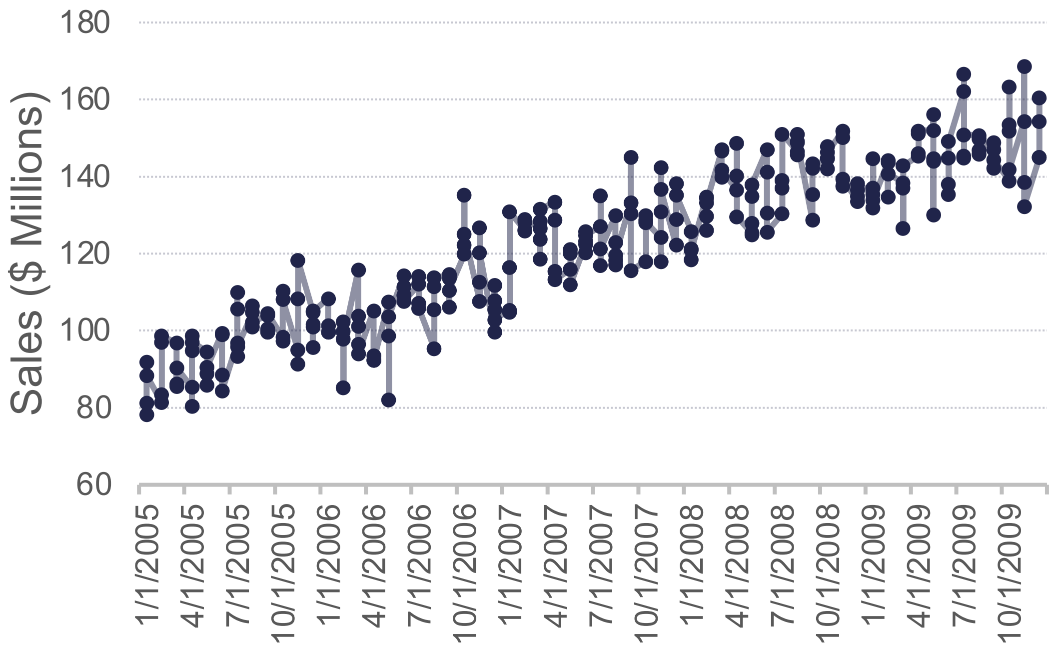
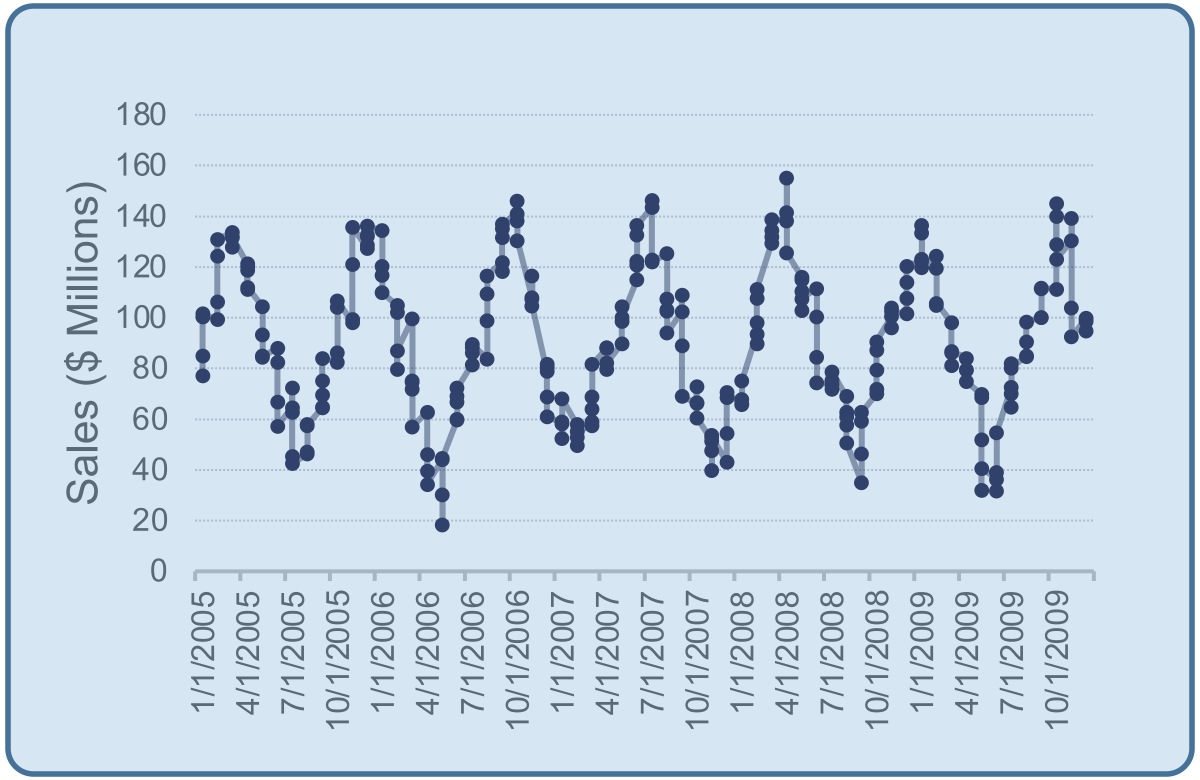
- What about changes in mean – trending, seasonality? **NOT** stationary.





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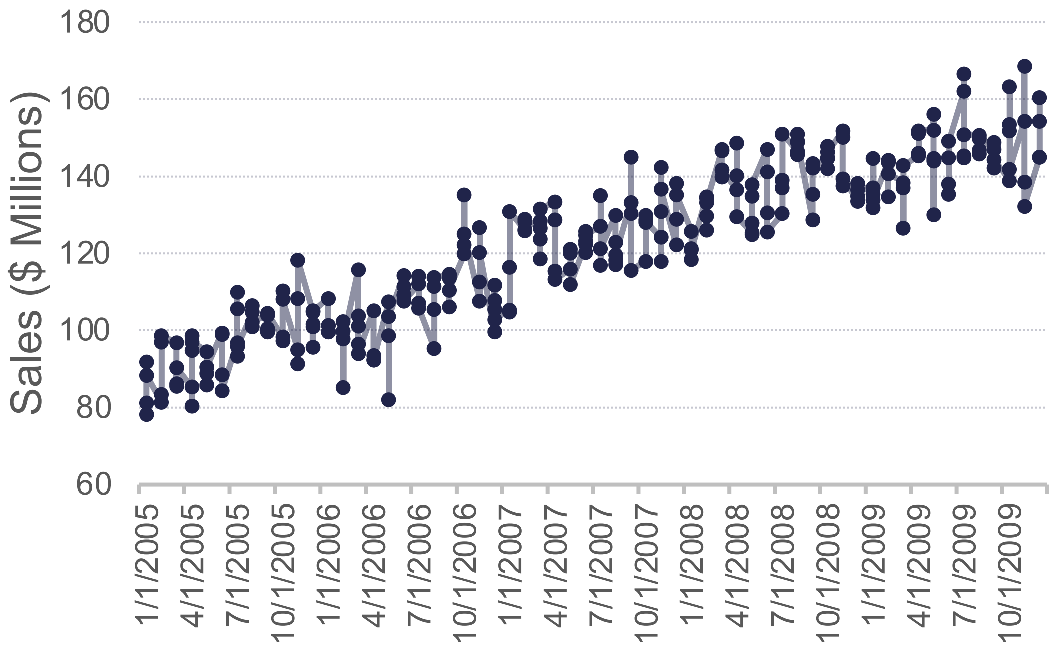
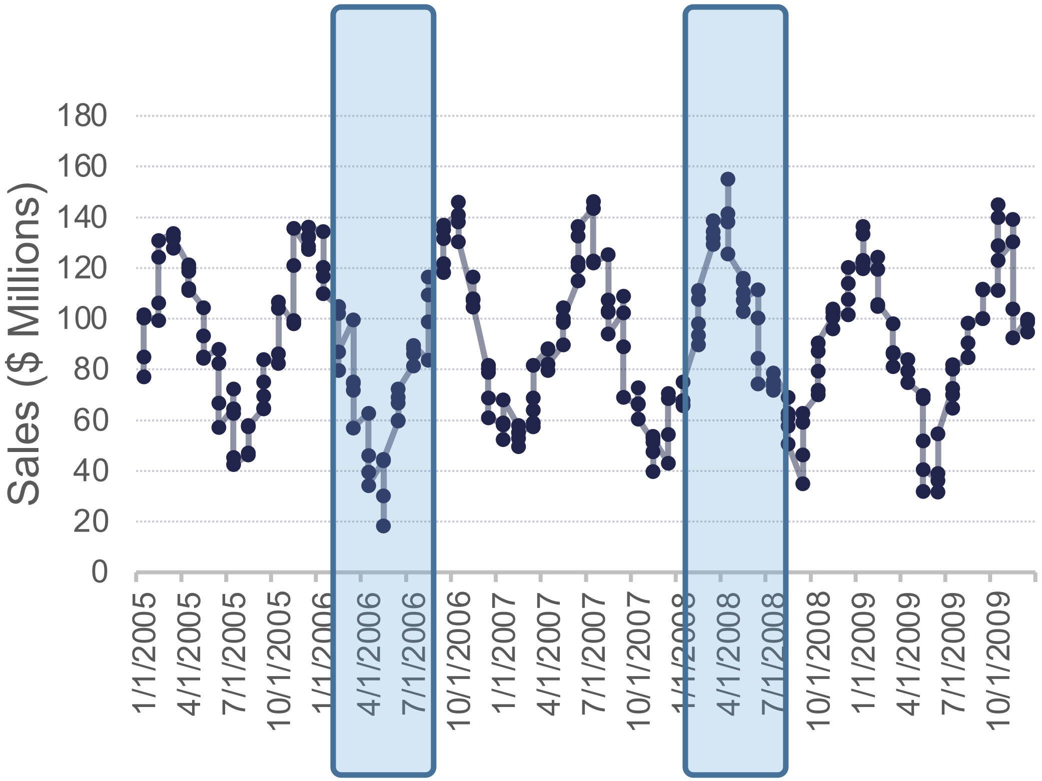
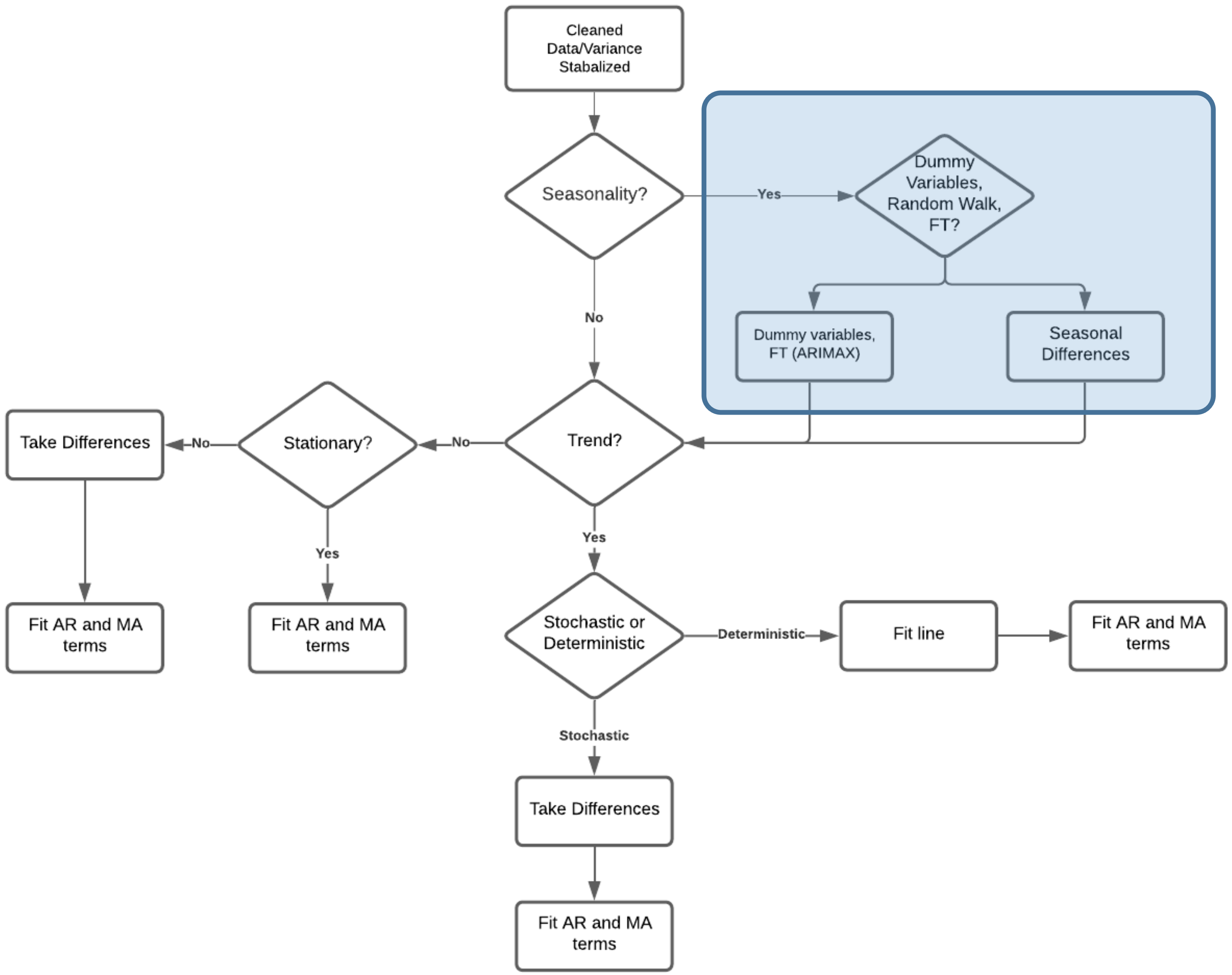




ARIMA Framework



Seasonal ARIMA Models

- Similar to trend, seasonality can be solved with a deterministic solution or a stochastic solution.
  - **Deterministic** – Seasonal dummy variables, Fourier transforms, predictor variables
  - **Stochastic** – Seasonal differences
- Once data is made stationary, we can model with traditional ARIMA approaches.

Seasonal Unit-Root Testing

- Similar to trend, we can perform statistical tests (Canova-Hansen test) for evaluating whether a unit root exists for seasonal data.
- Hypotheses:

$$H_0\text{: Deteministic Seasonality (Differencing not going to help)}$$

$$H_a\text{: Stochastic Seasonality (Differencing needed)}$$

Seasonal Unit-Root Testing

```
training %>% nsdiffs()

## [1] 1
```

Should take one **seasonal** difference

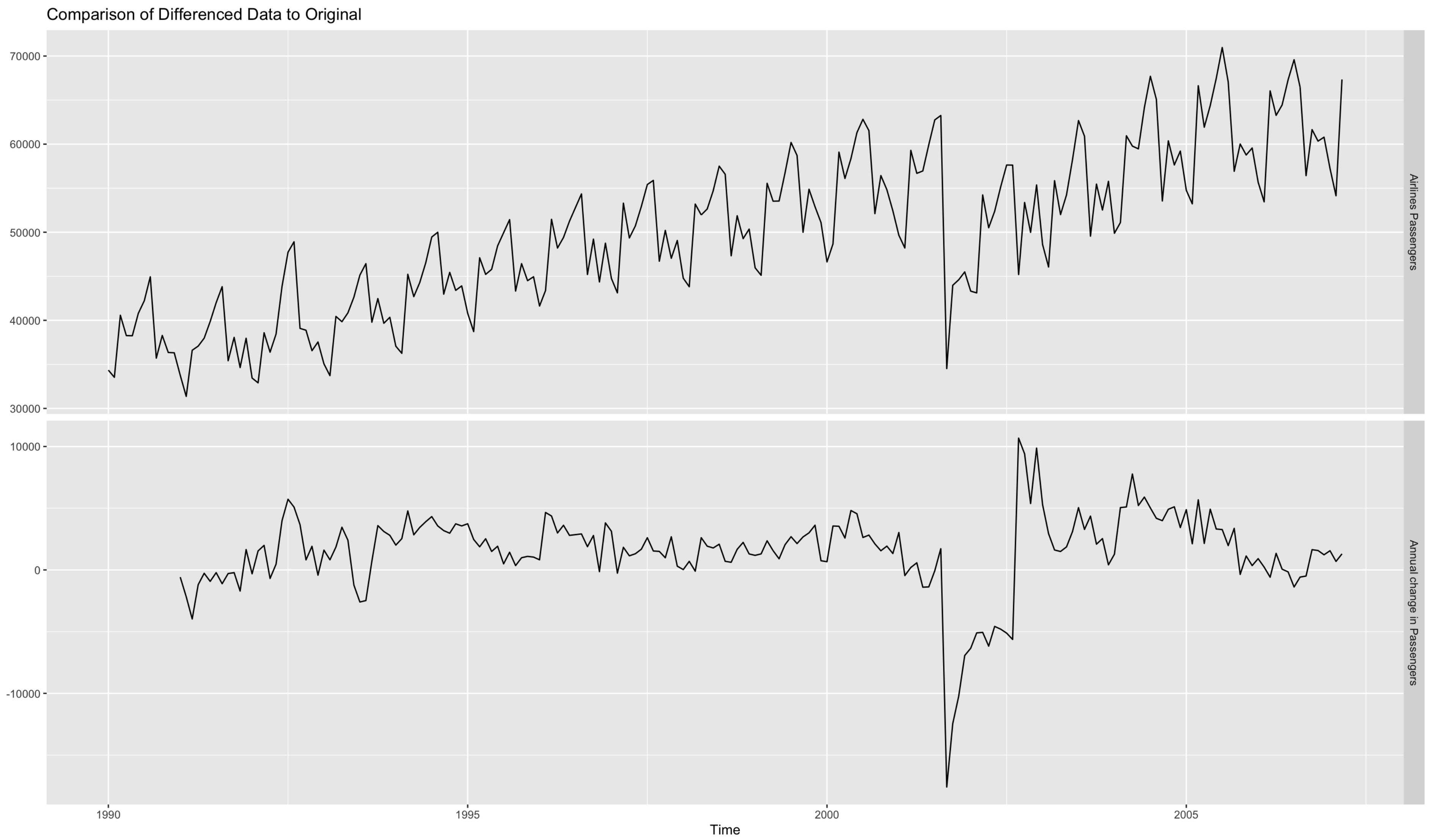
Seasonal Unit-Root Testing

```
training %>% nsdiffs()

## [1] 1
```

Not really good formal tests for seasons beyond 24
(Dickey-Fuller ends at season of 12)

Differenced Data



Unit-Root Testing

```
training %>% diff(lag = 12) %>% ndiffs()
## [1] 0
```

Should take 0 **regular** differences AFTER taking the seasonal difference

DETERMINISTIC SOLUTIONS

Which Deterministic Solution?

- Similar to trend, seasonality can be solved with a deterministic solution or a stochastic solution.
  - **Deterministic** – Seasonal dummy variables, Fourier transforms, predictor variables
  - **Stochastic** – Seasonal differences

- Once data is made stationary (model away the seasonality), we can model with traditional ARIMA approaches.

Seasonal Dummy Variables

- For a time series with *S* periods within a season, there will be *S-1* dummy variables, one for each period (and one accounted for with the intercept).
- Monthly Data:
  - One dummy variable for each month (S = 12)
- Weekly Data:
  - One dummy variable for each day of week (S = 7)
- Hourly Data:
  - One dummy variable for each hour (S = 24)

Seasonal Dummy Variables

- Example model with intercept:

$$Y_t = \beta_0 + \beta_1 JAN + \beta_2 FEB + \cdots + \beta_{11} NOV + e_t$$

$$\beta_0 + \beta_M = \text{effect of M}^{\text{th}} \text{ month}$$

$$\beta_0 = \text{effect of December}$$

Seasonal Dummy Variables

```
Month <- rep(0, length(training))
Month <- Month + 1:12

M <- factor(Month)
M <- relevel(M, ref="12")
```

Seasonal Dummy Variables

```
Season.Lin <- lm(training ~ M)
summary(Season.Lin)
```

```
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  48761.5     2002.3  24.353   <2e-16 ***
## M1           -4461.9     2792.1  -1.598   0.1116
## M2           -5433.9     2792.1  -1.946   0.0531 .
## M3            4099.8     2792.1   1.468   0.1436
## M4             814.9     2831.7   0.288   0.7738
## M5            1951.8     2831.7   0.689   0.4915
## M6            4844.6     2831.7   1.711   0.0887 .
## M7            7504.9     2831.7   2.650   0.0087 **
## M8            7297.4     2831.7   2.577   0.0107 *
## M9           -3242.5     2831.7  -1.145   0.2536
## M10           1064.1     2831.7   0.376   0.7075
## M11          -1268.2     2831.7  -0.448   0.6548
## ---
```

Seasonal Dummy Variables

```
M.Matrix <- model.matrix(~M)

Trend <- 1:length(training)

SD.ARIMA <- auto.arima(training, xreg = M.Matrix[,2:12], method="ML", seasonal = FALSE)
summary(SD.ARIMA)
```

Seasonal Dummy Variables

```
## Series: training
## Regression with ARIMA(1,1,1) errors
##
## Coefficients:
##          ar1      ma1     drift         M1         M2         M3         M4
##       0.4292  -0.7971  120.7148  -3947.9347  -5040.318  4373.0410  1776.4821
## s.e.  0.1142   0.0773   47.0832    485.1706    583.269   625.8909   653.3822
##              M5        M6          M7         M8         M9        M10
##       2774.5010  5539.126  8075.0713  7745.7280  -2913.554  1275.9094
## s.e.   664.7169   667.948   665.0425   655.1154    633.936   589.7366
##              M11
##       -1168.1437
## s.e.    486.9715
##
## sigma^2 estimated as 3751114:  log likelihood=-1844.41
## AIC=3718.82   AICc=3721.34   BIC=3768.74
##
## Training set error measures:
##                      ME     RMSE      MAE        MPE     MAPE      MASE
## Training set -13.60989 1865.287 1119.089 -0.1822502 2.399032 0.4156601
##                      ACF1
## Training set -0.002860136
```

Advantages and Disadvantages

Advantages

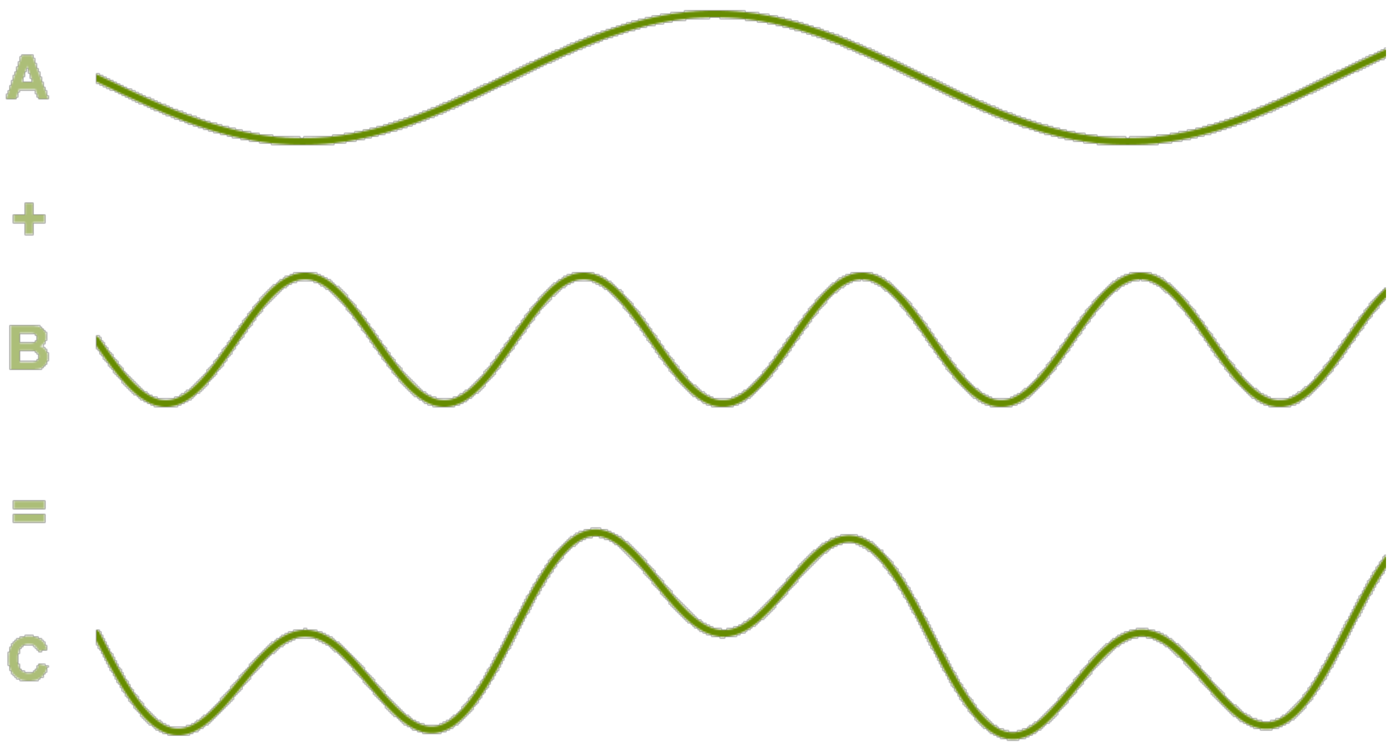
- Interpretation still holds.
  - Can easily measure and interpret effects from different parts of the season.
- Straight forward to implement.

Disadvantages

- Especially long or complex seasons are hard to deal with.
  - More than 24 periods in a season (365 days in year for example) is burdensome.
  - Some seasons are complex (365.25 days in a year, 52.17 weeks in a year, etc.).
- Seasonal effects remain constant.

Fourier Transforms (Harmonic Regression)

- Fourier showed that series of sine and cosine terms of the right frequencies approximate periodic series.



Fourier Transforms (Harmonic Regression)

- Add Fourier variables to a regression model predicting the target to remove the seasonal pattern.

$$X_{1,t} = \sin\left(\frac{2\pi t}{S}\right) \qquad X_{3,t} = \sin\left(2\times\frac{2\pi t}{S}\right) \qquad X_{5,t} = \sin\left(3\times\frac{2\pi t}{S}\right) \qquad \ldots$$

$$X_{2,t} = \cos\left(\frac{2\pi t}{S}\right) \qquad X_{4,t} = \cos\left(2\times\frac{2\pi t}{S}\right) \qquad X_{6,t} = \cos\left(3\times\frac{2\pi t}{S}\right) \qquad \ldots$$

$$Y_t = \beta_0 + \beta_1 X_{1,t} + \beta_2 X_{2,t} + \beta_3 X_{3,t} + \beta_4 X_{4,t} + \cdots + e_t$$

Fourier Transforms (Harmonic Regression)

- Add Fourier variables to a regression model predicting the target to remove the seasonal pattern.
- If you add the same number of Fourier variables as you have seasonal dummy variables, you will get the same predictions.
- However, typically do not need all the Fourier variables → especially with large values of *S*.

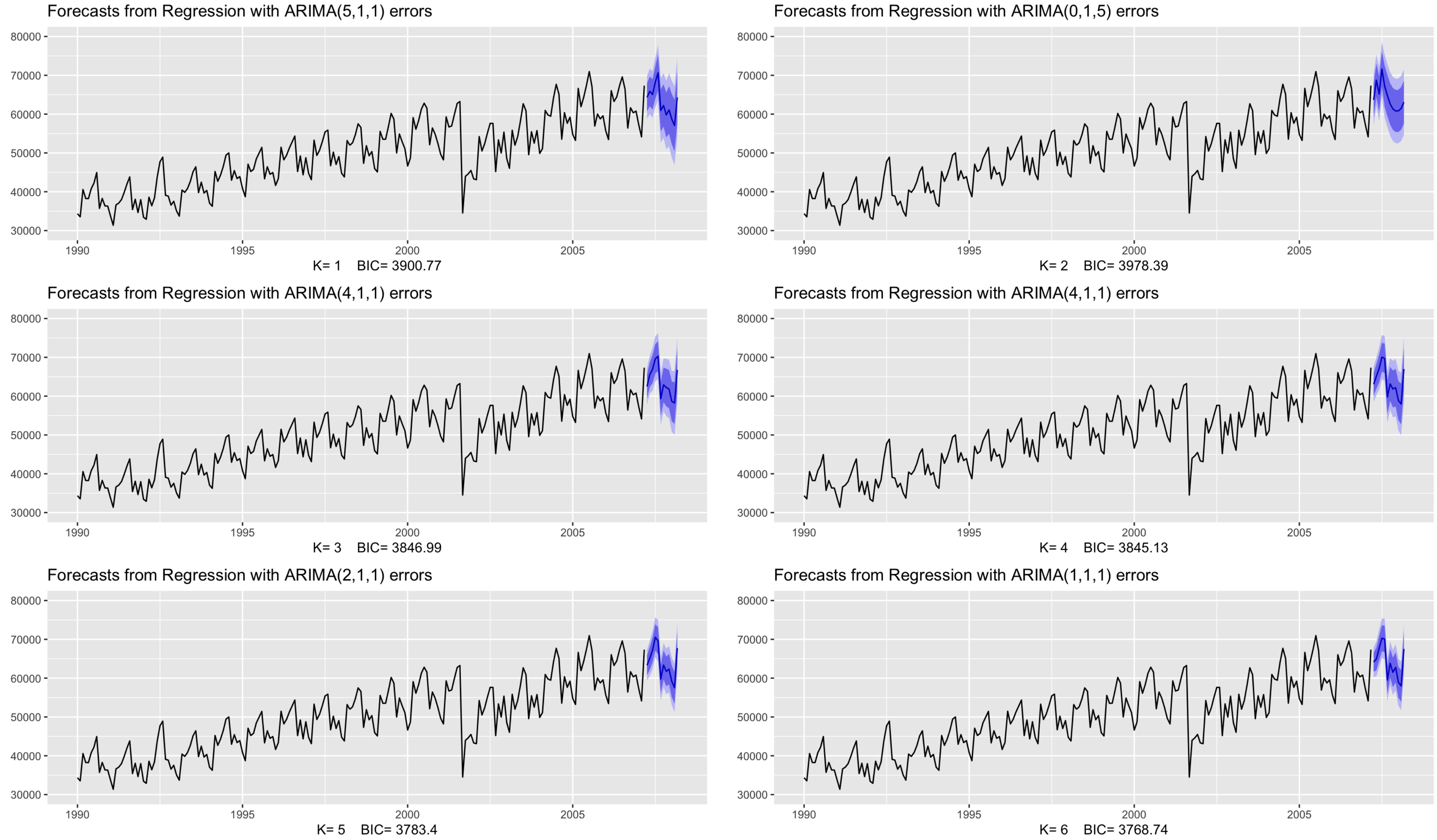
Fourier Transforms (Harmonic Regression)

```r
plots <- list()
for (i in seq(6)) {
  fit <- auto.arima(training, xreg = fourier(training, K = i),
                    seasonal = FALSE, lambda = NULL)

  plots[[i]] <- autoplot(forecast::forecast(fit,
                         xreg = fourier(training, K=i, h=12))) +
    xlab(paste("K=",i,"   BIC=",round(fit[["bic"]],2))) +
    ylab("") + ylim(30000,80000)
}

gridExtra::grid.arrange(
  plots[[1]],plots[[2]],plots[[3]],
  plots[[4]],plots[[5]],plots[[6]], nrow=3)
```

Fourier Transforms (Harmonic Regression)



Fourier Transforms (Harmonic Regression)

```
F.ARIMA <- auto.arima(training, xreg = fourier(training, K = 6), seasonal = FALSE)
```

```
## Series: training
## Regression with ARIMA(1,1,1) errors
##
## Coefficients:
##          ar1      ma1     drift      S1-12      C1-12     S2-12     C2-12
##       0.4289  -0.7970  120.6974  -1232.3084  -4334.8049  313.9197  677.9491
## s.e.  0.1142   0.0773   47.1030    270.2688    270.3805  194.1284  193.6296
##             S3-12      C3-12     S4-12     C4-12      S5-12     C5-12     C6-12
##       -2561.1131  1291.3900  413.8895  208.6503  2314.3804  274.0798  341.8763
## s.e.    152.7962   153.1557  130.4249  130.6610   118.9888  119.5082   81.8796
##
## sigma^2 estimated as 3751119:  log likelihood=-1844.41
## AIC=3718.82   AICc=3721.34   BIC=3768.74
##
## Training set error measures:
##                     ME     RMSE      MAE        MPE     MAPE      MASE
## Training set -13.55273 1865.288 1119.117 -0.1820955 2.399079 0.4156704
##                      ACF1
## Training set -0.002801581
```

Advantages and Disadvantages

Advantages

- Can handle long and complex seasonality.
  - If multiple seasons, just add more Fourier variables to account for them.

Disadvantages

- Trial and error for “right” amount of Fourier variables to use.
- No interpretable value.
- Effect of season remains constant.

Predictor Variables for Seasonality

- Last common approach to accounting for seasonality in data is to use other predictor variables that have matching season.
- Modeling these variables against the target might remove the seasonality.
- Example: Weather data and energy data
  - Hourly temperature correlates with hourly energy usage in the summer months (high heat → high energy usage)
  - Have same 24 hour cycle

Advantages and Disadvantages

Advantages

- Can handle long and complex seasonality.
  - If multiple seasons, just add more variables to account for them.
- Interpretation still holds.
  - Can easily measure and interpret effects from these variables.

Disadvantages

- Trial and error for “right” variables to use.
- Might not have predictor variables to use in this context.

What Next?

- After removing the seasonality through deterministic approaches, the remaining error term (residuals) are modeled with Seasonal ARIMA models.

$$Y_t = \beta_0 + \beta_1 X_{1,t} + \beta_2 X_{2,t} + \beta_3 X_{3,t} + \beta_4 X_{4,t} + \cdots + e_t$$

Seasonal ARIMA here!

- Still might need seasonal effects even though season is removed.

STOCHASTIC SOLUTION (DIFFERENCING)

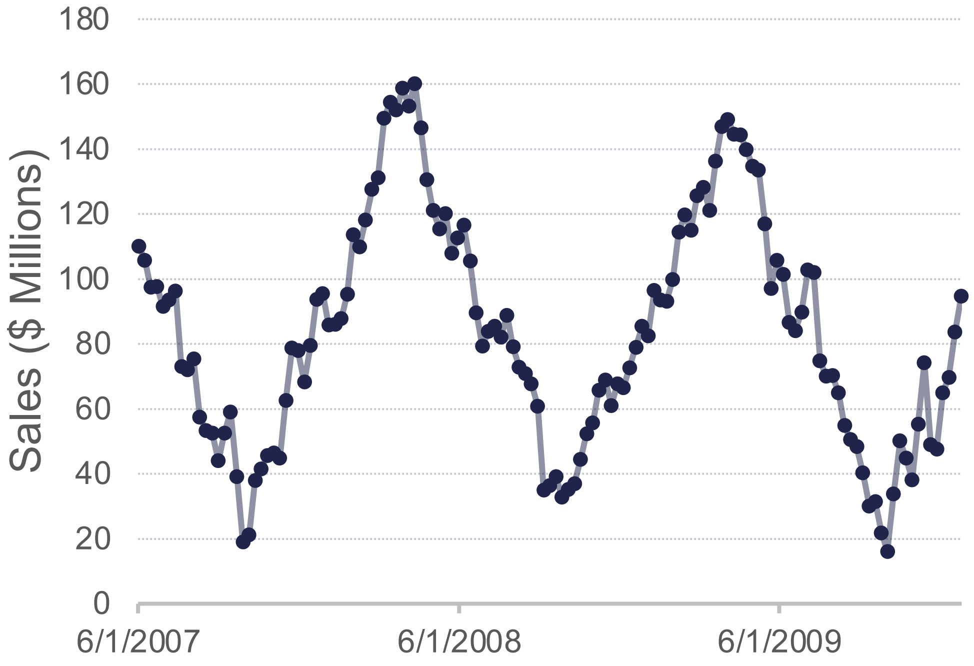
Stochastic Solution

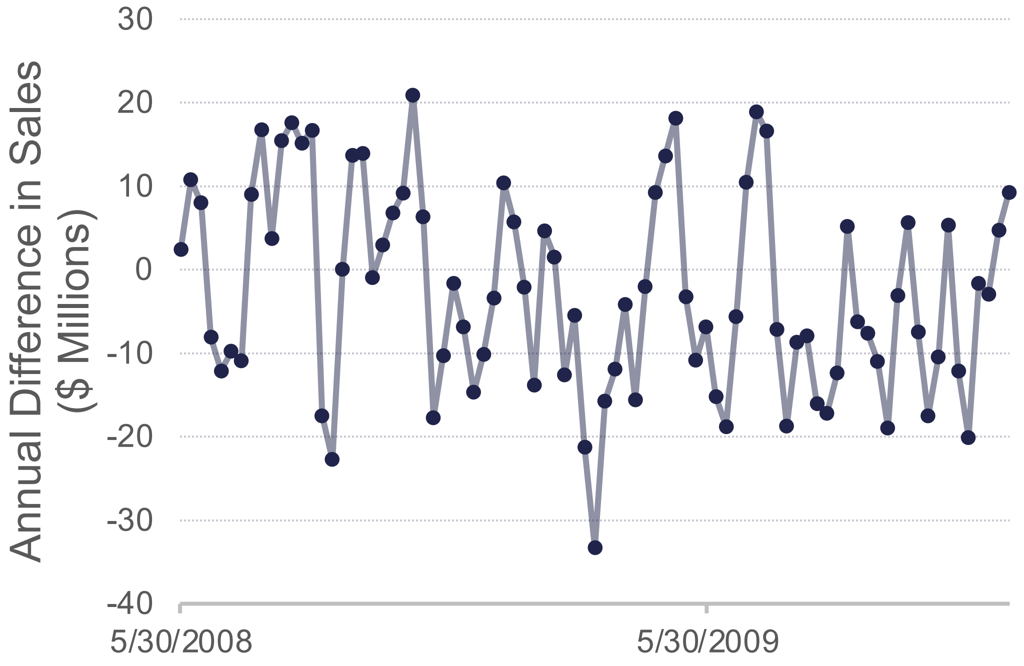
- Similar to trend, seasonality can be solved with a deterministic solution or a stochastic solution.
  - **Deterministic** – Seasonal dummy variables, Fourier transforms, predictor variables
  - **Stochastic** – Seasonal differences

- Once data is made stationary (model away the seasonality), we can model with traditional ARIMA approaches.

Seasonal Differencing

- Differencing on season → look at difference between current point and the same point in the previous season: $Y_t - Y_{t-S}$





What Next?

- After removing the seasonality through stochastic approaches, the remaining differences are modeled with Seasonal ARIMA models.

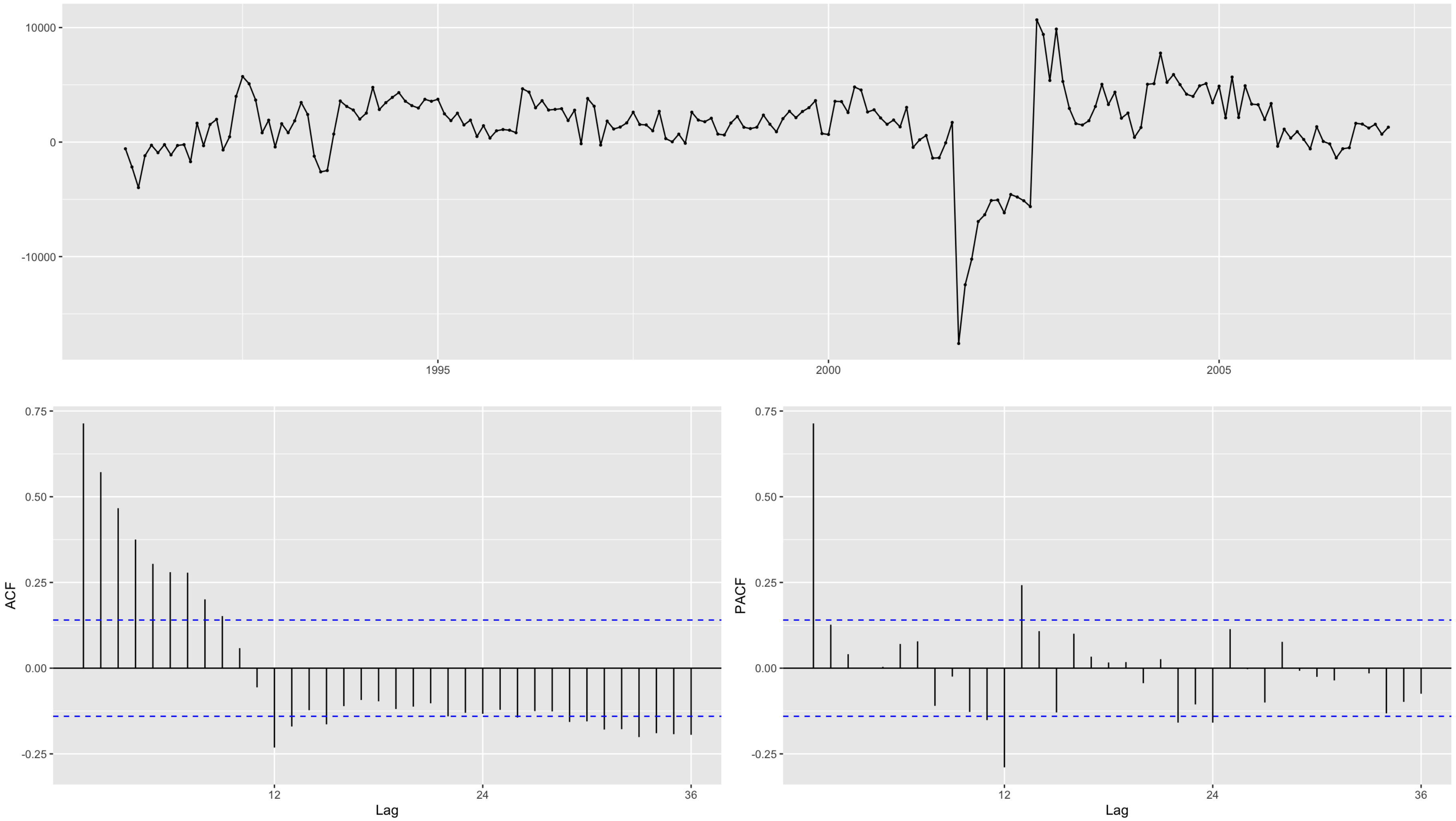
$$Y_t - Y_{t-s} = W_t$$

Seasonal ARIMA here!

- Still might need seasonal effects even though season is removed.

Seasonal Differencing

```
training %>% diff(lag = 12) %>% ggtsdisplay()
```



Limitations of Differencing

- Hard to evaluate stochastic effects for long and complex seasons.
- Most statistical tests for stochastic vs. deterministic can not handle past 12 or 24 periods in a season.
- Long/complex seasons → Best to just approach with deterministic solutions

SEASONAL ARIMA

More Complex ARIMA

- When extending to the Seasonal ARIMA framework, we add another set of terms – *P, D, Q*, and *S*.

$$\mathrm{ARIMA}(p, d, q)(P, D, Q)_S$$

# of AR Terms

# of MA Terms

# of First Differences

More Complex ARIMA

- When extending to the Seasonal ARIMA framework, we add another set of terms – *P, D, Q*, and *S*.

$$\mathrm{ARIMA}(p,d,q)(P,D,Q)_S$$

Length of Season

# of Seasonal AR Terms

# of Seasonal Differences

# of Seasonal MA Terms

Seasonal ARIMA

- **Seasonal** ARIMA models are typically written as the following:

$$\text{ARIMA}(1,0,1)(2,1,0)_{12}$$

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Seasonal ARIMA

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$$\text{ARIMA}(1,0,1)(2,1,0)_{12}$$

$$Y_t - Y_{t-12} = W_t$$

$$W_t = \omega + \phi_1 W_{t-1} + \phi_2 W_{t-12} + \phi_3 W_{t-24} + \theta_1 e_{t-1} + e_t$$

Seasonal ARIMA

- **Seasonal** ARIMA models are typically written as the following:

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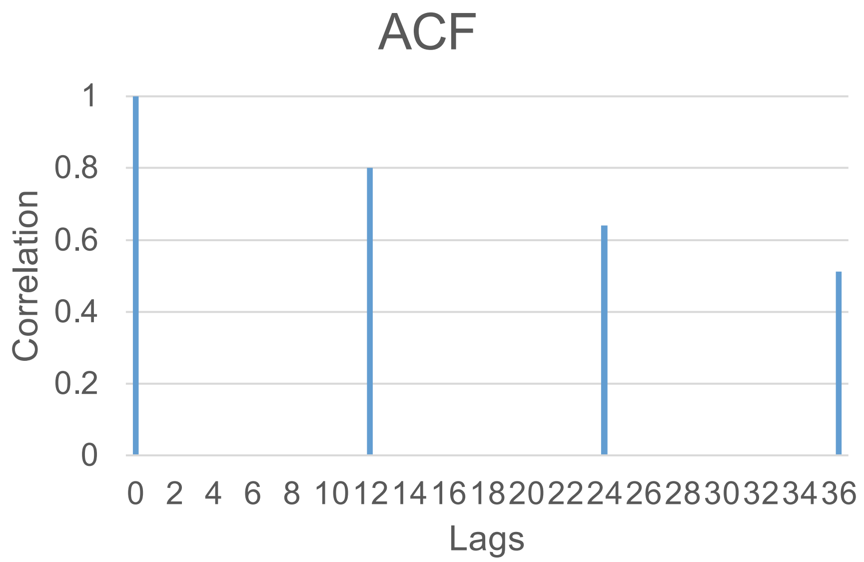
$$Y_t - Y_{t-12} = W_t$$

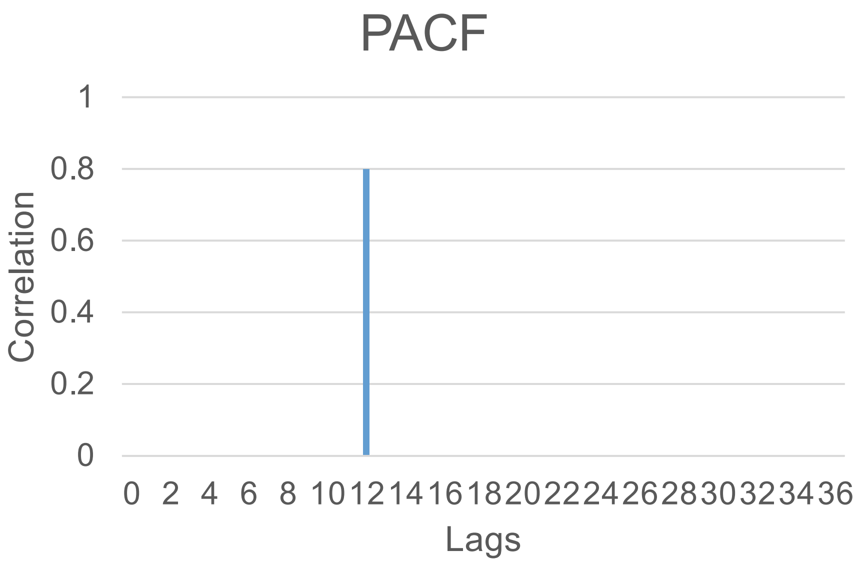
$$W_t = \omega + \phi_1 W_{t-1} + \phi_2 W_{t-12} + \phi_3 W_{t-24} + \theta_1 e_{t-1} + e_t$$

Seasonal ARIMA

- Seasonal ARIMA models have the same structure and approach as typical ARIMA models with AR and MA patterns in the PACF and ACF.
- The pattern is just on the seasonal lag instead of the individual lags.

$$\mathrm{ARIMA}(0,0,0)(1,0,0)_{12}$$





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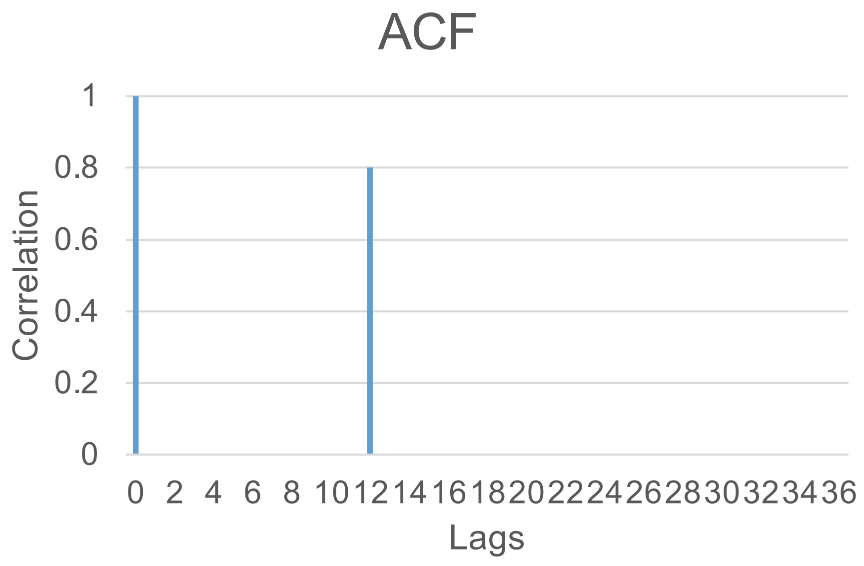
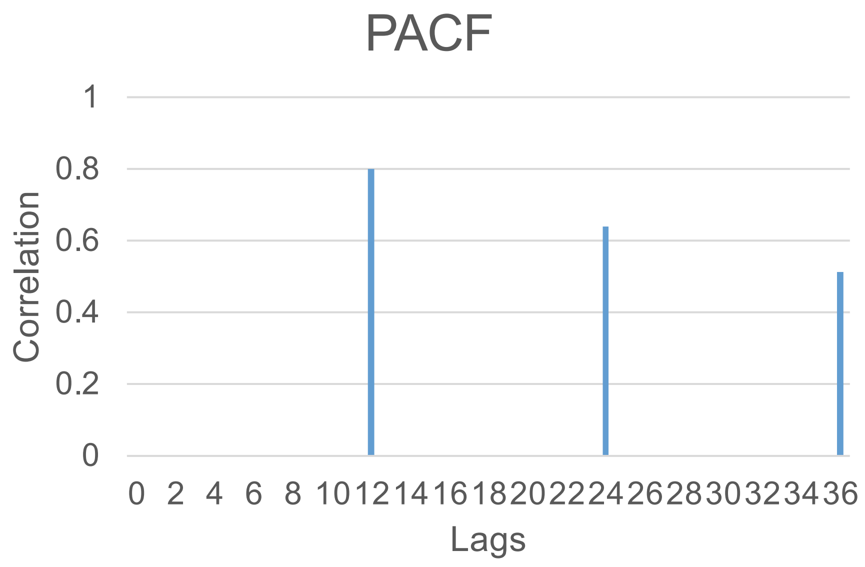
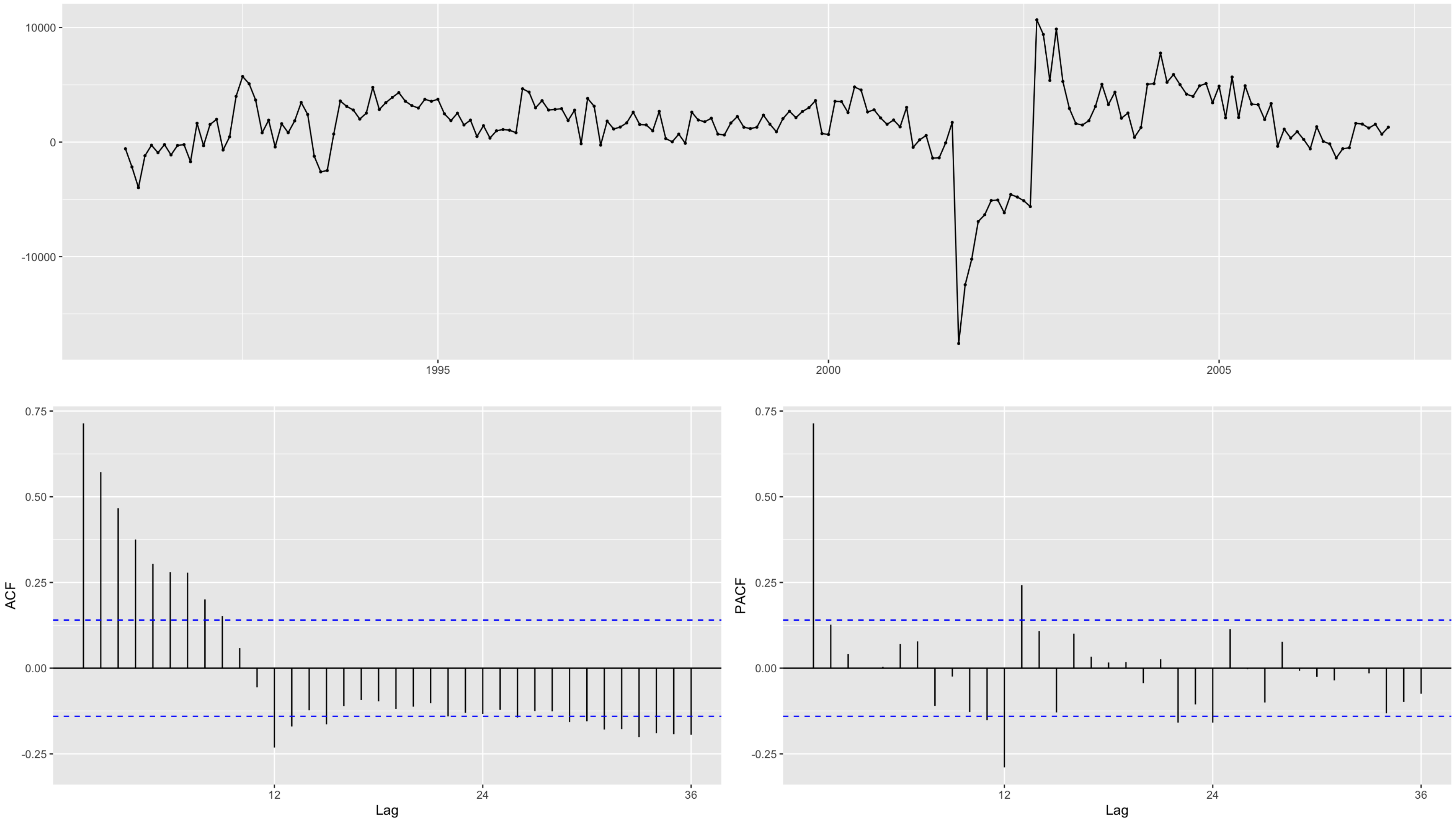
$$\text{ARIMA}(0,0,0)(0,0,1)_{12}$$





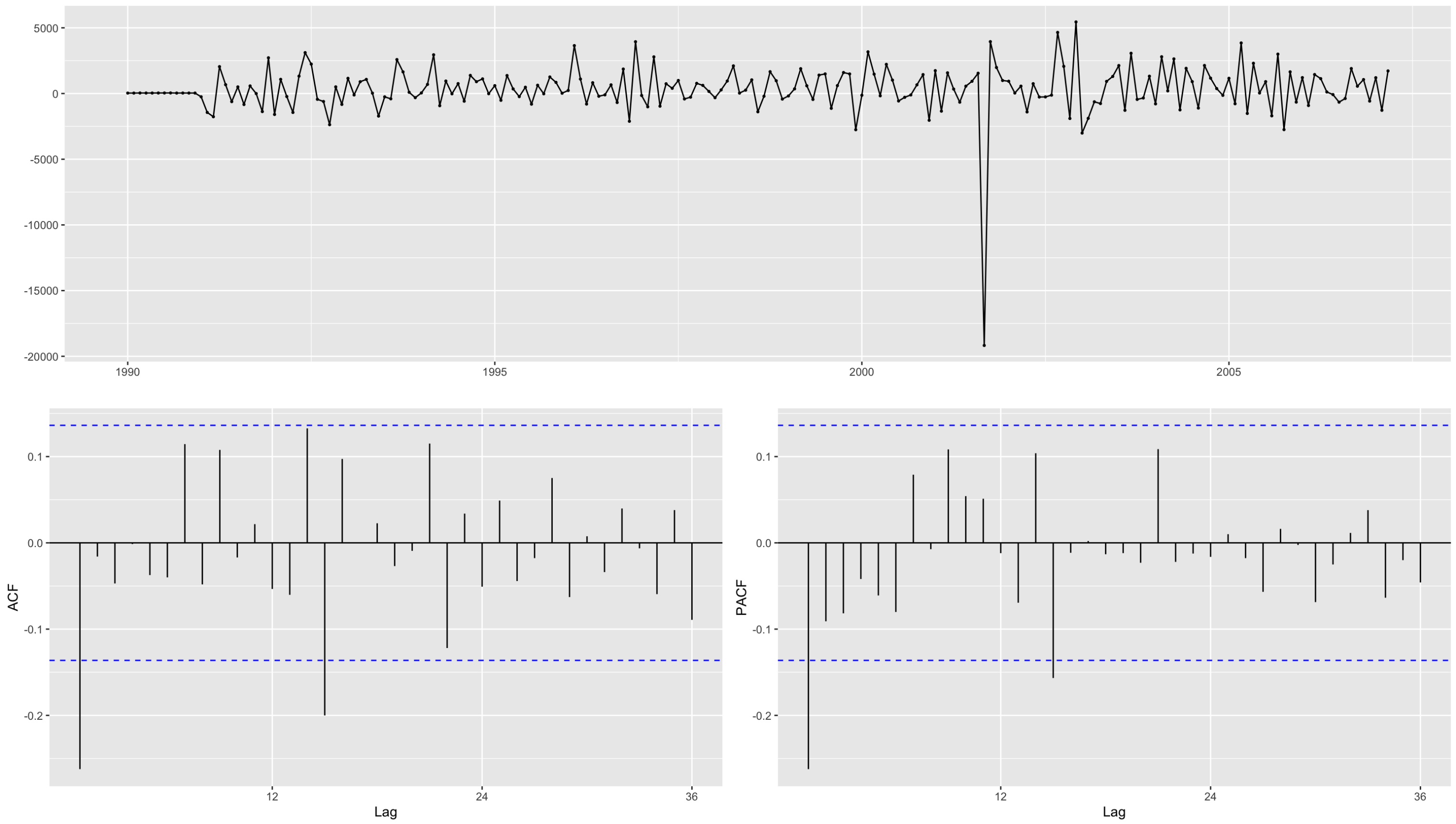
Seasonal Differencing

```
training %>% diff(lag = 12) %>% ggtsdisplay()
```



Seasonal ARIMA

```
training %>%
  Arima(order=c(1,0,0), seasonal=c(1,1,1)) %>%
  residuals() %>% ggtsdisplay()
```



Seasonal ARIMA

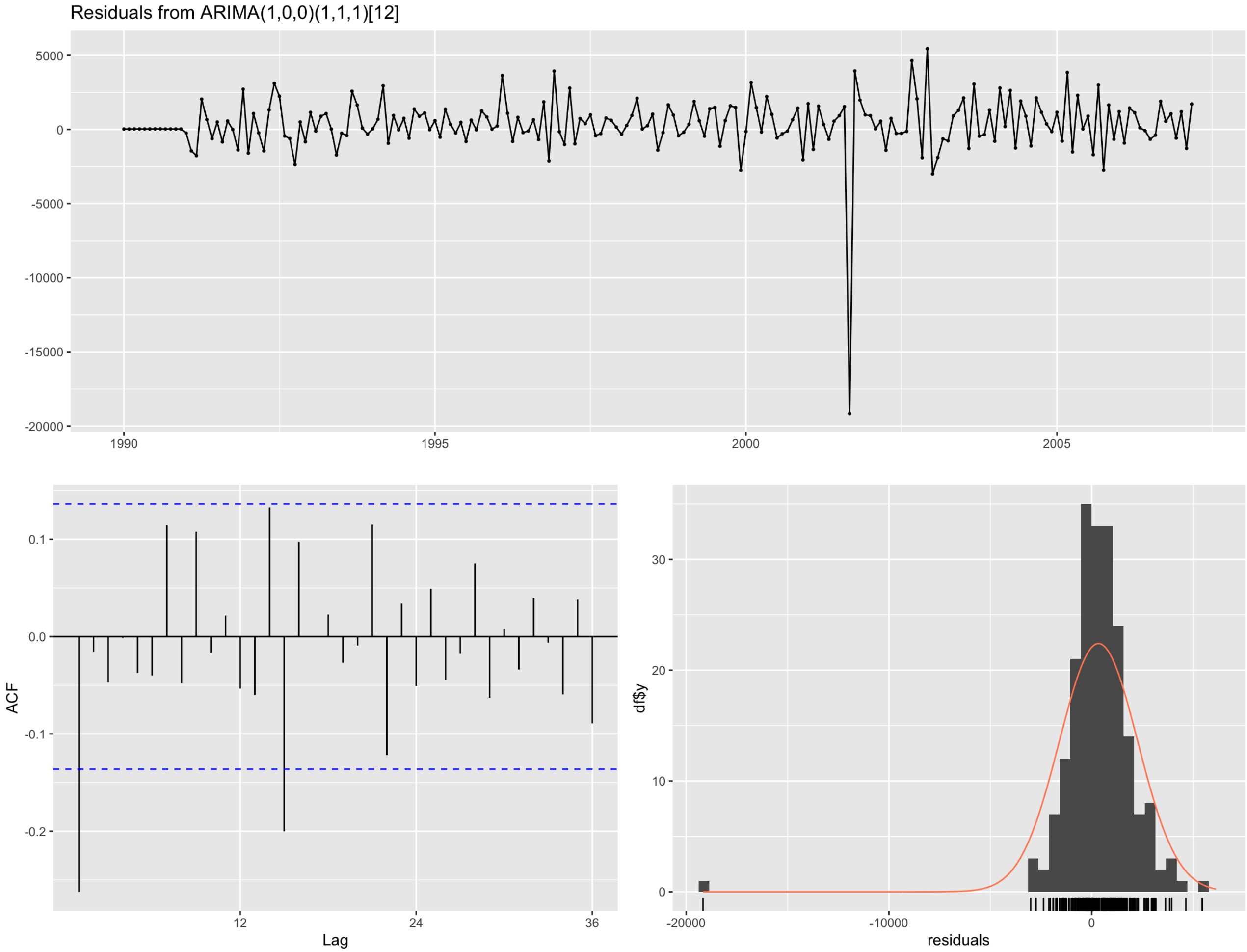
```
S.ARIMA <- Arima(training, order=c(1,0,0), seasonal=c(1,1,1))
summary(S.ARIMA)
```

```
## Series: training
## ARIMA(1,0,0)(1,1,1)[12]
##
## Coefficients:
##          ar1    sar1    sma1
##       0.9056  0.0917  -0.672
## s.e.  0.0364  0.1091   0.093
##
## sigma^2 estimated as 4126436:  log likelihood=-1763.94
## AIC=3535.88   AICc=3536.09   BIC=3548.97
##
## Training set error measures:
##                    ME     RMSE      MAE       MPE     MAPE      MASE       ACF1
## Training set 338.6503 1956.379 1156.221 0.5565257 2.418163 0.4294517 -0.2622466
```

Seasonal ARIMA

```
checkresiduals(S.ARIMA)
```

```
##  Ljung-Box test
##
## data:  Residuals from ARIMA(1,0,0)(1,1,1)[12]
## Q* = 45.934, df = 21, p-value = 0.001304
##
## Model df: 3.   Total lags used: 24
```



Seasonal ARIMA

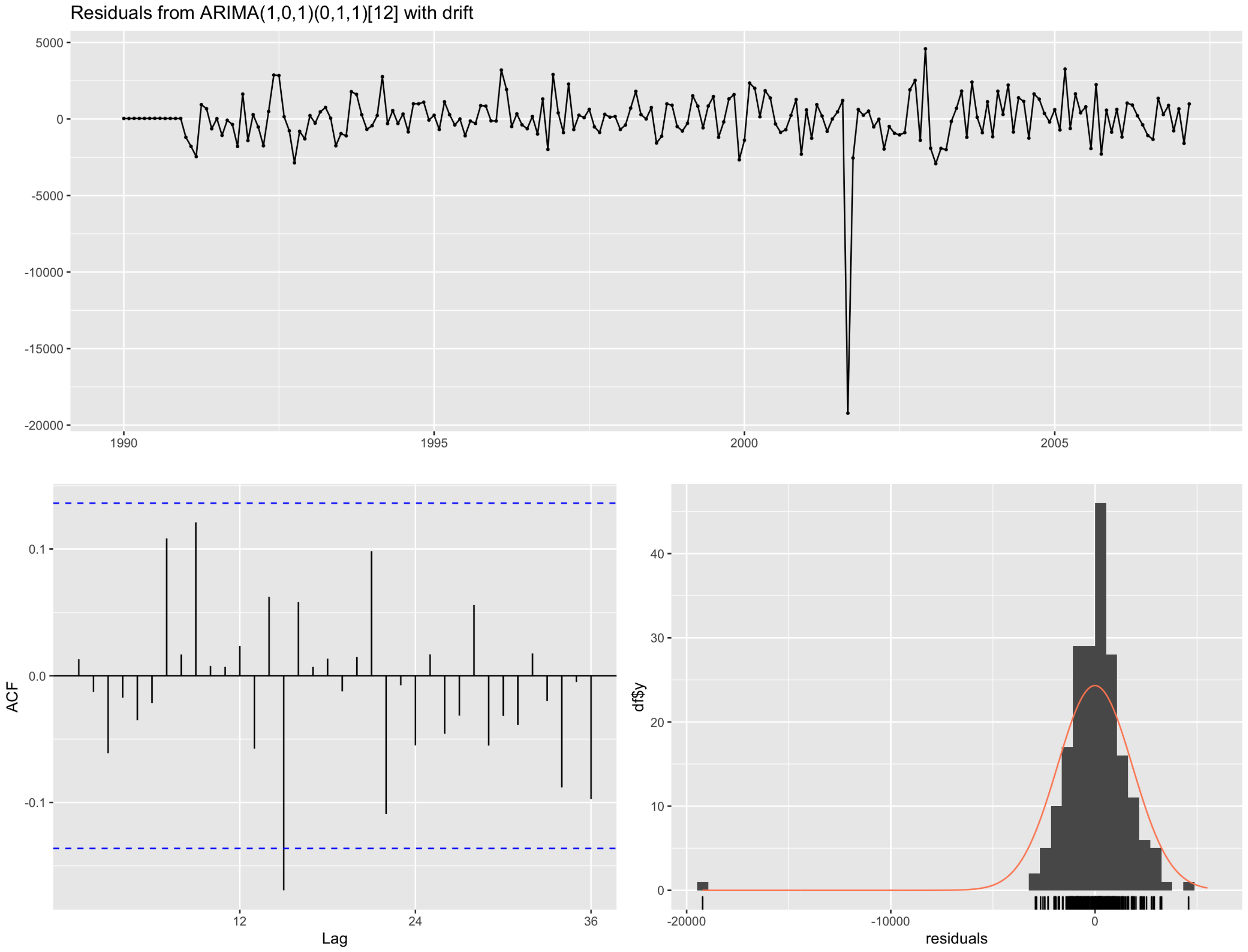
```
S.ARIMA <- auto.arima(training, method="ML", seasonal = TRUE)
summary(S.ARIMA)
```

```
## Series: training
## ARIMA(1,0,1)(0,1,1)[12] with drift
##
## Coefficients:
##          ar1      ma1     sma1     drift
##       0.8800  -0.2962  -0.6785  124.9788
## s.e.  0.0454   0.0950   0.0600   23.6330
##
## sigma^2 estimated as 3639517:  log likelihood=-1751.67
## AIC=3513.34   AICc=3513.66   BIC=3529.7
##
## Training set error measures:
##                       ME     RMSE      MAE        MPE      MAPE      MASE       ACF1
## Training set -4.332616 1832.54 1055.07 -0.1745474 2.217472 0.3918815 0.01300462
```

Seasonal ARIMA

```
checkresiduals(S.ARIMA)
```

```
##  Ljung-Box test
##
## data:  Residuals from ARIMA(1,0,1)(0,1,1)[12]
## Q* = 21.957, df = 20, p-value = 0.3428
##
## Model df: 4.   Total lags used: 24
```



Multiple Differences

- Models can contain both unit roots and seasonal unit roots.
- After removing the seasonal unit root through differencing to get W_t, ordinary differences can be calculated.

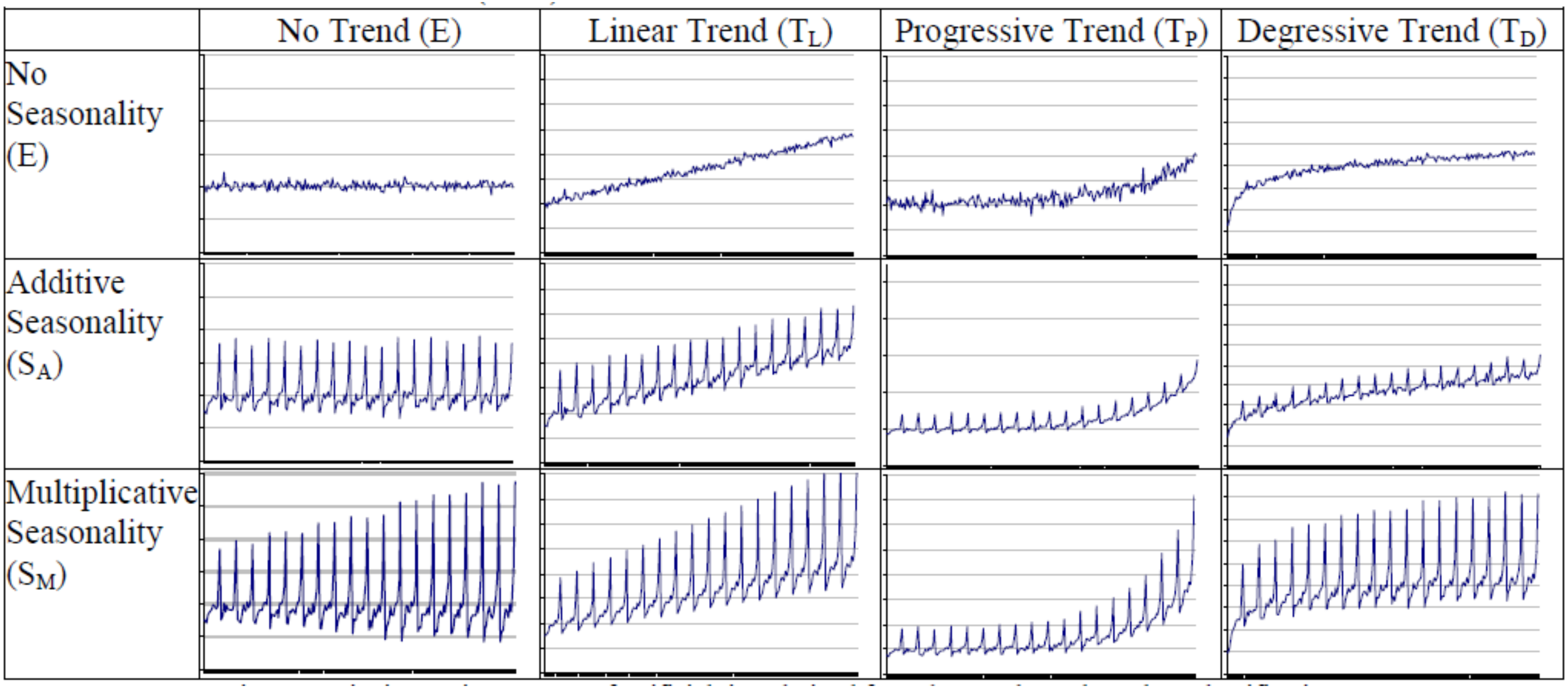
$$W_t = Y_t - Y_{t-12}$$
$$W_t = W_{t-1} + e_t - \beta e_{t-1}$$
$$W_t - W_{t-1} = e_t - \beta e_{t-1}$$
$$(Y_t - Y_{t-12}) - (Y_{t-1} - Y_{t-13}) = e_t - \beta e_{t-1}$$
$$Y_t = Y_{t-1} + Y_{t-12} - Y_{t-13} + e_t - \beta e_{t-1}$$

Limitations of Differencing

- Hard to evaluate stochastic effects for long and complex seasons.
- Most statistical tests for stochastic vs. deterministic can not handle past 12 or 24 periods in a season.
- Long/complex seasons → Best to just approach with deterministic solutions

MULTIPLICATIVE VS. ADDITIVE

Multiplicative vs. Additive



Backshift Operator – B

- The backshift operator is the mathematical operator to convert observations to their lags.
  - $\mathrm{B}(Y_t) = Y_{t-1}$

- This can be extended to any number of lags.
  - $\mathrm{B}^2(Y_t) = \mathrm{B}(Y_{t-1}) = Y_{t-2}$

Structures to Seasons

Additive

$$(1 - \alpha_1 \mathrm{B} - \alpha_2 \mathrm{B}^{12})Y_t = e_t$$

Multiplicative

$$(1 - \alpha_1 \mathrm{B})(1 - \alpha_2 \mathrm{B}^{12})Y_t = e_t$$

Structures to Seasons

Additive

$$(1 - \alpha_1 \mathrm{B} - \alpha_2 \mathrm{B}^{12})Y_t = e_t$$

$$Y_t - \alpha_1 \mathrm{B}(\mathrm{Y_t}) - \alpha_2 \mathrm{B}^{12}(Y_t) = e_t$$

Multiplicative

$$(1 - \alpha_1 \mathrm{B})(1 - \alpha_2 \mathrm{B}^{12})Y_t = e_t$$

Structures to Seasons

Additive

$$(1 - \alpha_1 \mathrm{B} - \alpha_2 \mathrm{B}^{12})Y_t = e_t$$

$$Y_t - \alpha_1 \mathrm{B}(\mathrm{Y_t}) - \alpha_2 \mathrm{B}^{12}(Y_t) = e_t$$

$$Y_t - \alpha_1 Y_{t-1} - \alpha_2 Y_{t-12} = e_t$$

Multiplicative

$$(1 - \alpha_1 \mathrm{B})(1 - \alpha_2 \mathrm{B}^{12})Y_t = e_t$$

Structures to Seasons

Additive

$$(1 - \alpha_1 \mathrm{B} - \alpha_2 \mathrm{B}^{12})Y_t = e_t$$

$$Y_t - \alpha_1 \mathrm{B}(\mathrm{Y_t}) - \alpha_2 \mathrm{B}^{12}(Y_t) = e_t$$

$$Y_t - \alpha_1 Y_{t-1} - \alpha_2 Y_{t-12} = e_t$$

$$Y_t = \alpha_1 Y_{t-1} + \alpha_2 Y_{t-12} + e_t$$

Multiplicative

$$(1 - \alpha_1 \mathrm{B})(1 - \alpha_2 \mathrm{B}^{12})Y_t = e_t$$

Structures to Seasons

Additive

$$(1 - \alpha_1 \mathrm{B} - \alpha_2 \mathrm{B}^{12})Y_t = e_t$$

Multiplicative

$$(1 - \alpha_1 \mathrm{B})(1 - \alpha_2 \mathrm{B}^{12})Y_t = e_t$$

$$\Downarrow$$

$$(1 - \alpha_1 \mathrm{B} - \alpha_2 \mathrm{B}^{12} + \alpha_1 \alpha_2 \mathrm{B}^{13})Y_t = e_t$$

Structures to Seasons

Additive

$$(1-\alpha_1\mathrm{B}-\alpha_2\mathrm{B}^{12})Y_t = e_t$$

$$(1-\alpha_1\mathrm{B}-\alpha_2\mathrm{B}^{12}-\boxed{\alpha_3}\mathrm{B}^{13})Y_t = e_t$$

Multiplicative

$$(1-\alpha_1\mathrm{B})(1-\alpha_2\mathrm{B}^{12})Y_t = e_t$$

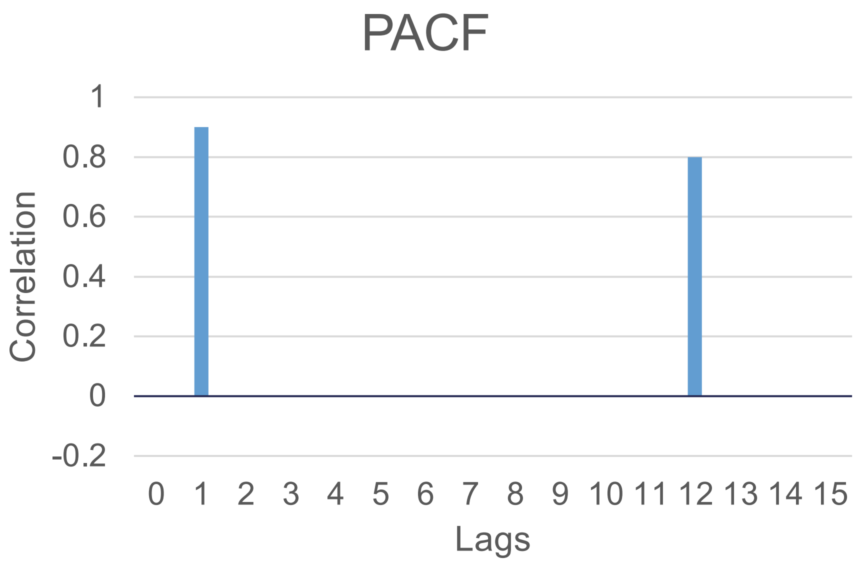
$$(1-\alpha_1\mathrm{B}-\alpha_2\mathrm{B}^{12}+\boxed{\alpha_1\alpha_2}\mathrm{B}^{13})Y_t = e_t$$

?

Structures to Seasons

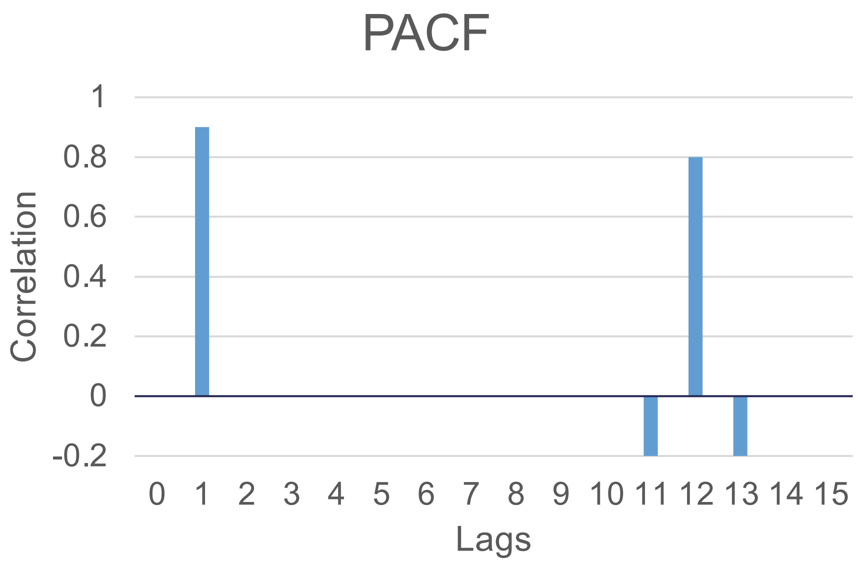
Additive

$$(1 - \alpha_1 \mathrm{B} - \alpha_2 \mathrm{B}^{12})Y_t = e_t$$



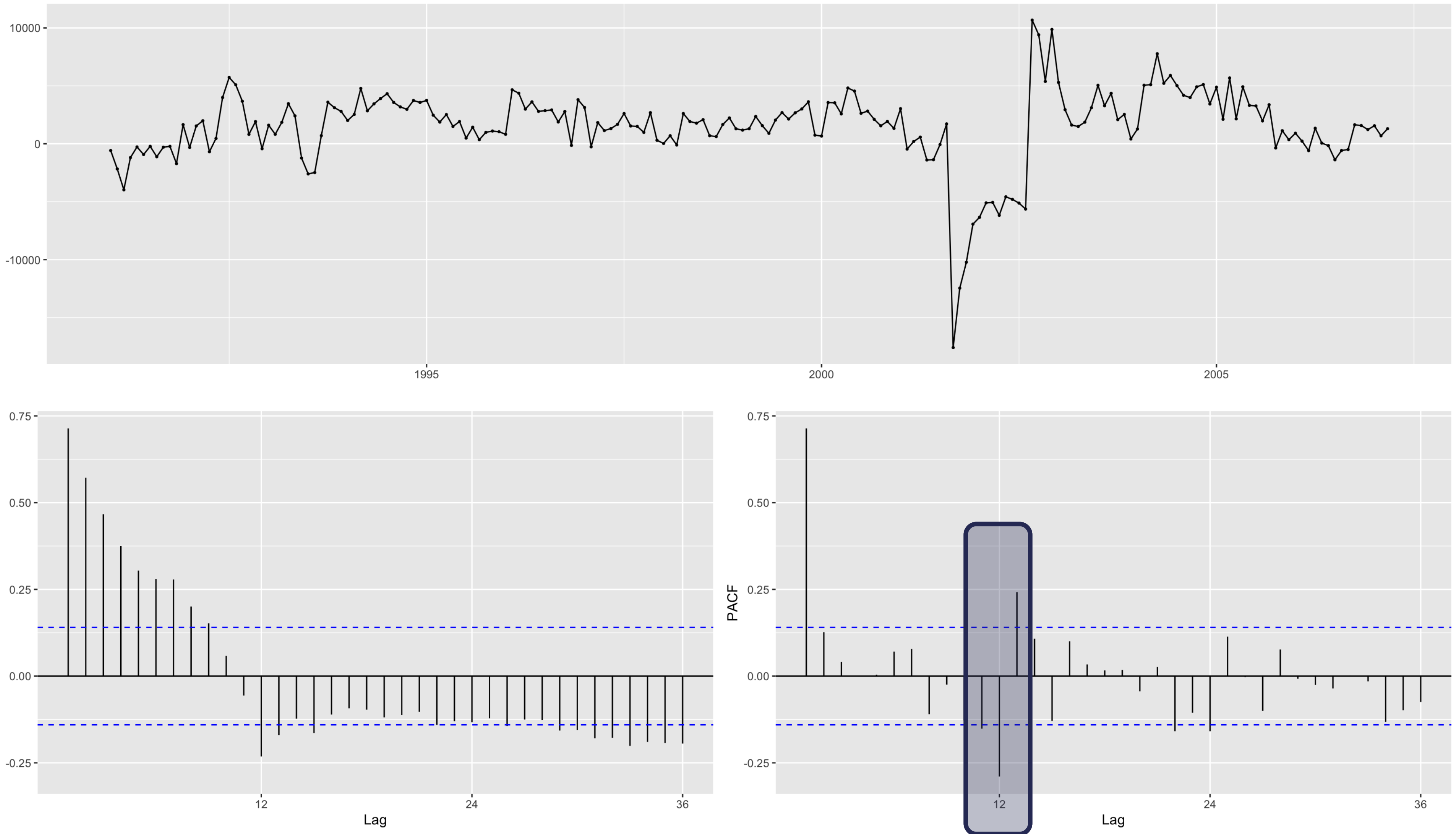
Multiplicative

$$(1 - \alpha_1 \mathrm{B})(1 - \alpha_2 \mathrm{B}^{12})Y_t = e_t$$



Seasonal Differencing

```
training %>% diff(lag = 12) %>% ggtsdisplay()
```



Structures to Seasons

Additive

```
S.ARIMA <- Arima(training, order=c(1,0,13),
                 seasonal=c(0,1,0),
                 fixed=c(NA,NA,0,0,0,0,0,0,0,0,0,0,NA,NA),
                 method="ML",)
summary(S.ARIMA)
```

```
## Coefficients:
##          ar1      ma1  ma2  ma3  ma4  ma5  ma6  ma7  ma8
##       0.9679  -0.3698    0    0    0    0    0    0    0
## s.e.  0.0237   0.0880    0    0    0    0    0    0    0
##       ma9   ma10   ma11     ma12    ma13
##         0      0      0  -0.6612  0.2490
## s.e.    0      0      0   0.0626  0.0766
```

Multiplicative

```
S.ARIMA <- auto.arima(training, method="ML",
                      seasonal = TRUE)
summary(S.ARIMA)
```

```
## Coefficients:
##          ar1      ma1     sma1     drift
##       0.8800  -0.2962  -0.6785  124.9788
## s.e.  0.0454   0.0950   0.0600   23.6330
```

Seasonal ARIMA Models

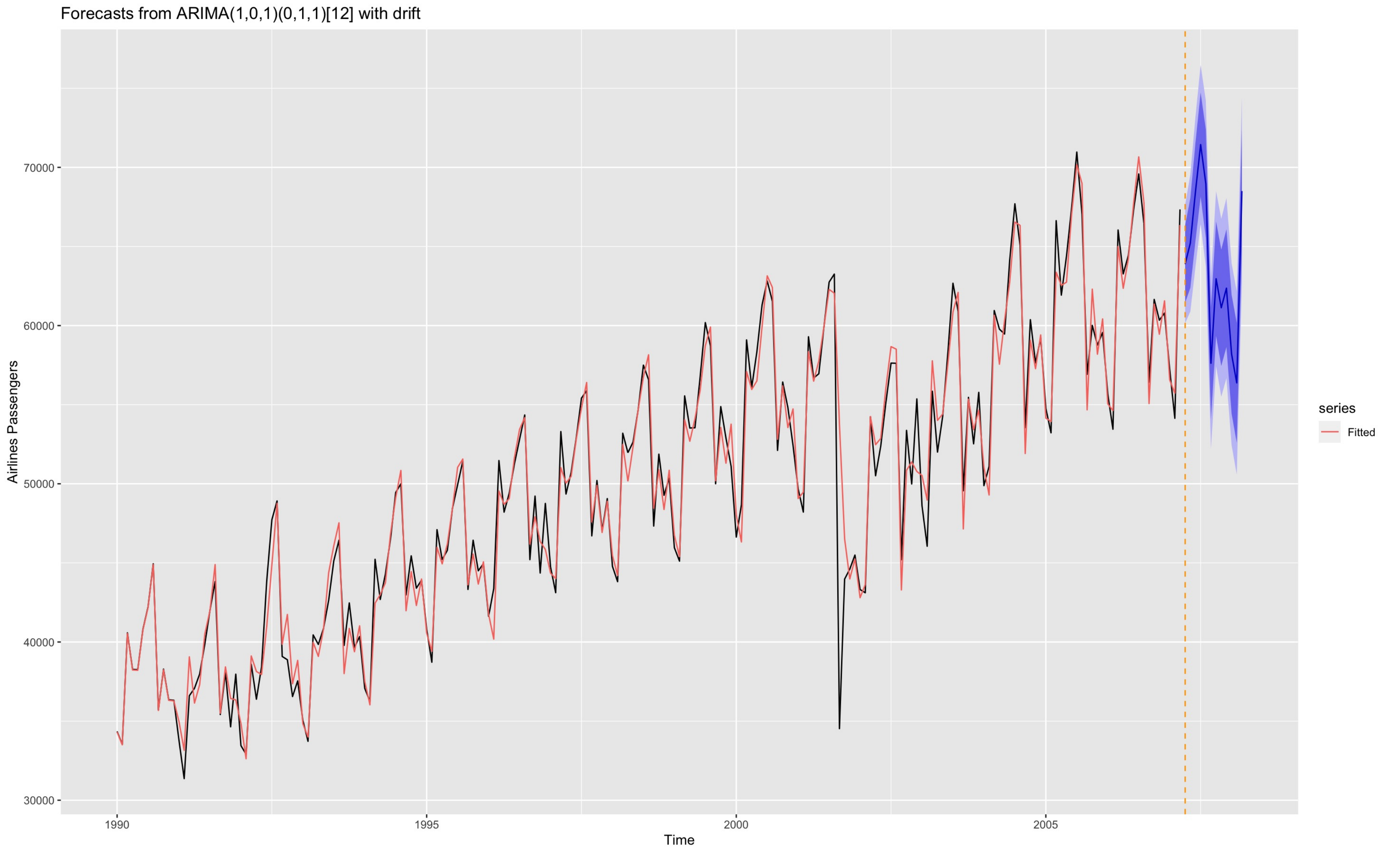
```
S.ARIMA <- auto.arima(training, method="ML", seasonal = TRUE)

autoplot(forecast::forecast(S.ARIMA, h = 12)) +
  autolayer(fitted(S.ARIMA), series="Fitted") +
  ylab("Airlines Passengers") +
  geom_vline(xintercept = 2007.25,color="orange",linetype="dashed")

S.ARIMA.error <- test - forecast::forecast(S.ARIMA, h = 12)$mean

S.ARIMA.MAE <- mean(abs(S.ARIMA.error))
S.ARIMA.MAPE <- mean(abs(S.ARIMA.error)/abs(test))*100
```

Seasonal ARIMA Models



Model Evaluation on Test Data

Model	MAE	MAPE
HW Exponential Smoothing	1134.58	1.76%
Seasonal ARIMA	1229.21	1.89%