Manish Pandar (NA17B111)

Ans. 3.

$$\bar{x} = \begin{bmatrix} 9 \\ 68 \\ 129 \end{bmatrix}, \quad S = \begin{bmatrix} 7 & 21 & 34 \\ 21 & 64 & 102 \\ 34 & 102 & 186 \end{bmatrix}$$

a) λ_3 (largest) $= 250.4$

eigen Vector corresponds to this $= (S - \lambda_3 I) V = 0$

$$V_3 = \begin{bmatrix} 0.9589 \\ 0.2830 \\ -0.0201 \end{bmatrix}$$

Remaining eigen Values.

$$\sum_{i=1}^{3} \lambda_i = \text{sum(diag of } S), \quad \prod_{i=1}^{7} \lambda_i = \det(S)$$

$\lambda_1 + \lambda_2 + \lambda_3 = 257$

$\lambda_1 \lambda_2 \lambda_3 = 146$

$\lambda_1 = 250.4$

$\lambda_2 = 6.5095, \quad \lambda_3 = 0.0896$

and Corresponding orthogonal Normalised Vector

$[S - \lambda_1 I] v_1 = 0, \quad (S - \lambda_2 I) V_2 = 0 \quad (S - \lambda_3 I) V_3 = 0$

$$V_1 = \begin{bmatrix} 0.1619 \\ 0.4877 \\ 0.8579 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 0.2330 \\ 0.8255 \\ -0.5135 \end{bmatrix} \quad V_3 = \begin{bmatrix} .9589 \\ -0.2830 \\ -0.0201 \end{bmatrix}$$

b) PCA,

$$\frac{\sum_{i=1}^{k} \lambda_i}{\sum_{i=1}^{N} \lambda_i} = \text{Data Variance}, \quad \frac{250.4}{257.0} > 95\%$$

Hence One PC should be enough.

c) For linear relationship, $V^T z_s$ among two two smallest eigen Values.

Solve $V_2^T(Z - \bar{Z}) = 0, \quad V_3^T(Z - \bar{Z}) = 0$

$\Rightarrow \quad 0.233 z_1 + 0.8259 z_2 - 0.5135 z_3 + 7.9 = 0$

$0.9589 z_1 - 0.2870 z_2 - 0.0201 z_3 + 12.206 = 0$

(d). $z = \begin{bmatrix} 101 \\ 73 \\ 135.5 \end{bmatrix}$

So for determining score -, will corespond to largest eigen value $\lambda_3 = 250.4$

score $= V_1^T z_s = 0$

$z_s = z - \bar{z}$, $\bar{z} = [9; 68; 125]$

score $= [0.1619 \quad 0.4877 \quad 0.8574] * [10.1 - 9; \; 73 - 61; \; 135.5 - 125]$

$= 8.1927$

(e) Estimate Mass of lizard, Where $SVL = 73mm$
~~HLS = 135.5mm~~

$z_2 = 73mm$

putin linear eq^n we got (c)

we get

$0.2332 z_1 - 0.5135 z_3 + 68.2706 = 0$

$0.9589 z_1 - 0.0201 z_3 - 7.4523 = 0$

We will get

$[z_1 = 10.66, \; z_3 = 137.7885]$

f) Estimate Mass, $SVL = 73mm$
$HLS = 135.5mm$

- Condition Where we have two eq^n & one unknown.

Elimate z_1 from eq^n

$z = [73 \quad 135.5]$ $\quad 2.8497 z_2 - 2.1828 z_3 + 20.4757 = 0$

so $X_3 = [2.8497 \quad -2.1828]$, $b = [-20.4757]$

objective fun. $f = \min_{\hat{z}, \lambda} \frac{1}{2}(z - \hat{z})^T (z - \hat{z}) + \lambda(X\hat{z} - b)$ $\quad \hat{z} \to$ estimates.

$\frac{df}{d\lambda} = (\hat{z} - z) + X^T \lambda$

Substitude $\to$ $\hat{z} = z - X_s^T (X_s X_s^T)^{-1} (X_s z - b)$

$\hat{z} = [71.87 \quad 136.13]$

In Final, $\boxed{z_1 = 11.01} \Rightarrow$ Estimated lizard Mass.