ME2 Computing- Coursework summary

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A) What physics are you trying to model and analyse? (Surely that is crazy!)

We are trying to analyse the price of European-style call & put options. An option is a non-linear financial derivative, whose price can be modelled over a certain time period using the Black-Scholes equation (shown in B). For a call option, the person has the right to buy the asset, and for a put option, the person has the right to sell the asset at an agreed price and time. The European style options ensures that the transaction can only be done at pre-determined date (called the maturity date). The purpose behind the equation is that we can hedge the option by buying and selling the underlying asset to reduce risk. The equation implies that there is only one right price for the option.
This equation can be rewritten as the heat-diffusion equation

B) What PDE are you trying to solve? (write the PDE)

$\frac{\partial V}{\partial t}+\frac{1}{2}S^2\sigma^2\frac{\partial^2 V}{\partial S^2}+rS\frac{\partial V}{\partial S}-rV=0$ for S > 0, t is an element of (0,T)

S(t): the relevant asset price at a time *t*
V(S,t): the option price as a function of the asset price *S* at a time *t*
C(s,t)/P(s,t): the price of the call/put option
K: the option's strike price (i.e. the price which the asset is agreed to be bought or sold at. Also called exercise price)
r: the interest rate which poses no financial risk
σ: the standard deviation of the asset's returns, represents the volatility of the stock
t: time in years

<u>Simplifying the equation using 'finance rationale' method.</u>

Time until maturity:	$\tau = T - t$
Future Value of Option:	$u = Ce^{r\tau}$
The asset price for given τ:	$x=\ln\left(\frac{S}{K}\right)+\left(r-\frac{1}{2}\sigma^2\right)\tau$

The final PDE becomes

$$\frac{\partial u}{\partial \tau}=\frac{1}{2}\sigma^2\frac{\partial^2 u}{\partial x^2}$$

This is a Parabolic PDE where $u = u(x,\tau), x\ \in(-\infty,\infty)\ and\ \tau\ \in(0,T)$

C) Boundary value and/or initial values for my specific problem: (be consistent with what you wrote in A)

For a **call option,** the option price V(S,t) for asset price S_T at maturity T, is maximum from [S_T-K (the payoff) , 0]. This is called the terminal condition; since not given for initial conditions. As the asset price increases (goes to infinity) for any time the option price also goes to infinity. When the asset price is 0 for any time, the option price is 0.

$$V(S_T,T)=\max\,[S_T-K,0]$$
$$V(0,t)=0$$
$$V(\infty,t)=\infty$$

These boundary conditions are then rewritten for the diffusion equation using 'finance rationale' method. The terminal condition now becomes an initial condition.

∴
$$u(x,0)=K(e^{\max\{x,0\}}-1)=\ K(e^x-1)H(x)$$
$$u(0,\tau)=0$$
$$u(\infty,\tau)=\infty$$

D) What numerical method are you going to deploy and why?

An implicit method is preferable to an explicit method, since even though an explicit approach is easy in appearance, we need to guarantee a convergence which leads to requiring small time steps and thus slow computing time.

An implicit method would require solving a system of linear equations for each time. The simpler implicit methods are only accurate for the first order in time and not accurate for larger time intervals. So, it is not much more efficient compared to an explicit method. The Crank-Nicolson method is second order accurate in time and space, which is what we implemented to solve the equation.

E) I am going to discretise my PDE as the following:

X is an unbounded interval (i.e. from minus infinity to positive infinity). However, to solve numerically this needs to be bounded within a certain interval [-L, L], where L is a large number which does not change for options, even with a different price. The boundary conditions are x = - L and x = L, where the value of x corresponds to stock prices close to zero for x = -L and approach infinity for x =L. L was selected to be twice the current asset (Stock) price. This will mean our model will be valid for asset prices lower than this threshold. The time steps will be discretised such that each step represents one week in terms of a year ie. dt = 1/52. When discretising the PDE, the value of r below had to be below 0.5 to improve the accuracy of the solving method.

$$r = \frac{k}{h^2}, r \leq 0.5$$

Where h is the step in the x direction and k is the step in the t direction. Then Crank-Nicolson method was used by having three arrays (matrices): u (the output array), A (the inverse matrix), and B (the matrix that multiplies A and includes initial and boundary conditions). A does not change for any time step so it was first determined using 'r' (the matrix is shown below for a single time interval). B had to be simultaneously made as u values were determined. **u = A^{-1}B.** The matrix B for output u at time interval j, where i is the spatial interval and N is the number of space intervals.

First 3 rows of matrix A

$$\begin{pmatrix} 2+2r & -r & 0 \\ -r & 2+2r & -r \\ -r & 2+2r & -r \end{pmatrix}$$

Last 3 rows of matrix A

$$\begin{pmatrix} -r & 2+2r & -r \\ -r & 2+2r & -r \\ 0 & -r & 2+2r \end{pmatrix}$$

Matrix B

$$\begin{matrix} u_{0,j} + u_{2,j} + u_{0,j+1} \\ u_{3,j} + u_{1,j} \\ \vdots \\ u_{i+1,j} + u_{i-1,j} \\ u_{N,j} + u_{N-2,j} + u_{N,j} \end{matrix}$$

F) Plot of results and comments (discuss how the results describe the physics and comment on any discrepancies or unexpected behaviours):

Figure 1 shows the option value against stock price for an inputted week for Apple and Microsoft stock. It should be noted that the value outputted is not a literal value but more figurative. It is possible to assign a literal currency value by scaling it however this depends on the method used by different banks, etc. Black-Scholes is commonly just used to compare the value of different options rather than an actual price. Also, it is quoted by brokers since it is a standard model anyone can refer to; even though the model does not consider market volatility and various other factors (explained in G). Figure 2 shows the option values for the following month of the mentioned stocks. The arrays have been sliced to exclude the boundary conditions since these are theoretically infinite conditions (though have been limited to a numerical value in the code) and therefore do not represent accurate option values. This is seen in the surface plot (figure 3). Figure 2 illustrates the Apple & Microsoft option prices at week 5, at a strike price of 50% of the stock price with an expiry time of two years. The Apple stock price = $320.30 and Microsoft stock price = $188.70 (from NASDAQ 1:55PM 20/02/2020). The plot below shows the same but for the entire month.

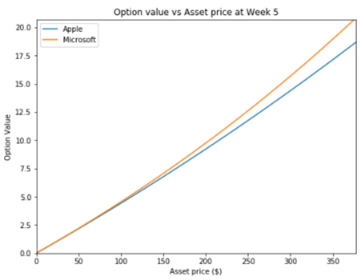

Figure 1

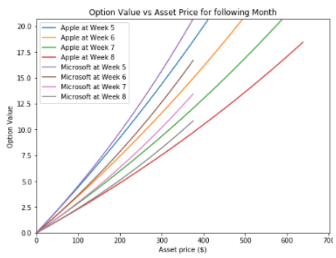

Figure 2

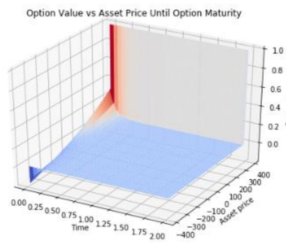

Figure 3

Figure 4

F) Plot of results and comments (continued):

See Figure 3. The value is not accurate at boundary conditions (in this case between an asset price of $400 and -$400), since theoretically these should be applied at an infinite grid, but we had to limit it to just twice the asset price. Also, the initial conditions are not as accurate as real financial models since they use significantly smaller time intervals where a smaller 'r' for Crank-Nicolson provide better models. However, with the limited computing power we had access to, this would take too much time. (>10 seconds).

G) Other remarks (limits of the model, convergence problems, possible alternative approaches, anything you find relevant and important to mention):

See Figure 4. The Black-Scholes equation itself has several setbacks. The original equation (which we used) is applicable for European-style financial systems, with no dividends and assumes a 'frictionless' market – no transaction costs (eg: commission). Also, the equation assumes no major volatility in the market which we demonstrated below in the following figure, showing the option values before and after coronavirus was confirmed as a pandemic by WHO. The call option value should go down, but our model shows it going up. This is because the model does not contemplate the possibility of financial externalities (eg: a bear/bull market, recession, federal interest rate fluctuations, etc.) In addition to these issues, our model has several other inaccuracies. To save computing time, we limited the spatial range to twice the stock price, so this means our model cannot predict large fluctuations in asset price. However, this is not a major issue since our model does not consider the high volatilities, which is how significant price changes occur.