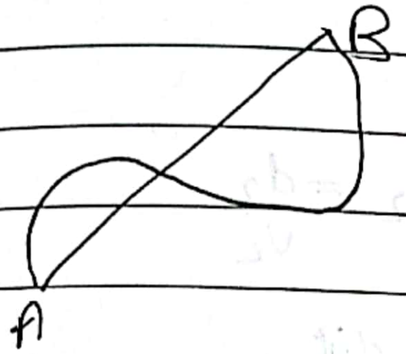
Kinematics I

Kinematics : I → study of motion



$\tilde{AB}$ = distance

$\vec{AB}$ = displacement

Average speed = $\langle v \rangle = V_{avg}$

$= \frac{\text{total distance}}{\text{total time}}$

Avg acceleration $= \langle \vec{a} \rangle = \vec{a}_{avg} = \frac{\Delta \vec{V}}{t}$

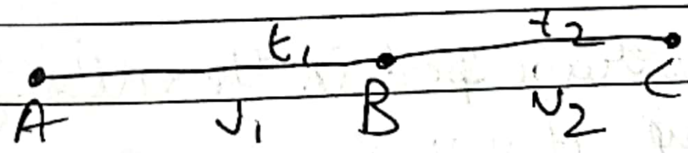
$= \frac{V_f - V_i}{t}$

Avg velocity $= \langle \vec{V} \rangle = \vec{V}_{avg} = \frac{\text{total dist}}{\text{total time}}$

$\vec{V_f}$ = final velocity

$\vec{V_i}$ = initial velocity

Q



Find the average speed.

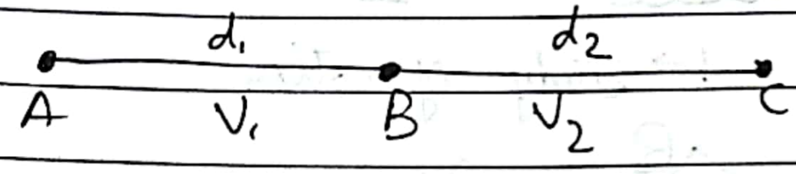
$V_{avg} = \frac{\text{total distance}}{\text{total time}}$

$= \frac{t_1 v_1 + v_2 t_2}{t_1 + t_2}$

$\because v_1 = \frac{d_1}{t_1}$

$d_1 = v_1 t_1$

Q



Find the average speed.

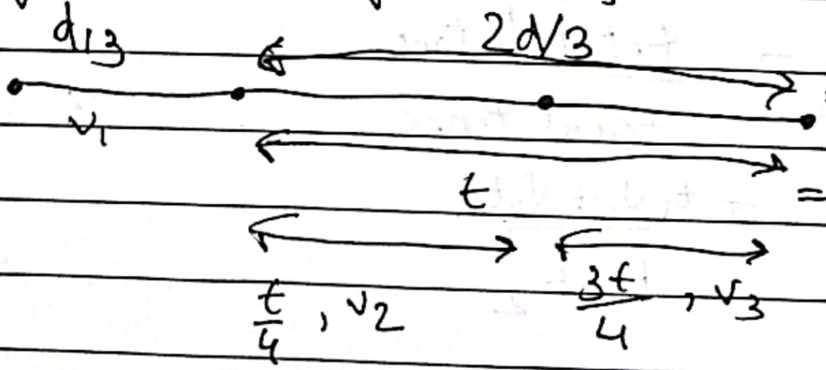
$$t_1 = \frac{d_1}{v_1} \quad , \quad t_2 = \frac{d_2}{v_2}$$

$$\therefore V_{avg} = \frac{\text{Total dist}}{\text{total time}}$$

$$= \frac{d_1 + d_2}{d_1 v_2 + d_2 v_1} \times v_1 v_2$$

Q A body is moving in a straight line $\frac{1}{3}^{rd}$ of the total distance is covered with speed v_1. For the remaining distance, $\frac{1}{4}^{th}$ of the time is covered with speed v_2 and the remaing with speed v_3.

Find the average speed in the entire journey ~~using~~ in terms of v_1, v_2, v_3.



$$= \frac{v_2 t}{4} + \frac{3 t v_3}{4}$$

$$= \frac{2d}{3} = \frac{(v_2 + 3v_3)t}{4}$$

$$t = \frac{8d}{3(v_2 + 3v_3)}$$

~~$v_2 \frac{t}{4} + \frac{3t}{4} v_3$~~

Date :

$$V_{avg} = \frac{\text{total distance}}{\text{total time}}$$

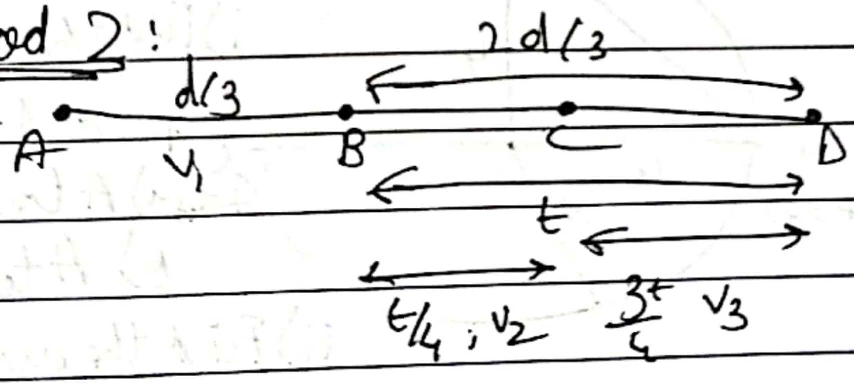
$$= \frac{\frac{d}{3} + \left(\frac{t v_2}{3} + \frac{3t v_3}{4}\right)}{\frac{d}{3v_1} + t}$$

$$= \frac{\frac{d}{3} + \frac{2d}{3}}{\frac{d}{3v_1} + \frac{8d}{3(v_2+3v_2)}}$$

$$= \frac{\cancel{d}}{\frac{\cancel{d}}{3}\left(\frac{1}{3v_1} + \frac{8}{\cancel{3}(v_2+3v_3)}\right)}$$

$$= \frac{3v_1(v_2+3v_3)}{v_2+3v_3+8v_1}$$

<u>Method 2</u>:



For BD

$$V_{avg} = \frac{2d/3}{t} = \frac{\frac{t v_2}{4} + \frac{3t v_3}{4}}{t} = \frac{\frac{\cancel{t}}{4}(v_2+3v_3)}{\cancel{t}}$$

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$$= \frac{v_2 + 3v_3}{4} = v'$$

For AD

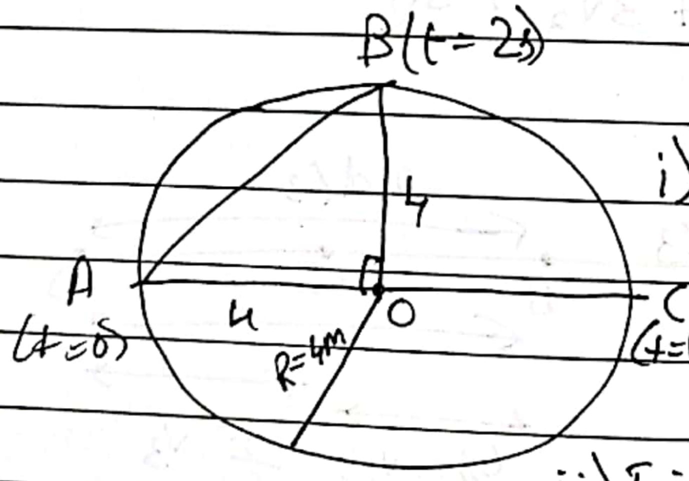
$$V_{avg} = \frac{d/3 + 2d/3}{\frac{d}{3v_1} + \frac{2d}{3v'}}$$

$$= \frac{d}{\frac{d}{3}\left(\frac{1}{v_1} + \frac{8}{v_2+3v_3}\right)}$$

$$= \frac{3(v_1 v_2 + v_1 3v_3)}{v_2+3v_3+8v_1}$$

$$= \frac{3v_1(v_2+3v_3)}{v_2+3v_3+8v_1}$$

Q



i) Find the average speed from a) A to B b) A to C

ii) Find the average velocity from: a) A to B , b) A to C

Date :

i) a) $\frac{\widetilde{AB}}{\pi} = \frac{2\pi}{2} = \pi$

b) $\frac{\widetilde{AC}}{8\pi} = \frac{4\pi}{\pi} = \pi$

ii) a) $\frac{\overrightarrow{AB}}{t} = \frac{4\sqrt{2}}{2} = 2\sqrt{2}\ ms^{-1}$

b) $\frac{\overrightarrow{AC}}{t} = \frac{8}{4} = 2ms^{-1}$

Equations of motion

When acceleration is constant

i) $v = u + at$

ii) $v^2 = u^2 + 2as$

iii) $s = ut + \frac{1}{2}at^2$

iv) $s = \left(\frac{u+v}{2}\right)t$

v) $S_{n^{th}} = u + \frac{a}{2}(2n-1)$

$u \rightarrow$ initial velocity

$v \rightarrow$ final velocity

$s \rightarrow$ displacement

$t \rightarrow$ time

$a \rightarrow$ acceleration (constant)

$S_{n^{th}} \rightarrow$ displacement in the n^{th} second

Q 2 particles P and Q start simultaneously from point A with velocities 15m/s and 20m/s in the same direction respectively. They move with different acceleration of constant magnitude. When P overtakes Q at point B, the velocity of P is $30ms^{-1}$. Find the velocity of Q at point B

$u_p = 15ms^{-1}$, $u_Q = 20ms$, $v_p = 30ms^{-1}$

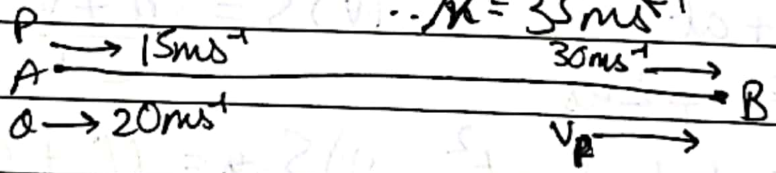
$\therefore \cancel{a_p = a_Q}$

$$\frac{v_p - u_p}{t} = \frac{v_Q - u_Q}{t}$$

$$30 - 15 = x - 20$$

$$35 = x$$

$$\therefore x = 35ms^{-1}$$



M1

→ Same dist → Same time

→ Diff. acceleration → Same displacement

$$S_p = 15t + \frac{1}{2}a_1t^2$$

$$S_Q = 20t + \frac{1}{2}a_2t^2$$

$$\left(15 + \frac{1}{2}a_1t\right)\cancel{t} = \left(20 + \frac{1}{2}a_2t\right)\cancel{t}$$

$$15 + 7.5 = 20 + \frac{1}{2}a_2t$$

$30 = 15 + a_1t$

$a_1t = 15$

$v_Q = 20 + a_2t$

$2.5 \times 2 = a_2t$

$a_2t = 5$

Date :

$\therefore v_0 = 20 + 5$

$\boxed{v_0 = 25 ms^{-1}}$

M2

$S = \left(\frac{u+v}{2}\right) t$, $S = \left(\frac{15+30}{2}\right) t$

$\therefore \frac{15+30}{\cancel{2}} = \frac{20+v}{\cancel{2}}$

$v = 45 - 20$

$\boxed{v = 25 ms^{-1}}$

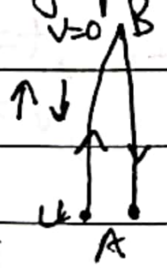
Motion under gravity

$g = 9.8 ms^{-2}$ $\boxed{g = 10 ms^{-2}}$

↳ If value not given

Sign convention → Your choice

1) If particle is projected upwards with speed u



→ Time of flight / Air resistance, negligible

→ Maximum height

Initial velocity = u, acceleration = $-g$

For time of flight

↑+ve $S = 0$

↓−ve $S = ut + \frac{1}{2}at^2$

$0 = t\left(u - \frac{1}{2}gt\right)$, $\therefore t = 0$ or $u - \frac{1}{2}gt = 0$

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$\boxed{\therefore t = \frac{2u}{g}}$, $\cancel{t=0}$ discarded

For Max height:

$v = 0$

$v^2 = u^2 + 2as$

$0 = -u^2 + 2gs$

$u^2 = 2gs$

$S = \frac{u^2}{2g}$

$$\boxed{\therefore h_{max} = S = \frac{u^2}{2g}}$$

Q

+↑ A particle is dropped from height h

−↓ i) Find the time of flight

ii) Find speed at bottom

$u = 0$, $S = -h$, $t = ?$

$a = -g$, $v = ?$

$\therefore v = u + at$

$v = 0 - gt$

$v^2 = u^2 + 2aS$

$\cancel{g^2t^2 = 0 - 2gs}$ $\quad v = \sqrt{2gh}$

$S = ut + \frac{1}{2}at^2$, $-H = 0 + \frac{1}{2}(-g)t^2$

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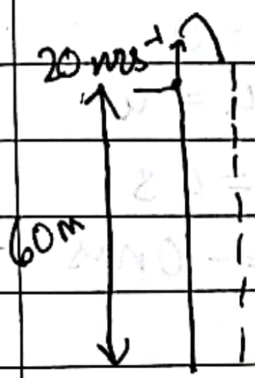
$\therefore h = \frac{1}{2} g t^2$, $t = \sqrt{\frac{2h}{g}}$

$\therefore v = \sqrt{2gh}$ and $t = \sqrt{\frac{2h}{g}}$

Q 

Take value of $g = 10 ms^{-2}$
i) Find time of flight
ii) Max height of object above the ground

$u = 20 ms^{-1}$
$a = -10 ms^{-2}$
$S = -60m$
$t = ?$

$h_{max} = \frac{u^2}{2g} = \frac{20^2}{20} = 20m + 60m = \underline{80 m}$

$S = ut + \frac{1}{2}at^2$

$-60 = 20t + \frac{1}{\cancel{2}}(-\cancel{10}^{5})t^2$

$5t^2 - 20t - 60 = 0$

$t^2 - 4t - 12 = 0$

$t^2 - 6t + 2t - 12 = 0$

$t(t-6) + 2(t-6) = 0$

$t = 6$ or $t = -2$

$\boxed{\therefore t = 6s}$

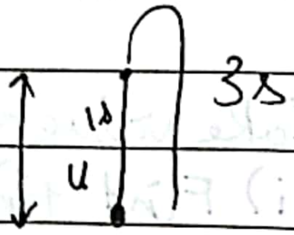
Date :

Q A particle is thrown upward with speed u. It reaches height h in 1s. Further it travels upward and takes another 3s to reach the ground. ($g = 10\,ms^{-2}$)

Find:

i) u

ii) h



$S = 0$

$u = u$

$t = 4s$

$a = -10\,ms^{-2}$

$$S = ut + \frac{1}{2}at^2$$

$$0 = 4u - 80$$

$$u = \frac{80}{4}, \quad \boxed{u = 20\,ms^{-1}}$$

For h, $t = 1s$

$$h = 20(1) + \frac{1}{2}(-10)1^2$$

$$= 20 - 5$$

$$\boxed{\therefore h = 15m}$$

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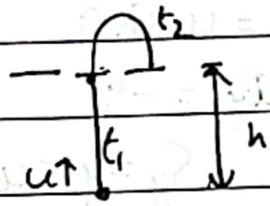
Q A particle is projected upward from the ground with ~~spectrum~~ speed u. It crosses height h at 2 instants, t_1 and t_2. Find:

a) u b) h

in terms of t_1, t_2 and g



$u = u$

$a = -g$

$h = ut_1 + \frac{1}{2}(-g)t_1^2$

$h = ut_2 + \frac{1}{2}(-g)t_2^2$

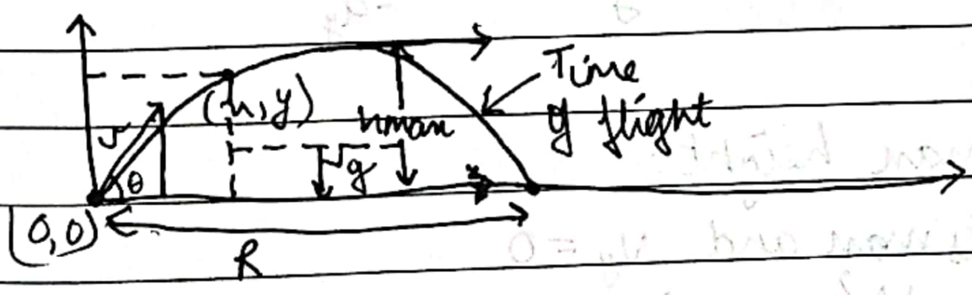
$$\therefore \; ut_1 - \frac{gt_1^2}{2} = ut_2 - \frac{gt_2^2}{2}$$

$$u\cancel{(t_1 - t_2)} = \frac{g}{2}\cancel{(t_1 - t_2)}(t_1 + t_2)$$

$$\boxed{u = \frac{g(t_1 + t_2)}{2}}$$

$$\boxed{\therefore h = \frac{g(t_1 t_2)}{2}}$$

Projectile Motion



$u_x = u\cos\theta$

$a_x = 0$

$S_x = x = u_x t + \frac{1}{2} a_x t^2$, $\boxed{x = u\cos\theta\, t}$

Date :

$v_x = u_x + a_x t$

$= u\cos\theta$

$u_y = u\sin\theta$

$a_y = -g$

$S_y = y = u_y t + \frac{1}{2} a_y t^2$

$y = u\sin\theta t - \frac{1}{2} g t^2$

$v_y = u_y + a_y t$

$= u\sin\theta - gt$

<u>For time of flight:</u>

Since A is on the x axis

$\therefore y = 0$

$0 = u\sin\theta t - \frac{1}{2} g t^2$

$0 = t\left(u\sin\theta - \frac{1}{2} g t\right)$

$t = \frac{2u\sin\theta}{g} = \frac{2u_y}{-a_y}$

<u>For max height:</u>

y is max and $v_y = 0$

$v_y^2 = u_y^2 + 2 a_y h_{max}$

$2g h_{max} = u^2 \sin^2\theta$

$h_{max} = \frac{u^2 \sin^2\theta}{2g} = \frac{u_y^2}{-2a_y}$

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For horizontal range:

$R = x$ at $t = \frac{2u\sin\theta}{g}$

$R = \frac{2u^2\cos\theta\sin\theta}{g} = \frac{u^2\sin 2\theta}{g} = \frac{2u_x u_y}{-a_y}$

$u \rightarrow$ initial velocity $\quad a \rightarrow$ acceleration

$\theta \rightarrow$ Angle of projection $\quad R \rightarrow$ horizontal range.

$T = \frac{2u\sin\theta}{g}$	$H_{max} = \frac{u^2\sin^2\theta}{2g}$	$R = \frac{u^2\sin 2\theta}{g}$
For max T $\sin\theta \rightarrow max$	For max H_{max}	For max Range
$\therefore \sin\theta = 1$	$\sin\theta \rightarrow max$	$\sin 2\theta \rightarrow max$
$\theta = 90°$	$\theta = 90°$	$2\theta = 90°$
$T_{max} = \frac{2u}{g}$	$(H_{max})_{max} = \frac{u^2}{2g}$	$\theta = 45°$
		$R_{max} = \frac{u^2}{g}$

Q An object is projected from the ground at an angle θ with the horizontal. The speed of projection is $20 ms^{-1}$

Find :

i) Range. ii) Max height.

take $g = 10 ms^{-2}$

For:

a) $\theta = 30°$ b) $\theta = 60°$ c) $\theta = 37°$ d) $\theta = 53°$

i) $R = \frac{u^2\sin 2\theta}{g}$

$= \frac{400\sin 2\theta}{10} = 40\sin 2\theta$

Date :

a) ~~ii) hmax~~ i) $R = \frac{u^2 \sin 2\theta}{g}$

$= \cancel{400}^{20} \times \frac{\sqrt{3}}{\cancel{20}}$

$= 20\sqrt{3}\ m$

ii) $h_{max} = \frac{u^2 \sin^2\theta}{2g}$

$= \cancel{400}^{\cancel{20}\,10} \times \frac{1}{\cancel{2}} \times \frac{1}{\cancel{20}}$ $= 10\ m$

b) i) $R = \frac{u^2 \sin 2\theta}{g} = \cancel{400}^{20} \times \frac{\sqrt{3}}{\cancel{20}}$

$= 20\sqrt{3}\ m$

ii) $h_{max} = \frac{u^2 \sin^2\theta}{2g} = \cancel{400}^{\cancel{20}\,5} \times \frac{3}{\cancel{4}} \times \frac{1}{\cancel{20}}$

$= 15 m$

c) i) Range $= \frac{u^2 \sin 2\theta}{g} = \frac{2 \times \cancel{400}^{\cancel{80}\,\cancel{16}\,3.2}}{\cancel{10}\,\cancel{8}} \times \frac{3}{\cancel{8}} \times \frac{4}{\cancel{8}}$

$= 38.4 m$

ii) $h_{max} = \frac{u^2 \sin^2\theta}{2g} = \frac{\cancel{400}^{\cancel{20}\,4} \times 9}{\cancel{20} \times \cancel{25}\,5}$

$= 7.2 m$

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d) i) $R = \frac{2u^2 \sin\theta\cos\theta}{g} = \frac{2\times 400 \times 3\times 4}{10\times 5\times 5}$

$= 38.4m$

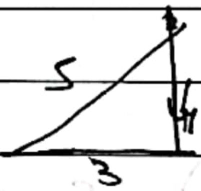
ii) $h_{max} = \frac{u^2\sin^2\theta}{2g} = 400 \times \frac{16}{25} \times \frac{1}{20}$

$= 12.8m$

Q $\vec{u} = 3\hat{i} + 4\hat{j}$ ms^{-1} (Take $g = 10ms^{-2}$)

i) Find H_{max}, R, θ, T

ii) Find the speed at $t = 0.4s$



i) $u = \sqrt{3^2+4^2} = 5ms^{-1}$

$H_{max} = \frac{u^2\sin^2\theta}{2g} = 25 \times \frac{16}{25} \times \frac{1}{20} = \boxed{0.8m}$

$R = \frac{2u\sin\theta\cos\theta}{g} = 2\times 5 \times \frac{4}{5} \times \frac{3}{5} \times \frac{1}{10} = \cancel{0.48m}$

$\boxed{R = 2.4m}$

$\boxed{\theta = 53^\circ}$

$T = \frac{2u\sin\theta}{g} = 2\times 5 \times \frac{4}{5} \times \frac{1}{10}$ $\boxed{T = 0.8m}$

ii) Speed is the magnitude of velocity

$v = \sqrt{v_x^2 + v_y^2}$

$v_x = u_x + a_x t$
$= 3+0 = 3$

$v_y = u_y + a_y t$
$= 4 + (-10)(0.4)$
$= 0$

$\boxed{V = 3ms^{-1}}$

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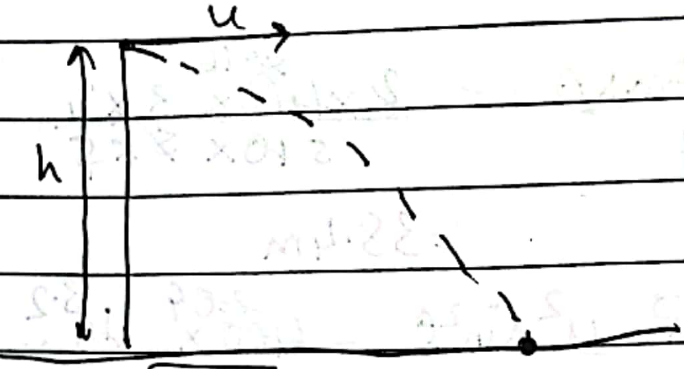
Q



Find :
i) time of flight
ii) R
iii) Find speed (just before hitting ground)

i) $\cancel{t = \sqrt{2h}}$ $\quad u_y = 0$

$$S_y = u_y t + \frac{1}{2} a_y t^2$$

$$S_y = \frac{1}{2} a_y t^2$$

$$h = \frac{1}{2} g t^2$$

$$t = \sqrt{\frac{2h}{g}}$$

ii) $S_x = u_x t + \frac{1}{2} a_x t^2 = u\sqrt{\frac{2h}{g}}$

iii) $v_x = u + 0(t)$ $\quad , \quad v = \sqrt{v_x^2 + u_y^2}$

$v_y = 0 - gt$ $\quad\quad v = \sqrt{u^2 + 2gh}$

$= 0 - g\sqrt{\frac{2h}{g}}$

$= -\sqrt{2gh}$

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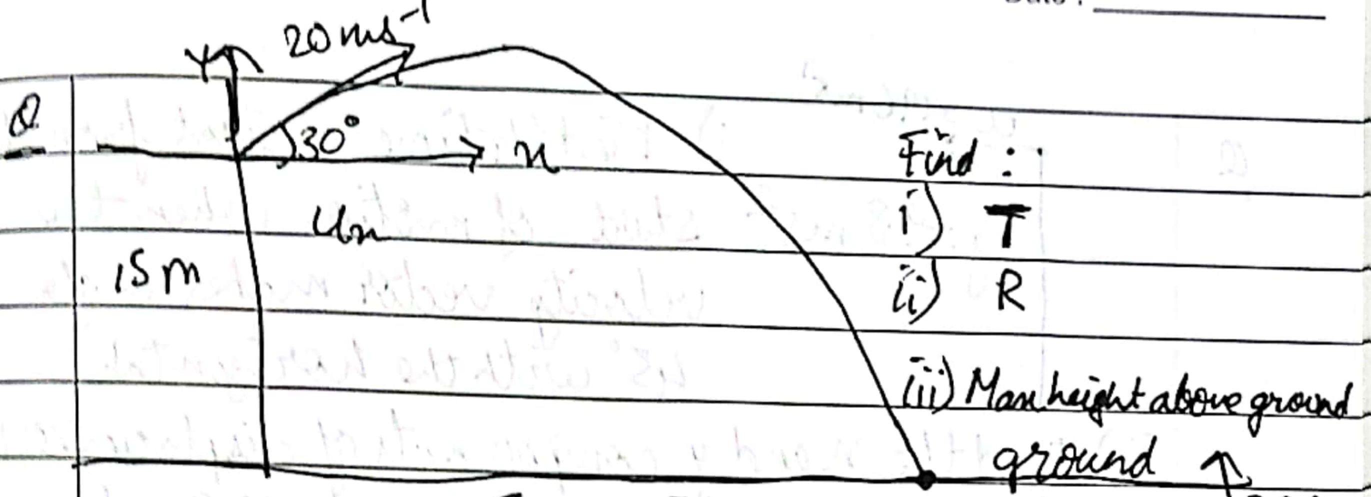
Q



Find :
i) T
ii) R
iii) Max height above ground

$S_y = -15$

$= u_y t + \frac{1}{2} a_y t^2$

$-15 = 10t + \frac{1}{2}(-10)t^2$

$-15 = 10t - 5t^2$

$5t^2 - 10t - 15 = 0$

$t^2 - 2t - 3 = 0$

$t^2 - 3t + t - 3 = 0$

$t(t-3) + 1(t-3) = 0$

$(t-3)(t+1) = 0$

$\boxed{\therefore t = 3}$ or $t = -1$

For range

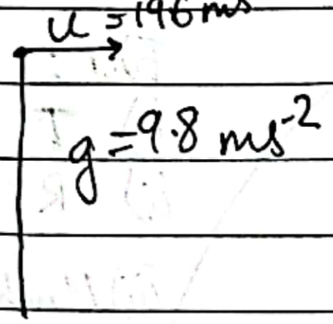
$R = S_x$

$S_x = u_x t + \frac{1}{2} a_x t^2$

$= \left(\frac{20\sqrt{3}}{2}\right) \times 3$

$= 30\sqrt{3}$ m

Q



i) Find the time instant from the start of motion when the velocity vector makes angle 45° with the horizontal.

ii) Find the x and y components of displacement till that instant.

$u_x = 19.6\,ms^{-1}$

$u_y = 0$

$a_x = 0$

$a_y = -9.8\,ms^{-2}$

$\tan 45 = \dfrac{|v_y|}{|v_x|}$

$v_x = u_x + a_x t$

$v_x = 19.6\,ms^{-1}$

$v_y = 0 - 9.8 \times 2$

$v_y = -19.6$

$v_y = u_y + a_y t$ $\quad a = \dfrac{v-u}{t}$

$-19.6 = 0 + (-9.8)t$

$t = \dfrac{-19.6}{-9.8}$ $\quad \cancel{t = \dfrac{-19.6}{-9.8}}$

$\boxed{t = 2\,s}$ $\quad \cancel{\boxed{t = 2\,s}}$

$S_x = u_x t + \frac{1}{2} a_x t^2$

$= \cancel{19.6}\ 39.2 + 0$

$= 39.2\,m \rightarrow$

$S_y = u_y t + \frac{1}{2} a_y t$

$= \cancel{19.6 - 9.8}\ 0 - 19.6$

$= \cancel{19.6\,m}\ -19.6\,m$

$= 19.6\,m \downarrow$

iii) Find the magnitude of average velocity in just 2s

$\cancel{v = \dfrac{39.2 - 19.6}{2} = \dfrac{19.6}{2}} = \cancel{}\ 9.8\,ms^{-1}$

Date :

$$v_{avg} = \frac{\text{Total displacement}}{\text{total time}}$$

$$= \frac{\sqrt{S_x^2 + S_y^2}}{2}$$

$$= \frac{\sqrt{39.2^2 + 19.6^2}}{2}$$

$$= \cancel{9.8\sqrt{5}} \; \frac{19.6}{2}\sqrt{2^2+1}$$

$$= 9.8\sqrt{5}\ ms^{-1}$$