

Problem Set 01: Thinking Logically

CS/MATH 113 Discrete Mathematics

Spring 2024

1. In an island there are two kinds of inhabitants, knights, who always tell the truth, and knaves, who always lie. You encounter two people A and B. Determine, if possible, what A and B are if they address you in the ways described.

(a) A says “At least one of us is a knave” and B says nothing.

Solution:

(b) A says “The two of us are both knights” and B says “A is a knave”.

Solution:

(c) Both A and B say “I am a knight.”

Solution:

(d) A says “We are both knaves” and B says nothing.

Solution:

2. Three friends are hanging out at a Cafe. The server approaches them and asks, “Does everyone want a slice of cake?” The first friend says “I don’t know”. The second friend says “I don’t know”. Finally, the third friend says “No, not everyone wants cake”. The server comes back and gives slices of cake to the friends who wanted it.

How did the server figure out who wanted the cake?

Solution:

3. An ancient Sicilian legend says that the barber in a remote town who can be reached only by traveling a dangerous mountain road shaves those people, and only those people, who do not shave themselves. Can there be such a barber? (*Hint*: think about who shaves the barber.)

Solution:

4. A father tells his two children, a boy and a girl, to play in their backyard without getting dirty. However, while playing, both children get mud on their foreheads. When the children stop playing, the father says “At least one of you has a muddy forehead,” and then asks the children to answer “Yes” or “No” to the question: “Do you know whether you have a muddy forehead?” The father asks this question twice. What will the children answer each time this question is asked, assuming that a child can see whether his or her sibling has a muddy forehead, but cannot see his or her own forehead? Assume that both children are honest and that the children answer each question simultaneously.

Solution:

5. At a fashion show, Mehwish would like to determine the relative salaries of three coworkers using two facts. First, she knows that if Zara is not the highest paid of the three, then Rubya is. Second, she knows that if Rubya is not the lowest paid, then Amna is paid the most. Is it possible to determine the relative salaries of Zara, Amna, and Rubya from what Mehwish knows? If so, who is paid the most and who the least? Explain your reasoning.

Solution: