Introduction to Pulsar Timing

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InPTA

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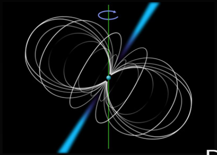
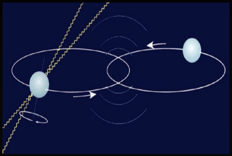
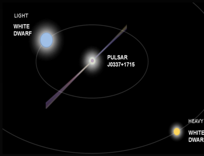
Radio Pulsars

Numbers on 08/April/2022

(3319)







Isolated

Binary ($\sim$344)

(i) NS-planet [$\sim$2]

(ii) NS-low mass [$\sim$ 40(bw)+22(rb)]

(iii) NS-MS [$\sim$28]

(iv) NS-WD [$\sim$189+12(?)]

(153 He + 48 CO)

(v) NS-NS [$\sim$22+4(?)]

(vi) NS-BH [0?]

Triple (2)

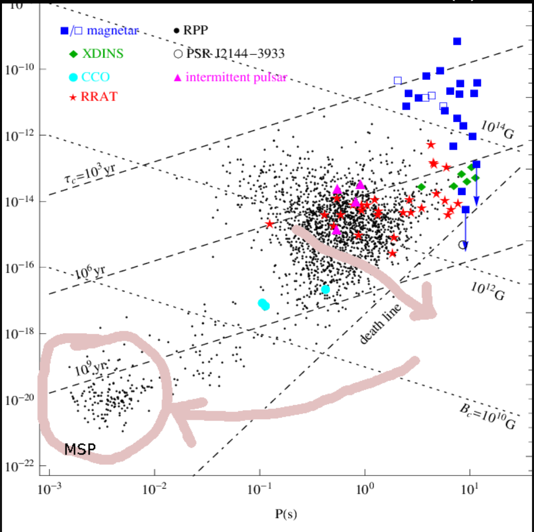
NS-WD-WD (1)

NS-planet-planet (1)

4-body: PSR B1257+12 with 3 planets (1992: 'first exoplanet'); BW: $M_c \simeq 0.02 - 0.05\ \mathrm{M}_\odot$, RB: $M_c \simeq 0.2 - 0.5\ \mathrm{M}_\odot$

Timing binary pulsars are more useful:

(i) more accurate models as 'old' pulsars are more stable, (ii) tests of gravity



Timing analysis of pulsars: Introduction

Let us have a 'stable' simple celestial light house in vacuum (complications will come)

- Record pulse Time of Arrivals (ToAs) at the observatory, say
  $T_{o1},\ T_{o2},\ T_{o3},\ T_{o4},\ \ldots$

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- Pulsar timing is 'mainly' finding $d1, d2, d3, d4,\ \cdots$ or in other words, finding $T_{p1},\ T_{p2},\ T_{p3},\ T_{p4},\ \cdots$; intelligent methods to check the correctness of these!

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- tempo2: Pulsar timing analysis software. Needs ToAs (tim) file + par file (pulsar parameters) + other info to convert $T_{oi} -> T_{pi}$ + statistical analysis tools, etc.

Timing analysis of pulsars: Complication-1

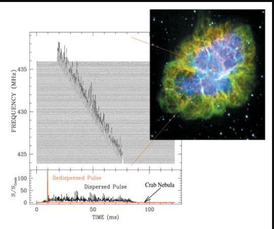
- Pulsars slow down, i.e., P_s not a constant, have positive $\dot{P}_\mathrm{s}$
- Taylor series: $P_\mathrm{s}(t) = P_\mathrm{s}(t_0) + \dot{P}_\mathrm{s}(t_0)(t - t_0) + \frac{1}{2!}\ddot{P}_\mathrm{s}(t_0)(t - t_0)^2 + \;\cdots$
  (all these times should be in pulsar frame)
- We have just measured ToAs, we do not know any of $P_\mathrm{s}(t)$, $P_\mathrm{s}(t_0)$, $\dot{P}_\mathrm{s}(t_0)$, etc. ... these are unknowns, model parameters. (Actually appoximate $\dot{P}_\mathrm{s}(t_0)$ is known from discovery)

  In reality we fit $f_s = P_\mathrm{s}^{-1}$ and its derivatives

  $f_\mathrm{s}(t) = f_\mathrm{s}(t_0) + \dot{f}_\mathrm{s}(t_0)(t - t_0) + \frac{1}{2!}\ddot{f}_\mathrm{s}(t_0)(t - t_0)^2 + \;\cdots$
- Model parameters: $f_s(t_0)$, $\dot{f}_s(t_0)$, $\ddot{f}_s(t_0)$ $\;\cdots$
- $f_s(t)$ is F0 and t in $f_s(t)$ is PEPOCH in the par file of tempo2.

Timing analysis of pulsars: Complication-2 (DM)

- It is not vacuum between the pulsar and the earth, there is ISM.
- So, the signal does not travel with the speed c, travel with the group velocity v_g.
- v_g depends on the frequency of the signal.
- pulsars are broadband emitter .. we observe at diffrent frequencies (different bands) and each band of about 200 MHz wide!
- Band is divided into N channels, data recorded at these channels, then added.
- Signals at different channels arrive at different times - addition do not give a good signal
- We need to correct for this dispersive delay to get a good pulse.
- pulse arrival times at the central frequency is repoted.



$$v_g = c\left[1-\left(\frac{\nu_p}{\nu}\right)^2\right]^{1/2},\ \nu_p = \sqrt{\frac{e^2 n_e}{\pi m_e}}$$

$$\nu\uparrow\ \Rightarrow v_g\uparrow\ \Rightarrow t_{travel} = \frac{\int_0^d dl}{v_g}\downarrow,$$

$$\Delta t_{\rm band} = \frac{e^2}{2\pi m_e c}\left(\frac{1}{\nu_{\rm up}^2}-\frac{1}{\nu_{\rm low}^2}\right)\int_0^d n_e(RA, DEC)dl.$$

Dedispersion: correction of Δt to get a good pulse

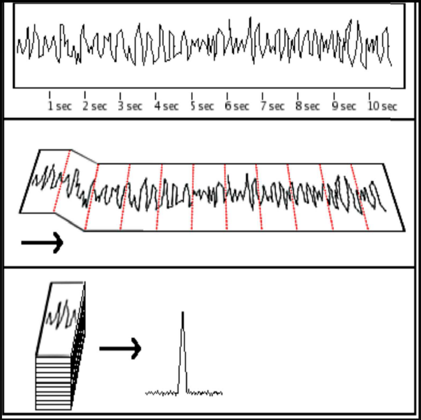
https://www.atnf.csiro.au/research/radio-school/2011/talks/pulsar_observations.pdf

Timing analysis of pulsars: Complication-2 (DM)

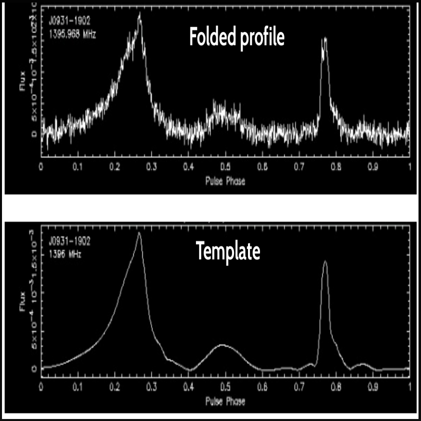
- $\Delta t_{\rm band} = \frac{e^2}{2\pi m_e c}(\frac{1}{\nu_{\rm up}^2} - \frac{1}{\nu_{\rm low}^2}) \int_0^d n_e(RA, DEC) dl = \mathcal{D}\,{\rm DM}\,(\frac{1}{\nu_{\rm up}^2} - \frac{1}{\nu_{\rm low}^2})$
- We know $\nu_{\rm low}$ and $\nu_{\rm up}$
- When we get a good pulse (through de-dispersion) we know DM.
- Dispersion Measure (DM)$= \int_0^d n_e(RA, DEC) dl$
- $\mathcal{D} = \frac{e^2}{2\pi m_e c} = 4.1488 \times 10^3 \ {\rm MHz\ pc^{-1}\ cm^3\ s}$
- n_e model known (RA, DEC needed): NE2001 model / YM16 model (demonstration?)
- Valuse of DM gives d
- Timing analysis improves DM!

Timing analysis of pulsars: Complication-3

We don't see each pulse! (rarely see)



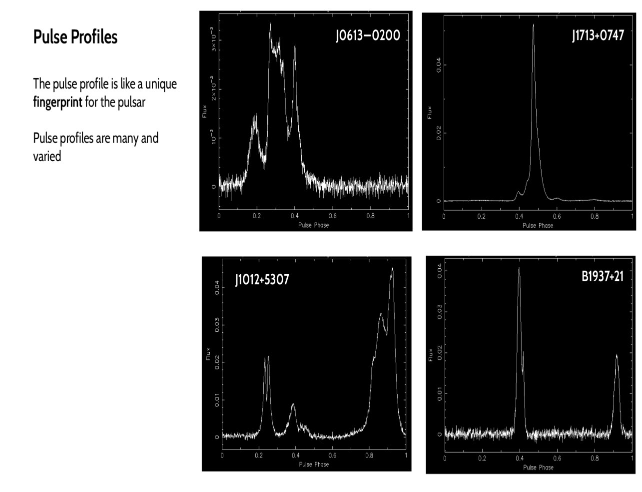
Ryan Lynch: Pulsar searching tutorial for PSC



James McKee: Pulsar timing tutorial, IPTA 2018 student week

(i) Make a template from the highest S/N profile, (ii) Use the template to get ToAs from other profiles.

Timing analysis of pulsars: Complication-1



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Timing analysis of pulsars: Why d is not constant?

- The earth is orbiting around the Sun: solution is to convert the ToAs to the barycenter (centre of mass) of the solar system (Römer delay)).
- You need to find the location of SSB - need masses and positions of the planets (including the earth). Data analysis software (tempo2) has the model for the solar system in 'tempo2/T2runtime/ephemeris/'

  Location of the telescope on the earth ('tempo2/T2runtime/observatory/')

  Need to know Earth's orientation (due to spin) ('tempo2/T2runtime/earth/')

  **Ephemeris and earth orientation need regular updates*
- Galactic rotation: Sun is moving around the Galactic centre (negligible)
- Galactic rotation: Pulsar is moving around the Galactic centre (neglible)
- The pulsar moves, kick imparted during the supernova.
- proper motion is a model parameter, proper motion is the angular motion in the sky RA, DEC come here too! ($\mu_{\rm ra}$, $\mu_{\rm dec}$) or (μ_l, μ_b, galactocentric) or (μ_{long}, μ_{lat} heliocentric ecliptic), μ_T total proper motion.
- $*$ Orbital motion for binary pulsars!

Timing analysis of pulsars: orbital motion (classical)

Kepler's third law:

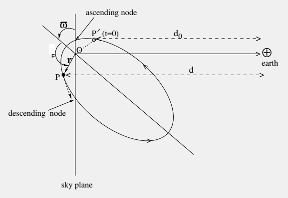
$$a_R = \left[\left(\frac{P_b}{2\pi}\right)^2 G\,(m_p + m_c)\right]^{1/3}$$

$$a_p = a_R\,\frac{m_c}{m_p + m_c}; \qquad a_c = a_R\,\frac{m_p}{m_p + m_c}$$

$$a_p^{'} = a_p \sin i = x$$

$d_0 \neq d$

we need $\Delta\, d = d - d_0$



Equation of an ellipse $\perp$ to the sky plane (the orbit projected $\perp$ to the sky):

$$r = a_p^{'}\left(1 - e^2\right)\left(1 + e\cos F\right)^{-1}$$

(Differentiating, we get $\dot{r} = v$, which comes in the expression of $\Delta_{\rm EB}$)

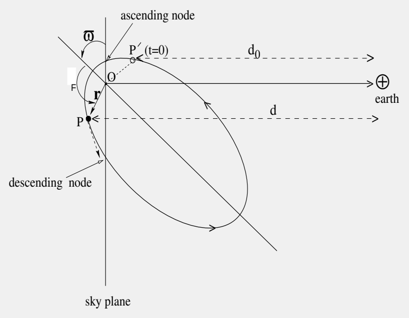
r projected along the line-of-sight is:

$$\Delta d = r_l = a_p^{'}\left(1 - e^2\right)\left(1 + e\cos F\right)^{-1}\sin\left(F + \varpi\right)$$

Fit e, ϖ, $a_p^{'}$, F to get $\Delta d(t)$ .... but instead of F, we fit P_b and T_0

Timing analysis of pulsars: orbital motion (classical)

F changes as the pulsar moves in the orbit. Kepler's equations give the variation of F with time t:



$$\boxed{\begin{aligned} M = n_b\,(t - T_0),\;\; n_b = \frac{2\pi}{P_b} \\ E - e\sin E = M \\ F = 2\tan^{-1}\left[\sqrt{\frac{1+e}{1-e}}\tan\frac{E}{2}\right] \end{aligned}}$$

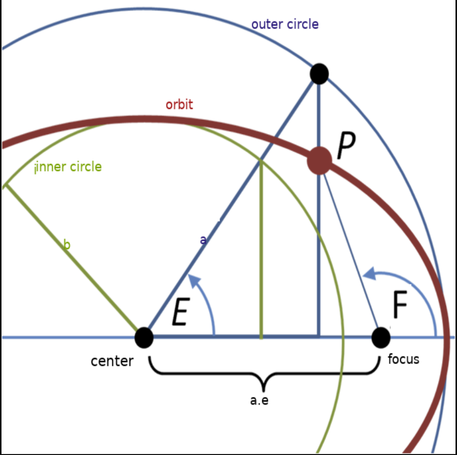
F: true anomaly, M: mean anomaly, E: eccentric anomaly and T_0: epoch of the periastron passage.

Fit e, P_b, T_0: get $M(t)$, $E(t)$, and $F(t)$

$*$ Keplerian orbital parameters to fit: P_b, $a_p^{'}$, e, ϖ, T_0

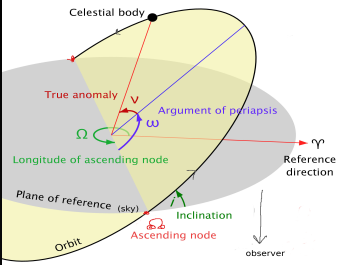
$*$ If e is small, then instead of e, ϖ, T_0; we fit $\epsilon_1\,(= e\sin\varpi)$, $\epsilon_2\,(= e\cos\varpi)$, $T_{\rm asc} = T_0 - \frac{\varpi}{n_b}$ (ELL1 model: Lange et al, 2001, MNRAS, 326, 274).

Orbital Geometry: Anomalies



(CheCheDaWaff: Wikipedia)

Orbital Geometry: Line of Node



(Wikipedia)

- Reference plane (sky plane): plane tangent to the celestial sphere at the point of interest.
- Node: either of the two points where the orbit intersects the reference plane.
- Line of nodes: the line joining the two nodes
- The 'ascending node' is the node where the orbiting secondary passes away from the observer, and the 'descending node' is the node where it moves towards the observer.
- Reference direction: The 'north', i.e., the perpendicular projection of the direction from the observer to the North Celestial Pole onto the sky plane

Wikipedia

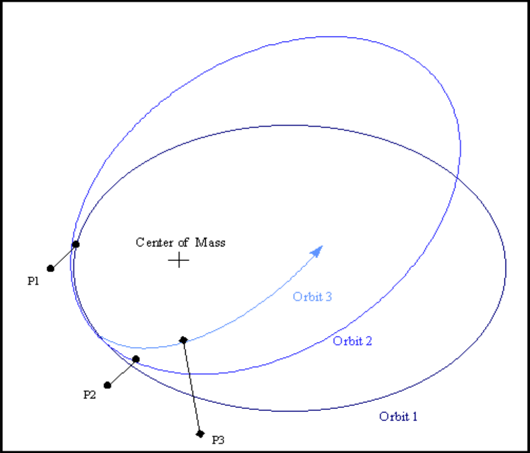
Timing analysis of pulsars: orbital motion (general relativistic)

* Keplerian orbital parameters to fit: P_b, $a_p^{'}$; e, ϖ, T_0 OR ϵ_1, ϵ_2, $T_{\rm asc}$
  * But P_b, e, ϖ NOT constant: we fit $\dot{P}_b$, $\dot{\varpi}$ ($\dot{e}$ is negligible small)
* $\dot{\varpi}$ negligible for small e binaries.
* 2 Post-Keplerian orbital parameters to fit: $\dot{P}_b$, $\dot{\varpi}$

- There are 3 more PK parameters (will discuss soon)!

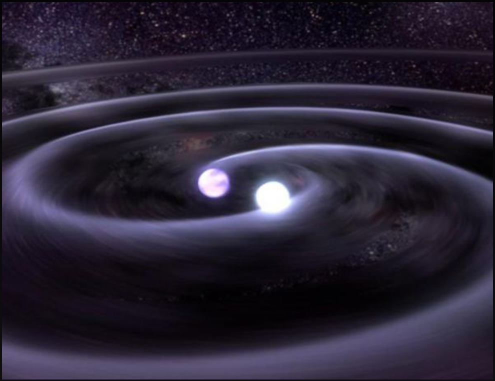
Post-Keplerian parameters for binary pulsars: $\dot{\varpi}$

$\dot{\varpi}$



http://hosting.astro.cornell.edu/academics/courses/astro201/psr1913.htm

Post-Keplerian parameters for binary pulsars: $\dot{P}_b$

$\dot{P}_b$, $\dot{e}$ 

$a_R = \left[\left(\frac{P_b}{2\pi}\right)^2 \, G\,(m_p + m_c)\right]^{1/3}$; So, $a_R \downarrow \implies P_b \downarrow$

(https://www.southampton.ac.uk/maths/research/projects/gravitational-waves.page)

Timing analysis of pulsars: More Relativistic Effects

Curvature of space due to mass (GR) affecting the value of d (Shapiro)

- planets in the solar system solar system ephemeris in tempo2 ('tempo2/T2runtime/ephemeris/')

  **You need to use the best ephemeris (DE438?) **
- companion of the pulsar (for a binary pulsar)

Special and General relativistic effects on time (Einstein)

- Moving clock runs slow. The pulsar and the earth are moving! (SR)
- A clock runs slow when it is in a gravitational potential (GR)

Timing analysis of pulsars: Usual Delays

All these effects in d and the clock are come as 'delays' in measured Time of Arrivals (ToAs) .. T_{oi}s

$*$ Except $\Delta\, d$ in the orbit that is taken care of by Keplerian parameters

- Delays common to solitary and binary pulsars
- Delays for binary pulsars

$$t_{\mathrm{SSB}} = t_{\mathrm{topo}} + t_{\mathrm{atm}} + t_{\mathrm{corr}} + \Delta_{\mathrm{R}\odot}$$

$$t_{\mathrm{psr}} = t_{\mathrm{SSB}} + t_{\mathrm{disp}} + \Delta_{\mathrm{S}\odot} + \Delta_{\mathrm{E}\odot} + (\Delta_{\mathrm{RB}} + \Delta_{\mathrm{SB}} + \Delta_{\mathrm{EB}} + \Delta_{\mathrm{AB}})$$

t_{psr}s are used in $f_s(t_i) = f_s(t_0) + \dot{f}_s(t_0)(t_i - t_0) + (1/2!)\,\ddot{f}_s(t_0)(t_i - t_0)^2\ \cdots$

Timing analysis of pulsars: Delays common to solitary and binary pulsars

- t_{topo}: Times at observatory (Initial: measured, T_{oi})
- t_{psr}: Times at pulsar (Final, T_{pi})
- t_{SSB}: Time at Solar System Barycentre
- t_{atm}: Atmospheric delay (usually not fitted)
- t_{corr}: clock corrections

  Which time? Indian time? Greenwich time?? UTC???

  International Time Standard 'Bureau international des Poids et Mesures' (leap year, leap second): MJD

  'tempo2/T2runtime/clock/' directory has files for clock corrections.
- t_{disp}: Dispersion delay
- $\Delta_{\mathrm{R}\odot}$: Römer delay
- $\Delta_{\mathrm{S}\odot}$: Shapiro delay
- $\Delta_{\mathrm{E}\odot}$: Einstein delay

Timing analysis of pulsars: Dispersion Delay

- We report ToAs for central frequency (ν_c) of the band (so dispersion delay within the band is taken care of by 'dedispersion')
- Better to convert ToAs as if it was all vacuum (no ISM)!

$$t_{\mathrm{disp}} = \frac{\mathcal{D}\ \mathrm{DM}}{\nu_c^2}$$

DM is a fit parameter $\Rightarrow$ improvement of DM

ISM is inhomogeneous, non-isotropic, and dynamic, pulsars are also moving:
$DM(t)$, $\frac{d\mathrm{DM}}{dt}$, $\frac{d^2\mathrm{DM}}{dt^2}$, $\cdots$ are model parameters

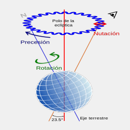
t in DM(t) is DMEPOCH in tempo2 par files. DMEPOCH and PEPOCH should be the same after a tempo2 fitting, but can be different if your initial DM and P_{s} values are taken from different measurements.

Timing analysis of pulsars: Dispersion Delay (derivation)

$$t_{\mathrm{ISM}} = \frac{\int_0^d dl}{v_g}, \quad t_{\mathrm{vac}} = \frac{d}{c}; \quad v_g = c\left[1 - \left(\frac{\nu_p}{\nu_c}\right)^2\right]^{1/2}$$

$$\begin{aligned}
t_{\mathrm{disp}} &= t_{\mathrm{ISM}} - t_{\mathrm{vac}} \\
&= \frac{1}{c}\int_0^d \left[1 - \left(\frac{\nu_p}{\nu_c}\right)^2\right]^{-1/2} dl - \frac{d}{c} \\
&= \frac{1}{c}\int_0^d \left[1 + \frac{1}{2}\left(\frac{\nu_p}{\nu_c}\right)^2\right] dl - \frac{d}{c} \\
&= \frac{d}{c} + \frac{1}{2}\int_0^d \left[\left(\frac{\nu_p}{\nu_c}\right)^2\right] dl - \frac{d}{c}; \quad \nu_p = \sqrt{\frac{e^2 n_e}{\pi m_e}} \\
&= \frac{e^2}{2\pi m_e}\frac{1}{\nu_c^2}\int_0^d n_e dl; \quad \mathcal{D} = \frac{e^2}{2\pi m_e c}; \quad \mathrm{DM} = \int_0^d n_e dl \\
&= \frac{\mathcal{D}\ \mathrm{DM}}{\nu_c^2}
\end{aligned}$$

Timing analysis of pulsars: Römer delay



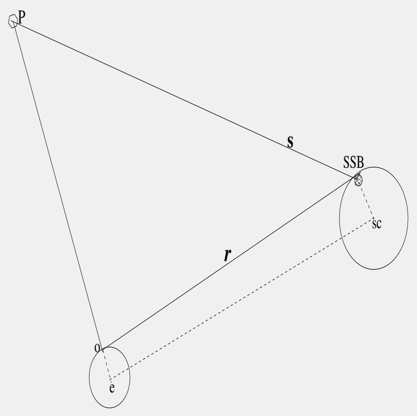
wikipedia

classical light travel time between the observatory and the SSB

$$\Delta_{\mathrm{R}\odot} = -\frac{\vec{r} \cdot \hat{\mathrm{s}}}{c}$$

$$\vec{r} = \vec{r}_{\mathrm{o,e}} + \vec{r}_{\mathrm{e,SC}} + \vec{r}_{\mathrm{SC,SSB}}$$

$$\vec{r}_{\mathrm{SC,SSB}} = \frac{\sum_i \frac{r_i}{m_i}}{1 + \sum_i m_i}$$



- Observatory location needed for $\vec{r}_{\mathrm{o,e}}$: geocentric coordinates of observatories in 'tempo2/T2runtime/observatory/'
- Earth's spin (precession and nutation) is needed 'tempo2/T2runtime/earth/'
- Planetary ephemeris (masses, locations) + sun's mass needed to determine $\vec{r}_{\mathrm{SC,SSB}}$ 'tempo2/T2runtime/ephemeris/' (DE438?)

Timing analysis of pulsars: Shapiro Delay

Earth location and planetary ephemeris needed Handbook of Pulsar Astronomy by Lorimer and Kramer

The *Shapiro delay*, $\Delta_{S\odot}$, is a relativistic correction that corrects for extra delays due to the curvature of space-time caused by the presence of masses in the solar system (Shapiro 1964). The delays are largest for a signal passing the limb of the Sun ($\sim 120\ \mu$s) while Jupiter can contribute as much as 200 ns. In principle, one has to sum over all bodies in the Solar System, yielding

$$\Delta_{S\odot} = -2 \sum_i \frac{GM_i}{c^3} \ln \left[\frac{\hat{s} \cdot \vec{r}_i^{\,E} + r_i^{\,E}}{\hat{s} \cdot \vec{r}_i^{\,P} + r_i^{\,P}} \right], \qquad (8.8)$$

where G is Newton's gravitational constant, M_i is the mass of body i, $\vec{r}_i^{\,P}$ is the pulsar position relative to it and $\vec{r}_i^{\,E}$ is the telescope position relative to that body at the time of closest approach of the photon. In practice, usually only the Sun, and in some cases Jupiter, need to be

Timing analysis of pulsars: Einstein Delay

The last term in Equation (8.6), $\Delta_{\mathrm{E}\odot}$, is called the *Einstein delay* and describes the combined effect of time dilation due to the motion of the Earth and gravitational redshift caused by the other bodies in the Solar System. This time varying effect takes into account the variation of an atomic clock on Earth in the changing gravitational potential as the Earth follows its elliptical orbit around the Sun. The delay amounts to an integral of the expression (Backer & Hellings 1986)

$$\frac{\mathrm{d}\Delta_{\mathrm{E}\odot}}{\mathrm{d}t} = \sum_i \frac{GM_i}{c^2 r_i^{\mathrm{E}}} + \frac{v_{\mathrm{E}}^2}{2c^2} - \text{constant}, \tag{8.9}$$

where the sum is again over all bodies in the Solar System but this time excluding the Earth. The distance r_i^{E} is the distance between the Earth and body i, while v_{E} is the velocity of the Earth relative to the Sun.

Handbook of Pulsar Astronomy by Lorimer and Kramer

Timing analysis of pulsars: Einstein Delay (derivation-1)

Moving clocks (clocks on the Earth) run slower!

$$t_{\rm moving} = \frac{t_{\rm rest}}{(1-\frac{v^2}{c^2})^{1/2}} = \frac{t_{\rm rest}}{(1-\frac{v^2}{2\,c^2})}; \quad t_{\rm rest} = t_{\rm moving}(1-\frac{v^2}{2\,c^2})$$

$$\Delta t = t_{\rm moving} - t_{\rm rest} = t_{\rm moving} - t_{\rm moving}(1-\frac{v^2}{2\,c^2}) = t_{\rm moving}\,\frac{v^2}{2\,c^2}$$

A clock runs slow when kept in a gravitational potential!

Line element in GR:

$$ds^2 = g_{\mu\,\nu}\,dx^\mu\,dx^\nu$$

Proper lengths

$$dl^0 = \sqrt{g_{00}}\,dx^0;\ dl^1 = \sqrt{g_{11}}\,dx^1;\ dl^2 = \sqrt{g_{22}}\,dx^2;\ dl^3 = \sqrt{g_{33}}\,dx^3$$

(^{0}, ^{1}, ^{2}, ^{3} are indices, NOT powers)

Timing analysis of pulsars: Einstein Delay (derivation-2)

In Schwarzschild (t, r, θ, ϕ):

$g_{00} = (1 - \frac{2GM}{c^2\, r})$, $g_{11} = -(1 - \frac{2GM}{c^2\, r})^{-1}$, $g_{22} = -r^2$, $g_{33} = -r^2 \sin\theta$

So,

$$dl^0 = \left(1 - \frac{2GM}{c^2\, r}\right)^{1/2} dx^0 = \left(1 - \frac{GM}{c^2\, r}\right) dx^0$$

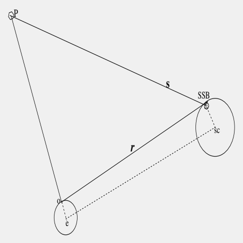
$$dx^0 - dl^0 = dx^0 - \left(1 - \frac{GM}{c^2\, r}\right) dx^0 = \frac{GM}{c^2\, r}\, dx^0$$

Timing analysis of pulsars: Delays in binary pulsars

- Δ_{RB}: Römer delay
- Δ_{SB}: Shapiro delay
- Δ_{EB}: Einstein delay
- Δ_{AB}: Aberration Delay

Timing analysis of pulsars: Delays for binary pulsars

- $\Delta_{\rm RB}$: Römer delay in binary



Remember,

$$\Delta_{\rm R\odot} = -\frac{\vec{r}\cdot\hat{\rm s}}{c}$$

As the pulsar moves in a binary orbit, $\hat{\rm s}$ changes continously.

$$\begin{aligned}\Delta_{\rm RB} &= x(\cos E - e)\sin\varpi + x\sin E\sqrt{1-e^2}\cos\varpi \quad \text{(classical)}\\ &= x(\cos E - e_r)\sin\varpi + x\sin E\sqrt{1-e_\theta^2}\cos\varpi \quad \text{(GR)}\end{aligned}$$

where

$$\begin{aligned}e_r &= e(1+\delta_r) \quad \text{(comes in the ellipse equation)}\\ e_\theta &= e(1+\delta_\theta) \quad \text{(comes in Kepler equations)}\end{aligned}$$

δ_r and δ_θ are two new parameters; rarely fitted as $\Delta_{\rm RB}$ is usually small

$*$ But we know that for small e pulsars; e and ϖ are not fitted!!

Timing analysis of pulsars: Delays for binary pulsars

- $\Delta_{\rm RB}$: Römer delay in small eccentrycity binary where e and ϖ are not fitted!!

$$\Delta_{\rm RB} = x\left(\sin\Phi - \frac{\epsilon_1}{2}\cos 2\Phi + \frac{\epsilon_2}{2}\sin 2\Phi\right)$$

where $\epsilon_1 = e\sin\varpi$ and $\epsilon_2 = e\cos\varpi$ (ELL1 model)

$\Phi = n_b(t - T_{\rm asc}), \quad T_{\rm asc} = T_0 - \frac{\varpi}{n_b}, \quad M = n_b\,(t - T_0), \quad n_b = \frac{2\pi}{P_b}$

Fit parameters: $\epsilon_1,\ \epsilon_1,\ T_{\rm asc}, P_b$

Timing analysis of pulsars: Delays for binary pulsars

- $\Delta_{\rm SB}$: Shapiro delay in binary (spacetime curvature due to the companion)

$$\Delta_{\rm SB} = -2r \ln \left[1 - e \cos E - s \left(\sin \varpi (\cos E - e) + \sqrt{1 - e^2} \cos \varpi \sin E\right)\right]$$

2 PK parameters to fit: $r = \frac{Gm_c}{c^3}$, $s = \sin i$.

(e and ϖ: already included Keplerian parameters. E can be calculated from e and P_b using Kepler equation. P_b is fitted as a Keplerian paramerer. Often we first fit Keplerian parameters, then Keplerian + PK parameters as contribution from PK parameters are smaller.)

For small e binaries, e and ϖ are not measured

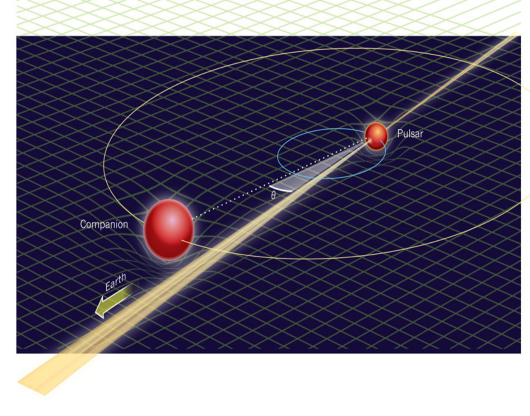
$$\Delta_{\rm SB} = -2r\, f(\Phi);\ \Phi = \frac{2\pi}{P_b}(t - T_{\rm asc});\ f(\Phi) = \sum_{k=1}^{\infty} a_k \cos(k\Phi) + \sum_{k=1}^{\infty} b_k \sin(k\Phi)$$

low-inclination: $k = 1, 2$; medium inclination: $k = 1, 2, 3$, high inclination: $k = 1, 2, 3, 4$, very high inclination ($\sin i \approx 1$) $k = 1, 2, 3, 4, 5$

$\bar{c} = |\cos i|$, $\varsigma = \frac{s}{1+\bar{c}}$, $h_3 = r\varsigma$, $h_4 = h_3\,\varsigma$. Often h_3 and ς are reported.

$a_0 = -2ln(1 + \varsigma^2)$, $a_k = (-1)^{\frac{k+2}{2}} \frac{2}{k} \varsigma^k$ for even k otherwise 0, $b_k = (-1)^{\frac{k+1}{2}} \frac{2}{k} \varsigma^k$ for odd k otherwise 0 (Freire and Wex, MNRAS 409, 2010, 199) (ELLH, DDH models)

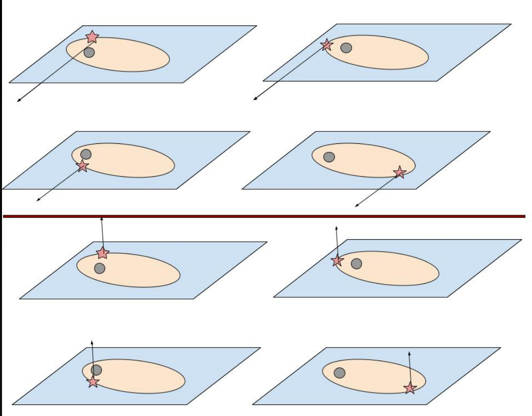
Timing analysis of pulsars: Shapiro delay in binary



$$s = \sin i, \ r = \frac{Gm_c}{c^3}$$

https://www.nature.com/articles/4671057a;   lager m_c $\Rightarrow$ more curvature

Timing analysis of pulsars: Shapiro delay in binary (II)



$$s = \sin i, \; r = \frac{Gm_c}{c^3}$$

understanding *i* dependence, curvature of space NOT shown for simplicity

Timing analysis of pulsars: Delays for binary pulsars

- Δ_{EB}: Einstein delay

  gravitational redshift due to companion's gravitational potential $\left(\frac{Gm_c}{c^2 r}\right)$ + time dilation due to the motion in the orbit $\left(\frac{v^2}{2c^2}\right)$, r, v are functions of a'_p, e, F (orbit eq.)

$$\Delta_{\mathrm{EB}} = \gamma \sin E$$

1 PK parameters to fit: γ (physically depends on e, P_b, m_p, m_c)

e comes as a multiplicative factor in γ, so for small e pulsars, Δ_{EB} is small - no need of alternate parametrization.

Unlike Δ_{RB} and Δ_{SB} where small e does not necessarily make these small.

(E can be calculated from e and P_b using Kepler equation. e and P_b are fitted as Keplerian parameters. Often we first fit Keplerian parameters, then Keplerian + PK parameters as contribution from PK parameters are smaller.)

Timing analysis of pulsars: Aberration Delay

$\Delta_{\rm AB}$ depends on the time-variable transverse component of the pulsar's orbital velocity.

$$\Delta_{\rm AB} = \mathcal{A}(t)\left[\sin(\varpi + F) + e\sin\varpi\right] + \mathcal{B}(t)\left[\cos(\varpi + F) + e\cos\varpi\right]$$

$$\mathcal{A}(t) = -\frac{P_s}{P_b}\,\frac{a_p^{'}/c}{\sin i\,(1-e^2)^{1/2}}\,\frac{\sin\eta}{\sin\lambda}$$

$$\mathcal{B}(t) = -\frac{P_s}{P_b}\,\frac{a_p^{'}/c}{\sin i\,(1-e^2)^{1/2}}\,\frac{\cos i\,\cos\eta}{\sin\lambda}$$

F: true-anomaly

λ: the angle between the spin axis and the line of sight

η: the angle between the spin axis and the line of nodes

(geodetic-precesion comes into play: arXiv:1606.02744)

Timing analysis of pulsars: Light Bending delay (rarely fitted)

Timing analysis of pulsars: Model Parameters

$$t_{\rm psr} = t_{\rm topo} + t_{\rm atm} + t_{\rm corr} + t_{\rm disp} + \Delta_{\rm R\odot} + \Delta_{\rm S\odot} + \Delta_{\rm E\odot} + (\Delta_{\rm RB} + \Delta_{\rm SB} + \Delta_{\rm EB} + \Delta_{\rm AB})$$

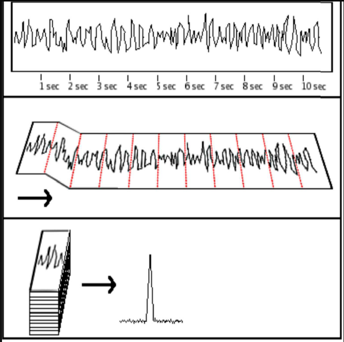
To calculate $t_{\rm psr}$ from $t_{\rm topo}$, we need the parameters listed below. But how do we now that our calculated $t_{\rm psr}$ are correct? We need to cross-check - fitting with tempo2!

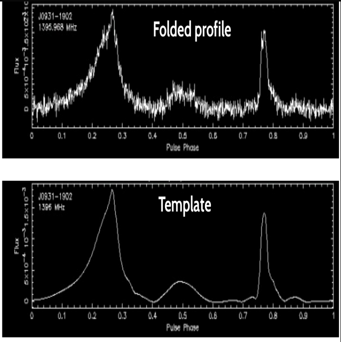
- Pulsar parameters: RA, DEC, $\mu_{\rm l}$, $\mu_{\rm b}$.
- Keplerian orbital parameters: $P_{\rm b}$, $x = a'_p = a_p \sin i$; e, ϖ, T_0 (or ϵ_1, ϵ_2, $T_{\rm asc}$)
  - Note: Often x/c is reported
- Post-Keplerian orbital parameters: $\dot{P}_b$, γ; $\dot{\varpi}$; r, s, (or h_3, ς, h_4)
- For specific pulsars, additional parameters might be fitted, like $\Omega_{\rm so}$ (geodetic precession due to the spin-orbit coupling), $\dot{a}'_p$, ...

  See Handbook of Pulsar Astronomy by Lorimer and Kramer for a list!
- More pulsar parameters: f_s, $\dot{f}_s$, $\ddot{f}_s$, etc.

Timing analysis of pulsars: Getting ToAs (t_{topo})

- Analyse raw data, fold and get (nice) integrated profiles
  (Use 'pinta' for uGMRT data)
- Use integrated profiles, make templates, find Time of Arrivals (ToAs) at the observatory. (Use 'psrchive')





Timing analysis of pulsars: ToA uncertainty

$$\sigma_{TOA} = \left(\frac{T_{sys}}{G}\right)\left(\frac{\eta}{S}\right)\frac{1}{\sqrt{\eta t B n_p}}\eta P$$

Typical Values

P	$= 0.004$ s	Pulse period
η	$= 0.05$	Duty cycle
T_{sys}	$= 20$K	System temperature
G	$= 2$ K/Jy	Telescope gain
S	$= 0.001$ Jy	Pulsar flux density
t	$= 1000$ s	Observation time
B	$= 10^8$ Hz	Bandwidth
n_p	$= 2$	Number of polarizations

$\rightarrow \sigma_{\mathrm{TOA}} = 200$ ns

From David Nice's talk, IPTA 2011 student week

Ideal: In reality uncertainties might be larger.
Use 'EFAC' (error factor $\sigma_{\mathrm{f}} = efac \times \sigma_{\mathrm{in}}$)
or 'EQUAD' (error quadrature; $\sigma_{\mathrm{f}} = \sqrt{\sigma_{\mathrm{in}}^2 + equad^2}$).

Timing analysis of pulsars: Fitting Procedure-1

- Timing analysis software needs ToA file (file.tim) plus a file containing initial values of parameters (file.par).
- If you are working with a known pulsar (as in InPTA), you have a par file from old fitting. Your fitting makes the parameters more accurate. You save the new parameters (newfile.par).
- If you are timing a new pulsar (say you discovered one), you can start with RA, DEC, f_s, DM obtained during the discovery.
- If you are adding ToA files from different observatories (or same observatory, different recording instruments), there might be 'delay' between the observatories/instruments. These are constants and known, should be added as 'JUMP's in the ToA file.

Timing analysis of pulsars: A sample tim file

* We have a file containing ToAs for a particular pulsar (file.tim):

file name	central freq	ToA in MJD	ToA uncertainty (μs)	observa name
filename1.fits	399.803986	59001.089293939020819	1.755	gmrt
filename2.fits	399.803986	59016.025058066739341	2.355	gmrt
filename3.fits	399.803986	59032.951215187515695	1.081	gmrt

** There might be more information (not-essential)

MJD: Continuous counting of time (days) – no complications of calendar (different months have different days, leap years etc)

MJD: Modified Julian date: The MJD therefore gives the number of days since midnight on 17-November, 1858.

JD: The number of days since noon on 01-January, 4713 BCE. It was proposed by J. J. Scaliger, so the name for this system derived from Julius Scaliger, NOT Julius Caesar.

https://scienceworld.wolfram.com/astronomy/JulianDate.html

A simple par file

PSRJ	J2145-0750		
RAJ	21:45:50.4639173		
DECJ	-07:50:18.53014		
F0	62.295887805089181947	1	0.00000000102503194795
F1	-1.8029995182581858378e-16	1	3.4231063733881641 95e-17
PEPOCH	58404		
POSEPOCH	58404		
DMEPOCH	58404		
DM	9.0106150628095362303		
PMRA	-9.5592174405136950202		
PMDEC	-8.9227666924681511866		
PX	1.7707489281518760195		
BINARY	T2		
PB	6.8389026359057731664	1	0.00000000467683565080
A1	10.164193812228306047	1	0.00005388980832090236
PBDOT	8.6585264952286113539e-14		

* There might be additional parameters
* You can model DM variation with time using DMMODEL/DMX technique

name of the parameter, value, tag, uncertainty

tag:
1: automatically fit, 0: NO fit, blank: fit by hand

Timing analysis of pulsars: Fitting Procedure-2

- $P_s \simeq t_{i+1} - t_i$: time difference between consecutive pulses (when $t_{\rm psr,i} = t_i$)

  $f_s = P_s^{-1}$ rate of change of pulse number.

$$f_s(t_i) = \frac{1}{t_{i+1} - t_i} = \frac{[N(t_{i+1}) - N(t_i)]}{(t_{i+1} - t_i)} \cdots (A)$$

$$N(t_{i+1}) = f_s(t_i) \times (t_{i+1} - t_i) + N(t_i) \cdots (\mathrm{B})$$

- RHS of Eq. (A) is valid for even non-consecutive pulses!
- We have a set of t_is that corresponds to N_is.
- We calculate $f_s(t_i) = f_s(t_0) + \dot{f}_s(t_0)(t_i - t_0) + (1/2!)\, \ddot{f}_s(t_0)(t_i - t_0)^2 \;\; \cdots (C)$ by fitting parameters in tempo2. We insert those in the RHS of the above equation (B) to obtain a value of the LHS. $\quad \mathcal{N}(t_{i+1})$.

  $*$ Pulse number should always be integer, say the nearest integer is $n(t_{i+1})$.
- We minimise χ^2 by changing model parameters . We use residuals (R_i) to understand the fitting!

$$\chi^2 = \sum_i \left(\frac{\mathcal{N}(t_{i+1}) - n(t_{i+1})}{\sigma(t_{i+1})} \right)^2 ; \quad R_i = \frac{\mathcal{N}(t_{i+1}) - n(t_{i+1})}{f_s(t_{i+1})}$$

- The best fit model parameters: timing solution!

Timing analysis of pulsars: fitting procedure explained

	N_i (no. of pulse)	ToA t_{0i}	uncert.	$t_{psr,i}$
$i=1$	11	t_{01} (MJD1)	σ_1	t_1
$i=2$	100	t_{02} (MJD2)	σ_2	t_2
$i=3$	250	t_{03} (MJD3)	σ_3	t_3
⋮	⋮	⋮		
NOT in tim file	NOT in tim file (we don't know!!)	in tim file		

Explanation of eqn(A)

Say $P_s = 2\,s$

$f_s = \frac{1}{2}\,s^{-1} = 0.5\,s^{-1}$

Say pulse 1 is at 5.5 sec
2 " " 7.5 "
⋮
7 " " 17.5 "

$$\boxed{f_s = \frac{1\ (=2-1)}{7.5-5.5} = \frac{7-1}{17.5-5.5}}$$

- $f_{s1} = f_{s,\text{initial at } t=t_0} + \dot{f}_{guess}(t_1 - t_0)$ (from par file)
- $f_{s2} = f_{s1} + \dot{f}_{s1}(t_2 - t_1) + \frac{\ddot{f}_{s1}}{2!}(t_2 - t_1)^2 + \dots$ (fit)
- $f_{s3} = f_{s2} + \dot{f}_{s2}(t_3 - t_2) + \frac{\ddot{f}_{s2}}{2!}(t_3 - t_2)^2 + \dots$ (fit)

Say · nearest integer

$f_{s\,\text{initial}} \times (t_1 - t_0) + N_0 = N_1 = 11.05$ · 11 (n_1)

(assumed the pulse at t_0 to be no 1)

$f_{s1} \times (t_2 - t_1) + N_1 = N_2 = 100.15$ · 100 (n_2)

$f_{s2} \times (t_3 - t_2) + N_2 = N_3 = 250.32$ · 250 (n_3)

$$R_1 = \frac{N_1 - n_1}{f_{s1}}, \quad R_2 = \frac{N_2 - n_2}{f_{s2}}, \quad R_3 = \frac{N_3 - n_3}{f_{s3}}$$

$$\chi^2 = \left(\frac{N_1 - n_1}{\sigma_1}\right)^2 + \left(\frac{N_2 - n_2}{\sigma_2}\right)^2 + \left(\frac{N_3 - n_3}{\sigma_3}\right)^2 + \dots$$

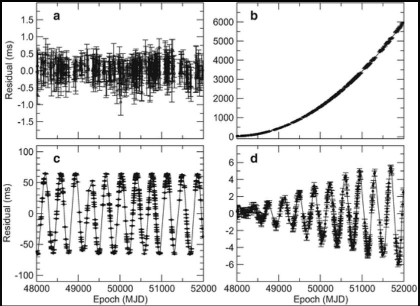
Timing analysis of pulsars: Jobs done by tempo2

1. You have $t_{\rm obs,i}$ in .tim file, initial parameters in .par files
2. Using those, convert $t_{\rm obs,i} \rightarrow t_{\rm psr,i}$
3. Calculate $f_s(t_i)$ using equation (C)
4. Calculate $\mathcal{N}(t_{i+1}), n(t_{i+1})$
5. Compute χ^2
6. Go to step (2) if χ^2 is not good with a change of parameters.
   You can select which parameters to fit (or change)

automatic

7. When minimum χ^2 is obtained, calculate R_i. Plot R_i Vs t_i if you wish.
8. Save the changed parameters in a new par file. "Timing solution"!

Timing analysis of pulsars: The good, the bad, and the ugly



(a) A good fit! (NOT so good - we get residuals in μs and want in nano-s!)

(b) $\dot{P}_s$ is underestimated (by 4%), i.e, $\dot{f}_s$ is overestimated (parabolic increase)

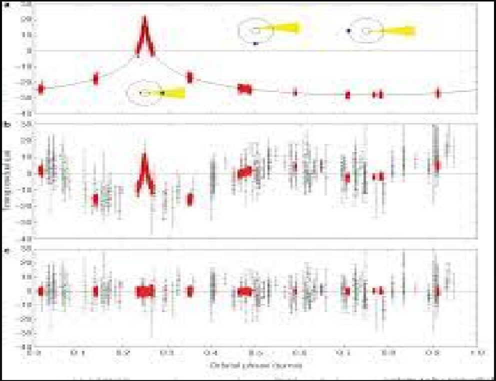
$\dot{P}_s$ underestimated $\Rightarrow$ P_s is smaller than the true value, calculated pulses numbers keep on arriving earlier and earlier

(c) error is position (DEC wrong by 1%): sinusoidal residual of 1 yr period

(d) proper motion NOT fitted ($\mu_T = 380\ \mathrm{milli\text{-}arcsec\,yr^{-1}}$)

Handbook of Pulsar Astronomy by Lorimer and Kramer

Timing analysis of pulsars: The good, the bad, and the ugly



(a) plot of residuals without Shapiro delay, other parameters are the best fit value (in c) .. no effect of Shapiro delay in some of the orbital phases

(b) fit without Shapiro delay, bad residuals and incorrect fitted values of the parameters

(c) best fit with Shapiro delay

PSR J1614−2230: Demorest et al., Nature 467, 1081-1083 (2010)

Timing Noise

- Noise: large residuals
- Structureless (equally bad everywhere) OR with structure(s)
- You have R_i, t_i. Take a fourier transform of t_i series.

  ($t_i \xrightarrow{\text{FT}}$ freq$_i$. These frequencies nothing to do with spin freq, or orbital freq, or freq of the radio waves!!)
- Structureless noise: white noise
- Noise at low frequency: red noise

- White noise
  - Template fitting error from radiometer noise (S/N)
  - Pulse jitter (independent of S/N)
  - Diffractive interstellar scintillations (finite scintle effect)
- Red noise

  Jim Cordes's slide from a talk in 2016
  - Neutron star spin noise
  - DM variations from ISM turbulence
    - Pre-fit: large, RMS ~ μs to tens x μs
    - Post-fit: small, RMS ~ 100 ns
      - » Frequency-dependent DMs, effects of profile evolution with ν
    - ISM refraction
  - Pulsar term GW perturbations (stochastic background)
- Other: clock errors, solar system ephemeris, instrumental polarization [not included in spectra]

Red noise in all pulsars: GWs?!

$$\boxed{\frac{A}{12\pi^2} \frac{f^{2\alpha-3}}{f_0^{2\alpha}}}$$

A : amplitude

f_0 : $1\ \mathrm{yr}^{-1}$

f : freq in yr^{-1},

α : -1 or $-2/3$ (model dependent)

Timing analysis of pulsars: Points not discussed

- Additional dispersion delay due to solar wind!
- tempo2 can do many more tasks, e.g., given parameters, it can generates ToAs for a hypothetical pulsar (‘fake’).
- There is another pulsar timing software ‘pint’ (arXiv:2012.00074). Not much documentation.
- ....
- ....

Timing analysis of pulsars: Summary

Multi-parameter fitting: needs patience

"Pulsar Timing is an art" – Bhal Chandra Joshi!

References (i) tempo2 papers by Hobbs et al, (ii) PhD thesis of Joris Verbiest, (iii) Handbook of pulsar astronomy, (iv) Gravitational Waves by Michele Maggiore ... many more!

PART-II

Applications of Pulsar Timing

Application of Pulsar Timing

- $B_{\mathrm{surf,min}}$, τ_c, n etc from f_s, $\dot{f}_s$, $\ddot{f}_s$
- Glitch in timing data (sudden increase of f_s): internal structure of the pulsar
- Orbital (Keplerian and Post-Keplerian) parameters: Measuring mass of the pulsar and the companion
- Tests of theories of gravity: Disagreement between PK parameters, need of additional terms (e.g. dipollar GW gives different expression for $\dot{P}_b$
- Detection of nan-Hz GW: Nature of timing residuals of an enseble of pulsars (Pulsar Timing Array)
- Measurement of moment of inertia of the pulsar from additional 'lense-Thirring effect' in ϖ: constraining of EoS

Use of Pulsar parameters P_s, $\dot{P}_\mathrm{s}$

$$\dot{E}_{rot} = -I\Omega_s\dot{\Omega}_s, \quad \Omega_s = 2\pi f_s = 2\pi\, P_s^{-1},\ \dot{\Omega}_s = -2\pi\, P_s^{-2}\, \dot{P}_s$$

$\dot{E}_{rad} = \frac{2}{3}\frac{m^2\sin^2\alpha}{c^3}\Omega_s^4$ (dipole; α: angle between the spin and the magnetic axes)

Equating, $\dot{\Omega}_s = -\frac{2}{3}\,\frac{m^2\sin^2\alpha}{Ic^3}\,\Omega_s^3, \quad \ddot{\Omega}_s = -2\,\frac{m^2\sin^2\alpha}{Ic^3}\,\Omega_s^3$

$\boxed{n = \frac{f_s\ddot{f}_s}{\dot{f}_s^2} = 3}$ 'breaking index' (obs. NOT 3 always, not perfect dipole?)

$$\boxed{B_{\mathrm{surf,min}} = \sqrt{\frac{3Ic^3}{8\pi^2R^6\sin^2\alpha}}\sqrt{P_s\dot{P}_s} = 3.2\times10^{19}\sqrt{P_s\dot{P}_s}} \quad (m = B_sR^3)$$

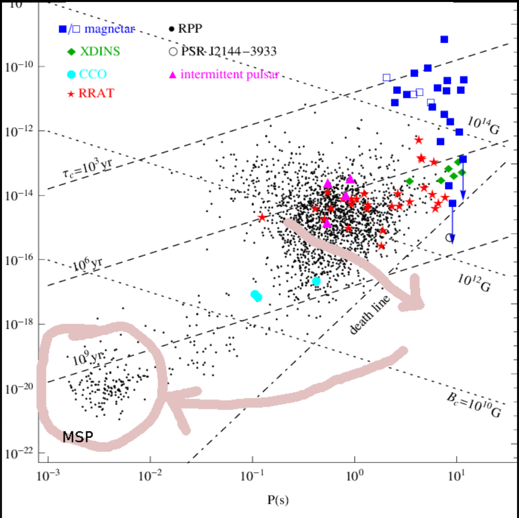
$(R = 10$ km, $I = 10^{45}\ \mathrm{gm\ cm}^2, \quad \alpha = 90^\circ$ giving $\sin\alpha = 1$ max.)

Also, $\frac{dP_s}{dt} = \frac{8\pi^2}{3}\frac{m^2\sin^2\alpha}{Ic^3}\,P_s^{-1} = K_1P_s^{-1} \quad \Longrightarrow \quad \int_{P_{s,0}}^{P_s}P_s\,dP_s = K_1\int_0^\tau dt$

$$K_1\tau = \frac{P^2-P_{s,0}^2}{2} = \frac{P_s^2}{2}\ \text{(assuming } P_{s,0} \ll P_s\text{)}.$$

Putting $K_1 = \dot{P}_s\,P_s$, $\boxed{\tau = \frac{P_s}{2\dot{P}_s}}$ 'characteristic age'

Use of Pulsar parameters P_s, $\dot{P}_s$



(τ lies between $10^3 - 7 \times 10^{10}$ years - age of the Universe 1.38×10^{10} years).

Death line: $\frac{B_{\rm surf,min}}{P_s^2} \geq 0.17 \times 10^{12}\ \mathrm{G\,s^{-2}}$

Keplerian parameters: Mass function

Kepler's third law:

$$a_R = \left[\left(\frac{P_b}{2\pi} \right)^2 \; G \left(m_p + m_c \right) \right]^{1/3}$$

$$a_p = a_R \, \frac{m_c}{m_p + m_c}; \qquad a_c = a_R \, \frac{m_p}{m_p + m_c}$$

$$a_p^{'} = a_p \sin i = x$$

Simple algebra gives:

$$\boxed{\frac{4\pi^2}{G} \, \frac{x^3}{P_b^2} = \frac{(m_c \sin i)^3}{(m_p + m_c)^2}}$$

If we only fit P_b and x (and other Keplerian parameters e, ϖ, T_0, not relavant here); we can evaluate the RHS of the above equation.

But can not find values of m_p, m_c, $\sin i$ separately!

PK parameters: Mass measurements

$$\dot{\omega} = 3 T_\odot^{2/3} \left(\frac{P_b}{2\pi}\right)^{-5/3} \frac{1}{1-e^2} (m_p + m_c)^{2/3} \implies \boxed{f1(P_b, e, \dot{\omega}, m_p, m_c) = 0}$$

$$\gamma = T_\odot^{2/3} \left(\frac{P_b}{2\pi}\right)^{1/3} e \, \frac{m_c(m_p + 2m_c)}{(m_p + m_c)^{4/3}} \implies \boxed{f2(P_b, e, \gamma, m_p, m_c) = 0}$$

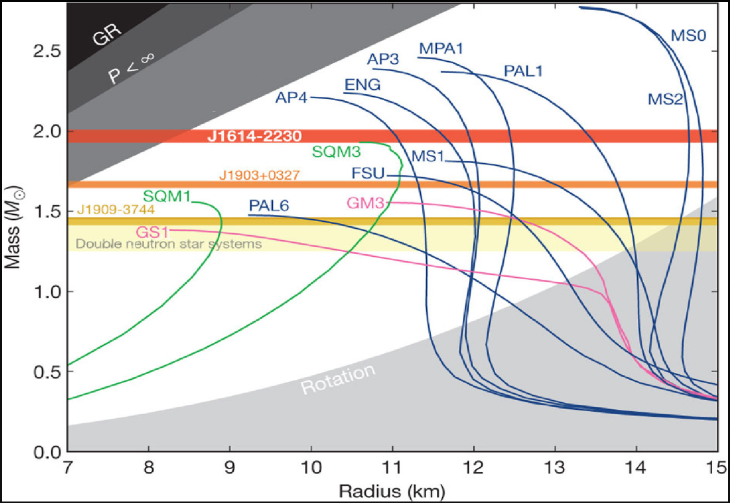
$$r = T_\odot m_c \implies \boxed{r/T_\odot - m_c = 0}$$

$$s = \sin i = T_\odot^{-1/3} \left(\frac{P_b}{2\pi}\right)^{-2/3} \mathcal{X} \, \frac{(m_p + m_c)^{2/3}}{m_c} \implies \boxed{f4(P_b, e, \mathcal{X}, s, m_p, m_c) = 0}$$

$$\dot{P}_b = -\frac{192\pi}{5} T_\odot^{5/3} \left(\frac{P_b}{2\pi}\right)^{-5/3} f(e) \frac{m_p m_c}{(m_p + m_c)^{1/3}} \implies \boxed{f5(P_b, e, \dot{P}_b, m_p, m_c) = 0}$$

$\mathcal{X} = \frac{a_p \sin i}{c}$, $a_p = a_R \frac{m_c}{(m_p+m_c)}$, $P_b^2 = \frac{4\pi^2 a_R^3}{G(m_p+m_c)}$, $f(e) = \frac{1+(73/24)e^2+(37/96)e^4}{(1-e^2)^7/2}$, $T_\odot = \frac{GM_\odot}{c^3} = 4.92540\ \mu\mathrm{s}$

Pulsar masses



$*$ LIGO-VIRGO observations of NS-NS merger gave some contraints in the $M - R$ space (modeling tidal disruption)

$**$ NICER also gave some values

PK parameters: Indirect proof of GR

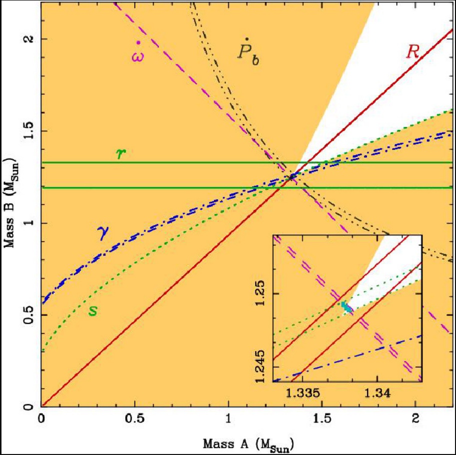
$$m_c = f1(P_b, e, m_p, \dot{\omega})$$

$$m_c = f2(P_b, e, m_p, \gamma)$$

$$m_c = r/T_\odot$$

$$m_c = f4(P_b, e, m_p, \mathcal{X}, s)$$

$$m_c = f5(P_b, e, m_p, \dot{P}_b)$$



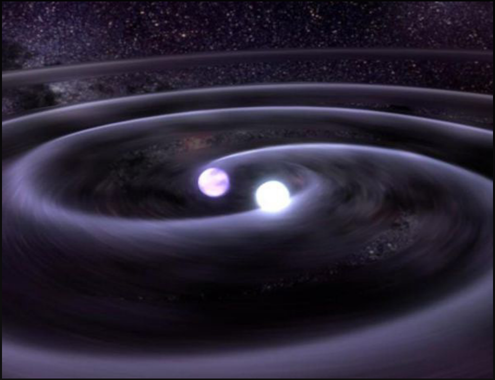
$R = \chi_1/\chi_2$

mass function: $\frac{(m_c \sin i)^3}{(m_p+m_c)^2} = \frac{4\pi^2}{G} \frac{(a_p \sin i)^3}{P_\mathrm{b}^2} =$ measured;

$|\sin i| \leq 1$ gives an allowed region (white) from the RHS of mass function (and the measured value)

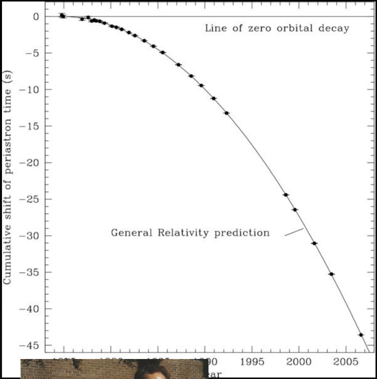
Existence of $\dot{P}_b$: Indirect detection of GWs

$\dot{P}_b$, $\dot{e}$



Existence of $\dot{P}_b$: Indirect detection of GWs

first *indirect* detection of gravitational wave (PSR B1913+16 by Hulse & Taylor, 1975; Nobel prize in 1993)



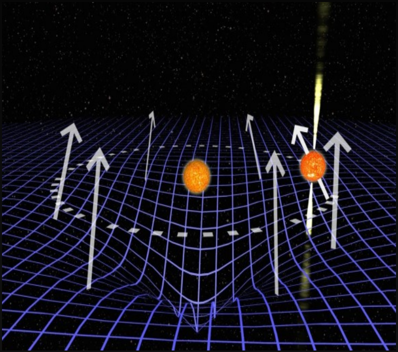
PSR B1913+16

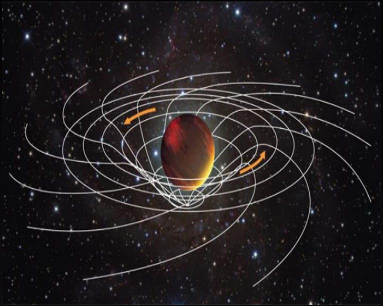
Discovery: Hulse & Taylor, 1975, ApJ, 195, L51
Figure:
Weisberg, Nice, & Taylor, 2010, ApJ, 722, 1030

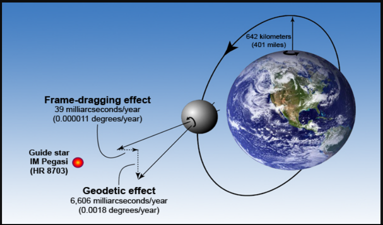
Nobel Prize in Physics 1993: "for the discovery of a new type of pulsar, a discovery that has opened up new possibilities for the study of gravitation"

Constraining EoS through $\dot{\omega}_{\rm obs}$: Lense-Thirring effect

$$\dot{\vec{S}}_i = \overrightarrow{\Omega}_{\rm S_i} \times \vec{S}_i, \qquad \dot{\vec{\mathcal{A}}} = \overrightarrow{\Omega}_{\rm b} \times \vec{\mathcal{A}} \;\; (\vec{\mathcal{A}} = \vec{p} \times \vec{L} - \mu k \hat{r}), \qquad \dot{\vec{L}} = \overrightarrow{\Omega}_{\rm b} \times \vec{L}$$







Constaring EoS through $\dot{\omega}_{\rm obs}$

Damour & Schäfer, 1988, Nuovo Cimento B, 101, 127

Bagchi, 2013, MNRAS, 428, 1201B; Bagchi, 2018, Universe, 4, 36

$$\boxed{\dot{\omega}_{\rm obs} \;=\; \dot{\omega}_{\rm 1PN} + \dot{\omega}_{\rm 2PN} + \dot{\omega}_{\rm LT} + \ldots}$$

$$\dot{\omega}_{\rm 1PN} = \frac{3\beta_0^2}{1-e^2}; \qquad \dot{\omega}_{\rm 2PN} = \frac{3\beta_0^3 f_0}{1-e^2}$$

$$\beta_0 \;=\; \frac{(GMn)^{1/3}}{c}$$

$$f_0 = \frac{1}{1-e^2}\left(\frac{39}{4}x_1^2 + \frac{27}{4}x_2^2 + 15x_1x_2\right) - \left(\frac{13}{4}x_1^2 + \frac{1}{4}x_2^2 + \frac{13}{3}x_1x_2\right)$$

$x_1 = \frac{m_1}{M}$, $x_2 = \frac{m_2}{M}$, $M = m_1 + m_2$, $n = \frac{2\pi}{P_{\rm orb}}$

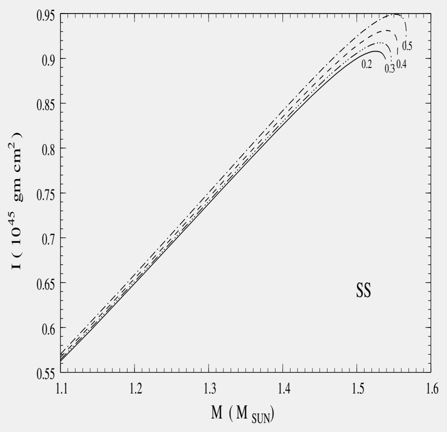
$\dot{\omega}_{\rm LT}$ should be $\gtrsim$ than the error in the observed value of $\dot{\omega}_{\rm obs}$

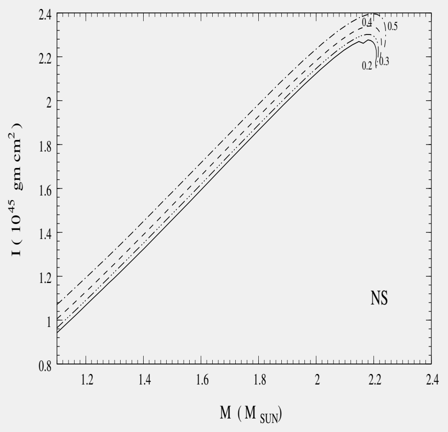
Best measurement of $\dot{\omega}_{\rm obs}$ (PSR J0737-3039A/B): $16.89947 \pm 0.00068 \ {\rm deg\ yr^{-1}}$

$\frac{\dot{\omega}_{\rm 2PN}}{\dot{\omega}_{\rm 1PN}} \sim 10^{-5}$ $\qquad$ $\frac{\dot{\omega}_{\rm LT}}{\dot{\omega}_{\rm 1PN}} \sim 10^{-6}$

$\dot{\omega}_{\rm LT}$ contains moment of inertia of the NS! $\qquad$ $\dot{\omega}_{\rm LT}(m_p, m_c, P_{orb}, e, P_s, I)$

Dense Matter Equations of State:





Bagchi, 2010, New Astronomy, 15, 126

Ω in units of $10^4\ \mathrm{s}^{-1}$ (for the fastest pulsar $\Omega = 0.45 \times 10^4\ \mathrm{s}^{-1}$)

Rapidly **R**otating **N**eutron **S**tar Code: http://www.gravity.phys.uwm.edu/rns/

Difficulty in measuring true values of period derivatives

Pathak and Bagchi, 2018, ApJ;
'GalDynPsr' at 'https://github.com/pathakdhruv/GalDynPsr

$$P_{\rm obs} = (c + \vec{v}_p . \widehat{n}_{sp})(c + \vec{v}_s . \widehat{n}_{sp})^{-1} P_{\rm int}$$

$$\frac{\dot{P}_{\rm int}}{P_{\rm int}} = \frac{(\vec{a}_s - \vec{a}_p) \cdot \widehat{n}_{sp}}{c} + \frac{1}{c}(\vec{v}_s - \vec{v}_p) \cdot \frac{d}{dt}(\widehat{n}_{sp}) + \frac{\dot{P}_{\rm obs}}{P_{\rm obs}}$$

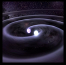
$P_{\rm obs} \simeq P_{\rm int} = P$

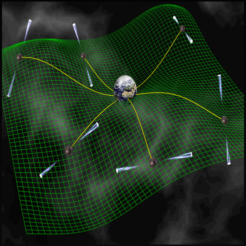
$$\left(\frac{\dot{P}}{P}\right)_{\rm obs} - \left(\frac{\dot{P}}{P}\right)_{\rm int} = -\left[\frac{(\vec{a}_s - \vec{a}_p) \cdot \widehat{n}_{sp}}{c} + \frac{1}{c}(\vec{v}_s - \vec{v}_p) \cdot \frac{d}{dt}(\widehat{n}_{sp})\right]$$

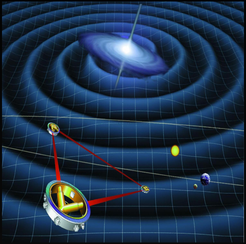
$$\vec{a}_{\parallel} = -\frac{\partial \Phi(R,z)}{\partial R}$$

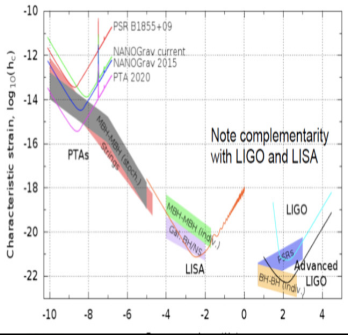
$$\vec{a}_{\perp} = -\frac{\partial \Phi(R,z)}{\partial z}$$

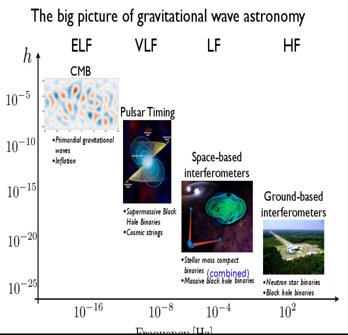
Pulsars and Gravitational Waves











Pulsar timing and gravitational waves:

As a first step, the power spectra of the pulsar timing residuals are analyzed. If they all show a very red power-law spectrum, the residuals may be dominated by a GW background. However, such red spectra can also be due to period noise intrinsic to the pulsar, uncorrected interstellar delays, inaccuracies in the Solar-System ephemeris, or variations in terrestrial time standards.

The presence of a stochastic GW background will cause the pulse TOAs to fluctuate randomly, but these fluctuations will be correlated between different pulsars. In order to detect the presence of a GW background, one needs to first calculate the correlation coefficient between the observed timing residuals of each pair of observed pulsars:

$$\mathcal{R}_{\rm obs} = \frac{1}{N}\sum_{i=0}^{N-1} R(t_i, \hat{k}_1)R(t_i, \hat{k}_2)$$

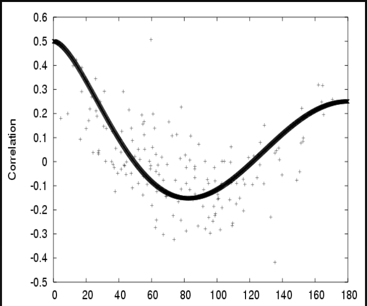
where $R(t_i, \hat{k})$ is the time series of N pulsar residuals sampled regularly in time, $\hat{k}_1$ and $\hat{k}_2$ are the directions to the two pulsars, and $\cos(\theta) = \hat{k}_1 \cdot \hat{k}_2$. It will be assumed that R has zero mean and that each pulsar pair has a unique angular separation. As the GW background is expected to be isotropic, we expect the theoretical expression of $\mathcal{R}$ to depend only on the angular separation between the pulsars.

Pulsar timing and gravitational waves:

In the presence of an isotropic GW background, the ensemble-averaged value of $r(\theta)$ is given by:

$$\langle \mathcal{R}(\theta) \rangle = \sigma_g^2 \zeta(\theta); \quad \zeta(\theta) = \frac{3}{2} x \log(x) - \frac{x}{4} + \frac{1}{2} + \frac{1}{2}\delta(x)$$

where $x = (1 - \cos(\theta))/2$, σ_g: RMS of the timing residuals induced by the stochastic GW background, and $\delta(x)$ equals 1 for $x = 0$ and 0 otherwise. The detection technique is to look for the presence of the function $\zeta(\theta)$ in the measured correlation coefficients $\mathcal{R}_{\rm obs}$.



Pulsar timing and gravitational waves:

As no ensemble average in reality, the measured statistic, $\mathcal{R}(\theta)$, will be of the form $\mathcal{R}(\theta) = \langle \mathcal{R}(\theta) \rangle + \Delta\mathcal{R}(\theta)$,
where $\Delta\mathcal{R}(\theta)$ is a "noise term" - a Gaussian random variable. The optimal way to detect the presence of a known functional form within random data is to calculate the correlation between the data and the known function. Hence, to detect the presence of the GW background one needs to calculate

$$\rho = \frac{\frac{1}{N_p}\sum_{i=0}^{N_p-1}(\mathcal{R}_{\mathrm{obs},i} - \bar{\mathcal{R}}_{\mathrm{obs}})(\zeta(\theta_i) - \bar{\zeta})}{\sigma_{\mathcal{R}}\sigma_{\zeta}}$$

where θ_i is the angle between the ith pair of pulsars and N_p is the number of distinct pairs of pulsars. $\bar{\mathcal{R}}$ and $\bar{\zeta}$ indicate the mean values over all pairs of pulsars and $\sigma^2_{\mathcal{R}}$ and σ^2_{ζ} are the variances of $\mathcal{R}_{\mathrm{obs}}$ and ζ respectively. For M pulsars, $N_p = M(M-1)/2$.

Low Frequency gravitational waves

ToAs: Time of Arrival (of pulses): Time series data

cross-power spectrum: Fourier trans. of cross-covariance between 2 time series

$$\text{Mean of } \mathbf{x} = \gamma_x = \frac{1}{N}\sum_{i=1}^{N} x_i$$

$$\text{Variance of } \mathbf{x} = \sigma_x^2 = \frac{1}{N}\sum_{i=1}^{N} \left|x_i - \gamma_x\right|^2$$

$$\text{Auto-Covariance function of } \mathbf{x} = c_{xx}[k] = \sum_{i=1}^{N-k+1} \left(x[i+k-1]-\gamma_x\right)\left(x[i]-\gamma_y\right)^*, \text{ for k=1:N}$$

$$\text{Cross-Covariance function of } \mathbf{x} \text{ and } \mathbf{y} = c_{xy}[k] = \sum_{i=1}^{N-k+1} \left(x[i+k-1]-\gamma_x\right)\left(y[i]-\gamma_y\right)^*, \text{ for k=1:N}$$

$$\text{Auto-Correlation function of } \mathbf{x} = r_{xx}[k] = \sum_{i=1}^{N-k+1} x[i+k-1]x^*[i], \text{ for k=1:N}$$

$$\text{Cross-Correlation function of } \mathbf{x} \text{ and } \mathbf{y} = r_{xy}[k] = \sum_{i=1}^{N-k+1} x[i+k-1]y^*[i], \text{ for k=1:N}$$

$$\zeta(\theta_{ij}) = \frac{3}{2}x\log x - \frac{x}{4} + \frac{1}{2}\,,$$

where $x = [1 - \cos(\theta_{ij})]/2$ and θ_{ij} is the angle between pulsars i and j subtended at the observer (Hellings & Downs 1983; Jenet et al. 2005). The function $\zeta(\theta_{ij})$ is independent of GW frequency, and is derived assuming GWs are described by general relativity;

$$P_g(f) = \frac{A^2}{12\pi^2}\left(\frac{f}{f_{1\,\mathrm{yr}}}\right)^{2\alpha-3}$$

The cross-power spectrum between the induced ToA perturbations in pulsars i and j is

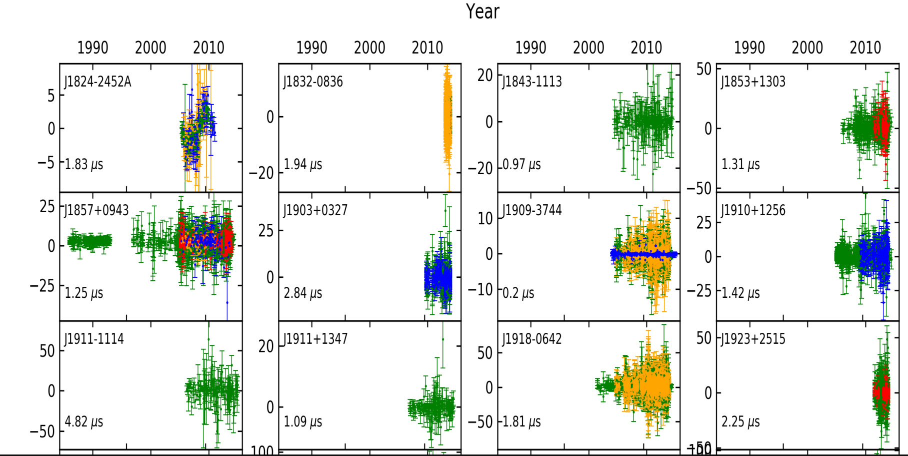
$$X_{ij}(f) = P_g(f)\zeta(\theta_{ij})$$

the timing residuals. Several models of the expected GWB from an ensemble of supermassive black hole binaries (SMBHBs) predict that the amplitude of the GWB will be in the range $5 \times 10^{-16} < A < 10^{-14}$ (Jaffe & Backer 2003; Wyithe & Loeb 2003; Sesana, Vecchio & Colacino 2008) with a spectral exponent $\alpha = -2/3$.

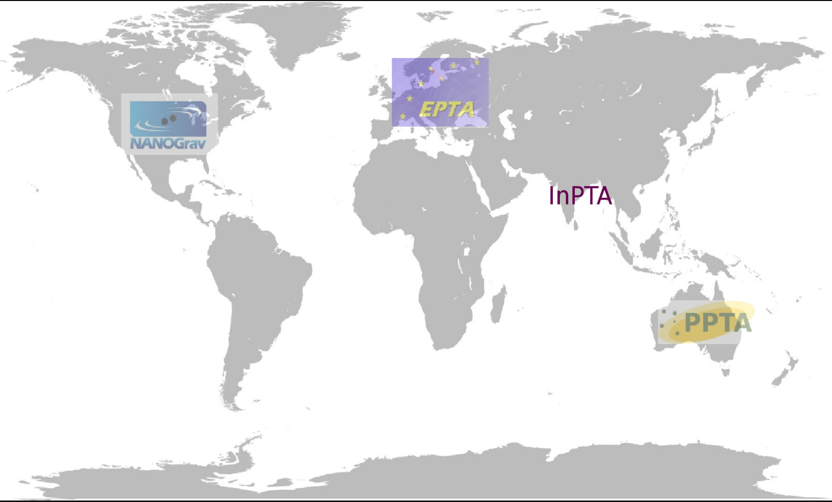
$$h_c(f) = A\left(\frac{f}{f_{1\mathrm{yr}}}\right)^{\alpha}$$

Yardley et al. 2011, MNRAS, 414, 1777.

No detection yet: IPTA second data relese Sept-2019



International Pulsar Timing Array



Alternative theories of gravity

In GR: (i) conservation of mass forbids monopolar gravitational wave emission, (ii) conservation of momentum forbids dipolar gravitational wave emission

Damour & Esposito-Farese, Class. Quant. Grav. 9 (1992) 2003

$$\dot{P}_b^{\mathrm{M,spin0}} = -\frac{\pi}{3}\frac{M_pM_c(3M_p+5M_c)^2}{(M_p+M_c)^{7/3}}\left(\frac{G^*}{c^3}\right)^{5/3}\left(\frac{2\pi}{P_b}\right)^{5/3}\frac{e^2(1+e^2/4)}{(1-e^2)^{7/2}}\alpha_p^2+\mathcal{O}(1/c^7)$$

$$\dot{P}_b^{\mathrm{D,spin0}} = -4\pi^2\frac{G^*}{c^3}\frac{1}{P_b}\frac{M_pM_c}{(M_p+M_c)}\frac{1+e^2/2}{(1-e^2)^{5/2}}(\alpha_p-\alpha_c)^2+\mathcal{O}(1/c^5)+\mathcal{O}(1/c^7)$$

$$\dot{P}_b^{\mathrm{Q,spin0}} = -\frac{32\pi}{5}\frac{M_pM_c^3}{(M_p+M_c)^{7/3}}\left(\frac{G^*}{c^3}\right)^{5/3}\left(\frac{2\pi}{P_b}\right)^{5/3}\frac{(1+73e^2/24+37e^4/96)}{(1-e^2)^{7/2}}\alpha_p^2+\mathcal{O}(1/c^7)$$

$$\dot{P}_b^{\mathrm{Q,spin2}} = -\frac{192\pi}{5}\frac{M_pM_c}{(M_p+M_c)^{1/3}}\left(\frac{G^*}{c^3}\right)^{5/3}\left(\frac{2\pi}{P_b}\right)^{5/3}\frac{(1+73e^2/24+37e^4/96)}{(1-e^2)^{7/2}}+\mathcal{O}(1/c^7)$$

$\varphi_0=\varphi(r\to\infty)$ is the value of the scalar field φ without the presence of any gravitating body.

$\alpha(\varphi)=\frac{\partial \ln A(\varphi)}{\partial\varphi}$; $\alpha_0=\alpha(\varphi_0)$, $G^*=G/(1+\alpha_0^2)$.

$\alpha_p(\varphi_{ap})=\frac{\partial \ln M_p}{\partial\varphi_{ap}}$, $\alpha_c(\varphi_{ac})=\frac{\partial \ln M_c}{\partial\varphi_{ac}}$, φ_{ap} is the value of φ at a large distance from the pulsar, and a combination of φ_0 and the scalar influence of the companion; while φ_{ac} is the value of φ at a large distance from the companion, and a combination of φ_0 and the scalar influence of the pulsar.

$\alpha_p=\alpha_p(\varphi_0)$, $\alpha_c=\alpha_c(\varphi_0)$.

For a NS-WD system $\alpha_c=\alpha_{wd}\to\alpha_0$, for a NS-NS system $\alpha_p\simeq\alpha_c$, and for a NS-BH system $\alpha_c=\alpha_{bh}=0$ ('no-scalar-hair' theorem for black holes).

Alternative theories of gravity

$\alpha_0 = 10^{-2}$	$\dot{P}_b^{\mathrm{M,spin0}}$ ($\alpha_p^{-2}\,\mathrm{ss}^{-1}$)	$\dot{P}_b^{\mathrm{D,spin0}}$ ($\alpha_p^{-2}\,\mathrm{ss}^{-1}$)	$\dot{P}_b^{\mathrm{Q,spin0}}$ ($\alpha_p^{-2}\,\mathrm{ss}^{-1}$)	$\dot{P}_b^{\mathrm{Q,spin2}}$ (ss^{-1})
J0348+0432 (NS-WD)	-1.28×10^{-25}	-3.53×10^{-9}	-2.70×10^{-16}	-2.59×10^{-13}
J1738+0333 (NS-WD)	-2.90×10^{-28}	-1.03×10^{-9}	-5.61×10^{-17}	-2.72×10^{-14}
NS-BH	-8.28×10^{-15}	-2.19×10^{-8}	-5.56×10^{-13}	-4.34×10^{-12}

	P_s (s)	P_{orb} (day)	e	M_c ($\mathrm{M}_\odot$)
J0348+0432 (NS-WD)	0.039	0.102	2.4×10^{-6}	0.097
J1738+0333 (NS-WD)	0.006	0.355	3.4×10^{-7}	0.104
NS-BH	0.080	0.1	0.1	10.000

Bagchi & Torres, JCAP 08 (2014) 055

Alternative theories of gravity

$\alpha_0 = 10^{-2}$

	$\dot{P}_b^{\mathrm{M,spin0}}$ $(\alpha_p^{-2}\,\mathrm{ss}^{-1})$	$\dot{P}_b^{\mathrm{D,spin0}}$ $(\alpha_p^{-2}\,\mathrm{ss}^{-1})$	$\dot{P}_b^{\mathrm{Q,spin0}}$ $(\alpha_p^{-2}\,\mathrm{ss}^{-1})$	$\dot{P}_b^{\mathrm{Q,spin2}}$ (ss^{-1})
J0348+0432 (NS-WD)	-1.28×10^{-25}	-3.53×10^{-9}	-2.70×10^{-16}	-2.59×10^{-13}
J1738+0333 (NS-WD)	-2.90×10^{-28}	-1.03×10^{-9}	-5.61×10^{-17}	-2.72×10^{-14}
NS-BH	-8.28×10^{-15}	-2.19×10^{-8}	-5.56×10^{-13}	-4.34×10^{-12}

NS-BH:

$\dot{P}_b^{\mathrm{M,spin0}}$ $(10^{-15}\,\alpha_p^{-2}\,\mathrm{ss}^{-1})$		$\dot{P}_b^{\mathrm{D,spin0}}$ $(10^{-8}\,\alpha_p^{-2}\,\mathrm{ss}^{-1})$		$\dot{P}_b^{\mathrm{Q,spin0}}$ $(10^{-13}\,\alpha_p^{-2}\,\mathrm{ss}^{-1})$		$\dot{P}_b^{\mathrm{Q,spin2}}$ $(10^{-12}\,\mathrm{ss}^{-1})$	
$\alpha_0 = 10^{-2}$	$\alpha_0 = 10^{-4}$	$\alpha_0 = 10^{-2}$	$\alpha_0 = 10^{-4}$	$\alpha_0 = 10^{-2}$	$\alpha_0 = 10^{-4}$	$\alpha_0 = 10^{-2}$	$\alpha_0 = 10^{-4}$
-8.28237	-8.28375	-2.19145	-2.19166	-5.56419	-5.56511	-4.33873	4.33945

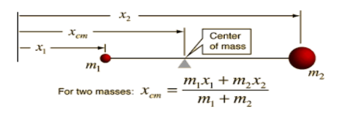
Bagchi & Torres, JCAP 08 (2014) 055

PART-III

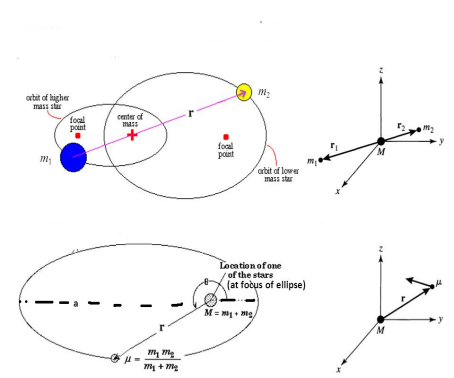
Procedures of Pulsar Timing: Making templates and producing ToAs (psrchive tutorials) + timing (tutorials on tempo2/tempo/pint)

Back-up slides

Binary Pulsars: Two body systems (classical description)



$$\vec{r} = \vec{r_2} - \vec{r_1},\ M = m_1 + m_2,\ \mu = \frac{m_1 m_2}{M};\quad P_b^2 = \frac{4\pi^2}{GM}\ a^3$$



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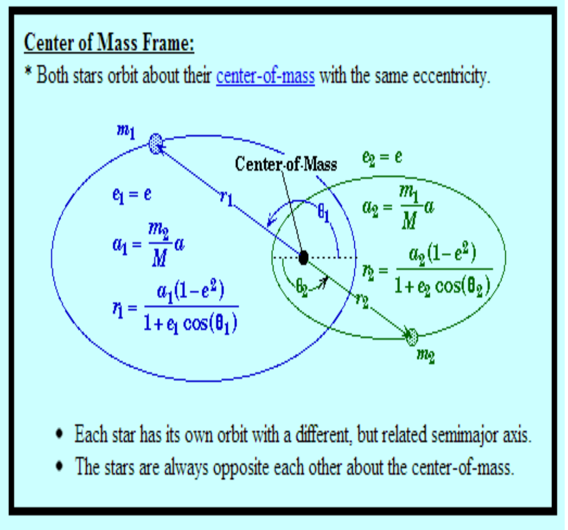

both same P_b