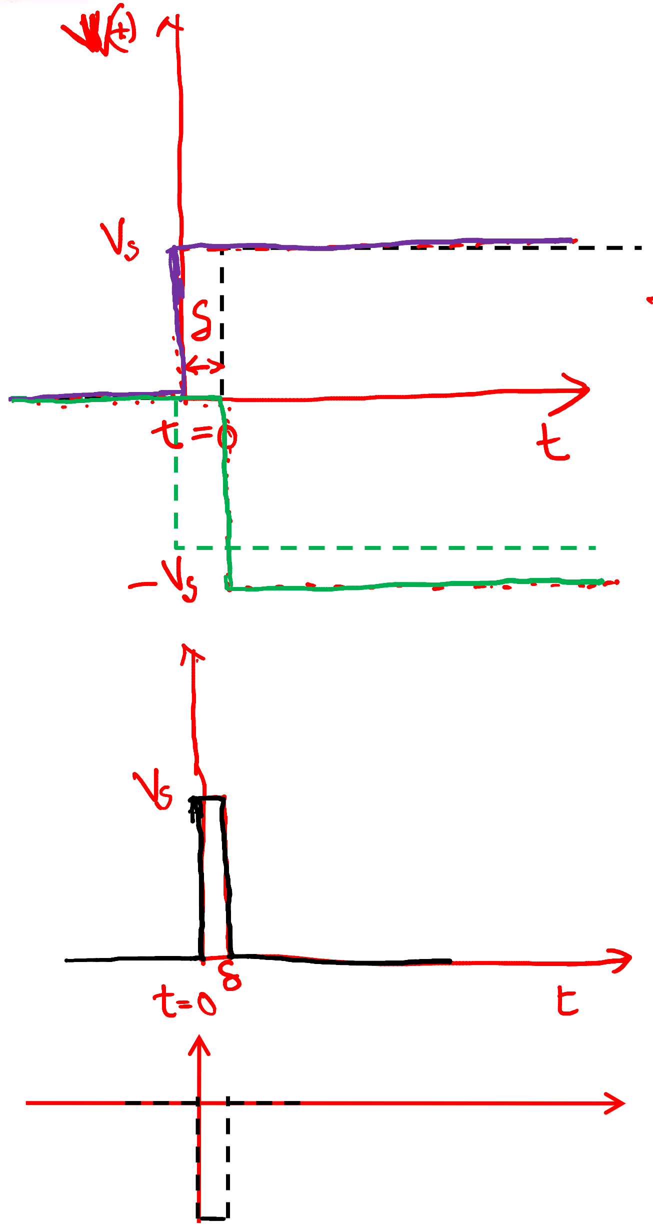
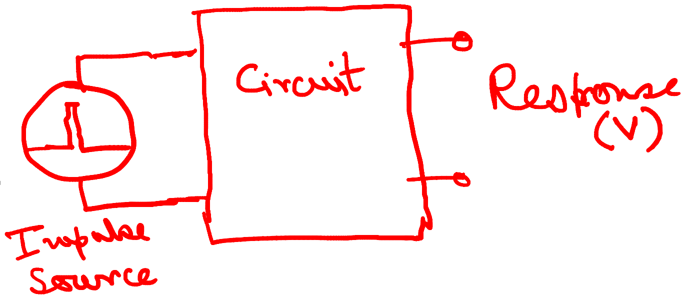
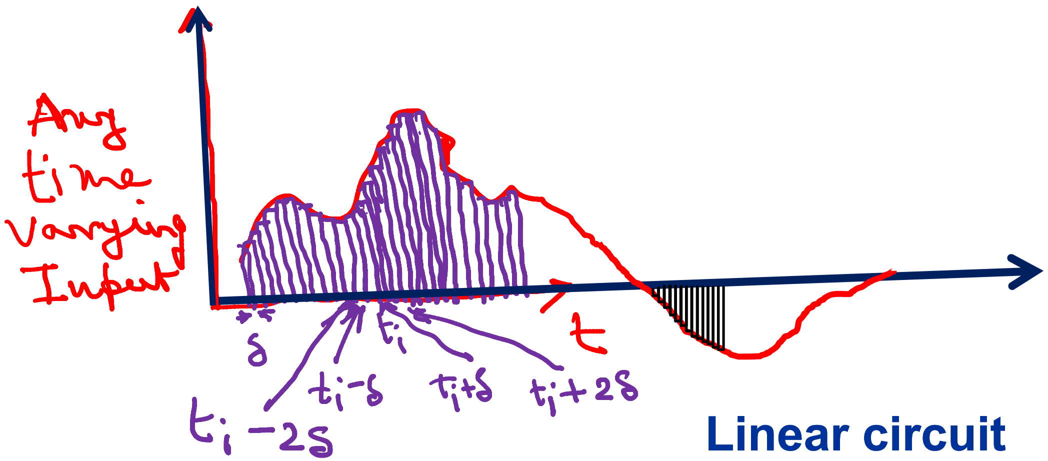
ESc201, Lecture 7: Transients

Impulse



Impulse Response





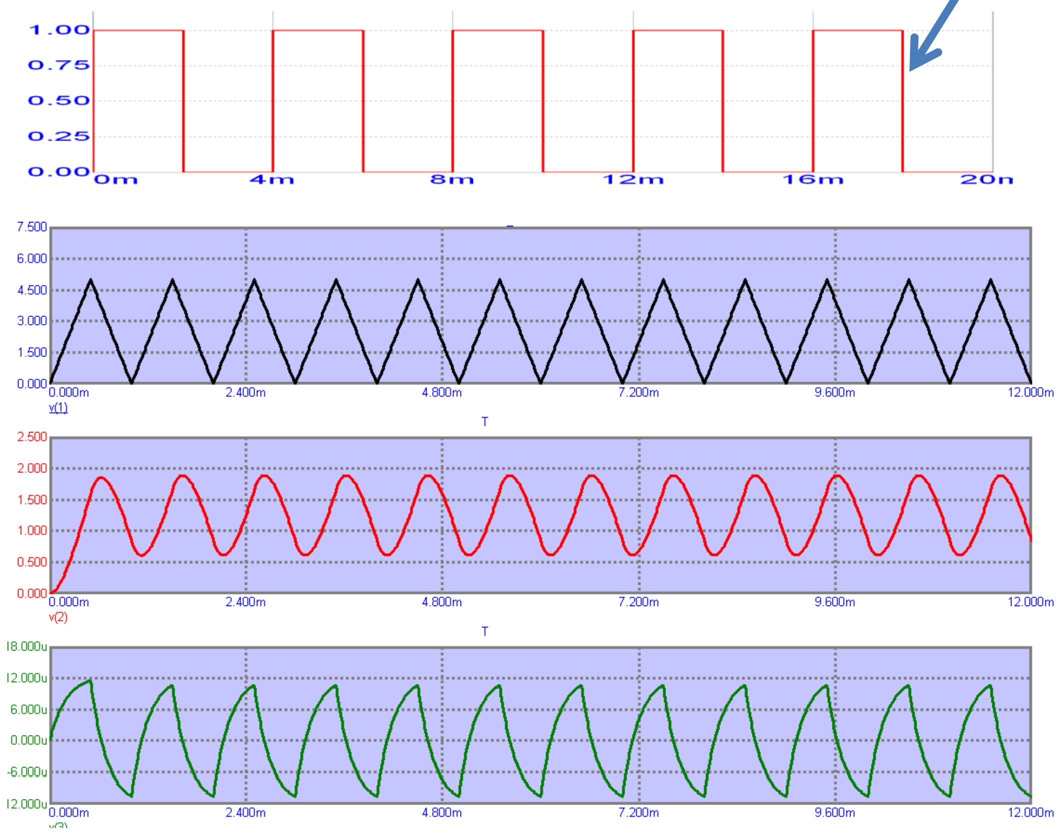
Linear circuit

We can find out the response of any circuit with input ~~of~~ any time varying input if we know the Impulse Response of the circuit.

ESc201, Lecture 8: Time varying, periodic, Sinusoidal

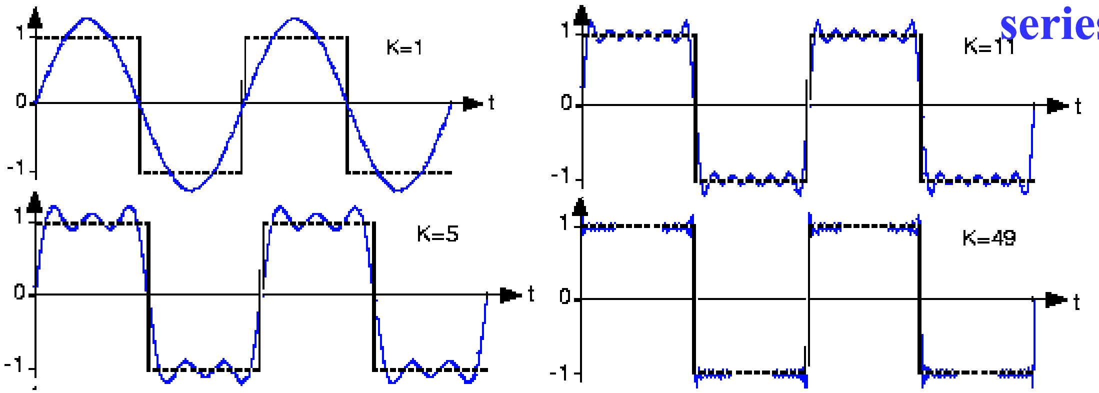
Periodic signals:

The Fourier transform of a function of time takes it to the frequency domain of the function. Some such transforms are that for a signal in time transformed to a description in the frequency domian. Another is the electron motion description in a crystal. From crystal spatial domain to the electron wave-vector domain.



The Fourier transform of a periodic signal is an impulse train where the impulse amplitudes are 2π times the Fourier coefficients of that signal. We need shifted impulses since they correspond to different frequency components in the Fourier series representation.

The periodic signal can be represented as a summation of a series of Sin functions



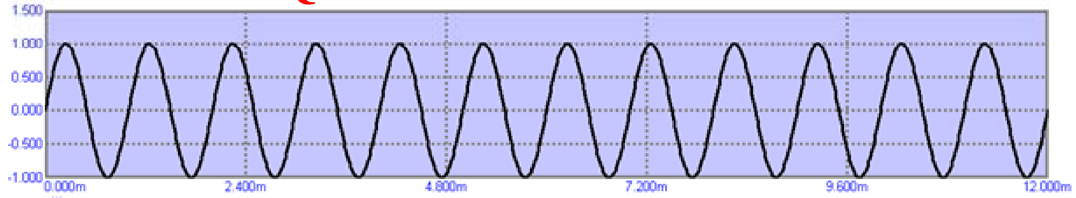
A square wave = sin(x) + sin(3x)/3 + sin(5x)/5 + ... (infinitely)

$$f(t)=\frac{4}{\pi}\sum_{1,3,5}^{\infty}\frac{1}{n}\mathrm{Sin}\left(\frac{n\pi t}{T}\right)$$

ESc201, Lecture 8: Time varying, periodic, Sinusoidal variation

What if the transform has only one frequency component the variation of which looks like the displacement of a frictionless pendulum? c.f. Walter Lewin's Last Lecture https://www.youtube.com/watch?reload=9&v=4a0FbQdH3dY

The summation has only one term !!

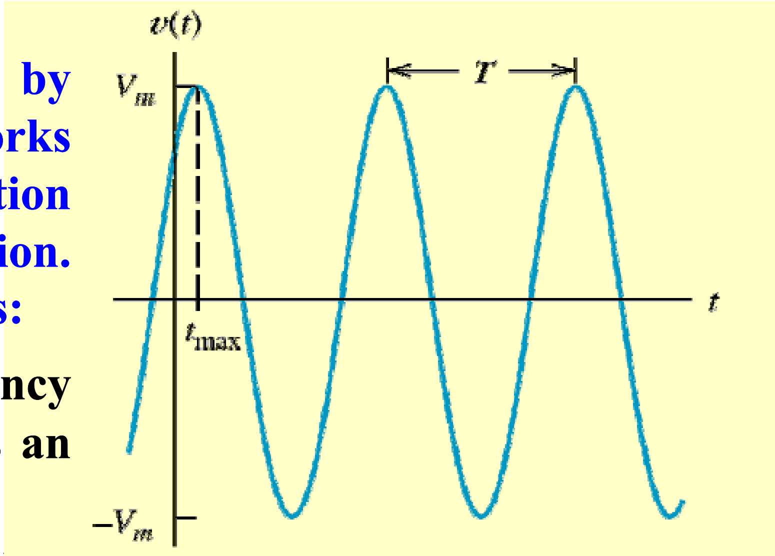


This is called Sinusoidal variation:

A *sinusoidal voltage source* (independent or dependent) produces a voltage that varies *sinusoidally* with *time*. A *sinusoidal current source* (independent or dependent) produces a current that varies *sinusoidally* with *time*.

A *sinusoidally* varying function can be represented by either Sine or a Cosine function. Although either works equally well, it is tradition to use the Cosine function which would be clear later in Phasor representation. Hence a sinusoidally varying voltage is represented as:

$v(t) = V_m.\cos(\omega t+\phi)$, where ω is the angular frequency (rad/s), ωt is has the dimension of phase angle, ϕ is an arbitrary phase angle, and $\omega = 2\pi f = 2\pi/T$.



Changing the phase ϕ ($t = \phi/\omega$) shifts the sinusoid on the time axis but has no effect on the amplitude.

Note: Average with time is always = 0

If ϕ is positive, sinusoid shifts left, and if negative the sinusoid shifts right on the time axis.

https://ocw.mit.edu/courses/electrical-engineering-and-computer-science/6-002-circuits-and-electronics-spring-2007/video-lectures/lecture-16/

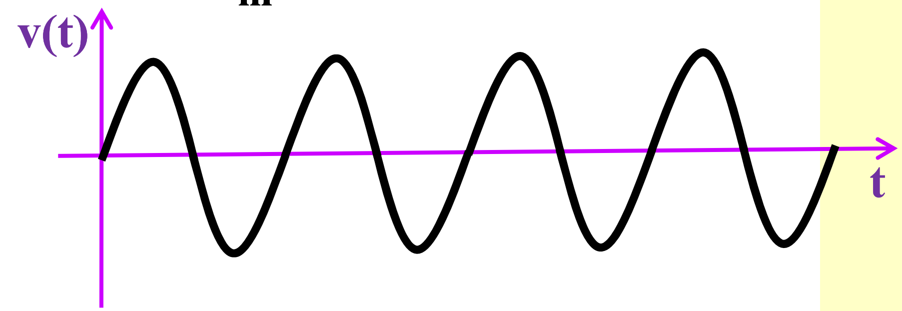
ESc201, Lecture 8: Time varying, periodic, Sinusoidal

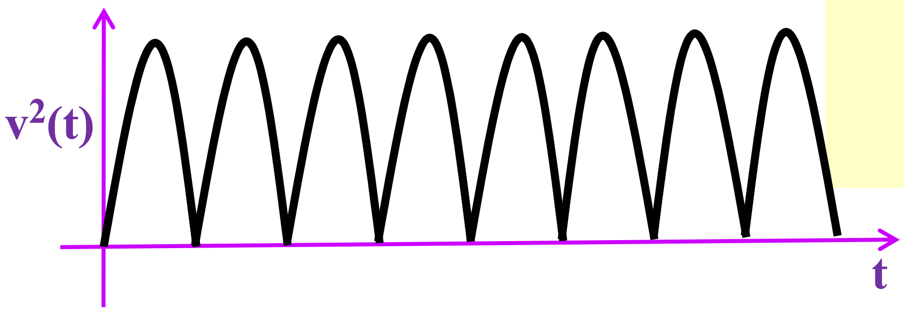
There is no ***transient disturbance (all perturbations have settled down) and LINEAR***
i.e. If the excitation is at frequency 'ω' then the response will also be in frequency 'ω'.

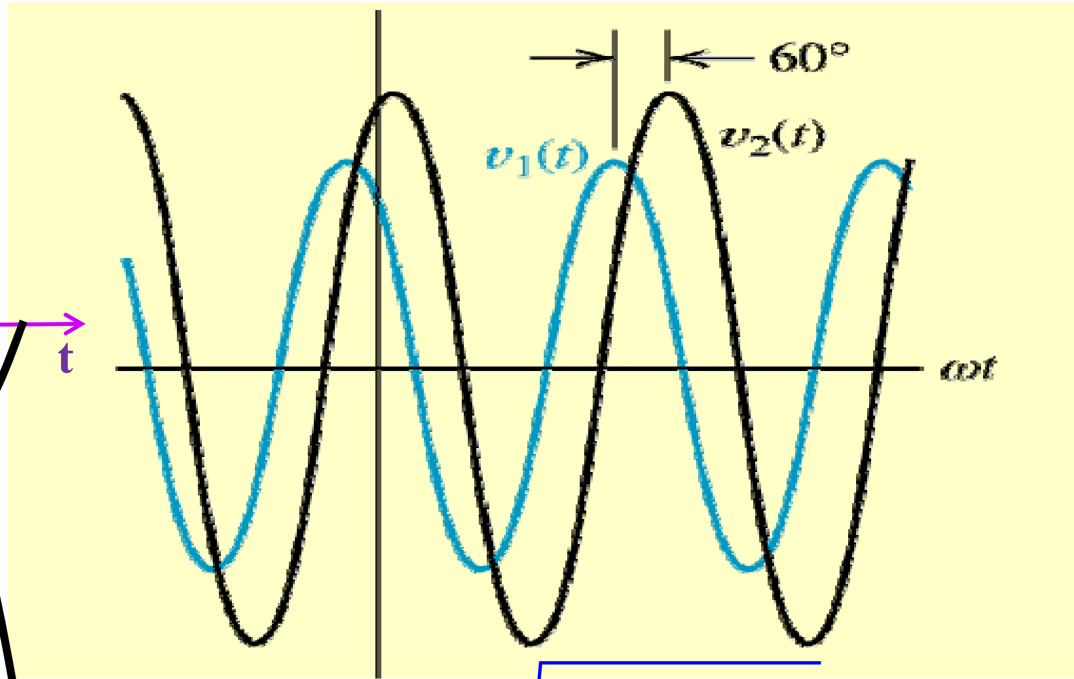
If the Average with time is always = 0, then we can at least compare the amplitudes (V_m) of two signals of the same frequency, but how does one compare the strength of two signals of different frequency? ***Use the rms voltage:***

$$V_{rms} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} v(t)^2\,dt} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} V_m^2 \cos^2(\omega t+\phi)dt} = \sqrt{\frac{1}{T} V_m^2 \left(\frac{T}{2}\right)} = \frac{V_m}{\sqrt{2}}$$

$$v_1(t) = v_{1_m}\cos(\omega t+\phi)$$
$$v_2(t) = v_{2_m}\cos(\omega t+\phi-60^0)$$







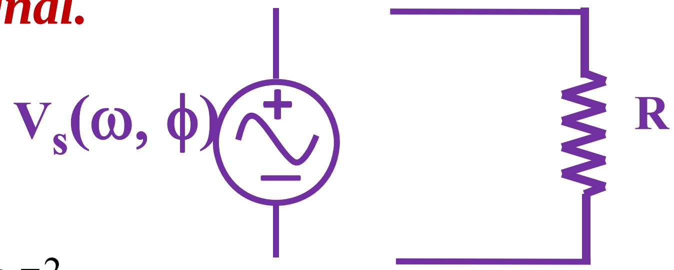
$$I_{rms} = \sqrt{\frac{1}{T}\int_{0}^{T} i(t)^2\,dt}$$

ESc201, Lecture 8: Time varying, periodic, Sinusoidal

Sinusoidal Steady-State Response

1. Deals with response of circuits under ***sinusoidal (sine or cosine functions)*** excitations (***voltage or current)***
2. The signals are ***periodic function of time***
3. There is no ***transient disturbance (all perturbations have settled down)***
4. ***Phasors***, ***Impedance***, ***Admittance***, ***and Response of RC and RL*** circuits to ***sinusoidal excitation***

Source for a sinusoidal signal.



$$p_{avg} = \frac{1}{T}\int_0^T \frac{v(t)^2}{R} dt = \frac{\left[\sqrt{\frac{1}{T}\int_0^T v(t)^2 dt}\right]^2}{R} = \frac{V_{rms}^2}{R} = RI_{rms}^2$$

Very similar to the dc expression except use the r.m.s. voltage.

$v(t) = V_m.\cos(\omega t+\phi_v)$, and similarly $i(t) = I_m.\cos(\omega t+\phi_i)$, but this is difficult to handle, so rewrite it in a different way that the v(t) and i(t) still has the same phase relationship between them.

Or $v(t) = V_m.\cos(\omega t)$ and $i(t) = I_m.\cos(\omega t+\phi_i - \phi_v)$. $\rightarrow p = V_m I_m \cos(\omega t) \cos(\omega t+\phi_i - \phi_v)$

using the $\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$.

$p=(1/2)V_m I_m \cos(\phi_i - \phi_v) + (1/2)V_m I_m \cos(\phi_i - \phi_v).\cos(2\omega t) - (1/2)V_m I_m \sin(\phi_i - \phi_v).\sin(2\omega t)$

Av. power is now $p_{avg}=(1/2)V_m I_m \cos(\phi_i - \phi_v)=(V_m/\sqrt{2})(I_m/\sqrt{2})\cos(\phi_i - \phi_v)=V_{rms}I_{rms}\cos(\theta)$

$\theta=(\phi_i - \phi_v)$, $\cos(\theta)$=power factor (pf). Similarly $\sin(\phi_i - \phi_v)$ is called the reactive factor (rf)

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Phasor Representation

As per Euler's identity

$$e^{\pm j\theta} = \cos\theta \pm j\sin\theta, \text{ therefore } \cos\theta = \mathrm{Re}\{e^{+j\theta}\} \text{ and } \sin\theta = \mathrm{Im}\{e^{+j\theta}\}$$

$$v(t) = V_m\cos(\omega t + \phi_v) = \mathrm{Re}\{V_m e^{j\phi_v}.e^{j\omega t}\} \qquad j=\sqrt{-1}$$

$$\mathbb{V} = V_m.e^{j\phi_v} = \mathscr{P}\{V_m\cos(\omega t+\phi_v)\}$$ **is the *phasor transform* of** $V_m\cos(\omega t+\phi_v)$

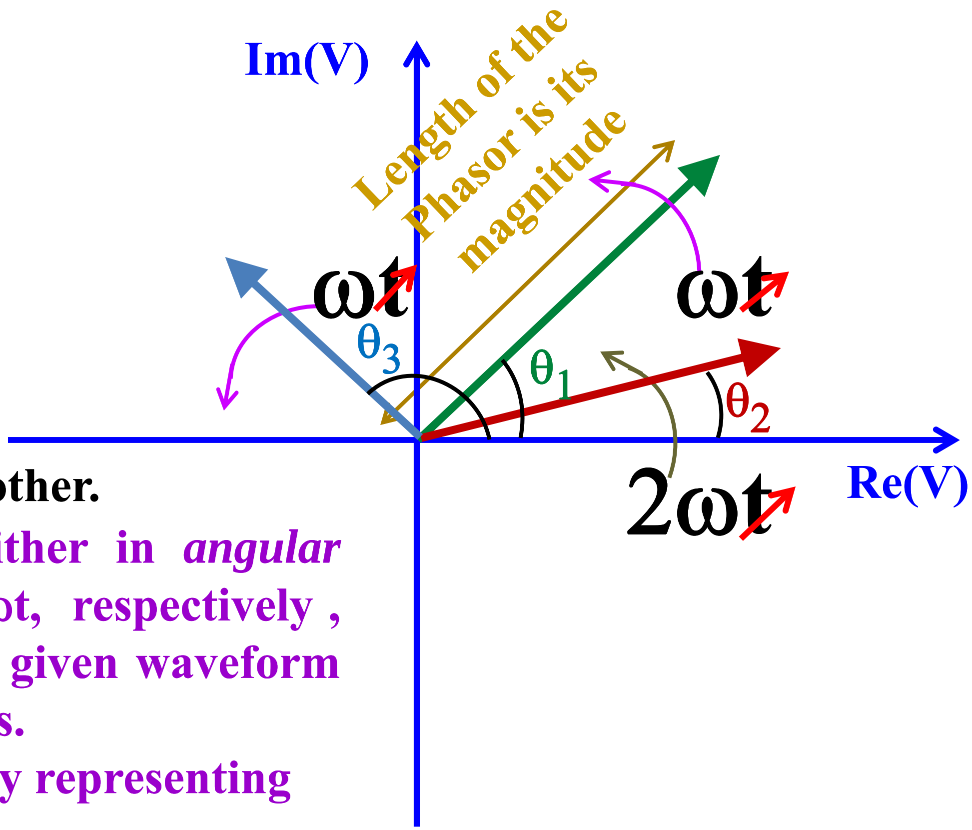
Thus the *phasor transform* transfers the sinusoidal function from the time domain to the complex domain.

i.e. $\mathbb{V} = V_m.e^{j\phi_v} = V_m\cos\phi_v + jV_m\sin\phi_v.$

$= V_m.\angle\phi_v$ **And visa-versa.**

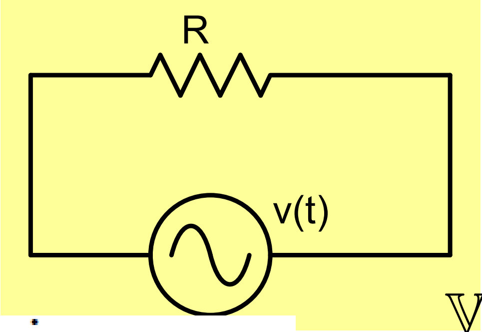
Note that $|e^{j\phi_v}| = \sqrt{\cos^2\phi_v + \sin^2\phi_v} = 1$

ϕ_v in this case is θ_1 for one and θ_2 for the other.



While representing sinusoidal signals either in *angular sweep domain* or *time* or *domain* i.e. t or ωt, respectively , confusion may arise regarding whether a given waveform is *leading or lagging* some other waveforms.

This confusion can largely be eliminated by representing the sinusoidal signals as *Phasors.*



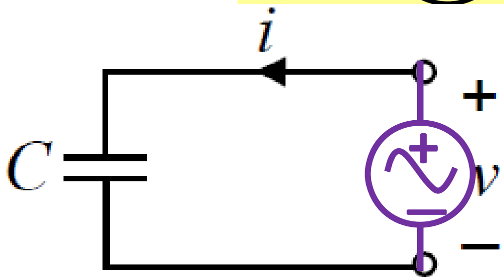
$$v(t) = V_m \cos(\omega t + \theta_v) \qquad v_R = V_m \angle \theta_v$$

$$i(t) = \frac{V_m}{R} \cos(\omega t + \theta_v) \qquad i_R = \frac{V_m}{R} \angle \theta_v$$

$$\mathbb{V}_R = R\mathbb{I}_R, \mathbb{I}_R = \frac{1}{R}\mathbb{V}_R = G\mathbb{V}_R$$

R is the Resistance

G is the Conductance



$$i_C = C\frac{dv}{dt} \qquad v(t) = V_m \cos(\omega t + \theta_v)$$

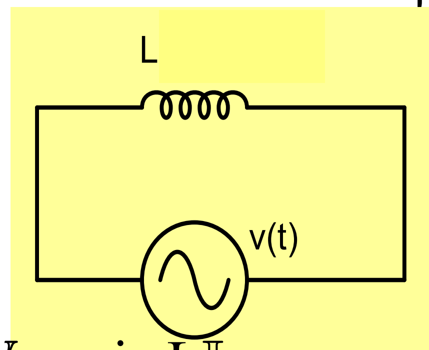
$$i_c = C\frac{d\left[V_m \cos(\omega t + \theta_v)\right]}{dt} = C\frac{d\left[\mathrm{Re}\{e^{j\phi_v}.e^{j\omega t}\}\right]}{dt}$$

$$= j\omega C.\mathrm{Re}\{e^{j\phi_v}.e^{j\omega t}\} = j\omega C v$$

$$\mathbb{I}_C = j\omega C\mathbb{V}_C \quad \text{or } \mathbb{V}_C = \frac{1}{j\omega C}\mathbb{I}_C = -j\left(\frac{1}{\omega C}\right)\mathbb{I}_C = j(-X_C)\mathbb{I}_C$$

$-X_c$ is the negative Reactance

$-1/X_c$ is the negative Susceptance



$$v_L = L\frac{di}{dt} = L\frac{d\left[\mathrm{Re}\{I_m e^{j\phi_i}.e^{j\omega t}\}\right]}{dt} = -\omega L I_m \sin(\omega t + \phi_i)$$

$$= -\omega L I_m \cos(\omega t + \phi_i - 90^o) = -\omega L\,\mathrm{Re}\{I_m e^{j\phi_i}.e^{-j90}.e^{j\omega t}\}$$

$$= j\omega L\,\mathrm{Re}\{I_m e^{j\phi_i}.e^{j\omega t}\}$$

$$\mathbb{V}_L = j\omega L\mathbb{I}_L$$

$$= jX_L\mathbb{I}_L$$

X_L is the positive Reactance **$1/X_L$ is the positive Susceptance**