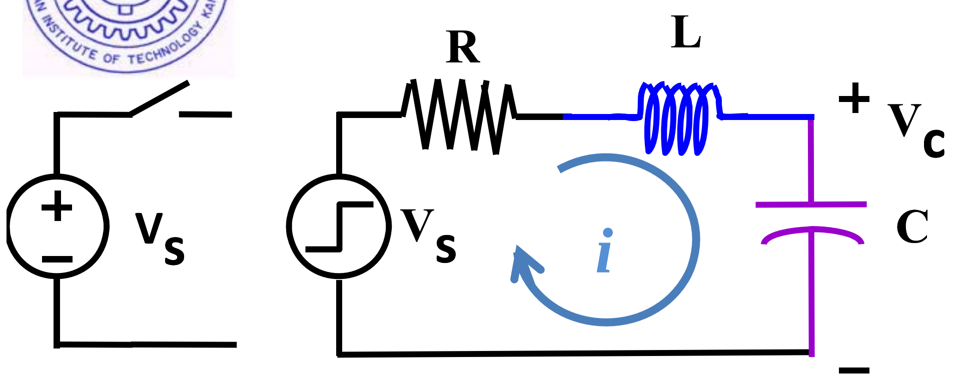
# ESc201, Lecture 12: Step and Freq response of RLC (Series)




$$Ri+L\frac{di}{dt}+\frac{1}{C}\int_0^t i\,d\tau+V_s=0$$

**Note that there would be an integration constant depending on the capacitor voltage at t=0$^+$**

**Differentiate with respect to time**

$$R\frac{di}{dt}+L\frac{d^2i}{dt^2}+\frac{i}{C}=0=L\frac{d^2i}{dt^2}+R\frac{di}{dt}+\frac{i}{C}$$

**In Laplace transform** $L\{f(t)\}=\int_0^\infty f(t)e^{-st}dt$ **form:** $\boldsymbol{i=Ie^{st}}$ $s^2+\frac{R}{L}s+\frac{1}{LC}=0$

$$s_{1,2}=-\frac{R}{2L}\pm\sqrt{\left(\frac{R}{2L}\right)^2-\frac{1}{LC}}$$ **or** $$s_{1,2}=-\alpha\pm\sqrt{\alpha^2-\omega_0^2}$$

**where** $\alpha=\frac{R}{2L}$ Np/sec $(s^{-1})$ , $\omega_0=\frac{1}{\sqrt{LC}}$ rad/s

**Three cases are possible:**

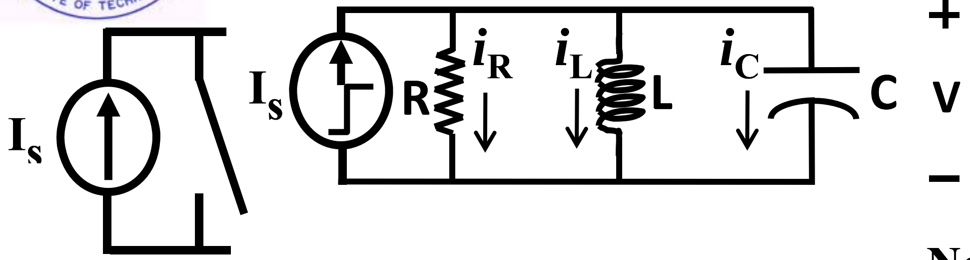
**(a) Overdamped case** $\alpha^2>\omega_0^2$ **(b) Critically Damped** $\alpha^2=\omega_0^2$ **(c) Undamped** $\alpha^2<\omega_0^2$

$$i(t)=A_1e^{s_1t}+A_2e^{s_2t} \qquad i(t)=D_1te^{-\alpha t}+D_2e^{-\alpha t} \qquad i(t)=e^{-\alpha t}\left(B_1\cos\omega_d t+B_2\sin\omega_d t\right)$$

$$i=C\frac{dv_c}{dt} \qquad \frac{di}{dt}=C\frac{d^2v_c}{dt^2} \qquad \frac{d^2v_c}{dt^2}+\frac{R}{L}\frac{dv_c}{dt}+\frac{v_c}{LC}i(t)=\frac{V_s}{LC}$$

**(a)** $v_c=V_\infty+A_1'e^{s_1t}+A_2'e^{s_2t}$ **(b)** $v_c=V_\infty+D_1'te^{-\alpha t}+D_2'e^{-\alpha t}$ **(c)** $v_c=V_\infty+e^{-\alpha t}\left(B_1'\cos\omega_d t+B_2'\sin\omega_d t\right)$

# ESc201, Lecture 12: Step and Freq response of RLC (Parallel)




$$i_L + C\frac{dv}{dt} + \frac{v}{R} = I_s \qquad v = L\frac{di}{dt}$$

$$i_L + \frac{L}{R}\frac{di_L}{dt} + LC\frac{d^2 i_L}{dt^2} = I_s$$

$$\int v\,dt = L\int di_L \qquad \frac{v}{R} + C\frac{dv}{dt} + \frac{1}{L}\int_0^t v\,d\tau = I_s$$

**Note that there would be an integration constant depending on the source current at t=0$^+$**

**Differentiate with respect to time**

$$\frac{d^2 v}{dt^2} + \frac{1}{RC}\frac{dv}{dt} + \frac{v}{LC} = 0 \qquad s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0 \qquad s_{1,2} = -\frac{1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$

$$s_{1,2} = -\alpha' \pm \sqrt{\alpha'^2 - \omega_0^2} \qquad \alpha' = \frac{1}{2RC}\ \text{Np/sec}\ (s^{-1}),\ \omega_0 = \frac{1}{\sqrt{LC}}\ \text{rad/s}$$

**Three cases are possible:**

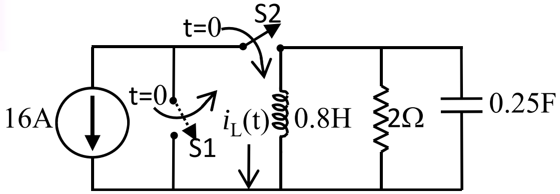
**(a) Overdamped case** $\alpha'^2 > \omega_0^2$ **(b) Critically Damped** $\alpha'^2 = \omega_0^2$ **(c) Undamped** $\alpha'^2 < \omega_0^2$

$$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} \qquad v(t) = D_1 t e^{-\alpha' t} + D_2 e^{-\alpha' t} \qquad v(t) = e^{-\alpha' t}\left(B_1 \cos\omega_d t + B_2 \sin\omega_d t\right)$$

$$v = L\frac{di_L}{dt} \qquad \frac{dv}{dt} = L\frac{d^2 i_L}{dt^2} \qquad \frac{d^2 i_L}{dt^2} + \frac{1}{CR}\frac{di_L}{dt} + \frac{i_L}{LC} = \frac{I_s}{LC}$$

$$i_L = I_\infty + A_1' e^{s_1 t} + A_2' e^{s_2 t} \quad \textbf{(b)}\ i_L = I_\infty + D_1' t e^{-\alpha' t} + D_2' e^{-\alpha' t} \quad \textbf{(c)}\ i_L = I_\infty + e^{-\alpha' t}\left(B_1' \cos\omega_d t + B_2' \sin\omega_d t\right)$$

## ESc201, Lecture 12: Step and Freq response of RLC (parallel)




$i_L(0^+) = 0$, $(di_L/dt)\big|_{t=0^+} = 0$, $i_L(\infty) = -16A$

$$\alpha' = \frac{1}{2RC} = \frac{1}{4\times 0.25} = 1 \text{ rad/s},\ \alpha'^2 = 1$$

$$\omega_0^2 = \frac{1}{LC} = \frac{1}{0.8\times 0.25} = 5,\ \omega_0^2 > \alpha'^2 \quad \text{Underdamped case.}$$

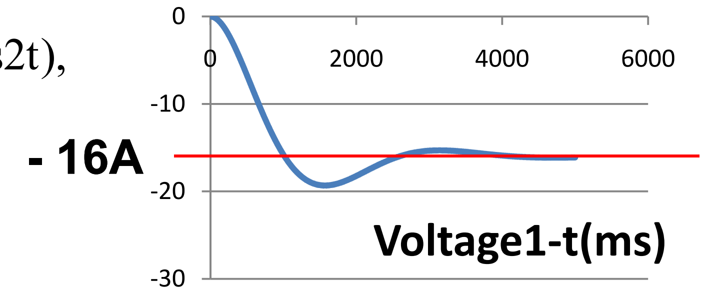
$$s_{1,2} = -\frac{1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}} \qquad s_{1,2} = -1 \pm \sqrt{1-5} = -1 \pm 2j \text{ rad/s}$$

$$\therefore\ i_L(t) = -16 + B_1'e^{-t}\cos 2t + B_2'e^{-t}\sin 2t \qquad i_L(0) = -16 + B_1' = 0 \text{ or } B_1' = 16 \text{ A}$$

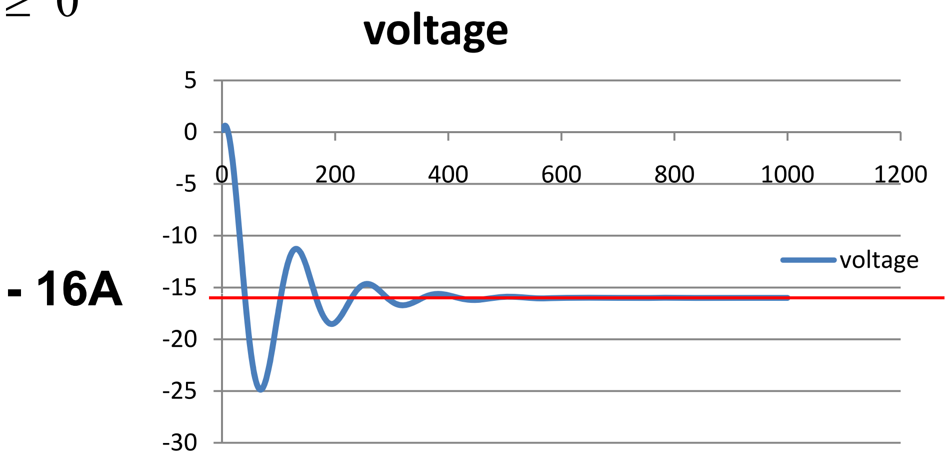
$$\frac{di_L(t)}{dt} = (-e^{-t})(B_1'\cos 2t + B_2'\sin 2t) + e^{-t}(-2B_1'\sin 2t + 2B_2'\cos 2t),$$

$$\left.\frac{di_L(t)}{dt}\right|_{t=0^+} = -B_1' + 2B_2' = 0 \quad \text{or } B_2' = \frac{B_1'}{2} = 8 \text{ A}$$

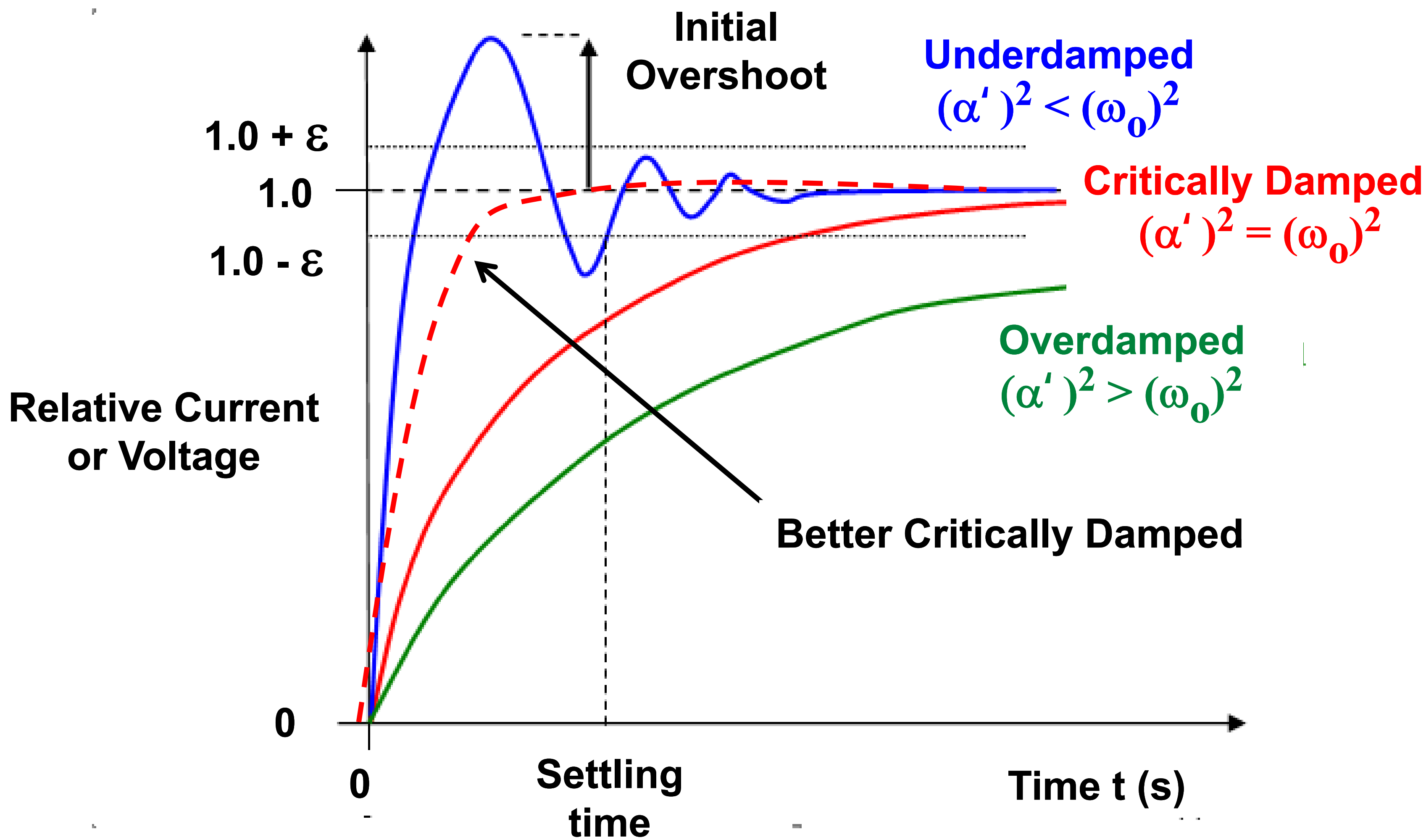
$$\therefore\ i_L(t) = -16 + 16e^{-t}\cos 2t + 8e^{-t}\sin 2t \quad \text{A for } t \geq 0$$




$$i_L(t) = -16 + 16e^{-10t}\cos 50t + 8e^{-10t}\sin 50t$$



ε is a small fraction, which tells us the tolerance within which the steady state value is required or measurements are done.




Note that as one goes from Underdamped to Overdamped case, the time required to attain a steady 1.0 level becomes longer. Ideally therefore one would want an underdamped case, where the Initial overshoot is < ε to get the fastest response.

**The most convenient way to plot a *Transfer Function is by Bode Plot.* Information regarding variation of *magnitude* and *phase* of any transfer function as a function of *frequency* can be made even simpler by a technique known as '*Asymptotic Bode Plot*'.**

**The *Transfer Function* |ℍ| <u>magnitude</u> is plotted in decibels (dB) & the <u>Phase</u> ∡θ in Deg.**

**If ℍ is in the in the complex frequency "*s*" domain, where $s = \alpha \pm j\omega$ , then for lumped R, L, C, etc. then ℍ can be expressed as a fraction of polynomials. i.e.**

$$H(s)=\frac{a_m s^m + \ldots\ldots + a_1 s + a_o}{b_n s^n + \ldots\ldots + b_1 s + a_o} = K'\frac{s(s+z_1)(s+z_2)\ \ldots\ldots(s+z_m)}{s(s+p_1)(s+p_2)\ \ldots\ldots(s+p_n)}$$

**Special cases:**

$z_i=0$, $|\mathbb{H}|\to\infty$, $p_i=0$, $|\mathbb{H}|\to 0$

$$=K\frac{\left(1+\frac{s}{z_1}\right)\left(1+\frac{s}{z_2}\right)\ldots\ldots\left(1+\frac{s}{z_m}\right)}{\left(1+\frac{s}{p_1}\right)\left(1+\frac{s}{p_2}\right)\ldots\ldots\left(1+\frac{s}{p_n}\right)} \xrightarrow{s=j\omega} K\frac{\left(1+j\frac{\omega}{z_1}\right)\left(1+j\frac{\omega}{z_2}\right)\ldots\ldots\left(1+j\frac{\omega}{z_m}\right)}{\left(1+j\frac{\omega}{p_1}\right)\left(1+j\frac{\omega}{p_2}\right)\ldots\ldots\left(1+j\frac{\omega}{p_n}\right)}$$

$$\mathbb{H} = |\mathbb{H}|\ \measuredangle\theta^o$$

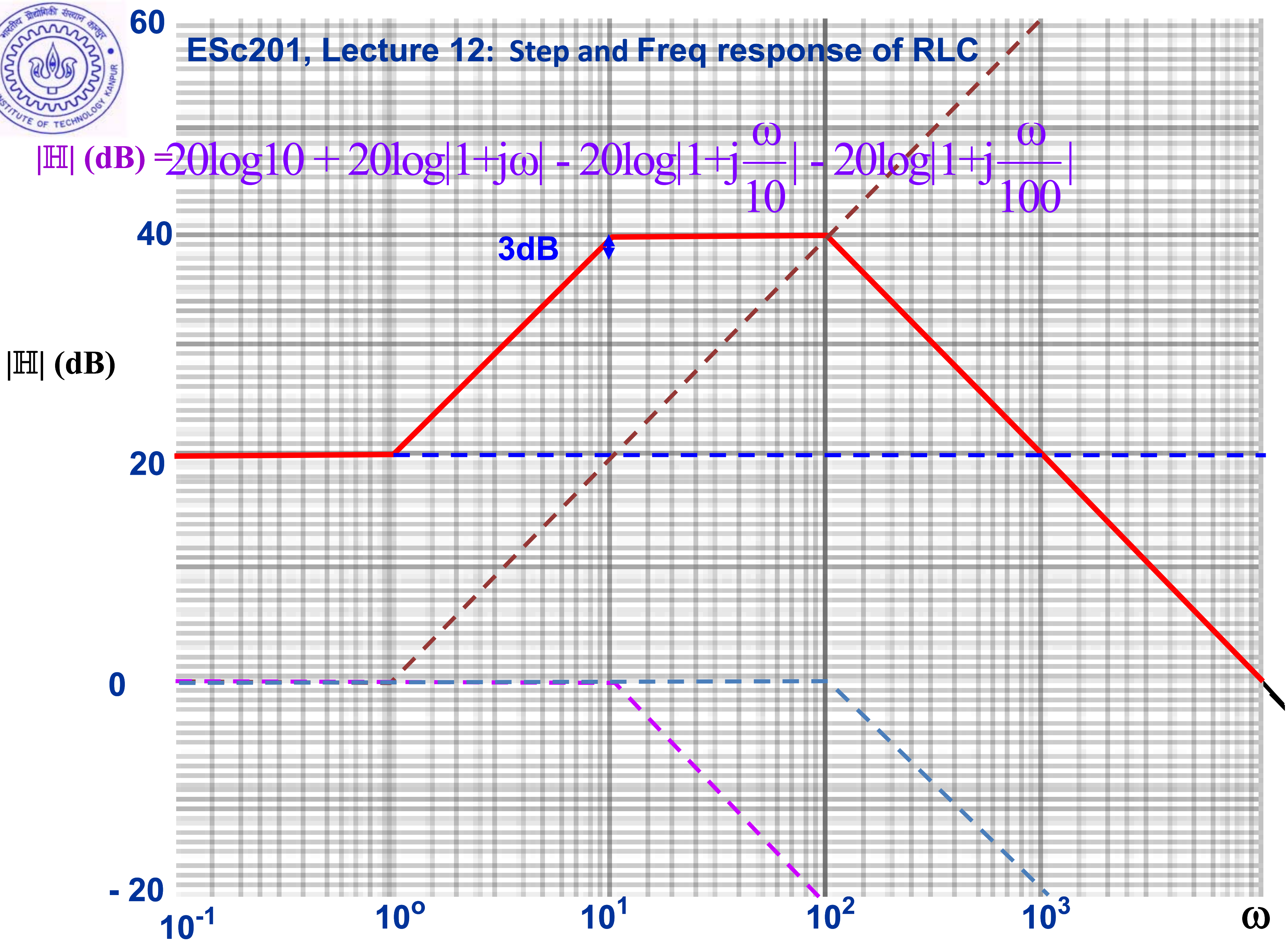
$$|\mathbb{H}|\ (\mathrm{dB}) = 20\log|\mathbb{H}|$$

$$H(j\omega)=\frac{10\left(1+j\frac{\omega}{1}\right)}{\left(1+j\frac{\omega}{10}\right)\left(1+j\frac{\omega}{100}\right)}$$

$$|\mathbb{H}|\ (\mathrm{dB}) = 20\log 10 + 20\log|1+j\omega| - 20\log\left|1+j\frac{\omega}{10}\right| - 20\log\left|1+j\frac{\omega}{100}\right|$$

$$\measuredangle\theta^o = \tan^{-1}(\omega/1) - \tan^{-1}(\omega/10) - \tan^{-1}(\omega/100)$$

$$|\mathbb{H}|\ (\mathrm{dB}) = 20\log 10 + 20\log|1+j\omega| - 20\log\left|1+j\frac{\omega}{10}\right| - 20\log\left|1+j\frac{\omega}{100}\right|$$



## ESc201, Lecture 12: Step and Freq response of RLC

$$\measuredangle\theta^{o} = \tan^{-1}(\omega/1) - \tan^{-1}(\omega/10) - \tan^{-1}(\omega/100)$$