



ESC201 UDAs SECTIONS on Brihaspati: Laboratory Days and Tutorial Rooms.

**M1, M2, M3,
Tu1, Tu2, Tu3,
W1, W2,
Th1, Th2, Th3,
F1, F2, F3.**

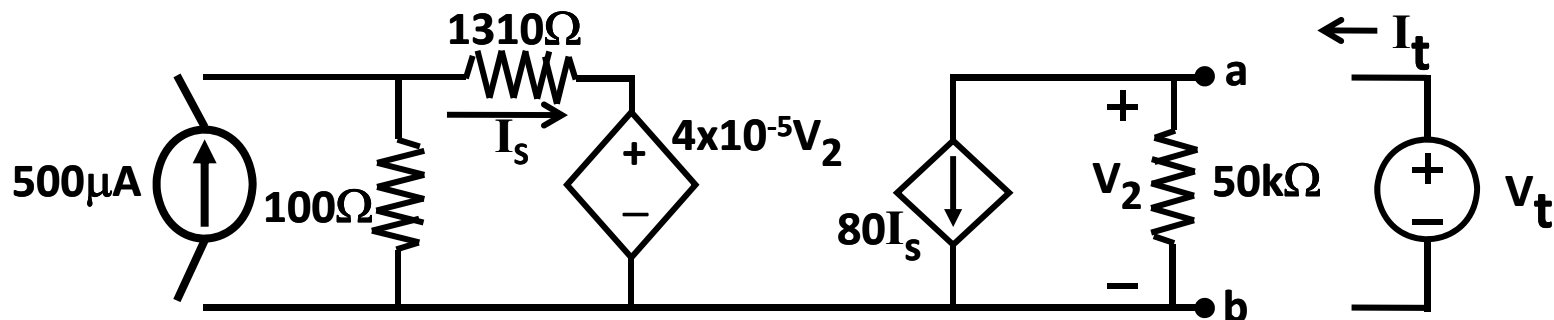


ESC201 UDas Lec6 Maximum Power Transfer

Test Voltage Method to Find R_{Th} when it is difficult to short the output end to find I_{sc} :

Procedure:

- i) Open circuit the load terminals and attach a test voltage source V_t .
- ii) Null all independent sources (short Voltage sources and open Current sources), keeping dependent source undisturbed.
- iii) Perform a circuit analysis, and find the current I_t drawn by the circuit from V_t .
- iv) Find R_{th} from the ratio of V_t / I_t .



$V_t = 50k(I_t - 80I_s)$ and $V_2 = V_t$. Again in the Controlled Voltage circuit

$$4 \times 10^{-5} V_t = -I_s(1310 + 100) = -1410 I_s.$$

$$\text{Or } V_t = 50kI_t + (50k \times 80 \times 4 \times 10^{-5} / 1410) V_t.$$

$$V_t - (160/1410)V_t = 50kI_t \text{ Which gives } V_t / I_t = 56.4k\Omega.$$



ESC201 UDas Lec6 Maximum Power Transfer condition

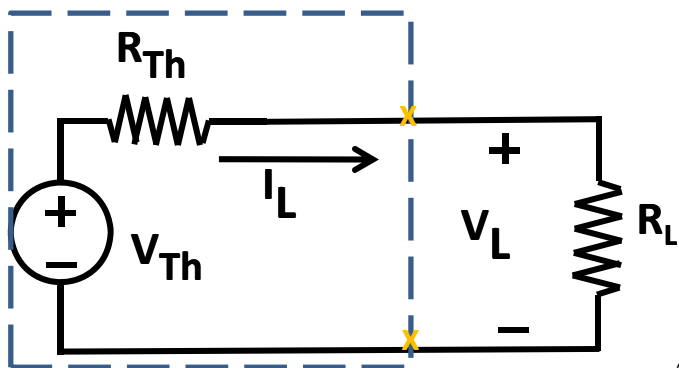
Maximum Power Transfer Theorem:

The Goal is to ensure that the maximum power is delivered to the load

- Extremely useful for audio applications : A speaker resistance can be tuned to ensure that the maximum power is transferred to it from the audio amplifier. Thus, the maximum possible level of sound is produced and minimum is wasted in the amplifier.
- One important condition for this to happen is given by the maximum power transfer theorem.

To find the value of R_L that would ensure *maximum power* to be transferred to it, and to find this maximum Power consumed by R_L which is $(P_L)_{\max}$.

Thevenin Equivalent representation of any circuit on the left is driving a load R_L .



$$I_L = \frac{V_{Th}}{R_{Th} + R_L} \quad \text{And} \quad P_L = R_L I_L^2 = \frac{V_{Th}^2 R_L}{(R_{Th} + R_L)^2}$$

Given V_{Th} and R_{Th} are fixed for the circuit it represents, the only way that P_L can be maximized is by varying R_L .

The condition at which P_L be maximum would be the condition $dP_L/dR_L = 0$.

$$\frac{dP_L}{dR_L} = 0 = \frac{(V_{Th})^2}{(R_{Th} + R_L)^2} - \frac{2(V_{Th})^2 R_L}{(R_{Th} + R_L)^3} \quad \text{or} \quad \frac{1}{2R_L} = \frac{1}{R_{Th} + R_L} \quad \text{or} \quad R_L = R_{Th}$$



ESC201 UDas Lec6 Transient Response of R, L, C circuits.

Time Domain (Transient) Response

For Inductors (L) the current, as mentioned earlier, cannot change instantaneously and for Capacitors (C) the voltage across it cannot change instantaneously.

These are Passive elements, capable of storing and delivering finite amounts of energy, but the average power cannot be greater than zero over an infinite time interval, as they are ideally not dissipative elements.

Therefore the current – voltage relationship for these two elements are a *function of time*.

To investigate the *time domain (transient)* response of RL and RC circuits

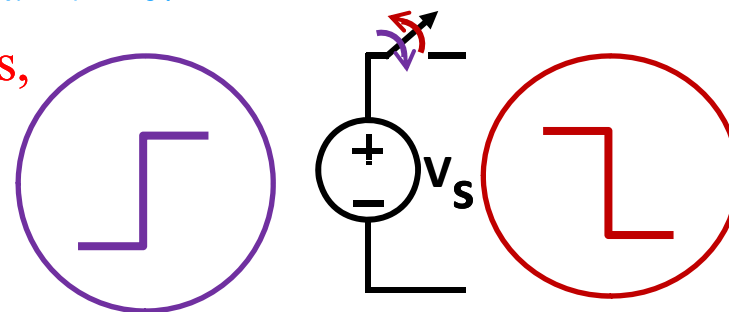
Transient Response:

Time response of RC and RL circuits, known as *1st-order circuits*, have to be found.

Circuits, having R, L, and C are known as *2nd-order circuits*, can be looked into later.

The input is assumed to be a *step function*, either going from zero to maximum or from maximum to zero within an infinitesimally small time.

The behaviour of the circuit on either of these inputs, which is a function of time, is known as the *transient response*.

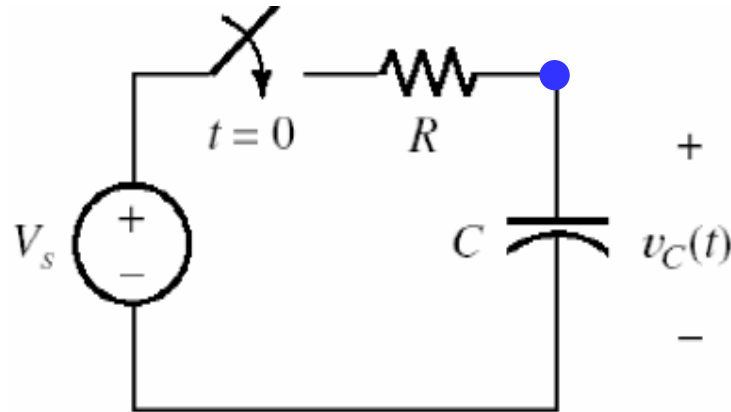




ESC201 UDas Lec6 Power Transfer and Transient Response

R-C Circuits:

V_s is a DC Voltage Source of magnitude V_s
Switch S was open for a long time and is closed at $t = 0$.



Application of KCL at the indicated node gives

$$C \frac{dv_c(t)}{dt} + \frac{v_c(t) - V_s}{R} = 0 \quad \text{For } t > 0^+$$

$$RC \frac{dv_c(t)}{dt} + v_c(t) = V_s$$

$v(t)$ for both $t = 0^-$ and 0^+ are zero, since the capacitor was initially discharged, and that the voltage across a capacitor cannot change instantly. i.e. $v_c(0^+) = v_c(0^-)$

$$\frac{dx}{dt} = -a_1 x + a_2$$

Solution:

$$x(t) = K_1 + K_2 e^{-a_1 t}$$

$$x(\infty) = K_1$$

$$x(t) = x(\infty) + K_2 e^{-a_1 t}$$

Using the initial condition at $t=0$:

$$x(0) = x(\infty) + K_2$$

$$x(t) = x(\infty) + \{x(0) - x(\infty)\} e^{-a_1 t}$$



ESC201 UDas Lec6 Power Transfer and Transient Response

$$RC \frac{dv_c(t)}{dt} + v_c(t) = V_s$$

$$v_c(t) = v_c(\infty) + \{v_c(0^+) - v_c(\infty)\} e^{-\frac{t}{RC}}$$

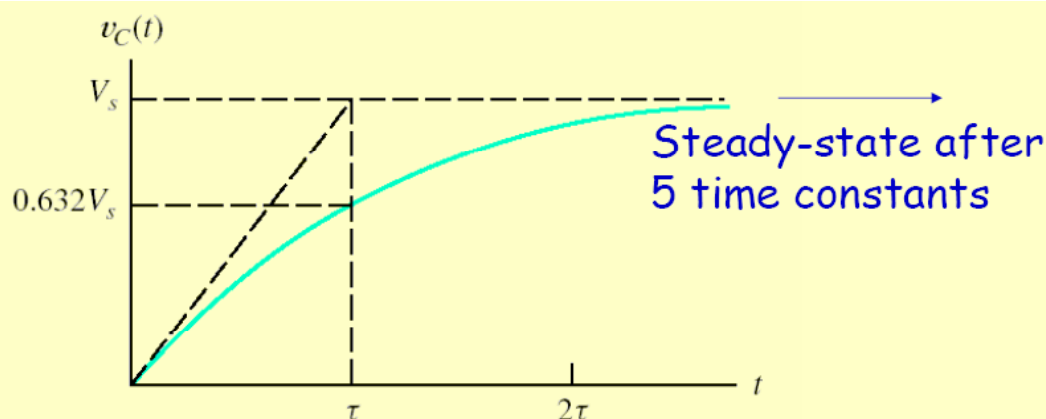
Where RC should have the dimension of time i.e. $\tau = RC$ is called the time constant of the R-C circuit.

If the capacitor was under discharged condition then

$$v_c(0^+) = v_c(0^-) = 0$$

and $v_c(\infty) = V_s$ Then

$$v_c(t) = V_s (1 - e^{-\frac{t}{\tau}})$$



Since the capacitor voltage varies exponentially with time, hence, it would take infinite time for this voltage to become exactly equal to V_s . However in $4-5\tau$ it is very close to V_s .



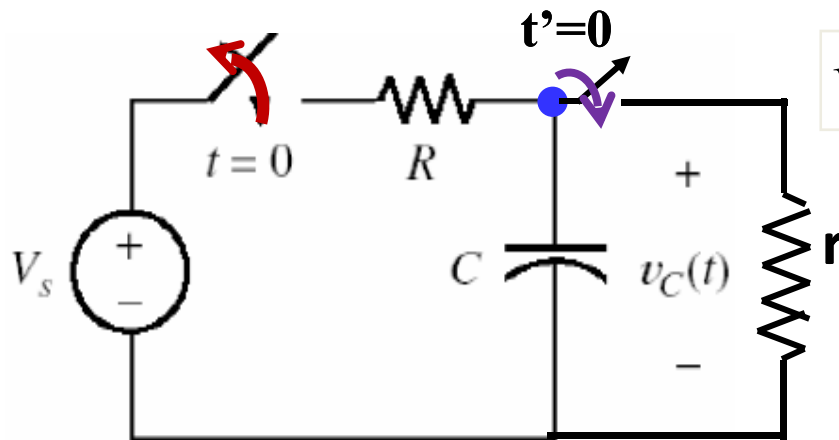
ESC201 UDas Lec6 Power Transfer and Transient Response

On the other hand if the switch is opened after the capacitor has fully charged to V_s :

$$v_C(t) = v_C(\infty) + \{v_C(0^+) - v_C(\infty)\} e^{-\frac{t}{RC}}$$

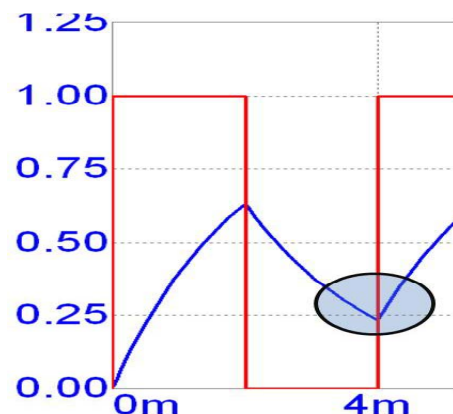
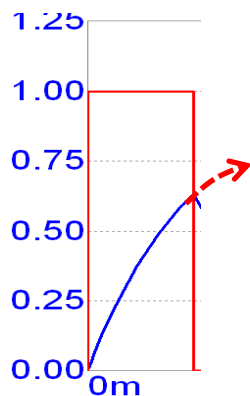
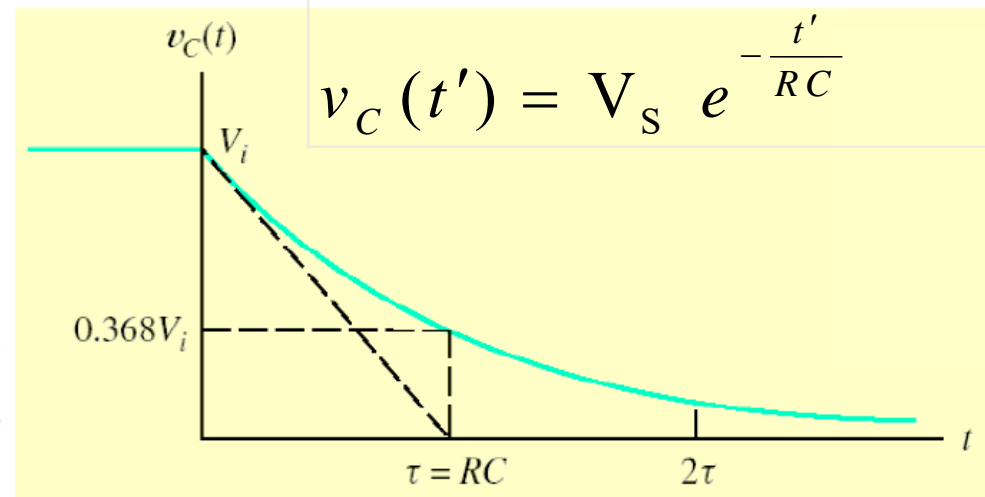
$$v_C(0^+) = v_C(0^-) = V_s$$

And the capacitor will ideally hold the charge for infinite time unless there is a discharge path, which can be provided by connecting a resistor “r” in parallel to the capacitor.



$$v_C(\infty) = 0, \quad v_C(t') = v_C(0^+) e^{-\frac{t'}{RC}}$$

$$v_C(t') = V_s e^{-\frac{t'}{RC}}$$



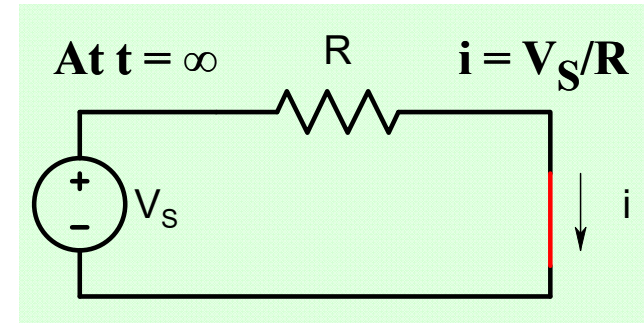
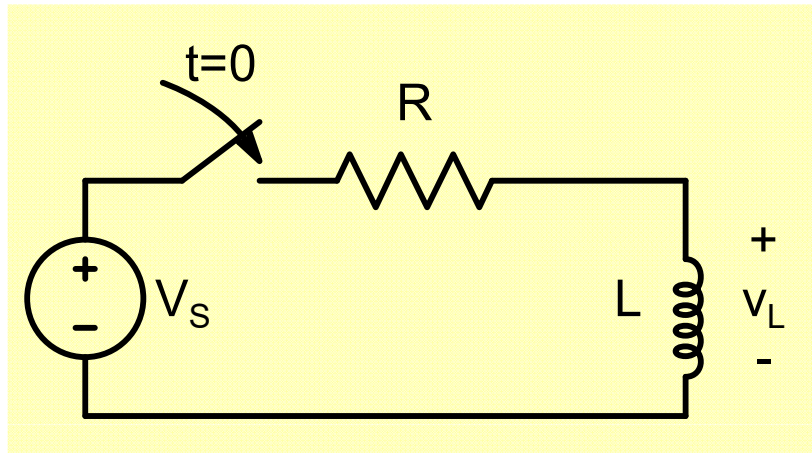
If the capacitor voltage at time $t'=0$ Has not reached, such as say $v_C(t)$ then

$$v_C(t' = 0^+) = v_C(t' = 0^-) = v_C(t)$$



ESC201 UDas Lec6 Power Transfer and Transient Response

R-L Circuits: V_s is a DC Voltage Source of magnitude V_s
Switch S was open for a long time and is closed at $t = 0$



$i_L(t)$ for both $t = 0^-$ and 0^+ are zero, since the Inductor was not carrying any current.

For $t > 0^+$

$$V_s = R i(t) + L \frac{di(t)}{dt}$$

$$x(t) = x(\infty) + \{x(0) - x(\infty)\} e^{-a_1 t}$$

Inductor current cannot change instantly.

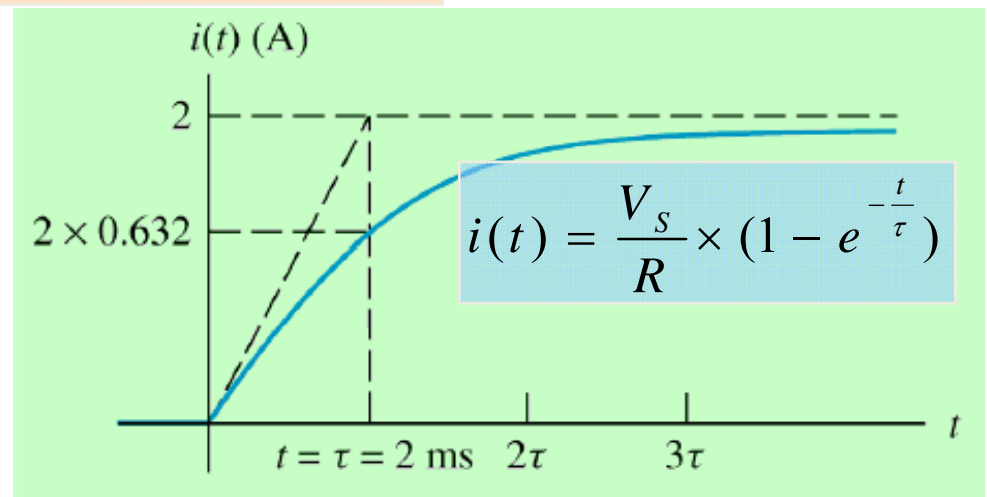
$$i(0^+) = i(0^-) = 0$$

$$i(t) = \frac{V_s}{R} + \left\{ i(0) - \frac{V_s}{R} \right\} e^{-\frac{R}{L}t}$$

Time Constant: $\tau = \frac{L}{R}$

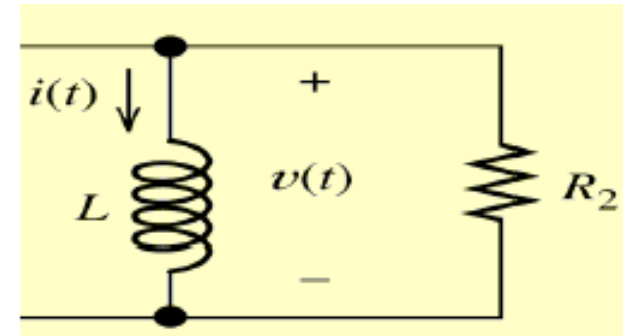
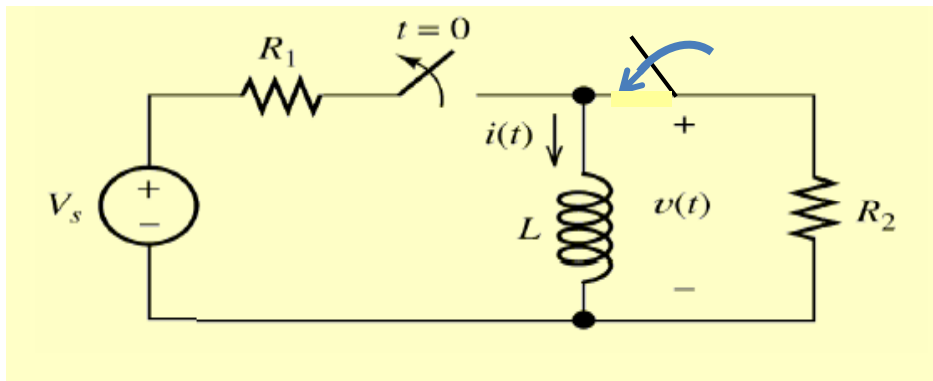
$$v_L(t) = V_s e^{-\frac{t}{\tau}}$$

As at $t=0^+$ $i_L t=0$ the whole of $V_s = v_L(t)$.





ESC201 UDas Lec6 Transient Response: Inductor Len's law.



$$i(t \rightarrow \infty) = 0$$

$$i(t) = i(\infty) + \{i(0^+) - i(\infty)\} \times e^{-\frac{t}{\tau}}$$

$$\tau = \frac{L}{R_2}$$

$$i(t) = i(0^+) \times e^{-\frac{t}{\tau}}$$

$$i(0^+) = i(0^-) = \frac{V_s}{R_1}$$

$$i(t) = \frac{V_s}{R_1} e^{-\frac{R_2}{L}t}$$

What happens to the voltage across the inductor now?

It was zero to start with. Think of Lenz's law and the inductor's application as a choke in a fluorescent tube light.