ESc201, Lecture 10: Sinusoidal steady state analysis - Power

$$p = (1/2)V_m I_m \cos(\phi_v - \phi_i)$$
$$+ (1/2)V_m I_m \cos(\phi_v - \phi_i).\cos(2\omega t) - (1/2)V_m I_m \sin(\phi_v - \phi_i).\sin(2\omega t)$$

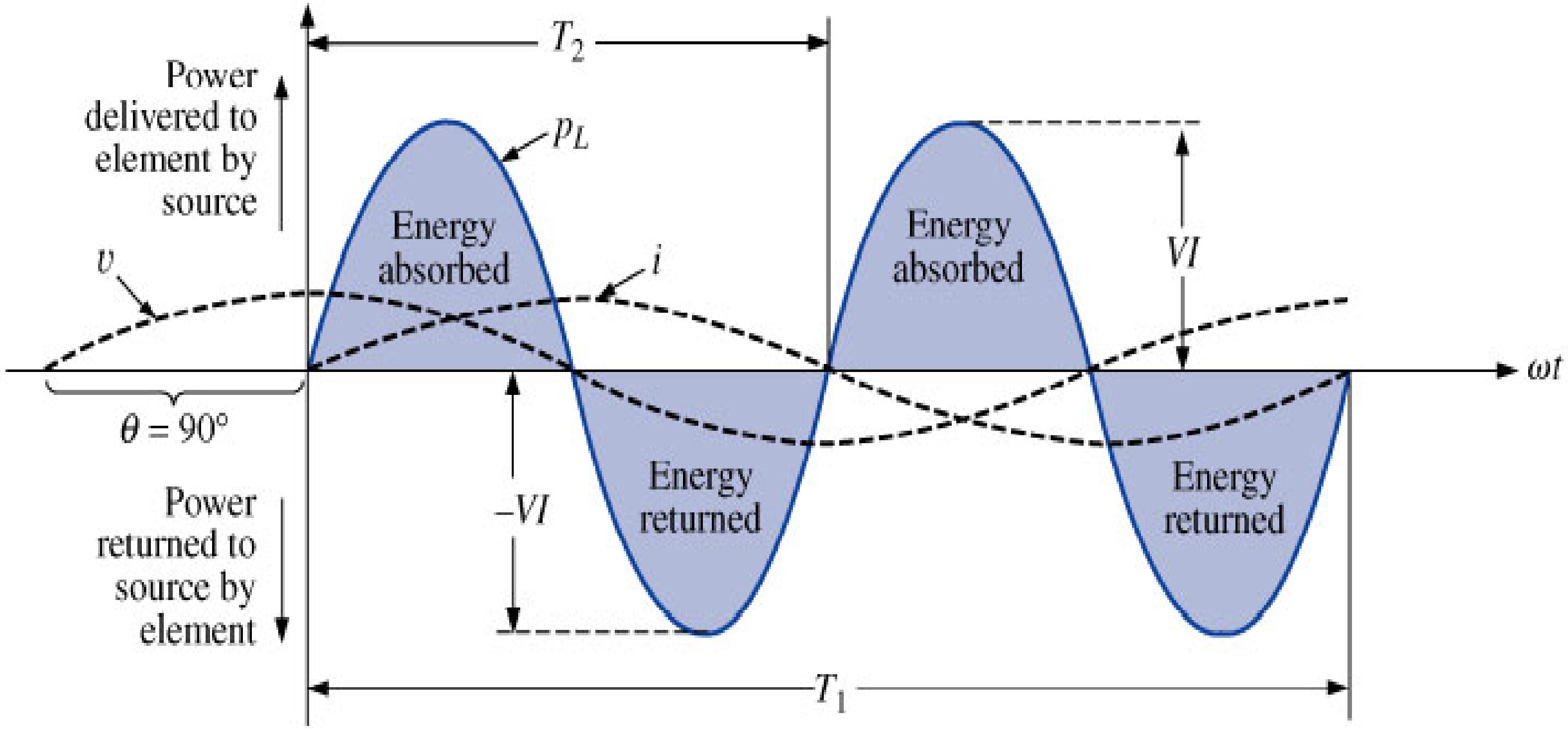
Hence p (Instantaneous Power) = $P + P.\cos(2\omega t) - Q.\sin(2\omega t)$

and Q (Reactive Power) = $V_{rms} I_{rms}.\sin(\theta)$

Complex Power is $\mathbb{S} = P + jQ = V_{rms} I^{*}_{rms} \measuredangle(\phi_v - \phi_i)$.

And $|\mathbb{S}| = \sqrt{(P^2 + Q^2)}$ is the (Apparent Power) in units of [V.A]

Inductive and Capacitive loads



From: Ketan Rajawat

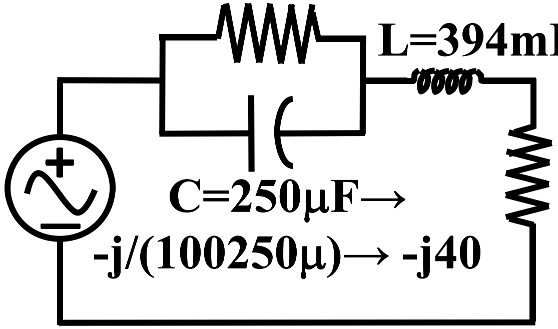
ESc201, Lecture 10: Sinusoidal steady state analysis - Power

Complex Power, Average Power, & Reactive Power delivered by the Source

ω = 100 rad/s

V_s=10cos(100t)

$\mathbb{V}_s = 10\angle 0^o$



Z seen by the source is
R||C +j39.4+5.8

Compute R||C

$$30\|-j40=\frac{30\times(-j40)}{30-j40}=\frac{1200\angle(-90^o)}{50\angle(-53.13^o)}=24\angle(-36.9^o)=19.2-j14.41\ \Omega$$

$$\tilde{Z}=19.2 - j14.41 + j39.4 + 5.8 = 25 + j25 = 35.36\angle 45^o$$

$$\tilde{I}_Z=\frac{10\angle 0^o}{35.36\angle 45^o}=0.283\angle-45^o=0.2\sqrt{2}\angle-45^o,\quad \tilde{I}_S=-\tilde{I}_Z$$

$$\tilde{S}_S=-\frac{1}{2}\tilde{V}_S\tilde{I}^*_S=-\frac{(10\angle 0^o)(0.2\sqrt{2}\angle-45^o)}{2}=-1.0\ -j1.0\ \ \text{VA}$$

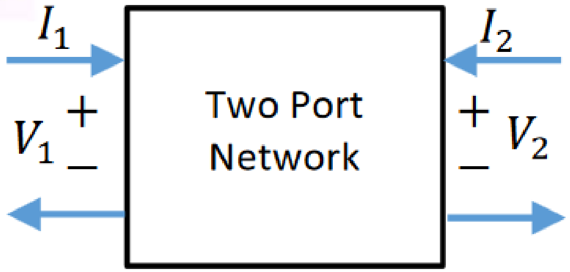
$$\tilde{S}_{S(\text{Supplying})}=1.0+j1.0\ \text{VA}\ \ (\text{Complex Power})$$

P (Average Power) = 1 W

Q (Reactive Power) = 1 VAR

$$\rightarrow\ |\tilde{S}_{S(\text{Supplying})}|=\sqrt{1^2+1^2}=\sqrt{2}=\sqrt{P^2+Q^2}$$

ESc201, Lecture 11: 2-Port Network



Most of the times one is only interested in finding out how the current/ voltage changes in a particular element when one input is varied. This is the underlying principle of a 2-port model. The most general description of a 2-port netork is carried out in the complex frequency "*s*" domain. Where $s = \alpha \pm j\omega$.

Z-Parameters $V_1 = z_{11} I_1 + z_{12} I_2$ $\quad V_2 = z_{21} I_1 + z_{22} I_2$ $\quad [Z][I] = [V]$

$$z_{11}\left(\begin{matrix}\text{Open circuit}\\ \text{Input Impedance}\end{matrix}\right) = \left.\frac{V_1}{I_1}\right|_{I_2=0} \ (\Omega)$$

For a 2-Port Network [Z] is a 2x2 matrix.

$$z_{22}\left(\begin{matrix}\text{Open circuit}\\ \text{Output Impedance}\end{matrix}\right) = \left.\frac{V_2}{I_2}\right|_{I_1=0} \ (\Omega)$$

$$z_{12}\left(\begin{matrix}\text{Reverse open circuit}\\ \text{Transimpedance}\end{matrix}\right) = \left.\frac{V_1}{I_2}\right|_{I_1=0} \ (\Omega)$$

$$z_{21}\left(\begin{matrix}\text{Forward open circuit}\\ \text{Transimpedance}\end{matrix}\right) = \left.\frac{V_2}{I_1}\right|_{I_2=0} \ (\Omega)$$

Y-Parameters [Y][V] = [I]

$I_1 = y_{11} V_1 + y_{12} V_2$
$I_2 = y_{21} V_1 + y_{22} V_2$

$$y_{11}\left(\begin{matrix}\text{Short Circuit}\\ \text{Input Admittance}\end{matrix}\right) = \left.\frac{I_1}{V_1}\right|_{V_2=0} \ (\mho)$$

$$y_{12}\left(\begin{matrix}\text{Reverse Short Circuit}\\ \text{Transconductance}\end{matrix}\right) = \left.\frac{I_1}{V_2}\right|_{V_1=0} \ (\mho)$$

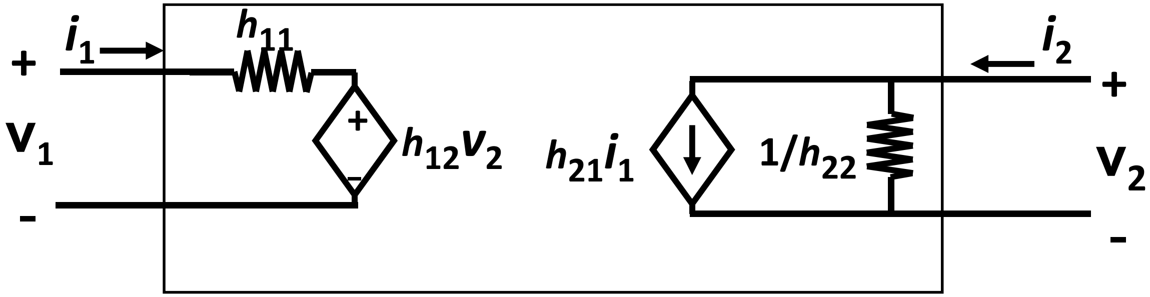
For a 2-Port Network [Y] is also a 2x2 matrix.

$$y_{21}\left(\begin{matrix}\text{Forward Short Circuit}\\ \text{Transconductance}\end{matrix}\right) = \left.\frac{I_2}{V_1}\right|_{V_2=0} \ (\mho)$$

$$y_{22}\left(\begin{matrix}\text{Short Circuit}\\ \text{Output Admittance}\end{matrix}\right) = \left.\frac{I_2}{V_2}\right|_{V_1=0} \ (\mho)$$

(HYBRID) h-Parameters – specially used for amplifiers

$$V_1 = h_{11} I_1 + h_{12} V_2$$
$$I_2 = h_{21} I_1 + h_{22} V_2$$



$$h_{11}\begin{pmatrix}\text{Short circuit}\\ \text{input Impedance}\end{pmatrix} = \left.\frac{V_1}{I_1}\right|_{V_2=0} \quad (\Omega)$$

$$h_{12}\begin{pmatrix}\text{Reverse open circuit}\\ \text{Voltage Gain}\end{pmatrix} = \left.\frac{V_1}{V_2}\right|_{I_1=0}$$

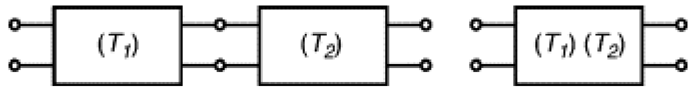
$$h_{21}\begin{pmatrix}\text{Forward short circuit}\\ \text{Current gain}\end{pmatrix} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$$

$$h_{22}\begin{pmatrix}\text{Open circuit}\\ \text{output Admitance}\end{pmatrix} = \left.\frac{I_2}{V_2}\right|_{I_1=0} \quad (\mho)$$

ABCD-Parameters

$$V_1 = AV_2 + B(-I_2)$$
$$I_1 = CV_2 + D(-I_2)$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}\begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$



$$V_{1a} = A_a V_{2a} + B_a(-I_{2a})$$
$$I_{1a} = C_a V_{2a} + D_a(-I_{2a})$$

$$V_{2a} = V_{1b}$$
$$-I_{2a} = I_{1b}$$

$$V_{1b} = A_b * V_{2b} + B_b * (-I_{2b})$$
$$I_{1b} = C_b * V_{2b} + D_b * (-I_{2b})$$

$$\begin{bmatrix} V_{1b} \\ I_{1b} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}_b \begin{bmatrix} V_{2b} \\ -I_{2b} \end{bmatrix}$$

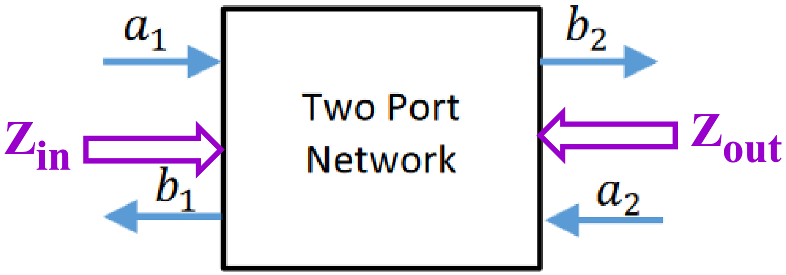
$$\begin{bmatrix} V_{1a} \\ I_{1a} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}_a \begin{bmatrix} V_{2a} \\ -I_{2a} \end{bmatrix}$$

$$\begin{bmatrix} V_{1a} \\ I_{1a} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}_a \begin{bmatrix} A & B \\ C & D \end{bmatrix}_b \begin{bmatrix} V_{2b} \\ -I_{2b} \end{bmatrix}$$

$$[ABCD]_{cascade} = [ABCD]_a * [ABCD]_b$$

S-Parameters

$$b_1 = S_{11} * a_1 + S_{12} * a_2$$
$$b_2 = S_{21} * a_1 + S_{22} * a_2$$



$$S_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0} \qquad S_{12} = \left.\frac{b_1}{a_2}\right|_{a_1=0}$$

$$S_{21} = \left.\frac{b_2}{a_1}\right|_{a_2=0} \qquad S_{22} = \left.\frac{b_2}{a_2}\right|_{a_1=0}$$

Easy to define but how to implement?

At "low" frequencies, one can completely characterize a linear device or network using an impedance matrix, which relates the currents and voltages at each device terminal to the currents and voltages at all other terminals. But, at microwave frequencies, it is difficult to measure total currents and voltages.

Instead, one can measure the magnitude and phase of each of the two transmission line waves $V^+(z)$ and $V^-(z)$.

OR can measure the input power at any port "a" and the reflected power at the same port as "b".

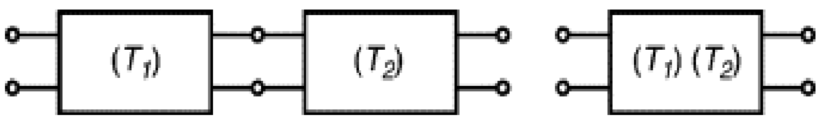
If a source is connected to port 1 and a load Z_L is connected to port 2 and $Z_{out} = Z_L$ then it is called a matched load and there is no reflection of the wave from the load, or $a_2 = 0$.

Similarly if the source is placed at port 2 and load at port 1 such that $Z_{in} = Z_L$ then $a_1 = 0$.

Typically, it is much more difficult to determine/measure the scattering parameters of the form S_{nn} , as opposed to scattering parameters of the form S_{mn} (where m≠n) where there is only an exiting wave from port m.

T-Parameters

$$b_1 = T_{11} * a_2 + T_{12} * b_2$$
$$a_1 = T_{21} * a_2 + T_{22} * b_2$$



$$[T]_{cascade} = [T]_a * [T]_b$$

a-Parameters

$$V_1 = a_{11}V_2 - a_{12}I_2,\ I_1 = a_{21}V_2 - a_{22}I_2$$

$$a_{11}\begin{pmatrix}\text{Voltage}\\ \text{Gain}\end{pmatrix} = \left.\frac{V_1}{V_2}\right|_{I_2=0} \qquad a_{22}\begin{pmatrix}\text{Current}\\ \text{Gain}\end{pmatrix} = -\left.\frac{I_1}{I_2}\right|_{V_2=0}$$

$$a_{12}\begin{pmatrix}\text{Trans}\\ \text{Impedance}\end{pmatrix} = -\left.\frac{V_1}{I_2}\right|_{V_2=0} \qquad a_{21}\begin{pmatrix}\text{Trans}\\ \text{Admittance}\end{pmatrix} = \left.\frac{I_1}{V_2}\right|_{V_2=0}$$

b-Parameters

$$V_2 = b_{11}V_1 - b_{12}I_1$$

$$I_2 = b_{21}V_1 - b_{22}I_1$$

$$b_{11} = \left.\frac{V_2}{V_1}\right|_{I_1=0} \qquad b_{12} = -\left.\frac{V_2}{I_1}\right|_{V_1=0}$$

$$b_{21} = \left.\frac{I_2}{V_1}\right|_{I_1=0} \qquad b_{22} = -\left.\frac{I_2}{I_1}\right|_{V_1=0}$$

g-Parameters

$$I_1 = g_{11}V_1 + g_{12}I_2$$

$$V_2 = g_{21}V_1 + g_{22}I_2$$

$$g_{11}\begin{pmatrix}\text{Open circuit}\\ \text{Input Admittance}\end{pmatrix} = \left.\frac{I_1}{V_1}\right|_{I_2=0} \qquad g_{12}\begin{pmatrix}\text{Reverse Short circuit}\\ \text{Current gain}\end{pmatrix} = \left.\frac{I_1}{I_2}\right|_{V_1=0}$$

$$g_{21}\begin{pmatrix}\text{Forward open circuit}\\ \text{Voltage gain}\end{pmatrix} = \left.\frac{V_2}{V_1}\right|_{I_2=0} \qquad g_{22}\begin{pmatrix}\text{Short circuit}\\ \text{Output Impedance}\end{pmatrix} = \left.\frac{V_2}{I_2}\right|_{V_1=0}$$

ESc201, Lecture 11: 2-Port Network

Parametric Conversion Table : **(Δ is the determinant of the matrix)**

$$z_{11}=\frac{y_{22}}{\Delta y}=\frac{a_{11}}{a_{21}}=\frac{b_{22}}{b_{21}}=\frac{\Delta h}{h_{22}}=\frac{1}{g_{11}}$$

$$z_{12}=-\frac{y_{12}}{\Delta y}=\frac{\Delta a}{a_{21}}=\frac{1}{b_{21}}=\frac{h_{12}}{h_{22}}=-\frac{g_{12}}{g_{11}}$$

$$z_{21}=-\frac{y_{21}}{\Delta y}=\frac{1}{a_{21}}=\frac{\Delta b}{b_{21}}=-\frac{h_{21}}{h_{22}}=\frac{g_{21}}{g_{11}}$$

$$z_{22}=\frac{y_{11}}{\Delta y}=\frac{a_{22}}{a_{21}}=\frac{b_{11}}{b_{21}}=\frac{1}{h_{22}}=\frac{\Delta g}{g_{11}}$$

$$y_{11}=\frac{z_{22}}{\Delta z}=\frac{a_{22}}{a_{12}}=\frac{b_{11}}{b_{12}}=\frac{1}{h_{11}}=\frac{\Delta g}{g_{22}}$$

$$y_{12}=-\frac{z_{12}}{\Delta z}=-\frac{\Delta a}{a_{12}}=-\frac{1}{b_{12}}=-\frac{h_{12}}{h_{11}}=\frac{g_{12}}{g_{22}}$$

$$y_{21}=-\frac{z_{21}}{\Delta z}=-\frac{1}{a_{12}}=-\frac{\Delta b}{b_{12}}=\frac{h_{21}}{h_{11}}=-\frac{g_{21}}{g_{22}}$$

$$y_{22}=\frac{z_{11}}{\Delta z}=\frac{a_{11}}{a_{12}}=\frac{b_{22}}{b_{12}}=\frac{\Delta h}{h_{11}}=\frac{1}{g_{22}}$$

(Whenever divided by Δ, $\Delta\neq 0$ is a must condition.

Conversion for the others are also there but not as frequently used.

$$h_{11}=\frac{\Delta z}{z_{22}}=\frac{1}{y_{11}}=\frac{a_{12}}{a_{22}}=\frac{b_{12}}{b_{11}}=\frac{g_{22}}{\Delta g}$$

$$h_{21}=-\frac{z_{21}}{z_{22}}=\frac{y_{21}}{y_{11}}=-\frac{1}{a_{22}}=-\frac{\Delta b}{b_{11}}=-\frac{g_{21}}{\Delta g}$$

$$h_{12}=\frac{z_{12}}{z_{22}}=-\frac{y_{12}}{y_{11}}=\frac{\Delta a}{a_{22}}=\frac{1}{b_{11}}=-\frac{g_{12}}{\Delta g}$$

$$h_{22}=\frac{1}{z_{22}}=\frac{\Delta y}{y_{11}}=\frac{a_{21}}{a_{22}}=\frac{b_{21}}{b_{11}}=\frac{g_{11}}{\Delta g}$$

$A = -\Delta h/h_{21}$, $B = -h_{11}/h_{21}$, $C = -h_{22}/h_{21}$, $D = -1/h_{21}$, **$[Y]=[Z]^{-1}\big|_{\Delta Z\neq 0}$, $[Z]=[Y]^{-1}\big|_{\Delta Y\neq 0}$**