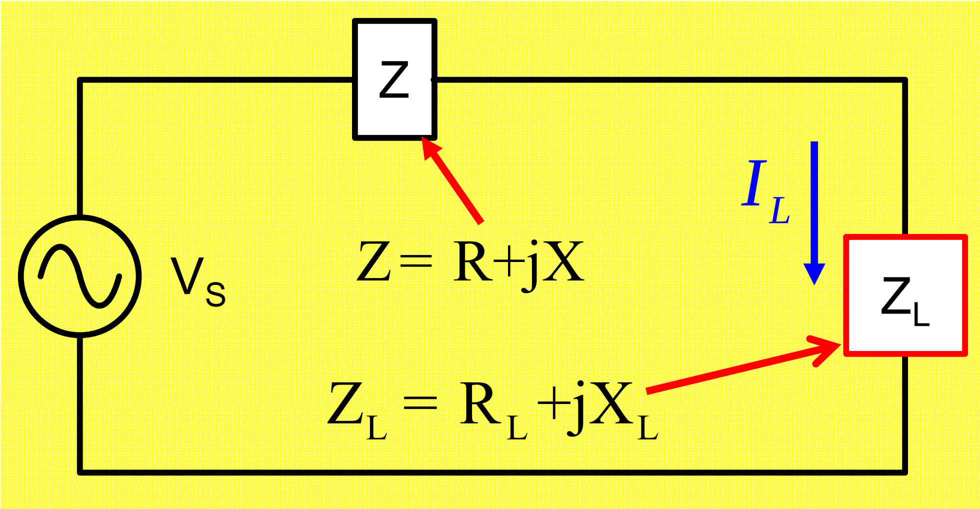
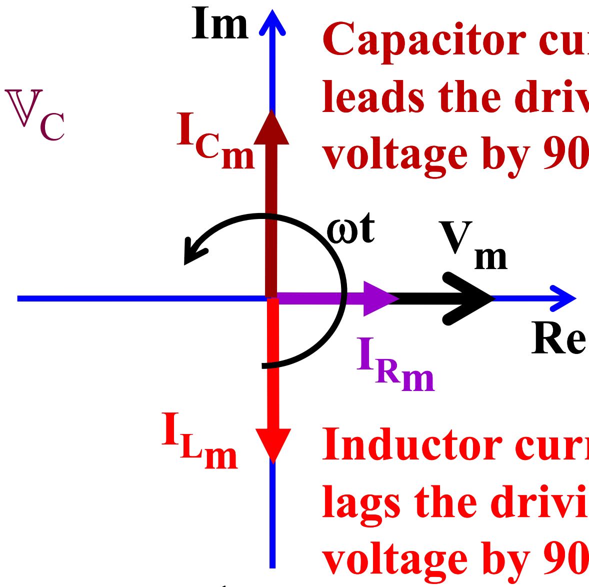
ESc201, Lecture 8: Sinusoidal steady state analysis

$$\mathbb{I}_R = \frac{1}{R}\mathbb{V}_R \qquad \mathbb{I}_L = -j\left(\frac{1}{X_L}\right)\mathbb{V}_L \qquad \mathbb{I}_C = j\left(\frac{1}{X_C}\right)\mathbb{V}_C$$





Capacitor current leads the driving voltage by 90°.

Inductor current lags the driving voltage by 90°.

$$P = I_{Rrms}^2 \times R = \frac{1}{2}\left|I_R\right|^2 R$$

$$I_L = \frac{V_S}{R + R_L + j(X + X_L)}$$

$$P_L = \frac{1}{2}\frac{V_S^2}{(R+R_L)^2 + (X+X_L)^2} R_L$$

For maximum load power : $X_L = -X$

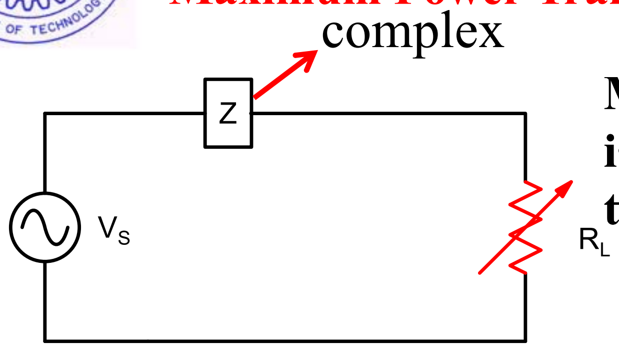
$$P_L = \frac{V_S^2}{(R+R_L)^2} R_L$$

Choose $R_L = R$ to maximize load power as done in the dc case and hence

$$Z_L = Z^* = R - jX$$

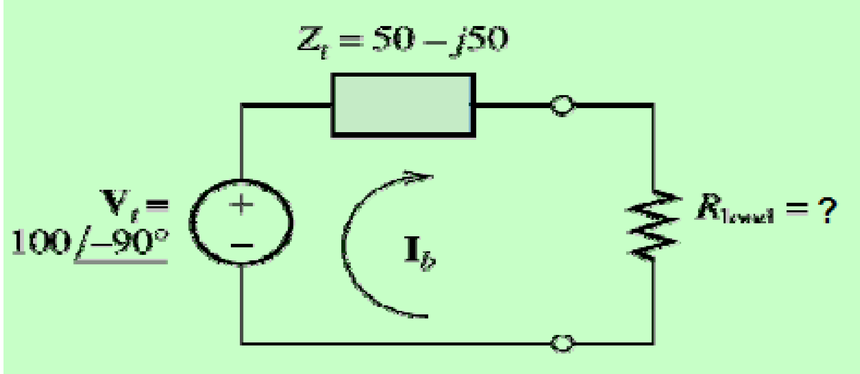
Maximum power is transferred to the load when load is complex conjugate of source impedance

Maximum Power Transfer for sinusoidal input when load is Resistive



Maximum power is transferred to the load when

$$R_L = |Z|$$



$$R_L = |Z| = \sqrt{50^2 + 50^2} = 70.71\Omega$$

$$Z_L = 50 + j50\,\Omega$$

Maximum power is transferred to the load when load is complex conjugate of source impedance

$$P = I_{brms}^2 R_{load}$$
$$= (\frac{0.7654}{\sqrt{2}})^2 \times 70.71 = 20.7$$

$$P = I_{arms}^2 R_{load}$$
$$= (\frac{1}{\sqrt{2}})^2 \times 50 = 25\text{W}$$

Power in steady state : Instantaneous power

$v(t) = V_m.\cos(\omega t)$ and $i(t) = I_m.\cos(\omega t+\phi_i - \phi_v)$. $\rightarrow$ $p = V_m I_m \cos(\omega t).\cos(\omega t+\phi_i - \phi_v)$

Or if the vgoltage is made the datum then one can write

$v(t) = V_m.\cos(\omega t+ \phi_v - \phi_i)$ and $i(t) = I_m.\cos(\omega t)$ $\rightarrow$ $p = V_m I_m \cos(\omega t+ \phi_v - \phi_i) \cos(\omega t)$

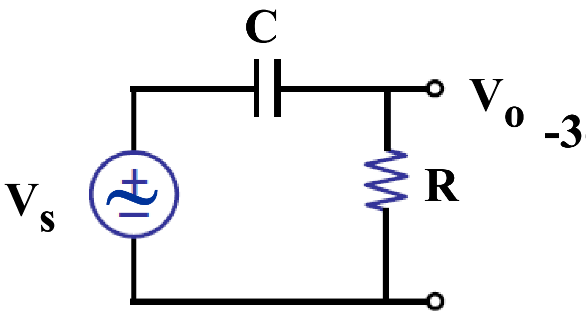
$p=(1/2)V_m I_m \cos(\phi_v - \phi_i) + (1/2)V_m I_m \cos(\phi_v - \phi_i).\cos(2\omega t) - (1/2)V_m I_m \sin(\phi_v - \phi_i).\sin(2\omega t)$

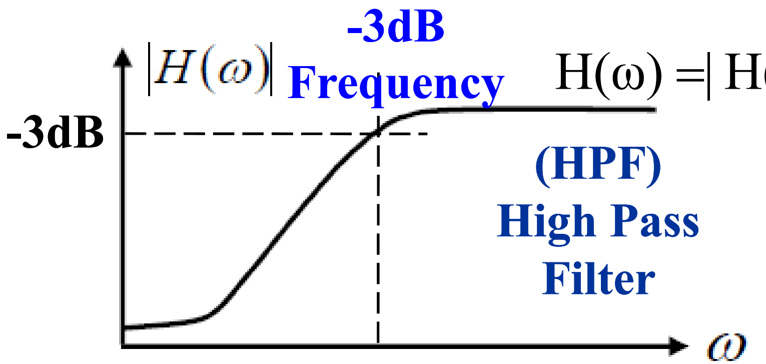
Av. power $p_{avg}=P = (1/2)V_m I_m \cos(\phi_v - \phi_i)=(V_m/\sqrt{2})(I_m/\sqrt{2})\cos(\phi_v - \phi_i)=V_{rms}I_{rms}\cos(\theta)$

$\theta=(\phi_v - \phi_i)$, $\cos(\theta)$=power factor (pf). Similarly $\sin(\theta)$ is called the reactive factor (rf)

ESc201, Lecture 8: Sinusoidal steady state analysis

$$\frac{\mathbb{V}_C}{\mathbb{I}_C} = Z_C = \frac{-j}{\omega C} \qquad \omega \to \infty,\ Z_c \to 0 \quad , \quad \omega \to 0,\ Z_c \to \infty$$

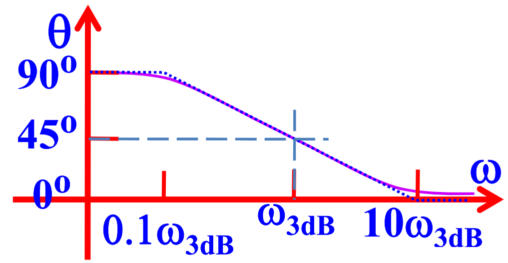


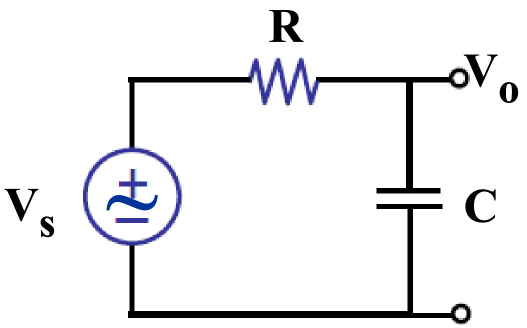


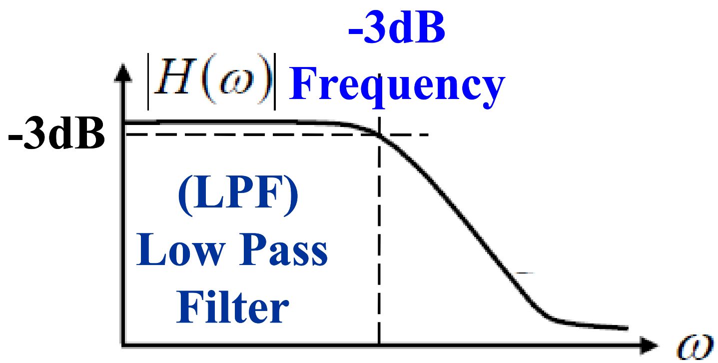
$$H(\omega) = |H(\omega)|\, e^{j\theta} = \frac{R e^{j\frac{1}{\omega RC}}}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}}$$

$$\theta \overset{\omega\to\infty}{=} 0^o, \quad \theta \overset{\omega\to 0}{=} 90^o$$

$$\frac{V_o(\omega)}{V_s(\omega)} = H(\omega) = \frac{R}{R + (-j/\omega C)} \overset{\omega\to\infty}{=} 1 \qquad H(\omega) = \frac{R}{R + (-j/\omega C)} \overset{\omega\to 0}{=} 0$$







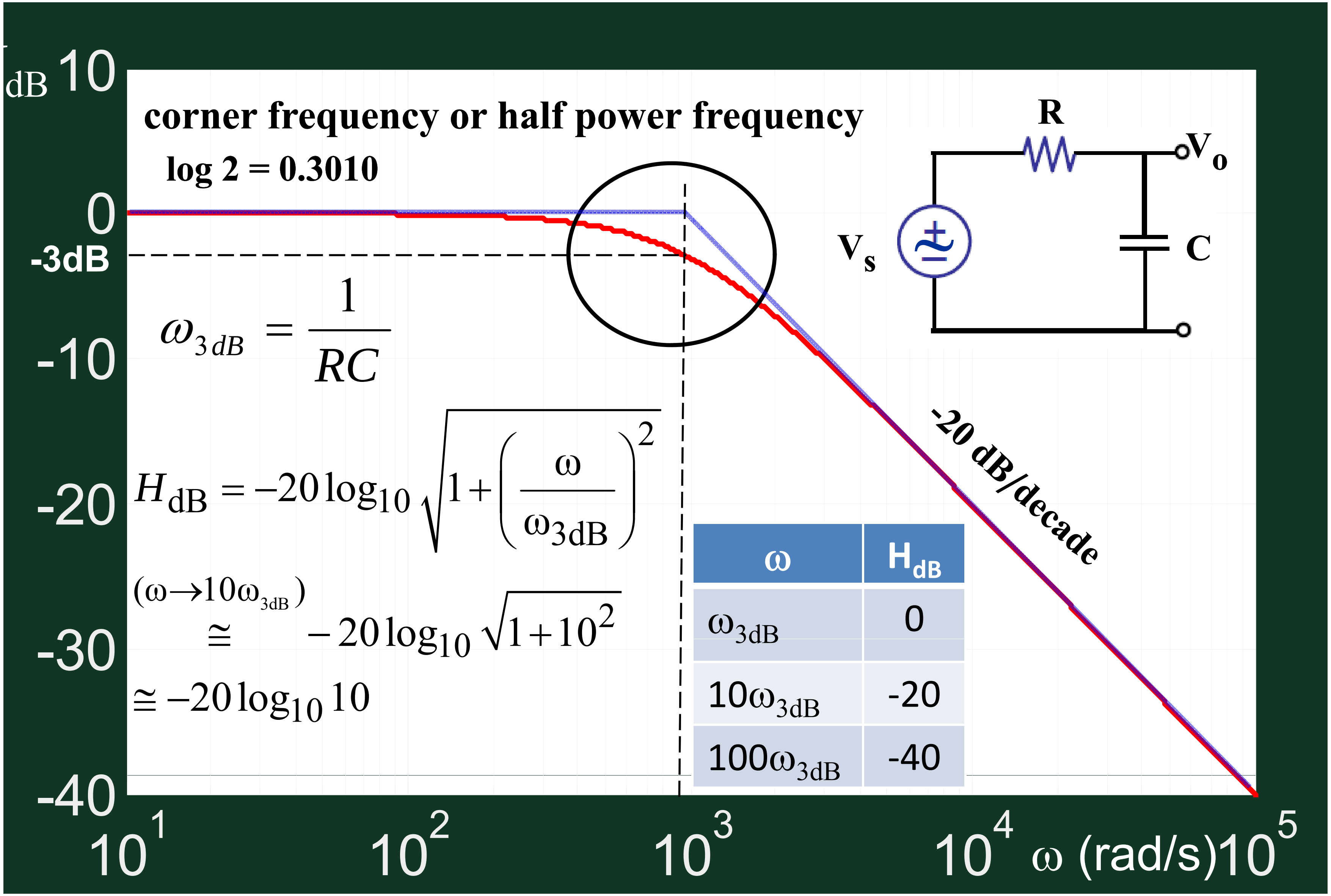
$$\theta = -\tan^{-1}\omega RC$$

$$\theta \overset{\omega\to 0}{=} 0^o, \quad \theta \overset{\omega\to\infty}{=} -90^o$$

$$\frac{V_o(\omega)}{V_s(\omega)} = H(\omega) = \frac{-j/\omega C}{R + (-j/\omega C)} = \frac{1}{1+j\omega RC} \overset{\omega\to\infty}{=} 0 \qquad H(\omega) = \frac{1}{1+j\omega RC} \overset{\omega\to 0}{=} 1$$

ESc201, Lecture 8: Sinusoidal steady state analysis

From: Ketan Rajawat



corner frequency or half power frequency

log 2 = 0.3010

$$\omega_{3\,dB} = \frac{1}{RC}$$

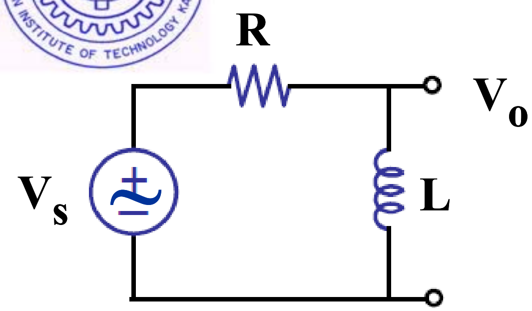
$$H_{dB} = -20\log_{10}\sqrt{1+\left(\frac{\omega}{\omega_{3dB}}\right)^2}$$

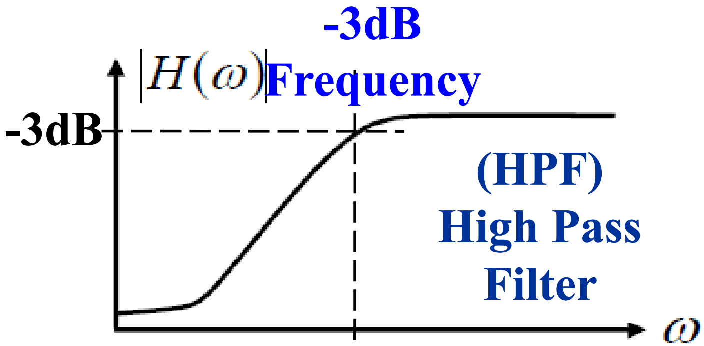
$$(\omega \to 10\omega_{3dB}) \quad \cong \; -20\log_{10}\sqrt{1+10^2}$$

$$\cong -20\log_{10} 10$$

ω	H_{dB}
ω_{3dB}	0
$10\omega_{3dB}$	-20
$100\omega_{3dB}$	-40

ESc201, Lecture 8: Sinusoidal steady state analysis





$$|H(\omega)|e^{j\theta} = \frac{e^{j\frac{R}{\omega L}}}{\sqrt{1+\left(\frac{R}{\omega L}\right)^2}}$$

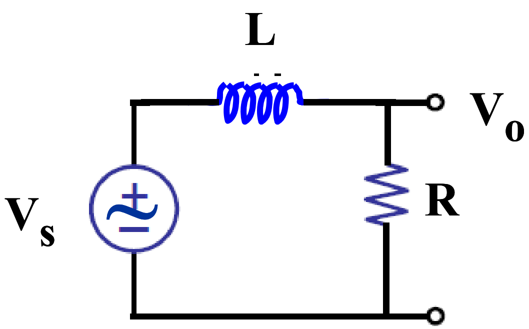
$$\theta \overset{\omega\to\infty}{=} 0^o,\ \theta \overset{\omega\to 0}{=} 90^o$$

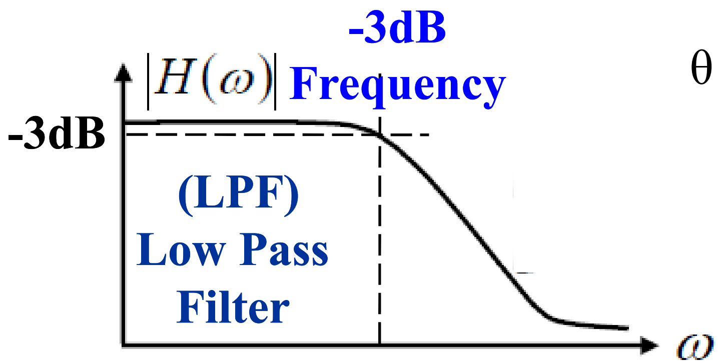
$$\frac{V_o(\omega)}{V_s(\omega)} = H(\omega) = \frac{j\omega L}{R + j\omega L} = \frac{1}{1\text{-}j(R/\omega L)} \overset{\omega\to\infty}{=} 1$$

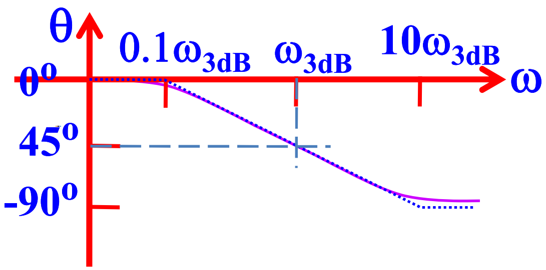
$$H(\omega) = \frac{j\omega L}{R + j\omega L} \overset{\omega\to 0}{=} 0$$

$$\theta = -\tan^{-1}\frac{\omega L}{R}$$

$$\theta \overset{\omega\to 0}{=} 0^o,\ \theta \overset{\omega\to\infty}{=} -90^o$$

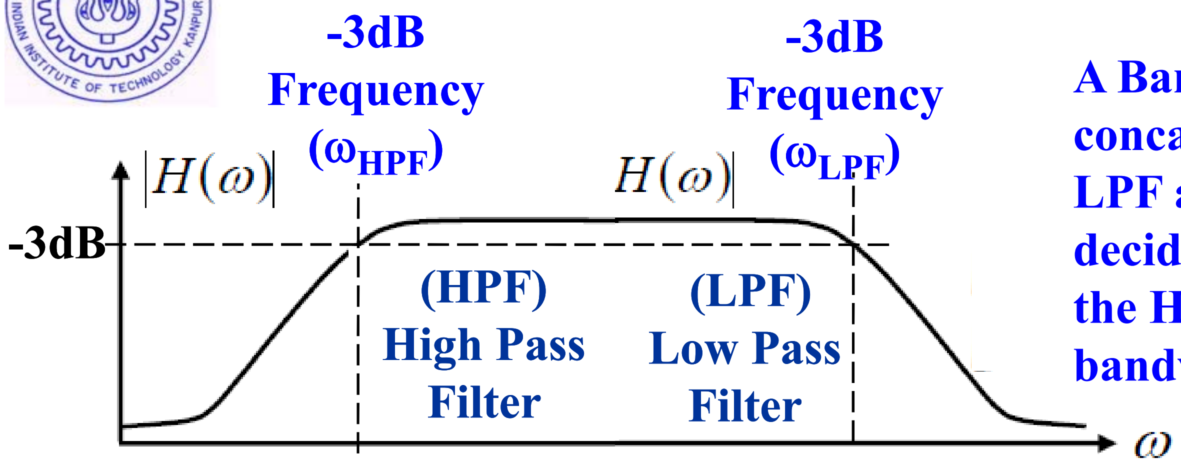




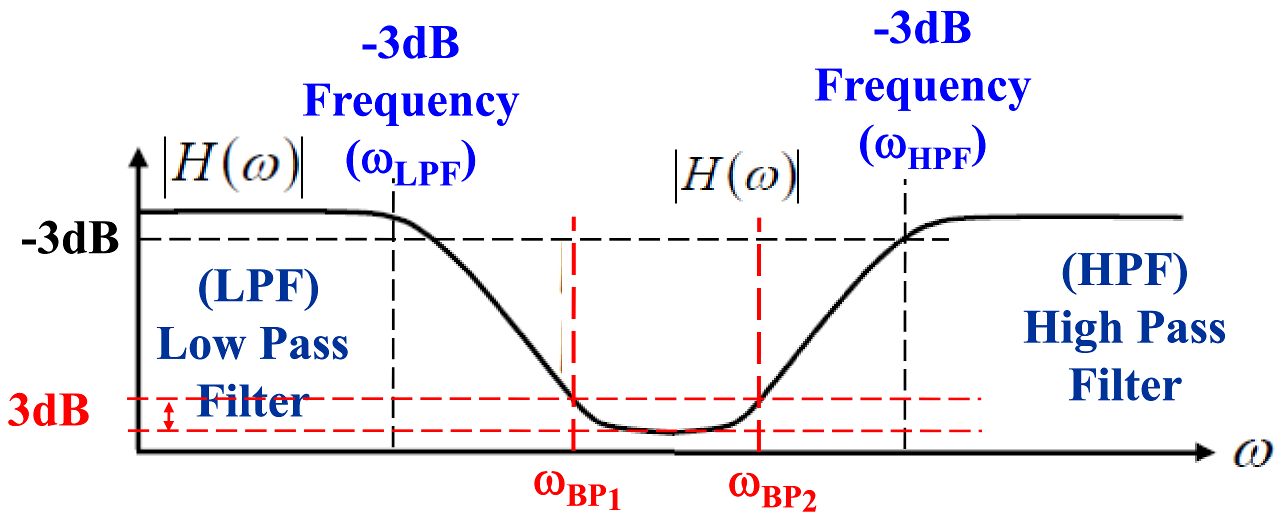


$$\frac{V_o(\omega)}{V_s(\omega)} = H(\omega) = \frac{R}{R + j\omega L} \overset{\omega\to\infty}{=} 0$$

$$H(\omega) = \frac{1}{1+j\omega L/R} \overset{\omega\to 0}{=} 1$$



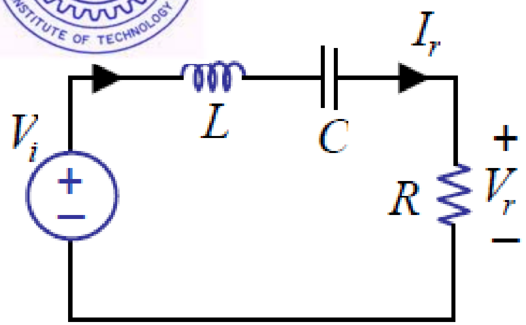
A Band Pass Filter can be made by concatenating two stages of HPF and LPF and the pass band would be decided by the 3dB frequencies of the HPF and the LPF. The pass bandwidth is the (ω_{HPF} - ω_{LPF}).



A Band Reject Filter can be made by concatenating two stages of LPF and HPF and the reject band would be decided by the 3dB frequencies of the HPF and the LPF. The pass bandwidth however is the ($\omega_{BP1} - \omega_{BP2}$).

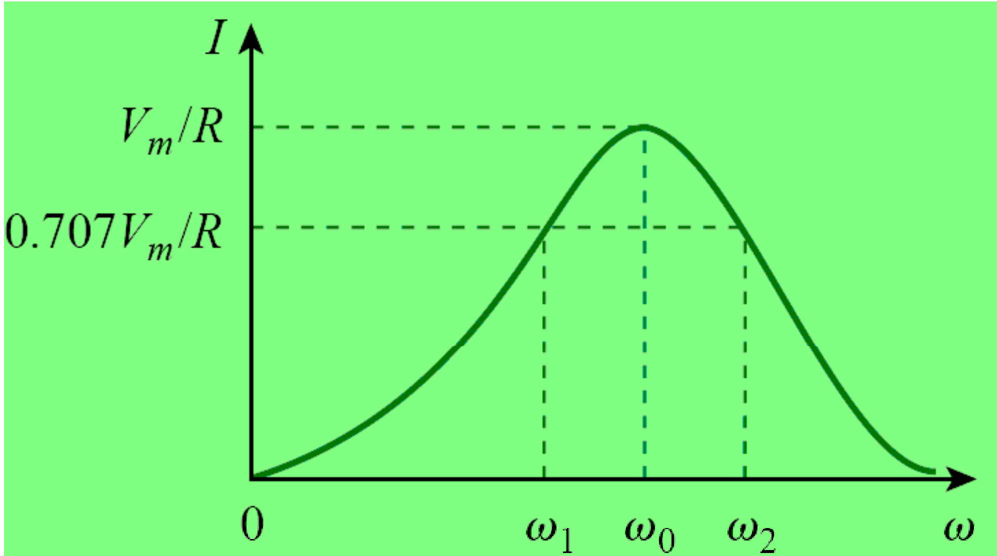
ESc201, Lecture 8: Sinusoidal steady state analysis

SERIES RESONANCE



$$Z_{eq} = R + j\omega L - j\frac{1}{\omega C}$$

$$|I(\omega)| = \frac{V_m}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$$



$$j\omega_O L - j\frac{1}{\omega_O C} = 0 \Rightarrow \omega_O = \frac{1}{\sqrt{LC}}$$

$$|I(\omega_1)| = \frac{V_m}{\sqrt{R^2 + (\omega_1 L - \frac{1}{\omega_1 C})^2}} = \frac{V_m}{\sqrt{2}R}$$

$$\omega_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

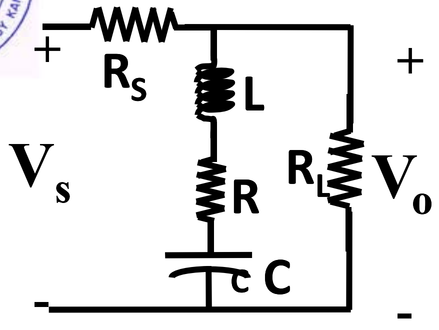
$$|I(\omega_2)| = \frac{V_m}{\sqrt{R^2 + (\omega_2 L - \frac{1}{\omega_2 C})^2}} = \frac{V_m}{\sqrt{2}R}$$

$$\omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$\omega_O = \sqrt{\omega_1 \omega_2} \quad \text{Bandwidth} = \Delta\omega = \omega_2 - \omega_1 = \frac{R}{L}$$

From: Ketan Rajawat

ESc201, Lecture 8: Sinusoidal steady state analysis



Let Z_{eq} be the impedance of the L-R_c-C section.

$$\frac{V_o}{V_s}=H(\omega)=\frac{Z_{eq}\parallel R_L}{R_S+Z_{eq}\parallel R_L} \qquad Z_{eq}=R_C+j(\omega L-\frac{1}{\omega C})$$

$$=\frac{R_CR_L+jR_L\left(\omega L-\frac{1}{\omega C}\right)}{R_S\left(R_C+R_L\right)+R_CR_L+jR_S\left(\omega L-\frac{1}{\omega C}\right)+jR_L\left(\omega L-\frac{1}{\omega C}\right)}$$

$$H(\omega)=\frac{R_CR_L+jR_L\left(\omega L-\frac{1}{\omega C}\right)}{R_S\left(R_C+R_L\right)+R_CR_L+j(R_S+R_L)\left(\omega L-\frac{1}{\omega C}\right)}=\frac{R_L\left[1+j\frac{1}{R_C}\left(\omega L-\frac{1}{\omega C}\right)\right]}{[R_S\left(R_C+R_L\right)+R_CR_L]\left[1+j\frac{(R_S+R_L)}{R_S\left(R_C+R_L\right)+R_CR_L}\left(\omega L-\frac{1}{\omega C}\right)\right]}$$

$$=\frac{R_L}{R_S\left(R_C+R_L\right)+R_CR_L}\cdot\frac{1+j\frac{1}{R_C}\left(\omega L-\frac{1}{\omega C}\right)}{1+j\frac{(R_S+R_L)}{R_S\left(R_C+R_L\right)+R_CR_L}\left(\omega L-\frac{1}{\omega C}\right)}=K\frac{1+jA\left(\omega L-\frac{1}{\omega C}\right)}{1+jB\left(\omega L-\frac{1}{\omega C}\right)}$$

$$\omega L=\frac{1}{\omega C},\ \omega_O=\frac{1}{\sqrt{LC}},$$

$$H(\omega_o)=K$$

$$K=\frac{R_L}{R_S\left(R_C+R_L\right)+R_CR_L}\text{(Not-}\omega\text{Function)} \quad A=\frac{1}{R_C}\text{(Not-}\omega\text{Function)} \quad B=\frac{R_S+R_L}{R_S\left(R_C+R_L\right)+R_CR_L}\text{(Not-}\omega\text{Function)}$$

$$\text{As } \omega\to 0, H(\omega)\to K\frac{A\left(\frac{1}{\omega C}\right)}{B\left(\frac{1}{\omega C}\right)}=K\frac{A}{B}=K\frac{R_S\left(R_C+R_L\right)+R_CR_L}{R_C(R_S+R_L)}\to\frac{R_L}{R_S+R_L}$$

$$K \ll \frac{R_L}{R_S+R_L}$$

$$\text{As } \omega\to\infty, H(\omega)\to K\frac{A\left(\omega L\right)}{B\left(\omega L\right)}=K\frac{A}{B}=K\frac{R_S\left(R_C+R_L\right)+R_CR_L}{R_C(R_S+R_L)}\to\frac{R_L}{R_S+R_L}$$

Therefor it acts as a Band Reject Filter.

Parallel RLC Circuit (Parallel Resonance): L and C are in parallel

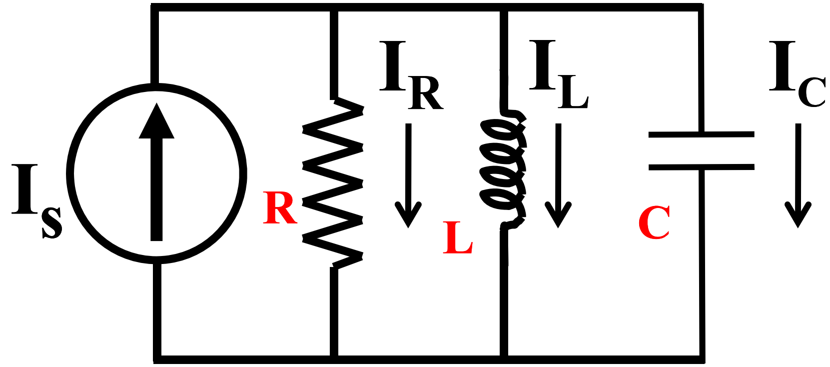
Admittance method works best for this (Tank) circuit

Net admittance $Y = G + j(B_C - |B_L|)$

$G = 1/R, \quad B_C = \omega C, \quad |B_L| = 1/(\omega L)$

$Y = G + j[\omega C - 1/(\omega L)]$

and Y is real when $\omega C = 1/(\omega L)$ **or for this condition** $\omega = \omega_o = 1/\sqrt{LC}$



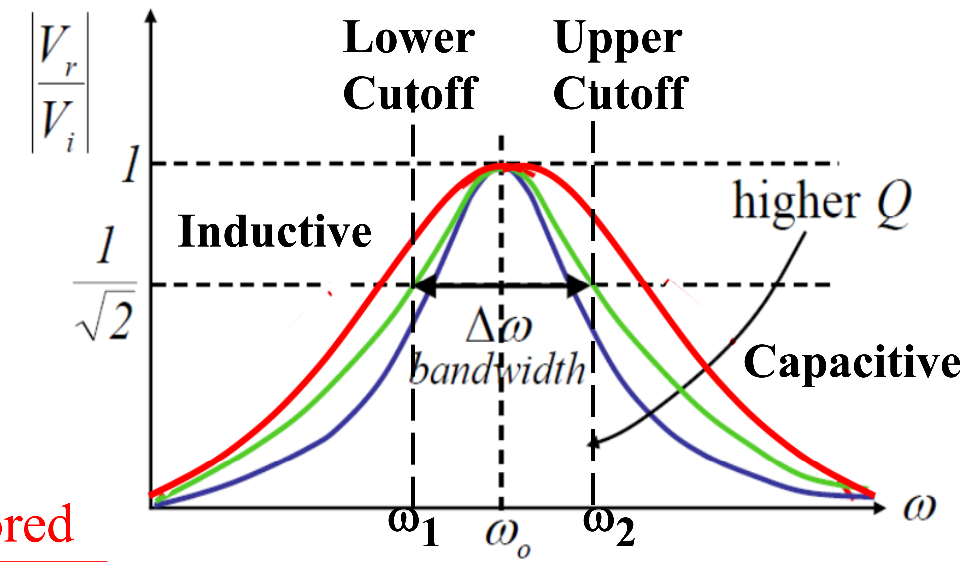
At $\omega = \omega_o = 1/\sqrt{LC}$**, note** $\omega_o L = 1/(\omega_o C)$

$$H(\omega) = \frac{I_R}{I_S} = \frac{\frac{1}{R}}{\frac{1}{R} + j\left(\frac{1}{\omega L}\right) + j\omega C}$$

$$= \frac{j\omega L}{R(1-\omega^2 LC) + j\omega C}$$

$$Q = \frac{\omega_o}{\Delta\omega} = \frac{\omega_o L}{R}$$

$$|H(\omega)| = \frac{\omega L}{\sqrt{R^2(1-\omega^2 LC)^2 + (\omega C)^2}} = 1$$

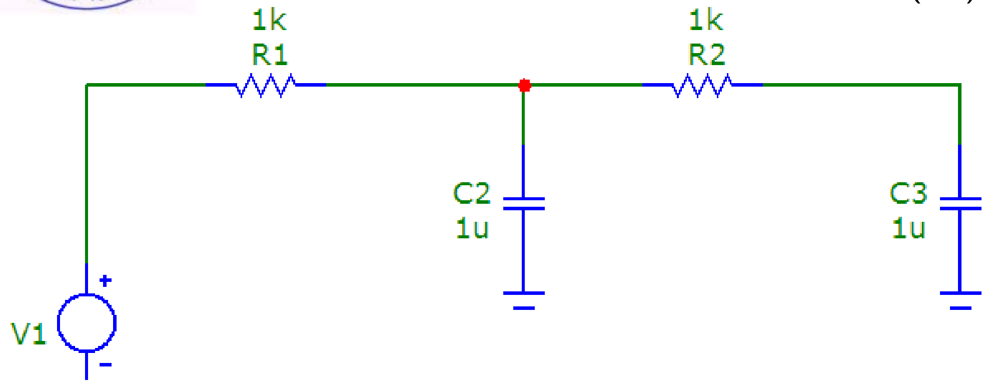


$$Q = \frac{\text{Maximum Energy Stored}}{\text{Total Enery Lost Per Period}}$$

ESc201, Lecture 8: Sinusoidal steady state analysis

Quick & dirty Bode plots

$$H(\omega) = \frac{10}{1 + j\dfrac{\omega}{10^3}} \times \frac{1}{1 + j\dfrac{\omega}{10^4}}$$



$$|H(\omega)| \approx \left(\frac{10^4}{\omega}\right)$$

$$20\log_{10}|H(\omega)| = 20 - 20\log_{10}\sqrt{1+\left(\frac{\omega}{10^3}\right)^2} - 20\log_{10}\sqrt{1+\left(\frac{\omega}{10^4}\right)^2}$$