MSO-203B ASSIGNMENT-02 **October 14, 2019.**

1. Rewrite the PDE's in their canonical forms and solve them.
   (a) $u_{xx} + 2\sqrt{3}u_{xy} + u_{yy} = 0$
   (b) $x^2u_{xx} - 2xyu_{xy} + y^2u_{yy} + xu_x + yu_y = 0$

2. (a) Find the general solution (in terms of arbitrary functions) of the first order PDE $2u_x(x,y) + 3u_y(x,y) + 8u(x,y) = 0$.
   (b) For the PDE given above, check for the characteristic property of the following curves
      (i) $y = x$ in the xy-plane
      (ii) $y = \frac{3x-1}{2}$.
   (c) Discuss the particular solutions of the above PDE, corresponding to
      (i) $u(x,x) = x^4$ on $y = x$
      (ii) $u(x,(3x-1)/2) = x^2$ on $y = (3x-1)/2$
      (iii) $u(x,(3x-1)/2) = e^{-4x}$.

3. In the lectures, you have seen that the linear transport equation in $\Omega := \mathbb{R} \times (0,\infty)$ with the Cauchy data given on $\Gamma := \mathbb{R} \times \{0\}$ is solvable on the entire Ω. Now, consider the linear transport equation

$$u_t + au_x = 0, \text{ in } \Omega := (0,\infty) \times (0,\infty)$$

and the constant $a \in \mathbb{R}$ is given.

   (a) Is the problem solvable in the entire domain if the Cauchy data is given on $\Gamma := (0,\infty) \times \{0\}$? For instance, observe the cases when $a < 0$ and $a > 0$.
   (b) Consider the linear transport equation

$$u_t + au_x = 0, \text{ in } \Omega := (0,L) \times (0,\infty)$$

   where both $L > 0$ and $a \in \mathbb{R}$ are given. Find the minimum part of the boundary of the domain where the Cauchy data has to be imposed for the Cauchy problem to be solvable in the entire domain.

4. (a) Solve the quasilinear Cauchy problem called the *Burgers' equation*

$$\begin{cases} u_t(x,t) + u(x,t)u_x(x,t) &= 0 \quad \text{in } \mathbb{R} \times (0,\infty) \\ u(x,0) &= u_0(x) \quad \text{on } \mathbb{R} \times \{0\}. \end{cases}$$

   (b) Compare the projected characteristics curves with the one obtained for linear transport equation in the lectures. What do you observe?
   (c) Show that if the Cauchy data u_0 is such that, for $r_1 < r_2$, $u_0(r_1) > u_0(r_2)$ then the characteristic curves passing through r_1 and r_2 will necessarily intersect. Also, show that u is multi-valued at the point of intersection.
   (d) Solve the *Burgers'* Cauchy problem with the non-decreasing initial data $u_0(x) = x$. Will the characteristic curves intersect giving rise to multi-valued solution? Is there a solution in the entire domain?