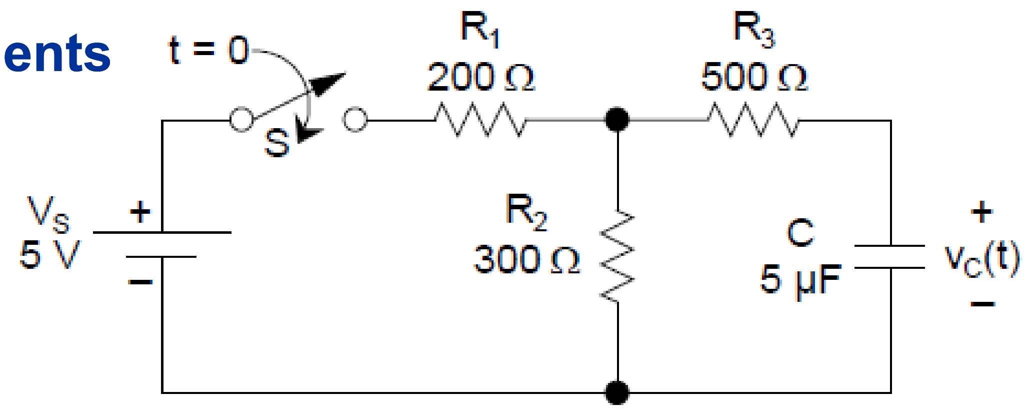
ESc201, Lecture 7: Transients



Example I-from A. K. Dutta

S was open for a long time, closes at $t=0$, and opens again at $t=5$ ms. Sketch $v_c(t)$. C will get completely discharged through R_2 & R_3. Given : $v_c=(0^-)=0$, At $t=0$, S closes, however, capacitor voltage cannot change instantly hence $v(0^+)=(0^-)=0$.

$$v_C(\infty)=\frac{R_1}{R_1+R_2}V_S=\frac{300\text{x}5}{200+300}=3\text{V}$$

For $t > 0$, the capacitor voltage would grow exponentially.

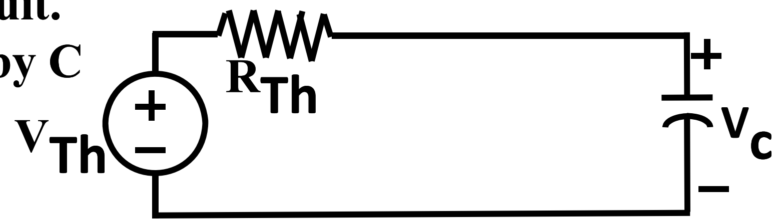
The problem has *two part transients* : one between 0 and 5ms, and the other beyond 5ms.

For t between 0 and 5 ms:

Need to find $v(\infty)$ and (time constant) τ_1

Since the capacitor would behave like an open-circuit.

For τ_1 find R_{eff1}, i.e. the Thevenin resistance seen by C by observation is:



$$R_{eff1} = R_3 + R_1||R_2 = 500 + 200||300 = 620\Omega$$

$\tau_1=R_{eff1}C=(620\Omega)\text{x}(5\mu F)=3.1\text{ms}$ **OR** $v_c(t)=v_c(\infty) + [v_c(0)-v_c(\infty)].e^{-t/\tau_1}=3 + (0-3)e^{-t/3.1m}$

$=3[1-e^{-t/3.1m}]$, **Therefore at $t=5$ms, $v_c(5\text{ms})=2.4$ V.**

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For t > 5 ms:

S opens again at t=5 ms, thus removing the source V_S from the circuit. Hence, C would now start to discharge and eventually as $t \to \infty$, $V_c(t) \to 0$.

For this part of the transient,

let t=5ms be time $t'=0$, i.e. new reference of time,

$v_c(0)= 2.4V$ and $v_c(\infty)=0$

The effective resistance seen by C for this case is $R_{eff2}=R_2+R_3=800\Omega$.

Since R_1 gets open-circuited time constant $\tau_2 = R_{eff2}C=800x5\mu F=4ms$

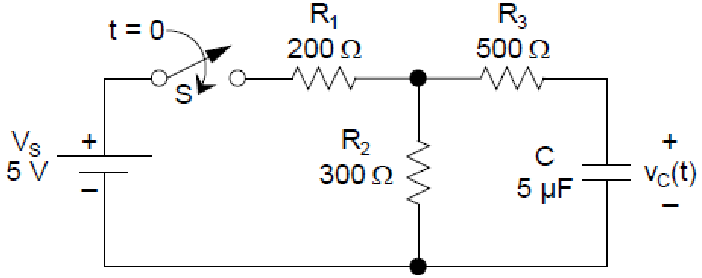
$$v_c(t)=v_c(\infty) + [v_c(0)-v_c(\infty)].e^{-t'/\tau_2}$$

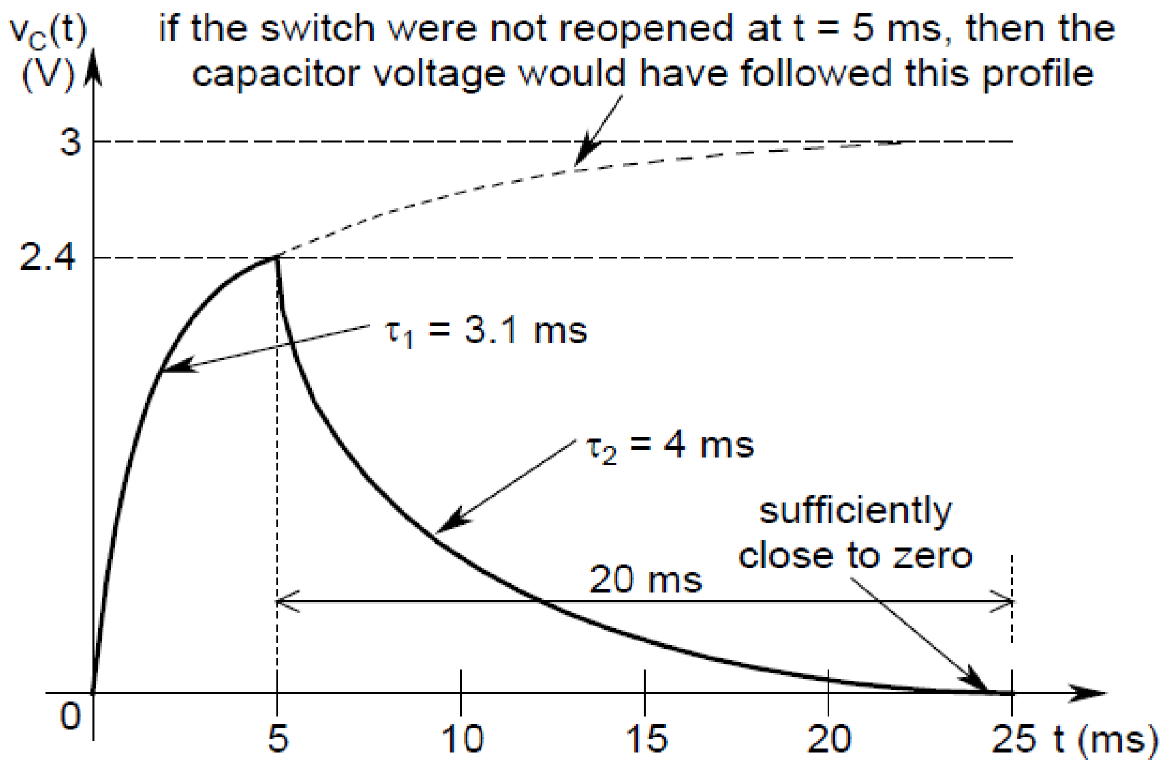
$$=2.4e^{-t'/4m} = 2.4e^{-(t-5ms)/4m}$$

At $t'=5\tau_2$,

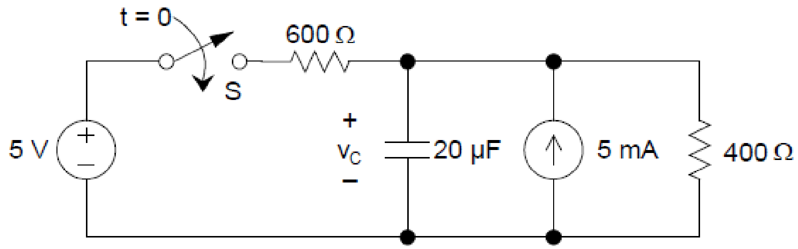
$v_c(t) = 0.7\%$ of $v_c(0)$,

quite small.



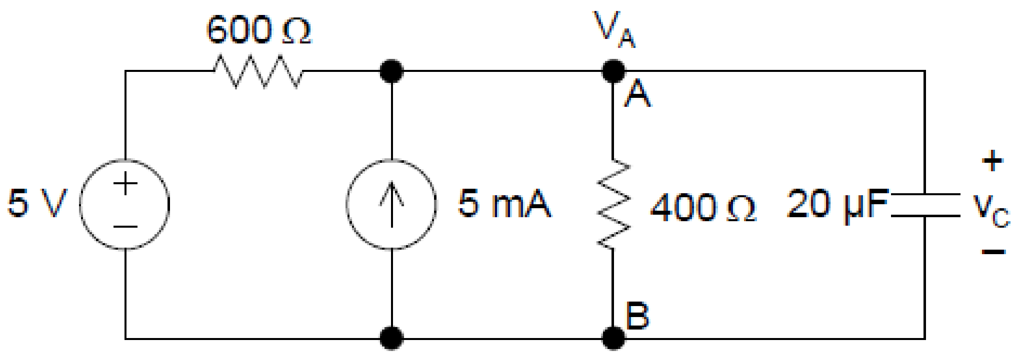


Example 2



S was open for a long time, and closes at t = 0.
Find v_C for $t = 0^-$, 0^+, ∞, and 10 ms.
With S open, 5 V source was disconnected from the circuit, and only 5mA source was active
$v_c(0^-)=v_c(0^+)=5mA \times 400\Omega = 2V$

With S closed at t = 0, we redraw the circuit :



Thevenin's equivalent at A-B after removing Capacitor. Taking B as the reference potential (ground), KCL

$$\frac{5-V_A}{600} + 5m = \frac{V_A}{400}$$

$V_A=V_{OC}=V_{Th}=3.2$ V and $R_{Th}= 600||400=240\Omega$.

Time constant $\tau= R_{Th}C= 240\Omega \times 20\mu F=4.8ms$.

$v_c(\infty)=3.2V$, $v_c(0)=2V$,

$v_c(t) = v_c(\infty) + [v_c(0)-v_c(\infty)].e^{-t/\tau}= 3.2 + (2-3.2).e^{-t/4.8m} == 3.2 - 1.2e^{-t/4.8m}$

Or $v_c(10ms)=3.05V$.

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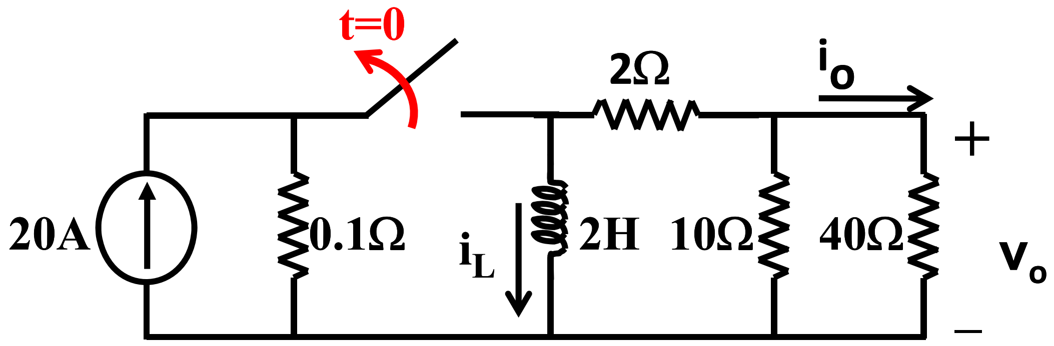
Example 3



As the switch has been closed for a long time, for $t < 0$ the voltage across the inductor must be $V_L=0$

$V_L(0^-)=0$, therefore $i_L(0^-)=20A$
As the current through the inductor cannot change instantaneously, $i_L(0^+)=20A$.

At t=0 when the switch opens, the equivalent resistance across the inductor is:

$R_{eq} = 2 + 10||40 = 10\Omega$. Or $\tau = L/R_{eq} = 2/10 = 0.2s$

$$i_L(t) = i_L(\infty) + \{i_L(0) - i_L(\infty)\}\, e^{-\frac{t}{L/R_{eq}}}$$

$I_L(0)=20A$.
and at t= or > 0

$i_L(\infty)=0$ as all the magnetic energy will dissipate in the resistors. Hence $i_L(t)=20e^{-5t}$ A.

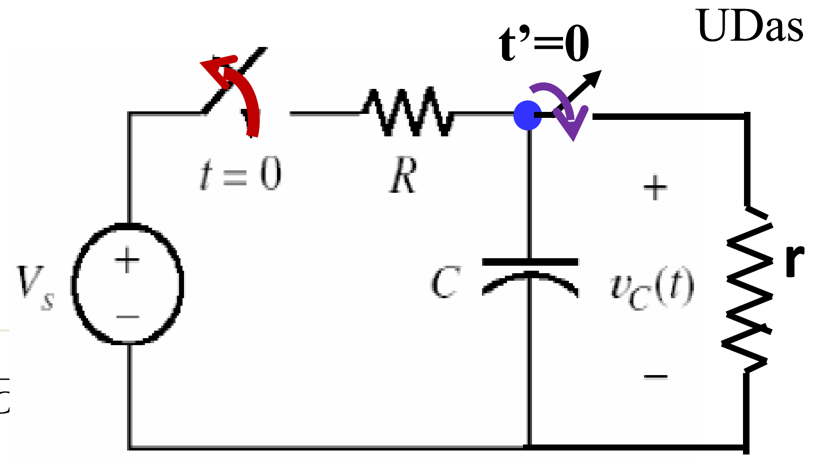
By current division then for $t \geq 0$, $i_o(t) = -[i_L(t)] \times 10/(10+40)$. So $i_o(t)$ changes instantaneously from $i_o(0^-)=0$ to $i_o(0^+) = -0.2 \times 20A = -4A$.
And $i_o(t \geq 0^+) = -0.2 i_L(t) = -4e^{-5t}$ A
Hence $v_o(t) = 40 i_o = -160e^{-5t}$ V for $t \geq 0^+$

The power dissipated in the 40Ω load is

$$W_{10\Omega} = \int_0^\infty \frac{v_o^2}{40} dt = \int_0^\infty \frac{(-160)^2 e^{-10t}}{40} dt = \int_0^\infty 640 e^{-10t} dt = \frac{640}{-10}\left[e^{-10t}\right]_0^\infty = 64J$$

UDas

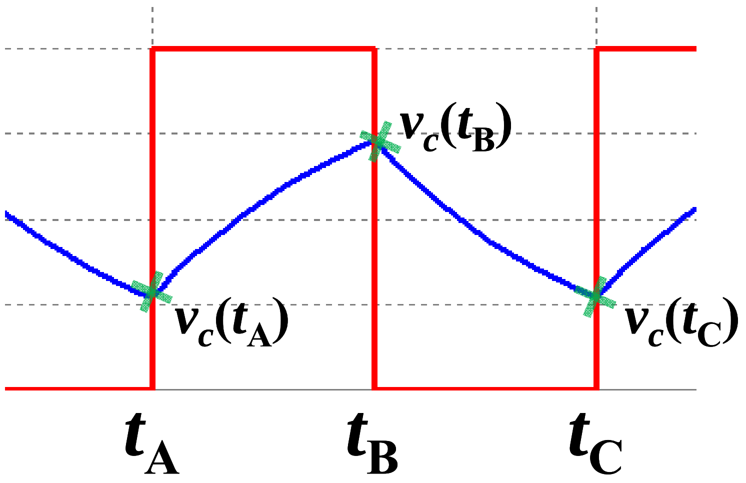
$$v_c(t) = v_c(\infty) + [v_c(0)-v_c(\infty)].e^{-t/\tau}$$



$$v_C(t) = v_C(\infty) + \{v_C(0^+) - v_C(\infty)\}\, e^{-\frac{t}{RC}}$$

$$v_c(t_B) = V_{\max} + \left[v_c(t_A) - V_{\max}\right] e^{-\frac{(t_B - t_A)}{\tau}}$$

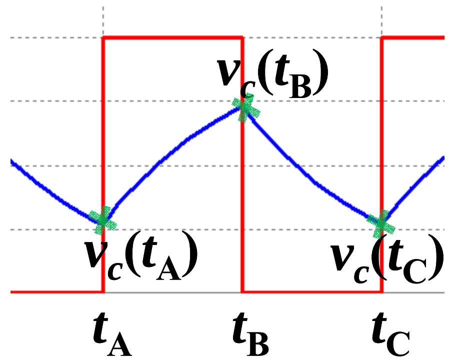
$$v_c(t_C) = V_{\min} + \left[v_c(t_B) - V_{\min}\right] e^{-\frac{(t_C - t_B)}{\tau}} = v_c(t_A)$$



$$\boxed{(t_B - t_A) = (t_C - t_B) = \frac{T}{2}}$$

Determine $v_c(t_A)$ and $v_c(t_B)$ in terms of V_{max} and V_{min}

$$v_c(t_B) = v_c(t_A)e^{-\frac{(t_B - t_A)}{\tau}} + V_{max}[1 - e^{-\frac{(t_B - t_A)}{\tau}}]$$



$$e^x = 1 + \frac{x^1}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots, e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \ldots$$

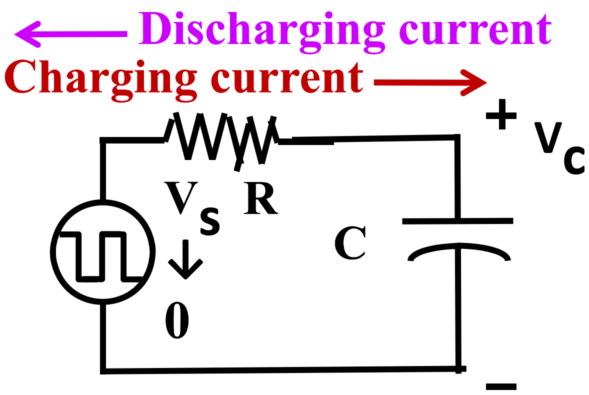
When RC=τ >> (t_B-t_A), x << 1

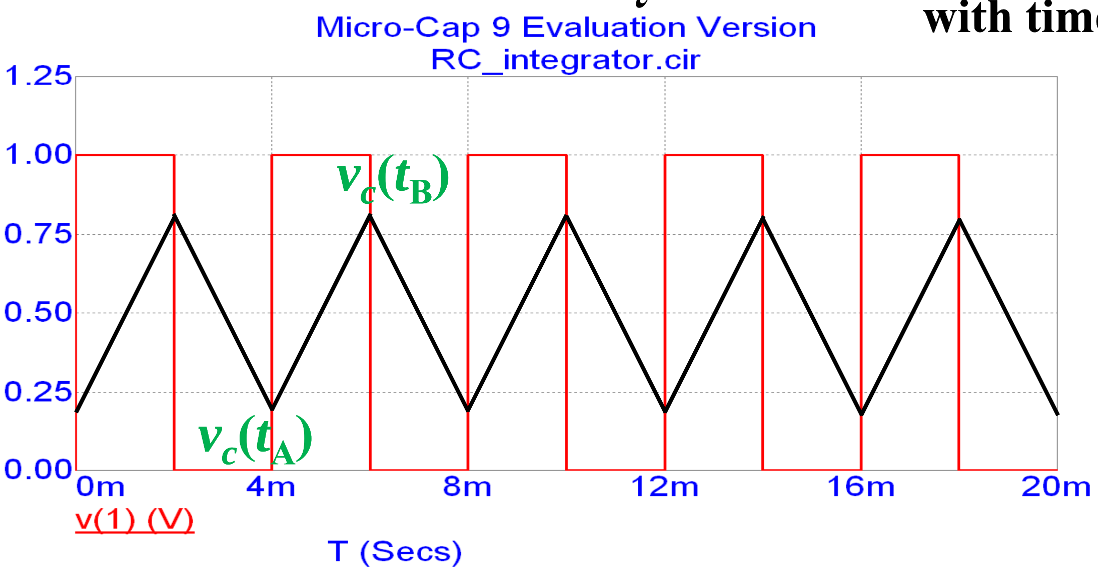
$$v_c(t_B) = v_c(t_A)[1 - 1 + \frac{(t_B - t_A)}{\tau}] + V_{max}[1 - 1 + \frac{(t_B - t_A)}{\tau}]$$

$$v_c(t_A) \ll V_{max}$$

$$v_c(t_B) \cong v_c(t_A) + V_{max}[\frac{(t_B - t_A)}{\tau}]$$

Linear variation with time



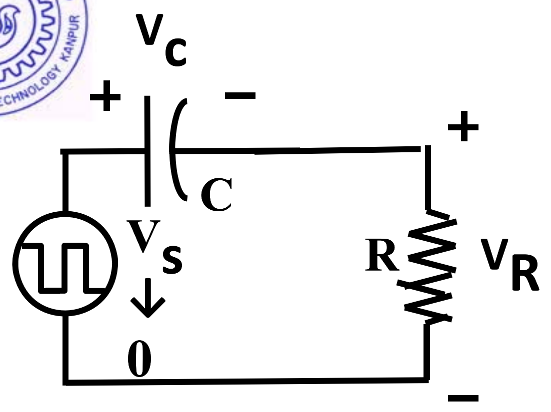


$$\int_{t_A}^{t_B} v_c(t_A)dt = v_c(t_A)t\,\Big|_{t_A}^{t_B}$$

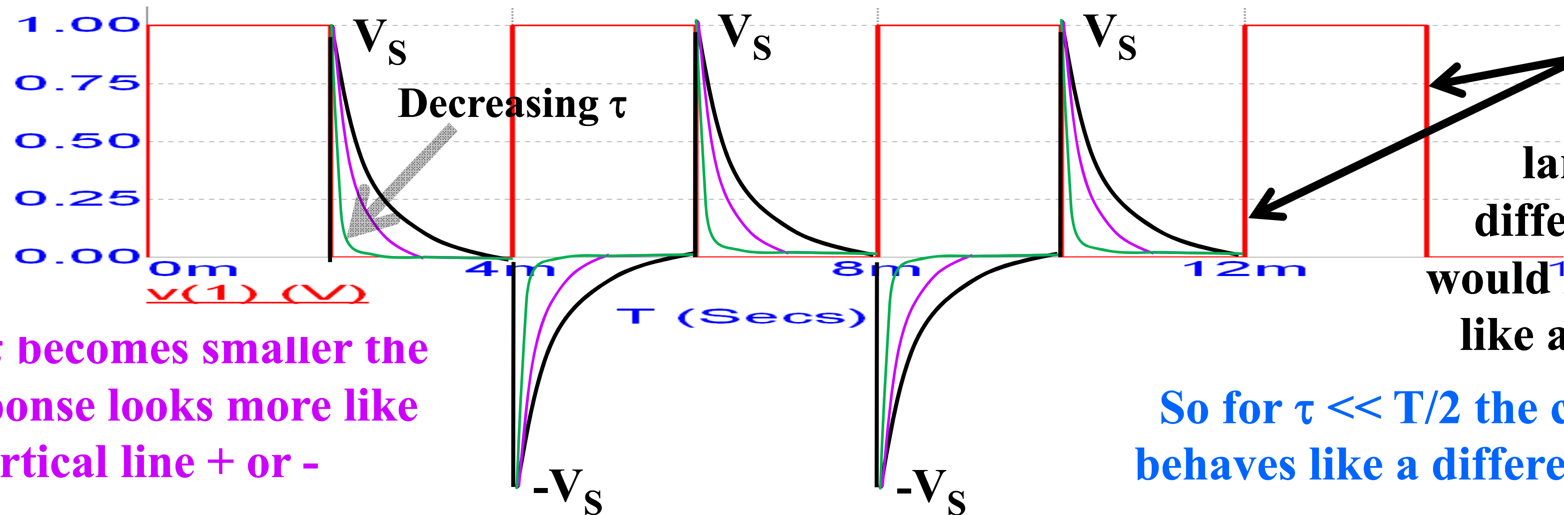
$$= v_c(t_A)[t_B - t_A]$$

The circuit performs as an integrator

ESc201, Lecture 7: Transients-Differentiator



When the source voltage goes from 0 to V_S, the voltage across the capacitor cannot change instantaneously, so $V_C(0^+) = V_C(0^-) = 0$
Or $V_R(0^+) = V_S$.
As time progresses the source is at V_S, C would charge as ~ $(1-e^{-t/\tau})$ to finally to a voltage V_S, thus decreasing the current to zero (given sufficient time, i.e. T is large compared to τ). Which means $V_R \to 0$



dv/dt is very large, so the differentiation would look more like an impulse

As τ becomes smaller the response looks more like a vertical line + or -

So for τ << T/2 the circuit behaves like a differentiator

When the source remains constant at V_S, V_R still remains at 0. But, when the source switches from V_S to 0, the source is shorted but V_C remains at V_S, and V_R is now - V_S. As C discharges through R exponentially, the voltage V_R has to come to 0. It remains at 0 till the supply remains at 0 and the same process repeats itself, when the supply again goes from 0 to V_S.

A similar exercise of differentiator/integrator can also be done for R-L circuits, however, in practical inductors it is avoided as losses are higher in real R-L circuits