Subarray is - continuous part of the array

{ 4, 1, 3, -2, 7, 9, 8 }

$$\left.\begin{array}{l} 3, 7, 9 \\ 1, 3, 9, 8 \\ 4, -2 \end{array}\right\} \text{subsequences.}$$

Subsequence → A sequence generated from deleting 0 or more ele from the given array.

-2	-3	6	2	4	-1	0	3	
✗	✗	✓	✗	✓	✓	✗	✓	[6, 4, -1, 3]
✗	✗	✗	✗	✓	✓	✓	✗	[4, -1, 0]
✗	✗	✓	✗	✗	✗	✗	✗	[6]
✓	✓	✓	✓	✓	✓	✓	✓	[-2, -3, 6, 2, 4, -1, 0, 3]
✗	✗	✗	✗	✗	✗	✗	✗	[ ]

[1, 2, 3, 4, 5]

a) 1, 2, 3, 4, 5 ✓

b) 4 ✓

c) 2, 3, 5 ✓

d) 5, 4, 3

* Order in subsequences should be maintained

$$\{\overset{0}{-1}, \overset{1}{4}, \overset{2}{3}, \overset{3}{9}\}$$

subarray

[-1] ✓ [4] ✓ [3, 9] ✓ All the subarrays

[-1, 4] ✓ [4,3] ✓ [9] ✓ are subsequences

[-1, 4, 3] ✓ [4, 3, 9] ✓

[-1, 4, 3, 9] ✓ [3] ✓

{-1, 4, 3, 9}

subsequence	subarrays
[-1, 4]	✓
[4, 3, 9]	✓
[4, 9]	✗
[-1, 3, 9]	✗
[3]	✓

All the subsequences are not subarrays.

$$\{\overset{0}{4}, \overset{1}{-1}, \overset{2}{2}\} \xrightarrow{\text{sort}} \{\overset{0}{-1}, \overset{1}{2}, \overset{2}{4}\}$$

	subsequence		subsequences	
0 ←	[ ]		[ ]	→ 0
4 ←	[4]		[-1]	→ -1
-1 ←	[-1]		[2]	→ 2
2 ←	[2]		[4]	→ 4
4 ←	[4, -1]	not same	[-1, 4]	→ 4
4 ←	[4, 2]		[-1, 2]	→ 2

2 ← [-1, 2] — not same — [2, 4] → 4

4 ← [4, -1, 2] — not same — [-1, 2, 4] → 4

Subsequences generated from given array may not be same as " " " sorted array.

Subsets : Exactly same as subsequences.

* Order does not matter

{4, -1, 2} $\xrightarrow{\text{sort}}$ {-1, 2, 4}

subsets		subsets
{ }		{ }
{4}		{-1}
{-1}		{2}
{2}		{4}
{4, -1}	same	{-1, 2}
{4, 2}	same	{2, 4}
{-1, 2}	same	{-1, 4}
{4, -1, 2}	same.	{-1, 2, 4}

subsets generated by original array are same as subsets " " sorted array.

In case of unique elements

Will the no. of subsets & subsequences be same?

$\{5, 2, 9\}$

subsequence (order is imp)	subsets (not order imp)
{ }	{ }
{5}	{5}
{2}	{2}
{9}	{9}
{5, 2}	{5, 2}
{5, 9}	{9, 5}
{2, 9}	{9, 2}
{5, 2, 9}	{5, 9, 2}

No. are same

x → 0

✓ → 1

Total no. of subsequences / subsets

$2 \times 2 \times 2 \rightarrow 8$

$2^3 \rightarrow \{a_0 \quad a_1 \quad a_2\}$

a_0	a_1	a_2		
X	X	X	0	{ }
X	X	✓	1	$\{a_2\}$
X	✓	X	2	$\{a_1\}$
X	✓	✓	3	$\{a_1, a_2\}$
✓	X	X	4	$\{a_0\}$
✓	X	✓	5	$\{a_0\ a_2\}$

A[n]

$a_0 \quad a_1 \quad a_2 \quad \ldots \quad a_n$

$\downarrow \quad \downarrow \quad \downarrow \quad \quad \downarrow$

$2 * 2 * 2 \ldots * 2$

2^n

→ No. of subseq / subset

✓ ✓ ✗ 6 $\{a_0 a_1\}$

✓ ✓ ✓ 7 $\{a_0, a_1, a_2\}$

a_0	a_1	
0 1	0 1	
0	0	{ }
0	1	$\{a_1\}$
1	0	$\{a_0\}$
1	1	$\{a_0\ a_1\}$

Multiple Companies

Que. 1. Given N array elements, check if there exists a subset sum = K.

$$\begin{array}{ccccccccc} & & 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ ar[7] = & \{ & 3, & -1, & 0, & 6, & 2, & -3, & 5\} \end{array}$$

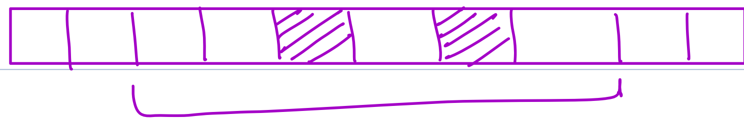
K = 10   {-1, 6, 5}

True   {3, 2, 5}

{3, -1, 6, 2}

⋮

K = 20   False



$$\{\overset{0}{4}, \overset{1}{1}, \overset{2}{\cancel{2}}, \overset{3}{5}, \overset{4}{\cancel{7}}, \overset{5}{3}, \overset{6}{4}\}$$

$$\{4, 5, 7, 12, 19, 22, 26\}$$

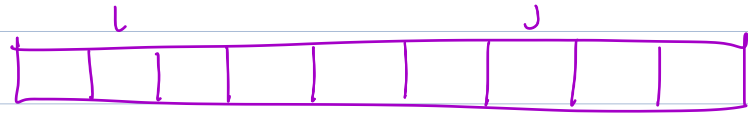
$$= \underline{PS[4] - PS[0]}$$

Brute Force

Generate all subsets.

How to generate all the subsets?

Can we use two for loops?



$$\underbrace{\{\overset{0}{-2}, \overset{1}{6}, \overset{2}{4}\}}_{2^N}$$

$2^3 = 8$ $[2^2 \quad 2^1 \quad 2^0]$

0 → Not taking
1 → Taking

0	0	0	0	{ }
1	0	0	1	{-2}
2	0	1	0	{6}
3	0	1	1	{-2, 6}
4	1	0	0	{4}

5	1	0	1	{4, -2}
6	1	1	0	{6, 4}
7	1	1	1	{-2, 6, 4}

$\xrightarrow{} \underline{(1 \ll N)}$

$\{\overset{0}{4}, \overset{1}{1}, \overset{2}{2}\}$

N = 3 → 2^3 → 8

```
for ( i = 0 ; i < 2^N ; i ++ ) {
    sum = 0
    for ( j = 0; j < N ; i++ ) {
        if ( checkBit ( i, j ) ) {
            // consider arr[j] into ans
            sum += arr[j]
        }
    }
    if (sum == K) return true
}
return false
```

i = 0   j → 0 to 2
checkBit(0, 0)
⋮

i = 1   j → 0 to 2
cB (1, 0)

Complexity

TC: $\underline{O(2^N * N)}$ ← N is very small. $\underline{\underline{N \leq 20}}$.

SC: $\underline{O(1)}$

Bit Masking

$2^N * N \xrightarrow{\text{Backtracking}} 2^N \xrightarrow{\text{DP (Dynamic Programming)}} N * K$

Zeta

Q2. Given all array elements, find the sum of subsets sum

$\{-2, 6, 4\}$ $\quad 2^3 \rightarrow 2^{3-1} = 2^2 = 4$

$\{\}$	$\Rightarrow$	0
$\{-2\}$	$\Rightarrow$	-2
$\{6\}$	$\Rightarrow$	6
$\{4\}$	$\Rightarrow$	4
$\{-2, 6\}$	$\Rightarrow$	4
$\{-2, 4\}$	$\Rightarrow$	2
$\{6, 4\}$	$\Rightarrow$	10
$\{-2, 6, 4\}$	$\Rightarrow$	8
		32

$$\underbrace{-2 \times 4 + 6 \times 4 + 4 \times 4}_{\Downarrow \text{ sum}}$$

Solution 1: Get all subsets sums $\rightarrow$ Exponential

Contribution Technique

The contribution of each ele in the answer.
For each ele, in how many subset it appears, then we're done.

$\underline{\ \ }\ \underline{\ \ }\ \underline{\ \ }\ \underline{\ \ }$

$1 \quad 2 \quad 2 \quad 2$

$-2, 6, 4$

$\{6\}$

$\{6, -2\}$

$\{6, 4\}$

$\{6, 2, 4\}$

$1, 3, 7, 4$

$\{1\}$ $\{1, 3, 4\}$

$\{1, 3\}$ $\{1, 7, 4\}$

$\{1, 7\}$ $\{1, 3, 7, 4\}$

$\{1, 4\}$

$\{1, 3, 7\}$

$\underset{1}{\_} \ \underset{2}{\_} \ \underset{2}{\_} \ \underset{2}{\_}$ 2^3

$\underset{2}{\_} \ \underset{1}{\_} \ \underset{2}{\_} \ \underset{2}{\_}$ 2^3

$\underset{2}{\_} \ \underset{2}{\_} \ \underset{1}{\_} \ \underset{2}{\_}$ 2^3

$\underset{2}{\_} \ \underset{2}{\_} \ \underset{2}{\_} \ \underset{1}{\_}$ 2^3

No. of subsets in which any ith ele is present = 2^{N-1}

$\Rightarrow$ $p = 2^{N-1}$ → $1 << (N-1)$

Can there be a possibility of overflow?

sum = 0

%

power(2, N-1, M)

```
for (i = 0; i < N; i++) {
    sum += (arr[i] * p)
}
return sum
```

power

$TC \rightarrow O(N) + O(N) = O(N)$

$SC \rightarrow O(1)$

Zelā

Q. Given N array elements, calc (sum of all subset sums) / 2^N

sum of all subset sums = $2^{N-1} * a_0 + 2^{N-1} * a_1 + \dots 2^{N-1} * a_n$

$2^{N-1}(a_0 + a_1 + a_2 + \dots a_n)$

$$\frac{2^{N-1}(a_0 + a_1 + a_2 + \dots a_n)}{2^N}$$

$2^{\cancel{N}-1-\cancel{N}} * (\text{sum of array elements})$

$$\Rightarrow \frac{\text{sum of arr ele}}{2}$$

Break ⇒ 5 min

9:55.

Que: Given N array elements, calc sum of MAX of every subsequences.

Fb

{3, 1, -4}

{ }	→	0
{3}	→	3
{1}	→	1
{-4}	→	-4
{3, 1}	→	3
{1, -4}	→	1
{3, -4}	→	3
{3, 1, -4}	→	3
		10

$3 \times 4 + 1 \times 2 + (-4) \times 1$

$12 + 2 - 4$

$\Rightarrow 10$

10

Brute Force

Generate all the subsequences

Exponential

Contribution Technique

In how many subsequences, an ele can be the max ele.

2 2 1 1 1

0 1 2 3 4

{1, 2, 3, 4, 5}

$2^0 \quad 2^1 \quad 2^2 \quad 2^3 \quad 2^4$

ans. $= 2^0 \times 1 + 2^1 \times 2 + 2^2 \times 3 + 2^3 \times 4 + 2^4 \times 5$

[1]
[2]
[4]
[1,2]
[1,4]
[2,4]
[1,2,4]

[1, 2, 4]

$2^0 \quad 2^1 \quad 2^2$

$\underbrace{1 \times 2^0 + 2 \times 2^1 + 4 \times 2^2}$

Generalization

Sorted

1 0 0 0 0

0 1 2 3 4 5 6 . . . i . . . N-1

2

ar[i]

[0 i-1]

$2^i \cdot ar[i]$

$i - \cancel{1} - 0 + \cancel{1}$

$= i$

2^i

$2 * a[i]$

① sort the array

② for(i = 0 ; i < N; i++) {

ans += ($\underbrace{arr[i] * 2^i}$)

$\Downarrow$

(arr[i] << i)

}

return ans.

sum of max of each subseq.

TC → O(N logN) + O(N)

= O(N log N)

SC → O(1)

sum of min of each subseq

Longest Palindromic Substring → Problem Solving Session