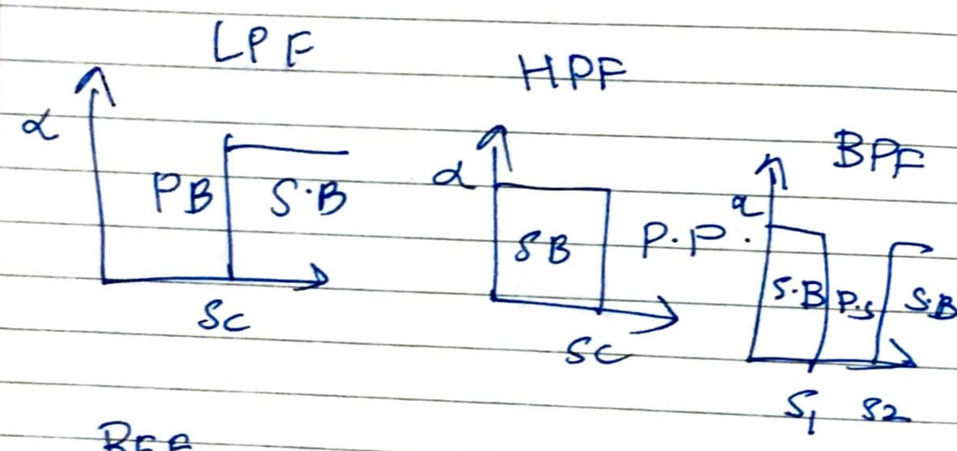
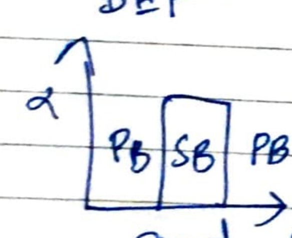
UNIT - III.

8/10. Network Components.

filter:- A filter is a 4 terminal network designed with reactive elements. It is used to separate the electrical signals on basis of their signals.

Definition:





Due to losses in coils and capacitors, non zero attenuation exists in pass band and finite attenuation exists in stop band.

(ii) Propagation const: (γ) for a symmetrical & reciprocal networks, γ is defined as natural logarithm of ratio of i/p to o/p current $P = \gamma = \log[I_1/I_2]$

$I_1 \Rightarrow$ i/p

$I_2 =$ o/p.

P is complex no. $= \alpha + j\beta$

(iii) α (Neper) :- magnitudes b/w i/p & o/p quantity.

(iv) β (Rad) :- Determines ϕ b/w i/p & o/p.

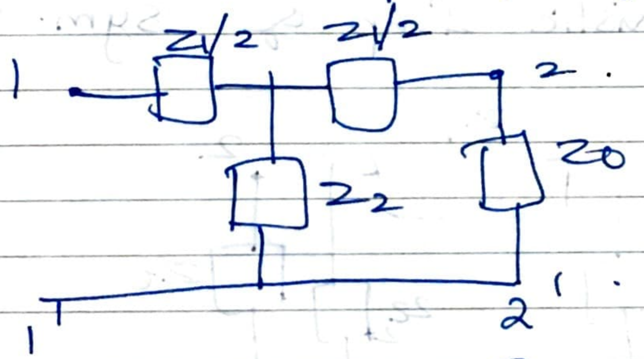
Characteristics :

The behavior of any filter is calculated at any based on :- Z_0, γ, α, β.

(i) Z_0 (Charac. Imp.) :

Z_0 of 4 terminal sym. network is defined as its i/p imp. when such networks are connected at tantom (cascade). For sym. network Z_0 is reffered as image impedance. For assym. network, Z_0 is reffered as iterative imp.

Charact. Imp. of sym. T-N/w.



$$Z_{0T} = \frac{Z_1}{2} + Z_2 \| \left(\frac{Z_1}{2} + Z_0\right)$$

$$= \text{"} + \frac{Z_2 \times \left[\frac{Z_1}{2} + Z_0\right]}{Z_2 + \frac{Z_1}{2} + Z_0}$$

$$Z_{0T} = \frac{\frac{Z_1 Z_2}{2} + \frac{Z_1^2}{4} + \frac{Z_1 Z_0}{2} + \frac{Z_1 Z_2}{2} + Z_2 Z_0}{Z_2 + \frac{Z_1}{2} + Z_0}$$

$$= \frac{Z_1 Z_2 + Z_2 Z_0 + \frac{Z_1 Z_0}{2} + \frac{Z_1^2}{4}}{Z_2 + Z_1/2 + Z_0}$$

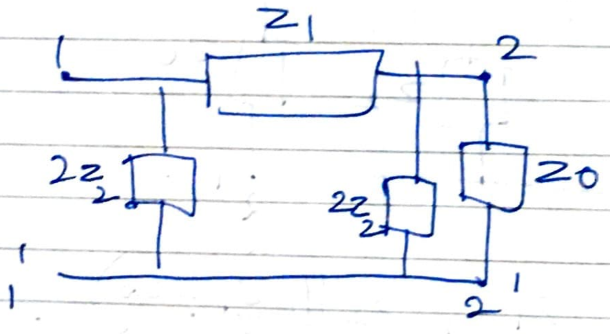
$\Rightarrow \quad Z_{0T}\left(Z_0 + \frac{Z_1}{2} + Z_2\right) = Z_0\left(Z_2 + \frac{Z_1}{2}\right) + \frac{Z_1^2}{4} + Z_1 Z_2.$

$\Rightarrow Z_0\left[1 - \left(Z_2 + \frac{Z_1}{2}\right)\right] = \frac{Z_1^2}{4} + Z_1 Z_2 - Z_{0T}\frac{Z_1}{2} + Z_2 Z_{0T}$

$Z_0\left[Z_0 + \frac{Z_1}{2} + Z_2 - Z_2 - \frac{Z_1}{2}\right] = \frac{Z_1^2}{4} + Z_1 Z_2$

$$\boxed{Z_{0T} = \sqrt{Z_1 Z_2 + \frac{Z_1^2}{4}}}$$

Characteristic Imp of sym. π-network



$2Z_2 \| Z_0 = \frac{2Z_2 \times Z_0}{2Z_2 + Z_0}.$

$\frac{2Z_2 \times Z_0 + Z_1(2Z_2 + Z_0)}{2Z_2 + Z_0} = \frac{2Z_2 Z_0 + 2Z_1 Z_2 + Z_1 Z_0}{2Z_2 + Z_0}.$

$$\frac{2Z_2 Z_0 + 2Z_1 Z_2 + Z_1 Z_0 + (2Z_2)^2 + 2Z_2 Z_0}{\frac{(2Z_2 Z_0 + 2Z_1 Z_2 + Z_1 Z_0)}{2Z_2 + Z_0} + 2Z_2} = Z_0$$

$$\Rightarrow 4Z_2^2 Z_0 + 4Z_1Z_2^2 + 2Z_2 Z_1 Z_0 + 8Z_2^3.$$

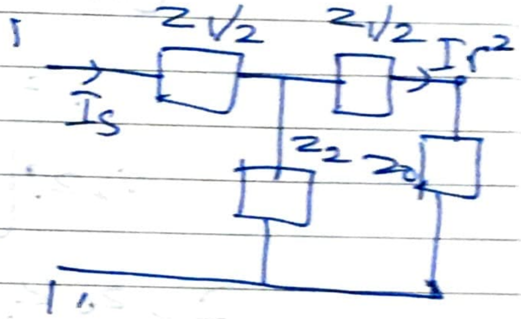
$$\boxed{Z_{0\pi} = \sqrt{\frac{Z_1Z_2}{1+\left(\frac{Z_1}{4Z_2}\right)}}}$$

$$\boxed{Z_{0T} \times Z_{0\pi} = Z_1Z_2}$$

Propagation const. of T n/w & π n/w.

T n/w.

$$P = \log_e\left(\frac{I_s}{I_R}\right)$$



$$I_R = I_s\left[\frac{Z_2}{Z_2+\frac{Z_1}{2}+Z_0}\right]$$

$$\frac{I_s}{I_R} = \frac{Z_2+\frac{Z_1}{2}+Z_0}{Z_2}.$$

$$= Z_2 + \frac{Z_1}{2} + \sqrt{Z_1Z_2+\frac{Z_1^2}{4}}.$$

$$\boxed{P = \log_e\left[\frac{Z_2+\frac{Z_1}{2}+\sqrt{Z_1Z_2+\frac{Z_1^2}{4}}}{Z_2}\right]}$$

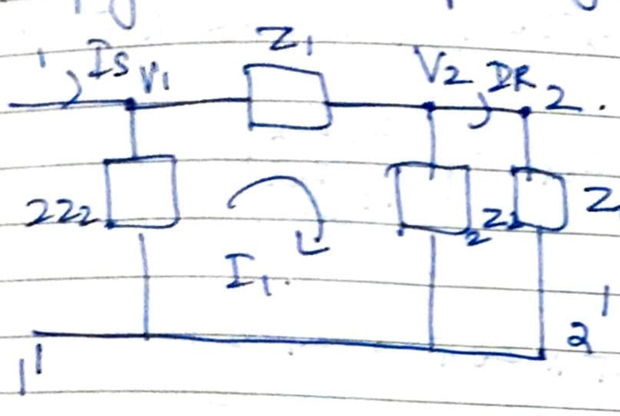
Let $x = \dfrac{Z_2+Z_1/2}{Z_2}.$

$$P = \log_e\left(x+\sqrt{x^2-1}\right).$$

$$\boxed{P = \cosh^{-1}(x)} \Rightarrow P = \cosh^{-1}\left[\frac{Z_2+Z_1/2}{Z_2}\right]$$

$$\Rightarrow P = \cosh^{-1}\left[1+\frac{Z_1}{2Z_2}\right.$$

Propagation const of a symmetrical π n/w



$$\frac{V_1}{2Z_2} + \frac{V_1 - V_2}{Z_1} = I_S \quad \Rightarrow \frac{V_2}{2Z_2} + \frac{V_2}{Z_0} + \frac{V_2 - V_1}{Z_1} = 0$$

$$V_2\left[\frac{1}{2Z_2} + \frac{1}{Z_0} + \frac{1}{Z_1}\right] = \frac{V_1}{Z_1}$$

$$V_1 = Z_1 V_2 \left[\frac{1}{2Z_2} + \frac{1}{Z_0} + \frac{1}{Z_1}\right].$$

$$V_2 Z_1\left[\frac{1}{2Z_2} + \frac{1}{Z_0}\right]\left[\frac{1}{2Z_2} + \frac{1}{Z_1}\right] - \frac{V_2}{Z_1} = I_S$$

$$I_R Z_0 Z_1\left[\frac{Z_0 + 2Z_2}{2Z_2 Z_0}\right]\left[\frac{Z_1 + 2Z_2}{2Z_2 Z_1}\right] - \frac{I_R Z_0}{Z_1} = I_S.$$

$$I_R\left\{\left[\frac{Z_0 Z_1[Z_0 + 2Z_2][Z_1 + 2Z_2]}{2Z_2 \cdot Z_0 \cdot 2Z_2 Z_1}\right] - \frac{Z_0}{Z_1}\right\} = I_S.$$

$$\frac{I_S}{I_R} = \quad ''$$

$$\frac{I_S}{I_R} = \frac{(Z_1^2 + 2Z_1Z_2)(Z_0^2 Z_1 + 2Z_2 Z_0 Z_1) - 4Z_0^2 Z_2^2}{4Z_1^2 Z_2^2 Z_0}$$

$$= Z_1^3 Z_0^2 + 2Z_2 Z_0 Z_1^3 + 2Z_1^2 Z_0^2 Z_2 + 4Z_2^2 Z_1^2 Z_0 - 4Z_0^2 Z_2^2 Z_1$$

$$\cancel{P = \log_e\left(\frac{I_S}{I_R}\right)}$$

$$P = \log_e \left(I_S / I_R \right)$$

$$= \log_e \left[\frac{4Z_2^2 + 2Z_1Z_2}{4Z_2^2} + \sqrt{\frac{Z_1Z_2}{1+\frac{Z_1}{4Z_2}} \left[\frac{Z_1+4Z_2}{4Z_2^2} \right]} \right]$$

$$= \log_e \left[\left(1 + \frac{Z_1}{2Z_2}\right) + \sqrt{\frac{Z_1Z_2}{1+\frac{Z_1}{4Z_2}} \left(\frac{4Z_2+Z_1}{4Z_2^2} \right)^2} \right]$$

$$= \log_e \left[\left(1 + \frac{Z_1}{2Z_2}\right) + \sqrt{\frac{Z_1(4Z_2+Z_1)}{4Z_2^2}} \right]$$

$$= \log_e \left[\left(1 + \frac{Z_1}{2Z_2}\right) + \frac{1}{Z_2}\sqrt{Z_2Z_1\left(1+\frac{Z_1}{4Z_2^2}\right)} \right]$$

$$= \log_e \left[\left(1 + \frac{Z_1}{2Z_2}\right) + \sqrt{\frac{Z_1Z_2\left(1+\frac{Z_1}{4Z_2^2}\right)}{Z_2}} \right]$$

Put $x = \dfrac{Z_2 + Z_1/2}{Z_2}$.

Put $\log_e \left[x + \sqrt{x^2-1} \right] = \cosh^{-1}(x)$.

$$P = R_k \cosh^{-1}\left[1 + \frac{Z_1}{2Z_2} \right]$$

Balanced n/w :- It is the one in which the corresponding series impedance elements are identical and these elements are symmetrical with respect to the ground

Symmetrical n/w :- It is the one in which the electrical properties are unaffected by interchanging the i/p and o/p terminals

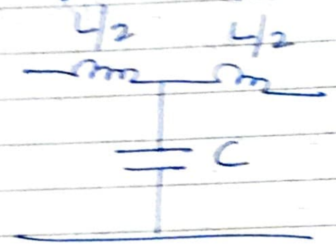
Equivalent n/ws :- Two n/ws having same characteristics impedance and same propagation constant are said to be equivalent networks.

Constant k filters :- A constant k filter is a T / π n/w in which the series and shunt imp Z_1 and Z_2 are connected by the relationships $\boxed{Z_1 Z_2 = R_k^2}$ where R_k = A real const. (i.e., resistance ~~imp~~ independance of frequency.

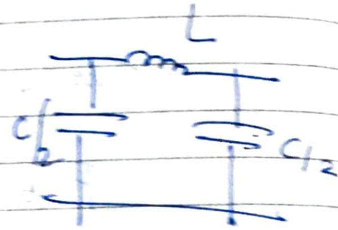
R_k is known as design / nominal impedance.

→ Low Pass (const k) filters:- Since series and shunt impedance of both T and π are same, they will have the same design impedance and cut-off frequency only.

T n/w



π n/w



$Z_1 = j\omega L \quad Z_2 = j/\omega C$

$Z_1 Z_2 = \frac{L}{C} = R_k^2 \, (R_0^2)$

$$\boxed{R_0 = \sqrt{L/C}}$$

To find the cut off freq:-

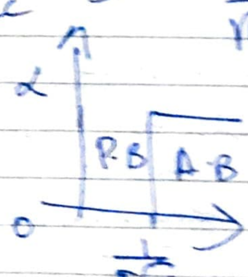
To find the lower cut-off frequency

$j\omega L - \frac{4j}{\omega C} = 0 \Rightarrow j\omega L = \frac{4j}{\omega C}$

$\omega^2 = \frac{4}{LC} \Rightarrow \omega = \frac{2}{\sqrt{LC}}$

$2\pi f_c = \frac{2}{\sqrt{LC}}$

$$\boxed{f_c = \frac{1}{\pi\sqrt{LC}}}$$



To find Z_0 (wh...

$Z_{0T} = \sqrt{Z_1Z_2 + \frac{Z_1^2}{4}}$ ...

$Z_0^2 - L/C =$ ...

$Z_{0T} =$ ...

$Z_{0T} = R_0$ ...

w.k.t

Design eqns ...

w.r.t Z_1 ...

At the ...

$\Rightarrow j\omega L$ ...

$\Rightarrow \omega$ ...

(1) $\Rightarrow L/C$ ...

(2) $\Rightarrow R_0$ ...

$L =$ ...

To find z_0 (characteristic imp):-

$$z_{0T} = \sqrt{z_1 z_2 + \frac{z_1^2}{4}} = \sqrt{\frac{L}{C} - \frac{\omega^2 L^2}{4}}$$

$$z_0^2 - L/C = \frac{\omega^2 L^2}{4}.$$

$$z_{0T} = \boxed{\sqrt{\frac{L}{C}\left[1 - \frac{LC\omega^2}{4}\right]}}$$

$$\boxed{z_{0T} = R_0\sqrt{1-\left(\frac{f}{f_c}\right)^2}}$$

w.k.t $z_{0T} \cdot z_{0\pi} = z_1 z_2$

$$z_{0\pi} = \left(\sqrt{\frac{L}{C}}\right)^2$$

$$= R_0\sqrt{1-\left(\frac{f}{f_c}\right)^2}$$

$$\boxed{z_{0\pi} = \frac{R_0}{\sqrt{1-\left(f/f_c\right)^2}}}$$

Design eqns. for LP.F

w.k.t $z_1 z_2 = \frac{L}{C} = R_k^2 \ (R_0^2)$.

$$\Rightarrow R_0 = \sqrt{L/C} \longrightarrow (1).$$

At the upper cut-off frequency, $z_1 = -4z_2$

$$\Rightarrow j\omega L = \frac{4j}{\omega C}.$$

$$\Rightarrow \omega^2 LC = 4 \longrightarrow (2).$$

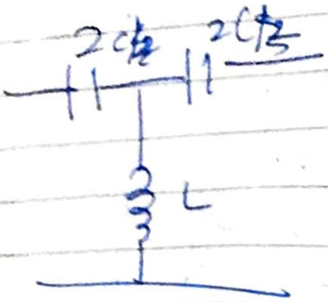
(1) $\Rightarrow L/C = R_0^2 \Rightarrow L = R_0^2 C$.

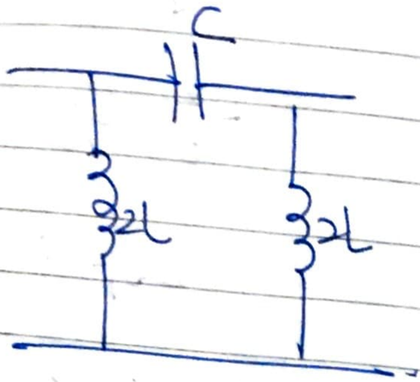
(2) $\Rightarrow R_0^2 C^2 = \frac{4}{\omega^2} \Rightarrow \boxed{C = \frac{2}{R_0\omega}}$

$$\boxed{L = R_0^{\cancel{2}} \frac{2}{\cancel{R_0}\omega}}$$

High Pass filter:

T n/w　　　π n/w.





$Z_1 = -\frac{j}{\omega C}$

$Z_2 = j\omega L.$

$Z_1 Z_2 = \frac{L}{C} = R_0^2 (R_k^2) \Rightarrow \boxed{R_0 = \sqrt{\frac{L}{C}}}$

$\frac{Z_1}{4Z_2} = \frac{-\cancel{j}}{\omega C(4\cancel{j}\omega L)} = \frac{-1}{4\omega^2 LC}.$

$$\boxed{\frac{Z_1}{4Z_2} = \frac{-1}{4\omega^2 LC}}$$

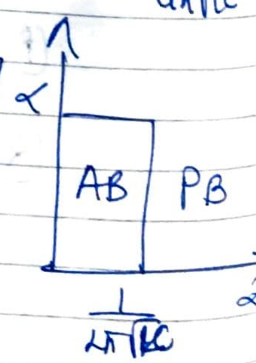
To find cut-off frequency.

By applying the condition of P.B, $\frac{Z_1}{4Z_2} = -1$ for f_1 (lower cut-off)

$\frac{Z_1}{4Z_2} = 0$ for f_2 (higher cut-off).

$\Rightarrow \frac{-1}{4\omega^2 LC} = -1 \Rightarrow \omega = \frac{1}{2\sqrt{LC}} \Rightarrow f_1 = \frac{1}{4\pi\sqrt{LC}}$

$\Rightarrow \frac{-1}{4\omega^2 LC} = 0. \Rightarrow \boxed{f_2 = \infty}$



To find Z_0:

$Z_{0T} = \sqrt{Z_1 Z_2 + \frac{Z_1^2}{4}} = \sqrt{\frac{L}{C} + \frac{-1}{4\omega^2 C^2}}.$

$Z_{0T} = \boxed{\frac{L}{C}\left[1 - \frac{1}{\omega^2 LC}\right]}$

$$\frac{1}{4 \times 4 \times \pi^2 \times f_c^2 \times LC}$$

$$\frac{1}{4\pi^2 \frac{4\pi^2 f_c^2}{LC}}$$

$Z_{0T} \cdot Z_{0\pi} = Z_1 Z_2$

$$Z_{0\pi} = \frac{\frac{L}{C}}{\sqrt{\frac{L}{C}}\left[\sqrt{1 - \left(\frac{f_c}{f}\right)^2}\right]} \Rightarrow Z_{0\pi} = \frac{R_0}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$$

Design eqn of h.p.f.

W.k.t. $R_0^2 = L/C \rightarrow (1)$.

At ~~the~~ lower cut off freq $\omega = \frac{1}{2\sqrt{LC}} \rightarrow (2)$

$(1) \rightarrow L = CR_0^2$

~~$(2) \rightarrow Z_1 = -4Z_2$~~

$\cancel{+\frac{1}{\omega C}} = \cancel{4\omega L}$

$\omega^2 = \frac{1}{4LC} \Rightarrow \omega = \frac{1}{2\sqrt{LC}}$

$$C = \frac{1}{2R_0\omega}$$

$C = \frac{1}{2R_0 2\pi f_c} \Rightarrow \boxed{C = \frac{1}{4\pi f_c R_0}}$

$$\boxed{L = \frac{R_0}{4\pi f_c}}$$

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$$\sin\frac{\gamma}{2} = \sqrt{\frac{Z_1}{4Z_2}}$$

Book chapter - 17. filters and attenuation

Att. factor and ϕ factor of a const. k low pass filter.

wkt: $\cos\beta = 1 + Z_1/2Z_2$.

$Z_1 = j\omega L$ + $Z_2 = -j/\omega C$.

$$\cos\beta = 1 + \frac{j\omega L \times \omega C}{2 - j}$$

$$1 - \frac{\omega^2 LC}{2}$$

$$= \frac{1 - \cancel{4}\pi^{\cancel{2}} \frac{1}{\cancel{\pi^2} LC} \cdot 4C}{\cancel{2}}$$

$$\cos\beta = 1 - 2\omega^2/\omega_c^2.$$

$$\beta = \cos^{-1}\left[1 - \frac{2\omega^2}{\omega_c^2}\right].$$

$$\beta = 2\sin^{-1}\frac{\omega}{\omega_c} \text{ rad } \left(\cos\beta = 1 - 2\sin^2\beta/2\right)$$

$$\cosh\alpha = -\left[1 + \frac{Z_1}{2Z_2}\right]$$

$$= -\left[1 + \frac{j\omega L \cdot \omega C}{2 - j}\right].$$

$$= -\left[1 - \frac{1}{2}\omega^2 LC\right]$$

$$= -\left[1 - \frac{2\omega^2}{\omega_c^2}\right] = \frac{2\omega^2}{\omega_c^2} - 1.$$

$$\boxed{\begin{aligned}\alpha &= \cosh^{-1}\left[2\left(\frac{\omega}{\omega_c}\right)^2 - 1\right] \\ \alpha &= 2\cosh^{-1}\left(\frac{\omega}{\omega_c}\right)\end{aligned}}$$ nepers.

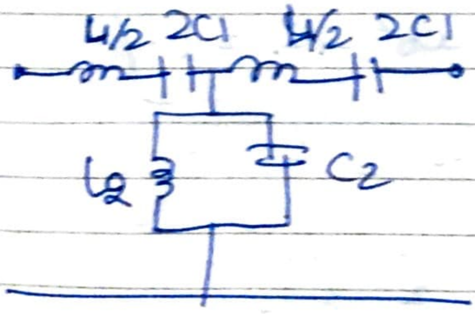
$\because \cosh x = -1 + 2\cosh^2\frac{x}{2}$

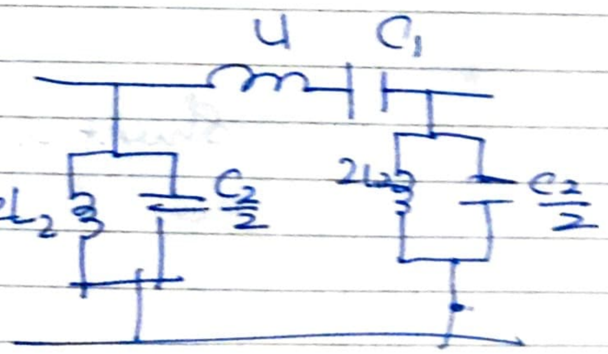
High P.F.

$Z_1 = \frac{-j}{\omega C}$ $\quad Z_2 = j\omega L.$

Band pass filter.

T- section. $\qquad\qquad$ π section.





$$Z_1 = j\omega L_1 + \frac{1}{j\omega C_1}$$

$$j\left(\omega L_1 - \frac{1}{\omega C_1}\right)$$

$$Z_2 = \frac{j\omega L_2 \times \frac{1}{j\omega C_2}}{j\omega L_2 + \frac{1}{j\omega C_2}} = \frac{\frac{L_2}{C_2}}{j\left(\omega L_2 - \frac{1}{\omega C_2}\right)}.$$

$$Z_1 Z_2 = j\left(\omega L_1 - \frac{1}{\omega C_1}\right) \cdot \frac{L_2}{C_2 j\left(\omega L_2 - \frac{1}{\omega C_2}\right)}$$

$$= \frac{[\omega^2 L_1 C_1 - 1] L_2}{\omega C_1 C_2 [\omega^2 L_2 C_2 - 1]} \cdot \frac{C_2}{C_1}.$$

~~At resonance $X_L = X_C$~~

~~Series arm: $j\omega L_1$~~

$$= \frac{\omega^2 L_1 L_2 C_1 - L_2}{C_1[\omega^2 L_2 C_2 - 1]}.$$

$$Z_1 Z_2 = \frac{L_2[\omega^2 L_1 C_1 - 1]}{C_1[\omega^2 L_2 C_2 - 1]}$$

At resonance $X_L = X_C$.

Series arm: $j\omega L_1 = \frac{1}{-j\omega C_1}$

$$\boxed{\omega_0^2 L_1 C_1 = 1.}$$

Shunt arm:- $j\omega L_2 = \frac{-1}{j\omega C_2}.$

$$\Rightarrow \boxed{\omega_0^2 L_2 C_2 = 1}$$

$\therefore L_1 C_1 = L_2 C_2$

$$\frac{L_2}{C_1} = \frac{L_1}{C_2} = R_0^2.$$

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$$\frac{Z_1}{4Z_2} = -1 \Rightarrow Z_1 = -4Z_2$$

$$Z_1^2 = -4Z_1Z_2$$

$$= -4R_0^2.$$

$$\boxed{Z_1 = \pm j2R_0}$$

Z_1 is +'ve from a particular frequency and is a -ve for some other frequency. W.K.T.

$$Z_1 = j\left[\frac{\omega^2 L_1 C_1 - 1}{\omega C_1}\right] = j2R_0.$$

$$2R_0\omega C_1 = \omega^2 L_1 C_1 - 1.$$

$$\omega^2 L_1 C_1 - 2R_0\omega C_1 - 1 = 0.$$

$$\therefore \omega = \frac{2R_0C_1 \pm \sqrt{4R_0^2C_1^2 + 4L_1C_1}}{2L_1C_1}$$

$$= \frac{2R_0C_1 \pm 2\sqrt{R_0^2C_1^2 + L_1C_1}}{2L_1C_1}$$

$$\boxed{f_1 = \frac{R_0 + \sqrt{R_0^2 + L_1/C_1}}{2\pi L_1}.}$$

Case(ii) when Z_1 acts a in-'ve imp.

$$-2jR_0 = j\left[\frac{\omega^2 L_1 C_1 - 1}{\omega C_1}\right]$$

$$\Rightarrow -2R_0\omega C_1 = \omega^2 L_1 C_1 - 1$$

$$\omega^2 L_1 C_1 + 2R_0\omega C_1 - 1 = 0.$$

$$\omega = \frac{-2R_0C_1 \pm \sqrt{4R_0^2C_1^2 + 4L_1C_1}}{2L_1C_1}.$$

$$\boxed{f_2 = \frac{-R_0 + \sqrt{R_0^2 + L_1/C_1}}{2\pi L_1}.}$$

$$f_1 f_2 = \frac{\left(R_0 + \sqrt{R_0^2 + 4/C_1}\right)\left(-R_0 + \sqrt{R_0^2 + 4/C_1}\right)}{4\pi^2 L_1^2}$$

$$f_0^2 = \frac{1}{4\pi^2 L_1 C_1} \qquad f_0 = f_1 f_2$$

$$\boxed{f_0 = \frac{1}{2\pi\sqrt{L_1 C_1}}}$$

Design equ. for. B.P.F.

$$1 - \omega_2^2 L_1 C_1 = 2R_0 \omega_2 C_1$$

$$1 - \frac{\omega_2^2}{\omega_0^2} = 2R_0 \omega_2 C_1$$

$$1 - \frac{f_2^2}{f_0^2} = 4\pi R_0 f_2 C_1$$

W.k.t. $f_0^2 = f_1 f_2$

$$1 - f_2/f_1 = 4\pi R_0 f_1 f_2 C_1$$

$$\boxed{C_1 = \frac{f_1 - f_2}{4\pi R_0 f_0^2}}$$

$$\boxed{L_2 = \frac{R_0^{\cancel{2}}(f_1 - f_2)}{4\pi \cancel{R_0} f_0^2}}$$

(ii) $L_1 C_1 = R_0^2$

$$L_1 = R_0^2 \frac{4\pi R_0 f_0^2}{f_1 - f_2}$$

$$L_1 C_1 = L_2 C_2 = R_0^2$$

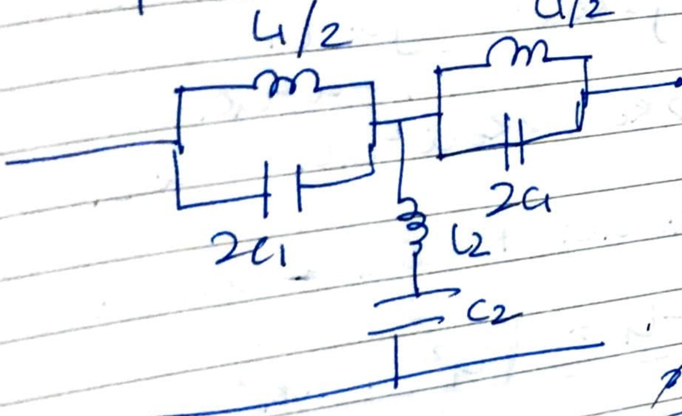
$$L_2 = \frac{R_0^3 \cdot \cancel{4\pi}\left(\frac{1}{\cancel{4\pi} L_1 C_1}\right)}{f_1 - f_2}$$

$$= \frac{R_0^3 / R_0^2}{\pi f_1 f_2} = \frac{R_0}{\pi(f_1 - f_2)}$$

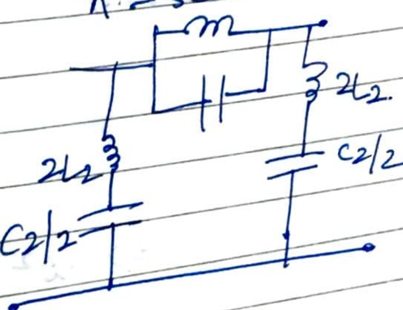
$$C_2 = \frac{1}{R_0 \pi (f_1 - f_2)}$$

Band stop filter

T section



π-se



$$Z_1 = \left[\frac{\frac{j\omega L_1}{\cancel{2}} \times \frac{\cancel{1}}{2\cancel{j}\omega C_1}}{j\omega \frac{L_1}{2} + \frac{1}{2j\omega C_1}}\right] = \frac{\frac{L_1}{4C_1}}{\frac{-2\omega^2 L_1 C_1 + 2}{4j\omega}}$$

$$\boxed{Z_1 = \frac{j\cancel{4} L_1 \omega}{\cancel{4}(1 - \omega^2 L_1 C_1)}}$$

$$\Rightarrow Z_1 = \frac{j\omega L_1}{1 - \omega^2 L_1 C_1}$$

$$Z_2 = j\omega L_2 + \frac{1}{j\omega C_2}$$

$$= \frac{-\omega^2 L_2 C_2 + 1}{j\omega C_2}$$

$$\therefore Z_1 Z_2 = \frac{j\omega L_1}{1 - \omega^2 L_1 C_1} \times \frac{1 - \omega^2 L_2 C_2}{j\omega C_2}$$

$$Z_1 Z_2 = \frac{L_1}{C_1}\left[\frac{\omega^2 L_2 C_2 - 1}{\omega^2 L_1 C_1 - 1}\right]$$

At resonance $X_L = X_C$

Series arm $j\omega L_1 = \frac{-1}{j\omega C_1}$

$L_1 C_1 = L_2 C_2 = R_0^2$

$\frac{Z_1}{4Z_2} = -1 \Rightarrow Z_1 = -4Z_2$

$Z_1^2 = -4Z_1 Z_2$

$\Rightarrow Z_1 = \pm j2R_0$

Case (i) Z_1 takes a +ve value

$$j2R_0 = \frac{j\omega L_1}{1-\omega^2 L_1 C_1}$$

$$2R_0 - 2R_0\omega^2 L_1 C_1 = \omega L_1$$

$$\Rightarrow \omega^2 2R_0 L_1 C_1 + \omega L_1 - 2R_0 = 0$$

$$\Rightarrow f_2 = \frac{-L_1 \pm \sqrt{L_1^2 + 16R_0^2 L_1 C_1}}{2\pi(4 L_1 C_1)R_0}$$

$$\frac{-1 \pm \sqrt{1 + 16R_0^2 C_1/L_1}}{8\pi R_0 C_1}$$

$$\therefore f_2 = \frac{-1 + \sqrt{1+16R_0^2 C_1^2}}{8\pi R_0 C_1}$$

$$f_1 f_2 = \frac{\cancel{1} + 16R_0^2 C_1/L_1 - \cancel{1}}{64\pi^2 R_0^2 C_1^2}$$

$$f_0^2 = \frac{1}{4\pi^2 L_1 C_1} \Rightarrow f_0 = \frac{1}{2\pi\sqrt{L_1 C_1}}$$

$$\omega = \frac{-L_1 \pm L_1\sqrt{1+16R_0^2 \frac{C_1}{L_1}}}{4R_0 L_1 C_1}$$

$$\boxed{f_1 = \frac{-1 + \sqrt{16 R_0^2 \frac{C_1}{L_1} + 1}}{8 \pi R_0 C_1}}$$

Design Equation :-

$$\omega_1 L_1 = 2R_0 - 2R_0\, \omega_1^2 L_1 C_1$$

$$\Rightarrow \omega_1 L_1 = 2R_0 - 2R_0 \left(\frac{\omega_1^2}{\omega_0^2}\right)$$

$$\omega_1 L_1 = 2R_0 \left[1 - \frac{\omega_1^2}{\omega_0^2}\right]$$

$$L_1 = \frac{2R_0}{\omega_1} \left[1 - \frac{f_1^2}{f_0^2}\right]$$

$$= \frac{R_0}{\pi f_1} \frac{(f_1 f_2 - f_1^2)}{f_1 f_2}$$

$$\boxed{L_1 = \frac{R_0}{\pi f_0^2} (f_1 f_2 - f_1^2)}$$

$$C_2 = \frac{f_2 - f_1}{R_0 \pi f_0^2}$$

$$f_0^2 = \frac{1}{4\pi^2 L_1 C_1}$$

$$C_1 = \frac{1}{4\pi^2 \frac{R_0 (f_2 - f_1) f_0^2}{\pi f_0^2}}$$

$$\boxed{C_1 = \frac{1}{4\pi R_0 (f_2 - f_1)}} \qquad \boxed{L_2 = \frac{R_0}{4\pi (f_2 - f_1)}}$$

TM mode $\Rightarrow$ magnetic fieel = 0.

$(H_z = 0)$

$$\nabla^2 E = \gamma^2 E$$

$$\Rightarrow \frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + h^2 E_z = 0$$

$$E_z(x,y,z) = E_z^0 (x,y) e^{-\gamma z}$$

$$E_z^0 = XY$$

$$\frac{\partial^2 E_{z0}}{\partial x^2} = \frac{\partial^2 \cancel{XY}}{\partial x^2}, \quad \frac{\partial^2 E_z}{\partial y^2} = \frac{X \partial^2 Y}{\partial y^2}$$

$$\frac{Y \partial^2 X}{\partial x^2} + \frac{X \partial^2 Y}{\partial y^2} + h^2 XY = 0$$

$\div$ by X & Y.

$$\Rightarrow \frac{1}{X} \frac{\partial^2 X}{\partial x^2} + \frac{1}{Y} \frac{\partial^2 Y}{\partial y^2} + h^2 = 0$$

$$\frac{1}{X} \frac{\partial^2 X}{\partial x^2} = -h^2 \underbrace{- \frac{1}{X} \frac{\partial^2 Y}{\partial y^2}}_{-A^2} = -h^2 + A^2$$

$$\frac{\partial^2 X}{\partial x^2} = h^2 - A^2$$

$$\frac{\partial^2 X}{\partial x^2} + B^2 X = 0 \quad B^2 = h^2 - A^2$$

$$\Downarrow$$

Sol:- $X = C_1 \cos Bx + C_2 \sin Bx.$

$$\frac{\partial^2 Y}{\partial y^2} + A^2 Y = 0 \Rightarrow Y = C_3 \cos Ay + C_4 \sin Ay$$

$$E_z^0 = XY = C_1 C_3 \cos Bx \cos Ay$$
$$+ C_1 C_4 \sin Ay \cos Bx$$
$$+ C_2 C_3 \cos Ay \sin Bx + C_2 C_4 \sin Bx \sin Ay.$$

Boundary condition:-

$$E_z = 0 \quad \text{at } x = 0, \; x = a \Rightarrow B = \frac{m\pi}{a}$$
$$y = 0 \quad y = b \qquad A = \frac{n\pi}{b}.$$
$$\Downarrow \qquad \Downarrow$$
$$C_1 = 0 \qquad C_3 = 0.$$

$$E_z^0 = C \sin \frac{m\pi}{a} x \sin \frac{n\pi}{b} y$$

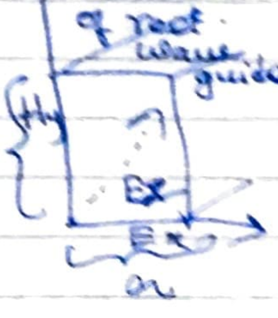
$$E_x^0 = -\frac{\gamma}{h^2} \frac{\partial E_z}{\partial x} - \cancel{\frac{j\omega\mu}{h^2}} \frac{\partial H_z}{\partial y} \rightarrow 0.$$ field component

$$= \frac{-\gamma}{h^2} \frac{\partial}{\partial x} \left[C \sin \frac{m\pi}{a} x \sin \frac{n\pi}{b} y \right]$$

$$= \frac{-j\bar{\beta}}{h^2} CB \sin \frac{n\pi}{b} y \cos \left(\frac{m\pi}{a} x \right)$$

$$H_y^0 = -\frac{j\omega\varepsilon C}{h^2} B \cos \left(\frac{m\pi}{a} x \right) \sin \left(\frac{n\pi}{b} y \right) b$$

TM mode of rect wave guide



other parameters.

$$h^2 = \bar{\gamma}^2 + \omega^2\mu\varepsilon.$$

$$h^2 = A^2 + B^2.$$

$$\bar{\gamma} = \sqrt{A^2 + B^2 - \omega^2\mu\varepsilon}$$

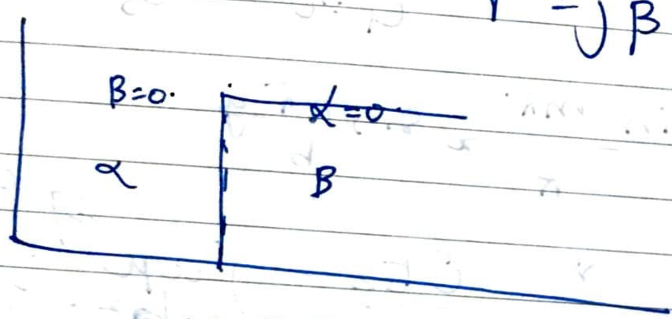
$$= \sqrt{\underbrace{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} - \omega^2\mu\varepsilon}$$

@ low freq $\bar{\gamma} = \alpha + j\beta$ = real p

$\bar{\gamma} = \alpha + 0.$

@ high freq,

$\bar{\gamma} = j\beta$ (no real component)



H.P.F.

finding cutoff frequency.

$$\omega^2\mu\varepsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2.$$

$$\Rightarrow f_c = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

cutoff λ.

$$\lambda_c = \frac{c}{f_c} = \frac{2\pi}{\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}}$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}.$$

$$v_p = \frac{\omega}{\beta} = \frac{\omega}{\sqrt{\omega^2\mu\epsilon - \left(\frac{m\pi}{a}\right)^2 \left(\frac{n\pi}{b}\right)^2}}$$

$$\lambda_g = \frac{2\pi}{\beta}$$

$$v_g = \frac{d\omega}{d\beta}$$

Attenuation of TE_{10} mode

H_x, H_z, E_y

$$H_x = \frac{j\beta a C}{\pi} \sin\left(\frac{\pi}{a}x\right)$$

$$H_z = C \cos\left(\pi/a\, x\right)$$

$$E_y = \frac{-j\omega\mu a C}{\pi} \sin\left(\pi/a\, x\right)$$

$$P_{avg} = \int \frac{1}{2} Re\left[\vec{E} \times \vec{H}^*\right] d\vec{s}$$

$$= \frac{1}{2}\,\omega\beta\mu\left(\frac{Ca}{\pi}\right)^2 \int_0^b\int_0^a \sin^2\left(\frac{\pi}{a}x\right) dx\, dy$$

$$P_{avg} = \left(\frac{Ca}{\pi}\right)^2 \omega\beta\mu\, ab$$

$$P_{avg} = 2\left[P_1(x=0) + P_2(y=0)\right]$$

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$$P_l(x=0) = \frac{1}{2}\int_0^b |H_y|^2 + |H_z|^2 R_s \, dy$$

$$= \frac{1}{2}\int_0^b C^2 R_s \, dy = \frac{b}{2} C^2 R_s$$

$$P_1(y=0) = \frac{1}{2}\int$$

$$= \frac{R_s}{2}\left[\frac{b^2 a}{2} + \left(\frac{\beta a C}{\pi}\right)^2 \frac{a}{2}\right]$$

$$\alpha = \frac{P_{loss}}{P_{avg}} = 2\left[\frac{b}{2} C^2 R_s + R_s/2\left(\frac{C^2 a}{2} + \right.\right.$$

$$1 + \left(\frac{\beta a}{\pi}\right)^2 = \left(\frac{f}{f_c}\right)^2$$

$$\alpha_{10} = \frac{R_s\left[1 + \frac{2b}{a}\left(\frac{f_c}{f}\right)^2\right]}{\eta b\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$$

Weber