1. Analog filter

$$\sum_{K=0}^{N} a_K \frac{d^K y(t)}{dt^K} = \sum_{K=0}^{M} b_K \frac{d^K x(t)}{dt^K}$$

$x(t)$ - I/p a_K, b_K :
$y(t)$ - o/p filter co-eff

Laplace transform

$$H_a(s) = \frac{\sum_{K=0}^{M} b_K \cdot s^K}{1 + \sum_{K=1}^{N} a_K s^K} \quad \boxed{s = j\omega}$$

freq res of Analog filter

2. Butterworth filter (LPF) (All pole)

$$|H(j\Omega)| = \frac{1}{\sqrt{1 + \left(\frac{\Omega}{\Omega_c}\right)^{2N}}} \rightarrow \text{order of filter}$$

$H(j\omega)$

flat passband
monotonic in PBand SB

As N↑ses response become ideal

To find the poles : (on left)

$s_k = e^{j\phi_k}(\Omega_c)$ for sum

$\phi_k = \frac{\pi}{2} + \frac{(2k-1)\pi}{2N}$

$s_k = e^{j(\ )}$

N-odd: $s_k = e^{j\pi K/N}$

N-even: $s_k = e^{j(2k-1)\pi/2N}$

All pole line on unit circle separated by (60°)

Normalised pole $\Omega_c = 1$
unnormalised pol $s_k' = s_k \Omega_c$

$s \to s/\Omega_c \Rightarrow$ TF of normalised Butterworth

N	denominator of H(s)
1	$s+1$
2	$s^2+\sqrt{2}s+1$
3	$(s+1)(s^2+s+1)$
4	$(s^2+0.76537s+1)(s^2+1.84775s+1)$
5	$(s+1)(s^2+0.618034s+1)(s^2+1.618034s+1)$

order of Butterworth filter (N)

$$|H(j\Omega)|^2 = \frac{1}{1+\varepsilon^2\left(\frac{\Omega}{\Omega_p}\right)^{2N}}$$

$$\boxed{\varepsilon = \sqrt{10^{0.1\alpha_p} - 1}}$$

$$N \geq \frac{\log \sqrt{\frac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}}}{\log(\Omega_s/\Omega_p)} = \log\left(\frac{\lambda}{\varepsilon}\right)$$

$$N \geq \frac{\log A}{\log(1/K)} \quad A = \lambda/\varepsilon \quad K = \frac{\Omega_p}{\Omega_s}$$

cut off freq: Ω_c

$$\Omega_c = \frac{\Omega_p}{[10^{0.1\alpha_p}-1]^{1/2N}} = \left[\frac{\Omega_p}{\varepsilon^{1/N}}\right]$$

$$= \frac{\Omega_s}{[10^{0.1\alpha_s}-1]^{1/2N}}$$

$\boxed{\text{when } \alpha_p = 3\text{dB} \quad \Omega_c = \Omega_p}$

To find transfer function $H_a(s)$

$H(s) \rightarrow$ from table

$H_a(s)$: replace s by $\left(\frac{s}{\Omega_c}\right)$

③ Tranformation of analog to digital filter

1. Approximation of derivative method

2. z transform method

Impulse invarient method!

$s = \sigma + j\Omega$ — σ: real, Ω: imag

$z = re^{j\omega}$ — r: radius, ω: phase

$z = e^{sT}$

$\boxed{\omega = \Omega T}$ (x)

$|z| = r \quad \angle z = \Omega T$

$= e^{\sigma T}$ $\boxed{\text{linear mapping}}$

$h_a(t)$ - impulse response of A.F

$H_a(s)$ - Transfer function of A.F

$h(n)$ - " " D.F

$H(z)$ - " D.F

$$H_a(s) = \sum_{K=1}^{N} \frac{C_K}{s - P_K} \quad \text{map to}$$

$$\frac{C_K}{s-P_K} = \frac{C_K}{1 - e^{P_K T} z^{-1}}$$

$$\boxed{H(z) = \sum_{K=1}^{N} \frac{T\, C_K}{1 - e^{P_K T} z^{-1}}}$$

poles with same real and imag differ by multiple of $\frac{2\pi}{T}$

So Multiples Mapping (many to one mapping)

due to $\boxed{\text{aliasing}}$

→ only for $\boxed{\text{LPF and BPF}}$

HPF BSF → high resonant freq

Bilinear transformation

$j\omega$ axis is mapped to unit circle in z plane only once, avoiding Aliasing

all their impulse and phase response of the analog filter is preserved in digital

$\boxed{\text{non linear mapping}}$

$$H(s) = \frac{b}{s+a} = \frac{Y(s)}{X(s)}$$

$$\boxed{s \Leftarrow \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

$$H(z) = \frac{b}{\frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right) + a}$$

$$\sigma = \frac{2}{T}\left[\frac{r^2-1}{1+r^2+2r\cos\omega}\right] = \frac{2}{T}$$

$$\Omega = \frac{2}{T}\left[\frac{2r\sin\omega}{1+r^2+2r\cos\omega}\right]$$

$$\boxed{\Omega = \frac{2}{T}\tan\frac{\omega}{2}}$$

$$\boxed{\omega = 2\tan^{-1}\left(\frac{\Omega T}{2}\right)}$$

Wrapping effect : freq compression at high freq

the graph is non linear results in distortion digital signal

Pre wrapping: the analog filter using $\Omega = \frac{2}{T}\tan\frac{\omega}{2}$ from A.F is designed using pre wrap the...

Scanned with CamScanner

④ z transform method

$$H(s) = \frac{\prod_{k=1}^{M} (s - z_k)}{\prod_{k=1}^{N} (s - p_k)}$$

$$H(z) = \frac{\prod_{k=1}^{M} (1 - e^{z_k T} z^{-1})}{\prod_{k=1}^{N} (1 - e^{p_k T} z^{-1})}$$

$$\boxed{(s-a) \rightarrow (1 - e^{aT} z^{-1})}$$

⑤ Frequency Transformation in Analog domain

Prototype LPF to LPF, HPF, BPF, BSF

(i) LPF → LPF

Ω_p → Ω_p'

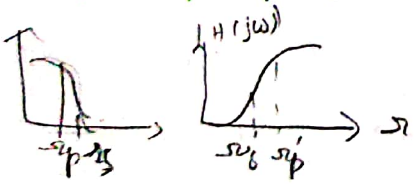
$$s \rightarrow \frac{\Omega_p}{\Omega_p'} s \qquad \Omega_p = 1 \text{ rad/sec}$$

$$H_p(s)\Big|_{s = \frac{\Omega_p s}{\Omega_p'}} = H_l(s)$$

(ii) LPF → HPF

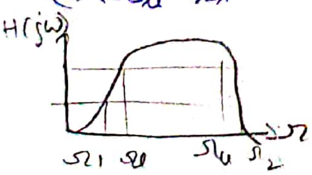
$$s \rightarrow \frac{\Omega_p \Omega_p'}{s} \qquad \Omega_p = 1 \text{ rad/sec}$$

$$H_h(s) = H_p(\Omega_p \Omega_p' / s)$$



(iii) LPF → BPF

$$s \rightarrow \left(\frac{s^2 + \Omega_l \Omega_u}{s(\Omega_u - \Omega_l)}\right) \Omega_p$$

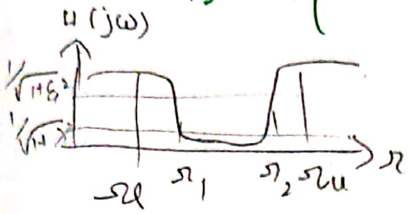


$$H_b(s) = H_p(s)\Big|_{s} = \Omega_p(\text{" "})$$

(iv) LPF → BSF $\Omega_p = 1$ rad/sec

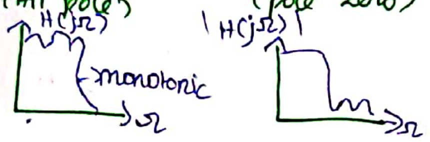
$$s \rightarrow \Omega_p\left(\frac{s(\Omega_u - \Omega_l)}{s^2 + \Omega_u \Omega_l}\right)$$

$$H_b(s) = H_p(s)\Big|_{s} = \Omega_p(\text{" "})$$



⑥ Analog lowpass chebychev

Type-I (All pole) | Type-2 (pole zero)



Type-I

$$|H(j\omega)|^2 = \frac{1}{1 + \varepsilon_p^2 C_N^2\left(\frac{\Omega}{\Omega_p}\right)}$$

Nth order chebychev polynomial

ripple in P.B

$$C_N(x) = \begin{cases} \cos(N \cos^{-1} x) & |x| \le 1 \\ \cosh(N \cosh^{-1} x) & |x| > 1 \end{cases}$$

$$\varepsilon = \sqrt{10^{0.1\alpha_p} - 1}$$

$$\boxed{N \geq \frac{\cosh^{-1}\sqrt{\frac{10^{0.1\alpha_s} - 1}{10^{0.1\alpha_p} - 1}}}{\cosh^{-1}(\Omega_s / \Omega_p)}} \quad (\text{or}) \ (1/k)$$

$$\boxed{K = \Omega_p / \Omega_s} \qquad A = \frac{\delta}{\varepsilon}$$

poles of location

$$S_k = r_1 \cos\phi_k + j r_2 \sin\phi_k$$

$$\phi_k = \frac{\pi}{2} + \frac{(2k-1)\pi}{2N}$$

$$\boxed{s_k = \sigma_k + j\Omega_k}$$

$$r_1 = \Omega_p\left[\frac{\mu^{1/N} - \mu^{-1/N}}{2}\right]$$

$$r_2 = \Omega_p\left[\frac{\mu^{1/N} + \mu^{-1/N}}{2}\right]$$

$$\boxed{\mu = \varepsilon^{-1} + \sqrt{1 + \varepsilon^{-2}}}$$

When N = even

Numerator: in denominator polynomial $s = 0$ divide by $\sqrt{1+\varepsilon_p^2}$

N = odd.

$s = 0$ is enough

⑦ Steps to design digital IIR filter (LPF)

(i) ~~Butter~~ BW (or) Chebby ⟹ impulse / Bilinear

(or)

(ii) Convert digital LPF to BPF, HPF, BSF

freq transn in digital domain

LPF impulse/digital LPF → HPF (digital)

⑥ Frequency Transformation in digital domain

LPF → LPF

$$z^{-1} \rightarrow \frac{z^{-1} - a}{1 - az^{-1}} \qquad a = \frac{\sin((\omega_p - \omega_p')/2)}{\sin[(\omega_p + \omega_p')/2]}$$

LPF → HPF

$$z^{-1} \rightarrow -\frac{z^{-1} + a}{1 + az^{-1}} \qquad a = -\frac{\cos[(\omega_p + \omega_p')/2]}{\cos[(\omega_p - \omega_p')/2]}$$

bandedge frequency

LPF → BPF

$$z^{-1} \rightarrow -\frac{z^{-2} - a_1 z^{-1} + a_2}{a_2 z^{-2} - a_1 z^{-1} + 1} \qquad a_1 = 2\alpha k/(k+1), \quad a_2 = (k-1)/(k+1)$$

$$\alpha = \frac{\cos[(\omega_u + \omega_l)/2]}{\cos[(\omega_u - \omega_l)/2]}$$

$$k = \cot\frac{\omega_u - \omega_l}{2} \tan\frac{\omega_p}{2}$$

LPF → BSF

$$z^{-1} \rightarrow \frac{z^{-2} - a_1 z^{-1} + a_2}{a_2 z^{-2} - a_1 z^{-1} + 1} \qquad k = \tan\frac{\omega_u - \omega_l}{2} \tan\frac{\omega_p}{2}$$

Unit - 3

① FIR filter

$$y(n) = \sum_{k=0}^{M} b_k x(n-k)$$

* non recursive
* linear phase when (order of filter) (N) is large
* need large memory and powerful computational facility

Properties :

$$H(\omega) = \sum_{n=0}^{N-1} h(n) e^{-j\omega n}$$

$h(n)$ → causal filter coefficient

$$\boxed{\angle H(\omega) = \phi(\omega)}$$

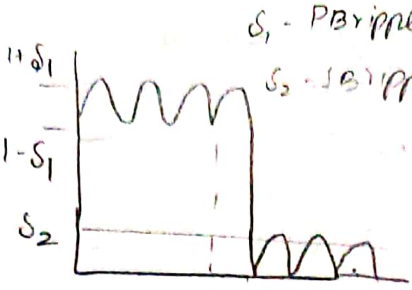
$$\boxed{\tau_p = -\frac{\phi(\omega)}{\omega}} \text{ phase delay}$$

$$\boxed{\tau_g = -\frac{d\phi(\omega)}{d\omega}} \text{ grp delay}$$

② characteristic of practical frequency selective filter

Ideal :
non causal, physically unrealizable, sharp cutoff from P.B to SB
It cannot drop from 1 to '0' abruptly.

Practical :
ripple in P.B is ok than constant |H(ω)|
ripple in SB is ok than "zero"



δ_1 - PB ripple
δ_2 - SB ripple

ω_p → P.B edge freq
ω_s → SB edge freq

transition band
$\boxed{\text{width} = \omega_s - \omega_p}$

width of P.B is usually called Band width of filter.

$\delta_1 \delta_2 \omega_p \omega_s$ → determines A_k B_k

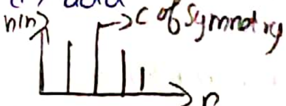
③ Linear phase characteristic of FIR

τ_p, τ_g → constant $\phi(\omega)$ linear

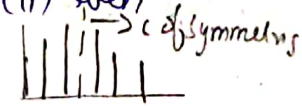
$$\boxed{\tau = \frac{N-1}{2} \quad h(n) = h(N-1-n) \quad 0 \le n \le N-1}$$

Symmetric: $h(n) = h(N-1-n)$

(i) odd

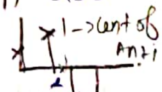


(ii) even

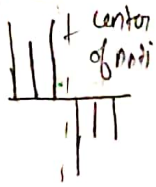


Anti symmetric: $h(n) = -h(N-1-n)$

(i) odd



(ii) even



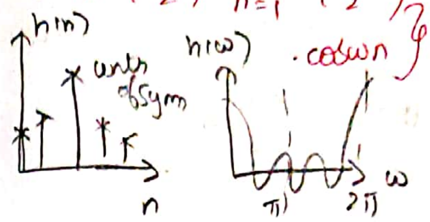
Application:

τ_g, τ_p const.	τ_g constant
linear phase.	Hilbert transf.
used to design LPF, HPF, BPF, BSF	& differentiator

condition for FIR to be linear phase
(i) symmetrical $h(n) = (N-1-n)$
(ii) τ_g τ_p → constant

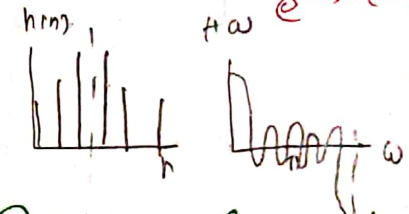
④ Symmetrical N is odd

$$H(\omega) = e^{-j\omega\left(\frac{N-1}{2}\right)} \left\{ h\left(\frac{N-1}{2}\right) + \sum_{n=1}^{N-1/2} 2\, h\left(\frac{N-1}{2}-n\right) \cos\omega n \right\}$$



N is even

$$H(\omega) = \sum_{n=1}^{N/2} 2h\left(\frac{N}{2}-n\right)\cos\omega\left(n-\tfrac{1}{2}\right) e^{-j\omega\left(\frac{N-1}{2}\right)}$$



⑤ Location of zero of linear phase FIR filter

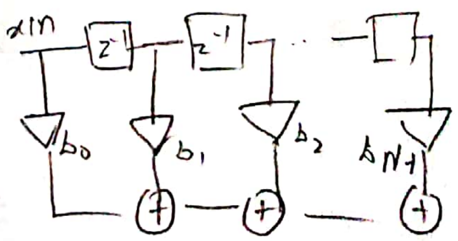
If z_0 is zero of $H(z)$ then z_0^{-1} is also a zero

⑥ Structures for Realising FIR.

1. Direct form Realisation
2. cascade Realisation
3. Linear phase realisation

① Direct method

$$y(n) = b_0 x(n) + b_1 x(n-1) + b_2 x(n-2) + \dots + b_{N-1} x(n-(N-1))$$



Cascade Realisation:

$$y(n) = \sum_{k=0}^{N-1} b_k x(n-k)$$

$$Y(z) = \sum_{k=0}^{N-1} b_k \cdot z^{-k} \cdot X(z) = \frac{Y(z)}{X(z)}$$

$$\boxed{H(z) = \sum_{k=0}^{N-1} b_k z^{-k}}$$

(i) if N is odd
$H(z) \div \left(\frac{N-1}{2}\right)$ second order factor

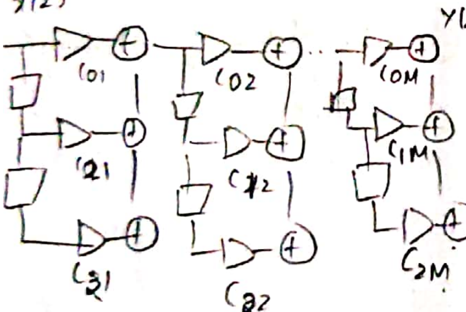
(ii) N is even
$H(z) \div \left(\frac{N-2}{2}\right)$ 1st order factor.

(i)
$$H(z) = H_1(z) H_2(z) \ldots H_{\frac{N-1}{2}}(z)$$

(ii)
$$H(z) = H_1(z) H_2(z)\, H_{\frac{N}{2}}(z)$$

$$H(z) = (c_{01} + c_{11} z^{-1}) \prod_{i=2}^{N/2} (c_{0i} + c_{1i} z^{-1} + c_{2i} z^{-2})$$

N = odd



$M = \frac{N-1}{2}$

N even: Same diagram

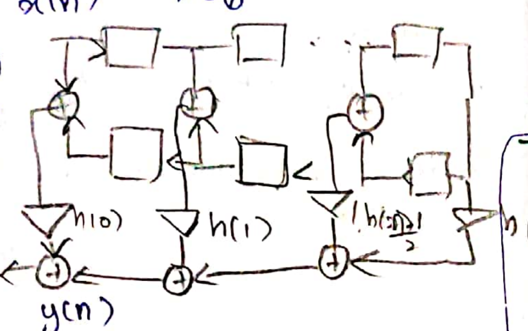
$H\left(\frac{N}{2}\right)$

Linear phase realisation
* To reduce the number of multipliers

$$H(z) = Z\{h(n)\} = \sum_{n=0}^{N-1} h(n) z^{-n}$$

N = odd

$$H(z) = h\left(\frac{N-1}{2}\right) z^{-\left(\frac{N-1}{2}\right)} + \sum_{n=0}^{(N-3)/2} h(n)\left[z^{-n} + z^{-(N-1-n)}\right]$$



N - even

$$H(z) = \sum_{n=0}^{\frac{N-2}{2}} h(n)\left[z^{-n} + z^{-(N-1-n)}\right]$$

Same diagram
$h\left(\frac{N-4}{2}\right)$ $h\left(\frac{N-2}{2}\right)$

(7) Frequency Sampling method for designing FIR Filter.

The samples are the DFT coefficient of the impulse response of the filter hence $h(n)$ is obtained by taking IDFT $\{H(K)\}$

$$\boxed{H(K) = H_d(\omega) \Big/_{\omega = \frac{2\pi(K+\alpha)}{N}}}$$

$K = 0, 1, \ldots, N-1$

Based on value of α there are 2 type
$\alpha = 0$ $\alpha = 1/2$

Type - I $(\alpha = 0)$

$$H(K) = H_d(\omega) \Big/_{\omega = \frac{2\pi K}{N}}$$

$$H(K) = H^*(N-K)$$

$$H(N-K) = H^*(K)$$

$|H(K)| = |H(N-K)| \rightarrow$ even

$\angle H(K) = -\angle H(N-K) \rightarrow$ odd

N = even $\left(\frac{N}{2}-1\right) \rightarrow \left(\frac{N-1}{2}\right) \rightarrow$ odd

$$h(n) = \frac{1}{N}\left\{H(0) + 2\sum_{K=1} Re\left[H(K) e^{\frac{j2\pi K n}{N}}\right]\right\}$$

$$\boxed{H(z) = \sum_{n=0}^{N-1} h(n) z^{-n}}$$

Type 2 $(\alpha = 1/2)$

n is even

$$h(n) = \frac{2}{N} \sum_{K=0}^{\frac{N}{2}-1} Re\left[H(K) e^{\frac{j2\pi n}{N}\left(\frac{K+1}{2}\right)}\right]$$

if n is odd
$\frac{N}{2} - 1 = \frac{N-3}{2}$.

(8) Design L.P FIR filter

(i) Fourier series (or) window method
(ii) Frequency Sampling method
(iii) optimal filter design method.

(i) Fourier series method

$$h(n) = h_d(n) - \left(\frac{N-1}{2}\right) to \left(\frac{N-1}{2}\right)$$

$$H(z) = z^{\left(-\frac{N-1}{2}\right)} \left\{h(0) + \sum_{n=1}^{\frac{N-1}{2}} h(n)\left[z^{n} + z^{-n}\right]\right\}$$

) TO 11

Disadvantage:

Gibbs phenomenon: due to abrupt truncation of the fourier series results in oscillation in the pass band and stop band .. these are due to slow convergence of fourier series in the point of discontinuity

How oscillation are reduced?

By multiply IIR $h_d(n)$ with finite weighting function (or) window function $w(n)$

$w(n) = w(-n) \neq 0$, $n = -\left(\frac{N-1}{2}\right)$ to $\left(\frac{N-1}{2}\right)$

$$\boxed{H(\omega) = H_d(\omega) * W(\omega)}$$

How to design refer pdf.

characteristic of window:

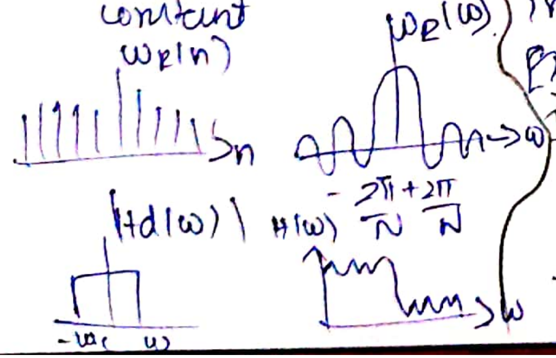
* the central lobe contain most of the energy, should be narrow
* the highest side lobe level of freq respons is ~~narrow~~ small
* the side lobe freq response should decrease in energy when ω to π

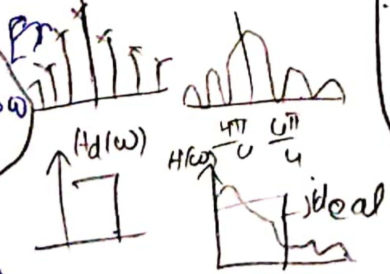
Raised cosine window (general hamming)

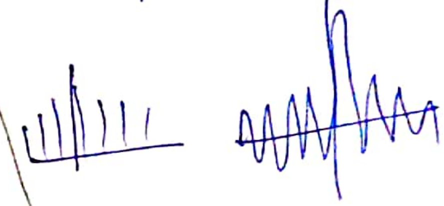
$$w_\alpha(n) = \begin{cases} \alpha + (1-\alpha)\cos\left(\frac{2\pi n}{N-1}\right) \\ 0 \end{cases}$$

$$W_\alpha(\omega) = \alpha \frac{\sin \omega N/2}{\sin \omega/2} - \frac{1-\alpha}{2} \frac{\sin\left(\frac{\omega N}{2} - \frac{\pi N}{N-1}\right)}{\sin\left(\frac{\omega}{2} - \frac{\pi}{N-1}\right)} + \frac{1-\alpha}{2} \frac{\sin\left(\frac{\omega N}{2} + \frac{\pi N}{N-1}\right)}{\sin\left(\frac{\omega}{2} + \frac{\pi}{N-1}\right)}$$

	Rectangular	Triangular	Hanning	hamming	Blackman window	Kaiser
function	$w_R(n) = 1 \quad -\left(\frac{N-1}{2}\right) \le n \le \left(\frac{N-1}{2}\right)$, 0 else; $w_R(\omega) = \frac{\sin \omega (N/2)}{\sin(\omega/2)}$	$w_T(n) = \begin{cases} 1 - \frac{2\lvert n \rvert}{N-1} \\ 0 \end{cases}$; $w_T(\omega) = \left[\frac{\sin\left(\frac{N-1}{4}\right)\omega}{\sin\left(\frac{\omega}{2}\right)}\right]^2$	$w_{Hn} = 0.5 + 0.5\cos\left(\frac{2\pi n}{N-1}\right)$; $W_{Hn}(\omega) = 0.5\frac{\sin \frac{\omega N}{2}}{\sin \omega/2} + \frac{0.25 \sin\left(\frac{\omega N}{2} - \frac{\pi N}{N-1}\right)}{\sin\left(\frac{\omega}{2} - \frac{\pi}{N-1}\right)} + \frac{0.25\left(\sin \frac{\omega N}{2} + \frac{\pi N}{N-1}\right)}{\sin\left(\frac{\omega}{2} + \frac{\pi}{N-1}\right)}$	$w_{Hn} = 0.54 + 0.46\cos\left(\frac{2\pi n}{N-1}\right)$; $w_{Hn}(\omega)$ 0.54, 0.23, 0.23	$w_B(n) = 0.42 + 0.5\cos\frac{2\pi n}{N-1} + 0.08\cos\frac{4\pi n}{N-1}$	$w_K(n) = \begin{cases} \frac{I_0(\beta)}{I_0(\alpha)}, & -\left(\frac{N-1}{2}\right) \end{cases}$; $\beta = \alpha\sqrt{1 - \left(\frac{2n}{N-1}\right)^2}$
MLW	$4\pi/N$	$8\pi/N$	$8\pi/N$	$8\pi/N$	$12\pi/N$	MLW under peak side lobe depends on N
Max side lobe mag	-13dB	-25dB	-31dB.	-41dB	-58dB	
Side lobe mag.	does not ↑ when ω↑es	↓es when ω↑s.	↓es when ω↑s	constant when ω↑s.	↓es when ↑ω	
feature	* width of transition ∝ width of MLW * gibbs in SB PB * attenuation in S.B is constant	Transition from PB to SB is not sharp, attenuation is less when compared to RW in SB (not used)	→ little taller; same		Side lobe attenuation is maximum, whereas achie. by ↑ MLW, but MLW by ↑N	MLW and width can be varied with N S&M depends on M







e(n)

unit - 4

1) Number n can be represented as

no of integer digit, ith digit of number

$$N = \sum_{i=n_1}^{n_2} d_i \, r^i$$

r → radix (or) base

n_1 → no of fraction digit

2) Floating point Representation

$F = M \times 2^{e}$ (e → +ve (or) -ve)

M $0.5 \le M \le 1$

3) Quantisation

Quantised error / Add noise e(n) $= x_q(n) - x(n)$

$(b+1)$ → bits

2^{b+1} → levels

2^{-b} → quantisation step size.

4) Error due to truncation of fixed point

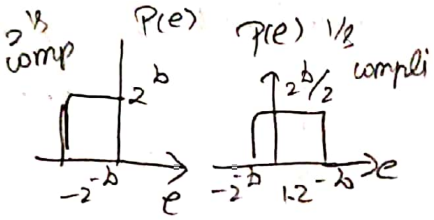
+ve number $0 \ge e > -2^{-b}$

Sign magnitude negative no $0 \le e < 2^{-b}$

1's complement $0 \le e \le 2^{-b}$

2's complement $0 \ge e > -2^{-b}$

PDF of fixed pt



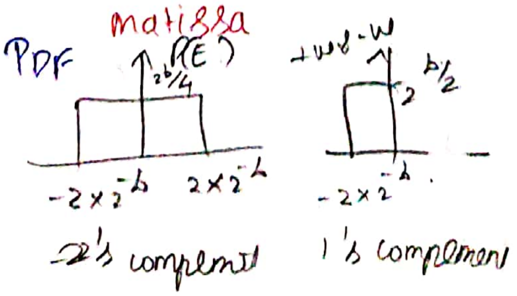
5) Error due to truncation of floating pt.

2's compliment +ve matissa $0 \ge \varepsilon > -2 \times 2^{-b}$

" " -ve matissa $0 \le \varepsilon < -2 \times 2^{-b}$

1's complement +ve and -ve $0 \le \varepsilon \le -2 \times 2^{-b}$

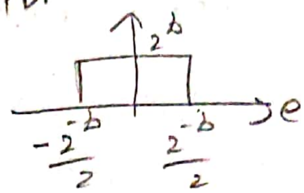
Sign magnitude +ve and -ve $0 \le \varepsilon < -2 \times 2^{-b}$



6) Error due to rounding off of fixed pt

$$-\frac{2^{-b}}{2} \le x_R - x \le \frac{2^{-b}}{2}$$

PDF

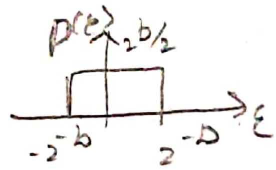


7) Error due to rounding " " floating pt

$$-\frac{2^{-b}}{2} \le \varepsilon . M \le \frac{2^{-b}}{2}$$

$M = 1/2$

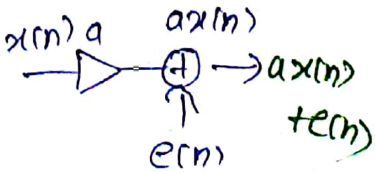
$-2^{-b} \le \varepsilon < 2^{-b}$



8) Product Quantisation error due to multiplication operation.

9) Noise transfer function

TF obtained by treating the noise source as actual i/p



$Q[a \cdot x(n)] = a \cdot x(n) + e(n)$

quantised o/p — unquantised product — product quantisation error.

(10) Coefficient Quantisation:

→ when digital filter are designed and filter coefficient are to be stored in registers for finite duration

→ the location of 0's and poles are ~~obtai~~ depends on value of filter coefficent.

→ when shifted, create the deviation in the frequency response of the system

→ desired filter are close to the unit circle, then those of the filter with quantised lie outside.

leading to instability

→ cascade realisation is preferred.

Limit cycle oscillation

Limit cycle:

when the I/p is given there will be not linearity and the o/p oscillates btw +ve and -ve value for increasing n (or) become constant for incringn

zero input limit cycle:-

If the o/p of the enter limit cycle, continute to remain in limit cycle even when I/p is zero

overflow limit cycle:

Oscillation produced when overflow occurs due to the sum of 2 binary number.

$y(n) = x(n) + \alpha y(n-1)$

$\alpha = \frac{1}{2}$ $x(n) = \begin{cases} 0.875 & n=0 \\ 0 & n \neq 0 \end{cases}$

$\alpha = -\frac{1}{2}$ $x(n) = \begin{cases} 0.875 & n=0 \\ 0 & n \neq 0. \end{cases}$

Dead Band:

The amplitude of the o/p during limit cycle are confined to range of value

$$\boxed{y(n-1) \leq \frac{\frac{1}{2} 2^{-b}}{1-|\alpha|}}$$

→ first order filter.