Unit- 4

Impulse Response & Stability :-

→ In a stable s/m, the response or o/p is predictable, finite & stable for a given i/p.

→ The different definitions of the stability are the following (ie) the s/m is stable if,

(a) If the o/p is ~~stable~~ bounded for the bounded i/p, then the s/m is stable.

(b) Asymptotically stable, if the o/p tends to zero when the i/p is zero.

(c) For a bounded disturbing i/p signal, if the o/p is tends to zero as $t \to \infty$, then the s/m is stable.

(d) the s/m is unstable, if the i/p is bounded disturbing signal & the o/p is oscillatory with infinite amplitude.

(e) If the o/p has constant oscillatory amplitude for the bounded i/p signal, then the s/m may be stable or unstable. Such s/ms are called as limitedly stable.

f) If a s/m o/p is stable for all variations of its parameters, then the s/m is <u>absolutely stable</u>.

g) If a s/m o/p is stable for a limited range of variations of its parameters, then the s/m is called <u>conditionally stable s/m</u>.

→ <u>Impulse Response of a s/m</u> :-

$$CLTF = \frac{C(s)}{R(s)} = M(s)$$

$$C(s) = M(s)\,R(s)$$

$$c(t) = L^{-1}\left[M(s)\,R(s)\right]$$

Impulse response ⇒ $c(t) = L^{-1}\left[M(s)\right]$

$\left[\because\ r(t) = \delta(t)\ \&\ R(s) = 1\right]$

$\therefore$ $\boxed{c(t) = m(t)}$ ⇒ Impulse response.

→ $C(s) = M(s)\,R(s)$.

By property of convolution,

$$c(t) = \left[m(t) * r(t)\right]$$

$$c(t) = \int_{-\infty}^{\infty} m(\tau)\, r(t-\tau)\, d\tau$$

→ Therefore, for any arbitary i/p, the o/p is obtained by the convolution of i/p & impulse response.

→ <u>BIBO - stability</u> :-

$$c(t) = \int_0^\infty m(\tau) r(t-\tau)\, d\tau.$$ ⇒ Relaxed s/m (ie) initial condn. are zero.

$$|c(t)| = \left| \int_0^\infty m(\tau) r(t-\tau) d\tau \right|.$$

$$= \int_0^\infty |m(\tau)| \, |r(t-\tau)| d\tau.$$

$$= \int_0^\infty |m(\tau)| \, A_1 \, d\tau.$$

∵ for a bounded i/p, $|r(t-\tau)| < \infty$

∴ $|r(t-\tau)| = A_1$ ⇒ finite value,

$$|c(t)| = \int_0^\infty |m(\tau) A_1 \, d\tau.$$

for a bounded o/p, $|c(t)| \leq A_2 < \infty$

$$\boxed{\therefore \; A_1 \int_0^\infty |m(\tau)| \, d\tau \leq A_2 < \infty}$$

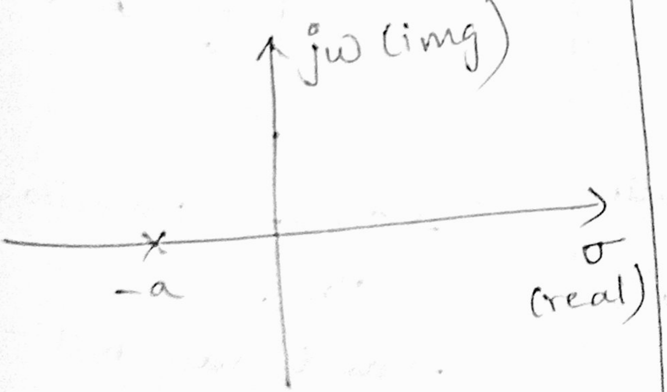
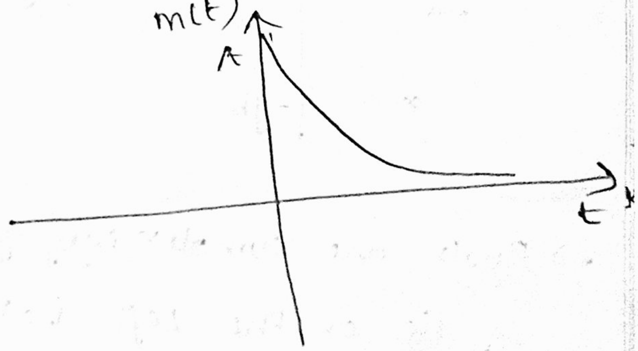
→ Hence for bounded o/p,

$$\int_0^{\infty} |m(t)|\, dt < \infty .$$

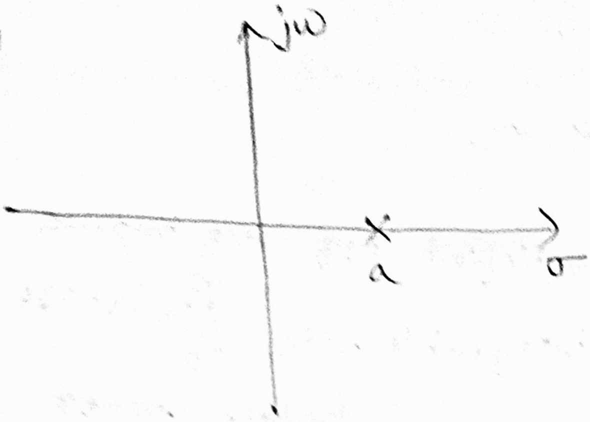
→ The above condt. is satisfied iff the impulse response is absolutelly integrable .(ie) $\int_0^{\infty} |m(t)|\, dt$ is finite . & hence the area under the curve from $t=0$ to ∞ is finite .

Location of poles on s-plane for stability .

$$M(s) = \frac{(s+z_1)(s+z_2)\cdots(s+z_m)}{(s+P_1)(s+P_2)\cdots(s+P_n)} .$$

Transfer function & location of poles $M(s)$	Impulse response $m(t)$.
1) $\boxed{M(s) = \frac{A}{s+a} .}$	$\boxed{\begin{aligned} m(t) &= L^{-1}[M(s)] \\ &= Ae^{-at} \end{aligned}}$
 	
→ Root on negative real axis.	→ exponentially decaying as $t \to \infty$, $m(t) = 0$ (finite) ∴ stable s/m .

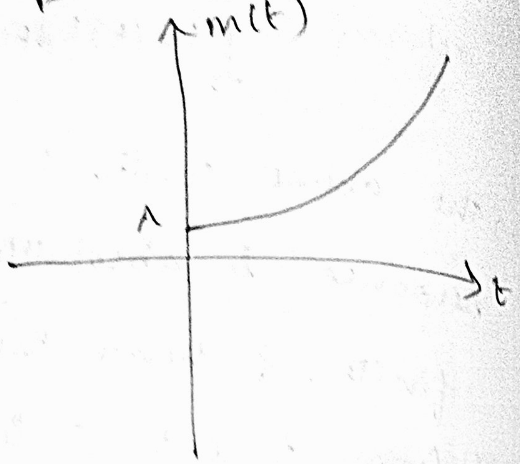
2)



$$\boxed{M(s) = \frac{A}{s-a}}$$

→ Root on positive real axis

$$\boxed{m(t) = Ae^{at}}$$



→ Exponentially increasing signal
as $t \to \infty$, $m(t) \to \infty$

∴ unstable s/m.

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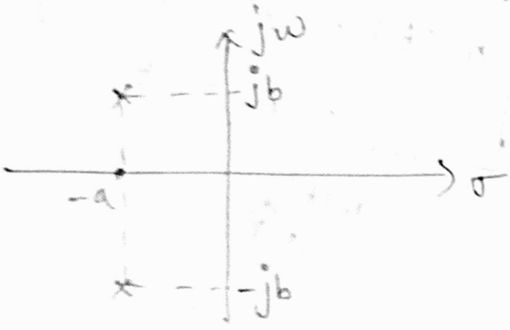
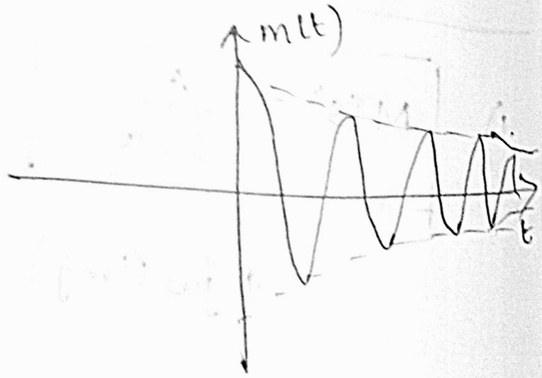
3) $$\boxed{M(s) = \frac{A}{s+a+jb} + \frac{A^*}{s+a-jb}}$$



→ Roots are complex conjugate & lie on the left half of s-plane.

$$\boxed{m(t) = Ae^{-(a+jb)t} + A^* e^{-(a-jb)t}}$$

$$= A'\left(2e^{-at} \cos bt\right)$$

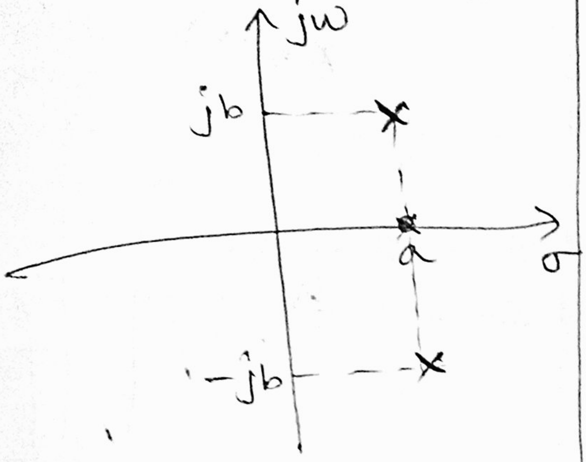


→ Damped sinusoidal s/m.

→ as $t \to \infty$, $m(t) = 0$.

→ ∴ s/m is stable.

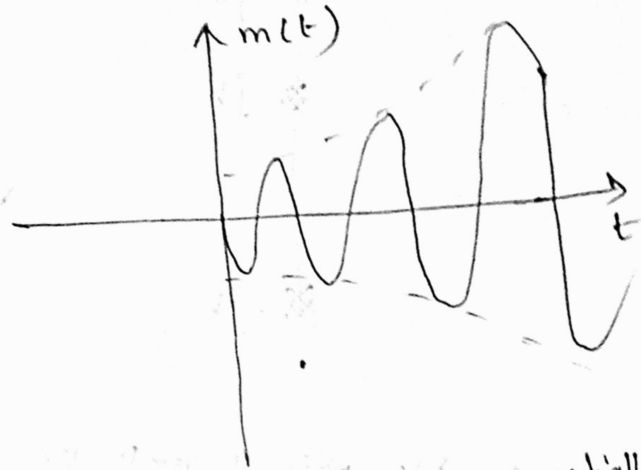
4) $M(s) = \frac{A}{s-a+jb} + \frac{A^*}{s-a-jb}$



→ Roots are complex conj, & lie on the Right half of s-plane.

$m(t) = A e^{(a-jb)t} + A^* e^{(a+jb)t}$

$= 2A' e^{at} \cos bt$.

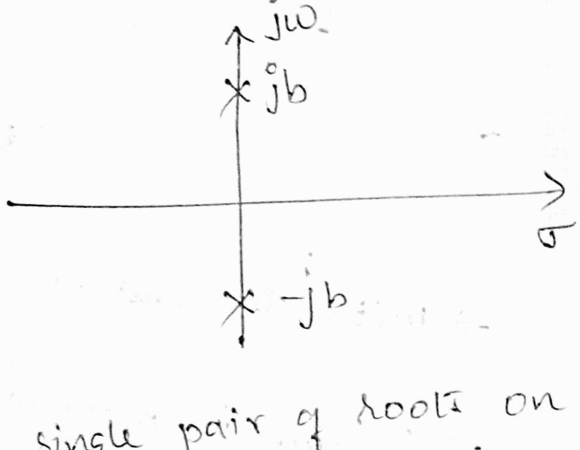


→ Undamped exponentially increasing sinusoidal signal.

→ $t \to \infty$, $m(t) \to \infty$.

→ ∴ s/m is unstable.

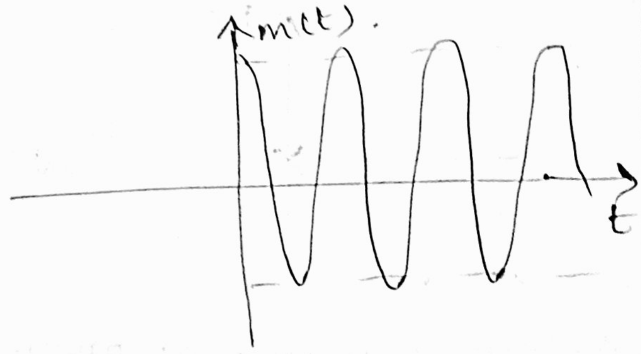
5) $M(s) = \frac{A}{s+jb} + \frac{A^*}{s-jb}$



→ single pair of roots on img. axis.

$m(t) = A e^{-jbt} + A^* e^{jbt}$

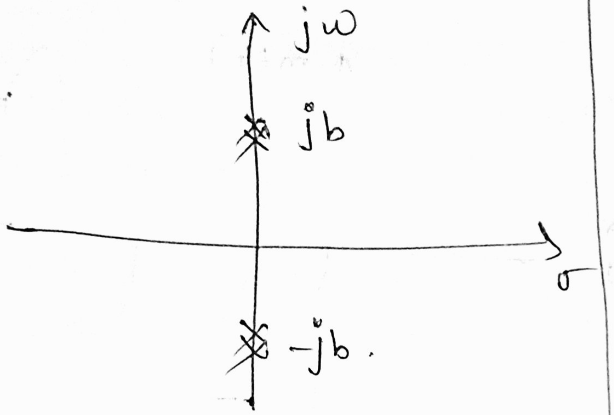
$= 2A' \cos bt$.



→ Impulse response is oscillatory with constant amplitude.

→ ∴ s/m is marginally stable.

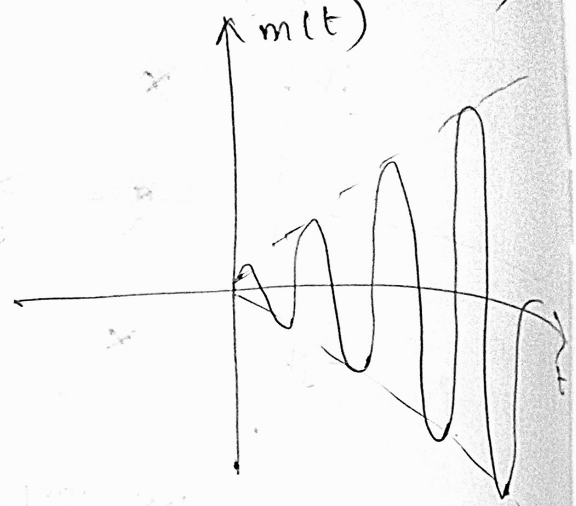
6) $M(s) = \dfrac{A}{(s+jb)^2} + \dfrac{A^*}{(s-jb)^2}$



→ Double pair of roots on img. axis

$m(t) = A t e^{-jbt} + A^* t e^{jbt}$

$m(t) = 2A' t (\cos bt)$



→ m(t) is linearly ↑ing sinusoidal
As $t \to \infty$, $m(t) \to \infty$.

→ ∴ s/m is unstable.

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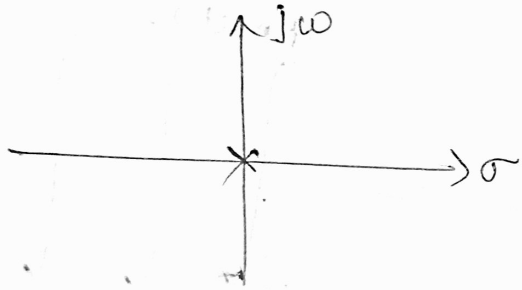
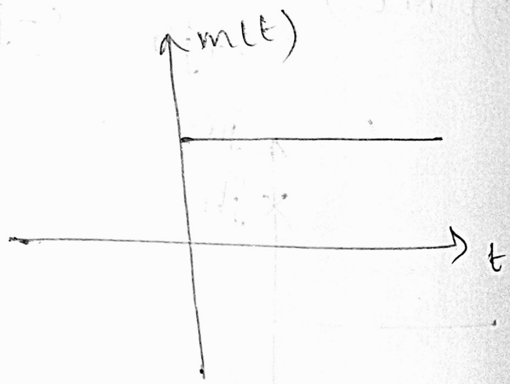
7) $M(s) = \dfrac{A}{s}$



→ single root at origin

$m(t) = A$

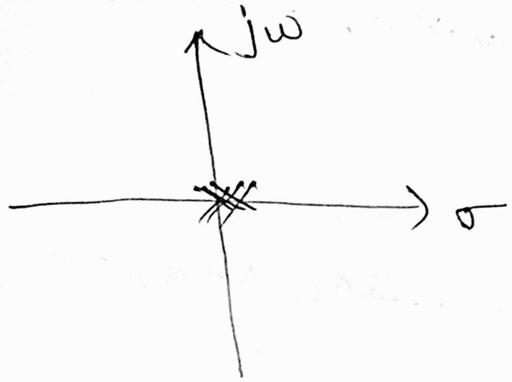


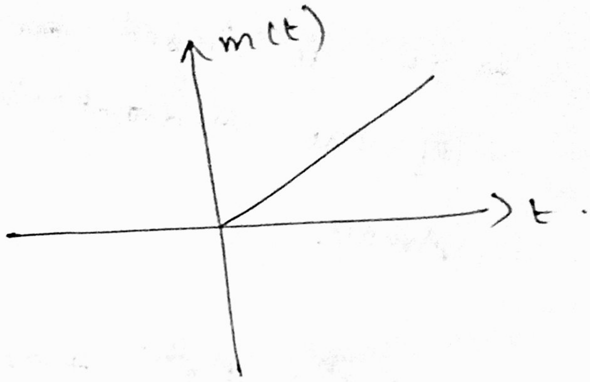
→ m(t) is constant.

→ ∴ s/m is marginally stable.

8) $M(s) = \frac{A}{s^2}$

$m(t) = At$.





→ Double pole (roots) at origin.

→ $m(t) \Rightarrow$ linearly increasing

→ As $t \to \infty$
$m(t) \to \infty$.

→ ∴ s/m is unstable.

→ <u>Note</u> :-

→ If all the co-efficients are positive & if no co-eff is zero, then all the roots are in the left half of s-plane.

→ If any co-eff. is equal to zero, then some of the roots may be on the imaginary axis. ~~or on the right h~~

→ If any co-efficient is negative then atleast one root is in the RH-splane.

Hence, the absence or negativeness of any of the co-efficients of a char. polynomial indicates that the s/m is either marginally stable or unstable.

Roots with negative Real parts $\xrightarrow{\text{always true}}$ All co-eff are positive

All co-eff are positive $\xrightarrow{\text{not always true [depends on img. part \& Real part]}}$ Roots with negative Real parts

not always true [depends on img. part & Real part]
$(0 \pm j\omega)$