(iv) P-D-controller :-

→ The proportional plus derivative controller produces an o/p signal consisting of two terms.

(a) proportional to error signal

(b) proportional to the derivative of the i/p error signal.

$$P_d(t) \propto \left[e(t) + \frac{d}{dt}e(t)\right]$$

→

$$P_d(t) = K_p e(t) + K_p T_d \frac{d}{dt}e(t).$$

K_p → proportional gain

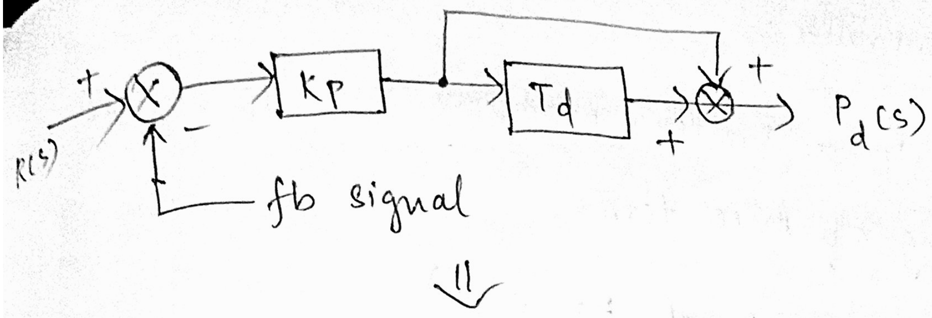
T_d → derivative time.

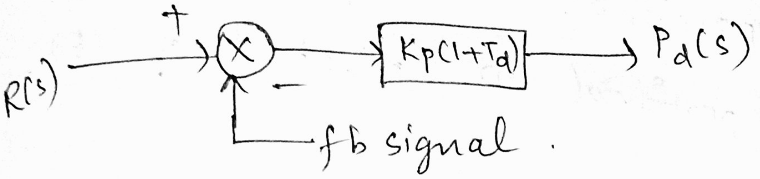
→ on, LT, $P_d(s) = K_p E(s) + K_p T_d\, s E(s)$

$$= E(s)\left[K_p + K_p T_d\right]$$

$$P_d(s) = E(s)\left[K_p(1+T_d)\right]$$

$$\frac{P_d(s)}{E(s)} = K_p(1+T_d).$$





→ The derivative control acts on the rate of change of error & its action is effective during transient periods and does not produce effective correction on constant error.

→ Hence the derivative controller is never used alone, but it is employed in association with proportional and integral controllers.

→ The derivative controller does not affect the steady state error directly but initiates an early correction action and tends to increase the stability of the s/m.

→ But it amplifies noise signals & may cause a saturation effect in the actuator.

→ The derivative control action is adjusted by varying the derivative time, hence it is called as rate control

(v) P-I-D - controller :-

→ The PID controller produces an o/p signal consisting of three terms:

(a) proportional to $e(t)$

(b) proportional to integral of $e(t)$

(c) proportional to derivative of $e(t)$.

→ $$P_{id}(t) \propto \left[e(t) + \int e(t)\,dt + \frac{d}{dt} e(t)\right]$$

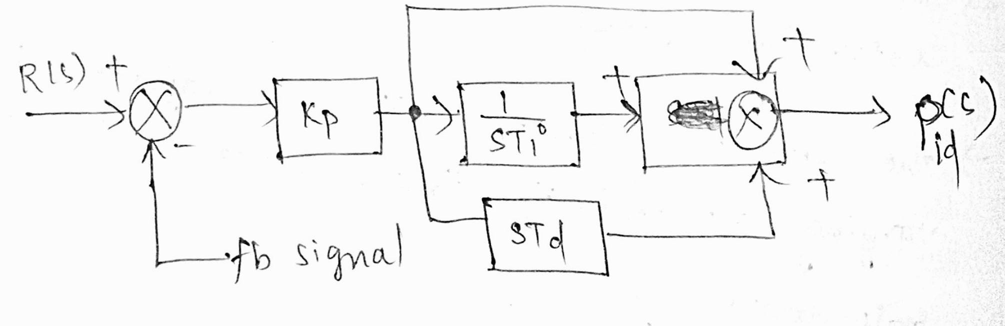
$$P_{id}(t) = K_p e(t) + \frac{K_p}{T_i}\int e(t)\,dt + K_p T_d \frac{d e(t)}{dt}$$

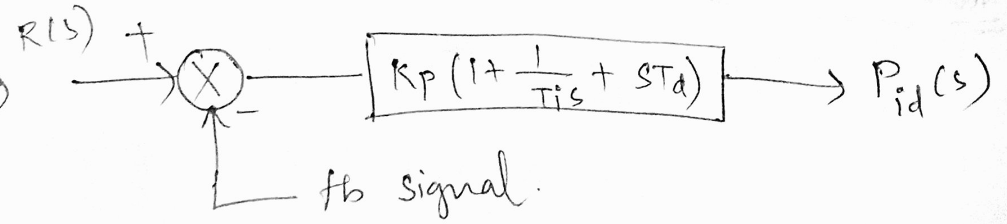
→ on, L.T

$$P_{id}(s) = K_p E(s) + \frac{K_p}{T_i}\frac{1}{s}E(s) + K_p T_d\, sE(s).$$

$$P_{id}(s) = E(s)\, K_p\left(1 + \frac{1}{sT_i} + sT_d\right)$$

$$\frac{P_{id}(s)}{E(s)} = K_p\left(1 + \frac{1}{sT_i} + sT_d\right)$$

→ 

→ 

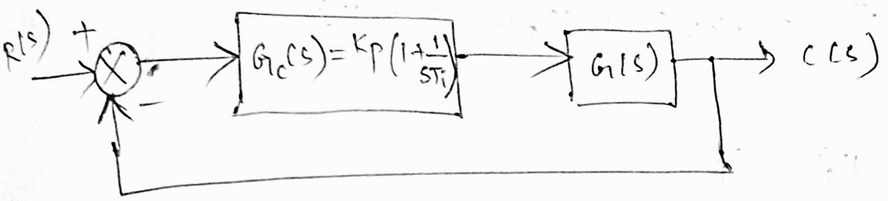
→ The proportional controller stabilizes the gain but produces a steady state error.

→ The Integral controller reduces or eliminates the steady state error.

→ The derivative controller reduces the rate of change of error.

Effect of PI & PD controllers :-

→ PI controller



$G_c(s)$ → controller transfer func.

$G(s)$ → open loop transfer func. $= \dfrac{\omega_n^2}{s(s+2\zeta\omega_n)}$.

$$\text{CLTF} \Rightarrow \frac{C(s)}{R(s)} = \frac{G_c(s)G(s)}{1+G_c(s)G(s)H(s)}$$

$$= \frac{K_p\left(1+\frac{1}{ST_i}\right)\left(\frac{\omega_n^2}{s^2+2\zeta\omega_n s}\right)}{1+K_p\left(\frac{ST_i+1}{ST_i}\right)\left(\frac{\omega_n^2}{s^2+2\zeta\omega_n s}\right)}$$

$$= \frac{K_p\left(\frac{ST_i+1}{ST_i}\right)\left(\frac{\omega_n^2}{s^2+2\zeta\omega_n s}\right)}{\dfrac{ST_i\left(s^2+2\zeta\omega_n s\right)+K_p\left(ST_i+1\right)\left(\omega_n^2\right)}{ST_i\left(s^2+2\zeta\omega_n s\right)}}$$

$$= \frac{K_p \omega_n^2 (1+sT_i)}{s^3 T_i + 2\zeta\omega_n s^2 T_i + K_p T_i s \omega_n^2 + K_p \omega_n^2}$$

$$= \frac{K_p \omega_n^2 (1+sT_i)}{T_i \left(s^3 + 2\zeta\omega_n s^2 + K_p \omega_n^2 s + \frac{K_p}{T_i} \omega_n^2 \right)}$$

$$\boxed{\frac{C(s)}{R(s)} = \frac{K_i \omega_n^2 (1+sT_i)}{s^3 + 2\zeta\omega_n s^2 + K_p \omega_n^2 s + K_i \omega_n^2}}$$

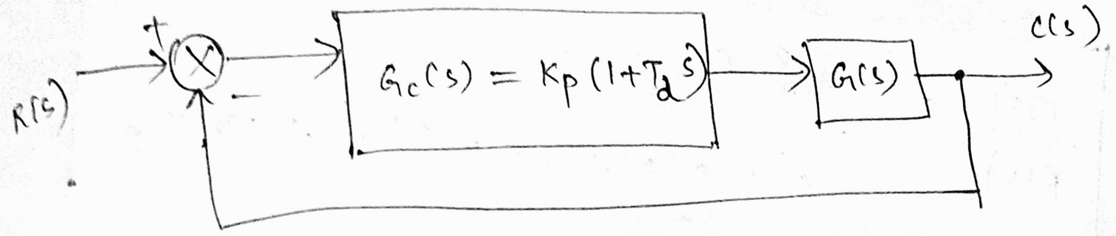
where $K_i = \frac{K_p}{T_i}$.

→ PI controller introduces one zero. Also ~~[illegible]~~ the order increases by one. Increase in the order of the s/m results in less stable s/m.

→ In $G_c(s)\,G(s)$, the PI controller increases the type number by one. The increase in type no. results in low steady state error.

→ PD controller :-

→ The proportional plus derivative controller, block diagram is,



$$G(s) = \frac{\omega_n^2}{s(s+2\zeta\omega_n)} \Rightarrow \text{OLTF}$$

→ $G_c(s)\,G(s) \Rightarrow$ Tf of PD

$$G_c(s)\,G(s) = K_p(1+T_d.s) \times \frac{\omega_n^2}{s(s+2\zeta\omega_n)}$$

$$= \frac{K_p\omega_n^2(1+T_d s)}{s(s+2\zeta\omega_n)}.$$

→ $$\text{CLTF} = \frac{G_c(s)\,G(s)}{1+G_c(s)\,G(s)}$$

$$= \frac{\left(K_p\omega_n^2(1+T_d s)\right)/s(s+2\zeta\omega_n)}{1+\dfrac{K_p\omega_n^2(1+T_d s)}{s(s+2\zeta\omega_n)}}$$

$$\text{CLTF} = \frac{K_p\omega_n^2(1+T_d s)}{s^2+2\zeta\omega_n s+K_p\omega_n^2(1+T_d s)}$$

$$\left[\frac{C(s)}{R(s)}\right] = \frac{\omega_n^2(K_p+K_pT_d s)}{s^2+2\zeta\omega_n s+\omega_n^2(K_p+K_pT_d s)}$$

$$= \frac{\omega_n^2 (K_p + K_d s)}{s^2 + 2\zeta\omega_n s + \omega_n^2 (K_p + K_d s)}$$

$$\boxed{\frac{C(s)}{R(s)} = \frac{\omega_n^2 (K_p + K_d s)}{s^2 + s(2\zeta\omega_n + \omega_n^2 K_d) + \omega_n^2 K_p}}$$

$2\zeta\omega_n s\left(\zeta + \frac{\omega_n K_d}{2}\right)$

where $K_d = K_p T_s$ & $\zeta' = \left(\zeta + \frac{\omega_n K_d}{2}\right)$

→ PD - controller introduces a zero in the s/m & increases the damping ratio. The addition of zero may increase the peak overshoot & reduce the rise time.

→ But ↑se in the damping ratio, ↓se the peak - overshoot & hence compensated.

$$\left[\because M_p = e^{-\zeta\pi/\sqrt{1-\zeta^2}} \right]$$
$$\& \ t_p = \pi/\omega_d$$

→ There is no increase in the type no. of the s/m in PD- controller. Hence PD- controller will not modify the steady state error.

pblms:

1) Find out the position, velocity and acceleration error co-efficients for the following unity fb s/ms having forward loop transfer function G(s) as

$$\frac{K}{s^2(s^2+8s+100)}$$

soln

$$G(s) = \frac{K}{s^2(s^2+8s+100)}.$$

$$H(s) = 1$$

$$K_p = \lim_{s\to 0} G(s)H(s).$$

$$= \lim_{s\to 0} \frac{K}{s^2(s^2+8s+100)}$$

$$\boxed{K_p = \infty}$$

$$K_v = \lim_{s\to 0} sG(s)H(s)$$

$$= \lim_{s\to 0} \frac{K}{s(s^2+8s+100)}$$

$$\boxed{K_v = \infty.}$$

$$K_a = \lim_{s\to 0} \frac{K}{(s^2+8s+100)}$$

$$\boxed{K_a = K/100.}$$

2) A unity fb s/m has $G(s) = \frac{10}{(s+1)}$. Find the steady state error and the generalized error co-efficient for $r(t) = t$.

Soln.

$$C_0 = \frac{1}{1+K_p}$$

$$C_1 = \frac{1}{K_v}$$

$$C_2 = \frac{1}{K_a}$$

$$K_p = \lim_{s \to 0} G(s)\,H(s)$$

$$= \lim_{s \to 0} \frac{10}{s+1}$$

$$\boxed{K_p = 10}$$

$$K_v = \lim_{s \to 0} s\,G(s)\,H(s)$$

$$K_v = 0.$$

$$K_a = \lim_{s \to 0} s^2 G(s)\,H(s)$$

$$\boxed{K_a = 0.}$$

$$\boxed{e_n = \lim_{s \to 0} \frac{d^n}{ds^n} F(s)}$$

$$F(s) = \frac{1}{1+G(s)\,H(s)}$$

$$= \frac{1}{1+\left(\frac{10}{s+1}\right)}$$

$$F(s) = \frac{(s+1)}{(s+11)}.$$

$$C_0 = \lim_{s \to 0} F(s) \Rightarrow \boxed{\frac{1}{11} = C_0}$$

$$C_1 = \lim_{s \to 0} \frac{d}{ds}\left(\frac{s+1}{s+11}\right) = \lim_{s \to 0} \frac{(s+11)(1) - (s+1)(1)}{(s+11)^2}$$

$$= \lim_{s \to 0} \frac{\cancel{s}+11-\cancel{s}-1}{(s+11)^2} = \lim_{s \to 0} \frac{10}{(s+11)^2}$$

$$\boxed{c_1 = \frac{10}{121}}$$

$$c_2 = \lim_{s \to 0} 10\left(-2 \times (s+11)^{-3}\right)$$

$$= \lim_{s \to 0} \frac{-20}{(s+11)^3}$$

$$= \frac{-20}{(11)^3}$$

$$\boxed{c_2 = -20/11^3.}$$

Steady state error

$$e_{ss} = \lim_{s \to 0} s\,E(s) \qquad E(s) = \frac{R(s)}{1+G(s)H(s)}$$

$$= \lim_{s \to 0} \frac{\cancel{s}(1/s^{\cancel{2}})}{1+\frac{10}{(s+1)}}$$

$$= \lim_{s \to 0} \frac{1\,(s+1)}{s(s+1+10)}$$

$$= \lim_{s \to 0} \frac{(s+1)}{s(s+11)}.$$

$$\boxed{e_{ss} = \infty}$$

3) The open loop tfr. function of a unity fb ctrl s/m is $G(s) = \frac{9}{(s+1)}$, using the generalized error series determine the error signal and steady state error of the s/m when the s/m is excited by $3t^2/2$.

soln.

$$G(s) = \frac{9}{(s+1)}$$

$$H(s) = 1$$

$$r(t) = \frac{3t^2}{2} \Rightarrow R(s) = \frac{3}{2} \times \frac{2!}{s^3} = \frac{3}{s^3}.$$

$$\boxed{e(t) = c_0\, r(t) + c_1 r'(t) + \frac{c_2}{2!} r''(t) + \dots}$$

$$c_0 = \lim_{s \to 0} F(s) = \lim_{s \to 0} \frac{1}{1+G(s)H(s)}$$

$$= \lim_{s \to 0} \frac{1}{1 + \left(\frac{9}{s+1}\right)}$$

$$= \lim_{s \to 0} \frac{s+1}{s+10}.$$

$$\boxed{c_0 = \frac{1}{10}.}$$

$$c_1 = \lim_{s \to 0} \frac{d}{ds} F(s) \Rightarrow \lim_{s \to 0} \frac{(s+10)-(s+1)}{(s+10)^2}$$

$$= \lim_{s \to 0} \frac{9}{(s+10)^2}$$

$$\boxed{c_1 = \frac{9}{100}.}$$

$$c_2 = \lim_{s \to 0} \frac{9(-2)}{(s+10)^3}$$

$$\boxed{c_2 = -\frac{18}{10^3}}$$

$r(t) = \frac{3t^2}{2}$ $\quad 6t$

$r'(t) = \frac{3}{\cancel{2}} \times \cancel{2}t = 3t$

$r''(t) = 3.$

$$e(t) = \frac{1}{10} \times \frac{3t^2}{2} + \frac{9}{100} \times 3t + \left(-\frac{\cancel{18}^{9}}{10^3} \times \frac{1}{\cancel{2}} \times 3\right)$$

$$e(t) = \frac{3}{20} t^2 + \frac{27}{100} t - \frac{27}{10^3}$$

$$\boxed{e(t) = 0.15t^2 + 0.27t - 0.027}$$

$$e_{ss} = \lim_{t \to \infty} e(t)$$

$$\boxed{e_{ss} = \infty}$$