Unit - 4

Impulse Response & Stability :-

→ In a stable s/m, the response or o/p is predictable, finite & stable for a given i/p.

→ The different definitions of the stability are the following (ie) the s/m is stable if,

(a) If the o/p is ~~stable~~ bounded for the bounded i/p, then the s/m is stable.

(b) Asymptotically stable, if the o/p tends to zero when the i/p is zero.

(c) For a bounded disturbing i/p signal, if the o/p is tends to zero as $t \to \infty$, then the s/m is stable.

(d) the s/m is unstable, if the i/p is bounded disturbing signal & the o/p is oscillatory with infinite amplitude.

(e) If the o/p has constant oscillatory amplitude for the bounded i/p signal, then the s/m may be stable or unstable. Such s/ms are called as limitedly stable.

f) If a s/m o/p is stable for all variations of its parameters, then the s/m is absolutely stable.

g) If a s/m o/p is stable for a limited range of variations of its parameters, then the s/m is called conditionally stable s/m.

→ Impulse Response of a s/m :-

$$CLTF = \frac{C(s)}{R(s)} = M(s)$$

$$C(s) = M(s)\,R(s)$$

$$c(t) = L^{-1}\left[M(s)\,R(s)\right]$$

Impulse response ⇒ $c(t) = L^{-1}\left[M(s)\right]$

$\left[\because r(t) = \delta(t) \;\&\; R(s) = 1\right]$

$\therefore \boxed{c(t) = m(t)}$ ⇒ Impulse response.

→ $C(s) = M(s)\,R(s)$.

By property of convolution,

$$c(t) = \left[m(t) * r(t)\right]$$

$$c(t) = \int_{-\infty}^{\infty} m(\tau)\, r(t-\tau)\, d\tau$$

→ Therefore, for any arbitrary i/p, the o/p is obtained by the convolution of i/p & impulse response.

→ **BIBO - stability :-**

$$c(t) = \int_0^\infty m(\tau) r(t-\tau)\, d\tau. \Rightarrow$$ Relaxed s/m (i.e.) initial condn are zero.

$$|c(t)| = \left| \int_0^\infty m(\tau) r(t-\tau) d\tau \right|.$$

$$= \int_0^\infty |m(\tau)|\, |r(t-\tau)| d\tau.$$

$$= \int_0^\infty |m(\tau)|\, A_1\, d\tau.$$

∵ for a bounded i/p, $|r(t-\tau)| < \infty$

∴ $|r(t-\tau)| = A_1 \Rightarrow$ finite value,

$$|c(t)| = \int_0^\infty |m(\tau) A_1\, d\tau.$$

for a bounded o/p, $|c(t)| \leq A_2 < \infty$

$$\boxed{\therefore A_1 \int_0^\infty |m(\tau)|\, d\tau \leq A_2 < \infty}$$

→ Hence for bounded o/p,

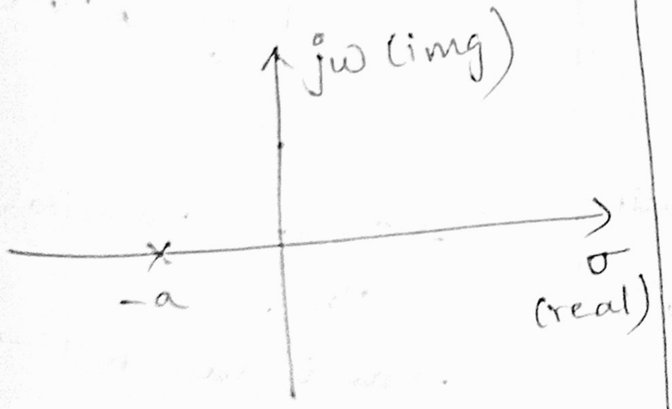
$$\int_0^\infty |m(t)|\, dt < \infty .$$

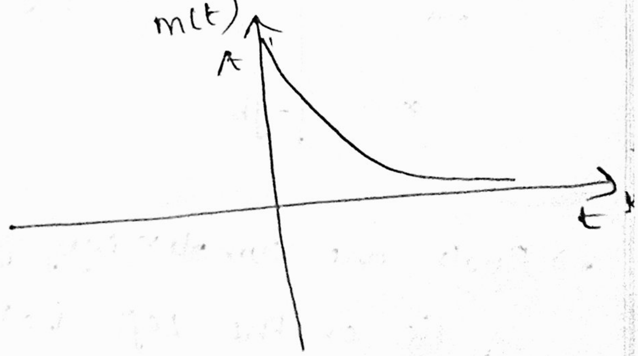
→ The above condt. is satisfied iff the impulse response is absolutely integrable (ie) $\int_0^\infty |m(t)|\,dt$ is finite & hence the area under the curve from $t = 0$ to ∞ is finite.

Location of poles on s-plane for stability.

$$M(s) = \frac{(s+z_1)(s+z_2)\cdots(s+z_m)}{(s+P_1)(s+P_2)\cdots(s+P_n)}.$$

Transfer function & location of poles $M(s)$	Impulse response $m(t)$
1) $\boxed{M(s) = \dfrac{A}{s+a}}$	$\boxed{\begin{aligned} m(t) &= L^{-1}[M(s)] \\ &= Ae^{-at} \end{aligned}}$





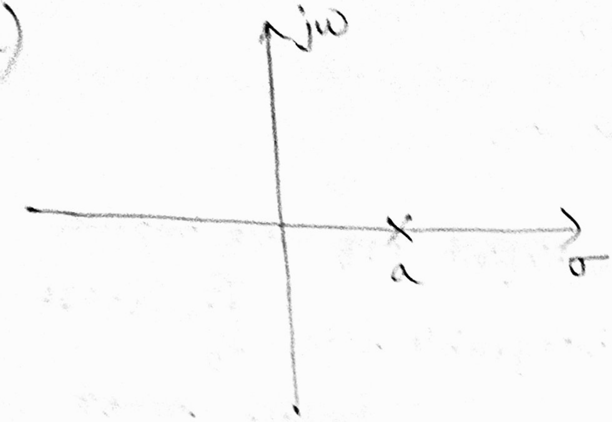
→ Root on negative real axis.

→ exponentially decaying

as $t \to \infty$, $m(t) = 0$ (finite)

∴ stable s/m.

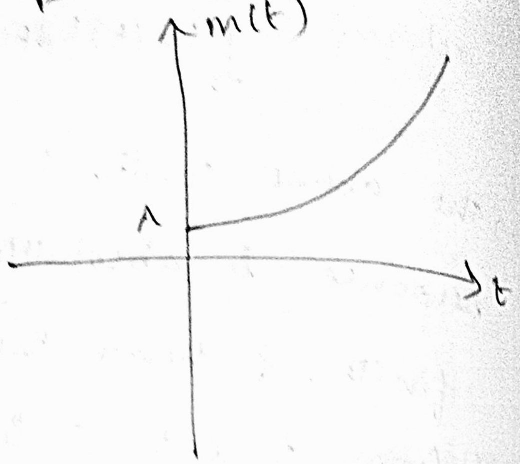
2)



$$\boxed{M(s) = \frac{A}{s-a}}$$

→ Root on positive real axis

$$\boxed{m(t) = A e^{at}}$$



→ Exponentially increasing signal as $t \to \infty$, $m(t) \to \infty$

∴ unstable s/m.

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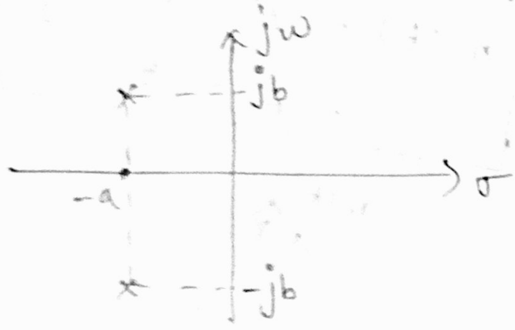
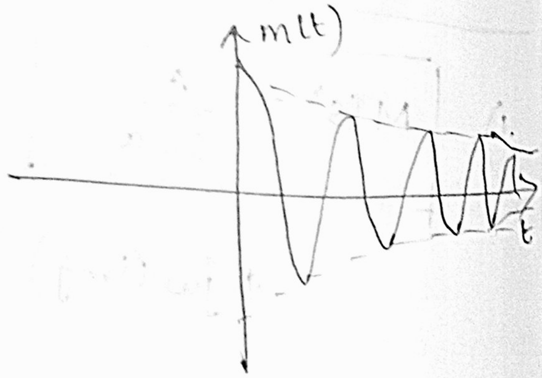
3) $$\boxed{M(s) = \frac{A}{s+a+jb} + \frac{A^*}{s+a-jb}}$$



→ Roots are complex conjugate & lie on the left half of s-plane.

$$\boxed{m(t) = A e^{-(a+jb)t} + A^* e^{-(a-jb)t}}$$

$$= A'\left(2e^{-at}\cos bt\right)$$

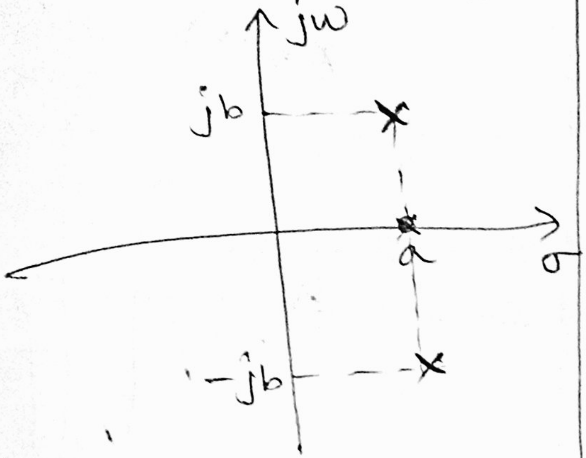


→ Damped sinusoidal s/m.

→ as $t \to \infty$, $m(t) = 0$.

→ ∴ s/m is stable.

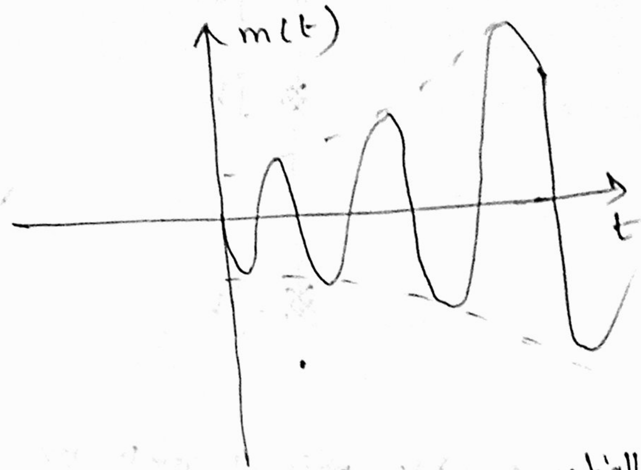
4) $M(s) = \dfrac{A}{s-a+jb} + \dfrac{A^*}{s-a-jb}$



→ Roots are complex conj, & lie on the Right half of s-plane.

$m(t) = A e^{(a-jb)t} + A^* e^{(a+jb)t}$

$= 2A' e^{at} \cos bt$.

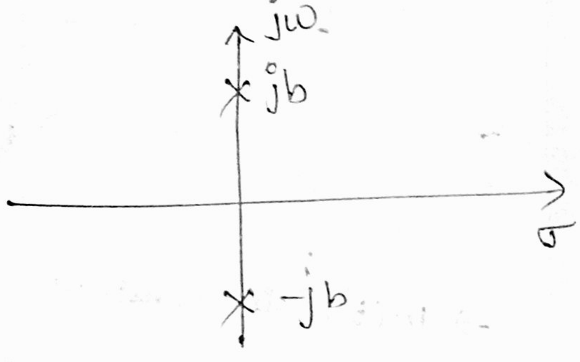


→ Undamped exponentially increasing sinusoidal signal.

→ $t \to \infty$, $m(t) \to \infty$.

→ ∴ s/m is unstable.

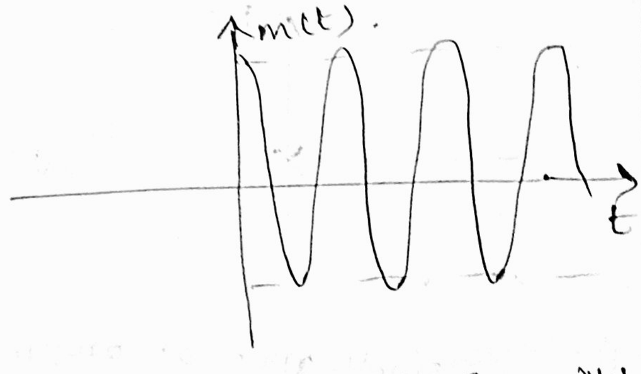
5) $M(s) = \dfrac{A}{s+jb} + \dfrac{A^*}{s-jb}$



→ single pair of roots on img. axis.

$m(t) = A e^{-jbt} + A^* e^{jbt}$

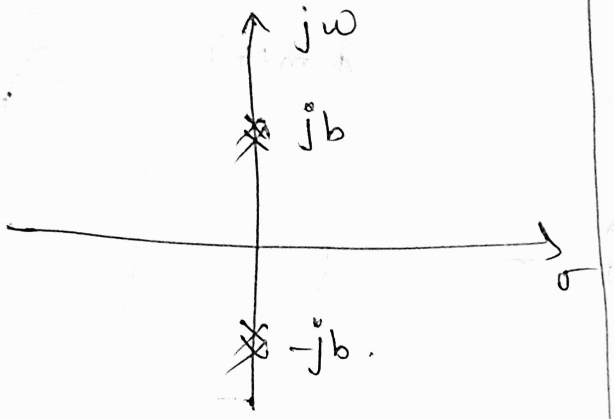
$= 2A' \cos bt$.



→ Impulse response is oscillatory with constant amplitude.

→ ∴ s/m is marginally stable.

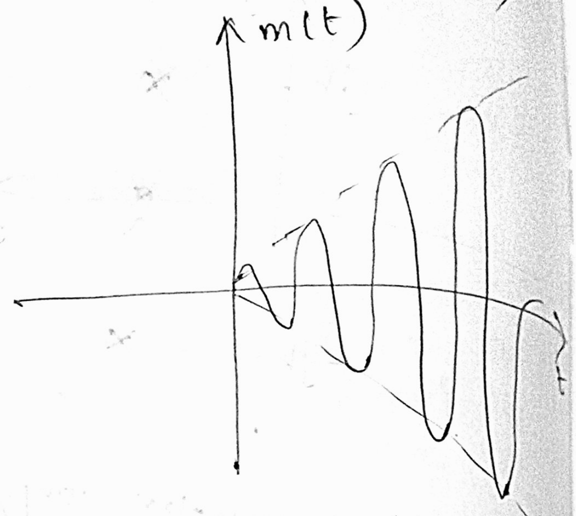
6) $M(s) = \dfrac{A}{(s+jb)^2} + \dfrac{A^*}{(s-jb)^2}$



→ Double pair of roots on img. axis

$m(t) = A t e^{-jbt} + A^* t e^{jbt}$

$m(t) = 2A' t (\cos bt)$



→ m(t) is linearly ↑ing sinusoidal
As $t \to \infty$, $m(t) \to \infty$.

→ ∴ s/m is unstable.

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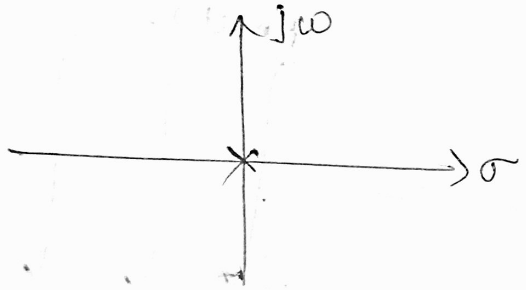
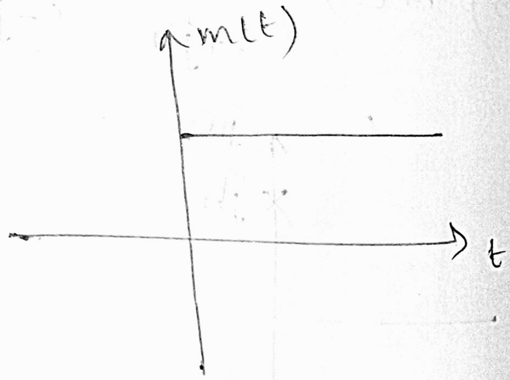
7) $M(s) = \dfrac{A}{s}$



→ single root at origin

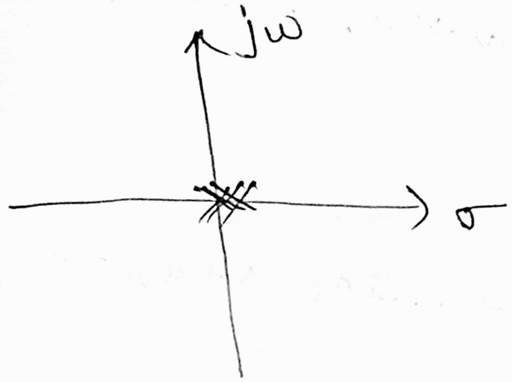
$m(t) = A$



→ m(t) is constant.

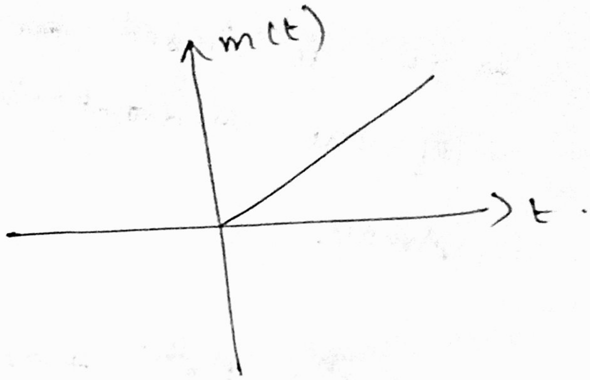
→ ∴ s/m is marginally stable.

8) $M(s) = \frac{A}{s^2}$



→ Double pole (roots) at origin.

$m(t) = At$.



→ m(t) ⇒ linearly increasing

→ As $t \to \infty$

$m(t) \to \infty$.

→ ∴ s/m is unstable.

→ Note :-

→ If all the co-efficients are positive & if no co-eff is zero, then all the roots are in the left half of s-plane.

→ If any co-eff. is equal to zero, then some of the roots may be on the imaginary axis. ~~or on the right h~~

→ If any co-efficient is negative then atleast one root is in the RH-splane.

Hence, the absence or negativeness of any of the co-efficients of a char. polynomial indicates that the s/m is either marginally stable or unstable.

Roots with negative Real parts $\xrightarrow{\text{always true}}$ All co-eff are positive

All co-eff are positive → Roots with negative Real parts: not always true [depends on img. part & Real part] $(0 \pm j\omega)$

Routh-Hurwitz Criterion:-

→ It is based on ordering the co-efficients of the characteristic equation, called as Routh Array.

→ $a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \dots + a_{n-1} s + a_n = 0$

where $a_0 > 0$.

s^n :	a_0	a_2	a_4	a_6	.....
s^{n-1} :	a_1	a_3	a_5	a_7	.....
s^{n-2} :	b_0	b_1	b_2	b_3	.....
s^{n-3} :	c_0	c_1	c_2	c_3	.....
⋮	⋮				
s^1 :	g_0				
s^0 :	h_0				

→ $b_0 = (-1)\begin{vmatrix} a_0 & a_2 \\ a_1 & a_3 \end{vmatrix} = \dfrac{a_2 a_1 - a_0 a_3}{a_1}$

$$b_1 = \frac{a_1 a_4 - a_0 a_5}{a_1}$$

$$b_2 = \frac{a_1 a_6 - a_0 a_7}{a_1}$$

$\rightarrow$ $c_0 = \dfrac{a_3 b_0 - a_1 b_1}{b_0}$.

$c_1 = \dfrac{b_0 a_5 - a_1 b_2}{b_0}$.

⋮

→ In the processing of contructing Routh array the missing terms are considered as zero.

→ The Routh stability criterion:-

→ All the elements in the first column of the Routh table should be positive, for the stable system.

→ If this condition is not met, the s/m is unstable and the no. of sign changes in the elements of the first column of the Routh array corresponds to the number of roots of char., eqn., in the right half of s-plane.

→ Cases of Routh array.

(1) Normal Routh Array.

(2) A row of all zeros.

(3) First element of a row is zero but some or other elements are not zero.

Eg problem

case (i) - Normal Routh array

1) Using Routh criterion, determine the stability of the s/m represented by the char. eqn.

$s^4 + 8s^3 + 18s^2 + 16s + 5 = 0$. comment on the location of the roots. & ~~stability~~

soln :)

$s^4 + 8s^3 + 18s^2 + 16s + 5 = 0$.

s^4	1	18	5
s^3	8	16	

s^4 :	1	18	5
s^3 :	1	2	
s^2 :	16	5	
s^1 :	1.7	0	
s^0 :	5		

$s^2 : \dfrac{8\times18-16}{8}$

$s^2 : \dfrac{1\times18-2\times1}{1} = 16$

$\dfrac{1\times5-0\times1}{1} = 5$

$s^1 : \dfrac{16\times2-5\times1}{16} = 1.665$

$\simeq 1.7$

$s^0 : \dfrac{1.7\times5-0\times16}{1.7}$

→ No change in sign in the 1st column of Routh array.

∴ All roots (4) are present on the L.H of the s plane & the s/m is stable.

Case (ii) - A row of all zeros.

2) Construct Routh-array and determine the stability of the s/m whose char. eqn is $s^6 + 2s^5 + 8s^4 + 12s^3 + 20s^2 + 16s + 16 = 0$. Also determine the no. of roots lying on right half of s-plane, left half of s-plane and on img. axis.

soln

s^6	1	8	20	16
s^5	2	12	16	

s^6	1	8	20	16
s^5	1	6	8	—
s^4	$\boxed{\cancel{2} \quad \cancel{12} \quad \cancel{16}}$ 1	6	8	—
s^3	0	0		
s^3	1	3		
s^2	3	8		
s^1	0.33	0		
s^0	8			

→ All zeros in s^3.

→ Auxillary eqn. A

$A = s^4 + 6s^2 + 8$

$\frac{dA}{ds} = 4s^3 + 12s.$

s^3	4	12
	$\boxed{1}$	$\boxed{3}$

→ There is no sign-change, but row with all zeros indicate the possibility of roots on Imaginary axis.

→ Hence the s/m is limitedly or
marginally stable.

→ Roots calculation

Auxillary polynomial.

$$s^4 + 6s^2 + 8 = 0.$$

$$s^2 = x$$

$$x^2 + 6x + 8 = 0$$

$$x = -3 \pm 1$$

$$x = -2 \ \& \ -4$$

$$s = \pm \sqrt{x}$$

$$= \pm\sqrt{-2} \ \& \ \pm\sqrt{-4}$$

$$= \pm j\sqrt{2} \ \& \ \pm j2.$$

$$s = +j\sqrt{2}, -j\sqrt{2}, +j2, -j2.$$

→ Out of 6 roots, 4 roots are present on the img. axis & the remaining 2 roots present on the L-H of S-plane.

→ No-roots present on R-H of S-plane.

case iii) First element of a row is zero.

3) Construct Routh array and determine the stability of the s/m represented by the char. eqn $s^5 + s^4 + 2s^3 + 2s^2 + 3s + 5 = 0$. Comment on the location of the roots of char. eqn.

Soln.

s^5	1	2	3
s^4	1	2	5
s^3	ε	-2	—
s^2	$\frac{2\varepsilon+2}{\varepsilon}$	5	—
s^1	$\frac{\varepsilon\left[\left(\frac{2\varepsilon+2}{\varepsilon}\right)(-2) - 5\varepsilon\right]}{(2\varepsilon+2)}$	—	—
s^0	5		

$s^3 \quad 0 \quad -2$

ε

$s^1 = \frac{-4\varepsilon - 4 - 5\varepsilon^2}{\varepsilon}$

$$\boxed{s^1 = \frac{-4\varepsilon - 4 - 5\varepsilon^2}{2\varepsilon + 2}}$$

sub $\varepsilon = 0$.

s^5	1	2	3
s^4	1	2	5
s^3	0	-2	
s^2	∞	5	
s^1	-2		
s^0	5		

No. of sign change = 2

$\therefore$ No. of Roots on RH of s plane is 2

Also the ~~stab~~ system is unstable.

4) char. eqn: $s^7 + 9s^6 + 24s^5 + 24s^4 + 24s^3 + 24s^2 + 23s + 15 = 0$.

Determine the location of roots on s-plane and hence the stability of the s/m.

s^7	1	24	24	23
s^6	9	24	24	15

s^7	1	24	24	23
s^6	3	8	8	5
s^5	21.33	21.33	21.33	
s^5	1	1	1	
s^4	5	5	5	
	1	1	1	
s^3	0	0	0	

Auxillary eqn. $s^4 + s^2 + 1 = A$.

$$\frac{dA}{ds} = 4s^3 + 2s$$

s^7	1	24	24	3
s^6	3	8	8	5
s^5	1	1	1	
s^4	1	1	1	
s^3	4	2		
s^2	0.5	1		
s^1	-6			
s^0	1			

No. of sign change = 2

$\therefore$ No. of Roots on R.H of s plane = 2

$\therefore$ s/m is unstable.

RH - roots from Auxillary eqn.

$$s^4 + s^2 + 1 = 0.$$

$$s^2 = x \Rightarrow x^2 + x + 1 = 0.$$

$$x = \frac{-1 \pm \sqrt{1-4}}{2}$$

$$= \frac{-1 \pm \sqrt{-3}}{2}$$

$$= \frac{-1 \pm \sqrt{3}j}{2} \Rightarrow -0.5 \pm j0.866$$

out of 7 roots $\Rightarrow$ 2 roots on RH of s-plane
5 roots on LH of s-plane

No roots present on img. axis.

<u>Note:</u> In Ist column of Routh-Array

→ If there is a sign change $\Rightarrow$ <u>S/m is unstable</u> and no. of sign change is equal to the no. of roots present on R-H of s-plane.

→ If there is no sign change & no row with all zeros $\Rightarrow$ <u>S/m is stable</u> and all roots are present on L-H of s-plane.

→ If there is a sign change & a row with all zeros $\Rightarrow$ some roots on RH & some roots on LH but no roots on img. axis. <u>S/m is unstable</u>

→ If 1st column consists of ∞, check only for the sign change & No comments on the presence of ∞.

→ If there is no sign change, but a row with all zeros, check for the auxillary roots & find the location of roots on s-plane.
If roots of auxillary equations are purely img, then the s/m is Marginally Stable

Root locus

→ Technique to adjust the closed loop poles to achieve the desired s/m performance by ~~varied~~ varying one or more s/m parameters.

→ let $G(s) = \dfrac{K}{S(S+P_1)(S+P_2)}$ ⇒ OLTF

for unity fb s/m, $\dfrac{C(s)}{R(s)} = \dfrac{G(s)}{1+G(s)}$

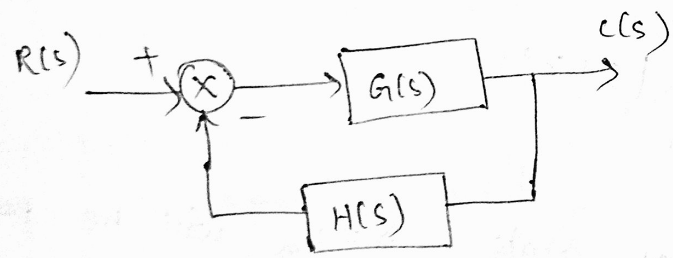
CLTF ⇒ $\dfrac{C(s)}{R(s)} = \dfrac{K}{S(S+P_1)(S+P_2)+K}$.

→ char. eqn $S(S+P_1)(S+P_2)+K=0$.

The 'K' value varied from 0 to ∞.
↳ OL gain

$K=0$ → roots of CLTF ⇒ roots of OLTF

$K=\infty$ → the roots of CLTF ⇒ zeros of OLTF.

→ 

$\dfrac{C(s)}{R(s)} = \dfrac{G(s)}{1+G(s)H(s)}$.

char. eqn. $1+G(s)H(s)=0$.

$G(s)H(s) = -1$.

→ By Evans condt.,

i) $|G(s)H(s)| = 1$. ⇒ Magnitude criterion.

ii) $\angle G(s)H(s) = \pm 180^\circ(2q+1)$, $q = 0, 1, 2, 3, \ldots$

↳ angle criterion.

→ $$|G(s)H(s)| = \frac{K\,|s+z_1| \times |s+z_2| \times |s+z_3| \ldots}{|s+P_1| \times |s+P_2| \times |s+P_3| \ldots}$$

$$= \frac{K \prod_{i=1}^{m} |s+z_i|}{\prod_{i=1}^{n} |s+P_i|}.$$

→ $|G(s)H(s)| = 1$

$$\therefore \frac{K \prod_{i=1}^{m} |s+z_i|}{\prod_{i=1}^{n} |s+P_i|} = 1.$$

Also $$K = \frac{\prod_{i=1}^{n} |s+P_i|}{\prod_{i=1}^{m} |s+z_i|}$$

→ The magnitude criterion states $s = s_a$ will be point on root locus for which $|G(s)H(s)|$ is equal to 1.

$\rightarrow \angle G(s)H(s) = \sum_{i=1}^{m} \angle(s+z_i) - \sum_{i=1}^{n} \angle(s+P_i)$

$\because \angle G(s)H(s) = \pm 180^\circ (2q+1)$.

$$\sum_{i=1}^{m} \angle(s+z_i) - \sum_{i=1}^{n} \angle(s+P_i) = \pm 180^\circ(2q+1)$$

$\rightarrow \therefore$ At $s = s_a$, will be a point on root locus if the angle criterion satisfies.

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construction of Root locus:-

1) The root locus is symmetrical about the real axis.

2) Each branch of the root locus originates from an open-loop pole corresponding to $K=0$ and terminates at either finite open loop zero $(K<\infty)$ or infinite open loop zero $(K=\infty)$.

$\rightarrow$ The no. of branches of the root-locus terminating on infinity is equal to $n-m$.

$n \rightarrow$ no. of open-loop poles.

$m \rightarrow$ no. of finite zeros.

3) Segments of the real axis having an odd no. of real axis open-loop poles plus zeros to their right are parts of the root locus.

4) The $n-m$ root locus branches that tend to infinity, do so along st. line asymptotes making angles with the real axis given by,

$$\boxed{\phi_A = \frac{180^\circ(2q+1)}{n-m}}\ ,\ q = 0, 1, \ldots n-m.$$

5) The point of inter-section of the asymptotes with the real axis is at $s = \sigma_A$ where,

$$\boxed{\sigma_A = \frac{\left(\text{sum of poles}\right) - \left(\text{sum of zeros}\right)}{n-m}.}$$

6) The breakaway and break-in points of RL are determined from the roots of the eqn.

$\frac{dk}{ds} = 0$. If 'r' no. of branches of root locus meet at a point, then they break away at an angle of $\pm 180^\circ/r$.

7) Angle of departure from a complex open-loop pole is given by,

$$\boxed{\phi_p = \pm 180^\circ(2q+1) + \phi}$$

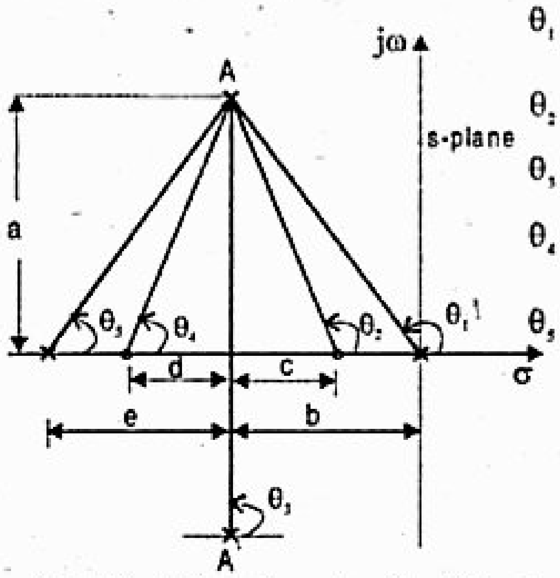
$\phi \rightarrow$ net angle (All ~~Ol~~ poles & zeros)

Angle of ~~Departure~~ Arrival, of a open-loop zero,

$$\boxed{\phi_r = \pm 180^\circ(2q+1) + \phi}$$

$$\left.\begin{array}{l}\text{Angle of Departure}\\ \text{(from a complex pole A)}\end{array}\right\} = 180^\circ - \left(\begin{array}{l}\text{Sum of angles of vector to the}\\ \text{complex pole A from other poles}\end{array}\right) + \left(\begin{array}{l}\text{Sum of angles of vectors to the}\\ \text{complex pole A from zeros}\end{array}\right)$$

Note : *The angles can be calculated as shown in fig 4.9 or they can be measured using protractor.*



$$\theta_1 = 180^\circ - \tan^{-1}\frac{a}{b}$$

$$\theta_2 = 180^\circ - \tan^{-1}\frac{a}{c}$$

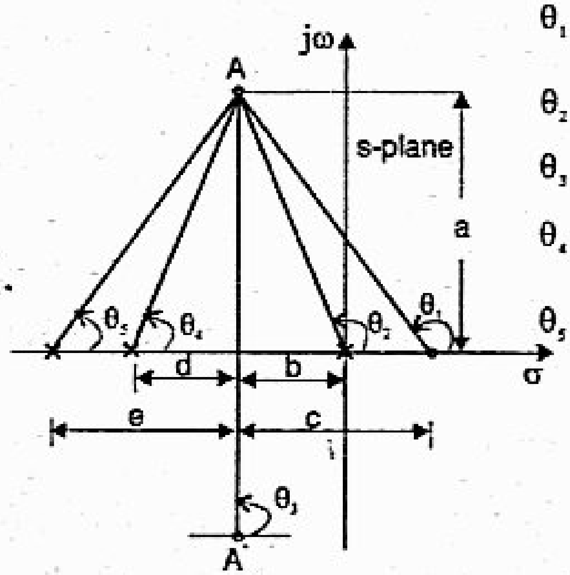
$$\theta_3 = 90^\circ$$

$$\theta_4 = \tan^{-1}\frac{a}{d}$$

$$\theta_5 = \tan^{-1}\frac{a}{e}$$

$$\left.\begin{array}{l}\text{Angle of arrival at a}\\ \text{complex zero A}\end{array}\right\} = 180^\circ - \left(\begin{array}{l}\text{Sum of angles of vectors to the}\\ \text{complex zero A from all other zeros}\end{array}\right) + \left(\begin{array}{l}\text{Sum of angles of vectors to the}\\ \text{complex zero A from poles}\end{array}\right)$$

Note : The angles can be calculated as shown in fig 4.10 or they can be measured using protractor.



$$\theta_1 = 180^\circ - \tan^{-1}\frac{a}{b}$$

$$\theta_2 = 180^\circ - \tan^{-1}\frac{a}{c}$$

$$\theta_3 = 90^\circ$$

$$\theta_4 = \tan^{-1}\frac{a}{d}$$

$$\theta_5 = \tan^{-1}\frac{a}{e}$$

8) The point of interaction of root locus branches with the img. axis can be determined by use of the Routh criterion.

9) open loop gain K,

$$K = \frac{\prod_{i=1}^{n} |s_a + P_i|}{\prod_{i=1}^{m} |s_a + z_i|}$$

Procedure for constructing Root locus:-

1) locate zeros & poles of $G(s)H(s)$. The RL-branch starts from open loop poles and terminates at zeros.

2) Determine RL on real axis

3) Determine the asymptotes of RL. and the meeting pt.

4) Find break-away & break-in pts.

5) If there is a complex pole then determine the angle of departure from the complex pole. If there is a complex zero then determine the angle of arrival at the complex zero.

6) Find the points where the root loci may cross the img. axis.

7) Take a series of test points and adjust the pts. to satisfy angle criterion.

8) Find K, by mag. criterion.

Eg1. A unity f/b ctrl s/m has an OLTF

$G(s) = \frac{K}{s(s^2+4s+13)}$. sketch the root locus.

Soln

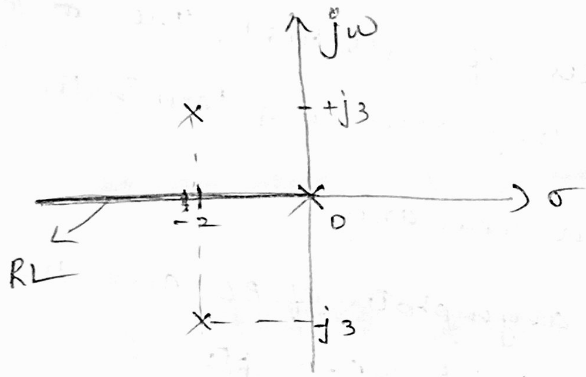
i) To locate poles & zeros.

$s(s^2+4s+13) = 0$.

$s = 0, -2+j3, -2-j3$.

$P_1 = 0, \quad P_2 = -2+j3, \quad P_3 = -2-j3$.

ii) To find the root locus.



select a point on the 'σ' axis, such that total no. of poles & zeros to the right of the point should be an odd no.

iii) To find angle of asymptotes & centroid

$$\text{Angle of Asymptotes} = \frac{\pm 180(2q+1)}{n-m}$$

$n = 3$ (no. of poles) $\qquad q = 0, 1, 2, \ldots n-m$

$m = 0$ (no. of zeros) $\qquad \therefore q = 0, 1, 2, 3$.

$n - m = 3$.

$q=0, \quad \angle A_1 = \frac{\pm 180^\circ}{3} = \pm 60^\circ$

$q=1, \quad \angle A_2 = \pm 180^\circ$

$q=2, \quad \angle A_3 = \pm 300^\circ \Rightarrow \mp 60^\circ.$

$q=3, \quad \angle A_4 = \pm 420^\circ.$

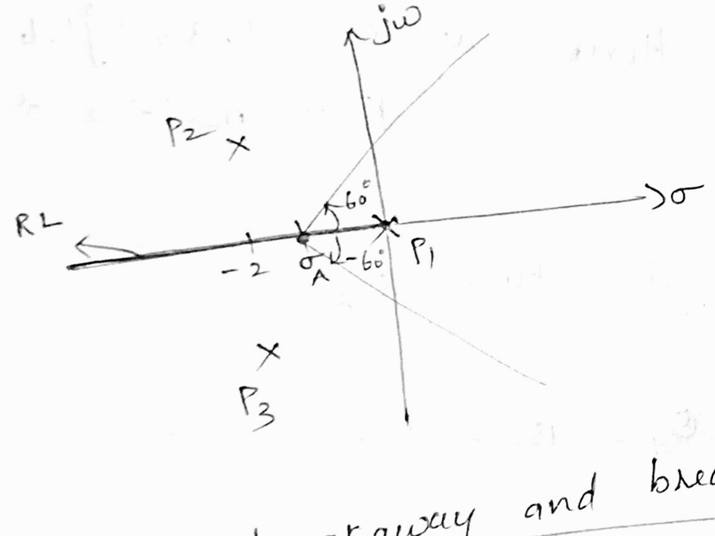
$= \pm(360 + 60^\circ)$

$= \pm 60^\circ.$

$$\text{centroid } (\sigma_A) = \frac{\left(\begin{matrix}\text{sum of}\\ \text{poles}\end{matrix}\right) - \left(\begin{matrix}\text{sum of}\\ \text{zeros}\end{matrix}\right)}{n-m}$$

$$= \frac{0 - 2 + j3 - 2 - j3 - 0}{3}$$

$$\sigma_A = -4/3 = -1.33.$$



(iv) To find the breakaway and breakin pt.

$$CLTF = \frac{G(s)}{1+G(s)H(s)}$$

$$= \frac{K}{s(s^2+4s+13)+K}.$$

$s(s^2+4s+13) + K = 0.$

$$K = -s^3 - 4s^2 - 13s$$

$$\frac{dK}{ds} = 0$$

$$-(3s^2 + 8s + 13) = 0.$$

$$3s^2 + 8s + 13 = 0.$$

$$s = -1.33 \pm j1.6$$

Sub $s = -1.33 \pm j1.6$ in K.

If K is real & positive, then RL has break-away & break-in pts.

But Here at $s = -1.33 \pm j1.6$,
$K \neq$ positive or real.

(v) <u>Angle of departure</u> (θ_A)

$$\theta_1 = 180^\circ - \tan^{-1}(3/2)$$

$$= 123.7^\circ.$$

$$\theta_2 = 90^\circ.$$

$\therefore$ Angle of departure (θ_A) $= 180^\circ - (\theta_1 + \theta_2)$
$= -33.7^\circ.$

Illy for P_3 pole, $\theta_A = +33.7^\circ$.

(vi) crossing pt.

$$s^3 + 4s^2 + 13s + K = 0.$$

$$s = j\omega$$

$$(j\omega)^3 + 4(j\omega)^2 + 13(j\omega) + K = 0.$$

$$-j\omega^3 - 4\omega^2 + 13j\omega + K = 0$$

on equating img. part to zero

$$-\omega^3 + 13\omega = 0$$

$$-\omega^3 = -13\omega$$

$$\omega^2 = 13$$

$$\omega = \pm\sqrt{13}$$

$$\boxed{\omega = \pm 3.6.}$$

$\Downarrow$

crossing pt $\Rightarrow \pm j3.6$

on eqt. real part to zero

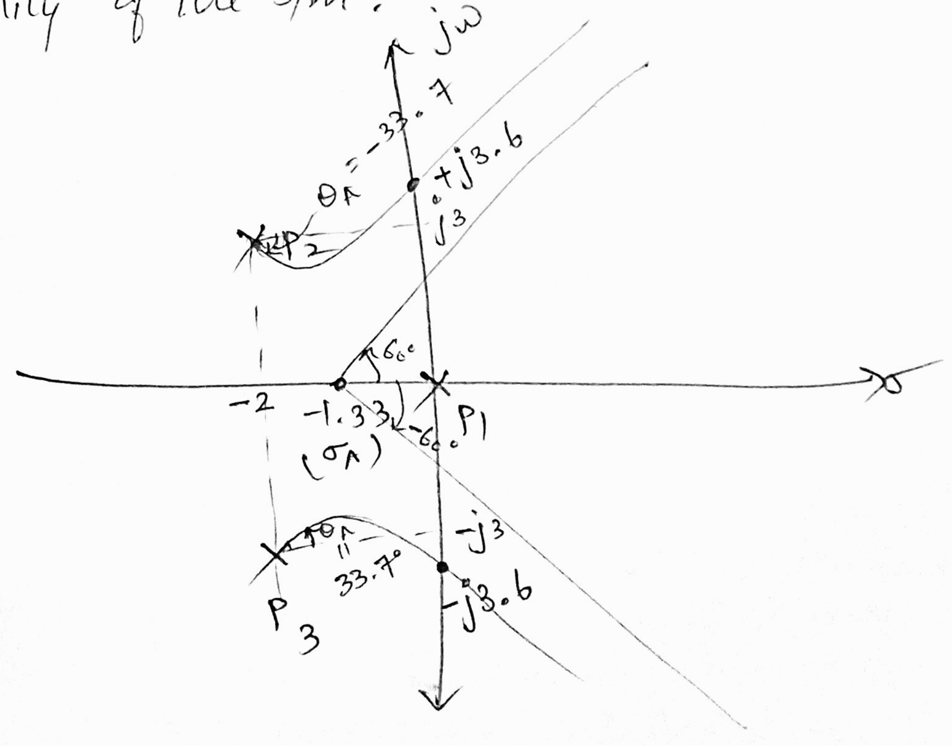
$$-4\omega^2 + K = 0$$

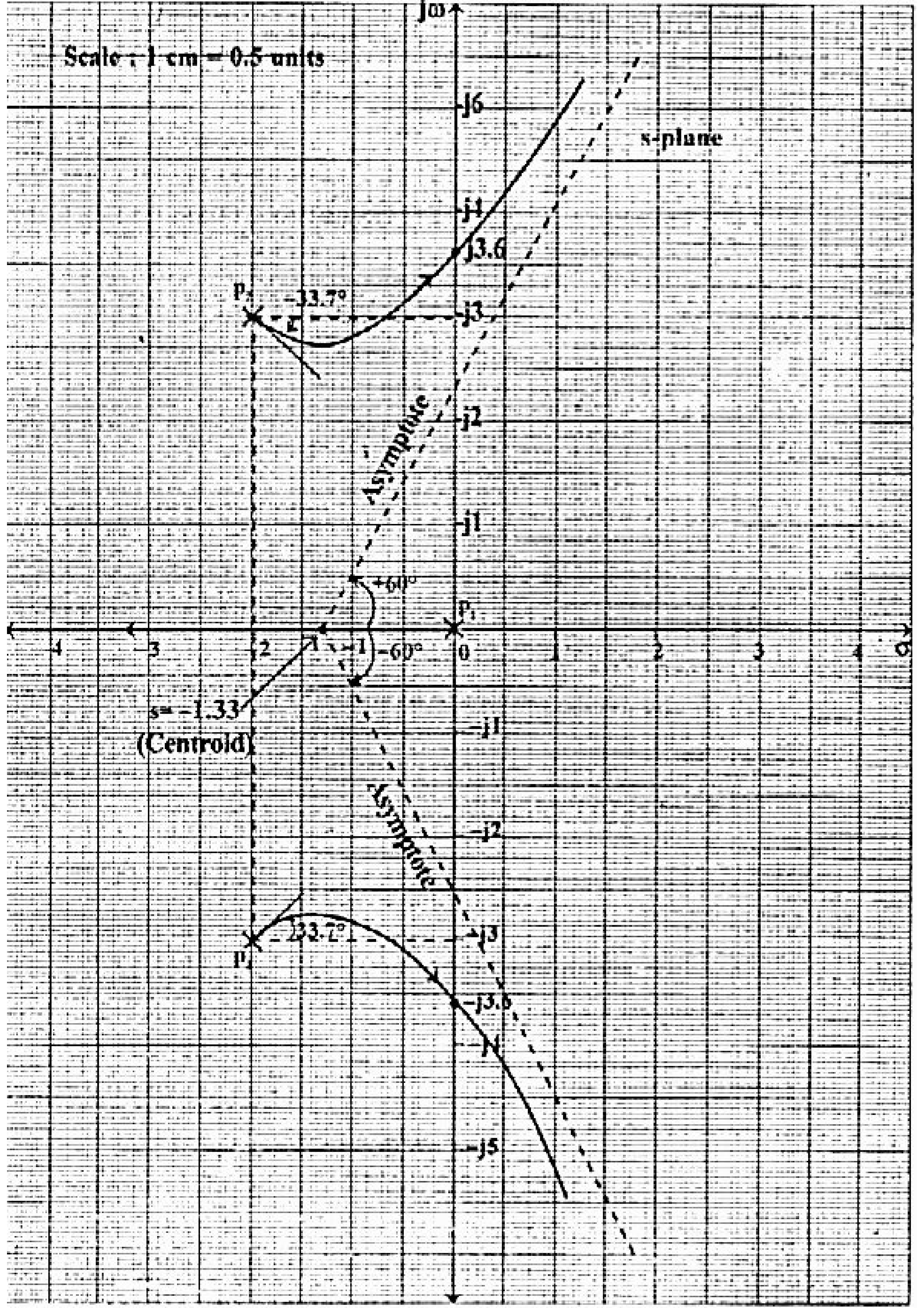
$$K = 4\omega^2$$

sub $\omega^2 = 13$

$$\boxed{K = 52.}$$

→ $K = 52$ is the limiting value of K for the stability of the s/m.





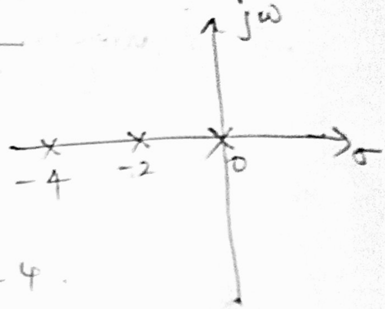
jω
Scale : 1 cm = 0.5 units
j6
s-plane
j4
j3.6
P_2
-33.7°
j3
j2
Asymptote
j1
+60°
P_1
-4
-3
-2
-1
0
1
2
3
4
σ
-60°
s= -1.33
(Centroid)
-j1
Asymptote
-j2
-j3
33.7°
P_3
-j3.6
-j4
-j5

2) Sketch RL for open loop tfr func. $\frac{K}{s(s+2)(s+4)}$

Find the value of K, so that the damping ratio of closed loop s/m is 0.5.

Soln.

(i) to locate poles & zeros of OLTF

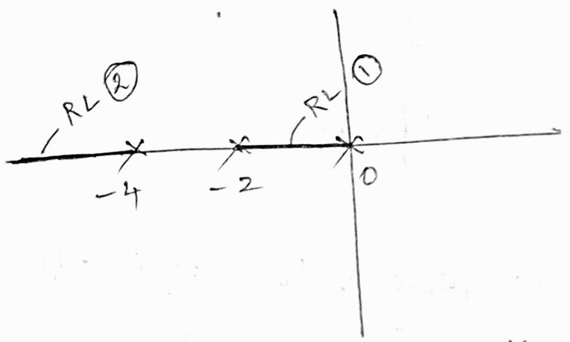


$s(s+2)(s+4)=0$

$s=0, -2, -4.$

$P_1=0, \quad P_2=-2, \quad P_3=-4.$

(ii) To find RL on real axis



Between $s=0$ & $s=-2$, the test pt is selected such that the right of the test pt should have an odd no. poles & zeros (totally)

Between $s=-2$ & $s=-4$, the total no. of OL zeros & poles is even. Hence there is no RL lies between $s=-2$ & -4

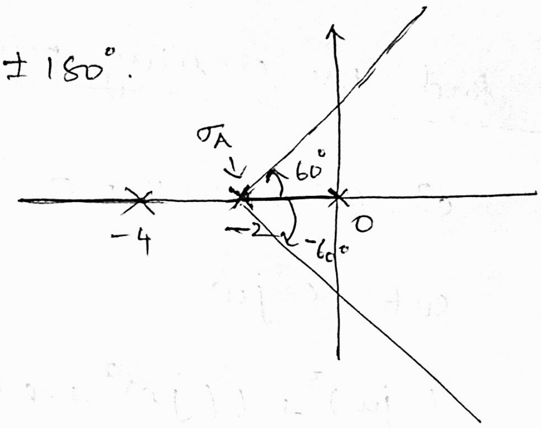
Between $s=-4$ to $-\infty$, the RL is present, since the total no. of poles & zeros to the right of the test point is an odd no.

iii) To find asymptotes & centroid

$$\theta_A = \frac{\pm 180^\circ (2q+1)}{n-m}, \quad q = 0, 1, 2, \dots n-m$$
$$= 0, 1, 2, 3.$$

$q = 0, \quad \theta_A = \pm 60^\circ$

$q = 1, \quad \theta_A = \pm 180^\circ.$



$$\sigma_A = \frac{-6-0}{3}$$

$\sigma_A = -2$

(iv) Breakaway & Break-in points.

$$CLTF = \frac{G(s)}{1+G(s)H(s)} = \frac{K}{s(s+2)(s+4)+K}.$$

Che. eqn. $s(s+2)(s+4) + K = 0$

$K = -s(s+2)(s+4)$

$K = -s^3 - 6s^2 - 8s$

$$\frac{dK}{ds} = 0$$

$-3s^2 - 12s - 8 = 0 \Rightarrow 3s^2 + 12s + 8 = 0.$

$s = -0.845, \quad -3.154.$

Sub s in K

$s = -0.845, \quad K \Rightarrow 3.08 \Rightarrow$ positive & real.

$s = -3.154, \quad K \Rightarrow -ve$ & real

$\therefore \boxed{s = -0.845}$ is the actual breakaway point.

v) To find angle of departure :-

since there are no complex poles or zeros, there is no need to find angle of departure or arrival.

vi) to find the crossing points of img. axis

$s^3 + 6s^2 + 8s + K = 0$

sub $s = j\omega$

$(j\omega)^3 + 6(j\omega)^2 + 8(j\omega) + K = 0$

$-j\omega^3 - 6\omega^2 + j8\omega + K = 0$

Equating img. part = 0

$(-\omega^3 + 8\omega)j = 0$

$\cancel{+}\omega^{\cancel{3}} = \cancel{+}8\cancel{\omega}$

$\omega^2 = 8$

$\omega = \pm\sqrt{8}$

$= \pm 2.8$

crossing pt $= \pm j2.8$

Eqn. Real part = 0.

$K - 6\omega^2 = 0$

$K = 6\omega^2$

$= 6(8)$

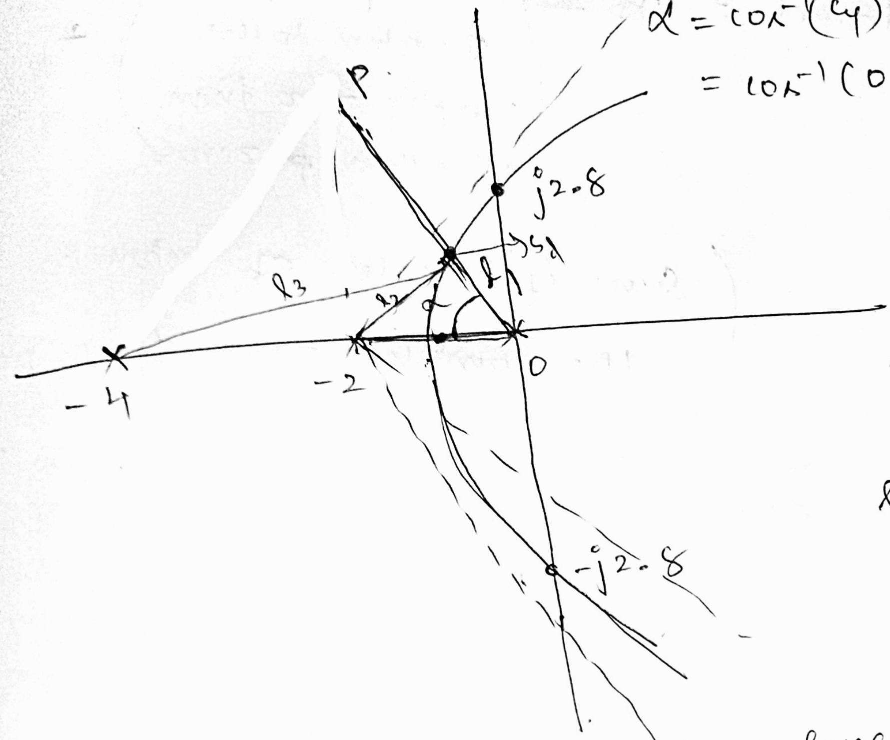
$\boxed{K = 42}$

→ RL ⇒ 3 branches.

i) from $s = -4$ to $-\infty$, RL 1 travels along -ve real axis to meet zero at infinity

ii) Other 2 RLs, start at $s = 0$ & $s = -2$ and travel thro' -ve real axis, & break away at $s = -0.845$ and crosses at $\pm j2.8$ and travels parallel to asymptotes to meet zero at infinity.

find K, for $\zeta = 0.5$



$\alpha = \cos^{-1}(\zeta)$
$= \cos^{-1}(0.5) = 60^\circ$

$l_1 = 2.6 \times 0.5$ cm
$= 1.3$

$l_2 = 3.5 \times 0.5$
$= 1.75$

$l_3 = 7 \times 0.5$
$= 3.5$

$$K = \frac{l_1 \times l_2 \times l_3}{1} = \frac{\text{pdt. of length of vectors from all poles to } s = s_d}{\text{pdt. of length of vectors from all poles to the pt } s = s_d}$$

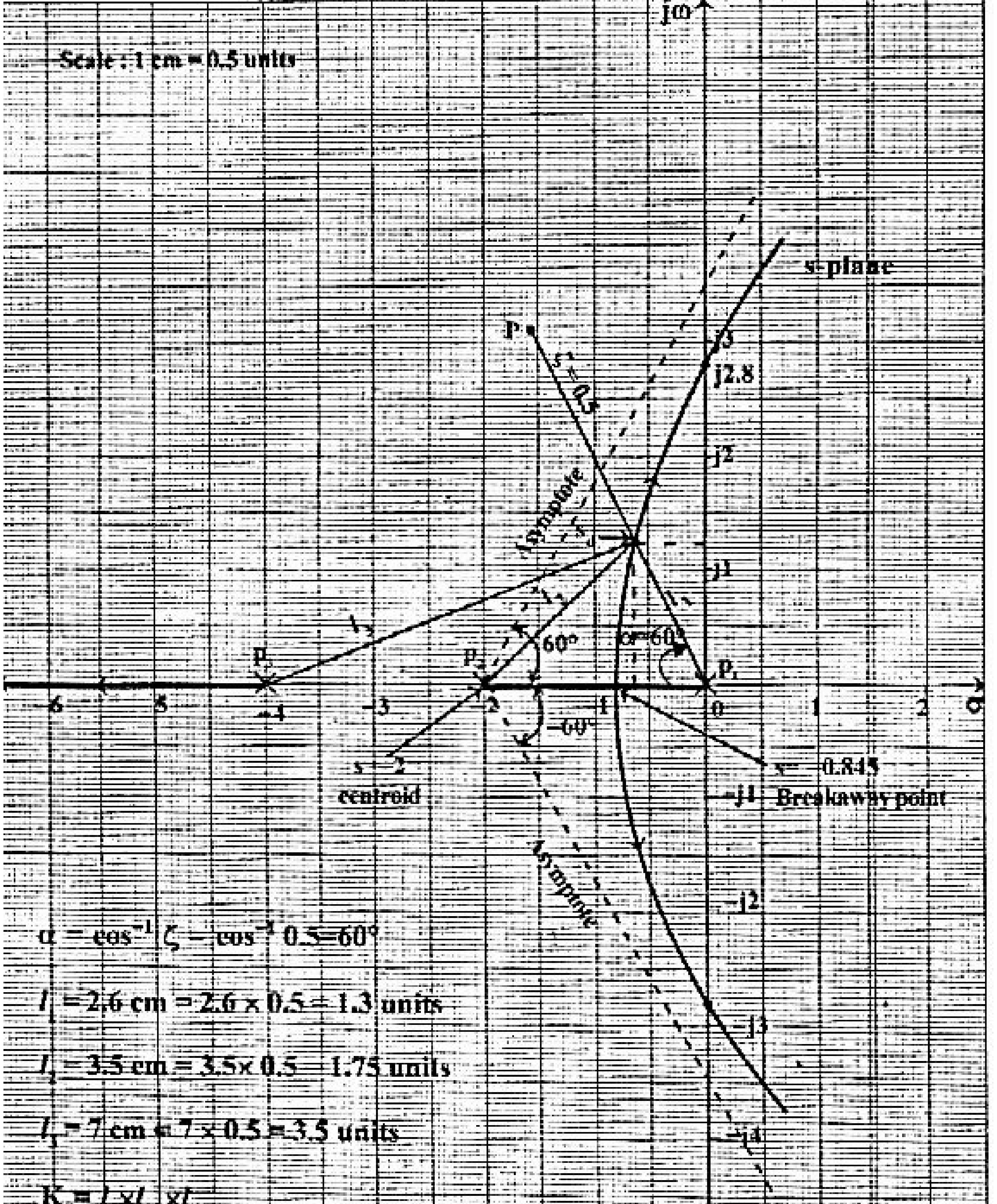
$$= \frac{1.3 \times 1.75 \times 3.5}{1}$$

$= 7.96$

$\simeq \underline{8}$

Angle of Departure :- $180^\circ - \left(\text{sum of angles of vector to the complex pole from other poles}\right)$

$+ \left(\text{sum of angles of vectors to the complex poles from other zeros}\right)$



$\alpha = \cos^{-1} \zeta = \cos^{-1} 0.5 = 60°$

$l_1 = 2.6 \text{ cm} = 2.6 \times 0.5 = 1.3$ units

$l_2 = 3.5 \text{ cm} = 3.5 \times 0.5 = 1.75$ units

$l_3 = 7 \text{ cm} = 7 \times 0.5 = 3.5$ units

$K = l_1 \times l_2 \times l_3$

$= 1.3 \times 1.75 \times 3.5 = 7.96 \approx 8$

Eg. 1) Sketch RL for unity f/b s/m whose OLTF is

$$G(s)H(s) = \frac{K(s+1.5)}{s(s+1)(s+5)}.$$

Soln

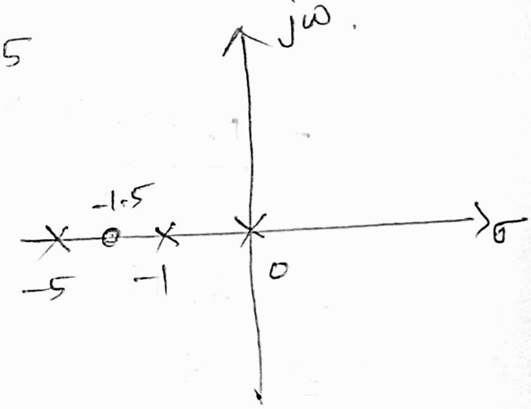
i) To locate zeros & poles.

$s(s+1)(s+5) = 0 \Rightarrow s = 0, -1, -5$

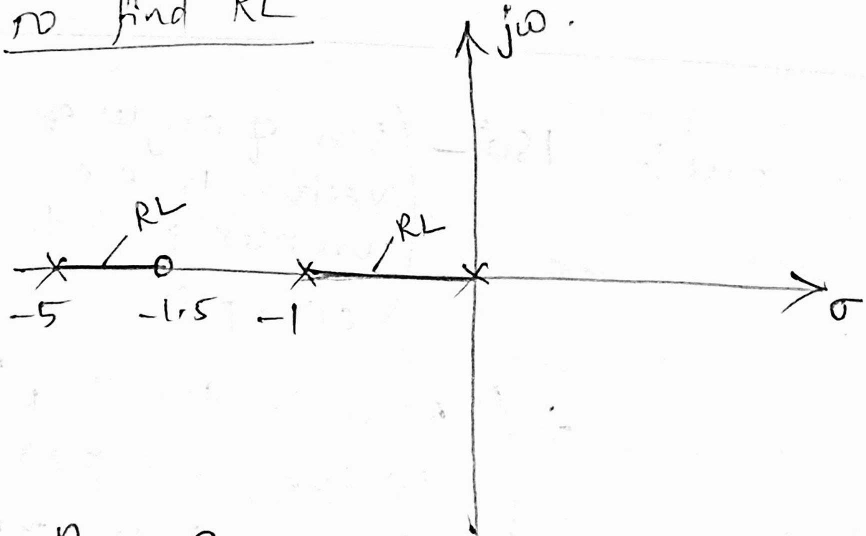
$P_1 = 0, \; P_2 = -1, \; P_3 = -5$

$(s+1.5) = 0 \Rightarrow s = -1.5$

$Z_1 = -1.5$



(ii) To find RL



$n = 3$

$m = 1$

$\boxed{n - m \Rightarrow 2}$

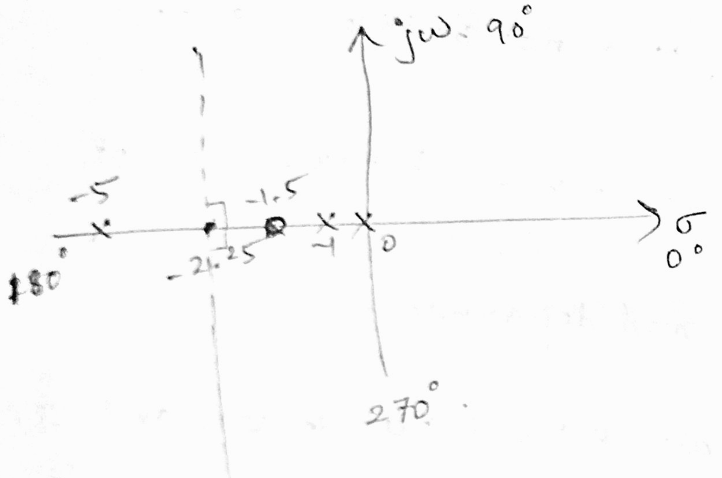
90
$\frac{3}{2} \times 180$

iii) To find Asymptotes & centroid.

$$\theta_A = \frac{\pm 180^\circ (2q+1)}{n-m}, \quad q = 0, 1, 2$$

$$q = 0 \Rightarrow \theta_A = \pm 90^\circ$$

$$q = 1 \Rightarrow \theta_A = \pm 270^\circ.$$



$$\sigma_A = \frac{\left(\text{sum of poles}\right) - \left(\text{sum of zeros}\right)}{n-m}$$

$$= \frac{-6 + 1.5}{2}.$$

$$= \frac{-4.5}{2}$$

$$\boxed{\sigma_A = -2.25}$$

iv) Breakaway & Break-in pt :-

$$CLTT = \frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)}$$

$$= \frac{K(s+1.5)}{s(s+1)(s+5) + K(s+1.5)}.$$

che. eqn $s(s+1)(s+5) + K(s+1.5) = 0.$

$$K = -\frac{s(s+1)(s+5)}{(s+1.5)}$$

$$= -\frac{(s^3 + 6s^2 + 5s)}{(s+1.5)}.$$

$$\frac{dK}{ds} = -2\frac{(s^3 + 5.25s^2 + 9s + 3.75)}{(s+1.5)^2}.$$

$\frac{dK}{ds} = 0 \Rightarrow s^3 + 5.25s^2 + 9s + 3.75 = 0.$

$s \Rightarrow -0.6, \ -2.3 \pm j0.89.$

$\boxed{s = -0.6}$, $K \rightarrow$ +ve & real $\Rightarrow 1.17$

$s = -2.3 \pm j0.89$, $K \rightarrow$ not positive & real.

$\therefore s = -0.6$ is the actual break-away pt.

(v) Angle of Arrival and departure :-

Since there are no complex poles and zeros,

$\therefore$ there is no angle of Arrival and departure.

(vi) To find crossing pts :-

$s(s+1)(s+5) + K(s+1.5) = 0.$

$s(s^2+6s+5) + Ks + 1.5K = 0.$

$s^3 + 6s^2 + 5s + Ks + 1.5K = 0.$

$s^3 + 6s^2 + (5+K)s + 1.5K = 0.$

$s = j\omega \Rightarrow (j\omega)^3 + 6(j\omega)^2 + j\omega(5+K) + 1.5K = 0.$

$-j\omega^3 - 6\omega^2 + j\omega(5+K) + 1.5K = 0$

Eq. img part $= 0$

$-\omega^3 + \omega(5+K) = 0.$

$\boxed{\omega^2 = 5+K}$

Eq. Real part $= 0.$

$-6\omega^2 + 1.5K = 0.$

$\omega^2 = \frac{1.5K}{6}$

$s^2 = 1.5 \cdot 15$

$-6(5+K) + 1.5K = 0$

$-30 - 6K + 1.5K = 0$

$-4.5K = 30$

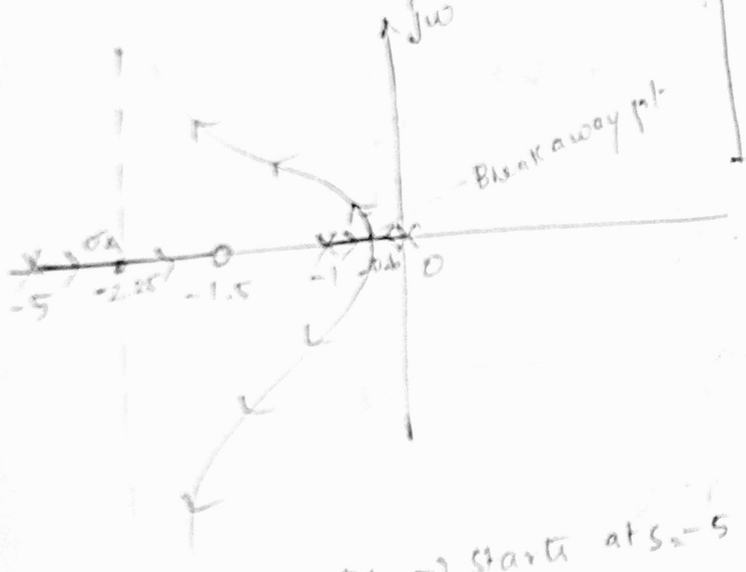
$$\boxed{K = -6.67}$$

$\omega^2 = 5 - 6.67$

$\omega^2 = -1.67 \Rightarrow$ -ve value.

∴ there is no crossing point on the img. axis.

Hence RL will not cross img. axis



RL starts from poles & ends at zeros

→ one branch of RL ⇒ starts at $s = -5$ & ends at $s = -1.5$

→ other two RL branch → starts at $s = 0$ & $s = -1$ and breakaway at $s = -0.6$, then travel parallel to asymptotes and meets zero at ∞.

2) sketch RL for unity fb s/m whose OLTF is

$$G(s) = \frac{K(s^2+6s+25)}{s(s+1)(s+2)}.$$

soln.

i) To locate poles & zeros.

zeros $\Rightarrow s^2+6s+25=0$.

$s = -3 \pm j4$.

$z_1 = -3+j4$

$z_2 = -3-j4$

poles $\Rightarrow s(s+1)(s+2)=0$.

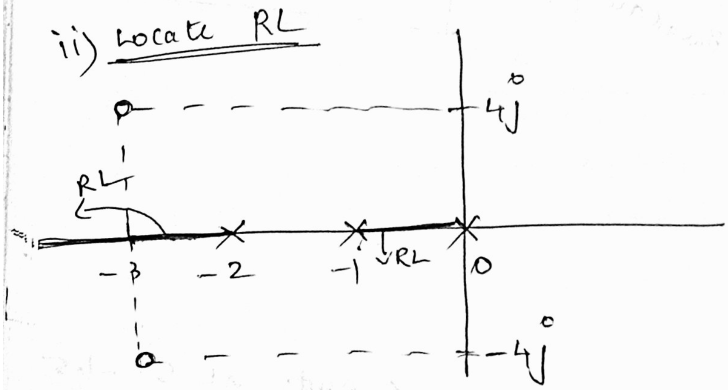
$s = 0, -1, -2$

$P_1 = 0$

$P_2 = -1$

$P_3 = -2$.

ii) Locate RL



iii) Asymptotes & centroid.

$$\theta_A = \pm\frac{180^\circ(2q+1)}{n-m}.$$

$n-m = 3-2 = 1.$

$q = 0, 1$

$q=0,\ \theta_A = \pm 180^\circ$.

$q=1,\ \theta_A = \pm 3\times 180^\circ$

$= \pm 540$

$= \pm(360+180^\circ)$

$= \pm 180^\circ$.

$$\sigma_A = \frac{\begin{pmatrix}\text{sum of} \\ \text{poles}\end{pmatrix} - \begin{pmatrix}\text{sum of} \\ \text{zeros}\end{pmatrix}}{n - m}$$

$$= \frac{(0 - 1 - 2) - (-3 + \cancel{j4} - 3 - \cancel{j4})}{1}$$

$$= \frac{-3 + 6}{1}$$

$$\boxed{\sigma_A = 3}$$

(iii) Break away pts :-

$$\frac{C(s)}{R(s)} = \frac{K(s^2 + 6s + 25)}{s(s+1)(s+2) + K(s^2 + 6s + 25)}.$$

char. eqn $\quad s(s+1)(s+2) + K(s^2 + 6s + 25) = 0.$

$$K = \frac{-s^3 - 3s^2 - 2s}{s^2 + 6s + 25}.$$

$$\frac{dK}{ds} = 0.$$

$$\frac{-(s^4 + 12s^3 + 91s^2 + 150s + 50)}{(s^2 + 6s + 25)^2} = 0$$

$$s^4 + 12s^3 + 91s^2 + 150s + 50 = 0.$$

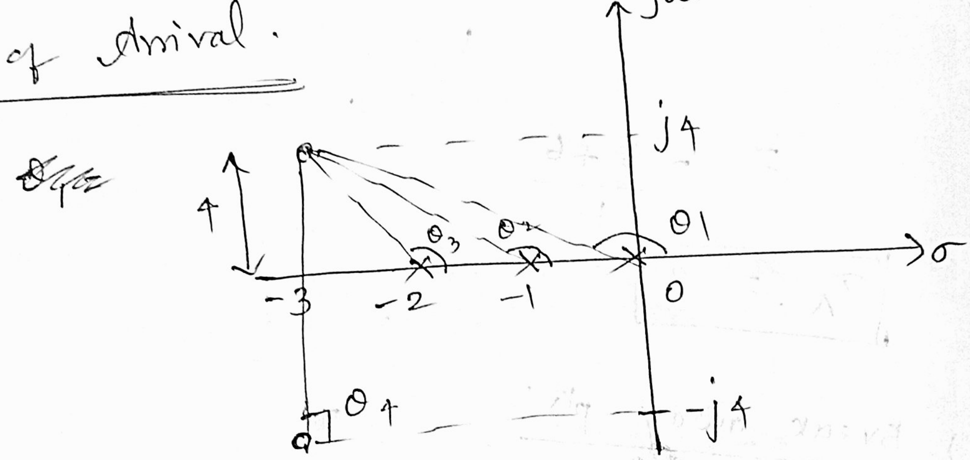
$$\boxed{s = -0.45, \ -1.55, \ -5 \pm j6.73}$$

→ $s = -0.45 \Rightarrow K$ is positive & real.

for all other values of $s \Rightarrow K$ is not real & +ve.

→ ∴ $\boxed{s = -0.45 \Rightarrow \text{Breakaway point}}$

v) Angle of Arrival.



Angle of arrival = $180° -$ (sum of angles of vector to the complex zero from all other zero)

+ (sum of angles of vectors to the complex zero from the poles)

$$= 180° - (\theta_4) + (\theta_1 + \theta_2 + \theta_3)$$

$\theta_4 = 90°$

$\theta_1 = 180 - \tan^{-1}\left(\frac{4}{3}\right)$

$= 126.9°.$

$\theta_2 = 180° - \tan^{-1}\left(\frac{4}{2}\right)$

$= 116.6°.$

$\theta_3 = 180° - \tan^{-1}\left(\frac{4}{1}\right)$

$= 104°.$

$\therefore$ Angle of arrival $= 180^\circ - 90^\circ + 126.9^\circ + 116.6^\circ + 104^\circ$

$= 77.5^\circ$.

IIIly for other zero, it is -77.5°.

vi) crossing pt :-

$s(s+1)(s+2) + K(s^2+6s+25) = 0$.

$s = j\omega$ & equating img & Real part $= 0$.

$\omega^2 = 2 + 6K$ & $K = 0.4 \pm j0.9$.

$\Rightarrow$ no crossing pt $\because$ K is img.

vii)