LIMIT CYCLE OSCILLATIONS:

* zero input limit cycle oscillation
* over flow " " "

LIMIT CYCLE: (only in IIR). (not present in FIR, ∵ no feedback)

In recursive system, when i/p is gn, there will (zero (or) non zero) be non-linearity in the system due to finite precision arithmetic operation. It results in periodic oscillations. ie o/p oscillates b/w a +ve & −ve value for increasing 'n', (or) become constant for increasing 'n'. Such oscillations are called limit cycles.

Zero input limit cycle:

If a system o/p enters a limit cycle, it will continue to remain in limit cycle even when the i/p is made zero.

overflow limit cycle:

Oscillations produced when overflow occurs due to the sum of 2 binary numbers.

FIRST ORDER IIR FILTER:

$$y(n) = x(n) + \alpha y(n-1)$$

let $\alpha = \frac{1}{2}$, $x(n) = \begin{cases} 0.875, & n=0 \\ 0, & n \neq 0 \end{cases}$ case (i)

$\alpha = -\frac{1}{2}$, $x(n) = \begin{cases} 0.875, & n=0 \\ 0, & n \neq 0 \end{cases}$ case (ii)

$\alpha = \frac{1}{2}$

n	$x(n)$	$y(n-1)$	$\alpha\, y(n-1)$	$Q[\alpha \cdot y(n-1)]$	$y_q(n)$
0	0.875	0.0	0.0	0.000	7/8
1	0	7/8	7/16	0.100	$\frac{1}{2} \simeq \frac{3}{8}$
2	0	1/2	1/4	0.010	$\frac{1}{4} = \frac{2}{8}$
3	0	1/4	1/8	0.001	1/8
4	0	1/8	1/16	0.001	1/8 limit cycle
5	0	1/8	1/16	0.001	1/8

where, $Q[\alpha \cdot y(n-1)] \rightarrow$ rounding operation (3-bit excluding sign-bit)

$y_q(n) \rightarrow$ o/p after Quantization.

$$y_q(n) = x(n) + Q[\alpha \cdot y(n-1)]$$

$\alpha = -\frac{1}{2}$

n	$x(n)$	$y(n-1)$	$\alpha \cdot y(n-1)$	$Q[\alpha \cdot y(n-1)]$	$y_q(n)$
0	0.875	0	0.000	0.000	7/8
1	0	7/8	-7/16	1.100	-1/2
2	0	-1/2	1/4	0.010	1/4
3	0	1/4	-1/8	1.001	-1/8
4	0	-1/8	1/16	0.001	1/8 limit cycle
5	0	1/8	-1/16	1.001	-1/8
6	0	-1/8	1/16	0.001	1/8

Even when i/p is made zero,

limit cycle remains [o/p → no change]

DEAD BAND:

The amplitude of the o/p during a limit cycle are confined to a range of values known as dead band of the filter. $y(n) = x(n) + \alpha y(n-1)$

$$-\frac{q}{2} \le (x_R - x) < q/2$$

$$-\frac{q}{2} \le \left(Q[\alpha y(n-1)] - [\alpha y(n-1)]\right) < q/2$$

$$\boxed{y(n-1) \le \frac{\frac{1}{2}\, 2^{-b}}{1-|\alpha|}}$$ $\Leftarrow$ first order filter

Ex: Explain the characteristics of a limit cycle oscillation w.r. to the system described by the difference equation

$$y(n) = 0.95\, y(n-1) + x(n)$$

Determine the dead band of the filter.

Soln: b-4 bits (excluding sign bit)

$$x(n) = \begin{cases} 0.875, & n=0 \\ 0, & n \neq 0 \end{cases}$$

After quantization, $y_q(n) = x(n) + Q[0.95\, y(n-1)]$

n=0, $y_q(0) = x(0) + Q[0.95\, y(-1)]$ $\because y(-1) = 0$

$= 0.875 + 0 = 0.875$

n=1 $y_q(1) = x(1) + Q[0.95\, y(0)]$

$= 0.000 + Q[0.95 \times 0.875] = 0 + Q[0.83125]$

$[0.83125]_{10} = (0.1101010\ldots)_2 \xrightarrow{r} (0.1101)_2 = (0.8125)_{10}$

n=2 $y_q(2) = x(2) + Q[0.95 \times 0.8125]$

$= 0 + Q[0.771875]$

$(0.771875)_{10} = (0.1100011\ldots)_2 \xrightarrow{r} (0.1100)_2 = (0.75)_{10}$

n=3

$$y_q(3) = x(3) + Q[0.95 \cdot y(2)] = 0 + Q[0.95 \times 0.75]$$
$$= 0 + Q[7125]$$

$$(0.7125)_{10} = (0.101101\ldots)_2 \xrightarrow{r} (0.1011)_2 = (0.6875)_{10}$$

n=4

$$y_q(4) = x(4) + Q[0.95 \times 0.6875]$$
$$= 0 + Q[0.653125]$$

$$(0.653125)_{10} = (0.101001\ldots)_2 \xrightarrow{r} (0.1010)_2 = (0.625)_{10}$$

n=5

$$y_q(5) = x(5) + Q[0.925 \times 0.625]$$
$$= 0 + Q[0.59375]$$

$$(0.59375)_{10} = (0.10011\ldots)_2 \xrightarrow{r} (0.1010)_2 = (0.625)_{10}$$

n=6

$$y_q(6) = x(6) + Q[0.925 \times 0.625]$$
$$= 0 + Q[0.59375]$$

$$(0.59375)_{10} = (0.10011\ldots)_2 \xrightarrow{r} (0.1010)_2 = (0.625)_{10}$$

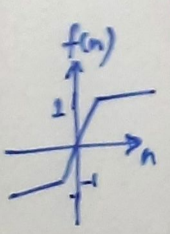
From the above calculation, $n \geq 5$ o/p remains constant at 0.625 causing limit cycle behaviour.

$$\text{Dead band} = \frac{\frac{1}{2} 2^{-b}}{1 - |\alpha|} = \frac{\frac{1}{2} 2^{-4}}{1 - 0.925} = 0.625$$

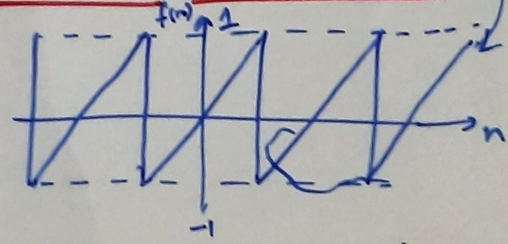
Dead band = 0.625.

transfer chars. of adder

TWO METHODS TO ELIMINATE OVERFLOW OSCILLATIONS:



* Saturation Arithmetic
* Scaling of $x(n)$ & $h(n)$



Saturation Arithmetic: when overflow is sensed, the o/p is set equal to max. allowable value & when underflow is sensed, the o/p is set equal to min. allowable value.

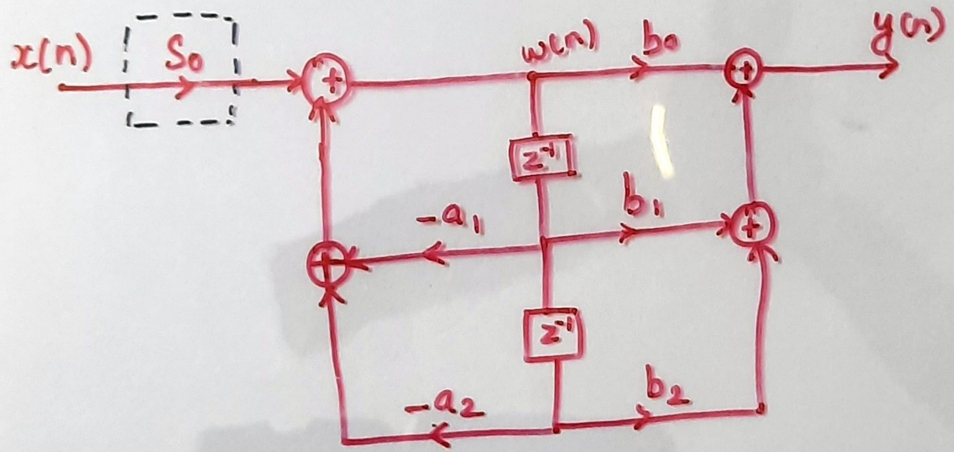
dis adv:

This saturation arithmatic introduces distortion that results in non-linear system. But when saturation occurs infrequently, distortion is less.

Scaling factor:

In order to reduce the oscillations in limit cycle, we have to scale the input signal & impulse response by introducing scaling factor (S_0) b/w i/p & any internal summing node in the system such that the overflow becomes a rare event.

let 2nd order IIR filter,



$$H(z) = S_0 \cdot \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}}$$

$$S_0^2 = \frac{1}{\frac{1}{2\pi j} \oint_c S(z) S(z^{-1}) z^{-1} dz}$$

(or)

$$S_0^2 = \frac{1}{\frac{1}{2\pi j} \oint_c \frac{z^{-1} dz}{D(z) \cdot D(z^{-1})}}$$

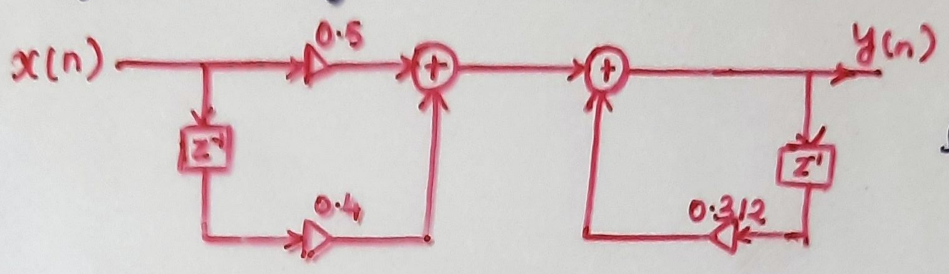
$\left[\because S(z) = \frac{1}{D(z)} \right.$

Ex: The transfer function of the digital filter is given by $H(z) = \frac{0.5 + 0.4z^{-1}}{1 - 0.312z^{-1}}$. Find the scalar factor s_0 to avoid overflow in adder 1 of the digital filter.

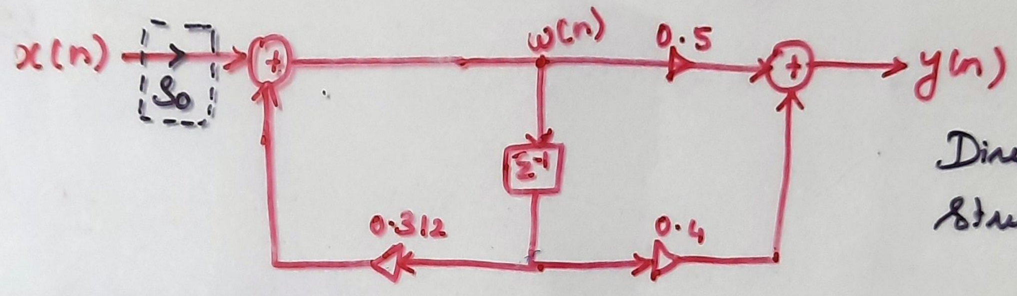
Soln: $\frac{Y(z)}{X(z)} = \frac{0.5 + 0.4z^{-1}}{1 - 0.312z^{-1}}$;

$$Y(z) - 0.312z^{-1}Y(z) = 0.5X(z) + 0.4z^{-1}X(z)$$

$$y(n) = 0.312\,y(n-1) + 0.5\,x(n) + 0.4\,x(n-1)$$



Direct form I



Direct form-II Structure.

$$S_0^2 = \frac{1}{\frac{1}{2\pi j}\oint_c s(z).s(z^{-1}).\,z^{-1}.\,dz}$$

$$w(n) = S_0 \cdot x(n) + 0.312\,w(n-1)$$

$$W(z) = s_0 \cdot X(z) + 0.312\,z^{-1}W(z)$$

$$\frac{W(z)}{X(z)} = \frac{S_0}{1 - 0.312z^{-1}} = \frac{S_0}{D(z)}$$

$$D(z) = 1 - 0.312z^{-1}$$

$$s(z) = \frac{1}{D(z)} = \frac{1}{1 - 0.312z^{-1}}$$

R.8 47 & 59

$$S_0^2 = \frac{1}{\frac{1}{2\pi j}\oint_c z^{-1} s(z)\cdot s(z^{-1})\cdot dz}$$

$$\left.\begin{array}{l}\frac{1}{2\pi j}\oint_c s(z)\cdot s(z^{-1}) \\ z^{-1}dz\end{array}\right\} = (1-\cancel{0.312z^{-1}})\cdot\frac{z^{-1}}{(1-\cancel{0.312z^{-1}})(1-0.312z)}\Bigg/_{z=0.312}$$

$$= \frac{(0.312)^{-1}}{1-(0.312\times0.312)} = \frac{3.2051}{0.90266} = 3.55073.$$

$$S_0^2 = \frac{1}{3.55073} \Rightarrow S_0 = \frac{1}{\sqrt{3.551}} = 0.5307$$

$$\boxed{S_0 = 0.5307}$$