$$S_0^2 = \frac{1}{\frac{1}{2\pi j}\oint_c z^{-1} s(z)\cdot s(z^{-1})\cdot dz}$$

$$\left.\frac{1}{2\pi j}\oint_c s(z)\cdot s(z^{-1}) z^{-1} dz\right\} = \cancel{(1-0.312z^{-1})}\cdot\frac{z^{-1}}{\cancel{(1-0.312z^{-1})}(1-0.312z)}\Bigg/_{z=0.312}$$

$$= \frac{(0.312)^{-1}}{1-(0.312\times 0.312)} = \frac{3.2051}{0.90266} = 3.55073.$$

$$S_0^2 = \frac{1}{3.55073} \Rightarrow S_0 = \frac{1}{\sqrt{3.551}} = 0.5307$$

$$\boxed{S_0 = 0.5307}$$

PRODUCT QUANTIZATION ERROR:

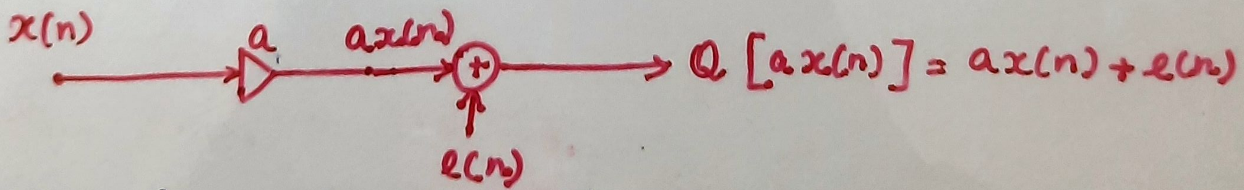
* Error due to quantization of the multiplier o/p

NOISE TRANSFER FUNCTION:

* defined as the transfer function from noise source to the filter o/p

(or)

* Transfer function obtained by treating the noise source as actual i/p.



statistical model of fixed pt product quantization.

$$Q[a \cdot x(n)] = a \cdot x(n) + e(n)$$

$Q[a \cdot x(n)]$ ⇓ quantized o/p

$a \cdot x(n)$ ⇓ unquantized product

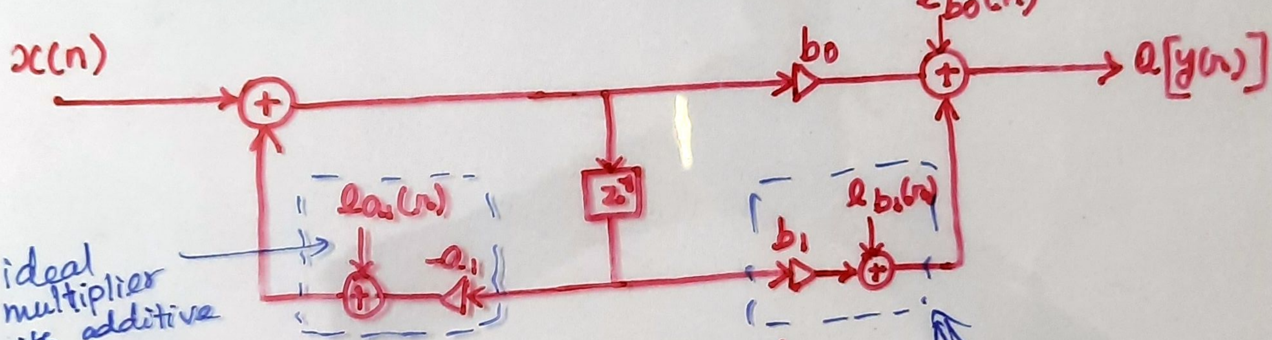
$e(n)$ ⇓ product quantization error

FIRST ORDER IIR FILTER:

$$y(n) = -a_1\, y(n-1) + b_0\, x(n) + b_1\, x(n-1)$$

After quantization,

$$Q[y(n)] = \underbrace{-a_1\, y(n-1) + e_{a_1}(n)}\ + \underbrace{b_0 x(n) + e_{b_0}(n)}\ + \underbrace{b_1\, x(n-1) + e_{b_1}(n)}$$



ideal multiplier with additive roundoff noise

finite precision multiplier ≡

direct form-II structure

Quantization noise model of IIR using DF-II structure

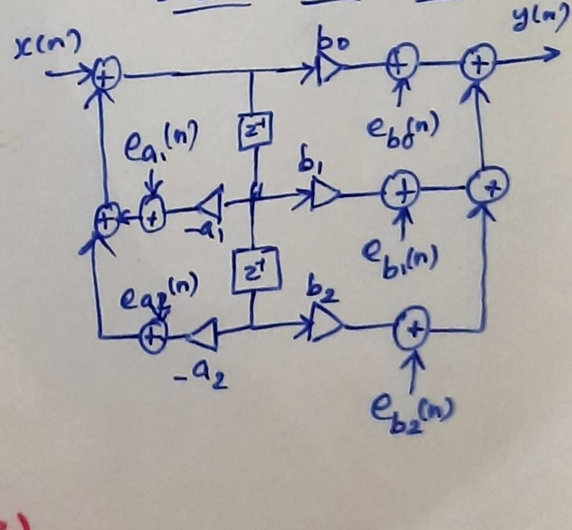
Output noise power,

$$\sigma_{0k}^2 = \sigma_e^2 \cdot \frac{1}{2\pi j} \oint_C H_k(z) \cdot H_k(z^{-1})\, z^{-1} dz$$

total, $$\sigma_0^2 = \sum_{k=0}^{\infty} \sigma_{0k}^2$$

$$\sigma_e^2 = \frac{2^{-2b}}{12}$$

second order IIR



Direct form realization.

$$NTF\{e_{a_i}(n)\} = \{e_{b_i}(n)\} = H(z)$$

In Cascade form realization.

$$NTF\{e_{a_i}(n)\} = H(z) = H_1(z) \cdot H_2(z)$$

$$e_{b_i}(n) = H_2(z)$$

Ex: In the IIR system given below the products are rounded to 4-bits (including sign bit)

$$H(z) = \frac{1}{(1-0.35z^{-1})(1-0.62z^{-1})}$$

Find the output round off noise power in
(a) direct form realization
(b) cascade form realization

Soln: $b = 4$-bits

$$\sigma_e^2 = \frac{2^{-2\times 4}}{12} = 2^{-8}/12 = 3.255\times10^{-4} = 0.3255\times10^{-3}$$

Direct form realization:

$$H(z) = \frac{1}{(1-0.35z^{-1})(1-0.62z^{-1})}$$

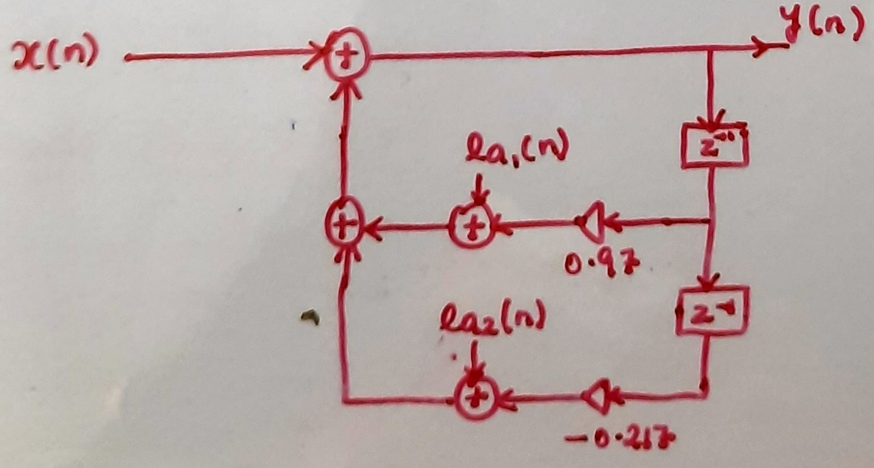
$$\frac{Y(z)}{X(z)} = \frac{1}{1-0.97z^{-1}+0.217z^{-2}}$$

$$Y(z) = X(z) + 0.97z^{-1}Y(z) - 0.217z^{-2}Y(z)$$

$$y(n) = x(n) + 0.97y(n-1) - 0.217y(n-2)$$

After quantization,

$$y(n) = x(n) + \underbrace{0.97y(n-1) + e_{a_1}(n)} \underbrace{-0.217y(n-2) + e_{a_2}(n)}$$



$$NTF\{e_{a1}(n)\} = T_1(z) = \frac{1}{(1-0.35z^{-1})(1-0.62z^{-1})} = H(z)$$

$$NTF\{e_{a2}(n)\} = T_2(z) = \quad " \quad " \quad = H(z)$$

$$\sigma_{01}^2 = \sigma_e^2 \frac{1}{2\pi j} \oint_c T_1(z) \cdot T_1(z^{-1}) \cdot z^{-1} dz$$

poles of $T_1(z)$, $P_1 = 0.35$, $P_2 = 0.62$

Poles of $T_1(z^{-1})$, $P_3 = 1/0.35$, $P_4 = 1/0.62$

$$\frac{1}{2\pi j} \oint_c T_1(z) \cdot T_2(z^{-1}) z^{-1} dz = \cancel{(1-0.35z^{-1})} \frac{z^{-1}}{\cancel{(1-0.35z^{-1})}(1-0.62z^{-1})(1-0.35z)(1-0.62z)} \Bigg|_{z=0.35}$$

$$+ \cancel{(1-0.62z^{-1})} \cdot \frac{z^{-1}}{(1-0.35z^{-1})\cancel{(1-0.62z^{-1})}(1-0.35z)(1-0.62z)} \Bigg|_{z=0.62}$$

$$= -5.3905 + 7.6838 = 2.2933$$

$$\sigma_{01}^2 = \sigma_e^2 \cdot 2.2933 = 0.3255 \times 10^{-3} \times 2.2933$$

$$\sigma_{01}^2 = 0.7465 \times 10^{-3}$$

$$\sigma_{02}^2 = 0.7465 \times 10^{-3}$$

$$\sigma_0^2 = \sigma_{01}^2 + \sigma_{02}^2$$

$$= (0.7465 \times 10^{-3}) + (0.7465 \times 10^{-3})$$

$$\boxed{\sigma_0^2 = 1.493 \times 10^{-3}}$$

Cascade Realization:

$$H(z) = \frac{1}{(1-0.35z^{-1})(1-0.62z^{-1})}$$

$$H_1(z) = \frac{1}{1-0.35z^{-1}} \; ; \; H_2(z) = \frac{1}{1-0.62z^{-1}} \Rightarrow \text{case (i)}$$

$$H_1(z) = \frac{1}{1-0.62z^{-1}} \; ; \; H_2(z) = \frac{1}{1-0.35z^{-1}} \Rightarrow \text{case(ii)}$$

Case (i): $H_1(z) = \dfrac{Y_1(z)}{X_1(z)} = \dfrac{1}{1-0.35z^{-1}}$

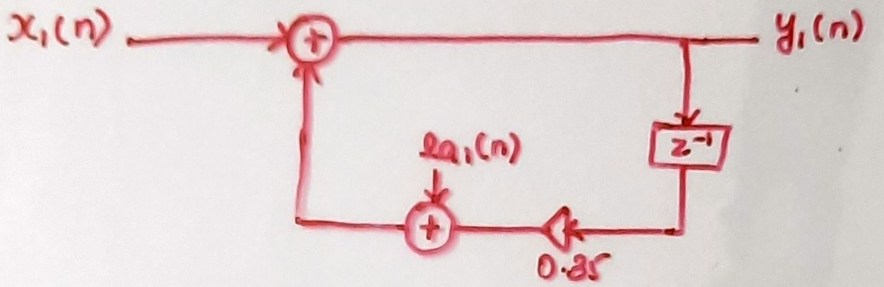
$$Y_1(z) = X_1(z) + 0.35z^{-1}\,Y(z)$$

$$y_1(n) = x_1(n) + 0.35\,y(n-1)$$

After quantization,

$$y_1(n) = x_1(n) + \underbrace{0.35y(n-1) + e_{a1}(n)}$$

Direct form structure of $H_1(z)$:



Case (ii): $H_2(z) = \dfrac{Y_2(z)}{X_2(z)} = \dfrac{1}{1-0.62z^{-1}}$

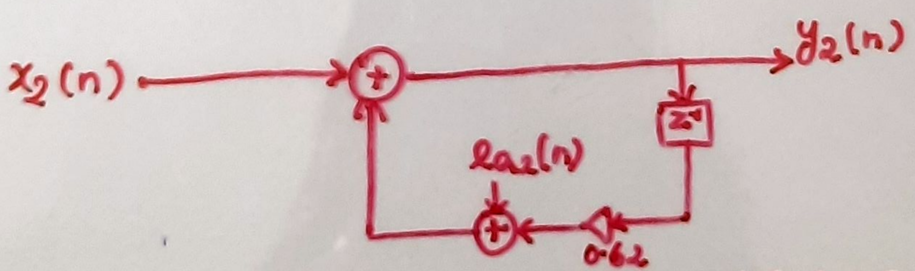
$$Y_2(z) = X_2(z) + 0.62z^{-1}\,X_2(z)$$

$$y_2(n) = x_2(n) + 0.62\,x_2(n-1)$$

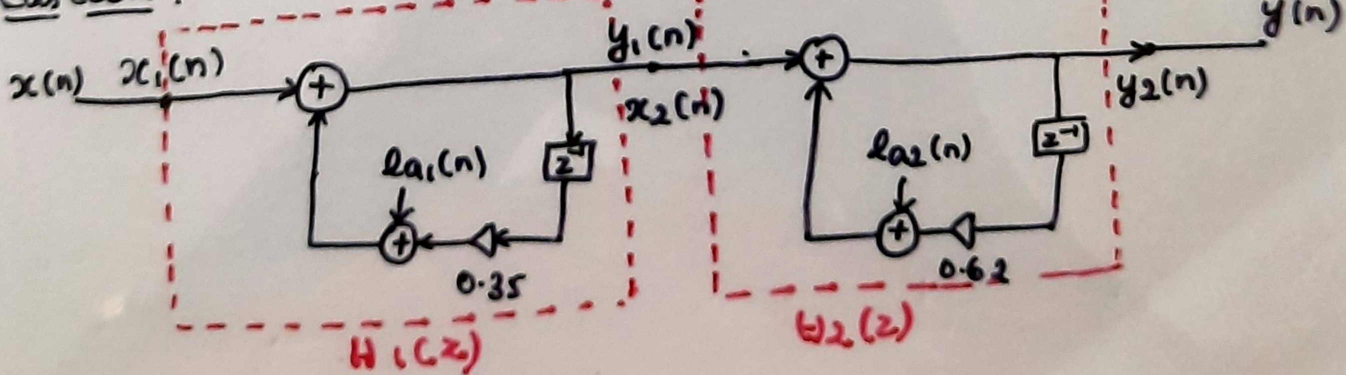
After quantization,

$$y_2(n) = x_2(n) + \underbrace{0.62x_2(n-1) + e_{a2}(n)}$$

Direct form structure of $H_2(z)$:



cascade:



$NTF\{e_{a1}(n)\} = T_1(z) = H(z)$

$NTF\{e_{a2}(n)\} = T_2(z) = H_2(z)$

$\sigma_{01}^2 = 0.7465 \times 10^{-3}$

$$\sigma_{02}^2 = \sigma_e^2 \cdot \frac{1}{2\pi j} \oint_c T_2(z) \cdot T_2(z^{-1}) \cdot z^{-1}\, dz$$

Poles of $T_2(z)$: $P_1 = 0.62$

Poles of $T_2(z^{-1})$: $P_2 = 1/0.62$

$$\frac{1}{2\pi j} \oint_c T_2(z) T_2(z^{-1}) z^{-1} dz = (1-\cancel{0.62z^{-1}}) \left. \frac{z^{-1}}{(1-\cancel{0.62z^{-1}})(1-0.62z)} \right|_{z=0.62}$$

$$= 2.6201$$

$\sigma_{02}^2 = \sigma_e^2 \times 2.6201$

$= 0.3255 \times 10^{-3} \times 2.6201$

$\sigma_{02}^2 = 0.8528 \times 10^{-3}$

$\sigma_0^2 = \sigma_{01}^2 + \sigma_{02}^2 = 0.7465 \times 10^{-3} + 0.8528 \times 10^{-3}$

$$\boxed{\sigma_0^2 = 1.59 \times 10^{-3}}$$

Case (ii): Solve it yourself.