3) Parabolic i/p.

$\rightarrow \quad e_{ss} = \lim_{s \to 0} \dfrac{sR(s)}{1 + G(s)H(s)}$.

parabolic i/p $\quad r(t) = t^2/2 \,, \quad R(s) = \dfrac{1}{s^3}$.

$\rightarrow \therefore \quad e_{ss} = \lim_{s \to 0} \dfrac{s \cdot 1/s^3}{1+G(s)H(s)} = \lim_{s \to 0} \dfrac{1}{s^2 + s^2 G(s)H(s)}$

$= \dfrac{1}{\lim s^2 G(s)H(s)}$

$e_{ss} = \dfrac{1}{Ka}$

→ (i) Type 0 s/m :-

$$K_a = \lim_{s \to 0} s^2 G(s) H(s)$$

$$= \lim_{s \to 0} s^2 \frac{K (s+z_1)(s+z_2)(s+z_3)\ldots}{(s+P_1)(s+P_2)(s+P_3)}$$

$$= 0.$$

$$\therefore \boxed{e_{ss} = \frac{1}{K_a} = \frac{1}{0} = \infty}$$

→ (ii) Type 1 - s/m

$$K_a = \lim_{s \to 0} s^2 G(s) H(s)$$

$$= \lim_{s \to 0} \frac{\cancel{s^2} K (s+z_1)(s+z_2)(s+z_3)\ldots}{\cancel{s}(s+P_1)(s+P_2)(s+P_3)\ldots}$$

$$= 0$$

$$\therefore \boxed{e_{ss} = \frac{1}{0} = \infty .}$$

→ (iii) Type 2 - s/m

$$K_a = \lim_{s \to 0} \frac{\cancel{s^2} K (s+z_1)(s+z_2)\ldots}{\cancel{s^2}(s+P_1)(s+P_2)\ldots}$$

⇒ constant

$$\therefore e_{ss} = \frac{1}{K_a} \Rightarrow \text{constant (finite)}.$$

(iv) Type 3 s/m

$\rightarrow \quad K_a = \lim_{s \to 0} \frac{\cancel{s^2}(s+z_1)(s+z_2)\ldots}{\cancel{s^3}(s+P_1)(s+P_2)\ldots}$

$K_a = \infty$

$\rightarrow \therefore \; e_{ss} = \frac{1}{K_a} = \frac{1}{\infty} = 0$.

$\boxed{e_{ss} = 0}$.

$\rightarrow \therefore$ for the type 3 & above, the unit parabolic i/p will have e_{ss} as 0.

Static error constants (K_p, K_v, K_a)

Error constant	Type no. of s/m			
	0	1	2	3
K_p	constant	∞	∞	∞
K_v	0	constant	∞	∞
K_a	0	0	constant	∞

Steady state Error for various Types of i/p:-

I/p signal	Type no. of s/m 0	1	2	3
unit step	$\frac{1}{1+K_p}$	0	0	0
unit Ramp	∞	$\frac{1}{K_v}$	0	0
unit parabolic	∞	∞	$\frac{1}{K_a}$	0.

Generalized Error Co-efficients / Dynamic Error co-efficients

→ $C_n = (-1)^n \int_0^t t^n f(t)\, dt$.

$C_0 = (-1)^0 \int_0^t f(t)\, dt = \int_0^t f(t)\, dt$.

$$\boxed{F(s) = L[f(t)] = \int_0^t f(t) e^{-st}\, dt = \frac{1}{1+G(s)H(s)}}$$

→ on taking $\lim_{s \to 0}$ on both sides,

$$\lim_{s\to 0} F(s) = \lim_{s\to 0} \int_0^t f(t) e^{-st}\, dt$$

$$= \lim_{s\to 0} \int_0^t f(t)\, dt$$

$$\boxed{\lim_{s\to 0} F(s) = C_0} \Rightarrow C_0 = \lim_{s\to 0} F(s).$$

$\rightarrow C_1 = \lim_{s \to 0} \frac{d}{ds} F(s)$

$\rightarrow C_2 = \lim_{s \to 0} \frac{d^2}{ds^2} F(s)$

$\rightarrow \therefore$ Generally, the dyanamic error-co-efficients are given by,

$$\boxed{C_n = \lim_{s \to 0} \frac{d^n}{ds^n} F(s)}$$

$\rightarrow$ correlation between Static & Dynamic error co-efficients

$$C_0 = \frac{1}{1+K_p}$$

$$\boxed{E(s) = \frac{R(s)}{1+G(s)H(s)}}$$

$$C_1 = \frac{1}{K_v}$$

$$C_2 = \frac{1}{K_a}$$

& $\boxed{e(t) = C_0 r(t) + C_1 r'(t) + \frac{C_2}{2!} r''(t) + \ldots}$

$r(t) \rightarrow$ i/p

$e(t) \rightarrow$ error

$C_n \rightarrow$ dynamic error constants