EC7552 - DISCRETE TIME SIGNAL PROCESSING

UNIT-1 DISCRETE FOURIER TRANSFORM

Review of discrete time (DT) signals and systems - DFT and its Properties - FFT Algorithms & its applications, overlap-add & Overlap-save method.

Review of DT Signals & Systems:

Signal: → any physical quantity that varies with time, space or any other independent variable.

(OR)

function of one or more indep. variables.

Signal Types:

1. Analog signal: Continuous in time and amplitude

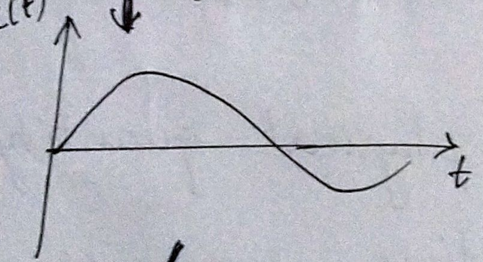
eg: Speech signal, ECG, EEG ...

2. Digital Signal: <u>discrete</u> or (dis continuous) in both time and amplitude.

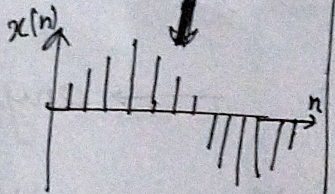
Eg: Attendance of the class.

<u>Signals</u>

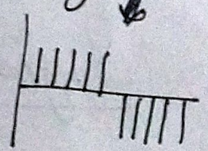
↙ Continuous-time ↓ Continuous value (amplitude) ↓ Analog ⬇



↘ Discrete - time
- ↙ Continuous-Value ↓ Discrete ⬇



- ↘ Discrete-value ↓ Digital ⬇



Analog ↓ Continuous function of an independent variable.

Discrete ⇓ function of discrete time instants that is defined by an integer.

$$x(t) \longrightarrow \boxed{ADC} \longrightarrow x(n)$$

(sampling).

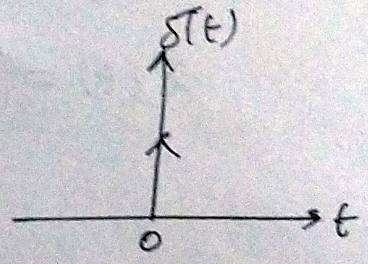
Elementary Signals:

→ to construct complex signals.
→ can be used as a test signals.

Continuous time signals:

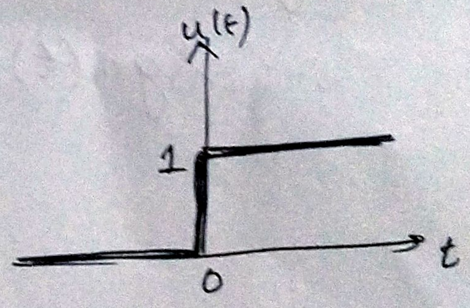
1. Unit Impulse signal:

$$\delta(t) = \begin{cases} \infty, & t = 0 \\ 0, & t \neq 0 \end{cases}$$



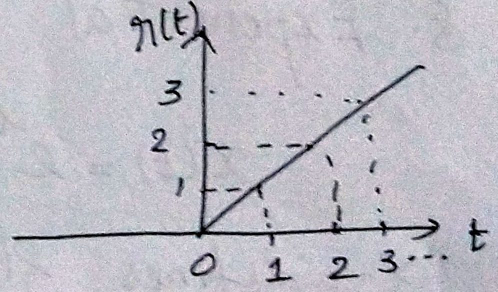
2. Unit-Step Signal:

$$u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$$



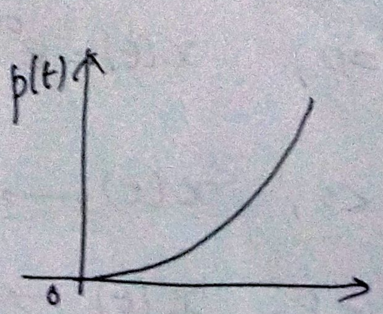
3. Unit-Ramp signal:

$$r(t) = \begin{cases} t, & t \geq 0 \\ 0, & t < 0 \end{cases}$$



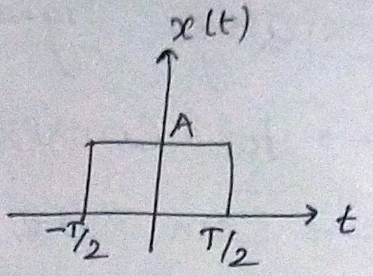
4. Parabolic Signal:

$$p(t) = \begin{cases} t^2/2, & t \geq 0 \\ 0, & t < 0 \end{cases}$$



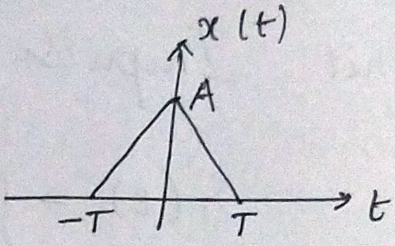
5. Rectangular Pulse :

$$x(t) = A \cdot \mathrm{rect}(t/T)$$



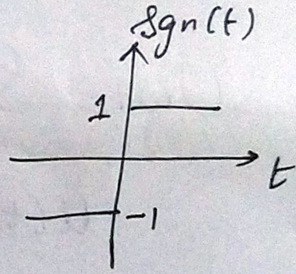
6. Triangular Pulse :

$$x(t) = A\left[1 - \frac{|t|}{T}\right]$$



7. Signum function :

$$\mathrm{sgn}(t) = \begin{cases} 1, & t > 0 \\ 0, & t = 0 \\ -1, & t < 0 \end{cases}$$



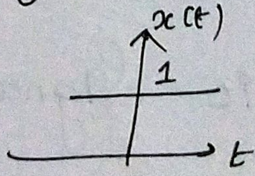
$$\boxed{\mathrm{sgn}(t) = 2u(t) - 1}$$

8. Exponential Signal :

$$x(t) = e^{\alpha t}$$

$\alpha \rightarrow$ defines shape of the exponential.

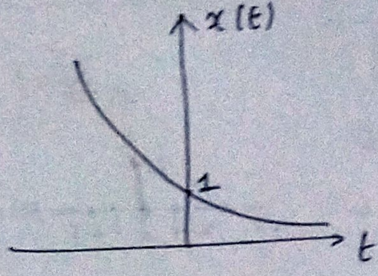
$\alpha = 0, \quad x(t) = e^{0t} = 1$

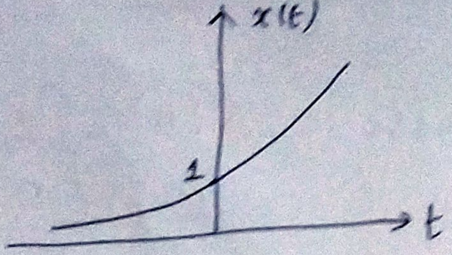


$\alpha < 0, \quad x(t) \rightarrow$ decaying exponential

$\alpha > 0, \quad x(t) \rightarrow$ raising exponential.

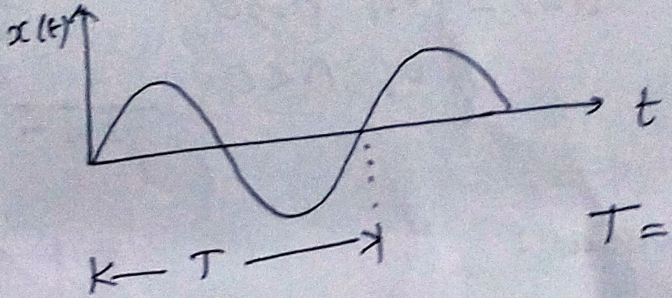
$\alpha < 0$ $\qquad\qquad$ $\alpha > 0$





9. Sinusoidal Signal :

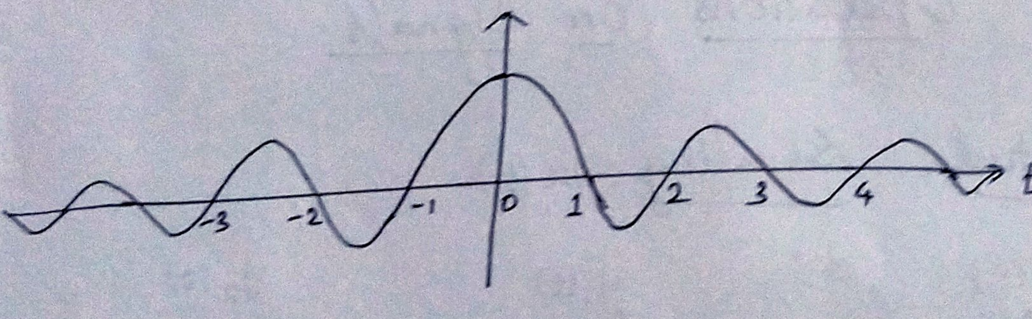
$$x(t) = A \sin(\omega_0 t \pm \phi) \quad \text{(or)} \quad A\cos(\omega_0 t + \phi)$$



$T = 2\pi/\omega_0$

10. Sinc function :

$$\text{sinc}(t) = \frac{\sin \pi t}{\pi t}$$

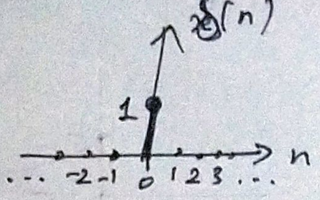


Relationship :

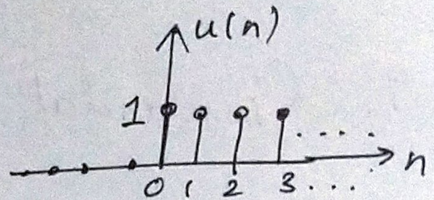
$$\delta(t) \underset{d/dt}{\overset{\int}{\rightleftarrows}} u(t) \underset{d/dt}{\overset{\int}{\rightleftarrows}} r(t) \underset{d/dt}{\overset{\int}{\rightleftarrows}} p(t)$$

Elementary DT Signals:

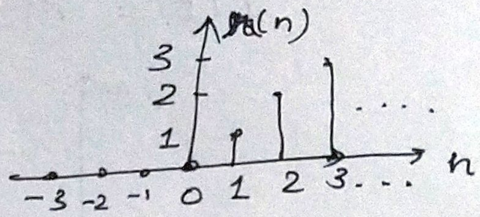
1. Impulse: $\delta(n) = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$



2. Step: $u(n) = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$



3. Ramp: $r(n) = \begin{cases} n, & n \geq 0 \\ 0, & n < 0 \end{cases}$



4. Exponential:

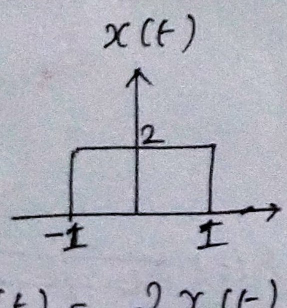
$x(n) = \begin{cases} a^n, & \text{for all } n \ \cancel{n \geq 0} \\ \cancel{0}, & \cancel{n < 0} \end{cases}$ see in later

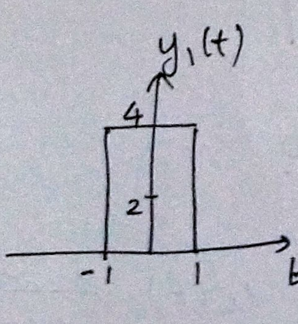
on page no: 11

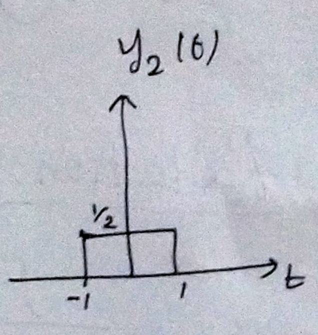
Basic Operations on Signals:

1. Amplitude Scaling: $Ax(t)$

Let





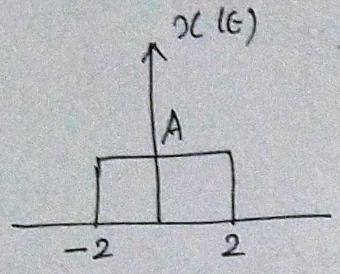


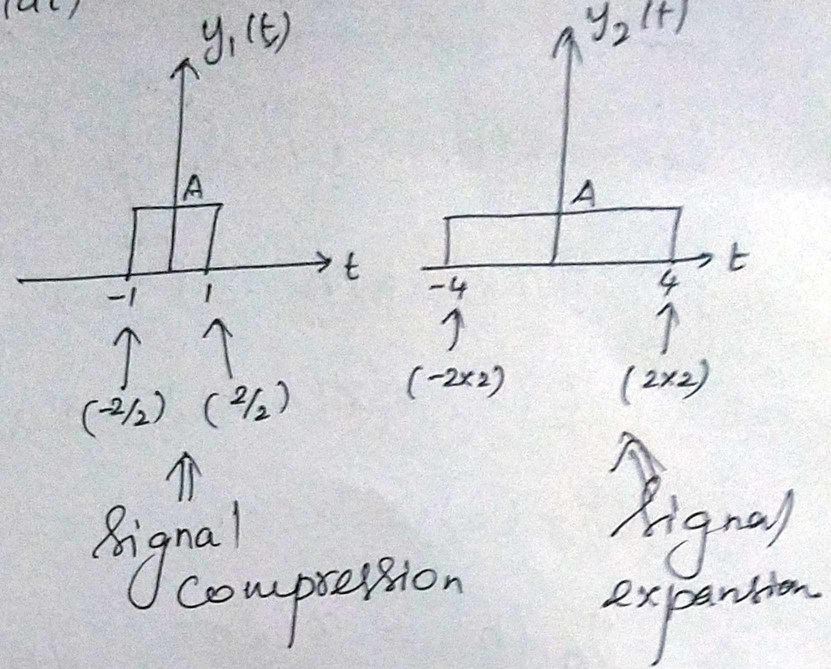
$y_1(t) = 2x(t)$

$y_2(t) = x(t)/2$ (or) $0.5x(t)$

2. Time Scaling : $x(at)$

Let



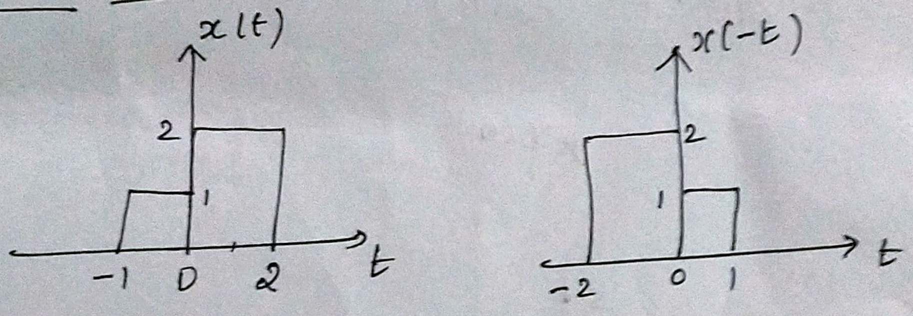


$y_1(t) = x(2t)$

$y_2(t) = x(t/2)$

3. Time Reversal (or) Reflection : $x(-t)$

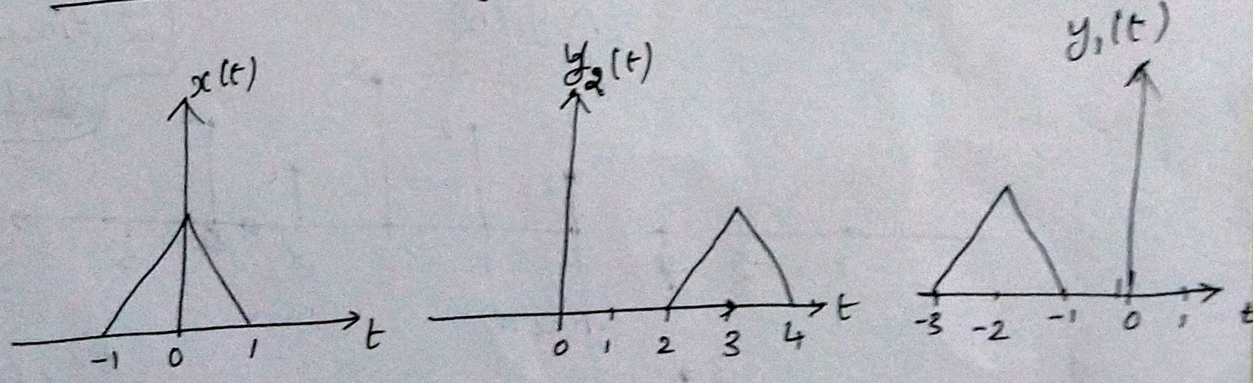
Let



$y(t) = x(-t)$

4. Time Shifting : $x(t \pm t_0)$

Let



$y_1(t) = x(t+2) \Rightarrow$ Left Shift.

$y_2(t) = x(t-3) \Rightarrow$ Right Shift

Basic Operations on DT Signals:

Let $x(n) = \{ \overset{(n=-2)}{1}, \overset{(n=-1)}{2}, \underset{\underset{n=0}{\uparrow}}{3}, \overset{(n=1)}{1}, \overset{(n=2)}{2} \}$

find: $y_1(n) = x(n-2) \Rightarrow$ delay the sequence by 2 units

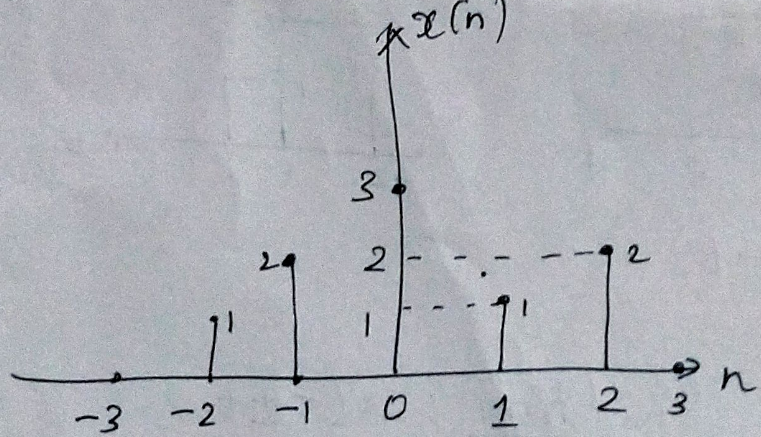
$y_2(n) = x(n+1) \Rightarrow$ advance by 1 unit.

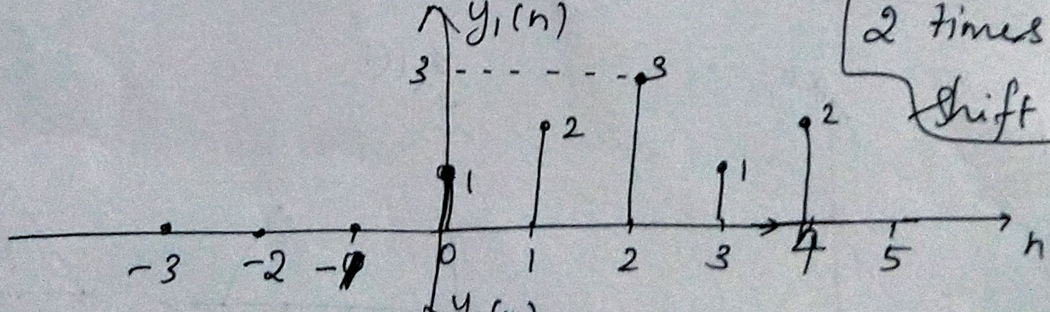
$y_3(n) = x(-n)$

$y_4(n) = x(2n)$, $y_5(n) = x(n/2)$

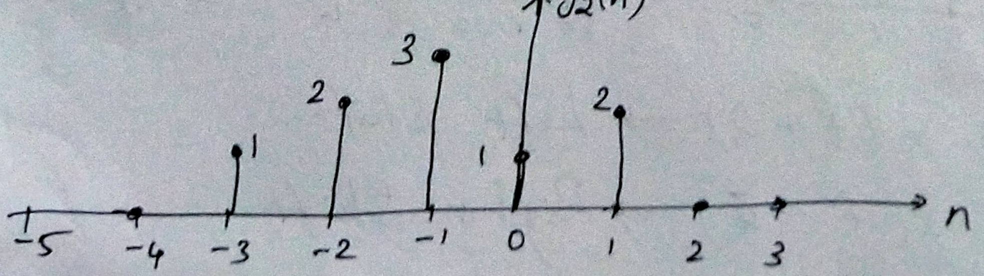
$y_6(n) = y_1(n) + y_2(n)$,

Soln:

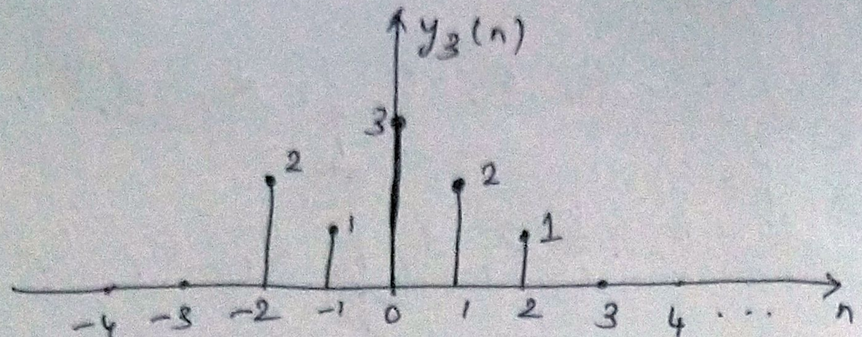


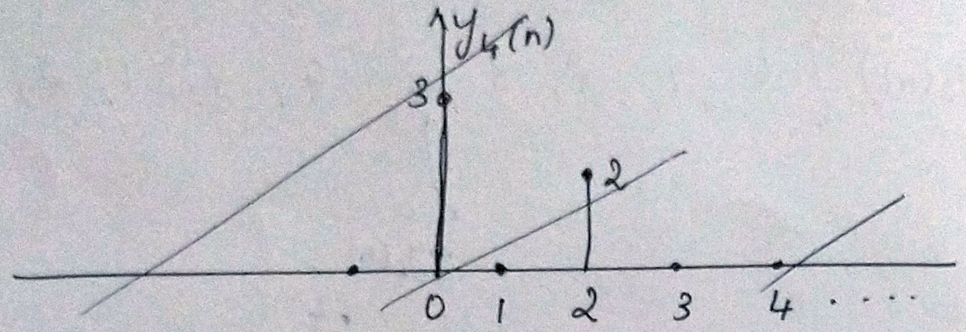


2 times right shift x(n)



Left shift x(n) by 1 time





$y_4(n) = x(2n)$

$n=0, \quad y_4(0) = x(0) = 3$

$n=1, \quad y_4(1) = x(2) = 2$

$n=2, \quad y_4(2) = x(4) = 0$

$n=3, \quad y_4(3) = x(6) = 0$

$\vdots$

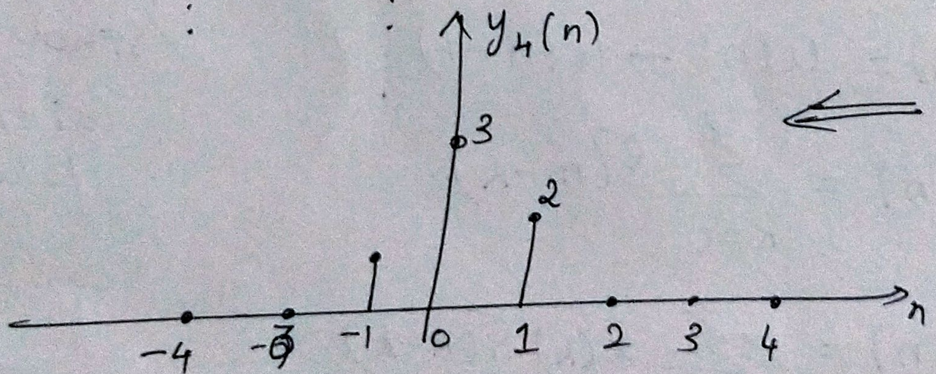
$n=-1,$

$y_4(-1) = x(-2) = \cancel{2}1$

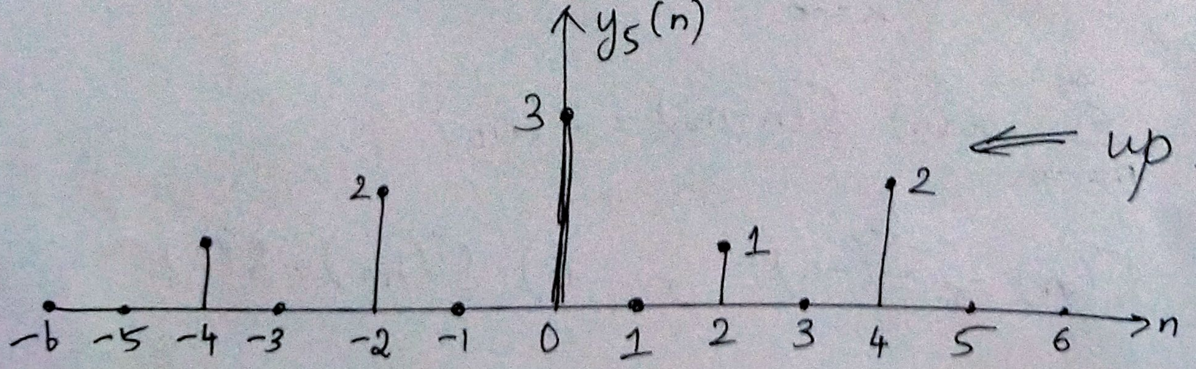
$n=-2 \quad y_4(-2) = x(-4) = 0$

$n=-3, \quad y_4(-3) = x(-6) = 0$

$\vdots$



$\Leftarrow$ down sampling

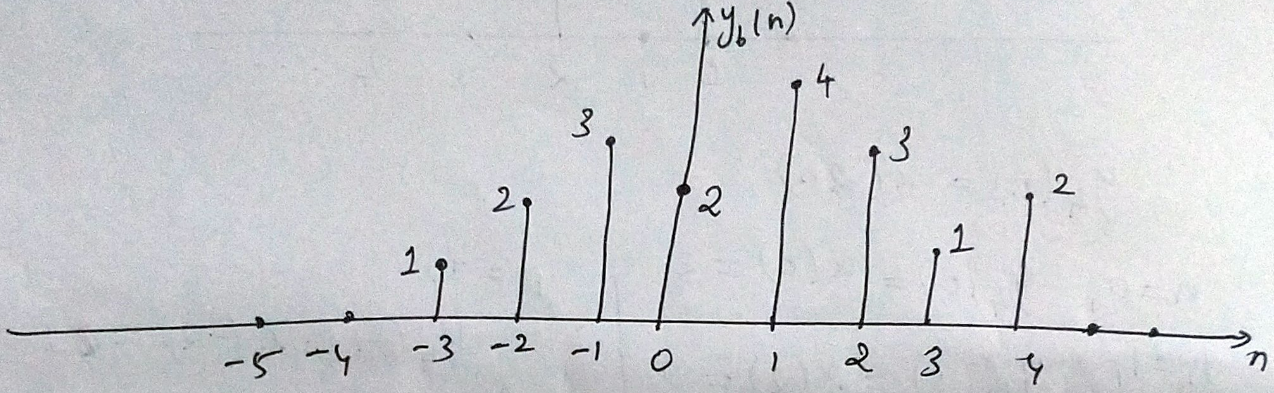


$\Leftarrow$ up sampling

$$y_1(n) = \{\ldots, 0, 0, 0, \underset{\substack{\uparrow \\ n=0}}{1}, 2, 3, 1, 2, 0 \ldots\}$$

$$y_2(n) = \{\ldots 0, 1, 2, 3, \underset{\substack{\uparrow \\ n=0}}{1}, 2, 0, 0, 0, 0, \ldots\}$$

$$y_b(n) = (y_1(n) + y_2(n)) \; \{\ldots 0, 1, 2, 3, \underset{\substack{\uparrow \\ n=0}}{2}, 4, 3, 1, 2, 0, \ldots\}$$



Properties of Unit Impulse Sequence:

Study with proof.

1) $\delta(n) = u(n) - u(n-1)$

2) $u(n) = \sum_{k=0}^{\infty} \delta(n-k)$

3) $x(n) = \sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k)$

4) $\sum_{n=-\infty}^{\infty} x(n)\,\delta(n-n_0) = x(n_0)$

5) $\delta(n) = \delta(-n)$, 6) $\delta(kn) = \delta(n)$

Exponential Sequence:

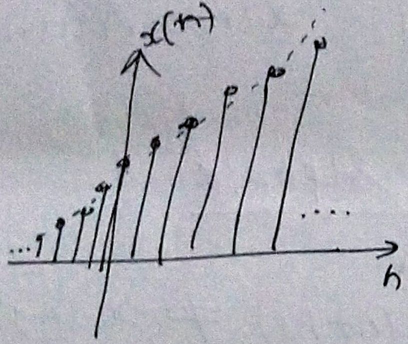
$$x(n) = a^n \text{ for all } n.$$

where, $a > 1 \Rightarrow$ sequence grows exponentially

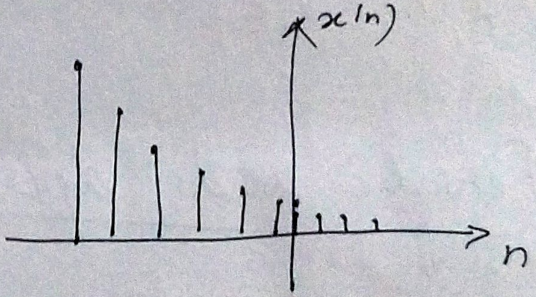
$0 < a < 1, \Rightarrow$ " decays "

$a < 0 \Rightarrow$ exponential signal takes alternate signs.

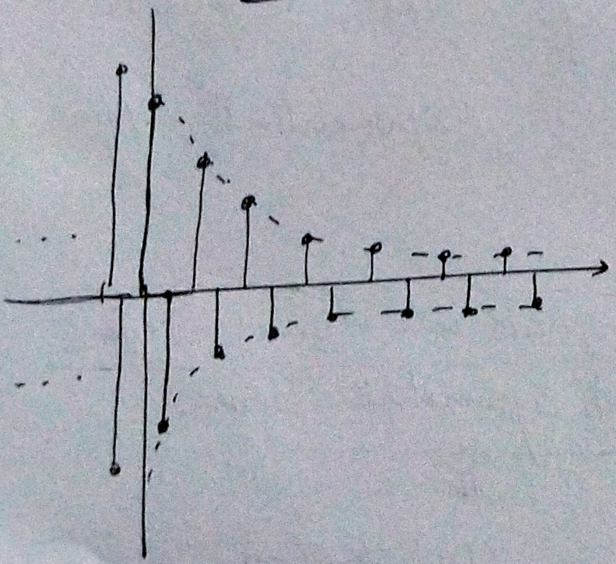
$a > 1$



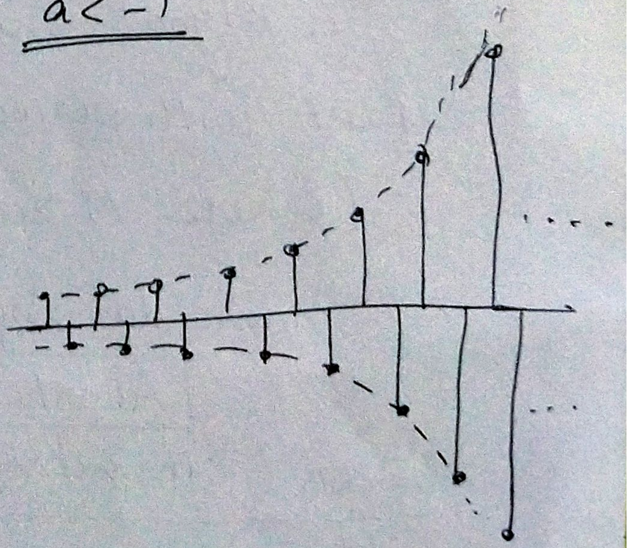
$0 < a < 1$



$-1 < a < 0$



$a < -1$



Classification of Signals:

1. Energy & Power Signal:

Energy signal:

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

If $E \rightarrow$ finite ($0 < E < \infty$)

then $x(n) \rightarrow$ energy signal

eg: $x(n) = Ae^{-an}$

Power signal:

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

If P is finite and non zero, then $x(n)$ is power signal.

eg: $x(n) = A u(n)$

2. Periodic and aperiodic Signal:

Periodic:

$x(n+N) = x(n), \forall n$

A signal should repeat with period "N". (where $N > 0$)

eg: Sinusoidal Signal

Aperiodic:

$x(n+N) \neq x(n)$

eg: Step, impulse, exponential signal.

$\boxed{\text{All periodic signals are Power Signals}}$

3. Even (Symmetric) & Anti Symmetric Signal:

Even (Symmetric):

$x(n) = x(-n)$

eg: $x(n) = a^{|n|}$

Anti Symmetric:

$x(n) \neq x(-n)$

(or)

$x(-n) = -x(n)$

eg: $x(n) = n$

Pbm: Check whether $x(n) = \sin\left(\frac{\pi}{3}n\right) + \cos\left(\frac{\pi n}{4}\right)$ is periodic or not. If periodic find the fundamental period.

$$x(n) = \underbrace{\sin\left(\frac{\pi}{3}n\right)}_{x_1(n)} + \underbrace{\cos\left(\frac{\pi n}{4}\right)}_{x_2(n)}$$

$$\Downarrow \qquad\qquad \Downarrow$$

$$N_1 = ? \qquad\qquad N_2 = ?$$

wkt, $\omega = 2\pi f = \frac{2\pi}{T}$ $\left[\because f = \frac{1}{T}\right.$

$T \rightarrow$ time period for CT signal

$N \rightarrow$ " " " DT "

$\therefore \omega = \frac{2\pi}{N} \Rightarrow N = \frac{2\pi}{\omega}$

from $x_1(n)$:

$N_1 = \frac{2\pi}{\pi/3}$ $\left[\because \omega = \pi/3\right.$

$N_1 = 6$ (periodic)

from $x_2(n)$:

$N_2 = \frac{2\pi}{\pi/4}$ $\left[\because \omega = \pi/4\right.$

$N_2 = 8$ (periodic)

Overall fundamental period $\Big\}$ $N = LCM(N_1, N_2)$

$= LCM(6, 8) = 24.$

eg: Check whether the signal

$x(n) = A \sin(6\pi n + \pi/6)$ is

Power signal or Energy signal.

Soln: Gn signal is periodic signal.

$\therefore$ Ans: Power signal.

---

eg: Find even and odd Component of the signal $x(n) = \{-2, 1, \underset{\uparrow}{2}, -4, 2\}$

$$\boxed{\begin{array}{l} \text{even Component } x_e(n) = \frac{1}{2}\{x(n) + x(-n)\} \\ \text{odd Component } x_o(n) = \frac{1}{2}\{x(n) - x(-n)\} \end{array}}$$

Ans: $x_e(n) = \{0, -1.5, \underset{\uparrow}{2}, -1.5, 0\}$

$x_o(n) = \{-2, 2.5, \underset{\uparrow}{0}, -2.5, 2\}$

(Or) You can solve it using graphical Representation.

Causal and Non-Causal sequence:

Causal:

$x(n) = 0, \; n < 0$

eg: $x(n) = \{0, \underset{\uparrow}{0}, 1, 2, 3\}$

Right sided sequence.

Non-Causal:

$x(n) = 0, \; n > 0$

eg: $x(n) = \{1, 2, \underset{\uparrow}{1}, 0, 0 \ldots\}$

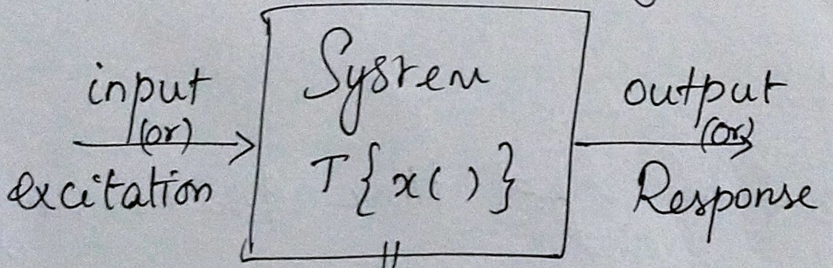
Left sided sequence.

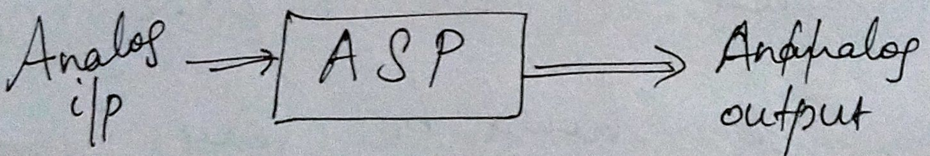
eg: $x(n) = \{1, \underset{\uparrow}{2}, 3, 1, 2, 0 \ldots\}$

2 sided sequence.

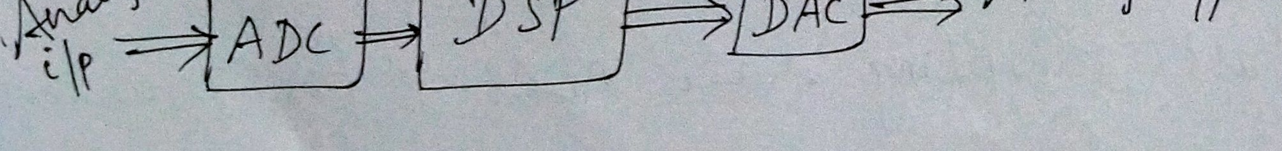
System:

→ physical device that performs an operation on the signal





(or)



Advantages of DSP:

* Flexibility: Reconfiguration by simply changing the program.
* Accuracy: better control of accuracy requirements.
* Easy Storage: in magnetic media so transport & process done in remote labs.
* Processing: Sophisticated than Analog.
* Cost effective: Cheaper.

Limitations:

⇒ to perform real time processing, Conversion speed of ADC & processing speed of DSP should be very high.

⇒ High Bandwidth signals require fast sampling rate ADCs & fast processors.

Applications of DSP:

* Speech processing : Compression & decompression for storage & txion & Rxion

* Communication : noise elimination by filter in txion channels. aswell as echo cancellation

* Bio medical : Spectrum Analysis of ECG & EEG signals....
  ECG ⇓ heart, EEG ⇓ brain

* Consumer electronics : Music Synthesis ie audio, Video.

* Seismology : to predict earthquake, Nuclear explosions, & earth movements.

* Image Processing : Compression & enhancement, finger print matching, to identify hidden images received by <u>Radars</u>.

Classification of DT Systems:

1. Static and dynamic systems.

Static or memoryless

$y(n) \propto x(n)$

(or)

$y(n) = f\{x(n)\}$

↑ present i/p.

eg: $y(n) = n\,x(n) + 6x^2(n)$

dynamic ⇓ have memory

$$y(n) = f\{x(n), \overbrace{x(n-1), x(n-2) \ldots}^{\text{past}} \; \underbrace{x(n+1), x(n+2) \ldots}_{\text{future}}\}$$

eg: $y(n) = x(n-1) + 2x(n)$

2. Time Invariant & Time Variant systems

If $y(n-k) = y'(n-k)$ — If not.

where

$y(n-k) \Rightarrow$ delaying input sequence

$y'(n-k) \Rightarrow$ delaying o/p sequence [ie put $n = n-k$

eg: $y(n) = x(n) - x(n-1)$

$y(n-k) = x(n-k) - x(n-1-k) = x(n-k) - x(n-k-1)$

$y'(n-k) = x(n-k) - x(n-k-1)$

$y(n-k) = y'(n-k)$, $\rightarrow$ Time Invariant.

pbm: $y(n) = n x(n)$

$y(n-k) = n \cdot x(n-k)$

$y'(n-k) = (n-k)\, x(n-k)$

$y(n-k) \neq y'(n-k)$, $\therefore$ Time Variant.

3. Linear and Non-linear Systems

Linear $\Downarrow$ Satisfies Superposition principle

Non-linear $\Downarrow$ do not Satisfy.

i.e. $\boxed{T[a_1 x_1(n) + a_2 x_2(n)] = a_1 T[x_1(n)] + a_2 T[x_2(n)]}$

where $y(n) = T\{x(n)\}$

eg: $y(n) = n x(n)$

$y_1(n) = T\{x_1(n)\} = n x_1(n)$

$y_2(n) = T\{x_2(n)\} = n x_2(n)$

$$T[a_1 x_1(n) + a_2 x_2(n)] = n[a_1 x_1(n) + a_2 x_2(n)]$$
$$= a_1 n x_1(n) + a_2 n x_2(n)$$
$$= a_1 T[x_1(n)] + a_2 T[x_2(n)]$$
$$= a_1 y_1(n) + a_2 y_2(n)$$

$\therefore$ Linear system.

Pbm: $y(n) = x^2(n^{*})$

$y_1(n) = T[x_1(n)] = x_1^2(n^{*})$

$y_2(n) = T[x_2(n)] = x_2^2(n^{*})$

$T[a_1 x_1(n) + a_2 x_2(n)] = [a_1 x_1(n) + a_2 x_2(n)]^2$

$= a_1^2 x_1^2(n) + a_2^2 x_2^2(n) + 2a_1 a_2 x_1(n) x_2(n)$

$\neq a_1 T[x_1(n)] + a_2 T[x_2(n)]$

Non-linear System

4. Causal and Non-Causal Systems:

Causal: $y(n) = f\{y(n-1), y(n-2) \dots, x(n), x(n-1), x(n-2) \dots\}$

eg: $y(n) = ax(n)$

$y(n) = x(n) - x(n-1)$

Non-Causal: also depends on future i/ps & o/ps. ie.

$y(n) = \{x(n), x(n-1), x(n-2) \dots, y(n-1), y(n-2) \dots, x(n+1), x(n+2) \dots, y(n+1), y(n+2) \dots\}$

eg: $y(n) = x(n) + 3x(n+4)$

$y(n) = x(n^2)$

5. Stable and Unstable Systems:

Stable $\Downarrow$ BIBO

$$\sum_{k=-\infty}^{\infty} |h(k)| < \infty$$

Unstable $\Downarrow$ If not

i.e. Unbounded output.

$h(n)$: ?

$$\boxed{\begin{aligned} \text{If } x(n) &= \delta(n) \\ y(n) &= h(n) \end{aligned}}$$

impulse response

eg: $y(n) = n\, x(n)$

$$h(n) = n \cdot \delta(n)$$

$$\sum_{n=-\infty}^{\infty} |h(n)| = \sum_{n=-\infty}^{\infty} |n \cdot \delta(n)|$$

$$= \dots |(-2)\cancel{\delta(-2)}^{0}| + |(-1)\cdot\cancel{\delta(-1)}^{0}| + |0 \cdot \overset{=1}{\delta(0)}| +$$

$$|(1)\cdot\underset{0}{\cancel{\delta(1)}}| + |(2)\,\underset{0}{\cancel{\delta(2)}}| + \dots$$

$$= 0+0+0+0+0 = 0$$

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty \quad \therefore \text{ Stable.}$$

eg: $y(n) = \cos[x(n)]$, $\Leftarrow$ check the Stability.

6. FIR and IIR systems

FIR ⇓ $h(n)$ has finite no. of samples

IIR ⇒ $h(n)$ has infinite no. of samples.

7. Recursive and Non-Recursive DT S/s:

Recursive ⇓

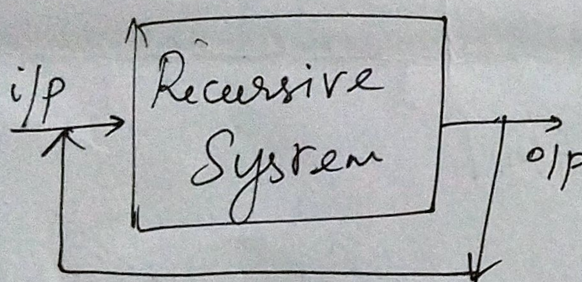
$$y(n) = f\{y(n-1), y(n-2) \ldots x(n), x(n-1) \ldots\}$$

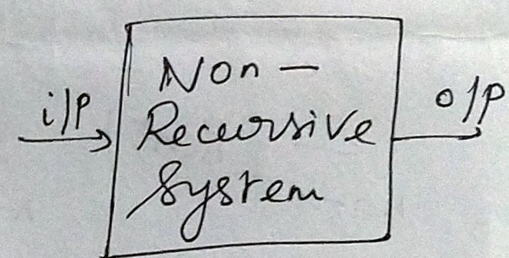
past o/ps, present & past i/ps

Non-Recursive ⇓

$$y(n) = f\{x(n), x(n-1), x(n-2) \ldots\}.$$

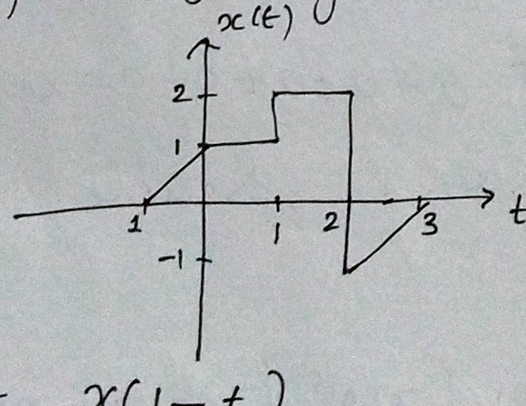
only present & past i/ps.





pbm:

If a plot of signal $x(t)$



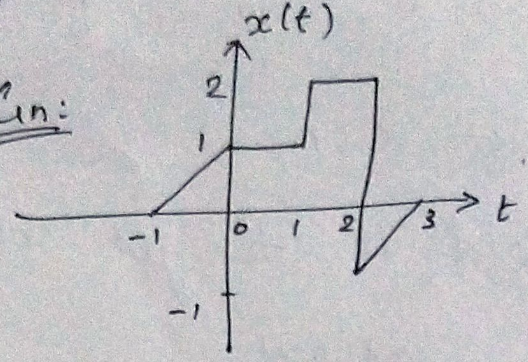
then plot $x(1-t)$.

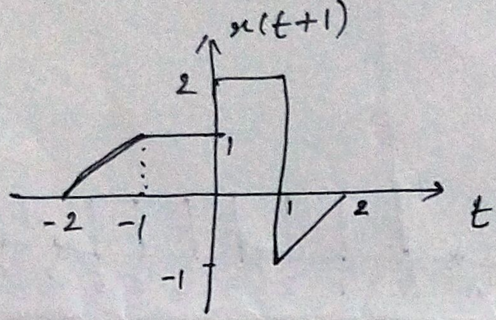
Soln: $x(1-t) = x(-t+1)$

Step 1 : plot $x(t+1)$

Step 2 : fold $x(t+1)$ to get $x(-t+1)$

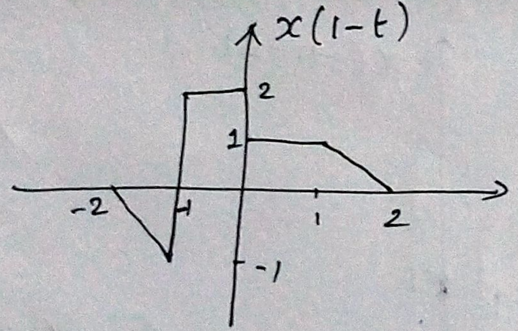
Step 1 : Gin:





left shift $x(t)$ by 1 time.

Step 2:

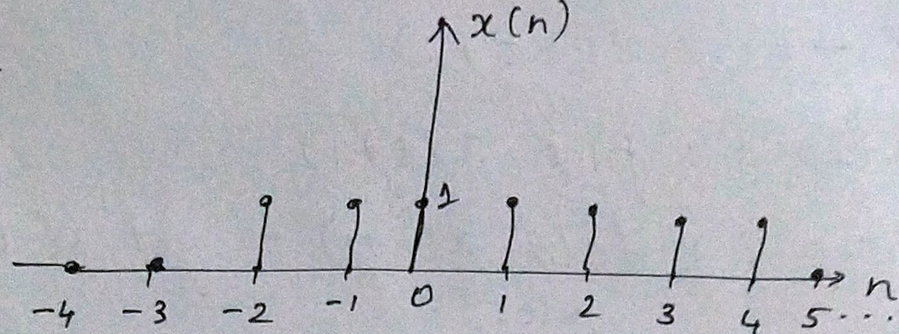


Pbm: $x(n)$ is defined as

$$x(n) = \begin{cases} 0, & \text{for } n < -2 \text{ or } n > 4 \\ 1 & \text{otherwise} \end{cases}$$

Determine the ~~fe~~ value of n for which $x(-n-2)$ is guaranteed to be zero.

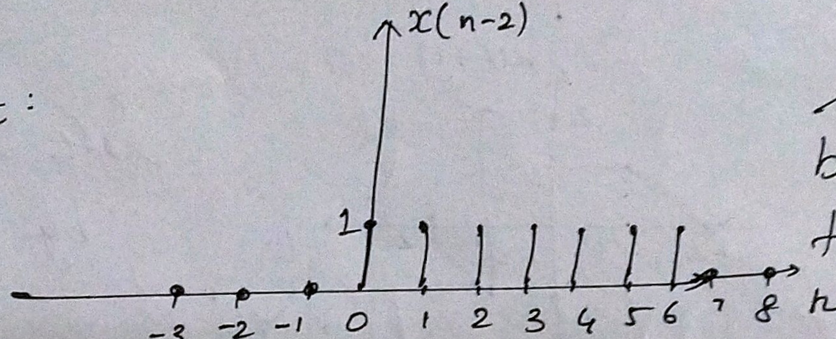
Soln:



Step 1 : plot $x(n-2)$

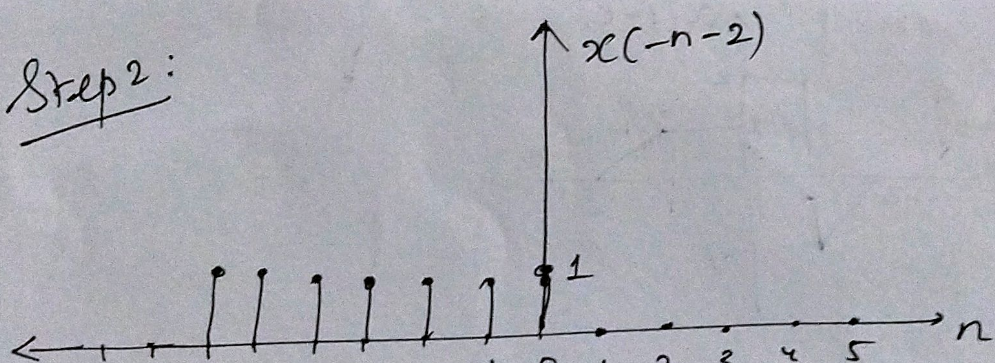
Step 2 : fold it to get $x(-n-2)$

Step 1 :



Shift $x(n)$ by 2 times towards Right.

Step 2 :



Ans:

$$x(-n-2) = \begin{cases} 0, & \text{for } n > 0 \text{ and } n < -6 \\ 1 & \text{otherwise} \end{cases}$$