REALIZATION OF IIR FILTER:

1. Direct form Realization → I, II
2. Cascade form Realization
3. Parallel form Realization

IIR FILTER

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1+\sum_{k=1}^{N} a_k z^{-k}}$$

Direct form-I Realization: $(M+N)$ additions & $(M+N+1)$ memory locations, $(M+N)$ multiplications

Similar to Block diagram Representation of the filter.

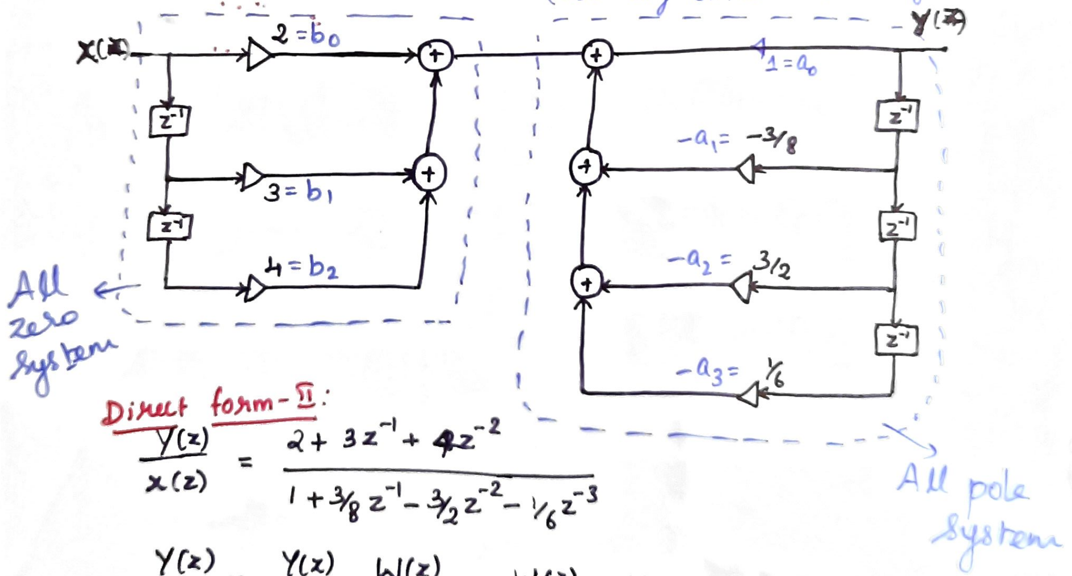
Direct form-II Realization: $(M+N)$ additions, $(M+N+1)$ multiplications & memory locations $= \max\{M, N\}$

(or) Canonic → to reduce the number of delay elements and memory locations

Ex: Obtain the direct form-I, II structures of LTI system governed by the eqn,

$$y(n) = \frac{-3}{8}y(n-1) + \frac{3}{2}y(n-2) + \frac{1}{6}y(n-3) + 2x(n) + 3x(n-1) + 4x(n-2)$$

Direct form-I Realization: (directly obtained from system function)



Direct form-II:

$$\frac{Y(z)}{X(z)} = \frac{2+3z^{-1}+4z^{-2}}{1+\frac{3}{8}z^{-1}-\frac{3}{2}z^{-2}-\frac{1}{6}z^{-3}}$$

$$\frac{Y(z)}{X(z)} = \frac{Y(z)}{X(z)}\cdot\frac{W(z)}{W(z)} = \frac{W(z)}{X(z)}\cdot\frac{Y(z)}{W(z)}$$

$$\frac{W(z)}{X(z)}\cdot\frac{Y(z)}{W(z)} = \left(2+3z^{-1}+4z^{-2}\right)\left(\frac{1}{1+\frac{3}{8}z^{-1}-\frac{3}{2}z^{-2}-\frac{1}{6}z^{-3}}\right)$$

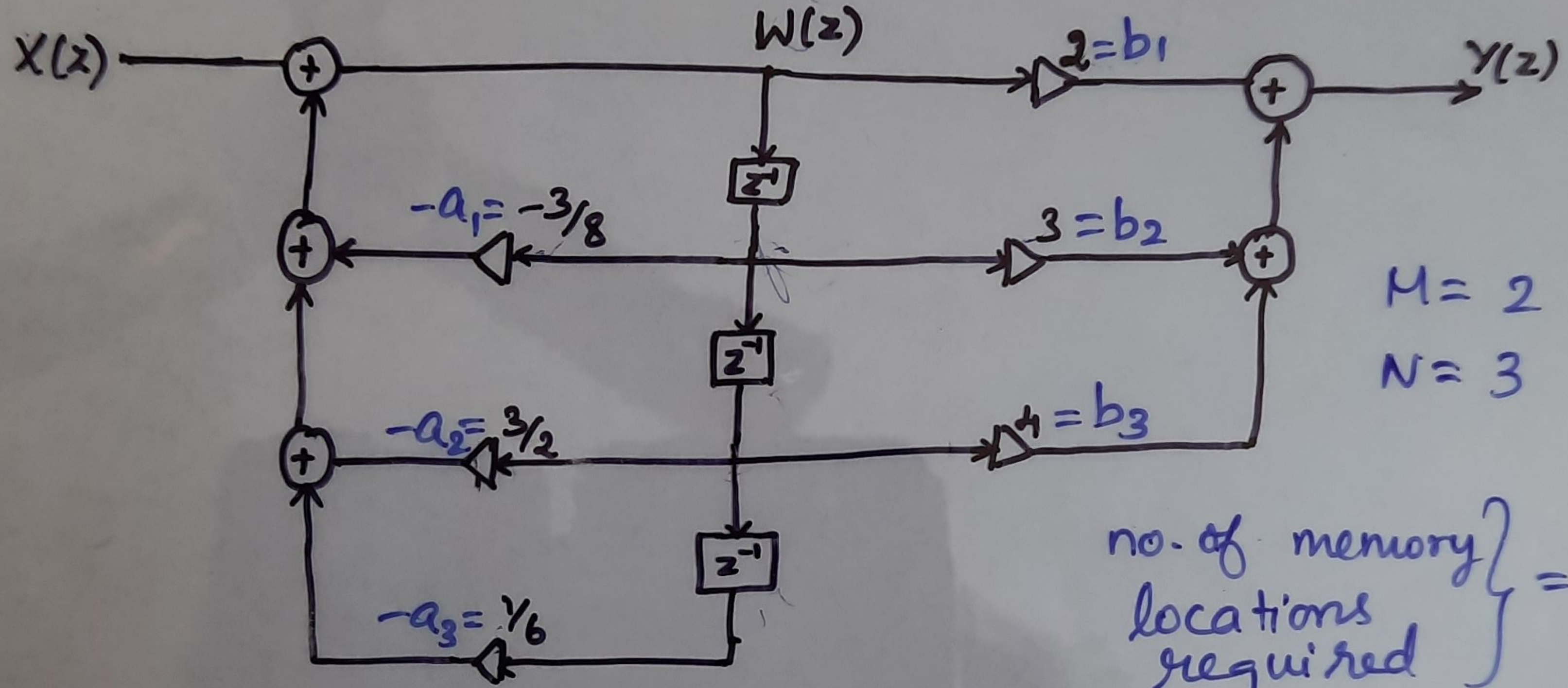
I.R.2

$$\frac{W(z)}{X(z)} = \frac{1}{1+\frac{3}{8}z^{-1}-\frac{3}{2}z^{-2}-\frac{1}{6}z^{-3}} \quad ; \quad X(z) = W(z) + \tfrac{3}{8}z^{-1}W(z) - \tfrac{3}{2}z^{-2}W(z) - \tfrac{1}{6}z^{-3}W(z)$$

$$\frac{Y(z)}{W(z)} = (2+3z^{-1}+4z^{-2})$$

$$W(z) = X(z) - \tfrac{3}{8}z^{-1}W(z) + \tfrac{3}{2}z^{-2}W(z) + \tfrac{1}{6}z^{-3}W(z)$$

$$Y(z) = 2W(z) + 3z^{-1}W(z) + 4z^{-2}W(z)$$

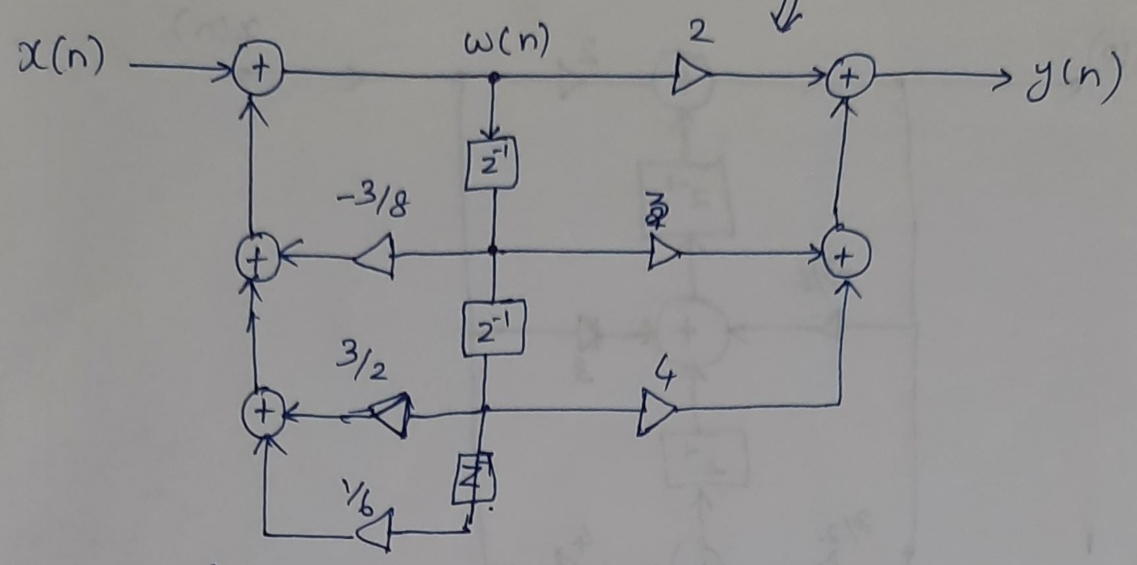


$M = 2$

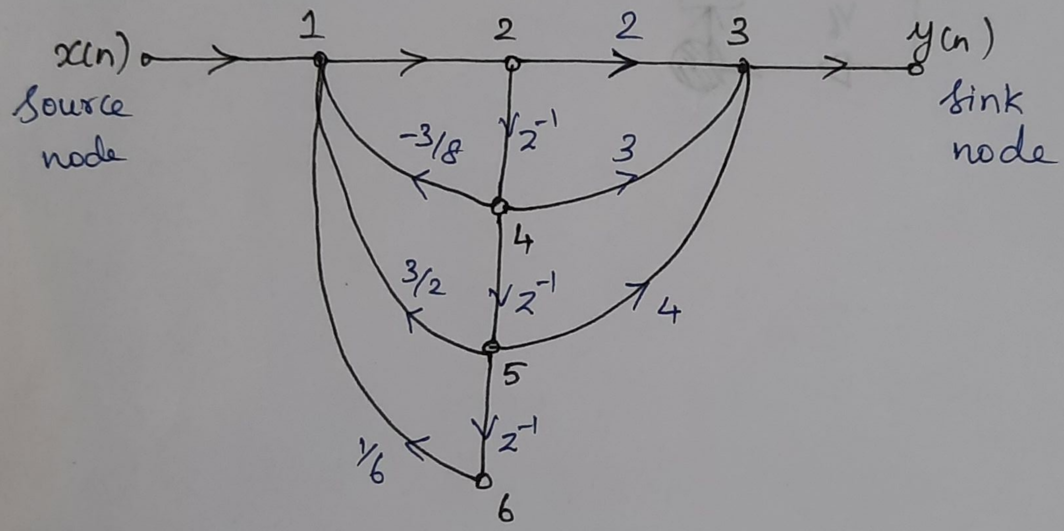
$N = 3$

no. of memory locations required $\Big\} = 3$

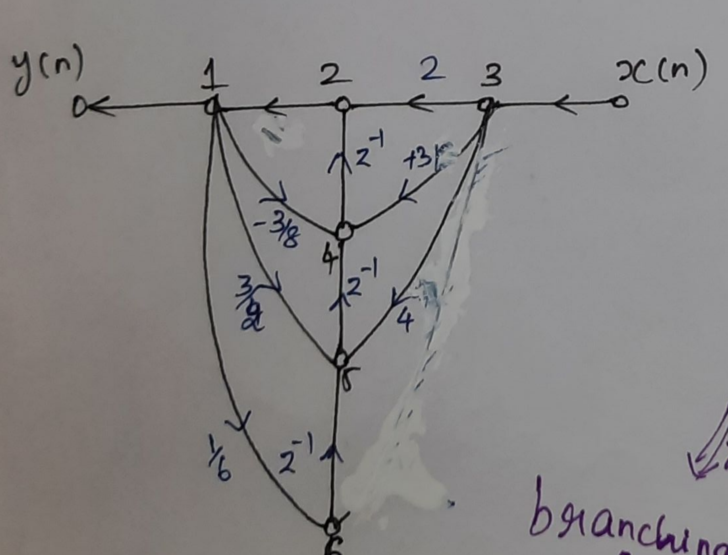
Signal flow graph of direct form II structure:



Signal flow graph:



Transposed structure: ⇒ using transposition or flow graph-reversal theorem.



⇓

→ Reverse directions of all branch transmittances

→ interchange i/p & o/p

→ system functions remains unchanged.

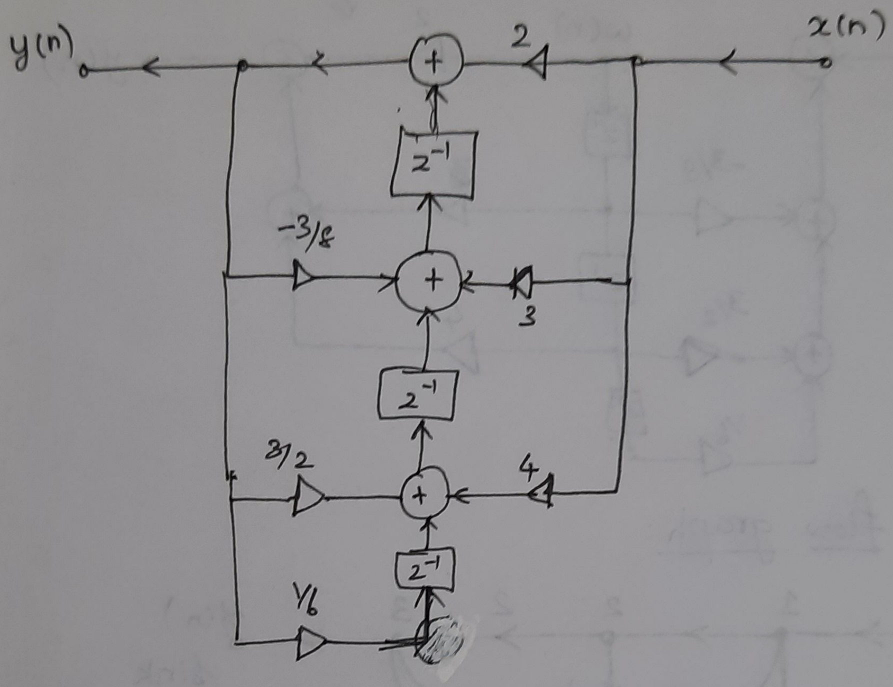
⇓ results in

branching nodes

↕

summing nodes

Transposed direct form-II Structure:



order sections i.e. $H(z) = H_1(z) \cdot H_2(z) \dots$

(ii) Realize each transfer function $H_1(z), H_2(z) \dots$ in direct form-II realization.

(iii) connect all the individual in cascade form (or) in series form as shown in fig.

Ex: Obtain a cascade realization of the system characterised by the transfer function

$$H(z) = \frac{2(z+2)}{z(z-0.1)(z+0.5)(z+0.3)}$$

Soln:

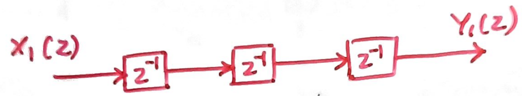
$$H(z) = \frac{2\cancel{z}\cdot(1+2z^{-1})\times z^{-1}}{\cancel{z}(1-0.1z^{-1})\, z^{-1}(1+0.5z^{-1})\, z^{-1}(1+0.4z^{-1})}$$

$$= \frac{2z^{-3}(1+2z^{-1})}{(1-0.1z^{-1})(1+0.5z^{-1})(1+0.4z^{-1})}$$

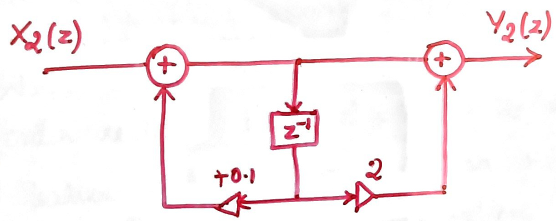
$$= 2z^{-3}\cdot\frac{1+2z^{-1}}{1-0.1z^{-1}}\cdot\frac{1}{1+0.5z^{-1}}\cdot\frac{1}{1+0.4z^{-1}}$$

$$H(z) = 2\, H_1(z)\cdot H_2(z)\cdot H_3(z)\cdot H_4(z)$$

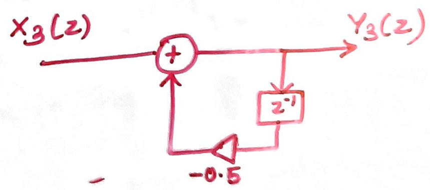
$H_1(z) = z^{-3}$



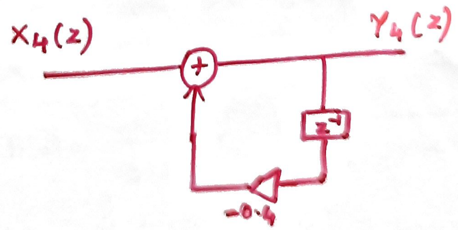
$H_2(z) = \dfrac{1+2z^{-1}}{1-0.1z^{-1}}$



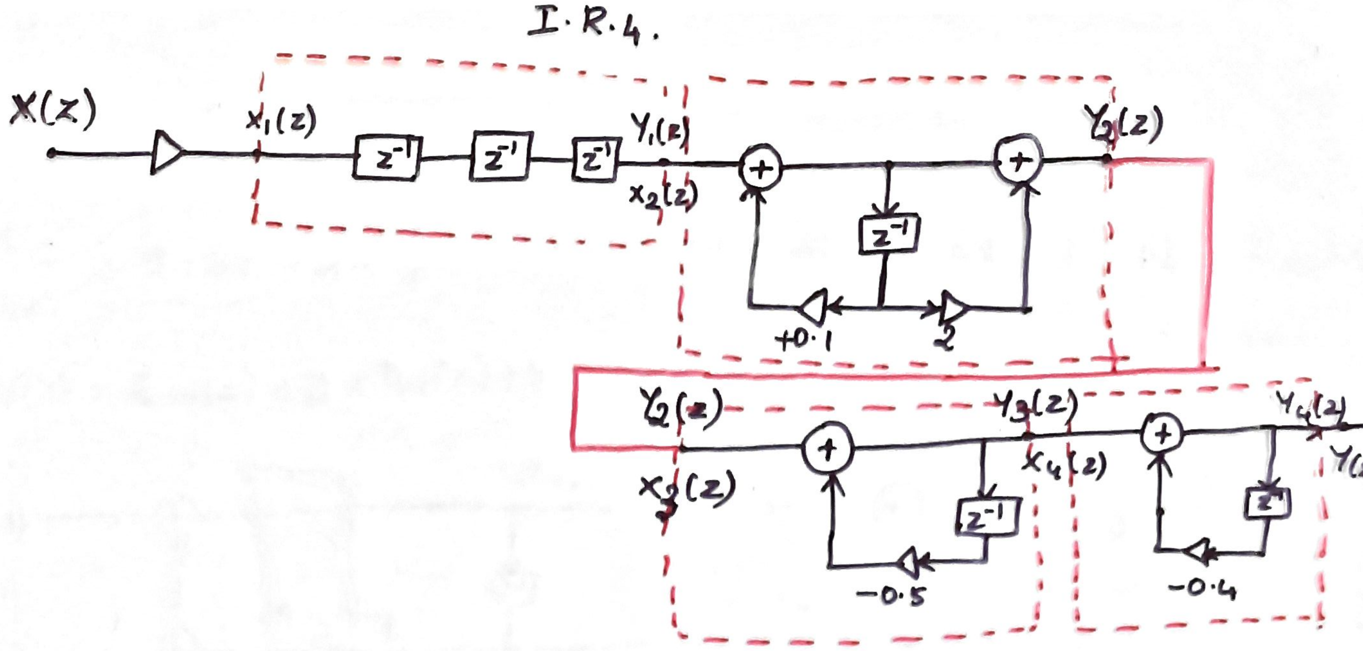
$H_3(z) = \dfrac{1}{1+0.5z^{-1}}$



$H_4(z) = \dfrac{1}{1+0.4z^{-1}}$



I.R.4.

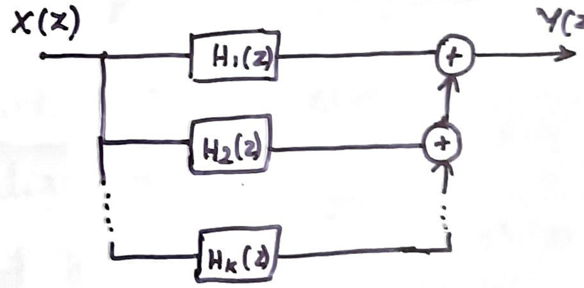


Parallel form structure:

Step (i): Write H(z) as the sum of first & second order sections using partial fraction expansion technique.

Step (ii): Realize each transfer function in Direct form-II structure.

Step (iii): Connect all the individual structures in parallel form as shown in fig.



Ex: Draw the parallel form structure of a DT system represented by transfer function,

$$H(z) = \frac{1 + 3z^{-1} + 2z^{-2}}{1 + \frac{3}{8}z^{-1} - \frac{3}{32}z^{-2} - \frac{1}{64}z^{-3}}$$

Soln:

$$H(z) = \frac{z^{-2}(z^2+3z+2)}{z^{-3}\left(z^3 + \frac{3}{8}z^2 - \frac{3}{32}z - \frac{1}{64}\right)} = \frac{z^{-2}(z+1)(z+2)}{z^{-3}\left(z+\frac{1}{8}\right)\left(z^2+\frac{2}{8}z-\frac{8}{64}\right)}$$

$\Downarrow$

$\frac{1}{8}$

$\frac{1}{8}$	1	3/8	−3/32	−1/64
	↓	1/8	2/64	−1/64
	1	2/8	−8/64	0

I.R.5

$$H(z) = \frac{z^{-1}(z+1)\cdot z^{-1}(z+2)}{z^{-1}(z+\frac{1}{8})\cdot z^{-2}(z^2+\frac{2}{8}z-\frac{1}{8})}$$

$$z^2+\frac{2}{8}z-\frac{1}{8} = (z-\tfrac{1}{4})(z+\tfrac{1}{2})$$

$$H(z) = \frac{z^{-1}(z+1)\cdot z^{-1}(z+2)}{z^{-1}(z+\frac{1}{8})z^{-1}(z-\frac{1}{4})z^{-1}(z+\frac{1}{2})}$$

$$H(z) = \frac{(1+z^{-1})(1+2z^{-1})}{(1+\frac{1}{8}z^{-1})(1-\frac{1}{4}z^{-1})(1+\frac{1}{2}z^{-1})}$$

$$H(z) = \frac{A}{1+\frac{1}{8}z^{-1}} + \frac{B}{1-\frac{1}{4}z^{-1}} + \frac{C}{1+\frac{1}{2}z^{-1}}$$

$$H(z) = \underbrace{\frac{-35/3}{1+\frac{1}{8}z^{-1}}}_{H_1(z)} + \underbrace{\frac{8/3}{1-\frac{1}{4}z^{-1}}}_{H_2(z)} + \underbrace{\frac{10}{1+\frac{1}{2}z^{-1}}}_{H_3(z)}$$

Parallel form structure: