Frequency Response Analysis

→ The sinusoidal transfer function is the frequency domain representation of the s/m & it is called as freq. domain transfer function.

→ $T(s) = \frac{C(s)}{R(s)}$

$s = j\omega \Rightarrow T(j\omega) = \frac{C(j\omega)}{R(j\omega)}$.

$$\boxed{T(j\omega) = |T(j\omega)| \angle T(j\omega)}$$

→ Freq. Response :- $T(j\omega)$ is separated into magnitude and phase function. They will be real functions of ω and are called as freq. Response.

OLTF $\Rightarrow G(s) \Rightarrow G(j\omega) \Rightarrow |G(j\omega)| \angle G(j\omega)$

loop Transfer function $\Rightarrow G(s)H(s) \Rightarrow G(j\omega)H(j\omega)$

$\Rightarrow |G(j\omega)H(j\omega)| \angle G(j\omega)H(j\omega)$

CLTF $\Rightarrow M(s) \Rightarrow M(j\omega) \Rightarrow |M(j\omega)| \angle M(j\omega)$.

→ Freq. Domain Specifications :-

(1) Resonant peak (M_r)
(2) Resonant freq. (ω_r)
(3) Bandwidth (ω_b)
(4) Cut-off rate
(5) Gain Margin (K_g)
(6) phase Margin (γ).

1) Resonant Peak (M_r) ⇒ The max. value of magnitude of CLTF is called resonant peak. A large resonat peak corresponds to a large overshoot in transient response.

$$\boxed{M_r = \frac{1}{2\zeta\sqrt{1-\zeta^2}}}$$

2) Resonant frequency (ω_r) → The freq. at which resonant peak (M_r) occurs is called resonant freq. It is the indicative of the speed of transient response.

$$\boxed{\omega_r = \omega_n\sqrt{1-2\zeta^2}}$$

3) Bandwidth ⇒ It is the range of freq. for which normalized gain of the s/m is more than -3dB. The freq. at which the gain is -3dB is called cut-off frequency. A large BW corresponds to a small rise time or fast response.

$$\omega_b = \omega_n\left[\sqrt{1-2\zeta^2+\sqrt{2-4\zeta^2+4\zeta^4}}\right]$$

$$= \omega_n\left[1-2\zeta^2+\sqrt{2-4\zeta^2+4\zeta^4}\right]^{1/2}$$

4) cut-off Rate :-

The slope of the log. magnitude curve near the cut-off freq. is called cut-off rate. This rate indicates the ability of the s/m to distinguish the signal from noise.

5) Gain Margin (K_g) :- Defined as the value of gain to be added to s/m in order to bring the s/m to the verge of instability.

$$\boxed{K_g = \frac{1}{|G(j\omega_{pc})|} \Rightarrow (K_g)_{dB} \Rightarrow 20 \log \frac{1}{|G(j\omega_{pc})|}}$$

$$G(j\omega_{pc}) \Rightarrow G(s)\Big|_{\omega=\omega_{pc}}.$$

$\omega_{pc} \rightarrow$ phase cross over freq. It is the freq at which the phase of OLTF is 180°.

6) Phase Margin (γ) :- Defined as the additional phase lag to be added at the gain cross over freq to bring the s/m to the verge of instability.

$$\boxed{\gamma = 180^\circ + \phi_{gc}}$$

$$\phi_{gc} \Rightarrow \angle G(j\omega_{gc})$$

$\omega_{gc} \rightarrow$ gain cross over freq. It is the freq at which the magnitude of OLTF is unity.

$$\phi = 90^\circ - \tan^{-1}\left[\frac{\left[-2\zeta^2 + \sqrt{4\zeta^4+1}\right]^{1/2}}{2\zeta}\right]$$

$\rightarrow$ $\omega_r = \omega_n\sqrt{1-2\zeta^2}$

$$M_r = \frac{1}{2\zeta\sqrt{1-\zeta^2}}.$$

$\zeta = 0 \Rightarrow \omega_r = \omega_n$

$M_r = \infty.$

@ $1-2\zeta^2 = 0 \Rightarrow \omega_r = 0 \Rightarrow$ no resonant peak at this condt.

$\zeta^2 = \frac{1}{2}$

$\zeta = \frac{1}{\sqrt{2}}$

$\boxed{0 < \zeta < 1/\sqrt{2} \Rightarrow \omega_r < \omega_n \text{ \& } M_r > 1}$

$\boxed{\zeta \geq 1/\sqrt{2} \Rightarrow \text{no resonant peak \& } M_r = 1}$

1) Bode plot.

General procedure for constructing Bode plots :-

1) Rewrite the transfer function in the time constant form.

2) Identify the corner frequencies associated with each factor of the transfer function.

3) Knowing the corner frequencies, draw the asymptotic magnitude plot.

for each corner freq $\Rightarrow$ slope $+20$ db/decade for a zero. &

slope -20 dB/dec for a pole.

→ m ~~Double~~ pole $\Rightarrow$ $-(20m)$ dB/dec, slope
m ~~Double~~ zero $\Rightarrow$ $+(20m)$ dB/dec slope.

→ m complex conj pole $\Rightarrow$ $(-40m)$ dB/dec. slope
m complex conj zero $\Rightarrow$ $(+40m)$ dB/dec slope.

magnitude Response of the plot.

1) choose an arbitrary frequency ω_l, which is lesser than the lowest corner freq. calculate the db magnitude of K or $K/(j\omega)^n$ or $K(j\omega)^n$ at ω_l and at lowest corner frequency.

2) Then calculate the gain (dB) at every corner frequency by using the formula.

$$\text{Gain at } \omega_y = (\text{change in gain from } \omega_x \text{ to } \omega_y) + (\text{Gain at } \omega_x)$$

$$= \left[(\text{slope from } \omega_x \text{ to } \omega_y) \times \log \frac{\omega_y}{\omega_x}\right] + (\text{gain at } \omega_x).$$

3) choose an arbitrary freq, ω_h, which is greater than the highest corner frequency. calculate the gain at ω_h

→ Determination of gain Margin & phase Margin from Bode plot :-

$$\rightarrow \quad K_g = \frac{1}{|G(j\omega_{pc})|} \Rightarrow 20 \log \frac{1}{|G(j\omega_{pc})|}$$

$$\Rightarrow 20\left[(\log 1) - \log(|G(j\omega_{pc})|)\right]$$

$$\boxed{K_g\,(dB) \Rightarrow -20 \log\left[|G(j\omega_{pc})|\right]}$$

$\rightarrow \gamma = 180^\circ + \phi_{gc}$

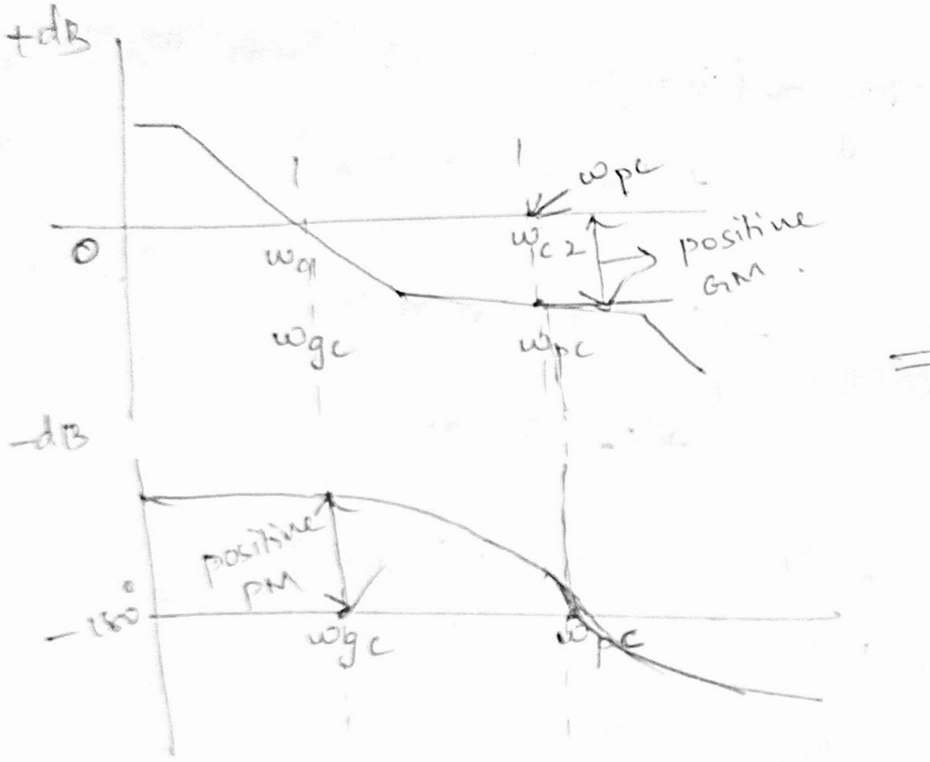
$\gamma = 180^\circ + \angle G(j\omega_{gc})$

$\omega_{gc} \rightarrow$ is the freq. at which $|G(j\omega)|$ is unity

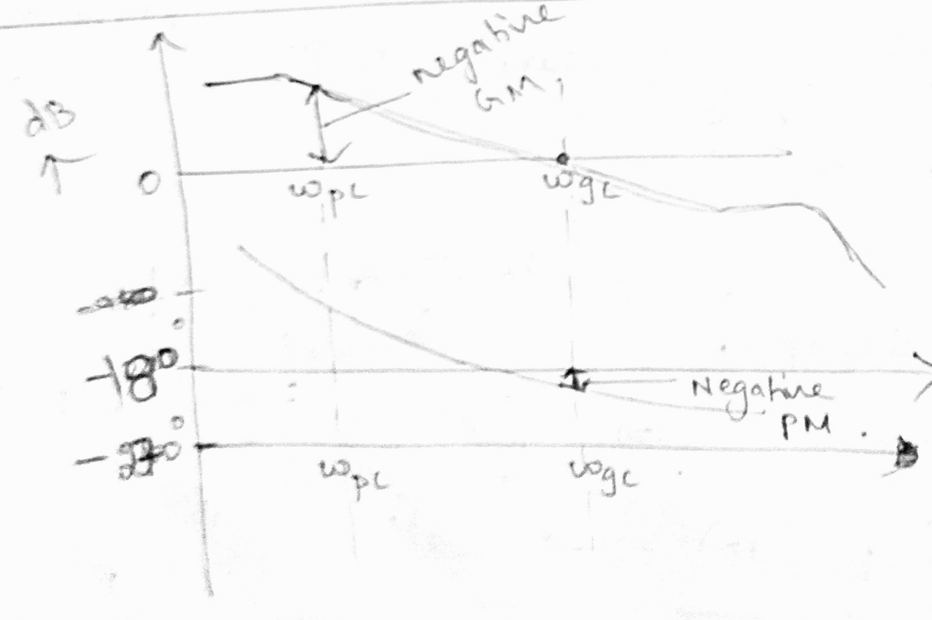
& hence $\left[|G(j\omega)|\right]_{dB}$ is zero.

γ is $-$ve, if $\phi_{gc} < -180^\circ$.

+ve, if $\phi_{gc} > -180^\circ$.



$\Rightarrow$ positive PM & GM



$\Rightarrow$ Negative PM & GM.

Q1) Sketch the Bode plot for the following, & determine the s/m gain K for the gain cross over freq. to be 5 rad/sec.

$$G(s) = \frac{Ks^2}{(1+0.2s)(1+0.02s)}$$

soln let $K=1$

i) $s \rightarrow j\omega$

$$G(j\omega) = \frac{K(j\omega)^2}{(1+0.2j\omega)(1+0.02j\omega)}$$

ii) corner frequencies & slope

$$\omega_{c1} = 1/0.2 \Rightarrow 5 \text{ rad/sec}$$

$$\omega_{c2} = \frac{1}{0.02} \Rightarrow 50 \text{ rad/sec}$$

Term	corner freq (rad/sec)	slope (db/dec)	change in slope (db/dec)
$(j\omega)^2$	–	+40	–
$\frac{1}{1+0.2j\omega}$	$\omega_{c1} = \frac{1}{0.2} = 5$	−20	$40-20 = 20$
$\frac{1}{1+0.02j\omega}$	$\omega_{c2} = \frac{1}{0.02} = 50$	−20	$20-20 = 0$

(iii) choose ω_l, such that less than lower ω_c, &
ω_h such that greater than ~~the~~ corner frequencies.

$\omega_l = 0.5$ rad/sec

$\omega_h = 100$ rad/sec.

(iv) calculation of $|G(j\omega)|$ in dB,

$$|G(j\omega)|_{\omega=\omega_l} \Rightarrow 20 \log |(j\omega)^2|$$

$$= 20 \log (\omega^2)$$

$$= 20 \log (0.5^2)$$

$$= \underline{\underline{-12 \text{ dB}}}.$$

$$|G(j\omega)|_{\omega=\omega_{c1}} \Rightarrow 20 \log |(j\omega_{c1})^2|$$

$$\Rightarrow 20 \log |5^2|$$

$$= \underline{\underline{28 \text{ dB}}}.$$

$$|G(j\omega)|_{\omega=\omega_{c2}} \Rightarrow \left[(\text{Slope from } \omega_{c1} \text{ to } \omega_{c2}) \times \log \frac{\omega_{c2}}{\omega_{c1}} \right]$$

$$+ A|_{\omega=\omega_{c1}}$$

$$= 20 \times \log \frac{50}{5} + 28$$

$$= \underline{\underline{48 \text{ dB}}}$$

$$|G(j\omega)|_{\omega=\omega_h} \Rightarrow (\text{slope from } \omega_{c2} \text{ to } \omega_h) \times \log \left(\frac{\omega_h}{\omega_{c2}}\right)$$

$$+ A|_{\omega=\omega_{c2}}$$

$$= 0 \times \log \frac{100}{50} + 48 \Rightarrow \underline{48 \text{ dB}}$$

(v) calculation of k.

given $\omega_{gc} = 5$ rad/sec.

At $\omega_{gc} \Rightarrow |G(j\omega_{gc})| = 1$

$\therefore \quad |G(j\omega_{gc})|_{dB} = 0.$

At $\omega = \omega_{gc} = 5$ rad/sec $\Rightarrow \omega_{c1}$

$|G(j\omega_{gc})|_{dB} = 28$ dB

To make it zero add (−28 dB) to it $\Rightarrow$ 28 dB − 28 dB $\Rightarrow$ 0 dB

$\therefore$ At each point of mag. plot, add −28 dB.

At $\omega = \omega_l \Rightarrow |G(j\omega)| = -12 - 28 = -40$ dB

$\omega = \omega_{c1} \Rightarrow |G(j\omega)| = 0 \Rightarrow 0$ dB

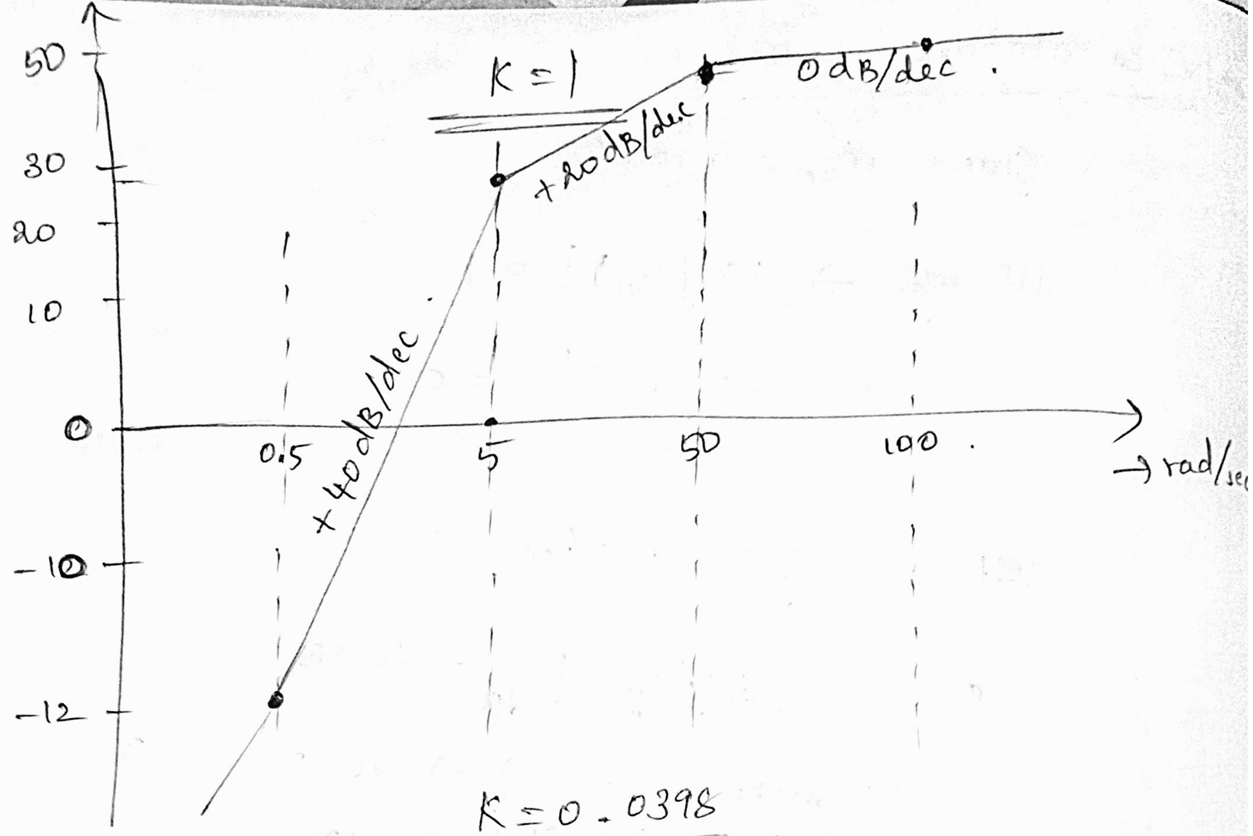
$\omega = \omega_{c2} \Rightarrow |G(j\omega)| = 48 - 28 = 20$ dB

$\omega = \omega_h \Rightarrow |G(j\omega)| = 48 - 28 = 20$ dB.

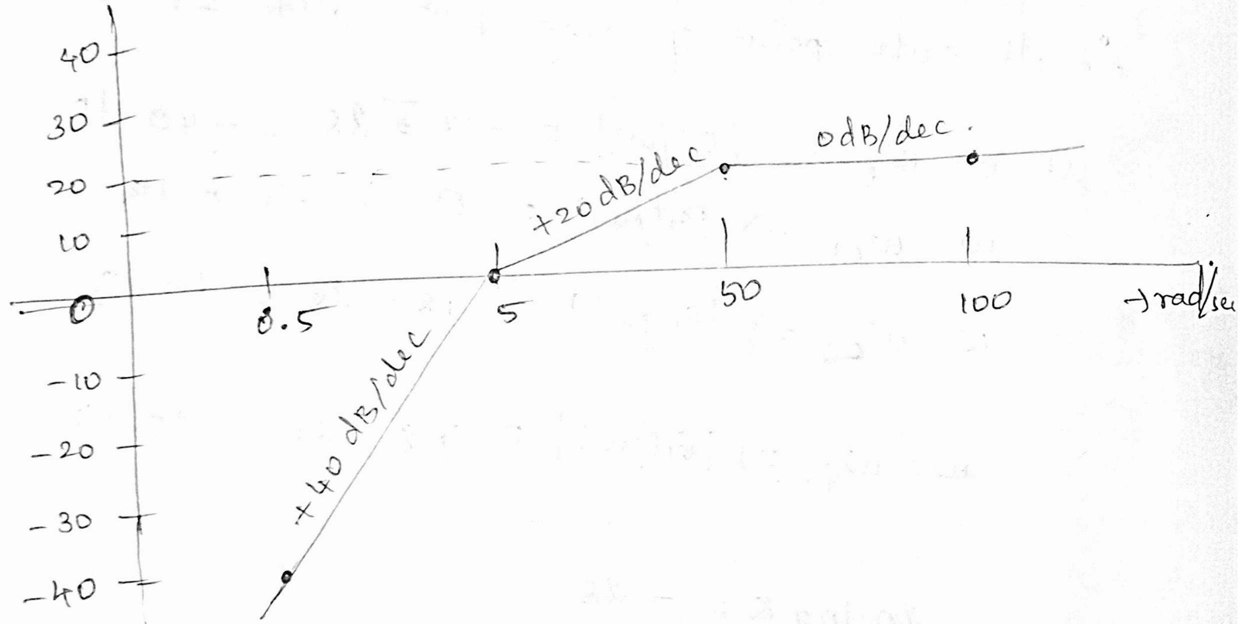
$$20 \log K = -28$$

$$\log K = \frac{-28}{20}$$

$$K = 10^{(-28/20)} = \underline{0.0398}$$



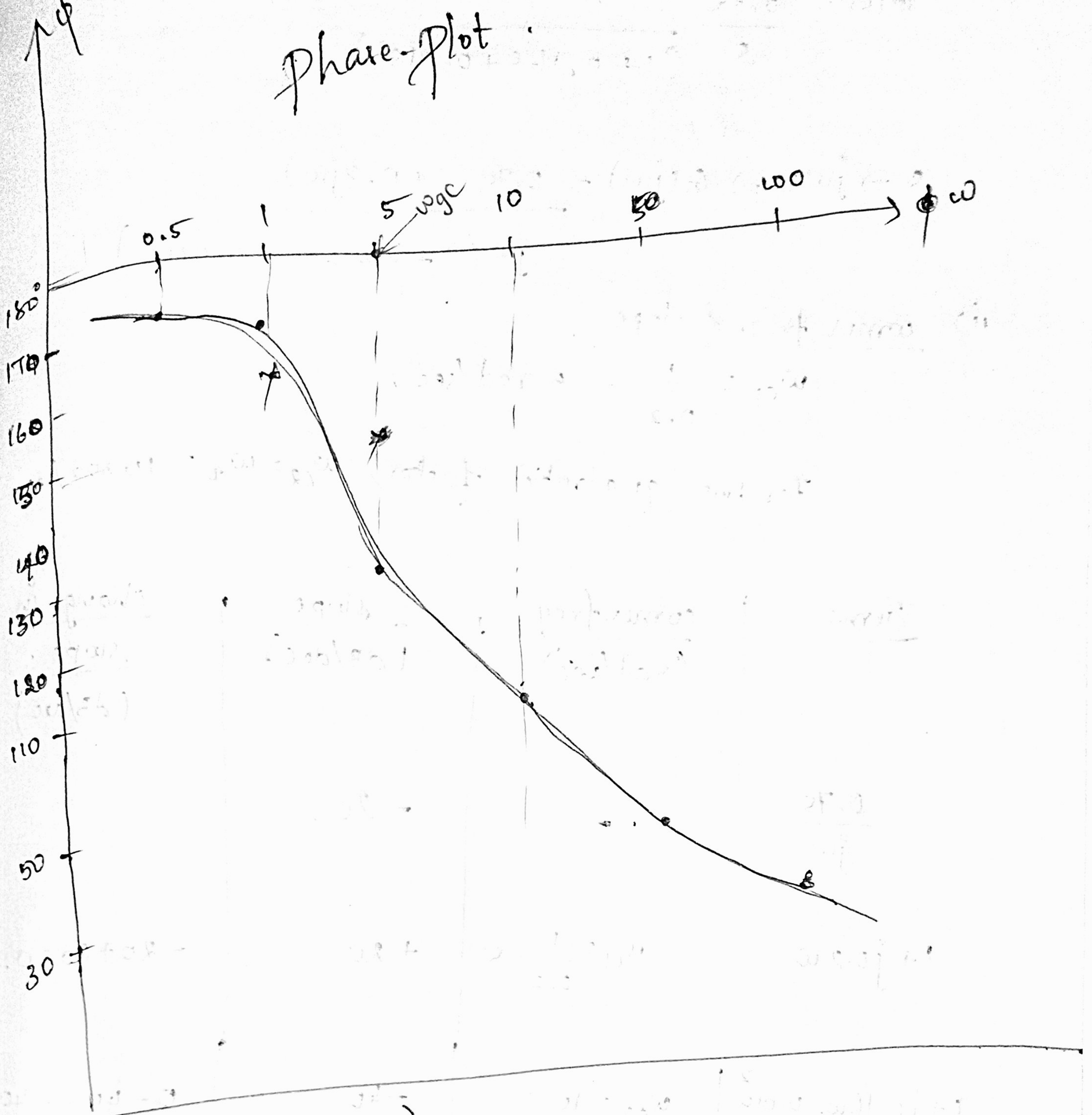
K = 0.0398



(vi) Phase Plot :- $\phi = \angle G(j\omega) = 180^\circ - \tan^{-1}(0.2\omega) - \tan^{-1}(0.02\omega)$

ω	$\tan^{-1} 0.2\omega$	$\tan^{-1} 0.02\omega$	$\phi = \angle G(j\omega)$
0.5	5.7	0.6	≃ 174
1	11.3	1.1	168
5	45	5.7	130
10	63.4	11.3	106
50	84.3	45	51
100	87.1	63.4	30

Phase-Plot



2) $G(s) = \dfrac{75(1+0.2s)}{s(s^2+16s+100)}$.

soln

$s^2+16s+100 \Rightarrow s^2 + 2\zeta\omega_n s + \omega_n^2$

$\omega_n^2 = 100 \Rightarrow \omega_n = 10$

$2\zeta\omega_n = 16 \Rightarrow \zeta = \dfrac{16}{2\times10} = 0.8$

→ i) convert the tf into time constant form.

$G(s) = \dfrac{75(1+0.2s)}{100s\left(s^2/100 + \frac{16}{100}s + 1\right)}$.

$$G(s) = \frac{0.75}{s} \frac{(1+0.2s)}{(1+0.16s+0.01s^2)}$$

$$s \to j\omega \Rightarrow G(j\omega) = \frac{0.75(1+0.2j\omega)}{j\omega(1+j0.16\omega - 0.01\omega^2)}$$

ii) corner freq. & slope

$$\omega_{c1} = \frac{1}{0.2} = 5 \text{ rad/sec.}$$

For the quadratic factor, $\omega_{c2} = \omega_n = 10$ rad/sec.

Term	corner freq. (rad/sec)	slope (dB/dec)	change in slope. (dB/dec)
$\frac{0.75}{j\omega}$	–	-20.	
$1+j0.2\omega$	$\omega_{c1} = \frac{1}{0.2} = 5$	$+20$	$-20+20=0$.
$1+j0.16\omega - 0.01\omega^2$	$\omega_{c2} = 10$	-40	$0-40=-40$.

iii) choosing ω_l & ω_h

$\omega_l = 0.5$ rad/sec.

$\omega_h = 20$ rad/sec.

iv) calculation of gain

$$\omega = \omega_l \Rightarrow |G(j\omega)| = 20 \log \left|\frac{0.75}{j\omega_l}\right| = 20 \log \left|\frac{0.75}{0.5}\right|$$

$$= 3.5 \text{ dB}$$

$$\omega = \omega_{c1} = |G(j\omega_{c1})| = 20 \log \left| \frac{0.75}{j\omega_{c1}} \right|$$

$$= 20 \log \left(\frac{0.75}{5} \right)$$

$$= -16.5 \, dB.$$

$$\omega = \omega_{c2}, \; |G(j\omega_{c2})| = \left[\text{slope from } \omega_{c1} \text{ to } \omega_{c2} \times \left(\log \frac{\omega_{c2}}{\omega_{c1}} \right) \right]$$

$$+ |G(j\omega)|_{\omega = \omega_{c1}}$$

$$= 0 \times \log \frac{10}{5} - 16.5$$

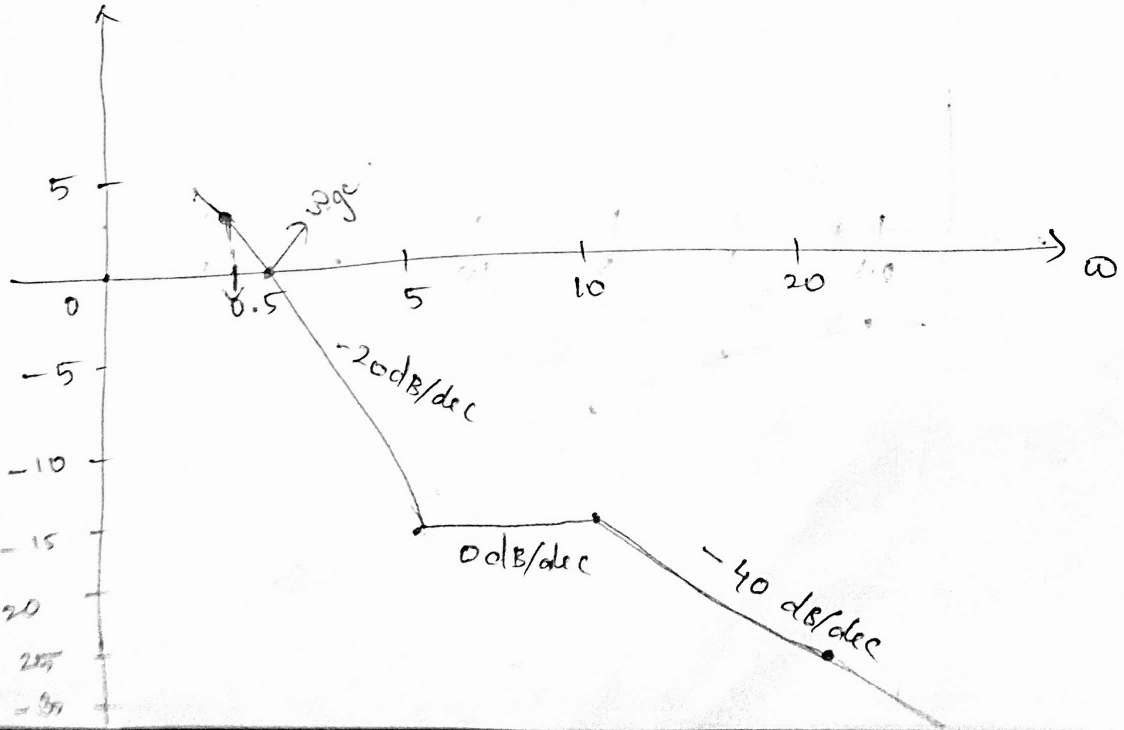
$$= -16.5 \, dB.$$

$$\omega = \omega_h \Rightarrow \left(\text{slope from } \omega_{c2} \text{ to } \omega_h \times \log \frac{\omega_h}{\omega_{c2}} \right) +$$

$$|G(j\omega)|_{\omega = \omega_{c2}}$$

$$= -40 \times \log \left(\frac{20}{10} \right) + (-16.5)$$

$$= -28.5 \, dB.$$



Phase plot

$$\phi = \angle G(j\omega) = \begin{cases} \tan^{-1} 0.2\omega - 90^\circ - \left(\tan^{-1} \dfrac{0.16\omega}{1-0.01\omega^2}\right), & \omega < 10 \\ \tan^{-1} 0.2\omega - 90^\circ - \left(\tan^{-1} \dfrac{0.16\omega}{1-0.01\omega^2} + 180^\circ\right), & \omega > 10 \end{cases}$$

ω	$\tan^{-1} 0.2\omega$	$\tan^{-1}\left(\dfrac{0.16\omega}{1-0.01\omega^2}\right)$	ϕ
0.5	5.7	4.6	-89
1	11.3	9.2	$\simeq -86$
5	45	46.8	-92
10	63.5	90	-116
20	75.9	$-46.8 + 180 = 133.2$	-148
50	84.3	$-18.4 + 180 = 161.6$	-168
100	87.1	$-92 + 180 = 170.8$	-174

phase margin (γ) $= 180^\circ + \phi_{gc}$

$= 180^\circ - 89$

$$\boxed{\gamma = 91^\circ}$$

Stability consideration:-

$$G(s) = \frac{K}{s(1+P_1 s)(1+P_2 s)}.$$

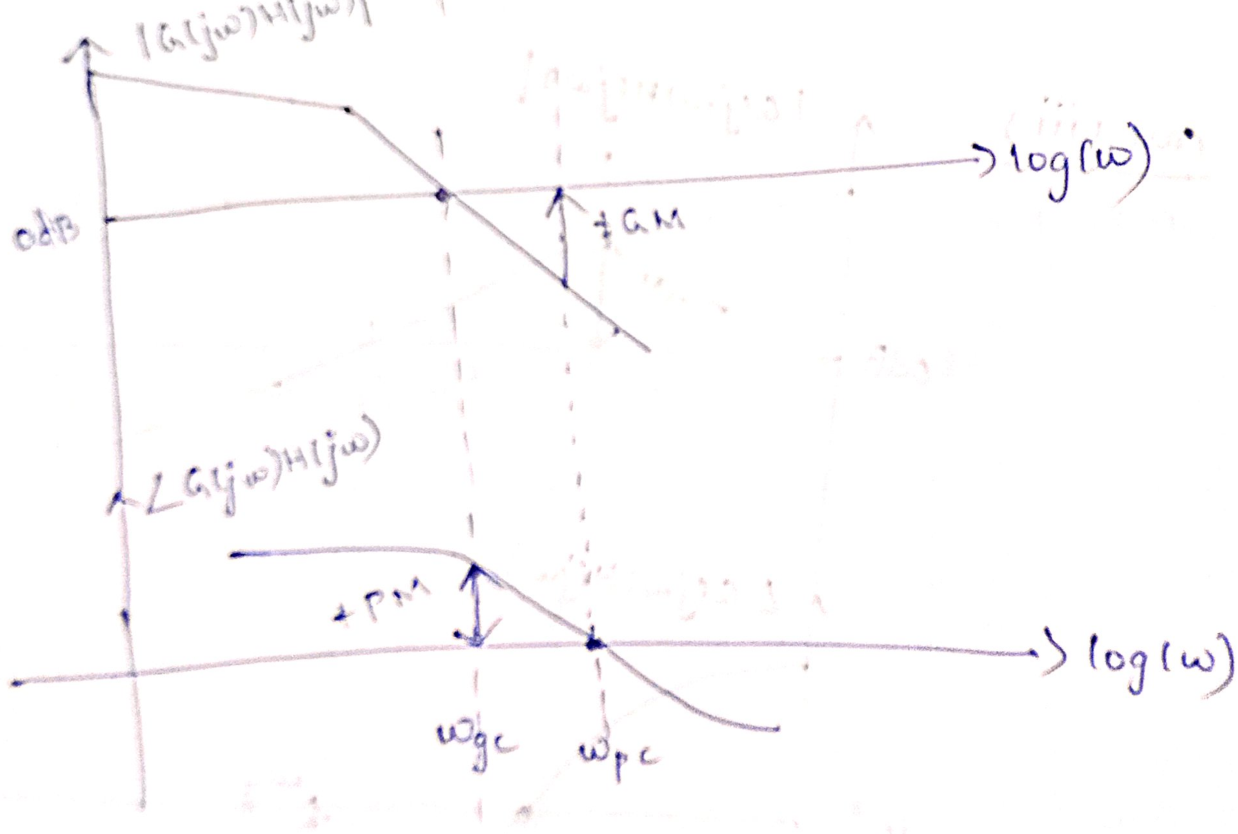
$$GM\,(K_g) = \frac{\text{Critical value of } K}{\text{Actual value of } K} \quad \text{——— (1)}$$

Also

$$K_g = \frac{1}{|G(j\omega)|}\Big|_{\omega=\omega_{pc}} \quad \text{——— (2)}$$

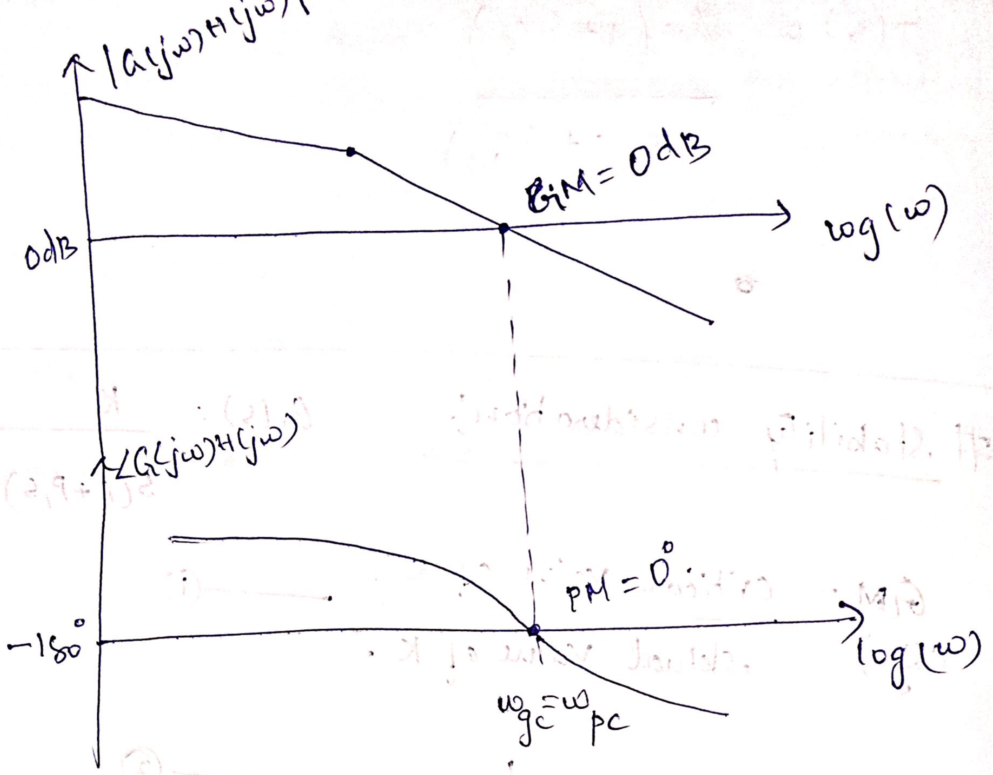
$$PM = 180^\circ + \underbrace{\angle[G(j\omega)H(j\omega)]}_{\phi}\Big|_{\omega=\omega_{gc}}. \quad \text{——— (3)}$$

Case (i)



$\omega_{gc} < \omega_{pc} \Rightarrow$ PM & GM positive & the s/m is stable.

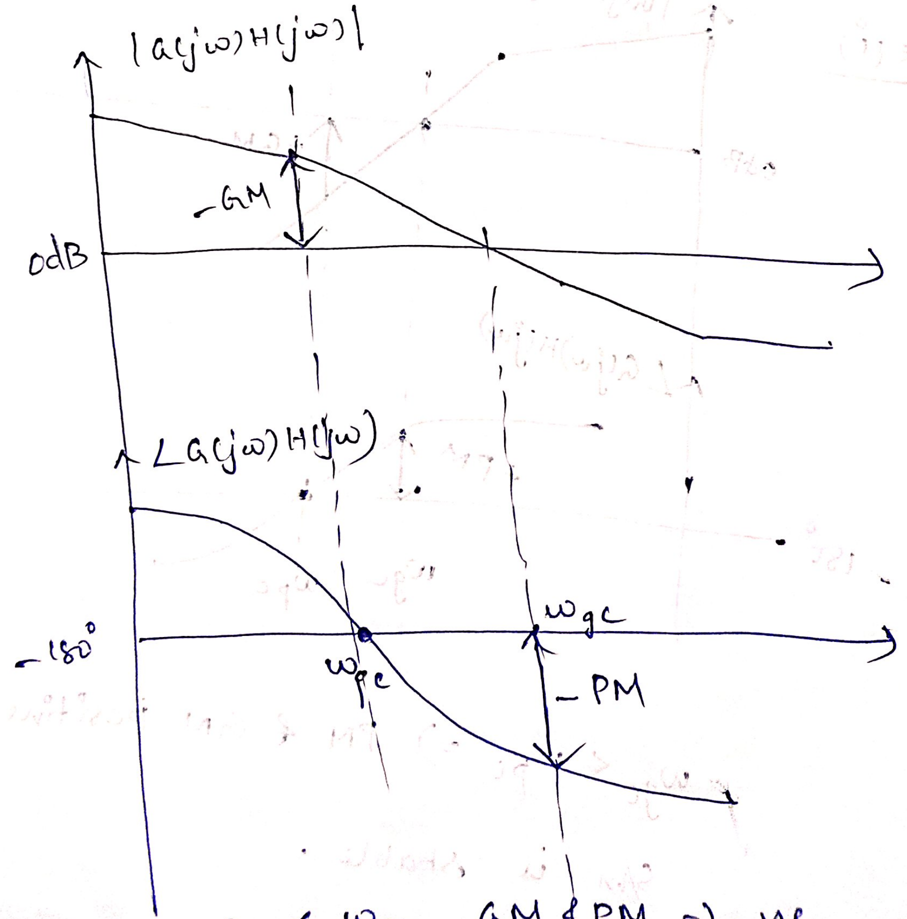
case (ii)



$\omega_{gc} = \omega_{pc} \Rightarrow$ Marginally stable

& $GM = 0dB$, $PM = 0°$.

case (iii)



$\omega_{pc} < \omega_{gc}$, GM & PM $\Rightarrow$ -ve

s/m is unstable

Minimum phase Functions:-

If all poles and zeros of any transfer func. lie on LH of s-plane, the tfr func. is known as the minimum phase transfer function.

All pass phase Functions:-

If all the zeros of a tfr func. lie on the R-H side of the s-plane, all poles lie on LH side of the s-plane. & the location of every pole-zero pair is symmetric about the $j\omega$ axis, then the transfer func is called as All pass phase func.

$$T(s) = \frac{(s-a)(s-b)}{(s+b)(s+a)}.$$

polar plot :-

→ plot of $|G(j\omega)|$ vs $\angle G(j\omega)$, $\omega \to 0$ to ∞.

→ polar plot ⇒ locus of vectors $|G(j\omega)| \angle G(j\omega)$.

→ polar graph → concentric circles ⇒ magnitude
Radial lines ⇒ phase angles.

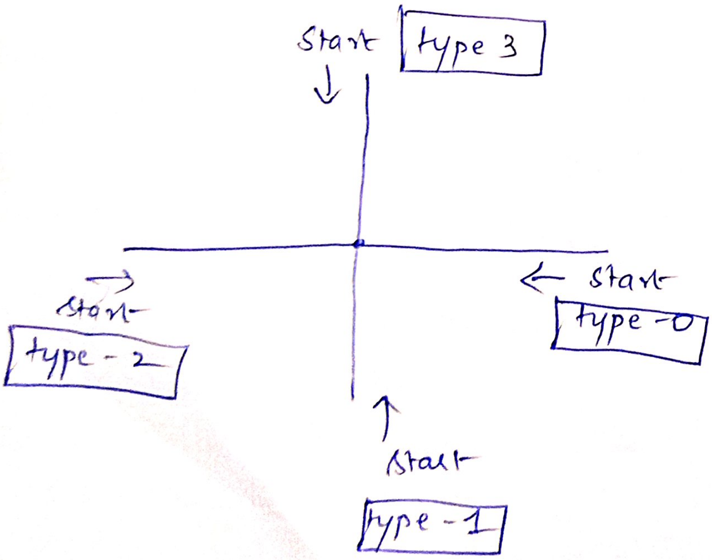
→ $G(j\omega) = G_R(j\omega) + j\,G_I(j\omega)$ [Rectangular co-ordinates].

→ The type no. determines the quadrant at which the polar plot starts and the order of the s/m determines the quadrant at which polar plot ends.

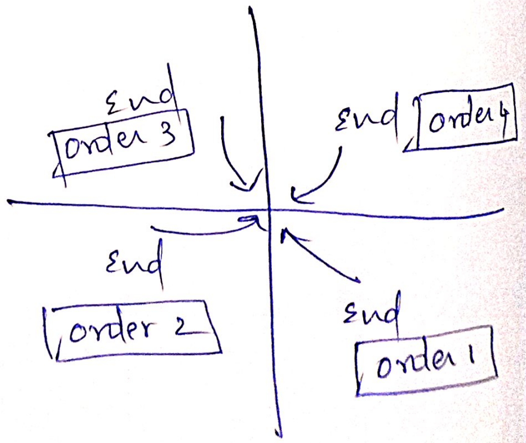
→ Minimum phase ⇒ All poles and zeros of the s/m are on the L-H of the s-plane.

→ when a pole is added to a s/m, the polar plot end point will shift by -90°.

→ when a zero is added to a s/m, the polar plot end pt will shift by $+90^\circ$.



Start of All pole minimum phase s/m.

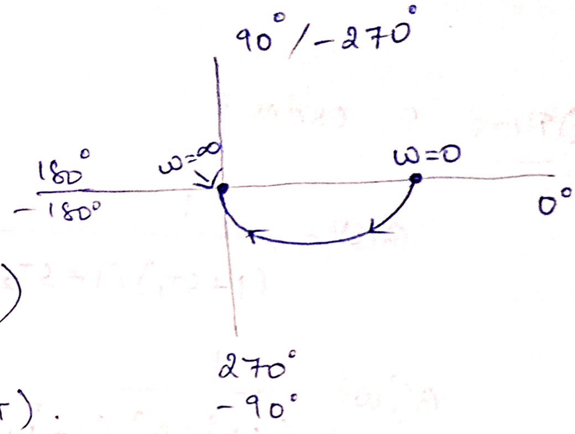


End of all pole minimum phase s/m.

Typical sketches of polar plot :-

1) Type 0, order 1

$$G(s) = \frac{1}{1+sT}.$$



$$G(j\omega) = \frac{1}{1+j\omega T}.$$

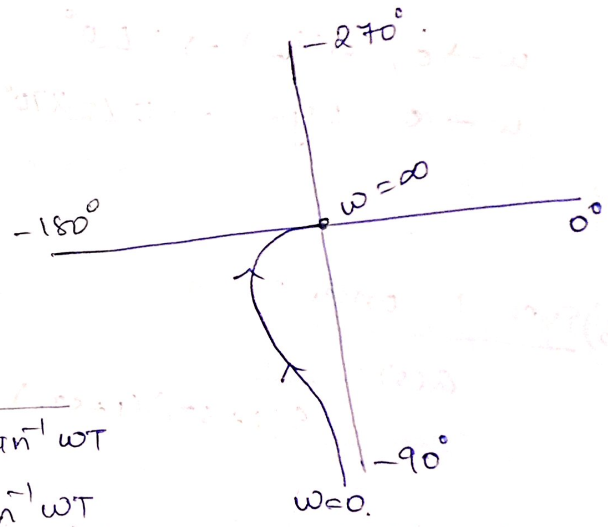
$$= \left|\frac{1}{j\omega T+1}\right| \angle -\tan^{-1}(\omega T)$$

$$G(j\omega) = \frac{1}{\sqrt{\omega^2T^2+1}} \angle -\tan^{-1}(\omega T).$$

As $\omega \to 0$, $G(j\omega) = 1\angle 0°$.

As $\omega \to \infty$, $G(j\omega) = 0\angle -90°$.

2) Type 1, order 2.



$$G(s) = \frac{1}{s(1+sT)}$$

$$G(j\omega) = \frac{1}{j\omega(1+j\omega T)}$$

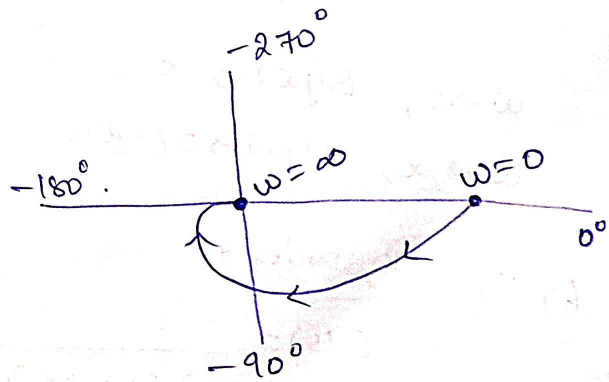
$$= \frac{1}{\omega\angle 90° \sqrt{1+\omega^2T^2} \angle \tan^{-1}\omega T}$$

$$= \frac{1}{\omega\sqrt{1+\omega^2T^2}} \angle -90° - \tan^{-1}\omega T$$

As $\omega \to 0$, $G(j\omega) \to \infty \angle -90°$

As $\omega \to \infty$, $G(j\omega) \to 0\angle -180°$.

3) Type 0, Order 2

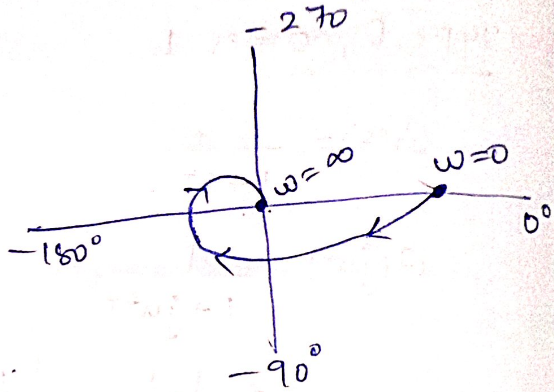


$$G(s) = \frac{1}{(1+sT_1)(1+sT_2)}$$

$$G(j\omega) = \frac{1}{(1+j\omega T_1)(1+j\omega T_2)}$$

$$= \frac{1}{\left(\sqrt{1+\omega^2T_1^2}\right)\left(\sqrt{1+\omega^2T_2^2}\right)\angle \tan^{-1}\omega T_1 \angle \tan^{-1}\omega T_2}$$

As $\omega \to 0$, $G(j\omega) \to 1\angle 0^\circ$
$\omega \to \infty$, $G(j\omega) \to 0^\circ \angle -180^\circ$.



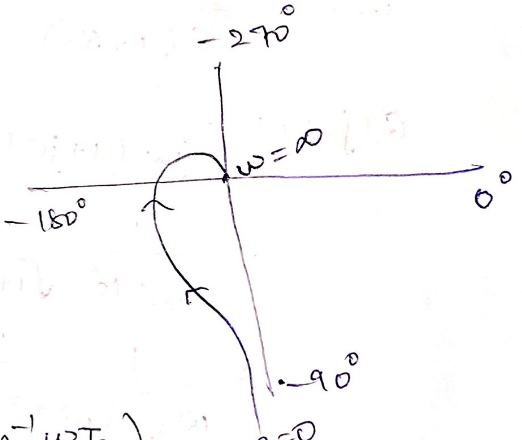
4) **Type 0, order 3**

$$G(s) = \frac{1}{(1+sT_1)(1+sT_2)(1+sT_3)}.$$

$$G(j\omega) = \frac{1}{(1+j\omega T_1)(1+j\omega T_2)(1+j\omega T_3)}.$$

$$G(j\omega) = \frac{1 \angle -\tan^{-1}(\omega T_1) - \tan^{-1}(\omega T_2) - \tan^{-1}(\omega T_3)}{\sqrt{1+\omega^2 T_1^2}\sqrt{1+\omega^2 T_2^2}\sqrt{1+\omega^2 T_3^2}}$$

$\omega \to 0$, $G(j\omega) \to 1\angle 0^\circ$
$\omega \to \infty$, $G(j\omega) \to 0\angle -270^\circ$.



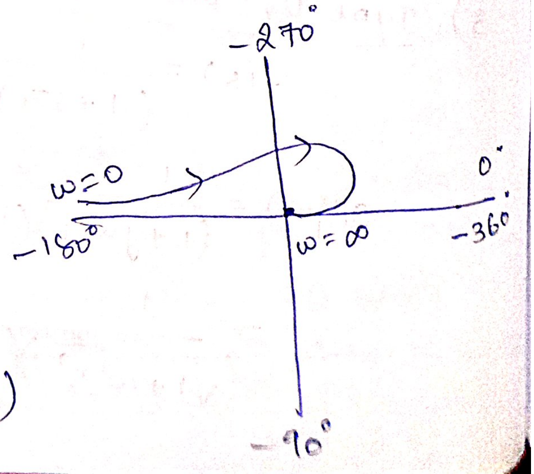
5) **Type 1, order 3**

$$G(s) = \frac{1}{s(1+sT_1)(1+sT_2)}$$

$$G(j\omega) = \frac{1}{j\omega(1+j\omega T_1)(1+j\omega T_2)}$$

$$= \frac{1 \angle (-90^\circ - \tan^{-1}\omega T_1 - \tan^{-1}\omega T_2)}{\omega\sqrt{1+\omega^2 T_1^2}\sqrt{1+\omega^2 T_2^2}}$$

$\omega \to 0$, $G(j\omega) \to \infty\angle -90^\circ$
$\omega \to \infty$, $G(j\omega) \to 0\angle -270^\circ$.



6) **Type 2, order 4**

$$G(s) = \frac{1}{s^2(1+sT_1)(1+sT_2)}$$

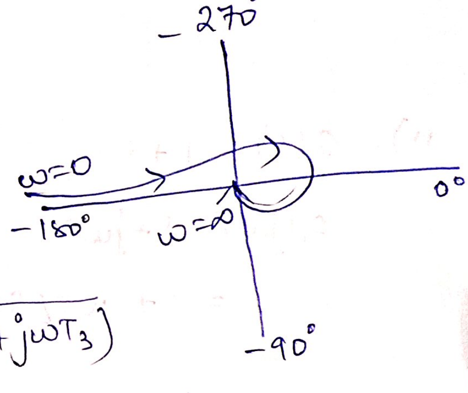
$$G(j\omega) = \frac{1}{(j\omega)^2(1+j\omega T_1)(1+j\omega T_2)}$$

$$G(j\omega) = \frac{1 \angle -180^\circ - \tan^{-1}\omega T_1 - \tan^{-1}\omega T_2}{\omega^2\sqrt{(1+\omega^2 T_1^2)(1+\omega^2 T_2^2)}}$$

As $\omega \to 0$, $G(j\omega) \to \infty \angle -180^\circ$

$\omega \to \infty$, $G(j\omega) \to 0 \angle -360^\circ$.

7) Type 2, order 5



$$G(s) = \frac{1}{s^2(1+sT_1)(1+sT_2)(1+sT_3)}$$

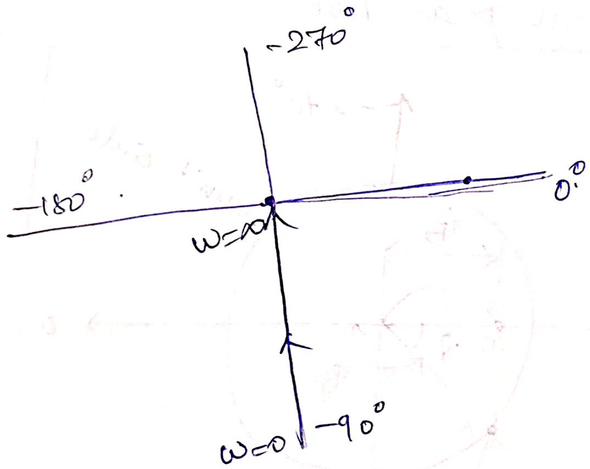
$$G(j\omega) = \frac{1}{(j\omega)^2(1+j\omega T_1)(1+j\omega T_2)(1+j\omega T_3)}$$

$$= \frac{1 \angle -180^\circ - \tan^{-1}\omega T_1 - \tan^{-1}\omega T_2 - \tan^{-1}\omega T_3}{\omega^2\sqrt{(1+\omega^2 T_1^2)(1+\omega^2 T_2^2)(1+\omega^2 T_3^2)}}.$$

As $\omega \to 0$, $\infty \angle -180^\circ$

$\omega \to \infty$, $G(j\omega) = 0 \angle -450^\circ \Rightarrow 0 \angle -90^\circ$.

8) $G(s) = 1/s$.



$$G(j\omega) = \frac{1}{j\omega}$$

$$= \frac{1 \angle -90^\circ}{\omega}.$$

$\omega \to 0$, $\infty \angle -90^\circ$

$\omega \to \infty$, $0 \angle -90^\circ$.

9) $$G(s) = \frac{1+sT}{sT}$$

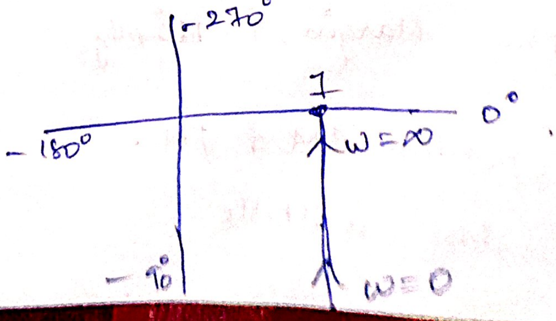
As $\omega \to 0$, $G(j\omega) = \infty \angle -90^\circ + 1$

$\omega \to \infty$, $G(j\omega) = 0 \angle -90^\circ + 1$

$$G(j\omega) = \frac{1+j\omega T}{j\omega T}$$

$$= \frac{1}{j\omega T} + 1$$

$$= \frac{1 \angle -90^\circ}{\omega T} + 1$$

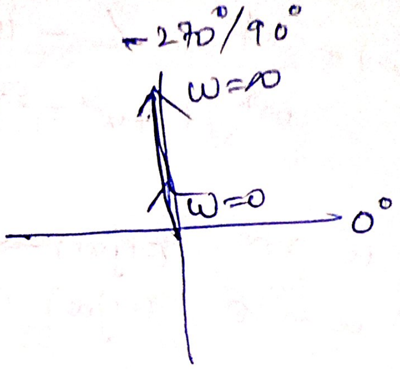


10) $G(s) = s$

$G(j\omega) = j\omega = \omega \angle 90^\circ$

$\omega \to 0, \; G(j\omega) \to 0 \angle 90^\circ$

$\omega \to \infty, \; G(j\omega) \to \infty \angle 90^\circ$.



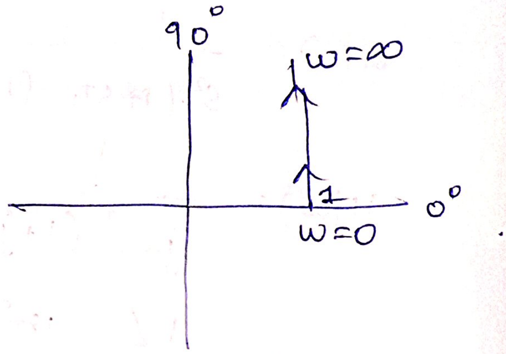
11) $G(s) = 1 + sT$

$G(j\omega) = 1 + j\omega T$

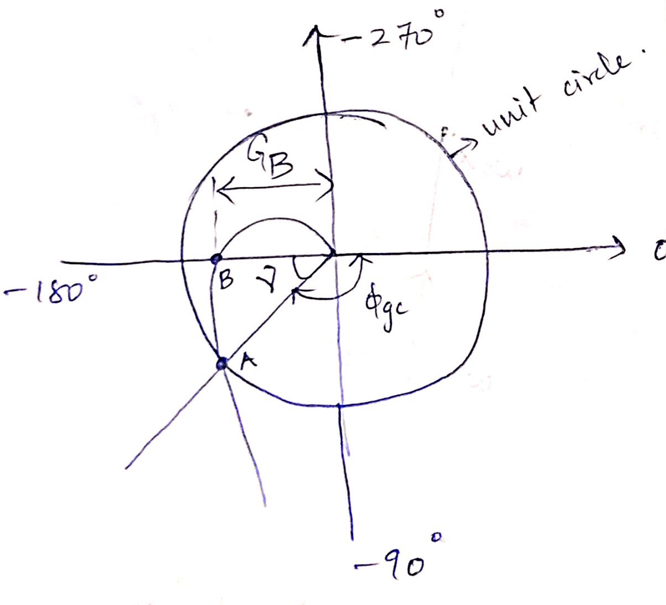
$= 1 + \omega T \angle 90^\circ$.

As $\omega \to 0, \; G(j\omega) = 1 + 0 \angle 90^\circ$

$\omega \to \infty, \; G(j\omega) = 1 + \infty \angle 90^\circ$.



Determination of Gain Margin and phase Margin from polar plot.

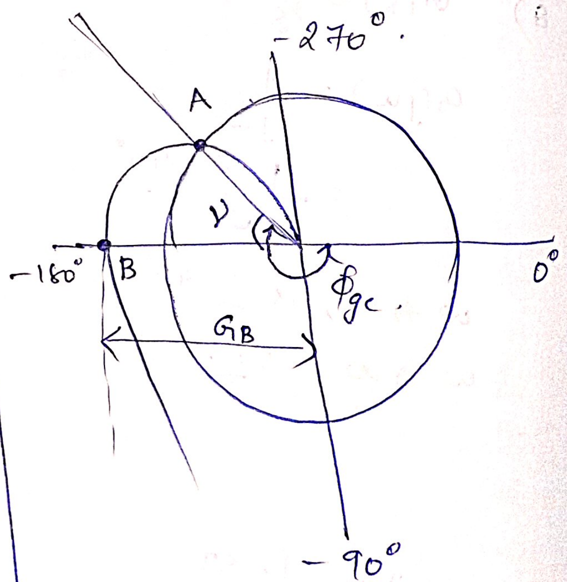


Gain Margin, $K_g = 1/G_B$.

phase Margin, $\gamma = 180^\circ + \phi_{gc}$. [$0^\circ$ to -180°]

Has positive GM & PM.

s/m is stable.



GM $\Rightarrow K_g = 1/G_B$

PM $\Rightarrow \gamma = 180^\circ + \phi_{gc}$ more negative than -180°

Has Negative GM & PM

s/m is unstable.

To determine K for specified GM.

1) Draw $G(j\omega)$ locus with $K=1$.

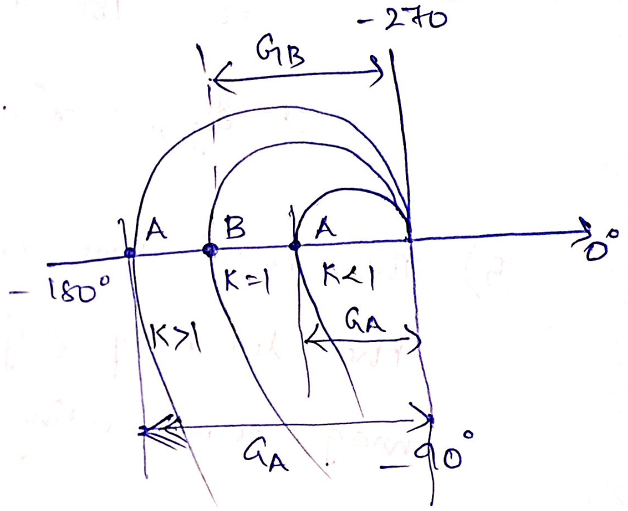
2) Let it cut the -180° axis at pt. B corresponding to a Gain of G_B & the specified gain Margin be 'x' dB.

3) For 'x' dB, $G(j\omega)$ locus will cut -180° at pt. A whose magnitude is G_A.

$$20 \log\left(\frac{1}{G_A}\right) = x$$

$$\frac{1}{G_A} = 10^{x/20}$$

$$G_A = \frac{1}{10^{x/20}}$$



$$\therefore \boxed{K = \frac{G_A}{G_B}}$$

4) If $K>1$, then the s/m gain should be increased.

If $K<1$, then the s/m gain should be reduced.

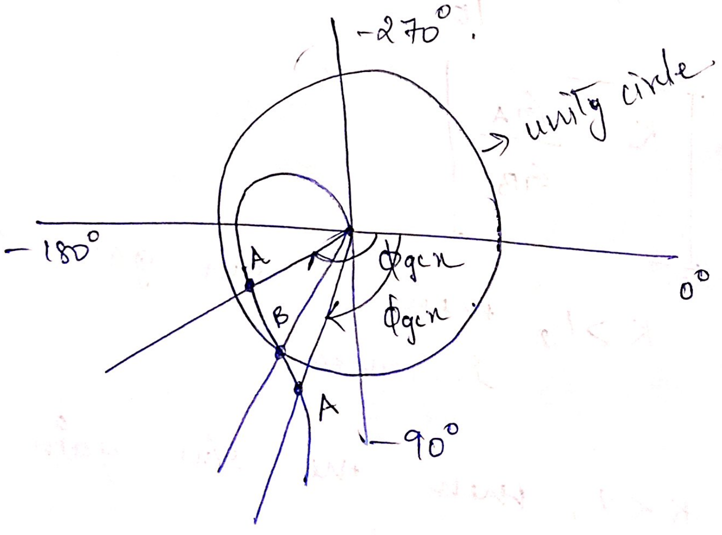
→ To determine K for specified PM.

1) Draw $G(j\omega)$ locus with $K=1$.
2) Let it cut the unity circle at pt. B. The Gain at point B is G_B & equal to unity.
3) Let the specified PM is x°.
4) For a x°, let ϕ_{gcx} be the phase angle of $G(j\omega)$ at gain cross-over freq.

$$x^\circ = 180^\circ + \phi_{gcx}.$$

$$\therefore \phi_{gcx} = x^\circ - 180^\circ.$$

5) The radial line corresponding to ϕ_{gcx} will cut the locus of $G(j\omega)$ with $K=1$ at pt A & the mag. corresponding to that point be G_A.



$$\boxed{K = \frac{G_B}{G_A} = \frac{1}{G_A}}$$ $\because G_B = 1.$

Q 1) The open loop transfer function of a unity fb slm is given by $G(s) = \dfrac{1}{s(1+s)(1+2s)}$, sketch the polar plot and determine the gain margin and phase Margin.

soln:)

$$G(s) = \frac{1}{s(1+s)(1+2s)}.$$

$$s \to j\omega \Rightarrow G(j\omega) = \frac{1}{j\omega(1+j\omega)(1+2j\omega)}.$$

corner frequencies $\omega_{c1} = \frac{1}{2} = 0.5$ rad/sec.

$\omega_{c2} = 1/1 = 1$ rad/sec.

→ Magnitude & phase of $G(j\omega)$.

$$|G(j\omega)| = \frac{1}{\omega\sqrt{\omega^2+1}\,\sqrt{1+4\omega^2}}.$$

$$= \frac{1}{\omega\sqrt{(\omega^2+1)(1+4\omega^2)}}$$

$$|G(j\omega)| = \frac{1}{\omega\sqrt{4\omega^4+5\omega^2+1}}$$

$$\angle G(j\omega) = -90^\circ - \tan^{-1}(\omega) - \tan^{-1}(2\omega).$$

→ in polar - co-ordinates

ω (rad/sec)	0.35	0.4	0.45	0.5	0.6	0.7	1.0
$\lvert G(j\omega)\rvert$	2.2	1.8	1.5	1.2	0.9	0.7	0.3
$\angle G(j\omega)$	-144°	-150°	-156°	-162°	-171°	$-179.5 \approx -180^\circ$	-198°

→ in rectangular co-ordinates :-

ω (rad/sec)	0.35	0.4	0.45	0.5	0.6	0.7	1.0
$G_R(j\omega)$	-1.78	-1.56	-1.37	-1.14	-0.89	-0.7	-0.29
$G_I(j\omega)$	-1.29	-0.9	-0.61	-0.37	-0.14	0	0.09

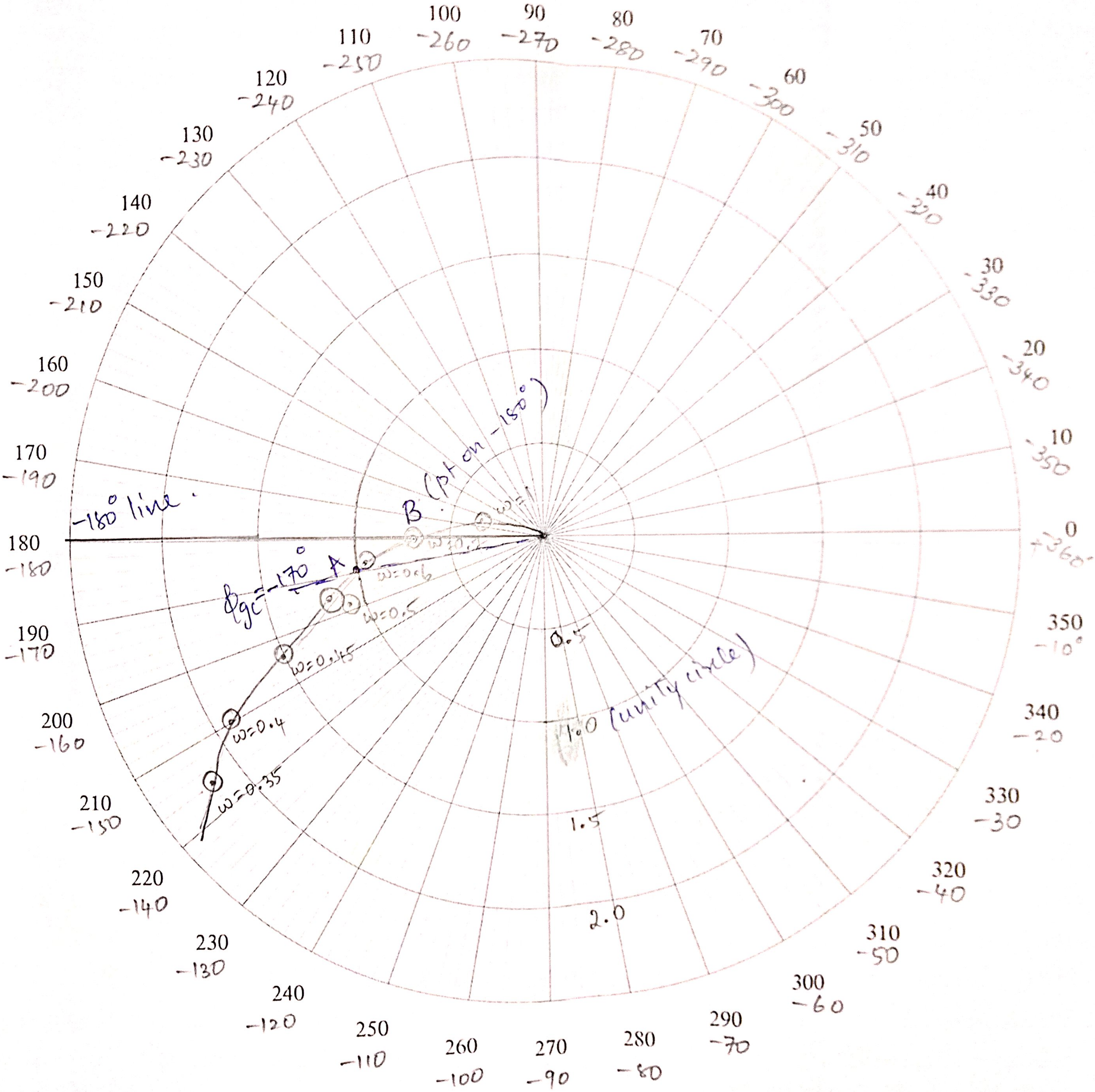
From graph i) $GM = \frac{1}{0.7} = 1.4286$

ii) $PM = 180 + \phi_{gc}$ | $= 180 + \phi_{gc}$

$= 180^\circ - 170$ | $= 180 - 168$

$= 10^\circ$ | $= 12^\circ$.

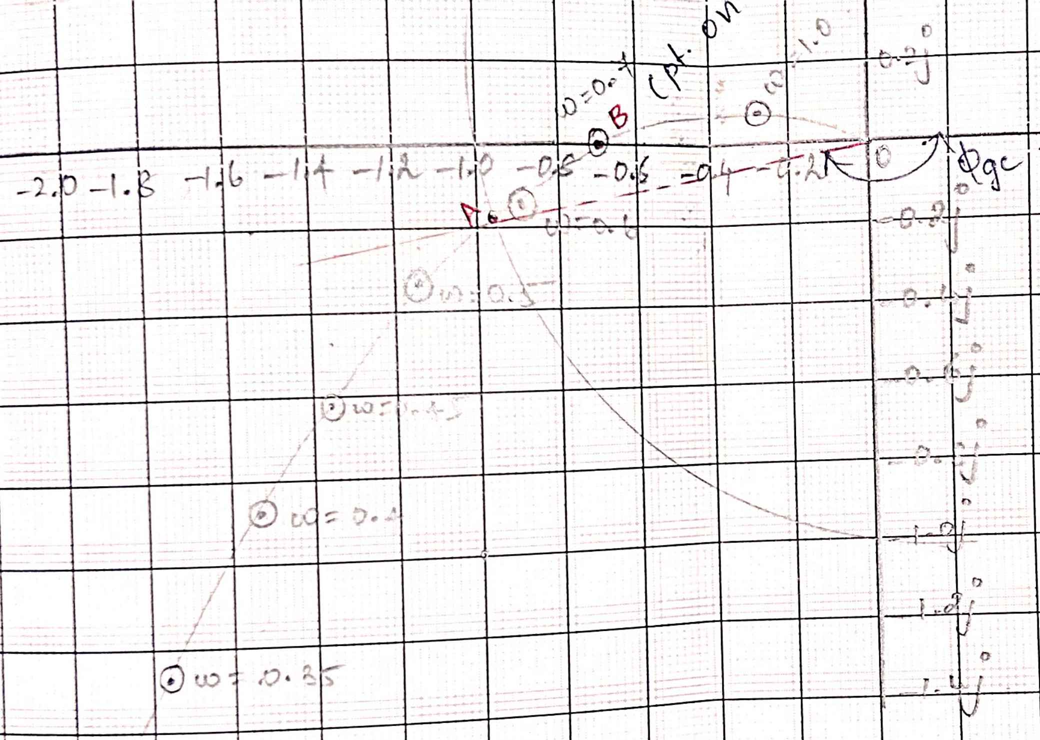
1) $G(s) = \dfrac{1}{s(1+s)(1+2s)}$.



$$GM = \frac{1}{0.7} = 1.4286.$$

$$PM = 180° - 170 = 10°.$$

$$G(s) = \frac{1}{s(1+s)(1+2s)}$$



$$\text{Gain Margin} = \frac{1}{|G(j\omega_{pc})|} = \frac{1}{0.7} = 1.4286$$

$$\begin{aligned}\text{Phase margin} &= 180^\circ + \phi_{gc} \\ &= 180^\circ - 168^\circ \\ &= 12^\circ.\end{aligned}$$

2) The OLTF of a unity fb s/m is given by $G(s) = \dfrac{1}{s^2(1+s)(1+2s)}$ ∴ Sketch the polar -- plot and determine the gain Margin and phase margin.

Soln:)

$$G(s) = \frac{1}{s^2(1+s)(1+2s)}.$$

$$G(j\omega) = \frac{1}{(j\omega)^2(1+j\omega)(1+2j\omega)}.$$

$$= \frac{1}{\omega^2 \angle 180^\circ \sqrt{1+\omega^2} \angle \tan^{-1}\omega \sqrt{1+4\omega^2} \angle \tan^{-1} 2\omega}$$

$$|G(j\omega)| = \frac{1}{\omega^2 \sqrt{1+5\omega^2+4\omega^4}}$$

$$\angle G(j\omega) = -180^\circ - \tan^{-1}\omega - \tan^{-1} 2\omega.$$

ω (rad/sec)	0.45	0.5	0.55	0.6	0.65	0.7	0.75	1.0
$\lvert G(j\omega)\rvert$	3.3	2.5	1.9	1.5	1.2	0.97	0.8	0.3
$\angle G(j\omega)$	-246	-251	-256	-261	-265	-269	-273	-288

→ Rect. co-ordinates.

ω	0.45	0.5	0.55	0.6	0.65	0.7	0.75	1.0
$G_R(j\omega)$	-1.34	-0.81	-0.46	-0.23	-0.1	-0.02	0.04	0.09
$G_I(j\omega)$	3.01	2.36	1.84	1.46	1.2	1.0	0.8	0.29

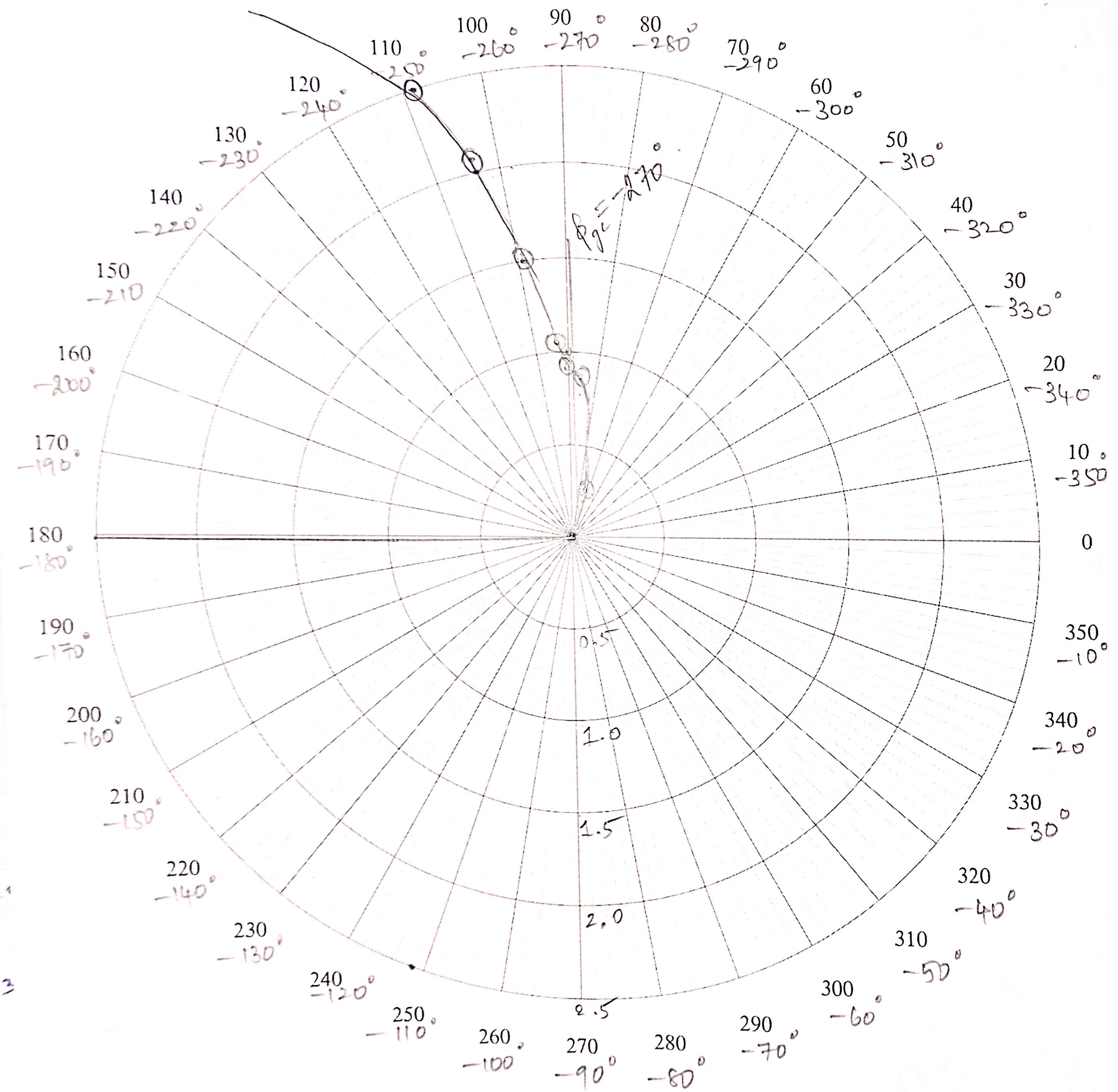
Gain Margin $(K_g) \Rightarrow \dfrac{1}{\lvert G(j\omega_{pc})\rvert} = \dfrac{1}{\infty} = 0.$

phase Margin $\Rightarrow \gamma = 180^\circ + \phi_{gc}.$

$= 180^\circ - 270^\circ$

$= -90^\circ.$

$$G(s) = \frac{1}{s^2(1+s)(1+2s)}$$



GM ⇒ No point for ω-values cuts the −180° line except origin

$\therefore \ k_g = \frac{1}{\infty} = 0$.

PM ⇒ The line cuts the unit circle at $\phi_{gc} = -270°$.

$\therefore \ \gamma = 180 - 270° = -90°$.

3) consider a unity feedback s/m having an open loop transfer function $G(s) = \dfrac{K}{s(1+0.2s)(1+0.05s)}$.

sketch the polar plot & determine the value of K so that (i) Gain margin is 18 db.

(ii) phase margin is 60°.

solu:)

$$G(s) = \frac{K}{s(1+0.2s)(1+0.05s)} \quad \text{with } K=1,$$

$$G(j\omega) = \frac{1}{j\omega(1+0.2j\omega)(1+0.05j\omega)}$$

$$|G(j\omega)| = \frac{1}{\left[\omega\sqrt{(1+(0.2\omega)^2)}\sqrt{1+(0.05\omega)^2}\right]}$$

$$\angle G(j\omega) = -90° - \tan^{-1}0.2\omega - \tan^{-1}0.05\omega$$

$$\omega_{c1} = \frac{1}{0.2} = 5 \text{ rad/sec}.$$

$$\omega_{c2} = \frac{1}{0.05} = 20 \text{ rad/sec}.$$

ω	$\lvert G(j\omega)\rvert$	$\angle G(j\omega)$	$G_R(j\omega)$	$G_I(j\omega)$
0.6	1.65	−98	−0.23	−1.63
0.8	1.23	−101	−0.23	−1.21
1	$\simeq$ $\boxed{1.0}$	$\boxed{-104}$	−0.24	−0.97
2	0.5	−117.5	−0.23	−0.44
3	0.3	−129.4	−0.19	−0.23
4	0.2	−140	−0.15	−0.13
5	0.14	−149	−0.120	−0.072

ω	$\lvert G(j\omega)\rvert$	$\angle G(j\omega)$	$G_R(j\omega)$	$G_I(j\omega)$
6	0.1	-157	-0.092	-0.039
7	0.07	-164	-0.067	-0.019
9	0.05	-176	-0.050	-0.0034
10	$\boxed{0.04}$	$\boxed{-180}$	$\boxed{-0.04}$	$\boxed{0}$
11	0.03	-184	-0.030	0.002
14	0.02	-195	-0.019	0.005

→ with $K = 1$

$$GM\ (k_g) = \frac{1}{0.04} = 25 \Rightarrow K_{g(dB)} = 20 \log 25 = \underline{\underline{28\ dB}}.$$

$$PM\ (\gamma) = 180^\circ - 104^\circ.$$

$$\underline{\underline{\gamma = 76^\circ}}$$

→ <u>case (i)</u>

Specified, $G_A = 18\,dB \Rightarrow 20 \log G_A = 18$

$$\log G_A = 18/20$$

$$G_A = \underline{\underline{7.943}}$$

$$K_g = \frac{1}{G_A}$$

$$= \frac{1}{7.943}$$

$$k_g = 0.125$$

$$K = \frac{G_A}{G_B}$$

$$= \frac{0.125}{0.04}$$

value $\boxed{K = 3.125}$

case (ii)

with $K = 1$, $PM(\gamma) = 76^\circ$.

Specified $\gamma = 60^\circ$

$$\gamma = 180^\circ + \phi_{gc}$$

$$\phi_{gc} = \gamma - 180^\circ$$
$$= 60^\circ - 180^\circ$$

$$\phi_{gc} = -120^\circ.$$

At $\phi_{gc} = -120$, $|G(j\omega)| = 0.425$ (G_A).

value $K = \frac{G_B}{G_A} = \frac{1}{G_A} = \frac{1}{0.425}$

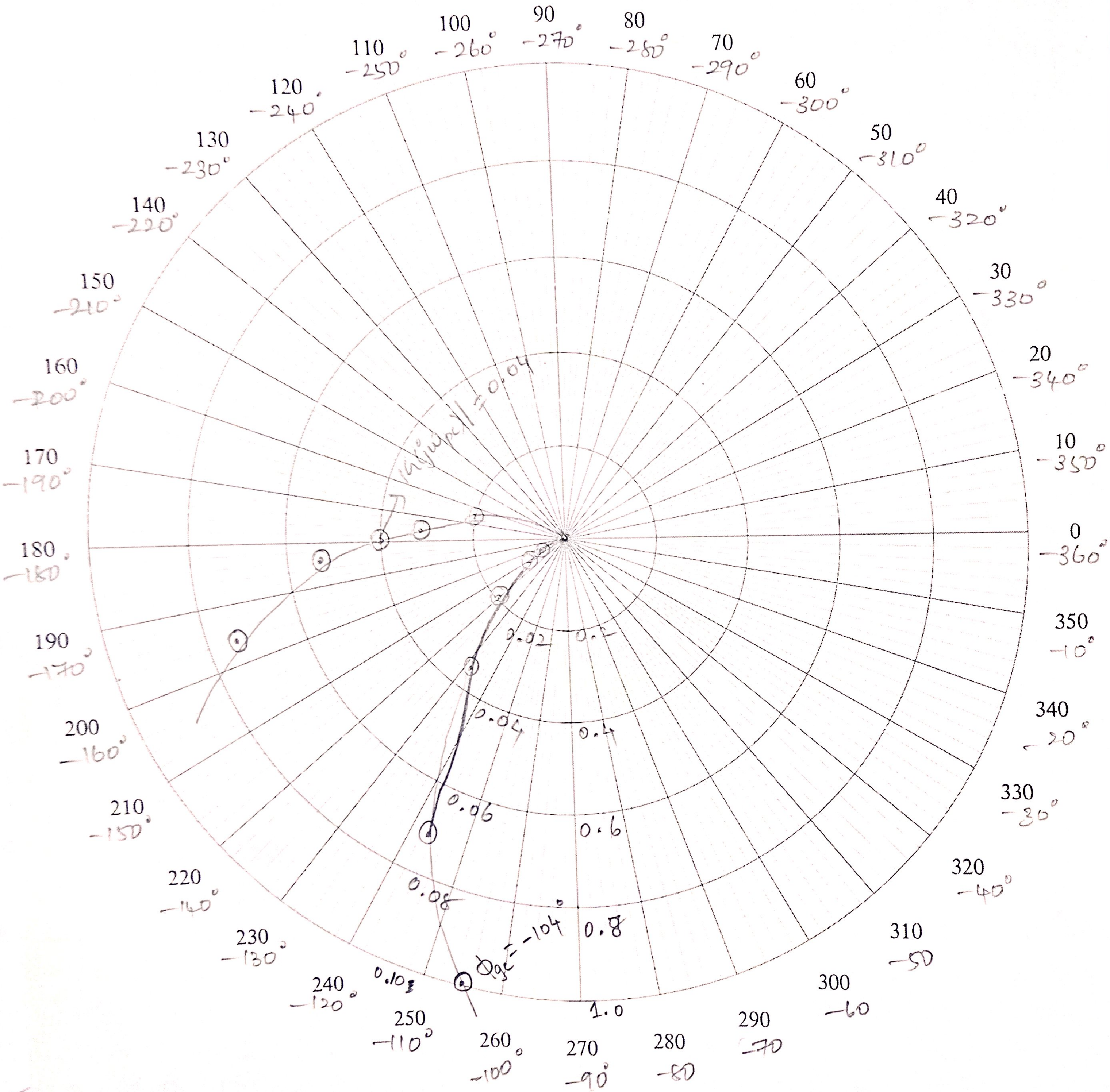
$$\boxed{K = 2.353}$$

1) consider a unity feedback s/m having an open loop transfer function, $G(s) = \frac{K}{s(1+0.5s)(1+4s)}$. sketch the polar plot and determine the value of K so that

(i) Gain margin is 20 dB

(ii) phase margin is 30°.

3) $G(s) = \dfrac{K}{s(1+0.2s)(1+0.05s)}$



$$GM_{(Kg)} = \frac{1}{|G(j\omega_{pc})|} = \frac{1}{0.04} = 25$$

$$PM_{(\gamma)} = 180° + \phi_{gc} = 180° - 104° = 76°.$$

Nichols plot

→ It is a frequency response plot of the open loop transfer function of a s/m.

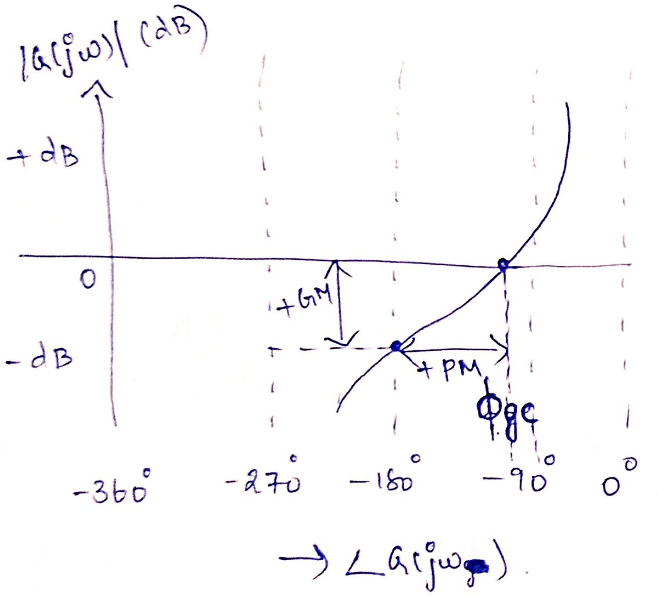
→ It is a plot b/w magnitude $|G(j\omega)|$ in dB and the phase $\angle G(j\omega)$ in degree, with ω as a varying parameter.

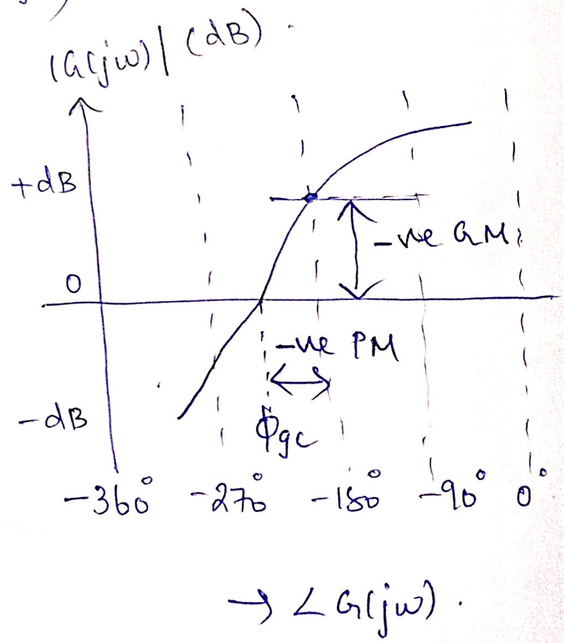
→ Usually the choice of frequencies are corner frequencies.

→ **Determination of Gain Margin and Phase Margin from Nichols plot**

$$GM \Rightarrow K_g = \frac{1}{|G(j\omega_{gc})|} \Rightarrow K_g\,(dB) = 20\log\left(\frac{1}{|G(j\omega_{pc})|}\right)$$

$$PM \Rightarrow \phi = 180^\circ + \phi_{gc}$$
$$= 180^\circ + \angle G(j\omega_{gc})$$





→ **Gain Adjustment in Nichols plot**

Let x = change in dB.

x → +ve, if plot is shifted up.

x → -ve, if plot is shifted down.

$20 \log K = x$

$\log K = \frac{x}{20}$

$$\boxed{K = 10^{x/20}}$$

pblm

1) consider a unity fb s/m. having an OLTF

$G(s) = \frac{K(1+10s)}{s^2(1+s)(1+2s)}$. sketch the Nichols plot and

determine the value of K so that i) gain margin is 10dB

ii) phase margin is 10°.

soln:)

$$G(s) = \frac{K(1+10s)}{s^2(1+s)(1+2s)}.$$

$K=1$, $$G(j\omega) = \frac{(1+10j\omega)}{(j\omega)^2(1+j\omega)(1+2j\omega)}.$$

$$G(j\omega) = \frac{\sqrt{1^2+(10\omega)^2}\ \angle\tan^{-1}(10\omega)}{\omega^2\angle 180^\circ\left(\sqrt{1+\omega^2}\right)\angle\tan^{-1}\omega\left(\sqrt{1+(2\omega)^2}\right)\angle\tan^{-1}2\omega}$$

$$|G(j\omega)| = \frac{\sqrt{1+100\omega^2}}{\omega^2\sqrt{1+\omega^2}\sqrt{1+4\omega^2}} \Rightarrow |G(j\omega)|_{dB} = 20\log\left[\frac{\sqrt{1+100\omega^2}}{\omega^2\sqrt{1+\omega^2}\sqrt{1+4\omega^2}}\right]$$

$$\angle G(j\omega) = \tan^{-1}10\omega - 180^\circ - \tan^{-1}\omega - \tan^{-1}2\omega.$$

ω (rad/sec)	0.2	0.4	0.6	0.8	1.0	1.5	2.0	3.0	4.0
$\lvert G(j\omega)\rvert_{dB}$	34.1	25.4	19.3	14.3	10	1.4	-5.3	-15.2	-22.5
$\angle G(j\omega)$ deg.	-150	-164	-181	-194	-204	-222	-232	-244	-250.

→ From plot $(K=1)$, $GM\ (k_g) = -19.5\,dB$

$PM\ (\gamma) = -45^\circ$.

(i) K, for GM = 10 dB.

if $k_g\ (dB) = 10\,dB$, $|G(j\omega_{pc})| = -10\,dB$, at -180°.

But at -180°, $|G(j\omega)| = +19.5\,dB$, To make $|G(j\omega)|$ to $-10\,dB$, add $-29.5\,dB$ at every point of phase.

$$20 \log k_1 = -29.5\,dB.$$

$$\log k_1 = -\frac{29.5}{20}$$

$$k_1 = 10^{(-29.5/20)} \Rightarrow \boxed{k_1 = 0.0335}$$

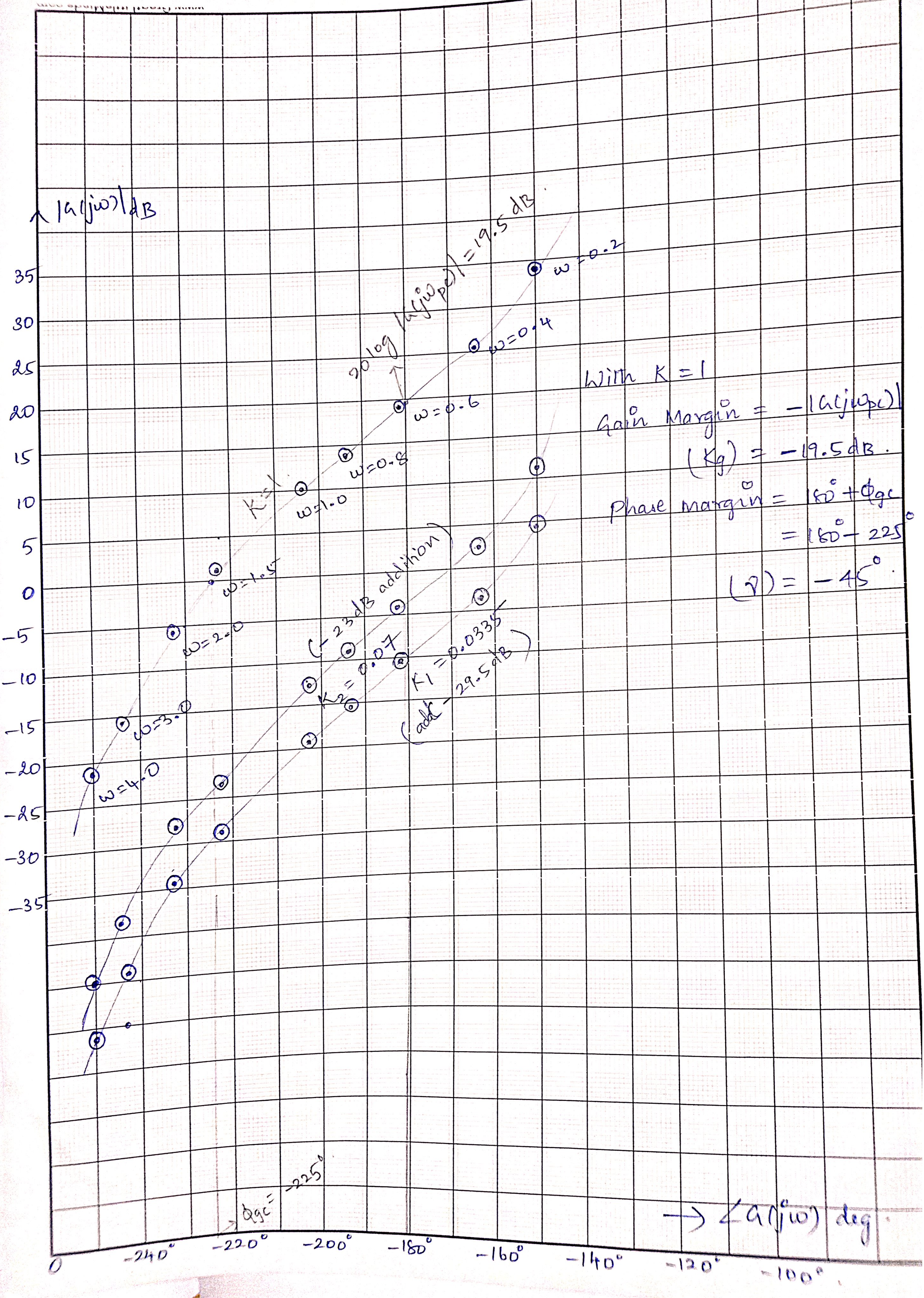
(ii) K, for PM = 10°.

$$\gamma = 180^\circ + \phi_{gc}.$$

$$10 = 180^\circ + \phi_{gc}$$

$$\phi_{gc} = -170^\circ.$$

At -170°, $|G(j\omega_{gc})| = +23\,dB$. But add $-23\,dB$ at every point of phase to have $|G(j\omega_{gc})| = 0\,dB$ at -170°.



With $K = 1$

Gain Margin $= -|G(j\omega_{pc})|$

$(K_g) = -19.5\,dB$.

Phase margin $= 180^\circ + \phi_{gc}$

$= 180^\circ - 225^\circ$

$(\gamma) = -45^\circ$.

$$20 \log K_2 = -23 \text{ dB}$$

$$\log K_2 = -23/20$$

$$K_2 = 10^{(-23/20)}$$

$$\boxed{K_2 = 0.07}$$

Lag Compensator :

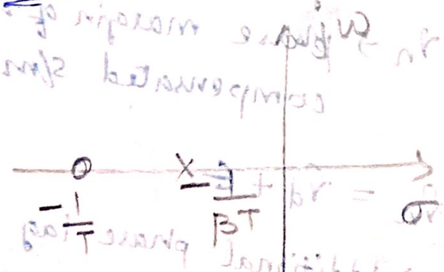
→ If a sinusoidal signal is applied to a lag n/w, then in steady state the o/p will have a phase lag wrt i/p.

→ gives large improvement in steady state performance but results in slower response due to reduced BW.

→ The attenuation due to the lag compensator will shift the gain crossover freq. to a lower freq. pt., where the phase margin is acceptable.

→ ∴ The lag compensator ⇒ ↓se BW and results in a slower transient response.

→ s-plane Repsn.



→ tfr func

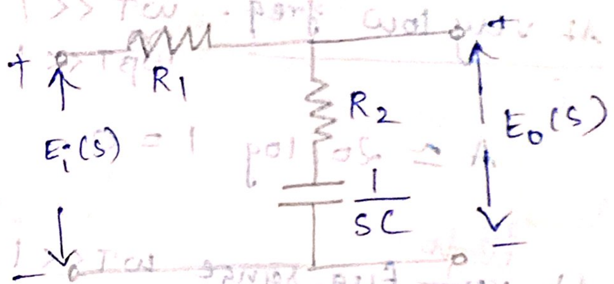
$$\boxed{G_c(s) = \frac{s+Z_c}{s+P_c} \qquad G_c(s) = \frac{s+\frac{1}{T}}{s+\frac{1}{\beta T}}}$$

$Z_c = \frac{1}{T}$ $P_c = \frac{1}{\beta T}$

$T>0$ $\beta>1$

→ $T = \frac{1}{Z_c}$ $\beta = \frac{Z_c}{P_c}$.

→ Realization using electrical n/w



$$E_o(s) = E_i(s)\left(\frac{R_2+\frac{1}{sC}}{R_1+R_2+\frac{1}{sC}}\right)$$

$$\frac{E_o(s)}{E_i(s)} = \frac{\left(s+\frac{1}{R_2C}\right)}{\left(\frac{R_1+R_2}{R_2}\right)\left(s+\frac{1}{\left(\frac{R_1+R_2}{R_2}\right)R_2C}\right)}$$

$$\frac{E_o(s)}{E_i(s)} = \frac{1}{\beta}\,\frac{\left(s+\frac{1}{T}\right)}{\left(s+\frac{1}{\beta T}\right)} \Rightarrow \text{general}$$

$\beta = \frac{R_1+R_2}{R_2}$ $T = R_2C$.

→ $G_c(s) = \frac{\left(s+\frac{1}{T}\right)}{\left(s+\frac{1}{\beta T}\right)} = \beta\frac{(1+sT)}{(1+s\beta T)}$

$G_c(j\omega) = \frac{\beta(1+j\omega T)}{(1+j\omega\beta T)}$

$\omega = 0$; $G_c(j\omega) = \beta$.

→ If β is neglected,

$$G_c(j\omega) = \frac{1+j\omega T}{1+j\omega\beta T}.$$

$$\boxed{G_c(j\omega) = \frac{\sqrt{1+(\omega T)^2}\ \angle \tan^{-1}\omega T}{\sqrt{1+(\omega\beta T)^2}\ \angle \tan^{-1}\omega\beta T}}$$

→ At very low freq. $\boxed{\omega T << 1,\ \omega\beta T << 1}$

$$\boxed{A \simeq 20 \log 1 = 0}$$

→ At ~~very~~ high freq. range $\boxed{\omega T << 1 \ \&\ \omega\beta T >> 1}$

$$\boxed{A = 20 \log \frac{1}{\sqrt{(\omega\beta T)^2}} = 20 \log \frac{1}{\omega\beta T}}$$

→ At very high freq. $\boxed{\omega T >> 1 \ \&\ \omega\beta T >> 1}$

$$A = 20 \log \frac{\sqrt{(\omega T)^2}}{\sqrt{(\omega\beta T)^2}}$$

$$= 20 \log \frac{\omega T}{\omega\beta T}$$

$$\boxed{A = 20 \log \frac{1}{\beta}}$$

→ $\phi = \angle G_c(j\omega)$

$$\boxed{\phi = \tan^{-1}(\omega T) - \tan^{-1}(\omega\beta T)}$$

→ Freq. of max phase lag $(\omega_m) = \sqrt{\omega_{c1} \cdot \omega_{c2}}$

$$= \sqrt{\frac{1}{\beta T} \cdot \frac{1}{T}}$$

$$\boxed{\omega_m = \frac{1}{T\sqrt{\beta}}}$$

$$\boxed{\phi_m = \tan^{-1}\left[\frac{1-\beta}{2\sqrt{\beta}}\right]}$$

max phase angle.

$$\boxed{\beta = \frac{1-\sin\phi_m}{1+\sin\phi_m}}$$

Design of Lag compensator using Bode plot

1) choose the value of K
2) sketch the bode plot
3) determine the phase margin of the Bode plot.
4) If phase margin does not satisfy the req. then lag compensation a req.
5) γ_d → phase margin as gn. in specification

γ_n → phase margin of the compensated s/m

$$\boxed{\gamma_n = \gamma_d + \epsilon}$$

ϵ ⇒ Additional phase lag compensate for shift gain crossover freq.

initial value of $\boxed{\epsilon = 5^\circ}$

6

Lead compensator

→ The lead compensation ↑ses BW, which improves the speed of response & also reduces the amt. of overshoot.

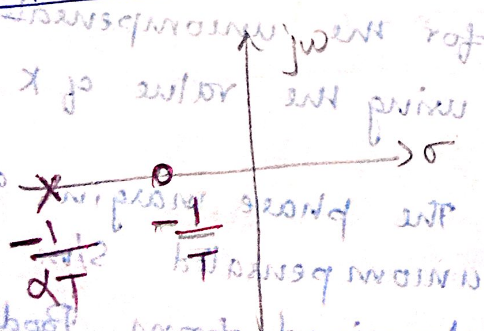
→ It appreciably improves the transient response, & there is a small change in steady state accuracy

→ lead compensation ⇒ makes unstable s/m to a stable s/m.

→ Lead compensator ⇒ HPF & hence it amplifies high frequencies signals.

→ If pole introduced by the compensation is not cancelled by zero in the s/m, then lead compensator ↑ses order of the s/m by one.

→ S-plane Reprsn:

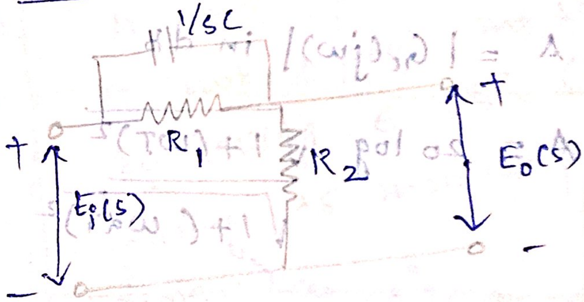


$$G_c(s) = \left(\frac{s+\frac{1}{T}}{s+\frac{1}{\alpha T}}\right)$$

$T > 0$, $\alpha < 1$

$$\boxed{Z_c = \frac{1}{T} \qquad P_c = \frac{1}{\alpha T} \qquad T = \frac{1}{Z_c} \qquad \alpha = \frac{Z_c}{P_c}}$$

→ Realisation of lead compensator using electrical n/w



$$E_o(s) = E_i(s) \cdot \frac{R_2}{R_2 + \left(\frac{R_1}{R_1 C s + 1}\right)}$$

$$\frac{E_o(s)}{E_i(s)} = \frac{\left(s + \frac{1}{R_1 C}\right)}{\left[s + \left(\frac{1}{R_2/(R_1+R_2)}\right)\frac{1}{R_1 C}\right]}$$

General form, $G_c(s) = \frac{\left(s+\frac{1}{T}\right)}{\left(s+\frac{1}{\alpha T}\right)}$

$T = R_1 C$

$\alpha = \frac{R_2}{R_1 + R_2}$.

Freq. Response of Lead compensator

$$\boxed{G_c(s) = \alpha \frac{(1+sT)}{(1+\alpha sT)}}$$

→ $G_c(j\omega) = \alpha \frac{(1+j\omega T)}{(1+\alpha j\omega T)}$

→ $A = |G_c(j\omega)|$ in db

$$\boxed{A = 20 \log \frac{\sqrt{1+(\omega T)^2}}{\sqrt{1+(\omega \alpha T)^2}}}$$

→ $G_c(j\omega) = \frac{(1+j\omega T)}{(1+\alpha j\omega T)}$

$$= \frac{\sqrt{1+(\omega T)^2} \angle \tan^{-1}(\omega T)}{\sqrt{1+(\alpha\omega T)^2} \angle \tan^{-1}(\alpha\omega T)}$$

→ At very low freq.

$$\boxed{A = 0 \text{ db}}$$

→ In freq. range, $\omega T >> 1$ & $\omega \alpha T << 1$

$$A = 20 \log \sqrt{(\omega T)^2}$$

$$\boxed{A = 20 \log(\omega T)}$$

→ At High freq.,

$$\boxed{A = 20 \log \frac{1}{\alpha}}$$

→ Freq. of max. phase lead,

$$\omega_m = \sqrt{\omega_{c_1} \cdot \omega_{c_2}}$$

$$= \sqrt{\frac{1}{T} \cdot \frac{1}{\alpha T}}$$

$$\boxed{\omega_m = \frac{1}{T\sqrt{\alpha}}}$$

→ $$\boxed{\phi_m = \tan^{-1}\left(\frac{1-\alpha}{2\sqrt{\alpha}}\right)}$$ max phase

→ $$\boxed{\alpha = \frac{1-\sin\phi_m}{1+\sin\phi_m}}$$

Procedure for design of lead compensator using Bode plot

1) The open loop gain K of the given s/m is determined to satisfy the req. of error constant.

2) The Bode plot is drawn for the uncompensated s/m using the value of K.

3) The phase margin of uncompensated s/m is determined from Bode plot.

4) Amt. of phase angle to be contributed by the lead n/w by $\phi_m = \gamma_d - \gamma + \epsilon$

→ Lead compensator

→ ↑ses BW & speeds up the response & decreases the max. overshoot in the step response.

→ Lag compensator

→ ↑ses the low freq. gain & thus improves the steady state accuracy of the s/m.

→ but reduces speed of responses due to reduced BW.

→ Need of Lag-Lead

→ If improvements in both transient and steady state response are desired.

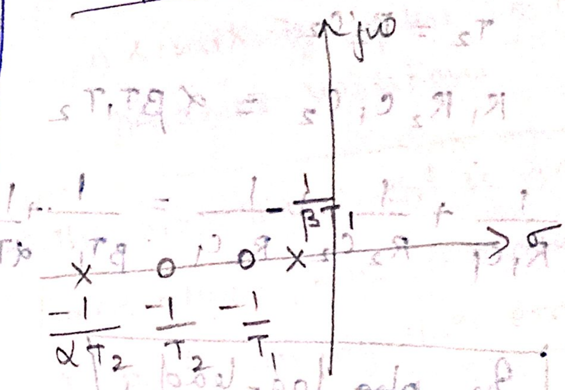
→ Single lag-lead is economical to use.

→ It combines the adv. of lag & lead comp.

→ lag-lead ⇒ posses 2 zeros and 2 poles

Hence ↑ses the order of the s/m by two, unless cancellation of poles & zeros occur.

→ S-plane Repr^n:

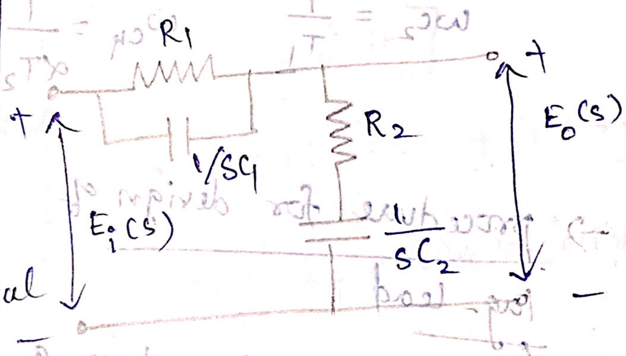


→ Tfr func.

$$G_c(s) = \frac{\left(s+\frac{1}{T_1}\right)\left(s+\frac{1}{T_2}\right)}{\left(s+\frac{1}{\beta T_1}\right)\left(s+\frac{1}{\alpha T_2}\right)}$$

$\beta > 1$, $0 < \alpha < 1$

→ Realization of lag-lead by electrical n/w.



$$\frac{E_o(s)}{E_i(s)} = \frac{\left(s+\frac{1}{R_1C_1}\right)\left(s+\frac{1}{R_2C_2}\right)}{s^2+s\left(\frac{1}{R_1C_1}+\frac{1}{R_2C_2}+\frac{1}{R_2C_1}\right)+\frac{1}{R_1R_2C_1C_2}}$$

general

$$G_c(s) = \frac{\left(s+\frac{1}{T_1}\right)\left(s+\frac{1}{T_2}\right)}{s^2+s\left(\frac{1}{\beta T_1}+\frac{1}{\alpha T_2}\right)+\frac{1}{\alpha\beta T_1T_2}}$$

$$T_1 = R_1 C_1$$

$$T_2 = R_2 C_2$$

$$R_1 R_2 C_1 C_2 = \alpha \beta T_1 T_2$$

$$\frac{1}{R_1 C_1} + \frac{1}{R_2 C_2} + \frac{1}{R_2 C_1} = \frac{1}{\beta T_1} + \frac{1}{\alpha T_2}$$

In ~~the~~ lag-lead,

$$\boxed{\alpha = \frac{1}{\beta}}$$

→ Freq - Response of Lag-Lead

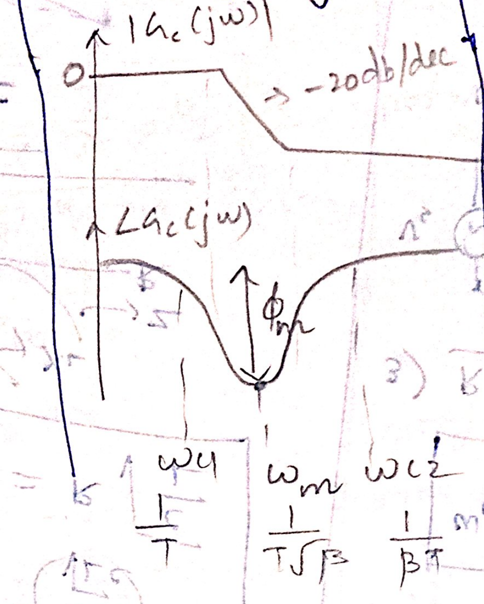
$$G_c(j\omega) = \frac{(1 + j\omega T_1)(1 + j\omega T_2)}{(1 + j\omega\beta T_1)(1 + j\omega\alpha T_2)}$$

$$\omega_{c1} = \frac{1}{\beta T_1} \qquad \omega_{c3} = \frac{1}{T_2}$$

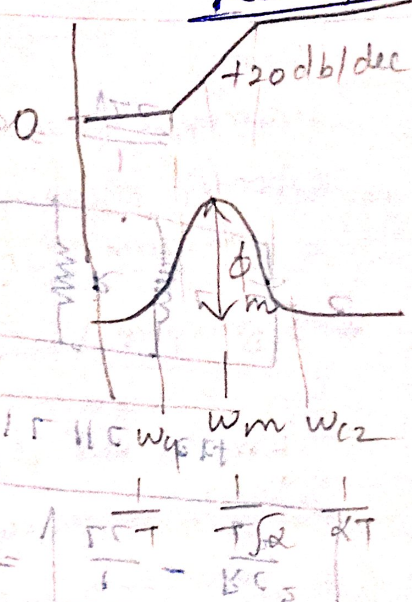
$$\omega_{c2} = \frac{1}{T_1}, \qquad \omega_{c4} = \frac{1}{\alpha T_2}$$

lag

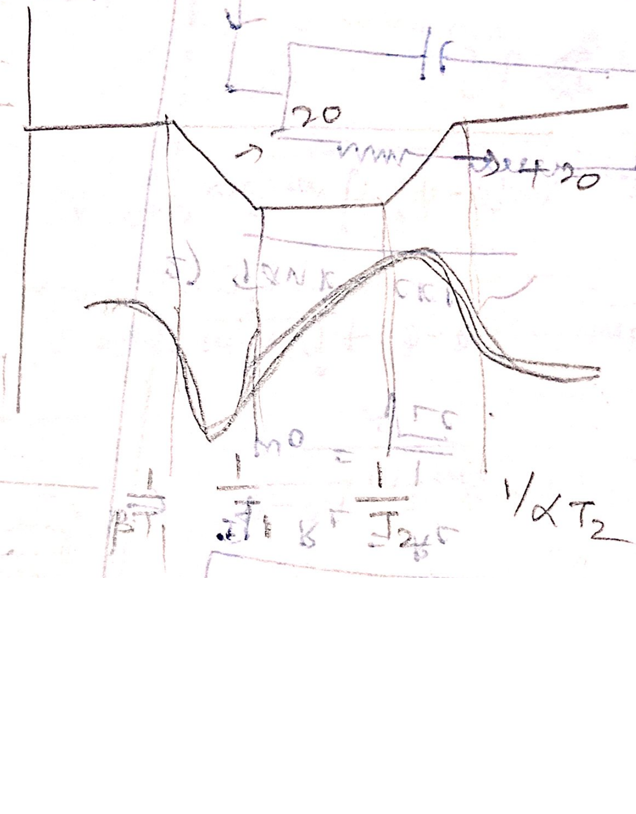
18). freq.
Responce-



Lead



lag-lead



$\alpha = \frac{1 - \sin\phi_m}{}$