INTRODUCTION TO FILTER:

* **FILTER:** → rejects unwanted frequencies from i/p & allows the desired frequencies to get o/p.

* **TYPES:**

1. Analog filter
   - LPF (Low Pass Filter)
   - HPF (High Pass Filter)
   - BPF (Band Pass Filter)
   - BSF (Band Stop Filter)
2. Digital filter
   - FIR (Finite impulse response)
   - IIR (Infinite Impulse Response)

* **Pass Band:** → Band (or) Range of frequencies that are passed through filter

* **Stop Band:** → " blocked by the filter

* **Cutoff-frequency (Ω_c):**
  → frequency between the 2 bands.
  at which magnitude is $1/\sqrt{2}$.

* **Comparison of analog & digital filters:**

Analog	Digital
1. process analog i/p & generate analog o/p	1. process & generate digital data.
2. constructed using active (or) passive components.	2. using adder, multiplier & delay elements.
3. defined by linear differential equation.	3. defined by linear difference equation.

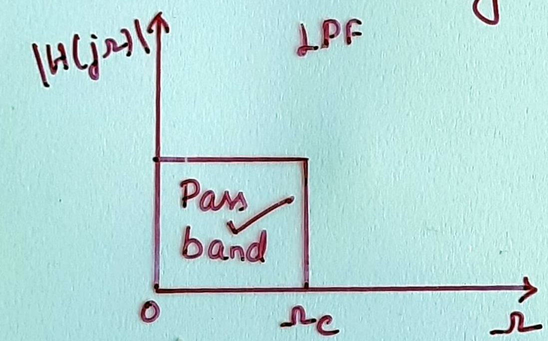
4. We can change the design by changing the components.	4. by changing the filter coefficients. (program)

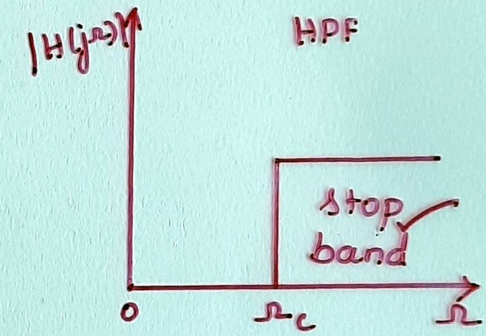
* <u>LPF</u>: allows low frequences ($\Omega = 0$ to $\Omega = \Omega_c$) & rejects high frequency ($\Omega > \Omega_c$)

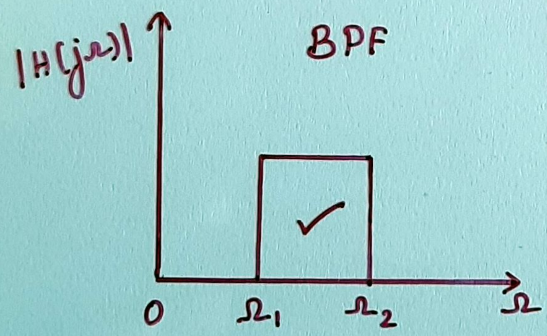
* <u>HPF</u>: allows high frequencies ($\Omega > \Omega_c$) & rejects low frequencies ($\Omega = 0$ to $\Omega = \Omega_c$)

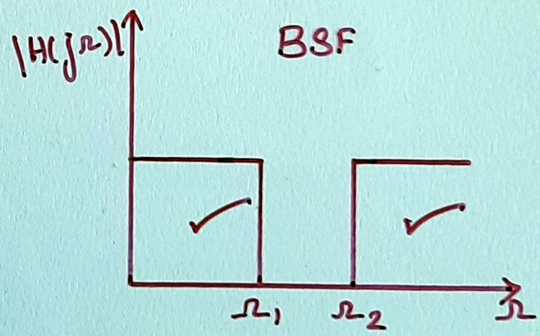
* <u>BPF</u>: allows only a band of frequencies (Ω_1 to Ω_2) & rejects remaining frequencies. ($\Omega < \Omega_1$ & $\Omega > \Omega_2$)

* <u>BSF</u>: rejects only a band of frequencies (Ω_1 to Ω_2) & allows remaining frequencies ($< \Omega_1$, & $> \Omega_2$)









* <u>Why we go for digital filters</u> ?

(or)

<u>Advantages of digital filters</u>:

→ have high thermal stability.

i.e., digital → donot change with temp, power

analog → changes with temp. & power supply variatiations

→ highly immune to noise.

→ operate over wide range of frequencies.

→ programmable (ie by changing pgm we can reconfigure the filter design).

→ Multiple filtering is possible

→ Registers are used to store filter coefficients.

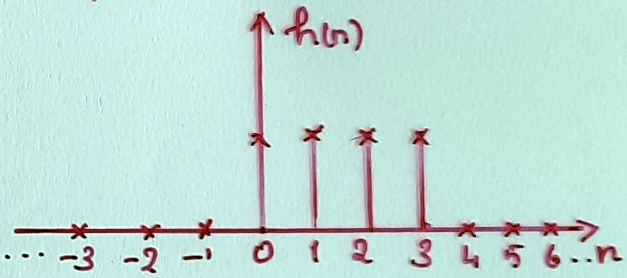
* Disadvantages of digital filters:

→ Quantisation error occurs.

→ BW of the discrete signal is limited by the sampling frequency.

* FIR FILTER: (preferred when linear phase characteristics of Filter is ~~preferred~~ required)

→ Impulse response of a digital filter is determined by finite number of impulse sequences (or) non-zero terms, then filter → FIR.



Eg: $h(n) = \begin{cases} 1, & 0 \le n \le 3 \\ 0, & \text{otherwise} \end{cases}$

→ FIR filter is defined by

$$y(n) = \sum_{K=0}^{M} b_K \, x(n-k)$$

o/p $y(n)$ is function of only i/p sequence ie there is no feed back [$y(n-1)$ $y(n-2)$ ....]. hence, It is also known as Non-recursive system.

→ No analog to digital conversion while designing the FIR filter.

→ feature: we can easily obtain exact linear phase at the o/p.

* **Advantages of FIR filter:**

→ design with exact linear phase.

→ obtain stable o/p (always).

→ noise is less (than IIR) due to absence of feedback.

→ obtain efficient realization (recursive & non-recursive form)

* **Disadvantage of FIR filter:**

→ To get exact linear phase, N should be large. (than IIR)

N → order of the filter.

→ need large memory & requires powerful computational facilities for implementation

* **Properties of FIR filter:**

→ Frequency response = F.T { h(n) }

$$\therefore H(\omega) = \sum_{n=-\infty}^{\infty} h(n) \cdot e^{-j\omega n} \rightarrow \text{infinite}$$

$$H(\omega) = \sum_{n=0}^{N-1} h(n) \cdot e^{-j\omega n}$$ [h(n) → causal sequence

h(n) → filter coefficients.

$|H(\omega)|$ → Amplitude Response

$\phi(\omega)$ (or) $\angle H(\omega)$ → phase response

* **phase delay (τ_p) :**

→ negative ratio of phase response $\phi(\omega)$ & frequency 'ω' of a filter, i.e.

$$\tau_p = -\frac{\phi(\omega)}{\omega}.$$

* **Group delay (τ_g):**

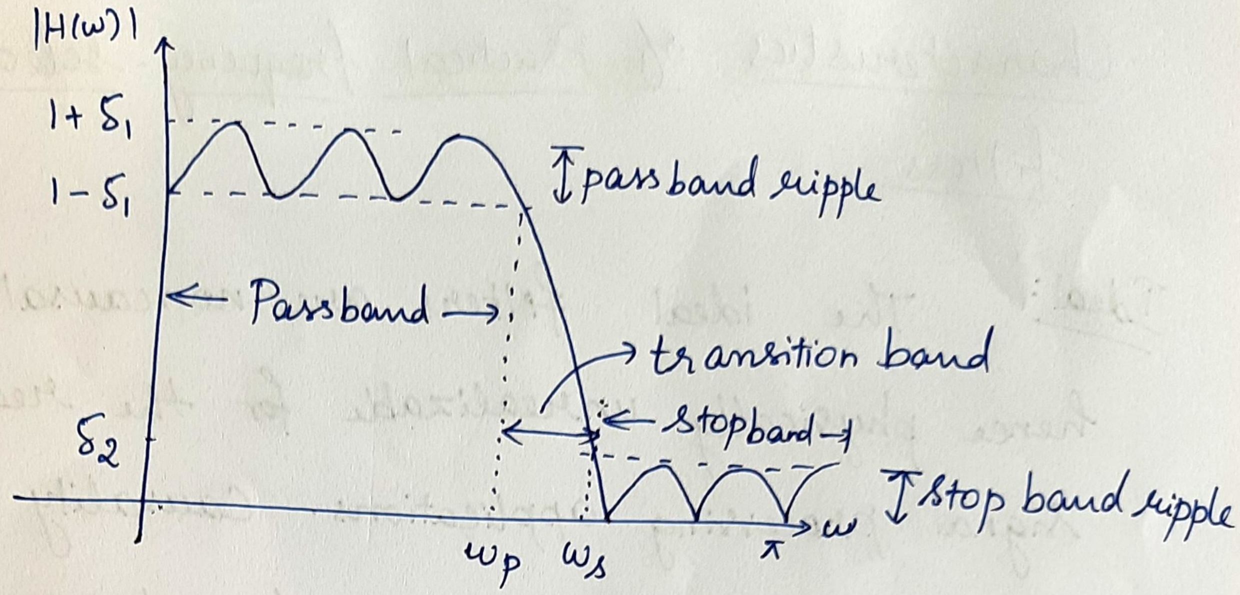
→ negative differentiation of $\phi(\omega)$ w.r.to 'ω' (or) rate of decrease of $\phi(\omega)$ w.r.to 'ω'.

i.e. $$\tau_g = -\frac{d\,\phi(\omega)}{d\omega}.$$

Characteristics of Practical frequency-selective filters:

Ideal: The ideal filters are noncausal and hence physically unrealizable for the real time signal processing applications. Causality implies that the frequency response characteristics $H(\omega)$ of the filter cannot be zero, except at a finite set of points in a frequency. In addition, $H(\omega)$ cannot have an infinitely sharp cutoff from passband to stopband, i.e., $H(\omega)$ cannot drop from unity to zero abruptly.

Practical: In practice, it is not necessary to insist that the magnitude $|H(\omega)|$ be constant in the entire passband of the filters. A small amount of ripple in the passband is usually tolerable. Similarly, it is not necessary for the filter response $|H(\omega)|$ to be zero in the stopband. A small, non-zero value or a small amount of ripple in the stopband is also tolerable.



Magnitude response of a practical LPF

where, $\delta_1 \rightarrow$ Passband ripple

$\delta_2 \rightarrow$ Stop band ripple

$\omega_p \rightarrow$ Passband edge frequency.

$\omega_s \rightarrow$ Stopband edge frequency.

The transition of the frequency response from passband to stopband defines the transition band or transition region of the filter. (width $= \omega_s - \omega_p$).

The band edge frequency ω_p defines the edge of the Passband, while the frequency ω_s denotes the beginning of the stopband.

The width of the passband is usually called the bandwidth of the filter. for eg. LPF with a passband edge freq ω_p, has bandwidth of ω_p.

If there is ripple in the passband of the filter, its value is denoted as δ_1 and the magnitude $|H(\omega)|$ varies between the limits $1 \pm \delta_1$. The ripples in the stopband of the filter is denoted as δ_2.

In any filter design problem, we can specify

1. maximum tolerable PB ripple.
2. " " SB "
3. PB edge freq ω_p.
4. SB " " ω_s.

Based on these specifications, we can select the parameters $\{a_k\}$ and $\{b_k\}$ in the frequency response characteristics which best approximates the desired specifications.

* LINEAR PHASE CHARCTERISTICS OF FIR FILTER:

To obtain linear phase τ_g & $\tau_p \rightarrow$ constant $\frac{\phi(\omega)}{\omega} = \text{constant}$

For constant τ_p & τ_g, $\phi(\omega)$ must be linear $\frac{\phi(\omega)}{\omega} = -\tau$

$$\therefore \phi(\omega) = -\tau . \omega$$

Proof:

wkt, Frequency Response $H(\omega) = \sum_{n=0}^{N-1} h(n). e^{-j\omega n}$

$$\angle H(\omega) \text{ (or) } \phi(\omega) = \tan^{-1}\left[\frac{H_i(\omega)}{H_r(\omega)}\right]$$

$$H(\omega) = \sum_{n=0}^{N-1} h(n)[\cos\omega n - j\sin\omega n] \quad [\because e^{-j\theta} = \cos\theta - j\sin\theta$$

$$H_r(\omega) = \sum_{n=0}^{N-1} h(n).\cos\omega n$$

$$H_i(\omega) = -\sum_{n=0}^{N-1} h(n).\sin\omega n$$

$$\therefore \phi(\omega) = \tan^{-1}\left[\frac{-\sum_{n=0}^{N-1} h(n).\sin\omega n}{\sum_{n=0}^{N-1} h(n).\cos\omega n}\right]$$

$$-\tau\omega = \tan^{-1}\left[\frac{-\sum_{n=0}^{N-1} h(n).\sin\omega n}{\sum_{n=0}^{N-1} h(n).\cos\omega n}\right]$$

$$\tan(\tau\omega) = \sum_{n=0}^{N-1} h(n).\sin\omega n \Big/ \sum_{n=0}^{N-1} h(n).\cos\omega n$$

$$\frac{\sin\tau\omega}{\cos\tau\omega} = \frac{\sum_{n=0}^{N-1} h(n).\sin\omega n}{\sum_{n=0}^{N-1} h(n).\cos\omega n}$$

$$\sum_{n=0}^{N-1} h(n).\cos\omega n.\sin\tau\omega = \sum_{n=0}^{N-1} h(n).\sin\omega n.\cos\tau\omega$$

$$\sum_{n=0}^{N-1} h(n).\cos wn.\sin \tau w - \sum_{n=0}^{N-1} h(n).\sin wn.\cos \tau w = 0$$

$$\sum_{n=0}^{N-1} h(n)\left[\cos wn.\sin \tau w - \sin wn.\cos \tau w\right] = 0$$

$$\sum_{n=0}^{N-1} h(n).\sin(w\tau - wn) = 0 \quad \left[\because \sin(A-B) = \sin A \cos B - \cos A \sin B\right.$$

$\Downarrow$

This is transcedental equation.

soln of above eqn is, h(n) is symmetrical

$$\boxed{\tau = \frac{N-1}{2}} \; ; \; \boxed{h(n) = h(N-1-n)} \text{ for } 0 \le n \le N-1$$

$\Downarrow$ condition for FIR filter to be linear.

From the above condition, for every value of 'N', there will be only one value of 'τ' for which linear phase can be obtained easily.

* TYPES OF h(n):

1. Symmetrical impulse response → N odd, N even

$$\boxed{h(n) = h(N-1-n)}$$

2. Anti-symmetrical impulse response → N odd, N even

$$h(n) \neq h(N-1-n) \text{ (or) } \boxed{h(n) = -h(N-1-n)}$$

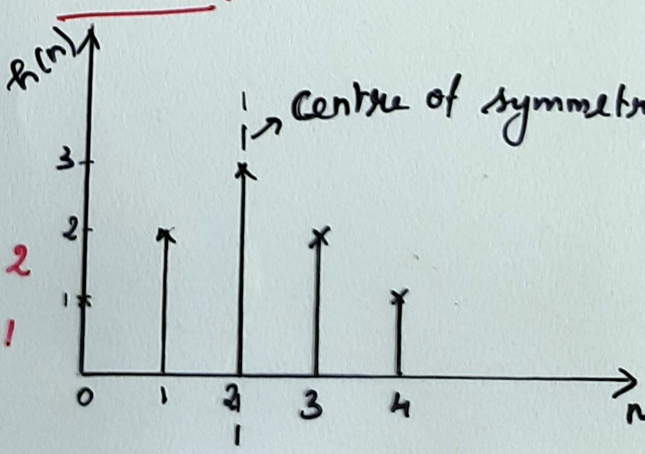
* Symmetrical h(n) for N → odd:

$N=5, \; \tau = \frac{5-1}{2} = 2$

eg: $h(n) = \{1, 2, 3\}$

$h(3) = h(5-1-3) = h(1) = 2$

$h(4) = h(5-1-4) = h(0) = 1$



Symmetrical impulse response for N→even:

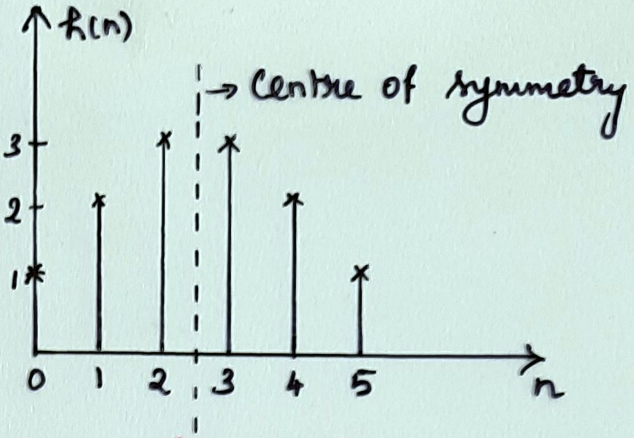
$N = 6$; $\tau = \frac{6-1}{2} = 2.5$

$h(n) = \{1, 2, 3\}$

$h(0) = h(6-1-0) = h(5)$

$h(1) = h(6-1-1) = h(4)$

$h(2) = h(6-1-2) = h(3)$



Anti-symmetrical impulse response for N-odd:

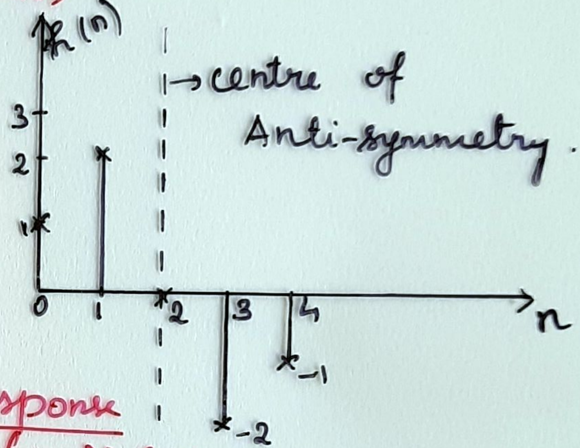
$N = 5,\quad \tau = 2.\quad h(n) = -h(N-1-n)$

$h(n) = \{1, 2, 3\}$

$h(4) = -h(5-1-4) = -h(0) = -1$

$h(3) = -h(5-1-3) = -h(1) = -2$

$h(2) = -h(2) = 0.$



Anti-symmetrical impulse response for N-even:

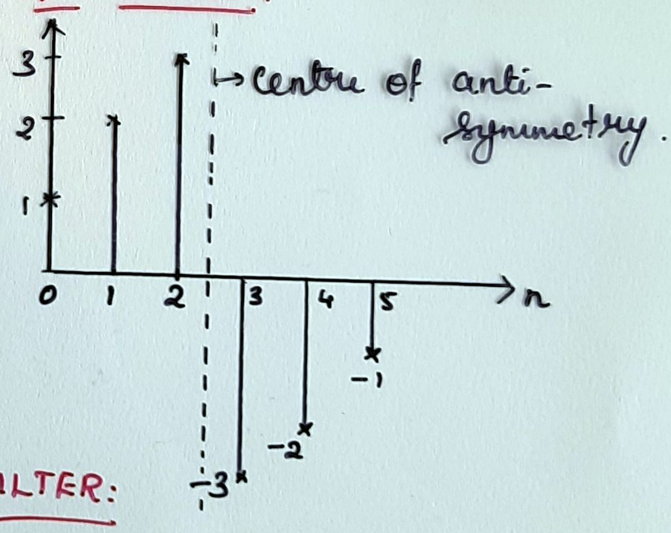
$N = 6,\quad \tau = 2.5$

$h(n) = \{1, 2, 3\}$

$h(3) = -h(6-1-3) = -h(2) = -3$

$h(4) = -h(6-1-4) = -h(1) = -2$

$h(5) = -h(6-1-5) = -h(0) = -1$



* FREQUENCY RESPONSE OF LINEAR PHASE FIR FILTER:

Depending upon N, & Symmetry conditions.

Case (i): Symmetrical impulse response when N is odd.

Case (ii): " " " " N is even.

Case (iii): Anti- " " " " N is odd

Case (iv): " " " " " N is even

Types of impulse response & its applications:

1. Symmetrical:

$$h(n) = h(N-1-n)$$

τ_p & τ_g $\Rightarrow$ constant

phase charateristics of FIR filter $\Rightarrow$ linear

Appln: to design linear phase LPF, HPF, BPF & BSF.

2) Anti- symmetrical:

$$h(n) = -h(N-1-n)$$

(alone) τ_g $\Rightarrow$ constant

Appln: to design Hilbert transformers & differentiators.

Condition for FIR filter to have linear phase:

i) $h(n)$ should be symmetrical

$$h(n) = h(N-1-n)$$

ii) $\left.\begin{array}{l}\tau_p \text{ (phase delay)} \\ \tau_g \text{ (group delay)}\end{array}\right\}$ should be constant (i.e., independent of freq.

Case (i): Frequency Response of Linear phase FIR filter when impulse response is symmetrical & N is odd

$N=7$

$$\begin{array}{ccccccc} 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ 0 & & \frac{N-3}{2} & \frac{N-1}{2} & \frac{N+1}{2} & & N-1 \end{array}$$

Frequency response for FIR Filter is,

$$H(\omega) = \sum_{n=0}^{N-1} h(n) \cdot e^{-j\omega n}$$

$$\sum_{n=0}^{N-1} = \sum_{n=0}^{(N-3)/2} \text{''} + \sum_{n=\frac{N-1}{2}} \text{''} + \sum_{n=\frac{N+1}{2}}^{N-1} \text{''}$$

$$H(\omega) = \sum_{n=0}^{(N-3)/2} h(n) \cdot e^{-j\omega n} + h\left(\frac{N-1}{2}\right) \cdot e^{-j\omega\left(\frac{N-1}{2}\right)} + \underbrace{\sum_{n=\frac{N+1}{2}}^{N-1} h(n) \cdot e^{-j\omega n}}$$

Apply Symmetrical condition,

$$\boxed{h(n) = h(N-1-n)}$$

Let, $m = N-1-n$; $n = N-1-m$

when $n = \frac{N+1}{2}$, $m = N-1-\left(\frac{N+1}{2}\right) = \frac{N-3}{2}$

$n = N-1$, $m = N-1-(N-1) = 0$

$$H(\omega) = h\left(\frac{N-1}{2}\right) \cdot e^{-j\omega\left(\frac{N-1}{2}\right)} + \sum_{n=0}^{(N-3)/2} h(n) e^{-j\omega n} + \sum_{m=\frac{N-3}{2}}^{0} h(N-1-m) \cdot e^{-j\omega(N-1-m)}$$

$$H(\omega) = \text{''} + \text{''} + \sum_{m=0}^{\frac{N-3}{2}} h(N-1-m) \cdot e^{-j\omega(N-1-m)}$$

Replace m by n,

$$H(\omega) = \text{''} + \text{''} + \sum_{n=0}^{(N-3)/2} h(N-1-n) \cdot e^{-j\omega(N-1-n)}$$

$[\because h(n) = h(N-1-n)]$

$$= e^{-j\omega\left(\frac{N-1}{2}\right)} \left\{ h\left(\frac{N-1}{2}\right) + \sum_{n=0}^{(N-3)/2} h(n) \cdot e^{-j\omega n} \cdot e^{j\omega\left(\frac{N-1}{2}\right)} + \sum_{n=0}^{N-3/2} h(n) \cdot e^{-j\omega(N-1)} \cdot e^{+j\omega n} \cdot e^{j\omega\left(\frac{N-1}{2}\right)} \right\}$$

$$H(\omega) = e^{-j\omega\left(\frac{N-1}{2}\right)} \left\{ h\left(\frac{N-1}{2}\right) + \sum_{n=0}^{(N-3)/2} h(n) \cdot e^{j\omega\left(\frac{N-1}{2} - n\right)} + \sum_{n=0}^{(N-3)/2} h(n) \cdot e^{-j\omega\left[(N-1) - \left(\frac{N-1}{2}\right) - n\right]} \right\}$$

$$H(\omega)= e^{-j\omega\left(\frac{N-1}{2}\right)}\left\{h\left(\frac{N-1}{2}\right)+\sum_{n=0}^{\left(\frac{N-3}{2}\right)} h(n)\left[e^{j\omega\left(\frac{N-1}{2}-n\right)}+e^{-j\omega\left(\frac{N-1}{2}-n\right)}\right]\right\}$$

$$\cos\theta = \frac{e^{j\theta}+e^{-j\theta}}{2}$$

$$H(\omega)= e^{-j\omega\left(\frac{N-1}{2}\right)}\left\{h\left(\frac{N-1}{2}\right)+\sum_{n=0}^{\frac{N-3}{2}} h(n)\left[2\cos\left(\frac{N-1}{2}-n\right)\omega\right]\right\}$$

let $k=\frac{N-1}{2}-n$, $n=\frac{N-1}{2}-k$

when $n=0$, $k=\frac{N-1}{2}$ & $n=\frac{N-3}{2}$; $k=\frac{N-1}{2}-\frac{N-3}{2}=1$

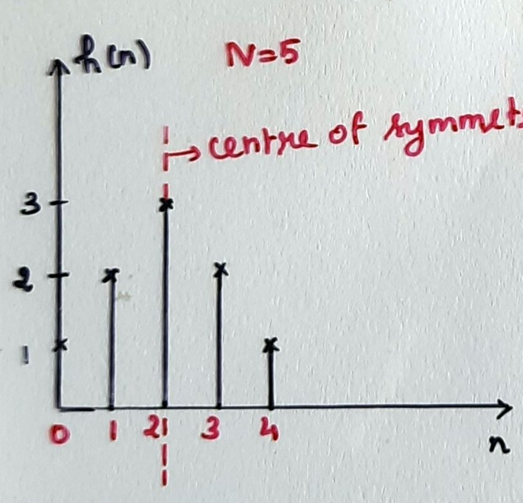
$$H(\omega)= e^{-j\omega\left(\frac{N-1}{2}\right)}\left\{h\left(\frac{N-1}{2}\right)+\sum_{k=1}^{N-1/2} 2\,h\left(\frac{N-1}{2}-k\right)\cos\omega k\right\}$$

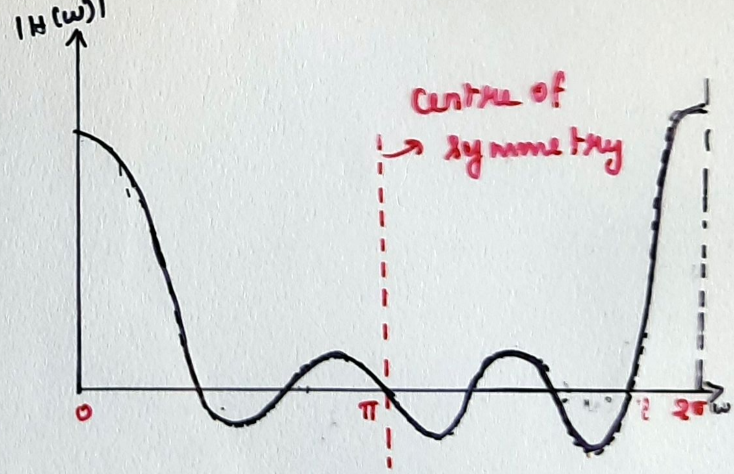
Replace k by n.

$$\boxed{H(\omega)= e^{-j\omega\left(\frac{N-1}{2}\right)}\left\{h\left(\frac{N-1}{2}\right)+\sum_{n=1}^{N-1/2} 2\,h\left(\frac{N-1}{2}-n\right)\cdot\cos\omega n\right\}}$$

$$|H(\omega)| = h\left(\frac{N-1}{2}\right)+\sum_{n=1}^{N-1/2} 2\,h\left(\frac{N-1}{2}-n\right)\cos\omega n$$

$$\angle H(\omega) = -\omega\left(\frac{N-1}{2}\right) = -\omega\tau \quad [\text{where } \tau=\frac{N-1}{2}$$

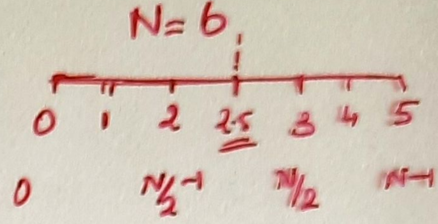




Case (ii): when impulse response is symmetrical & N is even:

$$H(\omega) = \sum_{n=0}^{N-1} h(n).e^{-j\omega n}$$

$$= \sum_{n=0}^{N/2-1} h(n).e^{-j\omega n} + \sum_{n=N/2}^{N-1} h(n).e^{-j\omega n}$$



let $m = N-1-n$; $n = N-1-m$

when $n = N/2$, $m = N-1-N/2 = N/2-1$

$n = N-1$, $m = N-1-(N-1) = 0$

$$H(\omega) = \sum_{n=0}^{N/2-1} h(n).e^{-j\omega n} + \sum_{m=N/2-1}^{0} h(N-1-m).\, e^{-j\omega(N-1-m)}$$

$$\sum_{m=N/2-1}^{0} \rightarrow \sum_{m=0}^{N/2-1}$$

Replace m by n,

$$H(\omega) = \sum_{n=0}^{N/2-1} h(n).e^{-j\omega n} + \sum_{n=0}^{N/2-1} h(N-1-n).\, e^{-j\omega(N-1-n)}$$

Apply symmetrical conditions, $h(n) = h(N-1-n)$

$$= \sum_{n=0}^{N/2-1} h(n).e^{-j\omega n} + \sum_{n=0}^{N/2-1} h(n).\, e^{-j\omega(N-1-n)}$$

$$= \sum_{n=0}^{N/2-1} h(n)\left[e^{-j\omega n} + e^{-j\omega(N-1-n)}\right]$$

$$= \sum_{n=0}^{N/2-1} h(n)\, e^{-j\omega\left(\frac{N-1}{2}\right)}\left\{e^{-j\omega n}.e^{j\omega\left(\frac{N-1}{2}\right)} + e^{-j\omega(N-1-n)}.e^{j\omega\left(\frac{N-1}{2}\right)}\right\}$$

$$= \sum_{n=0}^{N/2-1} h(n).e^{-j\omega\left(\frac{N-1}{2}\right)}\left\{e^{j\omega\left(\frac{N-1}{2}-n\right)} + e^{-j\omega\left(N-1-n-\frac{N-1}{2}\right)}\right\}$$

$$= \sum_{n=0}^{N/2-1} h(n).\, e^{-j\omega\left(\frac{N-1}{2}\right)}.\left\{e^{j\omega\left(\frac{N-1}{2}-n\right)} + e^{-j\omega\left(\frac{N-1}{2}-n\right)}\right\}$$

$$H(\omega) = \sum_{n=0}^{N/2-1} h(n).\, e^{-j\omega\left(\frac{N-1}{2}\right)}.\ 2\cos\omega\left(\frac{N-1}{2}-n\right)\Big\}$$

$$\frac{N-1}{2} - n = \frac{N}{2} - n - \frac{1}{2}$$

$$H(\omega) = \sum_{n=0}^{\frac{N}{2}-1} 2\,h(n)\,.\; e^{-j\omega\left(\frac{N-1}{2}\right)} \cdot \cos\omega\left(\frac{N}{2}-n-\frac{1}{2}\right)$$

let $N/2 - n = k$, $\quad n = N/2 - k$

when $n=0$, $k = N/2$ & $n = N/2 - 1$, $k = 1$

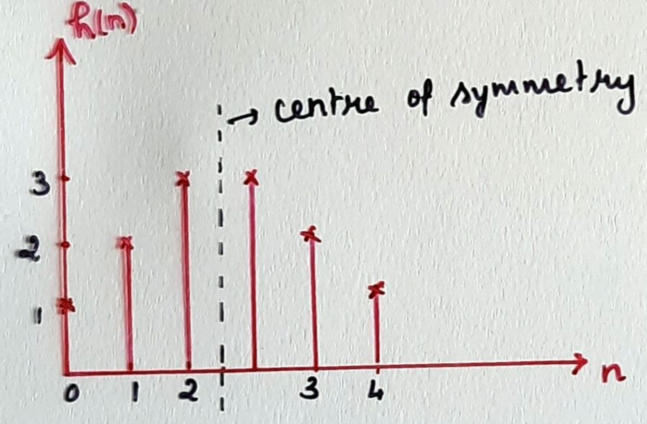
$$H(\omega) = \sum_{k=1}^{N/2} 2\,h\left(N/2 - k\right)\cos\omega\left(k-\tfrac{1}{2}\right) \cdot e^{-j\omega\left(\frac{N-1}{2}\right)}$$

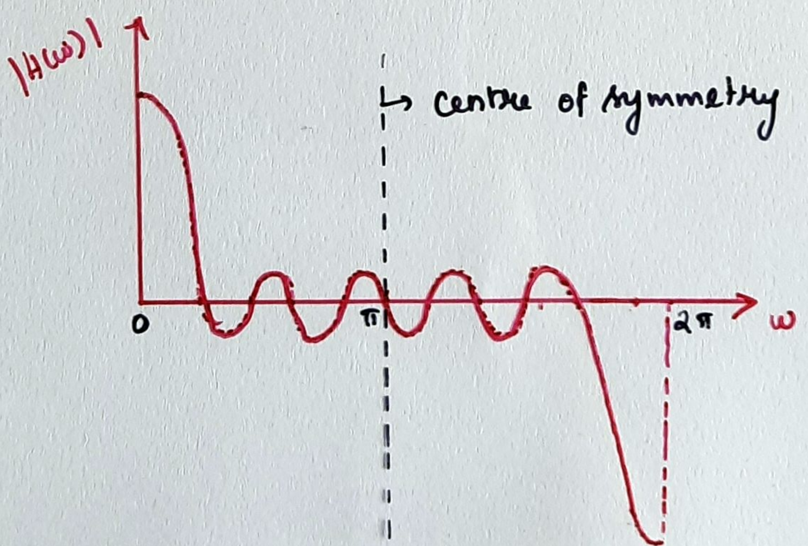
Replace k by n,

$$\boxed{H(\omega) = \sum_{n=1}^{N/2} 2\,h\left(N/2 - n\right)\cos\omega\left(n-\tfrac{1}{2}\right)\, e^{-j\omega\left(\frac{N-1}{2}\right)}}$$

$$|H(\omega)| = \sum_{k=1}^{N/2} 2\,h\left(N/2 - n\right)\cos\omega\left(n - \tfrac{1}{2}\right)$$

$$\angle H(\omega) = \phi(\omega) = -\omega\cdot\left(\frac{N-1}{2}\right) = -\omega\tau \quad \left[\because \tau = \frac{N-1}{2}\right.$$





Location of zeros of Linear phase FIR filters:

The transfer function of a Linear phase FIR filter is given by,

$$H(z) = \sum_{n=0}^{N-1} h(n).z^{-n} \longrightarrow ①$$

Let z_0 is a finite zero of $H(z)$ & $z_0 \neq 0$

then $$H(z)/_{z=z_0} = H(z_0) = \sum_{n=0}^{N-1} h(n)\, z_0^{-n} = 0$$

$$H(z_0) = h(0) + h(1) z_0^{-1} + h(2) z_0^{-2} + \dots + h(N-1).z_0^{-(N-1)} = 0 \quad ②$$

For a linear phase filter,

$$h(n) = h(N-1-n)$$

② ⇒

$$H(z_0) = h(N-1) + h(N-2) z_0^{-1} + \dots + h(1) z_0^{-(N-2)} + h(0) z_0^{-(N-1)} = 0$$

$$H(z_0) = z^{-(N-1)} \left[h(N-1) z_0^{N-1} + h(N-2) z_0^{N-2} + \dots + h_1 z_0^{1} + h(0) \right] = 0$$

$$= z^{-(N-1)} \sum_{n=0}^{N-1} h(n) z_0^{n} = 0$$

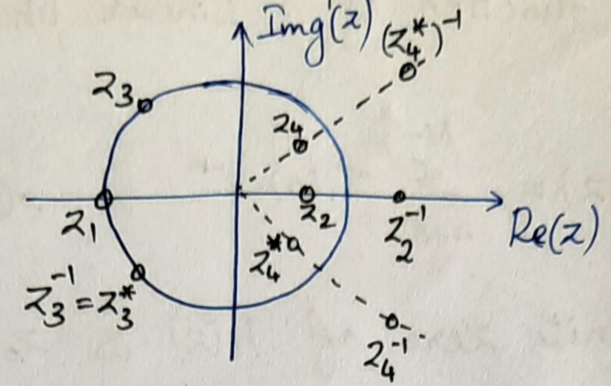
$$H(z_0) = z^{-(N-1)} \sum_{n=0}^{N-1} h(n) (z_0^{-1})^{-n} = z^{-(N-1)} H(z_0^{-1}) = 0$$

$$\therefore H(z_0^{-1}) = 0$$

$$H(z_0) = H(z_0^{-1}) = 0 \longrightarrow ③$$

From ③, We find that If 'z_0' is a zero of $H(z)$, then z_0^{-1} is also a zero.

From the result above, we can find the location of zeros in a linear phase FIR filter as follows.



⇒ If $z_1 = -1$, then $z_1^{-1} = \frac{1}{-1} = -1 = z_1$, the zero lies at $z_1 = -1$, (this group contains only one zero on the unit-circle)

⇒ If z_2 is real zero with $|z_2| < 1$ then z_2^{-1} is also a real zero & there are two zeros in this group.

eg: $z_2 = \frac{1}{2} + j0$
$z_2^{-1} = 2$

⇒ If z_3 is a complex zero with $|z_3| = 1$ then $z_3^{-1} = z_3^*$ & there are two zeros in this group.

eg: $z_3 = \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}$, $|z_3| = \sqrt{\frac{1}{2} + \frac{1}{2}} = 1$ (on the unit circle)

$z_3^{-1} = z_3^* = \frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}$ $|z_3^{-1}| = \sqrt{\frac{1}{2} + \frac{1}{2}} = 1$ (")

⇒ If z_4 is a complex zero with $|z_4| \neq 1$.

this group contains 4 zeros: z_4, z_4^{-1}, z_4^*, $(z_4^*)^{-1}$

eg: $z_4 = 1 + j1$, $|z_4| \neq 1$, $|z_4| = 1.414$; $z_4^* = 1 - j1$, $|z_4^*| = 1.414$

$z_4^{-1} = \frac{1}{1+j} = \frac{1}{2} - j\frac{1}{\sqrt{2}}$, $|z_4^{-1}| = 0.707$; $(z_4^*)^{-1} = \frac{1}{1-j1} = \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}$, $|(z_4^*)^{-1}| = 0.707$