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Transmission Lines and Wave Guides.

Unit-I

4/7/15

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Unit - I

Introduction to transmission Lines

* Tr-ion S/m's are used to transfer energy from one point to another.

* The energy may be
  - Sound
  - Electrical power
  - Digital / Analog / optical signal
  - or any combination of the above
  - Audio / video / Data.

* **General properties of tr-ion S/m's**

Incident waves – Waves emerging directly from the sender.

* The signal propagate along a tr-ion path down the length of the tunnel (tr-ion line)

* Impedance – opposition caused by the air mass in the tunnel.

* Imp. is determined by the physical characteristics of the tunnel. (width, height ...)

* characteristics imp (Z_0) → The imp. related to the characteristics of the tunnel.
* Bend or discontinuities along the line walls cause a change – reflections in the signal propagation path.
* The walls do absorb some energy – weaken – attenuated the propagated energy.
* amplitude attn./unit length → α
* frequency attenuation – dispersion
* propagation velocity – speed.

  velocity of sound waves – 331 m/sec

  velocity of electrical waves – 3×10^8 m/sec
* Tunnel is infinitely long → signal will totally attenuated or absorbed
* Tunnel is not infinitely long – signal will be reflected

$$\text{Reflection co-efficient} = \frac{\text{Reflected wave}}{\text{Incident wave}}$$

* To avoid this – put some good absorption material at the end of the tunnel.

* Creating matching termination — Date: at chap

the propagation characteristics of a long tunnel in a tunnel of finite length.

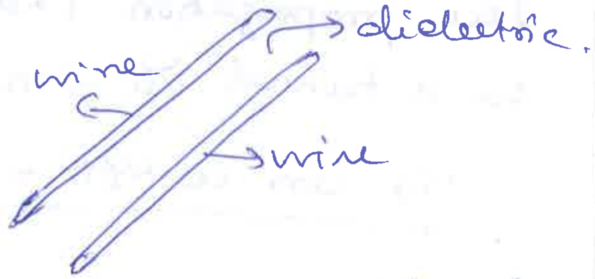
$$\text{Tr-ion co-eff} = \frac{\text{Received Signal}}{\text{Incident Signal}}$$

* Time propagation delay can be specified by,
  - seconds
  - Periodic Time (T) $= (1/f)$ Sec's
  - Phase delay $- 2\pi$ radians in a period Time T.

$$\text{Standing wave Ratio (SWR)} = \frac{V_{max}}{V_{min}}$$

Types of Electrical transmission line

1. Twin Lines



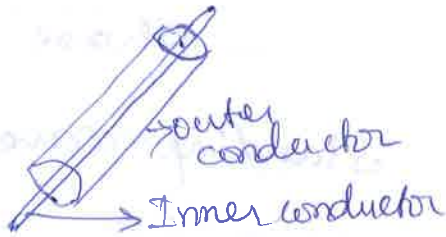
* Twin line spaced by a polyethylene dielectric.
* Used at relatively low frequencies.
* ch. imp. $Z_0 = 300\Omega$

freq = VHF

* TP lines & overhead power lines.

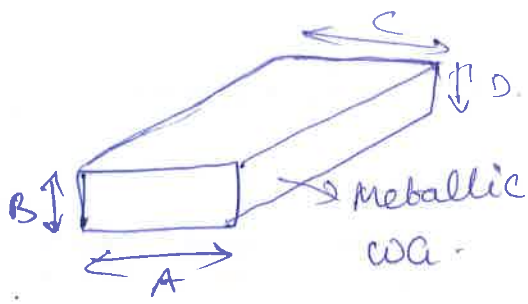
also used at Low & Medium freq radio tr-ers

2. Co-axial cable



* carrying HF signals
* Connects TV sets to antenna
* Two wire line, but the outer conductor forms a ~~clar~~ shield around the inner conductor to prevent radiation.

3. Metallic wave guide



* Tunnel or WG
* HF Tr-ion line ∴ Low attn. & radiation losses
* But expensive ∵ it is metallic construction (copper)
* Also relatively heavy + lacks flexibility

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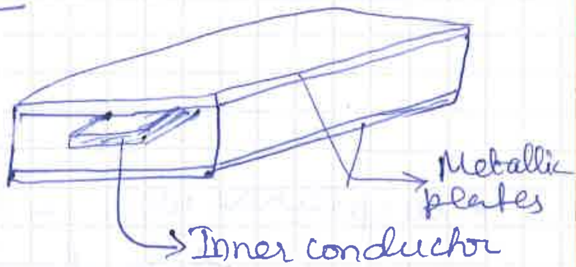
4. Strip line

* Similar to a flattened co-axial line.
* Advantage – ie it can be easily constructed with Integrated circuits.

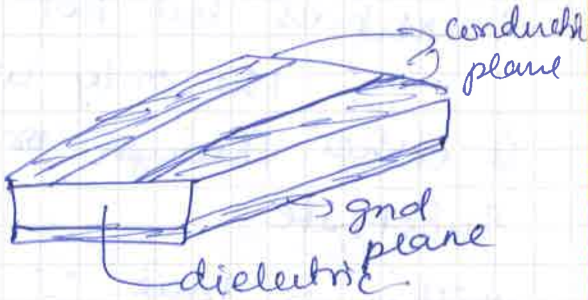


5) Micro strip line

* Varient of the strip line, with part of the "shield" removed.

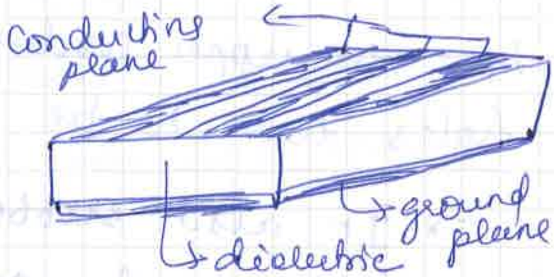
(6) Slot line

* Useful line for HF transmission



(7) Co-planar waveguide

* One varient of the WG.
* Used for MIC.



* For minimum loss
  - WG, co-axial line and strip line — find difficult in connecting external comp. to the inner conductor

* Co-planar WG → Better in this respect
  └ Finds favour in Monolithic MIC (MMIC)
* ∵ It allows easy series and ∥d connections to external Electrical components

* Microstrip line:- — Usefull for making series connections but not ∥d connections.
  * ∵ the only way through to the gnd. plane is either through or around the edge of the substrate.
* This is true ∵ when a SC is required b/w the upper conductor and the gnd plane, holes have to be drilled through the substrate.
  * It also suffers from radiation losses.
  * Can be made easily & conveniently & it is ∴ used extensively.

Transmission Line - General Solution

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* Lumped ckt elements
* Distributed elements
* Analysis of the circuit is also different.
* The voltage drop across each series increment of line, the voltage applied to each ^ment of shunt admittance is a variable.

∴ The shunted current is a variable along the line.

Hence, the cty.. around the loop is not a constant, but varies from pt to pt along the line.

Notations used will be defined as follows

R - series Resistance Ω/unit length of line

L - Series Inductance H/unit length " "

C capacitance b/w conductors Farads/ " " "

G - Shunt leakage conductance b/w conductors, ℧/unit " " " "

ωL - series reactance, Ω/unit " " " "

Z = R+jωL - series impedance, Ω/ " "

ωC - Shunt susceptance ℧/" " "

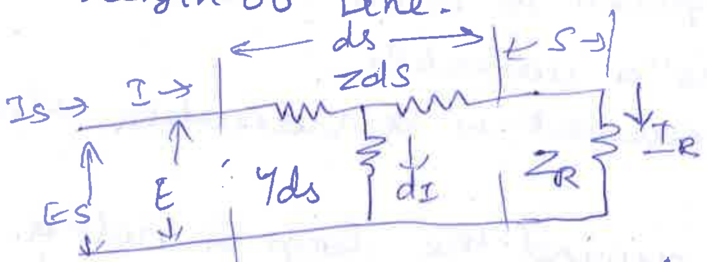
$Y = G + j\omega C$ = Shunt admittance, ℧/unit .. ...

S : Distance to the point of observation, measured from the receiving end of the line.

I - current in the line at any point

E - Voltage b/w conductors at any point

l - length of Line.



long line, with the elements of one of the infinitesimal section.

(line made up of infinitesimal T sections)

* This elemental section is of length ds and carries a current I.

* The series line imp. being z Ω/unit length

* Zds → series imp. of the element

* The voltage drop in the length ds is

$$dE = I \cdot z\,ds$$

$$\frac{dE}{ds} = IZ \quad \text{—— (1)}$$

* Y - Shunt admittance mho/unit length

Yds - admittance of the element of line

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$dI \rightarrow$ cty. flows across the line or from one conductor to the other.

$dI = EY\,ds$

$$\frac{dI}{ds} = EY \quad \text{——} \; (2)$$

Equ's (1) & (2) differentiated w.r.to s

$$\frac{d^2E}{ds^2} = Z\frac{dI}{ds} = ZYE \quad \text{——} \; (3)$$

$$\frac{d^2I}{ds^2} = Y\frac{dE}{ds} = ZYI \quad \text{——} \; (4)$$

(3) & (4) $\rightarrow$ differential equ's of the tr. on line.

E & I should be represented in the form of wave equ's.

$\therefore E = E_0 e^{j\omega t}$ & $I = I_0 e^{j\omega t}$.

say $\frac{d^2}{ds^2} = m^2 \rightarrow$ the operator,

(3) becomes $m^2E - ZYE = 0$; $[m^2 - (ZY)]E = 0$

$$m = \pm\sqrt{ZY} \quad \text{——} \; (5)$$

This result indicates 2 solutions ie + sign & - sign

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The solutions of the differential equ's are,

$$E = Ae^{\sqrt{ZY}s} + Be^{-\sqrt{ZY}s} \quad \text{—— (6)}$$

$$I = Ce^{\sqrt{ZY}s} + De^{-\sqrt{ZY}s} \quad \text{—— (7)}$$

where A, B, C & D ⇒ arbitrary constants of integration.

* Since distance is measured from the receiving end of the line, To assign the conditions;

$$s=0, \quad I = I_R \quad E = E_R.$$

∴ (6) & (7) becomes, $E_R = A+B$; $I_R = C+D$ —— (8)

* A second set of boundary conditions is not available.

* But the same set may be used over again if a new set of equ's are formed by differentiation of (6) & (7)

(6) → $\frac{dE}{ds} = A\sqrt{ZY}e^{\sqrt{ZY}s} - B\sqrt{ZY}\, e^{-\sqrt{ZY}s} = IZ$ (from (1))

(7) ⇒ $\therefore I = A\sqrt{\frac{Y}{Z}}e^{\sqrt{ZY}s} - B\sqrt{\frac{Y}{Z}}e^{-\sqrt{ZY}.s}$ —— (9)

$$\frac{dI}{ds} = C\sqrt{ZY}\,e^{\sqrt{ZY}s} - D\sqrt{ZY}e^{-\sqrt{ZY}.s} = EY \text{ (from (2))}$$

$$E = C\sqrt{\frac{Z}{Y}}e^{\sqrt{ZY}s} - D\sqrt{\frac{Z}{Y}}e^{-\sqrt{ZY}.s} \quad \text{—— (10)}$$

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At $s=0$, equ's (9) & (10) becomes,

$$I_R = A\sqrt{\frac{Y}{Z}} - B\sqrt{\frac{Y}{Z}} \quad \text{—— (11)}$$

$$E_R = C\sqrt{\frac{Z}{Y}} - D\sqrt{\frac{Z}{Y}} \quad \text{—— (12)}$$

$E_R = I_R Z_R$ & $\sqrt{\frac{Z}{Y}}$ has been identified as Z_0 of the line

The solutions for the constants of the above equt as, ~~$A \frac{E_R}{2}$~~

let $x = \sqrt{Z/Y}$ and $\frac{1}{x} = \sqrt{Y/Z}$

from (11) & (12) $I_R = \frac{A}{x} - \frac{B}{x} = \frac{1}{x}(A-B)$ —— (13)

$$E_R = Cx - Dx.$$

$$A+B = Cx - Dx \quad \text{—— (14)}$$

$C+D = \frac{1}{x}(A-B)$ (from (8))

$x(C+D) = A-B$ —— (13)

Add (13) & (16) we have,

$$\begin{aligned} A-B &= Cx + Dx \\ A+B &= Cx - Dx \\ \hline 2A &= 2Cx \end{aligned}$$

$\boxed{A = Cx}$ → (15)

Sub (13) & (16) we have,

$$\begin{aligned} A-B &= Cx + Dx \\ A+B &= Cx - Dx \\ \hline -2B &= 2Dx \end{aligned}$$

$\boxed{B = -Dx}$ → (15)

From (13) $xI_R = A - B = Cx + Dx$.

$E_R = Cx - Dx$

(add) $E_R + xI_R = 2Cx$ — (16)

(sub) $xI_R - E_R = 2Dx$ — (17)

$C = \frac{E_R}{2x} + \frac{I_R}{2}$ sub $x = \sqrt{\frac{Z}{Y}} = Z_0$

$E_R = I_R Z_R$

$\therefore C = \frac{I_R}{2} + \frac{E_R}{2}\sqrt{\frac{Y}{Z}}$

$= \frac{I_R}{2} + \frac{I_R}{2}\frac{Z_R}{Z_0} \Rightarrow \boxed{\frac{I_R}{2}\left[1 + \frac{Z_R}{Z_0}\right] = C}$ — (18)

(17) $\Rightarrow$

$xI_R - E_R = 2Dx$; $D = \frac{I_R}{2} - \frac{E_R}{2x}$

$D = \frac{I_R}{2} - \frac{E_R}{2}\sqrt{\frac{Y}{Z}} \Rightarrow \frac{I_R}{2} - \frac{E_R \cdot Z_R}{2 Z_0} \Rightarrow \boxed{\frac{I_R}{2}\left[1 - \frac{Z_R}{Z_0}\right] = D}$ — (19)

(15) $\Rightarrow A = Cx = \frac{I_R x}{2} + \frac{E_R}{2} = \frac{I_R}{2}\sqrt{Z/Y} + \frac{E_R}{2}$

$= \frac{E_R \cdot Z_0}{2Z_R} + \frac{E_R}{2} \Rightarrow \cancel{E_R}$

$= \frac{E_R}{2}\left[1 + \frac{Z_0}{Z_R}\right]$ — (20)

$-E_R = 2Dx$ — (17)

$\sqrt{\frac{Z}{Y}} = Z_0$

Z_R

$\boxed{= C}$ — (18)

$\boxed{\left[1 - \frac{Z_R}{Z_0} \right] = D}$ — (19)

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(15) $B = -Dx = \frac{E_R}{2} - \frac{I_R x}{2} = \frac{E_R}{2} - \frac{I_R}{2}\sqrt{\frac{Z}{Y}}$

$B = \frac{E_R}{2} - \frac{E_R}{2Z_R} \cdot Z_0 \Rightarrow \frac{E_R}{2}\left[1 - \frac{Z_0}{Z_R}\right]$ — (21)

$\therefore$ $A = \frac{E_R}{2}\left[1 + \frac{Z_0}{Z_R}\right]$ $\quad C = \frac{I_R}{2}\left[1 + \frac{Z_R}{Z_0}\right]$

$B = \frac{E_R}{2}\left[1 - \frac{Z_0}{Z_R}\right]$ $\quad D = \frac{I_R}{2}\left[1 - \frac{Z_R}{Z_0}\right]$

from (6)

$$E = Ae^{\sqrt{ZY}s} + Be^{-\sqrt{ZY}s}$$

$$= \frac{E_R}{2}\left(1 + \frac{Z_0}{Z_R}\right)e^{\sqrt{ZY}s} + \frac{E_R}{2}\left[1 - \frac{Z_0}{Z_R}\right] \cdot e^{-\sqrt{ZY} \cdot s}$$

$$= \frac{E_R}{2}\left[\left(1 + \frac{Z_0}{Z_R}\right)e^{\sqrt{ZY}s} + \left(1 - \frac{Z_0}{Z_R}\right) \cdot e^{-\sqrt{ZY} \cdot s}\right]$$ — (22)

From (7)

$$I = Ce^{\sqrt{ZY}s} + De^{-\sqrt{ZY} \cdot s}$$

$$= \frac{I_R}{2}\left[1 + \frac{Z_R}{Z_0}\right]e^{\sqrt{ZY} \cdot s} + \frac{I_R}{2}\left[1 - \frac{Z_R}{Z_0}\right] \cdot e^{-\sqrt{ZY}s}$$

$$= \frac{I_R}{2}\left[\left(1 + \frac{Z_R}{Z_0}\right)e^{\sqrt{ZY} \cdot s} + \left[1 - \frac{Z_R}{Z_0}\right]e^{-\sqrt{ZY} \cdot s}\right]$$ — (23)

From (22)

$$E = \frac{E_R}{2}\left[\left(\frac{Z_R+Z_0}{Z_R}\right)e^{\sqrt{ZY}\,s} + \left(\frac{Z_R-Z_0}{Z_R}\right)e^{-\sqrt{ZY}\,s}\right]$$

$$= \frac{E_R(Z_R+Z_0)}{2Z_R}\left[e^{\sqrt{ZY}\,s} + \frac{\left(\frac{Z_R-Z_0}{\cancel{Z_R}}\right)}{\left(\frac{Z_R+Z_0}{\cancel{Z_R}}\right)}\cdot e^{-\sqrt{ZY}\cdot s}\right]$$

$$E = \frac{E_R\left[Z_R+Z_0\right]}{2Z_R}\left[e^{\sqrt{ZY}\cdot s} + \left(\frac{Z_R-Z_0}{Z_R+Z_0}\right)e^{-\sqrt{ZY}\cdot s}\right] \quad \text{— (24)}$$

||ly from (23)

$$I = \frac{I_R}{2}\left[\left(\frac{Z_0+Z_R}{Z_0}\right)e^{\sqrt{ZY}\,s} + \left(\frac{Z_0-Z_R}{Z_0}\right)e^{-\sqrt{ZY}\,s}\right]$$

$$= \frac{I_R(Z_R+Z_0)}{2Z_0}\left[e^{-\sqrt{ZY}\,s} + \left(\frac{Z_0-Z_R}{Z_0+Z_R}\right)\cancel{\left(\frac{Z_R-Z_0}{Z_R+Z_0}\right)}e^{-\sqrt{ZY}\cdot s}\right] \quad \text{— (25)}$$

The equations (24) & (25) provide voltage and current at any point on a transmission line.

Eqn's (22) & (23) Also be arranged as, $\boxed{I_R = \frac{E_R}{Z_R}}$

$$E = \frac{E_R}{2}e^{\sqrt{ZY}\,s} + \frac{I_R\cdot Z_0}{2}e^{\sqrt{ZY}\cdot s} + \frac{E_R}{2}e^{-\sqrt{ZY}\cdot s} - \frac{I_R Z_0}{2}e^{-\sqrt{ZY}\,s}$$

$$= E_R\left[\frac{e^{\sqrt{ZY}\,s} + e^{-\sqrt{ZY}\,s}}{2}\right] + I_R Z_0\left[\frac{e^{\sqrt{ZY}\,s} - e^{-\sqrt{ZY}\cdot s}}{2}\right]$$

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$$E = E_R \cosh\sqrt{ZY}\cdot s + I_R Z_0 \sinh\sqrt{ZY}\cdot s \quad \text{--- (26)}$$

$$\text{Ily } (23) \Rightarrow I = \frac{I_R e^{\sqrt{ZY}s}}{2} + \frac{E_R}{2Z_0} e^{\sqrt{ZY}\cdot s} + \frac{I_R}{2} e^{-\sqrt{ZY}s} - \frac{E_R}{2Z_0} e^{-\sqrt{ZY}s}$$

$$= I_R \left[\frac{e^{\sqrt{ZY}s} + e^{-\sqrt{ZY}\cdot s}}{2}\right] + \frac{E_R}{Z_0}\left[\frac{e^{\sqrt{ZY}\cdot s} - e^{-\sqrt{ZY}\cdot s}}{2}\right]$$

$$I = I_R \cosh\sqrt{ZY}\cdot s + \frac{E_R}{Z_0} \sinh\sqrt{ZY}\cdot s \quad \text{--- (27)}$$

Equations (26) & (27) also useful form for the voltage and current values at any point on a tr-ion line.

Physical significance of the equations: The infinite line

Equ (27) may be written for the sending end current I_s of a line of length l as -

$E_R = I_R Z_R$

$$I_s = I_R\left[\cosh\sqrt{ZY}\,l + \frac{Z_R}{Z_0}\sinh\sqrt{ZY}\,l\right]$$

If the line is terminated in $Z_R = Z_0$, then

$$I_s = I_R\{\cosh\sqrt{ZY}\,l + \sinh\sqrt{ZY}\,l)$$

$$\frac{I_s}{I_R} = e^{\sqrt{ZY}\,l} = e^{\gamma l} \quad \text{——— (28)}$$

where, $\sqrt{ZY} = \gamma$ = propagation constant γ, per unit length of line. $\gamma = \alpha + j\beta$.

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α – amp. attn. constant

β – ~~propagation~~ phase const.

Division of Equ's (26) by (27) leads to an expression for the i/p impedance of line of length l,

$\div I_R$

$$Z_s = \frac{E_s}{I_s} = Z_0\left[\frac{Z_R\cosh\gamma l + Z_0\sinh\gamma l}{Z_0\cosh\gamma l + Z_R\sinh\gamma l}\right] \quad \text{—— (29)}$$

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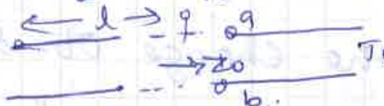
In different form, this result may be obtained by division of (26) by (25) & $\frac{E_R}{I_R} = Z_R$

$$Z_S = \frac{E_S}{I_S} = Z_0 \left[\frac{e^{\gamma l} + \left(\frac{Z_R - Z_0}{Z_R+Z_0}\right)e^{-\gamma l}}{e^{\gamma l} - \left(\frac{Z_R-Z_0}{Z_R+Z_0}\right)e^{-\gamma l}}\right] \quad \text{(30)}$$

$$\frac{E}{I} = \frac{\frac{E_R(Z_R+Z_0)}{2Z_R}\left(e^{\sqrt{ZY}.s} + \left(\frac{Z_R-Z_0}{Z_R+Z_0}\right)e^{-\sqrt{ZY}.s}\right)}{\frac{I_R(Z_R+Z_0)}{2Z_0}\left(e^{\sqrt{ZY}.s} - \left(\frac{Z_R-Z_0}{Z_R+Z_0}\right)e^{-\sqrt{ZY}s}\right)}$$

$$= \left(\frac{E_R}{I_R}\right)^{Z_R}\frac{Z_0}{Z_R}\frac{\left(e^{\sqrt{ZY}.s} + \left(\frac{Z_R-Z_0}{Z_R+Z_0}\right)e^{-\gamma l}\right)}{\left(e^{\sqrt{ZY}s} + \left(\frac{Z_R \pm Z_0}{Z_R+Z_0}\right)e^{-\gamma l}\right)}$$

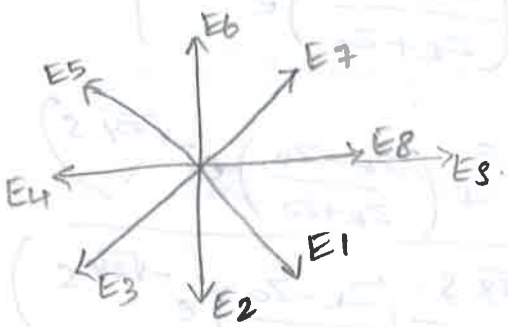
To find the value of sending end i/p imp. Z_S, when the line is terminated in its characteristics impedance ie $Z_R = Z_0$, equ (30) becomes, $Z_S = Z_0$.

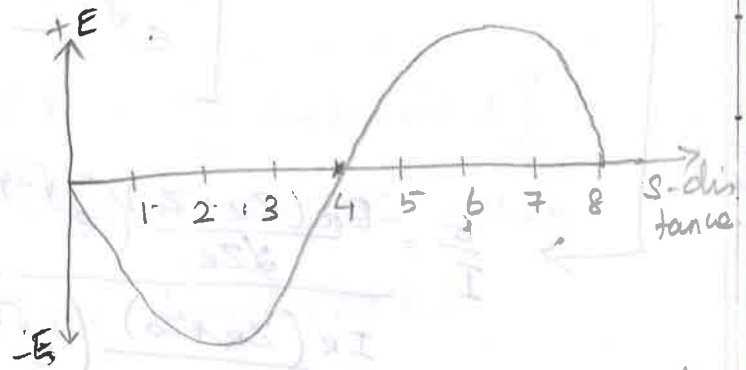


A line of infinite length, terminated in a load equivalent to its characteristics imp. appears to the sending end gen as a infinite line.

Wavelength & velocity of propagation

Wavelength :- The distance, the wave travels along the line, while the phase angle is changing through 2π radians is called wavelength.





1/8 WL points along a line at a particular time instant
(a)

Plot of the phasors of (a) showing voltage on the line as a function of distance.

Wavelength is represented by λ.

$\therefore$ distance $s = \lambda$

$\beta\lambda = 2\pi$; $\boxed{\lambda = \frac{2\pi}{\beta}}$ — ①

Since, the change of 2π in phase angle represents one cycle in time and occurs in a distance of one wavelength, then, $x = vt$

$\boxed{\lambda = \frac{v}{f}}$ — ②

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The velocity can be expressed in terms of line constants,

$$u = \lambda f = \frac{2\pi f}{\beta} \qquad \therefore u = \frac{\omega}{\beta} \quad \text{—— (3)}$$

This is velocity of propagation along the line.

$$Z = R + j\omega L \qquad Y = G + j\omega C.$$

$$\gamma = \alpha + j\beta = \sqrt{ZY} = \sqrt{RG - \omega^2 LC + j\omega(LG + CR)} \quad \text{—— (4)}$$

Squaring on both sides,

$$\alpha^2 + j2\alpha\beta - \beta^2 = RG - \omega^2 LC + j\omega(LG + CR)$$

equating the real and Imaginary parts,

$$\alpha^2 = \beta^2 + RG - \omega^2 LC \quad \text{—— (5)}$$

$$2\alpha\beta = \omega(LG + CR)$$

squaring, $\quad 4\alpha^2\beta^2 = \omega^2(LG + CR)^2 \quad$ —— (6)

Sub. (5) into (6), we have,

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$4(\beta^2 + RG - \omega^2 LC)\beta^2 = \omega^2(LG + CR)^2$$

$$\beta^4 + \beta^2(RG - \omega^2 LC) - \frac{\omega^2}{4}(LG + CR)^2 = 0$$

A solution for β, neglecting the -ve values,

$$\beta^2 = \frac{(\omega^2 LC - RG) \pm \sqrt{(RG - \omega^2 LC)^2 + \cancel{4}\,\omega^2(LG + CR)^2 / \cancel{4}}}{2}$$

$$\beta = \sqrt{\frac{w^2LC - RG + \sqrt{(RG - w^2LC)^2 + w^2(LG+CR)^2}}{2}} \quad \text{—} (7)$$

lly From equ (5)

$$\alpha = \sqrt{\frac{RG - w^2LC + \sqrt{(RG - w^2LC)^2 + w^2(LG+CR)^2}}{2}} \quad \text{—} (8)$$

In a perfect line $R = 0$; $G = 0$ then (7) becomes

$$\beta = w\sqrt{LC} \quad \text{—} (9) \qquad \sqrt{\frac{w^2LC + \sqrt{(w^2LC)^2}}{2}} = \sqrt{\frac{2w^2LC}{2}}$$

The velocity of propagation for such an ideal line is,

$$v = \frac{w}{\beta} = \frac{1}{\sqrt{LC}} \text{ m/sec.} \quad \text{—} (10)$$

which shows the line parameter values fix the velocity of propagation.

$$L = \frac{\lambda}{I} = \left(2\frac{\mu}{8\pi} + \frac{\mu_v}{2\pi} \ln\frac{d}{a}\right)$$

$$C = \frac{\pi \epsilon}{\ln\frac{d}{a}} \text{ farads/m}$$

$$LC = \frac{\pi \epsilon}{\ln\left(\frac{d}{a}\right)}\left(\frac{\mu}{4\pi} + \frac{\mu_v}{2\pi} \ln\left(\frac{d}{a}\right)\right]$$

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for a line of non-magnetic material with air spacing,

$$LC = \frac{\mu_u \varepsilon_u}{4\ln\left(\frac{d}{a}\right)} + \mu_u \varepsilon_u \quad \text{—— (11)}$$

$$\therefore \quad v = \frac{1}{\mu_u \varepsilon_u \left[\frac{1}{4\ln\left(\frac{d}{a}\right)} + 1\right]} \ m/sec \quad \text{—— (12)}$$

μ_u = Magnetic permeability of space

$= 4\pi \times 10^{-7}$ in KMS unit

a – radius of the conductor

d – distance b/w conductors of a $\parallel^{l}$ wire line

$\frac{1}{4\ln\frac{d}{a}}$ → Internal inductance of the conductors

This internal inductance is ~~fixed~~, as by skin effect, the velocity rises and reaches the limiting condition of,

$$u = \frac{1}{\sqrt{\mu_u \varepsilon_u}} \ m/sec$$

which in space becomes, 3×10^8 ~~m~~/sec $= c$

which is identified as velocity of light in space.

$\boxed{\text{wave form distortion}}$

The distortionless Line

If a line is to have neither frequency nor delay distortion, then α and the velocity of propagation cann't be fns of frequency.

$v = \frac{\omega}{\beta}$ $\beta \to$ Direct fn. of freq.

$$\beta = \sqrt{\frac{\omega^2 LC - RG + \sqrt{(RG - \omega^2 LC)^2 + \omega^2 (LG + CR)^2}}{2}}$$

Shows that if the term under the second radical be reduced to equal $(RG + \omega^2 LC)^2$ — ?

Then the required condition on β is obtained.

* Expanding the terms under the internal radical, $(RG - \omega^2 LC)^2 + \omega^2 (LG + CR)^2 = (RG + \omega^2 LC)^2$

$$\cancel{R^2G^2} - 2\omega^2 RGLC + \cancel{\omega^4 L^2 C^2} + \omega^2 L^2 G^2 + \omega^2 C^2 R^2 + 2\omega^2 LGCR = (RG + \omega^2 LC)^2$$

$$= \cancel{R^2G^2} + 2\omega^2 LCRG + \cancel{\omega^4 L^2 C^2}$$

$$\omega^2 L^2 G^2 + \omega^2 C^2 R^2 - 2\omega^2 LCRG = 0$$

$$\omega (LG - CR)^2 = 0$$

This condn. will make β, a direct fn. of frequency.

$$(LG - CR)^2 = 0$$

$$\boxed{LG = CR}$$

A hypothetical line might be built to fullfill this condn.

nos delay
ation

$(LG+CR)^2$

and radical

is obtained
internal
$=(RG+w^2LC)^2$
$+w^2C^2R^2+$
$=(RG+w^2LC)^2$
2

line might

$$u = \frac{w}{\beta} = \frac{1}{\sqrt{LC}} \quad ; \quad \boxed{\beta = w\sqrt{LC} \quad u = \frac{1}{\sqrt{LC}}}$$

Date:

which is same for all frequencies, thus eliminating delay distortions

$$\text{||}^{ly} \quad \alpha = \sqrt{\frac{RG - w^2LC + \sqrt{(RG-w^2LC)^2 + w^2(LG+CR)^2}}{2}}$$

The internal radial is forced to lead to,

$(RG+w^2LC)^2$

w.k.T $LG = CR$

$$\cancel{R^2G^2} - 2w^2L\cancel{C}RG + w^4\cancel{L^2}C^2 + w^2L^2G^2 + w^2C^2R^2 + 2w^2\cancel{L}GCR$$
$$= \cancel{R^2}G^2 + 2w^2LCRG + w^4\cancel{L^2}C^2$$

$$w^2(LG-CR)^2 = 0 \quad ; \quad (LG-CR)^2 = 0.$$

It is possible to make α

$\alpha = \sqrt{RG}$ and the velocity independent of free simultaneously.

$\hookrightarrow$ which is independent of freq

$LG = CR$

$\frac{L}{C} = \frac{R}{G}$ requires large value of L

Since G = small.

G is ↑sed α & attn. is ↑sed → resulting poor line efficiency.

To ↑se R raises the size & cost of conductors above economic limits, so the hypothetical results can't be achieved.

The telephone Cable

Consider ordinary TP cable. wires are insulated with Paper & twisted in pairs.

→ ∴ negligible values of inductance & conductance. $L \& G = 0$.

$$\therefore Z = R. \quad (1) \qquad Y = j\omega C \quad (2)$$

$$\gamma = \alpha + j\beta = \sqrt{ZY} = \sqrt{RG - \omega^2 LC + j\omega(LG + CR)}$$

when $L = G = 0$ $\quad \gamma = \sqrt{j\omega CR} = \sqrt{j\frac{2\omega CR}{2}} \quad (3)$

$$\gamma = \alpha + j\beta = (1+j1)\sqrt{\frac{\omega CR}{2}} \qquad (1+j1)^2 = 1 + j^2 + 2j = 2j$$

$$\alpha = \sqrt{\frac{\omega CR}{2}} \qquad \beta = \sqrt{\frac{\omega CR}{2}}$$

The velocity of propagation is

$$v = \frac{\omega}{\beta} = \frac{\omega}{\sqrt{\frac{\omega CR}{2}}} = \sqrt{\frac{2\omega}{CR}}$$

It is observed that both α & velocity are fn's of freq, ∴ the higher freq's are attenuated more ($\alpha \propto f$) & travel faster ($v \propto f$) than lower freq's. ∴ considerable ~~fed~~ freq & delay distortion is the result on TP cable.

Inductance Loading of TP cables

It is necessary to ~~Dec~~ the L ratio to achieve distortionless condn.

* consider a uniformly loaded cable circuit
* Assume $G = 0$

L has been ↑sed, so that ωL is large w.r.to R.

$$\boxed{\tan^{-1}\frac{1}{x} = \frac{\pi}{2} - \tan^{-1}x}$$

Then,

$$Z = R + j\omega L$$

$$Y = j\omega C$$

***

$$Z = \sqrt{R^2 + \omega^2 L^2}\ \angle\left(\frac{\pi}{2} - \tan^{-1}\frac{R}{\omega L}\right) \quad \text{— (1)}$$

$\gamma l^2 \rightarrow \frac{\text{angle}}{2}$

$$\gamma l = \sqrt{ZY} = \sqrt{\sqrt{R^2 + \omega^2 L^2}\ \angle\left(\frac{\pi}{2} - \tan^{-1}\frac{R}{\omega L}\right) \times \omega C \angle\frac{\pi}{2}}$$

$$= \sqrt{\sqrt{R^2\omega^2C^2 + \omega^4L^2C^2}\ \angle\left(\frac{\pi^2}{4} - \frac{\pi}{2}\tan^{-1}\frac{R}{\omega L}\right)}$$

$$= \sqrt{\sqrt{\omega^4L^2C^2\left\{\left(\frac{R^2}{\omega^2L^2}\right) + 1\right\}}\ \angle\left(\frac{\pi^2}{4} - \frac{\pi}{2}\tan^{-1}\frac{R}{\omega L}\right)}$$

$$= \sqrt{\omega^2 LC\sqrt{1 + \frac{R^2}{\omega^2L^2}}} + \angle\left(\frac{\pi^2}{4} - \frac{\pi}{2}\tan^{-1}\frac{R}{\omega L}\right)$$

$$= \omega\sqrt{LC}\ \sqrt[4]{1 + \left(\frac{R^2}{\omega^2L^2}\right)}\ \angle\left(\frac{\pi}{2} - \frac{1}{2}\tan^{-1}\frac{R}{\omega L}\right) \quad \text{— (2)}$$

If R is small w.r.to ωL, the term $\frac{R^2}{\omega^2L^2}$ may be dropped & γl becomes

$$\gamma = \omega\sqrt{LC}\ \boxed{\pi/2 - \tfrac{1}{2}\tan^{-1}\frac{R}{\omega L}} \quad \text{--- (3)}$$

if $\theta = \pi/2 - \frac{1}{2}\tan^{-1} R/\omega L$,

$$\cos\theta = \cos\left(\pi/2 - \tfrac{1}{2}\tan^{-1}\frac{R}{\omega L}\right) = \sin\left(\tfrac{1}{2}\tan^{-1}\frac{R}{\omega L}\right)$$

For a small angle,

$$\sin\theta = \tan\theta = \theta$$
$$\tan^{-1}\theta = \theta$$

so that $\cos\theta = \dfrac{R}{2\omega L}$ — (4)

Likewise, $\sin\theta = \sin\left(\pi/2 - \frac{1}{2}\tan^{-1}\frac{R}{\omega L}\right) = 1$ — (5)

$= \sin(\pi/2)$

∴ rewrite eqn (3)

$$\gamma = \omega\sqrt{LC}\,(\cos\theta + j\sin\theta) = \omega\sqrt{LC}\left(\frac{R}{2\omega L} + j\right)$$

$$\Rightarrow \frac{R\omega\sqrt{LC}}{2\omega L} + j\omega\sqrt{LC}$$

∴ for the uniformly loaded cable,

$$= \frac{R}{2}\sqrt{C/L} + j\omega\sqrt{LC}$$
$$= \alpha + j\beta$$

$$\alpha = \frac{R}{2}\sqrt{\frac{C}{L}}$$

$$\beta = \omega\sqrt{LC}$$

$$v = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}$$

* Under the assumption of $G = 0$, & ωL large w.r.t. to R,

(i) the attn α & velocity v are independent of freq (no ω term) & loaded cable will be distortionless.

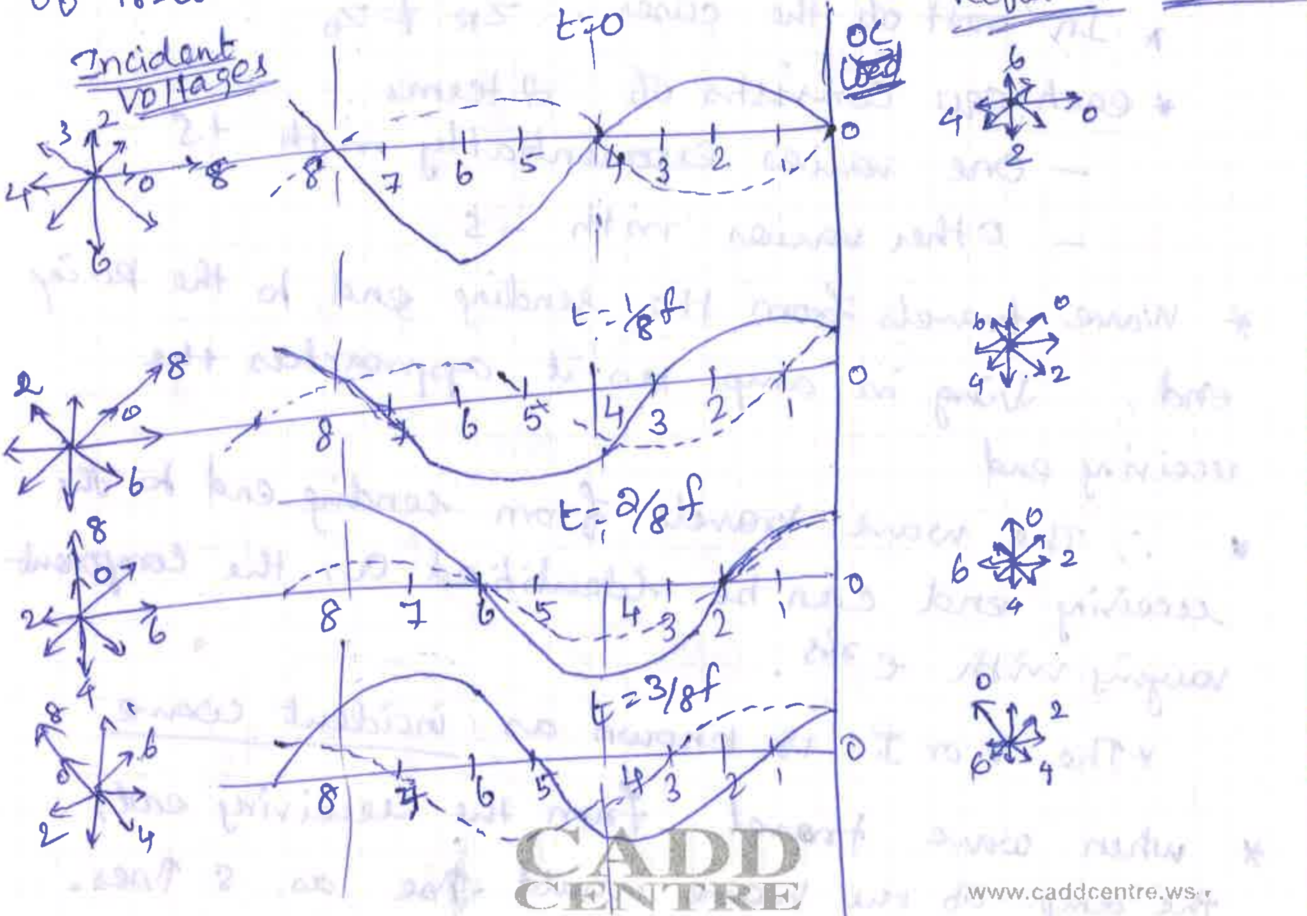
Date:

$\alpha = \frac{R}{2}\sqrt{C/L}$ shows that the attn. may be ↓ed by ↑ing L, provided that R is not also increased too greatly.

* continuous or uniform loading is expensive and achieves only a small ↑se in L per unit length.

* Lumped loading is preferred as a means of tr-con ↑ement for cables.

(*)(*) → Reflected Voltages



Reflection on a line, not terminated in Z_0

* The voltage and currents on the line, with S measured as positive from the receiving end, equs (24) & (25)

$$E = \frac{E_R(Z_R+Z_0)}{2Z_R}\left[e^{\gamma s} + \left(\frac{Z_R-Z_0}{Z_R+Z_0}\right)e^{-\gamma s}\right] \quad (1)$$

$$I = \frac{I_R(Z_R-Z_0)}{2Z_0}\left[e^{\gamma s} - \left(\frac{Z_R-Z_0}{Z_R+Z_0}\right)e^{-\gamma s}\right] \quad (2)$$

* In most of the cases, $Z_R \neq Z_0$.
* each equ consists of 2 terms.
  - One varies exponentially with $+S$
  - Other varies with $-S$.

* Wave travels from the sending end to the Ri-ing end, $\downarrow$ing in amp as it approaches the receiving end

* ∴ The wave travels from sending end to the receiving end can be identified as the component varying with $e^{\gamma s}$.
  * The V or I is known as <u>incident wave</u>

* when wave travel from the receiving end, the amp. of the wave would $\downarrow$se as S $\uparrow$ses.

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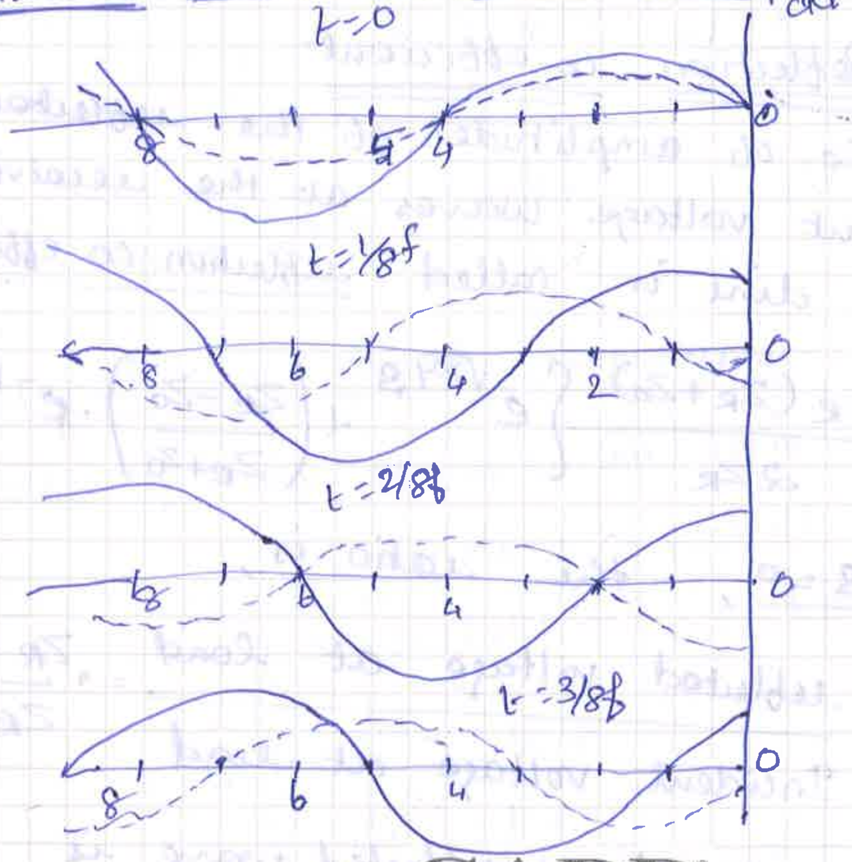
∴ The voltage or current is known as **reflected wave** & is represented by $e^{-\gamma s}$.

(*)(*) → For Reflected voltage, the initial value equal to the incident voltage at the load (for OC)

* The **total instantaneous voltage at any pt.** on the line is the **vector sum of the voltage of the** incident and reflected waves.

Incident and reflected current waves open ckt



* The only diff. b/w the curves for voltage and current (Z_R is infinite or OC) is in the reversed phase of the reflected current wave.

* From the fig, It is seen that, the 2 cty. waves are equal and opp. phase at the OC'ed receiving end.

* The total instantaneous cty. at that point always being zero as required by the open ckt.

Reflection co-efficient

The ratio of amplitudes of the reflected and incident voltage waves at the receiving end of the line is called reflection co-efficient.

$$E = \frac{E_R (Z_R + Z_0)}{2 Z_R} \left\{ e^{\sqrt{ZY} s} + \left(\frac{Z_R - Z_0}{Z_R + Z_0} \right) \cdot e^{-\sqrt{ZY} s} \right]$$

Sub. $s = 0$, the ratio is,

$$K = \frac{\text{reflected voltage at load}}{\text{Incident voltage at load}} = \frac{Z_R - Z_0}{Z_R + Z_0}$$

The polarity of the reflected wave is dependent on the angles & magnitudes of Z_R & Z_0.

∴ For termination of Z_0 (charac. imp.) the reflection co-efficient is zero.

From equ's (24) & (25)

$$E = \frac{E_R(Z_R+Z_0)}{2Z_R}\left(e^{\sqrt{ZY}\cdot s} + \frac{Z_R-Z_0}{Z_R+Z_0}\, e^{-\sqrt{ZY}\cdot s}\right)$$

$$= \frac{E_R(Z_R+Z_0)}{2Z_R}\left(e^{\gamma s} + K e^{-\gamma s}\right) \quad — (1)$$

$$I = \frac{I_R(Z_R+Z_0)}{2Z_0}\left[e^{\sqrt{ZY}\cdot s} - \left(\frac{Z_R-Z_0}{Z_R+Z_0}\right) e^{-\sqrt{ZY}\cdot s}\right]$$

$$= I_R \cdot \frac{Z_R+Z_0}{2Z_0}\left[e^{\gamma s} - K e^{-\gamma s}\right]. \quad — (2)$$

Input and Transfer impedances

The i/p imp of a tr-ion line is,

$$Z_s = Z_0\left[\frac{Z_R\cosh\gamma l + Z_0\sinh\gamma l}{Z_0\cosh\gamma l + Z_R\sinh\gamma l}\right] \quad — (1)$$

$$= Z_0\left[\frac{e^{\gamma l} + K e^{-\gamma l}}{e^{\gamma l} - K e^{-\gamma l}}\right] \quad — (2)$$

If the voltage at the sending-end is known, it is convenient to have the tr. impedance.

The sending end voltage is, E_S

$E_R = I_R Z_R$

$$E_S = \frac{E_R(Z_R+Z_0)}{2Z_R}\left(e^{\gamma l} + k e^{-\gamma l}\right)$$

$$= \frac{I_R(Z_R+Z_0)}{2}\left(e^{\gamma l} + k e^{-\gamma l}\right) \quad \text{③}$$

for which the transfer impedance is,

$$Z_T = \frac{E_S}{I_R} = \frac{(Z_R+Z_0)}{2}\left(e^{+\gamma l} + k e^{-\gamma l}\right) \quad \text{④}$$

For substituting the value of k, ④ becomes,

$$Z_T = \left(\frac{Z_R+Z_0}{2}\right)\left(e^{\gamma l} + \left(\frac{Z_R-Z_0}{Z_R+Z_0}\right)e^{-\gamma l}\right)$$

$$= \left(\frac{Z_R \cancel{+Z_0}}{2}\right)\left(\frac{(Z_R+Z_0)\, e^{\gamma l} + (Z_R-Z_0)e^{-\gamma l}}{Z_R \cancel{+Z_0}}\right)$$

$$= \frac{1}{2}\left[Z_R e^{\gamma l} + Z_0 e^{\gamma l} + Z_R e^{-\gamma l} - Z_0 e^{-\gamma l}\right]$$

$$= Z_R\left[\frac{e^{\gamma l} + e^{-\gamma l}}{2}\right] + Z_0\left[\frac{e^{\gamma l} - e^{-\gamma l}}{2}\right]$$

$$Z_T = Z_R \cosh \gamma l + Z_0 \sinh \gamma l \quad \text{⑤}$$

if the expression is desired in terms of the hyperbolic functions.

Open and short circuited lines

Date:

A line is terminated with $Z_R = \infty$ open circuit
$Z_R = 0$ short circuit.

The i/p imp. of a line of length l is,

$$Z_S = Z_0 \left[\frac{Z_R \cosh \gamma l + Z_0 \sinh \gamma l}{Z_0 \cosh \gamma l + Z_R \sinh \gamma l} \right] \quad \text{--- (1)}$$

For the short circuit case $Z_R = 0$. so that,

$$Z_S = Z_0 \frac{Z_0 \sinh \gamma l}{Z_0 \cosh \gamma l} = \boxed{Z_{SC} = Z_0 \tanh \gamma l} \quad \text{--- (2)}$$

Equ (1) may be rewritten as,

$$Z_S = Z_0 \left[\frac{\cosh \gamma l + (Z_0/Z_R) \sinh \gamma l}{\left(\frac{Z_0}{Z_R}\right) \cosh \gamma l + \sinh \gamma l} \right]$$

The i/p imp. of the open circuited line of length l, with $Z_R = \infty$ is,

$$Z_{OC} = Z_0 \coth \gamma l \quad \text{--- (3)}$$

$$Z_0 = \coth \gamma l \quad \text{--- (3)}$$

Multiplying equs (2) & (3) we have

$E_R = I_R Z_R$

--- (3)

$e^{-\gamma l}$)

--- (4)

becomes ;

$-\gamma l$)

$-\gamma l$)

$o e^{-\gamma l}$]

$^{-\gamma l}$]

(5)

e hyperbolic
functions.

$Z_0 = \sqrt{Z_{oc} \cdot Z_{sc}}$ —— (4) → Experimentally determining the value of Z_0 of a line.

(2) →

$$\tanh \gamma l = \frac{Z_{sc}}{Z_0} = \sqrt{\frac{Z_{sc} \cdot Z_{sc}}{Z_{sc} \cdot Z_{oc}}} = \sqrt{\frac{Z_{sc}}{Z_{oc}}}.$$

$$\gamma l = \tanh^{-1} \sqrt{\frac{Z_{sc}}{Z_{oc}}}.$$

Reflection factor and reflection Loss

* Assume $Z_1 \neq Z_2$. imp. mismatch will be the result.

* It causes,
  * a change in the ratio of V to I and
  * Energy tr.ed by the electric field to that tr.ed by magnetic field.
  * Portion of the energy is reflected by the load.
  * The energy delivered to the load, may be less than would be delivered from source. (comparatively if the imp's matched.)

It is said to be reflection loss.

*Impedance matching b/w a given genr & load might be obtained by insertion of an ideal transformer & a lossless phase shifter b/w source and load.

From the theory of ideal Transformer,

$$\frac{I_1}{I_2} = \sqrt{\frac{Z_2}{Z_1}} \quad \text{———} \; (1)$$

For imp. matching, * Z_2 may be adjusted to Z_1 by choosing proper tsfr. ratio.

* The phase angle of Z_2 may be adjusted to Z_1 by operation of the phase shifter.

Under these conditions Z_2 is imp. matched to Z_1.

Now the ct. flow through the genr would be,

$$I_1 = \frac{E}{2Z_1} \longleftarrow (2)$$

say I_2' — current flow thro' the load or secondary of the tsfr.

Under imp. matching condn, eqn (1) & (2) may be written as,

$$I_2 = I_1\sqrt{\frac{Z_1}{Z_2}}$$

$$I_2' = \frac{E}{2Z_1}\sqrt{\frac{Z_1}{Z_2}} = \frac{E}{2\sqrt{Z_1 Z_2}} \; \text{———} \; (3)$$

without imp, matching the ct. I_2 is given by,

$$|I_2| = \frac{|E|}{|Z_1+Z_2|} \quad \text{— (4)}$$

∴ The ratio of the ct. actually flowing in the load which might flow under imp. matched condition is,

$$\left|\frac{I_2}{I_2'}\right| = \frac{\frac{|E|}{|Z_1+Z_2|}}{\left|\frac{|E|}{2\sqrt{Z_1Z_2}}\right|} = \frac{|2\sqrt{Z_1Z_2}|}{|Z_1+Z_2|} \quad \text{— (5)}$$

This ratio indicates the change in current in the load due to reflection at the mismatched Jn 4 is called reflection factor K

$$K = \frac{|2\sqrt{Z_1Z_2}|}{|Z_1+Z_2|}$$

The Reflection loss is defined as the no. of nepers or decibels by which the ct. in the load matched conditions would exceed the current actually flowing in the load.

$$\frac{V_1}{V_2} = \frac{I_1}{I_2} = e^N$$

$$N \text{ neper} = \ln\left|\frac{V_1}{V_2}\right| = \ln\frac{I_1}{I_2}$$

$$dB = 10\log\frac{P_1}{P_2}$$

Date:

$$\text{Reflection loss nepers} = \ln \left| \frac{Z_1 + Z_2}{2\sqrt{Z_1 Z_2}} \right| \quad \text{— (6)}$$

$$\text{reflection loss, dB} = 20 \log \left| \frac{Z_1 + Z_2}{2\sqrt{Z_1 Z_2}} \right| \quad \text{— (7)}$$

Insertion Loss

At terminal 11',
$Z_g \neq Z_s$

The IL of a line or N/W is defined as the no. of nepers or decibels, by the ct. in the load is changed by the insertion.

The sending end ct. can be written as,

$$I_s = \frac{I_R (Z_R + Z_0)}{2 Z_0} \left(e^{\gamma l} - K e^{-\gamma l} \right) \quad \text{— (1)}$$

$$I_s = \frac{E}{Z_g + Z_s} \quad \text{— (2)} \qquad K = \frac{Z_R - Z_0}{Z_R + Z_0}$$

The i/p imp Z_s can be written as,

$$Z_s = Z_0 \left(\frac{e^{\gamma l} + K e^{-\gamma l}}{e^{\gamma l} - K e^{-\gamma l}} \right) \quad \text{— (3)}$$

sub. (3) into (2)

$$\therefore I_s = \frac{E}{Z_g + Z_0 \left(\frac{e^{\gamma l} + K e^{-\gamma l}}{e^{\gamma l} - K e^{-\gamma l}} \right)}$$

$$I_s = \frac{E\left(e^{\gamma l} - k e^{-\gamma l}\right)}{Z_g\left(e^{\gamma l} - k e^{-\gamma l}\right) + Z_0\left(e^{\gamma l} + k e^{-\gamma l}\right)} \quad (4)$$

Sub (4) into (1), we have

$$\frac{E\left(e^{+\gamma l} - \cancel{k e^{-\gamma l}}\right)}{Z_g\left(e^{\gamma l} - k e^{-\gamma l}\right) + Z_0\left(e^{\gamma l} + k e^{-\gamma l}\right)} = \frac{I_R\left(Z_R + Z_0\right)}{2Z_0}\left(e^{\gamma l} - \cancel{k e^{-\gamma l}}\right)$$

$$I_R = \frac{2Z_0 E}{\left(Z_R + Z_0\right)\left[Z_g\left(e^{\gamma l} - k e^{-\gamma l}\right) + Z_0\left(e^{\gamma l} + k e^{-\gamma l}\right)\right]}$$

$$= \frac{2Z_0 E}{\left(Z_R + Z_0\right)\left[Z_g e^{\gamma l} - Z_g k e^{-\gamma l} + Z_0 e^{\gamma l} + Z_0 k e^{-\gamma l}\right]}$$

$$= \frac{2Z_0 E}{\left(Z_R + Z_0\right)\left[\left(Z_g + Z_0\right)e^{\gamma l} + k e^{-\gamma l}\left[Z_0 - Z_g\right]\right]}$$

$$= \frac{2Z_0 E}{\left(Z_R + Z_0\right)\left(Z_g + Z_0\right)e^{\gamma l} + \cancel{\left(Z_R + Z_0\right)}\frac{\left(Z_R - Z_0\right)}{\cancel{\left(Z_R + Z_0\right)}}\left(Z_0 - Z_g\right)e^{-\gamma l}}$$

$$= \frac{2Z_0 E}{\left(Z_R + Z_0\right)\left(Z_g + Z_0\right)e^{\gamma l} + \left(Z_0 - Z_g\right)\left(Z_R - Z_0\right)e^{-\gamma l}} \quad (5)$$

Date:

Equ ⑤ is the value of current actually flowing in the load Z_R.

⑧ without the line present, the ct. I_R' flowing in the load would be, $I_R' = \frac{E}{Z_g + Z_0}$ —— ⑥

The ratio of I_R' & I_R ie the ct flow in the load, if gens & load were directly connected, to that flowing with the line inserted is,

$$\frac{I_R'}{I_R} = \frac{\frac{E}{Z_g + Z_R}}{\frac{2Z_0 E}{(Z_R+Z_0)(Z_g+Z_0)e^{\gamma l} + (Z_0 - Z_g)(Z_R - Z_0)e^{-\gamma l}}}$$

$$= \frac{(Z_R+Z_0)(Z_g+Z_0)e^{\alpha l}e^{j\beta l} + (Z_0-Z_g)(Z_R-Z_0)e^{-\alpha l}e^{-j\beta l}}{2Z_0(Z_g+Z_R)}$$ —— ⑦

if α is large or the line is long, the second term in the numerator may be neglected w.r.to 1st term

$$\frac{I_R'}{I_R} = \frac{(Z_R+Z_0)(Z_g+Z_0)e^{\alpha l}e^{j\beta l}}{2Z_0(Z_g+Z_R)}$$

both numerator & denominator $\times$ by $2\sqrt{Z_g Z_R}$,

$$\frac{I_R'}{I_R} = \frac{2\sqrt{Z_g Z_R}\cdot(Z_R+Z_0)(Z_g+Z_0)\,e^{\alpha l}\cdot e^{j\beta l}}{4\sqrt{Z_g Z_R Z_0^2}\cdot(Z_g+Z_R)} \quad \text{--- (8)}$$

The IL is to be calculated as a fn. of ck. magnitudes only. ∴ Taking absolute value & rearranging, phase angle may be neglected.

$$\left|\frac{I_R'}{I_R}\right| = \frac{\overset{\text{III}}{2\sqrt{Z_g Z_R}}}{|(Z_g+Z_R)|}\cdot\frac{\overset{\text{I}}{|Z_g+Z_0|}}{2\sqrt{Z_g Z_0}}\cdot\frac{\overset{\text{II}}{|Z_R+Z_0|}}{2\sqrt{Z_R Z_0}}\cdot\overset{\text{IV}}{e^{\alpha l}} \quad \text{--- (9)}$$

1

The reflection factor may be written as,

$$K_S = \frac{2\sqrt{Z_g\cdot Z_R}}{|Z_g+Z_0|}$$ → reflection factor at terminal 11'

where the gen may be mismatched at its jn with line

when α = large, or line is large, its i/p imp is Z_0.

$$\therefore K_R = \frac{2\sqrt{Z_R Z_0}}{|Z_R+Z_0|} = K_R$$ — reflection factor at terminal 22'

3rd term is $$\frac{2\sqrt{Z_g\cdot Z_R}}{|Z_g+Z_R|} = K_{SR}$$ — reflection factor occurring if the gen were directly connected to load.

Date:

The fourth term is, $e^{\alpha l}$ → loss in the line.

The current ratio is, $\left|\dfrac{I_R'}{I_R}\right| = \dfrac{k_{SR}}{k_S k_R} e^{\alpha l}$ — (10)

The Insertion loss may be obtained by taking the log of the current ratio,

$$IL \text{ (nepers)} = \ln\frac{1}{k_S} + \ln\frac{1}{k_R} - \ln\frac{1}{k_{SR}} + \alpha l \quad \text{— (11)}$$

$$IL \text{ dB} = 20\left[\log\frac{1}{k_S} + \log\frac{1}{k_R} - \log\frac{1}{k_{SR}} + 0.4343\,\alpha l\right] \quad \text{— (12)}$$

$\frac{1}{k_{SR}}$ → RL when gen directly connected to the load.

Hence this is subtracted, ∴ it is not due to insertion of the line.

$$\text{Insertion loss (dB)} = 10\log\left(\frac{P_i}{P_o}\right)$$

$$\text{Tr.ion loss or attn. (dB)} = 10\log\frac{P_i - P_r}{P_o}$$

$$\text{Reflection loss (dB)} = 10\log\frac{P_i}{P_i - P_r}$$

$$\text{Return loss (dB)} = 10\log\left(\frac{P_i}{P_r}\right)$$

(1) A lossless line has a charac. imp. of 400 Ω. Determine the standing wave ratio of the Rx-ing end impedance is 800+j0.0 Ω

$Z_0 = 400\,\Omega \quad Z_R = (800 + j0)\,\Omega$

Reflection co-eff Γ or $K = \dfrac{Z_R - Z_0}{Z_R + Z_0} = \dfrac{800-400}{800+400}$

$= \dfrac{1}{3} = 0.33$

SWR $S = \dfrac{1+\Gamma}{1-\Gamma} = \dfrac{1+0.33}{1-0.33} = 2.\ (1.985)$

2) A tr-ion line has the following unit length parameters, $L = 0.1\,\mu H$, $R = 5\,\Omega$, $G = 0.01\,\mho$, $C = 300\,pF$. Calculate the charac. imp & propagation const at 500 MHZ.

$Z = R + j\omega L = 5 + j(2\pi \times 500 \times 10^{6}) \times (0.1 \times 10^{-6})$

$= 5 + j314.15 = 314.199 \angle 89.01^\circ$

$Y = G + j\omega C = 0.01 + j(2\pi \times 500 \times 10^{6}) \times 300 \times 10^{-12}$

$= 0.01 + j0.9424 = 0.9425 \angle 89.39^\circ$

Charac. imp $Z_0 = \sqrt{\dfrac{Z}{Y}} = \sqrt{\dfrac{314.199 \angle 89.01}{0.9425 \angle 89.39^\circ}}$

$= 18.281 \angle -0.157^\circ\,\Omega$

prop - const $\gamma = \sqrt{ZY} = \sqrt{314.199 \angle 89.01 \times 0.9425 \angle 89.39^\circ}$

$\gamma = 17.201 \angle 89.24^\circ$

③ The characteristic imp. of a uniform tr-ion line is 2309.6 Ω at a freq of 800 MHZ. At this freq the propagation const is $0.054\,(0.0366 + j0.99)$. Determine R and L.

$\gamma = 0.054\,(0.0366 + j0.99) = 0.053 \angle 87.88^\circ$

$\omega = 2\pi f = 50.24 \times 10^6$

④ Find the attenuation and phase shift const. of a wave propagating along the line, whose prop. const is $1.048 \times 10^{-4} \angle 88.8^\circ$

$\gamma = 1.048 \times 10^{-4} \angle 88.8^\circ = 2.19 \times 10^{-6} + j1.048 \times 10^{-4}$

$\gamma = \alpha + j\beta$

$\alpha = 2.19 \times 10^{-6}$ Nepers/m

$\beta = 1.048 \times 10^{-4}$ radians/m

(5) Calculate the charac. imp. of a tr-ion line, if the following measurements have been made on the line. $Z_{oc} = \cancel{800}\ 500 \angle -60^\circ\ \Omega$ & $Z_{sc} = 500 \angle 30^\circ\ \Omega$

$$Z_0 = \sqrt{Z_{oc} \cdot Z_{sc}} = \sqrt{500 \angle -60^\circ \cdot 500 \angle 30^\circ}$$

$$= 526.404 \angle -30^\circ$$

(6) A generator of 1V, 1000 cycles, supplies power to a 100 mile open wire line terminated in Z_0 and having the following parameters.

$R = 10.4\ \Omega/\text{mile}$, $L = 0.00367\ H/\text{mile}$,

$G = 0.8 \times 10^{-8}\ \mho/\text{mile}$, $C = 0.00835\ \mu F/\text{mile}$.

Find out the received power and voltage.

The line constants are,

$$z = R + j\omega L = 10.4 + j23.0 = 25.2 \angle 66^\circ$$

$$y = G + j\omega C = (0.8 + j\,52.5) \times 10^{-6}$$

$$= 52.6 \times 10^{-6} \angle 90^\circ$$

$$Z_0 = \sqrt{\frac{z}{y}} = \sqrt{\frac{25.2 \angle 66^\circ}{52.6 \times 10^{-6} \angle 90^\circ}} = 692 \angle -12^\circ\ \Omega$$

$$\gamma = \sqrt{zy} = \sqrt{25.2 \angle 66^\circ \times 52.6 \times 10^{-6} \angle 90^\circ} = 0.0363 \angle 78^\circ$$

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$\alpha = 0.0363 \cos 78^\circ = 0.00755$ neper/mile.

$\beta = 0.0363 \sin 78^\circ = 0.0355$ radian per mile

$u = \frac{\omega}{\beta} = \frac{6280}{0.0355} = 177,000$ miles/second.

$\lambda = \frac{2\pi}{\beta} = 177$ miles

The line is terminated in Z_0, then $Z_S = Z_0$, so that

$I_S = \frac{E_S}{Z_0} = \frac{1}{692 \angle -12^\circ} = 0.00145 \angle 12^\circ$ amp.

$\frac{I_R}{I_S} = e^{-\gamma l} = e^{-\alpha l} \, e^{-j\beta l} = e^{-0.755} \, e^{-j3.55}$

$e^{-j3.35} \Rightarrow -3.55$ radians.

$I_R = I_S \cdot e^{-0.755} \angle -203.8^\circ$

$= 0.00145 \angle 12^\circ \times 0.472 \angle -203.8^\circ$

$= 0.000685 \angle -191.8^\circ$ amp [E_s reference]

The received voltage is, $E_R = I_R Z_0$

$= 0.000685 \angle -191.8^\circ \times 692 \angle -12^\circ$

$= 0.474 \angle -203.8^\circ$ volts (E_s reference.

The received power is,

$$P_R = E_R I_R \cos\theta = 318 \times 10^{-6} \text{ watts.}$$

$$\delta : \sqrt{Z Y} = \sqrt{\sqrt{R^2 + \omega^2 L^2}\ \angle \tfrac{\pi}{2} - \tan^{-1} R/\omega L \times \omega C \angle \tfrac{\pi}{2}}$$

$$= \sqrt{\sqrt{\omega^2 L^2 \left(1 + \frac{R^2}{\omega^2 L^2}\right)}\ \angle \tfrac{\pi}{2} - \tan^{-1} R/\omega L \ \ \omega C \angle \tfrac{\pi}{2}}$$

$$= \sqrt{\omega L \sqrt{\left(1 + \frac{R^2}{\omega^2 L^2}\right)}\ (\omega C)\ \angle \tfrac{\pi}{2} - \tan^{-1}(R/\omega L) \times \angle \tfrac{\pi}{2}}$$

$$= \omega \sqrt{LC} \cdot \sqrt{\left(1 + \frac{R^2}{\omega^2 L^2}\right)} \sqrt{\left(\tfrac{\pi}{2} + \tfrac{\pi}{2}\right) - \tan^{-1}\underbrace{\left(R/\omega L + \tfrac{\pi}{2}\right)}_{\tan\theta}}$$

$$= \omega \sqrt{LC} \sqrt{\left(1 + \frac{R^2}{\omega^2 L^2}\right)} \sqrt{\pi - \tan^{-1}(R/\omega L)}$$

$$= \omega \sqrt{LC} \sqrt{\left(1 + R^2/\omega^2 L^2\right)} \left(\pi - \tan^{-1} R/\omega L\right)^{1/2}$$

$$= \left(\quad \right) \left(\tfrac{\pi}{2} - \tfrac{1}{2} \tan^{-1} R/\omega L\right)$$

$$(r \cos\theta + i \sin\theta)^{1/2} = \sqrt{r} \left(\cos\theta/2 + i \sin\theta/2\right)$$

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Types of Modern Transmission Lines

* Modern TL's can be sub-divided into,

1) Microstrip lines.
2) Strip lines
3) Slot lines

Sub divided into

1) Balanced – Electric field distribution is symmetrical
2) UnBalanced – Electric field distribution is not uniform.

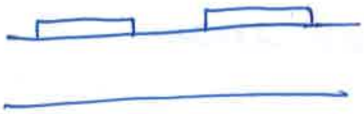
Planar Transmission Line

* A conducting metal strip that lie entirely in ||^l planes.

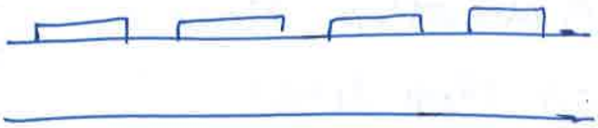
* One or more ||^l metal strips placed on a dielectric substrate, adjacent to a conducting ground plane.

* Widely used in μstrip line.

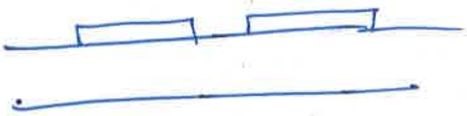
Types of planar transmission line



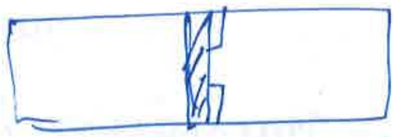
coupled µstrip line
ⓐ



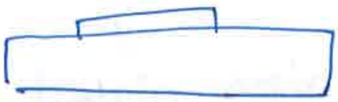
co-planar tr. con line
ⓑ



co-planar strip tr. con line ⓒ



Shield slot line
ⓓ



Slot line

ⓔ

ⓕ Strip line.

ⓐ → * used in directional couplers.

* Used two modes of operation.

(i) even mode → Has the same voltage and at on the two strips.

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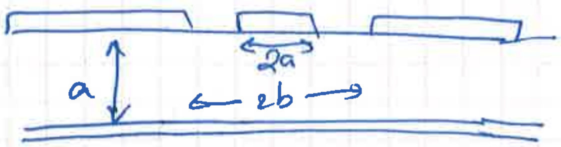
Odd Mode :- Has opposite voltage and currents on the two strips.

(5) consists of single strip mounted b/w two gnd planes on the same side of the dielectric substrate.

Adv. over μstrip : Shunt connection of components to the gnd plane can be made on the same side of the substrate.

* Also, allows series connection of components to be made with equal facility to that for μstrip lines.



* gnd plane & dielectric region exists.
* On the dielectric region, along center flat conductor, additionally there are two further metallic depositions separated from the center conductor by gaps.

The Electric field pattern exists b/w the Center conductor and the two co-planar gnd planes.

* Used rarely.

② It is less desirable than co-planar lines.

∴ it is not balanced w.r.t. to gnd plane