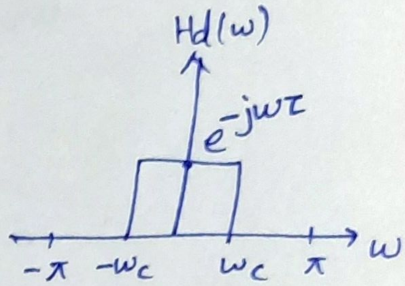
Pbm: Design a LPF using rectangular window by taking 9 samples of $w(n)$ and with a cut-off frequency of 1.2 radians/sec.

Soln:

Given: $w_c = 1.2$ rad/sec, $N = 9$



$$\boxed{\tau = \frac{N-1}{2}}$$

Step 1: $H_d(w) = ?$

if $H_d(w) = \begin{cases} 1, & -w_c \le w \le w_c \\ 0 & \text{otherwise} \end{cases}$

$\Updownarrow$

$h_d(n) \Rightarrow h(n)$ contains samples from $\frac{-(N-1)}{2}$ to $\frac{(N-1)}{2}$

$\Downarrow$

non-causal filter coefficients $\Rightarrow$ not physically realizable.

To get causal filter, ~~coefficients~~,

its coefficients should be shifted by 'τ' times towards right, which is obtained by multiplying $H_d(w)$ by $e^{-jw\tau}$ & it is given by,

$$H_d(w) = \begin{cases} 1 \cdot e^{-jw\tau}, & -w_c \le w \le w_c \\ 0, & \text{otherwise} \end{cases}$$

$\Uparrow$

desired frequency response of LPF.

Step 2: $h_d(n) = IFT\{H_d(\omega)\} = ?$

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(\omega)\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} e^{-j\omega\tau}\cdot e^{j\omega n}\, d\omega = \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} e^{j\omega(n-\tau)}\, d\omega$$

$$= \frac{1}{2\pi j(n-\tau)}\left[e^{j\omega(n-\tau)}\right]_{-\omega_c}^{\omega_c}$$

$$= \frac{1}{2\pi j(n-\tau)}\left[e^{j\omega_c(n-\tau)} - e^{-j\omega_c(n-\tau)}\right]$$

sinc function $\Leftarrow$ $\boxed{h_d(n) = \frac{1}{\pi(n-\tau)} \sin\left[\omega_c(n-\tau)\right]}$ $\quad \because \frac{e^{j\theta} - e^{-j\theta}}{2j} = \sin\theta$

when $n = \tau$, $\frac{\sin \omega_c(n-\tau)}{n-\tau} = \frac{0}{0}$ (indeterminate)

$\therefore h_d(n)$ is evaluated using L' Hospital rule,

$$h_d(n) = \underset{n\to\tau}{Lt} \frac{\sin \omega_c(n-\tau)}{\pi(n-\tau)}$$

$$= \frac{1}{\pi} \underset{n\to\tau}{Lt} \frac{\sin \omega_c(n-\tau)}{(n-\tau)}$$

$\boxed{\because \underset{\theta\to 0}{Lt} \frac{\sin A\theta}{\theta} = A}$

at $n = \tau$, $h_d(n) = \frac{1}{\pi}\omega_c$

at $n \neq \tau$ $h_d(n) = \frac{1}{\pi}\frac{\sin\omega_c(n-\tau)}{(n-\tau)}$, Step 3: $h(n) = ?$

$$\boxed{h(n) = h_d(n) \cdot w_R(n)}$$

$h(n)$ → finite impulse response

$h_d(n)$ → infinite impulse response

$w_R(n)$ → rectangular window sequence.

$$w_R(n) = \begin{cases} 1, & n = 0 \text{ to } (N-1) \\ 0, & \text{otherwise} \end{cases}$$

$N = 9$

$\tau = \frac{N-1}{2} = 4$

$\boxed{\tau = 4}$

$\omega_c = 1.2 \text{ rad/sec}$

$$\therefore h(n) = \begin{cases} h_d(n) \cdot 1, & n = 0 \text{ to } (N-1) \\ 0, & \text{otherwise} \end{cases}$$

n=0:

$$h(0) = h_d(0) = \frac{\sin(1.2)(0-4)}{\pi(0-4)} = -0.0793$$

n=1:

$$h(1) = h_d(1) = \frac{\sin(1.2)(1-4)}{\pi(1-4)} = -0.0470$$

n=2:

$$h(2) = h_d(2) = \frac{\sin(1.2)(2-4)}{\pi(2-4)} = 0.1075$$

n=3:

$$h(3) = h_d(3) = \frac{\sin(1.2)(3-4)}{\pi(3-4)} = 0.2967$$

n=4 $\boxed{n = \tau}$

$$h(4) = h_d(4) = \frac{\omega_c}{\pi} = \frac{1.2}{\pi} = 0.3820$$

n=5:

$$h(5) = h_d(5) = \frac{\sin(1.2)(5-4)}{\pi(5-4)} = 0.2967$$

$\underline{n=6}$: $h(6) = h_d(6) = \dfrac{\sin(1.2)(6-4)}{\pi(6-4)} = 0.1075$

$\underline{n=7}$: $h(7) = h_d(7) = \dfrac{\sin(1.2)(7-4)}{\pi(7-4)} = -0.047$

$\underline{n=8}$:
$h(8) = h_d(8) = \dfrac{\sin(1.2)(8-4)}{\pi(8-4)} = -0.0793$

From $h(n)$, we can observe that

filter coefficients

$$\left.\begin{aligned} h(0) &= h(8) \\ h(1) &= h(7) \\ h(2) &= h(6) \\ h(3) &= h(5) \end{aligned}\right\}$$ Satisfies symmetry condition $\boxed{h(n) = h(N-1-n)}$

$h(4)$

Step 4: $H(z) = ZT\{h(n)\} = ?$

$$ZT\{h(n)\} = \boxed{H(z) = \sum_{n=0}^{N-1} h(n) z^{-n}}$$

$$= h(0) + h(1)z^{-1} + h(2)z^{-2} + h(3)z^{-3} + h(4)z^{-4} +$$
$$h(5)z^{-5} + h(6)z^{-6} + h(7)z^{-7} + h(8)z^{-8}$$

Apply symmetrical condition of $h(n)$,

$$= h(4)z^{-4} + h(0)\left[z^{0} + z^{-8}\right] + h(1)\left[z^{-1} + z^{-7}\right]$$
$$+ h(2)\left[z^{-2} + z^{-6}\right] + h(3)\left[z^{-3} + z^{-5}\right]$$

$$H(z) = h(4)z^{-4} + \sum_{n=0}^{3} h(n)\left[z^{-n} + z^{-(8-n)}\right]$$ ⇐ from derivation of case(i)

$$H(z) = 0.3820\, z^{-4} + (-0.0793)[z^{0} + z^{-8}] + (-0.047)[z^{-1} + z^{-7}] + 0.1075[z^{-2} + z^{-6}] + 0.2967[z^{-3} + z^{-5}]$$

$\Downarrow$

$H(z)$ can be realized using any one of the structure of FIR filter. (discussed later)

Step 5: $H(\omega) = \text{DTFT}\{h(n)\} = ?$

$H(\omega) = |H(\omega)| \angle H(\omega)$

$$\boxed{H(\omega) = H(z)\big|_{z=e^{j\omega}} \quad (\text{or}) \quad H(\omega) = \sum_{n=0}^{N-1} h(n)\, e^{-j\omega n}}$$

$$H(\omega) = 0.3820\, e^{-j4\omega} - 0.0793[e^{j0} + e^{-j8\omega}] - 0.047[e^{-j\omega} + e^{-j7\omega}] + 0.1075[\cancel{k} e^{-j2\omega} + e^{-j6\omega}] + 0.2967[e^{-j3\omega} + e^{-j5\omega}]$$

$$H(\omega) = e^{-j4\omega} \underbrace{\Big\{ 0.3820 - 0.0793[e^{j4\omega} + e^{-j4\omega}] - 0.047[e^{j3\omega} + e^{-j3\omega}] + 0.1075[e^{j2\omega} + e^{-j2\omega}] + 0.2967[e^{j\omega} + e^{-j\omega}] \Big\}}_{|H(\omega)|}$$

$\Downarrow$

$\angle H(\omega) = -4\omega$

wkt $\cos\theta = \dfrac{e^{j\theta} + e^{-j\theta}}{2}$

$$|H(\omega)| = 0.3820 - 0.0793(2\cos 4\omega) - 0.047(2\cos 3\omega) + 0.1075(2\cos 2\omega) + 0.2967(2\cos\omega)$$

F.22(f)

$$|H(\omega)| = 0.3820 - 0.1586 \cos 4\omega - 0.094 \cos 3\omega + 0.215 \cos 2\omega + 0.5934 \cos \omega$$

Magnitude response of LPF is obtained by plotting the graph between $|H(\omega)|$ and ω for various values of ω from 0 to 2π.

F. 22. (g)

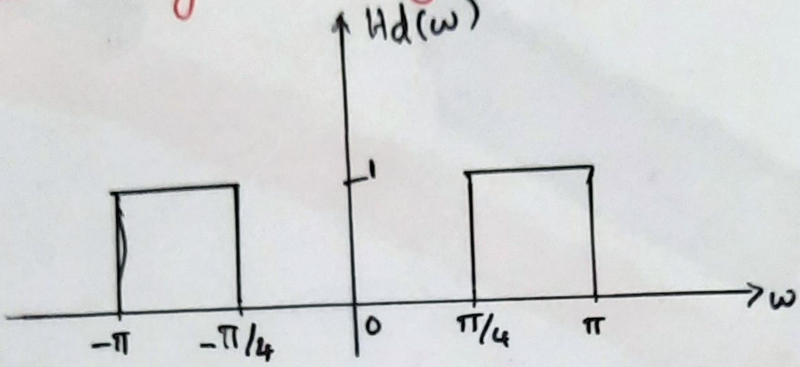
Pbm: Design an ideal HPF with a frequency Response

$$Hd(\omega) = \begin{cases} 1 \; ; & \pi/4 \le |\omega| \le \pi \\ 0 \; ; & |\omega| \le \pi/4 \end{cases} \quad \text{for } N = 11$$

Find the value of $h(n)$ & $H(z)$ & plot the magnitude response using hanning window.

Soln: (i)

Step 1:



$\omega_c = \pi/4$

(ii) $$hd(n) = \frac{1}{2\pi} \int_{-\omega_c}^{\omega_c} Hd(\omega) . e^{j\omega n} . d\omega$$

Step 2: $$hd(n) = \frac{\sin(n\pi) - \sin(n\pi/4)}{n\pi}$$

(iii) Truncation of $hd(n)$ into $h(n)$.

Step 3:

$$h(n) = \begin{cases} hd(n) . \omega_{Hn}(n) \; ; & -5 \le n \le 5 \\ 0 & ; \text{otherwise} \end{cases}$$

$$\omega_{Hn}(n) = \begin{cases} 0.5 + 0.5 \cos \frac{2\pi n}{N-1} \; ; & -5 \le n < 5 \\ 0 & ; \text{otherwise} \end{cases}$$

Sub $N = 11$,

$$\omega_{Hn}(n) = \begin{cases} 0.5 + 0.5 \cos\left(\frac{\pi n}{5}\right) \; ; & -5 \le n \le 5 \\ 0 & ; \text{otherwise} \end{cases}$$

$\omega_{Hn}(0) = 0.5 + 0.5 = 1$

$\omega_{Hn}(1) = \omega_{Hn}(-1) = 0.9045$

$\omega_{Hn}(2) = \omega_{Hn}(-2) = 0.655$

$\omega_{Hn}(3) = \omega_{Hn}(-3) = 0.345$

$\omega_{Hn}(4) = \omega_{Hn}(-4) = 0.0945$

$\omega_{Hn}(5) = \omega_{Hn}(-5) = 0$

F.22.(h)

$hd(0) = 0.75$.

$hd(1) = hd(-1) = -0.225$

$hd(2) = hd(-2) = -0.159$

$hd(3) = hd(-3) = -0.075$

$hd(4) = hd(-4) = 0$

$hd(5) = hd(-5) = 0.045$

Substitute $hd(n)$ & $w_H(n)$ in $h(n)$.

$h(0) = hd(0) \cdot w_{Hn}(0) = 0.75 \cdot$

$h(1) = hd(1) \cdot w_{Hn}(1) = -0.204 = h(-1)$

$h(2) = hd(2) \cdot w_{Hn}(2) = -0.104 = h(-2)$

$h(3) = hd(3) \cdot w_{Hn}(3) = -0.026 = h(-3)$

$h(4) = hd(4) \cdot w_{Hn}(4) = 0 = h(-4)$

$h(5) = hd(5) \cdot w_{Hn}(5) = 0 = h(-5)$

$h(n) \neq 0, n < 0$

$\therefore$ Anticausal

(iv) $H(z) = \left\{ h(0) + \sum_{n=1}^{5} h(n)\left[z^{n} + z^{-n}\right] \right\} z^{-5}$

Step 4: $H(z) = -0.026 z^{-2} - 0.104 z^{-3} - 0.204 z^{-4} + 0.75 z^{-5} - 0.204 z^{-6} - 0.104 z^{-7} - 0.026 z^{-8}$

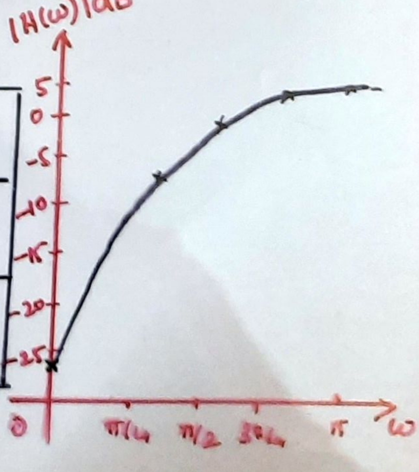
$h(n) = \{ \underset{\uparrow}{0}, 0, -0.026, -0.104, 0.75, -0.204, -0.104, -0.026, 0, 0 \}$

Causal Sequence

(v) $|H(\omega)| = h(5) + \sum_{n=1}^{5} 2h(5-n)\cos\omega n$

$|H(\omega)| = 0.75 - 0.408\cos\omega - 0.208\cos 2\omega - 0.052\cos 3\omega$

ω_{deg}	0	$\pi/4$	$\pi/2$	$3\pi/4$	π
$\lvert H(\omega)\rvert$	0.082	0.498	0.96	1.0017	1.002
$\lvert H(\omega)\rvert_{dB}$	-21.72	-6.05	-0.38	+0.015	0.017



F-22(i)

Pbm Design a linear phase FIR filter using Hamming ~~ar~~ window for the following desired ~~speci~~ frequency response

$$H_d(\omega) = \begin{cases} e^{-j3\omega} , & \pi/4 \le |\omega| < \pi \\ 0 , & \text{otherwise} \end{cases}$$

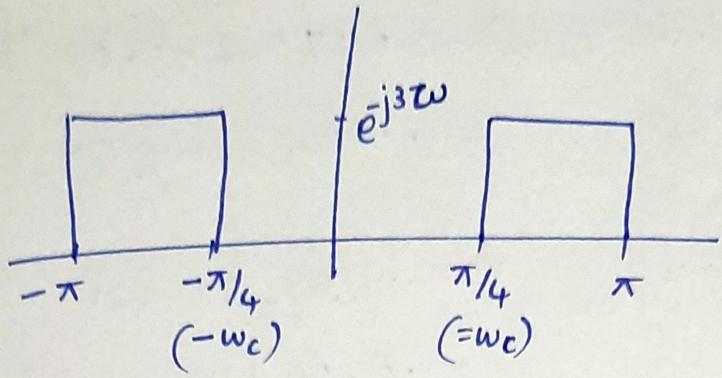
Homework 2. Same problem using Hanning window

Homework 3: Same problem using Rectangular window

Homework 1: Derivation for frequency Response of linear phase FIR filter for case (iii) and case (iv)

F-22 (j)

Given: $\tau = 3$, $\frac{N-1}{2} = 3$, $\boxed{N = 7}$



$\Rightarrow$ HPF

Step 1: given in problem (ie $H_d(\omega)$)

Step 2: $h_d(n) = ?$

$$h_d(n) = IFT\{H_d(\omega)\} = \begin{cases} \frac{\sin \omega_c(n-3)}{\pi(n-3)}, & n \neq \tau \\ 1 - \frac{\omega_c}{\pi}, & n = \tau \end{cases}$$

Step3: $h(n) = ?$

$$h(n) = h_d(n) \cdot w_{ham}(n)$$

where, $w_{ham}(n) = 0.54 - 0.46 \cos\left(\frac{2\pi n}{6}\right)$, $n = 0$ to $N-1$ $(=6)$

$w_{ham}(0) = w_{ham}(6) = 0.08$

$w_{ham}(1) = w_{ham}(5) = 0.31$

$w_{ham}(2) = w_{ham}(4) = 0.77$

$w_{ham}(3) = 1$

$h(n) = h_d(n) \cdot w_{ham}(n)$

n=0:

$h(0) = h_d(0) \cdot w_{ham}(0) = -0.07506 \times 0.08$

$= -0.006 = h(6)$

n=1:

$h(1) = h_d(1) \cdot w_{ham}(1) = -0.15923 \times 0.31$

$= -0.04936 = h(5)$

n=2:

$h(2) = h_d(2) \cdot w_{ham}(2) = -0.22586 \times 0.77$

$= -0.17391 = h(4)$

n=3: $n=\tau$

$h(3) = h_d(3) \cdot w_{ham}(3) = 0.75 \times 1$

$= 0.75$

$h(n) = \{\underset{\uparrow}{-0.006}, -0.04936, -0.17391, 0.75, -0.17391, -0.04936, -0.006\}$

Step 4: $H(z) = ZT\{h(n)\}$

Step 5: $H(\omega) = DTFT\{h(n)\} = |H(\omega)| \angle H(\omega)$

$H(\omega) = H(z)\big|_{z=e^{j\omega}}$

Pbm: Design a BPF for the following specifications

$f_{c_1} = 100Hz$, $f_{c_2} = 200Hz$, $F_s = 1000Hz$, filter length $= 9$

$\Downarrow$ sampling rate

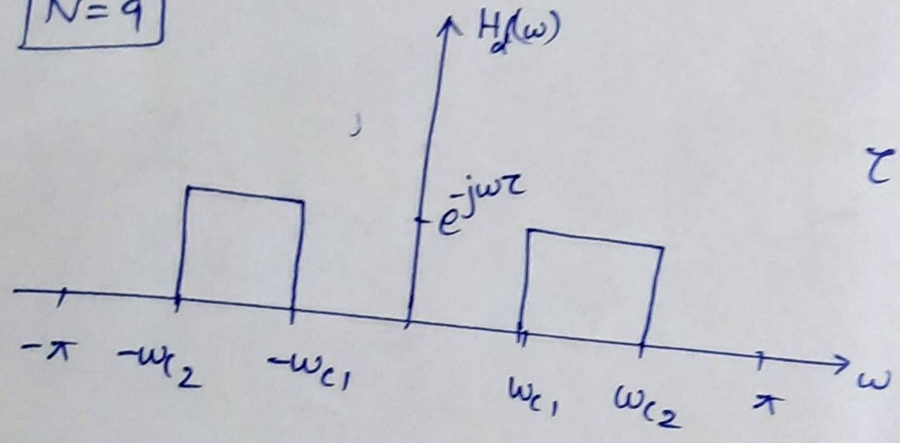
Soln:

normalised values

$$\omega_{c_1} = \frac{2\pi f_{c_1}}{F_s} = \frac{2\pi \times 100}{1000} = 0.2\pi \text{ rad/sec}$$

$$\omega_{c_2} = \frac{2\pi f_{c_2}}{F_s} = \frac{2\pi \times 200}{1000} = 0.4\pi \text{ rad/sec}$$

$\boxed{N = 9}$



$\tau = \frac{9-1}{2} = 4$

Step 1:
$$H_d(\omega) = \begin{cases} e^{-j\omega\tau}, & \omega_{c_1} \le |\omega| \le \omega_{c_2} \\ 0, & \text{otherwise } \omega_{c_2} \le |\omega| \pi \ \& \ |\omega| \le \omega_{c_1} \end{cases}$$

Step 2:
$$h_d(n) = \frac{1}{2\pi} \int_{-0.4\pi}^{-0.2\pi} e^{j\omega(n-4)}\, d\omega + \frac{1}{2\pi} \int_{0.2\pi}^{0.4\pi} e^{j\omega(n-4)}\, d\omega$$

Step 3: $h(n) = h_d(n) \cdot w(n)$

$h(n) \Downarrow H(z)$ &

$w(n) \Downarrow$ Choose suitable window

Step 4 & 5: $H(\omega)$

F.22(m)

Homework 4 Complete the above problem using any one of the window

Homework 5: Same above problem But BSF

with same window.