Unit - II Time Response Analysis

Time Response Analysis

→ Time Response of the S/m is the o/p of the closed loop system as a function of time [c(t)]

→ The response c(t) can be obtained from the transfer function & the i/p to the S/m.

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + H(s)G(s)} = M(s)$$

$$C(s) = R(s)M(s)$$

$$c(t) = L^{-1}[R(s)M(s)]$$

→ The time response of a control S/m consists of two parts.

(i) transient response
(ii) steady state response.

→ Transient response: is the response of the s/m when the i/p changes from one state to another.

→ steady state response: is the response as time (t) approaches infinity.

Standard test signals :-

Name of the signal (i/p)	Time domain equation of signal $r(t)$	$L[r(t)]$ (ie) $R(s)$
Step	A	A/s
unit Step	1	$1/s$
Ramp	At	A/s^2
unit Ramp	t	$1/s^2$
Parabolic	$At^2/2$	A/s^3
unit parabolic	$t^2/2$	$1/s^3$
Impulse	$\delta(t)$	1

→ Impulse Response ⇒ with i/p as impulse signal

$$R(s) = 1$$

$$M(s) = \frac{G(s)}{1+G(s)H(s)} = \frac{C(s)}{R(s)}$$

$$\therefore C(s) = R(s)\left[\frac{G(s)}{1+G(s)H(s)}\right]$$

$$C(s) = \frac{G(s)}{1+G(s)H(s)}$$

$$c(t) = L^{-1}\left[\frac{G(s)}{1+G(s)H(s)}\right]$$

∴ Impulse response is the inverse LT of transfer function.

Order of a s/m :-

→ The i/p & o/p relationship of a ctrl s/m can be expressed by n^{th} order differential equation,

$$a_0 \frac{d^n}{dt^n} p(t) + a_1 \frac{d^{n-1}}{dt^{n-1}} p(t) + \dots + a_n p(t) = b_0 \frac{d^m}{dt^m} q(t)$$

$$+ b_1 \frac{d^{m-1}}{dt^{m-1}} q(t) + \dots + b_m q(t).$$

$p(t) \rightarrow$ o/p / Response
$q(t) \rightarrow$ I/p / Excitation.

→ Also, order can be determined from the transfer function of the s/m.

$$T(s) = \frac{P(s)}{Q(s)} = \frac{b_0 s^m + b_1 s^{m-1} + \dots + b_{m-1} s + b_m}{a_0 s^n + a_1 s^{n-1} + \dots + a_{n-1} s + a_n}.$$

→ $P(s) \Rightarrow$ Numerator polynomial.
$Q(s) \Rightarrow$ Denominator polynomial.

→ The order of the s/m is given by the maximum power of 's' in the denominator polynomial [Q(s)]

$\therefore Q(s) = a_0 s^n + a_1 s^{n-1} + \dots + a_n$.

$n \rightarrow$ order of the s/m.
$n = 0 \Rightarrow$ zero order s/m
$n = 1 \Rightarrow$ 1st order s/m
$n = 2 \Rightarrow$ 2nd order s/m

→ Type of the s/m : The numerator and denominator polynomial can be expressed in the factor form as shown in,

$$T(s) = \frac{P(s)}{Q(s)} = \frac{(s+z_1)(s+z_2)\ldots\ldots(s+z_m)}{(s+P_1)(s+P_2)\ldots\ldots(s+P_n)}.$$

→ here 'n' is the no. of poles. Therefore order of the s/m is given by the number of poles of the t/f function.

→ The No. of poles at the origin gives the type of the s/m.

Eg: $Q(s) = s^2(s+1)(s+2)$

no. of poles at origin ⇒ 2.

∴ It is type 2 s/m.

If no pole present at the origin, then it is type-0 s/m.

Recall : Partial Fraction Expansion.

Case 1 : Function with separate/distinct poles.

$$T(s) = \frac{K}{s(s+P_1)(s+P_2)}.$$

$$\frac{K}{s(s+P_1)(s+P_2)} = \frac{A}{s} + \frac{B}{s+P_1} + \frac{C}{s+P_2}.$$

$$A = T(s) \times s \Big|_{s=0}$$

$$B = T(s) \times (s+P_1) \Big|_{s=-P_1}$$

$$C = T(s) \times (s+P_2) \Big|_{s=-P_2}$$

Case 2: Tfr function with Multiple poles.

$$T(s) = \frac{K}{s(s+P_1)(s+P_2)^2}$$

$$\frac{K}{s(s+P_1)(s+P_2)^2} = \frac{A}{s} + \frac{B}{s+P_1} + \frac{C}{(s+P_2)} + \frac{D}{(s+P_2)^2}$$

$$A = T(s) \times s \Big|_{s=0}$$

$$B = T(s) \times (s+P_1) \Big|_{s=-P_1}$$

$$C = \frac{d}{ds}\left[T(s) \times (s+P_2)^2\right] \Big|_{s=-P_2}$$

$$D = T(s) \times (s+P_2)^2 \Big|_{s=-P_2}$$

Case 3: Tfr function with complex conjugate poles.

$$T(s) = \frac{K}{(s+P_1)(s^2+bs+c)}$$

$$\frac{K}{(s+P_1)(s^2+bs+c)} = \frac{A}{(s+P_1)} + \frac{Bs+C}{(s^2+bs+c)} \quad \text{--- (1)}$$

$$A = T(s) \times (s+P_1) \Big|_{s=-P_1}$$

To solve for B & C, cross multiply the above equation (1), sub the value of (A) and the equate the like power of s.

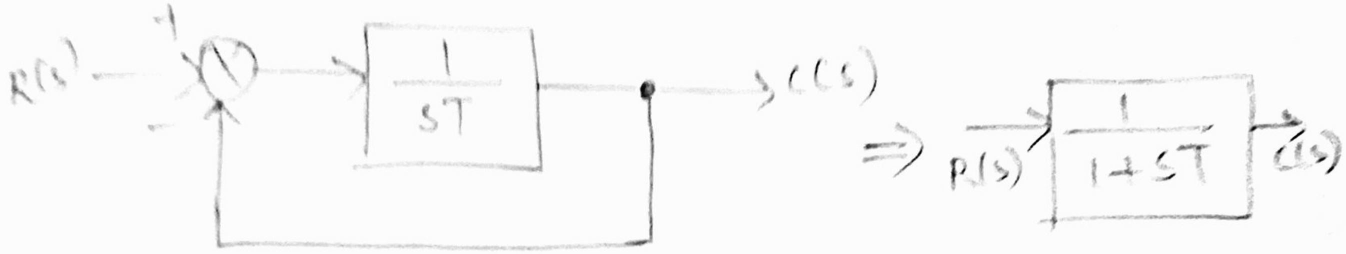
eg:
$$1 = (s^2+s+1) + Bs^2 + 2Bs + Cs + 2C$$
$$1 = (1+B)s^2 + (1+2B+C)s + (1+2C)$$

co-eff of s^2 $\Rightarrow$ $1+B = 0$

co-eff of s $\Rightarrow$ $1+2B+C = 0$

~~co-eff~~ constant $\Rightarrow$ $1+2C = 1$

Response of First order s/m for unit step i/p



$$\rightarrow \frac{C(s)}{R(s)} = \frac{1}{1+ST}.$$

→ unit step i/p, $r(t) = 1 \Rightarrow R(s) = 1/s$.

$$\rightarrow C(s) = R(s)\left[\frac{1}{1+ST}\right]$$

$$C(s) = \frac{1}{s}\left[\frac{1}{1+ST}\right]$$

$$C(s) = \frac{1}{s(1+ST)}$$

$$\rightarrow \frac{1}{s(1+sT)} = \frac{1}{sT\left(s+\frac{1}{T}\right)} = \frac{(1/T)}{s(s+1/T)}$$

$$\rightarrow \frac{1/T}{s(s+1/T)} = \frac{A}{s} + \frac{B}{(s+1/T)}$$

$$\rightarrow A = \frac{(1/T)}{\cancel{s}(s+1/T)} \times \cancel{s} \Bigg|_{s=0} = \frac{(1/T)}{(1/T)} = 1$$

$$\rightarrow B = \frac{(1/T)}{s\cancel{(s+1/T)}} \times \cancel{(s+1/T)} \Bigg|_{s=-1/T} = \frac{(1/T)}{(-1/T)} = -1$$

$$\rightarrow c(s) = \frac{1}{s(1+sT)} = \frac{A}{s} + \frac{B}{(s+1/T)}$$

$$= \frac{1}{s} + \frac{-1}{s+1/T}$$

$$\rightarrow c(t) = L^{-1}\left[\frac{1}{s} + \frac{(-1)}{s+1/T}\right]$$

$$\boxed{c(t) = \left(1 - e^{-t/T}\right)}$$ → unit step response.

$$\boxed{LT\left(e^{-at}\right) = \frac{1}{s+a}}$$

→ For step Response, (Amplitude $\neq 1$)

$$\boxed{c(t) = A\left(1 - e^{-t/T}\right)}$$, $T \rightarrow$ time constant

$$\rightarrow \quad c(t) = \begin{cases} (1 - e^{-t/T}), & \text{unit step } i/p \\ A(1 - e^{-t/T}), & \text{step } i/p. \end{cases}$$

$\rightarrow$ $t = 0$, $c(t) = 1 - 1 = 0$

$t = 1T$, $c(t) = 1 - e^{-1} = 0.632$

$t = 2T$, $c(t) = 1 - e^{-2} = 0.865$

$t = 3T$, $c(t) = 1 - e^{-3} = 0.95$

$t = \infty$, $c(t) = 1 - e^{-\infty} = 1$