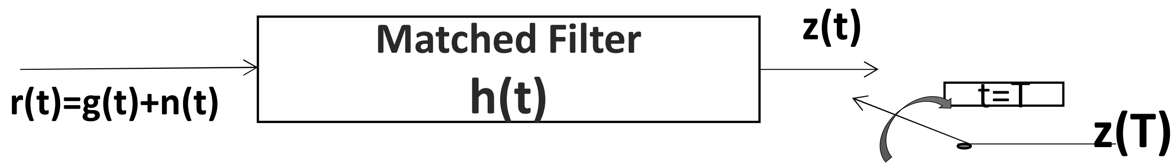
MAXIMUM LIKELIHOOD DECISION
(ML Decision)

MATCHED FILTER

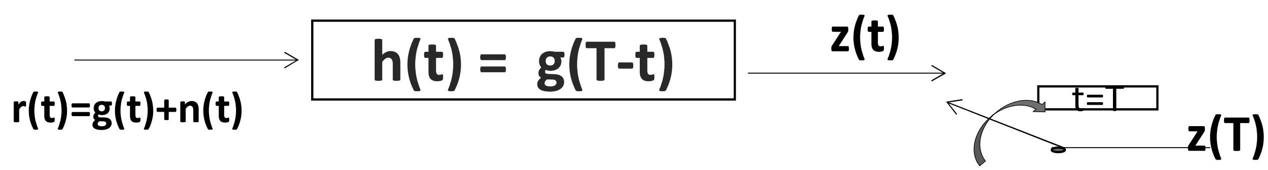
Received Signal $r(t) = g(t) + n(t)$ ← **AWGN** **(Additive White Gaussian Noise)**

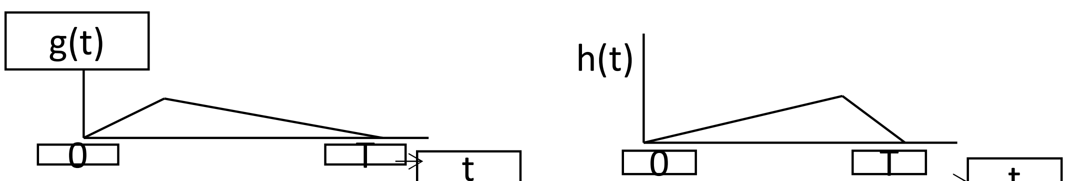
$g(t)$ is known to the receiver (exists for $0 \le t \le T$)



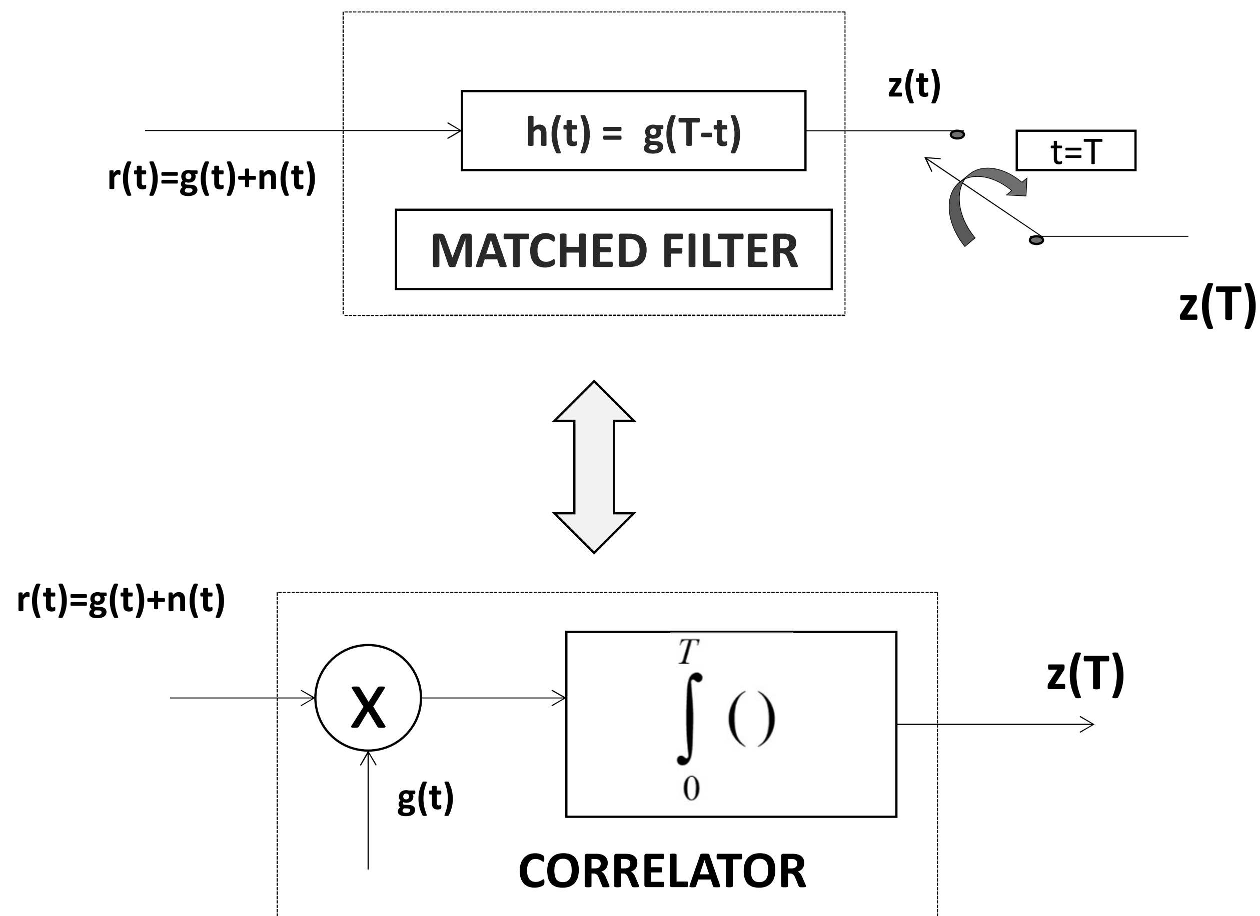
OPTIMAL *(FILTER)*

in the sense that the SIGNAL TO NOISE RATIO at the output is MAXIMIZED



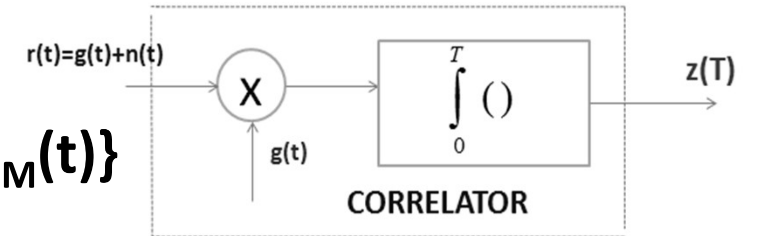


Matched filter & Correlator



Digital Communication Receiver

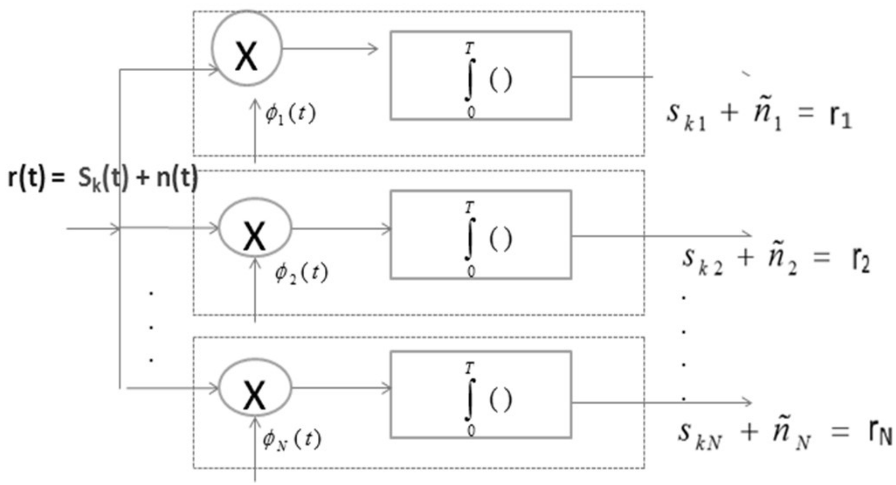
Received Signal $r(t) = g(t) + n(t)$
$g(t)$ is known to the receiver (exists for $0 \le t \le T$)



Symbol Set $\{S_1(t), S_2(t), \ldots\ldots S_M(t)\}$

$r(t) = S_k(t) + n(t)$

$$s_k(t) = s_{k1}\phi_1(t) + s_{k2}\phi_2(t) + \ldots + s_{kN}\phi_N(t)$$



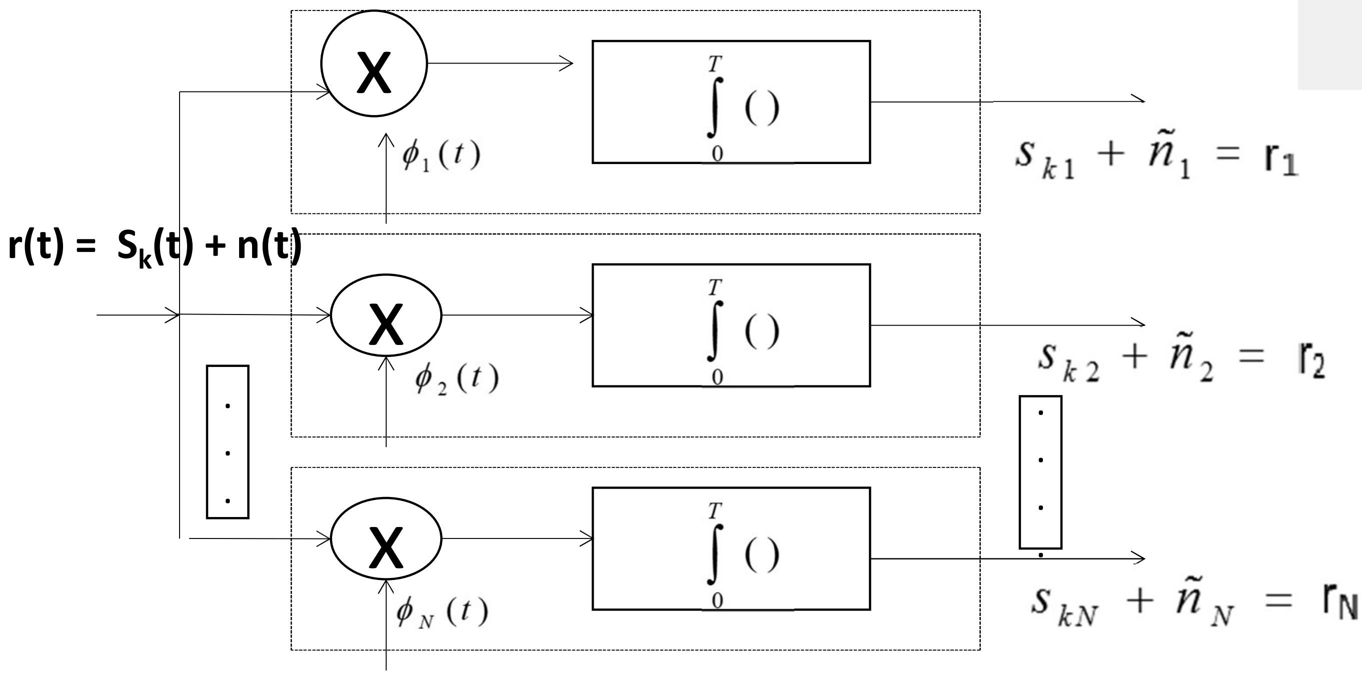
CORRELATOR

$$s_i(t)=\sum_{j=1}^{N} s_{ij}\phi_j(t)$$

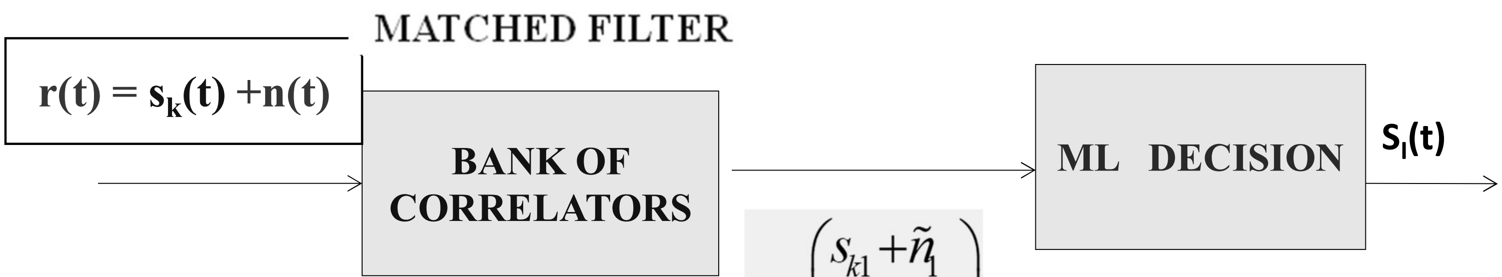
$$s_{ij}=\int_0^T s_i(t)\phi_j(t)\,dt$$

$$\bar{r}=\begin{pmatrix} s_{k1}+\tilde{n}_1 \\ s_{k2}+\tilde{n}_2 \\ . \\ . \\ s_{kN}+\tilde{n}_N \end{pmatrix}$$

$$s_k(t)=s_{k1}\phi_1(t)+s_{k2}\phi_2(t)+\ldots+s_{kN}\phi_N(t)$$



RECEIVER



$$\bar{r} = \begin{pmatrix} s_{k1} + \tilde{n}_1 \\ s_{k2} + \tilde{n}_2 \\ . \\ . \\ s_{kN} + \tilde{n}_N \end{pmatrix}$$

MAXIMUM LIKELIHOOD DECISION

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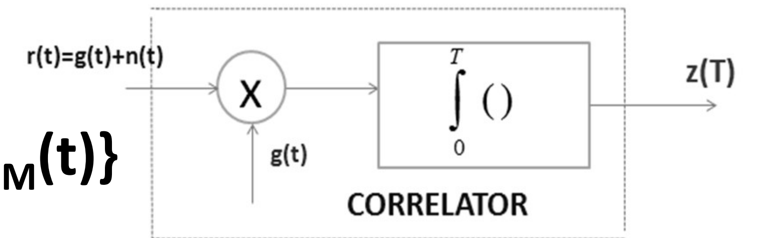
Digital Communication Receiver

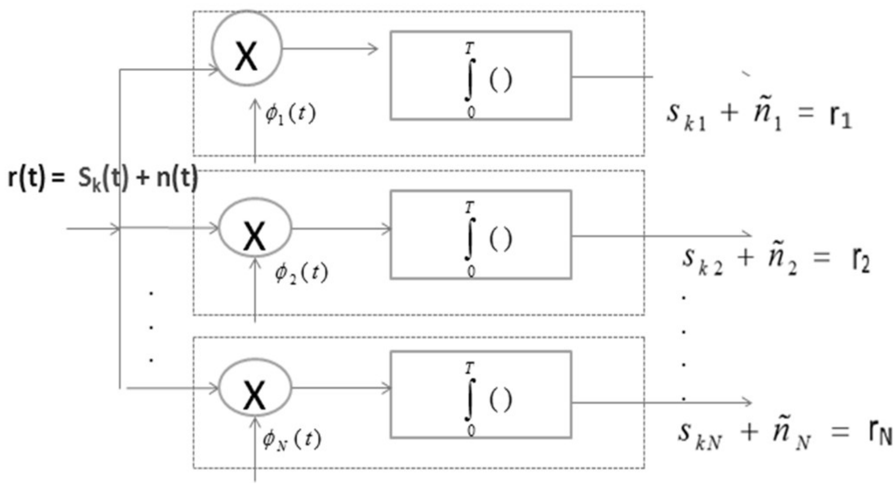
Received Signal   r(t) = g(t) + n(t)
g(t) is known to the receiver (exists for $0 \le t \le T$)



Symbol Set $\{S_1(t), S_2(t), \ldots\ldots S_M(t)\}$

r(t) = $S_k(t)$ + n(t)

$$s_k(t) = s_{k1}\phi_1(t) + s_{k2}\phi_2(t) + \ldots + s_{kN}\phi_N(t)$$



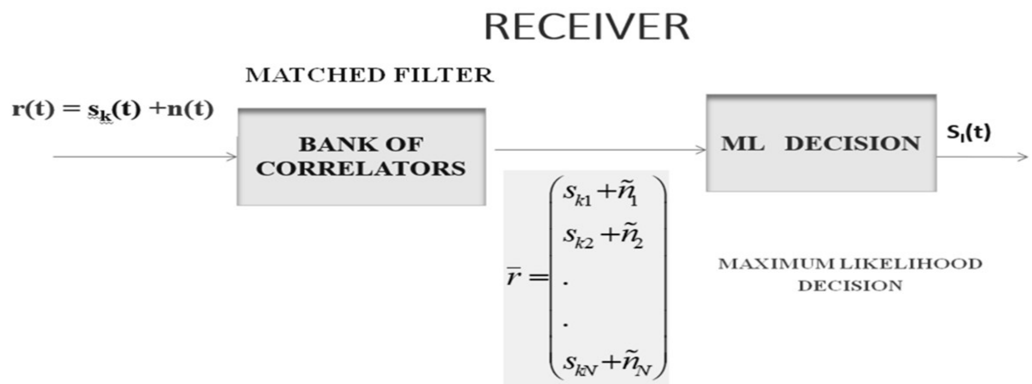
RECEIVER

$$\bar{r} = \begin{pmatrix} s_{k1}+\tilde{n}_1 \\ s_{k2}+\tilde{n}_2 \\ . \\ . \\ s_{kN}+\tilde{n}_N \end{pmatrix}$$

$$f_{\mathbf{R}}(\mathbf{r}\,|\,\mathbf{s}_k) = (\pi N_0)^{-N/2} \exp\left[-\frac{1}{N_0}\sum_{j=1}^{N}(r_j - s_{kj})^2\right] \qquad k = 1,\ 2, \ldots, M$$

$$\ln\left[f_{\mathbf{R}}(\mathbf{r}\,|\,\mathbf{s}_k)\right] = -\frac{N}{2}\ln(\pi N_0) - \frac{1}{N_0}\sum_{j=1}^{N}(r_j - s_{kj})^2 \qquad k = 1, 2, \ldots, M$$

$$P(\mathbf{s}_i \text{ sent} | \mathbf{r}) \geq P(\mathbf{s}_k \text{ sent} | \mathbf{r}), \text{ for all } k \neq i$$



P(s_m sent/ when 'r' is received) --- is maximum

Decision S_m - Transmitted symbol

Maximum a posteriori decision

$$P\,(s_m\ transmitted\ |'r'is\ received) = \frac{P('r'is\ received \mid s_m\ transmitted\,)\ P(s_m\ Transmitted)}{P\,('r'is\ received)}$$

P('r' is received) has no role to play in decision process

When the transmitted symbols are equiprobable

Find out S_m that maximises the conditional probability

ML Decision (*Maximum likelihood* decision)

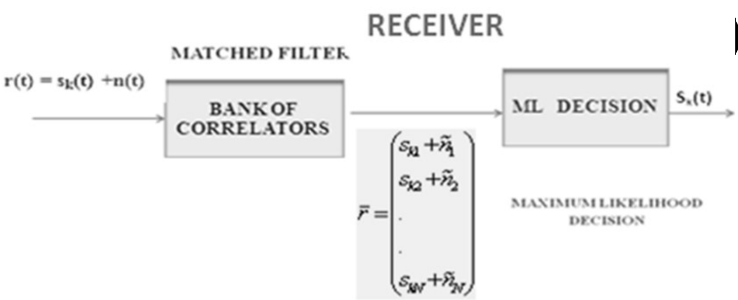
$$P\,('r'is\ received\ |s_m\ transmitted)$$

equivalently

Find out S_m that maximises the conditional probability density function

$$f_R\,('r'is\ received\ |s_m\ transmitted)$$

ECISION



$$\overline{r} = \begin{pmatrix} s_{k1} + \tilde{n}_1 \\ s_{k2} + \tilde{n}_2 \\ . \\ . \\ s_{kN} + \tilde{n}_N \end{pmatrix}$$

TRANSMITTED SYMBOL SET $\{ S_1(t), S_2(t), \ldots S_M(t) \}$

$r(t) = S_k(t) + n(t)$

Maximum a posteriori probability (MAP) Decision

MAP Decision $P[S_j(t) \text{ sent} / \overline{r}] > P[S_k(t) \text{ sent} / \overline{r}]$ ***for all*** $k \neq j$

MAP Decision Received Symbol is $S_j(t)$

ML Decision

If the **symbols are equiprobable** i.e. $P(s_1) = P(S_2) = \ldots = P(S_M)$

MAP Decision → ML Decision (*Maximum Likelihood* Decision)

ML Decision $f_{R/S_j}(\overline{r}/S_j(t)) > f_{R/S_K}(\overline{r}/S_k(t))$ ***for all*** $k \neq j$

ML Decision Received Symbol is $S_j(t)$

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ML Decision (*Maximum likelihood* decision)

equivalently

Find out S_m that maximises the conditional probability density function

$$f_R\,('r' \text{ is received } | s_m \text{ transmitted})$$

RECEIVER

$$\bar{r} = \begin{pmatrix} s_{k1} + \tilde{n}_1 \\ s_{k2} + \tilde{n}_2 \\ . \\ . \\ s_{kN} + \tilde{n}_N \end{pmatrix}$$

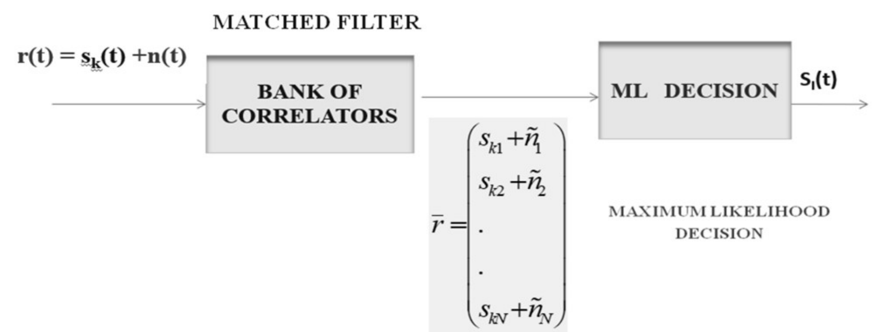
$$f_R(\mathbf{r} | \mathbf{s}_k) = (\pi N_0)^{-N/2} \exp\left[-\frac{1}{N_0} \sum_{j=1}^{N} (r_j - s_{kj})^2 \right] \qquad k = 1,\ 2, \ldots, M$$

Which s_m maximises this function?

Log Likelihood function

$$\ln\left[f_R(\mathbf{r} | \mathbf{s}_k) \right] = -\frac{N}{2} \ln(\pi N_0) - \frac{1}{N_0} \sum_{j=1}^{N} (r_j - s_{kj})^2 \qquad k = 1, 2, \ldots, M$$

RECEIVER



$$\bar{r} = \begin{pmatrix} s_{k1} + \tilde{n}_1 \\ s_{k2} + \tilde{n}_2 \\ . \\ . \\ s_{kN} + \tilde{n}_N \end{pmatrix}$$

$$f_{\mathrm{R}}(\mathrm{r}\,|\,\mathbf{s}_k) = (\pi N_0)^{-N/2} \exp\left[-\frac{1}{N_0} \sum_{j=1}^{N} (\mathrm{r}_j - s_{kj})^2 \right]$$

$$\ln\left[f_{\mathrm{R}}(\mathrm{r}\,|\,\mathbf{s}_k) \right] = -\frac{N}{2}\ln(\pi N_0) - \frac{1}{N_0}\sum_{j=1}^{N}(\mathrm{r}_j - s_{kj})^2 \qquad k = 1, 2, \ldots, M$$

Log likelihood function

$$\text{set } \hat{\mathbf{s}} = \mathbf{s}_i \text{ if}$$

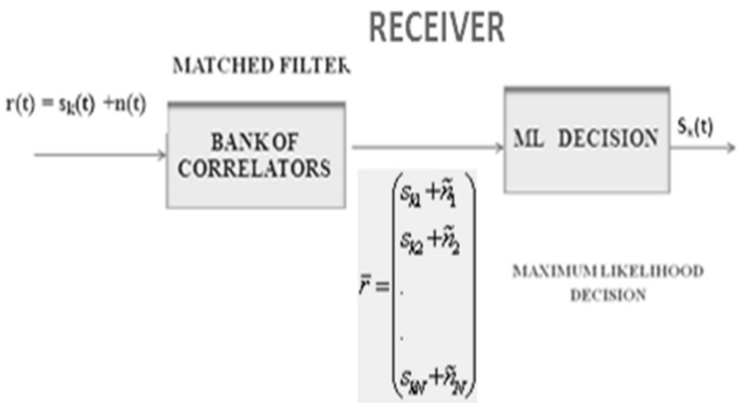
$$\ln[f_{\mathrm{R}}(\mathrm{r}\,|\,\mathbf{s}_k)] \text{ is maximum for } k = i$$

$$-\frac{1}{N_0}\sum_{j=1}^{N}(\mathrm{r}_j - s_{kj})^2 \text{ is maximum for } k = i$$

ML DECISION BOUNDARY

$$\sum_{j=1}^{N}(\mathrm{r}_j - s_{kj})^2 \text{ is } \textbf{minimum} \text{ for } k = i$$

MAXIMUM LIKELIHOOD (ML) DECISION



ML DECISION BOUNDARY

$$\sum_{j=1}^{N}(r_j - s_{kj})^2 \text{ is } \textbf{minimum} \quad \text{for } k = i$$

$$\bar{r} = \begin{pmatrix} s_{k1}+\tilde{n}_1 \\ s_{k2}+\tilde{n}_2 \\ . \\ . \\ s_{kN}+\tilde{n}_N \end{pmatrix}$$

ML Boundary **r(t) = sk(t) + n(t)**