

Stub  $\rightarrow$  piece of wire.

We go for S.C, then in O.C lots of energy gets wasted.

S.C two types of connection

(i) series

(ii) Parallel.

$\rightarrow$  Electrical length (wavelength) ~~will go~~

Physical length.

(Serially we are not going to connect the sub bcz of these two problems)

$\rightarrow$  Mechanical problem.

So we are going to connect the sub parallel.

Single - stub Matching  $\rightarrow$  use single sub for matching

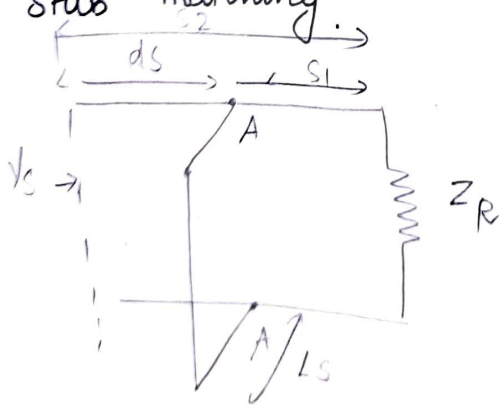
purposes

Double - stub Matching

$\rightarrow$  uses 2 subs for matching

purposes

Single Stub matching:



$L_s \rightarrow$  length of the stub

$s_2 \rightarrow$  dist from load to source end.

At AA' we are placing the stub.

(i) What is the length. (Electrical length)

(ii) Where to locate the stub. Physical length

(Because when frequency  $\uparrow$  Electrical length radiation will come into account)

$Y_p$  admittance at pt before the stub

$$(at A) \quad Y_s = G_0 \pm jB_0 \quad \begin{matrix} \text{conductance} \\ \text{susceptance of the line} \end{matrix}$$

Point A is located such that at that point

$$G_0 = 1/R_0$$

The stub is selected such that its

$Y_p$  susceptance is  $\mp jB_0$ .

(Stub must cancel the susceptance of the line)

Admittance at pt A after the stub

$$Y_s \text{ at A} = G_0 \pm jB_0 \mp jB_0$$

$$Y_s = G_0$$

$$Z_s = R_0$$

where

Thus the line from source to pt A acts as a smooth line. When line is terminated at  $Z_s = R_0$  (then no standing wave)  $\Rightarrow$  so we call it as smooth line. (no reflected waves)  
This characteristic is valid from source to the point A, not from A to load.

So to calculate (location, length) we measure standing wave ratio & voltage min nearest to load.

$$Z_B = \frac{R_0 (1 + |K| \angle \phi - 2\beta s)}{1 - |K| \angle \phi - 2\beta s}$$

$$Y_s = \frac{G_0 (1 - |K| \angle \phi - 2\beta s)}{1 - |K| \angle \phi - 2\beta s}$$

$$G_S + jB_S = G_{10} \frac{[1 - |K| \angle \phi - 2\beta S]}{[1 + |K| \angle \phi - 2\beta S]}$$

$$\frac{G_S + jB_S}{G_{10}} = \frac{1 - |K| \angle \phi - 2\beta S}{1 + |K| \angle \phi - 2\beta S}$$

$$\Rightarrow \frac{G_S}{G_{10}} + \frac{jB_S}{G_{10}} = \frac{1 - |K| e^{j\phi} e^{-j2\beta S}}{1 + |K| e^{j\phi} e^{-j2\beta S}}$$

Normalized conductance.

$$= \frac{1 - |K| [\cos \phi + j \sin \phi] [\cos 2\beta S - j \sin 2\beta S]}{1 + |K| [\cos \phi + j \sin \phi] [\cos 2\beta S - j \sin 2\beta S]}$$

$$= -2|K| [\cos(\phi - 2\beta s) + j \sin(\phi - 2\beta s)]$$

$$= \frac{1 - |K| [\cos(\phi - 2\beta s) + j \sin(\phi - 2\beta s)]}{1 + |K| [\cos(\phi - 2\beta s) + j \sin(\phi - 2\beta s)]}$$

$$= \frac{(1 - |K| \cos(\phi - 2\beta s) - j \sin(\phi - 2\beta s) K) (1 + K \cos(\phi - 2\beta s) - j \sin(\phi - 2\beta s) K)}{(1 + K \cos(\phi - 2\beta s) + j K \sin(\phi - 2\beta s)) (1 + K \cos(\phi - 2\beta s) - j K \sin(\phi - 2\beta s))}$$

$$= \frac{1 - K^2 \cos^2(\phi - 2\beta s) - j^2 (1 - j K \sin(\phi - 2\beta s))^2 - (K^2 \cos^2(\phi - 2\beta s))}{(1 + K \cos(\phi - 2\beta s))^2 - (j^2 K^2 \sin^2(\phi - 2\beta s))}$$

$$= \frac{1 - K^2 \cos^2(\phi - 2\beta s) - j^2 (1 - j K \sin(\phi - 2\beta s))^2 - (K^2 \cos^2(\phi - 2\beta s))}{(1 + K \cos(\phi - 2\beta s))^2 - (j^2 K^2 \sin^2(\phi - 2\beta s))}$$

$$= \frac{1 - 2j K \sin(\phi - 2\beta s) + K^2 \sin^2(\phi - 2\beta s) - K^2 \cos^2(\phi - 2\beta s)}{1 + K^2 \cos^2(\phi - 2\beta s) + 2K \cos(\phi - 2\beta s) + K^2 \sin^2(\phi - 2\beta s)}$$

$$= \frac{1 - K^2 - 2j K \sin(\phi - 2\beta s)}{1 + 2K \cos(\phi - 2\beta s) + K^2} = \frac{1 - K^2}{1 + 2K \cos(\phi - 2\beta s) + K^2} + \frac{j(-2K \sin(\phi - 2\beta s))}{1 + K^2 + 2K \cos(\phi - 2\beta s)}$$

$$\frac{G_S}{G_0} = \frac{1 - K^2}{1 + K^2 + 2K \cos(\phi - 2\beta s)}$$

$$\frac{B_S}{G_0} = \frac{-2K \sin(\phi - 2\beta s)}{1 + K^2 + 2K \cos(\phi - 2\beta s)}$$

To find the particular pt.  
 $G_S : G_0 = 1/R_0$   
 location of point A.

$$\frac{G_S}{G_0} = 1 \rightarrow$$

$$1 + k^2 + 2k \cos(\phi - 2\beta s) = 1 - k^2$$

$$2[k^2 + k \cos(\phi - 2\beta s)] = 0$$

$$k + \cos(\phi - 2\beta s) = 0.$$

$$\cos(\phi - 2\beta s) = -k.$$

$$\cos^{-1}(k) - \pi = x$$

$$\cos^{-1}(k) = (x + \pi)$$

$$k = \cos(x + \pi)$$

$$= -\cos x \Rightarrow$$

$$x = \cos^{-1}(-k) \rightarrow (1)$$

$$\text{But } x = \cos^{-1}|k| = -\pi.$$

$$\cos^{-1}(-k) = \cos^{-1}|k| - \pi$$

$$(\phi - 2\beta s) = \cos^{-1}(-k)$$

$$\phi - 2\beta s = \cos^{-1}|k| - \pi.$$

The point A lies at a distance of  $s_1$  from load

$$\Rightarrow \phi - 2\beta s_1 = \cos^{-1}|k| - \pi$$

$$s_1 = \frac{\pi - \cos^{-1}|k| + \phi}{2\beta}.$$

Thus the distance  $d$  from voltage minimum to the point of stub connection is given by  $d = s_2 - s_1$ .

Let the 1st voltage minima point at  $s_2$  and  $d$  is given by.

$$s_2 = \frac{\phi + \pi}{2\beta}.$$

$$d = \frac{\phi + \pi}{2\beta} - \frac{\pi + \cos^{-1}|k| - \phi}{2\beta}$$



$$d = \frac{\cos^{-1}|K|}{2\beta}$$

$$= 1 - [K \cos \phi + j \sin \phi] [\cos 2\beta s - j \sin 2\beta s]$$

$$1 + |K| [\cos \phi \cos 2\beta s - j \cos \phi \sin 2\beta s + j \sin \phi \cos 2\beta s + \sin \phi \sin 2\beta s]$$

$$= 1 - |K| [\cos(\phi - 2\beta s) + j \sin(\phi - 2\beta s)]$$

$$1 + |K| [\cos(\phi - 2\beta s) + j \sin(\phi - 2\beta s)]$$

$$= 1 - |K| [\cos(\phi - 2\beta s) + j \sin(\phi - 2\beta s)]$$

$$1 + |K| [\cos(\phi - 2\beta s) + j \sin(\phi - 2\beta s)]$$

$$= 1 - K \cos(\phi - 2\beta s) - j \sin(\phi - 2\beta s) K \times (1 + K \cos(\phi - 2\beta s) - j \sin(\phi - 2\beta s) K)$$

$$(1 + K \cos(\phi - 2\beta s) + (j \sin(\phi - 2\beta s) K) (1 + K \cos(\phi - 2\beta s) - j \sin(\phi - 2\beta s) K)$$

$$= 1^2 + K \cos(\phi - 2\beta s) - j \sin(\phi - 2\beta s) K - K \cos(\phi - 2\beta s) - K^2 \cos^2(\phi - 2\beta s)$$

$$- j K^2 \cos(\phi - 2\beta s) \sin(\phi - 2\beta s) + j K \sin(\phi - 2\beta s) + K^2 \cos(\phi - 2\beta s) \sin(\phi - 2\beta s) + K^2 \sin^2(\phi - 2\beta s)$$

$$+ K \cos(\phi - 2\beta s) - j \sin(\phi - 2\beta s) K + K \cos(\phi - 2\beta s) + K^2 \cos^2(\phi - 2\beta s)$$



7/14/20

To find length of line:

$$\frac{B_s}{G_{10}} = \frac{-2K \sin(\phi - 2\beta s)}{1 + |K|^2 + 2K \cos(\phi - 2\beta s)}$$

The point A is at distance  $S_1$  from load S-S1

$$S_1 = \frac{\pi + \phi - \cos^{-1}|K|}{2\beta}$$

$$\sin(\phi - 2\beta s_1) = \sin(-\pi + \cos^{-1}|K|) = -\sqrt{1-K^2}$$

$$\cos(\phi - 2\beta s_1) = \cos(\cos^{-1}|K| - \pi) = -K$$

$$\frac{B_s}{G_0} = \frac{+2K\sqrt{1-K^2}}{1+K^2-2K^2} = \frac{+2K\sqrt{1-K^2}}{1-K^2}$$

$$= \frac{+2K}{\sqrt{1-K^2}}$$

$$B_{s_1} = \left[ \frac{2K}{\sqrt{1-K^2}} \right] G_0$$

To cancel this susceptance, susceptance of stub should be equal to

$$B_{\text{stub}} = \left( -\frac{2K}{\sqrt{1-K^2}} \right) G_0$$

In general the i/p impedance of shorted line is given by  $Z_{sc} = jR_0 \tan(\beta s)$

$$B_{sc} = \frac{-jG_0}{\tan \beta s}$$

The stub connected is also a short-circuited TL and at a length of 'L'.

$$B_{sc} = \frac{-jG_0}{\tan \beta L}$$

$$G_0 \left[ \pm \frac{2K}{\sqrt{1-K^2}} \right] = \frac{G_0}{\tan \beta L}$$

$$\tan \beta L = \pm \frac{\sqrt{1-K^2}}{2K}$$

$$\beta L = \tan^{-1} \left( \frac{\pm \sqrt{1-K^2}}{2K} \right)$$

$$L = \frac{\lambda}{2\pi} \tan^{-1} \left[ \frac{\pm \sqrt{1-K^2}}{2K} \right]$$

In terms of  $S$ ,  $S = \frac{1-K}{1+K} \Rightarrow K = \frac{1+S}{-1+S}$

$$S + SK = 1 - K$$

$$S - 1 = -K - SK$$

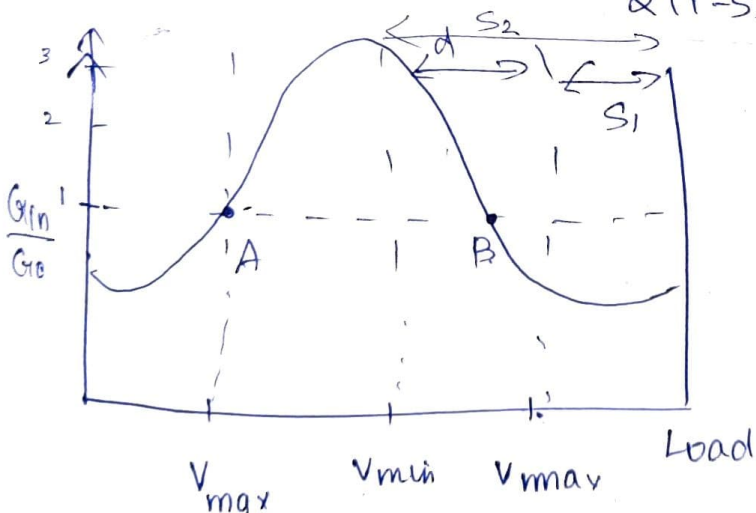
$$S - 1 = -K(-1 - S)$$

$$K = \frac{1-S}{1+S}$$

$$K = \frac{1-S}{1+S}$$

$$L = \frac{\lambda}{2\pi} \tan^{-1} \left[ \frac{\pm \sqrt{1 - \left(\frac{1-S}{1+S}\right)^2}}{2\left(\frac{1-S}{1+S}\right)} \right]$$

$$= \frac{\lambda}{2\pi} \tan^{-1} \left[ \frac{\pm \sqrt{1 - (1-S)^2}}{2(1-S)} \right]$$



We'll have two possible solutions.

PROB:

$$Z_0 = 50 \Omega$$

$$Z_L = 40 + j10 \Omega$$

$$f = 750 \text{ MHz}$$

Find the length & distance of stub.

$$K = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$= \frac{40 + j10 - 50}{40 + j10 + 50}$$

$$= \frac{-10 + j10}{90 + j10}$$

$$d = \frac{\cos^{-1} |K|}{2\beta}$$

$$= -0.1 + j0.1$$

$$= -0.097 + j0.121$$

$$= \frac{-1+j}{9+j} = \frac{(-1+j)(9-j)}{81+1} = \frac{-9+j+9j+1}{82} = \frac{-8+10j}{82}$$

$$\beta = \frac{2\pi}{\lambda}$$

$$= \frac{2\pi}{2.5}$$

$$= 2.512$$

$$f = 750 \text{ MHz}$$

$$\lambda = \frac{c}{f}$$

$$= \frac{3 \times 10^8}{750 \times 10^6}$$

$$= 0.4 \text{ m}$$

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$$d = \cos^{-1}(-0.097 + 0.121j)$$

$2\beta$

$$|K| = \sqrt{(0.097)^2 + (0.121)^2}$$

$$2\beta d = \cos^{-1}$$

$$= \sqrt{(9.409 \times 10^{-3})^2 + (0.0146)^2}$$

$$= 0.1550$$

$$d = \frac{\cos^{-1}(0.1550)}{2\beta}$$

~~Not needed~~

$$= \frac{81.07^\circ}{2\beta} \frac{1.415}{\lambda}$$

$$= \frac{1.415}{2 \times \frac{2\pi}{\lambda}}$$

$$= \frac{1.415}{2 \times 2.512} = 0.281$$

$$= \frac{1.415}{4\pi} = 0.112\lambda$$

Location:

$$L = \frac{\lambda}{2\pi} \tan^{-1} \left( \frac{\pm \sqrt{1-K^2}}{2K} \right)$$

$$= \frac{\lambda}{2\pi} \tan^{-1} \left( \frac{\pm \sqrt{1-(0.1550)^2}}{2 \times (0.1550)} \right) \Rightarrow \text{Length}$$

$$= \frac{\lambda}{2\pi} \tan^{-1}(3.186)$$

$$= \frac{\lambda}{2\pi} \times 1.266 = 0.201\lambda \text{ m} \checkmark$$

$$S_1 = \frac{\pi - \cos^{-1}|K| + \phi}{2\beta}$$

$$\frac{0.316\lambda}{\text{Ans}}$$

$$= \frac{\pi - 1.415 - 0.895}{2 \times \frac{2\pi}{\lambda}}$$

$$= \lambda \left( 1 \right) \Rightarrow \text{Location}$$