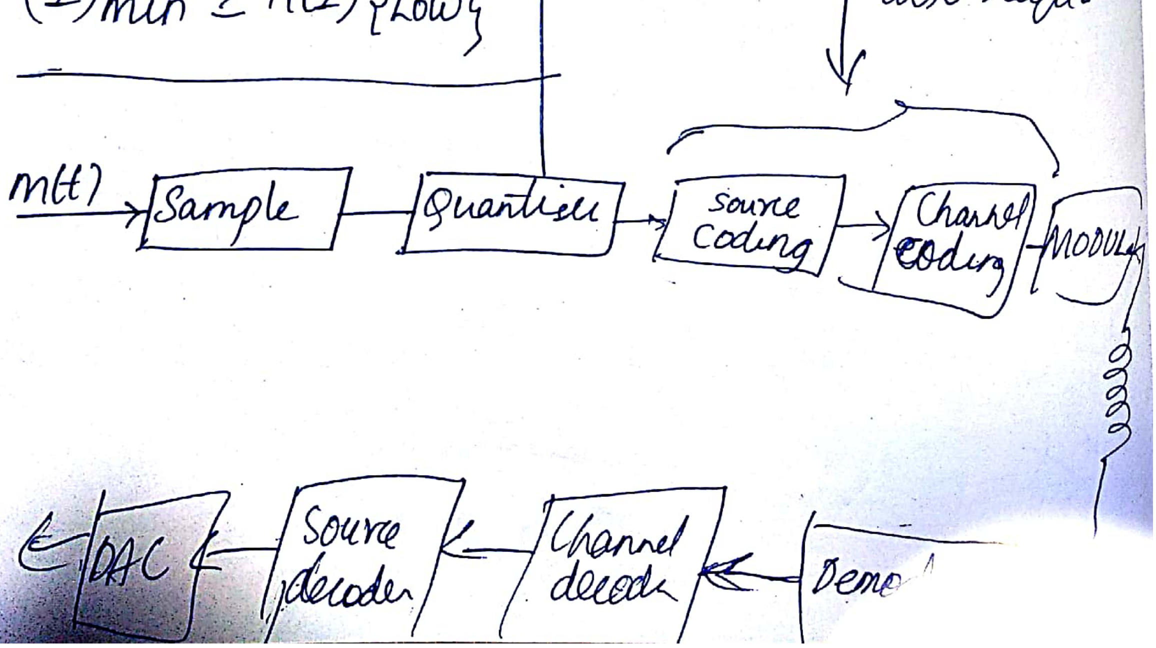
11/8/18 CHANNEL CODING

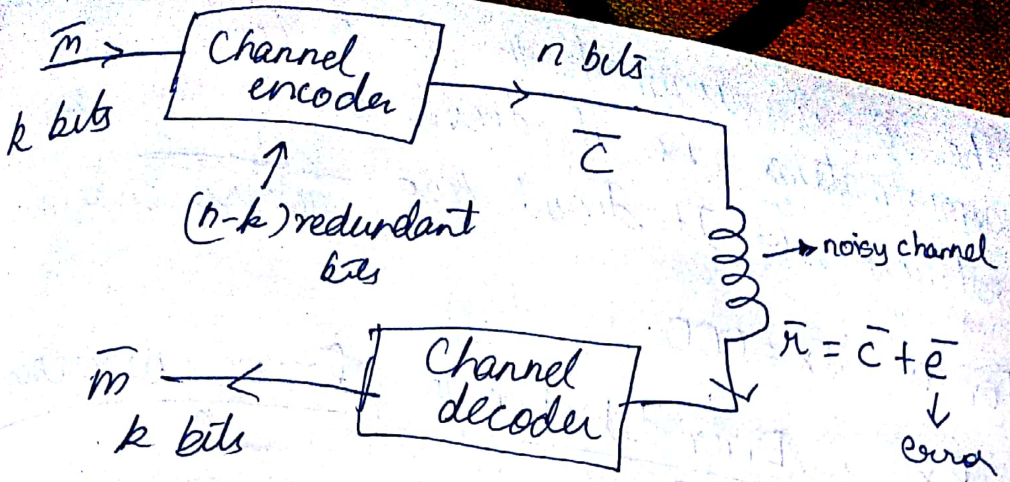
## INFORMATION THEORY - Channel Coding Theorem

Given a discrete memoryless source of entropy $H(I)$. It emits a symbol every $T_s$ second & the information is passed thro' a channel of capacity C bits/symbol which accepts a symbol every $T_c$ second.

If $\frac{H(I)}{T_s} \leq \frac{C}{T_c}$, it is possible to transmit information thro' the channel with arbitrarily low probability of error

| SOURCE CODING | Channel Coding |
| --- | --- |
| • Minimise the no. of bits/symbol | • Add redundant bits |
| Effic % := $\frac{H(I)}{I} \times 100\%$ <br> $(I)_{min} \geq H(I)$ {Low} (made) | • not necessary but if used decoder also reqd. |






Eg: Let $k = 1$, $n = 3$

$\bar{m} = 0 \Rightarrow \bar{c} = 000$

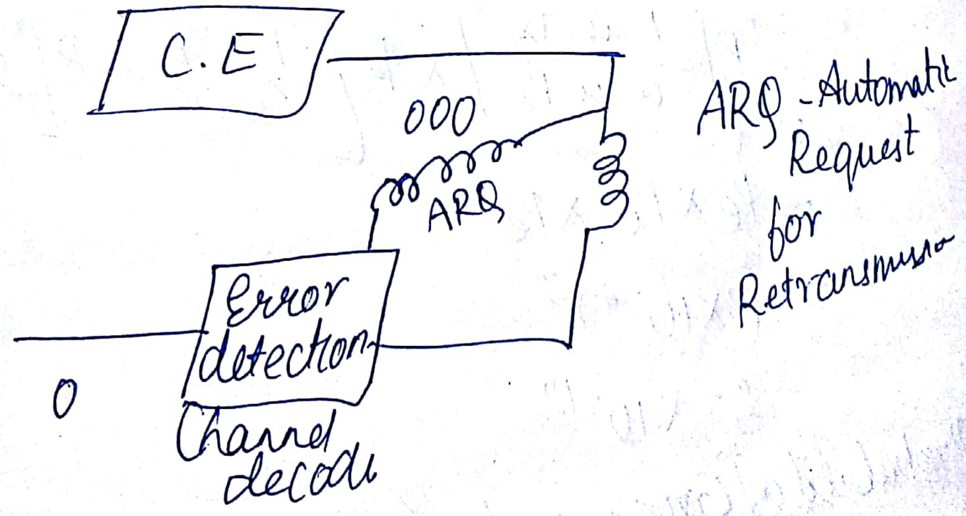
$\bar{m} = 1 \Rightarrow \bar{c} = 111$

Let $\bar{c} = 000$, $\bar{r} = \left.\begin{matrix} 000 \\ 001 \\ 010 \\ \vdots \\ 111 \end{matrix}\right\}$ due to error

When $\bar{r}$ is passed thro' channel decoder, it knows that $\bar{r}$ must be 000 (or) 111 to decode 0 (or) 1 respectively.

Eg: If $\bar{c} = 001$ for a Tx of 000,

In Error detection mode of channel decoder




In error correction mode,

the $\bar{c}$ is assumed to be 0 by approximating i.e it doesn't waste time for retransmission. It doesn't have ~~re channel~~ reverse channel.

For a $Tx = 000$

| $Rx$ | Error detection o/p | Error correction o/p |
|---|---|---|
| 0 0 0 | 0 | 0 |
| 0 0 1 | ARQ | 0 |
| 0 1 0 | ARQ | 0 |
| 0 1 1 | ARQ | 1 |
| 1 0 0 | ARQ | 0 |
| 1 0 1 | ARQ | 1 |
| 1 1 0 | ARQ | 1 |
| 1 1 1 | 1 | 1 |

Channel $P_e = 3 \times 10^{-4}$

Probability [Error detection failure] $= P\left[\bar{r} = 111 / \bar{c} = 000\right]$

$= P\left[\frac{1 \text{ is } Tx}{0 \text{ is } Rx}\right] \times P\left[1/0\right] \times P\left[1/0\right]$

$= P_e \times P_e \times P_e$

$= (3 \times 10^{-4})^3$

$= 27 \times 10^{-12}$

Probability [Error correction failure] $= P\left[\bar{r} = 111 / \bar{c} = 000\right] + P\left[\bar{r} = 110 / \bar{c} = 000\right]$

$$+P\left[\frac{\bar{r}=101}{\bar{c}=000}\right]+P\left[\frac{\bar{r}=011}{\bar{c}=000}\right]$$

$$=P\left[\frac{1}{0}\right]\times P\left[\frac{1}{0}\right]\times P\left[\frac{1}{0}\right]+P\left[\frac{1}{0}\right]\times P\left[\frac{1}{0}\right]\times P\left[\frac{0}{0}\right]+$$

$$P\left[\frac{1}{0}\right]\times P\left[\frac{0}{0}\right]\times P\left[\frac{1}{0}\right]+P\left[\frac{0}{0}\right]\times P\left[\frac{1}{0}\right]\times P\left[\frac{1}{0}\right]$$

$$=P_e^3+P_e^2\times(1-P_e)+P_e^2\times(1-P_e)+P_e^2(1-P_e)$$

$$\cancel{=27\times10^{-12}+3P_e^2(1-P_e)}$$

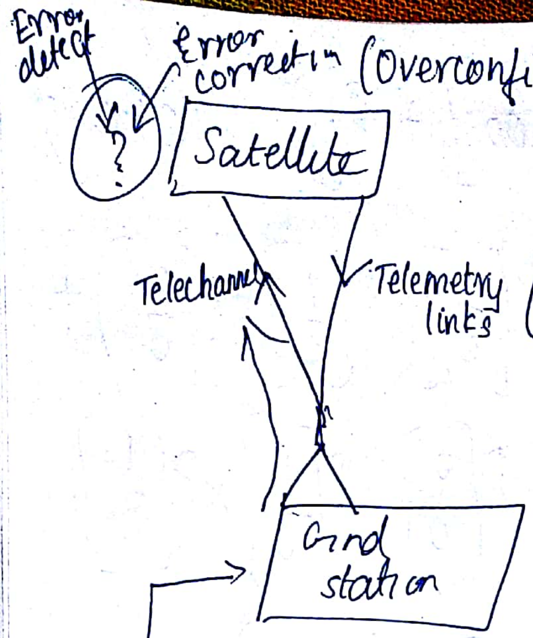
$$\cancel{=27\times10^{-12}+27\times10^{-}}$$

$$=P_e^3+3P_e^2(1-P_e)=3P_e^2-2P_e^3$$

$$=27\times10^{-8}-54\times10^{-12}$$

$$\simeq 2.69\times10^{-7}$$

| Error Detection | Error Correction |
| --- | --- |
| • Needs reverse channel for ARQ | • Doesn't need |
| • More robust against noise | • Less tolerance to noise. |




(used to send the status of satellite (e.g Battery capacity)) out of it, ~~only~~ one bit is alloted for ARQ

$\bar{m} \longrightarrow \bar{c}_1$

16 bits    31 bits

~~16 bit~~

$2^{16}$ channel $\rightarrow$ $2^{31}$ code words

but due to one-to-one mapping
only $2^{16}$ of $2^{31}$ code words are used

- For a very crucial and robust system where every info is vital, error detection preferred
- If minute errors doesn't matter, error correction can be used.

## CHANNEL ENCODING SCHEMES

- Linear block codes
- Convolutional codes
- Cylindrical codes
- Turbo codes
- Low Density Parity Check Codes

# LINEAR BLOCK CODES :-

$(n,k)$ - LBC

$$\underset{(m_1, m_2, \dots m_n)}{\bar{m}} \longrightarrow \boxed{\begin{matrix}(n,k)\\ \text{LBC}\end{matrix}} \longrightarrow \bar{c} = (c_1, c_2, \dots, c_n)$$

Systematic code

$$\bar{c} = (m_1, m_2, \dots, m_k, P_1, P_2 \dots P_{n-k})$$

(or)

$$(P_1, P_2, \dots P_{n-k}, m_1, m_2, \dots m_k)$$

Parity bits

$$(m_1, m_2, \dots m_k) \xrightarrow{\bar{m}} \underset{G}{\boxed{(n,k)}} \xrightarrow{\bar{c}} (m_1, m_2, \dots m_k, P_1, P_2, \dots P_{n-k})$$

$$\underset{1 \times n}{\bar{c}} = \underset{1 \times k}{\bar{m}}\ \underset{k \times n}{G}$$

$$G = (I_k / P_{k \times (n-k)})$$

$$G = \left[\begin{array}{cccccc|cccc} 1 & 0 & \dots & \dots & \dots & 0 & P_{11} & P_{12} & \dots & P_{1(n-k)} \\ 0 & 1 & \dots & \dots & \dots & 0 & P_{21} & P_{22} & \dots & P_{2(n-k)} \\ 0 & 0 & 1 & \dots & \dots & 0 & \vdots & & & \\ \vdots & & & & & & \vdots & & & \\ 0 & 0 & \dots & \dots & 0 & 1 & P_{k1} & \dots & \dots & P_{k(n-k)} \end{array}\right]$$

G → generator matrix

H → Parity Check matrix

$$H = \left(I_{n-k} \mid P_{(n-k)\times k}\right)$$

$$H^T = \begin{pmatrix} I_{n-k} \\ \hline P_{k\times(n-k)} \end{pmatrix}$$

$$\bar{r} \longrightarrow \boxed{H^T} \longrightarrow \bar{r}H^T = \bar{S}$$

↳ syndrome vector

(error)

<u>Eg:</u>

$$G = \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix} \Rightarrow (7,4) \text{ code}$$

$$\bar{m} \longrightarrow \boxed{\begin{matrix}(n,k)\\ \text{LBC}\end{matrix}} \longrightarrow \bar{c}\ (c_1, c_2, \ldots c_n)$$

$(m_1, m_2, \ldots m_k)$

$\bar{m}_1$ - 0 0 0 0     $\bar{c}_1$

0 0 0 1     $\bar{c}_2$

0 0 1 0     $\bar{c}_3$

    $\bar{c}_4$

⋮

$\bar{m}_{16}$ - 1 1 1 1     $\bar{c}_{16}$

Let $\bar{m}$

$\bar{c} = \bar{m}$

$= [$

$= [$

$m_{15} =$

$m_{16} =$

$\bar{c}_{15}$

$\bar{c}_{16}$

$\bar{c}_1$

$\bar{c}_2$

$\bar{c}_3$

$\bar{c}_4$

$\bar{c}_5$

$\bar{c}_6$

$\bar{c}_7$

Let $\bar{m} = (1001)$

$\bar{c} = \bar{m} G$

$= [1\ 0\ 0\ 1]_{1\times 4} [G]_{4\times 7}$

$= [1\ 0\ 0\ 1\ 1\ 0\ 0]_{1\times 7}$

$2 \overline{)2}$ $1-0$

$1+1=2$ $(1\times 0)$

mod $2 = 0$

$2 \overline{)3}$ $1-1$

$m_{15} = 1110$

$m_{16} = 1111$

$\bar{C}_{15} = [1\ 1\ 1\ 0] \times [G] = [1\ 1\ 1\ 0\ 1\ 0\ 0]$

$\bar{C}_{16} = [1\ 1\ 1\ 1] \times [G] = [1\ 1\ 1\ 1\ 1\ 1\ 1]$

$\bar{C}_1 = [0\ 0\ 0\ 0] \times [G] = [0\ 0\ 0\ 0\ 0\ 0\ 0]$

$\bar{C}_2 = [0\ 0\ 0\ 1] \times [G] = [0\ 0\ 0\ 1\ 0\ 1\ 1]$

$\bar{C}_3 = [0\ 0\ 1\ 0] \times [G] = [0\ 0\ 1\ 0\ 1\ 0\ 1]$

$\bar{C}_4 = [0\ 0\ 1\ 1] \times [G] = [0\ 0\ 1\ 1\ 1\ 1\ 0]$

$\bar{C}_5 = [0\ 1\ 0\ 0] \times [G] = [0\ 1\ 0\ 0\ 1\ 1\ 0]$

$\bar{C}_6 = [0\ 1\ 0\ 1] \times [G] = [0\ 1\ 0\ 1\ 1\ 0\ 1]$

$\bar{C}_7 = [0\ 1\ 1\ 0] \times [G] = [0\ 1\ 1\ 0\ 0\ 1\ 1]$

$\bar{C}_8 = [0\ 1\ 1\ 1] \times [G] = [0\ 1\ 1\ 1\ 0\ 0\ 0]$

$\bar{C_9} = [1\,0\,0\,0] \times G = [1\,0\,0\,0\,1\,1\,1]$

$\bar{C_{10}} = [1\,0\,0\,1] \times G = [1\,0\,0\,1\,1\,0\,0]$

$\bar{C_{11}} = [1\,0\,1\,0] \times G = [1\,0\,1\,0\,0\,1\,0]$

$\bar{C_{12}} = [1\,0\,1\,1] \times G = [1\,0\,1\,1\,0\,0\,1]$

$\bar{C_{13}} = [1\,1\,0\,0] \times G = [1\,1\,0\,0\,0\,0\,0]$

$\bar{C_{14}} = [1\,1\,0\,1] \times G = [1\,1\,0\,1\,0\,1\,0]$

For

$\bar{r} = [0\,0\,1\,1\;\;1\,1\,\underline{1}]$ → not a valid code word

nearest value $= [0\,0\,1\,1\;\;1\,1\,\underline{0}]$ — only 1 bit error

Error detection can be used.

For $\bar{r} = [1\,0\,1\,1\,\underline{1}\,\underline{1}\,1]$

nearest $= [1\,0\,1\,1\,\underline{0}\,\underline{0}\,1]$ — 2 bits error

Error detection can't be used

Error correction is preferred.

Hamming dist. b/n 2 code words $\Big\}$ = No. of bits by which two code words differ

Comparing $C_1$ with $C_2, \ldots C_{16}$

i.e $C_i$ with $C_j$ $i = j = 1, \ldots 16.$

we find that the

minimum Hamming dist. $= 3 = (d_H)$

for the above set.

Properties:

1) $C_i \oplus C_j = \overline{C}_k$

Adding 2 codes $\longrightarrow$ results in another code word.

2) All zero code word $\Rightarrow$ code word

3) Hamming weight of a code word $=$ No. of 1's in a code word

Proof for each

1) $\overline{C}_8 \oplus \overline{C}_{10}$

$= [0111000] \oplus [1001100]$

~~$= [1000100$~~ $= [1110100]$ ✓ $= \overline{C}_{15}$

(2) $C_1 = [0000000] \Rightarrow$ valid code word

(3)

$(d_H)_{min} = \min(\text{Hamming weight}) = \cancel{4}\,4$

Set of code word $= \cancel{4}\ 3$

Error correction capability = $\frac{d_{min} - 1}{2}$

Error detection capability = $d_{min} - 1$

Detect 2 bit error

Correct 1 bit error