Routh-Hurwitz Criterion:-

→ It is based on ordering the co-efficients of the characteristic equation, called as Routh Array.

→ $a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \dots + a_{n-1} s + a_n = 0$

where $a_0 > 0$.

s^n :	a_0	a_2	a_4	a_6	....
s^{n-1} :	a_1	a_3	a_5	a_7	....
s^{n-2} :	b_0	b_1	b_2	b_3	....
s^{n-3} :	c_0	c_1	c_2	c_3	....
⋮	⋮				
s^1 :	g_0				
s^0 :	h_0				

→ $b_0 = (-1)\begin{vmatrix} a_0 & a_2 \\ a_1 & a_3 \end{vmatrix} = \dfrac{a_2 a_1 - a_0 a_3}{a_1}$

$b_1 = \dfrac{a_1 a_4 - a_0 a_5}{a_1}$

$b_2 = \dfrac{a_1 a_6 - a_0 a_7}{a_1}$

$\rightarrow$ $c_0 = \dfrac{a_3 b_0 - a_1 b_1}{b_0}$.

$c_1 = \dfrac{b_0 a_5 - a_1 b_2}{b_0}$.

⋮

→ In the processing of constructing Routh array the missing terms are considered as zero.

→ <u>The Routh stability criterion:-</u>

→ All the elements in the first column of the Routh table should be positive, for the stable system.

→ If this condition is not met, the s/m is unstable and the no. of sign changes in the elements of the first column of the Routh array corresponds to the number of roots of char., eqn., in the right half of s-plane.

→ <u>Cases</u> of <u>Routh array</u>.

(1) Normal Routh Array.

(2) A row of all zeros.

(3) First element of a row is zero but some or other elements are not zero.

Eg problem

case (i) - Normal Routh array.

1) Using Routh criterion, determine the stability of the s/m represented by the char. eqn. $s^4 + 8s^3 + 18s^2 + 16s + 5 = 0$. comment on the location of the roots. & ~~stability~~

solu :)

$s^4 + 8s^3 + 18s^2 + 16s + 5 = 0.$

s^4 . 1 18 5

s^3 8 16

$s^2 : \frac{8 \times 18 - 16}{8}$

s^4 :	1	18	5
s^3 :	1	2	
s^2 :	16	5	
s^1 :	1.7	0	
s^0 :	5		

$s^2 : \frac{1 \times 18 - 2 \times 1}{1} = 16$

$\frac{1 \times 5 - 0 \times 1}{1} = 5$

$s^1 : \frac{16 \times 2 - 5 \times 1}{16} = 1.665$

$\simeq 1.7$

$s^0 : \frac{1.7 \times 5 - 0 \times 16}{1.7}$

→ No change in sign in the 1st column of Routh array.

→∴ All roots (4) are present on the L.H of the s plane & the s/m is stable.

Case (ii) - A row of all zeros.

2) Construct Routh-array and determine the stability of the s/m whose char. eqn is $s^6+2s^5+8s^4+12s^3+20s^2+16s+16=0$. Also determine the no. of roots lying on right half of s-plane, left half of s-plane and on img. axis.

soln

s^6	1	8	20	16
s^5	2	12	16	

s^6	1	8	20	16
s^5	1	6	8	—
s^4	$\cancel{2}$ 1	$\cancel{12}$ 6	$\cancel{16}$ 8	—
s^3	0	0		
s^3	1	3		
s^2	3	8		
s^1	0.33	0		
s^0	8			

→ All zeros in s^3.

→ Auxillary eqn. A

$A = s^4 + 6s^2 + 8$

$\frac{dA}{ds} = 4s^3 + 12s.$

s^3	4	12
	$\boxed{1}$	$\boxed{3}$

→ There is no sign-change, but row with all zeros indicate the possibility of roots on Imaginary axis.

→ Hence the s/m is limitedly or marginally stable.

→ Roots calculation

Auxillary polynomial.

$$s^4 + 6s^2 + 8 = 0.$$

$$s^2 = x$$

$$x^2 + 6x + 8 = 0$$

$$x = -3 \pm 1$$

$$x = -2 \text{ \& } -4$$

$$s = \pm\sqrt{x}$$

$$= \pm\sqrt{-2} \text{ \& } \pm\sqrt{-4}$$

$$= \pm j\sqrt{2} \text{ \& } \pm j2.$$

$$s = +j\sqrt{2},\ -j\sqrt{2},\ +j2,\ -j2.$$

→ Out of 6 roots, 4 roots are present on the img. axis & the remaining 2 roots present on the L-H of S-plane.

→ No-roots present on R-H of S-plane.

case iii) First element of a row is zero.

3) construct Routh array and determine the stability of the s/m represented by the char. eqn $s^5+s^4+2s^3+2s^2+3s+5=0$. comment on the location of the roots of char. eqn.

Soln.

s^5	1	2	3
s^4	1	2	5
s^3	ε	-2	—
s^2	$\frac{2\varepsilon+2}{\varepsilon}$	5	—
s^1	$\frac{\varepsilon\left[\left(\frac{2\varepsilon+2}{\varepsilon}\right)(-2)-5\varepsilon\right]}{(2\varepsilon+2)}$	—	—
s^0	5		

$s^3 \quad 0 \quad -2$

$\quad\;\; \varepsilon$

$$s^1 = \frac{-4\varepsilon - 4 - 5\varepsilon^2}{\varepsilon}$$

$$\boxed{s^1 = \frac{-4\varepsilon - 4 - 5\varepsilon^2}{2\varepsilon+2}}$$

sub $\varepsilon = 0$.

s^5	1	2	3
s^4	1	2	5
s^3	0	-2	
s^2	∞	5	
s^1	-2		
s^0	5		

No. of sign change = 2

∴ No. of Roots on RH of s plane is 2

Also the ~~s/m~~ system is unstable.

4) char. eqn: $s^7 + 9s^6 + 24s^5 + 24s^4 + 24s^3 + 24s^2 + 23s + 15 = 0$.

Determine the location of roots on s-plane and hence the stability of the s/m.

s^7	1	24	24	23
s^6	9	24	24	15

s^7	1	24	24	23
s^6	3	8	8	5
s^5	21.33	21.33	21.33	
s^5	1	1	1	
s^4	5	5	5	
	1	1	1	
s^3	0	0	0	

Auxillary eqn. $s^4 + s^2 + 1 = A$.

$$\frac{dA}{ds} = 4s^3 + 2s$$

s^7	1	24	24	3
s^6	3	8	8	5
s^5	1	1	1	
s^4	1	1	1	
s^3	4	2		
s^2	0.5	1		
s^1	-6			
s^0	1			

No. of sign change = 2

∴ No. of Roots on R.H of s plane = 2

∴ s/m is unstable.

RH – roots from Auxillary eqn.

$$s^4 + s^2 + 1 = 0.$$

$$s^2 = x \Rightarrow x^2 + x + 1 = 0.$$

$$x = \frac{-1 \pm \sqrt{1-4}}{2}$$

$$= \frac{-1 \pm \sqrt{-3}}{2}$$

$$= \frac{-1 \pm \sqrt{3}\,j}{2} \Rightarrow -0.5 \pm j0.866$$

out of 7 roots ⇒ 2 roots on RH of s-plane
5 roots on LH of s-plane

No roots present on img. axis.

Note: In I^{st} column of Routh-Array

→ If there is a sign change ⇒ S/m is unstable and no. of sign change is equal to the no. of roots present on R-H of s-plane.

→ If there is no sign change & no row with all zeros ⇒ S/m is stable and all roots are present on L-H of s-plane.

→ If there is a sign change & a row with all zeros ⇒ some roots on RH & some roots on LH but no roots on img. axis. S/m is unstable

→ If 1^{st} column consists of ∞, check only for the sign change & No comments on the presence of ∞.

→ If there is no sign change, but a row with all zeros, check for the auxillary roots & find the location of roots on s-plane.
If roots of auxillary equations are purely img, then the s/m is Marginally Stable