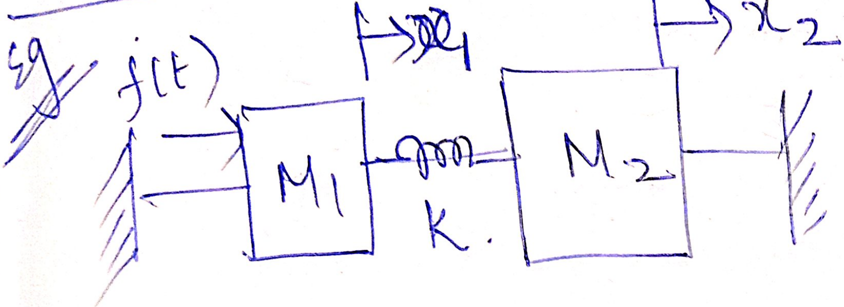
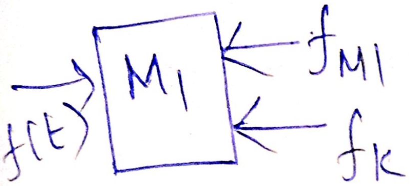
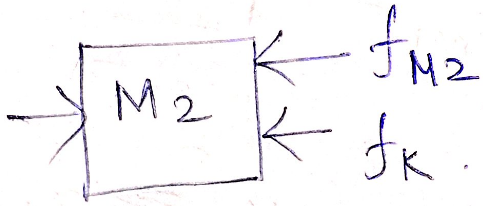
eg



Free-body diagram





$$f_{M1} = M_1 \frac{d^2 x_1}{dt^2}$$

$$f_K = K(x_1 - x_2)$$

$$f(t) = f_{M1} + f_K$$

$$f_{M2} + f_K = 0$$

$$f_{M2} = M_2 \frac{d^2 x_2}{dt^2}$$

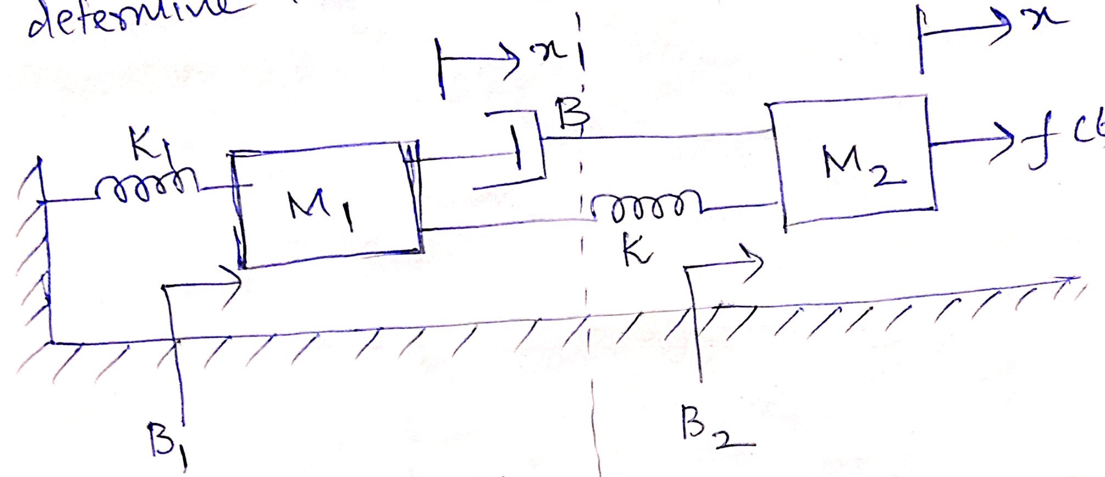
$$f_K = K(x_2 - x_1)$$

$$M_2 \frac{d^2 x_2}{dt^2} + K(x_2 - x_1) = 0$$

pblm

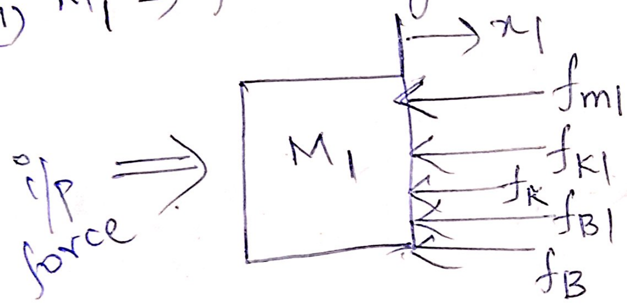
Mechanical-Translational s/m :-

1) Write the differential equations governing the mechanical s/m shown in fig., & determine the transfer function.



soln Each mass ⟺ ~~Each~~ one node.

(i) $M_1 \Rightarrow$ free body diagram.



$$f_{M1} = M_1 \frac{d^2 x_1}{dt^2}$$

$$f_{K1} = K_1 x_1 .$$

$$f_K = K(x_1 - x)$$

$$f_{B1} = B_1 \frac{dx_1}{dt}$$

$$f_B = B \frac{d(x_1 - x_2)}{dt} .$$

→ The i/p force should be equal to the sum of all opposing forces.

$\therefore$ i/p force $= f_{m_1} + f_{K_1} + f_K + f_{B_1} + f_B$

$$= M_1 \frac{d^2 x_1}{dt^2} + K_1 x_1 + K(x_1 - x_2) +$$

$$B_1 \frac{dx_1}{dt} + B \frac{d}{dt}(x_1 - x_2).$$

$\because$ i/p force $= 0$,

$$M_1 \frac{d^2 x_1}{dt^2} + K_1 x_1 + K(x_1 - x_2) + B_1 \frac{dx_1}{dt} + B \frac{d}{dt}(x_1 - x_2) = 0$$

$$\boxed{L\left[\frac{d^2x}{dt^2}\right] = s^2 X(s) - sx(0) - x'(0); \; L\left[\frac{dx}{dt}\right] = sX(s) - x(0)}$$

→ on taking laplace transform, [initial conditions $= 0$]

$$M_1 s^2 X_1(s) + B_1 s X_1(s) + Bs\left[X_1(s) - X(s)\right] +$$

$$K_1 X_1(s) + K\left[X_1(s) - X(s)\right] = 0.$$

→ separate, $X_1(s)$ & $X(s)$ variables,

$$X_1(s)\left[M_1 s^2 + B_1 s + Bs + K_1 + K\right] -$$

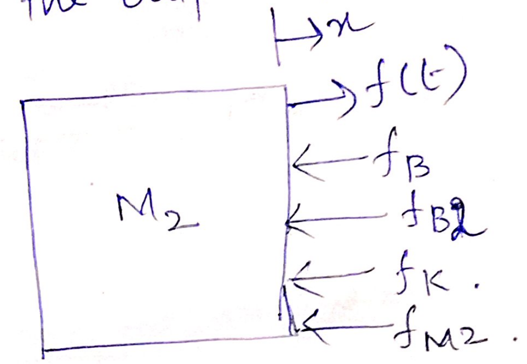
$$X(s)\left[Bs + K\right] = 0.$$

→ $X_1(s)\left[M_1 s^2 + s(B_1 + B) + K_1 + K\right] = X(s)\left[Bs + K\right]$

$$\rightarrow \boxed{X_1(s) = X(s)\,\frac{Bs+K}{M_1 s^2 + (B_1+B)s + (K_1+K)}} \quad \text{—— (1)}$$

(ii) $M_2 \Rightarrow$ free body diagram.

→ The net opposing forces on M_2 will be equal to the output force $f(t)$.



$$f_{M2} = M_2 \frac{d^2x}{dt^2} \qquad f_K = K(x - x_1)$$

$$f_{B2} = B_2 \frac{dx}{dt}$$

$$f_B = B \frac{d}{dt}(x - x_1)$$

→ Net force is equal to o/p force

$$\therefore f(t) = f_{M2} + f_{B2} + f_B + f_K$$

$$f(t) = M_2 \frac{d^2x}{dt^2} + B_2 \frac{dx}{dt} + B \frac{d}{dt}(x - x_1)$$

$$+ K(x - x_1).$$

→ on taking laplace transform,

$$F(s) = M_2\left[s^2 X(s)\right] + B_2\, sX(s) + B\, s\left[X(s) - X_1(s)\right]$$

$$+ K\left[X(s) - X_1(s)\right]$$

$$F(s) = M_2\left[s^2 X(s)\right] + B_2 s X(s) + BsX(s) + kX(s) - BsX_1(s) - kX_1(s).$$

$$F(s) = X(s)\left[M_2 s^2 + B_2 s + Bs + K\right] - X_1(s)\left[Bs + K\right]$$

→ sub eqn ① in $F(s)$,

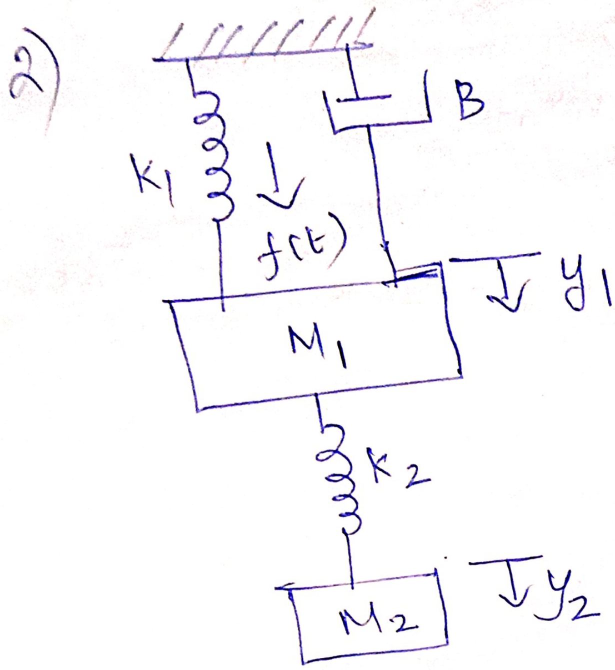
$$F(s) = X(s)\left[M_2 s^2 + B_2 s + Bs + K\right] - (Bs+K)\frac{X(s)(Bs+K)}{M_1 s^2 + (B+B_1)s + (K+K_1)}$$

$$F(s) = X(s)\left[\frac{\left\{\left[M_2 s^2 + B_2 s + Bs + K\right]\left[M_1 s^2 + (B+B_1)s + (K+K_1)\right]\right\} - \left\{(Bs+K)^2\right\}}{M_1 s^2 + (B+B_1)s + (K+K_1)}\right]$$

$$\rightarrow \frac{F(s)}{X(s)} = \frac{(M_2 s^2 + B_2 s + Bs + K)(M_1 s^2 + (B+B_1)s + (K+K_1)) - (Bs+K)^2}{M_1 s^2 + (B+B_1)s + (K+K_1)}.$$

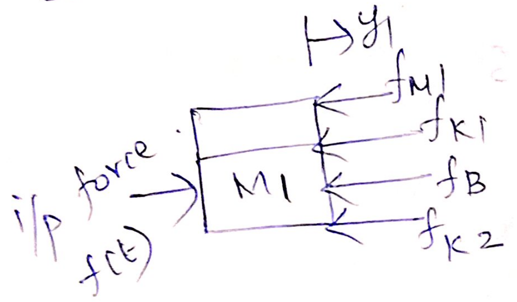
$$\Rightarrow T(s) = \frac{X(s)}{F(s)} = \frac{M_1 s^2 + (B+B_1)s + (K+K_1)}{\left[(M_2 s^2 + B_2 s + Bs + K)(M_1 s^2 + (B+B_1)s + (K+K_1)) - (Bs+K)^2\right]}$$

2)



soln

(i) $M_1 \Rightarrow$ free body diagram



$$f(t) = f_{M1} + f_{K1} + f_B + f_{K2}.$$

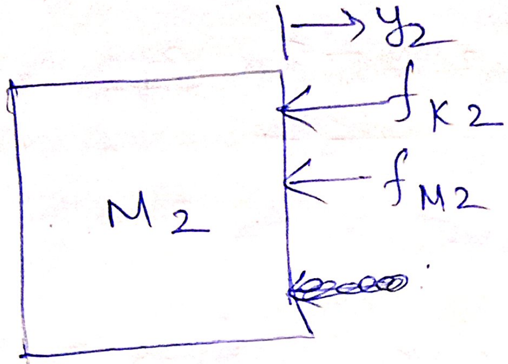
$$f(t) = M_1 \frac{d^2 y_1}{dt^2} + K_1 y_1 + B\frac{dy_1}{dt} + K_2(y_1 - y_2)$$

→ on Taking LT, with zero initial condt,.

$$F(s) = M_1 s^2 Y_1(s) + BsY_1(s) + K_1 Y_1(s) + K_2\left[Y_1(s) - Y_2(s)\right]$$

$$\boxed{F(s) = Y_1(s)\left[M_1 s^2 + Bs + K_1 + K_2\right] - K_2 Y_2(s)} \rightarrow (1)$$

(ii) $M_2 \Rightarrow$ free body diagram.



net opposing force $= f_{K2} + f_{M2} = 0$.

$$f_{M2} = M_2 \frac{d^2 y_2}{dt^2} \quad \& \quad f_{K2} = K_2(y_2 - y_1)$$

$$\therefore \quad M_2 \frac{d^2 y_2}{dt^2} + K_2(y_2 - y_1) = 0.$$

→ on taking LT,

$$M_2 \, s^2 Y_2(s) + K_2 \left[Y_2(s) - Y_1(s) \right] = 0$$

$$M_2 \, s^2 Y_2(s) = K_2 \left[Y_1(s) - Y_2(s) \right]$$

$$M_2 s^2 Y_2(s) + K_2 Y_2(s) = K_2 Y_1(s)$$

$$Y_2(s) \left[M_2 s^2 + K_2 \right] = K_2 Y_1(s)$$

$$\therefore \boxed{Y_1(s) = \frac{Y_2(s) \left[M_2 s^2 + K_2 \right]}{K_2}} \quad \text{— (2)}$$

→ sub ② in ①

$$F(s) = Y_2(s)\frac{[M_2 s^2 + K_2]}{K_2}[M_1 s^2 + Bs + K_1 + K_2] - K_2 Y_2(s)$$

$$F(s) = Y_2(s)\left[\frac{(M_2 s^2 + K_2)(M_1 s^2 + Bs + K_1 + K_2) - K_2^2}{K_2}\right]$$

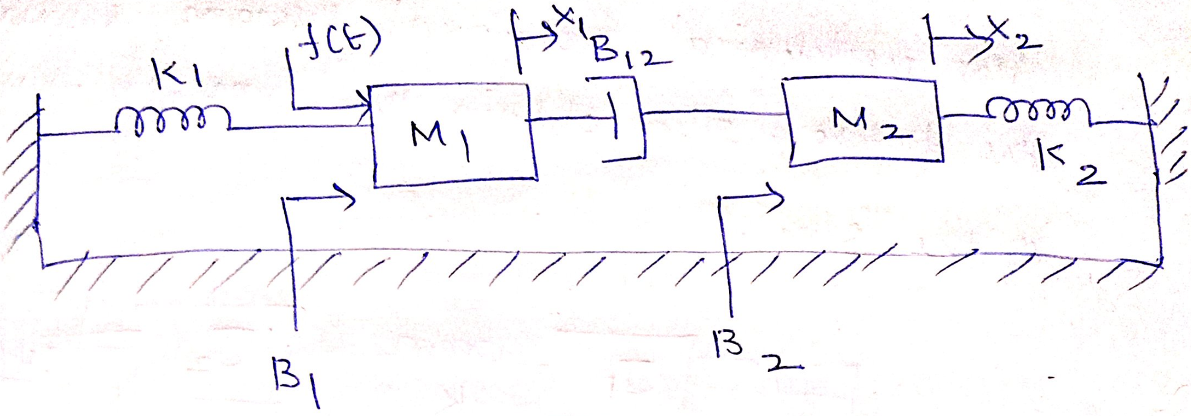
~~F(s)~~

→ Tfr function $= \frac{Y_2(s)}{F(s)}$
$T(s)$

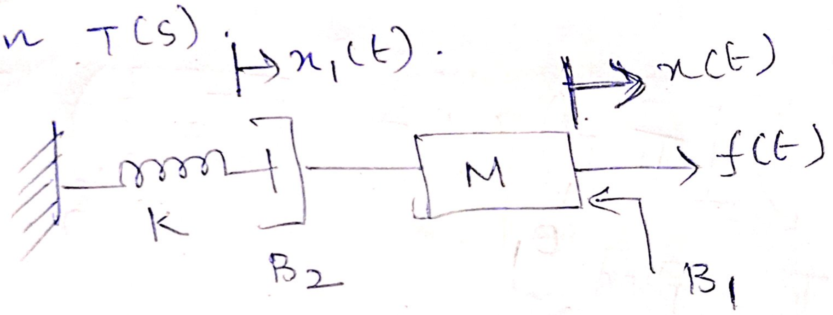
$$\boxed{T(s) = \frac{K_2}{(M_2 s^2 + K_2)(M_1 s^2 + Bs + K_1 + K_2) - K_2^2}}$$

Exercise problems

1) Determine $\frac{X_1(s)}{F(s)}$ & $\frac{X_2(s)}{F(s)}$ for the s/m.

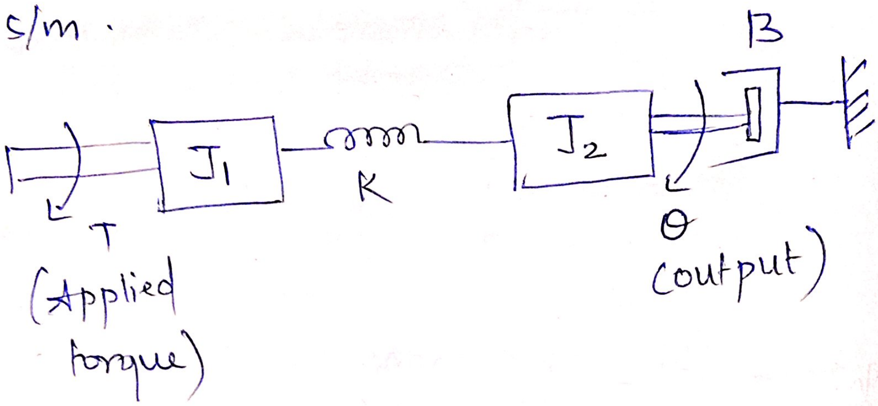


2) Write the equations of motion in s-domain for the s/m. and also determine the tfr function T(s).

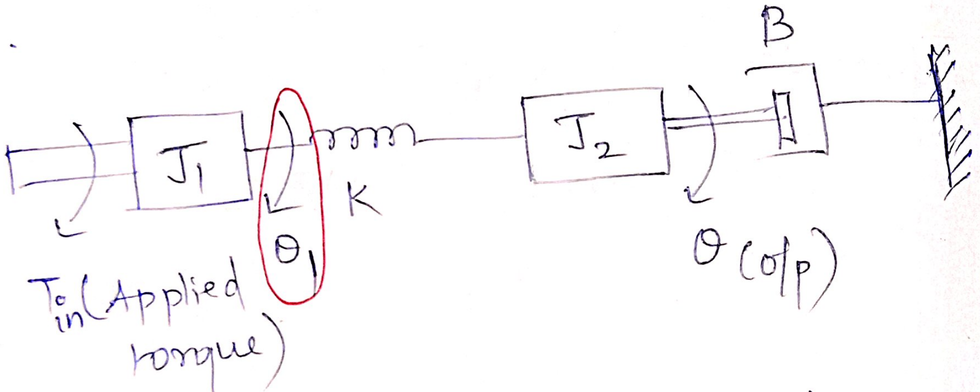


pblms on Mechanical Rotational s/m.

1) Write the differential equations governing the mechanical rotational system shown in the figure. Also obtain the transfer function of the s/m.

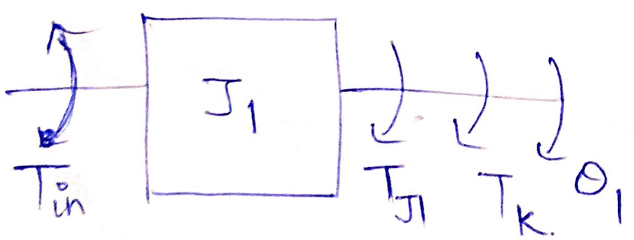


soln



Assume the J_1 s/m has the angular displacement of θ_1.

(i) $J_1 \Rightarrow$ free body diagram.



→ T_{in} = sum of opposing torques due to T_{J1} & T_K.

$$T_{J1} = J_1 \frac{d^2\theta_1}{dt^2} \qquad T_K = K(\theta_1 - \theta)$$

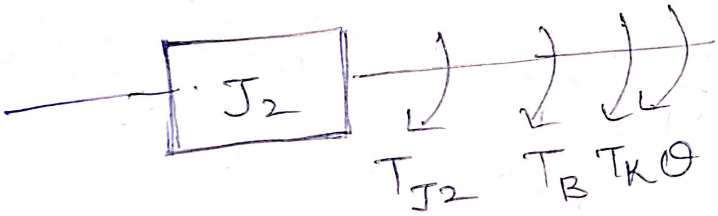
→ $\therefore T_{in} = T_{J1} + T_K$

$$T_{in} = J_1 \frac{d^2\theta_1}{dt^2} + K(\theta_1 - \theta)$$

→ LT of the above eqn will be,

$$\boxed{T_{in}(s) = J_1 s^2 \theta_1(s) + K\,\theta_1(s) - K\theta(s)} \quad (1)$$

(ii) J_2 ⇒ free body diagram.



$$T_{J2} = J_2 \frac{d^2\theta}{dt^2}$$

$$T_B = B\frac{d\theta}{dt}$$

$$T_K = K(\theta - \theta_1)$$

→ net torque is,

$$T_{J2} + T_B + T_K = 0$$

$$J_2 \frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} + K(\theta - \theta_1) = 0.$$

→ on taking L.T.,

$$J_2 s^2 \theta(s) + Bs\,\theta(s) + K[\theta(s) - \theta_1(s)] = 0$$

$$\theta(s)\left[J_2 s^2 + Bs + K\right] = K\,\theta_1(s).$$

$$\boxed{\theta_1(s) = \frac{\theta(s)\left[J_2 s^2 + Bs + K\right]}{K}.} \quad (2)$$

→ sub for $\theta_1(s)$ in (1)

$$(1) \Rightarrow \boxed{T_{in}(s) = J_1 s^2 \theta_1(s) + K\theta_1(s) - K\theta(s)}$$

$$= J_1 s^2 \theta_1(s) + \cancel{K}\left[\frac{\theta(s)(J_2 s^2 + Bs + K)}{\cancel{K}}\right] - K\theta(s)$$

$$= J_1 s^2 \theta(s) \left[\frac{J_2 s^2 + Bs + K}{K}\right] + \frac{\cancel{K}\theta(s)}{\cancel{K}}(J_2 s^2 + Bs + K)$$

$$- K\theta(s)$$

$$T_{in}(s) = \theta(s)\left[\frac{J_1(s^2)}{K}\left[J_2 s^2 + Bs + K\right] + (J_2 s^2 + Bs + K) - (K)\right]$$

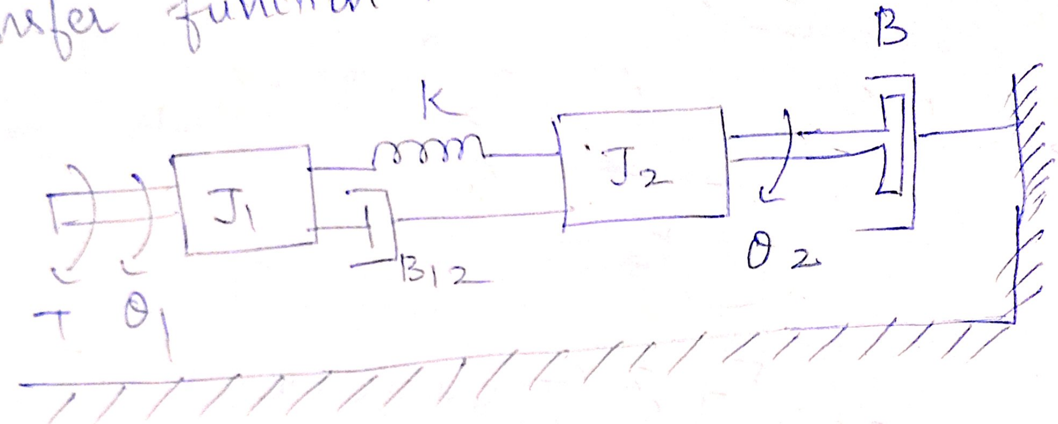
1 2 3

$$= \theta(s)\left[\frac{J_1 s^2 (J_2 s^2 + Bs + K) + K(J_2 s^2 + Bs + K) - (K^2)}{K}\right]$$

$$T_{in}(s) = \theta(s)\left[\frac{(J_2 s^2 + Bs + K)(J_1 s^2 + K) - K^2}{K}\right]$$

$$\boxed{\frac{\theta(s)}{T_{in}(s)} = \frac{K}{(J_2 s^2 + Bs + K)(J_1 s^2 + K) - K^2}}$$

2) Write the differential equations governing the mechanical rotational s/m & determine its transfer function.

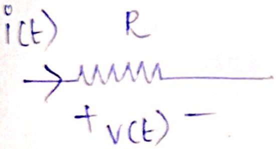
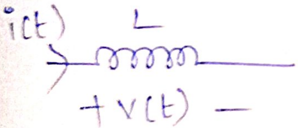
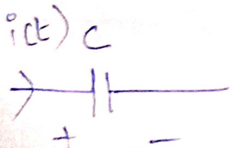


Electrical systems:-

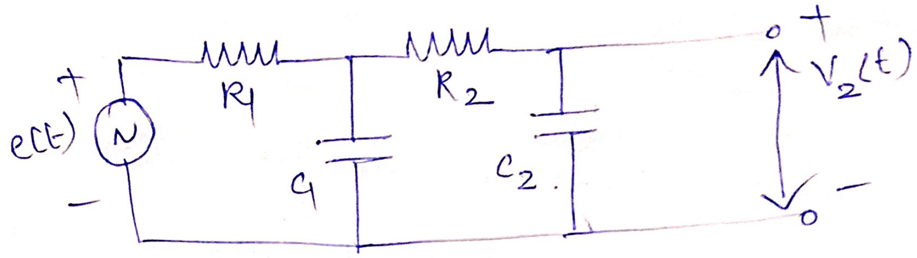
→ The models of electrical s/m can be obtained by using resistor, capacitor and inductor.

→ The differential equations governing the electrical s/ms can be formed by writing KCL or KVL & the transfer function is determined by taking L.T. of differential equations.

→ Current-Voltage Relation of R, L, C.

Element	Voltage across the Element	current thro' the element
	$v(t) = i(t) R.$	$i(t) = \frac{v(t)}{R}$
	$v(t) = L \frac{di(t)}{dt}$	$i(t) = \frac{1}{L}\int v(t)\,dt$
	$v(t) = \frac{1}{C}\int i(t)\,dt$	$i(t) = C\frac{dv(t)}{dt}$

→ Eg:- obtain the tfr function of the n/w shown below,



Soln

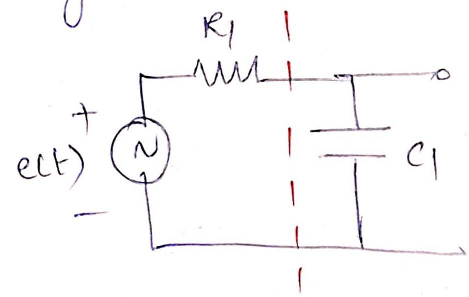
→ Tfr function = $\frac{o/p}{i/p}$ ⇒ $\frac{LT[V_2(t)]}{LT[e(t)]}$

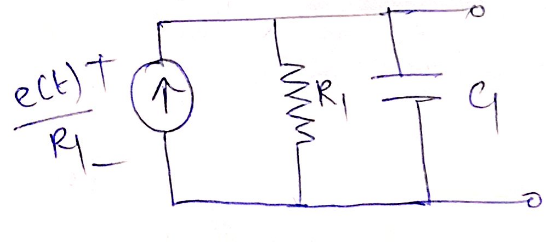
⇓

in-terms of voltage.

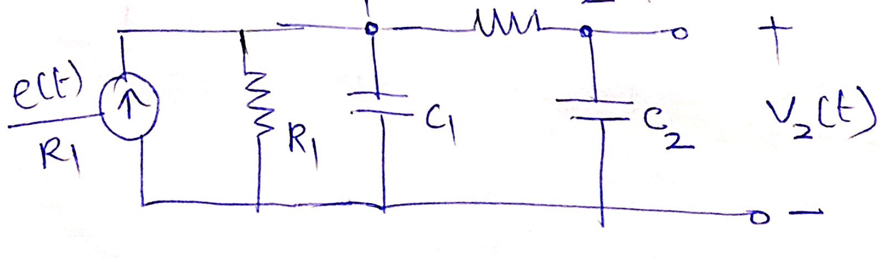
So go for nodal analysis.

→ By source-transformation theorem,





→



→ At node 1,

$$\frac{e(t)}{R_1} = \frac{V_1}{R_1} + C\frac{dV_1}{dt} + \frac{V_1 - V_2}{R_2}$$

→ LT of above eqn,

$$\boxed{\frac{E(s)}{R_1} = \frac{V_1(s)}{R_1} + CSV_1(s) + \frac{1}{R_2}\left[V_1(s) - V_2(s)\right]} \rightarrow (1)$$

→ At node 2,

$$\frac{V_1 - V_2}{R_2} = C_2\frac{dV_2}{dt}.$$

→ LT of above eqn;

$$\frac{1}{R_2}\left[V_1(s) - V_2(s)\right] = C_2 S V_2(s)$$

$$\frac{V_1(s)}{R_2} = C_2 S V_2(s) + \frac{1}{R_2} V_2(s).$$

$$V_1(s) = R_2\left[\frac{R_2 C_2 S V_2(s) + V_2(s)}{R_2}\right]$$

$$\boxed{\begin{aligned} V_1(s) &= R_2 C_2 S V_2(s) + V_2(s) \\ V_1(s) &= V_2(s)\left[1 + R_2 C_2 S\right] \end{aligned}} \rightarrow (2)$$

→ sub ② in ①.

$$\frac{E(s)}{R_1} = \frac{V_1(s)}{R_1} + csV_1(s) + \frac{1}{R_2}\left[V_1(s) - V_2(s)\right]$$

$$\frac{E(s)}{R_1} = V_1(s)\left[\frac{1}{R_1} + \frac{1}{R_2} + Cs\right] - \frac{1}{R_2}V_2(s)$$

$$\frac{E(s)}{R_1} = \left(\frac{1}{R_1} + \frac{1}{R_2} + Cs\right)\left(V_2(s)\left(1 + R_2C_2s\right)\right) - \frac{1}{R_2}V_2(s)$$

$$\frac{E(s)}{R_1} = \left[\left(\frac{R_1 + R_2 + R_1R_2Cs}{R_1R_2}\right)\left(1 + R_2C_2s\right) - \frac{1}{R_2}\right]V_2(s)$$

→ Tfr func $= \frac{V_2(s)}{E(s)}$

$$\frac{E(s)}{\cancel{R_1}} = \left[\frac{\left(R_1 + R_2 + R_1R_2Cs\right)\left(1 + R_2C_2s\right) - R_1}{\cancel{R_1}R_2}\right]V_2(s)$$

$$\therefore \boxed{\frac{V_2(s)}{E(s)} = \frac{R_2}{\left[\left(R_1 + R_2 + R_1R_2Cs\right)\left(1 + R_2C_2s\right) - R_1\right]}}$$