1) $R = 10.4\ \Omega/km \quad L = 0.00367\ H/km$

$G = 0.8 \times 10^{-6}\ mho/km \quad C = 0.00835 \times 10^{-6}\ F/km$

$\omega = 2\pi f = 2\pi \times 1\ kHz$

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = \sqrt{\frac{10.4 + j\omega\, 0.00367}{0.8\times10^{-6} + j\omega\, 0.00835\times10^{-6}}}$$

$$= \sqrt{\frac{10.4 + j2\pi 10^3 \times 0.00367}{0.8\times10^{-6} + j2\pi 10^3 \times 0.00835\times10^{-6}}}$$

$$= \sqrt{\frac{10.4+j23}{0.8+j52.5}} \times 10^3$$

$a+ib = r\angle\theta$

$\theta = \tan^{-1}\left(\frac{b}{a}\right)$

$r^2 = a^2+b^2$

$10.4 + j23 = 25.24\ \angle 65.66^\circ$

$0.8 + j52.5 = 52.5\ \angle 89.126^\circ$

$$Z_0 = \sqrt{\frac{25.24\ \angle 65.66^\circ}{52.5\ \angle 89.126^\circ}} \times 10^3$$

$$Z_0 = 692.8\ \angle -11.733^\circ = 67832 - j\,140.8$$

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}\times 10^{-6}$$

$$= \sqrt{(25.24\ \angle 65.66^\circ)(52.5\ \angle 89.126^\circ)}$$

$$= \sqrt{1325.1\times10^{-6}\ \angle 154.786^\circ}$$

$$\gamma = 0.036\ \angle 77.393^\circ = 0.0078 + j0.035$$

$\gamma = \alpha + j\beta$

$\alpha = 0.0078 \qquad \beta = 0.035$

$$\lambda = 2\pi/\beta = 2\pi/0.035 = 179.51\ km$$

Line terminated in Z_0 → i/p impedance is also Z_0.

$I_S = E_S/Z_S = \frac{1}{692.8 \angle -11.733} = 0.00144 \angle 11.733$ amp

$\frac{I_R}{I_S} = e^{-\gamma l} = e^{-(\alpha + j\beta)l}$ $\quad e^{j\theta} = \cos\theta - j\sin\theta$

$I_R/I_S = e^{-\alpha l} e^{-j\beta l}$

$= e^{-0.78} e^{-j3.5}$

$= 0.4584 \angle -200.535°$

$I_R = 0.00144 \angle 11.733 \times 0.4584 \angle -200.535$

$I_R = 0.00066 \angle -188.80$ A

$E_R = I_R Z_0 = 0.00066 \angle -188.802 \times 692.8 \angle -11.733$

$E_R = 0.4572 \angle -200.535°$ V

$f = 5$ kHz $\quad Z_{OC} = 141.9 \angle -84.1° \Omega$ $\quad Z_{SC} = 62.0 \angle 37.7 \Omega$

$l = 2$ km.

$Z_0 = ?$ $\quad \gamma = ?$ $\quad V =$ $\quad \lambda =$

$Z_0 = \sqrt{Z_{OC} Z_{SC}}$

$= \sqrt{141.9 \angle -84.1° \times 62.0 \angle 37.7}$

$= \sqrt{8797.8 \angle -46.4°}$

$Z_0 = 96.796 \angle -23.2$

$\tanh\gamma = \sqrt{\frac{Z_{SC}}{Z_{OC}}} = \sqrt{\frac{62.0 \angle 37.7}{141.9 \angle -84.1}}$

$= \sqrt{0.4369 \angle 121.8°}$

$\gamma = \tanh^{-1}(0.66 \angle 60.9°) = 0.321 + j0.577$

$\lambda = 2\pi/\beta$. $\quad V = \frac{\omega}{\beta}$.

$e^{2\gamma l} - 1 = 0.321 e^{2\gamma l} + j0.577 e^{2\gamma l} + 0.321 + j0.577$

$e^{2\gamma l}(0.679 - j0.577) = 1.321 + j0.577$

$e^{2\gamma l} = 1.321 + j0.577 / 0.6761 - j0.577$

$= 1.4415 \angle 23.595^\circ / 0.7\,9397 \angle -40.35^\circ$

$= 1.8155 \angle 63.945^\circ$

$2\gamma l = \ln(1.8155 \angle 63.945^\circ)$

$\ln(re^{j\theta}) = \ln r + j\theta \qquad 63.945^\circ = 1.116 \text{ rad}$

$\gamma = 0.5963 + 1.116j \,/\, 4$

$$\boxed{\gamma = 0.1490 + 0.279j}$$

$$\gamma = \omega/\beta = \frac{2\pi \times 5 \times 10^3}{0.279} = 1.126 \times 10^5 \text{ miles}^{-1}$$

(iv) $\lambda = 2\pi/\beta = 2\pi/0.279 = 22.52 \text{ miles}$

5) $R = 2.25\Omega$ $L = 1.0\mu H/m$.

$C = 1\times10^{-12}$ F/m $G = 0$ $f = 0.5\times10^{9}$ Hz.

$\alpha = ?$

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)} = \sqrt{2.25+j\omega 1\times10^{-2})(0+j\omega 1\times10^{-12})}$$

$\omega = \pi\times10^{9}$

$$= \sqrt{2.25+j(\pi\times10^{+3})(j\pi\times10^{-3})}$$

$$= \sqrt{-\pi^2 + j\,2.25\pi\times10^{-3}}$$

$$\gamma = \sqrt{9.869 \angle 0.0110^\circ - 0.0410^\circ} \quad \sqrt{179.959} = 3.1414 \angle 89.979$$

~~$\gamma = 3.141 + 0.001j$~~

~~$= \alpha + j\beta$~~

~~$\therefore \alpha = 3.141$~~

$= 1.151\times10^{-3} + j\,3.1413 = \alpha + j\beta$.

$\alpha = 1.151\times10^{-3}$ neper/m.

2) Reflection constant $= \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{(30 - j23) - 50}{(30 + j23) + 50}$

$= \frac{30.479 \angle 49^\circ}{83.24 \angle -16.04}$

$\beta = \frac{2\pi}{\lambda} = \frac{6.28}{0.45} = \boxed{14 \text{ rad m}^{-1}}$ $= 0.366 \angle 65.04^\circ$

$= \boxed{0.154 + j0.331}$

$C = 50 \times 10^{-12} \text{ F/m}$

$L = 200 \times 10^{-9} \text{ H/m}$

$f = 500 \text{ MHz}$
$= 500 \times 10^6 \text{ Hz}$

for lossless $\alpha = 0$, $R = 0$, $G = 0$

Find v, Z_0, β

$Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{200 \times 10^{-9}}{50 \times 10^{-12}}} = \sqrt{4 \times 10^3} = \boxed{63.2}$

$\beta = \omega\sqrt{LC} = 2 \times \pi \times 5 \times 10^8 \sqrt{200 \times 50 \times 10^{-21}}$

$= \pi \times 10^9 \sqrt{10^4 \times 10^{-21}} = \pi \times 10^9 \sqrt{10^{-18} \times 10}$

$= 3141.6 \times 10^6 \times 10^{-9} \times \sqrt{10}$

$\boxed{\beta = 9.9346 \text{ rad/m}^{-1}}$

$v = \frac{\omega}{\beta} = \frac{2\pi \times 500 \times 10^6}{2.16\pi} = \frac{10^9}{2.16} = \boxed{316.4 \times 10^6}$