UNIT- II

DESIGN of Infinite Impulse Response Filters (IIR)

Analog filters, - Butterworth filters, Chebyshev Type filters (upto 3rd order), Analog transformation of prototype LPF to BPF / BSF / HPF. Transformation of analog filters into equivalent digital filters using Impulse Invariant method and Bilinear Z- transform method - Realization Structures for IIR filters - direct, cascade, parallel forms.

Filter:

A filter is a device, which rejects unwanted frequencies from the input signal and allow the desired frequencies.

Filtering is one of the operation performed in DSP processor to remove the noises in the communication systems.

Filter classification:

- Filter
  - Analog (Based on frequency Response)
    - LPF
      - Butterworth (✓)
      - Chebyshev (✓)
      - Elliptical (x)
    - HPF
    - BPF
    - BSF
  - Digital (Based on no. of sample points used)
    - IIR
      - Butterworth (✓)
      - Chebyshev (✓)
    - FIR

~~Analog~~ Digital filter	~~Digital~~ Analog filter
1. Operates on digital samples of the signal.	1. Operates on analog (actual) signal.
2. defined by linear difference equation.	2. defined by linear differential equation.
3. contains adders, multi-pliers and delays imple-mented in digital logic (either in h/w or s/w or both)	3. contains electrical components like resistors, inductors and capacitors.
4. filter coefficients are designed to satisfy the desired frequency Response.	4. the approximation problem is solved to satisfy the desired frequency response.

Why we go for digital filters:

1. The values of resistors, capacitors and inductors, used in analog filters changes with temper-ature. Since the digital filters do not have these components, they have high thermal stability.

2. In digital filters, the precision of the filter depends on the length (or size) of the registers used to store the filter coefficients. Hence by increasing the register bit-length, the performance characteristics of the filter like accuracy, dynamic range, stability and frequency response tolerance can be enhanced.

3. The digital filters are programmable, hence it can be altered any time to obtain desired characteristics.

4. Digital filters can operate over a wide range of frequencies.

5. Digital filters are highly immune to noise and possess considerable parameter stability.

6. A single digital filter can be used to process multiple signals by using the techniques of multiplexing.

7. There are no problems of input or output impedance matching with digital filters.

Disadvantages of digital filters:

1. Since the performance of the digital filter depends on register-bit-length used to implement the filter, quantization error arises due to finite word length effect (round off the bits).

2. The bandwidth of the discrete time signal is limited by the sampling frequency.

IIR filter: (Recursive-type)

The filter designed by considering all the infinite (I) samples of impulse (I) response (R) is called IIR filter. The impulse response is obtained by taking inverse Fourier Transform of ideal frequency response.

$$h(n) \xrightleftharpoons{FT} H(\omega)$$

Recursive:

present output $y(n)$ sample depends on the present input $x(n)$, past input $x(n-1), x(n-2)\ldots$ & output samples $y(n-1), y(n-2)\ldots$.

Important features: (IIR)

1. The physically realizable IIR filters do not have linear phase.

2. The IIR filter specifications include the desired characteristics for the magnitude response only.

FIR filter: (Non-Recursive type)

The filters designed by selecting finite (F) number of samples of impulse (I) response (R) are called FIR filters. The impulse response of desired filter can be obtained by inverse Fourier Transform of ideal frequency response which consists of infinite samples.

$$h_d(n) \overset{FT}{\rightleftharpoons} H_d(\omega)$$

Non-Recursive:

The present output sample depends on the present input sample and previous (or past) input samples.

Important features: (FIR)

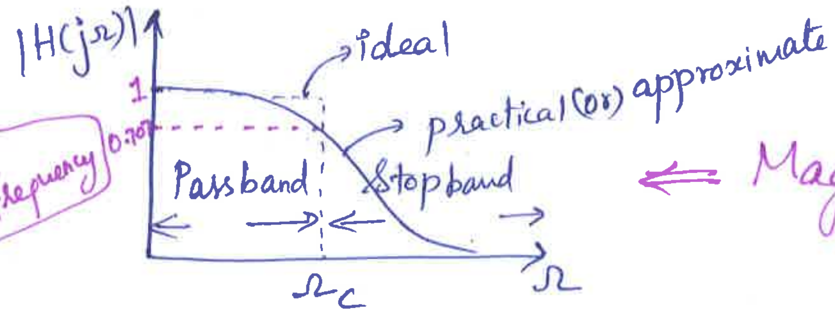
1. FIR filters can have precisely linear phase.
2. Always stable.
3. Errors due to round-off noise are less severe than in IIR filters.

Analog Filter Types:

1. Low Pass Filter (LPF):

LPF is one, which allows low frequencies in the passband $0 < \Omega < \Omega_c$, whereas the high frequencies in the stop band $\Omega > \Omega_c$ are blocked.

$\Omega \rightarrow$ Analog freq.
$\omega \rightarrow$ digital frequency



⟸ Magnitude Response of LPF.

Cutoff frequency (Ω_c): ⟶ frequency between the passband and the stop band, where the magnitude $|H(j\Omega)| = \frac{1}{\sqrt{2}} = 0.707$

Passband:

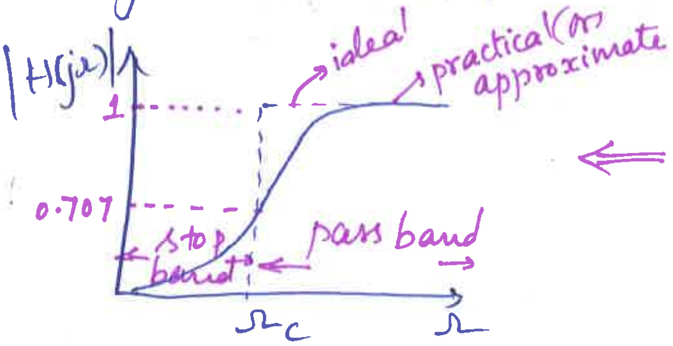
→ is the range of frequencies of signal that are passed through the filter.

Stop band:

→ is the range of frequencies of signal that are blocked by the filter.

2. High Pass Filter: (HPF)

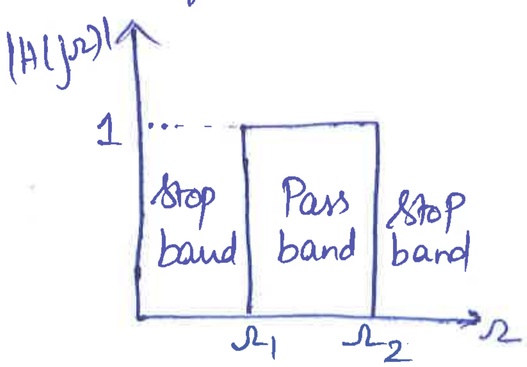
→ allows high frequencies above $\Omega > \Omega_c$ and rejects the frequencies between $\Omega = 0$ and $\Omega = \Omega_c$.



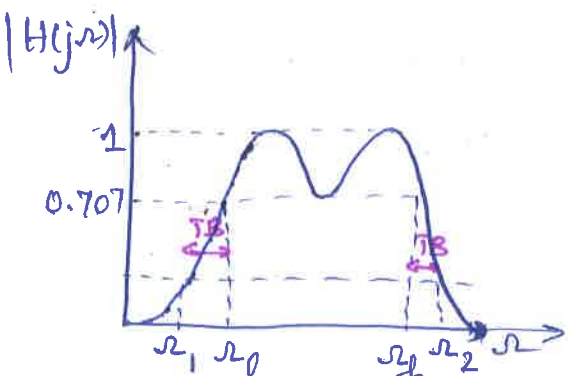
⇐ Magnitude Response of HPF.

3. Band Pass Filter: (BPF)

→ allows only a band of frequencies from Ω_1 to Ω_2 to pass and stops all other frequencies.



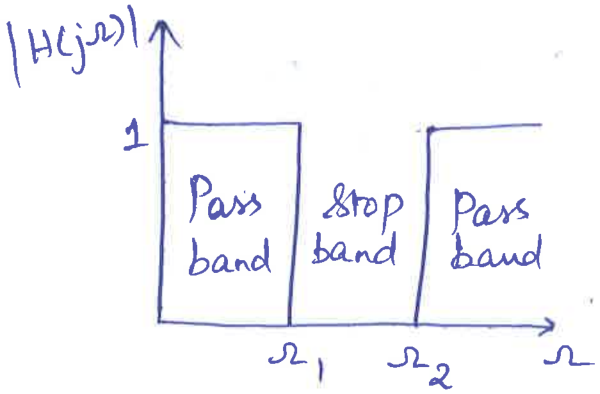
ideal Magnitude Response



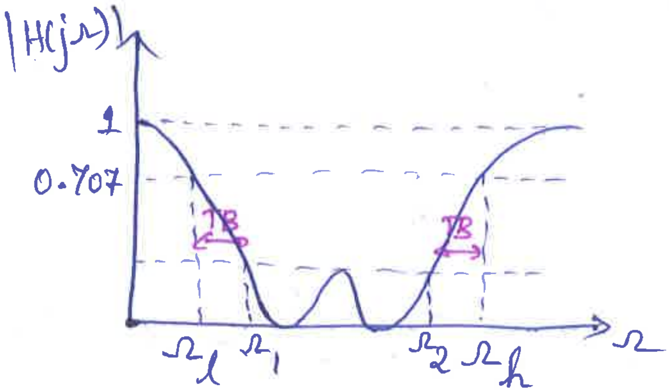
Approximate Magnitude Response

4. Band Stop Filter (or) Band Reject Filter: (BSF (or) BRF)

→ rejects a band of frequencies between Ω_1 and Ω_2 and allows the remaining frequencies.



ideal Magnitude Response



Approximate Magnitude Response.

from fig: TB → Transition Band

→ range of frequencies tha allows a transition between passband and stopband of a filter. It is defined by Pass band, Stop band and cut-off frequencies (or) corner frequency.

Ideal Filter:

→ transmits the signal under the passband without attenuation and completely suppress the signal in the stop band.

characteristics:

→ has constant gain in passband zero gain in stop band.

→ has linear phase response.

→ must be ~~to~~ causal.

Analog filter:

Let us describe the analog filter by linear constant-coefficient differential equation given by

$$\sum_{k=0}^{N} a_k \frac{d^k y(t)}{dt^k} = \sum_{k=0}^{M} b_k \frac{d^k x(t)}{dt^k}$$

where $x(t) \rightarrow$ input signal

$y(t) \rightarrow$ output of the filter.

$a_k, b_k \rightarrow$ filter-coefficients.

The impulse response of these filter coefficients is related to system function (or) transfer function by laplace transform,

$$H_a(s) = \frac{Y(s)}{X(s)} = LT\{h(t)\} = \int_{-\infty}^{\infty} h(t) \cdot e^{-st} \, dt.$$

$$H_a(s) = \frac{\sum_{k=0}^{M} b_k \cdot s^k}{\sum_{k=0}^{N} a_k \cdot s^k} = \frac{\sum_{k=0}^{M} b_k \cdot s^k}{1 + \sum_{k=1}^{N} a_k \cdot s^k}$$

$s = j\Omega$, $H_a(j\Omega) \rightarrow$ frequency Response of Analog filter.

$|H_a(j\Omega)| \rightarrow$ Magnitude " " "

$\angle H_a(j\Omega) \rightarrow$ Phase " " "

Design of IIR digital filters from analog filters:

The most common technique used for designing IIR digital filters is indirect method which requires 3 steps:

Step 1: Map the desired digital filter specifications into those for an equivalent analog filter.

Step 2: Derive the analog transfer function for the analog prototype.

Step 3: Transform the transfer function $H(s)$ of the analog prototype into an equivalent digital filter transfer function ($H(z)$)

Characteristics of Commonly used Analog filters:

Butterworth filter:

The Butterworth LPF has a magnitude response

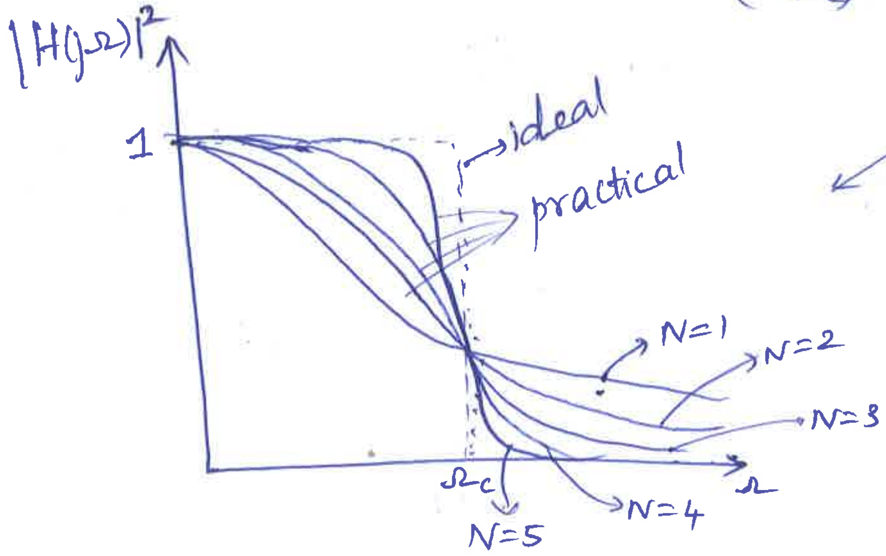
$$|H(j\Omega)| = \frac{1}{\sqrt{1+\left(\Omega/\Omega_c\right)^{2N}}}$$

$N \rightarrow$ order of the filter

$\Omega_c \rightarrow$ cut-off frequency.

(or)

$$|H(j\Omega)|^2 = |H(\Omega)|^2 = \frac{1}{1+\left(\Omega/\Omega_c\right)^{2N}}$$ ← Magnitude squared frequency Response.



The magnitude response has a maximally flat in the passband and monotonic in both passband and stop band.

From the figure above, it can be observed that, when N increases, the approximate response approaches ideal response.

<u>poles of a normalised Butterworth filter</u>:

$$|H(j\Omega)|^2 = \frac{1}{1+\left(\Omega/\Omega_c\right)^{2N}} \longrightarrow (1)$$

for a normalised filter, $\Omega_c = 1$

$$|H(j\Omega)|^2 = \frac{1}{1+\Omega^{2N}} \longrightarrow (2)$$

wkt, $s = j\Omega$, $\Omega = s/j$

Sub $\Omega = s/j$ in equation (2),

$$|H(j\Omega)|^2 = \frac{1}{1+(s/j)^{2N}}$$

The poles are obtained by ~~b~~ equating the denominator polynomial to zero.

$$1 + \left(\frac{s}{j}\right)^{2N} = 0$$

$$1 + (-s^2)^{N} = 0 \quad \text{(3)} \quad [\because j^2 = -1]$$

$$\cancel{(-s^2)^N = -1}$$

$$\cancel{(-1)^N \cdot s^{2N} = -1 = e^{+j(2k+1)\pi}}, \quad k = 0,1,2,3 \ldots \cancel{N-1}$$

Let $N \to$ odd:

eqn (3) reduces to $s^{2N} = 1$

ie $1 + (-1)^{N} \cdot s^{2N} = 0$

$$1 - s^{2N} = 0, \quad s^{2N} = 1 = e^{j2\pi k}$$

$$s^{2N} = e^{j2\pi k}$$

roots (or) poles: $s_k = e^{j2\pi k/2N} = e^{j\pi k/N}$, $k = 1, 2, \cdots 2N$ $\rightarrow$ (4)

$N \to$ even:

equation (3) reduces to $s^{2N} = -1 = e^{j(2k-1)\pi}$

$$s_k = e^{j(2k-1)\pi/2N}, \quad k = 1, 2, \cdots 2N \longrightarrow (5)$$

for $N=3$, eqn ④ becomes $s^6=1$

$$s_k = e^{j\pi k/3}, \quad k=1,2,\cdots 6$$

$$s_1 = e^{j\pi/3} = 0.5+j0.866$$
$$s_2 = e^{j2\pi/3} = -0.5+j0.866$$
$$s_3 = e^{j\pi} = -1$$
$$s_4 = e^{j4\pi/3} = -0.5-j0.866$$
$$s_5 = e^{j5\pi/3} = 0.5-j0.866$$
$$s_6 = 1$$

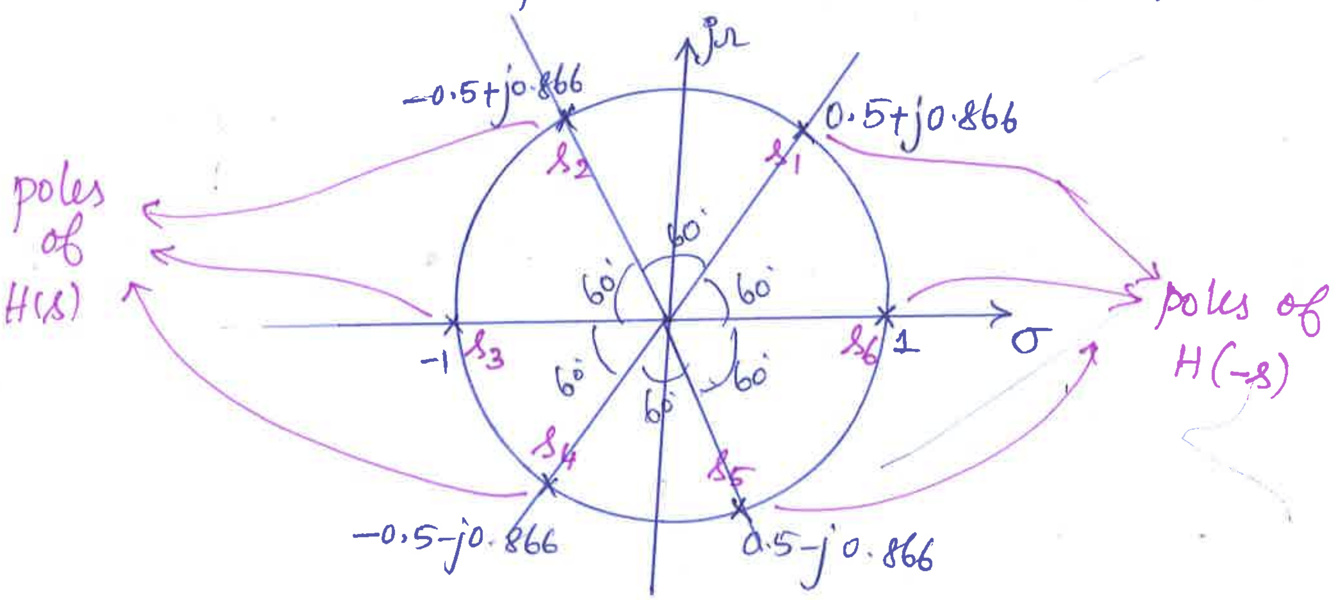
$\rightarrow$ poles of $|H(j\Omega)|^2$

$\Downarrow$

$= H(j\Omega)\cdot H(-j\Omega)$

$= H(s)\cdot H(-s)$

If all the poles are marked in s-plane



From the above fig, we can observe that,

All the poles of magnitude squared response of analog Butterworth filter lie on a unit-circle and they are separated by an angle $60°$ $\left(=\frac{360°}{2N}=\frac{360°}{6}\right)$

Here half of the poles lie on Right half of Splane that makes the filter unstable.

To <u>ensure stability</u>, let us consider only the poles that lie in left half of s-plane, hence we can write the denominator polynomial of the transfer function H(s) as,

$$(s+1)(s+0.5-j0.866)(s+0.5+j0.866)=0$$

$$(s+1)\{(s+0.5)^2+(0.866)^2\}=0$$

$$(s+1)(s^2+s+1)=0$$

∴ the transfer function of a 3rd order Butterworth filter for cut-off frequency $\Omega_c = 1$ rad/sec is

$$H(s) = \frac{1}{(s+1)(s^2+s+1)}$$

To find the poles that lie only in LH of s-plane,

$$s_k = e^{j\phi_k}$$

where $\phi_k = \frac{\pi}{2} + \frac{(2k-1)\pi}{2N}$, $k=1,2 \ldots N$

~~N=4~~

$$\boxed{s_k = e^{j\left(\pi/2 + \frac{(2k-1)\pi}{2N}\right)}}, \; k=1,2,\ldots N \longrightarrow (6)$$