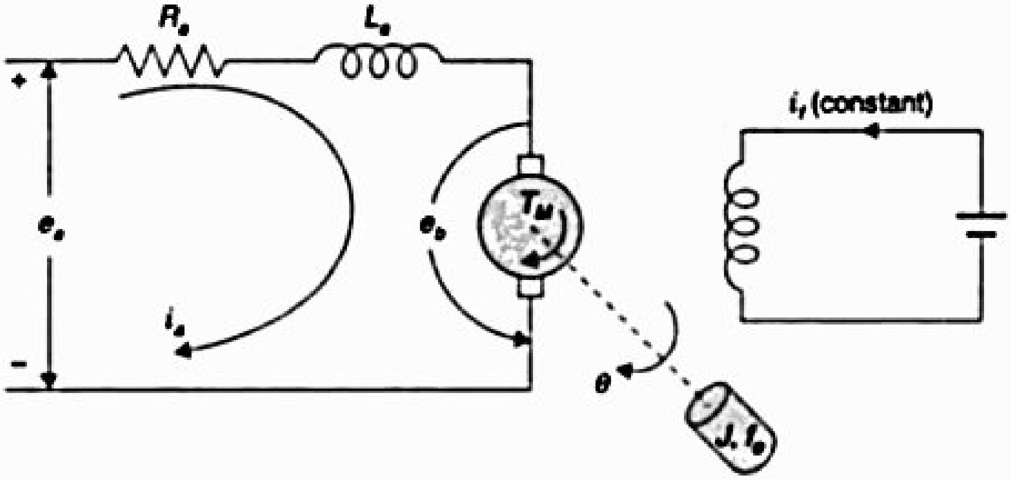

Fig. 2.21. Armature-controlled d.c. motor.

i_a = armature current (A).

i_f = field current (A).

e_a = applied armature voltage (V).

e_b = back emf (volts).

T_M = torque developed by motor (Nm).

θ = angular displacement of motor-shaft (rad).

J = equivalent moment of inertia of motor and load referred to motor shaft (kg-m²).

f_0 = equivalent viscous friction coefficient of motor and load referred to motor shaft $\left(\frac{\text{Nm}}{\text{rad/s}}\right)$.

In servo applications, the d.c. motors are generally used in the linear range of the magnetization curve. Therefore, the air gap flux ϕ is proportional of the field current, *i.e.*,

$$\phi = K_f i_f \quad ...(2.46)$$

where K_f is a constant.

The torque T_M developed by the motor is proportional to the product of the armature current and air gap flux, *i.e.*,

$$T_M = K_1 K_f i_f i_a \quad ...(2.47)$$

where K_1 is a constant.

In the armature-controlled d.c. motor, the field current is kept constant, so that eqn. (2.46) can be written as

$$T_M = K_T i_a \quad ...(2.48)$$

where K_T is known as the motor torque constant.

The motor back emf being proportional to speed is given as

$$e_b = K_b \frac{d\theta}{dt} \quad ...(2.49)$$

where K_b is the back emf constant.

The differential equation of the armature circuit is

$$L_a \frac{di_a}{dt} + R_a i_a + e_b = e_a \quad ...(2.50)$$

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The torque equation is

$$J\frac{d^2\theta}{dt^2} + f_0\frac{d\theta}{dt} = T_M = K_T i_a \qquad ...(2.51)$$

Taking the Laplace transforms of eqns. (2.48) to (2.50), assuming zero initial conditions, we get

$$E_b(s) = K_b s\theta(s) \qquad ...(2.52)$$

$$(L_a s + R_a)I_a(s) = E_a(s) - E_b(s) \qquad ...(2.53)$$

$$(Js^2 + f_0 s)\,\theta(s) = T_M(s) = K_T I_a(s) \qquad ...(2.54)$$

From eqns. (2.51) to (2.53), the transfer function of the system is obtained as

$$G(s) = \frac{\theta(s)}{E_a(s)} = \frac{K_T}{s[(R_a + sL_a)(Js + f_0) + K_T K_b]} \qquad ...(2.55)$$

The block diagram representation of eqn. (2.53) is shown in Fig. 2.22 (*a*) where the circular block representing the differencing action is known as the *summing point*. Equation (2.54) is represented by a block shown in Fig. 2.22 (*b*).

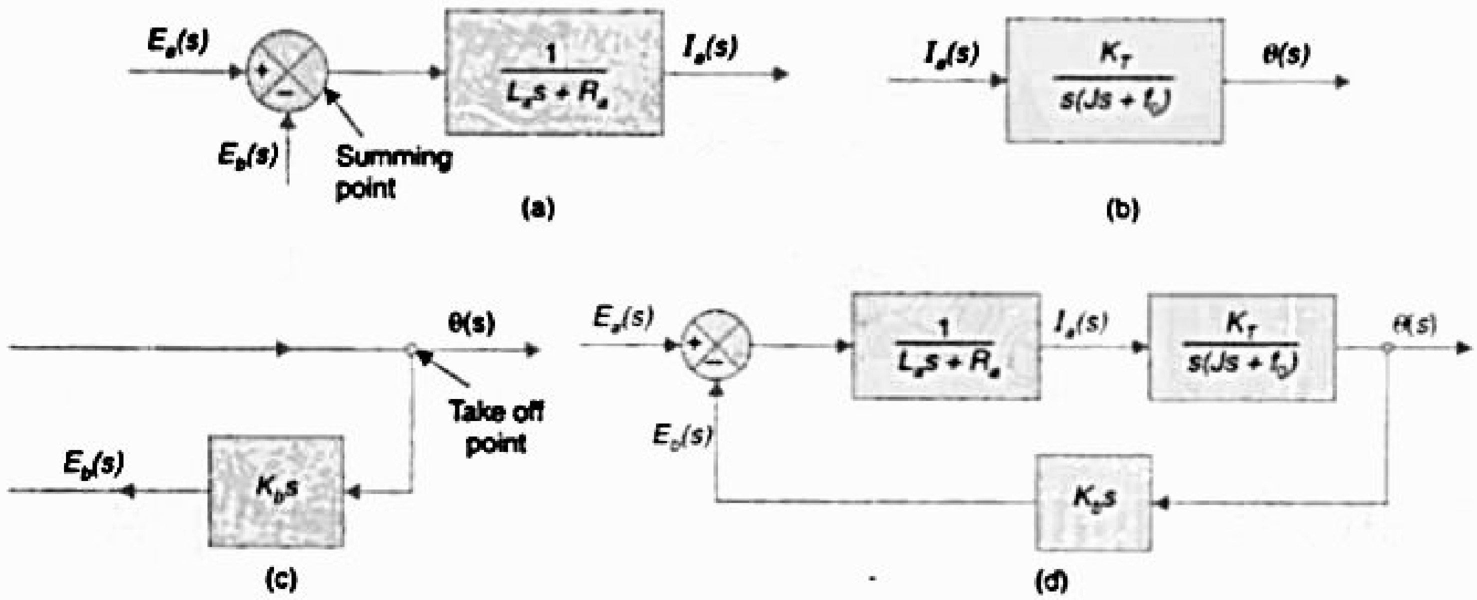

Fig. 2.22. Block diagram of armature-controlled d.c. motor.

Figure 2.22 (*c*) represents eqn. (2.52) where a signal is taken off from a *take-off* point and fed to the feedback block ($K_b s$). Fig. 2.22 (*d*) is the complete block diagram of the system under consideration, obtained by connecting the block diagram shown in Fig. 2.22 (*a*), (*b*) and (*c*). It may be pointed out here that when a signal is taken from the output of a block, this does not affect the output as per assumption 1 of the procedure for driving transfer functions advanced earlier.

However, it should be noted that the block diagram of the system under consideration can be directly obtained from the physical system of Fig. 2.21 by using the transfer functions of simple electrical and mechanical networks derived already. The voltage applied to the armature circuit is $E_a(s)$ which is opposed by the back emf ($E_b(s)$). The net voltage ($E_a - E_b$) acts on a linear circuit comprised of resistance and inductance in series, having the transfer function $1/(sL_a + R_a)$. The result is an armature current $I_a(s)$. For fixed field, the torque developed by the motor is $K_T I_a(s)$. This torque rotates the load at a speed $\dot{\theta}(s)$ against the moment of inertia J and viscous friction with coefficient f_0 [the transfer function is $1/(Js + f_0)$]. The back emf signal $E_b = K_b\, \dot{\theta}(s)$ is taken off from the shaft speed and fedback negatively to the summing point. The angle signal $\theta(s)$ is obtained by integrating (*i.e.*, $1/s$) the speed $\dot{\theta}(s)$. This results in the block diagram of Fig. 2.23, which is equivalent to that of Fig. 2.22 as can be seen by shifting the take off point from $\dot{\theta}(s)$ to $\theta(s)$.

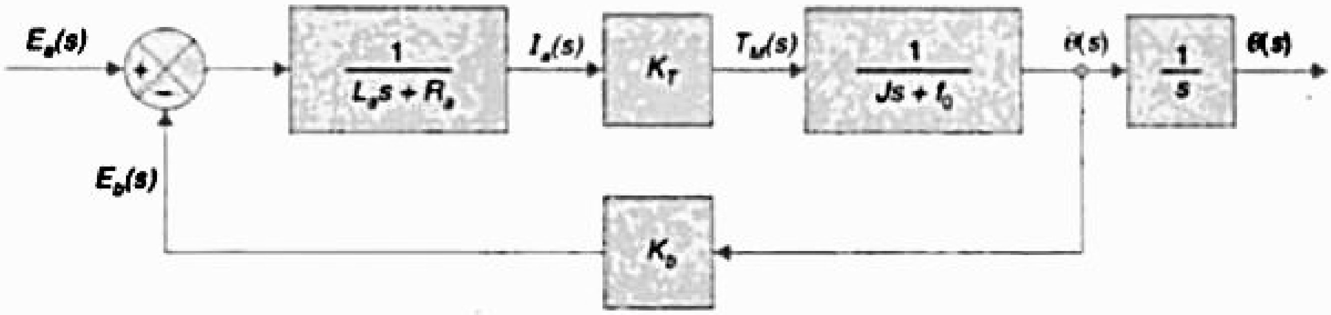

Fig. 2.23. Block diagram of armature-controlled d.c. motor.

The *armature circuit inductance* L_a *is usually negligible*. Therefore from eqn. (2.55), the transfer function of the armature controlled motor simplifies to

$$\frac{\theta(s)}{E_a(s)} = \frac{K_T/R_a}{Js^2 + s(f_0 + K_T K_b/R_a)} \qquad ...(2.56)$$

The term $(f_0 + K_T K_b/R_a)$ indicates that the back emf of the motor effectively increases the viscous friction of the system. Let

$$f = f_0 + K_T K_b/R_a$$

be the effective viscous friction coefficient. Then from eqn. (2.56)

$$\frac{\theta(s)}{E_a(s)} = \frac{K_T / R_a}{s(Js + f)} \qquad ...(2.57)$$

The transfer function given by eqn. (2.56) may be written in the form

$$\frac{\theta(s)}{E_a(s)} = \frac{K_m}{s(s\tau_m + 1)} \qquad ...(2.58)$$

where $K_m = K_T/R_a f$ = motor gain constant, and $\tau_m = J/f$ = motor time constant.

The motor torque and back emf constants K_T, K_b are interrelated. Their relationship is deduced below. In metric units, K_b is in V/rad/s and K_T is in Nm/A.

Electrical power converted to mechanical form $= e_b i_a = K_b \dot{\theta} i_a$ W

Power at shaft (in mechanical form) $= T\dot{\theta} = K_T i_a \dot{\theta}$ W

At steady speed these two powers balance. Hence

$$K_b \dot{\theta} i_a = K_T i_a \dot{\theta} \qquad \text{or} \qquad K_b = K_T \text{ (in MKS units)}$$

This result can be used to advantage in practice as K_b can be measured more easily and with greater accuracy than K_T.