Unit-II

Transmission lines & wave guides

Impedance matching and Transformation

Reflection phenomena – standing waves, $\lambda/8$, $\lambda/4$ and $\lambda/2$ lines – $\lambda/4$ Impedance transformers, stub matching – Single and Double stub – Smith chart and Applications – Numerical Examples.

Constants for the line of zero dissipation

For transmission of energy at high frequencies, where the power efficiency is high, it is possible to assume negligible losses or zero dissipation in the analysis of performance of tr-ion lines.

The line parameters for the line of zero dissipation are, $(R \,\&\, G = 0)$

$$Z = j\omega L \quad ; \quad Y = j\omega C$$

∴ The charac. imp Z_0 may be written as,

$$Z_0 = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{j\omega L}{j\omega C}} = \sqrt{\frac{L}{C}} \text{ ohms.} \quad \text{——} (1)$$

This value is wholly resistive & may be given by the symbol R_0,

$$Z_0 = R_0 = \sqrt{\frac{L}{C}} \quad \text{——} (2)$$

Voltages & currents on the dissipationless line

The voltage at any point distant s units from the receiving end of a tr-ion line is,

$$E = \frac{E_R(Z_R+Z_0)}{2Z_R}\left(e^{\gamma s} + K e^{-\gamma s}\right)$$

For the line of zero dissipation, attn $\alpha = 0$

$Z_0 = R_0$. So that,

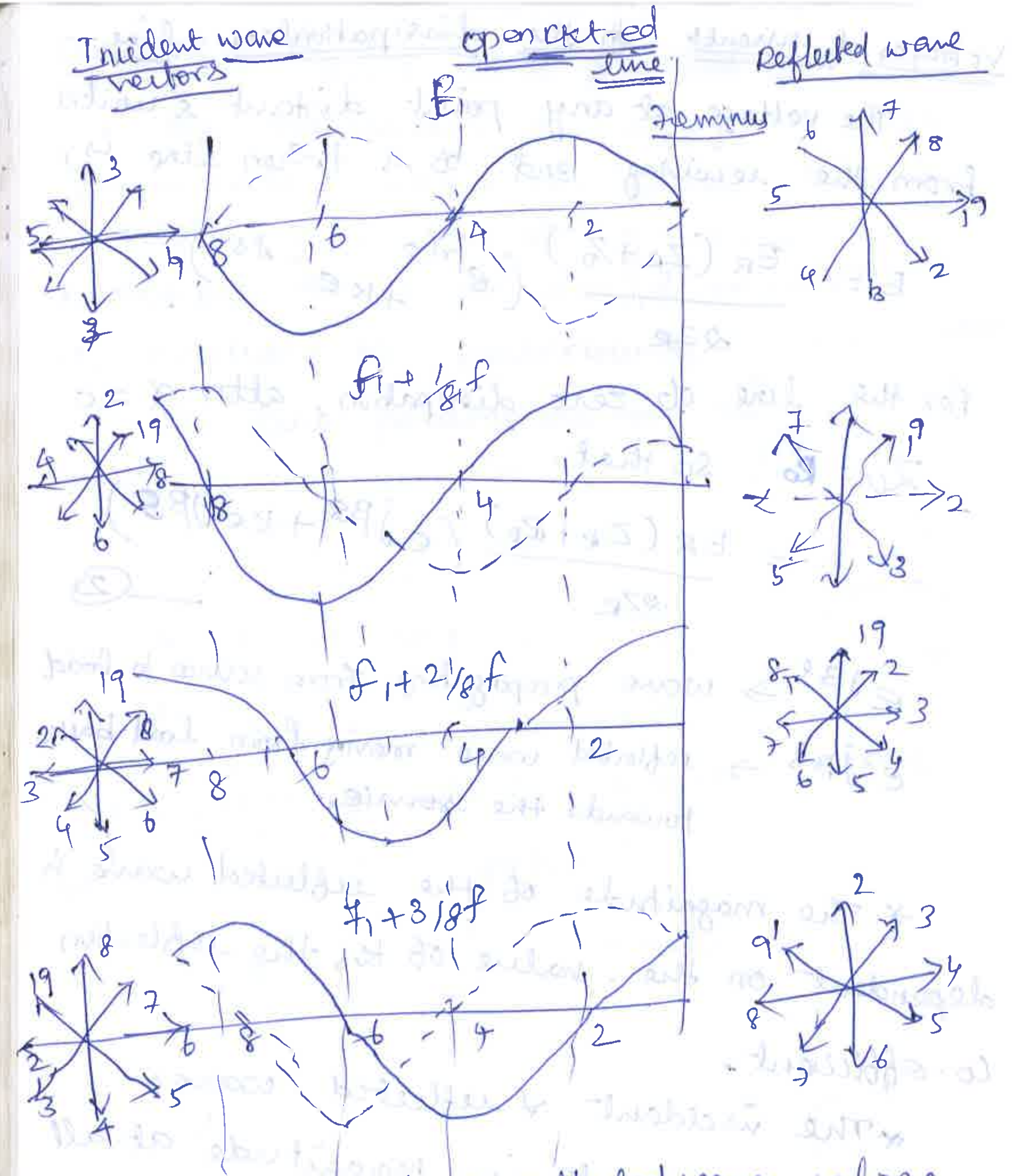
$$E = \frac{E_R(Z_R+R_0)}{2Z_R}\left(e^{j\beta s} + K e^{-j\beta s}\right) \quad \text{——} (2)$$

$e^{j\beta s}$ → wave propagating from source to load

$e^{-j\beta s}$ → reflected wave moving from load back towards the source.

* The magnitude of the reflected wave is dependent on the value of K, the reflection co-efficient.

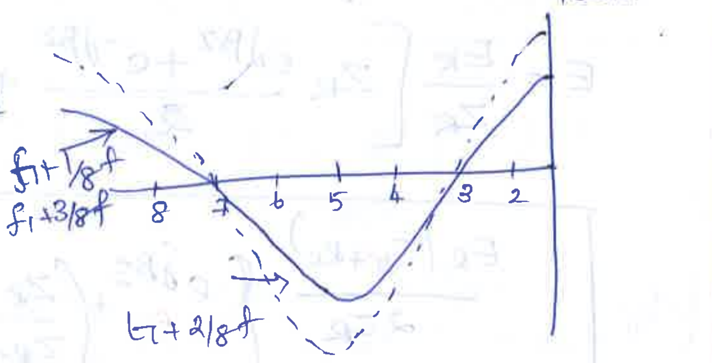
* The incident & reflected waves remaining constant in magnitude at all points on the line.



incident & reflected waves along dissipationless line for successive instants of time.

* The actual voltage at any point on the transmission line is the sum of the incident and reflected wave voltages at that point.

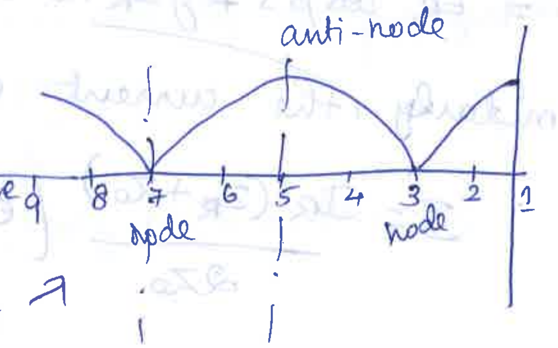
The sum voltage is plotted in fig → (open ckt-ed line)



* It can be seen that the resultant total voltage wave appears to stand still on the line, oscillating in magnitude with time but having fixed positions of maxima and minima.

* Such a wave is known as standing wave.

* If line voltages are measured with an effective voltmeter, the magnitude will appear like as in fig ↗



* Since the voltmeter does not distinguish b/w positive and negative values.

Equ ② may be reduced to,

$$E = \frac{E_R}{Z_R}\left[Z_R \frac{e^{j\beta z}+e^{-j\beta z}}{2} + jR_0 \frac{e^{j\beta z}-e^{-j\beta z}}{2j} \right]$$

$$= \frac{E_R(Z_R+R_0)}{2Z_R}\left\{ e^{j\beta s} + \left(\frac{Z_R-R_0}{Z_R+R_0}\right) e^{-j\beta s} \right\}$$

$$= \frac{E_R}{Z_R}\cdot\frac{1}{2}\left[Z_R e^{j\beta s} + R_0 e^{j\beta s} + Z_R e^{-j\beta s} - R_0 e^{-j\beta s} \right]$$

$$= \frac{E_R}{Z_R}\left[Z_R \left(\frac{e^{j\beta s}+e^{-j\beta s}}{2}\right) + jR_0 \left(\frac{e^{j\beta s}-e^{-j\beta s}}{2j}\right) \right]$$

$$E = E_R \cos\beta s + j I_R R_0 \sin\beta s \quad ③$$

Similarly, the current on the line is,

$$I = \frac{I_R(Z_R+Z_0)}{2Z_0}\left(e^{j\beta s} - k e^{-j\beta s}\right) \quad ④$$

The current at any point on a distortionless line is,

$$I = I_R \cos\beta s + j\frac{E_R}{R_0}\sin\beta s \quad ⑤$$

$$I = \frac{I_R(Z_R+R_0)}{2Z_R}\left[e^{j\beta s} - \left(\frac{Z_R-R_0}{Z_R+R_0}\right) e^{-j\beta s} \right]$$

$$= \frac{I_R}{2Z_R}\left[Z_R e^{j\beta s} + R_0 e^{j\beta s} - Z_R e^{-j\beta s} + R_0 e^{-j\beta s} \right]$$

$$= \frac{I_R}{Z_R}\left[Z_R \left(\frac{e^{j\beta s}-e^{-j\beta s}}{2}\right) + R_0 \left(\frac{e^{j\beta s}+e^{-j\beta s}}{2}\right) \right]$$

$$= I_R \cos\beta s + j\frac{E_R}{R_0}\sin\beta s$$

$$j\frac{I_R}{Z_R}R_0 = j\frac{E_R}{Z_R R_0}R_0 = j\frac{E_R}{R_0}$$

From the defn. of velocity of propagation,

$$\beta = \frac{2\pi f}{v} = \frac{2\pi}{\lambda}$$

The ct. & voltage may be written as,

$$E = E_R \cos\frac{2\pi s}{\lambda} + j I_R R_0 \sin\frac{2\pi s}{\lambda} \quad \text{—— (6)}$$

$$I = I_R \cos\frac{2\pi s}{\lambda} + j \frac{E_R}{R_0} \sin\frac{2\pi s}{\lambda} \quad \text{—— (7)}$$

The ct. & voltage distribution is seen as the sum of cosine & quadrature sine distributions.

* If the line is OC-ed, $I_R = 0$

$$E_{OC} = E_R \cos\frac{2\pi s}{\lambda} \quad \text{—— (8)}$$

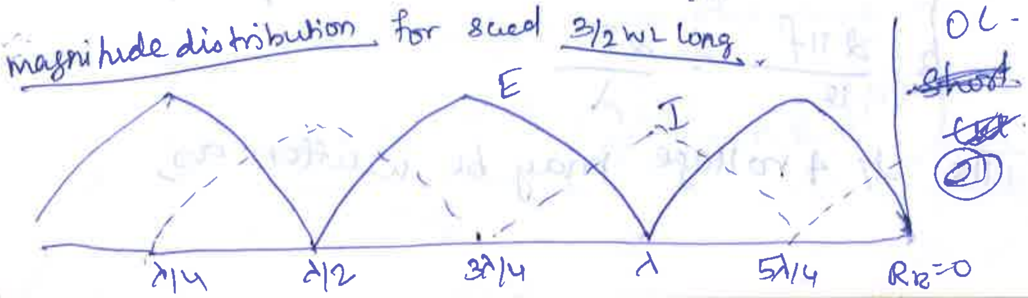
$$I_{OC} = \frac{jE_R}{R_0} \sin\frac{2\pi s}{\lambda} \quad \text{—— (9)}$$

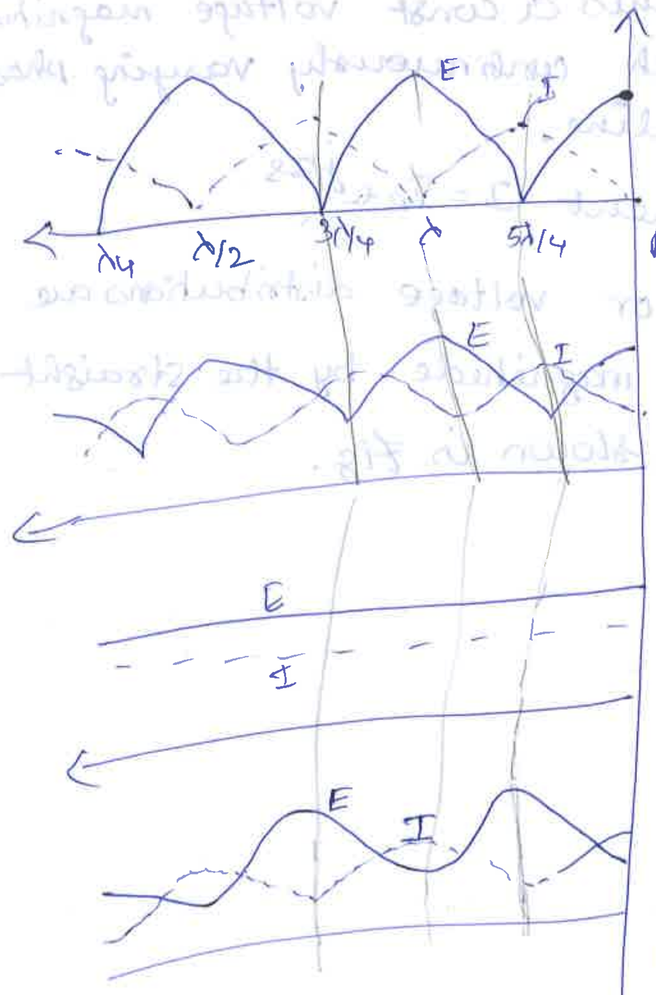
If the line is short circuited, then $E_R = 0$

equ (6) becomes, $E_{SC} = j I_R R_0 \sin\frac{2\pi s}{\lambda}$ —— (10)

(7) ⇒ $I_{SC} = I_R \cos\frac{2\pi s}{\lambda}$ —— (11)

magnitude distribution for seud 3/2 WL long.





(a) $R_R = \infty$

$R_R > R_0$, K = +ve

$K = \frac{1}{2}$

(b) $R_R = 3R_0$. incident amp twice that of the reflected wave. zeros of (a) replaced with minima.

(c) $R_R = R_0$

$R_R < R_0$, K = -ve

$K = -\frac{1}{2}$

(d) The phase of the reflected wave is reversed. $R_R = R_0/3$

Power being tr-ed.

If the line is terminated in $Z_R = Z_0 = R_0$ the reflection co-efficient and reflected wave become zero, & the voltage on the line is found → $E = E_R e^{j\beta s}$ ←—— (12)

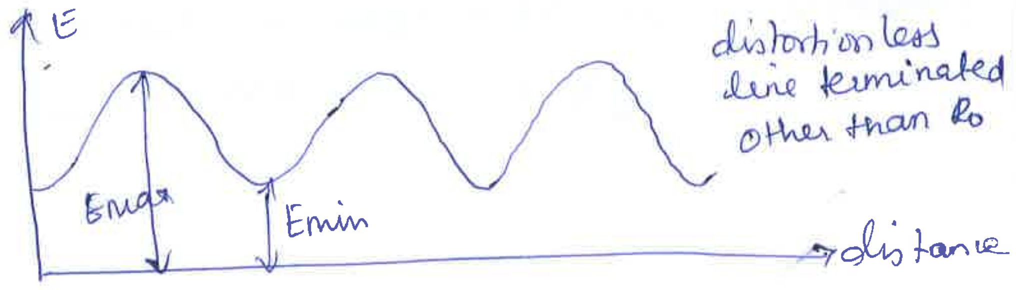
which represents a const voltage magnitude (no attn.) which continuously varying phase angle along the line.

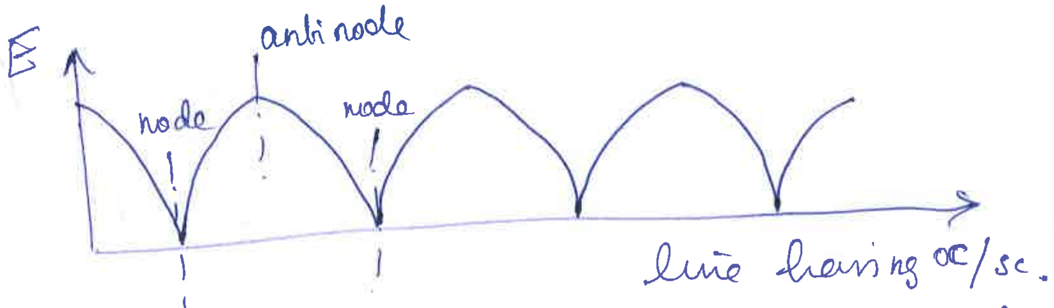
IIly for current $I = I_R e^{j\beta s}$.

Such ct.- or voltage distributions are represented in magnitude by the straight horizontal line shown in fig.

Standing waves, nodes, standing wave Ratio

The voltage magnitudes are measured along the length of a line terminated in a load other than R_0





* The maxima and minima values on a line are labeled as,

1) Nodes :- are pts of zero voltage or current in the standing wave system.
2) Anti nodes :- are pts of max. voltage or current.

A line terminated in R_0 has no standing waves, & called smooth line.

For open ckt

Voltage nodes occurs at :- $\lambda/4, 3\lambda/4, 5\lambda/4 \ldots$

ct. nodes occurs at :- $0, \lambda/2, \lambda, 3\lambda/2 \ldots$

For short ckt

Voltage nodes — $0, \lambda/2, \lambda \ldots$

current nodes — $\lambda/4, 3\lambda/4, 5\lambda/4 \ldots$

Standing wave Ratio $= S = \left|\frac{E_{max}}{E_{min}}\right| = \left|\frac{I_{max}}{I_{min}}\right|$

From the voltage eqn

$$E = \frac{E_R(Z_R+Z_0)}{2Z_R}\left(e^{j\beta s} + k e^{-j\beta s}\right) \quad \text{—(1)}$$

At the pts where the incident & reflected waves are in phase,

$$E_{max} = \frac{E_R(Z_R+Z_0)}{2Z_R}\left(1+|k|\right) \quad \text{—(2)}$$

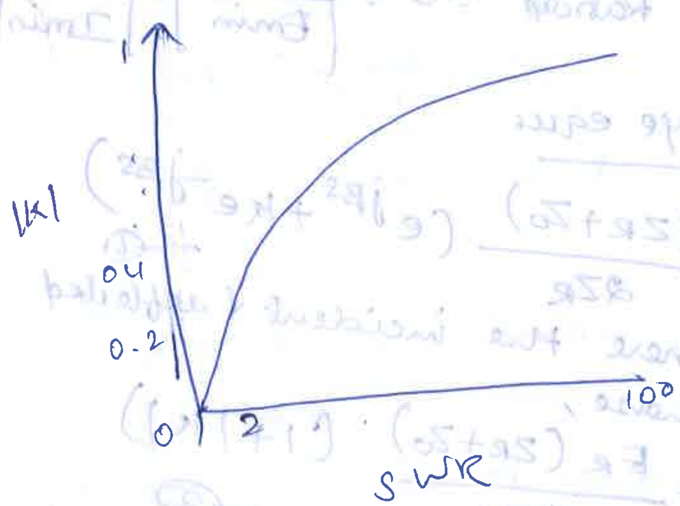
$||^{ly}$, the voltage minima occurs at pts at which the incident & reflected waves are out of phase

$$E_{min} = \frac{E_R(Z_R+Z_0)}{2Z_R}(1-|k|) \quad \text{—(3)}$$

∴ SWR in terms of ref. co-efficient :

$$S = \frac{1+|k|}{1-|k|} \quad \text{—(4)}$$

$$|k| = \frac{S-1}{S+1} = \frac{|E_{max}| - |E_{min}|}{|E_{max}| + |E_{min}|} \quad \text{—(5)}$$



I/P Impedance of the dissipationless line

The i/p imp of a dissipationless line is,

$$E = E_R \cos\beta s + j I_R R_0 \sin\beta s$$

$$I = I_R \cos\beta s + j \frac{E_R}{R_0} \sin\beta s$$

$$Z_s = \frac{E_s}{I_s} = \frac{E_R \cos\beta s + j I_R R_0 \sin\beta s}{I_R \cos\beta s + j \frac{E_R}{R_0} \sin\beta s}$$

$E_R = I_R Z_R$, Therefore, ÷ by $I_R \cos\beta s$

$$Z_s = \frac{Z_R + j R_0 \tan\beta s}{1 + j \frac{Z_R}{R_0} \tan\beta s}$$

$$\boxed{Z_s = R_0 \left[\frac{Z_R + j R_0 \tan\beta s}{R_0 + j Z_R \tan\beta s}\right]} \quad \text{—(1)}$$

~~From this~~ equ, imp is,

- Imp. is complex in general
- periodic with variation of βs, the period being π or $s = \lambda/2$.

Another form of i/p imp is

$$E = \frac{E_R(Z_R+R_0)}{2Z_R}\left(e^{j\beta s} + K e^{-j\beta s}\right)$$

$$I = \frac{I_R (Z_R + Z_0)}{2 Z_0} \left(e^{j\beta s} - k e^{-j\beta s} \right)$$

$$Z_s = \frac{E_s}{I_s} = \frac{I_R \left(\frac{Z_R + R_0}{2} \right) \left(e^{j\beta s} + k e^{-j\beta s} \right)}{\frac{I_R (Z_R + R_0)}{2 R_0} \left(e^{j\beta s} - k e^{-j\beta s} \right)}$$

$$\boxed{Z_s = R_0 \left(\frac{e^{j\beta s} + k e^{-j\beta s}}{e^{j\beta s} - k e^{-j\beta s}} \right)}$$

$re^{j\theta} = |r| \angle \theta$

$k = |k| \angle \phi$

$$= R_0 \left(\frac{1 \angle \beta s + |k| \angle \phi - \beta s}{1 \angle \beta s - |k| \angle \phi - \beta s} \right) \quad \text{— (2)}$$

$\phi \rightarrow$ angle of reflection co-efficient k.

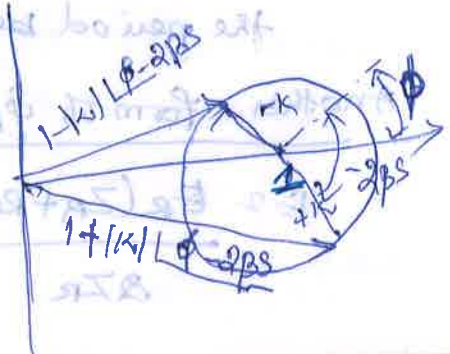
$\div$ by $1 \angle \beta s$,

$$\therefore Z_s = R_0 \left(\frac{1 + |k| \angle \phi - 2\beta s}{1 - |k| \angle \phi - 2\beta s} \right) \quad \text{— (3)}$$

* The numerator & denominator show by separate phasors.

* Adding Unity to + or − k

for $|k| = 0.5$ or $S = 3 \leftarrow \frac{1+0.5}{1-0.5}$



* At $S = \phi/2\beta + n\lambda/4$ the numerator & denominator terms are in-phase ($n = 0, 1, 2, \ldots$). At these pts, the i/p of the line is purely resistive.

* The angle of k vector in terms of $2\beta s$, half a revolution represents in terms of wavelength

The value of max i/p imp (resistive), occurring at $S = \phi/2\beta + n\lambda/2$;

$$R_{max} = R_0 \left(\frac{1 + |k|}{1 - |k|} \right)$$

$$= \underline{2 R_0} \quad \text{— (4)}$$

The min. i/p imp. also resistive, occurring at $S = \phi/2\beta + (2n-1)\lambda/4$,

$$R_{min} = R_0 \left(\frac{1 - |k|}{1 + |k|} \right) = \frac{R_0}{S} \quad \text{— (5)}$$

I/p imp. of open & short circuited line

The i/p imp. of a dissipationless line is;

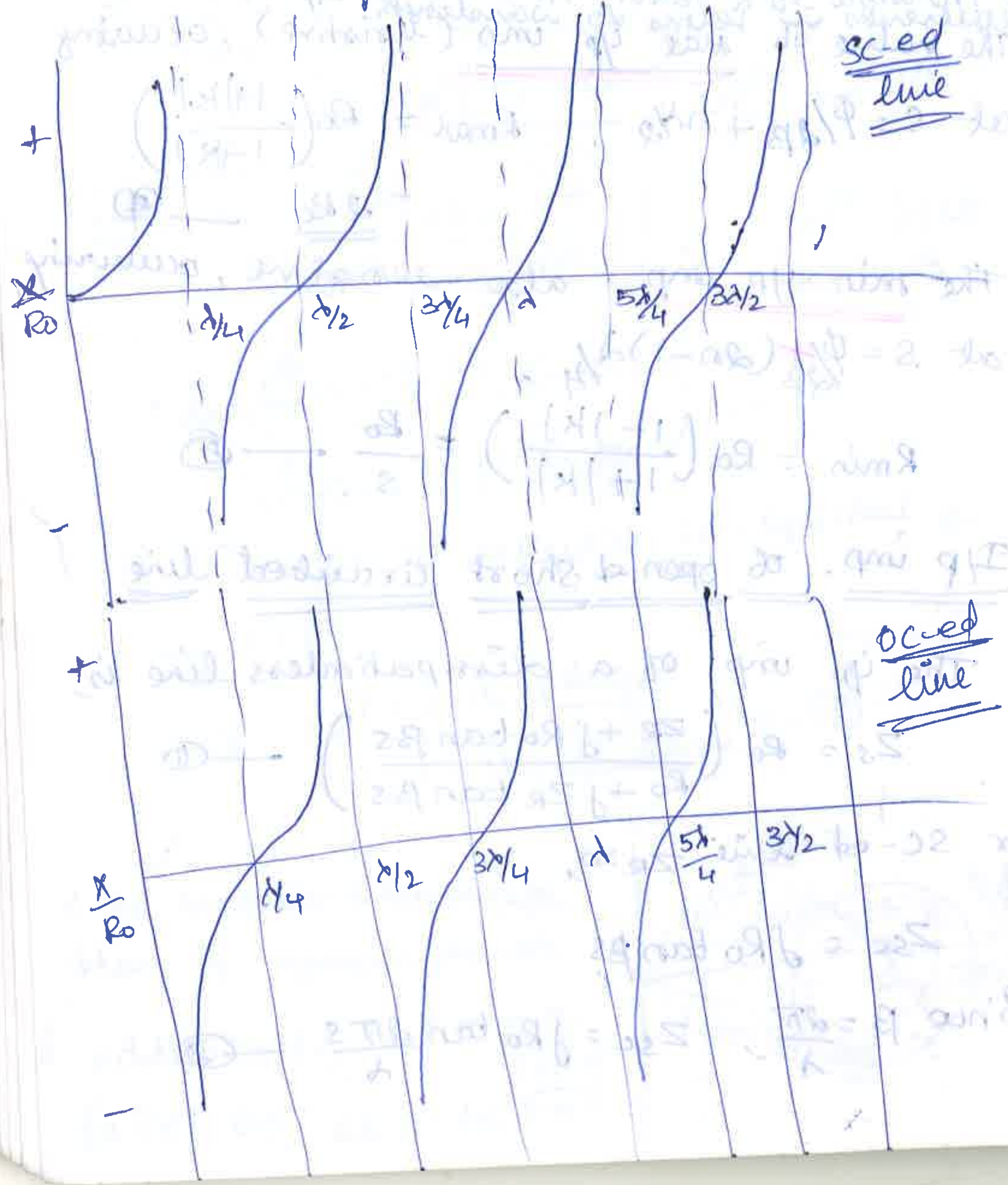
$$Z_s = R_0 \left(\frac{Z_R + j R_0 \tan \beta s}{R_0 + j Z_R \tan \beta s} \right) \quad \text{— (1)}$$

For SC-ed line $Z_R = 0$,

$$Z_{sc} = j R_0 \tan \beta s$$

Since $\beta = \frac{2\pi}{\lambda}$, $\quad Z_{sc} = j R_0 \tan \frac{2\pi s}{\lambda} \quad \text{— (2)}$

The variation $\frac{Z_{sc}}{R_0} = \frac{X}{R_0}$ with length of line s is plotted.



I/p imp for OC-ed line

From eqn ①,

$$Z_s = R_0 \left(\frac{1 + j\frac{R_0}{Z_R}\tan\beta s}{\frac{R_0}{Z_R} + j\tan\beta s} \right)$$

For oc-ed line $Z_R = \infty$, so that, $\frac{R_0}{j\tan\beta s}$

$$Z_{oc} = \frac{-jR_0}{\tan\beta s} = -jR_0 \cot\frac{2\pi s}{\lambda} \quad \text{③}$$

plot of $\frac{Z_{oc}}{R_0} = \frac{X}{R_0}$ as a fn. of length of line is shown in fig.

* for both cases i/p imp is pure reactance
* for I^{st} $\lambda/4$ WL,
  * SC line act as an Inductance
  * OC line act as a capacitance.
- These reactances reverse each $\lambda/4$ WL.
* i/p imp of $\lambda/4$ sc-ed → appear as infinite reactance
* " " " " oc-ed → " " zero "

In practical cases,

- always be a small resistance component of the i/p imp. → indicates power loss.
- Zero or ∞ infinite imp.'s are never achieved.

The Quarter wave line – Imp Matching

The expression for the i/p imp. of a dissipationless line may be rearranged as.

$$Z_s = R_0 \left[\frac{Z_R + jR_0 \tan \beta s}{R_0 + jZ_R \tan \beta s}\right]$$

$$= R_0 \left[\frac{\dfrac{Z_R}{\tan\left(\frac{2\pi}{\lambda} s\right)} + jR_0}{\dfrac{R_0}{\tan\left(\frac{2\pi s}{\lambda}\right)} + jZ_R}\right]$$

if the line is made a quarter wave long

$S = \lambda/4$

$$Z_s = \frac{R_0^2}{Z_R}$$

ie the i/p imp. of the line is equal to the square of R_0 of the line divided by the load impedance.

A quarter wave section of line may be thought of as a tsfr. to match a load imp Z_R Ω to a source of Z_s Ω's.

To obtain this, the charac. imp R_0' of the Quarter wave section of line is properly choosen according to,

$$R_0' = |\sqrt{Z_s Z_R}|$$

R_0' → Matching section. ie equal to geometric mean of the source & load imp's.

① $\lambda/4$ line – Impedance Inverter

- It can tsfom low imp. into a high imp & vice versa.

* $\lambda/4$ sc-ed line trfr-ing the zero imp SC termination to an apparent oc, & of the oc-ed $\lambda/4$ line is trfr-ing the ~~the~~ oc termination to a low value or an apparent SC.

Appln:- ① $\lambda/4$ matching section is to couple a tr-ion line to a resistive load such as an antenna.

The $\lambda/4$ line ~~line~~ matching section then must be designed to a charac. imp R_0', chosen that the ant. resistance R_A is trfr-ed to a value equal to the charac. imp R_0 of the transmission line.

$$R_0' = \sqrt{R_A R_0}$$

$R_0' \rightarrow$ just the value required to achieve critical coupling and max. power trfr from the tr. ion line to the load.

Trfr. is is a single freq or NB device

The Band width may be incr'd by using 2 or more $\lambda/4$ sections in series.

* Also used if the load is not pure resistance.

*It should be connected b/w pt's corresponding to Imax or Emax.

* At which places the tr-ion line has resistive imp. given by $\frac{R_0}{S}$ or SR_0

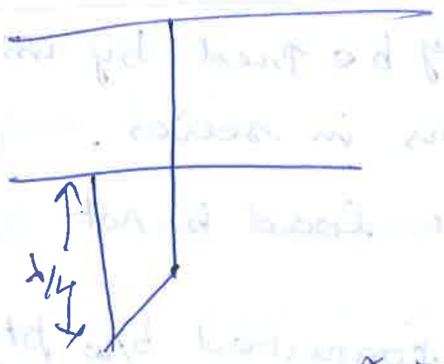
∴ Matching trfr. charac. imp is,

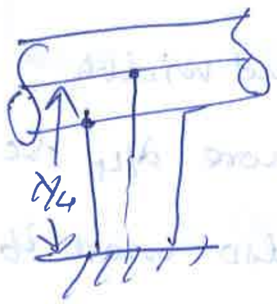
$$R_0' = \sqrt{R_0 \cdot \frac{R_0}{S}} = R_0 \sqrt{\frac{1}{S}}$$

② Appln: 2: sc-ed $\lambda/4$ line → Insulator

* To support open wire line or the center conductor of a co-axial line.

* The i/p imp of a $\lambda/4$ shorted line is very high → called copper insulator.





$\lambda/8$ wave line

i/p imp of a line, when $s = \lambda/8$

$$Z_S = R_0 \left[\frac{Z_R + jR_0 \tan(\pi/4)}{R_0 + jZ_R \tan(\pi/4)} \right] \quad (1)$$

line is terminated in a pure resis R_R

$$Z_S = R_0 \left(\frac{R_R + jR_0}{R_0 + jR_R} \right) \quad (2)$$

Numerator & deno. have identical magnitudes

$$|Z_S| = R_0$$

$\lambda/8$ line used to tsfrom any resistance to an imp with mag. = to R_0 of the line.

The $\lambda/2$ line – Half wave line

when the length of the line having $S = \lambda/2$ is, the i/p imp. is,

$$Z_S = R_0 \left(\frac{Z_R + jR_0 \tan\pi}{R_0 + jZ_R \tan\pi} \right)$$

$$Z_S = Z_R$$

* $\lambda/2$ line → considered as 1 to 1 tsfrmr.

* used to connect a load to source, where the load & source can't be made adjacent.

* A group of capacitors may be placed in $\|^{l}$ by connecting ~~them~~ with sections of line n - half waves in length.

∴ Insulators on a HF line should not be spaced at $\lambda/2$ intervals, since their effect would be cumulative, lowering the insulation resistance of the line.

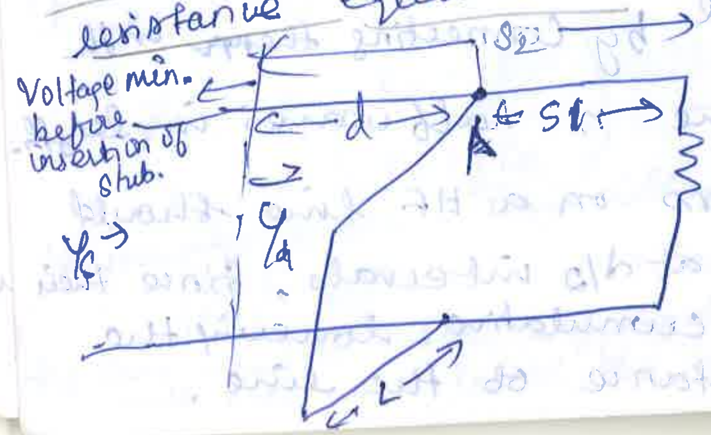
Single stub impedance Matching

For greatest efficiency & delivered max. power, a HF tr-lns line should be operated as a smooth line or with an R_0 termination (charac imp). L, T, π matching N/ws.

– Theory –

One way is, the use of an open or closed stub line of suitable length as a reactance shunted across the tr-lns line at a designated distance from the load.

* The length of line & the load to resonance with an anti resonant resistance equal to R_0.



* The i/p conductance of a line is $\frac{1}{SR_0}$ at voltage max.

* " " " " " S/R_0 at a voltage min.

S – SWR

∴ At some intermediate pt. A, the real part of the i/p admittance may have an intermediate value $1/R_0$.

The i/p admittance $Y_s = \frac{1}{R_0} \pm jB$.

$1/R_0 \rightarrow$ conductance $B \rightarrow$ susceptance.

* After placing the conductance $1/R_0$, a short stub line having i/p susceptance $\mp B$ may be connected across the tr-lns line.

$$Y_s = \frac{1}{R_0} \pm jB \mp jB = \frac{1}{R_0}.$$

The i/p imp of the tr-lns line at pt. A is $Z_s = R_0$

* The line from the source to A is then terminated in R_o, & is a smooth line.
* From A to load, reflection & standing waves occur.
* But the distance is always < than a wavelength, the losses are not severe.
* The min. nearest to the load.
* Voltage min is chosen rather than V_{max}.
* The location of the stub is fixed, w.r.to an V_{min}, no knowledge of the load imp. is needed.

From i/p imp. of a tr-con line

From ③

$$Z_s = R_o \frac{1+|k| \angle \phi - 2\beta s}{1-|k| \angle \phi - 2\beta s} \quad —①$$

$$Y_s' = \frac{1}{R_o}\left(\frac{1-|k| \angle \phi - 2\beta s}{1+|k| \angle \phi - 2\beta s}\right) \quad —②$$

$G_o = 1/R_o$; changing to Rectangular co-ordinates gives

$$Y_s = G_o\left[\frac{1-|k|\cos(\phi-2\beta s) - j|k|\sin(\phi-2\beta s)}{1+|k|\cos(\phi-2\beta s)+j|k|\sin(\phi-2\beta s)}\right]$$

$$\textcircled{*} \; Y_s = G_o\left\{\frac{1+|k|\cos\phi - jk\sin\phi}{1+k\cos\phi + jk\sin\phi} \times \frac{1+k\cos\phi - jk\sin\phi}{1+k\cos\phi - jk\sin\phi}\right.$$

$$Y_s = G_o\left[\frac{1-|k|^2 - 2j|k|\sin(\phi-2\beta s)}{1+|k|^2+2|k|\cos(\phi-2\beta s)}\right]$$

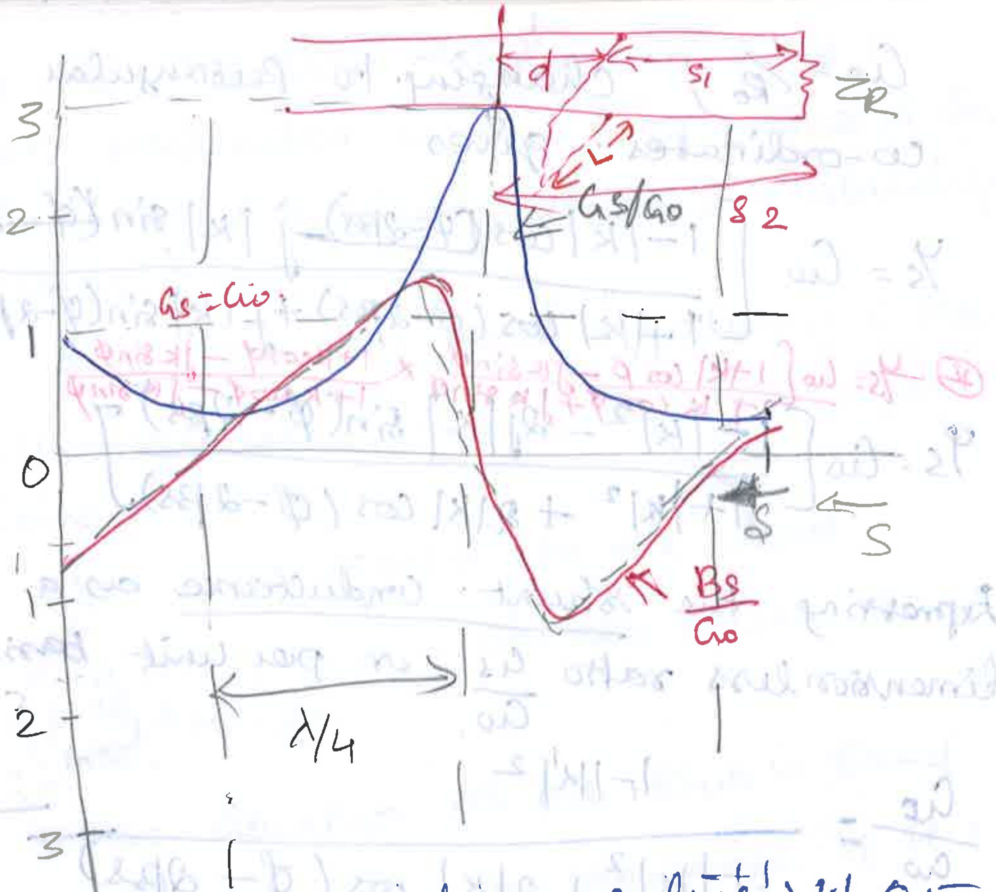
Expressing the shunt conductance as a dimensionless ratio $\frac{G_s}{G_o}$ or per unit basis $Y_s = G_s \pm jB_s$

$$\frac{G_s}{G_o} = \frac{1-|k|^2}{1+|k|^2+2|k|\cos(\phi-2\beta s)} \quad —③$$

Shunt susceptance on a per unit basis is

$$\frac{B_s}{G_o} = \left[\frac{(-1)\,-2|k|\sin(\phi-2\beta s)}{1+|k|^2+2|k|\cos(\phi-2\beta s)}\right] \quad —④$$

$$\Rightarrow G_o\left\{\frac{1+k\cos\phi - jk\sin\phi - k\cos\phi - k^2\cos^2\phi + jk^2\cos\phi\sin\phi - jk\sin\phi - jk^2\cos\phi\sin\phi - k^2\sin^2\phi}{1+k\cos\phi - jk\sin\phi + k^2\cos^2\phi + k\cos\phi - jk^2\sin\phi\cos\phi + jk\sin\phi + jk^2\sin\phi\cos\phi + k^2\sin^2\phi}\right\}$$



Admittance Condns on a line $|k| : 0.5$

from (4), $\phi - 2\beta s_2 = -\pi$

$$s_2 = \frac{\phi + \pi}{2\beta} \quad \text{—— (5)} \qquad \cancel{\frac{(1+k)(1-k)}{(1-k)^2}}$$

At this distance from the load,

$$(3) \rightarrow \frac{G_s}{G_o} = \frac{1 - |k|^2}{1 + |k|^2 - 2k} = \frac{1+|K|}{1-|K|} = S \quad \text{—— (6)}$$

$R_s = \frac{R_o}{S}$, The pt of max. $\frac{G_s}{G_o}$ is recognized as a pt of min. voltage at a distance s_2 from the load. at $G_s = G_o$, at which the stub is to be connected, the value of $\frac{G_s}{G_o} = 1$, ∴

$$1 = \frac{1 - |k|^2}{1 + |k|^2 + 2k\cos(\phi - 2\beta s_1)}.$$

from (6)
$\cos(\phi - 2\beta s_1) = -|K|$, $\cos^{-1}(-K) = -\pi + \cos^{-1}|k|$

$$s_1 = \frac{\phi + \pi - \cos^{-1}|k|}{2\beta} \quad \text{—— (7)}$$

Hence the distance d from the Vmin to pt of stub is,

$d = s_2 - s_1$, from (5) & (7), $d = \frac{\cos^{-1}|k|}{2\beta}$.

$$d = \frac{\cos^{-1}\left(\frac{S-1}{S+1}\right)\lambda/4}{\pi} \quad \text{—— (8)} \qquad \boxed{2\beta = \frac{2 \times 2\pi}{\lambda} = \frac{4\pi}{\lambda}}$$

from (4) & (7), $B_s = G_o\left[\frac{-2|k|\sin(\pi + \cos^{-1}|k|)}{1 + |k|^2 + 2|k|\cos(\pi + \cos^{-1}|k|)}\right]$

for an angle whose cosine is $|k|$, sine is $\sqrt{1 - |k|^2}$

$$\therefore B_s = G_o\left(\frac{2|k|\sqrt{1 - |k|^2}}{1 - |k|^2}\right) \quad \text{—— (9)}$$

The susceptance of the stub required to cancel the line susceptance, must be −ve of Bs ⇒

The circle diagram for the dissipationless line

– Solves the imp. equis

The i/p imp. of a dissipationless line may be written on per unit basis, is,

$$\frac{Z_s}{R_o} = \frac{1+|k|\angle\phi-2\beta s}{1-|k|\angle\phi-2\beta s} \quad (1)$$

↳ normalized source impedance applicable to all lines, Z/R_o is complex, may be write,

$$\frac{Z_s}{R_o} = r_a + jx_a \quad (2)$$

x_a ↳ reactance, r_a ↳ resistance

$$r_a + jx_a = \frac{1+|k|\angle\phi-2\beta s}{1-|k|\angle\phi-2\beta s} \quad (3)$$

∴ line quality is ~~most~~ ~~easily~~ measured. It is convenient to replace SWR S. $|k|$ in terms of S,

$$\boxed{k = \frac{s-1}{s+1}}$$

$$(r_a+1+jx_a)\left(\frac{S-1}{S+1}\right)\angle\phi-2\beta s = r_a - 1 + jx_a \quad (4)$$

$$\boxed{\begin{aligned} &r_a - r_a|k|\angle\phi-2\beta s + jx_a - jx_a|k|\angle\phi-2\beta s - 1 - |k|\angle\phi-2\beta s = 0 \\ &r_a - 1 + jx_a = r_a|k|\angle\phi-2\beta s + jx_a|k|\angle\phi-2\beta s + |k|\angle\phi-2\beta s \\ &r_a - 1 + jx_a = (r_a + 1 + jx_a)|k|\angle\phi-2\beta s \quad (5) \end{aligned}}$$

Equating the squares of the magnitudes of after clearing of fractions,

$$r_a^2 - r_a\left(\frac{S^2+1}{S}\right) + x_a^2 = -1$$

$$\frac{G_s}{G_o} = 1, \quad 1 = \frac{1-|k|^2}{1+|k|^2+2|k|\cos(\phi-2\beta s_1)}$$

$$\cos(\phi-2\beta s_1) = -|k| \rightarrow (1-|k|^2)$$

(F) $\cos^{-1}(-|k|) = -\pi + \cos^{-1}|k|$

$$s_1 = \frac{\phi+\pi-\cos^{-1}|k|}{2\beta}$$

By adding the term to complete the square,

$$r_a^2 - \frac{2r_a(s^2+1)}{2s} + \left(\frac{s^2+1}{2s}\right)^2 + x_a^2 = \left(\frac{s^2+1}{2s}\right)^2 - 1$$

$$\left[r_a - \left(\frac{s^2+1}{2s}\right)\right]^2 + x_a^2 = \left(\frac{s^2-1}{2s}\right)^2 \quad \text{— (5)}$$

ie (5) is in the form of,

$$(x-c)^2+y^2=r^2$$

r – radius c – center

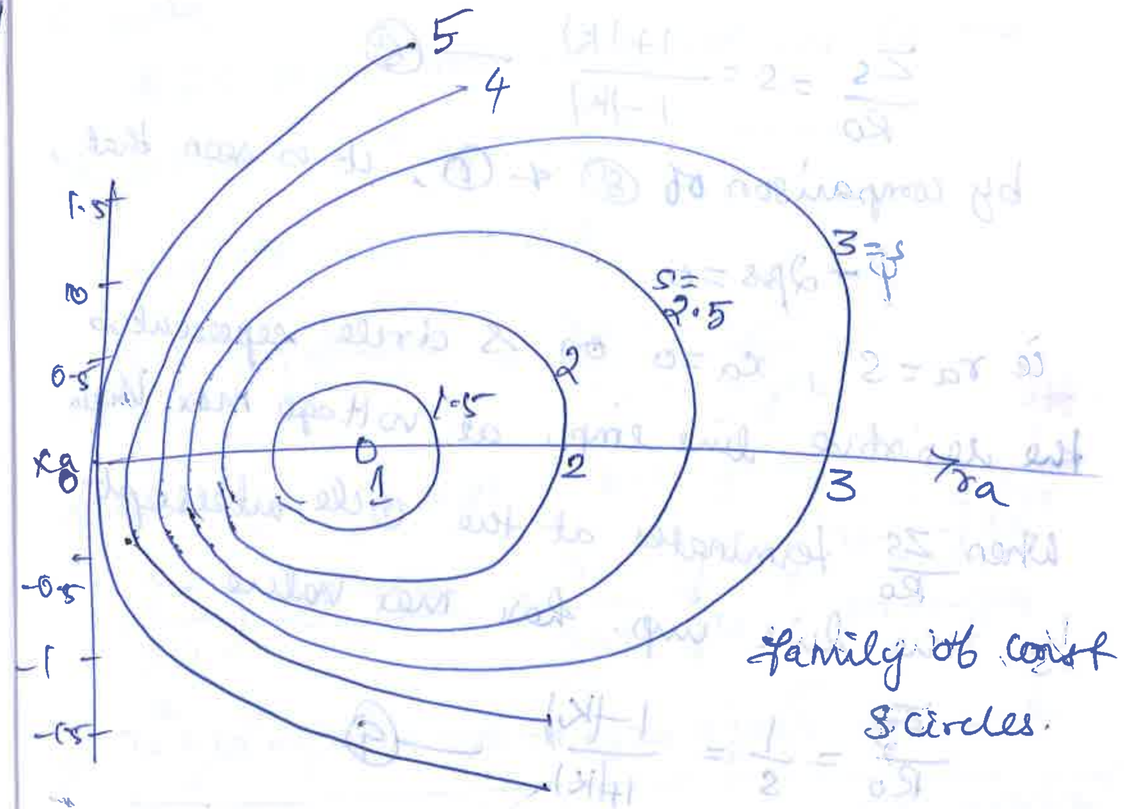
An actual circle will have a radius,

$$r = \frac{s^2-1}{2s} = \frac{s-1/s}{2} \quad \text{— (6)}$$

a shift of the center of the circle on the r_a axis is (abscissa)

$$c = \frac{s^2+1}{2s} = \frac{s+1/s}{2} \quad \text{— (7)}$$

The family of circles may be drawn for successive values of S



family of const S circles.

The minimum value for s is unity.

max value for s is infinity

S ↑ses → radius is ↑ses

→ centers moves to the right

S → ∞ the circle becomes the x_a axis.

when $\frac{Z_s}{R_o}$ lies on the abscissa, with magnitude S, the line impedance has max value,

$$\frac{Z_s}{R_o} = S = \frac{1+|K|}{1-|K|} \quad \text{— (8)}$$

by comparison of (8) & (1), it is seen that,

$$\phi - 2\beta s = 0$$

ie $r_a = S$, $x_a = 0$ on S circle represents the resistive line imp. at voltage max. V_{max}

When $\frac{Z_s}{R_o}$ terminates at the circle intercept $1/S$, the line imp. has max value.

$$\frac{Z_s}{R_o} = \frac{1}{S} = \frac{1-|K|}{1+|K|} \quad \text{— (9)}$$

ie for resistive load $\phi = 0$.

$$\beta s = \pi/2$$

Thus moving through $\beta s = \pi/2$ radians back along the line has caused the tip of the imp vector to travel over a distance on the circle of $-\pi$ radians. Hence it is seen advisable to place a βs scale on the S circle.

* βs scale increases clockwise, or in the direction of increasing negative angles.

rewrite the eqn (1),

$$\left(\frac{S-1}{S+1}\right) \angle \phi - 2\beta s = \frac{r_a - 1 + jx_a}{r_a + 1 + jx_a}$$

rationalizing the right side gives,

$$\left(\frac{S-1}{S+1}\right) \angle \phi - 2\beta s = \frac{r_a - 1 + jx_a}{r_a + 1 + jx_a} \times \frac{r_a + 1 - jx_a}{r_a + 1 - jx_a}$$

$$= \frac{r_a^2 + \cancel{r_a} - \cancel{jx_a r_a} - \cancel{r_a} - 1 + jx_a + \cancel{jx_a r_a} + jx_a + x_a^2}{r_a^2 + r_a - \cancel{jx_a r_a} + r_a + 1 - \cancel{jx_a} + \cancel{jx_a r_a} + \cancel{jx_a} + x_a^2}$$

$$= \frac{r_a^2 - 1 + 2jx_a + x_a^2}{r_a^2 + 2r_a + 1 + x_a^2} = \frac{r_a^2 - 1 + x_a^2 + j2x_a}{(r_a+1)^2 + x_a^2} \quad \text{(10)}$$

The angle ϕ may be made zero in order that the βs scale may start at $0°$ on the abscissa.

* Equating the tangents of the ~~gngles~~ angles in equ (10),

$$\tan(-2\beta s) = \frac{2x_a}{r_a^2 - 1 + x_a^2} \qquad (11)$$

sign? $\tan(-\theta) = -\tan\theta$

$$(r_a^2 - 1 + x_a^2)(-\tan 2\beta s) = 2x_a$$

$$(r_a^2 - 1 + x_a^2) + \frac{2x_a}{\tan 2\beta s} = 0 \Rightarrow r_a^2 + x_a^2 + \frac{2x_a}{\tan 2\beta s} = 1$$

After the square, this may be written as,

$$r_a^2 + \left(x_a + \frac{1}{\tan 2\beta s}\right)^2 = 1 + \frac{1}{\tan^2 2\beta s} = \frac{1}{\sin^2 2\beta s}$$ *

$$\left(x_a^2 + \frac{2x_a}{\tan 2\beta s} + \frac{1}{\tan^2 2\beta s}\right) \qquad (12)$$

$\frac{1}{\tan^2\theta}$ $\left(\frac{1}{\sin^2\theta}\right)$?

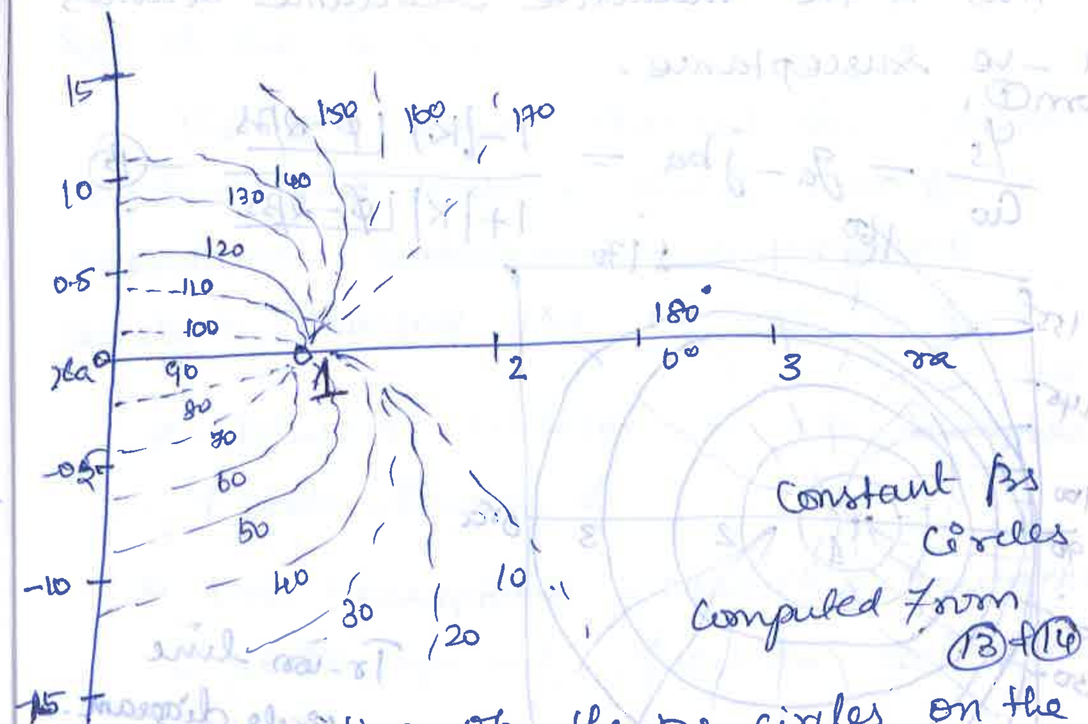
Lines of equal βs value are then seen to be circles of radius, $= \frac{1}{\sin 2\beta s}$ (13)

with a shift of center downward on the x_a axis (ordinate) $= \frac{1}{\tan 2\beta s}$ (14)

* $1 + \frac{\cos^2 2\beta s}{\sin^2 2\beta s} = 1 + \frac{1 - \sin^2\beta s}{\sin^2\beta s} = \cancel{1} + \frac{1}{\sin^2\beta s} - \cancel{\frac{\sin^2\beta s}{\sin^2\beta s}} = \frac{1}{\sin^2\beta s}$

A family of such circles is shown in Fig.

All the βs circle pass through the point $r_a = 1$ $x_a = 0$



Superposition of the βs circles on the S circles provides a scale of βs angles & results in the below circle diagram.

* computed from (13) & (14) interms of 2βs.

Per unit admittance may be written as,

$$\frac{Y_s}{G_0} = \frac{1}{r_a + jx_a} = \frac{r_a - jx_a}{(r_a + jx_a)(r_a - jx_a)}$$

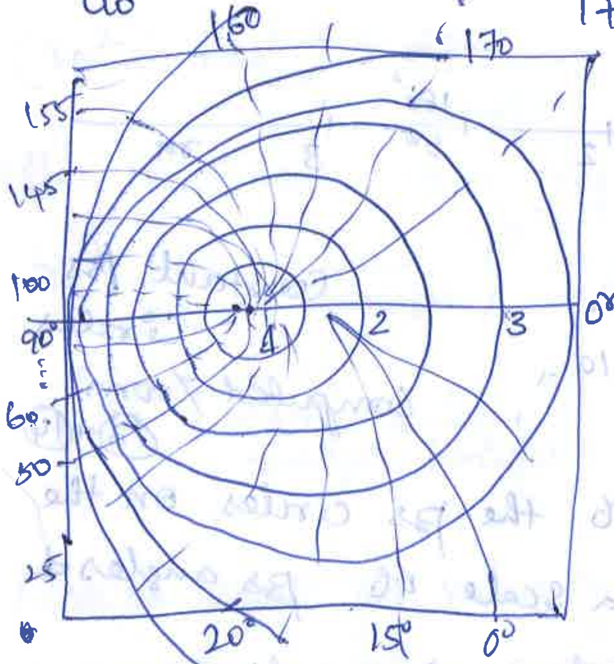
$$= \frac{r_a}{r_a^2 + x_a^2} - \frac{j x_a}{r_a^2 + x_a^2} \quad \text{——} (15)$$

$$= g_a - j b_a$$

Thus a +ve inductive reactance becomes a -ve susceptance.

from (1),

$$\frac{Y_s}{G_0} = g_a - j b_a = \frac{1 - |K| \angle \phi - 2\beta s}{1 + |K| \angle \phi - 2\beta s} \quad \text{——} (16)$$



Tr-ion line circle diagram.

replacing r_a by g_a

$r_a \to -b_a$

$+|K| \to -|K|$ sub. in (5) we have,

$$\left[g_a - \left(\frac{s^2 - 1}{2s}\right)\right]^2 + b_a^2 = \left(\frac{s^2 - 1}{2s}\right)^2 \quad \text{——} (17)$$

which is exactly similar to Equ (5).

The substitution of $-b_a$ for x_a need not be made, since equ (5) is independent of the sign of the b_a term.

The circle diagram obtained for impedance, resistance and reactance can be used for admittance, conductance & susceptance merely by changing the r_a scale to g_a.

* Inductive reactance is -ve susceptance & plots downward.
* +ve susceptance - capacitive reactance - plots upward from the axis of reals.

⇒ The susceptance of a SC-ed stub is,

$$B_{sc} = -G_0 \cot \beta L$$

where,
$L \rightarrow$ length of the SC-ed stub.

If stub & line have equal G_0, then,

$$\frac{\cancel{G_0}}{\tan \beta L} = \cancel{G_0}\left(\frac{2|K|\sqrt{1-|K|^2}}{1-|K|^2}\right)$$

$$\tan \beta L = \frac{1-|K|^2}{2|K|\sqrt{1-|K|^2}} \quad \left(\frac{\sqrt{1-|K|^2}}{2|K|}\right)$$

$$L = \frac{\lambda}{2\pi} \tan^{-1}\left(\frac{1-|K|^2}{2|K|\sqrt{1-|K|^2}}\right) \quad \text{—(10)}$$

By the use of SWR existing before connection of the stub, this equ. may be conveniently expressed as,

$$L = \frac{\lambda}{2\pi} \tan^{-1}\frac{\sqrt{S}}{S-1} \quad \text{—(11)}$$

This is the length of SC-ed stub to be placed d meters toward the load from a point at which a Vmin existed before the attachement of the stub.

Also possible to place the stub d meters toward the source from the voltage min.

The sign of the reactance is reversed on that side w.r.to the sign of the location nearer the load.

The stub length L' should be,

$$\boxed{L' = \lambda/2 - L} \quad \text{for SC-ed stub} \quad \text{—(12)}$$

The Smith Circle diagram

The Smith diagram is obtained from a transformation of equation (10).

$$\left(\frac{S-1}{S+1}\right)\angle\phi-2\beta s = |K|\,\angle\phi-2\beta s$$

$$= \frac{r_a^2-1+x_a^2+j2x_a}{(r_a+1)^2+x_a^2}$$

By introducing a new variable $U+jV$.

$$U+jV = \frac{r_a^2-1+x_a^2}{(r_a+1)^2+x_a^2} + j\frac{2x_a}{(r_a+1)^2+x_a^2}.$$

Equating the real and Imaginaries, we have,

$$U = \frac{r_a^2-1+x_a^2}{(r_a+1)^2+x_a^2} \quad \text{——(1)}$$

$$V = \frac{2x_a}{(r_a+1)^2+x_a^2} \quad \text{——(2)}$$

$$\Rightarrow \frac{Z_s}{R_0} = r_a+jx_a = \frac{1+|K|\angle\phi-2\beta s}{1-|K|\angle\phi-2\beta s} \quad \text{——(1)}$$

Say $|K|\angle\phi-2\beta s = u+jv$

$$\frac{Z_s}{R_0} = r_a+jx_a = \frac{1+(u+jv)}{1-(u+jv)} = \frac{(1+u)+jv}{(1-u)-jv} \quad \text{——(2)}$$

on rationalizing the above equation,

$$r_a+jx_a = \frac{(1+u)+jv}{(1-u)-jv} \times \frac{(1-u)+jv}{(1-u)+jv}$$

$$= \frac{(1+u)(1-u)+jv+juv+jv-juv-v^2}{(1-u)^2+v^2}$$

$$= \frac{(1-u^2-v^2)+j(2v)}{(1-u)^2+v^2}$$

$$= \frac{(1-u^2-v^2)}{(1-u)^2+v^2} + j\frac{2v}{(1-u)^2+v^2} \quad \text{——(3)}$$

$$r_a = \frac{1-u^2-v^2}{(1-u)^2+v^2} \quad \text{——(4)} \qquad x_a = \frac{2v}{(1-u)^2+v^2} \quad \text{——(5)}$$

Constant resistance circle

From equ (4),

$$r_a\left[(1-u)^2+v^2\right] = 1-u^2-v^2$$

$$r_a\left[(1-2u+u^2)+v^2\right] = 1-u^2-v^2$$

$$r_a - 2ur_a + u^2 r_a + v^2 r_a = 1-u^2-v^2$$

$$u^2\left[r_a+1\right] - 2ur_a + v^2(r_a+1) = 1-r_a$$

÷ by $1+r_a$,

$$u^2 - 2u\left(\frac{r_a}{1+r_a}\right) + v^2 = \frac{1-r_a}{1+r_a}$$

To complete the square by adding $\left(\frac{r_a}{1+r_a}\right)^2$ on both sides, we have,

$$u^2 - 2u\left(\frac{r_a}{1+r_a}\right) + \left(\frac{r_a}{1+r_a}\right)^2 + v^2 = \left(\frac{1-r_a}{1+r_a}\right) + \left(\frac{r_a}{1+r_a}\right)^2$$

$$\left(u - \frac{r_a}{1+r_a}\right)^2 + v^2 = \frac{(1+r_a)(1-r_a)+r_a^2}{(1+r_a)^2} = \frac{1}{(1+r_a)^2}$$

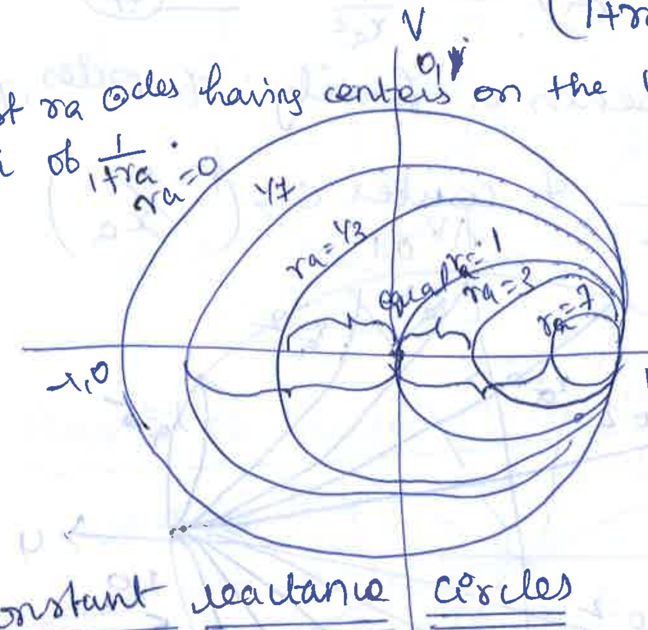
$$\left[u - \left(\frac{r_a}{1+r_a}\right)\right]^2 + v^2 = \left(\frac{1}{1+r_a}\right)^2 \quad \text{—— (6)}$$

ie in the form of $(x-c)^2+y^2=r^2$

∴ Circle of radius $r = \left(\frac{1}{1+r_a}\right)$

center $= \left(\frac{r_a}{1+r_a}, 0\right)$

Const r_a circles having centers on the U axis at $\frac{r_a}{r_a+1}$ and radii of $\frac{1}{1+r_a}$.



constant r_a circles
resistance circles

Constant reactance circles

From equ (5), $x_a = \dfrac{2v}{(1-u)^2+v^2}$ —— (5)

$$x_a\left[(1-u)^2+v^2\right] = 2v$$

$$x_a\left[1-2u+u^2+v^2\right] - 2v = 0$$

÷ by x_a, $1-2u+u^2+v^2-\frac{2v}{x_a} = 0$

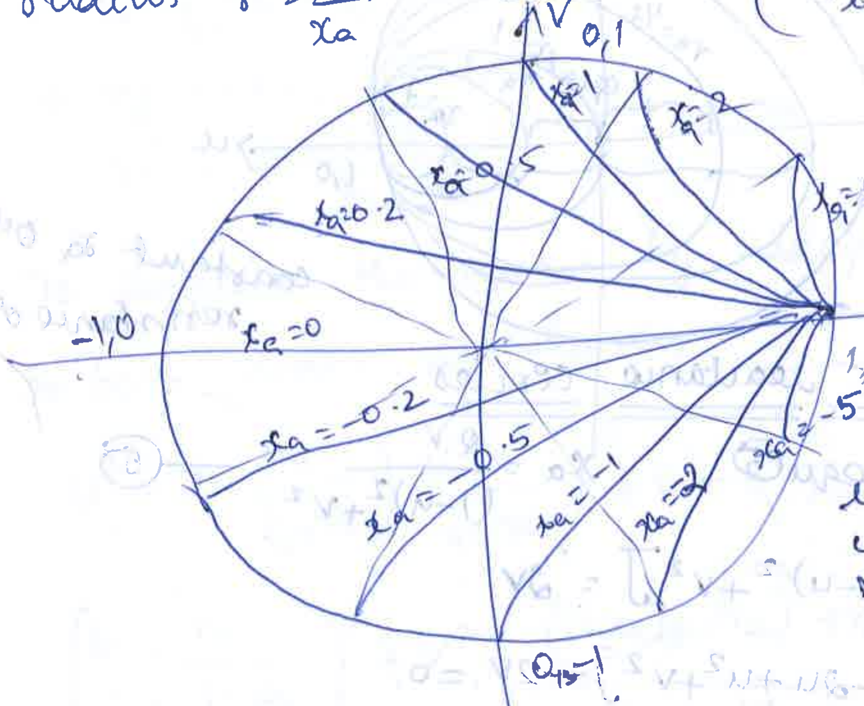
$$(u-1)^2+v^2-\frac{2v}{x_a} = 0$$

adding $\frac{1}{x_a^2}$ on both sides,

$$(u-1)^2 + v^2 - \frac{2v}{x_a} + \frac{1}{x_a^2} = \frac{1}{x_a^2}$$

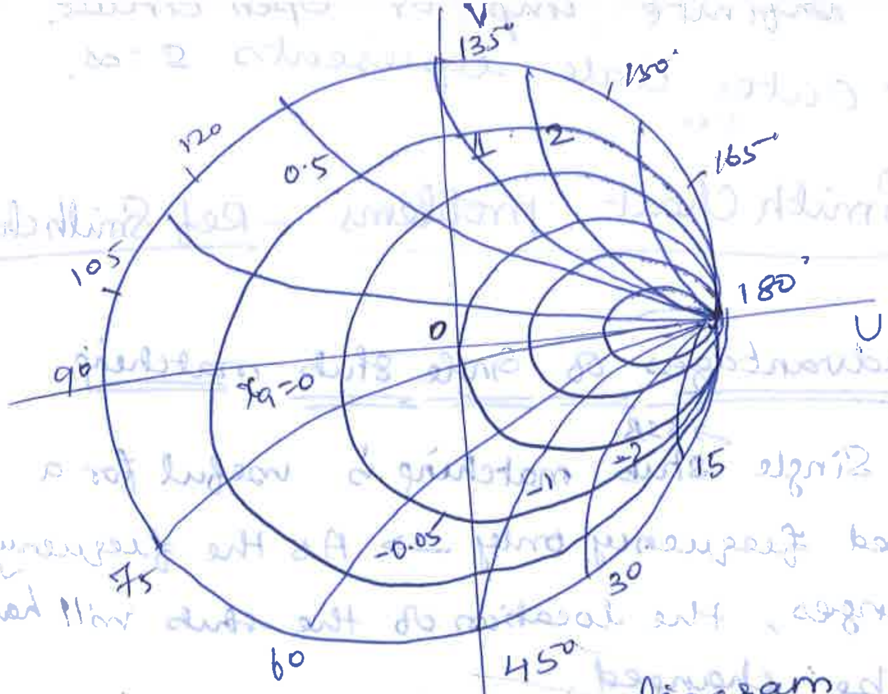
$$(u-1)^2 + \left(v - \frac{1}{x_a}\right)^2 = \frac{1}{x_a^2} \quad \text{--- (7)}$$

equ (7) represents a family of circles of radius $r = \frac{1}{x_a}$ & center $c = \left(1, \frac{1}{x_a}\right)$



Const. reactance circles. x_a-circles

* The superposition of constant resistant circle and const. reactance circles forms the Smith chart.



Basis of Smith circle diagram

* The imp. of a line may be read at any point on the appropriate SWR or S. circle.
* The point at the center of the chart represents the imp. of a line terminated in its charac. imp. $\frac{Z}{Z_0} = 1$
* The pt. at extreme left of r_a axis is zero imp or SC
* The pt. at extreme rt. of r_a axis

is infinite imp. or open circuit.

* Outer circle represents $S = \infty$.

Smith Chart problems - Ref Smith charts

Disadvantages of Single stub matching

1. Single stub (→ SSB) matching is useful for a fixed frequency only → As the frequency changes, the location of the stub will have to be changed.

— SSB → Narrow Band System.

2. For final adjustment, the stub has to be moved along the line slightly.

This is possible only in open wire lines and ∴ on co-axial lines, SSB may become inaccurate in practice.

Though it reduces the reflection losses to a considerable extent.

Unit - III

Network components

Filter ~~conn~~ fundamentals - filter design - lumped elements and distributed element approach to filter design - Design of Attenuators & equalizers - Lattice type, concept of inverse networks - Transients in transmission lines - Lattice diagram. Numerical examples.

Filters

* Resonant circuit that will select narrow bands of frequencies and reject the other frequencies.

ie freely pass desired bands of freq's & almost totally suppress other band of frequencies.

Such reactive N/w's called filters

Ideal filter:- would pass all freq's in a given band without reduction in magnitude and totally suppress all other frequencies.

Nepers:- The ratio of i/p voltage or ct. to o/p voltage & current is

$$\frac{V_1}{V_2} = \frac{I_1}{I_2} = e^N$$

$$\ln \frac{V_1}{V_2} = N$$

The unit of N has been given the name neper, & is defined as,

$$N \text{ nepers} = \ln \frac{V_1}{V_2} = \ln \frac{I_1}{I_2}$$

Two voltages or currents differ by one neper when one of them is e times as large as the other.

Also be expressed as,

ln → natural log.

$$\frac{P_1}{P_2} = e^{2N}$$

The no. of nepers represents a convenient measure of the power loss or gain of a given N/w.

log → log with base 10.

logarithms to the base 10, naming the unit of the bel

$$\text{no. of bels} = \log \frac{P_1}{P_2}$$

It is convenient that a unit 1/10th called decibels abbreviated "dB"

$$dB = 10 \log \frac{P_1}{P_2}.$$

For the case of equal imp in i/p & o/p circuits,

$$dB = 20 \log \frac{I_1}{I_2} = 20 \log \frac{V_1}{V_2}$$

Equating with power ratios,

$$e^{2N} = 10^{dB/10}$$

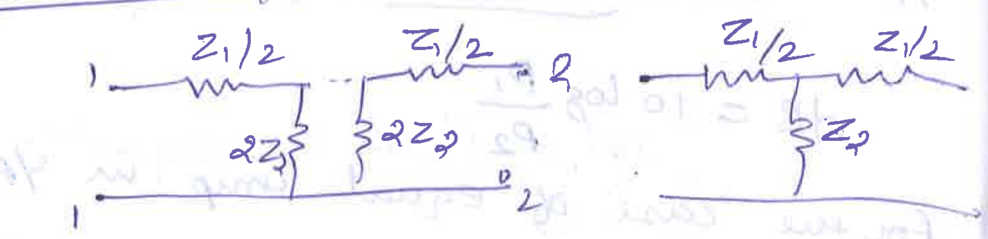
Taking log on both sides,

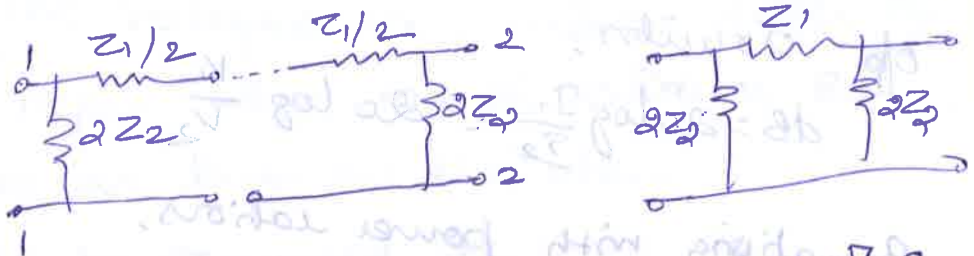
$$\boxed{\begin{array}{l} 8.686\,N = dB \\ 1 \text{ neper} = 8.686\,dB. \end{array}}$$

is the relationship b/w nepers & decibels.

Characteristics impedance of symmetrical N/w

Symmetrical N/w





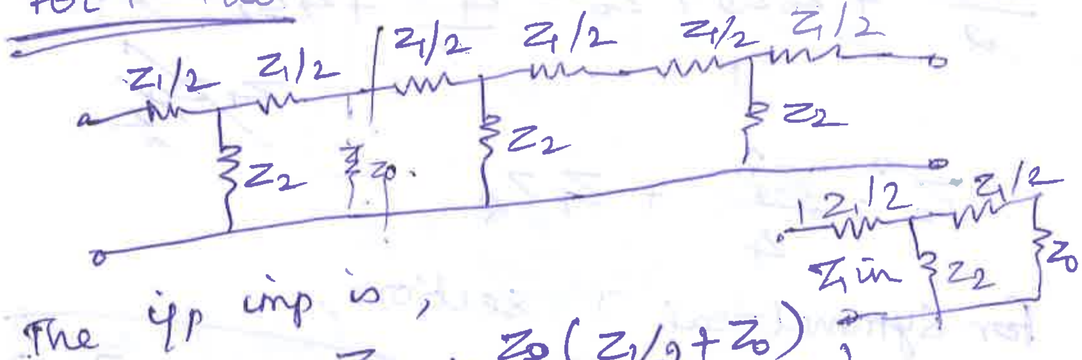
* when $Z_1 = Z_2$ or 2 series arms of a T N/w are equal or the shunt arms of a π N/w are equal the N/w is symmetrical.

* A series connection of several T or π N/w's are called "ladder N/w's"

* A symmetrical N/w is terminated in Z_0, i/p imp is also Z_0, the impedance transformation ratio is unity.

The value of Z_0 for a symmetrical N/w can be easily determined.

For T N/w: Terminated in an imp Z_0,



The i/p imp is,

$$Z_{in} = \frac{Z_1}{2} + \frac{Z_2(Z_1/2 + Z_0)}{\frac{Z_1}{2} + Z_2 + Z_0} \quad \text{--- (1)}$$

i/p is equal to Z_0,

$$Z_0 = \frac{(Z_2 Z_1/2) + Z_0 Z_2}{\frac{Z_1}{2} + Z_2 + Z_0} + \frac{Z_1}{2}$$

$$= \frac{\frac{Z_1^2}{4} + \frac{Z_1 Z_2}{2} + \frac{Z_0 Z_1}{2} + \frac{Z_1 Z_2}{2} + Z_0 Z_2}{Z_1/2 + Z_2 + Z_0}$$

$$= \frac{(Z_1^2/4) + Z_1 Z_2 + Z_0 Z_2 + (Z_1 Z_0/2)}{\frac{Z_1}{2} + Z_2 + Z_0}$$

$$\frac{Z_0 Z_1}{2} + \cancel{Z_0 Z_2} + Z_0^2 = \frac{Z_1^2}{4} + Z_1 Z_2 + \cancel{Z_2 Z_0} + \cancel{Z_1 Z_0/2}$$

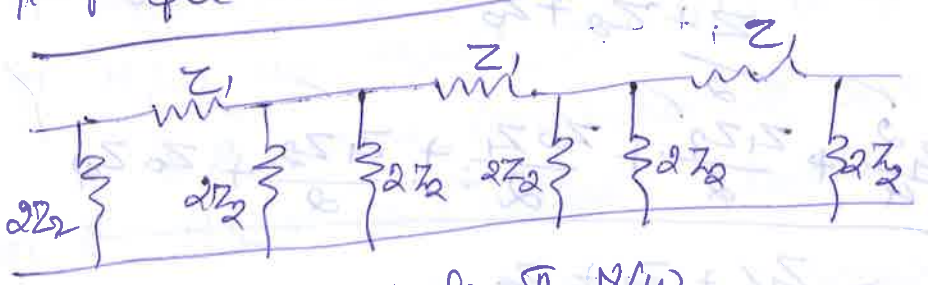
$$Z_0^2 = \frac{Z_1^2}{4} + Z_1 Z_2.$$

For symmetrical 'T' section,

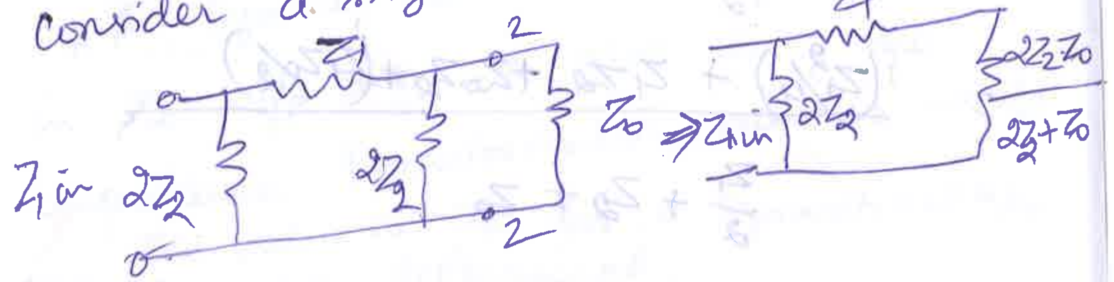
$$Z_{0T} = \sqrt{\frac{Z_1^2}{4} + Z_1 Z_2} = \sqrt{Z_1 Z_2\left(1 + \frac{Z_1}{4Z_2}\right)} \quad \text{--- (2)}$$

$Z_{0T} \rightarrow$ charac. imp for 'T' N/w.

||ly for 'π' matching N/w



Consider a single π N/w



$$Z_{in} = \frac{\left(\left(\frac{2Z_2Z_0}{2Z_2+Z_0}\right)+Z_1\right)2Z_2}{Z_1+\frac{2Z_2Z_0}{2Z_2+Z_0}+2Z_2}$$

Requiring that,

$Z_{in} = Z_0$ leads to

$$Z_{0\pi} = Z_0 = \frac{4Z_1Z_2^2+2Z_1Z_2Z_0+4Z_2^2Z_0}{2Z_1Z_2+Z_1Z_0+2Z_2Z_0+4Z_2^2+2Z_0Z_2}$$

$$4Z_1Z_2^2 + \cancel{2Z_1Z_2Z_0} + \cancel{4Z_2^2Z_0} = \cancel{2Z_0Z_1Z_2} + Z_1Z_0^2 + 2Z_2Z_0^2 + \cancel{4Z_0Z_2^2} + 2Z_0^2Z_2$$

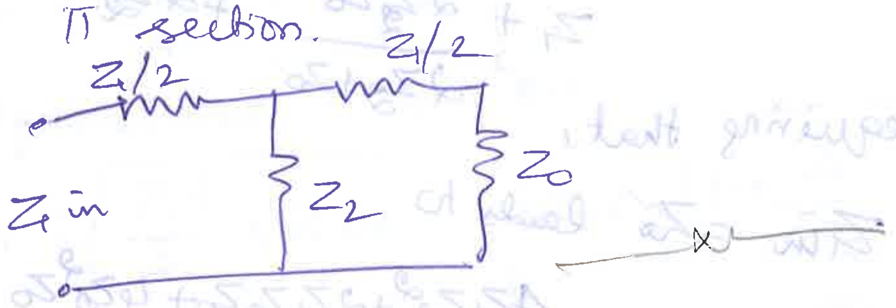
$$4Z_1Z_2^2 = Z_1Z_0^2 + 4Z_0^2Z_2 \qquad \div 4Z_2$$

$$Z_0^2\left[Z_1 + 4Z_2\right] = 4Z_1Z_2^2$$

$$Z_0^2 = \frac{4Z_1Z_2^2}{Z_1+4Z_2} = \left(\frac{Z_1Z_2}{\frac{Z_1}{4Z_2}+1}\right)$$

$$Z_{0\pi} = \sqrt{\frac{Z_1 Z_2}{1 + Z_1/4Z_2}} \quad \text{(B)}$$

$Z_{0\pi} \rightarrow$ charac. imp. of the symmetric π section.



$$Z_{1OC} = Z_{OC} = \frac{Z_1}{2} + Z_2$$

$$Z_{SC} = Z_{SC} = \frac{Z_1}{2} + \frac{Z_1 Z_2/2}{\frac{Z_1}{2} + Z_2}$$

$$Z_{oc} \cdot Z_{sc} = \frac{Z_1^2}{4} + \frac{Z_1^2 Z_2}{4}$$

Ct. & voltage ratios as exponentials:

The absolute value of the ratio of i/p ct. to o/p ct of a given symmetrical N/w was defined as an exponential fn.

$$\frac{I_1}{I_2} = e^{\gamma} \quad ; \gamma \to \text{complex no.} \Rightarrow \alpha + j\beta.$$

$$\frac{I_1}{I_2} = e^{\gamma} = e^{\alpha + j\beta}$$

$$\frac{I_1}{I_2} = A \angle \beta$$

$$A = \left|\frac{I_1}{I_2}\right| = e^{\alpha} \quad ; \quad \angle\beta = e^{j\beta}.$$

It is also true for $\frac{V_1}{V_2} = e^{\gamma}$

$\gamma \to$ propagation const.

$\alpha \to$ attenuation const. $\to$ Attn. produced in passing thro' the N/w.

$\alpha \to$ nepers.

$\beta \to$ phase const $\to$ phase angle b/w i/p and o/p Quantities or the shift in phase introduced by the N/w.

β Unit — radians.

For cascaded N/w

$$\frac{I_1}{I_2} \times \frac{I_2}{I_3} \times \cdots \times \frac{I_{n-1}}{I_n} = \frac{I_1}{I_n}$$

for which $e^{\gamma_1} \times e^{\gamma_2} \times e^{\gamma_3} \times \cdots = e^{\gamma_n}$

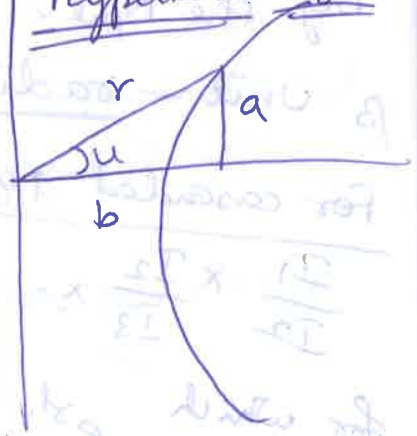
& taking natural log,

$$\gamma_1 + \gamma_2 + \gamma_3 + \cdots = \gamma_n.$$

ie the overall prop. const is equal to the sum of the individual propagation constants.

Hyperbolic trigonometry

Meaning of a hyperbolic angle



The hyperbola is the locus for the radius r.

$\sinh u = \frac{a}{r}$

$\cosh u = b/r$

$\tanh u = \frac{a}{b}$.

hyperbola fns simplify the writing of certain exponential relations.

Few properties are summarized & extended to the case of complex angles.

$$\sinh u = \frac{e^{u} - e^{-u}}{2}$$

$$\cosh u = \frac{e^{u} + e^{-u}}{2}$$

$$\tanh u = \frac{\sinh u}{\cosh u} = \frac{1}{\coth u}$$

$$\cosh^2 u - \sinh^2 u = 1$$

For u is large, $\sinh u = \cosh u$.

u is imaginary or $u = j\omega$.

$$\sinh j\omega = \frac{e^{j\omega} - e^{-j\omega}}{2} = j \sin \omega$$

$$\cosh j\omega = \frac{e^{j\omega} + e^{-j\omega}}{2} = \cos \omega.$$

For complex angles, $u = a + jb$

$$\sinh(a+jb) = \sinh a \cosh jb + \cosh a \sinh jb$$
$$= \sinh a \cos b + j \cosh a \sin b$$

$$\cos(a+jb) = \cosh a + \cosh jb + \sinh a \sinh jb$$
$$= \cosh a \cos b + j \sinh a \sin b$$

Few useful half angles are

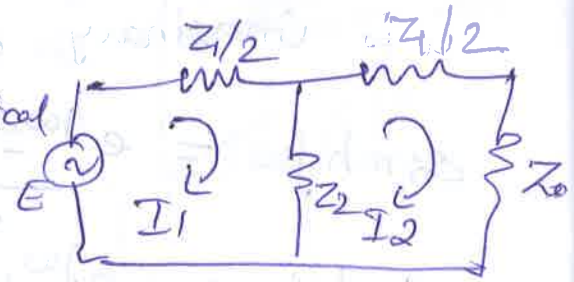
$$\sinh \frac{u}{2} = \sqrt{\frac{1}{2}(\cosh u - 1)}$$

$$\cosh \frac{u}{2} = \sqrt{\frac{1}{2}(\cosh u + 1)}$$

$$\sinh u = 2 \sinh \frac{u}{2} \cosh \frac{u}{2}.$$

Properties of symmetrical N/w's

Consider a symmetrical N/w with gen & load.



The mesh eqn's are

$$E = I_1\left(\frac{Z_1}{2} + Z_2\right) - Z_2 I_2 \quad \text{—— (1)}$$

$$0 = -I_1 Z_2 + I_2\left(\frac{Z_1}{2} + Z_0 + Z_2\right) \quad \text{—— (2)}$$

$$I_1 Z_2 = I_2\left(Z_1/2 + Z_0 + Z_2\right)$$

$$I_1/I_2 = \frac{1}{Z_2}\left(Z_1/2 + Z_0 + Z_2\right)$$

$$\frac{I_1}{I_2} = \frac{Z_1/2 + Z_0 + Z_2}{Z_2} = e^{\gamma} \quad \text{—— (3)}$$

$$\frac{Z_1}{2} + Z_0 + Z_2 = Z_2 e^{\gamma}$$

$$Z_0 = Z_2(e^{\gamma} - 1) - \frac{Z_1}{2} \quad \text{—— (4)}$$

from $Z_{0T} = \sqrt{\frac{Z_1^2}{4} + Z_1 Z_2} = Z_0$

$$Z_{0T}^2 = Z_0^2 = \frac{Z_1^2}{4} + Z_1 Z_2 \quad \text{—— (5)}$$

$$Z_2(e^{\gamma} - 1)^2 - Z_1 e^{\gamma} = 0$$

$$e^{2\gamma} - 2e^{\gamma} + 1 = \frac{Z_1}{Z_2} e^{\gamma}$$

$$\frac{e^{2\gamma} - 2e^{\gamma} + 1}{e^{\gamma}} = \frac{Z_1}{Z_2}$$

$$\frac{e^{2\gamma}}{e^{\gamma}} - 2 + \frac{1}{e^{\gamma}} = \frac{Z_1}{Z_2}$$

$$e^{\gamma} + e^{-\gamma} = 2 + \frac{Z_1}{Z_2}$$

$$\frac{e^{\gamma} + e^{-\gamma}}{2} = 1 + \frac{Z_1}{2Z_2}$$

$$\cosh\gamma = 1 + \frac{Z_1}{2Z_2}. \quad \text{—— (6)}$$

$$\cosh^2\gamma - \sinh^2\gamma = 1$$

$$\cosh^2\gamma - 1 = \sinh^2\gamma$$

Filter fundamentals

A Filter N/W,

* Transmit or pass – desired freq band
* Stop or attenuate – undesired freq.

(1) $\frac{I_1}{I_2} = e^{\alpha} . e^{j\beta}$. $\alpha = 0$ & $I_1 = I_2$

ie no attn., only phase shift, the operation is in pass band.

(2) when $\alpha \rightarrow +ve$ I_2 is smaller than I_1

Attn. has occurred & operation is in an stop band freq's

* The prop. const γ may be conveniently studied,

$$\sinh\frac{\gamma}{2} = \sqrt{\frac{1}{2}\left(1 + \frac{Z_1}{2Z_2} - 1\right)} = \sqrt{\frac{Z_1}{4Z_2}} \quad (1)$$

*Assume, the N/W contains only pure reactances.

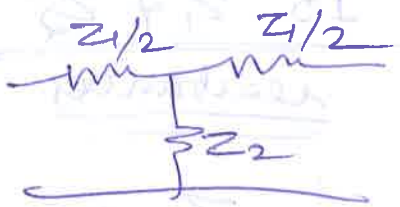
ie $\frac{Z_1}{4Z_2}$ will be real & either +ve or −ve depending on the type of reactance Z_1 & Z_2 & the N/W contains only pure reactance.

$$\sinh\frac{\gamma}{2} = \sinh\left(\frac{\alpha}{2} + \frac{j\beta}{2}\right)$$

$$= \sinh\frac{\alpha}{2}\cos\frac{\beta}{2} + j\cosh\frac{\alpha}{2}\sin\frac{\beta}{2} \quad (2)$$

Case 1

If Z_1 & Z_2 are same type of reactances,



$\frac{Z_1}{4Z_2} > 0$, → +ve & real

∴ $\sinh\frac{\gamma}{2}$ be real, imaginary term = 0

(1) $\cosh\frac{\alpha}{2}\sin\frac{\beta}{2} = 0$ — (3)

(2) $\sinh\frac{\alpha}{2}\cos\frac{\beta}{2} = \sqrt{\frac{Z_1}{4Z_2}}$ — (4)

From (3), $\sin\frac{\beta}{2} = 0$ $\beta = n\pi$, $n = 0, 2, 4 \ldots$

from (4) $\cos\frac{\beta}{2} = 1$ ∴ $\sinh\frac{\alpha}{2} = \sqrt{\frac{Z_1}{4Z_2}}$

Attn. $\alpha = 2\sinh^{-1}\sqrt{\frac{Z_1}{4Z_2}}$ — (5)

i.e. $\left|\frac{Z_1}{4Z_2}\right| \geq 0$ implies a stop or attn. band of freq's.

Case 2

If Z_1 & Z_2 are opposite types of reactances

$\frac{Z_1}{4Z_2}$ is -ve, $\left|\frac{Z_1}{4Z_2}\right| < 0$,

& $\sinh\frac{\gamma}{2} = \sqrt{\frac{Z_1}{4Z_2}}$ is imaginary.

$$\sinh\frac{\gamma}{2} = \sinh\frac{\alpha}{2}\cdot\cos\frac{\beta}{2} + j\cosh\frac{\alpha}{2}\sin\frac{\beta}{2}$$

the real term must be zero.

(3) $\sinh\frac{\alpha}{2}\cos\frac{\beta}{2} = 0$ —— (6)

(4) $\cosh\frac{\alpha}{2}\sin\frac{\beta}{2} = \sqrt{\frac{Z_1}{4Z_2}}$ —— (7)

must be satisfied.

two condns are possible.

(6) ⇒ I $\sinh\frac{\alpha}{2} = 0$, $\therefore \alpha = 0$ $\beta \neq 0$

$$\sin\frac{\beta}{2} = \sqrt{\frac{Z_1}{4Z_2}}$$

(7) ⇒ II $\cos\frac{\beta}{2} = 0$, $\therefore \sin\frac{\beta}{2} = \pm 1$

$\alpha \neq 0$ $\beta = (2n-1)\pi$

$$\cosh\frac{\alpha}{2} = \sqrt{\frac{Z_1}{4Z_2}}$$

From condn:- I leads to a PB, which is limited by the upper limit on the sine or by $\sin\frac{\beta}{2} = 1$ or

$$-1 < \frac{Z_1}{4Z_2} < 0$$

$$\boxed{\text{phase angle}\ \beta = 2\sin^{-1}\sqrt{\frac{Z_1}{4Z_2}}} \quad \text{—— (8)}$$

Condn:- II leads to stop or attn. band

Since $\alpha \neq 0$,

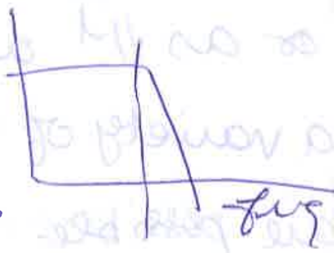
The phase angle is π, & attn. is given by $\boxed{\alpha = 2\cosh^{-1}\sqrt{\frac{Z_1}{4Z_2}}}$ — (9)

$\frac{Z_1}{4Z_2} < -1$ → ∵ The hyperbolic cosine has no value below unity.

Values of $\frac{Z_1}{4Z_2}$ classified into 3 regions corresponding to α & β, these regions being bounded by, $\frac{Z_1}{4Z_2}$ values of $+\infty$, -1 & $-\infty$

$\frac{Z_1}{4Z_2}$ =	$+\infty$ to 0	0 to -1	-1 to $-\infty$
Reactance type	same	opposite	opposite
Band	Stop	pass	stop
α	$2\sinh^{-1}\sqrt{\frac{Z_1}{4Z_2}}$	0	$2\cosh^{-1}\sqrt{\frac{Z_1}{4Z_2}}$
β	π	$2\sin^{-1}\sqrt{\frac{Z_1}{4Z_2}}$	π

The freq's at which the N/w changes from pass N/w to a stop N/w, or vice versa is called cutoff freq's.



These freq's occur when,

$\frac{Z_1}{4Z_2} = 0$ or $Z_1 = 0$

$\frac{Z_1}{4Z_2} = -1$ or $Z_1 = -4Z_2$

where Z_1 & Z_2 are opposite type reactances

Since Z_1 & Z_2 have no. of configurations, as L & C elements or as ||l and series combinations a variety of types of performances are possible.

Behavior of the charact. impedance

For symmetrical T n/w,

$$Z_{0T} = \sqrt{Z_1 Z_2 \left(1 + \frac{Z_1}{4Z_2}\right)}$$

if a N/w is entirely made up pure reactances, then this may be,

$$Z_{0T} = \sqrt{-X_1 X_2 \left(1 + \frac{X_1}{4X_2}\right)}$$

$-$ sign due to j^2

(1) X_1 & X_2 are same type reactance stop band exist.

$\frac{X_1}{4X_2}$ = real & positive.

$Z_0 \Rightarrow$ pure reactance in the attenuation region

(2) X_1 & X_2 were opposite reactances

$-1 < \frac{X_1}{4X_2} < 0 \Rightarrow$ pass band exist.

product $X_1 X_2 = -ve$.

$Z_0 \rightarrow$ real & able to absorb power from a source.

(3) X_1 & $X_2 \rightarrow$ opposite reactance.

$\frac{X_1}{4X_2} < -1$ stop band exist.

$X_1 X_2 = -ve$

$Z_0 \rightarrow$ pure reactances

From the above,

* pass band $Z_0 \rightarrow$ real & positive.

reactive N/w is terminated with a resistive $Z_0 = R_0$, i/p imp. $= Z_0$

The N/w can accept power & will transmit to the resistive load without loss or attenuation.

In stop band $Z_0 \rightarrow$ has been shown to be reactive.

if a N/w is terminated ~~is~~ in its reactive Z_0,

→ it will appear as a totally reactive circuit.

→ can't accept or transmit power,

Since there is no resistive element in which the power may be dissipated.

* The N/w may transmit V or I but with 90° phase angle b/w them & with considerable attenuation.

Hly for π N/w,

$$Z_{0\pi} = \frac{Z_1 Z_2}{Z_{0T}}$$

$Z_1 Z_2$ is always real for Z_1 & Z_2 as pure reactances.

The constant k Low Pass Filter

Z_1 & Z_2 → reactance arms

$$Z_1 Z_2 = k^2$$

where k → const. independent of freq.

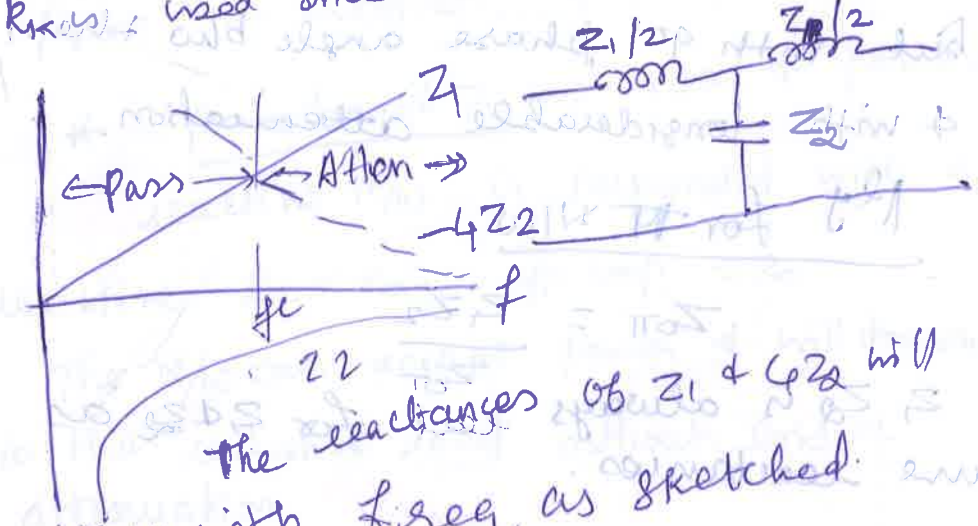
The N/w's or filter section → called const-k filters.

Let assume $Z_1 = j\omega L$

$$Z_2 = -j/\omega C.$$

Then the product $Z_1 Z_2 = \frac{L}{C} = R_k^2$

R_k is used since k must be real.



The reactances of Z_1 & $4Z_2$ will vary with freq as sketched.

The pass band starts at freq at which $Z_1 = 0$ & runs to the freq at which $Z_1 = -4Z_2$.

The reactance curves show that a pass band starts at $f = 0$ & continues to some higher freq f_c.

All freq's above f_c lie in stop band or attenuation band.

Thus the N/w is called LPF.

The cut off freq. may be determined, $\omega_c = 2\pi f_c$

$$Z_1 = -4Z_2$$

$$j\omega_c L = -4Z_2, \quad j\omega_c L = \frac{4j}{\omega_c C}$$

$$f_c^2 = \frac{4}{4\pi^2 LC}$$

$$f_c = \frac{1}{\pi\sqrt{LC}}$$

The $\sinh\frac{\gamma}{2}$ may be evaluated as,

$$\sinh\frac{\gamma}{2} = \sqrt{\frac{Z_1}{4Z_2}} = \sqrt{\frac{-\omega^2 LC}{4}} = \frac{j\omega}{2}\sqrt{LC}$$

$$\sinh\frac{\gamma}{2} = j\frac{2\pi f}{2}\sqrt{LC} = j\frac{f}{f_c}$$

f is passband $f/f_c < 1$

So that $-1 < \frac{Z_1}{4Z_2} < 0$

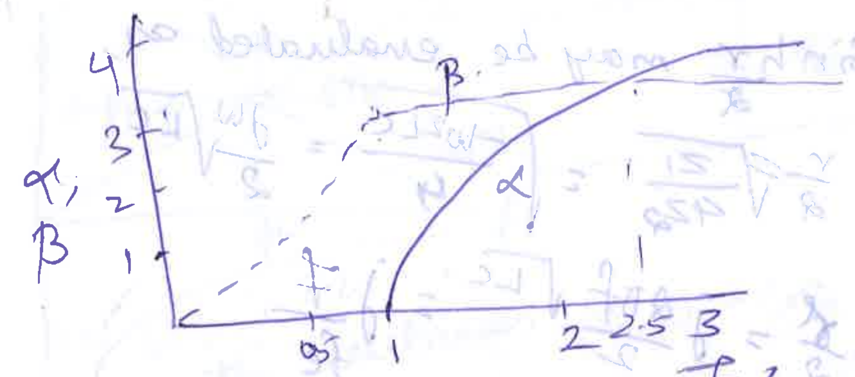
$\frac{f}{f_c} < 1$, $\alpha = 0$ $\beta = 2\sin^{-1}(f/f_c)$,

where as, f is in attn band or $f/f_c > 1$

So that $\frac{Z_1}{4Z_2} < -1$ then,

$\frac{f}{f_c} > 1$, $\alpha = 2\cosh^{-1}\left(\frac{f}{f_c}\right)$; $\beta = \pi$

The variation of α & β is plotted



α – neper
β – radians

α is zero throughout the passband.

* Raises gradually from f_c at i.e. cut-off freq at $f/f_c = 1$ to ∞ at infinite freq.

* β is zero at 0 freq.

* Ties gradually through the pass band, reaching π at f_c & remaining at π for all higher freq's.

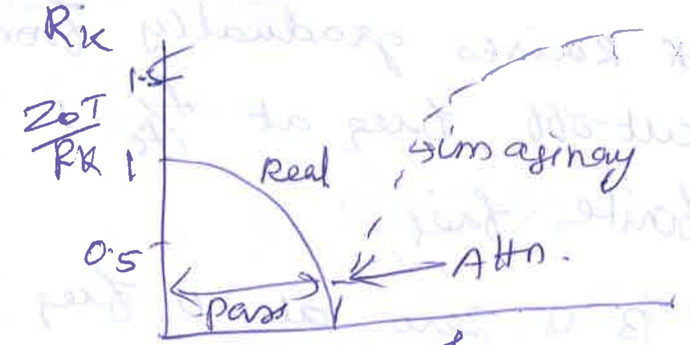
The charac. imp. for T-section

$$Z_{0T} = \sqrt{Z_1 Z_2 \left(1 + \frac{Z_1}{4Z_2}\right)}$$

$$Z_{0T} = \sqrt{\frac{L}{C}\left(1 - \frac{\omega^2 LC}{4}\right)} \qquad R_k^2 = \frac{L}{C}$$

$$= \sqrt{\frac{L}{C}\left(1 - \left(\frac{f}{f_c}\right)^2\right)} = R_k \sqrt{1 - \left(\frac{f}{f_c}\right)^2}$$

Values of $\frac{Z_{0T}}{R_K}$ = plotted against f/f_c



Z_{0T} varies throughout f/f_c the pass band, reaching zero at cut-off, then becomes imaginary in the attn. band, rising to infinite reactance at infinite freq.

$R = R_K = \sqrt{L/C}$

$Z_1 = -4Z_2$ $\quad \omega_c L = \frac{4}{\omega_c C}$

$f_c = \frac{1}{\pi\sqrt{LC}} \Rightarrow \pi^2 f_c^2 LC = 1$

Sub. $L = R^2 C \Rightarrow \pi^2 f_c^2 R^2 C^2 = 1$

$C^2 = \frac{1}{\pi^2 f_c^2 R^2} \Rightarrow \boxed{C = \frac{1}{\pi f_c R}}$

Hly sub $C = \frac{L}{R^2}$

$\therefore \pi^2 f_c^2 \frac{L^2}{R^2} = 1$

$L^2 = \frac{R^2}{\pi^2 f_c^2}$; $\boxed{L = \frac{R}{\pi f_c}}$

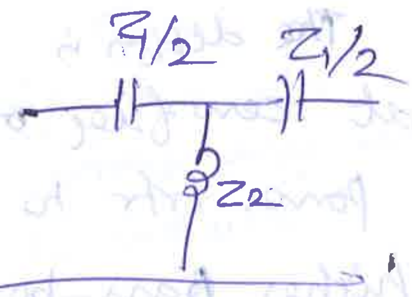
The design is based on an imp. match at zero freq only. Matched load power trfr to a ML will drop at higher pass-band freq's

Const K - HPF

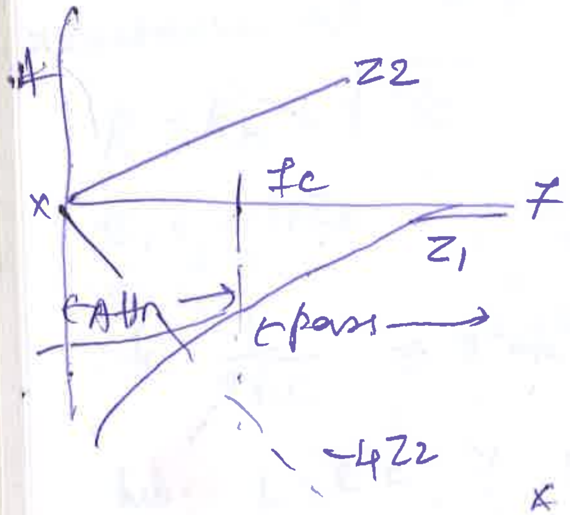
The positions of inductance & capacitance are interchanged to make

$Z_1 = \frac{-j}{\omega C}$; $Z_2 = j\omega L$

Then $Z_1 Z_2 = k^2$



Z_1 & Z_2 are sketched as a fn. of freq.



* Z_1 & $-4Z_2$ are compared.
* At cut of freq Z_1 equal $-4Z_2$
* pass band is from that freq to ∞

Cut-off freq. → determined as the freq at which, $Z_1 = -4Z_2$.

$$\frac{-j}{\omega_c C} = -j4\omega_c L$$

$$4\omega_c^2 LC = 1$$

$$\boxed{f_c = \frac{1}{4\pi\sqrt{LC}}}$$

Using this,

$$\therefore \sinh\frac{\gamma}{2} = \sqrt{\frac{Z_1}{4Z_2}} = \sqrt{\frac{-1}{4\omega^2 LC}}$$

$$= \frac{-j}{2\omega\sqrt{LC}} = \boxed{-j\frac{f_c}{f}}$$

The region which $\frac{f_c}{f} < 1$ in pass band

∴ the variations of γ inside & outside the pass band will be identical with the values of LPF

Design :- $R = R_K = \sqrt{\frac{L}{C}}$

$Z_1 = -4Z_2$; $4\omega_c^2 LC = 1$

then $\frac{L}{C} = R^2$

Sub $L = R^2 C$ the $4\omega_c^2 R^2 C^2 = 1$

$C = \frac{1}{2\omega_c R}$ $\boxed{\frac{1}{4\pi f_c R} = C}$

Sub $C = \frac{L}{R^2}$, then $\frac{4\omega_c^2 L^2}{R^2} = 1$

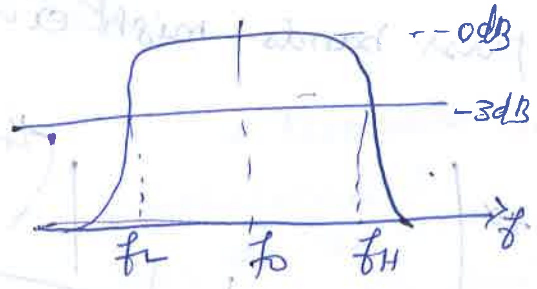
$L^2 = \frac{R^2}{4\omega_c^2}$ ⇒ $\boxed{L = \frac{R}{4\pi f_c}}$

Charac imp. of HPF =

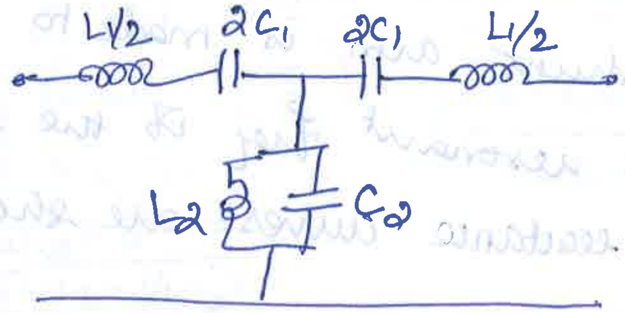
$$\boxed{Z_{0T} = R_K\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

Band pass Filters

Pass a band of frequencies & to attenuate frequencies on both sides of the pass band.



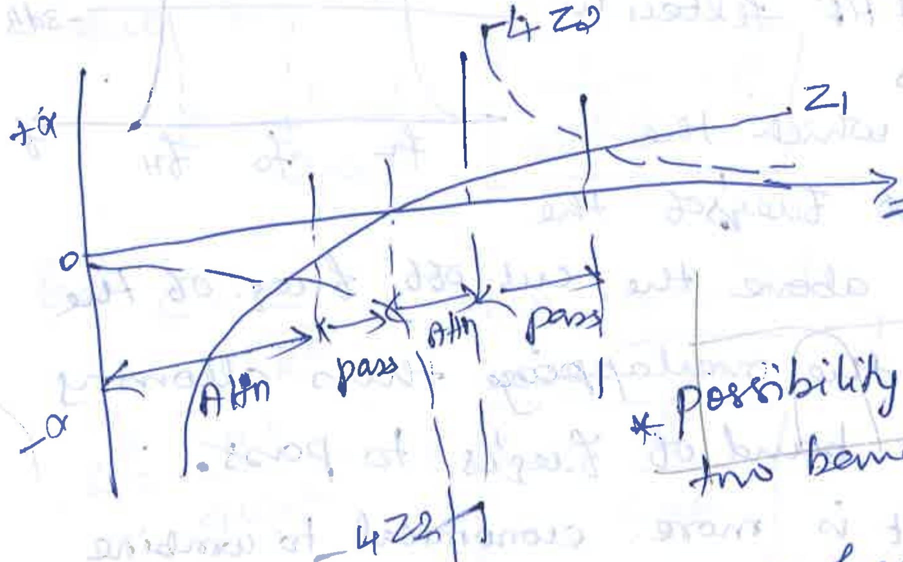
* LP & HF filters in series
* in which the cut-off freq's of the LPF is above the cut-off freq. of the HPF, the overlapping thus allowing only a band of freq's to pass.
* It is more economical to combine the LP & HP fn's into a single filter section.



BPF

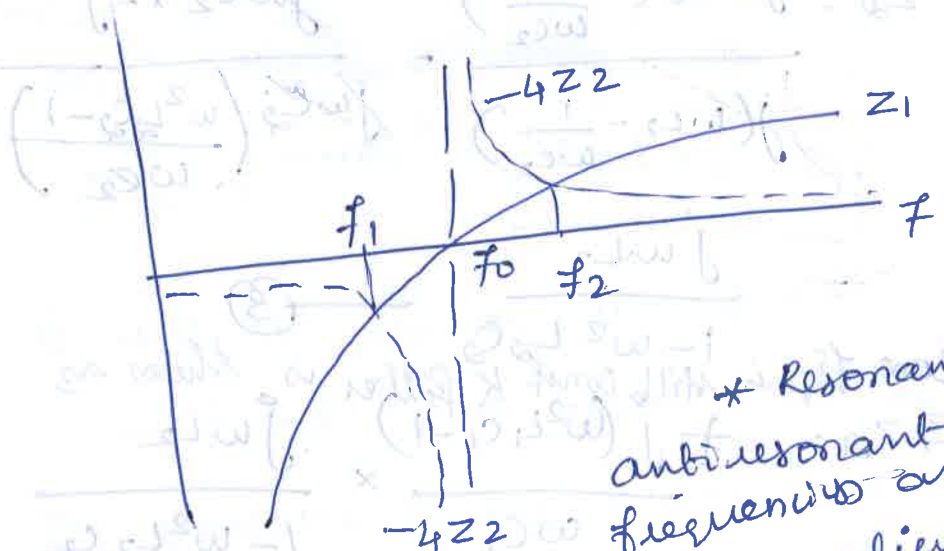
The BPF is designed with a series resonant series arm and an antiresonant shunt arm.

* The reactance curves show that two pass bands might exist.



* possibility of two bands.

* However the antiresonant freq. of the shunt arm is made to correspond to the resonant freq of the series arm the reactance curves are shown below.



* Resonant & antiresonant frequencies are properly adjusted

* For this condn, ~~the~~ of equal resonant freq's

$$\omega_0^2 L_1 C_1 = 1 = \omega_0^2 L_2 C_2 \quad \text{--- (1)}$$

$$L_1 C_1 = L_2 C_2$$

The imp. of the arms are

series $j\omega L_1 - j/\omega C_1$

$$Z_1 = j\left(\omega L_1 - \frac{1}{\omega C_1}\right) = j\frac{(\omega^2 L_1 C_1 - 1)}{\omega C_1} \quad \text{--- (2)}$$

$$Z_2 = \frac{j\omega L_2 \times j/\omega C_2}{(j\omega L_2 - j/\omega C_2)}$$

$$Z_2 = \frac{j\omega L_2\left(-\frac{j}{\omega C_2}\right)}{j\left(\omega L_2 - \frac{1}{\omega C_2}\right)} = \frac{j\omega L_2 \times -j}{\cancel{j}\omega C_2\left(\frac{\omega^2 L_2 C_2 - 1}{\omega C_2}\right)}$$

$$= \frac{j\omega L_2}{1-\omega^2 L_2 C_2} \quad \text{—— (3)}$$

The above fig is still const K filter is shown as,

$$Z_1 Z_2 = \pm j\frac{(\omega^2 L_1 C_1 - 1)}{\omega C_1} \times \frac{j\omega L_2}{1-\omega^2 L_2 C_2}$$

$$= \frac{-\cancel{\omega} L_2(\omega^2 L_1 C_1 - 1)}{\cancel{\omega} C_1(1-\omega^2 L_2 C_2)}$$

$$= \frac{-L_2(\omega^2 L_1 C_1 - 1)}{C_1(1-\omega^2 L_2 C_2)}$$

if $L_1 C_1 = L_2 C_2$ then

$$\frac{L_1}{C_2} = \frac{L_2}{C_1} = Z_1 Z_2 = R_k^2 \quad \text{—— (3a)}$$

At cut-off freq's, $Z_1 = -4Z_2$

$\times$ by Z_1, we have,

$$Z_1^2 = -4Z_1 Z_2 = -4R_k^2$$

$$Z_1 = \sqrt{-4R_k^2} = \boxed{\pm j 2 R_k = Z_1} \quad \text{—— (A)}$$

i.e. $+Z_1$ at lower cut-off f_1 = $-Z_1$ at upper cut-off f_2.

The reactance of the series arm at the cut-off freq's,

from (2)

$$-j\left(\omega_1 L_1 - \frac{1}{\omega_1 C_1}\right) = j\left(\omega_2 L_1 - \frac{1}{\omega_2 C_1}\right)$$

$$\frac{1}{\omega_1 C_1} - \omega_1 L_1 = \omega_2 L_2 - \frac{1}{\omega_2 C_1}$$

$$\frac{1-\omega_1^2 L_1 C_1}{\omega_1 C_1} = \frac{\omega_2^2 L_1 C_1 - 1}{\omega_2 C_1}$$

$$\omega_2 C_1 - \omega_1^2\omega_2 L_1 C_1^2 = \omega_2^2 \omega_1 L_1 C_1^2 - \omega_1 C_1$$

$\div \omega_2 C_1$, $\quad 1 - \omega_1^2 L_1 C_1 = \frac{\omega_1}{\cancel{\omega_2}}\omega_2 L_1 C_1 - \frac{\omega_1}{\omega_2}$

from (6) & (7) we have

$$L_1 C_1 = \frac{1}{\omega_0^2} \rightarrow L_1 = \frac{1}{\omega_0^2 C_1}$$

Sub C_1 from (8); $f_1 f_2 = f_0$

$$L_1 = \frac{\cancel{4\pi} R \cdot f_1 f_2}{\cancel{4}\pi^{\cancel{2}} \cancel{f_0^2} (f_2 - f_1)}$$

$$\boxed{L_1 = \frac{R}{\pi (f_2 - f_1)}} \quad \text{—— (9)}$$

||ly it is possible to obtain the values for the shunt arm,

$$\frac{L_2}{C_1} = \frac{L_1}{C_2} = R^2 \quad \text{ie} \quad \frac{L_2}{C_1} = R^2$$

$$L_2 = C_1 R^2 = \frac{(f_2 - f_1) R^2}{4\pi R f_1 f_2}$$

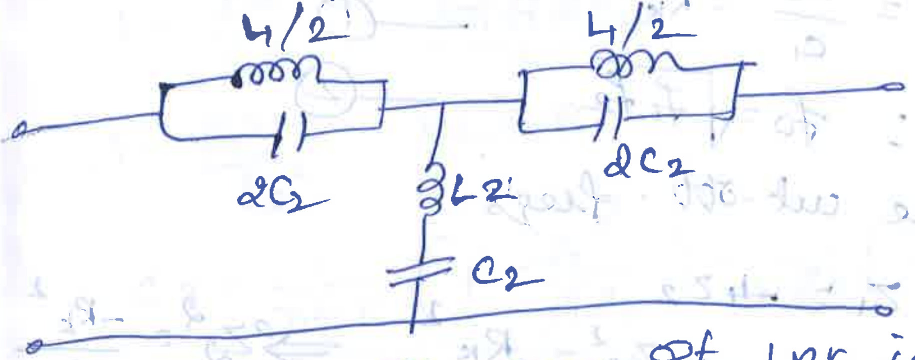
$$L_2 = \frac{R(f_2 - f_1)}{4\pi f_1 f_2} \quad \text{—— (10)}$$

$$C_2 = \frac{L_1}{R^2} = \frac{R}{\pi (f_2 - f_1) R^2} = \frac{1}{\pi R (f_2 - f_1)} \quad \text{—— (11)}$$

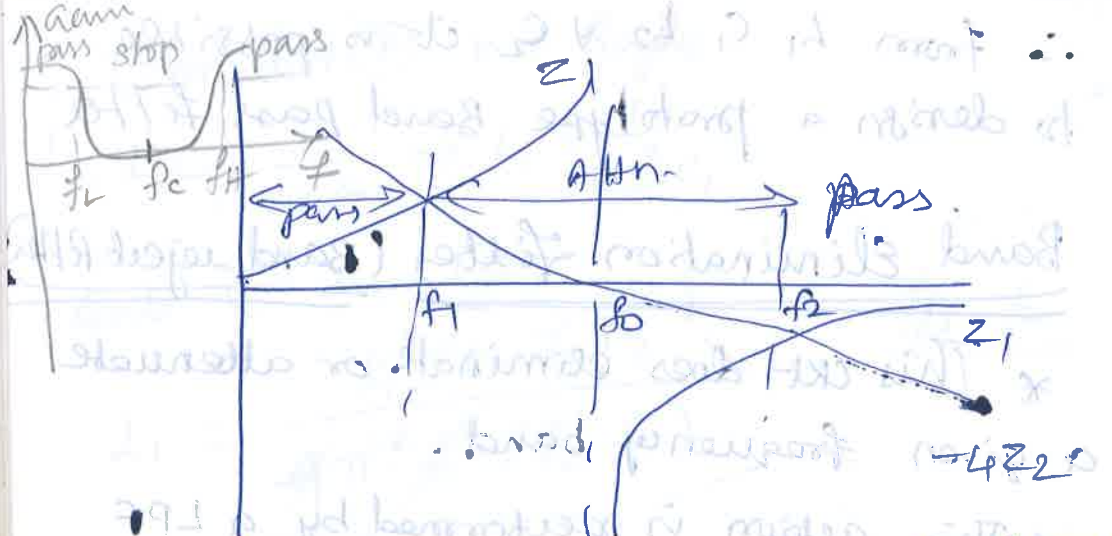
∴ from L_1 C_1 L_2 & C_2 it is possible to design a prototype Band pass filter

Band Elimination filter (Band reject filter)

* This ckt does eliminate or attenuate a given frequency band.
* This action is performed by a LPF in ||l with HPF section.
* The ckt is obtained by interchange the series & ||l tuned arms of the BPF



* The cut off freq of LPF is below that of HPF.



As for the BPF, the series & shunt arms are made antiresonant & resonant at the same freq f_0.

$$\frac{L_1}{C_2} = \frac{L_2}{C_1} = R_k^2 \quad \text{—— (1)}$$

$$\therefore \quad f_0 = \sqrt{f_1 f_2} \quad \text{—— (2)}$$

At the cut-off freq's:

$$Z_1 = -4Z_2$$

$$Z_1 Z_2 = -4Z_2^2 = R_k^2 \Rightarrow Z_2^2 = \frac{-R_k^2}{4}$$

$$Z_2 = \pm j\frac{R_k}{2} \quad \text{—— (3)}$$

if the filter is terminated in a load $R = R_k$

then at the lower cut-off freq,

$$\cancel{Z_2 = j\left(\omega_1 L_1 - \frac{1}{\omega C_1}\right)}$$

$$Z_2 = j\left(\frac{1}{\omega_1 C_2} - \omega_1 L_2\right) = j\frac{R}{2} \quad \text{—— (3)}$$

Since $L_2 C_2 = \frac{1}{\omega_0^2}$ $\qquad \omega_0^2 L_1 C_1 = \omega_0^2 L_2 C_2 = 1$

Anti resonant series arm is made correspond to the resonant freq of the series arm.

(a)

$$j\left(\frac{1-\omega_1^2 L_2 C_2}{\omega_1 C_2}\right) = j\frac{R}{2}$$

$$1 - \frac{\omega_1^2}{\omega_0^2} = \frac{\omega_1 R C_2}{2}$$

$$1 - \frac{f_1^2}{f_0^2} = \frac{2\pi f_1 R C_2}{2}$$

$$1 - \frac{f_1^2}{f_0^2} = \pi f_1 C_2 R$$

$$C_2 = \frac{1}{\pi f_1 R}\left(1 - \frac{f_1^2}{f_1 f_2}\right) = \frac{1}{\pi R}\left(\frac{f_2 - f_1}{f_1 f_2}\right)$$

$$\boxed{C_2 = \frac{1}{\pi R}\left(\frac{f_2 - f_1}{f_1 f_2}\right)} \quad \text{—— (4)}$$

$$f_0 = \sqrt{f_1 f_2} = \frac{1}{2\pi\sqrt{L_2 C_2}} \; ; \; f_0^2 = \frac{1}{4\pi^2 L_2 C_2}$$

(a) →

$$L_2 = \frac{1}{4\pi^2 f_0^2 C_2} \Rightarrow \frac{\cancel{\pi} \cdot R \cdot (\cancel{f_1 f_2})}{4\pi^{\cancel{2}} (\cancel{f_1 f_2})(f_2 - f_1)}$$

$$\boxed{L_2 = \frac{R}{4\pi (f_2 - f_1)}} \quad \text{— (5)}$$

from equ (1),

$$\frac{L_1}{C_2} = R^2 \; ; \; L_1 = R^2 \cdot C_2$$

$$L_1 = \frac{R^2}{\pi \cdot R} \frac{(f_2 - f_1)}{(f_1 f_2)} = \frac{R(f_2 - f_1)}{\pi \cdot f_1 f_2} \quad \text{— (6)}$$

$$\frac{L_2}{C_1} = R^2 \; ; \; C_1 = \frac{L_2}{R^2} = \frac{R}{R^2 \, 4\pi (f_2 - f_1)}$$

$$\boxed{C_1 = \frac{1}{4\pi R \cdot (f_2 - f_1)}} \quad \text{— (7)}$$

Attenuators. (4 terminal N/w)

Amplifier → used to ↑se the signal level by a given amount.

Attenuators → used to ↓se the signal level by a given amount

* Usually resistive N/w.

→ introduce a given loss b/w specified impedances.

classified into → Symmetrical &
Asymmetrical

→ Either fixed type or variable type.

Resistance attenuators — have only resistive components —

— Introduce no phase shift ∴ $\beta = 0$

& prop const $\gamma = \alpha$

→ Eg: volume control in Broad casting stations.

Attenuators must satisfy;

1) give correct i/p imp.

2) give correct o/p imp.

3) must provide specified attn.

Capacitance Attenuators — HF Applns.

Symmetrical Attenuators

* Used b/w two equal impedances

* Since the attenuators have only resistive components, its charac. imp will also resistive.

Attn is usually in decibels & nepers

Attn $D = 10 \log_{10} \frac{P_1}{P_2}$

$P_1 \rightarrow$ i/p power ; $P_2 \rightarrow$ o/p power.

say $N = \sqrt{\frac{P_1}{P_2}}$

$\therefore D = 10 \log_{10} N^2 = 20 \log_{10} N$

$N = \text{Anti}\log_{10}\left(\frac{D}{20}\right)$.

In case of Symmetrical Attn's both pair of terminals are matched to the same resistance — the chara. resistance of the attenuator.

$$\therefore \frac{P_1}{P_2} = \frac{R_0 I_S^2}{R_0 I_R^2} = \left(\frac{I_S}{I_R}\right)^2$$

$$N^2 = \left(\frac{I_S}{I_R}\right)^2 \quad \& \quad \boxed{N = \frac{I_S}{I_R}}$$

Types of Symmetrical attenuators

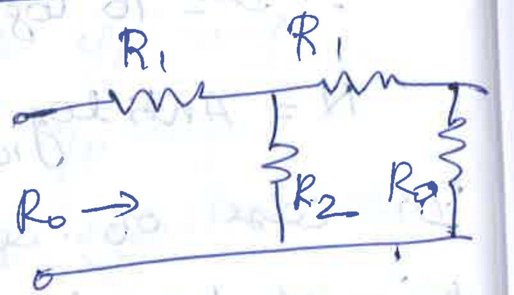
1) T

2) π

3) Lattice

4) Bridged T

1) Symmetrical T - attenuators



$$Z_2 = \frac{Z_0}{\sinh \gamma l}$$

$$Z_1 = \cancel{2Z_2} = Z_0\left(\frac{e^{\gamma l/2} - e^{-\gamma l/2}}{e^{\gamma l/2} + e^{-\gamma l/2}}\right) = Z_0 \tanh \frac{\gamma l}{2}$$

$R_1 = R_0 \tanh \alpha/2$

$R_2 = R_0/\sinh \alpha$ since $Z_0 = R_0$, $\gamma = \alpha$ for attenuator.

$e^{\gamma} = e^{P} = \frac{I_s}{I_R}$ $P = \alpha$ for resistance attenuator.

Hence $e^{\alpha} = \frac{I_s}{I_R} = N$,

$$R_1 = R_0\left(\frac{e^{\alpha/2} - e^{-\alpha/2}}{e^{\alpha/2} + e^{-\alpha/2}}\right)$$

× & ÷ by RHS by $e^{\alpha/2}$ $R_1 = R_0\left(\frac{e^{\alpha} - e^{0}}{e^{\alpha} + e^{0}}\right)$

$$R_1 = R_0 \frac{e^{\alpha} - 1}{e^{\alpha} + 1} = R_0 \frac{N-1}{N+1}\ ;\ e^{\alpha} = N \quad (1)$$

$$R_2 = \frac{R_0}{\frac{e^{\alpha} - e^{-\alpha}}{2}} = \frac{2R_0}{e^{\alpha} - e^{-\alpha}} = \frac{2R_0}{N - 1/N} \quad (2)$$

$$= R_0 \cdot \frac{2N}{N^2 - 1} \quad (2)$$

Eqn's (1) & (2) are called design eqns of the symmetrical T-attenuator.

The attn. introduced by a symmetrical T attenuator can be determined by,

$$\cosh \alpha = \left(1 + \frac{Z_1}{2Z_2}\right) \text{neper} = \left(1 + \frac{R_1}{R_2}\right) \text{neper}$$

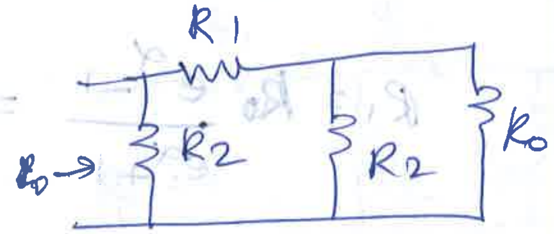
$$\alpha = \cosh^{-1}\left(1 + \frac{R_1}{R_2}\right) \text{neper}$$

to get dB, × by 8.686

Symmetrical π-attenuator

$R_1 = R_0 \sinh\alpha$

$R_2 = R_0 \coth\frac{\alpha}{2}$



$Z_0 = R_0$ & $P = \alpha$ for resistance attenuator

By defn. of propagation const,

$e^{\gamma} = e^{P} = \frac{I_S}{I_R}$ — But $P = \alpha$

$e^{\alpha} = \frac{I_S}{I_R} = N$.

The series and shunt arm may be written as,

$R_1 = R_0 \frac{e^{\alpha} - e^{-\alpha}}{2} = R_0 \frac{N - \frac{1}{N}}{2}$

$= R_0 \frac{N^2 - 1}{2N}$ — (3)

||ly $R_2 = R_0 \coth\frac{\alpha}{2} = R_0 \frac{e^{\alpha/2} + e^{-\alpha/2}}{e^{\alpha/2} - e^{-\alpha/2}} = R_0 \frac{e^{\alpha} + 1}{e^{\alpha} - 1}$

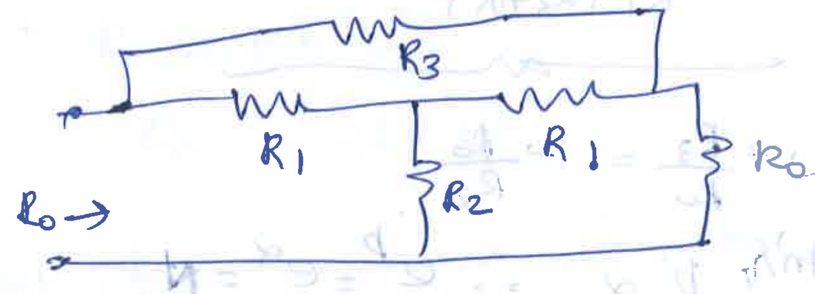
$R_2 = R_0 \left(\frac{N+1}{N-1}\right)$ — (4)

equs (3) & (4) are the design equ's for the symmetrical π-attenuator.

Symmetrical Bridged T attenuator

It may be designed to have a contact resistance R_0 but any desired attn. by making.

$R_3 R_2 = R_1^2 = R_0^2$ — (5)



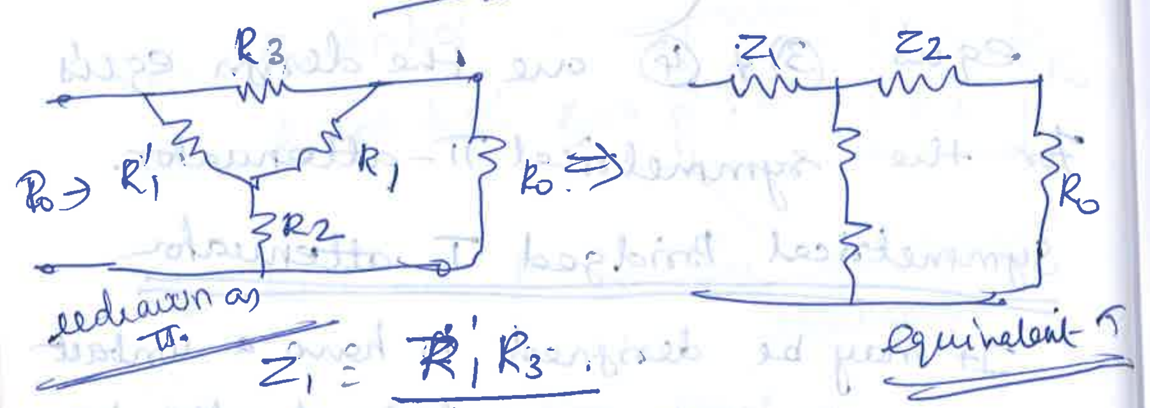
The propagation const. can be expressed as,

case1: when $R_1 = R_0$

$P = \log_e\left(1 + \frac{R_3}{R_0}\right)$ &

$P = \log_e\left(1 + R_0/R_2\right)$

$Z_3 = R_3 = Z_2 = R_2$ & $Z_0 = R_0$



$$Z_1 = \frac{R_1' R_3}{R_1' + R_3 + R_1}$$

$$Z_2 = \frac{R_3 R_1}{R_1' + R_3 + R_1}$$

$$e^P = 1 + \frac{R_3}{R_0} = 1 + \frac{R_0}{R_2}$$

For Attn, $P = \alpha$ $\therefore$ $e^P = e^\alpha = N$.

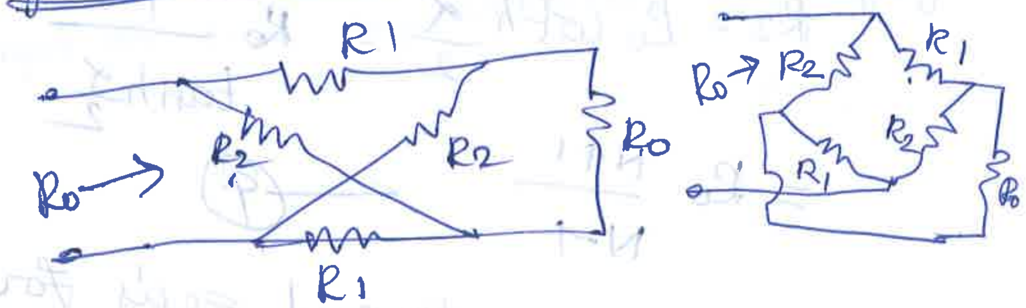
$$N = 1 + \frac{R_3}{R_0} = 1 + \frac{R_0}{R_2}$$

$$R_3 = R_0(N-1) \quad (6) \qquad R_2 = \frac{R_0}{N-1} \quad (7).$$

from (5), $R_1 = R_0$,

(6) & (7) are design eqns for bridged T symmetrical attenuator.

Symmetrical Lattice Attenuators



$$R_1 = Z_0 \tanh \frac{P}{2}$$

$$R_2 = Z_0 \coth \frac{P}{2}$$

But for resistance attenuator, $Z_0 = R_0$, and $P = \alpha$

The series arm R_1 & shunt arm R_2 can be written as,

$$R_1 = R_0 \tanh\frac{\alpha}{2} = R_0 \frac{e^{\alpha/2} - e^{-\alpha/2}}{e^{\alpha/2} + e^{-\alpha/2}}$$

$$= R_0 \frac{e^{\alpha}-1}{e^{\alpha}+1} = R_0 \frac{N-1}{N+1} \quad \text{--- (8)}$$

Ily $R_2 = R_0 \coth\frac{\alpha}{2} = R_0 \frac{1}{\tanh\frac{\alpha}{2}}$

$$= R_0 \frac{N+1}{N-1} \quad \text{--- (9)}$$

(8) & (9) are the designed eqn's for Symmetrical lattice attenuator.

Asymmetrical Attenuators

Attn. a connected b/w two imps of unequal value an asymmetrical attn. may be designed to have image imp's equal to the given imps...

when Na is the attn. of an asymmetrical attenuator.

$$N_a = \sqrt{\frac{P_1}{P_2}} = \sqrt{\frac{I_S^2}{I_R^2} \cdot \frac{R_{i1}}{R_{i2}}} = \frac{I_S}{I_R}\sqrt{\frac{R_{i1}}{R_{i2}}} \quad \text{--- (10)}$$

where R_{i1} & R_{i2} are the image imp's of the asymmetrical attenuator.

Functional classification of N/w's

1. Filters.
2. Attenuators
3. Equalizers — corrective N/w.
4. Matching N/w.