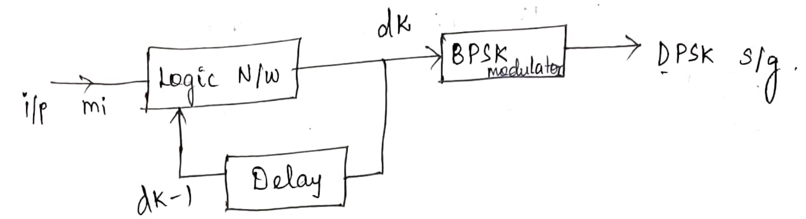
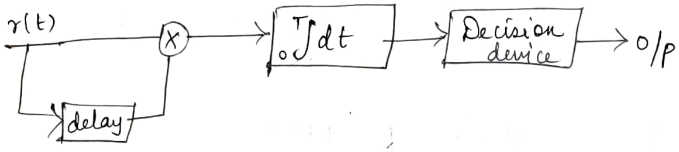
Txr:



Rxr:



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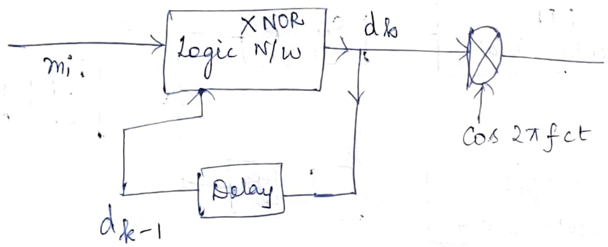

DPSK Transmitter

$$d_k = m_i d_{k-1} + \overline{m_i}\, \overline{d_{k-1}}$$

m_i	1	0	0	1	1	1	0	0
d_{k-1}	1	1	0	1	1	1	1	0
$m_i d_{k-1}$	1	0	0	1	1	1	0	0
$\overline{m_i}\, \overline{d_{k-1}}$	0	0	1	0	0	0	0	1
d_k	1	0	1	1	1	1	0	1
BPSK	0	π	0	0	0	0	π	0

UNIT-3 Information theory & Coding.

Probability is represented by P_k

Information is represented by I_k

If P_k is more I_k is less.

Information can be additive.

Prob1: In a village, it has 8 telephones how long must be the phone number?

Length = 3 (bcoz of binary) digital information

Channel capacity is the no. of bits it can allow in one second.

Source:

Source is a random variable denoted by S. which takes symbols $S = \{S_0, S_1, \dots S_{k-1}\}$ with probability

$$P(S = S_k) = P_k$$

$$\boxed{\sum_{k=0}^{k-1} P_k = 1}$$

The following assumptions are made about source:

i) The symbols are statistically independent that are produced by the source. Since it is statistically independent the channel is memory-less.

ii) $I_k = \log_2 \left(\frac{1}{P_k}\right)$

Properties of information:

i) Entropy:

Entropy is the average information generated by a discrete memoryless source.

It is defined as $H(S) = \sum_{k=0}^{K-1} P_k I_k$.

Properties of Entropy:

i) If K is the no. of symbols generated by a source then entropy is bounded by

$$\boxed{0 \leq H(S) \leq \log_2 K}$$

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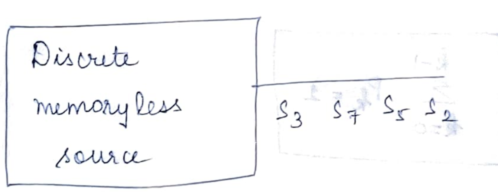
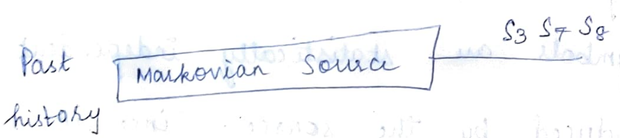
DMS



$$P(S_3 S_7 S_5 S_2) = P(S_3)\, P(S_7)\, P(S_5)\, P(S_2)$$



$$P(S_3 S_7 S_8) = P(S_3)\, P(S_7/S_3)\, P(S_8/S_3, S_7)$$

Let symbol set = $\{S_1, S_2, \ldots, S_K\}$

Source Probability = $\{P_1, P_2, \ldots, P_K\}$

Information associated with the source

$$I_K = \log_2 \left(\frac{1}{P_K}\right)$$

$$\sum_{i=1}^{K} P_i = 1$$

Average information associated with source symbol (entropy)

$$H(S) = \sum_{K=1}^{K-1} P_k \log_2 \left(\frac{1}{P_K}\right)$$

$$0 \leq H(S) \leq \log_2 k$$

where k is no. of symbols

Information Rate, $R = R_S\, H(S)$

(R_S ↑ symbol rate)

Prob 1: Consider a binary source for which symbol 0 occurs with a probability P_0 & symbol 1 with probability P_1. Find the source entropy. Assume source is memoryless.

$$P_1 = 1 - P_0$$

$$H(S) = \sum_{k=0}^{2-1} P_k \log_2 \left(\frac{1}{P_k}\right)$$

$$= P_1 \log_2 \left(\frac{1}{P_1}\right) + P_0 \log_2 \left(\frac{1}{P_0}\right)$$

$$H(S) = (1 - P_0) \log_2 \left(\frac{1}{1-P_0}\right) + P_0 \log_2 \left(\frac{1}{P_0}\right)$$

$$H(S) = -P_0 \log_2 P_0 - (1-P_0)\log_2(1-P_0)$$

i) If $P_0 = 0$

$$\boxed{H(S) = 0}$$

ii) If $P_0 = 1$

$$H(S) = -1(0) = 0.$$

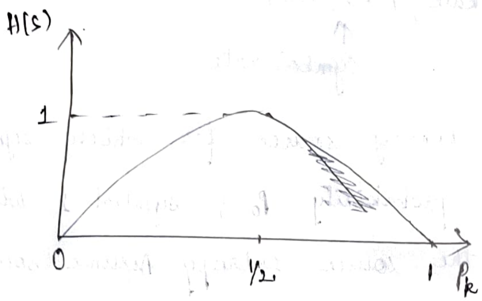
iii) If $P_0 = P_1 = 1/2$

$$\boxed{H(S) = max}$$

$$H(S) = -\frac{1}{2}\log_2(2)^{-1} - \left(\frac{1}{2}\right)\log_2(2)^{-1}$$

$$= +\frac{1}{2} + \frac{1}{2} = \frac{2}{2} = 1. \qquad \boxed{H(S) = 1}$$

Symbols are equiprobable. ∴ entropy is max



Prob 2: For a discrete memoryless source with source alphabet $S = \{S_0, S_1, S_2\}$ with probabilities $P(S_0) = 1/4$, $P(S_1) = 1/4$, $P(S_2) = 1/2$. i) Calculate the entropy of the source. ii) Calculate the entropy of second order extension (i.e) $H(S^2)$

$$H(S) = \sum_{K=0}^{2} P_k \log_2\left(\frac{1}{P_k}\right)$$

$$= \frac{1}{4}\log_2 2^2 + \frac{1}{4}\log_2 2^2 + \frac{1}{2}\log_2 2$$

$$= \frac{2}{4} + \frac{2}{4} + \frac{1}{2} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = 1 + \frac{1}{2}$$

$$\boxed{H(S) = 1\tfrac{1}{2}} \text{ or } \boxed{H(S) = 3/2}$$

Let symbols of $S^2 = \{\sigma_0, \sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5, \sigma_6, \sigma_7, \sigma_8\}$

$S^2 = \{S_0S_0, S_0S_1, S_0S_2, S_1S_0, S_1S_1, S_1S_2, S_2S_0, S_2S_1, S_2S_2\}$

$$P(S^2) = \left\{\frac{1}{16}, \frac{1}{16}, \frac{1}{8}, \frac{1}{16}, \frac{1}{16}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{4}\right\}$$

$$H(S^2) = \sum_{\sigma=0}^{8} P_\sigma \log_2\left(\frac{1}{P_\sigma}\right)$$

$$= \frac{4}{16}\log_2 2^4 + \frac{4}{8}\log_2 2^3 + \frac{1}{4}\log_2 2^2$$

$$= \frac{4\times 4}{16} + \frac{4\times 3}{8} + \frac{2}{4}$$

$$= 1 + \frac{1}{2} + \frac{3}{2} = 3.$$

$$\boxed{H(S^2) = 3}$$

$$\boxed{H(S^2) = 2H(S)}$$

$$\boxed{\therefore H(S^2) = 2H(S)}$$

$$\boxed{\text{In general } H(S^n) = nH(S)}$$

Source Coding theorem.

Requirements:

i) ~~Sou~~ Code words produced by the encoder are in binary form.

ii) The source code is uniquely ~~decodade~~ decodable so that original source sequence can be reconstructed perfectly from the encoded binary sequence.

Assumptions:

Let the binary code word assigned with symbol s_k by the encoder have length l_k.

The average codeword length $\bar{L} = \sum_{k=0}^{K-1} P_k l_k$

Source Coding Theorem / Shannon Theorem: $\boxed{\text{All coding schemes (x) 16m}}$

Given a discrete memoryless source of entropy $H(S)$. The average codeword length $\bar{L}$ for any source coding is bounded as $\boxed{\bar{L} \ge H(S)}$

Efficiency of the source encoder is given by

$$\boxed{\eta = \frac{H(S)}{\bar{L}}}$$

Prefix Coding:

A code in which no codeword is the prefix of any other codeword.

Example:

Source Symbol	Probability of occurence.	Code I	Code II	Code III
s_0	0.5	0	0	0
s_1	0.25	1	10	01
s_2	0.125	00	110	011
s_3	0.125	11	111	0111

prefix prefix prefix

Code-II is the acceptable codeword.

i) Prefix codes are always uniquely decodable.

ii) Decoding of a prefix code can be accomplished as soon as the binary sequence representing a source symbol is fully received.

Prefix codes are also known as instanteneous codes. It is defined as a code in which no code word is the prefix of any other codeword

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Shanon-Fano Code:

i) Order symbols in descending order of probability.

ii) Divide symbols into sub groups such that sub groups probabilities are as close as possible.

iii) Assign bit 0 to top sub group & bit 1 to bottom sub group.

iv) Iterate steps (ii) & (iii) as long as there is more than one symbol in any sub group.

v) Extract variable length codewords from the resulting tree (top down)

Prob1:) 8 symbols A, B, C, D, E, F, G, H with probabilities $1/2, 1/4, 1/8, 1/16, 1/32, 1/64, 1/128, 1/128$. are given using shannon fano code. Find coding efficiency.

$$\eta = \frac{H(S)}{\bar{L}}$$

Source symbol	Probability of occurence	Code Words	Bits/Symbol (l_k)
A	1/2	0	1
B	1/4	~~01~~ 10	2
C	1/8	110	3
D	1/16	1110	4
E	1/32	11110	5
F	1/64	111110	6
G	1/128	1111110	7
H	1/128	1111111	7

$$H(S) = \sum_{k=0}^{7} P_k \log_2\left(\frac{1}{P_k}\right)$$

$$= \frac{1}{2}\log_2 2 + \frac{2}{4}\log_2 2 + \frac{3}{8}\log_2 2 + \frac{4}{16}\log_2 2 + \frac{5}{32}\log_2 2 + \frac{6}{64}\log_2 2 + \frac{7\times 2}{128}\log_2 2.$$

$$= 1.9296875 + \frac{7}{128}$$

$$H(S) = 1.984375 = \frac{127}{64}$$

$$\bar{L} = \sum_{k=0}^{7} P_k l_k$$

$$= \left(\frac{1}{2}\right)\times 1 + \left(\frac{1}{4}\right)2 + \left(\frac{1}{8}\right)3 + \left(\frac{1}{16}\right)4 + \left(\frac{1}{32}\right)5 + \left(\frac{1}{64}\right)6 + \left(\frac{1}{128}\right)7 + \left(\frac{1}{128}\right)7$$

$$= \frac{127}{64}$$

$$\eta = \frac{\frac{127}{64}}{\frac{127}{64}} = 1$$

$$\boxed{\eta = 100\%}$$

Prob2:) 8 symbols with probability 0.04, 0.04, ~~0.06~~ 0.16, ~~0.06~~ 0.06, 0.17, 0.27, 0.20, 0.06. Find coding efficiency.

Symbol	Probability	Code word	Bits/symbol
A	0.27	00	2
B	0.20	01	2
C	0.17	100	3
D	0.16	101	3
E	0.06	1100	4
F	0.06	1101	4
G	0.04	~~10000~~ 1110	4
H	0.04	~~10001~~ 1111	4

Less probable → Longer codeword

3.703
5.882
16.667

$$H(S) = \sum_{k=0}^{7} P_k \log_2\left(\frac{1}{P_k}\right)$$

$= 0.27 \times 1.888 + 0.20 \times 2.32 + 0.17 \times 2.556$

$+ 0.16 \times 2.644 + 0.06 \times 4.059 \times 2$

$+ 0.04 \times 4.644 \times 2$

$= 0.51 + 0.4644 + 0.43452 + 0.42304 +$

$0.48708 + 0.37152$

$H(S) = 2.69056$

$$\bar{L} = \sum_{k=0}^{7} P_k l_k$$

$= 0.27 \times 2 + 0.20 \times 2 + 0.17 \times 3 + 0.16 \times 3$

$+ 0.06 \times 8 + 0.04 \times 8$

$= 2.73$

$$\eta = \frac{H(S)}{\bar{L}} = \frac{2.69056}{2.73}$$

$= 0.9855$

$\boxed{\eta = 98.55\%}$

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HUFFMANN CODING:

Procedure:

i) The source symbols are listed in the order of decreasing probabilities.

ii) Two source symbols of lowest probability are assigned as '0' & '1'.

ii) These two source symbols are being combined into a new source symbol with probability equal to the sum of the two original probabilities. The probability of the new symbol is placed in the list in accordance with its value.

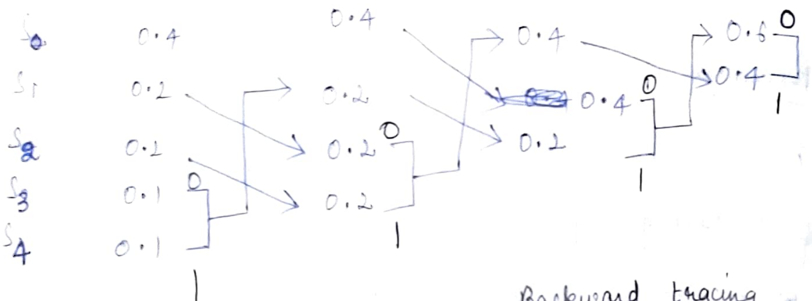
iii) The procedure is repeated until we are left with final list of source symbols of only two for which '0' & '1' are assigned. The code for each source symbol is found by working backward & tracing the sequence of '0's' & '1's' assigned to that symbol & as well as its successors.

16m

Prob 1:) Using Huffman coding, find the efficiency of given source symbols.

S_0	0.1
S_1	0.2
S_2	0.4
S_3	0.1

Soln:



Backward tracing

Symbol	Probability	Codeword	l_k
S_0	0.4	00	2
S_1	0.2	10	2
S_2	0.2	11	2
S_3	0.1	010	3
S_4	0.1	011	3

$$H(S) = \sum_{k=0}^{4} P_k \log_2\left(\frac{1}{P_k}\right)$$

$$= 0.4 \log_2\left(\frac{1}{0.4}\right) + 0.4 \log_2\left(\frac{1}{0.2}\right) + 0.2 \log_2\left(\frac{1}{0.1}\right)$$

$$= +0.5288 + 0.9288 + 0.6644$$

$$= +2.122$$

$$\bar{L} = \sum_{k=0}^{4} P_k l_k$$

$$= 0.4 \times 2 + 0.2 \times 4 + 0.1 \times 6$$

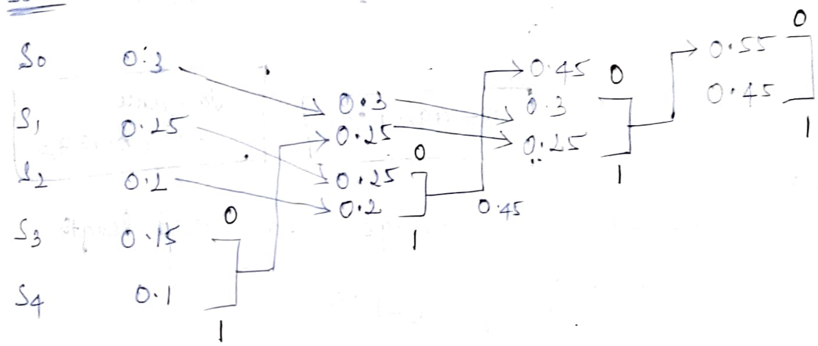
$$= 0.8 + 0.8 + 0.6 = 2.2$$

$$\eta = \frac{H(S)}{\bar{L}} = \frac{+2.122}{2.2} = +0.9645$$

$$\boxed{\eta = 96.45\%}$$

Prob 2: Find efficiency. Symbol probabilities 0.1, 0.2, 0.15, 0.3, 0.25

Soln:



Symbol	Probability	Codeword	l_k
S_0	0.3	00	2
S_1	0.25	10	2
S_2	0.2	11	2
S_3	0.15	010	3
S_4	0.1	011	3

$$H(S) = \sum_{k=0}^{4} P_k \log_2\left(\frac{1}{P_k}\right)$$

$$= 0.3 \log_2\left(\frac{1}{0.3}\right) + 0.25 \log_2\left(\frac{1}{0.25}\right) + 0.2 \log_2\left(\frac{1}{0.2}\right) + 0.15 \log_2\left(\frac{1}{0.15}\right) + 0.1 \log_2\left(\frac{1}{0.1}\right)$$

$$= 0.5210 + 0.5 + 0.4643 + 0.4105 + 0.3321$$

$$= 2.2279$$

$$\bar{L} = \sum_{k=0}^{4} P_k l_k$$

$= 0.3\times 2 + 0.25\times 2 + 0.2\times 2 + 0.15\times 3 + 0.1\times 3$

$= 0.6 + 0.5 + 0.4 + 0.45 + 0.3$

$= 2.25$

$$\eta = \frac{H(S)}{\bar{L}} = \frac{2.2279}{2.25} = 0.9901$$

$\boxed{\eta = 99.01\%}$ $\boxed{\text{Variance } \sigma^2 = 0.1875}$

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Variance of average code word length:

$$\sigma^2 = \sum_{k=1}^{K-1} P_k (l_k - \bar{L})^2$$

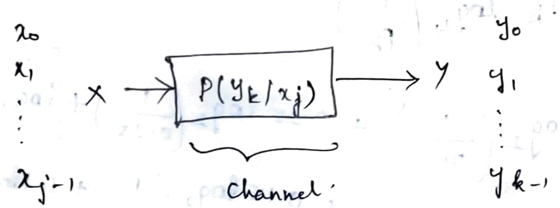
$\sigma^2 = (0.4)[2-2.2]^2 + (0.2)[2-2.2]^2 + 0.2[2-2.2]^2 + 0.1[3-2.2]^2 + 0.1[3-2.2]^2$

$= 0.016 + 8\times10^{-3} + 8\times10^{-3} + 0.064 + 0.64$

$= 0.144 \times 10^{-3}$ (16)

$\boxed{\sigma^2 = 0.16}$

Discrete Memoryless channels: (General)



X & Y → Random variables

$P(y_k/x_j)$ → Transition probability (or) Conditional probability

Channel matrix:

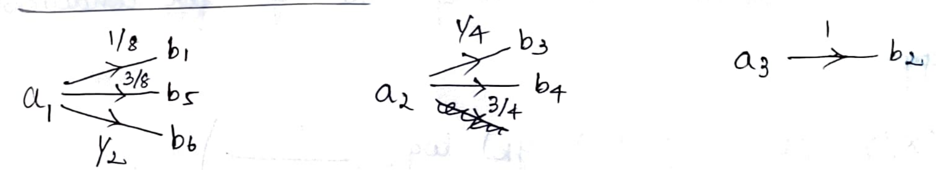
$$P(y/x) = \begin{bmatrix} P(y_0/x_0) & P(y_1/x_0) & \cdots & P(y_{k-1}/x_0) \\ P(y_0/x_1) & P(y_1/x_1) & \cdots & P(y_{k-1}/x_1) \\ \vdots & & & \\ P(y_0/x_{j-1}) & P(y_1/x_{j-1}) & \cdots & P(y_{k-1}/x_{j-1}) \end{bmatrix}$$

$$\boxed{\sum_{k=0}^{K-1} P(y_k/x_j) = 1}$$

Rows – input
Columns – output

TYPES OF CHANNEL:

i) Lossless channel:



$$P(y/x) = \begin{bmatrix} 1/8 & 0 & 0 & 0 & 3/8 & 1/2 \\ 0 & 0 & 1/4 & 3/4 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\cancel{P(x/y) = \begin{bmatrix} 1/8 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1/4 & 0 \\ 0 & 3/4 & 0 \\ 3/8 & 0 & 0 \\ 1/2 & 0 & 0 \end{bmatrix}}$$

$$
\begin{array}{cc}
 & \begin{array}{ccc} a_1 & a_2 & a_3 \end{array} \\
P(A/B) \begin{array}{c} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_6 \end{array} & \left|\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{array}\right|
\end{array}
$$

Mutual information:

Channel output 'y' has a noisy version of channel input x then entropy is uncertain about x. To avoid uncertainity we go for conditional entropy.

$$H(X/Y=y_k) = \sum_{j=0}^{J-1} P(x_j/y_k) \log_2 \left(\frac{1}{P(x_j/y_k)}\right)$$

The mean value is represented as

$$H(X/Y) = \sum_{k=0}^{K-1} \sum_{j=0}^{J-1} P(y_k)\, H(X/Y=y_k)$$

$$= \sum_{k=0}^{K-1} \sum_{j=0}^{J-1} \underbrace{P(x_j/y_k)\cdot P(y_k)} \log_2 \frac{1}{P(x_j/y_k)}$$

$$P(x_j, y_k) = P(x_j/y_k)\cdot P(y_k)$$

$$\boxed{H(X/Y) = \sum_{k=0}^{K-1} \sum_{j=0}^{J-1} P(x_j, y_k) \log_2 \frac{1}{P(x_j/y_k)}}$$

↓

Conditional entropy.

Because of noise we go for conditional entropy

Conditional entropy represents the amount of uncertainity remaining about the channel ~~output~~ input after the channel output has been observed

Mutual information has been defined as $I(X,Y)$

$$I(X,Y) = H(X) - H(X/Y)$$

Properties of Mutual Information:

→ Mutual information of a channel is symmetric

$$I(X,Y) = I(Y,X)$$

→ Mutual information is always non negative

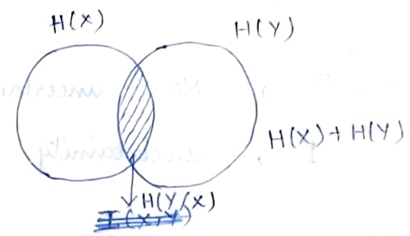
$$I(X,Y) \geq 0$$

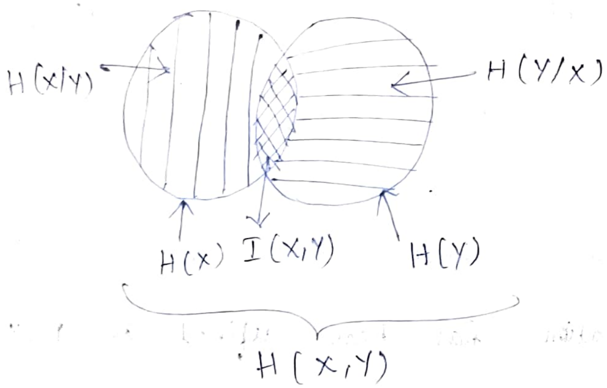
→ Mutual information can be represented as

$$I(X,Y) = H(Y) - H(Y/X)$$

→ Joint entropy

$$H(X,Y) = -I(X,Y) + H(X) + H(Y)$$





Channel capacity:

It is the maximum average mutual information is channel capacity (C)

$$C = \max I(X,Y)$$

$$I(X,Y) = \sum_{j=0}^{J-1} \sum_{k=0}^{K-1} P(x_j, y_k) \log \frac{P(y_k/x_j)}{P(y_k)}$$

For a lossless channel:

$$H(A/B) = -\sum_{j=1}^{J-1} \sum_{k=1}^{K-1} P(a_k, b_j) \log_2 P(a_k/b_j)$$

$$= -\sum_{j=1}^{J-1} \sum_{k=1}^{K-1} P(b_j)\, P(a_k/b_j) \log_2 P(a_k/b_j)$$

$$H(A/B) = \begin{cases} 0, & \text{No uncertainity} \\ 1, & \text{Uncertainity present} \end{cases}$$

$$I(A,B) = H(A) - H(A/B)$$

Channel capacity $C = \max(I(A,B)) = \max(H(A))$

$$= \max\left\{-\sum_{k=0}^{K-1} P(a_k) \log_2 P(a_k)\right\}$$

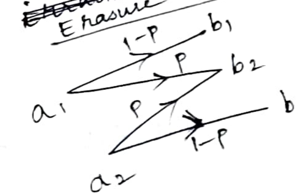
$$P(a_1) = P(a_2) = \cdots P(a_k) = \frac{1}{k}.$$

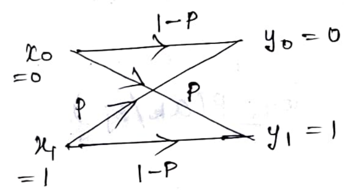
$$C = \max\left\{-\sum_{k=0}^{K-1} P(a_k) \log_2 \left(\frac{1}{k}\right)\right\}$$

$$\boxed{C = \frac{1}{k} \log_2 (k)}$$

ii) Binary Symmetric channel:

Study Binary ~~Erasure~~ Erasure channel



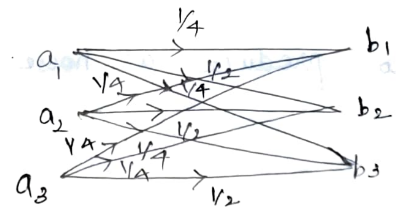


The channel is symmetric because the probability of receiving '1' if '0' is sent is same as that of probability of receiving '0' if '1' is sent.

Here the conditional probability of error is denoted by P.

(*) Capacity derivation read in Simon Haykin.

iii) Uniform channel:



$$P(B/A) = \begin{bmatrix} 1/4 & 1/2 & 1/4 \\ 1/4 & 1/2 & 1/4 \\ 1/4 & 1/4 & 1/2 \end{bmatrix}$$

$$P(A/B) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$H(A/B) = -\sum_{j=0}^{J-1} \sum_{k=0}^{K-1} P(a_k, b_j) \log_2 P(a_k/b_j)$$

iv) Ideal channel:

$a_1 \to b_1$

$a_2 \to b_2$

$\vdots$

$a_k \to b_j$

$$C = \max(I(X,Y))$$

$$= \frac{1}{k} \log_2(k) = \frac{1}{j} \log_2(j) \quad (\because k = j)$$

$$\boxed{C = \log_2(k) = \log_2(j)}$$

Channel coding theorem: (Shannon's IInd theorem)

→ It is used to produce a noise resistance communication.

Let a discrete memoryless source have entropy $H(S)$ and produce symbols once in every T_s seconds. Let a discrete memoryless channel have a capacity C & be used once in every T_c seconds.

Then $\frac{H(S)}{T_s} \le \frac{C}{T_c}$, if this condition is satisfied it is possible to transmit information over channel & reconstruct it with small probability of error.

$\frac{C}{T_c}$ ratio is called as critical rate

Conversely $\frac{H(S)}{T_s} \ge \frac{C}{T_c}$ it is not possible to transmit ~~transfer~~ information over the channel & reconstruct is also not possible.

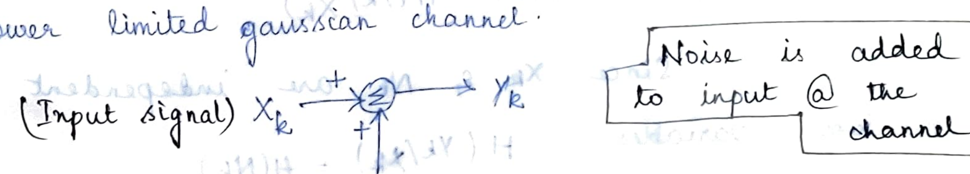
Code rate, $r = \frac{T_c}{T_s}$

If $r \le C$ then the code is capable of achieving low probability of error.

→ Shannon Hartley Law

Shannon's IIIrd theorem: Channel capacity theorem:

This theorem is defined for band limited, power limited gaussian channel.



N_k (Additive white gaussian noise)

Let X_k where $k = 0, 1, \ldots, n$ denote the continuous random variables obtained by uniform sampling of the zero mean stationary process $X(t)$ at a rate of $2B$ samples per second.

Stationary process: its statistical properties will not vary with time.

The no. of samples transmitted in T seconds over a noisy channel is given by $2BT$.

Let X_k has sample of the transmitted signal, the channel ~~output~~ i/p is added with additive white gaussian noise of zero mean & PSD = $\frac{N_0}{2}$. Y_k denotes the sample of the received signal

$$Y_k = X_k + N_k$$

where $k = 0, 1, 2, \ldots n$

The noise sample N_k is having the variance

$$\boxed{\sigma^2 = N_0 B}$$

$E[X_k^2] = P$ Avg. transmitted power.

Then maximum information is defined as

$$I(X_k, Y_k) = H(Y_k) - H(Y_k/X_k)$$

Since X_k & N_k are independent random variables

$$H(Y_k/X_k) = H(N_k)$$

So, $I(X_k, Y_k) = H(Y_k) - H(N_k)$ —— (1)

Variance of the sample Y_k of the signal $\sigma_{Y_k}^2 = P + \sigma_{N_k}^2$.

∴ Entropy of Y_k can be written as

$$H(Y_k) = \frac{1}{2}\log_2 2\pi e(P + \sigma_{N_k}^2) \quad \text{—— (2)}$$

Variance of N_k is $\sigma_{N_k}^2 = N_0 \cdot B$

Then entropy of noise

$$H(N_k) = \frac{1}{2}\log_2(2\pi e\sigma_{N_k}^2) \quad \text{—— (3)}$$

Sub. (2) & (3) in (1) we get

$$I(X_k, Y_k) = \frac{1}{2}\log_2 2\pi e(P + \sigma_{N_k}^2) - \frac{1}{2}\log_2(2\pi e$$

$$\boxed{C = \frac{1}{2}\log_2\left(1 + \frac{P}{\sigma_{N_k}^2}\right)}$$

$\sigma_{N_k}^2 \to$ noise power

$P \to$ signal power

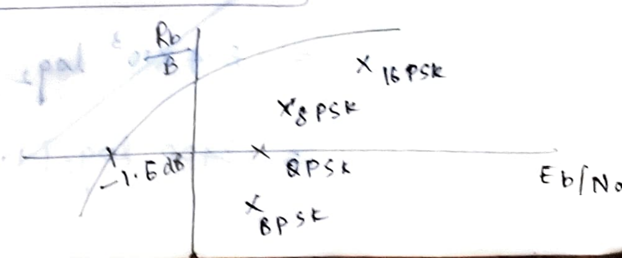
$\frac{P}{\sigma_{N_k}^2}$ = SNR

$C \Rightarrow$ depends on SNR

The channel is used 'n' times

$$\boxed{C = B\log_2\left(1 + \frac{P}{\sigma_{N_k}^2}\right)}$$

$$\boxed{C = B\log_2(1 + SNR)}$$



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Channel Capacity Theorem:

$$C = B \log_2 \left(1 + \frac{P}{N_0 B}\right) \text{ bits/sec}$$

Channel capacity theorem states that capacity of the channel of bandwidth B Hertz ~~per~~ perturbed by additive white Gaussian noise of PSD $N_0/2$ and limited in bandwidth to B is given by

$$C = B \log_2 \left(1 + \frac{P}{N_0 B}\right)$$

where P is the average transmitted Power.

Prob 1: A voice graded channel of a telephone network has a Bandwidth of 3.4 KHZ.

a.) Calculate the channel capacity of the telephone channel for a SNR of 31 dB

b.) Calculate the minimum SNR required to support information transmission through the telephone channel at a rate of 4800 bits/sec

a.) $C = B \log_2 \left(1 + \frac{P}{N_0 B}\right)$

$\cancel{3.4 \times 10^3 \log_2 \left(1 + \frac{2\,SNR}{B}\right)}$

$\cancel{= 3.4 \times 10^3 \log_2 \left(1 + \frac{62}{3.4K}\right)}$

$\cancel{= 3400 \cdot \log_2 (1.018235294)}$

$= B \log_2 \left(1 + \frac{P}{N_0 B}\right)$

$= B \log_2 (1 + 31)$

$= 3.4 \times 10^3 \log_2 (2^5)$

$= 17 \times 10^3$

$$\boxed{C = 17 \text{ Kbits/sec}}$$

b.) $C = 4800$ bits/sec.

$\frac{4800}{3.4 \times 10^3} = \log_2 (1 + SNR)$

$1.4117 = \frac{\log (1 + SNR)}{\log 2}$

$\log (1 + SNR) = 0.4249$

$1 + SNR = 2.660$

$SNR = 2.660 - 1$

$$\boxed{SNR = 1.66 \text{ dB}}$$

Ideal System:

i) Ideal system is a system which transmits data at a bit rate of $\boxed{R_b = C}$

ii) The average transmitted power is represented as $P = E_b \cdot C$

where E_b is the transmitted bit energy

The ideal system is defined by Shannon's equation.

$$C = B \log_2 \left(1 + \frac{P}{N_0 B}\right)$$

$$\frac{C}{B} = \log_2 \left(1 + \frac{E_b C}{N_0 \cdot B}\right)$$

$$0.3010 \frac{C}{B} = \log_{10} \left(1 + \frac{E_b C}{N_0 \cdot B}\right)$$

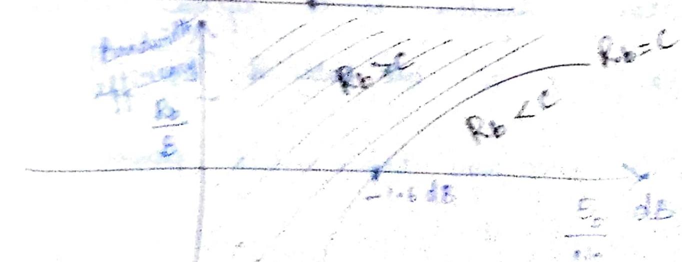
$$1 + \frac{E_b C}{N_0 B} = 10^{\left(0.3010 \frac{C}{B}\right)}$$

$$\boxed{\frac{E_b}{N_0} = \frac{B}{C}\left[10^{\left(0.3010 \frac{C}{B}\right)} - 1\right]}$$

$$\boxed{\frac{E_b}{N_0} = \frac{\left[\left(2^{C/B}\right) - 1\right]}{C/B}}$$

where $\frac{C}{B}$ is the bandwidth efficiency.

Capacity boundary curve:



For infinite BW: The signal to energy to noise ratio $\frac{E_b}{N_0}$ approaches

$$\left(\frac{E_b}{N_0}\right)_\infty = \lim_{B \to \infty} \left(\frac{E_b}{N_0}\right)$$

$$= \ln 2 = 0.6931 = 10 \log(0.693)$$

$$= -1.59$$

$$\simeq -1.6 \text{ dB}$$

The value -1.6 dB is known as Shannon limit.

The capacity boundary defined by the curve for the critical bit rate $\boxed{R_b = C}$ separates the combinations of system parameters that have the potential for supporting error free transmission $\boxed{R_b < C}$ and those for which the error free transmission is not possible for $\boxed{R_b > C}$.

Run length encoding

It is a simple form of lossless data compression in which the runs of data sequences in which the same data value occurs in many consecutive data elements are stored as a single data value.

Lampel Ziv Welch (LZW Algorithm) (X)

→ Algorithm used for compression. It is typically used to compress image files. It is lossless meaning no data is lost while compressing.

→ IDEA: rely on reoccurring patterns to save data space

→ Typically every character is stored with 8 binary bits, allowing upto 256 unique symbols for the data (ASCII)

→ This algorithm tries to extend this library to 9 to 12 bits per character. The new unique symbols are made up of combinations of symbols that occurred previously in the string.

LPC (Linear Predictive Coding).

→ Speech modelling

- Voiced (vocal cord & vibration)
- Unvoiced s, p, sh (without vocal cord vibration)

Vowels: a, e, i, o, u

Voiced source:

Sounds (vowels) are either voiced or unvoiced.

Voiced sounds are generated by vocal cords & vibration. These vibrations are periodic in time & are approximated by an impulse train.

where f_0 is the pitch

Unvoiced source:

Unvoiced sound (s, p, t) are generated without vocal cord vibration.

The excitation is modelled by a white gaussian noise source.

The unvoiced sounds have no pitch since they are excited by a non periodic signals.

1.) Silence
2.) LPC coefficients
3.) Gain
4.) Voiced / Unvoiced
5.) Pitch frequency

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Speech Modelling

20 - 30 ms

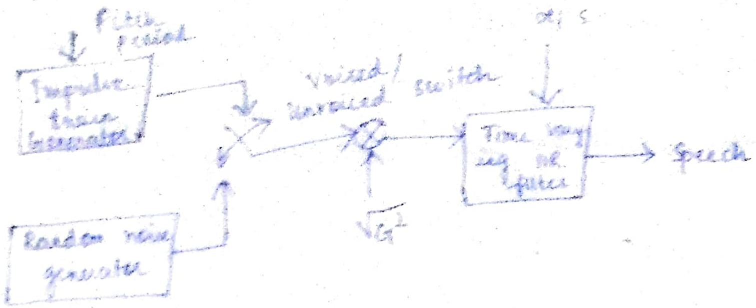
↳ short segment → frames

Hamming window

Apply a short duration (20-30 ms) overlapping window (normally hamming window) to speech signal in order to segment into frames.

LPC synthesizer

LPC Synthesizer



LPC Gain Coefficient:

$$G^2 = R(0) - \sum_{k=1}^{p} \alpha_k R(k) = \text{minimum mean square error prediction}$$

Speech difference equation for a p^{th} order filter

$$s(n) = \sum_{k=1}^{p} a_k s(n-k) + G u(n)$$

To minimize mean square error prediction:

$$e(n) = s(n) - \sum_{k=1}^{p} \alpha_k s(n-k)$$

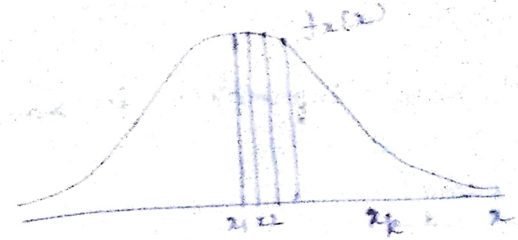
where for a single input impulse or a stationary white noise, the obtained coefficients are identical to a_k

Differential Entropy and mutual information for continuous ensembles:

Differential entropy is defined as

$$h(X) = \int_{-\infty}^{\infty} f_x(x) \log_2 \frac{1}{f_x(x)} dx$$

PDF ⇒ Probability density function.



$$\Rightarrow \int_{-\infty}^{\infty} f_x(x)\, dx = 1$$

$$x_1 - \frac{\Delta x}{2} < x < x_1 + \frac{\Delta x}{2} \to x_1$$

$$H(X) = \lim_{\Delta x \to 0} \sum_{k=-\infty}^{\infty} f_x(x_k) \cdot \Delta x \log_2 \left(\frac{1}{f_x(x_k)\Delta x} \right)$$

$$= \lim_{\Delta x \to 0} \left[\sum_{k=-\infty}^{\infty} f_x(x_k) \log_2 \left(\frac{1}{f_x(x_k)} \right) \Delta x - \log_2 \Delta x \sum_{k=-\infty}^{\infty} f_x(x_k) \Delta x \right]$$

$$= \int_{-\infty}^{\infty} f_x(x) \log_2 \frac{1}{f_x(x)} dx - \lim_{\Delta x \to 0} \log_2 \Delta x \int_{-\infty}^{\infty} f_x(x) \cdot dx$$

$$\boxed{H(X) = h(x) - \lim_{\Delta x \to 0} \log_2 \Delta x}$$

$$\int_{-\infty}^{\infty} f_x(x)\, dx = 1$$

Maximum differential entropy: It has to satisfy

1.) $\int_{-\infty}^{\infty} f_x(x)\, dx = 1$ 2.) $\int_{-\infty}^{\infty} (x-\mu)^2 f_x(x)\, dx = \sigma^2 = \text{constant}$

Using the method of Lagrange multiplier:

$$f_x(x) = \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left(\frac{-(x-\mu)^2}{\sigma^2} \right) \Leftarrow \text{Gaussian PDF}$$

$$h(x)_{max} = \frac{1}{2} \log_2 (2\pi\sigma^2 e)$$

For maximum $H(X)$, PDF should be Gaussian

$$H(X)_{max} = h(x) - \lim_{\Delta x \to 0} \log_2 \Delta x$$

$f_{\cdot}(\cdot)$ → Expansion

If uncertainity is more information is more

Advantage of Source Coding:

→ Compact representation

→ Redundancy $\xrightarrow{\text{ideas}}$ Bandwidth decreases

Channel coding

Add Redundancy → BW expansion

Adding noise immunity reduces BER. So in this we introduce Forward Error Correction

ARQ ⇒ Automatic

error detected

Technique to add noise immunity:

* Linear Block codes
*

Q. A channel encoder takes input at a rate of 1 Kbps & produces output at a rate of 3 kbps. The bit error rate of the channel is 2×10^{-4}. Assume that at the receiver and majority logic decoder is used.

$k=1$ → channel encoder → $n=3$

111 → 1   101 → 1   001 → 0

110 → 1   011 → 1   010 → 0

Assume 111 is transmitted

111 — No error

110 — Error

101 — Error

011 — Error

000 — Failure of error detection mechanism

Failure of Error correction

100 — Error

010 — Error

001 — Error

Probability of failure of error detection.

$$= P\left[\frac{000}{111}\right] \begin{matrix}\leftarrow \text{Received} \\ \leftarrow \text{Transmitted}\end{matrix}$$

$$= P(0/1) \cdot P(0/1) \cdot P(0/1)$$

$$BER = 2\times10^{-4}$$

$$= 2\times10^{-4} \times 2 \times 2\times10^{-4}$$

$$= \cancel{8\times10^{-12}}\ 8\times10^{-12}$$

Prob [error correction scheme fails]

$$P\left[\frac{001}{111}\right] = P[0/1]\cdot P[0/1]\cdot P[1/1]$$

Assume $P[0/1]$ to be p & $P[1/1]$ to be $1-p$

$$P\left(\frac{001}{111}\right) = p \cdot p \cdot (1-p) = p^2(1-p)$$