Overlap - Save method:

Let $L \rightarrow$ length of longer sequence

$M \rightarrow$ " " Smaller "

$N_1 \rightarrow$ " " each sectioned convolution sequence.

Step 1: Split the longer sequence into sequences of size equal to smaller sequence.

longer sequence

eg: $x(n) = \{\overset{n=0}{1}, \overset{n=1}{2} | \overset{n=2}{3}, \overset{n=3}{4} | \overset{n=4}{5}, \overset{n=5}{6} | \overset{n=6}{7}, \overset{n=7}{8}\}$

sectioned sequences

$\Rightarrow N_1 = 2$

smaller sequence

$h(n) = \{1, 2\}$

$L = 8,\ M = 2,$

$x_1(n) = \{\overset{n=0}{1}, \overset{n=1}{2}\}$, $x_2(n) = \{\overset{n=2}{3}, \overset{n=3}{4}\}$

$x_3(n) = \{\overset{n=4}{5}, \overset{n=5}{6}\}$, $x_4(n) = \{\overset{n=6}{7}, \overset{n=7}{8}\}$

Step 2: determine the no. of samples N_2 that will be obtained in the output of linear convolution of each section. ie $\boxed{\cancel{N = L + M - 1}} \neq \boxed{N_2 = M + N_1 - 1}$

$\cancel{N = 8 + 2}$

for above eg: $N_2 = 2 + 2 - 1 = 3$; $\boxed{N_2 = 3}$

Step 3: Convert the smaller sequence $h(n)$ into N_2-sample sequence by appending zeros at the end.

$N_2 = 3$

$h(n) = \{1, 2\}, \Rightarrow h(n) = \{1, 2, 0\}$

Step4: Convert the each sectioned sequence into N_2 – sample sequences using the samples of original longer sequence.

$x_1(n), x_2(n) \ldots$ — $N_2 \to 3$ — $x(n)$

Two methods of Step 4:

method 1:

The overlapping samples are placed at the beginning of the sections. i.e. first sample of $x_2(n)$ is placed as the overlapping sample at the end of $x_1(n)$. Similarly, the first sample of $x_3(n)$ is placed as the overlapping sample at the end of $x_2(n)$. and so on.

from eg:

$$x_1(n) = \{\overset{n=0}{1}, \overset{n=1}{2}, \overset{n=2}{3}\}$$

$$x_2(n) = \{\overset{n=2}{3}, \overset{n=3}{4}, \overset{n=4}{5}\}$$

$$x_3(n) = \{\overset{n=4}{5}, \overset{n=5}{6}, \overset{n=6}{7}\}$$

$$x_4(n) = \{\overset{n=6}{7}, \overset{n=7}{8}, \overset{n=8}{0}\}$$

no next sequence, so it is 0

Step5: perform circular convolution of each section with $h(n)$. ie

$$y_1(n) = x_1(n) \circledast h(n)$$
$$y_2(n) = x_2(n) \circledast h(n)$$
$$y_3(n) = x_3(n) \circledast h(n)$$
$$y_4(n) = x_4(n) \circledast h(n)$$

will contain N_2 - no. of samples

from eq:

$x_1(n) = \{1,2,3\}, \; h(n) = \{1,2,0\}$

$$\begin{bmatrix} 1 & 3 & 2 \\ 2 & 1 & 3 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 1+6+0 \\ 2+2+0 \\ 3+4+0 \end{bmatrix} = \begin{bmatrix} 7 \\ 4 \\ 7 \end{bmatrix}$$

$$\boxed{y_1(n) = \{7,4,7\}}$$

$x_2(n) = \{3,4,5\}, \; h(n) = \{1,2,0\}$

$$\begin{bmatrix} 3 & 5 & 4 \\ 4 & 3 & 5 \\ 5 & 4 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 3+10+0 \\ 4+6+0 \\ 5+8+0 \end{bmatrix} = \begin{bmatrix} 13 \\ 10 \\ 13 \end{bmatrix}$$

$$\boxed{y_2(n) = \{13,10,13\}}$$

$x_3(n) = \{5,6,7\}, \; h(n) = \{1,2,0\}$

$$\begin{bmatrix} 5 & 7 & 6 \\ 6 & 5 & 7 \\ 7 & 6 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 5+14+0 \\ 6+10+0 \\ 7+12+0 \end{bmatrix} = \begin{bmatrix} 19 \\ 16 \\ 19 \end{bmatrix}$$

$$\boxed{y_3(n) = \{19,16,19\}}$$

$x_4(n) = \{7,8,0\}, \; h(n) = \{1,2,0\}$

$$\begin{bmatrix} 7 & 0 & 8 \\ 8 & 7 & 0 \\ 0 & 8 & 7 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 7+0+0 \\ 8+14+0 \\ 0+16+0 \end{bmatrix} = \begin{bmatrix} 7 \\ 22 \\ 16 \end{bmatrix}$$

$$\boxed{y_4(n) = \{7,22,16\}}$$

Step6: Enter the output sequences $y_1(n), y_2(n)$ .... in the table as shown below. to combine the output of the convolution of each section.

n	0	1	2	3	4	5	6	7	8
$y_1(n)$	~~7~~	4	7						
$y_2(n)$			~~13~~	10	13				
$y_3(n)$					~~19~~	16	19		
$y_4(n)$							~~7~~	22	16
$y(n)$	*	4	7	10	13	16	19	22	16

overlapping region

Step7

Step 7:

In can be observed that, the last (N_2-1) samples in an output sequence overlaps with the first (N_2-1) samples of next output sequence in the table above. While combining the outputs, the first (N_2-1) samples of every output sequence is discarded and the remaining non-overlapping samples are simply saved as samples of $y(n)$.

Ans: $y(n) = \{*, 4, 7, 10, 13, 16, 19, 22, 16\}$

Ans is similar to that of overlap add method except first N_2-1 samples homework

Perform linear convolution of the following sequences by overlap-add method & overlap save method. (I & II)

$x(n) = \{1, 2, 3, -1, -2, -3, 4, 5, 6\}$, $h(n) = \{2, 1, -1\}$

method 2:

first 3-steps are same as that of overlap-save I method.

Step4: the overlapping samples are placed at the end of the section. ie. last sample of $x_1(n)$ is placed as overlapping sample at the end of $x_2(n)$. The last sample of $x_2(n)$ is placed as overlapping sample at the end of $x_3(n)$ and so on. Since there is no previous section for $x_1(n)$, the overlapping sample of $x_1(n)$ is taken as zero.

let $x(n) = \{1, 2, 3, 4, 5, 6, 7, 8\}$, $h(n) = \{1, 2\}$

Step1: $x_1(n) = \{1, 2\}$, $x_2(n) = \{3, 4\}$, $x_3(n) = \{5, 6\}$, $x_4(n) = \{7, 8\}$

Step2 : $N_2 = 2$, $L = 8$, $M = 2$, $N_1 = 2$

Step3 : $h(n) = \{1, 2, 0\}$

Step4: $x_1(n) = \{1, 2, 0\}$ no previous section

$x_2(n) = \{3, 4, 2\}$

$x_3(n) = \{5, 6, 4\}$

$x_4(n) = \{7, 8, 6\}$.

Step5: Circular convolution computation:

$$y_1(n) = x_1(n) \circledast h(n)$$

$$\begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 1+0+0 \\ 2+2+0 \\ 0+4+0 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ 4 \end{bmatrix}$$

$$\boxed{y_1(n) = \{1, 4, 4\}}$$

$$y_2(n) = x_2(n) \circledast h(n)$$

$$\begin{bmatrix} 3 & 2 & 4 \\ 4 & 3 & 2 \\ 2 & 4 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 3+4+0 \\ 4+6+0 \\ 2+8+0 \end{bmatrix} = \begin{bmatrix} 7 \\ 10 \\ 10 \end{bmatrix}$$

$$\boxed{y_2(n) = \{7, 10, 10\}}$$

$$y_3(n) = x_3(n) \circledast h(n)$$

$$\begin{bmatrix} 5 & 4 & 6 \\ 6 & 5 & 4 \\ 4 & 6 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 5+8+0 \\ 6+10+0 \\ 4+12+0 \end{bmatrix} = \begin{bmatrix} 13 \\ 16 \\ 16 \end{bmatrix}$$

$$\boxed{y_3(n) = \{13, 16, 16\}}$$

$$y_4(n) = x_4(n) \circledast h(n)$$

$$\begin{bmatrix} 7 & 6 & 8 \\ 8 & 7 & 6 \\ 6 & 8 & 7 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 7+12+0 \\ 8+14+0 \\ 6+16+0 \end{bmatrix} = \begin{bmatrix} 19 \\ 22 \\ 22 \end{bmatrix}$$

$$\boxed{y_4(n) = \{19, 22, 22\}}$$

Step6: Combine the output of the convolution of each section.

n	0	1	2	3	4	5	6	7	8
$y_1(n)$	1	4	~~4~~						
$y_2(n)$			7	10	~~10~~				
$y_3(n)$					13	16	~~16~~		
$y_4(n)$							19	22	~~22~~
Step 7: $y(n)$	1	4	7	10	13	16	19	22	*

Overlapping region

Step 7: From the above table, we can observe that the last (N_2-1) samples in output sequence overlaps with the first (N_2-1) samples of next output sequence. While combining the outputs, the last (N_2-1) samples of every output sequence is discarded and the remaining non-overlapping samples are simply saved as samples of $y(n)$

Ans: $y(n) = \{1, 4, 7, 10, 13, 16, 19, 22, *\}$

Note: Answer is similar to that of overlap-add method except last (N_2-1) samples.