3) DFT & IDFT method:

$$\left.\begin{array}{l} x_1(n) \circledast x_2(n) \\ = x_3(n) \end{array}\right\} \underset{N}{\overset{DFT}{\rightleftharpoons}} \left\{\begin{array}{l} X_1(k)\cdot X_2(k) \\ = X_3(k) \end{array}\right.$$

Step 1: find $X_1(k) = DFT\{x_1(n)\}$

Step 2: find $X_2(k) = DFT\{x_2(n)\}$

Step 3: Multiply $X_1(k)$ & $X_2(k)$ to get $X_3(k)$

Step 4: find $x_3(n) = IDFT\{X_3(k)\}$

Soln:

Step 1: $x_1(n) = \{1, 1, 2, 2\}$

$$X_1(k) = \sum_{n=0}^{3} x_1(n)\cdot e^{-j2\pi kn/4}$$

k=0: $X_1(0) = \sum_{n=0}^{3} x_1(n) = 1+1+2+2 = \boxed{6 = X_1(0)}$

k=1: $X_1(1) = \sum_{n=0}^{3} x_1(n)\cdot e^{-j\frac{2\pi n}{4_2}}$

$$= x_1(0)\cdot 1 + x_1(1)\, e^{-j\pi/2} + x_1(2)\, e^{-j\pi} + x_1(3)\, e^{-j3\pi/2}$$

$$= 1 + 1\cdot(\cancel{\cos\pi/2}^{\,0} - j\sin\pi/2) + 2\cdot(\cos\pi - j\cancel{\sin\pi}^{\,0})$$
$$+ 2(\cos 3\pi/2 - j\sin 3\pi/2)$$

$$= 1 + 1(-j) + 2(-1) + 2(+j)$$

$$= 1 - j - 2 + 2j = \boxed{-1+j = X_1(1)}$$

k=2: $X_1(2) = \sum_{n=0}^{3} x_1(n)\cdot e^{-j\frac{2\pi n}{4}\times 2}$

$$X_1(2) = x_1(0)\cdot 1 + x_1(1)\cdot e^{-j\pi} + x_1(2)\,e^{-j2\pi} + x_1(3)\cdot e^{-j3\pi}$$

$$= 1\cdot 1 + 1(\cos\pi - j\cancel{\sin\pi}^{0}) + 2(\cos 2\pi - j\cancel{\sin 2\pi}^{0})$$

$$+ 2(\cos 3\pi - j\cancel{\sin 3\pi}_{0})$$

$$= 1 + 1(-1) + 2(1) + 2(-1)$$

$$= \cancel{1} - \cancel{1} + \cancel{2} - \cancel{2} = \boxed{0 = X_1(2)}$$

<u>K=3</u>: $$X_1(3) = \sum_{n=0}^{3} x_1(n)\cdot e^{-j\frac{2\pi n}{4_2}\times 3}$$

$$= x_1(0)\cdot 1 + x_1(1)\cdot e^{-j3\pi/2} + x_1(2)\cdot e^{-j3\pi} + x_1(3)\cdot e^{-j9\pi/2}$$

$$= 1\cdot 1 + 1(\cancel{\cos 3\pi/2}^{0} - j\overset{(-1)}{\sin\frac{3\pi}{2}}) + 2\cdot(\cos 3\pi - j\cancel{\sin 3\pi}^{0})$$

$$+ 2(\cancel{\cos\frac{9\pi}{2}}_{0} - j\sin\frac{9\pi}{2})$$

$$= 1 + 1(+j) + 2(-1) + 2(j) = \boxed{-1-j = X_1(3)}$$

$$X_1(k) = \{6,\ -1+j,\ 0,\ -1-j\}$$

<u>Step2</u>: |||ly

$$X_2(k) = \text{DFT}\{x_2(n)\}$$

$$x_2(n) = \{1, 2, 3, 4\}$$

$$X_2(k) = \sum_{n=0}^{3} x_2(n)\cdot e^{-j\frac{2\pi n}{4_2}\cdot k}$$

$$X_2(k) = \{10,\ -2+2j,\ -2,\ -2-2j\}$$

Step3: $X_3(k) = X_1(k) \cdot X_2(k)$

$X_3(k) = \{60, -4j, 0, +4j\}$

Step4: $x_3(n) = \text{IDFT}\{X_3(k)\}$

$$x_3(n) = \frac{1}{N}\sum_{k=0}^{N-1} X_3(k) \cdot e^{+j\frac{2\pi kn}{N}}, \quad n=0 \text{ to } N-1$$

n=0: $x_3(0) = \frac{1}{4}\sum_{k=0}^{3} X_3(k) \cdot 1 = \frac{1}{4}\{60 \cancel{-4j} + 0 \cancel{+4j}\}$

$$= \boxed{15 = x_3(0)}$$

n=1: $$x_3(1) = \frac{1}{4}\sum_{k=0}^{3} X_3(k) \cdot e^{+j\frac{2\pi n}{\cancel{4}_2} \cdot 1}$$

$$= \frac{1}{4}\left\{X_3(0) \cdot 1 + X_3(1) \cdot e^{j\pi/2} + X_3(2) \cdot e^{+j\pi} + X_3(3) \cdot e^{j3\pi/2}\right\}$$

$$= \frac{1}{4}\left\{60 + (-4j)(\cancel{\cos \pi/2}^{0} + j\underbrace{\sin \pi/2}_{(=1)}) + 0 + (4j)(\underbrace{\cos 3\pi/2}_{0} + j\underbrace{\sin 3\pi/2}_{(-1)})\right\}$$

$$= \frac{1}{4}\{60 - 4j(j) + 4j(-j)\}$$

$$= \frac{1}{4}\{60 + 4 + 4\} = \frac{1}{4}\{68\} = 17$$

$$\boxed{x_3(1) = 17}$$

$n=2$: $$x_3(2) = \frac{1}{4} \sum_{k=0}^{3} X_3(k) \cdot e^{j\frac{2\pi n}{4} \times 2}$$

$$= \frac{1}{4}\left\{ X_3(0) + X_3(1) \cdot (\cos\pi + j\,\cancel{\sin\pi}^{0}) + \overset{0}{\cancel{X_3(2)}} + X_3(3) \cdot (\cos 3\pi + j\,\cancel{\sin 3\pi}^{0}) \right\}$$

$$= \frac{1}{4}\left\{ 60 + (-4j)(-1) + 0 + 4j(-1) \right\}$$

$$= \frac{1}{4}\left\{ 60 + \cancel{4j} - \cancel{4j} \right\} = \frac{1}{4} \times 60$$

$$\boxed{x_3(2) = 15}$$

$n=3$: $$x_3(3) = \frac{1}{4} \sum_{k=0}^{3} X_3(k)\, e^{j\frac{\cancel{2}\pi n}{\cancel{4}_2} \times 3}$$

$$= \frac{1}{4}\left\{ X_3(0)\cdot 1 + X_3(1)\left(\cancel{\cos\frac{3\pi}{2}}^{0} + j\underbrace{\sin\frac{3\pi}{2}}_{(-1)}\right) + \underset{X_3(2)}{0} + X_3(3)\cdot\left(\underbrace{\cancel{\cos\frac{9\pi}{2}}}_{0} + j\underbrace{\sin\frac{9\pi}{2}}_{(1)}\right) \right\}$$

$$= \frac{1}{4}\left\{ 60 + (-4j)(-j) + 0 + (4j)(j) \right\}$$

$$= \frac{1}{4}\left\{ 60 - 4 - 4 \right\} = \frac{1}{4} \times 52 = 13$$

$$\boxed{x_3(3) = 13}$$

Ans: $$x_3(n) = x_1(n) \circledast x_2(n) = \{15, 17, 15, 13\}$$

4) Tabular array method:

$$x_1(n) = \{1, 1, 2, 2\}, \quad x_2(n) = \{1, 2, 3, 4\}$$

$$x_3(n) = x_1(n) \circledast x_2(n)$$

n	-3	-2	-1	0	1	2	3
$x_1(n)$				1	1	2	2
$x_2(n)$				1	2	3	4
$x_2((-n))_4$	4	3	2	1	4	3	2
$x_2((1-n))_4$	1	4	3	2	1	4	3
$x_2((2-n))_4$			4	3	2	1	4
$x_2((3-n))_4$				4	3	2	1

n=0

n=1

n=2

n=3

multiply & add

$$x_3(l) = \sum_{n=0}^{3} x_1(n) \cdot x_2((l-n))_4$$

$l=0$:

$$x_3(0) = \sum_{n=0}^{3} x_1(n)\, x_2((-n))_4$$

$$= 1\cdot1 + 1\cdot4 + 2\cdot3 + 2\cdot2$$

$$= 1+4+6+4 = \boxed{15 = x_3(0)}$$

$l=1$:

$$x_3(1) = \sum_{n=0}^{3} x_1(n)\, x_2((1-n))_4$$

$$= 1\cdot2 + 1\cdot1 + 2\cdot4 + 2\cdot3$$

$$= 2+1+8+6 = \boxed{17 = x_3(1)}$$

$\underline{l=2}$: $$x_3(2) = \sum_{n=0}^{3} x_1(n)\cdot x_2((2-n))_4$$

$$= 1\cdot 3 + 1\cdot 2 + 2\cdot 1 + 2\cdot 4$$

$$= 3+2+2+8 = \boxed{15 = x_3(2)}$$

$\underline{l=3}$: $$x_3(3) = \sum_{n=0}^{3} x_1(n)\cdot x_2((3-n))_4$$

$$= 1\cdot 4 + 1\cdot 3 + 2\cdot 2 + 2\cdot 1$$

$$= 4+3+4+2 = \boxed{13 = x_3(3)}$$

$$\therefore\ x_3(n) = \{15, 17, 15, 13\}.$$

<u>homework</u>: Find <u>Circular</u> <u>convolution</u> of the following sequences.

1) $x_1(n) = \{1, -1, -2, 3, -1\}$, $x_2(n) = \{1, 2, 3\}$
Using matrix - multiplication method

2) $x_1(n) = \{1, 2, 3, 1\}$, $x_2(n) = \{4, 3, 2, 2\}$
using concentric circle method.

3) $x_1(n) = \{1, 1, 2, 1\}$, $x_2(n) = \{1, 2, 3, 2\}$
using DFT & IDFT method

4) $x_1(n) = \{\underset{\uparrow}{1}, -1, 2\}$, $x_2(n) = \{1, 4, 2, 3\}$
using Tabular array method.

means Convolution

Linear convolution from (or) Circular Convolution:

(Linear Convolution by DFT (or) Linear filtering by DFT)

Linear	circular
$y(n) = x_1(n) * x_2(n)$	$y(n) = x_1(n) \circledast x_2(n)$
L = length of $x_1(n)$	L = length of $x_1(n)$
M = length of $x_2(n)$	M = length of $x_2(n)$
N = length of $y(n)$	N = length of $y(n)$
After convolution, $\boxed{N = L + M - 1}$	After convolution, $\boxed{N = \max\{L, M\}}$
eg: if $x_1(n) = \{1,1,1,1,1\}$, $x_2(n) = \{1,1,1\}$	eg: If $x_1(n) = \{1,1,1,1,1\}$, $x_2(n) = \{1,1,1\}$
$L = 5$, $M = 3$	$L=5, M=3$
$y(n) = \{1,2,3,3,3,2,1\}$	before doing circular convolution make $L = M$ by adding $(L-M)$ zeros.
$N = 5+3-1 = 7$	$\therefore x_2 = \{1,1,1,0,0\}$
$\boxed{N=7}$	$y(n) = \{3,3,3,3,3\}$
	$\boxed{N=5}$

conclusion:

The circular convolution results in an L-point sequence, ie. $(M-1)$ points shorter than that given by linear convolution. ie. circular convolution will contain corrupted points due to time-domain aliasing.

drawback (not used to find response of LTI system)

Linear convolution output

$$y(n) = \{\overset{y(0)}{1}, \overset{y(1)}{2}, \overset{y(2)}{3}, \overset{y(3)}{3}, \overset{y(4)}{3}, \overset{y(5)}{2}, \overset{y(6)}{1}\}$$

$$\{\underset{y(0)}{\overset{2}{\underset{+}{1}}}, \underset{y(1)}{\overset{1}{\underset{+}{2}}}, \underset{y(2)}{3}, \underset{y(3)}{3}, \underset{y(4)}{3}\}$$

$$\{\underset{y(0)}{\overset{\overset{y(5)}{2}}{\underset{+}{1}}}, \underset{y(1)}{\overset{\overset{y(6)}{1}}{\underset{+}{2}}}, 3, 3, 3, 2, 1\}$$

aliasing

Circular convolution output $\Big\}$ $y(n) = \{\overset{2+1}{3}, \overset{1+2}{3}, 3, 3, 3\}$

$y(0)$ is aliased with $y(5)$

$y(1)$ " " $y(6)$.

Due to overcome aliasing, linear convolution from circular convolution is performed.

ie to get same output from both linear & circular convolution.

eg: $x_1(n) = \{1, 2, 3, 1\}$, $x_2(n) = \{1, 1, 1\}$

only Linear convolution:

$x_2(n)$ \ $x_1(n)$	1	2	3	1
1	1	2	3	1
1	1	2	3	1
1	1	2	3	1

$L = 4$, $M = 3$

$\boxed{N = 6}$

$y(n) = \{1, 3, 6, 6, 4, 1\}$

only circular convolution: $L = 4$, $M = 3$, to make $L = M$
add $(L-M)$ zeros (ie 1 zero) at the end of $x_2(n)$.

$x_1(n) = \{1, 2, 3, 1\}$, $x_2(n) = \{1, 1, 1, \boxed{0}\}$

$$\begin{array}{c} x_1(n) \\ \begin{bmatrix} 1 & 1 & 3 & 2 \\ 2 & 1 & 1 & 3 \\ 3 & 2 & 1 & 1 \\ 1 & 3 & 2 & 1 \end{bmatrix} \end{array} \begin{array}{c} x_2(n) \\ \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix} \end{array} = \begin{bmatrix} 1+1+3+0 \\ 2+1+1+0 \\ 3+2+1+0 \\ 1+3+2+0 \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \\ 6 \\ 6 \end{bmatrix}$$

$$y(n) = \{\underset{(1+4)}{5}, \underset{(3+1)}{4}, 6, 6\}$$

<u>Linear Convolution through Circular Convolution</u>:

$L \rightarrow$ length of $x_1(n)$

$M \rightarrow$ length of $x_2(n)$

Step 1: make L equal to $(N = L+M-1)$ by adding zeros at the ~~ep~~ end of $x_1(n)$

length of linear convolution output.

Step 2: make M equal to $(N = L+M-1)$ by adding zeros at the end of $x_2(n)$

Step 3: perform circular convolution of zero added sequences $x_1(n)$ & $x_2(n)$.

from pblm:

eg: $L=4$, $M=3$, $N = \overset{L+M-1}{4+3-1} = 6$

Step 1: $x_1(n) = \{1, 2, 3, 1, \boxed{0, 0}\}$

$x_2(n) = \{1, 1, 1, \boxed{0, 0, 0}\}$

$$\begin{array}{c} x_1(n) \\ \begin{bmatrix} 1 & 0 & 0 & 1 & 3 & 2 \\ 2 & 1 & 0 & 0 & 1 & 3 \\ 3 & 2 & 1 & 0 & 0 & 1 \\ 1 & 3 & 2 & 1 & 0 & 0 \\ 0 & 1 & 3 & 2 & 1 & 0 \\ 0 & 0 & 1 & 3 & 2 & 1 \end{bmatrix} \end{array} \begin{array}{c} x_2(n) \\ \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \end{array} = \begin{bmatrix} 1+0+0+0+0+0 \\ 2+1+0+0+0+0 \\ 3+2+1+0+0+0 \\ 1+3+2+0+0+0 \\ 0+1+3+0+0+0 \\ 0+0+1+0+0+0 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 6 \\ 6 \\ 4 \\ 1 \end{bmatrix}$$

$y(n) = \{1, 3, 6, 6, 4, 1\}$ ≡ linear convolution result.

Note: One more homework is in page no:

Sectioned Convolution (or) Fast Convolution: (or) Filtering of long data sequences.

While implementing linear convolution in FIR filters, the input signal $x(n)$ is much longer than the impulse response $h(n)$ of a DSP system. Circular convolution cannot be used to implement ~~implemented~~ linear convolution by padding zeros, due to the following reasons:

1. The entire sequence should be available before convolution can be carried out. hence there will be characteristic delay in getting the output.

2. Large amount of memory is required to store the sequences.

The above problems can be overcome by this sectioned convolution.

In this sectioned convolution, the longer sequence is sectioned (or splitted) into the size of smaller sequence. Then the linear convolution of each section of longer sequence and the smaller sequence is performed. The output sequences obtained from the convolutions of all the sections are combined to get the overall output sequence.

Linear filtering.

There are two methods of sectioned convolution.

1. Overlap-Add method.
2. Overlap-Save method.

Here Filtering is linear, successive blocks can be processed one at a time via the DFT, and the output blocks are fitted together to form the overall output signal sequence.

~~Overall-ad~~

Overlap-add method:

Steps to perform Overlap-add convolution:

Let $L \rightarrow$ length of longer sequence

$M \rightarrow$ length of smaller sequence.

$N \rightarrow$ length of each sectioned sequence.

Step 1: Split the longer sequence into the size of smaller sequence.

eg: $x(n) = \{1, 2 \mid 3, 4 \mid 5, 6 \mid 7, 8 \mid \cancel{z}\}$

$h(n) = \{1, 2\}$

$L = 8, \quad M = 2$

$x_1(n) = \{1, 2\}; \; x_2(n) = \{3, 4\}; \; x_3(n) = \{5, 6\}; \; x_4(n) = \{7, 8\}$

Step 2: Compute linear convolution (or) linear convolution through circular convolution between each section (N) and the smaller sequence (M) to produce output sequences of size $(M+N-1)$.

i.e. $y_1(n) = x_1(n) * h(n)$

$y_2(n) = x_2(n) * h(n)$

$y_3(n) = x_3(n) * h(n)$

$\vdots \qquad \vdots$

and so on.

from above example,

$M=2, \; N=2$

hence all the output sequences $y_1(n)\, y_2(n) \ldots$ will contain $(2+2-1) = 3$ samples

Step 3: find the starting value of n for all $y_1(n), y_2(n)$ etc:

eg: $x(n) = \{\overset{n=0}{1}, \overset{n=1}{2} \mid \overset{n=2}{3}, \overset{n=3}{4} \mid \overset{n=4}{5}, \overset{n=5}{6} \mid \overset{n=6}{7}, \overset{n=7}{8}\}$

by default

$h(n) = \{\overset{n=0}{1}, \overset{n=1}{2}\}$

$y_1(n) = \underset{\text{starts at } n=0}{x_1(n)} * \underset{\text{starts at } n=0}{h(n)}$

$0 + 0 = 0$

hence $y_1(n)$ will start at $n=0$

$y_2(n) = x_2(n) * h(n)$, hence $y_2(n)$ will start at $n=2$

$x_2(n)$ starts at $n=2$, $h(n)$ starts at $n=0$

$2 + 0 = 2$

$y_3(n) = x_3(n) * h(n)$, hence $y_3(n)$ will start at $n=4$

$x_3(n)$ starts at $n=4$, $h(n)$ starts at $n=0$

$4 + 0 = 4$

and so on.

Step 4: Enter all the output sequences $y_1(n), y_2(n) \ldots$ in the table shown below, to combine the output of the convolution of each section.

n	0	1	2	3	4	5	6	7	8
$y_1(n)$	$\boxed{1}$	$\boxed{3}$	$\boxed{2}$						
$y_2(n)$			$\boxed{3}$	$\boxed{10}$	$\boxed{8}$				
$y_3(n)$					$\boxed{5}$	$\boxed{16}$	$\boxed{12}$		
$y_4(n)$							$\boxed{7}$	$\boxed{22}$	$\boxed{16}$
⋮									
$y(n)$	1	3	5	10	13	16	19	22	16.

starting point of $y_1(n)$; start pt of $y_2(n)$; start pt of $y_3(n)$; start pt of $y_4(n)$; overlapping region; overlapping region; overlapping region

and so on.

Step 5: $y(n)$

Step 2 output:

eg: $y_1(n) = x_1(n) * h(n)$

$x_1(n) = \{1, 2\}$, $h(n) = \{1, 2\}$

$y_1(n) = \{1, 3, 2\}$

	1	2
1	1	2
2	1	2

+

From the table, It can be observed that, last (M-1) samples of each output sequence overlaps with the first (M-1) samples of next output sequence.

Step 5: Add the samples in the overlapped region and retain the non-overlapped samples as such, in the table shown in Step 4.

n	0	1	2	3	4	5	6	7	8
y(n)	1	3	(2+3)	10	(8+5)	16	(12+7)	22	16

↑ overall output

Ans: $y(n) = \{1, 3, 5, 10, 13, 16, 19, 22, 16\}$

pbm Perform the linear convolution of the following sequences using overlap-add method. Also sketch the output sequence.

$$x(n) = \{1, -1 \,|\, 2, -2 \,|\, 3, -3 \,|\, 4, -4\}, \quad h(n) = \{-1, 1\}.$$

Step 1: $L = 8$, $M = 2$

$x_1(n) = \{1, -1\}$, $x_2(n) = \{2, -2\}$, $x_3(n) = \{3, -3\}$, $x_4(n) = \{4, -4\}$

Step 2 & 3) $y_1(n) = x_1(n) * h(n)$

h(n) \ $x_1(n)$	n=0: 1	n=1: -1
n=0: -1	-1	1
n=1: 1	1	-1

$$y_1(n) = \{\underset{n=0}{-1}, \underset{n=1}{2}, \underset{n=2}{-1}\}$$

$y_2(n) = x_2(n) * h(n)$

$h(n)$ \ $x_2(n)$	$n=2$: 2	$n=3$: −2
$n=0$: −1	−2	2
$n=1$: 1	2	−2

+

$y_2(n) = \{\overset{n=2}{-2}, \overset{n=3}{4}, \overset{n=4}{-2}\}$

$y_3(n) = x_3(n) * h(n)$

$h(n)$ \ $x_3(n)$	$n=4$: 3	$n=5$: −3
$n=0$: −1	−3	3
$n=1$: 1	3	−3

+

$y_3(n) = \{\overset{n=4}{-3}, \overset{n=5}{6}, \overset{n=6}{-3}\}$

$y_4(n) = x_4(n) * h(n)$

$h(n)$ \ $x_4(n)$	$n=6$: 4	$n=7$: −4
$n=0$: −1	−4	4
$n=1$: 1	4	−4

+

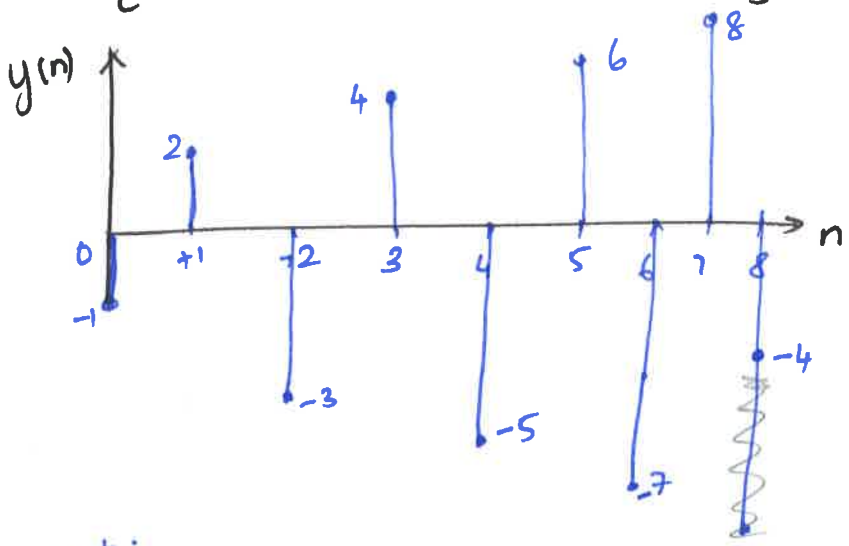
$y_4(n) = \{\overset{n=6}{-4}, \overset{n=7}{8}, \overset{n=8}{-4}\}$

overlapped region

Step 4 & 5:

n	0	1	2	3	4	5	6	7	8
$y_1(n)$	−1	2	−1						
$y_2(n)$			−2	4	−2				
$y_3(n)$					−3	6	−3		
$y_4(n)$							−4	8	−4
$y(n)$	−1	2	−3	4	−5	6	−7	8	−4

$$y(n) = \{-1, 2, -3, 4, -5, 6, -7, 8, -4\}$$



homework:

By means of DFT & IDFT, determine the response of the FIR filter with impulse response $h(n) = \{\underset{\uparrow}{1}, 2, 3\}$ to the input sequence

$$x(n) = \{\underset{\uparrow}{1}, 2, 2, 1\}.$$

hint: Use linear filtering by DFT.