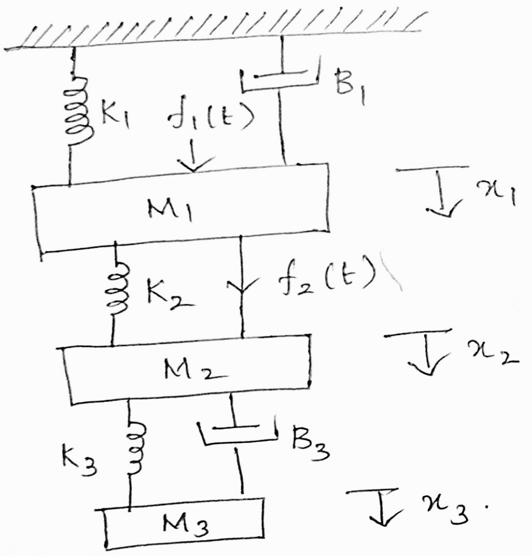
CONTROL SYSTEMS.

Assignment - I - Solutions.

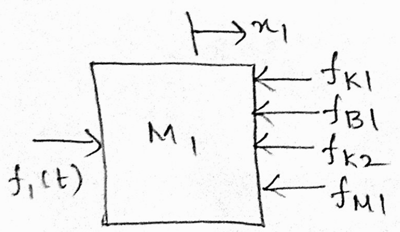
I) Write the differential equations governing the mechanical system shown in the fig. Draw the force - voltage and force - current electrical analogous circuits and verify by writing mesh and node equations.

(i)



solution.

M₁ - free body diagram.



$$f_{M1} = M_1 \frac{d^2x_1}{dt^2}$$

$$f_{K1} = K_1 x_1$$

$$f_{B1} = B_1 \frac{dx_1}{dt}$$

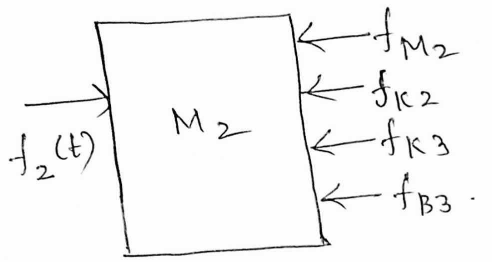
$$f_{K2} = K_2 (x_1 - x_2)$$

By Newton's II - law,

$$f_1(t) = f_{M1} + f_{K1} + f_{B1} + f_{K2}$$

$$\boxed{f_1(t) = M_1 \frac{d^2x_1}{dt^2} + K_1 x_1 + B_1 \frac{dx_1}{dt} + K_2(x_1 - x_2)} \quad \text{(1)}$$

$M_2 \rightarrow$ free body diagram



$$f_{M2} = M_2 \frac{d^2x_2}{dt^2}$$

$$f_{K2} = K_2(x_2 - x_1)$$

$$f_{K3} = K_3(x_2 - x_3)$$

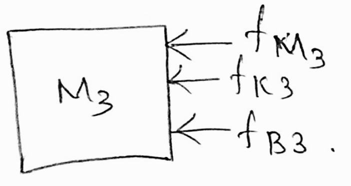
$$f_{B3} = B_3 \frac{d(x_2 - x_3)}{dt}$$

By Newton's II law,

$$f_2(t) = f_{M2} + f_{K2} + f_{K3} + f_{B3}.$$

$$\boxed{f_2(t) = M_2 \frac{d^2x_2}{dt^2} + K_2(x_2 - x_1) + K_3(x_2 - x_3) + B_3 \frac{d(x_2 - x_3)}{dt}.} \quad \text{(2)}$$

$M_3 \rightarrow$ free body diagram:



$$f_{M3} = M_3 \frac{d^2x_3}{dt^2}$$

$$f_{K3} = K_3(x_3 - x_2)$$

$$f_{B3} = B_3 \frac{d(x_3 - x_2)}{dt}.$$

By Newton's II - law

$$f_{M3} + f_{K3} + f_{B3} = 0$$

$$\boxed{M_3 \frac{d^2x_3}{dt^2} + K_3(x_3 - x_2) + B_3 \frac{d(x_3 - x_2)}{dt} = 0} \quad \text{(3)}$$

We know that, $\frac{d^2x}{dt^2} = \frac{dv}{dt}$, $\frac{dx}{dt} = v$ and $x = \int v\,dt$. (2)

On replacing the displacements by velocities in differential equations (1), (2) & (3),

$$f_1(t) = M_1 \frac{dv_1}{dt} + B_1 v_1 + K_2 \int (v_1 - v_2)\,dt + K_1 \int v_1\,dt \quad \text{—(4)}$$

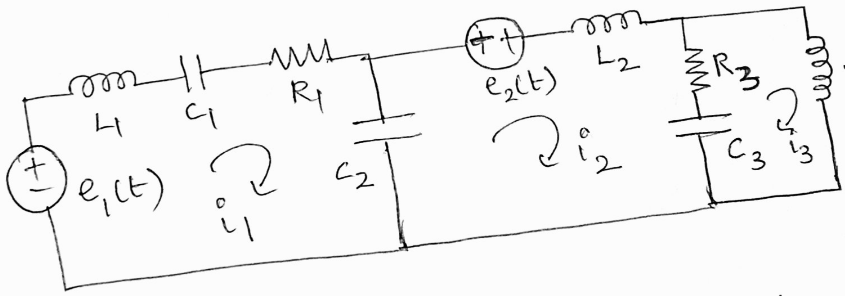
$$f_2(t) = M_2 \frac{dv_2}{dt} + B_3 \left[(v_2 - v_3)\right] + K_2 \int (v_2 - v_1)\,dt + K_3 \int (v_2 - v_3)\,dt \quad \text{—(5)}$$

$$0 = M_3 \frac{dv_3}{dt} + B_3\left[v_3 - v_2\right] + K_3 \int (v_3 - v_2)\,dt \quad \text{—(6)}$$

Force-voltage Analogy circuit:-

$M_1 \rightarrow L_1$	$B_1 \rightarrow R_1$	$K_1 \rightarrow 1/C_1$	$f_1(t) = e_1(t)$
$M_2 \rightarrow L_2$	$B_3 \rightarrow R_3$	$K_2 \rightarrow 1/C_2$	$f_2(t) = e_2(t)$
$M_3 \rightarrow L_3$		$K_3 \rightarrow 1/C_3$.	

$v_1 \rightarrow i_1$
$v_2 \rightarrow i_2$
$v_3 \rightarrow i_3$



Mesh equations:-

loop 1: $e_1(t) = L_1 \frac{di_1}{dt} + \frac{1}{C_1}\int i_1\,dt + R_1 i_1 + \frac{1}{C_2}\int (i_1 - i_2)\,dt$.

loop 2: $e_2(t) = L_2 \frac{di_2}{dt} + R_3(i_2 - i_3) + \frac{1}{C_3}\int (i_2 - i_3)\,dt + \frac{1}{C_2}\int (i_2 - i_1)\,dt$

loop 3: $R_3(i_3 - i_2) + \frac{1}{C_3}\int (i_3 - i_2)\,dt + L_3 \frac{di_3}{dt} = 0$.

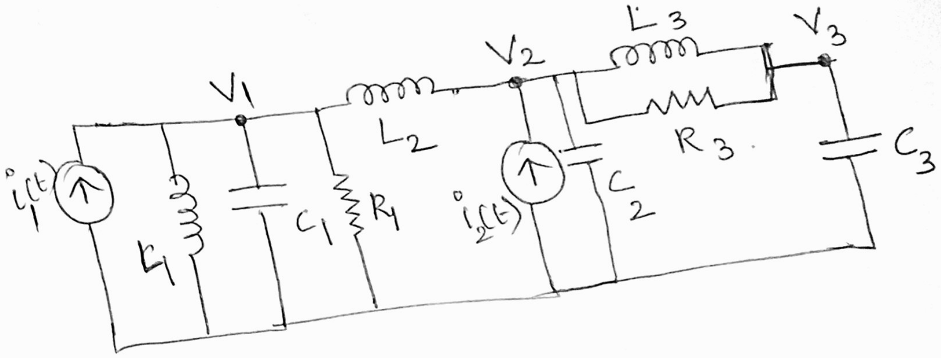
Force - current Analogy circuit :-

$M_1 \rightarrow C_1$	$B_1 \rightarrow 1/R_1$	$K_1 \rightarrow 1/L_1$	$f_1(t) \rightarrow i_1(t)$
$M_2 \rightarrow C_2$	$B_3 \rightarrow 1/R_3$	$K_2 \rightarrow 1/L_2$	$f_2(t) \rightarrow i_2(t)$.
$M_3 \rightarrow C_3$		$K_3 \rightarrow 1/L_3$	

$v_1 \rightarrow V_1$

$v_2 \rightarrow V_2$

$v_3 \rightarrow V_3$.



Node equations.

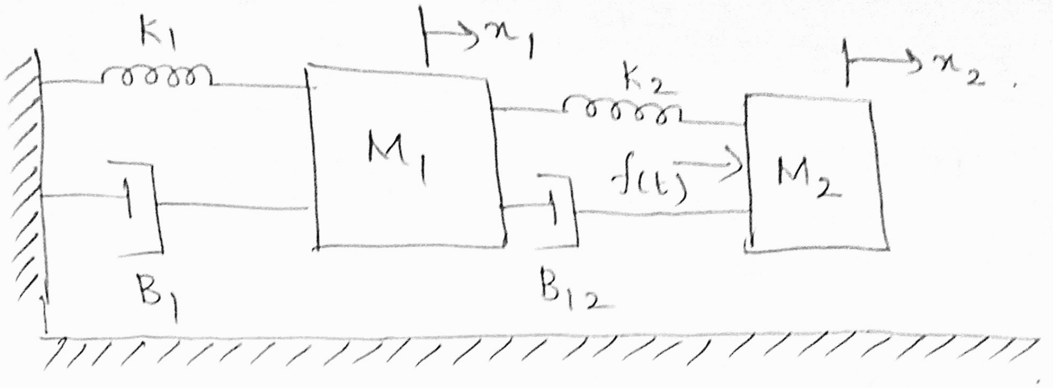
node 1 :
$$\frac{1}{L_1}\int V_1\, dt + C_1\frac{dV_1}{dt} + \frac{V_1}{R_1} + \frac{1}{L_2}\int (V_1 - V_2)\, dt = i_1(t)$$

node 2 :
$$\frac{1}{L_2}\int (V_2 - V_1)\, dt + C_2\frac{dV_2}{dt} + \frac{1}{L_3}\int (V_2 - V_3)\, dt + \frac{(V_2 - V_3)}{R_3} = i_2(t)$$

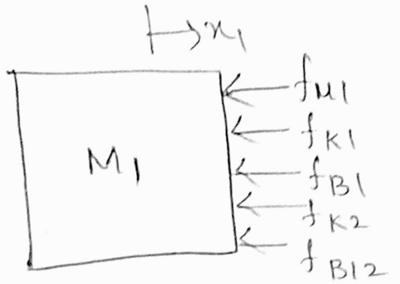
node 3 :
$$\frac{1}{L_3}\int (V_3 - V_1)\, dt + \frac{1}{R_3}(V_3 - V_2) + C_3\frac{dV_3}{dt} = 0.$$

It is inferred that the node equations and mesh equations are similar to ~~that~~ of the differential equations, (4), (5) & (6).

(ii)



Soln



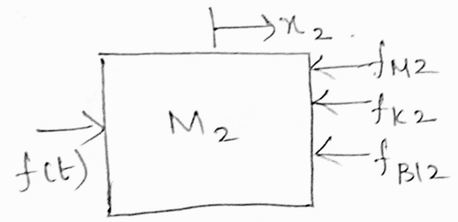
$$f_{M1} = M_1 \frac{d^2 x_1}{dt^2} \qquad f_{K1} = K_1 x_1$$

$$f_{B1} = B_1 \frac{dx_1}{dt} \qquad f_{K2} = K_2(x_1 - x_2)$$

$$f_{B12} = B_{12} \frac{d(x_1 - x_2)}{dt}$$

By Newton's II law,

$$\boxed{M_1 \frac{d^2 x_1}{dt^2} + B_1 \frac{dx_1}{dt} + B_{12} \frac{d(x_1 - x_2)}{dt} + K_1 x_1 + K_2(x_1 - x_2) = 0} \quad (1)$$



$$f_{M2} = M_2 \frac{d^2 x_2}{dt^2}$$

$$f_{K2} = K_2(x_2 - x_1)$$

$$f_{B12} = B_{12} \frac{d(x_2 - x_1)}{dt}$$

By Newton's II law,

$$\boxed{M_2 \frac{d^2 x_2}{dt^2} + B_{12} \frac{d(x_2 - x_1)}{dt} + K_2(x_2 - x_1) = f(t)} \quad (2)$$

→ Replace, the displacements by Velocities,

$$M_1 \frac{dv_1}{dt} + B_1 v_1 + B_{12}(v_1 - v_2) + K_1 \int v_1 \, dt + K_2 \int (v_1 - v_2) \, dt = 0. \quad (3)$$

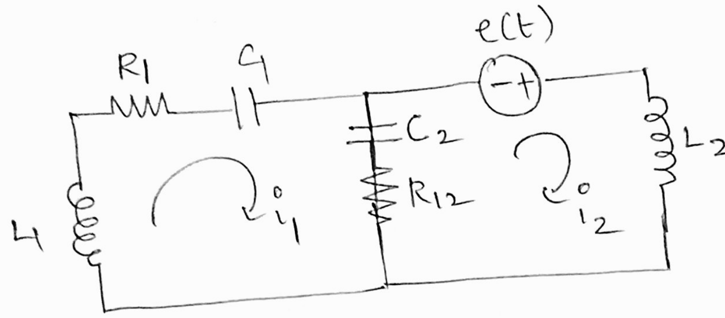
$$M_2 \frac{dv_2}{dt} + B_{12}(v_2 - v_1) + K_2 \int (v_2 - v_1) \, dt = f(t) \quad (4)$$

→ Force-voltage Analogy circuit.

$M_1 \rightarrow L_1$ $\quad B_1 \rightarrow R_1$ $\quad K_1 \rightarrow 1/C_1$ $\quad f(t) = e(t)$

$M_2 \rightarrow L_2$ $\quad B_{12} \rightarrow R_{12}$ $\quad K_2 \rightarrow 1/C_2$ $\quad v_1 = i_1$

$v_2 = i_2$



Mesh equations:-

loop 1: $$L_1 \frac{di_1}{dt} + R_1 i_1 + \frac{1}{C_1}\int i_1 dt + \frac{1}{C_2}\int (i_1 - i_2) dt + R_{12}(i_1 - i_2) = 0$$

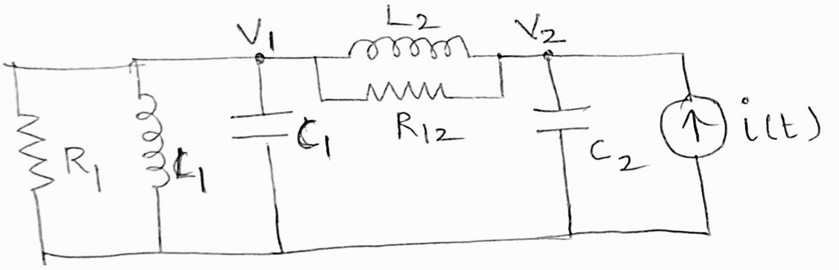
loop 2: $$L_2 \frac{di_2}{dt} + R_{12}(i_2 - i_1) + \frac{1}{C_2}\int (i_2 - i_1) dt = e(t).$$

→ Force-current Analogy circuit

$M_1 \rightarrow C_1$ $\quad B_1 \rightarrow 1/R_1$ $\quad K_1 \rightarrow 1/L_1$ $\quad f(t) = i(t)$

$M_2 \rightarrow C_2$ $\quad B_{12} \rightarrow 1/R_{12}$ $\quad K_2 \rightarrow 1/L_2$ $\quad v_1 = V_1$

$v_2 = V_2$.



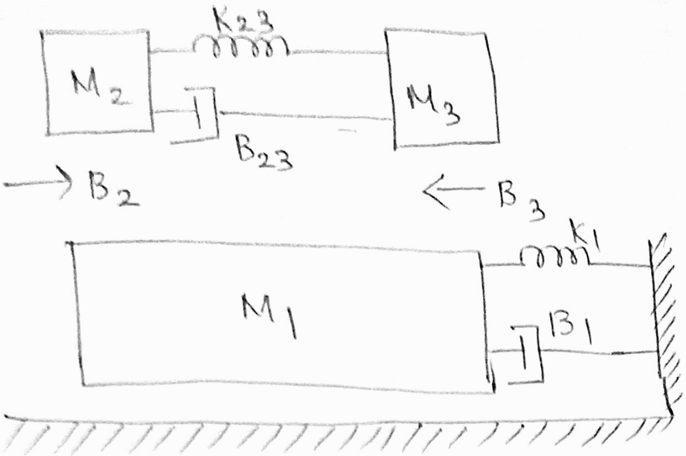
Node Equations:

node 1: $$\frac{V_1}{R_1} + \frac{1}{L_1}\int V_1 dt + C_1 \frac{dV_1}{dt} + \frac{1}{L_2}\int (V_1 - V_2) dt + \frac{(V_1 - V_2)}{R_{12}} = 0$$

node 2: $$C_2 \frac{dV_2}{dt} + \frac{1}{L_2}\int (V_2 - V_1) dt + \frac{V_2 - V_1}{R_{12}} = i(t)$$

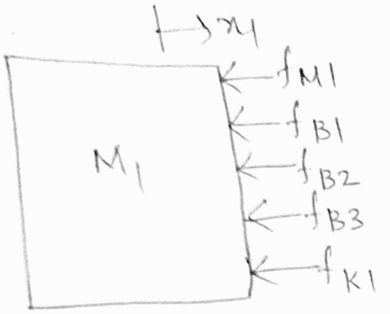
- The node equations & Mesh equations are similar to the differential equations (3) & (4).

(iii)



4

soln



$$f_{M1} = M_1 \frac{d^2 x_1}{dt^2}$$

$$f_{B1} = B_1 \frac{dx_1}{dt}$$

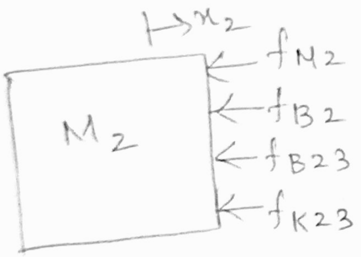
$$f_{B2} = B_2 \frac{d(x_1 - x_2)}{dt}$$

$$f_{B3} = B_3 \frac{d(x_1 - x_3)}{dt}$$

$$f_{K1} = K_1 x_1$$

By Newton's II-law,

$$\boxed{M_1 \frac{d^2 x_1}{dt^2} + B_1 \frac{dx_1}{dt} + B_2 \frac{d(x_1 - x_2)}{dt} + B_3 \frac{d}{dt}(x_1 - x_3) + K_1 x_1 = 0} \quad \text{--- (1)}$$



$$f_{M2} = M_2 \frac{d^2 x_2}{dt^2}$$

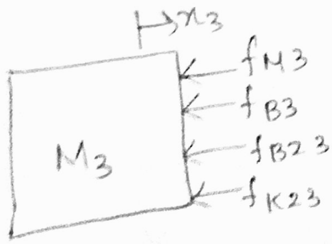
$$f_{B2} = B_2 \frac{d(x_2 - x_1)}{dt}$$

$$f_{B23} = B_{23} \frac{d}{dt}(x_2 - x_3)$$

$$f_{K23} = K_{23}(x_2 - x_3)$$

By Newtons - II law

$$\boxed{M_2 \frac{d^2 x_2}{dt^2} + B_{23} \frac{d}{dt}(x_2 - x_3) + B_2 \frac{d}{dt}(x_2 - x_1) + K_{23}(x_2 - x_3) = 0} \quad \text{--- (2)}$$



$$f_{M3} = M_3 \frac{d^2 x_3}{dt^2}$$

$$f_{B3} = B_3 \frac{d(x_3 - x_1)}{dt}$$

$$f_{B23} = B_{23} \frac{d(x_3 - x_2)}{dt}$$

$$f_{K23} = K_{23}(x_3 - x_2)$$

By Newton's-II law,

$$\boxed{M_3 \frac{d^2 x_3}{dt^2} + B_3 \frac{d(x_3 - x_1)}{dt} + B_{23} \frac{d(x_3 - x_2)}{dt} + K_{23}(x_3 - x_2) = 0} \quad \text{—(3)}$$

Replace displacements by velocities,

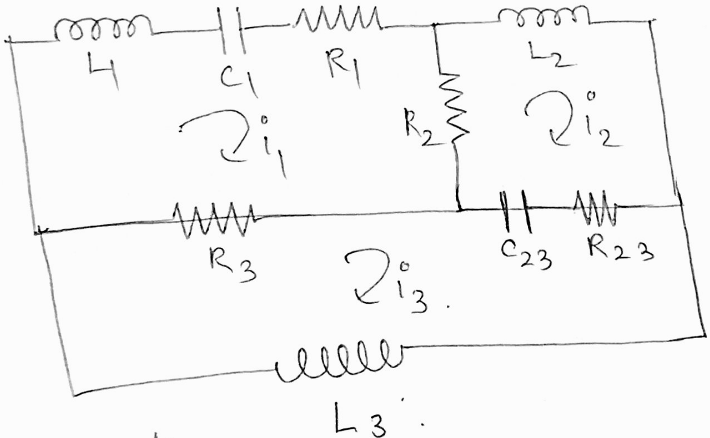
$$M_1 \frac{dv_1}{dt} + B_1 v_1 + B_2(v_1 - v_2) + B_3(v_1 - v_3) + K_1 \int v_1 \, dt = 0 \quad \text{—(4)}$$

$$M_2 \frac{dv_2}{dt} + B_{23}(v_2 - v_3) + B_2(v_2 - v_1) + K_{23} \int (v_2 - v_3) \, dt = 0 \quad \text{—(5)}$$

$$M_3 \frac{dv_3}{dt} + B_3(v_3 - v_1) + B_{23}(v_3 - v_2) + K_{23} \int (v_3 - v_2) \, dt = 0 \quad \text{—(6)}$$

→ Force - voltage analogy circuit

$M_1 \to L_1$	$B_1 \to R_1$	$K_1 \to 1/C_1$	$v_1 \to i_1$
$M_2 \to L_2$	$B_2 \to R_2$	$K_{23} \to 1/C_{23}$	$v_2 \to i_2$
$M_3 \to L_3$	$B_3 \to R_3$		$v_3 \to i_3$.
	$B_{23} \to R_{23}$		



Mesh equations:-

loop 1: $L_1 \frac{di_1}{dt} + \frac{1}{C_1} \int i_1 \, dt + R_1 i_1 + R_2(i_1 - i_2) + R_3(i_1 - i_3) = 0.$

loop 2: $L_2 \frac{di_2}{dt} + R_{23}(i_2 - i_3) + \frac{1}{C_{23}} \int (i_2 - i_3) \, dt + R_2(i_2 - i_1) = 0$

loop 3: $L_3 \frac{di_3}{dt} + R_3(i_3 - i_1) + \frac{1}{C_{23}} \int (i_3 - i_2) \, dt + R_{23}(i_3 - i_2) = 0.$

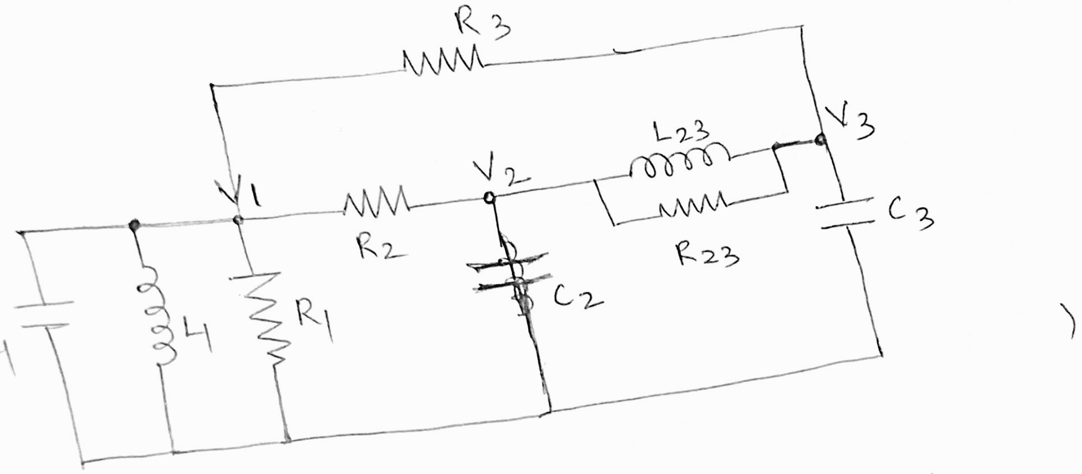
(3) Force - current Analogy circuit (5)

$M_1 \rightarrow C_1$ $\quad B_1 \rightarrow 1/R_1$ $\quad K_1 \rightarrow 1/L_1$ $\quad v_1 \rightarrow V_1$

$M_2 \rightarrow C_2$ $\quad B_2 \rightarrow 1/R_2$ $\quad K_{23} \rightarrow 1/L_{23}$ $\quad v_2 \rightarrow V_2$

$M_3 \rightarrow C_3$ $\quad B_3 \rightarrow 1/R_3$ $\quad v_3 \rightarrow V_3$.

$B_{23} \rightarrow 1/R_{23}$



Node Equations:-

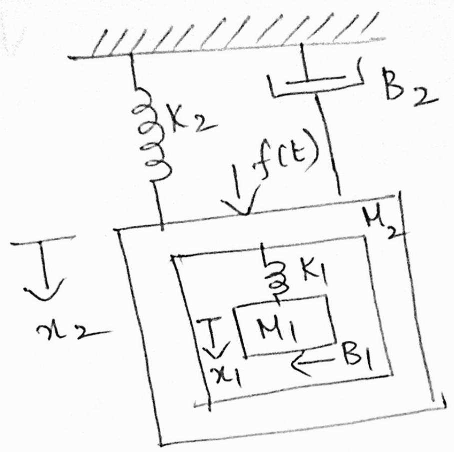
node 1: $C_1\frac{dV_1}{dt} + \frac{1}{L_1}\int V_1\,dt + \frac{V_1}{R_1} + \frac{(V_1-V_2)}{R_2} + \frac{(V_1-V_3)}{R_3} = 0.$

node 2: $C_2\frac{dV_2}{dt} + \frac{(V_2-V_1)}{R_2} + \frac{1}{L_{23}}\int (V_2-V_3)\,dt + \frac{V_2-V_3}{R_{23}} = 0.$

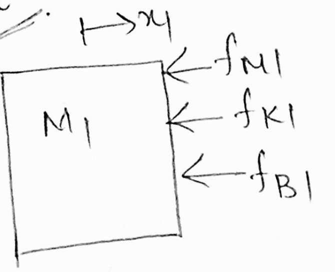
node 3: $C_3\frac{dV_3}{dt} + \frac{1}{L_{23}}\int (V_3-V_2)\,dt + \frac{(V_3-V_2)}{R_{23}} + \frac{(V_3-V_1)}{R_3} = 0.$

- The Mesh & node equations are similar to the differential equations (4), (5) & (6) of mechanical s/ms.

iv)



soln.



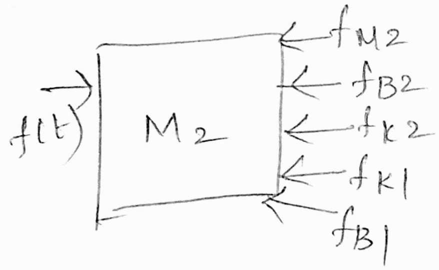
$$f_{M1} = M_1 \frac{d^2x_1}{dt^2}$$

$$f_{B1} = B_1 \frac{d(x_1 - x_2)}{dt}$$

$$f_{K1} = K_1(x_1 - x_2)$$

By Newton's II-law

$$\boxed{M_1 \frac{d^2x_1}{dt^2} + B_1 \frac{d}{dt}(x_1 - x_2) + K_1(x_1 - x_2) = 0} \quad (1)$$



$$f_{M2} = M_2 \frac{d^2x_2}{dt^2}$$

$$f_{B2} = B_2 \frac{dx_2}{dt}$$

$$f_{K2} = K_2 x_2$$

$$f_{K1} = K_1(x_2 - x_1)$$

$$f_{B1} = B_1 \frac{d(x_2 - x_1)}{dt}$$

By Newton's II-law.

$$\boxed{M_2 \frac{d^2x_2}{dt^2} + B_2 \frac{dx_2}{dt} + B_1 \frac{d(x_2 - x_1)}{dt} + K_1(x_2 - x_1) + K_2 x_2 = f(t)} \quad (2)$$

Replacing displacements by velocities,

$$M_1 \frac{dv_1}{dt} + B_1 (v_1 - v_2) + K_1 \int (v_1 - v_2)\, dt = 0 \quad \text{— (3)}$$

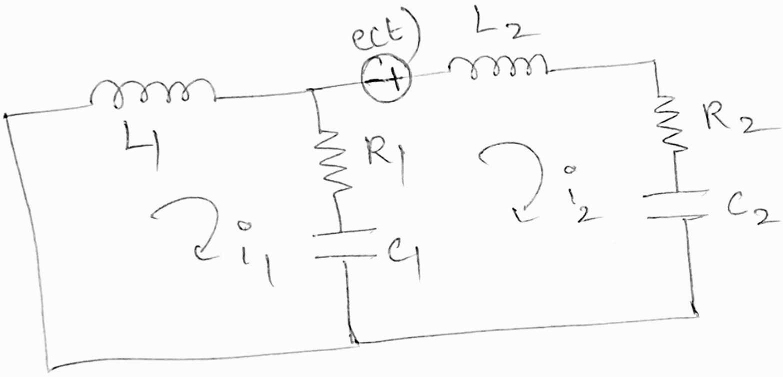
$$M_2 \frac{dv_2}{dt} + B_2 v_2 + B_1 (v_2 - v_1) + K_1 \int (v_2 - v_1)\, dt + K_2 \int v_2\, dt = f(t) \quad \text{— (4)}$$

→ Force-Voltage Analogous circuit:

$M_1 \to L_1$ $\quad B_1 \to R_1$ $\quad K_1 \to 1/C_1$ $\quad f(t) \to e(t)$

$M_2 \to L_2$ $\quad B_2 \to R_2$ $\quad K_2 \to 1/C_2$ $\quad v_1 \to i_1$

$v_2 \to i_2$



Mesh equations:-

loop 1: $L_1 \frac{di_1}{dt} + R_1(i_1 - i_2) + \frac{1}{C_1}\int (i_1 - i_2)\, dt = 0$

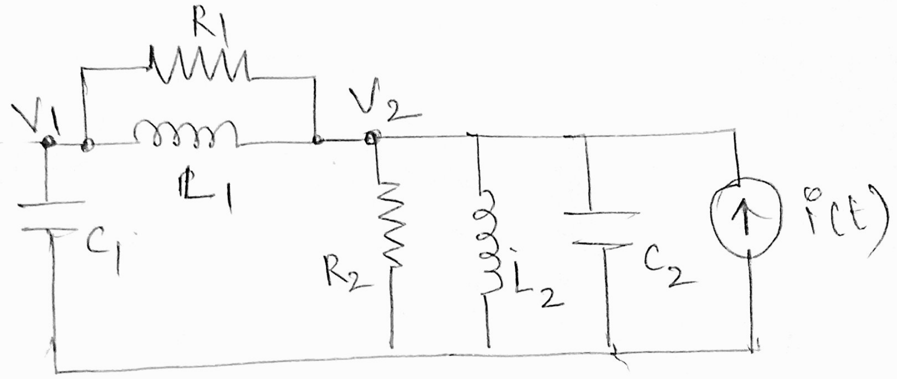
loop 2: $L_2 \frac{di_2}{dt} + R_2 i_2 + \frac{1}{C_2}\int i_2\, dt + \frac{1}{C_1}\int (i_2 - i_1)\, dt + R_1(i_2 - i_1) = \cancel{0}\ e(t)$

→ Force-current Analogy circuit

$M_1 \to C_1$ $\quad B_1 \to 1/R_1$ $\quad K_1 \to 1/L_1$ $\quad f(t) = i(t)$

$M_2 \to C_2$ $\quad B_2 \to 1/R_2$ $\quad K_2 \to 1/L_2$ $\quad v_1 \to V_1$

$v_2 \to V_2$



Node equations:

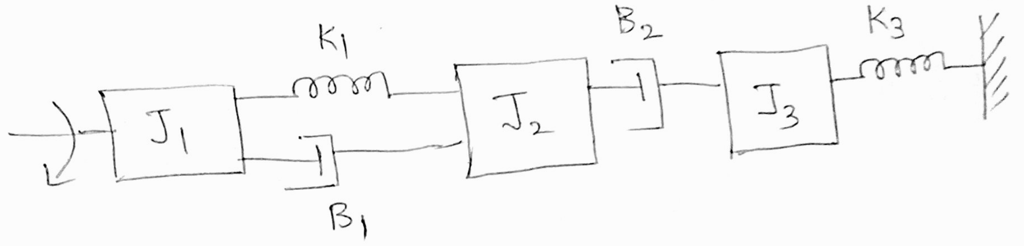
node 1: $C_1 \dfrac{dV_1}{dt} + \dfrac{1}{L_1} \int (V_1 - V_2)\, dt + \dfrac{V_1 - V_2}{R_1} = 0.$

node 2: $C_2 \dfrac{dV_2}{dt} + \dfrac{1}{L_2} \int V_2\, dt + \dfrac{V_2}{R_2} + \dfrac{(V_2 - V_1)}{R_1} + \dfrac{1}{L_1} \int (V_2 - V_1)\, dt = i(t)$

→ Thus the node & mesh equations are verified with the differential equations (3) & (4) of mechanical s/m.

i) Write the differential equations governing the mechanical rotational systems. Draw the Torque-voltage and Torque-current electrical analogous circuits and verify by writing mesh and node equations.

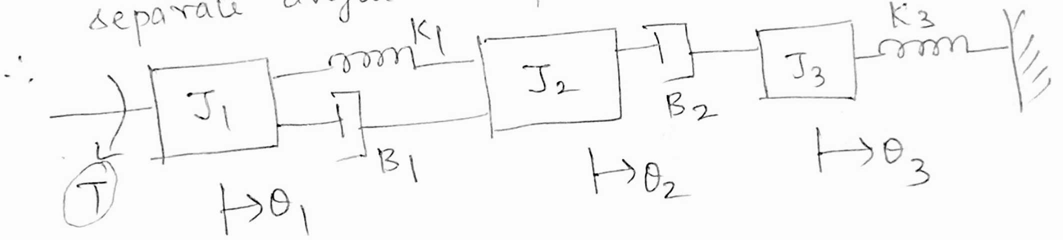
(i)



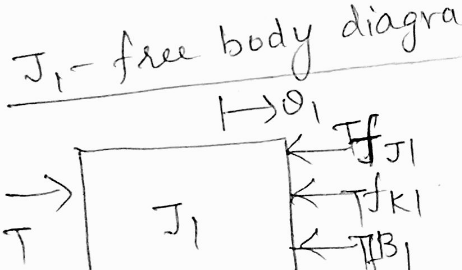
Soln:-

→ In the above diagram, there is one arrow mark indication is given. We have to assume that one i/p torque is given to J_1. Also assume separate angular displacements (θ) for each (J).

∴



J_1 - free body diagram.



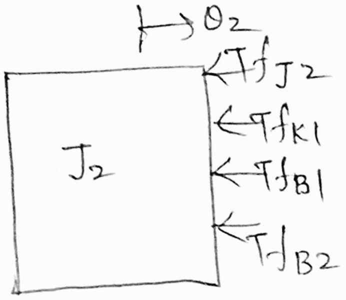
$$Tf_{J1} = J_1 \frac{d^2\theta_1}{dt^2} \qquad Tf_{B1} = B_1 \frac{d(\theta_1 - \theta_2)}{dt}$$

$$Tf_{K1} = K_1(\theta_1 - \theta_2)$$

By newton's II - law

$$\boxed{J_1 \frac{d^2\theta_1}{dt^2} + B_1 \frac{d(\theta_1 - \theta_2)}{dt} + K_1(\theta_1 - \theta_2) = T} \quad \text{(1)}$$

$J_2 \rightarrow$ free body diagram.



$$Tf_{J_2} = J_2 \frac{d^2(\theta_2)}{dt^2} \qquad Tf_{K_1} = K_1(\theta_2 - \theta_1)$$

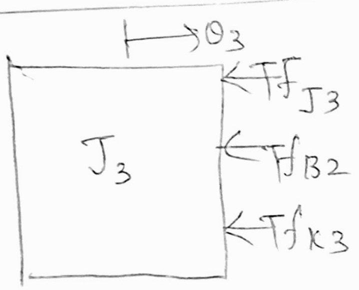
$$Tf_{B_1} = B_1 \frac{d(\theta_2 - \theta_1)}{dt}$$

$$Tf_{B_2} = B_2 \frac{d(\theta_2 - \theta_3)}{dt}$$

By newton's - II law,

$$\boxed{J_2 \frac{d^2\theta_2}{dt^2} + B_1 \frac{d(\theta_2 - \theta_1)}{dt} + B_2 \frac{d(\theta_2 - \theta_3)}{dt} + K_1(\theta_2 - \theta_1) = 0} \quad \text{--- (2)}$$

$J_3 \rightarrow$ free body diagram.



$$Tf_{J_3} = J_3 \frac{d^2\theta_3}{dt^2}$$

$$Tf_{B_2} = B_2 \frac{d(\theta_3 - \theta_2)}{dt}$$

$$Tf_{K_3} = K_3 \theta_3.$$

$$\therefore \boxed{J_3 \frac{d^2\theta_3}{dt^2} + B_2 \frac{d(\theta_3 - \theta_2)}{dt} + K_3\theta_3 = 0} \quad \text{--- (3)}$$

$\rightarrow$ Replace angular displacements (θ) with angular velocities (ω)

$$\frac{d^2\theta}{dt^2} = \frac{d\omega}{dt}, \quad \frac{d\theta}{dt} = \omega, \quad \theta = \int \omega \, dt.$$

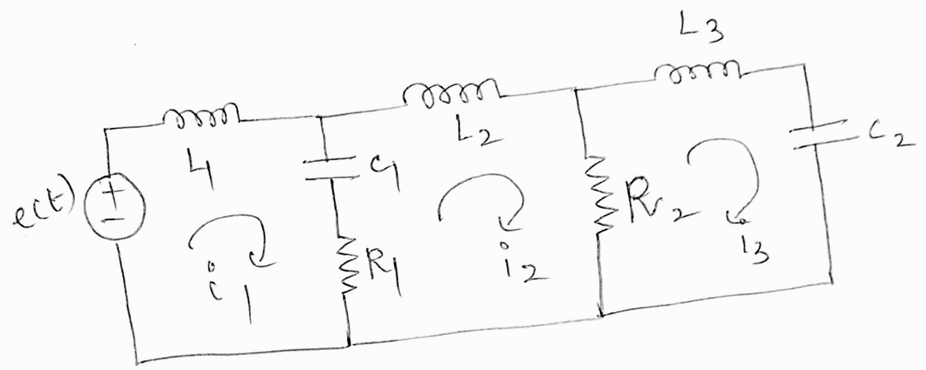
$$\rightarrow J_1 \frac{d\omega_1}{dt} + B_1(\omega_1 - \omega_2) + K_1 \int (\omega_1 - \omega_2)\, dt = T \quad \text{--- (4)}$$

$$J_2 \frac{d\omega_2}{dt} + B_1(\omega_2 - \omega_1) + B_2(\omega_2 - \omega_3) + K_1 \int (\omega_2 - \omega_1)\, dt = 0 \quad \text{--- (5)}$$

$$J_3 \frac{d\omega_3}{dt} + B_2 (\omega_3 - \omega_2) + K_3 \int \omega_3 \, dt = 0 \quad \text{--- (6)}.$$

→ **Torque - voltage Analogous circuit.**

$J_1 \to L_1$	$B_1 \to R_1$	$K_1 \to 1/C_1$	$T \to e(t)$
$J_2 \to L_2$	$B_2 \to R_2$	$K_2 \to 1/C_2$	$\omega_1 \to i_1$
$J_3 \to L_3$			$\omega_2 \to i_2$
			$\omega_3 \to i_3$.



Mesh equations :-

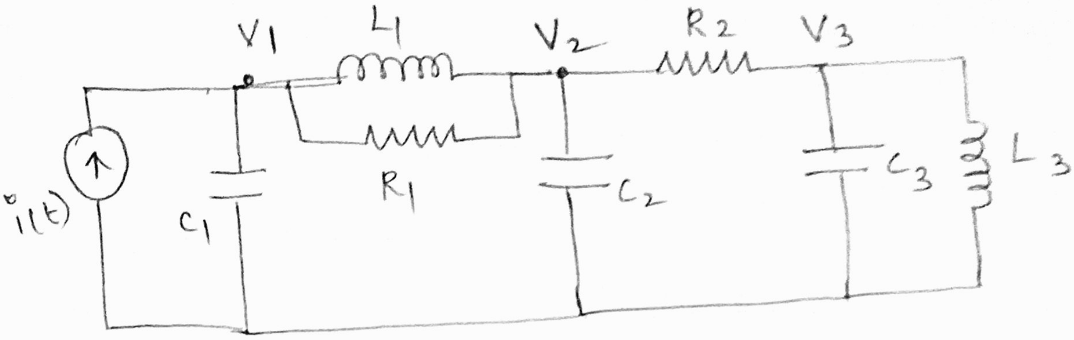
$$L_1 \frac{di_1}{dt} + \frac{1}{C_1} \int (i_1 - i_2) dt + R_1 (i_1 - i_2) = e(t).$$

$$L_2 \frac{di_2}{dt} + R_2 (i_2 - i_3) + R_1 (i_2 - i_1) + \frac{1}{C_1} \int (i_2 - i_1) dt = 0.$$

$$L_3 \frac{di_3}{dt} + \frac{1}{C_2} \int i_3 \, dt + R_2 (i_3 - i_2) = 0.$$

→ **Torque - current Analogous circuit**

$J_1 \to C_1$	$B_1 \to 1/R_1$	$K_1 \to 1/L_1$	$T = (i(t))$
$J_2 \to C_2$	$B_2 \to 1/R_1$	$K_2 \to 1/L_2$	$\omega_1 \to V_1$
$J_3 \to C_3$			$\omega_2 \to V_2$
			$\omega_3 \to V_3$.



node equations :-

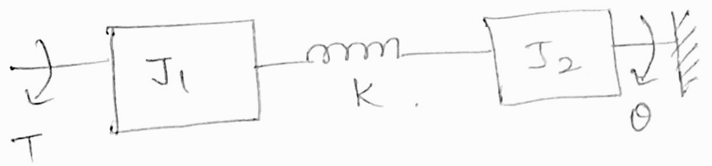
node 1 : $C_1 \frac{dV_1}{dt} + \frac{1}{L_1}\int (V_1 - V_2)\,dt + \left(\frac{V_1 - V_2}{R_1}\right) = i(t)$

node 2 : $C_2 \frac{dV_2}{dt} + \frac{1}{L_1}\int (V_2 - V_1)\,dt + \frac{V_2 - V_1}{R_1} + \frac{V_2 - V_3}{R_2} = 0.$

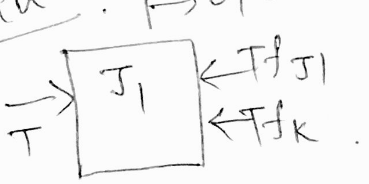
node 3 : $C_3 \frac{dV_3}{dt} + \frac{V_3 - V_2}{R_2} + \frac{1}{L_3}\int V_3\,dt = 0.$

→ Thus the node & mesh equations are verified with eqn (a), (c) & (b)

(ii)



soln. θ_1 (assume)

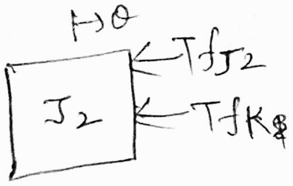


$Tf_{J1} = J_1 \frac{d^2\theta_1}{dt^2}$

$Tf_K = K(\theta_1 - \theta)$

$$\therefore \boxed{J_1 \frac{d^2\theta_1}{dt^2} + K(\theta_1 - \theta) = T} \quad (1)$$

θ



$Tf_{J2} = J_2 \frac{d^2\theta}{dt^2}$

$Tf\,K1 = K(\theta - \theta_1)$

$$\therefore \boxed{J_2 \frac{d^2\theta}{dt^2} + K(\theta - \theta_1) = 0} \quad (2)$$

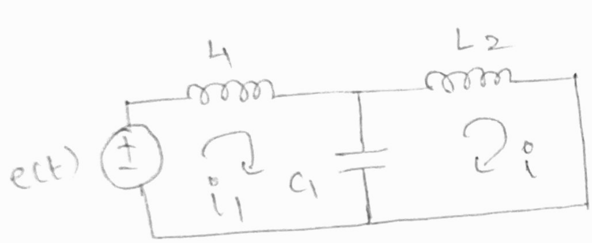
→ Replace the angular displacements by angular velocities.

$$J_1 \frac{d\omega_1}{dt} + K\int(\omega_1 - \omega)\,dt = T \quad \text{—— (3)}$$

$$J_2 \frac{d\omega}{dt} + K\int(\omega - \omega_1)\,dt = 0 \quad \text{—— (4)}.$$

→ Torque-Voltage analogy:

$J_1 \to L_1$	$K \to 1/C_1$	$T \to e(t)$
$J_2 \to L_2$		$\omega \to i$
		$\omega_1 \to i_1$



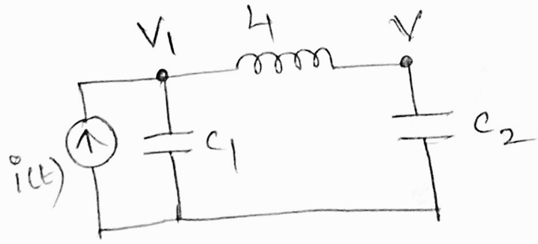
Mesh equations:-

loop 1 : $L_1 \frac{di_1}{dt} + \frac{1}{C_1}\int(i_1 - i)\,dt = e(t)$.

loop 2 : $L_2 \frac{di_2}{dt} + \frac{1}{C_1}\int(i - i_1)\,dt = 0$.

→ Torque-current analogy

$J_1 \to C_1$	$K \to 1/L_1$	$T \to i(t)$
$J_2 \to C_2$		$\omega \to V$
		$\omega_1 \to V_1$



node equations:-

node 1: $C_1 \frac{dV_1}{dt} + \frac{1}{L_1}\int(V_1 - V)\,dt = i(t)$

node 2: $C_2 \frac{dV_2}{dt} + \frac{1}{L_1}\int(V - V_1)\,dt = 0$.

Node & Mesh Equ. Thus Verified.