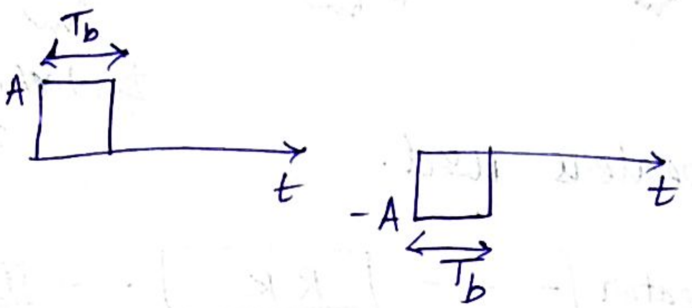
DIGITAL COMMUNICATION

2 books
Haykin
Clarke

Binary

0 $\rightarrow$ $S(0)$ is transmitted
1 $\rightarrow$ $S(1)$ " "



$T_b \rightarrow$ bit duration
$R_b = 1/T_b$

Distortion can occur before the signal ~~reach~~ reaches reciever $\rightarrow$ amplitude reduction / noise.

Reciever can still identify the signal

Digital modulation is equivalent to mapping.

M-ary modulation

$M = 4$

$00 \rightarrow S_1(t)$ $01 \rightarrow S_2(t)$ $10 \rightarrow S_3(t)$ $11 \rightarrow S_4(t)$

symbol set $\Rightarrow \{S_1(t), S_2(t), S_3(t), S_4(t)\}$
$\downarrow$
signal (waveform)

Waveform encoding

$\rightarrow$ Sampling
$\rightarrow$ Quantisation

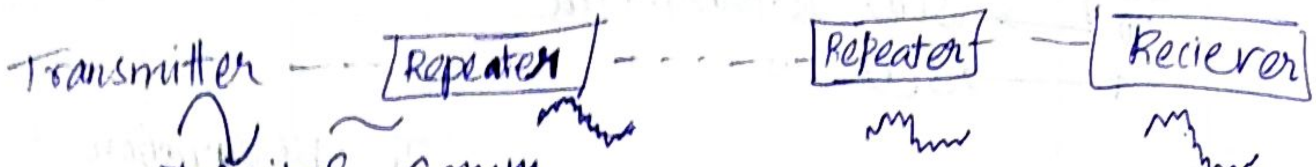
noise $\rightarrow$ white noise
thermal noise
$\hookrightarrow$ contains all freq

Ideal channel $\rightarrow$ only ~~id~~ amp will reduce during transmission ~~and~~ due to uniform attenuation being offered to all freq

shape doesn't change.

Analog comm.

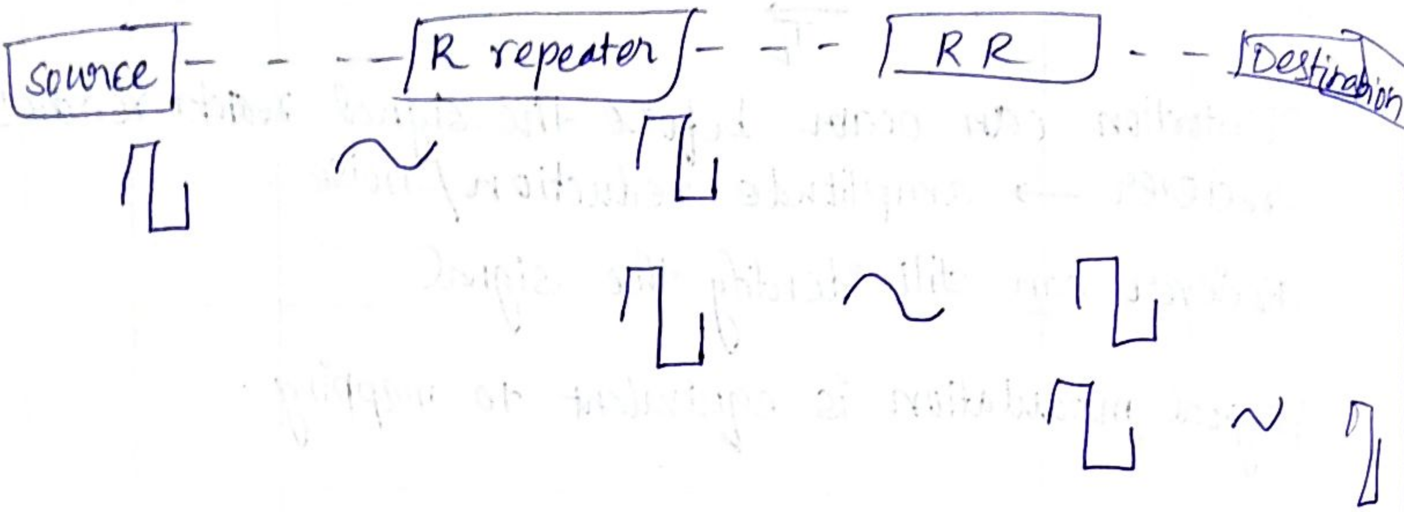
Repeater → removes noise outside of signal bandwidth and amplifies.



Distortion there slightly

Digital comm

Regenerative repeaters used.



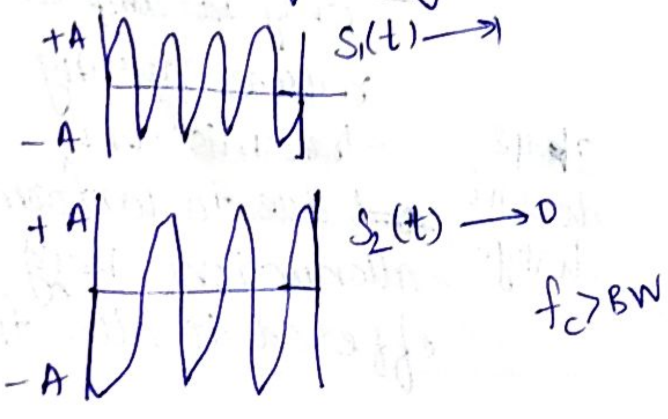
No distortion

UNIT-3

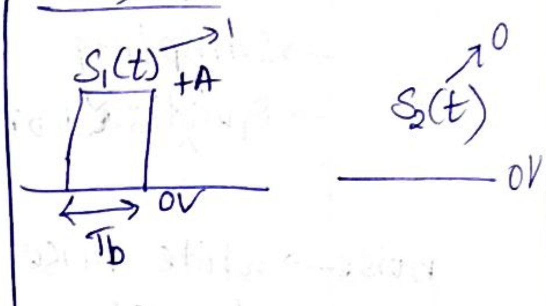
Line codes

To transmit bits through wire → they have to be turned to waveform. (signalling)

* Baseband signalling (Mapping)
* Band pass signalling.

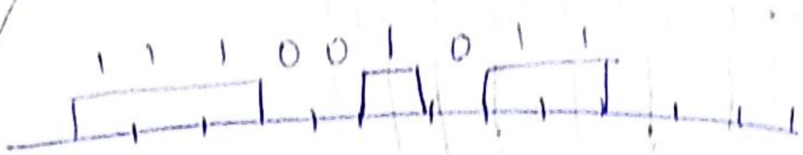


Base band



(f → BW) non return to zero

Unipolar NRZ → no negative amp.



Unipolar RZ → returns to zero after $T_b/2$.

Polar NRZ

Polar RZ

Bipolar / AMI (RZ) → Alternate mark inversion → has error detection capability

2 +a cannot be consecutive.

↓ alternate +a, −a for representing 1.

manchester → split phase code.

↳ for 1 → +a for $T_b/2$ → first half; −a for $T_b/2$ → second half

for 0 → reverse.

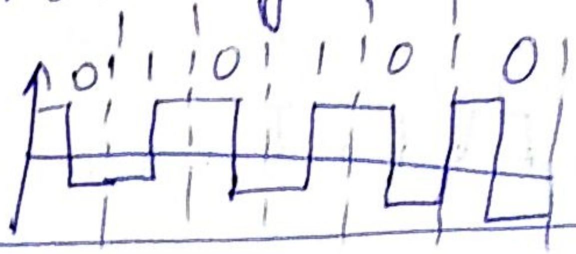
$$S_x(f) = |X(f)|^2$$ → Random process

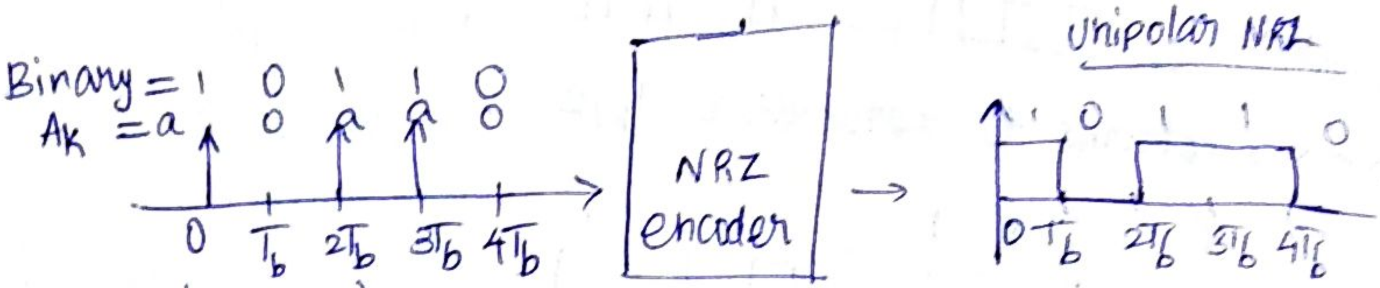
↳ W/Hz

↓ power spectral density → how power is distributed among diff freq.

$$S_x(f) = \int_{-\infty}^{\infty} R_x(\tau) e^{-j2\pi f\tau}\, d\tau$$

Manchester code → good for clock extraction.

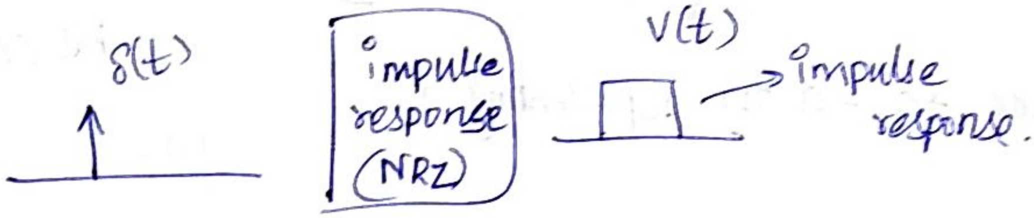




$a \times \delta(t)$ $\quad 0 \times \delta(t - T_b)$ $\quad a \times \delta(t - 2T_b)$

random sequence

i/p data ⇒ $x(t) = \sum_{K=-\infty}^{\infty} A_K \, \delta(t - KT_b)$



NRZ encoded form ⇒ $g(t) = \sum_{K=-\infty}^{\infty} A_K \, v(t - KT_b)$

convolution

$A(K) * v(t)$. $\qquad \sum A(KT_b) \, v(t - KT_b)$

$A(K)$ ↓ random

$v(t)$ ↓ continuous deterministic.

$X(t) \longrightarrow$ filter $h(t)$ $\longrightarrow y(t)$

$X(t)$: random Process

$y(t)$: random

o/p power spectral density $\quad S_{yy}(f) = |H(f)|^2 \, S_{xx}(f)$

Auto correlation

0 a 0 0 a 0 a a a 0 a 0 0

$A_k \; A_{k+1}$

Generally $R_{xx}(\tau) = E(x(t) * x(t+\tau))$, $E(X) = \int x \underbrace{f_x(x)}_{pdf} dx$

(prob density func)

Here, $R_A(n) = E(A_k \times A_{k+n})$.

$R_A(0) = E(A_k^2) = \sum_{k=1}^{2} (P_k A_k^2)$ — 2 possible value of $A_k = 0/1$

$= p(A_k=0)\, 0^2 + p(A_k=a) a^2$

probability

$= \frac{1}{2} \times 0 + \frac{1}{2} \times a^2 = \frac{a^2}{2}$

WKT, $R(1) = R(-1)$ → Autocorrelation func. is symmetric.

00
aa
a0
aa

~~when two~~

$R_A(1) = E(A_k \times A_{k+1})$

$= \sum_{k=1}^{4} p(A_k A_{k+1}) \; A_k A_{k+1}$

$= \sum_{k=1}^{4} p(A_k) p(A_{k+1}) \; A_k A_{k+1}$

$= \sum_{k=1}^{4} \frac{1}{2} \times \frac{1}{2} \; A_k A_{k+1}$

$= \sum_{k=1}^{4} \frac{1}{4} \; A_k A_{k+1}$

$= \frac{1}{4}[0 \times 0] + \frac{1}{4}[0 \times a] + \frac{1}{4}[a \times 0] + \frac{1}{4}[a \times a]$

$= 0 + 0 + 0 + \frac{a^2}{4}$

$= \frac{a^2}{4}$

when two statistically independent events are present, then joint prob. $p(A_k A_{k+1}) = p(A_k) p(A_{k+1})$

$R_A(2) = E(A_k \times A_{k+2})$

same 4 combi used.

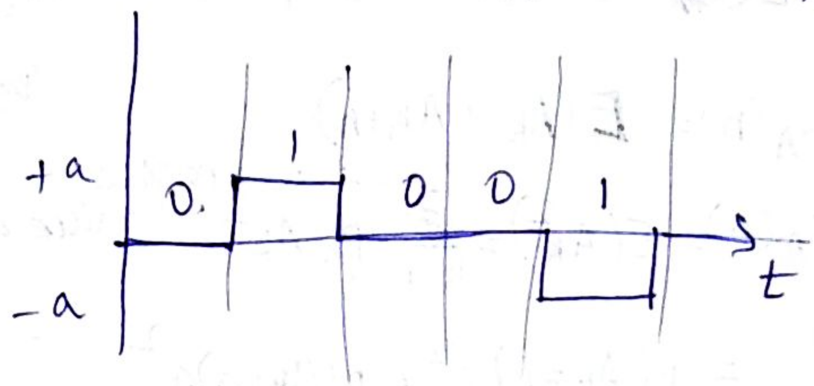
$\therefore R_A(2) = \frac{a^2}{4} = R_A(3)$

$\therefore R_A(n) = \frac{a^2}{4}$ for $n \neq 0$

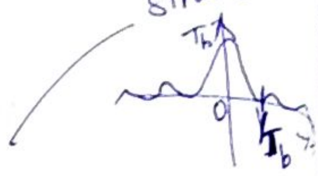
Line codes

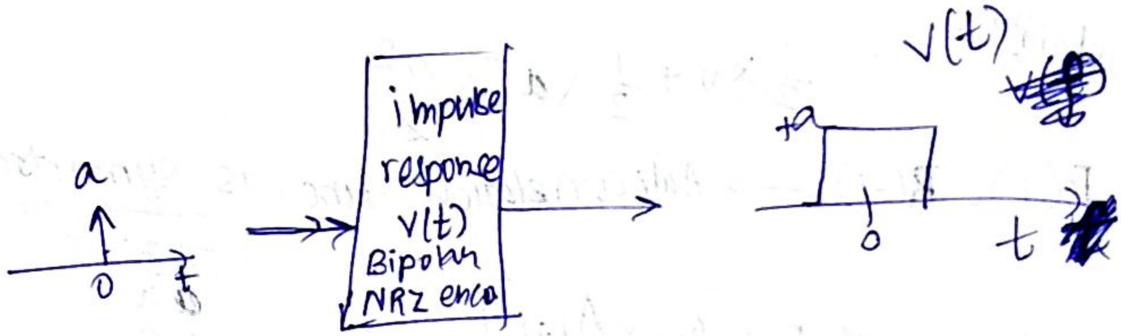
AMI Bipolar NRZ

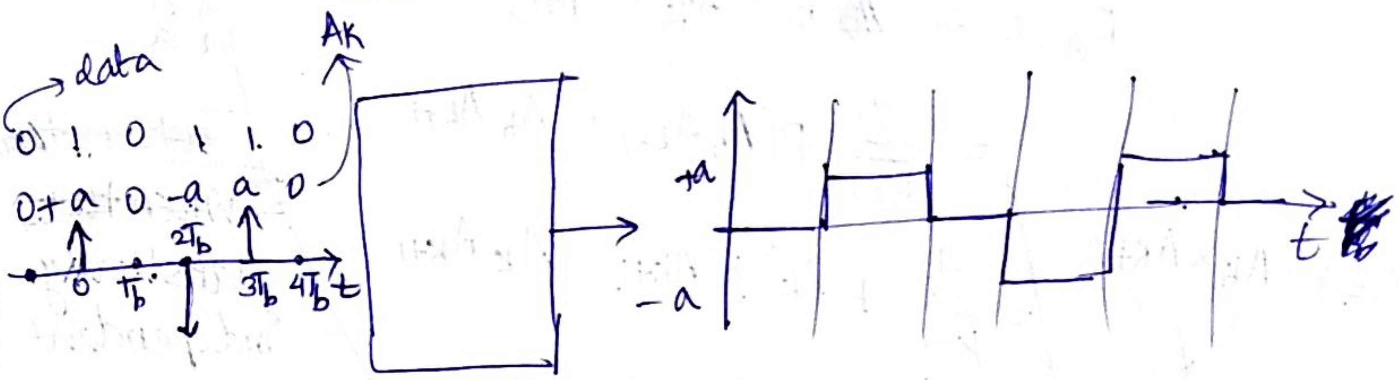
Alternate one's switch ~~from~~ btw +a ~~and~~ and -a.



V(f) = sinc θ







i/p data

$$x(t) = \sum_{K=-\infty}^{\infty} A_K \, \delta(t - KT_b)$$

NRZ encoded wave

$$g(t) = \sum_{K=-\infty}^{\infty} A_K \, v(t - KT_b)$$

$$R_A(n) = E(A_k A_{k+n})$$

$0 \longrightarrow \frac{P}{1/2}$ — 0 A_k 50% of t

$1 \longrightarrow 1/2$ < $+a$ 25%, $-a$ 25%

$0 \Rightarrow \frac{1}{2},\ +a \Rightarrow \frac{1}{4},\ -a \Rightarrow \frac{1}{4}$

$$R_A(0) = E(A_k)^2$$

$$= \sum_{k=1}^{3} P_k A_k^2$$

$$= \frac{1}{2} \times 0^2 + \frac{1}{4} \times (a)^2 + \frac{1}{4} \times (-a)^2$$

$$= \frac{a^2}{4} + \frac{a^2}{4} = \frac{2a^2}{4} = \frac{a^2}{2}$$

$$R_A(1) = E(A_k A_{k+1}).$$

$$\cancel{= \sum_{k=1}}$$

→ conditional probability.

A_k	A_{k+1}	$P(A_k A_{k+1}) = P(A_k)\,P\left(\frac{A_{k+1}}{A_k}\right)$	$A_k A_{k+1}$	$P(A_k A_{k+1})\,A_k A_{k+1}$
0	0	$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$	0	0
0	a	$\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$	0	0
0	$-a$	$\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$	0	0
a	0	$\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$	0	0
a	a	$\frac{1}{4} \times 0 = 0$	a^2	0
a	$-a$	$\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$	$-a^2$	$-a^2/8$
$-a$	0	$\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$	0	0
$-a$	a	$\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$	$-a^2$	$-a^2/8$
$-a$	$-a$	$\frac{1}{4} \times 0 = 0$	a^2	0

$-a$ can be followed by a or 0. Thus conditional probability is $\frac{1}{2}$

$$= \frac{-a^2}{8} - \frac{a^2}{8}$$

$$= \frac{-2a^2}{8} = \frac{-a^2}{4}$$

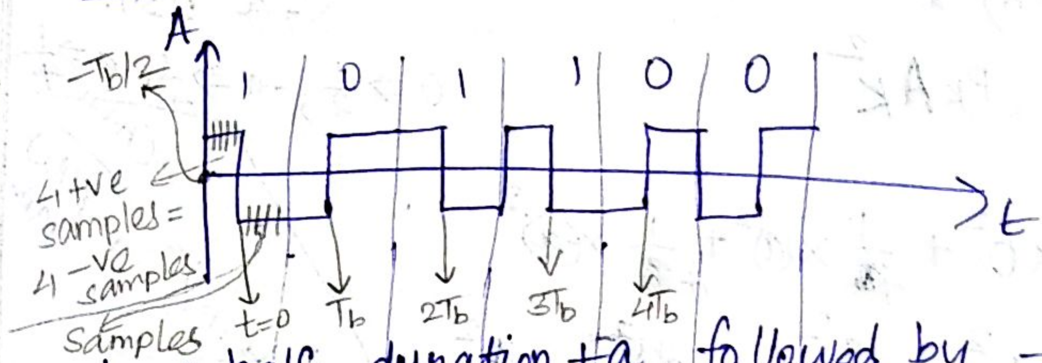
$$R_A(1) = \frac{-a^2}{4} = R_A(-1)$$

$$R_A(0) = \frac{a^2}{2}$$

(encoding) For $1 \longrightarrow \uparrow +a$

$0 \longrightarrow \downarrow -a$

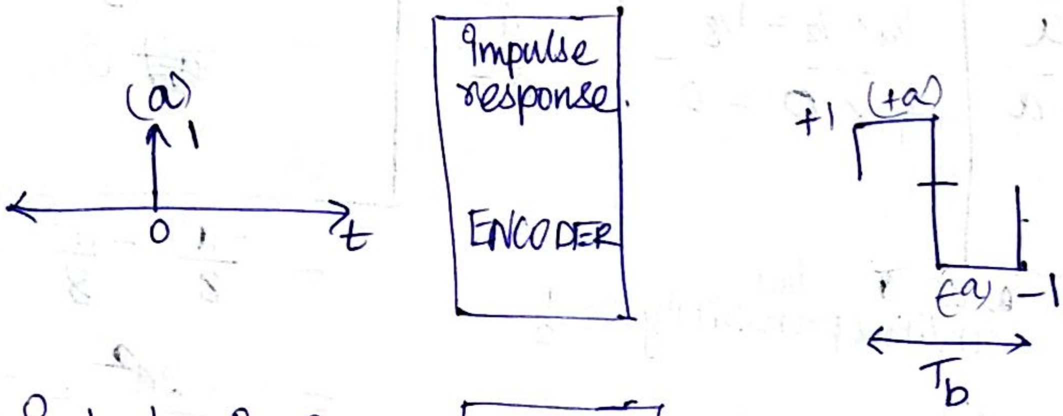
MANCHESTER ENCODING

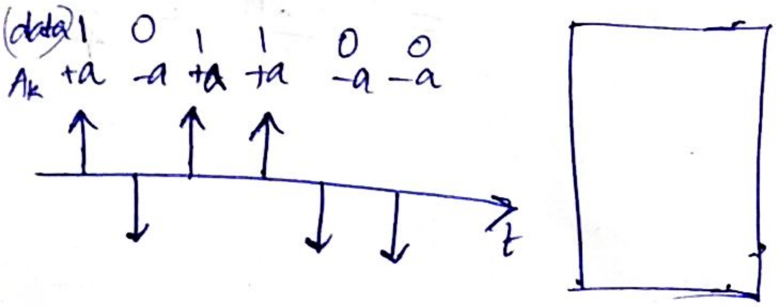


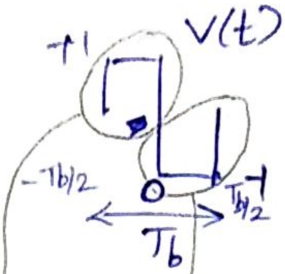
$1 \longrightarrow$ half duration $+a$ followed by $-a$

$0 \longrightarrow$ " " $-a$ " " $+a$

rising edge coincides with msg.
According to clock the values in msg signal are sampled.
In manchester, equal no. of positive and negative samples should be there. So if clock gets distorted or shifted also, error can be recognised.







$$\cancel{v(t) = p(t) *}$$

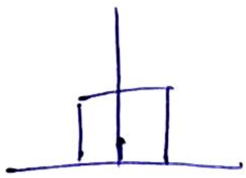
$$v(t) = p\left(t+\frac{T_b}{4}\right) - p\left(t-\frac{T_b}{4}\right).$$

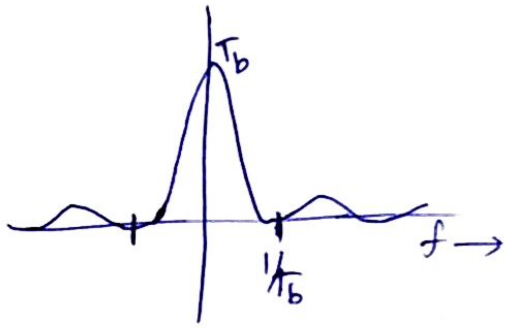
$$V(f) = P(f)\, e^{-j2\pi f \frac{T_b}{4}} - P(f)\, e^{j2\pi f \frac{T_b}{4}}$$

$$= P(f)\left[e^{-j2\pi f \frac{T_b}{4}} - e^{j2\pi f \frac{T_b}{4}} \right]$$

$T_b \operatorname{sinc}(fT_b)$

rect pulse





~~Auto correlation:~~

$$R_A(0) = E(A_k)^2$$

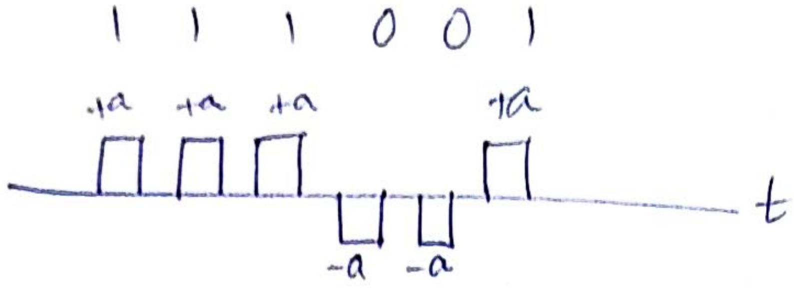
$$= \sum_{k=1}^{2} P_k A_k^2$$

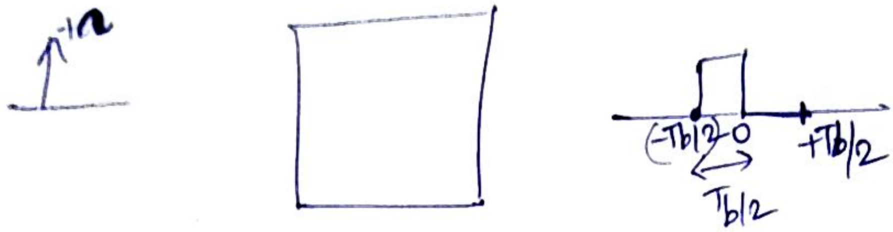
$$= \frac{1}{2} \times a^2 + \frac{1}{2} \times (-a)^2$$

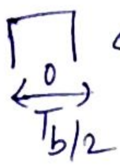
$$= \frac{\cancel{2}a^2}{\cancel{2}} = a^2$$

$$R_A(n) = 0,\ n \neq 0$$

Polar RZ







$\longleftrightarrow \frac{T_b}{2} \; \text{sinc}\left(\frac{fT_b}{2}\right)$

$R_A(0) = a^2$

$R_A(n) = 0 \quad n \neq 0$

Extracting clock

Take abs value of polar RZ

$r(t)$ 

$|r(t)|$  $\longrightarrow$ clock

Transparency

Bipolar RZ/ → error correction is possible
NRZ   +a, +a not possible consecutive.

If DC component is zero → no need to filter it.

Broader the pulse width → smaller BW
(in time domain)

Lossy and Deem.

[expanding the signal at time ~~freq~~ domain, shrinks the signal in freq domain]

spectrum of real signal is always symmetric about the y axis.

BW → take only +ve side.
Take main lobe BW.

Dig comm demands larger BW than Analog comm.

In BP-RZ, if long sequence of zeroes arise, clock extraction is difficult.

BNZS (HDB3) coding scheme

more than 3 0's cannot be consecutively transmitted.

If 4 0's appear it is replaced by

000V or B00V

V=1
B=1

B00V → used when even no. of 1's follow the spcl sequence.

000V → " " odd " " "

even 1's   even 1's   odd 1's

1 1 0 0 0 0 1 1 0 0 0 0 1 0 0 0 0

1 1 B 0 0 V 1 1 B 0 0 V 1 0 0 0 V

+a -a +a 0 0 +a -a +a -a 0 0 -a +a 0 0 0 +a

B → 1
V → 1

violates bipolar

on purpose

So that the reciever can differentiate btwn inherent 1001 and the converted B00V.

B3ZS signalling

↓

allow upto 2 zeroes to appear.

even one's → B0V

odd → 00V

V → shuld have bipolar violation

B8ZS signalling

allows ~~zz~~ 7 zeroes,

if 8^{th} zero appears replacement happens.

000 V B 0 V B — two violations happen

even/odd no. of ones doesn't matter

B6ZS

0V B0VB

---

0 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0 1 1

0 0 0 0 0 1 0 0 0 0 0 0 1 1 0 1 0 1

0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 0 0 1 0

HDB3

0 0 0 +a -a +a 0 0 +a -a 0 0 -a +a 0 0 +a -a +a

0 0 0 1 1 B 0 0 V B 0 0 V B 0 0 V 1 1

-a 0 0 -a 0 +a 0 0 0 +a 0 0 -a +a 0 -a 0 +a -a 0 0 -a

B 0 0 V 0 1 0 0 0 V 0 0 1 1 0 1 0 1 B 0 0 V

+a 0 0 +a -a 0 0 -a +a -a +a 0 0 -a 0

B 0 0 V B 0 0 V 1 1 1 0 0 1 0

B3ZS

+a 0 +a -a +a -a 0 -a +a 0 +a -a 0 -a +a 0 +a -a +a -a 0 -a 0 0 +a

B 0 V 1 1 B 0 V B 0 V B 0 V B 0 V 1 1 B 0 V 0 0 1

0 0 +a -a 0 -a +a -a 0 +a 0 -a +a 0 +a -a 0 -a +a 0 +a -a 0 -a

0 0 V B 0 V 1 1 0 1 0 1 B 0 V B 0 V B 0 V B 0 V

+a -a +a 0 0 -a 0

1 1 1 0 0 1 0

B8ZS

0 0 0 +a -a 0 0 0 -a +a 0 +a -a 0 0 0 0
0 0 0 1 1 0 0 0 V B 0 V B 0 0 0 0
+a -a 0 0 0 0 0 +a 0 0 0 0 0 0 -a +a 0
1 1 0 0 0 0 0 1 0 0 0 0 0 0 1 1 0 1
0 +a 0 0 0 +a -a 0 -a +a 0 0 0 0 -a +a -a
0 1 0 0 0 V B 0 V B 0 0 0 0 1 1 1
0 0 +a 0
0 0 1 0.

B6ZS

0 0 0 +a -a 0 -a +a 0 +a -a 0 -a +a 0 +a -a
0 0 0 1 1 0 V B 0 V B 0 V B 0 V B
+a -a 0 0 0 0 0 +a 0 +a -a 0 -a +a -a +a 0 -a 0 +a
1 1 0 0 0 0 0 1 0 V B 0 V B 1 1 0 1 0 1
0 +a -a 0 -a +a 0 +a -a 0 -a +a -a +a -a
0 V B 0 V B 0 V B 0 V B 1 1 1
0 0 +a 0
0 0 1 0

Interference

Channels of communication
- twisted pair
- coaxial cable
- optical fibre
- satellite communication (wireless channel)
- wireless communiation.

multipaths (reflections in transmission) not considered.

A single path is only considered.

This path is affected by ⇒

(i) AWGN (Additive white gaussian noise) channel.

* Noise is white
equal power in all freq
Auto correlation is impulse

* Noise is Gaussian

* Noise is added at front end of reciever.

(ii) channel Attenuation → leads to reduction in power.
shape of signal does not change

(iii) Bandwidth limitation

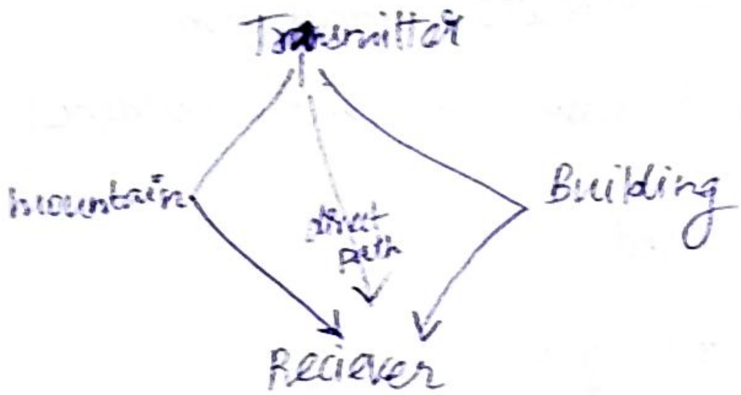
Bandwidth limitation
leads to signal distortion
(change in signal shape).

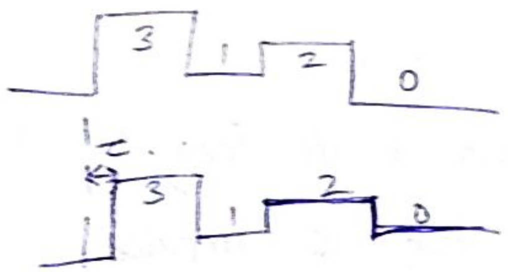
pulse becomes broader.

This change in shape and broadening in pulse width is called inter symbol interference (ISI)

To overcome this, channel equalization is used.

<u>Multipath effect</u> → this is seen in wireless communication.





signal in direct path

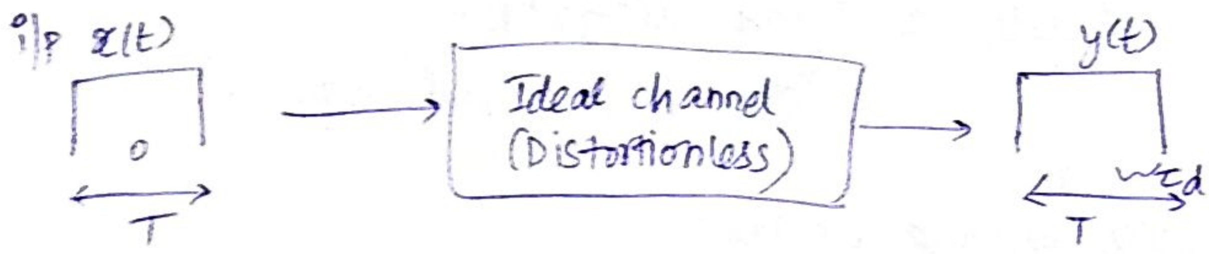
longer path

at reciever.

superposition of both signals.

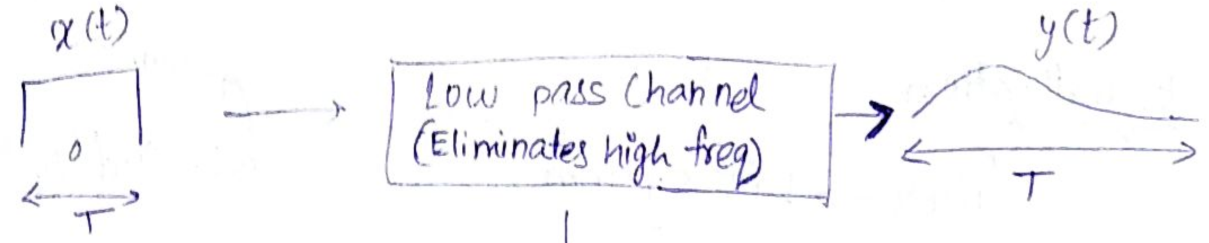
τ → pulse broadening.

~~Fon~~ <u>For our course only single path is considered. Due to Bandwidth limitation there is pulse broadening.</u>



Ideal channel → Time invariant system (LTI)

$H(f)$ is constant over desired frequency.
$f = -\infty$ to $+\infty$



↓
allows only freq close to zero
(side lobes truncate)

$X(f) \Rightarrow$ sinc funct
$f = -\infty$ to $+\infty$

when signal shrinks in freq domain, it expands in time domain → pulse broadening in $y(t)$

Pulse broadening → interference of pulses
↓
decision cannot be made by the reciever on a particular pulse.
other pulses also influence the ~~desi~~ decision.

This is called ISI.

Solution to ISI $\Rightarrow$

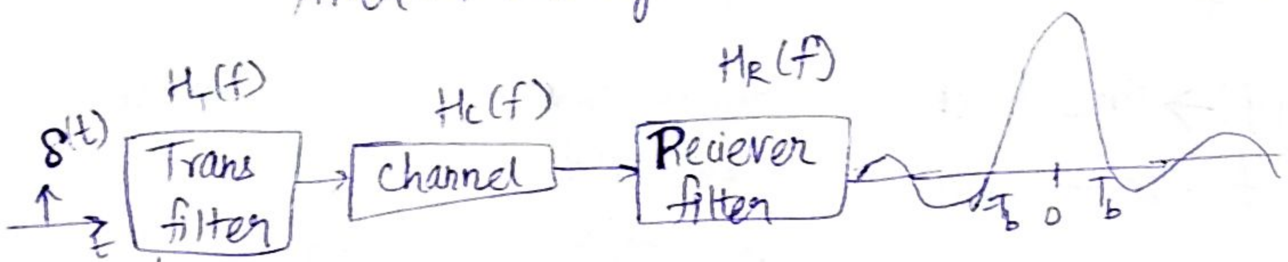
$\underbrace{\text{channel estimation}}_{\text{computing } H(f) \text{ of channel}}$, & channel equalization.

Equalization

filter $\rightarrow$ LTI system.

assume

$h(t) = \text{sinc}(t/T_b)$



describes line coding scheme used

Composite system transfer function $= H(f)$

$$H(f) = H_T(f)\, H_C(f)\, H_R(f)$$

$$H(f) \xrightarrow{FT^{-1}} h(t).$$

Ideal equalization $\rightarrow$ resulting $h(t)$ is sinc function.

(i.e)

$H_R(f)$ is a pulse.

$H_R(f) \rightarrow$ equalizing filter

Let $1 \to +1 \qquad b_k = \pm 1$

$0 \to -1$

Let m^{th} bit be $b_m = 1$

At reciever, waveform is $h(t - mT_b)$

General, recieved waveform $= \sum_{n=-\infty}^{\infty} b_n h(t - nT_b)$

$$= b_m h(t - mT_b) + \sum_{n=-\infty, n \neq m}^{\infty} b_n h(t - nT_b)$$

To take decision on m^{th} bit, sample it at $t = mT_b$.

$$r(mT_b) = b_m h(mT_b - mT_b) + \sum_{n=-\infty, n \neq m}^{\infty} b_n h(mT_b - nT_b)$$

Due to mth bit

$$r(mT_b) = b_m h(0) + \sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} b_k h(kT_b)$$

ISI from other bits

To avoid ISI,

$$r(mT_b) = b_m h(0)$$

Equivalently, $h(kT_b) = 0, \; k \neq 0$

$h(kT_b) \neq 0, \; k = 0$

consider $h_\Delta(t) \to$ discrete time version of $h(t)$

$$h_\Delta(t) = h(t) \sum_{k=-\infty}^{\infty} \delta(t - kT_b)$$

$$= \sum_{k=-\infty}^{\infty} h(kT_b)\, \delta(t - kT_b)$$

for zero ISI,

$$\sum_{k=-\infty}^{\infty} h(kT_b)\,\delta(t-kT_b) = h(0)\,\delta(t)$$

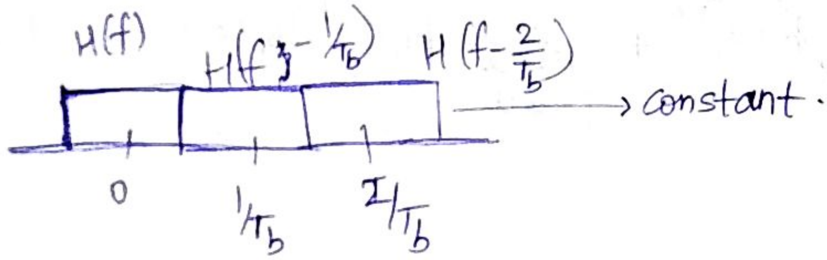
$$h(t) \sum_{k=-\infty}^{\infty} \delta(t-kT_b) = h(0)\,\delta(t)$$

Take FT on both sides

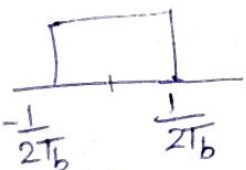
$$H(f) * \sum_{k=-\infty}^{\infty} \delta\left(f-\frac{k}{T_b}\right) = h(0) = \text{a constant}$$

$$\sum_{k=-\infty}^{\infty} H\left(f-\frac{k}{T_b}\right) = \text{constant.}$$

Sinc function has min. BW



$H(f) =$



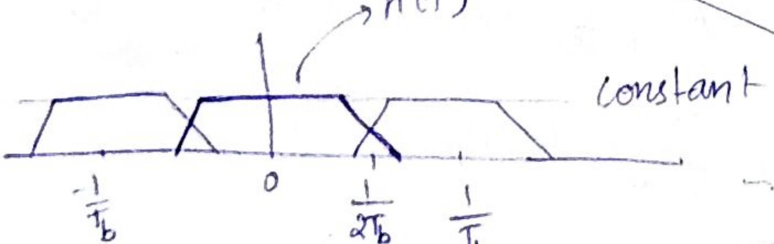
→ has min BW. Thus it is ideal.

$$BW = \frac{1}{2T_b} = \frac{f_s}{2}$$

$$\boxed{W = \frac{f_s}{2}}$$

$h(t) = \text{sinc}(t/T_b)$

Another solution ⇒



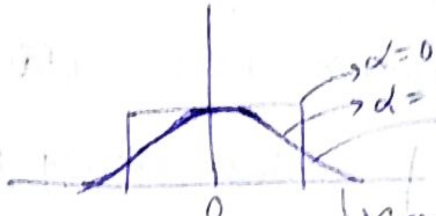
f_s = Bit rate $= R_b = \frac{1}{T_b}$

$BW = \frac{R_b}{2} = W \rightarrow$ Nyquist signalling rate (Diff from sampling rate)

$\boxed{W = \frac{f_s}{2}}$

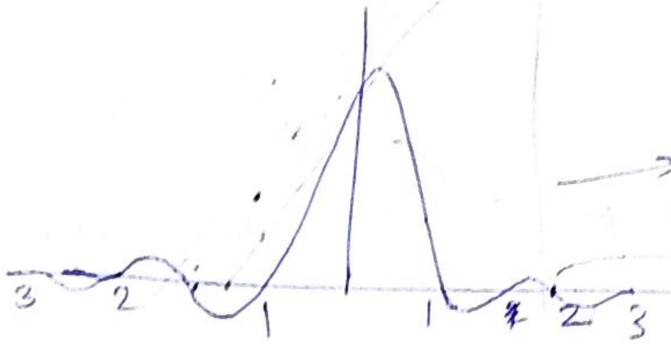
Theoretically ideal → but not practically as slight shift in time also causes ISI.
↓
from $\frac{1}{T_b}$

If side lobes decay faster → better for practical use.



raise cosine spectrum in time dom.

compromise in BW



→ decay faster.

→ more zero crossings.

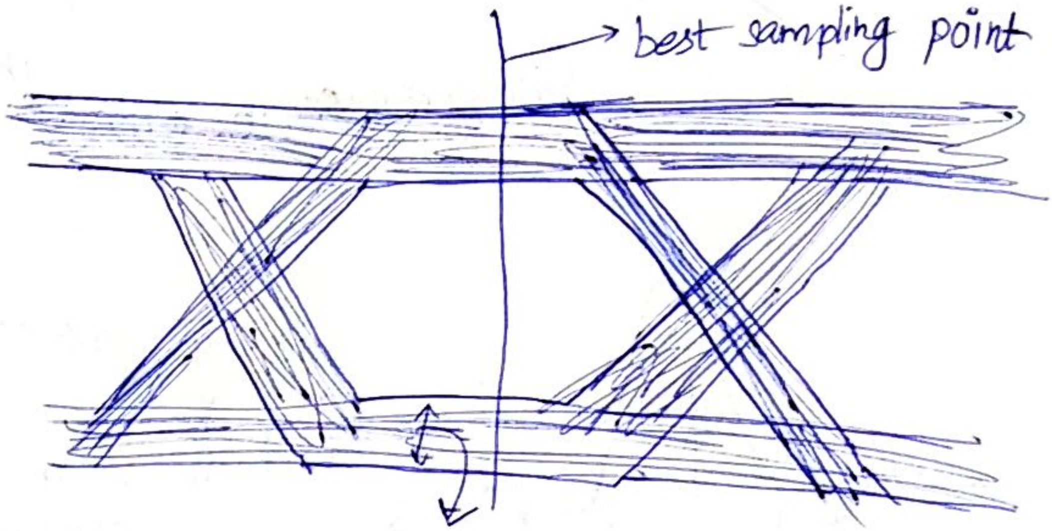
$$h(t) = \frac{\sin(\pi t/T)\cos(\pi \alpha t/T)}{(\pi t/T)(1 - 4\alpha^2 t^2/T^2)}$$

Bit error rate $= 10^{-6} = \frac{1}{10^6} \rightarrow$ 1 bit out of 10^6 bits is wrong.

~~God~~ Good value for communication.

Eye Pattern

The recieved pulse is affected by ISI, pulse broadening, amplitude reduction, noise addition.



jitter → delay in clk pulse.

Power $= \int S_{xx}(f)\, df$

Parsevals identity

$$\int |X(f)|^2\, df = \int |x(t)|^2\, dt$$

$\int \operatorname{sinc}^2 fT_b\, df \longrightarrow$ integration rectangular pulse

Avg. power

$$= \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt \qquad T \to \infty$$

$\int_{-T}^{T} |x(t)|^2 \, dt \to$ Energy.

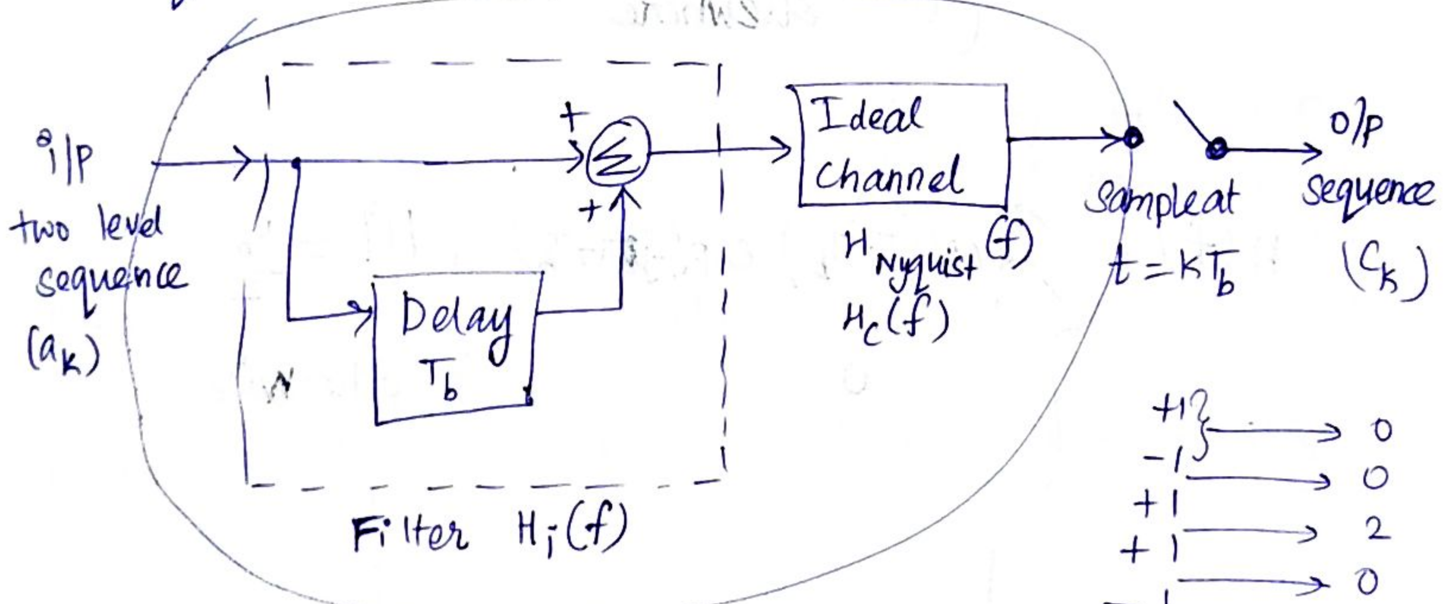
Compute BW required to transmit bits at the rate of 1 Mbps for each of the line coding schemes. Assume $a = 10V$. Compute power requirements for each of the coding schemes.

DUOBINARY ENCODING: (correlative encoding)

Allows ISI in a controlled way

↓

requires min BW



+1, −1 → 0
−1, +1 → 0
+1, +1 → 2
+1, −1 → 0
−1, −1 → −2

↓ using this, if we know first bit, we can make a decision

current bit exactly coincides with previous bit

↳ controlled ISI

$$x(t) \xrightarrow{FT} X(f)$$

$$x(t-t_0) \xrightarrow{FT} e^{-j2\pi f t_0} X(f)$$

Delay element →

$$h(t) = \delta(t - T_b)$$

$$H(f) = \exp(-j2\pi f T_b)$$

H(f) of total system =

$$H_c(f)\left[1 + \exp(-j2\pi f T_b)\right]$$

$$= H_c(f)\left[\exp(j\pi f T_b) + \exp(-j\pi f T_b)\right]\exp(-j\pi f T_b)$$

$$= 2H_c(f)\cos(\pi f T_b)\exp(-j\pi f T_b).$$

$$H_c(f) = \begin{cases} 1 & |f| \le \frac{R_b}{2} \\ 0 & \text{elsewhere} \end{cases}$$

$$H(f) = \begin{cases} 2\cos(\pi f T_b)\exp(-j\pi f T_b), & |f| \le \frac{R_b}{2} \\ 0, & \text{otherwise} \end{cases}$$

Data seq		1	0	1	0	1	0	0	1	1
polar b_k	Assume +1	+1	−1	+1	−1	+1	−1	−1	+1	+1

has to be agreed by reciever & transmitter.

$c_k = b_k + b_{k-1} = \quad +2 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad -2 \quad 0 \quad +2$

If this becomes 1·5 due to noise, one bit becomes wrong

This error is propogated due to seq of zeroes

At reciever.

If $c_k > +1 \longrightarrow$ Data 1

$c_k < -1 \longrightarrow 0$

$-1 < c_k < +1 \longrightarrow$ Current bit is the inverted version of the previous bit.

Decision at reciever $\rightarrow$	+1	-1	+1	-1	+1	-1	-1	+1	+1
Data $\rightarrow$	1	0	1	0	1	0	0	1	1

<u>disadvantage:</u>
There is propogation of error.

To overcome this, <u>precoder</u> is added before the duobinary encoder.

does encoding of bits even before line coding.

Data b_k		1	0	1	0	1	0	0	1	1
$a_k = b_k \oplus a_{k-1}$	1	0	0	1	1	0	0	0	1	0
		-1	-1	+1	+1	-1	-1	-1	+1	-1
$c_k = a_k + a_{k-1}$	+1	0	-2	0	+2	0	-2	-2	0	0

$\rightarrow$ this enters the duo binary encoder.

EX OR operation

If $c_k > +1 \longrightarrow$ data 0

$c_k < -1 \longrightarrow$ data 0

$-1 < c_k < +1 \longrightarrow$ data 1

Decision 1 0 1 0 1 0 0 1 1

error does not propogate.

because any decision doesn't depend on the previous bit.

Amplitude of pulse depends on Amp of msg.

PAM Pulse amplitude modulation

PPM position

PWM width.

width of pulse varies according to amplitude of msg.

the delay given to the pulse ~~de~~ varies according to amplitude of msg.