pblm.

1) For a unity fb control s/m, the open loop transfer function, $G(s) = \frac{10(s+2)}{s^2(s+1)}$. Find

(i) the position, velocity & acceleration error constants.

(ii) the steady state error when the i/p is R(s), where $R(s) = \frac{3}{s} - \frac{2}{s^2} + \frac{1}{3s^3}$.

soln :)

(i) $K_p \rightarrow$ positional error.

$$K_p = \lim_{s \to 0} G(s) H(s)$$

$\because H(s) = 1$

$$= \lim_{s \to 0} \frac{10(s+2)}{s^2(s+1)}$$

$$= \frac{20}{0}$$

$$\boxed{K_p = \infty.}$$

$K_v \rightarrow$ velocity error

$$K_v = \lim_{s \to 0} s\, G(s) H(s)$$

$$= \lim_{s \to 0} \frac{10(s+2)}{s(s+1)}$$

$$= \frac{20}{0}$$

$$\boxed{K_v = \infty.}$$

$K_a \rightarrow$ Acceleration error constant.

$$k_a = \lim_{s \to 0} s^2 \, G(s) H(s)$$

$$= \frac{10(s+2)}{(s+1)}$$

$$= \frac{20}{1}$$

$$\boxed{K_a = 20}$$

(ii) Steady state error (e_{ss})

$$E(s) = \frac{R(s)}{1 + G(s) H(s)}.$$

$$\boxed{e_{ss} = \lim_{t \to \infty} e(t) = \lim_{s \to 0} s \, E(s)}$$

$$E(s) = \frac{R(s)}{1 + G(s)\,H(s)}$$

$$= \frac{\left(\dfrac{9s^2 - 6s + 1}{3s^3}\right)}{\dfrac{s^3 + s + 10s + 20}{\cancel{s^2}(s+1)}}$$

$$= \frac{(9s^2 - 6s + 1)}{3s} \times \frac{(s+1)}{s^3 + s + 10s + 20}$$

$$sE(s) = \frac{(9s^2 - 6s + 1)(s+1)}{3(s^3 + s + 10s + 20)}$$

$$e_{ss} = \lim_{s \to 0} sE(s)$$

$$= \frac{(1)(1)}{3(20)}$$

$$\boxed{e_{ss} = \frac{1}{60}}$$

2) The open loop transfer function of a servo s/m with unity fb is $G(s) = \frac{10}{s(0.1s+1)}$.

Evaluate the static error constants of the s/m. obtain the steady state error of the s/m, when subjected to an i/p given by the polynomial, $r(t) = a_0 + a_1 t + \frac{a_2}{2} t^2$.

Soln:) Static error constants ⇒ K_p, K_v & K_a.

$$K_p = \lim_{s \to 0} G(s) H(s)$$

$$= \lim_{s \to 0} \frac{10}{s(0.1s+1)} = \frac{1}{0}$$

$$\boxed{K_p = \infty}.$$

$$K_v = \lim_{s \to 0} s\,G(s) H(s)$$

$$= \lim_{s \to 0} \frac{10}{(0.1s+1)} = 10$$

$$\boxed{K_v = 10}.$$

$$K_a = \lim_{s \to 0} s^2 G(s) H(s)$$

$$= \lim_{s \to 0} \frac{10s}{(0.1s+1)}$$

$$\boxed{K_a = 0}$$

→ Steady state error :-

$$e(t) = c_0 r(t) + c_1 r'(t) + \frac{c_2}{2!} r''(t) + \cdots$$

$$r(t) = a_0 + a_1 t + \frac{a_2}{2} t^2 .$$

$$r'(t) = a_1 + \frac{a_2}{\cancel{2}} \cdot \cancel{2} t$$

$$r'(t) = a_1 + a_2 t$$

$$r''(t) = a_2 .$$

$$c_0 = \frac{1}{1+K_p} = \frac{1}{\infty} = 0 .$$

$$c_1 = \frac{1}{K_v} = \frac{1}{10} = 0.1$$

$$c_2 = \frac{1}{K_a} = \frac{1}{0} = \infty .$$

$$e(t) = 0 + \frac{1}{10}(a_1 + a_2 t) + \infty .$$

$$e_{ss} = \lim_{t \to \infty} e(t)$$

$$\boxed{e_{ss} = \infty .}$$

3) A unity fb s/m has the forward transfer function $G(s) = \dfrac{K_1(2s+1)}{s(5s+1)(1+s)^2}$. when the i/p $r(t) = 1+6t$ determine the minimum value of K_1, so that the steady state error is less than 0.1.

solu,

$$G(s) = \frac{K_1(2s+1)}{s(5s+1)(1+s)^2}.$$

$$E(s) = \frac{R(s)}{1+G(s)H(s)}$$

$$r(t) = 1+6t$$

$$R(s) = 1/s + \frac{6}{s^2} = \left(\frac{s+6}{s^2}\right)$$

$$E(s) = \frac{(s+6)}{s^2\left[1 + \dfrac{K_1(2s+1)}{s[5s+1][1+s^2]}\right]}$$

$$= \frac{(s+6)(5s^2+s)(1+s^2)}{s^2\left[(5s^2+s)(1+s^2) + K_1(2s+1)\right]}$$

$$e_{ss} = \lim_{s\to 0} sE(s) = \lim_{s\to 0} \frac{\cancel{s}(s+6)(5s^2+s)(1+s^2)}{s^{\cancel{2}}\left[(5s^2+s)(1+s^2)+K_1(2s+1)\right]}$$

$$= \lim_{s\to 0} \frac{6}{(K_1)}$$

$$e_{ss} = 0.1$$

$$\therefore \frac{6}{k_1} = 0.1$$

$$k_1 = \frac{6}{0.1}$$

$$\boxed{k_1 = 60}$$

Controllers:-

→ A controller is a device introduced in the s/m to modify the error signal and to produce a control signal.

→ The controller modifies the transient response of the s/m.

→ Depending on the control actions provided the controllers can be classified as,

1) Two-position or ON-OFF controllers.
2) proportional (P) controller.
3) Integral (I) controller.
4) PI controller.
5) PD controller.
6) PID controller.

→ (i) proportional controllers :- (P)

→ It is a device that produces a control signal, $p(t)$ proportional to the i/p error signal. $e(t)$

$$p(t) \propto e(t).$$

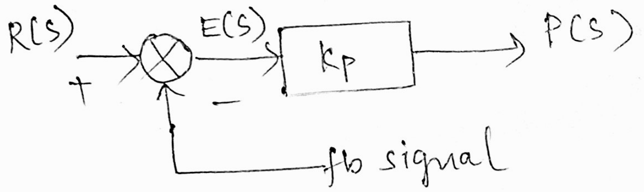
$$p(t) = K_p \, e(t).$$

$K_p \rightarrow$ proportional gain or constant.

→ on taking L.T.

$$P(s) = K_p E(s).$$

$\therefore$ Tfr function of p-controller, $\frac{P(s)}{E(s)} = K_p.$



→ P-controller amplifies the error signal by K_p and also increases the loop gain by the same amount of K_p.

→ ↑se in Loop gain improves the steady state tracking accuracy, disturbance signal rejection & makes the s/m less sensitive to parameter variations.

→ But increasing the gain to very large values may lead to instability of the s/m, which leads to constant steady state error.

ii) Integral-controllers:-(I)

→ The integral controller is a device the produces a control signal I(t) which is proportional to integral of the i/p error signal e(t).

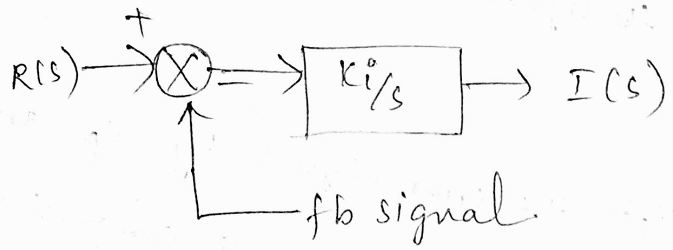
$$I(t) \propto \int e(t)\,dt$$

$$I(t) = K_i \int e(t)\,dt$$

K_i → Integral gain or constant.

→ LT ⇒ $I(s) = \frac{K_i}{s} E(s)$

$$\therefore \frac{I(s)}{E(s)} = \frac{K_i}{s}$$



→ Integral controller removes or reduces the steady state error without the manual reset. Hence it is referred to as automatic reset.

→ This controller may lead to oscillatory response & hence the s/m may become unstable.

(iii) P-I controller :-

→ The proportional plus Integral controller (PI) produces an o/p signal; proportional to error signal and also proportional to the integral of error signal.

$$P_i(t) \propto \left[e(t) + \int e(t)\,dt \right]$$

$$P_i(t) = K_p\, e(t) + \frac{K_p}{T_i} \int e(t)\,dt.$$

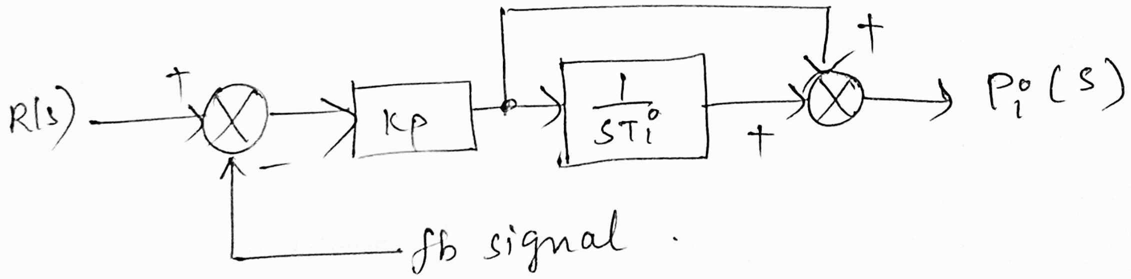
$K_p \rightarrow$ proportional gain

$T_i \rightarrow$ Integral time.

→ $$P_i(S) = K_p E(s) + \frac{K_p}{T_i} \frac{1}{s} E(s)$$

$$P_i(s) = E(s) \left[K_p + \frac{K_p}{sT_i} \right]$$

→ $$\frac{P_i(s)}{E(s)} = K_p \left(1 + \frac{1}{sT_i} \right)$$



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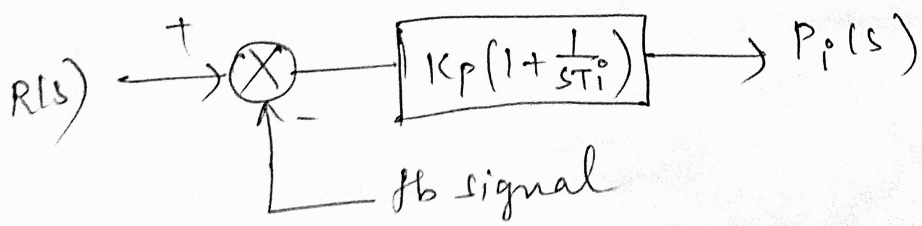

→ The proportional action increases the loop gain & makes the s/m less sensitive to s/m parameters and the Integral action eliminates the steady state error.

→ The Inverse of Integral time T_i is called as reset rate.