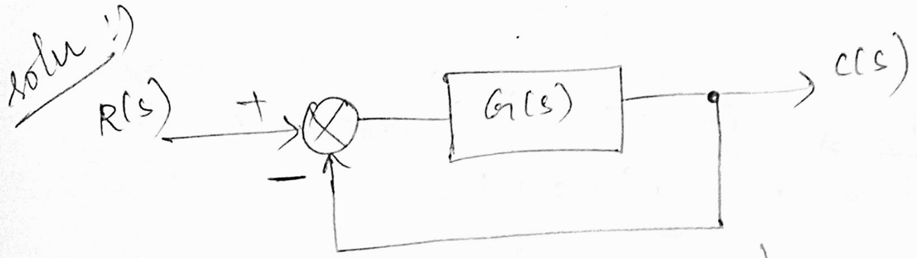
Eg:

1) A unity feedback control s/m is characterized by the following open loop transfer function $G(s) = \frac{(0.4s+1)}{s(s+0.6)}$

Determine its transient response for unit step i/p & sketch the response. Evaluate the maximum overshoot and corresponding peak time.

solu :)



$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)} = \frac{\frac{(0.4s+1)}{s(s+0.6)}}{1+\frac{(0.4s+1)}{s(s+0.6)}}$$

$$= \frac{0.4s+1}{s(s+0.6)+0.4s+1}$$

$$\boxed{\frac{C(s)}{R(s)} = \frac{0.4s+1}{s^2+s+1}.}$$

$\rightarrow r(t) = u(t) \Rightarrow R(s) = 1/s.$

$$C(s) = R(s)\left(\frac{0.4s+1}{s^2+s+1}\right) = \frac{1\,(0.4s+1)}{s(s^2+s+1)}$$

$$\frac{0.4s+1}{s(s^2+s+1)} = \frac{A}{s} + \frac{Bs+C}{s^2+s+1}.$$

$$A = C(s) \times s\Big|_{s=0} \Rightarrow \boxed{A=1}$$

$$\rightarrow \quad 0.4s + 1 = A(s^2+s+1) + (Bs+C)s$$

$$0.4s+1 = s^2+s+1 + Bs^2 + Cs.$$

$$1+0.4s = (1+B)s^2 + (1+C)s + 1$$

$\rightarrow$ equate $\Rightarrow$ $1+B = 0 \Rightarrow \boxed{B=-1}$

$1+C = 0.4 \Rightarrow \boxed{C = -0.6}$

$$\rightarrow C(s) = \frac{1}{s} + \frac{(-1)s + (-0.6)}{s^2+s+1}.$$

$$= \frac{1}{s} - \frac{(s+0.6)}{(s^2+s+1)}$$

$s^2+s+1 \Rightarrow$ to have a perfect square

$$s^2+s+1+0.25-0.25$$

$$(s^2+s+0.25)+0.75$$

$$(s+0.5)^2+0.75$$

$$\rightarrow C(s) = \frac{1}{s} - \frac{(s+0.5)}{(s+0.5)^2+0.75} - \frac{0.1}{(s+0.5)^2+0.75}$$

$$= \frac{1}{s} - \frac{s+0.5}{(s+0.5)^2+0.75} - \frac{0.1 \times \sqrt{0.75}}{\sqrt{0.75}\left[(s+0.5)^2+0.75\right]}$$

$$= \frac{1}{s} - \frac{s+0.5}{(s+0.5)^2+0.75} - \frac{0.1}{\sqrt{0.75}} \frac{\sqrt{0.75}}{(s+0.5)^2+0.75}$$

$$c(t) = \mathcal{L}^{-1}\left[C(s)\right]$$

$$= 1 - e^{-0.5t}\cos(\sqrt{0.75}\,t) - 0.115\, e^{-0.5t}\sin(\sqrt{0.75}\,t)$$

$$c(t) = 1 - e^{-0.5t}\left[\cos(\sqrt{0.75}\,t) + 0.115\sin(\sqrt{0.75}\,t)\right]$$

→ Transient response is the response that vanishes (is equal to zero) when $t \to \infty$.

In the above eqn, the 2nd term, as $t \to \infty$, becomes zero,

$$\therefore\ c(t)\Big|_{transient} = e^{-0.5t}\left[\cos(\sqrt{0.75}\,t) + 0.115\sin(\sqrt{0.75}\,t)\right]$$

Note:

→ $c(t) \Rightarrow$ Total Response

Total Response = Transient Response + Steady state Response

$$c(t) = c_t(t) + c_{ss}(t).$$

$$\boxed{c(t)\Big|_{t\to\infty} = c_{ss}(t)}$$

since $\boxed{\lim_{t\to\infty} c_t(t) = 0}$

→ Maximum overshoot (M_p)

$$\% M_p = e^{-\zeta\pi/\sqrt{1-\zeta^2}} \times 100\ \%$$

$$s^2+s+1 \Rightarrow s^2 + 2\zeta\omega_n s + \omega_n^2$$

$$\omega_n^2 = 1 \Rightarrow \omega_n = 1 \text{ rad/sec}.$$

$$2\zeta\omega_n = 1$$

$$\zeta = \frac{1}{2} = 0.5 \;//$$

$$\% M_p = e^{-0.5\pi/\sqrt{1-0.5^2}} \times 100\ \%$$

$$= e^{-1.813} \times 100\ \%$$

$$\boxed{\% M_p = 16.32\ \%}$$

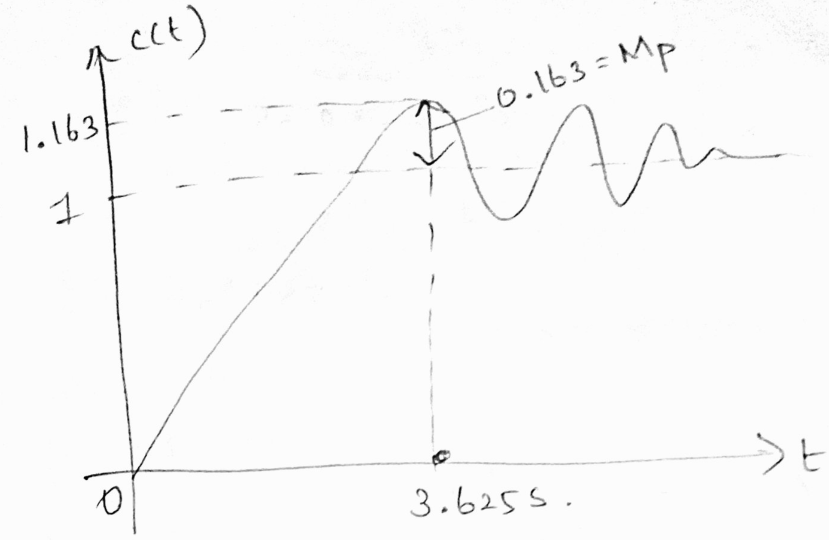
$$M_p = 0.163.$$

→ peak time $t_p = \dfrac{\pi}{\omega_d} = \dfrac{\pi}{\omega_n\sqrt{1-\zeta^2}}$

$$= \frac{\pi}{(1)\sqrt{1-0.25}}$$

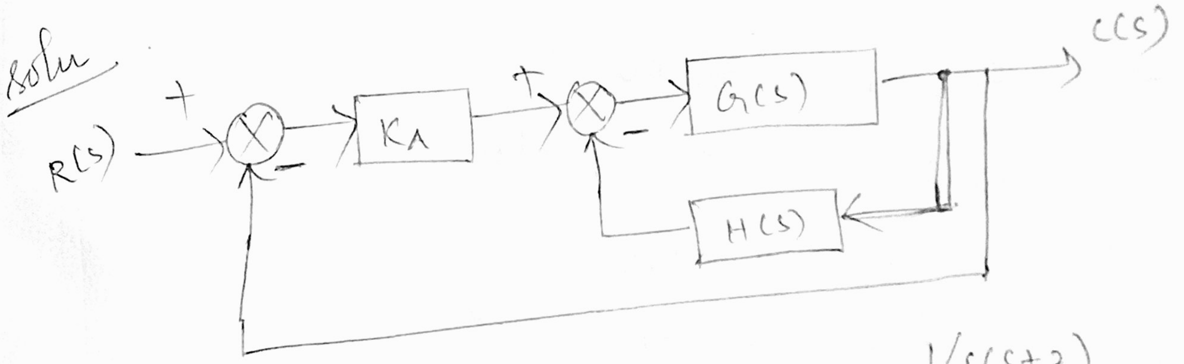
$$= \pi / 0.866$$

$$\boxed{t_p = 3.625 \text{ sec}}$$



2) A unity feedback control s/m has an amplifier with gain $K_A = 10$ & gain ratio $G(s) = 1/s(s+2)$ in the feed forward path. A derivative feedback, $H(s) = sK_0$ is introduced as a minor loop around $G(s)$. Determine the derivative feedback constant, (K_0) so that the s/m damping factor is 0.6.

Solu



Inner fb loop $\Rightarrow \dfrac{G(s)}{1+G(s)H(s)} = \dfrac{1/s(s+2)}{1+\dfrac{1}{s(s+2)} \cdot sK_0}$

$$= \frac{1}{s(s+2) + sK_0}$$

$$= \frac{1}{s^2 + 2s + sK_0}$$

Multiplied with $K_A = \dfrac{K_A}{s^2 + 2s + sK_0}$

$\rightarrow$ unity fb $\Rightarrow$ $$\frac{C(s)}{R(s)} = \frac{\dfrac{K_A}{s^2+2s+\cancel{2}K_o s}}{1+\dfrac{K_A}{s^2+2s+K_o s}}$$

$$\boxed{\frac{C(s)}{R(s)} = \frac{K_A}{s^2+(2\cancel{4}+K_o)s+K_A}}$$

$\rightarrow$ $K_A = 10$

$$\frac{C(s)}{R(s)} = \frac{10}{s^2+(2+K_o)s+10}.$$

$$\omega_n^2 = 10 \Rightarrow \omega_n = \sqrt{10} = \underline{\underline{3.162 \text{ rad/sec}}}$$

$\rightarrow$ $$2\zeta\omega_n = (2+K_o)$$

$$2\times 0.6\times 3.162 = 2+K_o$$

$$K_o = 3.7944-2$$

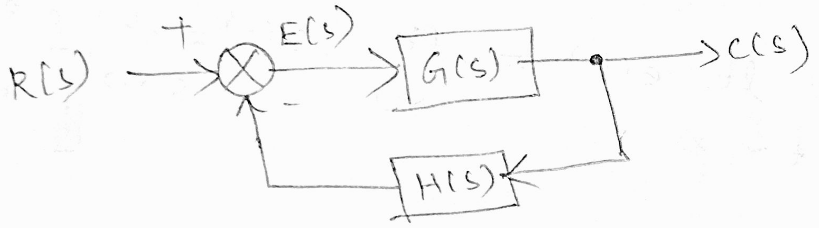
$$\boxed{K_o = 1.7944}$$

Steady State error :-

→ The steady state error is the value of error signal [e(t)], which t tends to infinity.

→ The error arise from the nature of i/ps, type of s/m & from non-linearity of s/m components.

→ The performance of the s/m is studied from the steady state error with step, ramp & parabolic i/ps.



→ $E(s) = R(s) - C(s)H(s)$.

& $C(s) = E(s)\,G(s)$.

→ ∴ $E(s) = R(s) - E(s)\,G(s)\,H(s)$

$E(s) + E(s)\,G(s)\,H(s) = R(s)$

$$\boxed{E(s) = \frac{R(s)}{1 + G(s)H(s)}}$$

→ $e(t)$ → error signal in time domain.

∴ $e(t) = L^{-1}[E(s)]$

$$= L^{-1}\left[\frac{R(s)}{1 + G(s)H(s)}\right]$$

→ e_{ss} ⇒ Steady state error. is defined as the Value of e(t) as t tends to infinity.

$$\boxed{e_{ss} = \lim_{t \to \infty} e(t)}$$

→ By final value theorem of Laplace transform,

$$\boxed{e_{ss} = \lim_{t \to \infty} e(t) \overset{LT}{\Longleftrightarrow} \lim_{s \to 0} sE(s)}$$

→ ∴ $$\boxed{e_{ss} = \lim_{t \to \infty} e(t) = \lim_{s \to 0} sE(s) = \lim_{s \to 0} \left[\frac{s\,R(s)}{1+G(s)H(s)}\right]}$$

Static Error constants:-

→ The steady state error value depends on the type number and the i/p signal.

→ Type-0 ⇒ constant steady state error (e_{ss}) with step i/p

Type-1 ⇒ constant e_{ss} for ramp i/p or velocity signal

Type-2 ⇒ constant e_{ss} for parabolic signal. or acceleration signal

→ errors:-

(i) positional error constant $(K_p) = \lim_{s \to 0} G(s)H(s)$

(ii) velocity error constant $(K_v) = \lim_{s \to 0} s\,G(s)H(s)$

(iii) Acceleration error constant $(K_a) = \lim_{s \to 0} s^2 G(s) H(s)$.

→ The error K_p, K_v & K_a are called as static error constants.

(1) <u>I/p :- unit step signal</u>.

$$e_{ss} = \lim_{s \to 0} s E(s)$$

$$= \lim_{s \to 0} \frac{s \cdot R(s)}{1 + G(s) H(s)}$$

$$= \lim_{s \to 0} \frac{\cancel{s} \cdot 1/\cancel{s}}{1 + G(s) H(s)}$$

$$= \frac{1}{1 + \lim_{s \to 0} G(s) H(s)}$$

$$e_{ss} = \frac{1}{1 + K_p}.$$

<u>case (i)</u> : Type '0' s/m

$$K_p = \lim_{s \to 0} G(s) H(s) = \lim_{s \to 0} \frac{K (s+Z_1)(s+Z_2) \ldots}{(s+P_1)(s+P_2) \ldots}$$

$$K_p = \frac{K (Z_1)(Z_2) \ldots}{(P_1)(P_2) \ldots}$$

$= \text{constant}$.

$$\therefore \boxed{e_{ss} = \frac{1}{1 + K_p}} \Rightarrow \text{constant}$$

case ii) Type - 1 s/m

$$K_p = \lim_{s \to 0} G(s) H(s)$$

$$= \lim_{s \to 0} K \frac{(s+z_1)(s+z_2)\cdots}{s(s+P_1)(s+P_2)\cdots}$$

$$K_p = \infty.$$

$$e_{ss} = \frac{1}{1+K_p} = \frac{1}{1+\infty} = 0$$

$$\boxed{\therefore e_{ss} = 0.}$$

→ ∴ the value of 'K_p' is infinity for ~~~~ Type '1' & above s/ms with unit step as i/p.

2) Ramp i/p

$$e_{ss} = \lim_{s \to 0} \frac{s R(s)}{1 + G(s) H(s)}$$

$$r(t) = t \Rightarrow R(s) = 1/s^2.$$

$$e_{ss} = \lim_{s \to 0} \frac{s \cdot 1/s^2}{1 + G(s) H(s)}$$

$$= \lim_{s \to 0} \frac{1}{s[1 + G(s) H(s)]}$$

$$e_{ss} = \lim_{s \to 0} \frac{1}{s + sG(s)H(s)}$$

$$= \lim_{s \to 0} \frac{1}{sG(s)H(s)}$$

$$\boxed{e_{ss} = \lim_{s \to 0} \frac{1}{K_v}}$$

case (i) - Type 0 s/m.

$$K_v = \lim_{s \to 0} sG(s)H(s)$$

$$= \lim_{s \to 0} \frac{sK(s+z_1)(s+z_2)\ldots}{(s+P_1)(s+P_2)\ldots}$$

$$K_v = 0$$

$$\therefore \boxed{e_{ss} = \lim_{s \to 0} \frac{1}{K_v} = \frac{1}{0} = \infty.}$$

case (ii) Type-1 s/m.

$$K_v = \lim_{s \to 0} sG(s)H(s)$$

$$= \lim_{s \to 0} \frac{\cancel{s}K(s+z_1)(s+z_2)\ldots}{\cancel{s}(s+P_1)(s+P_2)\ldots}$$

$K_v =$ constant value

$$\boxed{e_{ss} = \frac{1}{K_v} \Rightarrow \text{constant}}$$

Case (iii) – Type – 2 s/m.

$$K_v = \lim_{s \to 0} s\, G(s) H(s)$$

$$= \lim_{s \to 0} \frac{\cancel{s} \cdot K (s+z_1)(s+z_2) \ldots}{s^{\cancel{2}} (s+P_1)(s+P_2) \ldots}$$

$$= \infty.$$

$$\therefore \boxed{e_{ss} = 1/K_v = 1/\infty = 0.}$$

Above Type 2 – the e_{ss} will be zero for ramp i/p.