Design Of Infinite Impulse Response Filters

Filter Transformations:

Filter - on getting an i/p produces an o/p

design → Analog domain - $H(S) \Rightarrow$ i/p, o/p $\Rightarrow$ Ct; 't'
→ Digital domain - $H(Z) \Rightarrow$ i/p, o/p $\Rightarrow$ Dt; 'n'

LTI S/m

I/P → [FILTER] → O/P

Conti.
Time domain :- $x(t)$ $h(t)$ [impulse response] $y(t)$

Freq. domain :- stablem system has bounded i/p and bounded o/p

For fourier transform, to converge, Dirichlet's condition should be followed.

For Conti. time, $h(t)$ should be absolutely integrable. If they are not stable, we should use Laplace Transform, which having poles help us find stable and unstable

Laplace Transform -

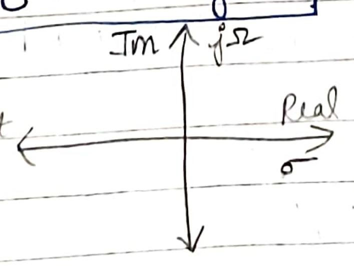
$$H(S) = \frac{\text{Num polynomial}}{\text{Denom polynomial}}$$

(Transfer function)

Num polynomial → zeros
Denom polynomial → poles (help us identify causal / stable)

$$\boxed{S = \sigma + j\Omega}$$

Unstable → Stable (position of pole movement is imp)



ROC:
→ Right of right most pole - causal s/m
→ Include $j\omega$ axis - stable s/m
→ All poles lie to left of $j\omega$ axis - Causal and stable s/m

Discrete –

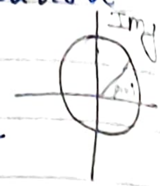
$x[n] \longrightarrow \boxed{h[n]} \longrightarrow y[n]$

DFT – sampling of fourier transformation

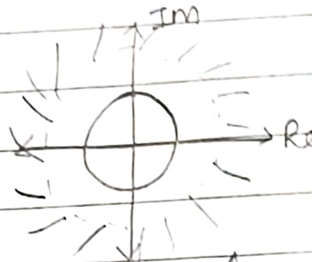
DTFT $\longrightarrow$ follows dirichlet's condition

$\Downarrow$ not satisfied

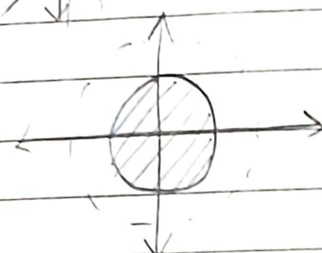
z – transform (poles) $\Rightarrow z = re^{j\omega}$



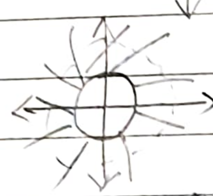
ROC → Causal : H(Z) –



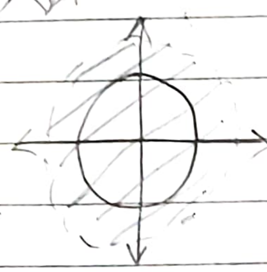
→ Non-causal : H(Z) –



→ Stable and Causal : H(Z) –



→ Stable and Non-causal : H(Z) –



Analog to Digital –

Analog Filter (H(S)) $\xrightarrow{\text{2 methods}}$ Digital Filter (H

→ DFI
→ DFII

implementation by adder, multiplier, delay elements

① impulse - invariant method

② bi-linear transformation

Impulse - Invariant method -

$$s \rightarrow z$$
$$H_a(s) \rightarrow H_a(z)$$
$$t \rightarrow n \quad \text{(sampling followed by quantisation)}$$

$$\boxed{H_a(s) \xrightarrow{ILT} h_a(t) \xrightarrow[T]{\text{sampling}} h_a(nT) \xrightarrow{\text{quantisation}} h[n] \xrightarrow{ZT} H(z)}$$

NOTE: $H_a(s)$ should be a simple one pole system (polynomial of degree 1)

Given, $$\boxed{H_a(s) = \sum_{K=1}^{N} \frac{C_K}{s - P_K}} \quad \text{— (a)}$$

$\Rightarrow$ ILT, $$h_a(t) = \sum_{K=1}^{N} C_K \, ILT\left\{\frac{1}{s - P_K}\right\}$$

$$h_a(t) = \sum_{K=1}^{N} C_K e^{P_K t}$$

$\Rightarrow$ Sampling and quantisation, $$h_a(nT) = h[n] = \sum_{K=1}^{N} C_K e^{P_K nT}$$

$\Rightarrow$ ZT, $$H(z) = \sum_{n=0}^{\infty}\left[\sum_{K=1}^{N} C_K e^{P_K nT}\right] \cdot z^{-n}$$

$$H(z) = \sum_{K=1}^{N} C_K \sum_{n=0}^{\infty}\left[e^{P_K T} \cdot z^{-1}\right]^{n}$$

$$\boxed{H(z) = \sum_{K=1}^{N} \frac{C_K}{1 - e^{P_K T} \cdot z^{-1}}} \quad \text{— (b)}$$

Comparing (a) and (b),

$$s - P_K \Longleftrightarrow 1 - e^{P_K T} \cdot z^{-1}$$

If $s - P_K = 0 \Rightarrow \boxed{s = P_K}$

and $\Rightarrow 1 - (e^{P_K T} \cdot z^{-1}) = 0 \Rightarrow \boxed{z = e^{P_K T}}$

$\therefore \boxed{Z = e^{ST}} \Rightarrow \boxed{Z = e^{(\sigma + j\Omega)T}}$

$S = \sigma + j\Omega$ and $Z = re^{j\omega}$

$\hookrightarrow |Z| = \boxed{r = e^{\sigma T}}$

$\angle Z = \boxed{\omega = \Omega T}$

	Real values		equal to	greater than	less than
Analog	σ	[0]	all poles on img axis	on right side of S-plane	on left side of S-plane
Digital	r	[1]	mapped on unit units	outside unit circle	inside unit circle

what happens to the poles

We know that $S_1 = \sigma_1 + j\Omega_1$ — (c)

$S_2 = \sigma_2 + j(\Omega_1 + 2\pi)$

$S_2 = \sigma_2 + j\Omega_1 + j2\pi$ — (d)

$$H_a(S) = \frac{C}{(S-S_1)(S-S_2)} \Rightarrow \frac{A}{S-S_1} + \frac{B}{S-S_2}$$

$$\Rightarrow \frac{A}{1-e^{S_1T}Z^{-1}} + \frac{B}{1-e^{S_2T}Z^{-1}}$$

(since $S - P_K = 1 - e^{P_KT} \cdot Z^{-1}$)

By observing (c) and (d), we can see that, irrespective different real (and same imaginary) parts, they are mapped to the same Z-plane.

This causes a major DRAWBACK, many to one mapping in impulse-invariant method (IIM).

This can be overcome in BI-LINEAR TRANSFORMATION method

1) Find $H(Z)$ using IIM for given $H_a(S) = \frac{2}{(S+1)(S+2)}$

(NOTE - when sampling interval is not given, T = 1 s)

$$H_a(S) = \frac{2}{(S+1)(S+2)} = \frac{A}{(S+1)} + \frac{B}{(S+2)}$$

$$\left.\begin{aligned} A + B &= 0 \\ 2A + B &= 2 \end{aligned}\right\} \Rightarrow \boxed{\begin{aligned} A &= 2 \\ B &= -2 \end{aligned}}$$

$$H_a(S) = \frac{2}{S+1} - \frac{2}{S+2}$$

$$\Rightarrow H(Z) = \frac{2}{1-e^{-1}.Z^{-1}} + \frac{-2}{1-e^{-2}.Z^{-1}}$$

$$= \frac{2Z}{Z-e^{-1}} + \frac{-2Z}{Z-e^{-2}}$$

$$= \frac{2Z}{Z-0.3678} - \frac{2Z}{Z-0.1353}$$

$$= 2Z\left[\frac{Z-0.1353-Z+0.3678}{Z^2-0.5032Z+0.0498}\right]$$

$$= \frac{2Z(0.2325)}{Z^2-0.5032Z+0.0498}$$

$$\boxed{H(Z) = \frac{0.4652Z}{Z^2-0.5032Z+0.0498}}$$

2) Find $H(Z)$ using IIM for given $H_a(S) = \frac{10}{S^2+7S+10}$ for T = 0.2 s

$$H_a(S) = \frac{10}{(S+5)(S+2)} = \frac{A}{S+5} + \frac{B}{S+2}$$

$A + B = 0$

$2A + 5B = 10$

$3A = 10$

$$= \frac{10}{3}\left(\frac{1}{S+5}\right) - \frac{10}{3}\left(\frac{1}{S+2}\right)$$

$\Rightarrow B = \frac{-10}{3}$ $\boxed{A = \frac{10}{3}}$

Date
Page

$$H(z) = \frac{10/3}{1-e^{-5*0.2}\cdot z^{-1}} - \frac{10/3}{1-e^{-2*0.2}\cdot z^{-1}}$$

$$= \frac{10/3\, z}{z-e^{-1}} - \frac{10/3\, z}{z-e^{-0.4}}$$

$$= \frac{10z}{3}\left[\frac{1}{z-0.3678} - \frac{1}{z-0.6703}\right]$$

$$= \frac{10z}{3}\left[\frac{-0.6703+0.3678}{z^2-1.0381z+0.2465}\right]$$

$$= \frac{10z}{3}\left[\frac{-0.3025}{z^2-1.0381z+0.2465}\right]$$

$$\boxed{H(z) = \frac{1.0083z}{z^2-1.0381z+0.2465}}$$

Bi-linear transformation method -

Drawback in IIM - many to one mapping

$\Downarrow$ overcome

Conformal mapping

$\Downarrow$

$\boxed{\text{One pole in 'S' plane} \rightarrow \text{one pole in 'z' plane}}$

$$s \rightarrow z$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{a}{s+b}$$

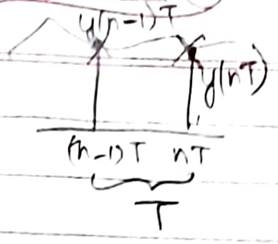
$$sY(s) + bY(s) = aX(s)$$

Apply Inverse - Laplace transform,

$$\frac{d\,y(t)}{dt} + by(t) = a\,x(t)$$

Take integration on both sides for "T", $(n-1)T \rightarrow nT$

$$\int_{(n-1)T}^{nT} \frac{d}{dt} y(t)\cdot dt + b\int_{(n-1)T}^{nT} y(t)\cdot dt = a\int_{(n-1)T}^{nT} x(t)\cdot dt$$



$t \xrightarrow{\text{Sampled}} (n)T \,/\, (n-1)T$

We integrate to find the sum and average $\Rightarrow \dfrac{y((n-1)T) + y(nT)}{2}$

$$\left[y(nT) - y((n-1)T)\right] + bT\left[\frac{y(nT) + y((n-1)T)}{2}\right] = \frac{aT}{2}\left[x[nT] + x[(n-1)T]\right]$$

Substituting $nT \equiv n$ and $(n-1)T = n-1$

$$\Rightarrow y(n) - y(n-1) + \frac{bT}{2}\left[y(n) + y(n-1)\right] = \frac{aT}{2}\left[x[n] + x[n-1]\right]$$

Taking Z-transform,

$$y(z) - z^{-1}y(z) + \frac{bT}{2}\left[Y(z) + z^{-1}Y(z)\right] = \frac{aT}{2}\left[X(z) + z^{-1}X(z)\right]$$

$$Y(z)\left[1 - z^{-1} + \frac{bT}{2}(1+z^{-1})\right] = \frac{aT}{2}X(z)\left[1 + z^{-1}\right]$$

$$H(z) = \frac{Y(z)}{X[z]} = \cancel{\frac{\frac{aT}{2}(1+z^{-1})}{1 - z^{-1} + \frac{bT}{2}(1+z^{-1})}}$$

$$= \frac{\frac{aT}{2}\left(\frac{1+z^{-1}}{1+z^{-1}}\right)}{\frac{1-z^{-1}}{1+z^{-1}} + \frac{bT}{2}}$$

$$H(z) = \frac{\frac{aT}{2}}{\frac{z-1}{z+1} + \frac{bT}{2}} = \frac{a}{\frac{2}{T}\left(\frac{z-1}{z+1}\right) + b}$$

$$s + b \Leftrightarrow \frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right] + b$$

$$\Rightarrow \boxed{s = \left(\frac{1-z^{-1}}{1+z^{-1}}\right)\frac{2}{T}}$$

We know that,

$$s = \sigma + j\Omega \qquad e^{j\omega} = \cos\omega + j\sin\omega$$

$$z = re^{j\omega}$$

$$\sigma + j\Omega = \frac{2}{T}\left[\frac{1-(re^{j\omega})^{-1}}{1+(re^{j\omega})^{-1}}\right]$$

$$\sigma + j\Omega = \frac{2}{T}\left[\frac{r(\cos\omega + j\sin\omega) - 1}{r(\cos\omega + j\sin\omega) + 1}\right]$$

$$= \frac{\cancel{2}}{T}\,\frac{(\cos\omega + j\sin\omega - 1)^2}{\cancel{2}\cos\omega\, j\sin\omega}$$

$$= \cancel{\cos^2\omega + \sin^2\omega}$$

$$\frac{(\cos\omega + j\sin\omega)^2 + 1 - 2\cos\omega\, j\sin\omega}{T}$$

$$= \frac{1 + 2\cos\omega\, j\sin\omega + 1 - 2\cos\omega\, j\sin\omega}{T\cos\omega\, j\sin\omega}$$

$$= \frac{2}{Tj\cos\omega\sin\omega}$$

$$=$$

$$\boxed{\sigma = \frac{2}{T}\left[\frac{(r^2-1)}{r^2+1+2r\cos\omega}\right] \qquad \Omega = \frac{2}{T}\left[\frac{2r\sin\omega}{r^2+1+2r\cos\omega}\right]}$$

	σ	S-plane		Z-plane
(i)	0	All poles on $j\omega$ axis	1	all poles on unit circle
(ii)	−ve	left side of plane	<1	inside unit circle
(iii)	+ve	right side of plane	>1	outside unit circle

Analog freq (Ω) $\longleftrightarrow$ Digital freq (ω)

relation

$\sigma = 0 ; \gamma = 1$

$$\therefore \Omega = \frac{2}{T}\left[\frac{\cancel{2}\sin\omega}{\cancel{2}(1+\cos\omega)}\right]$$

$$\boxed{\Omega = \frac{2\sin\omega}{T(1+\cos\omega)}}$$

Simplifying, $\Rightarrow$ $$\boxed{\underset{(Hz)}{\Omega} = \frac{2}{T_{(sec)}}\tan\left(\frac{\omega}{2}\right)^{rad/deg}}$$

Inverse

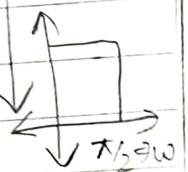
$$\therefore \boxed{\omega = 2\tan^{-1}\left(\frac{T\Omega}{2}\right)}$$

3)

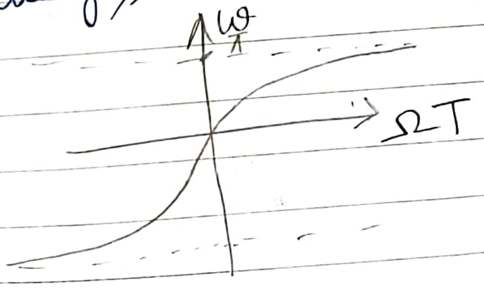
ΩT	(rad) ω	(degree) ω
1	0.927	53.1
5	2.38	136.39
10	2.74	157.38
20	2.94	168.58
40	3.04	174.28
80	3.09	177.14
800	3.13	179.71
1000	3.137	179.77
10,000	3.141	179.97
80,000	3.1415	179.997

large variations

Small variations



To ~~use~~ find what filter to use (accurate design), we should use analog freq (or convert to Ω)



Conversion of $\omega_{(d)}$ to $\Omega_{(a)}$ is called prewarping for filter design

classmate

Date

Page

4) $H(s) = \dfrac{2}{(s+1)(s+2)}$ and $T = 1s$. Convert it into $H(Z)$ using BLTM

$$\boxed{H(S) = \frac{A}{S+1} + \frac{B}{S+2}} \rightarrow \text{no need of partial fraction}$$

$$S = \frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]$$

$$\Rightarrow H(Z) = \frac{2}{\left(\frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]+1\right)\left(2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)+2\right)}$$

$$\frac{2(1+z^{-1})^2}{4(1-z^{-1})^2+6(1-(z^{-1})^2)+2(1+z^{-1})^2} = \frac{2}{4\left(\frac{1-z^{-1}}{1+z^{-1}}\right)^2+6\left(\frac{1-z^{-1}}{1+z^{-1}}\right)+2}$$

$$2(1+z^{-1})^2$$

$$= \frac{}{4+4z^{-2}-8z^{-1}+6-6z^{-2}+2+2z^{-2}+4z^{-1}} = \frac{0.166\,(1+z^{-1})^2}{(1-0.33)z^{-1}}$$

$$= \frac{}{10+12z^{-2}-4z^{-1}}$$

5) $H(S) = \dfrac{s^2+4.525}{s^2+0.629s+0.504}$ $\quad H(z) = ?$

$$H(Z) = \frac{2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)^2+4.525}{\left(\frac{1-z^{-1}}{1+z^{-1}}\right)^2+0.629\left(\frac{1-z^{-1}}{1+z^{-1}}\right)+0.504}$$

$$= \frac{4(1-z^{-1})^2+4.525(1+z^{-1})^2}{2(1-z^{-1})^2+0.629(1-(z^{-1})^2)+0.504(1+z^{-1})^2}$$

$$= 8.525 + 8.525z^{-2} + 1.05\,z^{-1}$$

$$5.762 + 3.246 \quad -6.992$$

$$= \frac{4(1+z^{-2}-2z^{-1}) + 4.525(1+z^{-2}+2z^{-1})}{4(1+z^{-2}-2z^{-1}) + 1.258(1-z^{-2}) + 0.504(1+z^{-2}+2z^{-1})}$$

$$= \frac{(4+4.525)+(4.525+4)z^{-2}+(-8+9.050)z^{-1}}{(4+1.258+0.504)+(4-1.258+0.504)z^{-2}+(-8+1.008)z^{-1}}$$

$$= \frac{8.525 + 8.525z^{-2} + 1.050z^{-1}}{5.762 + 3.246z^{-2} - 6.992z^{-1}}$$

$$= \frac{1.479 + 1.479z^{-2} + 0.1822z^{-1}}{1 + 0.563z^{-2} - 1.213z^{-1}}$$

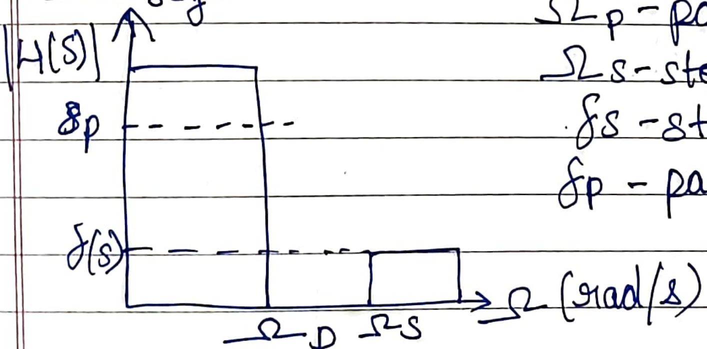
Butterworth Filter design -

All poles filter
[No Zeros]

$$H(S) = \frac{\text{constant}}{(S-S_1)(S-S_2)\ldots\ldots(S-S_K)}$$

LPF

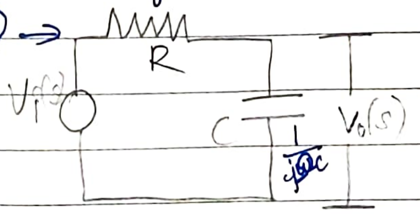
$S = j\Omega$



Ω_p - passband frequency
Ω_s - stopband frequency
δ_s - stopband attenuation
δ_p - passband attenuation

I order system ⟶ Single pole

(LPF) →



$$H(S) = \frac{V_o(S)}{V_i(S)} = ?$$

$$V_o(S) = V_i(S)\left[\frac{\frac{1}{SC}}{R + \frac{1}{SC}}\right]$$

$$\boxed{H_a(S) = \frac{1/RC}{S + 1/RC} = \frac{\Omega}{S+\Omega_c}}$$

$$\boxed{H_a(S) = \frac{\Omega_c}{S+\Omega_c}} \rightarrow \text{Denormalised}$$

Normalised s/m: $\Omega = 1$ rad/sec

$$\boxed{H_a(S) = \frac{1}{S+1}} \rightarrow \text{S/m func/ T.F of Ist order system}$$

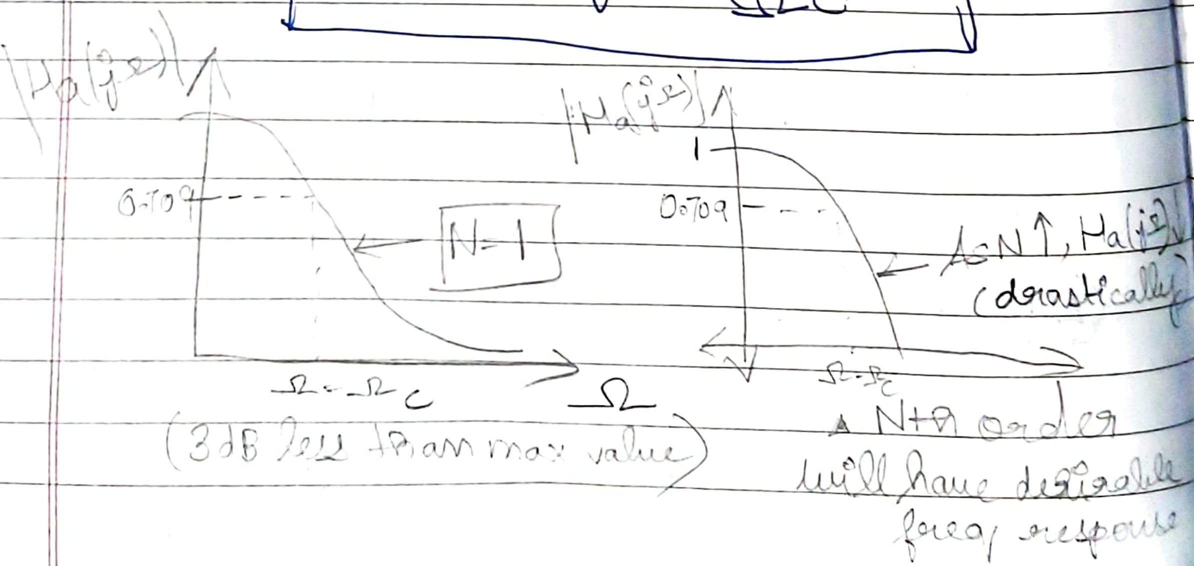
$S = j\Omega$

$$\Rightarrow H_a(j\Omega) = \frac{\Omega_c}{j\Omega + \Omega_c}$$

$$= \frac{1}{1 + j\left(\frac{\Omega}{\Omega_c}\right)}$$

1st order $\leftarrow$ $$\boxed{|H_a(j\Omega)| = \frac{1}{\sqrt{1+\left(\frac{\Omega}{\Omega_c}\right)^2}}}$$

Nth order $\leftarrow$ $$\boxed{H_a(j\Omega) = \frac{1}{\sqrt{1+\left(\frac{\Omega}{\Omega_c}\right)^{2N}}}}$$



Calculation

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Calculation of poles in BW expression:

$$|H_a(j\Omega)| = \frac{1}{\sqrt{1+\left(\frac{\Omega}{\Omega_c}\right)^{2N}}}$$

&

$$|H_a(\Omega)|^2 = \frac{1}{1+\left(\frac{\Omega}{\Omega_c}\right)^{2N}}$$

$$H_a(s)\cdot H_a(-s) = \frac{1}{1+\left(\frac{s}{j\Omega_c}\right)^{2N}}$$

For poles calculation, ~~pole~~ deno $=0$

$$\Rightarrow 1+\left(\frac{s}{j\Omega_c}\right)^{2N} = 0$$

$$\left[\frac{s^2}{-1(\Omega_c)^2}\right]^N = 1 = e^{j(2k-1)\pi}$$

where $k = 1$ to N

$$\left(\frac{-s^2}{\Omega_c^2}\right)^N = e^{j(2k-1)\pi}$$

$$s^2 = e^{j\frac{(2k-1)\pi}{N}} \cdot -\Omega_c^2$$

Poles for N-th order system $\leftrightarrow$ $\boxed{s_k = \pm j\,\Omega_c\, e^{j\left(\frac{2k-1}{2N}\right)\pi} \quad k = 1 \text{ to } N}$

Poles for 1st order system $\leftrightarrow$ $s_k = \pm j\,\Omega_c\, e^{j\frac{\pi}{2}}$

$$\boxed{s_k = -1\,\Omega_c}$$

$\updownarrow$

$H(s) \Rightarrow$ G.E $\Rightarrow \dfrac{\text{Constant}}{s+\Omega_c}$

Poles for 2nd order system $\Big\} = S_k = \pm j\Omega_c e^{j\left(\frac{2K-1}{2N}\right)\pi}$

$K = 1, 2$

$\boxed{\text{Normalised} \;\therefore \Omega_c = 1}$

$$\begin{cases} \dfrac{S_1}{\Omega_c} = \dfrac{-1}{\sqrt{2}} + j\dfrac{1}{\sqrt{2}} \\[2ex] \dfrac{S_2}{\Omega_c} = \dfrac{-1}{\sqrt{2}} - j\dfrac{1}{\sqrt{2}} \end{cases}$$

$$H_a(s) = \frac{\text{constant}}{(S - S_1)(S - S_2)}$$

$$= \frac{1}{\left(S - \left(\frac{-1}{\sqrt{2}} - \frac{j}{\sqrt{2}}\right)\right)\left(S - \left(\frac{-1}{\sqrt{2}} + \frac{j}{\sqrt{2}}\right)\right)}$$

$$= \frac{1}{\left[\left(S + \frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2\right]}$$

Normalised $\Omega_c = 1$ →

$$\boxed{H_a(s) = \frac{1}{(s^2 + \sqrt{2}s + 1)}}$$

Denormalised $\Rightarrow$ $s \rightarrow \dfrac{s}{\Omega_c}$

$$\boxed{H_a(s) = \frac{\Omega_c^2}{s^2 + \sqrt{2}\,\Omega_c s + \Omega_c^2}}$$

Poles for 3 rd order system $\Big\} \Rightarrow s_1 = \frac{-1}{2} + j\frac{\sqrt{3}}{2}$

$s_2 = -1$

$s_3 = \frac{-1}{2} - j\frac{\sqrt{3}}{2}$

$$H_a(s) = \frac{1}{(s+1)\left[\left(s+\frac{1}{2}\right)^2 - \left(\frac{\sqrt{3}j}{2}\right)^2\right]}$$

$$\boxed{H_a(s) = \frac{1}{(s+1)(s^2+s+1)}}$$ ← Normalised

$$\boxed{H_a(s) = \frac{\Omega_c^3}{(s+\Omega_c)(s^2+\Omega_c s+\Omega_c)^2}}$$

↳ denormalised $\left[s = \frac{s}{\Omega_c}\right]$

Normalised :

N	Poles	T.F
1	$s_1 = -1$	$1/s+1$
2	$s_1 = \frac{-1}{\sqrt{2}} + \frac{j}{\sqrt{2}}$ $s_2 = \frac{-1}{\sqrt{2}} - \frac{j}{\sqrt{2}}$	$\frac{1}{s^2+\sqrt{2}s+1}$
3	$s_1 = \frac{-1}{2} + \frac{j\sqrt{3}}{2}$ $s_2 = -1$ $s_3 = \frac{-1}{2} - \frac{\sqrt{3}j}{2}$	$\frac{1}{(s+1)(s^2+s+1)}$

N = odd ↓ real existing

N = even ↓ no real poles

For denormalised,

(LPF) $s = s/\Omega_c$

(HPS) $s = \frac{\Omega_c}{s}$

Page

Generalised T.F of Butterworth filter of Nth order system.

$N \rightarrow odd \Rightarrow$ one real poles, $\left(\frac{N-1}{2}\right)$ complex conjugate poles

$N \rightarrow even \Rightarrow \frac{N}{2}$ complex conjugate poles

$$H_a(S) = \begin{cases} \dfrac{\Omega_c^N}{(S+\Omega_c)\prod_{K=1}^{(N-1)/2}(S^2+b_K\Omega_c S+\Omega_c^2)} & \rightarrow \text{for odd} \\ \dfrac{\Omega_c^N}{\prod_{K=1}^{N/2}(S^2+b_K\Omega_c S+\Omega_c^2)} & \rightarrow \text{for even} \end{cases}$$

product

where $b_K = 2\sin\left(\frac{(2K-1)}{2N}\right)$, $K = 1$ to N

$$\boxed{\delta_p = -3dB \Rightarrow \Omega_p = \Omega_c \text{ (cutoff)}}$$

(3dB from max value)

$N \Rightarrow$ i) $\Omega_c \rightarrow$ known

ii) $\Omega_c \rightarrow$ unknown

(i) $\Omega_c \rightarrow$ known $\Rightarrow \Omega_c = \Omega_p$; $\delta_p = \frac{1}{\sqrt{2}}$

$|H_a(S)| = \delta_s \Rightarrow \Omega = \Omega_s$

$$|H_a(S)| = |H_a(j\Omega)| = \frac{1}{\sqrt{1+\left(\frac{\Omega_s}{\Omega_c}\right)^{2N}}} = \delta(s)$$

classmate
Date
Page

$$\Rightarrow (\delta_s)^2 = \frac{1}{1+\left(\frac{\Omega_s}{\Omega_c}\right)^{2N}}$$

$$N \log_{10}\left(\frac{\Omega_s}{\Omega_c}\right) = \log_{10}\left(\sqrt{\frac{1}{\delta_s^2}-1}\right)$$

$$N \geq \frac{\log_{10}\left(\sqrt{\frac{1}{\delta_s^2}-1}\right)}{\log_{10}\left(\frac{\Omega_s}{\Omega_c}\right)}$$

(order)

[round it off up to get it close to cut off freq with desirable characteristics]

(ii) $\Omega_c \rightarrow$ unknown

When $\Omega = \Omega_p \Rightarrow |H_a(j\Omega)| = \delta_p$

When $\Omega = \Omega_s = |H_a(j\Omega)| = \delta_s$

$$(\delta_p)^2 = \frac{1}{1+\left(\frac{\Omega_p}{\Omega_c}\right)^{2N}} \qquad \delta(s)^2 = \frac{1}{1+\left(\frac{\Omega_s}{\Omega_c}\right)^{2N}}$$

$$\left(\frac{\Omega_p}{\Omega_c}\right)^{2N} = \frac{1}{(\delta_p)^2} - 1 \qquad \left(\frac{\Omega_s}{\Omega_c}\right)^{2N} = \frac{1}{(\delta_c)^2} - 1$$

$$\boxed{N \geq \frac{\log_{10}\left(\sqrt{\dfrac{\frac{1}{\delta_s^2}-1}{\frac{1}{\delta_p^2}-1}}\right)}{\log_{10}\left(\frac{\Omega_s}{\Omega_p}\right)}}$$

$\Rightarrow \underbrace{\delta_s, \delta_p, \Omega_c, \Omega_p}_{\text{always known}}$

and when $\Omega_p \neq 3dB$, Ω_c is not known

For calculating Ω_c, compute order and using it find Ω_c

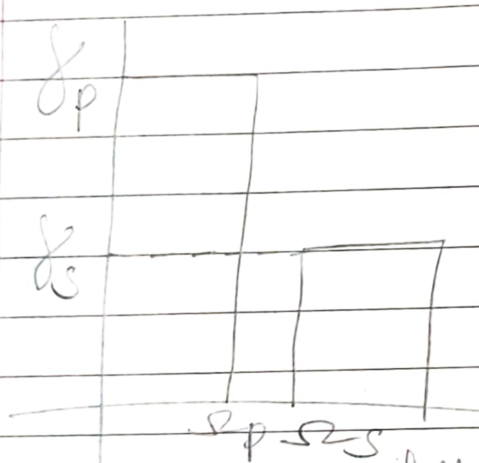
Procedure for BW Filter →

Given $\Omega_p, \Omega_s, \delta_p, \delta_s$

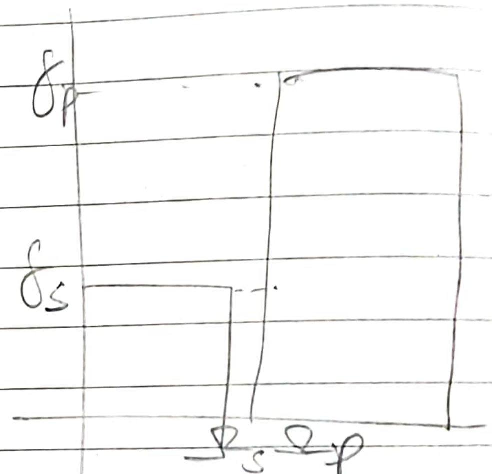
(i) $N \rightarrow$ order of filter $\begin{cases} \Omega_c \text{ is known} \\ \Omega_c \text{ is unknown} \end{cases}$

(ii) If Ω_c is known, skip this step and if it is not known, calculate Ω_c using N and any one above step

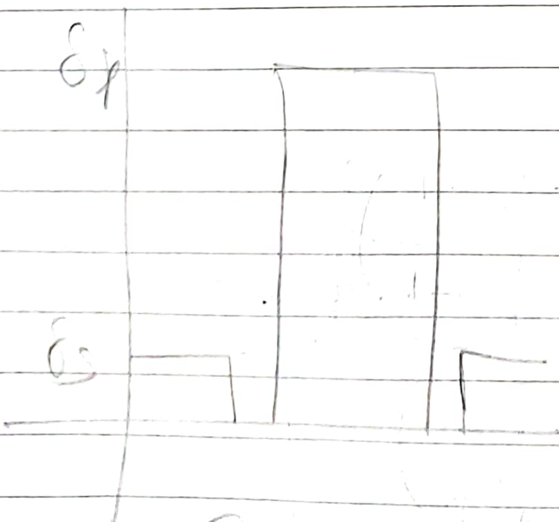
(iii) Calculate T.F



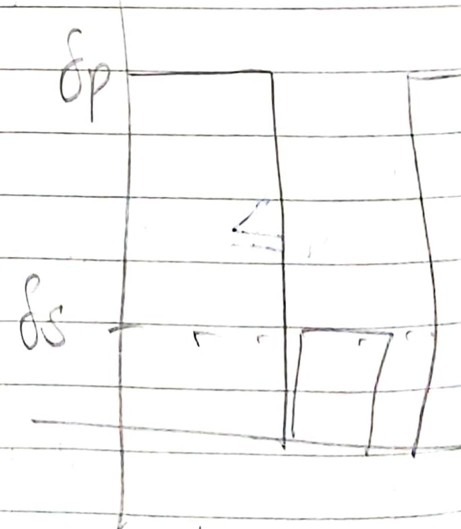
Low pass filter



High pass filter

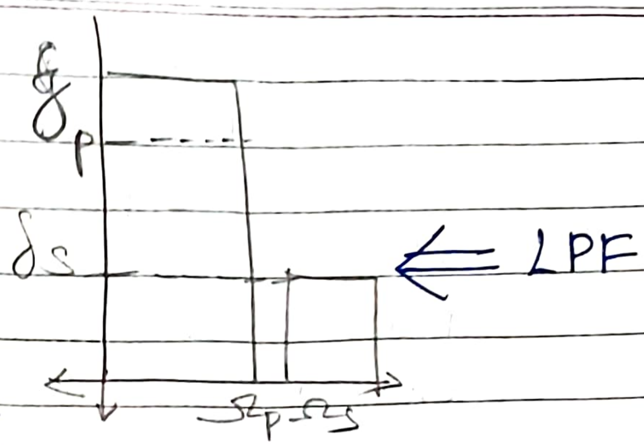


Bandpass



bandstop

6) BW. filter with



$\Omega_p = 1000\pi \ \frac{rad}{sec}$

$\Omega_s = 2000\pi \ \frac{rad}{sec}$

$\delta_p = -3 \text{ dB (from max value)} = 10^{-3/20} = 0.707$

$\delta_s = 40 \text{ dB} = 10^{-40/20} = 0.01$

$\Omega_c = \text{known} = 1000\pi$

$$N \geq \frac{\log_{10} \sqrt{\left(\frac{1}{\delta_s}\right)^2 - 1}}{\log_{10}\left(\frac{\Omega_s}{\Omega_c}\right)} = \frac{\log_{10}\left(\sqrt{\frac{1}{(0.01)^2} - 1}\right)}{\log_{10}\left(\frac{2000\pi}{1000\pi}\right)}$$

$$= \frac{1.99}{0.301} = 6.64$$

$\boxed{N = 7}$

$N \rightarrow$ odd

$$\Rightarrow H_a(s) = \frac{\Omega_c^N}{(s+\Omega_c) \prod_{k=1}^{(N-1)/2} (s^2 + b_k \Omega_c s + \Omega_c^2)}$$

where $b_k = 2 \sin\left(\frac{2k-1}{2N}\right)\pi$
$k = 1 \text{ to } N$

$$H_a(s) = \frac{3.0202 * 10^{24}}{(s + 3141.6)(s^2 + 1398 s + 9.869 * 10^6)(s^2 + 5660.8 s + 9.8 * 10^6)(s^2 + 3917.56 s + 9.86 * 10^6)}$$

classmate
Date
Page

$$\Omega = \frac{2}{T}\tan\left(\frac{\omega}{2}\right) \Rightarrow \text{BLTM}$$

$$\omega = \Omega T \quad \Rightarrow \text{IIM}$$

Incase, ~~for~~ we are given ω_p, ω_s, we should use any of the above 2 methods according to specification

7) ~~Des~~ B.W. filter $\Rightarrow H(Z)$ using BLTM

$$\left.\begin{array}{l}\Omega_p = 500\,rad/s \\ \Omega_s = 1000\,rad/s\end{array}\right\} LPF$$

$$\delta_p = -3dB = 0.707$$

$$\delta_s = -15dB = 10^{-15/20} = 0.1778$$

$$\Omega_s > \Omega_p \Rightarrow LPF$$

$$\delta_p = -3dB \Rightarrow \Omega_c = \Omega_p$$

$$\Rightarrow N \geq \frac{\log_{10}\left(\sqrt{\frac{1}{\delta_s^2} - 1}\right)}{\log_{10}\left(\frac{\Omega_s}{\Omega_c}\right)}$$

$$\geq \frac{\log\left(\sqrt{\frac{1}{(0.1778)^2} - 1}\right)}{\log\left(\frac{1000}{500}\right)}$$

$$N = 2.46 \Rightarrow \boxed{N = 3}$$

classmate

Date

Page

$N = 3$

$$H_a(s) = \frac{(\Omega_c)^N}{(s+\Omega_c)\prod_{k=1}^{(N-1)/2} s^2 + b_k \Omega_c s + \Omega_c^2}$$

$$b_k = 2 \sin \frac{(2k-1)\pi}{2N} \qquad k = 1 \text{ to } N$$

$$\Rightarrow k = 1, 2, 3$$

$$\Rightarrow b_1 = 2 \sin \frac{\pi}{6} \qquad b_2 = 2 \sin \frac{\pi}{2} \qquad b_3 = 2 \sin \frac{7\pi}{6}$$

$$b_1 = 0.0182 \qquad b_2 = 0.0548 \qquad b_3 = 0.1279$$

$$H_a(s) = \frac{1.25 \times 10^8}{(s+500)(s^2 + 0.0182 s + 2.5 \times 10^5)}$$

$$s = \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right) = 2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)$$

$$H(z) = \frac{1.25 \times 10^8}{\left(2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)+500\right)\left[4\left(\frac{1-z^{-1}}{1+z^{-1}}\right)^2 + 0.0182 * 2\left(\frac{1-z^{-1}}{1+z^{-1}}\right) + 2.5 \times 10^5\right]}$$

$$= \frac{1.25 \times 10^8 (1+z^{-1})^2}{\left[2(1-(z^{-1})^2) + 500(1+z^{-1})^2\right]\left[4(1-z^{-1})^2 + 0.0364(1-(z^{-1})^2) + 2.5 * 10^5 (1+z^{-1})^2\right]}$$