Lossless line

Cond perfect $\alpha = 0$

$R = 0 \quad G = 0$

ln dielectric are less

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$$

$$= \sqrt{(j\omega L)(j\omega C)}$$

$$\boxed{\gamma = j\omega\sqrt{LC}}$$

$\gamma = \alpha + j\beta \quad \alpha = 0 \quad \beta = \omega\sqrt{LC}$

phase velocity $V_p = \frac{D}{t}$

$$V = \nu\lambda$$

$$= \frac{f\,2\pi}{\beta}$$

$$\boxed{V = \frac{\omega}{\beta}}$$

$$v_p = \frac{\omega}{\omega\sqrt{LC}} = \frac{1}{\sqrt{LC}}$$

$$\boxed{v_p = \frac{1}{\sqrt{LC}}}$$

$$Z_0 = \sqrt{\frac{(R+j\omega L)}{(G+j\omega C)}} = \sqrt{\frac{j\omega L}{j\omega C}} = \sqrt{\frac{L}{C}}$$

$$\boxed{Z_0 = \sqrt{\frac{L}{C}}}$$

Distortion less line

α Indep of freq

$$\beta = \sqrt{\frac{(\omega^2 LC - RG) + \sqrt{(RG - \omega^2 LC)^2 + \omega^2 (LG + CR)^2}}{2}}$$

$$(RG - \omega^2 LC)^2 + \omega^2 (RC + LG)^2$$

$$\cancel{(RG)^2} + (\omega^2 LC)^2 \; \rho - 2RG\,\omega^2 LC$$

$$+ \omega^2 \left[(RC)^2 + (LG)^2 + 2RCLG \right]$$

$$= (RG)^2 + (\omega^2 LC)^2 + 2RG\omega^2 LC$$

$$-2RGLC + (RC)^2 + (LG)^2 = 0$$

$$(RC - LG)^2 = 0$$

$$RC = LG$$

$$\boxed{R/L = \frac{G}{C}}$$

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$$

$$= \sqrt{R\left(1+\frac{j\omega L}{R}\right)G\left(1+\frac{j\omega C}{G}\right)} \qquad \frac{L}{R} = \frac{C}{G}$$

$$G = \frac{R}{L}C$$

$$\alpha + j\beta = \sqrt{RG} \times \left(1 + \frac{j\omega C}{G}\right) \qquad \left(\frac{L}{R} = \frac{G}{C}\right)$$

$$\alpha = \sqrt{RG} \qquad \beta = \frac{\omega C}{G}\sqrt{RG}$$

$$= \frac{\omega C}{\sqrt{G}}\sqrt{R}$$

$$= \frac{\omega C}{\sqrt{G}} \times \sqrt{\frac{GL}{C}}$$

$$\boxed{\beta = \omega\sqrt{LC}}$$

$$Z_0 = \sqrt{\frac{(R+j\omega L)}{(G+j\omega C)}} = \sqrt{\frac{R\left(1+\frac{j\omega L}{R}\right)}{G+\left(1+\frac{j\omega C}{G}\right)}}$$

R, G negl

$$= \sqrt{\frac{R}{G}}$$

$$= \sqrt{\frac{j\omega L}{j\omega C}} = \sqrt{\frac{L}{C}}$$

$$R_0 = \sqrt{\frac{L}{C}}$$

$$V_p = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{LC}} = \frac{1}{\sqrt{LC}}$$

$$V_P = \frac{1}{\sqrt{LC}}$$

(V_P ind of freq) but (β = dep on freq)

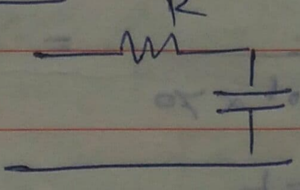
$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$ $\qquad Z_0 = R_0 + jX_0$

$\gamma = \alpha + j\beta$ $\qquad Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$

lossless $\gamma = j\omega\sqrt{LC}$ $\qquad \sqrt{L/C}$

Dist $\sqrt{RG} + j\omega\sqrt{LC}$ $\qquad \sqrt{L/C}$

Telephone Cable. Insulated wire with L & G negligible



$G=0$ $\quad$ $L=0$ $\qquad \sqrt{2j}$

$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$ $\qquad (1+j1)$

$\gamma = \sqrt{jR\omega C}$ $\qquad (1+j)^2 =$

$\qquad 1+j^2+2j$

$$= \sqrt{\frac{j2\omega CR}{2}} = (1+j)\sqrt{\frac{\omega CR}{2}}$$

$$\alpha = \sqrt{\frac{\omega C R}{2}} \qquad \beta = \sqrt{\frac{\omega C R}{2}}$$

$$v_p = \frac{\omega}{\beta} = \frac{\sqrt{\omega}\sqrt{\omega}}{\sqrt{\frac{\cancel{\omega} C R}{2}}}$$

$$\boxed{v_p = \sqrt{\frac{2\omega}{RC}}}$$

→ High freq faster than low freq components

α, β dep on freq.

Higher freq are attenuated more (α)

An airline characteristic Impedance of 70Ω, and phase constant of 3 rad/m at 100 MHz, Calc Inductance per meter and C per meter of line.

$R = G = 0 \qquad \alpha \to 0 \qquad \sigma_c \to \infty$

$$Z_0 = \sqrt{\frac{L}{C}} \qquad \beta = \omega\sqrt{LC}$$

$$\frac{Z_0}{\beta} = \sqrt{\frac{L}{C}} \times \frac{1}{\omega\sqrt{LC}}$$

$$\frac{Z_0}{\beta} = \frac{1}{\omega C}$$

$$C = \frac{\beta}{\omega Z_0} = \frac{3}{2\pi \times 100 \times 10^6 \times 70} = 68.2 \text{ PF/m}$$

$$R_0 = \sqrt{\frac{L}{C}} = R_0^2 C = L$$

$$L = (70)^2 \times (68.2 \times 10^{-12})$$

$$\boxed{L = 334.2 \text{ nH/m}}$$

Distortionless line has $Z_0 = 60\Omega$, $\alpha = 20\,mNp/m$
$v = 0.6\,C$ where c is speed of light in a vacuum.
Find R, L, C, G, C λ at 100MHz

$1\,Np = 8.685\,dB$

$1\,dB = 0.115129\,dB$

$$\frac{R}{L} = \frac{G}{C} \quad G = \frac{RC}{L}$$

$$\boxed{Z_0 = \sqrt{\frac{L}{C}}} \rightarrow (A)$$

$$\alpha = \sqrt{RG} = R\sqrt{\frac{C}{L}} = \frac{R}{Z_0}$$

$$R = \alpha Z_0$$

$$\boxed{V = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}} \rightarrow (B)$$

$$R = \alpha Z_0 = (20\times10^{-3}) \times 60$$

$$\boxed{R = 1.2\,\Omega/m}$$

$$\frac{Z_0}{V} = \frac{\sqrt{L/C}}{1/\sqrt{LC}} = \frac{60}{0.6\times3\times10^8} = 333\,nH/m$$

$$\boxed{L = 333\,nH/m}$$

$$G = \frac{\alpha^2}{R} = \frac{400\times10^{-6}}{1.2} = 333\,\mu S/m$$

$$\boxed{G = 333\,\mu S/m}$$

$$VZ_0 = \frac{1}{\sqrt{LC}} \times \sqrt{\frac{L}{C}} = \frac{1}{C}$$

$$C = \frac{1}{VZ_0} = \frac{1}{0.6\times3\times10^8\times60} = 92.59\,pF/m$$

$$\lambda = \frac{v}{f} = \frac{0.6\times3\times10^8}{10^8} = 1.8\,m$$

Characteristic Impedence

Max. power $Z_S = Z_L$

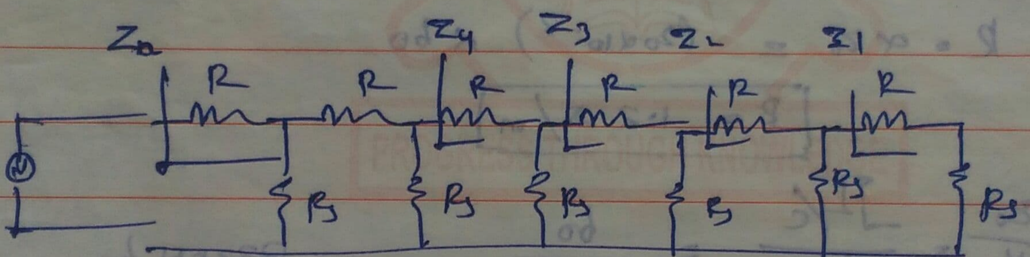
$Z_0 \rightarrow$ Impedance looking into long line.

(or)

Impedance looking line into finite length that is terminated in a purely resistive load with R_S equal to characteristic Impedance of line instead

∞ line (or) finite line terminated in purely resistive load $= Z_0$

$Z_0 = \sqrt{\frac{R}{G}}$ $\quad Z_0 = \sqrt{\frac{L}{C}}$ Independent freq also.



$R = 10\,\Omega$

$R_S = 100\,\Omega$

$Z_1 = R + R_S = 10 + 100 = 110$

Second section

$$Z_2 = R + \frac{R_S \times Z_1}{R_S + Z_1} = 10 + \frac{100 \times 110}{100 + 110} = 62.38\,\Omega$$

3rd

$$Z_3 = R + \frac{R_S \times Z_2}{R_S + Z_2} = 10 + \frac{100 \times 62.38}{100 + 62.38} = 48.3$$

4th

$$Z_4 = R + \frac{R_S \times Z_3}{R_S + Z_3} = 10 + \frac{10 \times 48.32}{100 + 48.32} = 42.6$$

node at last $37\,\Omega$

PART - B

Impedence looking into line 37Ω

$$Z_0 = Z_1 = R + \frac{R_s \times Z_L}{R_s + Z_L}$$

$$= 10 + \frac{100 \times 37}{100 + 37} = 37\Omega$$

$$Z_0 = Z_2 = R + \frac{R_s \times Z_1}{R_s + Z_1}$$

$$= 10 + \frac{100 \times 37}{100 + 37} = 37\Omega$$

physical significance of Tx line

$$I = I_R \left(\cosh \sqrt{Zy}\, l + \frac{Z_R}{Z_0} \sinh \sqrt{Zy}\, l \right)$$

If line Termi $Z_R = Z_0$

$$I_s = I_R \left(\cosh \sqrt{Zy}\, l + \sinh \sqrt{Zy}\, l \right)$$

$$\frac{I_s}{I_R} = e^{\sqrt{Zy}\, l} = e^{\gamma l}$$

Similarly

$$E = E_R \left[\cosh \sqrt{Zy}\, l + \frac{Z_0}{Z_R} \sinh \sqrt{Zy}\, l \right]$$

$$\frac{E(l)}{I(l)} = \frac{E_R \left[\cosh \sqrt{Zy}\, l + \frac{Z_0}{Z_R} \sinh \sqrt{Zy}\, l \right]}{I_R \left[\cosh \sqrt{Zy}\, l + \frac{Z_R}{Z_0} \sinh \sqrt{Zy}\, l \right]}$$

$$\frac{E(l)}{I(l)} = Z(l) = Z_R \frac{\left[\frac{Z_R \cosh \sqrt{Zy}\, l + Z_0 \sinh \sqrt{Zy}\, l}{Z_R} \right]}{\left[\frac{Z_0 \cosh \sqrt{Zy}\, l + Z_R \sinh \sqrt{Zy}\, l}{Z_0} \right]}$$

$$Z_s = \frac{E_s}{I_s} = Z_0 \left[\frac{Z_R \cosh \sqrt{ZY} l + Z_0 \sinh \sqrt{ZY} l}{Z_0 \cosh \sqrt{ZY} l + Z_R \sinh \sqrt{ZY} l} \right]$$

From Earlier

$$E = \frac{E_R (Z_R + Z_0)}{2Z_R} \left[e^{\sqrt{ZY} l} + \left(\frac{Z_R - Z_0}{Z_R + Z_0}\right) e^{-\sqrt{ZY} l} \right]$$

$\div$

$$I = \frac{I_R (Z_0 + Z_R)}{2Z_0} \left[e^{\sqrt{ZY} l} - \left(\frac{Z_R - Z_0}{Z_R + Z_0}\right) e^{-\sqrt{ZY} l} \right]$$

$$Z_s = \frac{E_s}{I_s} = \frac{E_R}{2Z_R} \times \frac{2Z_0}{I_R}$$

$$= Z_0 \left[\frac{e^{\gamma l} + \left(\frac{Z_R - Z_0}{Z_R + Z_0}\right) e^{-\gamma l}}{e^{\gamma l} - \left(\frac{Z_R - Z_0}{Z_R + Z_0}\right) e^{-\gamma l}} \right] \Rightarrow (A)$$

line Terminated in its characteristic impedance

$$Z_R = Z_0$$

$$Z_s = Z_0$$

another $l = \infty$ $\quad e^{\infty} = \infty \quad e^{-\infty} = 0$

$$Z_s = Z_0 \left[\frac{e^{\infty} + 0}{e^{\infty} - 0} \right]$$

$$\boxed{Z_s = Z_0}$$

Infinite line Impedance Z_0

Small line terminated with load Impedance equal to Characteristic Impedance.

$$E = \frac{E_R (Z_R + Z_0)}{2Z_R}\left[e^{\gamma l} + \left(\frac{Z_R - Z_0}{Z_R + Z_0}\right)e^{-\gamma l}\right]$$

$$I = \frac{I_R (Z_R + Z_0)}{2Z_0}\left[e^{\gamma l} - \left(\frac{Z_R - Z_0}{Z_R + Z_0}\right)e^{-\gamma l}\right]$$

$\because Z_R = Z_0$

$E = E_R e^{\gamma l}$ $\qquad E_R = E_s e^{-\gamma l}$

$I = I_R e^{\gamma l}$ $\qquad I_R = I_s e^{-\gamma l}$

$\boxed{\gamma = \alpha + j\beta}$

$$\boxed{\begin{aligned} E &= E_s e^{-\alpha l} e^{-j\beta l} \\ I &= I_s e^{-\alpha l} e^{-j\beta l} \end{aligned}}$$

Input Impedence

$$Z_s = Z_0\left[\frac{Z_R \cosh \gamma l + Z_0 \sinh \gamma l}{Z_0 \cosh \gamma l + Z_R \sinh \gamma l}\right]$$

$$(A) \Rightarrow \quad Z_s = Z_0\left[\frac{e^{\gamma l} + \left(\frac{Z_R - Z_0}{Z_R + Z_0}\right)e^{-\gamma l}}{e^{\gamma l} - \left(\frac{Z_R - Z_0}{Z_R + Z_0}\right)e^{-\gamma l}}\right]$$

$$\boxed{Z_s = Z_0\left(\frac{e^{\gamma l} + k\, e^{-\gamma l}}{e^{\gamma l} - k\, e^{-\gamma l}}\right)}$$

Sending end voltage

$$E_s = \frac{E_R (Z_R + Z_0)}{2Z_R} \left(e^{\gamma l} + K e^{-\gamma l} \right)$$

$$E_s = \frac{I_R (Z_R + Z_0)}{2} \left(e^{\gamma l} + K e^{-\gamma l} \right)$$

Transfer Impedance

$$Z_T = \frac{E_s}{I_R} = \frac{Z_R + Z_0}{2} \left(e^{\gamma l} + K e^{-\gamma l} \right)$$

$$Z_T = \left(\frac{Z_R + Z_0}{2} \right) \left(e^{\gamma l} + \frac{Z_R - Z_0}{Z_R + Z_0} e^{-\gamma l} \right)$$

$$= \frac{Z_R + Z_0}{2} e^{\gamma l} + \frac{Z_R - Z_0}{2} e^{-\gamma l}$$

$$\boxed{Z_T = Z_R \left[\frac{e^{\gamma l} + e^{-\gamma l}}{2} \right] + Z_0 \left[\frac{e^{\gamma l} - e^{-\gamma l}}{2} \right]}$$

Open and Short Ckt line

$Z_R = \infty$ open

$Z_R = 0$ Short

I/P Impedance of line

$$Z_s = Z_0 \left(\frac{Z_R \cosh \gamma l + Z_0 \sinh \gamma l}{Z_0 \cosh \gamma l + Z_R \sinh \gamma l} \right)$$

Short $Z_R = 0$

$$Z_{sc} = Z_0 \frac{Z_0 \sinh \gamma l}{Z_0 \cosh \gamma l} = Z_0 \tanh \gamma l$$

$$\boxed{Z_{sc} = Z_0 \tanh \gamma l}$$