Analog Transformation of prototype LPF to HPF/BPF/BSF:

(or)

Frequency Transformation in Analog domain:

So far we concentrated on designing a LPF for the given specifications. In this section, we discuss the frequency transformations that can be used to design LPFs with different pass band frequencies such as HPF, BPF, BSF from a normalized Low Pass Analog filter ($\Omega_c = 1\,\text{rad/sec}$).

LPF $\longrightarrow$ LPF:

LPF with Passband edge frequency Ω_p $\longrightarrow$ LPF with passband edge frequency Ω_p'

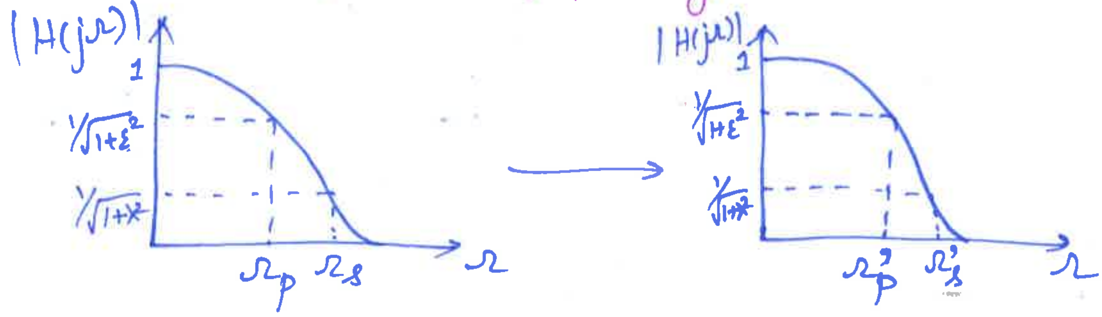
$$\boxed{s \longrightarrow \frac{\Omega_p \cdot s}{\Omega_p'}}$$

sub $\Omega_p = 1\,\text{rad/sec}$

$$H_p(s)\Big|_{s=\Omega_p s/\Omega_p'} = H_l(s)$$

$H_l(s)$ → Transfer fn. of transformed LPF

$H_p(s)$ ↓ Transfer function of prototype filter



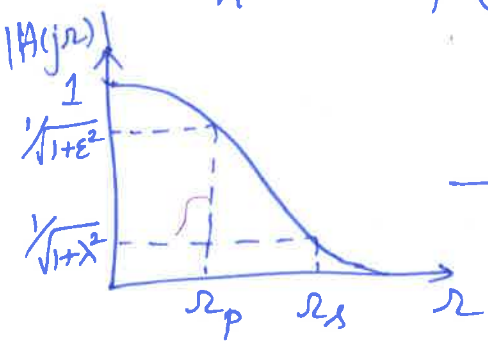
LPF → HPF :

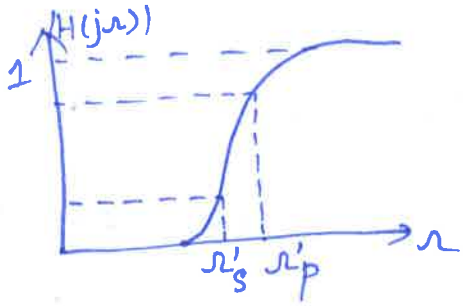
LPF with Passband edge frequency Ω_p ⟶ HPF with Passband edge frequency Ω_p'

$$\boxed{s \longrightarrow \frac{\Omega_p \cdot \Omega_p'}{s}}$$

Sub $\Omega_p = 1$ rad/sec

$$H_h(s) = H_p\left(\Omega_p \cdot \Omega_p' / s\right)$$





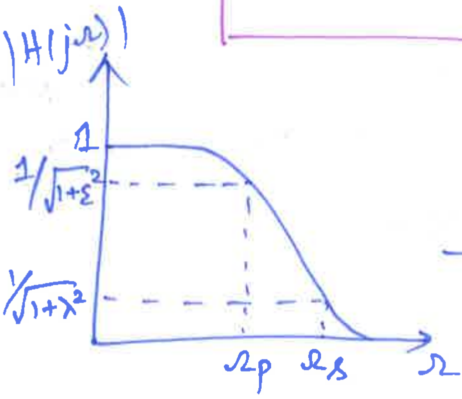
$\Omega_s' = \Omega_p, \quad \Omega_p' = \Omega_s$

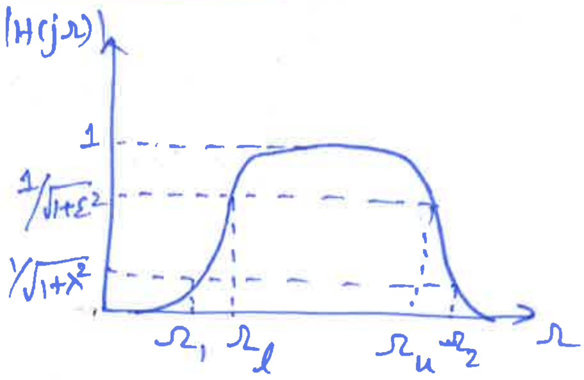
LPF → BPF :

LPF with Passband edge frequency Ω_p (or) Ω_c ⟶ BPF with lower band edge frequency Ω_l and an upper band edge frequency Ω_h (or) Ω_u

$$\boxed{s \longrightarrow \left(\frac{s^2 + \Omega_l \Omega_u}{s(\Omega_u - \Omega_l)}\right)\Omega_p}$$

Substitute $\Omega_p = 1$ rad/sec





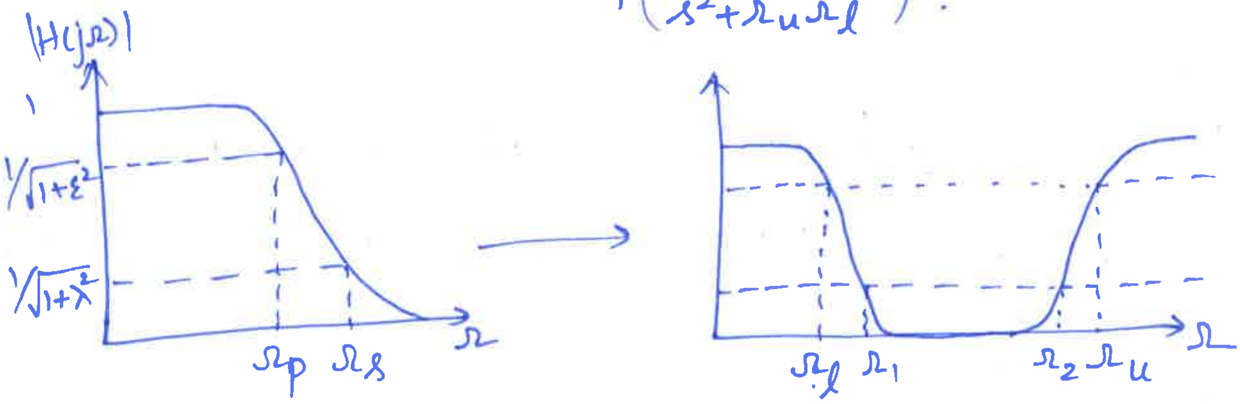
$$H_b(s) = H_p(s)\Big|_{s = \Omega_p \left(\frac{s^2 + \Omega_l \Omega_u}{s(\Omega_u - \Omega_l)}\right)}$$

LPF to BSF:

LPF with passband edge frequency Ω_p or Ω_c $\longrightarrow$ BSF with lower band edge frequency Ω_l

$$\boxed{s \longrightarrow \Omega_p\left(\frac{s(\Omega_u-\Omega_l)}{s^2+\Omega_u\Omega_l}\right)}$$ Sub $\Omega_p = 1\,rad/sec$

$$H_{bs}(s) = H_p(s)\Big/_{s=\Omega_p\left(\frac{s(\Omega_u-\Omega_l)}{s^2+\Omega_u\Omega_l}\right)}.$$



Pbm: Transform the single-pole Low Pass Butterworth filter with system function $H(s) = \dfrac{\Omega_p}{s+\Omega_p}$ into a BPF with upper & lower band edge frequencies Ω_u & Ω_l respectively.

Soln:

LPF $\longrightarrow$ BPF

$$s \longrightarrow \cancel{\frac{s(\Omega_u-\Omega_l)}{s^2+\Omega_u\Omega_l}} \quad \frac{s^2+\Omega_u\Omega_l}{s(\Omega_u-\Omega_l)}$$

Substitute 's' in $H(s)$,

$\Omega_p = 1\,rad/sec$

$$H(s) = \frac{1}{\left(\dfrac{s^2+\Omega_u\Omega_l}{s(\Omega_u-\Omega_l)}\right)+1}$$

$$H(s) = \frac{s(\Omega_u-\Omega_l)}{s^2+s(\Omega_u-\Omega_l)+\Omega_u\Omega_l}$$

zero of $H(s)$ is at $s=0$.

pole of $H(s)$ is at,

$$s = \frac{-(\Omega_u-\Omega_l) \pm \sqrt{(\Omega_u-\Omega_l)^2-4(\Omega_u\Omega_l)}}{2}$$

$$s = \frac{-(\Omega_u-\Omega_l) + \sqrt{\Omega_u^2+\Omega_l^2-6\Omega_u\Omega_l}}{2}$$

prbm: For the following specifications design a HPF.

$\alpha_p = 3dB$, $\alpha_s = 15dB$, $\Omega_p = 1000$ rad/sec, $\Omega_s = 500$ rad/sec

Design Steps:

1) Design ~~an~~ prototype LPF from the given specifications. (i.e. $H(s) = ?$) for $\Omega_p = \Omega_c = 1$ rad/sec

2) Transform LPF into HPF by substituting $s \rightarrow \frac{\Omega_p'}{s}$ to get $H_h(s)$.

Solution:

given: for HPF $\alpha_p = 3dB$, $\alpha_s = 15dB$, $\Omega_p' = 1000$ rad/sec

$\Omega_s' = 500$ rad/sec

$\therefore$ Specifications for LPF (prototype) $\Rightarrow$ $\alpha_p = 3dB$, $\Omega_p = 500$ rad/sec

$\alpha_s = 15dB$, $\Omega_s = 1000$ rad/sec

Step1: $N = ?\,,\quad H(s) = ?$

$$N \geq \frac{\log \lambda/\varepsilon}{\log 1/k}\,, \qquad \lambda = \sqrt{10^{0.1\alpha_s} - 1} = 5.533$$

$$\varepsilon = \sqrt{10^{0.1\alpha_p} - 1} = 1$$

$$\geq \frac{\log 5.533}{\log\left(\frac{1}{0.5}\right)} \qquad k = \Omega_p/\Omega_s = 0.5$$

$$\geq 2.468 \qquad \boxed{N=3}$$

from table of denominator polynomial of BW filter,

$$H(s) = \frac{1}{(s+1)\,(s^2+s+1)}$$

Step2:

$$H_h(s) = H(s)\Big/_{s \to \frac{1000}{s}}$$

$$H_h(s) = \frac{1}{\left(\frac{1000}{s}+1\right)\left(\left(\frac{1000}{s}\right)^2 + \left(\frac{1000}{s}\right)+1\right)}$$

$$\boxed{H_h(s) = \frac{s^3}{(s+1000)\,(s^2+1000s+(1000)^2)}}$$