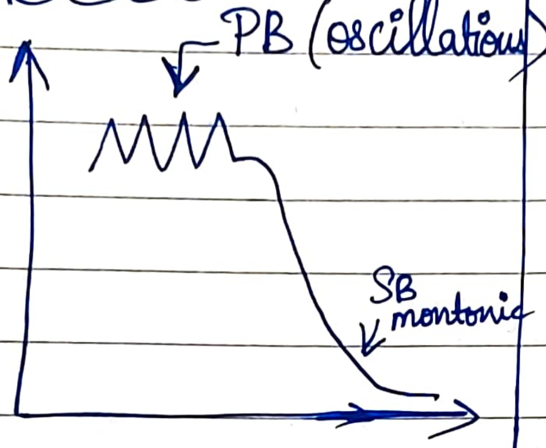
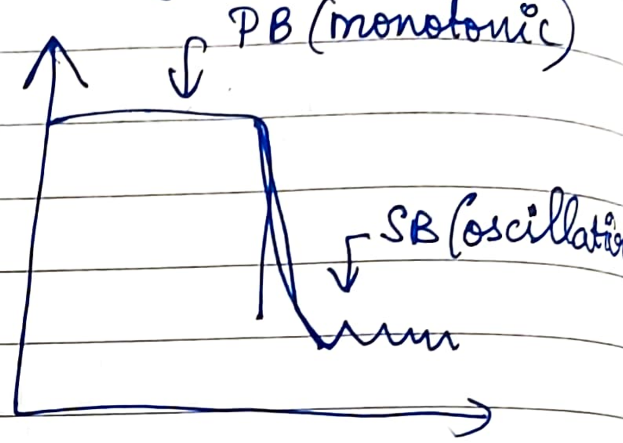
Chebyshev Filter design:

Types and General Transfer function-

Type - I



Type - II



General Transfunction

Butterworth Nth order $\Big\}$: $|H_a(j\Omega)| = \dfrac{1}{\sqrt{1 + \left(\dfrac{\Omega}{\Omega_c}\right)^{2N}}}$

Chebyshev Nth order $\Big\}$: $|H_a(j\Omega)| = \dfrac{1}{\sqrt{1 + \varepsilon^2 C_N^2\left(\dfrac{\Omega}{\Omega_p}\right)}}$

ε - typically < 1 (lesser than (or) smaller value)

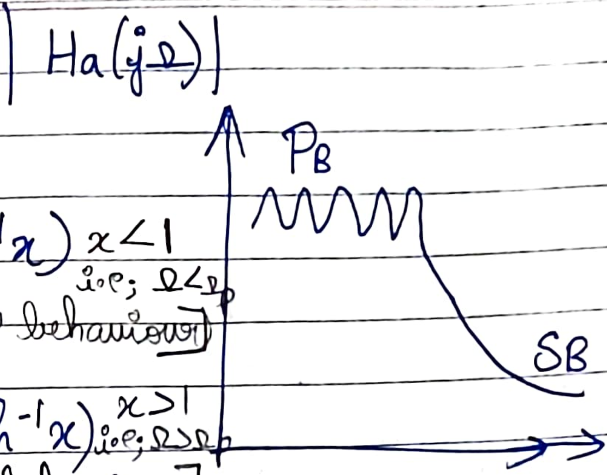
$C_N\left(\dfrac{\Omega}{\Omega_p}\right)$ — is called chebyshev polynomial

Let $x = \dfrac{\Omega}{\Omega_p} \Longrightarrow \boxed{C_N(x)}$

(ii) Chebyshev polynomial $C_N[x]$:- $\left(x = \dfrac{\Omega}{\Omega_p}\right)$

We are interested in 'Type I' filter only.

Type I -



$$C_N[x] = \begin{cases} \cos(N\cos^{-1}x) & x<1 \text{ i.e; } \Omega<\Omega_p \\ \text{[oscillator behaviour]} & \\ \cosh(N\cosh^{-1}x) & x>1 \text{ i.e; } \Omega>\Omega_p \\ \text{[monotonic behaviour]} & \end{cases}$$

$N=0, \quad C_0(x) = \cos 0 = 1$

$N=1, \quad C_1(x) = \cos(\cos^{-1}x) = x$

$N=2, \quad C_2(x) = \cos(2\cos^{-1}x)$

$= \cos(2\theta) \quad$ where $\theta = \cos^{-1}x$, $x = \cos\theta$

$= 2\cos^2\theta - 1$

$C_2(x) = 2x^2 - 1$

$N=3, \quad C_3(x) = \cos(3\cos^{-1}x)$

$= \cos 3\theta$

$= 4\cos^3\theta - 3\cos\theta$

$C_3(x) = 4x^3 - 3x$

Recursive Relation -

Let $A = (N-1)\theta \; ; \; B = \theta$

$2\cos A \cos B = \cos(A+B) + \cos(A-B)$

$2\cos(N-1)\theta\cos\theta = \cos(N\theta) + \cos[(N-2)\theta]$

$\Rightarrow \cos N\theta = 2\cos[(N-1)\theta]\cos\theta - \cos[(N-2)\theta]$

WKT, $\theta = \cos^{-1}x$

$\Rightarrow \cos[N\cos^{-1}x] = 2\cos[(N-1)\cos^{-1}x]\cos[\cos^{-1}x] - \cos[(N-2)\cos^{-1}x]$

$\Longrightarrow \boxed{C_N(x) = 2x\, C_{N-1}(x) - C_{N-2}(x)}$

Replace nos for various values of N,

$C_2(x) = 2x^2 - 1$

$C_3(x) = 4x^3 - 3x \qquad C_4(x) = 8x^4 - 8x^2 + 1$

N	$C_N(x)$
0	1
1	x
2	$2x^2-1$
3	$4x^3-3x$
4	$8x^4-8x^2+1$

Note: 1) (i) N - odd $\Rightarrow$ polynomial power is odd

(ii) N - even $\Rightarrow$ polynomial power is even

2) $C_N(0) = \begin{cases} 0, & \text{if } N \text{ is odd} \\ (-1)^{N/2}, & \text{if } N \text{ is even} \end{cases}$

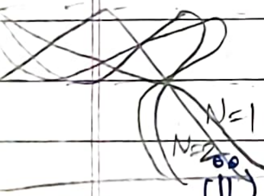
i.e; at origin ($\Omega = 0$)

T.F : $$|H_a(j\Omega)| = \frac{1}{\sqrt{1+\varepsilon^2 C_N^2\left(\frac{\Omega}{\Omega_p}\right)}}$$

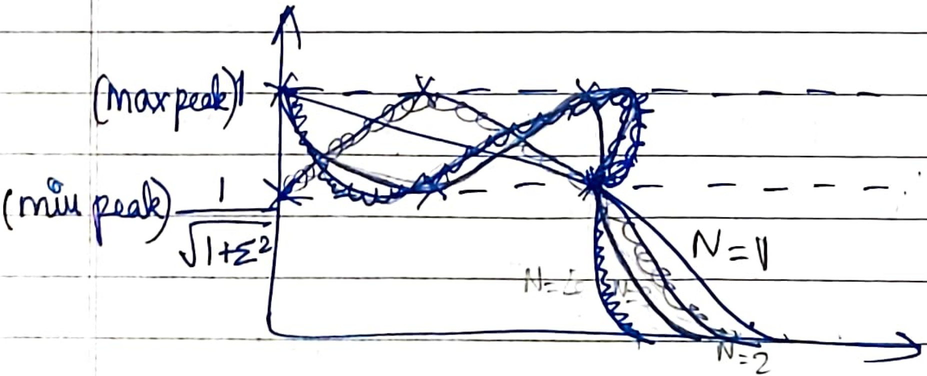
At origin, $\Omega = 0$

$\Rightarrow$ (i) N - odd : $\boxed{|H_a(j\Omega)| = 1}$

and $C_N(0) = 0$



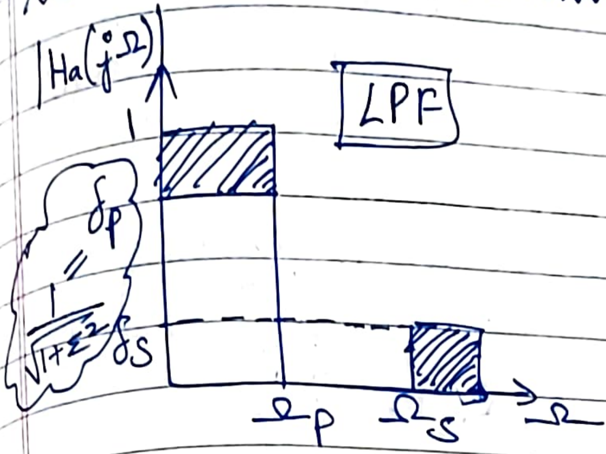
(ii) N - even and $C_N(0) = \pm 1$ } : $\boxed{|H_a(j\Omega)| = \frac{1}{\sqrt{1+\varepsilon^2}}}$



No of maxima & minima in P_B $= N$ - order of the Filter

Starting pt at origin $= \begin{cases} 1, & N\text{-odd} \\ 1/\sqrt{1+\varepsilon^2}, & N\text{-even} \end{cases}$

(iii) 3 dB Cutoff frequency (Ω_{3dB}) -



At $\Omega = \Omega_{3dB}$

$$\delta_p = \frac{1}{\sqrt{1+\varepsilon^2}} = \frac{1}{\sqrt{2}}$$

(since $\varepsilon = \pm 1$ at $\Omega_{3dB} = \Omega$)

At $\Omega = \Omega_{3dB}$ $$\left|H_a(j\Omega)\right|^2 = \frac{1}{1+\varepsilon^2 C_N^2\left(\frac{\Omega}{\Omega_p}\right)} = \frac{1}{2}$$

$$\Rightarrow \frac{1}{1+\varepsilon^2 C_N^2\left(\frac{\Omega}{\Omega_p}\right)} = \frac{1}{2}$$

$$\Rightarrow \varepsilon^2 C_N^2\left(\frac{\Omega}{\Omega_p}\right) = 1$$

$$\boxed{C_N\left(\frac{\Omega_3}{\Omega_p}\right) = \frac{1}{\varepsilon}}$$

$\Downarrow$

SB is active

We know that in SB, $C_N(x)$ is given as:

$$\cosh\left[N\cosh^{-1}\left(\frac{\Omega_3}{\Omega_p}\right)\right] = \frac{1}{\varepsilon}$$

Simplifying 3dB cutoff frequency $\leftarrow$ $$\boxed{\Omega_3 = \Omega_p \cdot \cosh\left[\frac{1}{N}\cosh^{-1}\left(\frac{1}{\varepsilon}\right)\right]}$$

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(iv) Poles and TF -

$$|H_a(j\Omega)| = \frac{1}{\sqrt{1+\varepsilon^2 C_N^2\left(\frac{\Omega}{\Omega_p}\right)}}$$

N-odd
real and other complex conjugate poles

poles $\Rightarrow 1+\varepsilon^2 C_N^2\left(\frac{\Omega}{\Omega_p}\right)=0$

N-even
All poles are complex conjugate

TF $\rightarrow$ constants C_0, C_K, b_K unknown:

$$H_a(s) = \begin{cases} \dfrac{C_0 \cdot \Omega_p^N \left[\prod_{K=1}^{(N-1)/2} C_K\right] \cdot 1}{(s+\Omega_p C_0) \cdot \prod_{K=1}^{(N-1)/2} (s^2+b_K\Omega_p s+C_K\Omega_p^2)} & N\text{-odd} \\ \dfrac{\Omega_p^N \left[\prod_{K=1}^{N/2} (C_K)\right] \cdot \left(\frac{1}{\sqrt{1+\varepsilon^2}}\right)}{\prod_{K=1}^{N/2} (s^2+b_K\Omega_p s+C_K\Omega_p^2)} & N\text{-even} \end{cases}$$

where $C_0 = y_N$

$$C_K = y_N^2 + \cos^2\left[\frac{(2K-1)\pi}{2N}\right]$$

$$b_K = 2y_N \sin\left[\frac{(2K-1)\pi}{2N}\right]$$

$$y_N = \frac{1}{2}\left\{\left[\sqrt{1+\frac{1}{\varepsilon^2}}+\frac{1}{\varepsilon}\right]^{1/N} - \left[\sqrt{1+\frac{1}{\varepsilon^2}}+\frac{1}{\varepsilon}\right]^{-1/N}\right\}$$

$\downarrow$
shrinkage factor

(v) Calculation of N and ε -

$\Omega = \Omega_p$

$$\delta_p = \frac{1}{\sqrt{1+\varepsilon^2}} \Rightarrow \varepsilon = \sqrt{\frac{1}{\delta_p^2}-1}$$

$\underline{\underline{N}}$ at $\Omega = \Omega_s$: $|H_a(j\Omega)|^2 = \delta_s^2$

$$\delta_s^2 = \frac{1}{1+\varepsilon^2 C_N^2\left(\frac{\Omega_s}{\Omega_p}\right)}$$

Simplifing,
$$C_N\left(\frac{\Omega_s}{\Omega_p}\right) = \frac{\sqrt{\frac{1}{\delta_s^2}-1}}{\sqrt{\frac{1}{\delta_p^2}-1}}$$

$\Downarrow$

SB

$$\cosh\left[N\cosh^{-1}\left(\frac{\Omega_s}{\Omega_p}\right)\right] = \frac{\sqrt{\frac{1}{\delta_s^2}-1}}{\sqrt{\frac{1}{\delta_p^2}-1}}$$

$$\Rightarrow \boxed{N \geq \frac{\cosh^{-1}\frac{\sqrt{\frac{1}{\delta_s^2}-1}}{\sqrt{\frac{1}{\delta_p^2}-1}}}{\cosh^{-1}\left(\frac{\Omega_s}{\Omega_p}\right)}}$$

Procedure for designing-

Given δ_p, δ_s, Ω_p, Ω_s

(i) Find N and ε

(ii) Find y_N

(iii) Find C_0, C_K, b_K

(iv) Find T.F (N-odd, N-even)

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IIR filter structure design-

IIR - infinite impulse response

independant Variable → infinite $\boxed{n\,\infty}$

$y[n] \xrightarrow[\text{on}]{\text{depends}}$ present and past input
$x[n]$ $x[n-k]$

$$\boxed{\sum_{k=0}^{N} a_k\, y[n-k] = \sum_{k=0}^{M} b_k\, x[n-k]}$$

$$a_0\, y[n] + a_1\, y[n-1] + a_2\, y[n-2] \ldots\ldots a_N\, y[n-N] = b_0\, x[n] + b_1 x[n-1] + \ldots + b_M x[n-M]$$

$a_0 = 1$ ~~and~~ taking Z transform, $\left(\frac{Y(z)}{X(z)}\right) = H(z)$

$$Y(z) + a_1 z^{-1} Y(z) + \ldots\ldots + a_N z^{-N} Y(z) = b_0 X(z) + b_1 z^{-1} X(z) + \ldots + b_N z^{-N} X(z)$$

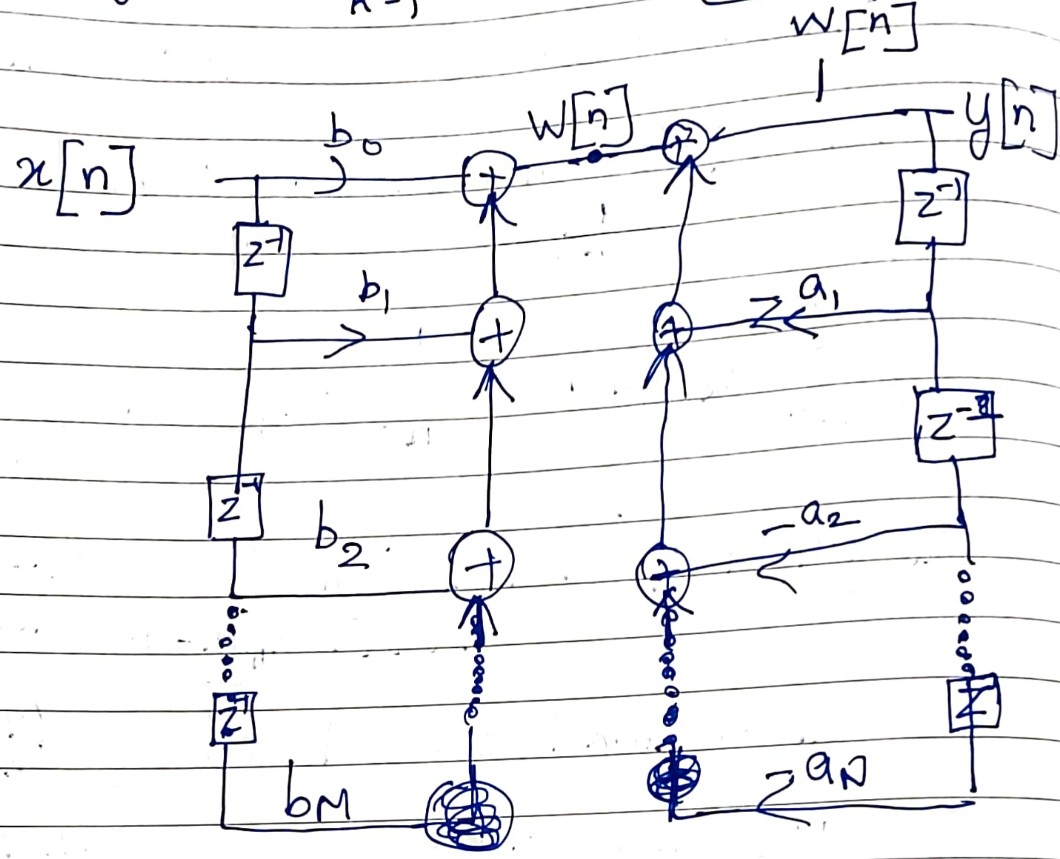
$$\boxed{H(z) = \frac{Y(z)}{X(z)} = \frac{b_0 + b_1 z^{-1} + \ldots\ldots + b_N z^{-N}}{1 + a_1 z^{-1} + \ldots\ldots + a_N z^{-N}}}$$

$$\Rightarrow \boxed{H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{N} b_k z^{-k}}{1 + \sum_{k=1}^{N} a_k z^{-k}}}$$

We have, Direct form I Cascaded
Direct form II Parallel

Direct form I :-

$$y[n] = -\sum_{K=1}^{N} a_K\, y[n-k] + \underbrace{\sum_{K=0}^{M} b_K\, x[n-k]}_{W[n]}$$



We use adders (M+N), multipliers (M+N+2) and delay elements (M+N)

Direct form II :-

For better performance, we need to have less number of delay elements and hence, we use delays elements = max[M, N]

$$H(Z) = \frac{Y(Z)}{X(Z)} = \frac{\sum_{K=0}^{M} b_K z^{-K}}{1+\sum_{K=0}^{N} a_K z^{-K}}$$

$$H(Z) = \underbrace{\frac{Y(Z)}{W(Z)}}_{Num} \cdot \underbrace{\frac{W(Z)}{X(Z)}}_{Dem} \qquad \frac{\sum_{K=0}^{M} b_K z^{-K}}{1+\sum_{K=1}^{N} a_K z^{-K}}$$

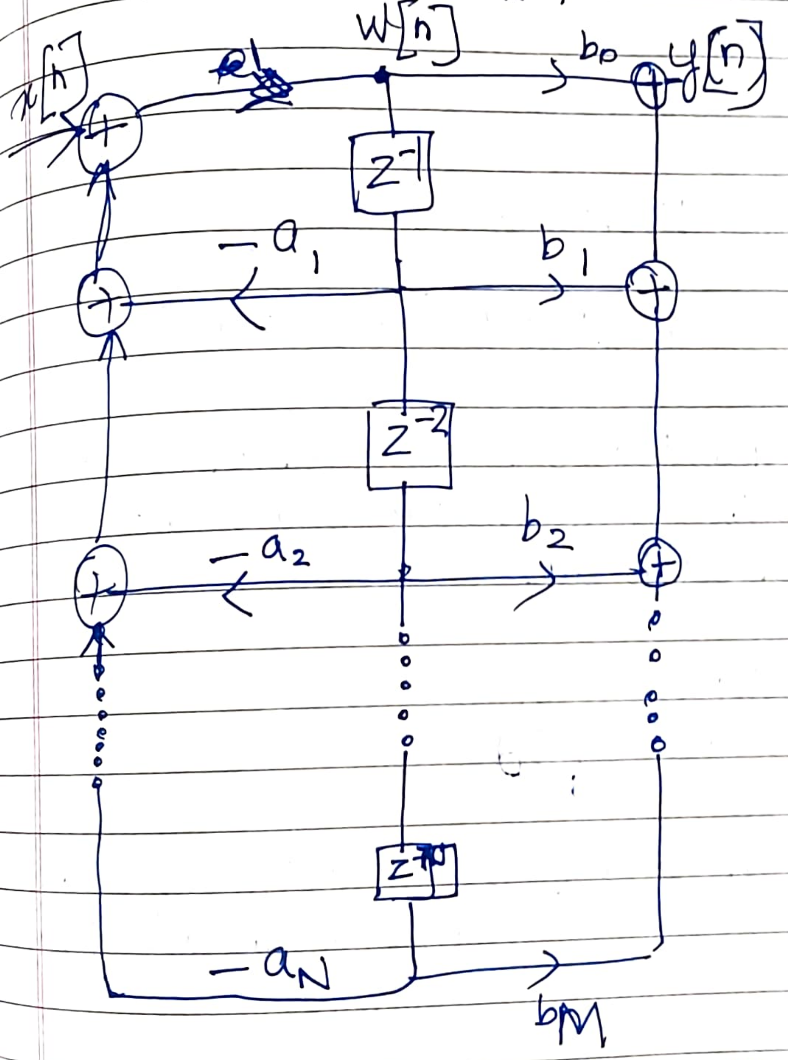
$$y(z) = W(z) \cdot \sum_{k=0}^{M} b_k z^{-k}$$

$$\boxed{y[n] = \sum_{k=0}^{M} b_k \, W[n-k]}$$

$$X(z) = W[z] + \sum_{k=1}^{N} a_k z^{-k} \, W[z]$$

$$\boxed{x[n] = W[n] + \sum_{k=1}^{N} a_k W[n-k]}$$

$$\Rightarrow W[n] = x[n] - \sum_{k=1}^{N} a_k \left[W[n-k]\right]$$

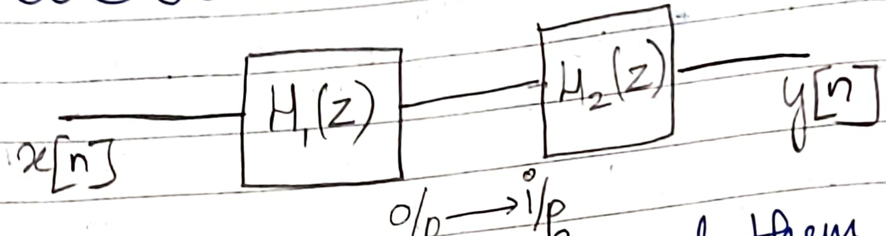


Assuming M and N are same

We use adders (M+N), multipliers (M+N+2) and delay elements (Max [M, N])

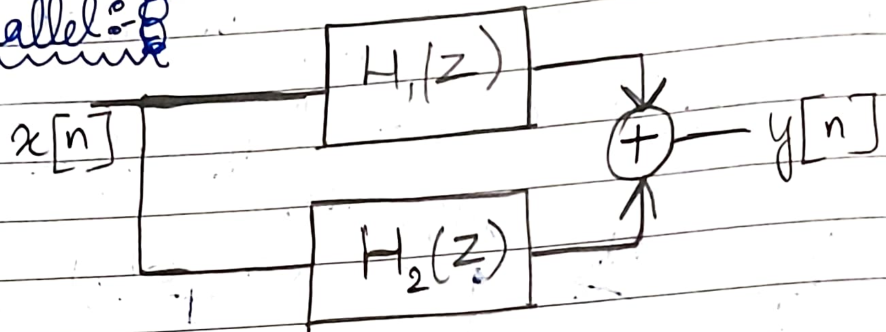
▲ Efficient to design

Cascaded (Series interconnection) :-



We take D.F II and connect them in series

Parallel :-



$$\boxed{H(z) = \frac{\text{Nr polynomial of } z}{\text{Dr polynomial of } z}}$$

Depending on numerator and denominator polynomial we should have partial fraction split up.

Nr degree $\rightarrow D_N$
Dr degree $\rightarrow D_d$

When $D_N \geq D_d \Rightarrow$ we will have one constant atleast + some fractions

When $D_N < D_d \Rightarrow$ we will have only fractions

After getting fractions, we should apply partial fraction method

1

$H(z) \rightarrow H_1(z) + H_2(z) \dots\dots + H_k(z)$ and then we should implement functions as D.F II and the respective inputs as $x[n]$

DF II   DF I

$x[n]$

q) DF I, DF II, Cascaded, Parallel with

$$y[n] = -0.1y[n-1] + 0.2y[n-2] + 3x[n] + 3.6[x[n-1]] + 0.6x[n-2]$$

Parallel: $\longrightarrow$ $H(z) = \dfrac{Y(z) \rightarrow ?}{X(z) \rightarrow ?}$

$\Downarrow$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{3 + 3.6z^{-1} + 0.6z^{-2}}{+0.1z^{-1} + 1 - 0.2z^{-2}}$$

$\boxed{D_N = 2 \quad D_d = 2} \Rightarrow$ constant + fraction

$H(z) = H_1(z) + H_2(z)$ [since $K = D_d$]
+ constant

$$
\begin{array}{r|l}
 & -3 \\
1+0.1z^{-1}-0.2z^{-2} & 3 + 3.6z^{-1} + 0.6z^{-2} \\
 & -3 - 0.3z^{-1} + 0.6z^{-2} \\
\hline
 & 6 + 3.9z^{-1}
\end{array}
$$

$$H(z) = -3 + \frac{3.9z^{-1} + 6}{1 + 0.1z^{-1} - 0.2z^{-2}}$$

$$= -3 + \frac{6 + 3.9z^{-1}}{(1+0.5z^{-1})(1-0.4z^{-1})}$$

$$= -3 + \frac{A}{(1+0.5z^{-1})} + \frac{B}{(1-0.4z^{-1})}$$

$A(1-0.4z^{-1}) + B(1+0.5z^{-1})$

$A + B = 6$

$-0.4A + 0.5B = 3.9$

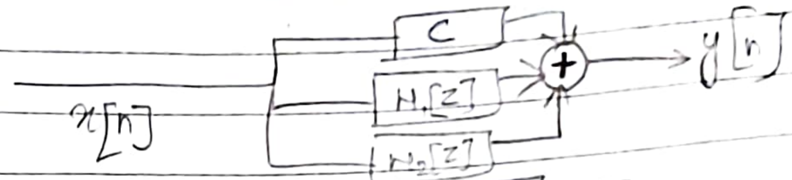
$0.4A + 0.4B = 2.4$

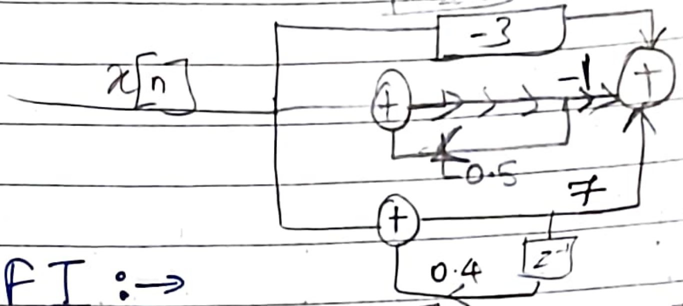
$0.9B = 6.3$

$B = 7$

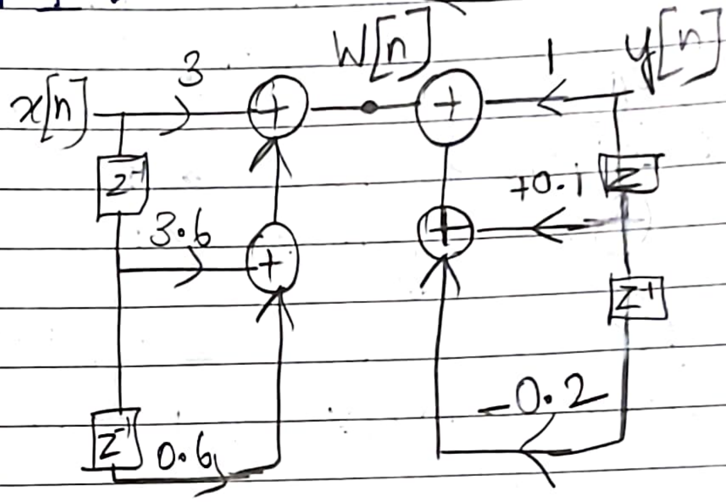
$A = -1$

$$H(z) = -3 + \frac{-1}{1+0.5z^{-1}} + \frac{7}{1-0.4z^{-1}}$$

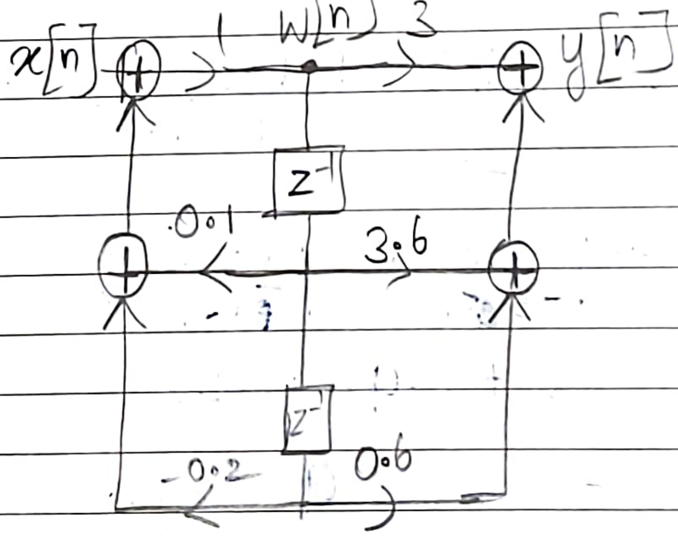




D.F I :→



D.F II :→



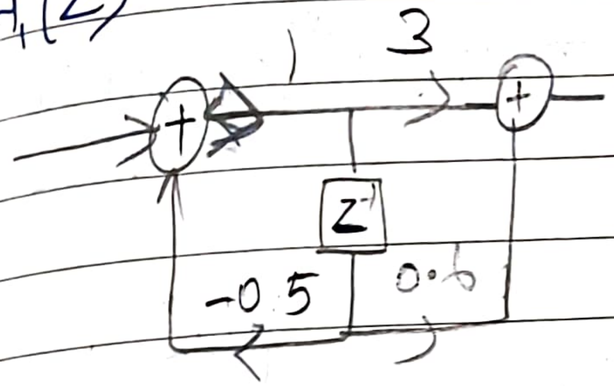
Cascaded :→

$$H(z) = \frac{3 + 3.6z^{-1} + 0.6z^{-2}}{1 + 0.1z^{-1} - 0.2z^{-2}}$$

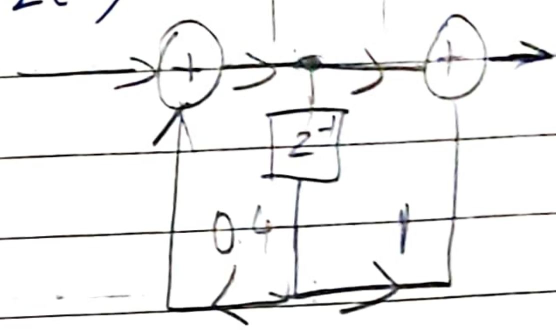
$$H(z) = \frac{(3+0.6z^{-1})(1+z^{-1})}{(1+0.5z^{-1})(1-0.4z^{-1})}$$

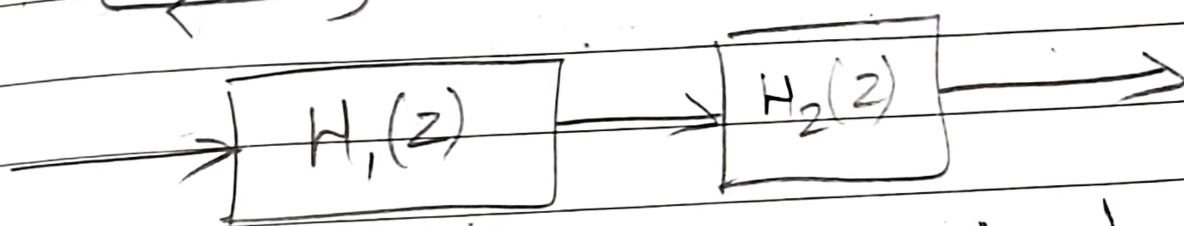
$$= \frac{3+0.6z^{-1}}{1+0.5z^{-1}} \cdot \frac{1+z^{-1}}{1-0.4z^{-1}}$$

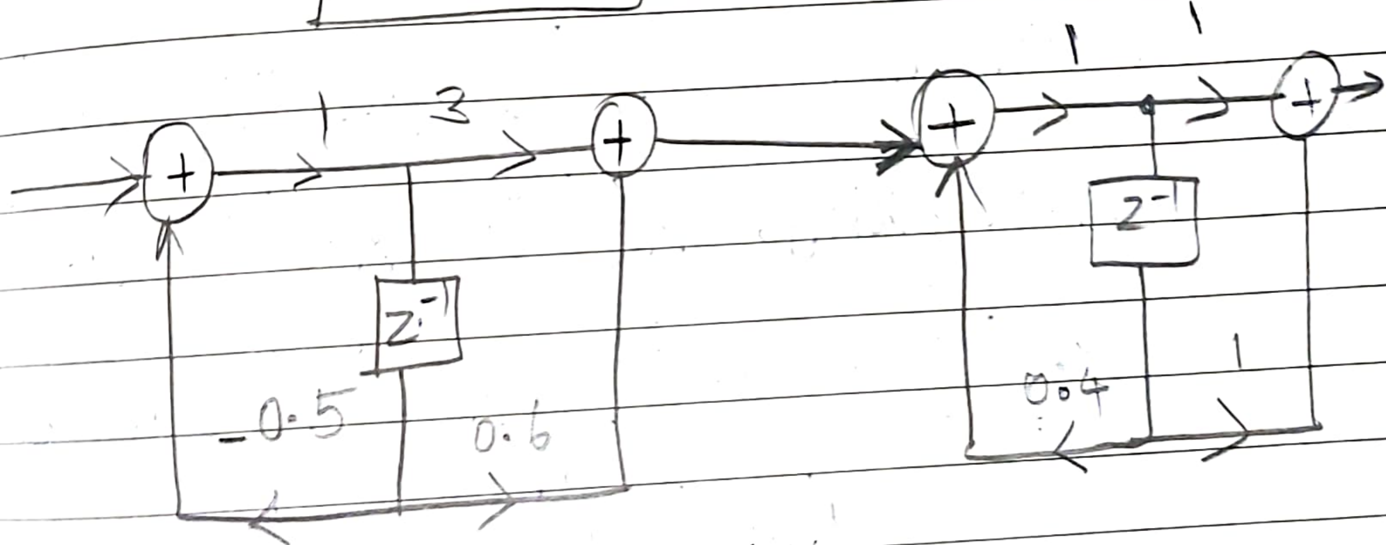
$H_1(z)$



$H_2(z)$







FIR FILTER

(aka Linear Phase Filter)

DT LTI System:-

$$x[n] \longrightarrow \boxed{h[n]} \longrightarrow y[n]$$

DTFT of $\{h[n]\} \longrightarrow H(e^{j\omega})$ [Magnitude and Phase part]

Polar $\longrightarrow re^{j\omega}$

s/m $\longrightarrow$ Stable $\longrightarrow$ BIBO

$y[n] \propto x(n)$ ✓

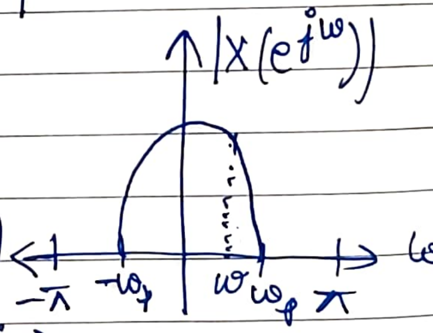
$x(n-k)$ ✓

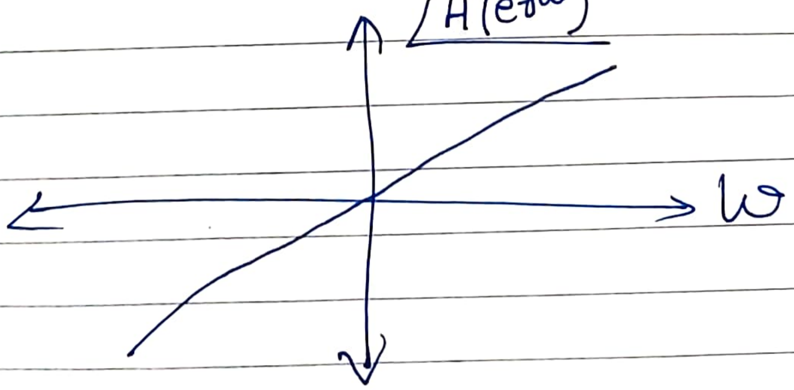
$x(n+k)$ ✗

$$\text{DTFT}\{h[n]\} = H(e^{j\omega}) = |H(e^{j\omega})| \angle H(e^{j\omega})$$

$$\text{DTFT}\{x[n]\} = X(e^{j\omega}) = |X(e^{j\omega})| \angle X(e^{j\omega})$$

$$Y(e^{j\omega}) = |H(e^{j\omega})|\,|X(e^{j\omega})| \;\angle H(e^{j\omega}) + \angle X(e^{j\omega})$$





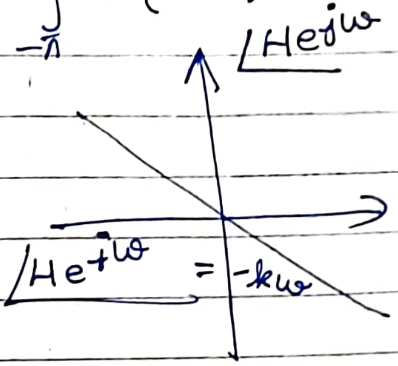
$$\angle H(e^{j\omega}) = K\omega$$

$$Y(e^{j\omega}) = |H(e^{j\omega})||X(e^{j\omega})| \angle H(e^{j\omega}) + X(e^{j\omega})$$

$$Y(e^{j\omega}) = A \cdot |X(e^{j\omega})| \angle e^{j\omega k}$$

$$\text{IDTFT} \Rightarrow y[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} Y(e^{j\omega}) e^{j\omega n} d\omega$$

$$y[n] = A \cdot \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega}) e^{jk\omega} e^{j\omega n} d\omega$$



$$\Rightarrow \boxed{y[n] = A \cdot x[n+k]}$$

$$y[n] \propto x[n-k]$$

$$\angle H(e^{j\omega}) = -k\omega$$

$$\boxed{H(e^{j\omega}) = A \cdot e^{-jk\omega}}$$

|| Stability of IIR : depends on past o/p

Linear Phase Filter :-

$h[n] \longrightarrow$ length $\rightarrow N$
(finite)

N : odd

$$h(0) \quad h(1) \ldots \ldots h\left(\frac{N-3}{2}\right) \ldots h\left(\frac{N-1}{2}\right) \ldots h\left(\frac{N+1}{2}\right) \ldots h(N-2) \quad h(N-1)$$

middle sample

In general, $h(r) = h[N-1-r]$

$$h[n] = h[0] + h[1] + \ldots\ldots + h[r] + \ldots\ldots + h\left(\frac{N-3}{2}\right) + h\left(\frac{N-1}{2}\right) + h\left(\frac{N+1}{2}\right) + \ldots\ldots + h(N-1-r) + \ldots\ldots + h(N-2) + h(N-1)$$

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DTFT of $h[n]$:

$$H(e^{j\omega}) = h(0)e^{j0} + h(1)e^{-j\omega} + \ldots + h(r)e^{-j\omega r}$$
$$+ h\left(\frac{N-3}{2}\right)e^{-j\omega\left(\frac{N-3}{2}\right)} + h\left(\frac{N-1}{2}\right)e^{-j\omega\left(\frac{N-1}{2}\right)}$$
$$+ h\left(\frac{N+1}{2}\right)e^{-j\omega\left(\frac{N+1}{2}\right)} + \ldots + h(N-1-r)e^{-j\omega(N-1-r)}$$
$$+ \ldots + h(N-1)e^{-j\omega(N-1)}$$

Replace by general formula,

$$H(e^{j\omega}) = h\left(\frac{N-1}{2}\right)e^{-j\omega\left(\frac{N-1}{2}\right)} + \sum_{r=0}^{\frac{N-3}{2}} h(r)\left(e^{-j\omega r} + e^{-j\omega\left(\frac{N-1}{2}-r\right)}\right)$$

$$= e^{-j\omega\left(\frac{N-1}{2}\right)}\left[h\left(\frac{N-1}{2}\right) + \sum_{r=0}^{N-3/2} h(r)\left[e^{\left(r+\frac{N-1}{2}\right)j\omega} + e^{-j\omega\left(\frac{N-1}{2}-r\right)}\right]\right]$$

$$= \underbrace{e^{-j\omega\left(\frac{N-1}{2}\right)}}_{\text{Phase}}\underbrace{\left[h\left(\frac{N-1}{2}\right) + \sum_{r=0}^{N-3/2} h(r)\cdot 2\cos\left[\left(\frac{N-1}{2}-r\right)\omega\right]\right]}_{|H(e^{j\omega})|}$$

$$\boxed{H(e^{j\omega}) = A\,e^{-jk\omega}}$$

$$\boxed{K = \frac{N-1}{2} + \pi}$$

(i) windowing

$\hookrightarrow h(n) \times w(n)$

$\downarrow$

$h_j(n)$

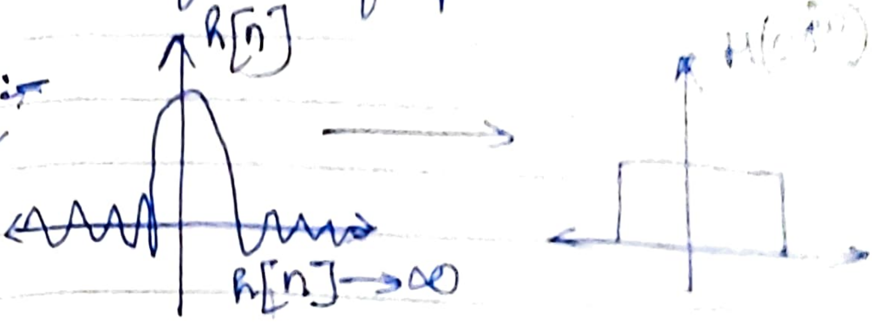
$\downarrow$ DTFT

$H_d(e^{j\omega})$

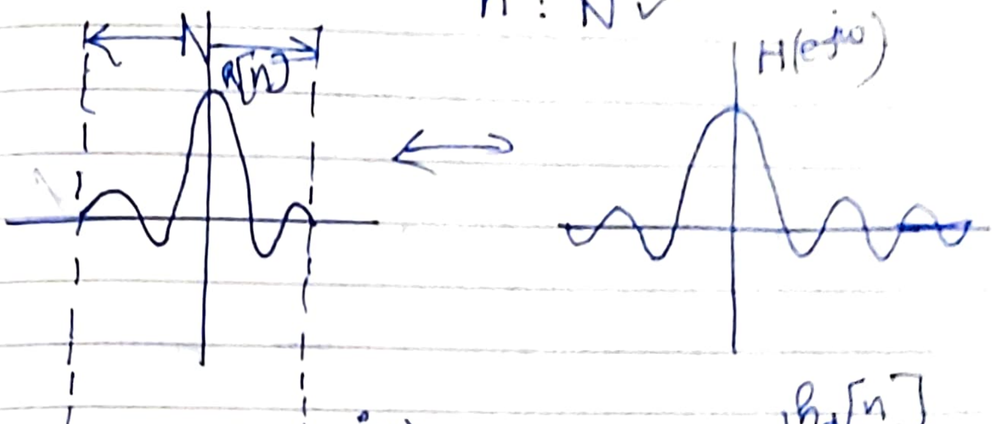
(ii) freq Sampling method

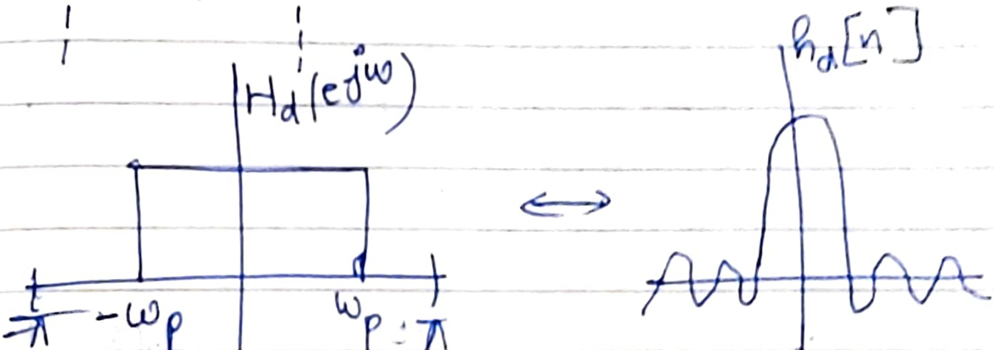
IIR cannot be utilised when we require btwn 2 finite frequencies

IIR:-



FIR:- $h[n] \longrightarrow n : \infty$ ✗
$n : N$ ✓





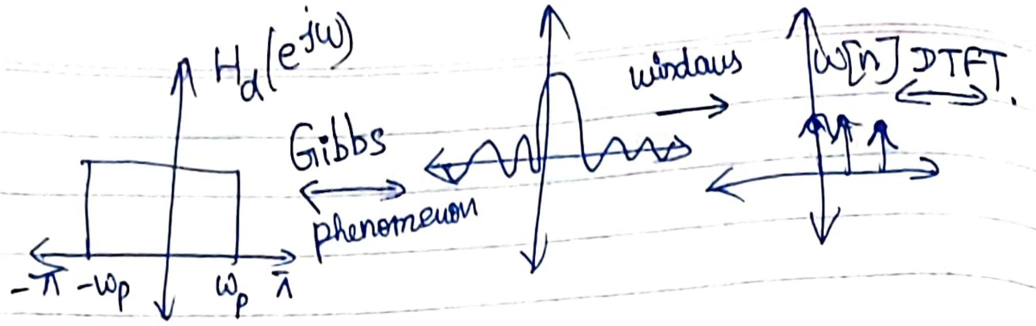
$h_d[n]$ - must be causal $h_d[n]$ - finite causal $w[n]$ - 0 to N-1

$$h_d[n] \times w[n] = h[n] \ (0 \text{ to } N-1)$$

DTFT: $Y(e^{j\omega}) = X(e^{j\omega}) H(e^{j\omega})$

$$h[n] = h_d[n] w[n]$$

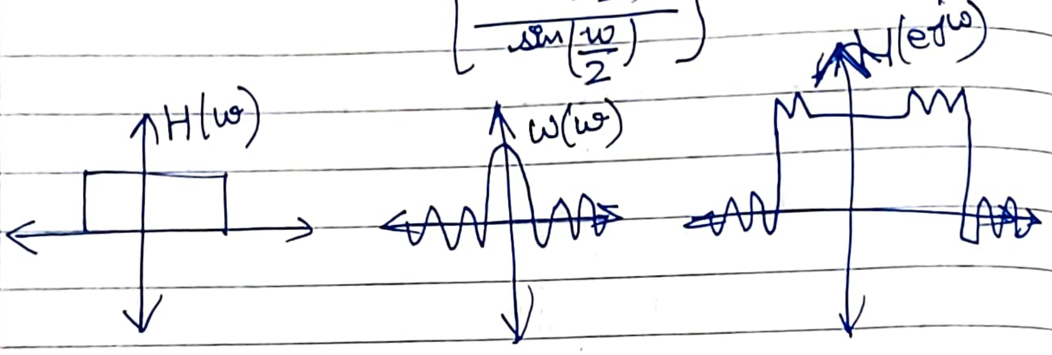
$$H(e^{j\omega}) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H(e^{j\omega}) e^{j\omega n} w[n] \, d\omega$$



$$W[e^{j\omega}] = \sum_{n=0}^{N-1} w[n] e^{-j\omega n}$$

$$= \sum_{n=0}^{N-1} e^{-j\omega n} = \frac{1 - e^{-j\omega N}}{1 - e^{-j\omega}} = \frac{e^{-j\omega \frac{N}{2}}}{e^{-j\frac{\omega}{2}}} \left[\frac{e^{\frac{j\omega N}{2}} - e^{\frac{-j\omega N}{2}}}{e^{\frac{j\omega}{2}} - e^{\frac{-j\omega}{2}}} \right]$$

$$= e^{-j\omega\left[\frac{N-1}{2}\right]} \left\{ \frac{\sin\left(\frac{\omega N}{2}\right)}{\sin\left(\frac{\omega}{2}\right)} \right\}$$



$$H(\omega) \cdot W(\omega) = \sum H(\theta) \cdot W(\theta - \omega)$$

Having oscillation is called GIBBS PHENOMENON

Steps - FIR: $H_d[e^{j\omega}] \xrightarrow{IDFT} h_d[n] * W[n] \rightarrow h[n] \xrightarrow[ZT]{DTFT} H[z]$

Linear Phase: LPF HPF BPF BSF :-

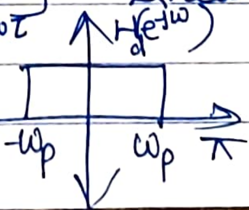
$N \rightarrow$ odd

(1) LPF -

$$H_d(e^{j\omega}) = \underbrace{e^{-j\omega\left(\frac{N-1}{2}\right)}}_{\text{Phase}} \underbrace{|1|}_{\text{mag}} \rightarrow \text{Ideal will have mag } 1$$

$H_d(e^{j\omega}) \longrightarrow h_d[n] \rightarrow$ Linear phase LPF

$e^{-j\omega\tau}$



$$\tau = \frac{N-1}{2}$$

$$h_d[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\omega_p}^{\omega_p} e^{-j\omega\tau}\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\omega_p}^{\omega_p} e^{j\omega(n-\tau)}\, d\omega$$

$$= \frac{1}{2\pi}\left[\frac{e^{j\omega(n-\tau)}}{j(n-\tau)}\right]_{-\omega_p}^{\omega_p}$$

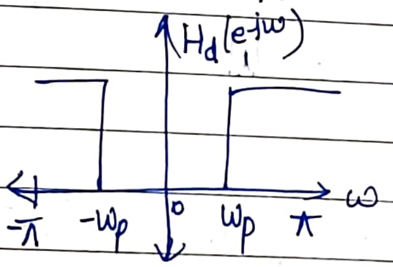
$$= \frac{1}{2\pi j(n-\tau)}\left[e^{+j\omega_p(n-\tau)} - e^{-j\omega_p(n-\tau)}\right]$$

$$\boxed{h_d[n] = \frac{\sin[\omega_p(n-\tau)]}{\pi(n-\tau)}}$$

$n \to -\infty$ to ∞

$$h_d[n] = \begin{cases} \dfrac{\sin(\omega_p(n-\tau))}{\pi(n-\tau)}, & n \neq \tau \\ \dfrac{\omega_p}{\pi}, & n = \tau \end{cases}$$

(2) HPF -



$$H_d(e^{j\omega}) = \begin{cases} e^{-j\omega\tau}, & \omega_p < \omega < \tau \\ 0, & \text{otherwise} \end{cases}$$

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})\, e^{j\omega n}\, d\omega$$

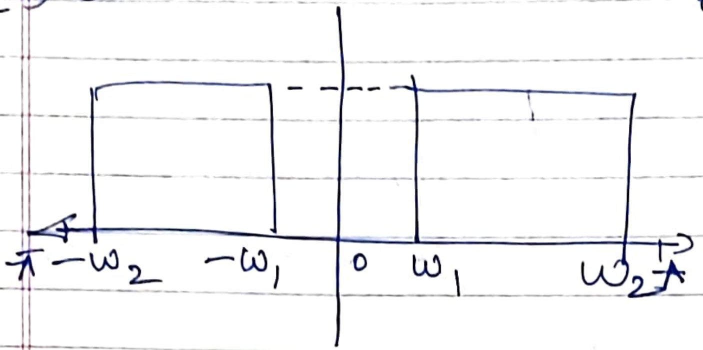
$$= \frac{1}{2\pi}\left[\int_{-\infty}^{-\omega_p} e^{-j\omega\tau} e^{j\omega n}\, d\omega + \int_{\omega_p}^{\pi} e^{-j\omega\tau} e^{j\omega n}\, d\omega\right]$$

$$= \frac{1}{2\pi}\left[\left[\frac{e^{j\omega(n-\tau)}}{j(n-\tau)}\right]_{-\pi}^{-\omega_p} + \left[\frac{e^{j\omega(n-\tau)}}{j(n-\tau)}\right]_{\omega_p}^{\pi}\right]$$

$$h_d[n] = \frac{\sin((n-\tau)\pi) - \sin((n-\tau)\omega_p)}{\pi(n-\tau)}$$

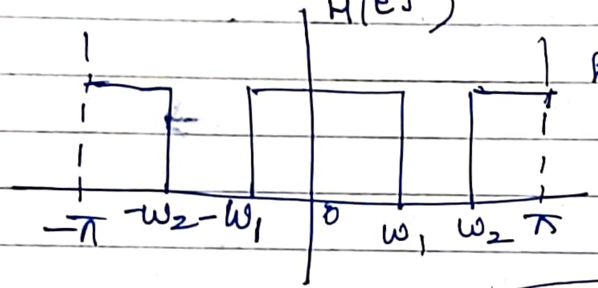
$$h_d[n] = \begin{cases} \dfrac{\sin(n-\tau)\pi - \sin(n-\tau)\omega_p}{\pi(n-\tau)}, & n \neq \tau \\ 1 - \dfrac{\omega_p}{\pi}, & n = \tau \end{cases}$$

(3) BPF -



$$h_d[n] = \begin{cases} \dfrac{\sin((n-\tau)\omega_2) - \sin((n-\tau)\omega_1)}{\pi n - \tau}, & n \neq \tau \\ \dfrac{\omega_2 - \omega_1}{\pi}, & n = \tau \end{cases}$$

(4) Linear phase Band Reject as Band Stop filter BSP-



$$h_d[n] = \begin{cases} \dfrac{\sin((n-\tau)\omega_1) + \sin((n-\tau)\pi) - \sin((n-\tau)\omega_2)}{\pi(n-\tau)}, & n \neq \tau \\ \dfrac{\omega_1 + \pi - \omega_2}{\pi}, & n = \tau \end{cases}$$

Window Types:-

$\boxed{\text{DRAWBACK:- oscillations in output}}$

(i) Rectangular window: $w[n] = \begin{cases} 1, & 0 \leq n \leq N-1 \\ 0, & \text{otherwise} \end{cases}$

(ii) Hanning window: $w_{Hann}[n] = \begin{cases} \frac{1}{2} - \frac{1}{2}\left(\cos\left(\frac{2\pi n}{N-1}\right)\right), & 0 \leq n \leq N-1 \\ 0, & \text{otherwise} \end{cases}$

$\boxed{MLW_{Hamm} < MLW_{Hann}}$

(iii) Hamming window: $w_{hamming}[n] = \begin{cases} 0.54 - 0.46\cos\left(\frac{2\pi n}{N-1}\right), & 0 \leq n \leq N-1 \\ 0, & \text{otherwise} \end{cases}$

(iv) Bartlett's window: $w_{Bartlett}[n] = \begin{cases} \frac{2n}{N-1}, & 0 \leq n \leq \frac{N-1}{2} \\ 2 - \frac{2n}{N-1}, & \frac{N-1}{2} \leq n \leq N-1 \end{cases}$

(v) Blackman window: $w[n] = 0.42 - 0.5\cos\frac{2\pi n}{N-1} + 0.08\cos\frac{4\pi n}{N-1}$

1) Design linear phase LPF with length N=7 and cut-off frequency $\omega_p = 1 rad/sec$. Use rectangular window and hann window.

(i) $H_d(e^{j\omega}) \rightarrow h_d(n)$

(ii) $h_d(n) \times w(n) \rightarrow h(n)$

(iii) DTFT of $h(n) \rightarrow H(e^{j\omega})$

$$\tau = \frac{N-1}{2} = 3$$

1)(i)
$$h_d[n] = \begin{cases} \frac{\sin(n-\tau)\omega_p}{(n-\tau)\pi}, & n \neq \tau \\ \frac{\omega_p}{\pi}, & n = \tau \end{cases}$$

$h[r] = h[N-1-r]$

$n=0$ $\quad h_d(0) = h_d(6) = \frac{\sin(0-3)1}{(0-3)\pi} = 0.0150$

$$h_d(1) = h_d(5) = \frac{\sin(1-3)1}{(1-3)\pi} = 0.1447$$

$$h_d(2) = h_d(4) = \frac{\sin(2-3)}{(2-3)\pi} = 0.2678$$

$$h_d(3) = 0.3183$$

$$w[n] = \begin{cases} 1, & 0 \leq n \leq 6 \\ 0, & \text{otherwise} \end{cases}$$

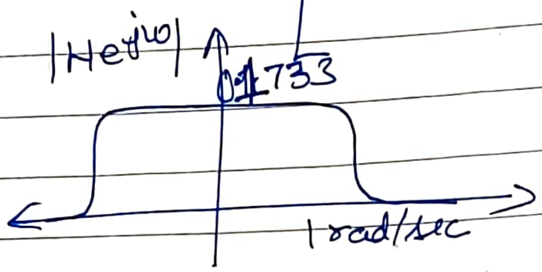
(ii) $h_d[n] \times w[n] = h[n]$

$$h_d[n] = h[n]$$

(iii) $H(e^{j\omega}) = e^{-j\omega\left(\frac{N-1}{2}\right)} \left[|He^{j\omega}| \right]$

$$= e^{-j\omega 3}\left[h\left(\frac{N-1}{2}\right) + 2\sum_{r=0}^{\frac{N-1}{2}} h(r)\cos\left(\left(\frac{N-1}{2} - r\right)\omega\right)\right]$$

$$= e^{j\omega 3}\left[0.3183 + 0.03\cos 3\omega + 0.2894\cos 2\omega + 0.5356\cos\omega\right]$$



b) Hann Window :

(i) $w[n] = \begin{cases} \frac{1}{2} - \frac{1}{2}\cos\left(\frac{2\pi n}{N-1}\right), & 0 \le n \le N-1 \\ 0, & \text{otherwise} \end{cases}$

$w[n] = \frac{1}{2} - \frac{1}{2}\cos\left(\frac{2\pi n}{\cancel{6}\,3}\right)$

$W[0] = 0$

$W[1] = \frac{1}{2} - \frac{1}{2}\cos\left(\frac{\pi}{3}\right)$

$= 0.25$

$W[2] = \frac{1}{2} - \frac{1}{2}\cos\left(\frac{2\pi}{3}\right)$

$= 0.75$

$W[3] = \frac{1}{2} - \frac{1}{2}\cos\pi = 1$

$W[4] = \frac{1}{2} - \frac{1}{2}\cos\frac{4\pi}{3} = 0.75$

$W[5] = \frac{1}{2} - \frac{1}{2}\cos\frac{5\pi}{3} = 0.25$

$$W[6] = \frac{1}{2} - \frac{1}{2}\cos\left(\frac{6\pi}{3}\right)$$
$$= 0$$

(ii) $h[n] = h_d[n] \times W[n]$

$$h[0] = h[6] = 0$$

$$h[1] = h[5] = 0.036175$$

$$h[2] = h[4] = 0.20085$$

$$h[3] = 0.3183$$

(iii) $H(e^{j\omega}) = e^{-j\omega\left(\frac{N-1}{2}\right)}\left[|H(e^{j\omega})|\right]$

$$H(e^{j\omega}) = e^{-3j\omega}\left[|H(e^{j\omega})|\right]$$

$$H(e^{j\omega}) = e^{-3j\omega}\left[h\left(\frac{N-1}{2}\right) + 2\left[h(0)\cos 3\omega + h(1)\cos 2\omega + h(2)\cos\omega\right]\right]$$

$$H(e^{j\omega}) = e^{-3j\omega}\left[0.3183 + 2\left[0 + 0.036175\cos 2\omega + 0.20085\cos\omega\right]\right]$$

$$H(e^{j\omega}) = e^{-3j\omega}\left[0.3183 + 0.07235\cos 2\omega + 0.4017\cos\omega\right]$$

$$\boxed{H(e^{j\omega}) = e^{-j\omega 3}\left[0.3183 + 0.07235\cos 2\omega + 0.4017\cos\omega\right]}$$

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Page

2) HPF $N = 7, \omega_p = 2$ rads
$\Rightarrow$ hamming window

(i) $$h_d[n] = \begin{cases} \dfrac{\sin[(n-\tau)\pi] - \sin[(n-\tau)\omega_p]}{(n-\tau)\pi} & \text{for } n \neq \tau \\ 1 - \dfrac{\omega_p}{\pi} & \text{for } n = \tau \end{cases}$$

$n \Rightarrow 0$ to $N-1$ $\rightarrow$ Linear phase
$h(r) = h[\cancel{N}-1-r]$

$$h_d(0) = \frac{\sin((0-3)\pi) - \sin[(0-3)2]}{(-3)\pi}$$

$$= \frac{-\sin 3\pi + \sin(6)}{3\pi}$$ [radian mode]

$$\boxed{h_d(0) = 0.0296 = h_d(6)}$$

$$h_d(1) = h_d(5) = \frac{\sin((1-3)\pi) - \sin[(1-3)2]}{-3\pi}$$

$$= \frac{-\sin 2\pi + \sin(4)}{-\cancel{3}\pi}$$

$$= 0.1204$$

$h_d[2] = h_d[4] = -0.2894$

$h_d[3] = 0.3634$

(ii) $w[n] \Rightarrow$ hamming window
$n \rightarrow 0$ to 6

$n \rightarrow 0$ to 6

$$w[n] = 0.54 - 0.46 \cos \frac{2\pi n}{N-1}$$

$w(0) = w(6) = 0.08$

$w(1) = w[5] = 0.31$

$w[2] = w[3] = 0.77$

$w[3] = 1$

(iii) $h[n] = h_d[n] \times W[n] \quad n - 0 \text{ to } 6$

$h[0] = h[6] = 0.0024$

$h[1] = h[5] = 0.0373$

$h[2] = h[4] = -0.228$

$h[3] = 0.3634$

(iv) $H(e^{j\omega}) = e^{-j\omega\left(\frac{N-1}{2}\right)} \left[h\left(\frac{N-1}{2}\right) + 2\sum_{r=0}^{\frac{N-3}{2}} h[r] \cos\left[\left(\frac{N-1}{2} - r\right)\omega\right]\right]$

$$He^{j\omega} = e^{-j\omega 3}\left[0.3634 + 2\sum_{r=0}^{2} h[r]\cos[(3-r)\omega]\right]$$

$$\boxed{He^{j\omega} = e^{-j\omega 3}\left[0.3634 + 0.0048\cos 3\omega + 0.0746\cos 2\omega - 0.4456\cos\omega\right]}$$

3) BPF $\quad N = 11, \; \omega_1 = 1 \text{ rads}, \; \omega_2 = 3 \text{ rads}$

$\Rightarrow$ Blackmann Window

4) BSF $\quad N = 11, \; \omega_1 = 1 \text{ rads}, \; \omega_2 = 2 \text{ rads}$

$\Rightarrow$ Bartlett window

3) $h_d[n] = \begin{cases} \dfrac{\sin(n-\tau)\omega_2 - \sin(n-\tau)\omega_1}{(n-\tau)\pi}, & n \neq \tau \\ \dfrac{\omega_2 - \omega_1}{\pi}, & n \neq \tau \end{cases}$

$N = 11$

(i) $\Rightarrow h(\gamma) = h[N-1-\gamma]$

$h_d[0] = h_d[10] = 0.0264$

$h_d[1] = h_d[9] = 0.1390$

$h_d[2] = h_d[8] = -0.04462$

$h_d[3] = h_d[7] = -0.2652$

$h_d[4] = h_d[6] = +0.21585$

$h_d[5] = 0.3183$

(ii) $W[n] = 0.42 - 0.5\cos\left(\dfrac{2\pi n}{N-1}\right) + 0.08\cos\left(\dfrac{4\pi n}{N-1}\right)$

$W[0] = W[10] = 0$

$W[1] = W[9] = 0.0402$

$W[2] = W[8] = 0.\,[illegible]8$

$W[3] = W[7] = 0.5098$

$W[4] = W[6] = 0.8492$

$W[5] = 1$

(iii) $h[n] = h_d[n] \times w[n]$

$h[0] = h[10] = 0$

$h[1] = h[9] = 0.00585$

$h[2] = h[8] = -0.00895$

$h[3] = h[7] = -0.13514$

$h[4] = h[6] = +0.1833$

$h[5] = 0.3183$

(iv) $$H(e^{j\omega}) = e^{-j\omega\left(\frac{N-1}{2}\right)}\left[h\left[\frac{N-1}{2}\right] + 2\big[h[0]\cos(5\omega) + h[1]\cos(4\omega) + h[2]\cos(3\omega) + h[3]\cos(2\omega) + h[4]\cos(\omega)\big]\right]$$

$$H(e^{j\omega}) = e^{-j\omega(5)}\left[-0.3183 + 0\cos 5\omega + 0.0107\cos 4\omega - 0.0179\cos 3\omega - 0.2702\cos 2\omega + 0.3666\cos\omega\right]$$

H.W

4) $$h_d[n] = \begin{cases} \dfrac{\sin((n-\tau)\omega_1) + \sin((n-\tau)\pi) - \sin((n-\tau)\omega_2)}{(n-\tau)\pi}, & n \neq \tau \\ \dfrac{\omega_1 + \pi - \omega_2}{\pi}, & n = \tau \end{cases}$$

(i) $$h_d[0] = h_d[10] = \frac{\sin((0-5)1) + \sin(0-5)\pi - \sin((0-5)2)}{(0-5)\pi}$$

$$= \frac{\sin -5\cdot 1 + \sin -5\pi + \sin 10)}{-5\pi}$$

$$h_d[0] = h_d[10] = -0.0264$$

$$h_d[1] = h_d[9] = -0.1389$$

$$h_d[2] = h_d[8] = 0.0446$$

$$h_d[8] = h_d[7] = 0.2652$$

$$h_d[4] = h_d[6] = -0.0216$$

$$h_d[5] = 0.6816$$

(ii) $$W[n] = \begin{cases} \frac{2n}{N-1}, & 0 \le n \le \frac{N-1}{2} \\ 2 - \frac{2n}{N-1}, & \frac{N-\phi}{2} \le n \le N-1 \end{cases}$$

$$W[0] = 0 = W[10]$$
$$W[1] = 0.2 = W[9]$$
$$W[2] = 0.4 = W[8]$$
$$W[3] = 0.6 = W[7]$$
$$W[4] = 0.8 = W[6]$$
$$W[5] = 1$$

(iii) $$h_d[n] \times W[n] = h[n]$$

$$h[0] = h[10] = 0$$
$$h[1] = h[9] = -0.02778$$
$$h[2] = h[8] = 0.01784$$
$$h[8] = h[7] = 0.15912$$
$$h[4] = h[6] = -0.01728$$
$$h[5] = 0.6816$$

(iv) $H(e^{j\omega}) = e^{-j\omega\left(\frac{N-1}{2}\right)} \left[h\left[\frac{N-1}{2}\right] + 2\left[h[0]\cos 5\omega + h[1]\cos 4\omega + h[2]\cos 3\omega + h[3]\cos 2\omega + h[4]\cos\omega \right] \right]$

$$H(e^{j\omega}) = e^{-j\omega 5}\left[0.6816 + 0\cos 5\omega + -0.0556\cos 4\omega + 0.03568\cos 3\omega + 0.31824\cos 2\omega + -0.03456\cos\omega\right]$$

FT of DT $\Longrightarrow$ DTFT

DFT $\longrightarrow$ Sampling of DFT

<u>FIR filters</u> →

$h[n] \longrightarrow$ finite length
Causal
Stability

<u>linear phase</u> →
$h[n] \rightarrow$ symmetric :- $h[r] = h[N-1-r]$

<u>frequency sampling</u> →

$H(e^{j\omega}) \longrightarrow$ sampling $\longrightarrow h[n] \rightarrow$ Symmetry → IDFT
$\updownarrow$
$K = 0$ to $N-1$

$H(e^{j\omega})$:- Freq response of desired filter
⇓ sampling "N" ; $K = 0$ to $N-1$
$H(K) \Rightarrow [\text{DFT}(h_d[n])]$
⇓ IDFT
$h[n] \Rightarrow$ Symmetry
"linearphase"

$$h[n] \Rightarrow H(K) = \sum_{n=0}^{N-1} h[n] e^{-j\frac{2\pi K}{N}n}$$

$\Downarrow$ IDFT

$$h[n] = \frac{1}{N}\sum_{K=0}^{N-1} H[K] e^{j\frac{2\pi K}{N}n}$$

$$H(e^{j\omega}) \xrightarrow[\text{"N"}]{\text{Sampling}} H(K) = |H(K)| e^{j\theta_K}$$

$$\boxed{H(e^{j\omega}) = |H(e^{j\omega})| e^{j\theta}}$$

Phase $\Rightarrow \theta = +k\omega$

$$k = -\frac{(N-1)}{2}$$

$\therefore$ $\theta_K = -\left(\frac{N-1}{2}\right)\omega$

$$= -\left(\frac{N-1}{2}\right)\left(\frac{2\pi K}{N}\right)$$

$$\boxed{\theta_K = -\frac{(N-1)\pi K}{N}}$$ $\Big\}\rightarrow$ phase component for FIR filter

$K = 0$ to $N-1$

For H(K) to be a linear phase FIR filter,

$$\boxed{\begin{array}{l} |H(K)| = |H(N-K)| \rightarrow \text{even func} \\ \theta(K) = -\theta(N-K) \rightarrow \text{odd func} \end{array}}$$

$$\theta(K) = -\theta(N-K)$$

$$\theta(N-K) = -\left(\frac{N-1}{N}\right)\pi(N-K)$$

$$\boxed{\theta(N-K) = -(N-1)\pi + \frac{(N-1)}{N}\pi K}$$

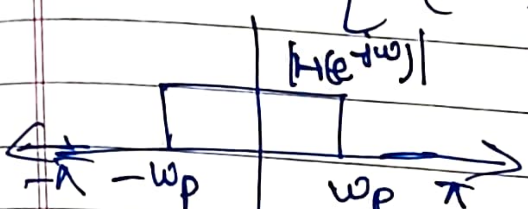
Mostly, we will use N-odd:-

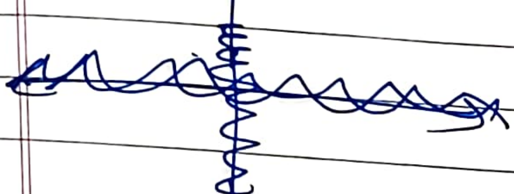
N-odd:

$$\theta(K) = \begin{cases} -\left(\frac{N-1}{N}\right)\pi K, & K = 0 \ldots \frac{N-1}{2} \\ (N-1)\pi - \left(\frac{N-1}{N}\right)\pi K, & K = \frac{N+1}{2}, \ldots N-1 \end{cases}$$

N-even:

$$\theta(K) = \begin{cases} -\left(\frac{N-1}{N}\right)\pi K, & 0 \le N \le \frac{N}{2} - 1 \\ (N-1)\pi - \left(\frac{N-1}{N}\right)\pi K, & \frac{N}{2}+1 \le N \le N-1 \end{cases}$$



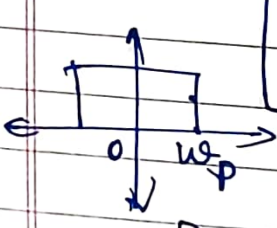


For N-odd,

$$H(k) = \begin{cases} |H(K)|\, e^{-j\left(\frac{N-1}{N}\right)\pi K}, & 0 \le K \le \frac{N-1}{2} \\ |H(K)|\, e^{j\left[(N-1)\pi - \left(\frac{N-1}{N}\right)\pi K\right]}, & \frac{N+1}{2} \le K \le N-1 \end{cases}$$

For N-even,

$$H(K) = \begin{cases} |H(K)|\, e^{-j\left(\frac{N-1}{N}\right)\pi K}, & 0 \le K \le \frac{N}{2} - 1 \\ |H(K)|\, e^{-j\left[(N-1)\pi - \left(\frac{N-1}{N}\right)\pi K\right]}, & \frac{N}{2} + 1 \le N \le N-1 \\ 0, & K = \frac{N}{2} \end{cases}$$



$IDFT[H(K)] \Rightarrow h[n]$

$$h[n] = \frac{1}{N} \sum_{K=0}^{N-1} H(K) \cdot e^{\frac{j2\pi K}{N} n}$$

N-odd:

$$h[n] = \frac{1}{N}\left[H(0) + 2\sum_{k=1}^{\frac{N-1}{2}} Re\left(H(k)\, e^{j\frac{2\pi k n}{N}}\right)\right]$$

N-even:

$$h[n] = \frac{1}{N}\left[H(0) + 2\sum_{k=1}^{\frac{N}{2}-1} Re\left(H(k)\, e^{j\frac{2\pi k n}{N}}\right)\right]$$

5)

$N=7$

	$\omega \Rightarrow \frac{2\pi}{N}(K)$ [$K = 0$ to $N-1$]
$K=0$	$\omega = 0$
1	$\frac{2\pi}{7}$
2	$\frac{4\pi}{7}$
3	$\frac{6\pi}{7}$
4	$\frac{8\pi}{7}$
5	$\frac{10\pi}{7}$
6	$\frac{12\pi}{7}$

$$|H(K)| = \begin{cases} 1, & K = 0, 1, 6 \\ 0, & K = 2, 3, 4, 5 \end{cases}$$

Phasor: N - odd

$$\theta_K = \begin{cases} -\left(\frac{N-1}{N}\right)\pi K, & K = 0 \ldots \frac{N-1}{2} \\ (N-1)\pi - \left(\frac{N-1}{N}\right)\pi K, & K = \frac{N+1}{2} \ldots N-1 \end{cases}$$

$$\theta_K = \begin{cases} -\frac{6}{7}\pi K, & K = 0, 1, 2, 3 \\ 6\pi - \frac{6}{7}\pi K, & K = 4, 5, 6 \end{cases}$$

$$H(K) = |H(K)|\, e^{+j\theta_K}$$

$$H(K) = \begin{cases} 1 \cdot e^{+j\frac{6\pi}{7}K}, & K = 0, 1 \\ 0, & K = 2, 3, 4, 5 \\ e^{-j\frac{6\pi}{7}(K-7)}, & K = 6 \end{cases}$$

$$h[n] = \frac{1}{N}\left[H(0) + 2\sum_{K=1}^{\frac{N-1}{2}} \mathrm{Re}\left[H(K)\, e^{j\frac{2\pi K n}{N}}\right]\right]$$

$$h[n] = \frac{1}{7}\left[1 + 2\,\mathrm{Re}\left[H(1)\, e^{j\frac{2\pi n}{7}}\right]\right]$$

$$= \frac{1}{7}\left[1 + 2\,\mathrm{Re}\left[e^{-\frac{6\pi}{7}j} \cdot e^{j\frac{2\pi n}{7}}\right]\right]$$

$$= \frac{1}{7}\left[1 + 2^{-j\frac{2\pi}{7}(3-n)}\right]$$

$$\boxed{h[n] = \frac{1}{7}\left[1 + 2\cos\left(\frac{2\pi}{7}(3-n)\right)\right]}$$

linear phase

$n = 0$ to 6 $\quad h[r] = h[N-1-r]$

$h[0] = h[6] = -0.8019/7 = -0.1145$

$h[1] = h[5] = 0.5549/7 = 0.07928$

$h[2] = h[4] = 2.2469/7 = 0.3210$

$h[3] = 3/7 = 0.42857$

$H(e^{j\omega}) \longrightarrow \mathrm{DTFT}[h[n]]$

$$H(e^{j\omega}) = e^{-j\omega\left(\frac{N-1}{2}\right)}\left[h\left(\frac{N-1}{2}\right) + 2\sum_{r=0}^{\frac{N-3}{2}} h(r)\cos\left[\frac{N-1}{2} - r\right]\omega\right]$$

$$H(e^{j\omega}) = e^{-j\omega 3}\left[0.42857 - 0.22912\cos 3\omega + 0.15856\cos 2\omega + 0.6420\cos\omega\right]$$

b) $|H(K)| = \begin{cases} 1, & 0 \le K \le 3,\ 12 \le K \le 14 \\ 0, & 4 \le K \le 11 \end{cases}$

$N = 15$

$$\Rightarrow \theta_K = \begin{cases} -\frac{14}{15}\pi K, & K = 0 \ldots 7 \\ 14\pi - \frac{14}{15}\pi K, & K = 8 \ldots 14 \end{cases}$$

$$H(K) = \begin{cases} 1\, e^{-\frac{14}{15}\pi K}, & K = 0, 1, 2, 3 \\ 0, & 4 \le K \le 11 \\ e^{-\frac{14}{15}j(\pi K - 15)}, & K = 12, \ldots, 14 \end{cases}$$

$$h[n] = \frac{1}{N}\left[H(0) + 2\sum_{k=1}^{(N-1)/2} Re\left(H(k)e^{\frac{2\pi kn}{N}j}\right)\right]$$

$$h[n] = \frac{1}{15}\left[1 + 2\sum_{k=1}^{3} Re\left(H(k)e^{\frac{2\pi n}{7}j}\right)\right]$$

$$= \frac{1}{15}\left[1 + 2 * \left[e^{-\frac{14}{15}\pi} e^{+\frac{2\pi n}{15}j} + e^{-\frac{28}{15}\pi} e^{+\frac{4\pi n}{15}j} + e^{-\frac{42}{15}\pi} e^{+\frac{6\pi n}{15}j}\right]\right]$$

$$h[n] = \frac{1}{15}\left[1 + 2\left[\cos\left[\frac{2\pi}{15}(n-7)\right] + \cos\left[\frac{4\pi}{15}[n-7]\right] + \cos\left[\frac{6\pi}{15}[n-7]\right]\right]\right]$$

$h[0] = h[14] = -0.05678$

$h[1] = h[13] = 0.041$

$h[2] = h[12] = 0.0666$

$h[3] = h[11] = -0.0365$

$h[4] = h[10] = -0.1078$

$h[5] = h[9] = 0.0341$

$h[6] = h[8] = 0.3188$

$h[7] = 0.46667$

$$H(e^{j\omega}) = e^{-j\omega\left(\frac{N-1}{2}\right)}\left[h\left(\frac{N-1}{2}\right) + 2\sum_{2}^{\frac{N-3}{2}} h(\gamma)\cos\left[\frac{N-1}{2} - \gamma\right]\omega\right]$$

Realisation -

$h[n] \longrightarrow H(Z)$

N = odd

$H(Z) = ?$

⇓

$$h[r] = h[N-1-r]$$

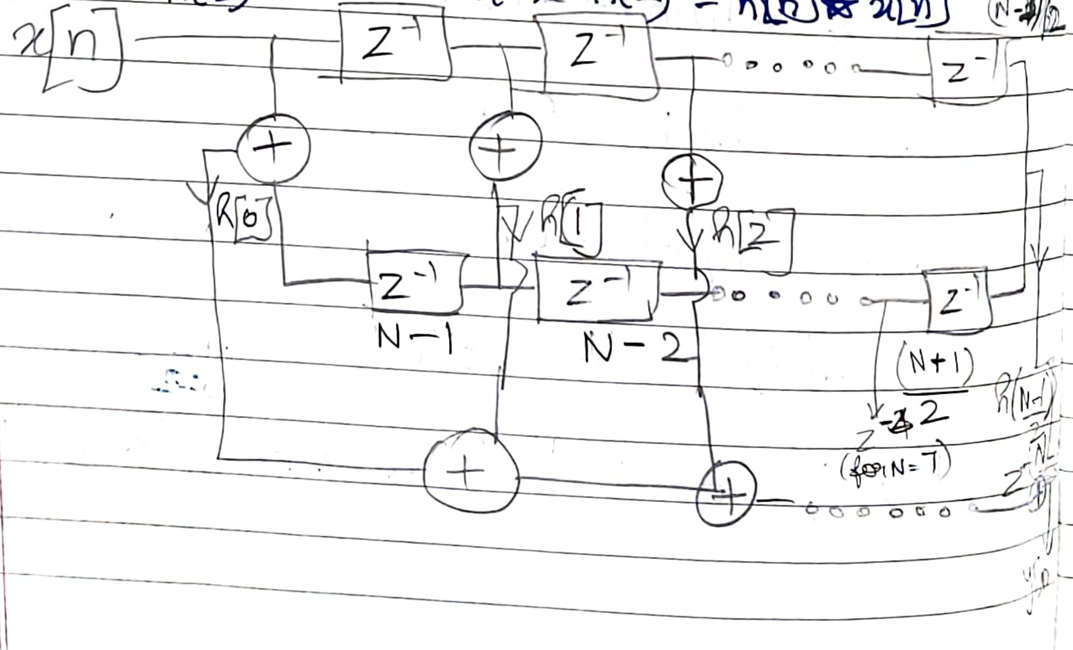
$$H(Z) = h(0) + h[1]Z^{-1} + h(2)z^{-2} + \dots + h[N-1]\cdot z^{-(N-1)}$$

$$= h[0] + h[1]z^{-1} + h[2]z^{-2} + \dots + h[0]z^{-(N-1)}$$

$$= h[0]\left[1 + z^{-(N-1)}\right] + h[1]\left[z^{-1} + z^{-(N-2)}\right] + \dots + h\left[\frac{N-1}{2}\right] z^{-\left(\frac{N-1}{2}\right)}$$

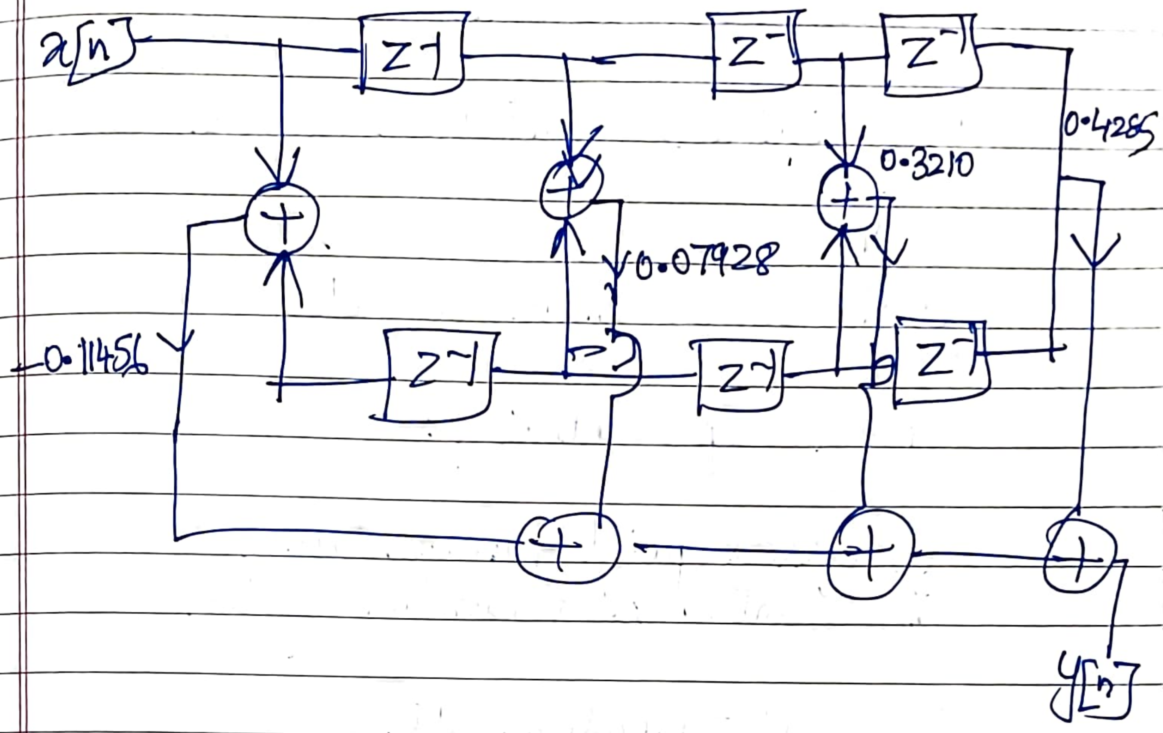
$$\boxed{H(Z) = h\left(\frac{N-1}{2}\right) z^{-\left(\frac{N-1}{2}\right)} + \sum_{r=0}^{\frac{N-3}{2}} h[r]\left[z^{-r} + z^{-(N-1-r)}\right]}$$

$$H(Z) = \frac{Y(Z)}{X(Z)} \Rightarrow Y(Z) = H(Z) * X(Z) = h[n] * x[n]$$



7) Obtain H(Z) from h[n] (from prob (5))

$$H(z) = -0.01145 + 0.07928 z^{-1} + 0.3210 z^{-2} + 0.4285 z^{-3} + 0.3210 z^{-4} + 0.07928 z^{-5} - 0.01145 z^{-6}$$



8) Obtain H(Z) from h[n] (from prob (6))