Electrical Analogous of Mechanical-Translational s/m.

→ The three basic elements mass, dash-pot & spring that are used in modelling mechanical-translational s/ms are analogous to resistance, inductance and capacitance of electrical s/m.

→ (i) Force - voltage Analogy.

(a) Elements having same current ⇒ series connection.

Elements having same velocity ⇒ series connection.

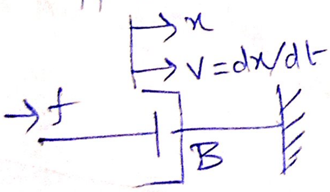
(b) Each node in mechanical s/m ~~should~~ ~~have the~~ corresponds to a closed loop in electrical s/m. A mass is considered as a node.

(c) No. of mesh currents = No. of velocities of nodes.

→ **Mechanical s/m**

i/p: force o/p: velocity

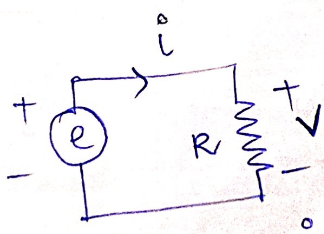
1)



$$f = B\frac{dx}{dt} = Bv.$$

Electrical s/m.

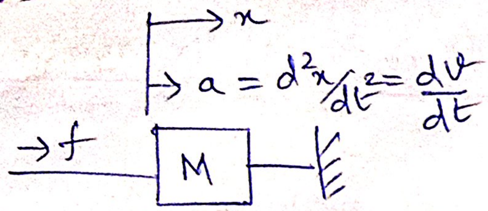
i/p: voltage o/p: current thru the element



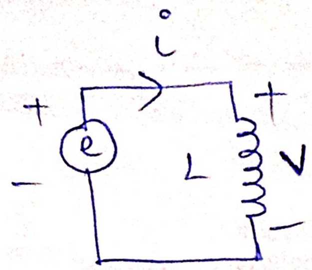
$$e = V = Ri$$

$$i = \frac{V}{R}$$

2)



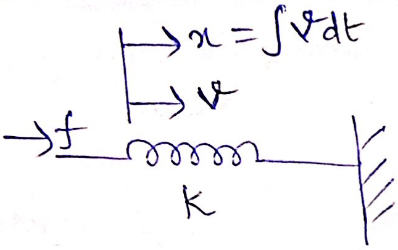
$$f = M\frac{d^2x}{dt^2} = M\frac{dv}{dt}$$



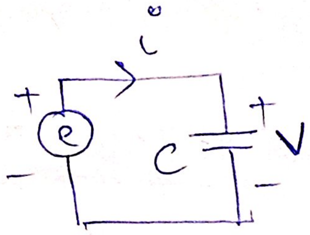
$$V = e = L\frac{di}{dt}.$$

$$i = \frac{1}{L}\int v\,dt$$

3)



$$f = Kx = K\int v\,dt.$$



$$V = e = \frac{1}{C}\int i\,dt.$$

$$i = C\frac{dv}{dt}$$

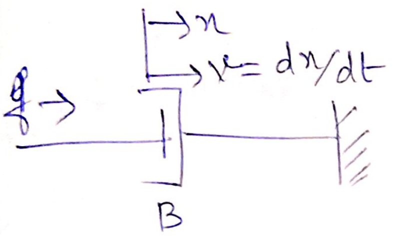
→ Analogous Quantities in Force-Voltage Analogy:
(i/p)

Item	Mechanical s/m	Electrical s/m
Independent variable (I/p)	Force, (f)	voltage (e, V)
Dependent variable (o/p)	Velocity, (v) Displacement, (x)	current, (i) charge, (q)
Dissipative element	Frictional co-eff of dash-pot (B)	Resistance (R)
Storage element	Mass, (M) stiffness of spring, (K)	Inductance (L) Inverse of capacitance (1/C)

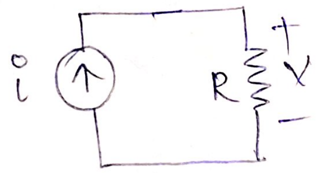
physical law	Newton's second law $\Sigma f = 0$	KVL $\Sigma V = 0$

(ii) Force - current Analogy.

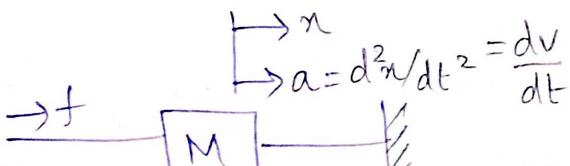
Mechanical s/m	Electrical s/m
I/p: force o/p: velocity	i/p: current source o/p: voltage across the element.



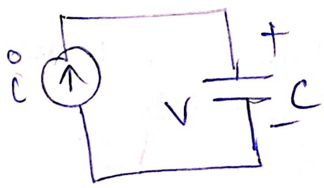
$$f = B\frac{dx}{dt} = BV$$



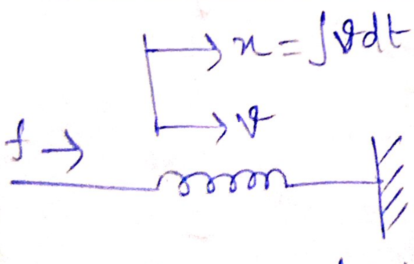
$$V = iR$$
$$i = V/R$$



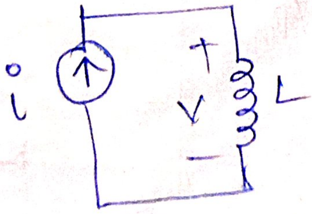
$$f = M\frac{d^2x}{dt^2} = M\frac{dv}{dt}$$



$$i = C\frac{dv}{dt}$$
$$V = \frac{1}{C}\int i\,dt$$



$$f = Kx = \int v\,dt$$



$$i = \frac{1}{L}\int v\,dt$$
$$V = L\frac{di}{dt}$$

→ Analogous Quantities in force-current (i/p) analogy

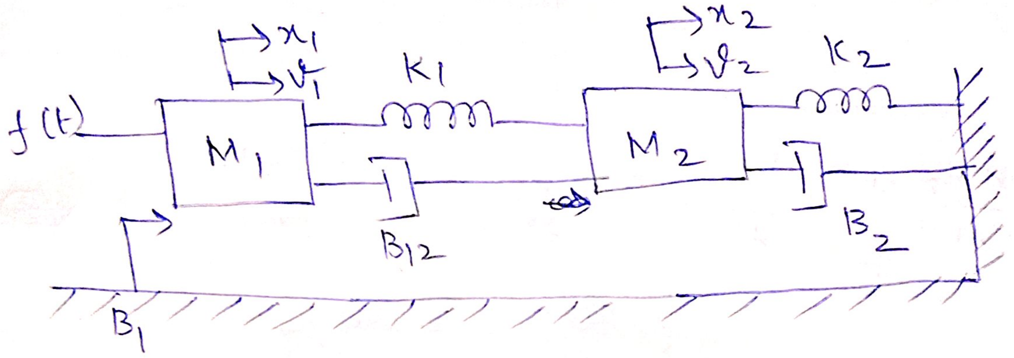
Item	Mechanical s/m	Electrical s/m
Independent Variable (i/p)	force (f)	current, i
Dependent Variable (o/p)	velocity (v) Displacement, (x)	Voltage (V) Flux (ϕ)
Dissipative element	Frictional co-eff of Dashpot (B)	conductance ($G = 1/R$)
Storage element	Mass (M) Stiffness of spring (K)	capacitance (C) Inverse of Inductance, (1/L)
physical law	Newton's II law $\Sigma f = 0$	KCL $\Sigma i = 0$

→ comparison:-

M-s/m	F-V	F-C
f (i/p)	V (i/p)	i (i/p)
v (o/p)	i (o/p)	V (o/p)
x	q	ϕ
B	R	1/R
M	L	C
K	1/C	1/L

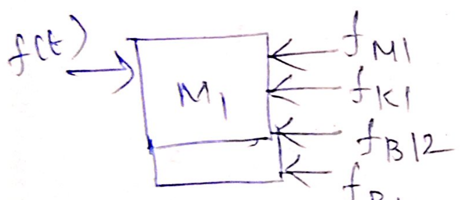
pblm

1) Write the differential equations governing the mechanical s/m. Draw the F-V & F-C electrical analogous circuits & verify by writing Mesh and node equations.



Soln

$M_1 \rightarrow$ free body diagram.



$$f_{M1} = M_1 \frac{d^2 x_1}{dt^2}$$

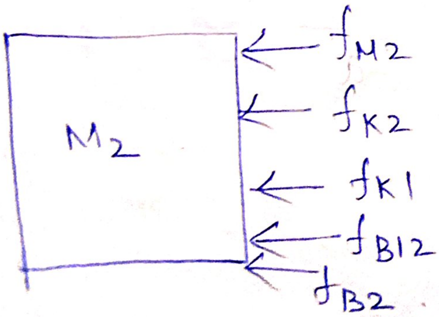
$$f_{K1} = K_1 (x_1 - x_2)$$

$$f_{B1} = B_1 \frac{dx_1}{dt}$$

$$f_{B12} = B_{12} \frac{d(x_1 - x_2)}{dt}.$$

$$\boxed{f(t) = M_1 \frac{d^2 x_1}{dt^2} + B_1 \frac{dx_1}{dt} + B_{12} \frac{d(x_1 - x_2)}{dt} + K_1 (x_1 - x_2)} \quad \text{—— (1)}$$

$M_2 \rightarrow$ free body diagram.



$$f_{M2} = M_2 \frac{d^2 x_2}{dt}$$

$$f_{K2} = K_2 (x_2 - x_1)$$

$$f_{B12} = B_{12} \frac{d(x_2 - x_1)}{dt}$$

$$f_{B2} = B_2 \frac{d(x_2)}{dt}$$

$$f_{K1} = K_1 (x_2 - x_1)$$

We know that

$$\left.\begin{aligned} \frac{d^2x}{dt^2} &= \frac{dv}{dt} \\ \frac{dx}{dt} &= v \\ x &= \int v\,dt. \end{aligned}\right\} \Rightarrow \text{I}$$

$$\boxed{M_2 \frac{d^2 x_2}{dt^2} + B_2 \frac{dx_2}{dt} + B_{12} \frac{d}{dt}(x_2 - x_1) + K_2 x_2 + K_1 (x_2 - x_1) = 0} \rightarrow (2)$$

→ sub (I) in (1) & (2).

$$(1) \Rightarrow M_1 \frac{dv_1}{dt} + B_1 v_1 + B_{12}(v_1 - v_2) + K_1\left(\int (v_1 - v_2)\,dt\right) = f(t) \rightarrow (3)$$

$$(2) \Rightarrow M_2 \frac{dv_2}{dt} + B_2 v_2 + K_2 \int v_2\,dt + B_{12}(v_2 - v_1) + K_1 \int (v_2 - v_1)\,dt = 0 \rightarrow (4)$$

(i) F-V Analogous ckt

no. of currents $\boxed{v_1 \rightarrow i_1 \text{ \& } v_2 \rightarrow i_2}$.

$f(t)$ = voltage V or e,

M_1 = Inductance L_1, $M_2 = L_2$.

B_1 = Resistance R_1, $B_2 = R_2$.

B_{12} = Resistance Between 2 mesh or 2 node R_{12}

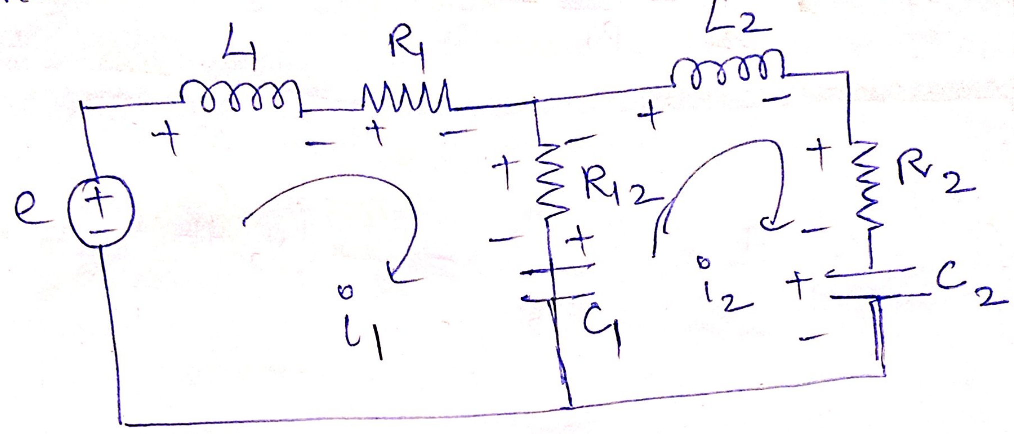
K_1 = Inverse of capacitance $(1/C_1)$

$K_2 = 1/C_2$.

2 → ~~nodes~~ Mass ⇒ 2 - loop/mesh.

⇑

~~2-mass~~.



$$e(t) = L_1 \frac{di_1}{dt} + R_1 i_1 + R_{12}(i_1 - i_2) + \frac{1}{C_1}\int (i_1 - i_2)\,dt \quad \text{—— (5)}$$

$$R_{12}(i_2 - i_1) + L_2 \frac{di_2}{dt} + R_2 i_2 + \frac{1}{C_2}\int i_2\,dt + \frac{1}{C_1}\int (i_2 - i_1)\,dt = 0 \quad \text{—— (6)}$$

(ii) F-C Analogous circuit

$f(t) = i(t)$, $\boxed{v_1 = V_1 \ \& \ v_2 = V_2}$ no. of voltages.

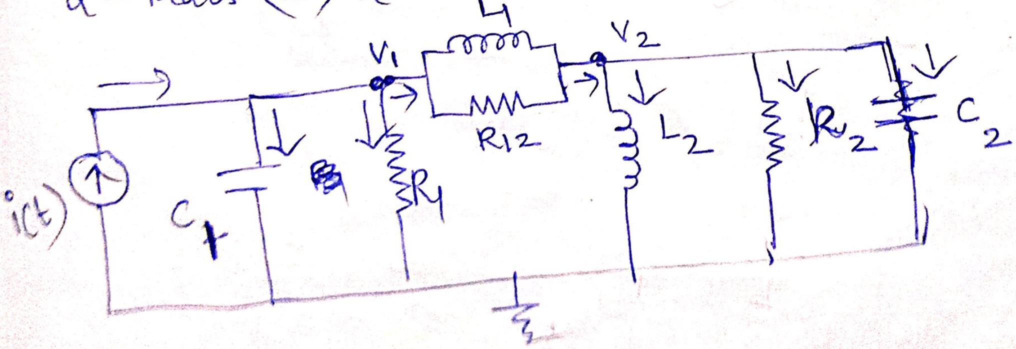
$K_1 = 1/L_1 \Rightarrow$ llel conn

$M_1 \rightarrow C_1 \quad M_2 \rightarrow C_2$

$K_2 = 1/L_2 \Rightarrow$ llel conn.

$B_1 = 1/R_1 \quad B_2 = 1/R_2 \quad B_{12} = 1/R_{12}$

2-Mass ⇔ 2 nodes in the electric ckt.



→ By KCL at node 1 & 2.

node ① ⇒ $$\boxed{i(t) = C_1 \frac{dV_1}{dt} + \frac{V_1}{R_1} + \frac{1}{L_1}\int (V_1 - V_2)\,dt + \frac{V_1 - V_2}{R_{12}}}$$ → (7)

node ② ⇒ $$\frac{1}{L_1}\int (V_1 - V_2)\,dt + \frac{V_1 - V_2}{R_2} = \frac{1}{L_2}\int V_2\,dt + \frac{V_2}{R_2} + C_2\frac{dV_2}{dt}$$

$$\boxed{C_2\frac{dV_2}{dt} + \frac{V_2}{R_2} + \frac{1}{L_2}\int V_2\,dt - \frac{1}{L_1}\int (V_1 - V_2)\,dt + \frac{V_2 - V_1}{R_2} = 0}$$ → (8).