F.22(m)

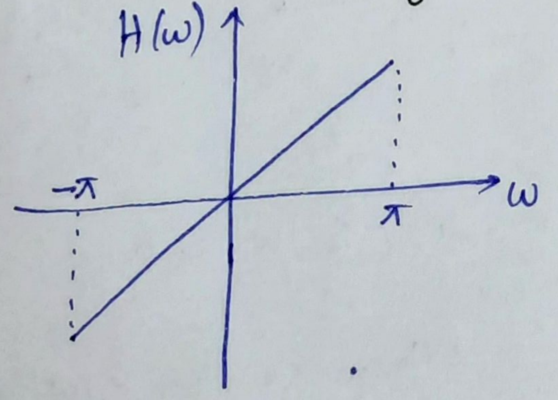
Homework 4 Complete the above problem using any one of the window

Homework 5: Same above problem But **BSF** with same window.

Homework 6: ⇒ (application of FIR filter when $h(n)$ is Anti-symmetrical

Design an ideal differentiator with frequency response $H(e^{j\omega}) = j\omega, \; -\pi \le \omega \le \pi$ using rectangular and Hanning window, with $N=8$. & plot frequency response for both windows.

Given



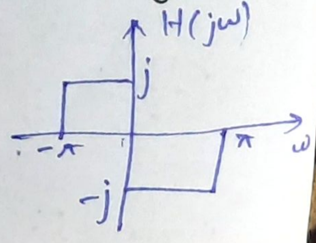
complete this pbm

Soln: $$H_d(\omega) = \begin{cases} j\omega \cdot e^{-j\tau\omega}, & -\pi \le \omega \le \pi \\ 0, & \text{otherwise} \end{cases}$$

Homework 7:

Design an ideal Hilbert transformer having frequency response $H(e^{j\omega}) = \begin{cases} j, & \text{for } -\pi \le \omega \le 0 \\ -j, & \text{for } 0 \le \omega \le \pi \end{cases}$

using a) rectangular window b) Blackman window for $N=11$.



Frequency sampling method of designing FIR filters:

FIR filters are designed by sampling the frequency response (desired) at sufficient number of points (i.e., 'N'-points). These samples are the DFT coefficients of the impulse response of the filter. Hence $h(n)$ is obtained by taking $IDFT\{H(k)\}$.

$$H(K) = H_d(\omega)\Big/_{\omega = \frac{2\pi}{N}(k+\alpha)}, \quad k = 0,1,\ldots(N-1)$$

Based on the value of α, there are two types of design procedure. ($\alpha=0$, $\alpha=\frac{1}{2}$)

Procedure for type-I design: ($\alpha=0$)

Step 1: choose the ideal (desired) frequency response $H_d(\omega)$

Step 2: Sample $H_d(\omega)$ at N-points by taking $\omega = \omega_k = \frac{2\pi k}{N}$, $k=0,1,2\ldots N-1$ to generate $H(K)$.

ie $H(K) = H_d(\omega)\Big/_{\omega=\frac{2\pi k}{N}}$, $k=0,1,2\ldots(N-1)$

For practical realizability, DFT samples of $h(n)$ should be real.

For $h(n)$ to be real sequence,

$H(k)$ should satisfy complex-conjugate symmetry

ie $$\boxed{\begin{gathered} H(k) = H^*(N-k) \\ \text{(or)} \\ H(N-k) = H^*(k) \end{gathered}}$$ with

$|H(k)| = |H(N-k)| \Rightarrow$ magnitude response is <u>even</u>

&

$\angle H(k) = -\angle H(N-k) \Rightarrow$ phase response is <u>odd</u> function.

<u>Step 3:</u> Compute $h(n)$ by taking IDFT$\{H(k)\}$

ie $$h(n) = \frac{1}{N} \sum_{k=0}^{N-1} H(k) \cdot e^{j2\pi kn/N}, \quad n = 0, 1, \cdots N-1$$

By applying symmetry property of DFT in $H(k)$,

when N is odd,

$$h(n) = \frac{1}{N}\left\{ H(0) + 2\sum_{k=1}^{\frac{N-1}{2}} \mathrm{Re}\left[H(k) e^{j2\pi kn/N}\right] \right\}$$

when N is even,

$$h(n) = \frac{1}{N}\left\{ H(0) + 2\sum_{k=1}^{\frac{N}{2}-1} \mathrm{Re}\left[H(k) \cdot e^{j2\pi kn/N}\right] \right\}.$$

Step 4: Take z-transform of $h(n)$ to get the filter transfer function $H(z)$.

$$H(z) = \sum_{n=0}^{N-1} h(n) \cdot z^{-n}.$$

Procedure for type II design: $(\alpha = 1/2)$

Step 1: Choose ideal frequency response $H_d(\omega)$.

Step 2: Sample $H_d(\omega)$ at N-points by taking $\omega = \omega_k = \frac{2\pi}{N}(k+(1/2))$, where $k = 0, 1, 2, \dots N-1$ to generate $H(k)$.

$$H(k) = H_d(\omega)\Big|_{\omega = \frac{2\pi}{N}(k+(1/2))}, \text{ for } k = 0, 1, 2 \dots (N-1)$$

Step 3: Compute DFT samples of $h(n)$ by taking IDFT of $H(k)$.

[After applying symmetry property of $H(k)$ to get real $h(n)$].

when N is odd,

$$h(n) = \frac{2}{N} \sum_{k=0}^{\frac{N-3}{2}} Re\left[H(k) \cdot e^{j\frac{2\pi n}{N}(k+\frac{1}{2})}\right]$$

when N is even,

$$h(n) = \frac{2}{N} \sum_{k=0}^{\frac{N}{2}-1} Re\left[H(k) \cdot e^{j\frac{2\pi n}{N}(k+\frac{1}{2})}\right]$$

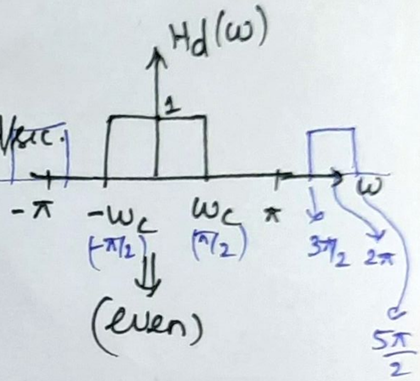
Step 4: Take z-transform of $h(n)$ to get filter transfer function $H(z)$.

$$H(z) = \sum_{n=0}^{N-1} h(n).\, z^{-n}.$$

Pbm: Determine the filter coefficients $h(n)$ obtained by Sampling $H_d(\omega) = \begin{cases} e^{-j\omega\tau}, & 0 \le |\omega| \le \pi/2 \\ 0, & \pi/2 \le |\omega| \le \pi \end{cases}$ for $N=7$

Given: $N=7$, $\tau = \frac{N-1}{2} = 3$

filter : LPF with $\omega_c = \pi/2$ rad/sec.



Solution:

Step 1: $H_d(\omega) = \begin{cases} e^{-j3\omega}, & 0 \le |\omega| \le \pi/2 \\ 0, & \pi/2 \le |\omega| \le \pi \end{cases}$

Step 2: $H(k) = H_d(\omega) \Big/_{\omega = \frac{2\pi k}{7}}$, $k = 0, 1, 2 \ldots 6$

$k=0, \quad \omega = 0$

$k=1, \quad \omega = \frac{2\pi}{7} < \frac{\pi}{2}$

rad : (0.897) $\quad (1.5707)$

$k=2, \quad \omega = \frac{4\pi}{7} > \pi/2 \ \& < \pi$

$(1.7951) \quad (1.5707) \quad (3.14)$

$k=3, \quad \omega = \frac{6\pi}{7} > \pi/2 \ \& < \pi$

$(2.692) \quad (1.5707) \quad (3.14)$

$k=4, \quad \omega = \frac{8\pi}{7} > \pi \ \& < 3\pi/2$

$(3.59) \quad (3.14) \quad (4.712)$

$k=5, \quad \omega = \frac{10\pi}{7} < 3\pi/2$

$(4.487) \quad (4.712)$

$k=6, \quad \omega = \frac{12\pi}{7} > 3\pi/2 < 5\pi/2$

$(5.385) \quad (4.712) \quad (7.853)$

$$H(0) = H_d(0) = e^{j0} = 1 \left(e^{-j3\times\frac{2\pi\times 0}{7}}\right)$$

$$H(1) = H_d(2\pi/7) = e^{-j3\times 2\pi/7}$$

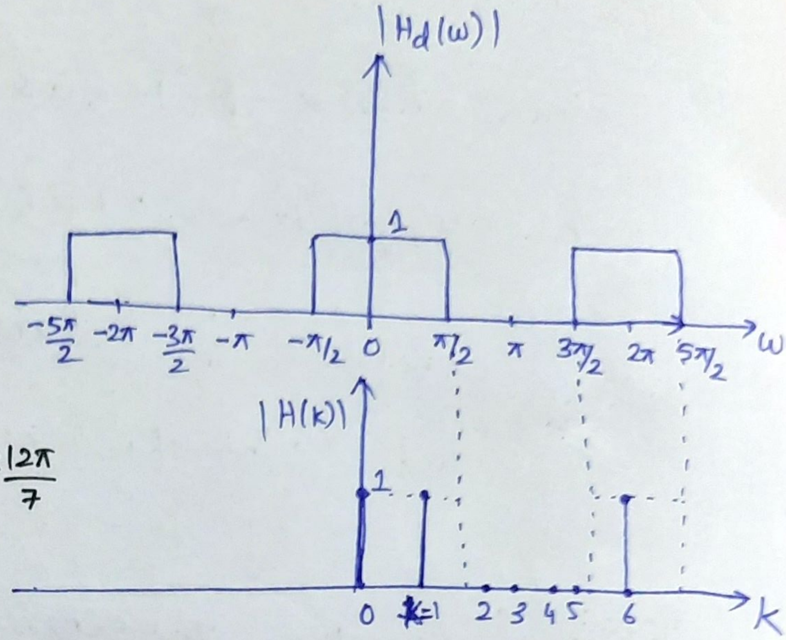
$$H(2) = H_d(4\pi/7) = \cancel{1}\, 0$$

$$H(3) = H_d(6\pi/7) = 0$$

$$H(4) = H_d(8\pi/7) = 0$$

$$H(5) = H_d(10\pi/7) = 0$$

$$H(6) = H_d(12\pi/7) = e^{-j3\times\frac{12\pi}{7}}$$



$$H(k) = \begin{cases} e^{-j6\pi k/7}, & k=0,1 \\ 0, & k=2,3,4,5 \\ e^{-36\pi/7}, & k=6 \end{cases}$$

<u>Step 3:</u> $h(n) = \text{IDFT}\{H(k)\}$

when N is odd $(N=7)$,

$$h(n) = \frac{1}{7}\left\{H(0) + 2\sum_{k=1}^{3} \text{Re}\left[H(k)\cdot e^{j2\pi kn/7}\right]\right\}, \quad n=0,1,2\ldots 6$$

$$= \frac{1}{7}\left\{1 + 2\text{Re}\left[e^{-j6\pi/7}\cdot e^{j2\pi \overset{(1)}{k} n/7}\right]\right\}$$

$$= \frac{1}{7}\left\{1 + 2\,\text{Re}\left[e^{j2\pi/7(\cancel{k}n-3)}\right]\right\}$$

$$= \frac{1}{7}\left\{1 + 2\cos\left[\frac{2\pi}{7}(n-3)\right]\right\}$$

$$\left.\begin{aligned} h(0) = h(6) &= \frac{1}{7}\left(1+2\cos\frac{6\pi}{7}\right) = -0.11456 \\ h(1) = h(5) &= \frac{1}{7}\left(1+2\cos\frac{4\pi}{7}\right) = 0.07928 \\ h(2) = h(4) &= \frac{1}{7}\left(1+2\cos\frac{2\pi}{7}\right) = 0.321 \\ h(3) &= \frac{1}{7}\left(1+2\cos 0\right) = 0.42857. \end{aligned}\right\} \text{filter coefficients.}$$

Homework 8: Complete the same problem for $N=15$ and plot the magnitude response.

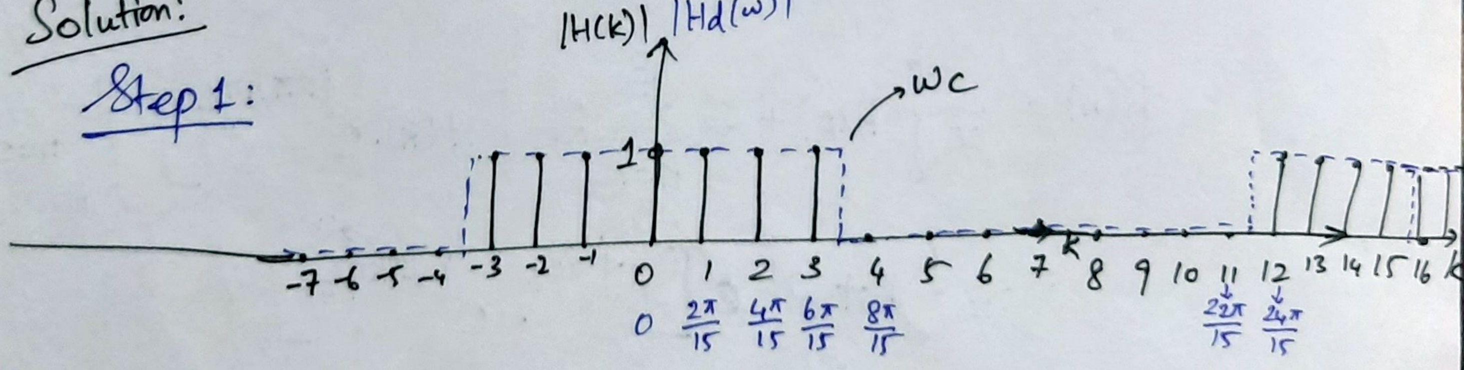
Pbm: Determine the coefficients of a linear phase FIR filter of length $M=15$ has a symmetric unit sample response and a frequency response that satisfies the conditions

$$H\left(\frac{2\pi k}{15}\right) = \begin{cases} 1; & k=0,1,2,3 \\ 0; & k=4,5,6,7 \end{cases}$$

Given $\boxed{N=15}$

Solution:

Step 1:



$k=0, \quad \omega=0$

$k=3, \quad \omega = \frac{2\pi\times 3}{15} = \frac{6\pi}{15}$

$k=4, \quad \omega = \frac{8\pi}{15}$

$k=7, \quad \omega = \frac{14\pi}{15}$

Step 2: $\{H(k)\} = \begin{cases} 1 \cdot e^{-j\frac{14\pi k}{15}} & \text{for } k = 0 \text{ to } 3 \\ 0 & \text{for } k = 4 \text{ to } 11 \\ 1 \cdot e^{-j\frac{14\pi k}{15}} & \text{for } k = 12 \text{ to } 14 \end{cases}$ $\boxed{\because N=15}$ $(k = 0 \text{ to } 14)$.

Step 3: $N=15$, $N \to$ odd

$$h(n) = \frac{1}{15}\left\{ H(0) + 2\sum_{k=1}^{7} Re\left(e^{-j\frac{14\pi k}{15}} \cdot e^{j2\pi kn/15} \right) \right\}$$

$$= \frac{1}{15}\left\{ 1 + 2\sum_{k=1}^{7} Re\left(e^{j\frac{2\pi k}{15}(n-7)} \right) \right\}$$

$$= \frac{1}{15}\left\{ 1 + 2\sum_{k=1}^{3} \cos\left[\frac{2\pi k}{15}(n-7)\right] \right\} \quad \because H(k)=0,\ k=4 \text{ to } 11$$

$$h(n) = \frac{1}{15}\left\{ 1 + 2\cos\frac{2\pi}{15}(n-7) + 2\cos\frac{4\pi}{15}(n-7) + 2\cos\frac{6\pi}{15}(n-7) \right\}$$

$h(0) = h(14) = -0.05$

$h(1) = h(13) = 0.041$

$h(2) = h(12) = 0.0666$

$h(3) = h(11) = -0.0365$

$h(4) = h(10) = -0.1078$

$h(5) = h(9) = 0.034$

$h(6) = h(8) = 0.3188$

$h(7) = 0.466$

filter coefficients

Pbm: Using frequency sampling method, design a BPF with the following specifications.

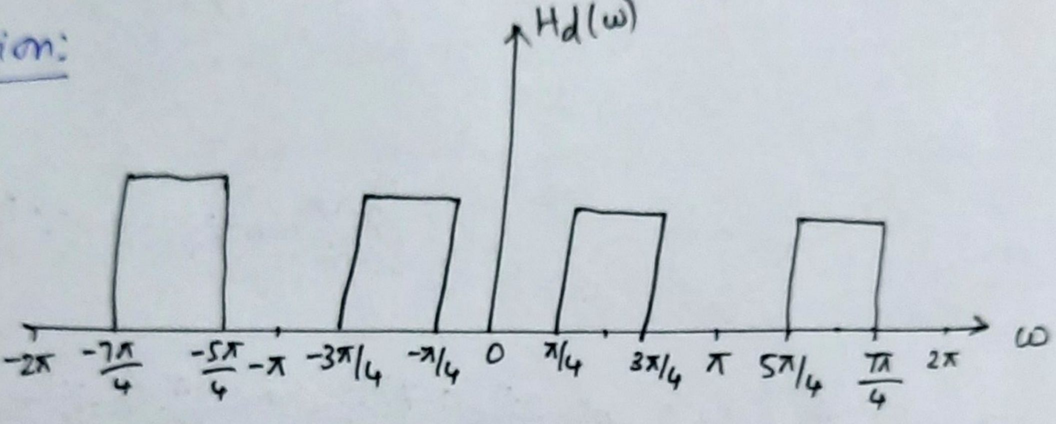
Sampling frequency $F_s = 8000\ Hz$

Cut off frequencies: $f_{c1} = 1000 Hz$ & $f_{c2} = 3000\ Hz$

Determine the filter coefficients for $N=7$.

Solution:

Step 1:



normalized cut off frequencies:

$$\omega_{c1} = \frac{2\pi f_{c1}}{F_s} = \frac{2\pi \times 1000}{8000} = \frac{\pi}{4} \text{ rad/sec}$$

$$\omega_{c2} = \frac{2\pi f_{c2}}{F_s} = \frac{2\pi \times 3000}{8000} = \frac{3\pi}{4} \text{ rad/sec}$$

$$H_d(\omega) = \begin{cases} e^{-j\omega\tau} & ; \ \pi/4 \le |\omega| \le 3\pi/4 \\ 0 & ; \ 3\pi/4 \le |\omega| \le \pi \end{cases}$$

Step 2:

$$H(k) = H_d(\omega)\Big|_{\omega = \frac{2\pi k}{7}}$$

$\omega_{c_1} = \pi/4 = 0.785$

$\pi = 3.14$ $\quad \omega_{c_1} = \frac{5\pi}{4} = 3.926$

$\omega_{c_2} = 3\pi/4 = 2.356$

$\omega_{c_2} = \frac{7\pi}{4} = 5.497$

$N=7$

$\tau = \frac{N-1}{2} = 3$

$H(0) = H_d(\omega)\big/_{\omega=0} = 0$ (Stop band)

$H(1) = H_d(\omega)\big/_{\omega=\frac{2\pi\times1}{7}=0.897} = e^{-j3\omega} = e^{-j3\times\frac{2\pi}{7}}$

$H(2) = H_d(\omega)\big/_{\omega=\frac{4\pi}{7}=1.7951} = e^{-j3\omega} = e^{-j3\times\frac{4\pi}{7}}$

$H(3) = H_d(\omega)\big/_{\omega=\frac{6\pi}{7}=2.692} = 0$ (within Stop band)

$H(4) = H_d(\omega)\big/_{\omega=\frac{8\pi}{7}=3.59} \cancel{= e^{-j3\omega}} = \cancel{e^{-j3\times\frac{8\pi}{7}}} = 0$ (Stopband)

$H(5) = H_d(\omega)\big/_{\omega=\frac{10\pi}{7}=4.487} = e^{-j3\omega} = e^{-j3\times\frac{10\pi}{7}}$

$H(6) = H_d(\omega)\big/_{\omega=\frac{12\pi}{7}=5.385} = e^{-j3\omega} = e^{-j3\times\frac{12\pi}{7}}$

Step3: $N=7$, $h(n) = \frac{1}{7}\left\{H(0) + 2\sum_{k=1}^{3} \mathrm{Re}\left[H(k)\cdot e^{j2\pi kn/7}\right]\right\}$

$h(0) = h(6) = -0.07928$

$h(1) = h(5) = 0.321$

$h(3) = 0.57$

$h(2) = h(4) = 0.11456$