Shannon-Fano Code: INF The-Rev.pdf

example Consider a Symbol set A to J with prob 1/4, 1/4, $\frac{1}{16}$, $\frac{1}{4}$, $\frac{1}{32}$, $\frac{1}{32}$, $\frac{1}{32}$. Calculate coding efficiency using Shanon Fano code

Symbol.	Prob.	q_x.	Length.	Shannon Code
A	1/4	0	2	00
B	1/4	1/4	2	01
C	1/8	1/2	3	100
D	1/8	5/8	3	101
E	1/16	3/4	4	1100
F	1/16	13/16	4	1101
G	1/32	7/8	5	11100
H	1/32	29/32	5	11101
I	1/32	15/16	5	11110
J	1/32	31/32	5	11111

$$\text{Entropy} = \sum_{k=1}^{9} P_k \log_2\left(\frac{1}{P_k}\right)$$

$$= \frac{2}{4}\log_2\left(\frac{4}{1}\right) + \frac{2}{8}\log_2\left(\frac{8}{1}\right) + \frac{2}{16}\log_2\left(\frac{1}{16}\right) + \frac{4}{32}\log_2\left(\frac{1}{32}\right)$$

$$= \frac{2}{4}(2) + \frac{2}{8}(3) + \frac{2}{16}(4) + \frac{4}{32}(5)$$

$$= \frac{4}{4} + \frac{3}{4} + \frac{8}{16} + \frac{4 \times 5}{32} =$$

$= 1 + \frac{3}{4} + \frac{1}{2} + \frac{5}{16} = 2.5625$

$L = \sum P_k L_i$
$= 2 \times \frac{1}{4} \times 2 +$
$3 \times \frac{1}{8} \times 2 +$
$4 \times \frac{1}{16} \times 2 +$
$4 \times 4 \times \frac{1}{32} =$
2.87

$$\text{Entropy} = \sum_{K=0}^{9} P_k \log_2\left(\frac{1}{P_K}\right)$$

$$= \frac{2}{4}\log_2(4) + \frac{2}{8}\log_2(8) + \frac{2}{16}\log_2(16) + \frac{4}{32}\log_2(32)$$

$$= \frac{2}{4}(2) + \frac{2}{8}(3) + \frac{2}{16}(4) + \frac{4}{32}(5)$$

$$= 1 + \frac{3}{4} + \frac{1}{2} + 0.3125$$

$$= 2.5625$$

$$\eta = \frac{H(s)}{L} = \frac{2.5625}{L} = \frac{2.56}{2.8} = 91.4\%$$

example prob:

Info. Rate = Entropy x Data Rate.

X_i	$P(X_i)$
A	0.30
B	0.10
C	0.02
D	0.15
E	0.40
F	0.03

~~Entro~~

Ans: 97.9

H.W

x_i	Prob.	q_x	Length	Shannon code.
A	0.02		2	00
B.	0.03		2	01
C.	0.10		3	100
D	0.15		3	101
E.	0.30		4	1100
F	0.40		4.	1101

$$\text{Entropy} = \sum_{k=1}^{6} P_k \log_2\left(\frac{1}{P_k}\right).$$

$$= 0.02\left(\log_2\left(\frac{1}{0.02}\right)\right) + 0.03\left(\log_2\left(\frac{1}{0.03}\right)\right) + 0.10\left(\log_2\left(\frac{1}{0.10}\right)\right) + 0.15\left(\log_2\left(\frac{1}{0.15}\right)\right) + 0.30\left(\log_2\left(\frac{1}{0.30}\right)\right) + 0.40\left(\log_2\left(\frac{1}{0.40}\right)\right)$$

$$= 0.02(5.64) + 0.03(5.05) + 0.10(3.32) + 0.15(2.73) + 0.30(1.73) + 0.40(1.32)$$

$$= 0.1128 + 0.1515 + 0.332 + 0.4095 + 0.519 + 0.528$$

$$= 2.0528 \;//$$

$$L = \sum_{k=1}^{6} P_k l_k$$

$$= 0.02(2) + 0.03(2) + 0.10(3) + 0.15(3) + 0.30(4) + 0.40(4)$$

$$= 3.65$$

$$\eta = \frac{H(x)}{L} = \frac{2.0528}{3.65} =$$

Scanned with CamScanner

H.W

x_i	Prob.	q_x.	Length	Shannon code.
E	0.40	0	2	00
A	0.30	0.40	2	01
D	0.15	0.30	3	100
B	0.10	0.15	3	101
F	0.03	0.10	4	1100
C	0.02	0.03	4.	1101.

$$\text{Entropy} = \sum_{k=1}^{6} P_k \log_2\left(\frac{1}{P_k}\right).$$

$$= 0.02\left(\log_2\left(\frac{1}{0.02}\right)\right) + 0.03\log_2\left(\frac{1}{0.03}\right) +$$
$$0.10\left(\log_2\left(\frac{1}{0.10}\right)\right) + 0.15\left(\log_2\left(\frac{1}{0.15}\right)\right)$$
$$+ 0.30\left(\log_2\left(\frac{1}{0.30}\right)\right) + 0.40\left(\log_2\left(\frac{1}{0.40}\right)\right)$$

$$= 0.1128 + 0.1515 + 0.332 + 0.4095 +$$
$$0.519 + 0.528 = 2.0528 //$$

$$L = \sum_{k=1}^{6} P_k l_k = (0.40)2 + (0.30)2 + (0.15)3 + (0.10)3 + (0.03)4 + (0.02)4.$$

$$= 2.35$$

$$\eta = \frac{H(s)}{L} = \frac{2.05}{2.35} = 87.23\%.$$

1/10/20 Refer Mam notes → Sep 30, 2020.

We consider binary data, so we consider log (base 2).

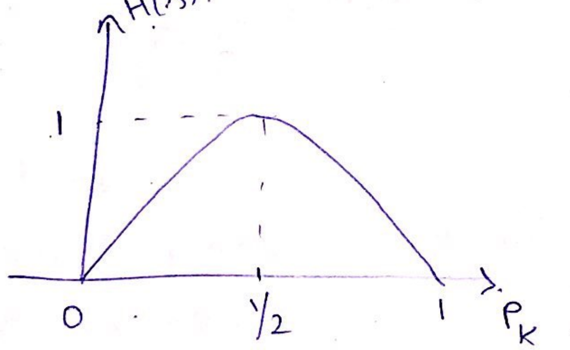
prob 1:

$$P_1 + P_0 = 1$$
$$P_1 = 1 - P_0$$
$$H(s) = \sum_{k=0}^{1} P_k \log_2\left(\frac{1}{P_k}\right).$$
$$= P_1 \log_2\left(\frac{1}{P_1}\right) + P_0 \log_2\left(\frac{1}{P_0}\right).$$
$$= (1-P_0)\log_2\left(\frac{1}{1-P_0}\right) + P_0 \log_2\left(\frac{1}{P_0}\right).$$
$$= -(1-P_0)\log_2(1-P_0) - P_0 \log_2(P_0).$$

Put $P_0 = 0$.
$$H(s) = 0$$
Put $P_0 = 1$
$$H(s) = 0$$

Put $P_0 = P_1 = \frac{1}{2}$.
$$H(s) = -\frac{1}{2}\log_2(2)^{-1} - \frac{1}{2}(2)^{-1}$$
$$= \frac{1}{2} + \frac{1}{2} = 1.$$
$$H(s) = 1$$

※ If Symbols are equiprobable, entropy is max



Prob 2:

S	P(S)
S_0	1/4
S_1	1/4
S_2	1/2

$$H(s) = \sum_{K=0}^{2} P_K \log_2\left(\frac{1}{P_K}\right)$$

$$= \frac{1}{4}\log_2(4) + \frac{1}{4}\log_2(4) + \frac{1}{2}\log_2(2)$$

$$= \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = 1.5 //$$

$S^2 =$	$P(s^2)$
$\sigma_0 = S_0 S_0$	1/16
$\sigma_1 = S_0 S_1$	1/16
$S_0 S_2$	1/8
$S_1 S_0$	1/16
$S_1 S_1$	1/16
$S_1 S_2$	1/8
$S_2 S_0$	1/8
$S_2 S_1$	1/8
$\sigma_8 = S_2 S_2$	1/4

$$\Rightarrow H(s^2) = \sum_{\sigma=0}^{8} P_\sigma \log_2\left(\frac{1}{P_\sigma}\right)$$

$$= 4 \times \frac{1}{16} \times \log(16) + 4 \times \frac{1}{8} \times \log(8) + \frac{1}{4} \times \log(4)$$

$$= 1 + \frac{12}{8} + \frac{2}{4}$$

$$= 1 + \frac{3}{2} + \frac{1}{2} = 3 //$$

$$\Rightarrow H(s^2) = 2H(s).$$

In general, $H(s^n) = n H(s)$ //

Scanned with CamScanner

Prob 1:

Source Symbol	Prob	Code	Bits/symbol (l).
A	1/2	0	1
B	1/4	10	2
C	1/8	110	3
D	1/16	1110	4
E	1/32	11110	5
F	1/64	111110	6
G	1/128	1111110	7
H	1/128	1111111	7

$$H(S) = \sum_{k=0}^{\cancel{8}\,7} P_k \log_2\left(\frac{1}{P_k}\right).$$

$$= \frac{1}{2}(1) + \frac{1}{4}(2) + \frac{1}{8}(3) + \frac{1}{16}(4) + \frac{1}{32}(5) + \frac{1}{64}(6) + \frac{1}{128}(7) + \frac{1}{128}(7).$$

$$H(S) = 1.9843.$$

$$L = \sum_{k=0}^{\cancel{8}\,7} P_k l_k.$$

$$= \frac{1}{2}(1) + \frac{1}{4}(2) + \frac{1}{8}(3) + \frac{1}{16}(4) + \frac{1}{32}(5) + \frac{1}{64}(6) + \frac{1}{128}(7) + \frac{1}{128}(7)$$

$$= \frac{127}{64} = 1.9843.$$

$$\eta = \frac{H(S)}{L} = 1$$

$$\boxed{\eta = 100\%}$$

Huffman Coding

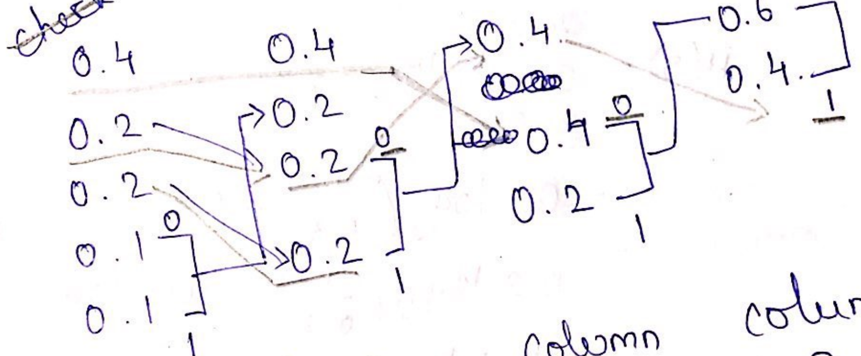
X-16M

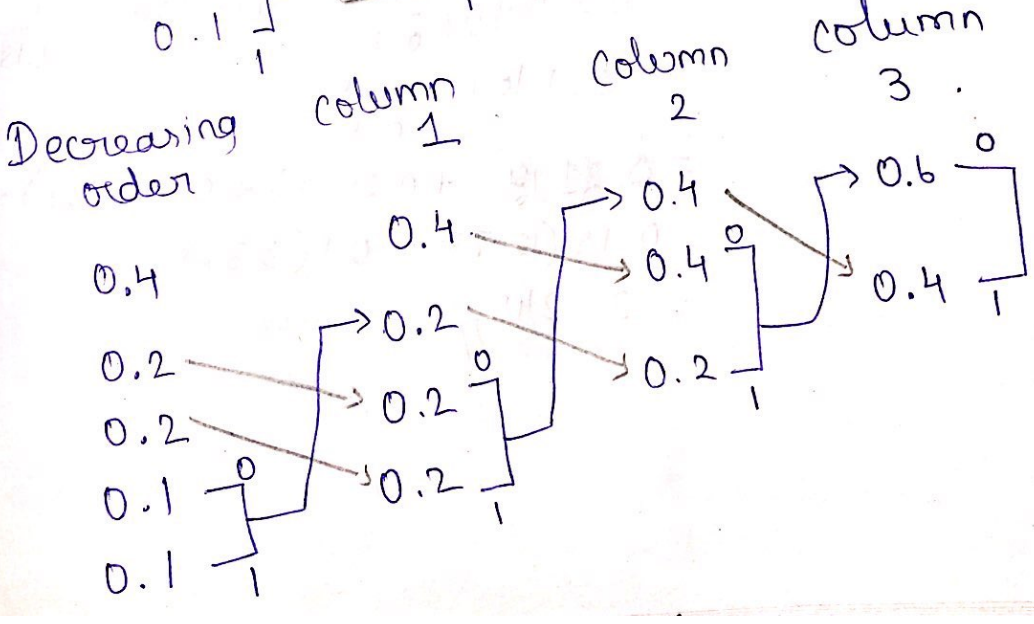
S	P(S)
S_0	0.1
S_1	0.2
S_2	0.4
S_3	0.1
S_4	0.2

Find the efficiency using Huffman coding.

now,

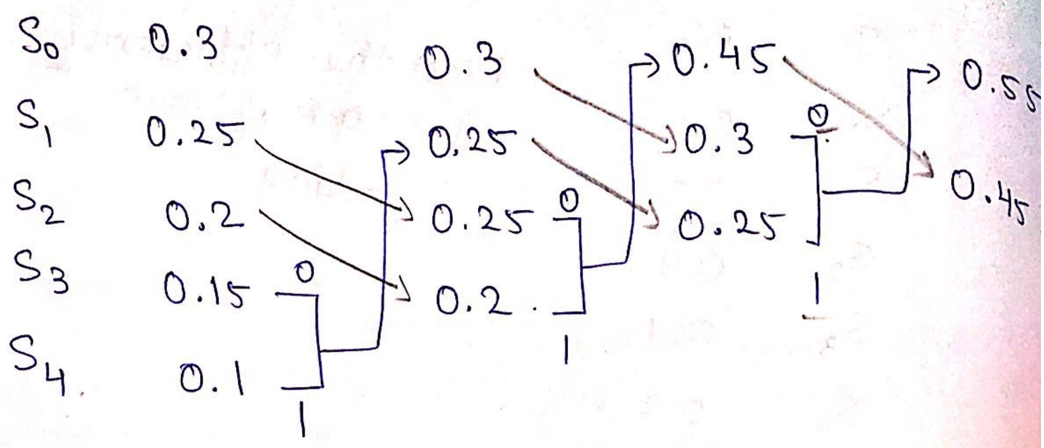
S	P(S)	codeword	l_K
S_0	0.4	00	2
S_1	0.2	10	2
S_2	0.2	11	2
S_3	0.1	010	3
S_4	0.1	011	3





Prob 2

Find efficiency using Huffman Coding



Symbol (s)	P(s)	Code	length
S_0	0.3	00	2
S_1	0.25	~~01~~ 10	2
S_2	0.2	11	2
S_3	0.15	010	3
S_4	0.1	011	3

$$H(s) = \sum_{k=0}^{4} P_k \log_2\left(\frac{1}{P_k}\right)$$

$$= 0.3\log_2\left(\frac{1}{0.3}\right) + 0.25\log_2\left(\frac{1}{0.25}\right) + 0.2\log_2\left(\frac{1}{0.2}\right) + 0.15\log_2\left(\frac{1}{0.15}\right) + 0.1\log_2\left(\frac{1}{0.1}\right)$$

$$= 0.3(1.73) + 0.25(2) + 0.2(2.32) + 0.15(2.73) + 0.1(3.32)$$

$$= 2.2245$$

$$\bar{L} = \sum_{k=0}^{4} P_k l_k.$$

$$= (0.3)(2) + 0.25(2) + (0.2)2 + 0.15(3) + 0.1(3)$$

$$= 2.25.$$

$$\eta = \frac{H(s)}{\bar{L}} = 0.9901.$$

$$\eta = 99\%$$

3/10/20

Mutual Information

→ Due to noise we go for conditional entropy.

Properties:

→ Symmetric.

→ Non negative.

Channel Capacity:

Max. avg mutual info.

Channel Coding Theorem. (Shannon's IInd theorem).

→ low error

→ Max channel capacity

$$\frac{H(s)}{T(s)} \leq \frac{C}{T_c}.$$

$\frac{C}{T_c}$: Critical rate.

Code rate : $r = \frac{T_c}{T_s}$.

$$r \leq C.$$

Shannon's III rd Theorem: Channel Capacity theorem.
(Shannon-Hartley).

→ for bandlimited, power limited gaussian channel.

$$C = B \log_2\left(1 + \frac{P}{\sigma_{NK}^2}\right).$$

$$\text{w.k.t} \quad SNR = \frac{P}{\sigma_{NK}^2}.$$

$$\underset{\substack{\downarrow \\ B.W}}{C = B} \log_2(1 + SNR).$$

(Or)

$$C = B\log_2\left(1 + \frac{P}{N_0 B}\right)$$

N_0: Noise P.S.D.

1. $B = 3.4\,KHz$

$SNR = 31$

(a)

$$C = B\log_2(1 + SNR)$$
$$= 3.4k \log_2(1+31)$$
$$= 3.4k \log_2(32)$$
$$= 3.4K \times 5$$
$$C = 17\,k\,bits/sec$$

(b)

$C = 4800$ bits/sec.

$$4800 = B\log_2(1+SNR).$$

$$\frac{4800}{3.4K} = \log_2(1+SNR).$$

$$1.411 = \log_2(1+SNR)$$

$$2^{1.411} = (1+SNR)$$

$2.65 = 1 + SNR$

$\boxed{SNR = 1.65 \text{ dB}}$

→ Derivation of Shannon Hartley

→ Mutual info derivation

for error free transmission $\boxed{R_b < C}$

Differential Entropy & mutual information for continuous ensemble:

$$h(x) = \int_{-\infty}^{\infty} f_x(x) \log_2 \frac{1}{f_x(x)} \, dx .$$

Area under PDf is 1.

ADV of Source Coding:

- → Compact transmission
- → Reduction in B.W

also in Nam notes pg.15.

Linear Prediction Coding → PPT/PDF- Nimrod Peleg

→ Modeling of Vocal track.
→ LPC

Voiced: w vocal cord vibration (a, e, i)
Unvoiced: wo "
(sh, p)

Voiced use Impulse train generator
Unvoiced " White noise "

prediction of 10 order is enough for Unvoiced.

8/10/20

RUN LENGTH ENCODING (Wikipedia)

→ Data Compression

Channel Coding (Slideshare Linearblock coding - 40500442)

Linear block Codes: Systematic codes

$k \rightarrow$ Encoder $\rightarrow n$

modulo-2-sum: no carry.

$$C_{1 \times n} = M_{1 \times k} G_{k \times N}$$

G: Generator Matrix: we have parity bits followed by Identity matrix. & vice versa.

Parity Check Matrix(H)

$H = I_{N-K} \; P^T$.

↓

Parity matrix Transpose.

$1 \oplus 0 = 1$

$0 \oplus 1 = 1$

$0 \oplus 0 = 0$

$1 \oplus 1 = 0$

Syndrome:

$s = r H^T = (S_1, S_2, \ldots S_{n-k})$.

↓

To check if there is error.

Problem: (mam notes).

For (7,4) code, find all the possible code words & parity check matrix for gn G matrix, if received vector is

$R = [1011001]$ & $R = [1001001]$

find error Syndrome.

Ans:

$n = 7$

$k = 4$.

Code vector $C = D * G$

$$G = \left[\begin{array}{cccc|ccc} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{array}\right]$$

$\underbrace{\hspace{3em}}_{\text{Identity matrix}}$ $\underbrace{\hspace{2em}}_{\text{Parity matrix}}$

$C = 0000$ to 1111

$(k = 4)$.

$C_0 = [0\;0\;0\;0\;0\;0\;0]$

$C_1 = [0\;0\;0\;1] \left[\; G \; \right] = [0\;0\;0\;1\;0\;1\;1]$

$$C_2 = [0\,0\,1\,0]\begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

$$C_2 = [0\,0\,1\,0\,1\,0\,1]$$

$$C_3 = [0\,0\,1\,1]\,G = [0\,0\,1\,1\,1\,1\,0]$$

Modulo-2 addition so carry is omitted

$$C_4 = [0\,1\,0\,0]\,G = [0\,1\,0\,0\,1\,1\,0]$$

$$C_5 = [0\,1\,0\,1]\,G = [0\,1\,0\,1\,1\,0\,1]$$

$$C_6 = [0\,1\,1\,0]\,G = [0\,1\,1\,0\,0\,1\,1]$$

$$C_7 = [0\,1\,1\,1]\,G = [0\,1\,1\,1\,0\,0\,0]$$

$$C_8 = [1\,0\,0\,0]\,G = [1\,0\,0\,0\,1\,1\,1]$$

$$C_9 = [1\,0\,0\,1]\,G = [1\,0\,0\,1\,1\,0\,0]$$

$$C_{10} = [1\,0\,1\,0]\,G = [1\,0\,1\,0\,0\,1\,0]$$

Modulo-2 omit carry 1.

$$C_{11} = [1\,0\,1\,1]\,G = [1\,0\,1\,1\,0\,0\,1]$$

$1+1 = 10$
$+1$
omit $\leftarrow$ 1 1

$$C_{12} = [1\,1\,0\,0]\,G = [1\,1\,0\,0\,0\,0\,1]$$

$$C_{13} = [1\,1\,0\,1]\,G = [1\,1\,0\,1\,0\,1\,0]$$

$$C_{14} = [1\,1\,1\,0]\,G = [1\,1\,1\,0\,1\,0\,0]$$

$C_{15} = [1\ 1\ 1\ 1]\,G = [1\ 1\ 1\ 1\ 1\ 1\ 1]$.

so first k bits are Msg code itself, remaing n-k bits have to be calculated.

$$\text{Parity Matrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

(from Generator Matrix)

$$P^T = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}.$$

$$H = I_{N-K}\,P^T$$

$$H = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$H^T = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$s = rH^T$$

$$= [1\,0\,1\,1\ 0\,0\,1] \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

since no error same message is received. i.e 1011.

$$= [(1+1+0+0)\ \ (1+0+1+0)\ \ (1+1+1+1)]$$

$$= [0\ \ 0\ \ 0] \Rightarrow \text{No errors.}$$

for R_2.

$$s = [\underbrace{100}1001]\begin{bmatrix}1 & 1 & 1\\1 & 1 & 0\\1 & 0 & 1\\0 & 1 & 1\\1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

$$= \begin{bmatrix}1 & 0 & 1\end{bmatrix}$$

⇒ for this syndrome, check in H^T matrix
so 3rd row so 3rd bit has error,
so original bits are

$10\underline{0}1 \rightarrow 1011$

Hamming Codes:

→ Hamming distance: no. of bits by which two codewords differ

→ " weight: no. of 1's in the code

D	C	Hamming Weight
000		

min Hamming weight : other than '0', the min weight is to be considered.

Error Detection Capability :

$$\text{Error correction} = \frac{dH_{min} - 1}{2}.$$

$$\text{Error Detection} = dH_{min} - 1$$

10/10/20

Cyclic Code

- ↳ Sys. Polyn.
- ↳ Shift register.

```
6 5 4 3 2 1 0
1 0 1 0 0 1 1
```

$\Rightarrow x^6 + x^4 + x + 1$

(n, k) Cyclic code can be done by.

polynomial with degree n-k.

→ Poly. with degree n-k.

→ factor $x^n - 1$.

ex. (7,4) : $G(x) = x^3 + x + 1$.

$m(x) = x^3 + 1$

~~$\Rightarrow c(x) = G(x)\, m(x)$~~

1. Premultiply by x^{n-k} with msg $U(x)$.
2. Obtain remainder from div. $x^{n-k} U(x)$ by $G(x)$.
3. Combine $b(x)$ & x

Pblm 1:

Consider (7,4) Cyclic code generated by $g(x) = 1 + x + x^3$, & $U(x) = 1 + x^3$. Obtain corr. code vector by Sys. encoding.

$$b(x) = x + x^2$$

Code polynomial $v(x) = b(x) + x^3 U(x)$

$= x + x^2 + x^3(1 + x^3)$

$= x + x^2 + x^3 + x^6$

$= \underline{x^6 + 0x^5 + 0x^4 + x^3 +}$
$\underline{x^2 + x + 0}$

$$
\begin{array}{r}
x^3 + x \\
x^3+x+1 \overline{\smash{)}\, x^6 \qquad + \quad x^3} \\
x^6 \quad - x^4 + x^3 \\
\hline
x^4 \\
x^4 \quad + x^2 + x \\
\hline
x^2 + x \\
\hline
\end{array}
$$

$\Rightarrow$ ~~Code = {1 0 0 1 1 1 0}~~

Code = {0 1 1 1 0 0 1}

Pblm 2: (7,4) Cyclic code generated by $g(x) = 1 + x + x^3$.

Message. 110.

Find the code polynomial & code vector using Sys. Encoding.

$U(x) =$ ~~0~~ $1 + x$.

$7 - 4 = 3$.

$\therefore x^3 U(x) = x^3 + x^4$.

$G(x) = x^3 + x + 1$.

$$
\begin{array}{r}
x + 1 \\
x^3+x+1 \overline{\smash{)}\, x^4 \qquad + \qquad x^3} \qquad \times . \\
x^4 \quad + \qquad + x^2 + x . \\
\hline
x^3 - x^2 - x . \\
(-)\ x^3 + 0 + x + 1 \\
\hline
-x^2 - 2x - 1
\end{array}
$$

$b(x) = x^2+1$

$V(x) = b(x) + x^3 U(x)$

$= x^2+1 + x^4+x^3$.

$= 1 + 0x + x^2+x^3+x^4+0x^5+0x^6$

$= \{1011100\}$

Check.

H.W

(7,4) cyclic code | $g(x) = 1+x+x^3$ | $U(x) = 0001$

Assignment

PSD of polar (RZ & NRZ) code derivation & BPSK scheme PSD.

$U(x) = x^3$.

$x^3 U(x) = x^6$.

$$\begin{array}{r|l} & x^3 \\ x^3+x+1 & x^6 \\ & x^6 + x^4 + x^3 \\ \hline & +x^4 + x^3 \end{array}$$

$V(x) = b(x) + x^3 u(x)$

$= x^4+x^3+x^6$.

$= \{1011000\}$.