DESIGN TECHNIQUES FOR LINEAR PHASE FIR FILTERS:

1. Fourier series method & window method
2. Frequency sampling method
3. Optimal filter design method.

FOURIER SERIES METHOD:

$$H(\omega) = \sum_{n=-\infty}^{\infty} h(n)\, e^{-j\omega n}$$

$$H_d(\omega T) = H(\omega)\Big/_{\omega=\omega T} = \sum_{n=-\infty}^{\infty} h_d(n)\, e^{-j\omega n T}$$

$$h_d(n) = \text{IFT}\{H_d(\omega T)\} = \frac{1}{\omega_s}\int_{-\omega_s/2}^{\omega_s/2} H_d(\omega T)\, e^{jn\omega T}\, d\omega.$$

where, $\omega_s \rightarrow$ sampling frequency in rad/sec

$\omega_s = 2\pi f_s$, $f_s \rightarrow$ sampling rate (or) frequency in Hz.

$T_s = 1/f_s$, $T_s \rightarrow$ Sampling period in sec.

Infinite duration sequence $h_d(n)$ can be converted into finite duration $h(n)$ by truncating $h_d(n)$ to 'N' samples.

$\therefore h(n) = h_d(n)$ for $n = 0$ to $N-1$

(or)

$n = -\left(\frac{N-1}{2}\right)$ to $\left(\frac{N-1}{2}\right)$

$$H(z) = Z\{h(n)\} = \sum_{n=-\left(\frac{N-1}{2}\right)}^{\frac{N-1}{2}} h(n) \cdot z^{-n}$$

$\Rightarrow$ non-causal filter

To get causal filter, multiply $H(z)$ by $z^{-\left(\frac{N-1}{2}\right)}$

$$H(z) = z^{-\left(\frac{N-1}{2}\right)} \left\{ h(0) + \sum_{n=1}^{\frac{N-1}{2}} h(n)\left[z^{n} + z^{-n}\right] \right\}$$

DRAWBACK OF FOURIER SERIES METHOD:

* Gibbs phenomenon

The abrupt truncation of the fourier series results in oscillations in the pass band & stop band. These oscillations are due to slow convergence of the fourier series particularly near the points of discontinuity. This effect is known as Gibbs phenomenon.

DESIGN OF FIR FILTERS USING WINDOWS:

NEED: To reduce oscillations due to the truncation of $h_d(n)$ to 'N' samples in fourier series method, we go for this window method.

HOW OSCILLATIONS ARE REDUCED?

By Multiplying the infinite impulse response $h_d(n)$ with a finite weighting function (or) window function $w(n)$.

where $w(n) = w(-n) \neq 0$ for $n = 0$ to $N-1$ (or) $\frac{-(N-1)}{2}$ to $\left(\frac{N-1}{2}\right)$

$= 0$; otherwise

finite duration sequence, $h(n) = h_d(n) \cdot w(n)$ for $\frac{-(N-1)}{2} \leq n \leq \frac{N-1}{2}$

$= 0$; otherwise.

In frequency domain,

$$H(\omega) = H_d(\omega) * W(\omega)$$

TYPES OF WINDOWS:

1. Rectangular window
2. Triangular (or) Bartlett window
3. Raised cosine window
4. Hanning window
5. Hamming window
6. Blackman window
7. kaiser window.

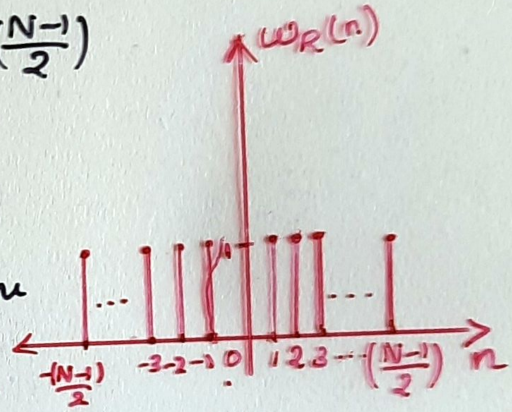
CHARACTERISTICS OF WINDOW:

* The central lobe of the frequency response of the window should contain most of the energy of the window & should be narrow.
* The highest side lobe level of the frequency response should be small.
* The side lobes of the frequency response should decrease in energy rapidly as ω tends to π.

RECTANGULAR WINDOW:

The N-pt rectangular window function is defined by,

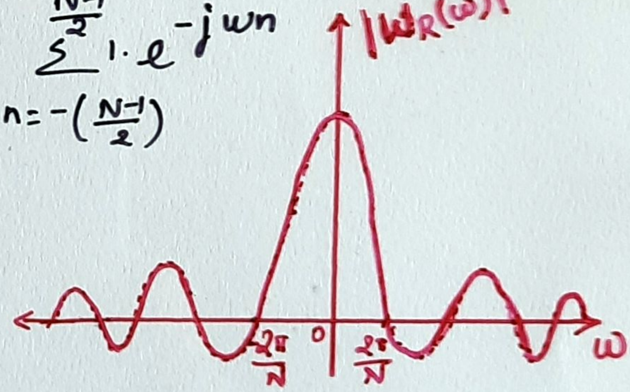
$$w_R(n) = \begin{cases} 1 & \text{for } -\left(\frac{N-1}{2}\right) \le n \le \left(\frac{N-1}{2}\right) \\ 0 & \text{for otherwise} \end{cases}$$



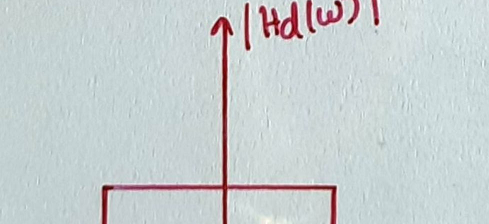
Frequency response of rectangular window is equal to fourier transform of $w_R(n)$ ie

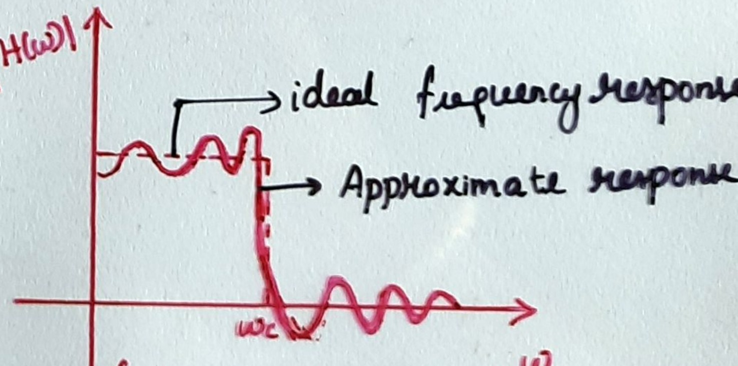
$$W_R(\omega) = FT\{w_R(n)\} = \sum_{n=-\left(\frac{N-1}{2}\right)}^{\frac{N-1}{2}} 1 \cdot e^{-j\omega n}$$

$$W_R(\omega) = \frac{\sin \omega N/2}{\sin \omega/2}$$



ideal frequency response of LPF:





frequency response of LPF approximated using $w_R(n)$

main lobe width: → distance between the 2 points closest to $\omega=0$. where $|W_R(\omega)|=1$ (or) $|W_R(\omega)|_{dB}=0$

For rectangular window, main lobe width is $\frac{4\pi}{N}$.

Characteristic feature of rectangular window:

* main lobe width is $4\pi/N$
* Maximum sidelobe magnitude is $-13\,dB$.
* Side lobe magnitude does not increase when ω ↑es.

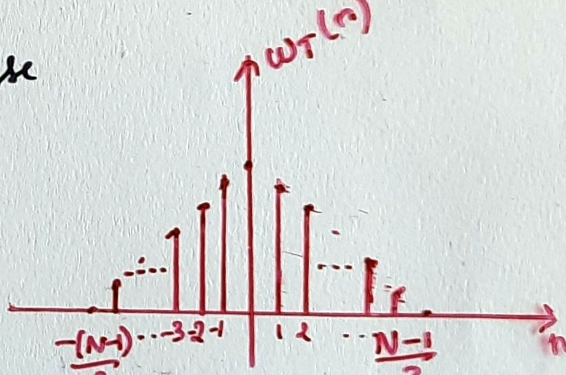
FEATURES OF FIR FILTER DESIGN USING R.W:

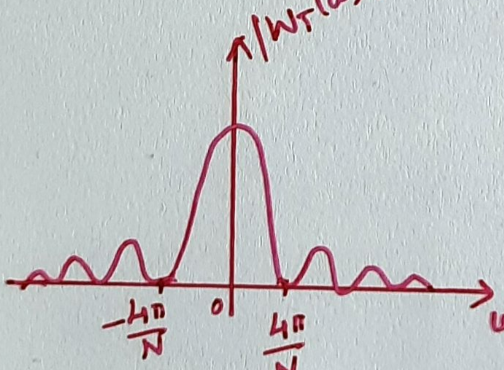
* The width of the transition region is related to the width of the mainlobe of the window
* Gibb's oscillations are noticed in pass band & stop band.
* The attenuation in stop band is constant & cannot be varied.

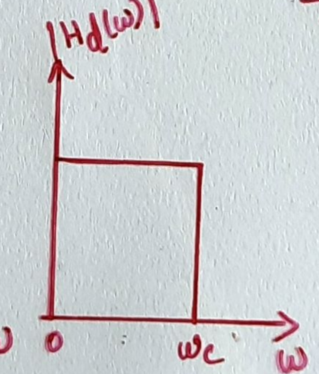
TRIANGULAR (OR) BARTLETT WINDOW:

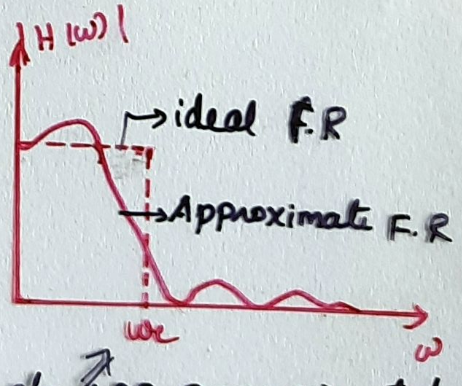
$$w_T(n) = \begin{cases} 1 - \dfrac{2|n|}{N-1} & ; -\left(\dfrac{N-1}{2}\right) \le n \le \left(\dfrac{N-1}{2}\right) \\ 0 & ; \text{otherwise} \end{cases}$$

$$W_T(\omega) = \left[\frac{\sin\left(\frac{N-1}{4}\right)\omega}{\sin(\omega/2)}\right]^2$$









magnitude response of LPF approximated using this window.

Characteristics features of TW:

* MLW is $8\pi/N$
* Max. side lobe magnitude is -25 dB.
* side lobe magnitude slightly $\downarrow$es when $\omega \uparrow$es

Disadvantages of TW:

* Transition from pass band to stop band is not sharp & In stop band, attenuation is less when compared to RW. So, TW is not usually a good choice for design.

RAISED COSINE WINDOW:

$$w_\alpha(n) = \begin{cases} \alpha + (1-\alpha)\cos\left(\frac{2\pi n}{N-1}\right) & ; -\left(\frac{N-1}{2}\right) \le n \le \left(\frac{N-1}{2}\right) \\ 0 & ; \text{otherwise} \end{cases}$$

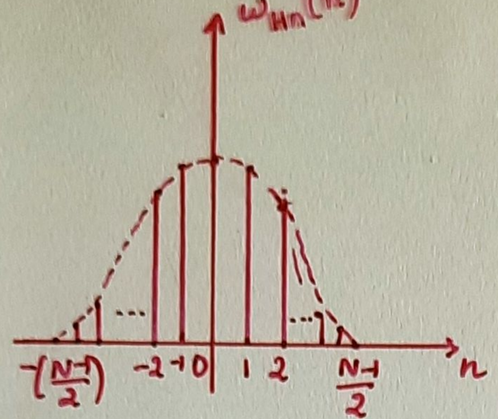
$$W_\alpha(\omega) = \alpha \cdot \frac{\sin \omega N/2}{\sin \omega/2} - \frac{1-\alpha}{2} \frac{\sin\left(\frac{\omega N}{2} - \frac{\pi N}{N-1}\right)}{\sin\left(\frac{\omega}{2} - \frac{\pi}{N-1}\right)} + \frac{1-\alpha}{2} \frac{\sin\left(\frac{\omega N}{2} + \frac{\pi N}{N-1}\right)}{\sin\left(\frac{\omega}{2} + \frac{\pi}{N-1}\right)}$$

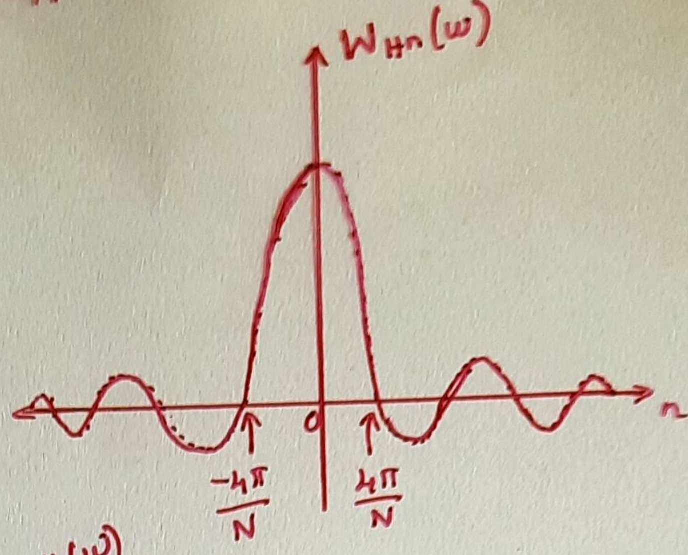
It is otherwise known as generalized hamming window.

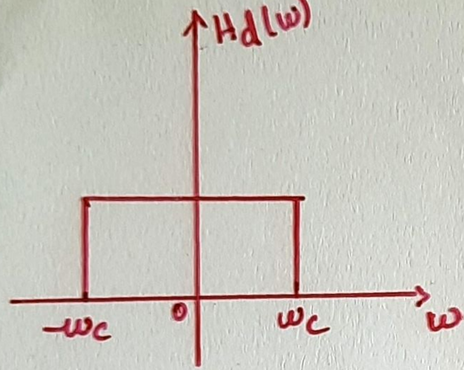
HANNING WINDOW: $\alpha = 0.5$

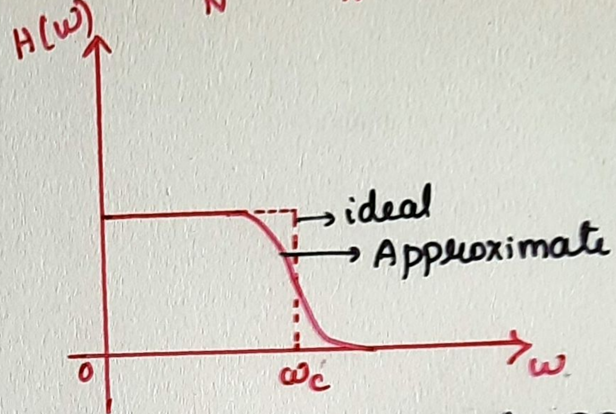
$$w_{Hn}(n) = \begin{cases} 0.5 + 0.5\cos\left(\frac{2\pi n}{N-1}\right) & ; -\left(\frac{N-1}{2}\right) \le n \le \left(\frac{N-1}{2}\right) \\ 0 & ; \text{otherwise} \end{cases}$$

$$W_{Hn}(\omega) = 0.5 \frac{\sin \omega N/2}{\sin \omega/2} + 0.25 \frac{\sin\left(\frac{\omega N}{2} - \frac{\pi N}{N-1}\right)}{\sin\left(\frac{\omega}{2} - \frac{\pi}{N-1}\right)} + 0.25 \cdot \frac{\sin\left(\frac{\omega N}{2} + \frac{\pi N}{N-1}\right)}{\sin\left(\frac{\omega}{2} + \frac{\pi}{N-1}\right)}$$









frequency response of LPF approximated using $\omega_{Hn}(n)$.

* Characteristics feature :

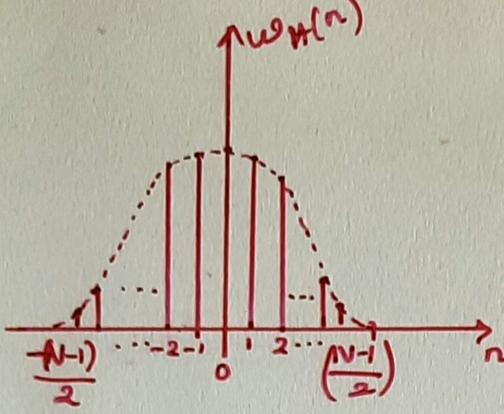
* Main lobe width is $8\pi/N$.
* Max. side lobe magnitude is $-31dB$.
* Side lobe magnitude ↓es with ↑ing ω.

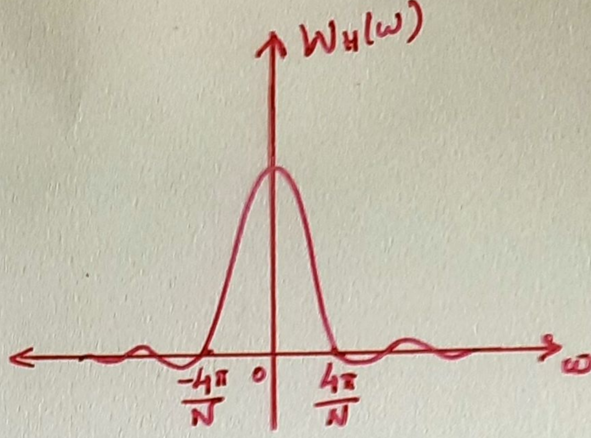
HAMMING WINDOW: $\alpha = 0.54$

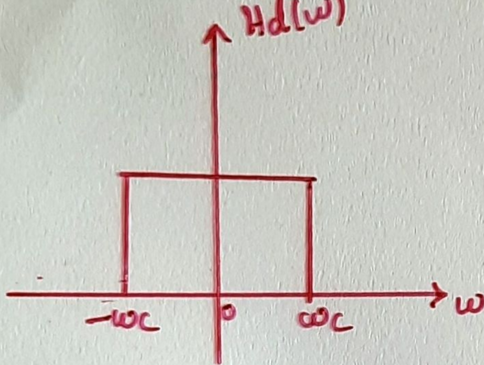
$$\omega_H(n) = \begin{cases} 0.54 + 0.46\cos\left(\frac{2\pi n}{N-1}\right) & ; -\left(\frac{N-1}{2}\right) \leq n \leq \left(\frac{N-1}{2}\right) \\ 0 & ; \text{otherwise} \end{cases}$$

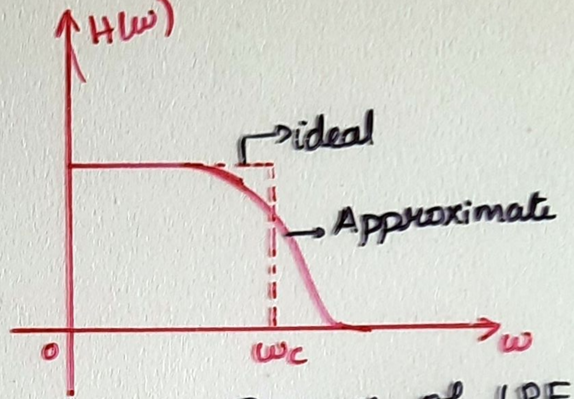
$$W_H(\omega) = 0.54\,\frac{\sin \omega N/2}{\sin \omega/2} + 0.23\,\frac{\sin\left(\frac{\omega N}{2} - \frac{\pi N}{N-1}\right)}{\sin\left(\omega/2 - \pi/N-1\right)}$$

$$+ 0.23\,\frac{\sin\left(\frac{\omega N}{2} + \frac{\pi N}{N-1}\right)}{\sin\left(\frac{\omega}{2} + \frac{\pi}{N-1}\right)}.$$









Frequency Response of LPF approximated using $w_H(n)$.

Characteristics features:

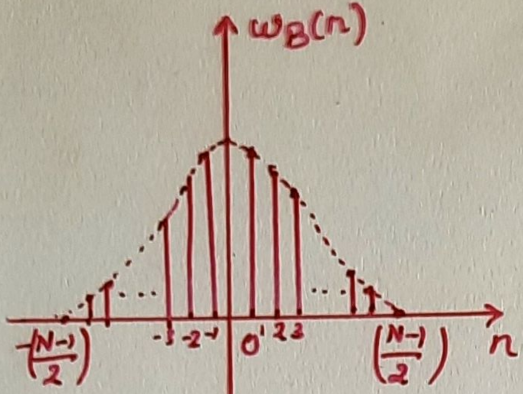
* MLW is $8\pi/N$
* Max. SL Mag is -41dB
* SL. Mag remains constant for ↑ing ω.

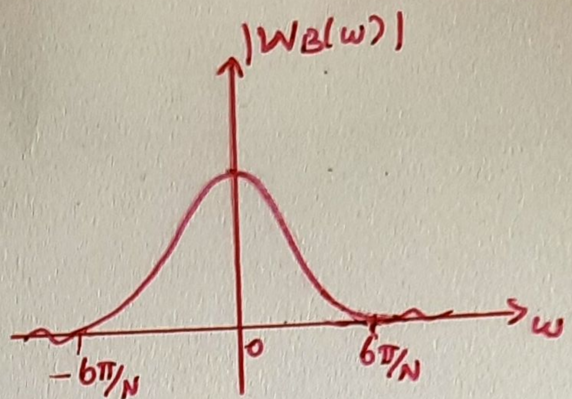
BLACKMAN WINDOW:

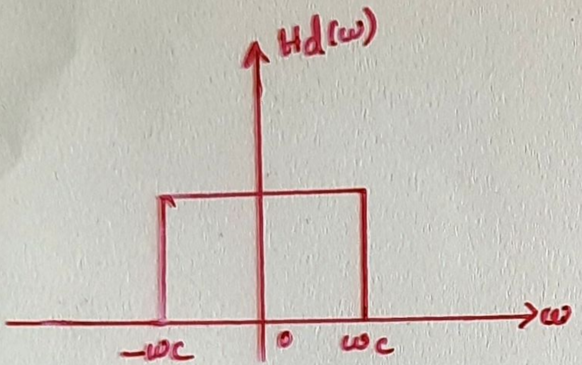
$$w_B(n) = \begin{cases} 0.42 + 0.5\cos\frac{2\pi n}{N-1} + 0.08\cos\frac{4\pi n}{N-1} ; & -\left(\frac{N-1}{2}\right) \le n \le \left(\frac{N-1}{2}\right) \\ 0 & ; \text{otherwise} \end{cases}$$

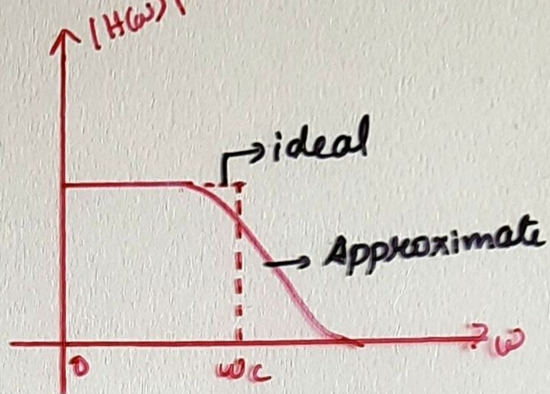
Characteristics feature:

* MLW is $12\pi/N$
* -58dB
* ↓es with ↑ing ω.
* side lobe attenuation is highest among windows, which is achieved at the expense of increased MLW. However, the MLW is reduced by ↑ing the value of N.









KAISER WINDOW:

$$w_K(n) = \begin{cases} \dfrac{I_0(\beta)}{I_0(\alpha)} & ; -\left(\dfrac{N-1}{2}\right) \le n \le \left(\dfrac{N-1}{2}\right) \\ 0 & ; \text{otherwise} \end{cases}$$

where $\beta = \alpha \sqrt{1-\left(\dfrac{2n}{N-1}\right)^2}$

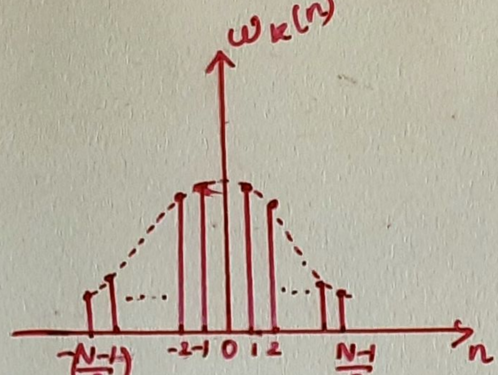
where α is an independent variable that can be varied to control side lobe levels w.r.to the main lobe peak.

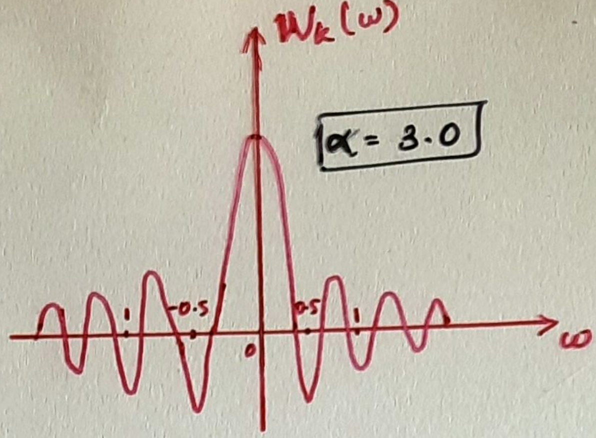
$I_0(x) \rightarrow$ zero order modified Bessel functions of the first kind.

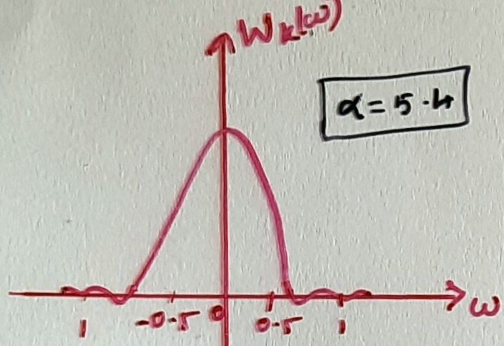
$$I_0(x) = 1 + \sum_{k=1}^{\infty}\left[\frac{1}{k!}\left(\frac{x}{2}\right)^k\right]^2$$

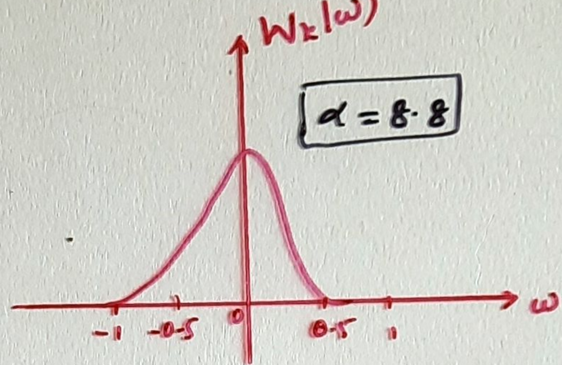
$$= 1 + \sum_{k=1}^{\infty}\frac{\left[(0.5\,x)^2\right]^k}{(k!)^2} = 1 + \sum_{k=1}^{\infty}\frac{(0.25\,x^2)^k}{(k!)^2}$$

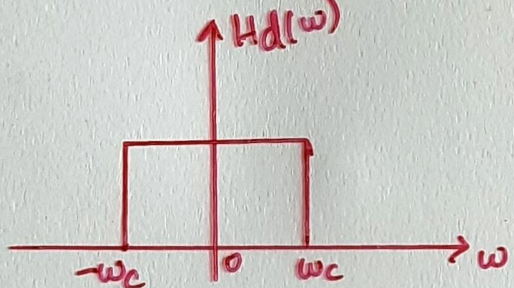
$$I_0(x) = 1 + \frac{0.25x^2}{(1!)^2} + \frac{(0.25x^2)^2}{(2!)^2} + \frac{(0.25x^2)^3}{(3!)^2} + \cdots$$

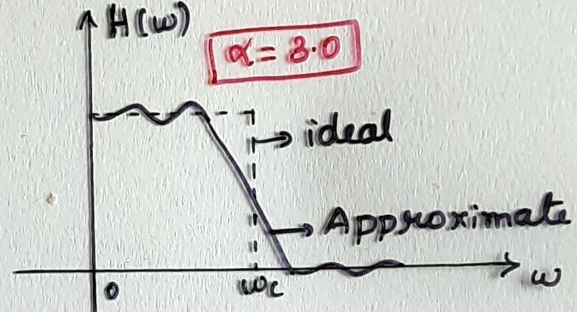


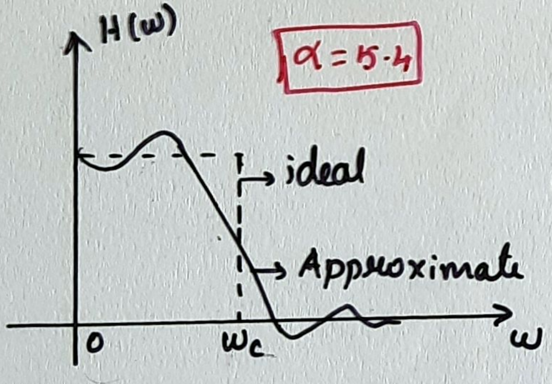


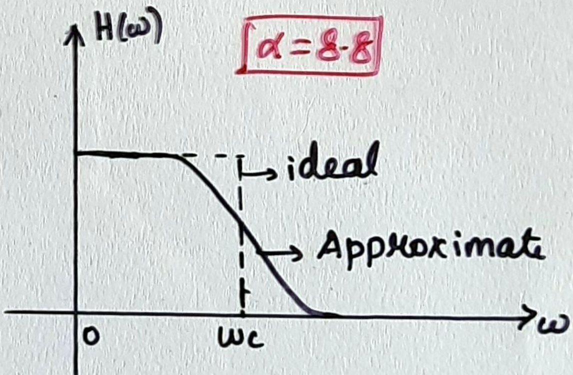












Features of Kaiser widow Spectrum:

* Width of mainlobe & peak side lobe are available depends upon α & N.
* Magnitude of side lobe can be varied by varying α.
* Main lobe width can be varied by varing the length of 'N'.