Power spectral Density of line codes

* Line code may be treated as random process

$$x(t) = \sum_{k=-\infty}^{\infty} A_K \, v(t-KT)$$ where $v(t)$ is the symbol pulse shape and T is the duration of one symbol.

* General expression of PSD of a digital signal is described as

$$S_X(f) = \frac{1}{T} |V(f)|^2 \sum_{n=-\infty}^{\infty} R_A(n) \, e^{-j2\pi f n T}$$

$V(f)$ is the Fourier transform of pulse shaping function $v(t)$ and $R_A(n)$ is the autocorrelation function of the data.

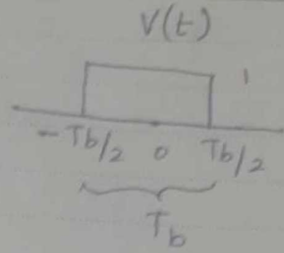
$$R_A(n) = E[A_K \cdot A_{K-n}] = \sum_{i=1}^{I} [A_K \cdot A_{K-n}]_i \, P_i$$

A_K and A_{K-n} are voltage levels of data pulses at kth and $(k-n)$th symbol positions respectively.

P_i is the probability of the ith product $A_K \cdot A_{K-n}$
I is number of possible products $A_K \cdot A_{K-n}$.

2020 Meeting Think To Do

Unipolar NRZ format



$$A_K = \begin{cases} a & \rightarrow \text{for } 1 \\ 0 & \rightarrow \text{for } 0 \end{cases}$$

$$R_A(n) = E[A_K \cdot A_{K-n}]$$

put $n = 0$

$$R_A(0) = E[A_K \cdot A_K] = E[A_K^2]$$

$$E[A_K^2] = \sum A_K^2 \cdot P(A_K)$$

$$R_A(0) = P(A_K = a) \cdot a^2 + P(A_K = 0) \cdot 0^2$$

$$\boxed{R_A(0) = \frac{1}{2} a^2}$$

put $n = 1$ $\quad R_A(1) = E[A_K \cdot A_{K+1}]$

$$= \sum A_K \cdot A_{K+1} \; P(A_K) \cdot P(A_{K+1})$$

A_K	A_{K+1}	$P(A_K)$	$P(A_{K+1})$	$A_K \cdot A_{K+1}$	$P(A_K) \cdot P(A_{K+1})$
0	0	$1/2$	$1/2$	0	$1/4$
0	a	$1/2$	$1/2$	0	$1/4$
a	0	$1/2$	$1/2$	0	$1/4$
a	a	$1/2$	$1/2$	a^2	$1/4$

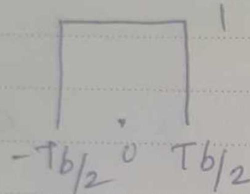
$$R_A(1) = a^2 \times \frac{1}{4} = \frac{a^2}{4}$$

put $n = 2$ $\quad R_A(2) = E[A_k \cdot A_{k+2}] = \frac{a^2}{4}$

for n values from 1 to ∞ $\quad R_A(n) = \frac{a^2}{4}$

$$R_A(n) = \begin{cases} \frac{a^2}{4} & n \neq 0 \\ \frac{a^2}{2} & n = 0 \end{cases}$$

$v(t) =$



$$V(f) = \int_{-T_b/2}^{T_b/2} 1 \cdot e^{-j2\pi f t} \, dt$$

$$V(f) = T_b \operatorname{sinc}(f T_b)$$

PSD, $S_X(f) = \frac{1}{T_b} \cdot T_b^2 \operatorname{sinc}^2(f T_b) \left[\frac{a^2}{2} + \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} \frac{a^2}{4} \exp(-j2\pi f n T_b) \right]$

$$= T_b \operatorname{sinc}^2(f T_b) \left(\frac{a^2}{2} + \frac{a^2}{4} \left[\sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} e^{-j2\pi f n T_b} \right] \right)$$

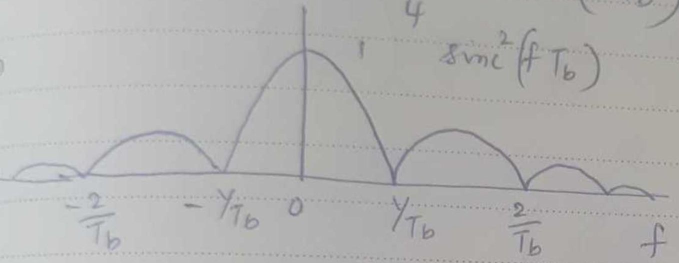
$$= T_b \operatorname{sinc}^2(f T_b) \left(\frac{a^2}{2} + \frac{a^2}{4} \left[\sum_{n=-\infty}^{\infty} e^{-j2\pi f n T_b} - 1 \right] \right)$$

to remove $n \neq 0$

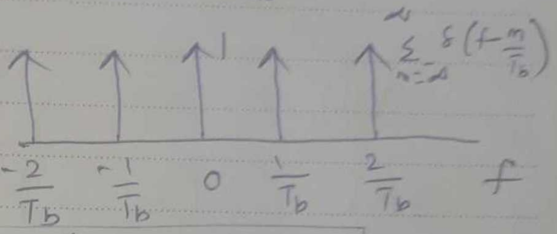
$$= \frac{a^2 T_b}{4} \operatorname{sinc}^2(f T_b) + \frac{a^2}{4} T_b \operatorname{sinc}^2(f T_b) \sum_{n=-\infty}^{\infty} e^{-j2\pi f n T_b}$$

$$\sum_{n=-\infty}^{\infty} e^{-j2\pi f n T_b} = \frac{1}{T_b}\sum_{m=-\infty}^{\infty} \delta\left(f-\frac{m}{T_b}\right)$$

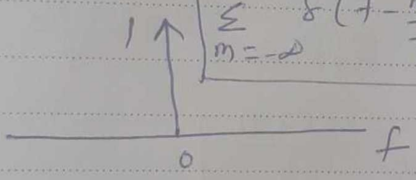
$$\therefore \; S_x(f) = \frac{a^2 T_b}{4}\,\text{sinc}^2(fT_b) + \frac{a^2 T_b}{4}\,\text{sinc}^2(fT_b)\cdot\frac{1}{T_b}\sum_{m=-\infty}^{\infty}\delta\left(f-\frac{m}{T_b}\right)$$



$\times$

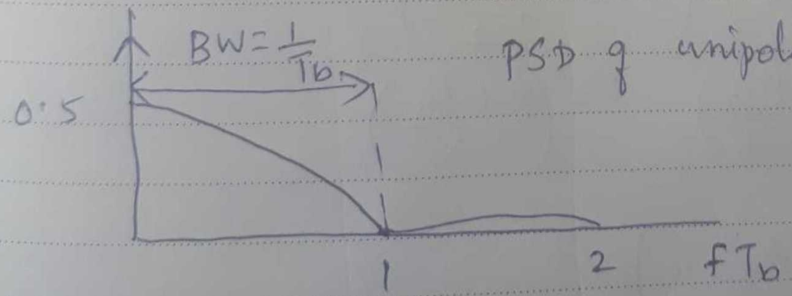


$=$



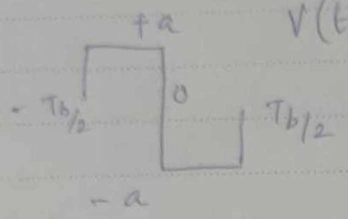
$$\boxed{\sum_{m=-\infty}^{\infty}\delta\left(f-\frac{m}{T_b}\right)\cdot\text{sinc}^2(fT_b) = \delta(f)}$$

$$\boxed{S_x(f) = \frac{a^2 T_b}{4}\,\text{sinc}^2(fT_b) + \frac{a^2}{4}\,\delta(f)}$$



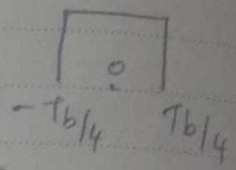
PSD of unipolar NRZ code

Manchester Coding



$v(t)$

Assume $q(t)$ be



$$Q(f) = \frac{T_b}{2} \operatorname{sinc}\left(\frac{f T_b}{2}\right)$$

$v(t)$ in terms of $q(t)$ as

$$v(t) = q\left(t + \frac{T_b}{4}\right) - q\left(t - \frac{T_b}{4}\right)$$

$$V(f) = Q(f)\, e^{\frac{j 2\pi f T_b}{4}} - Q(f) = e^{\frac{-j 2\pi f T_b}{4}}$$

$$= Q(f) \left[e^{\frac{j 2\pi f T_b}{4}} - e^{\frac{-j 2\pi f T_b}{4}} \right]$$

$$V(f) = \frac{T_b}{2} \operatorname{sinc} \frac{f T_b}{2} \left[2j \sin \frac{2\pi f T_b}{4} \right)$$

$$= T_b \operatorname{sinc}\left(\frac{f T_b}{2}\right) \left(j \sin \frac{\pi f T_b}{2}\right)$$

$$|V(f)|^2 = T_b^2 \operatorname{sinc}^2\left(\frac{f T_b}{2}\right) \sin^2\left(\frac{\pi f T_b}{2}\right)$$

$$R_A(n) = E[A_k \cdot A_{k+n}]$$

put $n = 0$ $\quad R_A(0) = E[A_k^2] = \sum A_k^2\, p(A_k)$

$= P(A_k = -a) \cdot a^2 + P(A_k = a) \cdot a^2$

$= \frac{1}{2} a^2 + \frac{1}{2} a^2 = \frac{2a^2}{2} = \underline{\underline{a^2}}$

put $n=1$ $\quad R_A(1) = E[A_k \cdot A_{k+1}]$

$= \Sigma A_k \cdot A_{k+1} \; P(A_k) \, P(A_{k+1})$

A_k	A_{k+1}	$P(A_k)$	$P(A_{k+1})$	$A_k \cdot A_{k+1}$	$P(A_k) \cdot P(A_{k+1})$
$+a$	$+a$	$1/2$	$1/2$	a^2	$1/4$
$+a$	$-a$	$1/2$	$1/2$	$-a^2$	$1/4$
$-a$	$+a$	$1/2$	$1/2$	$-a^2$	$1/4$
$-a$	$-a$	$1/2$	$1/2$	a^2	$1/4$

$= \frac{a^2}{4} - \frac{a^2}{4} - \frac{a^2}{4} + \frac{a^2}{4} = 0$

$$R_A(n) = \begin{cases} a^2, & n = 0 \\ 0, & \text{elsewhere} \end{cases}$$

Power spectral Density

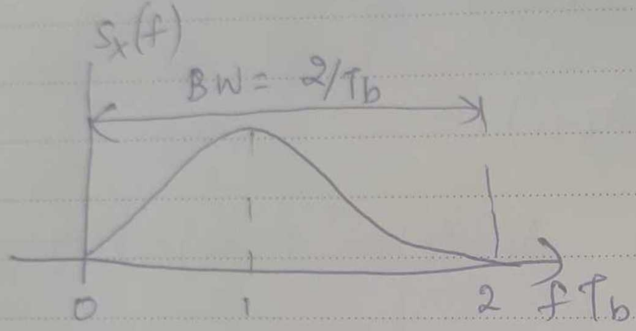
$$S_{xx}(f) = \frac{1}{T_b} \sum_{n=-\infty}^{\infty} R_A(n) \exp(-j2\pi f n T_b) \, |V(f)|^2$$

$$= \frac{a^2}{T_b} T_b^2 \, \text{sinc}^2\left(\frac{fT_b}{2}\right) \sin^2 \frac{f\pi T_b}{2}$$

$$= a^2 T_b \, \text{sinc}^2\left(\frac{fT_b}{2}\right) \sin^2\left(\frac{\pi f T_b}{2}\right)$$

$$\frac{\pi f T_b}{2} = \pi$$

$$\boxed{f = \frac{2}{T_b}}$$

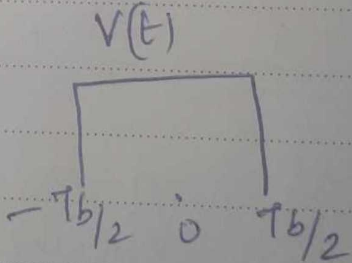


PSD of Manchester code

Bipolar NRZ scheme

A_k = '0' — 0

'1' $\begin{cases} +a \\ -a \end{cases}$



$$V(f) = T_b \, \text{sinc}(fT_b) \Rightarrow |V(f)|^2 = T_b^2 \, \text{sinc}^2 fT_b$$

$$R_A(n) = E[A_k \cdot A_{k+n}]$$

put $n=0$ $\quad R_A(0) = E[A_k^2]$

$$R_A(0) = P(A_k=0)\cdot A_k^2 + P(A_k=+a)\cdot A_k^2 + P(A_k=-a)\cdot A_k^2$$

$$= \frac{1}{2}(0) + \frac{1}{4}a^2 + \frac{1}{4}a^2$$

$$\boxed{R_A(0) = \frac{2}{4}a^2 = \frac{a^2}{2}}$$

put $n=1$ $\quad R_A(1) = E[A_k \cdot A_{k+1}]$

$$\boxed{\begin{array}{l} P(A\cdot B) = P(A)\cdot P(B/A) \\ P\left(\dfrac{A_{k+1}=-a}{A_k=+a}\right) = 1/2 \end{array}}$$

A_k	A_{k+1}	$P(A_k)$	$P(A_{k+1}) = P(A_{k+1}/A_k)$	$P(A_k)\cdot P(A_{k+1}/A_k)$	$A_k \cdot A_{k+1}$
0	a	$1/2$	$1/4$	$1/8$	0
0	$-a$	$1/2$	$1/2$	$1/8$	0
0	0	$1/2$	$1/2$	$1/4$	0
a	0	$1/4$	$1/2$	$1/8$	0
a	$-a$	$1/4$	$1/2$	$1/8$	$-a^2$
$-a$	0	$1/4$	$1/2$	$1/8$	0
$-a$	$+a$	$1/4$	$1/2$	$1/8$	$-a^2$

$$R_A(1) = 0 - \frac{a^2}{8} - \frac{a^2}{8} = -\frac{a^2}{4}$$

put $n = 2$ $\quad R_A(2) = 0$

$$R_A(n) = \begin{cases} \frac{a^2}{2}, & n = 0 \\ -\frac{a^2}{4}, & n = 1 \text{ and } -1 \\ 0 & \text{for } n \neq 0, 1 \end{cases}$$

$$\boxed{R_A(n) = R_A(-n)}$$

$$S_X(f) = \frac{1}{T_b} \sum_{n=-\infty}^{\infty} R_A(n) \exp(-j2\pi f T_b n) \, |V(f)|^2$$

$$= \frac{1}{T_b} T_b^2 \operatorname{sinc}^2(fT_b) \left[R_A(-1) e^{j2\pi f T_b} + R_A(0) + R_A(1) e^{-j2\pi f T_b} \right]$$

$$= T_b \operatorname{sinc}^2(fT_b) \left[2 R_A(1) \cos 2\pi f T_b + R_A(0) \right]$$

$$= T_b \operatorname{sinc}^2(fT_b) \left[2 \cdot \left(-\frac{a^2}{4}\right) \cos 2\pi f T_b + \frac{a^2}{2} \right]$$

$$= \frac{a^2}{2} T_b \operatorname{sinc}^2(fT_b) \left[1 - \cos 2\pi f T_b \right]$$

$$= \frac{a^2 T_b}{2} \operatorname{sinc}^2(fT_b) \left(2 \sin^2 \pi f T_b \right)$$

$$S_x(f) = a^2 T_b \, \mathrm{sinc}^2(f T_b) \, \sin^2(\pi f T_b)$$