13/08

Transmission line theory and parameters

Transmission Line:

A medium for propagating or guiding energy from ~~po~~ one point to another.

why propagation of energy:

→ Power transfer

→ Information transfer

KHz range → pair of wire

why twisted pair? → Reduce interference

Optical → 10^{12} Hz (Higher freq)

$$T > t$$

$T \rightarrow$ Time period

$t \rightarrow$ Transit time

$$\frac{1}{f} > \frac{l}{V}$$

$$\frac{V}{f} > l$$

$$\lambda = \frac{v}{f} > l$$

If $\lambda < l$, transit effect cannot be ignored.

Lumped and Distributive elt

Lumped → can be physically realised

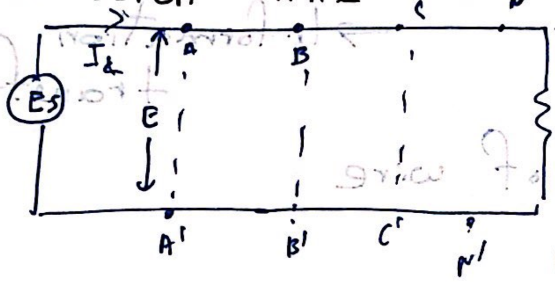
Distributive elt → cannot be seen

$Z_0 \rightarrow$ char impedance

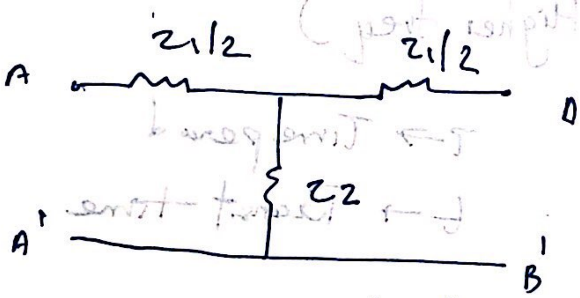
Z_0 = High freq $\rightarrow$ Highly R

Loss (dB/meter) is min at $\underline{\underline{77\,\Omega}}$
(Cu wires)
Power is max at $30\,\Omega$
$50\,\Omega$ is a $\underline{\underline{\text{compromise}}}$.

Transmission line as cascaded T section



we divide transmission line into uniform T sections



series imp $= Z_1$
shunt imp $= Z_2$

- AB, BC ... $\rightarrow$ identical no. of section
- Es $\rightarrow$ voltage applied at transmitted end and terminated at Z_L ($\underline{Z_0}$)
- Due to distributed elts, there will be phasor diff

$$\frac{I_S}{I_L} = e^{\gamma} \quad (\gamma \rightarrow \text{Propagation constant per unit length})$$

Primary constants of transmission line

- Primary constants ~~Resistance~~ $\rightarrow$ R, L, G and C
- secondary constants $\rightarrow$ $Z_0 = \sqrt{\frac{\text{series imp}}{\text{Shunt imp}}} = \sqrt{\frac{R + j\omega_0 L}{G + j\omega_0 C}}$
- Propagation constant $\rightarrow$ Propagation constant (γ or T)

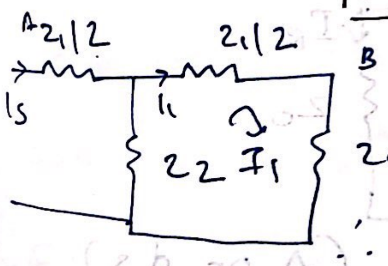
$$\gamma = \sqrt{\text{series imp} \times \text{shunt imp}}$$

[series impedance is denoted by 'Z' and shunt impedance by 'Y']

γ (or) $P = \alpha + j\beta$ $\quad$ $\alpha \rightarrow$ attenuation constant

$\beta \rightarrow$ ~~Propagation~~ phase constant

In general, $\boxed{\frac{I_{n-1}}{I_n} = e^{\gamma}}$ $\rightarrow$ (1) $\quad$ n = no. of subsections



$$I_1 = I_s \cdot \frac{Z_2}{Z_1/2 + Z_2 + Z_0}$$

$$\therefore \frac{I_s}{I_1} = 1 + \frac{Z_1}{2Z_2} + \frac{Z_0}{Z_2} \rightarrow (2)$$

From (1) and (2) $\quad \frac{I_s}{I_1} = e^{\gamma}$

$$e^{\gamma} = 1 + \frac{Z_1}{2Z_2} + \frac{Z_0}{Z_2}$$

$$\Rightarrow \boxed{\gamma = \ln\left[1 + \frac{Z_1}{2Z_2} + \frac{Z_0}{Z_2}\right]}$$

Also $\frac{I_S}{I_1} = e^{\gamma}$

$$\frac{I_S}{I_2} = \frac{I_S}{I_1} \cdot \frac{I_1}{I_2} = e^{\gamma} \cdot e^{\gamma} = e^{2\gamma}$$

$\frac{I_S}{I_3} = e^{3\gamma}$. Hence in general

$$\boxed{\frac{I_S}{I_N} = e^{N\gamma}}$$

$$\boxed{I_n = I_S e^{-n\gamma}}$$

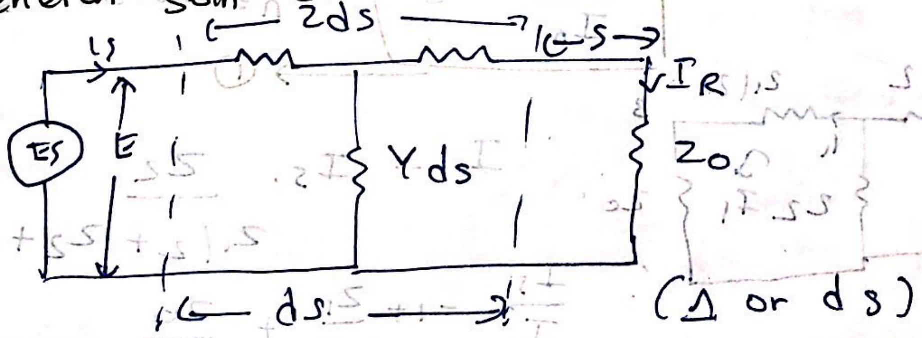
Illy $\frac{E_S}{I_S} = Z_0 = \frac{E_1}{I_1} = \frac{E_2}{I_2} \dots = \frac{E_n}{I_n}$

$$\frac{E_S}{I_S} = \frac{E_N}{I_N}$$

$$\frac{E_S}{E_N} = \frac{I_S}{I_N} = e^{N\gamma}$$

$$\boxed{E_N = E_S e^{-N\gamma}}$$

General soln of transmission line



General soln gives the variation of I and V, I along the transmission line

length $l \rightarrow$ infinite T sections

$s \rightarrow$ Distance from receiving end ds $\rightarrow$ length of sections

series imp : $z\,ds$ shunt imp : $Y\,ds$

Elemental voltage drop in length ds

$$dE = I \cdot z\,ds$$

$$\frac{dE}{ds} = I \cdot Z \longrightarrow (1)$$

Leakage current flowing through shunt admittance from one conductor to other,

$$dI = E \cdot Y\,ds$$

$$\frac{dI}{ds} = E \cdot Y \longrightarrow (2)$$

Diff (1) and (2) w.r.t 's'

$$\frac{d^2E}{ds^2} = Z \cdot \frac{dI}{ds} \longrightarrow (3)$$

$$\frac{d^2I}{ds^2} = \frac{dE}{ds} \cdot Y \longrightarrow (4)$$

sub (2) in (3) $\frac{d^2E}{ds^2} = Z \cdot E \cdot Y \longrightarrow (5)$

sub (1) in (4) $\frac{d^2I}{ds^2} = I \cdot Z \cdot Y \longrightarrow (6)$

(5) and (6) are second order diff eqns:

$$\frac{d^2E}{ds^2} - ZEY = 0$$

$$(m^2 - ZY)E = 0 \Rightarrow \boxed{m = \pm\sqrt{ZY}}$$

$$\frac{d^2I}{ds^2} - ZIY = 0$$

$$(m^2 - ZY)I = 0 \qquad \boxed{m = \pm\sqrt{ZY}}$$

General soln for eqn for E and I :

$$E = Ae^{(\sqrt{ZY})s} + Be^{(-\sqrt{ZY})s}$$

$$I = Ce^{(\sqrt{ZY})s} + De^{(-\sqrt{ZY})s}$$

$$\boxed{\begin{aligned} E &= Ae^{\gamma s} + Be^{-\gamma s} \quad \longrightarrow (7) \\ I &= Ce^{\gamma s} + De^{-\gamma s} \quad \longrightarrow (8) \end{aligned}}$$

A, B, C, D are constants, we must find their values

At the receiving end $E = E_R$ $I = I_R$ $s = 0$

$$E_R = Ae^0 + Be^0 = A + B \longrightarrow (9)$$

$$I_R = Ce^0 + De^0 = C + D \longrightarrow (10)$$

Again differentiating (7) and (8) w.r.t 's' :

$$\frac{dE}{ds} = \gamma Ae^{\gamma s} - \gamma Be^{-\gamma s}$$

$$IZ = \gamma Ae^{\gamma s} - \gamma Be^{-\gamma s}$$

$$\frac{dI}{ds} \Rightarrow I = \frac{A\sqrt{ZY}}{Z} e^{\gamma s} - \frac{B\sqrt{ZY}}{Z} e^{-\gamma s}$$

$$\boxed{I = A\sqrt{\frac{Y}{Z}}\, e^{\gamma s} - B\sqrt{\frac{Y}{Z}}\, e^{-\gamma s}} \longrightarrow (11)$$

$$\frac{dI}{ds} = C\gamma e^{\gamma s} - D\gamma e^{-\gamma s}$$

$$EY = C\sqrt{ZY}\, e^{\gamma s} - D\sqrt{ZY}\, e^{-\gamma s}$$

Scanned with CamScanner

$$\boxed{E = C\sqrt{\frac{Z}{Y}}\, e^{rs} - D\sqrt{\frac{Z}{Y}}\, e^{-rs}} \quad \text{(12)}$$

substituting boundary conditions in (11) and (12)

$$I_R = A\sqrt{\frac{Y}{Z}} - B\sqrt{\frac{Y}{Z}} \quad \text{(13)}$$

$$E_R = C\sqrt{\frac{Z}{Y}} - D\sqrt{\frac{Z}{Y}} \quad \text{(14)}$$

solving (9) and (14)

$$E_R = A + B$$

$$I_R = A\sqrt{\frac{Y}{Z}} - B\sqrt{\frac{Y}{Z}}$$

$$E_R\sqrt{\frac{Y}{Z}} = A\sqrt{\frac{Y}{Z}} + B\sqrt{\frac{Y}{Z}}$$

(2) ~~$I_R = A\sqrt{\frac{Y}{Z}} - B\sqrt{\frac{Y}{Z}}$~~

$$A = \frac{E_R}{2} + \frac{I_R}{2}\sqrt{\frac{Z}{Y}}$$

$B = E$

Scanned with CamScanner

$$\boxed{A = \frac{E_R}{2}\left[1+\frac{Z_0}{Z_R}\right]} \qquad \boxed{B = \frac{E_R}{2}\left[1-\frac{Z_0}{Z_R}\right]}$$

$$\boxed{C = \frac{I_R}{2}\left[1+\frac{Z_R}{Z_0}\right]} \qquad \boxed{D = \frac{I_R}{2}\left[1-\frac{Z_R}{Z_0}\right]}$$

$\therefore$

$$E = \frac{E_R}{2}\left[1+\frac{Z_0}{Z_R}\right]e^{\gamma s} + \frac{E_R}{2}\left[1-\frac{Z_0}{Z_R}\right]e^{-\gamma s}$$

$$I = \frac{I_R}{2}\left[1+\frac{Z_R}{Z_0}\right]e^{\gamma s} + \frac{I_R}{2}\left[1-\frac{Z_R}{Z_0}\right]e^{-\gamma s}$$

$$E = \frac{E_R(Z_R+Z_0)}{2Z_R}\left[e^{\gamma s} + \frac{(Z_R-Z_0)}{(Z_R+Z_0)}e^{-\gamma s}\right] \quad (15)$$

$$I = \frac{I_R(Z_R+Z_0)}{2Z_0}\left[e^{\gamma s}\right.$$

$$\boxed{E = E_R \cosh \gamma s + I_R Z_0 \sinh \gamma s} \rightarrow (17)$$

$$I = \frac{I_R Z_R e^{\gamma s}}{2Z_0} + \frac{I_R Z_0 e^{\gamma s}}{2Z_0} - \frac{I_R Z_R e^{-\gamma s}}{2Z_0} + \frac{I_R Z_0 e^{-\gamma s}}{2Z_0}$$

$$\boxed{I = I_R \cosh \gamma s + \frac{E_R}{Z_0} \sinh \gamma s} \rightarrow (18)$$

(15), (16), (17), (18) $\rightarrow$ General soln of TL

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Physical significance of the eqn:

$s = l$

Case 1: Sending in current

$l \rightarrow$ total length of transmission line

$$I_S = I_R \cosh \gamma l + \frac{E_R}{Z_0} \sinh \gamma l$$

$$= I_R \left[\cosh \gamma l + \frac{Z_R}{Z_0} \sinh \gamma l \right]$$

Case 2: Sending end voltage

$$E_S = E_R \cosh \gamma l + I_R Z_0 \sinh \gamma l$$

$$E_S = E_R \left[\cosh \gamma l + \frac{Z_0}{Z_R} \sinh \gamma l \right]$$

$$E_S = I_R \left[Z_R \cosh \gamma l + Z_0 \sinh \gamma l \right]$$

Case 3: i/p imp $Z_S = \frac{E_S}{I_S}$

$$Z_S = \frac{I_R [Z_R \cosh \gamma l + Z_0 \sinh \gamma l]}{I_R [\cosh \gamma l + \frac{Z_R}{Z_0} \sinh \gamma l]}$$

$$Z_S = \frac{Z_0 [Z_R \cosh \gamma l + Z_0 \sinh \gamma l]}{Z_0 (\cosh \gamma l + Z_R \sinh \gamma l)}$$

Note 1: when line is terminated in Z_0,

$Z_R = Z_0$

$$\boxed{Z_S = Z_0}$$

It shows that line terminated in characteristic impedance, its i/p impedance is also its characteristic impedance.

Note 2:

consider an infinite length, $l \to \infty$

$$Z_S = \lim_{l \to \infty} Z_0 \left[\frac{Z_R + Z_0 \tanh \gamma l}{Z_0 + Z_R \tanh \gamma l} \right]$$

$$= \frac{Z_0 [Z_R + Z_0]}{(Z_0 + Z_R]}$$

$$\boxed{Z_S = Z_0}$$

Finite line terminated in its characteristic impedance behaves like an infinite line to the sending end.

Wavelength and velocity of propagation

The distance the wave travels along the line when phase angle is changing through 2π radians is called wavelength

$\lambda \to$ wave length $\beta \to$ phase constant

$s = \lambda$ $\quad \beta\lambda = 2\pi$ $\quad \boxed{\lambda = \frac{2\pi}{\beta}}$

velocity of propagation along the line, based on observations of change in phase along the line

$$\boxed{v = \frac{\omega}{\beta}} \qquad \omega = 2\pi f$$

condition for distortionless line:

Propagation constant

$$\gamma = \alpha + j\beta = \sqrt{ZY}$$

series imp ~~(R+jω~~

$$= \sqrt{(R+j\omega L)(G+j\omega C)}$$

Ideal line: $R = 0$ and $G = 0$

$$\gamma = \alpha + j\beta = \sqrt{j^2\omega^2 LC}$$

$$\alpha + j\beta = j\omega\sqrt{LC}$$

$$\boxed{\beta = \omega\sqrt{LC}}$$

$$\boxed{\alpha = 0}$$

condition for minimum attenuation

d

$$\boxed{RC = LG}$$

$$CR = LG$$

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condition for minimum attenuation:

$$\gamma = \alpha + j\beta$$

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$$

$$(\alpha+j\beta)^2 = (R+j\omega L)(G+j\omega C)$$

$$\alpha^2 - \beta^2 + 2j\alpha\beta = RG - \omega^2 LC + j\omega RC + j\omega GL$$

Equating real and imaginary terms

$$\alpha^2 - \beta^2 = RG - \omega^2 LC \qquad 2\alpha\beta = \omega(RC+GL)$$

$$\alpha^2 = \beta^2 + RG - \omega^2 LC \qquad \alpha\beta = \frac{\omega}{2}(RC+GL)$$

$$\alpha^2\beta^2 = \beta^4 + RG\beta^2 - \omega^2 LC\beta^2 \qquad \alpha^2\beta^2 = \frac{\omega^2}{2^2}(RC+GL)^2$$

$$\beta^4 + RG\beta^2 - \omega^2 LC\beta^2 = \frac{\omega^2}{2^2}(RC+GL)^2$$

$$\beta^4 + \beta^2(RG - \omega^2 LC) - \frac{\omega^2}{4}(RC+GL)^2 = 0$$

$$\beta^2 = \frac{-(RG-\omega^2 LC) \pm \sqrt{(RG-\omega^2 LC)^2 + \omega^2(RC+GL)^2}}{2}$$

Neglect -ve term

$$\beta = \sqrt{\frac{(\omega^2 LC - RG) + \sqrt{(RG-\omega^2 LC)^2 + \omega^2(RC+GL)^2}}{2}}$$

$$\alpha = \sqrt{\frac{(RG - \omega^2 LC) + \sqrt{(RG-\omega^2 LC)^2 + \omega^2(RC+GL)^2}}{2}}$$

If a line is to have neither frequency nor delay distortion α and v cannot be functions of frequency

$$v = \frac{\omega}{\beta}$$

β must be a factor frequency

$\therefore$ $(RG-\omega^2LC)^2 + \omega^2(RC+GL)^2$ must be equal to $(RG+\omega^2LC)^2$

$$R^2G^2 + \omega^4L^2C^2 - 2\omega^2LCRG + \omega^2L^2G^2 + \omega^2C^2R^2 + 2\omega^2LCRG = R^2G^2 + \omega^4L^2C^2 + 2\omega^2LCRG$$

$$\omega^2L^2G^2 + \omega^2C^2R^2 = 2\omega^2LCRG$$

$$\omega^2L^2G^2 + \omega^2C^2R^2 - 2\omega^2LCRG = 0$$

$$(LG - CR)^2 = 0$$

$$\boxed{LG = CR}$$

$$\frac{R}{L} = \frac{G}{C}$$

I/P impedance interms of Z_{SC} and Z_{OC}

Z_{OC} $\infty \Rightarrow I_R = 0, \quad Z_R = \infty$

Z_{SC} $\Rightarrow E_R = 0, Z_R = 0$

$$Z_S = \frac{Z_0(Z_R \cosh \gamma l + Z_0 \sinh \gamma l)}{Z_0 \cosh \gamma l + Z_R \sinh \gamma l}$$

Z_{SC}, Z_{OC}

Z_{OC} $\Rightarrow I_R = 0, \quad Z_R = \infty$

Z_{SC} $E_R = 0, \quad Z_R = 0$

$$Z_{OC} = \lim_{Z_R \to \infty} \frac{Z_0 Z_R (\cosh \gamma l + \frac{Z_0}{Z_R} \sinh \gamma l)}{Z_R (\frac{Z_0}{Z_R} \cosh \gamma l + \sinh \gamma l)}$$

$$= \lim_{Z_R \to \infty} \frac{Z_0 (1 + \frac{Z_0}{Z_R} \tanh \gamma l)}{\frac{Z_0}{Z_R} + \tanh \gamma l}$$

$$= \frac{Z_0}{\tanh \gamma l}$$

$$\boxed{Z_{OC} = Z_0 \coth \gamma l}$$

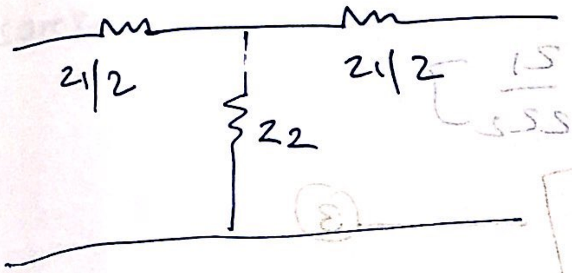
$$Z_s \; (Z_{in}) = \frac{Z_0 (Z_R \cosh\gamma l + Z_0 \sinh\gamma l)}{Z_0 \cosh\gamma l + Z_R \sinh\gamma l} \quad \text{— (1)}$$

$Z_R = 0$

$$Z_{sc} = \frac{Z_0^2 \sinh\gamma l}{Z_0 \cosh\gamma l} = Z_0 \tanh\gamma l \quad \text{— (3)}$$

$$\boxed{Z_{sc} Z_{oc} = Z_0^2} \qquad \boxed{Z_0 = \sqrt{Z_{sc} Z_{oc}}}$$

Propagation constant interms of Z_{sc} and Z_{oc}



$$e^{\gamma} = 1 + \frac{Z_1}{2Z_2} + \frac{Z_0}{Z_2} \quad \text{— (1)}$$

$$e^{-\gamma} = \left[1 + \frac{Z_1}{2Z_2} + \frac{Z_0}{Z_2}\right]^{-1}$$

we know that for a T section

$$Z_0 = \sqrt{\frac{Z_1^2}{4} + Z_1 Z_2}$$

$$e^{-\gamma} = \left[1 + \frac{Z_1}{2Z_2} + \sqrt{\frac{Z_1^2}{4Z_2^2} + \frac{Z_1}{Z_2}}\right]^{-1} \qquad \sqrt{\frac{Z_1^2}{4Z_2^2} + \frac{Z_1 Z_2}{Z_2^2}}$$

$$= \left[1 + \frac{Z_1}{2Z_2} + \sqrt{\frac{Z_1^2 + 4Z_1 Z_2}{4Z_2^2}}\right]^{-1}$$

By using binomial series and eliminating higher order terms

$$= 1 - \frac{z_1}{2z_2} - \sqrt{\frac{z_1^2}{4z_2^2} + \frac{z_1}{z_2} + \frac{z_1^2}{4z_2^2} + \frac{z_1^2}{4z_2^2} + \frac{z_1}{z_2}}$$

$$e^{-\gamma} = 1 + \frac{z_1}{2z_2} - \frac{z_0}{z_2} \quad \text{——} (2)$$

(1) + (2)

$$e^{\gamma} + e^{-\gamma} = \left[1 + \frac{z_1}{2z_2} + \frac{z_0}{z_2}\right] + 1 + \frac{z_1}{2z_2} - \frac{z_0}{z_2}$$

$$\cancel{2}\cosh\gamma = \cancel{2}\left[1 + \frac{z_1}{2z_2}\right]$$

$$\boxed{\cosh\gamma = 1 + \frac{z_1}{2z_2}} \quad \text{——} (3)$$

$$e^{\gamma} - e^{-\gamma} = \frac{2z_0}{z_2}$$

$$\boxed{\sinh\gamma = \frac{z_0}{z_2}} \quad \text{——} (4)$$

(4) ÷ (3) $\tanh\gamma = \frac{z_0}{z_2} \times \frac{2z_2}{2z_2 + z_1}$

$$\tanh\gamma = \frac{z_0}{z_2 + z_1/2}$$

$$\tanh\gamma = \frac{\sqrt{z_{oc} \cdot z_{sc}}}{z_{oc}}$$

$$\boxed{\tanh \gamma = \sqrt{\frac{Z_{sc}}{Z_{oc}}}}$$

- variation of char impedence

- ~~when there is variation~~ when freq changes, each ~~different~~ ~~vari~~ component will undergo different attenuation

- $\beta \rightarrow$ phase distortion

  when all freq does not arrive at the same time

$$\beta = \omega\sqrt{LC}$$

$$\nu = \frac{\omega}{\beta}$$

$$\nu = \frac{\omega}{\omega\sqrt{LC}} = \frac{1}{\sqrt{LC}}$$

$$\alpha = \sqrt{RG}$$

$$Z_0 = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$$

Since,

$$LG = CR$$

$$Z_0 = \sqrt{\frac{L}{C}}$$

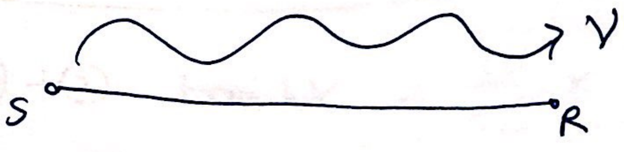
No freq component is present in α →

Uniform velocity velocity → No phase distortion

~~Absence~~ Reflection coefficient

→ Ratio of voltage of reflected wave to that of incident wave

$$E = \underbrace{\frac{E_R}{2}\left[1+\frac{Z_0}{Z_R}\right]e^{\gamma s}}_{\text{Incident wave}} + \underbrace{\frac{E_R}{2}\left[1-\frac{Z_0}{Z_R}\right]e^{-\gamma s}}_{\text{Reflected wave}}$$



Reflection coefficient k

$$= \frac{E_R/2\left[1-\frac{Z_0}{Z_R}\right]e^{-\gamma s}}{E_R/2\left[1+\frac{Z_0}{Z_R}\right]e^{\gamma s}}$$

Reflection occurs at $s=0$ (reflection end)

$$k = \frac{Z_R - Z_0}{Z_R + Z_0}$$

<u>Cases:</u>

1st case:

$\cancel{Z_R = Z_0}$

$\boxed{\cancel{k=0}}$

1st case

$Z_0 = Z_R$

$\cancel{k=0}$

<u>Cases:</u>

2nd case

Case 1: when $Z_R = Z_0$

$k=0$

Case 2: when $Z_R = 0$ (S.C)

$k=-1$

case 3: when $Z_R = \infty$

$k=1$

∴ The reflection will be more when the line is <u>O.C</u>

1109 I/P impedance in term of reflection coefficient

$$k = \frac{Z_R - Z_0}{Z_R + Z_0}$$

$$Z_S = \frac{E}{I} = \frac{\frac{E_R}{2Z_R}(Z_R+Z_0)e^{\gamma l} + \frac{E_R}{2Z_R}(Z_R-Z_0)e^{-\gamma l}}{\frac{I_R}{2Z_0}[Z_R+Z_0]e^{\gamma l} - \frac{I_R}{2Z_0}(Z_R-Z_0)e^{-\gamma l}}$$

$$= \frac{\frac{E_R}{2Z_R}(Z_R+Z_0)e^{\gamma l} + \frac{E_R}{2Z_R}k(Z_R+Z_0)e^{-\gamma l}}{\frac{I_R}{2Z_0}(Z_R+Z_0)e^{\gamma l} - \frac{I_R}{2Z_0}k(Z_R+Z_0)e^{-\gamma l}}$$

$$= Z_0 \frac{I_R(Z_R+Z_0)e^{\gamma l} + I_R k(Z_R+Z_0)e^{-\gamma l}}{I_R(Z_R+Z_0)e^{\gamma l} - I_R k(Z_R+Z_0)e^{-\gamma l}}$$

$$\boxed{Z_S = \frac{Z_0[e^{\gamma l} + ke^{-\gamma l}]}{e^{\gamma l} - ke^{-\gamma l}}}$$ (final eqn)

Reflection loss

Defined as no. of ~~net~~ nepers or ~~of~~ decibels by which ~~the~~ current in load under matched condition would exceed the current actually flowing in the load

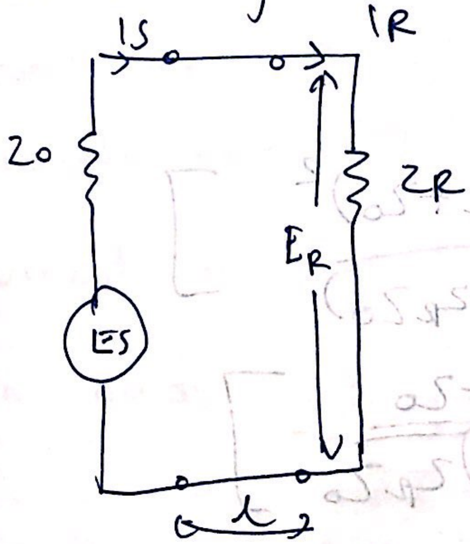
Reflection loss $= \ln \left| \frac{I_2'}{I_2} \right|$ neper

(K) $= 20 \log \left| \frac{I_2'}{I_2} \right|$ dB

Reflection factor

$$k = \frac{2\sqrt{Z_R Z_0}}{Z_R + Z_0}$$

This ratio indicates the change in current in the load due to reflection at a mismatched junction



Let P_1 be the power at receiving end due to the incident wave

$P_2 \to$ power absorbed by the load

$P_3 \to$ Power reflected down the line

$$P_1 = P_2 + P_3$$

$P \propto I^2$

$I \propto (P)^{1/2}$

Reflection Loss: $P_1 = E_R I_R$

$P_3 = |k| E_R \cdot |k| I_R$

$P_3 = |k^2| E_R I_R$

$= |k^2| P_1$

$P_2 = P_1 - P_3$

$P_2 = P_1 (1 - |k|^2)$

Reflection loss $= 10 \log \left[\frac{P_1}{P_2} \right] dB$

$= 10 \log \left[\frac{1}{1 - |k|^2} \right] dB$

w.k.t: $k = \frac{Z_R - Z_0}{Z_R + Z_0}$

final eqn:

$R.L = 10 \log \left[\frac{(Z_R + Z_0)^2}{4 (Z_R Z_0)} \right]$

$= 20 \log \left[\frac{Z_R + Z_0}{2 \sqrt{Z_R Z_0}} \right]$

$$\boxed{R.L = 20 \log \left[\frac{1}{|k|} \right]}$$

Return loss:

Ratio of power at the receiving end due to incident wave to the power reflected by the load.

$$\text{Return loss} = 10 \log \left| \frac{P_1}{P_3} \right| dB$$

$$= 10 \log \left| \frac{P_1}{K^2 P_1} \right| dB$$

$$= \cancel{10 \log \left| \frac{1}{K^2} \right|} dB$$

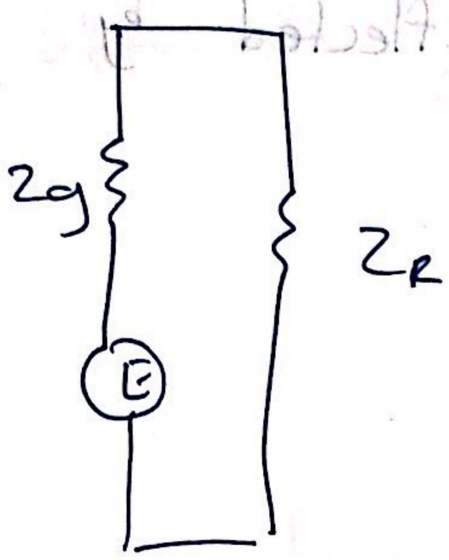
$$= 10 \log \left| \frac{1}{K^2} \right| dB$$

$$= 20 \log \left| \frac{1}{K} \right| dB$$

$$\text{Return loss} = 20 \log \left[\frac{Z_R + Z_0}{Z_R - Z_0} \right] dB$$

Insertion loss:

In order to achieve matched conditions, a phase shifter or a transformer is introduced. Ratio of current in load without any insertion in load current at load w/o insertion

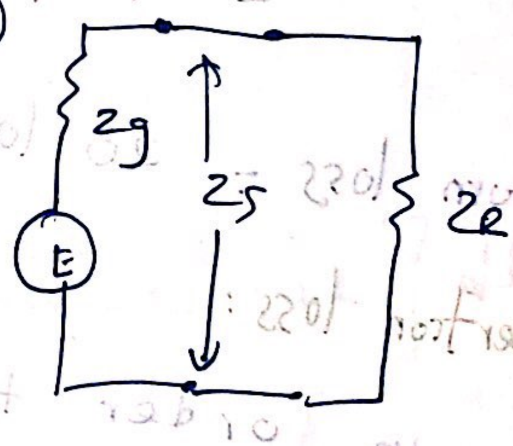


$$I = \frac{E}{Z_g + Z_R} \longrightarrow (1)$$

current at load with insertion:-

sending end current is given by

$$I_s = \frac{E}{Z_g + Z_s} \quad (Z_g \mp Z_s) \longrightarrow (2)$$



$$Z_s = \frac{Z_0(e^{\gamma l} + ke^{-\gamma l})}{e^{\gamma l} - ke^{-\gamma l}} \longrightarrow (3)$$

$$I_s = \frac{E}{Z_g + Z_0 \dfrac{(e^{\gamma l} + ke^{-\gamma l})}{e^{\gamma l} - ke^{-\gamma l}}}$$

$$= E(e^{\gamma l} - ke^{-\gamma l}) \longrightarrow (4)$$

$$I_s = \frac{I_R}{2Z_0}(Z_R+Z_0)\left[e^{\gamma l} - k e^{-\gamma l}\right] \quad \text{--- } (5)$$

By equating (4) and (5)

$$\frac{E(e^{\gamma l} - \cancel{k} e^{-\gamma l})}{Z_g(e^{\gamma l} - k e^{-\gamma l}) + Z_0(e^{\gamma l} + k e^{-\gamma l})} = \frac{I_R}{2Z_0}(Z_R+Z_0)\left[e^{\gamma l} \cancel{-} k e^{-\gamma l}\right]$$

$$I_R = \frac{E \cdot 2Z_0}{(Z_R+Z_0)\left[Z_g(e^{\gamma l} - k e^{-\gamma l}) + Z_0(e^{\gamma l} + k e^{-\gamma l})\right]}$$

$$= \frac{2Z_0(Z_g + Z_R)\, I_0}{(Z_R+Z_0)(Z_0+Z_g)e^{\gamma l} + (Z_0-Z_g)(-Z_0+Z_R)e^{-\gamma l}}$$

$$\frac{I_0}{I_R} = \frac{(Z_R+Z_0)(Z_0+Z_g)e^{\gamma l} + (Z_0-Z_g)(-Z_0+Z_R)e^{-\gamma l}}{2Z_0(Z_g+Z_R)}$$

If we consider a very long line ($l >>> 1$)

$$\frac{I_0}{I_R} = \frac{(Z_R+Z_0)(Z_0+Z_g)e^{\gamma l}}{2Z_0(Z_g+Z_R)}$$

multiplying and dividing by $2\sqrt{Z_g \cdot Z_R}$

$$\frac{I_0}{I_R} = \frac{(Z_R+Z_0)(Z_g+Z_0)\,e^{\gamma l}}{2Z_0(Z_g+Z_R)} \cdot \frac{2\sqrt{Z_g Z_R}}{2\sqrt{Z_g Z_R}}$$

$$= \frac{Z_R+Z_0}{2\sqrt{Z_0 Z_R}} \cdot \frac{(Z_g+Z_0)}{2\sqrt{Z_g Z_0}} \cdot \frac{2\sqrt{Z_g Z_R}}{Z_g+Z_R} \cdot e^{\gamma l}$$

(load end) (source end) (Line)

$$= \left(\frac{1}{k_R}\right)\left(\frac{1}{k_S}\right)(k_{RS})\,e^{\gamma l}$$

$k \rightarrow$ Reflection factor

from the definition of insertion loss, it is given by

$$I = \ln\left[\frac{I_0}{I_R}\right]$$

$$I = \ln\left\{\left[\frac{1}{k_R} \cdot \frac{1}{k_S} \cdot k_{RS}\right] e^{\gamma l}\right\}$$

$$\boxed{I = \ln(k_{RS}) - \ln(k_R) - \ln(k_S) + \gamma l}$$

Impedance Matching and Transformation
Unit - V

03/09

voltage and current eqn on a zero dissipation line :

$$E = E_R \cosh \gamma s + I_R Z_0 \sinh \gamma s$$

$$I = I_R \cosh \gamma s + \frac{E_R}{Z_0} \sinh \gamma s$$

when $s = l$

$$E = E_R \cosh \gamma l + I_R Z_0 \sinh \gamma l$$

$$I = I_R \cosh \gamma l + \frac{E_R}{Z_0} \sinh \gamma l$$

$Z_0 = R_0$, $\gamma = \alpha + j\beta = j\beta$

$$\Rightarrow E = E_R \cosh j\beta l + I_R R_0 \sinh j\beta l$$

$$E = E_R \cos \beta l + \left[\begin{matrix} 0 \\ j \end{matrix}\right] I_R R_0 \sin \beta l$$ (sin odd fn) cos

$$I = I_R \cos \beta l + \left[\begin{matrix} 0 \\ j \end{matrix}\right] \frac{E_R}{R_0} \sin \beta l$$ cos $\rightarrow$ i will be removed

Case (i):

When line is open circuited at the receiving end, the voltage and current eqn is given by $I_R = 0$

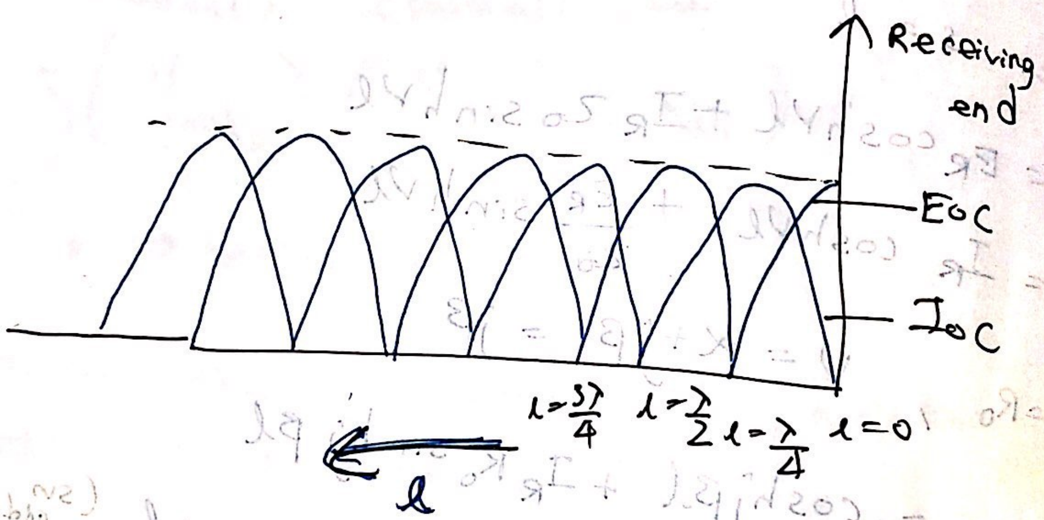
$$E_{oc} = E_R \cos \beta l$$

$$I_{OC} = +j \frac{E_R}{R_0} \sin \beta l$$

we know that $\beta = \frac{2\pi}{\lambda}$

$$\boxed{E_{OC} = E_R \cos \frac{2\pi}{\lambda} l}$$

$$\boxed{I_{OC} = j \frac{E_R}{R_0} \sin \frac{2\pi}{\lambda} l}$$



Case-ii) When line is short circuited at the receiving end, the voltage and current is given by eqn $E_R = 0$

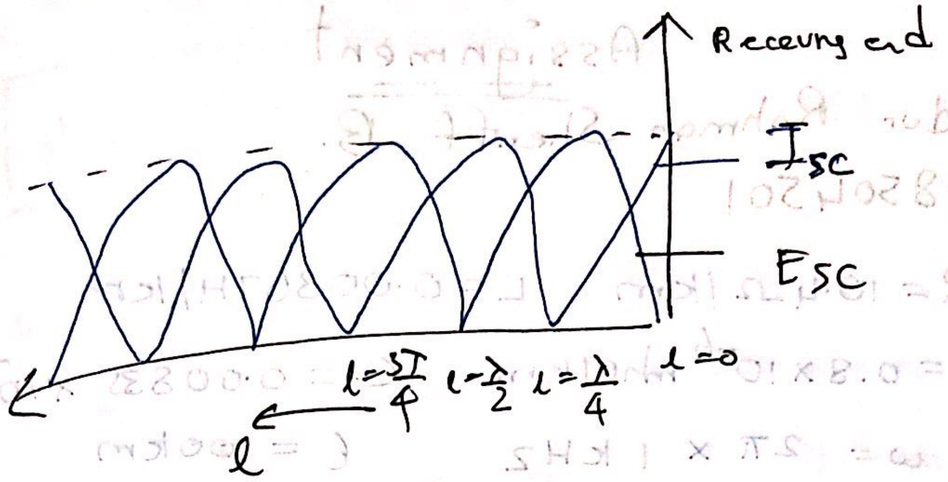
$$E_{SC} = j I_R R_0 \sin \beta l$$

$$I_{SC} = I_R \cos \beta l$$

$$\beta = \frac{2\pi}{\lambda}$$

$$E_{SC} = j I_R R_0 \sin \frac{2\pi}{\lambda} l$$

$$I_{SC} = I_R \cos \frac{2\pi}{\lambda} l$$



Case (iii):-

When the line is terminated in an reflected wave is absent

$(K=0)$

voltage and current eqn. in terms of k:

$$E = \frac{E_R}{2}\left(1+\frac{Z_0}{Z_R}\right)e^{\gamma s} + \frac{E_R}{2}\left[1-\frac{Z_0}{Z_R}\right]e^{-\gamma s}$$

$$= \frac{E_R}{2}\left[\frac{Z_R+Z_0}{Z_R}\right]e^{\gamma s} + \frac{E_R}{2}\left[\frac{Z_R-Z_0}{Z_R}\right]e^{-\gamma s}$$

$$= \frac{E_R(Z_R+Z_0)}{2Z_R}\left[e^{\gamma s} + |k|\,e^{-\gamma s}\right]$$

Ily

$$I = \frac{I_R(Z_R+Z_0)}{2Z_0}\left[e^{\gamma s} - |k|\,e^{-\gamma s}\right]$$

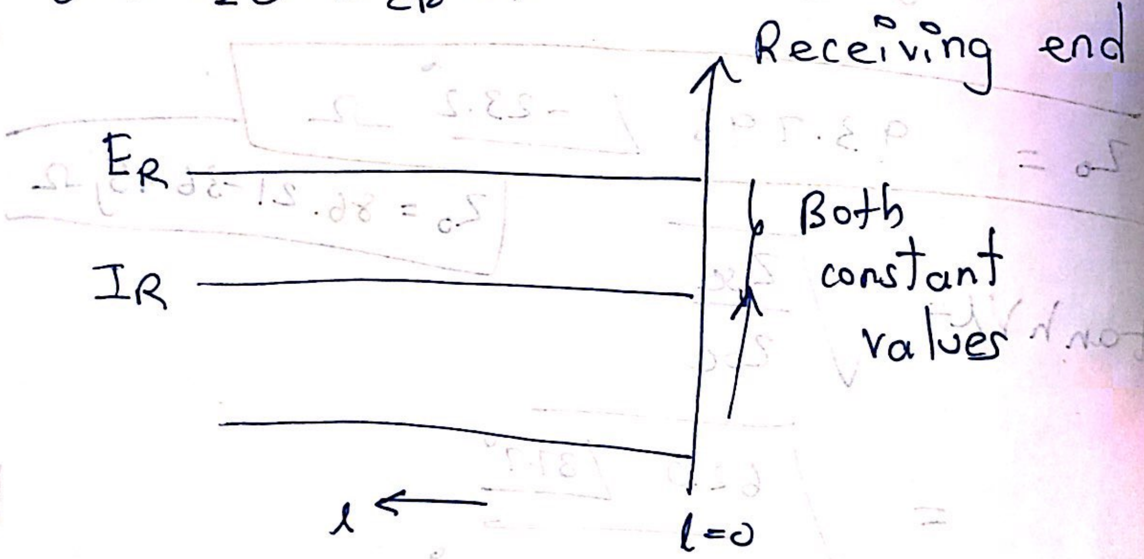
when there is no reflection, it will be ideal case

$$Z_R = Z_0$$

$$\Rightarrow \alpha = 0$$

$E = E_R\, e^{\gamma s}$ similarly $I = I_R e^{\gamma s}$

$\cancel{E = \frac{E_R}{2}\left(1 + \frac{Z_0}{Z_R}\right) e^{\gamma s}}$



Note: In case of open circuit, current is minimum at load point at a distance of $\frac{\lambda}{4}$ from the load point we have current maximum

[Distance between two successive min or max $= \frac{\lambda}{2}$

Distance btw maxima and minima $= \frac{\lambda}{4}$]

08/09

Standing Waves

→ stationary wave

A → Antinodes

Maximum current or voltage in standing wave

N → nodes

0 voltage or current in standing wave

When no standing wave is present, it means there isn't any reflection.

That line is called smooth line:

standing wave ratio (SWR):

Ratio of max to min magnitudes of V or I on a line having standing waves is called standing wave ratio and it is denoted by 'S'.

$$S = \left|\frac{E_{max}}{E_{min}}\right| \quad \text{or} \quad S = \left|\frac{I_{max}}{I_{min}}\right|$$

Consider

$|E^+| \rightarrow$ magnitude of incident voltage

$|E^-| \rightarrow$ magnitude of reflected voltage

$$E_{max} = |E^+| + |E^-|$$

$$E_{min} = |E^+| - |E^-|$$

$$S = \frac{|E^+| + |E^-|}{|E^+| - |E^-|}$$

$\div$ throughout by $|E^+|$

$$S = \frac{1 + \left|\frac{E^-}{E^+}\right|}{1 - \left|\frac{E^-}{E^+}\right|}$$

$$\boxed{S = \frac{1+k}{1-k}}$$

k = reflection coefficient

k in terms of S

$$S - Sk = 1 + k$$

$$S - 1 = Sk + k$$

$$\cancel{k = \frac{S-1}{S+1}} \approx \boxed{k = \frac{S-1}{S+1}}$$

Case:

(i) when $Z_R = Z_0$

$k = 0 \Rightarrow \boxed{S = 1}$

(ii) $k = 1$

$1 = \frac{S-1}{S+1}$

$S+1 = S-1$

$\boxed{S = \infty}$

(iii) $k = -1$

$-1 = \frac{S-1}{S+1}$

$-S-1 = S-1$

$\boxed{S = 0}$

S will range from 0 to ∞

$\boxed{0 \leq S \leq \infty}$

(ii) when $Z_R > Z_0$

$k = \frac{Z_R - Z_0}{Z_R + Z_0} \Rightarrow 0 < k < 1 \Rightarrow 1 < S < \infty$

$\boxed{S = \frac{R_R}{R_0}}$

R_o or Z_o

(iii) $Z_R < Z_o$

$$k = \frac{Z_R - Z_o}{Z_R + Z_o} \Rightarrow -1 < k < 0$$

$$0 < S < 1$$

$$\boxed{S = \frac{R_o}{R_R}}$$

Voltage eqn of zero dissipation line in terms of 'k':

$$E = E_R e^{\gamma l}$$

$$E = E_R \cosh(j\beta l) + I_R Z_o \sinh(j\beta l)$$

$$= E_R\left(\frac{e^{j\beta l} + e^{-j\beta l}}{2}\right) + I_R Z_o \left(\frac{e^{j\beta l} - e^{-j\beta l}}{2}\right)$$

$$= \frac{e^{j\beta l}}{2}\left[I_R Z_R + I_R Z_o\right] + \frac{e^{-j\beta l}}{2}\left[I_R Z_R - I_R Z_o\right]$$

$$= \frac{e^{j\beta l}}{2} I_R \left[(Z_R + Z_o) + e^{-2j\beta l}(Z_R - Z_o)\right]$$

$$\cancel{= \frac{I_R (Z_R + Z_o)}{2} e^{j\beta l}\left[1 + |k| e^{j\phi} e^{-}\right]}$$

$$= \frac{I_R (Z_R + Z_o)}{2} e^{j\beta l}\left[1 + \frac{Z_R - Z_o}{Z_R + Z_o} e^{-j2\beta l}\right]$$

$\frac{Z_R - Z_o}{Z_R + Z_o}$ = Reflection coeff

Magnitude of $k = \phi$

$$= \frac{I_R (Z_R+Z_0)}{2} e^{j\beta l} \left[1 + |k| e^{j\phi} e^{-j2\beta l} \right]$$

$$= \frac{I_R (Z_R+Z_0)}{2} e^{j\beta l} \left[1 + |k| e^{j(\phi - 2\beta l)} \right]$$

Xradians to degree

$$E = \frac{I_R}{2} (Z_R+Z_0) e^{j\beta l} \left[1\angle 0^\circ + |k| \angle \phi - 2\beta l \right]$$

First term represents the incident wave + second term represents the reflected wave.

Voltage will be maximum when both incident and reflected wave are in phase

$$\phi - 2\beta l = 0$$

$$\Rightarrow \boxed{E_{max} = \frac{I_R}{2} (Z_R+Z_0) e^{j\beta l} \left[1\angle 0^\circ + |k| \angle 0^\circ \right]}$$

For voltage minimum, IW and RW will be out of phase

$$\phi - 2\beta l = \pi$$

$$\boxed{E_{min} = \frac{I_R}{2} (Z_R+Z_0) e^{j\beta l} \left[1\angle 0^\circ + |k| \angle \pi \right]}$$

$$E_{min} = \frac{I_R}{2}(Z_R + Z_0)e^{j\beta l}[1 - |k|]$$

$$S = \frac{E_{max}}{E_{min}} = \frac{1+|k|}{1-|k|}$$

Input impedance of a zero dissaption line: $Z_s = \frac{E_s}{I_s}$

Distortionless $V = j\beta l$

$\cosh j = \cos\beta l$

$\sinh j = j\sin$

$$\frac{E_s}{I_s} = \frac{E_R\cos\beta l + j I_R R_0 \sin\beta l}{I_R\cos\beta l + j\frac{E_R}{R_0}\sin\beta l}$$

$$= \frac{E_R + j I_R R_0 \tan\beta l}{I_R + j\frac{E_R}{R_0}\tan\beta l}$$

$$Z_s = \frac{\cancel{I_R}[Z_R + j R_0\tan\beta l]}{\cancel{I_R}[1 + j\frac{Z_R}{R_0}\tan\beta l]}$$

$$\boxed{Z_{in} = \frac{R_0[Z_R + jR_0\tan\beta l]}{R_0 + j Z_R\tan\beta l}}$$

I/p imp interms of k

$$Z_s = Z_0 \left\{ \frac{[Z_R \cosh j\beta l + Z_0 \sinh j\beta l]}{[Z_0 \cosh \beta l + Z_R \sinh \beta l]} \right.$$

$$= Z_0 \cdot \frac{\left[Z_R \cdot \frac{e^{j\beta l} + e^{-\beta l}}{2} + Z_0 \frac{e^{j\beta l} - e^{-j\beta l}}{2}\right]}{\left[Z_0 \frac{e^{j\beta l} + e^{-\beta l}}{2} + Z_R \frac{e^{\beta l} - e^{-j\beta l}}{2}\right]}$$

$$= Z_0 \frac{\left[\frac{e^{j\beta l}}{2}(Z_R + Z_0) + \frac{e^{-j\beta l}}{2}(Z_R - Z_0)\right]}{\left[\frac{e^{j\beta l}}{2}(Z_0 + Z_R) - \frac{e^{-j\beta l}}{2}(Z_R - Z_0)\right]}$$

$$= Z_0 \left\{ \div \frac{e^{j\beta l}}{2}(Z_R + Z_0) \right.$$

$$Z_s = Z_0 \frac{\left[1 + \frac{e^{-2j\beta l}}{2}\left(\frac{Z_R - Z_0}{Z_R + Z_0}\right)\right]}{\left[1 - e^{-2j\beta l}\left(\frac{Z_R - Z_0}{Z_R + Z_0}\right)\right]}$$

$$k = \frac{Z_R - Z_0}{Z_R + Z_0}$$

$$Z_s = Z_0 \frac{\left[1 + |k| e^{j\phi} e^{-j2\beta l}\right]}{\left[1 - |k| e^{j\phi} e^{-j2\beta l}\right]}$$

$$Z_s = Z_0 \left[\frac{1 + |k| e^{j(\phi - 2\beta l)}}{1 - |k| e^{j(\phi - 2\beta l)}}\right]$$

$$Z_s = Z_0 \left[\frac{1 + |k| \angle \phi - 2\beta l}{1 - |k| \angle \phi - 2\beta l}\right]$$

$$\cancel{Z_s = \frac{1+k\angle\phi-2\beta l}{1-k\angle\phi-2\beta l}}$$

~~Zin~~

$$\boxed{Z_{in} = R_0\left[\frac{1+k\angle\phi-2\beta l}{1-k\angle\phi-2\beta l}\right]}$$

case-I:

I/p imp will be maximum when they are in phase (i.e) $\phi-2\beta l = 0$ or 2π.

$$Z_{in(max)} = R_0\left[\frac{1+|k|}{1-|k|}\right] \qquad s = \frac{\phi}{2\beta}$$

distance

$$\boxed{Z_{in(max)} = R_0 S} \qquad S = SWR$$

case-ii):

I/p impedance will be minimum at a distance of $\frac{\lambda}{4}$, from maximum point.

$$s = \frac{\phi}{2\beta} + \frac{\lambda}{4} \Rightarrow s = \frac{\phi}{2\left(\frac{2\pi}{\lambda}\right)} + \frac{\lambda}{4}$$

$$s = \frac{\phi\lambda}{4\pi} + \frac{\lambda}{4} = \frac{\lambda(\phi+\pi)}{4\pi}$$

$$\boxed{s = \frac{\phi+\pi}{2\beta}}$$

$$Z_{in_{min}} = R_0 \left[\frac{1 + |k| \angle \phi - 2\beta l}{1 - |k| \angle \phi - 2\beta l} \right]$$

$$= R_0 \left[\frac{1 + |k| \angle \pi}{1 - |k| \angle \pi} \right]$$

$$= R_0 \left[\frac{1 - |k|}{1 + |k|} \right]$$

$$\phi - 2\beta l = \pi$$

$$\phi = \pi + 2\beta S$$? ?

Input impedance of open and short circuited line:

i) Short circuited line:

$$Z_S = R_0 \left[\frac{Z_R + j R_0 \tan\beta S}{R_0 + j Z_R \tan\beta S} \right]$$

For S.C $Z_R = 0$ $\Rightarrow$ $\boxed{Z_{SC} = j R_0 \tan\beta S}$

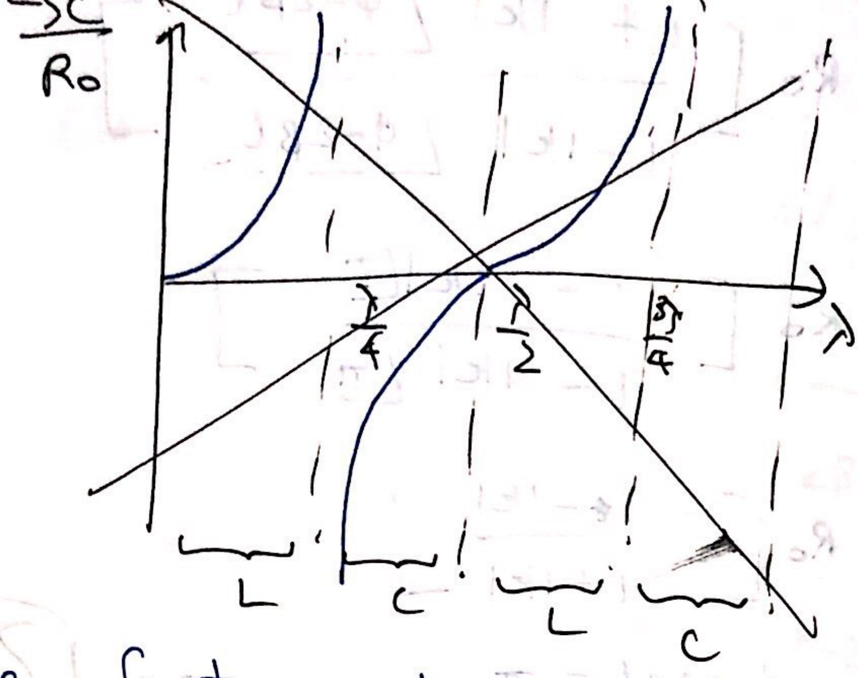
$$\frac{Z_{SC}}{R_0} = j \tan\beta S$$

$s = 0 \Rightarrow j \tan 0 = 0$

$s = \frac{\lambda}{4} \Rightarrow j \tan \frac{\lambda}{4} \cdot \frac{2\pi}{\lambda} = j\infty$

$s = \frac{\lambda}{2} \Rightarrow j \tan \frac{\lambda}{2} \cdot \frac{2\pi}{\lambda} = 0$

$s = \frac{3\lambda}{4} \Rightarrow j \tan \frac{3\lambda}{4} \cdot \frac{2\pi}{\lambda} = -j\infty$



For the first quarter wavelength, a sc line acts as an inductance.
For the next quarter it acts as a capacitance

ii) Open circuited line:

$$Z_S = R_o\left[\frac{Z_R + jR_o\tan\beta s}{R_o + jZ_R\tan\beta s}\right]$$

$Z_R = \infty$ for OC

$$Z_{S_{oc}} = \cancel{Z_R}\ \cancel{R_o}\left[\cancel{1}\right]$$

$$= R_o\ \frac{Z_R}{Z_R}\left[\frac{1 + j\frac{R_o}{Z_R}\tan\beta s}{\frac{R_o}{Z_R} + j\tan\beta s}\right]$$

$$Z_{R_{oc}} = R_o\left[\frac{1}{j\tan\beta s}\right]$$

$$\boxed{Z_{oc} = -jR_o\cot\beta s}$$

At infinity impedance → parallel LC ckt

Low impedance → series LC ckt

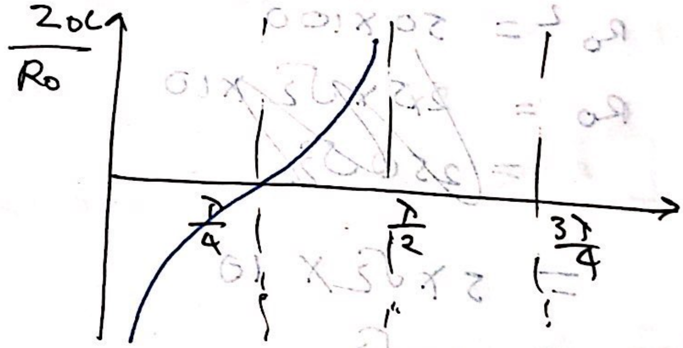
$= -j\cot 0 = -j\infty$

$s = 0$

$s = \frac{\lambda}{4} \quad = -j\cot \frac{\lambda}{4}\cdot\frac{2\pi}{\lambda} = 0$

$s = \frac{\lambda}{2} \quad \Rightarrow -j\cot \frac{\lambda}{2}\cdot\frac{2\pi}{\lambda} = j\infty$

$s = \frac{3\lambda}{4} \quad \Rightarrow -j\cot \frac{3\lambda}{4}\cdot\frac{2\pi}{\lambda} = 0$



Impedance Matching

→ In order to have max power transfer

~~what are waveline~~ ?

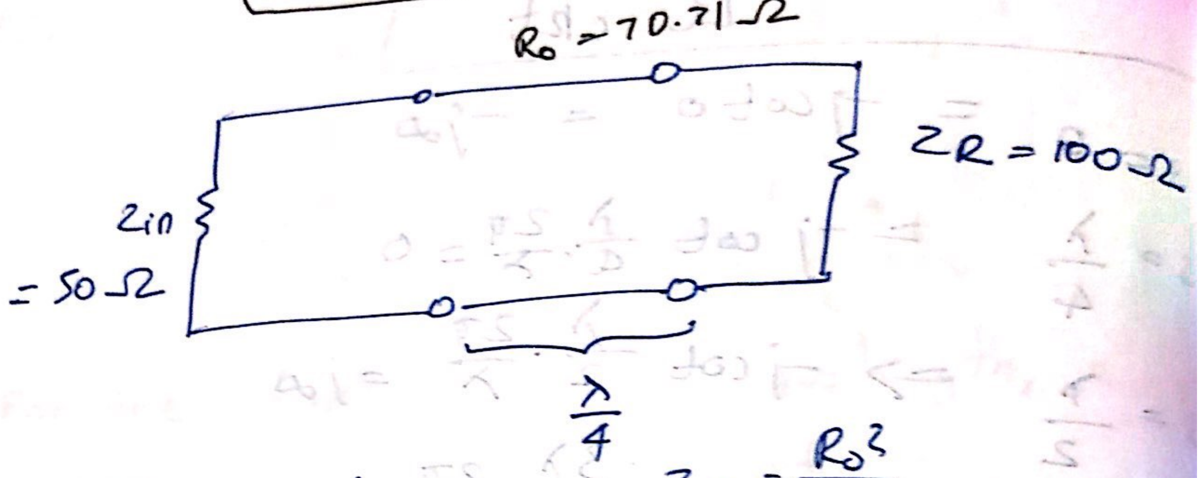
i) Quarter waveline : $\left[\frac{\lambda}{4}\right]$

$$Z_{in} = R_0\left[\frac{Z_R + jR_0\tan\left(\frac{2\pi}{\lambda}\right)\left(\frac{\lambda}{4}\right)}{R_0 + jZ_R\tan\left(\frac{2\pi}{\lambda}\right)\left(\frac{\lambda}{4}\right)}\right]$$

$$= R_0\left[\frac{Z_R/\infty + jR_0}{R_0/\infty + jZ_R}\right] \Rightarrow \frac{R_0^2}{Z_R}$$

Scanned with CamScanner

$$\boxed{Z_{in} = \frac{R_o^2}{Z_R}}$$



$$Z_{in} = \frac{R_o^2}{Z_R}$$

$$50 = \frac{R_o^2}{100}$$

$$R_o^2 = 50 \times 100$$

$$R_o = \cancel{2 \times 5 \times \sqrt{2} \times 10}$$

$$= \cancel{250\sqrt{2}}$$

$$= 5 \times \sqrt{2} \times 10$$

$$= 50\sqrt{2}$$

$$\boxed{R_o = 70.71\,\Omega}$$

ii) Half waveline: $\left(\frac{\lambda}{2}\right)$

$$Z_{in} = R_o\left[\frac{Z_R + jR_o \tan\left(\frac{2\pi}{\lambda}\right)\left(\frac{\lambda}{2}\right)}{R_o + jZ_R \tan\left(\frac{2\pi}{\lambda}\right)\left(\frac{\lambda}{2}\right)}\right]$$

$$Z_{in} = R_o\left(\frac{Z_R}{R_o}\right) \quad \boxed{Z_{in} = Z_R}$$

when no match is possible, half waveline is used as a one to one transformer

(iii) Eigth waveline:

$$Z_{in} = R_0 \left[\frac{Z_R + jR_0 \tan\left(\frac{2\pi}{\lambda}\right)\left(\frac{\lambda}{8}\right)}{R_0 + jZ_R \tan \frac{2\pi}{\lambda} \cdot \frac{\lambda}{8}} \right]$$

$$Z_{in} = R_0 \left[\frac{Z_R + jR_0}{R_0 + jZ_R} \right] \quad |Z_{in}| = R_0 \frac{\sqrt{Z_R^2 + R_0^2}}{\sqrt{R_0^2 + Z_R^2}}$$

$$\boxed{|Z_{in}| = R_0}$$

It transforms any resistance to an impedance with a magnitude equal to R_0 of the line.

[Quarter waveline is usually used]

problem:

A ~~loseless~~ lossy transmission line with $Z_0 = 50\Omega$ is terminated in a load $50 + j50\Omega$. Find the following:

(i) Reflection at load

(ii) VSWR

(iii) ~~k at a distance~~ Admittance at load

$$Z_L = 50 + j50\,\Omega$$

$$Z_0 = 50\,\Omega$$

$$Z_L' = \frac{Z_L}{Z_0} = 1 + j$$

(normalized load imp)

$$K = \frac{Z_R - Z_0}{Z_R + Z_0} = \frac{50 + 50j - 50}{50 + 50j + 50}$$

$$= \frac{j50}{100 + 50j}$$

$$= \frac{j}{2+j} = \frac{1}{1-2j} \cdot \frac{1+2j}{1+2j}$$

$$K = \frac{1+2j}{5}$$

$K = 0.2 + 0.4j$ $\quad |K| = \cancel{(0.64)}$

$K = 0.44 \angle 63.43^\circ$ $\quad |K| = 0.44$

(ii) VSWR

$$S = \frac{1 - |K|}{1 + |K|}$$

$$= \frac{1 + 0.44}{1 - 0.44} = \frac{\cancel{0.56}}{\cancel{1.44}} \frac{1.44}{0.56}$$

$$\boxed{S = \cancel{2.28} \ 2.571}$$

$$\boxed{\begin{array}{l} \text{Range} \\ \text{VSWR} \rightarrow 1 - \infty \\ K \rightarrow 0 - 1 \end{array}}$$

(iii) $Y_L' = \dfrac{1}{Z_L'}$

(Normalized load admittance)

$$= \frac{1}{1+j} = \frac{1-j}{(1+j)(1-j)}$$

$$= \frac{1-j}{2} = 0.5 - 0.5j$$

$\cancel{= 0.707 - 0.707j}$

$Y_L' \times 50 = Y_L$

$Y_L = 25 - 25j$ (load admittance)