control systems :-

→ when a number of elements or components are connected in a sequence to perform a specific function, the group thus formed is called a system.

→ When a o/p quantity is controlled by varying the i/p quantity, the s/m is said to be control s/m.

o/p quantity ⇒ controlled variable or Response.

i/p quantity ⇒ command signal or excitation.

→ For example,

(i) In a driving s/m, the speed is controlled by position of the accelerator.

Hence the i/p quantity ⇒ position of the accelerator

ctrlled o/p qty ⇒ speed

(ii) Traffic lights control s/m, a sequence of i/p signal is applied to the s/m to have o/p as one of the three lights, for particular duration of time.

→ **Types of control systems:-**

Classification of control s/ms

- Based on Type of the signal
  - Continuous — If the i/p signal of the s/m is continuous
  - Discrete — If the i/p signal is discrete.
- Based on no. of i/ps & o/ps
  - SISO
  - MIMO.
- Based on feedback.
  - open loop
  - closed loop

Open loop control s/ms. (OLS)

→ The s/m which does not automatically correct the variation in its o/p is called OLS.

→ The o/p quantity has no effect upon the i/p quantity. (i.e) there will be no feedback to the i/p for correction.

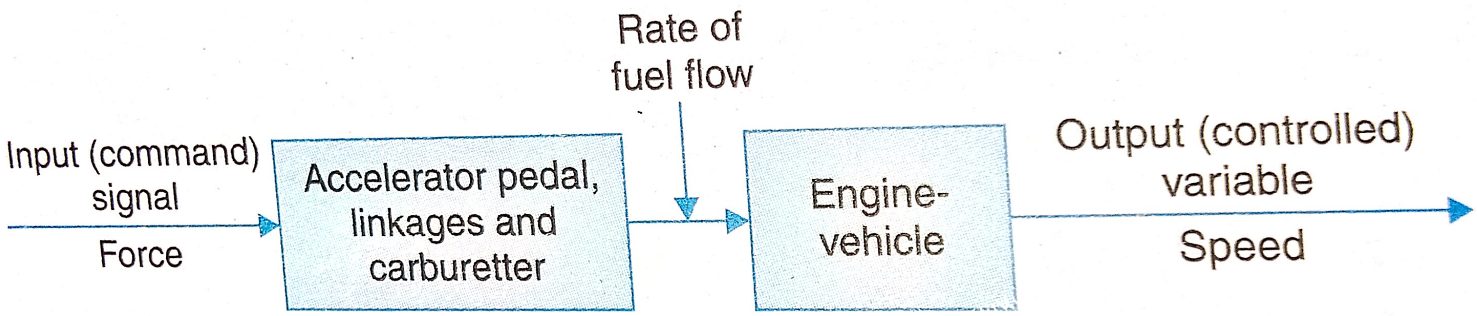
s(t) i/p → controller → Processor → o/p c(t).

→ For a driving s/m, controller → accelerator position.

Processor → Automobile S/m.

o/p → speed

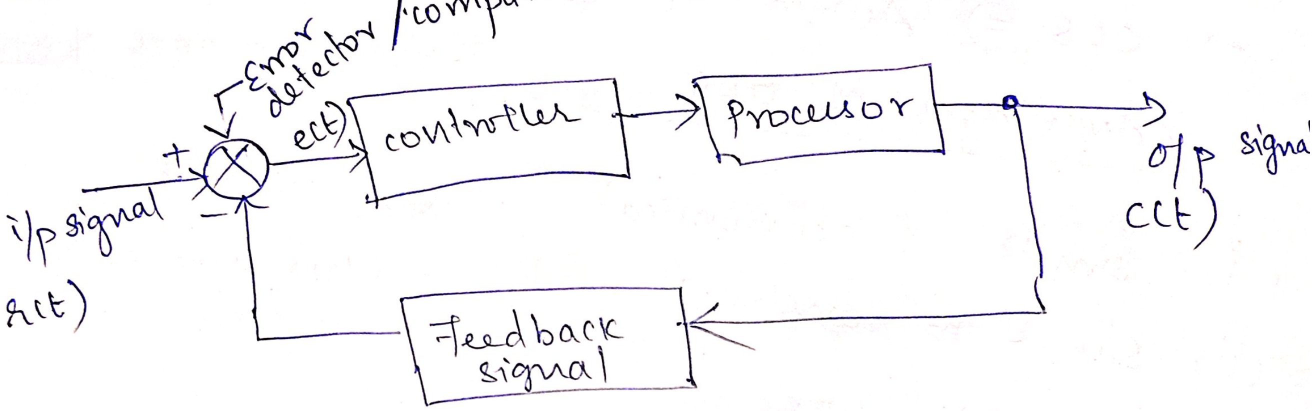
engine bringing the engine vehicle sp



→ With no restrictions in the level ~~[illegible]~~ of speed,
By ~~[illegible]~~ ~~[illegible]~~ adjusting the accelerator position & corresponding speed ie attained as o/p quantity.

closed loop control s/m.

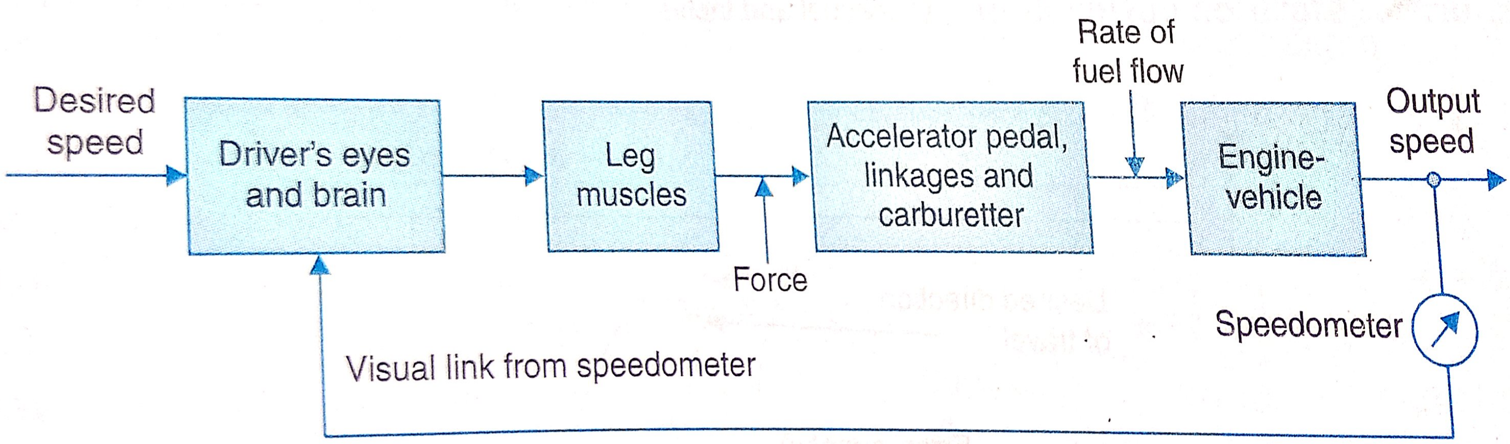
→ The control s/ms in which the o/p has an effect upon the i/p quantity in order to maintain the desired o/p is called CLS.

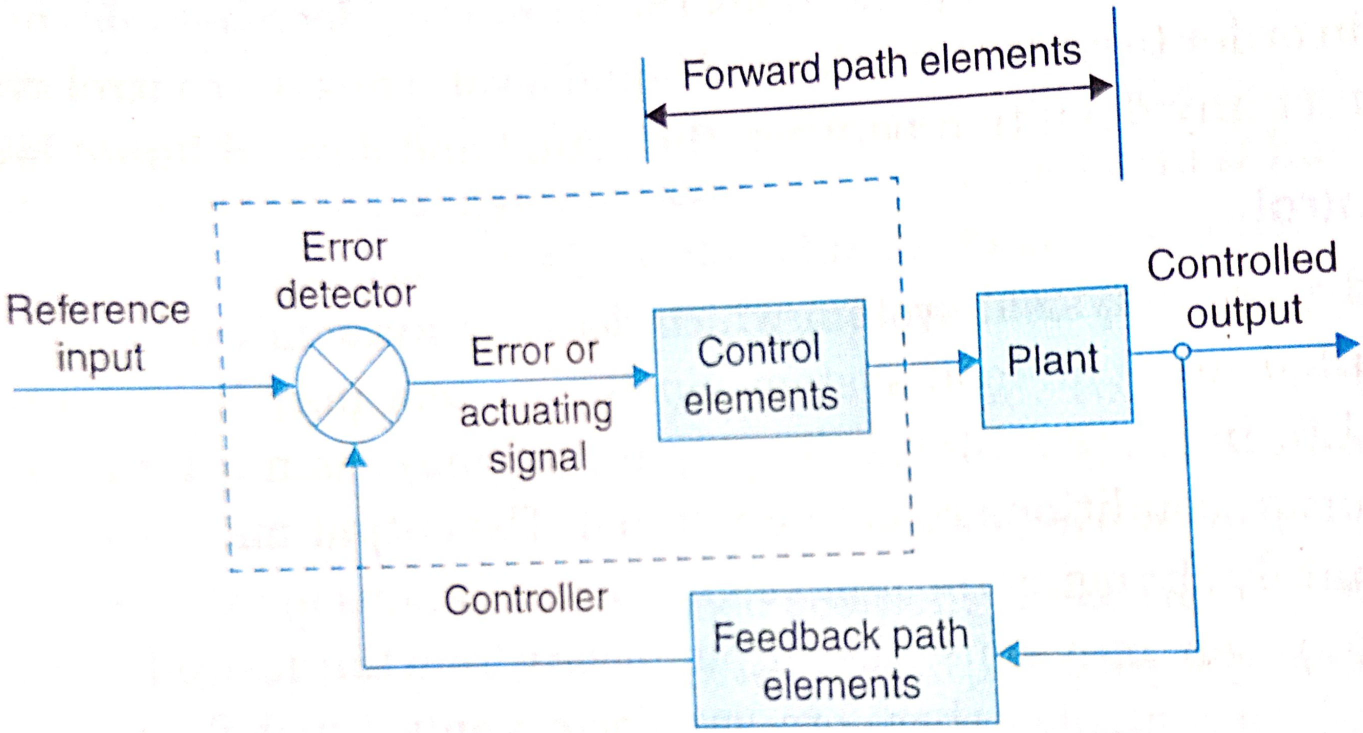


→ consider the same driving s/m. The speed & acceleration of the vehicle are determined & controlled by the driver by observing the traffic and road conditions.

→ If the vehicle is need to be maintained at a speed of 50 km/hr, then with accelerator position, the desired speed can be attained.

→ But if suddenly the situation changes, & there should be a change in the speed level [(ie) 50 km/hr to 20 km/hr] the state of o/p is feedback to the i/p & it is used to modify the controlling s/m to get the desired o/p.


Desired speed
Driver's eyes and brain
Leg muscles
Force
Accelerator pedal, linkages and carburetter
Rate of fuel flow
Engine-vehicle
Output speed
Speedometer
Visual link from speedometer


Forward path elements
Error
detector
Reference
input
Error or
actuating
signal
Control
elements
Plant
Controlled
output
Controller
Feedback path
elements

→ The reference signal or i/p signal corresponds to the desired o/p. The error signal generated by the error detector & the difference between the reference signal and feedback signal.

→ Then the controller modifies & amplifies the error signal to get a better control action on the o/p.

→ CLS ⇒ Automatic control s/m.

→ Advantages of OLS

(i) simple & economical.
(ii) Easier to construct.
(iii) stable.

→ Advantages of CLS

(i) accurate
(ii) less affected by noise.

→ Disadvantages of OLS

(i) Inaccurate & unreliable.
(ii) No automatic correction with respect to ~~changes in~~ external conditions.

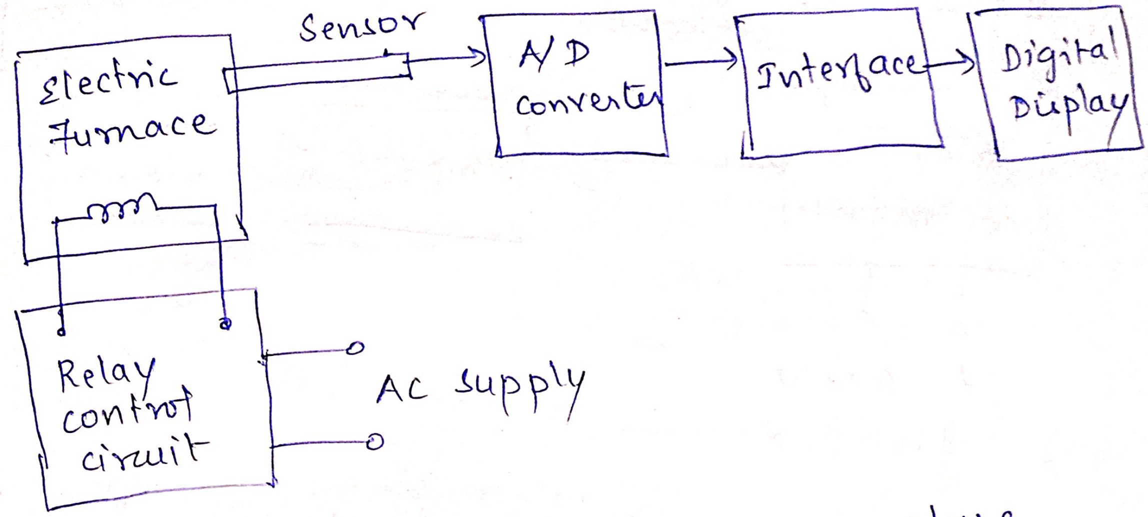
→ Disadvantages of CLS

(i) complex & costly.
(ii) The feedback may lead to oscillations.
(iii) Reduces the overall gain.
(iv) less stable.

Examples of control s/ms.

→ Temperature control s/m.

(i) open loop s/m.



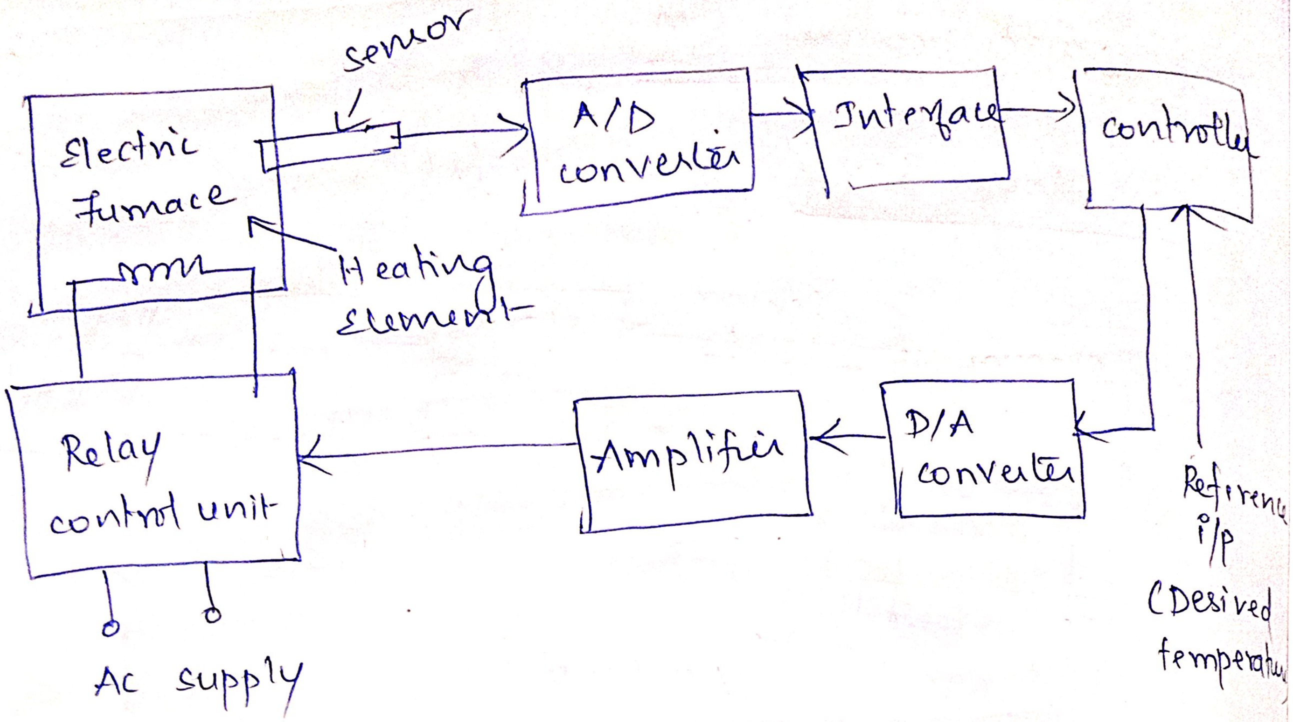
→ The o/p in the s/m is the temperature which depends on the time during which the supply to heater remains ON.

→ The temperature is measured by the sensor which gives the analog value corresponding to the temperature of the furnace.

→ The analog signal is converted to a digital signal by A/D converter and the display is used to display the value of the temperature.

→ so, in this s/m, if there is any change in the o/p temperature, the ON time of the relay is not automatically altered.

(ii) closed loop s/m :-



→ Here, the ON & OFF time of relay is controlled by a controller.

→ The actual temperature is sensed by the sensor and converted to digital by A/D.

→ The computer (ie) controller unit reads the actual temperature and compares with desired temperature.

→ ~~[illegible]~~ If there is any difference in the temperature, then it sends signal to switch ON or OFF the relay thro' D/A converter and Amplifier.

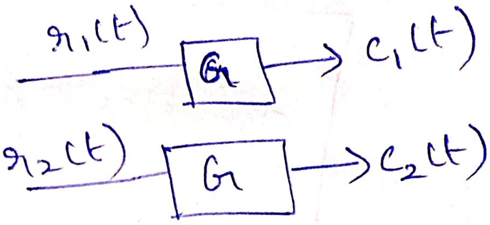
→ Thus the s/m automatically corrects any changes in the o/p.

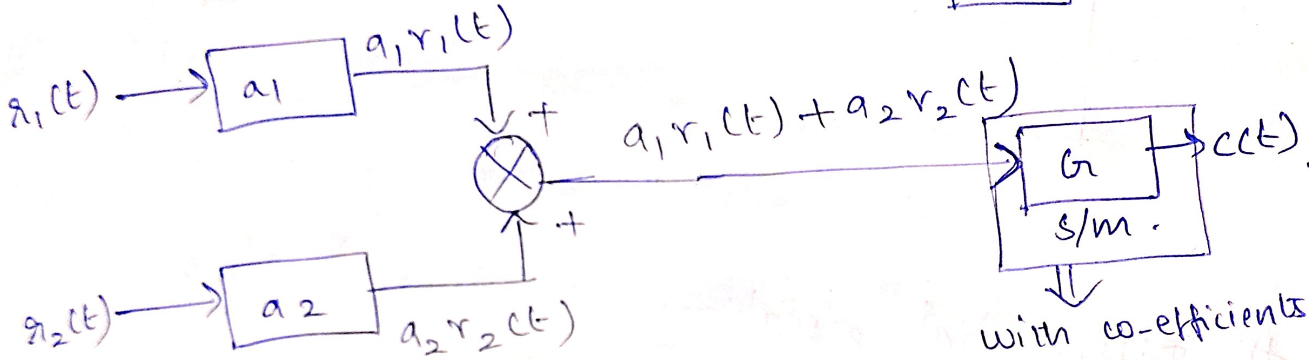
Mathematical Models of control s/ms.

→ The i/p - o/p relations of various physical components of a s/ms are governed by the set of differential equations.

→ These differential equations are solved by using transforms (Laplace transforms).

→ Linear ~~time~~ ~~Invariant~~ s/ms.





→ Linear Time Invariant s/ms:-

→ The s/m is said to be linear time Invariant if the co-efficients of the differential equation describing the s/m are constants.

→ Linear - time varying s/ms.

→ The s/m is said to be Linear time varying if the co-efficients of the differential equations are functions of time.

→ Transfer function :-

→ The transfer function of a s/m is defined as the ratio of laplace transform of o/p to the laplace transform of i/p with zero initial conditions.

$$\boxed{\text{Transfer func.} = \left.\frac{LT(o/p)}{LT(i/p)}\right|_{\text{zero initial conditions.}}}$$

Differential equations of physical s/ms :-

(1) Mechanical-translational s/ms.

→ The model of mechanical translational s/ms can be obtained by using three basic elements

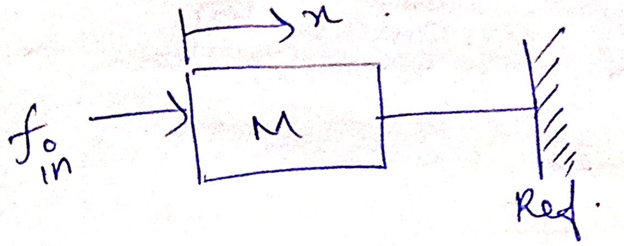
(i) Mass ⇒ weight of Mechanical s/m

(ii) Spring ⇒ elastic deformation

(iii) dash-pot ⇒ Friction.

→ When a force is applied to the translational mechanical s/m, it is opposed by opposing forces due to mass, friction & elasticity of the s/m.

(i) Mass element



f – force

M – Mass

x – displacement.

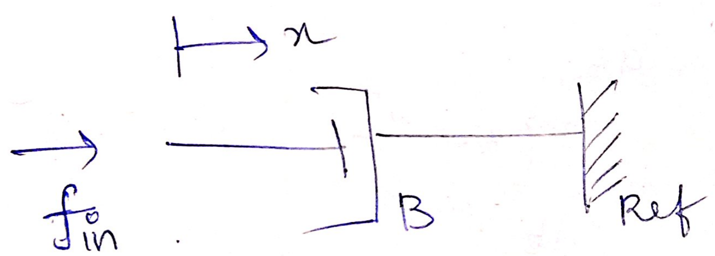
→ When a force is applied on a body of Mass M, the mass will offer a opposing force which is proportional to acceleration of the body.

→ The opposing force is denoted by (f_m)

$$\therefore f_m \propto \frac{d^2x}{dt^2}.$$ [Due to mass]

$$\boxed{f_{in} = f_m = M\frac{d^2x}{dt^2}.}$$

(ii) Dash-pot / Damper.



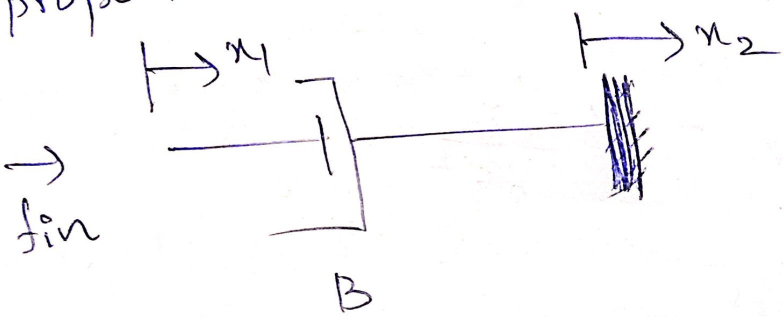
→ ~~With~~ To a frictional element of negligible mass and elasticity, the force is applied, then that dash-pot (frictional element) will offer a opposing force proportional to the velocity of the body.

→ The opposing force due to friction, denoted by f_b is given by

$$f_b \propto \frac{dx}{dt}$$

$$\boxed{f_{in} = f_b = B\frac{dx}{dt}.}$$

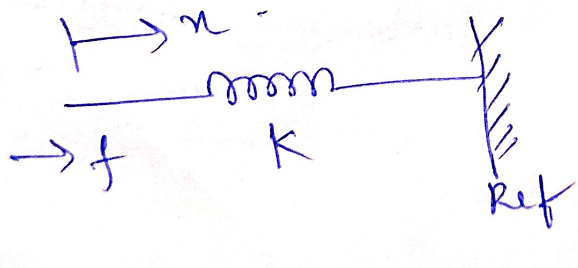
→ If the dashpot has the displacement at both ends, then the opposing force is proportional to differential velocity.



$$f_b \propto \frac{d(x_1 - x_2)}{dt}$$

$$\boxed{f_{in} = f_b = B\frac{d(x_1 - x_2)}{dt}.}$$

(iii) <u>Spring</u>

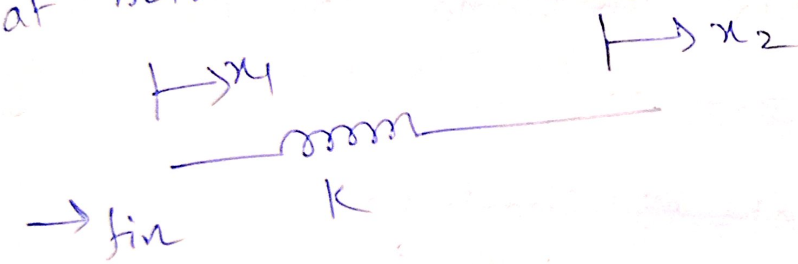


→ When a force applied on the spring, it will offer an ~~force~~ opposing forcing which is proportional to the displacement of the body.

$$f_K \propto x$$

$$\boxed{f_{in} = f_K = Kx.}$$

→ With the spring that has displacemente at both ends



$$f_K \propto (x_1 - x_2)$$

$$\boxed{f_K = K(x_1 - x_2)} = f_{in}$$

(2) Mechanical - Rotational s/ms.

→ The Model of rotational-mechanical s/ms can be obtained by using three basic elements.

(i) moment of Inertia (J) of mass

(ii) dash-pot with rotational frictional co-efficient (B)

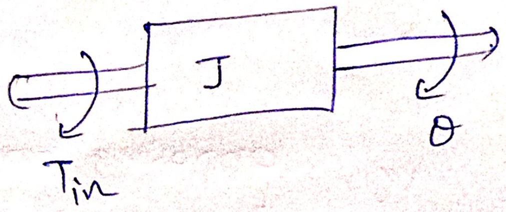
(iii) torsional spring with stiffness (K)

→ ~~(i) Moment of Inertia.~~

→ when a torque is applied to a rotational mechanical s/m, it is opposed by opposing torques due to moment of inertia, friction and elasticity of the system.

(i) Moment of Inertia (J)

consider a mass element, when a torque is applied, ~~to~~ it will produce an opposing torque ~~[illegible]~~ which is proportional to the angular acceleration.



opposing torque → T_j

θ → angular displacement

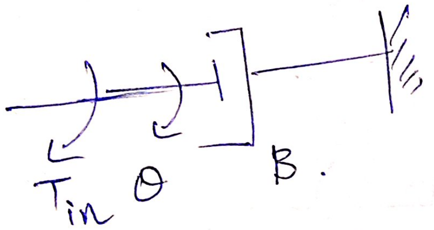
T_{in} → i/p Torque.

$$T_j \propto \frac{d^2\theta}{dt^2}.$$

$$\boxed{T_{in} = T_j = J\frac{d^2\theta}{dt^2}}$$

(ii) <u>dash-pot</u> $T_b \propto$ angular velocity.

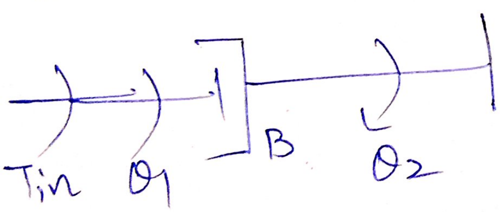
opposing Torque → T_b.



$$T_b \propto \frac{d\theta}{dt}$$

$$\boxed{T_{in} = T_b = B\frac{d\theta}{dt}}$$

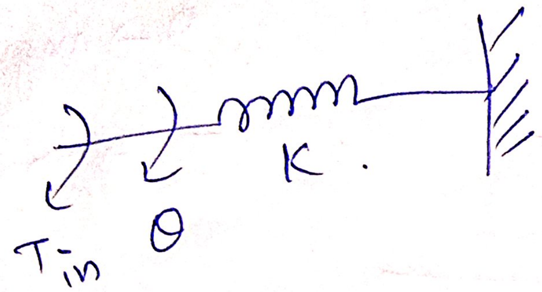
If the dash-pot has angular displacement at both ends, then



$$T_b \propto \frac{d(\theta_1 - \theta_2)}{dt}$$

$$\boxed{T_{in} = T_b = B\frac{d}{dt}(\theta_1 - \theta_2)}$$

(iii) Elastic-element spring.

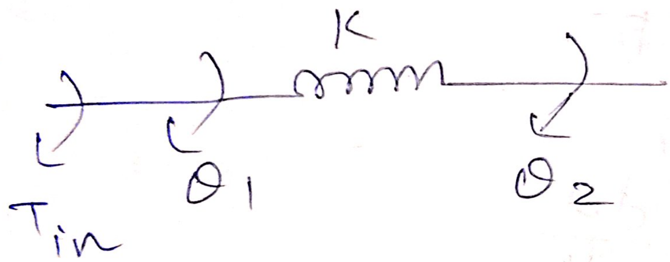


Opposing torque T_K is proportional to angular displacement

$$T_K \propto \theta$$

$$T_{in} = \boxed{T_K = K\theta}.$$

If the element has angular displacement at both the ends, then

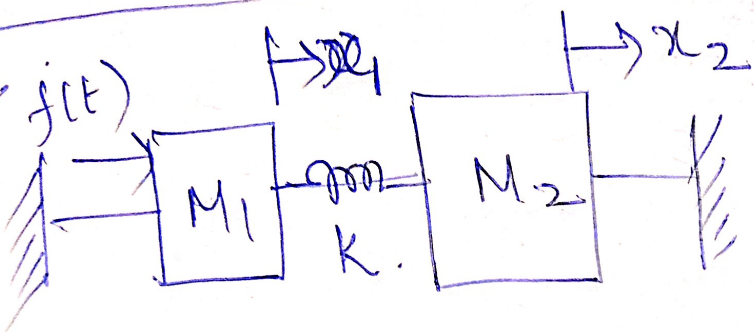


$$T_K \propto (\theta_1 - \theta_2)$$

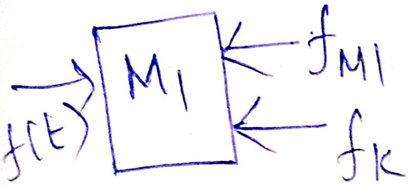
$$T_{in} = \boxed{T_K = K(\theta_1 - \theta_2)}$$

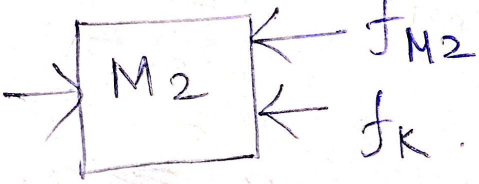
Rea.

eg



Free-body diagram





$$f_{M1} = M_1 \frac{d^2 x_1}{dt^2}.$$

$$f_K = K(x_1 - x_2)$$

$$f(t) = f_{M1} + f_K.$$

$$f_{M2} + f_K = 0$$

$$f_{M2} = M_2 \frac{d^2 x_2}{dt^2}$$

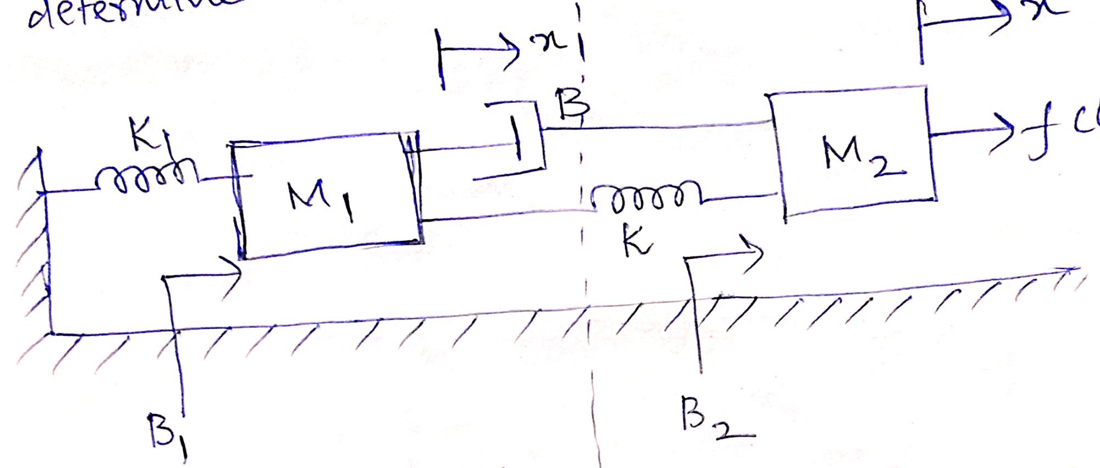
$$f_K = K(x_2 - x_1).$$

$$M_2 \frac{d^2 x_2}{dt^2} + K(x_2 - x_1) = 0$$

pblm

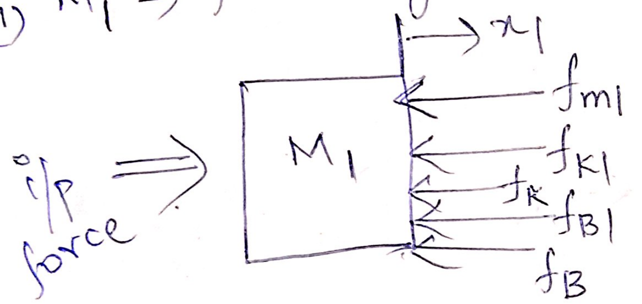
Mechanical-Translational s/m:-

1) Write the differential equations governing the mechanical s/m shown in fig., & determine the transfer function.



soln Each mass ⟺ ~~each~~ one node.

(i) $M_1 \Rightarrow$ free body diagram.



$$f_{M1} = M_1 \frac{d^2 x_1}{dt^2}$$

$$f_{K1} = K_1 x_1$$

$$f_K = K(x_1 - x)$$

$$f_{B1} = B_1 \frac{dx_1}{dt}$$

$$f_B = B \frac{d(x_1 - x_2)}{dt}$$

→ The i/p force should be equal to the sum of all opposing forces.

$\therefore$ i/p force $= f_{m_1} + f_{K_1} + f_K + f_{B_1} + f_B$

$$= M_1 \frac{d^2 x_1}{dt^2} + K_1 x_1 + K(x_1 - x_2) +$$

$$B_1 \frac{dx_1}{dt} + B \frac{d}{dt}(x_1 - x_2)$$

$\because$ i/p force $= 0$,

$$M_1 \frac{d^2 x_1}{dt^2} + K_1 x_1 + K(x_1 - x_2) + B_1 \frac{dx_1}{dt} + B \frac{d}{dt}(x_1 - x_2) = 0$$

$$\boxed{L\left[\frac{d^2x}{dt^2}\right] = s^2 X(s) - s x(0) - x'(0);\ L\left[\frac{dx}{dt}\right] = sX(s) - x(0)\ [\text{initial conditions} = 0]}$$

→ on taking laplace transform,

$$M_1 s^2 X_1(s) + B_1 s X_1(s) + Bs[X_1(s) - X(s)] +$$

$$K_1 X_1(s) + K[X_1(s) - X(s)] = 0.$$

→ separate, $X_1(s)$ & $X(s)$ variables,

$$X_1(s)\left[M_1 s^2 + B_1 s + Bs + K_1 + K\right] -$$

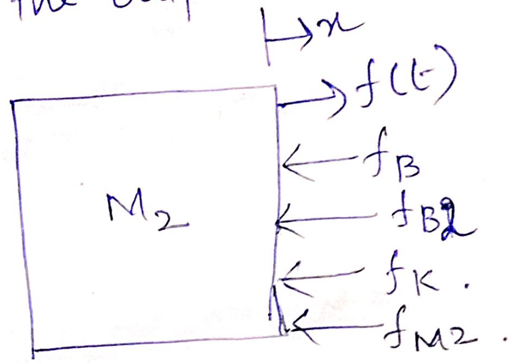
$$X(s)\left[Bs + K\right] = 0.$$

→ $X_1(s)\left[M_1 s^2 + s(B_1 + B) + K_1 + K\right] = X(s)\boxed{[Bs + K]}$

$$\boxed{X_1(s) = X(s) \frac{Bs+K}{M_1 s^2 + (B_1+B)s + (K_1+K)}} \quad \text{---} \; (1)$$

(ii) $M_2 \Rightarrow$ free body diagram.

→ The net opposing forces on M_2 will be equal to the output force $f(t)$.



$$f_{M2} = M_2 \frac{d^2x}{dt^2} \qquad f_K = K(x - x_1)$$

$$f_{B2} = B_2 \frac{dx}{dt}$$

$$f_B = B \frac{d}{dt}(x - x_1)$$

→ Net force is equal to o/p force

$$\therefore f(t) = f_{M2} + f_{B2} + f_B + f_K$$

$$f(t) = M_2 \frac{d^2x}{dt^2} + B_2 \frac{dx}{dt} + B \frac{d}{dt}(x - x_1)$$

$$+ K(x - x_1).$$

→ on taking laplace transform,

$$F(s) = M_2 \left[s^2 X(s)\right] + B_2 s X(s) + B s\left[X(s) - X_1(s)\right]$$

$$+ K\left[X(s) - X_1(s)\right]$$

$$F(s) = M_2\left[s^2 X(s)\right] + B_2 s X(s) + BsX(s) + kX(s) - BsX_1(s) - kX_1(s).$$

$$F(s) = X(s)\left[M_2 s^2 + B_2 s + Bs + K\right] - X_1(s)\left[Bs + K\right]$$

→ sub eqn (1) in $F(s)$,

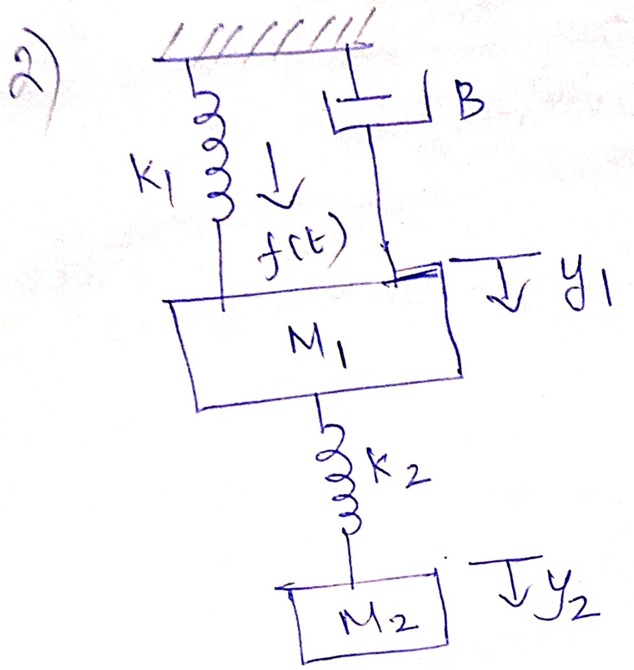
$$F(s) = X(s)\left[M_2 s^2 + B_2 s + Bs + K\right] - (Bs+K)\frac{X(s)\,(Bs+K)}{M_1 s^2 + (B+B_1)s + (K+K_1)}$$

$$F(s) = X(s)\left[\frac{\left\{\left[M_2 s^2 + B_2 s + Bs + K\right]\left[M_1 s^2 + (B+B_1)s + (K+K_1)\right]\right\} - \left\{(Bs+K)^2\right\}}{M_1 s^2 + (B+B_1)s + (K+K_1)}\right]$$

$$\rightarrow \frac{F(s)}{X(s)} = \frac{(M_2 s^2 + B_2 s + Bs + K)(M_1 s^2 + (B+B_1)s + (K+K_1)) - (Bs+K)^2}{M_1 s^2 + (B+B_1)s + (K+K_1)}.$$

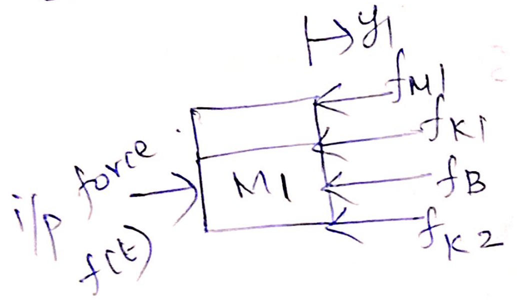
$$\Rightarrow T(s) = \frac{X(s)}{F(s)} = \frac{M_1 s^2 + (B+B_1)s + (K+K_1)}{\left[(M_2 s^2 + B_2 s + Bs + K)(M_1 s^2 + (B+B_1)s + (K+K_1)) - (Bs+K)^2\right]}$$

2)



soln

(i) $M_1 \Rightarrow$ free body diagram.



$$f(t) = f_{M1} + f_{K1} + f_B + f_{K2}.$$

$$f(t) = M_1 \frac{d^2 y_1}{dt^2} + K_1 y_1 + B \frac{dy_1}{dt} + K_2(y_1 - y_2)$$

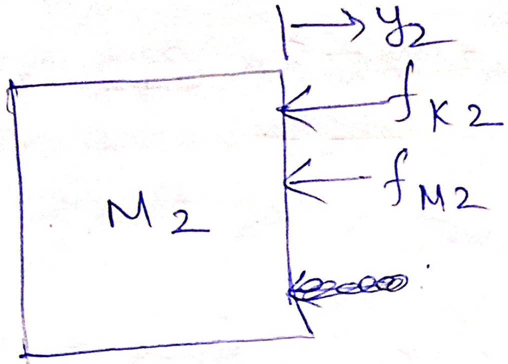
→ on Taking LT, with zero initial condt,.

$$F(s) = M_1 s^2 Y_1(s) + BsY_1(s) + K_1 Y_1(s) + K_2[Y_1(s) - Y_2(s)]$$

$$\boxed{F(s) = Y_1(s)[M_1 s^2 + Bs + K_1 + K_2] - K_2 Y_2(s)}$$

→ 1

(ii) $M_2 \Rightarrow$ free body diagram.



net opposing force $= f_{K2} + f_{M2} = 0$.

$$f_{M2} = M_2 \frac{d^2 y_2}{dt^2} \quad \& \quad f_{K2} = K_2(y_2 - y_1)$$

$$\therefore M_2 \frac{d^2 y_2}{dt^2} + K_2(y_2 - y_1) = 0.$$

→ on taking LT,

$$M_2 s^2 Y_2(s) + K_2\left[Y_2(s) - Y_1(s)\right] = 0$$

$$M_2 s^2 Y_2(s) = K_2\left[Y_1(s) - Y_2(s)\right]$$

$$M_2 s^2 Y_2(s) + K_2 Y_2(s) = K_2 Y_1(s)$$

$$Y_2(s)\left[M_2 s^2 + K_2\right] = K_2 Y_1(s)$$

$$\therefore \boxed{Y_1(s) = Y_2(s)\frac{\left[M_2 s^2 + K_2\right]}{K_2}} \quad (2)$$

→ sub ② in ①

$$F(s) = Y_2(s)\frac{\left[M_2 s^2 + K_2\right]}{K_2}\left[M_1 s^2 + Bs + K_1 + K_2\right] - K_2 Y_2(s)$$

$$F(s) = Y_2(s)\left[\frac{(M_2 s^2 + K_2)(M_1 s^2 + Bs + K_1 + K_2) - K_2^2}{K_2}\right]$$

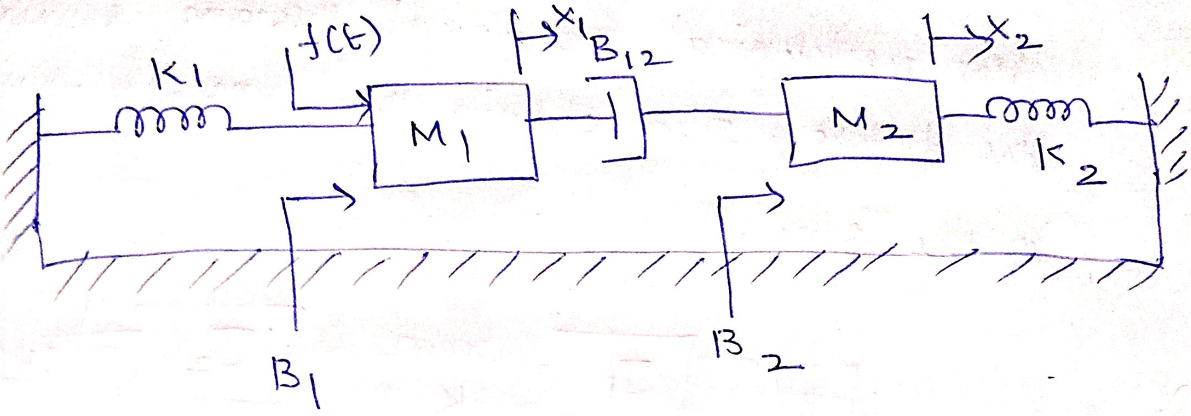
~~F(s)~~

→ Tfr function $T(s) = \dfrac{Y_2(s)}{F(s)}$

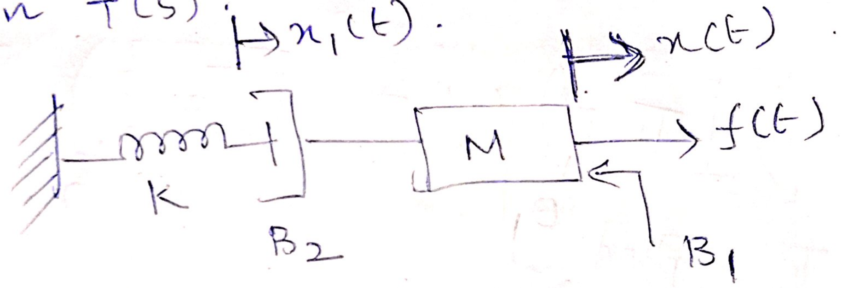
$$\boxed{T(s) = \frac{K_2}{(M_2 s^2 + K_2)(M_1 s^2 + Bs + K_1 + K_2) - K_2^2}}$$

Exercise problems

1) Determine $\frac{X_1(s)}{F(s)}$ & $\frac{X_2(s)}{F(s)}$ for the s/m.

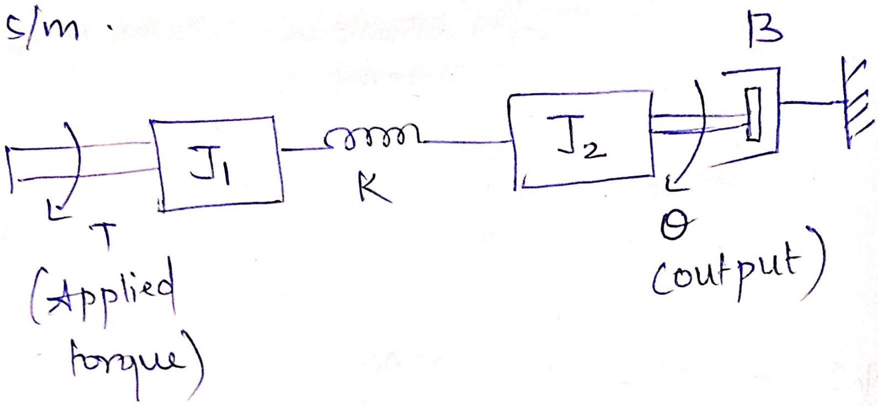


2) Write the equations of motion in s-domain for the s/m. and also determine the tfr function $T(s)$.

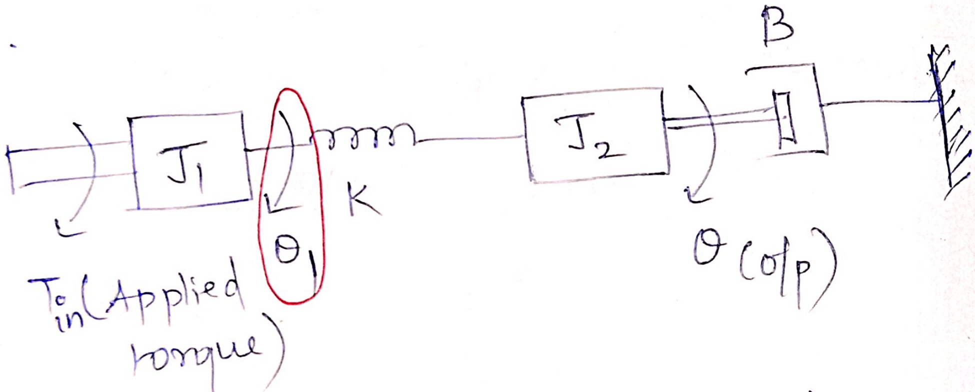


Pblms on Mechanical Rotational s/m.

1) Write the differential equations governing the mechanical rotational system shown in the figure. Also obtain the transfer function of the s/m.

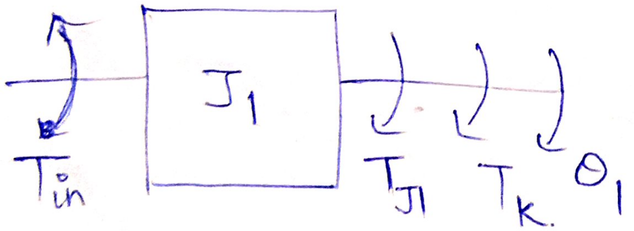


soln



Assume the J_1 s/m has the angular displacement of θ_1.

(i) $J_1 \Rightarrow$ free body diagram.



→ T_{in} = sum of ~~tot~~ opposing torques due to T_{J1} & T_K.

$$T_{J1} = J_1 \frac{d^2\theta_1}{dt^2} \qquad T_K = K(\theta_1 - \theta)$$

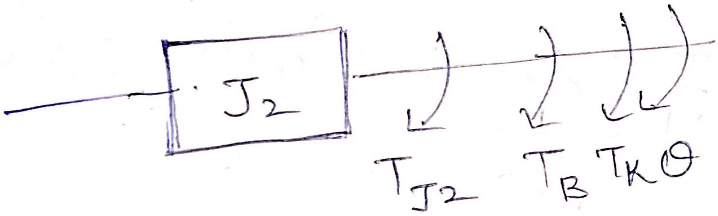
→ ∴ $T_{in} = T_{J1} + T_K$

$$T_{in} = J_1 \frac{d^2\theta_1}{dt^2} + K(\theta_1 - \theta)$$

→ LT of the above eqn will be,

$$\boxed{T_{in}(s) = J_1 s^2 \theta_1(s) + K\,\theta_1(s) - K\theta(s)} \quad (1)$$

(ii) J_2 ⇒ free body diagram.



$$T_{J2} = J_2 \frac{d^2\theta}{dt^2}$$

$$T_B = B\frac{d\theta}{dt}$$

$$T_K = K(\theta - \theta_1)$$

→ net torque is,

$$T_{J2} + T_B + T_K = 0$$

$$J_2 \frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} + K(\theta - \theta_1) = 0 .$$

→ on taking L.T.,

$$J_2 s^2 \theta(s) + Bs\,\theta(s) + K[\theta(s) - \theta_1(s)] = 0$$

$$\theta(s)\left[J_2 s^2 + Bs + K\right] = K\theta_1(s) .$$

$$\boxed{\theta_1(s) = \frac{\theta(s)\left[J_2 s^2 + Bs + K\right]}{K} .} \quad (2)$$

→ sub for $\theta_1(s)$ in (1)

$$(1) \Rightarrow \boxed{T_{in}(s) = J_1 s^2 \theta_1(s) + K\theta_1(s) - K\theta(s)}$$

$$= J_1 s^2 \theta_1(s) + K\left[\frac{\theta(s)(J_2 s^2 + Bs + K)}{K}\right] - K\theta(s)$$

$$= J_1 s^2 \theta(s) \left[\frac{J_2 s^2 + Bs + K}{K}\right] + \frac{\cancel{K}\theta(s)}{\cancel{K}}\left(J_2 s^2 + Bs + K\right)$$

$$- K\theta(s)$$

$$T_{in}(s) = \theta(s)\left[\frac{J_1(s^2)\left[J_2 s^2 + Bs + K\right]}{K} + \left(J_2 s^2 + Bs + K\right) - (K)\right]$$

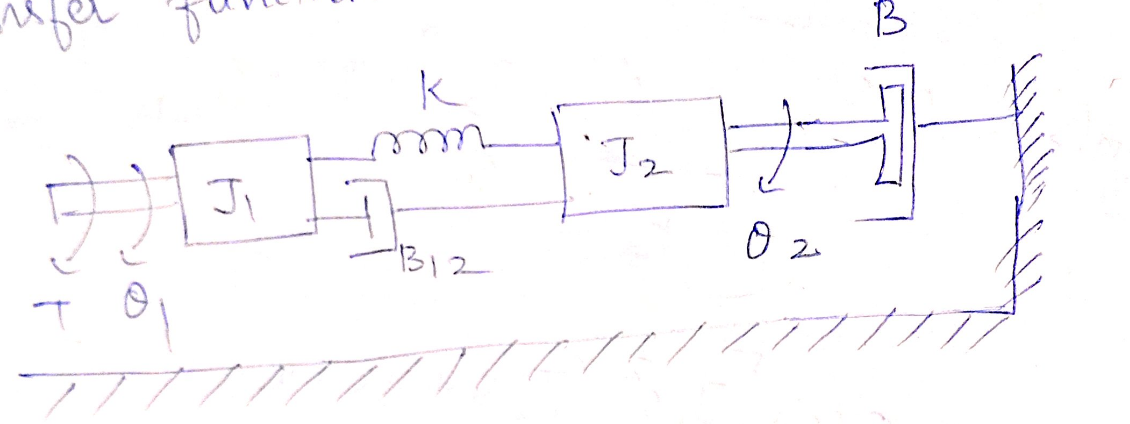
1 2 3

$$= \theta(s)\left[\frac{J_1 s^2\left(J_2 s^2 + Bs + K\right) + K\left(J_2 s^2 + Bs + K\right) - (K^2)}{K}\right]$$

$$T_{in}(s) = \theta(s)\left[\frac{\left(J_2 s^2 + Bs + K\right)\left(J_1 s^2 + K\right) - K^2}{K}\right]$$

$$\boxed{\frac{\theta(s)}{T_{in}(s)} = \frac{K}{\left(J_2 s^2 + Bs + K\right)\left(J_1 s^2 + K\right) - K^2}}$$

2) Write the differential equations governing the mechanical rotational s/m & determine its transfer function.

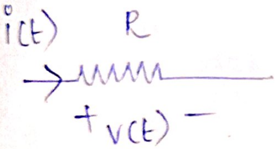
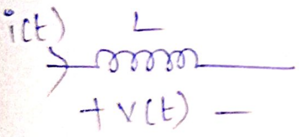
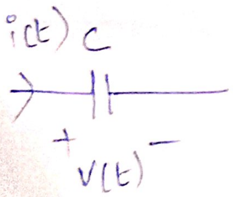


Electrical systems :-

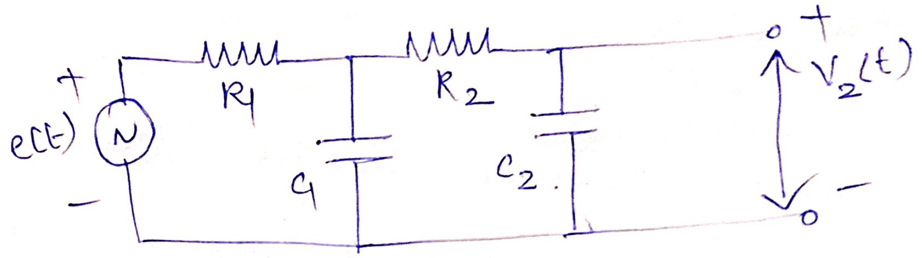
→ The models of electrical s/m can be obtained by using resistor, capacitor and inductor.

→ The differential equations governing the electrical s/ms can be formed by writing KCL or KVL & the transfer function is determined by taking L.T. of differential equations.

→ Current-Voltage Relation of R, L, C.

Element	Voltage across the Element	current thro' the element
	$v(t) = i(t)R$.	$i(t) = \frac{v(t)}{R}$
	$v(t) = L\frac{di(t)}{dt}$	$i(t) = \frac{1}{L}\int v(t)\,dt$
	$v(t) = \frac{1}{C}\int i(t)\,dt$	$i(t) = C\frac{dv(t)}{dt}$

→ Eg:- obtain the tfr function of the n/w shown below,



Soln

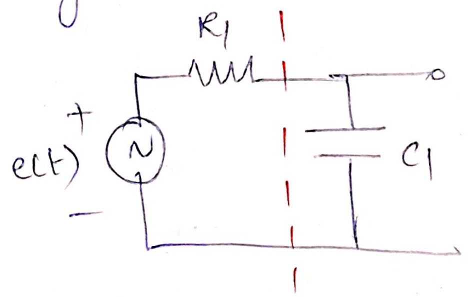
→ Tfr function = $\frac{o/p}{i/p}$ ⇒ $\frac{LT[V_2(t)]}{LT[e(t)]}$

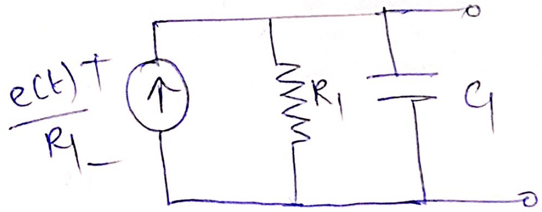
⇓

in-terms of voltage.

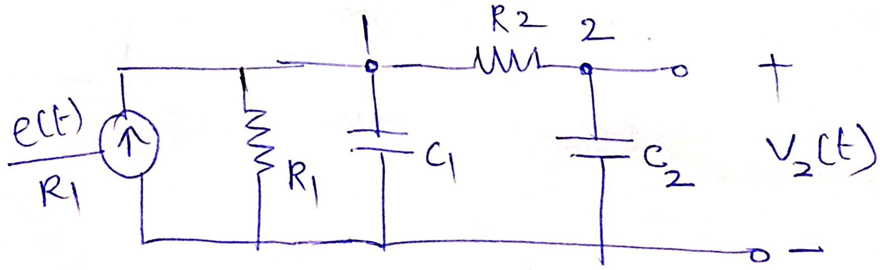
So go for nodal analysis.

→ By source-transformation theorem,





→



→ At node 1,

$$\frac{e(t)}{R_1} = \frac{V_1}{R_1} + \frac{C dV_1}{dt} + \frac{V_1 - V_2}{R_2}$$

→ LT of above eqn,

$$\boxed{\frac{E(s)}{R_1} = \frac{V_1(s)}{R_1} + C s V_1(s) + \frac{1}{R_2}\left[V_1(s) - V_2(s)\right]} \rightarrow (1)$$

→ At node 2,

$$\frac{V_1 - V_2}{R_2} = C_2 \frac{dV_2}{dt}.$$

→ LT of above eqn;

$$\frac{1}{R_2}\left[V_1(s) - V_2(s)\right] = C_2 s V_2(s)$$

$$\frac{V_1(s)}{R_2} = C_2 s V_2(s) + \frac{1}{R_2} V_2(s).$$

$$V_1(s) = R_2\left[\frac{R_2 C_2 s V_2(s) + V_2(s)}{R_2}\right]$$

$$\boxed{\begin{aligned} V_1(s) &= R_2 C_2 s V_2(s) + V_2(s) \\ V_1(s) &= V_2(s)\left[1 + R_2 C_2 s\right] \end{aligned}} \rightarrow (2)$$

→ sub ② in ①.

$$\frac{E(s)}{R_1} = \frac{V_1(s)}{R_1} + CsV_1(s) + \frac{1}{R_2}\left[V_1(s) - V_2(s)\right]$$

$$\frac{E(s)}{R_1} = V_1(s)\left[\frac{1}{R_1} + \frac{1}{R_2} + Cs\right] - \frac{1}{R_2}V_2(s)$$

$$\frac{E(s)}{R_1} = \left(\frac{1}{R_1} + \frac{1}{R_2} + Cs\right)\left(V_2(s)\left(1 + R_2C_2s\right)\right) - \frac{1}{R_2}V_2(s)$$

$$\frac{E(s)}{R_1} = \left[\left(\frac{R_1 + R_2 + R_1R_2Cs}{R_1R_2}\right)\left(1 + R_2C_2s\right) - \frac{1}{R_2}\right]V_2(s)$$

→ Tfr func $= \dfrac{V_2(s)}{E(s)}$

$$\frac{E(s)}{\cancel{R_1}} = \left[\frac{\left(R_1 + R_2 + R_1R_2Cs\right)\left(1 + R_2C_2s\right) - R_1}{\cancel{R_1}R_2}\right]V_2(s)$$

$$\therefore \boxed{\frac{V_2(s)}{E(s)} = \frac{R_2}{\left[\left(R_1 + R_2 + R_1R_2Cs\right)\left(1 + R_2C_2s\right) - R_1\right]}}$$

Electrical Analogous of Mechanical-Translational s/m.

→ The three basic elements mass, dash-pot & spring that are used in modelling mechanical-translational s/ms are analogous to resistance, inductance and capacitance of electrical s/m.

→ (i) Force - Voltage Analogy.

(a) Elements having same current ⇒ series connection.

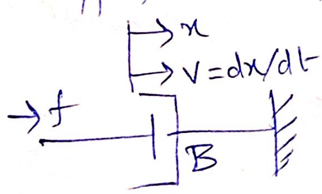
Elements having same velocity ⇒ series connection.

(b) Each node in mechanical s/m ~~should have the~~ corresponds to a closed loop in electrical s/m. A mass is considered as a node.

(c) No. of mesh currents = No. of velocities of nodes.

→ Mechanical s/m

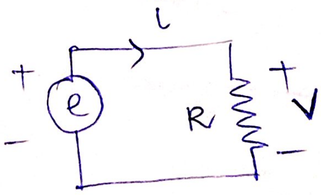
i/p: force o/p: velocity

1) 

$$f = B\frac{dx}{dt} = Bv.$$

Electrical s/m.

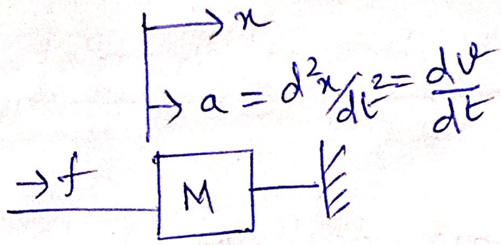
i/p: voltage o/p: current thru the element



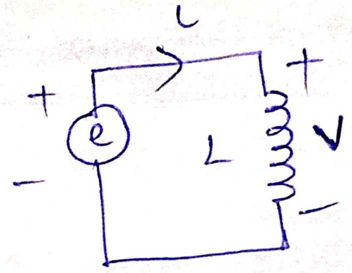
$$e = V = Ri$$

$$i = \frac{V}{R}$$

2)



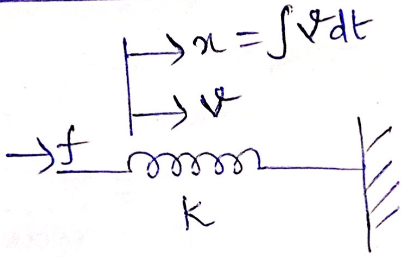
$$f = M\frac{d^2x}{dt^2} = M\frac{dv}{dt}$$



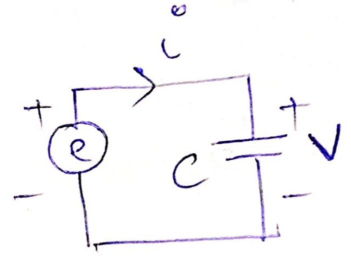
$$V = e = L\frac{di}{dt}.$$

$$i = \frac{1}{L}\int v\,dt$$

3)



$$f = Kx = K\int v\,dt.$$



$$V = e = \frac{1}{C}\int i\,dt.$$

$$i = C\frac{dv}{dt}$$

→ Analogous Quantities in Force-Voltage Analogy:
(i/p)

Item	Mechanical s/m	Electrical s/m
Independent variable (I/p)	Force, (f)	Voltage (e, V)
Dependent variable (o/p)	Velocity, (v) Displacement, (x)	current (i) charge (q)
Dissipative element	Frictional co-eff of dash-pot (B)	Resistance (R)
Storage element	Mass, (M) stiffness of spring, (K)	Inductance (L) Inverse of capacitance ($\frac{1}{C}$)

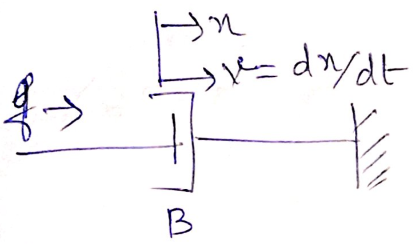
Physical law	Newton's second law $\Sigma f = 0$	KVL $\Sigma V = 0$.

(ii) Force - current Analogy.

Mechanical s/m

I/p: force

o/p: velocity

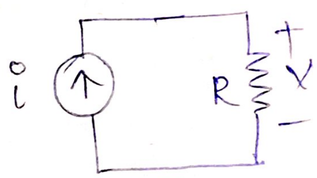


$$f = B\frac{dx}{dt} = Bv$$

Electrical s/m

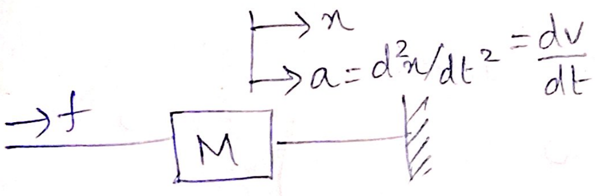
i/p: current source

o/p: voltage across the element.

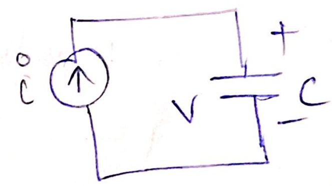


$$V = iR$$

$$i = V/R$$

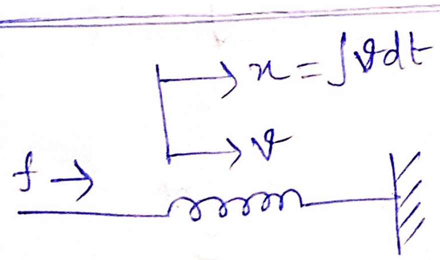


$$f = M\frac{d^2x}{dt^2} = M\frac{dv}{dt}$$

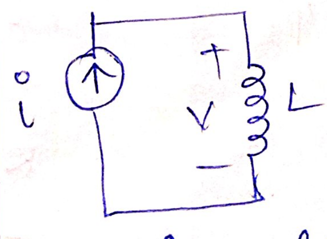


$$i = C\frac{dv}{dt}$$

$$V = \frac{1}{C}\int i\,dt$$



$$f = Kx = \int v\,dt$$



$$i = \frac{1}{L}\int v\,dt$$

$$V = L\frac{di}{dt}$$

→ Analogous Quantities in force-current analogy (i/p)

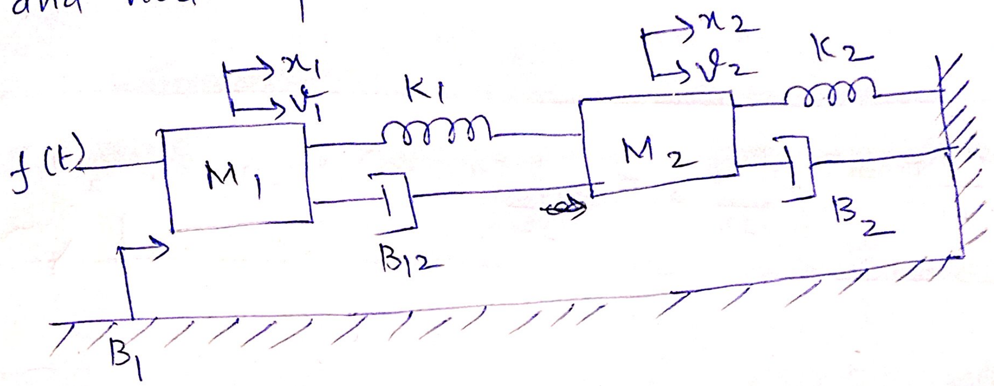
Item	Mechanical s/m	Electrical s/m.
Independent Variable (i/p)	force (f)	current, i
Dependent Variable (o/p)	Velocity (v) Displacement, (x)	Voltage (V) Flux (ϕ)
Dissipative element	Frictional co-eff of Dashpot (B)	conductance ($G = 1/R$)
storage element.	Mass (M) Stiffness of spring (K)	capacitance (C) Inverse of Inductance, ($1/L$)
physical law	Newton's II law $\Sigma f = 0$	KCL $\Sigma i = 0$

→ comparison:-

M-s/m	F-V	F-C
f (i/p)	V (i/p)	i (i/p)
v (o/p)	i (o/p)	V (o/p).
x	q	ϕ
B	R	$1/R$
M	L	C
K	$1/C$	$1/L$

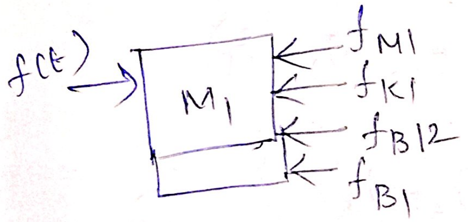
pblm.

1) Write the differential equations governing the mechanical s/m. Draw the F-V & F-C electrical analogous circuits & verify by writing Mesh and node equations.



Soln

$M_1 \to$ free body diagram.



$$f_{M1} = M_1 \frac{d^2 x_1}{dt^2}$$

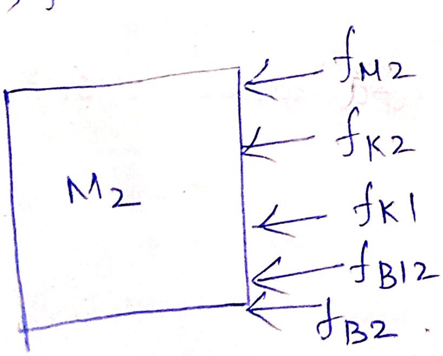
$$f_{K1} = K_1 (x_1 - x_2)$$

$$f_{B1} = B_1 \frac{dx_1}{dt}$$

$$f_{B12} = B_{12} \frac{d(x_1 - x_2)}{dt}.$$

$$\boxed{f(t) = M_1 \frac{d^2 x_1}{dt^2} + B_1 \frac{dx_1}{dt} + B_{12} \frac{d(x_1 - x_2)}{dt} + K_1 (x_1 - x_2)} \quad \text{--- (1)}$$

$M_2 \to$ free body diagram.



$$f_{M2} = M_2 \frac{d^2 x_2}{dt}$$

$$f_{K2} = K_2 (x_2 - x_1)$$

$$f_{B12} = B_{12} \frac{d(x_2 - x_1)}{dt}$$

$$f_{B2} = B_2 \frac{d(x_2)}{dt}$$

$$f_{K1} = K_1 (x_2 - x_1)$$

We know that

$$\left.\begin{aligned} \frac{d^2x}{dt^2} &= \frac{dv}{dt} \\ \frac{dx}{dt} &= v \\ x &= \int v\,dt. \end{aligned}\right\} \Rightarrow \text{I}$$

$$\boxed{M_2 \frac{d^2 x_2}{dt^2} + B_2 \frac{dx_2}{dt} + B_{12} \frac{d}{dt}(x_2 - x_1) + K_2 x_2 + K_1(x_2 - x_1) = 0 \quad \to (2)}$$

→ sub (I) in (1) & (2).

$$(1) \Rightarrow M_1 \frac{dv_1}{dt} + B_1 v_1 + B_{12}(v_1 - v_2) + K_1\left(\int (v_1 - v_2)\,dt\right) = f(t) \quad \to (3)$$

$$(2) \Rightarrow M_2 \frac{dv_2}{dt} + B_2 v_2 + K_2 \int v_2\,dt + B_{12}(v_2 - v_1) + K_1 \int (v_2 - v_1)\,dt = 0 \quad \to (4)$$

(i) F-V

$v_1 \to i_1$ & $v_2 \to i_2$.

$f(t)$ = Voltage V or e,

M_1 = Inductance L_1, $M_2 = L_2$.

B_1 = Resistance R_1, $B_2 = R_2$.

B_{12} = Resistance Between 2 mesh or 2 node R_{12}.

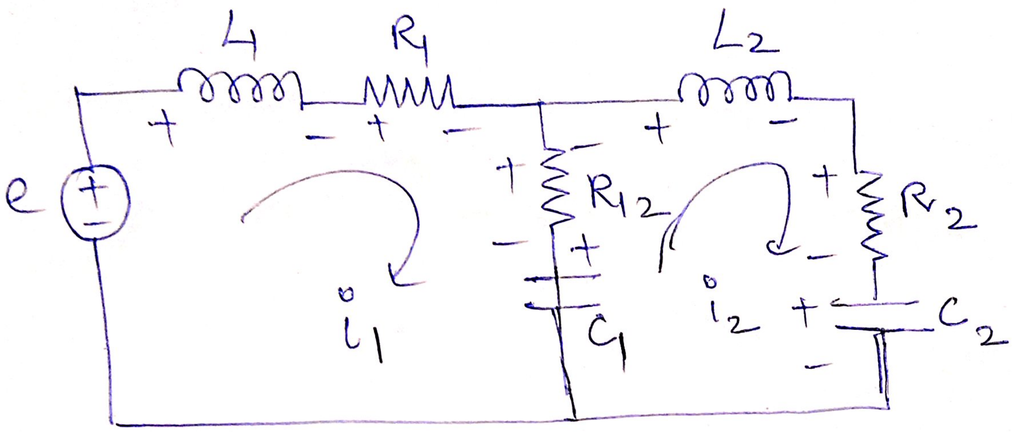
K_1 = Inverse of capacitance $(1/C_1)$

$K_2 = 1/C_2$.

2 → nodes ⇒ 2 - loop/mesh.

↑

2-mass.



$$e(t) = L_1 \frac{di_1}{dt} + R_1 i_1 + R_{12}(i_1 - i_2) + \frac{1}{C_1}\int (i_1 - i_2)\,dt \quad \text{--- (5)}$$

$$R_{12}(i_2 - i_1) + L_2 \frac{di_2}{dt} + R_2 i_2 + \frac{1}{C_2}\int i_2\,dt$$

$$+ \frac{1}{C_1}\int (i_2 - i_1)\,dt = 0 \quad \text{--- (6)}$$

Electrical Analogous of Mechanical-Translational s/m.

→ The three basic elements mass, dash-pot & spring that are used in modelling mechanical-translational s/ms are analogous to resistance, inductance and capacitance of electrical s/m.

→ (i) Force - voltage Analogy.

(a) Elements having same current ⇒ series connection.

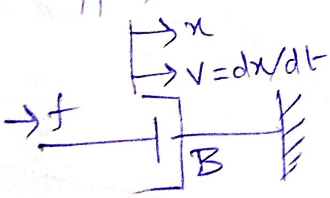
Elements having same velocity ⇒ series connection.

(b) Each node in mechanical s/m ~~should have the~~ corresponds to a closed loop in electrical s/m. A mass is considered as a node.

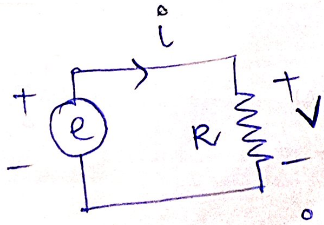
(c) No. of mesh currents = No. of velocities of nodes.

→

Mechanical s/m	Electrical s/m
i/p: force o/p: velocity	i/p: voltage o/p: current thru the element

1) 

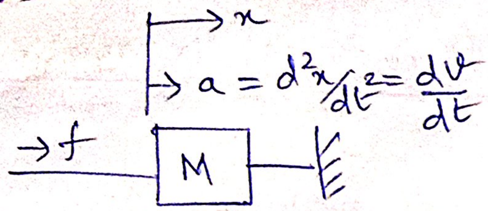
$f = B\frac{dx}{dt} = Bv.$



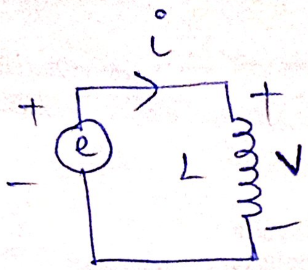
$e = V = Ri$

$i = \frac{V}{R}$

2)



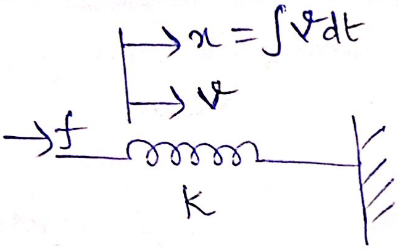
$$f = M\frac{d^2x}{dt^2} = M\frac{dv}{dt}$$



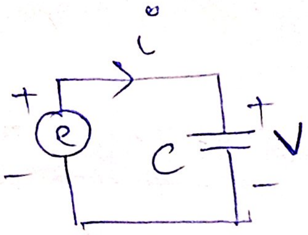
$$V = e = L\frac{di}{dt}.$$

$$i = \frac{1}{L}\int v\,dt$$

3)



$$f = Kx = K\int v\,dt.$$



$$V = e = \frac{1}{C}\int i\,dt.$$

$$i = C\frac{dv}{dt}$$

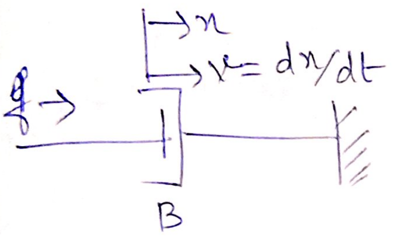
→ Analogous Quantities in Force-Voltage Analogy:
(i/p)

Item	Mechanical s/m	Electrical s/m
Independent variable (I/P)	Force, (f)	voltage (e, V)
Dependent variable (o/p)	Velocity, (v) Displacement, (x)	current (i) Charge (q)
Dissipative element	Frictional co-eff of dash-pot (B)	Resistance (R)
Storage element	Mass, (M) stiffness of spring (K)	Inductance (L) Inverse of capacitance (1/C)

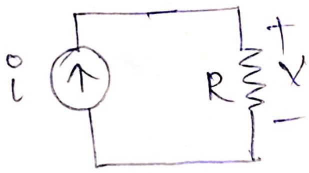
physical law	Newton's second law $\Sigma f = 0$	KVL $\Sigma V = 0$

(ii) Force - current Analogy.

Mechanical s/m	Electrical s/m
I/p: force o/p: velocity	i/p: current source o/p: voltage across the element.

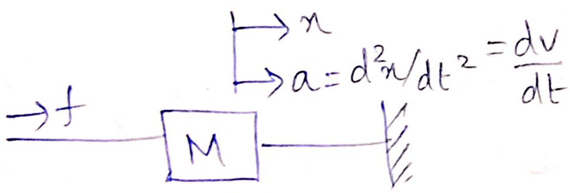


$$f = B\frac{dx}{dt} = BV$$

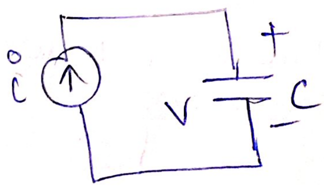


$$V = iR$$

$$i = V/R$$

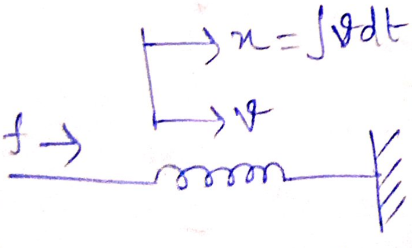


$$f = M\frac{d^2x}{dt^2} = M\frac{dv}{dt}$$

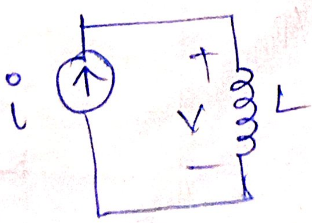


$$i = C\frac{dv}{dt}$$

$$V = \frac{1}{C}\int i\,dt$$



$$f = Kx = \int v\,dt$$



$$i = \frac{1}{L}\int V\,dt$$

$$V = L\frac{di}{dt}$$

→ Analogous Quantities in force-current analogy (i/p)

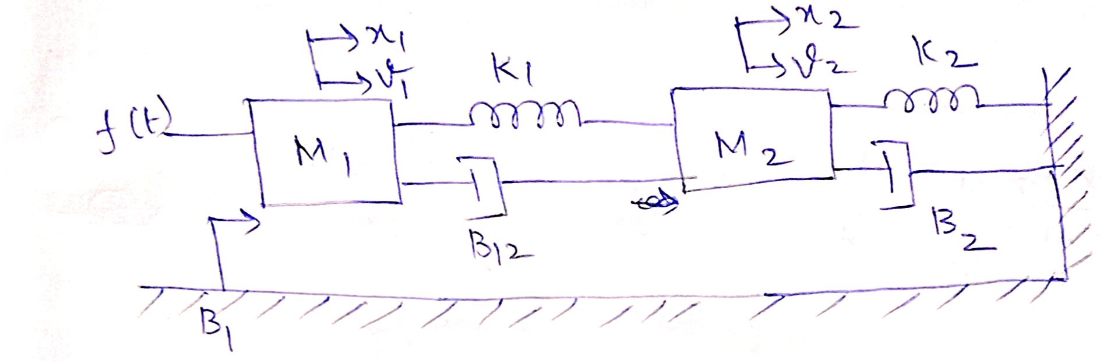
Item	Mechanical s/m	Electrical s/m.
Independent Variable (i/p)	force (f)	current, i
Dependent Variable (o/p)	Velocity (v) Displacement, (x)	Voltage (V) Flux (ϕ)
Dissipative element	Frictional co-eff of Dash pot (B)	conductance ($G = 1/R$)
Storage element.	Mass (M) Stiffness of spring (K)	capacitance (C) Inverse of Inductance, (1/L)
physical law	Newton's II law $\Sigma f = 0$	KCL $\Sigma i = 0$

→ comparison:-

M-s/m	F-V	F-C
f (i/p)	V (i/p)	i (i/p)
v (o/p)	i (o/p)	V (o/p).
x	q	ϕ
B	R	1/R
M	L	C
K	1/C	1/L

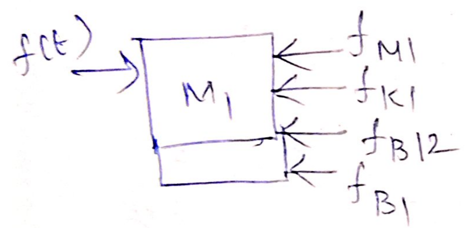
pblm

1) Write the differential equations governing the mechanical s/m. Draw the F-V & F-C electrical analogous circuits & verify by writing Mesh and node equations.



Soln

$M_1 \to$ free body diagram.



$$f_{M1} = M_1 \frac{d^2 x_1}{dt^2}$$

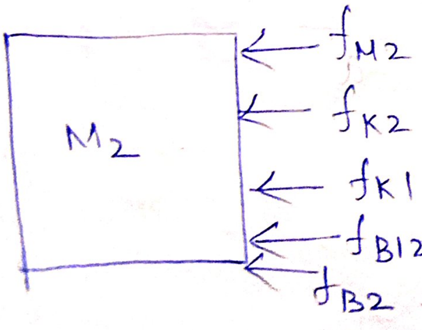
$$f_{K1} = K_1 (x_1 - x_2)$$

$$f_{B1} = B_1 \frac{dx_1}{dt}$$

$$f_{B12} = B_{12} \frac{d(x_1 - x_2)}{dt}.$$

$$\boxed{f(t) = M_1 \frac{d^2 x_1}{dt^2} + B_1 \frac{dx_1}{dt} + B_{12} \frac{d(x_1 - x_2)}{dt} + K_1 (x_1 - x_2) \quad \text{—— (1)}}$$

$M_2 \to$ free body diagram.



$$f_{M2} = M_2 \frac{d^2 x_2}{dt}$$

$$f_{K2} = K_2 (x_2 - x_1)$$

$$f_{B12} = B_{12} \frac{d(x_2 - x_1)}{dt}$$

$$f_{B2} = B_2 \frac{d(x_2)}{dt}$$

$$f_{K1} = K_1 (x_2 - x_1)$$

We know that

$$\left.\begin{aligned} \Rightarrow \frac{d^2x}{dt^2} &= \frac{dv}{dt} \\ \frac{dx}{dt} &= v \\ x &= \int v\,dt . \end{aligned}\right\} \Rightarrow \text{(I)}$$

$$\boxed{M_2\frac{d^2x_2}{dt^2} + B_2\frac{dx_2}{dt} + B_{12}\frac{d}{dt}(x_2-x_1) + K_2x_2 + K_1(x_2-x_1) = 0} \rightarrow (2)$$

$\rightarrow$ sub (I) in (1) & (2).

(1) $\Rightarrow M_1\frac{dv_1}{dt} + B_1v_1 + B_{12}(v_1-v_2) + K_1\left(\int(v_1-v_2)dt\right) = f(t)$ $\rightarrow$ (3)

(2) $\Rightarrow M_2\frac{dv_2}{dt} + B_2v_2 + K_2\int v_2\,dt + B_{12}(v_2-v_1)$

$+ K_1\int(v_2-v_1)\,dt = 0.$ $\rightarrow$ (4)

(i) F-V Analogous ckt

no. of currents

$f(t)$ = voltage V or e, $\boxed{v_1 \to i_1 \;\&\; v_2 \to i_2}$.

M_1 = Inductance L_1, $M_2 = L_2$.

B_1 = Resistance R_1, $B_2 = R_2$.

B_{12} = Resistance Between 2 mesh or 2 node R_{12}

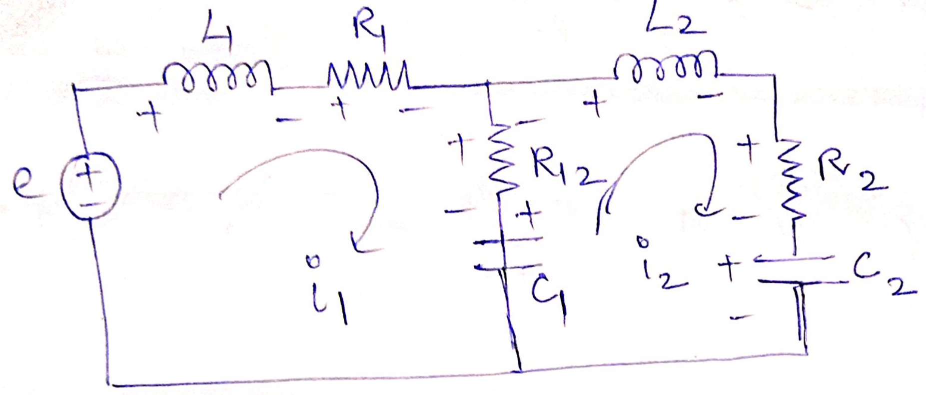
K_1 = Inverse of capacitance ($1/C_1$)

$K_2 = 1/C_2$.

$2 \to$ ~~nodes~~ Mass $\Rightarrow$ 2 - loop/mesh.

$\Uparrow$

~~2-mass~~.



$$e(t) = L_1 \frac{di_1}{dt} + R_1 i_1 + R_{12}(i_1 - i_2) + \frac{1}{C_1}\int (i_1 - i_2)\,dt \quad \text{—— (5)}$$

$$R_{12}(i_2 - i_1) + L_2 \frac{di_2}{dt} + R_2 i_2 + \frac{1}{C_2}\int i_2\,dt + \frac{1}{C_1}\int (i_2 - i_1)\,dt = 0 \quad \text{—— (6)}$$

(ii) F-C Analogous circuit

no. of voltages.

$f(t) = i(t)$, $\boxed{v_1 = V_1 \;\&\; v_2 = V_2}$

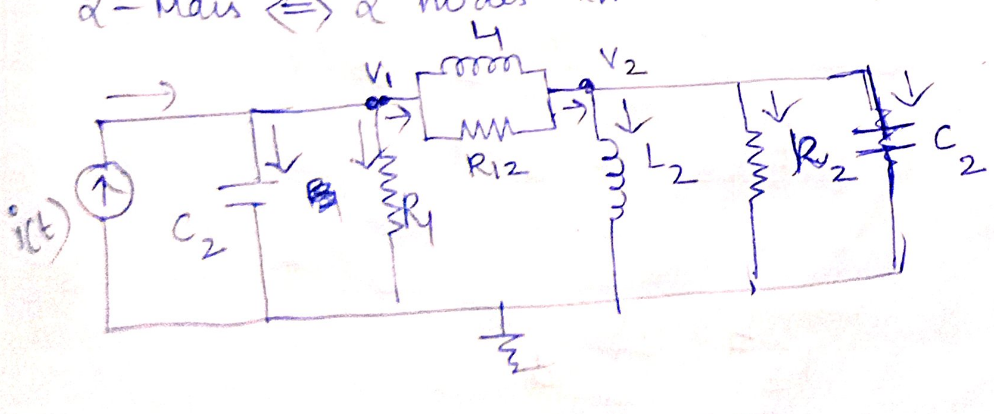
$K_1 = 1/L_1 \Rightarrow$ ||el conn

$M_1 \to C_1$ $M_2 \to C_2$

$K_2 = 1/L_2 \Rightarrow$ ||el conn.

$B_1 = 1/R_1$ $B_2 = 1/R_2$ $B_{12} = 1/R_{12}$

2-Mass $\Leftrightarrow$ 2 nodes in the electric ckt.



→ By KCL at node 1 & 2.

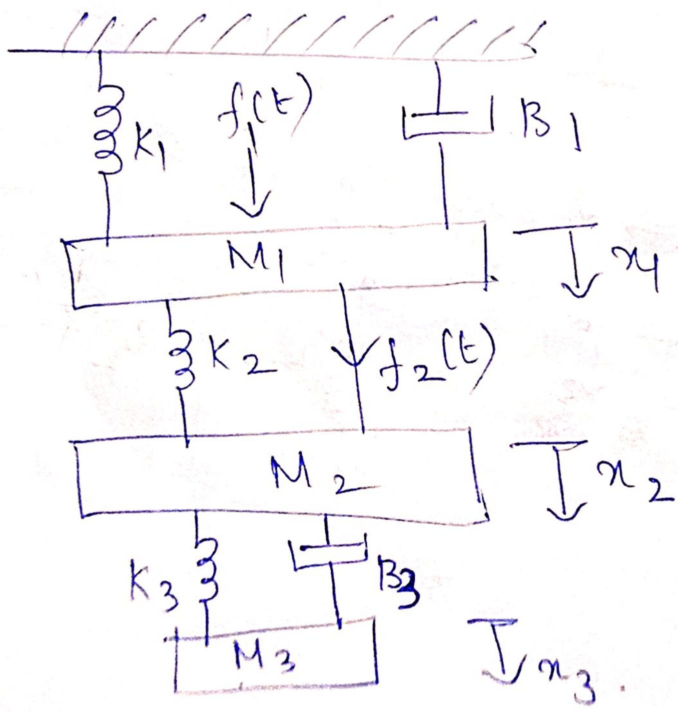
node (1) ⇒ $$\boxed{i(t) = C_2\frac{dV_2}{dt} + \frac{V_1}{R_1} + \frac{1}{L}\int (V_1 - V_2)\,dt + \frac{V_1 - V_2}{R_{12}}}$$ → (7)

node (2) ⇒ $$\frac{1}{L}\int (V_1 - V_2)\,dt + \frac{V_1 - V_2}{R_2} = \frac{1}{L_2}\int V_2\,dt + \frac{V_2}{R_2} + C_2\frac{dV_2}{dt}$$

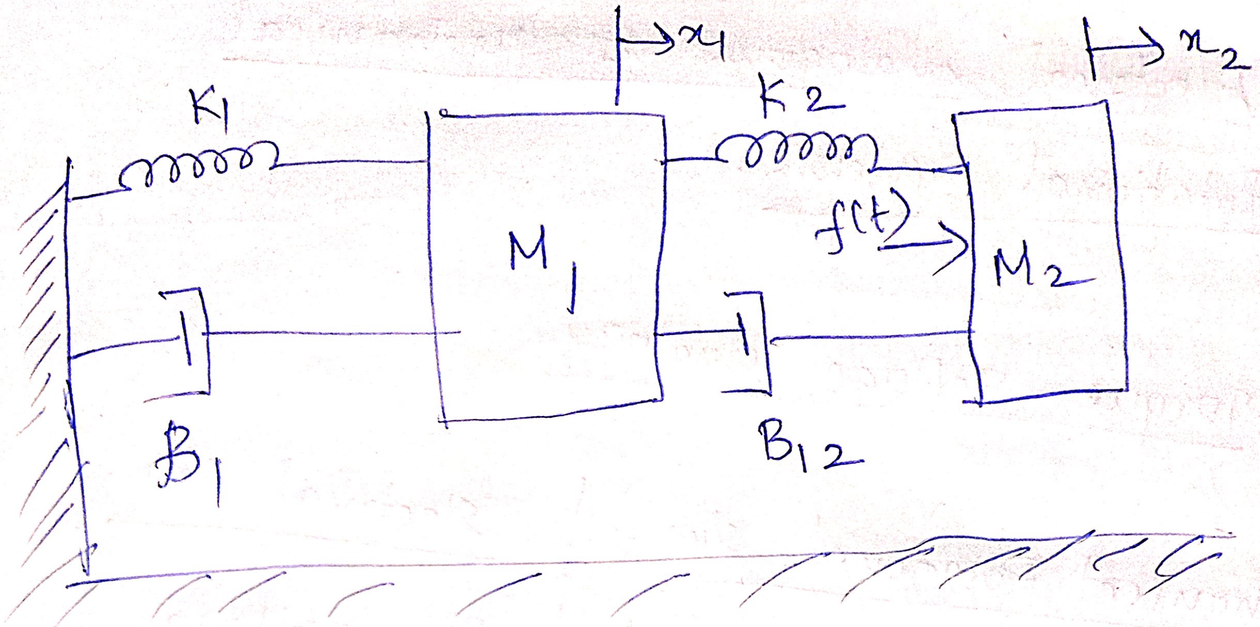
$$\boxed{C_2\frac{dV_2}{dt} + \frac{V_2}{R_2} + \frac{1}{L_2}\int V_2\,dt - \frac{1}{L}\int (V_1 - V_2)\,dt + \frac{V_2 - V_1}{R_2} = 0}$$ → (8).

Exercise.

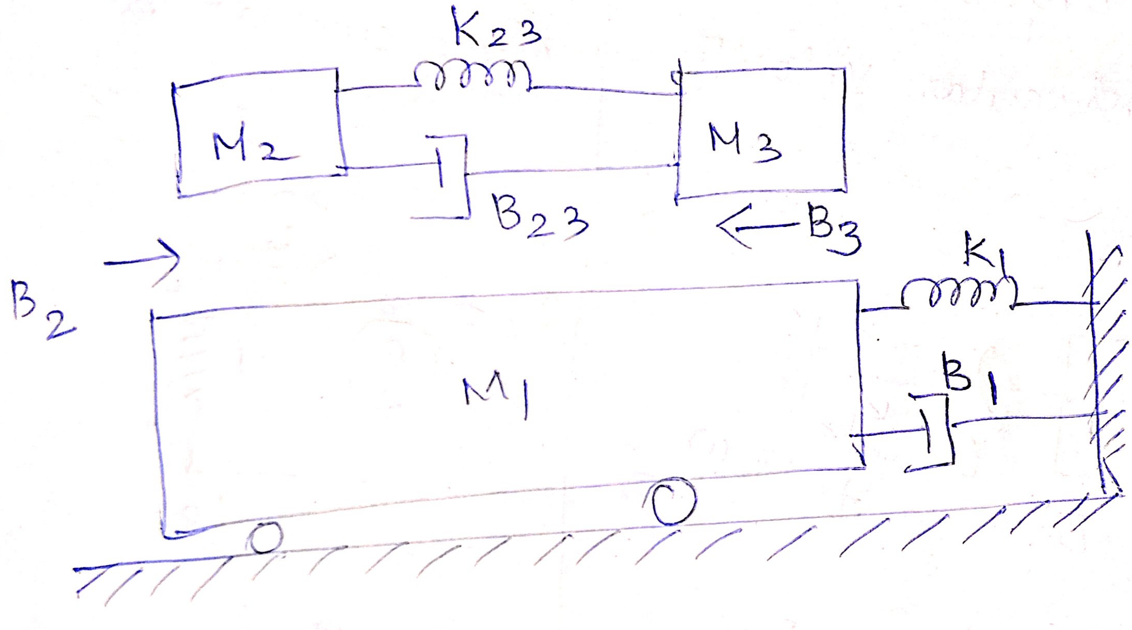
1) Write the differential equations governing the mechanical s/m shown in the fig. Draw the F-V & F-C electrical analogous circuits, and verify by writing mesh & node equations.



2)



3)



Electrical analogous of Mechanical Rotational systems.

(1) Torque - Voltage Analogy.

Mechanical Rotational s/m.

i/p : Torque

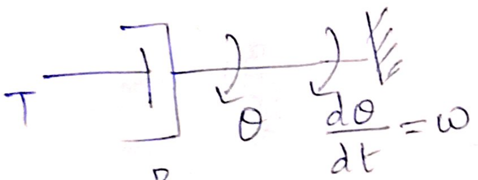
o/p : Angular velocity

Electrical s/m.

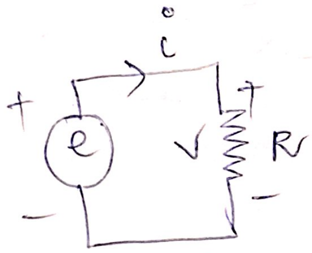
i/p : Voltage

o/p : current.

1)



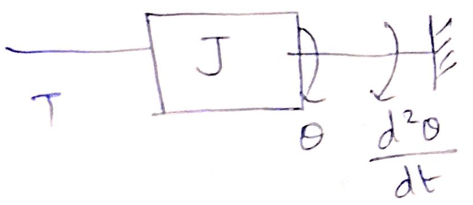
$$T = B\frac{d\theta}{dt} = B\omega$$



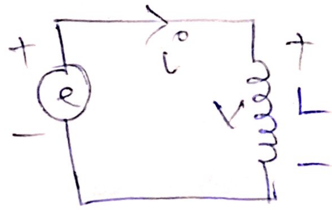
$$V = e = iR$$

$$i = V/R.$$

2)



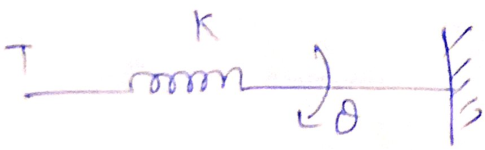
$$T = J\frac{d^2\theta}{dt^2} = J\frac{d\omega}{dt}$$



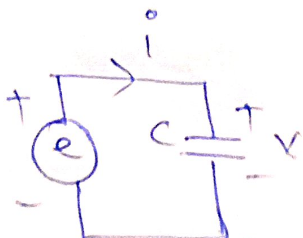
$$V = e = L\,di/dt$$

$$i = 1/L \int V\,dt$$

3)



$$T = K\theta = K\int \omega\,dt.$$



$$V = e = 1/C \int i\,dt$$

$$i = C(dV/dt)$$

→ Torque-Voltage Analogy - Quantities

Item	Mechanical Rotational s/m	Electrical s/m.
Independent variable (i/p)	Torque, T	Voltage (e, V)
Dependent variable (o/p)	Angular velocity (ω)	current (i)
	Angular displacement (θ)	charge (q)
Dissipative element	Rotational co-eff of dash-pot (B)	Resistance (R)
Storage element	Moment of Inertia (J)	Inductance (L)
	Stiffness of spring (K)	Inverse of capacitance ($1/C$)
Physical law	Newton's second law $\Sigma T = 0$.	KVL $\Sigma V = 0$.

2) Torque - current Analogy.

Mechanical rotational s/m

i/p : Torque

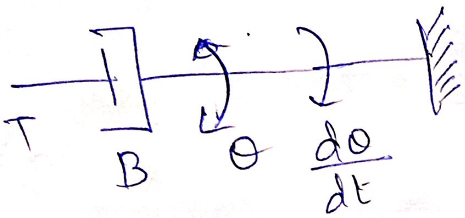
o/p : Angular velocity (ω)

Electrical s/m

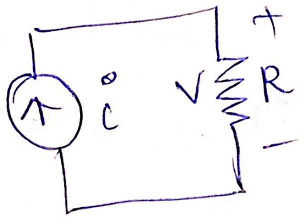
I/p : current

o/p : voltage.

1)



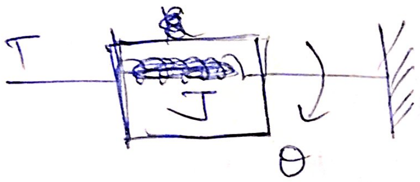
$$T = B\frac{d\theta}{dt} = B\omega$$



$$V = iR.$$

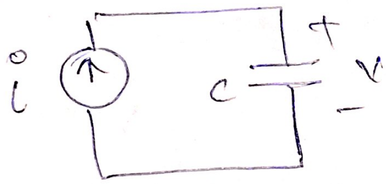
$$i = V/R$$

2)



$$T = J\frac{d^2\theta}{dt^2}.$$

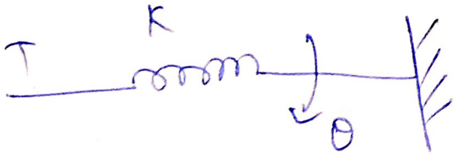
$$T = J\frac{d\omega}{dt}$$



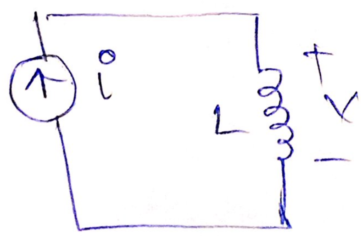
$$i = c\frac{dv}{dt}.$$

$$V = \frac{1}{C}\int i\,dt.$$

3)



$$T = K\theta$$
$$= K\int \omega\,dt.$$



$$V = L\frac{di}{dt}$$

$$i = \frac{1}{L}\int v\,dt$$

→ Analogous Quantities in Torque - current Analogy.

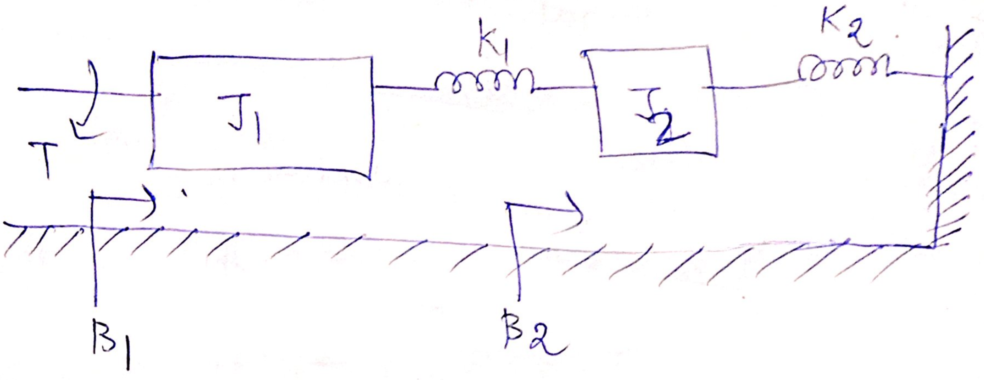
Item	Mechanical Rotational S/m	Electrical S/m.
Independent variable. (i/p)	Torque (T)	current (i)
Dependent variable. (o/p)	Angular velocity (ω)	voltage (V)
	Angular displacement (θ)	flux (φ)
Dissipative element.	Dashpot (B)	conductance (G) = 1/R.
Storage element	Moment of Inertia (J)	capacitance (C)
	Stiffness of Spring (K)	Inverse of inductance (1/L)
Physical law	Newton's II. law $\Sigma T = 0$.	KCL $\Sigma i = 0$.

comparison

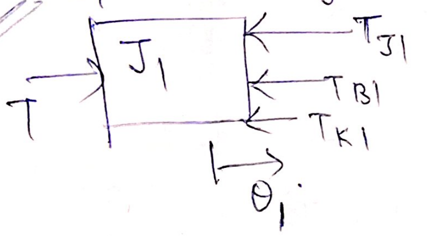
M-R S/m	F-V	F-C
T (i/p)	V	i
ω (o/p)	i	V
θ	q	φ
B	R	1/R
J	L	C
K	1/C	1/L

eg:-

1) Write the differential equation of the Mechanical-Rotational s/m, Draw T-V & T-C analogous circuits and verify using mesh and node equations.



Solution: J_1 - free body diagram

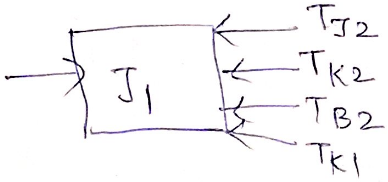


$$T_{J1} = J_1 \frac{d^2\theta_1}{dt^2}$$

$$T_{B1} = B_1 \frac{d\theta_1}{dt}$$

$$T_{K1} = K_1(\theta_1 - \theta_2)$$

$J_2 \rightarrow$ free-body diagram



$$T_{J2} = J_2 \frac{d^2\theta_2}{dt^2}$$

$$T_{K_2} = K_2 \theta_2$$

$$T_{B2} = B_2 \frac{d\theta_2}{dt}$$

$$T_{K1} = K_1(\theta_2 - \theta_1)$$

$$1 \Rightarrow J_1 \frac{d^2\theta_1}{dt^2} + B_1 \frac{d\theta_1}{dt} + K_1(\theta_1 - \theta_2) = T$$

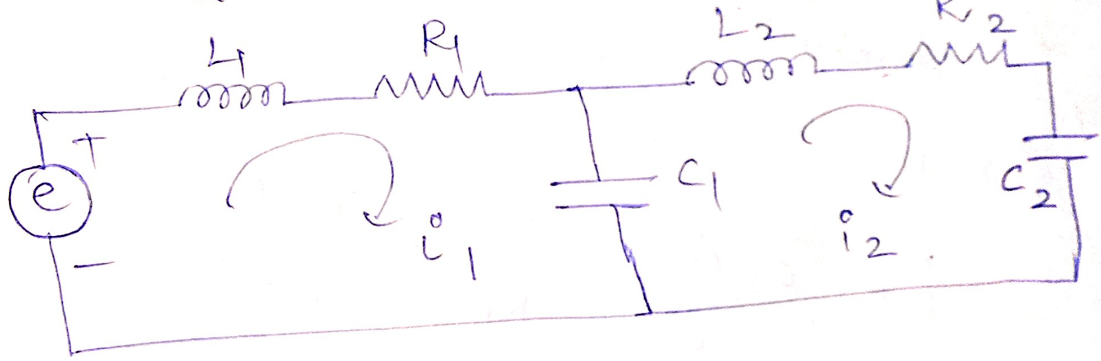
$$2 = J_2 \frac{d^2\theta_2}{dt^2} + B_2 \frac{d\theta_2}{dt} + K_2\theta_2 + K_1(\theta_2 - \theta_1) = 0.$$

→ Rewrite everything in terms of angular velocity.

wkt $\theta = \int \omega \, dt$ $\quad \frac{d^2\theta}{dt^2} = \frac{d\omega}{dt}$ $\quad \frac{d\theta}{dt} = \omega$

$$\therefore \boxed{\begin{aligned} & J_1 \frac{d\omega_1}{dt} + B_1\omega_1 + K_1 \int(\omega_1 - \omega_2)\,dt = T \quad (3) \\ & J_2 \frac{d\omega}{dt} + B_2\omega_2 + K_2 \int \omega_2 \, dt + K_1 \int(\omega_2 - \omega_1)\,dt = 0 \quad (4) \end{aligned}}$$

(i) – T – V . $J_1 \to L_1$, $T \to V$ or e, $J_2 \to L_2$
$B_1 \to R_1$, $B_2 \to R_2$, $K_1 \to 1/C_1$
$K_2 \to 1/C_2$ & $\omega_1 = i_1$, $\omega_2 = i_2$.



KVL

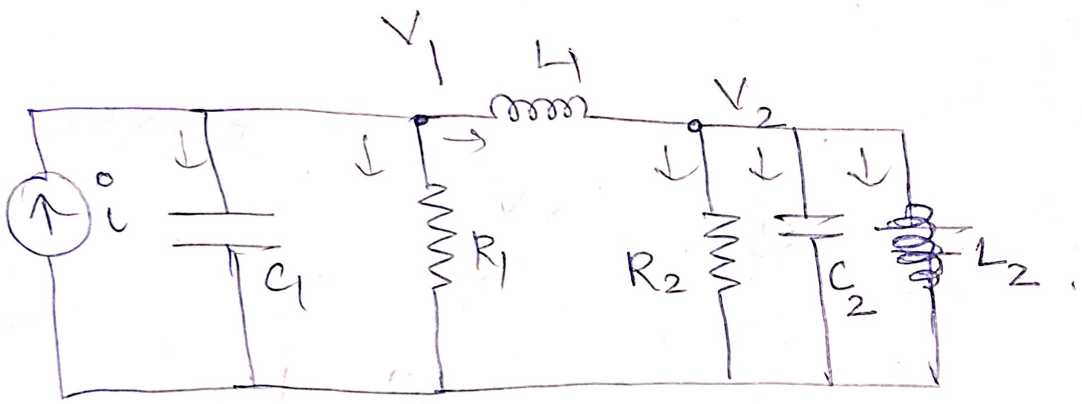
$$\boxed{\begin{aligned} & L_1 \frac{di_1}{dt} + R_1 i_1 + \frac{1}{C_1}\int(i_1 - i_2)\,dt = e \quad (5) \\ & L_2 \frac{di_2}{dt} + i_2 R_2 + \frac{1}{C_2}\int i_2 \, dt + \frac{1}{C_1}\int(i_2 - i_1)\,dt = 0 \quad (6) \end{aligned}}$$

(ii) T-C Analogy

$J_1 \rightarrow C_1$ $\quad B_1 = 1/R_1$ $\quad K_1 = 1/L_1$

$J_2 \rightarrow C_2$ $\quad B_2 = 1/R_2$ $\quad K_2 = 1/L_2$

$T \rightarrow i$ $\quad \omega_1 \rightarrow V_1.$ $\quad \omega_2 \rightarrow V_2.$



node equations

KCL

$$i = C_1 \frac{dV_1}{dt} + \frac{V_1}{R_1} + \frac{1}{L_1}\int (V_1 - V_2)\,dt \quad \text{——} (7)$$

$$\frac{1}{L_1}\int (V_1 - V_2)\,dt = \frac{V_2}{R_2} + C_2\frac{dV_2}{dt} + \frac{1}{L_2}\int V_2\,dt.$$

$$C_2\frac{dV_2}{dt} + \frac{V_2}{R_2} + \frac{1}{L_2}\int V_2\,dt + \frac{1}{L_1}\int (V_2 - V_1)\,dt = 0 \quad \text{——} (8)$$

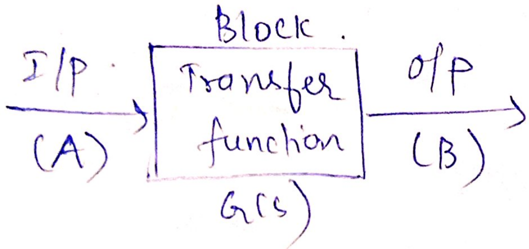
6/9/20

Block Diagrams.

→ A block diagram of a system is a pictorial representation of the functions performed by each component and the flow of signals.

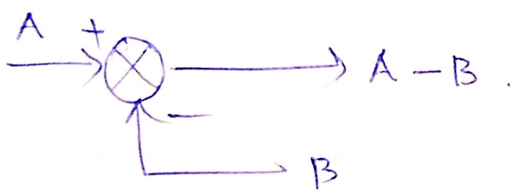
→ The elements of a block diagram are block, branch point and summing point.

→ **Block**

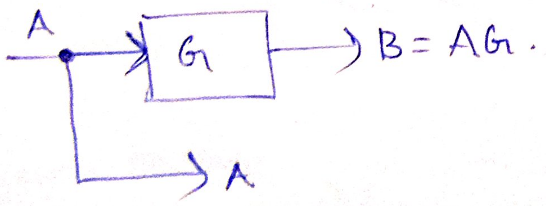


$$B = A(G(s))$$

→ **Summing point** → used to add two or more



→ **Branch point** → the signal from a block goes con-currently to other blocks.



Eg 1) construct the block diagram ~~of~~ for the following differential equation.

$$(1) \rightarrow V_a = i_a R_a + L_a \frac{di_a}{dt} + e_b$$

$$(2) \rightarrow T = K_t i_a$$

$$(3) \rightarrow T = J \frac{d\omega}{dt} + B\omega$$

$$(4) \rightarrow e_b = K_b \omega$$

$$(5) \rightarrow \omega = \frac{d\theta}{dt}$$

soln. Take LT for all equations.

$$V_a(s) = I_a(s) R_a + L_a s I_a(s) + E_b(s) \text{ —— } (1)$$

$$T(s) = K_t I_a(s) \text{ —— } (2)$$

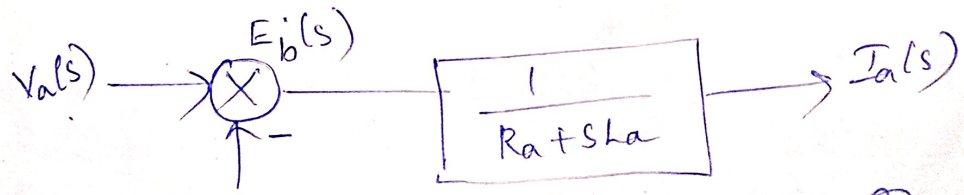
$$T(s) = J s \omega(s) + B\omega(s) \text{ —— } (3)$$

$$E_b(s) = K_b \omega(s) \text{ —— } (4)$$

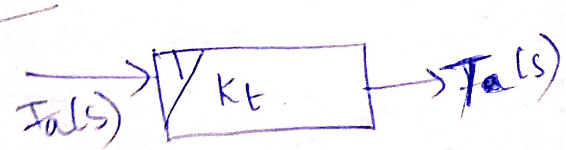
$$\omega(s) = s\theta(s) \text{ —— } (5)$$

from (1)

$$V_a(s) - E_b(s) = I_a(s)(R_a + sL_a) \Rightarrow I_a(s) = \frac{V_a(s) - E_b(s)}{R_a + sL_a}$$

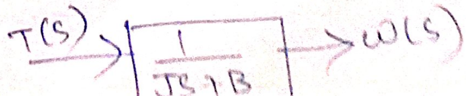


from (2)



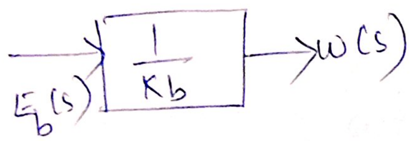
from (3)

$$\omega(s) = \frac{1}{Js + B} T(s)$$



④ $E_b(s) = K_b \omega(s)$

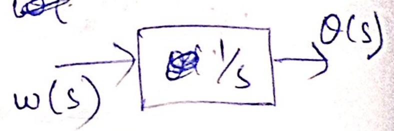
$$\omega(s) = \frac{1}{K_b}\left[E_b(s)\right]$$



⑤

$$\omega(s) = s(\theta(s))$$

$$\theta(s) = \frac{1}{s}\,\omega(s)$$

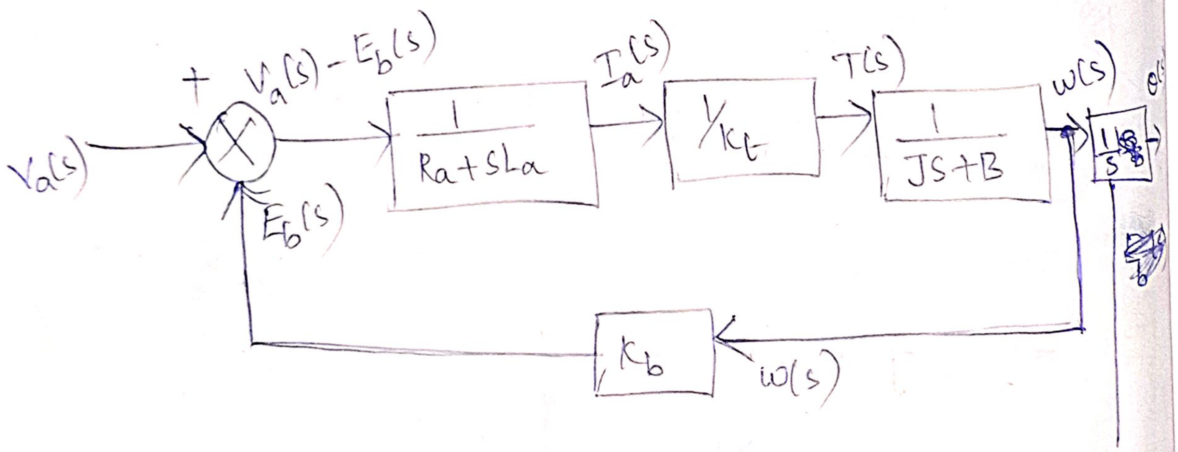


By comparing all the blocks,

$V_a(s) \rightarrow$ i/p

o/p $\rightarrow \theta(s)$

all other blocks are intermediate blocks.



---

2) construct the block diagram for the following diff. eqns.

$$V_f = R_f i_f + L_f \frac{di_f}{dt}$$

$$T = K_{tf}\, i_f$$

$$T = J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt}.$$

Soln

→ Take L.T for all equations.

① → $V_f(s) = R_f I_f(s) + L_f s I_f(s)$

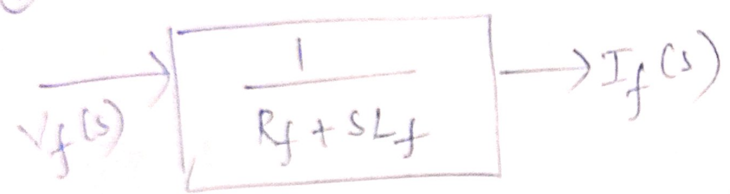
② → $T(s) = K_{tf} I_f(s)$

③ → $T(s) = J s^2 \theta(s) + B s \theta(s)$.

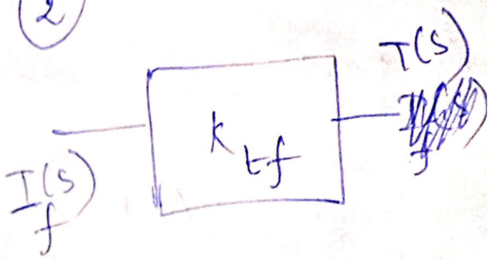
$T(s) = \theta(s)\left[J s^2 + B s\right]$

$\theta(s) = \dfrac{1}{J s^2 + B s} T(s)$

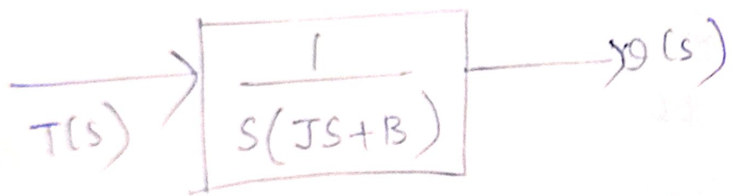
①

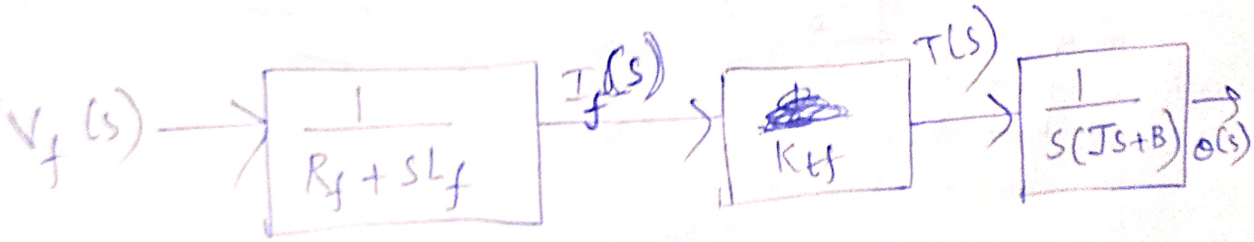


②



③

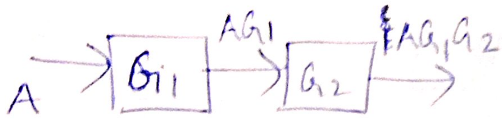


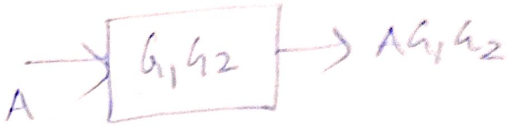


Rules of Block diagram Alagebra :-

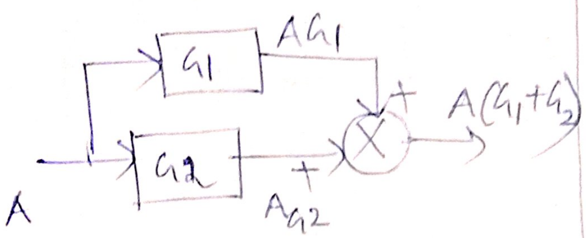
Rules – Blocks	Reduced/modified Blocks

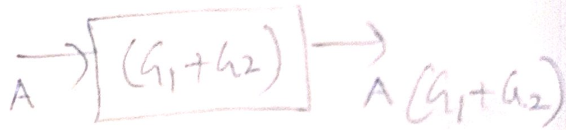
1) cascade blocks



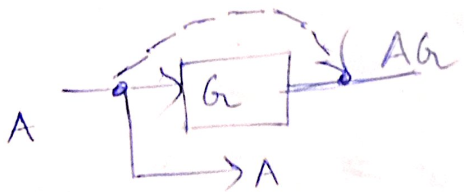


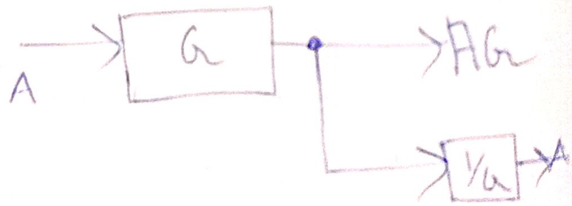
2) parallel blocks



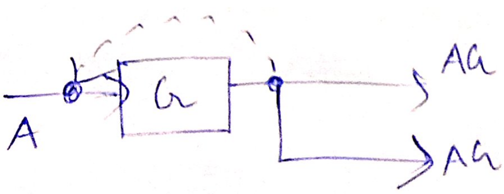


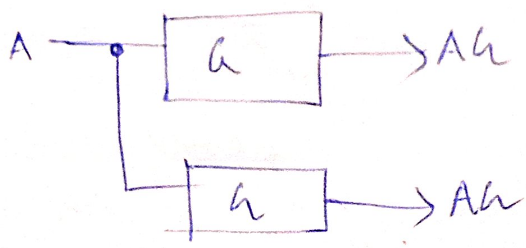
3) Moving branch point ahead of the block



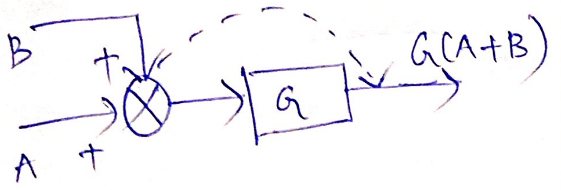


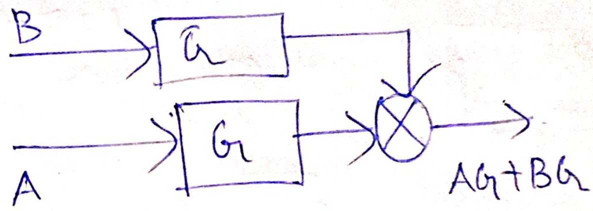
4) Moving the branch point before the block



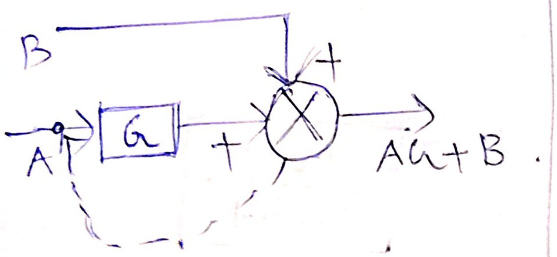


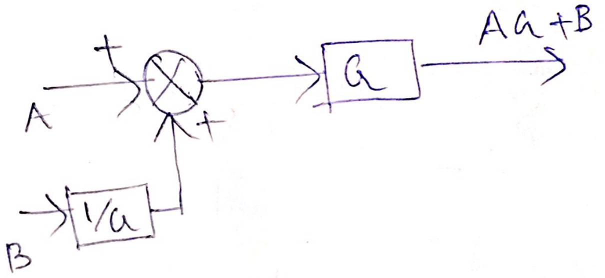
5) Moving the summing point ahead of the block



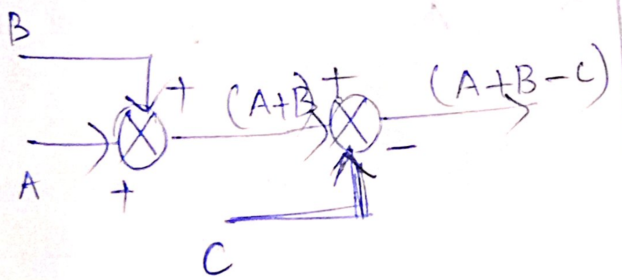


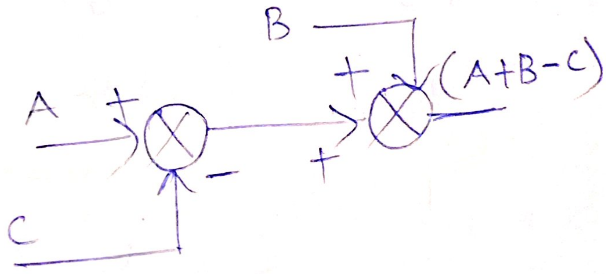
6) Moving the summing point before the block



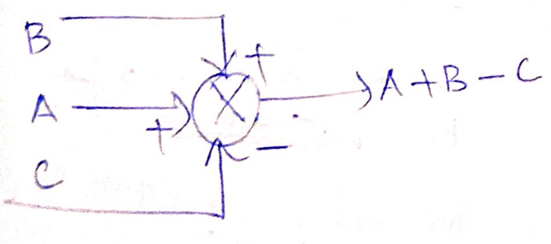


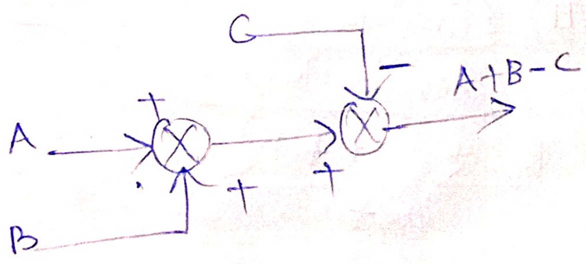
7) Interchanging summing point



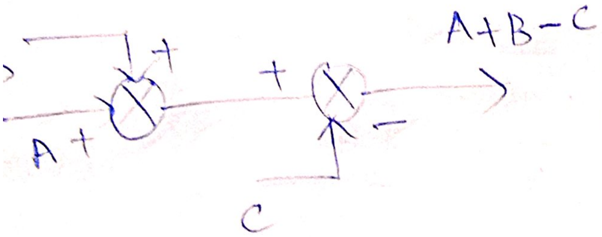


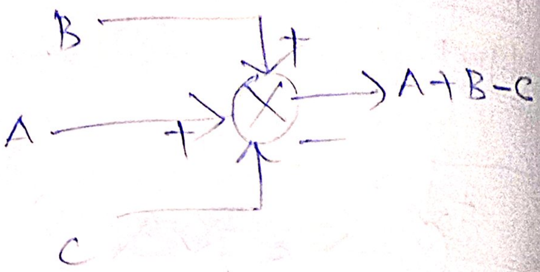
8) splitting summing points



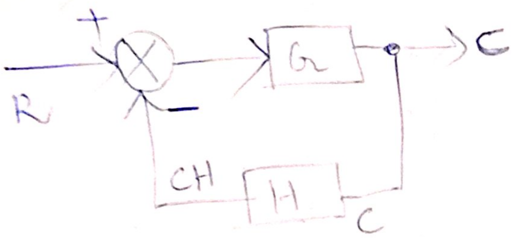


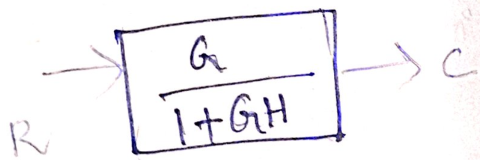
9) combining summing points





10) Eliminating negative feedback loop





$(R-CH)G = C$

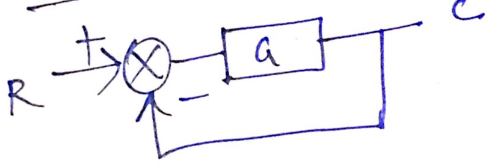
$RG - CHG = C$

$RG = C + CHG$

$RG = C(1+HG)$

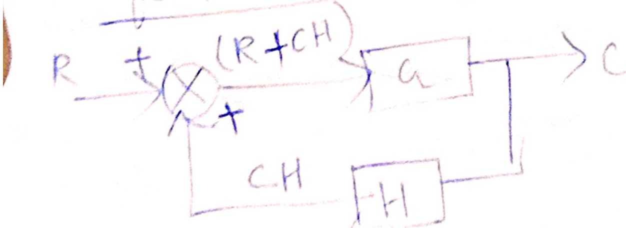
$C = R\left(\frac{G}{1+GH}\right)$

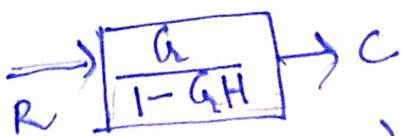
For unity fb (-ve)



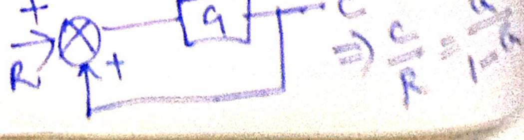
$C/R = \frac{G}{1+G}$

11) Eliminating positive feedback loop



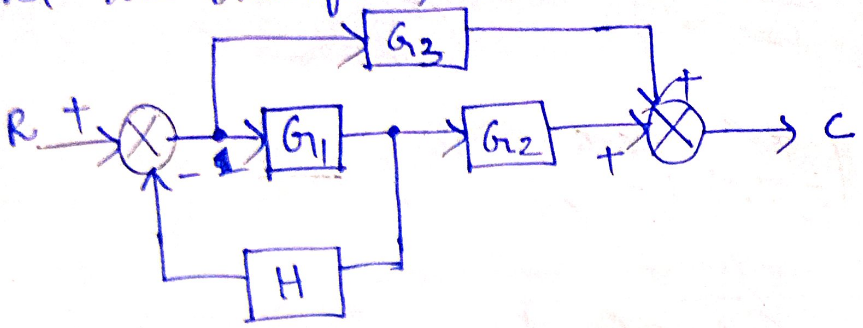


For unity fb (+ve)



$\Rightarrow \frac{C}{R} = \frac{G}{1-G}$

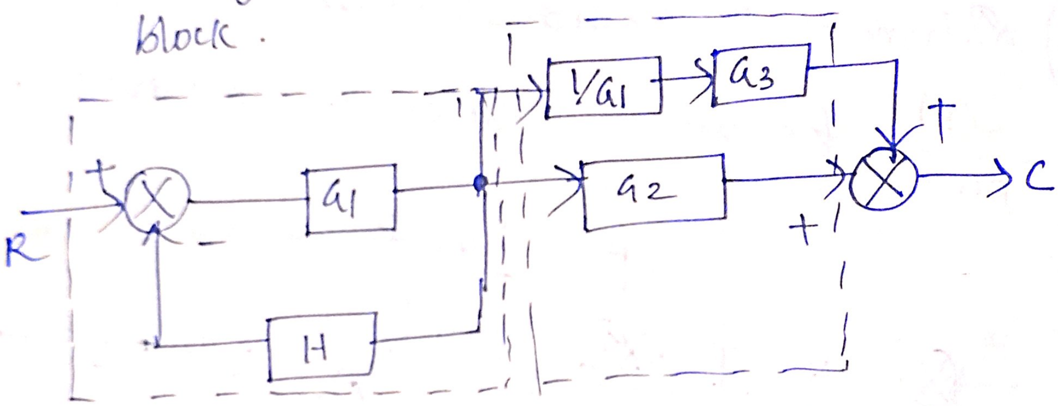
Q1) Reduce the block diagram shown in figure & find the transfer function.

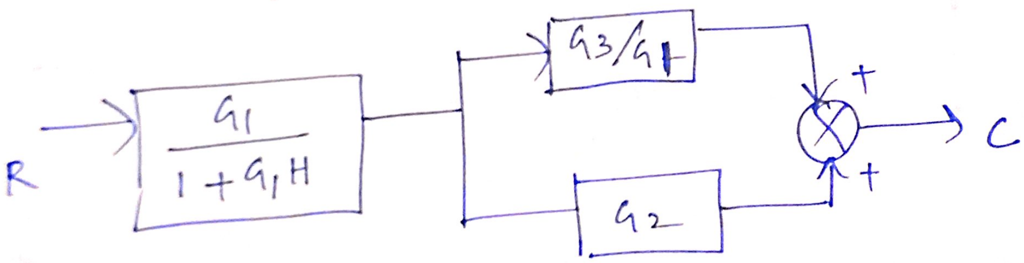


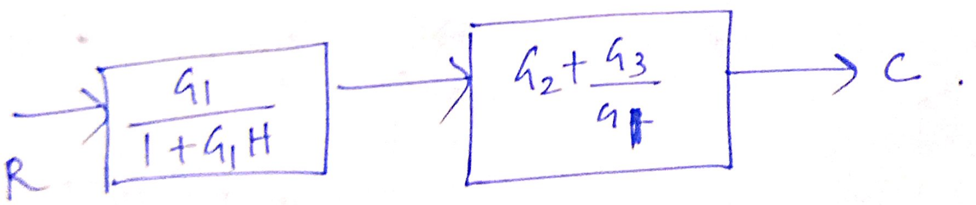
Solu:-

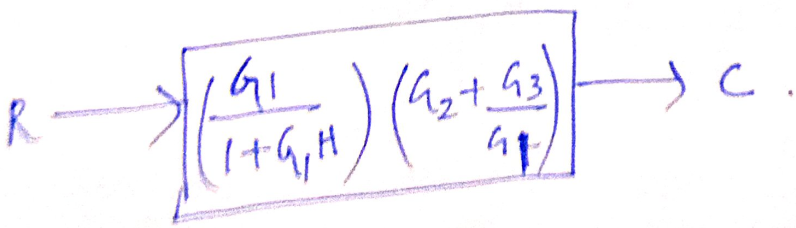
step 1

Moving the branch point ahead of the block.









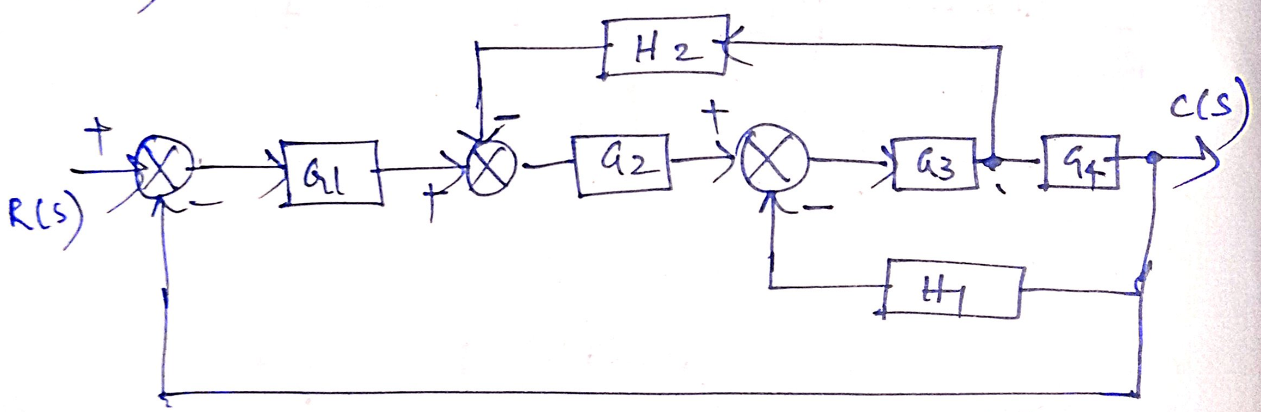
$$\frac{C}{R} = \left(\frac{G_1}{1+G_1 H}\right)\left(G_2 + G_3/G_1\right)$$

$$= \left(\frac{\cancel{G_1}}{1+G_1 H}\right)\left(\frac{G_1 G_2 + G_3}{\cancel{G_1}}\right)$$

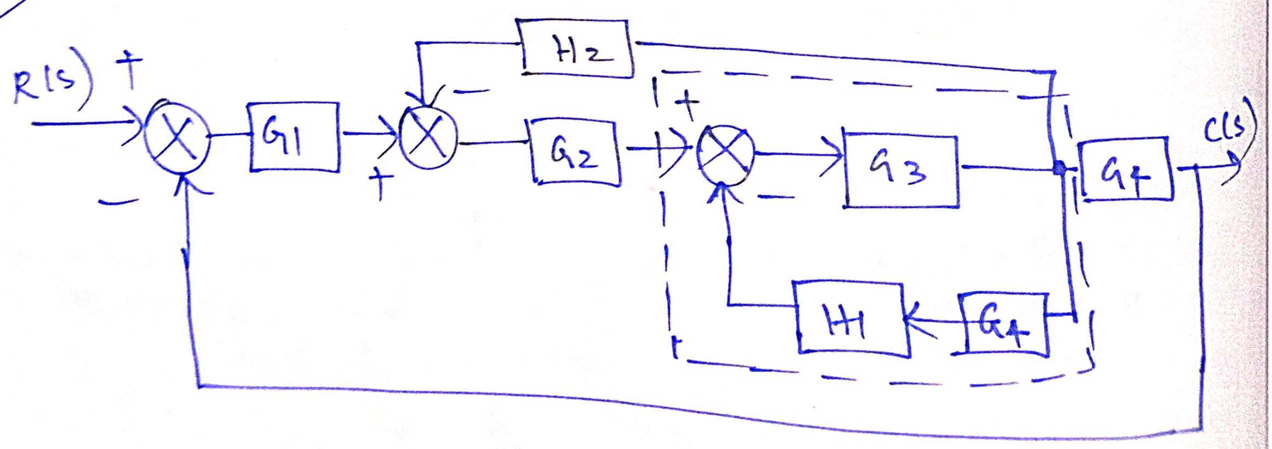
$$\boxed{\frac{C}{R} = \frac{G_1 G_2 + G_3}{1+G_1 H}}$$

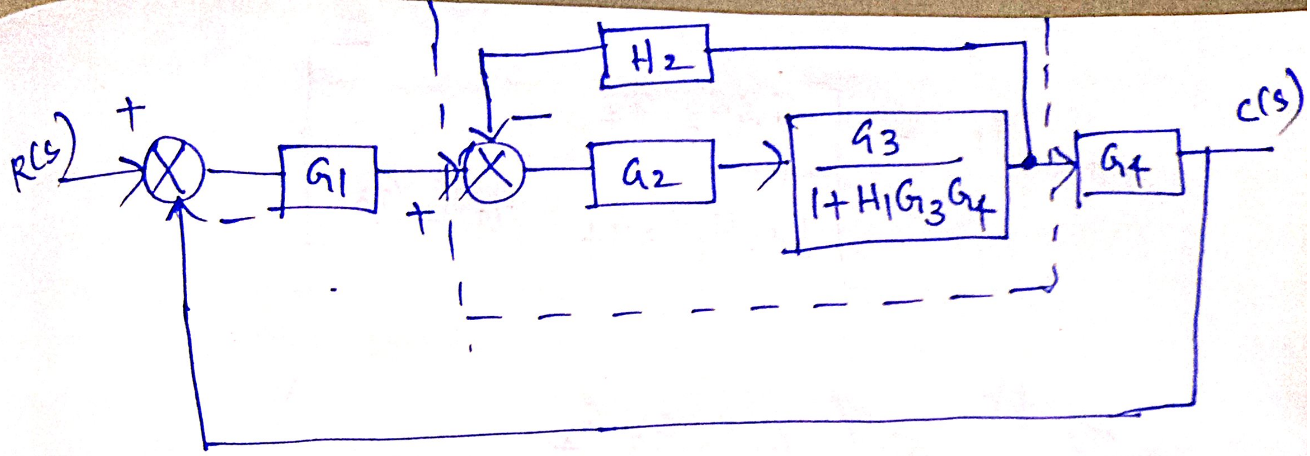
---

2) Determine the overall transfer function $\frac{C(s)}{R(s)}$ for the S/m shown in the figure



Soln.



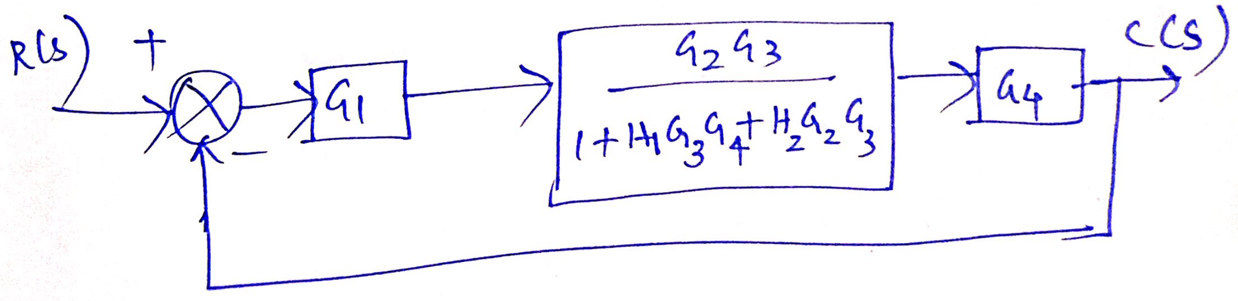


$$\text{cascade} \Rightarrow G_2\left(\frac{G_3}{1+H_1G_3G_4}\right)$$

$$\text{n/w fb} \Rightarrow \frac{\dfrac{G_2G_3}{1+H_1G_3G_4}}{1+H_2\left(\dfrac{G_2G_3}{1+H_1G_3G_4}\right)}$$

$$\Rightarrow \frac{\dfrac{G_2G_3}{\cancel{(1+H_1G_3G_4)}}}{\dfrac{(1+H_1G_3G_4)+H_2G_2G_3}{\cancel{(1+H_1G_3G_4)}}}$$

$$\Rightarrow \frac{G_2G_3}{1+H_1G_3G_4+H_2G_2G_3}$$



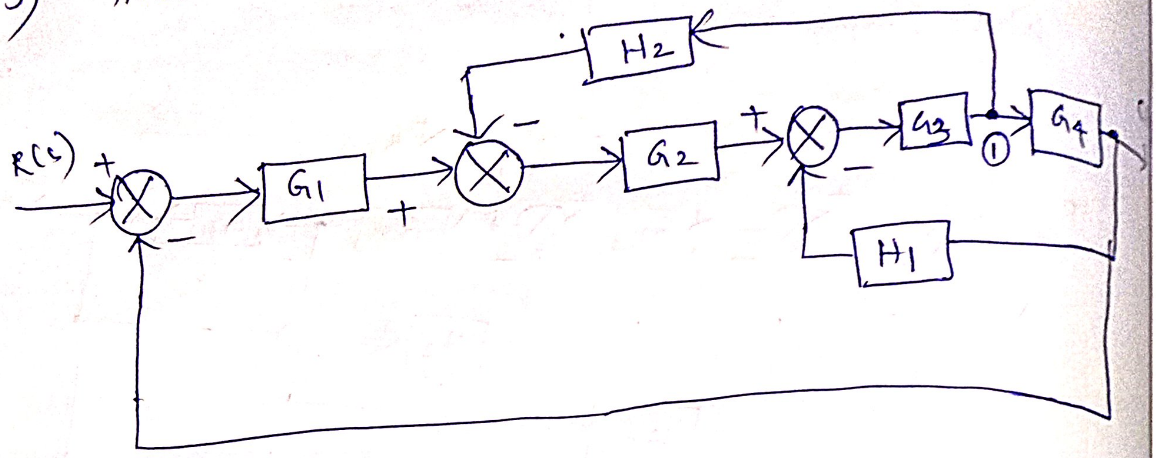
cascade $\Rightarrow \left[\dfrac{G_1 G_2 G_3 G_4}{1 + H_1 G_3 G_4 + H_2 G_2 G_3} \right]$

unity fb $\Rightarrow \dfrac{\left(\dfrac{G_1 G_2 G_3 G_4}{1 + H_1 G_3 G_4 + H_2 G_2 G_3} \right)}{1 + \left(\dfrac{G_1 G_2 G_3 G_4}{1 + H_1 G_3 G_4 + H_2 G_2 G_3} \right)}$

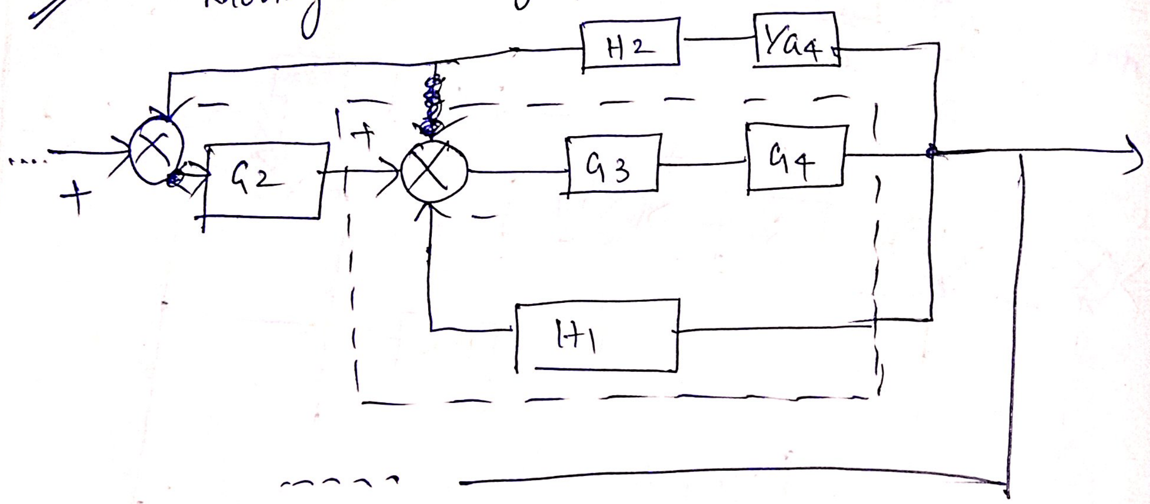
$$\therefore \frac{C(s)}{R(s)} = \frac{G_1 G_2 G_3 G_4}{1 + H_1 G_3 G_4 + H_2 G_2 G_3 + G_1 G_2 G_3 G_4}.$$

$$R(s) \longrightarrow \boxed{\dfrac{G_1 G_2 G_3 G_4}{1 + H_1 G_3 G_4 + H_2 G_2 G_3 + G_1 G_2 G_3 G_4}} \longrightarrow C(s)$$

3) Find $C(s)/R(s)$.

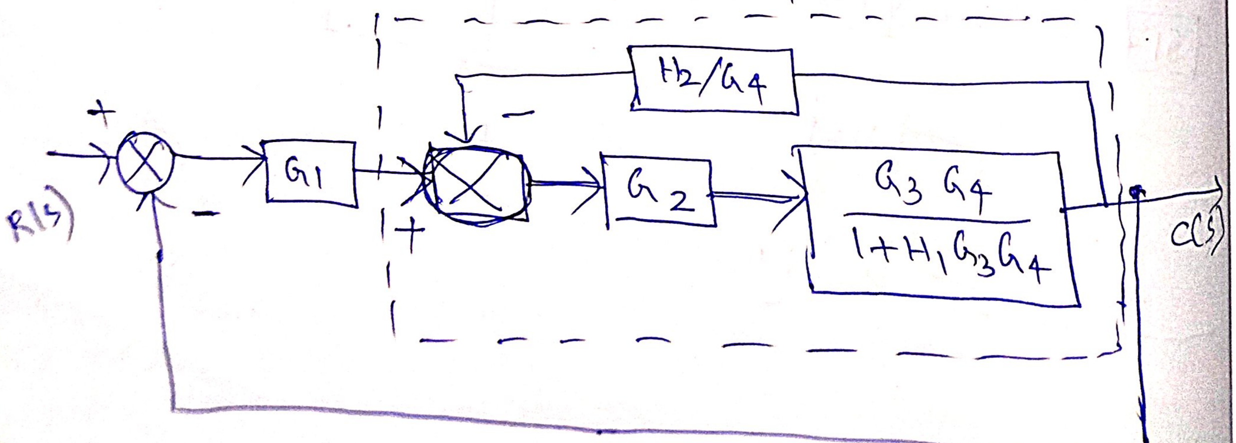


Soln. Moving Branching point 1 after G4



cascade $\Rightarrow G_3 G_4$.

−ve fb $\Rightarrow \dfrac{G_3 G_4}{1 + H_1 G_3 G_4}$.

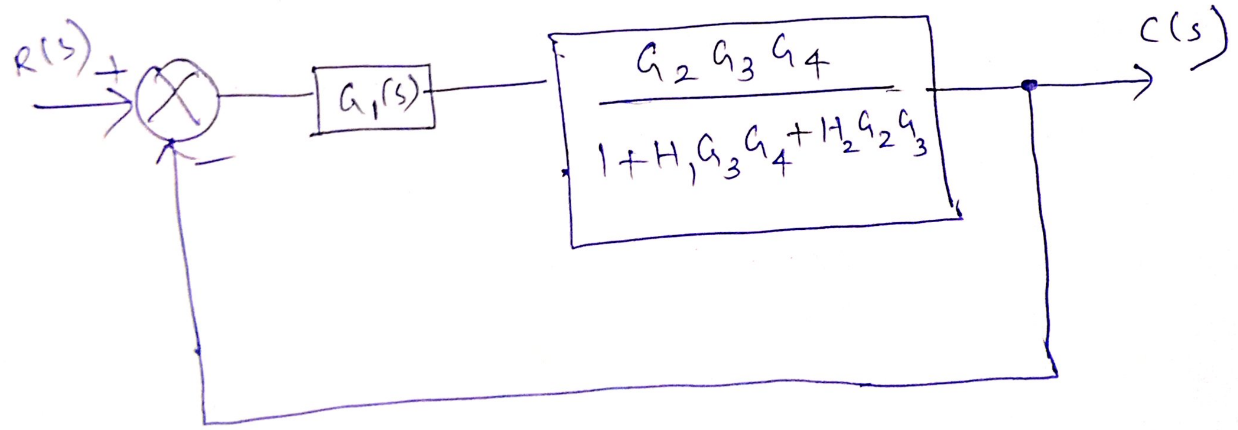


cascade $\Rightarrow \dfrac{G_2 G_3 G_4}{1 + H_1 G_3 G_4}$

$-$ve fb. $\Rightarrow \dfrac{\dfrac{G_2 G_3 G_4}{1 + H_1 G_3 G_4}}{1 + \dfrac{H_2}{\cancel{G_4}}\left(\dfrac{G_2 G_3 \cancel{G_4}}{1 + H_1 G_3 G_4}\right)}$

$$= \dfrac{\dfrac{G_2 G_3 G_4}{\cancel{(1 + H_1 G_3 G_4)}}}{\dfrac{1 + H_1 G_3 G_4 + H_2 G_2 G_3}{\cancel{(1 + H_1 G_3 G_4)}}}$$

$$= \dfrac{G_2 G_3 G_4}{1 + H_1 G_3 G_4 + H_2 G_2 G_3}.$$

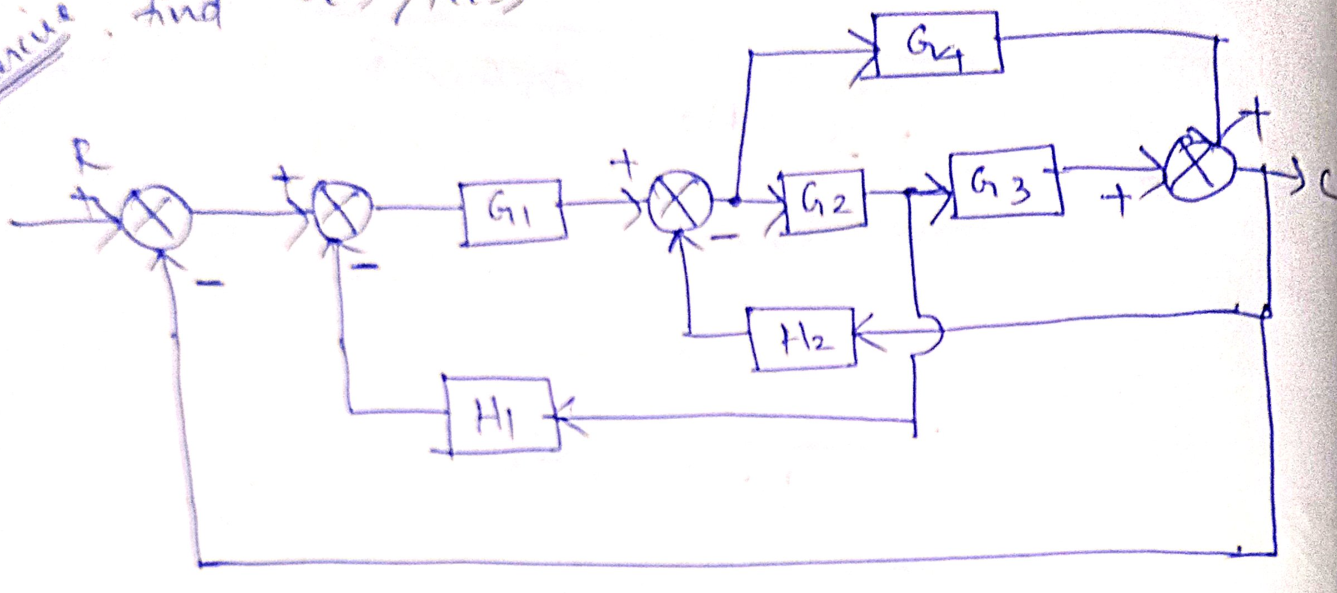


cascade $\Rightarrow \dfrac{G_1 G_2 G_3 G_4}{1 + H_1 G_3 G_4 + H_2 G_2 G_3}.$

overall unity fb $\Rightarrow \dfrac{\left[G_1 G_2 G_3 G_4 / (1 + H_1 G_3 G_4 + H_2 G_2 G_3)\right]}{1 + \left(G_2 G_1 G_3 G_4 / (1 + H_1 G_3 G_4 + H_2 G_2 G_3)\right)}$

$$\frac{C(s)}{R(s)} = \frac{G_1 G_2 G_3 G_4}{1 + H_1 G_3 G_4 + H_2 G_2 G_3 + G_1 G_2 G_3 G_4}$$

Exercise. Find $C(s)/R(s)$.



Signal flow graph.

→ It depicts the flow of signals from one point of a s/m to another and gives the relationships among the signals.

→ It consists of a n/w in which nodes are connected by directed branches.

→ It uses Mason's gain formula to compute the transfer function & it is simple compared to the block-reduction technique.

→ Elements / terms in Signal flow graph:-

1) Node: A pt. representing a variable or signal.

2) Branch: directed line segment joining two nodes.
   Arrow on the branch ⇒ direction of signal flow.
   gain of the branch ⇒ transmittance.

3) Transmittance: gain acquired by the signal when it travels from one node to another.

4) I/p node: source node, has only outgoing branches.

5) O/p node: sink node, has only incoming branches

6) mixed node: Has both incoming & outgoing branches.

7) path: It is a traversal of connected branches in the direction of the branch arrows. It should not cross a node more than once.

8) open path : path that starts at one node & ends ~~at~~ another node.

9) closed path : start & end nodes are same.

10) Forward path : path from i/p node to o/p node & does not cross any node more than once.

11) Forward path gain : It is product of the branch gain of a forward path.

12) Individual loop : closed path & no crossing of any node more than once.

13) loop gain : product of gains of a loop.

14) Non-touching loop : If there is no common node between any loops, then the loops are non-touching loop.

Mason's Gain formula :

$$T(s) = \frac{C(s)}{R(s)}$$

$$\boxed{T = \frac{1}{\Delta} \sum_{K} P_K \Delta_K}$$

$P_K \Rightarrow$ forward path gain of K^{th} forward ~~loop~~ path.

$K \Rightarrow$ no. of forward paths

$\Delta = 1 -$ (sum of individual loop gain)

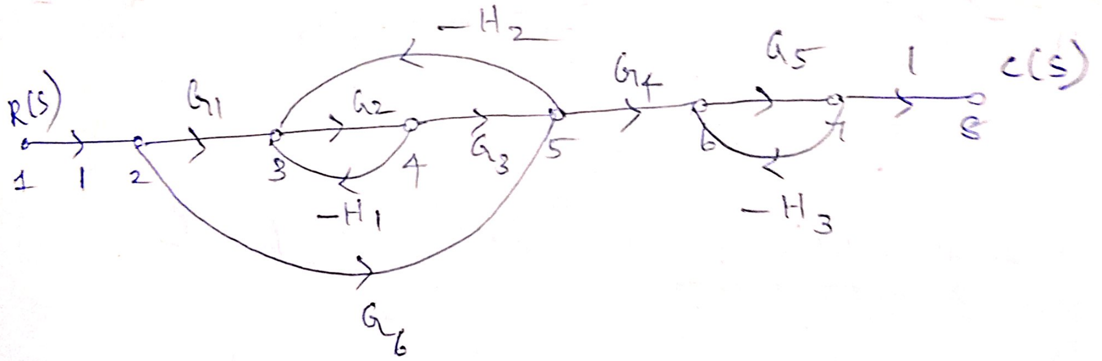
$+$ (sum of gain path of all possible combinations of two non-touching loops)

$-$ (sum of gain path of all possible combinations of three non-touching loops)

$+ \ldots$

$\Delta_K = \Delta$ for that part of the graph which is not touching K^{th} forward path.

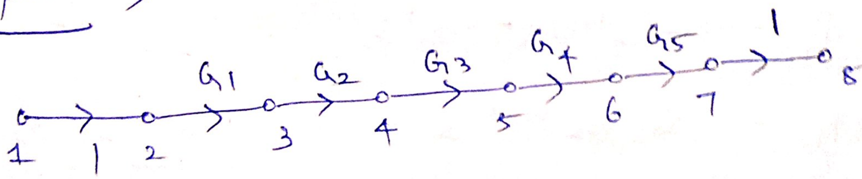
Eg: Find the over-all transfer function of the s/m whose signal flow graph.



Soln

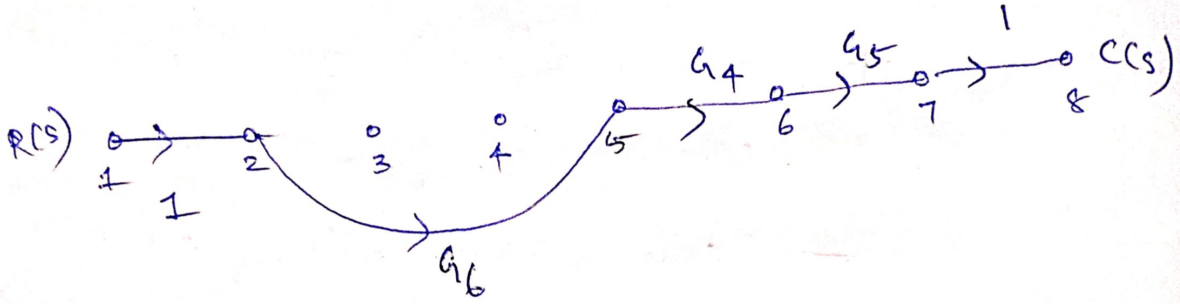
① No. of forward paths. $(K) = 2$.

② 1^{st} path $\Rightarrow$ 1 − 2 − 3 − 4 − 5 − 6 − 7 − 8



path gain $P_1 = G_1 G_2 G_3 G_4 G_5$

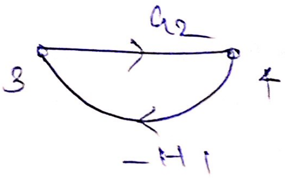
2^{nd} forward path $\Rightarrow$ 1 – 2 – 5 – 6 – 7 – 8



Frwd path Gain $P_2 = G_6 G_4 G_5$.

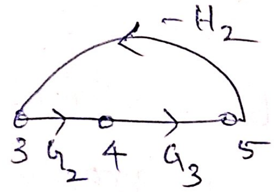
③ Individual loop gain.

loop 1



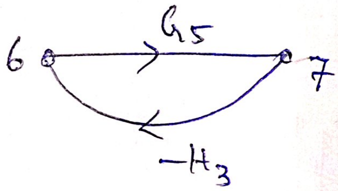
$P_{11} = -G_2 H_1$

loop 2



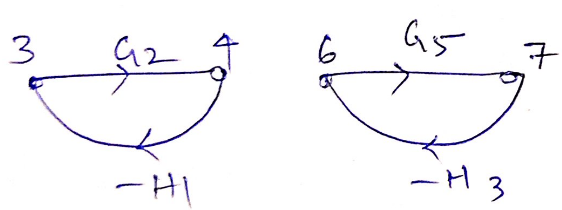
$P_{21} = -G_2 G_3 H_2$

loop 3



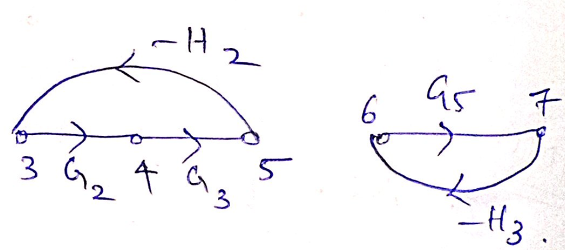
$P_{31} = -G_5 H_3$.

④ Gain products of two non-touching loops



$P_{12} = (-G_2 H_1)(-G_5 H_3)$

$P_{12} = G_2 G_5 H_1 H_3$



$P_{22} = (-G_2 H_2)(-G_5 H_3)$

$P_{22} = G_2 G_5 H_2 H_3$

⑤ calculation of Δ & Δ_K:

$$\Delta = 1 - (P_{11} + P_{21} + P_{31}) + (P_{12} + P_{22})$$

$$= 1 - \left[-G_2H_1 - G_2G_3H_2 - G_5H_3\right] + \left[G_2G_5H_1H_3 + G_2G_5H_2H_3\right]$$

$$= 1 + G_2H_1 + G_2G_3H_2 + G_5H_3 + G_2G_5H_1H_3 + G_2G_5H_2H_3.$$

$\Delta_1 = 1$, there is no loop which is not in contact with the 1st forward path.

$\Delta_2 = 1 - $ (loop gain that is not in contact with 2nd forward path)

$$= 1 - P_{11}$$

$$= 1 - (-G_2H_1)$$

$$= 1 + G_2H_1.$$

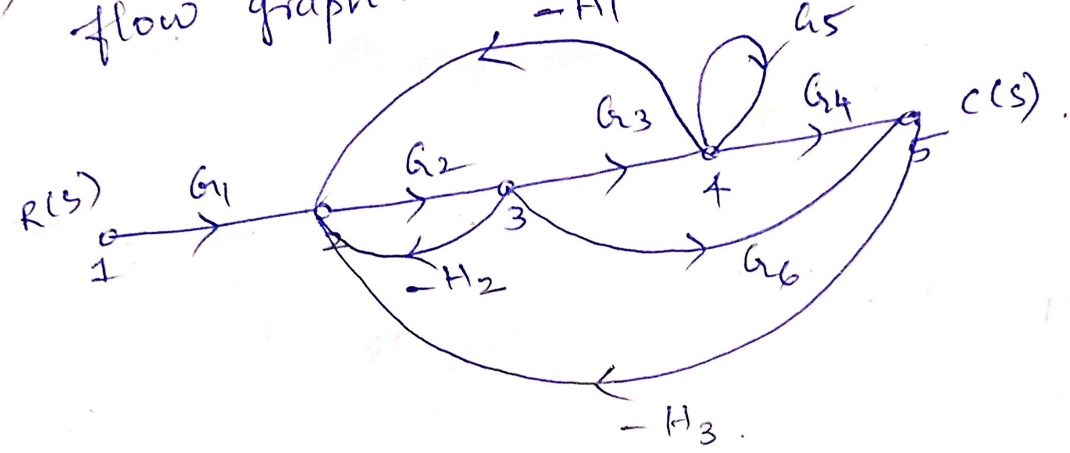
$$T(s) = \frac{1}{\Delta} \sum_K P_K \Delta_K = \frac{1}{\Delta} \sum_2 P_K \Delta_K.$$

$$= \frac{1}{\Delta}\left[P_1\Delta_1 + P_2\Delta_2\right]$$

$$= \frac{1\left[(G_1G_2G_3G_4G_5)1 + G_4G_5G_6(1 + G_2H_1)\right]}{1 + G_2H_1 + G_2G_3H_2 + G_5H_3 + G_2G_5H_1H_3 + G_2G_5H_2H_3}$$

$$T(s) = \frac{G_4 G_5 \left[G_1 G_2 G_3 + G_6 + G_2 G_6 H_1 \right]}{1 + G_2 H_1 + G_2 G_3 H_2 + G_5 H_3 + G_2 G_5 H_1 H_3 + G_2 G_3 G_5 H_2 H_3}$$

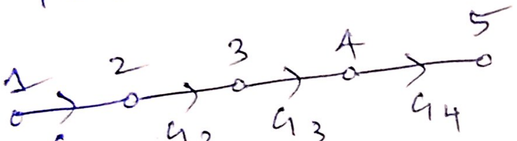
2) Find the overall gain for the signal flow graph.



Soln ① No. of forward paths $(K) = 2$.

② path - 1

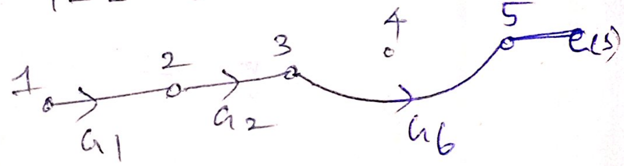
1 - 2 - 3 - 4 - 5



Fwd Path gain $P_1 = G_1 G_2 G_3 G_4$

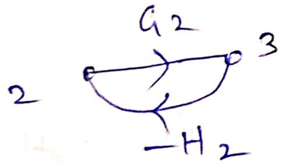
path - 2

1 - 2 - 3 - 5

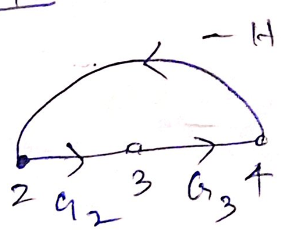


$P_2 = G_1 G_2 G_6$.

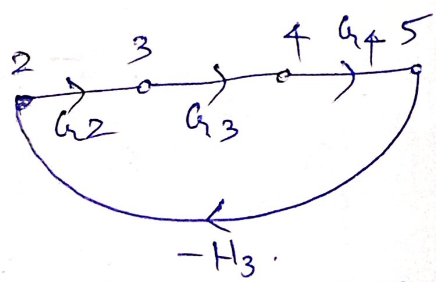
③ Individual loops & its gain



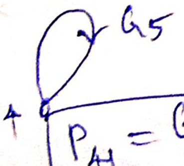
$\boxed{P_{11} = -G_2 H_2}$



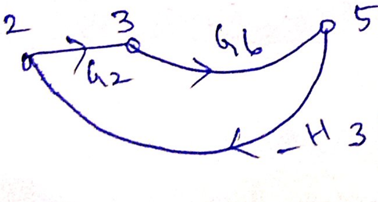
$\boxed{P_{21} = -G_2 G_3 H_1}$



$\boxed{P_{31} = -G_2 G_3 G_4 H_3}$

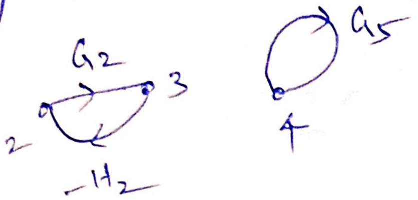


$\boxed{P_{41} = G_5}$

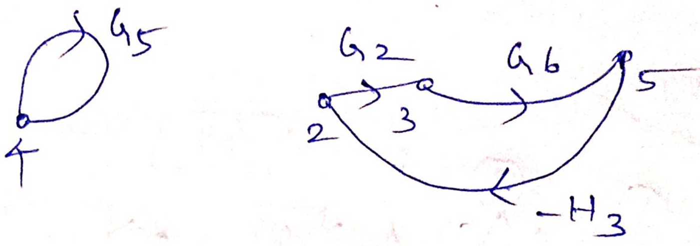


$\boxed{P_{51} = -G_2 G_6 H_3}$

4) Non-touching loops.



$P_{12} = -G_2G_5H_2$



$P_{22} = -G_2G_5G_6H_3.$

5) calculation of Δ & Δ_K

$$\Delta = 1 - (P_{11} + P_{21} + P_{31} + P_{41} + P_{51}) + (P_{12} + P_{22})$$

$$= 1 + H_3G_2G_6 + G_2H_2 + G_2G_3H_1 + G_2G_3G_4H_3 - G_5$$

$$- G_2G_5H_2 - G_2G_5G_6H_3.$$

$$\Delta_1 = 1 - 0 = 1$$

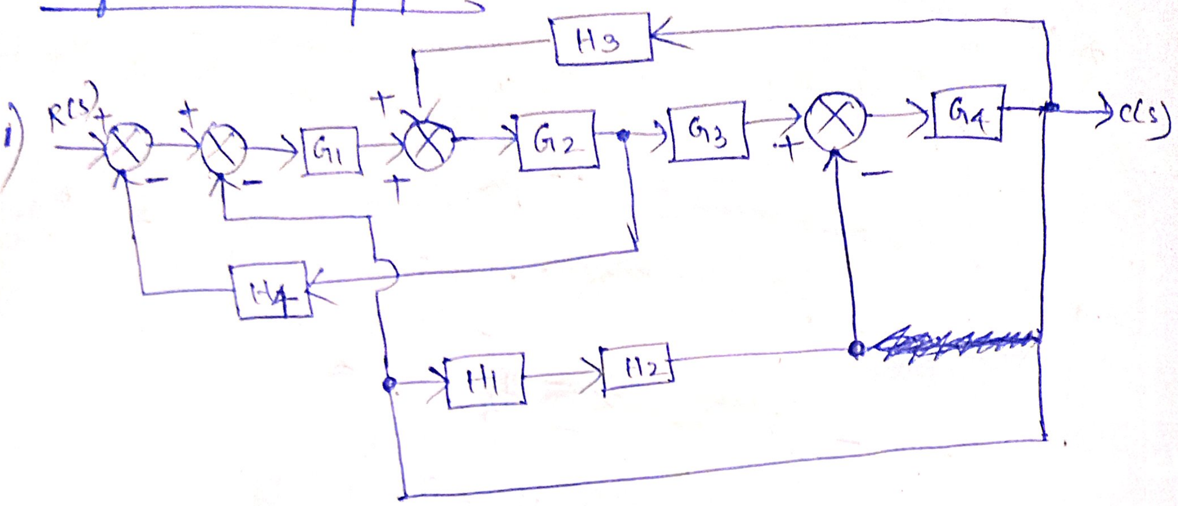
$$\Delta_2 = 1 - P_{41} = 1 - G_5$$

$$T(s) = \frac{1}{\Delta} \sum_{K=2} P_K \Delta_K = \frac{1}{\Delta}\left[P_1\Delta_1 + P_2\Delta_2\right]$$

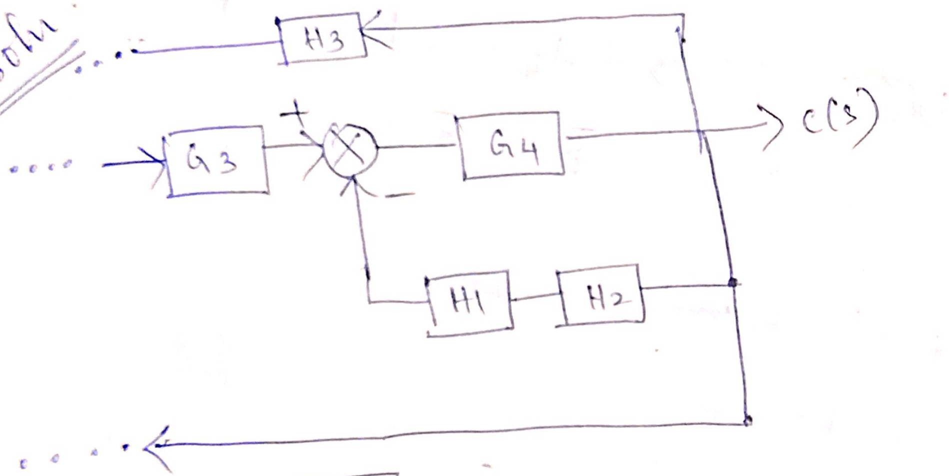
$$\boxed{T(s) = \frac{G_1G_2G_3G_4 + G_1G_2G_6 - G_1G_2G_5G_6}{(1 + H_3G_2G_6 + G_2H_2 + G_2G_3H_1 + G_2G_3G_4H_3 - G_5 - G_2G_5H_2 - G_2G_5G_6H_3)}}$$

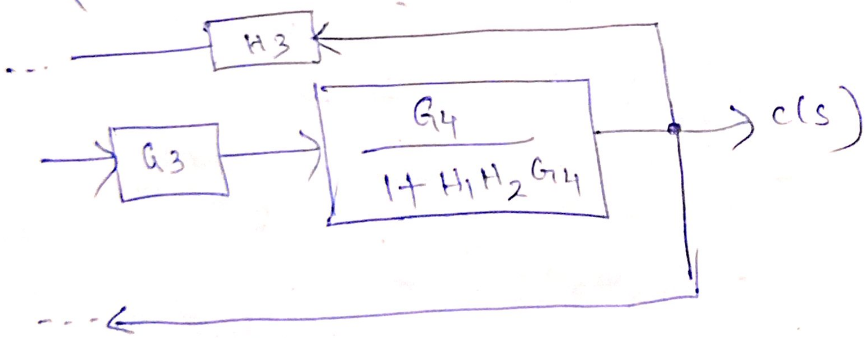
problems on Block Reduction Technique & Signal Flow Graph.

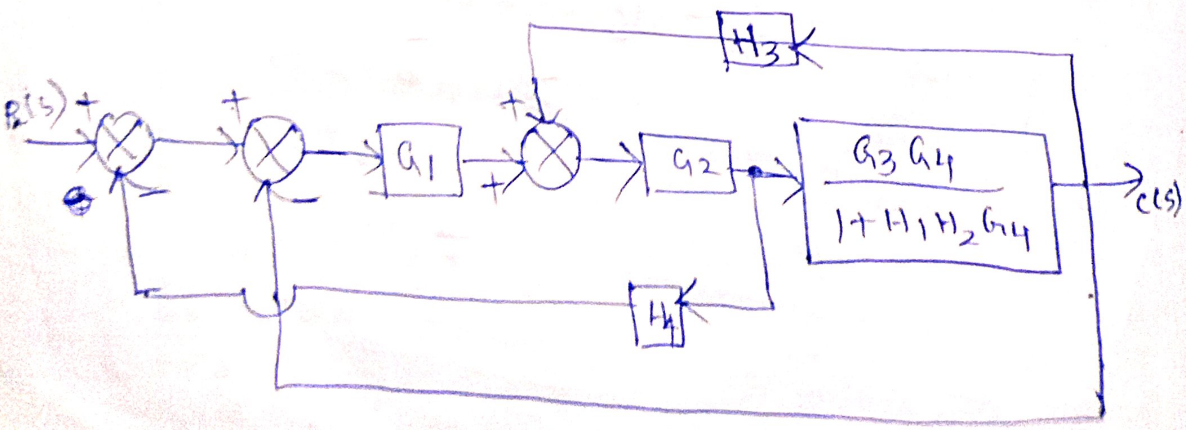
1)

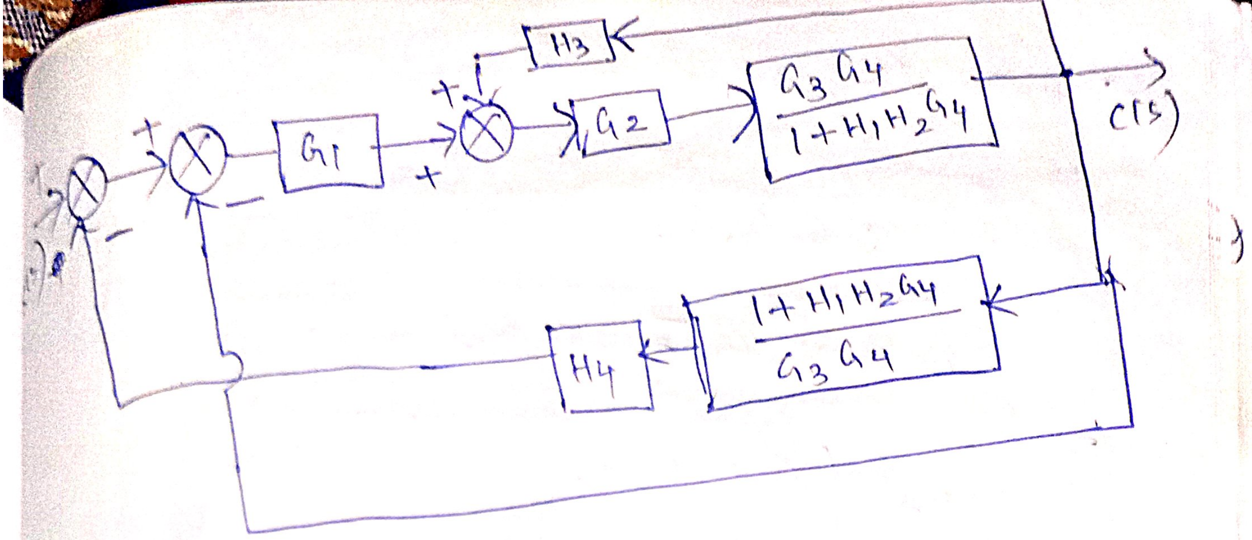


soln:









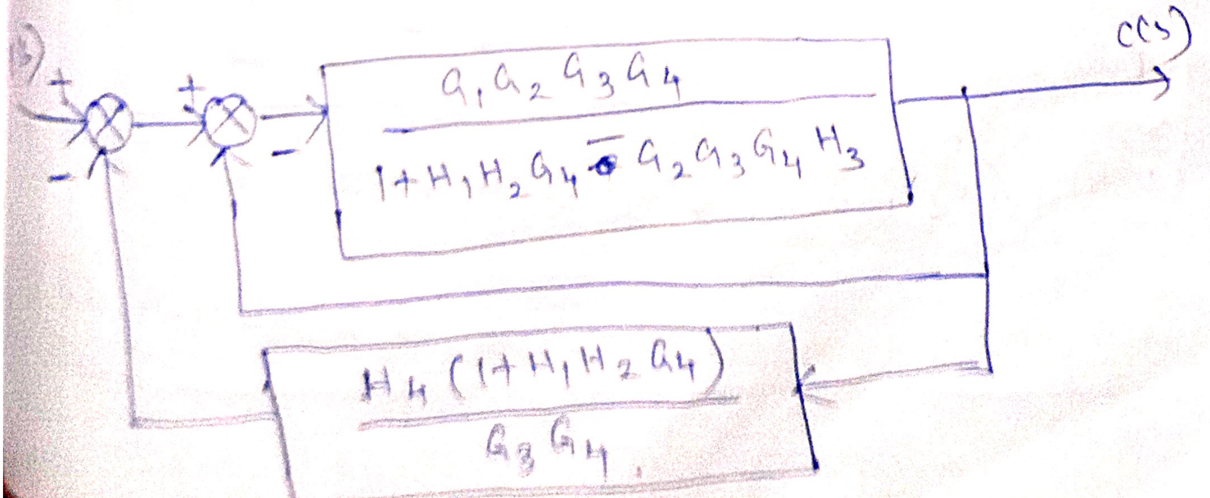
→ upper side ⇒ +ve fb.

$$= \frac{\left(\dfrac{G_2 G_3 G_4}{1+H_1H_2H_4}\right)}{1 - H_3 \dfrac{G_2G_4G_3}{1+H_1H_2G_4}}$$

$$= \frac{G_2G_3G_4}{1 + H_1H_2G_4 - H_3G_2G_3G_4}.$$

⇒ cascaded with G_1

$$\therefore = \frac{G_1G_2G_3G_4}{1+H_1H_2G_4 - G_2G_3G_4H_3}.$$



$\Rightarrow$ unity fb,

$$= \frac{\dfrac{G_1 G_2 G_3 G_4}{1 + H_1 H_2 G_4 - G_2 G_3 G_4 H_3}}{1 + \left(\dfrac{G_1 G_2 G_3 G_4}{1 + H_1 H_2 G_4 - G_2 G_3 G_4 H_3}\right)}$$

$$= \frac{G_1 G_2 G_3 G_4}{1 + H_1 H_2 G_4 - G_2 G_3 G_4 H_3 + G_1 G_2 G_3 G_4}.$$

final fb o/p.

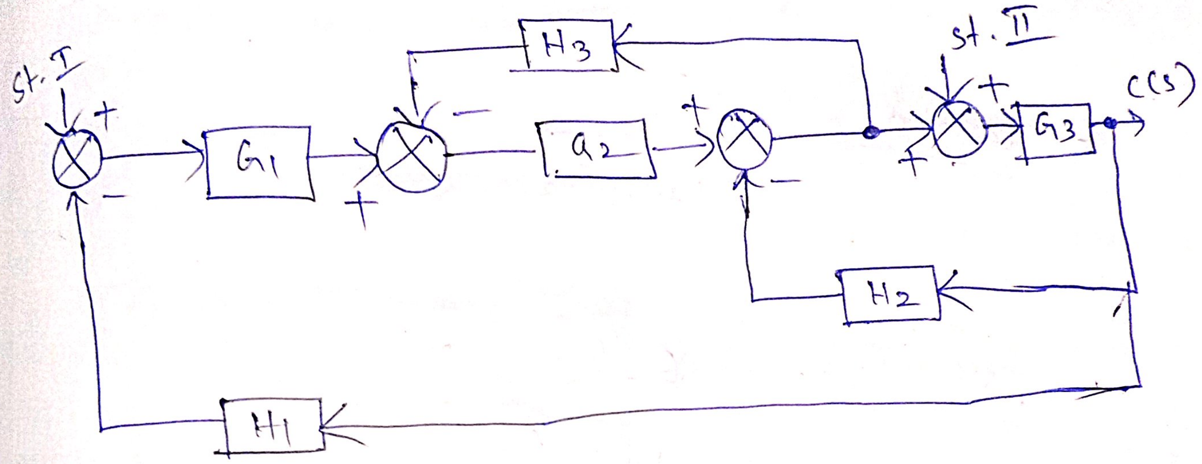
$$= \frac{\left(\dfrac{G_1 G_2 G_3 G_4}{1 + H_1 H_2 G_4 - G_2 G_3 G_4 H_3 + G_1 G_2 G_3 G_4}\right)}{1 + \left(\dfrac{(G_1 G_2 \cancel{G_3} \cancel{G_4})}{(1 + H_1 H_2 G_4 - G_2 G_3 G_4 H_3 + G_1 G_2 G_3 G_4)}\right) \dfrac{H_4 (1 + H_1 H_2 G_4)}{\cancel{G_3} \cancel{G_4}}}$$

$$\boxed{\frac{C(s)}{R(s)} = \frac{G_1 G_2 G_3 G_4}{(1 + H_1 H_2 G_4 - G_2 G_3 G_4 H_3 + G_1 G_2 G_3 G_4 + G_1 G_2 H_4 + G_1 G_2 G_4 H_1 H_2 H_4)}}$$

For the s/m represented by the block diagram, Evaluate the closed loop transfer function when the R is

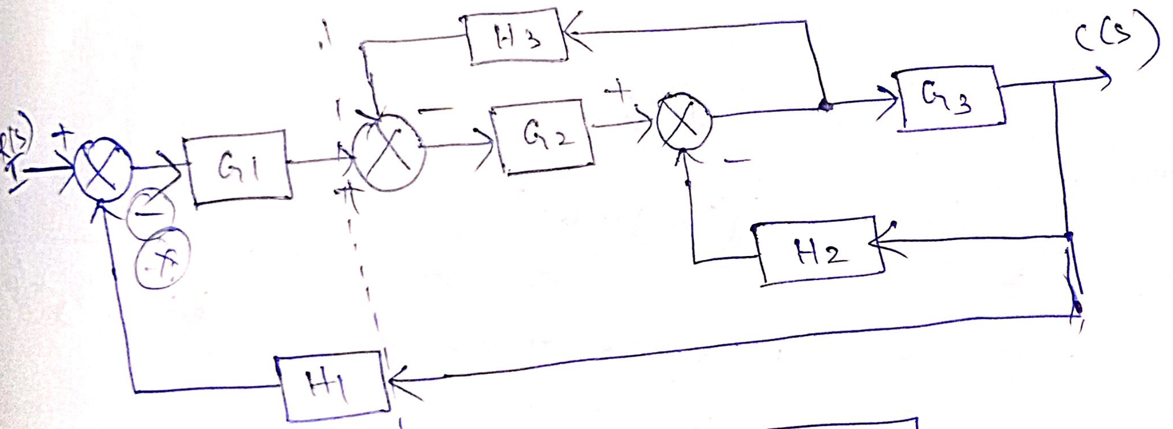
(i) at station - I

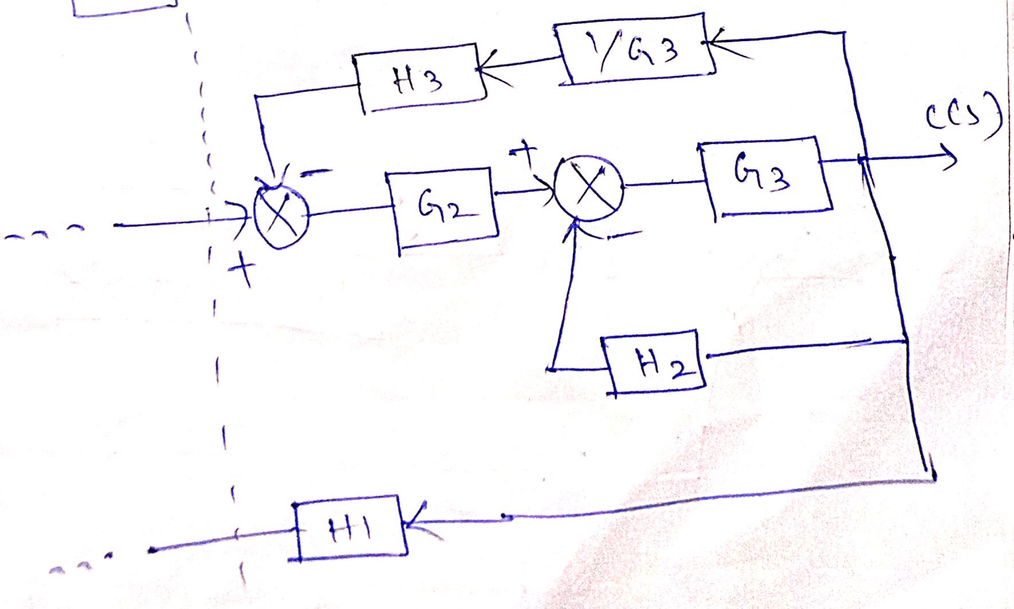
(ii) at station - II.

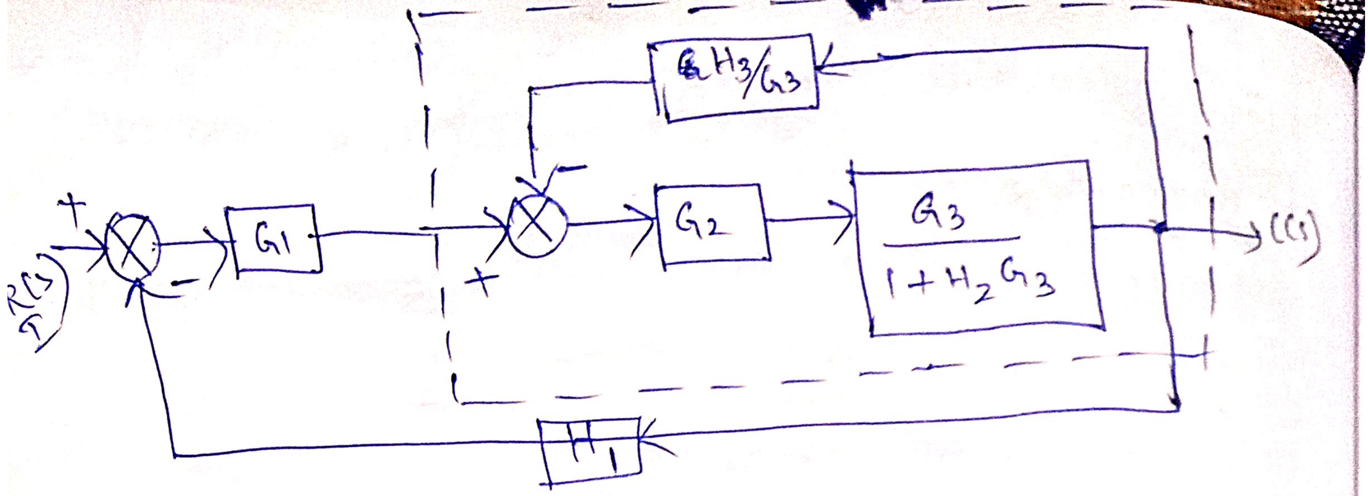


soln

(i) R at St. I





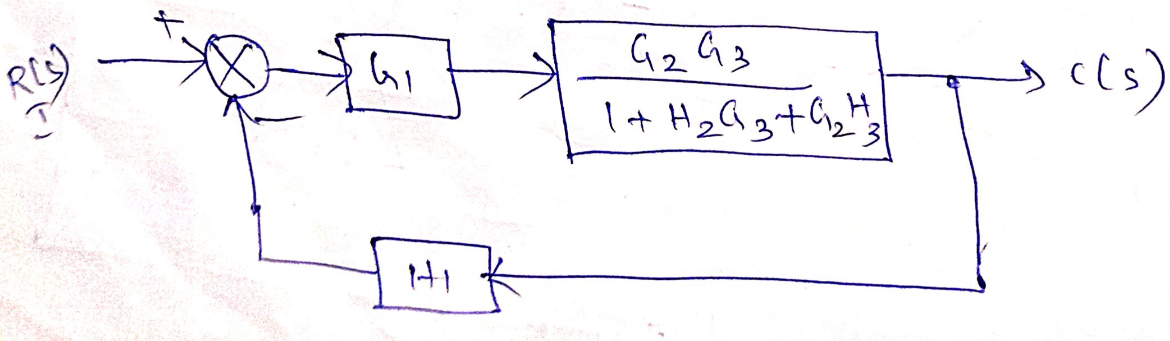


$\Rightarrow$ Cascade $\dfrac{G_2G_3}{1+H_2G_3}$

-ve fb

$$= \frac{\left(\dfrac{G_2G_3}{1+H_2G_3}\right)}{1+\dfrac{H_3}{\cancel{G_3}}\left(\dfrac{G_2\cancel{G_3}}{1+H_2G_3}\right)}$$

$$= \frac{\dfrac{G_2G_3}{\cancel{(1+H_2G_3)}}}{\dfrac{1+H_2G_3+G_2H_3}{\cancel{(1+H_2G_3)}}} = \frac{G_2G_3}{1+H_2G_3+G_2H_3}.$$

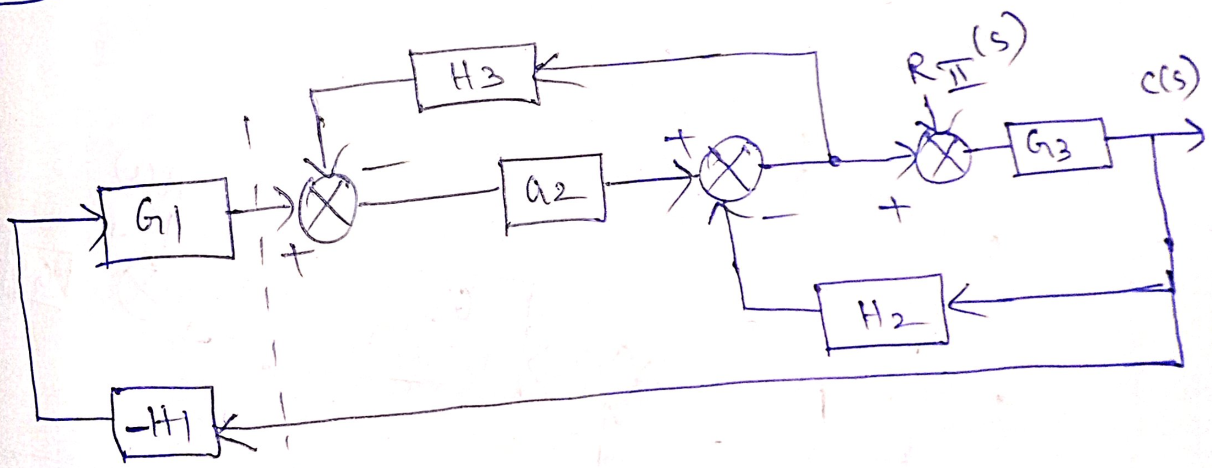


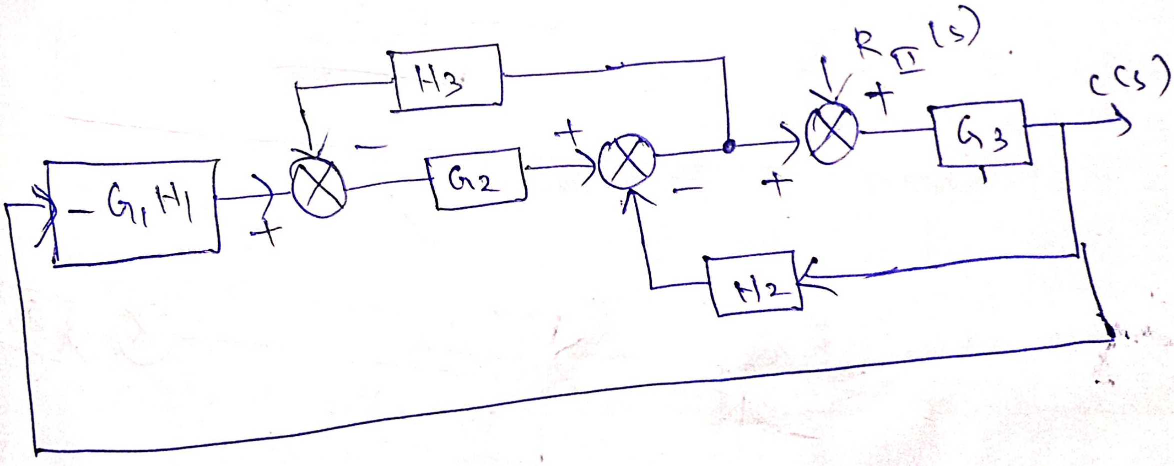
cascade $\Rightarrow \dfrac{G_1 G_2 G_3}{1 + H_2 G_3 + G_2 H_3}$

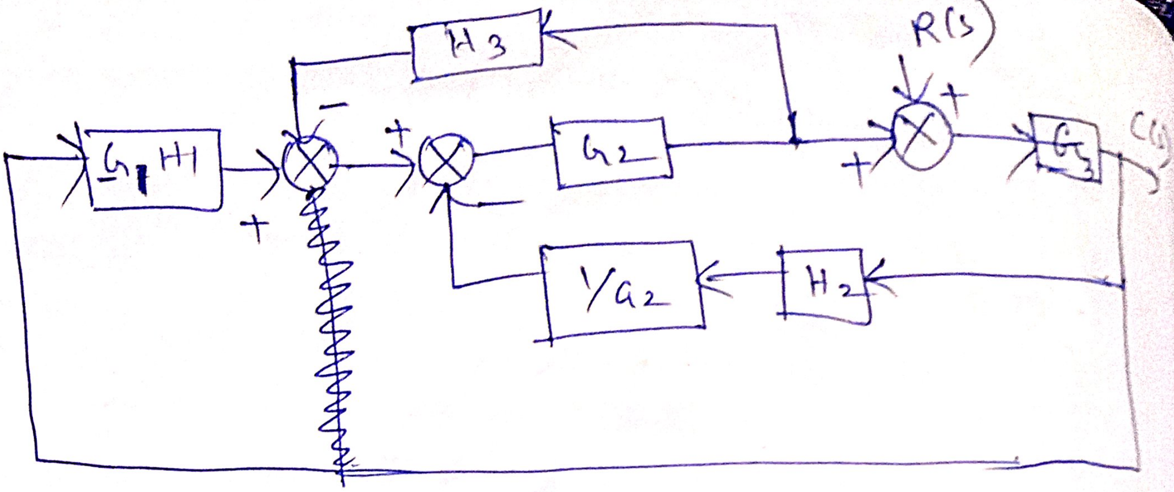
-ve fb. $\Rightarrow \dfrac{\left(\dfrac{G_1 G_2 G_3}{1 + H_2 G_3 + G_2 H_3}\right)}{1 + H_1 \dfrac{G_1 G_2 G_3}{1 + H_2 G_3 + G_2 H_3}}$

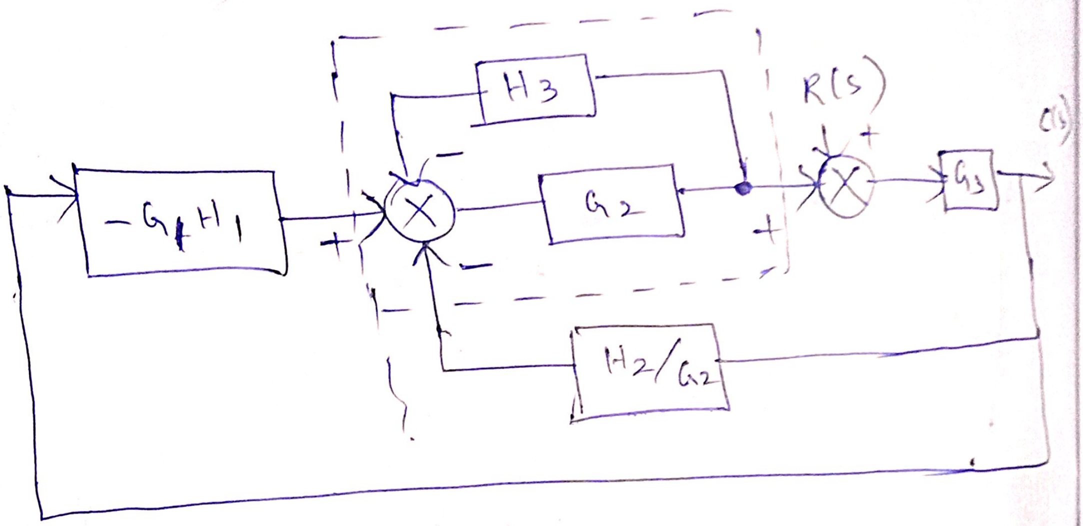
$$\boxed{\frac{C(s)}{R_I(s)} = \frac{G_1 G_2 G_3}{1 + H_2 G_3 + G_2 H_3 + H_1 G_1 G_2 G_3}}$$

1)

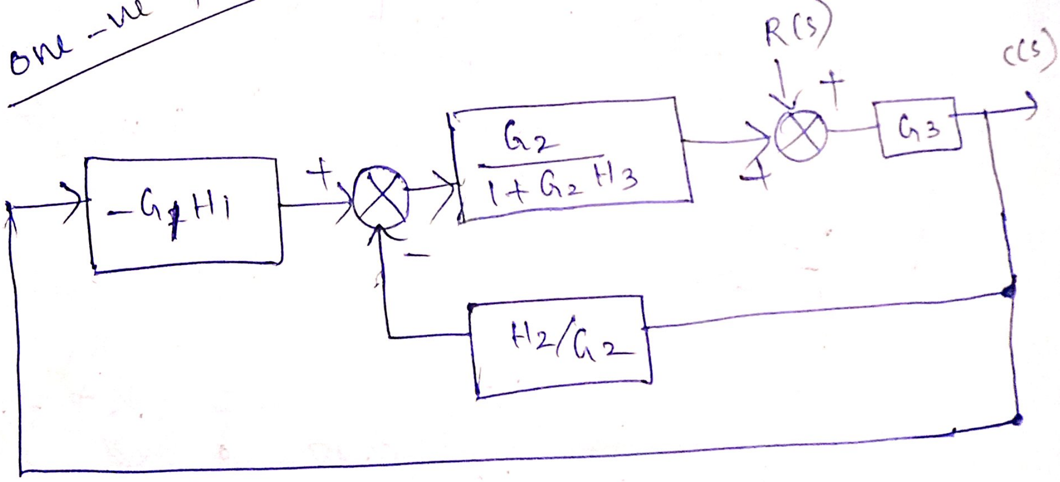


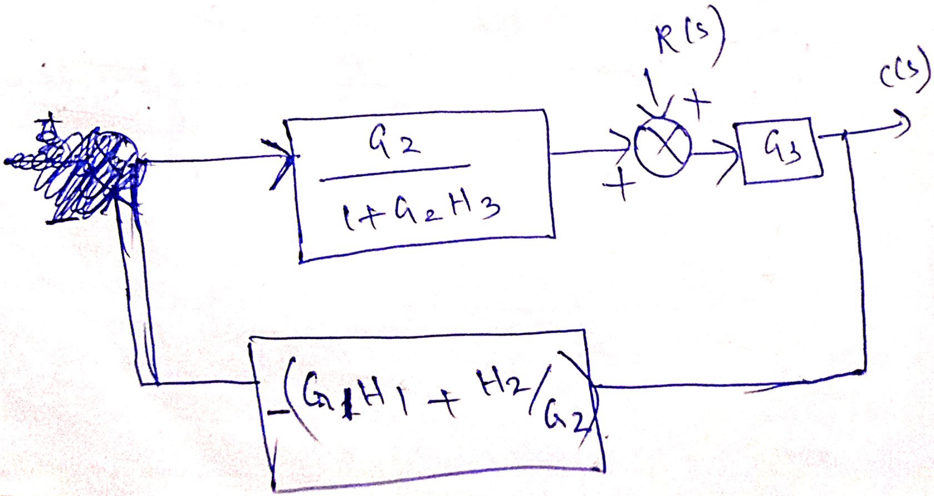


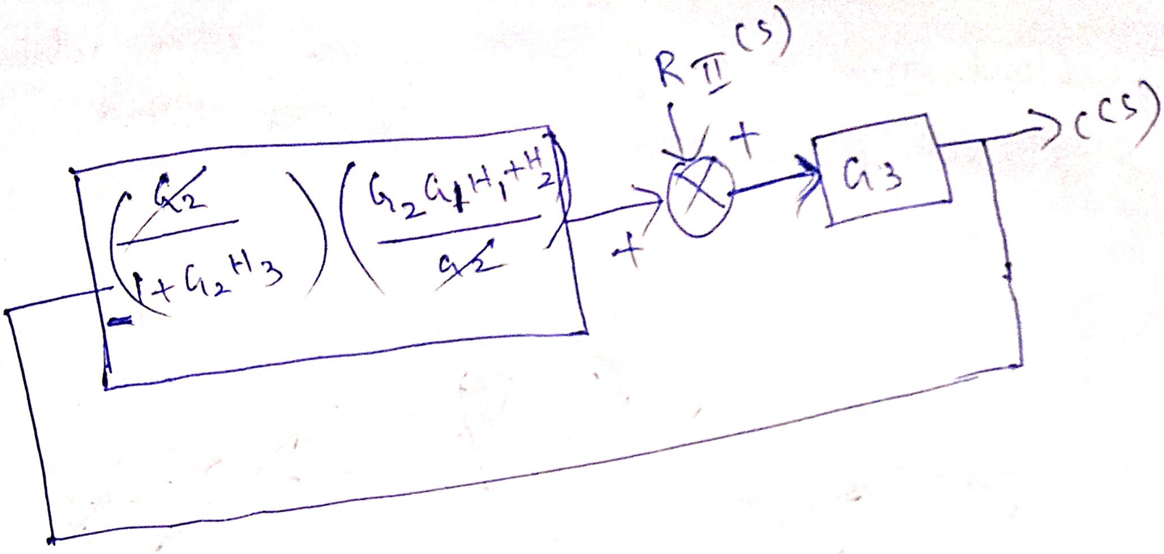

H3
R(s)
+
−
+
G1H1
G2
G3
C(s)
+
−
+
1/G2
H2


H3
R(s)
+
C(s)
−
−G1H1
G2
G3
+
+
−
H2/G2

one -ve fb.


R(s)
+
C(s)
G2
1+G2H3
G3
−G1H1
+
−
+
H2/G2


R(s)
C(s)
+
G2
1+G2H3
G3
+
−(G1H1 + H2/G2)



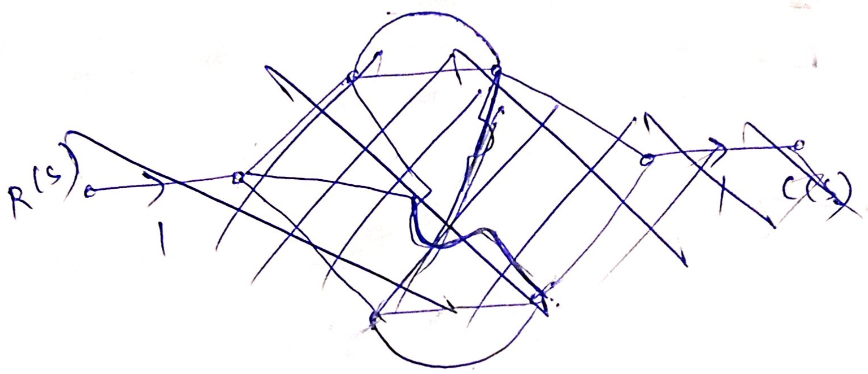
+ve fb

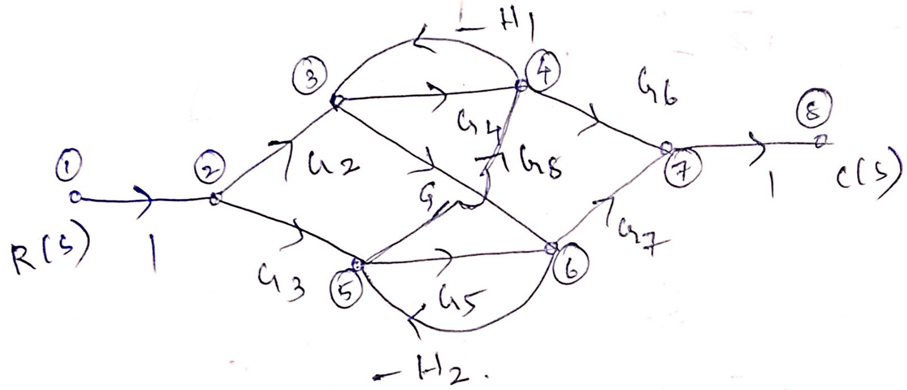
$$= \frac{G_3}{1 - \left[-G_3\left(\frac{G_2G_1H_1+H_2}{1+G_2H_3}\right)\right]}$$

$$= \frac{G_3}{1 + \left[G_3\left(\frac{G_2G_1H_1+H_2}{1+G_2H_3}\right)\right]}$$

$$\boxed{\frac{C(s)}{R_{II}(s)} = \frac{G_3(1+G_2H_3)}{1+G_2H_3+G_1G_2G_3H_1+H_2G_3}}$$

3) Find the overall gain of the s/m whose signal flow graph is shown below.

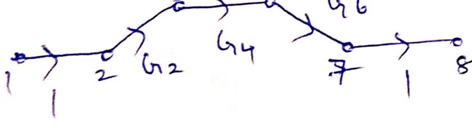




Soln ① No. of forward paths (K) = 6

Path 1

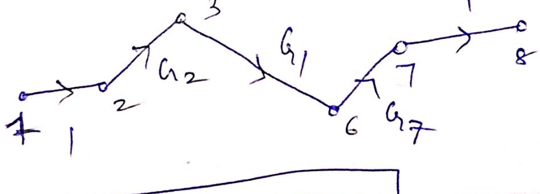
1 - 2 - 3 - 4 - 7 - 8



$$\boxed{P_1 = G_2 G_4 G_6}$$

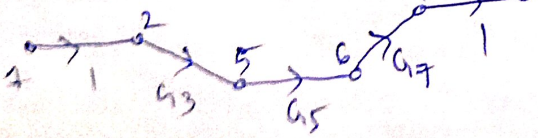
Path-2

1 - 2 - 3 - 6 - 7 - 8



$$\boxed{P_2 = G_1 G_2 G_7}$$

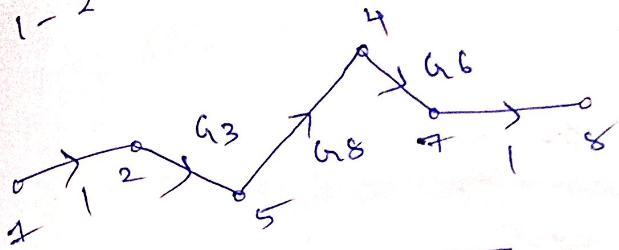
Path-3. 1 - 2 - 5 - 6 - 7 - 8



$$\boxed{P_3 = G_3 G_5 G_7}$$

Path-4

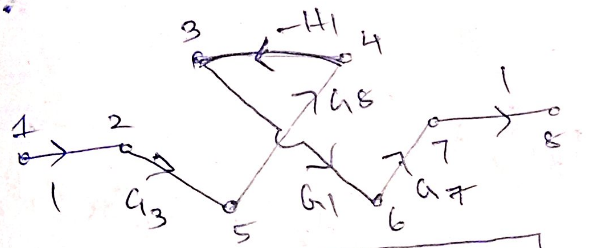
1 - 2 - 5 - 4 - 7 - 8



$$\boxed{P_4 = G_3 G_6 G_8}$$

Path 5

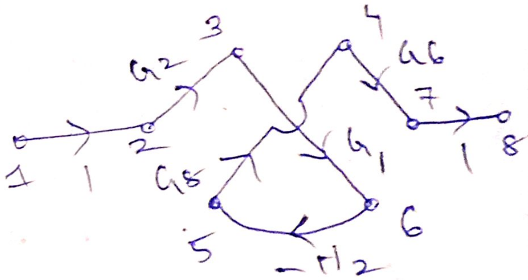
1-2-5 - 4 - 3 - 6 - 7 - 8



$$\boxed{P_5 = -G_1 G_3 G_7 G_8 H_1}$$

Path 6

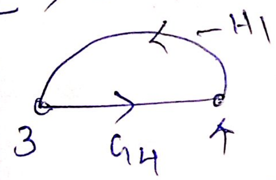
1 - 2 - 3 - 6 - 5 - 4 - 7 - 8



$$\boxed{P_6 = -G_1 G_2 G_6 G_8 H_2}$$

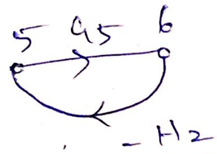
① Individual loops

loop 1 → 3 - 4



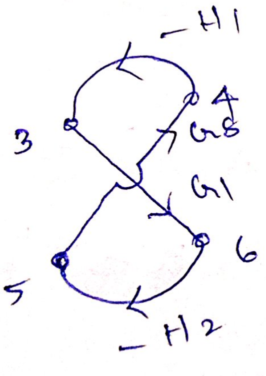
$$\boxed{P_{11} = -G_4 H_1}$$

loop 2



$$\boxed{P_{21} = -G_5 H_2}$$

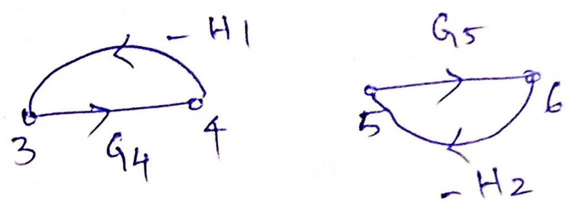
loop-3.



$$\boxed{P_{31} = G_1 G_8 H_1 H_2}$$

3) Non-touching loops.

- 2-non-touching loop.



$$P_{12} = G_4 G_5 H_1 H_2$$

④ calculation of Δ & Δ_K

$$\Delta = 1 - (P_{11} + P_{21} + P_{31}) + (P_{12})$$

$$= 1 + G_4 H_1 + G_5 H_2 - G_1 G_8 H_1 H_2 + G_4 G_5 H_1 H_2$$

$$\Delta_1 = 1 - (P_{21}) = 1 + G_5 H_2$$

$$\Delta_2 = 1$$

$$\Delta_3 = 1 - (P_{11}) = 1 + G_4 H_1$$

$$\Delta_4 = 1$$

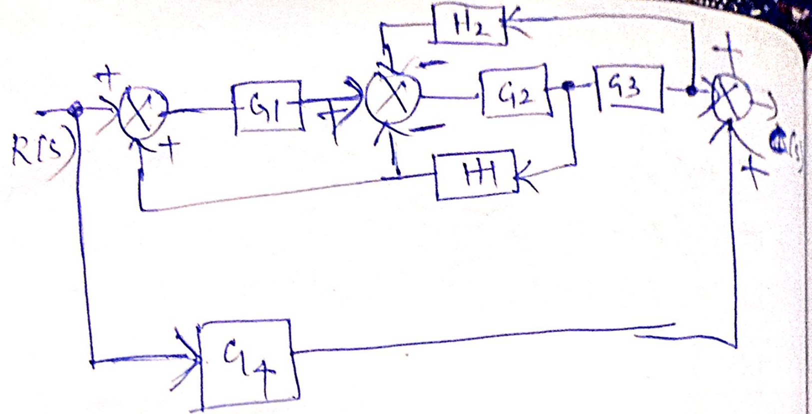
$$\Delta_5 = 1$$

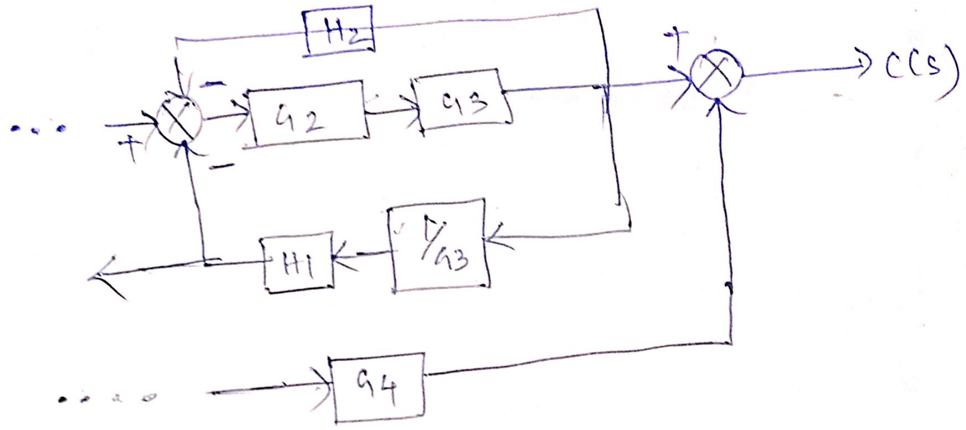
$$\Delta_6 = 1$$

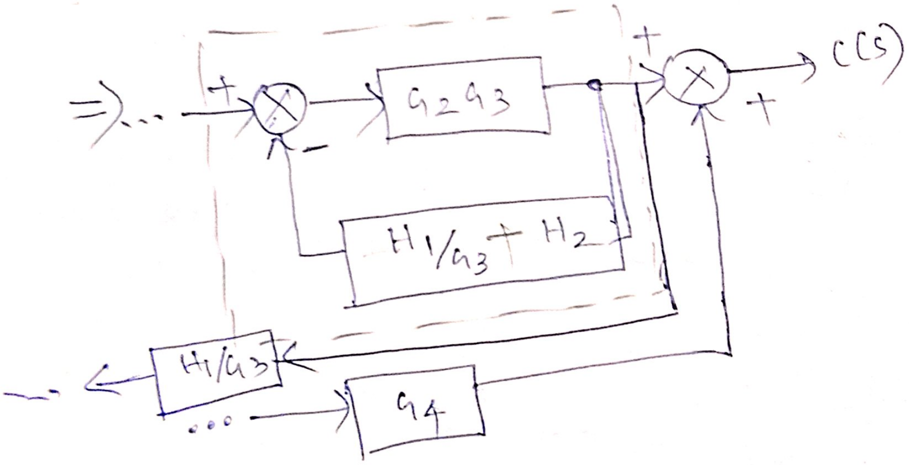
$$T(s) = \frac{1}{\Delta} \sum_{K=6} P_K \Delta_K = \frac{1}{\Delta}\left[P_1 \Delta_1 + P_2 \Delta_2 + P_3 \Delta_3 + P_4 \Delta_4 + P_5 \Delta_5 + P_6 \Delta_6\right]$$

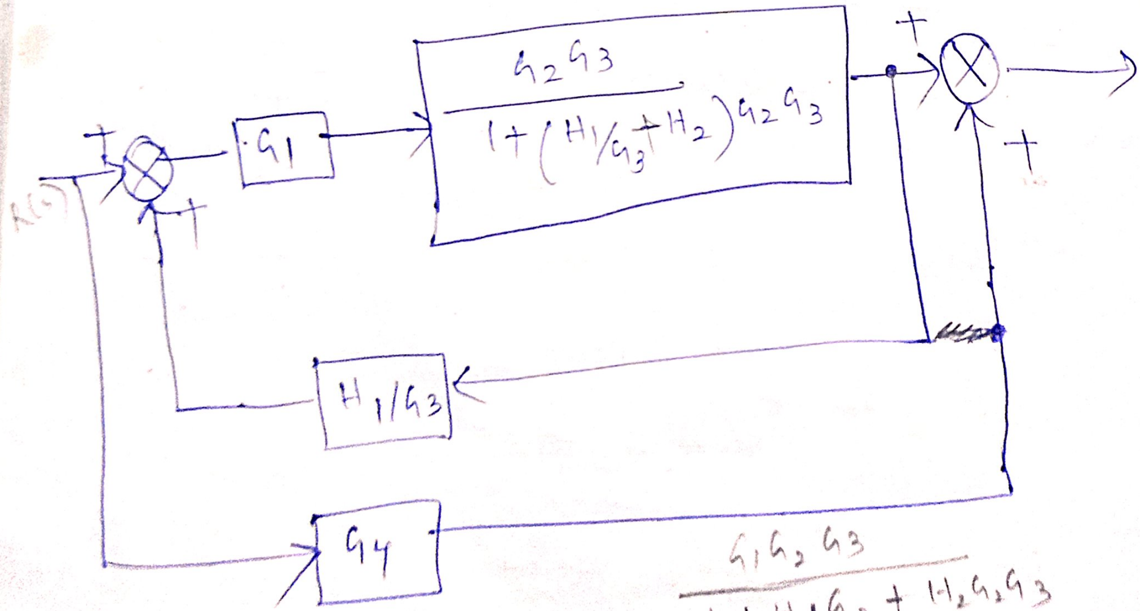
$$T(s) = \frac{\left\{ (G_2 G_4 G_6)(1 + G_5 H_2) + G_1 G_2 G_7 + (G_3 G_5 G_7)(1 + G_4 H_1) + G_3 G_6 G_8 - G_1 G_3 G_7 G_8 H_1 - G_1 G_2 G_6 G_8 H_2 \right\}}{1 + G_4 H_1 + G_5 H_2 - G_1 G_8 H_1 H_2 + G_4 G_5 H_1 H_2}$$

2) Find $\frac{C(s)}{R(s)}$ for





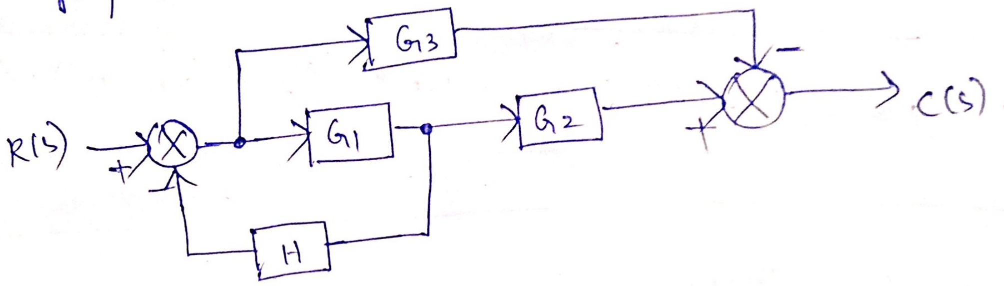




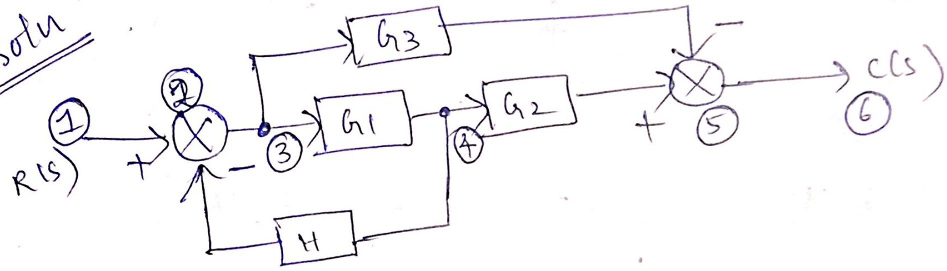
$$G_4 + \frac{G_1G_2G_3}{1+H_1G_2+H_2G_2G_3-H_1G_1G_2} \Leftarrow \frac{\dfrac{G_1G_2G_3}{1+H_1G_2+H_2G_2G_3}}{1 - \dfrac{H_1}{G_3}\dfrac{G_2G_3G_1}{(1+H_1G_2+H_2G_2G_3)}}$$

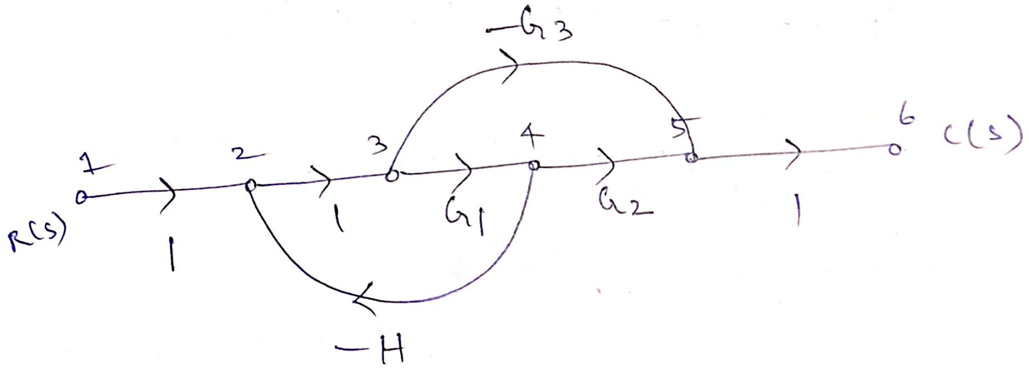
Block Diagram to Signal flow Graph.

1) convert the given block diagram to signal flow graph and determine $C(s)/R(s)$.



soln

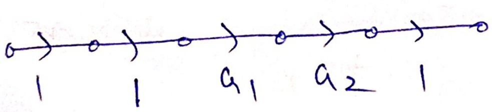




① No. of forward paths (K) = 2

Path 1

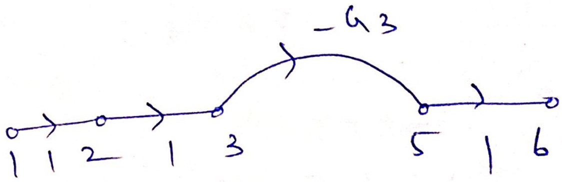
1 - 2 - 3 - 4 - 5 - 6



$P_1 = G_1 G_2$

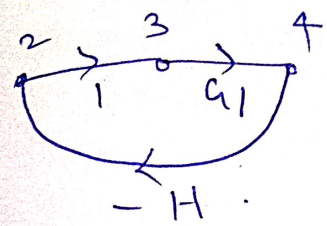
Path 2

1 - 2 - 3 - 5 - 6



$P_2 = -G_3$

Individual loops



$P_{11} = -G_1 H$.

calculation of Δ & Δ_K.

$\Delta = 1 - P_{11}$ $\quad\quad \Delta_1 = 1$

$= 1 + G_1 H$. $\quad\quad \Delta_2 = 1$

$$T(s) = \frac{1}{\Delta} \sum_K P_K \Delta_K.$$

$$= \frac{1}{\Delta}\left[P_1 \Delta_1 + P_2 \Delta_2\right]$$

$$\boxed{T(s) = \frac{G_1 G_2 - G_3}{1 + G_1 H}.}$$