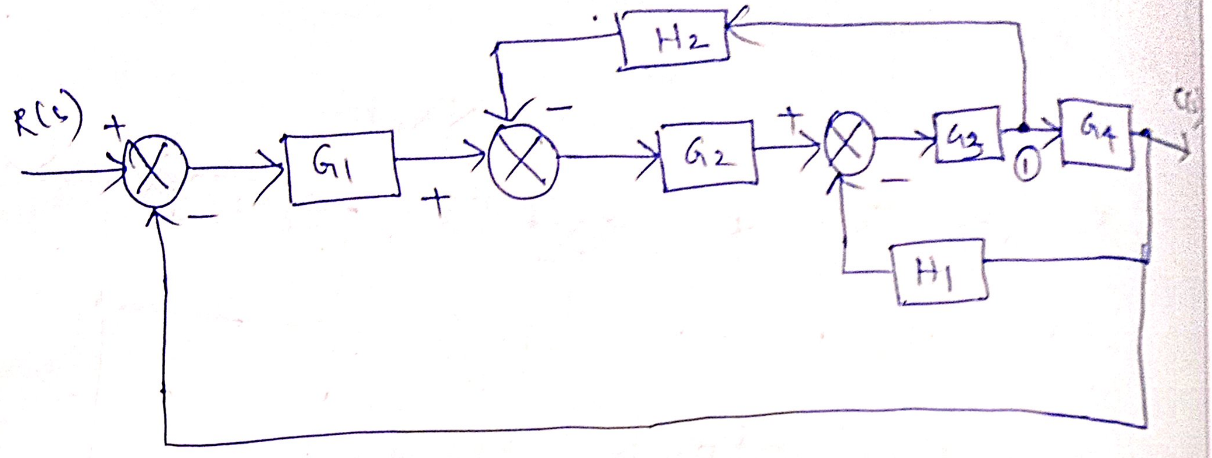
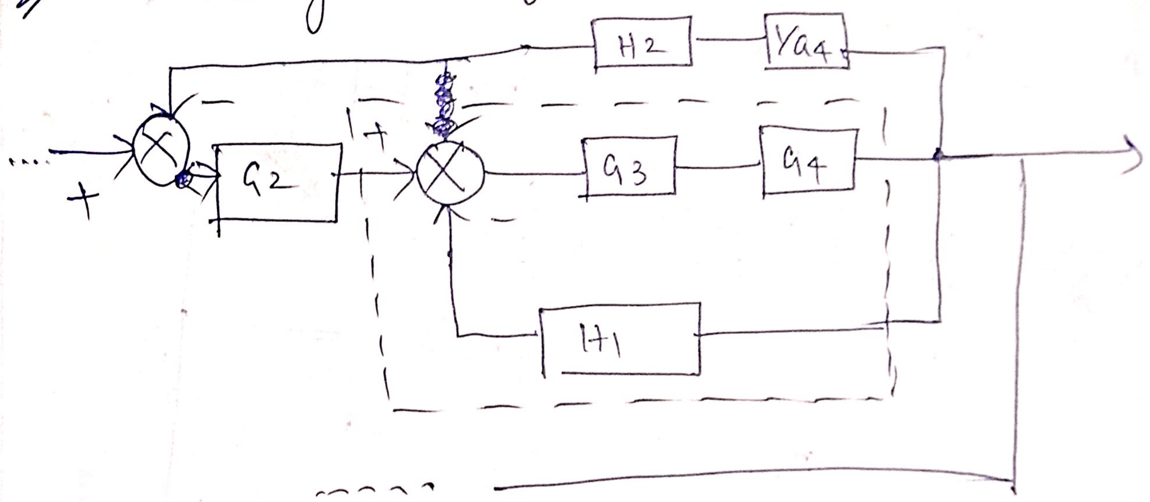
3) Find $C(s)/R(s)$.



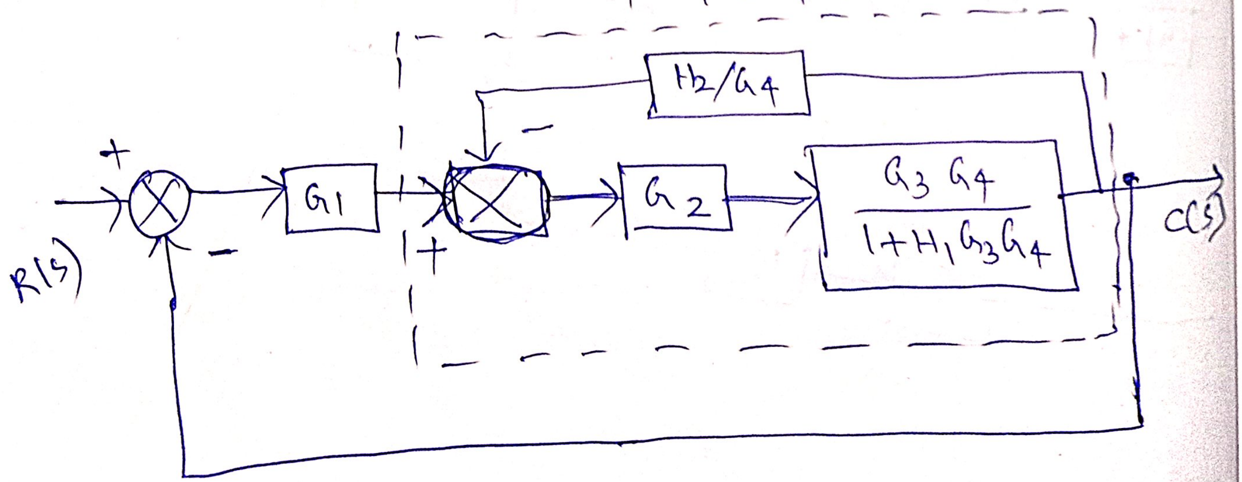
Soln.

Moving Branching point 1 after G4



cascade $\Rightarrow G_3G_4$.

$-ve$ fb $\Rightarrow \dfrac{G_3G_4}{1+H_1G_3G_4}$.

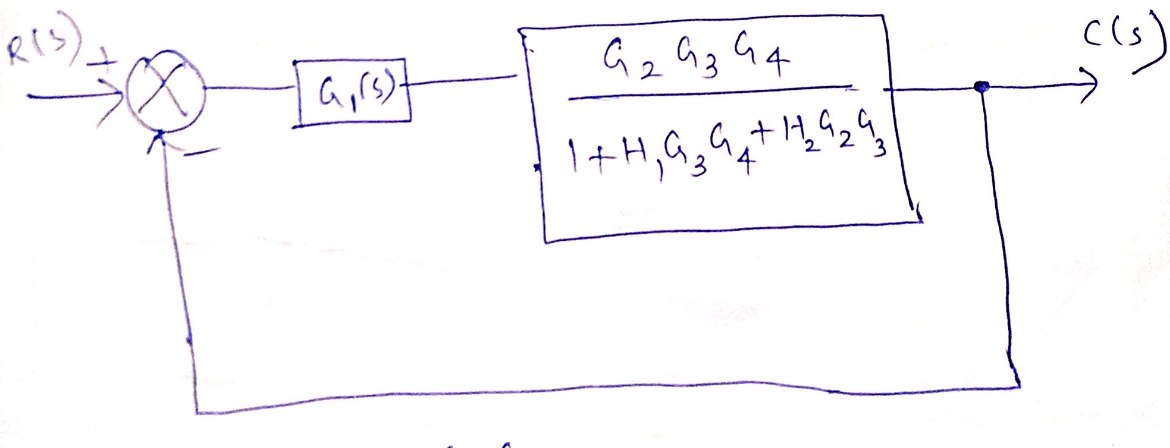


cascade $\Rightarrow \dfrac{G_2 G_3 G_4}{1 + H_1 G_3 G_4}$

$-$ve fb. $\Rightarrow \dfrac{\dfrac{G_2 G_3 G_4}{1 + H_1 G_3 G_4}}{1 + \dfrac{H_2}{\cancel{G_4}}\left(\dfrac{G_2 G_3 \cancel{G_4}}{1 + H_1 G_3 G_4}\right)}$

$$= \frac{\dfrac{G_2 G_3 G_4}{(\cancel{1 + H_1 G_3 G_4})}}{\dfrac{1 + H_1 G_3 G_4 + H_2 G_2 G_3}{(\cancel{1 + H_1 G_3 G_4})}}$$

$$= \frac{G_2 G_3 G_4}{1 + H_1 G_3 G_4 + H_2 G_2 G_3}.$$



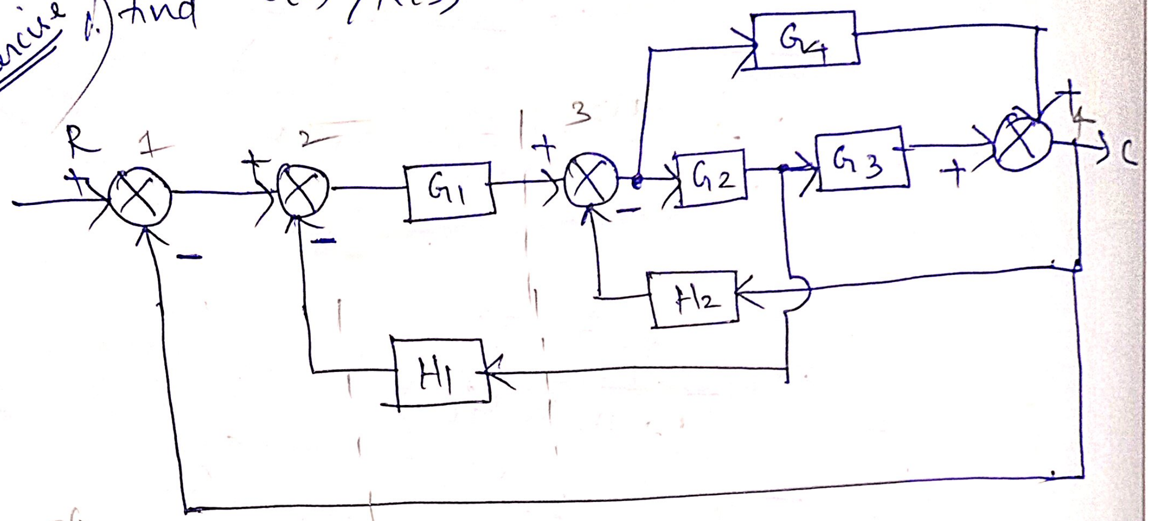
cascade $\Rightarrow \dfrac{G_1 G_2 G_3 G_4}{1 + H_1 G_3 G_4 + H_2 G_2 G_3}.$

overall - unity fb $\Rightarrow \dfrac{\left[G_1 G_2 G_3 G_4 / (1 + H_1 G_3 G_4 + H_2 G_2 G_3)\right]}{1 + \left(G_2 G_1 G_3 G_4 / (1 + H_1 G_3 G_4 + H_2 G_2 G_3)\right)}$

$$\frac{C(s)}{R(s)} = \frac{G_1 G_2 G_3 G_4}{1 + H_1 G_3 G_4 + H_2 G_2 G_3 + G_1 G_2 G_3 G_4}$$

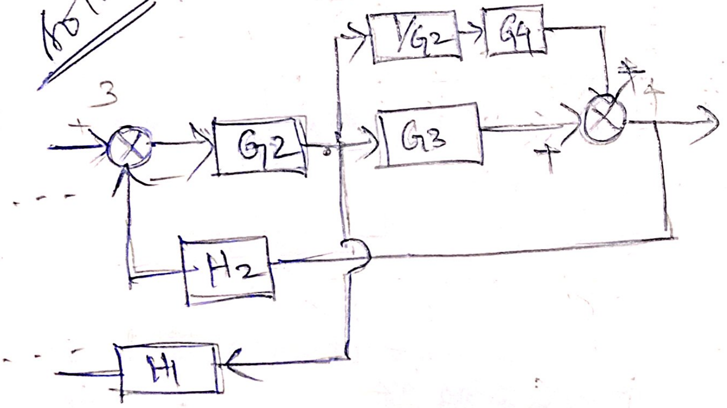
No 61

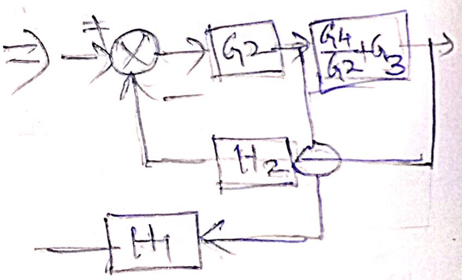
Exercise 1) Find $C(s)/R(s)$.

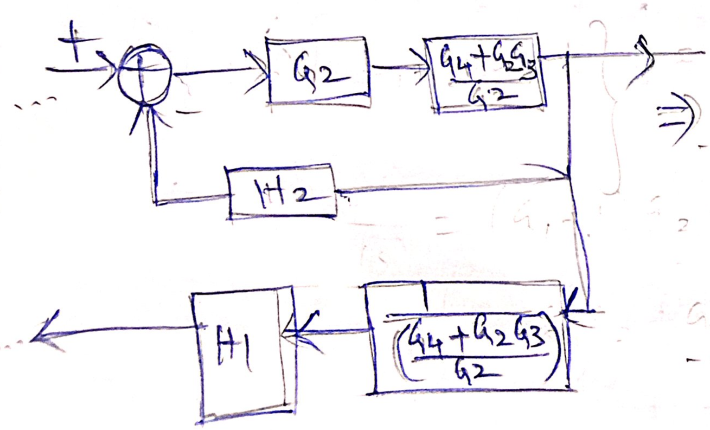


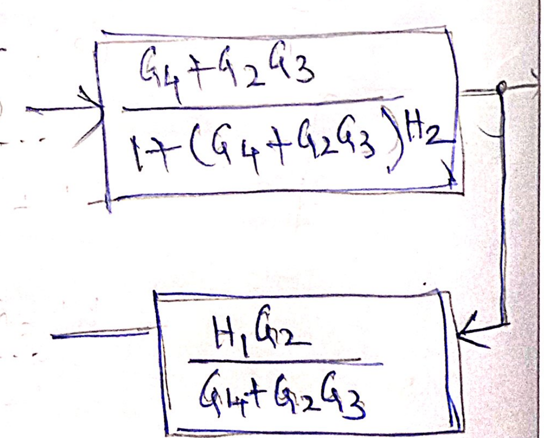
BD to SFG

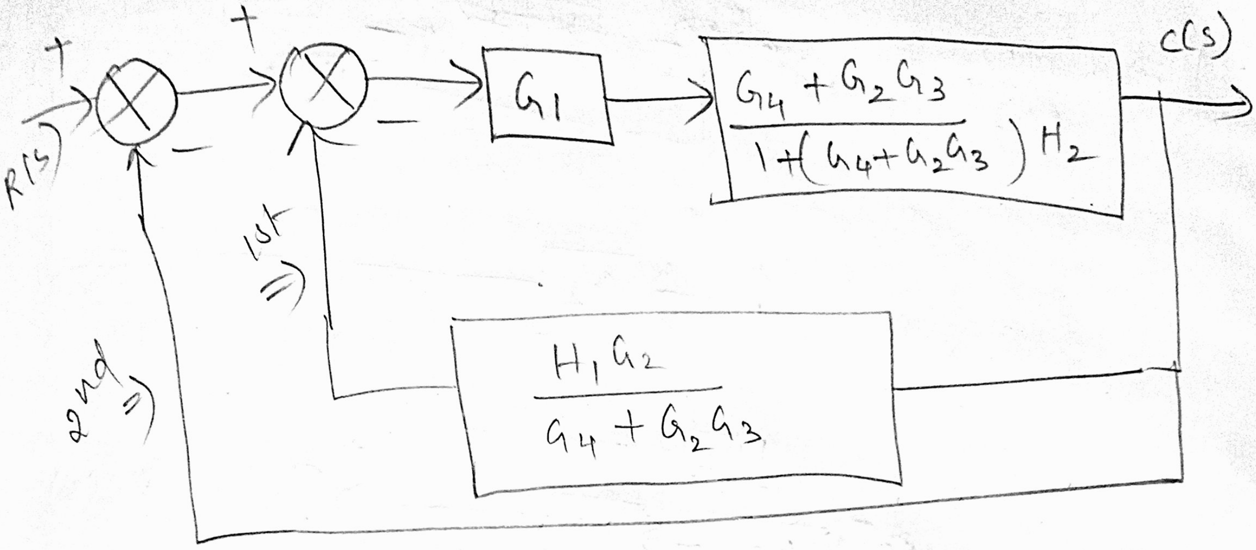
Soln





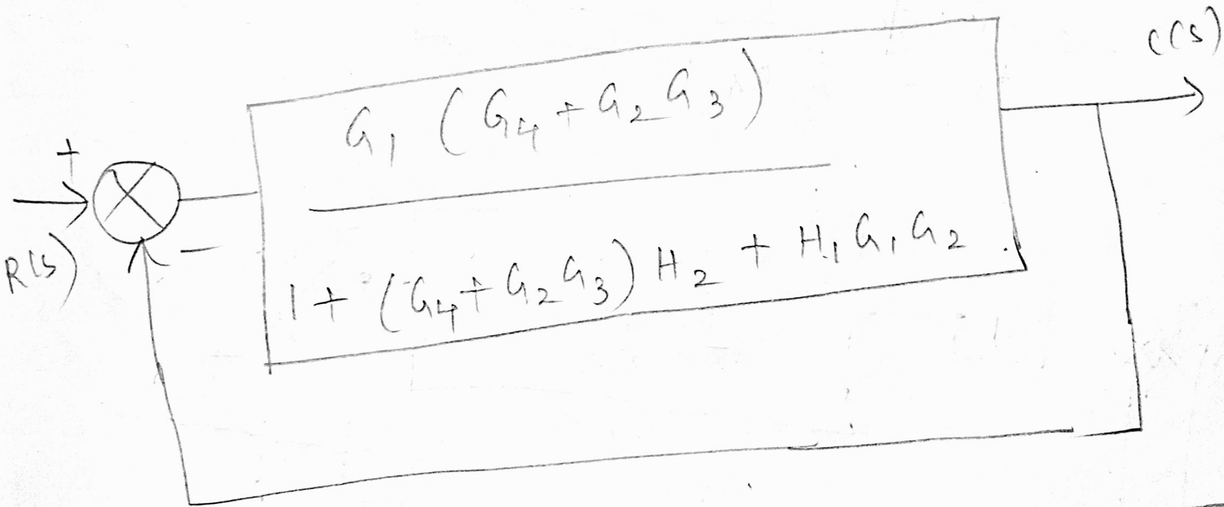






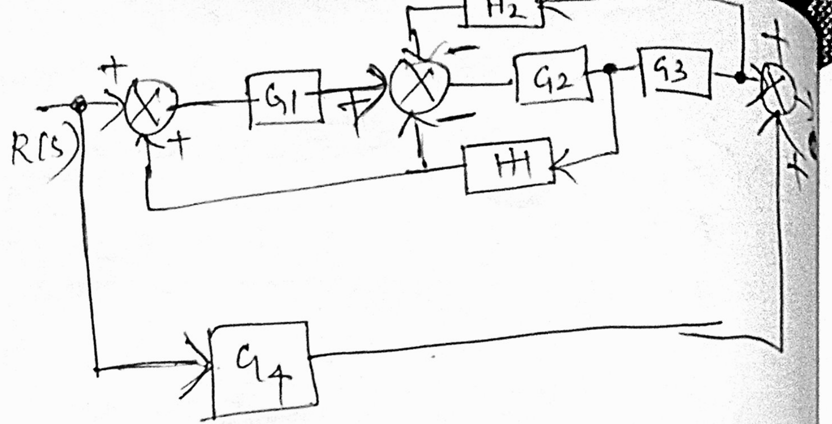
<u>1st −ve loop</u>

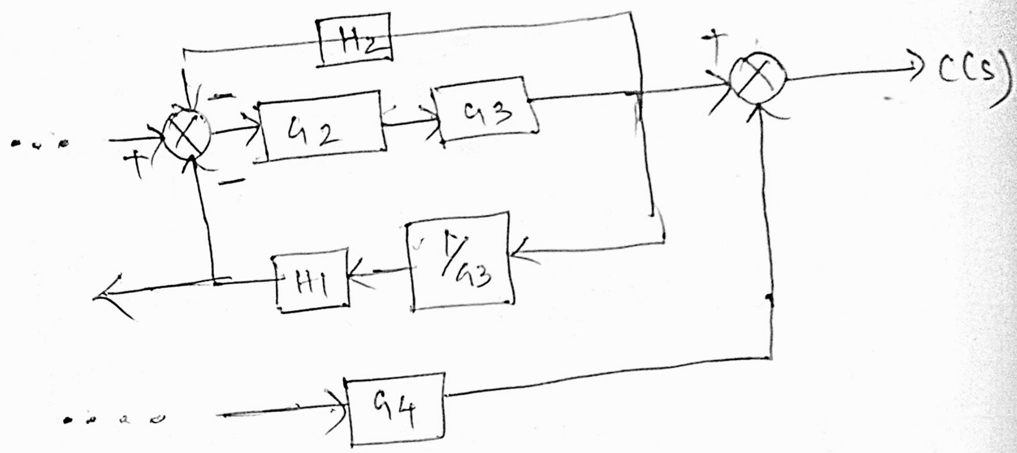
$$= \frac{\dfrac{G_1(G_4+G_2G_3)}{1+(G_4+G_2G_3)H_2}}{1 + \left(\dfrac{H_1G_2}{\cancel{G_4+G_2G_3}}\right)\left(\dfrac{G_1(\cancel{G_4+G_2G_3})}{1+(G_4+G_2G_3)H_2}\right)}$$

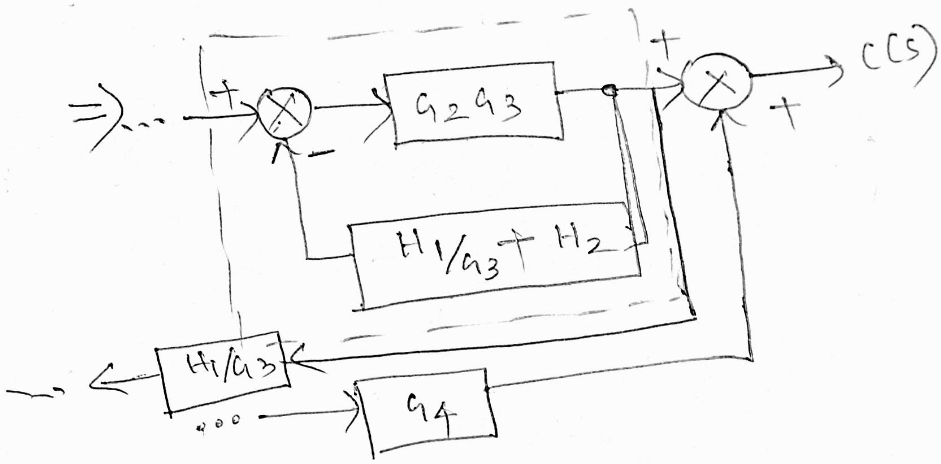


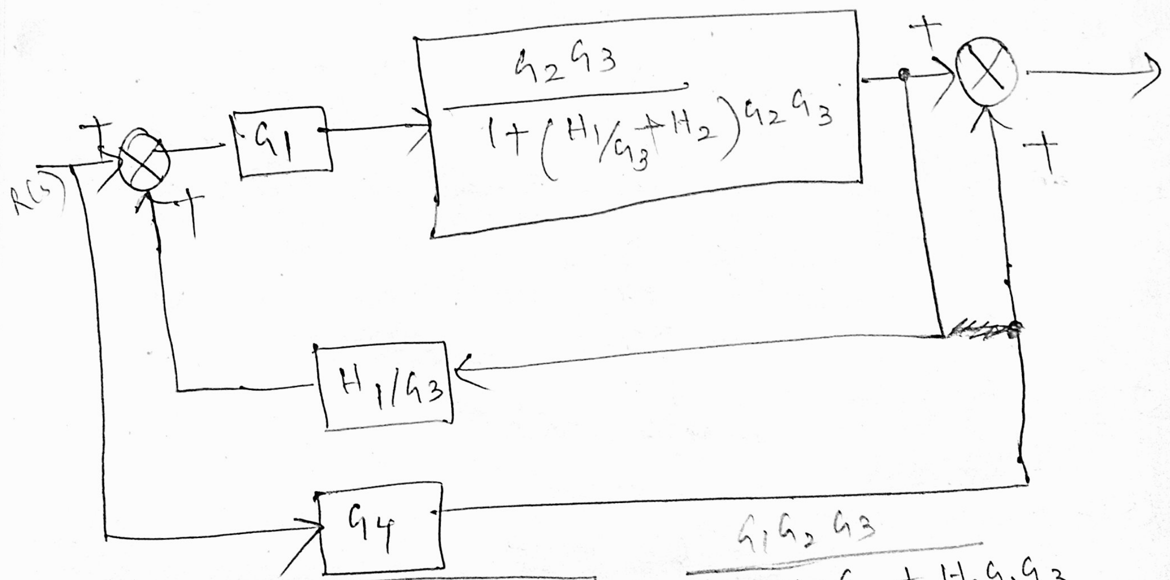
$$\boxed{\frac{C(s)}{R(s)} = \frac{G_1G_2G_3 + G_1G_4}{1+(G_4+G_2G_3)H_2 + H_1G_1G_2 + G_1(G_4+G_2G_3)}}$$

2) Find $\frac{C(s)}{R(s)}$ for



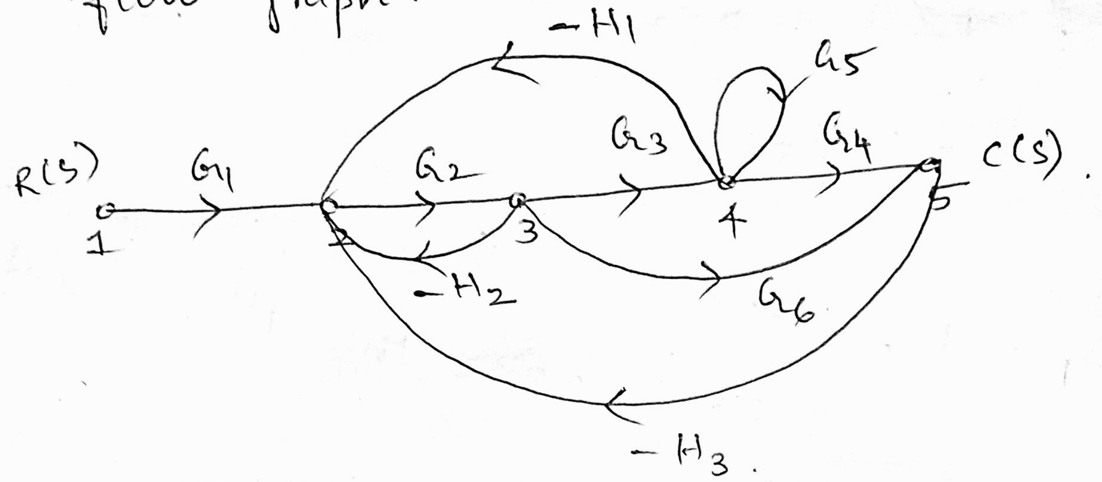






$$\boxed{G_4 + \frac{G_1G_2G_3}{1+H_1G_2+H_2G_2G_3-H_1G_1G_2}} \Leftarrow \frac{\dfrac{G_1G_2G_3}{1+H_1G_2+H_2G_2G_3}}{1 - \dfrac{H_1}{\cancel{G_3}}\,\dfrac{G_2\cancel{G_3}G_1}{(1+H_1G_2+H_2G_2G_3)}}$$

2) Find the overall gain for the signal flow graph.

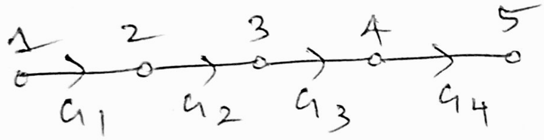


Soln

① No. of forward paths (K) = 2.

② **path - 1**

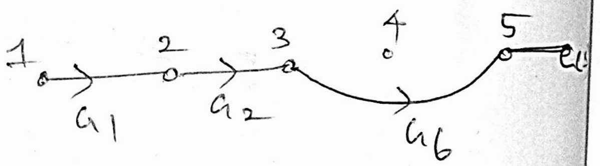
1 - 2 - 3 - 4 - 5



Frwd Path gain $P_1 = G_1 G_2 G_3 G_4$

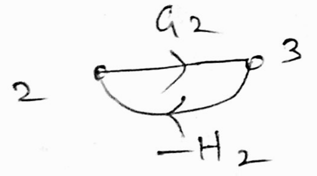
path - 2

1 - 2 - 3 - 5

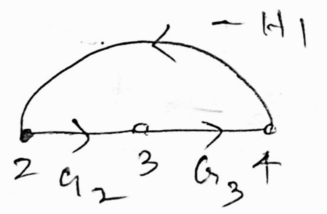


$P_2 = G_1 G_2 G_6$.

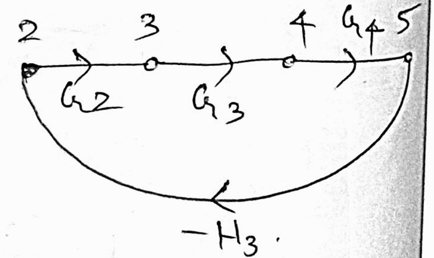
③ Individual loops & its gain



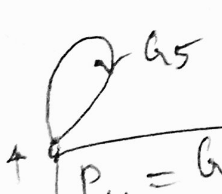
$\boxed{P_{11} = -G_2 H_2}$



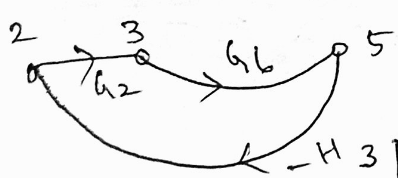
$\boxed{P_{21} = -G_2 G_3 H_1}$



$\boxed{P_{31} = -G_2 G_3 G_4 H_3}$

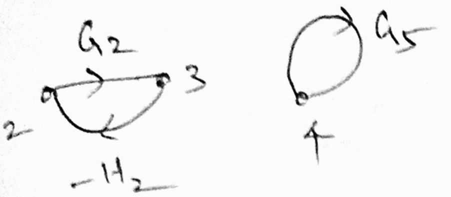


$\boxed{P_{41} = G_5}$

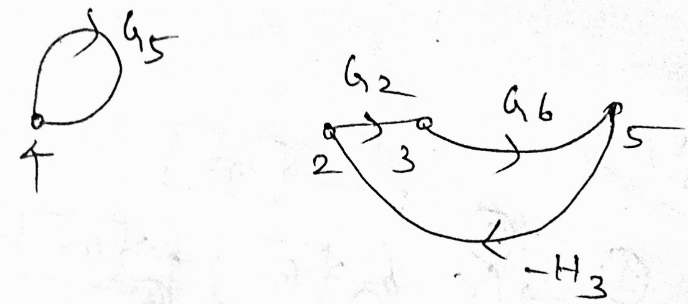


$\boxed{P_{51} = -G_2 G_6 H_3}$

(7) <u>Non-touching loops.</u>



$P_{12} = -G_2 G_5 H_2$



$P_{22} = -G_2 G_5 G_6 H_3.$

(8) <u>calculation of Δ & Δ_K</u>

$$\Delta = 1 - (P_{11} + P_{21} + P_{31} + P_{41} + P_{51}) + (P_{12} + P_{22})$$

$$= 1 + H_3 G_2 G_6 + G_2 H_2 + G_2 G_3 H_1 + G_2 G_3 G_4 H_3 - G_5 - G_2 G_5 H_2 - G_2 G_5 G_6 H_3.$$

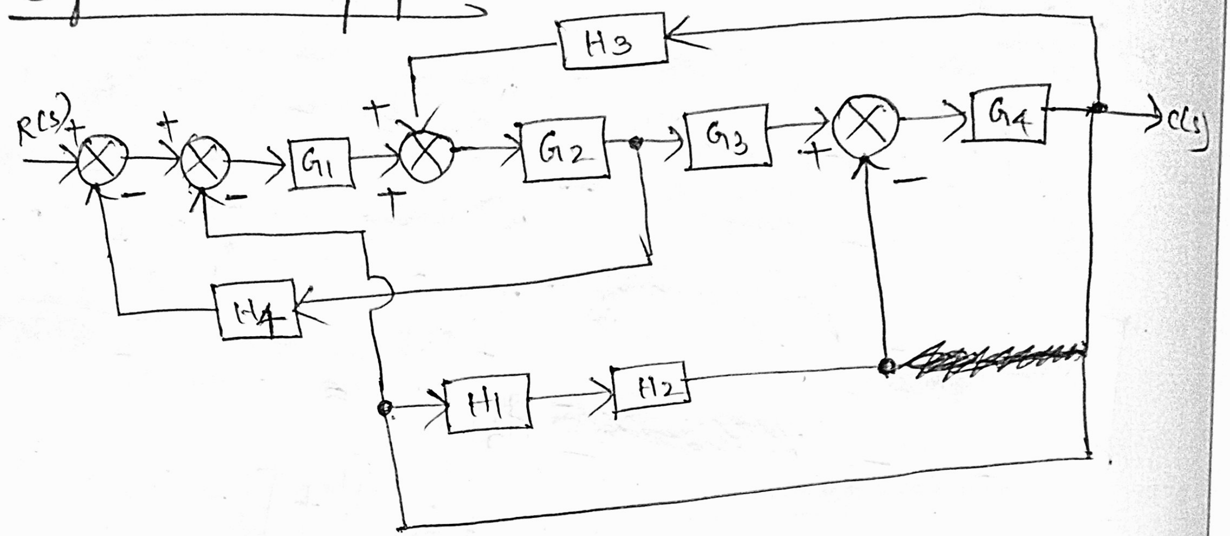
$$\Delta_1 = 1 - 0 = 1$$

$$\Delta_2 = 1 - P_{41} = 1 - G_5$$

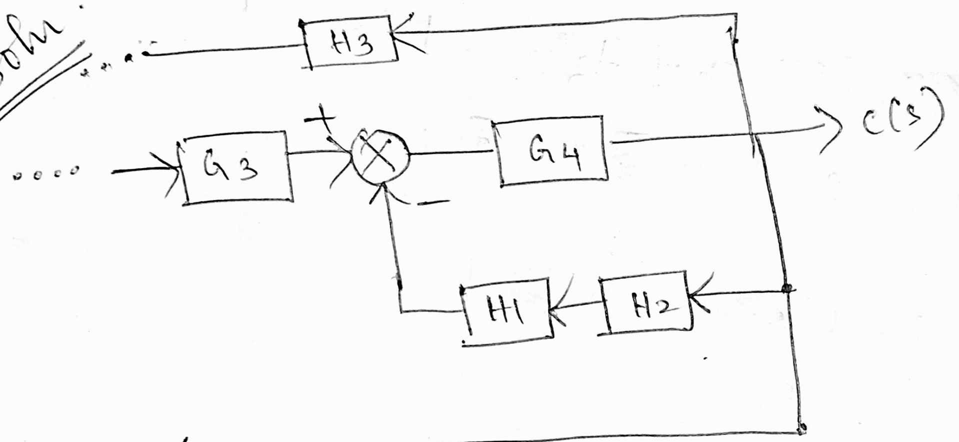
$$T(s) = \frac{1}{\Delta} \sum_{K=2} P_K \Delta_K = \frac{1}{\Delta}\left[P_1 \Delta_1 + P_2 \Delta_2\right]$$

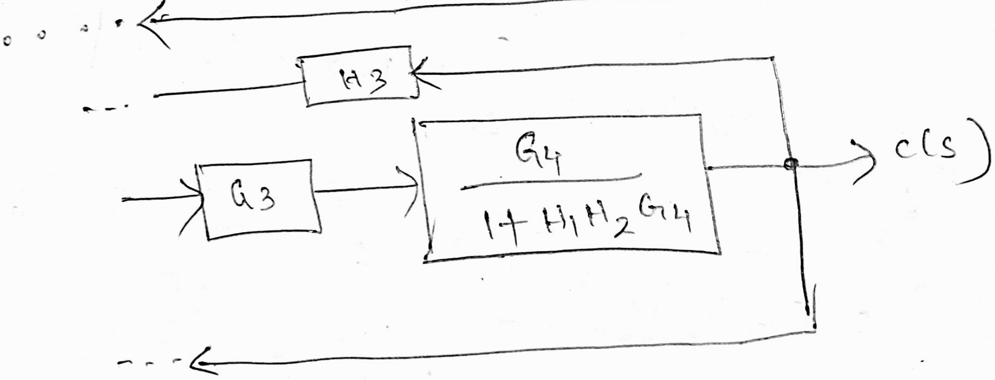
$$\boxed{T(s) = \frac{G_1 G_2 G_3 G_4 + G_1 G_2 G_6 - G_1 G_2 G_5 G_6}{\left(1 + H_3 G_2 G_6 + G_2 H_2 + G_2 G_3 H_1 + G_2 G_3 G_4 H_3 - G_5 - G_2 G_5 H_2 - G_2 G_5 G_6 H_3\right)}}$$

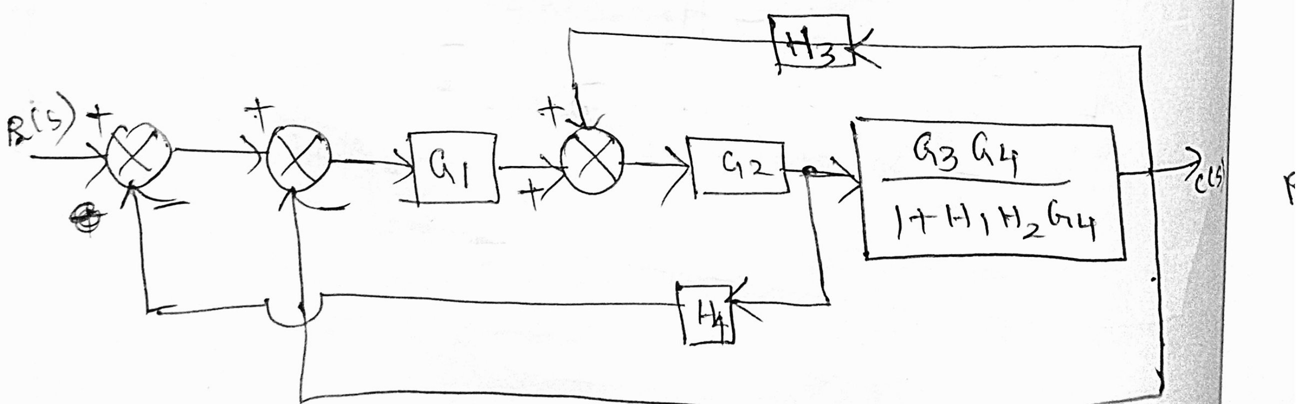
problems on Block Reduction Technique + Signal Flow Graph.

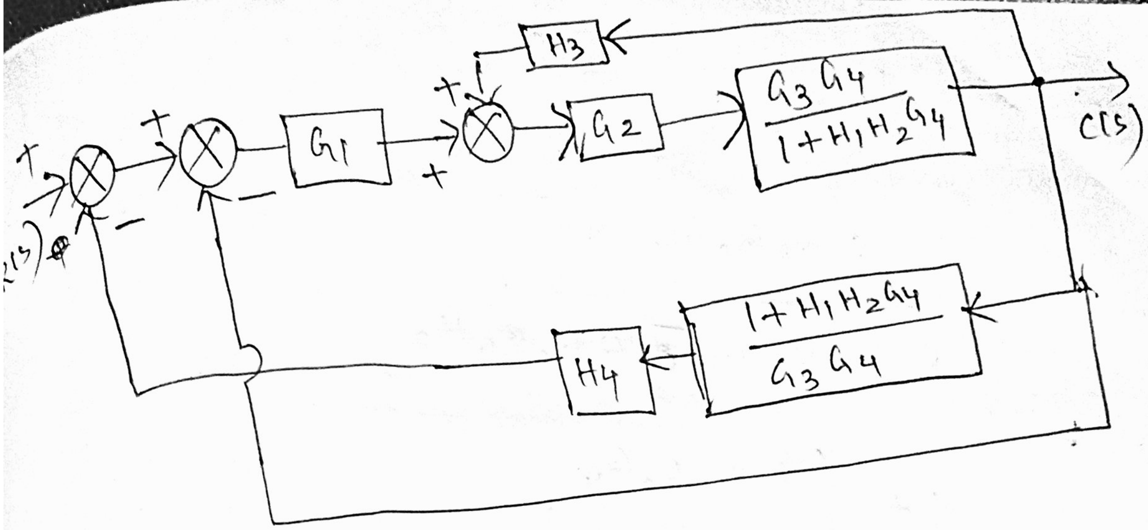
i) 

soln:









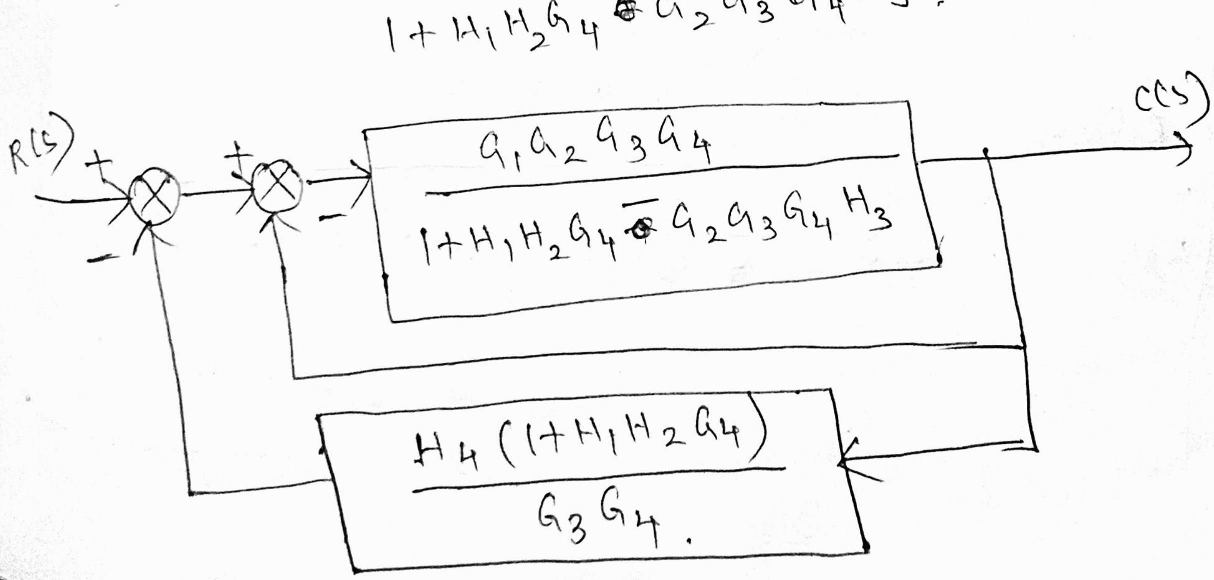
→ upper side ⇒ +ve fb.

$$= \frac{\left(\dfrac{G_2 G_3 G_4}{1 + H_1 H_2 H_4}\right)}{1 - H_3 \dfrac{G_2 G_4 G_3}{1 + H_1 H_2 G_4}}$$

$$= \frac{G_2 G_3 G_4}{1 + H_1 H_2 G_4 - H_3 G_2 G_3 G_4}.$$

⇒ cascaded with G_1

$$\therefore = \frac{G_1 G_2 G_3 G_4}{1 + H_1 H_2 G_4 - G_2 G_3 G_4 H_3}.$$



$\Rightarrow$ unity fb,

$$= \frac{\dfrac{G_1 G_2 G_3 G_4}{1 + H_1 H_2 G_4 \mp G_2 G_3 G_4 H_3}}{1 + \left(\dfrac{G_1 G_2 G_3 G_4}{1 + H_1 H_2 G_4 \mp G_2 G_3 G_4 H_3} \right)}$$

$$= \frac{G_1 G_2 G_3 G_4}{1 + H_1 H_2 G_4 \mp G_2 G_3 G_4 H_3 + G_1 G_2 G_3 G_4}.$$

final fb o/p

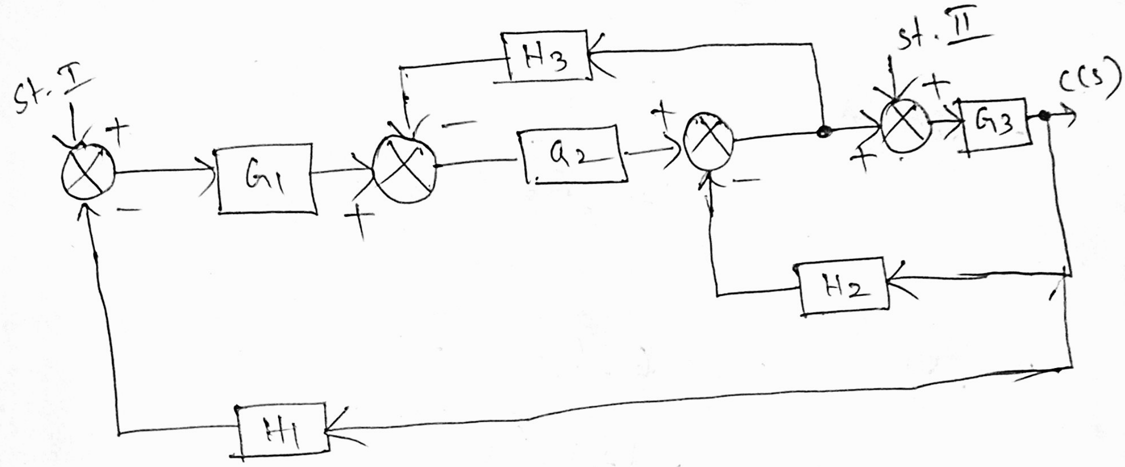
$$= \frac{\left(\dfrac{G_1 G_2 G_3 G_4}{1 + H_1 H_2 G_4 \mp G_2 G_3 G_4 H_3 + G_1 G_2 G_3 G_4} \right)}{1 + \left(\dfrac{(G_1 G_2 \cancel{G_3} \cancel{G_4})}{1 + H_1 H_2 G_4 \mp G_2 G_3 G_4 H_3 + G_1 G_2 G_3 G_4} \right) \dfrac{H_4 (1 + H_1 H_2 G_4)}{\cancel{G_3} \cancel{G_4}}}$$

$$\boxed{\frac{C(s)}{R(s)} = \frac{G_1 G_2 G_3 G_4}{\left(1 + H_1 H_2 G_4 \mp G_2 G_3 G_4 H_3 + G_1 G_2 G_3 G_4 + G_1 G_2 H_4 + G_1 G_2 G_4 H_1 H_2 H_4 \right)}}$$

2) For the s/m represented by the block diagram, Evaluate the closed loop transfer function when the R is

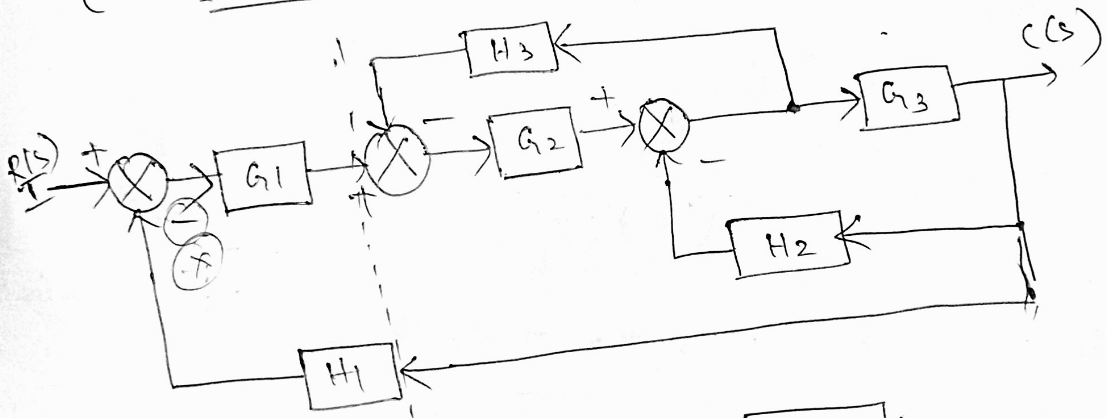
(i) at station - I

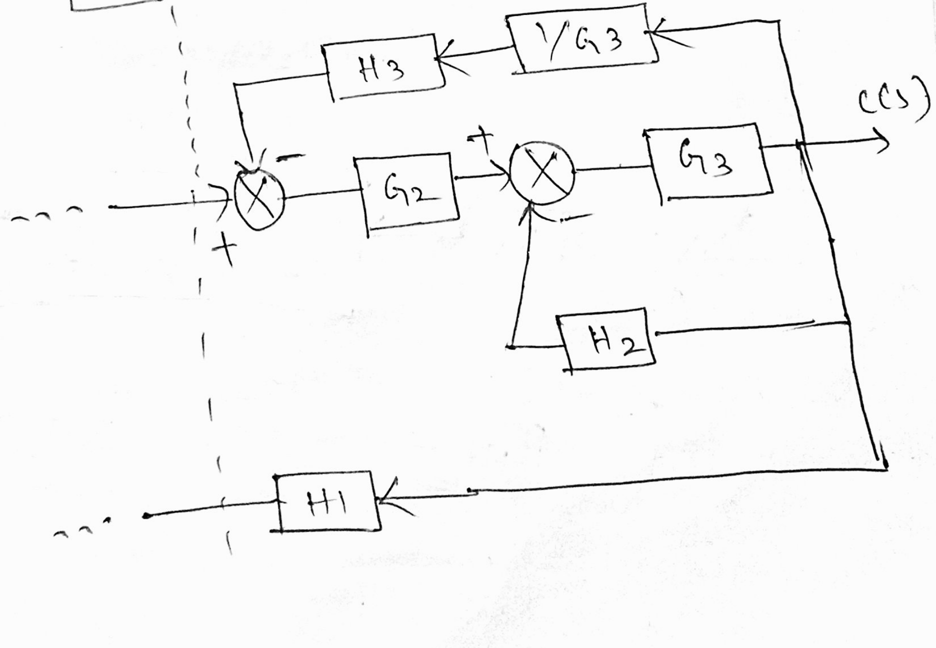
(ii) at station - II.

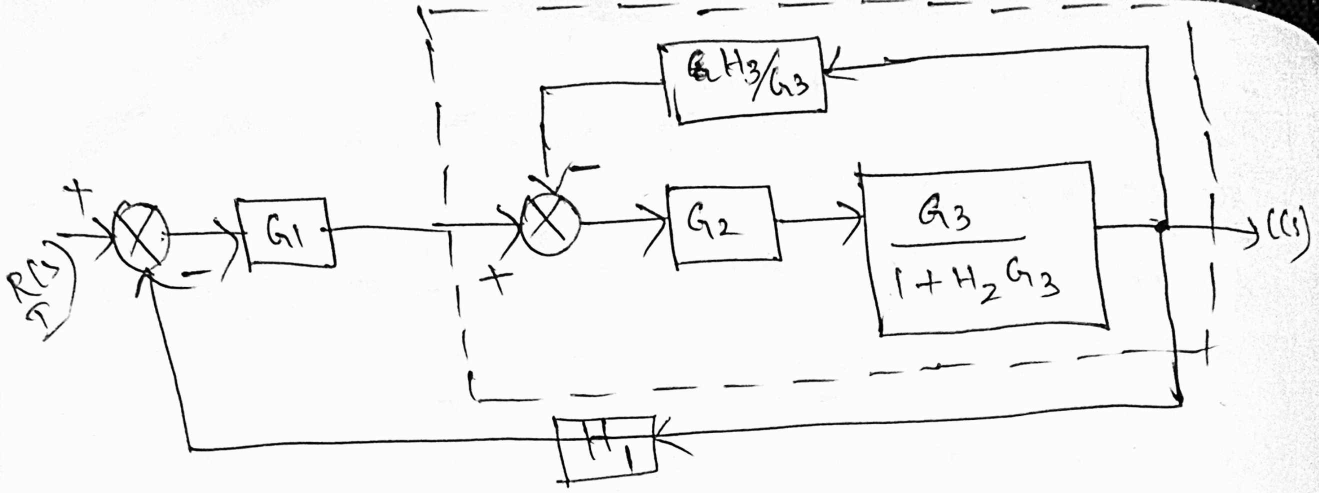


soln

(i) R at St. I





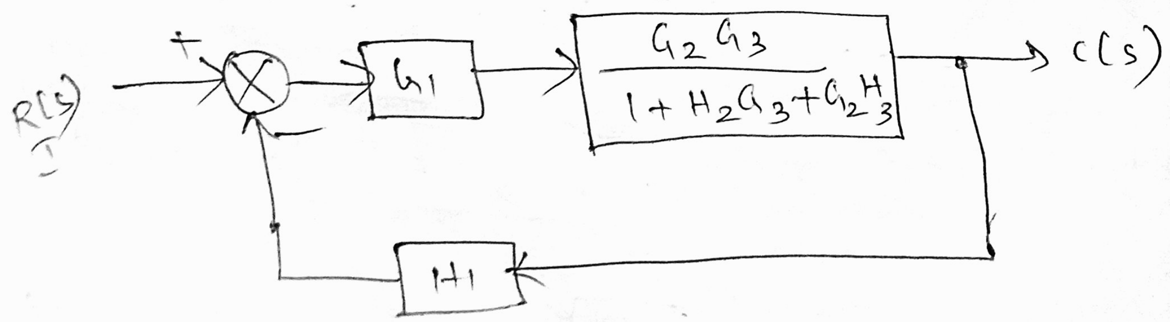


$\Rightarrow$ ~~cas~~cascade $\dfrac{G_2G_3}{1+H_2G_3}$

-ve fb

$$= \frac{\left(\dfrac{G_2G_3}{1+H_2G_3}\right)}{1+\dfrac{H_3}{\cancel{G_3}}\left(\dfrac{G_2\cancel{G_3}}{1+H_2G_3}\right)}$$

$$= \frac{\dfrac{G_2G_3}{\cancel{(1+H_2G_3)}}}{\dfrac{1+H_2G_3+G_2H_3}{\cancel{(1+H_2G_3)}}} = \frac{G_2G_3}{1+H_2G_3+G_2H_3}$$

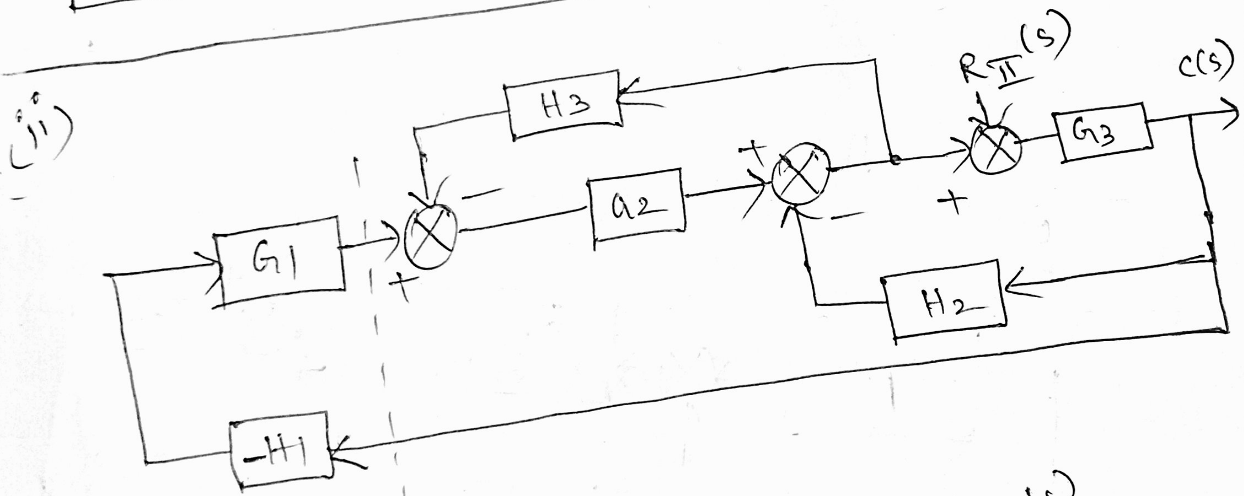


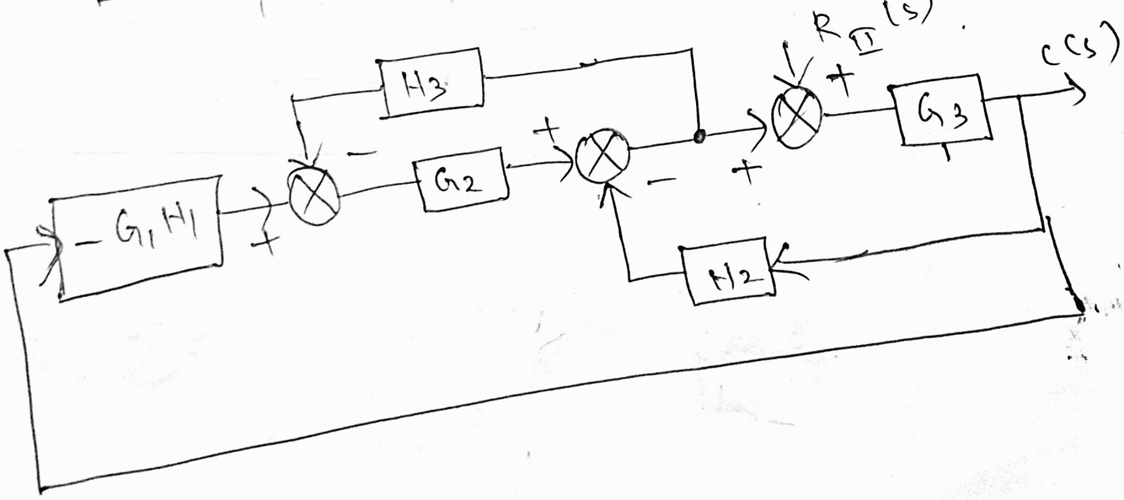
cascade $\Rightarrow \dfrac{G_1 G_2 G_3}{1 + H_2 G_3 + G_2 H_3}$

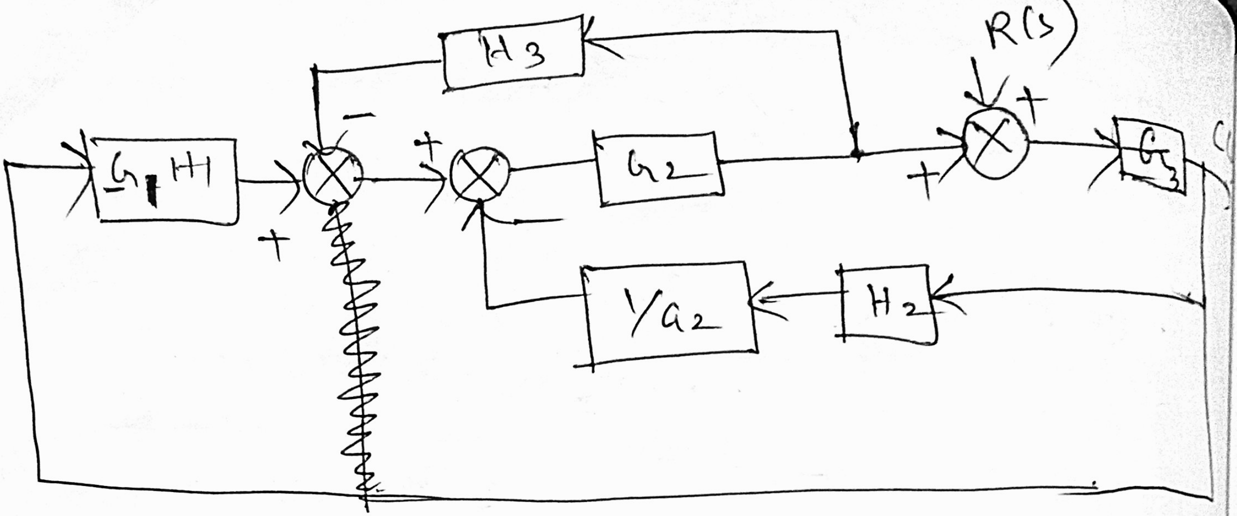
-ve fb. $\Rightarrow \dfrac{\left(\dfrac{G_1 G_2 G_3}{1 + H_2 G_3 + G_2 H_3}\right)}{1 + H_1 \dfrac{G_1 G_2 G_3}{1 + H_2 G_3 + G_2 H_3}}$

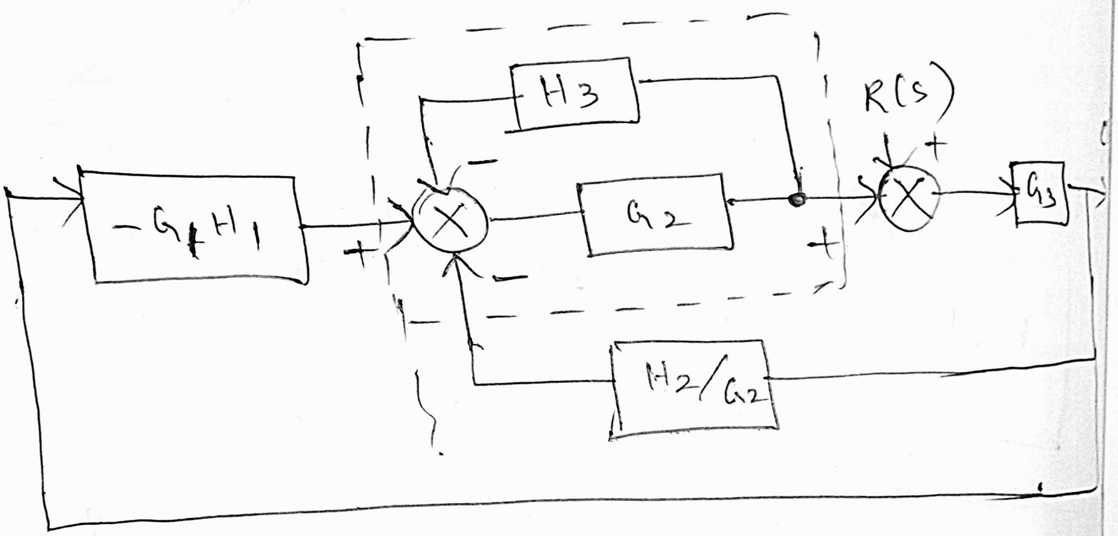
$$\boxed{\frac{C(s)}{R_I(s)} = \frac{G_1 G_2 G_3}{1 + H_2 G_3 + G_2 H_3 + H_1 G_1 G_2 G_3}}$$

(ii)

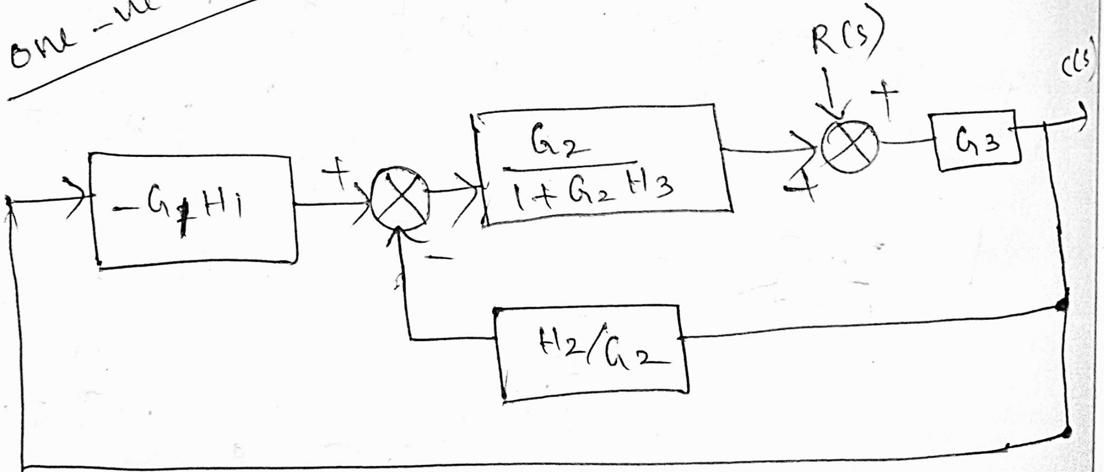


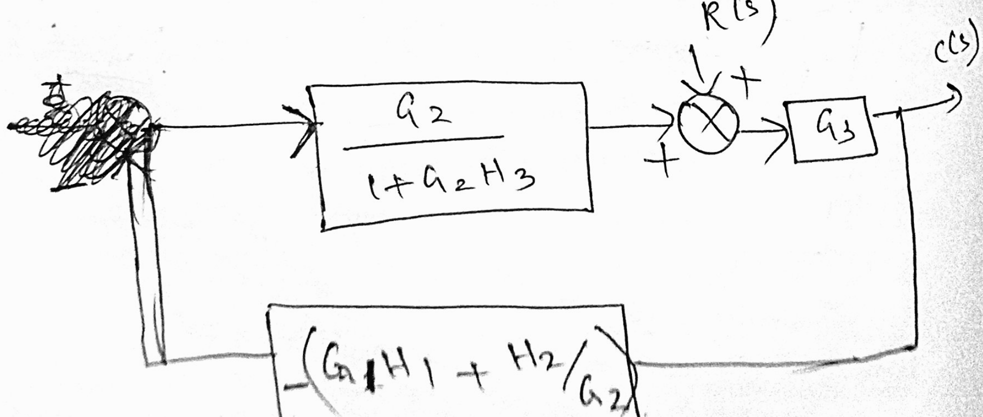


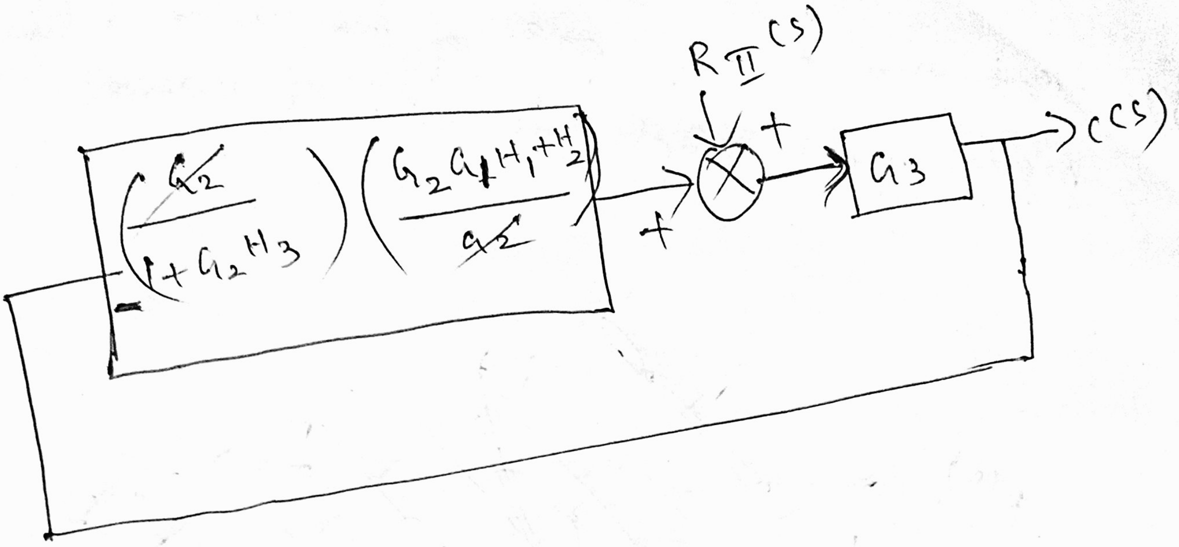

H3
R(s)
G1H1
G2
1/G2
H2
C(


H3
R(s)
-G1H1
G2
G3
H2/G2

one -ve fb.


R(s)
C(s)
-G1H1
G2
1+G2H3
G3
H2/G2


R(s)
C(s)
G2
1+G2H3
G3
-(G1H1 + H2/G2)



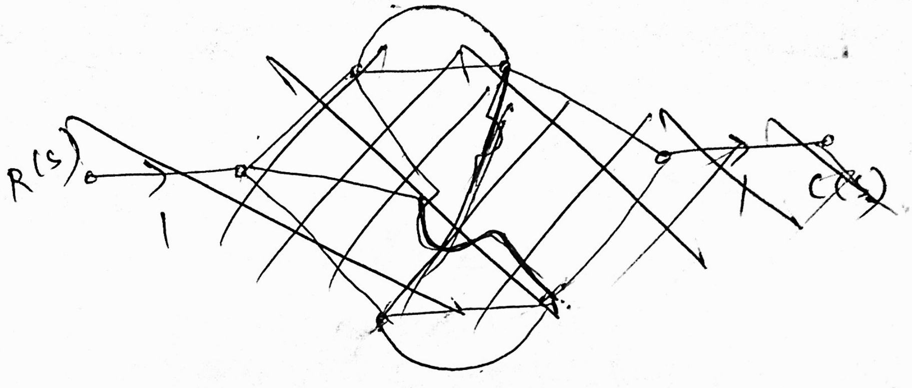
+ve fb

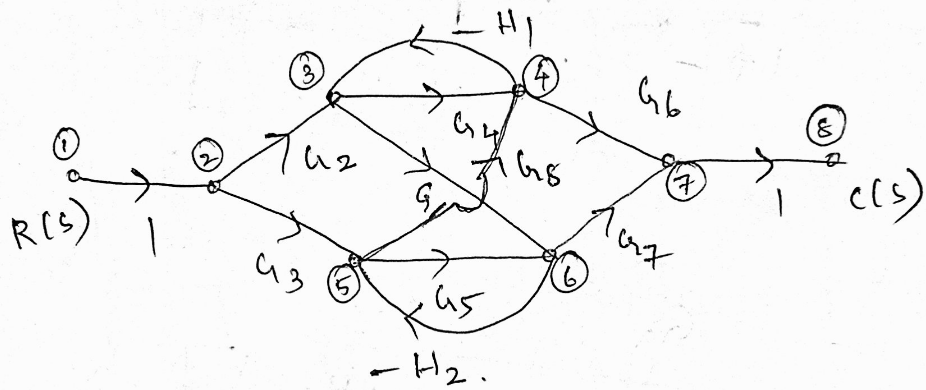
$$= \frac{G_3}{1 - \left[-G_3\left(\frac{G_2G_1H_1+H_2}{1+G_2H_3}\right)\right]}$$

$$= \frac{G_3}{1 + \left[G_3\left(\frac{G_2G_1H_1+H_2}{1+G_2H_3}\right)\right]}$$

$$\boxed{\frac{C(s)}{R_{II}(s)} = \frac{G_3(1+G_2H_3)}{1+G_2H_3+G_1G_2G_3H_1+H_2G_3}}$$

3) Find the overall gain of the s/m whose signal flow graph is shown below.

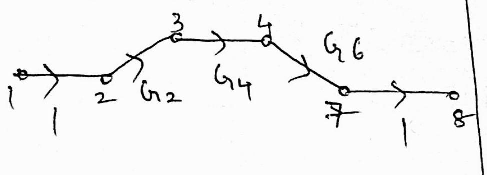




Soln ① No. of forward paths (K) = 6

Path 1

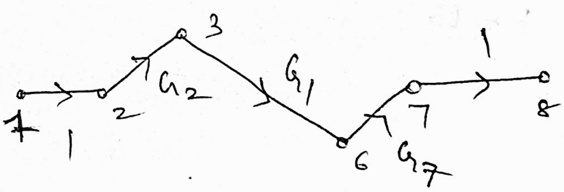
1 – 2 – 3 – 4 – 7 – 8



$\boxed{P_1 = G_2 G_4 G_6}$

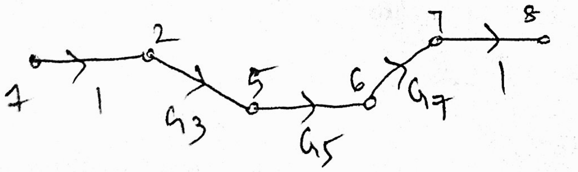
Path – 2

1 – 2 – 3 – 6 – 7 – 8



$\boxed{P_2 = G_1 G_2 G_7}$

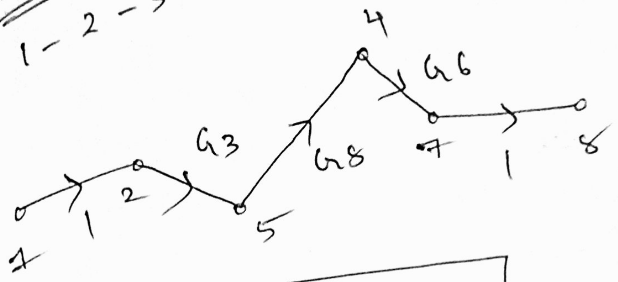
Path – 3 . 1 – 2 – 5 – 6 – 7 – 8



$\boxed{P_3 = G_3 G_5 G_7}$

Path-4

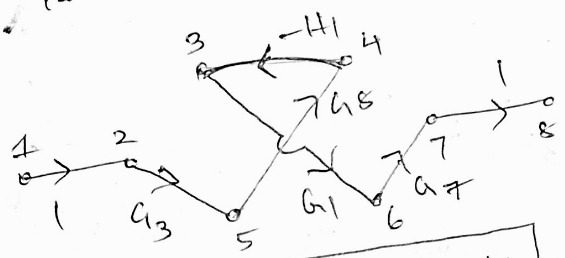
1 - 2 - 5 - 4 - 7 - 8



$$\boxed{P_4 = G_3 G_6 G_8}$$

Path 5

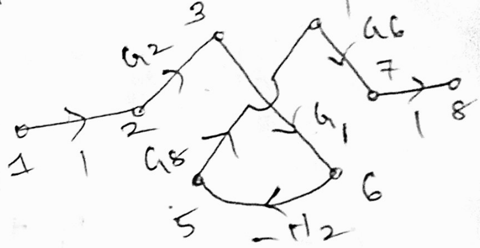
1-2-5 - 4 - 3 - 6 - 7 - 8



$$\boxed{P_5 = -G_1 G_3 G_7 G_8 H_1}$$

Path 6

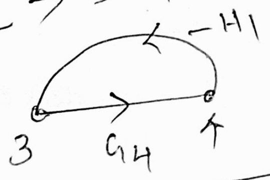
1 - 2 - 3 - 6 - 5 - 4 - 7 - 8



$$\boxed{P_6 = -G_1 G_2 G_6 G_8 H_2}$$

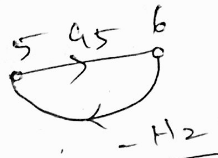
① Individual loops

loop 1 → 3 - 4



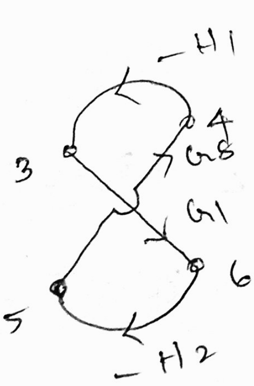
$$\boxed{P_{11} = -G_4 H_1}$$

loop 2



$$\boxed{P_{21} = -G_5 H_2}$$

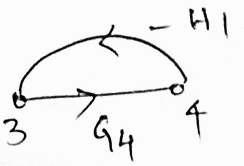
loop-3

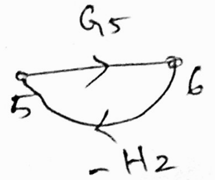


$$\boxed{P_{31} = G_1 G_8 H_1 H_2}$$

③ Non-touching loops.

– 2-non-touching loop:





$$P_{12} = G_4 G_5 H_1 H_2$$

④ calculation of Δ & Δ_K

$$\Delta = 1 - (P_{11} + P_{21} + P_{31}) + (P_{12})$$

$$= 1 + G_4 H_1 + G_5 H_2 - G_1 G_8 H_1 H_2 + G_4 G_5 H_1 H_2$$

$$\Delta_1 = 1 - (P_{21}) = 1 + G_5 H_2$$

$$\Delta_2 = 1$$

$$\Delta_3 = 1 - (P_{11}) = 1 + G_4 H_1$$

$$\Delta_4 = 1$$

$$\Delta_5 = 1$$

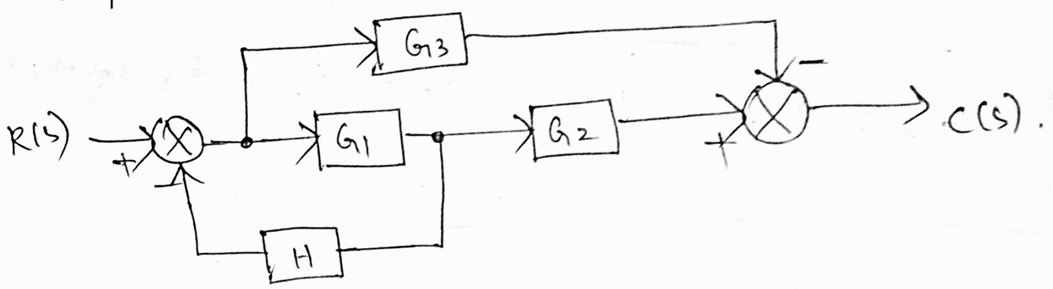
$$\Delta_6 = 1$$

$$T(s) = \frac{1}{\Delta} \sum_{K=6} P_K \Delta_K = \frac{1}{\Delta}\left[P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3 + P_4\Delta_4 + P_5\Delta_5 + P_6\Delta_6\right]$$

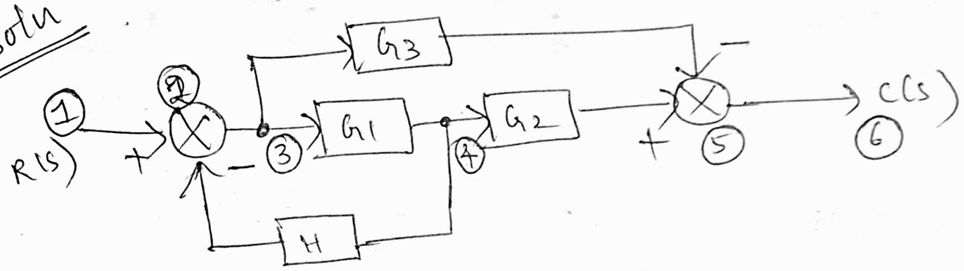
$$T(s) = \frac{\left\{ (G_2 G_4 G_6)(1 + G_5 H_2) + G_1 G_2 G_7 + (G_3 G_5 G_7)(1 + G_4 H_1) + G_3 G_6 G_8 - G_1 G_3 G_7 G_8 H_1 - G_1 G_2 G_6 G_8 H_2 \right\}}{1 + G_4 H_1 + G_5 H_2 - G_1 G_8 H_1 H_2 + G_4 G_5 H_1 H_2}$$

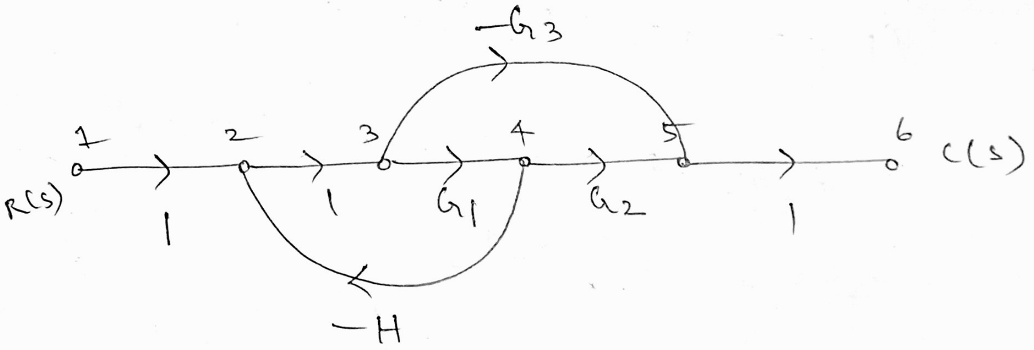
Block Diagram to Signal flow Graph.

1) convert the given block diagram to signal flow graph and determine $C(s)/R(s)$.



soln

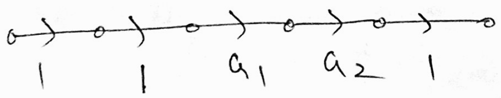




① No. of forward paths (K) = 2

Path 1

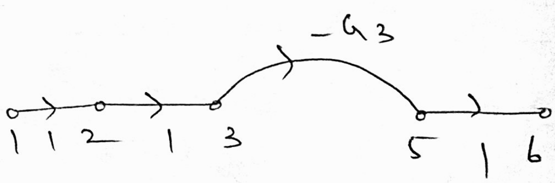
1 − 2 − 3 − 4 − 5 − 6



$P_1 = G_1 G_2$

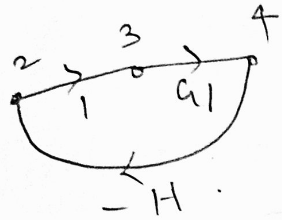
Path 2

1 − 2 − 3 − 5 − 6



$P_2 = -G_3$

Individual loops



$P_{11} = -G_1 H$.

calculation of Δ & Δ_K.

$\Delta = 1 - P_{11}$

$= 1 + G_1 H$.

$\Delta_1 = 1$

$\Delta_2 = 1$

$$T(s) = \frac{1}{\Delta} \sum_K P_K \Delta_K .$$

$$= \frac{1}{\Delta}\left[P_1 \Delta_1 + P_2 \Delta_2\right]$$

$$\boxed{T(s) = \frac{G_1 G_2 - G_3}{1 + G_1 H}.}$$