STEPS TO DESIGN FIR FILTER USING WINDOWS:

1. Write the desired frequency response $H_d(\omega)$ of filter.
2. Find $h_d(n)$ by finding $IFT\{H_d(\omega)\}$

   ie $h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(\omega)\, e^{j\omega n}\, d\omega$.

3. Truncate Infinite duration sequence $h_d(n)$ into 'N' samples ie $h(n)$.

   $h(n) = h_d(n) \cdot w(n)$

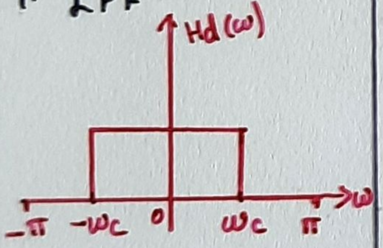
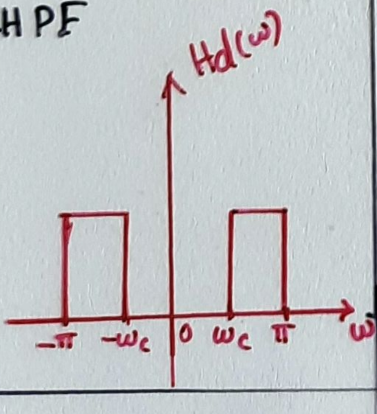
4. Find z-transform of $h(n)$ ie $H(z) = z^{-\left(\frac{N-1}{2}\right)} \left\{ h(0) + \sum_{n=1}^{\frac{N-1}{2}} h(n)\left[z^{n} + z^{-n}\right] \right\}$
5. Find frequency response of $h(n)$ ie $|H(\omega)|$

   Find $|H(\omega)|$ for various values of ω from 0 to π.

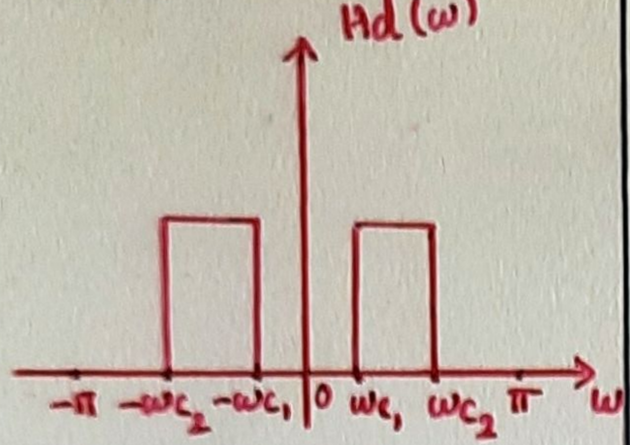
   & Sketch the graph between $|H(\omega)|$ & ω.

   $|H(\omega)| = h\left(\frac{N-1}{2}\right) + \sum_{n=1}^{\left(\frac{N-1}{2}\right)} 2\, h\left(\frac{N-1}{2} - n\right) \cos \omega n$

$H_d(\omega)$ & $h_d(n)$ for FIR filter design using windows:

TYPES OF FILTER	$H_d(\omega)$	$h_d(n)$
1. LPF 	$H_d(\omega) = e^{-j\omega\tau}\ ; -\omega_c \le \omega \le \omega_c$ $0\ ; -\pi \le \omega < -\omega_c$ $0\ ; \omega_c < \omega \le \pi$	$h_d(n) = \frac{1}{2\pi} \int_{-\omega_c}^{\omega_c} e^{-j\omega\tau} \cdot e^{j\omega n} \cdot d\omega$
2. HPF 	$H_d(\omega) = e^{-j\omega\tau}\ ; -\pi \le \omega \le -\omega_c$ $0\ ; -\omega_c < \omega < \omega_c$ $e^{-j\omega\tau}\ ; \omega_c \le \omega \le \pi$	$h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{-\omega_c} e^{-j\omega\tau} \cdot e^{j\omega n}\, d\omega + \frac{1}{2\pi} \int_{\omega_c}^{\pi} e^{-j\omega\tau} \cdot e^{j\omega n}\, d\omega$.

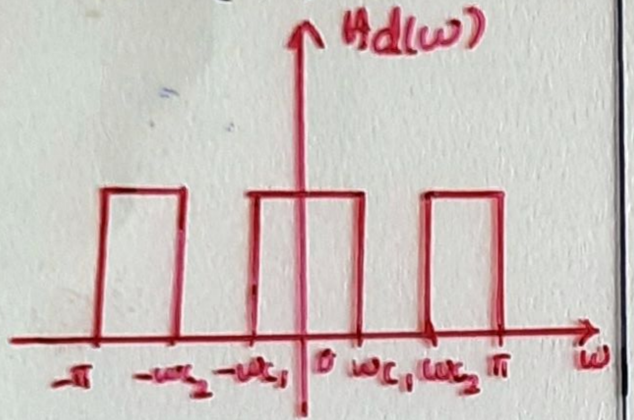
3. BPF:



$$H_d(\omega) = \begin{cases} 0 & ; -\pi \le \omega < -\omega_{c_2} \\ e^{-j\omega\tau} & ; -\omega_{c_2} \le \omega \le -\omega_{c_1} \\ 0 & ; -\omega_{c_1} < \omega < \omega_{c_1} \\ e^{-j\omega\tau} & ; \omega_{c_1} \le \omega \le \omega_{c_2} \\ 0 & ; \omega_{c_2} < \omega \le \pi \end{cases}$$

$$h_d(n) = \frac{1}{2\pi} \int_{-\omega_{c_2}}^{-\omega_{c_1}} e^{-j\omega\tau} \cdot e^{j\omega n} \, d\omega + \frac{1}{2\pi} \int_{\omega_{c_1}}^{\omega_{c_2}} e^{-j\omega\tau} \cdot e^{j\omega n} \cdot d\omega.$$

4. BSF (or) BRF:



$$h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{-\omega_{c_1}} e^{-j\omega\tau} \cdot e^{j\omega n} \, d\omega + \frac{1}{2\pi} \int_{-\omega_{c_1}}^{\omega_{c_1}} e^{-j\omega\tau} \cdot e^{j\omega n} \, d\omega + \frac{1}{2\pi} \int_{\omega_{c_2}}^{\pi} e^{-j\omega\tau} \cdot e^{j\omega n} \, d\omega$$

$$H_d(\omega) = \begin{cases} e^{-j\omega\tau} & ; -\pi \le \omega \le -\omega_{c_2} \\ 0 & ; -\omega_{c_2} \le \omega \le -\omega_{c_1} \\ e^{-j\omega\tau} & ; -\omega_{c_1} \le \omega \le \omega_{c_1} \\ 0 & ; \omega_{c_1} \le \omega < \omega_{c_2} \\ e^{-j\omega\tau} & ; \omega_{c_2} \le \omega \le \pi \end{cases}$$