Unit-5

State Space Analysis :-

→ It is a s/m Analysis Method.

→ s/m Analysis Method.

- Classical or Transfer function Based method.
  - RL
  - polar
  - Nyquist
  - polar plot
- State variable method.

→ Limitations of classical method :-

→ valid only for LTI s/m

→ Valid only for SISO s/m.

→ do not describe about the initial conditions / state of the s/m. Gives only zero state Response.

State Space formulation :-

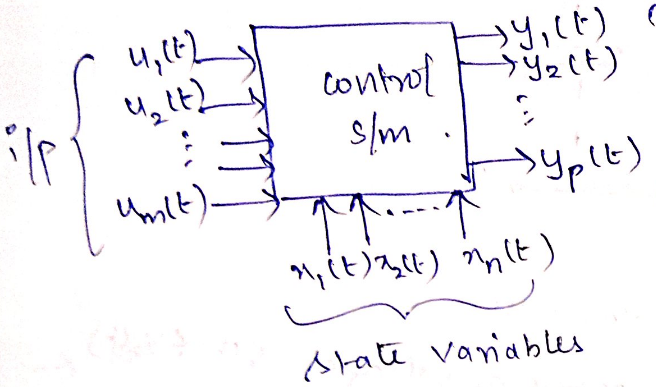
→ The state of a dynamic s/m is a minimal set of variables at $t = t_0$ of i/ps for $t \geq t_0$, completely determines the behaviour of the s/m for $t > t_0$.

→ A set of variables which describes the s/m at any time instant are called state variables.

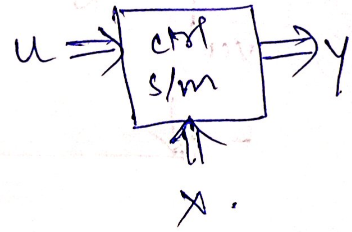
→ Let State variables ⇒ $x_1(t), x_2(t), \ldots, x_n(t)$

I/p variables ⇒ $u_1(t), u_2(t), \ldots, u_m(t)$

o/p variables ⇒ $y_1(t), y_2(t), \ldots, y_n(t)$.



⇒



→ i/p vector $U(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \\ \vdots \\ u_m(t) \end{bmatrix}$

State variable vector $X(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{bmatrix}$

o/p vector $y(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \\ \vdots \\ y_p(t) \end{bmatrix}$

→ State Model

$$\underset{n\times 1}{[\dot{X}]} = \underset{n\times n}{[A]}\,\underset{n\times 1}{[X]} + \underset{n\times m}{[B]}\,\underset{m\times 1}{[U]}$$

$$\underset{p\times 1}{[Y]} = \underset{p\times n}{[C]}\,\underset{n\times 1}{[X]} + \underset{p\times m}{[D]}\,\underset{m\times 1}{[U]}$$

$[\dot{X}]$ → derivatives of state variables.

$[A]$ → s/m matrix

$[B]$ → i/p matrix

$[C]$ → o/p matrix

$[D]$ → Txn matrix

$\left.\begin{matrix} \\ \\ \\ \end{matrix}\right\}$ (co-efficients)

3) With direct differential equation :-

$$\frac{d^3y}{dt^3} + 6\frac{d^2y}{dt^2} + 11\frac{dy}{dt} + 6y + u = 0.$$

where $u \to i/p$
$y \to o/p$.

Find the state transition matrix.

Soln

$y = x_1$

$\frac{dy}{dt} = x_2 = \dot{x}_1 \Rightarrow \boxed{\dot{x}_1 = x_2}$ —— (1)

$\frac{d^2y}{dt^2} = x_3 = \dot{x}_2 \Rightarrow \boxed{\dot{x}_2 = x_3}$ —— (2)

$\frac{d^3y}{dt^3} = \dot{x}_3$

$\dot{x}_3 + 6\dot{x}_2 + 11\dot{x}_1 + 6x_1 + u = 0.$

$\dot{x}_3 + 6x_3 + 11x_2 + 6x_1 + u = 0.$

$\boxed{\dot{x}_3 = -6x_1 - 11x_2 - 6x_3 - u}$ —— (3)

$\boxed{y = x_1}$ —— (4)

i/p - state transition matrix

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} u$$

o/p $$y = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}.$$

→ transfer function to differential eqn :

$$\frac{Y(s)}{U(s)} = \frac{10}{s^3 + 4s^2 + 2s + 1}.$$

soln $$Y(s)\left[s^3 + 4s^2 + 2s + 1\right] = 10\left[U(s)\right]$$

$$s^3 Y(s) + 4s^2 Y(s) + 2s Y(s) + Y(s) = 10\,U(s).$$

$$\frac{d^3y}{dt^3} + 4\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y(t) = 10\,u(t).$$

$y = x_1$

$\frac{dy}{dt} = \dot{x}_1 = x_2$

$\frac{d^2y}{dt^2} = \dot{x}_2 = x_3$

$\frac{d^3y}{dt^3} = \dot{x}_3$

$\dot{x}_3 + 4x_3 + 2x_2 + x_1 = 10u.$

$\dot{x}_3 = -x_1 - 2x_2 - 4x_3 + 10u.$

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 10 \end{pmatrix} u$$

o/p $\Rightarrow$ $y = x_1$

$$y = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}.$$

→ Using signal flow graph.

$$\frac{Y(s)}{U(s)} = \frac{10}{s^3 + 4s^2 + 2s + 1}.$$

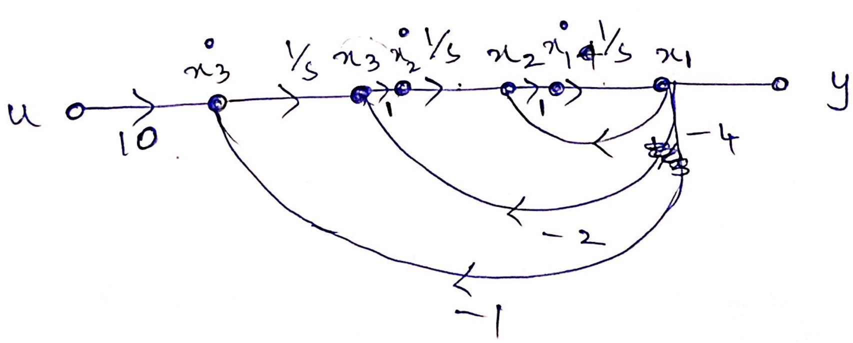
soln.

$$\frac{Y(s)}{U(s)} = \frac{10}{s^3 + 4s^2 + 2s + 1}.$$

$$= \frac{10}{s^3\left(1 + 4/s + 2/s^2 + 1/s^3\right)}$$

$$= \frac{10/s^3}{1 + \left(4/s + 2/s^2 + 1/s^3\right)}.$$

$$= \frac{10/s^3}{1 - \left(-4/s - 2/s^2 - 1/s^3\right)}$$



$x_1 = y$

$\dot{x}_1 = x_2$

$\dot{x}_2 = x_3$

From SFG,

$\dot{x}_3 = 10u - x_1$

$\dot{x}_2 = x_3 - 2x_1$

$\dot{x}_1 = +x_2 - 4x_1$

$\dot{x}_1 = -4x_1 + x_2$

$\dot{x}_2 = -2x_1 + x_3$

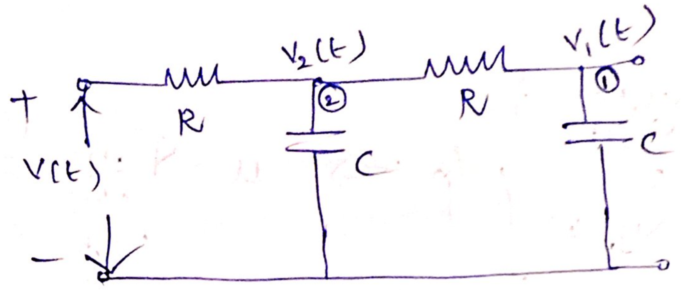
$\dot{x}_3 = -x_1 + 10u$

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} -4 & 1 & 0 \\ -2 & 0 & 1 \\ -1 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 10 \end{pmatrix} u$$

$y = x_1$

$$y = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

Eg: 1 Obtain the state model of the electrical n/w shown in fig, by choosing $V_1(t)$ & $V_2(t)$ as state variable.



Soln. $V_1(t)$ & $V_2(t)$ ⇒ State variables.

∴ ~~par~~ Eqn. should be in terms of voltages $V_1(t)$ & $V_2(t)$

→ Therefore by nodal Analysis,

At node 1,

$$\frac{V_1 - V_2}{R} + C\frac{dV_1}{dt} = 0 \quad \text{— (i)}$$

$V_1 \Rightarrow x_1$ $\qquad \frac{dV_1}{dt} = \dot{x}_1$

$V_2 \Rightarrow x_2$

$$\frac{x_1 - x_2}{R} + C\dot{x}_1 = 0.$$

$$\dot{x}_1 = \frac{x_2}{CR} - \frac{x_1}{CR}$$

$$\dot{x}_1 = -\frac{1}{RC}(x_1) + \frac{1}{RC}x_2 \quad \text{— (2)}$$

At node 2

$$\frac{V_2(t) - V(t)}{R} + \frac{V_2(t) - V_1(t)}{R} + C\frac{dV_2(t)}{dt} = 0 \quad \text{— (3)}$$

$$\frac{x_2 - V(t)}{R} + \frac{x_2 - x_1}{R} + C\dot{x}_2 = 0$$

$\rightarrow v(t) = i/p \Rightarrow u(t)$.

$$\dot{x}_2 = \frac{x_1}{RC} - \frac{2x_2}{RC} + \frac{u(t)}{RC} \quad \text{—— (4)}$$

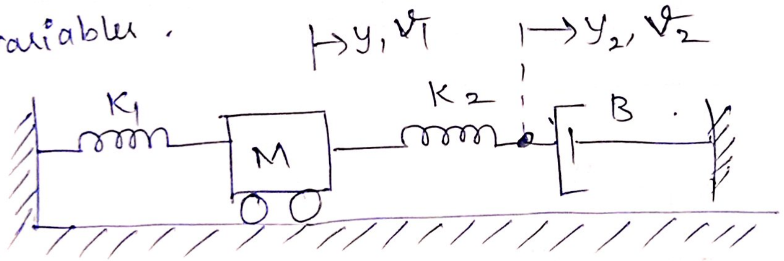
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1/RC & 1/RC \\ 1/RC & -2/RC \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1/RC \end{bmatrix} u(t).$$

$\rightarrow$ o/p: $v_1(t)$.

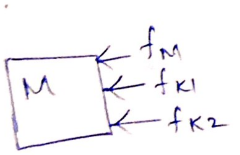
$$y = v_1(t) = x_1$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

2) obtain the state model of the mechanical s/m shown in fig by choosing a minimum of three state variables.



Solu.



$$f_M + f_{K1} + f_{K2} = 0.$$

$$M\frac{d^2y_1}{dt^2} + K_1 y_1 + K_2(y_1 - y_2) = 0.$$

Three state variables.

$y_1 = x_1$ | $y_2 = x_2$ | $\frac{d^2y_1}{dt^2} = \dot{x}_3$

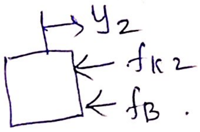
$v_1 = \frac{dy_1}{dt} = x_3$

$$M\dot{x}_3 + x_1(K_1+K_2) - K_2 x_2 = 0.$$

$$M\dot{x}_3 = -(K_1+K_2)x_1 + K_2 x_2$$

$$\boxed{\dot{x}_3 = -\frac{(K_1+K_2)}{M}x_1 + \frac{K_2}{M}x_2} \quad (1).$$

→ At Intermediate pt,



$$f_{K2} + f_B = 0$$

$$K_2(y_2 - y_1) + B\frac{dy_2}{dt} = 0.$$

$$K_2(x_2 - x_1) + B\dot{x}_2 = 0.$$

$$\boxed{\dot{x}_2 = \frac{K_2}{B}x_1 - \frac{K_2}{B}x_2.} \quad (2)$$

→ Also,

$$x_1 = y_1$$

$$\frac{dx_1}{dt} = \frac{dy_1}{dt}$$

$$\boxed{\dot{x}_1 = x_3.} \quad (3).$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ K_2/B & -K_2/B & 0 \\ -\frac{(K_1+K_2)}{M} & K_2/M & 0 \end{bmatrix}$$

→ o/p ⇒ displacements ⇒ y_1 & y_2.

$y_1 = x_1$ $\quad$ $y_2 = x_2$.

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

2) $\dfrac{Y(s)}{U(s)} = \dfrac{10(s+4)}{s(s+1)(s+3)}$

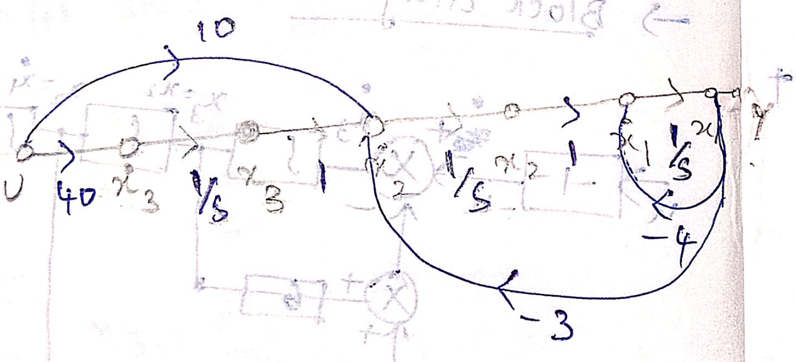
$= \dfrac{10s+40}{s(s^2+3s+s+3)}$

$= \dfrac{10s+40}{s^3+3s^2+s^2+3s}$

$= \dfrac{10s+40}{s^3+4s^2+3s}$

$= \dfrac{(10s+40)}{s^3\left(1+\frac{4}{s}+\frac{3}{s^2}\right)}$

$= \dfrac{\frac{10}{s^2}+\frac{40}{s^3}}{1-\left(-\frac{4}{s}-\frac{3}{s^2}\right)}$



$\dot{x}_1 = x_2 - 4x_1$

$\dot{x}_2 = 10u + x_3 - 3x_1$

$\dot{x}_3 = 40u$

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} -4 & 1 & 0 \\ -3 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} 0 \\ 10 \\ 40 \end{pmatrix} u$$

IV partial fraction

$$\frac{Y(s)}{U(s)} = \frac{10(s+4)}{s(s+1)(s+3)}$$

$$\frac{10(s+4)}{s(s+1)(s+3)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+3}.$$

$$A = \frac{10(s+4)}{\cancel{s}(s+1)(s+3)} \cdot \cancel{s} \Big|_{s=0}$$

$$= \frac{40}{3}$$

$$B = \frac{10(s+4)}{s(s+3)} \Big|_{s=-1}$$

$$= \frac{30}{-2} = -15$$

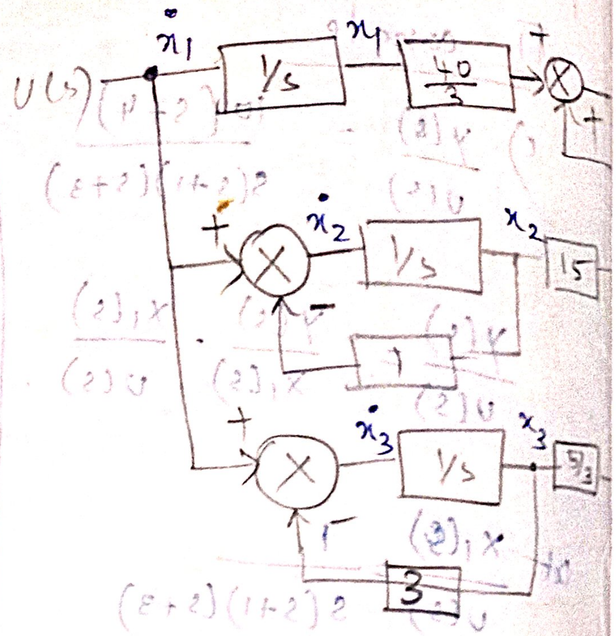
$$C = \frac{10(s+4)}{s(s+1)} \Big|_{s=-3}$$

$$= \frac{10(1)}{6} = \frac{5}{3}.$$

$$\frac{Y(s)}{U(s)} = \frac{40}{3\,s} - \frac{15}{s+1} + \frac{5}{3(s+3)}$$

$$= \left(\frac{1}{s} \times \frac{40}{3}\right) - \frac{15}{s(1+1/s)} + \frac{5/3}{s(1+3/s)}$$

$$Y(s) = \left(\frac{1}{s} \times \frac{40}{3} - \frac{1/s \times 15}{1+\frac{1}{s}\times 1} + \frac{1/s \times 5/3}{1+\frac{3}{s}}\right) U(s)$$



$$\dot{x}_1 = u$$

$$\dot{x}_2 = u - x_2$$

$$\dot{x}_3 = u - 3x_3.$$

$$y = \frac{40}{3}x_1 + \frac{5}{3}x_3 - 15x_2$$

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} (u)$$

$$y = \begin{pmatrix} \frac{40}{3} & -15 & \frac{5}{3} \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

pblms

State Transition Matrix & Transfer function

$$\Rightarrow \boxed{T(s) = C\left[(sI-A)^{-1} B\right] + D}$$

↳ OLTF/CLTF.

$\phi(s) = (sI-A)^{-1} \Rightarrow$ State Transition matrix in freq. domain.

$$= \frac{1}{|sI-A|} \text{adj}\left[sI-A\right]$$

pblm.

1) The state space equation of a s/m is described by

$$\dot{x} = Ax + Bu$$
$$y = Cx.$$

where x is state vector, u is i/p & y is o/p.

$$A = \begin{bmatrix} 0 & 1 \\ 0 & -2 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

Find the transfer function of the s/m & also the steady state error for unit step i/p with unity fb.

soln:)

$$T(s) = C\left[(sI-A)^{-1} B\right] + D.$$

$$sI - A = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 0 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} s & -1 \\ 0 & s+2 \end{bmatrix}$$

$$|sI-A| = s(s+2) = s^2 + 2s.$$

$$\text{Adj}\,[sI - A] = \begin{bmatrix} s+2 & 1 \\ 0 & s \end{bmatrix}$$

$$(sI - A)^{-1} = \frac{1}{s^2+2s} \begin{bmatrix} s+2 & 1 \\ 0 & s \end{bmatrix}$$

$$(sI - A)^{-1} B = \frac{1}{s^2+2s} \begin{bmatrix} s+2 & 1 \\ 0 & s \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= \frac{1}{s^2+2s} \begin{bmatrix} 1 \\ s \end{bmatrix}$$

$$C\left[(sI - A)^{-1} B\right] = \frac{1}{s^2+2s} \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ s \end{bmatrix}$$

$$= \frac{1}{s^2+2s}.$$

$$G(s) = T(s) = \frac{1}{s(s+2)}$$

→ <u>Steady state error</u> ⇒ $\boxed{\text{CLTF}}$ ⇒ steady state error concept is valid only for CLTF.

$$\text{CLTF} = \frac{G(s)}{1+G(s)}$$

$$\text{CLTF} = \frac{1/s(s+2)}{1+1/s(s+2)} = \frac{1}{s^2+2s+1}$$

$\Rightarrow$ It is a 2nd order, type 0 s/m.

$$e_{ss} = \frac{1}{1+K_p}.$$

$$K_p = \lim_{s\to 0} G(s)\,H(s)$$

$$= \lim_{s\to 0} G(s)$$

$$= \lim_{s\to 0} \left[\frac{1}{s(s+2)}\right]$$

$$= 1/0$$

$$= \infty.$$

$$e_{ss} = \frac{1}{\infty}$$

$$e_{ss} = 0$$

---

2) A state variable s/m $\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ 0 & -3 \end{bmatrix} x(t) + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u(t)$

with initial condt. $x(0) = [-1 \quad 3]^T$ and the unit step i/p $u(t)$. Find the state transition matrix and state transition equation.

Soln.

$$x(0) = \begin{pmatrix} -1 \\ 3 \end{pmatrix}.$$

State transition Matrix $\phi(t) = L^{-1}[\phi(s)]$

$$\phi(s) = [sI - A]^{-1}.$$

$$sI - A = \begin{pmatrix} s & -1 \\ 0 & s+3 \end{pmatrix}$$

$$(sI-A)^{-1} = \begin{pmatrix} 1/s & \frac{1}{s(s+3)} \\ 0 & \frac{1}{s+3} \end{pmatrix}$$

$$\phi(s) = \begin{pmatrix} \frac{1}{s} & \frac{1}{3s} - \frac{1}{3(s+3)} \\ 0 & 1/s+3 \end{pmatrix}$$

$$\phi(t) = \begin{pmatrix} 1 & \frac{1}{3} - \frac{1}{3} e^{-3t} \\ 0 & e^{-3t} \end{pmatrix}.$$

→ State - transition equation.

$$x(t) = x(t)\Big|_{ZIR} + x(t)\Big|_{ZSR}.$$

ZIR ⇓ Zero Initial Response

ZSR ⇓ Zero state Response,

$$x(t)\Big|_{ZIR} = [\phi(t)][x(0)]$$

$$x(t)\Big|_{ZSR} = L^{-1}\Big[[\phi(s)][B\,U(s)]\Big]$$

$$x(t)\big|_{ZIR} = [\phi(t)][x(0)]$$

$$= \begin{pmatrix} 1 & \frac{1}{3}(1-e^{-3t}) \\ 0 & e^{-3t} \end{pmatrix} \begin{pmatrix} -1 \\ 3 \end{pmatrix}$$

$$= \begin{pmatrix} -e^{-3t} \\ 3e^{-3t} \end{pmatrix} \text{ --- } (1)$$

$$x(t)\big|_{ZSR} \Rightarrow$$

$$B\,U(s) = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} 1/s \end{pmatrix}$$

$$= \begin{pmatrix} 1/s \\ 0 \end{pmatrix}$$

$$\phi(s)[B\,U(s)] = \begin{bmatrix} \frac{1}{s} & \frac{1}{3}\left(\frac{1}{s} - \frac{1}{s+3}\right) \\ 0 & \frac{1}{s+3} \end{bmatrix}$$

$$\begin{bmatrix} 1/s \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1/s^2 \\ 0 \end{bmatrix}$$

$$L^{-1}\left[\phi(s)[B\,U(s)]\right] = \begin{bmatrix} t \\ 0 \end{bmatrix} \text{ --- } (2)$$

$$x(t) = x(t)\big|_{ZIR} + x(t)\big|_{ZSR}$$

$$= \begin{pmatrix} -e^{-3t} \\ 3e^{-3t} \end{pmatrix} + \begin{pmatrix} t \\ 0 \end{pmatrix}$$

$$\boxed{x(t) = \begin{pmatrix} t - e^{-3t} \\ 3e^{-3t} \end{pmatrix}}$$

Observability & controllability

i) observable if Rank of $\begin{bmatrix} C & CA & CA^2 & \cdots & CA^{n-1} \end{bmatrix}^T = n$

$n \rightarrow$ order.

ii) controllable if Rank of $\begin{bmatrix} B & AB & A^2B & \cdots & A^{n-1}B \end{bmatrix} = n$

Eg 1) s/m repr^n. by $\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} -3 & 0 \\ 0 & -2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 4 \\ 1 \end{pmatrix} u$

$$y = \begin{pmatrix} 2 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

check for controllability and observability.

soln

$$A = \begin{pmatrix} -3 & 0 \\ 0 & -2 \end{pmatrix} \qquad C = \begin{pmatrix} 2 & 0 \end{pmatrix}$$

$$B = \begin{pmatrix} 4 \\ 1 \end{pmatrix} \qquad D = \begin{pmatrix} 0 & 0 \end{pmatrix}$$

(i) observable. $n-1 = 2-1 = 1$

$$CA = \begin{pmatrix} 2 & 0 \end{pmatrix}_{1\times 2} \begin{pmatrix} -3 & 0 \\ 0 & -2 \end{pmatrix}_{2\times 2}$$

$$= \begin{pmatrix} -6 & 0 \end{pmatrix}$$

$$\cancel{CA^2 = \begin{pmatrix} 2 & 0 \end{pmatrix} \begin{pmatrix} -3 & 0 \\ 0 & -2 \end{pmatrix} \begin{pmatrix} -3 & 0 \\ 0 & -2 \end{pmatrix}}$$

$$\cancel{= \begin{pmatrix} 2 & 0 \end{pmatrix} \begin{pmatrix} 9 & 0 \\ 0 & 4 \end{pmatrix}}$$

$$\cancel{= \begin{pmatrix} 18 & 0 \end{pmatrix}}$$

$$\begin{bmatrix} C & CA \end{bmatrix}^T = \begin{bmatrix} 2 & 0 \\ -6 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -3 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\text{Rank} \begin{bmatrix} C & CA^T \end{bmatrix} \neq 2.$$

$\therefore$ It is not observable.

(ii) <u>controllable</u>

$$B = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$$

$$AB = \begin{pmatrix} -3 & 0 \\ 0 & -2 \end{pmatrix} \begin{pmatrix} 4 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} -12 \\ -2 \end{pmatrix}$$

$$\begin{bmatrix} B & AB \end{bmatrix} = \begin{bmatrix} 4 & -12 \\ 1 & -2 \end{bmatrix}$$

<u>Rank</u>

$$= \begin{bmatrix} 1 & -2 \\ 4 & -12 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -2 \\ 0 & -4 \end{bmatrix} \quad R_2 \to R_2 - 4R_1$$

$$= \begin{bmatrix} 1 & -2 \\ 0 & -4 \end{bmatrix}$$

Rank $= 2$.

$\therefore$ controllable.