UNIT-II

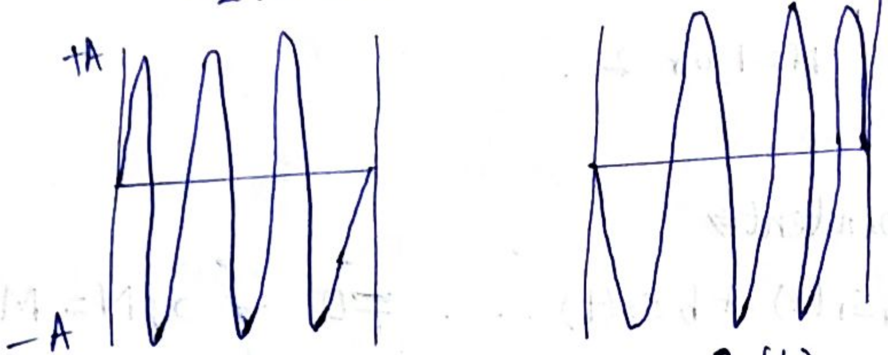
In freq spectrum,

Base band $\rightarrow$ freq centred abt 0.

Band Pass $\rightarrow$ Centre freq $\gg$ BW
$\downarrow$
around f_c.

BAND PASS SIGNALLING

$S_1(t) \rightarrow 1$
$S_2(t) \rightarrow 0$ } $\rightarrow$ symbols



$S_1(t)$ $S_2(t)$.

Both Baseband and band pass signal consists of a symbol set of 2 symbols.

For m-ary signalling scheme, symbol set consists many symbols.

Symbol set is finite.

4 - ary

2 bits.

$S_1(t) \rightarrow 00$ $S_2 \rightarrow 01$ $S_3 \rightarrow 10$ $S_4 \rightarrow 11$
$\downarrow$
Time $= 2T_b$

SIGNAL SPACE REPRESENTATION / SIGNAL CONSTELLATION

Assume a symbol set,

$$\{s_1(t), s_2(t), \ldots, s_M(t)\}$$

Let set of orthonormal (Real) basis functions

↳ can be complex also.

$$\{\phi_1(t), \phi_2(t), \phi_3(t), \phi_4(t) \ldots \phi_N(t)\}$$ $\boxed{N \leq M}$

For binary, $N = 1$ or 2.

Linearly independent ⇒

$$a_1 s_1(t) + b\, s_2(t) \ldots \neq 0 \longrightarrow N = M$$

(or else)

$$N < M$$

Orthonormality ⇒

$$\int_0^T \phi_i(t)\, \phi_j(t)\, dt = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$$

Basis functions have same time period as the symbol set.

If $i = j = 2$,

$$\underbrace{\boxed{\int_0^T \phi_i^2(t)\, dt}}_{\text{energy}} = 1$$

∴ Basis func must have unit energy.

orthogonal basis func $\Rightarrow$ Energy $\neq 1$

$$\boxed{s_i(t) = \sum_{j=1}^{N} \underbrace{s_{ij}}_{\text{value}} \underbrace{\phi_j(t)}_{\text{func}}}$$

$0 \leq t \leq T$

$i = 1, 2 \ldots M$

All the symbols should be expressed as linear combination of basis func $\longrightarrow$ If this is true, all the waveforms are in the space spanned by basis func.

$\downarrow$

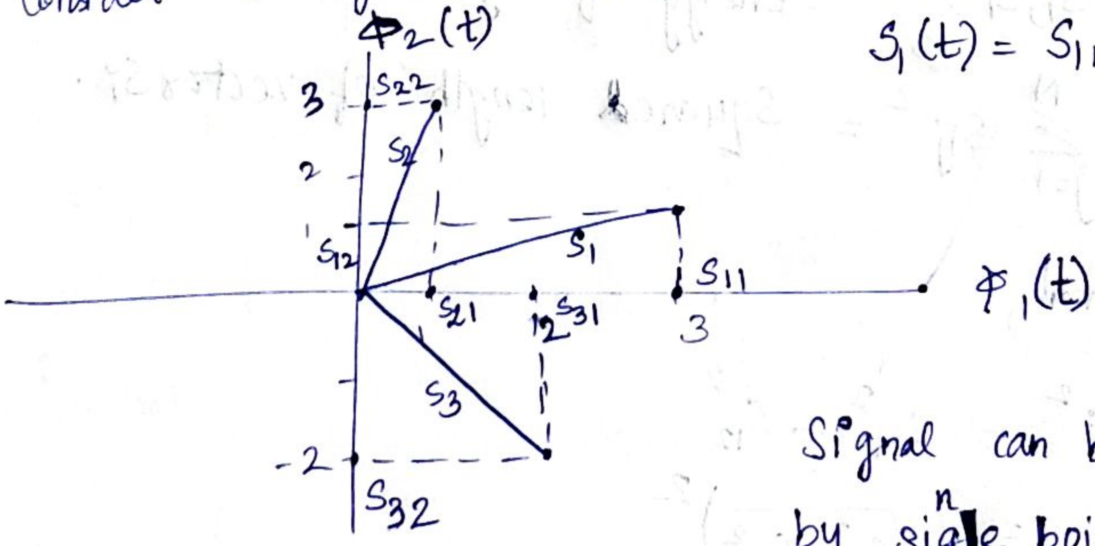
the signal can be represented by a set of numbers $[s_{ij}]$

$\downarrow$

N numbers are used else infinite number (points) should be used to described the conti time signal.

$$\boxed{s_{ij} = \int_0^T s_i(t)\,\phi_j(t)\,dt}$$

Consider 2 ~~symbols~~ basis



$s_1(t) = s_{11}\,\phi_1(t) + s_{12}\,\phi_2(t)$

Signal can be represented by single point $\rightarrow$ signal constellation

$$s_i = \begin{bmatrix} s_{i1} \\ s_{i2} \\ \vdots \\ s_{iN} \end{bmatrix} \qquad i = 1, 2, \ldots M$$

Energy $\Rightarrow$ $E_i = \int_0^T s_i^2(t)\,dt$

$$= \int_0^T \left[\sum_{j=1}^{N} s_{ij}\phi_j(t)\right]\left[\sum_{k=1}^{N} s_{ik}\phi_k(t)\right] dt$$

$$= \sum_{j=1}^{N}\sum_{k=1}^{N} s_{ij}\, s_{ik} \underbrace{\left(\int_0^T \phi_j(t)\,\phi_k(t)\,dt\right)}_{=1 \text{ only when } j=k}$$

$$= \sum_{j=1}^{N} s_{ij}\, s_{ij}\,[1]$$

$$\boxed{E_i = \sum_{j=1}^{N} s_{ij}^2}$$

$\underbrace{\|s_i\|^2}_{\text{norm}} = (s_i, s_i) \rightarrow$ inner product = Energy of signal.

$$= \sum_{j=1}^{N} s_{ij}^2 = \text{Squared length of vector } s_i.$$

For prev diagram,

$$E_1 = \sum_{j=1}^{2} s_{ij}^2 = s_{11}^2 + s_{12}^2$$

$$= \left(\sqrt{s_{11}^2 + s_{12}^2}\right)^2$$

= square of length of s_1 vector.

GRAM SCHMIDT ORTHOGONALISATION PROCESS

when symbol set is given, this process is used to find orthonormal basis.

If symbol set is linearly independent.

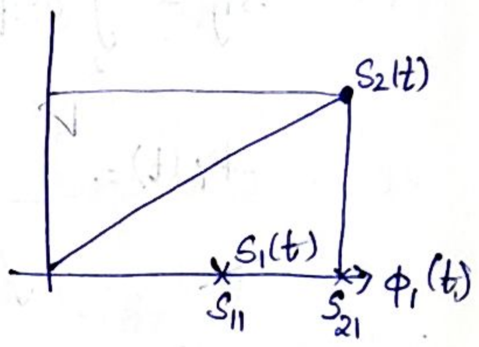
$$a_1 S_1 + a_2 S_2 + \cdots \neq 0 \longrightarrow N = M$$

So basis set can be directly found by normalization.

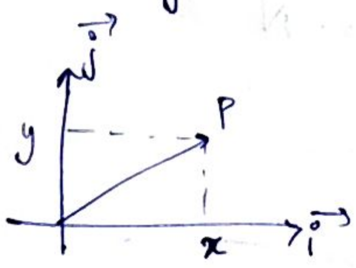
$$\boxed{\phi_1(t) = \frac{S_1(t)}{\sqrt{E_1}}}$$

$E_1 \rightarrow$ energy of S_1 $= \int |S_1(t)|^2 dt$

$$S_1(t) = \sqrt{E_1}\,\phi_1(t)$$
$$= S_{11}\,\phi_1(t)$$



Generally



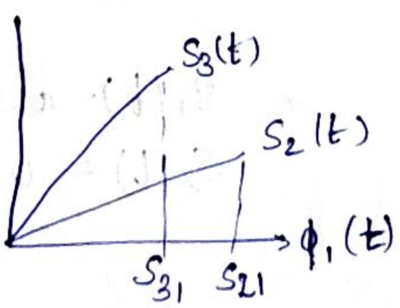
$$\vec{P} = x\vec{i} + y\vec{j}$$

$$\vec{P} \cdot \vec{i} = x$$

$$\langle \vec{P}, \vec{i} \rangle = x$$

inner product

$$S_{21} = \langle S_2(t), \phi_1(t) \rangle$$
$$= \int_0^T S_2(t)\,\phi_1(t)\,dt$$



$S_2(t)$ and $S_3(t)$ have same length (energy)

$S_{21} > S_{31}$

$\therefore S_2$ is more resembling to $\phi_1(t)$.

$$S_{21} = \int_0^T s_2(t)\,\phi_1(t)\,dt$$

$$g_2(t) = s_2(t) - s_{21}\,\phi_1(t)$$

$$\phi_2(t) = \frac{g_2(t)}{\sqrt{E_2}} = \frac{g_2(t)}{\sqrt{\int_0^T g_2^2(t)\,dt}} \longrightarrow \text{energy of } g_2.$$

$$\Downarrow$$

$$\phi_2(t) = \frac{S_2(t) - S_{21}\,\phi_1(t)}{\sqrt{E_2 - S_{21}}} \cdot \sqrt{\int_0^T S_2(t)\,dt - \int_0^T S_{21}^{\,2}\,\phi_1}$$

$$g_i(t) = s_i(t) - \sum_{j=1}^{i-1} s_{ij}\,\phi_j(t)$$

$$S_{ij} = \int_0^T s_i(t)\,\phi_j(t)\,dt$$

$$\phi_i(t) = \frac{g_i(t)}{\sqrt{\int_0^T g_i^2(t)\,dt}} \qquad i = 1, 2, \ldots N$$

Unipolar NRZ $\Rightarrow$

$\phi_1(t)$

$S_{11} = a$

$S_{21} = 0$

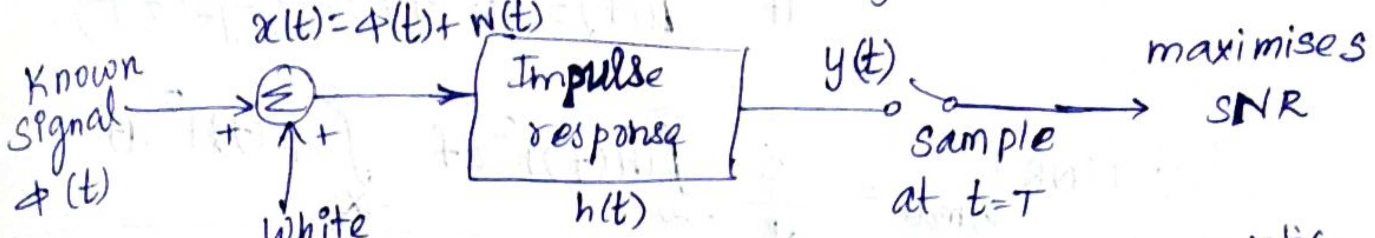
$S_1(t) = a\,\phi_1(t)$

$S_2(t) = 0 \times \phi_1(t)$

MATCHED FILTER

Optimal filter.

$y(t) = \phi_0(t) + n(t)$ — coloured noise.



White noise $w(t)$ → has all frequencies
↓
~~rando~~

known signal → i/p buried in noise

deterministic signal

$$\boxed{\begin{aligned}\phi_0(t) &= \phi(t) * h(t)\\ n(t) &= w(t) * h(t)\end{aligned}}$$

$n(t)$ → random process

$$(SNR)_0 = \frac{|\phi_0(T)|^2}{E[n^2(t)]}$$

$|\phi_0(T)|^2$ → variance

$E[n^2(t)]$ → expectation

Signal power ⇒

~~$\phi_0(T)$~~ $|\phi_0(T)|^2 = \left|\int_{-\infty}^{\infty} H(f)\,\phi(f)\exp(j2\pi f T)\,df\right|^2$

$\phi(t) * h(t) \Rightarrow$ $\phi_0(t) = \int_{-\infty}^{\infty} H(f)\,\phi(f)\exp(j2\pi f t)\,df$

$$S_N(f) = \frac{N_0}{2}|H(f)|^2 \longrightarrow \text{PSD } (W/Hz)$$

$$E[n^2(t)] = \int_{-\infty}^{\infty} S_N(f)\,df$$

↓
power of noise at matched filter.

$$(SNR)_0 = \frac{\left|\int_{-\infty}^{\infty} H(f)\,\phi(f)\exp(j2\pi f T)\,df\right|^2}{\frac{N_0}{2}\int_{-\infty}^{\infty}|H(f)|^2\,df}$$

Schwarz's inequality ⇒

$$\left|\int_{-\infty}^{\infty} x(t)\,y(t)\,dt\right|^2 \le \int_{-\infty}^{\infty}|x(t)|^2\,dt \int_{-\infty}^{\infty}|y(t)|^2\,dt$$

if $y(t) = k\,x^*(t)$.

$$\left| \int_{-\infty}^{\infty} H(f)\, \Phi(f) \exp(j2\pi f T)\, df \right|^2 \le \int_{-\infty}^{\infty} |H(f)|^2 df \int_{-\infty}^{\infty} |\Phi(f)|^2 df$$

if $H(f) = \Phi^*(f) \exp(-j2\pi f T)$

$$\therefore (SNR)_{0,max} = \frac{\cancel{\frac{2}{N_0}}\ \cancel{\int_{-\infty}^{\infty} |H(f)|^2 df} \int_{-\infty}^{\infty} |\Phi(f)|^2 df}{\frac{N_0}{2} \cancel{\int_{-\infty}^{\infty} |H(f)|^2 df}}$$

$$\boxed{(SNR)_{0,max} = \frac{2}{N_0} \int_{-\infty}^{\infty} |\Phi(f)|^2 df}$$

when $\boxed{H_{opt}(f) = \Phi^*(f) \exp(-j2\pi f T)}$ $\rightarrow$ matched filter

duration of $\Phi(t) = T$.

$\Phi(t)$ and $\Phi(f)$ is known.

If $\Phi(t) \rightarrow$ real,

$$\Phi^*(f) = \Phi(-f)$$

$$H_{opt}(f) = \Phi^*(f) \exp(-j2\pi f T)$$

$$\cancel{= \Phi(-f) \exp}$$

$$h_{opt}(t) = \int_{-\infty}^{\infty} \Phi^*(f) \exp(-j2\pi f T) \exp(j2\pi f t)\, df$$

$$= \int_{-\infty}^{\infty} \Phi(-f) \exp[-j2\pi f (T-t)]\, df$$

$\boxed{h_{opt}(t) = \Phi(T-t)}$ $\longrightarrow$ optimal matched filter

$S_{13} = \oint \langle S_1(t), \phi_3(t) \rangle$

$S_{13} = \int S_1(t)\, \phi_3(t)\, dt$.

$\downarrow$

value decides resemblence, (correlation btwn $S_1(t)$ & $\phi_3(t)$)

$S_{13} \to$ +ve & large $\to$ +ve correlation

MATCHED FILTER VS CORRELATOR

$x(t) = \phi(t) + w(t)$ $\longrightarrow \boxed{h(t)} \longrightarrow$ $y(t) = \phi_0(t) + n(t)$

$h(t) = \phi(T-t)$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$h(t-\tau) = \phi(T-(t-\tau))$$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, \phi(T-t+\tau)\, d\tau$$

$\to$ Matched filter o/p

WKT,

$\phi(t) \neq 0 \qquad 0 \le t \le T$

$\quad = 0 \qquad$ elsewhere

At $t=T$,

$$y(T) = \int_{-\infty}^{T} x(\tau)\, \phi(\tau)\, d\tau$$

$\to$ correlator o/p.

$\therefore$ At $\boxed{t=T}$

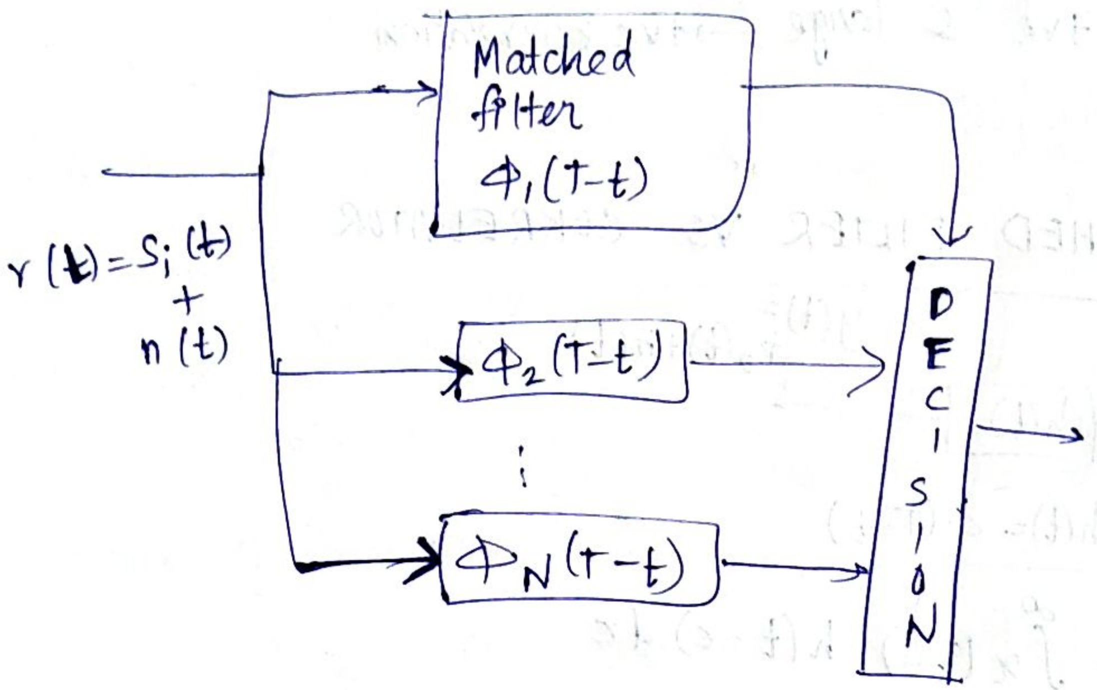
matched filter o/p = correlator o/p.

$\therefore$ matched filter and correlator are identical.

$$S_i(t) = \sum_{j=1}^{N} S_{ij}\,\phi_j(t)$$

$i = 1, 2, \ldots M$

Incoming signal:



Equivalently