LINE CODES

(BASEBAND SIGNALLING)

HOW DO WE TRANSMIT BITS OVER A TWISTED WIRE PAIR / CABLE/WIRELESS (SAY RF) ?

MAP THE BITS TO WAVEFORMS ---- SIGNALLING

1 → $S_1(t)$

0 → $S_2(t)$

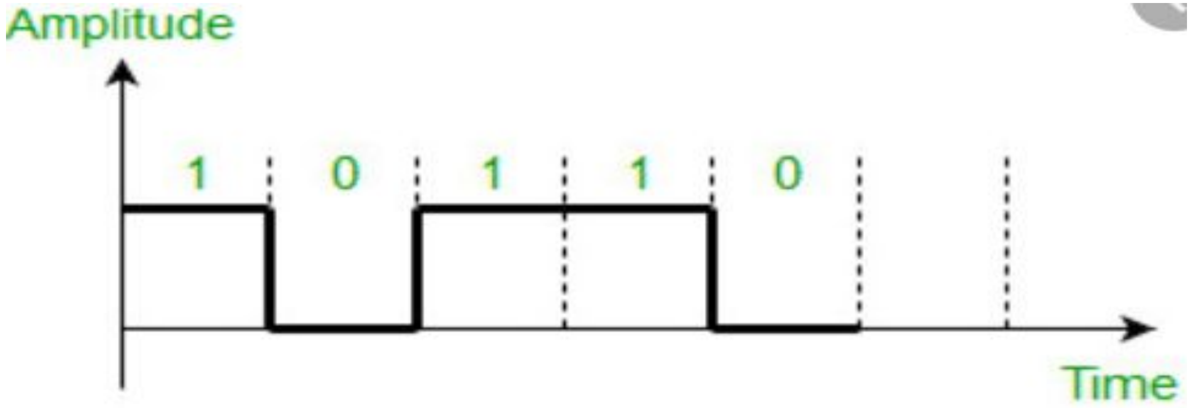
a. BASEBAND SIGNALLING (LINE CODES)

b. BANDPASS SIGNALLING

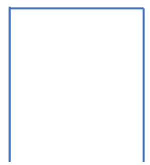
BASE BAND SIGNALLING (Mapping)
LINE CODES

1 → $S_1(t)$
0 → $S_2(t)$

Unipolar NRZ



$S_1(t)$



+A

0 V

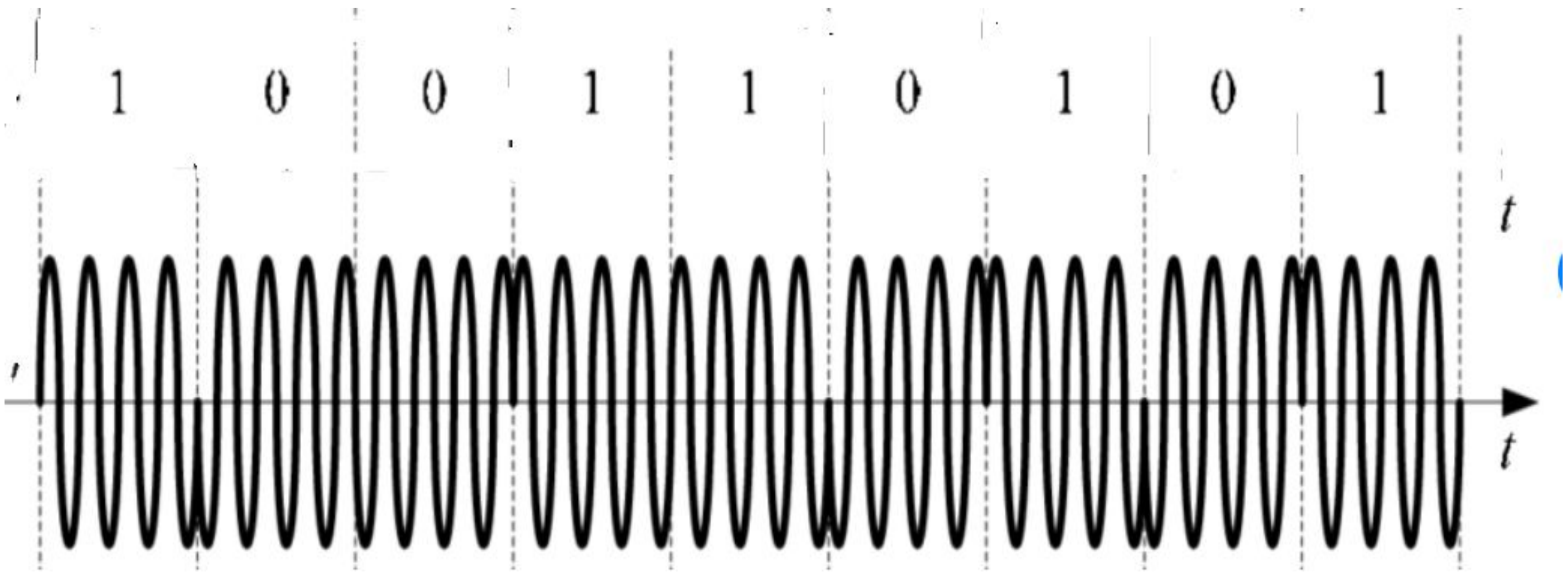
$S_2(t)$

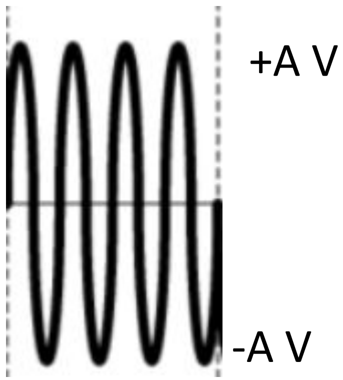
0 V

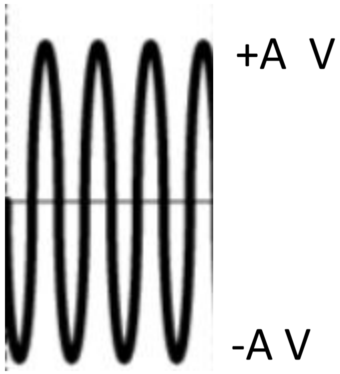
BAND PASS SIGNALLING (Mapping)

1 → $S_1(t)$
0 → $S_2(t)$

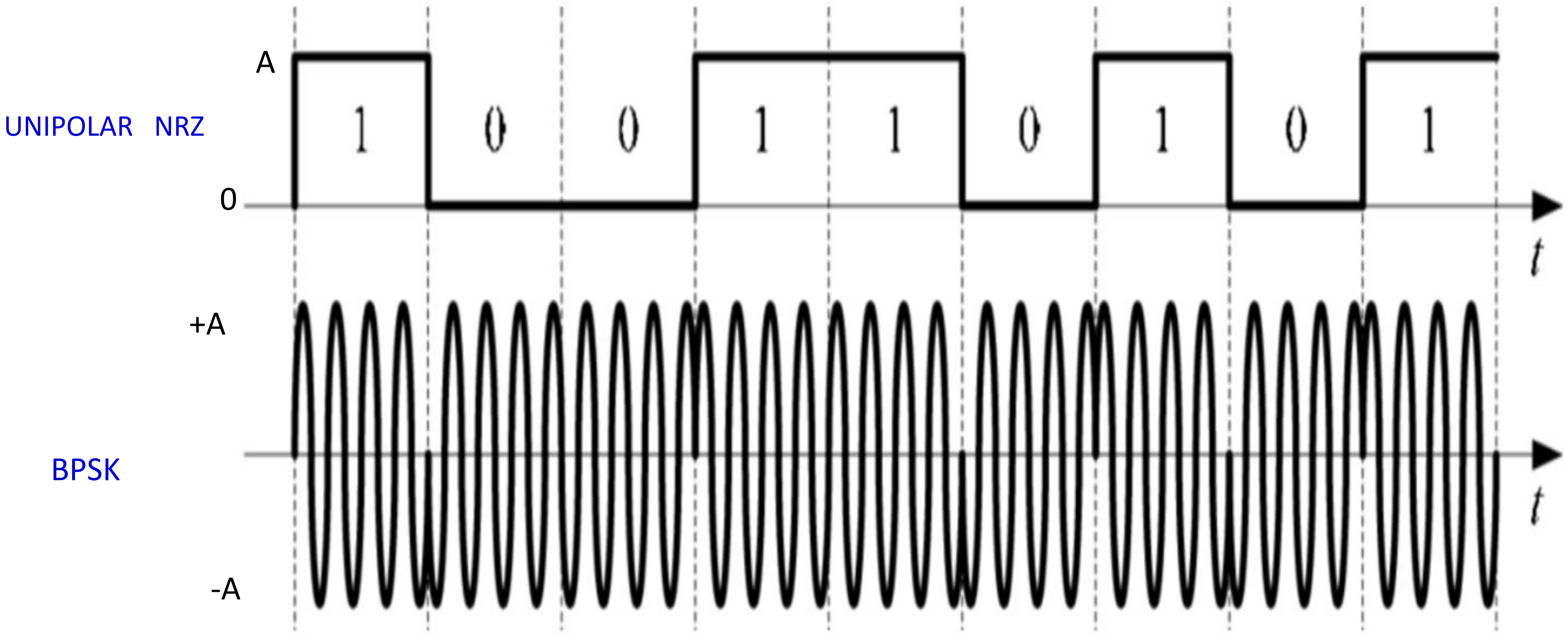
BPSK







BASEBAND SIGNALLING vs BANDPASS SIGNALLING



BASEBAND SIGNAL VS PASSBAND SIGNAL

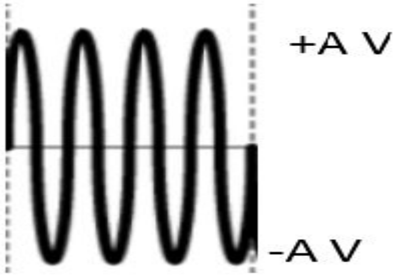
$1 \rightarrow S_1(t)$
$0 \rightarrow S_2(t)$

$S_1(t)$

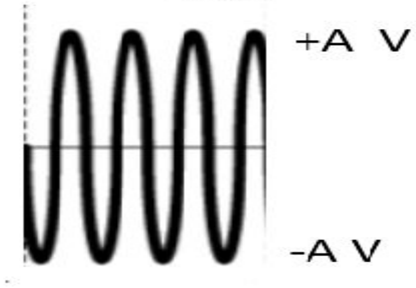
$S_2(t)$

$1 \rightarrow S_1(t)$
$0 \rightarrow S_2(t)$

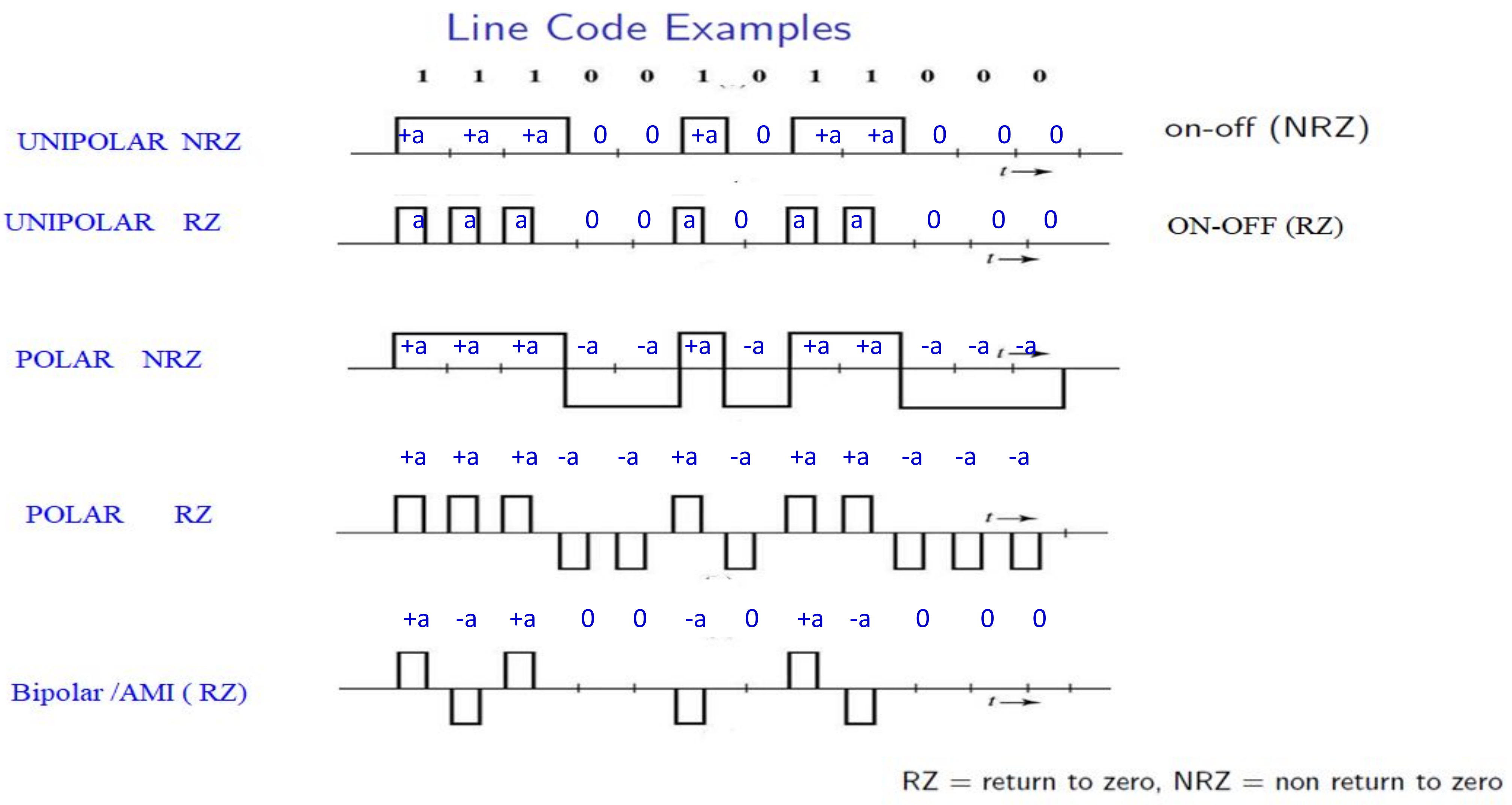
$S_1(t)$



$S_2(t)$

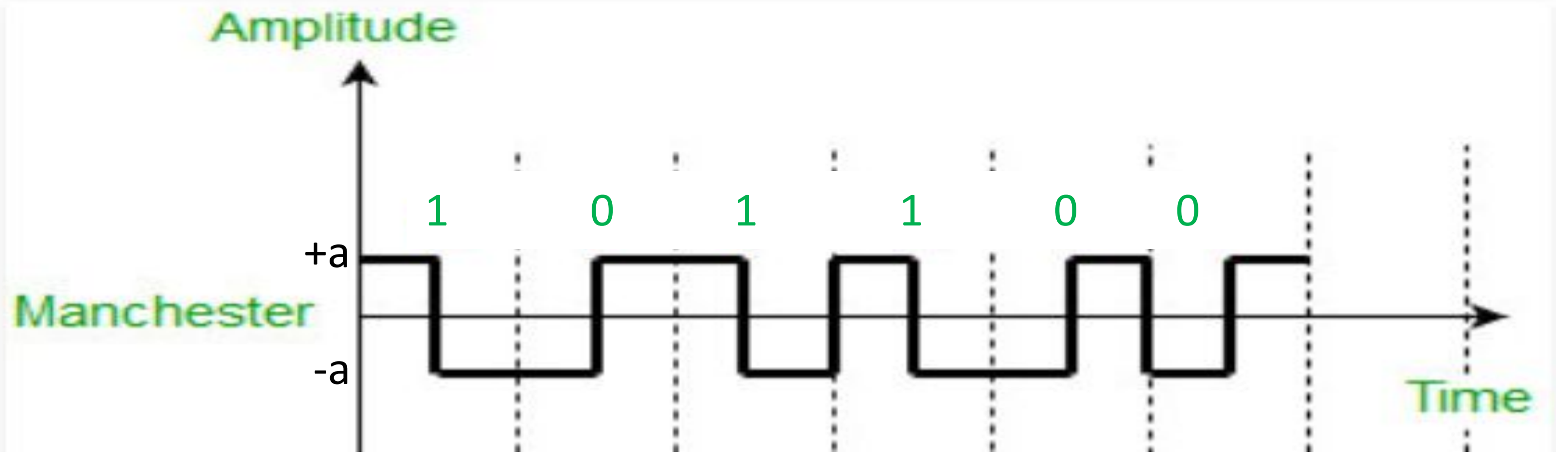


LINE CODES (BASEBAND SIGNALLING)

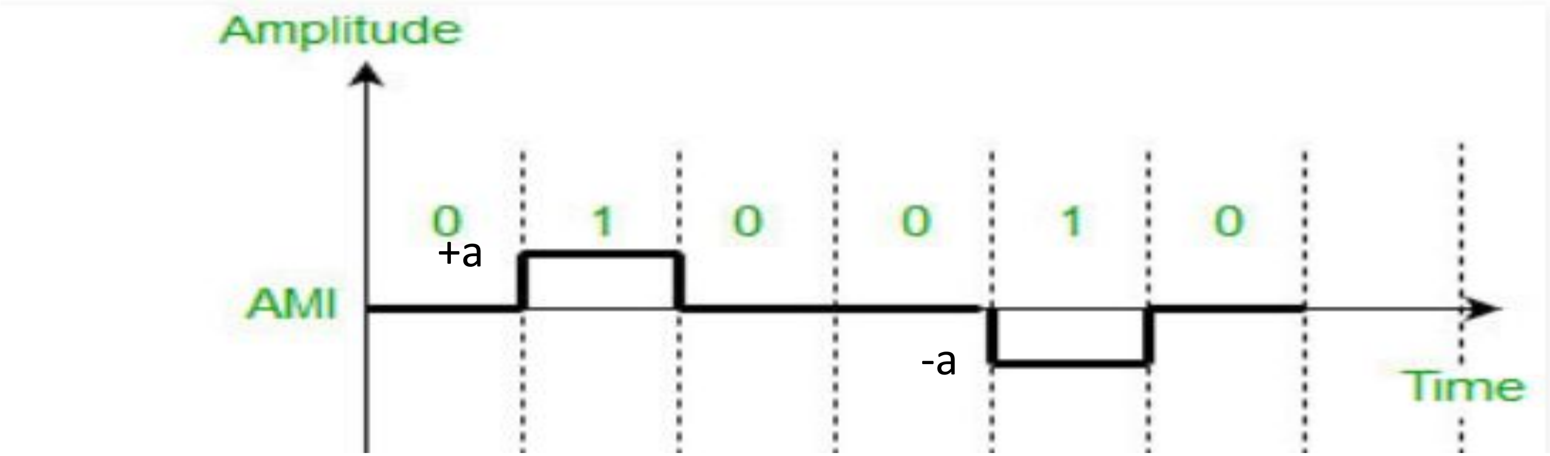


Line Codes contd ....

MANCHESTER



BIPOLAR (AMI) NRZ

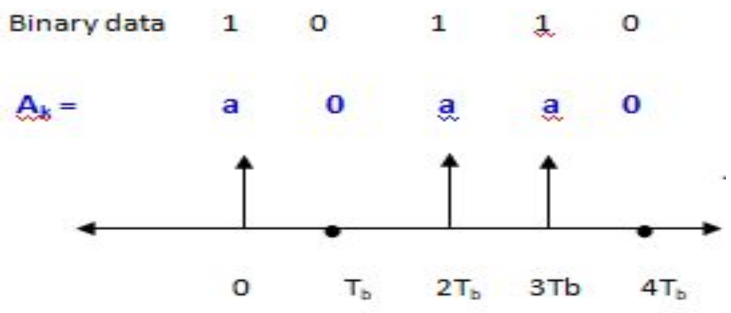


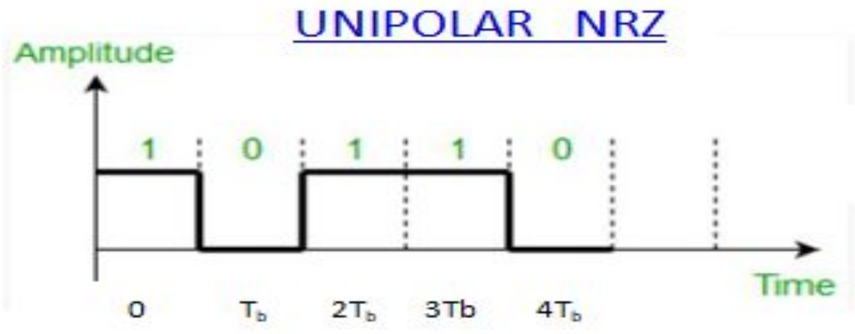
DESIRABLE PROPERTIES OF LINE CODES

- **LOW Transmission Bandwidth**
- **Favourable Power Spectral Density**
- **HIGH Power Efficiency**
- **Error Detection and Correction Capability**
- **Adequate Timing Content**
- **Transparency**

UNIPOLAR NRZ

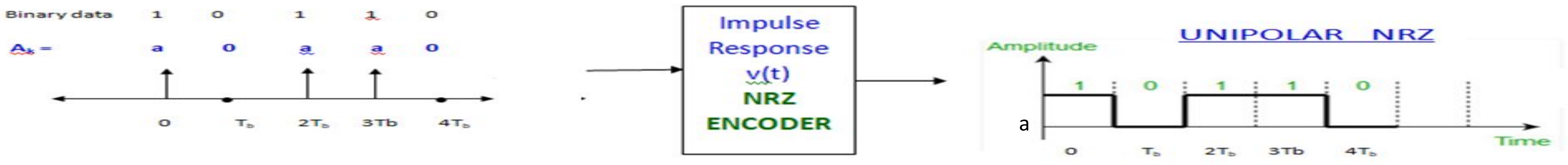
UNIPOLAR NRZ





HOW TO OBTAIN THE POWER SPECTRAL DENSITY?

Unipolar NRZ



$$Input\ \ data\ \ \boldsymbol{x(t)} = \sum_{k=-\infty}^{\infty} A_K\ \ \boldsymbol{\delta}(t - kT_b)$$

$$NRZ\ encoded\ waveform\ \boldsymbol{g(t)} = \sum_{k=-\infty}^{\infty} A_K\ \ \boldsymbol{v}(t - kT_b)$$

$$\text{Power spectral density of } \mathbf{g(t)} \ = \ S_{gg}(f) = |V(f)|^2\ S_{xx}(f)$$

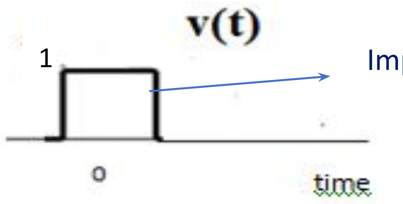
Here, the input is a random (binary) sequence

$$S_{gg}(f) = \frac{1}{T_b}|V(f)|^2 \sum_{n=-\infty}^{\infty} R_A(n) \exp(-j2\pi n f T_b)$$

$$S_{xx}(f) = \text{Fourier Transform of } R_A(n)$$

$R_A(n)$ is the autocorrelation of the input random sequence

$$|V(f)|^2 \ = \ T_b^2\ sinc^2(\pi f T_b)$$



v(t)

Dr. Mala John Department of Electronics Engineering MIT Campus of Anna University malajohn@annauniv.edu

AUTOCORELATION SEQUENCE $R_A(n)$

$A_k \quad A_{k+1}$

0 a 0 0 a 0 a a a 0 a 0 0

$$R_A(n) = E(A_k A_{k+n})$$

$$R_A(0) = E(A_k^2) = \sum_{k=1}^{2}(p_k\ A_k^2)$$

$$= \text{p}(A_k = 0)0^2 + \text{p}(A_k = a)a^2 = \frac{1}{2} \times 0 + \frac{1}{2} \times a^2 = \frac{a^2}{2}$$

AUTOCORELATION SEQUENCE $R_A(n)$

,

$$R_A(n) = E(A_k A_{k+n})$$

$$R_A(1) = E(A_k A_{k+1}) = \sum P(A_k A_{k+1})\, A_k A_{k+1} =$$

A_k	A_{k+1}	$P(A_k A_{k+1})$ $=P(A_k)P(A_{k+1})$	$A_k A_{k+1}$	$P(A_k A_{k+1})\, A_k A_{k+1}$
0	0	½ x ½ = ¼	0	0
0	a	½ x ½ = ¼	0	0
a	0	½ x ½ = ¼	0	0
a	a	½ x ½ = ¼	a^2	$\frac{a^2}{4}$
$\sum P(A_k A_{k+1})\, A_k A_{k+1}$				$\frac{a^2}{4}$

$$R_A(1) = \frac{a^2}{4}$$

Similarly,

$$R_A(n) = \frac{a^2}{4} \qquad n \neq 0$$

Power Spectral Density of UNIPOLAR NRZ waveform

$$R_A(n) = \frac{a^2}{4}, \; n \neq 0$$

$$R_A(0) = \frac{a^2}{2}$$

$$|V(f)|^2 = T_b^2 \, sinc^2(\pi f T_b)$$

$$S_{gg}(f) = \frac{1}{T_b} |V(f)|^2 \sum_{n=-\infty}^{\infty} R_A(n) \exp(-j2\pi n f T_b)$$

$$S_{gg}(f) = \frac{1}{T_b} T_b^2 \, sinc^2(f T_b) \quad [\; \frac{a^2}{2} + \frac{a^2}{4} \sum_{n=-\infty, n\neq 0}^{\infty} \exp(-j2\pi f n T_b)]$$

$$S_{gg}(f) = T_b \; sinc^2(f T_b) \quad [\; \frac{a^2}{4} + \frac{a^2}{4} \sum_{n=-\infty}^{\infty} \exp(-j2\pi f n T_b)]$$

$$\sum_{n=-\infty} \exp(-j2\pi f n T_b) = \frac{1}{T_b} \sum_{n=-\infty} \delta(f - \frac{n}{T_b})$$

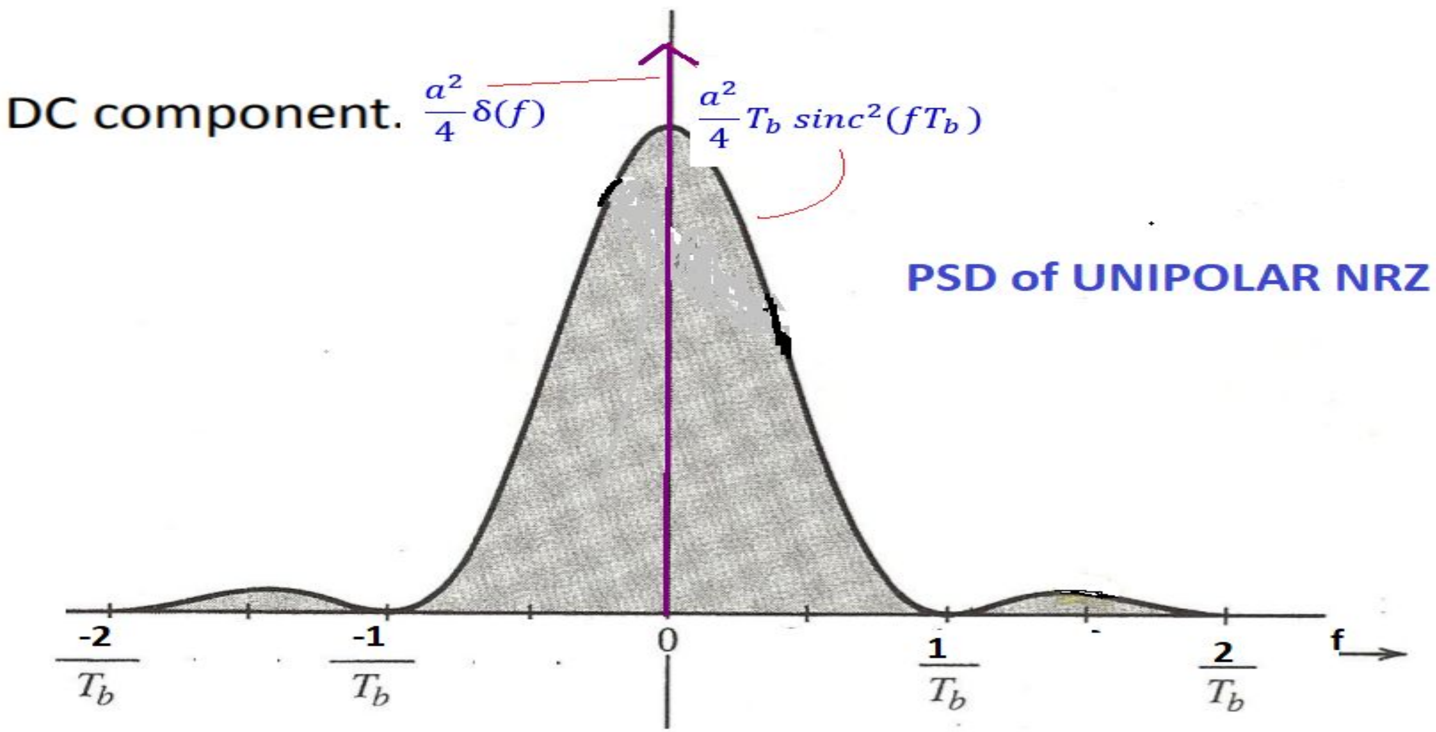
$$S_{gg}(f) = \frac{a^2}{4} T_b \; sinc^2(f T_b) + \frac{a^2}{4} sinc^2(f T_b) \sum_{n=-\infty}^{\infty} \delta(f - \frac{n}{T_b})$$

$$= \frac{a^2}{4} T_b \, sinc^2(f T_b) + \frac{a^2}{4} \delta(f)$$

How?

Power Spectral Density of UNIPOLAR NRZ waveform

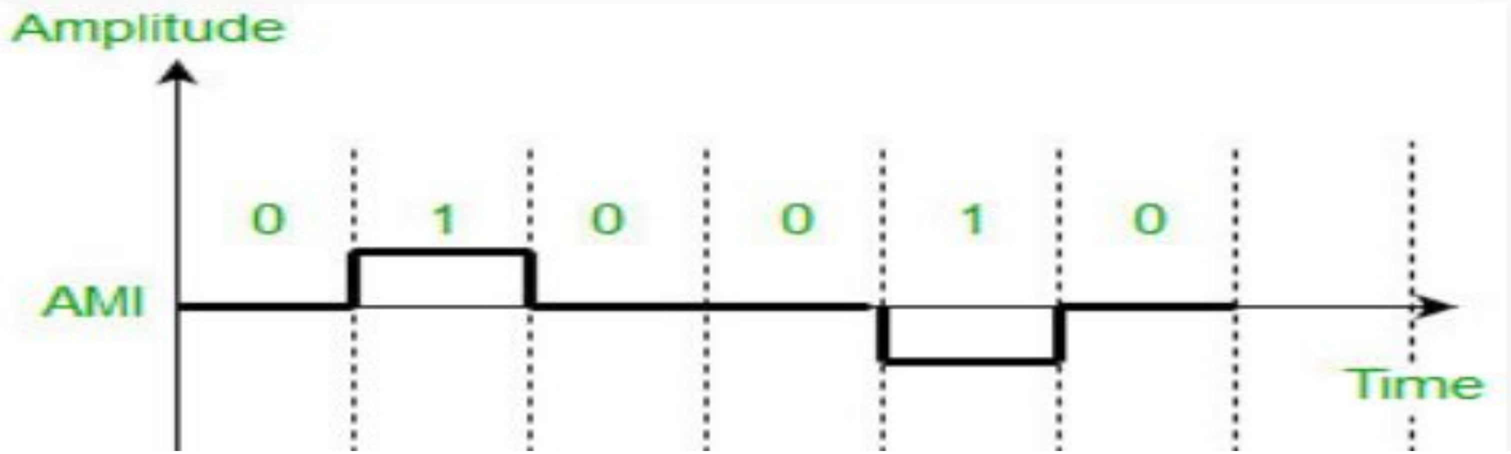
- PSD of UNIPOLAR NRZ$= \frac{a^2}{4} T_b \, sinc^2(fT_b) + \frac{a^2}{4}\delta(f)$



LINE CODES

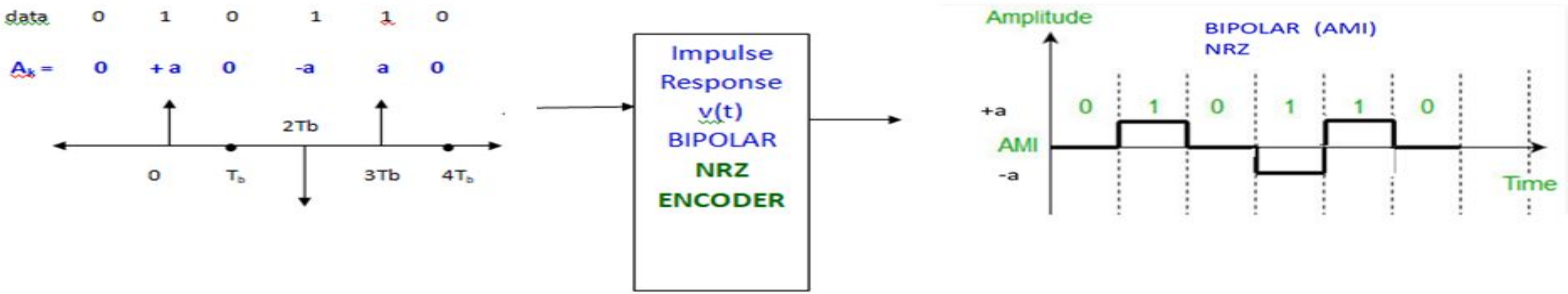
BIPOLAR NRZ

BIPOLAR (AMI) NRZ



HOW TO OBTAIN THE POWER SPECTRAL DENSITY?

BIPOLAR NRZ



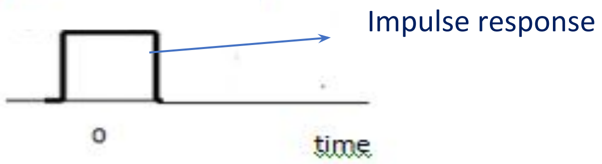
Here, the input is a random (binary) sequence

$$S_{gg}(f) = \frac{1}{T_b}|V(f)|^2 \sum_{n=-\infty}^{\infty} R_A(n)\exp(-j2\pi n f T_b)$$

$S_{xx}(f)$ = Fourier Transform of $R_A(n)$

$R_A(n)$ is the autocorrelation of the input random sequence

$$|V(f)|^2 = T_b^2\, sinc^2(\pi f T_b)$$



AUTOCORELATION SEQUENCE $R_A(n)$

A_k A_{k+1}

0 a 0 0 a 0 a a a 0 a 0 0

$$R_A(n) = E(A_k A_{k+n})$$

$$R_A(0) = E(A_k^2) = \sum_{k=1}^{2}(p_k A_k^2)$$

$$= \mathrm{p}(A_k = 0)0^2 + \mathrm{p}(A_k = a)a^2 = \frac{1}{2} \times 0 + \frac{1}{2} \times a^2 = \frac{a^2}{2}$$

AUTOCORELATION SEQUENCE $R_A(n)$

,

$$R_A(n) = E(A_k A_{k+n})$$

$$R_A(1) = E(A_k A_{k+1}) = \sum A_k A_{k+1} P(A_k A_{k+1})$$

A_k	A_{k+1}	$P(A_k A_{k+1})$ $=P(A_k) P(\frac{A(k+1)}{A_k})$	$A_k A_{k+1}$	$A_k A_{k+1} P(A_k A_{k+1})$
0	0	½ x ½ = ¼	0	0
0	+a	½ x ¼ = $\frac{1}{8}$	0	0
0	-a	½ x ¼ = $\frac{1}{8}$	0	0
+a	0	¼ x ½ = ¼	0	0
+a	+a	¼ x 0 = 0	a^2	0
+a	-a	¼ x ½ = $\frac{1}{8}$	$-a^2$	$\frac{-a^2}{8}$
-a	0	¼ x ½ = ¼	0	0
-a	+a	¼ x ½ = $\frac{1}{8}$	$-a^2$	$\frac{-a^2}{8}$
-a	-a	¼ x 0 = 0	a^2	0
	$\sum P(A_k A_{k+1}) A_k A_{k+1}$			$\frac{-a^2}{4}$

$$R_A(1) = -\frac{a^2}{4} = R_A(-1)$$

Similarly,

$$R_A(n) = 0, \qquad n \neq 0, \qquad n \neq \pm 1$$

$$R_A(0) = \frac{a^2}{2}, \quad R_A(\pm 1) = \frac{a^2}{4}$$

$$R_A(n) = 0, \qquad n \neq 0, \qquad n \neq \pm 1$$

Power Spectral Density of BIPOLAR NRZ waveform

$$R_A(0) = \frac{a^2}{2}, \quad R_A(\pm 1) = \frac{a^2}{4}$$

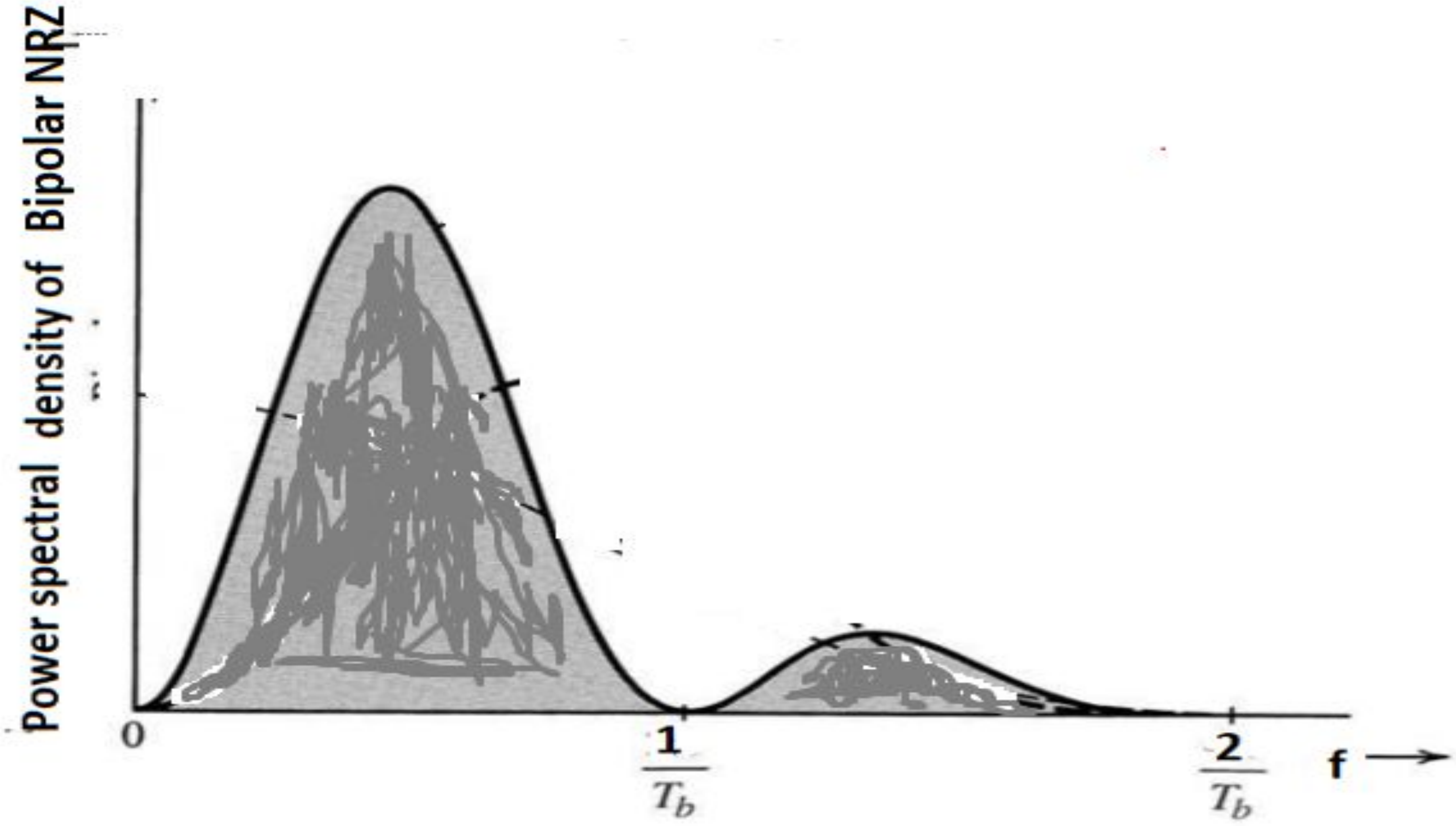
$$R_A(n) = 0, \qquad n \neq 0, \qquad n \neq \pm 1$$

$$|V(f)|^2 = T_b^2 \ sinc^2(\pi f T_b)$$

$$S_{gg}(f) = \frac{1}{T_b} |V(f)|^2 \sum_{n=-\infty}^{\infty} R_A(n) \exp(-j2\pi n f T_b)$$

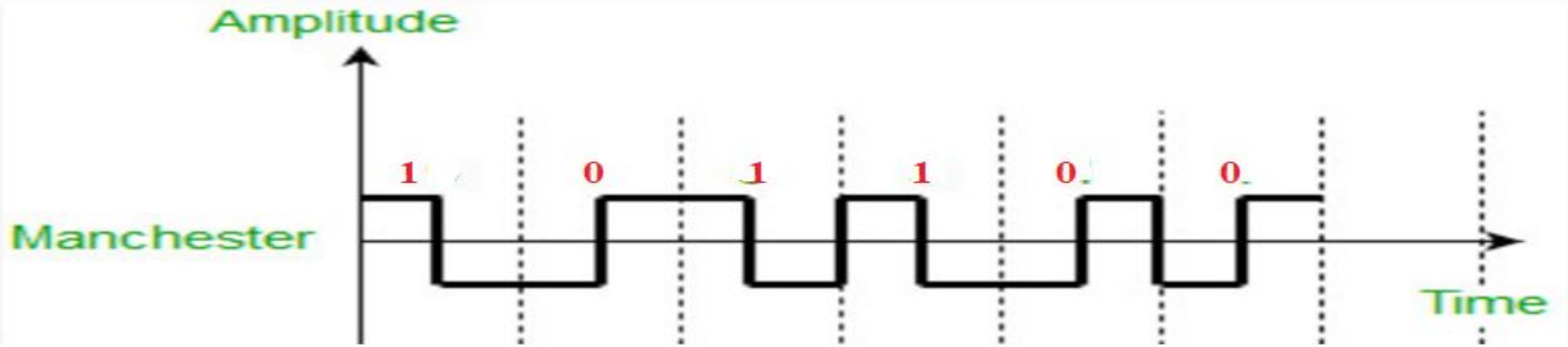
$$S_{gg}(f) = \frac{1}{T_b} T_b^2 \ sinc^2(f T_b) \quad [\ \frac{a^2}{2} - \frac{a^2}{4} [\exp(+j2\pi f T_b) + \exp(-j2\pi f T_b)]$$

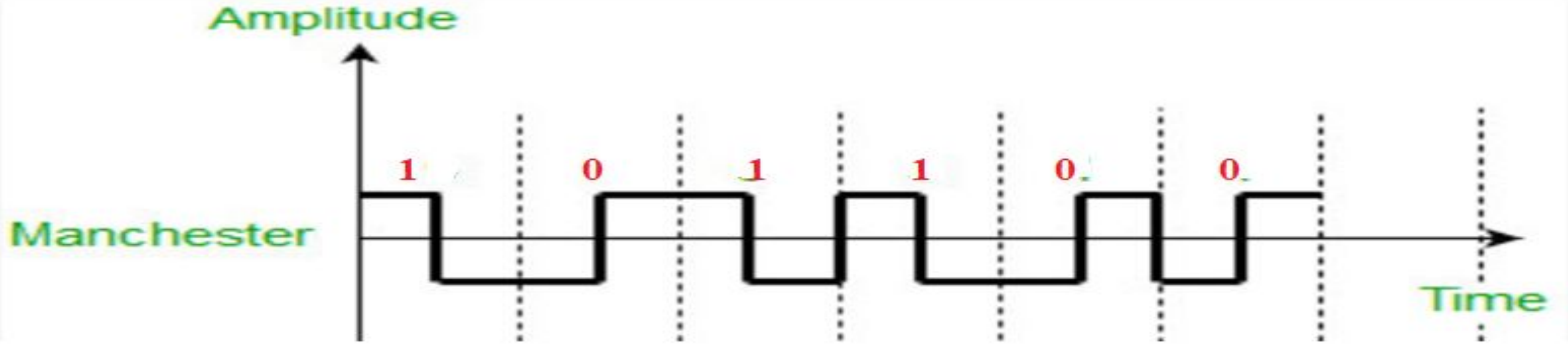
$$S_{gg}(f) = a^2 T_b \ \ sinc^2(f T_b) \ \ \sin^2(\pi f T_b)$$

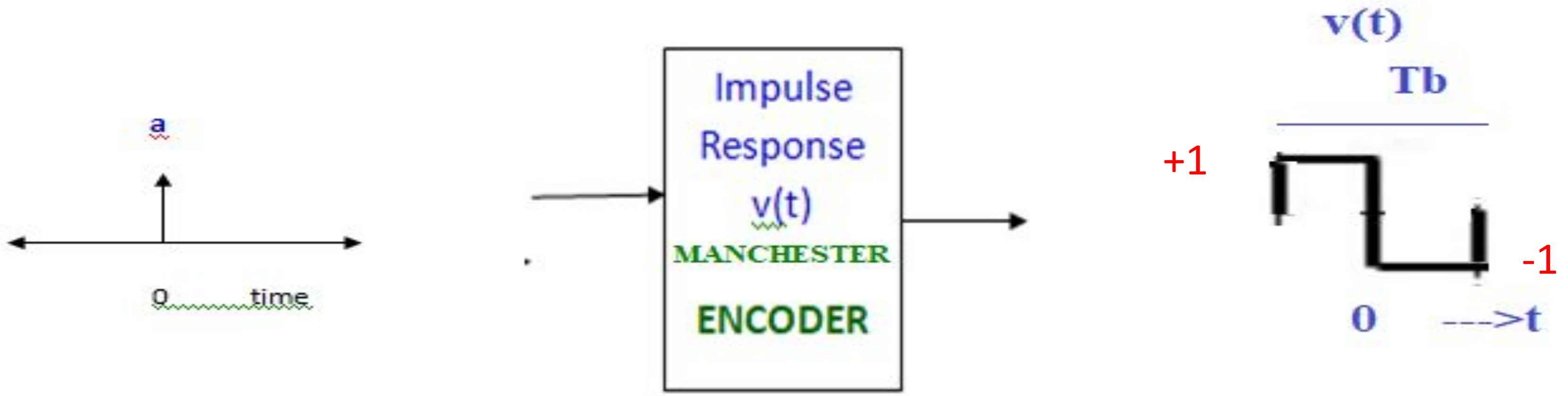


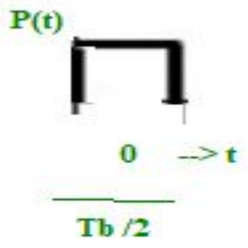
Power spectral density of Bipolar NRZ
0
$\frac{1}{T_b}$
$\frac{2}{T_b}$
f

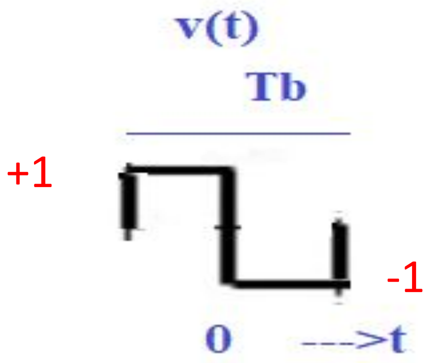
MANCHESTER ENCODING


Amplitude
1
0
1
1
0
0
Manchester
Time

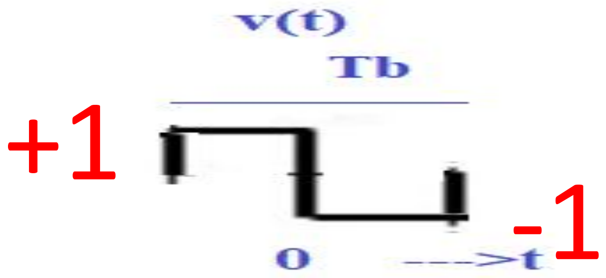

Amplitude
1
0
1
1
0.
0.
Manchester
Time

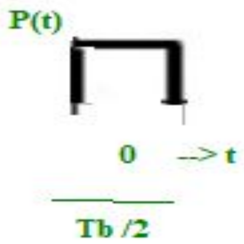

a
0 time
Impulse
Response
v(t)
MANCHESTER
ENCODER
v(t)
Tb
+1
-1
0 --->t





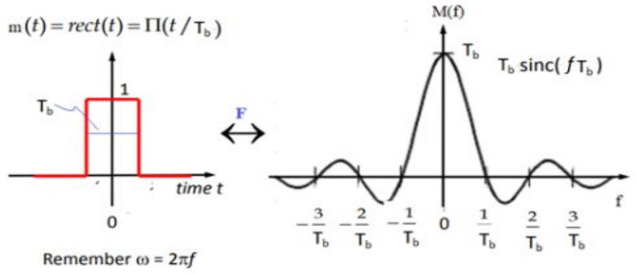
$$v(t) = p\left(t + \frac{T_b}{4}\right) - p\left(t - \frac{T_b}{4}\right)$$





$$v(t) = p(t + \frac{T_b}{4}) - p(t - \frac{T_b}{4})$$

$$P(f) = \frac{T_b}{2}\, sinc\,(\frac{fT_b}{2})$$



$$V(f) = P(f) exp\left(\frac{j2\pi f T_b}{4}\right) - P(f) exp\,(-\frac{j2\pi f T_b}{4})$$

$$S_{gg}(f) = \frac{1}{T_b}|V(f)|^2 \sum_{n=-\infty}^{\infty} R_A(n)\exp(-j2\pi n f T_b)$$

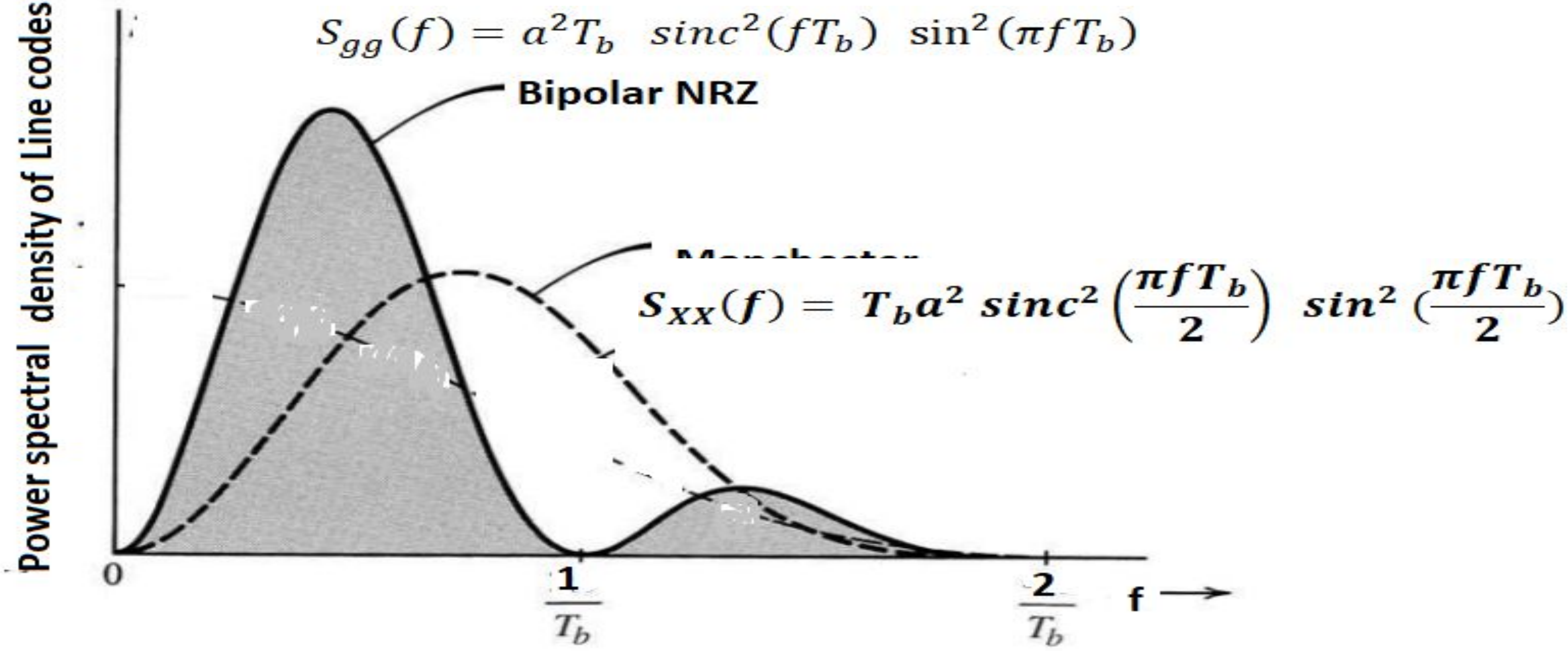
$$R_A(0) = a^2 \times 1/2 + (-a)^2 \times 1/2 = a^2$$

$$R_A(n) = 0, \quad n \neq 0$$

$$P(f) = \frac{T_b}{2}\ sinc\left(\frac{fT_b}{2}\right)$$

$$V(f) = P(f)exp\left(\frac{j2\pi fT_b}{4}\right) - P(f)exp\left(-\frac{j2\pi fT_b}{4}\right)$$

$$S_{XX}(f) = T_b a^2\ sinc^2\left(\frac{\pi fT_b}{2}\right)\ sin^2\left(\frac{\pi fT_b}{2}\right)$$

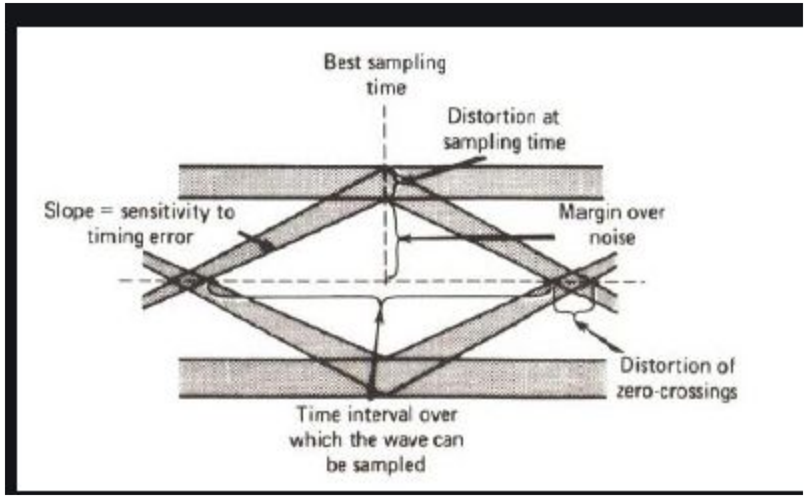


$S_{gg}(f) = a^2 T_b \ \ sinc^2(fT_b) \ \ \sin^2(\pi f T_b)$
Bipolar NRZ
$S_{XX}(f) = \ T_b a^2 \ sinc^2\left(\frac{\pi f T_b}{2}\right) \ \ sin^2\left(\frac{\pi f T_b}{2}\right)$
Power spectral density of Line codes
0
$\frac{1}{T_b}$
$\frac{2}{T_b}$
f

ASSIGNMENT

- POLAR RZ
- POLAR NRZ
- UNIPOLAR RZ

EYE PATTERN



Best sampling time
Distortion at sampling time
Slope = sensitivity to timing error
Margin over noise
Distortion of zero-crossings
Time interval over which the wave can be sampled

INTER SYMBOL INTERFERENCE

COMBATING CHANNEL LIMITATIONS

BANDWIDTH LIMITATION

SIGNAL DISTORTION

Received Pulse shape is different from that of (broader than) the Transmitted pulse

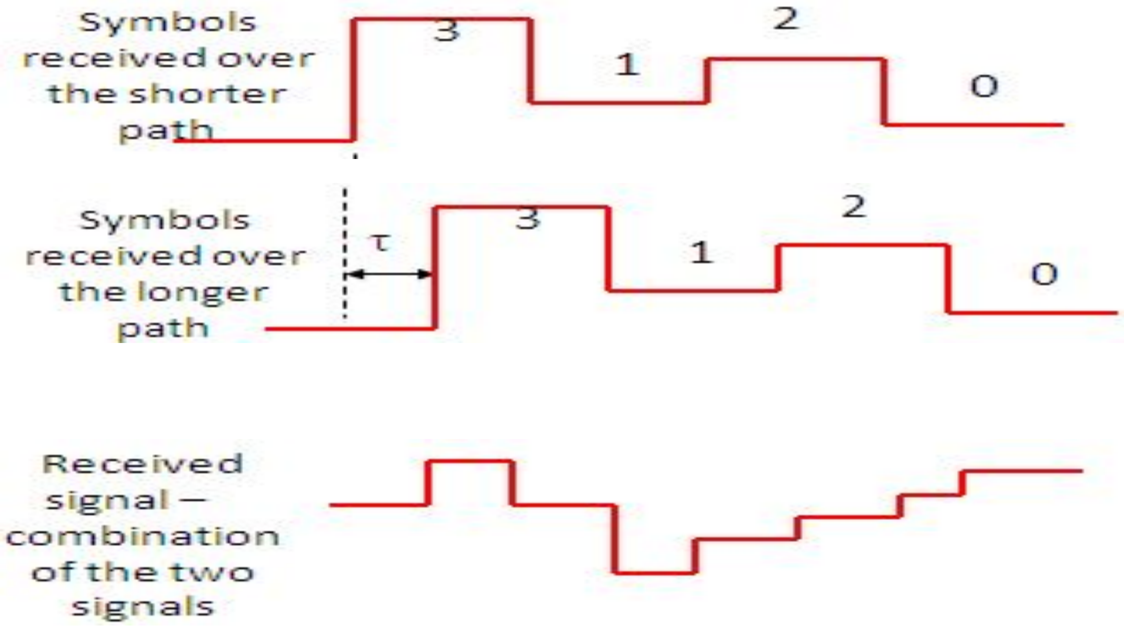
INTER SYMBOL INTERFERENCE (ISI)

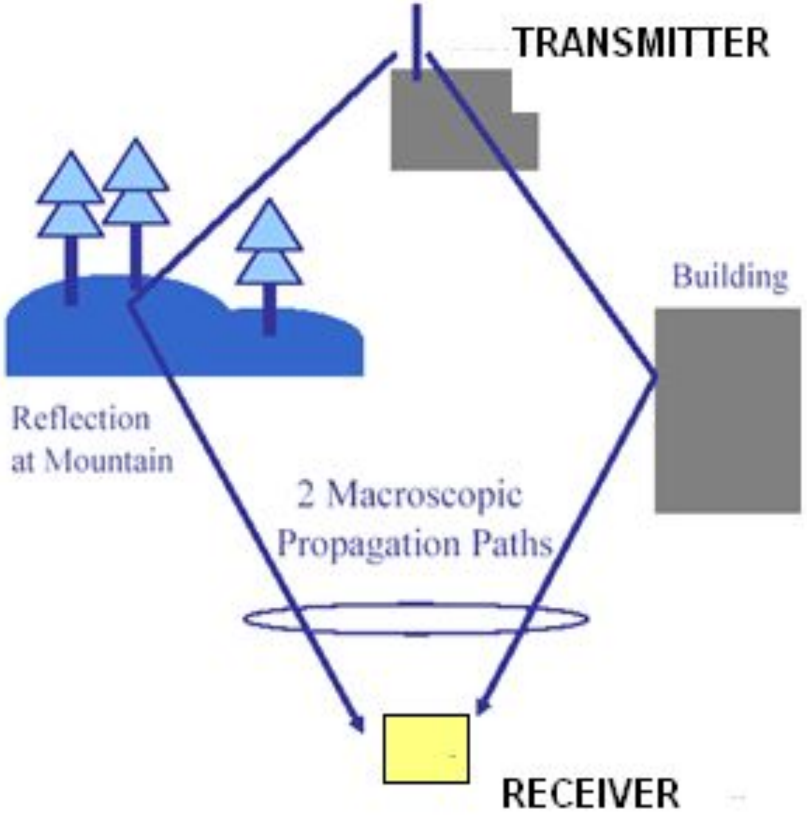
SOLUTION

CHANNEL EQUALIZATION

ISI
due to
MULTIPATH EFFECT
(WIRELESS CHANNEL)

ISI due to Multipath effect in Wireless Channel





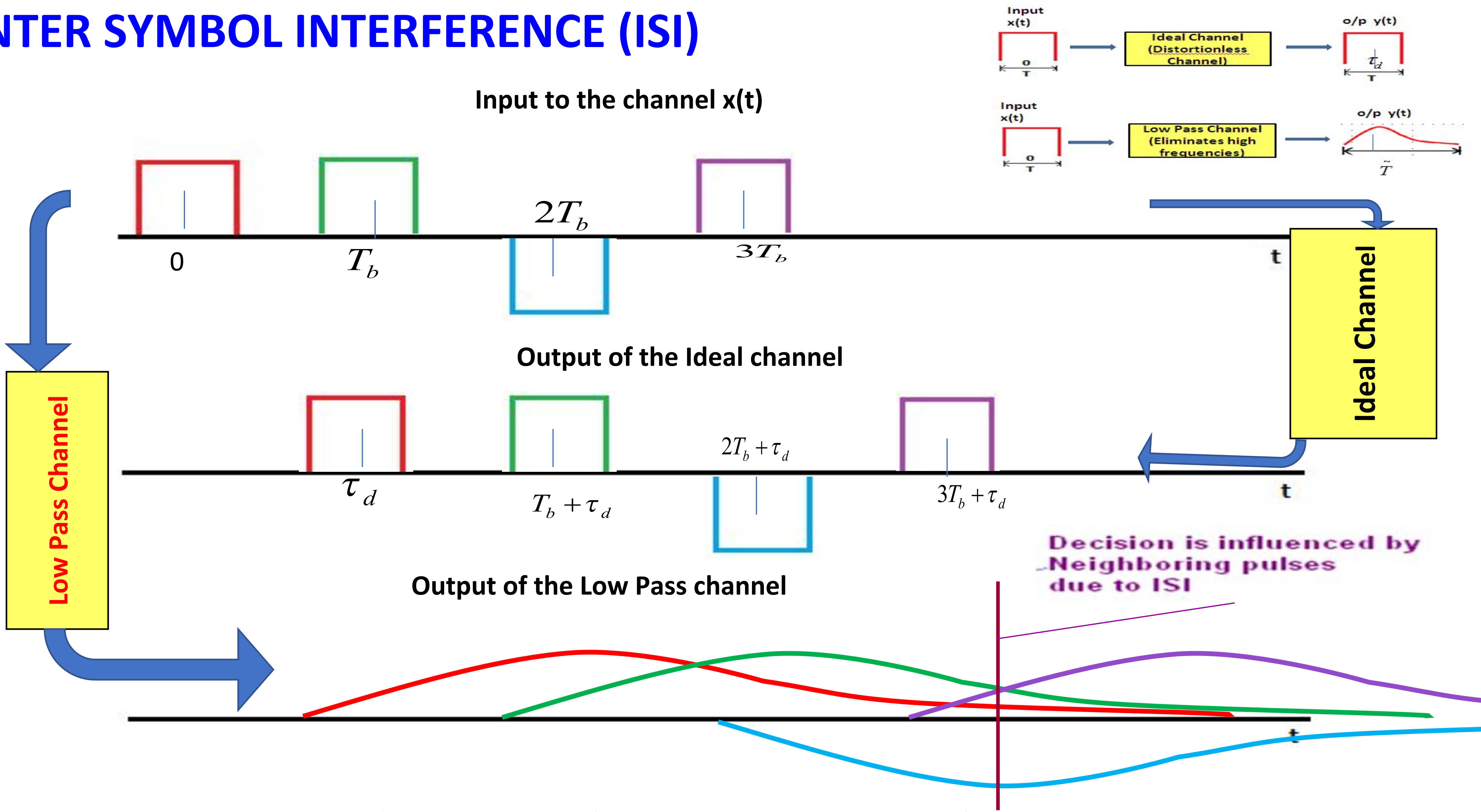
ISI
(Inter Symbol Interference)

due to

CHANNEL BANDWIDTH LIMITATION

CHANNEL BANDWIDTH LIMITATION & PULSE BROADENING

INTER SYMBOL INTERFERENCE (ISI)



ISI

due to

CHANNEL BANDWIDTH LIMITATION

SOLUTION

CHANNEL ESTIMATION

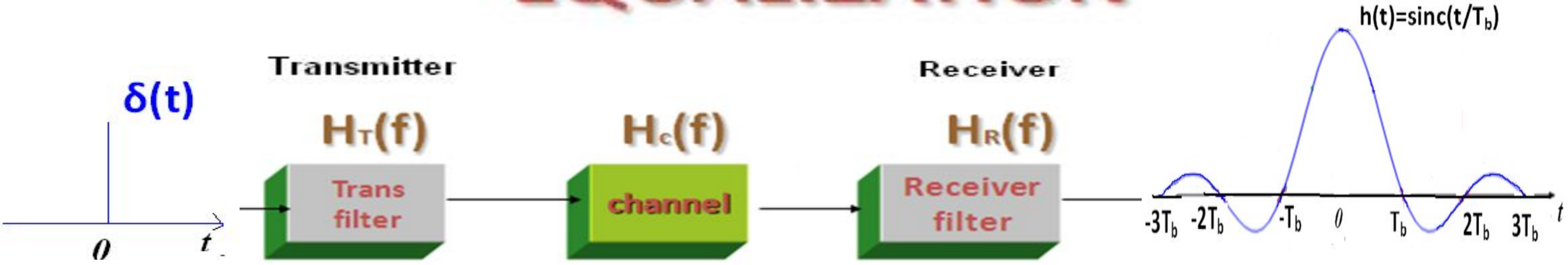
&

CHANNEL EQUALIZATION

Solution to ISI

CHANNEL EQUALIZATION

EQUALIZATION



Composite System Transfer Function = H(f)

FT

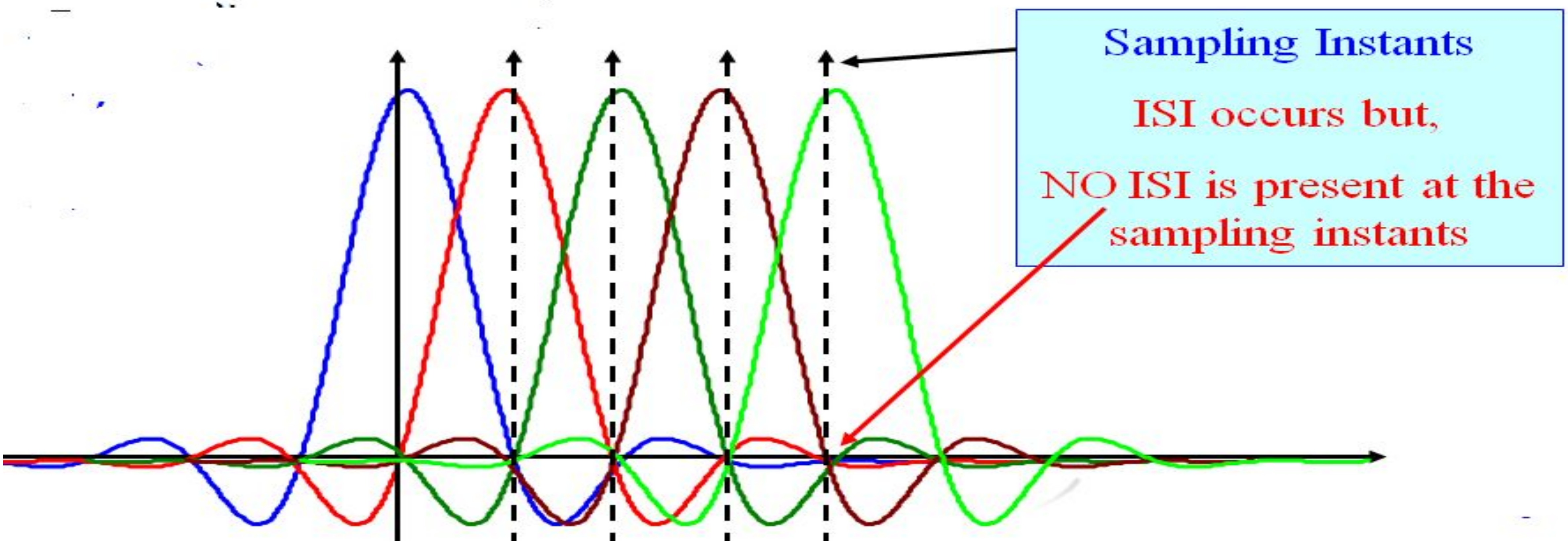
$$H(f) \leftrightarrow h(t)$$

$$H(f) = H_T(f)\, H_c(f)\, H_R(f)$$

$H_R(f)$ ▯ Equalizing Filter –

Compensates for the effects of

Transmit Filter and the Channel



Sampling Instants
ISI occurs but,
NO ISI is present at the sampling instants

$Transmitted\ bit\ \ say\ m^{th}\ bit\ \ 1\ encoded\ as\ \ +1$

$b_m = 1$

In general say

$b_k = \pm 1 \quad (BIT\ 0/1)$

$Bit\ is\ Received\ as\ waveform\ h(t - mT_b),$

Received waveform r(t) $= \sum_{n=-\infty}^{\infty} b_n h(t - nT_b)$

$$= b_m h(t - mT_b) + \sum_{\substack{n=-\infty \\ n \neq m}}^{\infty} b_n\ h(t - nT_b)$$

$To\ take\ decision\ on\ the\ m^{th}\ bit,\ sampling\ it\ at\ t = mT_b$

$$\boldsymbol{r(mT_b) = \underbrace{b_m h(mT_b - mT_b)}_{\text{Due to } m^{th} \text{ bit}} + \underbrace{\sum_{\substack{n=-\infty \\ n \neq m}}^{\infty} b_n h(mT_b - nT_b)}_{\text{ISI from OTHER BITS}}}$$

$$r(mT_b) = b_m h(0) + \sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} b_k h(kT_b)$$

To avoid ISI,

$$r(mT_b) = b_m h(0)$$

Equivalently, $h(kT_b) = 0, \quad k \neq 0$

$$h(kT_b) \neq 0, \quad k = 0$$

CONDITION for ABSENCE of ISI

$$h(kT_b) = 0, \quad k \neq 0$$

$$h(kT_b) \neq 0, \quad k = 0$$

Considering h_Δ(t) =the discrete-time version of h(t)

$$h_\Delta(\mathrm{t}) = \mathrm{h(t)} \sum_{k=-\infty}^{\infty} \delta(\mathrm{t} - \mathrm{kT_b})$$

$$= \sum_{k=-\infty}^{\infty} \mathrm{h(kT_b)}\delta(\mathrm{t} - \mathrm{kT_b})$$

For zero ISI

$$\sum_{k=-\infty}^{\infty} \mathrm{h(kT_b)}\delta(\mathrm{t} - \mathrm{kT_b}) = \mathrm{h(0)}\ \delta(\mathrm{t})$$

$$\mathrm{h(t)} \sum_{k=-\infty}^{\infty} \delta(\mathrm{t} - \mathrm{kT_b}) = \mathrm{h(0)}\delta(\mathrm{t})$$

Taking Fourier Transform on both sides,

$$H(f) * \sum_{k=-\infty}^{\infty} \delta\left(\mathrm{f} - \frac{\mathrm{k}}{\mathrm{T_b}}\right) = \mathrm{h(0)} = \text{a constant}$$

Activate W

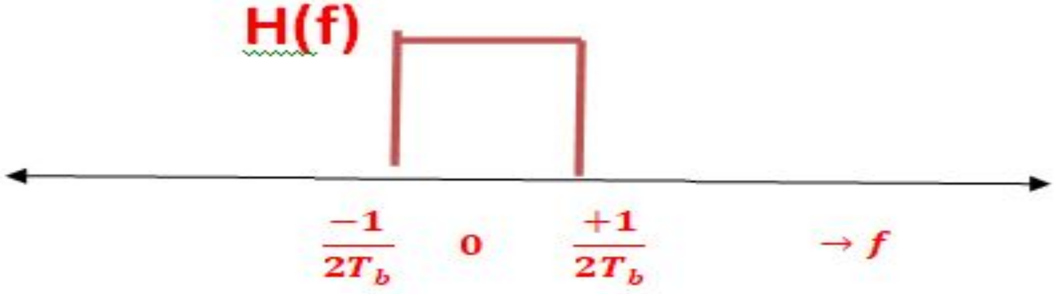
$$\sum_{k=-\infty}^{\infty} \mathrm{H}\left(\mathrm{f} - \frac{\mathrm{k}}{\mathrm{T_b}}\right) = \text{constant}$$

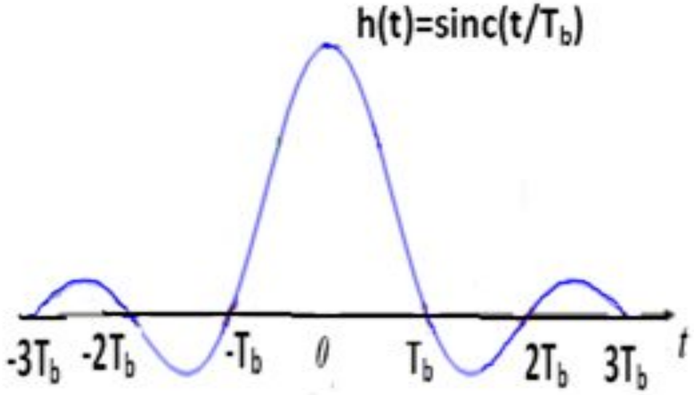
CONDITION for ABSENCE of ISI

$$h(kT_b) = 0, \quad k \neq 0$$

$$h(kT_b) \neq 0, \quad k = 0$$

$$\sum_{k=-\infty}^{\infty} H\left(f - \frac{k}{T_b}\right) = \text{constant}$$

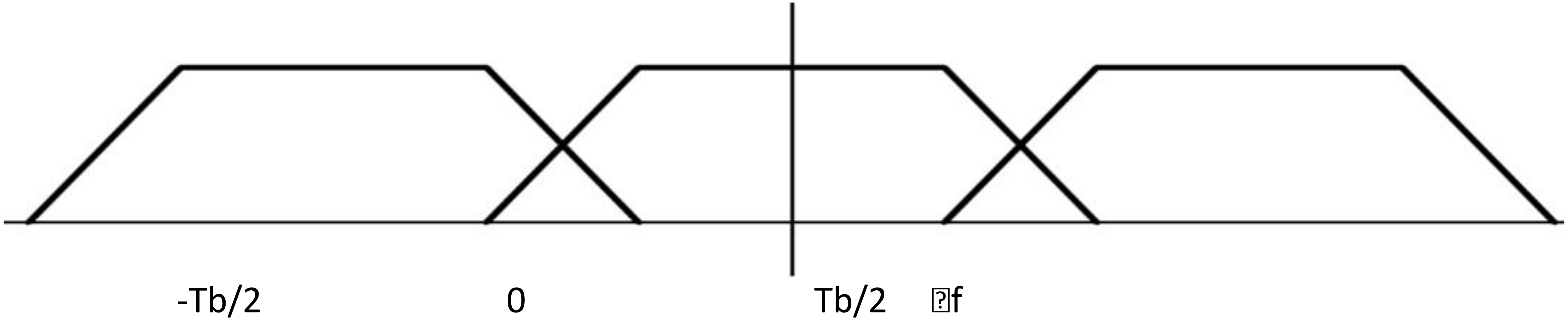




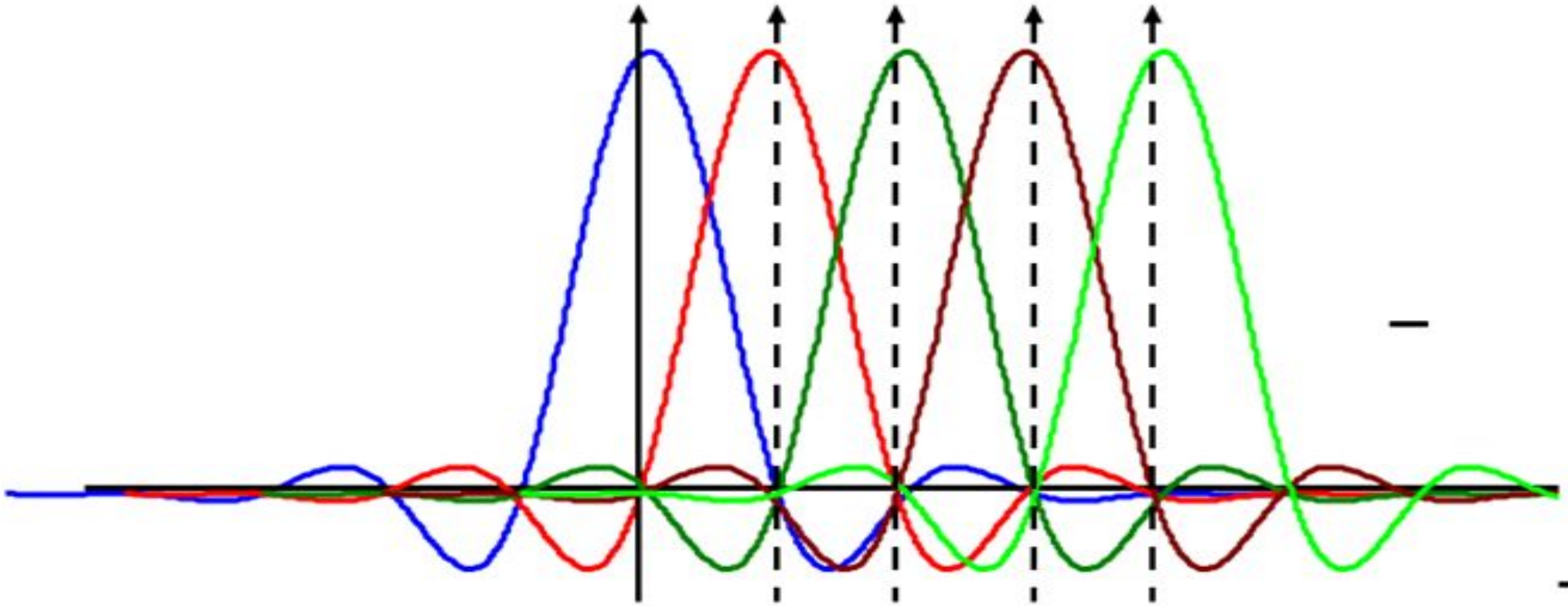
MINIMUM BANDWIDTH SOLUTION

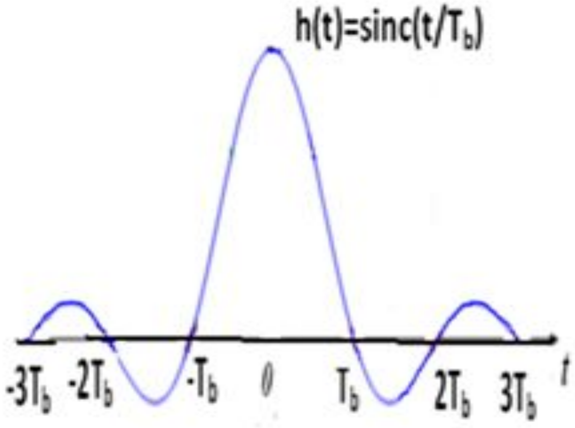
The SOLUTION is NOT UNIQUE

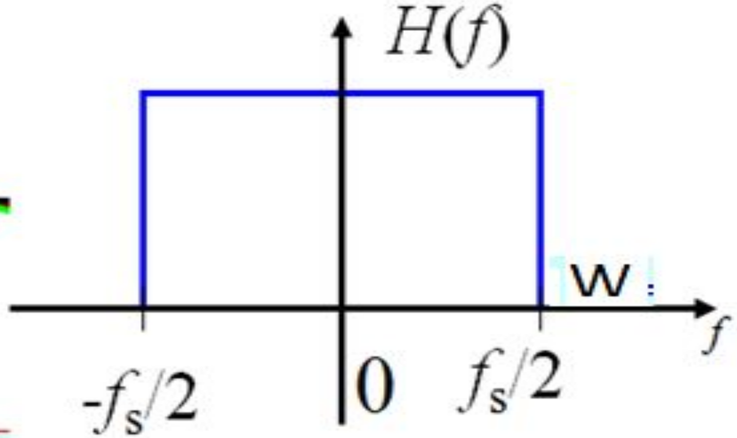
H(f) -another choice satisfying the condition



- (SYMBOL RATE) fs $=1/T_s=2.W$

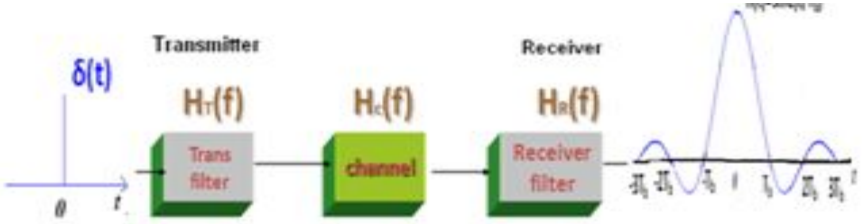


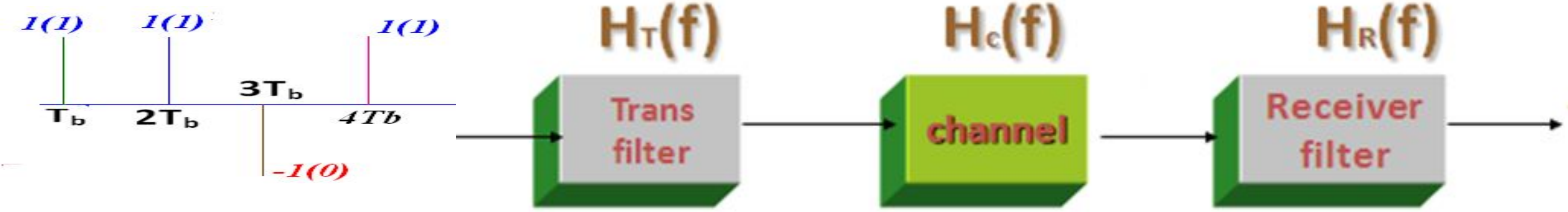


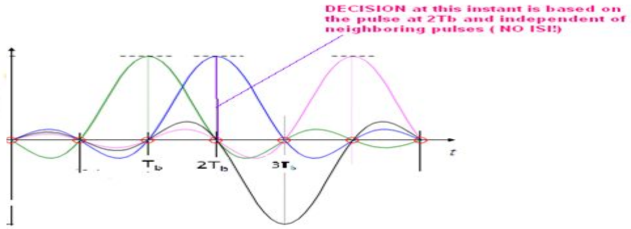


Absolute bandwidth $= W = \frac{f_s}{2}$ MINIMUM BANDWIDTH

EQUALIZATION







Look for
Realizable Solution
Less Sensitive to Timing Error

Minimum Bandwidth Solution
Zero ISI
Non_Causal →Not Physically Realizable
Decays as 1/ltl for large ltl
Requires Perfect Sample Timings

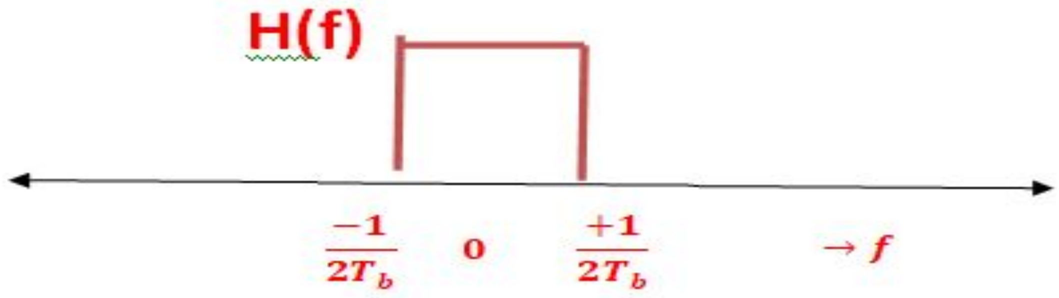
CONDITION for ABSENCE of ISI

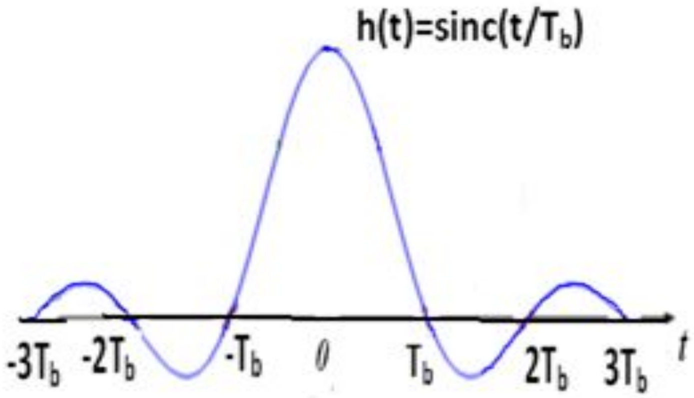
$$h(kT_b) = 0, \quad k \neq 0$$

$$h(kT_b) \neq 0, \quad k = 0$$

The SOLUTION is NOT UNIQUE

$$\sum_{k=-\infty}^{\infty} H\left(f - \frac{k}{T_b}\right) = \text{constant}$$





CONDITION for ABSENCE of ISI

$$h(kT_b) = 0, \quad k \neq 0$$
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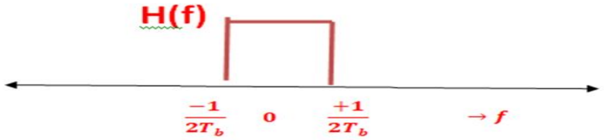
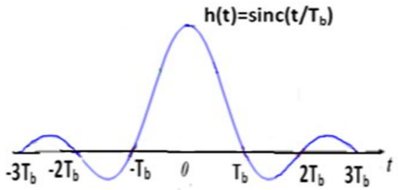
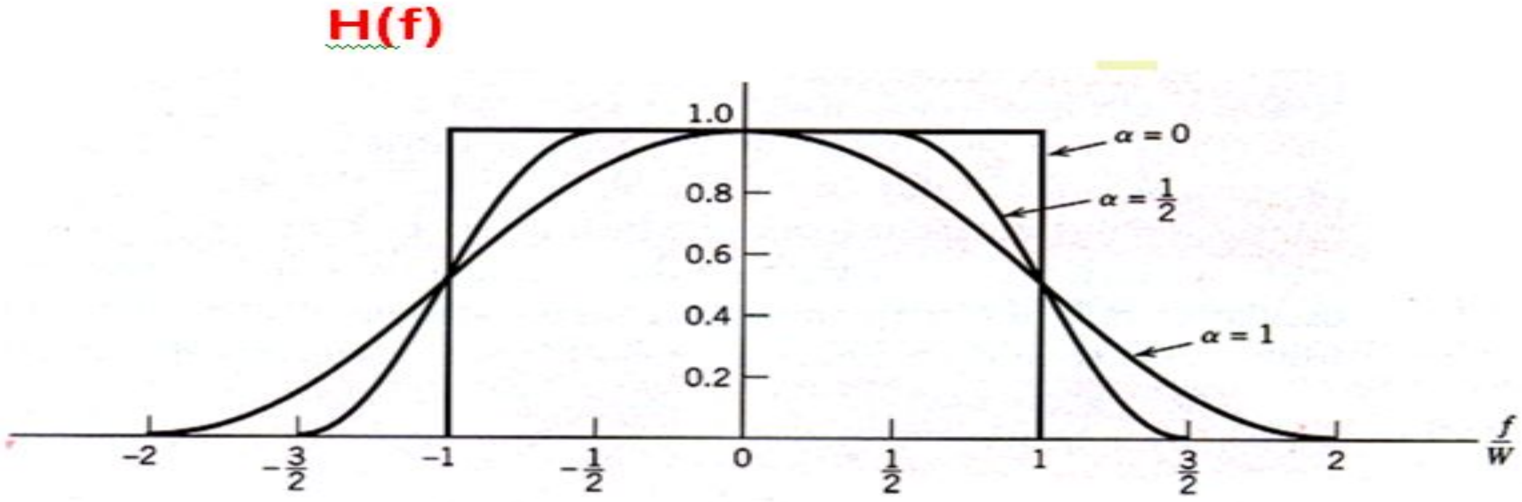
$$\sum_{k=-\infty}^{\infty} H\left(f - \frac{k}{T_b}\right) = \text{constant}$$







$W = 1/2T_b.$

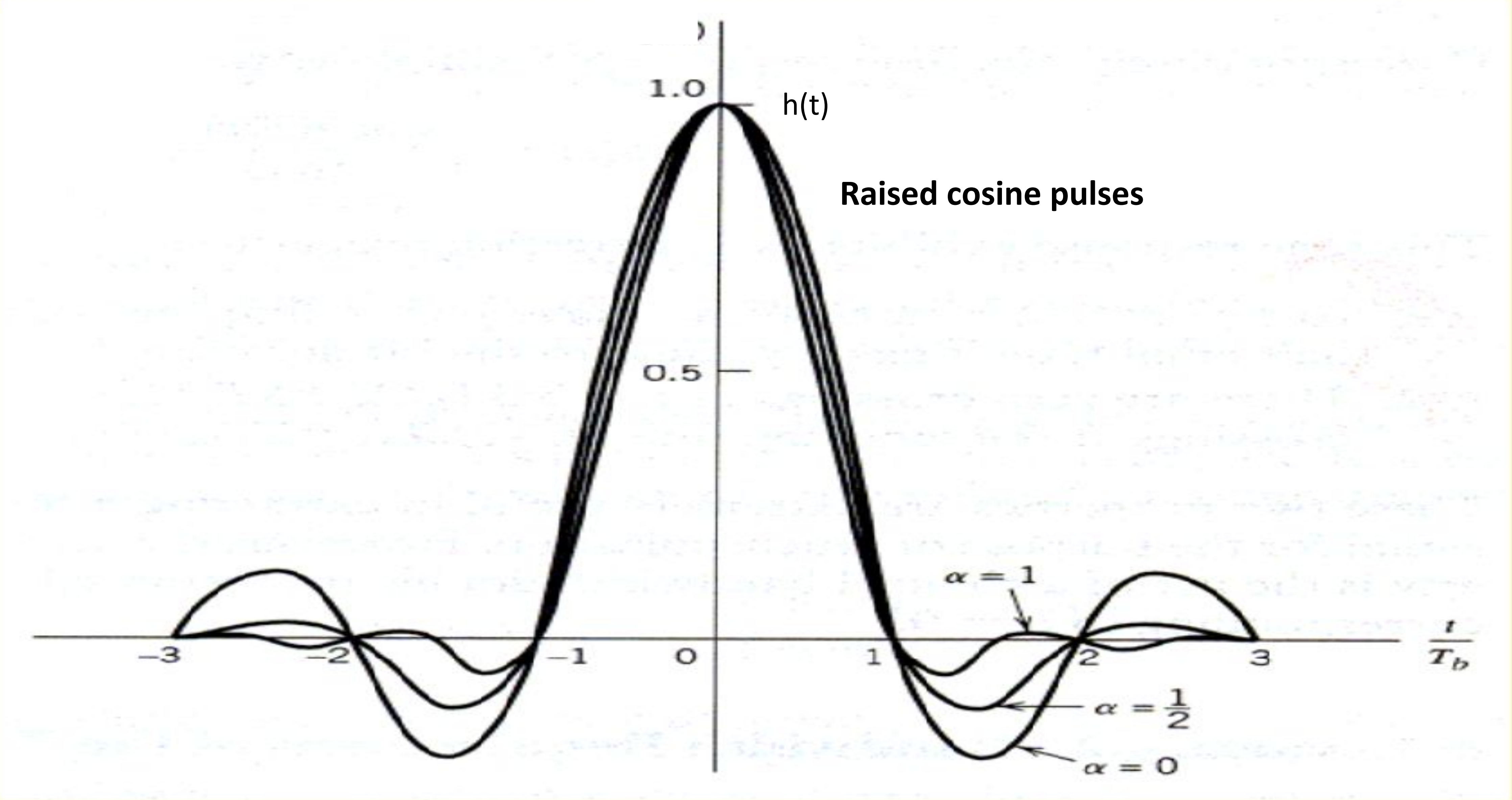
Raised Cosine Spectrum

$$H(f) = \begin{cases} \dfrac{1}{2W} & 0 \le |f| < f_1 \\ \dfrac{1}{4W}\left\{1 - \sin\left[\dfrac{\pi(|f| - W)}{2W - 2f_1}\right]\right\} & f_1 \le |f| < 2W - f_1 \\ 0 & |f| > 2W - f_1 \end{cases}$$

α is the rolloff factor. $W = 1/2T_b$

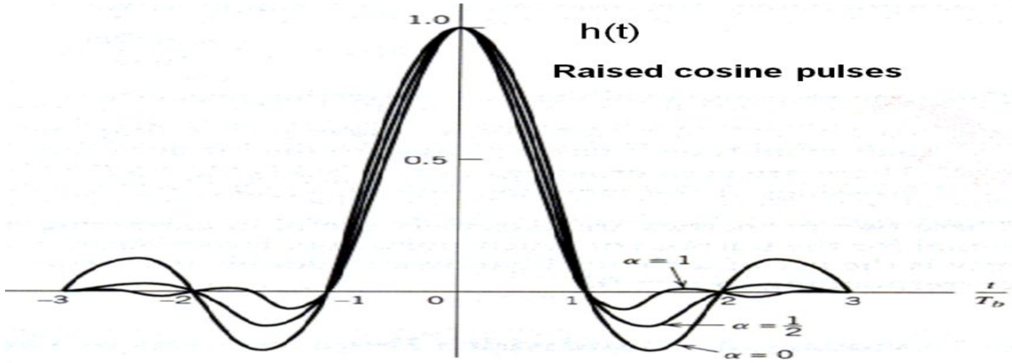
$$\alpha = 1 - f_1 / W$$

The transmission bandwidth is $(1+\alpha)W$



1.0
h(t)
Raised cosine pulses
0.5
α = 1
α = 1/2
α = 0
−3
−2
−1
0
1
2
3
t/Tb

$$\textbf{h(t)} = \frac{\sin(\pi t/T)}{\pi t/T}\frac{\cos(\pi\alpha t/T)}{1-4\alpha^2t^2/T^2}$$



$\alpha = 0$, **ideal solution - h(t) is sinc function**

$\alpha = 0$, (sinc pulse) decays as $1/t$.

for $\alpha > 0$, **raised cosine pulses**

for $\alpha > 0$, $x(t)$ decays as $1/t^3$

Hence, the raised cosine spectrum is much less sensitive to timing errors than the sinc pulse.

Just like all other bandlimited pulses, raised cosine spectrum is not timelimited.

Therefore, truncation and delay is required for realization.

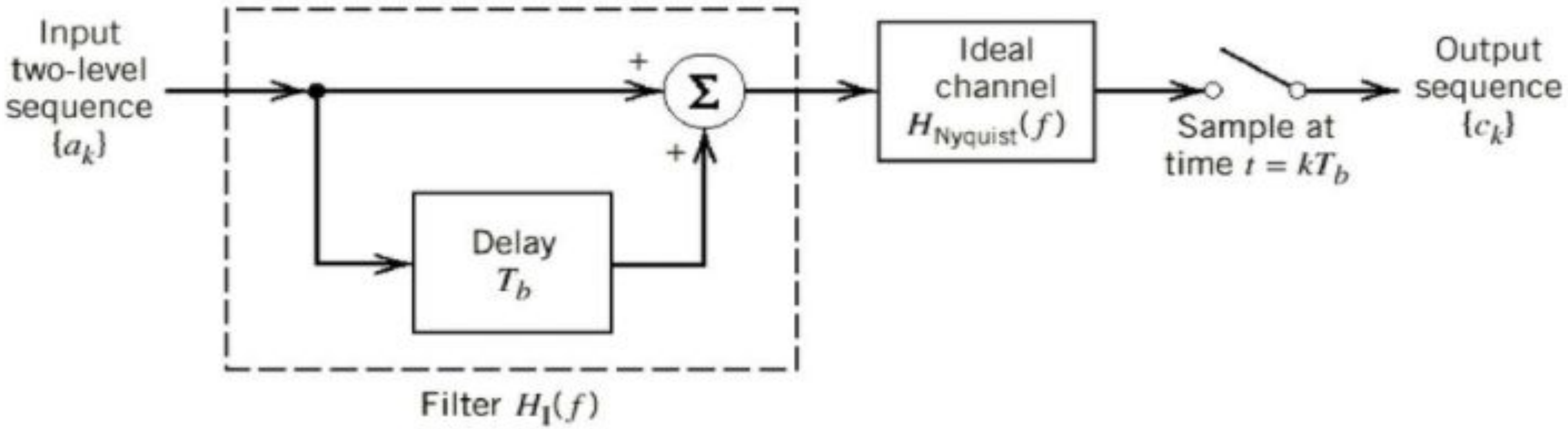
DUOBINARY ENCODING

CORRELATIVE ENCODING

CONTROLLED ISI

REQUIRES MINIMUM BANDWIDTH

Duobinary signaling scheme.



$$H(f) = H_C(f)[1 + \exp(-j2\pi f T_b)]$$

$$= H_C(f)[\exp(j\pi f T_b) + \exp(-j\pi f T_b)]\exp(-j\pi f T_b)$$

$$= 2H_C(f)\cos(\pi f T_b)\exp(-j\pi f T_b),$$

$$H_c(f) = 1, \quad |f| \leq \frac{R_b}{2}$$

$$= 0, ELSEWHERE$$

$$H(f) = 2\cos(\pi f T_b)\ \exp(-2\pi f T_b), \qquad |f| \leq \frac{R_b}{2}$$

$$= 0, \qquad otherwise$$

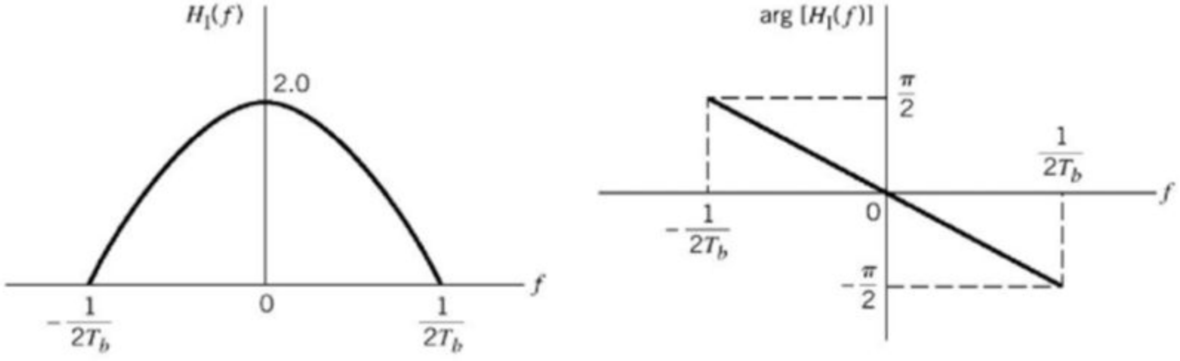
$$H(f) = 2\cos(\pi f T_b)\,\exp(-2\pi f T_b), \quad |f| \le \frac{R_b}{2}$$

$$= 0, \quad otherwise$$

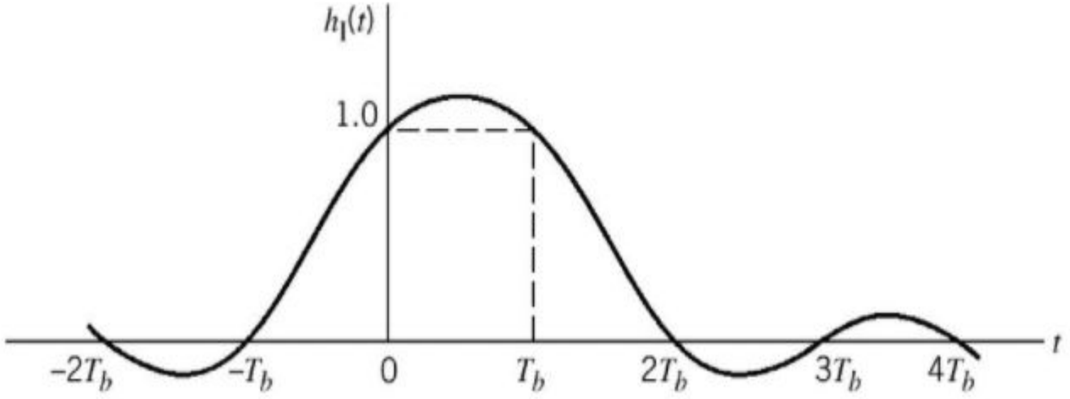
$$h(t) = \frac{\sin(\pi t/T_b)}{\pi t/T_b} + \frac{\sin[\pi(t - T_b)/T_b]}{\pi(t - T_b)/T_b}$$

$$= \frac{\sin(\pi t/T_b)}{\pi t/T_b} - \frac{\sin(\pi t/T_b)}{\pi(t - T_b)/T_b}$$

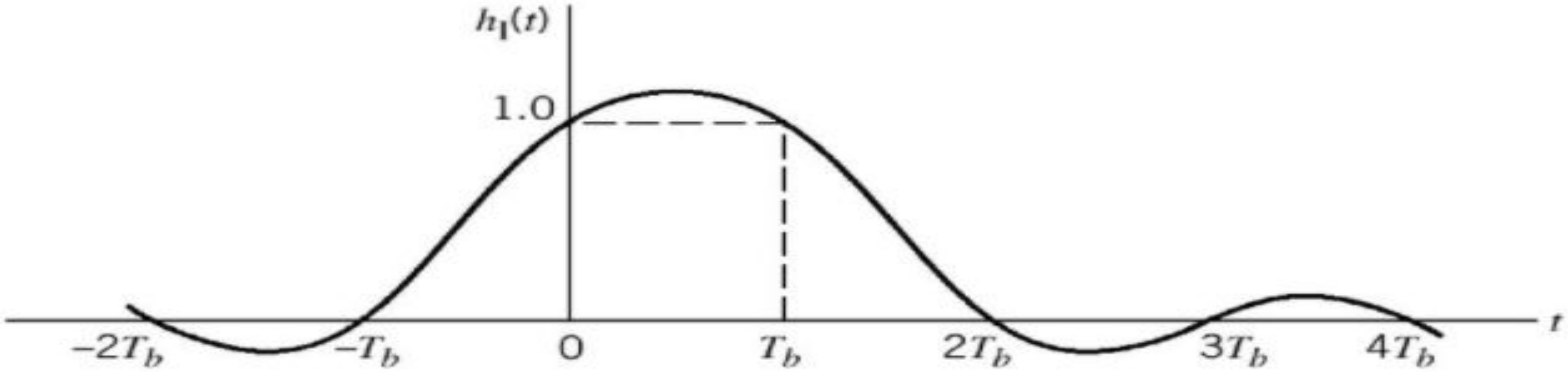
$$= \frac{T_b^2 \sin(\pi t/T_b)}{\pi t(T_b - t)}.$$



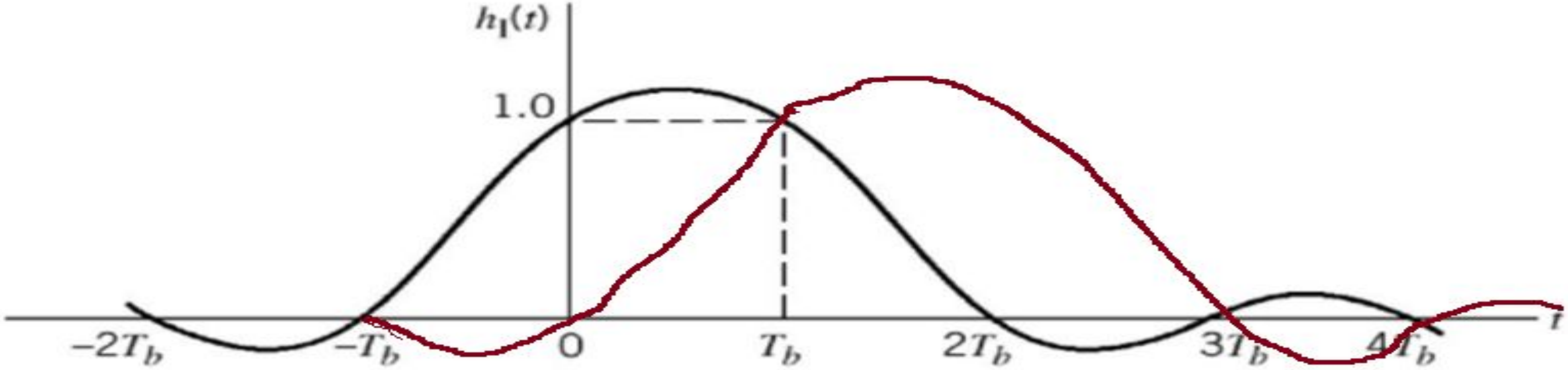
(a) (b)



Impulse response of the duobinary conversion filter.

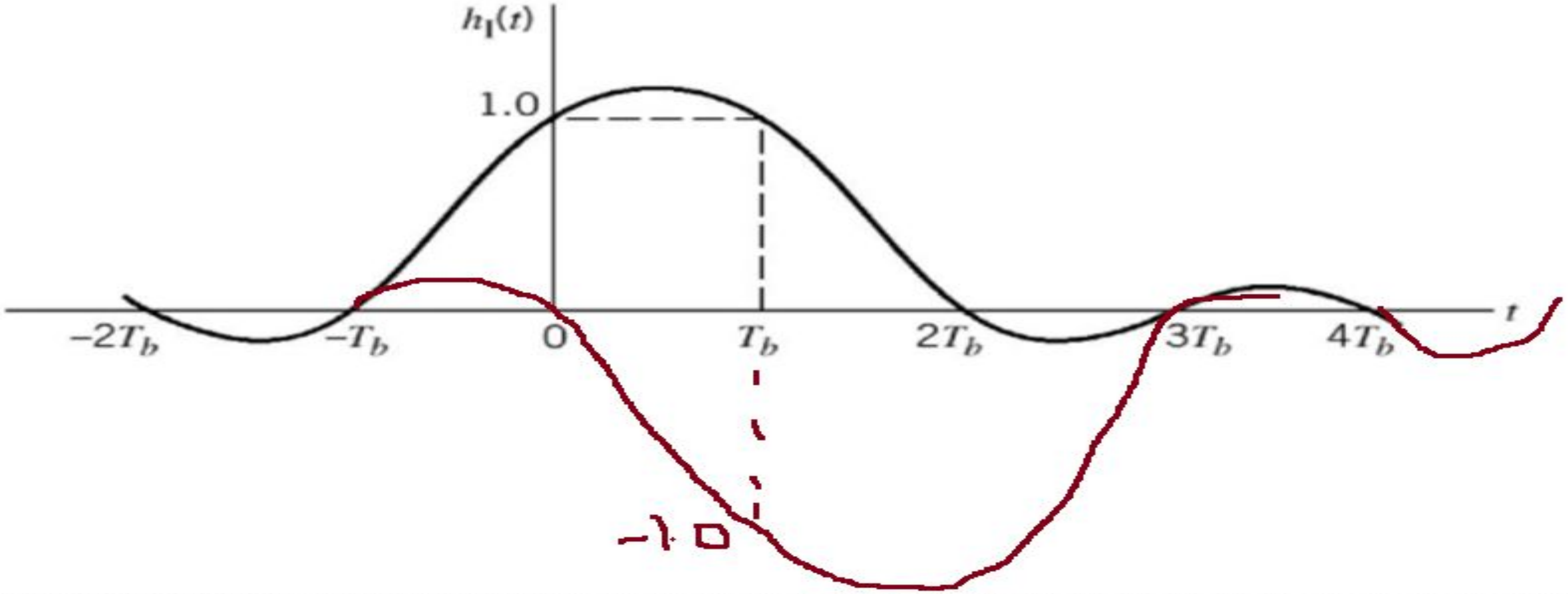


CONTROLLED ISI



CONTROLLED ISI

Impulse response of the duobinary conversion filter.



Duobinary encoding

Data sequence 1 0 1 0 1 0 0 1 1

b_k + 1 +1 -1 +1 -1 +1 -1 -1 +1 +1

$C_k = b_k + b_{k-1}$ +2 0 0 0 0 0 -2 0 +2

Decision 1 0 1 0 1 0 0 1 1

If $C_k > +1$ ---→ Data 1

$C_k < -1$ ---→ Data 0

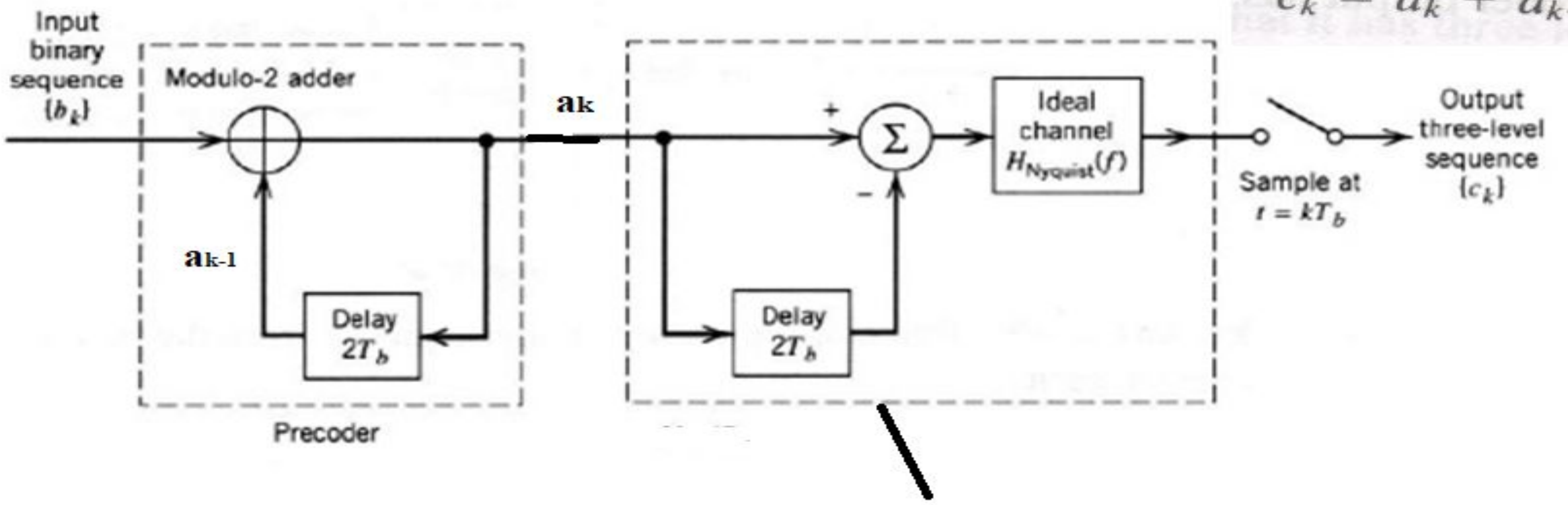
$-1 < C_k < 1$ --→ Current bit is the inverted version of the previous bit

PROPAGATION OF ERROR

DUOBINARY ENCODER WITH PRECODER

$$a_k = b_k + a_{k-1} \quad \text{modulo-2}$$

$$c_k = a_k + a_{k-1}$$



DUOBINARY ENCODER

$$a_k = b_k + a_{k-1} \quad \text{modulo-2}$$

DUOBINARY ENCODER WITH A PRECODER

Data sequence b_k 1   0   1   0   1   0   0   1   1

$a_k = b_k \oplus a_{k-1}$ 1   0   0   1   1   0   0   0   1   0

+1   -1   -1   +1   +1   -1   -1   -1   +1   -1

$C_k = a_k + a_{k-1}$ 0   -2   0   +2   0   -2   -2   0   0

Decision   1   0   1   0   1   0   0   1   1

If $C_k > +1$ --- Data 0
$C_k < -1$ --- Data 0
$-1 < C_k < 1$ -- Data 1

ERROR DOES NOT PROPAGATE