Signal flow graph.

→ It depicts the flow of signals from one point of a s/m to another and gives the relationships among the signals.

→ It consists of a n/w in which nodes are connected by directed branches.

→ It uses Mason's gain formula to compute the transfer function & it is simple compared to the block-reduction technique.

→ Elements / terms in Signal flow graph:-

1) Node: A pt. representing a variable or signal.

2) Branch: directed line segment joining two nodes.
Arrow on the branch ⇒ direction of signal flow.
gain of the branch ⇒ transmittance.

3) Transmittance: gain acquired by the signal when it travels from one node to another.

4) I/p node: source node, has only outgoing branches.

5) o/p node: sink node, has only incoming branches

6) mixed node: Has both incoming & outgoing branches.

7) path: It is a traversal of connected branches in the direction of the branch arrows.
It should not cross a node more than once.

8) open path : path that starts at one node & ends ~~at~~ another node.

9) closed path : start & end nodes are same.

10) Forward path : path from i/p node to o/p node & does not cross any node more than once.

11) Forward path gain: It is product of the branch gain of a forward path.

12) Individual loop : closed path & no crossing of any node more than once.

13) loop gain : product of gains of a loop.

14) Non-touching loop : If there is no common node between any loops, then the loops called non-touching loop.

Mason's Gain formula :

$$T(s) = \frac{C(s)}{R(s)}$$

$$\boxed{T = \frac{1}{\Delta} \sum_{K} P_K \Delta_K}$$

$P_K \Rightarrow$ forward path gain of K^{th} forward ~~loop~~ path.

$K \Rightarrow$ no. of forward paths

$\Delta = 1 -$ (sum of individual loop gain)

$+$ (sum of gain prdt of all possible combinations of two non-touching loops)

$-$ (sum of gain prdt of all possible combinations of three non-touching loops)

$+ \cdots$

$\Delta_K = \Delta$ for that part of the graph which is not touching K^{th} forward path.

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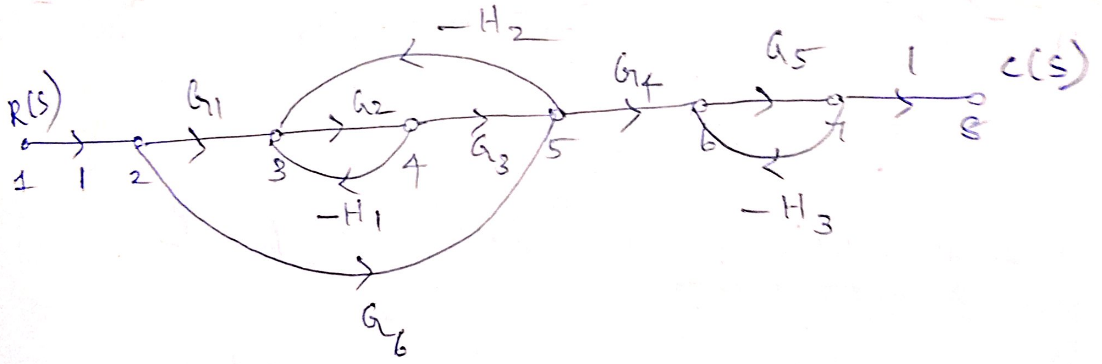
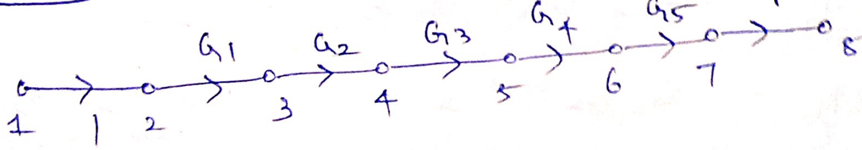
eg: Find the over-all transfer function of the s/m whose signal flow graph.



soln

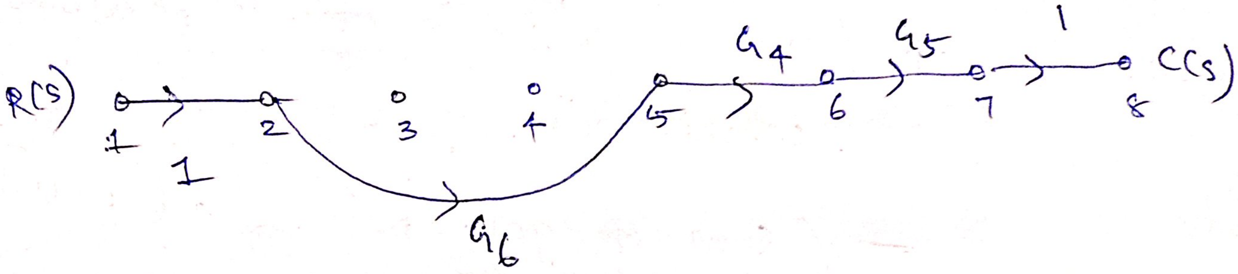
① No. of forward paths $(K) = 2$.

② 1^{st} path $\Rightarrow$ 1 - 2 - 3 - 4 - 5 - 6 - 7 - 8



path gain $P_1 = G_1 G_2 G_3 G_4 G_5$

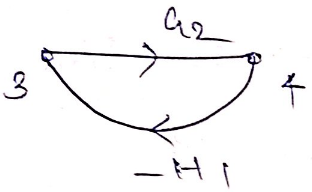
2^{nd} forward path $\Rightarrow$ 1 – 2 – 5 – 6 – 7 – 8



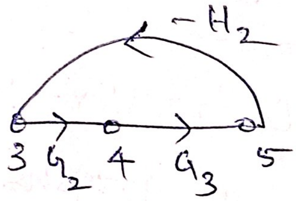
Frwd path Gain $P_2 = G_6 G_4 G_5$.

③ Individual loop gain.

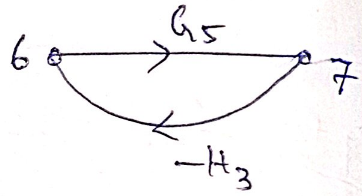
loop 1



loop 2

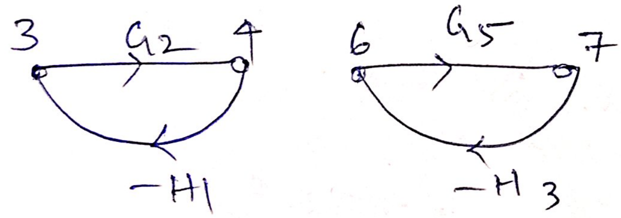


loop 3



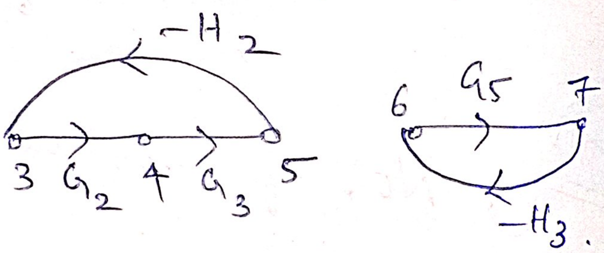
$P_{11} = -G_2 H_1$ $\qquad$ $P_{21} = -G_2 G_3 H_2$ $\qquad$ $P_{31} = -G_5 H_3$.

④ Gain products of two non-touching loops



$P_{12} = (-G_2 H_1)(-G_5 H_3)$

$P_{12} = G_2 G_5 H_1 H_3$



$P_{22} = (-G_2 G_3 H_2)(-G_5 H_3)$

$P_{22} = G_2 G_3 G_5 H_2 H_3$

5) calculation of Δ & Δ_K:

$$\Delta = 1 - (P_{11} + P_{21} + P_{31}) + (P_{12} + P_{22})$$

$$= 1 - [-G_2H_1 - G_2G_3H_2 - G_5H_3] + [G_2G_5H_1H_3 + G_3G_2G_5H_2H_3]$$

$$= 1 + G_2H_1 + G_2G_3H_2 + G_5H_3 + G_2G_5H_1H_3 + G_3G_2G_5H_2H_3.$$

$\Delta_1 = 1$, there is no loop which is not in contact with the 1st forward path.

$\Delta_2 = 1 - $ (loop gain that is not in contact with 2nd forward path)

$$= 1 - P_{11}$$

$$= 1 - (-G_2H_1)$$

$$= 1 + G_2H_1.$$

$$T(s) = \frac{1}{\Delta} \sum_K P_K \Delta_K = \frac{1}{\Delta} \sum_2 P_K \Delta_K.$$

$$= \frac{1}{\Delta}[P_1\Delta_1 + P_2\Delta_2]$$

$$= \frac{1[(G_1G_2G_3G_4G_5)1 + G_4G_5G_6(1 + G_2H_1)]}{1 + G_2H_1 + G_2G_3H_2 + G_5H_3 + G_2G_5H_1H_3 + G_3G_2G_5H_2H_3}$$

$$\boxed{T(s) = \frac{G_4 G_5 \left[G_1 G_2 G_3 + G_6 + G_2 G_6 H_1 \right]}{1 + G_2 H_1 + G_2 G_3 H_2 + G_5 H_3 + G_2 G_5 H_1 H_3 + G_2 G_3 G_5 H_2 H_3}}$$