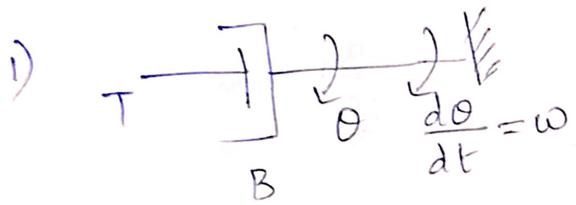
Electrical analogous of Mechanical Rotational systems.

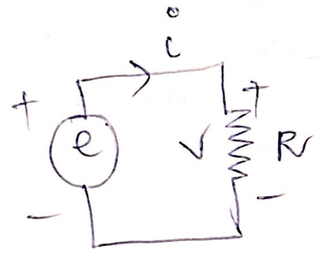
(1) Torque - Voltage Analogy.

Mechanical Rotational s/m.	Electrical s/m.
i/p : Torque	i/p : Voltage
o/p : Angular velocity	o/p : current.

1)



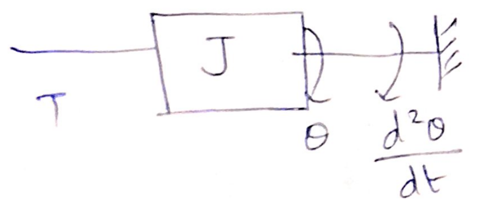
$$T = B\frac{d\theta}{dt} = B\omega$$



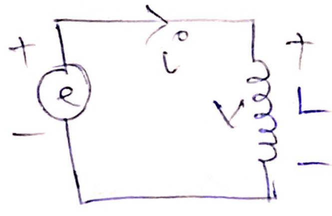
$$V = e = iR$$

$$i = V/R.$$

2)



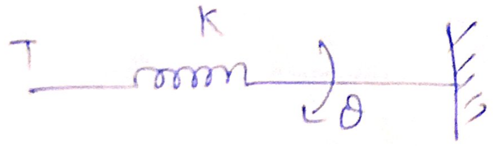
$$T = J\frac{d^2\theta}{dt^2} = J\frac{d\omega}{dt}$$



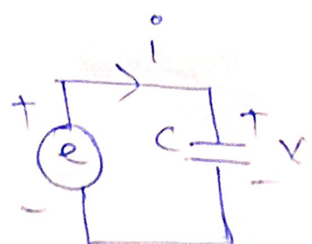
$$V = e = L\,di/dt$$

$$i = 1/L \int V\,dt$$

3)



$$T = K\theta = K\int \omega\,dt.$$



$$V = e = 1/C \int i\,dt$$

$$i = C(dV/dt)$$

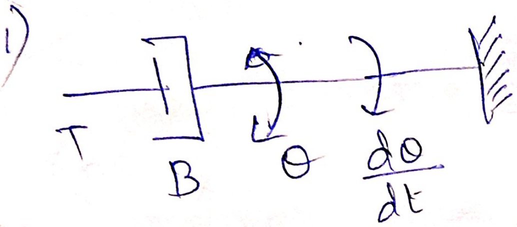
→ Torque - Voltage Analogy - Quantities

Item	Mechanical Rotational s/m	Electrical s/m.
Independent variable (i/p)	Torque, T	voltage (e, V)
Dependent variable (o/p)	Angular velocity (ω)	current (i)
	Angular displacement (θ)	charge (q)
Dissipative element	Rotational co-eff of dash-pot (B)	Resistance (R)
Storage element	Moment of Inertia (J)	Inductance (L)
	Stiffness of spring (K)	Inverse of capacitance ($1/C$)
Physical law	Newton's second law $\Sigma T = 0$.	KVL $\Sigma V = 0$.

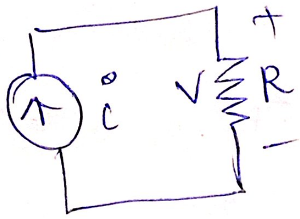
2) Torque - current Analogy.

Mechanical rotational s/m	Electrical s/m
i/p: Torque	I/p: current
o/p: Angular velocity (ω)	o/p: voltage.

1)



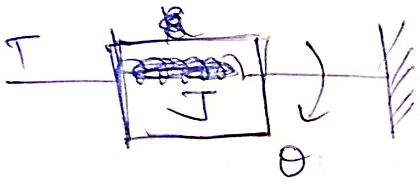
$$T = B\frac{d\theta}{dt} = B\omega$$



$$V = iR.$$

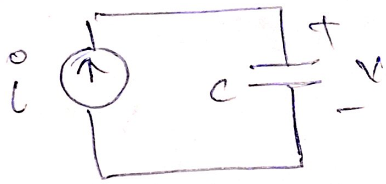
$$i = V/R$$

2)



$$T = J\frac{d^2\theta}{dt^2}.$$

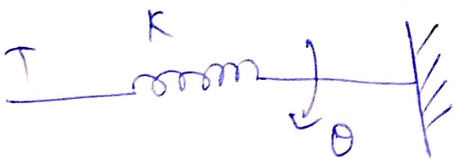
$$T = J\frac{d\omega}{dt}$$



$$i = C\frac{dV}{dt}.$$

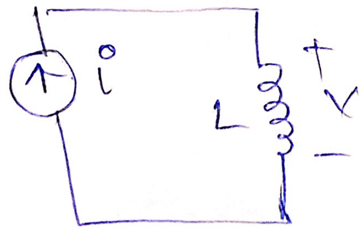
$$V = \frac{1}{C}\int i\,dt.$$

3)



$$T = K\theta$$

$$= K\int \omega\, dt.$$



$$V = L\frac{di}{dt}$$

$$i = 1/L \int V dt$$

→ Analogous Quantities in Torque – current Analogy

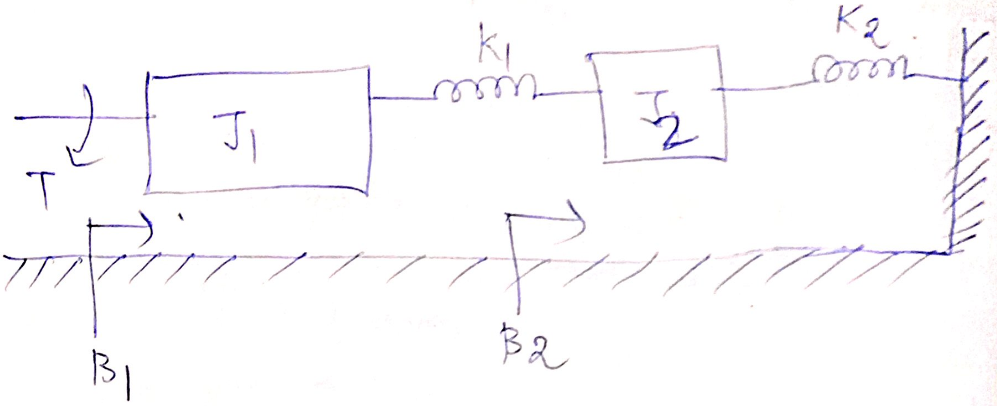
Item	Mechanical Rotational S/m	Electrical S/m.
Independent variable. (i/p)	Torque (T)	current (i)
Dependent variable. (o/p)	Angular velocity (ω)	voltage (v)
	Angular displacement (θ)	flux (φ)
Dissipative element.	Dashpot (B)	conductance (G) $= 1/R$.
Storage element	Moment of Inertia (J)	capacitance (C)
	Stiffness of spring (K)	Inverse of inductance ($1/L$)
Physical law	Newton's II law $\Sigma T = 0$.	KCL $\Sigma i = 0$.

comparison

M-R S/m	F-V	F-C
T (i/p)	V	i
ω (o/p)	i	V
θ	q	φ
B	R	$1/R$
J	L	C
K	$1/C$	$1/L$

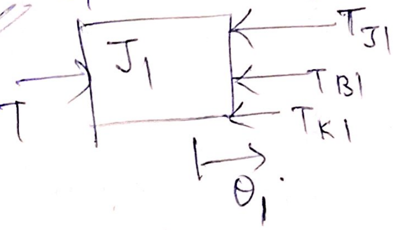
eg:

1) Write the differential equation of the Mechanical Rotational s/m, Draw T-V & T-C analogous circuits and verify using mesh and node equations.



Solution:

J_1 – free body diagram

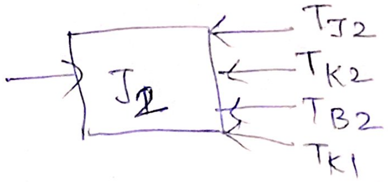


$$T_{J1} = J_1 \frac{d^2\theta_1}{dt^2}$$

$$T_{B1} = B_1 \frac{d\theta_1}{dt}$$

$$T_{K1} = K_1(\theta_1 - \theta_2)$$

$J_2 \rightarrow$ free-body diagram



$$T_{J2} = J_2 \frac{d^2\theta_2}{dt^2}$$

$$T_{K_2} = K_2 \theta_2$$

$$T_{B2} = B_2 \frac{d\theta_2}{dt}$$

$$T_{K1} = K_1(\theta_2 - \theta_1)$$

$$① \Rightarrow J_1 \frac{d^2\theta_1}{dt^2} + B_1 \frac{d\theta_1}{dt} + K_1(\theta_1 - \theta_2) = T$$

$$② = J_2 \frac{d^2\theta_2}{dt^2} + B_2 \frac{d\theta_2}{dt} + K_2\theta_2 + K_1(\theta_2 - \theta_1) = 0.$$

→ Rewrite everything in terms of angular velocity.

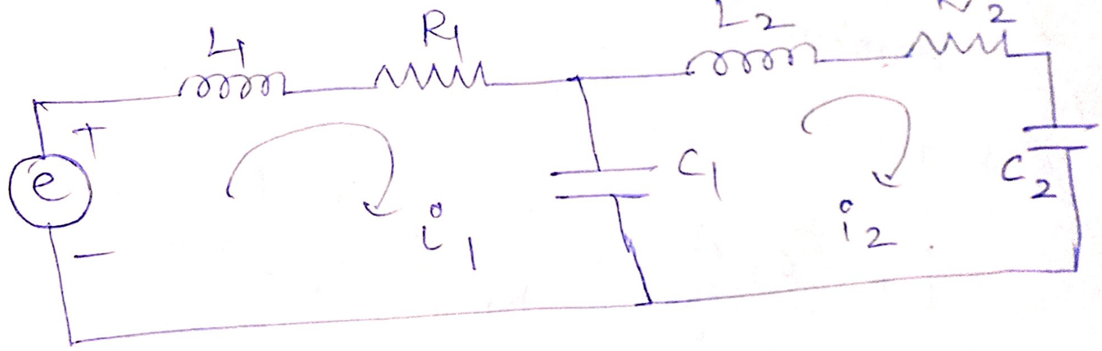
wkt $\theta = \int \omega \, dt$ $\quad \frac{d^2\theta}{dt^2} = \frac{d\omega}{dt}$ $\quad \frac{d\theta}{dt} = \omega$

$$\therefore \boxed{\begin{aligned} & J_1 \frac{d\omega_1}{dt} + B_1\omega_1 + K_1 \int (\omega_1 - \omega_2)\, dt = T \quad (3) \\ & J_2 \frac{d\omega}{dt} + B_2 \omega_2 + K_2 \int \omega_2 \, dt + K_1 \int (\omega_2 - \omega_1) dt = 0 \quad (4) \end{aligned}}$$

(i) – T–V . $J_1 \to L_1$, $T \to V$ or e, $J_2 \to L_2$

$B_1 \to R_1$, $B_2 \to R_2$, $K_1 \to 1/C_1$

$K_2 \to 1/C_2$ & $\omega_1 = i_1$, $\omega_2 = i_2$.



KVL

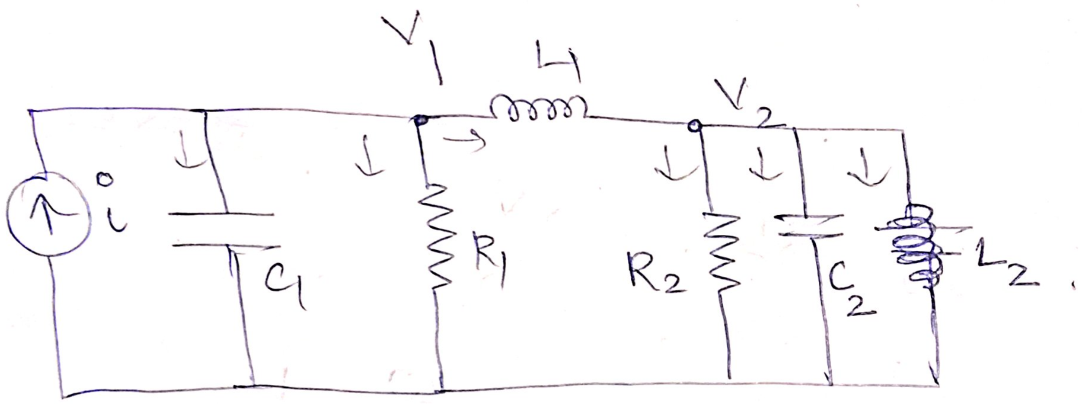
$$\boxed{\begin{aligned} & L_1 \frac{di_1}{dt} + R_1 i_1 + \frac{1}{C_1} \int (i_1 - i_2)\, dt = e \quad (5) \\ & L_2 \frac{di_2}{dt} + i_2 R_2 + \frac{1}{C_2} \int i_2 \, dt + \frac{1}{C_1} \int (i_2 - i_1) dt = 0. \quad (6) \end{aligned}}$$

(ii) T-C Analogy

$J_1 \rightarrow C_1$ $\quad B_1 = 1/R_1$ $\quad K_1 = 1/L_1$

$J_2 \rightarrow C_2$ $\quad B_2 = 1/R_2$ $\quad K_2 = 1/L_2$

$T \rightarrow i$ $\quad \omega_1 \rightarrow V_1.$ $\quad \omega_2 \rightarrow V_2.$



node equations

KCL

$$i = C_1 \frac{dV_1}{dt} + \frac{V_1}{R_1} + \frac{1}{L_1}\int (V_1 - V_2)\,dt \quad \text{—— (7)}$$

$$\frac{1}{L_1}\int (V_1 - V_2)\,dt = \frac{V_2}{R_2} + C_2 \frac{dV_2}{dt} + \frac{1}{L_2}\int V_2\,dt.$$

$$C_2 \frac{dV_2}{dt} + \frac{V_2}{R_2} + \frac{1}{L_2}\int V_2\,dt + \frac{1}{L_1}\int (V_2 - V_1)\,dt = 0 \quad \text{—— (8)}$$