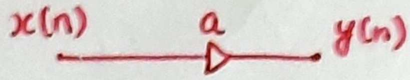
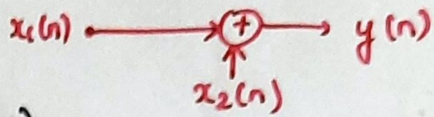
STRUCTURES FOR REALIZING FIR FILTERS:

BASIC ELEMENTS:

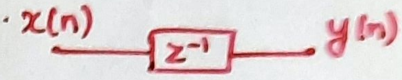
1) Multipliers ie $y(n) = a \cdot x(n)$



2) Adder ie $y(n) = x_1(n) + x_2(n)$



3) unit - delay element ie $y(n) = x(n-1)$



4) unit - advance element ie $y(n) = x(n+1)$.

TYPES OF STRUCTURES:

1. Direct form Realization
2. cascade realization

} Transversal Structure

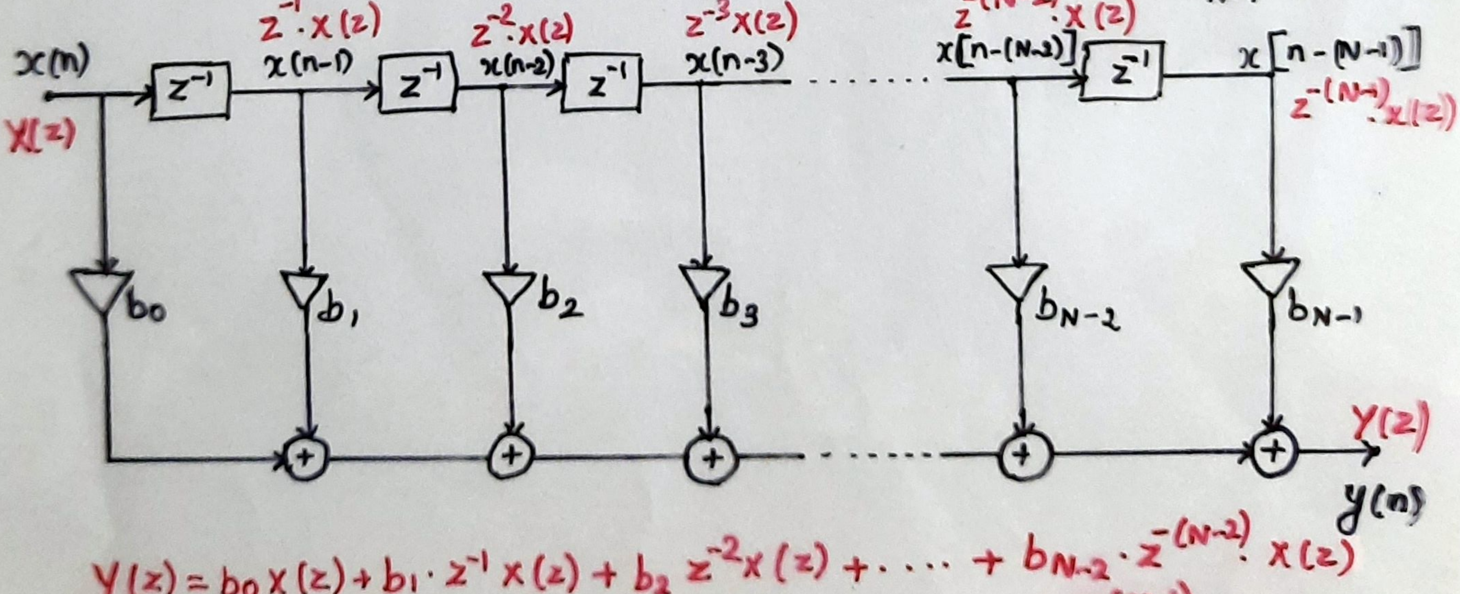
3. Linear phase Realization.

N^{th} order linear Difference equation for FIR:

$$y(n) = \sum_{k=0}^{N-1} b_k \, x(n-k)$$

Direct Form Structure:

$$y(n) = b_0 x(n) + b_1 x(n-1) + b_2 x(n-2) + b_3 x(n-3) \dots + b_{N-2} x[n-(N-2)] + b_{N-1} x[n-(N-1)].$$



$$Y(z) = b_0 X(z) + b_1 \cdot z^{-1} X(z) + b_2 z^{-2} X(z) + \dots + b_{N-2} \cdot z^{-(N-2)} \cdot X(z) + b_{N-1} \cdot z^{-(N-1)} \cdot X(z)$$

* Draw the direct form structure of the FIR system described by the transfer function $H(z) = 1 + \frac{1}{2}z^{-1} + \frac{3}{4}z^{-2} + \frac{1}{4}z^{-3} + \frac{1}{2}z^{-4} + \frac{1}{8}z^{-5}$.

Ex Pbm:1

Homework

q.

$$\frac{Y(z)}{X(z)} = 1 + \frac{1}{2}z^{-1} + \frac{3}{4}z^{-2} + \frac{1}{4}z^{-3} + \frac{1}{2}z^{-4} + \frac{1}{8}z^{-5}$$

$$Y(z) = X(z) + \frac{1}{2}z^{-1}X(z) + \frac{3}{4}z^{-2}X(z) + \frac{1}{4}z^{-3}X(z) + \frac{1}{4}z^{-4}X(z) + \frac{1}{8}z^{-5}.X(z)$$

$b_0 = 1$; $b_1 = \frac{1}{2}$; $b_2 = \frac{3}{4}$; $b_3 = \frac{1}{4}$; $b_4 = \frac{1}{4}$; $b_5 = \frac{1}{8}$.

* **CASCADE REALIZATION:**

$$y(n) = \sum_{k=0}^{N-1} b_k\, x(n-k)$$

$$Y(z) = \sum_{k=0}^{N-1} b_k \cdot z^{-k} \cdot X(z). \Rightarrow \frac{Y(z)}{X(z)} = \sum_{k=0}^{N-1} b_k \cdot z^{-k}$$

$$H(z) = \sum_{k=0}^{N-1} b_k z^{-k}$$

N → ODD: (i) $H(z)$ is divided into $\left(\frac{N-1}{2}\right)$ second order factors.

(ii) draw direct form structure for each second order factor. ie $H_1(z)$. $H_2(z)$ ....

(iii) Connect all the second order systems in cascade form.

$$X(z) \rightarrow \boxed{H_1(z)} \rightarrow \boxed{H_2(z)} \rightarrow \cdots \rightarrow \boxed{H_{\frac{N-1}{2}}(z)} \rightarrow Y(z)$$

$$H(z) = \sum_{k=0}^{(N-1)} b_k z^{-k} = \prod_{i=1}^{\frac{(N-1)}{2}} \left(C_{0i} + C_{1i} z^{-1} + C_{2i} z^{-2}\right)$$

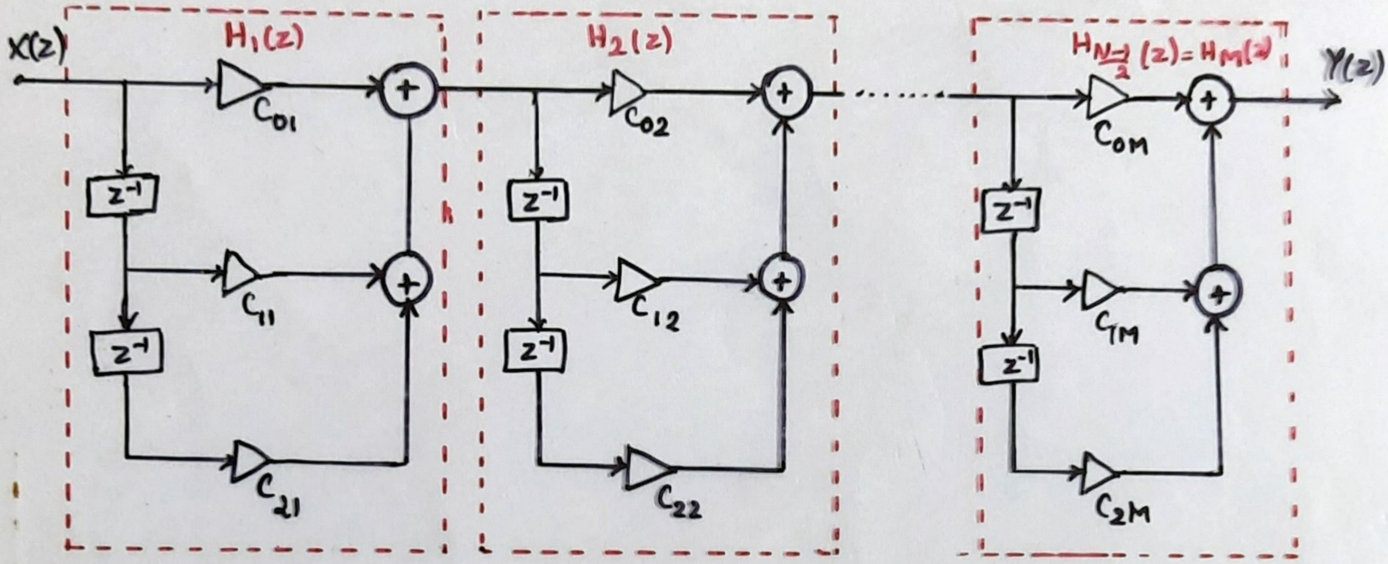
$$= H_1(z).\, H_2(z) \ldots H_{\frac{N-1}{2}}(z)$$

N → EVEN: $H(z)$ is divided into one first order factor and $\frac{N-2}{2}$ second order factors.

$$H(z) = \sum_{k=0}^{N-1} b_k \cdot z^{-k} = \left(c_{01} + c_{11} z^{-1}\right) \prod_{i=2}^{N/2} \left(c_{0i} + c_{1i} z^{-1} + c_{2i} z^{-2}\right)$$

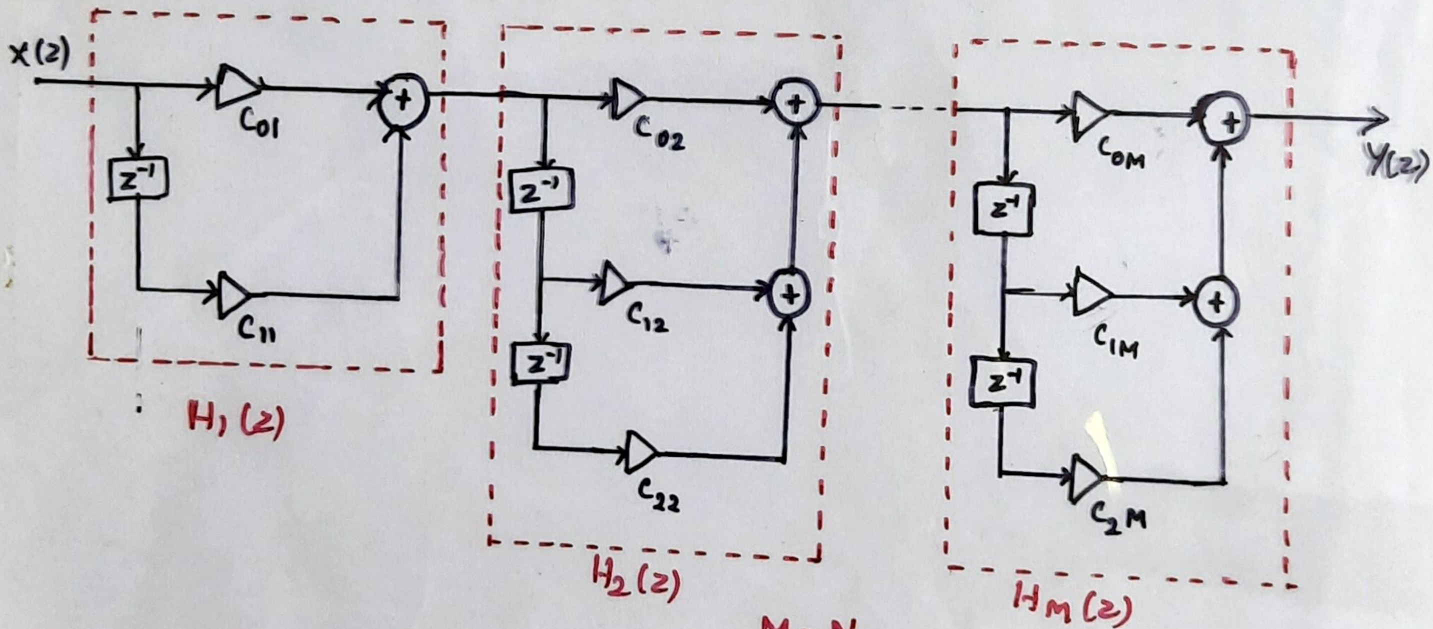
$$= H_1(z) \cdot H_2(z) \cdot H_3(z) \cdots H_{N/2}(z)$$

N→ODD:



$M = \frac{N-1}{2}$

N→EVEN



$M = \frac{N}{2}$

Obtain the cascade form realization of system function $H(z) = \underbrace{(1 + 2z^{-1} - z^{-2})}_{H_1(z)} \underbrace{(1 + z^{-1} - z^{-2})}_{H_2(z)}$

Homework 10

$H_1(z) \Rightarrow c_{01} = 1$; $c_{11} = 2$; $c_{21} = -1$

$c_{02} = 1$; $c_{12} = 1$; $c_{22} = -1$

LINEAR PHASE REALIZATION:

* TO REDUCE NUMBER OF MULTIPLIERS

For a linear phase FIR filter, $h(n) = h(N-1-n)$

$$H(z) = Z\{h(n)\} = \sum_{n=0}^{N-1} h(n) z^{-n}$$

N → even: $$H(z) = \sum_{n=0}^{\frac{N-2}{2}} h(n) z^{-n} + \sum_{n=N/2}^{N-1} h(n) z^{-n}$$

$m = N-1-n \; ; \quad n = N-1-m$

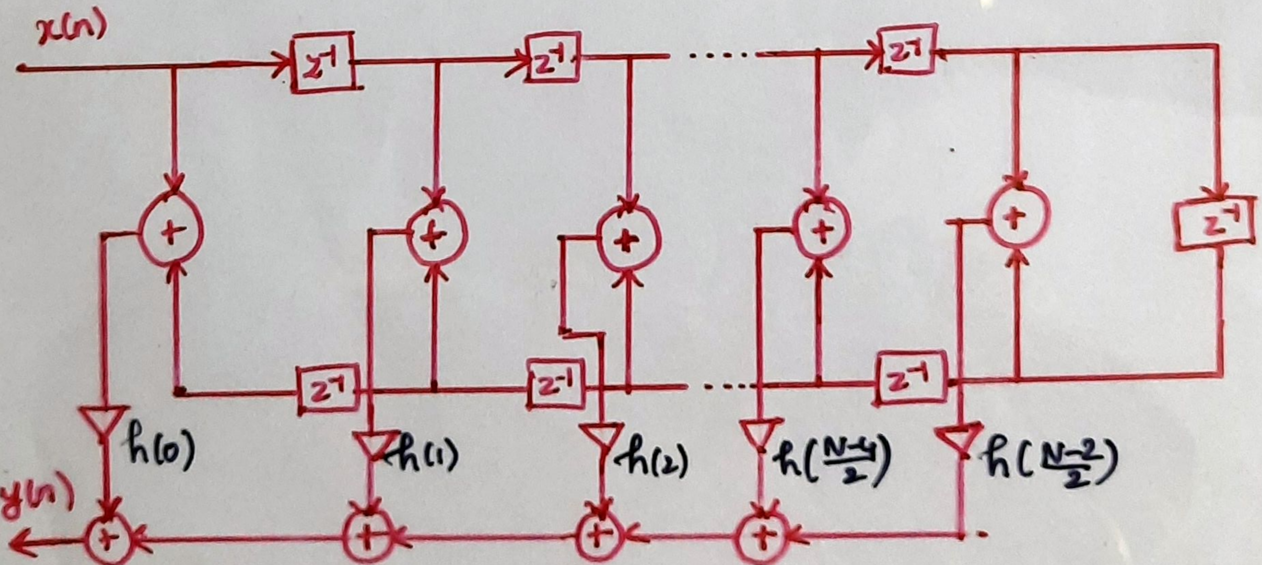
if $n = N/2$, $\quad m = N-1-N/2 = N/2 - 1 = \frac{N-2}{2}$

if $n = N-1$, $\quad m = N-1-(N-1) = 0$

$$H(z) = \sum_{n=0}^{\frac{N-2}{2}} h(n) z^{-n} + \sum_{m=0}^{\frac{N-2}{2}} h(N-1-m) . z^{-(N-1-m)}$$

$$= \sum_{n=0}^{\frac{N-2}{2}} h(n) z^{-n} + \sum_{n=0}^{\frac{N-2}{2}} h(n) . z^{-(N-1-n)}$$

$$= \sum_{n=0}^{\frac{N-2}{2}} h(n) \left[z^{-n} + z^{-(N-1-n)} \right]$$



N → ODD:

$$H(z) = \sum_{n=0}^{N-1} h(n)\, z^{-n}$$

$$= \sum_{n=0}^{(N-3)/2} h(n) \cdot z^{-n} + \sum_{n=\frac{N-1}{2}} h(n) \cdot z^{-n} + \sum_{n=\frac{N+1}{2}}^{N-1} h(n) \cdot z^{-n}$$

Let $m = N-1-n$; $n = N-1-m$

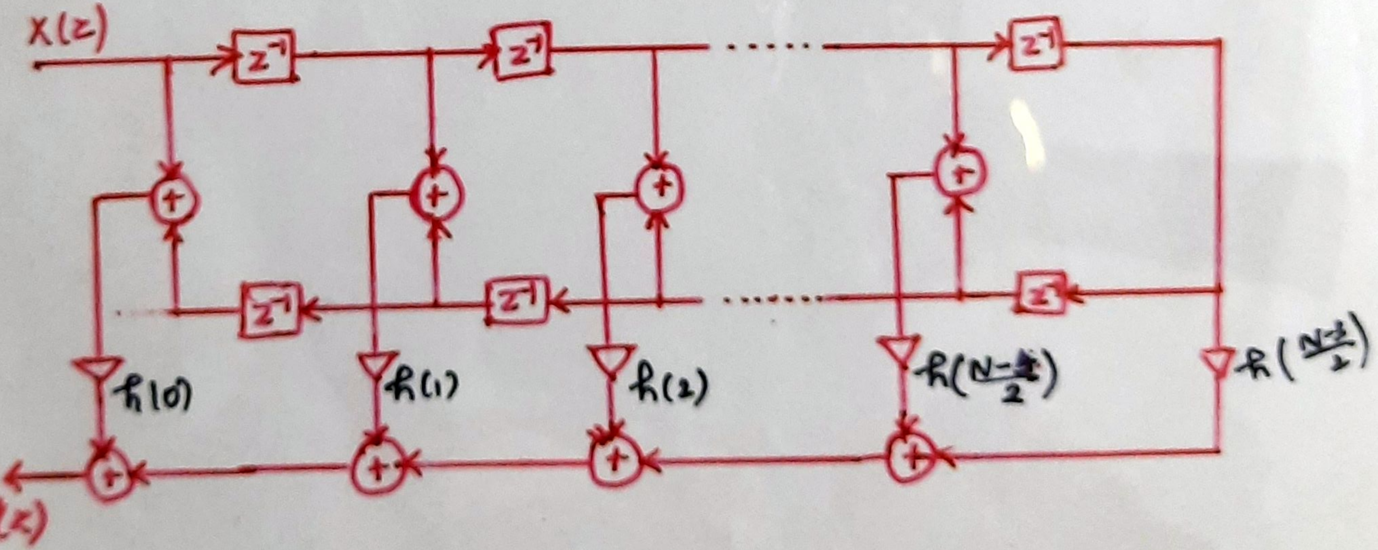
If $n = \frac{N+1}{2}$; $m = N-1-\frac{N+1}{2} = \frac{N-3}{2}$

If $n = N-1$; $m = N-1-(N-1) = 0$

$$H(z) = \sum_{n=0}^{(N-3)/2} h(n) \cdot z^{-n} + h\left(\frac{N-1}{2}\right) \cdot z^{-\left(\frac{N-1}{2}\right)} + \sum_{m=0}^{(N-3)/2} h(N-1-m) \cdot z^{-(N-1-m)}$$

$$H(z) = \quad " \quad + \quad " \quad + \sum_{n=0}^{(N-3)/2} h(n) \cdot z^{-(N-1-n)}$$

$$H(z) = h\left(\frac{N-1}{2}\right) \cdot z^{-\left(\frac{N-1}{2}\right)} + \sum_{n=0}^{(N-3)/2} h(n) \left[z^{-n} + z^{-(N-1-n)}\right]$$



P6) Realize the system function

$$H(z) = \tfrac{1}{2} + \tfrac{1}{3} z^{-1} + z^{-2} + \tfrac{1}{4} z^{-3} + z^{-4} + \tfrac{1}{3} z^{-5} + \tfrac{1}{2} z^{-6}$$

Home work 11

$h(0) = \frac{1}{2}$; $h(1) = \frac{1}{3}$; $h(2) = 1$; $h(3) = \frac{1}{4}$

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1) Obtain Cascade realization with minimum number of multipliers for the system function

Ex: Homework 12

$$H(z) = \left(\tfrac{1}{2} + z^{-1} + \tfrac{1}{2} z^{-2}\right)\left(1 + \tfrac{1}{3} z^{-1} + z^{-2}\right)$$

$\Downarrow$ linear $\qquad$ cascade $\qquad$ $\Downarrow$ linear phase structure

Soln: $h(0) = \frac{1}{2}$; $h(1) = 1$ $\Rightarrow H_1(z)$; $h(0) = 1$; $h(1) = \frac{1}{3}$ $\Rightarrow H_2(z)$

2) Realize the following system function using minimum number of multipliers

Homework 12

(i) $H(z) = 1 + \frac{1}{3} z^{-1} + \frac{1}{4} z^{-2} + \frac{1}{4} z^{-3} + \frac{1}{3} z^{-4} + z^{-5}$

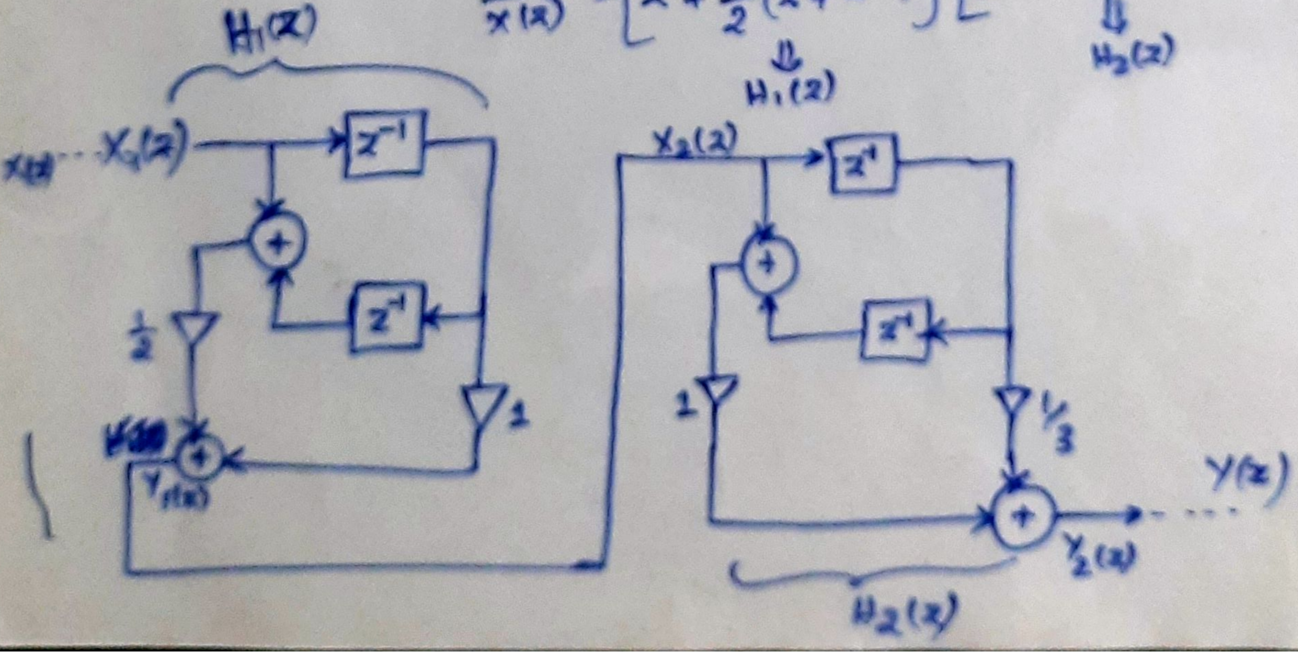
(ii) $H(z) = (1 + z^{-1})\left(1 + \frac{1}{2} z^{-1} + \frac{1}{2} z^{-2} + z^{-3}\right)$

$\Downarrow$ linear $\qquad$ cascade $\qquad$ $\Downarrow$ linear

Soln: $h(0) = 1$ in $H_1(z)$; $h(0) = 1$; $h(1) = \frac{1}{2}$ in $H_2(z)$

Soln: $H(z) = \left(\frac{1}{2} + z^{-1} + \frac{1}{2} z^{-2}\right)\left(1 + \frac{1}{3} z^{-1} + z^{-2}\right)$

$$\frac{Y(z)}{X(z)} = \underbrace{\left[z^{-1} + \frac{1}{2}\left(z^{0} + z^{-2}\right)\right]}_{H_1(z)} \underbrace{\left[\frac{1}{3} z^{-1} + 1 \cdot \left(z^{0} + z^{-2}\right)\right]}_{H_2(z)}$$



Comparison between FIR & IIR filters

FIR filter	IIR filter
1) The impulse response of this filter is restricted to finite number of samples.	1) The impulse response of this filter extends over an infinite duration.
2) These filters can be easily designed to have perfectly linear phase	2) These filters do not have linear phase.
3) FIR filters can be realized recursively and non-recursively.	3) IIR filters are easily realized recursively.
4) closed form design equations do not exist.	4) Closed form design equations are used to design variety of frequency selective filters.
5) Greater flexibility to control the shape of their magnitude response.	5) Less flexibility, usually limited to specific kind of filters.
6) These filters are always stable	6) These filters are not always stable.

FIR filter	IIR filter
7) In these filters, the poles are fixed at the origin, high selectivity can be achieved by using a relatively high order for transfer function.	7) The poles are placed anywhere inside the unit-circle, high selectivity can be achieved with low-order transfer function.
8) Errors due to round-off noise are less severe in FIR filters, mainly due to the absence of feedback.	8) IIR filters are more susceptible to errors due to round off noise.