DAY -10 - Summary and Key Takeaways - Hypothesis testing

HYPOTHESIS - Tentative explanation/Methodology used to prove or disprove a statement using statistical analysis.

Process of Hypothesis Testing -

Step -1 Formulate Hypothesis-

- NULL Hypothesis(H0) : What we are assuming. Always contains-"=","<=" or ">=". It is awlyas abt population parameter.
- Alternative Hypothesis(H1) : Opposite of H0. Never contains "=","<=" or ">=".

Step - 2. Select the appropriate test statistics.

- Two -Tail Test - Used when there is a rejection region in both tails.

z-test	**used when *σ is known***	**$z = (X - \mu) / [\ \sigma/\sqrt{(n)}\ ].$**
t-test	**used when *σ is not known***	**$t = (X - \mu) / [\ s/\sqrt{(n)}\ ].$**

- One - Tail Test - Used when there is a rejection region in either left tail or right tail.

Left tail test

- Contains ">=" in null hypothesis.
- Rejection region in left tail.

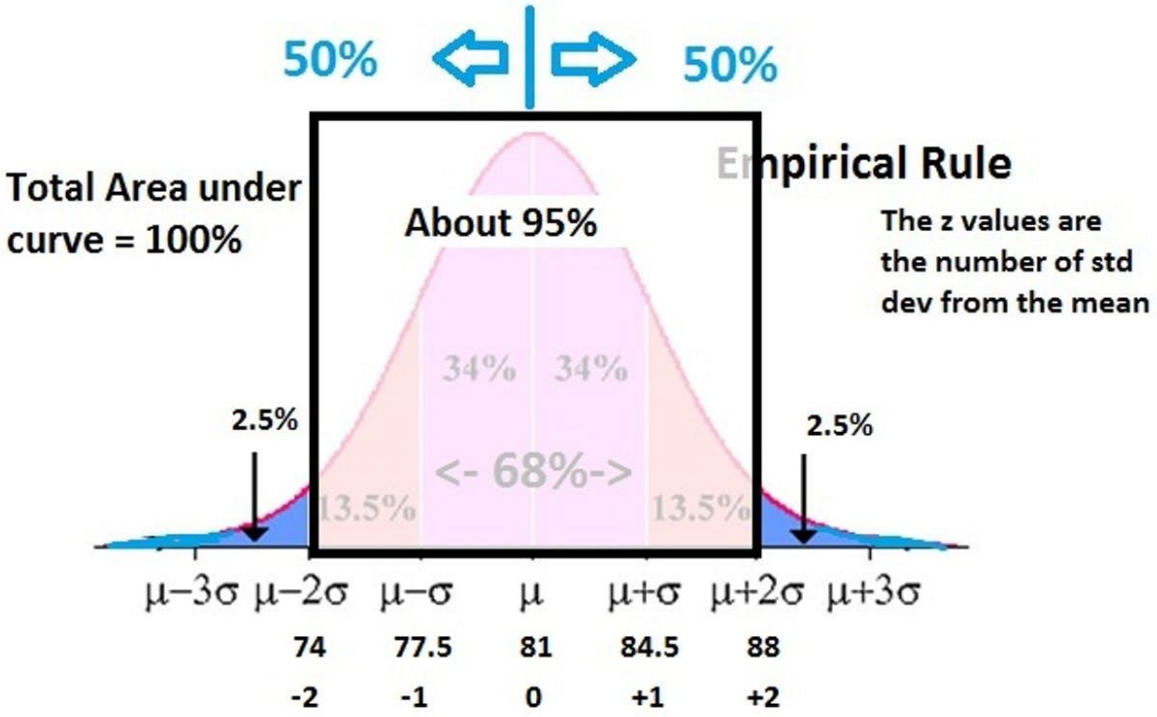
Right tail test

- Contains "<=" in null hypothesis.
- Rejection region in right tail.

Step-3. Choose the level of significance(**α),** confidence interval and degree of freedom

- **α:** error rate present in the sample. e.g - If you are doing a research on junior Horlicks and while testing the product, you consider a sample of 100 people out of which 95 are children and 5 are adults. So, there is 5% error in the sample. Thus, α = 0.05.
- **Confidence Interval** : range of lower and upper bounds of accepted region. There are 2 ways of finding this-

1. Empirical rule-



Thus according to the rule, approximately 95% of a normal distribution falls within 2 standard deviations of the mean.

e.g - Mean energy consumption in house is 200 and the std dev of sample taken is 20.

So, 68% of the people consume energy between (200-20)=180 to 220(200+20). Similarly, 95% consume between 160 and 240..

2. Formula -

C.I -- Upper value = mean + (Table value)*std dev , Lower value=mean - (Table value)*std dev

- **Degree of Freedom** - no. of rows- no of columns

In the above example, we have taken 200 samples of energy consumption i.e no. of rows=200 and no of column =1. So, d.f = 200-1=199.

Step-4 Compute the calculated test value of test statistic. (Using formula given in step -2)

Step 5 -Compute the table test value of test statistic. -(**α on right and d.f downwards)**

cum. prob	$t_{.50}$	$t_{.75}$	$t_{.80}$	$t_{.85}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$	$t_{.995}$	$t_{.999}$	$t_{.9995}$
one-tail	0.50	0.25	0.20	0.15	0.10	0.05	0.025	0.01	0.005	0.001	0.0005
two-tails	1.00	0.50	0.40	0.30	0.20	0.10	0.05	0.02	0.01	0.002	0.001
df											
1	0.000	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	318.31	636.62
2	0.000	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	0.000	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	0.000	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.000	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.000	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.000	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.000	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.000	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.000	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.000	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.000	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.000	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.000	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.000	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.000	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.000	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.000	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.000	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.000	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.000	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.000	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.000	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.000	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.000	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.000	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.000	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.000	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.000	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.000	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	0.000	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	0.000	0.679	0.848	1.045	1.296	1.671	2.000	2.390	2.660	3.232	3.460
80	0.000	0.678	0.846	1.043	1.292	1.664	1.990	2.374	2.639	3.195	3.416
100	0.000	0.677	0.845	1.042	1.290	1.660	1.984	2.364	2.626	3.174	3.390
1000	0.000	0.675	0.842	1.037	1.282	1.646	1.962	2.330	2.581	3.098	3.300
z	0.000	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	3.090	3.291
	0%	50%	60%	70%	80%	90%	95%	98%	99%	99.8%	99.9%
	Confidence Level										

Step-6- Compare the calculated and table values

Step-7 State the statistical decision

- If it is a two-tail test, if (- table value < calculated value < + table value) ---> It lies in accepted region and thus the null hypothesis is accepted. Else, H0 is rejected.
- If it is a one -tail test -

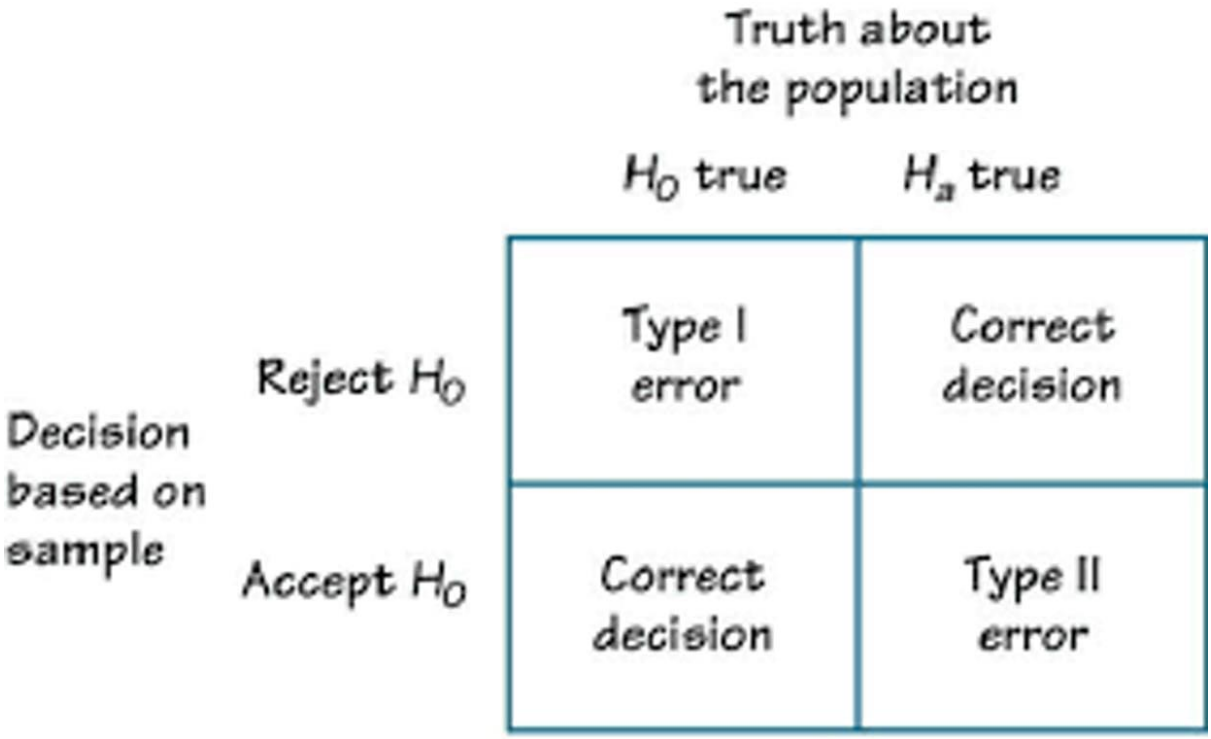
1. Left tail test -- (-table value < calculated value) --> It lies in accepted region and thus the null hypothesis is accepted. Else, H1 is accepted.
2. Right tail test -- (calculated value < + table value) --> It lies in accepted region and thus the null hypothesis is accepted. Else, H1 is accepted.

The above was a " critical value" approach for hypothesis testing.

"Probability " approach is the "p-value" approach-

- p-value < **α** --> Reject H0
- p-value >= **α** --> Accept H0

ERRORS IN HYPOTHESIS TESTING -



e.g - If H0 = A person does not have covid.

	H0 is true (No covid)	Ha is true (Covid)
Reject H0 (Diagnosed with covid)	Type -1 (**α**)	Correct Decision
Accept H0 (Not diagnosed with covid)	Correct decision	Type-2 (**β**)

The Type -2 error here is dangerous and irreparable. So, it can be controlled by increasing the value of **α** , decreases the value of **β.**

Example for hypothesis testing -

A hotel in NY costs $168 per night. A random sample of 25 hotels is taken with $\bar{x}$ = 172.50 and std. dev(s) of 15.40.

Step -1: Hypothesis formulation

H0 : μ =168

H1:μ ≠ 168

Step -2 : Deciding the Test

As " H0" contains "=", the rejection region lies in both tails. So, the "**Two Tail Test**". and as σ is unknown , **t-test** is chosen**.**

Step-3 : As **α is** not mentioned, **α** =0.05.

d.f = n-1 = 25-1=24.

Step-4 Calculate test value -

t = (X – μ) / [s/√(n)]. = (172.50 - 168)/[15.40/√25] = 1.46

Step -5 - Table test value -

cum. prob	$t_{.50}$	$t_{.75}$	$t_{.80}$	$t_{.85}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$	$t_{.995}$	$t_{.999}$	$t_{.9995}$
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	0%	50%	60%	70%	80%	90%	95%	98%	99%	99.8%	99.9%
	Confidence Level										

t table(0.05,24) = 2.064

Step - 6-Compare

-2.064 < 1.46 < 2.064

Step-7 - Conclusion

As the calculated value, lies in the accepted region, **The null hypothesis is accepted.**

Thus, **A hotel in NY costs $168 per night.** is proven.