The Quantitative Analytics of the Basic Neomonetarist Model

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A GREAT DEAL OF MACROECONOMIC RESEARCH in the last decade or two has been motivated by a lively debate over whether or not prices are sticky and whether and how much money matters. What are the lessons those who have not been active partisans can take away from this debate? I cannot claim true impartiality, because of my saltwater graduate school background on one side and my attraction to the elegance of the Real Business Cycle picture of an economy of optimizing agents responding to the fundamental economic forces of preferences and technology on the other. But like a bystander to the battle who moves among the dead and wounded to see what valuables he can gather, I have tried to collect things of worth from both sides. Thus, this paper is meant to stimulate discussion, not to be the final word on anything.

I will argue that a hybrid model should be taken much more seriously—a model following the Real Business Cycle paradigm as closely as possible except for adding what is logically necessary in order to graft in sticky prices. In order to avoid implicating either the New Keynesians or the New Classicals in my crimes, I will call such a hybrid model—grafting sticky prices into what would otherwise be a Real Business Cycle model—a "Neomonetarist" model. Such models are not unknown in the literature—for example, King (1991) and Cho and Cooley (1992) study models with both imperfect price flexibility and Real Business Cycle elements—but they have not been studied by as many people with as great intensity as other models. To justify more attention to such hybrid models, I must defend the assumption of sticky prices *and* defend the essence of the Real Business Cycle approach.

The main reason to be interested in sticky prices is that they provide one of the few means of generating large monetary nonneutralities. The basic evidence for the

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existence of large monetary nonneutralities is the evidence assembled by Friedman and Schwartz (1963), together with events after the end of their sample, such as the Volcker deflation. The details of the historical record discussed by Friedman and Schwartz (1963) have persuaded most macroeconomists that there has been a substantial exogenous component to the timing of monetary policy and sometimes even the direction of monetary policy based on the personalities and the inner workings of the Federal Open Market Committee and the Federal Reserve System more generally. The most exogenous shifts in monetary policy seem to have the clearest effects. Such historical evidence for the potency of monetary policy is also available for a host of other countries as well.

Despite considerable effort, there has been little success at generating large monetary nonneutralities in flexible-price models using plausible parameters for anything short of a hyperinflationary situation. The basic problem is that, if prices are perfectly flexible, the monetary services subsector (which is a small part of the financial services sector) looks a lot like any other type of service subsector in the economy. Short of massive bank failures or hyperinflation, the fraction of a percent share monetary services have in GDP[1] is simply too small for what happens to this sector to be a big deal if prices are perfectly flexible. (During hyperinflations, monetary services as a share of GDP do become substantial, sometimes rising above 10 percent of GDP.) By contrast, if prices are sticky, what happens in the monetary services sector can distort the decisions of firms in all sectors of the economy that have sticky prices.[2]

Real business cycle research has sometimes (though not always) been pursued in opposition to the idea of sticky prices, but this approach has many important strengths even if one is willing to entertain the idea that prices are sticky. Real Business Cycle research has consistently taken dynamics, sensible anticipation of the future, quantitative calibration and the welfare implications of models very seriously. As a result, even simple real business cycle models are rich in quantitative dynamic implications. The agreement on the basic real business cycle model of Prescott (1986) as a benchmark allows researchers to compare their results and communicate in a productive way as they modify one or two features of the basic model. Explicit, disciplined modeling opens the possibility of unexpected results.

Real business cycle models may be oversimplified, but it is hard to dismiss the forces identified in such models as important contributors to what is going on. Firms may not always minimize costs, but a tendency toward cost minimization surely governs much of what happens in the economy. All households may not be rationally forward looking, but an important fraction of households surely do condition

1. The amount paid for monetary services in proportion to GDP is equal to the rate of foregone interest times $M/PY = 1/V$. An M1 velocity of 6, coupled with an average foregone interest of 5 percent for the components of M1 would lead to a share of less than 1 percent ($5/6$ percent in this case) for monetary services in GDP.

2. The literature does give some examples of theoretical models that have large monetary nonneutralities with perfectly flexible prices and no information problems, but these are models with multiple equilibria or very fragile equilibria in which small costs of price adjustment would have an even easier time generating large monetary nonneutralities.

their decisions in important ways on expectations about the future. Households may not be on their labor supply curves at every instant, but they surely resist fluctuating too far off their labor supply curves for too long. Thus, it seems folly to ignore the economic forces highlighted in real business cycle models even if one wishes to add other mechanisms to a model as well. It is particularly difficult to think about investment in a sensible way without borrowing some of the rational expectations apparatus.

In addition to presenting a hybrid model, I will try to hint at a joint research program for New Keynesians and real business cyclists. In both approaches, the key to explaining economic fluctuations is constructing a model with plausible parameters that allows for a strong response to shocks. In both approaches the key model elements generating responsiveness of the economy to shocks are things like favorably shaped firm demand and cost curves, positive externalities, and a fairly large macroeconomic elasticity of labor supply. The challenge of adequately justifying such model elements provides plenty of scope for a joint research agenda.

1. THE MONETARY AND PRICE-ADJUSTMENT SIDE OF THE NEOMONETARIST MODEL

In this model I will use two kinds of approximations in order to focus on the most important effects and to have a model that is easy to understand. First, I will make a certainty-equivalence approximation throughout, and focus on linearizing or log-linearizing around steady-state values.[3] I will use an asterisk (*) to denote a steady-state value, a tilde (˜) to denote the deviation of a variable from its steady-state value, and a haček (ˇ) to denote the *logarithmic* deviation of a variable from its steady-state value.

Second, I will look at a Taylor expansion based on the typical length of time a price is fixed being relatively short.[4] This greatly simplifies many expressions in a very intuitive way. In fact, this fast-price-adjustment approximation is a formal way to justify something akin to the classical dichotomy—a way to justify separating slower-moving growth and fluctuations induced by real factors from higher-frequency fluctuations induced by monetary factors. I will call the aspects of the model that determine its short-run fluctuations in response to monetary factors the "monetary and price adjustment" side of the neomonetarist model. In the light of this approximate classical dichotomy, I will call the other side of the model based on

3. Although this is adequate for a first pass at studying the responses of variables to shocks in the Neomonetarist model, it precludes a serious welfare analysis of changes that affect the variance of output. The fact that under imperfect competition more output is typically beneficial socially means that the influence of departures from certainty equivalence on mean output matters for such a welfare analysis. For instance, it is not inconceivable that more variable output could induce enough extra saving through precautionary effects that the resulting larger mean capital stock and higher mean output could more than compensate for the direct cost of higher variance on welfare.

4. One advantage of working in continuous time is that it highlights the question of how long a price is likely to be fixed. In discrete time models it is all too tempting to dodge this issue by assuming that prices are set for "one period" without any serious thought given to whether "one period" is in fact the likely length of time for which a price is fixed.

real relationships that would hold among symmetrically acting firms and households the "classical" side of the model.

1.1. Money Demand and Supply

With velocity V assumed exogenous in the monetarist spirit, the identity

$$MV = PY$$

gets some real bite. M, of course, is the money supply, Y is aggregate output, and P is the aggregate price level. This equation determines aggregate output as

$$Y = \frac{MV}{P} ,$$

or in logarithmic deviations,

$$\check{Y} = \check{M} + \check{V} - \check{P} . \tag{1}$$

I assume that the share of money services in GDP is negligible, so that the money market only really matters because of its interaction with sticky prices. This is a reasonable assumption in non-hyperinflationary situations.

At this point, we already have an aggregate demand curve, (1), and a short-run aggregate supply curve, which is horizontal at the price P. The point of this paper is to derive the dynamics of these curves and to study the action going on behind the scenes of the aggregate supply–aggregate demand graph.

1.2. Price Adjustment Dynamics

I will use Calvo's (1982) model of price adjustment in continuous time. The biggest single reason for doing so is that here, in a model that has other important complicating features, it seems wise to focus on the time-dependent setting of fixed prices for the sake of tractability.

Let me at least explain why I emphasize sticky prices rather than sticky wages. The less important reason is that wage stickiness tends to yield counter-cyclical real wages that are not apparent in the data. A deeper and much more important reason for the focus on price stickiness as opposed to wage stickiness is the view I share with many other macroeconomists (both New Keynesian and New Classical) that most employment takes place as part of a long-term implicit contract in which the wage observed at any given time is only an installment payment on what the implicit contract says is due to a worker. Therefore, *the observed wage does not accurately represent a firm's labor costs*. True marginal labor costs are a matter of the additional amount a firm is implicitly promising to pay a worker *someday* in return for working an additional hour. Given long-term implicit contracts, sticky-looking wages are a convenience for both workers and firms and can even have an insurance role to play, but they have little bearing on true labor costs. Traditional models of sticky wages require not just sticky observed wages, which are easy to justify, but sticky labor costs, which are very difficult to justify.

The Evolution of the Aggregate Price Level. In Calvo's (1982) model, each firm gets the chance to consider and adjust its price at an interval determined by a Poisson process, with the chance to adjust the firm's price arriving with probability α per unit time. Because all firms are identical *except for the fixed prices they have at the moment,* each firm that gets the chance to change its price at a given time will choose the same new price, which I will call the *optimal reset price* and denote $\wp_t$.

Firms that have fixed prices and can only change them at idiosyncratic random intervals will unavoidably end up with *different* prices whenever there are any fluctuations. The appropriate global aggregate for these different prices can be quite complex. But because of the underlying symmetry between the various firms, even a complex price aggregate is equal, to a first-order approximation, to a simple, unweighted arithmetic average (and to a simple, unweighted geometric average) of all the prices. The interaction of a firm's change in its price and any departure of the quantity weight for that price from symmetry with the quantity weights for all of the other firms' prices is a second-order effect. Thus,

$$\dot{\tilde{P}}_t = \alpha[\tilde{\wp}_t - \tilde{P}_t] \ , \tag{2}$$

since price changes take place at the Poisson rate α from various prices averaging P_t to the optimal reset price $\wp_t$. If the initial steady state has zero inflation,[5] then $\wp^* = P^*$, and all of the deviations in (2) can be converted to logarithmic deviations by dividing (2) through by P^*:

$$\dot{\check{P}}_t = \alpha[\check{\wp}_t - \check{P}_t] \ . \tag{3}$$

The Optimal Reset Price. Given the chance, a firm chooses its reset price $\wp$ at time t by maximizing the expected present value of profits:

$$\max_{\wp_t} \ \mathrm{E}_t \int_t^{\infty} e^{-\int_t^{t'} (r_{t''}+\alpha)\,dt''}\, \Pi\left(\frac{\wp_t}{P_{t'}}\,,\, t'\right) dt' \ ,$$

where E_t is an expectation over all the events *other than* the arrival of the chance to reset one's price, conditional on information at time t, $r_{t''}$ is the real interest rate at time t'', and Π is the flow of real profits as a function of the firm's price relative to the aggregate price at time t'. The dependence of real profits on everything else is represented by the second (time) argument in Π. The profits are discounted by α as well as r because the current reset price will last for a length of time $t' - t$ only with probability $e^{-\alpha(t'-t)}$.

5. Trend inflation could result in interactions between price changes and asymmetric quantity weights which are technically first order, but these effects would vanish as $\alpha \to \infty$.

Using Π' to denote the derivative of Π with respect to its first (relative price) argument, the first-order condition for the optimal reset price is

$$0 = \mathrm{E}_t \int_t^\infty e^{-\int_t^{t'} (r_{t''}+\alpha) dt''} \frac{\Pi'\left(\frac{\wp_t}{P_{t'}}, t'\right)}{P_{t'}} dt' . \tag{4}$$

The certainty-equivalence approximation makes the impulse-response functions the same as in a perfect-foresight model, so I will drop the expectation sign E_t. Linearizing marginal profits around a relative price of 1,

$$\Pi'\left(\frac{\wp_t}{P_{t'}}, t'\right) \approx \Pi'(1, t') + \Pi''(1, t') \left[\frac{\wp_t}{P_{t'}} - 1\right] . \tag{5}$$

It is helpful to define the "wished for" or "desired" price $p_{t'}^{\#}$—the price that would be optimal at a particular instant if the firm could costlessly choose a price for just that instant without affecting what its price would be at any other time. The desired price $p_{t'}^{\#}$ satisfies the instantaneous first-order condition

$$0 = \Pi'\left(\frac{p_{t'}^{\#}}{P_{t'}}, t'\right) \approx \Pi'(1, t') + \Pi''(1, t') \left[\frac{p_{t'}^{\#}}{P_{t'}} - 1\right] . \tag{6}$$

Subtracting (6) from (5) yields the approximation

$$\Pi'\left(\frac{\wp_t}{P_{t'}}, t'\right) \approx \Pi''(1, t') \left(\frac{\wp_t - p_{t'}^{\#}}{P_{t'}}\right) . \tag{7}$$

Substituting (7) into the first-order condition (4) with the expectation sign omitted, one can then solve for the optimal reset price $\wp_t$:

$$\wp_t = \frac{\displaystyle\int_t^\infty e^{-\int_t^{t'} (r_{t''}+\alpha) dt''} \frac{\Pi''(1, t')}{P_{t'}^2} p_{t'}^{\#} dt'}{\displaystyle\int_t^\infty e^{-\int_t^{t'} (r_{t''}+\alpha) dt''} \frac{\Pi''(1, t')}{P_{t'}^2} dt'} . \tag{8}$$

In words, the optimal reset price is (to a linear approximation) a weighted average of the desired price over the length of time until the next price decision, appropriately discounted. Other than the discounting, the weight is the curvature of the profit function with respect to the relative price, divided by the square of the aggregate price level. The aggregate price level is part of the weight because the higher the aggregate price level, the less effect the firm's own price will have on its relative price level. Given the simplifying assumption of zero inflation in the initial steady state, one can differentiate (4) with respect to time using Leibniz' rule and divide by $\wp^* = (p\#)^* = P^*$ to get the following differential equation in logarithmic deviations:

$$\dot{\check{\wp}}_t = (\alpha + r^*)[\check{\wp}_t - \check{p}^{\#}_t] \,, \tag{9}$$

where r^* is the steady-state real interest rate.[6]

Equation (3) indicates that what matters for price dynamics is the ratio $\wp/P$. Combining (9) with (3) implies in turn that the growth rate of $\wp/P$ is given by

$$\begin{aligned} \frac{d}{dt} \ln(\wp/P) &= \dot{\check{\wp}}_t - \dot{\check{P}} \\ &= r^*(\check{\wp} - \check{P}) + (\alpha + r^*)(\check{P} - \check{p}^{\#}) \,. \end{aligned} \tag{10}$$

To interpret (10), combine it with (3) and integrate using the steady state as a terminal condition to get

$$\begin{aligned} \dot{\check{P}}_t &= \alpha(\check{\wp}_t - \check{P}_t) \\ &= \alpha(\alpha + r^*) \int_t^\infty e^{-r^*(t'-t)}[\check{p}^{\#}_{t'} - \check{P}_{t'}] \, dt' \,. \end{aligned} \tag{11}$$

That is, to a first-order approximation, the rate of inflation is proportional to a present discounted value of the (logarithmic) "inflationary price gap" $\check{p}^{\#} - \check{P} \approx \ln(p^{\#} / P)$. Since the discounting in (11) is only at the steady-state rate of interest r^*, the Calvo model of price adjustment implies that the expected size of the inflationary price gap in the fairly distant future (say ten years down the road) can matter a lot for inflation now. Thus, the model would allow the aggregate price level to move in apparently mysterious ways that might not seem to accord with the current state of the economy, much as stock prices often move in mysterious ways because of their dependence on expectations of the fairly distant future.

1.3. Market Structure and the Desired Price

The next order of business is to look at what determines the desired price $p^{\#}$ and the (logarithmic) inflationary price gap $\check{\wp}^{\#} - \check{P}$ between the desired price and the actual aggregate price.

In addition to depending on the level of output in relation to "full-employment" output, the desired price $p^{\#}$ depends on the model's market structure, broadly construed.

Models with sticky prices also tend to have imperfect competition and increasing returns for good reason. Sticky prices require imperfect competition in order for

6. See Kimball (1995) for more details.

firms to be price-setters. (Under perfect competition, each firm is a price-taker and the only price-setter is the Walrasian auctioneer.)

Imperfect competition, in turn, is intimately connected by economic logic with internal increasing returns to scale. Internal increasing returns to scale makes firms likely to get large enough that perfect competition becomes doubtful. Conversely, unless entry is artificially restricted, constant returns to scale would allow firms to enter imperfectly competitive markets (which are attractive because price is above cost) until those markets approach being perfectly competitive.

The Role of Imperfect Competition in Business Cycle Models. Besides making sticky prices possible, imperfect competition has a number of other important and attractive implications for business cycle models. First, for some, the fact that imperfect competition makes increasing returns to scale possible is a virtue.

Second, imperfect competition captures the intuition that, over some very relevant range, more output is better. As Weitzman (1984) notes, the eagerness of firms to sell extra units of their products (as evidenced by the considerable efforts devoted to marketing) is prima facie evidence of price being above marginal cost and therefore of imperfect competition in a wide range of industries. The eagerness of firms to provide more at the given price avoids, for the most part, rationing of supply by firms. The eagerness of firms to provide more at the given price (a price at which households and other firms are not averse to buying) indicates that output is likely to be socially too low.

Third, the existence of a markup of price over marginal cost raises the possibility that the markup might vary over the business cycle.

A countercyclical *actual* markup is a likely consequence of sticky prices if marginal cost is procyclical and prices (because of their stickiness) are fairly acyclical. The importance of even these undesired fluctuations in the *actual* markup cannot be overemphasized because the only important effects of nominal things on the real equations of a neomonetarist model operate through the fluctuations in the actual markup. In other words, to a first approximation, knowing the actual markup alone tells one enough to solve the model in real terms without knowing anything else from the monetary side of the model.

The Flexible Variety Aggregator. In large part because of the things discussed above, macroeconomists have been quite interested in the effects of market structure on economic fluctuations—"market structure" including both the shape of an individual firm's demand curve and how the shape of the firm's demand curve changes in response to aggregate forces. Specific macroeconomic investigations of the effects of market structure abound in the literature. Rather than try to deal directly with the great variety of models in the literature, I will use a model of market structure for which the main virtues are simplicity and flexibility in a few important dimensions. I will not make any particular claim to realism for this model of market structure in and of itself. I will merely express the hope that it can represent reasonably well the kind of effects one would see in a more realistic, more cumbersome model.

First, for simplicity, I will assume no entry and exit of the differentiated firms producing varied products.

Second, I will assume that the differentiated goods can be combined into an aggregate in essentially the same way in investment and government purchases as in consumption. The consumption variety aggregator that relates the quantities of the differentiated consumption goods to overall consumption can be seen as a reflection of the household's utility function. Similarly, the (mathematically identical) government purchases variety aggregator can be seen as a reflection of the government's objective function. But the (mathematically identical) investment variety aggregator has to be interpreted as a reflection of the production function for assembling capital from differentiated intermediate investment goods.

Having these three variety aggregators be mathematically identical is only a convenience. There is no reason in principle not to have a different degree and shape of imperfect competition for consumption, investment and government purchases. If the consumption, investment, and government purchases variety aggregators were allowed to differ, the composition of aggregate output would interact with the differing nature of imperfect competition for each type of good. Moreover, one could then expect to find a different rate of price adjustment in each of these three sectors, requiring one to keep track of three different imperfectly flexible prices.

Third, in order to finesse the thorny question of how one would determine the scale at which substitution among differentiated goods operates, I will assume constant returns to scale for the variety aggregator. This simplifying assumption means that I cannot model a desired markup that depends on the overall scale of the economy. But my setup *does* allow the elasticity of demand—and therefore the desired markup—to depend on the ratio of an individual firm's output to aggregate output.

In order to have a symmetric, constant-returns-to-scale variety aggregator that is otherwise quite flexible, let the aggregate quantity Y be given by the implicit relationship

$$1 = \int_0^1 \Upsilon(y_\ell/Y)\, d\ell \ , \tag{12}$$

where the quantities y of the differentiated goods are indexed by ℓ on the continuum from 0 to 1, and the function Υ satisfies $\Upsilon(1) = 1$, $\Upsilon'(\xi) > 0$ and $\Upsilon''(\xi) < 0$, for all $\xi \geq 0$. By construction, this variety aggregator is symmetric in the differentiated quantities y_ℓ, so that the aggregate quantity Y is determined in a symmetric way. The variety aggregator has constant returns to scale since if the equation is satisfied for one vector of quantities y_ℓ and Y, then the equation will continue to be satisfied after the vector of quantities y_ℓ and aggregate quantity Y are all multiplied by the same constant.

The Demand Curve Facing the Firm. The demand curve each differentiated firm faces can be derived by looking at the optimal choice of differentiated quantities by the household, government, or investment good assembler. The household, government, or investment good assembler minimizes the cost of the intermediate goods necessary to produce any given amount:

$$\min_{y_\ell} \int_0^1 p_\ell y_\ell \, d\ell$$

s.t.

$$1 = \int_0^1 \Upsilon(y_\ell / Y) \, d\ell \; ,$$

for given Y. The first-order condition for this problem is

$$p_\ell = \frac{\Lambda}{Y} \Upsilon'(y_\ell / Y) \tag{13}$$

for some value of the Lagrange multiplier Λ that is constant for all ℓ.

The importance of (13) is that the model allows one to generate any desired shape of demand curve facing the individual firm. The only restrictions are that the firm demand curve must be downward sloping and that the firm must get its normal market share at a relative price of 1 (symmetric with all of the other firms). With an eye to generating sufficient "real rigidity" in the sense of Ball and Romer (1990) to make sticky prices plausible, I will be especially interested in firm demand curves that have a smoothed-out kink at the firm's normal market share—making it easier for the firm to lose customers by raising its relative price above 1 than it is to gain customers by lowering its relative price below 1. By contrast, the commonly used model of Dixit and Stiglitz (1977) restricts one to constant-elasticity firm demand curves.

The key aspects of the firm demand curve can be summarized by the behavior of the implied elasticity of demand at various points. Writing $\xi = y_\ell / Y$, the inverse elasticity of demand for a differentiated firm is

$$\begin{aligned} \frac{1}{\epsilon(\xi)} &= -y_\ell \frac{d \ln(p_\ell)}{dy_\ell} \\ &= \frac{y_\ell}{Y} \frac{\Upsilon''(y_\ell / Y)}{\Upsilon'(y_\ell / Y)} \\ &= -\frac{\xi \Upsilon''(\xi)}{\Upsilon'(\xi)} \; . \end{aligned} \tag{14}$$

The elasticity calculation depends on the firm being an infinitesimal part of the economy, so that a change in its own price has a negligible effect on Y and Λ. It is easy to find a variety aggregator Y to match any desired dependence of the elasticity of demand ϵ on the firm's relative output (market share) ξ. Viewed as a differential equation for the function $\Upsilon'(\xi)$, (14) has the particular solution

$$\Upsilon'(\xi) = \exp\left(-\int_1^{\xi} \frac{1}{\zeta\epsilon(\zeta)}\, d\zeta\right) .$$

The elasticity of demand matters for two reasons. First, the desired markup $\mu(y_\ell/Y)$ that the firm would use in calculating its desired price is given by

$$\mu(y_\ell/Y) = \frac{\epsilon(y_\ell/Y)}{\epsilon(y_\ell/Y) - 1} . \tag{15}$$

Second, the elasticity relates movements in relative output y_ℓ/Y to movements in the relative price p_ℓ/P. Log-linearizing (13) around the steady state, where $y_\ell = Y$ and $p_\ell = P$,

$$\ln(y_\ell/Y) \approx -\epsilon^* \ln(p_\ell/P) , \tag{16}$$

with $\epsilon^* = \epsilon(y_\ell^*/Y^*) = \epsilon(1)$. Using logarithmic deviations from the steady state (treated like differentials from calculus) one can rewrite (16) as the locally exact equation

$$\check{y}_\ell - \check{Y} = -\epsilon^*(\check{p}_\ell - \check{P}) . \tag{17}$$

1.4. Full Employment Output and Desired Output

At this stage, it is helpful to represent a firm's real marginal cost as a function of a firm's own output y, aggregate output Y, and everything else "$\cdot$":

$$\frac{\mathrm{MC}}{P} = \Phi(y, Y, \cdot) ,$$

Since, in the model, capital is rented rather than being a fixed factor, marginal cost need not depend on firm output y, though it may. Aggregate output Y affects a firm's marginal cost through factor prices and perhaps through production externalities as well. In this paper "everything else" affecting marginal cost is the capital stock and the marginal value of capital, but in principle "everything else" could include exogenous variables such as technology.

For stability, a key assumption will be that at the steady state, the elasticity

$$\Omega = \left.\frac{y\Phi_y + Y\Phi_Y}{\Phi}\right|_{(y,Y,\cdot)=(Y^*,Y^*,\cdot^*)}$$

is positive; that is, factor price pressures are sufficient to make real marginal cost increase when aggregate output and firm output increase by the same percentage (such as when there is a balanced expansion of every firm's output at the same time). The elasticity of real marginal cost with respect to a balanced expansion of output is important because, in the short run, with all firms having given prices, the homotheticity of the variety aggregator guarantees that a monetary expansion causes just such a balanced expansion of output. If the elasticity were sometimes negative, it would mean that a monetary expansion could lower real marginal cost—a situation that would make it easy to generate multiple equilibria, with Ω negative at unstable equilibria and positive at stable equilibria. Even low positive values of Ω could have dramatic implications for the behavior predicted by the model, but I will argue that Ω is likely to be a substantial positive number.

The desired relative price $p^{\#}/P$, and the desired relative output $y^{\#}/Y$ that goes along with it, must satisfy the condition that marginal revenue $p^{\#}/\mu(y^{\#}/Y)$ equals marginal cost $P\Phi(y^{\#}, Y, \cdot)$. Thus, the desired relative price must equal the desired mark-up ratio times real marginal cost:

$$\frac{p^{\#}}{P} = \mu(y^{\#}/Y)\Phi(y^{\#}, Y, \cdot) \ . \tag{18}$$

In conjunction with (16), which must hold in particular for $p^{\#}/P$ and $y^{\#}/Y$, (18) determines $p^{\#}/P$ and $y^{\#}$ as functions of aggregate output Y and the other arguments of Φ summarized by "$\cdot$". Let me define "full employment output" Y^f to be the level of aggregate output at which the desired relative price $p^{\#}/P$ is 1. If the desired relative price is 1, all firms could have the desired price without altering the aggregate price. By (16), if the desired relative price $p^{\#}/P$ is 1, then the desired relative output $y^{\#}/Y$ is also 1. Thus, full employment output Y^f is the solution to

$$1 = \mu(1)\Phi(Y^f, Y^f, \cdot) \ . \tag{19}$$

Clearly, full employment output is a function of "$\cdot$"—variables like the capital stock and the marginal value of capital.

Dividing (18) by (19), log-linearizing around the steady-state, then combining the result with (17) yields the equation

$$\begin{aligned} 0 &= \frac{1}{\epsilon^*}[\check{y}^{\#} - \check{Y}] + \check{p}^{\#} - \check{P} \\ &= \omega[\check{y}^{\#} - \check{Y}] + \Omega[\check{Y} - \check{Y}^f] \ , \end{aligned}$$

where

$$\omega = \left(\frac{1}{\epsilon(y/Y)} + \frac{(y/Y)\mu'(y/Y)}{\mu(y/Y)} + \frac{y\Phi_y}{\Phi}\right)\Bigg|_{(y,Y,\cdot)=(Y^*,Y^*,\cdot^*)}$$

$$= \frac{1}{\epsilon^*} + \frac{\mu'(1)}{\mu(1)} + \frac{Y^*\Phi_y(Y^*, Y^*, \cdot^*)}{\Phi(Y^*, Y^*, \cdot^*)} .$$

Solving for $\check{y}^\# - \check{Y}$ (the logarithmic deviation of desired relative output) in terms of $\check{Y} - \check{Y}^f$ (the logarithmic deviation of actual output from full employment output),

$$\check{y}^\# - \check{Y} = -\frac{\Omega}{\omega} [\check{Y} - \check{Y}^f] . \tag{20}$$

1.5. The Desired Price

The Notional Short-Run Aggregate Supply Curve. As noted above, the short-run aggregate supply curve is just a fixed aggregate price P. However, (20) and (17) together yield the equation

$$\check{p}^\# - \check{P} = \frac{\Omega}{\epsilon^*\omega} [\check{Y} - \check{Y}^f] . \tag{21}$$

In words, the inflationary price gap is equal to $\frac{\Omega}{\epsilon^*\omega}$ times the "inflationary output gap" $\check{Y} - \check{Y}^f$. Since the inflationary price gap is proportional to the inflationary output gap, the rate of inflation is proportional to a present discounted value of either inflationary gap. Substituting (21) into (11),

$$\dot{\check{P}}_t = \alpha(\alpha + r^*) \frac{\Omega}{\epsilon^*\omega} \int_t^\infty e^{-r^*(t'-t)}[\check{Y}_{t'} - \check{Y}^f_{t'}]\, dt' . \tag{22}$$

Graphically, one can think of (21) as a notional short-run aggregate supply curve, giving the *desired* price $p^\#$ as a function of output. With the logarithms of output and price on the axes, the *notional* short-run aggregate supply curve intersects the *actual* short-run aggregate supply curve at (log) full employment output $\ln(Y^f)$, but the notional SRAS has an upward slope of $\frac{\Omega}{\epsilon^*\omega}$ instead of being flat. The vertical distance between the notional and actual short-run aggregate supply curves is the inflationary price gap. The rate of inflation is $\alpha(\alpha + r^*)$ times a present discounted value of this vertical distance.

Interpreting the Effects of Market Structure on the Desired Price. To interpret (20) and (21), think of the firm's marginal revenue and marginal cost curves.[7] Near the

7. For the sake of interpretation, it is also helpful to put the curves for desired output and desired price into the framework of Cooper and John (1988), making allowances for the fact that Cooper and John (1988) is essentially static. Equation (20) implies that the "reaction function" for desired firm output as a function of actual aggregate output is

$$\check{y}^* = \left[1 - \frac{\Omega}{\omega}\right] \check{Y} + \frac{\Omega}{\omega} \check{Y}^f .$$

(*continued*)

steady state, $(1/\epsilon^*)$ is the rate at which the demand curve falls, expressed as an elasticity. Since the desired markup ratio μ tells how far the marginal revenue curve is below the demand curve, $(1/\epsilon^*) + [\mu'(1)/\mu(1)]$ is the rate at which the *marginal revenue curve* falls, expressed as an elasticity. On the marginal cost side, $(Y^*\Phi_y/\Phi)$ is the rate at which the firm's marginal cost curve rises, expressed as an elasticity. Thus, ω is the difference in the slopes of the marginal cost and marginal revenue curves expressed as an elasticity—or equivalently, ω is the elasticity of MC/MR with respect to y. A 1 percent increase in firm output y makes the ratio of marginal revenue to marginal cost fall by ω percent.

At the macroeconomic level, $\Omega = [Y^*(\Phi_y + \Phi_Y)/\Phi]$ is the elasticity with which a proportional increase in both firm and aggregate output raises the firm's real marginal cost. On the marginal revenue side, the firm's real marginal revenue is not affected at all by a proportional increase in both aggregate output and firm output. By (16) and (15), the real price at which a firm can sell its output, *and* the markup that tells how much lower marginal revenue is, are both functions only of relative output y/Y. Thus, Ω is the elasticity of MC/MR with respect to the proportional increase in both aggregate output and firm output caused by a monetary expansion.

Now, consider an average firm with $y = Y$ that has marginal cost equal to marginal revenue when aggregate output is at the full employment level. If monetary forces cause a 1 percent increase in aggregate output above full employment output, with the firm's output going up proportionally, the average firm's marginal cost will be pushed Ω percent above its marginal revenue. Because the firm's price is fixed, there is no paradox in this gap between marginal cost and marginal revenue. However, by definition, desired output must equate marginal revenue and marginal cost. Since the elasticity of MC/MR with respect to the firm's output is ω, firm output Ω/ω percent *below what one would get from a balanced expansion* would restore equality between marginal revenue and marginal cost. As for the desired relative price, firm output Ω/ω percent below what one would get from a balanced expansion would be associated with an $\frac{\Omega}{\epsilon^*\omega}$ percent increase in the firm's price relative to the actual aggregate price.

The verbal definitions of ω (as the elasticity of MC/MR with respect to firm output, holding aggregate output fixed) and Ω (as the elasticity of MC/MR with respect to aggregate output when firm output moves proportionally) are more general than the particular model at hand. It is worth paying attention to these elasticities in a wide variety of models with imperfectly flexible prices.

"Real Rigidity" and the Output Elasticities of MC/MR. Following Ball and Ro-

The coefficient of $\check{Y}$ is the slope of the reaction function, and implies a multiplier of $1/(1 - \text{slope}) = \omega/\Omega$. To get the "reaction function" for desired price as a function of the actual aggregate price, define $P^f = MV/Y^f$ or $\check{P}^f = \check{M} + \check{V} - \check{Y}^f$, so that $\check{P} - \check{P}^f = -(\check{Y} - \check{Y}^f)$. Then

$$\check{p}^{\#} = \left[1 - \frac{\Omega}{\epsilon^*\omega}\right]\check{P} + \frac{\Omega}{\epsilon^*\omega}\check{Y}^f ,$$

with a corresponding multiplier of $\epsilon^*\omega/\Omega$. Since the optimality of the firm's price in steady state implies that $\epsilon^* > 1$, the slope of the "reaction function" and the multiplier (both of which measure the degree of strategic complementarity) are greater for a price game than for an output game.

mer (1990), it is easy to relate the elasticities Ω and ω to the cost of price rigidity. Ball and Romer's phrase "real rigidity" refers most directly to a change in aggregate output having little effect on a firm's desired price, and so can be identified here with a small value of $\frac{\Omega}{\epsilon^*\omega}$, the elasticity of the desired price with respect to aggregate output.

The "dead weight loss" triangle giving the approximate private flow cost of being away from desired output has a base of length approximately $[\check{y}_\ell - \check{y}^{\#}]Y^*$ and a height of approximately $[\Omega(\check{Y} - \check{Y}^f) + \omega(\check{y}_\ell - \check{Y})]\mathrm{MR}^*$. Using (20) and averaging over all firms, the average private flow cost of price rigidity (APFCPR) is approximately

$$APFCPR \approx \frac{.5P^*Y^*}{\mu^*}\left[\frac{\Omega^2}{\omega}(\check{Y} - \check{Y}^f)^2 + \omega\int_0^1 (\check{y}_\ell - \check{Y})^2\, d\ell\right]. \tag{23}$$

Of course, the average private flow cost of price rigidity is second-order—the one second-order quantity that I will examine in this paper. The second part of the APFCPR, involving the cross-sectional variance of output, depends on the history of the economy and so is difficult to analyze fully.[8] However, I conjecture that at least part of the cross-sectional variance in output will be related to the variance of movements in $\check{y}^{\#} - \check{Y}$, which is proportional to $(\Omega/\omega)^2$ times the variance of $[Y - Y^f]$. Thus, the second part of the APFCPR is likely to depend on Ω and ω in much the same way as the first part.[9] In any case, a substantial part of the average private flow cost of price rigidity is proportional to Ω^2/ω. As Ball and Romer suggest, this means that a small private cost of price rigidity is typically associated with "real rigidity"—a small elasticity of the desired price with respect to aggregate output, $\frac{\Omega}{\epsilon^*\omega}$.

One fact that would not be apparent from reading Ball and Romer (1990) is the way in which a high initial elasticity of demand ϵ^* lowers the elasticity of the desired price with respect to output. But in their own terms, Ball and Romer do make clear the dependence of both "real rigidity" and the private cost of price rigidity on Ω and ω.

The macroeconomic elasticity Ω incorporates all of the effects of factor price pressure on marginal cost as output increases. Ball and Romer (1990) look at the possibility of lowering Ω by increasing the macroeconomic labor supply elasticity and thereby reducing factor price pressure caused by increased output.

In general, the elasticity Ω may also include a negative effect from production

8. Ball and Romer (1990) make the second part of the APFCPR involving the cross-sectional variance of output zero by using a static model in which all firms start off at the same place.

9. Steady-state inflation would introduce an additional reason for cross-sectional variance in prices and outputs. The part of the cross-sectional variation in output due to steady-state inflation should be roughly proportional to the elasticity of demand ϵ times the amount of inflation that would take place in the average length of time a price is fixed, all squared. This part of the cross-sectional variance of output could vary independently from the two elasticities Ω and ω.

externalities and a negative effect from mechanisms that generate countercyclical desired markups. In the particular model at hand, the desired markup μ is acyclical, but many models exist that generate countercyclical desired markups.

The microeconomic elasticity ω incorporates all aspects of increasing marginal costs and decreasing marginal revenue that are internal to the firm. Ball and Romer (1990) show that having workers strongly attached to firms ("the yeoman farmer model")—instead of being hired on the spot market—leads to a lower private cost of price rigidity. This is because when workers are strongly attached to firms, labor costs depend on *firm* output—a dependence that adds to ω (as well as Ω). If workers are always hired on the spot market, labor costs depend only on aggregate output—a dependence that adds *only* to Ω, not ω. The same principle would say that having capital strongly attached to a particular firm rather than being hired on the spot market would also reduce the private cost of having sticky prices.

The microeconomic elasticity ω also includes any dependence of the desired markup on the firm's relative output or market share y/Y. Ball and Romer (1990) look at the possibility of using Woglom's (1982) model of customer markets to make the elasticity of demand (and therefore the marginal revenue curve) fall sharply with the firm's market share (since it is harder to get new customers than to drive away old ones). This corresponds to a desired markup that is sharply increasing in the firm's market share y/Y—something that could make ω quite large. In the Neomonetarist model at hand, it is easy to find a variety aggregator Y that yields a desired markup that is sharply increasing in each differentiated firm's market share, reproducing the most essential implication of Woglom's (1982) model of customer markets.

2. THE SHORT-RUN DYNAMICS OF THE NEOMONETARIST MODEL

Combining (10) with (21) and the Quantity Equation $\check{Y} = \check{M} + \check{V} - \check{P}$ yields

$$\begin{aligned} \dot{\check{\wp}}_t - \dot{\check{P}} &= r^*(\check{\wp} - \check{P}) + (\alpha + r^*)(\check{P} - \check{p}^{\#}) \\ &= r^*(\check{\wp} - \check{P}) + (\alpha + r^*)\,\frac{\Omega}{\epsilon^*\omega}\,[\check{P} + \check{Y}^f - \check{M} - \check{V}]\,. \qquad (24) \end{aligned}$$

The fast-price-adjustment or high-α approximation implies that the endogenous movements in full employment output Y^f are slow in comparison to the price adjustment dynamics. Therefore, treating full employment Y^f as exogenous yields a reasonably good approximation for the short-run dynamics of the model. The detailed analysis of the fast-price-adjustment approximation can be found in Kimball (1995).[10]

With Y^f, M and V treated as exogenous, (3) and (24) together form a dynamic system in $\check{P}$ and the (log) relative reset price $\check{\wp} - \check{P}$. In matrix form,

10. Calculated to a higher order of approximation than is needed here, it involves taking the derivative of eigenvectors with respect to changes in the dynamic matrix (in order to make a Taylor approximation).

$$\begin{bmatrix} \dot{\check{P}} \\ \dot{\check{\wp}} - \dot{\check{P}} \end{bmatrix} = \begin{bmatrix} 0 & \alpha \\ (\alpha + r^*) \dfrac{\Omega}{\epsilon^* \omega} & r^* \end{bmatrix} \begin{bmatrix} \check{P} \\ \check{\wp} - \check{P} \end{bmatrix} + \begin{bmatrix} 0 \\ (\alpha + r^*) \dfrac{\Omega}{\epsilon^* \omega} (\check{Y}^f - \check{M} - \check{V}) \end{bmatrix} . \tag{25}$$

The absolute value of the negative eigenvalue of this matrix gives the convergence rate κ:

$$\kappa = \sqrt{\frac{(r^*)^2}{4} + \frac{\Omega}{\epsilon^* \omega} \alpha(\alpha + r^*)} - \frac{r^*}{2}$$
$$\approx \alpha \sqrt{\frac{\Omega}{\epsilon^* \omega}} .$$

The approximation for κ on the second line ignores everything that is not at least of order α. The most interesting aspect of this equation is that the convergence rate at which the economy returns to full employment after a permanent change in M depends approximately on the square root of $\frac{\Omega}{\epsilon^* \omega}$, the elasticity of desired price with respect to aggregate output. Thus, a high degree of "real rigidity" in Ball and Romer's (1990) sense makes price adjustment slow in comparison to the frequency α at which individual firms adjust their prices. If $\frac{\Omega}{\epsilon^* \omega}$ is less than 1, aggregate price adjustment will be slower than α, the microeconomic rate at which firms get a chance to adjust their prices.[11]

If α is varied to hold the macroeconomic rate of price adjustment κ fixed, then a fall in Ω or an increase in ω makes price stickiness more plausible not only by reducing the private flow cost of price rigidity caused by a given departure of output from full employment output, but also by reducing the length of time a given firm incurs that private flow cost for each instance of fixing its price. Even an increase in ϵ^* for given Ω and ω (which has no direct effect on the private flow cost of price rigidity in (23)) would require an increase in α to hold κ fixed, thus implying a shorter period of price-fixity at the firm level.

3. THE CLASSICAL SIDE OF THE NEOMONETARIST MODEL

The "classical" side of the neomonetarist model is important for three reasons. First, the classical side of the model indicates what happens to variables other than P and Y. Second, the classical side of the model makes it possible to determine full

11. This result is reminiscent of Blanchard (1983).

employment output Y^f. Third, the classical side of the model determines the elasticity Ω of marginal cost with respect to aggregate output.

3.1. Investment Adjustment Costs

Investment adjustment costs are important to business cycle analysis in order to allow a distinction between the real interest rate and the net marginal product of capital. In the absence of some mechanism in a model to allow the relative price of capital relative to consumption goods to vary, the real interest rate and the rental rate for capital net of depreciation must always be equal. Without a distinction between the real interest rate and the net marginal product of capital, virtually *any* stimulus to the economy—including a monetary stimulus—must be associated with an *increase* in the real interest rate, since virtually every business cycle model implies a procyclical rental rate for capital.

Due to space limitations, I will present the model here *without* investment adjustment costs, then discuss what the effects of investment adjustment costs would be. The details of the model *with* investment adjustment costs can be found in Kimball (1995).

3.2. The Household

Although I am pushing trend growth of the economy into the background, I will use the King-Plosser-Rebelo (1988) form for the utility function in order to make the model consistent with trend growth. An especially convenient way of expressing the King-Plosser-Rebelo utility function is

$$\int_0^{\infty} e^{-\rho t} u(C, N)\, dt ,$$

where

$$u(C, N) = \frac{C^{1-\beta}}{1-\beta}\, e^{(\beta-1)\upsilon(N)} , \tag{26}$$

or for $\beta = 1$

$$u(C, N) = \ln(C) - \upsilon(N) . \tag{27}$$

This is the only type of additively time-separable utility function that allows income and substitution effects on labor supply to cancel out in the long run so that the trend in the real wage W does not induce a trend in per capita labor N.

For simplicity, I will treat households as owning all of the capital and renting it out to the firms at the real rental rate R,[12] and assume that each household holds an

12. Whether firms or households do the investing does not matter for outcomes as long as firms would also have to buy goods for investment purposes at the retail, marked up price.

identical cross-section of the shares of the residual profits of the firms. (The households are identical in every other way, too.)

In addition to capital, a household can hold a real quantity of bonds B earning interest at the real rate r. In addition to paying for consumption C and investment I, it must pay lump-sum taxes T.

Putting everything together, the representative household's problem is

$$\max_{C,N,I} \int_0^\infty e^{-\rho t} \frac{C^{1-\beta}}{1-\beta} e^{(\beta-1)\upsilon(N)}\, dt \ ,$$

s.t.

$$K = I - \delta K \tag{28}$$

$$\dot{B} = rB + WN + RK + \Pi - C - I - T \ , \tag{29}$$

where $\upsilon' \geq 0$, $\upsilon'' \leq 0$. If $\beta < 1$, an additional restriction is needed to guarantee concavity of $u(C,N)$, but I will argue that the labor-constant intertemporal elasticity of substitution $s = (1/\beta)$ is less than 1, making $\beta \geq 1$. Π is residual profits.

The current-value Hamiltonian H is

$$H = \frac{C^{1-\beta}}{1-\beta} e^{(\beta-1)\upsilon(N)} + \lambda[I - \delta K]$$
$$+ \nu[rB + WN + RK + \Pi - C - I - T] \ ,$$

where λ is the marginal value of a unit of capital in utils and ν is the marginal value in utils of one real dollar's worth of bonds. The marginal value of capital in dollars, Q, is

$$Q = \frac{\lambda}{\nu} \ . \tag{30}$$

The first-order condition for I is

$$\nu = \lambda \tag{31}$$

—or equivalently, $Q = 1$. Therefore, the first-order conditions for C and N boil down to

$$C^{-\beta} e^{(\beta-1)\upsilon(N)} = \lambda \tag{32}$$

$$\nu'(N) C^{1-\beta} e^{(\beta-1)\upsilon(N)} = W\lambda \tag{33}$$

—together implying

$$W = C\upsilon'(N) \; . \tag{34}$$

The Euler equations for λ and ν are

$$\dot{\lambda} = [\rho + \delta - R]\lambda \tag{35}$$

$$\dot{\nu} = (\rho - r)\nu$$

—which, combined with the first-order condition $\lambda = \nu$ imply that

$$r = \rho - \frac{\dot{\nu}}{\nu} = \rho - \frac{\dot{\lambda}}{\lambda} = R - \delta \; . \tag{36}$$

3.3. The Production Function

I will assume for each firm the same generalized Cobb-Douglas production function:

$$y_\ell = F(x_\ell, Y) = F(k_\ell^\theta n_\ell^{1-\theta}, Y) \; , \tag{37}$$

where x is a Cobb-Douglas composite of the two inputs with a cost share of θ for capital and $1 - \theta$ for labor as shown. The parameters θ and $1 - \theta$ are the cost shares for capital and labor because for any given composite input x_ℓ, the firm minimizes costs by solving

$$\min_{k_\ell, n_\ell} Wn_\ell + Rk_\ell$$

s.t.

$$k_\ell^\theta n_\ell^{1-\theta} = x_\ell$$

—implying among other things that

$$\frac{Wn_\ell}{Rk_\ell} = \frac{1-\theta}{\theta} \; .$$

In the form $n_\ell = (R(1 - \theta)/W\theta)k_\ell$, this relationship can be neatly summed up to the corresponding aggregate relationship. Thus, in the aggregate,

$$R = \frac{WN}{K} \frac{\theta}{1-\theta} \; . \tag{38}$$

In words, capital-labor substitution ensures that the real rental rate R is determined in a straightforward way by the capital stock and what is going on in the labor market, even when the economy is away from full employment.

The function F allows the possibility of both internal and external increasing returns to scale (including fixed costs, as long as the factor composition of the fixed costs is the same as the factor composition of the variable costs). Omitting the subscript on y_ℓ, the real total cost function for each firm is of the form

$$\frac{TC}{P} = R^{\theta}W^{1-\theta}C(y, Y) .$$

The real marginal cost $MC/P = \Phi(y, Y, \cdot)$ at the steady-state market share $y/Y = 1$ (where the key elasticities are determined) is therefore

$$\phi = \Phi(Y, Y, \cdot) = [R(Y, \cdot)]^{\theta}[W(Y, \cdot)]^{1-\theta}C_y(Y, Y) . \tag{39}$$

The size of the effect of factor price pressure on real marginal cost will be calculated below. In addition, there is a direct effect of aggregate output on marginal cost through both arguments of $C_y(y, Y)$. In the neighborhood of the steady state, with $y = Y = Y^*$, the elasticity $\aleph$ with which marginal cost *for given factor prices* falls as aggregate output and firm output increase in the same proportion is

$$\aleph = -\frac{Y^*[C_{yy}(Y^*, Y^*) + C_{yY}(Y^*, Y^*)]}{C_y(Y^*, Y^*)} .$$

The degree of internal returns to scale γ in the neighborhood of the steady state depends only on the *level* of marginal cost relative to total cost, not to the derivative of marginal cost:

$$\gamma = \frac{TC}{Y^*MC} = \frac{C(Y^*, Y^*)}{Y^*C_y(Y^*, Y^*)} .$$

Similarly, the degree of returns to scale Γ to a balanced expansion of output by all firms is

$$\Gamma = \frac{C(Y^*, Y^*)}{Y^*[C_y(Y^*, Y^*) + C_Y(Y^*, Y^*)]} .$$

3.4. *Steady-State Relationships Useful for Log-Linearizing around the Steady State*

Investment and the Rental Rate in Steady State. In the steady state, (28) becomes

$$I^*/K^* = \delta ,$$

and (36) becomes

$$R^* = r^* + \delta = \rho + \delta \ .$$

The Steady-State Composition of Output. Let $\zeta_C = C^*/Y^*$, $\zeta_I = I^*/Y^*$ and $\zeta_G = G^*/Y^*$ be the steady-state shares of consumption, investment, and government purchases in output. In the steady state, the real wage and the rental rate are equal to their marginal revenue products. Since internal increasing returns raise the marginal product, while a markup lowers it, the steady-state payments to capital as a share of output are equal to γ^*/μ^* times capital's share in costs:

$$\frac{R^*K^*}{Y^*} = \theta \frac{\gamma^*}{\mu^*} \ . \tag{40}$$

(This relationship can be derived by using the fact that

$$\mathrm{TR} = P^*Y^* = \mu^* MC \cdot Y^* = \mu^* \frac{\mathrm{TC}}{\gamma} \ .)$$

Thus,

$$\zeta_I = \frac{I^*}{Y^*} = \frac{I^*}{K^*}\frac{1}{R^*}\frac{R^*K^*}{Y^*} = \frac{\delta}{\rho + \delta}\frac{\gamma}{\mu^*}\theta \ . \tag{41}$$

Also,

$$\zeta_C = \frac{C^*}{Y^*} = \left(\frac{C^*}{W^*N^*}\right)\left[\frac{W^*N^*}{Y^*}\right] = \left(\frac{1}{\tau}\right)\left[\frac{\gamma}{\mu^*}(1 - \theta)\right] \tag{42}$$

and

$$\frac{\zeta_C}{\zeta_I} = \frac{\rho + \delta}{\delta}\frac{1 - \theta}{\theta\tau} \ ,$$

where the notation τ for the ratio of labor income to consumption

$$\tau = \frac{W^*N^*}{C^*}$$

will be used below as well.

3.5. *Log-Linearizing around the Steady State*

Expressing some of the key equations above in terms of logarithmic deviations from the steady state yields the following block of key equations.

For the representative household's first-order conditions, one obtains

$$\check{C} = s(\check{Q} - \check{\lambda}) + (1 - s)\tau\check{N} \tag{43}$$

$$\check{W} = \check{C} + \frac{\check{N}}{\eta} \tag{44}$$

where

$$s = \frac{1}{\beta}$$

is the labor-constant elasticity of intertemporal substitution for consumption and

$$\eta = \frac{\upsilon'(N^*)}{N^*\upsilon''(N^*)}$$

is the consumption-constant elasticity of labor supply.

The Euler equation for λ and the accumulation equation for K yield

$$\begin{aligned} \dot{\check{\lambda}} &= \left(\frac{\dot{\lambda}}{\lambda}\right) \\ &= -\check{r} \\ &= -(\rho + \delta)\check{R} \end{aligned} \tag{45}$$

$$\begin{aligned} \dot{\check{K}} &= (I^*/K^*)[\check{I} - \check{K}] \\ &= \delta[\check{I} - \check{K}] \; . \end{aligned} \tag{46}$$

From the production function,

$$\check{Y} = \Gamma[\theta\check{K} + (1 - \theta)\check{N}] \; . \tag{47}$$

From the constant shares,

$$\check{R} = \check{W} + \check{N} - \check{K} \; , \tag{48}$$

and from (39), the logarithmic deviation of real marginal cost ϕ is given by

$$\check{\phi} = \theta\check{R} + (1 - \theta)\check{W} - \aleph Y \; . \tag{49}$$

Finally, with government purchases assumed constant, $\check{G} = 0$ and the closed-economy accounting identity $Y = C + I + G$ can be log-linearized as

$$\check{Y} = \zeta_C\check{C} + \zeta_I\check{I} \; . \tag{50}$$

Certain other equations are peripheral to the model. For example, given the material balance condition and the Ricardian equivalence implied by the model, it is not necessary to keep track of the household stock of bonds or the government's budget constraint.

3.6. *The Short-Run Behavior of the Classical Side of the Model*

Recall the three things to be learned from the classical side of the model: the behavior of variables other than P and Y in response to a monetary expansion, the determinants of full employment output, and the determinants of Ω, the elasticity of marginal cost with respect to a balanced expansion in output by all firms.

Using the equations above, it is a straightforward matter to express the behavior of the key variables in terms of $\breve{Y}$, $\breve{K}$, and λ as follows:

$$\begin{bmatrix} \breve{N} \\ \breve{C} \\ \breve{I} \\ \breve{W} \\ \breve{R} \\ \breve{\phi} \\ \dot{\breve{K}} \\ \dot{\breve{\lambda}} \end{bmatrix} = \begin{bmatrix} \frac{1}{\Gamma(1-\theta)} & -\frac{\theta}{(1-\theta)} & 0 \\ \frac{\tau(1-s)}{\Gamma(1-\theta)} & -\frac{\tau(1-s)\theta}{(1-\theta)} & -s \\ \frac{\rho+\delta}{\Gamma\delta\theta}\left[\frac{\mu^*\Gamma}{\gamma} - 1 + s\right] & \frac{\rho+\delta}{\delta}(1-s) & s\,\frac{\zeta_C}{\zeta_I} \\ \frac{\tau(1-s)+\frac{1}{\eta}}{\Gamma(1-\theta)} & -\frac{\theta\left[\tau(1-s)+\frac{1}{\eta}\right]}{1-\theta} & -s \\ \frac{\tau(1-s)+\frac{1}{\eta}+1}{\Gamma(1-\theta)} & -\frac{\theta\left[\tau(1-s)+\frac{1}{\eta}+\frac{1}{\theta}\right]}{1-\theta} & -s \\ \frac{\tau(1-s)+\frac{1}{\eta}+\theta}{\Gamma(1-\theta)} - \aleph & -\frac{\theta}{1-\theta}\left[\tau(1-s)+\frac{1}{\eta}+1\right] & -s \\ \frac{(\rho+\delta)}{\Gamma\theta}\left[\frac{\mu^*\Gamma}{\gamma} - 1 + s\right] & -[s(\rho+\delta)-\rho] & \delta s\,\frac{\zeta_C}{\zeta_I} \\ -\frac{(\rho+\delta)\left[\tau(1-s)+\frac{1}{\eta}+1\right]}{\Gamma(1-\theta)} & \frac{\theta(\rho+\delta)\left[\tau(1-s)+\frac{1}{\eta}+\frac{1}{\theta}\right]}{(1-\theta)} & s(\rho+\delta) \end{bmatrix} \begin{bmatrix} \breve{Y} \\ \breve{K} \\ \breve{\lambda} \end{bmatrix} \quad (51)$$

Because of the log-linearizing, each element of the above matrix is in elasticity form. Since $\breve{Y} = \breve{M} + \breve{V} - \breve{P}$, in the short-run, and P cannot jump, the first column of elasticities with respect to Y are also the short-run elasticities with respect to M and V. Moreover, in the high-α approximation in which prices adjust quickly, a change in the money supply M or in velocity V will have only a small effect on K and λ. Therefore, as an approximation, we can take K and λ fixed as we vary M or V. That approximation is a formal version of the Classical Dichotomy.

Note that if the elasticity of intertemporal substitution in consumption s is 1, as many real business cycle models assume, (51) simplifies quite a bit. However, I will argue below that $s < 1$.

Here is the logic behind the effects of a monetary expansion predicted by the first column of the matrix in (51): (0) Output is determined by the quantity equation. With marginal revenue above marginal cost in the neighborhood of the steady state, firms are willing to supply all of the output that is demanded at their fixed prices. (1) Firms employ enough labor to produce the additional output. (2) If $s < 1$, complementarity in the utility function between labor and consumption leads the household to increase consumption when it provides more labor. (3) Saving and therefore investment increases because the additional consumption is less than the increase in output produced by the additional labor input. (4) The increased quantity of labor moves the representative household up its labor supply curve, forcing the real wage to rise. This increase in the real wage—in conjunction with the increased efficiency of the economy as output expands from its suboptimal steady-state value—is what guarantees that labor will increase enough in relation to consumption that saving will increase. (5) The rental rate of capital increases (even more than the real wage) because the increase in the real wage makes firms want to substitute capital for labor, but capital actually becomes scarcer relative to labor as N increases. In the absence of investment adjustment costs, the increase in the rental rate implies that the real interest rate increases, since $r = R - \delta$. (6) Unless productive externalities or downward-sloping marginal cost at the firm level make $\aleph$ an elasticity to be reckoned with, the increase in the real wage and the rental rate cause an increase in real marginal cost, squeezing firms' actual markups below their desired markups. (7) The increased investment results in capital accumulation. (8) The above-normal rental rate is associated with a temporarily high—and therefore falling—marginal value of capital λ.

Full employment output is the level of output at which real marginal cost is at its steady-state value—at which $\check{\phi} = 0$. Thus,

$$\check{Y}f = \frac{\dfrac{\theta}{1-\theta}\left[\tau(1-s) + \dfrac{1}{\eta} + 1\right]\check{K} + s\check{\lambda}}{\dfrac{\tau(1-s) + \dfrac{1}{\eta} + \theta}{\Gamma(1-\theta)} - \aleph}.$$

Since K and λ determine full employment output, the elasticity of marginal cost ϕ with respect to Y, holding K and λ constant, is equal to Ω:

$$\Omega = \frac{\tau(1-s) + \dfrac{1}{\eta} + \theta}{\Gamma(1-\theta)} - \aleph .$$

Everything in Ω other than the possible productive externality effects represented by $\aleph$ is a matter of the real wage increasing as the economy moves up along the labor supply curve and the rental rate increasing as firms try to substitute capital that is in fixed supply for more expensive labor. A high consumption-constant labor supply elasticity η or a high intertemporal elasticity of consumption s reduce Ω because they make the labor supply curve flatter. A high degree of economy-wide returns-to-scale Γ and a high labor's share $1 - \theta$ reduce Ω because they reduce the amount of extra labor needed to produce a given amount of extra output.

4. CALIBRATION

4.1. Parameter Values

Because the model has been solved algebraically, it is easy to plug in different sets of parameter values. Based on microeconomic evidence, I have a serious disagreement with two of the parameter values traditional in the real business cycle literature—the traditional values for the elasticity of intertemporal substitution and for the labor supply elasticity. These parameters I discuss further below.

Based on Basu (1993), Basu and Fernald (1994a, b), and Basu and Kimball (1995), I choose $\gamma = 1.1$ and $\mu^* = 1.1$, implying $\epsilon^* = 11$. (With $\gamma/\mu^* = 1$ the steady state has zero pure profits.) In the absence of strong evidence against constant *marginal* cost, and the absence of any strong evidence of productive externalities, I choose $\aleph = 0$ and $\Gamma = \gamma = 1.1$. Thus, in these parameters, I assume only a modest departure from constant returns to scale and perfect competition.

The other, less controversial parameter values are assigned as follows: $\theta = (\mu^*/\gamma)(\rho + \delta)K^*/Y^* = .3$, $1 - \theta = (\mu^*/\gamma)W^*N^*/Y^* = .7$, $\delta = .08/\text{year}$, $\rho = .02/\text{year}$, and $\tau = W^*N^*/C^* = 1$. These imply $\zeta_C = C^*/Y^* = (1 - \theta)/\tau = .7$, $\zeta_I = I^*/Y^* = \delta\theta/(\rho + \delta) = .24$ and $\zeta_G = G^*/Y^* = 1 - \zeta_C - \zeta_I = .06$. (The value .06 is too low for the share of *all* government purchases in output, but seems reasonable for the share of government purchases that are not good substitutes for either private consumption or private investment.)

The two parameters most difficult to calibrate are the microeconomic rate of price adjustment α and $\mu'(1)/\mu(1)$, the elasticity of the desired markup with respect to a firm's market share—which is the key to calibrating ω. Given the assumption of constant marginal costs, making $\Phi y = 0$,

$$\omega = \frac{1}{\epsilon^*} + \frac{\mu'(1)}{\mu(1)} = .09 + \frac{\mu'(1)}{\mu(1)} .$$

Therefore, α and ω should be viewed as unknown parameters. It is useful, however, to remember that in order to make sticky prices plausible, large values of ω are needed. To get some sense of the possible values of ω, note that a value of 4.28 for $\mu'(1)/\mu(1)$, making $\omega = 4.37$, would require a 1% increase in market share ξ to cause a fall in the elasticity of demand from 11 to about 8, so that μ would increase from $^{11}/_{10} = 1.1$ to $^{8}/_{7} \approx 1.1428$.

The Elasticity of Intertemporal Substitution and Additive Nonseparability between Consumption and Labor. A large share of real business cycle models assume an elasticity of intertemporal substitution s equal to 1, which is implausibly high.[13]

13. Some papers, notably King, Plosser, and Rebelo (1988) depart from this default value of 1, but the traditional value of one continually reasserts itself.

Empirically, no one has been able to establish conclusively that the real interest rate has *any* effect on the timing of consumption, much less that a one percentage point increase in the real interest rate increases the expected growth rate of consumption by 1 percent, as it would if the elasticity of intertemporal substitution were equal to one. Given the standard errors in, say, Hall's (1988) regressions, the failure to find intertemporal substitution empirically is quite consistent with some positive elasticity of intertemporal substitution, but it is not consistent with an elasticity of intertemporal substitution equal to 1. Responses to survey questions based on choices in hypothetical situations confirm these econometric results. (See Barsky, Juster, Kimball, and Shapiro 1995). Even if the Permanent Income Hypothesis or its auxiliary assumptions are failing to some extent, an elasticity of intertemporal substitution as large as one should be enough to cut through a lot of model noise.

The primary justification for assuming an elasticity of intertemporal substitution equal to one is that this is the only value for which the King-Plosser-Rebelo (1988) utility function yields additive separability between consumption and labor. But I see no compelling reason to assume additive separability between consumption and labor. Indeed, when tested, additive separability is often rejected. (See, for example, Garcia, Lusardi, and Ng 1994.) Within the class of King-Plosser-Rebelo preferences, an elasticity of intertemporal substitution less than one implies that for any given level of consumption, laboring more raises the marginal utility of consumption. To me, this seems quite plausible. It means that, even in a permanent income setting, additional labor induced by a temporary increase in the shadow real wage tends to induce additional consumption during that period of additional labor. The positive effect of labor on consumption implied by a King-Plosser-Rebelo utility function with elasticity of substitution less than one gives it a fighting chance at explaining the business cycle fact that consumption and labor tend to move in the same direction. In a model with a smaller elasticity of intertemporal substitution, interest rate effects become less important and effects induced by fluctuations in the shadow real wage become more important.

Therefore, to be consistent with Hall (1988), let me try $s = .2$.

The Consumption-Constant Elasticity of Labor Supply. Typical microeconomic estimates of the labor supply elasticity are quite low. However, the regressions behind these estimates either use high-frequency variations in observed wages, confounding the observed real wage with the shadow real wage, or they rely on life-cycle variations in the real wage. The effects of life-cycle variations in the real wage are difficult to disentangle from other age-linked effects.

In the absence of clear microeconomic evidence, the real business cycle literature has traditionally chosen quite high values of the labor supply elasticity because those large values make it easier to fit the macroeconomic data. There is too little space here to present the critique by Hall (1987) and others of the Rogerson (1984) justification for high macroeconomic labor supply elasticities. Instead, let me suggest a disciplined way of judging the appropriate size of the consumption-constant labor supply elasticity which is robust to the presence of Rogersonian nonconvexities at the microeconomic level.

Whatever its microeconomic underpinnings, to be consistent with steady-state growth, the macroeconomic labor supply relation should imply that the income and substitution effects of permanently higher real wages cancel. This allows one to gauge the size of the substitution effect by thinking about the size of the income effect. Therefore, if one can construct a representative household, it should have a King-Plosser-Rebelo (1988) utility function, as above.

Given a King-Plosser-Rebelo utility function for the representative household, the slope of the income expansion path can be identified by looking at how consumption and labor are related when the (shadow) real wage is held constant. By (44), when $\check{W} = 0$,

$$\check{N} = -\eta\check{C} .$$

As income increases, the additional expenditure on consumption is

$$\tilde{C} = C^*\check{C}$$

while the additional expenditure on leisure is

$$-W^*\tilde{N} = -C^*\left(\frac{W^*N^*}{C^*}\right)\check{N} = -C^*\tau\check{N} = C^*\tau\eta\check{C} .$$

Thus, the marginal expenditure share of consumption h is

$$h = \frac{\tilde{C}}{\tilde{C} - W^*\tilde{N}} = \frac{1}{1 + \tau\eta} ,$$

and the marginal expenditure share of leisure $1 - h$ is

$$1 - h = \frac{\tau\eta}{1 + \tau\eta} .$$

Given either of these marginal expenditure shares, the consumption-constant labor supply elasticity can be calculated as

$$\eta = \frac{1 - h}{\tau h} .$$

With the calibration $\tau = W^*N^*/C^* = 1$, the labor supply elasticity η is equal to the ratio of the marginal expenditure share of leisure $1 - h$ to the marginal expenditure share of consumption h.[14]

14. As a note on the history of thought, this method of calibrating the labor supply elasticity from the marginal expenditure shares of leisure and consumption is closely akin to the way in which Prescott (1986) calibrates the labor supply elasticity. However, Prescott (1986) calibrates the labor supply elasticity using the *average* expenditure shares of leisure and consumption, with the implicit assumption that the

The thought experiment to get these marginal expenditures shares is to think how one would spend the proceeds of lottery winnings. What percentage would be spent on increased consumption and what percentage would be spent on reduced work hours? If income and substitution effects are to cancel, anyone who wishes to argue for a high labor supply elasticity must also be prepared to defend the idea that people would spend a substantial fraction of such windfalls on reduced work. For what it is worth, a sample of students asked to imagine a future situation in which they were working had an average marginal expenditure share of leisure equal to about .25. Even though the modal response was zero, many indicated they would spend a substantial fraction of a windfall on reducing work hours. This implies a consumption-constant labor supply elasticity of about $.25/.75 = .33$. A marginal expenditure share of leisure of .5, which is as high as seems at all plausible to me, would yield a consumption-constant labor supply elasticity of about 1. The only way to get the infinite labor supply elasticity that appears in some Real Business Cycle models and still have income and substitution effects cancel is to have a marginal expenditure share of leisure equal to 1. That is, those models imply that a household would spend 100 percent of a windfall on reducing labor supply. Or, to the extent that the high labor supply elasticity arises from mechanisms at a higher level than that of the individual household, they imply that 100 percent of a windfall to the class of workers as a whole would be spent on reducing labor. This is a typically unnoticed feature of these models. In order to get the highest plausible value of η, take the somewhat extreme case in which half of a windfall is spent on consumption and half on leisure ($h = .5$), so that $\eta = 1$.

4.2. *General Equilibrium Elasticities Implied by These Parameter Values*

Substituting in the chosen parameter values, the short-run behavior of the model is given by

$$\begin{bmatrix} \check{N} \\ \check{C} \\ \check{I} \\ \check{W} \\ \check{R} \\ \check{\phi} \\ \dot{\check{K}} \\ \dot{\check{\lambda}} \end{bmatrix} = \begin{bmatrix} 1.30 & -.43 & 0 \\ 1.04 & -.34 & -.20 \\ 1.14 & 1 & .58 \\ 2.34 & -.77 & -.20 \\ 3.64 & -2.20 & -.20 \\ 2.73 & -1.20 & -.20 \\ .091 & 0 & .047 \\ -.363 & .220 & .020 \end{bmatrix} \begin{bmatrix} \check{Y} \\ \check{K} \\ \check{\lambda} \end{bmatrix} . \tag{52}$$

average and marginal expenditure shares are equal. This gave rise to the question of how to deal with sleep in calculating the average expenditure share of leisure, an issue which made an enormous difference in the implied labor supply elasticity. But the question of how to deal with sleep can be seen for the red herring it is once one focuses proper attention on the marginal expenditure shares as opposed to the average expenditure shares.

The elasticity of ϕ with respect to output, 2.73, is equal to Ω. Taking a stab in the dark, if $\mu'(1)/\mu(1)$ is equal to the value 4.28 which I entertain above, then $\omega = 4.37$ and

$$\sqrt{\frac{\Omega}{\epsilon^* \omega}} \approx .24 \; ,$$

implying that macroeconomic price adjustment would take more than four times as long as it takes an individual firm to adjust its price.

The Elasticity of Intertemporal Substitution and the Response of Saving and Investment to a Monetary Stimulus. There is not space here to make a detailed comparison of the general equilibrium elasticities above with the data, but one elasticity that seems at variance with the data is the elasticity of saving and investment with respect to output (in the context of a monetary stimulus): 1.14. The problem is that with $s = .2$, labor and consumption are such strong complements that most of the additional output produced by additional labor input is eaten up as consumption, so that there is little extra saving to be invested. Indeed, one can calculate that in response to a monetary stimulus, the general equilibrium marginal propensity to consume in this model is $(1 - s)\,\frac{\gamma}{\mu^*\Gamma}$ and the general equilibrium marginal propensity to save is $\frac{\gamma}{\mu^*\Gamma}\left[\frac{\mu^*\Gamma}{\gamma} - 1 + s\right]$. Increasing either the elasticity of intertemporal substitution s or the steady-state markup ratio μ^* can have a big effect in raising the marginal propensity to save from the value of .27 implied by the parameters above. Consider an increase in the elasticity of intertemporal substitution to $s = .5$. Then the general equilibrium marginal propensity to save becomes .55, and the elasticity of investment with respect to output (in the context of a monetary stimulus) becomes 2.27. The elasticity of consumption with respect to output falls to .65, but neither the elasticities of N, W, R, and ϕ with respect to output nor the convergence rate change much.

5. DOES THE REAL INTEREST RATE FALL IN RESPONSE TO A PERMANENT INCREASE IN THE MONEY SUPPLY?

As mentioned above, as long as the relative price of capital is fixed at 1, so that $r = R - \delta$, the real interest rate r must go up with output because the real rental rate R does. Therefore, in the model above, a monetary expansion *raises* the real interest rate, just as in King's (1991) model, and for essentially the same reason.

To use the language of intermediate macroeconomics, a relative price of capital fixed at 1 leads to an upward-sloping IS curve, due to the positive effect of aggregate output on the real rental rate of capital. The quantity equation yields a vertical LM curve, but one could easily combine a more general LM curve with this upward-sloping IS curve.

When investment adjustment costs allow the relative price of capital Q to vary, it becomes possible for a monetary expansion to cause the real interest rate to fall. But movements in Q must be substantial to overcome the positive effect of the rental rate on the real interest rate. This section is devoted to discussing that quantitative issue.

Either of two different technical tricks allow me to discuss the conditions for a monetary expansion to lower the real interest rate without going through the entire Q-theory neomonetarist model laid out in Kimball (1995). First, some simple inequalities relate the model with investment adjustment costs to the model without investment adjustment costs. Second, I look at an approximation in which the investment adjustment cost parameter j goes to zero, but $j\alpha$ has a finite positive limit. Then, in the limit, the behavior of the classical side of the model becomes the same as in the absence of investment adjustment costs, but the interaction between the investment adjustment costs and the convergence of the model to full employment (which takes place at a rate proportional to α) remains something to be reckoned with.

5.1. *Why the Answer Depends on the Size of the Investment Adjustment Cost and the Rate of Price Adjustment*

Q theory implies that the increase in investment induced by a monetary expansion must be associated with an increase in Q, the relative price of capital. As the aggregate price adjusts and the economy returns to full employment, investment I—and therefore Q—falls back to its normal level at the rate of price convergence κ. The capital loss from the falling price of capital lowers the rate of return on capital, which by arbitrage must equal the real interest rate r. The greater the investment adjustment costs, the more Q will rise and then fall, and the bigger the downward pressure on the real interest rate. The faster the economy returns to full employment, the greater the rate of capital loss, and the more likely it is that the real interest determined by arbitrage will fall.

To describe the same interest-rate-determination mechanism in levels instead of rates of change, consider the interaction between saving supply and investment demand. If—with no change in the real interest rate—the increase in investment demand would exceed the increase in saving supply induced by the expansion in output, then the real interest rate must rise to choke off investment demand and restore equilibrium. If, on the other hand, the increase in saving supply would exceed the increase in investment demand without a change in the real interest rate, then the real interest rate must fall to stimulate investment demand and restore equilibrium.

In response to a monetary expansion, saving supply depends almost entirely on current output. To a first approximation, the elasticity of saving supply with respect to a money-driven increase in output does not depend on the rate at which the economy returns to full employment and is also not very sensitive to modest changes in the size of the adjustment cost. But the elasticity of investment demand with respect to a permanent increase in the money supply depends strongly on both the size of the adjustment cost and the rate at which the economy returns to full employment. A larger investment adjustment cost reduces the elasticity of investment demand with respect to Q. Quicker convergence back to full employment means that the rental rate will only be high for a short time, leading to a smaller movement in Q. Thus, high investment adjustment costs and quick convergence rates make investment demand increase less in relation to the increase in saving supply, making it more likely for the real interest rate to fall in response to a permanent increase in the money supply.

5.2. *Investment Adjustment Costs and the Real Interest Rate*

To make the logic of arbitrage or investment demand and saving supply quantitative, it is necessary to derive an equation relating the real interest rate to the real rental rate R and the relative price of capital Q when there are investment adjustment costs.

With investment adjustment costs modeled as in Hayashi (1982), the owner of capital, who rents it out to firms, solves

$$\max_{I} \int_0^\infty e^{\int_0^{t'} r\, dt''} [RK - I]\, dt' ,$$

s.t.

$$\dot{K} = KJ(I/K) . \tag{53}$$

The function J giving the rate of increase in the capital stock for a given investment/capital ratio satisfies $J'(\cdot) \geq 0$, $J''(\cdot) \leq 0$, $J(\delta) = 0$, and $J'(\delta) = 1$, where δ is defined as I^*/K^*.[15]

The current value Hamiltonian is

$$H = RK - I + QKJ(I/K) .$$

The first-order condition for investment is

$$1 = QJ'(I/K) ,$$

Since $I^*/K^* = \delta$, in steady state the first-order condition implies $Q^* = 1$. The first-order condition can be log-linearized as

$$\check{Q} = j[\check{I} - \check{K}] , \tag{54}$$

where

$$j = \frac{-(I^*/K^*)J''(I^*/K^*)}{J'(I^*/K^*)}$$

15. The model without investment adjustment costs has $J(I/K) = (I/K) - \delta$, so that $J''(\cdot) = 0$.

After using the first-order condition and dividing through by Q, the Euler equation is

$$\frac{\dot{Q}}{Q} = r - J(I/K) + \frac{(I/K) - R}{Q} . \tag{55}$$

In the steady state, $I^*/K^* = \delta$, $Q^* = 1$ and $J(I^*/K^*) = J(\delta) = 0$ imply $r^* + \delta = R^*$ as in the absence of investment adjustment costs. In log-linearizing the Euler equation, since $J'(I^*/K^*) = (1/Q^*) = 1$, the derivative of the second line of (55) with respect to I/K is zero, and

$$\dot{\check{Q}} = \tilde{r} - \tilde{R} + r^*\check{Q} . \tag{56}$$

In integral form, using the steady state as a terminal condition,

$$\check{Q}_t = \int_t^\infty e^{-r^*(t'-t)}[\tilde{R} - \tilde{r}] , \tag{57}$$

implying that expectations of fairly distant future events have an important impact on Q and therefore on investment.

Finally, given $(I^*/K^*) = \delta$ and $J'(\delta) = 1$, the accumulation equation (53) can be log-linearized to

$$\dot{\check{K}} = \delta[\check{I} - \check{K}] . \tag{58}$$

Note that (58) is identical to (46), since investment adjustment costs play only a second-order role in the accumulation equation itself.

5.3. *What Is Needed for a Permanent Increase in the Money Supply to Lower the Real Interest Rate?*

Since $\tilde{R} = R^*\check{R} = (r^* + \delta)\check{R}$, and $\check{Q} = j[\check{I} - \check{K}]$, equation (56) can be rewritten

$$\tilde{r} = (r^* + \delta)\check{R} + j[\dot{\check{I}} - \dot{\check{K}}] - r^*j[\check{I} - \check{K}] . \tag{59}$$

As the rate of price adjustment α gets large, the one big component of $\dot{\check{I}}$ becomes

$$\dot{\check{I}} \approx -\kappa\check{I} \approx -\alpha \sqrt{\frac{\Omega}{\epsilon^*\omega}}\, \check{I} .$$

Furthermore, $\dot{\check{K}} = \delta[\check{I} - \check{K}]$, while at time zero, $\check{K} = 0$. Making these substitutions,

$$\tilde{r} = (r^* + \delta)\check{R} - [r^* + \delta + \kappa]j\check{I} .$$

Therefore, an approximate necessary condition for a permanent increase in the money supply to lower the real interest rate is

$$[r^* + \delta + \kappa]j > (r^* + \delta)\left.\frac{\check{R}}{\check{I}}\right|_{j>0} > (r^* + \delta)\left.\frac{\check{R}}{\check{I}}\right|_{j=0}. \tag{60}$$

Why do I claim in (60) that the ratio $\check{R}/\check{I}$ of the rental rate elasticity to the investment elasticity is higher when $j > 0$ than when $j = 0$? First, investment adjustment costs lead to less of an output expansion being devoted to investment. Second, since more of an output expansion is therefore spent on consumption, the relationship $W = Cv'(N)$ guarantees that the real wage goes up more and capital/labor substitution guarantees that the real rental rate goes up more as well.[16]

More formally, as $j \to 0$ and $\alpha \to \infty$, the only thing in (59) that is not zeroed out when multiplied by j is the revision of investment to its normal level as the economy goes back to full employment after the monetary stimulus In the limit as $j \to 0$ and $\alpha \to \infty$ the necessary condition for a permanent increase in the money supply to lower the real interest rate is

$$\lim_{\alpha\to\infty, j\to 0} j\kappa > (r^* + \delta)\left.\frac{\check{R}}{\check{I}}\right|_{j=0} = \frac{\delta\theta\left[\tau(1 - s) + \frac{1}{\eta} + 1\right]}{(1 - \theta)\left[\frac{\mu^*\Gamma}{\gamma} - 1 + s\right]} \tag{61}$$

using on the right-hand side the movements in R and I predicted by the model without investment adjustment costs—the elasticities given in the first column of (51). With $j \to 0$, (61) is consistent with (60). Using the parameter values given above, the condition (61) becomes $j\kappa > .32$/year when $s = .2$ and $j\kappa > .14$/year when $s = .5$.

Condition (60) looks the same as (61), but with $j(r^* + \delta + \kappa)$ in place of $j\kappa$. Conditions (61) and (60) are related in much the same way as a χ^2 test based on asymptotic theory and the corresponding F test which may be a better approximation even though it does not have as clean a formal justification.

5.4. *Calibrating* j *and* κ

By (54), the elasticity of I/K with respect to Q is $1/j$. Regressions of I/K on Q using stock and bond prices to measure Q have often found quite low coefficients, implying quite high values of j. However, these high values of j give an implausibly slow partial equilibrium adjustment rate for the capital stock. Using a new method

16. Kimball (1995) provides more details.

of identification based on statutory tax changes, Cummins, Hassett, and Hubbard (1994) find much larger elasticities of I/K with respect to Q—on the order of 5, implying $j = .2$. This value for j is broadly consistent with the findings of Shapiro (1986), who estimates adjustment costs using an Euler equation that implicitly defines Q in a way akin to (57) rather than using volatile stock and bond prices.

If $j = .2$, condition (61) for a monetary stimulus to lower the real interest rate becomes $\kappa > 1.60$/year when $s = .2$ and $\kappa > .70$/year when $s = .5$. Using (60) would subtract 10 percent per year from these necessary convergence rates. In other words, for a permanent increase in the money supply to lower the real interest rate, the half-life for macroeconomic price-adjustment must be less than 2 quarters when $s = .2$ or less than about 4 quarters when $s = .5$. This seems too fast if monetary shocks are to be an important cause of business cycles. Thus, by my judgment, plausible parameter values imply that a monetary expansion should lead to an increase in the real interest rate.[17]

To make things more concrete, even with $s = .5$, the model above says that a 1 percent increase in output raises the real rental rate by about .32 percent/year (32 basis points), while the supply of saving and therefore the level of investment increases by only about 2.27 percent. Given $j = .2$, Q needs to rise by only about .47 percent to yield this increase in investment demand. Therefore, most of the increase in Q must disappear in less than a year in order to have a capital loss sufficient to make the rate of return on capital and the real interest rate fall.

In the language of intermediate macroeconomics, the IS curve is stubbornly upward sloping. It may be necessary to model any real-world tendency for the real interest rate to fall in response to a monetary stimulus as a result of output being temporarily off the IS curve.

6. CONCLUSIONS

Though there is much more that could be said about the neomonetarist model above, several things are apparent.

First, a kind of classical dichotomy separating fluctuations caused by monetary forces from movements caused by real forces can be justified as an appropriate approximation when prices adjust quickly.

Second, an elasticity of demand falling sharply with a firm's market share is both easy to model and effective at reducing the private cost of price rigidity, making price stickiness more plausible.

Third, mechanisms that increase real rigidity and thereby make sticky prices more plausible also tend to slow down the general equilibrium adjustment of prices. This general equilibrium adjustment rate is not the same as the rate α at which indi-

17. However, as can be seen from (61), increasing any of j, η, or μ^* would slow down the necessary convergence rate considerably. Increasing either j or η would make investment demand less sensitive to money-driven movements in output, while saving supply would become more sensitive to money-driven movements in output if μ^* increased.

vidual firms are assumed to change their prices. If there is enough real rigidity to make sticky prices plausible, the general equilibrium adjustment rate is likely to be significantly slower than α.

Fourth, the exact nature of technology and preferences (including the elasticities of labor supply and intertemporal substitution) has an important effect even on the short-run behavior of the model before prices adjust.

Fifth, even when investment adjustment costs are introduced to give the real interest rate a fighting chance to go down in response to a monetary expansion, plausible parameter values imply that the real interest rate will *increase* in response to a monetary expansion.

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