Micro II, Dominic Rohner, Spring 2005

Problem Set 1

1. We analyze a contract between an employer (the Principal) and an employee (the Agent) in a context of moral hazard. The agent can exert two levels of effort, e^H and e^L, which have costs $c^H = 1$ and $c^L = 0$. The employee's reservation utility is $\overline{U} = 1$. The Principal is risk-neutral, and the Agent is risk-averse, with a utility function $u(w) = \sqrt{w}$.

   There are two possible outcomes, $x^L = 2$ and $x^H = 10$. The probabilities of these outcomes depending on the Agent's effort levels are given in the following table:

	x^L	x^H
e^L	1	0
e^H	$\frac{1}{2}$	$\frac{1}{2}$

   (a) First, assume that the Principal can observe the Agent's effort level and chooses to pay a wage w^H for effort level e^H and w^L for effort level e^L.

      i. Write the expected profit of the Principal when the agent chooses effort level e^H, Π^H, and when they choose effort level e^L, Π^L.

      Solution:

$$\begin{aligned}\Pi^H &= 2\frac{1}{2} + 10\frac{1}{2} - w^H, \\ &= 6 - w^H, \\ \Pi^L &= 2 - w^L.\end{aligned}$$

ii. Write the participation constraints of the Agent when choosing effort level e^H and effort level e^L.

Solution: We find:

$$\sqrt{w^H} - 1 \geq \overline{U} = 1$$
$$\sqrt{w^L} \geq 1$$

iii. What are the wages paid by the Principal in the optimal contract? What is the expected profit of the Principal when choosing to implement effort level e^L and effort level e^H? Show that the Principal prefers to implement effort level e^H.

Solution: From the participation constraints, we find $w^H = 4$ and $w^L = 1$. Thus, $\Pi^H = 6 - 4 = 2$ whereas $\Pi^L = 2 - 1 = 1$.

(b) Now assume that the Principal cannot observe the Agent's effort and pays wage w^H when the *outcome* is x^H and wage w^L when the *outcome* is x^L. First, assume that the Principal chooses to implement effort level e^H.

i. Write the expected profit of the Principal

Solution: We calculate

$$\Pi^H = 6 - \frac{1}{2}w^H - \frac{1}{2}w^L$$

ii. Write the participation constraint of the Agent choosing effort e^H

Solution: We find

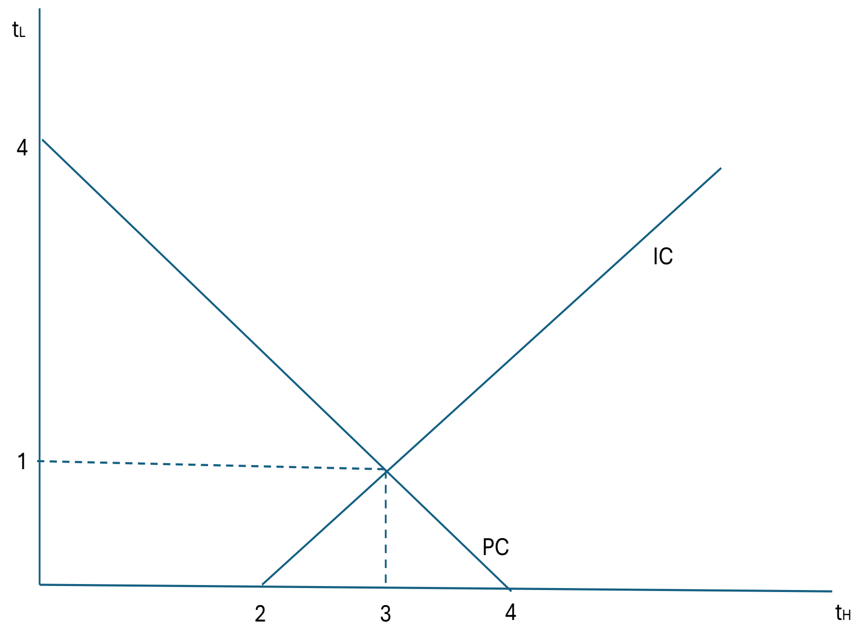
$$\frac{1}{2}\sqrt{w^H} + \frac{1}{2}\sqrt{w^L} - 1 \geq 1.$$

iii. Write the incentive constraint of the Agent

We find
$$\frac{1}{2}\sqrt{w^H} + \frac{1}{2}\sqrt{w^L} - 1 \geq \sqrt{w^L}.$$

iv. Plot the participation and incentive constraints of the Agent and the Principal's isoprofit curves in the space (t^H, t^L) where $t^H = \sqrt{w^H}, t^L = \sqrt{w^L}$.

Solution:



v. What is the optimal contract? Compute the expected profit of the Principal.

Solution: Optimal contract is $w^H = 9, w^L = 1$ (intersection in the figure). Profit is $\Pi^H = 6 - \frac{9}{2} - \frac{1}{2} = 1$.

(c) Now, suppose that the Principal chooses to implement the effort level e^L.

   i. Write the expected profit of the Principal:

      Solution: $\Pi^L = 2 - w^L$.

   ii. Write the participation constraint of the Agent who chooses the effort level e^L:

      Solution: We obtain:

$$\sqrt{w^L} \geq 1.$$

   iii. Deduce the optimal wage and the expected profit of the Principal. Show that the Principal is indifferent between the two effort levels.

      Solution: With $w^L = 1$, the expected profit is equal to $\Pi^L = 2 - 1 = \Pi^H$.

2. We now study a contract between a seller and a buyer. The buyer has a utility function given by $\theta q - pq$, where q is the quantity purchased and p is the unit price. There are two types of buyers: low-demand buyers ($\theta = 1$) and high-demand buyers ($\theta = 2$). It is assumed that there is a fraction $\frac{2}{3}$ of low-demand buyers and a fraction $\frac{1}{3}$ of high-demand buyers. Each buyer has a reservation utility of 0. Finally, the seller has a quadratic production cost given by $c(q) = q^2$.

   The seller cannot distinguish between low-demand and high-demand buyers and offers two contracts, (p^H, q^H) and (p^L, q^L), to separate them.

   (a) Write the expected profit of the seller when buyers of type H and type L self-select:

      Solution:

$$\Pi = \frac{2}{3}(p^L q^L - (q^L)^2) + \frac{1}{3}(p^H q^H - (q^H)^2).$$

(b) Write the participation constraints for both types of agents:

Solution:

$$\begin{aligned} q^L - p^L q^L &\geq 0, \\ 2q^H - p^H q^H &\geq 0. \end{aligned}$$

(c) Write the incentive constraints for both types of agents:

Solution:

$$\begin{aligned} q^L - p^L q^L &\geq q^H - p^H q^H, \\ 2q^H - p^H q^H &\geq 2q^L - p^L q^L. \end{aligned}$$

(d) Use the participation constraint of L-type agents and the incentive constraint of H-type agents to express $p^L q^L$ and $p^H q^H$ only as functions of q^L and q^H.

Solution: We obtain:

$$\begin{aligned} p^L q^L &= q^L, \\ p^H q^H &= 2q^H - q^L. \end{aligned}$$

(e) By substituting these expressions into the seller's expected profit, determine the optimal contract quantities q^H and q^L.

Solution: We obtain:

$$\Pi = \frac{2}{3}(q^L - (q^L)^2) + \frac{1}{3}(2q^H - q^L - (q^H)^2).$$

By differentiating with respect to q^L and q^H, we find:

$$q^L = \frac{1}{4}, q^H = 1.$$

(f) Verify that the quantity q^H corresponds to the efficient quantity (which maximizes total surplus), whereas the quantity q^L is below the efficient level.

Solution: The total surplus for H-type agents is $2q^H - (q^H)^2$, which is maximized when $q^H = 1$. The total surplus for L-type agents is $q^L - (q^L)^2$, which is maximized at $q^L = \frac{1}{2} > \frac{1}{4}$.

(g) Which of the two prices, p^H or p^L, is higher?

We find that $p^L = 1$, and:

$$p^H = 2 - \frac{q^L}{q^H} = 2 - \frac{1}{4} = \frac{7}{4} > 1 = p^L.$$

Multiple Choice Question
Tick all boxes with correct answers.

- ☐ Adverse selection is about hidden actions
- ☐ In the Spence signalling model the Principal moves first.
- ✓ In Adverse Selection, the bad type is pushed to the reservation utility level, whereas the good type collects an informational rent.
- ☐ In Moral Hazard models, the relation between effort and the result is deterministic.
- ✓ In the context of Expected Utility Theory, strict risk-aversion and strict concavity of the utility function are equivalent

Short Question: Describe the Independence Axiom of Expected Utility Theory

Solution: See slides 12-13 of the Behavioral Economics slides.