Macroeconomics A: Review Session V

Ramsey Model

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Ramsey vs. Solow

- The Solow model assumes an exogenous saving rate
- Ramsey model features a representative household that chooses the saving rate optimally
  - Explains long-run economic growth rather than business cycle fluctuations
  - Solved in continuous time typically (math is more pleasant)
- Household makes a choice between consumption and saving
  - It cares about future consumption
  - Take that $C = Y - I$ and that $Y = F(K, L)$
  - Capital depreciates at some rate, population grows
  - Household saves because it has to replace depreciated capital
  - Also, if $K < K^*$ then MPK is high and saving increases output
  - Model describes path from initial starting point to C^* and K^*
- Transversaility condition is important: defines beginning and end states and rules out absurd solutions

Ramsey Model in Continuous Time

- Let's solve the Ramsey model in continious time

$$U(0) = L(0) \int_0^\infty e^{-(\rho-n)t} \frac{c(t)^{1-\theta}}{1-\theta} dt$$

- The household budget constraint is

$$c(t) + \dot{k}(t) + (n+\delta)k(t) = w(t) + r(t)k(t) \tag{1}$$

- We can write the Hamiltonian as

$$\hat{\mathcal{H}} = e^{-(\rho-n)t} \left[\frac{c(t)^{1-\theta}}{1-\theta} + \lambda(t)(w(t) - c(t) - (n+\delta-r(t))k(t)) \right]$$

- The FOCs for c and k are

$$c(t)^{-\theta} = \lambda(t) \tag{2}$$

$$\lambda(t)(r(t) - n - \delta) = (\rho - n)\lambda(t) - \dot{\lambda}(t) \tag{3}$$

Solving the Ramsey Model

- Taking the time derivative of (2)

$$-\theta c(t)^{-\theta-1}\dot{c}(t) = \dot{\lambda}(t)$$

- Plugging this into (3)

$$c(t)^{-\theta}(r(t) - n - \delta) = (\rho - n)c(t)^{-\theta} + \theta c(t)^{-\theta-1}\dot{c}(t)$$

- Simplifying

$$\frac{\dot{c}(t)}{c(t)} = \frac{1}{\theta}(r(t) - (\delta + \rho))$$

- Under perfect competition $r(t) = f'(k(t))$
- When $\dot{c}(t) = 0$ then

$$f'(k(t)) = \delta + \rho$$

The Firm Problem

- Firms can pay either
  - Labor (wages)
  - Capital (rent/interest and sometimes depreciation)
  - Profits (dividends)
- Under perfect competition, profits are zero and all payments go to capital or labor
- With a Cobb-Douglas production function $Y = K^\alpha L^{1-\alpha}$ payments to capital are αY and payments to labor $(1-\alpha)Y$
- In other words, by the FOCs

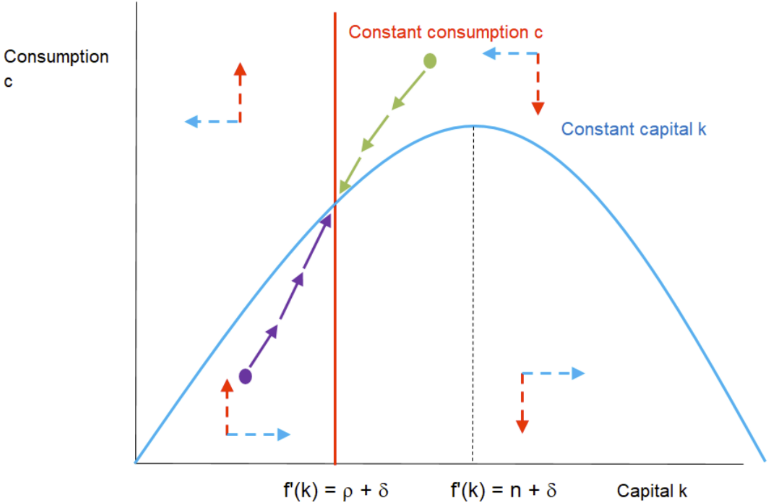
$$rK = \alpha Y \quad \text{and} \quad wL = (1-\alpha)Y$$

- Therefore $Y = rK + wL$ and putting this in the budget constraint (1)

$$\dot{k}(t) = \underbrace{f(k(t))}_{y(t)} - (n+\delta)k(t) - c(t)$$

- Here investment is $\dot{k}(t) + (n+\delta)k(t)$ so we have $y = c + inv$

Saddle Path



Consumption
c
Constant consumption c
Constant capital k
$f'(k) = \rho + \delta$
$f'(k) = n + \delta$
Capital k