Macroeconomics A, EI056

Class 7

The New Keynesian model

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What you will get from today class

- What is the effect of **monetary shocks**? Evidence from US (Romer and Romer 2023).
- **Strategic complementarity**: how a firm's choice depends on the choice made by other firms.
  - How to get persistent real effect of monetary policy.
- New Keynesian model: **sticky prices** in the **R**eal **B**usiness **C**ycle model (in other words, IS-TR with optimization).
  - Consumption of several **varieties** of goods.
  - New Keynesian Phillips curve, **NKPC**, the forward-looking nature of inflation.
- **Trade-off** in monetary policy and a new gain of commitment.
- Recent **developments**: financial frictions, limit on interest rate, heterogeneous agents.

A question to start

When a country faces a decrease in productivity, the central bank should adopt an expansionary monetary policy to support growth and employment.

The New Keynesian model calls for the opposite. It is therefore mis-specified.

Do you agree? Why or why not?

EVIDENCE ON EFFECT OF

MONETARY SHOCKS

Identifying shocks

- Do change in monetary policy affect GDP and inflation?
- Challenge in setting causality: if a recession leads to lower interest rate, one should not read lower rates as causing the recession.
- Variety of approaches to isolate a shock coming from monetary policy.
  - VAR with sign restrictions.
  - High-frequency events (changes of long-term rates in the minutes surrounding a policy decision).
  - Narrative approach from the minutes of the policy makers meetings.

TABLE 1—REQUIREMENTS FOR RIGOROUS NARRATIVE ANALYSIS

1. A reliable narrative source
- Real time
- Consistent over time
- Detailed and accurate

2. A clear idea of what one is looking for in the source
- Specify criteria in detail

3. Approach the source dispassionately and consistently
- Resist the temptation to see what you want to see in the source
- Compare classifications with another reader
- Read from beginning to end
- Don't use previous knowledge to focus on certain periods

4. Document the narrative evidence carefully
- Force yourself to explain your reasoning
- Make it easy for others to check your work

Romer, Christina, and David Romer (2023). "Presidential Address: Does Monetary Policy Matter? The Narrative Approach after 35 Years", *American Economic Review* 113 (6), pp. 1395-1423.

Narrative approach for the Federal Reserve

- Use the minutes from the meetings of the FOMC committee.
  - Detailed and open discussion on the economic situation.
- Times where policy changes not just because the economic cycle has moved.
  - Dummy variable for such meetings (no fine-tuning of magnitude).
  - No shocks post-1988, as policy making became more rule-based.

TABLE 2—MONETARY POLICY SHOCKS, 1946–2016

New dates			Original dates		
October	1947	(–)	October	1947	(–)
August	1955	(–)	September	1955	(–)
September	1958	(–)			
December	1968	(–)	December	1968	(–)
January	1972	(+)			
April	1974	(–)	April	1974	(–)
August	1978	(–)	August	1978	(–)
October	1979	(–)	October	1979	(–)
May	1981	(–)			
December	1988	(–)	December	1988	(–)

Notes: Contractionary shocks are denoted (–) and expansionary shocks are denoted (+). In setting our original dates, we did not have a classification for expansionary shocks.

Romer, Christina, and David Romer (2023). "Presidential Address: Does Monetary Policy Matter? The Narrative Approach after 35 Years", *American Economic Review* 113 (6), pp. 1395-1423.

- Tightening of monetary policy reduces GDP growth, and inflation (with some delay).

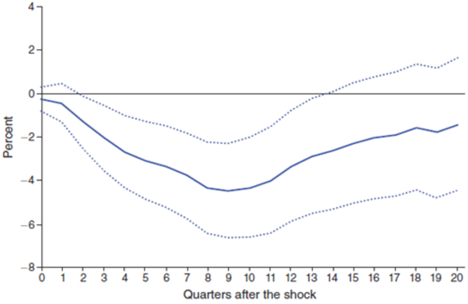

Figure 3. Response of Real GDP to a Monetary Policy Shock

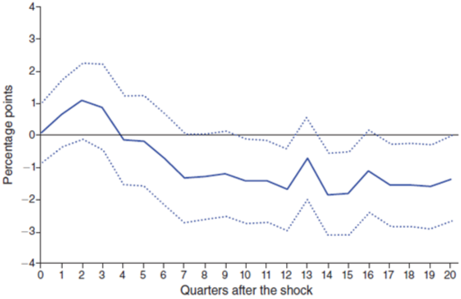

Figure 5. Response of GDP Price Index Inflation to a Monetary Policy Shock

Romer, Christina, and David Romer (2023). “Presidential Address: Does Monetary Policy Matter? The Narrative Approach after 35 Years”, *American Economic Review* 113 (6), pp. 1395-1423.

STRATEGIC COMPLEMENTARITY

Nominal and real rigidities.

- Models with **sticky** prices and wages: only change at infrequent intervals.
  - Time-dependent pricing: adjustment at fixed intervals (every month, quarter, year...). Simpler and standard, we focus on this.
  - State-dependent pricing: adjustment when the current price is too different from what the firm would set if it could choose (more appealing, but more complex to handle).
- **Nominal rigidity**: a firm that does not adjusting its price may face only a small amount of foregone profits, so it does not bother.
- **Real rigidity**: even when a firm can change its price, it does not adjust it all the way (in response to the shock).
  - Incomplete macroeconomic adjustment, even when all firms have adjusted at least once.
  - Due to strategic complementarity between firms.

Strategic complementarity

- There are multiple goods, which are **imperfect substitutes**.
- Price set by a firm j (p_j) is a function of:
  - **Aggregate** conditions: higher p_j following higher aggregate demand, for instance because of a monetary expansion (m).
  - **Competition**: keep p_j not to far from aggregate p (price of competitors).

$$p_j = am + (1-a)\,p \qquad ; \qquad p = \int_s p_s ds \qquad ; \qquad a < 1$$

- Can monetary expansion raise output persistently?
  - In other words: is there a real impact of a monetary shock even when **all** firms have adjusted their price at least once?

What is needed for persistence

- Price **stickiness**: no real effect when prices are fully flexible (all p_s move right away).
- **Staggering**: not all firms adjust at the same time (otherwise immediate adjustment once p_s can move) . Different prices coexist.
  - Example 1: each month 50% of first reset the price for two months.
  - Example 2: **"Calvo**" pricing, reset with some probability and keep the price in place until the next random opportunity.
- **Fixed** prices: choose a price applying to several periods, which is the **same** in each period (strong period-to-period link).
  - Not enough to choose a price for several periods, if we can set (today) different prices for different future periods (effectively disconnects the periods).
- Under Calvo price setting,
  - The share of firms that never had a chance to adjust converges to zero.
  - The real effect also converges to zero, but more slowly (even firms that adjust do not do so fully right away).

HOUSEHOLD'S CHOICE

Utility and budget constraint

- Need to have different goods with different prices → cannot have a model with just one good.
- Identical households (representative) value consumption C and dislike effort N:

$$U_t = E_t \sum_{i=0}^{\infty} \beta^i \left[\frac{(C_{t+i})^{1-\sigma}}{1-\sigma} - \chi \frac{(N_{t+i})^{1+\eta}}{1+\eta} \right] \qquad ; \qquad \beta \in (0,1)$$

- Income: wage WN and profits Π (taken as given) from the firms owned by the household. Saving in a **nominal bond** B with nominal interest rate i:

$$P_t C_t + B_t = (1 + i_{t-1}) B_{t-1} + W_t N_t + \Pi_t$$

- $B = 0$ in equilibrium (all households are identical), only role is to generate an **equilibrium interest rate**.

Varieties of goods

- Consumption is across different **varieties** of goods (not a single good) indexed by j.
  - Varieties (brands) are **imperfect substitutes**. So firms can set different prices for different varieties.
- Overall consumption C is a"constant elasticity of substitution" (**CES**) index of each variety consumption C_j (total number of varieties is 1).

$$C_t = \left[\int_0^1 [C_{j,t}]^{\frac{\theta - 1}{\theta}} dj \right]^{\frac{\theta}{\theta - 1}}$$

- $\theta > 1$ is the **elasticity of substitution** across brands (perfect competition is $\theta \to \infty$).

Allocation of consumption across varieties

- Minimize the expenditure required to purchase a given basket, subject to a target for C (derivation in the short technical problems):

$$\min_{C_j\text{'s}} \int_0^1 P_j C_j dj \qquad \text{subject to} \qquad \left[\int_0^1 [C_j]^{\frac{\theta-1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}} = C$$

- Demand for variety j reflects aggregate consumption and the **relative price** of the variety (price **elasticity** is the degree of substitutability θ):

$$C_j = \left[\frac{P_j}{P}\right]^{-\theta} C$$

- Consumer price index P: minimum expenditure required to purchase $C = 1$:

$$P = \left[\int_0^1 [P_j]^{1-\theta} dj\right]^{\frac{1}{1-\theta}}$$

Labor supply and dynamics choice (similar to RBC)

- Labor-consumption choice gives the **labor supply**:

$$\frac{W_t}{P_t} = \chi \left(N_t\right)^{\eta} \left(C_t\right)^{\sigma}$$

- Intertemporal allocation(consumption-savings) gives the **Euler condition**:

$$\frac{1}{P_t} \left(C_t\right)^{-\sigma} = \beta \left(1 + i_t\right) E_t \left[\frac{1}{P_{t+1}} \left(C_{t+1}\right)^{-\sigma} \right]$$

- Expected real interest rate = nominal rate adjusted by expected inflation: $E_t \left(1 + i_t\right) P_t / P_{t+1}$.
- Household's choice summarized by demands for C_j, labor supply, and the Euler.

PRICING BY FIRMS

Technology and cost

- Continuum of firms, each is the **monopoly** producer of a variety.
  - Takes account of demand for $C_{j,t}$ as a function of $P_{j,t}$ when making its choice.
- Technology uses only labor, with decreasing returns ($a \leq 1$). Output fully consumed (no capital). Productivity Z common to all firms (Walsh focuses on $a = 1$):

$$Y_{j,t} = C_{j,t} = Z_t \left(N_{j,t}\right)^a$$

- Real **marginal cost** Ψ_j :

$$\Psi_{j,t} = \frac{\partial}{\partial Y_{j,t}} \left[\frac{W_t}{P_t} N_{j,t} \right] = \frac{1}{a} \frac{W_t}{P_t} \left(Y_{j,t}\right)^{\frac{1-a}{a}} \left(\frac{1}{Z_t} \right)^{\frac{1}{a}}$$

- The firm chooses a price $P_{j,t}$ to maximize profits (possibly expected profits over several periods).

Solution under flexible prices

- Set price $P^*_{j,t}$ to maximize profits, subject to the demand from households:

$$\max\ P^*_{j,t} Y_{j,t} - W_t \left(\frac{Y_{j,t}}{Z_t} \right)^{\frac{1}{a}} \qquad \text{subject to} \qquad Y_{j,t} = \left[\frac{P^*_{j,t}}{P_t} \right]^{-\theta} C_t$$

- Price = **markup** over marginal cost (steps in technical problems). High markup when varieties are poor substitutes (θ is low):

$$P^*_{j,t} = \frac{\theta}{\theta - 1} P_t \Psi_{j,t}$$

- Monopolistic **inefficiency**, as price exceeds marginal cost ($\theta > \theta - 1$).
- In equilibrium $P^*_{j,t} = P_t$ (all firms are identical), hence $\Psi_{j,t} = (\theta - 1)/\theta$. This combined with the labor supply gives output.

Natural rate of output

- Approximation around a steady state (lower case letters are logs deviations):

$$
\begin{aligned}
y_t^{\text{flex}} &= \frac{1+\eta}{1+\eta+(\sigma-1)\,a} z_t \\
r_t^{\text{flex}} &= \sigma \frac{1+\eta}{1+\eta+(\sigma-1)\,a} \left(E_t z_{t+1} - z_t\right)
\end{aligned}
$$

- Higher productivity raises output, and future increase in productivity raises the real interest rate (RBC solution).
- y_t^{flex} and r_t^{flex}: **natural rates**, output and interest rate in the absence of price rigidities.
- **Gaps**: difference between the value of a variable and the value it would take under flexible prices:

$$
x_t = y_t - y_t^{\text{flex}} \qquad ; \qquad \tilde{r}_t = \left(i_t - E_t \pi_{t+1}\right) - r_t^{\text{flex}}
$$

Optimization under sticky prices: profits

- **Calvo** pricing: each period probability $1-\omega$ of being able to reset the price (to a level $P^*_{j,t}$). Otherwise price kept unchanged.
- Maximizes expected discounted profits:

$$\sum_{i=0}^{\infty} E_t \left[\omega^i \Delta_{t,t+i} \left[P^*_{j,t} Y_{j,t+i|t} - W_{t+i} \left(\frac{Y_{j,t+i|t}}{Z_{t+i}} \right)^{\frac{1}{a}} \right] \right]$$

- $j, t+i|t$ index: variable at time $t+i$ for firm j that reset its price at time t and hasn't changed since. $Y_{j,t+i|t}$ is output, $\Delta_{t,t+i} = \beta^i \left(C_{t+i}/C_t \right)^{-\sigma}$ is the discount factor.
- The profits reflect that:
  - Price can be in place for **several periods** (sum & expectation).
  - Discount by β and the **probability that the price still applies** ω^i (degree of price stickiness).
  - Profits weighted by the **marginal utility** of consumption through Δ: extra profit when poor (high Δ) is more valuable than when rich.

Optimal price

- **Markup** over **expected discounted marginal cost**, over the entire horizon when the price applies:

$$
\begin{aligned}
& P_{j,t}^{*} \sum_{i=0}^{\infty} E_t \left[\omega^i \Delta_{t,t+i} Y_{j,t+i|t} \right] \\
= & \frac{\theta}{\theta - 1} \sum_{i=0}^{\infty} E_t \left[\omega^i \Delta_{t,t+i} Y_{j,t+i|t} P_{t+i} \Psi_{j,t+i|t} \right]
\end{aligned}
$$

- Similar intuition as for the flexible price:
  - Include **current and future** expected prices and costs (discounted).
  - **Weight future states of nature** by a) probability that the price still applies ω^i, b) marginal utility, $\Delta_{t,t+i}$, c) strength of demand, $Y_{j,t+i|t}$.
- All firms that reset choose the same price: $P_{j,t}^{*} = P_t^{*}$, and $Y_{j,t+i|t} = Y_{t+i|t}$, and $\Psi_{j,t+i|t} = \Psi_{t+i|t}$.

Linear approximation of optimal price

- Firms' decisions give a new **individual price**, p_t^*, as a function of the CPI, p_{t+i}, and **firm-specific marginal cost**, $\varphi_{t+i|t}$. Derivation of NKPC
- We want a relation in terms of aggregate variables.
  - Individual marginal cost, $\varphi_{t+i|t}$ can be linked to the aggregate cost across all firms φ_t.
  - **Aggregate marginal cost** φ_t can be linked to the output gap:

$$\varphi_t = \left(\sigma + \frac{1 - a + \eta}{a}\right) x_t$$

  - High output raises cost:
    - Need more workers, hence to pay higher wages to raise labor supply
    - Direct higher marginal cost in the presence of decreasing returns to scale ($a < 1$).

New Keynesian Phillips Curve (NKPC)

- Linearized price and link between cost $\varphi_{t+i|t}$ and output gap x_t leads to the NKPC, which is an AS curve.
- **Current inflation** depend on **future expected inflation**, and the current **output gap**:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t \quad ; \quad \kappa = \frac{(1-\omega\beta)(1-\omega)}{\omega} \frac{1+\eta+(\sigma-1)a}{a+(1-a)\theta}$$

- Inflation is high if:
  - Expected **future inflation** is high. If competitors will charge higher prices tomorrow, take the chance of **adjust today**.
  - Positive **output gap raises firms' cost** (high wages or decreasing returns to scale), so firms want to increase prices.
  - Impact of output gap depends on parameters, including the degree of substitution θ (more substitutability lowers the sensitivity).

OVERALL SOLUTION AND POLICY

Model in three relations

- Aggregate supply: New Keynesian Phillips curve.
- Aggregate demand: Euler relation, expressed in terms of gaps:

$$
\begin{aligned}
y_t &= E_t y_{t+1} - \frac{1}{\sigma}\left(i_t - E_t \pi_{t+1}\right) \\
x_t &= E_t x_{t+1} - \frac{1}{\sigma}\tilde{r}_t
\end{aligned}
$$

- Policy rule for the interest rate (**Taylor rule**), with possible shocks:

$$
i_t = \delta_\pi \pi_t + \delta_x x_t + v_t
$$

- Output gap, inflation, and the interest rate are the endogenous variables.

The forward looking nature of the results

- **NKPC** implies that inflation reflects expected future output gaps (key role of expectations in policy):

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t = \kappa \sum_{i=0}^{\infty} \beta^i E_t x_{t+i}$$

- Initial empirical tests point towards the usual backward looking Phillips curve. Better measures of output gap and marginal costs support the forward looking NKPC.
- Euler implies that the **output gap** reflects expected **future deviation from the natural interest rate**:

$$x_t = E_t x_{t+1} - \frac{1}{\sigma} \tilde{r}_t = -\frac{1}{\sigma} \sum_{i=0}^{\infty} E_t \tilde{r}_{t+i}$$

- Interest rates far in the future matter a lot (questionable, but refined in more recent papers).

Welfare

- Utility of the representative agent to **assess alternative policies**.
- Linear expansion of the utility useless: expectations of all variables (functions of the shocks) are zero.
- **Quadratic** expansion of the utility (more complex): loss function with the variance of the output gap and the variance of inflation (weights are functions of the model parameters):

$$L = \frac{1}{2}\left[\lambda\left(x_t\right)^2 + \left(\pi_t\right)^2\right]$$

- A loss of zero requires **full stabilization around the natural rate**: $x_t = \pi_t = 0$.
- Optimal rule complex, so focus on simpler Taylor rule of the form:

$$i_t = \delta_\pi \pi_t + \delta_x x_t + v_t$$

Taylor principle

- Simple model: NKPC and AD show **no trade-off** between the output gap and inflation.
- Reason: output gaps are the only cause of inflation:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t \qquad ; \qquad x_t = E_t x_{t+1} - \frac{1}{\sigma} \tilde{r}_t$$

- While $\pi = x = 0$ is an equilibrium, is it a **stable** one? The model is a dynamic system in two control variables π and x.
- **Taylor principle**: stability if the central bank raises the real interest rate in response to inflation: $\delta_\pi > 1$.:

$$\kappa \left(\delta_\pi - 1\right) + \left(1 - \beta\right) \delta_x > 0$$

- A shock to the interest rate ($v_t > 0$) leads to a recession ($x < 0$) and negative inflation.

Impulse response

- Impact of a tightening shock of monetary policy (higher interest rate).

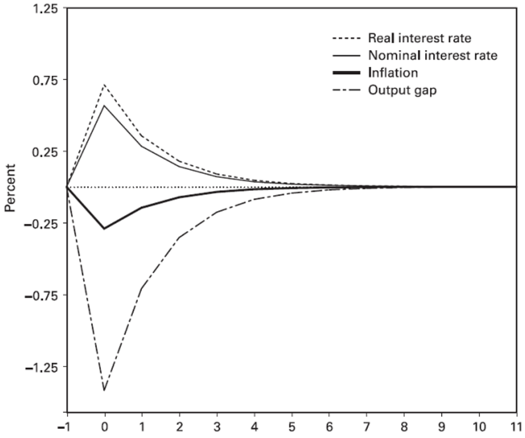

Figure 8.1
Response of output, inflation, and real interest rate to a policy shock in the new Keynesian model.

Walsh, chapter 8

TRADE – OFF AND COMMITMENT

Introducing a trade-off

- **Cost-push shock** e_t to the NKPC: output gaps are not the only drivers of inflation:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + e_t$$

- $\pi = x = 0$ is not an equilibrium. $e > 0$ leads to positive inflation and / or a recession ($x < 0$).
- **Sticky wages** are a form of cost-push shocks.
  - No problem if wages **or** prices are flexible, policy can produce efficient movements in real wages.
  - Problem if wages **and** prices are sticky.
- Minimize the discounted value of inflation and output gaps:

$$E_t \sum_{i=0}^{\infty} \beta^i \left[\lambda x_{t+i}^2 + \pi_{t+i}^2 \right]$$

- **No inflation bias** as the central bank aims for the natural rate of output. Yet, **commitment** is better.

New value of commitment

- Shocks are persistent (impact on future, hence role of expectations), $e_{t+1} = \rho e_t + \varepsilon_{t+1}$. Consider an inflationary shock $\varepsilon_t > 0$.
- **Discretion** vs. simple **commitment** rule of the form $x_t = -\xi e_t$ that the central bank follows (the optimal policy is more complex). Details of analysis
- Discretion:policy cannot commit to future inflation $\rightarrow$ no impact on expectation. Inflation or some recession, the NKPC effectively boils down to:

$$\pi_t = \kappa x_t + e_t$$

- Commitment: policy can promise negative future output gaps $\rightarrow$ affects future inflation: $E_t \pi_{t+1} < 0$. Output gap adjustment can be smoothed (commitment leads to a lower net present value of the loss):

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + e_t < \kappa x_t + e_t$$

Intuition for the gain of commitment

- Under commitment, the central bank **inflates by less than under discretion** (but gets a more negative output gap).
- Commitment leads to a higher welfare, as future policy affects the current gap and inflation.
- Smoothing the negative output gap after an inflationary shock, the central bank keeps **future output gaps (hence marginal cost) low**.
- Low future marginal costs moderate current price increases, improving the inflation - output trade off.
- Commitment allows to **spread the adjustment**, which is always a good idea with quadratic losses as large movements are particularly costly.

LIMITS AND RECENT DEVELOPMENTS

All is well?

- Dynamic Stochastic General Equilibrium (DSGE) models with price rigidities to analyze the design of optimal monetary policy.
  - Simple versions to get the intuition behind the transmission of policy.
- Two broad lines of criticism emerged with the crisis.
- Excessive "**mathiness**". Too much emphasis on the internal coherence and sophistication, at the expense of relevant aspects.
  - What if we are away from the steady state, because of a crisis?
- Neglect of **financial factors**.
  - What about banks and financial intermediaries?

Recent lines of research

- Models have been (and are being) expanded along many lines
  - JEP 2018 articles by Gali, Christiano-Eichenbaum-Trabandt, and Kaplan-Violante.
- **Banking sector** amplifies shocks, and is itself a source of shocks.
- Richer characteristics leading to more realism (habits in consumption, cost of adjusting capital).
- Introduction of **heterogeneous agents**: HANK, TANK, RANK (Heterogenous Agent New Keynesian, or Two Agents New Keynesian, instead of Representative Agent New Keynesian).
  - Keep track of the cross-section distribution of variables (can be hard).
  - Simpler versions with two agents go a long way (some agents do intertemporal optimization, others don't).
- Inclusion of **Zero Lower Bound on interest rate**. This makes commitment particularly useful.
  - Recent advance make the magnitude of the channel more realistic.

Use in policy

- Criticism that policy makers have focused on DSGE at the expense of other useful simpler frameworks.
- Central banks have developed DSGE and use them in formulating policy.
- Models used in specific ways (Gürkaynak and Tille 2017).
    - Develop internally coherent models for **"what if" policy scenario** analysis.
    - Support forecasting.
- Central banks hardly rely only on DSGE for policy making (and shouldn't).
    - Broad range of inputs from economic indicators, financial markets.

EXTRA : DERIVATION OF NKPC

GAIN FROM COMMITMENT

NKPC step 1: individual and aggregate marginal costs

- The (linearized) real marginal cost for a specific firm entails aggregate elements (wage, price, aggregate demand) and the firm's own price:

$$
\begin{aligned}
\varphi_{t+i|t} &= w_{t+i} - p_{t+i} - \frac{1}{a} z_{t+i} + \frac{1-a}{a} y_{t+i|t} \\
&= w_{t+i} - p_{t+i} - \frac{1}{a} z_{t+i} + \frac{1-a}{a} c_{t+i} - \frac{1-a}{a} \theta \left(p_t^* - p_{t+i} \right)
\end{aligned}
$$

- The aggregate marginal cost across all firms corresponds to the first three terms. We get a link between the average marginal cost and the cost for the resetting firms:

$$
\varphi_{t+i|t} = \varphi_t - \frac{1-a}{a} \theta \left(p_t^* - p_{t+i} \right)
$$

Pricing rule with aggregate marginal cost

- Linearization of firm's optimal condition:

$$p_t^* = (1 - \omega\beta) \sum_{i=0}^{\infty} (\beta\omega)^i E_t \left[p_{t+i} + \varphi_{t+i|t} \right]$$

- Combine the individual pricing rule with the relation between individual and aggregate marginal costs:

$$\begin{aligned} p_t^* &= (1 - \omega\beta) \sum_{i=0}^{\infty} (\beta\omega)^i E_t \left[p_{t+i} + \varphi_{t+i|t} \right] \\ &= (1 - \omega\beta) \sum_{i=0}^{\infty} (\beta\omega)^i E_t \left[p_{t+i} + \frac{a}{a + (1-a)\theta} \varphi_{t+i} \right] \end{aligned}$$

NKPC step 2: individual price and aggregate inflation

- The CPI dynamics gives the new price as a function of aggregate prices:

$$\pi_t = p_t - p_{t-1} = (1-\omega)\left(p_t^* - p_{t-1}\right)$$
$$\Rightarrow p_t^* = \frac{1}{1-\omega}\pi_t + p_{t-1}$$

- The pricing rule is then in terms of aggregate CPI and inflation. We can transform it to get:

$$\begin{aligned}
\pi_t &= \beta E_t \pi_{t+1} + \frac{(1-\omega\beta)(1-\omega)}{\omega}\frac{a}{a+(1-a)\theta}\varphi_t \\
&= \beta E_t \pi_{t+1} + \tilde{\kappa}\varphi_t
\end{aligned}$$

NKPC step 3: use the output gap

- Gaps are the difference between the value of a variable and the value it would take under flexible prices. The output and interest rate gaps are:

$$x_t = y_t - y_t^{\text{flex}} \qquad ; \qquad \tilde{r}_t = (i_t - E_t\pi_{t+1}) - r_t^{\text{flex}}$$

- The real marginal cost under sticky prices can be linked to the output gap.
- High output boosts wages through the labor supply, and the cost directly through decreasing returns to scale: Return

$$\varphi_t = \left(\sigma + \frac{1 - a + \eta}{a}\right) x_t$$

Allocation under discretion

- Under discretion the central bank minimizes the one period loss taking expectations as given:

$$\min \left[\lambda x_t^2 + \pi_t^2\right] \qquad \text{subject to} \qquad \pi_t = \kappa x_t + \left(\beta E_t \pi_{t+1} + e_t\right)$$

- The solution under discretion is:

$$x_t^{\text{disc}} = \frac{-\kappa}{\lambda\left(1-\beta\rho\right)+\kappa^2} e_t \qquad ; \qquad \pi_t^{\text{disc}} = \frac{\lambda}{\lambda\left(1-\beta\rho\right)+\kappa^2} e_t$$

- There is a trade-off following a cost push shocks. Policy tightens, leading to a small pickup in inflation and a recession (a negative output gap).

Allocation under commitment

- The central bank is committed to a rule linking the output gap and shocks: $x_t = -\xi u_t$. Conditional on this rule we can solve for the output gap and inflation.
- The central bank chooses ω to minimize:

$$\frac{1}{2} E_t \sum_{i=0}^{\infty} \beta^i \left(\lambda x_{t+i}^2 + \pi_{t+i}^2 \right)$$

- Compared to discretion, inflation reacts by less to the shock, while the output gap reacts by more (the overall loss is reduced): Return

$$x_t^{\text{comm}} = \frac{-\kappa}{\lambda \left(1 - \beta\rho\right)^2 + \kappa^2} e_t \quad ; \quad \pi_t^{\text{comm}} = \frac{\lambda \left(1 - \beta\rho\right)}{\lambda \left(1 - \beta\rho\right)^2 + \kappa^2} e_t$$