Intermediate Microeconomics Assignment 1

Due on October 24, 2021

Name Student ID

1. UMP: Consider the Cobb-Douglas utility function $U(x, y) = x^a y^b$ where $a > 0$ and $b > 0$. The price vector is $\boldsymbol{p} = (p_x, p_y)$, and the income is I.

   (1) Derive the Marshallian demand $x^*(p_x, p_y, I)$, $y^*(p_x, p_y, I)$, and the indirect utility (value function) $V(p_x, p_y, I)$.

   (2) If price p_x or p_y increases, is the consumer better-off or worse-off? If income increases, whether the consumer is better-off or worse-off.

2. EMP: Consider $U(x, y) = x^a y^{1-a}$.

   (1) Derive the Hicksian demand for $U(x, y) = x^a y^b$, and the expenditure function $E(p_x, p_y, u)$.

   (2) If price p_x or p_y increases, will the total expenditure increase or decrease?

   (3) Verify $x^*(p_x, p_y, I) = h_x(p_x, p_y, u)$ when $u = V$ where V is solved by 1 (1).

   (4) Verify $h_x(p_x, p_y, u) = x^*(p_x, p_y, I)$ when $I = E$ where E is solved by 2 (1).

3. Consider the quasi-linear utility $U(x, y) = u(x) + y$, where $u'(\cdot) > 0$, $u''(\cdot) < 0$ and $p_y = 1$. Assume that when the price of x increases from p_1 to p_2, the Mashallian demand changes from $x_1^*(p_1, p_y, I)$ to $x_2^*(p_2, p_y, I)$, where both x_1^* and x_2^* are interior solutions. Show that at p_2, the Hicksian demand $h_x(p_2, p_y, u_1)$ where u_1 is the original utility level before price increase must be an interior solution.