Macroeconomics A, EI056

Class 4

Long run growth

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What you will get from today class

- Long run growth: some **stylized facts**.
- The workhorse **Solow** model.
    - Focus on **capital** accumulation, and supply side.
    - The **Golden rule** and dynamic (in)efficiency.
- Limits to the Solow model.
    - Brief discussion on endogenous growth.
    - Two current challenges: climate change and reduced potential growth.
    - Role for policy.
- Extra slides: the model with **human** capital.

A question to start

Saving allows us to invest and raise future capital, hence growth. A society with a high saving rate will thus reach a higher standard of living.

Do you agree? Why or why not?

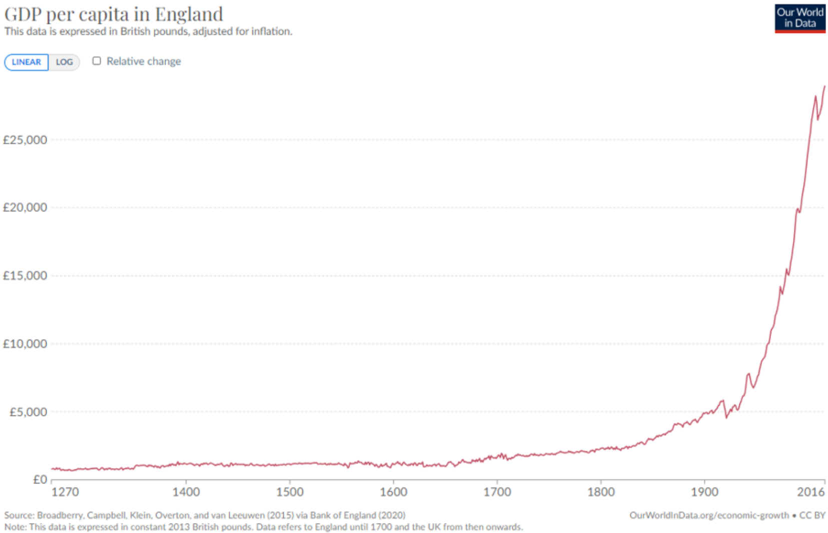
SOME SYLIZED FACTS ON GROWTH

Evidence on long run growth

- Historical evidence shows steady GDP per capita until the late 17th century.
  - Growth since picked up, especially since the **industrial revolution**.
  - Broad phenomenon, key role of **productivity**.
- **Sizable** growth, with role of capital accumulation .
  - U.S. output per worker tripled between 1950 and 2020, the capital stock per worker quintupled.
- Different paths of payments to factors.
  - The payment to **labor** (real wage) has similarly increased.
  - The payment to **capital** (real return) shows no trend.
- The **distribution** of income across factors of production is (broadly) stable.

A very long view: UK since 13th century

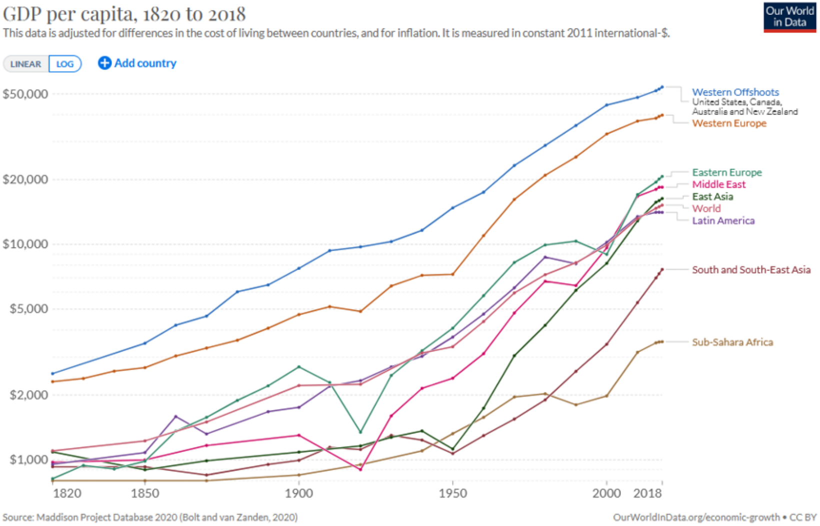
- Steady GDP per capita until 1700, then pick-up, especially since 1850.



https://ourworldindata.org/grapher/gdp-per-capita-in-the-uk-since-1270?time=1270..latest

Catching up

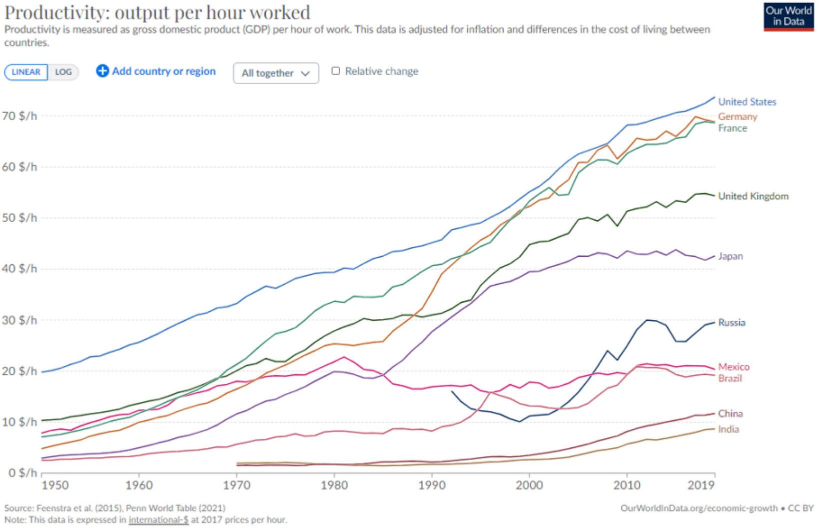
- Growth started in Europe, but has since broadened.



https://ourworldindata.org/grapher/average-real-gdp-per-capita-across-countries-and-regions

Heterogeneous rising productivity

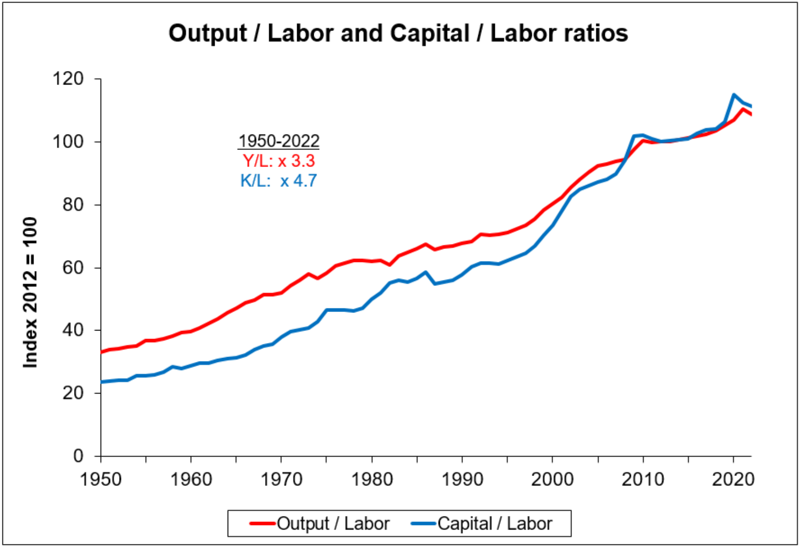
- Output per hour worked has increased, but unevenly.



https://ourworldindata.org/grapher/labor-productivity-per-hour-pennworldtable?tab=chart

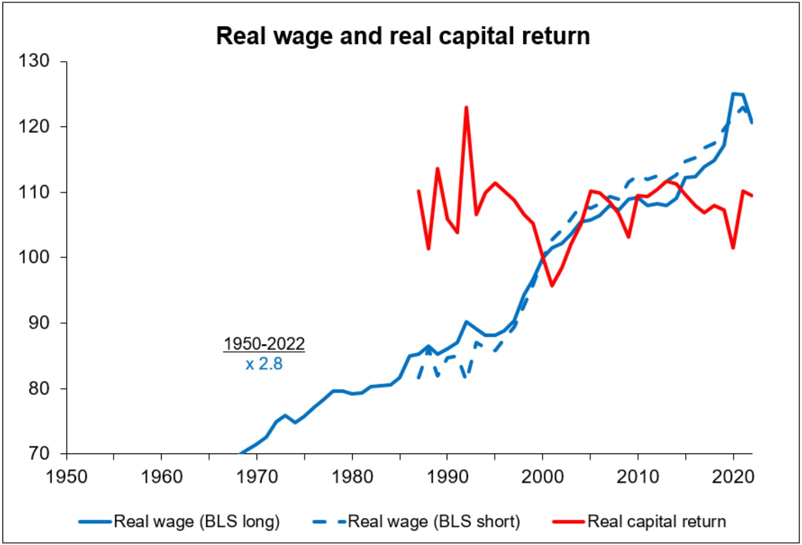
Capital deepening

- Higher output per worker associated with higher capital per worker.



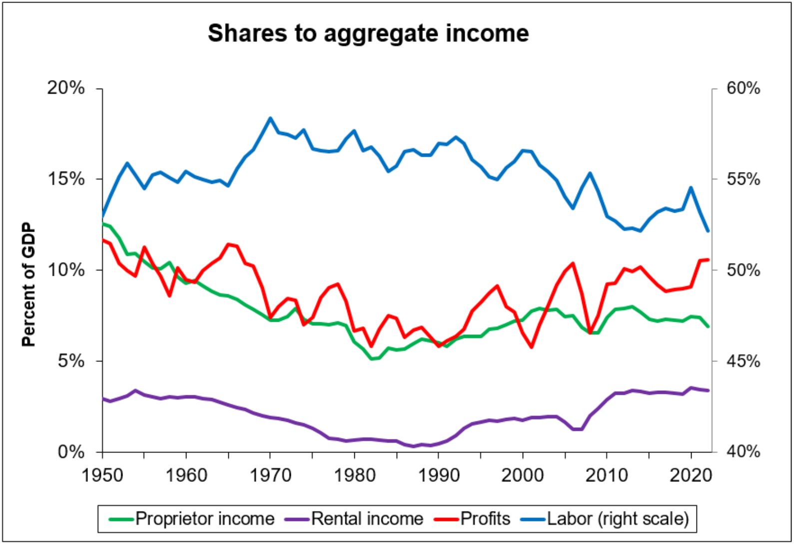
Different factor prices' paths

- Rising payment to labor, flat payment to capital.



Steady income share

- Share of GDP paid to factors steady (some shift since 2000's).



SOLOW GROWTH MODEL

Production technology

- Focus on the accumulation of production factors and the technology.
- A **supply-side** model. Demand will come with the Ramsey model next week.
- Output Y is produced using capital K and labor L through a production function (A is **productivity**):

$$Y_t = F\left(K_t, A_t L_t\right)$$

- **Decreasing** returns to scale for individual inputs: $\partial Y_t / \partial L_t > 0$ and $\partial^2 Y_t / \left(\partial L_t\right)^2 < 0$.
- **Constant** returns to scale with respect to all (technology is scalable):

$$cY_t = F\left(cK_t, cA_t L_t\right)$$

Technology in "per effective labor" terms

- Re-express variables in **"per effective labor"** terms. Divide throughout by all $A_t L_t$ (multiply by $c = 1/(A_t L_t)$):

$$\frac{Y_t}{A_t L_t} = F\left(\frac{K_t}{A_t L_t}, 1\right) \Rightarrow y_t = f(k_t)$$

  - Lower-case letters correspond to upper case letters divided by $A_t L_t$.
- Decreasing marginal productivity: $f'(k_t) > 0$, $f''(k_t) < 0$. First unit of capital is highly productive: $f'(0) = \infty$. No production without capital: $f(0) = 0$.
- Standard Cobb-Douglas specification ($0 < \alpha < 1$):

$$Y_t = (K_t)^{\alpha} (A_t L_t)^{1-\alpha} \Rightarrow y_t = (k_t)^{\alpha}$$

Capital accumulation

- Analysis in **continuous** time (parallel with Romer book, discrete time in technical exercices).
- The growth rate of a variable X is:

$$g_X = \frac{\dot{X}_t}{X_t} = \frac{1}{X_t} \lim_{\Delta \to 0} \frac{X_{t+\Delta} - X_t}{\Delta}$$

- Two **types** of factors:
  - Labor (L) cannot be accumulated. Working less today does not allow you to work more tomorrow.
  - Capital (K) can be accumulated.
- Output is either consumed or saved (**invested**).
  - Fraction s_K is invested. It is constant and exogenous (no optimization).
  - Capital **depreciates** at a rate δ.

Labor, productivity and capital growth

- **Labor** grows at an exogenous rate n, and **productivity** grows at an exogenous rate g:

$$\dot{L}_t = nL_t \qquad \dot{A}_t = gA_t$$

- Useful relation between the growth rates of capital ($\dot{K}_t$) and of scaled capital ($\dot{k}_t$).
  - Variables as functions of time. Take a differential of k (capital per effective labor) with respect to time.
  - Growth of k: growth of capital - growth of technology and labor (which lowers k_t):

$$\begin{aligned}
\dot{k}_t &= \frac{\partial k_t}{\partial t} = \frac{\partial}{\partial t}\left(\frac{K_t}{A_t L_t}\right) \\
&= \frac{1}{A_t L_t}\frac{\partial K_t}{\partial t} - \frac{K_t}{A_t (L_t)^2}\frac{\partial L_t}{\partial t} - \frac{K_t}{L_t (A_t)^2}\frac{\partial A_t}{\partial t} \\
&= \frac{1}{A_t L_t}\dot{K}_t - (n+g)\, k_t
\end{aligned}$$

Dynamics of capital

- Capital increases with savings, net of depreciation:

$$\dot{K}_t = s_K Y_t - \delta K_t$$

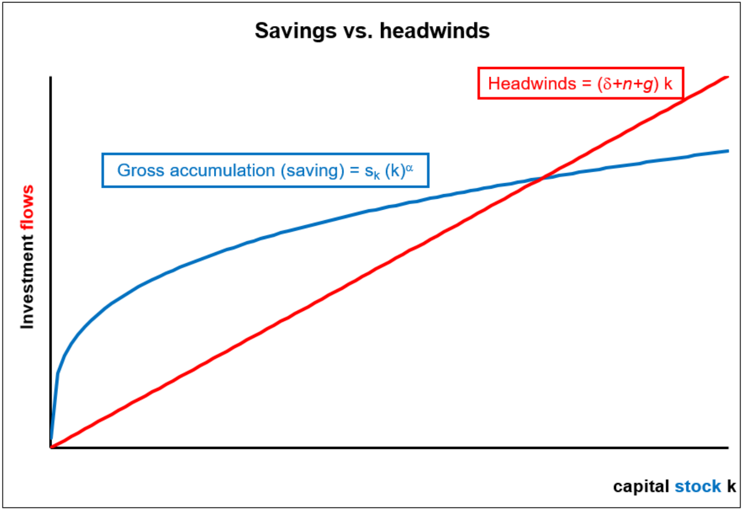
- Dynamics of **scaled** capital $\dot{k}_t$ (use the relation between $\dot{K}_t$ and $\dot{k}_t$,)

$$\begin{aligned}
\dot{K}_t &= s_K Y_t - \delta K_t \\
A_t L_t \left(\dot{k}_t + (n+g) k_t \right) &= s_K Y_t - \delta K_t \\
\dot{k}_t &= s_K \frac{Y_t}{A_t L_t} - \delta \frac{K_t}{A_t L_t} - (n+g) k_t \\
\dot{k}_t &= s_K y_t - (\delta + n + g) k_t
\end{aligned}$$

- Concave **savings** component, $s_K (k_t)^\alpha$, vs. linear **"headwinds"**, $\delta + n + g$.

Two lines

- Savings line (higher capital = higher output = higher savings) and headwinds line (higher capital = higher dilution and depreciation).



The steady state

- For low k_t, savings exceeds headwinds capital increases ($\dot{k}_t > 0$). Opposite for high k_t.
- In the steady state **capital is constant** ($\dot{k}_t = 0$):

$$\dot{k}_t = 0 = s_K (k_t)^\alpha - (\delta + n + g) k_t$$

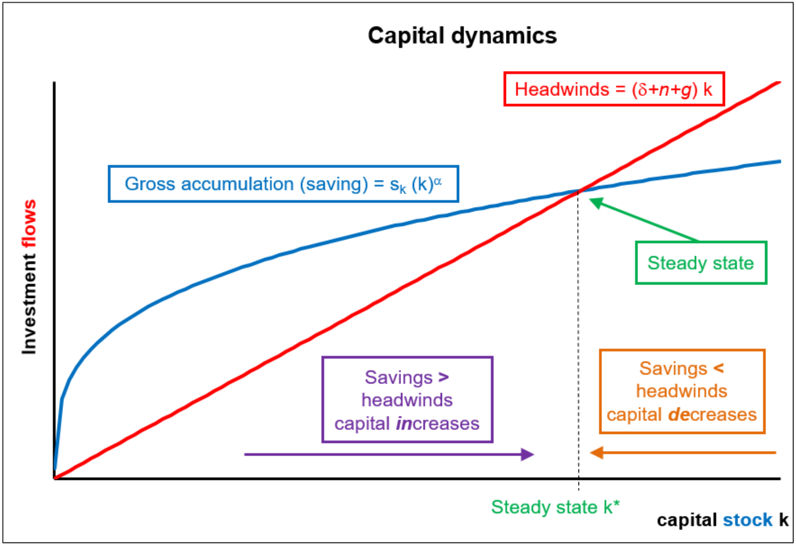
- This gives capital (asterisk denotes steady state values):

$$k^* = \left(\frac{s_K}{\delta + n + g} \right)^{\frac{1}{1-\alpha}}$$

- Keeping k constant requires 'break-even investment' to compensate the headwinds of depreciation, labor and technology growth.

Phase diagram

- Shows the steady state and capital dynamics.



Characteristics of steady state

- k^* and y^* are constant. Output Y grows at a rate $g + n$.
  - **Output per worker** Y/L grows at the rate of output $(g + n)$ net of population (n), i.e. the rate of technology g.
- The **real wage** is the marginal product of labor and grows at a rate g, the **rental rate of capital** is constant:

$$w_t = \frac{\partial Y_t}{\partial L_t} = (1-\alpha)\frac{Y_t}{L_t} \qquad ; \qquad r_t = \frac{\partial Y_t}{\partial K_t} = \alpha\frac{Y_t}{K_t}$$

- Each factor gets a **constant share** of output:

$$w_t \frac{L_t}{Y_t} = (1-\alpha)\frac{Y_t}{L_t}\frac{L_t}{Y_t} = 1-\alpha$$

- The model thus replicates the main stylized facts of growth.

GOLDEN RULE AND

DYNAMIC EFFICIENCY

How much to save?

- A higher savings rate s_K increases capital and output:

$$\frac{\partial k^*}{\partial s_K} = \frac{1}{1-\alpha}\frac{k^*}{s_K} > 0 \qquad ; \qquad \frac{\partial y^*}{\partial s_K} = \frac{\alpha}{1-\alpha}\frac{y^*}{s_K} > 0$$

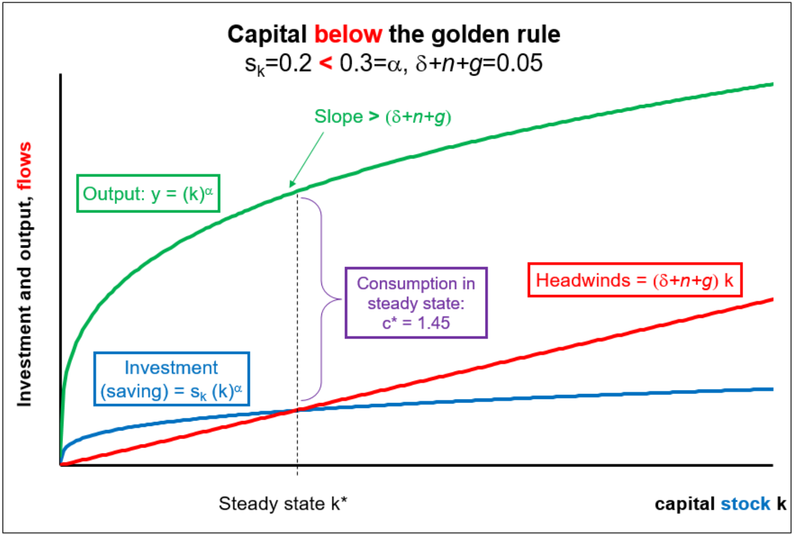
- Ambiguous impact on **consumption**: higher savings raises output, but also the needed break-even investment.

$$\begin{aligned} c^* &= (k^*)^\alpha - (\delta + n + g)\, k^* \\ \frac{\partial c^*}{\partial s_K} &= \left(\frac{\alpha}{s_K} - 1\right)\frac{(k^*)^\alpha}{1-\alpha} \end{aligned}$$

- Impact of s_K on c^* depends on whether the saving rate s_K exceeds the share of capital in the technology α.
  - Golden Rule sets $s_K = \alpha$ to maximize consumption (**save more** if you use a **capital intensive** production).

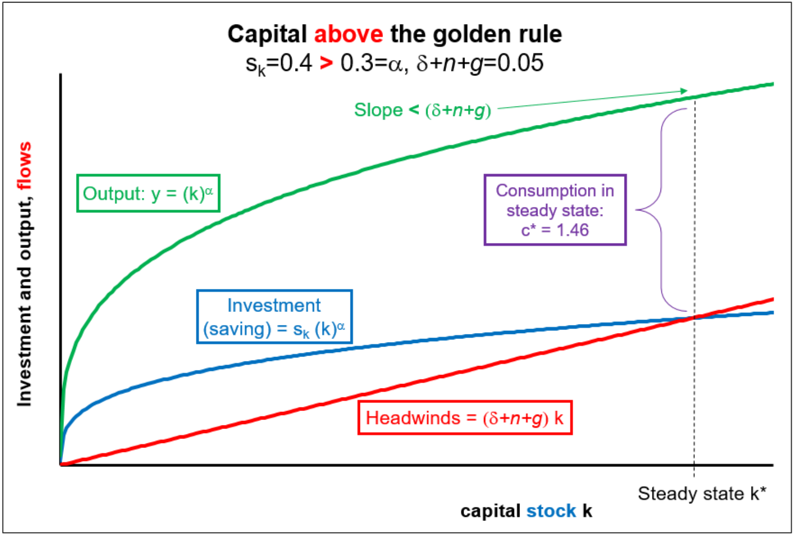
Low savings rate

- Phase diagram with $s_K < \alpha$.



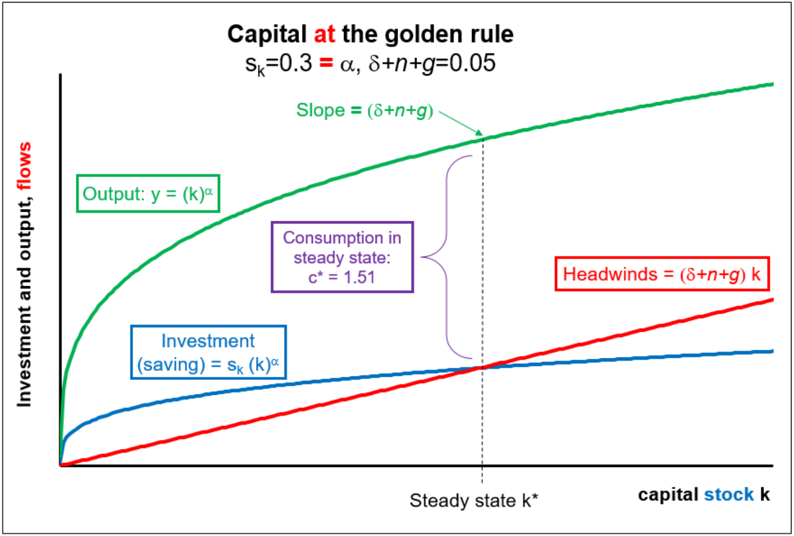
High savings rate

- Phase diagram with $s_K > \alpha$.



Golden rule savings rate

- Phase diagram with $s_K = \alpha$.



What happens if we save less?

- Start at a steady state:

$$k_0 = k^* (s_K) = (s_K / (\delta + n + g))^{\frac{1}{1-\alpha}}$$

- Permanent **reduction in the savings rate**: $ds_K < 0$.
- **Initial consumption** increases (initial capital k_0 is fixed):

$$c_0 = (1 - s_K) (k_0)^\alpha \Rightarrow \frac{\partial c_0}{\partial s_K} < 0$$

- Convergence to a new steady state with lower capital and output:

$$\frac{\partial k^* (s_K)}{\partial s_K} > 0 \qquad ; \qquad \frac{\partial y^* (s_K)}{\partial s_K} > 0$$

Long run impact

- What about long run consumption:

$$c^* = (k^*)^\alpha - (\delta + n + g)\, k^*$$

- **Offsetting effects** through savings and headwinds:

$$\frac{\partial c^*}{\partial s_K} = \left[\alpha\,(k^*)^{\alpha-1} - (\delta + n + g)\right] \frac{\partial k^*}{\partial s_K}$$

- Higher long run consumption if $\alpha\,(k^*)^{\alpha-1} - (\delta + n + g) < 0$, initial capital was **above** the Golden rule (s_K was above α). Figures of dynamics
- **Dynamically inefficient** economy: initial savings were too high, and reducing them increases consumption at all horizons (a free lunch).

BROADENING CAPITAL

Introducing human capital

- Technology includes human capital H $(0 < \beta < 1)$:

$$Y_t = (K_t)^\alpha (H_t)^\beta (A_t L_t)^{1-\alpha-\beta} \Rightarrow y_t = (k_t)^\alpha (h_t)^\beta$$

- Share s_H of output invested in human capital. Dynamics of **two capital stocks** k and h.:

$$\begin{aligned} \dot{k}_t &= s_K (k_t)^\alpha (h_t)^\beta - (\delta + n + g) k_t \\ \dot{h}_t &= s_H (k_t)^\alpha (h_t)^\beta - (\delta + n + g) h_t \end{aligned}$$

- Graphical solution with two lines in $k; h$ space: one where $\dot{k}_t = 0$, another where $\dot{h}_t = 0$. Derivations
- Key intuition: human capital implies that some of the **labor** input (L and H) can be **accumulated** as capital.
  - More "accumulable" factors in production (similar to assuming a larger share α).

Effect of broader capital

- Broadening capital by introducing human capital makes the model more realistic.
- **Slower convergence** to the steady state, more realistic. ▸ Derivations
  - Getting half-way to the steady state takes 17 years without human capital (too fast empirically) and 35 years with it.
- **Output differences** with realistic capital differences.
  - Ten-fold gap in output between two countries, $y_2^*/y_1^* = 10$, requires capital gap $(k_2^*/k_1^*)^{\alpha+\beta}$ (assuming $s_K = s_H$ in all countries).
  - Capital ratio: $1'000$ if only physical capital ($\alpha = 1/3$ and $\beta = 0$), 30 with both capitals ($\alpha = \beta = 1/3$).

LIMITS TO SOLOW,

ENDOGENOUS GROWTH,

CURRENT CHALLENGES,

AND POLICY

Limits of the Solow model

- **Convergence**: all countries will go towards the same steady state (in units of "per effective labor").
  - This is not the case empirically, as some countries are stuck in low growth traps.
  - Convergence is thus partial ("club convergence" as countries cluster in groups).
  - Productivity A can differ across countries.
    - Public **infrastructure** capital which varies across countries.
    - "**Institutional** infrastructure", such as the reliability of the public institutions and rule of law.
    - Causality is tricky to establish.
- Long run growth rate is fully exogenous, and **not a function of the savings** rate.
  - Growth is due to sheer luck. Policy and institutions play no role
- The endogenous growth literature considers models where this is not the case.

Evidence of convergence: countries

- Countries with initially low output per capita should grow faster. This is (imperfectly) the case.

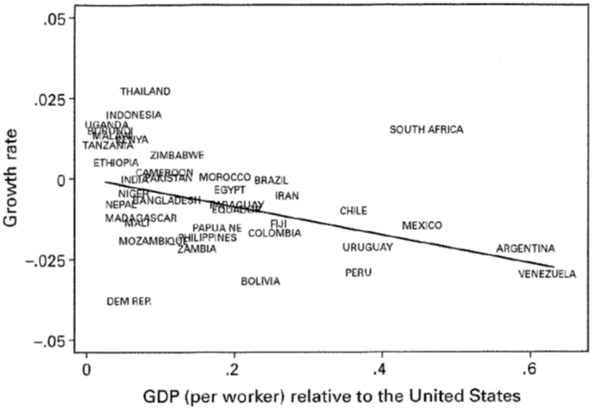


Figure I.1
Cross-country convergence
Aghion, Philippe, and Peter Howitt (2009), *The Economics of growth*, MIT press

Evidence of convergence: US states

- Stronger evidence of convergence across regions within a country (this has slowed in recent years).

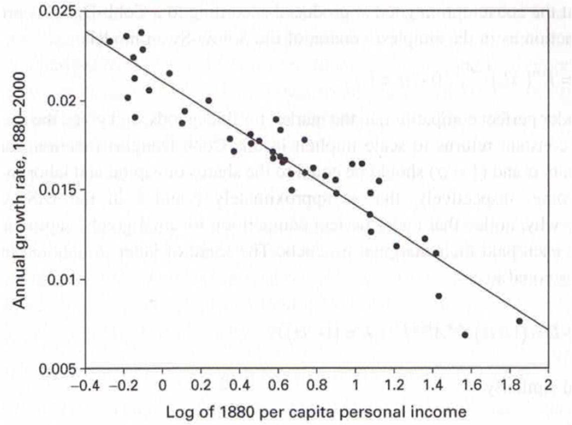


Figure 2.1
Convergence of personal income across U.S. states

Aghion, Philippe, and Peter Howitt (2009), *The Economics of growth*, MIT press

Endogenous growth

- The **AK model** considers that **output is linear in capital**, i.e. constant returns to scale. With an exogenous savings rate:

$$Y_t = A_t K_t \qquad ; \qquad \dot{K}_t = sY_t - \delta K_t$$

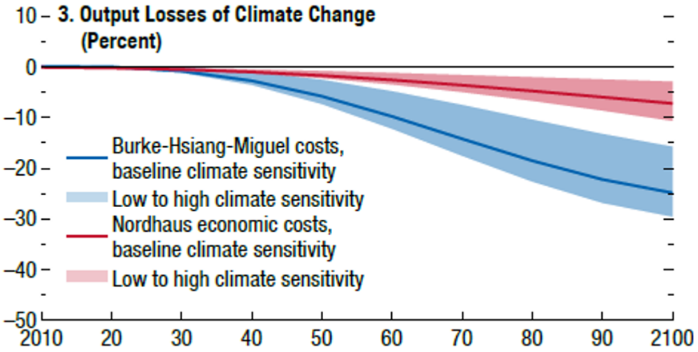
- **Savings boost growth**, even when A is constant:

$$\frac{\dot{Y}_t}{Y_t} = \frac{\dot{K}_t}{K_t} = sA_t - \delta$$

- Endogenous growth literature considers models where A is produced endogenously (Romer ch 3).
  - Labor and capital produce output, but also new technology: $\dot{A}_t = F\left(K_t; L_t; A_t\right)$.
  - Models with endogenous choice by agents of whether to put resources in output production or research and development.
  - Models with endogenous innovation: invest in producing better types of capital.

Current challenge 1: climate change

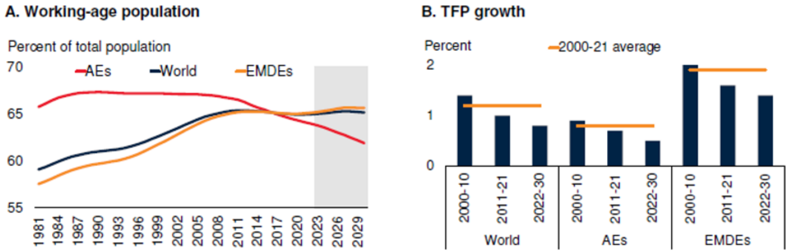
- Global warming hinders growth through several channels: destruction of capital, reduced productivity, uncertainty.
- Sizable effect (uncertain estimates), but prompt policy can handle it if adopted rapidly (IMF 2020, 2022).



IMF (2022). "Mitigating climate change – growth and distribution friendly strategies", *World Economic Outlook* chapter 3, October. https://www.imf.org/-/media/Files/Publications/WEO/2020/October/English/ch3.ashx

Current challenge 2: declining population

- Rapid decrease in the growth rate of population, which could turn negative.
- Along with slowdown in Total Factor Productivity and investment, this reduces potential growth in the 2020's (World Bank 2023).
- Declining population in an endogenous growth framework can depress innovation and lead to stagnating standards of living (Jones 2022).



World Bank (2023). "Falling Long-Term Growth Prospects"
https://www.worldbank.org/en/research/publication/long-term-growth-prospects

Can policy help?

- Solow model: higher savings is not necessarily a good idea, as it lowers consumption. Policy should intervene only in the presence of **externalities**.
- Non-excludability of knowledge. If everyone can use a new technique, the social benefit of developing it exceeds the private benefit that the developer gets.
- Productivity A can be a function of the **total capital** of the economy. An individual firm's investment boosts not only its own output directly, but also the productivity of others.
    - R&D and investment should be subsidized. Similar argument for human capital and education.
- Specific type of policy intervention depends on the specific conditions.

What to subsidize?

- A country can be at the **frontier** of innovation, or instead **catching up** from behind the frontier (following classes vs. writing a thesis).
  - Support the type of investment that is most adapted to the stage of development.
- This matters for the optimal policy. Example: education.
  - In a country far from the frontier promote primary / secondary education to implement the existing technologies.
  - In a country close, promote higher education to allow for innovation.

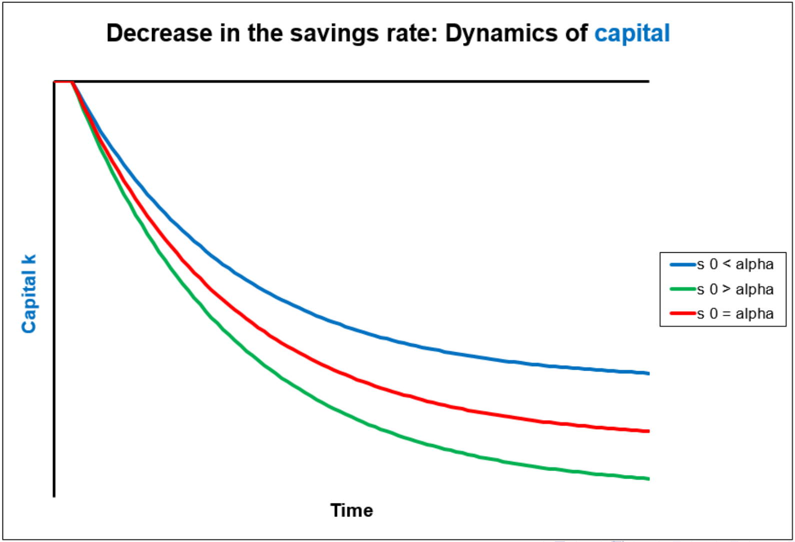
Should we protect innovators or foster competition?

- **Patents** can stimulate innovation by guaranteeing a sizable return to the innovator.
  - But once a technology is invented, it is optimal to make it freely available.
- Balance between fostering innovation and ensuring its spread. Optimal policy depends on **how close to the frontier** firms are.
  - If firms are far from the frontier, competition discourages innovation. An incumbent will lose his spot when a new firm enter regardless of what it does, so why bother innovating.
  - Closer to the frontier, the incumbent has a chance. He can stay at the frontier by innovating, in which case there is no room for a new firm.
- Can explain the slowdown of Europe growth relative to the US. Initially limited competition did not hurt Europe, now it does as it is at the frontier.

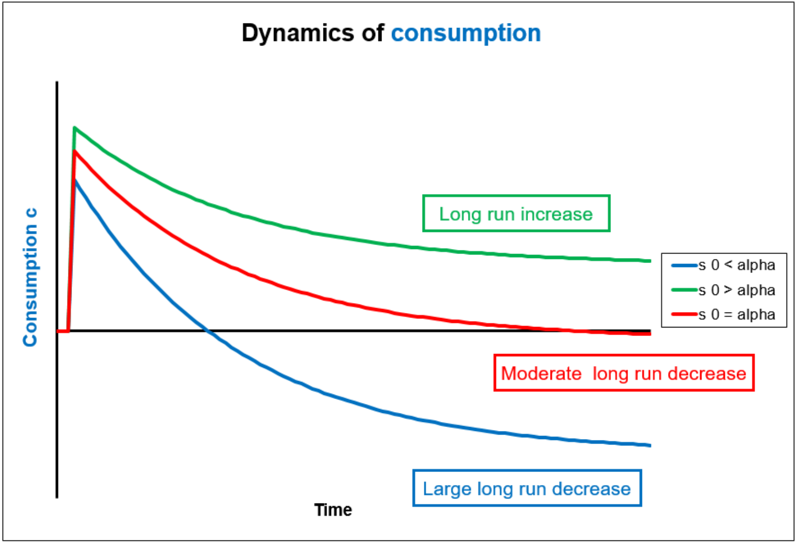
ADDITIONAL SLIDES

Dynamics of capital

- Long run decrease, especially if $s_K > \alpha$ initially.



- Consumption is **permanently** increased if $s_K > \alpha$ initially. Return



Solow model with human capital

- Cobb-Douglas technology with physical and human capital $(0 < \alpha, \beta < 1)$:

$$Y_t = (K_t)^{\alpha} (H_t)^{\beta} (A_t L_t)^{1-\alpha-\beta} \Rightarrow y_t = (k_t)^{\alpha} (h_t)^{\beta}$$

- Capital increases with savings, net of depreciation:

$$\dot{K}_t = s_K Y_t - \delta K_t \qquad ; \qquad \dot{H}_t = s_H Y_t - \delta H_t$$

- Dynamics of scaled capital physical and human capital:

$$\begin{aligned} \dot{k}_t &= s_K y_t - (\delta + n + g) k_t \\ \dot{h}_t &= s_H y_t - (\delta + n + g) h_t \end{aligned}$$

Phase diagram with both capitals

- Two laws of motion:

$$\begin{aligned}\dot{k}_t &= s_K (k_t)^{\alpha} (h_t)^{\beta} - (\delta + n + g) k_t \\ \dot{h}_t &= s_H (k_t)^{\alpha} (h_t)^{\beta} - (\delta + n + g) h_t\end{aligned}$$

- Constant physical capital defines a function of k as a function of h, that is increasing and concave

$$\dot{k}_t = 0 \Rightarrow k_t = \left(\frac{s_K}{\delta + n + g} \right)^{\frac{1}{1-\alpha}} (h_t)^{\frac{\beta}{1-\alpha}}$$

  We can see that:

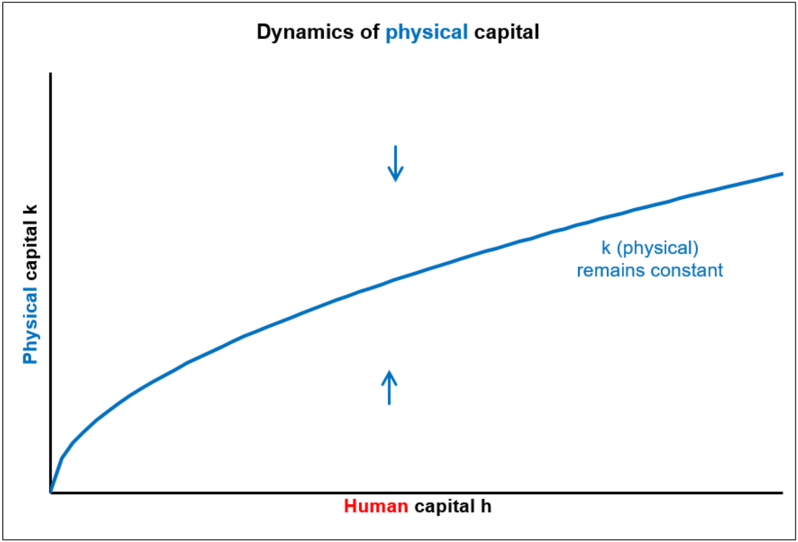
$$\frac{\partial k_t}{\partial h_t} > 0 \qquad ; \qquad \frac{\partial^2 k_t}{\partial h_t \partial h_t} < 0$$

- From any point above that line k is decreasing:

$$\left.\frac{\partial \dot{k}_t}{\partial k_t}\right|_{\dot{k}_t = 0} = \alpha s_K (k_t)^{\alpha - 1} (h_t)^{\beta} - (\delta + n + g) = (\alpha - 1)(\delta + n + g) < 0$$

Phase diagram: dynamics of K

- Physical capital decreases if initially high relative to human capital.



Dynamics of human capital

- Constant human capital also defines a function of k as a function of h, that is increasing and convex:

$$\dot{h}_t = 0 \Rightarrow k_t = \left(\frac{\delta + n + g}{s_H}\right)^{\frac{1}{\alpha}} (h_t)^{\frac{1-\beta}{\alpha}}$$

We can see that:

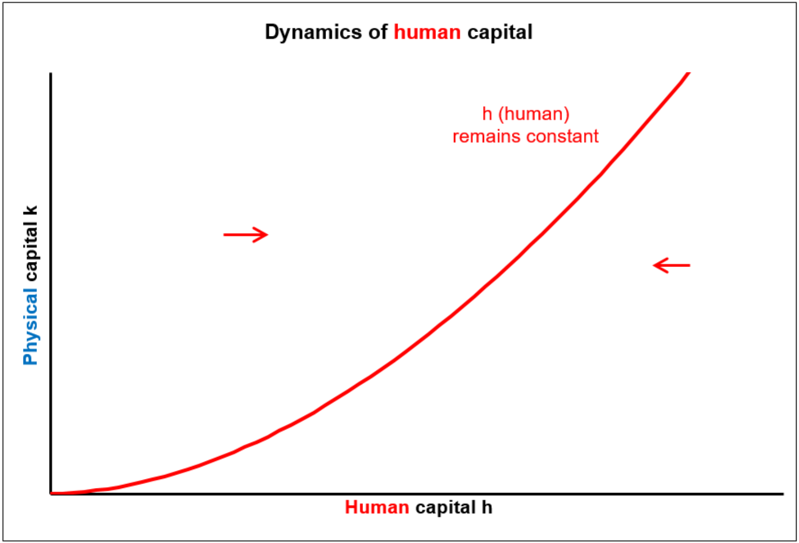
$$\frac{\partial k_t}{\partial h_t} > 0 \qquad ; \qquad \frac{\partial^2 k_t}{\partial h_t \partial h_t} > 0$$

- From any point above that line h is increasing:

$$\left.\frac{\partial \dot{h}_t}{\partial k_t}\right|_{\dot{h}_t=0} = \alpha s_H (k_t)^{\alpha-1} (h_t)^{\beta} > 0$$

Phase diagram: dynamics of H

- Human capital decreases if initially high relative to physical capital.



Steady state

- The steady state corresponds to the intersection of the curves, where all capitals are constant: $\dot{k}_t = \dot{h}_t = 0$

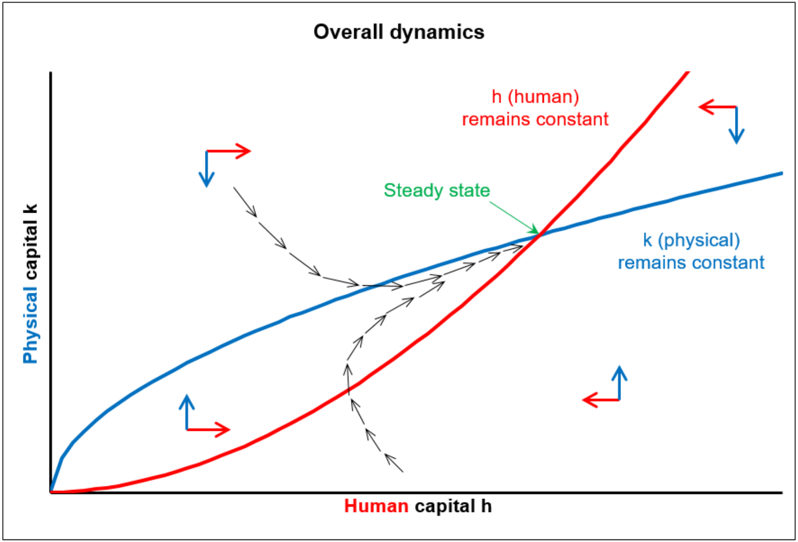
$$
\begin{aligned}
k^* &= (s_K)^{\frac{1-\beta}{1-\beta-\alpha}} (s_H)^{\frac{\beta}{1-\beta-\alpha}} \left(\frac{1}{\delta + n + g} \right)^{\frac{1}{1-\beta-\alpha}} \\
h^* &= (s_K)^{\frac{\alpha}{1-\beta-\alpha}} (s_H)^{\frac{1-\alpha}{1-\beta-\alpha}} \left(\frac{1}{\delta + n + g} \right)^{\frac{1}{1-\beta-\alpha}}
\end{aligned}
$$

- The output is then:

$$
\begin{aligned}
y^* &= (k^*)^{\alpha} (h^*)^{\beta} \\
&= (s_K)^{\frac{\alpha}{1-\beta-\alpha}} (s_H)^{\frac{\beta}{1-\beta-\alpha}} \left(\frac{1}{\delta + n + g} \right)^{\frac{\alpha+\beta}{1-\beta-\alpha}}
\end{aligned}
$$

Overall dynamics

- Dynamic path of both capitals. ◂ Return



Speed of convergence

- If we start away from the steady state, we **converge** to it. But how fast?
- Linear expansion around the steady state. We can write the speed of convergence of output (see appendix for details) as:

$$\begin{aligned} \dot{y}_t &= -(1-\beta-\alpha)(\delta+n+g)(y_t-y^*) \\ \frac{y_t-y^*}{y_0-y^*} &= \exp\{-(1-\beta-\alpha)(\delta+n+g)t\} \end{aligned}$$

- Convergence is **slower** with **high capital share**. Set $\delta+n+g=0.06$, and $\alpha=1/3$, compute the **half life values** (number of years to erase half the gap).
  - Without human capital ($\beta=0$): $t=17$, which is empirically too fast.
  - With human capital ($\beta=1/3$): $t=35$ which is more realistic. .

Illustration of speed of convergence

- Around the steady state, the accumulation speed of capital is the difference between its marginal productivity and the depreciation:

$$s_K (k_t)^\alpha \qquad \text{vs} \qquad (\delta + n + g)\, k_t$$

- The model with human capital behaves like a model with physical capital and a large α.
- The function $s_K (k_t)^\alpha$ is then steep. If capital is below the steady state it increases, but slowly as $s_K (k_t)^\alpha$ is fairly close to $(\delta + n + g)\, k_t$, while they are wider apart without human capital.

Phase diagram with different α

- The higher α, the steeper the savings line. Return