Macroeconomics B, EI060

Class 9

New open economy macroeconomics

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What you will get from today class

- Micro-founded model with nominal rigidities (Obstfeld and Rogoff 10.1).
  - Keynesian model (Mundell-Fleming), with optimizing behavior.
  - Encompasses both the short run (set prices) and the long run (flexible prices).
  - Allows for welfare analysis.
- Extensions.
  - Different elasticities of substitution across countries and within countries.
  - Partial transmission of exchange rates to import prices.
  - Different dimensions driving the current account.
  - Welfare: beggar-thy-neighbor, or beggar-thyself.
  - Overshooting.

A question to start

Starting from a situation where output is too low, a monetary expansion that stimulates economic activity is always beneficial.

Do you agree? Why or why not?

TWO COUNTRIES OPTIMIZING MODEL

Optimization and frictions

- “Now open economy macroeconomics”.
- Go beyond Mundell-Flemming by introducting optimization, while keeping frictions.
  - Optimal intertemporal choice by households and firms, robust to Lucas critique.
  - Welfare analysis using the household's utility.
- Maintain frictions of the Keynesian model.
  - Prices don't adjust immediately, so monetary policy has real effects.
  - Imperfect competition implies that output is too low (suboptimal use of resources, similar to unemployment), so policy has welfare effects.

Structure

- Two countries, Home (size n) and Foreign (size $1-n$).
- Each inhabited by a household who consumes a range of brands that are imperfect substitutes (n Home brands, $1-n$ Foreign brands).
- Each brand is produced by a unique firm, which has monopoly power (imperfect competition).
- Some changes compared to OR book (for practicality).
  - 2 levels of consumption basket: Home - Foreign baskets, and brands within each basket. OR has one level.
  - Household provides labor to firms. Cost of effort, and linear technology. OR by contrast have each different households, each producing a brand.
- We look at the effect of monetary shocks.

Consumption structure

- Home household's basket:

$$
\begin{aligned}
C &= \left[(n)^{\frac{1}{\lambda}} \left[C\left(h\right) \right]^{\frac{\lambda-1}{\lambda}} + (1-n)^{\frac{1}{\lambda}} \left[C\left(f\right) \right]^{\frac{\lambda-1}{\lambda}} \right]^{\frac{\lambda}{\lambda-1}} \\
C\left(h\right) &= \left[\left(\frac{1}{n}\right)^{\frac{1}{\theta}} \int_0^n \left[C\left(z,h\right) \right]^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}} \\
C\left(f\right) &= \left[\left(\frac{1}{1-n}\right)^{\frac{1}{\theta}} \int_n^1 \left[C\left(z,f\right) \right]^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}}
\end{aligned}
$$

- $\lambda > 0$: elasticity of substitution between Home goods and Foreign goods baskets.
- $\theta > 1$: elasticity of substitution between brands. In the book $\theta = \lambda$.

Consumption demand (1)

- Static allocation of consumption: minimize expenditure, subject to target level of basket.

$$
\begin{aligned}
PC &= P(h)\,C(h) + P(f)\,C(f) \\
&= \int_0^n P(z,h)\,C(z,h)\,dz + \int_n^1 P(z,f)\,C(z,f)\,dz
\end{aligned}
$$

- Consumption of the Home or Foreign goods basket reflects relative prices, time the elasticity of substitution, and overall demand:

$$
C(h) = n\left[\frac{P(h)}{P}\right]^{-\lambda} C \qquad ; \qquad C(f) = (1-n)\left[\frac{P(f)}{P}\right]^{-\lambda} C
$$

- The price index is:

$$
P = \left[n\,[P(h)]^{1-\lambda} + (1-n)\,[P(f)]^{1-\lambda}\right]^{\frac{1}{1-\lambda}}
$$

Consumption demand (2)

- Similar allocation at the level of brands:

$$C(z,h) = \frac{1}{n}\left[\frac{P(z,h)}{P(h)}\right]^{-\theta} C(h) = \left[\frac{P(z,h)}{P(h)}\right]^{-\theta}\left[\frac{P(h)}{P}\right]^{-\lambda} C$$

$$C(z,f) = \frac{1}{1-n}\left[\frac{P(z,f)}{P(f)}\right]^{-\theta} C(f) = \left[\frac{P(z,f)}{P(f)}\right]^{-\theta}\left[\frac{P(f)}{P}\right]^{-\lambda} C$$

- The price indices are:

$$P(h) = \left[\frac{1}{n}\int_0^n [P(z,h)]^{1-\theta}\,dz\right]^{\frac{1}{1-\theta}}$$

$$P(f) = \left[\frac{1}{1-n}\int_0^n [P(z,f)]^{1-\theta}\,dz\right]^{\frac{1}{1-\theta}}$$

Foreign variables

- Similar choice in the Foreign country.
  - The Foreign Household also has weights n on Home goods and $1-n$ on Foreign goods, so baskets are similar.
  - Foreign variables, and prices in Foreign currency, denoted by $*$.

$$
\begin{aligned}
C^{*}(z,h) &= \frac{1}{n}\left[\frac{P^{*}(z,h)}{P^{*}(h)}\right]^{-\theta} C^{*}(h) = \left[\frac{P^{*}(z,h)}{P^{*}(h)}\right]^{-\theta}\left[\frac{P^{*}(h)}{P^{*}}\right]^{-\lambda} C^{*} \\
C^{*}(z,f) &= \frac{1}{1-n}\left[\frac{P^{*}(z,f)}{P^{*}(f)}\right]^{-\theta} C^{*}(f) = \left[\frac{P^{*}(z,f)}{P^{*}(f)}\right]^{-\theta}\left[\frac{P^{*}(f)}{P^{*}}\right]^{-\lambda} C^{*}
\end{aligned}
$$

Utility and budget

- Utility of consumption, real balance (to get a money demand, small weight χ), and disutility of labor.

$$U_t = \sum_{s=t}^{\infty} \beta^{s-t} \left[\ln(C_s) + \chi \ln\left(\frac{M_s}{P_s}\right) - \frac{\kappa_s}{2} (L_s)^2 \right]$$

- Purchases domestic money, a bond paying off in Home currency (could have Foreign currency). Income from wages, profits of domestic firms (lump sum), net of tax:

$$B_{t+1} + M_t + P_t C_t = (1 + i_t) B_t + M_{t-1} + W_t L_t + \Pi_t - T_t$$

- Government rebates money creation through a transfer (in appendix, we look at government spending). Bonds are between Home and Foreign households (no government debt):

$$0 = (M_t - M_{t-1}) + T_t$$

Intertemporal optimization

- Maximize utility subject to the budget constraints of each period. Condition with respect to consumptions, bond, labor, and money:

$$\begin{aligned} \lambda_t &= \frac{1}{P_t C_t} \quad ; \quad \lambda_{t+1} = \frac{1}{P_{t+1} C_{t+1}} \\ \lambda_t &= \beta \left(1 + i_{t+1}\right) \lambda_{t+1} \quad ; \quad \lambda_t W_t = \kappa_t L_t \\ \lambda_t - \beta \lambda_{t+1} &= \chi \left(\frac{M_t}{P_t}\right)^{-1} \frac{1}{P_t} \end{aligned}$$

- Combining gives the Euler conditions, the money demand, and the labor supply:

$$\begin{aligned} C_{t+1} &= \beta \left(1 + i_{t+1}\right) \frac{P_t}{P_{t+1}} C_t \\ \frac{M_t}{P_t} &= \chi C_t \frac{1 + i_{t+1}}{i_{t+1}} \\ \kappa_t L_t C_t &= \frac{W_t}{P_t} \end{aligned}$$

Foreign optimization

- Similar steps for the Foreign household.

$$
\begin{aligned}
C_{t+1}^{*} &= \beta \left(1+i_{t+1}\right) \frac{\mathcal{E}_t P_t^{*}}{\mathcal{E}_{t+1} P_{t+1}^{*}} C_t^{*} \\
\frac{M_t^{*}}{P_t^{*}} &= \chi C_t^{*} \frac{\left(1+i_{t+1}\right) \mathcal{E}_t}{\left(1+i_{t+1}\right) \mathcal{E}_t - \mathcal{E}_{t+1}} \\
\kappa_t^{*} L_t^{*} C_t^{*} &= \frac{W_t^{*}}{P_t^{*}}
\end{aligned}
$$

- $\mathcal{E}$: exchange rate in terms of units of Home currency per unit of Foreign currency (an increase is a depreciation of the Home currency)

Firm

- Each brand z is produced by a firm using a linear technology in labor: $Y(z,h) = L(z,h)$.
- Demand faced by the firm computed from allocation of consumption by Home and Foreign Households:

$$Y(z,h) = nC(z,h) + (1-n)\, C^*(z,h)$$

- Profits (note that the firm's prices $P(z,h)$ and $P^*(z,h)$ matter with elasticity θ):

$$\begin{aligned}\Pi(z,h) = {} & (P(z,h) - W)\left[\frac{P(z,h)}{P(h)}\right]^{-\theta}\left[\frac{P(h)}{P}\right]^{-\lambda} nC \\ & + (\mathcal{E}P^*(z,h) - W)\left[\frac{P^*(z,h)}{P^*(h)}\right]^{-\theta}\left[\frac{P^*(h)}{P^*}\right]^{-\lambda} (1-n)\, C^*\end{aligned}$$

Optimal price

- If the firm can set the price, it chooses a markup over marginal cost, and the law of one price:

$$P(z,h) = \mathcal{E} P^*(z,h) = \frac{\theta}{\theta - 1} W$$

- Similarly for the Foreign firm:

$$\frac{P(f)}{\mathcal{E}} = P^*(f) = \frac{\theta}{\theta - 1} W^*$$

- Note that the elasticity θ drives the markup (not λ).

Aggregate output

- All firms within a country make the same choice, so for instance $P(z,h) = P(h)$.
- Home and Foreign outputs are:

$$\begin{aligned} Y &= n\left[\frac{P(h)}{P}\right]^{-\lambda} C + (1-n)\left[\frac{P^*(h)}{P^*}\right]^{-\lambda} C^* \\ Y^* &= n\left[\frac{P(f)}{P}\right]^{-\lambda} C + (1-n)\left[\frac{P_t^*(f)}{P^*}\right]^{-\lambda} C^* \end{aligned}$$

- Note that the elasticity λ enters.

- Bonds are in zero net supply: $nB_t + (1-n)\,B_t^* = 0$. Home current account:

$$\begin{aligned} B_{t+1} + P_t C_t &= (1+i_t)\,B_t + P_t(h)\,n\left[\frac{P_t(h)}{P_t}\right]^{-\lambda} C_t \\ &\quad + \mathcal{E}_t P_t^*(h)\,(1-n)\left[\frac{P_t^*(h)}{P_t^*}\right]^{-\lambda} C_t^* \end{aligned}$$

- Foreign current account:

$$\begin{aligned} -\frac{n}{1-n}\frac{B_{t+1}}{\mathcal{E}_t} + P_t^* C_t^* &= -\frac{n}{1-n}(1+i_t)\,\frac{B_t}{\mathcal{E}_t} + n\frac{P_t(f)}{\mathcal{E}_t}\left[\frac{P_t(f)}{P_t}\right]^{-\lambda} C_t \\ &\quad + (1-n)\,P_t^*(f)\left[\frac{P_t^*(f)}{P_t^*}\right]^{-\lambda} C_t^* \end{aligned}$$

LINEARIZED SOLUTION

Symmetric steady state

- Take linear approximations around a steady state where all countries are identical, with no bond holdings, $B_0 = B_0^* = 0$.
- The interest rate offsets the discount: $1 = \beta(1 + i_0)$.
- Optimal pricing, the labor supply, and the current account gives output and consumption:

$$C_0 = Y_0 = \left(\frac{\theta - 1}{\theta \kappa_0}\right)^{\frac{1}{2}} < \left(\frac{1}{\kappa_0}\right)^{\frac{1}{2}}$$

- The monopolistic distortion ($\theta < \infty$) implies that output is inefficiently low.
- Prices are given by the money demand, and the exchange rate reflects the relative monetary stances:

$$\mathcal{E}_0 = \frac{M_0}{M_0^*}$$

Shocks, and two periods adjustment

- Use log-linear deviations around this steady state: $x_t = \ln X_t - \ln X_0 = (X_t - X_0)/X_0$.
- The economy is initially at the symmetric steady state.
- In period t permanent monetary shocks occur, $\overline{m}$ and $\overline{m}^*$ (fiscal and productivity shocks presented in the appendix).
- Prices cannot adjust in period t (the short run), and are set in the currency of the firm.
  - Prices of domestic goods don't changes, prices of imports change with the exchange rate: $p(h) = p^*(f) = 0$, $p^*(h) = -e$, and $p(f) = e$.
  - A depreciation ($e > 0$) raises the competitiveness of Home goods. PPP holds: $p - p^* = e$.
- Prices fully adjust at period $t+1$. From then on the economy is in a long run steady state, with variables denoted by upper bars.
  - Differ from the original steady state if bond holdings have changed: $\overline{b} = \overline{B}/(P_0 C_0) \neq 0$.
- Convenient to express results in worldwide averages, $x^W = nx + (1-n)x^*$, and cross-country differences (which we focus on), $x - x^*$.

Long run solution

- Solution conditional on the bonds accumulated in the short run, $\overline{b}$.
- If Home accumulated bonds in the short run ($\overline{b} > 0$), the Home households is better off in the long run:
  - Consumes more.
  - Works less.
  - Benefits from higher terms-of-trade (or lower price competitively to tilt world demand towards Foreign goods).

$$
\begin{aligned}
\overline{p}(h) - \overline{p}^*(f) - \overline{e} &= \frac{1}{2\lambda}\frac{1-\beta}{\beta}\frac{\overline{b}}{1-n} \\
\overline{c} - \overline{c}^* &= \frac{1+\lambda}{2\lambda}\frac{1-\beta}{\beta}\frac{\overline{b}}{1-n} \\
\overline{y} - \overline{y}^* &= -\frac{1}{2}\frac{1-\beta}{\beta}\frac{\overline{b}}{1-n}
\end{aligned}
$$

Exchange rate dynamics

- Euler conditions in terms of cross-country difference:

$$(\bar{c} - \bar{c}^*) = (c - c^*) + (p - p^* - e) - (\bar{p} - \bar{p}^* - \bar{e})$$

- Purchasing power parity holds in the short and long run: $\bar{p} - \bar{p}^* = \bar{e}$, $p - p^* = e$. The real interest rate is then the same in both countries, and the consumption difference is constant: $(\bar{c} - \bar{c}^*) = (c - c^*)$.
- Money market equilibrium in the short and long run:

$$\begin{aligned}\bar{m} - \bar{m}^* - (p - p^*) &= (c - c^*) + \frac{\beta}{1-\beta}(e - \bar{e}) \\ \bar{m} - \bar{m}^* - (\bar{p} - \bar{p}^*) &= (\bar{c} - \bar{c}^*)\end{aligned}$$

- Euler implies that there is no overshooting (with our parameters):

$$\begin{aligned}\bar{m} - \bar{m}^* - \bar{e} &= \bar{m} - \bar{m}^* - e - \frac{\beta}{1-\beta}(e - \bar{e}) \\ e - \bar{e} &= -\frac{\beta}{1-\beta}(e - \bar{e}) \\ e &= \bar{e}\end{aligned}$$

Short run solution: the MM line

- Short run money demand gives a relation between relative consumption and the exchange rate.

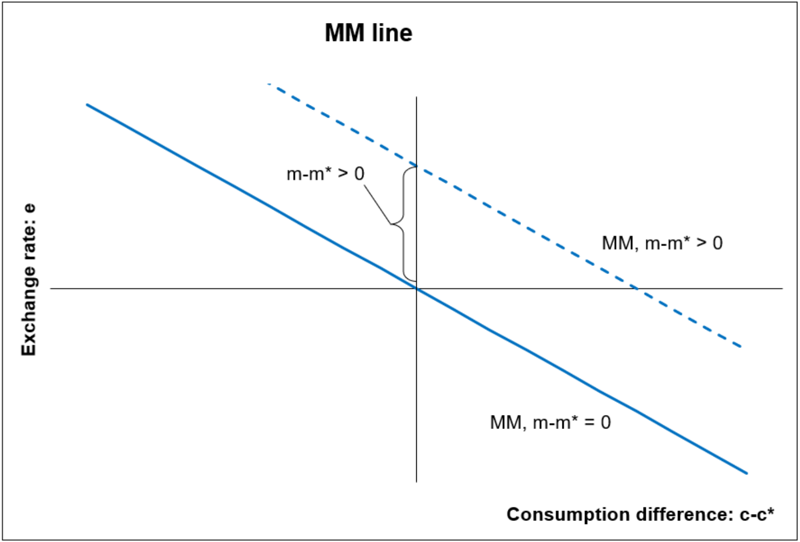
$$(c - c^*) = (\overline{m} - \overline{m}^*) - (p - p^*) = (\overline{m} - \overline{m}^*) - e$$

$$\Rightarrow e = (\overline{m} - \overline{m}^*) - (c - c^*)$$

- Changes in the money supply shift this relation.
- Higher money supply has to be matched with a higher nominal consumption, either through higher consumption or higher prices (i.e. a depreciated home currency).

MM line

- Negative relation between $c - c^*$ and e.



Short run solution: the GG line

- The current account and output demand in the short run imply (using $b = 0$):

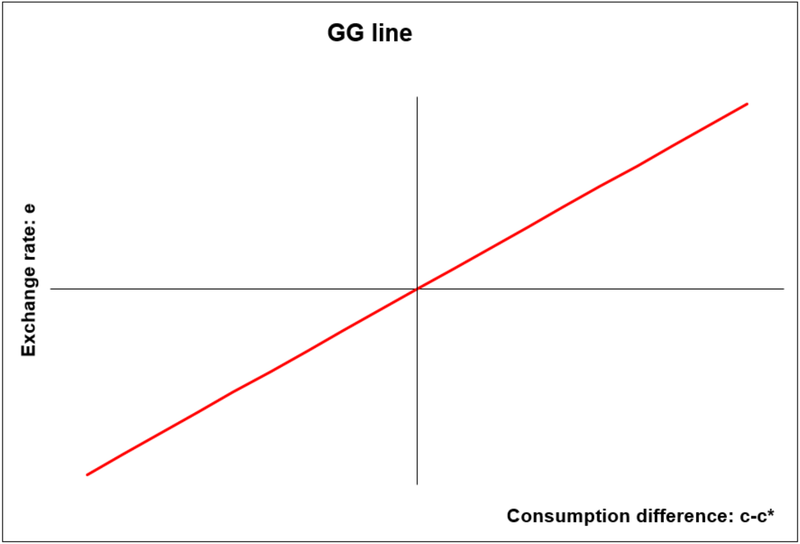
$$\frac{\bar{b}}{1-n} + (c - c^*) = e(\lambda - 1)$$

- Depreciation increases output (as $(y - y^*) = \lambda e$), and worsens the terms of trade (as $(p(h) - p^*(f)) - e = -e$).
  - First effect dominates, so revenue increases by $e(\lambda - 1)$.
  - Additional revenue can finance consumption and savings.
- Long run solution and constant consumption difference, we get a relation between consumption and the exchange rate:

$$e = \left[1 + \frac{\beta}{1-\beta}\frac{2\lambda}{1+\lambda}\right]\frac{1}{\lambda - 1}(c - c^*)$$

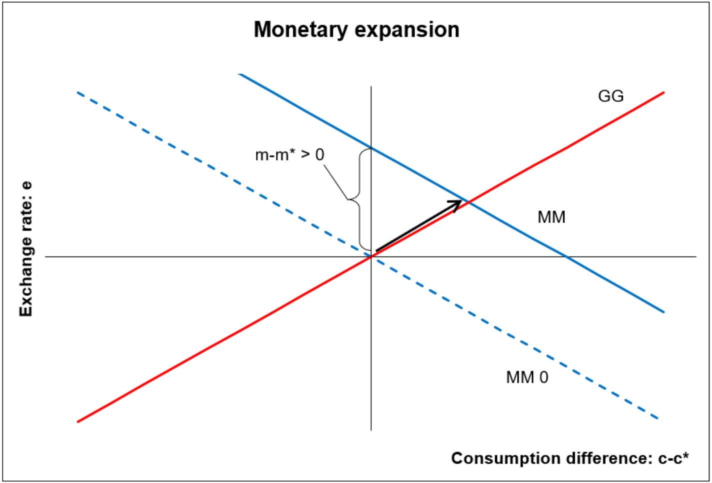
GG line

- Positive relation between $c - c^*$ and e.



Monetary expansion

- An expansion in the Home country, $\overline{m} - \overline{m}^* > 0$, depreciates the exchange rate and raises relative consumption.



Analytical solution

- Combining the two lines, we solve for the exchange rate, consumption and the current account:

$$
\begin{aligned}
e &= \frac{2\lambda + (1+\lambda)\frac{1-\beta}{\beta}}{2\lambda + \lambda(1+\lambda)\frac{1-\beta}{\beta}} (\overline{m} - \overline{m}^*) \\
c - c^* &= \frac{(\lambda^2 - 1)\frac{1-\beta}{\beta}}{2\lambda + \lambda(1+\lambda)\frac{1-\beta}{\beta}} (\overline{m} - \overline{m}^*) \\
\frac{\overline{b}}{1-n} &= \frac{2}{2 + (1+\lambda)\frac{1-\beta}{\beta}} (\lambda - 1)(\overline{m} - \overline{m}^*)
\end{aligned}
$$

- A monetary expansion raises consumption, depreciates the currency, and leads to a current account surplus provided $\lambda > 1$.

RICHER EXCHANGE RATE EFFECT

Incomplete pass-through

- Import prices reflect the exchange rate only to an extent s. We considered $s = 1$. If $s = 0$ import prices are not affected by the exchange rate.
- Incomplete pass-through of the exchange rate implies that PPP does not hold in the short run:

$$\mathrm{p} - \mathrm{p}^* = s\mathrm{e}$$

- The real interest rate then differs between the two countries. A depreciation of the Home currency reduces its real interest rate, relative to the Foreign country:

$$\begin{aligned}
& [\mathrm{i} - (\overline{\mathrm{p}} - \mathrm{p})] - [(\mathrm{i} + \mathrm{e} - \overline{\mathrm{e}}) - (\overline{\mathrm{p}}^* - \mathrm{p}^*)] \\
= & -(\overline{\mathrm{p}} - \mathrm{p}) - (\mathrm{e} - \overline{\mathrm{e}}) + (\overline{\mathrm{p}}^* - \mathrm{p}^*) \\
= & -(\overline{\mathrm{p}} - \overline{\mathrm{p}}^* - \overline{\mathrm{e}}) + (\mathrm{p} - \mathrm{p}^* - \mathrm{e}) \\
= & -(1 - s)\,\mathrm{e}
\end{aligned}$$

Current account

- Consider a more general utility for consumption: $\frac{(C)^{1-\sigma}}{1-\sigma}$. We can show that the current account is given by:

$$\frac{\overline{b}}{1-n} = \frac{\sigma(\lambda-1)+(1+\lambda)}{\sigma(\lambda-1)+(1+\lambda)\frac{1}{\beta}}\left[\frac{\sigma-1}{\sigma}(1-s)+(\lambda-1)s\right]e$$

- With full pass-through ($s=1$), driven by the elasticity λ.
  - If $\lambda>1$ a depreciation leads to a large shift of demand towards home goods ($y-y^*=\lambda e$).
  - Enough to offset the worsening of the home terms of trade, Home can afford more consumption, smooths through savings.
- Without pass-through ($s=0$) no demand switching ($y-y^*=0$). Current account reflects the intertemporal elasticity σ.
  - Depreciation lowers the Home real interest rate, shifts Home consumption towards the short run.
  - Boosts Home exports earnings. This dominates if consumption is not sensitive to the real interest rate ($\sigma>1$), hence a current account surplus.

Welfare analysis

- Expand the utility of the household:

$$\mathrm{u}_t = \mathrm{c} - \frac{\theta - 1}{\theta}\mathrm{y} + \frac{\beta}{1-\beta}\left[\overline{\mathrm{c}} - \frac{\theta - 1}{\theta}\overline{\mathrm{y}}\right]$$

- Consumption raises welfare, output (effort) lowers it.
  - Smaller weight of effort due to monopolistic distortion $\frac{\theta-1}{\theta}$.
  - Output is too low. Increase both output and consumption leads to a larger consumption utility gain (we get closer to the competitive efficient level).
- A monetary expansion raises global welfare when $\theta < \infty$:

$$\mathrm{u}_t^W = \frac{1}{\theta\sigma}\overline{\mathrm{m}}^W$$

Welfare difference

- Cross country welfare difference, using the current account relations:

$$\mathrm{u}_t - \mathrm{u}_t^* = (1-s)\,\mathrm{e} + \frac{\lambda - \theta}{\lambda\theta}\left[(\mathrm{y} - \mathrm{y}^*) + \frac{\beta}{1-\beta}(\bar{\mathrm{y}} - \bar{\mathrm{y}}^*)\right]$$

- With limited pass-through ($s < 1$), depreciation raises Home welfare.
  - Higher earnings on exports, with limited reduction of export quantity (beggar-thy-neighbor).
- If $\lambda < \theta$, Home suffers from an output expansion (beggar-thyself).
  - Selling the additional output requires a lower price of Home goods.
  - Cost from worsening of the terms of trade high, extra consumption does not offset the cost of effort.
- No difference if $\lambda = \theta$. Everyone benefits from the global gain.

Overshooting

- If we consider a richer utility of real balances, $\frac{(M/P)^{1-\varepsilon}}{1-\varepsilon}$, we can get overshooting. The Euler, and short and long run money demands are:

$$\begin{aligned}
(\bar{c} - \bar{c}^*) &= (c - c^*) - (1 - s)\, e \\
\varepsilon\,(\overline{m} - \overline{m}^*) - \varepsilon s e &= (c - c^*) + \frac{\beta}{1-\beta}\,(e - \bar{e}) \\
\varepsilon\,(\overline{m} - \overline{m}^*) - \varepsilon \bar{e} &= (\bar{c} - \bar{c}^*)
\end{aligned}$$

- There is overshooting ($e - \bar{e} > 0$) if pass-through is limited ($s < 1$) and the utility of money is very concave ($\varepsilon > 1$):

$$e - \bar{e} = \frac{1-\beta}{\beta + (1-\beta)\,\varepsilon}\,(\varepsilon - 1)\,(1 - s)\, e$$

- The lower real interest in Home brings relative consumption forward $(c - c^*) > (\bar{c} - \bar{c}^*)$ which raises relative money demand.
- If $\varepsilon > 1$ money demand does not increase much, so a difference is needed in nominal returns, i.e. overshooting.

Main takeaways

- Model combining optimization with price rigidities.
  - Consistent short- and long-run solution.
  - Effects in both countries, can be of different sizes.
- Shocks can have long-run effects if they lead to changes in bond holdings.
  - New steady state is not necessarily the initial symmetric one ("unit root" feature).
- Pass-through of exchange rate to import prices matters.
  - Deviation from PPP, exchange rate overshooting.
  - Different channels for the current account.
- Welfare analysis using the household's utility.
  - Higher Home output does not necessarily raise Home welfare (relative to Foreign).