Lecture Notes:
International Trade I

Based on lectures by **Monika Mrazova** in Spring semester, 2025

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These lecture notes were taken in the course *International Trade I* taught by **Monika Mrazova** at Graduate of International and Developoment Studies, Geneva as part of the International Economics program (Semester II, 2024).

Currently, these are just drafts of the lecture notes. There can be typos and mistakes anywhere. So, if you find anything that needs to be corrected or improved, please inform at `jingle.fu@graduateinstitute.ch`.

Contents

Lecture 1.

Comparative Advantage and Gains from Trade

1.1 Internatioanl trade: Standard Assumptions

What distinguished trade theory from general-equilibrium analysis is the exiatence of a **hierarchical market structure**.

- 'International' good markets
- 'Domestic' factor markets.

Question.

- How does the integration of good markets affect good prices?
- How do changes in good prices, in turn, affect factor prices, factor allocation, production and welfare?

While these assumptions are less fundamental, we will also often assume that:

Assumption 1.1.1.

- Consumers have identical homothetic preferences in each country (representative agent)
- Model is static.

1.2 Neoclassical trade

"Neoclassical trade theory" is characterized by three key assumptions:

1. **Perfect competition** in all markets
2. **Constant returns to scale** in production
3. **No distortions**

Note.
Increasing returns to scale (IRS) are a much more severe issue addressed by "New" trade theory

Let's first stick to the general case and show how simple revealed preference arguments can be used to establish two important results:

1. *Gains from trade*(Samuelson 1939)
2. *Law of comparative advantage*(Deardorff 1980)

1.2.1 Basic environment

Consider a world economy with $n = 1, 2, \cdots, N$ countries, each populated by $h = 1, \cdots, H$ households. There are $g = 1, \cdots, G$ goods:

- $y^n \equiv (y_1^n, \cdots, y_G^n) \equiv$ Output vector in country n
- $c^{nh} \equiv (c_1^{nh}, \cdots, c_G^{nh}) \equiv$ Consumption vector of household h in country n
- $p^n \equiv (p_1^n, \cdots, p_G^n) \equiv$ Price vector in country n

There are $f = 1, \cdots, F$ factors:

- $v^n \equiv (v_1^n, \cdots, v_F^n) \equiv$ Factor endowment vector in country n
- $w^n \equiv (w_1^n, \cdots, w_F^n) \equiv$ Factor price vector in country n

Supply side

We denote by Ω^n the set of combinations (y, v) feasible in country n, our assumption of constant returns to scale implies that Ω^n is a convex set.

Definition 1.2.1 (Revenue function).

The **revenue function** in country n of a firm producing output y using factors v is a function $r^n(y, v)$ such that:

$$r^n(y, v) \equiv \max_y \{py | (y, v) \in \Omega^n\} \tag{1.1}$$

Note (see Dixit-Norman pp. 31-36 for details).

- Revenue function summarizes all relevant properties of technology;
- Under perfect competition, y^n maximizes the value of output in country n:

$$r^n(p^n, v^n) = p^n y^n.$$

Demand side

We denote by u^{nh} the utility function of household h in country n.

Definition 1.2.2 (Expenditure function).

The **expenditure function** of household h in country n is a function $e^{nh}(p^n, u^{nh})$ such that:

$$e^{nh}(p, u) \equiv \min_c \{pc | u^{nh}(c) \geq u\} \tag{1.2}$$

Note (see Dixit-Norman pp. 59-64 for details).

- Here factor endowments are in fixed supply, but easy to generalize to case where households choose factor supply optimally
- Holding p fixed, $e^{nh}(p, u)$ is increasing in u.

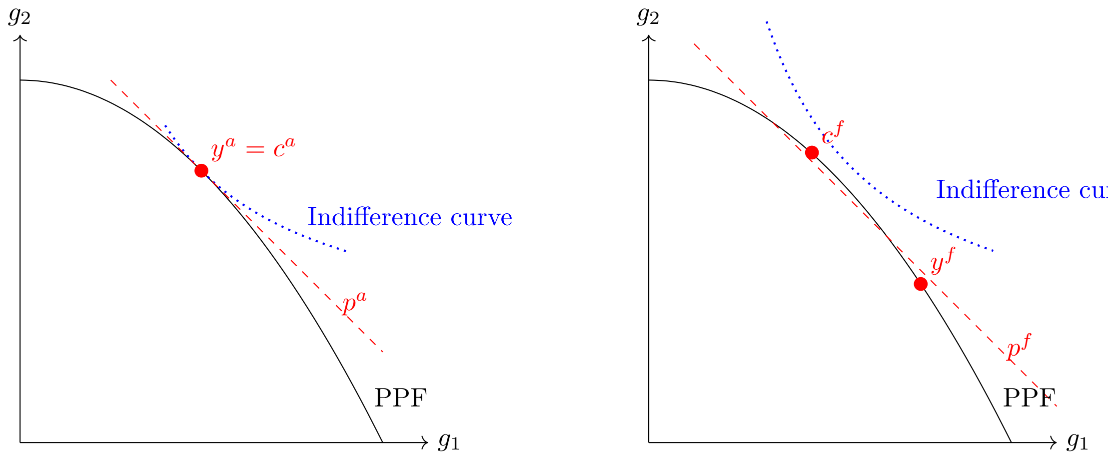
- Household's optimization implies:

$$e^{nh}(p^n, u^{nh}) = p^n c^{nh}$$

where c^{nh} and u^{nh} are the consumption and utility level of the household h in country n in equilibrium, respectively.

1.2.2 Gains from Trade

In the next propositions, when we say "in a neoclassical trade model", we mean in a model where equations (1.1) and (1.2) hold in any equilibrium.



(a) Autarky Equilibrium

(b) Free Trade Equilibrium

Figure 1.1: Equilibria for a Small Country

Formula of Gains from Trade

Arkolakis, Costinot, Rodriguez-Clare (AER, 2012)

Assumption 1.2.1.

- CES utility function(Dixit-Stiglitz)

$$U = \left[\int q(\omega)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$

- One factor production(labor) and constant RS
- 'Iceberg' trade costs
- Import demand system is CES: artial equilibrium (given wages) elasticity of aggregate bilateral trade flow relative to domestic demand is ε w.r.t. trade costs τ_{ij} fo any i, j.

Define a "foreign shock" as any change in (foreign) endowments and trade costs that do not affect a country's endowment or its ability to serve its own market. Define $\hat{W} = \frac{W'}{W}$ and $\hat{\lambda}_{jj} = \frac{\lambda'_{jj}}{\lambda_{jj}}$

Proposition 1.2.1. The change in country j's real income associated with any foreign shock can be computed as $\hat{W}_j = \hat{\lambda}_{jj}^{\frac{1}{\varepsilon}}$, where λ_{ij} is the share of country j's spending on country i's goods.

Corollary 1.2.1. Gains from TRade relative to autarky can be computed as $\hat{W}_j = \lambda_{jj}^{-\frac{1}{\varepsilon}}$.

One household per country

Consider first the case where there is just one household per country, $H = 1$. Without risk of confusion, we drop h and n from all variables.

We denote by:

- (y^a, c^a, p^a) the vector of output, consumption and good prices under autarky;
- (y^f, c^f, p^f) the vector of output, consumption and good prices under free trade.
- u^a and u^f the utility levels under autarky and free trade.

Proposition 1.2.2. *In a neoclassical trade model with one household per country, free trade makes all households (weakly) better off.*

Proof.

Under free trade, households can consume at prices p^f. By definition of the expenditure function, we have:

$$\begin{aligned} e(p^f, u^a) &\leq p^f c^a \\ &= p^f y^a \\ &\leq r(p^f, v^f) \\ &= e(p^f, u^f) \end{aligned}$$

Since $e(p, \cdot)$ is increasing, we get $u^f \geq u^a$. □

Note.

- Two inequalities in the previous proof correspond to consumption and production gains from trade.
- Previous inequalities are weak. Equality if kinks in IC or PPF.
- Previous proposition only establishes that households always prefer "free trade" to "autarky." It **does not** say anything about the comparisons of trade equilibria.

Multiple households per country: domestic lump-sum transfers

With multiple households per country, moving away from autarky is likely to create winners and losers. In order to establish the Pareto-superiority of trade, we will need to allow for **policy instruments**. We start with *domestic lump-sum transfers* and then *commodity taxes*.

We now reintroduce the index h and denote by:

- c^{ah} and c^{fh} the consumption vectors of household h under autarky and free trade;

- v^{ah} and v^{fh} the endowment vectors of country h under autarky and free trade;
- u^{ah} and u^{fh} the utility levels of household h under autarky and free trade;
- τ^h the lump-sum transfer from the government to household h under free trade. [1]

Proposition 1.2.3.

In a neoclassical trade model with multiple households per country, there exist domestic lump-sum transfers such that free trade is (weakly) Pareto superiority than autarky in all countries.

Proof.

For any h, set the lump-sum transfer τ^h such that:

$$\tau^h = (p^f - p^a)c^{ah} - (w^f - w^a)v^{fh}.$$

Budget constraint unde rautarky implies that: $p^a c^{ah} \leq w^a v^{fh}$. Therefore, we have:

$$p^f c^{ah} \leq w^f v^{fh} + \tau^h.$$

Thus c^{ah} is still int he budget set of household h under free trade.

By definition, the government revenue is given by:

$$\begin{aligned} -\sum \tau^h &= (p^a - p^f)\sum c^{ah} - (w^a - w^f)\sum v^{fh} \\ &= (p^a - p^f)y^a - (w^a - w^f)v^f \\ &= -p^f y^a + w^f v^f \\ &\geq -r(p^f, v^f) + w^f v^f \\ &= -(p^f y^f - w^f v^f) = 0. \end{aligned}$$

□

So, each household can buy its autarky consumption bundle at free trade prices and still have some money left. But, the government must know individual preferences to implement the transfers.

If it does not, households can manipulate mechanism by altering their announcements or autarky behavior. In other words, lump-sum transfers typically are not incentive compatible.

Multiple households per country: commodity taxes

We now restrict the set of instruments to commodity taxes/subsidies.

Suppose that the government can affect the porices faced by households under free trade by setting τ^{good} and τ^{factor}:

$$\begin{aligned} p^h &= p^f + \tau^{good} \\ w^h &= w^f + \tau^{factor}. \end{aligned}$$

Proposition 1.2.4.

In a neoclassical trade model with multiple households per country, there exist commodity taxes/subsidies such that free trade is (weakly) Pareto superior to autarky in all countries.

[1] $\tau^h \leq 0 \Leftrightarrow$ lump-sum tax and $\tau^h \geq 0 \Leftrightarrow$ lump-sum subsidy.

Proof.

Consider two following taxes:

- $\tau^{good} = p^a - p^f$
- $\tau^{factor} = w^a - w^f$

By construction, household is indifferent between autarky and free trade. Now consider the government revenue:

$$\begin{aligned}
-\sum \tau^h &= \sum \tau^{good} c^{ah} - \sum \tau^{factor} v^{fh} \\
&= (p^a - p^f) \sum c^{ah} - (w^a - w^f) \sum v^{fh} \\
&= (p^a - p^f) y^a - (w^a - w^f) v^f \\
&= -p^f y^a + w^f v^f \\
&\geq -r(p^f, v^f) + w^f v^f \\
&= -(p^f y^f - w^f v^f) = 0.
\end{aligned}$$

□

Tax revenue is non-negative. If all households are on the same side of the market for at least for at least one good ot factor, government can cut a tax or raise a subsidy to generate Pareto improvement.

This scheme sacrifices the consumer gains from trade, but preserves the gains from reorganizing production.

Note.

The previous proof only relies on existence of *production gains* from trade.

- It's closely related to Diamod and Mirrlees (1971) result on the production efficiency
- When only commodity taxes are available, DM sho that production should remain efficient at a social optimum
- Thus, trade, acting as an expansion of PPF, should remain free(ignore issues of market power)

But, if there's a kink in the PPF, there are no production gains. [a]

Factor taxation still informationally intensive: need to know endowments in efficiency units, may lead to different business taxes

[a] Similar problem with "moving costs", see Feenstra p. 185

1.2.3 Law of Comparative Advantage

The previous results have focused on normative predictions. Let's take a look at how the same revealed preference argument can be used to make positive predictions about patterns of trade.

Theorem 1.2.1 (Law of Comparative Advantage)**.**

Countries tend to export goods in which they have a CA, i.e. lower relative autarky prices compared to other countries

Lecture 2.

Ricardian Model

2.1 Introduction

2.1.1 Reasons of trade

- **Countries' differences: comparative advantage**
  - Productivity: Ricardo
  - Endowments: Heckscher-Ohlin
- **Countries' similarities: economies of scale**

2.1.2 Comparative advantage

Note.

Stanislaw Ulam's challenge to Paul Samuelson: "name me one proposition in all of the social sciences which is both true and non-trivial".

Samuelson's answer: Comparative advantage. "That it is logically true need not be argued before a mathematician; that is is not trivial is attested by the thousands of important and intelligent men who have never been able to grasp the doctrine for themselves or to believe it after it was explained to them.".[a]

[a] P.A. Samuelson (1969), "The Way of an Economist," in P.A. Samuelson, ed., International Economic Relations: Proceedings of the Third Congress of the International Economic Association, Macmillan: London, pp. 1-11.

In the simplest and earliest complete model of production and trade, the reason for trade is **comparative advantage**. And the source of comparative advantage is **differences in production technologies**.

These are the differnces in production functions, not the differences in labor productivities due to different endowments of capital which is the type of Heckscher-Ohlin model.

2.1.3 Basic assumptions

- Labor is the only factor of production
- Constant returns to scale
- Perfect competition
- Full employment
- Endowments given, confined to country but intersectorally mobile within each country
- Two countries with different technologies (production functions)
- Number of goods: $n \geq 2$

2.1.4 Technology

Question. How is efficiency measured?

With only one factor, we measure by the usage of the factor: **labbor requirement per unit output**.

We define the quantity of labour (e.g. number of hours) necessary to produce one unit of good i as a_i. If $a_i = 2$, it means that 2 hours of labor are needed to produce one unit of good i.

So, the labor productivity is the inverse of the labor requirement a_i, the higher the labor requirement, the lower the productivity.

2.1.5 Production Possibility Frontier: Refresher

PPF is the maximum possible production level for a given technology and factor endowment.

- Constraint: points outside the PPF are not feasible;
- Efficiency: points inside the PPF are inefficient, those on the PPF are efficient;
- Opportunity cost: the slope of the PPF is the opportunity cost of producing one more unit of good i in terms of good j.
- Concavity: the PPF is usually concave to the origin due to the law of diminishing returns.

Technology progress shifts the PPF outwards.

2.2 The two-sector Model

2.2.1 A Simple Numerical Example

Let's begin with a simple example: US and India producing corn and auto.

	US	India
Labor force	$L = 200$	$L^* = 800$
Labor per unit corn	$a_c = 8$	$a_C^* = 50$
Labor per unit auto	$a_A = 10$	$a_A^* = 40$

Specific assumption of this example : US is more efficient/productive at producing both goods, meaning US has an absolute advantage in both goods.

Production Possibility Frontier

US

The full-employment condition is:

$$L = a_c Q_c + a_A Q_A$$

The PPF is a straight line connecting these two points:

$$Q_A = \frac{L}{a_A} - \frac{a_C}{a_A} Q_C$$

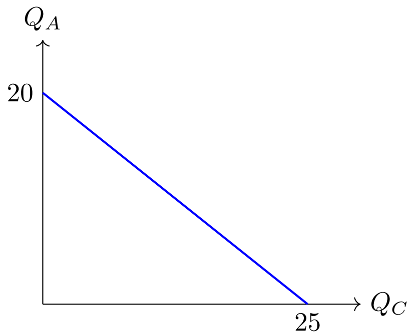


Figure 2.1: US PPF

India

The full-employment condition is:

$$L^* = a_C^* Q_C + a_A^* Q_A$$

The PPF is a straight line connecting these two points:

$$Q_A^* = \frac{L^*}{a_A^*} - \frac{a_C^*}{a_A^*} Q_C$$

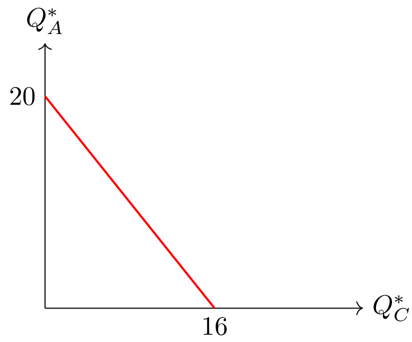


Figure 2.2: India PPF

Relative price and Technology

> **Notation.**
>
> Relative price of corn: $P = \frac{P_C}{P_A}$;
>
> Relative quantity of corn: $Q = \frac{Q_C}{Q_A}$;
>
> L and L^* are the endowments of labor(in efficient units) in the two countries;
>
> w and w^* are the wage(in efficiency units) in the two countries.

Workers are paid the value of their marginal products: $w_C = \frac{P_C}{a_C}$ and $w_A = \frac{P_A}{a_A}$.

Perfectly mobile labor: If both goods are produced, the wage in both sectors must be the same.

$$w_C = w_A \Leftrightarrow P = \frac{a_C}{a_A}.$$

Production and Supply in the US

If $P = \frac{a_C}{a_A} = 0.8$, the international price(relative price) is equal to the domestic opportunity cost of producing corn in terms of auto. Trade cannot bring extra gains. Hence the production set doesn't affect the consumption set, and the production set can be anywhere on the PPF. $1 \leq \frac{Q_c}{Q_A} < \infty$.

If $P < \frac{a_C}{a_A} = 0.8$, international price of C is lower than the domestic opportunity cost of producing C in terms of A. US will produce only A and import C. $Q_C = 0$, $Q_A = 20$, and $\frac{Q_C}{Q_A} = 0$.

If $P > \frac{a_C}{a_A} = 0.8$, international price of C is higher than the domestic opportunity cost of producing C in terms of A. US will produce only C and export A. $Q_A = 0$, $Q_C = 25$, and $\frac{Q_C}{Q_A} = \infty$.

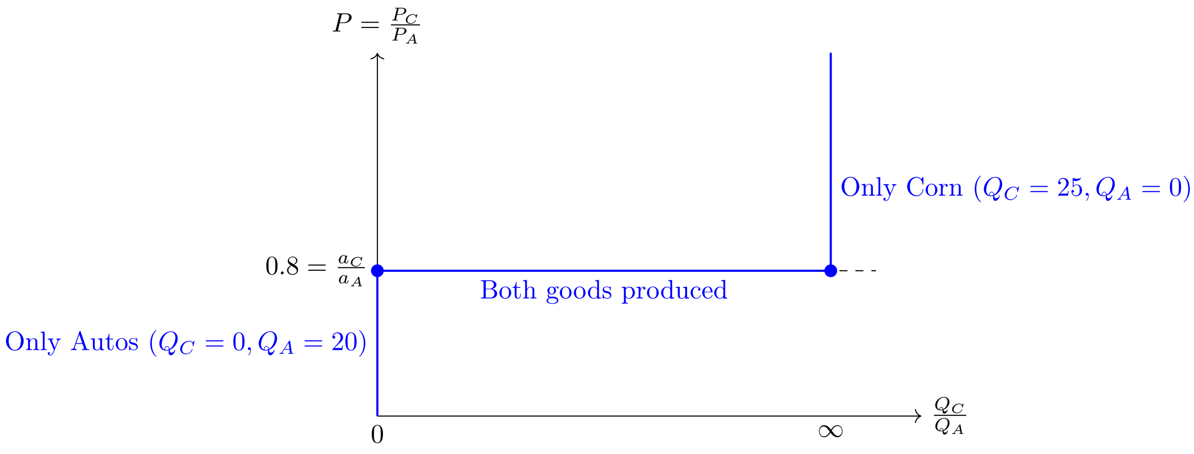


Figure 2.3: US Relative supply

Autarky equilibria in the two countries

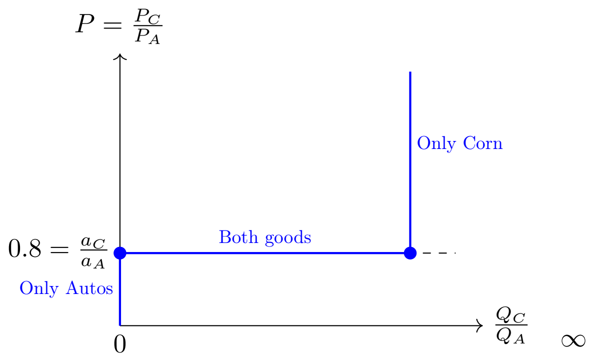


Figure 2.4: US Relative Supply

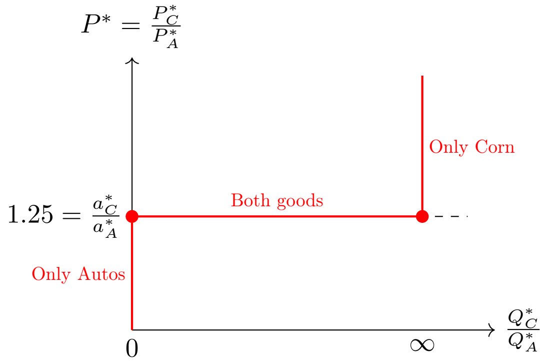


Figure 2.5: India Relative Supply

Trade: Relative supply

When we combine the two countries to form the world market, we get the following relative supply curve:

- When $P < 0.8$, both countries produce only autos, world ratio $R = 0$.

- When $P = 0.8$, US can vary production between 20 autos, no corn (world $R = 0$), and 0 autos, 25 corn (world $R = 1.25$) while India produces only autos, so world $R \in [0, 1.25]$.

- When $0.8 < P < 1.25$, US produces only corn, India produces only autos, so $R = \frac{25}{20} = 1.25$.

- When $P = 1.25$, India can vary production while US produces only corn, so world $R \in [1.25, \infty)$.

- When $P > 1.25$, both countries produce only corn, so world $R = \infty$.

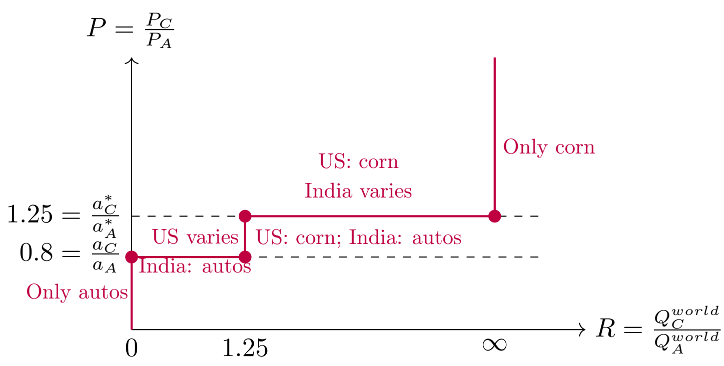


Figure 2.6: World Relative Supply Curve

Trading Equilibrium

Depending on position of relative demand, the trading equilibrium can be one of three types:

A. $P = 0.8$, US produces both goods, India produces only autos;

B. $0.8 < P < 1.25$, US produces only corn, India produces only autos;

C. $P = 1.25$, US produces only corn, India produces both goods.

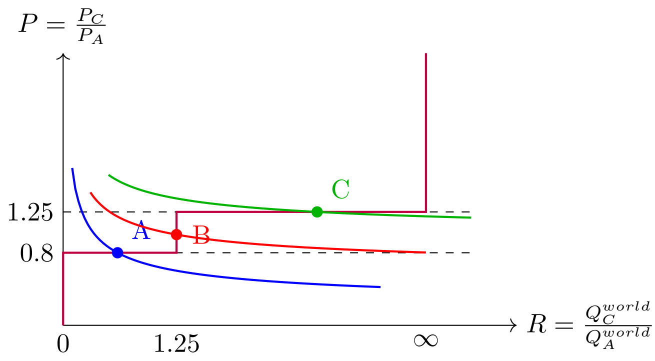


Figure 2.7: Trading Equilibrium

Efficient production and world PPF

Suppose initially all labor produc autos in both: 40 in all. If any corn is produced, it's better to do so by switching sone labor in the US: because wach auto not produced releases 10 labor which can then produce 1.25 corn, while in India, each less auto yields only 0.8 more corn.

Only when all US labor has been diverted to producing corn should any Indian labor be switched.

Conversely, starting with all corn: 41 units, to produce any autos, Indian labor should be switched.

This despite the US producing autos more efficiently than India: only 10 units of labor against 40. The reason: the US produces corn even more efficiently: only 8 units of labor against 50. What matters is the ratio (opportunity cost): $\frac{10}{8} > \frac{40}{50}$, or $\frac{10}{40} > \frac{8}{50}$.

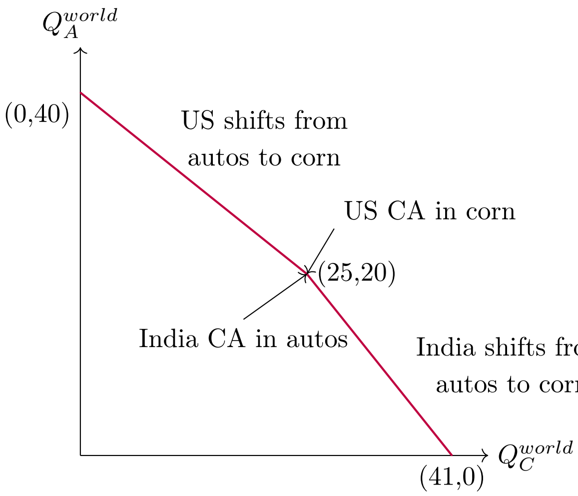


Figure 2.8: World Production Possibility Frontier

TRading equilibrium and world PPF

If preferences are identical and homthetic everywhere, we can draw the world indifference curves. Depending on their shape, there are three types of outcomes:

A. US produces both goods, India produces only autos;

B. US produces only corn, India produces only autos;

C. US produces only corn, India produces both goods.

Relative price of corn = slope of PPF: 0.8 in A, 1.25 in C, between 0.8 and 1.25 in B.

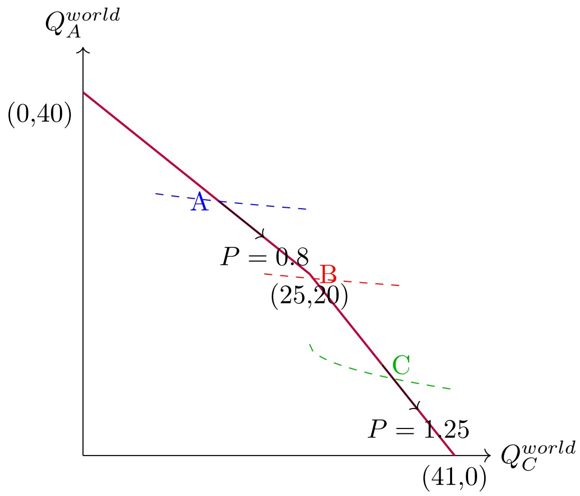


Figure 2.9: Trading Equilibrium on the World PPF

2.2.2 Continuum of Goods: Dornbusch, Fischer and Samuelson(1997)

Appendix

Recommended Resources

Books

[1] James H. Stock and Mark W. Watson. *Introduction to Econometrics*. 4th ed. New York: Pearson, 2003

[2] Jeffrey M. Wooldridge. *Introductory Econometrics: A Modern Approach*. 7th ed. Cengage Learning, 2020

[3] Bruce E. Hansen. *Econometrics*. Princeton, New Jersey: Princeton University Press, 2022

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[6] Joshua Chan et al. *Bayesian Econometric Methods*. 2nd ed. Cambridge, United Kingdom: Cambridge University Press, 2019

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