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FINANCIAL INTEGRATION AND CRISES 2021

Lecture 4

Lecture 4 - Introduction

The Intertemporal Approach to the Current Account

- The dynamics of the CA as determined by 'Consumption smoothing' against output and investment fluctuations
- Shocks: Temporary vs. Permanent Anticipated vs. Unanticipated

It shows the role of financial integration for

- Absorption of domestic shocks and consumption smoothing
- Investment financing without decreasing consumption

A benchmark to evaluate global imbalances

References:

Obstfeld, M. and K. Rogoff (1996) Foundations of International Macroeconomics, Chap. 2 SUW (2021) Chap. 3, 4, 5

Model assumptions

Consumption and investment are chosen by agents/countries who behave optimally given their objectives and constraints.

- Small open economy; world prices and interest rates are 'exogenous'
- Prices are fully flexible Output is supply determined
- The nominal exchange rate does not matter
- □ No valuation effects: $B_t B_{t-1} = CA_t = rB_{t-1} + TB_t$
- One type of real bond internationally traded
- $lue{}$ Bonds are freely traded while K is held within the country
- Infinite horizon, i.e. parents care about their children:
- $U_t = u(C_t^{\mathcal{Y}}; C_{t+1}^o) + \beta E_t U_{t+1}$
- $U_t = u(C_t^y; C_{t+1}^o) + \beta E_t u(C_{t+1}^y; C_{t+2}^o) + \beta^2 E_t U_{t+2}$

Household's maximization problem

The representative dynasty maximizes the Utility

$$U_t = E_t \sum_{i=0}^{\infty} \beta^i u(C_{t+i})$$

subject to the flow resource constraints (and the No-Ponzi game conditions)

•
$$B_{t+i} + C_{t+i} = (1+r_t)B_{t-1+i} + Y_{t+i} - G_{t+i} - I_{t+i}$$
 $i = 1, 2, 3, ...$

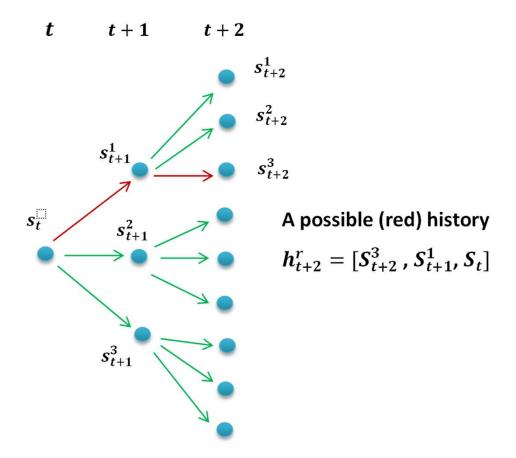
Set the Lagrangian

• Max
$$E_t \sum_{i=0}^{\infty} \beta^i u(C_{t+i}) - \lambda_t [B_t + C_t - (1+r_t)B_{t-1} - Y_t + G_t + I_t] + -E_t \beta \lambda_{t+1} [B_{t+1} + C_{t+1} - (1+r_{t+1})B_t - Y_{t+1} + G_{t+1} + I_{t+1}] - E_t \beta^2 \lambda_{t+2} [.]$$

- Note that expectations are taken over all possible future realizations of our variables, that is, $E_t X_{t+N} = \sum_{h_N=1}^H \pi_t^{h_N} X_{t+N}^{h_N}$ (for discrete distributions)
- These realizations depend on sequences of events, more precisely, on histories of states of nature up to time t+N, here denoted by h_N .

Histories of states of nature

At t+1 there are 3 states/histories: 3 consumption goods and 3 constraints At t+2 there are 9 histories and thus 9 consumption goods; 9 constraints



A more careful maximization problem

Consider the **Utility Function** that is maximized:

- $U_t = E_t \sum_{i=0}^{\infty} \beta^i u(C_{t+i})$ Note that it can be written as
- $U_t = u(C_t) + \sum_{h_1=1}^{H_1} \pi_t^{h_1} \beta u(C_{t+1}^{h_1}) + \sum_{h_2=1}^{H_2} \pi_t^{h_2} \beta^2 u(C_{t+2}^{h_2}) + \cdots$

where $h_j=1, 2, ... H_j$ denote the H_j possible histories at time t+j, so that at t+1 there are H_1 different possible consumptions : $C_{t+1}^1, C_{t+1}^2, ...$ depending on the realization of the state of nature; H_2 at t+2, etc.

- In maximizing the utility function we must thus consider each $C_{t+1}^{h_1}$ as a different good $C_{t+1}^1 \neq C_{t+1}^2 \neq \cdots \neq C_{t+1}^{H_1}$
- □ The resource constraint would also differ depending on the history:

While there is just one constraint at time t, instead at t+1 there are H_1 different budget constraints, and at t+2 there are H_2 different budget constraints, one for each history of states of nature.

Utility maximization

Lagrangian

•
$$Max \ u(C_t) + \sum_{h_1=1}^{H_1} \pi_t^{h_1} \beta u(C_{t+1}^{h_1}) + \sum_{h_2=1}^{H_2} \pi_t^{h_2} \beta^2 u(C_{t+2}^{h_2}) + \cdots$$

$$-\lambda_t [B_t + C_t - (1 + r_t)B_{t-1} - Y_t + G_t + I_t] +$$

$$-\beta \sum_{h_1=1}^{H_1} \pi_t^{h_1} \lambda_{t+1}^{h_1} [B_{t+1}^{h_1} + C_{t+1}^{h_1} - (1 + r_{t+1})B_t - Y_{t+1}^{h_1} + G_{t+1}^{h_1} + I_{t+1}^{h_1}] + \dots$$

In extensive notation is

•
$$Max \ u(C_t) + \pi_t^1 \beta u(C_{t+1}^1) + \pi_t^2 \beta u(C_{t+1}^2) + \pi_t^3 \beta u(C_{t+1}^3) \dots$$

 $-\lambda_t [B_t + C_t - (1 + r_t)B_{t-1} - Y_t + G_t + I_t] +$
 $-\beta \pi_t^1 \lambda_{t+1}^1 [B_{t+1}^1 + C_{t+1}^1 - (1 + r_{t+1})B_t - Y_{t+1}^1 + G_{t+1}^1 + I_{t+1}^1] +$
 $-\beta \pi_t^2 \lambda_{t+1}^2 [B_{t+1}^2 + C_{t+1}^2 - (1 + r_{t+1})B_t - Y_{t+1}^2 + G_{t+1}^2 + I_{t+1}^2] +$
 $-\beta \pi_t^3 \lambda_{t+1}^3 [B_{t+1}^3 + C_{t+1}^3 - (1 + r_{t+1})B_t - Y_{t+1}^3 + G_{t+1}^3 + I_{t+1}^3] +$
 $+ \dots +$

Note: B_t is known and enters all constraints of time t+1 (with the same return)

First order conditions

Assume for the moment that Y_t and I_t are given.

Take the derivative with respect to $oldsymbol{\mathcal{C}}_t$ and each $oldsymbol{\mathcal{C}}_{t+i}^{h_i}$

$$\bullet \quad \beta \pi_t^{h_i} u' \left(C_{t+i}^{h_i} \right) = \beta \pi_t^{h_i} \lambda_{t+i}^{h_i} \quad \rightarrow \quad u' \left(C_{t+i}^{h_i} \right) = \lambda_{t+i}^{h_i}$$

• $u'(C_{t+1}^{h_1}) = \lambda_{t+1}^{h_1}$ (1) one FOC for each history h_1 of states of nature

Take the derivative with respect to B_t

$$-\lambda_t + \beta \pi_t^1 \lambda_{t+1}^1 (1 + r_{t+1}) + \beta \pi_t^2 \lambda_{t+1}^2 (1 + r_{t+1}) + \beta \pi_t^3 \lambda_{t+1}^3 (1 + r_{t+1}) + \dots = 0$$

$$-\lambda_t + \beta E_t \lambda_{t+1} (1 + r_{t+1}) = 0$$
 (2)

Combine equations (0), (1) and (2)

•
$$u'(C_t) = \beta E_t u'(C_{t+1})(1 + r_{t+1})$$
 Euler Equation

• $\lim_{T \to \infty} \pi_t^J \beta^T \lambda_{t+T}^J \, B_{t+T}^J = 0$ Transversality Conditions - one for each history J

Equilibrium conditions

Along the optimal path we have the

Euler Equation(s)

•
$$u'(C_t) = \beta E_t u'(C_{t+1})(1 + r_{t+1})$$
 (3)

Transversality Conditions

$$-\lim_{T\to\infty} \pi_t^J \beta^T u'(C_{t+T}^J) B_{t+T}^J = 0 \tag{4}$$

- Although No-Ponzi game conditions are usually assumed, note that the No-Ponzi game conditions are implied by the Tranversality conditions of foreign consumers that bind our debt growth.
- This provides a model-based justification of No-Ponzi game conditions.
- Note that the correct discount factors to be used are marginal rates of substitution between current and future history-contingent consumption.

Intertemporal Budget Constraint

- lacktriangle Assume a constant $r_t=r$ and note that the constraint
- $B_t + C_t = (1+r)B_{t-1} + Y_t G_t I_t$ is equal to
- $B_t = (1+r)B_{t-1} + TB_t$. Then, we have the
- Intertemporal Budget Constraint

$$B_{t-1} = \frac{E_t(C_t - Y_t + G_t + I_t)}{(1+r)} + \frac{E_t(C_{t+1} - Y_{t+1} + G_{t+1} + I_{t+1})}{(1+r)^2} \dots \frac{E_t(C_{\infty} - Y_{\infty} + G_{\infty} + I_{\infty})}{(1+r)^{\infty}}$$

or

$$\sum_{i=0}^{\infty} \frac{1}{(1+r)^i} E_t C_{t+i} = (1+r) B_{t-1} + \sum_{i=0}^{\infty} \frac{1}{(1+r)^i} E_t [Y_{t+i} - G_{t+i} - I_{t+i}]$$
 (5)

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Consumption Smoothing

- \square Assume $\beta(1+r)=1$ and a quadratic utility:
- $u(C_t) = C_t \frac{b}{2}C_t^2$ so that $u'(C_t) = 1 bC_t$

and the Euler Equations (3) imply that

- $E_t C_{t+1} = C_t$ consumption is a random walk (6)
- $E_t C_{t+i} = C_t$ for any $i \ge 1$ (7)
- ullet Substituting C_t for $E_t C_{t+i}$ in the IBC yields
- $C_t \sum_{i=0}^{\infty} \frac{1}{(1+r)^i} = (1+r) B_{t-1} + \sum_{i=0}^{\infty} \frac{1}{(1+r)^i} E_t [Y_{t+i} G_{t+i} I_{t+i}]$ (8)

Permanent Income Consumption

•
$$C_t = r B_{t-1} + \frac{r}{1+r} \sum_{i=0}^{\infty} \frac{1}{(1+r)^i} E_t [Y_{t+i} - G_{t+i} - I_{t+i}]$$
 (9)

Consumption is determined according to the certainty equivalence principle: consumer behaves as if future stochastic variables turned out to be equal to expected values.

Dynamics of the Current Account

Define the permanent level of a variable X_t as the constant value \bar{X}_t that yields the same present value of X_t

$$\bar{X}_t = \frac{r}{1+r} \sum_{i=0}^{\infty} \frac{1}{(1+r)^i} E_t X_{t+i}$$
 (11)

□ This allows us to write **permanent-income consumption** (9) as

•
$$C_t = r B_{t-1} + \bar{Y}_t - \bar{G}_t - \bar{I}_t$$
 (12)

• Recalling the $CA_t = rB_{t-1} + Y_t - C_t - G_t - I_t$ we have

Current Account Dynamics

•
$$CA_t = (Y_t - \bar{Y}_t) - (G_t - \bar{G}_t) - (I_t - \bar{I}_t)$$
 (13)

Current Account determination with consumption smoothing

Fin. Int. allows to absorb domestic specific shocks and to smooth consumption

- $CA_t = B_t B_{t-1} = (Y_t \overline{Y}_t) (G_t \overline{G}_t) (I_t \overline{I}_t)$
 - In good times when Y_t is above its permanent level $CA_t > 0$, people save and accumulate foreign assets (if $I_t \approx \bar{I}_t$)
 - In bad times people sell foreign assets or borrow internationally to maintain \mathcal{C}_t
 - When investment needs, I_t , are high, then ${\it CA}_t < 0$: people finance I_t on world capital markets
 - When G_t is high (i.e. wars) it is financed with foreign capital
- $lue{}$ Note that, as consumption is at its permanent level, $\mathcal{C}_t = ar{\mathcal{C}}_t$:
- $CA_t = (Y_t \bar{Y}_t) (G_t \bar{G}_t) (I_t \bar{I}_t) (C_t \bar{C}_t) =$
- $CA_t \equiv rB_{t-1} + TB_t = TB_t \overline{TB}_t \rightarrow rB_{t-1} = -\overline{TB}_t$
- Net investment income finances a permanent trade deficit (or a permanent surplus finances payments on foreign liabilities).

Output Shocks and the Current Account

lacksquare For simplicity, set $I_t=0$ and $G_t=0$. Then

Permanent Income Consumption

(here \overline{Y} is not permanent income)

•
$$C_t = r B_{t-1} + \overline{Y} + \frac{r}{1+r} \sum_{i=0}^{\infty} \frac{1}{(1+r)^i} E_t[Y_{t+i} - \overline{Y}]$$
 (1)

 $lue{}$ Consider an autoregressive stochastic process for Y_t :

•
$$Y_{t+1} - \overline{Y} = \rho(Y_t - \overline{Y}) + \varepsilon_{t+1}$$
 with $0 \le \rho \le 1$; $E_t \varepsilon_{t+1} = 0$

and substituting it in the consumption function above

•
$$C_t = r B_{t-1} + \overline{Y} + \frac{r}{1+r-\rho} [Y_t - \overline{Y}]$$
 (3)

•
$$C_t = r B_{t-1} + \overline{Y} + \frac{r \rho}{1 + r - \rho} [Y_{t-1} - \overline{Y}] + \frac{r}{1 + r - \rho} \varepsilon_t$$
 (4)

 $f \square$ and substituting this equation for C_t in $CA_t = r \ B_{t-1} + Y_t - C_t$

Current Account determination

In an economy without capital where $I_t=0$ and with $G_t=0$

•
$$CA_t = \frac{(1-\rho)}{1+r-\rho} \rho [Y_{t-1} - \bar{Y}] + \frac{1-\rho}{1+r-\rho} \varepsilon_t$$
 (5)

- lacktriangle Temporary output shocks, ho < 1, improve the CA_t
- The change in CA_t decreases with the persistence, ρ , of the shock; the more persistent is the shock the greater is the change in C_t
- Permanent shocks, $\rho=1$, do not affect the CA_t

If future output increases more than current output as in the case:

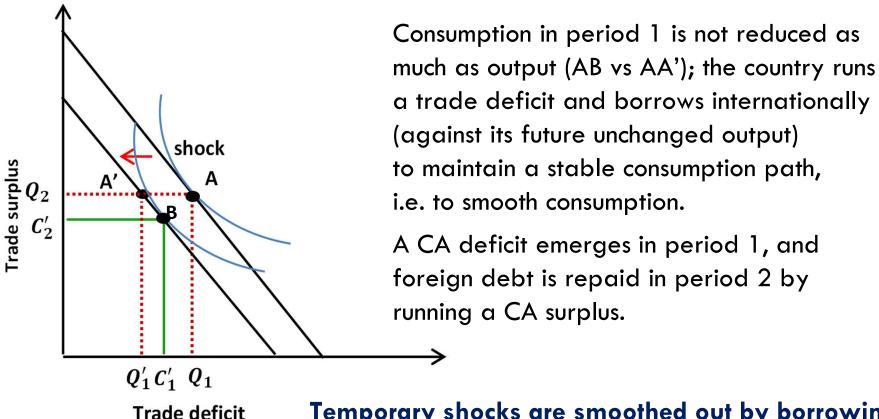
•
$$Y_{t+1} - Y_t = \rho(Y_t - Y_{t-1}) + \varepsilon_{t+1}$$
 (6)

lacksquare C_t would rise more than Y_t and CA_t would worsen.

The introduction of capital may change the first prediction.

Example: Temporary bad output shock

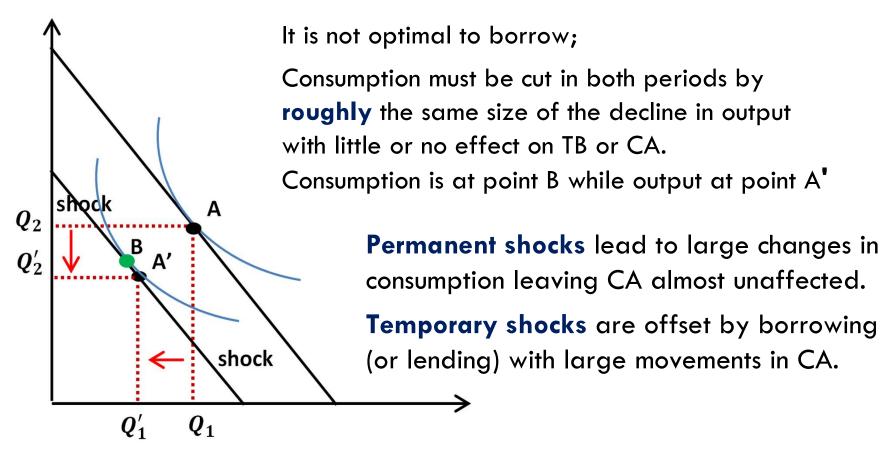
Consider a temporary bad output shock that reduces output only in period 1, eg because of bad weather conditions (a drought reduces coffee crops in Ethiopia).



Temporary shocks are smoothed out by borrowing; a benefit of financial integration.

Example: Permanent bad output shock

The bad shock is permanent: $\,Q_2$ falls as much as $\,Q_1$



Output shocks — Role of Expectations

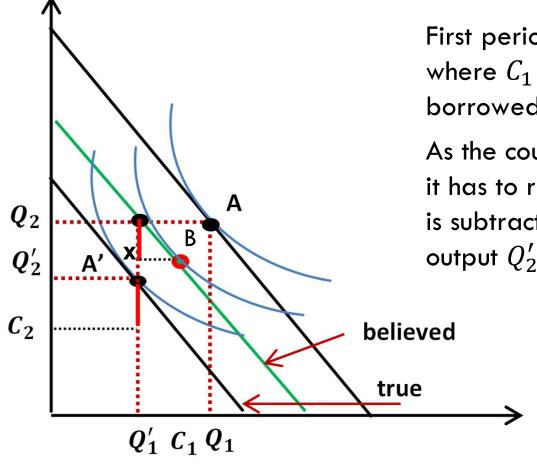
- The effects of output shocks on consumption and the CA depend on whether shocks are:
 - Temporary or Permanent

Temporary output shocks should not much affect consumption (they affect saving) and thus lead to greater changes in the CA balance.

- Importantly, to the extent that the duration of shocks is uncertain, their effects on consumption and the CA depend on whether shocks are:
 - Expected to be Temporary
 - Expected to be Permanent
- Positive output shocks that are **erroneously expected to be permanent** may lead to large increase in consumption and CA deficits that are not sustainable in the long run when shocks reveal to be temporary.

Wrong expectations about shock duration

Suppose the (bad) output shock is permanent leading to point A' but **people believe it is temporary** (green resource constraint).



First period Consumption is at point B where $C_1 > Q_1'$ so that $C_1 - Q_1'$ is borrowed internationally.

As the country enters period 2 with debt, it has to repay the red segment x Q_2 that is subtracted from unexpectedly low output Q_2' to find consumption C_2 .

Note that C_2 is much lower than C_1 A crisis may break up!

Investment Decision

Introduce capital

- $Y_t = A_t F(K_{t-1})$ and $I_t = K_t K_{t-1}$
- The resource constraint becomes

•
$$B_t + K_t + C_t = (1 + r_t)B_{t-1} + K_{t-1} + A_tF(K_{t-1}) - G_t$$

and the representative dynasty maximizes:

$$E_t \sum_{i=0}^{\infty} \beta^i u(C_{t+i}) - E_t \lambda_t [B_t + K_t + C_t - (1+r_t)B_{t-1} - K_{t-1} - A_t F(K_{t-1}) + G_t] +$$

$$-E_{t}\beta\lambda_{t+1}[B_{t+1}+K_{t+1}+C_{t+1}-(1+r_{t+1})B_{t}-K_{t}-A_{t+1}F(K_{t})+G_{t+1}]-E_{t}\beta^{2}\lambda_{t+2}[$$

FOCs include additional conditions (5) and (6)

•
$$u'(C_t) = \lambda_t$$
; $u'(C_{t+1}^{h_1}) = \lambda_{t+1}^{h_1}$

(1) one for each history h_1

$$-\lambda_t + E_t \lambda_{t+1}^{h_1} (1 + A_{t+1}^{h_1} F'(K_t)) = 0$$
 (5)

$$\lim_{T \to \infty} \pi_t^J \beta^T \lambda_{t+T}^J K_{t+T}^J = 0$$

(6) Transversality (one for each history J)

The risk premium

 \square Along the optimal path, from (1) and (5) we have the

Euler Equation (we drop the superscripts h_1 for ease of notation)

•
$$u'(C_t) = \beta E_t u'(C_{t+1})(1 + A_{t+1}F'(K_t))$$
 (7)

lacktriangle Then, recalling the Euler Equation for B_t with $r_{t+1}=r$

•
$$u'(C_t) = \beta E_t u'(C_{t+1})(1+r) = \beta E_t u'(C_{t+1})(1+A_{t+1}F'(K_t))$$
 (8)

•
$$rE_t u'(C_{t+1}) = E_t u'(C_{t+1}) E_t A_{t+1} F'(K_t) + Cov_t (u'(C_{t+1}) A_{t+1} F'(K_t))$$
 (9)

•
$$E_t A_{t+1} F'(K_t) = r - \frac{\text{Co} v_t (u'(C_{t+1}) A_{t+1} F'(K_t))}{E_t u'(C_{t+1})}$$
 (10)

$$r - \frac{(1+r)\beta \text{Co}v_t(u'(C_{t+1})A_{t+1}F'(K_t))}{u'(C_t)} \tag{11}$$

• $E_t A_{t+1} F'(K_t) = r + p_t$ (12) where $p_t > 0$ is the risk premium

Investment function

- Note that capital at the end of period t, K_t , is in the information set at t since it is determined by investment in period t:
- $E_t A_{t+1} F'(K_t) = F'(K_t) E_t A_{t+1} = r + p_t$ (13)
- ullet Hence, K_t and investment, $I_t = K_t K_{t-1}$, are functions of $E_t A_{t+1}$
- $K_t = F'^{-1} \left(\frac{r + p_t}{E_t A_{t+1}} \right)$ (14) with K_t increasing with $E_t A_{t+1}$

where F'^{-1} is the inverse of F' . Example: $F(K_t) = \frac{1}{\alpha}K_t^{\alpha}$

- $K_t = \left(\frac{E_t A_{t+1}}{r + p_t}\right)^{\frac{1}{1 \alpha}}$
- lacksquare Now, assume that $p_t>0$ is constant or negligible and
- $A_{t+1} \bar{A} = \rho[A_t \bar{A}] + \varepsilon_{t+1}$ with $E_t \varepsilon_{t+1} = 0$ and $0 \le \rho \le 1$

Productivity Shocks and the Current Account

Note that a shock ε_t at t implies $E_t A_{t+1} = \bar{A} + \rho \varepsilon_t$, $E_t A_{t+2} = \bar{A} + \rho^2 \varepsilon_t$ etc.

- Hence persistent (ρ >0) productivity shocks not only increase current output $A_tF(K_{t-1})$ but also future productivity and thus investment and future output.
- In fact, to the extent that $\rho>0$ a shock $\varepsilon_t\uparrow\to K_t\uparrow\to A_{t+1}F(K_t)\uparrow\uparrow$ Recall: $C_t=rB_{t-1}+\frac{r}{1+r}\sum_{i=0}^{\infty}\frac{1}{(1+r)^i}\,E_t[Y_{t+i}-G_{t+i}-I_{t+i}]$
- $CA_t = r B_{t-1} + Y_t C_t G_t I_t$

Impact on Current Account

- If $\rho=1$, then $I_t\uparrow$, $I_{t+1}\leftrightarrow$, and $K_t\uparrow\to Y_{t+1}\uparrow\uparrow$ and forever As $Y_{t+1}\uparrow>Y_t\uparrow\to C_t\uparrow>Y_t\uparrow\to CA_t\downarrow$ both because $C_t\uparrow$ and $I_t\uparrow$
- $\blacksquare \ \ \, \text{If} \,\, 0<\rho<1 \,\, \text{there exists} \,\, \rho>\overline{\rho} \,\, \text{ such that} \,\, \mathcal{C}_t \,\, \uparrow +I_t \,\, \uparrow>Y_t \,\, \uparrow \to CA_t \,\, \downarrow \,\,$
- lacktriangle The more persistent productivity shocks the worse the impact on the CA_t

Evidence on Productivity Shocks

- Glick and Rogoff (JME 1995) look at the evidence on the effects of productivity shocks for G-7 countries by estimating:

where superscript C stands for country-specific shock and W for global shocks.

- lacktriangle As productivity shocks appear to be permanent in the data, i.e. ho=1, the intertemporal-approach model predicts that:
- H_0 : $a_1 > 0$; $b_1 < 0$; $|b_1| > a_1$ stronger effect on CA_t vs I_t as $C_t > Y_t$
- Glick and Rogoff find
- $\hat{a}_1 > 0$; $\hat{b}_1 < 0$ but hypothesis $|b_1| > a_1$ is rejected.
- However, this is consistent with the perception of persistent but temporary shocks, i.e. $\bar{\rho}<\rho<1$, that lead $C_t\uparrow< Y_t\uparrow$ and CA_t fall but by less than the increase in I_t .

Evidence on Output Shocks

Kraay and Ventura (QJE 2000) argue that output shocks may have any effect on CA_t , even if $\rho=0$ because output shocks increase saving S_t that is invested in domestic assets, K_t , and foreign assets, B_t , so as to keep their shares constant in the portfolio.

Noting that $S_t = I_t + CA_t$ and $\Delta I_t = \Delta K_t$ and $\Delta CA_t = \Delta B_t$ then the rule is:

- \Box CA_t worsens in debtor countries with $B_{t-1}<0$; i.e. a share >1 of ΔS_t is used to finance ΔI_t

Intuition: consider $F'(K_t)E_tA_{t+1}=r+p_t$ with $\rho=0\to E_tA_{t+1}=\bar{A}$

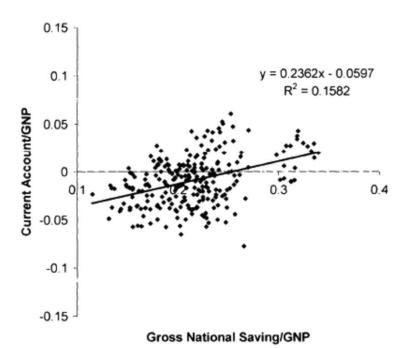
If p_t falls in good times when $\Delta S_t > 0$ (and Δ wealth >0), then K_t increases.

This theory requires weak diminishing returns and a significant p_t (high investment risk), i.e. a strong preference for holding a diversified portfolio.

Evidence of portfolio effect in CA determination 13 industrial countries – 1973-1995

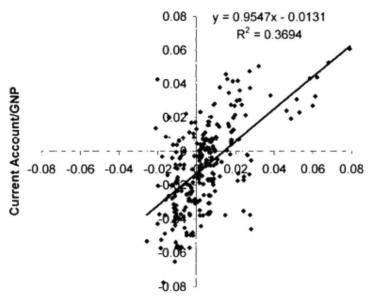
Traditional Rule with ho=0

$$\Delta CA_t = \Delta S_t$$



Kray-Ventura Rule

$$\Delta CA_t = \frac{B}{K+B} \Delta S_t$$



(Gross National Saving/GNP) x (Foreign Assets/Total Assets)