Problem Set I

EI037 Microeconomics I

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1. Preference-based Approach

In the preference-based approach, the objectives of the decision maker are summarized by the preference relation $\succeq$. For individual preferences to be "rational", we impose two basic assumptions about $\succeq$: completeness and transitivity.

1.a. Write down the definitions of completeness and transitivity. Can you provide one example of a preference relation $\succeq$ that satisfies completeness but not transitivity?

Answer: *The preference relation $\succeq$ is rational if it possesses the two following properties: (i) completeness - for all $x, y \in X$, we have that $x \succeq y$ or $y \succeq x$ (or both); and (ii) transitivity - for all $x, y, z \in X$, if $x \succeq y$ and $y \succeq z$, then $x \succeq z$.*
The assumption that $\succeq$ is complete says that the individual has a well-defined preference between any two possible alternatives. Transitivity implies that it is impossible to face the decision maker with a sequence of pairwise choices in which their preferences appear to cycle.
One example that a preference relation $\succeq$ satisfies completeness but not transitivity is: $X = \{x, y, z\}$; $x \succeq y$, $y \succeq z$, and $z \succeq x$.

1.b. Is it possible that a preference relation $\succeq$ satisfies transitivity but not completeness? If you think it is possible, please provide one example. If not, please explain why.

Answer: *It is possible. $X = \{x, y, z, v\}$; $x \succeq y$, $y \succeq z$, and $z \succeq x$. In this case, transitivity is satisfied for all alternatives to which a preference relation is assigned, but not all alternatives have been assigned a preference relation.*

1.c. Do you think the transitivity assumption is reasonable? Can you give a real-world example that violates transitivity, but seems to make common sense?

Answer: *The answer should be that it is reasonable. Examples to justify it include perceptible differences or the framing problem offered on p7. of MWG.*

1.d. From $\succeq$, we can derive another important relation on X, the *strict preference relation* $\succ$:

$$x \succ y \iff x \succeq y \text{ but not } y \succeq x.$$

Show that $\succ$ is transitive but not complete.

Answer: *If a preference relation (denote generally as "V") is complete, it requires that for all $x, y \in X$, we have that $x\ V\ y$ or $y\ V\ x$ (or both). Clearly, $\succ$ is not a complete preference relation, since for any $x, y \in X$ that are equally preferable, we cannot use $\succ$ to characterize the relationship between them.*

2. Choice Rules

In the choice-based approach, the objectives of the decision maker are summarized by the choice structure $(\mathcal{B}, C(\cdot))$. For individual choices to be "reasonable", we impose some restrictions regarding an individual's choice behavior. Apart from the fact that an individual's choice cannot be empty nor out of their budget set, we require that their choice satisfy the weak axiom of revealed preference (or weak axiom in short).

2.a. What is a choice structure $(\mathcal{B}, C(\cdot))$? Provide an example of $(\mathcal{B}, C(\cdot))$.

Answer: *A choice structure $(\mathcal{B}, C(\cdot))$ consists of two elements: $\mathcal{B}$ is a set of non-empty subsets of X, also called budget sets B. The budget sets B contained in $\mathcal{B}$ should be an exhaustive list of the choice sets that can be available to a decision maker. Then, $C(\cdot)$ is a choice correspondence, which assigns a non-empty set of chosen elements $C(B)$ for every budget set $B \in \mathcal{B}$.*

For example, let $X = \{x, y, z\}$ and $\mathcal{B} = \{\{x, y\}, \{x, y, z\}\}$. One possible choice structure $(\mathcal{B}, C(\cdot))$ could be $C(\{x, y\}) = \{x\}$.

2.b. What is the weak axiom (WA)? Provide an example of $(\mathcal{B}, C(\cdot))$ that violates WA.

Answer: *The weak axiom of revealed preference (WA) explains that if for some $B \in \mathcal{B}$ with $x, y \in B$ we have $x \in C(B)$, then for any $B' \in \mathcal{B}$ with $x, y \in B'$ and $y \in C(B')$, we must also have $x \in C(B')$. In other words, the weak axiom tells us that if x is ever chosen when y is available, then there can be no budget set B' containing both alternatives for which y is chosen and x is not.*
For example, let $X = \{x, y, z\}$ and $\mathcal{B} = \{\{x, y\}, \{x, y, z\}\}$. One possible choice structure $(\mathcal{B}, C(\cdot))$ that would violate the weak axiom could be $C_1(\{x, y\}) = \{x\}$ and $C_2(\{x, y, z\}) = \{y\}$.

2.c. We defined a revealed preference relation $\succeq^*$ from the observed individual choice behavior in $C(\cdot)$ in class. What is this definition? How does it differ from $\succeq$ defined in the preference-based approach?

Answer: *Given a choice structure $(\mathcal{B}, C(\cdot))$ the revealed preference relation $\succeq^*$ is defined by*

$$x \succeq^* y \iff \text{there is some } B \in \mathcal{B} \text{ such that } x, y \in B \text{ and } x \in C(B).$$

We read $x \succeq^* y$ as "x is revealed at least as good as y". The revealed preference relation $\succeq^*$ does not need to be either complete or transitive. In particular, for any pair of alternatives x and y to be comparable, it is necessary that, for some $B \in \mathcal{B}$, we have $x, y \in B$ and either $x \in C(B)$ or $y \in C(B)$, or both.

2.d. Provide an example of a preference for which an induced choice structure satisfies WA, but the preference itself does not satisfy transitivity nor completeness.

Answer: *One example could be* $X = \{x, y, z, w\}$*, with the underlying preference being* $\{x \succ y,\ y \succ z,\ z \succ x\}$*. The budget sets observed are* $\mathcal{B} = \{\{x, y\}, \{y, z\}, \{\{z, x\}\}$*, with the choice structure being* $C(\{x, y\}) = \{x\}$*,* $C(\{y, z\}) = \{y\}$*,* $C(\{x, z\}) = \{z\}$*.*

Clearly, the choice structure $(\mathcal{B}, C(\cdot))$ *satisfies WA. However, the underlying preference violates transitivity. Moreover, it also violates completeness, since any preference relations between* w *and* x, y, z *are not given.*

3. Consumer Choice

3.a. Some definitions

In the entire world, there are 10 apples, 10 bananas, 20 bottles of wine, and 10 Gruyere cheese traded in the market economy at the price of 1, 2, 3, and 4 CHF respectively. You as an individual have 60 CHF. Use the notation we studied in Lecture 2.

(i) What is the set of commodities in the economy? What does L equal to?

Answer: *The set of commodities is* $\{apple, banana, wine, cheese\}$*. There are* $L = 4$ *commodities in the economy.*

(ii) Give an example of a commodity vector (i.e., consumption bundle).

Answer: *A commodity vector is any vector containing the quantity information of these three commodities* $x = (x_A, x_B, x_W, x_C) \in \mathbb{R}^{\mathbb{L}}$*. For example* $x = (5, 8, 10, 7)$ *is a commodity vector.*

(iii) What is the commodity set?

Answer: *The commodity set is a subset of commodity space* $\mathbb{R}^L$ *which reflect physical and/or institutional constraints that limit the consumer's choice. The commodity set here is:* $X = \{x \equiv (x_A, x_B, x_W, x_C) \mid x_A \in [0, 10], x_B \in [0, 10], x_W \in [0, 20], x_C \in [0, 10]\}$*.*

(iv) What is the budget set?

Answer: *The budget set is given by* $B = \{x \in X \mid (1 \times x_A) + (2 \times x_B) + (3 \times x_W) + (4 \times x_C) \leq 60\}$

(v) Plot out your budget plane.

Answer: *{Budget plane plot}*

3.b. Weak axiom of revealed preference.

Consider a consumer who consumes only two goods and satisfies Walras' law. When prices are (2,4), they demand (y,10). When prices are (5,5), they demand (9,3). Nothing else of significance has changed between the two situations.

(i) Suppose that $y = 5$. Do these consumption plans satisfy the weak axiom of revealed preference?

Answer: *WA requires that if* $p^0x^1 \leq w^0$*, it must be that* $p^1x^0 > w^1$*. In words, if consumption bundle* x^1 *is available at* (p^0, w^0) *but* x^0 *is chosen, then when* x^1 *is chosen at* (p^1, w^1)*, it must be that* x^0 *is not affordable under* (p^1, w^1)*. (Otherwise WA requires the individual to choose* x^0*, as his choice under* (p^0, w^0) *revealed that he prefers* x^0 *over* x^1*).*

We have $p^0x^0 = 2 \times y + 4 \times 10 = 2y + 40$ *and* $p^1x^1 = 5 \times 9 + 5 \times 3 = 60$*. When* $y = 5$*:*

$$p^0x^1 = 2 \times 9 + 4 \times 3 = 30 \leq 50 = p^0x^0 \equiv w^0$$

In other words, consumption choice (9,3) is affordable in the first scenario when (5,10) was chosen. Hence we require $p^1x^0 > w^1 \equiv p^1x^1$ *for WA to hold. In other words, in the second scenario, when (9,3) is chosen over (5,10), it must be that (5,10) is not affordable.*

$$p^1x^0 = 5 \times 5 + 5 \times 10 = 75 > 60 = p^1x^1 \equiv w^1$$

The weak axiom is therefore satisfied.

(ii) Suppose that $y = 2$. Do these consumption plans satisfy the weak axiom of revealed preference?

Answer: *When $y = 2$, we have $p^0x^0 = 2\times2+4\times10 = 44$ and $p^1x^1 = 5\times9+5\times3 = 60$. Then,*

$$p^0x^1 = 2 \times 9 + 4 \times 3 = 30 \leq 44 = p^0x^0 \equiv w^0$$

In other words, consumption choice (9,3) is affordable in the first scenario when (2,10) was chosen. Moreover,

$$p^1x^0 = 5 \times 2 + 5 \times 10 = 60 = 60 = p^1x^1 \equiv w^1$$

In other words, consumption choice (2,10) is affordable in the scenario when (9,3) was chosen. So the weak axiom is NOT satisfied.

(iii) For which range of y do these consumption plans violate the weak axiom?

Answer: *The weak axiom is violated if*

$$p^0x^1 \leq w^0 \quad \textit{and} \quad p^1x^0 \leq w^1$$

We then have

$$\begin{aligned} 2 \times 9 + 4 \times 3 \leq 2y + 40 \quad &\textit{and} \quad 5 \times y + 5 \times 10 \leq 60 \\ \Rightarrow \ 30 \leq 2y + 40 \quad &\textit{and} \quad 5y + 50 \leq 60 \\ \Rightarrow \ -10 \leq 2y \quad &\textit{and} \quad 5y \leq 10 \\ \Rightarrow \ -5 \leq y \quad &\textit{and} \quad y \leq 2 \end{aligned}$$

Since y cannot be a negative number, the left-handside condition is equivalent to $0 \leq y$. Therefore, we have that the weak axiom is violated if $0 \leq y \leq 2$.

4. Classical Demand Theory

Show that if a consumer has a Cobb-Douglas preference

$$U = x^{\beta} y^{1-\beta}; \text{ with } \beta \in (0,1),$$

their demand function is homogeneous of degree one w.r.t. wage w, i.e., $x(p, \alpha w) = \alpha x(p, w)$ for all $\alpha > 0$.

Answer: *We begin by solving the consumer's utility maximization problem (UMP), in order to find the consumer's (Walrasian) demand function.*

$$\max_{x_1, x_2 \geq 0} \quad x^{\beta} y^{1-\beta} \quad s.t. \; p_1 x_+ p_2 y \leq w$$

The problem's Lagrangian is therefore

$$\mathcal{L} = x^{\beta} y^{1-\beta} - \lambda(p_1 x_+ p_2 y - w)$$

Since the consumer's utility function is of the Cobb-Douglas form, we can directly derive the optimal consumer demand

$$x^*(p, w) = \frac{\beta w}{p_1} \quad and \quad y^*(p, w) = \frac{(1-\beta) w}{p_2}.$$

We then verify that their demand function is homogeneous of degree one w.r.t. w for any $\alpha > 0$, as follows

$$x(p_1, \alpha w) = \frac{\beta \alpha w}{p_1} = \alpha x(p_1, w) \quad and \quad y(p, \alpha w) = \frac{(1-\beta)\alpha w}{p_2} = \alpha y(p_2, w).$$