Macroeconomics A; EI060

Short problems

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1 Exchange rate with flexible prices

Question: In class, we showed that the nominal exchange rate can be written as a function of the current real exchange rate gap and the stream of current and future money supplies:

$$e_t - \bar{q} = \frac{1-\phi\delta}{1+\eta\psi\delta}(q_t - \bar{q}) + \frac{1}{1+\eta}\sum_{s=t}^{\infty}\left(\frac{\eta}{1+\eta}\right)^{s-t} m_s$$

Consider that the money supply is growing at a rate μ, so $m_s = m_t + \mu(s-t)$.

When prices are flexible, the real exchange rate is always at its long-run value, as nominal variables have no impact on real variables. Show that:

$$e_t^{flex} = \bar{q} + m_t + \mu\eta$$

It is useful to recall that:

$$\sum_{s=t}^{\infty}\left[\left(\frac{\eta}{1+\eta}\right)^{s-t}(s-t)\right] = \eta(1+\eta)$$

2 Nominal exchange rate: sticky vs. flexible prices

Question: Consider that until time 0 we are in a steady state with money growing at a rate μ. At time 0 the central bank announces, unexpectedly, that the money growth rate will now be $\mu' > \mu$.

The flexible exchange rate at time 0 just before and just after the announcement are:

$$e_0^{flex} = \bar{q} + m_0 + \eta\mu$$
$$\left(e_0^{flex}\right)' = \bar{q} + m_0 + \eta\mu'$$

Before the announcement the real exchange rate is $\bar{q}$, and the nominal exchange rate is equal to its flexible price value e_0^{flex}. The price level is then $p_0 = e_0^{flex} - \bar{q}$ (we set the foreign price level at zero).

The price level is sticky and does not vary after the announcement. Show that the real exchange rate is then:

$$q_0 - \bar{q} = e_0 - e_0^{flex}$$

Using this result, show that:

$$e_0 - e_0^{flex} = \frac{1+\eta\psi\delta}{\phi\delta+\eta\psi\delta}\left[\left(e_0^{flex}\right)' - e_0^{flex}\right]$$

3 Exchange rate dynamics

Question: Recall the following difference between the exchange rate and the level that prevails under flexible prices:

$$e_t - \left(e_t^{flex}\right)' = \frac{1-\phi\delta}{1+\eta\psi\delta}\left(q_t - \bar{q}\right)$$

Show that:

$$e_0 - \left(e_0^{flex}\right)' = \frac{1-\phi\delta}{1+\eta\psi\delta}\left(q_0 - \bar{q}\right)$$

Using these results, and the autoregressive dynamics for the real exchange rate seen in class, show that:

$$e_t - \left(e_t^{flex}\right)' = (1-\psi\delta)^t\left(e_0 - \left(e_0^{flex}\right)'\right)$$

as well as:

$$e_0 - \left(e_0^{flex}\right)' = \frac{1-\phi\delta}{\phi\delta+\eta\psi\delta}\eta\left(\mu' - \mu\right)$$

4 Interest rates

Question: Recall the autoregressive relation of the nominal exchange rate gap:

$$e_t - \left(e_t^{flex}\right)' = (1-\psi\delta)^t\left(e_0 - \left(e_0^{flex}\right)'\right)$$

Show that:

$$e_{t+1} - e_t = \left(e_{t+1}^{flex}\right)' - \left(e_t^{flex}\right)' - \psi\delta\left(1-\psi\delta\right)^t\left(e_0 - \left(e_0^{flex}\right)'\right)$$

Using the interest parity, show that:

$$i_{t+1} - (i^* + \mu) = (\mu' - \mu) - \psi\delta\left(1-\psi\delta\right)^t\left(e_0 - \left(e_0^{flex}\right)'\right)$$

and:

$$i_1 - (i^* + \mu) = \phi\delta\frac{1+\eta\psi\delta}{\phi\delta+\eta\psi\delta}\left(\mu' - \mu\right)$$