Macroeconomics A; EI060

Technical appendix: Nominal exchange rate

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1 Interest parity (Harms VIII.3.2 - 3.4)

1.1 Parities

At time t an investor in the Home country can purchase a local bond and get the interest rate i_{t+1}^H at time $t+1$. She can also invest in the Foreign country and get the interest rate i_{t+1}^F, in Foreign currency. These interest rates are known at time t.

The exchange rate at time t is E_t, expressed in units of Home currency per unit of Foreign currency, so an increase is a depreciation of the Home currency. The spot exchange rate at time $t+1$ is E_{t+1}, which is uncertain. At time t there is the possibility to transact at a forward exchange rate F_t, with the transaction taking place at time $t+1$ but at a price set at time t.

If the Home investor invests in Foreign currency, she buys $1/E_t$ units of Foreign currency at time t, with a certain payoff of $\left(1+i_{t+1}^F\right)/E_t$ at time $t+1$ in terms of Foreign currency. If she sells these at the uncertain spot rate E_{t+1}, she gets a payoff of $\left(1+i_t^F\right)\left(E_{t+1}/E_t\right)$ of Home currency at time $t+1$. If she had resold the Foreign currency on the forward market to start with, she gets a certain payoff of $\left(1+i_{t+1}^F\right)\left(F_t/E_t\right)$ at time $t+1$.

In equilibrium, investors are investing in both currencies, and must therefore be indifferent between them. This links the returns on the Home and Foreign investments.

If the Home investor takes the risk of the moving exchange rate, the no-arbitrage is the uncovered interest parity condition that equalizes expected returns ($E_{t+1}^{\text{expected}}$ is the expected exchange rate from the point of view of period t):

$$1+i_{t+1}^H = \left(1+i_{t+1}^F\right)\frac{E_{t+1}^{\text{expected}}}{E_t}$$

If the Home investor had resold the Foreign currency immediately using the forward exchange rate market, there is no risk and we have the covered interest rate parity:

$$1+i_{t+1}^H = \left(1+i_{t+1}^F\right)\frac{F_t}{E_t}$$

1.2 Linearized expressions

We linearize the relations around a situation where both interest rates are equal to i_0 and the exchange rate is constant at E_0. We denote $e_t = (E_t - E_0)/E_0$ and $i^H_{t+1} = \left(i^H_{t+1} - i_0\right)/(1+i_0)$ (the textbook notation can unfortunately bring some confusion between the level of the interest rate and its deviation from the steady state level.

The uncovered interest rate parity is expanded as:

$$\begin{aligned}
1 + i^H_{t+1} &= \left(1 + i^F_{t+1}\right) \frac{E^{\text{expected}}_{t+1}}{E_t} \\
1 + i_0 + i^H_{t+1} - i_0 &= (1+i_0) + \left(i^F_{t+1} - i_0\right) + (1+i_0) \frac{E^{\text{expected}}_{t+1} - E_0}{E_0} - (1+i_0) \frac{E_0}{(E_0)^2} (E_t - E_0) \\
i^H_{t+1} - i_0 &= \left(i^F_{t+1} - i_0\right) + (1+i_0) e^{\text{expected}}_{t+1} - (1+i_0) e_t \\
\frac{i^H_{t+1} - i_0}{1+i_0} &= \frac{i^F_{t+1} - i_0}{1+i_0} + e^{\text{expected}}_{t+1} - e_t \\
i^H_{t+1} &= i^F_{t+1} + e^{\text{expected}}_{t+1} - e_t
\end{aligned}$$

If we go to a second-order expansion, we get the variance of the future exchange rate appearing. A risk averse agent would then ask for a risk premium ρ_t leading to: $i^H_{t+1} = i^F_{t+1} + e^{\text{expected}}_{t+1} - e_t - \rho_t$.

Following similar steps for the covered interest parity, we get:

$$i^H_{t+1} = i^F_{t+1} + f_t - e_t$$

2 Monetary model (Harms VIII.4)

2.1 Dynamic optimization and money demand

The representative agent gets utility from consumption, C, and from real money balances, M/P (both with a log utility), over and infinite horizon. The expected utility is

$$U_t = \sum_{s=t}^{\infty} \beta^s \left[ln\left(C_s\right) + \chi ln\left(\frac{M_s}{P_s}\right)\right]$$

The agent invests in Home money, a bond in Home currency, and a bond in Foreign currency. The flow budget constraint is:

$$B^H_{t+1} + E_t B^F_{t+1} + M_t = \left(1 + i^H_t\right) B^H_t + E_t \left(1 + i^F_t\right) B^F_t + M_{t-1} + P_t \left(Y_t - C_t\right)$$

Focus on periods t and $t+1$ for brevity. We use the notation of state of nature k with probability $\pi_{k,+1}$, and recall that M_t, B^H_{t+1}, B^F_{t+1}, i^H_t and i^F_t are know at time t. The Lagrangian is:

$$\mathcal{L} = ln\left(C_t\right) + \chi ln\left(\frac{M_t}{P_t}\right) + \beta \sum_k \pi_{k,t+1} \left[ln\left(C_{k,t+1}\right) + \chi ln\left(\frac{M_{k,t+1}}{P_{k,t+1}}\right)\right]$$

$$+\Lambda_t \begin{bmatrix} \left(1+i_t^H\right) B_t^H + E_t \left(1+i_t^F\right) B_t^F + M_{t-1} \\ +P_t \left(Y_t - C_t\right) - B_{t+1}^H - E_t B_{t+1}^F - M_t \end{bmatrix}$$

$$+\sum_k \Lambda_{k,t+1} \begin{bmatrix} \left(1+i_{t+1}^H\right) B_{t+1}^H + E_{k,t+1} \left(1+i_{t+1}^F\right) B_{t+1}^F + M_t \\ +P_{k,t+1} \left(Y_{k,t+1} - C_{k,t+1}\right) - B_{k,t+2}^H - E_{k,t+1} B_{k,t+2}^F - M_{k,t+1} \end{bmatrix}$$

The optimality conditions with respect to consumption, money, and bonds are:

$$\begin{aligned}
0 &= \frac{\partial L}{\partial C_t} = \frac{1}{C_t} - \Lambda_t P_t \\
0 &= \frac{\partial L}{\partial C_{k,t+1}} = \beta \pi_{k,t+1} \frac{1}{C_{k,t+1}} - \Lambda_{k,t+1} P_{k,t+1} \\
0 &= \frac{\partial L}{\partial M_t} = \chi \frac{1}{M_t} - \Lambda_t + \sum_k \Lambda_{k,t+1} \\
0 &= \frac{\partial L}{\partial B_{t+1}^H} = -\Lambda_t + \left(1+i_{t+1}^H\right) \sum_k \Lambda_{k,t+1} \\
0 &= \frac{\partial L}{\partial B_{t+1}^F} = -\Lambda_t E_t + \left(1+i_{t+1}^F\right) \sum_k \Lambda_{k,t+1} E_{k,t+1}
\end{aligned}$$

The condition for the Home bond gives the Euler. The expectation sign is $\mathbb{E}_t$, to avoid confusion with the exchange rate E_t:

$$\begin{aligned}
\Lambda_t &= \left(1+i_{t+1}^H\right) \sum_k \Lambda_{k,t+1} \\
\frac{1}{P_t C_t} &= \left(1+i_{t+1}^H\right) \sum_k \beta \pi_{k,t+1} \frac{1}{P_{k,t+1} C_{k,t+1}} \\
\frac{1}{P_t C_t} &= \beta \left(1+i_{t+1}^H\right) \mathbb{E}_t \left(\frac{1}{P_{t+1} C_{t+1}} \right)
\end{aligned}$$

Doing the same steps with condition for the Foreign bond, and combining gives the uncovered interest parity with payoffs evaluated using the marginal utility of consumption (pricing kernel):

$$\begin{aligned}
\Lambda_t E_t &= \left(1+i_{t+1}^F\right) \sum_k \Lambda_{k,t+1} E_{k,t+1} \\
\frac{1}{P_t C_t} E_t &= \left(1+i_{t+1}^F\right) \sum_k \beta \pi_{k,t+1} \frac{E_{k,t+1}}{P_{k,t+1} C_{k,t+1}} \\
\frac{1}{P_t C_t} E_t &= \beta \left(1+i_{t+1}^F\right) \mathbb{E}_t \left(\frac{E_{t+1}}{P_{t+1} C_{t+1}} \right) \\
\beta \left(1+i_{t+1}^H\right) \mathbb{E}_t \left(\frac{1}{P_{t+1} C_{t+1}} \right) E_t &= \beta \left(1+i_{t+1}^F\right) \mathbb{E}_t \left(\frac{E_{t+1}}{P_{t+1} C_{t+1}} \right) \\
\left(1+i_{t+1}^H\right) \mathbb{E}_t \left(\omega_{t+1}\right) &= \left(1+i_{t+1}^F\right) \mathbb{E}_t \left(\frac{E_{t+1}}{E_t} \omega_{t+1} \right) \\
\left(1+i_{t+1}^H\right) \mathbb{E}_t \left(\omega_{t+1}\right) &= \left(1+i_{t+1}^F\right) \left[\mathbb{E}_t \left(\frac{E_{t+1}}{E_t} \right) \mathbb{E}_t \left(\omega_{t+1}\right) + \mathbb{CV}_t \left(\frac{E_{t+1}}{E_t}, \omega_{t+1} \right) \right]
\end{aligned}$$

If we abstract from covariance (which we do when taking linear approximations) this implies:

$$
\begin{aligned}
\left(1+i_{t+1}^{H}\right)\mathbb{E}_t\left(\omega_{t+1}\right) &= \left(1+i_{t+1}^{F}\right)\mathbb{E}_t\left(\frac{E_{t+1}}{E_t}\right)\mathbb{E}_t\left(\omega_{t+1}\right)\\
\left(1+i_{t+1}^{H}\right) &= \left(1+i_{t+1}^{F}\right)\mathbb{E}_t\left(\frac{E_{t+1}}{E_t}\right)\\
\left(1+i_{t+1}^{H}\right) &= \left(1+i_{t+1}^{F}\right)\frac{\mathbb{E}_t\left(E_{t+1}\right)}{E_t}
\end{aligned}
$$

Combing the optimality condition for money and Home bond, we get the money demand:

$$
\begin{aligned}
0 &= \chi\frac{1}{M_t}-\Lambda_t+\sum_k\Lambda_{k,t+1}\\
0 &= \chi\frac{1}{M_t}-\frac{1}{P_tC_t}+\sum_k\beta\pi_{k,t+1}\frac{1}{P_{k,t+1}C_{k,t+1}}\\
0 &= \chi\frac{1}{M_t}-\frac{1}{P_tC_t}+\frac{1}{P_tC_t}\frac{1}{1+i_{t+1}^{H}}\\
0 &= \chi\frac{1}{M_t}-\frac{1}{P_tC_t}\frac{i_{t+1}^{H}}{1+i_{t+1}^{H}}\\
\chi\frac{1}{M_t} &= \frac{1}{P_tC_t}\frac{i_{t+1}^{H}}{1+i_{t+1}^{H}}\\
M_t &= \chi P_tC_t\frac{1+i_{t+1}^{H}}{i_{t+1}^{H}}\\
\frac{M_t}{P_t} &= \chi C_t\frac{1+i_{t+1}^{H}}{i_{t+1}^{H}}
\end{aligned}
$$

The money demand is linearized as follows:

$$
\begin{aligned}
\frac{M_t}{P_t} &= \chi C_t\frac{1+i_{t+1}^{H}}{i_{t+1}^{H}}\\
\frac{M_0}{P_0}+\frac{M_t-M_0}{P_0}-\frac{M_0}{\left(P_0\right)^2}\left(P_t-P_0\right) &= \chi C_0\frac{1+i_0}{i_0}+\chi\frac{1+i_0}{i_0}\left(C_t-C_0\right)+\chi C_0\frac{i_0-\left(1+i_0\right)}{\left(i_0\right)^2}\left(i_{t+1}^{H}-i_0\right)\\
\frac{M_0}{P_0}\frac{M_t-M_0}{M_0}-\frac{M_0}{P_0}\frac{P_t-P_0}{P_0} &= \chi C_0\frac{1+i_0}{i_0}\frac{C_t-C_0}{C_0}+\chi C_0\frac{-\left(1+i_0\right)}{\left(i_0\right)^2}\frac{i_{t+1}^{H}-i_0}{1+i_0}\\
\frac{M_0}{P_0}\left(\frac{M_t-M_0}{M_0}-\frac{P_t-P_0}{P_0}\right) &= \chi C_0\frac{1+i_0}{i_0}\left(\frac{C_t-C_0}{C_0}-\frac{1}{i_0}\frac{i_{t+1}^{H}-i_0}{1+i_0}\right)\\
m_t-p_t &= c_t-\frac{1}{i_0}i_{t+1}^{H}\\
m_t-p_t &= c_t-\lambda i_{t+1}^{H}
\end{aligned}
$$

2.2 Exchange rate solution

We take the linear approximation of the uncovered interest parity and the money demand:

$$
i_{t+1}^{H} = i_{t+1}^{F}+\mathbb{E}_t\left(e_{t+1}\right)-e_t
$$

$$m_t - p_t = c_t - \lambda i^H_{t+1}$$

We also consider that purchasing parity holds: $P_t = E_t P^F_t$, which implies:

$$p_t = e_t + p^F_t$$

Use this to substitute for the price level in the money demand, and use the money demand to substitute out for the interest rate in the uncovered interest parity:

$$\begin{aligned}
i^H_{t+1} &= i^F_{t+1} + \mathbb{E}_t\left(e_{t+1}\right) - e_t \\
\frac{1}{\lambda}\left(c_t - m_t + p_t\right) &= i^F_{t+1} + \mathbb{E}_t\left(e_{t+1}\right) - e_t \\
\frac{1}{\lambda}\left(c_t - m_t + e_t + p^F_t\right) &= i^F_{t+1} + \mathbb{E}_t\left(e_{t+1}\right) - e_t \\
\frac{1}{\lambda}\left(c_t - m_t + p^F_t\right) + \left(\frac{1}{\lambda} + 1\right) e_t &= i^F_{t+1} + \mathbb{E}_t\left(e_{t+1}\right) \\
\left(c_t - m_t + p^F_t\right) + (1+\lambda) e_t &= \lambda i^F_{t+1} + \lambda \mathbb{E}_t\left(e_{t+1}\right) \\
e_t &= \frac{\lambda}{1+\lambda}\mathbb{E}_t\left(e_{t+1}\right) + \frac{1}{1+\lambda}\left[\lambda i^F_{t+1} + m_t - \left(c_t + p^F_t\right)\right] \\
e_t &= \frac{\lambda}{1+\lambda}\mathbb{E}_t\left(e_{t+1}\right) + \frac{1}{1+\lambda} m_t + \frac{1}{1+\lambda}\left[\lambda i^F_{t+1} - \left(c_t + p^F_t\right)\right]
\end{aligned}$$

For simplicity set the last bracket to zero, and iterate forward:

$$\begin{aligned}
e_t &= \frac{\lambda}{1+\lambda}\mathbb{E}_t\left(e_{t+1}\right) + \frac{1}{1+\lambda} m_t \\
e_t &= \frac{1}{1+\lambda} m_t + \frac{\lambda}{1+\lambda}\mathbb{E}_t\left(\frac{1}{1+\lambda} m_{t+1} + \frac{\lambda}{1+\lambda} e_{t+2}\right) \\
e_t &= \frac{1}{1+\lambda} m_t + \frac{\lambda}{1+\lambda}\frac{1}{1+\lambda}\mathbb{E}_t\left(m_{t+1}\right) \\
&\quad + \left(\frac{\lambda}{1+\lambda}\right)^2 \mathbb{E}_t\left(\frac{1}{1+\lambda} m_{t+2} + \frac{\lambda}{1+\lambda} e_{t+3}\right) \\
e_t &= \frac{1}{1+\lambda}\left(m_t + \frac{\lambda}{1+\lambda}\mathbb{E}_t\left(m_{t+1}\right) + \left(\frac{\lambda}{1+\lambda}\right)^2 \mathbb{E}_t\left(m_{t+2}\right)\right) \\
&\quad + \left(\frac{\lambda}{1+\lambda}\right)^3 \mathbb{E}_t\left(e_{t+3}\right) \\
e_t &= \frac{1}{1+\lambda}\sum_{s=t}^{\infty}\left(\left(\frac{\lambda}{1+\lambda}\right)^s \mathbb{E}_t\left(m_s\right)\right) + lim_{s\to\infty}\left(\frac{\lambda}{1+\lambda}\right)^s \mathbb{E}_t\left(e_s\right)
\end{aligned}$$

The transversality condition implies that the last term is zero, hence:

$$e_t = \frac{1}{1+\lambda}\sum_{s=t}^{\infty}\left[\left(\frac{\lambda}{1+\lambda}\right)^{s-t}\mathbb{E}_t\left(m_s\right)\right]$$

2.3 Specific examples

2.3.1 Constant money growth

Consider that the money supply grows at a constant rate μ:

$$m_s = m_t + \mu (s - t)$$

The exchange rate is then:

$$\begin{aligned}
e_t &= \frac{1}{1+\lambda} \sum_{s=t}^{\infty} \left[\left(\frac{\lambda}{1+\lambda} \right)^{s-t} (m_t + \mu (s-t)) \right] \\
e_t &= \frac{1}{1+\lambda} m_t \sum_{s=t}^{\infty} \left[\left(\frac{\lambda}{1+\lambda} \right)^{s-t} \right] + \frac{1}{1+\lambda} \mu \sum_{s=t}^{\infty} \left[\left(\frac{\lambda}{1+\lambda} \right)^{s-t} (s-t) \right] \\
e_t &= \frac{1}{1+\lambda} m_t \sum_{s=0}^{\infty} \left[\left(\frac{\lambda}{1+\lambda} \right)^{s} \right] + \frac{1}{1+\lambda} \frac{\lambda}{1+\lambda} \mu \sum_{s=0}^{\infty} \left[\left(\frac{\lambda}{1+\lambda} \right)^{s-1} s \right]
\end{aligned}$$

Recall that:

$$\begin{aligned}
\sum_{s=0}^{\infty} \left[\left(\frac{\lambda}{1+\lambda} \right)^{s} \right] &= \frac{1}{1 - \frac{\lambda}{1+\lambda}} \\
\sum_{s=0}^{\infty} \left[\left(\frac{\lambda}{1+\lambda} \right)^{s} \right] &= 1 + \lambda
\end{aligned}$$

and:

$$\begin{aligned}
\sum_{s=0}^{\infty} \left[\left(\frac{\lambda}{1+\lambda} \right)^{s-1} s \right] &= \sum_{s=0}^{\infty} \left[\frac{\partial \left(\frac{\lambda}{1+\lambda} \right)^{s}}{\partial \left(\frac{\lambda}{1+\lambda} \right)} \right] \\
\sum_{s=0}^{\infty} \left[\left(\frac{\lambda}{1+\lambda} \right)^{s-1} s \right] &= \frac{\partial}{\partial \left(\frac{\lambda}{1+\lambda} \right)} \left[\sum_{s=0}^{\infty} \left[\left(\frac{\lambda}{1+\lambda} \right)^{s} \right] \right] \\
\sum_{s=0}^{\infty} \left[\left(\frac{\lambda}{1+\lambda} \right)^{s-1} s \right] &= \frac{\partial}{\partial \left(\frac{\lambda}{1+\lambda} \right)} (1+\lambda) \\
\sum_{s=0}^{\infty} \left[\left(\frac{\lambda}{1+\lambda} \right)^{s-1} s \right] &= \frac{\partial (1+\lambda)}{\partial \lambda} \frac{\partial \lambda}{\partial \left(\frac{\lambda}{1+\lambda} \right)} \\
\sum_{s=0}^{\infty} \left[\left(\frac{\lambda}{1+\lambda} \right)^{s-1} s \right] &= \frac{1}{\partial \left(\frac{\lambda}{1+\lambda} \right) / \partial \lambda} \\
\sum_{s=0}^{\infty} \left[\left(\frac{\lambda}{1+\lambda} \right)^{s-1} s \right] &= \frac{1}{\frac{1+\lambda-\lambda}{(1+\lambda)^2}} \\
\sum_{s=0}^{\infty} \left[\left(\frac{\lambda}{1+\lambda} \right)^{s-1} s \right] &= (1+\lambda)^2
\end{aligned}$$

We therefore get:

$$
\begin{aligned}
e_t &= \frac{1}{1+\lambda} m_t \sum_{s=0}^{\infty} \left[\left(\frac{\lambda}{1+\lambda} \right)^s \right] + \frac{1}{1+\lambda} \frac{\lambda}{1+\lambda} \mu \sum_{s=0}^{\infty} \left[\left(\frac{\lambda}{1+\lambda} \right)^{s-1} s \right] \\
e_t &= \frac{1}{1+\lambda} m_t (1+\lambda) + \frac{1}{1+\lambda} \frac{\lambda}{1+\lambda} \mu (1+\lambda)^2 \\
e_t &= m_t + \lambda \mu
\end{aligned}
$$

2.3.2 Autoregressive money growth

Consider that the money supply follows an autoregressive process around $m = 0$:

$$
m_{t+1} = \rho m_t + \epsilon_{t+1}
$$

The exchange rate is then:

$$
\begin{aligned}
e_t &= \frac{1}{1+\lambda} \sum_{s=t}^{\infty} \left[\left(\frac{\lambda}{1+\lambda} \right)^{s-t} \mathbb{E}_t (m_s) \right] \\
e_t &= \frac{1}{1+\lambda} \sum_{s=t}^{\infty} \left[\left(\frac{\lambda}{1+\lambda} \right)^{s-t} \rho^{s-t} m_t \right] \\
e_t &= m_t \frac{1}{1+\lambda} \sum_{s=t}^{\infty} \left[\left(\frac{\lambda \rho}{1+\lambda} \right)^{s-t} \right] \\
e_t &= m_t \frac{1}{1+\lambda} \frac{1}{1 - \frac{\lambda \rho}{1+\lambda}} \\
e_t &= m_t \frac{1}{1+\lambda-\lambda\rho} \\
e_t &= m_t \frac{1}{1+\lambda(1-\rho)}
\end{aligned}
$$

2.3.3 Future monetary expansion

Consider that the money supply is constant at $\underline{m}$. At time t the central bank announces that at time $T \geq 1$ it will increase the money supply permanently to $\overline{m}$.

Before the change is announced, the exchange rate is:

$$
\begin{aligned}
e_t &= \frac{1}{1+\lambda} \sum_{s=t}^{\infty} \left[\left(\frac{\lambda}{1+\lambda} \right)^{s-t} \underline{m} \right] \\
e_t &= \underline{m} \frac{1}{1+\lambda} \sum_{s=t}^{\infty} \left[\left(\frac{\lambda}{1+\lambda} \right)^{s-t} \right] \\
e_t &= \underline{m} \frac{1}{1+\lambda} (1+\lambda) \\
e_t &= \underline{m}
\end{aligned}
$$

From period T onward the exchange rate is $e_t = \overline{m}$.

Between the announcement and the actual implementation, we have:

$$
\begin{aligned}
e_t &= \frac{1}{1+\lambda}\sum_{s=t}^{\infty}\left[\left(\frac{\lambda}{1+\lambda}\right)^{s-t}\mathbb{E}_t\left(m_s\right)\right] \\
e_t &= \frac{1}{1+\lambda}\sum_{s=t}^{T-1}\left[\left(\frac{\lambda}{1+\lambda}\right)^{s-t}\mathbb{E}_t\left(m_s\right)\right]+\frac{1}{1+\lambda}\sum_{s=T}^{\infty}\left[\left(\frac{\lambda}{1+\lambda}\right)^{s-t}\mathbb{E}_t\left(m_s\right)\right] \\
e_t &= \frac{1}{1+\lambda}\underline{m}\sum_{s=t}^{T-1}\left[\left(\frac{\lambda}{1+\lambda}\right)^{s-t}\right]+\frac{1}{1+\lambda}\overline{m}\sum_{s=T}^{\infty}\left[\left(\frac{\lambda}{1+\lambda}\right)^{s-t}\right] \\
e_t &= \frac{1}{1+\lambda}\underline{m}\left(\sum_{s=t}^{\infty}\left[\left(\frac{\lambda}{1+\lambda}\right)^{s-t}\right]-\sum_{s=T}^{\infty}\left[\left(\frac{\lambda}{1+\lambda}\right)^{s-t}\right]\right)+\frac{1}{1+\lambda}\overline{m}\sum_{s=T}^{\infty}\left[\left(\frac{\lambda}{1+\lambda}\right)^{s-t}\right] \\
e_t &= \frac{1}{1+\lambda}\underline{m}\sum_{s=t}^{\infty}\left[\left(\frac{\lambda}{1+\lambda}\right)^{s-t}\right]+\frac{1}{1+\lambda}\left(\overline{m}-\underline{m}\right)\sum_{s=T}^{\infty}\left[\left(\frac{\lambda}{1+\lambda}\right)^{s-t}\right] \\
e_t &= \underline{m}+\frac{1}{1+\lambda}\left(\overline{m}-\underline{m}\right)\left(\frac{\lambda}{1+\lambda}\right)^{T-t}\sum_{s=T}^{\infty}\left[\left(\frac{\lambda}{1+\lambda}\right)^{s-T}\right] \\
e_t &= \underline{m}+\frac{1}{1+\lambda}\left(\overline{m}-\underline{m}\right)\left(\frac{\lambda}{1+\lambda}\right)^{T-t}(1+\lambda) \\
e_t &= \underline{m}+\left(\overline{m}-\underline{m}\right)\left(\frac{\lambda}{1+\lambda}\right)^{T-t} \\
e_t &= \overline{m}-\overline{m}+\underline{m}+\left(\overline{m}-\underline{m}\right)\left(\frac{\lambda}{1+\lambda}\right)^{T-t} \\
e_t &= \overline{m}-\left(\overline{m}-\underline{m}\right)\left[1-\left(\frac{\lambda}{1+\lambda}\right)^{T-t}\right]
\end{aligned}
$$