Macroeconomics A: EI056

Final exam

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1 General instructions

The exam consists of 4 questions. Section 2 has 3 short subquestions that mostly connect with what we saw in class, and sections 3-4-5 go beyond what we saw in class.

The weight of each question in the grade is indicated, so you can allocate your time accordingly.

A good strategy is to first read through the questions, and then start with the easiest one before proceeding to the harder ones.

Short "bullet points" answers stressing the main points are fine, you don't have to write long paragraphs.

Best of luck!

2 Short questions (25 % of grade)

2.1 Inflation-unemployment trade-off

Imagine that you are sitting in a policy meeting where someone makes a careful econometric presentation showing that times of high inflation are also times of low unemployment.

One policy maker then points that given this correlation, the central bank should be more relaxed about inflation to reduce unemployment, and asks for your opinion. What would you reply?

Answer: The correlation cannot be systematically exploited by policy. The empirical pattern reflects the fact that inflation surprises lead to higher output, as summarized by the aggregate supply curve. Any systematic policy to raise inflation will however raise both expected inflation and actual inflation, and thus not lead to any inflation surprise. Therefore, the economy would shift towards higher inflation levels with no gain in unemployment.

This point is the Lucas critique that reduced form correlations between endogenous variables are conditional on expectations and the policy regime. A different policy regime will lead to a different correlation. It is thus important to reason in terms of structural form (taking the formation of expectations into account) rather than reduced form.

2.2 Bank panic

In the class on financial intermediaries we saw that depositors' panics could bring down a healthy bank. Explain the mechanism.

How can public deposit insurance, and "lending of last resort" by the central bank help?

Answer: The bank engages in maturity transformation. It is funded by short term deposits that can be withdrawn without notice, and invests in long term project. This structure allows society to get the high return on long term projects while still having the possibility for depositors who need cash early to get their money.

The contract offered by the bank is that depositors get an amount c_1 if they withdraw early and $c_2 > c_1$ if they wait. This is calibrated under the assumption that a fraction t of depositors withdraw early.

The system can break down when depositors who don't need the money right away think, for some reasons, that more than t depositors will withdraw the amount c_1 in the first period. The bank will have to pay for the extra withdrawal by liquidating some of the long term investment early on, and then there will not be enough money left in the long term investments for the bank to pay c_2 to the depositors who waited, who may not even get c_1 or anything. The rational decision is then for a patient depositors to get his money out early, so as to be sure to get anything. If enough depositors do so, the bank gets bankrupt even though it fundamentally does not have a problem. In other words, the bank is illiquid, but not insolvent.

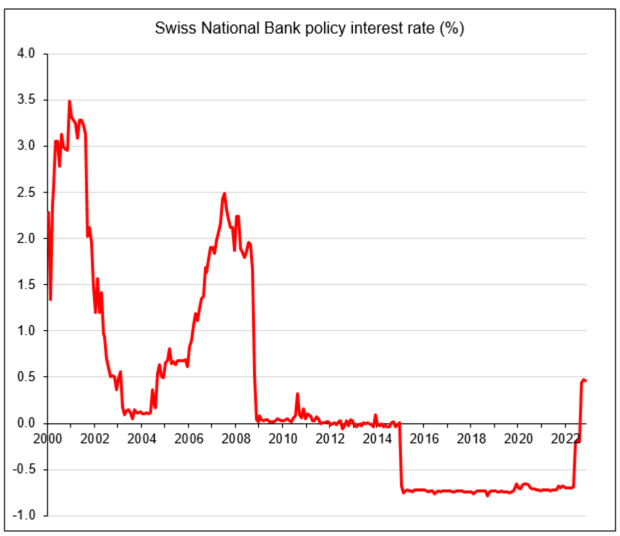
Deposit insurance can solve this. The government ensures patient depositors that they will get c_2 if the wait, regardless of what happens. The depositors do not have an incentive to withdraw early, and the bank faces no problem. In fact, the promise of government support is enough and the government does not have to ever deliver on the promise.

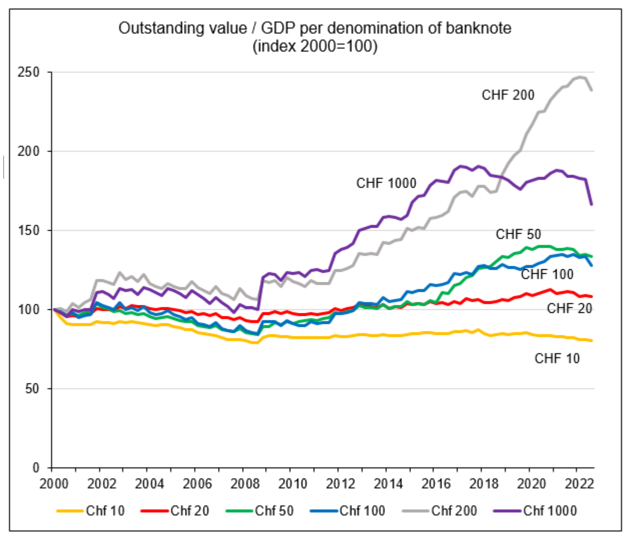
The lender of last resort funding from the central bank can also help. If the bank faces more withdrawals than expected, it does not need to liquidate the long term investment, and thus will be able to pay its promised c_2 in the final period. Instead, the bank borrows cash from the central bank to pay for the extra early withdrawals, and will repay this cash tomorrow. The central bank is willing to grant such a loan because it knows that the bank is fundamentally solvent thanks to its good long term investment.

2.3 Cash usage

The figures below show the policy interest rate of the Swiss National Bank, and the volumes of cash in circulation (scaled by GDP, expressed as an index 2000=100 for simplicity) distinguishing between the different banknotes denomination (the lowest is the 10 franc banknote, the highest the 1'000 franc note).

1. Explain the co-movements between the volume of cash and the interest rate. How does this relate to models seen in class?
2. Is the pattern the same for all banknotes? Discuss in relation to the different roles of money (in class we discussed the role of "mean of exchange" and "store of value".





Answer: The charts show that interest rates matter for cash demand, but the situation is contrasted across banknotes.

1. Until 2008 interest rates were positive, and cash holding low and stable. After that, interest rates were low and negative, and cash holdings surged. In recent months, interest rates have increased, and cash holdings have started going down. This is in line with the money demand we saw in class which shows that money is inversely related to the interest rate, because when interest rates are high bank accounts and bonds are more interesting than money which gives no interest rate.

2. The relation is quite different across denominations.

   (a) For low denominations (CHF 10 and 20) we see that holdings of cash have been quite steady. They increased more for intermediate denomination (Chf 50 and 100), and much more for the largest banknotes (CHF 200 and 1000).

   (b) Low denomination notes are mostly used for transactions. This relates to money as a mean of exchange. In this role, the interest rate does not matter so much as notes are used to make purchases, and missing an interest rate on a couple of 10 franc notes does not represent a material cost.

   (c) High denomination notes are not used so much for transaction, but more for keeping money aside (i.e. a form of saving). This is the store of value role. For this the interest rate matters, as keeping a couple of thousand francs in banknotes means foregoing a non-negligible.

4 Patience in overlapping generations model (25 % of grade)

4.1 Structure and steady state

Consider the overlapping generations (OLG) model.

An individual lives for two periods and maximizes a log utility of consumption. An important parameter is the discount factor ρ. A higher ρ means the agent is more **impatient**.

$$U_t = \ln(c_{1,t}) + \frac{1}{1+\rho}\ln(c_{2,t+1})$$

There are L_t young agents at time t, with population growing at a rate n. As we saw in class, an agent born at time t supplies one unit of labor when young, getting a wage w_t, and saves resources into old age earning a rate of return r_{t+1}.

The agent pays a tax $\tau_{1,t}$ when young and $\tau_{2,t+1}$ when old.

We present the model to "set the stage" (don't try to derive the equations). Output is produced using labor and capital, $Y_t = (K_t)^{\alpha}(L_t)^{1-\alpha}$. Scale capital is $k_t = K_t/L_t$. The wage and the interest rate are (take this as given):

$$w_t = (1-\alpha)(k_t)^{\alpha} \qquad ; \qquad r_t = \alpha (k_t)^{\alpha-1}$$

As in class, the young agents invest in capital. This gives a dynamic of capital (take this as given):

$$k_{t+1} = \frac{1}{(2+\rho)(1+n)}\left((w_t - \tau_{1,t}) + (1+\rho)\frac{\tau_{2,t+1}}{1+r_{t+1}}\right)$$

The clearing of the market for goods is (take this as given):

$$(k_t)^{\alpha} = c_{1,t} + \frac{1}{1+n}c_{2,t} + (1+n)k_{t+1} - k_t$$

We assume that there is not government spending or debt, so the total taxes add up to zero (remember that population is growing):

$$0 = \tau_{1,t} + \frac{1}{1+n}\tau_{2,t}$$

We first solve for the steady state of the model when taxes are zero and the discount factor is equal to ρ. Do **not** do the derivations. Instead, consider the numbers in the table below. We contrast two cases. In case 1 agents are impatient (high ρ), while they are patient in case 2 (low ρ). In both cases $\alpha = 0.1$ and $n = 0.32$.

		Case 1	Case 2
Discount	ρ	0.5	0
Wage	w^*	0.779	0.799
Capital	k^*	0.236	0.302
Interest rate	r^*	0.367	0.293

A useful result (take this as given) is that consumption in the steady state is:

$$c_1^* + \frac{1}{1+n^*} c_2^* = (k^*)^\alpha - nk^*$$

1. Discuss the relative values of the wage, capital and interest rates across the two cases.
2. Is consumption maximized in either case? Hint: think of the sensitivity of consumption with respect to capital.

Answer: We first present the derivations of the various equations. The consumption levels reflects the Euler condition and the intertemporal budget constraint:

$$c_{1,t} + \frac{c_{2,t+1}}{1+r_{t+1}} = w_t - \left(\tau_{1,t} + \frac{\tau_{2,t+1}}{1+r_{t+1}}\right) \qquad ; \qquad \frac{c_{2,t+1}}{c_{1,t}} = \frac{1+r_{t+1}}{1+\rho}$$

which implies the consumptions:

$$\begin{aligned}
c_{1,t} &= \frac{1+\rho}{2+\rho}\left[w_t - \left(\tau_{1,t} + \frac{\tau_{2,t+1}}{1+r_{t+1}}\right)\right] \\
c_{2,t+1} &= \frac{1+r_{t+1}}{2+\rho}\left[w_t - \left(\tau_{1,t} + \frac{\tau_{2,t+1}}{1+r_{t+1}}\right)\right]
\end{aligned}$$

The capital accumulation equation is:

$$\begin{aligned}
K_{t+1} &= L_t\left(w_t - \tau_{1,t} - c_{1,t}\right) \\
K_{t+1} &= L_t\left(\frac{1}{2+\rho}\left(w_t - \tau_{1,t}\right) + \frac{1+\rho}{2+\rho}\frac{\tau_{2,t+1}}{1+r_{t+1}}\right) \\
\frac{K_{t+1}}{L_{t+1}} &= \frac{L_t}{L_{t+1}}\left(\frac{1}{2+\rho}\left(w_t - \tau_{1,t}\right) + \frac{1+\rho}{2+\rho}\frac{\tau_{2,t+1}}{1+r_{t+1}}\right) \\
k_{t+1} &= \frac{1}{(2+\rho)(1+n)}\left(\left(w_t - \tau_{1,t}\right) + (1+\rho)\frac{\tau_{2,t+1}}{1+r_{t+1}}\right)
\end{aligned}$$

The total taxes are constant:

$$\begin{aligned}
0 &= L_t\tau_{1,t} + L_{t-1}\tau_{2,t} \\
0 &= \tau_{1,t} + \frac{L_{t-1}}{L_t}\tau_{2,t} \\
0 &= \tau_{1,t} + \frac{1}{1+n}\tau_{2,t}
\end{aligned}$$

The clearing of the goods market is:

$$\begin{aligned}
Y_t &= L_t c_{1,t} + L_{t-1}c_{2,t} + K_{t+1} - K_t \\
(K_t)^\alpha (L_t)^{1-\alpha} &= L_t\left[c_{1,t} + \frac{L_{t-1}}{L_t}c_{2,t} + \frac{L_{t+1}}{L_t}\frac{K_{t+1}}{L_{t+1}} - \frac{K_t}{L_t}\right] \\
(k_t)^\alpha &= c_{1,t} + \frac{1}{1+n}c_{2,t} + (1+n)k_{t+1} - k_t
\end{aligned}$$

In the steady state taxes are zero, hence the consumption, factor payment, and capital dynamics are:

$$
\begin{aligned}
c_1^* &= \frac{1+\rho}{2+\rho} w^* \quad ; \quad c_2^* = \frac{1+r^*}{2+\rho} w^* \\
w^* &= (1-\alpha)(k^*)^{\alpha} \quad ; \quad r^* = \alpha (k^*)^{\alpha-1} \\
k^* &= \frac{1}{(2+\rho)(1+n)} w^*
\end{aligned}
$$

The capital dynamics imply:

$$
\begin{aligned}
k^* &= \frac{1}{(2+\rho)(1+n)} w^* \\
k^* &= \frac{1}{(2+\rho)(1+n)} (1-\alpha)(k^*)^{\alpha} \\
k^* &= \left[\frac{1-\alpha}{(2+\rho)(1+n)}\right]^{\frac{1}{1-\alpha}}
\end{aligned}
$$

The real interest rate is then:

$$
\begin{aligned}
r^* &= \alpha (k^*)^{\alpha-1} \\
r^* &= \frac{\alpha}{1-\alpha} (2+\rho)(1+n)
\end{aligned}
$$

We now turn to the question that was asked.

1. When agents are impatient (case 1), the real interest rate is high and capital is low, as is the wage. The opposite is true when they are patient (case 2).

2. Looking at whether consumption is maximized or not, the clearing of the goods market implies that output is equal to overall consumption minus the investment needed to keep k constant:

$$
\begin{aligned}
(k^*)^{\alpha} &= c_1^* + \frac{1}{1+n^*} c_2^* + nk^* \\
c_1^* + \frac{1}{1+n^*} c_2^* &= (k^*)^{\alpha} - nk^*
\end{aligned}
$$

   The derivative of consumption with respect to capital is $\alpha (k^*)^{\alpha-1} - n = r^* - n$. The maximization of consumption requires $r^* = n$, which is the Golden Rule. Consumption is not maximized in either case 1 or 2, as the Golden rule capital is such that $r^* = n = 0.32$, in which case $k^* = 0.274$. When agents are patient (case 2), output is higher. This does not translate into consumption as a lot of the extra output goes towards keeping the capital stock in line with the population growth. The economy has a lot of capital and is dynamically inefficient.war

4.2 Tax and transfer: impatient economy

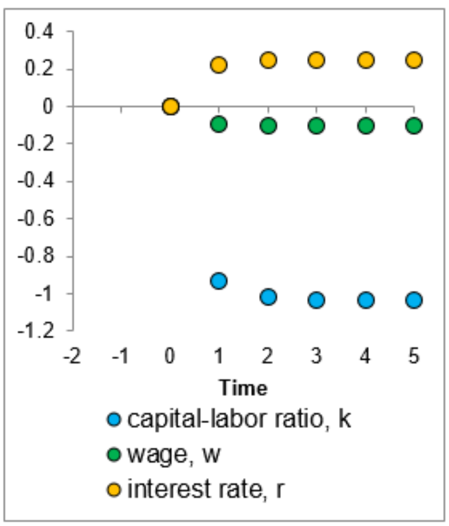
The economy starts in the steady state. At time $t = 0$ the government announces a tax-transfer scheme.

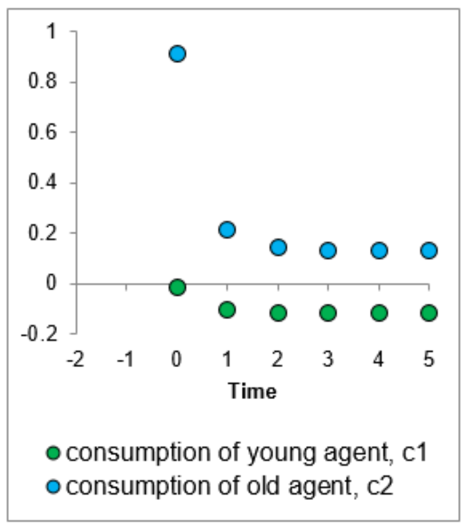
- Each young agent is taxed.
- The proceeds are fully used to give a transfer to the old agents in the same period.
- The scheme remains in place forever.

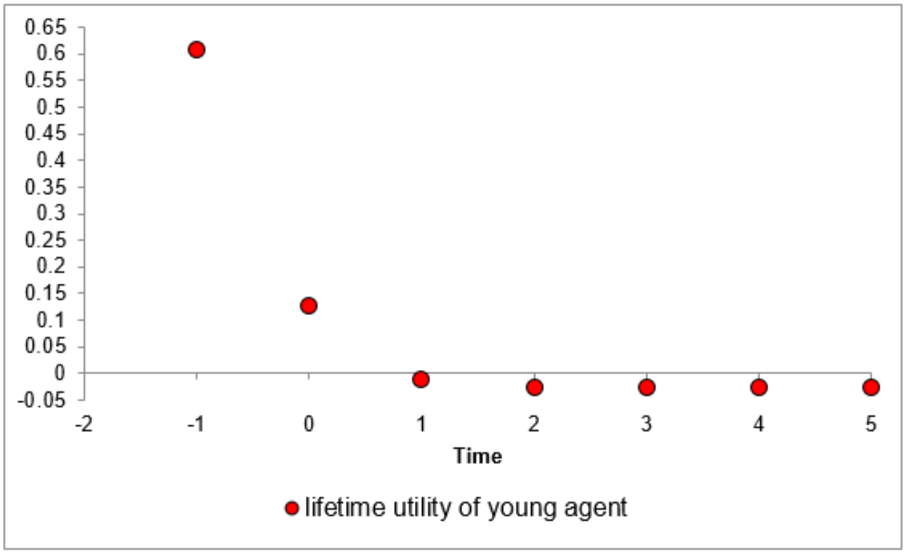
The figure below shows the path of the various variables in a log-linearized version of the model, considering case 1 ($\rho = 0.5$) in the table above.

The top-left panel presents the capital, the wage and the real interest rate. The top right panel presents the movements in the consumption of young and old agents. The bottom panel presents the lifetime utility of a newborn agent (while the policy starts at $t = 0$, there is an effect on the ex-post utility of the agent borne at time $t = -1$).

1. Describe the effect of the tax-transfer.
2. Is it welfare improving? Why or why not?







Answer: We first present the computation of the log-linear version of the model. Deviations are denoted by hatted values (specifically: $\hat{c}_{1,t} = (c_{1,t} - c_1^*)/c_1^*$, $\hat{c}_{2,t} = (c_{2,t} - c_2^*)/c_2^*$, $\hat{k}_t = (k_t - k^*)/k^*$, and $\hat{w}_t = (w_t - w^*)/w^*$). We also define: $\hat{r}_{t+1} = (r_{t+1} - r^*)/(1+r^*)$, $\hat{\tau}_{1,t} = \tau_{1,t}/w^*$, $\hat{\tau}_{2,t} = \tau_{2,t}/w^*$.

The system of equations is:

$$\begin{aligned}
c_{1,t} &= \frac{1+\rho}{2+\rho}\left[w_t - \left(\tau_{1,t} + \frac{\tau_{2,t+1}}{1+r_{t+1}}\right)\right] \\
c_{2,t+1} &= \frac{1+r_{t+1}}{2+\rho}\left[w_t - \left(\tau_{1,t} + \frac{\tau_{2,t+1}}{1+r_{t+1}}\right)\right] \\
w_t &= (1-\alpha)(k_t)^{\alpha} \\
r_t &= \alpha (k_t)^{\alpha-1} \\
k_{t+1} &= \frac{1}{(2+\rho)(1+n)}\left((w_t - \tau_{1,t}) + (1+\rho)\frac{\tau_{2,t+1}}{1+r_{t+1}}\right) \\
0 &= \tau_{1,t} + \frac{1}{1+n}\tau_{2,t}
\end{aligned}$$

This is expanded around the steady state as follows:

$$\begin{aligned}
\hat{c}_{1,t} &= \hat{w}_t - \hat{\tau}_{1,t} - \frac{\hat{\tau}_{2,t+1}}{1+r^*} \\
\hat{c}_{2,t+1} &= \hat{w}_t - \hat{\tau}_{1,t} - \frac{\hat{\tau}_{2,t+1}}{1+r^*} + \hat{r}_{t+1} \\
\hat{w}_t &= \alpha \hat{k}_t \\
\hat{r}_t &= \frac{r^*}{1+r^*}(\alpha - 1)\hat{k}_t \\
\hat{k}_{t+1} &= \hat{w}_t - \hat{\tau}_{1,t} + (1+\rho)\frac{\hat{\tau}_{2,t+1}}{1+r^*} \\
\hat{\tau}_{1,t} &= -\frac{1}{1+n}\hat{\tau}_{2,t}
\end{aligned}$$

Using the relation between $\hat{\tau}_{1,t}$ and $\hat{\tau}_{2,t}$ we can write the system as:

$$\begin{aligned}
\hat{c}_{1,t} &= \hat{w}_t - \hat{\tau}_{1,t} + \frac{1+n}{1+r^*}\hat{\tau}_{1,t+1} \\
\hat{c}_{2,t+1} &= \hat{w}_t - \hat{\tau}_{1,t} + \frac{1+n}{1+r^*}\hat{\tau}_{1,t+1} + \hat{r}_{t+1} \\
\hat{w}_t &= \alpha\hat{k}_t \\
\hat{r}_t &= \frac{r^*}{1+r^*}(\alpha-1)\hat{k}_t \\
\hat{k}_{t+1} &= \hat{w}_t - \hat{\tau}_{1,t} - \frac{(1+\rho)(1+n)}{1+r^*}\hat{\tau}_{1,t+1}
\end{aligned}$$

The last relation, along with $\hat{w}_t = \alpha\hat{k}_t$, gives the dynamics of capital as a function of shocks. All the other variables follow.

$$\hat{k}_{t+1} = \alpha\hat{k}_t - \hat{\tau}_{1,t} - \frac{(1+\rho)(1+n)}{1+r^*}\hat{\tau}_{1,t+1}$$

The clearing of the goods market is written as:

$$\begin{aligned}
\alpha(k)^{\alpha}\hat{k}_t &= c_{1,t}\hat{c}_{1,t} + \frac{1}{1+n}c_{2,t}\hat{c}_{2,t} + (1+n)k_{t+1}\hat{k}_{t+1} - k_t\hat{k}_t \\
\frac{1+r^*}{1+n}\hat{k}_t &= (1+\rho)\hat{c}_{1,t} + \frac{1+r^*}{1+n^*}\hat{c}_{2,t} + \hat{k}_{t+1}
\end{aligned}$$

A subtlety arises in the first period of the shocks, as these are not anticipated. In that first period capital does not change: $\hat{k}_t = 0$. Consumption of the old agents can only change thanks to movements in the interest rate and transfer shocks:

$$\hat{c}_{2,t} = \hat{r}_t - \frac{2+\rho}{1+r^*}\hat{\tau}_{2,t}$$

The utility is:

$$\hat{U}_t = \hat{c}_{1,t} + \frac{1}{1+\rho}\hat{c}_{2,t+1}$$

With the technicalities answered for, we now turn to the question that was asked.

1. The tax-transfer scheme leads to a reduction of the capital stock, as it effectively offers another way to save (reduction in after tax income when young, offset by an increase when old). The reduced capital lowers the marginal product of labor (and the wage) and raises the marginal product of capital (and the real interest rate). In other words, the tax-transfer scheme crowds out private savings in capital.

2. How does this affect agents?

   (a) The old agents at time 0 are happy as they can consume more. The young agents at time 0 have to reduce their consumption when young. They get some additional consumption when old, but that is not enough and their overall welfare is lower. Agents borne at time 1 or after are clearly worse off as their consumption is reduced at all times. This is because their income when young is reduces due to the lower capital accumulation.

(b) The policy is therefore not a good idea. This is because the implicit rate of return that agents get through the redistribution is the rate of population growth, which in case 1 is lower than the real interest rate. The policy diverts savings from a high return option to a low return one, which is not efficient.

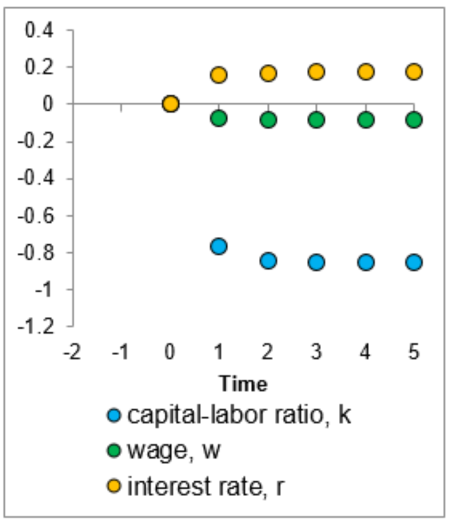
4.3 Tax and transfer: patient economy

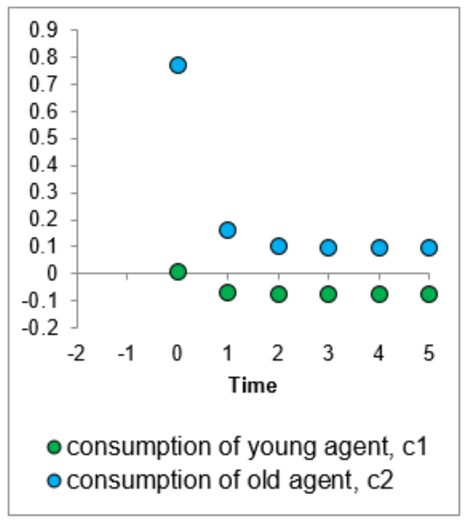
We consider the same tax and transfer scheme, but now in the patient economic of case 2 ($\rho = 0$).

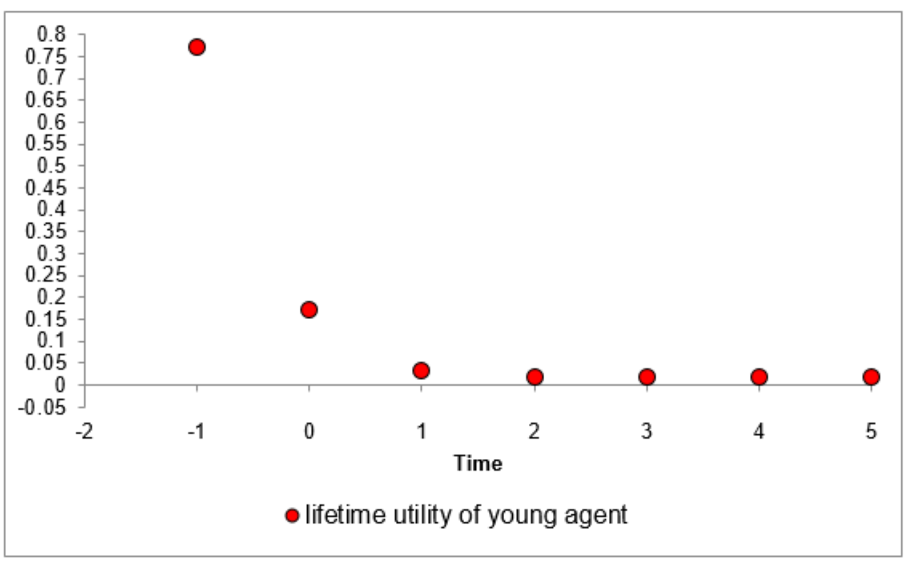
The figure below shows the path of the various variables.

1. Describe the effect of the tax-transfer.

2. Is it welfare improving? Why or why not?







Answer: The answers of the various points are as follows.

1. The top-left panel is similar to case 1: the transfer scheme represents an alternative saving channel that reduces savings through capital, leading to lower capital, a lower wage, and a higher real interest rate. The impact on consumption is also similar (top-right panel).

2. The welfare impact is very different from case 1.

   (a) Again, the old agents at time 0 are happy as they can consume more. The young agents at time 0 get a higher consumption when young as well as when old, and are thus happy. What is very different is that agents borne at time 1 or after are better off. While their consumption when young decrease a bit, this is more than offset by their higher consumption when old.

   (b) This is because the economy is dynamically inefficient. The very patient agents save so much into capital that they drive the real interest rate below the population growth rate. Savings through capital then delivers a lower return than the implicit return on the transfer scheme (the population growth rate). The policy thus offers a better saving channel and raises welfare.

4.4 Application to recent trends

Until recently, the world economy has been faced with a persistent decrease of real interest rates over the last 15 years.

In light of this model, what are the implications for the funding of retirement schemes. Specifically, should we ask people to acquire assets or do transfers across generations?

Answer: When the interest rate was high, savings through capital was better. This explains why retirement schemes through capitalization (the income when retired comes from the capital that each person accumulated) were better that redistribution "pay as you go" schemes where money is transferred from young to old persons.

The reduction in the interest rates however leads to a different trade-off, and provides an argument why pensions schemes could be re-oriented towards transfer schemes.

5 Timing of interest rate policy (25 % of grade)

5.1 Dynamics of inflation and output

Consider a model where inflation is driven by a Phillips curve where inflation, π_t, increases ($\dot{\pi}_t > 0$) when output is above equilibrium ($y_t > 0$):

$$\dot{\pi}_t = \lambda y_t \qquad ; \qquad \lambda > 0$$

Output reflects the real interest rate, as seen in the New Keynesian model (this is coming from the Euler condition), where i_t is the nominal interest rate set by the central bank:

$$y_t = -b\,(i_t - \pi_t) \qquad ; \qquad b > 0$$

Using this framework:

1. How can the central bank stabilize inflation?
2. How can it bring inflation down?

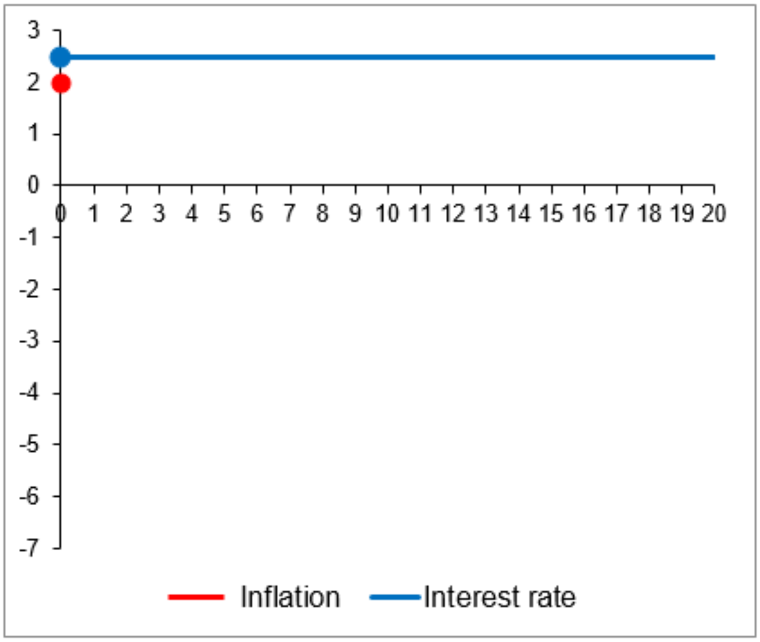
Answer: To stabilize inflation at some value π^*, the central bank needs to have $\dot{\pi}_t = 0$ which requires $y_t = 0$. To achieve this, the central bank should simply set i constant and equal to π^*.

If initially inflation is above the target π^*, the central bank needs to deliver $\dot{\pi}_t < 0$ which requires a recession $y_t < 0$. It is achieved by setting a positive real interest rate, that is a nominal interest rate above the inflation target, $i_t > \pi^*$.

5.2 Impact of constant interest rate

Suppose that inflation is initially positive $\pi_0 > 0$.

Consider that the central bank keeps the interest rate at a value $i_0 > \pi_0 > 0$ and always keeps the interest rate at that value. The chart below shows the path of the interest rate (blue line, with initial value at the blue point), and the initial inflation value (red point).



1. What is the path of inflation through time? Hint: use the charts provided at the end of the question to draw the various variables.

2. What is the path of output?

3. Is the policy a good idea?

Answer: The paths are drawn in the figures below. The first figure shows the interest rate (blue line) and inflation (red line), and the second output (green line).

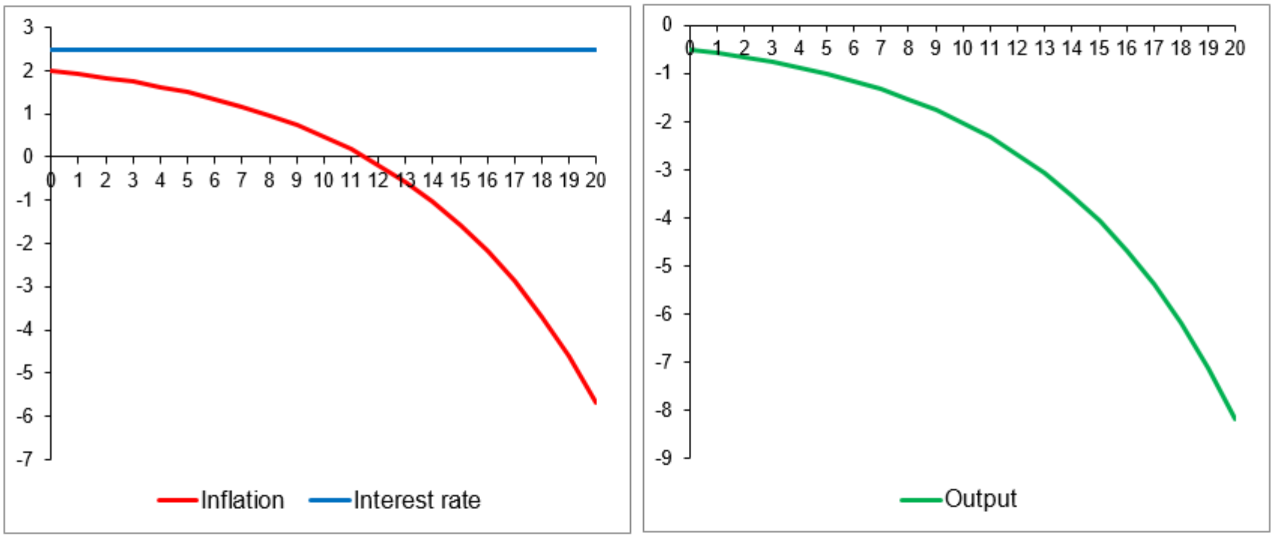
At time 0, we have $i_0 > \pi_0$ which gives a low output $y_0 < 0$. This translates into a reduction in inflation, $\dot{\pi}_0 < 0$. Therefore inflation decreases going forward.

The decrease in inflation makes the gap $i_0 > \pi$ even larger, which lowers output further. This pushes inflation further down, at an accelerating rate.

1. Inflation goes through a declining path, at an ever increasing path. This means that it goes into negative territory at some point.

2. As the nominal interest rate is constant, the ever lower values of inflation lead to ever lower values of output.

3. The policy is terrible, as the economy spirals into a situation of deflation and collapsing output.


Inflation
Interest rate
Output