

# International Trade I

## The Ricardian Model<sup>1</sup>

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<sup>1</sup> These lecture notes are based on materials from A. Costinot, A. Dixit, R. Feenstra, Feenstra&Taylor, and J. P. Neary.

# Outline of the Lecture

- 1 Introduction
- 2 The Two-Sector Model
- 3 Continuum of Goods: Dornbusch, Fischer and Samuelson (1977)
- 4 Many countries: Eaton and Kortum (2002)
- 5 Weak points of the Ricardian model

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- 1 **Introduction**
- 2 The Two-Sector Model
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- 5 Weak points of the Ricardian model

# Reasons for trade

## Countries' differences

→ **comparative advantage**

- **Productivity (Ricardo)**
- Endowments (H-O)

## Countries' similarities

- Economies of scale

# Comparative advantage

- Stanislaw Ulam's challenge to Paul Samuelson: "name me one proposition in all of the social sciences which is both true and non-trivial"
- **Comparative advantage:** "That it is logically true need not be argued before a mathematician; that it is not trivial is attested by the thousands of important and intelligent men who have never been able to grasp the doctrine for themselves or to believe it after it was explained to them."<sup>2</sup>

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<sup>2</sup>P.A. Samuelson (1969), "The Way of an Economist," in P.A. Samuelson, ed., International Economic Relations: Proceedings of the Third Congress of the International Economic Association, Macmillan: London, pp. 1-11

# Background

- Simplest and earliest (1817) complete model of production and trade
- Reason for trade: **comparative advantage**
- Source of comparative advantage: **differences in production technologies** across countries
- Note: these are differences in production functions, not differences in labor productivities due to different endowments of capital (that type of model (Heckscher-Ohlin) will be discussed later)

# Main Assumptions

- Perfect competition
- Labor only factor of production (“labour theory of value”)
- Endowments given, confined to country but intersectorally mobile within each country
- Full employment
- Two countries with different technologies (production functions)
- Constant returns to labor
- Number of goods:  $n \geq 2$

# Technology

How is efficiency measured?

- With only one factor, simply by the usage of that factor: **Labour Requirement per unit Output**
- The technology is hence modelled with a coefficient,  $a_i$ , which measures the quantity of labour (e.g. number of hours) necessary to produce one unit of good  $i$ 
  - ▶ E.g. if  $a_X = 2$ , 2 hours of labour are needed to produce one unit of good  $X$ .
- Note: **Labour productivity** is the inverse of the labour requirement per unit output  $a_i$ . The higher  $a_i$ , the less labour is productif.



# Production Possibility Frontier: Refresher

PPF = maximum possible production level for a given technology and factor endowment

- **Constraint:** points on the outside of the PPF are not feasible
- **Efficiency:** points inside the PPF are not efficient, those on the frontier are
- **Opportunity cost:** the slope of the PPF indicates the opportunity cost of the two goods
- **Concavity:** the PPF is often concave (the opportunity cost of one good increases with the produced quantity of this good), but not always
- Technological progress shifts the PPF outwards

# Outline of the Lecture

## 1 Introduction

## 2 The Two-Sector Model

- The Basic Set-up
- Autarkic equilibria
- Free trade equilibrium
- Wages and gains from trade
- Summing up

## 3 Continuum of Goods: Dornbusch, Fischer and Samuelson (1977)

## 4 Many countries: Eaton and Kortum (2002)

## 5 Weak points of the Ricardian model

# A Simple Numerical Example

- 2 countries: US and India
- 2 goods: Autos (A) and Corn (C)

	US	India
<b>Labour force</b>	$L = 200$	$L^* = 800$
<b>Labour per unit Corn</b>	$a_C = 8$	$a_C^* = 50$
<b>Labour per unit Auto</b>	$a_A = 10$	$a_A^* = 40$

- Specific assumption of this example : US is more efficient/productive at producing both goods
- US have an **absolute advantage** in producing both goods

# Production Possibility Frontiers

## US

Full-employment condition:

$$L = a_C Q_C + a_A Q_A$$

$$\Rightarrow \text{PPF: } Q_A = \frac{L}{a_A} - \frac{a_C}{a_A} Q_C$$

## India

Full-employment condition:

$$L^* = a_C^* Q_C^* + a_A^* Q_A^*$$

$$\Rightarrow \text{PPF: } Q_A^* = \frac{L^*}{a_A^*} - \frac{a_C^*}{a_A^*} Q_C^*$$

# Relative Price and Technology

- Notation:

- ▶ Relative price of Corn:  $P = \frac{P_C}{P_A}$
- ▶ Relative quantity of Corn:  $R = \frac{Q_C}{Q_A}$

- Workers are paid the value of their marginal products:

$$w_C = \frac{P_C}{a_C} \quad \text{and} \quad w_A = \frac{P_A}{a_A}$$

- Perfectly mobile labour: If both goods produced

$$\Rightarrow w_C = w_A \Leftrightarrow P = \frac{a_C}{a_A}$$

# Production and supply in the US

- If  $P = \frac{a_C}{a_A} = 0.8$ :
  - ▶  $Q_C$  and  $Q_A$  anywhere on PPF
  - ▶  $0 \leq \frac{Q_C}{Q_A} \leq \infty$
- If  $P < \frac{a_C}{a_A} = 0.8$ :
  - ▶  $Q_C = 0$
  - ▶  $Q_A = 20$
  - ▶  $\frac{Q_C}{Q_A} = 0$
- If  $P > \frac{a_C}{a_A} = 0.8$ :
  - ▶  $Q_C = 25$
  - ▶  $Q_A = 0$
  - ▶  $\frac{Q_C}{Q_A} = \infty$

# Autarkic equilibria in the two countries

## Autarky in the US

## Autarky in India

- In autarky, relative supply (technology) determines relative prices, and together with relative demand, the quantities
- Relative price of corn is lower in the US, relative quantity consumed is higher

## Trade: Relative supply

- When  $P < 0.8$ , both countries produce only autos, world ratio  $R = 0$
- When  $0.8 < P < 1.25$ , US produces 25 corn, India produces 20 autos,  $R = 1.25$
- When  $P > 1.25$ , both countries produce only corn,  $R = \infty$
- When  $P = 0.8$ , India produces 20 autos. US varies between 20 autos, no corn (world  $R = 0$ ), and 0 autos, 25 corn (world  $R = 1.25$ )
- When  $P = 1.25$ , similarly world  $R$  can be anywhere between 1.25 and  $\infty$ .



# Trading equilibrium

Depending on position of relative demand, the trading equilibrium can be one of three types: complete specialization (B) or incomplete (A, C).

- A:  $P = 0.8$ . US produces both goods, India autos
- B:  $0.8 < P < 1.25$ , US produces only corn, India only autos
- C:  $P = 1.25$ . US produces only corn, India produces both goods

## Efficient production and world PPF: construction

Suppose initially all labor produces autos in both (40 in all). If any corn is to be produced, it is better to do so by switching some labor in the US: each auto not produced releases 10 labor which can then produce  $10/8 = 1.25$  corn. In India each less auto yields only  $40/50 = 0.8$  more corn. Only when all US labor has been diverted to producing corn should any Indian labor be switched.

Conversely, starting with all corn (41 units), to produce any autos, Indian labor should be switched.

This despite the US producing autos more efficiently than India: only 10 units of labor against 40. The reason: the US produces corn *even more* efficiently: only 8 units of labor against 50. What matters is the ratio (opportunity cost):  $10/8 > 40/50$ , or  $10/40 > 8/50$ .

# Efficient production and world PPF: graph

- World PPF: juxtapose the country PPFs as the outer lines
- “Wrong” assignment of production would yield the less efficient inner lines

# Trading equilibrium and world PPF

If preferences are identical and homothetic everywhere, we can draw world indifference curves. Depending on their shape, three types of outcomes:

- A: US produces both goods, India produces only autos
- B: US produces only corn, India produces only autos
- C: US produces only corn, India produces both goods

Relative price of corn = slope of PPF: 0.8 in A, 1.25 in C, between 0.8 and 1.25 in B

# Absolute vs comparative advantage

- Equilibrium/efficient pattern of production determined by **comparative**, not **absolute** advantage: in equilibrium countries specialise in the production of the good in which they have comparative advantage
- **Absolute advantage**: Lower labor requirement per unit of output
- **Comparative advantage**: Country A has comparative advantage over country B in good X as against good Y if the ratio of unit labor requirement for X to that for Y is smaller in country A than in country B.  
In our example:  $a_C/a_A < a_C^*/a_A^*$  (lower opportunity cost)

# Equilibrium trade patterns

- In the trading equilibrium, each country has to satisfy its *budget constraint*: the value of production = the value of consumption, i.e.

$$P_A Q_A + P_C Q_C = P_A D_A + P_C D_C$$

where  $D_i$  represents the demand for good  $i$ ; or in terms of relative price

$$Q_A + \frac{P_C}{P_A} Q_C = D_A + \frac{P_C}{P_A} D_C$$

- With a specific preference structure, we can solve for imported and exported quantities.

# Wages in autarky

Under constant returns to scale,  $P = MC = AC$  for each good produced,  $<$  if not produced

## Autarky:

- US produces both goods:  $P_C^a = a_C w^a = 8w^a$ ,  
 $P_A^a = a_A * w^a = 10w^a$

Each US worker can buy  $\frac{w^a}{P_C^a} = \frac{1}{8}$  corn and  $\frac{w^a}{P_A^a} = \frac{1}{10}$  auto

- India produces both goods:  $P_C^{*a} = a_C^* w^{*a} = 50w^{*a}$ ,  
 $P_A^{*a} = a_A^* w^{*a} = 40w^{*a}$

Each Indian worker can buy  $\frac{w^{*a}}{P_C^{*a}} = \frac{1}{50}$  corn and  $\frac{w^{*a}}{P_A^{*a}} = \frac{1}{40}$  auto

# Wages in free trade

**Free Trade:** Consider type B equilibrium. For definiteness, let  $\frac{P_C}{P_A} = 1$  so  $P_C = P_A$  Important: No country label here; world market for goods, same price

- US produces only corn:  $P_C = 8w$ ,  $P_A < 10w$

Each US worker can buy  $\frac{w}{P_C} = \frac{1}{8}$  corn and  $\frac{w}{P_A} > \frac{1}{10}$  auto

- India produces only autos:  $P_C < 50w^*$ ,  $P_A = 40w^*$

Each Indian worker can buy  $\frac{w^*}{P_C} > \frac{1}{50}$  corn and  $\frac{w^*}{P_A} = \frac{1}{40}$  auto



# Gains from trade

## Rise in wages

- In type B equilibrium, labor in both countries is better off in trade than in autarky: can buy more of the good no longer produced. Benefit of trade: ability to buy cheaper imports!
- In type A/C equilibria, the country producing both goods has no gain, but no loss either.

## Increase in consumption

- Trade enables countries to consume outside their PPF.

# Main findings

- Rival claims:
  - ▶ US: We can't compete with low-wage Indian labor
  - ▶ India: We can't compete with highly productive US laborare both wrong: Wages adjust; US continues to compete in corn, India in autos.
- This model has only one factor in each country, and mobile across sectors: therefore no distributive conflict over trade.  
Reminder: different models have different uses. This one is useful for clarifying the idea of comparative advantage in the simplest way.

# Main findings (continued)

- Absolute advantage does matter: it affects the standard of living. Here each US worker can buy much more than each Indian worker, whether we compare the two in autarky, or compare them with trade. But it does not matter for determining which goods to produce where, and therefore for the pattern of trade, nor for existence of gains from trade. Distinct comparisons: (a) Is US richer than India? Absolute advantage is relevant. (b) Does US gain from trade with India? Comparative advantage is relevant.
- Different types A, B, C of equilibria also can arise for supply side reasons: (one country much larger or much more productive than the other)

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# Recall basic assumptions

- Perfect competition
- 2 countries: Home and Foreign (\*) with different technologies
- One factor of production: labor
- Notation:
  - ▶  $L$  and  $L^*$ : endowments of labor in the two countries
  - ▶  $w$  and  $w^*$ : wages in the two countries
- Full employment
- Constant returns to scale (CRS)

# Supply side assumptions

- There is a **continuum** of goods indexed by  $z \in [0, 1]$
- Since there are CRS, we can define the (constant) unit labor requirements in both countries:  $a(z)$  and  $a^*(z)$ 
  - ▶  $a(z)$  and  $a^*(z)$  capture all we need to know about technology in the two countries
- W.l.o.g., we order goods such that  $A(z) \equiv \frac{a^*(z)}{a(z)}$  is decreasing
  - ▶ Hence Home has a comparative advantage in the low- $z$  goods
  - ▶ For simplicity, we'll assume strict monotonicity

# Efficient international specialization: Proposition

- Previous supply-side assumptions are all we need to make qualitative predictions about pattern of trade
- Let  $p(z)$  denote the price of good  $z$  under free trade
- Profit-maximization requires

$$p(z) \leq wa(z), \text{ with equality if } z \text{ is produced at Home} \quad (1)$$

$$p(z) \leq w^*a^*(z), \text{ with equality if } z \text{ is produced in Foreign} \quad (2)$$

- **Proposition:** There exists  $\tilde{z} \in [0, 1]$  such that Home produces all goods  $z < \tilde{z}$  and Foreign produces all goods  $z > \tilde{z}$ .

## Efficient international specialization: Proof

- By contradiction: Suppose that there exists  $z' < z$  such that  $z$  produced at Home and  $z'$  is produced abroad. (1) and (2) imply

$$\begin{array}{lll} p(z) = wa(z) & \text{and} & p(z') \leq wa(z') \\ p(z) \leq w^*a^*(z) & \text{and} & p(z') = w^*a^*(z') \end{array}$$

This implies

$$wa(z)w^*a^*(z') = p(z)p(z') \leq wa(z')w^*a^*(z)$$

which can be rearranged as

$$a^*(z')/a(z') \leq a^*(z)/a(z)$$

This contradicts  $A$  strictly decreasing.



## Efficient international specialization: Remarks

- Proposition simply states that Home should produce and specialize in the goods in which it has a CA
- Note that:
  - ▶ Proposition does not rely on continuum of goods
  - ▶ Continuum of goods + continuity of  $A$  is important to derive

$$A(\tilde{z}) = \frac{w}{w^*} \equiv \omega \quad (3)$$

- Equation (3) is the first of DFS's two equilibrium conditions:
  - ▶ Conditional on wages, goods should be produced in the country where it is cheaper to do so
- To complete characterization of free trade equilibrium, we need look at the demand side to pin down the relative wage  $\omega$

## Demand side assumptions

- Consumers have **identical Cobb-Douglas** pref around the world
- Denote by  $b(z) \in (0, 1)$  the share of expenditure on good  $z$ :

$$b(z) = \frac{p(z)c(z)}{wL} = \frac{p(z)c^*(z)}{w^*L^*}$$

where  $c(z)$  and  $c^*(z)$  are consumptions at Home and Abroad

- By definition, share of expenditure satisfy:  $\int_0^1 b(z)dz = 1$

## Trade balance

- $\theta(\tilde{z}) \equiv \int_0^{\tilde{z}} b(z) dz$ : share of income spent (in both countries) on goods produced at Home
- Trade balance requires

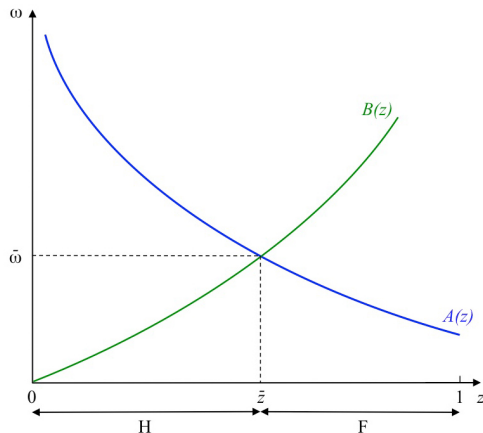
$$\underbrace{\theta(\tilde{z})w^*L^*}_{\text{Home exports}} = \underbrace{(1 - \theta(\tilde{z}))wL}_{\text{Home imports}}$$

- Which can be rearranged as

$$\omega = \frac{\theta(\tilde{z})}{1 - \theta(\tilde{z})} \left( \frac{L^*}{L} \right) \equiv B(\tilde{z}) \quad (4)$$

- ▶  $B' > 0$ : an increase in  $\tilde{z}$  leads to a trade surplus at Home, which must be compensated by an increase in Home's relative wage  $\omega$

# Equilibrium



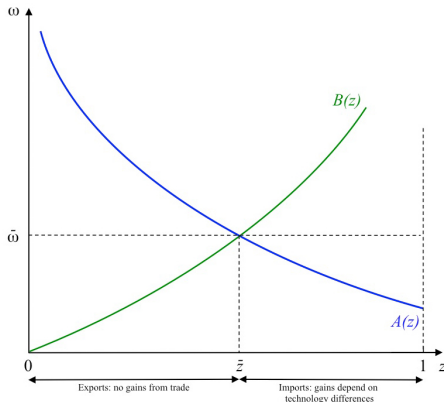
$$\frac{\theta(\bar{z})}{1 - \theta(\bar{z})} \left( \frac{L^*}{L} \right) = \bar{\omega} = A(\bar{z})$$

- Efficient international specialization, Equation (3), and trade balance, (4), jointly determine  $(\bar{z}, \bar{\omega})$

# Gains from trade: the argument

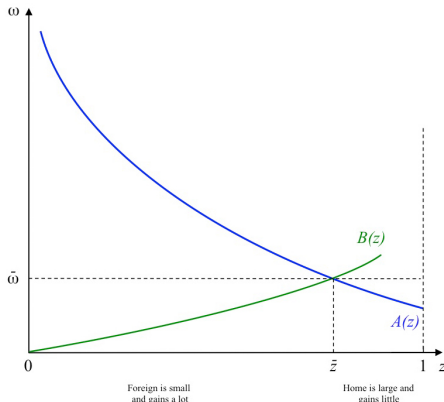
- Set  $w = 1$  under autarky and free trade
- Indirect utility of Home representative household only depends on  $p(\cdot)$
- For goods  $z$  produced at Home under free trade: no change compared to autarky
- For goods  $z$  produced in Foreign under free trade:  
 $p(z) = w^* a^*(z) < a(z)$
- Since all prices go down, indirect utility must go up

# Gains from trade: the intuition



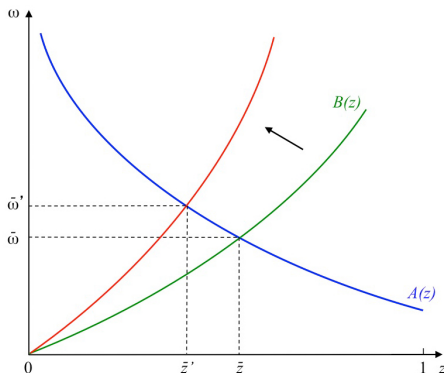
- For home, compare free trade with “autarky”: self-sufficiency
- Goods exported after trade also produced in autarky  $\Rightarrow$  no gains accrue on them
- Gains arise on *imported* goods only
- Gains are greater the larger the gap between Home and Foreign technology

# Gains from trade: who gains more?



- Implication: Small countries gain more from trade
- Non-economists find this very hard to understand/believe
- Only qualification: the small country has to adjust by more
- But, its adjustment is the source of its gains

# Effects of an increase in Foreign labor force



- Suppose that  $L^*/L$  goes up (rise of China):
  - ▶  $\omega$  goes up and  $\tilde{z}$  goes down
  - ▶ At initial wages, an increase in  $L^*/L$  creates a trade deficit in Foreign, which must be compensated by an increase in  $\omega$



# Effects of an increase in Foreign labor force

- Increase in  $L^*/L$  raises indirect utility, i.e. real wage, of representative household at Home and lowers it in Foreign
  - ▶ Set  $w = 1$  before and after the change in  $L^*/L$
  - ▶ For goods  $z$  whose production remains at Home: no change in  $p(z)$
  - ▶ For goods  $z$  whose production remains in Foreign:
 
$$\omega \uparrow \Rightarrow w^* \downarrow \Rightarrow p(z) = w^* a^*(z) \downarrow$$
  - ▶ For goods  $z$  whose production moves to Foreign:
 
$$w^* a^*(z) \leq a(z) \Rightarrow p(z) \downarrow$$
  - ▶ So Home gains. Similar logic implies Foreign welfare loss.
- Comments:
  - ▶ In spite of CRS at the industry-level, everything is as if we had DRS at the country-level
  - ▶ As Foreign's size increases, it specializes in sectors in which it is relatively less productive (compared to Home), which worsens its terms-of-trade, and so, lowers real GDP per capita
  - ▶ The flatter the  $A$  schedule, the smaller this effect

# Effects of technological change

- Improvement in Home technology  $\Rightarrow$  upward shift of the  $A(z)$  schedule:
  - ▶ Higher home relative wage: i.e., higher income
  - ▶ Increase in home's share of world production and trade
- International transfer of the most efficient technology: for all  $z$ ,  $a(z) = a^*(z)$  (offshoring?)
  - ▶ If Home has the most efficient technology,  $a(z) < a^*(z)$  initially, then it will lose from international transfer (no gains from trade)

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# Ricardian Model: Random Productivities

- Random productivity shocks can “soften” predictions of the Ricardian model and make it more amenable to empirical implementation
- Model generates predictions about industry trade flows

# Ricardian Model: Basic Assumptions

- $N$  countries  $i = 1, \dots, N$
- All markets perfectly competitive
- Continuum of goods of measure one as in DFS (1977):  $u \in [0, 1]$
- CES preferences with elasticity of substitution  $\sigma > 0$

$$U_i = \left( \int_0^1 q_i(u)^{\frac{\sigma-1}{\sigma}} du \right)^{\frac{\sigma}{\sigma-1}}$$

- One factor of production (labor)
- Technology: CRS
  - ▶  $a_i(u)$  vary by country  $i$  and good  $u$  (as in DFS)
  - ▶  $a_i(u)$  are the realization of random variables (new)
  - ▶ Note: if distribution is smooth then  $a_i(u) \rightarrow$  no need to keep track of individual  $a_i(u)$ s, but only of the key summary statistics of the distribution

# Ricardian Model: Technology

- $Z_i(u)$ : productivity of (any) firm producing  $u$  in country  $i$  ( $1/a_i(u)$ )
- $Z_i(u)$  drawn independently (across goods and countries) from a **Fréchet** distribution:

$$\Pr(Z_i \leq z) = F_i(z) = e^{-T_i z^{-\theta}}$$

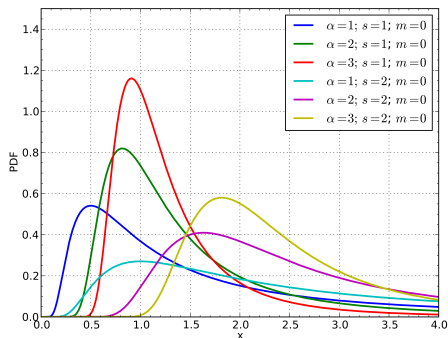
with  $\theta > \sigma - 1$  (important restriction, see below)

- ▶  $T_i > 0$ : country  $i$ 's state of technology (*absolute advantage*)
- ▶  $\theta > 1$ : measures intra-industry heterogeneity, idiosyncratic variation in know-how across varieties  $\rightarrow$  drives intra-industry trade (*comparative advantage*)
  - ★ High  $\theta \Rightarrow$  less variability  $\Rightarrow$  less intra-industry trade
  - ★ Important assumption that  $\theta$  is common across industries
  - ★  $\theta$  guides impact of changes in fundamental productivity on aggregate trade flows

# Fréchet Distribution

$$\Pr(z \leq x) = e^{-\left(\frac{x-m}{s}\right)^{-\alpha}}$$

$\alpha$ : shape - inverse measure of dispersion;  $m$ : location;  $s$ : scale



# Fréchet Distribution: Useful Properties

- Take independent Fréchet random variables  $Z_i, i = 1, \dots, N$  such that

$$\Pr(Z_i \leq z) = F_i(z) = e^{-T_i z^{-\theta}}$$

then:

- For any  $B_i > 0$ ,  $B_i Z_i$  is also Fréchet with  $\Pr(B_i Z_i \leq z) = e^{-T_i B_i^\theta z^{-\theta}}$
- $Z = \max\{Z_1, \dots, Z_N\}$  is also Fréchet with  $\Pr(Z \leq z) = e^{-[\sum_k T_k] z^{-\theta}}$
- $\Pr[Z_i = \max\{Z_1, \dots, Z_N\}] = \frac{T_i}{\sum_k T_k}$
- $E[Z_i^\zeta] = \Gamma\left(\frac{\theta-\zeta}{\theta}\right) T_i^{\frac{\zeta}{\theta}}$ , where  $\Gamma(\cdot)$  is the Gamma function

$$\Gamma(x) \equiv \int_0^\infty e^{-t} t^{x-1} dt$$



# Trade Costs

- Iceberg trade costs: For every unit shipped from country  $i$  to country  $j$ , only  $1/d_{ij}$  units arrive (with  $d_{ii} = 1$ )
- Cost of producing one unit of  $u$  in  $i$  and delivering it to  $n$

$$P_{ni}(u) = d_{ni}w_i/Z_i(u)$$

- In any country  $n$ , the price  $p_n(u)$  paid by buyers of variety  $u$  is

$$P_n(u) = \min [P_{ni}(u)]$$

# Distribution of Prices with Fréchet

- Country  $i$  presents country  $n$  with a distribution of (potential) prices:

$$\begin{aligned} G_{ni}(p) &= \Pr[P_{ni} \leq p] = \Pr[Z_i \geq w_i d_{ni}/p] \\ &= 1 - F_i(d_{ni}w_i/p) = 1 - e^{-[T_i(d_{ni}w_i)^{-\theta}]p^\theta} \end{aligned}$$

- The distribution of minimum prices in country  $n$  is

$$G_n(p) = 1 - \prod_{i=1}^N [1 - G_{ni}(p)]$$

- So, distribution of minimum prices takes the same form as distribution of price offers:

$$G_n(p) = 1 - e^{-\Phi_n p^\theta} \quad \text{with} \quad \Phi_n \equiv \sum_{i=1}^N T_i (d_{ni}w_i)^{-\theta}$$

# Allocation of Purchases

- Probability that  $i$  is the lowest cost source for  $n$ :

$$\begin{aligned}
 \pi_{ni} &= \Pr(P_{ni} \leq \min_{s \neq i} P_{ns}) \\
 &= \int_0^\infty \Pi_{s \neq i} [1 - G_{ns}(p)] dG_{ni}(p) \\
 &= \frac{T_i(w_i d_{ni})^{-\theta}}{\Phi_n}
 \end{aligned}$$

- By law of large numbers, this is also the fraction of goods that country  $n$  buys from  $i$

# Conditional Price Distribution

- Price distribution of a good that  $n$  actually buys from  $i$  is independent of  $i$ , and it has the form of  $G_n(p)$ .
  - ▶ A source country with a high fundamental productivity (or lower wage or lower geographic barriers) exploits its advantage by selling a wider range of goods (expansion along extensive margin), but the average price charged is the same
  - ▶ The share of spending by country  $n$  on goods from country  $i$  is the same as the probability  $\pi_{ni}$

# The Price Index

- The exact price index for a CES utility with elasticity of substitution  $\sigma < 1 + \theta$ , defined as

$$p_n \equiv \left( \int_0^1 p_n(u)^{1-\sigma} du \right)^{1/(1-\sigma)}$$

is given by

$$p_n = \gamma \Phi_n^{-1/\theta}$$

where

$$\gamma = \left[ \Gamma \left( \frac{\theta + 1 - \sigma}{\theta} \right) \right]^{1/(1-\sigma)}$$

with  $\sigma - 1 < \theta$  for Gamma function to be well defined

# Trade Flows and Gravity

- Total spending in country  $n$  on goods from country  $i$ :  $X_{ni}$
- Country  $n$ 's total spending:  $X_n \equiv \sum_i X_{ni}$
- We know that  $X_{ni}/X_n = \pi_{ni}$ , so

$$X_{ni} = \frac{T_i(w_i d_{ni})^{-\theta}}{\Phi_n} X_n \quad (5)$$

- Letting  $Y_i = \sum_n X_{ni}$  be country  $i$ 's total sales, then

$$Y_i = \sum_n \frac{T_i(w_i d_{ni})^{-\theta} X_n}{\Phi_n} = T_i w_i^{-\theta} \Omega_i^{-\theta}$$

where

$$\Omega_i^{-\theta} \equiv \sum_n \frac{d_{ni}^{-\theta} X_n}{\Phi_n}$$

## Trade Flows and Gravity (2)

- Solving  $T_i w_i^{-\theta}$  from  $Y_i = T_i w_i^{-\theta} \Omega_i^{-\theta}$  and plugging into (5):

$$X_{ni} = \frac{X_n Y_i d_{ni}^{-\theta} \Omega_i^{\theta}}{\Phi_n}$$

- Using  $p_n = \gamma \Phi_n^{-1/\theta}$  we get

$$X_{ni} = \gamma^{-\theta} X_n Y_i d_{ni}^{-\theta} (p_n \Omega_i)^{\theta}$$

This is the **Gravity Equation** with bilateral resistance  $d_{ni}$  and multilateral resistance terms  $p_n$  (inward) and  $\Omega_i$  (outward)

# Trade and Geography

- From (5) we also get that country  $i$ 's share in country  $n$ 's expenditures normalized by its own share is

$$S_{ni} \equiv \frac{X_{ni}/X_n}{X_{ii}/X_i} = \frac{\Phi_i}{\Phi_n} d_{ni}^{-\theta} = \left( \frac{p_i d_{ni}}{p_n} \right)^{-\theta}$$

- ▶ This shows the importance of trade costs and comparative advantage in determining trade volumes
- ▶ Note: no trade barriers  $\Rightarrow S_{ni} = 1$



# Trade and Geography (2)

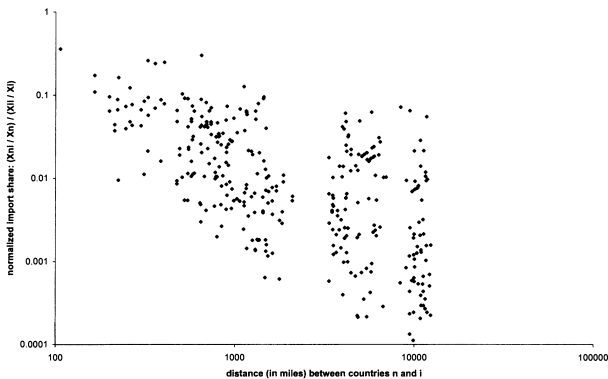


FIGURE 1.—Trade and geography.

# Gains from Trade

- From (5), we know that

$$\pi_{nn} = \frac{X_{nn}}{X_n} = \frac{T_n w_n^{-\theta}}{\Phi_n}$$

- We also know that  $p_n = \gamma \Phi_n^{-\theta}$  so

$$\omega_n \equiv w_n/p_n = \gamma^{-1} T_n^{1/\theta} \pi_{nn}^{-1/\theta}$$

- Under autarky, we have  $\omega_n^A = \gamma^{-1} T_n^{1/\theta}$ , hence the **gains from trade** are given by

$$GT_n \equiv \omega_n/\omega_n^A = \pi_{nn}^{-1/\theta}$$

Trade elasticity  $\theta$  and share of expenditure on domestic goods  $\pi_{nn}$  are sufficient statistics to compute GT

## Gains from Trade (2)

- A typical value for  $\pi_{nn}$  (manufacturing) is 0.7. With  $\theta = 5 \Rightarrow GT_n = 0.7^{-1/5} = 1.074$  or 7.4% gains
- Belgium:  $\pi_{nn} = 0.2 \Rightarrow GT_n = 0.2^{-1/5} = 1.38$  or 38% gains

# Outline of the Lecture

- 1 Introduction
- 2 The Two-Sector Model
- 3 Continuum of Goods: Dornbusch, Fischer and Samuelson (1977)
- 4 Many countries: Eaton and Kortum (2002)
- 5 Weak points of the Ricardian model**

# Assumed technological differences

- Technological differences (labor input coefficients) are simply assumed → How do they persist when technology is a recipe which anyone can use?
- Possible answer: No; some other skills are required in using recipes. Some countries can't organize production, make and enforce contracts etc.
- Another possible answer: Labor productivity differences are due to different amounts of other factors, especially capital.
  - ▶ Consider correlation between exports and relative productivities across industries in bilateral US-China trade. If US has more capital relative to labor, US labor productivity could be higher in all industries, but relatively more so in capital-intensive ones (agriculture!) Then the correlation actually supports quite a different theory: Comparative advantage governed by relative factor endowments.

# No distributive conflict

- With just one factor mobile across sectors, there is just one wage  
→ No distributive conflict.
- But such conflict is an important aspect of reality
- Other special features:
  - ▶ PPF is straight line
  - ▶ Supply responds suddenly at one critical relative price

# Next lecture...

- Better approach to reality needs some intermediate settings and combinations:
  - ▶ Multiple factors with different degrees of mobility across sectors  
If one US state is hit with a regional shock, unemployment rate falls back to national level within 6 years. In comparison, capital depreciates over 15-20 years, and structures over 30-50 years. (But there may be easier shift of some kinds of workers, equipment, structures across uses.)
  - ▶ Regular bowed-out PPF; gradual supply response.
- The next two models depict such situations
  - ▶ Ricardo-Viner: Capital specific to sectors, labor mobile across sectors
  - ▶ Heckscher-Ohlin: Both capital and labor mobile across sectors