ECON 39: Undergraduate International Trade
Specific Factors Examples
Professor Treb Allen

1. Consider a country (the U.S.) and two goods (footballs and soccer balls). Suppose that the country is inhabited by L_{US} workers, F_{US} units of a specific factor used in the production of footballs and S_{US} units of a specific factor used in the production of soccer balls. Suppose that the total quantity of soccer balls that can be produced is determined by the production function $Q_{US}^{SB} = (S_{US})^{\frac{1}{2}} \left(L_{US}^{SB}\right)^{\frac{1}{2}}$, where L_{US}^{SB} is the labor allocated to the production of soccer balls. Similarly, suppose that the total quantity of footballs that can be produced is determined by the production function $Q_{US}^{FB} = (F_{US})^{\frac{1}{2}} \left(L_{US}^{FB}\right)^{\frac{1}{2}}$, where L_{US}^{FB} is the labor allocated to the production of footballs.

   (a) Find an equation that depends only on exogenous model parameters for the Production Possibility Frontier in the U.S.

   - *Start with the production function for footballs (or soccer balls, it doesn't matter):*

$$\begin{aligned}
Q_{US}^{FB} &= (F_{US})^{\frac{1}{2}} \left(L_{US}^{FB}\right)^{\frac{1}{2}} \iff \\
Q_{US}^{FB} &= (F_{US})^{\frac{1}{2}} \left(\bar{L}_{US} - L_{US}^{SB}\right)^{\frac{1}{2}} \iff \\
\left(Q_{US}^{FB}\right)^2 &= F_{US} \left(\bar{L}_{US} - L_{US}^{SB}\right) \iff \\
L_{US}^{SB} &= \bar{L}_{US} - \frac{\left(Q_{US}^{FB}\right)^2}{F_{US}}
\end{aligned}$$

   Then substitute this into the production function for soccer balls:

$$\begin{aligned}
Q_{US}^{SB} &= (S_{US})^{\frac{1}{2}} \left(L_{US}^{SB}\right)^{\frac{1}{2}} \iff \\
Q_{US}^{SB} &= (S_{US})^{\frac{1}{2}} \left(\bar{L}_{US} - \frac{\left(Q_{US}^{FB}\right)^2}{F_{US}}\right)^{\frac{1}{2}}
\end{aligned}$$

   (b) Suppose that the representative agent in the U.S. has preferences $U = \min\left\{C_{US}^{SB}, C_{US}^{FB}\right\}$, where C_{US}^{SB} and C_{US}^{FB} are the quantity consumed of soccer balls and footballs, respectively. Find an equation for the equilibrium autarkic relative price of footballs to soccer balls (i.e. $\frac{p_{US}^{FB}}{p_{US}^{SB}}$) that depends only on exogenous model parameters.

   - *Because the representative agent wants to consume both goods, we need that both goods are produced. From the first equilibrium condition, this requires that workers are indifferent between producing in both sectors:*

$$\begin{aligned}
p_{US}^{FB} \times \frac{\partial Q^{FB}}{\partial L_{US}^{FB}} &= p_{US}^{SB} \times \frac{\partial Q^{SB}}{\partial L_{US}^{SB}} \iff \\
p_{US}^{FB} \times \frac{1}{2} \times F_{US}^{\frac{1}{2}} \times \left(L_{US}^{FB}\right)^{-\frac{1}{2}} &= p_{US}^{SB} \times \frac{1}{2} \times S_{US}^{\frac{1}{2}} \times \left(L_{US}^{SB}\right)^{-\frac{1}{2}} \iff \\
\frac{p_{US}^{FB}}{p_{US}^{SB}} &= \frac{S_{US}^{\frac{1}{2}} \times \left(L_{US}^{SB}\right)^{-\frac{1}{2}}}{F_{US}^{\frac{1}{2}} \times \left(L_{US}^{FB}\right)^{-\frac{1}{2}}} \iff \\
\frac{p_{US}^{FB}}{p_{US}^{SB}} &= \left(\left(\frac{S_{US}}{F_{US}}\right) \times \left(\frac{L_{US}^{FB}}{L_{US}^{SB}}\right)\right)^{\frac{1}{2}}. \qquad (1)
\end{aligned}$$

   From the second equilibrium condition (utility maximization), we know that:

$$C_{US}^{SB} = C_{US}^{FB}.$$

From the third equilibrium condition (production = consumption), we know that:

$$
\begin{aligned}
Q_{US}^{SB} &= Q_{US}^{FB} \iff \\
(S_{US})^{\frac{1}{2}} \left(L_{US}^{SB}\right)^{\frac{1}{2}} &= (F_{US})^{\frac{1}{2}} \left(L_{US}^{FB}\right)^{\frac{1}{2}} \iff \\
\frac{S_{US}}{F_{US}} &= \frac{L_{US}^{FB}}{L_{US}^{SB}}
\end{aligned}
\tag{2}
$$

Substituting equation (2) into equation (1) yields:

$$
\begin{aligned}
\frac{p_{US}^{FB}}{p_{US}^{SB}} &= \left(\left(\frac{S_{US}}{F_{US}}\right) \times \left(\frac{L_{US}^{FB}}{L_{US}^{SB}}\right)\right)^{\frac{1}{2}} \iff \\
\frac{p_{US}^{FB}}{p_{US}^{SB}} &= \left(\left(\frac{S_{US}}{F_{US}}\right) \times \left(\frac{S_{US}}{F_{US}}\right)\right)^{\frac{1}{2}} \iff \\
\frac{p_{US}^{FB}}{p_{US}^{SB}} &= \frac{S_{US}}{F_{US}}.
\end{aligned}
$$

(c) Suppose now that there is another country (Mexico), where we assume that $\frac{S_{MEX}}{F_{MEX}} > \frac{S_{US}}{F_{US}}$. If the two countries began trading, how would the labor allocation across the sectors change compared to the autarkic equilibrium? Show your answer using figures. Who would lose from trade in the U.S.?

- *If $\frac{S_{MEX}}{F_{MEX}} > \frac{S_{US}}{F_{US}}$ then $\frac{p_{MEX}^{FB}}{p_{MEX}^{SB}} > \frac{p_{US}^{FB}}{p_{US}^{SB}}$ from the previous question, hence we would expect that the relative price of footballs in the U.S. would rise with trade. This would cause the U.S. to produce more footballs, requiring labor to move from the soccer ball sector to the football sector. The losers would be the owners of the specific factor for soccer balls. The figure is the standard PPF figure (with Leontief indifference curves) showing the autarkic equilibrium, the new production in the U.S. and the new consumption in the U.S.*

(d) Suppose that in anticipation of losing from trade, those that would lose from trade in the U.S. (that you identified in part (c)) lobby the government to impose a domestic tax (or subsidy) on footballs so that the equilibrium autarkic relative price with the subsidy is $\frac{\tilde{p}_{US}^{FB}}{\tilde{p}_{US}^{SB}} = \tau \frac{p_{US}^{FB}}{p_{US}^{SB}}$, where $\tau > 1$ indicates that the government has imposed a tax on footballs that has made footballs relatively more expensive and $\tau < 1$ indicates that the government has imposed a subsidy on footballs that makes the footballs relatively less expensive. What is the smallest tax or subsidy (i.e the τ closest to one) such that the losers identified in part (c) do not lose from trade?

- *We know that the producer of the good that is imported after trade loses out. We also know that what good is imported depends is whichever good becomes less expensive with trade. Finally, we know that the equilibrium world price is somewhere between the autarkic prices in each one of the countries. Hence, if the government taxed footballs so that soccer balls did not become less expensive after trade, then the owners of the soccer balls would not be hurt by trade. Since the autarkic relative price of footballs in the U.S. is $\frac{p_{US}^{FB}}{p_{US}^{SB}} = \frac{S_{US}}{F_{US}}$ and in Mexico is $\frac{p_{MEX}^{FB}}{p_{MEX}^{SB}} = \frac{S_{MEX}}{F_{MEX}}$, the goal would be to choose τ such that:*

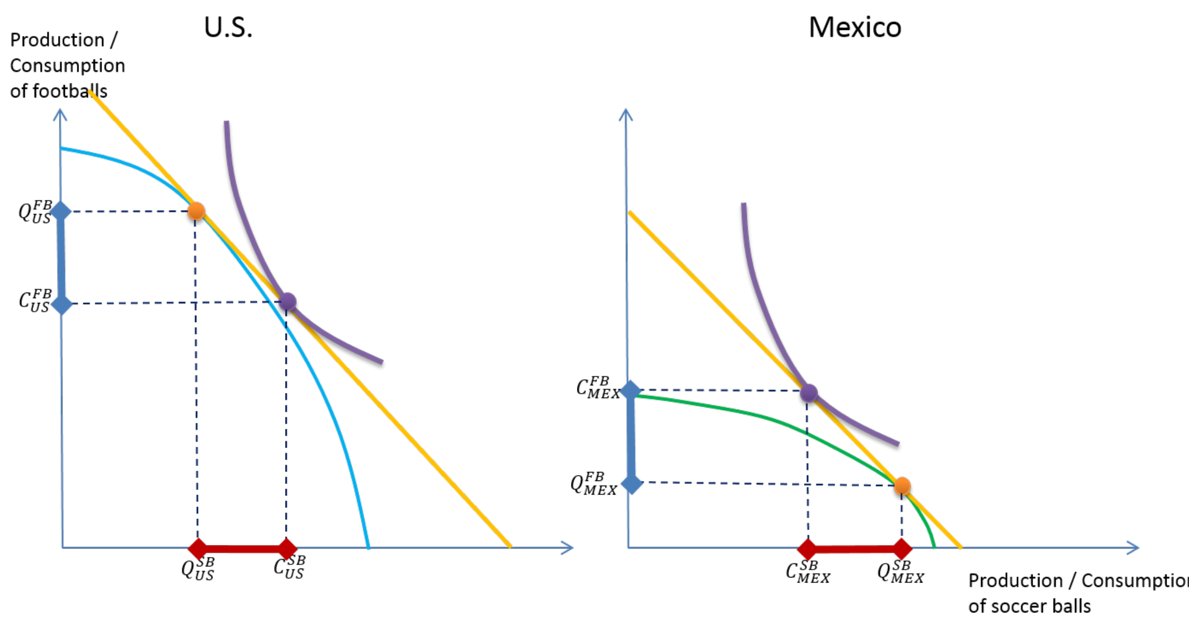
$$
\begin{aligned}
\frac{\tilde{p}_{US}^{FB}}{\tilde{p}_{US}^{SB}} &= \frac{p_{MEX}^{FB}}{p_{MEX}^{SB}} \iff \tau \frac{p_{US}^{FB}}{p_{US}^{SB}} \\
\tau \frac{p_{US}^{FB}}{p_{US}^{SB}} &= \frac{p_{MEX}^{FB}}{p_{MEX}^{SB}} \iff \\
\tau \left(\frac{S_{US}}{F_{US}}\right) &= \left(\frac{S_{MEX}}{F_{MEX}}\right) \iff \\
\tau &= \left(\frac{S_{MEX}}{F_{MEX}}\right) / \left(\frac{S_{US}}{F_{US}}\right) > 1
\end{aligned}
$$

so that τ would be a tax on footballs.

2. Consider a world with two countries and two goods. As in class, let us call the countries U.S. and Mexico and the goods footballs and soccer balls. In each country i, suppose that footballs are produced combining labor L_i^{FB} and a football specific factor F_i, soccer balls are produced combining labor L_i^{SB} and a soccer ball specific factor S_i. Assume that the representative agent in country i has preferences $U_i = \min\left\{C_i^{FB}, C_i^{SB}\right\}$, where C_i^{FB} is the consumption of footballs and C_i^{SB} is the consumption of soccer balls. In each country i, assume that the production function for footballs is $Q_i^{FB} = \left(L_i^{FB}\right)^{\frac{1}{2}} (F_i)^{\frac{1}{2}}$ and the production function for soccer balls is $Q_i^{SB} = \left(L_i^{SB}\right)^{\frac{1}{2}} (S_i)^{\frac{1}{2}}$. Finally, assume that both countries have identical populations (i.e. $L_{US} = L_{MEX}$), identical football specific factors (i.e. $F_{US} = F_{MEX}$) and the only difference between the two countries is that Mexico has more of the soccer ball specific factor (i.e. $S_{US} < S_{MEX}$).

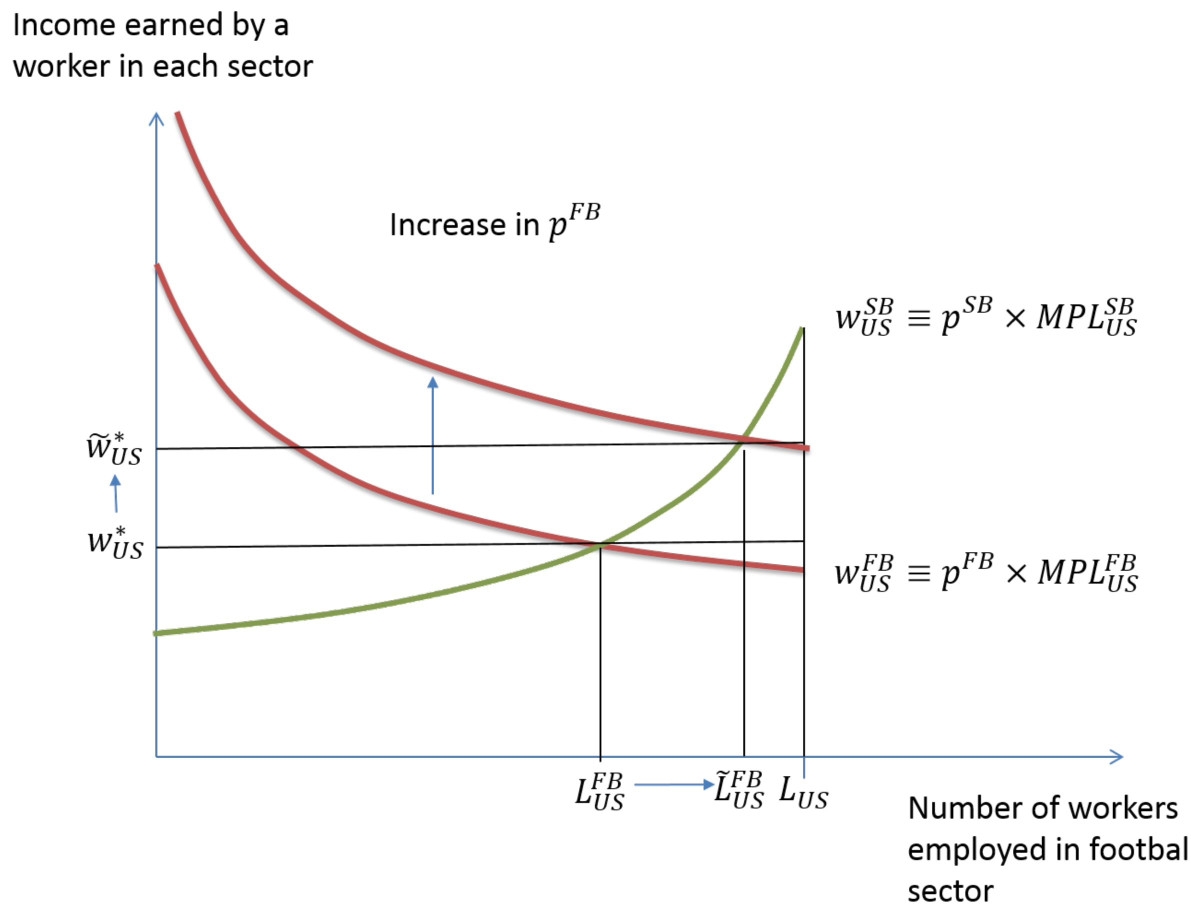
   (a) Draw the production possibility frontiers of both countries and show the equilibrium production and consumption of both countries in free trade. How can we see from your figure that the markets clear?

   - *This is basically straight from the lecture slide reproduced below. The only differences are as follows: (1) I have specified a Leontief utility function, so that the purple lines below should be kinked with the kink on the forty-five degree line; and (2) the PPF should have the same y intercept in both figures (since both countries can produce an equal amount of footballs) but Mexico should have a larger x intercept (since Mexico has a greater endowment of the soccer balls). The easiest way to show that the markets clear is to argue that the triangle in both figures created by the blue segment, the red segment, and the CPF are the same. (It's fine if they are not exactly the same on picture).*



   (b) Suppose that Mexico's endowment of the soccer ball specific factor increases. How would the world price change? Who in the U.S. would benefit and who would lose from this change? Explain your answer using figures.

   - *An increase in the endowment of Mexico's soccer ball specific factor should shift its PPF to the right (leaving the y-axis unchanged). This will increase the relative world price of footballs. In the U.S., this price change would benefit the owners of the football specific factor, hurt the owners of the soccer ball specific factor, and have an ambiguous effect on workers. The correct answer should include both the change in the PPF figure as well as a figure showing the returns to the difference factors in the U.S. (see below).*



(c) Suppose now that we have the following actual numbers: $L_{US} = L_{MEX} = 2$, $F_{US} = F_{MEX} = 1$, $S_{US} = 1$ and $S_{MEX} = 2$. What is the equilibrium quantity produced and consumed in each location as a function of the world price? What is the equation that (implicitly) defines the world price?

- *From producer maximization, we have that the wage must be equalized across the two sectors in both countries:*

$$p^{FB} MPL_i^{FB} = p^{SB} MPL_i^{SB}$$

for all $i \in \{US, MEX\}$. Taking partial derivatives of the production functions gives us the marginal product of labor in each country and sector: $MPL_{US}^{FB} = MPL_{MEX}^{FB} = \frac{1}{2}\left(L_{US}^{FB}\right)^{-\frac{1}{2}}$, $MPL_{US}^{SB} = \frac{1}{2}\left(L_{US}^{SB}\right)^{-\frac{1}{2}}$, and $MPL_{MEX}^{SB} = \frac{1}{2}\left(L_{MEX}^{SB}\right)^{-\frac{1}{2}} 2^{\frac{1}{2}}$. Normalize $p^{SB} = 1$ and define $p \equiv \frac{p^{FB}}{p^{SB}}$. Then we can use the wage equalization and the equations for the MPL in the U.S. to yield:

$$p = \frac{MPL_{US}^{SB}}{MPL_{US}^{FB}} \iff$$
$$p = \frac{\frac{1}{2}\left(L_{US}^{SB}\right)^{-\frac{1}{2}}}{\frac{1}{2}\left(L_{US}^{FB}\right)^{-\frac{1}{2}}} \iff$$
$$L_{US}^{FB} = p^2 L_{US}^{SB}$$

We can then combine this result with the labor market clearing condition that $L_{US}^{FB} + L_{US}^{SB} = 2$

to solve for the equilibrium distribution of labor across the two sectors in the U.S.:

$$\begin{aligned} 2 - L_{US}^{SB} &= p^2 L_{US}^{SB} \iff \\ L_{US}^{SB} &= \frac{2}{1+p^2} \end{aligned}$$

and hence:

$$L_{US}^{FB} = \frac{2p^2}{1+p^2} L_{US}^{SB}$$

We can do a similar thing for Mexico. We start by using the profit maximization (wage equalization) equation:

$$\begin{aligned} p &= \frac{MPL_{MEX}^{SB}}{MPL_{MEX}^{FB}} \iff \\ p &= \frac{\frac{1}{2}\left(L_{MEX}^{SB}\right)^{-\frac{1}{2}} 2^{\frac{1}{2}}}{\frac{1}{2}\left(L_{MEX}^{FB}\right)^{-\frac{1}{2}}} \iff \\ L_{MEX}^{FB} &= \frac{p^2}{2} L_{MEX}^{SB} \end{aligned}$$

and then combine the result with labor market clearing:

$$\begin{aligned} 2 - L_{MEX}^{SB} &= \frac{p^2}{2} L_{MEX}^{SB} \iff \\ L_{MEX}^{SB} &= \frac{4}{2+p^2} \end{aligned}$$

and:

$$L_{MEX}^{FB} = \frac{2p^2}{2+p^2}.$$

Since we now know the labor allocated to each sector (as a function of the world relative price), we can then use the production function to figure out the quantity produced of each good in each country:

$$\begin{aligned} Q_{US}^{FB} &= \left(\frac{2p^2}{1+p^2}\right)^{\frac{1}{2}} \\ Q_{US}^{SB} &= \left(\frac{2}{1+p^2}\right)^{\frac{1}{2}} \\ Q_{MEX}^{FB} &= \left(\frac{2p^2}{2+p^2}\right)^{\frac{1}{2}} \\ Q_{MEX}^{SB} &= \left(\frac{2}{2+p^2}\right)^{\frac{1}{2}} 2 \end{aligned}$$

We can then figure out the income in each country:

$$\begin{aligned} Y_{US} &= pQ_{US}^{FB} + Q_{US}^{SB} \iff \\ Y_{US} &= p\left(\frac{2p^2}{1+p^2}\right)^{\frac{1}{2}} + \left(\frac{2}{1+p^2}\right)^{\frac{1}{2}} \iff \\ Y_{US} &= \left(2\left(1+p^2\right)\right)^{\frac{1}{2}} \end{aligned}$$

and:

$$
\begin{aligned}
Y_{MEX} &= pQ^{FB}_{MEX} + Q^{SB}_{MEX} \iff \\
Y_{MEX} &= p\left(\left(\frac{2p^2}{2+p^2}\right)^{\frac{1}{2}}\right) + \left(\left(\frac{4}{2+p^2}\right)^{\frac{1}{2}} 2^{\frac{1}{2}}\right) \iff \\
Y_{MEX} &= \left(2\left(2+p^2\right)\right)^{\frac{1}{2}}
\end{aligned}
$$

Given preferences, we know that $C_i^{FB} = C_i^{SB}$. Combined with the budget constraint $pC_i^{FB} + C_i^{SB} = Y_i$, this implies that equilibrium consumption is:

$$C_i^{FB} = \frac{Y_i}{1+p} = C_i^{SB}.$$

Combined with the income equation, we then have:

$$C^{FB}_{US} = C^{SB}_{US} = \frac{\left(2\left(1+p^2\right)\right)^{\frac{1}{2}}}{1+p}$$

and:

$$C^{FB}_{MEX} = C^{SB}_{MEX} = \frac{\left(2\left(2+p^2\right)\right)^{\frac{1}{2}}}{1+p}$$

Finally, we can solve for the prices but relying on market clearing in one of the sectors:

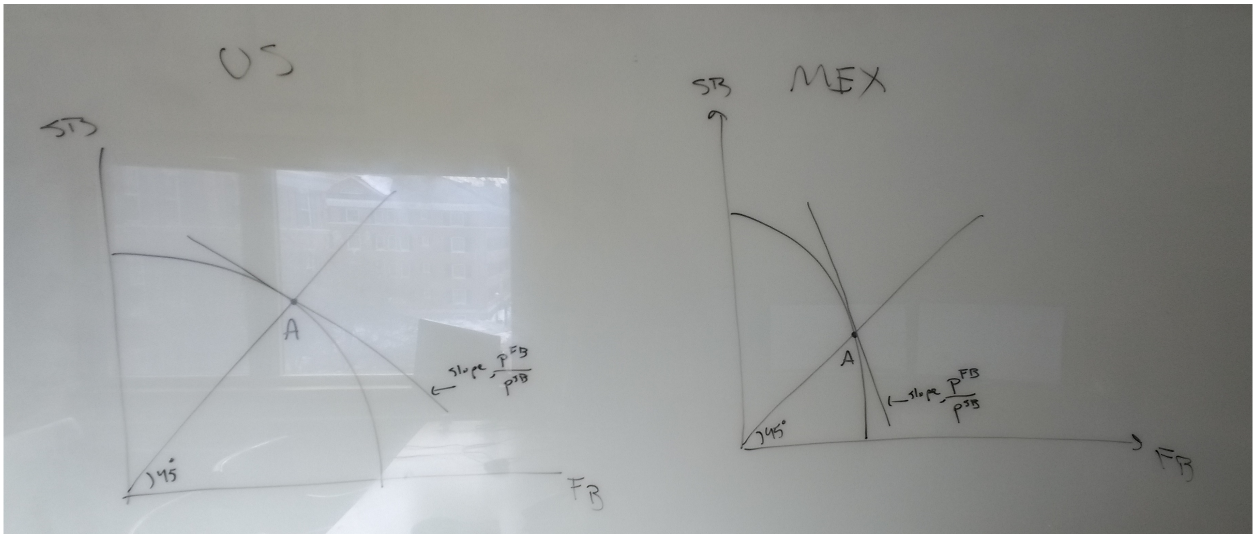
$$
\begin{aligned}
Q^{FB}_{US} + Q^{FB}_{MEX} &= C^{FB}_{US} + C^{FB}_{MEX} \iff \\
\left(\frac{2p^2}{1+p^2}\right)^{\frac{1}{2}} + \left(\frac{2p^2}{2+p^2}\right)^{\frac{1}{2}} &= \frac{\left(2\left(1+p^2\right)\right)^{\frac{1}{2}}}{1+p} + \frac{\left(2\left(2+p^2\right)\right)^{\frac{1}{2}}}{1+p} \iff \\
p\left(\left(1+p^2\right)^{-\frac{1}{2}} + \left(2+p^2\right)^{-\frac{1}{2}}\right) &= \left(\frac{1}{1+p}\right)\left(\left(1+p^2\right)^{\frac{1}{2}} + \left(2+p^2\right)^{\frac{1}{2}}\right)
\end{aligned}
$$

This is as far as you would have to go.

3. The Specific Factors Model (40 points total). Consider a world with two countries and two goods. As in class, let us call the countries U.S. and Mexico and the goods footballs and soccer balls. Suppose that the U.S. is endowed with 16 units of labor (L_{US}), 16 units of the specific factor used in the production of footballs (F_{US}) and 16 units of the specific factor used in the production of soccer balls (S_{US}). Suppose that Mexico is endowed with 16 units of labor (L_{MEX}), 4 units of the specific factor used in the production of footballs (F_{MEX}) and 16 units of the specific factor used in the production of soccer balls (S_{MEX}). The representative agent in each country $i \in \{US, MEX\}$ has preferences $U_i = \min\left\{C_i^{FB}, C_i^{SB}\right\}$ where C_i^{FB} is the consumption in country i of footballs and C_i^{SB} is the consumption in country i of footballs. In each country $i \in \{US, MEX\}$, the production function of footballs is $Q_i^{FB} = \left(L_i^{FB}\right)^{\frac{1}{2}}\left(F_i\right)^{\frac{1}{2}}$, where L_i^{FB} is the labor allocated to the production of footballs in country i. Finally, in each country $i \in \{US, MEX\}$, the production function of soccer balls is $Q_i^{SB} = \left(L_i^{SB}\right)^{\frac{1}{2}}\left(S_i\right)^{\frac{1}{2}}$, where L_i^{SB} is the labor allocated to the production of soccer balls in country i.

   (a) Suppose that both the U.S. and Mexico are initially in autarky. Draw the production possibilities frontier for both countries (with footballs on the x-axis and soccer balls on the y-axis). Indicate the equilibrium autarkic production and consumption of both goods in both countries (no need to solve for the actual numbers). In which country is the autarky relative price of footballs to soccer balls higher? What is the economic intuition for this difference in prices?

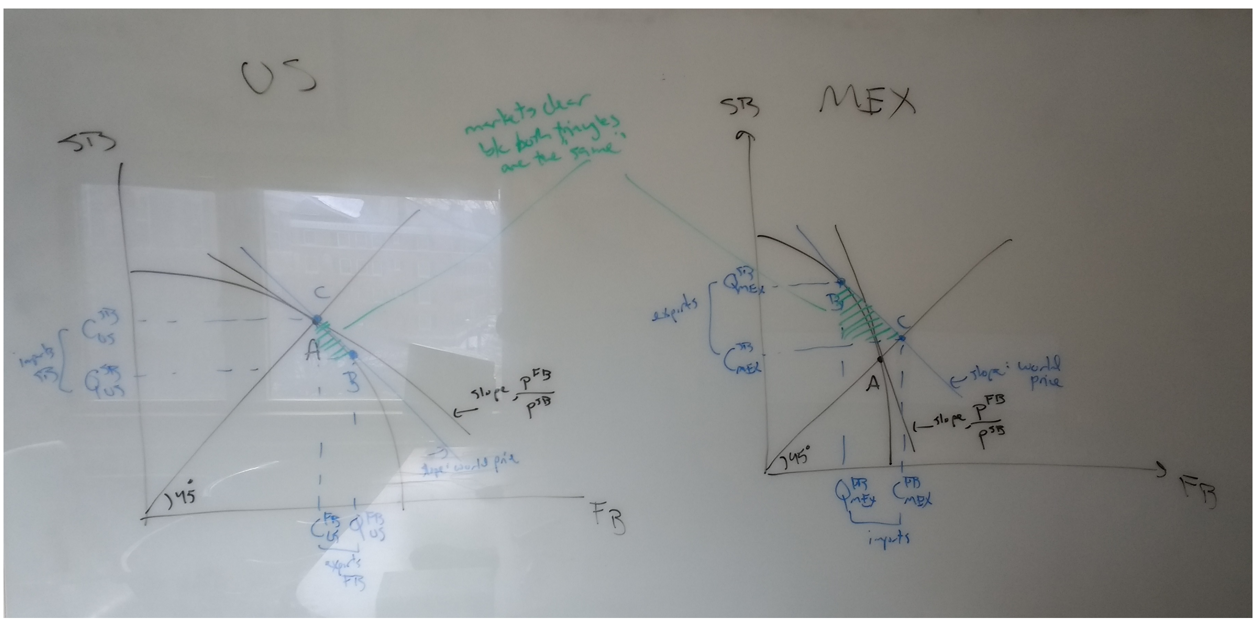
   - *See the attached figure:*



The autarkic relative price of footballs is higher in Mexico because Mexico has fewer of the football specific factor, so it cannot produce as many.

(b) Now suppose that the two countries open to trade. Draw a new figure showing the equilibrium production and consumption of both goods in both countries (again, no need to solve for the actual numbers). Which country exports what? How do we see on the figure that markets clear?

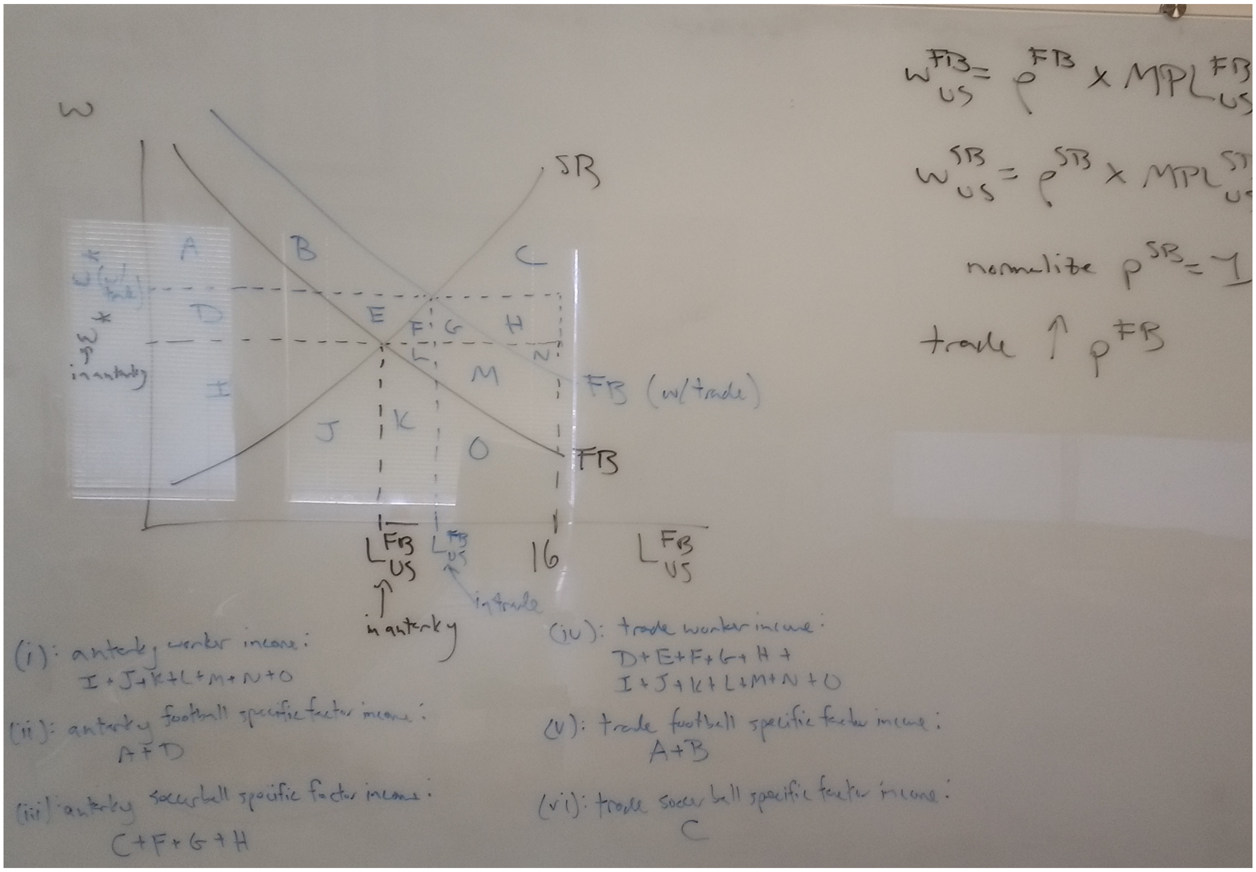
- *See the attached figure:*



The U.S. exports footballs and Mexico exports soccer balls. We see that the markets clear because the trade triangles are equal.

(c) Use a (new) figure to show how trade changed the income of each of the three factors in the U.S. (workers, football specific factor, soccer ball specific factor). Carefully label on the figure the following six things: (i) the original income of workers; (ii) the original income of the soccer ball specific factor; (iii) the original income of the football specific factor; (iv) the new income of the soccer ball specific factor; (v) the new income of the football specific factor; and (vi) the new income of workers.

- *See the attached figure.*



The key thing to note is that the relative price of footballs to soccer balls increased in the U.S. when it opens to trade. This increases the relative wage of workers and causes labor to move into the football sector, increasing the income of the football specific factor and decreasing the returns to the soccer ball specific factor. See the attached figure.

(d) With trade, are U.S. workers able to consume more or fewer soccer balls? Are they able to consume more or fewer footballs? What is the economic intuition?

- *Recall that we have* $w_{US} = p^{FB} \times MPL^{FB}_{US} = p^{SB} \times MPL^{SB}_{US}$. *The amount of footballs a worker can consume is* $\frac{w_{US}}{p^{FB}} = MPL^{FB}_{US}$*; conversely, the amount of soccer balls a worker can consume is* $\frac{w_{US}}{p^{SB}} = MPL^{SB}_{US}$. *From part (c), we see that opening up to trade caused more labor to move into the production of footballs. Because of diminishing returns to labor, this increases the marginal product of labor in the soccer ball sector and decreases the marginal product of labor in the football sector. As a result, workers can consume more soccer balls but fewer footballs.*