14.581 International Trade
— Lecture 14: Firm Heterogeneity (Theory) —

Firm Heterogeneity and Trade
What's wrong with previous theories?

- Nineties have seen a boom in the availability of micro-level data
- **Problem:** previous theories are at odds with (or cannot account for) many micro-level facts:
  1. Within a given industry, there is *firm-level heterogeneity*
  2. *Fixed costs matter in export* related decisions
  3. Within a given industry, more productive firms are more likely to export
  4. Trade liberalization leads to intra-industry reallocation across firms
  5. These reallocations are correlated with productivity and export status

Firm Heterogeneity and Trade
What does Melitz (2003) do about it?

- Melitz (2003) will develop a model featuring facts 1 and 2 that can explain facts 3, 4, and 5
- This is by far the most influential trade paper in the last 10 years
- **Two building blocks:**
  1. Krugman (1980): CES, IRS technology, monopolistic competition
  2. Hopenhayn (1992): equilibrium model of entry and exit
- From a normative point of view, Melitz (2003) may also provide "new" source of gains from trade if trade induces reallocation of labor from least to most productive firms (more on that later)

- Like in Krugman (1980), representative agent has CES preferences:

$$U = \left[\int_{\omega \in \Omega} q\left(\omega\right)^{\frac{\sigma-1}{\sigma}} d\omega \right]^{\frac{\sigma}{\sigma-1}}$$

where $\sigma > 1$ is the elasticity of substitution

- Consumption and expenditures for each variety are given by

$$q\left(\omega\right) = Q \left[\frac{p\left(\omega\right)}{P} \right]^{-\sigma} \qquad (1)$$

$$r\left(\omega\right) = R \left[\frac{p\left(\omega\right)}{P} \right]^{1-\sigma} \qquad (2)$$

where:

$$P \equiv \left[\int_{\omega \in \Omega} p\left(\omega\right)^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}}, \ R \equiv \int_{\omega \in \Omega} r\left(\omega\right), \text{ and } Q \equiv R/P$$

Melitz (2003)
Production

- Like in Krugman (1980), labor is the only factor of production
  - $L \equiv$ total endowment, $w = 1 \equiv$ wage
- Like in Krugman (1980), there are IRS in production

$$l = f + q/\varphi \tag{3}$$

- Like in Krugman (1980), monopolistic competition implies

$$p(\varphi) = \frac{1}{\rho\varphi} \tag{4}$$

- CES preferences with monopoly pricing, (2) and (4), imply

$$r(\varphi) = R(P\rho\varphi)^{\sigma-1} \tag{5}$$

- These two assumptions, (3) and (4), further imply

$$\pi(\varphi) \equiv r(\varphi) - l(\varphi) = \frac{r(\varphi)}{\sigma} - f$$

- **Comments:**

  1. Higher productivity φ in the model implies higher *measured* productivity

  $$\frac{r(\varphi)}{l(\varphi)} = \frac{1}{\rho}\left[1 - \frac{f}{l(\varphi)}\right]$$

  2. More productive firms produce more and earn higher revenues

  $$\frac{q(\varphi_1)}{q(\varphi_2)} = \left(\frac{\varphi_1}{\varphi_2}\right)^{\sigma} \text{ and } \frac{r(\varphi_1)}{r(\varphi_2)} = \left(\frac{\varphi_1}{\varphi_2}\right)^{\sigma-1}$$

  3. φ can also be interpreted in terms of quality. This is isomorphic to a change in units of account, which would affect prices, but nothing else

- By definition, the CES price index is given by

$$P = \left[\int_{\omega \in \Omega} p(\omega)^{1-\sigma} \, d\omega \right]^{\frac{1}{1-\sigma}}$$

- Since all firms with productivity φ charge the same price $p(\varphi)$, we can rearrange CES price index as

$$P = \left[\int_0^{+\infty} p(\varphi)^{1-\sigma} \, M\mu(\varphi) \, d\varphi \right]^{\frac{1}{1-\sigma}}$$

where:

- $M \equiv$ mass of (surviving) firms in equilibrium
- $\mu(\varphi) \equiv$ (conditional) pdf of firm-productivity levels in equilibrium

- Combining the previous expression with monopoly pricing (4), we get

$$P = M^{\frac{1}{1-\sigma}} / \rho \widetilde{\varphi}$$

where

$$\widetilde{\varphi} \equiv \left[\int_0^{+\infty} \varphi^{\sigma-1} \mu(\varphi) \, d\varphi \right]^{\frac{1}{\sigma-1}}$$

- One can do the same for all aggregate variables

$$R = M r(\widetilde{\varphi}), \ \Pi = M \pi(\widetilde{\varphi}), \ Q = M^{\frac{\sigma}{\sigma-1}} q(\widetilde{\varphi})$$

- **Comments:**

  1. These are the same aggregate variables we would get in a Krugman (1980) model with a mass M of identical firms with productivity $\widetilde{\varphi}$
  2. But productivity $\widetilde{\varphi}$ now is an *endogenous* variable which may respond to changes in trade cost, leading to *aggregate* productivity changes

- In order to determine how $\mu(\varphi)$ and $\widetilde{\varphi}$ get determine in equilibrium, one needs to specify the entry and exit of firms
- Timing is similar to Hopenhayn (1992):
  1. There is a large pool of identical potential entrants deciding whether to become active or not
  2. Firms deciding to become active pay a fixed cost of entry $f_e > 0$ and get a productivity draw φ from a cdf G
  3. After observing their productivity draws, firms decide whether to remain active or not
  4. Firms deciding to remain active exit with a constant probability δ

Melitz (2003)
Aside: Pareto distributions

- In variations and extensions of Melitz (2003), most common assumption on the productivity distribution G is Pareto:

$$G\left(\varphi\right) \equiv 1 - \left(\frac{\underline{\varphi}}{\varphi}\right)^{\theta} \text{ for } \varphi \geq \underline{\varphi}$$

$$g\left(\varphi\right) \equiv \theta\underline{\varphi}^{\theta}\varphi^{-\theta-1} \text{ for } \varphi \geq \underline{\varphi}$$

- Pareto distributions have two advantages:
  1. Combined with CES, it delivers closed form solutions
  2. Distribution of firm sizes remains Pareto, which is not a bad approximation empirically (at least for the upper tail)

- But like CES, Pareto distributions will have very strong implications for equilibrium properties (more on this later)

- In a stationary equilibrium, a firm either exits immediately or produces and earns the same profits $\pi(\varphi)$ in each period

- In the absence of time discounting, expected value of a firm with productivity φ is

$$v(\varphi) = \max\left\{0, \sum_{t=0}^{+\infty} (1-\delta)^t \pi(\varphi)\right\} = \max\left\{0, \frac{\pi(\varphi)}{\delta}\right\}$$

- There exists a unique productivity level $\varphi^* \equiv \inf\left\{\varphi \geq 0 : \frac{\pi(\varphi)}{\delta} > 0\right\}$

- Productivity cutoff φ^* can also be written as:

$$\pi(\varphi^*) = 0$$

- Once we know φ^*, we can compute the pdf of firm-productivity levels

$$\mu\left(\varphi\right) = \begin{cases} \frac{g(\varphi)}{1-G(\varphi^*)} & \text{if } \varphi \geq \varphi^* \\ 0 & \text{if } \varphi < \varphi^* \end{cases}$$

- Accordingly, the measure of aggregate productivity is given by

$$\widetilde{\varphi}\left(\varphi^*\right) = \left[\frac{1}{1-G\left(\varphi^*\right)}\int_{\varphi^*}^{+\infty}\varphi^{\sigma-1}g\left(\varphi\right)d\varphi\right]^{\frac{1}{\sigma-1}}$$

- Let $\overline{\pi} \equiv \Pi/M$ denote average profits per period for surviving firms
- Free entry requires the total expected value of profits to be equal to the fixed cost of entry

$$0 \times G\left(\varphi^*\right) + \frac{\overline{\pi}}{\delta} \times \left[1 - G\left(\varphi^*\right)\right] = f_e$$

- **Free Entry Condition (FE):**

$$\overline{\pi} = \frac{\delta f_e}{1 - G\left(\varphi^*\right)} \tag{6}$$

- Holding constant the fixed costs of entry, if firms are less likely to survive, they need to be compensated by higher average profits

Melitz (2003)
Zero cutoff profit condition

- Definition of φ^* can be rearranged to obtain a second relationship between φ^* and $\overline{\pi}$
- By definition of $\overline{\pi}$, we know that

$$\overline{\pi} = \Pi/M = \pi\left[\widetilde{\varphi}\left(\varphi^*\right)\right] \Leftrightarrow \overline{\pi} = f\left[\frac{r\left[\widetilde{\varphi}\left(\varphi^*\right)\right]}{\sigma f} - 1\right]$$

- By definition of φ^*, we know that
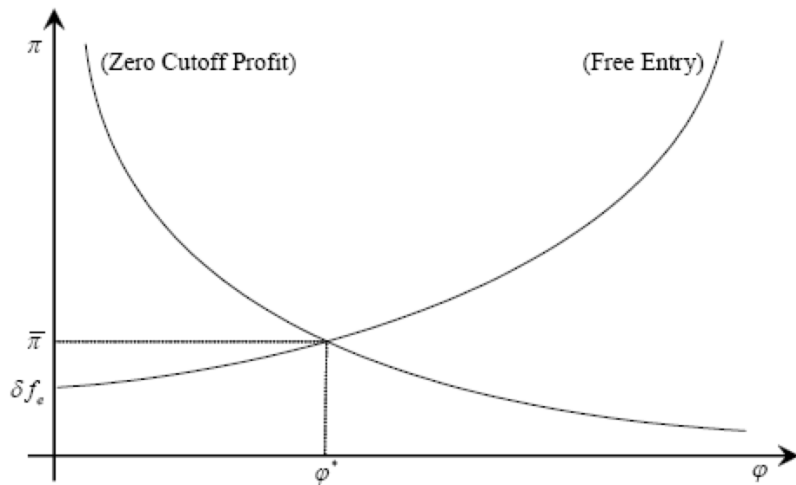
$$\pi\left(\varphi^*\right) = 0 \Leftrightarrow r\left(\varphi^*\right) = \sigma f$$

- Two previous expressions imply **ZCP condition:**

$$\overline{\pi} = f\left[\frac{r\left[\widetilde{\varphi}\left(\varphi^*\right)\right]}{r\left(\varphi^*\right)} - 1\right] = f\left[\left(\frac{\widetilde{\varphi}\left(\varphi^*\right)}{\varphi^*}\right)^{\sigma-1} - 1\right] \tag{7}$$

- FE and ZCP, (6) and (7), determine a unique $(\overline{\pi}, \varphi^*)$, and therefore $\widetilde{\varphi}$, independently of country size L

  - the only variable left to compute is M, which can be done using free entry and labor market clearing as in Krugman (1980)

- However, ZCP is not necessarily downward sloping:

  - it depends on whether $\widetilde{\varphi}$ or φ^* increases relatively faster
  - ZCP is downward sloping for most common distributions

- In the Pareto case, it is easy to check that $\widetilde{\varphi}/\varphi^*$ is constant:

  - So ZCP is flat and average profits are independent of φ^*

- Free entry and labor market clearing imply

$$L = R = \overline{r}M$$

- We can rearrange the previous expression

$$M = \frac{L}{\overline{r}} = \frac{L}{\sigma(\overline{\pi} + f)}$$

- Like in Krugman (1980), welfare of a representative worker is given by

$$U = 1/P = M^{\frac{1}{\sigma-1}}\rho\widetilde{\varphi}$$

- Since $\widetilde{\varphi}$ and $\overline{\pi}$ are independent of L, growth in country size and costless trade will also have the same impact as in Krugman (1980):
  - welfare $\nearrow$ because of $\nearrow$ in total number of varieties in each country

Melitz (2003)
Open economy model

- In the absence of trade costs, we have seen trade integration does not lead to any intra-industry reallocation ($\widetilde{\varphi}$ is fixed)

- In order to move away from such (counterfactual) predictions, Melitz (2003) introduces two types of trade costs:

    1. **Iceberg trade costs:** in order to sell 1 unit abroad, firms need to ship $\tau \geq 1$ units

    2. **Fixed exporting costs:** in order to export abroad, firms must incur an additional fixed cost f_{ex} (information, distribution, or regulation costs) *after learning their productivity φ*

- In addition, Melitz (2003) assumes that $c = 1, ..., n$ countries are symmetric so that $w_c = 1$ in all countries

- Monopoly pricing now implies

$$p_d\left(\varphi\right) = \frac{1}{\rho\varphi}, \ p_x\left(\varphi\right) = \frac{\tau}{\rho\varphi}$$

- Revenues in the domestic and export markets are

$$r_d\left(\varphi\right) = R_d\left[P_d\rho\varphi\right]^{\sigma-1}, \ r_x\left(\varphi\right) = \tau^{1-\sigma}R_x\left[P_x\rho\varphi\right]^{\sigma-1}$$

- Note that by symmetry, we must have

$$P_d = P_x = P \text{ and } R_d = R_x = R$$

- Let $f_x \equiv \delta f_{ex}$. Profits in the domestic and export markets are

$$\pi_d\left(\varphi\right) = \frac{r_d\left(\varphi\right)}{\sigma} - f, \ \pi_x\left(\varphi\right) = \frac{r_x\left(\varphi\right)}{\sigma} - f_x$$

- Expected value of a firm with productivity φ is

$$v\left(\varphi\right) = \max\left\{0, \sum_{t=0}^{+\infty}\left(1-\delta\right)^{t}\pi\left(\varphi\right)\right\} = \max\left\{0, \frac{\pi\left(\varphi\right)}{\delta}\right\}$$

- But total profits of are now given by

$$\pi\left(\varphi\right) = \pi_{d}\left(\varphi\right) + \max\left\{0, \pi_{x}\left(\varphi\right)\right\}$$

- Like in the closed economy, we let $\varphi^{*} \equiv \inf\left\{\varphi \geq 0 : \frac{\pi(\varphi)}{\delta} > 0\right\}$

- In addition, we let $\varphi_{x}^{*} \equiv \inf\left\{\varphi \geq \varphi^{*} : \frac{\pi_{x}(\varphi)}{\delta} > 0\right\}$ be the export cutoff

- In order to have both exporters and non-exporters in equilibrium, $\varphi_{x}^{*} > \varphi^{*}$, we assume that:

$$\tau^{\sigma-1}f_{x} > f$$
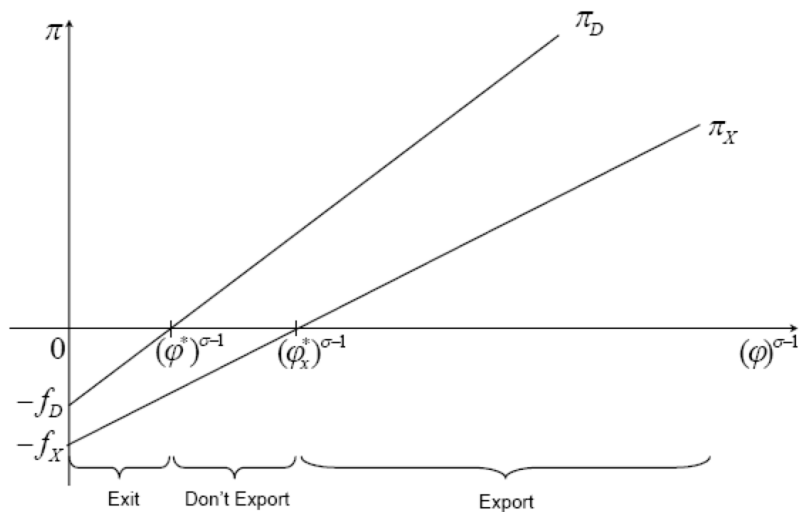
- In the model, more productive firms (higher φ) select into exports
- Empirically, this directly implies larger firms (higher $r(\varphi)$)
- **Question:** Does that also mean that firms with higher *measured* productivity select into exports?
- **Answer:**
    - Exporters are larger. This tends to raise their measured productivity
    - But exporters also pay additional fixed exporting costs. This tends to lower their measured productivity
- **Comments:**
    - Around φ_x^*, second negative effect must dominate
    - In the data, productivity advantage of exporters hold both unconditionally and after conditioning on size

- In the open economy, aggregate productivity is now given by

$$\widetilde{\varphi}_t = \left\{ \frac{1}{M_t} \left[M \widetilde{\varphi}^{\sigma-1} + n M_x \left(\widetilde{\varphi}_x / \tau \right)^{\sigma-1} \right] \right\}^{\frac{1}{\sigma-1}}$$

where:

- $M_t \equiv M + n M_x$ is the total number of varieties
- $\widetilde{\varphi} = \left[\frac{1}{1-G(\varphi^*)} \int_{\varphi^*}^{+\infty} \varphi^{\sigma-1} g(\varphi) \, d\varphi \right]^{\frac{1}{\sigma-1}}$ is the average productivity across all firms
- $\widetilde{\varphi}_x = \left[\frac{1}{1-G(\varphi_x^*)} \int_{\varphi_x^*}^{+\infty} \varphi^{\sigma-1} g(\varphi) \, d\varphi \right]^{\frac{1}{\sigma-1}}$ is the average productivity across all exporters

- Once we know $\widetilde{\varphi}_t$, we can still compute all aggregate variables as:

$$
\begin{aligned}
P &= M_t^{\frac{1}{1-\sigma}} / \rho \widetilde{\varphi}_t, \\
R &= M_t r\left(\widetilde{\varphi}_t\right), \\
\Pi &= M_t \pi\left(\widetilde{\varphi}_t\right), \\
Q &= M_t^{\frac{\sigma}{\sigma-1}} q\left(\widetilde{\varphi}_t\right)
\end{aligned}
$$

- **Comment:**
  - Like in the closed economy, there is a tight connection between welfare $(1/P)$ and average productivity $(\widetilde{\varphi}_t)$
  - But in the open economy, this connection heavily relies on symmetry: welfare depends on the productivity of *foreign*, not *domestic* exporters

- The condition for free entry is unchanged
- **Free Entry Condition (FE):**

$$\overline{\pi} = \frac{\delta f_e}{1 - G\left(\varphi^*\right)} \tag{8}$$

- The only difference is that average profits now depend on export profits as well

$$\overline{\pi} = \pi_d\left(\widetilde{\varphi}\right) + n p_x \pi_x\left(\widetilde{\varphi}_x\right)$$

where:

- $p_x = \frac{1 - G(\varphi_x^*)}{1 - G(\varphi^*)}$ is probability of exporting conditional on successful entry

- By definition of the cut off productivity levels, we know that

$$
\begin{aligned}
\pi_d \left(\varphi^* \right) &= 0 \ \Leftrightarrow \ r_d \left(\varphi^* \right) = \sigma f \\
\pi_x \left(\varphi_x^* \right) &= 0 \ \Leftrightarrow \ r_x \left(\varphi_x^* \right) = \sigma f_x
\end{aligned}
$$

- This implies

$$
\frac{r_x \left(\varphi_x^* \right)}{r_d \left(\varphi^* \right)} = \frac{f_x}{f} \Leftrightarrow \varphi_x^* = \varphi^* \tau \left(\frac{f_x}{f} \right)^{\frac{1}{\sigma-1}}
$$

- By rearranging $\overline{\pi}$ as a function of φ^*, we get new **ZCP condition**:

$$
\overline{\pi} = f \left[\left(\frac{\widetilde{\varphi} \left(\varphi^* \right)}{\varphi^*} \right)^{\sigma-1} - 1 \right] + n p_x f_x \left[\left(\frac{\widetilde{\varphi}_x \left(\varphi^* \right)}{\varphi_x^* \left(\varphi^* \right)} \right)^{\sigma-1} - 1 \right]
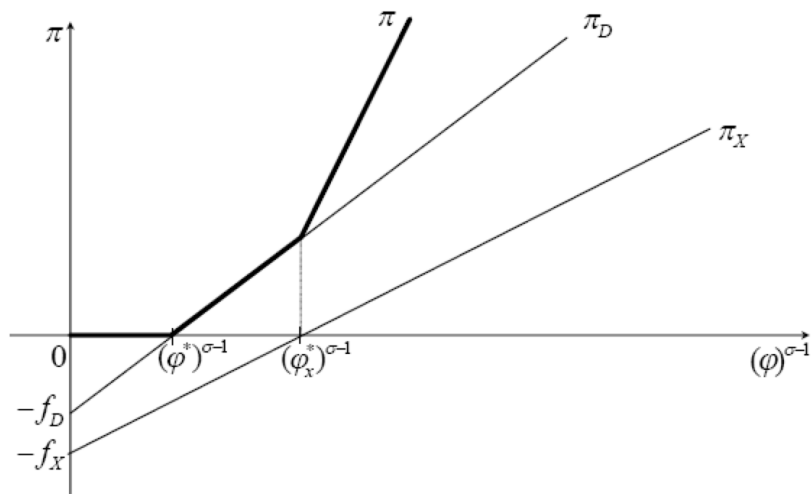$$

- In line with empirical evidence, exposure to trade forces the least productive firms to exit: $\varphi^* > \varphi_a^*$

- **Intuition:**
  - *For exporters:* Profits $\nearrow$ due to export opportunities, but $\searrow$ due to the entry of foreign firms in the domestic market ($P \searrow$)
  - *For non-exporters:* only the negative second effect is active

- **Comments:**
  - The $\nearrow$ in φ^* is not a new source of gains from trade. It's *because* there are gains from trade ($P \searrow$) that φ^* $\nearrow$increases
  - Welfare unambiguously $\nearrow$ though number of domestic varieties $\searrow$

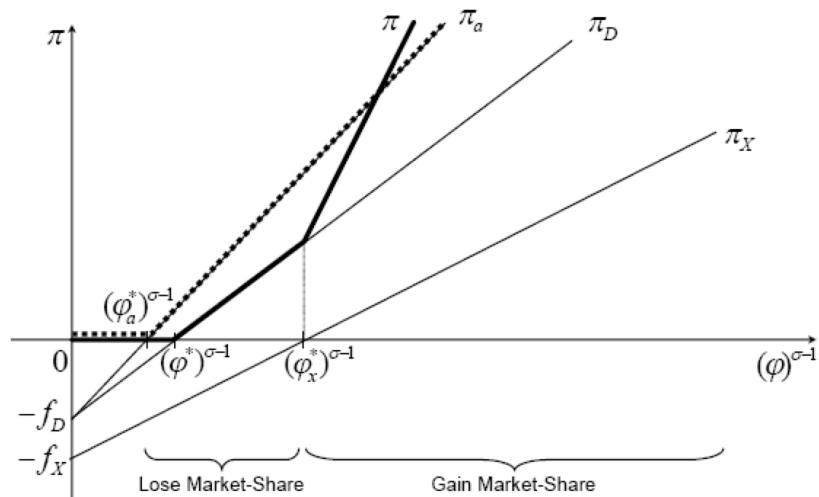$$M = \frac{R}{\bar{r}} = \frac{L}{\sigma \left(\bar{\pi} + f + p_x n f_x \right)} < M_a$$

Melitz (2003)
The Impact of Trade

Melitz (2003)
Other comparative static exercises

- Starting from autarky and moving to trade is theoretically standard, but not empirically appealing

- Melitz (2003) also considers:

  1. Increase in the number of trading partners n
  2. Decrease in iceberg trade costs τ
  3. Decrease in fixed exporting costs f_x

- Same qualitative insights in all scenarios:

  - Exit of least efficient firms
  - Reallocation of market shares from less from more productive firms
  - Welfare gains

- Melitz and Redding (2013) Handbook Chapter offers an excellent overview of various extensions of Melitz (2003)

Melitz (2003)
Back to the empirical evidence

- Consider the following statement: "In response to trade liberalization, measured productivity at the industry-level increases, whereas measured productivity at the firm-level does not change"
- Do you think that this is consistent with Melitz (2003)?
    - Measured productivity at the firm-level depends on q, which varies!
    - Measured productivity at the industry-level depends on total fixed costs paid, which may not (e.g. with Pareto distributions of productivity; see Burstein and Cravino 2014)
- What about the finding of Trefler (2004) that larger changes in Canadian tariffs leads to greater industry reallocations?
    - Segerstrom and Sogita (2014) show that with two initially symmetric sectors and two countries, the exact opposite must happen
    - Can you see why?