Macroeconomics B: EI060

Midterm exam

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April 9, 2025

1 General instructions

The exam consists of 4 questions. Section 2 has 3 short subquestions, and sections 3-4-5 require more thinking. The weight of each question in the grade is indicated, so you can allocate your time accordingly.

A good strategy is to first read through the questions, and then start with the easiest one before proceeding to the harder ones.

Short "bullet points" answers stressing the main points are fine, you don't have to write long paragraphs.

Best of luck!

2 Short questions (30 % of grade)

2.1 Effect of income shocks

Consider the model of a small economy with exogenous endowment, and with a single world good (hence no real exchange rate of terms-of-trade).

You take part in a roundtable when someone states that "if an economy sees its GDP increases, it is wealthier and thus should not save so much through a current account surplus". What is your assessment of this?

Answer: We saw in class that the current account reflects an intertemporal choice. A country saves (runs a surplus) if its endowment today is lower than its "permanent" endowment, that is the level of a constant endowment that would give the same net present value of current and future endowments that the country actually has.

If the current endowment is larger than the permanent one, the positive news is temporary and the country should save (run a current account surplus) to smooth consumption.

If the current endowment is equal to the permanent one, the positive news is permanent and today is not different from the future. The country should then not change its savings and current account.

If the current endowment is lower than the permanent one, the positive news is indicating even better news later one, so endowment is actually lower than in the future (even though both current and future endowments are now better than what we thought previously). The country should borrow (run a current account deficit) to smooth consumption.

2.2 Complete markets

Do complete international financial market imply that consumption levels are equalized across countries?

Answer: No. Complete markets imply that consumption moves in step, i.e. that relative consumption is constant. Relative consumption is however not equal to one, as a country with a larger average income, or a less volatile one, will have higher consumption.

Markets allow for the sharing of idiosyncratic risk, i.e. movements in a country's income that are uncorrelated with the incomes of other countries, but not of aggregate income risk which affects everybody.

2.3 Savings in the presence of uncertainty

Consider the model we discussed in class where a small economy has a stochastic endowment, of expected value Y and can interact with an external insurer who is risk neutral.

We expand the setting by taking two periods. Period 2 is the one we saw in class, and in period 1 the country gets with certainty an income equal to Y. It can save (but not borrow) some of that income in a world bond that gives a set interest rate. The lender can seize these savings if the country defaults on its contract. We take our usual assumption that the interest rate on the bonds exactly mirrors the country's discount factor $\beta(1+r)=1$.

1. If there is no default risk on the contract between the country and the insurer, does the possibility to save helps?

2. Does your answer changes if there is a risk of default, and in case of default the insurance can seize a share $\eta < 1$ of the period 2 income?

3. Can there be some insurance if $\eta = 0$?

Answer: The insurance contract calls for the insurer to make a payment to the country when income is low, and receive a payment from the country when income is high. In expected terms, the insurer makes zero profits.

1. Without default is not an issue, the possibility to save adds nothing. As $\beta(1+r) = 1$ the country wants to smooth consumption, and as the income in period 1 and period 2 (after insurance payments / receipts) are the same, smoothing happens automatically. You may wonder whether the presence of risk implies that there are precautionary savings, as seen in class. This is not the case because the insurance contracts gets rid of the risk, unlike the case in class where there was only one bond and no insurance.

2. When default is possible, some insurance is still feasible. When income is high, the country is tempted to default on its payments, so the contract calls for a payment that is only a fraction η of the income, i.e. the amount that the insurer could seize on his own. The country then bears some of the income risk. In the states of nature where income is low, the country receives money and does not default. It thus gets full insurance, but at a level of consumption lower than in the case where default is absent. The possibility of savings helps, as it allows the country to build a collateral that the insurer can seize in case of default. This allows the insurer to provide insurance over a broader range of states of nature, as the collateral can be thought of as a higher value of η (not exactly, but the idea is similar).

3. If the insurer cannot seize any share of the output, the insurance market collapses when there are no savings option. However, when the country saves in the bond, it provides the insurer with something to seize in case of default. Partial insurance is then restored.

3 Monetary policy and exchange rate (20 % of grade)

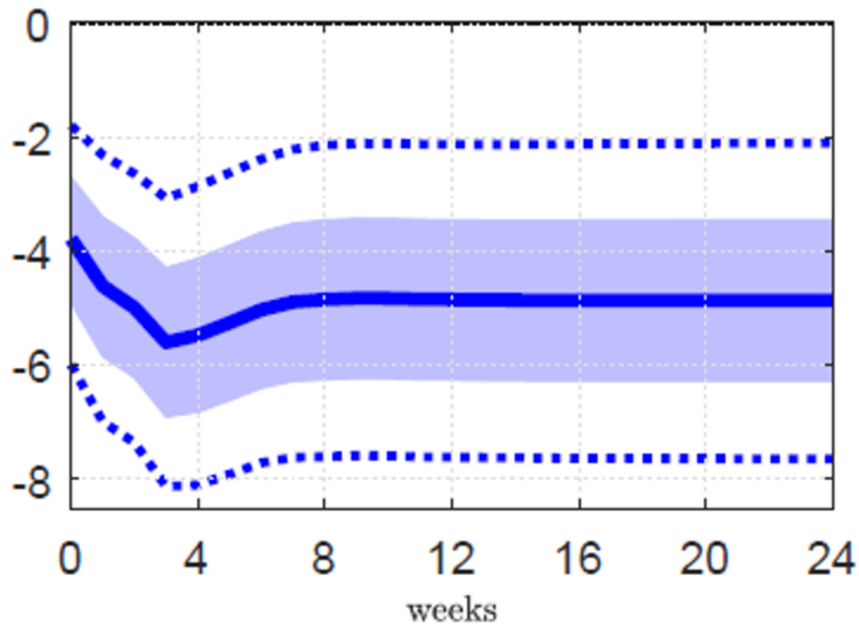
3.1 Impact of Swiss monetary policy

In class, we saw the monetary model of the exchange rate. We focused on a small open economy, but if we explicitly consider the money market abroad we get the same expression as we saw in class, simply replacing the expected domestic money at time $s \geq t$, m_s, with the expected difference between the domestic and foreign monetary stances, $m_s - m_s^F$.

We now consider the exchange rate between a small economy (Switzerland) and a large economy (the Euro area). The figure below shows the impact of a tightening of Swiss monetary policy (higher Swiss interest rate, which corresponds to $m_t < 0$). A positive value is a depreciation of the Swiss franc. The dotted lines show the statistical confidence bands.

Do you find this result surprising or not? Explain the economic intuition.

Effect of an increase of Swiss interest rate on
Chf / Euro exchange rate (positive value = Swiss franc **de**preciation).



Answer: We first derive the exchange rate solution, for reference. We use the uncovered interest parity condition, purchasing power parity, and the money demand in the domestic and foreign countries:

$$
\begin{aligned}
i_{t+1}^H &= i_{t+1}^F + \mathbb{E}_t(e_{t+1}) - e_t \\
p_t &= e_t + p_t^F \\
m_t - p_t &= c_t - \lambda i_{t+1}^H \\
m_t^F - p_t^F &= c_t^F - \lambda i_{t+1}^F
\end{aligned}
$$

Take the different of the two money demands, and abstract from c_t and c_t^F which reflect the real side:

$$\begin{aligned}\left(m_t - m_t^F\right) - \left(p_t - p_t^F\right) &= -\lambda\left(i_{t+1}^H - i_{t+1}^F\right)\\ \left(m_t - m_t^F\right) - e_t &= -\lambda\left(\mathbb{E}_t\left(e_{t+1}\right) - e_t\right)\\ \left(m_t - m_t^F\right) - (1+\lambda)\, e_t &= -\lambda \mathbb{E}_t\left(e_{t+1}\right)\\ (1+\lambda)\, e_t &= \left(m_t - m_t^F\right) + \lambda \mathbb{E}_t\left(e_{t+1}\right)\\ e_t &= \frac{\lambda}{1+\lambda}\mathbb{E}_t\left(e_{t+1}\right) + \frac{1}{1+\lambda}\left(m_t - m_t^F\right)\end{aligned}$$

If we iterate forward, and use the transversality condition $(im_{s\to\infty}\left(\frac{\lambda}{1+\lambda}\right)^s \mathbb{E}_t\left(e_s\right) = 0)$ we get:

$$e_t = \frac{1}{1+\lambda}\sum_{s=t}^{\infty}\left[\left(\frac{\lambda}{1+\lambda}\right)^s \mathbb{E}_t\left(m_s - m_s^F\right)\right]$$

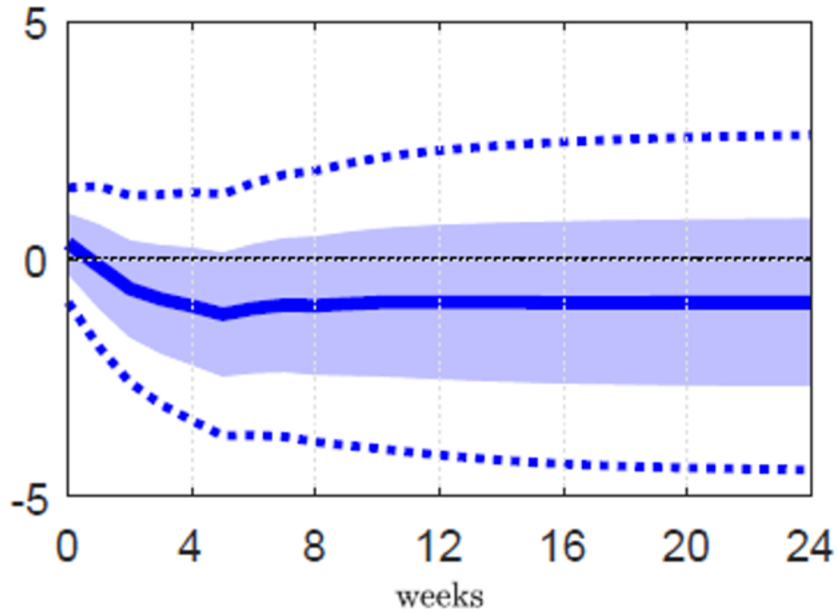
The figure shows that a higher Swiss interest rate ($m_t < 0$) leads to a statistically significant appreciation of the Swiss franc, i.e. $e_t < 0$. This is what the model predicts, especially if the decrease is persistent so $\mathbb{E}\left(m_s\right) < 0$ for quite a few periods.

3.2 Impact of Euro monetary policy

The figure below presents a similar exercise as previously, but this time for an increase in the Euro area interest rate (corresponding to $m_t^F < 0$).

Do you find this result surprising or not? Explain the economic intuition.

Effect of an increase of Euro area interest rate on
Chf / Euro exchange rate (positive value = Swiss franc **de**preciation).



Answer: The figure shows that a higher European interest rate leads to a small, and statistically insignificant, appreciation of the Swiss franc.

This is not what we expect, as $m_t^F < 0$ should lead to $e_t > 0$, i.e. we should have the mirror image of the previous question.

Remember however that Switzerland is a small economy that pays a lot of attention to what is happening in its large neighbor. If the Swiss National Bank was expected to do nothing in reaction of the European interest rate change, then we should indeed see a Swiss franc depreciation. However, it could be that the Swiss do not like to let the exchange rate move too much in response to the policy of the large neighbor, and the markets expect that the Swiss National Bank will follow the European Central Bank and raise its own interest rate in the near future, that is $\mathbb{E}(m_s) < 0$ in a few periods. The move $m_t^F < 0$ does not not lead to a one-for-one move in $\mathbb{E}_t\left(m_s - m_s^F\right)$, and instead the expected gap could be close to zero soon. In that case, the exchange rate shouldn't move much.

The figures are taken from the following paper: Christian Christian (2020). « The effect of monetary policy on the Swiss franc: an SVAR approach», SNB working paper 2/2020

https://www.snb.ch/n/mmr/reference/working_paper_2020_02/source/working_paper_2020_02.n.pdf

4 Impact of future endowment changes (25 % of grade)

4.1 Consumption dynamics

Consider and infinite horizon economy with world traded good (with price 1) and a local non-traded good (with price P_t^N). The demands and price index are (take this as given):

$$\begin{aligned} C_t^T &= \gamma \left[\frac{1}{P_t}\right]^{-\theta} C_t \quad ; \quad C_t^N = (1-\gamma)\left[\frac{P_t^N}{P_t}\right]^{-\theta} C_t \\ P_t &= \left[\gamma + (1-\gamma)\left(P_t^N\right)^{1-\theta}\right]^{\frac{1}{1-\theta}} \end{aligned}$$

where θ reflect the sensitivity of demand to the price. The higher θ, the *less* substitutable are the two goods.

The outputs are endowments. The agent can trade a bond giving a return r in units of traded good, which exactly mirrors the impatience (utility discount factor): $\beta(1+r) = 1$. The Euler consumption giving the dyamics of traded good consumption is (take this as given):

$$C_{t+1}^T = \left[\frac{P_t}{P_{t+1}}\right]^{\sigma-\theta} C_t^T$$

where σ reflects the sensitivity of intertemporal allocation to the interest rate. The higher σ, the *lower* is the curvature of the consumption utility function.

<u>Explain why the impact of the price dynamics on the dynamics on consumption depends on $\sigma - \theta$.</u>

Answer: We first present the building blocks in more details. The consumption basket of traded and non-traded goods is given by:

$$C_t = \left[(\gamma)^{\frac{1}{\theta}}\left(C_t^T\right)^{\frac{\theta-1}{\theta}} + (1-\gamma)^{\frac{1}{\theta}}\left(C_t^N\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}$$

The static optimization gives the consumption allocation:

$$\begin{aligned} C_t^T &= \gamma \left[\frac{1}{P_t}\right]^{-\theta} C_t \\ C_t^N &= (1-\gamma)\left[\frac{P_t^N}{P_t}\right]^{-\theta} C_t \\ P_t &= \left[\gamma + (1-\gamma)\left(P_t^N\right)^{1-\theta}\right]^{\frac{1}{1-\theta}} \end{aligned}$$

The intertemporal utility of consumption is:

$$U_t = \sum_{s=t}^{\infty} (\beta)^{s-t}\, \beta \frac{(C_s)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}$$

The intermporal allocation with respect to the consumption basket is:

$$C_{t+1} = \left[\beta\left(1+r\right)\frac{P_t}{P_{t+1}}\right]^{\sigma} C_t$$

Using the consumption allocation, we write:

$$\begin{aligned} C_{t+1} &= \left[\beta\left(1+r\right)\frac{P_t}{P_{t+1}}\right]^{\sigma} C_t \\ C_{t+1}^T\left(\gamma\right)^{-1}\left[\frac{1}{P_{t+1}}\right]^{\theta} &= \left[\beta\left(1+r\right)\right]^{\sigma}\left[\beta\frac{P_t}{P_{t+1}}\right]^{\sigma} C_t^T\left(\gamma\right)^{-1}\left[\frac{1}{P_t}\right]^{\theta} \\ C_{t+1}^T &= \left[\frac{P_t}{P_{t+1}}\right]^{\sigma-\theta} C_t^T \end{aligned}$$

We now discuss the impact of a rising price of the non-traded good, $P_{t+1}^N > P_t^N$, which translates into overall inflation, $P_{t+1} > P_t$.

- The first effect of CPI inflation is to reduce the real interest in terms of the consumption basket. This leads the consumer to borrow more, and moves overall consumption towards the present. The move of overall consumption is also occurring with traded good consumption, leading to a higher C_t^T relative to C_{t+1}^T. The sensitivity of this channel reflects the willingness to substitute through time, given by σ.
- The second effect of a rising price for the non-traded good is to make the non-traded good relatively cheap today. The composition of the consumption basket shifts away from the temporarily expensive traded good in period t. The inflation then leads to a lower C_t^T relative to C_{t+1}^T. The sensitivity of this channel reflects the willingness to substitute between the two goods, given by θ.

4.2 Traded consumption and endowment

In a steady state where endowments and prices are constant, we have (take this as given):

$$C_t^T = rB_t + Y^T$$

where B_t are the initial holdings of bonds.

If we log linearize the model around a steady state with no bond holdings, the demands for goods imply (take this as given):

$$\begin{aligned} c_t^T &= \theta\left(1-\gamma\right)p_t^N + c_t \\ c_t^N &= -\theta\gamma p_t^N + c_t \end{aligned}$$

Show that the price of the non-traded goods is:

$$p_t^N = \frac{1}{\theta}\left(c_t^T - y_t^N\right)$$

Combining the Euler condition with this property, we can show (take this as given):

$$c_{t+1}^T - c_t^T = \frac{\frac{\sigma-\theta}{\theta}(1-\gamma)}{1+\frac{\sigma-\theta}{\theta}(1-\gamma)}\left(y_{t+1}^N - y_t^N\right)$$

Explain why the dynamics of the endowment of the non-traded good affect the intertemporal path of consumption, but the dynamics of the endowment of the traded good does not.

Answer: The intertemporal budget constraint is given by:

$$\begin{aligned}
\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} P_s C_s &= (1+r) B_t + \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}\left[Y_s^T + P_s^N Y_s^N\right] \\
\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}\left[C_s^T + P_s^N C_s^N\right] &= (1+r) B_t + \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}\left[Y_s^T + P_s^N Y_s^N\right] \\
\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} C_s^T &= (1+r) B_t + \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} Y_s^T
\end{aligned}$$

Using the Euler condition, we write:

$$C_s^T = \left[\frac{P_t}{P_s}\right]^{\sigma-\theta} C_t^T$$

which implies:

$$C_t^T \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}\left[\frac{P_t}{P_s}\right]^{\sigma-\theta} = (1+r) B_t + \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} Y_s^T$$

In a steady state$P_t = P_s$:

$$\begin{aligned}
C_t^T \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} &= (1+r) B_t + Y^T \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} \\
C_t^T \frac{1+r}{r} &= (1+r) B_t + Y^T \frac{1+r}{r} \\
C_t^T &= r B_t + Y^T
\end{aligned}$$

In terms of log linear approximations, the CPI is $p_t = (1-\gamma) p_t^N$ and the demands for goods are:

$$\begin{aligned}
c_t^T &= \theta p_t + c_t = \theta(1-\gamma) p_t^N + c_t \\
c_t^N &= \theta\left(p_t - p_t^N\right) + c_t = -\theta\gamma p_t^N + c_t
\end{aligned}$$

Taking the difference, we get:

$$\begin{aligned}
c_t^T - c_t^N &= \theta(1-\gamma) p_t^N + \theta\gamma p_t^N + c_t - c_t \\
c_t^T - y_t^N &= \theta p_t^N + \theta\gamma p_t^N
\end{aligned}$$

$$p_t^N = \frac{1}{\theta}\left(c_t^T - y_t^N\right)$$

The price of the non-traded good relfects the relative consumption, with the consumption of the non-traded good reflecting the endowment. A more abundant supply of the non-traded good reduces its price, everything else equal.

Using this, the Euler condition is:

$$\begin{aligned}
c_{t+1}^T &= (\sigma-\theta)\left(p_t - p_{t+1}\right) + c_t^T \\
c_{t+1}^T &= (\sigma-\theta)(1-\gamma)\left(p_t^N - p_{t+1}^N\right) + c_t^T \\
c_{t+1}^T &= \frac{\sigma-\theta}{\theta}(1-\gamma)\left(\left(c_t^T - y_t^N\right) - \left(c_{t+1}^T - y_{t+1}^N\right)\right) + c_t^T \\
\left(1+\frac{\sigma-\theta}{\theta}(1-\gamma)\right)c_{t+1}^T &= \frac{\sigma-\theta}{\theta}(1-\gamma)\left(y_{t+1}^N - y_t^N\right) + \left(1+\frac{\sigma-\theta}{\theta}(1-\gamma)\right)c_t^T \\
c_{t+1}^T &= \frac{\frac{\sigma-\theta}{\theta}(1-\gamma)}{1+\frac{\sigma-\theta}{\theta}(1-\gamma)}\left(y_{t+1}^N - y_t^N\right) + c_t^T
\end{aligned}$$

An increase in the endowment of the non-traded good through time reduces its price. If the intertemporal dimension dominates ($\sigma-\theta > 0$) the decreasing price of the non-traded good translates into deflation, which raises the real interest rate and pushes consumption into the future, leading to a higher $c_{t+1}^T - c_t^T$. It is true that the changing traded consumption partially offsets the impact of endowment (both non-traded endowment and traded consumption increase through time) on the price of the non-traded good, but this is partial. The dynamics of the non-traded endowment affects the dynamics of the traded consumption because it changes the real interest rate.

An increase in the endowment of the traded good through time does not lead to a changing dynamics of the traded consumption (it can affect its level in the two periods in parallel) because it does lead to a change in prices and the rel interest rates. This is because movements in the traded endowment can simply be offset through international trade.

4.3 Impact of higher future endowment

The economy starts at a steady state with no bond holdings. The endowment of traded good always remains at the steady state level.

In period t we learn that from period $T > t$ onwards the endowment of the non-traded good will permanently increase by an amount χ. In response to this news, consumption of the traded good and bond savings may change.

In the following, do not derive algebra, but think using the results presented in previous parts of the question. Consider that $\sigma - \theta > 0$.

1. Does consumption of the traded good change between $T-1$ and T?
2. What will happen to consumption of the traded good from T onwards?
3. What happens to the consumption of the traded good between t and T?
4. What is the path of the real exchange rate?

5. <u>What changes for points 1-3 if we assume $\sigma - \theta < 0$ instead?</u>

Answer: Recall that the Euler implies:

$$c_{t+1}^T - c_t^T = \frac{\frac{\sigma-\theta}{\theta}(1-\gamma)}{1+\frac{\sigma-\theta}{\theta}(1-\gamma)}\left(y_{t+1}^N - y_t^N\right)$$

1. Consumption of the traded good increases by a step between $T-1$ and T, as the non-traded endowment increases: $y_T^N - y_{T-1}^N = \chi$.

2. From T onwards, the non-traded endowment is constant at the new higher level, and the Euler implies that the traded good consumption is also constant. It's level is increased of the economy enters T with assets.

3. Between t and T the non-traded endowment is constant at the initial level, and the Euler implies that the traded good consumption is also constant. We thus have a pattern when after the news is announced, traded consumption is constant, then moves up when the change actually occurs, and remains constant. As in net present value terms the consumption must be equal to the unchanged traded endowments, we must have an initial period of negative deviation of consumption from the initial steady state, followed by a period of positive deviation. The country therefore runs a current account surplus (traded consumption is lower than the endowment) accumulating assets, followed by a permanent current account deficit financed by the earnings on the assets.

4. Between t and T the lower traded consumption, combined with the unchanged non-traded endowment, leads to a real depreciation, as the non-traded goods becomes cheaper in adjustment to the lower traded consumption ($p_t^N < 0$). From T onwards, consumption of the traded good increases, but so does the endowment of the non-traded good. The relation above however shows that the change of consumption is lower, $\frac{\frac{\sigma-\theta}{\theta}(1-\gamma)}{1+\frac{\sigma-\theta}{\theta}(1-\gamma)} < 1$, so the increased supply of the non-traded good dominates and pushes the price down, leading to a further depreciation.

5. If $\sigma - \theta < 0$, the impact of the changing price of the non-traded good on the dynamics of the traded is dominated by the substitution effect between the two goods. The effects on traded consumption and the current account are then reversed (initial current account deficit, followed by a surplus).

Specifically, the analysis proceeds as follows. The jump in consumption at time T is:

$$c_{t+T}^T - c_{t+T-1}^T = \frac{\frac{\sigma-\theta}{\theta}(1-\gamma)}{1+\frac{\sigma-\theta}{\theta}(1-\gamma)}\chi$$

Before this jump (but after the announcement), consumption is constant at $c^{T,low}$. After the jump, it is constant at $c^{T,low} + \frac{\frac{\sigma-\theta}{\theta}(1-\gamma)}{1+\frac{\sigma-\theta}{\theta}(1-\gamma)}\chi$.

To compute $c^{T,low}$, start from intertemporal budget constraint, combined with the Euler condition:

$$C_t^T \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} \left[\frac{P_t}{P_s}\right]^{\sigma-\theta} = (1+r)\, B_t + \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} Y_s^T$$

In period t, there are no initial assets, $B_t = 0$, and output is constant. We express this in as a log linear approximation:

$$\begin{aligned}
0 &= c^{T,low} \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} + \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} (\sigma-\theta)\left[p_t - p_s\right] \\
0 &= \frac{1+r}{r} c^{T,low} + \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} (\sigma-\theta)(1-\gamma)\left[p_t^N - p_s^N\right] \\
0 &= \frac{1+r}{r} c^{T,low} + \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} \frac{\sigma-\theta}{\theta}(1-\gamma)\left[\left(c_t^T - y_t^N\right) - \left(c_s^T - y_s^N\right)\right] \\
0 &= \frac{1+r}{r} c^{T,low} + \frac{\sigma-\theta}{\theta}(1-\gamma)\left[\sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} c_t^T - \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} \left(c_s^T - y_s^N\right)\right] \\
0 &= \frac{1+r}{r} c^{T,low} + \frac{\sigma-\theta}{\theta}(1-\gamma)\frac{1+r}{r} c^{T,low} \\
&\quad - \frac{\sigma-\theta}{\theta}(1-\gamma)\left[\sum_{s=t}^{t+T-1} \left(\frac{1}{1+r}\right)^{s-t} \left(c_s^T - y_s^N\right) + \sum_{s=t+T}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} \left(c_s^T - y_s^N\right)\right] \\
0 &= \frac{1+r}{r}\left(1 + \frac{\sigma-\theta}{\theta}(1-\gamma)\right) c^{T,low} \\
&\quad - \frac{\sigma-\theta}{\theta}(1-\gamma)\left[\sum_{s=t}^{t+T-1} \left(\frac{1}{1+r}\right)^{s-t} c^{T,low} + \sum_{s=t+T}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} \left(c^{T,high} - \chi\right)\right] \\
0 &= \frac{1+r}{r}\left(1 + \frac{\sigma-\theta}{\theta}(1-\gamma)\right) c^{T,low} \\
&\quad - \frac{\sigma-\theta}{\theta}(1-\gamma)\left[\sum_{s=t}^{t+T-1} \left(\frac{1}{1+r}\right)^{s-t} c^{T,low} + \sum_{s=t+T}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} \left(c^{T,low} - \frac{1}{1+\frac{\sigma-\theta}{\theta}(1-\gamma)}\chi\right)\right] \\
0 &= \frac{1+r}{r}\left(1 + \frac{\sigma-\theta}{\theta}(1-\gamma)\right) c^{T,low} \\
&\quad - \frac{\sigma-\theta}{\theta}(1-\gamma)\left[\sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} c^{T,low} - \frac{1}{1+\frac{\sigma-\theta}{\theta}(1-\gamma)}\chi \sum_{s=t+T}^{\infty} \left(\frac{1}{1+r}\right)^{s-t}\right] \\
0 &= \frac{1+r}{r}\left(1 + \frac{\sigma-\theta}{\theta}(1-\gamma)\right) c^{T,low} \\
&\quad - \frac{\sigma-\theta}{\theta}(1-\gamma)\left[\frac{1+r}{r} c^{T,low} - \frac{1}{1+\frac{\sigma-\theta}{\theta}(1-\gamma)}\chi \left(\frac{1}{1+r}\right)^{T} \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t}\right] \\
0 &= \frac{1+r}{r} c^{T,low} + \frac{\frac{\sigma-\theta}{\theta}(1-\gamma)}{1+\frac{\sigma-\theta}{\theta}(1-\gamma)}\chi \left(\frac{1}{1+r}\right)^{T} \frac{1+r}{r} \\
c^{T,low} &= -\frac{\frac{\sigma-\theta}{\theta}(1-\gamma)}{1+\frac{\sigma-\theta}{\theta}(1-\gamma)}\chi \left(\frac{1}{1+r}\right)^{T}
\end{aligned}$$

Consumption decreases if $\sigma - \theta > 0$, leading to a currrent account deficit.

The real exchange rate before the change is:

$$
\begin{aligned}
p_t^N &= \frac{1}{\theta}\left(c^{T,low} - 0\right) \\
p_t^N &= -\frac{1}{\theta}\frac{\frac{\sigma-\theta}{\theta}\left(1-\gamma\right)}{1+\frac{\sigma-\theta}{\theta}\left(1-\gamma\right)}\chi\left(\frac{1}{1+r}\right)^T
\end{aligned}
$$

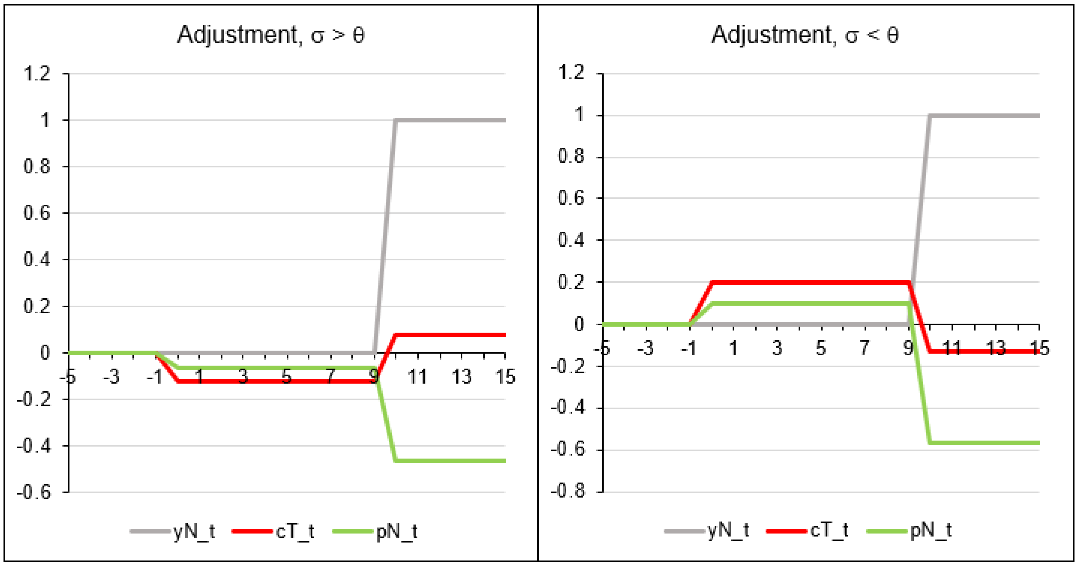
and after the change:

$$
\begin{aligned}
p_T^N &= \frac{1}{\theta}\left(c^{T,higher} - \chi\right) \\
p_T^N &= \frac{1}{\theta}\left(c^{T,low} + \frac{\frac{\sigma-\theta}{\theta}\left(1-\gamma\right)}{1+\frac{\sigma-\theta}{\theta}\left(1-\gamma\right)}\chi - \chi\right) \\
p_T^N &= \frac{1}{\theta}\left(-\frac{\frac{\sigma-\theta}{\theta}\left(1-\gamma\right)}{1+\frac{\sigma-\theta}{\theta}\left(1-\gamma\right)}\chi\left(\frac{1}{1+r}\right)^T - \frac{1}{1+\frac{\sigma-\theta}{\theta}\left(1-\gamma\right)}\chi\right) \\
p_T^N &= -\frac{1}{\theta}\frac{\frac{\sigma-\theta}{\theta}\left(1-\gamma\right)}{1+\frac{\sigma-\theta}{\theta}\left(1-\gamma\right)}\chi\left(\frac{1}{1+r}\right)^T - \frac{1}{\theta}\frac{1}{1+\frac{\sigma-\theta}{\theta}\left(1-\gamma\right)}\chi
\end{aligned}
$$

If $\sigma - \theta > 0$, the exchange rate depreciates at the time of announcement, and even more when the change happens. If $\sigma - \theta < 0$, the exchange rate appreciates initially, but ultimately depreciates if the interval T is long enough. Specifically, the threshold at which $p_T^N = 0$ is:

$$
\begin{aligned}
0 &= -\frac{1}{\theta}\frac{\frac{\sigma-\theta}{\theta}\left(1-\gamma\right)}{1+\frac{\sigma-\theta}{\theta}\left(1-\gamma\right)}\chi\left(\frac{1}{1+r}\right)^T - \frac{1}{\theta}\frac{1}{1+\frac{\sigma-\theta}{\theta}\left(1-\gamma\right)}\chi \\
0 &= \frac{\sigma-\theta}{\theta}\left(1-\gamma\right)\left(\frac{1}{1+r}\right)^T + 1 \\
\left(\frac{1}{1+r}\right)^T &= \frac{\theta}{-\left(\sigma-\theta\right)\left(1-\gamma\right)} \\
T &= \ln\left(\frac{\theta}{-\left(\sigma-\theta\right)\left(1-\gamma\right)}\right)/\ln\left(\frac{1}{1+r}\right)
\end{aligned}
$$

The figure below shows the impact of an 1 unit increase in non-traded endowment ($\chi = 1$) at time $T = 10$, announced at $t = 0$. In both we set $\gamma = 0.5$ and $\theta = 2$. The left panel is the case where $\sigma = 3$ and the right panel corresponds to $\sigma = 1$. There is a temporary reduction of consumption in the left panel, leading to a surplus that ultimately supports higher permanent consumption. The pattern is mirrored in the right panel. In both panels, the real exchange rate depreciates in the new steady state (lower price of the non-traded good thanks to higher supply). In the left panel, there is already a depreciation in the short run, but the right panel shows an initial appreciation.


Adjustment, σ > θ
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-5
-3
-1
1
3
5
7
9
11
13
15
yN_t
cT_t
pN_t
Adjustment, σ < θ
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-5
-3
-1
1
3
5
7
9
11
13
15
yN_t
cT_t
pN_t

5 Hedging of different endowments via portfolio (25 % of grade)

5.1 Portfolio Euler conditions

Consider a two-country model with the following features:

- There is one good consumed, so no real exchange rate or terms-of-trade.
- The Home agent gets an income in two stochastic segments, $Y^A + Y^L$.
  - It is possible to diversify Y^A by issuing some equity claims and sell them to the Foreign agent. This is an income on dividends for instance.
  - Y^A cannot be diversified, and the Home agent is fully exposed to it. This is an income on labor for instance.
- Similarly, the Foreign agent gets an income in two stochastic segments, $Y^{A*} + Y^{L*}$.
- The agents make a portfolio choice, with a one-period horizon.
  - The Home agent hold domestic equity and gets a fraction $1-x$ of Y^A, as well as equity on the other endowment getting a fraction x of Y^{A*}. The portfolio share of foreign assets in the Home agent's portfolio x is set before income shocks are realized. We assume symmetry, so the Foreign agent also puts a share x in Foreign (domestic) assets.

The Home agent sets x to maximize her expected utility of consumption, which is $\frac{\mathbb{E}(C)^{1-\rho}}{1-\rho}$. We can show that the optimal portfolio choice is such that (take this as given):

$$0 = E(C)^{-\rho}\left(Y^A - Y^{A*}\right)$$

What is the intuitive interpretation of this relation?

Answer: We first derive the results, allowing the share put in domestic assets by Home and Foreign agents, x and x^*, to differ. The consumptions of the Home and Foreign agents are:

$$\begin{aligned} C &= (1-x)\,Y^A + Y^L + xY^{A*} \\ C^* &= (1-x^*)\,Y^{A*} + Y^{L*} + x^*Y^A \end{aligned}$$

The Home agent maximizes:

$$\frac{\mathbb{E}\left((1-x)\,Y^A + Y^L + xY^{A*}\right)^{1-\rho}}{1-\rho}$$

The first-order condition is:

$$0 = \mathbb{E}\left[\left((1-x)\,Y^A + Y^L + xY^{A*}\right)^{-\rho}\frac{\partial\left((1-x)\,Y^A + Y^L + xY^{A*}\right)}{\partial x}\right]$$

$$
\begin{aligned}
0 &= \mathbb{E}\left[(C)^{-\rho}\left(-Y^{A}+Y^{A*}\right)\right] \\
0 &= \mathbb{E}\left[(C)^{-\rho}\left(Y^{A}-Y^{A*}\right)\right] \\
\mathbb{E}\left[(C)^{-\rho}Y^{A}\right] &= \mathbb{E}\left[(C)^{-\rho}Y^{A*}\right]
\end{aligned}
$$

This condition indicates that the agent chooses to set the expected discounted return to be equal across the two assets. Y^A and Y^{A*} are the returns (dividends) on the Home and Foreign assets. The agent weights the return in a specific state of nature by the marginal utility of consumption $(C)^{-\rho}$. The portfolio is chosen (recall that x enters C) so that these weighted payoffs are equalized in expected terms.

5.2 Optimal portfolio choice: step 1

We solve the model by taking approximations around a steady state where: $Y_0^A = Y_0^{A*} = Y_0$ and $Y_0^L = Y_0^{L*} = zY_0$. We denote log approximations by lower case letters.

We can show that the share the Home agent invests in Foreign assets, x, is (take this as given, as the proof requires a quadratic approximation of the optimality condition):

$$
x = \frac{1+z}{\rho}\frac{\mathbb{E}\left(y^{A*}-y^{A}\right)}{\mathbb{E}\left[\left(y^{A*}-y^{A}\right)^{2}\right]} - \frac{\mathbb{E}\left[\left(y^{A*}-y^{A}\right)\left(y^{A}+zy^{L}\right)\right]}{\mathbb{E}\left[\left(y^{A*}-y^{A}\right)^{2}\right]}
$$

How do you interpret this portfolio share in terms of economic intuition (notice that $y^{A*}-y^{A}$ is the return differential – also called "excess return" – between the Foreign asset and the Home asset)?

Answer: The optimality condition is approximated as:

$$
\begin{aligned}
0 &= \mathbb{E}\left[(C)^{-\rho}\left(Y^{A}-Y^{A*}\right)\right] \\
0 &= \mathbb{E}\left[\left((1-x)Y^{A}+Y^{L}+xY^{A*}\right)^{-\rho}\left(Y^{A}-Y^{A*}\right)\right] \\
0 &= (C_0)^{-\rho}(Y_0-Y_0) \\
&\quad +\mathbb{E}\left[-\rho(C_0)^{-\rho-1}(Y_0-Y_0)\left(\begin{array}{c}(1-x)\left(Y^{A}-Y_0\right)\\ +\left(Y^{L}-zY_0\right)+x\left(Y^{A*}-Y_0\right)\end{array}\right)\right] \\
&\quad +\mathbb{E}\left[(C_0)^{-\rho}\left[\left(Y^{A}-Y_0\right)-\left(Y^{A*}-Y_0\right)\right]\right] \\
&\quad +\frac{1}{2}\mathbb{E}\left[-\rho(-\rho-1)(C_0)^{-\rho-2}(Y_0-Y_0)\left(\begin{array}{c}(1-x)\left(Y^{A}-Y_0\right)\\ +\left(Y^{L}-zY_0\right)+x\left(Y^{A*}-Y_0\right)\end{array}\right)^{2}\right] \\
&\quad +\frac{1}{2}\mathbb{E}\left[-2\rho(C_0)^{-\rho-1}\left[\left(Y^{A}-Y_0\right)-\left(Y^{A*}-Y_0\right)\right]\left(\begin{array}{c}(1-x)\left(Y^{A}-Y_0\right)\\ +\left(Y^{L}-zY_0\right)+x\left(Y^{A*}-Y_0\right)\end{array}\right)\right] \\
&\quad 0 = \mathbb{E}\left[(C_0)^{-\rho}Y_0\left(y^{A}-y^{A*}\right)\right] \\
&\quad +\frac{1}{2}\mathbb{E}\left[-2\rho(C_0)^{-\rho-1}(Y_0)^{2}\left(y^{A}-y^{A*}\right)\left((1-x)y^{A}+zy^{L}+xy^{A*}\right)\right] \\
0 &= (C_0)^{-\rho}Y_0\mathbb{E}\left(y^{A}-y^{A*}\right)
\end{aligned}
$$

$$
\begin{aligned}
& -\rho\left(C_0\right)^{-\rho-1}\left(Y_0\right)^2 \mathbb{E}\left[\left(y^A-y^{A*}\right)\left((1-x) y^A+z y^L+x y^{A*}\right)\right] \\
0 \quad = \quad & \left(C_0\right)^{-\rho} Y_0 \mathbb{E}\left(y^A-y^{A*}\right) \\
& -\rho\left(C_0\right)^{-\rho-1}\left(Y_0\right)^2 \mathbb{E}\left[\left(y^A-y^{A*}\right)\left((1-x) y^A+z y^L+x y^{A*}\right)\right] \\
0 \quad = \quad & \mathbb{E}\left(y^A-y^{A*}\right)-\frac{\rho}{1+z} \mathbb{E}\left[\left(y^A-y^{A*}\right)\left((1-x) y^A+z y^L+x y^{A*}\right)\right]
\end{aligned}
$$

Re-arranging the terms we write:

$$
x \quad = \quad \frac{1+z}{\rho} \frac{\mathbb{E}\left(y^{A*}-y^A\right)}{\mathbb{E}\left[\left(y^{A*}-y^A\right)^2\right]}-\frac{\mathbb{E}\left[\left(y^{A*}-y^A\right)\left(y^A+z y^L\right)\right]}{\mathbb{E}\left[\left(y^{A*}-y^A\right)^2\right]}
$$

The share of the Foreign asset in the Home agent's portfolio reflects two elements.

1. The first is the expected excess return, $\mathbb{E}\left(y^{A*}-y^A\right)$. The investor puts more of her portfolio in the Foreign asset when it is expected to deliver a higher return as the Home asset. As a positive excess return is not guaranteed, the investor scales the expected excess return by risk, which is the variance of the excess return $\mathbb{E}\left[\left(y^{A*}-y^A\right)^2\right]$. Also, the investor is less sensitive to the expected excess return when she is more risk averse (ρ is higher).

2. The second element reflects the hedging property of the Foreign asset. If the Foreign assets givens a higher return than the Home asset ($y^{A*}-y^A>0$) in states of nature when the Home investor has a high income (y^A+zy^L is high), the Foreign asset is less appealing that the Home asset because it pays off when the investor does not need the money that much. Hence the share x is lower when the covariance $\mathbb{E}\left[\left(y^{A*}-y^A\right)\left(y^A+z y^L\right)\right]$ is positive.

5.3 Optimal portfolio choice: step 2

Building on the steps above, we can show (take this as given):

$$
x=\frac{1}{2}-\frac{z}{2} \frac{\mathbb{E}\left[\left(y^{A*}-y^A\right)\left(y^L-y^{L*}\right)\right]}{\mathbb{E}\left[\left(y^{A*}-y^A\right)^2\right]}
$$

1. Explain the intuition behind the second ratio (containing $\mathbb{E}\left[\left(y^{A*}-y^A\right)\left(y^L-y^{L*}\right)\right]$).

2. Consider that production uses only labor, but their is imperfect competition so y^A represents the return on firms' profits. We also consider that Home income components (y^A and y^L) may be correlated with each other, but they are uncorrelated with Foreign income components y^{A*} and y^{L*}). Discuss the intuition behind the portfolio share x held by an investors in assets abroad when:

   (a) There is no labor income ($z=0$).

   (b) The economies are subjected to productivity shocks. Prices and wages are fully flexible, and a productivity decrease lowers the return on factors of production, so asset (dividend) income in a country is low when the labor income is low.

(c) The economies are subjected to productivity shocks, but output in each country is constant (this is what happens when the prices of goods don't adjust, take this as given). This implies that productivity only affects the input used: with lower productivity, we produce the same output, but need more labor to do so.

Answer: Recall that the quadratic expansion of the Home agent's optimal portfolio condition gives:

$$0 = \mathbb{E}\left(y^A - y^{A*}\right) - \frac{\rho}{1+z}\mathbb{E}\left[\left(y^A - y^{A*}\right)\left((1-x)\,y^A + zy^L + xy^{A*}\right)\right]$$

Following similar steps for the optimal condition for the Foreign agent, we get:

$$\begin{aligned}
0 &= \mathbb{E}\left[(C^*)^{-\rho}\left(Y^A - Y^{A*}\right)\right] \\
0 &= \mathbb{E}\left[\left(x^*Y^A + Y^{L*} + (1-x^*)\,Y^{A*}\right)^{-\rho}\left(Y^A - Y^{A*}\right)\right] \\
0 &= \mathbb{E}\left(y^A - y^{A*}\right) - \frac{\rho}{1+z}\mathbb{E}\left[\left(y^A - y^{A*}\right)\left(x^*y^A + zy^{L*} + (1-x^*)\,y^{A*}\right)\right]
\end{aligned}$$

In a symmetric allocation, $x = x^*$. Taking the difference between the two optimality conditions, we write:

$$\begin{aligned}
0 &= \mathbb{E}\left(y^A - y^{A*}\right) - \mathbb{E}\left(y^A - y^{A*}\right) \\
&\quad -\frac{\rho}{1+z}\mathbb{E}\left[\left(y^A - y^{A*}\right)\left((1-x)\,y^A + zy^L + xy^{A*}\right)\right] \\
&\quad +\frac{\rho}{1+z}\mathbb{E}\left[\left(y^A - y^{A*}\right)\left(xy^A + zy^{L*} + (1-x)\,y^{A*}\right)\right] \\
0 &= \mathbb{E}\left[\left(y^A - y^{A*}\right)\left((2x-1)\left(y^A - y^{A*}\right) + z\left(y^{L*} - y^L\right)\right)\right] \\
0 &= 2x\mathbb{E}\left[\left(y^{A*} - y^A\right)^2\right] - \mathbb{E}\left[\left(y^{A*} - y^A\right)^2\right] \\
&\quad +z\mathbb{E}\left[\left(y^A - y^{A*}\right)\left(y^{L*} - y^L\right)\right] \\
x &= \frac{1}{2} - \frac{z}{2}\frac{\mathbb{E}\left[\left(y^{A*} - y^A\right)\left(y^L - y^{L*}\right)\right]}{\mathbb{E}\left[\left(y^{A*} - y^A\right)^2\right]}
\end{aligned}$$

We now turn to the analysis in terms of economic intuitition.

1. The first term reflects the world portfolio, where each agent holds half her wealth abroad. The second term reflects the hedging of labor income. If the Foreign asset pays off well when Home labor income is low ($E\left(y^{A*} - y^A\right)\left(y^L - y^{L*}\right) < 0$), the Foreign asset is a good hedge of labor income risk and the agent tilts her portfolio towards the Foreign asset: $x > 0.5$.

2. We now consider the special cases.

   (a) If $z = 0$ there is no labor income, and thus the hedging motive disappears. The agent simply evenly splits her portfolio across the two assets.

   (b) If prices are flexible, a productivity decrease in the Home country reduces output and wages. The gap between the value of output and the wage bill (that is firms' profits) also decreases. The decrease in Home productivity implies that $y^{A*} - y^{A} > 0$ and $y^{L} - y^{L*} < 0$, so $\mathbb{E}\left[\left(y^{A*} - y^{A}\right)\left(y^{L} - y^{L*}\right)\right] < 0$. This leads to a higher portfolio share x as the Foreign asset is a good hedge for the domestic labor income.

(c) When prices are sticky, demand does not change and so output is given. The only impact of a productivity shock is to change the amount of labor used, hence the labor income, a change that is mirrored in profits. A productivity decrease in the Home country raises the labor income. As output is unchanged, he gap between the value of output and the wage bill (that is firms' profits) decreases. The decrease in Home productivity implies that $y^{A*} - y^{A} > 0$ and $y^{L} - y^{L*} > 0$, so $\mathbb{E}\left[\left(y^{A*} - y^{A}\right)\left(y^{L} - y^{L*}\right)\right] > 0$. This leads to a lower portfolio share x as the Foreign asset is a not a good hedge for the domestic labor income, and instead the domestic asset is a good hedge.

To elaborate further, consider that the returns on assets (y^{A} and y^{A*}) each have a a variance σ^2. The endowments on which assets cannot be written (y^{L} and y^{L*}) each have a a variance $\mu^2\sigma^2$. Home and Foreign endowments are independent from each other ($\mathbb{E}\left(y^{A}y^{A*}\right) = \mathbb{E}\left(y^{L}y^{L*}\right) = \mathbb{E}\left(y^{A}y^{L*}\right) = \mathbb{E}\left(y^{A*}y^{L}\right) = 0$), and the correlation between Home endowments y^{A} and y^{L} is equal to λ (that is $\lambda = \frac{\mathbb{E}\left(y^{A}y^{L}\right)}{\sqrt{\mathbb{E}(y^{A})^2\mathbb{E}(y^{L})^2}}$) and similarly for Foreign endowments. The portfolio share is then:

$$
\begin{aligned}
x &= \frac{1}{2} - \frac{z}{2}\frac{\mathbb{E}\left[\left(y^{A*} - y^{A}\right)\left(y^{L} - y^{L*}\right)\right]}{\mathbb{E}\left[\left(y^{A*} - y^{A}\right)^2\right]} \\
x &= \frac{1}{2} - \frac{z}{2}\frac{-2\lambda\mu\sigma^2}{2\sigma^2} = \frac{1 + z\lambda\mu}{2}
\end{aligned}
$$

With price flexibility and productivity shocks, $y^{A} = y^{L}$ and $y^{A*} = y^{L*}$, implying that $\lambda = \mu = 1$. The portfolio exhibits foreign bias, as it mostly consists of assets abroad:

$$x = \frac{1+z}{2} > \frac{1}{2}$$

When prices are sticky, the sum of endowments is constant. This implies:

$$\bar{Y} = \exp\left[\ln Y^{A}\right] + \exp\left[\ln Y^{L}\right]$$

which we express as a linear expansion:

$$0 = Y_0 y^{A} + zY_0 y^{L} \Rightarrow y^{L} = -\frac{1}{z}y^{A}$$

The two endowments are perfectly negatively correlated ($\lambda = -1$). In terms of variance, we get: $\mu = 1/z$. This implies that there is full domestic bias, as the portfolio only consists of domestic assets:

$$x = 0$$