Macroeconomics A; EI060

Technical appendix: Obstfeld-Rogoff “new open economy macroeconomics” general equilibrium model

Cédric Tille

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1 Introduction (based on Obstfeld and Rogoff 10.1 and 10.4)

This technical appendix presents the computations, based on chapter 10 of the Obstfeld and Rogoff textbook.

I make two small changes from the textbook.

- I allow the elasticity of substitution between the baskets of Home and Foreign goods to differ from the elasticity of substitution between brands within each basket.
- Instead of considering that their are many households, each producing a brand with a quadratic utility cost, I consider that there is one household supplying labor (with a quadratic utility cost) to many firms, with each firm producing a brand using a linear technology. This does not change the message of the model, and is more in line with the standard approach in other papers.

Section presents the building blocks.

2 Building blocks of the model

2.1 Household's consumption allocation

2.1.1 Baskets

The household in the Home country consumes a CES basket in two steps. First, the basket consists of two subbaskets of goods produced in the Home country, $C(h)$, and of goods produced in the Foreign country, $C(f)$:

$$C = \left[(n)^{\frac{1}{\lambda}}\left[C(h)\right]^{\frac{\lambda-1}{\lambda}} + (1-n)^{\frac{1}{\lambda}}\left[C(f)\right]^{\frac{\lambda-1}{\lambda}}\right]^{\frac{\lambda}{\lambda-1}}$$

where $\lambda > 0$ is the elasticity of substitution between the two baskets, and n is the size of the Home country. The second step of the allocation splits the consumption of each subbasket across a continuum of brands indexed by z:

$$\begin{aligned} C(h) &= \left[\left(\frac{1}{n}\right)^{\frac{1}{\theta}} \int_0^n [C(z,h)]^{\frac{\theta-1}{\theta}} dz\right]^{\frac{\theta}{\theta-1}} \\ C(f) &= \left[\left(\frac{1}{1-n}\right)^{\frac{1}{\theta}} \int_n^1 [C(z,f)]^{\frac{\theta-1}{\theta}} dz\right]^{\frac{\theta}{\theta-1}} \end{aligned}$$

where $\theta > 1$ is the elasticity of substitution between brands.

2.1.2 Allocation across Home and Foreign made subbaskets

The Household allocates demand to minimize the cost, subject to a target value of the basket:

$$\mathcal{L} = P(h)C(h) + P(f)C(f) - \varphi\left[\left[(n)^{\frac{1}{\lambda}}[C(h)]^{\frac{\lambda-1}{\lambda}} + (1-n)^{\frac{1}{\lambda}}[C(f)]^{\frac{\lambda-1}{\lambda}}\right]^{\frac{\lambda}{\lambda-1}} - C\right]$$

The first order conditions are:

$$\begin{aligned} 0 &= \frac{\partial \mathcal{L}_C}{\partial C(h)} \\ 0 &= P(h) - \varphi\left[(n)^{\frac{1}{\lambda}}[C(h)]^{\frac{\lambda-1}{\lambda}} + (1-n)^{\frac{1}{\lambda}}[C(f)]^{\frac{\lambda-1}{\lambda}}\right]^{\frac{\lambda}{\lambda-1}-1}(n)^{\frac{1}{\lambda}}[C(h)]^{\frac{\lambda-1}{\lambda}-1} \\ 0 &= P(h) - \varphi[C]^{\frac{1}{\lambda}}(n)^{\frac{1}{\lambda}}[C(h)]^{\frac{-1}{\lambda}} \end{aligned}$$

and:

$$\begin{aligned} 0 &= \frac{\partial \mathcal{L}_C}{\partial C(f)} \\ 0 &= P(f) - \varphi\left[(n)^{\frac{1}{\lambda}}[C(h)]^{\frac{\lambda-1}{\lambda}} + (1-n)^{\frac{1}{\lambda}}[C(f)]^{\frac{\lambda-1}{\lambda}}\right]^{\frac{\lambda}{\lambda-1}-1}(1-n)^{\frac{1}{\lambda}}[C(f)]^{\frac{\lambda-1}{\lambda}-1} \\ 0 &= P(f) - \varphi[C]^{\frac{1}{\lambda}}(1-n)^{\frac{1}{\lambda}}[C(f)]^{\frac{-1}{\lambda}} \end{aligned}$$

Combining these we get:

$$\begin{aligned} P(h)C(h) + P(f)C(f) &= \varphi[C]^{\frac{1}{\lambda}}\left[(n)^{\frac{1}{\lambda}}[C(h)]^{\frac{\lambda-1}{\lambda}} + (1-n)^{\frac{1}{\lambda}}[C(f)]^{\frac{\lambda-1}{\lambda}}\right] \\ PC &= \varphi[C]^{\frac{1}{\lambda}}[C]^{\frac{\lambda-1}{\lambda}} \\ P &= \varphi \end{aligned}$$

The demands are therefore:

$$\begin{aligned} P(h) &= P[C]^{\frac{1}{\lambda}}(n)^{\frac{1}{\lambda}}[C(h)]^{\frac{-1}{\lambda}} \\ \frac{P(h)}{P} &= [C]^{\frac{1}{\lambda}}(n)^{\frac{1}{\lambda}}[C(h)]^{\frac{-1}{\lambda}} \end{aligned}$$

$$\begin{aligned}
\left[\frac{P(h)}{P}\right]^{-\lambda} &= [C]^{-1}(n)^{-1}C(h) \\
C(h) &= n\left[\frac{P(h)}{P}\right]^{-\lambda}C
\end{aligned}$$

and:

$$\begin{aligned}
P(f) &= P[C]^{\frac{1}{\lambda}}(1-n)^{\frac{1}{\lambda}}[C(f)]^{\frac{-1}{\lambda}} \\
\frac{P(f)}{P} &= [C]^{\frac{1}{\lambda}}(1-n)^{\frac{1}{\lambda}}[C(f)]^{\frac{-1}{\lambda}} \\
\left[\frac{P(f)}{P}\right]^{-\lambda} &= [C]^{-1}(1-n)^{-1}C(f) \\
C(f) &= (1-n)\left[\frac{P(f)}{P}\right]^{-\lambda}C
\end{aligned}$$

Combining these and the consumption index, we get the price index:

$$\begin{aligned}
C &= \left[(n)^{\frac{1}{\lambda}}[C(h)]^{\frac{\lambda-1}{\lambda}}+(1-n)^{\frac{1}{\lambda}}[C(f)]^{\frac{\lambda-1}{\lambda}}\right]^{\frac{\lambda}{\lambda-1}} \\
C &= \left[(n)^{\frac{1}{\lambda}}\left[n\left[\frac{P(h)}{P}\right]^{-\lambda}C\right]^{\frac{\lambda-1}{\lambda}}+(1-n)^{\frac{1}{\lambda}}\left[(1-n)\left[\frac{P(f)}{P}\right]^{-\lambda}C\right]^{\frac{\lambda-1}{\lambda}}\right]^{\frac{\lambda}{\lambda-1}} \\
C &= \left[n\left[\frac{P(h)}{P}\right]^{1-\lambda}+(1-n)\left[\frac{P(f)}{P}\right]^{1-\lambda}\right]^{\frac{\lambda}{\lambda-1}}C \\
1 &= n\left[\frac{P(h)}{P}\right]^{1-\lambda}+(1-n)\left[\frac{P(f)}{P}\right]^{1-\lambda} \\
[P]^{1-\lambda} &= n[P(h)]^{1-\lambda}+(1-n)[P(f)]^{1-\lambda} \\
P &= \left[n[P(h)]^{1-\lambda}+(1-n)[P(f)]^{1-\lambda}\right]^{\frac{1}{1-\lambda}}
\end{aligned}$$

2.1.3 Allocation across brands within subbaskets

The Household allocates demand across Home brands to minimize the cost of the Home, subject to a target value of the basket:

$$\mathcal{L} = \int_0^n P(z,h)\,C(z,h)\,dz-\varphi\left[\left[\left(\frac{1}{n}\right)^{\frac{1}{\theta}}\int_0^n [C(z,h)]^{\frac{\theta-1}{\theta}}\,dz\right]^{\frac{\theta}{\theta-1}}-C(h)\right]$$

The first order condition for a specific brand is:

$$\begin{aligned}
0 &= \frac{\partial\mathcal{L}_C}{\partial C(z,h)} \\
0 &= P(z,h)-\varphi\left[\left(\frac{1}{n}\right)^{\frac{1}{\theta}}\int_0^n [C(z,h)]^{\frac{\theta-1}{\theta}}\,dz\right]^{\frac{\theta}{\theta-1}-1}\left(\frac{1}{n}\right)^{\frac{1}{\theta}}[C(z,h)]^{\frac{\theta-1}{\theta}-1}
\end{aligned}$$

$$0 = P(z,h) - \varphi [C(h)]^{\frac{1}{\theta}} \left(\frac{1}{n}\right)^{\frac{1}{\theta}} [C(z,h)]^{\frac{-1}{\theta}}$$

The expenditure is then:

$$\begin{aligned}
\int_0^n P(z,h)\, C(z,h)\, dz &= \varphi [C(h)]^{\frac{1}{\theta}} \left[\left(\frac{1}{n}\right)^{\frac{1}{\theta}} \int_0^n [C(z,h)]^{\frac{\theta-1}{\theta}} dz\right] \\
P(h)\, C(h) &= \varphi [C(h)]^{\frac{1}{\theta}} [C(h)]^{\frac{\theta-1}{\theta}} \\
P(h) &= \varphi
\end{aligned}$$

The demand for a brand is then:

$$\begin{aligned}
P(z,h) &= \varphi [C(h)]^{\frac{1}{\theta}} \left(\frac{1}{n}\right)^{\frac{1}{\theta}} [C(z,h)]^{\frac{-1}{\theta}} \\
\frac{P(z,h)}{P(h)} &= [C(h)]^{\frac{1}{\theta}} \left(\frac{1}{n}\right)^{\frac{1}{\theta}} [C(z,h)]^{\frac{-1}{\theta}} \\
\left[\frac{P(z,h)}{P(h)}\right]^{-\theta} &= [C(h)]^{-1}\, nC(z,h) \\
C(z,h) &= \frac{1}{n}\left[\frac{P(z,h)}{P(h)}\right]^{-\theta} C(h)
\end{aligned}$$

Combining these and the consumption index, we get the price index:

$$\begin{aligned}
C(h) &= \left[\left(\frac{1}{n}\right)^{\frac{1}{\theta}} \int_0^n [C(z,h)]^{\frac{\theta-1}{\theta}} dz\right]^{\frac{\theta}{\theta-1}} \\
C(h) &= \left[\left(\frac{1}{n}\right)^{\frac{1}{\theta}} \int_0^n \left[\frac{1}{n}\left[\frac{P(z,h)}{P(h)}\right]^{-\theta} C(h)\right]^{\frac{\theta-1}{\theta}} dz\right]^{\frac{\theta}{\theta-1}} \\
C(h) &= \left[\frac{1}{n}\int_0^n \left[\frac{P(z,h)}{P(h)}\right]^{1-\theta} dz\right]^{\frac{\theta}{\theta-1}} C(h) \\
1 &= \frac{1}{n}\int_0^n \left[\frac{P(z,h)}{P(h)}\right]^{1-\theta} dz \\
[P(h)]^{1-\theta} &= \frac{1}{n}\int_0^n [P(z,h)]^{1-\theta} dz \\
P(h) &= \left[\frac{1}{n}\int_0^n [P(z,h)]^{1-\theta} dz\right]^{\frac{1}{1-\theta}}
\end{aligned}$$

Following similar steps, the demand for a Foreign brand is:

$$\begin{aligned}
C(z,f) &= \frac{1}{1-n}\left[\frac{P(z,f)}{P(f)}\right]^{-\theta} C(f) \\
P(f) &= \left[\frac{1}{1-n}\int_0^n [P(z,f)]^{1-\theta} dz\right]^{\frac{1}{1-\theta}}
\end{aligned}$$

2.1.4 Summary of consumption allocation

The various demands and price indices for the Home household are:

$$
\begin{aligned}
C(h) &= n\left[\frac{P(h)}{P}\right]^{-\lambda} C && (1)\\
C(f) &= (1-n)\left[\frac{P(f)}{P}\right]^{-\lambda} C && (2)\\
C(z,h) &= \frac{1}{n}\left[\frac{P(z,h)}{P(h)}\right]^{-\theta} C(h) = \left[\frac{P(z,h)}{P(h)}\right]^{-\theta}\left[\frac{P(h)}{P}\right]^{-\lambda} C && (3)\\
C(z,f) &= \frac{1}{1-n}\left[\frac{P(z,f)}{P(f)}\right]^{-\theta} C(f) = \left[\frac{P(z,f)}{P(f)}\right]^{-\theta}\left[\frac{P(f)}{P}\right]^{-\lambda} C && (4)\\
P(h) &= \left[\frac{1}{n}\int_0^n [P(z,h)]^{1-\theta} dz\right]^{\frac{1}{1-\theta}} && (5)\\
P(f) &= \left[\frac{1}{1-n}\int_0^n [P(z,f)]^{1-\theta} dz\right]^{\frac{1}{1-\theta}} && (6)\\
P &= \left[n[P(h)]^{1-\lambda} + (1-n)[P(f)]^{1-\lambda}\right]^{\frac{1}{1-\lambda}} && (7)
\end{aligned}
$$

The allocation for the household in the Foreign country is similar. The consumption baskets are:

$$
\begin{aligned}
C^* &= \left[(n)^{\frac{1}{\lambda}}[C^*(h)]^{\frac{\lambda-1}{\lambda}} + (1-n)^{\frac{1}{\lambda}}[C^*(f)]^{\frac{\lambda-1}{\lambda}}\right]^{\frac{\lambda}{\lambda-1}}\\
C^*(h) &= \left[\left(\frac{1}{n}\right)^{\frac{1}{\theta}}\int_0^n [C^*(z,h)]^{\frac{\theta-1}{\theta}} dz\right]^{\frac{\theta}{\theta-1}}\\
C^*(f) &= \left[\left(\frac{1}{1-n}\right)^{\frac{1}{\theta}}\int_n^1 [C^*(z,f)]^{\frac{\theta-1}{\theta}} dz\right]^{\frac{\theta}{\theta-1}}
\end{aligned}
$$

where $*$ denotes Foreign country variables. The demands and prices indices are ($*$ denotes prices in Foreign currency):

$$
\begin{aligned}
C^*(h) &= n\left[\frac{P^*(h)}{P^*}\right]^{-\lambda} C^* && (8)\\
C^*(f) &= (1-n)\left[\frac{P^*(f)}{P^*}\right]^{-\lambda} C^* && (9)\\
C^*(z,h) &= \frac{1}{n}\left[\frac{P^*(z,h)}{P^*(h)}\right]^{-\theta} C^*(h) = \left[\frac{P^*(z,h)}{P^*(h)}\right]^{-\theta}\left[\frac{P^*(h)}{P^*}\right]^{-\lambda} C^* && (10)\\
C^*(z,f) &= \frac{1}{1-n}\left[\frac{P^*(z,f)}{P^*(f)}\right]^{-\theta} C^*(f) = \left[\frac{P^*(z,f)}{P^*(f)}\right]^{-\theta}\left[\frac{P^*(f)}{P^*}\right]^{-\lambda} C^* && (11)\\
P^*(h) &= \left[\frac{1}{n}\int_0^n [P^*(z,h)]^{1-\theta} dz\right]^{\frac{1}{1-\theta}} && (12)
\end{aligned}
$$

$$P^*(f) = \left[\frac{1}{1-n}\int_0^n \left[P^*(z,f)\right]^{1-\theta} dz\right]^{\frac{1}{1-\theta}} \tag{13}$$

$$P^* = \left[n\left[P^*(h)\right]^{1-\lambda} + (1-n)\left[P^*(f)\right]^{1-\lambda}\right]^{\frac{1}{1-\lambda}} \tag{14}$$

2.2 Household's intertemporal allocation

2.2.1 Home country

The Home country Household maximizes the following utility:

$$U_t = \sum_{s=t}^{\infty} \beta^{s-t}\left[\frac{(C_s)^{1-\sigma}}{1-\sigma} + \chi \ln\left(\frac{M_s}{P_s}\right) - \frac{\kappa_s}{2}(L_s)^2\right] \tag{15}$$

where M denotes money holdings and L denotes labor.

The household can purchase domestic currency and a one-period bond denominated in Home currency (considering a bond in foreign currency is equivalent). The household's income consists of the revenue from wages WL, the profits from all firms in the country Π (treated as a lump sum amount), the interest income on her bond holdings, net of any lump sum taxes T. The budget constraint for a Home household is:

$$B_{t+1} + M_t + P_t C_t = (1+i_t) B_t + M_{t-1} + W_t L_t + \Pi_t - T_t \tag{16}$$

The Lagrangian of the Home household is :

$$\begin{aligned}
\mathcal{L} &= \sum_{s=t}^{\infty} \beta^{s-t}\left[\frac{\left(C_s^j\right)^{1-\sigma}}{1-\sigma} + \chi \ln\left(\frac{M_s}{P_s}\right) - \frac{\kappa_s}{2}(L_s)^2\right] \\
&\quad -\sum_{s=t}^{\infty} \beta^{s-t}\lambda_s\left[B_{s+1} + M_s + P_s C_s - (1+i_s) B_s - M_{s-1} - W_s L_s - \Pi_s + T_s\right]
\end{aligned}$$

The first order conditions with respect to consumption, bond holdings, cash balances, and labor are:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial C_t} &= 0 \Rightarrow \frac{1}{P_t (C_t)^{\sigma}} = \lambda_t \\
\frac{\partial \mathcal{L}}{\partial C_{t+1}} &= 0 \Rightarrow \frac{1}{P_{t+1} (C_{t+1})^{\sigma}} = \lambda_{t+1} \\
\frac{\partial \mathcal{L}}{\partial M_t} &= 0 \Rightarrow \chi \left(\frac{M_t}{P_t}\right)^{-1} \frac{1}{P_t} = \lambda_t - \beta\lambda_{t+1} \\
\frac{\partial \mathcal{L}}{\partial B_{t+1}} &= 0 \Rightarrow \lambda_t = \beta(1+i_{t+1})\lambda_{t+1} \\
\frac{\partial \mathcal{L}}{\partial L_t} &= 0 \Rightarrow \kappa_t L_t = \lambda_t W_t
\end{aligned}$$

Re-arranging these relations, we derive the Euler relation, the money demand, and the labor supply:

$$\begin{aligned}
(C_{t+1})^{\sigma} &= \beta\left(1+i_{t+1}\right)\frac{P_t}{P_{t+1}}\left(C_t\right)^{\sigma} && (17)\\
\frac{M_t}{P_t} &= \chi\left(C_t\right)^{\sigma}\frac{1+i_{t+1}}{i_{t+1}} && (18)\\
\kappa_t L_t\left(C_t\right)^{\sigma} &= \frac{W_t}{P_t} && (19)
\end{aligned}$$

2.2.2 Foreign country

The Foreign country Household maximizes the following utility:

$$U_t = \sum_{s=t}^{\infty}\beta^{s-t}\left[\frac{\left(C_s^*\right)^{1-\sigma}}{1-\sigma}+\chi\ln\left(\frac{M_s^*}{P_s^*}\right)-\frac{\kappa_s^*}{2}\left(L_s^*\right)^2\right] \tag{20}$$

subject to the following budget constraint, where $\mathcal{E}$ is the exchange rate in terms of units of Home currency per unit of Foreign currency (so an increase is a depreciation of the Home currency):

$$\frac{B_{t+1}^*}{\mathcal{E}_t}+M_t^*+P_t^*C_t^* = \left(1+i_t\right)\frac{B_t^*}{\mathcal{E}_t}+M_{t-1}^*+W_t^*L_t^*+\Pi_t^*-T_t^* \tag{21}$$

Following the same steps as for the Home households, we get the Euler relation, the money demand, and the labor supply:

$$\begin{aligned}
\left(C_{t+1}^*\right)^{\sigma} &= \beta\left(1+i_{t+1}\right)\frac{\mathcal{E}_t P_t^*}{\mathcal{E}_{t+1}P_{t+1}^*}\left(C_t^*\right)^{\sigma} && (22)\\
\frac{M_t^*}{P_t^*} &= \chi\left(C_t^*\right)^{\sigma}\frac{\left(1+i_{t+1}\right)\mathcal{E}_t}{\left(1+i_{t+1}\right)\mathcal{E}_t-\mathcal{E}_{t+1}} && (23)\\
\kappa_t^* L_t^*\left(C_t^*\right)^{\sigma} &= \frac{W_t^*}{P_t^*} && (24)
\end{aligned}$$

2.3 Government spending

The government in each country purchases goods following the same allocation as the household. The purchases are financed by money creation and taxes:

$$\begin{aligned}
P_t G_t &= \left(M_t - M_{t-1}\right) + T_t\\
P_t^* G_t^* &= \left(M_t^* - M_{t-1}^*\right) + T_t^*
\end{aligned}$$

where variables are expressed in per capita term.

2.4 Demand and optimal price setting

2.4.1 Home country

Each brand is produced by a firm using a linear technology that is linear in labor:

$$Y(z,h) = L(z,h)$$

The demand faced by the firm is computing using (3) and (10), as well as their equivalent for government spending. Recall that consumption and government spending are expressed in per capita terms, so we need to scale by the sizes of the two countries, n and $1-n$ respectively:

$$\begin{aligned}
Y(z,h) &= n\left(C(z,h)+G(z,h)\right)+(1-n)\left(C^*(z,h)+G^*(z,h)\right) \\
Y(z,h) &= n\left[\frac{P(z,h)}{P(h)}\right]^{-\theta}\left[\frac{P(h)}{P}\right]^{-\lambda}(C+G)+(1-n)\left[\frac{P^*(z,h)}{P^*(h)}\right]^{-\theta}\left[\frac{P^*(h)}{P^*}\right]^{-\lambda}(C^*+G^*)
\end{aligned}$$

If the firm can set prices, it chooses $P(z,h)$ for domestic sales and $P^*(z,h)$ for exports to maximize its profits:

$$\begin{aligned}
\Pi(z,h) &= (P(z,h)-W)\,n\left[\frac{P(z,h)}{P(h)}\right]^{-\theta}\left[\frac{P(h)}{P}\right]^{-\lambda}(C+G) \\
&\quad+(\mathcal{E}P^*(z,h)-W)(1-n)\left[\frac{P^*(z,h)}{P^*(h)}\right]^{-\theta}\left[\frac{P^*(h)}{P^*}\right]^{-\lambda}(C^*+G^*)
\end{aligned}$$

The first-order conditions are:

$$\begin{aligned}
0 &= \frac{\partial\Pi(z,h)}{\partial P(z,h)} \\
0 &= n\left[\frac{P(z,h)}{P(h)}\right]^{-\theta}\left[\frac{P(h)}{P}\right]^{-\lambda}(C+G) \\
&\quad-\theta(P(z,h)-W)\,n\left[\frac{P(z,h)}{P(h)}\right]^{-\theta-1}\frac{1}{P(h)}\left[\frac{P(h)}{P}\right]^{-\lambda}(C+G) \\
0 &= n\frac{P(z,h)}{P(h)}-\theta(P(z,h)-W)\,n\frac{1}{P(h)} \\
0 &= P(z,h)-\theta(P(z,h)-W) \\
P(z,h) &= \frac{\theta}{\theta-1}W
\end{aligned}$$

and:

$$\begin{aligned}
0 &= \frac{\partial\Pi(z,h)}{\partial P^*(z,h)} \\
0 &= \mathcal{E}(1-n)\left[\frac{P^*(z,h)}{P^*(h)}\right]^{-\theta}\left[\frac{P^*(h)}{P^*}\right]^{-\lambda}(C^*+G^*) \\
&\quad-\theta(\mathcal{E}P^*(z,h)-W)(1-n)\left[\frac{P^*(z,h)}{P^*(h)}\right]^{-\theta-1}\frac{1}{P^*(h)}\left[\frac{P^*(h)}{P^*}\right]^{-\lambda}(C^*+G^*)
\end{aligned}$$

$$
\begin{aligned}
0 &= \mathcal{E}(1-n)\frac{P^*(z,h)}{P^*(h)} - \theta\left(\mathcal{E}P^*(z,h) - W\right)(1-n)\frac{1}{P^*(h)} \\
0 &= \mathcal{E}P^*(z,h) - \theta\left(\mathcal{E}P^*(z,h) - W\right) \\
\mathcal{E}P^*(z,h) &= \frac{\theta}{\theta-1}W
\end{aligned}
$$

The optimal prices are the same for all firms, hence $P(z,h) = P(h)$ and $P^*(z,h) = P^*(h)$. The optimal flexible prices for domestic and exports are the same, in the same currency:

$$
P(h) = \mathcal{E}P^*(h) = \frac{\theta}{\theta-1}W \tag{25}
$$

2.4.2 Foreign country

The demand faced by the firm is computing using (4) and (11), as well as their equivalent for government spending:

$$
\begin{aligned}
Y^*(z,f) &= n\left(C(z,f) + G(z,f)\right) + (1-n)\left(C^*(z,f) + G^*(z,f)\right) \\
Y^*(z,f) &= n\left[\frac{P(z,f)}{P(f)}\right]^{-\theta}\left[\frac{P(f)}{P}\right]^{-\lambda}(C+G) + (1-n)\left[\frac{P^*(z,f)}{P^*(f)}\right]^{-\theta}\left[\frac{P^*(f)}{P^*}\right]^{-\lambda}(C^*+G^*)
\end{aligned}
$$

The profit maximization with flexible prices leads to:

$$
\frac{P(f)}{\mathcal{E}} = P^*(f) = \frac{\theta}{\theta-1}W^* \tag{26}
$$

Under flexible prices, purchasing parity holds: $\mathcal{E}P^* = P$.

2.5 Global equilibrium

As all firms in a country are identical, the per capita output is $Y_t = Y_t(z,h)$ and $Y_t^* = Y_t^*(z,f)$. The bonds traded internationally are in zero net supply:

$$
nB_t + (1-n)B_t^* = 0
$$

The output demands imply:

$$
Y_t = n\left[\frac{P_t(h)}{P_t}\right]^{-\lambda}(C_t+G_t) + (1-n)\left[\frac{P_t^*(h)}{P_t^*}\right]^{-\lambda}(C_t^*+G_t^*) \tag{27}
$$

$$
Y_t^* = n\left[\frac{P_t(f)}{P_t}\right]^{-\lambda}(C_t+G_t) + (1-n)\left[\frac{P_t^*(f)}{P_t^*}\right]^{-\lambda}(C_t^*+G_t^*) \tag{28}
$$

Profits are equal to sales net of wage costs:

$$
\begin{aligned}
\Pi_t &= P_t(h)\,n\left[\frac{P_t(h)}{P_t}\right]^{-\lambda}(C_t+G_t) + \mathcal{E}_tP_t^*(h)(1-n)\left[\frac{P_t^*(h)}{P_t^*}\right]^{-\lambda}(C_t^*+G_t^*) - W_tY_t \\
\Pi_t^* &= \frac{P_t(f)}{\mathcal{E}_t}n\left[\frac{P_t(f)}{P_t}\right]^{-\lambda}(C_t+G_t) + P_t^*(f)(1-n)\left[\frac{P_t^*(f)}{P_t^*}\right]^{-\lambda}(C_t^*+G_t^*) - W_t^*Y_t^*
\end{aligned}
$$

Combining the households' and governments' budget constraint, the per capita resource constraints are:

$$\begin{aligned}
B_{t+1}+M_t+P_tC_t &= (1+i_t)\,B_t+M_{t-1}+W_tL_t+\Pi_t-T_t \\
B_{t+1}+P_t\,(C_t+G_t) &= (1+i_t)\,B_t+W_tY_t+\Pi_t \\
B_{t+1}+P_t\,(C_t+G_t) &= (1+i_t)\,B_t+P_t\,(h)\,n\left[\frac{P_t\,(h)}{P_t}\right]^{-\lambda}(C_t+G_t) \\
&\quad +\mathcal{E}_tP_t^*\,(h)\,(1-n)\left[\frac{P_t^*\,(h)}{P_t^*}\right]^{-\lambda}(C_t^*+G_t^*)
\end{aligned} \tag{29}$$

and:

$$\begin{aligned}
-\frac{n}{1-n}\frac{B_{t+1}}{\mathcal{E}_t}+P_t^*\,(C_t^*+G_t^*) &= -\frac{n}{1-n}\,(1+i_t)\,\frac{B_t}{\mathcal{E}_t}+n\frac{P_t\,(f)}{\mathcal{E}_t}\left[\frac{P_t\,(f)}{P_t}\right]^{-\lambda}(C_t+G_t) \\
&\quad +(1-n)\,P_t^*\,(f)\left[\frac{P_t^*\,(f)}{P_t^*}\right]^{-\lambda}(C_t^*+G_t^*)
\end{aligned} \tag{30}$$

where we used $nB_t+(1-n)\,B_t^*=0$.

3 Log linear approximations

3.1 Symmetric steady state

Consider that all variables are constant and that there are no cross-country bond holdings: $B_0=B_0^*=0$, no government spending: $G_0=G_0^*=0$, and that the cost of effort is the same in the two countries, $\kappa_0^*=\kappa_0$.

The Euler conditions (17) or (22) pin down the nominal interest rate, which is also the real interest rate as inflation is zero:

$$1=\beta\,(1+i_0)$$

As purchasing power parity holds, (29) and (30) imply:

$$\begin{aligned}
C_0 &= \left[\frac{P_0\,(h)}{P_0}\right]^{1-\lambda}(nC_0+(1-n)\,C_0^*) \\
C_0^* &= \left[\frac{P_0\,(f)}{P_0}\right]^{1-\lambda}(nC_0+(1-n)\,C_0^*)
\end{aligned}$$

Combining the optimal Home price (25) and the labor supply (19) implies:

$$\begin{aligned}
P_0\,(h) &= \frac{\theta}{\theta-1}W_0 \\
P_0\,(h) &= \frac{\theta\kappa_0}{\theta-1}P_0L_0\,(C_0)^{\sigma} \\
\frac{P_0\,(h)}{P_0} &= \frac{\theta\kappa_0}{\theta-1}\,(C_0)^{\sigma}\,Y_0
\end{aligned}$$

$$
\begin{aligned}
\frac{P_0(h)}{P_0} &= \frac{\theta\kappa_0}{\theta-1}(C_0)^{\sigma}\left[\frac{P_0(h)}{P_0}\right]^{-\lambda}(nC_0+(1-n)C_0^*) \\
\frac{P_0(h)}{P_0} &= \frac{\theta\kappa_0}{\theta-1}\left[\left[\frac{P_0(h)}{P_0}\right]^{1-\lambda}(nC_0+(1-n)C_0^*)\right]^{\sigma}\left[\frac{P_0(h)}{P_0}\right]^{-\lambda}(nC_0+(1-n)C_0^*) \\
1 &= \frac{\theta\kappa_0}{\theta-1}\left[\frac{P_0(h)}{P_0}\right]^{(1-\lambda)\sigma-(1+\lambda)}(nC_0+(1-n)C_0^*)^{1+\sigma}
\end{aligned}
$$

We proceed similarly using (26) and (24):

$$
\begin{aligned}
P_0^*(f) &= \frac{\theta}{\theta-1}W_0^* \\
P_0^*(f) &= \frac{\theta\kappa_0}{\theta-1}P_0^*L_0^*(C_0^*)^{\sigma} \\
\frac{P_0^*(f)}{P_0^*} &= \frac{\theta\kappa_0}{\theta-1}(C_0^*)^{\sigma}Y_0^* \\
\frac{P_0(f)}{P_0} &= \frac{\theta\kappa_0}{\theta-1}(C_0^*)^{\sigma}\left[\frac{P_0(f)}{P_0}\right]^{-\lambda}(nC_0+(1-n)C_0^*) \\
\frac{P_0(f)}{P_0} &= \frac{\theta\kappa_0}{\theta-1}\left[\left[\frac{P_0(f)}{P_0}\right]^{1-\lambda}(nC_0+(1-n)C_0^*)\left[\frac{P_0(f)}{P_0}\right]^{-\lambda}\right]^{\sigma}(nC_0+(1-n)C_0^*) \\
1 &= \frac{\theta\kappa_0}{\theta-1}\left[\frac{P_0(f)}{P_0}\right]^{(1-\lambda)\sigma-(1+\lambda)}(nC_0+(1-n)C_0^*)^{1+\sigma}
\end{aligned}
$$

Combining these relations, we get $P_0(h) = P_0(f)$, which is also equal to P_0. This implies $C_0 = C_0^* = nC_0 + (1-n)C_0^*$, and we write:

$$
\begin{aligned}
1 &= \frac{\theta\kappa_0}{\theta-1}\left[\frac{P_0(h)}{P_0}\right]^{(1-\lambda)\sigma-(1+\lambda)}(nC_0+(1-n)C_0^*)^{1+\sigma} \\
1 &= \frac{\theta\kappa_0}{\theta-1}(C_0)^{1+\sigma} \\
C_0 &= \left(\frac{\theta-1}{\theta\kappa_0}\right)^{\frac{1}{1+\sigma}}
\end{aligned}
$$

Output is also equal to consumption: $Y_0 = C_0$.

The price level is given by the money market equilibrium (18):

$$
\begin{aligned}
\frac{M_0}{P_0} &= \chi(C_0)^{\sigma}\frac{1+i_0}{i_0} \\
\frac{M_0}{P_0} &= \chi(C_0)^{\sigma}\frac{(1+i_0)}{(1+i_0)-1} \\
\frac{M_0}{P_0} &= \chi(C_0)^{\sigma}\frac{\beta(1+i_0)}{\beta(1+i_0)-\beta} \\
\frac{M_0}{P_0} &= \chi\left(\frac{\theta-1}{\theta\kappa_0}\right)^{\frac{\sigma}{1+\sigma}}\frac{1}{1-\beta} \\
P_0 &= M_0\left(\frac{\theta\kappa_0}{\theta-1}\right)^{\frac{\sigma}{1+\sigma}}\frac{1-\beta}{\chi}
\end{aligned}
$$

Following similar steps on (23) we get:

$$P_0^* \quad = \quad M_0^* \left(\frac{\theta\kappa_0}{\theta-1}\right)^{\frac{\sigma}{1+\sigma}} \frac{1-\beta}{\chi}$$

Purchasing power parity implies that the exchange is the ratio of money supplies:

$$\mathcal{E}_0 = \frac{M_0}{M_0^*}$$

3.2 Linear approximations

We express the various relations in terms of log deviations from the symmetric steady state. We denote these deviations by San Serif letter. For instance:

$$\mathsf{c}_t \quad = \quad \ln C_t - \ln C_0 = \frac{C_t - C_0}{C_0}$$

The consumer price indexes (7) and (14) are:

$$\mathsf{p}_t = n\mathsf{p}_t\,(h) + (1-n)\,\mathsf{p}_t\,(f) \qquad ; \qquad \mathsf{p}_t^* = n\mathsf{p}_t^*\,(h) + (1-n)\,\mathsf{p}_t^*\,(f) \tag{31}$$

The Euler relations (17) and (22) are expanded as follows. Start with the Foreign country:

$$\begin{aligned} \left(C_{t+1}^*\right)^\sigma &= \beta\,(1+i_{t+1})\,\frac{\mathcal{E}_t P_t^*}{\mathcal{E}_{t+1} P_{t+1}^*}\,(C_t^*)^\sigma \\ (C_0)^\sigma\left[1+\sigma\mathsf{c}_{t+1}^*\right] &= (C_0)^\sigma + (C_0)^\sigma\,\sigma\mathsf{c}_t^* + \beta\,(C_0)^\sigma\,(i_{t+1}-i_0) \\ &\quad + (C_0)^\sigma\left(\mathsf{e}_t + \mathsf{p}_t^* - \mathsf{e}_{t+1} - \mathsf{p}_{t+1}^*\right) \\ \sigma\mathsf{c}_{t+1}^* &= \sigma\mathsf{c}_t^* + \beta\mathsf{i}_{t+1} + \mathsf{e}_t + \mathsf{p}_t^* - \mathsf{e}_{t+1} - \mathsf{p}_{t+1}^* \end{aligned} \tag{32}$$

Similarly for the home country:

$$\sigma\mathsf{c}_{t+1} \quad = \quad \sigma\mathsf{c}_t + \beta\mathsf{i}_{t+1} + \mathsf{p}_t - \mathsf{p}_{t+1} \tag{33}$$

The foreign money demand (23) is expanded as follows:

$$\begin{aligned} \frac{M_t^*}{P_t^*} &= \chi\,(C_t^*)^\sigma\,\frac{(1+i_{t+1})\,\mathcal{E}_t}{(1+i_{t+1})\,\mathcal{E}_t - \mathcal{E}_{t+1}} \\ \frac{M_0^*}{P_0^*}\,(1+\mathsf{m}_t^* - \mathsf{p}_t^*) &= (C_0)^\sigma\,\frac{\chi}{1-\beta} + (C_0)^\sigma\,\frac{\chi}{1-\beta}\sigma\mathsf{c}_t^* \\ &\quad + \chi\,(C_0)^\sigma\,\frac{[(1+i_0)-1]-(1+i_0)}{[(1+i_0)-1]^2}\,(i_{t+1}-i_0) \\ &\quad + \chi\,(C_0)^\sigma\,\frac{(1+i_0)\,[(1+i_0)-1]-(1+i_0)^2}{[(1+i_0)-1]^2}\mathsf{e}_t \\ &\quad + \chi\,(C_0)^\sigma\,\frac{(1+i_0)}{[(1+i_0)-1]^2}\mathsf{e}_{t+1} \end{aligned}$$

$$
\begin{aligned}
\frac{M_0^*}{P_0^*}\left(1+\mathsf{m}_t^*-\mathsf{p}_t^*\right) &= (C_0)^\sigma \frac{\chi}{1-\beta}+(C_0)^\sigma \frac{\chi}{1-\beta}\sigma \mathsf{c}_t^* \\
&+\chi (C_0)^\sigma \left[\frac{\frac{1-\beta}{\beta}-\frac{1}{\beta}}{\left[\frac{1-\beta}{\beta}\right]^2}\mathsf{i}_{t+1}+\frac{\frac{1}{\beta}\frac{1-\beta}{\beta}-\left(\frac{1}{\beta}\right)^2}{\left[\frac{1-\beta}{\beta}\right]^2}\mathsf{e}_t+\frac{\frac{1}{\beta}}{\left[\frac{1-\beta}{\beta}\right]^2}\mathsf{e}_{t+1}\right] \\
\mathsf{m}_t^*-\mathsf{p}_t^* &= \sigma \mathsf{c}_t^*-\frac{\beta}{1-\beta}\left(\beta \mathsf{i}_{t+1}+\mathsf{e}_t-\mathsf{e}_{t+1}\right) \qquad (34)
\end{aligned}
$$

Using similar steps on the home money demand (18) we get:

$$
\mathsf{m}_t-\mathsf{p}_t = \sigma \mathsf{c}_t-\frac{\beta}{1-\beta}\beta \mathsf{i}_{t+1} \qquad (35)
$$

The output demand of a representative home firm (27) is written as:

$$
\begin{aligned}
Y_t &= n\left[\frac{P_t(h)}{P_t}\right]^{-\lambda}(C_t+G_t)+(1-n)\left[\frac{P_t^*(h)}{P_t^*}\right]^{-\lambda}(C_t^*+G_t^*) \\
C_0\left[1+\mathsf{y}_t\right] &= C_0-\lambda n C_0\left[\mathsf{p}_t(h)-\mathsf{p}_t\right]-\lambda(1-n)C_0\left[\mathsf{p}_t^*(h)-\mathsf{p}_t^*\right] \\
&+C_0\left[n\left(\mathsf{c}_t+\mathsf{g}_t\right)+(1-n)\left(\mathsf{c}_t^*+\mathsf{g}_t^*\right)\right] \\
\mathsf{y}_t &= -\lambda n\left(\mathsf{p}_t(h)-\mathsf{p}_t\right)-\lambda(1-n)\left(\mathsf{p}_t^*(h)-\mathsf{p}_t^*\right) \qquad (36) \\
&+\left(n\mathsf{c}_t+(1-n)\mathsf{c}_t^*\right)+\left(n\mathsf{g}_t+(1-n)\mathsf{g}_t^*\right)
\end{aligned}
$$

where: $\mathsf{g}_t=\frac{G_t}{C_0}$ and $\mathsf{g}_t^*=\frac{G_t^*}{C_0}$. Similarly the foreign output (28) is:

$$
\begin{aligned}
\mathsf{y}_t^* &= -\lambda n\left(\mathsf{p}_t(f)-\mathsf{p}_t\right)-\lambda(1-n)\left(\mathsf{p}_t^*(f)-\mathsf{p}_t^*\right) \qquad (37) \\
&+\left(n\mathsf{c}_t+(1-n)\mathsf{c}_t^*\right)+\left(n\mathsf{g}_t+(1-n)\mathsf{g}_t^*\right)
\end{aligned}
$$

The foreign current account (30) is written as:

$$
\begin{aligned}
-\frac{n}{1-n}B_{t+1}+\mathcal{E}_t P_t^*\left(C_t^*+G_t^*\right) &= -\frac{n}{1-n}(1+i_t)B_t+nP_t(f)\left[\frac{P_t(f)}{P_t}\right]^{-\lambda}(C_t+G_t) \\
&+(1-n)\mathcal{E}_t P_t^*(f)\left[\frac{P_t^*(f)}{P_t^*}\right]^{-\lambda}(C^*+G^*) \\
-\frac{n}{1-n}P_0C_0\mathsf{b}_{t+1} &= -P_0C_0-P_0C_0\left(\mathsf{c}_t^*+\mathsf{g}_t^*\right)-P_0C_0\left(\mathsf{e}_t+\mathsf{p}_t^*\right) \\
&-\frac{1}{\beta}\frac{n}{1-n}P_0C_0\mathsf{b}_t+nP_0C_0+(1-n)P_0C_0 \\
&+nP_0C_0\left[\mathsf{c}_t+\mathsf{g}_t+\mathsf{p}_t(f)-\lambda\left(\mathsf{p}_t(f)-\mathsf{p}_t\right)\right] \\
&+(1-n)P_0C_0\left[\mathsf{c}_t^*+\mathsf{g}_t^*+\mathsf{e}_t+\mathsf{p}_t^*(f)-\lambda\left(\mathsf{p}_t^*(f)-\mathsf{p}_t^*\right)\right] \\
-\frac{n}{1-n}\mathsf{b}_{t+1}+\mathsf{c}_t^*+\mathsf{g}_t^* &= -\frac{1}{\beta}\frac{n}{1-n}\mathsf{b}_t+n\mathsf{p}_t(f)+(1-n)\left(\mathsf{e}_t+\mathsf{p}_t^*(f)\right) \qquad (38) \\
&-\left(\mathsf{e}_t+\mathsf{p}_t^*\right)+\mathsf{y}_t^*
\end{aligned}
$$

where $\mathsf{b}_t = \frac{B_t}{P_0 C_0}$. Following similar steps, we linearize (29) as:

$$\mathsf{b}_{t+1} + \mathsf{c}_t + \mathsf{g}_t = \frac{1}{\beta}\mathsf{b}_t + n\mathsf{p}_t(h) + (1-n)(\mathsf{e}_t + \mathsf{p}_t^*(h)) - \mathsf{p}_t + \mathsf{y}_t \tag{39}$$

Combining the optimal price (25) with the labor supply (19) in the Home country, we write:

$$\begin{aligned} P_t(h) &= \frac{\theta\kappa_t}{\theta-1} P_t Y_t (C_t)^{\sigma} \\ P_0[1+\mathsf{p}_t(h)] &= \frac{\theta\kappa_0}{\theta-1} P_0 Y_0 (C_0)^{\sigma}[1-\mathsf{a}_t + \mathsf{p}_t + \mathsf{y}_t + \sigma\mathsf{c}_t] \\ \mathsf{p}_t(h) - \mathsf{p}_t &= -\mathsf{a}_t + \mathsf{y}_t + \sigma\mathsf{c}_t \end{aligned} \tag{40}$$

where we defined a productivity gains as $\mathsf{a}_t = -\kappa_t$. In addition $\mathsf{p}_t(h) = \mathsf{e}_t + \mathsf{p}_t^*(h)$. This equation only applies when prices are flexible. Following similar steps, (26) and (24) imply:

$$\mathsf{p}_t^*(f) - \mathsf{p}_t^* = -\mathsf{a}_t^* + \mathsf{y}_t^* + \sigma\mathsf{c}_t^* \tag{41}$$

4 Solution of the model

4.1 General approach

We solve the model assuming that prices don't adjust for one period (the short run), and then fully adjust from the following period onwards (the long run), bring the economy to a new steady state.

We consider that the economy is initially at the symmetric steady state. It is then hit by unexpected and permanent shocks in the money supply, $\bar{\mathsf{m}}$ and $\bar{\mathsf{m}}^*$, government spending, $\bar{\mathsf{g}}$ and $\bar{\mathsf{g}}^*$, and productivity, $\bar{\mathsf{a}}$ and $\bar{\mathsf{a}}^*$.

It is convenient to first write the results taking worldwide averages : $\mathsf{x}_t^W = n\mathsf{x}_t + (1-n)\mathsf{x}_t^*$, and then in terms of cross-country differences: $\mathsf{x}_t - \mathsf{x}_t^*$. The results for each country are then computed as $\mathsf{x}_t = \mathsf{x}_t^W + (1-n)(\mathsf{x}_t - \mathsf{x}_t^*)$ and $\mathsf{x}_t^* = \mathsf{x}_t^W - n(\mathsf{x}_t - \mathsf{x}_t^*)$.

For convenience, we rewrite the equations used in the analysis, namely the consumer price indices, the Euler conditions, the money market equilibria, the outputs, the current accounts, and the optimal prices (which only apply when prices are flexible)

$$\begin{aligned} \mathsf{p}_t &= n\mathsf{p}_t(h) + (1-n)\mathsf{p}_t(f) \\ \mathsf{p}_t^* &= n\mathsf{p}_t^*(h) + (1-n)\mathsf{p}_t^*(f) \\ \sigma\mathsf{c}_{t+1} &= \sigma\mathsf{c}_t + \beta\mathsf{i}_{t+1} + \mathsf{p}_t - \mathsf{p}_{t+1} \\ \sigma\mathsf{c}_{t+1}^* &= \sigma\mathsf{c}_t^* + \beta\mathsf{i}_{t+1} + \mathsf{e}_t + \mathsf{p}_t^* - \mathsf{e}_{t+1} - \mathsf{p}_{t+1}^* \\ \mathsf{m}_t - \mathsf{p}_t &= \sigma\mathsf{c}_t - \frac{\beta}{1-\beta}\beta\mathsf{i}_{t+1} \\ \mathsf{m}_t^* - \mathsf{p}_t^* &= \sigma\mathsf{c}_t^* - \frac{\beta}{1-\beta}(\beta\mathsf{i}_{t+1} + \mathsf{e}_t - \mathsf{e}_{t+1}) \\ \mathsf{y}_t &= -\lambda n(\mathsf{p}_t(h) - \mathsf{p}_t) - \lambda(1-n)(\mathsf{p}_t^*(h) - \mathsf{p}_t^*) \end{aligned}$$

$$
\begin{aligned}
& +\left(n\mathsf{c}_t+(1-n)\,\mathsf{c}_t^*\right)+\left(n\mathsf{g}_t+(1-n)\,\mathsf{g}_t^*\right) \\
\mathsf{y}_t^* &= -\lambda n\left(\mathsf{p}_t(f)-\mathsf{p}_t\right)-\lambda(1-n)\left(\mathsf{p}_t^*(f)-\mathsf{p}_t^*\right) \\
& +\left(n\mathsf{c}_t+(1-n)\,\mathsf{c}_t^*\right)+\left(n\mathsf{g}_t+(1-n)\,\mathsf{g}_t^*\right) \\
\mathsf{b}_{t+1}+\mathsf{c}_t+\mathsf{g}_t &= \frac{1}{\beta}\mathsf{b}_t+n\mathsf{p}_t(h)+(1-n)\left(\mathsf{e}_t+\mathsf{p}_t^*(h)\right)-\mathsf{p}_t+\mathsf{y}_t \\
-\frac{n}{1-n}\mathsf{b}_{t+1}+\mathsf{c}_t^*+\mathsf{g}_t^* &= -\frac{1}{\beta}\frac{n}{1-n}\mathsf{b}_t+n\mathsf{p}_t(f)+(1-n)\left(\mathsf{e}_t+\mathsf{p}_t^*(f)\right)-\left(\mathsf{e}_t+\mathsf{p}_t^*\right)+\mathsf{y}_t^* \\
\mathsf{p}_t(h)-\mathsf{p}_t &= -\mathsf{a}_t+\mathsf{y}_\mathsf{t}+\sigma\mathsf{c}_t=\mathsf{e}_t+\mathsf{p}_t^*(h)-\mathsf{p}_t \\
\mathsf{p}_t^*(f)-\mathsf{p}_t^* &= -\mathsf{a}_t^*+\mathsf{y}_t^*+\sigma\mathsf{c}_t^*=\mathsf{p}_t(f)-\mathsf{e}_t-\mathsf{p}_t^*
\end{aligned}
$$

4.2 Long run

4.2.1 Worldwide averages

Once the economy reaches a new steady state, all variables are constant. We denote long run variables with an upper bar. The Euler conditions imply that $\bar{\imath}=0$.

We start with worldwide averages. The price indices are:

$$
\bar{\mathsf{p}}^W = n\left[n\bar{\mathsf{p}}(h)+(1-n)\,\bar{\mathsf{p}}(f)\right]+(1-n)\left[n\bar{\mathsf{p}}^*(h)+(1-n)\,\bar{\mathsf{p}}^*(f)\right]
$$

As the optimal price setting implies $\bar{\mathsf{p}}(h)=\bar{\mathsf{e}}+\bar{\mathsf{p}}^*(h)$ and $\bar{\mathsf{p}}^*(f)=\bar{\mathsf{p}}(f)-\bar{\mathsf{e}}$, the expression simplifies to:

$$
\begin{aligned}
\bar{\mathsf{p}}^W &= n\left[n\bar{\mathsf{p}}(h)+(1-n)\,\bar{\mathsf{p}}^*(f)+(1-n)\,\bar{\mathsf{e}}\right] \\
& +(1-n)\left[n\bar{\mathsf{p}}(h)-n\bar{\mathsf{e}}+(1-n)\,\bar{\mathsf{p}}^*(f)\right] \\
\bar{\mathsf{p}}^W &= n\bar{\mathsf{p}}(h)+(1-n)\,\bar{\mathsf{p}}^*(f)
\end{aligned}
$$

The money demand is:

$$
\bar{\mathsf{m}}^W-\bar{\mathsf{p}}^W = \sigma\bar{\mathsf{c}}^W
$$

The outputs are (the current accounts give the same result):

$$
\begin{aligned}
\bar{\mathsf{y}}^W &= -\lambda n\left[n\left(\bar{\mathsf{p}}(h)-\bar{\mathsf{p}}\right)+(1-n)\left(\bar{\mathsf{p}}^*(h)-\bar{\mathsf{p}}^*\right)\right] \\
& -\lambda(1-n)\left[n\left(\bar{\mathsf{p}}(f)-\bar{\mathsf{p}}\right)+(1-n)\left(\bar{\mathsf{p}}^*(f)-\bar{\mathsf{p}}^*\right)\right] \\
& +\bar{\mathsf{c}}^W+\bar{\mathsf{g}}^W \\
\bar{\mathsf{y}}^W &= -\lambda n\left[n\left(\bar{\mathsf{p}}(h)-\bar{\mathsf{p}}\right)+(1-n)\left(\bar{\mathsf{p}}(h)-\bar{\mathsf{e}}-\bar{\mathsf{p}}^*\right)\right] \\
& -\lambda(1-n)\left[n\left(\bar{\mathsf{p}}^*(f)+\bar{\mathsf{e}}-\bar{\mathsf{p}}\right)+(1-n)\left(\bar{\mathsf{p}}^*(f)-\bar{\mathsf{p}}^*\right)\right] \\
& +\bar{\mathsf{c}}^W+\bar{\mathsf{g}}^W \\
\bar{\mathsf{y}}^W &= -\lambda n\left[\bar{\mathsf{p}}(h)-n\bar{\mathsf{p}}-(1-n)\,\bar{\mathsf{p}}^*\right] \\
& -\lambda(1-n)\left[\bar{\mathsf{p}}^*(f)-n\bar{\mathsf{p}}-(1-n)\,\bar{\mathsf{p}}^*\right] \\
& +\bar{\mathsf{c}}^W+\bar{\mathsf{g}}^W
\end{aligned}
$$

$$
\begin{aligned}
\bar{\mathsf{y}}^{W} &= -\lambda\left[n\bar{\mathsf{p}}(h)+(1-n)\bar{\mathsf{p}}^{*}(f)-\bar{\mathsf{p}}^{W}\right] \\
&\quad +\bar{\mathsf{c}}^{W}+\bar{\mathsf{g}}^{W} \\
\bar{\mathsf{y}}^{W} &= \bar{\mathsf{c}}^{W}+\bar{\mathsf{g}}^{W}
\end{aligned}
$$

The optimal price setting relations are:

$$
\begin{aligned}
n\bar{\mathsf{p}}(h)+(1-n)\bar{\mathsf{p}}^{*}(f)-\bar{\mathsf{p}}^{W} &= -\bar{\mathsf{a}}^{W}+\bar{\mathsf{y}}^{W}+\sigma\bar{\mathsf{c}}^{W} \\
0 &= -\bar{\mathsf{a}}^{W}+\bar{\mathsf{y}}^{W}+\sigma\bar{\mathsf{c}}^{W}
\end{aligned}
$$

Combining the optimal prices and output gives the consumption:

$$
\begin{aligned}
0 &= -\bar{\mathsf{a}}^{W}+\bar{\mathsf{y}}^{W}+\sigma\bar{\mathsf{c}}^{W} \\
0 &= -\bar{\mathsf{a}}^{W}+\bar{\mathsf{c}}^{W}+\bar{\mathsf{g}}^{W}+\sigma\bar{\mathsf{c}}^{W} \\
\bar{\mathsf{c}}^{W} &= \frac{1}{1+\sigma}\left(\bar{\mathsf{a}}^{W}-\bar{\mathsf{g}}^{W}\right) \qquad (42)
\end{aligned}
$$

Output is:

$$
\begin{aligned}
\bar{\mathsf{y}}^{W} &= \bar{\mathsf{c}}^{W}+\bar{\mathsf{g}}^{W} \\
\bar{\mathsf{y}}^{W} &= \frac{1}{1+\sigma}\bar{\mathsf{a}}^{W}+\frac{\sigma}{1+\sigma}\bar{\mathsf{g}}^{W} \qquad (43)
\end{aligned}
$$

The money market equilibrium gives the price level:

$$
\begin{aligned}
\bar{\mathsf{m}}^{W}-\bar{\mathsf{p}}^{W} &= \sigma\bar{\mathsf{c}}^{W} \\
\bar{\mathsf{p}}^{W} &= \bar{\mathsf{m}}^{W}+\frac{1}{1+\sigma}\left(\bar{\mathsf{g}}^{W}-\bar{\mathsf{a}}^{W}\right) \qquad (44)
\end{aligned}
$$

4.2.2 Cross-country differences

Purchasing power parity holds: $\bar{\mathsf{p}}-\bar{\mathsf{p}}^{*}=\bar{\mathsf{e}}$. The difference of outputs is:

$$
\begin{aligned}
\bar{\mathsf{y}}-\bar{\mathsf{y}}^{*} &= -\lambda\left[n\left(\bar{\mathsf{p}}(h)-\bar{\mathsf{p}}\right)+(1-n)\left(\bar{\mathsf{p}}^{*}(h)-\bar{\mathsf{p}}^{*}\right)\right] \\
&\quad +\lambda\left[n\left(\bar{\mathsf{p}}(f)-\bar{\mathsf{p}}\right)+(1-n)\left(\bar{\mathsf{p}}^{*}(f)-\bar{\mathsf{p}}^{*}\right)\right] \\
\bar{\mathsf{y}}-\bar{\mathsf{y}}^{*} &= -\lambda\left[n\bar{\mathsf{p}}(h)+(1-n)\left(\bar{\mathsf{p}}(h)-\bar{\mathsf{e}}\right)\right] \\
&\quad +\lambda\left[n\left(\bar{\mathsf{p}}^{*}(f)+\bar{\mathsf{e}}\right)+(1-n)\bar{\mathsf{p}}^{*}(f)\right] \\
\bar{\mathsf{y}}-\bar{\mathsf{y}}^{*} &= -\lambda\left[\bar{\mathsf{p}}(h)-\bar{\mathsf{p}}^{*}(f)-\bar{\mathsf{e}}\right]
\end{aligned}
$$

The difference of current accounts is:

$$
\frac{1}{1-n}\bar{\mathsf{b}}+\left(\bar{\mathsf{c}}-\bar{\mathsf{c}}^{*}\right)+\left(\bar{\mathsf{g}}-\bar{\mathsf{g}}^{*}\right) = \frac{1}{\beta}\frac{1}{1-n}\bar{\mathsf{b}}+\left[\bar{\mathsf{p}}(h)-\bar{\mathsf{p}}^{*}(f)-\bar{\mathsf{e}}\right]+\left(\bar{\mathsf{y}}-\bar{\mathsf{y}}^{*}\right)
$$

The difference of optimal prices is:

$$
\bar{\mathsf{p}}(h)-\bar{\mathsf{p}}^{*}(f)-\bar{\mathsf{e}} = -\left(\bar{\mathsf{a}}-\bar{\mathsf{a}}^{*}\right)+\left(\bar{\mathsf{y}}-\bar{\mathsf{y}}^{*}\right)+\sigma\left(\bar{\mathsf{c}}-\bar{\mathsf{c}}^{*}\right)
$$

Combining the output and optimal prices we obtain:

$$\begin{aligned}
\bar{\mathsf{p}}(h)-\bar{\mathsf{p}}^*(f)-\bar{\mathsf{e}} &= -(\bar{\mathsf{a}}-\bar{\mathsf{a}}^*)+(\bar{\mathsf{y}}-\bar{\mathsf{y}}^*)+\sigma(\bar{\mathsf{c}}-\bar{\mathsf{c}}^*) \\
\bar{\mathsf{p}}(h)-\bar{\mathsf{p}}^*(f)-\bar{\mathsf{e}} &= -(\bar{\mathsf{a}}-\bar{\mathsf{a}}^*)-\lambda[\bar{\mathsf{p}}(h)-\bar{\mathsf{p}}^*(f)-\bar{\mathsf{e}}]+\sigma(\bar{\mathsf{c}}-\bar{\mathsf{c}}^*) \\
(1+\lambda)[\bar{\mathsf{p}}(h)-\bar{\mathsf{p}}^*(f)-\bar{\mathsf{e}}] &= -(\bar{\mathsf{a}}-\bar{\mathsf{a}}^*)+\sigma(\bar{\mathsf{c}}-\bar{\mathsf{c}}^*) \\
\bar{\mathsf{c}}-\bar{\mathsf{c}}^* &= \frac{1+\lambda}{\sigma}[\bar{\mathsf{p}}(h)-\bar{\mathsf{p}}^*(f)-\bar{\mathsf{e}}]+\frac{1}{\sigma}(\bar{\mathsf{a}}-\bar{\mathsf{a}}^*)
\end{aligned}$$

The current account is then:

$$\begin{aligned}
\frac{1}{1-n}\bar{\mathsf{b}}+(\bar{\mathsf{c}}-\bar{\mathsf{c}}^*)+(\bar{\mathsf{g}}-\bar{\mathsf{g}}^*) &= \frac{1}{\beta}\frac{1}{1-n}\bar{\mathsf{b}}+[\bar{\mathsf{p}}(h)-\bar{\mathsf{p}}^*(f)-\bar{\mathsf{e}}]+(\bar{\mathsf{y}}-\bar{\mathsf{y}}^*) \\
\frac{1+\lambda}{\sigma}[\bar{\mathsf{p}}(h)-\bar{\mathsf{p}}^*(f)-\bar{\mathsf{e}}] &= \frac{1-\beta}{\beta}\frac{1}{1-n}\bar{\mathsf{b}}+(1-\lambda)[\bar{\mathsf{p}}(h)-\bar{\mathsf{p}}^*(f)-\bar{\mathsf{e}}]-(\bar{\mathsf{g}}-\bar{\mathsf{g}}^*)-\frac{1}{\sigma}(\bar{\mathsf{a}}-\bar{\mathsf{a}}^*) \\
\frac{1-\sigma+\lambda(1+\sigma)}{\sigma}[\bar{\mathsf{p}}(h)-\bar{\mathsf{p}}^*(f)-\bar{\mathsf{e}}] &= \frac{1-\beta}{\beta}\frac{1}{1-n}\bar{\mathsf{b}}-(\bar{\mathsf{g}}-\bar{\mathsf{g}}^*)-\frac{1}{\sigma}(\bar{\mathsf{a}}-\bar{\mathsf{a}}^*) \\
\bar{\mathsf{p}}(h)-\bar{\mathsf{p}}^*(f)-\bar{\mathsf{e}} &= \frac{\sigma}{\sigma(\lambda-1)+(1+\lambda)}\left[\frac{1-\beta}{\beta}\frac{\bar{\mathsf{b}}}{1-n}-(\bar{\mathsf{g}}-\bar{\mathsf{g}}^*)-\frac{1}{\sigma}(\bar{\mathsf{a}}-\bar{\mathsf{a}}^*)\right] \qquad (45)
\end{aligned}$$

Output and consumption are then:

$$\begin{aligned}
\bar{\mathsf{y}}-\bar{\mathsf{y}}^* &= \frac{\sigma\lambda}{\sigma(\lambda-1)+(1+\lambda)}\left[-\frac{1-\beta}{\beta}\frac{\bar{\mathsf{b}}}{1-n}+(\bar{\mathsf{g}}-\bar{\mathsf{g}}^*)+\frac{1}{\sigma}(\bar{\mathsf{a}}-\bar{\mathsf{a}}^*)\right] \qquad (46) \\
\bar{\mathsf{c}}-\bar{\mathsf{c}}^* &= \frac{1+\lambda}{\sigma(\lambda-1)+(1+\lambda)}\left[\frac{1-\beta}{\beta}\frac{\bar{\mathsf{b}}}{1-n}-(\bar{\mathsf{g}}-\bar{\mathsf{g}}^*)\right]+\frac{\lambda-1}{1-\sigma+\lambda(1+\sigma)}(\bar{\mathsf{a}}-\bar{\mathsf{a}}^*) \qquad (47)
\end{aligned}$$

The money market gives the exchange rate:

$$\begin{aligned}
\bar{\mathsf{m}}-\bar{\mathsf{m}}^*-(\bar{\mathsf{p}}-\bar{\mathsf{p}}^*) &= \sigma(\bar{\mathsf{c}}-\bar{\mathsf{c}}^*) \\
\bar{\mathsf{p}}-\bar{\mathsf{p}}^* &= (\bar{\mathsf{m}}-\bar{\mathsf{m}}^*)-\sigma(\bar{\mathsf{c}}-\bar{\mathsf{c}}^*) \\
\bar{\mathsf{e}} &= (\bar{\mathsf{m}}-\bar{\mathsf{m}}^*)-\frac{\sigma(1+\lambda)}{\sigma(\lambda-1)+(1+\lambda)}\left[\frac{1-\beta}{\beta}\frac{\bar{\mathsf{b}}}{1-n}-(\bar{\mathsf{g}}-\bar{\mathsf{g}}^*)\right] \qquad (48) \\
&\quad -\frac{\sigma(\lambda-1)}{\sigma(\lambda-1)+(1+\lambda)}(\bar{\mathsf{a}}-\bar{\mathsf{a}}^*)
\end{aligned}$$

4.3 Short run

4.3.1 Price adjustment

In the short run, prices are preset and the optimal price setting does not hold. The prices of domestic sales do not change. We assume that a fraction s of exchange rate movements is transmitted to the price of imported goods:

$$\begin{aligned}
0 &= \mathsf{p}(h)=\mathsf{p}^*(f) \\
\mathsf{p}^*(h) &= -s\mathsf{e}
\end{aligned}$$

$$\mathsf{p}(f) = s\mathsf{e}$$

The price indices are then:

$$\begin{aligned} \mathsf{p} &= (1-n)\, s\mathsf{e} \\ \mathsf{p}^* &= -ns\mathsf{e} \\ \mathsf{p}-\mathsf{p}^* &= s\mathsf{e} \end{aligned}$$

4.3.2 Exchange rate dynamics

In terms of cross-country differences, the Euler conditions are:

$$\begin{aligned} \sigma(\bar{\mathsf{c}}-\bar{\mathsf{c}}^*) &= \sigma(\mathsf{c}-\mathsf{c}^*) + (\mathsf{p}-\mathsf{p}^*-\mathsf{e}) - (\bar{\mathsf{p}}-\bar{\mathsf{p}}^*-\bar{\mathsf{e}}) \\ \sigma(\bar{\mathsf{c}}-\bar{\mathsf{c}}^*) &= \sigma(\mathsf{c}-\mathsf{c}^*) - (1-s)\,\mathsf{e} \end{aligned}$$

The money market equilibrium implies:

$$\begin{aligned} \bar{\mathsf{m}}-\bar{\mathsf{m}}^*-(\mathsf{p}-\mathsf{p}^*) &= \sigma(\mathsf{c}-\mathsf{c}^*) + \frac{\beta}{1-\beta}(\mathsf{e}-\bar{\mathsf{e}}) \\ \bar{\mathsf{m}}-\bar{\mathsf{m}}^*-s\mathsf{e} &= \sigma(\mathsf{c}-\mathsf{c}^*) + \frac{\beta}{1-\beta}(\mathsf{e}-\bar{\mathsf{e}}) \end{aligned}$$

Using the money market equilibrium and its long run equivalent, we rewrite the Euler as:

$$\begin{aligned} \sigma(\bar{\mathsf{c}}-\bar{\mathsf{c}}^*) &= \sigma(\mathsf{c}-\mathsf{c}^*) - (1-s)\,\mathsf{e} \\ \bar{\mathsf{m}}-\bar{\mathsf{m}}^*-(\bar{\mathsf{p}}-\bar{\mathsf{p}}^*) &= \bar{\mathsf{m}}-\bar{\mathsf{m}}^*-s\mathsf{e}-\frac{\beta}{1-\beta}(\mathsf{e}-\bar{\mathsf{e}}) - (1-s)\,\mathsf{e} \\ -\bar{\mathsf{e}} &= -s\mathsf{e}-\frac{\beta}{1-\beta}(\mathsf{e}-\bar{\mathsf{e}}) - (1-s)\,\mathsf{e} \\ \mathsf{e}-\bar{\mathsf{e}} &= -\frac{\beta}{1-\beta}(\mathsf{e}-\bar{\mathsf{e}}) \\ 0 &= -\frac{1}{1-\beta}(\mathsf{e}-\bar{\mathsf{e}}) \\ \mathsf{e} &= \bar{\mathsf{e}} \qquad (49) \end{aligned}$$

4.3.3 Worldwide averages

In worldwide terms, the price index is zero: $\mathsf{p}^W = 0$. The Euler condition is, using the long run solution $\bar{\mathsf{m}}^W = \bar{\mathsf{p}}^W + \bar{\mathsf{c}}^W$:

$$\begin{aligned} \sigma\bar{\mathsf{c}}^W &= \sigma\mathsf{c}^W + \beta\mathsf{i} + \mathsf{p}^W - \bar{\mathsf{p}}^W \\ \bar{\mathsf{m}}^W &= \sigma\mathsf{c}^W + \beta\mathsf{i} \end{aligned}$$

Using (49) the money market equilibrium is:

$$\begin{aligned}\bar{\mathsf{m}}^W - \mathsf{p}^W &= \sigma \mathsf{c}^W - \frac{\beta}{1-\beta}\beta \mathsf{i} \\ \bar{\mathsf{m}}^W &= \sigma \mathsf{c}^W - \frac{\beta}{1-\beta}\beta \mathsf{i}\end{aligned}$$

Combining the Euler and the money demands, we get:

$$\begin{aligned}\sigma \mathsf{c}^W + \beta \mathsf{i} &= \sigma \mathsf{c}^W - \frac{\beta}{1-\beta}\beta \mathsf{i} \\ \mathsf{i} &= 0 \qquad (50)\end{aligned}$$

hence:

$$\mathsf{c}^W = \frac{1}{\sigma}\bar{\mathsf{m}}^W \qquad (51)$$

The ouptut is:

$$\begin{aligned}\mathsf{y}^W &= -\lambda n \left[n\left(-\mathsf{p}\right) + (1-n)\left(-s\mathsf{e} - \mathsf{p}^*\right)\right] \\ &\quad -\lambda (1-n)\left[n\left(s\mathsf{e} - \mathsf{p}\right) + (1-n)\left(-\mathsf{p}^*\right)\right] \\ &\quad +\mathsf{c}^W + \bar{\mathsf{g}}^W \\ \mathsf{y}^W &= -\lambda n \left[-n(1-n)s\mathsf{e} - (1-n)^2 s\mathsf{e}\right] \\ &\quad -\lambda (1-n)\left[n^2 s\mathsf{e} + (1-n) n s\mathsf{e}\right] \\ &\quad +\mathsf{c}^W + \bar{\mathsf{g}}^W \\ \mathsf{y}^W &= \lambda n (1-n) s\mathsf{e} - \lambda n (1-n) s\mathsf{e} + \mathsf{c}^W + \bar{\mathsf{g}}^W \\ \mathsf{y}^W &= \mathsf{c}^W + \bar{\mathsf{g}}^W \\ \mathsf{y}^W &= \frac{1}{\sigma}\bar{\mathsf{m}}^W + \bar{\mathsf{g}}^W \qquad (52)\end{aligned}$$

4.3.4 Cross-country differences

We have already shown that the money market equilibrium is:

$$\bar{\mathsf{m}} - \bar{\mathsf{m}}^* - s\mathsf{e} = \sigma\left(\mathsf{c} - \mathsf{c}^*\right)$$

The difference in output is:

$$\begin{aligned}\mathsf{y} - \mathsf{y}^* &= -\lambda \left[n\left(-\mathsf{p}\right) + (1-n)\left(-s\mathsf{e} - \mathsf{p}^*\right)\right] \\ &\quad +\lambda \left[n\left(s\mathsf{e} - \mathsf{p}\right) + (1-n)\left(-\mathsf{p}^*\right)\right] \\ \mathsf{y} - \mathsf{y}^* &= -\lambda \left[-n - (1-n)\right](1-n) s\mathsf{e} \\ &\quad +\lambda \left[n + (1-n)\right] n s\mathsf{e} \\ \mathsf{y} - \mathsf{y}^* &= \lambda s\mathsf{e}\end{aligned}$$

The current account is (recalling that the initial b is zero):

$$\begin{aligned}
\frac{\bar{\mathsf{b}}}{1-n}+(\mathsf{c}-\mathsf{c}^*)+(\bar{\mathsf{g}}-\bar{\mathsf{g}}^*) &= (1-n)(1-s)\,\mathsf{e}+n(1-s)\,\mathsf{e}-(\mathsf{p}-\mathsf{p}^*)+\mathsf{y}-\mathsf{y}^* \\
\frac{\bar{\mathsf{b}}}{1-n}+(\mathsf{c}-\mathsf{c}^*)+(\bar{\mathsf{g}}-\bar{\mathsf{g}}^*) &= (1-s)\,\mathsf{e}-s\mathsf{e}+\mathsf{y}-\mathsf{y}^* \\
\frac{\bar{\mathsf{b}}}{1-n}+(\mathsf{c}-\mathsf{c}^*)+(\bar{\mathsf{g}}-\bar{\mathsf{g}}^*) &= (1-s)\,\mathsf{e}-\frac{1}{\lambda}(\mathsf{y}-\mathsf{y}^*)+(\mathsf{y}-\mathsf{y}^*) \\
\frac{\bar{\mathsf{b}}}{1-n}+(\mathsf{c}-\mathsf{c}^*)+(\bar{\mathsf{g}}-\bar{\mathsf{g}}^*) &= (1-s)\,\mathsf{e}+\frac{\lambda-1}{\lambda}(\mathsf{y}-\mathsf{y}^*)
\end{aligned}$$

Using our results so far (including the Euler condition and (47)), this implies:

$$\begin{aligned}
(1-s)\,\mathsf{e}+\frac{\lambda-1}{\lambda}(\mathsf{y}-\mathsf{y}^*) &= \frac{\bar{\mathsf{b}}}{1-n}+(\mathsf{c}-\mathsf{c}^*)+(\bar{\mathsf{g}}-\bar{\mathsf{g}}^*) \\
(1-s)\,\mathsf{e}+(\lambda-1)\,s\mathsf{e} &= \frac{\bar{\mathsf{b}}}{1-n}+(\bar{\mathsf{c}}-\bar{\mathsf{c}}^*)+\frac{1}{\sigma}(1-s)\,\mathsf{e}+(\bar{\mathsf{g}}-\bar{\mathsf{g}}^*) \\
\frac{\sigma-1}{\sigma}(1-s)\,\mathsf{e}+(\lambda-1)\,s\mathsf{e} &= \frac{\bar{\mathsf{b}}}{1-n}+\frac{1+\lambda}{\sigma(\lambda-1)+(1+\lambda)}\left[\frac{1-\beta}{\beta}\frac{\bar{\mathsf{b}}}{1-n}-(\bar{\mathsf{g}}-\bar{\mathsf{g}}^*)\right] \\
&\quad+\frac{\lambda-1}{\sigma(\lambda-1)+(1+\lambda)}(\bar{\mathsf{a}}-\bar{\mathsf{a}}^*)+(\bar{\mathsf{g}}-\bar{\mathsf{g}}^*) \\
\frac{\sigma-1}{\sigma}(1-s)\,\mathsf{e}+(\lambda-1)\,s\mathsf{e} &= \left(1+\frac{1+\lambda}{\sigma(\lambda-1)+(1+\lambda)}\frac{1-\beta}{\beta}\right)\frac{\bar{\mathsf{b}}}{1-n} \\
&\quad+\frac{(\lambda-1)\,\sigma}{\sigma(\lambda-1)+(1+\lambda)}\left[(\bar{\mathsf{g}}-\bar{\mathsf{g}}^*)+\frac{1}{\sigma}(\bar{\mathsf{a}}-\bar{\mathsf{a}}^*)\right]
\end{aligned}$$

which implies:

$$\begin{aligned}
\frac{\bar{\mathsf{b}}}{1-n} &= \frac{\sigma(\lambda-1)+(1+\lambda)}{\sigma(\lambda-1)+(1+\lambda)\frac{1}{\beta}}\left[\frac{\sigma-1}{\sigma}(1-s)\,\mathsf{e}+(\lambda-1)\,s\mathsf{e}\right] \\
&\quad-\frac{(\lambda-1)\,\sigma}{\sigma(\lambda-1)+(1+\lambda)\frac{1}{\beta}}\left[(\bar{\mathsf{g}}-\bar{\mathsf{g}}^*)+\frac{1}{\sigma}(\bar{\mathsf{a}}-\bar{\mathsf{a}}^*)\right]
\end{aligned}$$

Using this result, and the fact that $\mathsf{e}=\bar{\mathsf{e}}$, (48) becomes:

$$\begin{aligned}
\bar{\mathsf{e}} &= (\bar{\mathsf{m}}-\bar{\mathsf{m}}^*)-\frac{\sigma(1+\lambda)}{\sigma(\lambda-1)+(1+\lambda)}\left[\frac{1-\beta}{\beta}\frac{\bar{\mathsf{b}}}{1-n}-(\bar{\mathsf{g}}-\bar{\mathsf{g}}^*)\right] \\
&\quad-\frac{\sigma(\lambda-1)}{\sigma(\lambda-1)+(1+\lambda)}(\bar{\mathsf{a}}-\bar{\mathsf{a}}^*) \\
\mathsf{e} &= (\bar{\mathsf{m}}-\bar{\mathsf{m}}^*)-\frac{1-\beta}{\beta}\frac{\sigma(1+\lambda)}{\sigma(\lambda-1)+(1+\lambda)\frac{1}{\beta}}\left[\frac{\sigma-1}{\sigma}(1-s)\,\mathsf{e}+(\lambda-1)\,s\mathsf{e}\right] \\
&\quad+\frac{\sigma(1+\lambda)}{\sigma(\lambda-1)+(1+\lambda)\frac{1}{\beta}}\frac{1}{\beta}(\bar{\mathsf{g}}-\bar{\mathsf{g}}^*)-\frac{\sigma(\lambda-1)}{\sigma(\lambda-1)+(1+\lambda)\frac{1}{\beta}}(\bar{\mathsf{a}}-\bar{\mathsf{a}}^*) \\
D\mathsf{e} &= (\bar{\mathsf{m}}-\bar{\mathsf{m}}^*)+\frac{\sigma}{\sigma(\lambda-1)+(1+\lambda)\frac{1}{\beta}}\left[\frac{1+\lambda}{\beta}(\bar{\mathsf{g}}-\bar{\mathsf{g}}^*)-(\lambda-1)(\bar{\mathsf{a}}-\bar{\mathsf{a}}^*)\right]
\end{aligned}$$

where:

$$D = 1+\frac{1-\beta}{\beta}\frac{\sigma(1+\lambda)}{\sigma(\lambda-1)+(1+\lambda)\frac{1}{\beta}}\left[\frac{\sigma-1}{\sigma}(1-s)+(\lambda-1)s\right]$$

The exchange rate is then:

$$\begin{aligned}\mathsf{e} &= \frac{1}{D}(\bar{\mathsf{m}}-\bar{\mathsf{m}}^*)+\frac{1}{D}\frac{\sigma(1+\lambda)}{\sigma(\lambda-1)+(1+\lambda)\frac{1}{\beta}}\frac{1}{\beta}(\bar{\mathsf{g}}-\bar{\mathsf{g}}^*) \\ &\quad -\frac{1}{D}\frac{\sigma(\lambda-1)}{\sigma(\lambda-1)+(1+\lambda)\frac{1}{\beta}}(\bar{\mathsf{a}}-\bar{\mathsf{a}}^*)\end{aligned} \tag{53}$$

Consumption follows from the money demand:

$$\mathsf{c}-\mathsf{c}^* = \frac{1}{\sigma}(\bar{\mathsf{m}}-\bar{\mathsf{m}}^*)-\frac{1}{\sigma}s\mathsf{e}$$

4.3.5 MM and GG lines

The money market equilibrium gives the MM line:

$$\begin{aligned}\bar{\mathsf{m}}-\bar{\mathsf{m}}^*-s\mathsf{e} &= \sigma(\mathsf{c}-\mathsf{c}^*) \\ \mathsf{e} &= \frac{1}{s}(\bar{\mathsf{m}}-\bar{\mathsf{m}}^*)-\frac{\sigma}{s}(\mathsf{c}-\mathsf{c}^*)\end{aligned}$$

The lower the extent of pass-through s, the steeper the line.

The current account and the output difference imply:

$$\begin{aligned}\frac{\bar{\mathsf{b}}}{1-n}+(\mathsf{c}-\mathsf{c}^*)+(\bar{\mathsf{g}}-\bar{\mathsf{g}}^*) &= (1-s)\,\mathsf{e}+\frac{\lambda-1}{\lambda}(\mathsf{y}-\mathsf{y}^*) \\ \frac{\bar{\mathsf{b}}}{1-n}+(\mathsf{c}-\mathsf{c}^*)+(\bar{\mathsf{g}}-\bar{\mathsf{g}}^*) &= (1-s)\,\mathsf{e}+\frac{\lambda-1}{\lambda}\lambda s\mathsf{e} \\ \frac{\bar{\mathsf{b}}}{1-n}+(\mathsf{c}-\mathsf{c}^*)+(\bar{\mathsf{g}}-\bar{\mathsf{g}}^*) &= [1-s+(\lambda-1)s]\,\mathsf{e}\end{aligned}$$

We solve $\frac{\bar{\mathsf{b}}}{1-n}$ as a function of $\mathsf{c}-\mathsf{c}^*$ starting from (47):

$$\begin{aligned}\bar{\mathsf{c}}-\bar{\mathsf{c}}^* &= \frac{1+\lambda}{\sigma(\lambda-1)+(1+\lambda)}\left[\frac{1-\beta}{\beta}\frac{\bar{\mathsf{b}}}{1-n}-(\bar{\mathsf{g}}-\bar{\mathsf{g}}^*)\right]+\frac{\lambda-1}{1-\sigma+\lambda(1+\sigma)}(\bar{\mathsf{a}}-\bar{\mathsf{a}}^*) \\ \frac{\sigma(\lambda-1)+(1+\lambda)}{1+\lambda}(\bar{\mathsf{c}}-\bar{\mathsf{c}}^*) &= \frac{1-\beta}{\beta}\frac{\bar{\mathsf{b}}}{1-n}-(\bar{\mathsf{g}}-\bar{\mathsf{g}}^*)+\frac{\lambda-1}{1+\lambda}(\bar{\mathsf{a}}-\bar{\mathsf{a}}^*) \\ \frac{1-\beta}{\beta}\frac{\bar{\mathsf{b}}}{1-n} &= \frac{\sigma(\lambda-1)+(1+\lambda)}{1+\lambda}(\bar{\mathsf{c}}-\bar{\mathsf{c}}^*)+(\bar{\mathsf{g}}-\bar{\mathsf{g}}^*)-\frac{\lambda-1}{1+\lambda}(\bar{\mathsf{a}}-\bar{\mathsf{a}}^*) \\ \frac{\bar{\mathsf{b}}}{1-n} &= \frac{\beta}{1-\beta}\frac{\sigma(\lambda-1)+(1+\lambda)}{1+\lambda}(\bar{\mathsf{c}}-\bar{\mathsf{c}}^*)+\frac{\beta}{1-\beta}\left[(\bar{\mathsf{g}}-\bar{\mathsf{g}}^*)-\frac{\lambda-1}{1+\lambda}(\bar{\mathsf{a}}-\bar{\mathsf{a}}^*)\right]\end{aligned}$$

Next, we use the Euler condition:

$$\sigma(\bar{\mathsf{c}}-\bar{\mathsf{c}}^*) = \sigma(\mathsf{c}-\mathsf{c}^*)-(1-s)\,\mathsf{e}$$

$$(\bar{\mathsf{c}}-\bar{\mathsf{c}}^*) = (\mathsf{c}-\mathsf{c}^*) - \frac{(1-s)}{\sigma}\mathsf{e}$$

this gives $\frac{\bar{\mathsf{b}}}{1-n}$ as a function of $\mathsf{c}-\mathsf{c}^*$:

$$\begin{aligned}
\frac{\bar{\mathsf{b}}}{1-n} &= \frac{\beta}{1-\beta}\frac{\sigma(\lambda-1)+(1+\lambda)}{1+\lambda}(\bar{\mathsf{c}}-\bar{\mathsf{c}}^*) + \frac{\beta}{1-\beta}\left[(\bar{\mathsf{g}}-\bar{\mathsf{g}}^*) - \frac{\lambda-1}{1+\lambda}(\bar{\mathsf{a}}-\bar{\mathsf{a}}^*)\right] \\
\frac{\bar{\mathsf{b}}}{1-n} &= \frac{\beta}{1-\beta}\frac{\sigma(\lambda-1)+(1+\lambda)}{1+\lambda}(\mathsf{c}-\mathsf{c}^*) - \frac{\beta}{1-\beta}\frac{\sigma(\lambda-1)+(1+\lambda)}{1+\lambda}\frac{(1-s)}{\sigma}\mathsf{e} \\
&\quad + \frac{\beta}{1-\beta}\left[(\bar{\mathsf{g}}-\bar{\mathsf{g}}^*) - \frac{\lambda-1}{1+\lambda}(\bar{\mathsf{a}}-\bar{\mathsf{a}}^*)\right]
\end{aligned}$$

The current account then implies:

$$\begin{aligned}
[1-s+(\lambda-1)s]\,\mathsf{e} &= \frac{\bar{\mathsf{b}}}{1-n} + (\mathsf{c}-\mathsf{c}^*) + (\bar{\mathsf{g}}-\bar{\mathsf{g}}^*) \\
[1-s+(\lambda-1)s]\,\mathsf{e} &= \frac{\beta}{1-\beta}\frac{\sigma(\lambda-1)+(1+\lambda)}{1+\lambda}(\mathsf{c}-\mathsf{c}^*) \\
&\quad - \frac{\beta}{1-\beta}\frac{\sigma(\lambda-1)+(1+\lambda)}{1+\lambda}\frac{(1-s)}{\sigma}\mathsf{e} \\
&\quad + \frac{\beta}{1-\beta}\left[(\bar{\mathsf{g}}-\bar{\mathsf{g}}^*) - \frac{\lambda-1}{1+\lambda}(\bar{\mathsf{a}}-\bar{\mathsf{a}}^*)\right] \\
&\quad + (\mathsf{c}-\mathsf{c}^*) + (\bar{\mathsf{g}}-\bar{\mathsf{g}}^*) \\
\left[\left(1+\frac{\beta}{1-\beta}\frac{\sigma(\lambda-1)+(1+\lambda)}{1+\lambda}\frac{1}{\sigma}\right)(1-s)+(\lambda-1)s\right]\mathsf{e} &= \left[1+\frac{\beta}{1-\beta}\frac{\sigma(\lambda-1)+(1+\lambda)}{1+\lambda}\right](\mathsf{c}-\mathsf{c}^*) \\
&\quad + \frac{1}{1-\beta}(\bar{\mathsf{g}}-\bar{\mathsf{g}}^*) - \frac{\beta}{1-\beta}\frac{\lambda-1}{1+\lambda}(\bar{\mathsf{a}}-\bar{\mathsf{a}}^*)
\end{aligned}$$

which is the GG line. If $\sigma = s = 1$ (as in class), it simplifies to:

$$\mathsf{e} = \left[1+\frac{\beta}{1-\beta}\frac{2\lambda}{1+\lambda}\right]\frac{1}{\lambda-1}(\mathsf{c}-\mathsf{c}^*)$$

5 Welfare effect

Abstracting from the direct utility effect of real monetary balances (setting χ to a very small value), and using the fact that labor is equal to output, the term in (15) is expanded as:

$$\begin{aligned}
\frac{(C)^{1-\sigma}}{1-\sigma} - \frac{\kappa}{2}(Y)^2 &= \frac{(C_0)^{1-\sigma}}{1-\sigma} - \frac{\kappa_0}{2}(Y_0)^2 \\
&\quad + (C_0)^{1-\sigma}\mathsf{c} - \kappa_0 (Y_0)^2\mathsf{y} + \frac{\kappa_0}{2}(Y_0)^2\mathsf{a} \\
\frac{(C)^{1-\sigma}}{1-\sigma} - \frac{\kappa}{2}(Y)^2 &= \frac{(C_0)^{1-\sigma}}{1-\sigma} - \frac{\kappa_0}{2}(Y_0)^2 \\
&\quad + \left(\left(\frac{\theta-1}{\theta\kappa_0}\right)^{\frac{1}{1+\sigma}}\right)^{1-\sigma}\mathsf{c} - \kappa_0\left(\left(\frac{\theta-1}{\theta\kappa_0}\right)^{\frac{1}{1+\sigma}}\right)^2\mathsf{y} + \frac{\kappa_0}{2}\left(\left(\frac{\theta-1}{\theta\kappa_0}\right)^{\frac{1}{1+\sigma}}\right)^2\mathsf{a}
\end{aligned}$$

$$
\begin{aligned}
\frac{(C)^{1-\sigma}}{1-\sigma} - \frac{\kappa}{2}(Y)^2 &= \frac{(C_0)^{1-\sigma}}{1-\sigma} - \frac{\kappa_0}{2}(Y_0)^2 \\
&\quad + \left(\frac{\theta-1}{\theta\kappa_0}\right)^{\frac{1-\sigma}{1+\sigma}} \left[\mathsf{c} - \kappa_0 \frac{\theta-1}{\theta\kappa_0}\mathsf{y} + \frac{\kappa_0}{2}\frac{\theta-1}{\theta\kappa_0}\right]\mathsf{a} \\
\frac{(C)^{1-\sigma}}{1-\sigma} - \frac{\kappa}{2}(Y)^2 &= \frac{(C_0)^{1-\sigma}}{1-\sigma} - \frac{\kappa_0}{2}(Y_0)^2 + \left(\frac{\theta-1}{\theta\kappa_0}\right)^{\frac{1-\sigma}{1+\sigma}} \left[\mathsf{c} - \frac{\theta-1}{\theta}\mathsf{y} + \frac{\theta-1}{2\theta}\right]\mathsf{a}
\end{aligned}
$$

(15) is the expanded as:

$$
\begin{aligned}
U_t &= \sum_{s=t}^{\infty}\beta^{s-t}\left[\frac{(C_s)^{1-\sigma}}{1-\sigma} - \frac{\kappa_s}{2}(L_s)^2\right] \\
(U_t - U_0)\left(\frac{\theta-1}{\theta\kappa_0}\right)^{-\frac{1-\sigma}{1+\sigma}} &= \sum_{s=t}^{\infty}\beta^{s-t}\left[\mathsf{c}_s - \frac{\theta-1}{\theta}\mathsf{y}_s + \frac{\theta-1}{2\theta}\mathsf{a}_s\right] \\
\mathsf{u}_t &= \mathsf{c} - \frac{\theta-1}{\theta}\mathsf{y} + \frac{1}{2}\frac{\theta-1}{\theta}\bar{\mathsf{a}} + \sum_{s=t+1}^{\infty}\beta^{s-t}\left[\bar{\mathsf{c}} - \frac{\theta-1}{\theta}\bar{\mathsf{y}} + \frac{\theta-1}{2\theta}\bar{\mathsf{a}}\right] \\
\mathsf{u}_t &= \mathsf{c} - \frac{\theta-1}{\theta}\mathsf{y} + \frac{\beta}{1-\beta}\left[\bar{\mathsf{c}} - \frac{\theta-1}{\theta}\bar{\mathsf{y}}\right] + \frac{1}{1-\beta}\frac{\theta-1}{2\theta}\bar{\mathsf{a}}
\end{aligned}
$$

Similarly, we write:

$$
\mathsf{u}_t^* = \mathsf{c}^* - \frac{\theta-1}{\theta}\mathsf{y}^* + \frac{\beta}{1-\beta}\left[\bar{\mathsf{c}}^* - \frac{\theta-1}{\theta}\bar{\mathsf{y}}^*\right] + \frac{1}{1-\beta}\frac{\theta-1}{2\theta}\bar{\mathsf{a}}^*
$$

The worldwide average utility is:

$$
\begin{aligned}
\mathsf{u}_t^W &= \mathsf{c}^W - \frac{\theta-1}{\theta}\mathsf{y}^W + \frac{\beta}{1-\beta}\left[\bar{\mathsf{c}}^W - \frac{\theta-1}{\theta}\bar{\mathsf{y}}^W\right] + \frac{1}{1-\beta}\frac{\theta-1}{2\theta}\bar{\mathsf{a}}^W \\
\mathsf{u}_t^W &= \mathsf{c}^W - \frac{\theta-1}{\theta}\left(\mathsf{c}^W + \bar{\mathsf{g}}^W\right) + \frac{\beta}{1-\beta}\left[\bar{\mathsf{c}}^W - \frac{\theta-1}{\theta}\left(\bar{\mathsf{c}}^W + \bar{\mathsf{g}}^W\right)\right] + \frac{1}{1-\beta}\frac{\theta-1}{2\theta}\bar{\mathsf{a}}^W \\
\mathsf{u}_t^W &= \frac{1}{\theta}\left(\mathsf{c}^W + \frac{\beta}{1-\beta}\bar{\mathsf{c}}^W\right) - \frac{1}{1-\beta}\frac{\theta-1}{\theta}\bar{\mathsf{g}}^W + \frac{1}{1-\beta}\frac{\theta-1}{2\theta}\bar{\mathsf{a}}^W \\
\mathsf{u}_t^W &= \frac{1}{\theta}\left(\frac{1}{\sigma}\bar{\mathsf{m}}^W + \frac{\beta}{1-\beta}\frac{1}{1+\sigma}\left(\bar{\mathsf{a}}^W - \bar{\mathsf{g}}^W\right)\right) - \frac{1}{1-\beta}\frac{\theta-1}{\theta}\bar{\mathsf{g}}^W + \frac{1}{1-\beta}\frac{\theta-1}{2\theta}\bar{\mathsf{a}}^W \\
\mathsf{u}_t^W &= \frac{1}{\theta\sigma}\bar{\mathsf{m}}^W - \frac{1}{\theta}\left(\frac{\beta}{1-\beta}\frac{1}{1+\sigma} + \frac{\theta-1}{1-\beta}\right)\bar{\mathsf{g}}^W + \frac{1}{\theta}\left[\frac{\beta}{1-\beta}\frac{1}{1+\sigma} + \frac{1}{1-\beta}\frac{\theta-1}{2}\right]\bar{\mathsf{a}}^W \qquad (54)
\end{aligned}
$$

Turning to cross-country difference, recall that the long run and short run outputs are:

$$
\begin{aligned}
\bar{\mathsf{y}} - \bar{\mathsf{y}}^* &= -\lambda\left[\bar{\mathsf{p}}(h) - \bar{\mathsf{p}}^*(f) - \bar{\mathsf{e}}\right] \\
\mathsf{y} - \mathsf{y}^* &= \lambda s \mathsf{e}
\end{aligned}
$$

The current accounts are then:

$$
\frac{1}{1-n}\bar{\mathsf{b}} + (\bar{\mathsf{c}} - \bar{\mathsf{c}}^*) + (\bar{\mathsf{g}} - \bar{\mathsf{g}}^*) = \frac{1}{\beta}\frac{1}{1-n}\bar{\mathsf{b}} + \left[\bar{\mathsf{p}}(h) - \bar{\mathsf{p}}^*(f) - \bar{\mathsf{e}}\right] + (\bar{\mathsf{y}} - \bar{\mathsf{y}}^*)
$$

$$
(\bar{\mathsf{c}} - \bar{\mathsf{c}}^*) + (\bar{\mathsf{g}} - \bar{\mathsf{g}}^*) = \frac{1-\beta}{\beta}\frac{1}{1-n}\bar{\mathsf{b}} + \frac{\lambda - 1}{\lambda}(\bar{\mathsf{y}} - \bar{\mathsf{y}}^*)
$$

and:

$$
\frac{\bar{\mathsf{b}}}{1-n} + (\mathsf{c} - \mathsf{c}^*) + (\bar{\mathsf{g}} - \bar{\mathsf{g}}^*) = (1-s)\,\mathsf{e} + \frac{\lambda - 1}{\lambda}(\mathsf{y} - \mathsf{y}^*)
$$

The cross-country utility is then:

$$
\begin{aligned}
\mathsf{u}_t - \mathsf{u}_t^* &= (\mathsf{c} - \mathsf{c}^*) - \frac{\theta - 1}{\theta}(\mathsf{y} - \mathsf{y}^*) + \frac{\beta}{1-\beta}\left[(\bar{\mathsf{c}} - \bar{\mathsf{c}}^*) - \frac{\theta - 1}{\theta}(\bar{\mathsf{y}} - \bar{\mathsf{y}}^*)\right] + \frac{1}{1-\beta}\frac{\theta - 1}{2\theta}(\bar{\mathsf{a}} - \bar{\mathsf{a}}^*) \\
\mathsf{u}_t - \mathsf{u}_t^* &= (1-s)\,\mathsf{e} + \frac{\lambda - 1}{\lambda}(\mathsf{y} - \mathsf{y}^*) - \frac{\theta - 1}{\theta}(\mathsf{y} - \mathsf{y}^*) - (\bar{\mathsf{g}} - \bar{\mathsf{g}}^*) - \frac{\bar{\mathsf{b}}}{1-n} \\
&\quad + \frac{\beta}{1-\beta}\left[\frac{1-\beta}{\beta}\frac{1}{1-n}\bar{\mathsf{b}} + \frac{\lambda - 1}{\lambda}(\bar{\mathsf{y}} - \bar{\mathsf{y}}^*) - \frac{\theta - 1}{\theta}(\bar{\mathsf{y}} - \bar{\mathsf{y}}^*) - (\bar{\mathsf{g}} - \bar{\mathsf{g}}^*)\right] + \frac{1}{1-\beta}\frac{\theta - 1}{2\theta}(\bar{\mathsf{a}} - \bar{\mathsf{a}}^*) \\
\mathsf{u}_t - \mathsf{u}_t^* &= (1-s)\,\mathsf{e} + \frac{\lambda - \theta}{\lambda\theta}\left[(\mathsf{y} - \mathsf{y}^*) + \frac{\beta}{1-\beta}(\bar{\mathsf{y}} - \bar{\mathsf{y}}^*)\right] \qquad (55) \\
&\quad - \frac{1}{1-\beta}(\bar{\mathsf{g}} - \bar{\mathsf{g}}^*) + \frac{1}{1-\beta}\frac{\theta - 1}{2\theta}(\bar{\mathsf{a}} - \bar{\mathsf{a}}^*)
\end{aligned}
$$