Macroeconomics A; EI056

Technical appendix: Alternative ordering of the matrix solution

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1 Introduction

This appendix presents an alternative ordering of the matrix system that is closer to the review session.

2 Linearized system

The system is a simple version of Blanchard-Kahn. $\hat{k}_t$ is the predetermined state variable, and $\hat{c}_t$ the control variable. The two equations are:

$$\begin{aligned}\hat{k}_{t+1} &= \frac{1+r^*}{1+n}\hat{k}_t - \frac{1}{\alpha}\frac{r^*-\alpha n}{1+n}\hat{c}_t \\ \hat{c}_{t+1}-\hat{c}_t &= -\frac{1}{\theta}\frac{r^*}{1+r^*}(1-\alpha)\hat{k}_{t+1}\end{aligned}$$

We write this in a matrix form exactly as in the mains appendix:

$$\left|\begin{array}{c}\hat{k}_{t+1} \\ \hat{c}_{t+1}\end{array}\right| = A\left|\begin{array}{c}\hat{k}_t \\ \hat{c}_t\end{array}\right|$$

The eigenvalue-eigenvector decomposition of A is $A = C^{-1}\Lambda C$:

$$\begin{aligned}A &= C^{-1}\Lambda C \\ CA &= \left|\begin{array}{cc}J_1 & 0 \\ 0 & J_2\end{array}\right|\left|\begin{array}{cc}C_{11} & C_{12} \\ C_{21} & C_{22}\end{array}\right|\end{aligned}$$

Here we take a different ordering of the eigenvalues matrix: $J_1 < 1 < J_2$. This is the one used in the review session and the Blanchard-Kahn paper. This ordering requires some adjusting in the

Matlab program.

We then write the system as:

$$\begin{array}{rcl}
\begin{vmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{vmatrix} \begin{vmatrix} \hat{k}_{t+1} \\ \hat{c}_{t+1} \end{vmatrix} & = & \begin{vmatrix} J_1 & 0 \\ 0 & J_2 \end{vmatrix} \begin{vmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{vmatrix} \begin{vmatrix} \hat{k}_t \\ \hat{c}_t \end{vmatrix} \\
\begin{vmatrix} C_{11}\hat{k}_{t+1} + C_{12}\hat{c}_{t+1} \\ C_{21}\hat{k}_{t+1} + C_{22}\hat{c}_{t+1} \end{vmatrix} & = & \begin{vmatrix} J_1 & 0 \\ 0 & J_2 \end{vmatrix} \begin{vmatrix} C_{11}\hat{k}_t + C_{12}\hat{c}_t \\ C_{21}\hat{k}_t + C_{22}\hat{c}_t \end{vmatrix} \\
\begin{vmatrix} z_{t+1} \\ q_{t+1} \end{vmatrix} & = & \begin{vmatrix} J_1 & 0 \\ 0 & J_2 \end{vmatrix} \begin{vmatrix} z_t \\ q_t \end{vmatrix}
\end{array}$$

The reasoning is similar as in the main appendix, but the order is reverted. The second row $q_{t+s} = (J_2)^s q_t$ goes to infinity as $J_2 > 1$. It thus must be that $q_t = 0$, which implies:

$$\begin{array}{rcl}
0 & = & C_{21}\hat{k}_t + C_{22}\hat{c}_t \\
\hat{c}_t & = & -(C_{22})^{-1} C_{21}\hat{k}_t
\end{array}$$

The first row of the matrix system is then:

$$\begin{array}{rcl}
z_{t+1} & = & J_1 z_t \\
C_{11}\hat{k}_{t+1} + C_{12}\hat{c}_{t+1} & = & J_1 \left(C_{11}\hat{k}_t + C_{12}\hat{c}_t \right) \\
C_{11}\hat{k}_{t+1} - C_{12}(C_{22})^{-1} C_{21}\hat{k}_{t+1} & = & J_1 \left(C_{11}\hat{k}_t - C_{12}(C_{22})^{-1} C_{21}\hat{k}_t \right) \\
\hat{k}_{t+1} & = & \left(C_{11} - C_{12}(C_{22})^{-1} C_{21} \right)^{-1} J_1 \left(C_{11} - C_{12}(C_{22})^{-1} C_{21} \right) \hat{k}_t
\end{array}$$