

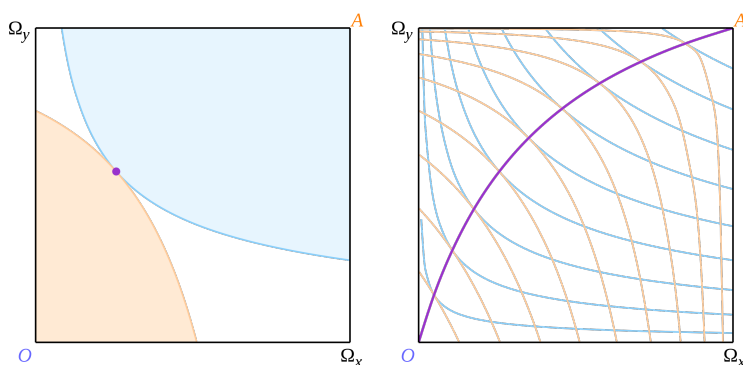
Final Exam

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EI037 Microeconomics

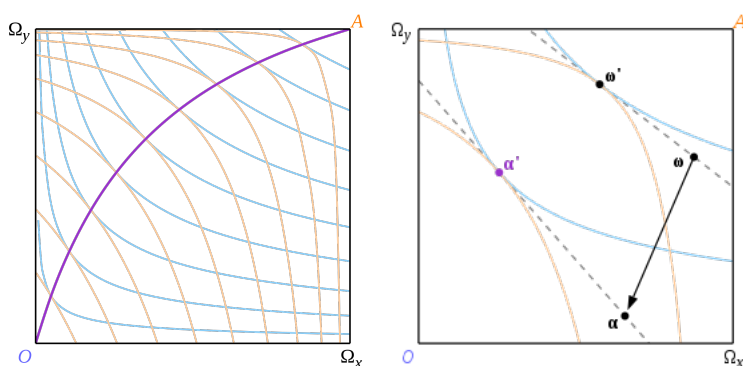
This is a closed book exam. You need to solve this exam alone and independently. Your answers should be legible, clear and concise. In order to get full credit you have to provide complete answers, including how answers are derived. Partial answers will lead to partial credit. Wrong additional statements (i.e., guessing) might reduce the given credit. The exam is 1 hour 50 minutes. Good luck!

1. Fundamental Welfare Theorems (1 pt)

- a. Explain the first fundamental welfare theorem in words.
- b. Explain the first fundamental welfare theorem using the following Edgeworth boxes.



- c. Explain the second fundamental welfare theorem in words.
- d. Explain the second fundamental welfare theorem using the following Edgeworth boxes.



2. Consumption, Production, and Competitive Equilibrium (1.5 pt)

Consider an economy with L identical consumers, where each have Cobb-Douglas preferences over two goods x and y :

$$U = x^\alpha y^{1-\alpha}.$$

The total amount of good x is fixed in the economy and is equal to \bar{X} . Each consumer owns a share $\frac{1}{L}$ of \bar{X} . Good y is produced by firms using good x and labor l . The firms' technology is Cobb-Douglas:

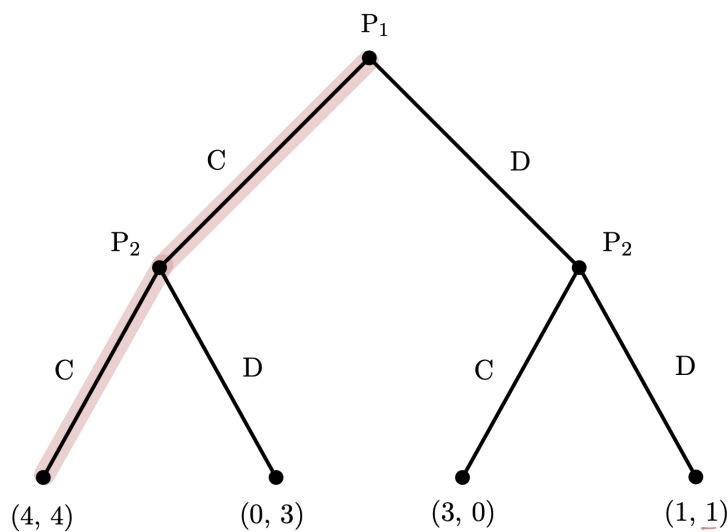
$$y = x^\beta l^{1-\beta}.$$

Each consumer supplies one unit of labor. The wage is normalized to be equal to 1. Denote the price of x and y by p_x and p_y , respectively.

- Solve for the consumer's demand of x and y as functions of prices p_x , p_y and income.
(Hint: first write down the consumer's income as a function of wage and the return from owning units of x .)
- Solve for the firms' marginal cost of production (MC) of y as a function of p_x and wage. How does the firms' supply of y depend on p_y and MC? (Hint: to solve for MC, it is equivalent to solve for a representative firm's cost minimization problem subject to this firm producing one unit of y .)
- Write down the market clearing condition(s) and find the equilibrium values of p_x and p_y .

3. Strategic Interactions (3.5 pt)

Consider the following dynamic game with perfect information (which is a variation of the Prisoners' Dilemma). Two players, P_1 and P_2 , sequentially choose to play confess (C) or defect (D).



- Write down the normal form representation of this game.
- Find the Nash equilibrium(s) of this game.
- How many subgames does this game have? 3
- Find the subgame perfect Nash equilibrium(s) of this game.

Consider the following dynamic game with *incomplete* information.

Nature randomly draws between options A and B with probability p and $(1 - p)$, respectively. Then, two players, P_1 and P_2 , move sequentially as displayed on the game tree. P_1 does not observe Nature's move.

$$P_C = P \times 3 + (1-P) \times 4 = 4-P$$

$$P_D = P \times 0 + (1-P) \times 10 = 10-10P$$

$$P_C = P_D$$

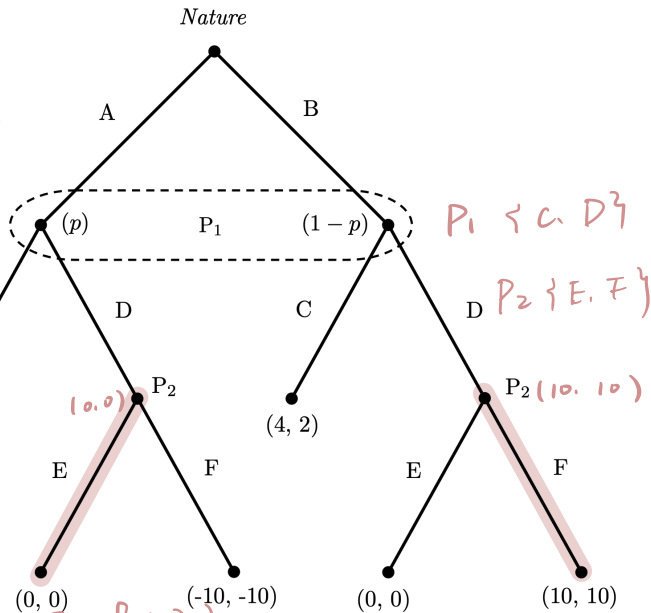
$$4-P = 10-10P$$

$$9P = 6 \quad P = \frac{6}{9} = \frac{2}{3}$$

$$\text{if } P < \frac{2}{3} \quad P_C < P_D \Rightarrow C$$

$$\text{if } P > \frac{2}{3} \quad P_C > P_D \Rightarrow D$$

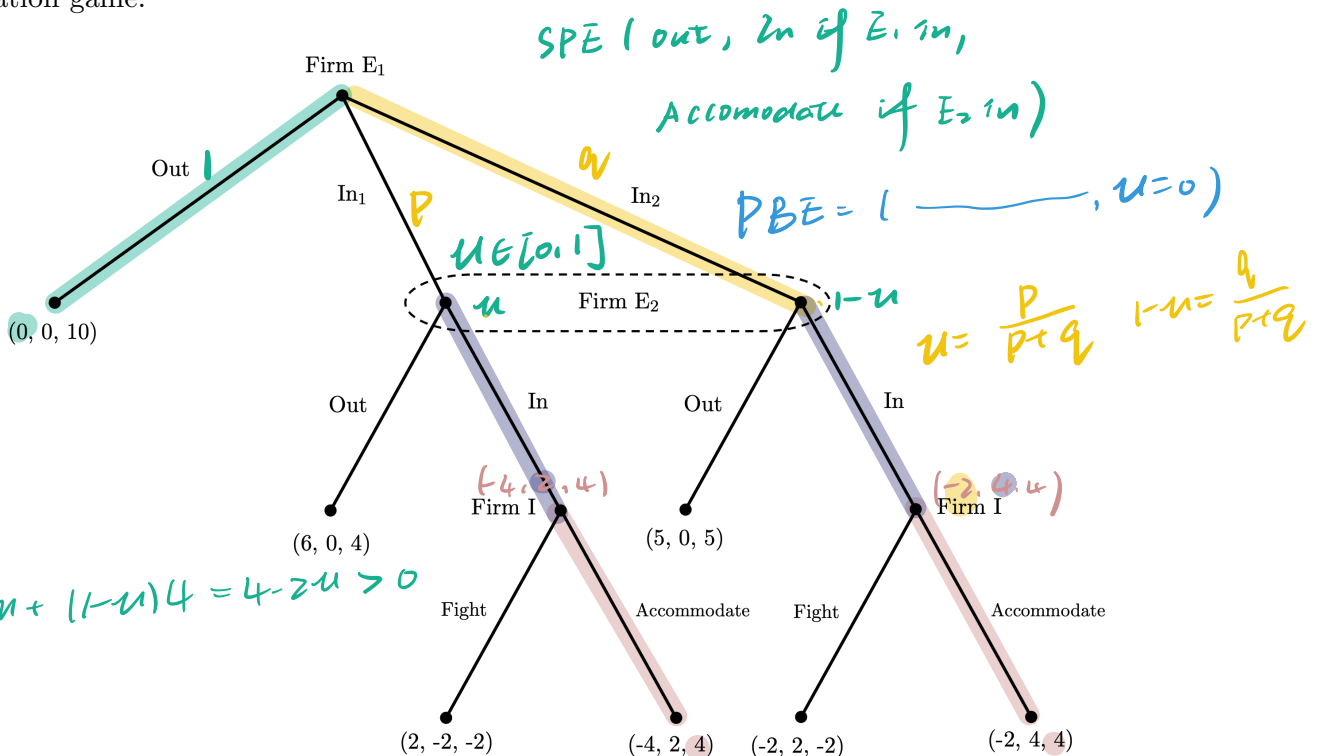
$$\text{if } P = \frac{2}{3} \quad (C, IE \text{ if } A, F \text{ if } B)$$



e. Find the Bayesian Nash equilibrium(s) of this game. How does your answer depend on the value of p ?

$$P > \frac{2}{3} \quad (D, IE \text{ if } A, F \text{ if } B).$$

Consider the following dynamic game with *imperfect* information, which is a variation of the predation game.



- f. Find the subgame perfect Nash equilibrium(s) of this game.
- g. Find the weak perfect Bayesian equilibrium(s) of this game.