L4. Welfare Economics and Aggregate Demand

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Literature

- MWG (1995), Chapters 3.I and 4
- Kreps (1990), Chapter 2, Varian (1992), Chapter 10

Welfare Evaluations of Economic Changes

How can we evaluate the welfare effect of an economic change on an individual?

For simplicity we will focus on a price change from p to p', but many other economic changes could be considered as well.

An obvious measure of the welfare change involved in moving from p^0 to p^1 is just the difference in indirect utility for the consumer under consideration:

$$v\left(p^1, w\right) - v\left(p^0, w\right).$$

If this difference is positive, then the consumer is better off with the new prices p^1. But, of course, the utility difference cannot be used to **measure the size** of the welfare effect.

If we want to measure this in monetary terms, there are two possible approaches.

$$\begin{pmatrix} u^0 \\ p^0 \end{pmatrix} \begin{matrix} \to u^1 \\ \to p^1 \end{matrix}$$

First Approach: Equivalent Variation

Suppose the government considers a new policy that leads to lower prices. We could ask the consumer, how much money he has to get in the situation with the old prices, so that he is just indifferent to the situation with the new prices.

- EV tells us to which amount of money at current prices the policy change is equivalent to.

- Of course, if the policy change yields an increase in prices, EV will be negative. In this case, EV tells us how much the consumer is willing to pay in order to avoid the price change.

$$EV = e(p^0, u^1) - e(p^0, u^0)$$

To define EV more formally, let $v\left(p^0, w\right) = u^0$ and $v\left(p^1, w\right) = u^1$. EV is implicitly defined by

$$v\left(p^0, w + EV\right) = v\left(p^1, w\right) = u^1$$

Note that $e\left(p^0, u^0\right) = w$ and $e\left(p^0, u^1\right) = w + EV$. Hence, we have:

$$EV = e\left(p^0, u^1\right) - w = e\left(p^0, u^1\right) - e\left(p^0, u^0\right)$$

u^0
$\uparrow$
p^0

u^1
$\uparrow$
p^1

Second Approach: Compensating Variation

We could ask the consumer, how much money we can take away from him in the situation with the new prices so that he is just indifferent to the situation with the old prices. This is called the **Compensating Variation, CV**. Again, CV may be negative.

More formally, CV is implicitly defined by

$$v\left(p^0, w\right) = v\left(p^1, w - CV\right) = u^0.$$

Let $v\left(p^1, w\right) = u^1$. Note that $e\left(p^1, u^1\right) = w$ and $e\left(p^1, u^0\right) = w - CV$. Hence,

$$CV = w - e\left(p^1, u^0\right) = e\left(p^1, u^1\right) - e\left(p^1, u^0\right)$$

Illustrate graphically.

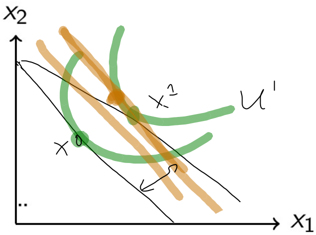


Figure: Figure 4.1: Equivalent Variation

Figure: Figure 4.2: Compensating Variation

If only one price (p_1) changes from p_1^0 to p_1^1 and all other prices remain constant, then we can express EV and CV by using the Hicksian demand curve. Recall that $e\left(p^0, u^0\right) = w = e\left(p^1, u^1\right)$:

$$
\begin{aligned}
EV &= e\left(p^0, u^1\right) - e\left(p^0, u^0\right) \\
&= e\left(p^0, u^1\right) - e\left(p^1, u^1\right) \\
&= \int_{p_1^1}^{p_1} \frac{\partial e\left(p, u^1\right)}{\partial p_1} dp_1 \\
&= \int_{p_1^1}^{p_1^0} h_1(p, u^1) dp_1
\end{aligned}
$$

The last step follows from Shephard's Lemma.

Similarly,

$$
\begin{aligned}
CV &= e\left(p^1, u^1\right) - e\left(p^1, u^0\right) \\
&= e\left(p^0, u^0\right) - e\left(p^1, u^0\right) \\
&= \int_{p_1^1}^{p_1} \frac{\partial e\left(p, u^0\right)}{\partial p_1} dp_1 \\
&= \int_{p_1^1}^{p_1^0} h_1\left(p, u^0\right) dp_1
\end{aligned}
$$

Thus, EV and CV can be interpreted as the area below the Hicksian demand function.

Illustrate graphically.

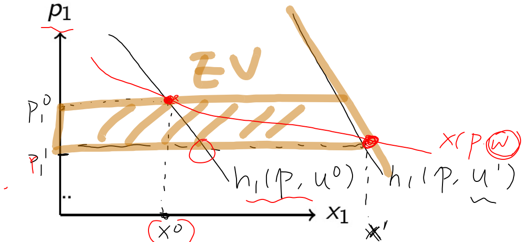
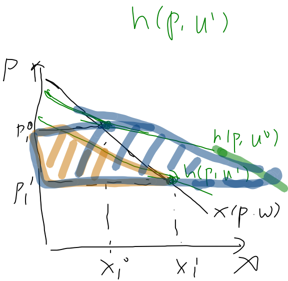


Figure: Figure 4.3: Demand Curves and Equivalent Variation



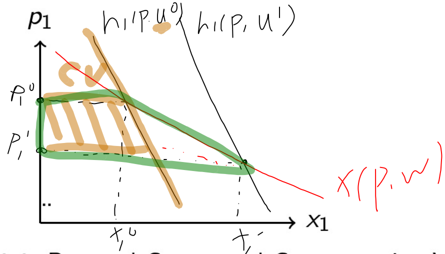


Figure: Figure 4.4: Demand Curves and Compensating Variation

The Hicksian demand function is not directly observable. This is why the area below the (Walrasian) demand function (consumer surplus) is often used as an approximation.

Note:

- $\frac{\partial x(p,w)}{\partial w} = 0 \Rightarrow$ Hicksian and Walrasian demand are identical and EV and CV coincide.
- $\frac{\partial x(p,w)}{\partial w} > 0 \rightarrow$ hormal good.
  $\Rightarrow$ change of Walrasian demand due to a price reduction is bigger than the change of Hicksian demand. $\Rightarrow$ Consumer's surplus underestimates EV.
  $\Rightarrow$ Consumer's surplus overestimates CV.
- $\frac{\partial x(p,w)}{\partial w} < 0$.
  $\Rightarrow$ change of Walrasian demand due to a price reduction is smaller than the change of Hicksian demand.
  $\Rightarrow$ Consumer's surplus overestimates EV.
  $\Rightarrow$ Consumer's surplus underestimates CV.

The two approaches differ:

- EV takes the old prices p^0 as the reference point.
- CV takes the new prices p^1 as the reference point.

If there are income effects, EV and CV yield different welfare while the CV uses different base prices for each project. Hence, with CV it is not possible to compare the welfare effects across projects.

Which criterion to use depends on the question under consideration.

Examples:

1. The government considers several new policies that hurt consumers. It wants to know, how much people are "willing to pay" in order to avoid the policy change. It selects the policy change that is least costly for consumers.
2. The government has introduced the new policy measure already and now wants to decide on a compensation scheme (at the new prices) to make sure that no consumer is worse of in the new situation than he was in the old situation. In this case CV is the appropriate measure.

If it is not entirely clear which measure to use, then there are two arguments in favor of the equivalent variation:

1. EV measures the welfare change in monetary units at current prices. It is much easier to evaluate the value of a "Swiss Franken" at current prices than at some hypothetical prices.
2. If you are comparing more than one proposed policy change, the EV uses the same base prices for all projects, while the CV uses different base prices for each project. Hence, with CV it is not possible to compare the welfare effects across projects.

Aggregate Demand

Aggregate demand is simply the sum of all individual's demand functions:

$$D(p, w_1, w_2, \ldots, w_I) = \sum_{i=1}^{I} x_i(p, w_i)$$

In this section we will deal with two questions:

1. In general aggregate demand depends on the vector of wealth levels $(w_1, \ldots, w_I)$. However, it would be very convenient if aggregate demand could be written like individual demand as a function of prices and only one wealth level. Thus, under what conditions can aggregate demand be expressed as a function of prices and **aggregate wealth**?
2. Under what conditions can we apply the results of individual demand theory to aggregate demand? More precisely, when does aggregate demand satisfy the weak axiom?

We will briefly discuss these questions in turn.

Aggregate Demand and Aggregate Wealth

Aggregate demand can be written as a function of aggregate wealth $w = \Sigma_{i=1}^{I} w_i$ if and only if for any $(w_1, \dots, w_I)$ and $(w_1', \dots, w_I')$ with $\Sigma_{i=1}^{I} w_i = \Sigma_{i=1}^{I} w_i'$ we have

$$D\left(p, w_1, \dots, w_I\right) = D\left(p, w_1', \dots, w_I'\right).$$

The following proposition offers a necessary and sufficient condition for this to be the case:

Proposition 4.1 A necessary and sufficient condition for aggregate demand to be a function of prices and aggregate wealth is that preferences of all individuals admit indirect utility functions of the **Gorman form** with the coefficients on w_i the same for every consumer i, i.e.

$$v_i\left(p, w_i\right) = a_i(p) + b(p) w_i.$$

Remarks:

1. You are asked to prove sufficiency yourself in one of the exercises. Necessity is much more difficult.
2. These indirect utility function represent the consumers' preferences only locally: If w_i becomes very small, a corner solution would obtain.
3. This type of preferences is clearly quite restrictive. It assumes that the wealth expansion paths of all consumers are parallel straight lines.

So far we assumed that individual wealth is exogenously given. If individual wealth is explained endogenously in the model and depends on prices and aggregate wealth, then our chances to get an aggregate demand function that depends only on aggregate wealth are much better. All we need is that individual wealth is a function of prices and aggregate wealth only.

Definition 4.1 A **wealth distribution rule** is a set of functions $(w_1(p, w), \ldots, w_I(p, w)))$ with $\sum_{i=1}^{I} w_i(p, w) = w$ for all (p, w).

If the income distribution is determined by a wealth distribution rule, then trivially aggregate demand is a function of (p, w) only:

$$D\left(p, w_1(p, w), \ldots, w_I(p, w)\right) = \tilde{D}(p, w).$$

Aggregate Demand and the Weak Axiom

Note first that if individual demand functions are continuous, homogeneous of degree 0 and satisfy Walras' law, then so does the aggregate demand function. Why?

Suppose now that all individual demand functions satisfy the weak axiom of revealed preferences. Does this imply that the aggregate demand function $D(p,w)$ also satisfies the weak axiom, i.e., if $p \cdot D(p', w') \leq w$ and $D(p,w) \neq D(p', w')$, then $p' \cdot D(p,w) > w'$ for any (p,w) and (p', w')?

The answer to this question is no.

To see this consider the following counter example:

There are two consumers, each of whom has the same wealth w. Consider two situations, one with price vector p, the other with price vector p'. The wealth level of the two consumers is the same in both situations. For each of the two consumers the weak axiom holds, i.e., if $p \cdot x_i\,(p', w) \leq w$ and $x_i(p, w) \neq x\,(p', w)$, then $p' \cdot x(p, w) > w$ for any (p, w) and (p', w).

- Illustrate this graphically.
- Show graphically that the WA does not hold if we aggregate the demand functions of the two consumers.

The intuitive reason for why the WA may fail to hold in the aggregate is the existence of wealth effects. Recall that the WA is equivalent to the compensated law of demand, i.e., if we compensate a consumer such that he can still afford the same utility level under the new prices, we have that

$$(p' - p)\left[x_i\left(p', w_i'\right) - x_i\left(p, w_i\right)\right] \leq 0$$

i.e. the price change and the quantity change go in opposite direction. If we add up these inequalities over all i, then this inequality also holds in the aggregate.

Now, if we look at a group of consumers with aggregate demand function $D(p, w)$ (assuming that demand can be written as a function of total wealth), we have to compensate this group of consumers such that the old aggregate demand vector is still affordable at the new prices. However, we do not have to compensate each individual consumer such that he can still afford his old consumption bundle. Thus, depending on how consumers are compensated, there will be additional wealth effects that may go in any direction. These wealth effects are responsible for that the compensated law of demand may fail to hold in the aggregate.

Remarks:

1. This result is very disappointing. It tells us that the central structural property of individual demand, the weak axiom (or the negative semi-definiteness of the substitution matrix) need not hold in the aggregate. Thus, it seems that almost any aggregate demand function that satisfies homogeneity of degree zero and Walras' Law is consistent with rationality.

2. However, while aggregation may destroy structural properties of the demand function, it may also add new structural properties. Examples:
   - If each individual demand function is discontinuous, aggregate demand may nevertheless become a continuous function, if there are sufficiently many consumers (a continuum).
   - Hildebrand (1983, 1996): If the distribution of wealth. satisfies some statistical properties, then the aggregate demand function must satisfy not only the weak axiom but also the (uncompensated) law of demand.