Problem Set I

EI037 Microeconomics I

Prof. Yuan Zi

1. Preference Relations

In the preference-based approach, the objectives of the decision maker are summarized by the preference relation $\succeq$*. For individual preferences to be "rational", we impose two basic assumptions about* $\succeq$*.*

1.a. What are these two basic assumptions? Also explain their intuition.

1.b. Consider a rational preference relation $\succeq$. Show that if $u(x) = u(y)$ implies $x \sim y$ and if $u(x) > u(y)$ implies $x \succ y$, then $u(.)$ is a utility function representing $\succeq$.

1.c. Show that if $f : \mathbb{R} \to \mathbb{R}$ is a strictly increasing function and $u : X \to \mathbb{R}$ is a utility function representing the preference relation $\succeq$, then the function $v : X \to \mathbb{R}$ defined by $v(x) = f(u(x))$ is also a utility function representing the preference relation $\succeq$. (*PS:* $g : A \to B$ *is just the formal notation for the fact of* g *being a function from the set* A *to the set* B*. For example,* $f : \mathbb{R} \to \mathbb{R}$ *simply says that the domain and co-domain of f are both the set of real numbers.*)

2. Choice Rules

In the choice-based approach, the objectives of the decision maker are summarized by the choice structure $(\mathcal{B}, C(\cdot))$*. For individual choices to be "reasonable", we impose some restrictions regarding an individual's choice behavior. Apart from the fact that an individual's choice cannot be empty nor out of his budget set, we require that his choice satisfy the weak axiom of revealed preference (or weak axiom in short).*

2.a. What is the weak axiom? Also explain its intuition.

2.b. We defined a revealed preference relation $\succeq^*$ from the observed individual choice behavior in $C(\cdot)$ in class. What is this definition? How does it differ from $\succeq$ defined in the preference-based approach?

2.c. Suppose that the choice structure $(\mathcal{B}, C(\cdot))$ satisfies the weak axiom. Consider the following two possible revealed preferred relations, $\succ^*$ and $\succ^{**}$:

$$x \succ^* y \iff \text{there is some } B \in \mathcal{B} \text{ such that } x, y \in B, x \in C(B), \text{and } y \notin C(B)$$

$$x \succ^{**} y \iff x \succeq^* y \text{ but not } y \succeq^* x$$

(i) Show that $\succ^*$ and $\succ^{**}$ give the same relation over X; that is, for any $x, y \in X$, $\succ^* \iff \succ^{**}$.

(ii) Must $\succ^*$ be transitive? Why?

(iii) [Bonus] Show that if $\mathcal{B}$ includes all subsets of X of *up to* three elements, then $\succ^*$ must be transitive.

3. Consumer Choice

3.a. Some definitions

In the entire world, there are 10 apples, 20 bottles of wine, and 15 Gruyere cheese traded in the market economy at the price of 1, 2, and 4 CHF respectively. You as an individual have 52 CHF. Use the notation we studied in Lecture 2.

(i) What is the set of commodities in the economy? What does L equal to?

(ii) Give an example of a commodity vector (i.e., consumption bundle).

(iii) What is the commodity set?

(iv) What is the budget set?

(v) Plot out your budget plane.

3.b. Weak axiom of revealed preference

Consider a consumer who consumes only two goods and satisfies Walras' Law. When prices are (2,4), he demands (5,10). When prices are (6,3) he demands (10, y). Nothing else of significance has changed between the two situations.

(i) Suppose that $y = 5$. Do these consumption plans satisfy the weak axiom of revealed preference?

(ii) Suppose that $y = 10$. Do these consumption plans satisfy the weak axiom of revealed preference?

(iii) For which range of y do these consumption plans violate the weak axiom?

3.c. Show that if $x(p, w)$ is homogeneous of degree one w.r.t. w - i.e., $x(p, \alpha w) = \alpha x(p, w)$ for all $\alpha > 0$ - and satisfies Walras's law, then $\varepsilon_{lw}(p, w) = 1$, where $\epsilon_{lw}(p, w) \equiv \frac{\partial x_l(p,w)}{\partial w} \frac{w}{x_l(p,w)}$ for every l. Interpret. Can you say something about $D_w x(p, w)$ and the form of the Engle curves and wealth expansion path in this case? (*Hint:* you can use the two-commodity graph to assist your explanation.)