**VARs** 

Other Approaches

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# Empirical Evidence on Role of Money

- Big questions in monetary economics:
  - How is an economy with money different from an economy without money?

- What effects do change in monetary policy have on real activity and inflation?
- Before we add money to canonical RBC model, turn to empirical evidence.
  - Will provide facts any successful model must match and motivate features we will add to model.
- Estimating effect of monetary policy is hard because policy is endogenous and expectations affect equilibrium today.
  - In fact, if monetary policy is successful and countercyclical, estimates of its effect would be biased towards zero.
  - Intuition: Monetary policy would move around but output would be stable.

# Evidence for Nominal Rigidity

- 1. Vector Autoregressions
- 2. VAR Evidence for Non-Neutrality
- 3. Other Approaches
- 4. Natural Experiments

#### Introduction to VARs

- Before we get started, introduce a key econometric tool: Vector Autoregression or VAR.
  - Proposed by Sims (1980), who won the Nobel Prize for it.
  - Wanted a way to describe economic time series with minimal theoretical restrictions.

- This is a key tool in macro to summarize relationships between macroeconomic time series.
  - To motivate / test models.
  - Examine response to structural shocks.
  - Frequently used at central banks.

#### Stationarity and White Noise

- Cannot find regularities if things do not repeat themselves.
- Leads to concept of stationarity.
  - A time series  $\{x_t\}$  is stationary if the mean, variance, and autocorrelation can be well approximated by sufficiently long time averages.

Other Approaches

• In other words,  $\{x_t\}$  is covariance stationary if:

$$E\{x_t\} = \mu \ \forall \ t \ \text{and} \ E\{(x_t - \mu)(x_{t-k} - \mu)\} = g_k \ \forall \ t, k$$

- Sometimes not a great assumption (e.g., economies in transition), but for post-war US GDP, it works.
- Otherwise, detrend or difference.
- A white noise process has mean zero, a constant variance, and is serially uncorrelated.

## Autoregressions

• An autoregression is a regression of a time series  $\{x_t\}$  on lags of itself.

Other Approaches

• Example: AR(1)

$$x_t = \beta_0 + \beta_1 x_{t-1} + \varepsilon_t$$

- Stationary and stable if  $|\beta_1| < 1$
- Otherwise goes off to infinity and never mean reverts.
- Can estimate AR(p)

$$x_t = \beta_0 + \beta_1 x_{t-1} + \beta_2 x_{t-2} + \dots + \beta_p x_{t-p} + \varepsilon_t$$

by OLS if  $\{x_t\}$  is stationary and  $\varepsilon_t$  is a white noise process.

#### Vector Autoregression

- A vector autoregression is a generalization of an autoregression in which  $x_t$  is a vector of time series.
- Simple example we will use:

$$x_t = \left[ \begin{array}{c} y_t \\ z_t \end{array} \right]$$

- However can be of arbitrary size n.
- The *reduced-form* single-lag VAR of  $x_t$  is then:

$$x_t = A_0 + A_1 x_{t-1} + e_t$$

where  $A_0$  is an  $n \times 1$  vector and  $A_1$  is an  $n \times n$  matrix.

#### Reduced-Form VAR: Estimation

More generally, for a VAR of size n with p lags,

$$x_t = A_0 + A_1 x_{t-1} + A_2 x_{t-2} + \dots + A_p x_{t-p} + e_t$$

Other Approaches

 $x_t$ ,  $A_0$ , and  $e_t$  are  $n \times 1$  vectors and  $A_i$  are  $n \times n$  matrices.

- There are thus  $n + pn^2$  coefficients and (n + 1) n/2 in the variance-covariance matrix.
- The right hand side only contains predetermined variables of a stationary process, and the error terms are assumed to be serially uncorrelated with constant variance (can relax).
  - So can estimate each equation by OLS.
  - Application of seemingly unrelated regression.

### Forecasting

Can forecast using VAR:

$$E_t x_{t+1} = A_0 + A_1 x_t$$
  

$$E_t x_{t+2} = A_0 + A_1 E_t x_{t+1} = A_0 + A_1 [A_0 + A_1 x_t]$$

- Often-used diagnostic tool is the forecast error variance decomposition (FEVD).
  - Tells us proportion of variance of moments in  $\{y_t\}$  or  $\{z_t\}$  due to  $e_{1,t}$  and  $e_{2,t}$ .
  - Like a partial R<sup>2</sup> of forecast error by forecast horizon.
  - See econometrics class for derivation.

### Impulse Responses

• Theorem: VAR has a vector moving average representation.

Other Approaches

 Example with one lag (and a zero constant term, to keep things simple):

$$x_{t} = Ax_{t-1} + e_{t}$$

$$= A(Ax_{t-2} + e_{t-1}) + e_{t}$$

$$= e_{t} + Ae_{t-1} + A^{2}e_{t-2} + A^{3}e_{t-3} + \dots$$

• The response of  $x_t$  to a one unit shock to  $e_{1,t}$  in period t after n periods with no other e shocks is:

$$IR(n) = A^n \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

## Impulse Response Functions

$$IR(n) = A^n \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

- This is called an *impulse response function to e*<sub>1+</sub> and is a convenient way to represent how shocks  $\{e_t\}$  affect  $\{x_t\}$ .
  - Can plot graphically and create standard error bands.
- Intuitively, this is the difference between two processes  $\{x_t\}$ that are made up of identical shocks  $\{e_t\}$  except in period t, where an additional unit one shock is added to  $e_{1,t}$ .

#### Structural VARs

- Unfortunately, reduced form VARs are restrictive.
  - 1. No simultaneous causality
  - 2. Shocks have to be uncorrelated and white noise.
  - Note: These are the same problem written different ways.

Other Approaches

- Generalize to a structural VAR.
- Tackle problem 1 first and allow for simultaneous causality:

$$y_t = b_{10} - b_{12}z_t + \gamma_{11}y_{t-1} + \gamma_{12}z_{t-1} + \varepsilon_{yt}$$
  

$$z_t = b_{20} - b_{21}y_t + \gamma_{21}y_{t-1} + \gamma_{22}z_{t-1} + \varepsilon_{zt}$$

where  $\varepsilon_{v,t}$  and  $\varepsilon_{z,t}$  are independent white noise processes.

• Cannot directly estimate because  $y_t$  is correlated with  $\varepsilon_{z,t}$  and vice-versa, violating exclusion restriction.

#### Reduced-Form Representation

• Write structural VAR as a matrix:

$$\begin{bmatrix} 1 & b_{12} \\ b_{21} & 1 \end{bmatrix} \begin{bmatrix} y_t \\ z_t \end{bmatrix} = \begin{bmatrix} b_{10} \\ b_{20} \end{bmatrix} + \begin{bmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{bmatrix} \begin{bmatrix} y_{t-1} \\ z_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{yt} \\ \varepsilon_{zt} \end{bmatrix}$$

$$Bx_t = \Gamma_0 + \Gamma_1 x_{t-1} + \varepsilon_t$$

• Premultiply by  $B^{-1}$  to get reduced-form representation:

$$x_t = A_0 + A_1 x_{t-1} + e_t$$

where 
$$A_0 = B^{-1}\Gamma_0$$
,  $A_1 = B^{-1}\Gamma_1$ , and  $e_t = B^{-1}\varepsilon_t$ .

• Note reduced form errors  $e_t$  are of form:

$$e_{1t} = (\varepsilon_{yt} - b_{12}\varepsilon_{zt})/(1 - b_{12}b_{21})$$

- Stationary white noise, but correlated with one another.
- For IRFs and FEVDs, want responses to  $\varepsilon_{zt}$  not  $e_{zt}$ .

#### The Identification Problem

 Cannot invert from reduced form to structural VAR unless add restrictions.

Other Approaches

 Reduced form has 9 unknowns, six as plus 3 terms of var-covar matrix:

$$y_t = a_{10} + a_{11}y_{t-1} + a_{12}z_{t-1} + e_{1t}$$
  

$$z_t = a_{20} + a_{21}y_{t-1} + a_{22}z_{t-1} + e_{2t}$$

• Structural form has 10 unknowns, 8 bs and  $\gamma$ s plus 2 terms of var-covar matrix (uncorrelated shocks):

$$y_t = b_{10} - b_{12}z_t + \gamma_{11}y_{t-1} + \gamma_{12}z_{t-1} + \varepsilon_{yt}$$
  

$$z_t = b_{20} - b_{21}y_t + \gamma_{21}y_{t-1} + \gamma_{22}z_{t-1} + \varepsilon_{zt}$$

- Fundamentally under-identified.
  - Intuitively, es depend on both  $\varepsilon_{vt}$  and  $\varepsilon_{zt}$  so cannot invert from es to  $\varepsilon$ s. Extra parameters determine this relationship.

#### Recursive VARs and Identification

- Solution is recursive system:
  - Assume  $y_t$  has contemporaneous effect on  $z_t$ , but  $z_t$  has no contemporaneous effect on  $y_t$ .

Other Approaches

- Jargon: "order"  $y_t$  first, in the sense that it is "causally prior."
- System is:

$$y_t = b_{10} + \gamma_{11}y_{t-1} + \gamma_{12}z_{t-1} + \varepsilon_{yt}$$
  

$$z_t = b_{20} - b_{21}y_t + \gamma_{21}y_{t-1} + \gamma_{22}z_{t-1} + \varepsilon_{zt}$$

SO

$$e_{1t} = \varepsilon_{yt}$$
 and  $e_{2t} = \varepsilon_{zt} - b_{21}\varepsilon_{yt}$ 

- Exactly identified because one parameter  $(b_{12})$  is now a zero. 9 parameters in both structural VAR and reduced form.
- Intuition: Can now distinguish  $\varepsilon_{vt}$  and  $\varepsilon_{zt}$  shocks.
  - Only ε<sub>vt</sub> shocks affect contemporaneous values of y<sub>t</sub>.
  - $e_{1t}$  attributed completely to  $\varepsilon_{vt}$ ; can invert es to get  $\varepsilon$ s.

# Cholesky Decomposition

- Lower triangular assumption on the structural residuals is called a *Cholesky decomposition*.
  - Most common identification scheme for VAR.
- Generalize this to *n* variable and *p* lag VAR.
  - B is then an  $n \times n$  matrix.
  - Exact identification requires  $(n^2 n)/2$  restrictions between the regression residuals and structural innovations.
  - Cholesky does this by setting exactly  $(n^2 n)/2$  values of the B matrix to zero.

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#### Frror Correlation Version

 Assume instead there is no simultaneous causality but  $e_t = C\varepsilon_t$  where  $\varepsilon_t$  are the independent structural shocks and C is an  $n \times n$  matrix:

$$x_t = A_0 + A_1 x_{t-1} + C \varepsilon_t$$

Other Approaches

Equivalent to reduced form VAR

$$C^{-1}x_t = C^{-1}A_0 + C^{-1}A_1x_{t-1} + \varepsilon_t$$

- Same as before. Recursive VAR if  $C^{-1}$  is lower triangular.
- Same problem cannot tell apart shocks.
- Now direct relationship between es and  $\varepsilon$ s instead of relationship arising through simultaneous causality.
- Alternate interpretation of Cholesky: Assume  $\varepsilon_{zt}$  affects both  $y_t$  and  $z_t$  but  $\varepsilon_{vt}$  has no contemporaneous effect on  $z_t$ .

## Cholesky Decomposition: Key Assumptions

- Cholesky is a STRONG assumption.
  - No reverse-causality
  - No omitted variables correlated with "lower ordered" variables and "higher ordered" variables.
  - Strong exclusion restriction.
- "Ordering" sounds innocuous. It's not.
  - n! possible orderings!

#### Stock and Watson: VAR Criticisms in Practice

- 1. What really are the VAR "shocks?"
  - Concern: Shocks reflect factors omitted from model. If correlated with included variables, then OVB.
  - In practice: imagine you order policy last and thus statistically model effect of variables on policy.
    - Assuming regression captures all channels through which policy responds to developments in economy.
    - Omitted channels may lead to correlation between policy and outcomes.

- Ex: "Price puzzle" of why inflation rises with negative monetary shock.
  - One answer is Fed is forward looking and rises rates when it (correctly) anticipated inflation.
  - VAR omits variables that predict this inflation.
- 2. Parameter instability.
- Timing assumptions do not reflect real-time data availability, causing misspecification.

#### Other SVAR Identification Schemes

- VARs criticized for being too reduced form.
- Structural VAR (SVAR) approach uses economic theory rather than Cholesky decomposition invert reduced form VAR to structural VAR (that is, to recover structural innovations from reduced form residuals).

- Must impose  $(n^2 n)/2$  restrictions.
- Examples:
  - Gali (1999) splits Solow residual into tech and non-tech shocks by assuming that only tech shocks affect long-run productivity.
  - Sign restrictions (Uhlig).
  - Assuming cross-sectional regression holds (e.g., Beraja, Hurst, and Ospina, 2016).

# Local Projections

- Jorda (2005) proposes more robust estimate of impulse response by local projection.
- Assume you have a shock S<sub>t</sub> and an outcome Y<sub>t</sub>. Idea is to directly and non-parametrically estimate the impulse response:

$$Y_{t+h} = \beta_0 + \beta_h S_t + \gamma X_t + \varepsilon_t^h$$

- Then the  $\beta_h$ s for h = 0, ..., T is the estimated T-period impulse response.
- Controls X<sub>t</sub> often include lags of outcome and shock to control for autocorrelation and anticipatory effects.
- Local projections increasingly popular.
  - Less extrapolation of model at long horizons.
  - More robust to misspecification, which compounds at long horizons.
  - But much wider standard errors. Bias-variance tradeoff between LP and SVAR (Plagborg-Moller and Wolf, 2021; Li, Plagborg-Moler, and Wolf, 2022).

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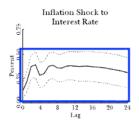
# VAR Evidence on Non-Neutrality

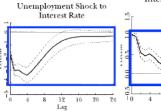
- Apply VARs to study the effects of monetary shocks.
- Key challenge is endogeneity.
  - Changes in monetary policy occur for good reasons.
  - Error term  $\varepsilon_t$  correlated with outcome:

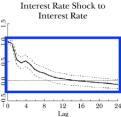
$$\Delta y_t = \alpha + \beta \Delta i_t + \varepsilon_t$$

- Start with simple VAR from Stock and Watson (2001).
  - 3 variables: inflation, unemployment, and Federal Funds interest rate.
  - Order  $\pi_t$ ,  $u_t$ ,  $R_t$  in recursive VAR.
    - $\pi_t$  affects  $u_t$  and  $R_t$  contemporaneously but not vice-versa.
    - $u_t$  affects  $R_t$  contemporaneously but not vice-versa.
  - See paper for data description (FEVD, Granger Causality tests).

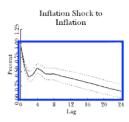
# IRFs: Taylor Rule



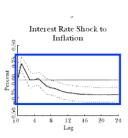




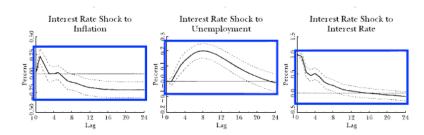
# IRFs: Phillips Curve







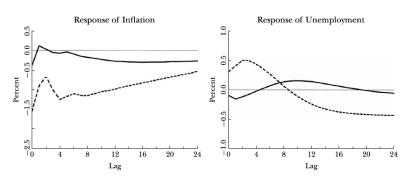
# IRFs: Monetary Non-Neutrality



## IRFs: Two Structural Approaches

• Stock and Watson then use a structural VAR in which impose a Taylor rule for identification rather than recursive VAR.

- In solid: backward-looking Taylor rule.
- In dashed: forward-looking Taylor rule.
- Structural assumptions (and hence ordering) not innocuous.

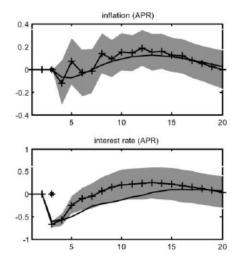


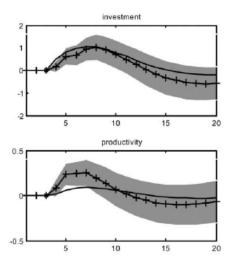
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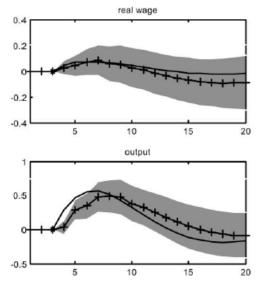
# Christiano, Eichenbaum, and Evans (2005)

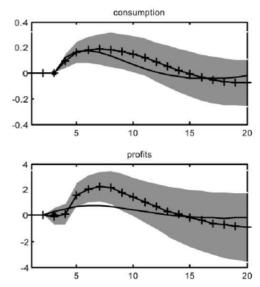
Paper does a lot. For now just focus on VAR evidence.

- Run an 9-variable VAR. Ordering:
  - 1. Real GDP
  - 2. Real Consumption
  - GDP Deflator
  - 4. Real Investment
  - 5. Real Wage
  - 6. Labor Productivity
  - 7. Federal Funds Rate
  - 8. Real Profits
  - M2 Growth
- Economic conditions can affect monetary policy, but monetary policy only affects economic conditions with a lag.
  - Trying to get around endogeneity of monetary policy by statistically modeling it. But still Stock-Watson concerns.









# Christiano, Eichenbaum and Evans (2005) Summary

- 1. Hump-shaped response of output, consumption and investment, peaking at  $1\frac{1}{2}$  years and returning to trend after 3.
- 2. Hump-shaped response of inflation, peaking after two years.
- 3. Interest rate falls for one year
- 4. Real profits, wages, and labor prod rise.
- 5. Growth rate of money rises immediately.
- Phillips curve and Taylor rule as in Stock and Watson still hold.
- Consistent with significant monetary non-neutrality
  - $\Rightarrow$  money affects real outcomes.

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## Other Approaches

- We may not like recursive VAR approach to identifying monetary shocks.
- Five other approaches that try to deal with causality more directly of which I want you to be aware.

Other Approaches

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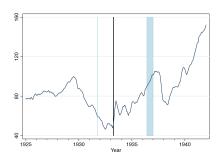
- 1. Large Shocks
- 2. Discontinuity-Based Approach
- 3. Narrative Approach
- 4. High Frequency Approach
- 5. Natural Experiments
- Good summary: Section 4 of Nakamura and Steinsson (2018) "Identification in Macroeconomics"

 Friedman and Schwartz (1963) famously argue that Fed made Great Depression worse.

Other Approaches

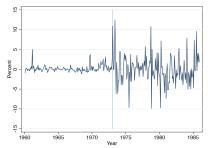
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- Focus on policy actions that are "of major magnitude," not caused by other developments, sharp results that they compare to science experiment.
  - But others have questioned since.



## Breakdown of Bretton Woods: Mussa (1986)

- In 1973 Bretton Woods fixed exchange rate system breaks.
  - Discontinuous and purely monetary change.
  - If money is neutral, should not affect real variables like real exchange rates.
- Monthly change in real Mark-Dollar exchange rate:



 But Itskhoki and Mukhin (2022) argue this is not evidence of monetary non-neutrality but instead financial frictions in foreign exchange markets.

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# Narrative Approach

- Narrative approach of Romer and Romer (1989) updated by Romer and Romer (2023)
  - Identify exogenous monetary shocks by using historical record.
  - Go through meeting transcripts, historical material, etc. to find a change in monetary policy unrelated to state of the economy e.g. a change in the Fed's preferences.

#### Examples:

- In December 1988, change view of what level of inflation is acceptable and raise rates.
- January 1972 think unemployment settled too high and lower.
- No monetary shocks 1988-2016! Only one expansionary shock!
- Impulse responses to these "exogenous" policy dates show non-neutrality.

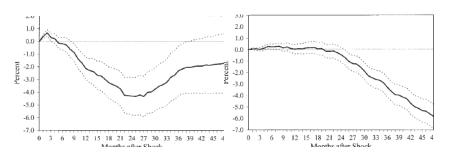
### Narrative Approach

- More quantitative method in Romer and Romer (2004):
  - Determine intended FFR at meeting based on Fed's internal staff "Greenbook" forecast.
    - Regression controlling for level and change in forecasts of output, inflation, and unemployment.

Other Approaches

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- Difference from FFRI agreed upon at meeting to obtain shock.
- IRFs show non-neutrality:

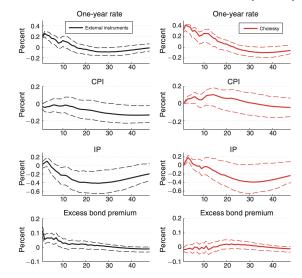


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# High-Frequency Approach

- Shock series based on response of Fed funds futures in short window around Fed announcements.
  - Captures verbal communication in addition to rate
  - Gürkaynak-Sack-Swanson (2005): Forward rates at long horizons affected by MP contrary to standard models.
  - Would like to look at responses of macro variables to show non-neutrality, but those are not at daily frequency!
- Two solutions to get at non-neutrality:
  - Gertler and Karadi (2015) time aggregate high frequency shocks and use as external instruments in VAR with low-frequency outcomes (e.g. inflation and output).
  - Nakamura and Steinsson (2018) compare responses of high frequency nominal and inflation-indexed Treasuries to separate real interest rate response and inflation expectations.
  - Issue with high frequency approach: Shocks are very small, lack of precision in local projection without VAR structure.

## High-Frequency Approach: Gertler-Karadi (2015)



### High-Frequency Approach Finds Non-Neutrality

- Gertler-Karadi (2015): Non neutrality, no price puzzle, credit spreads respond to monetary policy.
- Nakamura-Steinsson (2018)
  - Monetary shocks have large and persistent effects on real interest rates.
  - Monetary shocks have small effects on expected inflation at short horizons (< 1 year) and grows to a large effect over 2-3 years (hump shaped response).
  - Argue results imply that Fed announcements provide information that affects beliefs about economic fundamentals beyond interest rates.
- Which do we prefer?
  - Some issues of time aggregation here. N-S is more convincing for real interest rates and inflation expectations.
  - But sometimes we want to look at outcomes that are not as high frequency (e.g. output, realized inflation, credit spreads) and have to bite the time aggregation bullet.

#### Natural Experiments

- Ideal Evidence: Experiment where randomly change money supply in some places.
- Problem: Central banks are run by economists.
  - Changes in money supply are not random!
- Solution: Natural experiments. Examples:
  - Hyperinflations: inflation tracks money supply.
  - U.S. Great Depression (Freidman and Schwartz, 1963).
  - Gold Standard and Great Depression (Eichengreen and Sachs, 1985).
  - Breakdown of Bretton Woods (Mussa, 1986).
  - Volcker disinflation in early 1980s.

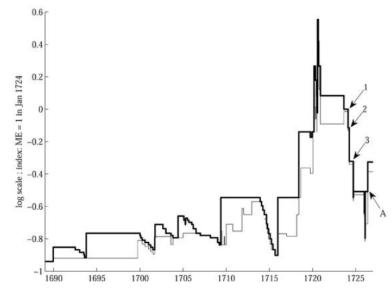
- Money: coins with no face value.
  - Government sets nominal value by decree, can change it overnight and without warning.

- Velde (2009) examines an episode where three times in 1724, French cut value of currency overnight by a cumulative 45%.
  - Ex: September 22, 1724 at 8am, all 5 livre coins are now 4 livre coins.
  - "The high price level reduced the real value of soldiers' wages and harmed government creditors."
  - Why? King and his misters wanted to (before economists!).
  - Revalue some in 1726.
- Expectations:

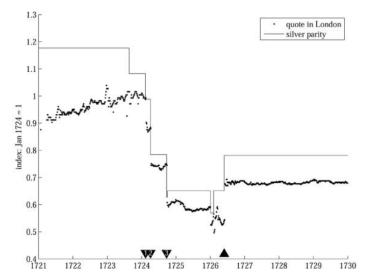
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- Had done before, but always fast inflations and gradual deflations.
- Velde argues these three deflations were "unforetold." Kept secret to reduce capital losses by state.

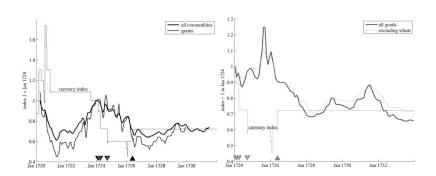
# Value of a Coin

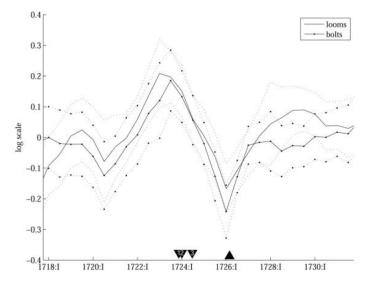


# Foreign Exchange Prices Adjust Instantaneously



## Commodities and Goods Prices Fall Slowly





## Summary: Strong Evidence of Non-Neutrality

- Introduced tool of VAR.
  - Recursive and structural.
  - Discussed assumptions, flaws, and benefits.
- Looked at VAR, narrative, high-frequency, and natural experiment evidence for monetary non-neutrality.
  - Strong evidence that money is non-neutral: it has effects on real economy.
  - Strong evidence of hump-shaped inflation responses.
- Next class: Introduce money and add it to RBC framework.
  - Can it explain the facts presented here?
  - Read Gali Ch. 2.