Geneva Graduate Institute (IHEID)

Econometrics I (EI035), Fall 2024

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Final Exam

Tuesday, 10 December

- You have 1h 30min.
- There are 42 points in total.
- Prepare concise answers.
- Write your answers on separate sheets, not on the exam copy.
- State clearly any additional assumptions, if needed.
- For full credit, you need to explain your answers.

Problem 1 (42 points)

Suppose you observe a sample of n unemployed invidiuals. Let y_i denote the time (in weeks) that individual i spent in unemployment, and let $\bar{y} \equiv \frac{1}{n}\sum_{i=1}^{n} y_i$ be the average unemployment span in your sample. You can assume that your observations are independent.

One can model y_i using an expontential distribution:

$$p(y_i|\ \lambda) = \lambda exp\{-\lambda y_i\} \quad , \quad \lambda > 0 \; .$$

The parameter λ is the job-finding rate. It tells you how many (acceptable) job offers per week an individual gets. For example, $\lambda = 3$ would tell you that (on average) an individual receives three offers every week, while $\lambda = 1/2$ would tell you that (on average), an individual receives an offer every two weeks. For now, we assume that this λ is the same for all individuals in the sample.

(a) (4 points) The mean and variance of y_i are given by

$$\mathbb{E}[y_i] = \frac{1}{\lambda} \quad \text{and} \quad \mathbb{V}[y_i] = \frac{1}{\lambda^2} \; .$$

Interpret these expressions, relying on the two examples $\lambda = 3$ and $\lambda = 1/2$.

(b) (3 points) Derive the log-likelihood $\ell(\lambda|Y)$ and find the Maximum Likelihood (ML) estimator

$$\hat{\lambda} \equiv \arg\max_{\lambda} \ell(\lambda|Y) \; .$$

(c) (3 points) What is the probability limit of the average unemployment span in your sample, $\bar{y}$? Based on that result, is $\hat{\lambda}$ consistent?

(d) (4 points) What is the approximate distribution of $\bar{y}$ for large n? Based on that result, what is the approximate distribution of $\hat{\lambda}$ for large n?

(e) (3 points) Describe another, numerical approach to approximate the distribution of $\hat{\lambda}$.

Problem 1 Continued

Now let's make the job-finding rate heterogeneous. Specifically, suppose x_i denotes the number of applications per week that individual i sent out, and let

$$\lambda_i = \exp\{\alpha + \beta x_i\} \ .$$

Note that this implies that

$$\mathbb{E}[y_i|x_i] = \frac{1}{\lambda_i} = \exp\{-(\alpha + \beta x_i)\} \quad \text{and} \quad \mathbb{V}[y_i|x_i] = \frac{1}{\lambda_i^2} = \exp\{-2(\alpha + \beta x_i)\} \ . \tag{1}$$

For simplicity, let's suppose you know α (so it's just a constant) and you only need to estimate β. The ML estimator can be defined as

$$\hat{\beta} = \arg\min_{\beta} Q_n(\beta) \ , \quad Q_n(\beta) = \frac{1}{n}\sum_{i=1}^{n} -(\alpha + \beta x_i) + y_i \exp\{\alpha + \beta x_i\} \ .$$

(f) (5 points) How do you interpret the parameters α and β?

(g) (4 points) How can you find $\hat{\beta}$? Derive the first-order condition associated with the above optimization problem.

(h) (6 points) Is $\hat{\beta}$ a consistent estimator for β_0, the true value for β?
Hint: Note that $\mathbb{E}[y_i|x_i] = \exp\{-(\alpha + \beta_0 x_i)\}$, *and remember the Law of Iterated Expectations (LIE). Also, note that* $x_i \geq 0$ *and a function like* $\mathbb{E}[x_i exp\{x_i \beta\}]$ *is strictly increasing in* β.[1]

(i) (6 points) Show that the asymptotic distribution of $\hat{\beta}$ is given by

$$\sqrt{n}(\hat{\beta} - \beta) \xrightarrow{d} N\left(0, \mathbb{E}[x_i^2]^{-1}\right) \ .$$

Hint: To simplify notation, write λ_{i0} *for* $exp\{\alpha + \beta_0 x_i\}$. *Also, your formula for the asymptotic variance simplifies thanks to the LIE and the results in Eq.* (1).

(j) (4 points) Construct a hypothesis test with size $\alpha = 0.05$ for testing

$$\mathscr{H}_0 : \mathbb{E}[y_i|x_i = 10] = \frac{1}{2}\mathbb{E}[y_i|x_i = 5] \quad \text{vs.} \quad \mathscr{H}_0 : \mathbb{E}[y_i|x_i = 10] \neq \frac{1}{2}\mathbb{E}[y_i|x_i = 5] \ ,$$

i.e. testing whether an individual submitting 10 applications per week spends (in expectation) exactly half as long in unemployment than a person sending out only 5 applications per week. More concretely, definining the test as $\varphi = \mathbf{1}\{T(X) < c_\alpha\}$, define the test-statistic $T(X)$ and find the critical value c_α.

[1] Strictly speaking, this holds provided that $\mathbb{E}[x_i] > 0$.