Micro II
Problem Set 2
SOLUTIONS

Dominic Rohner, Geneva Graduate Institute, Spring 2025

Question 1: Single-crossing condition

Take the simple model of public finance from class and show that the policy preferences fulfill the single-crossing condition (SCC).

Recall the policy preferences from the simple model of public finance:

$$W^i(g) = (y - g)\frac{y^i}{y} + H(g)$$

There are two ways to show that SCC holds:

Solution 1:

We know that $H(g)$ is concave and increasing. The quasi-linear structure of the policy preferences implies concavity also in $W^i(g)$. Concavity in $W^i(g)$ tells us that there can only be a single peak. Further, single-peakedness always implies that SCC holds (but not vice versa). Hence, SCC holds for the given policy preference.

Solution 2:

The policy preferences fulfill the Gans-Smart SCC under the following conditions:

If $g_1 > g_0$ and $y_1^i > y_0^i$ then, the SCC requires that

$$W^i(g_1; y_1^i) \geq W^i(g_0; y_1^i) \Rightarrow W^i(g_1; y_0^i) \geq W^i(g_0; y_0^i). \tag{1}$$

I.e. if the rich want more public spending, then *a fortiori* so do the poor.

Or if $g_2 < g_0$ and $y_2^i < y_0^i$ then, the SCC requires that

$$W^i(g_2; y_2^i) \geq W^i(g_0; y_2^i) \Rightarrow W^i(g_2; y_0^i) \geq W^i(g_0; y_0^i) \tag{2}$$

I.e. if the poor want less public spending, then *a fortiori* so do the rich. In other words, SCC enables us to project preferences over g on a set of voter types (here rich and poor).

Next, consider equation (1): Now we plug in the policy preference and rearrange the inequality:

$$W^i(g_1; y_1^i) \geq W^i(g_0; y_1^i) \Rightarrow W^i(g_1; y_0^i) \geq W^i(g_0; y_0^i)$$

Write the first inequality as

$$(y - g_1)\frac{y_1^i}{y} + H(g_1) \geq (y - g_0)\frac{y_1^i}{y} + H(g_0)$$

which can be rewritten as

$$\frac{H(g_1) - H(g_0)}{g_1 - g_0} \geq \frac{y_1^i}{y}$$

Realize that the LHS is independent of income. Since $y_0^i < y_1^i$ it is *a fortiori* true that

$$\frac{H(g_1) - H(g_0)}{g_1 - g_0} > \frac{y_0^i}{y}$$

Hence, we have shown that as long as the inequality holds, $W^i(g_1; y_0^i) > W^i(g_0; y_0^i)$ is true.

The proof is analogous for (2).

Hence, the policy preferences satisfy the Gans-Smart SCC property.

Question 2: Downsian competition in a simple public-good model

Consider the following economy. Agent i's preferences over a publicly provided good g and a privately provided good c^i are expressed by

$$w^i = c^i + \alpha^i \ln g \tag{3}$$

where α^i is the intrinsic parameter of agent i that is drawn from a distribution $F(\cdot)$ with mean α, i.e. some individuals appreciate public goods (like e.g. a public park) more than others. There is a unit measure of individuals in the society. They have initial resources only in the private good, i.e. $y^i = 1$ for all i, and one unit of private good is required to produce one unit of public good. To finance the public good production, the government raises a tax τ on each individual so that agent i's budget constraint is $c^i \leq y^i - \tau = 1 - \tau$.

a) Derive the Utilitarian optimum of a social planner (i.e. the level of g^* that maximises the society's Utilitarian welfare)

Equation (3) is the direct utility function of agent i. The indirect utility function replaces the agent's resource constraint, $c^i = 1 - \tau$, in (3):

$$\tilde{W}\left(\tau, g; \alpha^i\right) = 1 - \tau + \alpha^i \ln g.$$

In order to obtain the agent's policy preferences, we need to consider also the restrictions imposed by the government (in this case, by the government's resourse constraint). Since the public good provision is financed entirely from taxes on income, we have that:

$$\int_{\alpha^i} \tau dF\left(\alpha^i\right) = \int_{\alpha^i} g dF\left(\alpha^i\right) \iff \tau = g.$$

We now use this relation between τ and g to replace g for τ in the agent's preferences and derive the agent's policy preferences:

$$\tilde{W}(\tau, g\left(\tau\right); \alpha^i) \equiv W\left(\tau; \alpha^i\right) = 1 - \tau + \alpha^i \ln \tau.$$

The (Utilitarian) social optimum maximizes the sum (integral) of the citizens' policy preferences:

$$\max_{\tau} \int_{\alpha^i} \left[1 - \tau + \alpha^i \ln \tau\right] dF(\alpha^i) \Leftrightarrow \max_{\tau} \left[1 - \tau + \alpha \ln \tau\right]$$

where $\alpha = \int_{\alpha^i} \alpha^i dF(\alpha^i)$ is the mean of α^i.

FOC:

$$\begin{aligned} -1 + \alpha \frac{1}{\tau} &= 0 \Rightarrow \\ \tau^* &= \alpha, \\ g^* &= \tau^* = \alpha. \end{aligned}$$

So, the Utilitarian social optimum sets $g^* = \tau^* = \alpha$.

Now suppose that two politicians $P = A, B$ select platforms τ^A and τ^B. Assume that each maximizes the expected value of some exogenous rent R of being in office. Call π_P the vote share for politician P; then P's probability of winning the election is $p_P = Prob(\pi_P \geq \frac{1}{2})$ and her expected utility is then $p_P \cdot R$. First, the two candidates announce their platforms simultaneously and noncooperatively. Then the elections are held. Last, the elected politician implements her announced policy.

b) Assume that $\alpha^i = \alpha$ (homogenous voters). Determine the candidates' probability of winning. What are the announced platforms and which one is implemented? Discuss.

$$p_A = \begin{cases} 0 & if \quad W(\tau^A; \alpha) < W(\tau^B; \alpha) \\ \frac{1}{2} & if \quad W(\tau^A; \alpha) = W(\tau^B; \alpha) \\ 1 & if \quad W(\tau^A; \alpha) > W(\tau^B; \alpha) \end{cases}$$

Politicians maximize the probability of winning. They converge to the platform that maximizes $W(\tau; \alpha)$:

$$\tau^A = \tau^B = \tau^* = \alpha$$

Each politician will win with probability $\frac{1}{2}$ and will implement the announced policy $\tau^* = \alpha$.

c) Determine each candidate's probability of winning when agents are heterogeneous. What are the selected platforms in that case? Which one is implemented?

Since $\ln \tau$ is concave, it can be verified that individual α^i's policy preferences $W(\tau; \alpha^i) = 1 - \tau + \alpha^i \ln \tau$ are single peaked and maximized at $\tau^*(\alpha^i) = \alpha^i$ as follows:

$$\begin{aligned} &\max_{\tau} 1 - \tau + \alpha^i \ln \tau \Rightarrow \\ &\text{FOC:} -1 + \alpha^i \frac{1}{\tau} = 0 \\ &\Rightarrow \tau^*\left(\alpha^i\right) = \alpha^i. \end{aligned}$$

$$\text{SOC:} \ -\alpha^i \frac{1}{\tau^2} < 0.$$

The second-order conditions establish the strict concavity of $W(\tau; \alpha^i)$, with a single global max (single peak) at $\tau(\alpha^i) = \alpha^i$. Applying the median voter theorem, $\tau^A = \tau^B = \tau^m = \alpha^m$ where $\tau^m = \alpha^m$ is the tax preferred by the median voter with α^m. Each politician will win with probability $\frac{1}{2}$ and will implement τ^m.

d) What are the model's economic predictions? Discuss.

Economic predictions: both politicians' platforms converge to the median voter's optimum, which may deviate from the (utilitarian) social optimum.

If $\alpha^m > \alpha$, then there is "over-provision" of the public good relative to the utilitarian benchmark.

If $\alpha^m < \alpha$, then there is under-provision of the public good relative to the utilitarian benchmark.

Only in the case when for the distribution of tastes over public good provision mean and median coincide do we get that the utilitarian optimum corresponds to the winning policy.

Recall that, here, $y^i = y$, $\forall i$, and so there is no link between how the α^is vary in the population and income levels.

Question 3: Explain the concept of ”The Curse of Education”

Answer: See Slide 75 of Section II.