Macroeconomics I: Macroeconomic Principles Midterm Exam Long Question 2: capital dynamics (25% of grade)

Cédric Tille

October 28, 2020

1 Part a)

Take the models of capital dynamics seen in class. We assume that:

i) Population (and labor) is constant and set to a value of 1.

ii) Productivity has no trend.

iii) There is no capital depreciation.

Output Y is produced using capital K, with an exogenous productivity A:

$$Y_t = A_t (K_t)^{\alpha} \quad ; \quad \alpha < 1$$

Capital increases with investment, which corresponds to output minus consumption C:

$$K_{t+1} - K_t = Y_t - C_t$$

Consider a steady state with productivity equal to $\bar{A} = 1$. What is the steady state in a Solow model with saving rate s_K? Explain the intuition.

What is the steady state in a Ramsey model with a utility discount factor ρ? What is the saving rate at that point? Explain the intuition.

Answer: There is no steady state in the Solow model (this was a trick question), because we have no "headwinds" as depreciation, population growth, and productivity growth are all zero. As labor and productivity are constant at 1, the overall capital K is equal to the effective capital k. The dynamics of capital imply:

$$\begin{aligned} K_{t+1} - K_t &= s_K Y_t \\ K_{t+1} - K_t &= s_K (K_t)^\alpha > 0 \end{aligned}$$

Therefore, capital always increases. Intuitively, as we assume that savings are always positive and there as no headwinds, nothing stops the accumulation of capital that proceeds forever.

In the Ramsey model, the Euler equation implies that the real interest rate (equal to the marginal product of capital) is equal to the discount factor in the steady state (as consumption is constant):

$$\begin{aligned} \rho &= \bar{r} = \frac{\partial \bar{Y}}{\partial \bar{K}} \\ \rho &= \alpha \left(\bar{K}\right)^{\alpha-1} \\ \bar{K} &= \left(\frac{\alpha}{\rho}\right)^{\frac{1}{1-\alpha}} \end{aligned}$$

which is well defined. The savings rate at the steady state is zero. Intuitively, there is now a headwind in the form of the cost of waiting, ρ. At the steady state, the marginal gain from investing exactly offsets this headwind, and savings stop (as there is no need to offset a depreciation).

2 Part b)

We now focus on a Ramsey model with shocks (equivalent to a real business cycle model with constant labor) where the consumer maximizes the following intertemporal utility of consumption:

$$U_t = E_t \sum_{s=0}^{\infty} \left(\frac{1}{1+\rho}\right)^s D_{t+s} \ln\left(C_{t+s}\right)$$

where $\rho > 0$ is the rate of time preference and E_t denotes expectations from the point of view of period t. D_{t+s} is a stochastic time discount term that you can think of as a demand shocks. It fluctuates around a value of 1:

$$\ln\left(D_t\right) = \phi^D \ln\left(D_{t-1}\right) + \varepsilon_t^D$$

where ε_t^D is a random variable ($E_t \varepsilon_{t+1}^D = 0$) and $\phi^D \in [0,1]$ is the persistence. Productivity also fluctuates around a value of 1:

$$\ln(A_t) = \phi^A \ln(A_{t-1}) + \varepsilon_t^A$$

where ε_t^A is a random variable ($E_t \varepsilon_{t+1}^A = 0$) and $\phi^A \in [0,1]$ is the persistence.

We can show that (take this as given, r_{t+1} is the real interest rate $r_{t+1} = \alpha A_{t+1} (K_{t+1})^{\alpha-1}$):

$$\frac{D_t}{C_t} = \frac{1}{1+\rho} E_t \left(\frac{D_{t+1}}{C_{t+1}} (1 + r_{t+1}) \right)$$

Provide an intuitive interpretation of this expression.

Answer: We first derive the equations. As output is consumed or invested, $Y_t = C_t + K_{t+1} - K_t$, the consumer maximizes the following Lagrangian, where x_{t+s} is the state of nature at time $t+s$ that occurs with probability $\pi(x_{t+s})$:

$$\begin{aligned}
\mathcal{L}_t &= \sum_{s=0}^{\infty} \left(\frac{1}{1+\rho} \right)^s \sum_{x_{t+s}} \pi(x_{t+s}) D(x_{t+s}) \ln(C(x_{t+s})) \\
&\quad + \sum_{s=0}^{\infty} \left(\frac{1}{1+\rho} \right)^s \sum_{x_{t+s}} \pi(x_{t+s}) \lambda(x_{t+s}) \left[\begin{array}{c} A(x_{t+s}) (K(x_{t+s}))^{\alpha} \\ -C(x_{t+s}) - K(x_{t+s+1}) + K(x_{t+s}) \end{array} \right]
\end{aligned}$$

The first-order conditions with respect to C_t and $C(x_{t+1})$ are:

$$\begin{aligned}
0 &= \frac{\partial \mathcal{L}_t}{\partial C_t} \Rightarrow D_t \frac{1}{C_t} = \lambda_t \\
0 &= \frac{\partial \mathcal{L}_t}{\partial C(x_{t+1})} \Rightarrow D(x_{t+1}) \frac{1}{C(x_{t+1})} = \lambda(x_{t+1})
\end{aligned}$$

The first order condition with K_{t+1}, which is known at time t (so $K(x_{t+1}) = K_{t+1}$ in all states) is:

$$\begin{aligned}
0 &= \frac{\partial \mathcal{L}_t}{\partial K_{t+1}} \\
0 &= -\lambda_t + \frac{1}{1+\rho} \sum_{x_{t+1}} \pi(x_{t+1}) \lambda(x_{t+1}) \left[A(x_{t+1}) (K_{t+1})^{\alpha-1} + 1 \right]
\end{aligned}$$

Combining the conditions, we get:

$$
\begin{aligned}
\lambda_t &= \frac{1}{1+\rho}\sum_{x_{t+1}} \pi\left(x_{t+1}\right)\lambda\left(x_{t+1}\right)\left[A\left(x_{t+1}\right)\left(K_{t+1}\right)^{\alpha-1}+1\right] \\
\frac{D_t}{C_t} &= \frac{1}{1+\rho}\sum_{x_{t+1}} \pi\left(x_{t+1}\right)\frac{D\left(x_{t+1}\right)}{C\left(x_{t+1}\right)}\left[A\left(x_{t+1}\right)\left(K_{t+1}\right)^{\alpha-1}+1\right] \\
\frac{D_t}{C_t} &= \frac{1}{1+\rho}E_t\left(\frac{D_{t+1}}{C_{t+1}}\left[1+\alpha A_{t+1}\left(K_{t+1}\right)^{\alpha-1}\right]\right) \\
\frac{D_t}{C_t} &= \frac{1}{1+\rho}E_t\left(\frac{D_{t+1}}{C_{t+1}}\left(1+r_{t+1}\right)\right)
\end{aligned}
$$

This is the Euler expressions. If we get one unit of extra resource today, we can consume it today which gives a marginal utility of D_t/C_t. We can also invest it, and tomorrow it will give a gross return $1+r\left(x_{t+1}\right)$, depending on the state, with a marginal value of $D\left(x_{t+1}\right)/C\left(x_{t+1}\right)$. In addition, waiting is costly, as $\rho>0$. The optimal consumption choice is such that the value of consuming the resource today is equal to the expected value of investing it and consuming it tomorrow.

3 Part c)

The model can be written as log approximations around the steady state. We denote log deviations by hatted values: $\hat{C}_t = \left(C_t-\bar{C}\right)/\bar{C}$. The real interest rate is denoted as $\hat{r}_{t+1}=r_{t+1}-\rho$.

The system boils down to the Euler condition and the clearing of the good market (take this as given):

$$
\begin{aligned}
E_t\left(\hat{C}_{t+1}-\hat{C}_t\right) &= \left(E_t\hat{D}_{t+1}-\hat{D}_t\right)+\frac{\rho}{1+\rho}\left(E_t\hat{A}_{t+1}-(1-\alpha)\hat{K}_{t+1}\right) \\
\hat{A}_t+\alpha\hat{K}_t &= \hat{C}_t+\frac{\alpha}{\rho}\left(\hat{K}_{t+1}-\hat{K}_t\right)
\end{aligned}
$$

The solution for consumption and investment takes the form:

$$
\begin{aligned}
\hat{C}_t &= \eta_{CK}\hat{K}_t+\eta_{CA}\hat{A}_t+\eta_{CD}\hat{D}_t \\
\hat{K}_{t+1} &= \eta_{KK}\hat{K}_t+\eta_{KA}\hat{A}_t+\eta_{KD}\hat{D}_t
\end{aligned}
$$

Where we can solve for the various $\eta_{..}$ coefficients. I do so by assuming $\alpha = 1/3$ and $\rho = 0.04$.

Figure 1 illustrates the impact of a positive shock on $\hat{A}$ in period $t = 0$ (specifically $\varepsilon_0^A = 1$). The red line shows the case where shocks are not persistent ($\phi^A = 0$) and the blue line shows the case with persistence ($\phi^A = 0.5$).

Start with the situation without persistence ($\phi^A = 0$). Provide the intuition behind the dynamics, in particular that of the capital stock and the real interest rate.

Answer: For reference, we first show how the model is solved. We start with the steady state where $\bar{A} = \bar{D} = 1$. The clearing of the good market implies that $\bar{C} = \bar{Y} = \left(\bar{K}\right)^{\alpha}$, and the Euler implies that $\rho = \bar{r} = \alpha\left(\bar{K}\right)^{\alpha-1}$ which gives the capital stock:

$$\bar{K} = \left(\frac{\alpha}{\rho}\right)^{\frac{1}{1-\alpha}}$$

Note that the capital output ratio is:

$$\frac{\bar{K}}{\bar{Y}} = \frac{\alpha}{\rho}$$

We can write log-linear approximations of the system. The productivity and discount dynamics are:

$$\begin{aligned} \hat{D}_t &= \ln\left(D_t\right) - \ln\left(1\right) = \phi^D \hat{D}_{t-1} + \varepsilon_t^D \\ \hat{A}_t &= \ln\left(A_t\right) - \ln\left(1\right) = \phi^A \hat{A}_{t-1} + \varepsilon_t^A \end{aligned}$$

The production function is approximated as follows:

$$\begin{aligned} Y_t &= A_t\left(K_t\right)^{\alpha} \\ Y_t - \bar{Y} &= \left(A_t - 1\right)\left(\bar{K}\right)^{\alpha} + \alpha\left(\bar{K}\right)^{\alpha-1}\left(K_t - \bar{K}\right) \\ \bar{Y}\hat{Y}_t &= \left(\bar{K}\right)^{\alpha}\hat{A}_t + \alpha\left(\bar{K}\right)^{\alpha}\hat{K}_t \\ \hat{Y}_t &= \hat{A}_t + \alpha\hat{K}_t \end{aligned}$$

The dynamics of capital are:

$$
\begin{aligned}
Y_t &= C_t + K_{t+1} - K_t \\
Y_t - \bar{Y} &= \left(C_t - \bar{C}\right) + \left(K_{t+1} - \bar{K}\right) - \left(K_t - \bar{K}\right) \\
\bar{Y}\hat{Y}_t &= \bar{C}\hat{C}_t + \bar{K}\hat{K}_{t+1} - \bar{K}\hat{K}_t \\
\hat{Y}_t &= \hat{C}_t + \frac{\alpha}{\rho}\left(\hat{K}_{t+1} - \hat{K}_t\right)
\end{aligned}
$$

Combining with the production function, we get our first relation:

$$
\hat{A}_t + \alpha\hat{K}_t = \hat{C}_t + \frac{\alpha}{\rho}\left(\hat{K}_{t+1} - \hat{K}_t\right)
$$

The Euler condition is approximated as (recall that $E_t\hat{K}_{t+1} = \hat{K}_{t+1}$):

$$
\begin{aligned}
D_t\frac{1}{C_t} &= \frac{1}{1+\rho}E_t D_{t+1}\frac{1}{C_{t+1}}\left(1 + \alpha A_{t+1}\left(K_{t+1}\right)^{\alpha-1}\right) \\
\frac{1}{\bar{C}}\left(D_t - 1\right) - \frac{1}{\left(\bar{C}\right)^2}\left(C_t - \bar{C}\right) &= \frac{1}{1+\rho}E_t\frac{1}{\bar{C}}\left(1+\bar{r}\right)\left(D_{t+1} - 1\right) \\
&\quad - \frac{1}{1+\rho}E_t\left(1+\bar{r}\right)\frac{1}{\left(\bar{C}\right)^2}\left(C_{t+1} - \bar{C}\right) \\
&\quad + \frac{1}{1+\rho}E_t\frac{1}{\bar{C}}\alpha\left(\bar{K}\right)^{\alpha-1}\left(A_{t+1} - 1\right) \\
&\quad + \frac{1}{1+\rho}E_t\frac{1}{\bar{C}}\alpha\left(\alpha-1\right)\left(\bar{K}\right)^{\alpha-2}\left(K_{t+1} - \bar{K}\right) \\
\frac{1}{\bar{C}}\hat{D}_t - \frac{1}{\bar{C}}\hat{C}_t &= \frac{1}{1+\rho}E_t\frac{1}{\bar{C}}\left(1+\rho\right)\hat{D}_{t+1} - \frac{1}{1+\rho}E_t\left(1+\rho\right)\frac{1}{\bar{C}}\hat{C}_{t+1} \\
&\quad + \frac{1}{1+\rho}E_t\frac{1}{\bar{C}}\alpha\left(\bar{K}\right)^{\alpha-1}\hat{A}_{t+1} + \frac{1}{1+\rho}E_t\frac{1}{\bar{C}}\alpha\left(\alpha-1\right)\left(\bar{K}\right)^{\alpha-1}\hat{K}_{t+1} \\
\hat{D}_t - \hat{C}_t &= E_t\hat{D}_{t+1} - E_t\hat{C}_{t+1} + \frac{\rho}{1+\rho}\left(E_t\hat{A}_{t+1} - \frac{\rho}{1+\rho}\left(1-\alpha\right)\hat{K}_{t+1}\right)
\end{aligned}
$$

The real interest rate is:

$$
\begin{aligned}
r_{t+1} &= \alpha A_{t+1}\left(K_{t+1}\right)^{\alpha-1} \\
r_{t+1} - \rho &= \alpha\left(\bar{K}\right)^{\alpha-1}\left(A_{t+1} - 1\right) + \alpha\left(\alpha-1\right)\left(\bar{K}\right)^{\alpha-2}\left(K_{t+1} - \bar{K}\right) \\
r_{t+1} - \rho &= \alpha\left(\bar{K}\right)^{\alpha-1}\hat{A}_{t+1} + \alpha\left(\alpha-1\right)\left(\bar{K}\right)^{\alpha-1}\hat{K}_{t+1} \\
\hat{r}_{t+1} &= \rho\left(\hat{A}_{t+1} - \left(1-\alpha\right)\hat{K}_{t+1}\right)
\end{aligned}
$$

We conjecture the following relation between the state and control variables:

$$\begin{aligned}\hat{C}_t &= \eta_{CK}\hat{K}_t + \eta_{CA}\hat{A}_t + \eta_{CD}\hat{D}_t \\ \hat{K}_{t+1} &= \eta_{KK}\hat{K}_t + \eta_{KA}\hat{A}_t + \eta_{KD}\hat{D}_t\end{aligned}$$

Putting this into the good market clearing condition, we write:

$$\begin{aligned}\hat{A}_t + \alpha\hat{K}_t &= \hat{C}_t + \frac{\alpha}{\rho}\left(\hat{K}_{t+1} - \hat{K}_t\right) \\ \hat{A}_t + \alpha\hat{K}_t &= \eta_{CK}\hat{K}_t + \eta_{CA}\hat{A}_t + \eta_{CD}\hat{D}_t + \frac{\alpha}{\rho}\left(\eta_{KK}\hat{K}_t + \eta_{KA}\hat{A}_t + \eta_{KD}\hat{D}_t - \hat{K}_t\right)\end{aligned}$$

The coefficients on $\hat{A}_t$ imply:

$$\begin{aligned}1 &= \eta_{CA} + \frac{\alpha}{\rho}\eta_{KA} \\ \eta_{CA} &= 1 - \frac{\alpha}{\rho}\eta_{KA}\end{aligned}$$

The coefficients on $\hat{D}_t$ imply:

$$\begin{aligned}0 &= \eta_{CD} + \frac{\alpha}{\rho}\eta_{KD} \\ \eta_{CD} &= -\frac{\alpha}{\rho}\eta_{KD}\end{aligned}$$

The coefficients on $\hat{K}_t$ imply:

$$\begin{aligned}\alpha &= \eta_{CK} + \frac{\alpha}{\rho}\left(\eta_{KK} - 1\right) \\ \eta_{CK} &= \alpha\frac{1+\rho}{\rho} - \frac{\alpha}{\rho}\eta_{KK}\end{aligned}$$

We next use the conjecture in the Euler condition:

$$\begin{aligned}
0 &= -\hat{D}_t + \hat{C}_t + E_t\hat{D}_{t+1} - E_t\hat{C}_{t+1} + \frac{\rho}{1+\rho}\left(E_t\hat{A}_{t+1} - (1-\alpha)\hat{K}_{t+1}\right) \\
0 &= -\hat{D}_t + \eta_{CK}\hat{K}_t + \eta_{CA}\hat{A}_t + \eta_{CD}\hat{D}_t + E_t\hat{D}_{t+1} \\
&\quad -E_t\left[\eta_{CK}\hat{K}_{t+1} + \eta_{CA}\hat{A}_{t+1} + \eta_{CD}\hat{D}_{t+1}\right] + \frac{\rho}{1+\rho}\left(E_t\hat{A}_{t+1} - (1-\alpha)\hat{K}_{t+1}\right) \\
0 &= -(1-\eta_{CD})\hat{D}_t + \eta_{CK}\hat{K}_t + \eta_{CA}\hat{A}_t + (1-\eta_{CD})E_t\hat{D}_{t+1} \\
&\quad +\left(\frac{\rho}{1+\rho} - \eta_{CA}\right)E_t\hat{A}_{t+1} - \left[\eta_{CK} + \frac{\rho}{1+\rho}(1-\alpha)\right]\hat{K}_{t+1} \\
0 &= -(1-\eta_{CD})\hat{D}_t + \eta_{CK}\hat{K}_t + \eta_{CA}\hat{A}_t + (1-\eta_{CD})\phi^D\hat{D}_t \\
&\quad +\left(\frac{\rho}{1+\rho} - \eta_{CA}\right)\phi^A\hat{A}_t - \left[\eta_{CK} + \frac{\rho}{1+\rho}(1-\alpha)\right]\left(\eta_{KK}\hat{K}_t + \eta_{KA}\hat{A}_t + \eta_{KD}\hat{D}_t\right)
\end{aligned}$$

The coefficients on $\hat{A}_t$ imply:

$$\begin{aligned}
0 &= \eta_{CA} + \left(\frac{\rho}{1+\rho} - \eta_{CA}\right)\phi^A - \left[\eta_{CK} + \frac{\rho}{1+\rho}(1-\alpha)\right]\eta_{KA} \\
0 &= \left(1-\phi^A\right)\eta_{CA} + \frac{\rho}{1+\rho}\phi^A - \left[\eta_{CK} + \frac{\rho}{1+\rho}(1-\alpha)\right]\eta_{KA} \\
0 &= \left(1-\phi^A\right) + \frac{\rho}{1+\rho}\phi^A - \left[\alpha\frac{1+\rho-\phi^A}{\rho} + \frac{\rho}{1+\rho}(1-\alpha) + \frac{\alpha}{\rho}(1-\eta_{KK})\right]\eta_{KA} \\
\eta_{KA} &= \frac{1 - \frac{1}{1+\rho}\phi^A}{\alpha\frac{1+\rho-\phi^A}{\rho} + \frac{\rho}{1+\rho}(1-\alpha) + \frac{\alpha}{\rho}(1-\eta_{KK})}
\end{aligned}$$

The coefficients on $\hat{D}_t$ imply:

$$\begin{aligned}
0 &= -(1-\eta_{CD}) + (1-\eta_{CD})\phi^D - \left[\eta_{CK} + \frac{\rho}{1+\rho}(1-\alpha)\right]\eta_{KD} \\
0 &= -\left(1-\phi^D\right) - \left[\alpha\frac{1+\rho-\phi^D}{\rho} + \frac{\rho}{1+\rho}(1-\alpha) + \frac{\alpha}{\rho}(1-\eta_{KK})\right]\eta_{KD} \\
\eta_{KD} &= \frac{-\left(1-\phi^D\right)}{\alpha\frac{1+\rho-\phi^D}{\rho} + \frac{\rho}{1+\rho}(1-\alpha) + \frac{\alpha}{\rho}(1-\eta_{KK})}
\end{aligned}$$

The coefficients on $\hat{K}_t$ imply:

$$\begin{aligned}
0 &= \eta_{CK} - \left[\eta_{CK} + \frac{\rho}{1+\rho}(1-\alpha)\right]\eta_{KK} \\
0 &= \alpha\frac{1+\rho}{\rho} - \frac{\alpha}{\rho}\eta_{KK} - \left[\alpha\frac{1+\rho}{\rho} - \frac{\alpha}{\rho}\eta_{KK} + \frac{\rho}{1+\rho}(1-\alpha)\right]\eta_{KK} \\
0 &= \alpha\frac{1+\rho}{\rho} + \frac{\alpha}{\rho}(\eta_{KK})^2 - \left[\alpha\frac{2+\rho}{\rho} + \frac{\rho}{1+\rho}(1-\alpha)\right]\eta_{KK}
\end{aligned}$$

This polynomial has two roots, an explosive one $\eta_{KK} > 1$ and a stable one $.0 < \eta_{KK} < 1$

Let's now discuss the dynamics. The shock leads to a persistent increase in capital and consumption. Output increases a lot on impact, and then remains persistently positive but at a much lower level. The real interest rate is persistently reduced.

We may think that the investment (the increase in capital) is done to take advantage of the higher productivity. It is however not the case: as $\phi^A = 0$ the fact that $\hat{A}_t > 0$ does not imply that we expect a positive $\hat{A}_{t+1}$. There is therefore no reason to invest to take advantage of higher productivity.

Why do we invest then? The higher productivity $\hat{A}_t$ directly leads to a sharp increase in output. If the higher output was entirely consumed, consumption would be much higher today than tomorrow ($\hat{C}_t = \hat{A}_t > E_t\hat{C}_{t+1} = 0$). Such a consumption path would require a negative real interest rate from the Euler condition, $E_t\hat{r}_{t+1} < 0$. To get this negative real interest rate, we need the marginal product of capital to decrease (as $E_t\hat{r}_{t+1} = E_t\hat{A}_{t+1} - (1-\alpha)\hat{K}_{t+1}$), which is achieved by increasing investment.

Therefore, we invest not because capital will be more productive tomorrow, but because the front-loading of consumption, $\hat{C}_t > E_t\hat{C}_{t+1}$, requires a low real interest rate, and thus financing investment is cheap. The increase in output is then directed in part to investment.

In subsequent periods the productivity gain is gone, but we are left with a higher capital stock. This explains why output remains positive for several periods. As the capital stock remains high, the real interest rate remains negative. The Euler condition then implies that consumption is higher in period $t+s$ than in period $t+s+1$. How can we produce enough consumption? While output is higher, this is not enough. We therefore need to gradually dismantle the capital stock to get the desired front-loading of consumption (investment becomes negative after period t) which brings the economy gradually back to the steady state.

4 Part d)

Consider now the case with persistence ($\phi^A = 0.5$). Provide the intuition behind the dynamics, in particular that of the capital stock. You can focus on why the dynamics differ from the ones in point c).

Answer: As productivity will remain persistently high, $E_t \hat{A}_{t+1} = \phi^A \hat{A}_t > 0$, investment is appealing as the extra capital will be more productive for several periods. The incentive to invest is then stronger that the mere need to get a low real interest rate discussed in point c). In fact, the higher future productivity is large enough to lead to an increase in the real interest rate. Investing is expensive, but so productive that it is worth investing a lot.

The increase in output is persistent for two reasons. First, productivity is persistent. Second, the capital accumulation lasts longer than in point c).

We therefore get the capital stock to increase over several periods, after which the productivity gains become to small to warrant more capital accumulation. At that point, we shift to the same intuition as under point c) for the periods after t: the high capital leads to a low interest rate ($\hat{r} < 0$), which leads to a front-loading of consumption. As output is not large enough, the consumption is achieved by gradually dismantling the capital.

5 Part e)

Figure 2 illustrates the impact of a negative shock on $\hat{D}$ in period $t = 0$ (specifically $\varepsilon_0^D = -1$). The red line shows the case where shocks are not persistent ($\phi^D = 0$) and the blue line shows the case with persistence ($\phi^D = 0.5$).

Start with the situation without persistence ($\phi^D = 0$). Provide the intuition behind the dynamics, in particular that of the capital stock and the real interest rate.

Answer: The decrease in $\hat{D}_t$ can be understood as an increase in patience, as the consumer puts less weight on the current period relative to the future. We thus get a reduction in current consumption.

We may expect the lower value of current consumption relative to future one to generate an increase in the real interest rate in the Euler condition. This is not the case, because the Euler condition is about the marginal utility of consumption which is directly affected by the shock (i.e. what matters is $\hat{D}_t - \hat{C}_t$, not $-\hat{C}_t$). We can see if figure 2 that the fall in consumption is smaller than the shock, so $\hat{D}_t - \hat{C}_t$ increases. In other words, the direct increase in saving supply thanks to patience, $\hat{D}_t$, dominates the indirect effect through consumption $\hat{C}_t$. The consumer's valuation of consumption at time t decreases, relative to the one at time $t+1$, and the Euler condition implies a decrease in the real interest rate.

The lower interest rate makes investment cheaper and capital increase, thereby lowering the marginal product of capital. Investment is thus not fuelled by higher productivity, but by an increase in saving. While there is not change in the output at period t, the increase in capital leads to a persistent output increase in the future.

Once the shock is passed, the consumer becomes as impatient as she was initially. As we are left with a high capital stock, we can produce more and output and consumption are positive. The high capital stock also implies a low marginal productivity, and therefore the real interest rate remains negative. This leads to a front-loading of consumption. Output is not large enough to provide enough consumption, and we therefore need to gradually dismantle the capital stock to make for the difference between output and consumption.

6 Part f)

Consider now the case with persistence ($\phi^D = 0.5$). Provide the intuition behind the dynamics. You can focus on why the dynamics differ from the ones in point e).

Answer: The dynamics are similar to the ones without persistence, and the effect is simply larger and longer lasting. The increase in patience is now spread over several periods, leading to a longer reduction in consumption that is mirrored in higher capital (hence a low interest rate), and thus more output.

7 Part g)

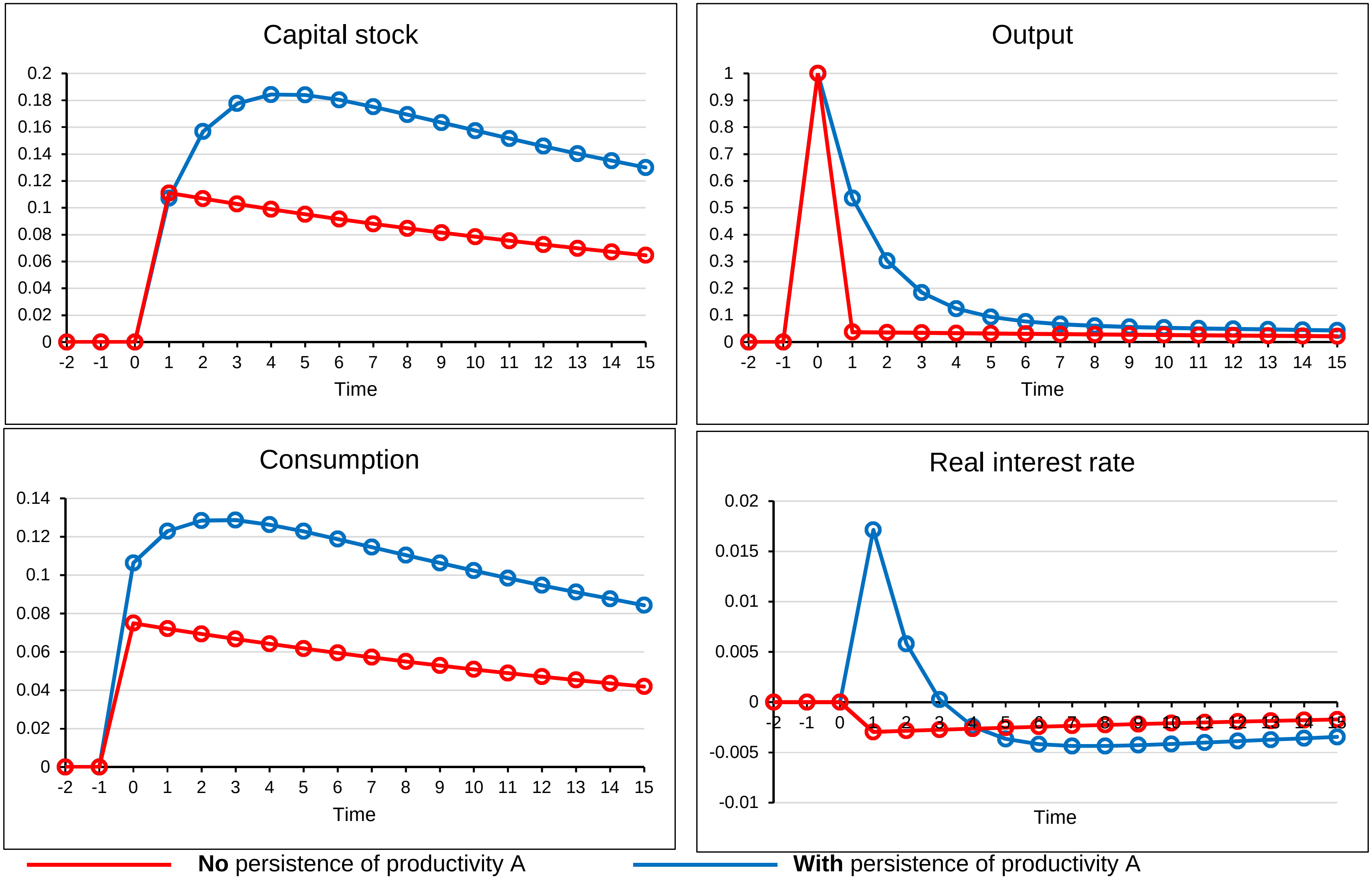
From this exercise, what do we learn about the correlation between the real interest rate and output?

Answer: With demand shocks $\hat{D}$ the correlation is clearly negative, irrespective of the persistence of the shock. Output increases only thanks to higher investment, which is fuelled by a temporary decrease in consumption. The higher patience reduces the real interest rate.

Things are more subtle with productivity shocks $\hat{A}$. Without persistence (or only a small persistence), productivity does not directly raise the appeal of investment. In fact, investment only increases because the front-loading of consumption leads to a low real interest rate. We thus get a negative correlation between the real interest rate and output.

If persistence is high enough however, the productivity directly impacts the marginal product of capital. This amplifies the increase in output, and also leads to a higher real interest rate. We therefore get a positive correlation between the real interest rate and output.

Long question 2, Figure 1: Impact of a positive shock on productivity A at period 0



Long question 2, Figure 2: Impact of a <u>negative</u> shock on <u>discount D</u> at period 0