Macroeconomics A:
Review Session X

Modeling the Financial Sector

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Outline

Bank Runs: Diamond-Dybvig

- Questions:
  - Why are runs so prevalent?
  - Why do people hold deposits if they are susceptible to runs?
  - What type of policies can be used to prevent/reduce/mitigate runs?
- Contributions of the model
  - Provides a precise definition of liquidity
  - Shows why financial intermediaries are needed
  - Demonstrates how liquidity transformation can lead to runs

The following exposition is based on Eric Sim's notes

Basic Setup

- There are three periods, indexed by T starting at $T = 0$
- $T = 0$ is the 'present' and $T = \{1, 2\}$ is the 'future'
- Households are (ex-ante) identical and receive \$1 in $T = 0$ and will need to consume in either $T = 1$ or $T = 2$
- Idiosyncratic uncertainty: individual does not know whether she will be type 1 (consumes in $T = 1$) or type 2 (consume in $T = 2$)
- Type revealed in $T = 1$ and a fixed fraction $t \in [0, 1]$ of households will be type 1 and $(1 - t)$ type 2
- There are two assets
  - Cash: save \$1 and have \$1 available in either $T = 1$ or $T = 2$
  - Investment: save \$1 and have r_1 in $T = 1$ and $r_2 \geq r_1$ in $T = 2$

Household Utility

- An individual household has utility:

$$U(c) = 1 - \frac{1}{c}$$

- Expected utility is simply the probability-weighted sum over types:

$$\mathbb{E}[U] = tU(c_1) + (1-t)U(c_2)$$

- Where c_1 and c_2 are consumption at each date depending on type
- Consumption allocations
  - Cash: $c_1 = c_2 = 1$

$$\implies \mathbb{E}[U_{cash}] = 0$$

  - Investment: $c_1 = r_1$ and $c_2 = r_2$

$$\implies \mathbb{E}[U_{inv}] = tU(r_1) + (1-t)U(r_2)$$

- Household prefers investment if $\mathbb{E}[U_{inv}] > 0$ (e.g. $r_1 = 1$ and $r_2 = 2$)

Liquidity

- We can think about liquidity as the discount for early liquidation of an investment
$$\ell = \frac{r_1}{r_2}$$
- Since $r_2 \geq r_1$ (by assumption) then $\ell \leq 1$ for investment
- The further ℓ is from 1, the less liquid is the asset
- Cash is perfectly liquid since $\ell = 1$
- Households still prefer the less liquid investment whenever expected utility is higher than cash

Liquidation Cost

- Suppose early liquidation incurs a cost τ where $\tau \geq 0$
  - Household gets $(1-\tau)r_1$ for early liquidation
- If $r_1 = 1$ and $r_2 = 2$, the liquidity of investment is then

$$\ell = (1-\tau)\frac{1}{2}$$

- Given that $t = 1/4$, utility becomes

$$\begin{aligned}\mathbb{E}[U_{inv}] &= \frac{1}{4}U(1-\tau) + \frac{3}{4}U(2) \\ &= \frac{1}{4}\left(1 - \frac{1}{1-\tau}\right) + \frac{3}{4} \times \frac{1}{2}\end{aligned}$$

- $\mathbb{E}[U_{inv}] < 0$ when $\tau > 3/5$
- In this case, household will prefer cash

Liquidation Cost and Returns

- Suppose $\tau = 2/3$
- This liquidation cost is high enough to discourage households from investment
- Now let's compute the expected returns to cash and investment

$$\mathbb{E}[R_{cash}] = 1$$
$$\mathbb{E}[R_{inv}] = \frac{1}{4}\left(1 - \frac{2}{3}\right) \times 1 + \frac{3}{4} \times 2 = \frac{19}{12} > 1$$

- The household will not directly invest if the project is sufficiently illiquid, even when net returns are positive
- This is because of the idiosyncratic (individual) risk the household will be type 1 and will need to liquidate the investment early
  - Risk aversion plays a role here

Introducing Banks

- An individual household is uncertain about when she will need to consume
  - This gives rise to a preference for liquidity
- There is no uncertainty in aggregate
  - Exactly fraction t of households will be type 1 and $1 - t$ will be type 2
- Bank can pool resources from many households exploiting this lack of aggregate uncertainty
- They do this by offering households an asset that is more liquid than the investment project

Worked Example

- The bank offers the following payout structure (assume $\tau = 0$)

$$r_{b,1} = 1.28 \quad \text{and} \quad r_{b,2} = 1.813$$

- This is more liquid than the investment

$$\frac{1.28}{1.813} = 0.706 > \frac{1}{2}$$

- Out of 100 households, 25 will need to withdraw in period $T = 1$
- The bank puts \$100 in investment at $T = 0$
- It will need to liquidate $25 \times 1.28 = 32$ units of the investment to raise necessary funds in $T = 1$, leaving 68 invested
- The remaining investment generates 136 in income in $T = 2$, which can be distributed to the remaining 75 deposit holders for

$$r_{b,2} = \frac{136}{75} = 1.813$$

Which Utility Is Highest?

- Expected utilities:

$$
\begin{aligned}
\mathbb{E}[U_{cash}] &= 0 \\
\mathbb{E}[U_{inv}] &= \frac{1}{4} \times 0 + \frac{3}{4} \times \frac{1}{2} = \frac{3}{8} \\
\mathbb{E}[U_{bank}] &= \frac{1}{4}\left(1 - \frac{1}{1.28}\right) + \frac{3}{4}\left(1 - \frac{1}{1.813}\right) = 0.391 > \frac{3}{8}
\end{aligned}
$$

- Utility is highest from bank deposits :)
- Household willing to tolerate lower expected returns on deposits because they are more liquid than investment
- This is even starker when $\tau > 0$

Adding Bank Runs to the Example

- Let's focus on multiplicity of equilibria given expectations
  - Good equilibrium: what we just described
  - Bad equilibrium: type 2s withdraw early in $T = 1$ because they expect other type 2s to withdraw early as well
- The bad equilibrium will cause the bank to fail and make everyone worse off
- Say that fraction f of depositors withdraw in $T = 1$ where $f \geq t$
- With \$100 in deposits, bank must liquidate $128 \times f$ of the investment
- The return is distributed to the remaining depositors at $T = 2$

$$r^*_{b,2} = \frac{(100 - 128f) \times 2}{(1 - f) \times 100}$$

- If $f > t$ (some type 2s withdraw), then $r^*_{b,2} < 1.813$.

Good Equilibrium

- Let f^* be the expectation of each household for f
- Suppose $f^* = 1/2$, meaning that 1/3 of the type 2s withdraw earlier than needed
- In this case

$$r^*_{b,2} = \frac{(100 - 64) \times 2}{50} = 1.44$$

- This is less than what was promised, but better than withdrawing at $T = 1$

$$r_{b,2} > r^*_{b,2} > r_{b,1}$$

- It is not optimal for type 2s to withdraw given this forecast
- Hence, $f^* = 1/2$ is not self-fulfilling even if believed by everyone

Bad Equilibrium

- Suppose instead $f^* = 3/4$
- Then households believe they get

$$r^*_{b,2} = \frac{(100 - 96) \times 2}{25} = 0.32$$

- This is (significantly) worse than r_1 and best to "get out now"
- But then $f^* = 3/4$ is not self-fulfilling
  - If everyone believes it, then everyone should withdraw
- So $f^* = f = 1$ is another Nash equilibrium
- It is completely rational to withdraw in $T = 1$ given this belief
- If everyone withdraws, the bank fails
  - It can pay \$1 per depositor in $T = 1$, less than the r_1 it promised
- There is some value of f^* that divides the good and bad equilibria

Dealing with Runs

- “First come, first served"
  - The first to line up are “made whole” and the last get nothing
  - This increases incentive to withdraw and withdraw early
- Suspension of convertibility
  - Simply refuse (temporarily) to convert deposits into cash
  - Economically costly and doesn’t stop runs, but keeps banks solvent
- Lender of last resort
  - Banks could go to the central bank if they need cash
  - Stigma: borrowing makes banks appear weak
- Deposit insurance
  - Has more or less eliminated traditional banking panics
  - People know deposits are safe, so no reason to run
  - Banks usually pay a (small) fee for insurance, but this is much less costly than bank runs

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Adverse Selection: Stiglitz and Weiss

- Two types of borrowers:
  - type G repays with probability q_g
  - type B repays with probability $q_b < q_g$
- Take r as the opportunity cost of funds, if lender can discriminate
  - type-G can borrow at rate r/q_g and type B at r/q_b
- Now assume the two types appear <u>identical</u> to lenders
- Changes in the terms of a loan (interest rate, collateral, amount) affect the mix of borrower types
- Assume a fraction g are of type G and the lender charges r_l so that

$$gq_g r_l + (1-g)q_b r_l = r \quad \iff \quad r_l = r/[gq_g + (1-g)q_b]$$

- The lender attracts more type B borrowers since $r/q_g < r_l < r/q_b$
- If a higher interest rate on loans increases the type-B share, the expected return to the lender declines

Moral Hazard: Stiglitz and Weiss (Again)

- Before, borrowers differed in their underlying riskiness
- Suppose instead each borrower can choose between several projects of differing risk
- If the lender cannot monitor this choice, a moral hazard results
- Higher loan rates lead the borrower to invest in riskier projects and lower the expected return to the lender
- Suppose the borrower can invest in
  - project A, which paying R^a in the good state and 0 in the bad
  - project B, which pays $R^b > R^a$ in the good state and 0 in the bad
- The probabilities of success for projects A and B are $p^a > p^b$
- Project B is the riskier project
- The expected payoff from A is higher: $p^a R^a > p^b R^b$

Moral Hazard (Cont.)

- By investing in A, the borrower's expected return is

$$\mathbb{E}[\pi^a] = p^a[R^a - (1 + r_l)L] - (1 - p^a)C$$

- Here, L is the loan amount and C is collateral
- By investing in B, the borrower's expected return is

$$\mathbb{E}[\pi^b] = p^b[R^b - (1 + r_l)L] - (1 - p^b)C$$

- It is straightforward to show that

$$\mathbb{E}[\pi^a] > \mathbb{E}[\pi^b] \iff \frac{p^a R^a - p^b R^b}{p^a - p^b} > (1 + r_l)L - C$$

- Assume there is some r_l^* where borrower is indifferent
  - Only when $r_l < r_l^*$ does borrower prefer to invest in A
  - What about collateral?