THE GREEN METAMORPHOSIS OF A SMALL OPEN ECONOMY*

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Abstract

We design a canonical model of green transition for a small open economy with endogenous energy efficiency. Production combines energy and traditional factors with low short-run substitutability and efficient technology adoption. Increases in brown energy taxation reduce brown energy usage and improve energy efficiency in the long run but lead to an increase in firms' marginal costs with inflationary effects and persistent output losses in the short run. Green public investment or subsidies induce a transition with no inflationary or output costs but with no energy efficiency improvements and no significant reduction in brown energy usage unless they increase substantially, exerting considerable fiscal pressure. Evaluating welfare along the transition, we show that policies that combine subsidies or public investment with carbon taxes can reduce the costs of the green transition.

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1 Introduction

We build a dynamic general equilibrium climate model for a small open economy with nominal rigidities and endogenous energy efficiency and growth to assess the role of economic policy in dealing with the green transition. We define the green transition as a transition to efficient energy use based on non-polluting and renewable sources. Even though small open economies may not individually make a sizeable contribution to global CO2 emissions, they certainly affect them as a whole, and studying the transition in such economies provides useful insights for portraying the macroeconomic dynamics of the green transition.

Our model builds on the standard New Keynesian model for a small open economy. We incorporate an endogenous supply of green energy and allow green and brown energy to be substitutes in energy production. Departing from the existing models and following Hassler et al. (2021a) and Hassler et al. (2022), we assume that intermediate goods production is characterized by low substitutability between energy and traditional inputs in the short run that firms can alter over longer periods through directed input-saving technical change. Differently from the previous authors, we consider nominal frictions to study the direct impact of the green transition on inflation as well as its indirect impact through the response of fiscal and monetary policies. Our model also captures the main features of Emerging Economies: these economies take international prices and risk-free rates as given; they are typically subject to financial constraints, which are represented by a positive premium on external debt, and have a higher average inflation rate than developed economies. Moreover, we assume a domestic exogenous supply of brown energy, a type of energy that many emerging markets may also produce and export. As such, the small open economy model we propose is useful to understand both short and long-run movements in macroeconomic aggregates along the green transition.

Following the Pigouvian approach, climate change policy has focused predominantly on carbon taxes or excise taxes on fuel (see, for example, Golosov et al. (2014), Aghion et al. (2016), Hassler et al. (2021b) and Angelopoulos et al. (2010) among others). It has also been suggested that taxes on brown energy should not be imposed in isolation but in combination with R&D subsidies (See, for example, Acemoglu et al. (2012)). Yet, apart from the work of Acemoglu et al. (2012), few authors have studied how other fiscal instruments can affect the green transition. Our work aims to close this gap by investigating not only how carbon taxes but also how green subsidies and public investment in green capital can affect the transitional dynamics of both real and nominal variables.

We calibrate the model to Chile, a representative emerging country that has become a stable economy for the last 30 years and is taking measures towards the green transition. Chile's Climate Action Plan 2017–2022 includes a reduction in the intensity of its CO2 emissions by at least 30% by 2030 and the advancement of non-conventional renewable energies by promoting an energy efficiency law. The plan specifies an increase in carbon taxes, moving from $5/t to at least $35/t. In the baseline scenario, we start from the initial steady state and assume a transition involving an increase in carbon taxation from 5% to 30%, according to the Chilean plan.

In the model we define the green transition as a fall in brown energy use. We show that carbon taxes can accelerate the green transition but at the cost of inflationary pressures and output losses in the short and medium run. The mechanism is the following. Given that agents foresee the increases in brown energy taxes, firms accumulate more green capital and smoothly decrease the usage of brown energy. However, in the short and medium run, as the economy is building the capacity to produce green energy, the green transition induced by increases in carbon taxes implies surges in both brown and green energy prices. To adjust to the increase in the price of energy, firms allocate more researchers to efficient energy usage increasing productivity and reducing the energy usage in total in the production function. Yet, this comes at the cost of reducing traditional factors' efficiency and capital demand and generating persistent output losses. Quantitatively, the permanent brown energy tax rise decreases brown energy usage by 33% in the new steady state, doubles inflation in the first two years, and reduces output by approximately one percent for several years. We show that monetary policy can ameliorate the short-run inflationary cost of the transition by reacting strongly to inflationary pressures but cannot ameliorate the associated output losses.

We next study the role of green subsidies and public investment. If a policy relies exclusively on green subsidies, it could attain a reduction in brown energy usage comparable to the one obtained by increases in carbon taxes, only when subsidies are raised at 300%. This is because subsidies do not induce changes in the firms' marginal costs and, if anything, motivate further inefficient energy usage. On the other hand, if a policy relies exclusively on financing public green investment, it would have to be around 7% of GDP and generate significant fiscal pressures for several years for it to be as successful in reducing brown energy usage as carbon taxation. Green public investment increases the productivity of the green sector and reduces green energy prices in the long run. Subsidies also reduce the price of green energy. However, in the short run, the subsidies also decrease the price of brown energy since they generate a fall in their demand. On the other hand, green public investment, by having a lagged effect on the price of green energy, does not reduce the price of brown energy, and firms find it optimal to reduce the usage of brown energy faster. In the medium run, the expectations of higher future productivity of the green sector induce firms to invest relatively more heavily in traditional factors, increasing capital demand and output. Hence, the induced transition is deflationary and expansionary.

A natural measure to rank the different policy choices is welfare. We compute welfare metrics using consumption equivalent measures with different assumptions regarding utility. We consider two cases: i) welfare depends on consumption only (no externality case); ii) welfare depends on consumption, but there is a pollution externality generated by the utilization of brown energy that results in detrimental effects on households' health, modeled as consumption losses (See also Hassler et al. (2021a)). The green transition is welfare-improving only when there is a pollution externality. Although effective in decreasing brown energy usage and improving energy efficiency, carbon taxes imply welfare costs that can be reduced if one combines carbon taxation with green subsidies or green public infrastructure investments.

Carbon taxes are generally well accepted in countries with significant experience thereof, but there is still public resistance to raising them. Ewald et al. (2022) show that lack of trust in the government and belief in the Pigouvian mechanism are important determinants for protesters' opposition. Given the documented resistance against raises in carbon taxes, our results suggest that policy mixes that combine a moderate increase in carbon taxation with increases in green subsidies or green public investment can attain similar reductions in brown energy usage with lower welfare costs. Yet, we highlight that such a policy mix will not increase substantially energy efficiency.

Our model belongs to the class of general equilibrium models with environmental features often tagged as E-DSGE models (See, e.g., Annicchiarico and Di Dio (2015), Annicchiarico and Di Dio (2017), Carattini et al. (2021) and Economides and Xepapadeas (2019), among others). Yet, we are interested in investigating how economic transformation affects the macroeconomy along the transitional dynamics. Many recent studies use E-DSGE models to investigate the consequences of the green transition. Ferrari and Nispi Landi (2022) employ a framework with abatement technology and no distinct energy usage and show that an increase in carbon taxation may be deflationary. Nakov and Thomas (2023) assume green and fossil energy as inputs in production, with fossil energy contributing to carbon emissions and global warming, and show that carbon taxes pose no trade-offs for monetary policy if set at their socially optimal level. Olovsson and

Vestin (2023) and Del Negro et al. (2023) use a similar framework and focus on monetary policy's role in the green transition. Finally, Coenen et al. (2023) build a large-scale macroeconomic model to analyze the role of policy along the green transition. Apart from focusing on the role of fiscal policy relative to the existing studies, our model economy incorporates energy efficiency in the production function and allows firms to react to relative energy price movements, adjusting available resources to improve energy efficiency, resulting in possible non-monotone medium-run transitional dynamics. In other words, in our proposed model, changes in the fiscal policy instruments induce supply-side effects that the existing studies do not assimilate. Moreover, unlike in the existing studies, in our model, we can evaluate the effectiveness of different policies in improving energy efficiency, providing an angle of analysis that is absent in other studies of the green transition.

The rest of the paper is organized as follows: Section 2 presents the model. Section 3 discusses the calibration and solution method employed. Section 4 presents transitional dynamics and discusses different policy experiments. Section 5 quantifies welfare along the green transition, and Section 6 concludes.

2 The Model

We adapt a small open economy New-Keynesian model to incorporate energy efficiency in production, directed technology change, and production of green energy. The domestic economy is populated by households, final domestic good producers, intermediate goods producers, producers of green energy, a government that determines fiscal policy, and an independent monetary authority. In turn, the rest of the world determines the demand for home exports and provides imports of final goods and brown energy fully elastically at internationally given prices and trades an asset with the small open economy that pays the risk-free rate plus a spread.

2.1 Households

The representative household in the domestic economy supplies a fixed share of its available time to work, $\bar{h}$, chooses consumption c_t, assets, and two types of investment: in capital goods, i_t, and in green technology, i_t^G.[1] The investment in green technology adds to the green capital stock, s_{t+1}^G, which evolves according to the following law of motion:

$$s_{t+1}^G = (1-\delta)\, s_t^G + \Phi_s(s_{t+1}^G, s_t^G) s_t^G + i_t^G. \tag{1}$$

Capital, k_t, follows a standard law of motion:

$$k_{t+1} = (1-\delta)\, k_t + \Phi_k(k_{t+1}, k_t) k_t + i_t, \tag{2}$$

where parameter δ represents a constant depreciation rate that we assume is the same in both specific capital stocks. Similarly, both capital stocks are subject to adjustment costs represented by function Φ_i, for $i = s, k$.[2]

On top of investing in capital and green technology, the household can save in two different assets: a domestic public bond, B_{t+1}, that pays a nominal return, R_t, after one period or a foreign bond, B_{t+1}^*, with a one-period return in foreign currency, $R_t^* \Phi_{t+1}^A(A_{t+1}^f)$, where $\Phi_{t+1}^A(A_{t+1}^f)$ is the spread on domestic bonds. In addition, the representative agent pays lump-sum taxes, τ_t, to the government and receives profits from the firms in the economy, Γ_t.

The household consumption is a bundle composed of domestic, $c_{H,t}$, and foreign goods,

[1] We do not model a labor supply choice since it does not provide the analysis with additional insights and it is harder to solve numerically.

[2] We have also considered a specification with time-to-built in investment for both capital stocks. The results we present here are qualitatively similar. Since our calibration is annual, we have decided to keep the specification with standard capital adjustment costs as our baseline.

$c_{F,t}$, given by:

$$c_t = \left[(1-\chi)^{\frac{1}{\theta}} c_{H,t}^{\frac{\theta-1}{\theta}} + \chi^{\frac{1}{\theta}} c_{F,t}^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}, \tag{3}$$

and their prices are $P_{H,t}$ and $P_{F,t}$, respectively.

The household chooses, c_t, B_{t+1}, B^*_{t+1}, i^G_t, i_t, s^G_{t+1}, k_{t+1} to maximize:

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U\left(c_t\right),$$

s.t.

$$i^G_t + i_t + c_t + \frac{B_{t+1}}{P_t} + FX_t \frac{B^*_{t+1}}{P_t} = \frac{B_t}{P_t} R_{t-1} + FX_t \frac{B^*_t}{P_t} R^*_{t-1} \Phi^A_t(\tilde{A}^f_t) + w_t \bar{h} + \frac{R^k_t}{P_t} k_t + \frac{R^G_t}{P_t} s^G_t + \Gamma_t - \tau_t,$$

$$s^G_{t+1} = (1-\delta)\, s^G_t + \Phi_s(s^G_{t+1}, s^G_t) s^G_t + i^G_t,$$

$$k_{t+1} = (1-\delta) k_t + \Phi_k(k_{t+1}, k_t) k_t + i_t.$$

Here FX_t is the nominal exchange rate, P_t is the domestic CPI, and $\Phi^A_t(\tilde{A}^f_t)$ is a debt-elastic interest rate spread given by:

$$\Phi^A_t(\tilde{A}^f_t) = \exp\left\{-\phi^A \tilde{A}^f_t\right\}.$$

$\tilde{A}^f_t$ is the real outstanding foreign debt in domestic currency to output ratio as in Justiniano and Preston (2010):

$$\tilde{A}^f_t = \frac{FX_t}{P_t \bar{Y}} \tilde{B}^*_t.$$

2.2 Domestic final good producer

A representative firm produces the domestic final good, $y_{H,t}$, from varieties, $y_{H,i,t}$, for i $\in [0,1]$ using the following technology:

$$y_{H,t} = \left[\int_0^1 y_{H,i,t}^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}}.$$

Here, ε is the elasticity of substitution between varieties. The optimization problem of the representative firm is the following:

$$\max_{y_t, \{y_{H,i,t}\}_{i \in [0,1]}} P_{H,t} y_{H,t} - \int_0^1 P_{H,i,t} y_{H,i,t} di,$$
$$\text{s.t } y_{H,t} = \left[\int_0^1 y_{H,i,t}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}.$$

2.3 Intermediate-good producers

Each firm in the intermediate-goods sector produces a variety $y_{H,i,t}$ in a monopolistic competition environment, facing a downward demand function, $a_{i,H,t}$. The technology for producing variety i uses fixed labor $\bar{h}_i$, physical capital $k_{i,t}$, and energy $e_{i,t}$ as productive inputs. Following Hassler et al. (2021a) and Hassler et al. (2022), we assume the following technology:

$$y_{H,i,t} = \left[\left(A_{i,t} k_{i,t}^{\alpha} h_{i,t}^{(1-\alpha)} \right)^{\frac{\epsilon-1}{\epsilon}} + \left(A_{e,i,t} e_{i,t} \right)^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}}$$

$A_{i,t}$ and $A_{e,i,t}$ are input-augmenting productivity factors, that are non-stationary. To account for changes in A and A_e, we assume that each firm disposes of a fixed stock of researchers equal to one, a fraction n of which can be allocated to improve the productivity of capital/labor, and the remaining fraction $(1 - n)$ is used to improve the efficiency of energy services, as in Hassler et al. (2022). Relative to the former authors, we assume that n is endogenous and determined at the firm level and use it as a way for the firms in the domestic economy to adjust to changes in the relative price of energy.

Using firm-level data, Alam et al. (2019) find that R&D investment improves the firm's environmental performance consistent with the theoretical argument of natural resource-based view (NRBV). In particular, NRBV postulates that corporate R&D investment helps attain technological development, intensifying production speed without the demand for increased energy. Hence, corporate R&D investment results in the reduction of energy intensity (energy/output ratio). Also, corporate R&D fosters the development and deployment of new and improved clean energy technologies that play a central role in the transition towards cleaner energy sources in the energy system. Moreover, new technologies enable a shift in the trajectory of energy consumption with improved energy efficiency. Thus, the authors conclude that the nourishment of corporate R&D activities can significantly improve environmental performance, particularly in reducing energy and carbon emission intensities. Thus, supported by the evidence in Alam et al. (2019), we assume that firms in our model economy choose R&D investment in energy efficiency that reduces the energy intensity in equilibrium and shifts the trajectory of energy consumption at the firm level.

We also assume that fixing the stock of researchers dedicated to improving the efficiency in the energy versus the capital/labor services is actually fixing the relative movements of the two production trends.[3] More precisely, the proportion of researchers in each sector affects the corresponding growth rates of productivity $A_{e,t}$, A_t, in the following fashion:

$$g_{i,t}^{A} = \frac{A_{i,t}}{A_{i,t-1}} = 1 + Bn_{i,t}^{\phi},$$

$$g_{i,t}^{Ae} = \frac{A_{e,i,t}}{A_{e,i,t-1}} = 1 + B_e(1 - n_{i,t})^{\phi}.$$

As is clearly demonstrated by the determinants of the two growth rates, firms face a trade-off in the allocation of researchers. A rise in R&D in one sector increases its productivity growth rate while decreasing the one in the other sector. We assume that the firms optimally choose n to balance this trade-off. Notice that a key parameter of the production function is the elasticity of substitution between traditional inputs and energy. Although we can assume this elasticity to be near zero in the short run, i.e., almost Leontief, di-

[3] Obviously, a single firm cannot change the economy's trends. Given that all firms behave symmetrically in equilibrium, we find this to be an innocuous assumption.

rected input-saving technical change can alter this elasticity at longer horizons. That is, by choosing n, the producer can move resources from A_t to $A_{e,t}$, increasing the efficiency (or intensity) of the use of energy compared to that of labor and capital, allowing for medium-run increasing resource use.

With regards to the energy used in the intermediate production sector, we assume that it is an aggregate of the polluting/brown ($e^B_{i,t}$) and clean/green energy ($e^G_{i,t}$), given by

$$e_{i,t} = \bar{E}\left[(1-\zeta)\left(e^G_{i,t}\right)^{\xi} + \zeta\left(e^B_{i,t}\right)^{\xi}\right]^{\frac{1}{\xi}}, \tag{4}$$

where ζ characterizes the relative importance of brown over total energy resources and ξ determines the elasticity of substitution between brown and green energy.

The optimization problem of firm i consists in choosing the allocation of researchers $n_{i,t}$, price $P_{H,i,t}$, and inputs of production $e^B_{i,t}$, $e^G_{i,t}$, $k_{i,t}$, $h_{i,t}$ taking as given the demand function $a_{i,H,t}$, and input prices P^G_t, P^B_t, W_t and fiscal policy τ^e.

When the firm changes the variety price, it has to pay an adjustment cost given by:

$$\Phi^{PH}(.) = P_{H,t}\frac{\kappa_{PH}}{2}\left(\frac{P_{H,i,t}}{P_{H,i,t-1}} - \bar{\pi}_H\right)^2 a_{H,t}$$

in nominal terms, as in Rotemberg (1982). Additionally, the firm has to pay a government tax τ^e proportional to the purchases of brown energy $P^B_t e^B_t$.

The firm's optimization problem, in nominal terms, is the following:

$$\max \mathbb{E}_0\left\{\sum_{t=0}^{\infty}\beta^t\frac{\lambda_t}{\lambda_0}\left[P_{H,t}\left(\frac{P_{H,i,t}}{P_{H,t}}\right)^{-\varepsilon}a_{H,t} - P^G_t e^G_{i,t} - P^B_t e^B_{i,t}\left(1+\tau^e\right) - W_t h_{i,t} - P_{H,t} - R^k_t k_t - \right.\right.$$
$$\left.\left.\frac{\kappa_{PH}}{2}\left(\frac{P_{H,i,t}}{P_{H,i,t-1}} - \bar{\pi}_H\right)^2 a_{H,t} + MC_t\left(F(e_{i,t},h_{i,t}) - \left(\frac{P_{H,i,t}}{P_{H,t}}\right)^{-\varepsilon}a_{H,t}\right)\right]\right\}.$$

Here, λ_t is the discount factor of the firm that coincides with the Lagrange multiplier of the consumer's problem. We solve for a symmetric equilibrium, where all intermediate firms make the same decisions. Therefore, in what follows, we present the aggregated variables for all i.

Using $\mu_{A,t}$ and $\mu_{A_e,t}$ for the Lagrange multipliers of the law of motions of efficiency, the optimal decision for $n_{i,t}$ is given by:

$$\mu_{A,t} g_{t-1}^{\alpha} B n^{\phi-1} A_{t-1} = \mu_{A_e,t} B_e (1-n)^{\phi-1} g_{t-1}^{\mu} A_{e,t-1},$$

and the optimal pricing decision results in:

$$\pi_{H,t}\left(\pi_{H,t}-\bar{\pi}_H\right) = \beta \mathbb{E}_t\left[\frac{\lambda_{t+1}}{\lambda_t}\pi_{H,t+1}^2\left(\pi_{H,t+1}-\bar{\pi}_H\right)\frac{a_{H,t+1}}{a_{H,t}}\right] + \frac{\varepsilon}{\kappa_{PH}}\left(\frac{mc_t}{p_{H,t}}-\frac{\varepsilon-1}{\varepsilon}\right), \tag{5}$$

which is the New Keynesian Phillips Curve, where $p_{H,t} = \frac{P_{H,t}}{P_t}$ is the relative price of domestically produced goods to the price level in the economy and real marginal cost is $mc_t = \frac{MC_t}{P_t}$. We present the rest of the optimality conditions in the Appendix.

2.4 Energy sectors

2.4.1 Green energy production

Green energy is produced domestically by combining specific green technology capital stock, s_t^G, and the stock of public green capital $s^{G,P}$, that firms take as given.

$$e_t^G = \Omega[(1-\gamma)(s_t^G)^{\omega} + \gamma(s^{G,P})^{\omega}]^{(\mu/\omega)}. \tag{6}$$

Ω denotes the level of productivity in clean energy production. Parameters ω and γ determine the elasticity of substitution between private and public stocks of green capital and the share of each in green energy production. μ is the green capital share.

This sector solves a static optimization problem consisting of choosing how much green capital to rent s_t^G to maximize the period profits, taking prices P_t^G, R_t^G, technology and a green subsidy from the government, s, as given.[4] Their profits for period t are given by the following expression.

$$\Gamma_t^G = (1+s)P_t^G e_t^G - R_t^G s_t^G$$

In the benchmark calibration, we assume that public and private green capital are substitutes and investigate the sensitivity of our results to this assumption later in the analysis.

2.4.2 Brown energy endowment

To simplify the model, we assume there is no production of brown energy in the economy. The economy receives an endowment of brown energy, $e^{B,d}$, that we assume is traded internationally and can be exported or imported at the international price $P^{B,*}$. Since e_t^B is the domestic demand for brown energy, the imports of brown energy, $e_t^{B,*}$, are given by:

$$e_t^{B,*} = e_t^B - e^{B,d}.$$

We assume the law of one price holds for the brown energy market, and thus, the domestic price of brown energy is the following:

$$P_t^B = FX_t P^{B,*}.$$

From the previous expression, note that

$$\frac{P_t^B}{P_t} = \frac{FX_t P^{B,*} P^*}{P_t P^*},$$

[4]We also consider the case of subsidizing purchases of green energy from the intermediate good producer and transitional dynamics are similar.

and

$$p_t^B = rer_t p^{B,*}$$

holds. rer_t is the real exchange rate. The international price $p^{B,*}$ is assumed to be fully exogenous and invariant over time.

2.5 Final goods imports

The economy imports foreign differentiated goods $y_{F,i,t}$, for which the law of one price holds. This means $P_{F,i,t} = FX_t P^*_{F,i,t}$. In addition, assuming a small open economy implies $P^*_{F,t} = P^*_t$. Integrating over all varieties, we obtain $P_{F,t} = FX_t P^*_t$, that is the price level of imported goods. Dividing by the domestic price level, we get the real exchange rate:

$$p_{F,t} = rer_t = FX_t \frac{P_t^*}{P_t}.$$

2.6 Final goods exports

The following expression gives the foreign demand for domestically produced goods:

$$c^*_{H,t} = \left(\frac{P^*_{H,t}}{P^*} \right)^{-\lambda} y^*,$$

where λ is the elasticity of substitution of foreign and domestic goods in the foreign economy. As for the case of the foreign price level P^*, the foreign output y^* is exogenous from the point of view of the small open economy.

2.7 Monetary Authority

The central bank sets the domestic interest rate R_t following a Taylor rule that depends on inflation and output deviations from their steady-state value:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_R} \left[\left(\frac{\pi_t}{\bar{\pi}}\right)^{\phi_\pi} \left(\frac{y_t}{y}\right)^{\phi_y}\right]^{1-\rho_R}. \tag{7}$$

2.8 Fiscal Authority

The fiscal authority satisfies the following period budget constraint:

$$\tau_t + \tau^e p_t^B e_t^B + b_{t+1} = s p_t^G e_t^G + \frac{b_t}{\pi_t} R_{t-1} + i^{G,P}, \tag{8}$$

where b_{t+1} is real debt with one-period maturity purchased by domestic households and τ_t are lump-sum taxes to the households that follow a fiscal rule:

$$\tau_t - \tau^* = \phi_\tau \left(b_t - \bar{b}\right).$$

Here, τ^* is the steady state value of this tax, and ϕ_τ is the elasticity of taxes to debt deviations from the steady state. When this elasticity is high enough, the fiscal authority adjusts taxes to ensure debt sustainability. $i^{G,P}$ is public investment in green capital that enhances the accumulation of the green public capital $s^{G,P}$.[5]

2.9 Aggregation

Aggregating all domestic and foreign agents, we derive the market clearing condition for home-produced goods, the NIPA equation, and the definition of net exports. These

[5]We assume that public green capital follows a law of motion analogous to 2, with the same parameters.

expressions are as follows:

$$
\begin{aligned}
y_{H,t} &= (1-\chi)p_{h,t}^{-\theta}\left(c_t + i_t + i_t^G + i^{G,P}\right) + c_{H,t}^*, \\
p_{H,t}y_{H,t} &= c_t + i_t + i_t^G + i^{G,P} + \frac{\kappa_P}{2}\left(\pi_t^H - \bar{\pi}_t^H\right)^2 p_{H,t}y_{H,t} + nx_t + p_t^B e_t^{B,*}, \\
nx_t &= \frac{FX_t}{P_t}P^*\frac{B_{t+1}^*}{P^*} - \frac{FX_t}{P_t}P^*\frac{B_t^*}{P^*}R_{t-1}^*\Psi^A\left(\tilde{A}_t^f\right).
\end{aligned}
$$

Then,

$$
nx_t = rer_t b_{t+1}^* - rer_t\frac{b_t^*}{\pi^*}R_{t-1}^*\Psi^A\left(\tilde{A}_t^f\right).
$$

Profits transferred to households are given by:

$$
\Gamma_t = \Gamma_t^I + \Gamma_t^G.
$$

2.10 Balance growth path assumptions

As mentioned earlier, the directed technical change affects the long-run energy share and economic growth (See also Hassler et al. (2021a)). In particular, define X_{t-1} as the output trend at period t, at which $y_{H,t}$ grows. We define:

$$
X_t = A_t k_t^\alpha \tag{9}
$$

such that

$$
g_t = \frac{X_t}{X_{t-1}} = \frac{A_t k_t^\alpha}{X_{t-1}^{1-\alpha}X_{t-1}^\alpha} = \tilde{A}_t\tilde{k}_t^\alpha, \tag{10}
$$

is the growth rate of the economy. Since the stock of capital's trend is X_{t-1}, its productivity factor A_t grows at $X_{t-1}^{1-\alpha}$. $\tilde{A}_t$ and $\tilde{k}_t$ are the stationarized counterparts of A_t and k_t.

To have a balanced growth path, given the functional form of the production function, we need the two additive parts of it to grow at the same rate. It means that we need:

$$X_{t-1} = X_{t-1}^{Ae} X_{t-1}^{e},$$

and from equation (4)

$$X_t^e = X_t^{eG} = X_t^{eB}.$$

Thus, all energy sources grow at the same rate for all t. Then, from the production function of green energy, we get the following condition:

$$X_{t-1}^{eG} = X_{t-1}^{\mu}.$$

Hence,

$$X_{t-1}^{eB} = X_{t-1}^{e} = X_{t-1}^{\mu}$$

and

$$X_{t-1}^{Ae} = X_{t-1}^{1-\mu}.$$

Finally, from the first order condition of intermediate producers to energy inputs, prices p_t^G and p_t^B grow at $X_{t-1}^{1-\mu}$. In the appendix, we present the complete set of stationarized equations.

3 Calibration and solution

3.1 Numerical implementation and function forms

As our economy has perfect foresight, we can use standard methods to approximate the transition between an initial and a final steady state non-linearly. Our calibration strategy intends to target business cycle first-order moments on national accounts and energy production for Chile and use them as the initial steady state.

We assume standard functions for utility and adjustment costs. In particular, we assume the Utility function is CRRA given by

$$U = \frac{c_t^{1-\sigma}}{1-\sigma},$$

and we assume quadratic adjustment costs for capital and for green capital as:

$$\Phi\left(\frac{k_{t+1}}{k_t}\right) = \frac{\kappa_K}{2}\left(\frac{k_{t+1}}{k_t} - \bar{g}\right)^2,$$

and

$$\Phi\left(\frac{s_{t+1}^G}{s_t^G}\right) = \frac{\kappa_S}{2}\left(\frac{s_{t+1}^G}{s_t^G} - \bar{g}\right)^2,$$

where g is the average growth rate of the economy g_t at the steady state.

3.2 Calibration

We calibrate the model to standard values in the literature or to match the first moments for Chile after the 90s, when the country implemented inflation targeting, and most of its macroeconomic indicators stabilized. The frequency is annual, and Tables 1 and 3 report the parameter values we use in our numerical exercise.

Some parameters take standard values in the literature, such as the intertemporal elasticity of substitution, σ, set to 1, and the annual depreciation of capital, δ, to 0.12. We assume that the specific green capital depreciates at the same rate as the traditional capital stock. We set the capital shares in the production of the intermediate goods and in the green energy production, α and μ, at 0.26 and 0.33.

Table 1: Calibrated Parameter Values I

	Parameter	Target/source	Value
β	Discount factor	Av. Inflation Chile	0.987
σ	CES elasticity in utility	Standard	1
θ	Subst. H & F in consumption	JP (2011)	0.85
λ	Foreign Subst.	JP (2011)	0.85
χ	Share F goods in consumption	JP (2011)	0.24
δ	Depreciation capital	Standard	0.12
κ_P	Adj. cost of prices	Standard	19
ϵ_P	Elasticity between varieties	Av. Markup 11%	10
α	Capital share in production	Standard	0.26
R^*	Gross risk free rate	3 months Tbill USA	1.03
$\bar{b}$	Public debt at initial steady state	Debt-to-GDP 16%	0.14
τ^*	Lump sum taxes at initial SS	Public spending/GDP 12%	0.11
ρ_R	Interest rate smoothing parameter	Standard	0.9
ϕ_π	Interest rate response to inflation	Martinez et al (2020)	1.12
ϕ_y	Interest rate response to output	Standard	0.25
ϕ_τ	Tax response to debt	Standard	0.07
ϕ_A	Sovereign spread parameter	Country spread Chile	0.009
κ	Adjustment cost capital K and G		0.005
$e^{B,d}$	e^B Domestic endowment	Imported/total energy	0.5
ξ	Subst. energy inputs	Papageorgiou et al (2015)	0.67
ζ	Share brown energy	Data Chile	0.3
μ	Green capital share in e^G	Standard	0.33
Be	Prod. coef researchers	Av. Growth 2.5%	0.11
ϕ_N	Prod. coef researchers	Hassler et al (2021)	0.92
γ	Green public and private K	An et al (2019)	0.44
ω	Public inv. share in e^G	An et al (2019)	0.66

The adjustment cost of prices parameter, κ_P, is set to 19. The value of κ_P determines the degree of nominal rigidity. One can relate this value to the average price contract length by exploiting the relationship between the log-linearized NK Phillips curve in the Calvo model and the one implied by the Rotemberg model. In particular, the slope of the Phillips curve to the real marginal costs is equal to ϵ/κ_P, while the corresponding value in the Calvo model is $[(1-\Psi)(1-\Psi\beta)/\Psi]$ where $1/(1-\Psi)$ is the average contract length. According to this relationship, the value of κ_P we chose corresponds roughly to an average contract length of one year.

The Taylor rule coefficient that indicates the interest rate response to inflation deviations from the target, ϕ_π, is equal to 1.12 as in Martínez et al. (2020), who perform a Bayesian estimation of a NK model for Chile. The Taylor rule coefficients for interest rate persistence, ρ_R, and reaction to output deviations, ϕ_y, take standard values in the literature: 0.9 and 0.25, respectively. The fiscal rule coefficient, ϕ_τ is 0.07, in line with papers that study fiscal-monetary policy interactions, such as Bianchi and Melosi (2017) and Bianchi (2021).

Following Justiniano and Preston (2010), we set the elasticity of substitution between domestic and foreign goods, in the domestic and foreign country, θ and λ, equal to 0.85 and the share of foreign goods in consumption, χ to 0.24. The elasticity between domestic varieties, ϵ_P, is set so that the steady-state markup is 11%.

Some parameters are calibrated to match first-order moments in Chilean data in the initial steady state of the model. For instance, the discount factor, β, is calibrated to 0.987 to get the average inflation rate of 4%, given the average nominal risk-free rate of 3%. The steady-state values of real debt $\bar{b}$ and taxes τ^* are calibrated to reproduce the average public debt-to-GDP ratio of 16% and government spending of 12%. The parameter that characterizes the sovereign spread, ϕ_A, takes the value 0.009 to generate a consistent sovereign spread for Chile.

We normalized the steady state values of the rest of world inflation rate (π^*), the real exchange rate (rer), the international price of brown energy ($p^{B,*}$) and labor ($\bar{h}$) to 1, and set the adjustment cost of traditional and green capitals, κ, to 0.005.

Regarding parameters related to the energy sector, we set the substitution of energy inputs in energy production to 0.67, following the estimates of Papageorgiou et al. (2017), assuming that the two energy inputs are substitutes. The domestic endowment of brown energy, $e^{B,d}$ is set to 0.5 to match the imported to total energy ratio in Chile, which is a 50%.[6] The coefficient Be in the evolution of energy efficiency, Ae, is calibrated to reproduce the average real per capita GDP growth rate of 1.025, according to the data, and ϕ_N is set to 0.92 as in estimation results from Hassler et al. (2021a). Parameters in the CES aggregator of public and private green capital, γ, and ω take values 0.44 and 0.66 as in An et al. (2019). Thus, we assume that public and private capital are substitutes in production. An et al. (2019) estimate a nested-CES production function, whereas the two types of capital are considered separately along with labor as inputs. Due to a lack of data availability, we assumed the substitution between public and private capital in production also holds for green energy production. We examine the sensitivity of our results to this assumption.

To finalize the parametrization of the model, we set the share of brown energy, ζ, to 0.3 and jointly calibrate the elasticity of substitution between physical capital and energy, ϵ, the energy CES coefficient $\bar{E}$, the productivity level in green energy production, Ω, and the coefficient B in the traditional factors TFP, to match first order moments for Chile in the initial steady state. To be more specific, we calibrated those parameters to minimize the distance between model and data values for the ratio of brown to total energy (e^B/e), the ratios of total investment ($i+i^G+i^{G,P}$) and green capital investment to GDP ($i^G+i^{G,P}$), and to ensure that the sum of energy ratios equals one ($e^B/e+e^G/e = 1$). The target values for these values are presented in Table 2.

[6]In figure A.6 in the appendix, we compare the benchmark calibration with a counterfactual where the small open economy is a net importer of brown energy.

Table 2: Data moments

e^B/e	$(i+i^G+i^{G,P})/y$	$(i^G+i^{G,P})/y$	$(e^B/e+e^G/e)$
0.72	0.20	0.01	1.00

Table 3 presents the resulting parameter values.

Table 3: Calibrated Parameter Values II

	Energy parameters	**Target/source**	**Value**
ϵ	Subst. energy and K	Jointly calibrated	0.4834
$\bar{E}$	CES energy	Jointly calibrated	2.514
Ω	TFP in e^G	Jointly calibrated	0.029
B	Prod. coef researchers	Jointly calibrated	0.021

4 Transitional Dynamics

We focus on a perfect foresight model for the transitional dynamics as our interest is in the long-run dynamics for which short-run uncertainty is irrelevant. We assume the economy is in equilibrium consistent with a steady state before period 1. Starting with this initial condition, we simulate a Green Transition where the economy moves from high use of brown energy to lower use. Given that the economy is a perfect foresight one, the agents in the model have known the whole transition since period 1.

4.1 A transition induced by increases in brown energy taxes

Following the existing literature, we investigate the transitional dynamics induced by increases in brown energy taxes. Chile has made important climate commitments, such as updating in 2020 its Nationally Determined Contribution target to reduce carbon dioxide

(CO2) emissions by up to 45% by 2030 from 2016 levels and reach carbon neutrality before 2050. Chile introduced a US$5 tax per ton of CO2 in 2014, making it the first Latin American country to introduce green taxes on CO2 emissions and local pollutants as part of its climate strategy. However, this tax has remained at low levels since then. Chile's Climate Action Plan 2017-2022 specifies a further substantial gradual increase in carbon taxes that will multiply the actual carbon tax seven times. Motivated by the Chilean Action Plan, we first assume that the secular dynamics around the green transition are solely driven by increases in brown energy taxes from 5% in the initial steady state to 35% in the new steady state, assuming a 200-year transition.[7] To be precise, starting from the initial steady state, we assume that brown energy taxes increase similarly to the one announced by Chile's action plan after twenty years and that the economy slowly transitions to the new steady state in 200 years. Figure 1 presents the transitional dynamics for the model economy in the first 40 years.[8]

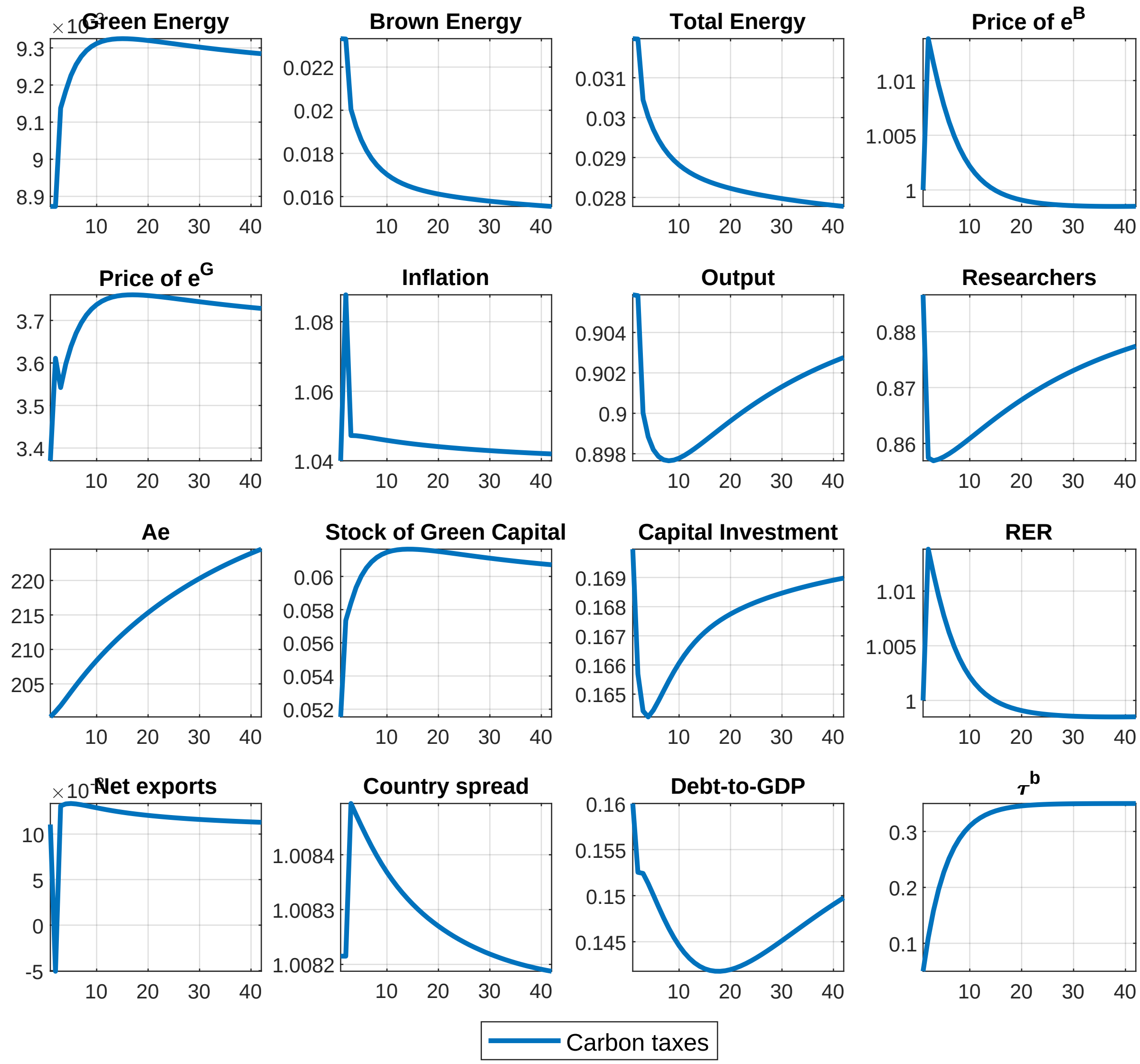
The increase in carbon taxes accelerates the use of green energy and reduces the use of brown energy. In the new steady state, brown energy usage decreases by 33% and total energy usage by 16%, while energy efficiency increases by 12%. The fall in energy usage is because firms reallocate researchers to improve energy efficiency in the short run, significantly affecting production dynamics in the long run.

In the short run, the increase in the taxation of brown energy translates into increases in the intermediate firms' marginal costs that push up inflation. The increase in the cost of using brown energy drives up domestic brown energy prices and, given the relatively low substitutability between brown and green energy in production, also the price of green energy. This results in a surge in inflation in the short and medium run that smooths out gradually as the firms allocate more researchers to efficient energy usage, increasing productivity.

[7] We assume 200 years for the transition to guarantee a proper convergence to the new steady state. However, Figure 1 shows that the carbon tax stabilizes in a new value in 20 years.

[8] The results for the whole transitional period are presented in the appendix Figure A.1

Figure 1: Impact of a Quantitative Easing shock conditional on a regime



Note: Dynamics for the first 40 years of transition from initial to a new steady state where carbon taxes increase from 5 to 35%.

Given the reallocation of researchers along the transition, the productivity of traditional inputs slows down shortly, discouraging physical capital accumulation. However, the complementarity between energy and traditional inputs in the short run, coupled with the slow dynamics in the energy accumulation and the increase in the cost of brown energy, render the green transition recessionary. In other words, since firms perfectly foresee long-term increases in brown taxation, they adjust their production by decreasing traditional factors, particularly investment demand, generating a recession in the short run. Actually, the dynamics of the economy resemble those of a sudden stop, with output falling and net exports reverting in the short run.

The policy has substantial output and inflationary costs. Inflation increases along the transition in the short run from 4 to more than 8%, while output falls persistently with a maximum fall of around 1% that lasts longer than three years. Hence, our analysis suggests that such a transition induces ”greenflation” in the first few years and significant and persistent output losses. The effects on inflation are moderate because the negative demand effect from the fall in private investment moderates the inflationary consequences of the transition.

Finally, given that we are interested in analyzing small open economies often subject to sovereign debt crises, we analyze in the last row of Figure 1 how the public debt-to-GDP ratio and the spreads behave along the green transition. The fall in output induces a persistent increase in the country spreads. On the other hand, the increase in revenues from the taxation of brown energy coupled with the low substitutability of energies in the short run results in a slight reduction in the debt-to-GDP level, originating from the increases in tax revenues.

Our numerical exercise, thus, suggests that a substantial increase in carbon taxation should result in an effective reduction of brown energy usage by 33% and an increase in energy efficiency in the long run, which would not involve substantial fiscal costs but is accompanied by ”greenflation” and persistent output losses in the short run.

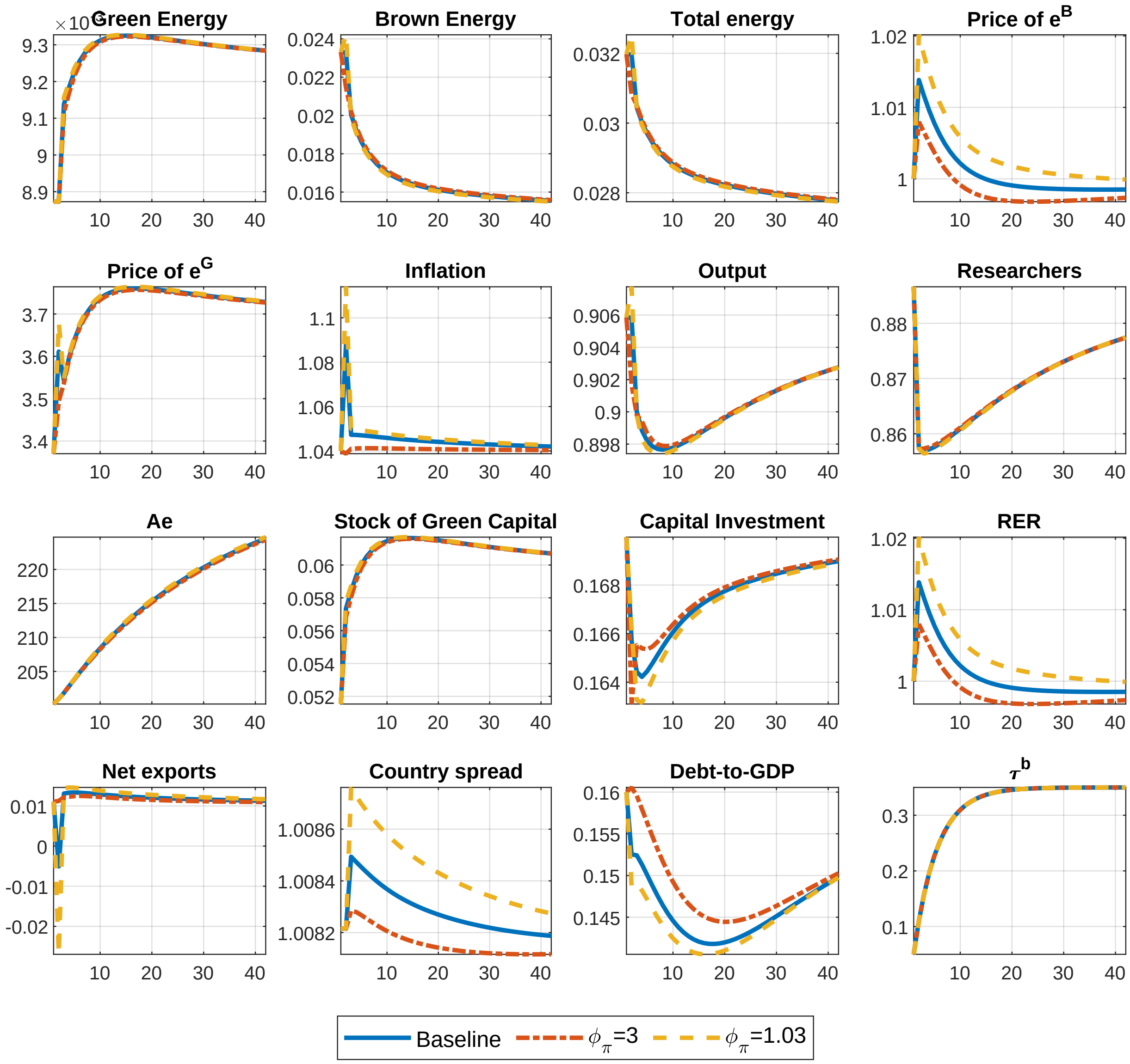
4.2 The role of monetary policy

Many recent papers investigate the role of monetary policy along the green transition assuming additional short-run frictions, such as different degrees of rigidity in the brown and green sector as in Del Negro et al. (2023) or price and wage rigidities as in Olovsson and Vestin (2023). Our model differs from those models as we are interested in also analyzing energy efficiency and concentrating on the role played by fiscal policy during the transition.

Since the role of the monetary policy stance during the green transition is at the center of academic and policy debates, in Figure 2, we repeat the baseline exercise by changing the stance of monetary policy. In the benchmark economy, we have assumed a Taylor coefficient for inflation of 1.12, following the estimation in Martínez et al. (2020). We now reduce this value to 1.03 (dashed yellow lines in Figure 2) and increase it to 3 (red dashed-dotted lines in Figure 2). The monetary policy stance does not significantly affect the mechanism behind the green transition as far as real variables and energy efficiency are concerned, apart from capital investment that falls more under the stricter policy rule. On the other hand, the stance of monetary policy significantly affects inflation dynamics. By reacting strongly to inflation deviations, monetary policy can control the relative changes in the price of brown energy, decreasing the supply side effects of the rise in carbon taxation and limiting greenflation. Hence, similar to Nakov and Thomas (2023), carbon taxes pose no trade-offs for monetary policy in our framework.

Monetary policy choices are crucial for determining the external balance along the green transition. As a result, a looser policy induces a higher real depreciation, significantly affecting the country's spread. More importantly, the monetary policy stance has short-run consequences on the fiscal space. The looser the monetary policy stance, the higher the increase in brown energy prices due to the higher real exchange rate depreciation and the larger the revenues from brown taxes. Hence, a more lax monetary policy implies a bigger reduction in the debt-to-GDP ratio that comes at the cost of higher inflation. In other words, our model economy generates interactions between monetary and fiscal policy, which are absent in the standard NK model along the green transition.

Figure 2: Transitional Dynamics: The role of monetary policy



Note: Dynamics for the first 40 years of transition from initial to new steady state where carbon taxes increase from 5 to 35% for different parameters of the monetary policy rule. Blue lines represent the baseline calibration; red dashed lines are the case of $\phi_\pi = 3$, and yellow discontinuous lines are the case of $\phi_\pi = 1.03$.

Hence, we would conclude that targeting inflation could be beneficial for moderating greenflation with no output losses in our small open economy. On the other hand, a loose monetary policy brings benefits in terms of fiscal space.

4.3 The role of supply frictions

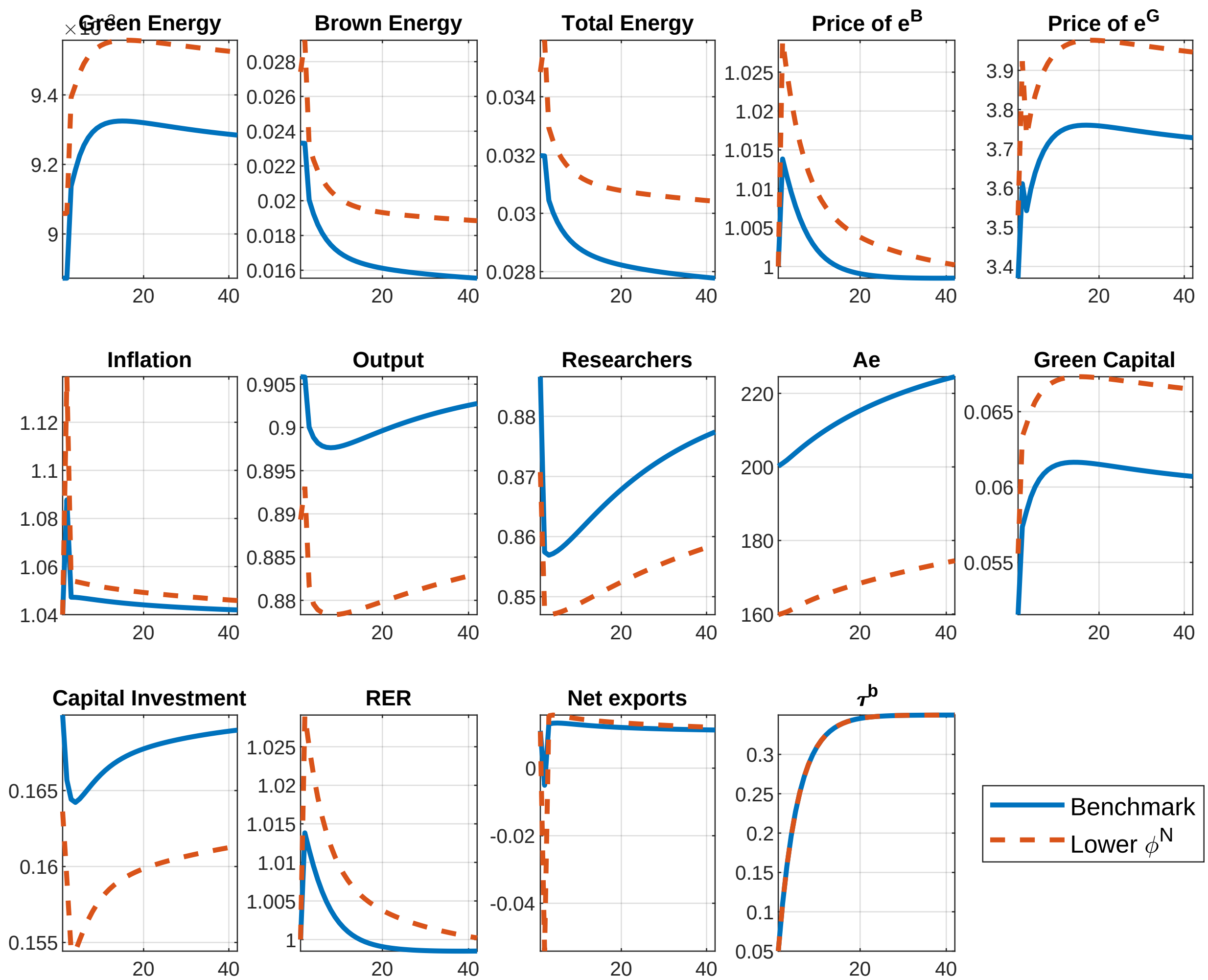
Supply frictions play a key role in shaping firms' responses to relative energy price changes in the short and long run. We have assumed various frictions that could significantly affect the transitional dynamics and the initial and terminal steady states.[9]

We start by considering how rigidities in the allocation of researchers shape the transitional dynamics. We consider a lower ability of firms to move researchers to increase energy efficiency by decreasing parameter ϕ^N (we reduce ϕ^N to 0.8 in this case). The transitional dynamics of the economy with lower ϕ^N are depicted with red dashed-dotted lines in Figure 3. Supply frictions play a crucial role in determining steady states and transitional dynamics. When firms cannot easily switch researchers through directed search, the economy starts from a steady state with higher total energy usage and higher usage of both types of energy and a lower energy efficiency and output level. Yet, for the same increase in taxation, the relative fall in brown energy usage is similar and equals 33%. Yet, the short-run dynamics for inflation and output differ. Inflation increases much more in the short run, and output falls relatively more during the green transition. Since the switch of researchers to produce energy efficiency is less effective (compare the slope of A_e in the graph for the two scenarios), firms have to invest more in green capital, crowding out relatively more the accumulation of traditional capital, inducing larger output losses in the short run. The implied smaller substitutability between traditional factors and energy deprives the firms of an extra margin of adjustment and leads to elevated marginal costs in the short run that translate into large increases in inflation. Inflation increases from 4 to almost 14% in the short run and remains above 5% for almost 20 years, while

[9] From the demand side, we have considered the role of price stickiness in shaping the transitional dynamics. Similarly, with the monetary policy stance, demand frictions affect the movements of the real exchange rate, for a given monetary policy rule, generating differences in the increases of brown energy prices that result in more inflation when prices are more rigid and higher fiscal gains from the increases in the price of brown energy.

output falls by almost 1.25% in the short run and remains persistently low.

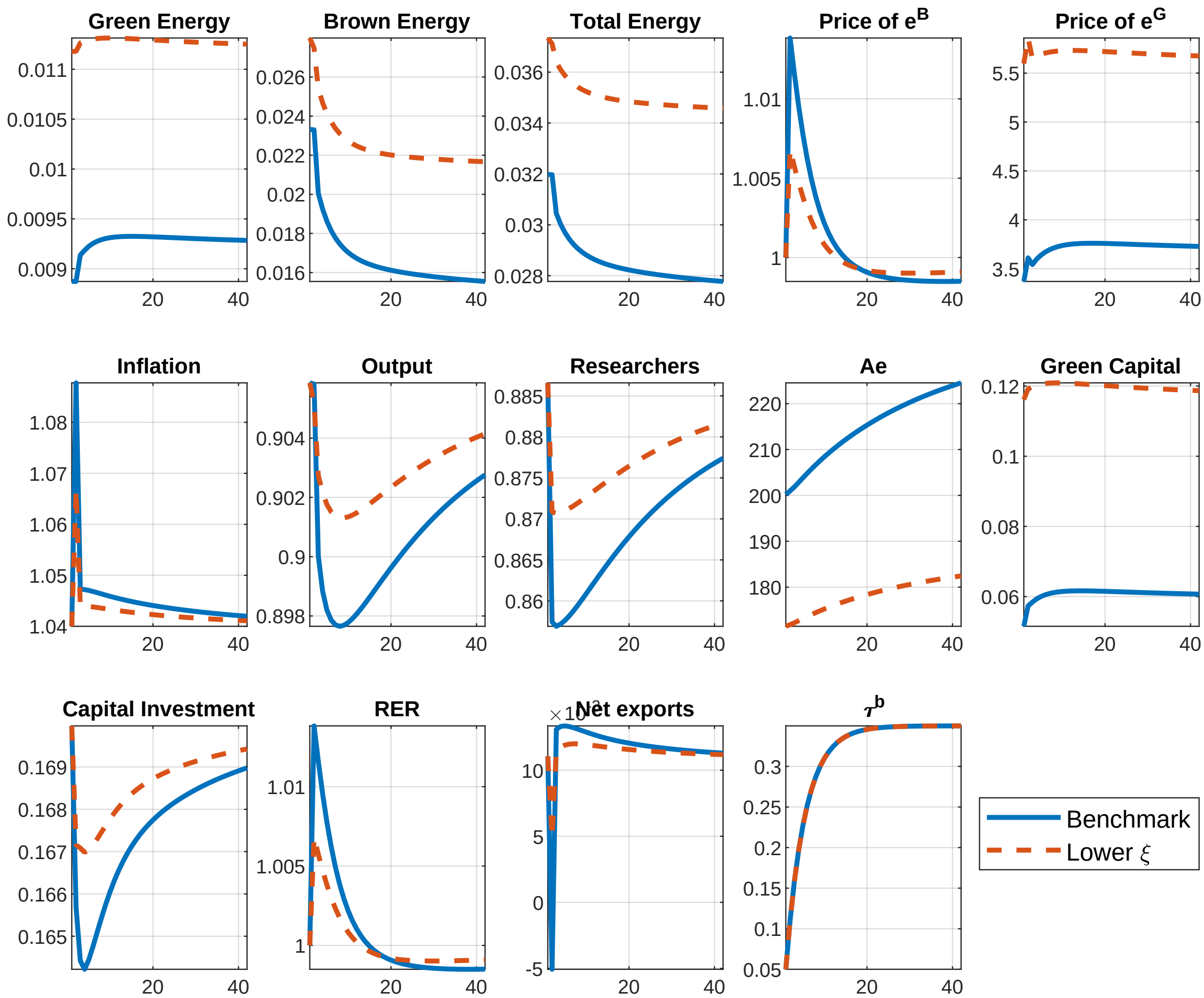
Figure 3: Transitional Dynamics: The role of substitutability between traditional factors and energy



Note: Dynamics for the first 40 years of transition from initial to a new steady state where carbon taxes increase from 5 to 35%. Blue lines represent the baseline calibration, and red dashed lines represent the case of $\phi^N = 0.8$.

We also investigate how changes in the substitutability between green and brown energy in production affect the transitional dynamics. Figure 4 presents transitional dynamics when we reduce ξ from 0.67 to 0.1 (red crossed lines in the figure). Again, changing ξ also alters the steady state values of green energy and prices and transitional dynamics.

Figure 4: Transitional Dynamics: The role of substitutability between energy inputs



Note: Dynamics for the first 40 years of transition from initial to a new steady state where carbon taxes increase from 5 to 35%. Blue lines represent the baseline calibration, and red dashed lines represent the case of $\xi = 0.1$.

A lower value of ξ alters the energy value in the initial steady state. Given the low substitutability between energy inputs, higher brown and green energy levels are needed to produce the same output. Furthermore, the same increase in carbon taxes is less efficient in bringing down the level of brown energy in the new steady state. That is a change in taxes from 5 to 35% in the economy with lower substitutability of energy inputs reduces the brown energy usage by only 23%, compared to 33% in the benchmark economy.

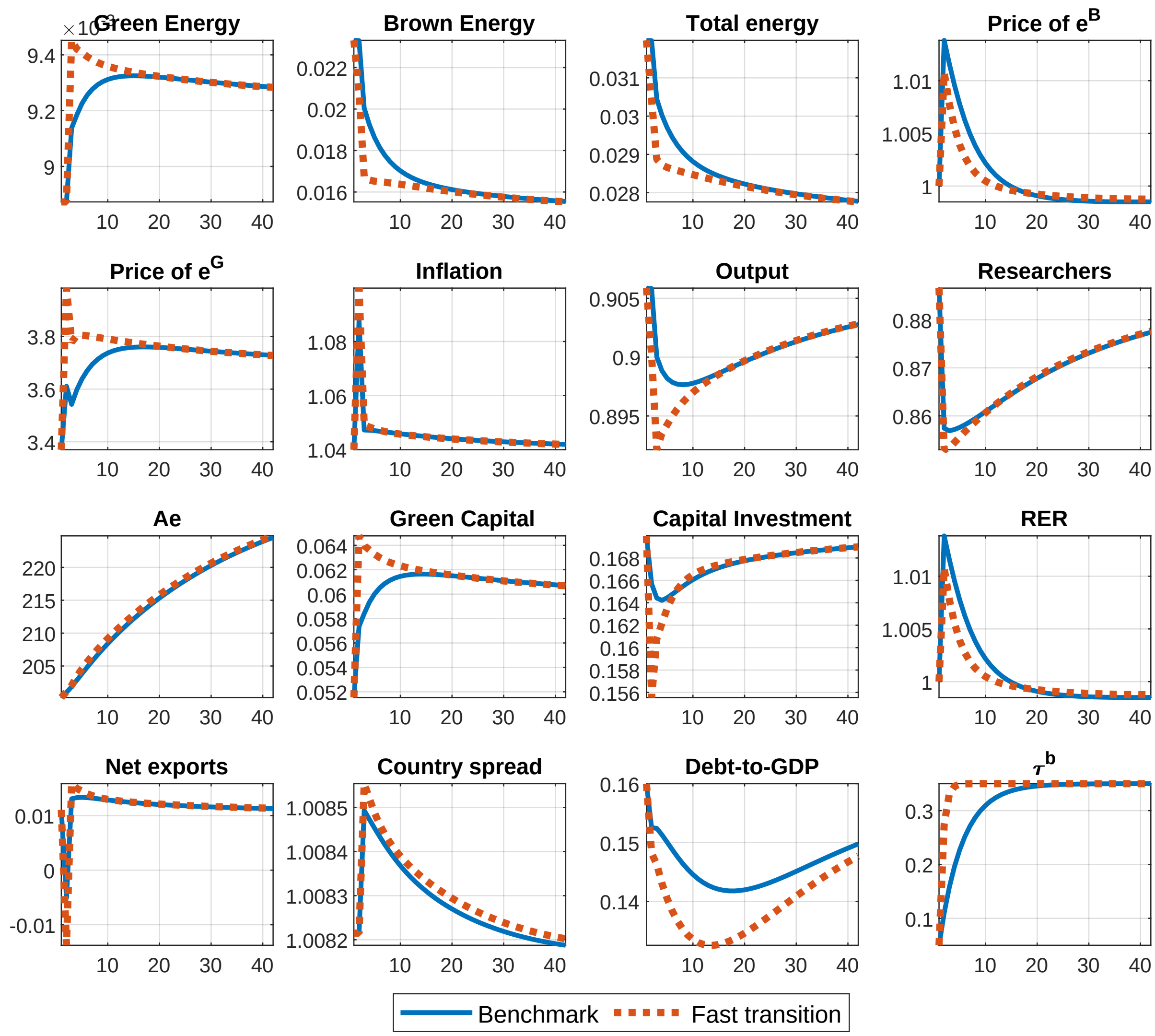
Firms, aware that green energy cannot substitute for brown energy in production, change less energy efficiency along the transition by moving fewer researchers in the R&D sector. This generates a smaller fall in traditional input demand and lower output costs in the short run. Given that the amount of green energy in the initial steady state is higher for this parameterization, the increase in brown taxes generates smaller pressure in marginal costs and, hence, less inflationary pressures along the green transition.

Overall, our sensitivity analysis suggests that every economy's idiosyncrasies should be considered when designing possible green transitions, as supply frictions are important for shaping the transitional dynamics and determining the success of the green transition.

4.4 The role of the speed of transition

Given the recent policy discussions, it is important to highlight how the transition speed might affect the economy's transitional dynamics. In Figure 5, we compare the transition in our baseline economy with the transition that would emerge if the fiscal authorities were to accelerate the increase in carbon taxation so that the new carbon tax target would be met after five years instead of twenty. Since firms would have to adjust faster to the rise in marginal costs, increasing brown taxes would lead to larger switches of researchers to attain energy efficiency in the short run, translating to higher output and inflationary costs in the short run. The dynamics of the model economy suggest that a more gradual transition is preferable if one were to moderate greenflation and output costs.

Figure 5: Transitional Dynamics: The speed of transition



Note: Dynamics for the first 40 years of transition from initial to a new steady state where carbon taxes increase from 5 to 35%. Blue lines represent the baseline calibration, and red dashed lines represent a faster transition.

4.5 Alternative fiscal policy tools

Most of the existing literature concentrates on the role of carbon taxes. A recent literature review by Timilsina (2022) lists three types of policies to reduce greenhouse gas (GHG) emissions: a) fiscal or pricing policies, b) regulatory policies, and c) direct public investment. Relative to the policies analyzed in the literature, we now investigate the role of another pricing policy - green subsidies - and direct public investment.

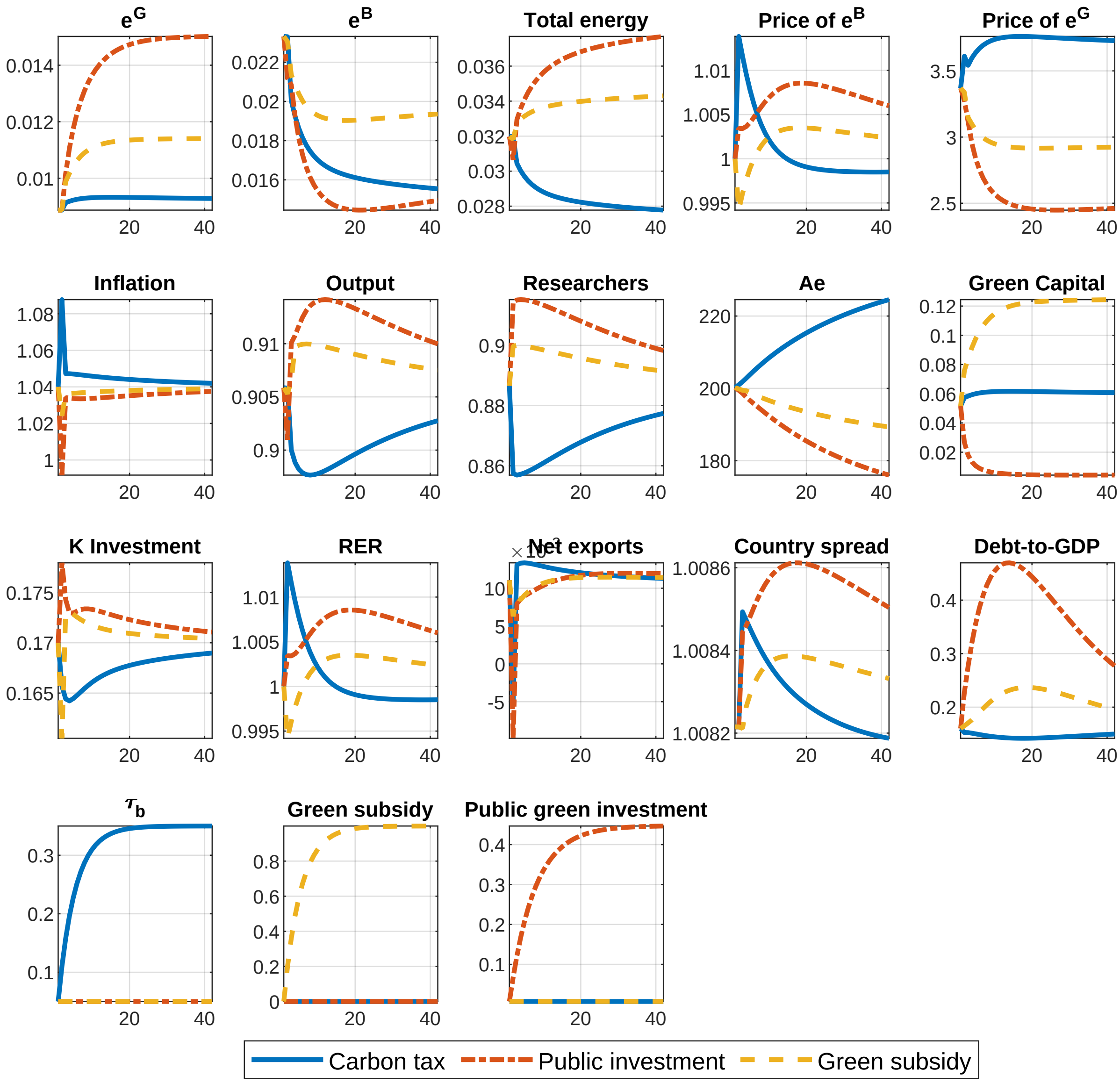
4.5.1 Green Subsidies

An alternative way to accelerate the green transition is through a subsidy to the price of green energy[10]. We investigate such a transition by considering that subsidies increase along a 200-year transition from zero in the initial steady state to 100% in the new steady state. The transition is depicted in Figure 6 with yellow dashed lines.

The increase in subsidies that we assume is purposefully exaggerated to highlight the impotence of subsidies as a fiscal tool to attain an effective reduction in brown energy usage. Green subsidies cannot reduce brown energy usage comparable to carbon taxes even if set to 100. This is because raising subsidies does not compel the firms to substitute brown for green energy, and little effort is placed into increasing the energy efficiency in production. Given that green energy is subsidized, relative demand for brown energy falls, hence its price. Subsidies further reduce the price of green energy, resulting in a fall in marginal costs that translate into deflationary pressures in the short run. The fall in the price of energy induces firms to invest more in the efficiency of traditional factors, and the policy leads to a steady state with relatively less energy efficiency and higher total energy usage. Moreover, the increase in traditional TFP increases the demand for capital, generating an output increase along the transition. The real exchange rate appreciates, and country spreads move less than in the case of carbon taxes. Finally, since subsidies must be financed partly with debt, the debt-to-GDP ratio increases moderately, generating fiscal pressures for the small open economy.

[10] We have also investigated the case of a subsidy to the acquisition of green energy from intermediate good producers. The results we present here are similar and available under request.

Figure 6: Transition using different fiscal instruments



Note: Dynamics for the first 40 years of transition from initial to a new steady state where carbon taxes increase from 5 to 35% (Blue continuous lines); subsidies change from 0 to 100% (yellow discontinuous lines) and public investment changes from 0 to 7% of GDP (red dashed lines).

4.5.2 Public green investment

An alternative way to induce the green transition is through green public investment. For example, the German government intends to accelerate the green transition through public green investment infrastructure. The German finance minister, Christian Lindner, recently announced the launching of 200 billion euros to fund industrial transformation between March 2022 and 2026, including climate protection, hydrogen technology, and electric vehicle charging network expansion. At the same time, Germany plans to boost energy production investment in renewables. In Figure 6, we plot with red dashed-dotted lines a green transition in which public investment in green infrastructure increases from almost 0 to 7% of GDP from the initial to the new steady state to achieve a similar drop in brown energy usage in the new steady state after 40 years as in the case of increases in carbon taxes. We consider such fiscal expansion to be very large, but we adopt it here to compare the dynamics induced by the two policy choices having the reduction in brown energy usage obtained as a common denominator.

The boost in green public investment crowds out green private investment, as we have assumed that the two are substitutes in green energy production.[11] Given the production structure of the green sector, the increase in green public capital operates as an increase in the productivity of the green energy sector, generating a fall in its price and an increase in green energy usage. Brown energy is substituted for cheaper green energy. As the change in the energy market pushes down energy costs for firms, they invest relatively more in the TFP of traditional factors, shifting researchers to improve traditional inputs' energy efficiency. Hence, energy efficiency deteriorates. As with green subsidies, this boosts capital investment and output in the short run and reduces inflation. However, given the size of the expansion, the fiscal costs associated with such a policy are considerable. Overall, green public investment encourages using green energy at a lower cost without substan-

[11] In Figure A.3 in the appendix, we present results for the case in which the two inputs are complements. In this case, green public investment becomes a more efficient fiscal policy tool as it incentivizes firms to invest more in green capital and reduce brown energy usage.

tially crowding out the usage of brown energy sources. A substantial green energy price fall must occur to make a difference. Relative to subsidies, such a policy can achieve the transition goals in terms of brown energy usage reduction quickly with no inflationary costs but cannot attain improvements in energy efficiency in the long run.

Several comments are in order after completing this exercise: First, transitions due to increases in taxes on brown energy are the most painful ones in terms of inflation and output losses, but at the same time, increases in brown energy prices are the ones that achieve the desired transformation for reasonable fiscal policy changes. Second, green public investment can effectively and timely bring about the green transition by reducing the price of green energy in the long run. Third, subsidies can substantially reduce brown energy usage only if raised to 300% (See Figure A.2 in the appendix). This is because subsidies decrease the price of brown energy as firms substitute green for brown energy in production and find it optimal to keep energy consumption at inefficiently high levels. Fourth, green public investment and subsidies increase green energy usage but do not lead to higher efficient energy use. Finally, given the differential paths of inflation and output when using the different policy tools, in the next section, we examine whether a policy mix with increases in both taxation and subsidies or green investment can ameliorate the short-run costs of the green transition.

4.5.3 Fiscal Policy Mix

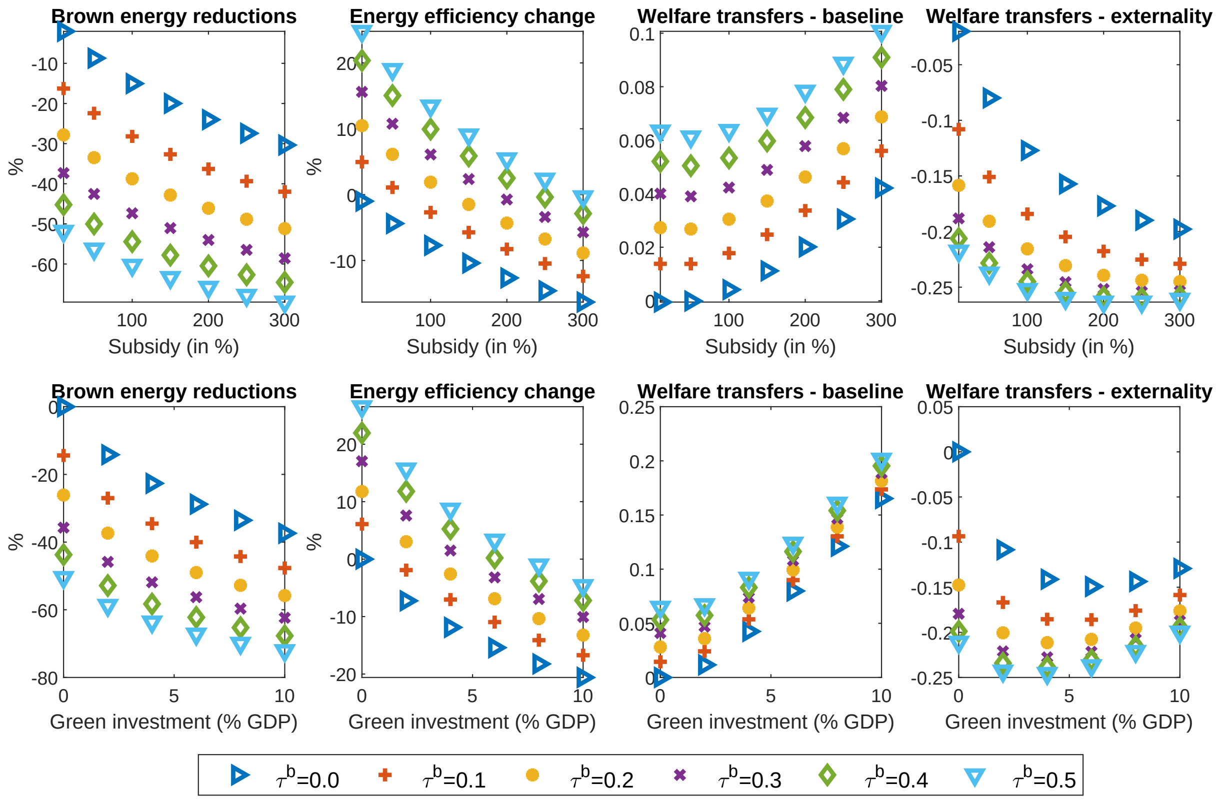
The public opposition to carbon taxation is well documented in the literature. For example, Carattini et al. (2018), revising the existing literature, lists personal costs, regressivity, damage to the wider economy, lack of efficiency, and the self-interest of the state as the main concerns the public has against carbon taxation. Given the political difficulties in increasing carbon taxes, we investigate alternative policy combinations of this policy tool with green subsidies or increases in green capital investment to attain a similar reduction in brown energy usage as in the benchmark economy. On the left panel of Figure 7, we present how different combinations of carbon tax increases and green subsidy increases (top panel) or public investment increases (bottom panel) can reduce the usage of brown energy in the new steady state[12]

For low values of subsidies, carbon taxes need to increase a lot to attain a reduction in brown energy usage. Subsidies complement carbon taxes as a policy tool for reducing brown energy usage in production. For example, a government can attain a further reduction of brown energy of 7 to 8% by increasing subsidies from 0 to 50% rather than using taxes alone. The picture looks similar in the combination of increases in public infrastructure and carbon taxes in the bottom left panel of Figure 7. Yet, in both cases, the use of mix policies results in a fall in the energy efficiency in the new steady state as depicted in the second column of Figure 7. Using the same example as before, an increase in subsidies by 50% for a given tax rate reduces energy efficiency in the new steady state by approximately 5%. Hence, the reduction of brown energy usage comes at the cost of lower energy efficiency when subsidies or green public investments are used as additional tools to attain a fall in brown energy usage.

But how do these policy combinations affect welfare? We answer this question in the next session.

[12]Figures A.4 and A.5 in the appendix present transitional dynamics when we consider a policy mix in which taxes on brown energy increase from 5 to 25% while green subsidies increase from 0 to 40% and a policy experiment in which we increase green public investment from almost 0 to 2.8% of GDP and increase taxation from 5 to 15% to achieve a similar decrease in the brown energy usage in the new steady state as the one obtained in the benchmark experiment, respectively. Given the dynamics of pure policies, it's not surprising that both policy mix scenaria are associated with lower output and inflation losses in the short run and bear a lighter fiscal burden in terms of increases in the debt-to-GDP ratio and spreads.

Figure 7: Brown energy reductions and welfare losses from different policy mix exercises



Note: First column: reductions in brown energy usage for different carbon tax changes depicted with different lines and different subsidy levels x-axis (top) or different public investment levels (bottom). Second column: changes in energy efficiency depicted with different lines and different subsidy levels x-axis (top) or different public investment levels (bottom). Third column: welfare transfers for keeping households indifferent between living in initial SS vs. greener SS with no externality in utility for different tax changes and subsidy levels and (top) or different public investment levels (bottom). Last column: welfare transfers for keeping households indifferent between living in initial SS vs. greener SS with externality in utility for different tax changes and subsidy levels and (top) or different public investment levels (bottom). Brown energy reductions and energy efficiency change are the percentage change in brown energy (e_B) or energy efficiency (A_e) along the transition, i.e., between period 1 and period 200. Green investment is in percentages of GDP.

5 The welfare costs of the green transition

A natural measure to rank the different policy choices is to look at welfare. In this section, we calculate the welfare costs of the green transition under different transition scenarios. First, we recover the trend along the transitions using (9) and (10):

$$X_t = \tilde{A}_t \tilde{k}_t^{\alpha} X_{t-1},$$

for a given initial condition X_0, common to all scenarios. Without loss of generality, we normalize X_0 Second, we recover the path of consumption in levels along the transition,

$$c_t = \tilde{c}_t X_{t-1},$$

where $\tilde{c}_t$ is the detrended value of consumption.

We calculate welfare using a consumption equivalence measure. We adopt as a benchmark the consumption in the initial steady state and compute how much consumers are willing to give up on the initial steady state consumption to reach a level of welfare along the transition that is comparable to their initial steady state, that is,

$$W_k = \sum_{t=1}^{T} \beta^t ln \left(c_{t,k} + \Lambda_k \right), \tag{11}$$

with

$$W_k = \sum_{t=1}^{T} \beta^t ln \left(c_{0,k} \right), \tag{12}$$

where T is equal to 200 periods, the length of the transition. k is the correspondent scenario: i) an increase in carbon taxes from 5% to 35%; ii) an increase in Green Public Infrastructure from 0 to 7% of GDP; iii) a policy mix with increases in brown taxes from 5% to 25% and of green subsidies from zero to 40% and iv) a policy mix with increases in brown taxes from 5% to 15% and of green public investment from zero to 2.8% of GDP. We do not include in this section the case of a 100% rise in subsidies, as it cannot attain the same level of brown energy usage in the steady state and is not comparable to the other policies considered. Notice $c_{0,k}$ is the consumption level at the initial steady state, and it is the same for all k.

The value of Λ_k determines the welfare gains or losses compared to the initial steady state. Positive values of Λ_k imply that consumers are worse off along the transition to the new steady state than with the initial steady state consumption level, and negative values, instead, represent welfare improvements. The first column of Table 4 presents the Λ_ks for the scenarios considered. The green transition is costly in terms of welfare, as we have not included any term in the utility function that could account for factors such as health or probability of survival that would have made the transition beneficial for the agents of our model economy. The best fiscal policy strategy to obtain the same level of green transition in terms of reduction in brown energy is the mix of subsidy and tax increases, while the least welfare-improving tool is the sole increase of green public infrastructure. This is because the increase in green public infrastructure spurs private investment and crowds out private consumption along the transition. Of course, this result depends on the assumption we have adopted on the production of green energy, such as the substitutability between private and public green capital and the share of public green capital in total green energy production.[13]

Generally speaking, if we exclude pure public green investment, the differences in welfare among the rest of the policy strategies considered are not economically significant. Yet, this conclusion is not general. In the third column of Figure 7, we calculate the welfare costs for various policy scenarios, mixing tax increases with green subsidies in the top

[13] In exercises we do not present here for the economy of space, we show that when we assume that public and private green capital are complements, the investment in public infrastructure delivers slightly better welfare outcomes because it does not crowd out private consumption (Figure A.3 presents transitional dynamics for this case).

Table 4: Welfare Comparisons

	No externality	Low Externality e^B	High Externality e^B
Carbon Tax	0.041	-0.023	-0.179
Green Subsidy 300%	0.042	-0.023	-0.198
Public Infrastructure	0.101	0.034	-0.148
Carbon Tax-Subsidy Mix	0.028	-0.036	-0.194
Carbon Tax-Green Capital Mix	0.040	-0.027	-0.194

panel and green public investment in the bottom panel. Regarding welfare, the best policy scenario is no tax changes and an increase in subsidies by 30% that produces a reduction of brown energy of approximately 5%.

5.1 Negative Externality

Given that our analysis concerns a small open economy, we have considered that a possible externality from brown energy usage in the small open economy should be minimal to the global economy and assumed that utility depends only on consumption. In this section, we present an extension of the welfare calculations when we assume an externality derived from the utilization of brown energy in the production of domestic goods, e_t^B. Following Hassler et al. (2021a), we assume there exists a damage function $D\left(e_t^B\right)$ that affects household consumption. The intuition for this externality is that the pollution generated by the utilization of brown energy as an input of production generates detrimental effects on households' health, and this is reflected, for simplicity, as a fall in consumption. The functional form for the damages function is given by:

$$D_t\left(e^B\right) = \tilde{\gamma}(e_t^B)^2.$$

The consumption net of the externality is given by:

$$\hat{c}_t = c_t - D\left(e_t^B\right).$$

We calibrate $\tilde{\gamma}$ to get the total consumption loss due to the externality as a 5% or a 20% GDP loss in the initial steady state. The second column of Table 4 presents welfare calculations for this measure for a low value of the externality and the third column for a high externality value. Assuming a negative externality from the utilization of brown energy in consumption, independently from its size, alters the welfare rankings of the different fiscal policy options and the conclusions determining the desirability of the green transition. With the externality (low or high), all green strategies considered are welfare-improving, apart from public green investment. When the externality is low, the combination of carbon taxes and green subsidies provides the best welfare outcome, while when the externality is high, both policy mix options are preferable to pure increases in taxes. This can also be seen in the last column of Figure 7 where we plot welfare transfers for the case of high γ. Now, welfare improvements correlate with energy reductions. Yet, policy conclusions differ with the metric one wants to use. Although an increase in the tax rate by 50% and an increase in subsidies by 300% attain the highest reduction in brown energy usage. Consumers would prefer a mix where subsidies increase by 200% and would be indifferent if the tax rate would hike to 40 or 50%. For the combination of taxes and green investment, hiking the tax rate by 50% and increasing green public infrastructure by 10% of GDP would attain a fall in brown energy usage of around 70% as in the previous mix. Yet, consumers would prefer a policy that increases green public infrastructure by 4% of GDP and would be indifferent if the tax rate would be hiked by 50 or 40% for this policy mix.

To sum up, Figure 7 suggests that for a high externality associated with using brown energy in production, a policy mix strategy is always preferable to just raising carbon taxes for reducing the usage of brown energy although this implies losses in terms of energy efficiency usage in the long run. According to our previous results, the choice of the different policy mix should depend on their fiscal sustainability aspect and repercussions on the international financial conditions of the small open economy.

6 Conclusions

We study the transitional dynamics of green transformation through fiscal policy actions in emerging markets. We have alternative policy instruments or a combination to move to a greener economy. Increases in brown taxes decrease the usage of brown energy but do not expand the green sector. They simply improve energy efficiency usage, leading to surges in inflation and output losses. Green subsidies are unable to reduce substantially brown energy usage even if raised to 100%. Green public investment does not substantially enhance the stock of green capital but does increase green energy usage and decrease green energy prices in the long run. However, both green subsidies and public investment require a substantial fiscal cost to attain similar decreases in brown energy levels as with carbon taxation and imply losses in terms of energy efficiency usage. Monetary policy can shape greenflation in the short run at the cost of higher fiscal stress. The specific characteristics of the economy and supply frictions should be considered when designing transitional policies as they imply different macroeconomic dynamics, and wrong choices can be very costly.

Finally, we provided different welfare metrics for assessing the various alternatives. Increasing green public infrastructure is the least preferable strategy as it implies a crowding out of private consumption. If the goal is to reduce brown energy usage because it induces negative externalities, then a mix of fiscal policy that combines increases in taxes with increases in subsidies or green public investment can be welfare improving.

To conclude, our analysis suggests that there is no easy way towards the green metamorphosis of a small open economy, and one has to decide which sacrifice to make to secure a greener planet.

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A Appendix

A.1 Equilibrium equations

A.1.1 Household

$$\lambda_t = c_t^{-\sigma}$$

$$k_{t+1} = (1-\delta)k_t + i_t + \frac{\kappa_K}{2}\left(\frac{k_{t+1}}{k_t} - \bar{g}\right)^2 k_t$$

$$\lambda_t \frac{1}{P_t} = \beta \mathbb{E}_t \left[\frac{\lambda_{t+1} R_t}{P_{t+1}}\right]$$

$$\lambda_t = \beta \mathbb{E}_t \left[\lambda_{t+1} \frac{R_t^*}{\pi_t^*} \frac{rer_{t+1}}{rer_t} \Phi_{t+1}^A(\tilde{A}_{t+1}^f)\right]$$

$$\lambda_t q_t \left(1 - \kappa_K \left(\frac{k_{t+1}}{k_t} - \bar{g}\right)\right) = \beta \mathbb{E}_t \left[\lambda_{t+1} \frac{R_{t+1}^k}{P_{t+1}} + \lambda_{t+1} q_{t+1} \left(1 - \delta + \left(-\kappa_K \left(\frac{k_{t+2}}{k_{t+1}} - \bar{g}\right) \frac{k_{t+2}}{k_{t+1}} + \frac{\kappa_K}{2}\left(\frac{k_{t+2}}{k_{t+1}} - \bar{g}\right)^2\right)\right)\right]$$

$$s_{t+1}^G = (1-\delta)\, s_t^G + \Phi_s(s_{t+1}^G, s_t^G) s_t^G + i_t^G$$

$$\lambda_t q_t^G \left(1 - \kappa_S \left(\frac{s_{t+1}^G}{s_t^G} - \bar{g}\right)\right) = \beta \mathbb{E}_t \left[\lambda_{t+1} \frac{R_{t+1}^s}{P_{t+1}} + \lambda_{t+1} q_{t+1}^G \left(1 - \delta + \left(-\kappa_S \left(\frac{s_{t+2}^G}{s_{t+1}^G} - \bar{g}\right) \frac{s_{t+2}^G}{s_{t+1}^G} + \frac{\kappa_S}{2}\left(\frac{s_{t+2}^G}{s_{t+1}^G} - \bar{g}\right)^2\right)\right)\right]$$

$$\lambda_t q_t^G = \lambda_t$$

$$p_{H,t} y_{H,t} = c_t + i_t + i_t^G + i_t^{G,P} + \frac{\kappa_P}{2}\left(\pi_t^H - \bar{\pi}_t^H\right)^2 p_{H,t} y_{H,t} + nx_t + p_t^B e_t^{B,*}$$

A.1.2 Intermediate goods producers

$$y_{H,t} = \left[\left(A_t k_t^\alpha \bar{h}^{(1-\alpha)}\right)^{\frac{\epsilon-1}{\epsilon}} + (A_{e,t} e_t)^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}}$$

$$e_t = \bar{E}\left[(1-\zeta)\left(e_t^G\right)^{\xi} + \zeta\left(e_t^B\right)^{\xi}\right]^{\frac{1}{\xi}}$$

$$r_t^k = mc_t y_{H,t}^{1/\epsilon}\left(A_t k_t^{\alpha}\bar{h}^{1-\alpha}\right)^{-1/\epsilon} A_t \alpha k_t^{\alpha-1}\bar{h}^{1-\alpha}$$

$$w_t = mc_t y_{H,t}^{1/\epsilon}\left(A_t k_t^{\alpha}\bar{h}^{1-\alpha}\right)^{-1/\epsilon} A_t(1-\alpha)k_t^{\alpha}\bar{h}^{-\alpha}$$

$$p_t^G(1-s) = mc_t y_{H,t}^{1/\epsilon}\left(A_{e,t}e_t\right)^{-1/\epsilon} A_e(1-\zeta)\left(\frac{e_t}{e_t^G}\right)^{1-\xi}$$

$$p_t^B(1+\tau^e) = mc_t y_{H,t}^{1/\epsilon}\left(A_{e,t}e_t\right)^{-1/\epsilon} A_e\zeta\left(\frac{e_t}{e_t^B}\right)^{1-\xi}$$

$$\pi_{H,t}\left(\pi_{H,t}-\bar{\pi}_H\right) = \beta\mathbb{E}_t\left[\frac{\lambda_{t+1}}{\lambda_t}\pi_{H,t+1}\left(\pi_{H,t+1}-\bar{\pi}_H\right)\frac{y_{H,t+1}}{y_{H,t}}\right] + \frac{\varepsilon_t}{\kappa_{PH}}\left(\frac{mc_t}{p_{H,t}} - \frac{\varepsilon_t - 1}{\varepsilon_t}\right)$$

$$\mu_{A,t}Bn^{\phi-1}A_{t-1} = \mu_{A_e,t}B_e(1-n)^{\phi-1}A_{e,t-1}$$

$$\mu_{A,t} = \beta\frac{\lambda_{t+1}}{\lambda_t}(1+Bn_{t+1}^{\phi})\mu_{A,t+1} + mc_t y_{h,t}^{1/\epsilon}(A_t k_{t-1}^{\alpha}H^{1-\alpha})^{-1/\epsilon}k_{t-1}^{\alpha}H^{1-\alpha}$$

$$\mu_{A_e,t} = \beta\frac{\lambda_{t+1}}{\lambda_t}(1+B_e(1-n_{t+1})^{\phi})\mu_{A_e,t+1} + mc_t y_{h,t}^{1/\epsilon}A_{e,t}^{-1/\epsilon}e_t^{(\epsilon-1)/\epsilon}$$

A.1.3 Green energy producer

$$e_t^G = \Omega L^{1-\mu}[(1-\gamma)(s_t^G)^{\omega} + \gamma(s_t^{G,P})^{\omega}]^{(\mu/\omega)}$$

$$\Omega\mu[(1-\gamma)(s_t^G)^{\omega} + \gamma(s_t^{G,P})^{\omega}]^{(\mu/\omega)-1}(1-\gamma)\mu(s_t^G)^{\omega-1} = \frac{R_t^s}{P_t^G}$$

A.1.4 Brown energy sector

$$p_t^B = rer_t p_t^{B,*}$$

A.1.5 Government

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_R}\left[\left(\frac{\pi_t}{\bar{\pi}}\right)^{\phi_\pi}\left(\frac{y_t}{y}\right)^{\phi_y}\right]^{1-\rho_R}$$

$$\tau_t + \tau^e e_t^b + b_{t+1} = sp_t^G e_t^G + \frac{b_t}{\pi_t}R_{t-1} + i_t^{G,P}$$

$$\tau_t - \tau^* = \phi_\tau\left(b_t - \bar{b}\right) + \sigma^\tau\epsilon_t^\tau$$

A.1.6 Definitions

$$1 = \left[(1-\chi)(p_{H,t})^{1-\theta} + \chi(rer_t)^{1-\theta}\right]^{\frac{1}{1-\theta}}$$

$$nx_t = rer_t b^*_{t+1} - rer_t \frac{b^*_t}{\pi^*_t} R^*_{t-1} \Psi^A \left(\tilde{A}^f_t\right)$$

$$\pi_{H,t} = \frac{p_{H,t}}{p_{H,t-1}} \pi_t$$

$$y_{H,t} = (1-\chi) p_{h,t}^{-\theta} \left(c_t + i_t + s^G_t + i^{G,P}_t\right) + c^*_{H,t}$$

$$c^*_{H,t} = \left(\frac{P^*_{H,t}}{rer_t}\right)^{-\lambda^*} y^*_t$$

$$\frac{A_t}{A_{t-1}} = 1 + B n_t^{\phi}$$

$$\frac{A_{e,t}}{A_{e,t-1}} = 1 + B_e (1-n_t)^{\phi}$$

$$X_t = A_t k_t^{\alpha}$$

$$e^{B,*}_t = e^B_t - e^{B,d}_t$$

A.2 Stationarized equilibrium equations

Define X_{t-1} as the gdp trend, and define: $g_t = \frac{X_t}{X_{t-1}}$ as the growth rate. We assume variables at t are stationarized by X_{t-1}. For instance, $\tilde{c}_t = \frac{c_t}{X_{t-1}}$.

Define: $\tilde{\lambda}_t = \frac{\lambda_t}{X_{t-1}^{-\sigma}}$.

In this section, we present the stationarized equilibrium equations.

A.3 Equilibrium equations

A.3.1 Household

$$\tilde{\lambda}_t = \tilde{c}_t^{-\sigma}$$

$$g_t\tilde{k}_{t+1} = (1-\delta)\tilde{k}_t + \tilde{i}_t + \frac{\kappa_K}{2}\left(\frac{\tilde{k}_{t+1}g_t}{\tilde{k}_t} - \bar{g}\right)^2 \tilde{k}_t$$

$$\tilde{\lambda}_t = \beta g_t^{-\sigma}\mathbb{E}_t\left[\frac{\tilde{\lambda}_{t+1}R_t}{\pi_{t+1}}\right]$$

$$\tilde{\lambda}_t = \beta g_t^{-\sigma}\mathbb{E}_t\left[\tilde{\lambda}_{t+1}\frac{R_t^*}{\pi_t^*}\frac{rer_{t+1}}{rer_t}\Phi_{t+1}^A(\tilde{A}_{t+1}^f)\right]$$

$$\tilde{\lambda}_t q_t\left(1-\kappa_K\left(\frac{\tilde{k}_{t+1}g_t}{\tilde{k}_t} - \bar{g}\right)\right) = \beta g_t^{-\sigma}\mathbb{E}_t\left[\tilde{\lambda}_{t+1}r_{t+1}^k + \tilde{\lambda}_{t+1}q_{t+1}\left(1-\delta+\left(-\kappa_K\left(\frac{\tilde{k}_{t+2}g_{t+1}}{\tilde{k}_{t+1}} - \bar{g}\right)\frac{\tilde{k}_{t+2}g_{t+1}}{\tilde{k}_{t+1}} + \frac{\kappa_K}{2}\left(\frac{\tilde{k}_{t+2}g_{t+1}}{\tilde{k}_{t+1}} - \bar{g}\right)^2\right)\right)\right]$$

$$g_t\tilde{s}_{t+1}^G = (1-\delta)\tilde{s}_t^G + \tilde{i}_t^G + \frac{\kappa_S}{2}\left(\frac{\tilde{s}_{t+1}^G g_t}{\tilde{s}_t^G} - \bar{g}\right)^2 \tilde{s}_t^G$$

$$\tilde{\lambda}_t q_t^G\left(1-\kappa_S\left(\frac{\tilde{s}_{t+1}^G g_t}{\tilde{s}_t^G} - \bar{g}\right)\right) = \beta g_t^{-\sigma}\mathbb{E}_t\left[\tilde{\lambda}_{t+1}r_{t+1}^s + \tilde{\lambda}_{t+1}q_{t+1}^G\left(1-\delta+\left(-\kappa_S\left(\frac{\tilde{s}_{t+2}^G g_{t+1}}{\tilde{s}_{t+1}^G} - \bar{g}\right)\frac{\tilde{s}_{t+2}^G g_{t+1}}{\tilde{s}_{t+1}^G} + \frac{\kappa_S}{2}\left(\frac{\tilde{s}_{t+2}^G g_{t+1}}{\tilde{s}_{t+1}^G} - \bar{g}\right)^2\right)\right)\right]$$

$$p_{H,t}\tilde{y}_{H,t} = \tilde{c}_t + \tilde{i}_t + \tilde{i}_t^G + \tilde{i}_t^{G,P} + \frac{\kappa_P}{2}\left(\pi_t^H - \bar{\pi}_t^H\right)^2 p_{H,t}\tilde{y}_{H,t} + \tilde{nx}_t + \tilde{p}_t^B e_t^{B,*}$$

A.3.2 Intermediate goods producers

$$\tilde{y}_{H,t} = \left[\left(\tilde{A}_t\tilde{k}_t^\alpha\bar{h}^{(1-\alpha)}\right)^{\frac{\epsilon-1}{\epsilon}} + \left(\tilde{A}_{e,t}\tilde{e}_t\right)^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}}$$

$$\tilde{e}_t = \bar{E}\left[(1-\zeta)\left(\tilde{e}_t^G\right)^\xi + \zeta\left(\tilde{e}_t^B\right)^\xi\right]^{\frac{1}{\xi}}$$

$$r_t^k = mc_t\tilde{y}_{H,t}^{1/\epsilon}\left(\tilde{A}_t\tilde{k}_t^\alpha h^{1-\alpha}\right)^{-1/\epsilon}\tilde{A}_t\alpha\tilde{k}_t^{\alpha-1}h^{1-\alpha}$$

$$\tilde{w}_t = mc_t\tilde{y}_{H,t}^{1/\epsilon}\left(\tilde{A}_t\tilde{k}_t^\alpha h^{1-\alpha}\right)^{-1/\epsilon}\tilde{A}_t(1-\alpha)\tilde{k}_t^\alpha h^{-\alpha}$$

$$\tilde{p}_t^G(1-s) = mc_t \tilde{y}_{H,t}^{1/\epsilon}\left(\tilde{A}_{e,t}e_t\right)^{-1/\epsilon}\tilde{A}_{e,t}(1-\zeta)\left(\frac{\tilde{e}_t}{\tilde{e}_t^G}\right)^{1-\xi}$$

$$\tilde{p}_t^B(1+\tau^e) = mc_t \tilde{y}_{H,t}^{1/\epsilon}\left(\tilde{A}_{e,t}e_t\right)^{-1/\epsilon}\tilde{A}_{e,t}\zeta\left(\frac{\tilde{e}_t}{\tilde{e}_t^B}\right)^{1-\xi}$$

$$\pi_{H,t}\left(\pi_{H,t}-\bar{\pi}_H\right) = \beta\mathbb{E}_t\left[\frac{\lambda_{t+1}}{\lambda_t}\pi_{H,t+1}\left(\pi_{H,t+1}-\bar{\pi}_H\right)\frac{\tilde{y}_{H,t+1}}{\tilde{y}_{H,t}g_t}\right]+\frac{\varepsilon_t}{\kappa_{PH}}\left(\frac{mc_t}{p_{H,t}}-\frac{\varepsilon_t-1}{\varepsilon_t}\right)$$

$$\mu_{A,t}g_{t-1}^{\alpha}Bn^{\phi-1}A_{t-1} = \mu_{A_e,t}B_e(1-n)^{\phi-1}g_{t-1}^{\mu}A_{e,t-1}$$

$$\mu_{A,t} = \beta\frac{\tilde{\lambda_{t+1}}}{\tilde{\lambda}_t}g_t^{-\sigma}(1+Bn_{t+1}^{\phi})\mu_{A,t+1}g_t^{\alpha}+mc_t\tilde{y}_{h,t}^{1/\epsilon}(\tilde{A}_t\tilde{k}_{t-1}^{\alpha}H^{1-\alpha})^{-1/\epsilon}\tilde{k}_{t-1}^{\alpha}H^{1-\alpha}$$

$$\mu_{A_e,t} = \beta\frac{\tilde{\lambda}_{t+1}}{\tilde{\lambda}_t}g_t^{-\sigma}(1+B_e(1-n_{t+1})^{\phi})\mu_{A_e,t+1}g_t^{\mu}+mc_t\tilde{y}_{h,t}^{1/\epsilon}\tilde{A}_{e,t}^{-1/\epsilon}\tilde{e}_t^{(\epsilon-1)/\epsilon}$$

A.3.3 Green energy producer

$$\tilde{e}_t^G = \Omega L^{1-\mu}[(1-\gamma)(\tilde{s}_t^G)^{\omega}+\gamma(\tilde{s}_t^{G,P})^{\omega}]^{(\mu/\omega)}$$

$$\Omega\mu[(1-\gamma)(\tilde{s}_t^G)^{\omega}+\gamma(\tilde{s}_t^{G,P})^{\omega}]^{(\mu/\omega)-1}(1-\gamma)\mu(\tilde{s}_t^G)^{\omega-1} = \frac{r_t^s}{\tilde{p}_t^G}$$

A.3.4 Brown energy sector

$$\tilde{p}_t^B = rer_t\tilde{p}_t^{B,*}$$

A.3.5 Government

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_R}\left[\left(\frac{\pi_t}{\bar{\pi}}\right)^{\phi_\pi}\left(\frac{y_t}{y}\right)^{\phi_y}\right]^{1-\rho_R}$$

$$\tilde{\tau}_t+\tau^e\tilde{e}_t^b+\tilde{b}_{t+1}g_t = s\tilde{p}_t^G\tilde{e}_t^G+\frac{\tilde{b}_t}{\pi_t}R_{t-1}+\tilde{i}_t^{G,P}$$

$$\tau_t-\tau^* = \phi_\tau\left(b_t-\bar{b}\right)+\sigma^\tau\epsilon_t^\tau$$

A.3.6 Definitions

$$1 = \left[(1-\chi)(p_{H,t})^{1-\theta} + \chi(rer_t)^{1-\theta}\right]^{\frac{1}{1-\theta}}$$

$$\tilde{nx}_t = rer_t \tilde{b}^*_{t+1} g_t - rer_t \frac{\tilde{b}^*_t}{\pi^*_t} R^*_{t-1} \Psi^A \left(\tilde{A}^f_t\right)$$

$$\pi_{H,t} = \frac{p_{H,t}}{p_{H,t-1}} \pi_t$$

$$\tilde{y}_{H,t} = (1-\chi) p_{h,t}^{-\theta} \left(\tilde{c}_t + \tilde{i}_t + \tilde{s}^G_t + \tilde{i}^{G,P}_t\right) + \tilde{c}^*_{H,t}$$

$$\tilde{c}^*_{H,t} = \left(\frac{p^*_{H,t}}{rer_t}\right)^{-\lambda^*} \tilde{y}^*_t$$

$$\frac{A_t}{A_{t-1}} = 1 + B n_t^{\phi}$$

$$\frac{A_{e,t}}{A_{e,t-1}} = 1 + B_e (1-n_t)^{\phi}$$

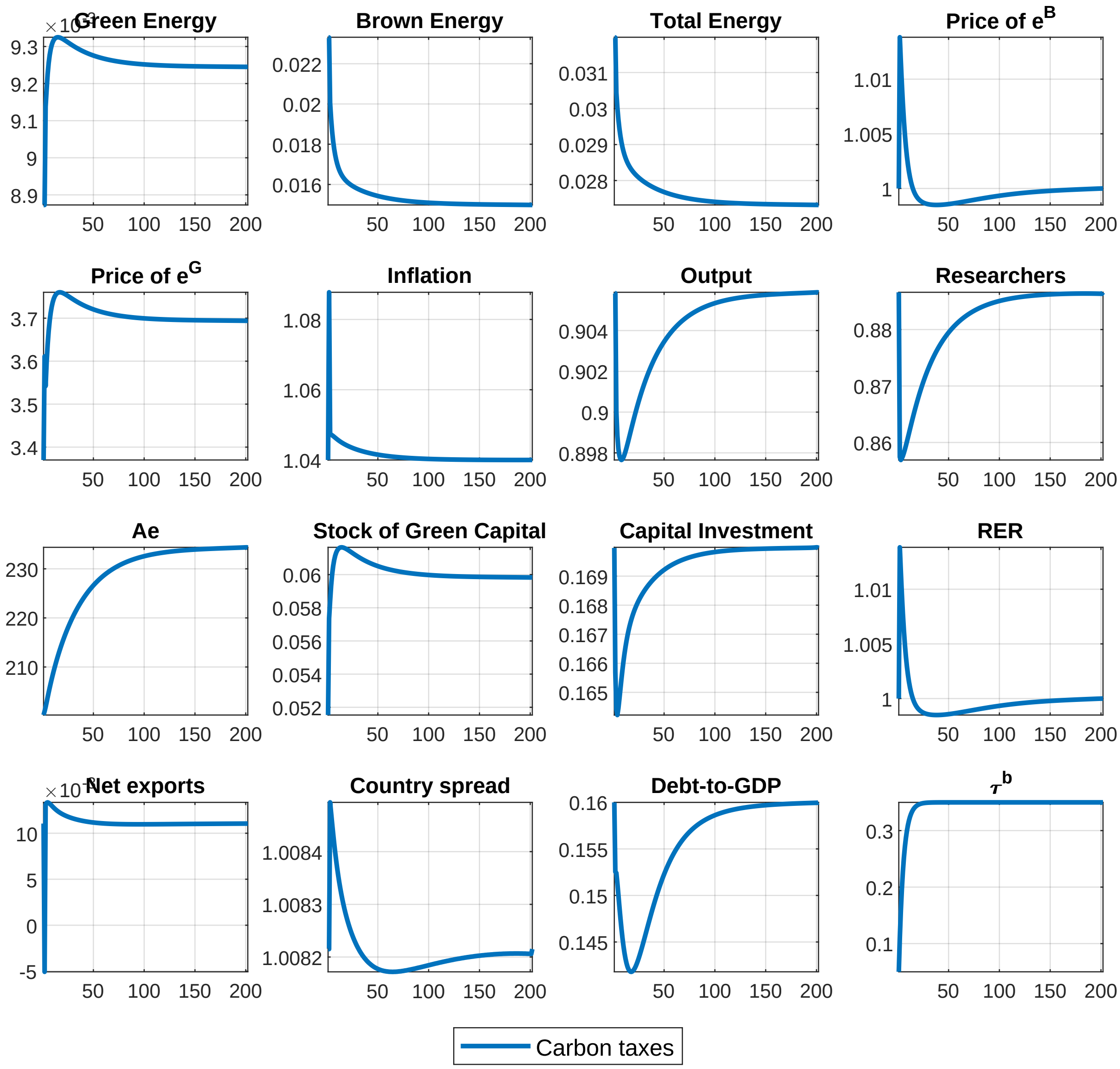
$$g_t = \tilde{A}_t \tilde{k}^{\alpha}_t$$

$$\tilde{e}^{B,*}_t = \tilde{e}^B_t - \tilde{e}^{B,d}_t$$

A.4 Transitional dynamics for the whole transition period

In the main text, we simulate a transition of 200 periods and show transitional dynamics for the first 40 years. However, some variables reach the final steady state at T=200. Although all policies are fully implemented within the 40 periods, some variables continue to adjust until they finally converge at T=200.

Figure A.1: Transitional Dynamics: 200 years

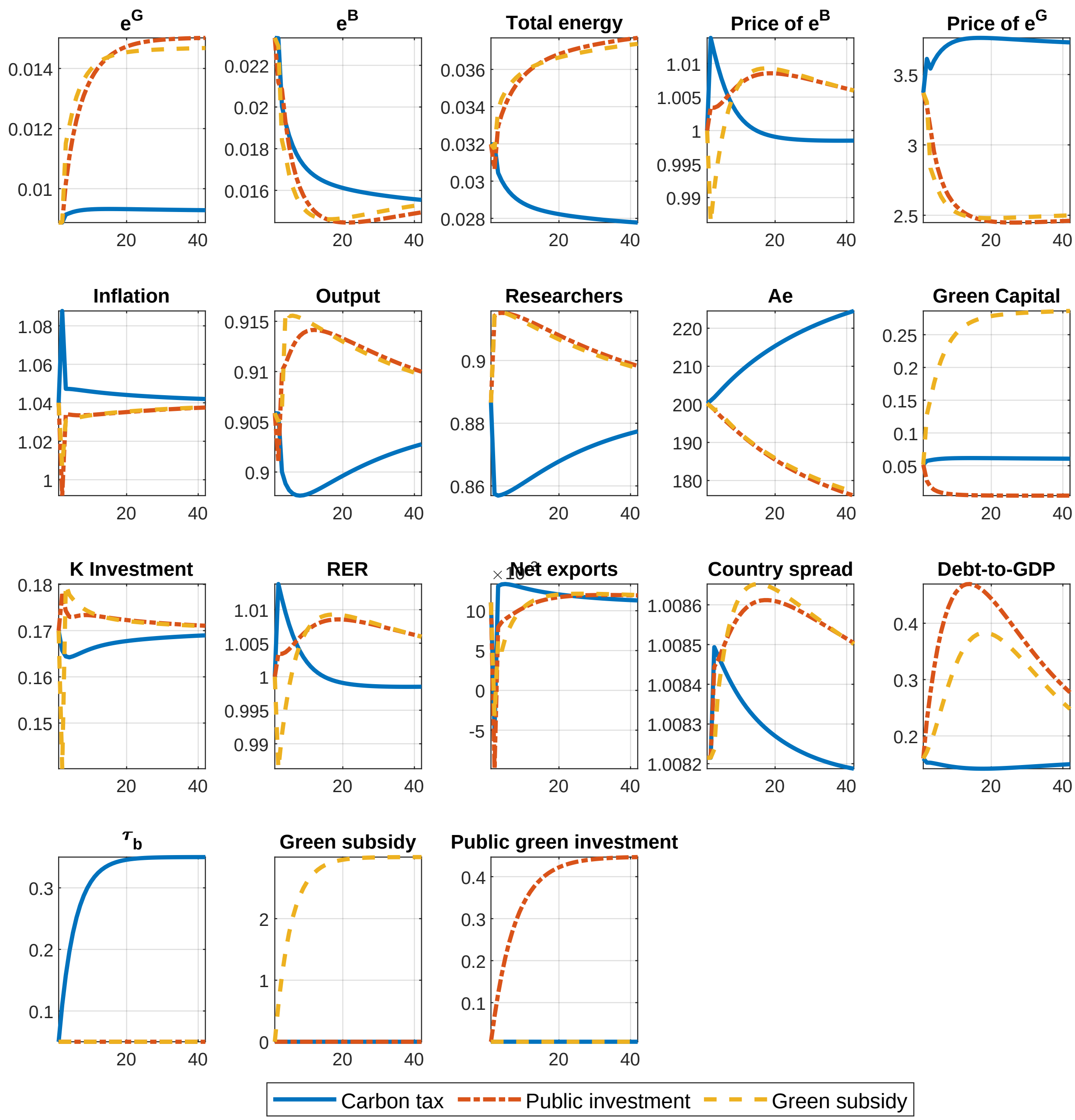


Note: Dynamics for the transition period from initial to a new steady state where carbon taxes increase from 5 to 35%.

A.5 Green subsidy increases of 300%

The next figure compares transitional dynamics for changes in carbon taxation from 5 to 35%, changes in green subsidies from 0 to 300% and changes in green public investment from 0 to 7% of GDP.

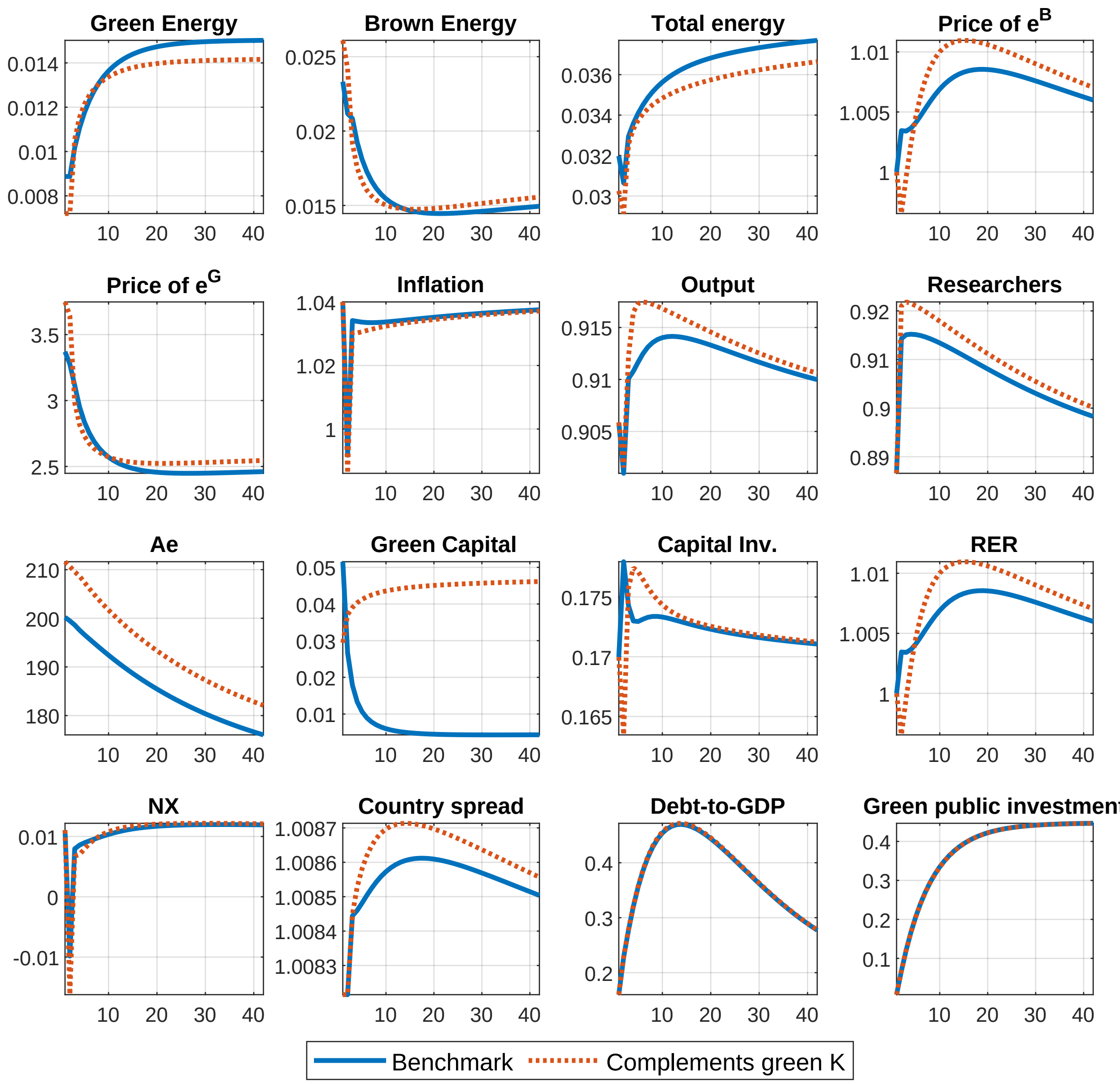
Figure A.2: Comparison of Transitional Dynamics when subsidies increase by 300%



Note: Dynamics for the first 40 years of transition from initial to a new steady state where carbon taxes increase from 5 to 35% (Blue continuous lines); subsidies change from 0 to 300% (red dashed lines) and public investment changes from 0 to 7% of GDP (yellow discontinuous line).

A.6 When green public and private capital are complements

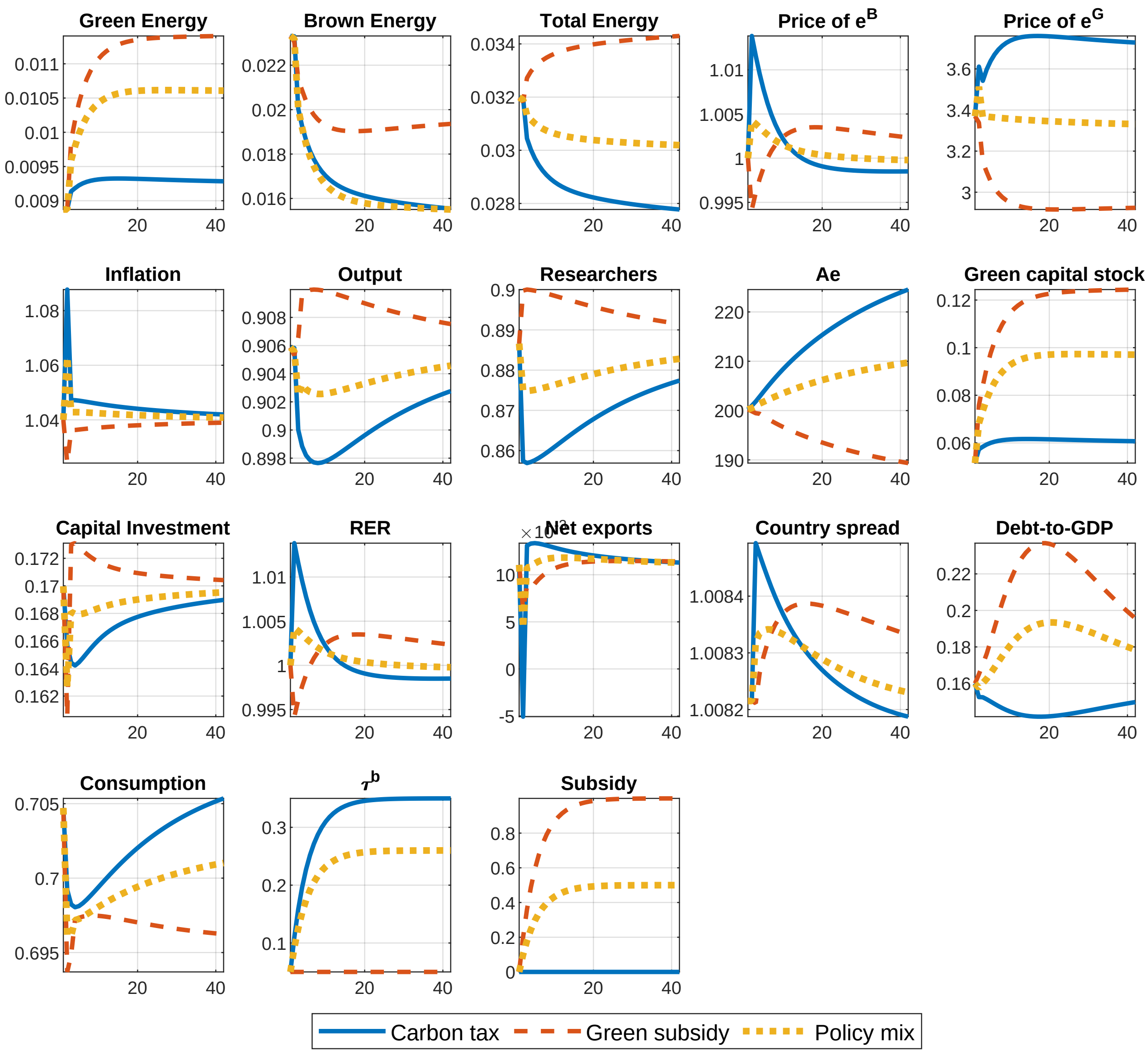
Figure A.3: Transitional Dynamics: Complementarity between private and public green capital



Note: Dynamics for the first 40 years of transition from initial to a new steady state where carbon taxes increase from 5 to 35%. Blue continuous lines show the transition with baseline calibration, and red dashed lines show the transition when $\omega = -0.1$.

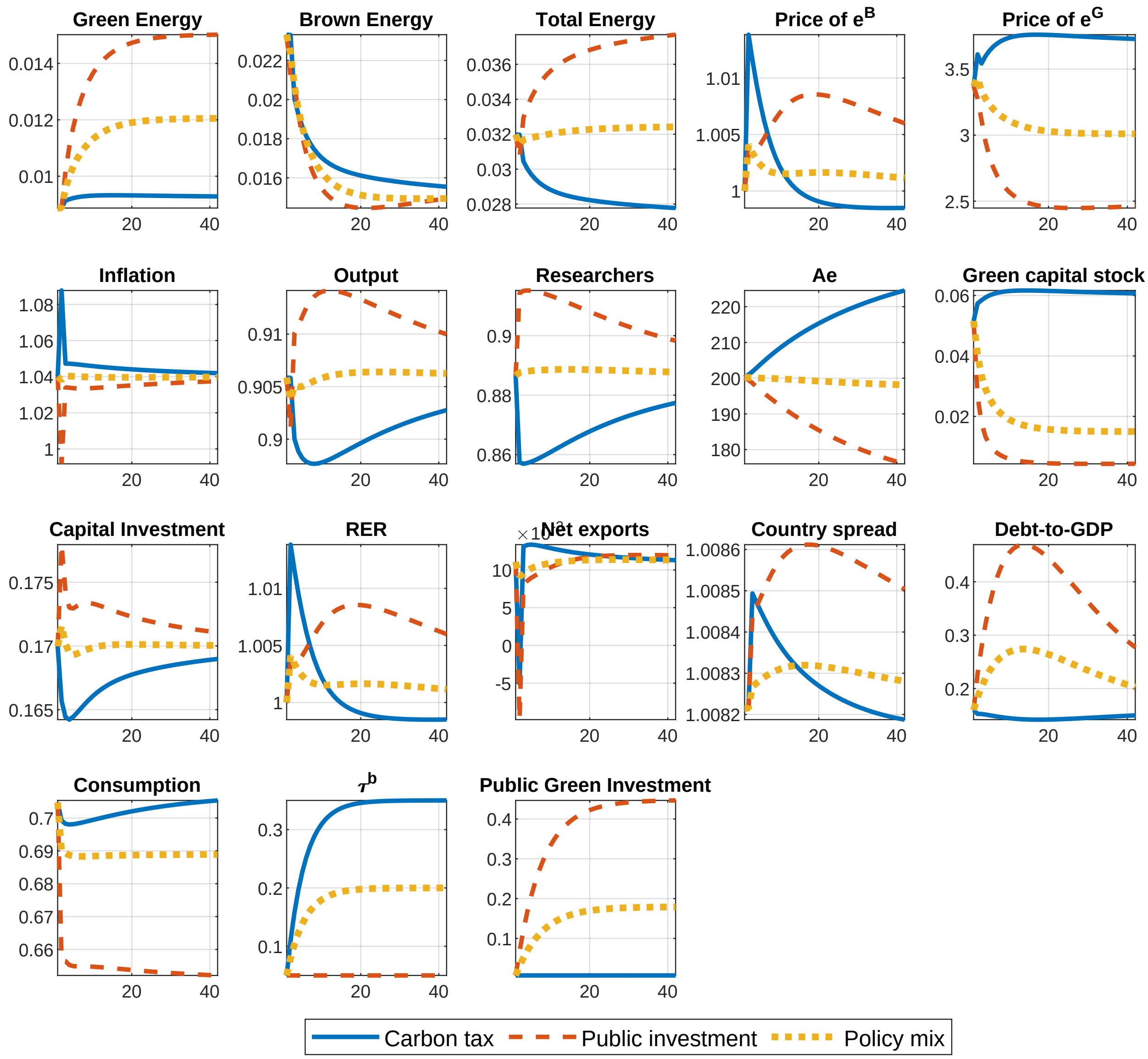
A.7 Policy mix transitions

Figure A.4: Transition tax and subsidy mix



Note: Dynamics for the first 40 years of transition from initial to a new steady state where carbon taxes increase from 5 to 35% (Blue continuous lines); subsidies change from 0 to 100% (red dashed lines) and policy mix with taxes changing from 5 to 25% and subsidies from 0 to 40% (yellow discontinuous lines).

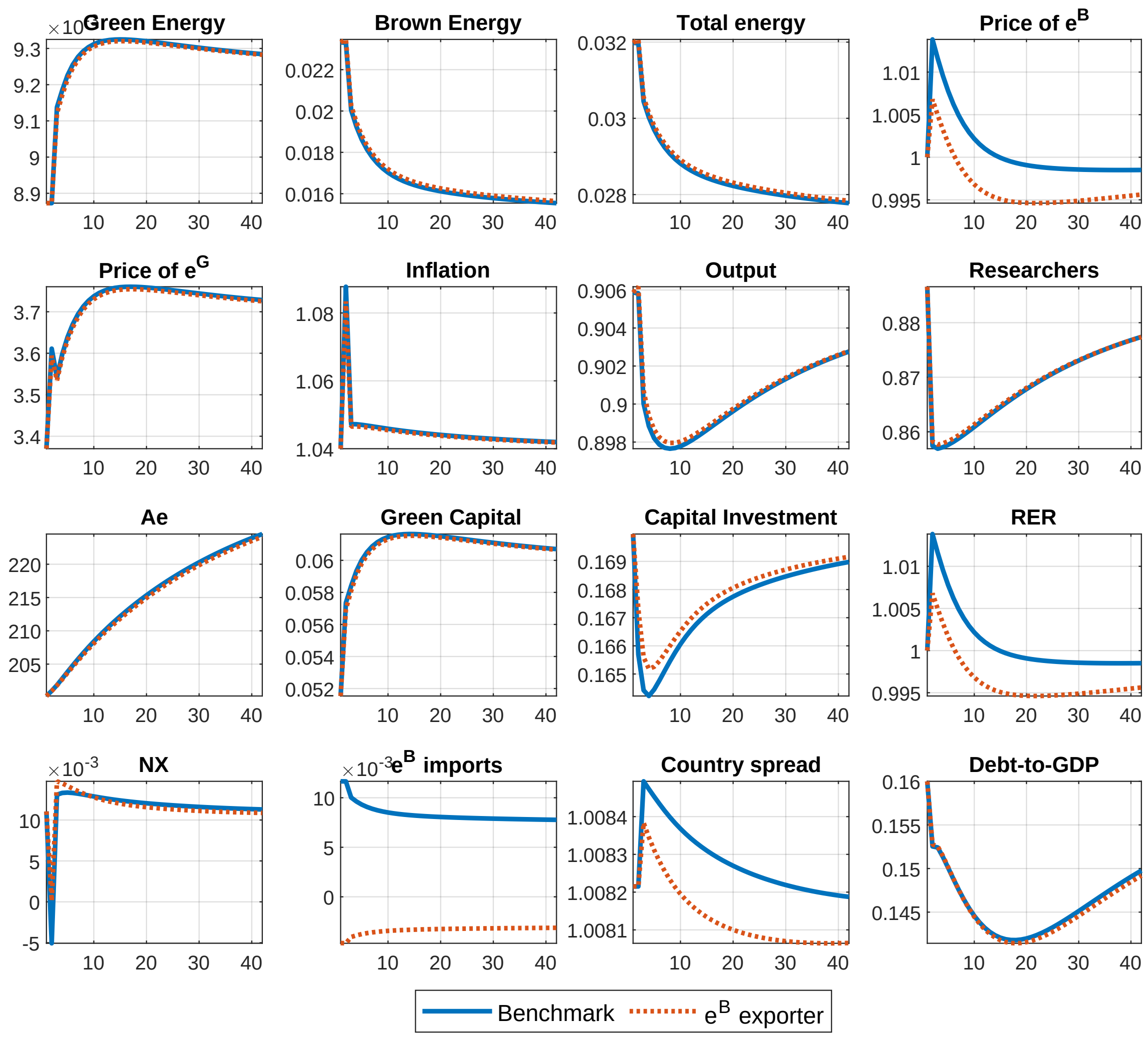
Figure A.5: Transition tax and green capital mix



Note: Dynamics for the first 40 years of transition from initial to a new steady state where carbon taxes increase from 5 to 35% (Blue continuous lines); public green investment changes from 0 to 7% (red dashed lines) and policy mix with taxes changing from 5 to 15% and subsidies from 0 to 2.8% (yellow discontinuous lines).

A.8 The case of an energy exporter

Figure A.6: Transitional Dynamics: Energy exporter



Note: Dynamics for the first 40 years of transition from initial to a new steady state where carbon taxes increase from 5 to 35%. Blue continuous lines show the transition with baseline calibration, and red dashed lines show the transition when the SOE is calibrated as a net brown energy exporter.