Macroeconomics A

Lecture 1 - Growth and Development Accounting & the Solow Model

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EI056 Macroeconomics A: Organization

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- Office hours: TBA
- I will upload course material (slides, readings) to Moodle

Prerequisites & Assessment

Prerequisites:

- Calculus and Statistics
- Constrained and unconstrained optimization techniques
- Basic regression analysis
- Intermediate micro, intermediate macro
- Some knowledge of programming (Matlab, R, Python, etc) would help you make the most of the course

Assessment:

- Two hand-in problem sets (25% each), one final exam in the last week of the course (50%)

What is Macroeconomics?

L. Robbins (1932) on economics (overall):

Economics is the science which studies human behaviour as a relationship between ends and scarce means which have alternative uses.

So what is Macro? My answer:

- The study of the mechanisms by which economic *aggregates* are determined (e.g. income, wealth, consumption of large groups of agents)

How large do these groups of agents need to be?

- At least as large as cities — probably even larger.

Note that getting those mechanisms right often requires a good understanding of the *microeconomic* mechanisms. This has become increasingly important.

Course content

Long-run macro (growth), 2-3 sessions:

- Why are some countries rich and some are poor?
- What determines the growth rate of an economy?
- How can we stimulate growth?

Short-run macro (business cycles), 9-10 weeks:

- Why do we have booms and recessions? What should we do to smooth out fluctuations?
- Theories of the labor market, goods market
- Monetary theory and policy
- Amplification mechanisms, in particular through financial markets
- Consumption and Risk

Economic Growth

Perhaps *the* biggest question in economics:

Why are some countries so much richer than other countries?

Empirical questions:

- When did income differences emerge?
- What are the empirical correlates of income differences?

Theoretical questions:

- Under what conditions does and economy feature economic growth?
- How should we model technological progress?

This week and next week: look at most important data and canonical growth models:

- Basic neoclassical growth models (Solow, Ramsey)
- Endogenous growth models

What are we talking about? a two-slide primer on national accounts & terminology

- Most basic "window" into the economy is through national accounts (developed in particular by Colin Clark, Simon Kuznets, Richard Stone).
- "Gross domestic product": total *value* of all goods and services produced within a country:

$$Y = \sum_{\text{productive units } j} VA_j = \sum_{\text{productive units } j} (Y_j - M_j)$$

where VA_j is value added of productive unit j (typically firms, but could be informal firms or households too), Y_j is the value of the output produced (at market prices), M_j are intermediate inputs that are being consumed as part of the production process. This identity is the basis of the **production approach** to measuring GDP.

- Everything that is being produced and is valuable finds a use somewhere. This is the basis for the second identity:

$$Y = C + I + G + (X - I)$$

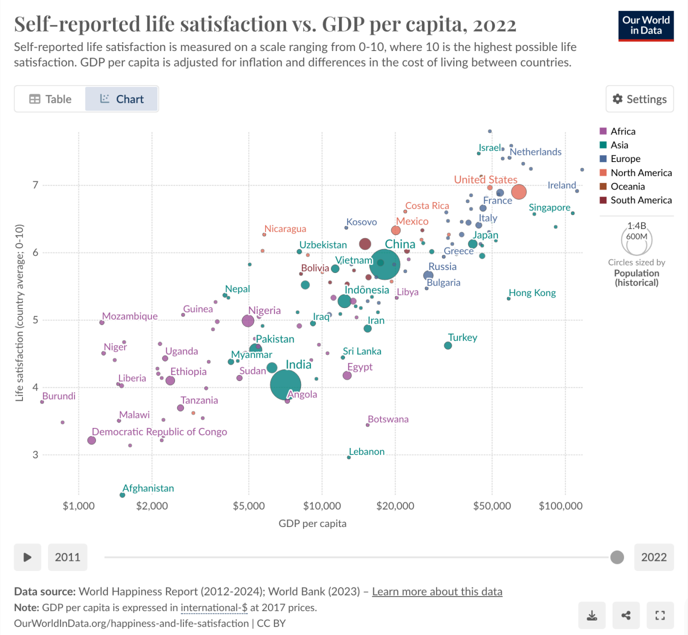
with Y gross nominal GDP, C (private) consumption, I investment, G government (public) consumption, X exports, I imports. This is the basis for the **expenditure approach** to measuring GDP.

- Everything that is being produced and is valuable must be owned by someone. Assume we consider two *factors* of production, capital and labor:

$$Y = wL + rK$$

where wL is labor income, and rK is capital income. This is the basis of the **income approach** to measuring GDP.

Is high GDP per capita a good thing?



GDP/capita and life expectancy

GDP/capita is obviously not welfare. But it's strongly correlated with other measures of welfare. Here: life expectancy

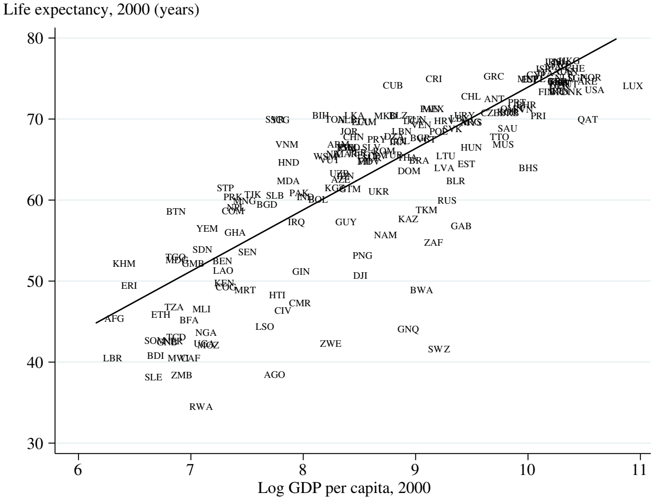
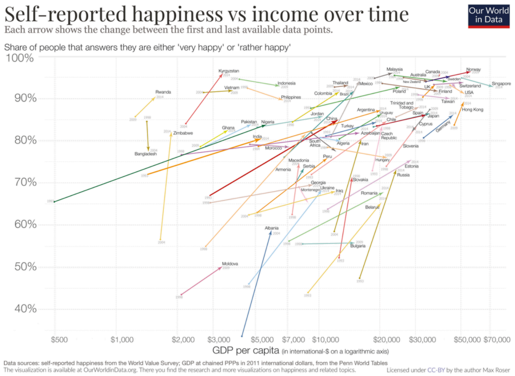


FIGURE 1.6 The association between income per capita and life expectancy at birth in 2000.

GDP per capita and happiness, #2

- The cross-sectional patterns are very robust. The time-series relationship (does happiness go up when gdp per capita goes up?) is debated.
- The "Easterlin Paradox" argued that there is no positive relationship in the time dimension, based on some older data in the US and Japan. Here is data from the World Values Survey:



- At shorter horizons: self-reported wellbeing is lower in recessions (Wolfers, 2003), similar for average self-reported health
- Unemployment (in particular female unemployment) and economic hardship is associated with domestic violence (Anderberg et al, 2015, Schneider et al., 2016)
- Pierce and Schott (2020) show impact of trade liberalization shock on mortality of blue collar workers in the US

Bottom line: economic wellbeing is an important part of overall wellbeing. Of course it's not the only determinant of wellbeing.

(Keep this in mind for the rest of the course. Also think about it when you hear people talking about "degrowth")

Huge income differences across countries

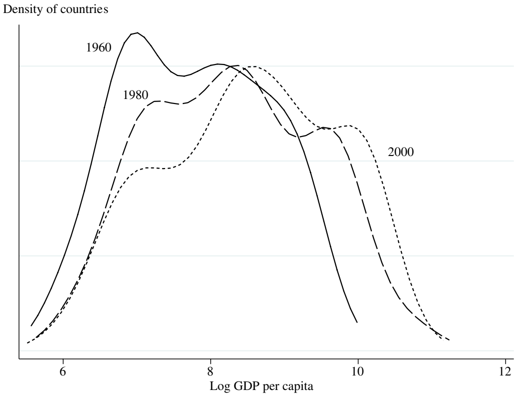


FIGURE 1.2 Estimates of the distribution of countries according to log GDP per capita (PPP adjusted) in 1960, 1980, and 2000.

- The US are 30 times as rich as Nigeria
- Based on some measures of inequality, inequality across countries has increased between 1960 and 2000.

Note: income $\neq$ value of domestic production, because you can get income from abroad.

But for most countries, the difference is small.

Income differences across countries, weighted by population

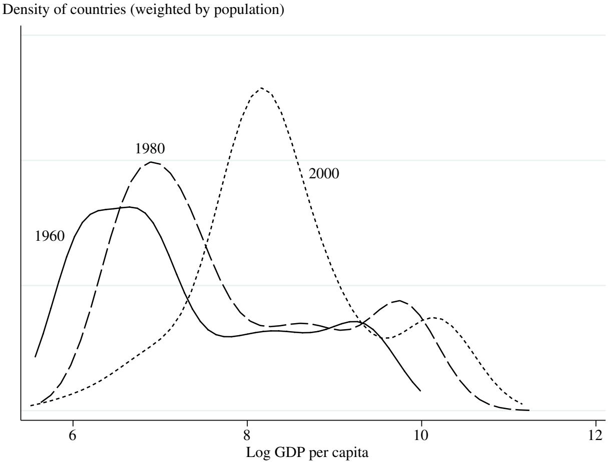


FIGURE 1.3 Estimates of the population-weighted distribution of countries according to log GDP per capita (PPP adjusted) in 1960, 1980, and 2000.

- … but some formerly very poor countries have grown much faster (notably, India and China)

Origins of Income Differences?

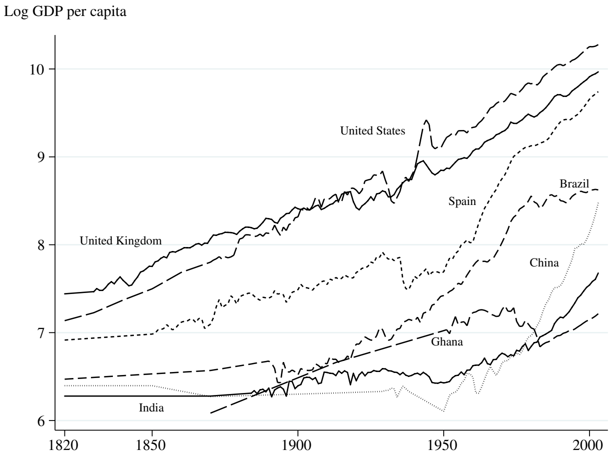


FIGURE 1.12 The evolution of income per capita in the United States, the United Kindgom, Spain, Brazil, China, India, and Ghana, 1820–2000.

- Rich countries are rich because they started to grow at an earlier point in time

Origins of Income Differences?

Over the very long run:

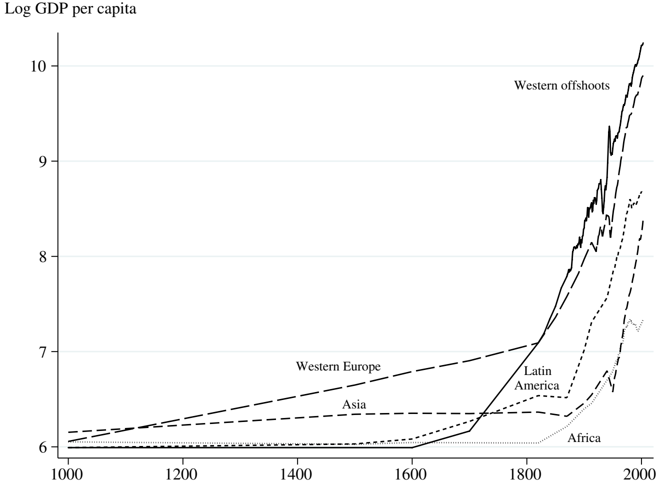


FIGURE 1.11 The evolution of average GDP per capita in Western offshoots, Western Europe, Latin America, Asia, and Africa, 1000–2000.

Understanding Modern Growth

- Important distinction: do we want to understand the *proximate* or the *fundamental* reasons for economic growth?
- Proximate reasons: US is richer than Nigeria because
  - it has more capital, skills? (higher levels of production factors)
  - better infrastructure? (lower transaction costs)
  - higher investment/schooling rates? (faster accumulation of factors)
- Fundamental reasons: *Why* is this the case?
  - Geography?
  - Institutions?
  - Culture?
  - History?

Proximate causes: investment rate?

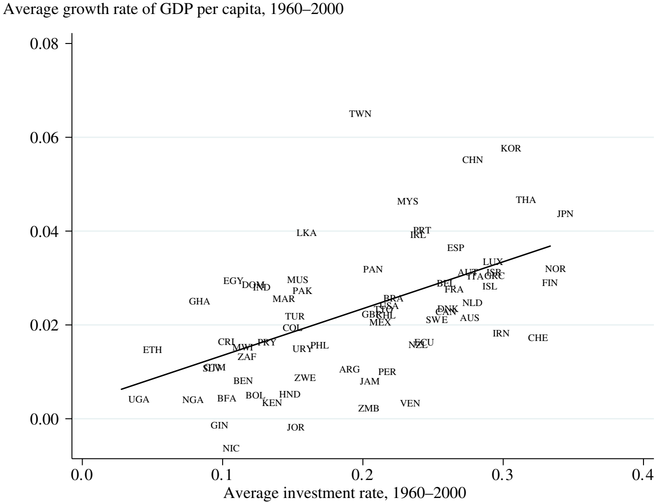


FIGURE 1.15 The relationship between average growth of GDP per capita and average growth of investments to GDP ratio, 1960–2000.

Countries that have had higher investment rates had on average also higher average GDP growth rates

Proximate causes: schooling?

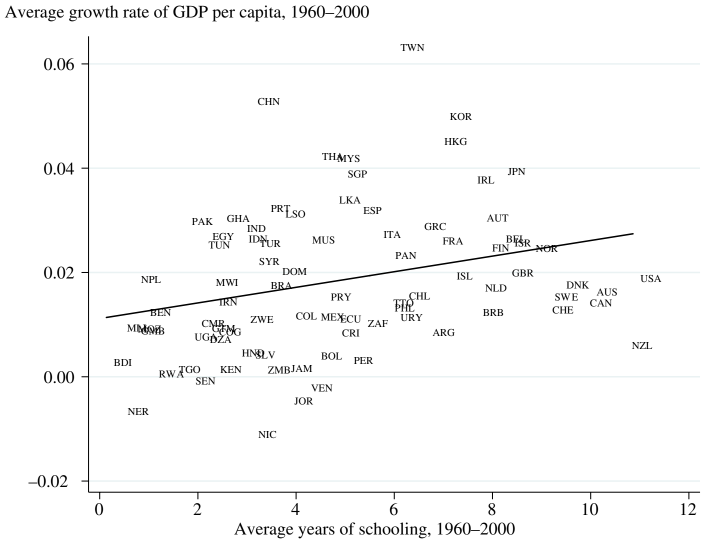


FIGURE 1.16 The relationship between average growth of GDP per capita and average years of schooling, 1960–2000.

The Solow model

- First general equilibrium model of growth
- Mechanism:
  - Firms: produce output using capital
  - Consumers: save fraction of their income $\rightarrow$ future capital $\rightarrow$ produce $\rightarrow$ save $\rightarrow$ ...
- Result: a theory of growth
- Questions we can ask:
  - Is there sustained growth in the long run, or does growth run out of steam?
  - What is the relationship between growth and investment/saving?
  - Can compare the growth experience of countries:
    - Do countries differ in the parameters that underlie growth?
    - Or are countries just at different points of a common trajectory?

Assumptions

- Time is discrete: $t = 0, 1, 2, \ldots$
- Closed economy, a single good is used both as consumption good and as capital input to production
- Two actors in the economy:
  - Representative firm: uses labor and capital (rented from households) for production
  - Representative household:
    - receives labor and capital income, and profits from owning the firms $(= 0)$
    - consumes and saves in the form of capital
  - Three markets: labor market, capital market, market for the final good

Households

- Households do not optimize ($\rightarrow$ main difference to Ramsey model)
- Receive income:

$$Y(t) = \underbrace{w(t)L^s(t)}_{\text{labor inc}} + \underbrace{R(t)K^s(t)}_{\text{capital inc}} + \underbrace{\Pi(t)}_{\text{profits}}$$

- Consume and save according to a simple rule of thumb ("Keynesian consumption function"):

$$S(t) = sY(t) \quad \text{and} \quad C(t) = (1-s)Y(t)$$

  i.e. they save a fraction s of their income, a fixed parameter
- No preferences specified! Hence, cannot make welfare statements
- In Ramsey model, we'll add an optimal consumption decision

Technology

- Firms produce according to the the production function

$$Y(t) = F(K_t), L(t), A(t))$$

  where L and K are labor and capital, and A is technology (a free publicly available, non-rival and non-excludable production shifter)
- $F(\cdot)$ is a *neoclassical production function*, i.e.
  - has constant returns to scale in K and L,

$$\lambda F(K, L) = F(\lambda K, \lambda L)$$

  - has positive and diminishing marginal returns to K and L,

$$\frac{\partial F}{\partial K} > 0, \quad \frac{\partial F^2}{\partial^2 K} < 0$$

    and likewise for L
  - satisfies the *Inada conditions*:

$$\lim_{K \to 0} F_K(K, L, A) = \infty, \quad \lim_{K \to \infty} F_K(K, L, A) = 0$$

    and likewise for L.

Market structure, endowments, market clearing

- All markets are perfectly competitive
- Labor market clearing: endowment of labor (inelastically supplied) equals the labor demanded by firms:

$$L^s(t) = L^d(t) =: L(t)$$

- Capital market clearing: supply of capital (stock of accumulated capital) equals firms' demand for capital:

$$K^s(t) = K^d(t) =: K(t)$$

- Goods market clearing: total supply (production) equals consumption plus investment:

$$Y(t) = C(t) + I(t)$$

- **Law of motion for capital**:

$$K(t+1) = (1-\delta)K(t) + I(t)$$

  δ is the depreciation rate, the fraction of capital that depreciates every period. $K(0)$ is the (exogenous) initial capital endowment.

Firm's profit maximization problem

- Static problem (why?):

$$\max_{K,L} F(K, L, A(t)) - w(t)L(t) - R(t)K(t)$$

  First-order optimality requires:

$$R(t) = F_K(K, L, A) \quad \text{and} \quad w(t) = F_L(K, L, A)$$

- From the fact that F has CRS,

$$F(K, L, A) = F_K(K, L, A)K + F_L(K, L, A)L$$

  (proof: differentiate $\lambda F(K, L, A)$ wrt λ and set $\lambda = 1$), we get that profits are zero:

$$\Pi = F(K, L, A) - R(t)K - w(t)L = 0$$

Equilibrium in the Solow Model

An equilibrium in the Solow model is a series of price $\{w(t), R(t)\}_t$ and quantities $\{C(t), S(t), Y(t), K(t), L(t)\}_t$ given $\{A(t), L(t)\}_t$ and $K(0)$ such that

- Firms maximize profits given prices
- Factor markets clear
- $C(t) = (1-s)Y(t)$ and $S(t) = sY(t)$ for all $t \geq 0$
- Capital evolves according to $K(t+1) = (1-\delta)K(t) + S(t)$

Fundamental Law of Motion

- From the law of motion for the capital stock:

$$
\begin{aligned}
K(t+1) &= (1-\delta)K(t) + I(t) && (1)\\
&= (1-\delta)K(t) + sY(t) && (2)\\
&= (1-\delta)K(t) + sF(K(t), L(t), A(t)) && (3)
\end{aligned}
$$

- The equilibrium of the Solow model is described by this non-linear difference equation and the paths of $A(t), L(t)$, and $K(0)$.

The canonical Solow model

- Extra assumptions: no population growth, no technological progress:

$$L(t) = L \quad \text{and} \quad A(t) = A$$

- Transform everything into per-capita terms: $k(t) := K(t)/L(t)$

$$\begin{aligned}
k(t+1) = \frac{K(t+1)}{L(t+1)} &= (1-\delta)\frac{K(t)}{L(t)} + s\frac{F(K(t), L, A)}{L(t)} && (4) \\
&= (1-\delta)k(t) + sF(k(t), 1, A) && (5) \\
&= (1-\delta)k(t) + sf(k(t)) && (6)
\end{aligned}$$

where $f(k(t) := F(K(t)/L(t), 1, A)$ and $k(0) = K(0)/L(0)$.

The canonical Solow model

Other equilibrium variables:

- Output per capita:

$$y(t) := \frac{Y(t)}{L(t)} = \frac{F(K(t), L, A)}{L} = f(k(t))$$

- Rental rate of capital:

$$R(t) = F_K(K(t), L, A) = f'(k(t))$$

- Wage rate:

$$w(t) = F_L(K(t), L, A) = f(k(t)) - f'(k(t))k(t)$$

- **Important property of CRS production functions:** marginal product of capital and labor only depend on the capital-labor ratio

Dynamic evolution of the economy

- We characterized everything in terms of the unique state variables $k(t)$, i.e.
$$k(t+1) = (1-\delta)k(t) + sf(k(t))$$
with $k(0)$ given.
- This difference equation implies a sequence $\{k(t)\}_{t=0}^{\infty}$
- Questions:
  - Is there a steady state k^* such that $k(t) = k^* \to k(t+1) = k^*$?
  - Is the system locally stable, i.e. if $|k(0) - k^*| < \varepsilon$ then $k(t) \to k^*$ as $t \to \infty$?
  - Is the system globally stable, i.e. $k(t) \to k^*$ for all $k(0)$?

The steady state of the Solow model

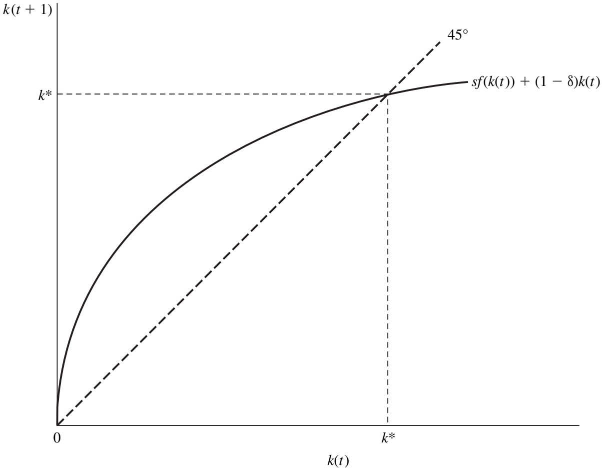


FIGURE 2.2 Determination of the steady-state capital-labor ratio in the Solow model without population growth and technological change.

- Two steady states, but ignore the one at $k(t) = 0$ (not interesting)

Steady-state analysis

- A steady state k^* satisfies

$$\underbrace{\delta k^*}_{\text{Depreciation}} = \underbrace{sf(k^*)}_{\text{Investment}}$$

- Hence, k^* defined by

$$\frac{f(k^*)}{k^*} = \frac{\delta}{s}$$

- Results: k^* exists and is unique (apart from $k^* = 0$) since (L'Hopital's rule)

$$\lim_{k\to 0} \frac{f(k)}{k} = \lim_{k\to 0} f'(k) = \infty$$

$$\lim_{k\to \infty} \frac{f(k)}{k} = \lim_{k\to \infty} f'(k) = 0$$

$$\frac{\partial f(k)/k}{\partial k} = \frac{f'(k)k - f(k)}{k^2} = -\frac{w(t)}{k^2} < 0$$

A different representation

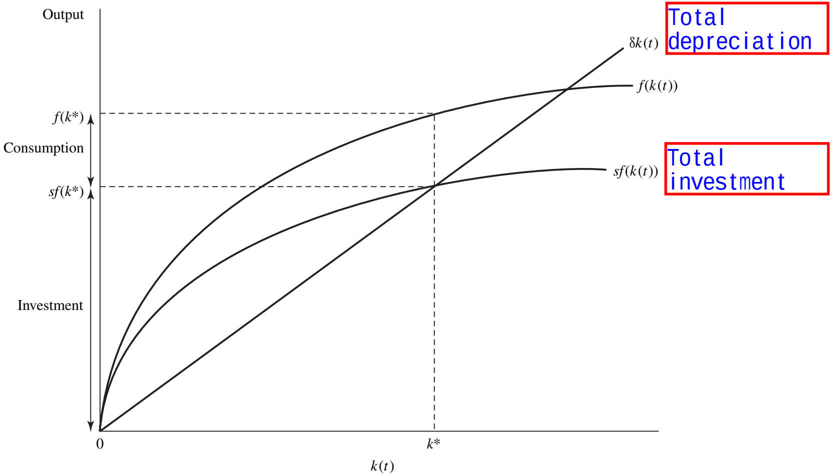


FIGURE 2.4 Investment and consumption in the steady-state equilibrium.

- In the SS, the amount of actual investment is exactly such that it compensates for depreciation ($\rightarrow$ "break-even" investment)

Things that could go wrong

... if F was not a neoclassical production function

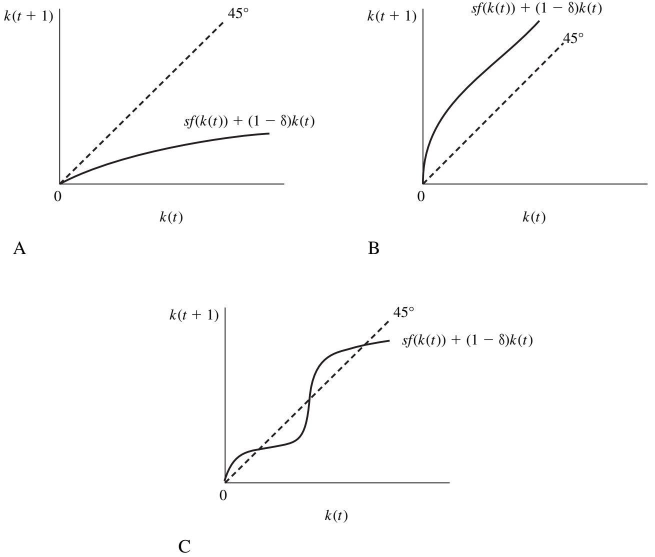


FIGURE 2.5 Examples of nonexistence and nonuniqueness of interior steady states when Assumptions 1 and 2 are not satisfied.

Comparative statics

- k^* satisfies
$$\frac{f(k^*(s,\delta,A),A)}{k^*(s,\delta,A)} = \frac{\delta}{s}$$
- Differentiation yields the comparative statics:
$$\frac{\partial k^*(s,\delta,A)}{\partial \delta} < 0$$
$$\frac{\partial k^*(s,\delta,A)}{\partial s} > 0$$
$$\frac{\partial k^*(s,\delta,A)}{\partial A} > 0$$
- Note that higher productivity A has "multiplier" effect on output (direct, and via k^*)

Steady-state consumption

- Steady-state consumption per capita:

$$c^* = c(k^*(s, \delta, A)) = (1 - s)f(k^*(s, \delta, A))$$

- s has two opposing effects
  - higher $s \Rightarrow$ higher $k^* \Rightarrow$ higher y^*
  - higher $s \Rightarrow$ lower $1 - s \Rightarrow$ lower fraction of output consumed
- Let $\hat{s}$ be the savings rate that maximizes consumption, i.e.

$$\hat{s} = \arg\max_s \{(1-s)f(k^*(s, \delta, A)\} = \arg\max_s \{f(k^*(s, \delta, A) - \delta k^*(s, \delta, A)\}$$

- Hence $\hat{s}$ implicitly defined by

$$f'(k^*(\hat{s}, \delta)) = \delta$$

- $\hat{s}$ is called the *golden rule saving rate* and $k^{\text{gold}} = f'^{-1}(\delta)$ is the golden rule capital stock
  - Savings are excessive if depreciation is higher than the marginal product of capital
  - By saving less, consumption would go up.
  - If $k > k^{\text{gold}}$, the economy is "dynamically inefficient"

Transitional dynamics: global stability

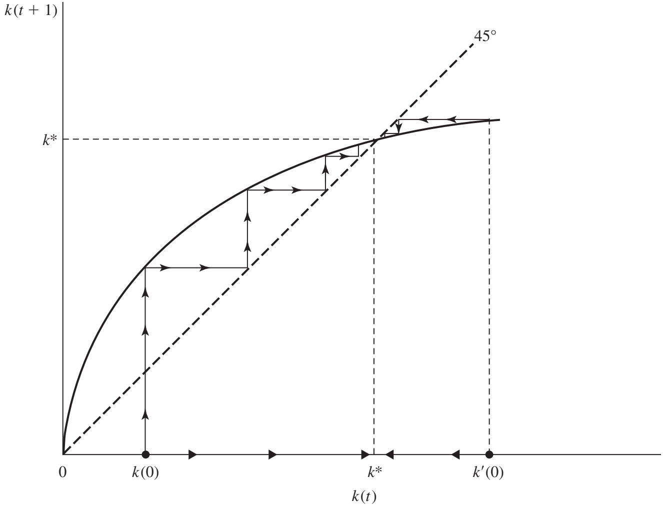


FIGURE 2.7 Transitional dynamics in the basic Solow model.

- Best way to show stability: Banach fixed point theorem

Growth in the Solow model

- Dynamics fully determined by the evolution of capital:

$$g_k(t) = \frac{k(t+1) - k(t)}{k(t)} = \frac{sf(k(t)) - \delta k(t)}{k(t)} \tag{7}$$

$$= s\frac{f(k(t))}{k(t)} - \delta = g_k(k(t)) \tag{8}$$

- Implications:
  - No growth in the long run: as $k \to k^*$, $y \to y^*$
  - Growth during transition, i.e. as long as $k(t) < k^*$
  - Growth rate is declining over time

Does this match up with reality?

Nicholas Kaldor (1958) characterized modern growth (i.e. post-industrial revolution growth) by the following facts:

1. The rate of growth of the capital stock per worker is roughly constant over long periods of time
2. The rate of growth of output per worker is roughly constant over long periods of time
3. The capital/output ratio is roughly constant over long periods of time
4. The rate of return to capital is roughly constant over long periods of time
5. The capital and labor shares in national income are roughly constant over long periods of time
6. The real wage grows over time

"Fact" 4 seems to be inconsistent with more recent evidence, and "Fact" 5 is contested for the last 30 years (see Grossman and Oberfield, 2022)

Augmented Solow model

- Change baseline Solow model by assuming positive constant (and exogenous) population growth and technological progress
- But to ensure that we match the Kaldor facts, we need labor-augmenting technology:

$$F(K(t), L(t), A(t)) = F(K(t), A(t)L(t))$$

- Assume continuous time (makes life a bit easier)

$$\dot{L}(t)/L(t) = n$$

$$\dot{A}(t)/A(t) = g$$

with $n > 1$, $g > 1$.

- The dynamic equation for capital in continuous time is

$$\dot{K}(t) = sF(K(t), A(t)L(t)) - \delta K(t)$$

- Define

$$k(t) = \frac{K(t)}{A(t)L(t)}$$

and take the log derivative w.r.t. time

$$\frac{\dot{k}(t)}{k(t)} = \frac{\dot{K}(t)}{K(t)} - g - n$$

- Output per efficiency unit of labor is

$$\hat{y}(t) := \frac{Y(t)}{A(t)L(t)} = F\left(\frac{K(t)}{A(t)L(t)}, 1\right) =: f(k(t))$$

- Hence:

$$\frac{\dot{k}(t)}{k(t)} = \frac{sF(K(t), A(t)L(t)) - \delta K(t)}{K(t)} - g - n$$
$$= \frac{sF(K(t), A(t)L(t))}{K(t)} - (\delta + g + n)$$
$$= \frac{sf(k(t)}{k(t)} - (\delta + g + n)$$

  and therefore

$$\dot{k}(t) = sf(k(t)) - (\delta + g + n)k(t)$$

- Same structure as in the baseline model, with effective depreciation rate $\delta + g + n$.

Balanced growth and steady state

- Steady state is given by

$$\frac{f(k^*)}{k^*} = \frac{\delta + g + n}{s}$$

- Existence, uniqueness, and stability properties as before
- In this steady state:
  - k is constant at k^*, hence $K(t)/L(t)$ grows at rate g
  - Y/L grows at rate g:

$$y(t) = \frac{Y(t)}{L(t)} = A(t)f(k^*)$$

  - Wages grow at rate g:

$$w(t) = A(t)(f(k(t)) - f'(k(t))k(t))$$

  - Hence $Y(t)$ grows at rate $g + n$
- Balanced growth path

Summary of the Solow Model

- Baseline model:
  - No long-run growth, i.e. capital accumulation alone does not make the economy grow in the long run
  - Conditional convergence: if countries have the same parameters s, δ, A and the same production function F, they will converge to the same steady state. Countries further from the steady state will grow faster.
  - Crucial mechanism: decreasing marginal product of capital
- Extended model:
  - Long-run growth possible if productivity grows: $g > 0$
  - Is consistent with the Kaldor facts if $g > 0$
- But: s and g, which are the most interesting, are exogenous
- Simple theory which can be confronted with data (Mankiw, Romer, and Weil, 1992)

Development Accounting

Development Accounting

- *Development accounting* asks whether the factors of production can explain income *levels* (see Caselli 2005)
- Use Cobb-Douglas production function in physical and human capital ($\rightarrow$ more general than labor):

$$Y_j = A_j K_j^{\alpha} (L_j h_j)^{1-\alpha} \qquad \text{or}$$

$$y_j = F(k_j, h_j, A_j) = A_j k_j^{\alpha} h_j^{1-\alpha} =: A_j y_j^{KH}$$

  Hence: for some α we can use observed k and h to ask how well we can predict y_j
- Caselli's measure of success:

$$\text{success} = \frac{Var(\log(y_j^{KH}))}{Var(\log(y_j))}$$

- If all countries had the same technology A_j, then success $= 1$

Measurement

- In growth/development regressions, people typically use the Penn World Tables (Heston, Summers, and Aten)
  - This includes values and deflators for output and capital, and the labor force
  - Capital stock constructed from investment (observed) and depreciation (calibrated) and law of motion ("perpetual inventory method")
- How to measure human capital? Not directly observable. But: can see distribution of schooling in the population, and can see individual's wage and schooling
- Regress log wage on schooling, that tells you how schooling raises the marginal product $\Rightarrow$ estimates of human capital at individual level, then aggregate it up.

Factors versus Productivity

var[log(y)]	1.297	y^{90}/y^{10}	21
var[log(y_{KH})]	0.500	y_{KH}^{90}/y_{KH}^{10}	7
$success_1$	0.39	$success_2$	0.34

Table 1: Baseline Success of the Factor-Only Model

(Table from Caselli's Handbook chapter)

- 40% of variation in income per capita driven by variation in factors
- Rich countries have more physical and human capital... but they're still too rich given these endowments

Productivity differences across the world

- From the production function

$$Y_j = K_j^{\alpha}(A_j H_j)^{1-\alpha}$$

  we can calibrate the relative productivity differences (relative to US) as

$$\frac{A_j}{A_{US}} = \left(\frac{Y_j}{Y_{US}}\right)^{\frac{1}{1-\alpha}} \left(\frac{K_{US}}{K_j}\right)^{\frac{\alpha}{1-\alpha}} \frac{H_{US}}{H_j}$$

- For $\alpha = 1/3$ we can measure this residual in the data.

Productivity differences across the world

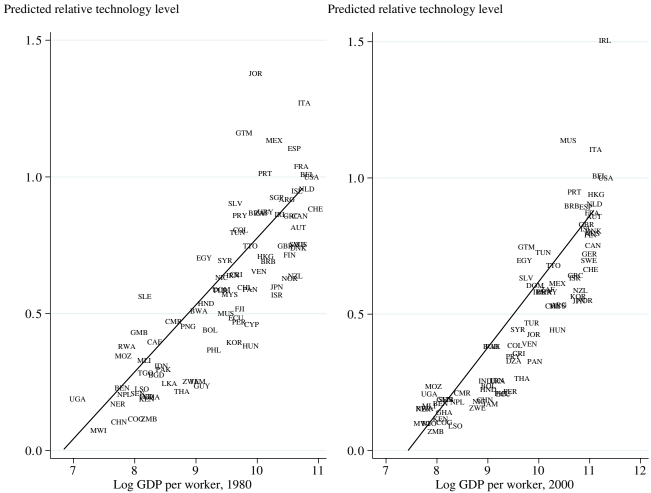


FIGURE 3.3 Calibrated technology levels relative to U.S. technology (from the Solow growth model with human capital) versus log GDP per worker, 1980 and 2000.

- If technologies were equal, we had $A_j / A_{US} = 1$
- Poor countries have low productivity – not obvious!

Taking stock

- Conceptually: 2 dimensions of growth and long-run development
  - factors: capital, human capital, ...
  - productivity: technology, capital quality, general efficiency, ...
- Both are important, with roughly a 50/50 split
- Neoclassical (and Solow) Model: rich theory how capital (and human capital) evolves
- Endogenous Growth Models: how does productivity A evolve?
- Growth models tend to be mostly theoretical. But huge empirical literature ("macro-development") on the determinants of the level of A!
- More and more data on firms available, very active research area

Factor Misallocation and Productivity

Explaining productivity differences across countries

- With more and more firm-level data available, why not look at productivity at the micro level?
- Hard to compare micro-productivity levels across countries: quality differences, demand differences, etc.
- But we can compare firms *within* each country!

Results:

- Even in developing countries (at least the better ones), the most productive firms have productivity levels comparable to those in Western Europe and the US.
- But: in developing countries there is a **long tail of very unproductive firms!**
  - Could this be important? Yes—because we know aggregate productivity is lower!
  - Why do they not upgrade their productivity?
  - Why do they survive?

Hsieh & Klenow 2009, QJE

Propose a particular empirical framework for studying firm productivity

- Very parametric, many extremely problematic assumptions
- Still, extremely influential and managed to convince people that issue is important
- Interesting patterns in the data

Preferences and Technology

- Consider a static economy of S sectors, with a representative consumer that consumes a Cobb-Douglas bundle of aggregates from different sectors s:

$$Y = \prod_{s=1}^{S} Y_s^{\theta_s} \quad \text{with} \quad \sum_{s=1}^{S} \theta_s = 1$$

- Each sector is a CES aggregate of (many) differentiated products:

$$Y_s = \left(\sum_{j=1}^{M_s} Y_{sj}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

  and each firm in sector s has production function

$$Y_{sj} = A_{sj} K_{sj}^{\alpha_s} L_{sj}^{1-\alpha_s}.$$

  These firms are monopolistically competitive in their sector (meaning: they are monopolists for their variety j, and are "small" in the sense that they don't take the impact of their pricing behavior on the price index into account)

- Utility maximization yields constant expenditure shares equal to θ_s on baskets of goods from each sector s:

$$\frac{P_s Y_s}{PY} = \theta_s$$

where $P_s = \left(\sum_j P_{sj}^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$ and $P = \prod_s P_s^{\theta_s}$.

- Demand for each variety

$$Y_{sj} = \left(\frac{P_{sj}}{P_s}\right)^{-\sigma} Y_s$$

for the producers of the varieties in each sector. P_s is the price of one unit of sector s output ($=$ Lagrange multiplier in Y_s's cost minimization problem).

Derivation of the demand curve

First problem:

$$\max \prod_s Y_s^{\theta_s} \qquad \text{s.t.} \quad \sum P_s Y_s = PY$$

Second problem: choose the composition of each s basket to minimize the cost of the basket:

$$\min \sum P_{sj} Y_{sj}$$

$$\text{s.t.} \quad \left(\sum_j^{M_s} Y_{sj}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} = Y_s$$

Lagrangian:

$$L = \sum P_{sj} Y_{sj} - \lambda \left[\left(\sum_j^{M_s} Y_{sj}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} - Y_s \right]$$

The first-order conditions are:

$$\frac{\partial L}{\partial Y_{sj}} = P_{sj} - \lambda \left[\left(\sum_j^{M_s} Y_{sj}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}-1} Y_{sj}^{-\frac{1}{\sigma}} \right] = 0$$

$$\Leftrightarrow \qquad P_{sj} = \lambda \left[\left(\sum_j^{M_s} Y_{sj}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}-1} Y_{sj}^{-\frac{1}{\sigma}} \right]$$

$$\Leftrightarrow \qquad P_{sj} Y_{sj}^{\frac{1}{\sigma}} = \lambda \left[\left(\sum_j^{M_s} Y_{sj}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{1}{\sigma-1}} \right] = \lambda Y_s^{\frac{1}{\sigma}}$$

$$\Leftrightarrow \qquad Y_{sj} = \left(\frac{P_{sj}}{\lambda} \right)^{-\sigma} Y_s$$

Derivation of the demand curve

To show that $\lambda = P_s$, plug this into the budget constraint:

$$
\begin{aligned}
\left(\sum_{j}^{M_s} \left(\left(\frac{P_{sj}}{\lambda} \right)^{-\sigma} Y_s \right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} &= Y_s \\
\Leftrightarrow \quad \left(\sum_{j}^{M_s} P_{sj}^{1-\sigma} \lambda^{\sigma-1} \right)^{\frac{\sigma}{\sigma-1}} &= 1 \\
\Leftrightarrow \quad \left(\sum_{j}^{M_s} P_{sj}^{1-\sigma} \right)^{\frac{\sigma}{\sigma-1}} &= \lambda^{-\sigma} \\
\Leftrightarrow \quad \left(\sum_{j}^{M_s} P_{sj}^{1-\sigma} \right)^{\frac{1}{1-\sigma}} &= \lambda
\end{aligned}
$$

Preferences and Technology (ctd.)

- Hsieh and Klenow (2009) assume (following a practice that has become popular) that there are firm-specific "distortions" affecting total production and capital, essentially modeled as "taxes".
- As a result of these wedges, firms produce different amounts than what would be dictated by their productivity and also may have different capital-labor ratios.
- Assume that
  - $\tau_{Y,js}$ is a distortion to the marginal product of capital and labor
  - $\tau_{K,js}$ is a distortion to the marginal product of capital relative to labor

  so that profits for the firm are

$$\pi_{is} = (1 - \tau_{Y,js})P_{js}Y_{js} - wL_{js} - (1 + \tau_{K,js})rK_{js}$$

  which the firm maximizes subject to the isoelastic demand function above.

Factor shares under cost minimization

- Cost minimization of the firms yields factor demands L_{js}, K_{js} such that

$$1 + \tau_{K,js} = \frac{\alpha_s}{1-\alpha_s} \frac{wL_{js}}{rK_{js}}$$

- Likewise, we get that

$$1 - \tau_{Y,js} = \frac{\sigma}{\sigma - 1} \left(\frac{1}{1-\alpha_s} \right) \frac{wL_{js}}{P_{js} Y_{js}}$$

- Note that both capital/labor ratio and labor share (first and second eqns, respectively) are readily observed in firm-level data
- Simple idea: with factor-neutral productivity and common factor prices, factor shares should only differ across firms because of wedges.
  - Of course, this assumes that firms have the same production function...

How important are these distortions/differences in factor shares?

A discussion of productivity

- We are interested in physical productivity A_{js} but we can typically only measure *revenue productivity*
- Let
$$TFPQ_{js} := \frac{Y_{js}}{K_{js}^{\alpha_s} L_{js}^{1-\alpha_s}} = A_{js}$$
denote physical productivity and
$$TFPR_{js} := \frac{P_{js} Y_{js}}{K_{js}^{\alpha_s} L_{js}^{1-\alpha_s}} = P_{js} A_{js}$$
denote revenue productivity.
- In an undistorted benchmark, TFPQ naturally varies across firms with A_{js} but *TFPR* would be constant across firms (higher productivity firms would charge proportionally lower prices): cost minimization gives
$$P_{js} = \frac{\sigma}{1-\sigma} \left(\frac{r}{\alpha}\right)^{\alpha} \left(\frac{w}{1-\alpha}\right)^{1-\alpha} \frac{1}{A_{js}}$$

- With distortion, the firm-level TFPR becomes

$$P_{js} A_{js} = \frac{\sigma}{1-\sigma} \left(\frac{r}{\alpha}\right)^{\alpha} \left(\frac{w}{1-\alpha}\right)^{1-\alpha} \underbrace{\frac{(1+\tau_{K,js})^{\alpha}}{1-\tau_{Y,js}}}_{\text{distortion}}$$

- Hence, the *variance of TFPR* can tell you something about the magnitude of distortions
- Indeed, Hsieh and Klenow show that you can write sectoral TFP as a weighted average of firm-level TFP, where the weights are determined by the deviation of TFPR from the mean:

$$A_s = \left(\sum_{j=1}^{M_s} \left(A_{js} \frac{\overline{TFPR}_s}{TFPR_{js}} \right)^{\sigma-1} \right)^{1/(\sigma-1)}$$

- For TFPQ, they use the demand elasticity to get prices, i.e.

$$A_{si} = \kappa_s \frac{(P_{js} Y_{js})^{\frac{\sigma}{\sigma-1}}}{K_{js}^{\alpha_s} L_{js}^{1-\alpha_s}}$$

Then they perform a counterfactual where they remove all distortions (equalize all TFPR's).

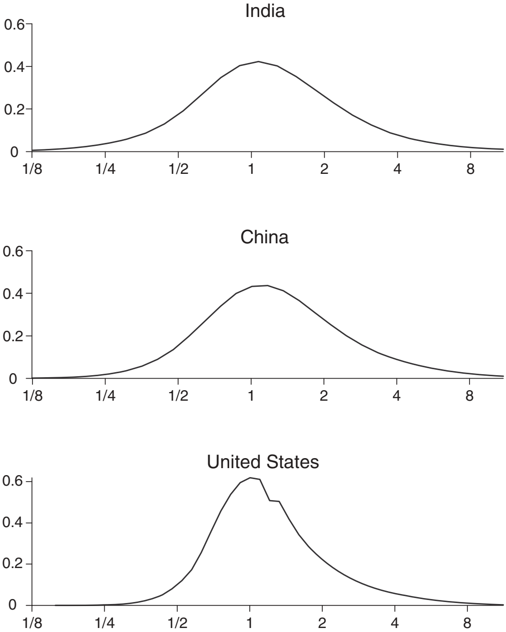


FIGURE II
Distribution of TFPR

TABLE IV
TFP GAINS FROM EQUALIZING TFPR WITHIN INDUSTRIES

China	1998	2001	2005
%	115.1	95.8	86.6
India	1987	1991	1994
%	100.4	102.1	127.5
United States	1977	1987	1997
%	36.1	30.7	42.9

- Productivity gains from equalizing TFP are large! (approx. doubling productivity for China and India!)
- Empirical moment that TFPR variation is larger in poorer countries has been taken as indication that firms face "distortions" in their factor choices and production choices
- Viewed by many people as the most promising "explanation" for low productivity in developing countries.

But many caveats

- Measurement:
  - Everything relies on Cobb-Douglas
  - Constant markups/isoelastic demand
  - Perhaps wrong demand elasticity
  - Adjustment costs (!) (see Asker, De Loecker, and Collard-Wexler, 2014)
  - Assumption that all firms use the same production function within industry (see Boehm & Oberfield, 2020)
  - Heterogeneity of factors
  - Measurement error (!)
- Interpretation. What are these "wedges"? How do we get rid of them?