8 New Keynesian Monetary Economics

8.1 Introduction

In the 1970s, 1980s, and early 1990s, models used for monetary policy analysis combined the assumption of nominal rigidity with a simple structure that linked the quantity of money to aggregate spending. Although the theoretical foundations of these models were weak, the approach proved remarkably useful in addressing a wide range of monetary policy topics.[1] Today, the standard approach in monetary economics and monetary policy analysis incorporates nominal wage or price rigidity into a dynamic stochastic general equilibrium (DSGE) framework that is based on optimizing behavior by the agents in the model.

These modern DSGE models with nominal frictions are commonly labeled *new Keynesian models* because, like older versions of models in the Keynesian tradition, aggregate demand plays a central role in determining output in the short run, and there is a presumption that some fluctuations both can be and should be dampened by countercylical monetary or fiscal policy.[2] Early examples of models with these properties include those of Yun (1996); Goodfriend and King (1998); Rotemberg and Woodford (1995; 1998); and McCallum and Nelson (1999). Galí (2002) discusses the derivation of the model's equilibrium conditions, and book-length treatments of the new Keynesian model are provided by Woodford (2003a) and Galí (2008).

The first section of this chapter shows how a basic money-in-the-utility function (MIU) model, combined with the assumption of monopolistically competitive goods markets and price stickiness, can form the basis for a simple linear new Keynesian model.[3] The model is a consistent general equilibrium model in which all agents face well-defined decision problems and behave optimally, given the environment in

1. Chapter 7 provided a taste of the many interesting insights obtained from these models.

2. Goodfriend and King (1998) proposed the name "the new neoclassical synthesis" to emphasize the connection with neoclassical rather than Keynesian traditions.

3. See chapter 2 for a discussion of money-in-the-utility function (MIU) models.

which they find themselves. To obtain a canonical new Keynesian model, three key modifications will be made to the MIU model of chapter 2. First, endogenous variations in the capital stock are ignored. This follows McCallum and Nelson (1999), who showed that, at least for the United States, there is little relationship between the capital stock and output at business cycle frequencies. Endogenous capital stock dynamics play a key role in equilibrium business cycle models in the real business cycle tradition, but as Cogley and Nason (1995) showed, the response of investment and the capital stock to productivity shocks actually contributes little to the dynamics implied by such models. For simplicity, then, the capital stock will be ignored.[4]

Second, the single final good in the MIU model is replaced by a continuum of differentiated goods produced by monopolistically competitive firms. These firms face constraints on their ability to adjust prices, thus introducing nominal price stickiness into the model. In the basic model, nominal wages will be allowed to fluctuate freely, although section 8.3.6 explores the implications of assuming that both prices and wages are sticky.

Third, monetary policy is represented by a rule for setting the nominal rate of interest. Most central banks today use a short-term nominal interest rate as their instrument for implementing monetary policy. The nominal quantity of money is then endogenously determined to achieve the desired nominal interest rate. Important issues are involved in choosing between money supply policy procedures and interest rate procedures; these are discussed in chapter 11.

These three modifications yield a new Keynesian framework that is consistent with optimizing behavior by private agents and incorporates nominal rigidities yet is simple enough to use for exploring a number of policy issues. It can be linked directly to the more traditional aggregate supply-demand (AS-IS-LM) model that long served as one of the workhorses for monetary policy analysis and is still common in most undergraduate texts. Once the basic framework has been developed, section 8.4 considers optimal policy as well as a variety of policy issues.

8.2 The Basic Model

The model consists of households and firms. Households supply labor, purchase goods for consumption, and hold money and bonds, and firms hire labor and produce and sell differentiated products in monopolistically competitive goods markets. The basic model of monopolistic competition is drawn from Dixit and Stiglitz (1977).

4. However, Dotsey and King (2001) and Christiano, Eichenbaum, and Evans (2005) emphasized the importance of variable capital utilization for understanding the behavior of inflation. Firm-specific capital in a new Keynesian framework was analyzed by Altig et al. (2005).

The model of price stickiness is taken from Calvo (1983).[5] Each firm sets the price of the good it produces, but not all firms reset their price in each period. Households and firms behave optimally; households maximize the expected present value of utility, and firms maximize profits. There is also a central bank that controls the nominal rate of interest. Initially, the central bank, in contrast to households and firms, is not assumed to behave optimally; optimal policy is explored in section 8.4.

8.2.1 Households

The preferences of the representative household are defined over a composite consumption good C_t, real money balances M_t/P_t, and the time devoted to market employment N_t. Households maximize the expected present discounted value of utility:

$$\mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \left[\frac{C_{t+i}^{1-\sigma}}{1-\sigma} + \frac{\gamma}{1-b} \left(\frac{M_{t+i}}{P_{t+i}} \right)^{1-b} - \chi \frac{N_{t+i}^{1+\eta}}{1+\eta} \right]. \tag{8.1}$$

The composite consumption good consists of differentiated products produced by monopolistically competitive final goods producers (firms). There is a continuum of such firms of measure 1, and firm j produces good c_j. The composite consumption good that enters the household's utility function is defined as

$$C_t = \left[\int_0^1 c_{jt}^{(\theta-1)/\theta}\, dj \right]^{\theta/(\theta-1)}, \qquad \theta > 1. \tag{8.2}$$

The household's decision problem can be dealt with in two stages. First, regardless of the level of C_t the household decides on, it will always be optimal to purchase the combination of individual goods that minimizes the cost of achieving this level of the composite good. Second, given the cost of achieving any given level of C_t, the household chooses C_t, N_t, and M_t optimally.

Dealing first with the problem of minimizing the cost of buying C_t, the household's decision problem is

$$\min_{c_{jt}} \int_0^1 p_{jt} c_{jt}\, dj$$

subject to

$$\left[\int_0^1 c_{jt}^{(\theta-1)/\theta}\, dj \right]^{\theta/(\theta-1)} \geq C_t, \tag{8.3}$$

5. See section 6.2.3.

where p_{jt} is the price of good j. Letting ψ_t be the Lagrangian multiplier on the constraint, the first-order condition for good j is

$$p_{jt} - \psi_t \left[\int_0^1 c_{jt}^{(\theta-1)/\theta}\, dj \right]^{1/(\theta-1)} c_{jt}^{-1/\theta} = 0.$$

Rearranging, $c_{jt} = (p_{jt}/\psi_t)^{-\theta} C_t$. From the definition of the composite level of consumption (8.2), this implies

$$C_t = \left[\int_0^1 \left[\left(\frac{p_{jt}}{\psi_t} \right)^{-\theta} C_t \right]^{(\theta-1)/\theta} dj \right]^{\theta/(\theta-1)} = \left(\frac{1}{\psi_t} \right)^{-\theta} \left[\int_0^1 p_{jt}^{1-\theta}\, dj \right]^{\theta/(\theta-1)} C_t.$$

Solving for ψ_t,

$$\psi_t = \left[\int_0^1 p_{jt}^{1-\theta}\, dj \right]^{1/(1-\theta)} \equiv P_t. \tag{8.4}$$

The Lagrangian multiplier is the appropriately aggregated price index for consumption. The demand for good j can then be written as

$$c_{jt} = \left(\frac{p_{jt}}{P_t} \right)^{-\theta} C_t. \tag{8.5}$$

The price elasticity of demand for good j is equal to θ. As $\theta \to \infty$, the individual goods become closer and closer substitutes, and consequently individual firms will have less market power.

Given the definition of the aggregate price index in (8.4), the budget constraint of the household is, in real terms,

$$C_t + \frac{M_t}{P_t} + \frac{B_t}{P_t} = \left(\frac{W_t}{P_t} \right) N_t + \frac{M_{t-1}}{P_t} + (1 + i_{t-1}) \left(\frac{B_{t-1}}{P_t} \right) + \Pi_t, \tag{8.6}$$

where M_t (B_t) is the household's nominal holdings of money (one-period bonds). Bonds pay a nominal rate of interest i_t. Real profits received from firms are equal to Π_t.

In the second stage of the household's decision problem, consumption, labor supply, money, and bond holdings are chosen to maximize (8.1) subject to (8.6). This leads to the following conditions, which, in addition to the budget constraint, must hold in equilibrium:

$$C_t^{-\sigma} = \beta(1 + i_t)\mathrm{E}_t \left(\frac{P_t}{P_{t+1}} \right) C_{t+1}^{-\sigma} \tag{8.7}$$

$$\frac{\gamma\left(\frac{M_t}{P_t}\right)^{-b}}{C_t^{-\sigma}} = \frac{i_t}{1+i_t} \tag{8.8}$$

$$\frac{\chi N_t^{\eta}}{C_t^{-\sigma}} = \frac{W_t}{P_t}. \tag{8.9}$$

These conditions represent the Euler condition for the optimal intertemporal allocation of consumption, the intratemporal optimality condition setting the marginal rate of substitution between money and consumption equal to the opportunity cost of holding money, and the intratemporal optimality condition setting the marginal rate of substitution between leisure and consumption equal to the real wage.[6]

8.2.2 Firms

Firms maximize profits, subject to three constraints. The first is the production function summarizing the available technology. For simplicity, capital is ignored, so output is a function solely of labor input N_{jt} and an aggregate productivity disturbance Z_t:

$$c_{jt} = Z_t N_{jt}, \qquad \mathrm{E}(Z_t) = 1,$$

where constant returns to scale have been assumed. The second constraint on the firm is the demand curve each firm faces. This is given by (8.5). The third constraint is that in each period some firms are not able to adjust their price. The specific model of price stickiness used here is due to Calvo (1983). Each period, the firms that adjust their price are randomly selected, and a fraction $1-\omega$ of all firms adjust while the remaining ω fraction do not adjust. The parameter ω is a measure of the degree of nominal rigidity; a larger ω implies that fewer firms adjust each period and that the expected time between price changes is longer. Those firms that do adjust their price at time t do so to maximize the expected discounted value of current and future profits. Profits at some future date $t+s$ are affected by the choice of price at time t only if the firm has not received another opportunity to adjust between t and $t+s$. The probability of this is ω^s.[7]

Before analyzing the firm's pricing decision, consider its cost minimization problem, which involves minimizing $W_t N_{jt}$ subject to producing $c_{jt} = Z_t N_{jt}$. This problem can be written, in real terms, as

6. See chapter 2 for further discussion of these first-order conditions in an MIU model.

7. In this formulation, the degree of nominal rigidity, as measured by ω, is constant, and the probability that a firm has adjusted its price is a function of time, not of the current state. State-dependent pricing models were discussed in section 6.2.5.

$$\min_{N_t} \left(\frac{W_t}{P_t}\right) N_t + \varphi_t(c_{jt} - Z_t N_{jt}),$$

where φ_t is equal to the firm's real marginal cost. The first-order condition implies

$$\varphi_t = \frac{W_t/P_t}{Z_t}. \tag{8.10}$$

The firm's pricing decision problem then involves picking p_{jt} to maximize

$$\mathrm{E}_t \sum_{i=0}^{\infty} \omega^i \Delta_{i,t+i} \left[\left(\frac{p_{jt}}{P_{t+i}}\right) c_{jt+i} - \varphi_{t+i} c_{jt+i}\right],$$

where the discount factor $\Delta_{i,t+i}$ is given by $\beta^i (C_{t+i}/C_t)^{-\sigma}$. Using the demand curve (8.5) to eliminate c_{jt}, this objective function can be written as

$$\mathrm{E}_t \sum_{i=0}^{\infty} \omega^i \Delta_{i,t+i} \left[\left(\frac{p_{jt}}{P_{t+i}}\right)^{1-\theta} - \varphi_{t+i}\left(\frac{p_{jt}}{P_{t+i}}\right)^{-\theta}\right] C_{t+i}.$$

While individual firms produce differentiated products, they all have the same production technology and face demand curves with constant and equal demand elasticities. In other words, they are essentially identical except that they may have set their current price at different dates in the past. However, all firms adjusting in period t face the same problem, so all adjusting firms will set the same price. Let p_t^* be the optimal price chosen by all firms adjusting at time t. The first-order condition for the optimal choice of p_t^* is

$$\mathrm{E}_t \sum_{i=0}^{\infty} \omega^i \Delta_{i,t+i} \left[(1-\theta)\left(\frac{p_t^*}{P_{t+i}}\right) + \theta\varphi_{t+i}\right]\left(\frac{1}{p_t^*}\right)\left(\frac{p_t^*}{P_{t+i}}\right)^{-\theta} C_{t+i} = 0. \tag{8.11}$$

Using the definition of $\Delta_{i,t+i}$, (8.11) can be rearranged to yield

$$\left(\frac{p_t^*}{P_t}\right) = \left(\frac{\theta}{\theta-1}\right) \frac{\mathrm{E}_t \sum_{i=0}^{\infty} \omega^i \beta^i C_{t+i}^{1-\sigma} \varphi_{t+i} \left(\frac{P_{t+i}}{P_t}\right)^{\theta}}{\mathrm{E}_t \sum_{i=0}^{\infty} \omega^i \beta^i C_{t+i}^{1-\sigma} \left(\frac{P_{t+i}}{P_t}\right)^{\theta-1}}. \tag{8.12}$$

Consider the case in which all firms are able to adjust their price every period ($\omega = 0$). When $\omega = 0$, (8.12) reduces to

$$\left(\frac{p_t^*}{P_t}\right) = \left(\frac{\theta}{\theta-1}\right)\varphi_t = \mu\varphi_t. \tag{8.13}$$

Each firm sets its price p_t^* equal to a markup $\mu > 1$ over its nominal marginal cost $P_t\varphi_t$. This is the standard result in a model of monopolistic competition. Because price exceeds marginal cost, output will be inefficiently low. When prices are flexible, all firms charge the same price. In this case, $p_t^* = P_t$ and $\varphi_t = 1/\mu$. Using the definition of real marginal cost, this means $W_t/P_t = Z_t/\mu < Z_t$ in a flexible-price equilibrium. However, the real wage must also equal the marginal rate of substitution between leisure and consumption to be consistent with household optimization. This condition implies, from (8.9), that

$$\frac{\chi N_t^\eta}{C_t^{-\sigma}} = \frac{W_t}{P_t} = \frac{Z_t}{\mu}. \tag{8.14}$$

Goods market clearing and the production function imply that $C_t = Y_t$ and $N_t = Y_t/Z_t$. Using these conditions in (8.14), and letting Y_t^f denote equilibrium output under flexible prices, Y_t^f is given by

$$Y_t^f = \left(\frac{1}{\chi\mu}\right)^{1/(\sigma+\eta)} Z_t^{(1+\eta)/(\sigma+\eta)}. \tag{8.15}$$

When prices are flexible, output is a function of the aggregate productivity shock, reflecting the fact that in the absence of sticky prices, the new Keynesian model reduces to a real business cycle model.

When prices are sticky ($\omega > 0$), output can differ from the flexible-price equilibrium level. Because a firm will not adjust its price every period, (8.12) shows it must take into account expected future marginal cost as well as current marginal cost whenever it has an opportunity to adjust its price.

The aggregate price index is an average of the price charged by the fraction $1 - \omega$ of firms setting their price in period t and the average of the remaining fraction ω of all firms that do not change their price in period t. However, because the adjusting firms were selected randomly from among all firms, the average price of the non-adjusters is just the average price of all firms that prevailed in period $t - 1$. Thus, from (8.4), the average price in period t satisfies

$$P_t^{1-\theta} = (1-\omega)(p_t^*)^{1-\theta} + \omega P_{t-1}^{1-\theta}. \tag{8.16}$$

To summarize, (8.7)–(8.10), (8.12), (8.14), and (8.16) represent a system in C_t, N_t, M/P_t, Y_t, φ_t, P_t, p_t^*, W_t/P_t, and i_t that can be combined with the aggregate production function, $Y_t = Z_tN_t$, and a specification of monetary policy to determine the economy's equilibrium.

8.3 A Linearized New Keynesian Model

One reason for the popularity of the new Keynesian model is that it allows for a simple linear representation in terms of an inflation adjustment equation, or Phillips curve, and an output and real interest rate relationship that corresponds to the IS curve of undergraduate macroeconomics. To derive this linearized version of the model, let $\hat{x}_t$ denote the percentage deviation of a variable X_t around its steady state and let the superscript f denote the flexible-price equilibrium. The equilibrium conditions in the model will be linearized around a steady state in which the inflation rate is zero.

8.3.1 The Linearized Phillips Curve

Equations (8.12) and (8.16) can be approximated around a zero average inflation, steady-state equilibrium to obtain an expression for aggregate inflation (see section 8.6.1 of the chapter appendix) of the form

$$\pi_t = \beta E_t \pi_{t+1} + \tilde{\kappa} \hat{\varphi}_t, \tag{8.17}$$

where

$$\tilde{\kappa} = \frac{(1-\omega)(1-\beta\omega)}{\omega}$$

is an increasing function of the fraction of firms able to adjust each period and $\hat{\varphi}_t$ is real marginal cost, expressed as a percentage deviation around its steady-state value.[8]

Equation (8.17) is often referred to as the *new Keynesian Phillips curve*. Unlike more traditional Phillips curve equations, the new Keynesian Phillips curve implies that real marginal cost is the correct driving variable for the inflation process. It also implies that the inflation process is forward-looking, with current inflation a function of expected future inflation. When a firm sets its price, it must be concerned with inflation in the future because it may be unable to adjust its price for several periods. Solving (8.17) forward,

$$\pi_t = \tilde{\kappa} \sum_{i=0}^{\infty} \beta^i \mathrm{E}_t \hat{\varphi}_{t+i},$$

which shows that inflation is a function of the present discounted value of current and future real marginal costs.

8. Ascari (2004) showed that the behavior of inflation in the Calvo model can be significantly affected if steady-state inflation is not zero.

The new Keynesian Phillips curve also differs from traditional Phillips curves in having been derived explicitly from a model of optimizing behavior on the part of price setters, conditional on the assumed economic environment (monopolistic competition, constant elasticity demand curves, and randomly arriving opportunities to adjust prices). This derivation reveals how $\tilde{\kappa}$, the impact of real marginal cost on inflation, depends on the structural parameters β and ω. An increase in β means that the firm gives more weight to future expected profits. As a consequence, $\tilde{\kappa}$ declines; inflation is less sensitive to current marginal costs. Increased price rigidity (a rise in ω) reduces $\tilde{\kappa}$; with opportunities to adjust arriving less frequently, the firm places less weight on current marginal cost (and more on expected future marginal costs) when it does adjust its price.

Equation (8.17) implies that inflation depends on real marginal cost and not directly on a measure of the gap between actual output and some measure of potential output or on a measure of unemployment relative to the natural rate, as is typical in traditional Phillips curves.[9] However, real marginal costs can be related to an output gap measure. The firm's real marginal cost is equal to the real wage it faces divided by the marginal product of labor (see (8.10)). In a flexible-price equilibrium, all firms set the same price, so (8.13) implies that real marginal cost will equal its steady-state value of $1/\mu$. Because nominal wages have been assumed to be completely flexible, the real wage must, according to (8.9), equal the marginal rate of substitution between leisure and consumption. Expressed in terms of percentage deviations around the steady state, (8.9) implies that $\hat{w}_t - \hat{p}_t = \eta\hat{n}_t + \sigma\hat{y}_t$. Recalling that $\hat{c}_t = \hat{y}_t$ and $\hat{y}_t = \hat{n}_t + \hat{z}_t$, the percentage deviation of real marginal cost around its steady-state value is

$$\begin{aligned}\hat{\varphi}_t &= (\hat{w}_t - \hat{p}_t) - (\hat{y}_t - \hat{n}_t)\\ &= (\sigma+\eta)\left[\hat{y}_t - \left(\frac{1+\eta}{\sigma+\eta}\right)\hat{z}_t\right].\end{aligned}$$

To interpret the term involving $\hat{z}_t$, linearize (8.15) giving flexible-price output to obtain

$$\hat{y}_t^f = \left(\frac{1+\eta}{\sigma+\eta}\right)\hat{z}_t. \tag{8.18}$$

Thus, (8.18) can be used to express real marginal cost as

$$\hat{\varphi}_t = \gamma(\hat{y}_t - \hat{y}_t^f), \tag{8.19}$$

9. See Ravenna and Walsh (2008) and Blanchard and Gali (2008) for models of labor market frictions that relate inflation to unemployment.

where $\gamma = \sigma + \eta$. Using this result, the inflation adjustment equation (8.17) becomes

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t, \tag{8.20}$$

where $\kappa = \gamma\tilde{\kappa} = \gamma(1-\omega)(1-\beta\omega)/\omega$ and $x_t \equiv \hat{y}_t - \hat{y}_t^f$ is the gap between actual output and flexible-price equilibrium output.

The preceding assumed that firms face constant returns to scale. If, instead, each firm's production function is $c_{jt} = Z_t N_{jt}^a$, where $0 < a \leq 1$, then the results must be modified slightly. When $a < 1$, firms with different production levels will face different marginal costs, and real marginal cost for firm j will equal

$$\varphi_{jt} = \frac{W_t/P_t}{aZ_t N_{jt}^{a-1}} = \frac{W_t/P_t}{ac_{jt}/N_{jt}}.$$

Linearizing this expression for firm $j's$ real marginal cost and using the production function yields

$$\hat{\varphi}_{jt} = (\hat{w}_t - \hat{p}_t) - (\hat{c}_{jt} - \hat{n}_{jt}) = (\hat{w}_t - \hat{p}_t) - \left(\frac{a-1}{a}\right)\hat{c}_{jt} - \left(\frac{1}{a}\right)\hat{z}_t. \tag{8.21}$$

Marginal cost for the individual firm can be related to average marginal cost, $\varphi_t = (W_t/P_t)/(aC_t/N_t)$, where

$$N_t = \int_0^1 N_{jt}\,dj = \int_0^1 \left(\frac{c_{jt}}{Z_t}\right)^{1/a} dj = \left(\frac{C_t}{Z_t}\right)^{1/a} \int_0^1 \left(\frac{p_{jt}}{P_t}\right)^{-\theta/a} dj.$$

When this last expression is linearized around a zero inflation steady state, the final term involving the dispersion of relative prices turns out to be of second order,[10] so one obtains

$$\hat{n}_t = \left(\frac{1}{a}\right)(\hat{c}_t - \hat{z}_t)$$

and

$$\hat{\varphi}_t = (\hat{w}_t - \hat{p}_t) - (\hat{c}_t - \hat{n}_t) = (\hat{w}_t - \hat{p}_t) - \left(\frac{a-1}{a}\right)\hat{c}_t - \left(\frac{1}{a}\right)\hat{z}_t. \tag{8.22}$$

Subtracting (8.22) from (8.21) gives

10. When linearized, the last term becomes

$$-\left(\frac{\theta}{a}\right)\int(\hat{p}_{jt} - \hat{p}_t)\,dj,$$

but to a first-order approximation, $\int \hat{p}_{jt}\,dj = \hat{p}_t$, so the price dispersion term is approximately equal to zero.

$$\hat{\varphi}_{jt} - \hat{\varphi}_t = -\left(\frac{a-1}{a}\right)(\hat{c}_{jt} - \hat{c}_t).$$

Finally, employing the demand relationship (8.5) to express $\hat{c}_{jt} - \hat{c}_t$ in terms of relative prices,

$$\hat{\varphi}_{jt} = \hat{\varphi}_t - \left[\frac{\theta(1-a)}{a}\right](\hat{p}_{jt} - \hat{p}_t).$$

Firms with relatively high prices (and therefore low output) will have relatively low real marginal costs. In the case of constant returns to scale ($a = 1$), all firms face the same marginal cost. Sbordone (2002) and Galí, Gertler, and López-Salido (2001) showed that when $a < 1$, the new Keynesian inflation adjustment equation becomes[11]

$$\pi_t = \beta E_t \pi_{t+1} + \tilde{\kappa}\left[\frac{a}{a + \theta(1-a)}\right]\hat{\varphi}_t.$$

In addition, the labor market equilibrium condition under flexible prices becomes

$$\frac{W_t}{P_t} = \frac{aZ_t N_t^{a-1}}{\mu} = \frac{\chi N_t^{\eta}}{C_t^{-\sigma}},$$

which implies flexible-price output is

$$\hat{y}_t^f = \left[\frac{1+\eta}{1+\eta+a(\sigma-1)}\right]\hat{z}_t.$$

When $a = 1$, this reduces to (8.18).

8.3.2 The Linearized IS Curve

Equation (8.20) relates output to inflation in the form of the deviation around the level of output that would occur in the absence of nominal price rigidity. It forms one of the two key components of an optimizing model that can be used for monetary policy analysis. The other component is a linearized version of the household's Euler condition, (8.7). Because consumption is equal to output in this model (there is no government or investment because capital has been ignored), (8.7) can be approximated around the zero inflation steady state as[12]

$$\hat{y}_t = \mathrm{E}_t \hat{y}_{t+1} - \left(\frac{1}{\sigma}\right)(\hat{\imath}_t - \mathrm{E}_t \pi_{t+1}), \tag{8.23}$$

11. See the chapter appendix for further details on the derivation.

12. See the chapter 2 appendix for details on linearizing the Euler condition; $\hat{c}_t = \hat{y}_t$ is used in (8.23).

where $\hat{\imath}_t$ is the deviation of the nominal interest rate from its steady-state value. Expressing this in terms of the output gap $x_t = \hat{y}_t - \hat{y}_t^f$,

$$x_t = \mathrm{E}_t x_{t+1} - \left(\frac{1}{\sigma}\right)(\hat{\imath}_t - \mathrm{E}_t \pi_{t+1}) + u_t, \tag{8.24}$$

where $u_t \equiv \mathrm{E}_t \hat{y}_{t+1}^f - \hat{y}_t^f$ depends only on the exogenous productivity disturbance (see (8.18)). Combining (8.24) with (8.20) gives a simple two-equation, forward-looking, rational-expectations model for inflation and the output gap measure x_t, once the behavior of the nominal rate of interest is specified.[13] This two-equation model consists of the equilibrium conditions for a well-specified general equilibrium model. The equations appear broadly similar, however, to the types of aggregate demand and aggregate supply equations commonly found in intermediate-level macroeconomics textbooks. Equation (8.24) represents the demand side of the economy (an expectational, forward-looking IS curve), and the new Keynesian Phillips curve (8.20) corresponds to the supply side. In fact, both equations are derived from optimization problems, with (8.24) based on the Euler condition for the representative household's decision problem and (8.20) derived from the optimal pricing decisions of individual firms.

There is a long tradition of using two-equation, aggregate demand–aggregate supply (AD-AS) models in intermediate-level macroeconomic and monetary policy analysis. Models in the AD-AS tradition are often criticized as "starting from curves" rather than starting from the primitive tastes and technology from which behavioral relationships can be derived, given maximizing behavior and a market structure (Sargent 1982). This criticism does not apply to (8.24) and (8.20). The parameters appearing in these two equations are explicit functions of the underlying structural parameters of the production and utility functions and the assumed process for price adjustment.[14] And (8.24) and (8.20) contain expectations of future variables; the absence of this type of forward-looking behavior is a critical shortcoming of older AD-AS frameworks. The importance of incorporating a role for future income was emphasized by Kerr and King (1996).

Equations (8.24) and (8.20) contain three variables: the output gap, inflation, and the nominal interest rate. The model can be closed by assuming that the central bank implements monetary policy through control of the nominal interest rate.[15] Alternatively, if the central bank implements monetary policy by setting a path for the nom-

13. With the nominal interest rate treated as the monetary policy instrument, (8.8) simply determines the real quantity of money in equilibrium.

14. The process for price adjustment, however, has not been derived from the underlying structure of the economic environment.

15. Important issues of price level determinacy arise under interest rate–setting policies (see chapter 11).

inal supply of money, (8.24) and (8.20), together with the linearized version of (8.8), determine x_t, π_t, and $\hat{\imath}_t$.[16]

8.3.3 Uniqueness of the Equilibrium

If a policy rule for the nominal interest rate is added to the model, this must be done with care to ensure that the policy rule does not render the system unstable or introduce multiple equilibria. For example, suppose monetary policy is represented by the following purely exogenous process for $\hat{\imath}_t$:

$$\hat{\imath}_t = v_t, \tag{8.25}$$

where v_t is a stationary stochastic process. Combining (8.25) with (8.24) and (8.20), the resulting system of equations can be written as

$$\begin{bmatrix} 1 & \sigma^{-1} \\ 0 & \beta \end{bmatrix} \begin{bmatrix} \mathrm{E}_t x_{t+1} \\ \mathrm{E}_t \pi_{t+1} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\kappa & 1 \end{bmatrix} \begin{bmatrix} x_t \\ \pi_t \end{bmatrix} + \begin{bmatrix} \sigma^{-1} v_t - u_t \\ 0 \end{bmatrix}.$$

Premultiplying both sides by the inverse of the matrix on the left produces

$$\begin{bmatrix} \mathrm{E}_t x_{t+1} \\ \mathrm{E}_t \pi_{t+1} \end{bmatrix} = M \begin{bmatrix} x_t \\ \pi_t \end{bmatrix} + \begin{bmatrix} \sigma^{-1} v_t - u_t \\ 0 \end{bmatrix}, \tag{8.26}$$

where

$$M = \begin{bmatrix} 1 + \frac{\kappa}{\sigma\beta} & -\frac{1}{\sigma\beta} \\ -\frac{\kappa}{\beta} & \frac{1}{\beta} \end{bmatrix}.$$

Equation (8.26) has a unique stationary solution for the output gap and inflation if and only if the number of eigenvalues of M outside the unit circle is equal to the number of forward-looking variables, in this case, two (Blanchard and Kahn 1980). However, only the largest eigenvalue of this matrix is outside the unit circle, implying that multiple bounded equilibria exist and that the equilibrium is locally indeterminate. Stationary sunspot equilibria are possible.

This example illustrates that an exogenous policy rule—one that does not respond to the endogenous variables x and π—introduces the possibility of multiple equilibria. To understand why, consider what would happen if expected inflation were to rise. Since (8.25) does not allow for any endogenous feedback from this rise in expected inflation to the nominal interest rate, the real interest rate must fall. This decline in the real interest rate is expansionary, and the output gap increases. The rise

16. An alternative approach (see section 8.4) specifies an objective function for the monetary authority and then derives the policymaker's decision rule for setting the nominal interest rate.

in output increases actual inflation, according to (8.20). Thus, a change in expected inflation, even if due to factors unrelated to the fundamentals of inflation, can set off a self-fulfilling change in actual inflation.

This discussion suggests that a policy that raised the nominal interest rate when inflation rose, and raised $\hat{\imath}_t$ enough to increase the real interest rate so that the output gap fell, would be sufficient to ensure a unique equilibrium. For example, suppose the nominal interest rate responds to inflation according to the rule

$$\hat{\imath}_t = \delta\pi_t + v_t. \tag{8.27}$$

Combining (8.27) with (8.24) and (8.20), $\hat{\imath}_t$ can be eliminated, and the resulting system is written as

$$\begin{bmatrix} \mathrm{E}_t x_{t+1} \\ \mathrm{E}_t \pi_{t+1} \end{bmatrix} = N \begin{bmatrix} x_t \\ \pi_t \end{bmatrix} + \begin{bmatrix} \sigma^{-1} v_t - u_t \\ 0 \end{bmatrix}, \tag{8.28}$$

where

$$N = \begin{bmatrix} 1 + \frac{\kappa}{\sigma\beta} & \frac{\beta\delta - 1}{\sigma\beta} \\ -\frac{\kappa}{\beta} & \frac{1}{\beta} \end{bmatrix}.$$

Bullard and Mitra (2002) showed that a unique stationary equilibrium exists as long as $\delta > 1$.[17] Setting $\delta > 1$ is referred to as the *Taylor principle*, because John Taylor was the first to stress the importance of interest rate rules that called for responding more than one-for-one to changes in inflation.

Suppose that instead of reacting solely to inflation, as in (8.27), the central bank responds to both inflation and the output gap according to

$$\hat{\imath}_t = \delta_\pi \pi_t + \delta_x x_t + v_t.$$

This type of policy rule is called a *Taylor rule* (Taylor 1993a), and variants of it have been shown to provide a reasonable empirical description of the policy behavior of many central banks (Clarida, Galí, and Gertler 2000).[18] With this policy rule, Bullard and Mitra (2002) showed that the condition necessary to ensure that the economy has a unique stationary equilibrium becomes

17. If the nominal interest rate is adjusted in response to expected future inflation (rather than current inflation), multiple solutions again become possible if $\hat{\imath}_t$ responds too strongly to $\mathrm{E}_t\hat{\pi}_{t+1}$. See Clarida, Galí, and Gertler (2000).

18. Sometimes the term *Taylor rule* is reserved for the case in which $\delta_\pi = 1.5$ and $\delta_x = 0.5$ when inflation and the interest rate are expressed at annual rates. These are the values Taylor (1993a) found matched the behavior of the federal funds rates rate during the Greenspan period.

$$\kappa(\delta_\pi - 1) + (1 - \beta)\delta_x > 0. \tag{8.29}$$

Determinacy now depends on both the policy parameters δ_π and δ_x. A policy that failed to raise the nominal interest rate sufficiently when inflation rose would lead to a rise in aggregate demand and output. This rise in x could produce a rise in the real interest rate that served to contract spending if δ_x were large. Thus, a policy rule with $\delta_\pi < 1$ could still be consistent with a unique stationary equilibrium. At a quarterly frequency, however, β is about 0.99, so δ_x would need to be very large to offset a value of δ_π much below 1.

The Taylor principle is an important policy lesson that has emerged from the new Keynesian model. It has been argued that the failure of central banks such as the Federal Reserve to respond sufficiently strongly to inflation during the 1970s provides an explanation for the rise in inflation experiences at the time (Lubik and Schorfheide 2004). Further, Orphanides (2001) argued that estimated Taylor rules for the Federal Reserve are sensitive to whether real-time data are used, and he found a much weaker response to inflation in the 1987–1999 period based on real-time data.[19] Because the Taylor principle is based on the mapping from policy response coefficients to eigenvalues in the state space representation of the model, one would expect that the exact restrictions the policy responses must satisfy to ensure determinacy will depend on the specification of the model. Two aspects of the model have been explored that lead to significant modifications of the Taylor principle.

First, Ascari and Ropele (2007) and Kiley (2007b) found that the Taylor rule can be insufficient to ensure determinacy when trend inflation is positive rather than zero as assumed when obtaining the standard linearized new Keynesian inflation equation. For example, Coibion and Gorodnichenko (2008) showed, in a calibrated model, that the central bank's response to inflation would need to be over ten-to-one to ensure determinacy if steady-state inflation exceeded 6 percent. However, many models assume some form of indexation (see chapter 6), and for these models the Taylor principle would continue to hold even in the face of a positive steady-state rate of inflation.

Second, the Taylor principle can be significantly affected when interest rates have direct effects on real marginal cost. Such an effect, usually referred to as the cost channel of monetary policy, is common in models in which firms need to finance wage payments, as in Christiano, Eichenbaum, and Evans (2005) or Ravenna and Walsh (2006), or in which search frictions in the labor market introduce an intertemporal aspect to the firm's labor demand condition (Ravenna and Walsh 2008). For

19. Other papers employing real-time data to estimate policy rules include Rudebusch (2006) for the United States and Papell, Molodtsova, and Nikolsko-Rzhevskyy (2008) for the United States and Germany.

example, Llosa and Tuesta (2006), for a model with a cost channel, and Kurozumi and Van Zandwedge (2008), for a model with search and matching frictions in the labor market, found that satisfying the standard Taylor principle of responding more than one-for-one to inflation need not ensure determinacy.

Finally, note that if v_t and u_t are zero for all t, the solution to (8.28) would be $\pi_t = x_t = 0$ for all t. In this case, the parameter δ in the policy rule (8.27) could not be identified. As Cochrane (2007) emphasized, determinacy relies on assumptions about how the central bank would respond to movements of inflation out of equilibrium. Estimated Taylor rules may not reveal how policy would react in circumstances that are not observed.

8.3.4 The Monetary Transmission Mechanism

The model consisting of (8.24) and (8.20) assumes that the impact of monetary policy on output and inflation operates through the real rate of interest. As long as the central bank is able to affect the real interest rate through its control of the nominal interest rate, monetary policy can affect real output. Changes in the real interest rate alter the optimal time path of consumption. An increase in the real rate of interest, for instance, leads households to attempt to postpone consumption. Current consumption falls relative to future consumption.[20]

Figure 8.1 illustrates the impact of a monetary policy shock (an increase in the nominal interest rate) in the model consisting of (8.24), (8.20), and the policy rule (8.27). The parameter values used in constructing the figure are $\beta = 0.99$, $\sigma = \eta = 1$, $\delta = 1.5$, and $\omega = 0.8$. In addition, the policy shock v_t in the policy rule is assumed to follow an AR(1) process given by $v_t = \rho_v v_{t-1} + \varepsilon_t$, with $\rho_v = 0.5$. The rise in the nominal rate causes inflation and the output gap to fall immediately. This reflects the forward-looking nature of both variables. In fact, all the persistence displayed by the responses arises from the serial correlation introduced into the process for the monetary shock v_t. If $\rho_v = 0$, all variables return to their steady-state values in the period after the shock.[21]

To emphasize the interest rate as the primary channel through which monetary influences affect output, it is convenient to express the output gap as a function of an *interest rate gap*, the gap between the current interest rate and the interest rate consistent with the flexible-price equilibrium. For example, let $\hat{r}_t \equiv \hat{\imath}_t - \mathrm{E}_t\pi_{t+1}$ be the real interest rate, and write (8.24) as

20. Estrella and Fuhrer (2002) noted that the forward-looking Euler equation implies counterfactual dynamics; (8.24) implies that $\mathrm{E}_t\hat{c}_{t+1} - \hat{c}_t = \sigma^{-1}(\hat{\imath}_t - \mathrm{E}_t\pi_{t+1})$, so that a rise in the real interest rate means that consumption must *increase* from t to $t+1$.

21. See Galí (2002) for a discussion of the monetary transmission mechanism incorporated in the basic new Keynesian model.

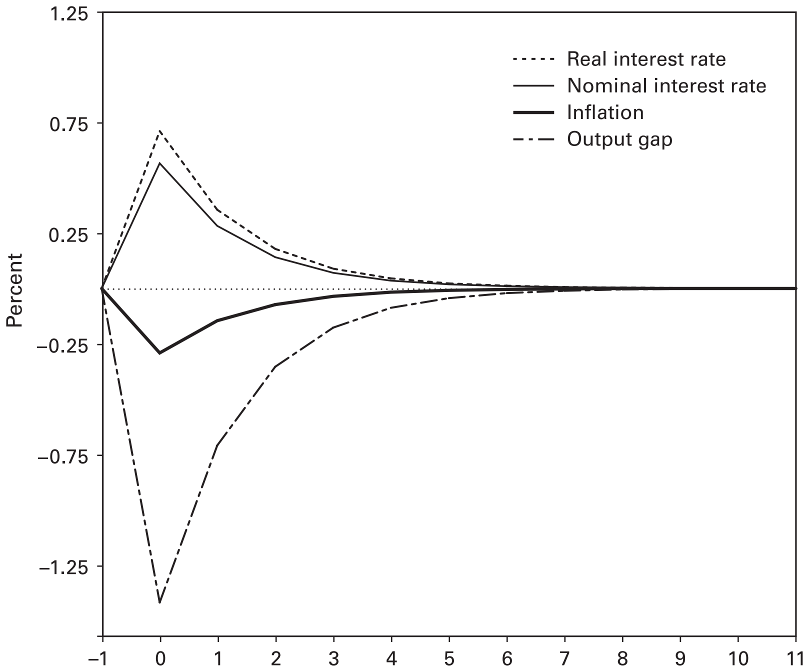

Figure 8.1
Response of output, inflation, and real interest rate to a policy shock in the new Keynesian model.

$$x_t = \mathrm{E}_t x_{t+1} - \left(\frac{1}{\sigma}\right)(\hat{r}_t - \tilde{r}_t),$$

where $\tilde{r}_t \equiv \sigma u_t$. Woodford (2000) labeled $\tilde{r}_t$ the *Wicksellian real interest rate*. It is the interest rate consistent with output equaling the flexible-price equilibrium level. If $\hat{r}_t = \tilde{r}_t$ for all t, then $x_t = 0$ and output is kept equal to the level that would arise in the absence of nominal rigidities. The interest rate gap $\hat{r}_t - \tilde{r}_t$ then summarizes the effects on the actual equilibrium that are due to nominal rigidities.[22]

The presence of expected future output in (8.24) implies that the future path of the one-period real interest rate matters for current demand. To see this, recursively solve (8.24) forward to yield

$$x_t = -\left(\frac{1}{\sigma}\right)\sum_{i=0}^{\infty} \mathrm{E}_t(\hat{r}_{t+i} - \tilde{r}_{t+i}).$$

22. Neiss and Nelson (2001) used a structural model to estimate the real interest rate gap $\hat{r}_t - \tilde{r}_t$ and found that it has value as a predictor of inflation.

Changes in the one-period rate that are persistent will influence expectations of future interest rates. Therefore, persistent changes should have stronger effects on x_t than more temporary changes in real interest rates.

The basic interest rate transmission mechanism for monetary policy could be extended to include effects on investment spending if capital were reintroduced into the model (Christiano, Eichenbaum, and Evans 2005; Dotsey and King 2001). Increases in the real interest rate would reduce the demand for capital and lead to a fall in investment spending. In the case of both investment and consumption, monetary policy effects are transmitted through interest rates.

In addition to these interest rate channels, monetary policy is often thought to affect the economy either indirectly through credit channels or directly through the quantity of money. Real money holdings represent part of household wealth; an increase in real balances should induce an increase in consumption spending through a wealth effect. This channel is often called the *Pigou effect* and was viewed as generating a channel through which price level declines during a depression would eventually increase real balances and household wealth sufficiently to restore consumption spending. During the Keynesian/monetarist debates of the 1960s and early 1970s, some monetarists argued for a direct wealth effect that linked changes in the money supply directly to aggregate demand (Patinkin 1965). The effect of money on aggregate demand operating through interest rate effects was viewed as a Keynesian interpretation of the transmission mechanism, whereas most monetarists argued that changes in monetary policy lead to substitution effects over a broader range of assets than Keynesians normally considered. Since wealth effects are likely to be small at business cycle frequencies, most simple models used for policy analysis ignore them.[23]

Direct effects of the quantity of money are not present in the model used here; the quantity of money appears in neither (8.24) nor (8.20). The underlying model was derived from an MIU model, and the absence of money in (8.24) and (8.20) results from the assumption that the utility function is separable (see (8.1)). If utility is not separable, then changes in the real quantity of money alter the marginal utility of consumption. This would affect the model specification in two ways. First, the real money stock would appear in the household's Euler condition and therefore in (8.24). Second, to replace real marginal cost with a measure of the output gap in (8.20), the real wage was equated to the marginal rate of substitution between leisure and consumption, and this would also involve real money balances if utility were nonseparable (see problem 7 at the end of this chapter). Thus, the absence of money constitutes a special case. However, McCallum and Nelson (1999) and Woodford

23. For an analysis of the real balance effect, see Ireland (2001b).

(2001b) argued that the effects arising with nonseparable utility are quite small, so that little is lost by assuming separability. Ireland (2001c) finds little evidence for nonseparable preferences in a model estimated on U.S. data.

The quantity of money is not totally absent from the underlying model, since (8.8) also must hold in equilibrium. Linearizing this equation around the steady state yields[24]

$$\hat{m}_t - \hat{p}_t = \left(\frac{1}{bi^{ss}}\right)(\sigma\hat{y}_t - \hat{\imath}_t). \tag{8.30}$$

Given the nominal interest rate chosen by the monetary policy authority, this equation determines the nominal quantity of money. Alternatively, if the policymaker sets the nominal quantity of money, then (8.20), (8.24), and (8.30) must all be used to solve jointly for x_t, π_t, and $\hat{\imath}_t$.

Chapter 10 discusses the role of credit channels in the monetary transmission process.

8.3.5 Adding Economic Disturbances

As the model consisting of (8.20) and (8.24) stands, there are no underlying nonpolicy disturbances that might generate movements in either the output gap or inflation other than the productivity disturbance that affects the flexible-price output level. It is common, however, to include in these equations stochastic disturbances arising from other sources.

Suppose the representative household's utility from consumption is subject to random shocks that alter the marginal utility of consumption. Specifically, let the utility function in (8.1) be modified to include a taste shock ψ:

$$\mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \left[\frac{(\psi_{t+i}C_{t+i})^{1-\sigma}}{1-\sigma} + \frac{\gamma}{1-b}\left(\frac{M_{t+i}}{P_{t+i}}\right)^{1-b} - \chi\frac{N_{t+i}^{1+\eta}}{1+\eta}\right]. \tag{8.31}$$

The Euler condition (8.7) becomes

$$\psi_t^{1-\sigma}C_t^{-\sigma} = \beta(1+i_t)\mathrm{E}_t(P_t/P_{t+1})(\psi_{t+1}^{1-\sigma}C_{t+1}^{-\sigma}),$$

which, when linearized around the zero inflation steady state yields

$$\hat{c}_t = \mathrm{E}_t\hat{c}_{t+1} - \left(\frac{1}{\sigma}\right)(\hat{\imath}_t - \mathrm{E}_t\pi_{t+1}) + \left(\frac{\sigma-1}{\sigma}\right)(\mathrm{E}_t\psi_{t+1} - \psi_t). \tag{8.32}$$

24. See the chapter 2 appendix.

If, in addition to consumption by households, the government purchases final output G_t, the goods market equilibrium condition becomes $Y_t = C_t + G_t$. When this is expressed in terms of percentage deviations around the steady state, one obtains

$$\hat{y}_t = \left(\frac{C}{Y}\right)^{ss}\hat{c}_t + \left(\frac{G}{Y}\right)^{ss}\hat{g}_t.$$

Using this equation to eliminate $\hat{c}_t$ from (8.32) and then replacing $\hat{y}_t$ with $x_t + \hat{y}_t^f$ yields an expression for the output gap $(\hat{y}_t - \hat{y}_t^f)$,

$$x_t = \mathrm{E}_t x_{t+1} - \tilde{\sigma}^{-1}(\hat{\imath}_t - \mathrm{E}_t\pi_{t+1}) + \xi_t, \tag{8.33}$$

where $\tilde{\sigma}^{-1} = \sigma^{-1}(C/Y)^{ss}$ and

$$\xi_t \equiv \left(\frac{\sigma-1}{\sigma}\right)\left(\frac{C}{Y}\right)^{ss}(\mathrm{E}_t\psi_{t+1} - \psi_t) - \left(\frac{G}{Y}\right)^{ss}(\mathrm{E}_t\hat{g}_{t+1} - \hat{g}_t) + (\mathrm{E}_t\hat{y}_{t+1}^f - \hat{y}_t^f).$$

Equation (8.33) represents the Euler condition consistent with the representative household's intertemporal optimality condition linking consumption levels over time. It is also consistent with the resource constraint $Y_t = C_t + G_t$. The disturbance term arises from taste shocks that alter the marginal utility of consumption, shifts in government purchases, and shifts in the flexible-price equilibrium output. In each case, it is expected changes in ψ, g, and $\hat{y}^f$ that matter. For example, an expected rise in government purchases implies that future consumption must fall. This reduces current consumption.

The source of a disturbance term in the inflation adjustment equation is both more critical for policy analysis and more controversial (section 8.4 takes up policy analysis). It is easy to see why exogenous shifts in (8.20) can have important implications for policy. Two commonly assumed objectives of monetary policy are to maintain a low and stable average rate of inflation and to stabilize output around full employment. These two objectives are often viewed as presenting central banks with a tradeoff. A supply shock, such as an increase in oil prices, increases inflation and reduces output. To keep inflation from rising calls for contractionary policies that would exacerbate the decline in output; stabilizing output calls for expansionary policies that would worsen inflation. However, if the output objective is interpreted as meaning that output should be stabilized around its flexible-price equilibrium level, then (8.20) implies that the central bank can always achieve a zero output gap (i.e., keep output at its flexible-price equilibrium level) and simultaneously keep inflation equal to zero. Solving (8.20) forward yields

$$\pi_t = \kappa\sum_{i=0}^{\infty}\beta^i \mathrm{E}_t x_{t+i}.$$

By keeping current and expected future output equal to the flexible-price equilibrium level, $\mathrm{E}_t\hat{x}_{t+i} = 0$ for all i, and inflation remains equal to zero. Blanchard and Galí (2007) described this as the "divine coincidence." However, if an error term is added to the inflation adjustment equation so that

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t + e_t, \tag{8.34}$$

then

$$\pi_t = \kappa \sum_{i=0}^{\infty} \beta^i \mathrm{E}_t x_{t+i} + \sum_{i=0}^{\infty} \beta^i \mathrm{E}_t e_{t+i}.$$

As long as $\sum_{i=0}^{\infty} \beta^i \mathrm{E}_t e_{t+i} \neq 0$, maintaining $\sum_{i=0}^{\infty} \beta^i \mathrm{E}_t x_{t+i} = 0$ is not sufficient to ensure that inflation always remains equal to zero. A trade-off between stabilizing output and stabilizing inflation can arise. Disturbance terms in the inflation adjustment equation are often called *cost shocks* or *inflation shocks*. Since these shocks ultimately affect only the price level, they are also called *price shocks*.

Clarida, Galí, and Gertler (2001) suggested one means of introducing a stochastic shock into the inflation adjustment equation. They added a stochastic *wage markup* to represent deviations between the real wage and the marginal rate of substitution between leisure and consumption. Thus, the labor supply condition (8.9) becomes

$$\left(\frac{\chi N_t^{\eta}}{C_t^{-\sigma}}\right) e^{\mu_t^w} = \frac{W_t}{P_t},$$

where μ_t^w is a random disturbance.[25] This could arise from shifts in tastes that affect the marginal utility of leisure. Or, if labor markets are imperfectly competitive, it could arise from stochastic shifts in the markup of wages over the marginal rate of substitution (Clarida, Galí, and Gertler 2002). Having linearized around the steady state, one obtains

$$\eta \hat{n}_t + \sigma \hat{c}_t + \mu_t^w = \hat{w}_t - \hat{p}_t. \tag{8.35}$$

The real marginal cost variable becomes

25. With the utility function given in (8.31), this becomes

$$\left(\frac{\chi N_t^{\eta}}{C_t^{-\sigma}}\right)\left(\frac{e^{\mu_t^w}}{\psi_t^{1-\sigma}}\right) = \frac{W_t}{P_t},$$

showing that μ_t^w affects the labor market condition in a manner similar to a taste shock.

$$\varphi_t = (\eta \hat{n}_t + \sigma \hat{c}_t) - (\hat{y}_t - \hat{n}_t) + \mu_t^w,$$

and this suggests that the inflation adjustment equation becomes

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \gamma \tilde{\kappa} x_t + \tilde{\kappa} \mu_t^w. \tag{8.36}$$

In this formulation, μ_t^w is the source of inflation shocks.

Although this approach appears to provide an explanation for a disturbance term to appear in the inflation adjustment equation, if μ_t^w reflects taste shocks that alter the marginal rate of substitution between leisure and consumption, then μ_t^w also affects the flexible-price equilibrium level of output. The same would be true if μ_t^w were a markup due to imperfect competition in the labor market. Thus, if the output gap variable in the inflation adjustment equation is correctly measured as the deviation of output from the flexible-price equilibrium level, μ_t^w no longer has a separate, independent impact on π_t.

Benigno and Woodford (2005) showed that a cost shock arises in the presence of stochastic variation in the gap between the welfare-maximizing level of output and the flexible-price equilibrium level of output. In the model developed so far, only two distortions are present, one due to monopolistic competition and one due to nominal price stickiness. The first distortion implies that the flexible-price output level is below the efficient output level even when prices are flexible. However, this wedge is constant, so when the model is linearized, percent deviations of the flexible-price output and the efficient output around their respective steady-state values are equal. If there are time-varying distortions such as would arise with stochastic variation in distortionary taxes, then fluctuations in the two output concepts will differ. In this case, if x_t^w is the percent deviation of the welfare-maximizing output level around its steady state (the welfare gap),

$$x_t = x_t^w + \delta_t,$$

where δ_t represents these stochastic distortions. Since policymakers would be concerned with stabilizing fluctuations in x_t^w, the relevant constraint the policymaker will face is obtained by rewriting the Phillips curve (8.20) as

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t^w + \kappa \delta_t. \tag{8.37}$$

In this formulation, δ_t acts as a cost shock; stabilizing inflation in the face of nonzero realizations of δ cannot be achieved without creating volatility in the welfare gap x_t^w. One implication of (8.37) is that the variance of the cost shock will depend on κ^2. Thus, if the degree of price rigidity is high, implying that κ is small, cost shocks will also be less volatile (see Walsh 2005a; 2005b).

Recent models, particularly those designed to be taken to the data, introduce a disturbance in the inflation equation by assuming that individual firms face random variation in the price elasticity of demand, that is, θ_t becomes time-varying (see 8.13). This modification raises similar issues to those arising with the introduction of a stochastic wage markup.

8.3.6 Sticky Wages and Prices

Erceg, Henderson, and Levin (2000) employed the Calvo specification to incorporate sticky wages *and* sticky prices into an optimizing framework.[26] The goods market side of their model is identical in structure to the one developed in section 8.3.2. However, they assumed that in the labor market individual households supply differentiated labor services; firms combine these labor services to produce output. Output is given by a standard production function, $F(N_t, K_t)$, but the labor aggregate is a composite function of the individual types of labor services:

$$N_t = \left[\int_0^1 n_{jt}^{(\gamma-1)/\gamma}\, dj\right]^{\gamma/(\gamma-1)}, \qquad \gamma > 1,$$

where n_{jt} is the labor from household j that the firm employs. With this specification, households face a demand for their labor services that depends on the wage they set relative to the aggregate wage rate. Erceg, Henderson, and Levin assumed that a randomly drawn fraction of households optimally set their wage each period, just as the models of price stickiness assume that only a fraction of firms adjust their price each period (see also Christiano, Eichenbaum, and Evans 2005; Sbordone 2001).

The model of inflation adjustment based on the Calvo specification implies that inflation depends on real marginal cost. In terms of deviations from the flexible-price equilibrium, real marginal cost equals the gap between the real wage (ω) and the marginal product of labor (*mpl*). Similarly, wage inflation (when linearized around a zero inflation steady state) responds to a gap variable, but this time the appropriate gap depends on a comparison between the real wage and the household's marginal rate of substitution between leisure and consumption. With flexible wages, as in the earlier sections where only prices were assumed to be sticky, workers are always on their labor supply curves; nominal wages can adjust to ensure that the real wage equals the marginal rate of substitution between leisure and consumption (*mrs*). When nominal wages are also sticky, however, ω_t and mrs_t can differ. If $\omega_t < mrs_t$, workers will want to raise their nominal wage when the opportunity to adjust arises. Letting π_t^w denote the rate of nominal wage inflation, Erceg, Henderson, and Levin showed that

26. Other models incorporating both wage and price stickiness include those of Guerrieri (2000); Ravenna (2000); Christiano, Eichenbaum, and Evans (2001); and Sbordone (2001; 2002). This is now standard in models being taken to the data.

$$\pi_t^w = \beta \mathrm{E}_t \pi_{t+1}^w + \kappa^w (mrs_t - \omega_t). \tag{8.38}$$

From the definition of the real wage,

$$\omega_t = \omega_{t-1} + \pi_t^w - \pi_t. \tag{8.39}$$

Equations (8.38) and (8.39), when combined with the new Keynesian Phillips curve in which inflation depends on $\omega_t - mpl_t$, constitute the inflation adjustment block of an optimizing model with both wage and price rigidities.

8.4 Monetary Policy Analysis in New Keynesian Models

During the ten years after its introduction, the new Keynesian model became the standard framework for monetary policy analysis. Clarida, Galí, and Gertler (1999), Woodford (2003a), McCallum and Nelson (1999), and Svensson and Woodford (1999; 2005), among others, popularized this simple model for use in monetary policy analysis. Galí (2002) and Galí and Gertler (2007) discussed some of the model's implications for monetary policy, and Galí (2008) provided an excellent treatment of the model and its implications for policy.

As noted in section 8.3, the basic new Keynesian model takes the form

$$x_t = \mathrm{E}_t x_{t+1} - \left(\frac{1}{\sigma}\right)(i_t - \mathrm{E}_t \pi_{t+1}) + u_t \tag{8.40}$$

and

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t + e_t, \tag{8.41}$$

where x is the output gap, defined as output relative to the equilibrium level of output under flexible prices, i is the nominal rate of interest, and π is the inflation rate. All variables are expressed as percentage deviations around their steady-state values. The demand disturbance u can arise from taste shocks to the preferences of the representative household, fluctuations in the flexible-price equilibrium output level, or shocks to government purchases of goods and services. The e shock is a *cost shock*. In this section, (8.40) and (8.41) are used to address issues of monetary policy design.

8.4.1 Policy Objectives

Given the economic environment that leads to (8.40) and (8.41), what are the appropriate objectives of the central bank? There is a long history in monetary policy analysis of assuming that the central bank is concerned with minimizing a quadratic loss function that depends on output and inflation. Models that assume this were discussed in chapter 7. Although such an assumption is plausible, it is ultimately ad hoc. In the new Keynesian model, the description of the economy is based on an approximation to a fully specified general equilibrium model. Can one therefore develop a policy objective function that can be interpreted as an approximation to the utility of the representative household? Put differently, can one draw insights from the general equilibrium foundations of (8.40) and (8.41) to determine the basic objectives central banks should pursue? Woodford (2003a), building on earlier work by Rotemberg and Woodford (1998), provided the most detailed analysis of the link between a welfare criterion derived as an approximation to the utility of the representative agent and the types of quadratic loss functions common in the older literature.

Woodford assumed that there is a continuum of differentiated goods c_{it} defined on the interval $[0, 1]$ and that the representative household derives utility from consuming a composite of these individual goods. The composite consumption good is defined as

$$Y_t = C_t = \left[\int_0^1 c_{jt}^{(\theta-1)/\theta}\, dj\right]^{\theta/(\theta-1)}. \tag{8.42}$$

In addition, each household produces one of these individual goods and experiences disutility from production. Suppose labor effort is proportional to output. Woodford assumed that the period utility of the representative agent is then

$$V_t = U(Y_t, z_t) - \int_0^1 v(c_{jt}, z_t)\, dj, \tag{8.43}$$

where $v(c_{jt}, z_t)$ is the disutility of producing good c_{jt}, and z_t is a vector of exogenous shocks.[27] Woodford demonstrated that deviations of the expected discounted utility of the representative agent around the level of steady-state utility can be approximated by

$$\mathrm{E}_t \sum_{i=0}^{\infty} \beta^i V_{t+i} \approx -\Omega \mathrm{E}_t \sum_{i=0}^{\infty} \beta^i [\pi_{t+i}^2 + \lambda(x_{t+i} - x^*)^2] + \text{t.i.p.}, \tag{8.44}$$

where t.i.p. indicates terms independent of policy. The derivation of (8.44) and the values of Ω and λ are given in section 8.6.2 of the chapter appendix. In (8.44), x_t is the gap between output and the output level that would arise under flexible prices, and x^* is the gap between the steady-state efficient level of output (in the absence of the monopolistic distortions) and the actual steady-state level of output.

27. Woodford considered a cashless economy, so real money balances do not appear in the utility function as they did in (8.1).

Equation (8.44) looks like the standard quadratic loss function employed in chapter 7 to represent the objectives of the monetary policy authority. There are, however, two critical differences. First, the output gap is measured relative to equilibrium output under flexible prices. In the traditional literature the output variable was more commonly interpreted as output relative to trend or output relative to the natural rate of output, which in turn was often defined as output in the absence of price surprises (see section 6.2.1).

A second difference between (8.44) and a standard quadratic loss function arises from the reason inflation variability enters the loss function. When prices are sticky, and firms do not all adjust simultaneously, inflation results in an inefficient dispersion of relative prices and production among individual producers. The representative household's utility depends on its consumption of a composite good; faced with a dispersion of prices for the differentiated goods produced in the economy, the household buys more of the relatively cheaper goods and less of the relatively more expensive goods. Because of diminishing marginal utility, the increase in utility derived from consuming more of some goods is less than the loss in utility due to consuming less of the more expensive goods. Hence, price dispersion reduces utility. Similarly, if one assumes diminishing returns to labor in the production process rather than constant returns to scale, dispersion on the production side will also be costly. The increased cost of producing more of some goods is greater than the cost saving from reducing production of other goods. For these reasons, price dispersion reduces utility, and when each firm does not adjust its price every period, price dispersion is caused by inflation. These welfare costs can be eliminated under a zero inflation policy.

In chapter 7, the efficiency distortion represented by x^* was used to motivate an overly ambitious output target in the central bank's objective function. The presence of $x^* > 0$ implies that a central bank acting under discretion to maximize (8.44) would produce a positive average inflation bias. However, with average rates of inflation in the major industrialized economies remaining low during the 1990s, many authors now simply assume that $x^* = 0$. In this case, the central bank is concerned with stabilizing the output gap x_t, and no average inflation bias arises.[28] If tax subsidies can be used to offset the distortions associated with monopolistic competition, one could assign fiscal policy the task of ensuring that $x^* = 0$. In this case, the central bank has no incentive to create inflationary expansions, and average inflation will be zero under discretion. Dixit and Lambertini (2002) showed that when both the monetary and fiscal authorities are acting optimally, the fiscal authority will use its tax

28. In addition, the inflation equation was derived by linearizing around a zero inflation steady state. It would thus be inappropriate to use it to study situations in which the average is positive.

instruments to set $x^* = 0$, and the central bank then ensures that inflation remains equal to zero.

In the context of the linear-quadratic model, (8.44) represents a second-order approximation to the welfare of the representative agent around the steady state. Expanding the period loss function,

$$\pi_{t+i}^2 + \lambda(x_{t+i} - x^*)^2 = \pi_{t+i}^2 + \lambda x_{t+i}^2 - 2\lambda x^* x_{t+i} + \lambda(x^*)^2.$$

Employing a first-order approximation for the structural equations will be adequate for evaluating the π_{t+i}^2 and x_{t+i}^2 terms, because any higher-order terms in the structural equations would become of order greater than 2 when squared. However, this is not the case for the $2\lambda x^* x_{t+i}$ term, which is linear in x_{t+i}. Hence, to approximate this correctly to the required degree of accuracy would require second-order approximations to the structural equations rather than the linear approximations derived in (8.40) and (8.41). Thus, assume the fiscal authority employs a subsidy to undo the distortion arising from imperfect competition so that $x^* = 0$. In this case, the linear approximations to the structural equations will allow correct evaluation of the second-order approximation to welfare. See Benigno and Woodford (2005) for a discussion of optimal policy when $x^* > 0$.

8.4.2 Policy Trade-offs

The basic new Keynesian inflation adjustment equation given by (8.20) did not include a disturbance term, such as the e_t that was added to (8.41). The absence of e implies that there is no conflict between a policy designed to maintain inflation at zero and a policy designed to keep the output gap equal to zero. If $x_{t+i} = 0$ for all $i \geq 0$, then $\pi_{t+i} = 0$. In this case, a central bank that wants to maximize the expected utility of the representative household will ensure that output is kept equal to the flexible-price equilibrium level of output. This also guarantees that inflation is equal to zero, thereby eliminating the costly dispersion of relative prices that arises with inflation. When firms do not need to adjust their prices, the fact that prices are sticky is no longer relevant. Thus, a key implication of the basic new Keynesian model is that price stability is the appropriate objective of monetary policy.[29]

The optimality of zero inflation conflicts with the Friedman rule for optimal inflation. M. Friedman (1969) concluded that the optimal inflation rate must be negative to make the nominal rate of interest zero (see chapter 4). The reason a different conclusion is reached here is the absence of any explicit role for money when the utility approximation given by (8.44) is derived. In general, zero inflation still generates

29. Notice that the conclusion that price stability is optimal is independent of the degree of nominal rigidity (see Adao, Correia, and Teles 1999).

a monetary distortion. With zero inflation, the nominal rate of interest will be positive and the private opportunity cost of holding money will exceed the social cost of producing it. A. Khan, King, and Wolman (2000) and Adao, Correia, and Teles (2003) considered models that integrate nominal rigidities and the Friedman distortion. Khan, King, and Wolman introduced money into a sticky price model by assuming the presence of cash and credit goods, with money required to purchase cash goods. If prices are flexible, it is optimal to have a rate of deflation such that the nominal interest rate is zero. If prices are sticky, price stability is optimal in the absence of the cash-in-advance (CIA) constraint. With both sticky prices and the monetary inefficiency associated with a positive nominal interest rate, the optimal rate of inflation is less than zero but greater than the rate that yields a zero nominal interest rate. Khan, King, and Wolman conducted simulations in a calibrated version of their model and found that the relative price distortion dominates the Friedman monetary inefficiency. Thus, the optimal policy is close to the policy that maintains price stability.

In the baseline model with no monetary distortion and with $x^* = 0$, the optimality of price stability is a reflection of the presence of only one nominal rigidity. The welfare costs of a single nominal rigidity can be eliminated using the single instrument provided by monetary policy. As discussed in section 8.3.6, Erceg, Henderson, and Levin (2000) introduced nominal wage stickiness into the basic new Keynesian framework as a second nominal rigidity. Nominal wage inflation with staggered adjustment of wages causes distortions of relative wages and reduces welfare. Erceg, Henderson, and Levin showed that in this case the approximation to the welfare of the representative agent will include a term in wage inflation as well as the inflation and output gap terms appearing in (8.44). Wage stability is desirable because it eliminates dispersion of hours worked across households. With two distortions—sticky prices and sticky wages—the single instrument of monetary policy cannot simultaneously offset both distortions. With sticky prices but flexible wages, the real wage can adjust efficiently in the face of productivity shocks, and monetary policy should maintain price stability. With sticky wages and flexible prices, the real wage can still adjust efficiently to ensure that labor market equilibrium is maintained in the face of productivity shocks, and monetary policy should maintain nominal wage stability. If both wages and prices are sticky, a policy that stabilizes either prices or wages will not allow the real wage to move so as to keep output equal to the flexible-price output. Productivity shocks will lead to movements in the output gap, and the monetary authority will be forced to trade off stabilizing inflation, wage inflation, and the output gap.

Galí, Gertler, and López-Salido (2002) defined the *inefficiency gap* as the gap between the household's marginal rate of substitution between leisure and consumption (mrs_t) and the marginal product of labor (mpl_t). This inefficiency gap can be divided into two parts: the wedge between the real wage and the marginal rate of substitution, labeled the *wage markup*, and the wedge between the real wage and the marginal product of labor, labeled the *price markup*. Based on U.S. data, they concluded that the wage markup accounts for most of the time series variation in the inefficiency gap. Levin et al. (2006) estimated a new Keynesian general equilibrium model with both price and wage stickiness. They found that the welfare costs of nominal rigidity are primarily generated by wage stickiness rather than by price stickiness. This finding is consistent with Christiano, Eichenbaum, and Evans (2005), who concluded that a model with flexible prices and sticky wages does better at fitting impulse responses estimated on U.S. data than a sticky price–flexible wage version of their model. Sbordone (2001) also suggested that nominal wage rigidity is more important empirically than price rigidity. Huang and Liu (2002) argued that wage stickiness is more important than price stickiness for generating output persistence.

In contrast, Goodfriend and King (2001) argued that the long-term nature of employment relationships reduces the effects of nominal wage rigidity on real resource allocations. Models that incorporate the intertemporal nature of employment relationships based on search and matching models of unemployment include Walsh (2003a; 2005b); Trigari (2009); Krause, López-Salido, and Lubik (2007); Thomas (2008); Ravenna and Walsh (2008); and Sala, Söderström, and Trigari (2008).

8.4.3 Optimal Commitment and Discretion

Suppose the central bank attempts to minimize a quadratic loss function such as (8.44), defined in terms of inflation and output relative to the flexible-price equilibrium.[30] Assume that the steady-state gap between output and its efficient value is zero (i.e., $x^* = 0$). In this case, the central bank's loss function takes the form

$$L_t = \left(\frac{1}{2}\right) \mathrm{E}_t \sum_{i=0}^{\infty} \beta^i (\pi_{t+i}^2 + \lambda x_{t+i}^2). \tag{8.45}$$

Two alternative policy regimes can be considered (see chapter 7). In a discretionary regime, the central bank behaves optimally in each period, taking as given the current state of the economy and private sector expectations. Given that the public knows that the central bank optimizes each period, any promises the central bank makes about future inflation will not be credible—the public knows that whatever may have been promised in the past, the central bank will do what is optimal at the time it sets policy. The alternative regime is one of commitment. In a commitment regime, the central bank can make credible promises about what it will do in the

30. Svensson (1999b; 1999d) argued that there is widespread agreement among policymakers and academics that inflation stability and output gap stability are the appropriate objectives of monetary policy.

future. By promising to take certain actions in the future, the central bank can influence the public's expectations about future inflation. When forward-looking expectations play a role, as in (8.41), discretion will lead to what is known as a *stabilization bias*.

Commitment

A central bank able to precommit chooses a path for current and future inflation and the output gap to minimize the loss function (8.45) subject to the expectational IS curve (8.40) and the inflation adjustment equation (8.41). Let θ_{t+i} and ψ_{t+i} denote the Lagrangian multipliers associated with the period $t+i$ IS curve and the inflation adjustment equation. The central bank's objective is to pick i_{t+i}, π_{t+i}, and x_{t+i} to minimize

$$\mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \left\{ \left(\frac{1}{2}\right)(\pi_{t+i}^2 + \lambda x_{t+i}^2) + \theta_{t+i}[x_{t+i} - x_{t+i+1} + \sigma^{-1}(i_{t+i} - \pi_{t+i+1}) - u_{t+i}] \right.$$
$$\left. + \psi_{t+i}(\pi_{t+i} - \beta\pi_{t+i+1} - \kappa x_{t+i} - e_{t+i}) \right\}.$$

The first-order condition for i_{t+i} takes the form

$$\sigma^{-1}\mathrm{E}_t(\theta_{t+i}) = 0, \qquad i \geq 0.$$

Hence, $\mathrm{E}_t\theta_{t+i} = 0$ for all $i \geq 0$. This result implies that (8.40) imposes no real constraint on the central bank as long as there are no restrictions on, or costs associated with, varying the nominal interest rate. Given the central bank's optimal choices for the output gap and inflation, (8.40) will simply determine the setting for i_t necessary to achieve the desired value of x_t. For that reason, it is often more convenient to treat x_t as if it were the central bank's policy instrument.

Setting $\mathrm{E}_t\theta_{t+i} = 0$, the remaining first-order conditions for π_{t+i} and x_{t+i} can be written as

$$\pi_t + \psi_t = 0 \tag{8.46}$$

$$\mathrm{E}_t(\pi_{t+i} + \psi_{t+i} - \psi_{t+i-1}) = 0, \qquad i \geq 1 \tag{8.47}$$

$$\mathrm{E}_t(\lambda x_{t+i} - \kappa\psi_{t+i}) = 0, \qquad i \geq 0. \tag{8.48}$$

Equations (8.46) and (8.47) reveal the dynamic inconsistency that characterizes the optimal precommitment policy. At time t, the central bank sets $\pi_t = -\psi_t$ and promises to set $\pi_{t+1} = -(\psi_{t+1} - \psi_t)$. But when period $t+1$ arrives, a central bank that reoptimizes will again obtain $\pi_{t+1} = -\psi_{t+1}$ as its optimal setting for inflation. That is, the first-order condition (8.46) updated to $t+1$ will reappear.

An alternative definition of an optimal precommitment policy requires that the central bank implement conditions (8.47) and (8.48) for all periods, including the current period. Woodford (2003a; 2003b) has labeled this the *timeless perspective* approach to precommitment. One can think of such a policy as having been chosen in the distant past, and the current values of the inflation rate and output gap are the values chosen from that earlier perspective to satisfy the two conditions (8.47) and (8.48). McCallum and Nelson (2000b) provided further discussion of the timeless perspective and argued that this approach agrees with the one commonly used in many studies of precommitment policies.

Combining (8.47) and (8.48), under the timeless perspective optimal commitment policy inflation and the output gap satisfy

$$\pi_{t+i} = -\left(\frac{\lambda}{\kappa}\right)(x_{t+i} - x_{t+i-1}) \tag{8.49}$$

for all $i \geq 0$. Using this equation to eliminate inflation from (8.41) and rearranging, one obtains

$$\left(1 + \beta + \frac{\kappa^2}{\lambda}\right)x_t = \beta \mathrm{E}_t x_{t+1} + x_{t-1} - \frac{\kappa}{\lambda}e_t. \tag{8.50}$$

The solution to this expectational difference equation for x_t will be of the form $x_t = a_x x_{t-1} + b_x e_t$. To determine the coefficients a_x and b_x, note that if $e_t = \rho e_{t-1} + \varepsilon_t$, the proposed solution implies $\mathrm{E}_t x_{t+1} = a_x x_t + b_x \rho e_t = a_x^2 x_{t-1} + (a_x + \rho) b_x e_t$. Substituting this into (8.50) and equating coefficients, the parameter a_x is the solution less than 1 of the quadratic equation

$$\beta a_x^2 - \left(1 + \beta + \frac{\kappa^2}{\lambda}\right)a_x + 1 = 0,$$

and b_x is given by

$$b_x = -\left\{\frac{\kappa}{\lambda[1 + \beta(1 - \rho - a_x)] + \kappa^2}\right\}.$$

From (8.49), equilibrium inflation under the timeless perspective policy is

$$\pi_t = \left(\frac{\lambda}{\kappa}\right)(1 - a_x)x_{t-1} + \left[\frac{\lambda}{\lambda[1 + \beta(1 - \rho - a_x)] + \kappa^2}\right]e_t. \tag{8.51}$$

Woodford (2003b) stressed that even if $\rho = 0$, so that there is no natural source of persistence in the model itself, $a_x > 0$ and the precommitment policy introduces inertia into the output gap and inflation processes. Because the central bank responds to the lagged output gap (see (8.49)), past movements in the gap continue to affect current inflation. This commitment to inertia implies that the central bank's actions at date t allow it to influence expected future inflation. Doing so leads to a better trade-off between gap and inflation variability than would arise if policy did not react to the lagged gap. Equation (8.41) implies that the inflation impact of a positive cost shock, for example, can be stabilized at a lower output cost if the central bank can induce a fall in expected future inflation. Such a fall in expected inflation is achieved when the central bank follows (8.49).

A condition for policy such as (8.49) that is derived from the central bank's first-order conditions and only involves variables that appear in the objective function (in this case, inflation and the output gap) is generally called a *targeting rule* (e.g., Svensson and Woodford 2005). It represents a relationship among the targeted variables that the central bank should maintain, because doing so is consistent with the first-order conditions from its policy problem.

Because the timeless perspective commitment policy is not the solution to the policy problem under optimal commitment, the policy rule given by (8.49) may be dominated by other policy rules. For instance, it may be dominated by the optimal discretion policy (see next section). Under the timeless perspective, inflation as given by (8.49) is the same function each period of the current and lagged output gap; the policy displays the property of continuation in the sense that the policy implemented in any period continues the plan to which it was optimal to commit in an earlier period. Blake (2001); Damjanovic, Damjanovic, and Nolan (2008); and C. Jensen and McCallum (2008) considered *optimal* continuation policies that require that the policy instrument, in this case x_t, be a time-invariant function, as under the timeless perspective, but rather than ignoring the first-period conditions, as is done under the timeless perspective, they focused on the optimal unconditional continuation policy to which the central bank should commit. This policy minimizes the unconditional expectation of the objective function, so that the Lagrangian for the policy problem becomes

$$\tilde{\mathrm{E}}\mathcal{L} = \tilde{\mathrm{E}}\left\{ \mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \left[\frac{1}{2}(\pi_{t+i}^2 + \lambda x_{t+i}^2) + \theta_{t+i}(\pi_{t+i} - \beta \mathrm{E}_t \pi_{t+i+1} - \kappa x_{t+i} - e_{t+i}) \right] \right\},$$

where $\tilde{\mathrm{E}}$ denotes the unconditional expectations operator. Because

$$\tilde{\mathrm{E}}\mathrm{E}_t \theta_{t+i} \pi_{t+i+1} = \tilde{\mathrm{E}} \theta_{t-1} \pi_t,$$

the unconditional Lagrangian can be expressed as

$$\tilde{\mathrm{E}}\mathcal{L} = \left(\frac{1}{1-\beta}\right) \tilde{\mathrm{E}}\left\{ \left[\frac{1}{2}(\pi_t^2 + \lambda x_t^2) + \theta_t \pi_t - \beta \theta_{t-1} \pi_t - \kappa \theta_t x_t - \theta_t e_t \right] \right\}.$$

The first-order conditions then become

$$\pi_t + \theta_t - \beta\theta_{t-1} = 0 \tag{8.52}$$

$$\lambda x_t - \kappa\theta_t = 0.$$

Combining these to eliminate the Lagrangian multiplier yields the optimal unconditional continuation policy:

$$\pi_t = -\left(\frac{\lambda}{\kappa}\right)(x_{t+i} - \beta x_{t+i-1}). \tag{8.53}$$

Comparing this to (8.49) shows that rather than giving full weight to past output gaps, the optimal unconditional continuation policy discounts the past slightly (recall $\beta \approx 0.99$).

Discretion

When the central bank operates with discretion, it acts each period to minimize the loss function (8.45) subject to the inflation adjustment equation (8.41). Because the decisions of the central bank at date t do not bind it at any future dates, the central bank is unable to affect the private sector's expectations about future inflation. Thus, the decision problem of the central bank becomes the single-period problem of minimizing $\pi_t^2 + \lambda x_t^2$ subject to the inflation adjustment equation (8.41).

The first-order condition for this problem is

$$\kappa\pi_t + \lambda x_t = 0. \tag{8.54}$$

Equation (8.54) is the optimal targeting rule under discretion. Notice that by combining (8.46) with (8.48) evaluated at time t, one obtains (8.54); thus, the central bank's first-order condition relating inflation and the output gap at time t is the same under discretion or under the fully optimal precommitment policy (but not under the timeless perspective policy). The differences appear in subsequent periods. For $t+1$, under discretion $\kappa\pi_{t+1} + \lambda x_{t+1} = 0$, whereas under precommitment (from (8.47) and (8.48)), $\kappa\pi_{t+1} + \lambda(x_{t+1} - x_t) = 0$.

The equilibrium expressions for inflation and the output gap under discretion can be obtained by using (8.54) to eliminate inflation from the inflation adjustment equation. This yields

$$\left(1 + \frac{\kappa^2}{\lambda}\right)x_t = \beta \mathrm{E}_t x_{t+1} - \left(\frac{\kappa}{\lambda}\right)e_t. \tag{8.55}$$

Guessing a solution of the form $x_t = \delta e_t$, so that $\mathrm{E}_t x_{t+1} = \delta\rho e_t$, one obtains

$$\delta = -\left[\frac{\kappa}{\lambda(1 - \beta\rho) + \kappa^2}\right].$$

Equation (8.54) implies that equilibrium inflation under optimal discretion is

$$\pi_t = -\left(\frac{\lambda}{\kappa}\right)x_t = \left[\frac{\lambda}{\lambda(1-\beta\rho)+\kappa^2}\right]e_t. \tag{8.56}$$

According to (8.56) the unconditional expected value of inflation is zero; there is no average inflation bias under discretion. However, there is a stabilization bias in that the response of inflation to a cost shock under discretion differs from the response under commitment. This can be seen by comparing (8.56) to (8.51).

Discretion versus Commitment

The impact of a cost shock on inflation and the output gap under the timeless perspective optimal precommitment policy and optimal discretionary policy can be obtained by calibrating (8.41) and (8.49) and solving them numerically. Four unknown parameters appear in the model: β, κ, λ, and ρ. The discount factor, β, is set equal to 0.99, appropriate for interpreting the time interval as one quarter. A weight on output fluctuations of $\lambda = 0.25$ is used. This value is also used by H. Jensen (2002) and McCallum and Nelson (2000b).[31] The parameter κ captures both the impact of a change in real marginal cost on inflation and the co-movement of real marginal cost and the output gap and is set equal to 0.05. McCallum and Nelson reported that the empirical evidence is consistent with a value of κ in the range $[0.01, 0.05]$. Roberts (1995) reported higher values; his estimate of the coefficient on the output gap is about 0.3 when inflation is measured at an annual rate, so this translates into a value for κ of 0.075 for inflation at quarterly rates. Jensen used a baseline value of $\kappa = 0.1$, whereas Walsh (2003b) used 0.05.

The solid lines in figures 8.2 and 8.3 show the response of the output gap and inflation to a transitory, one standard deviation cost push shock under the optimal precommitment policy.[32] Despite the fact that the shock itself has no persistence, the output gap displays strong positive serial correlation. By keeping output below potential (a negative output gap) for several periods into the future after a positive cost shock, the central bank is able to lower expectations of future inflation. A fall in $E_t\pi_{t+1}$ at the time of the positive inflation shock improves the trade-off between inflation and output gap stabilization faced by the central bank.

Outcomes under optimal discretion are shown by the dashed lines in the figures. There is no inertia under discretion; both the output gap and inflation return to their steady-state values in the period after the shock occurs. The difference in the stabilization response under commitment and discretion is the stabilization bias due to discretion. The intuition behind the suboptimality of discretion can be seen by con-

31. If (8.45) is interpreted as an approximation to the welfare of the representative agent, the implied value of λ would be much smaller.

32. The programs used to obtain these figures are available at ⟨http://people.ucsc.edu/~walshc/mtp3e/⟩.

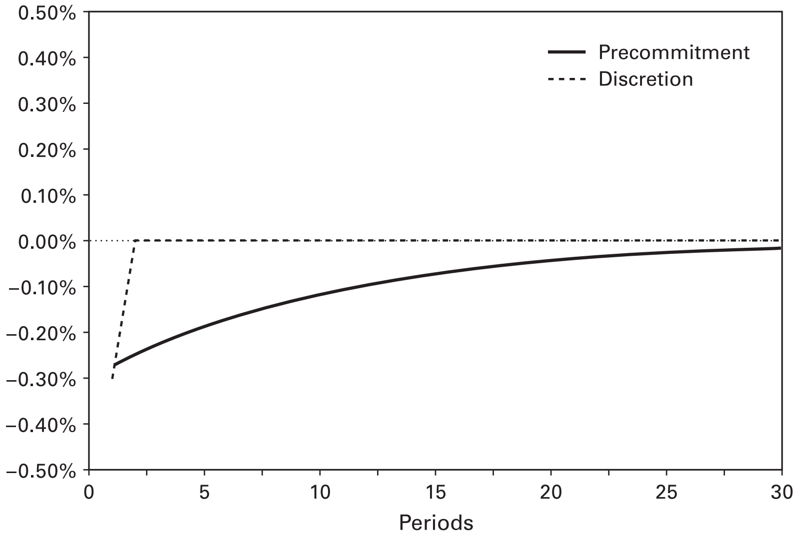

Figure 8.2
Response of output gap to a cost shock: timeless precommitment and pure discretion.

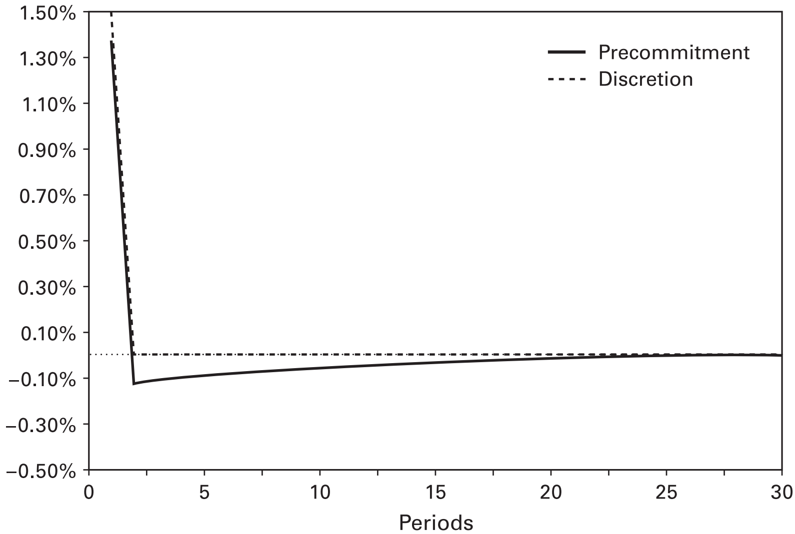

Figure 8.3
Response of inflation to a cost shock: timeless precommitment and pure discretion.

sidering the inflation adjustment equation given by (8.41). Under discretion, the central bank's only tool for offsetting the effects on inflation of a cost shock is the output gap. In the face of a positive realization of e_t, x_t must fall to help stabilize inflation. Under commitment, however, the central bank has two instruments; it can affect both x_t and $\mathrm{E}_t\pi_{t+1}$. By creating expectations of a deflation at $t+1$, the reduction in the output gap does not need to be as large. Of course, under commitment a promise of future deflation must be honored, so actually inflation falls below the baseline beginning in period $t+1$ (see figure 8.3). Consistent with producing a deflation, the output gap remains negative for several periods.

The analysis so far has focused on the goal variables, inflation and the output gap. Using (8.40), the associated setting for the interest rate can be derived. For example, under optimal discretion, the output gap is given by

$$x_t = -\left[\frac{\kappa}{\lambda(1-\beta\rho)+\kappa^2}\right]e_t,$$

and inflation is given by (8.56). Using these to evaluate $\mathrm{E}_t x_{t+1}$ and $\mathrm{E}_t\pi_{t+1}$ and then solving for i_t from (8.40) yields

$$\begin{aligned} i_t &= \mathrm{E}_t\pi_{t+1} + \sigma(\mathrm{E}_t x_{t+1} - x_t + u_t) \\ &= \left[\frac{\lambda\rho + (1-\rho)\sigma\kappa}{\lambda(1-\beta\rho)+\kappa^2}\right]e_t + \sigma u_t. \end{aligned} \tag{8.57}$$

Equation (8.57) is the reduced-form solution for the nominal rate of interest. The nominal interest rate is adjusted to offset completely the impact of the demand disturbance u_t on the output gap. As a result, it affects neither inflation nor the output gap. Section 8.3.3 illustrated how a policy that commits to a rule that calls for responding to the exogenous shocks renders the new Keynesian model's equilibrium indeterminate. Thus, it is important to recognize that (8.57) describes the *equilibrium* behavior of the nominal interest rate under optimal discretion; (8.57) is not an instrument rule (see Svensson and Woodford 1999).

8.4.4 Commitment to a Rule

In the Barro-Gordon model (examined in chapter 7) optimal commitment was interpreted as commitment to a policy that was a (linear) function of the state variables. In the present model, consisting of (8.40) and (8.41), the only state variable is the current realization of the cost shock e_t. Suppose then that the central bank can commit to a rule of the form[33]

33. This commitment does not raise the same uniqueness of an equilibrium problem that would arise under a commitment to an instrument rule of the form $i_t = b_i e_t$. See problem 9 at the end of this chapter.

$$x_t = b_x e_t. \tag{8.58}$$

What is the optimal value of b_x? With x_t given by (8.58), inflation satisfies

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa b_x e_t + e_t,$$

and the solution to this expectational difference equation is[34]

$$\pi_t = b_\pi e_t, \qquad b_\pi = \frac{1 + \kappa b_x}{1 - \beta\rho}. \tag{8.59}$$

Using (8.58) and (8.59), the loss function can now be written as

$$\left(\frac{1}{2}\right) \mathrm{E}_t \sum_{i=0}^{\infty} \beta^i (\pi_{t+i}^2 + \lambda x_{t+i}^2) = \left(\frac{1}{2}\right) \sum_{i=0}^{\infty} \beta^i \left[\left(\frac{1 + \kappa b_x}{1 - \beta\rho} \right)^2 + \lambda b_x^2 \right] e_t^2.$$

This is minimized when

$$b_x = -\left[\frac{\kappa}{\lambda(1 - \beta\rho)^2 + \kappa^2} \right].$$

Using this solution for b_x in (8.59), equilibrium inflation is given by

$$\pi_t = \left(\frac{1 + \kappa b_x}{1 - \beta\rho} \right) e_t = \left[\frac{\lambda(1 - \beta\rho)}{\lambda(1 - \beta\rho)^2 + \kappa^2} \right] e_t. \tag{8.60}$$

Comparing the solution for inflation under optimal discretion, given by (8.56), and the solution under commitment to a simple rule, given by (8.60), one notes that they are identical if the cost shock is serially uncorrelated ($\rho = 0$). If $0 < \rho < 1$, there is a stabilization bias under discretion relative to the case of committing to a simple rule.

Clarida, Galí, and Gertler (1999) argued that this stabilization bias provides a rationale for appointing a Rogoff-conservative central banker—a central bank that puts more weight on inflation objectives than is reflected in the social loss function—when $\rho > 0$, even though in the present context there is no average inflation bias.[35] A

34. To verify this is the solution, note that

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa b_x e_t + e_t = \beta b_\pi \rho e_t + \kappa b_x e_t + e_t$$
$$= [\beta b_\pi \rho + \kappa b_x + 1] e_t,$$

so that $b_\pi = \beta b_\pi \rho + \kappa b_x + 1 = (\kappa b_x + 1)/(1 - \beta\rho)$.

35. Rogoff (1985) proposed appointing a conservative central banker as a way to solve the average inflation bias that can arise under discretionary policies (see chapter 7). There is no average inflation bias in the present model because it is assumed that $x^* = 0$, ensuring that the central bank's loss function depends on output only through the gap between actual output and flexible-price equilibrium output.

Rogoff-conservative central banker places a weight $\hat{\lambda} < \lambda$ on output gap fluctuations (see section 7.3.2). In a discretionary environment with such a central banker, (8.56) implies that inflation will equal

$$\pi_t = \left[\frac{\hat{\lambda}}{\hat{\lambda}(1-\beta\rho)+\kappa^2}\right]e_t.$$

Comparing this with (8.60) reveals that if a central banker is appointed for whom $\hat{\lambda} = \lambda(1-\beta\rho) < \lambda$, the discretionary solution will coincide with the outcome under commitment to the optimal simple rule. Such a central banker stabilizes inflation more under discretion than would be the case if the relative weight placed on output gap and inflation stability were equal to the weight in the social loss function, λ. Because the public knows inflation will respond less to a cost shock, future expected inflation rises less in the face of a positive e_t shock. As a consequence, current inflation can be stabilized, with a smaller fall in the output gap. The inflation-output trade-off is improved.

Recall, however, that the notion of commitment used here is actually suboptimal. As noted earlier, fully optimal commitment leads to inertial behavior in that future inflation depends not on the output gap but on the change in the gap.

8.4.5 Endogenous Persistence

Empirical research on inflation (see section 6.3.2) has generally found that when lagged inflation is added to (8.41), its coefficient is statistically and economically significant. If lagged inflation affects current inflation, then even under discretion the central bank faces a dynamic optimization problem; decisions that affect current inflation also affect future inflation, and this intertemporal link must be taken into account by the central bank when setting current policy. Svensson (1999d) and Vestin (2006) illustrated how the linear-quadratic structure of the problem allows one to solve for the optimal discretionary policy in the face of endogenous persistence.

To analyze the effects introduced when inflation depends on both expected future inflation and lagged inflation, suppose (8.41) is replaced by

$$\pi_t = (1-\phi)\beta \mathrm{E}_t\pi_{t+1} + \phi\pi_{t-1} + \kappa x_t + e_t. \tag{8.61}$$

The coefficient ϕ measures the degree of backward-looking behavior exhibited by inflation.[36] If the central bank's objective is to minimize the loss function given by

36. Galí and Gertler (1999), Woodford (2003), and Christiano, Eichenbaum, and Evans (2005) developed inflation adjustment equations in which lagged inflation appears by assuming that some fraction of firms do not reset their prices optimally (see section 6.3.2).