## Macroeconomics A: EI056

# Quizz

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## 1 Filtering

**Question**: The real business cycle literature uses "Hodrick-Prescott" filtering of the data. What does this procedure do?

Answer: The literature is interested in business cycles as opposed to growth, and thus we need to take the long-run growth out of the data. A simple way is to remove a linear trend, but this assumes that long-run growth is constant, which is unappealing. Instead they are long waves, such as the growth slowdown of the 1970's, that should not be interpreted as business cycles. The filter computes the long-run growth component by trading off a stable component (the most stable one is a linear trend) and fitting long waves.

The standard specification of the filter considers that output movements of a periodicity above 8 years or so are growth, and shorter components are the business cycle. An extension is the bypass filter method that also removes high frequency movements, for instance assuming that cycles of a quarterly frequency reflects data noise.

#### 2 Solow residuals

**Question**: What are the Solow residuals?

**Answer**: The Solow residuals are the part of output movements that cannot be accounted for by movements in capital and labor. Consider a Cobb-Douglas technology:

$$Y_t = \left(K_t\right)^{\alpha} \left(A_t L_t\right)^{1-\alpha}$$

In logs we have:

$$\ln\left(Y_{t}\right) = \alpha \ln\left(K_{t}\right) + (1 - \alpha) \ln\left(L_{t}\right) + (1 - \alpha) \ln\left(A_{t}\right)$$

The growth rate of a variable between period t and t+1 is the difference between its log values:  $g(Y_t) = \ln(Y_t) - \ln(Y_{t-1})$ , thus:

$$g(Y_t) = \alpha g(K_t) + (1 - \alpha) g(L_t) + (1 - \alpha) g(A_t)$$

The Solow residuals are the part of  $g(Y_t)$  that is not accounted by capital or labor:

(Solow Residual)<sub>t</sub> = 
$$(1 - \alpha) g(A_t) = g(Y_t) - [\alpha g(K_t) + (1 - \alpha) g(L_t)]$$

The Solow residuals are really a "measure of our ignorance", i.e. the part of output that we cannot explain with labor and capital. While the baseline RBC treats them as a measure of productivity (though richer RBC models do not literally do so), this is not appealing. The Solow residuals are very volatile, with large negative values. It is hard to believe that productivity moves that much, and that there are episodes where the economy literally becomes substantially worse at producing.

The residuals are affected by the intensity of factor use. Consider labor hoarding. A firm faces a temporary slowdown in demand. It could fire workers, but instead choose to keep them even though there is less work to do. This allows the firm to have the workers ready when business picks up in the future, instead of having to hire new workers and train them. We therefore see a constant labor input and lower output, which looks like a fall in productivity. This is true in the sense that output per worker is down, but not in the sense that workers became less competent at their task.

### 3 Stylized facts

**Question**: What are the main business cycle facts, and the great ratios, that a model should match?

Answer: The various components of output are not equally volatile. Consumption (especially of non-durable) is less volatile than output. Investment (and consumption of durables such as cars) is more volatile. Total hours worked are as volatile as output, driven primarily by the number of workers with only little changes in hours per worker. Productivity (or more exactly the Solow residuals) move in step with output.

The great ratios show that over long samples the labor share of output (i.e. the total wage bill as a fraction of nominal output) is stable, the composition of output between investment and consumption is stable, and hours per workers are stable.

## 4 Intertemporal labor supply

Question: Explain how the labor supply can be understood in an intertemporal way.

Answer: The labor supply gives a relation between the current labor, current wage, and current consumption. Given current consumption, the equation is a positive relation between labor and the wage, and thus it can be seen as a static labor supply where a higher wage raises labor supply. This static view however is conditional on the level of consumption, and so we cannot just say that

higher wages raise the labor supply. In fact, if higher wages raised labor we would have a problem in the long run because the real wage shows a trend (reflecting productivity) but the number of hours worked per person does not. The influence of consumption in the labor supply is thus key, as it is the fact that rising consumption offsets the real wage (both grow at the trend rate of productivity) that keeps labor per capita constant in the long run.

The static labor supply is thus conditional on the level of consumption. But consumption reacts to shocks, and this reaction reflects the intertemporal allocation through the Euler condition. Combining the labor supply today and tomorrow and the Euler condition, we get a relation involving the current and future labor supply, the current and future wages, and the real interest rate. This dynamic representation shows that the relative wage (the ratio between current and future wage, adjusted by the real interest rate) affects the relative labor supply (the ratio between current and future hours worked). The relative labor supply increases (i.e. the number of hours worked today increases) only if the relative wage increases, i.e. the wage today is high relative to the wage tomorrow, so today is a good time to work. An increase in the relative wage can reflect a higher current wage, a lower future wage, or a higher real interest rate (which reduces the discounted value of the future wage).

The sensitivity of the labor reallocation in response to the real wage is called the elasticity of the labor supply. It plays a key role: to match the data the model requires a high value of this elasticity. But this is empirically questionable.

### 5 Key aspects of the RBC view

Question: What is the view of business cycles developed in the RBC framework?

**Answer**: The RBC framework views business cycles as efficient movements of the economy in response to productivity shocks (or other shocks). Sure this volatility is costly, but given that there are shocks the best response of the economy involves a volatile output.

A stable output would not be efficient, as it would imply producing similar amounts whether the economy is productive or not.

# 6 Extensions (extra)

**Question**: Is the baseline model successful? What are the extensions of the "high elasticity" model compared to the baseline one? Why are they useful?

Answer: The baseline model generates output volatility in line with the data, but does not generate enough movements in employment (because the elasticity of the labor supply with respect to wages is limited). Generating enough volatility of labor requires a high intertemporal substitutability, which is not empirically realistic based on micro studies. In addition the Solow residuals imply that there are frequent periods where the economy's productivity actually declines, which is hard to believe.

The high volatility model makes aggregate labor more elastic than individual labor, and allows for a variable intensity of use of capital. This generates a more appealing level of employment volatility, and variable utilization amplifies the impact of productivity shocks, so the model does not require variations of productivity that are empirically unrealistic.

One way to make labor supply more elastic is to go beyond the representative agent. To generate realistic movements, the RBC model require a high elasticity of labor supply. Micro studies however find that the elasticity at the individual level is much lower. A model that literally considers a representative agent then has a hard time fitting the data, as our representative individual is not that responsive to wages.

To get around this problem, we take a model where the aggregate individual is not identical to a specific individual. If individual agents can choose the hours worked as a function of the wage, they are not much responsive to the wage, i.e. the individual elasticity of labor supply is low, as the micro studies show. This limited elasticity reflects the fact that the marginal cost of working increases quickly as labor increases, thus a moderately higher wage does not lead to the worker supplying a large amount of additional labor.

Agents however cannot choose how many hours they work. Instead, an individual either work a set amount of hours, H, or not at all. Variations in the labor input are then done through the number of workers (the so-called "extensive margin") instead of the amount of hours per worker (the so-called "intensive margin"). Aggregating the situation across individuals, we get an aggregate utility that is linear in labor. A linear utility implies that the marginal cost of working does not increase as the labor input increases, thus in the aggregate the economy behaves like a representative agent that has a higher elasticity of labor supply than individual workers.