Macroeconomics B, EI060

Class 4

Extra slides on portfolio allocation

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Introduction

- Alternative presentation of the portfolio choice, starting at slide 21 in the class.
- The first three slides below as in the class (slides 22-24).
- The following slides present the solution in a different and simpler way than in class.
- The last two slides are the same as in class (slides 30-31).

Trading in bond and equity

- Endowment in period 1: output Y_1^H and today's value of the future output V_1^H.
- Invest in a bond and equity: claims on each country's output. Home households buys B_2^Hbonds and:
  - x_2^{HH} units of the Home equity. Price V_1^H, each unit pays of the endowment Y_2^H.
  - x_2^{HF} units of the Foreign equity. Price V_1^F, pays of the endowment Y_2^F.
- The Foreign household purchases $x_2^{FH} = 1 - x_2^{HH}$ and $x_2^{FF} = 1 - x_2^{HF}$ units of equity, and $B_2^F = -B_2^H$ bonds.
- Budget constraints (before we imposed $x_2^{HH} = 1$ and $x_2^{HF} = 0$):

$$\begin{aligned} C_1^H &= Y_1^H + V_1^H - B_2^H - x_2^{HH} V_1^H - x_2^{HF} V_1^F \\ C_2^H(k) &= (1+r) B_2^H + x_2^{HH} Y_2^H(k) + x_2^{HF} Y_2^F(k) \end{aligned}$$

Optimal allocation

- Three Euler conditions: bond, Home equity, Foreign equity:

$$\begin{aligned} u'\left(C_1^H\right) &= \beta\left(1+r\right)E\left[u'\left(C_2^H\right)\right] \\ u'\left(C_1^H\right) &= \beta E\left[u'\left(C_2^H\right)\frac{Y_2^H}{V_1^H}\right] \\ u'\left(C_1^H\right) &= \beta E\left[u'\left(C_2^H\right)\frac{Y_2^F}{V_1^F}\right] \end{aligned}$$

- Expected discounted excess return of an asset is zero:

$$0 = E\left[m_{1,2}^H\left(\frac{Y_2^H}{V_1^H}-\left(1+r\right)\right)\right] \;\; ; \;\; 0 = E\left[m_{1,2}^H\left(\frac{Y_2^F}{V_1^F}-\left(1+r\right)\right)\right]$$

Asset pricing

- Asset price indicates a hedging property:

$$V_1^H = E\left[m_{1,2}^H Y_2^H\right] \quad ; \quad V_1^F = E\left[m_{1,2}^H Y_2^F\right]$$

- As $E(ab) = E(a)E(b) + Cov(a,b)$ we write:

$$V_1^F = E\left[m_{1,2}^H\right] E\left[Y_2^F\right] + Cov\left[m_{1,2}^H, Y_2^F\right]$$

- Foreign equity is more valuable if:
  - Foreign output is expected to be high on average
  - Foreign output is abundant when consumption is valued ($m_{1,2}^H$ is high), it is a good hedge.

Full risk sharing

- There are two equities, each linked to a specific output. We thus have complete asset markets.
- This means that Home and Foreign consumption move in parallel across time and states of nature.
- Home consumption is a constant share μ^H of world output:

$$\begin{aligned} C_2^H(k) &= \mu^H \left(C_2^H(k) + C_2^F(k) \right) \\ C_2^H(k) &= \mu^H Y_2^W(k) \end{aligned}$$

- This implies that $C_2^H(k)/C_1^H = Y_2^W(k)/Y_1^W = C_2^F(k)/C_1^F$.
- We will have to solve for μ^H.

Asset prices

- CRRA utility, $u(C) = \frac{(C)^{1-\sigma}}{1-\sigma}$. The Euler conditions give the asset prices:

$$\left(C_1^H\right)^{-\sigma} = \beta E\left[\left(C_2^H\right)^{-\sigma} \frac{Y_2^H}{V_1^H}\right] \quad \Rightarrow \quad V_1^H = E\left[\beta \left(\frac{Y_2^W}{Y_1^W}\right)^{-\sigma} Y_2^H\right]$$

$$\left(C_1^H\right)^{-\sigma} = \beta E\left[\left(C_2^H\right)^{-\sigma} \frac{Y_2^F}{V_1^F}\right] \quad \Rightarrow \quad V_1^F = E\left[\beta \left(\frac{Y_2^W}{Y_1^W}\right)^{-\sigma} Y_2^F\right]$$

$$\left(C_1^H\right)^{-\sigma} = \beta (1+r) E\left[\left(C_2^H\right)^{-\sigma}\right] \quad \Rightarrow \quad 1+r = \frac{1}{\beta} \frac{\left(Y_1^W\right)^{-\sigma}}{E\left[\left(Y_2^W\right)^{-\sigma}\right]}$$

- Equity prices reflect the expected discounted future output, evaluated with the utility pricing kernel.
  - States where output grows a lot are less valued, as future output is then abundant and the marginal utility of consumption low.
- The real interest rate is high when future output tends to be high, relative to current output.

Portfolio shares

- Home consumption in period 2 is:

$$\begin{aligned} C_2^H(k) &= (1+r)\,B_2^H + x_2^{HH}\,Y_2^H(k) + x_2^{HF}\,Y_2^F(k) \\ \mu^H\left(Y_2^H(k) + Y_2^F(k)\right) &= (1+r)\,B_2^H + x_2^{HH}\,Y_2^H(k) + x_2^{HF}\,Y_2^F(k) \\ 0 &= (1+r)\,B_2^H + \left(x_2^{HH} - \mu^H\right) Y_2^H(k) \\ &\quad + \left(x_2^{HF} - \mu^H\right) Y_2^F(k) \end{aligned}$$

- This can only be true (in general) if.

$$B_2^H = 0 \qquad ; \qquad x_2^{HH} = x_2^{HF} = \mu^H$$

- The Home investor holds the same share of each stock market, and this share corresponds to her share of world consumption.

Home consumption share

- Home consumption in period 1 is:

$$\begin{aligned} C_1^H &= Y_1^H + V_1^H - B_2^H - x_2^{HH} V_1^H - x_2^{HF} V_1^F \\ \mu^H \left(Y_1^H + Y_1^F \right) &= Y_1^H + V_1^H - \mu^H V_1^H - \mu^H V_1^F \\ \mu^H &= \frac{Y_1^H + V_1^H}{Y_1^W + V_1^W} \end{aligned}$$

- The country's share of consumption is its share of world wealth: current endowment, and value of future endowment.
- The current account is output minus consumption:

$$\begin{aligned} CA_1 &= Y_1^H - C_1^H \\ CA_1 &= \frac{V_1^W}{Y_1^W + V_1^W} Y_1^H - \frac{Y_1^W}{Y_1^W + V_1^W} V_1^H \end{aligned}$$

- A country with high initial endowment or low value of future endowment runs a surplus.

More general formulation

- Recall that expected discounted excess return of an asset is zero. For instance, comparing the two equities:

$$0 = E\left[m_{1,2}^H\left(\frac{Y_2^H}{V_1^H} - \frac{Y_2^F}{V_1^F}\right)\right] \; ; \; 0 = E\left[m_{1,2}^F\left(\frac{Y_2^H}{V_1^H} - \frac{Y_2^F}{V_1^F}\right)\right]$$

- Difference implies that covariance between cross-country difference in pricing kernel and difference in returns is zero:

$$0 = E\left[\left(m_{1,2}^H - m_{1,2}^F\right)\left(\frac{Y_2^H}{V_1^H} - \frac{Y_2^F}{V_1^F}\right)\right]$$

- $m_{1,2}^H - m_{1,2}^F$ reflect future consumptions, hence portfolio shares (through the budget constraint of period 2).
- To solve a general model, linearizing is not enough.
  - Linear approximation gives everything conditional on portfolio shares.
  - The Euler difference gives the portfolio shares. It is a covariance, so it has to be approximated with a quadratic approximation.

Portfolio Home bias

- Can it be that a country holds a share of domestic assets in its portfolio that exceed its share in the world?
  - Empirically it is clearly the case
- It can be the case only if the domestic asset is a better hedge than the foreign one.
  - Domestic assets has a higher return than the foreign one when the investors consumption is low (marginal utility is high).
  - Relation with labor income: invest more in the asset that pays off better (than the other asset) when labor income is low.
- Not easy to get. If all goods are traded, higher productivity at home leads to high consumption. Makes holding foreign equity more appealing.
  - With labor income: high productivity raises both labor income and domestic asset return. The domestic asset is a bad hedge.

Explaining the bias

- Rich literature on generating a tilt towards domestic assets.
- Introduce sticky prices. Output is then driven by demand (price do not fall after a productivity gain).
  - Higher productivity allows for output to be produced with less labor.
  - Revenue is then paid more as profits (dividends) than wages.
  - Higher productivity boosts dividends and lowers wage income. Domestic equity is a good hedge.
- Introduce bonds in different currencies in addition to equity.
  - Demand shocks (such as monetary policy) leads to real exchange rate risk. Equity does not connect well to it.
  - Bonds handles demand shocks. Equity can then be used to hedge other shocks.
  - Side benefit: equity portfolio is very sensitive to the parameters when it is the only asset. Bonds solve this problem.
- Asymmetric information, as local investors know their own assets better.