PS1 Solutions

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Solution 1. Preference Relations

1.a. Two basic consumptions are: **Completeness** and **Transitivity**.

1. **Completeness:** This means that for $x, y \in X$, we have either $x \succeq y$ or $y \succeq x$, or both.
2. **Transitivity:** This means that for $x, y, x \in X$, if $x \succeq y$ and $y \succeq z$, then $x \succeq z$.

By saying that $\succeq$ is complete, we mean that for each individual, there is a well-defined preference between two goods. Transitivity implies that we will not face the decision with a series of choices such that the preferences form a cycle/loop.

1.b. By definition, $u(\cdot)$ is a utility function representing the preference $\succeq$ if, for all $x, y \in X$,

$$x \succeq y \Leftrightarrow u(x) \geq u(y).$$

Now, we know that $u(x) = u(y) \Leftrightarrow x \sim y$ and $u(x) > u(y) \Leftrightarrow x \succ y$. If $x \succeq y$, then if $y \succeq x$, we have $x \sim y$, showing $u(x) = u(y)$, or if $y \succeq x$ doesn't hold, then $x \succ y$, implying that $u(x) > u(y)$, thus if $x \succeq y$, $u(x) \geq u(y)$.

If $u(x) \geq u(y)$, then $u(x) > u(y)$ or $u(x) = u(y)$, implying that $x \succ y$ or $x \sim y$, both giving that $x \succeq y$.

1.c. By definition, $u : X \to \mathbb{R}$ is a utility function representing preference relation $\succeq$ means that for all $x, y \in X$,

$$x \succeq y \Leftrightarrow u(x) \geq u(y).$$

1. If $x \succeq y$, then $u(x) \geq u(y)$, as $f : \mathbb{R} \to \mathbb{R}$ is strictly increasing, $f(u(x)) \geq f(u(y))$ for all $u(x) \geq u(y)$. Thus, $x \succeq y \Rightarrow f(u(x)) \geq f(u(y))$.

2. If $f(u(x)) \geq f(u(y))$, as f is strictly increasing, this implies that $u(x) \geq u(y)$, which by definition gives that $x \succeq y$. Hence, we have

$$f(u(x)) \geq f(u(y)) \Leftrightarrow x \succeq y,$$

which shows that $v = f \circ u$ is also a utility function representing preference relation $\succeq$.

Solution 2. Choice Rules

2.a Weak Axiom: Given choice structure $(\mathcal{B}, C(\cdot))$, for some $B \in \mathcal{B}$ and $x, y \in B$, we have $x \in C(\mathcal{B})$. Then, for some other $B' \in \mathcal{B}$, with $x, y \in B'$, and $y \in C(B')$, we must also have $x \in C(B')$.

The WA means that if x is chosen when y is available, then there's no budget set B' containing both x and y that gives the result that y is chosen while x is not.

2.b Revealed Preference Relation: Given choice structure $(\mathcal{B}, C(\cdot))$,

$$x \succeq^* y \Leftrightarrow \text{There is some } B \in \mathcal{B} \text{ and } x, y \in B \text{ s.t. } x \in C(B)$$

Difference from $\succeq$: The revealed preference relation $\succeq^*$ is based on the decision-maker's actual choices, while $\succeq$ is based on the decision-maker's subjective preferences. $\succeq^*$ reflects observable behavior, and due to the lack of sufficient observations, it may not always align with the result using the preference approach.

2.c. (i) We prove from both sides. Firstly, $\Rightarrow$. If $x \succ^* y$, then we know that for some $B \in \mathcal{B}$ and $x, y \in B$, we have $x \in C(B)$ and $y \notin C(B)$. Thus $x \succeq^* y$. Suppose that $y \succeq^* x$, then there exists $B \in \mathcal{B}$, such that $x, y \in B$ and $x \in C(B)$. The Weak Axiom implies that $y \in C(B)$, which is a contradiction. Hence if $x \succ^* y$, $y \succeq^* x$ doesn't hold, which gives $x \succ^{**} y$.

Secondly, $\Leftarrow$.If $x \succ^{**} y$, then $x \succeq^* y$ but not $y \succeq^* x$. Still, by the definition of revealed preference, there is some $B \in \mathcal{B}$ and $x, y \in B$, we have $x \in C(B)$, but $y \notin C(B)$. This gives $\succ^{**} \Rightarrow \succ^*$.

(ii) The $\succ^*$ need not to be transitive. For example we have three alternatives x, y, z. Let $\mathcal{B} = \{(x, y), (y, z)\}$, and $C(x, y) = x$ and $C(y, z) = y$, then $x \succ^* y$ and $y \succ^* z$, but since (x, z) is not in any subsets of $\mathcal{B}$, we don't have $x \succ^* z$.

(iii) Bonus Let $x, y, z \in X$, $x \succ^* y$, and $y \succ^* z$.Then $x, y, z \in \mathcal{B}$ and by (i), $x \succ^{**} y$ and $y \succ^{**} z$. Hence, we have neither $y \succeq^* x$ nor $z \succeq^* y$. Since $\succeq^*$ rationalizes $(\mathcal{B}, C(\cdot))$, this implies that $y \notin C(\{x, y, z\})$ and $z \notin C(\{x, y, z\})$. Since $C(\{x, y, x\}) \neq \emptyset$, $C(\{x, y, x\}) = x$, we have $x \succ^* z$.

Solution 3. Consumer Choice

3.a (i) **Set of commodities**: $\{\text{apple}, \text{wine}, \text{cheese}\}$. There are $L = 3$ commodities in the set.

(ii) **A commodity vector** is $x = (x_A, x_W, x_C) \in \mathbb{R}^L_+$. Example: $x = (2, 15, 0)$, meaning 2 apples, 15 bottles of wine, and 0 piece of cheese.

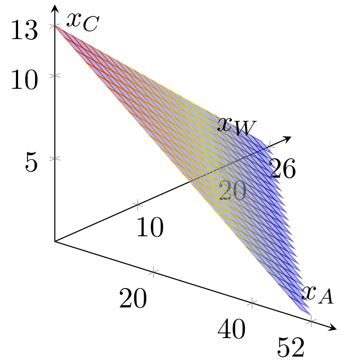
(iii) **Commodity set:** A subset of $\mathbb{R}^L$, showing the constraint to consumer's choice. In this question, the commodity set is

$$X = \{\{x_A, x_W, x_C\} | x_A \in [0, 10], x_W \in [0, 20], x_C \in [0, 15].\}$$

(iv) Budget set is set by commodity vector, price vector and a budget limit. In this question, the budger set is given by:

$$B = \{x \in X | x_A + 2x_W + 4x_C \leq 52\}$$

(v) It's given by the equation $x_1 + 2x_2 + 4x_3 = 52$.



3.b (i) Suppose we have two budget and price bundles: (p^0, x^0, w^0) and (p^1, x^1, w^1), the weak axiom says that, if $p^0x^1 \leq w^0$, then $p^1x^0 \geq w^1$. Otherwise the weak axiom implies that under (p^1, w^1), x^0 is chosen rather than x^1, for the condition gives that x^0 is preferred to x^1.

Right now, we have $p^0x^0 = 2 \times 5 + 4 \times 10 = 50$ and $p^1x^1 = 60 + 3y$. When $y = 5$,

$$p^0x^1 = (2, 4) \times (10, y) = 20 + 4y = 40 < 50 = p^0x^0 = w^0.$$

Hence if the WA is satisfied, we'll have $p^1x^0 \geq w^1$. However,

$$P^1x^0 = (6,3) \times (5,10) = 30 + 30 = 60 < 75 = p^1x^1 = w^1.$$

If $y = 5$, the consumption plan doesn't satisfy Weak Axiom.

(ii) If $y = 10$,

$$p^0x^1 = (2,4) \times (10,y) = 20 + 4y = 60 > 50 = p^0x^0 = w^0.$$

$$p^1x^0 = (6,3) \times (5,10) = 60 < 90 = p^1x^1 = w^1$$

We have: $p^0x^1 < w^1$ and $p^1x^0 < w^1$. This gives that: under (p^1, w^1), x_0 is affordable, and x_1 is preferred to x_0. Under (p^0, w^0), we cannot afford x^1. The consumption plan satisfies Weak Axiom.

(iii) For the general question, we have $w^0 = p^0x^0 = 50$, $w^1 = p^1x^1 = 60 + 3y$, $p^0x^1 = (2,4) \times (10,y) = 20 + 4y$ and $p^1x^0 = (6,3) \times (5,10) = 30 + 30 = 60$.

If the weak axiom is violated, we will have:

$$p^0x^1 \leq w^0 \text{ and } p^1x^0 \leq w^1$$

which is:

$$\begin{aligned}
&20 + 4y \leq 50 \text{ and } 60 \leq 60 + 3y \\
\Rightarrow &4y \leq 30 \text{ and } 0 \leq 3y \\
\Rightarrow &0 \leq y \leq 7.5
\end{aligned}$$

Therefore, we have the weak axiom violated if $0 \leq y \leq 7.5$.

3.c. As we know that $x(p, \alpha w) = \alpha x(p, w)$, we differentiate both sides with respect to α and then let $\alpha = 1$, and we have:

$$x(p, w) = wD_w x(p, w).$$

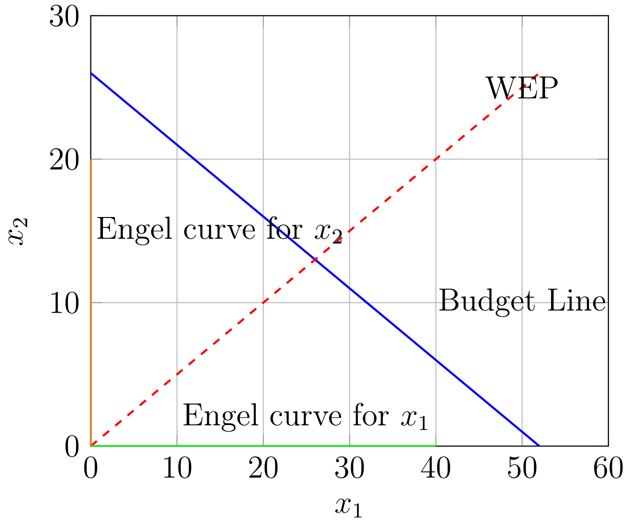
Hence,

$$\epsilon_{lw} = \frac{\partial x_l(p,w)}{\partial w} \frac{w}{x_l(p,w)} = \frac{x_l(p,w)}{w} \frac{w}{x_l(p,w)} = 1$$

Interpretation

This implies that for every l, the income elasticity is 1, which means a 1% increase in income leads to a 1% increase in the demand for good l.

Illustration with graph: plot the budget line $p_1x_1 + p_2x_2 = w$, along with the wealth expansion path and Engel curves for both goods.



As given in the text, the income elasticity is 1 for all goods and the total expenditure equals the consumer's income(wealth). Thus, as shown in the graph, we can get the conclusion as follows:

The Engel curves are linear, indicating that demand increases proportionally with income. The wealth expansion path shows that the consumer increases consumption of all goods proportionally with income.