Macroeconomics A; EI056

Short problems

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1 Effectiveness of policy in IS-TR

1.1 Solution of IS-TR

Question: Consider the IS-TR model (in a simpler form):

$$Y = c_1 Y - i_1 i + G \qquad i = \bar{i} + bY$$

What are the slopes of the IS and TR curves (in a figure with output on the horizontal axis)? Write the solution for output?

Answer: The IS curve is:

$$Y = -\frac{i_1}{1-c_1} i \Rightarrow i = -\frac{1-c_1}{i_1} Y$$

So the slope, in absolute value, is $(1-c_1)/i_1$. A steep IS corresponds to a low c_1 or a low i_1, that is an investment not sensitive to the interest rate.

The slope of TR is simply b. A steep TR reflect a monetary policy that is very reactive to output.

In equilibrium, output is:

$$Y = c_1 Y - i_1 bY - i_1 \bar{i} + G$$
$$Y = \frac{1}{1-c_1+i_1 b} G - \frac{i_1}{1-c_1+i_1 b} \bar{i}$$

1.2 Impact of fiscal policy

Question: How does the impact of fiscal policy on GDP depends on the slopes of IS and TR? Hint: think of how b and i_1 affect the results.

Provide a brief discussion in terms of economic intuition.

Answer: The impact of fiscal policy is:

$$\frac{\partial Y}{\partial G} = \frac{1}{1-c_1+i_1 b}$$

This is higher if IS is steep (a low value of i_1) or TR is flat (a low value of b):

$$\frac{\partial}{\partial i_1}\left(\frac{\partial Y}{\partial G}\right)=\frac{-b}{\left(1-c_1+i_1 b\right)^2}<0 \qquad \frac{\partial}{\partial b}\left(\frac{\partial Y}{\partial G}\right)=\frac{-i_1}{\left(1-c_1+i_1 b\right)^2}<0$$

A fiscal expansion tends to increase the interest rate, which crowds out investment. If i_1 is low, this crowding out effect is moderate and thus the stimulus from government spending does not "leak out" through lower investment.

The higher interest rate following a rise in government spending is due to the central bank reacting to output. If the central bank does little (b is low), it does not stand in the way of fiscal policy. This can be the case in a situation where the central bank was initially in a situation where it wanted a lower interest rate, but couldn't get it (for instance because it is hard to push the interest rate below zero). In such a case, the last thing it wants to do is increase the interest rate.

1.3 Impact of monetary policy

Question: How does the impact of monetary policy on GDP depends on the slopes of IS and TR? Again, provide a brief discussion in terms of economic intuition.

Answer: The impact of monetary policy is:

$$\frac{\partial Y}{\partial \bar{i}}=-\frac{i_1}{1-c_1+i_1 b}<0$$

This is larger (more negative) if IS is flat (a high value of i_1) or TR is flat (a low value of b):

$$\frac{\partial}{\partial i_1}\left(\frac{\partial Y}{\partial \bar{i}}\right)=\frac{-\left(1-c_1\right)}{\left(1-c_1+i_1 b\right)^2}<0 \qquad \frac{\partial}{\partial b}\left(\frac{\partial Y}{\partial \bar{i}}\right)=\frac{\left(i_1\right)^2}{\left(1-c_1+i_1 b\right)^2}>0$$

A monetary expansion (a reduction in $\bar{i}$) directly reduces the interest rate. This stimulates investment and output, especially when investment is very reactive to the interest rate (i_1 is high).

As the central bank offsets it own policy to some extent (it raises the interest rate in reaction to a boom in output, which is the positive slope of TR), the ultimate impact of its action is stronger if this offsetting (the slope of TR) does not undo the initial policy shifting the line TR (the change in) too much. This is the case when b is low.

1.4 Constraint on monetary policy

Question: Can you see a case where the central bank becomes powerless to affect output (think of the situation central banks faced since the early 2010 until last year)?

What can be done do get around the issue?

Answer: The central bank may face a limit about how low the interest rate can go. If it reaches this point, it cannot adopt an expansionary policy any longer.

Being faced with a constraint means that TR is flat, i.e. $b=0$. You may find the statement that policy is powerless puzzling, because we saw above that monetary policy is powerful when b is low. But remember that the policy we considered is a decrease in $\bar{i}$. If the interest rate is constraint, such a decrease cannot actually be implemented.

The solution to the problem is to move IS, for instance through a fiscal expansion. This is why there has been calls for more coordination of fiscal and monetary policies in recent years.

2 Slope of SRAS

2.1 Sticky wages

Question: Consider that firms produces using labor. A firm's profitability is a function of its cost, where P_t is the log price and Ww_t the log wage:

$$Profit_t = \Omega + \gamma (P_t - W_t)$$

Wage for a year are set at the end of the previous year. Workers want to preserve their purchasing power (i.e. their expected real wage is set at $\bar{W}$). How does this link wages, prices and output?

Answer: We can deduct yesterday price from both prices and wages in the profit:

$$Profit_t = \Omega + \gamma ((P_t - P_{t-1}) - (W_t - P_{t-1}))$$

The worker has set a wage so that the expected real wage is $\bar{\bar{W}} = W_t - P_t^e$ where e superscript denotes expectations. We thus get:

$$\begin{aligned}
Profit_t &= \Omega + \gamma \left((P_t - P_{t-1}) - \left(\bar{W} + P - P_{t-1}\right)\right) \\
Profit_t &= \Omega + \gamma \left(\pi_t - \pi_t^e - \bar{W}\right)
\end{aligned}$$

where $\pi_t = P_t - P_{t-1}$ is inflation.

Assuming that higher profits raise output, $Y_t = \beta Profit_t$, we get:

$$\begin{aligned}
Y_t &= \beta\gamma \left(\pi_t - \pi_t^e - \bar{W}\right) + \beta\Omega \\
Y_t &= \beta\gamma (\pi_t - \pi_t^e) + \beta\Omega - \beta\gamma\bar{W}
\end{aligned}$$

which is SRAS where $\beta\gamma = \eta$ and $\beta\Omega - \beta\gamma\bar{W} = \bar{Y}$.

2.2 Sticky prices

Question: Consider that n firms can always adjust their prices and keep output at a level $\bar{Y}$. If conditions are better than they expected yesterday, they increase their price above the level that they had expected yesterday: $P_t^{flex} = P_t^e (1 + \kappa)$ where e superscript denotes expectations and κ is a measure of economic condition ($\kappa > 0$ is a boom), P_{t-1} is the price yesterday, and $flex$ denotes that we are talking about firms with a flexible price.

The other $1 - n$ firms have set their price yesterday (based on the economic conditions they expected for today) and cannot change them:$P_t^{sticky} = P_t^e$. If conditions are better than expected, they are willing to produce more: $Y_t^{sticky} = \bar{Y} (1 + \kappa)$.

Aggregate output and prices are:

$$P_t Y_t = n\bar{Y} + (1 - n) Y_t^{sticky} \qquad P_t = nP_t^{flex} + (1 - n) P_t^{sticky}$$

Answer: The total output is:

$$\begin{aligned}
Y_t &= n\bar{Y} + (1 - n) \bar{Y} (1 + \kappa) \\
Y_t &= \bar{Y} + (1 - n) \bar{Y}\kappa
\end{aligned}$$

The aggregate price level is:

$$\begin{aligned}
P_t &= nP_t^{flex} + (1 - n) P_t^{sticky} \\
P_t &= nP_t^e (1 + \kappa) + (1 - n) P_t^e
\end{aligned}$$

Writing in terms of inflation:

$$\begin{aligned}P_t - P_{t-1} &= n\left[P_t^e(1+\kappa) - P_{t-1}\right] + (1-n)\left[P_t^e - P_{t-1}\right]\\ \pi_t &= n\left[\pi_t^e + P_t^e\kappa\right] + (1-n)\,\pi_t^e\\ \pi_t &= \pi_t^e + nP_t^e\kappa\end{aligned}$$

We combine the two to substitute for κ:

$$\begin{aligned}Y_t &= \bar{Y} + (1-n)\,\bar{Y}\frac{\pi_t - \pi_t^e}{nP_t^e}\\ Y_t &= \bar{Y} + \frac{(1-n)\,\bar{Y}}{nP_t^e}\left(\pi_t - \pi_t^e\right)\end{aligned}$$

which is the SRAS

2.3 Signalling

Question: Consider that when firms see higher demand across the economy they choose to raise their prices, but if they see higher demand for just their product they take advantage of their special popularity to raise price a bit and also increase output.

Firms however do not know exactly what is going on in the economy. All they know is the probability that shocks are aggregate (consumers have more money) or specific (consumer like the particular product of a specific firm).

How can this give raise to a SRAS? Hint: when shocks are specific to firms, they cancel out overall (some are more popular, others less).

Answer: The SRAS links aggregate output to aggregate prices. An aggregate shock means consumers have more money and go spend more on all firms.

If firms have perfect information, aggregate output never raises. Either we have an aggregate shock – but then all firm fully adjust their prices – or shocks are specific to firms – but then they cancel out.

If firms are uncertain, they think that the bigger spending from consumers could be due to the firm being popular. Each firm thinks so, and thus each firms accepts to produce more. But as all firms get more consumers, the extra production is done by all of them. We therefore see higher prices (firm use their popularity to increase price a bit, but not fully) and higher output. This looks like a short run aggregate supply.

3 Adjusting to AD shocks

3.1 Long run equilibrium

Question: Consider a simple version of the AS-AD model:

$$\begin{aligned}AD: Y_t &= \bar{Y} - \pi_t + d_t\\ AS: Y_t &= \bar{Y} + \left(\pi_t - \pi_t^e\right)\\ Expectations: \pi_t^e &= \pi_{t-1}^e + \varsigma\left(\pi_{t-1} - \pi_{t-1}^e\right)\end{aligned}$$

where d is a demand shift. The parameter $\varsigma \in (0,1)$denotes the sensitivity of expectations to past errors.

What is the long term equilibrium?

Answer: In the long term, inflation is equal to expectations and is constant: $\pi_t^e = \pi_{t-1}^e = \pi_t = \pi_{t-1} = \pi$.

Output is equal to the natural rate, $Y_t = \bar{Y}$, and the value of inflation comes from AD (d is constant in the long run): $\pi = d$

3.2 A permanent AD shock

Question: Initially we have $d = 0$. Consider a permanent demand shock where at time $t = 0$ the demand increases to $\overline{d} > 0$ and stays there forever ($\overline{d} = d_0 = d_1 = d_2 = ...$).

Show that the system can be written in general as:

$$\begin{aligned} \pi_t^e &= \pi_{t-1}^e + \varsigma\left(\pi_{t-1} - \pi_{t-1}^e\right) \\ \pi_t &= \frac{1}{2}\left(\pi_t^e + d_t\right) \\ Y_t &= \bar{Y} + \frac{1}{2}\left(d_t - \pi_t^e\right) \end{aligned}$$

Show that from time $t = 1$ onwards, the solution takes the form:

$$\begin{aligned} \left(\pi_t^e - \overline{d}\right) &= -\left(1 - \frac{\varsigma}{2}\right)^t \overline{d} \\ \left(\pi_t - \overline{d}\right) &= \frac{1}{2}\left(\pi_t^e - \overline{d}\right) \\ \left(Y_t - \bar{Y}\right) &= -\frac{1}{2}\left(\pi_t^e - \overline{d}\right) \end{aligned}$$

How do inflation, inflation expectations, and output evolve?

What drives the speed of the adjustment process?

Answer: It is useful to solve for AS-AD conditional on expectations:

$$\pi_t = \frac{1}{2}\left(\pi_t^e + d_t\right) \qquad Y_t = \bar{Y} + \frac{1}{2}\left(d_t - \pi_t^e\right)$$

We just get the following general system:

$$\begin{aligned} \pi_t^e &= \pi_{t-1}^e + \varsigma\left(\pi_{t-1} - \pi_{t-1}^e\right) \\ \pi_t &= \frac{1}{2}\left(\pi_t^e + d_t\right) \\ Y_t &= \bar{Y} + \frac{1}{2}\left(d_t - \pi_t^e\right) \end{aligned}$$

At time $t = -1$ we are in the initial steady state where $Y_{-1} = \bar{Y}$ and $\pi_0^e = \pi_{-1} = \pi = d = 0$.

At time $t = 0$ we get:

$$\pi_0 = \frac{1}{2}\overline{d} \qquad Y_0 = \bar{Y} + \frac{1}{2}\overline{d}$$

From time $t = 1$ onwards, it is useful to write the system in the following form (recalling that $d_t = \overline{d}$ for $t \geq 0$):

$$\begin{aligned} \left(\pi_t^e - \overline{d}\right) &= (1 - \varsigma)\left(\pi_{t-1}^e - \overline{d}\right) + \varsigma\left(\pi_{t-1} - \overline{d}\right) \\ \left(\pi_t - \overline{d}\right) &= \frac{1}{2}\left(\pi_t^e - \overline{d}\right) \\ \left(Y_t - \bar{Y}\right) &= -\frac{1}{2}\left(\pi_t^e - \overline{d}\right) \end{aligned}$$

which implies the expectations dynamics:

$$\left(\pi_t^e - \overline{d}\right) = \left(1 - \frac{\varsigma}{2}\right)\left(\pi_{t-1}^e - \overline{d}\right)$$

At time $t = 1$ we get:

$$\left(\pi_1^e - \overline{d}\right) = -\left(1 - \frac{\varsigma}{2}\right)\overline{d}$$
$$\left(\pi_1 - \overline{d}\right) = -\frac{1}{2}\left(1 - \frac{\varsigma}{2}\right)\overline{d}$$
$$\left(Y_t - \bar{Y}\right) = \frac{1}{2}\left(1 - \frac{\varsigma}{2}\right)\overline{d}$$

At time $t = 2$ we get:

$$\left(\pi_2^e - \overline{d}\right) = -\left(1 - \frac{\varsigma}{2}\right)^2\overline{d}$$
$$\left(\pi_2 - \overline{d}\right) = -\frac{1}{2}\left(1 - \frac{\varsigma}{2}\right)^2\overline{d}$$
$$\left(Y_2 - \bar{Y}\right) = \frac{1}{2}\left(1 - \frac{\varsigma}{2}\right)^2\overline{d}$$

At time $t = 3$ we get:

$$\left(\pi_3^e - \overline{d}\right) = -\left(1 - \frac{\varsigma}{2}\right)^3\overline{d}$$
$$\left(\pi_3 - \overline{d}\right) = -\frac{1}{2}\left(1 - \frac{\varsigma}{2}\right)^3\overline{d}$$
$$\left(Y_3 - \bar{Y}\right) = \frac{1}{2}\left(1 - \frac{\varsigma}{2}\right)^3\overline{d}$$

And so on. Notice that $1 - \frac{\varsigma}{2} < 1$ so the limit of $\left(1 - \frac{\varsigma}{2}\right)^t$ goes to zero as t goes to infinity

Inflation expectations increase starting at time $t = 1$ ($\pi_1^e = \frac{\varsigma}{2}\overline{d} > 0$), but remainsbelow $\overline{d}$ ($\pi_1^e - \overline{d} < 0$). The gap relative to $\overline{d}$ gradually converges to zero, hence ultimately expectations reach $\overline{d}$.

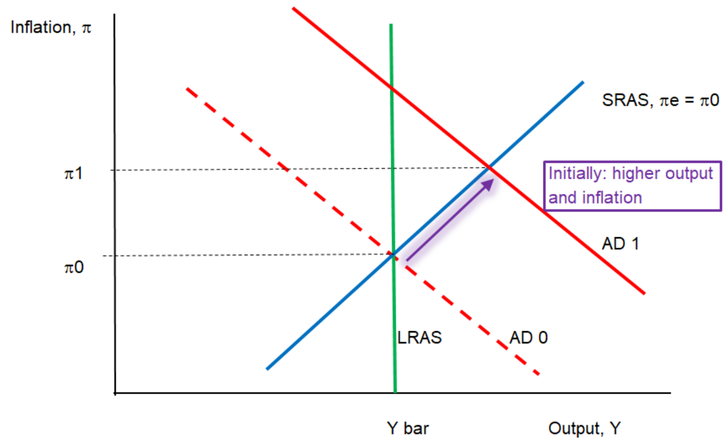
Inflation initially increases ($\pi_0 = \frac{1}{2}\overline{d} > 0$) but remains below $\overline{d}$ ($\left(\pi_0 - \overline{d}\right) = -\frac{1}{2}\overline{d}$). The gap between inflation and $\overline{d}$ gradually shrinks to zero, so in the end inflation matches expectations at $\overline{d}$.

The demand increase boosts output at $t = 0$ ($Y_0 - \bar{Y} = \frac{1}{2}\overline{d} > 0$). The gap between output and $\bar{Y}$ gradually goes to zero.

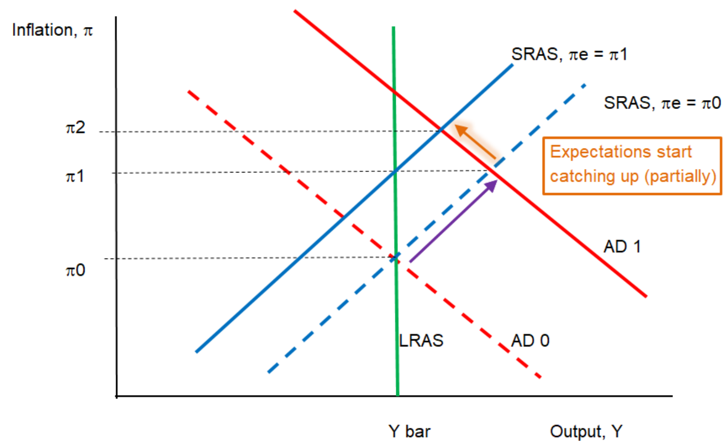
We thus get a temporary increase in output and a permanent increase in inflation. The speed of convergence is proportional to ς. The faster agents learn about inflation, the shorter the time of higher output.

The path is presented in the figure belows. The first shows the impact of the demand, before inflation expectations adjusted, with an increase in output. The second shows the first adjustment of expectations, which shifts SRAS up and leads to further inflation increase and an output decrease. The final chart show the subsequent adjustment (over many periods) where SRAS moves up until SRAS, LRAS., and AD all interect at the long run output and higher inflation.

AD shock



AD shock



AD shock