PROBLEM SET 5

EXERCISE 1:

$$\Delta d_t = \frac{i^d_t - g_t}{1+g_t} d_{t-1} + \frac{e_t(1+i^d_t)}{1+g_t} d_{t-1} - Tb_t$$

a) $d_{t-1} = 70\%$ $g_t = 3\%$ $i^d_t = 5\%$ $e_t = \frac{\Delta S_t}{S_t} = 20\%$. Increase in debt?

valuation effect = already a growth rate

$$\frac{0.20 * (1,05)}{(1,03)} = \frac{0.21}{1,03} = 0,2038$$

$0.2038 \times 0.70 = 0.14266 \Rightarrow$ D / GDP increases by 14.3 pp $\Rightarrow$ 84 % due to devaluation effects.

b) $\Delta d_t = 0 \iff Tb_t = \frac{i^d_t - g_t}{1+g_t} d_{t-1} + \frac{e_t(1+i^d_t)}{1+g_t} d_{t-1}$ $= 0$

$$= \frac{0.05 - 0.03}{1.03} \times 0.70 = 0.0135$$

No, the Tb_t of 1 % is not enough, it should be 1,35 %.

$$0.01 = 0.7 \times \frac{0.05 - g_t}{1+g_t} \Rightarrow g_t = 0.0352.$$

It means that GDP has to increase by $0.0352 - 0.03 = 0.0052$: 0.52 pp.

c) $g_t = 3\% \rightarrow g_t = 1\%$. Assume no evaluation / devaluation effect and $Tb_t = 0$

$$\Delta d_t = \frac{i^d_t - g_t}{1+g_t} \cdot d_{t-1}$$

70 % $\Rightarrow \frac{0.05 - 0.01}{1,01} \times 0.70 = 0.027$

40 % $\Rightarrow \frac{0.05 - 0.01}{1.01} \times 0.40 = 0.0158$

100 % $\Rightarrow \frac{0.05 - 0.01}{1.01} \times 1 = 0.0396$

It becomes less sustainable when the starting level is higher.

EXERCISE 2:

$d_t = 40\%$ $i = 0.03$ [Local currency]

a) $g^N = 0.02$ $\Delta d_t = 0$ b/c debt stable → No evaluation/devaluation effects

$$Tb_t = \frac{0.03 - 0.02}{1.02} \times 0.40 = 0,0039 \Rightarrow 0.4\% \text{ of GDP}$$

Investment = 2% of GDP over 5 years. ⇒ Tb_t worsens by 2% every year.
GDP increases after 6 years only. (i.e.) Every year it is 2% worse than the stabilizing one found in point a)

b) D/GDP at the end of 5th year?

We approximate the debt dynamics as follows:

$$d_{t+1} = (1+i-g)d_t - Tb_{t+1}$$ ← This is because $\frac{1+i}{1+g} \approx 1+i-g$

In our case we know that the Tb_t worsens every year by 2% b/c $\sim S - I \uparrow = TB \downarrow$. From which level of Tb do we start? The one found in point a), i.e the debt stabilizing one.

$D_{t+1} = (1+0.03-0.02)0.40 - [0.004 - 0.02] = 0.42$

$D_{t+2} = (1.02)0.42 - [0.004 - 0.02] = 0.4402$

$D_{t+3} = (1.01)0.4402 - [0.004 - 0.02] = 0.4606$

$D_{t+4} = (1.01)0.4606 - [0.004 - 0.02] = 0.4812$

$D_{t+5} = (1.01)0.4812 - [0.004 - 0.02] = 0.502$

It exceeds 50% b/c of the interest payment. Interests paid on higher debt than the beginning.

c) $\Delta d_{t+6} = 0 \Rightarrow \frac{i-g}{1+g} \times 0.502 = \underbrace{Tb_{t+6}}_{= Tb_t \text{ b/c it is always the debt-stabilizing one.}}$

$$\frac{0.03-g}{1+g} = \frac{0.004}{0.502} \Rightarrow g = 0.022$$

EXERCISE 3:

$$\Delta d_t = i_t d_{t-1} - g_t^N d_{t-1} - Tb_t$$

a) $Tb_t^* \mid \Delta d_t = 0$

$$Tb_t^* = (i_t - g_t^N) d_{t-1} = i_t d_{t-1} - g_t^N d_{t-1}$$

$Tb_t' = i_t d_{t-1} - g_t^{N'} d_{t-1}$
$Tb_t = i_t d_{t-1} - g_t^N d_{t-1}$

$$\Delta Tb_t^* = (-g^{N'} + g^N) d_{t-1} = -(g^{N'} - g^N) d_{t-1} = -(-0.03) \times 0,50$$

b) $d_t = 50\%$ $g_t^N \downarrow$ 3pp. Effect on Tb_t^*?

$$\Delta Tb_t^* = -\Delta g_t^N d_{t-1} = 0.03 \times 0,50 = 1.5\% \text{ or } 0.03 \times \frac{100}{100} = 3\%$$

The stabilizing trade balance has to increase if the growth rate declines.

c) $g_t^N = \bar{g} - 0.5 Tb_t$

$$Tb_t = i_t d_{t-1} - g_t^N d_{t-1} = i_t d_{t-1} - (\bar{g} - 0.5 Tb_t) d_{t-1}$$

$$Tb_t (1 - 0.5 d_{t-1}) = i_t d_{t-1} - \bar{g} d_{t-1}$$

$$\Delta Tb_t = -\frac{\Delta g_t^N}{1 - 0.5 d_{t-1}} \cdot d_{t-1} = \frac{-0.03}{1 - 0.5 \times 0.5} \times 0.5 = 0.02$$

EXERCISE 4:

$i = 5\%$ $i^d = 2\%$

UIP $\Rightarrow$ $(1 + i_t) = (1 + r^*) \frac{E_{t+1}}{E_t}$

$(1 + i_t)$: Return from investing one unit of domestic currency in one unit of domestically denominated bond

$(1 + r^*) \frac{E_{t+1}}{E_t}$: one unit of domestic bonds buys $1/E_t$ units of foreign bonds. And $1/E_t$ pays $\frac{(1+r^*)}{E_t}$ in $(t+1)$ that can be exchanged for $\left(\frac{1+r^*}{E_t}\right) E_{t+1}$ units of domestic currency.

! Liquidity = your currency is so bad that if you have it you can't get rid of it bc nobody wants it

a) $P_t = 0$

cost of domestic currency financing | Expected cost of foreign financing (by market (M))

$$(1 + i_t) = (1 + i_t^d)(1 + E_{t-1}^M e_t)(1 + P_t)$$

$$(1,05) = (1,02)(1 + E_{t-1}^M e_t) \Rightarrow \frac{1,05}{1,02} - 1 = E_{t-1}^M e_t \Rightarrow E_{t-1}^M e_t = 0.029 \sim 3\%$$

Expected depreciation by the mkt

If expected depreciation of the gov't is less than 3%, then the gov't will issue debt in foreign currency.

b) $P_t = 1\%$.

$$(1,05) = (1,02)(1,01)(1 + E^M_{t-1} e_t) \Rightarrow E^M_{t-1} e_t = 0.019 \sim 2\%$$

↑ Expected depreciation by the mkt.

The gov't will keep on issuing in foreign currency if $E^G_{t-1} e_t < 3\%$, bc the cost of financing in domestic currency will always be 5%.

For example, if the gov't expects $E^G_{t-1} e_t = 2.5\% > E^M_{t-1} e_t = 2.0\%$ anyway it prefers issuing debt in foreign currency bc even if there is a loss of 0.5% on the exchange rate there is a 1% gain at the higher liquidity of debt in foreign currency.

c) If the gov't is risk-averse the debt in local currency is less risky ⊛
In fact if debt is in local currency, the gov't has an incentive to depreciate
The lenders, then, don't trust the debt in local currency and ask higher interest rates. ⊛ If debt is in foreing currency, debt is higher when you depreciate

EXERCISE 5

$$Tb_t = P(1+R) d_{t-1}$$

a) $$d_t = (1+R) d_{t-1} - P(1+R) d_{t-1}$$
$$= (1+R)(1-P) d_{t-1}$$

b) $$(1+R)(1-P) = 1 \rightarrow 1 - P = \frac{1}{1+R} \Rightarrow \bar{P} = 1 - \frac{1}{1+R} = 1 - \frac{1+g}{1+i}$$
$$= \frac{1+i-1+g}{1+i}$$
$$= \frac{i-g}{1+i}$$

c) For the No-Ponzi to be respected it is important that $P > 0$ however the Tb cannot grow w/o a bound ⇒ debt grows at a rate lower than R_t if $P > 0$ + bound on the debt for sustainability.

↑ $Tb_t = P(1+R) d_{t-1}$ ↑

EXERCISE 6

$$s_t = \frac{i_t - g_t^N}{1+g_t^N} d_{t-1}$$

$T^G = G^G$: if $\frac{G}{GDP}\uparrow$ also $\frac{T}{GDP}\uparrow$ to keep $\Delta s_t = 0$ → Difficult to implement b/c the tax base (GDP) gets smaller and smaller, when $\frac{G}{GDP}\downarrow$ = or tax base stays the same $\frac{G}{GDP}=\uparrow$ but then you have to $\uparrow T$.

(A) EXERCISE 4:

Debt issued in domestic currency → Signalling Effect = I, as a govt, expect an actually higher depreciation than the mkt (otherwise I would have issued in foreign currency)

! currency depreciation increases debt burden if debt issued in foreign currency. Govt doesn't care about depreciation when debt is in domestic currency

Original sin → Situation in which countries cannot borrow abroad in their own currency, i.e. they have to pay a premium to be able to use their own currency.

Further clarification:

Exercise 4.b:

If I issue domestic currency debt my cost is 5% (with certainty) independently of whether this cost comes from a risk premium or from expected depreciation or both or from anything else.

On the other hand, if I issue in foreign currency my expected cost is equal to 2% plus the rate of foreign currency appreciation that I expect. In other words, with domestic currency debt, I do not care whether the 5% comes from investors' expected depreciation or premium or anything; I always have pay 5%.

Exercise 5.c:

if rho>0 at a rate lower than R thus satisfying the No-ponzi gam condition.

If rho is lower than R/(1+R), then the debt increases indefinitely in real terms (the Trade balance also increases according to the rule) So the No-Ponzi condition is satisfied but the debt may get too large. The Trade balance rho(1+R)d increases indefinitely and may exceed output if the latter grows at a slower pace which is a problem given that

Tb = Y - C - G –I.

Exercise 6:

Given any stabilizing surplus X= T – G achieving such surplus can be easier or more challenging depending on the level of G because the level of G determines the tax rate to be set because X = tY - G where t is the tax rate.

If the costs of taxation increase more than proportionately with t (You can think of the cost of distortionary taxation or social costs) then raising the tax rate starting from an already high level due to high G is more costly and thus more difficult than doing it when the tax rate is low because G is low.

The point here is that debt stabilization and its success also depends on the weight of the government in the economy, on whether you increase taxes or reduce G. On the nature of G (as certain expenditures are easier to cut), etc. etc. The point is that it does not make sense just to look at X and at the level of debt. It depends on so many things that, say, indicating a threshold level of debt is stupid.