Problem Set II

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EI037 Microeconomics I

1. Production Theory

a. Draw two production sets: one that violates irreversibility and one that satisfies this property.

b. Derive the cost function $c(w, q)$ and conditional factor demand functions $z(w, q)$ for each of the following single-output constant return technologies with production function given by

(i) $f(z) = z_1 + z_2$ (perfect substitutable inputs)

(ii) $f(z) = \min\{z_1, z_2\}$ (Leontief technology)

(iii) $f(z) = (z_1^\rho + z_2^\rho)^{\frac{1}{\rho}}, \ \ \rho \leq 1$ (constant elasticity of substitution technology)

c. Let $f(x)$ be the production of a firm with constant returns to scale technology. Suppose that each factor x_i is paid its value of marginal product, $w_i = p\frac{\partial f(x)}{\partial x_i}$. Show that profits must be zero.

d. Suppose $f(z)$ is a concave production function with $L-1$ inputs $(z_1, ..., z_{L-1})$. Suppose also that $\frac{\partial f(z)}{\partial z_l} \geq 0$ for all l, $z \geq 0$, and that the matrix $D^2 f(z)$ is negative definite for all z. Use the firm's first order conditions and the implicit function theorem to prove the following statements:

(i) An increase in the output price always increases the profit-maximizing level of output.

(ii) An increase in the output price always increases the demand for *some* input.

(iii) An increase in the price of an input leads to a reduction in the demand of the input.

2. Competitive Equilibrium and Welfare Theorems

a. The concept of Pareto efficiency we studied in class is sometimes known as *strong Pareto efficiency.* An outcome is *weakly Pareto efficient* if there is no alternative feasible allocation that makes *all* individuals *strictly* better off.

(i) Show that if an outcome is strongly Pareto efficient, then it is weakly Pareto efficient as well.

(ii) Show that if all consumers' preferences are continuous and strongly monotone, then these two notions of Pareto efficiency are equivalent for any *interior* outcome (i.e., an outcome in which each consumer's consumption lies in the interior of his consumption set). Assume for simplicity that $X_i = \mathbb{R}^L_+$ for all i.

(iii) Construct an example where the two notations are not equivalent. Why is the strong monotonicity assumption important in (ii)? What about interiority?

b. Consider a two-good quasi-linear model with one consumer and one firm. The initial endowment of the numeraire is $w_m > 0$, and the initial endowment for good l is 0. Let the consumer's quasi-linear utility function be $u(x) = \alpha + \beta ln(x) + m$, where $(\alpha, \beta) >> 0$. Let the firm use m to produce l, and its cost function be $c(q) = \sigma q$ for some scalar $\sigma > 0$. Assume that the consumer receives all the profits of the firm. Both the firm and the consumer act as price takers. Normalize the price of good m to equal 1, and denote the price of good l by p.

(i) Derive the consumer's and firm's first-order conditions.

(ii) Derive the competitive equilibrium price and output of good l. How do these vary with α, β, and σ?

3. Strategic Interactions

a. There are l firms in an industry. Each can try to convince Congress to give the industry a subsidy. Let h_i denote the number of hours of effort put in by firm i, and let $c_i(h_i) = w_i(h_i)^2$ be the cost of this effort to firm i, where w_i is a positive constant. When the effort levels of the firms are $(h_1, ..., h_l)$, the value of the subsidy that gets approved is $\alpha \sum_i h_i + \beta\,(\Pi_i h_i)$, where α and β are constants.

Consider a game in which the firms decide simultaneously and independently how many hours they will each devote to this effort. Show that each firm has a strictly dominant strategy if and only if $\beta = 0$. What is firm i's strictly dominant strategy when this is so?

b. Consider a game Γ_N with players 1, 2 and 3 in which $S_1 = \{L, M, R\}$, $S_2 = \{U, D\}$ and $S_3 = \{\ell, r\}$. Player 1's payoffs from each of his three strategies conditional on the strategy choices of players 2 and 3 are depicted as (u_L, u_M, u_R) in each of the four boxes shown below, where $(\pi, \varepsilon, \eta) >> 0$. Assume that $\eta < 4\varepsilon$.

		Player 3's Strategy ℓ	Player 3's Strategy r
Player 2's Strategy	U	$\pi + 4\varepsilon,\ \pi - \eta,\ \pi - 4\varepsilon$	$\pi - 4\varepsilon,\ \pi + \frac{\eta}{2},\ \pi + 4\varepsilon$
	D	$\pi + 4\varepsilon,\ \pi + \frac{\eta}{2},\ \pi - 4\varepsilon$	$\pi - 4\varepsilon,\ \pi - \eta,\ \pi + 4\varepsilon$

(i) Argue that (pure) strategy M is never a best response for player 1 to any independent randomizations by players 2 and 3.

(ii) Show that (pure) strategy M is not strictly dominated.

(iii) Show that (pure) strategy M can be a best response if player 2's and player 3's randomizations are allowed to be correlated.

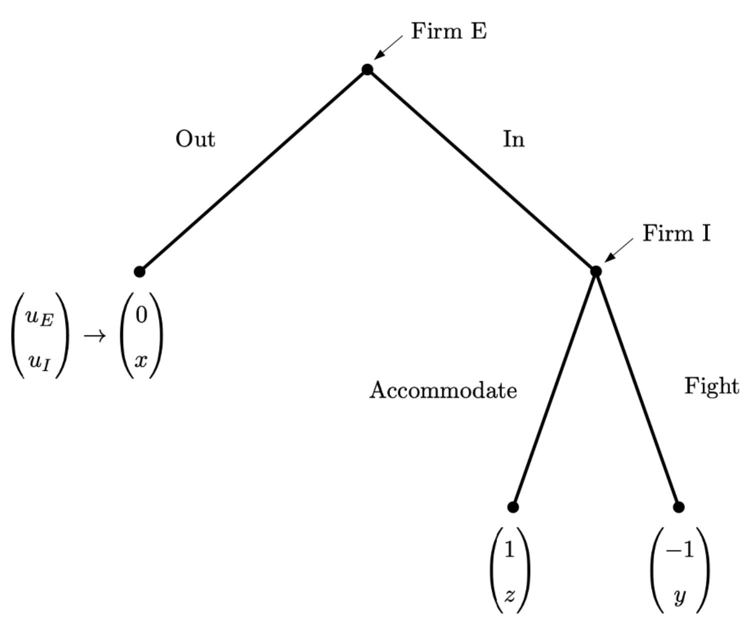
c. Consider a bargaining situation in which two individuals are considering undertaking a business venture that will earn them 100 dollars in profit, but they must agree on how to split the 100 dollars. Bargaining works as follows: the two individuals each make a demand simultaneously. If their demands sum to more than 100 dollars, then they fail to agree, and each gets nothing. If their demands sum to less than 100 dollars, they do the project, each gets his demand, and the rest goes to charity.

(i) What are each player's strictly dominated strategies?

(ii) What are each player's weakly dominated strategies?

(iii) What are the pure strategy Nash equilibria of this game?

d. At time 0, an incumbent firm (firm I) is already in the widget market, and a potential entrant firm (firm E) is considering entry. In order to enter, firm E must incur a cost of $K > 0$. Firm E's only opportunity to enter is at time 0. There are three production periods. In any period in which both firms are active in the market, the game in figure below is played. Firm E moves first, deciding whether to stay in or exit the market. If it stays in, firm I decides whether to fight (the upper payoff is for firm E). Once firm E plays "out", it is out of the market forever; firm E earns zero in any period during which it is out of the market, and firm I earns x. The discount factor for both firms is δ.



Assume that:

- $x > z > y$;
- $y + \delta x > (1 + \delta)z$;
- $1 + \delta > K$.

(i) What is the (unique) subgame perfect Nash equilibrium of this game?

(ii) Suppose now that firm E faces a financial constraint. In particular, if firm I fights *once* against firm E (in any period), firm E will be forced out of the market from that point on. Now what is the (unique) subgame perfect Nash equilibrium of this game? (*If the answer depends on the values of parameters beyond the three assumptions, indicate how.*)