Macroeconomics A: EI056

Midterm exam

Cédric Tille

November 8, 2022

1 General instructions

The exam consists of 4 questions. Section 2 has 3 short subquestions, and sections 3-4-5 require more thinking. The weight of each question in the grade is indicated, so you can allocate your time accordingly.

A good strategy is to first read through the questions, and then start with the easiest one before proceeding to the harder ones.

Short "bullet points" answers stressing the main points are fine, you don't have to write long paragraphs.

Best of luck!

2 Short questions (25 % of grade)

2.1 Forward guidance in IS-TR

In class we saw the IS-TR model. IS reflects the good market. Output Y is high when the real interest rate (nominal interest rate i net of expected inflation π^e) is low, or when households' wealth Ω is high (α and β are positive coefficients):

$$Y = -\alpha \left(i - \pi^e \right) + \beta \Omega$$

TR reflects the Taylor rule that sets the nominal interest rate as a function of a long run target $\bar{\imath}$, current inflation π, and output:

$$i = \bar{\imath} + \delta \pi + \lambda Y$$

Consider that the central bank has driven the interest rate as low as it can, for instance to a value zero (in the IS-TR diagram with Y on the horizontal axis and i on the vertical TR is flat). Faced with this situation, central banks have tried two things:

1. Forward guidance: communicate that you will accept some inflation in the future.
2. Asset purchases: purchases of assets (including risky ones) by the central bank.

How would you model these policies in the IS-TR diagram?

Answer: Forward guidance indicates that when the economy recovers monetary policy will be "behind the curve" for a while, leading to a pickup in inflation. This is expected so π^e increases and IS shifts to the right. As TR is flat this effect is powerful. In other words, the central bank reduces the real interest rate via expected inflation instead of via the nominal interest rate (which is what it usually does).

Asset purchases raise the price of assets (or more exactly prevents a collapse). This avoids a sharp contraction in Ω and therefore prevents IS from moving to the left.

2.2 Solow residuals

In the real business cycle class we discussed the Solow residuals.

1. Explain how they are constructed.
2. They are often used as a measure of productivity. What motivates this?
3. Are there shortcomings with that view, and if so how can they be addressed?

Answer: The Solow residuals are the growth rate of GDP that cannot be accounted for by the growth rate of capital and labor.

1. They are constructed as follows. Consider the following production function of output Y using capital K, labor L, productivity A:

$$Y = A\left(Z_K K\right)^{\alpha}\left(Z_L L\right)^{1-\alpha}$$

where Z_K and Z_L measure how hard the machines and people in the economy work. In terms of growth rates (denoted by lower case letters) we have:

$$y = a + \alpha z_K + \alpha k + (1-\alpha) z_L + (1-\alpha) l$$

which implies:

$$a = y - \alpha z_K - \alpha k - (1-\alpha) z_L - (1-\alpha) l$$

If we observe y, k, l, z_K and z_L we get a as a residual.

2. Abstracting from z_K and z_L, the Solow residual measure productivity a. Empirically, we see that they are quite volatile, and highly correlated with GDP. This motivates the analysis of productivity shocks in the RBC literature.

3. A problem is to control for z_K and z_L. If we don't and only consider the growth rate of output, capital and labor, the Solow residuals are:

$$SR = y - \alpha k - (1-\alpha)\, l = a + \alpha z_K + (1-\alpha)\, z_L$$

which measures true productivity as well as the intensity of factor uses. This is problematic:

   (a) The intensity is often pro-cyclical: when productivity is low, firms do not start selling their machines and firing their workers that much as they understand that after a while they will need to buy the machines and hires the worker again. It is more efficient to keep machines and workers idle to some extent so as to have them ready when demand picks up (this is called labor hoarding). Periods of low a thus also tend to be periods with low z_K and z_L, and thus low SR. But the magnitude of the Solow residuals do not directly reflect the magnitude of the productivity, as they also reflect the intensity changes.

   (b) A statistician that disregards the intensity of factor uses would thus read the sharp decrease in SR during a recession as a decrease in productivity a itself. This is not a realistic interpretation, as for instance it implies that technology literally regresses on a fairly regular basis. If instead the statistician is careful to control for the intensity of factor uses, he will infer that a is less volatile than SR, which makes more sense.

2.3 Expectations in AS-AD

When discussing the AS-AD model we consider backward-looking expectations and rational expectations.

1. How do the two differ?

2. Consider a policy of permanently higher government spending in the AS-AD. What is the impact on output in the short and long run? How does it depend on the way expectations are formed?

3. Developing a complete understanding of the economy is costly for agents (think of the effort involved). With this in mind, would you expect a small increase in government spending to have a different effect than a large one?

Answer:

1. Under backward-looking (or adaptive) expectations agent form their expectation of future inflation by looking at past values. This can include simply taking the last value of inflation and expecting it to continue, and correct the previous expectations depending on the current gap between actual and expected inflation. Rational expectations have no connection to the past behavior of inflation. Instead, agents fully understand the model and solve the equilibrium irrespective of what happened yesterday.

2. In the AS-AD diagram (output on the horizontal axis, inflation on the vertical), AD is a downward sloping line, Long Run AS is vertical, and Short Run AS is positively. Initially all three lines intersect at the same point. The higher government spending shifts AD to the right permanently. In the short run we move along the SRAS and output increases with higher inflation.

   (a) With adaptive expectations, agents gradually revise their inflation expectations upwards, and SRAS gradually shift to the left, so output gradually decreases. This goes on until SRAS, LRAS and AD (with higher spending) cross at the same point. At that point output is back to its initial value and inflation higher. The higher spending brings a temporary increase in output, with some persistence.

   (b) Under rational expectations, once the higher spending has taken place agents immediately compute the new equilibrium, and expect the long run inflation. The economy goes right away to the point where SRAS, LRAS and AD (with higher spending) cross. We therefore only have a temporary increase in output that rapidly goes away. In fact, if the higher spending policy was announced before taking place, the shift in expectations would be immediate and there would be no impact at all on output.

3. When shocks are small, going through the effort of figuring out equilibrium is not worth the cost. Agents then take some simple rule-of-thumb and expectations are backward looking (for instance survey of consumers in low inflation economies, such as Switzerland, show that expected inflation for the next 12 months is pretty much the inflation over the previous 12 months). If shocks are large, backward-looking expectations will lead to very large mistake, and then it is worth going through the trouble of understanding the model.

4 Central bank contract with punishment (25 % of grade)

4.1 Policy under discretion

Consider the time inconsistency model. In period t output is given by the aggregate supply:

$$y_t = \pi_t - \pi_t^e$$

where y is output, π is inflation and π^e is expected inflation. For simplicity I normalize the natural output (the level reached when inflation is fully expected) to zero.

The loss function of the central bank is:

$$L_t = \frac{1}{2}\left[\lambda\left(y_t - k\right)^2 + \left(\pi_t\right)^2\right]$$

where k is the target bias, as the central bank would like to reach a level of output higher than the natural rate).

We can show that (take this as given) that if the central bank takes expectations as a parameter it sets (the $disc$ superscript denotes "discretion"):

$$\pi_t^{disc} = \frac{\lambda}{1+\lambda}\pi_t^e + \frac{\lambda}{1+\lambda}k$$

Show that if the bank operates under discretion and agents form their expectations rationally, we get:

$$\pi^{disc} = \lambda k$$

Briefly explain the intuition.

Answer: Under discretion the central bank takes π_t^e as given, and minimizes:

$$L_t = \frac{1}{2}\left[\lambda\left(\pi_t - \pi_t^e - k\right)^2 + \left(\pi_t\right)^2\right]$$

The first order condition is:

$$\begin{aligned} 0 &= \lambda\left(\pi_t^{disc} - \pi_t^e - k\right) + \pi_t^{disc} \\ \left(1+\lambda\right)\pi_t^{disc} &= \lambda\pi_t^e + \lambda k \end{aligned}$$

which is the decision rule.

Under rational expectations, as there are no shocks, inflation is equal to expectation in equilibrium:

$$\begin{aligned} \left(1+\lambda\right)\pi_t^{disc} &= \lambda\pi_t^{disc} + \lambda k \\ \pi^{disc} &= \lambda k \end{aligned}$$

Equilibrium inflation is positive because the central bank cares about output (λ), it aims at an output above the natural rate (k) and inflation surprises have an effect on output.

4.2 Two periods horizon

We now consider that there are two periods, t and $t+1$. The central bank's loss is a discounted value of the two period-specific losses:

$$\begin{aligned}
\mathcal{L}_t &= L_t + \beta L_{t+1} \\
\mathcal{L}_t &= \frac{1}{2}\left[\lambda\left(y_t - k\right)^2 + \left(\pi_t\right)^2\right] + \frac{\beta}{2}\left[\lambda\left(y_{t+1} - k\right)^2 + \left(\pi_{t+1}\right)^2\right]
\end{aligned}$$

In the past, the central bank had always delivered inflation equal to π^*. Agents thus enter period t with their expectations set at this level, so we take $\pi_t^e = \pi^*$ as given.

At period t there is a new central bank president who decides whether or not to continue this policy.

- If she continues the past policy, she delivers $\pi_t = \pi_{t+1} = \pi^*$, then agents keep expecting π^* in all periods.
- If the central bank delivers a different inflation in period t, that is $\pi_t \neq \pi^*$, agents expect inflation to be $\pi_{t+1} = \pi^{disc}$ (derived above) for period $t+1$. In other words, any failure by the central bank to deliver on its promise triggers a change in expectation.

Consider that the central bank breaks its promise (we think later whether or not it should). What inflation does it choose in period $t+1$, and in period t (Hint: use the analysis of the previous point)?

Answer: If the central bank has broken its promise, agents expect $\pi_{t+1}^e = \pi^{disc} = \lambda k$. The central bank knows its credibility is gone, and reverts to discretion. The loss function is then:

$$\begin{aligned}
L_{t+1}^{deviate} &= \frac{1}{2}\left[\lambda\left(\pi_{t+1} - \pi_{t+1}^e - k\right)^2 + \left(\pi_{t+1}\right)^2\right] \\
L_{t+1}^{deviate} &= \frac{1}{2}\left[\lambda\left(\pi_{t+1} - \lambda k - k\right)^2 + \left(\pi_{t+1}\right)^2\right]
\end{aligned}$$

The first order condition is:

$$\begin{aligned}
0 &= \lambda\left(\pi_{t+1} - \lambda k - k\right) + \pi_{t+1} \\
0 &= \lambda\left(-\lambda k - k\right) + (1+\lambda)\,\pi_{t+1} \\
(1+\lambda)\,\pi_{t+1} &= \lambda(1+\lambda)\,k \\
\pi_{t+1} &= \lambda k
\end{aligned}$$

The central bank thus delivers the expected inflation: $\pi_{t+1} = \pi^{disc} = \lambda k$. A simpler way to have the results is to use the result of the previous section:

$$\begin{aligned}
\pi_{t+1}^{disc} &= \frac{\lambda}{1+\lambda}\pi_{t+1}^e + \frac{\lambda}{1+\lambda}k \\
\pi_{t+1}^{disc} &= \frac{\lambda}{1+\lambda}\lambda k + \frac{\lambda}{1+\lambda}k \\
\pi_{t+1}^{disc} &= \frac{\lambda}{1+\lambda}\left(\lambda+1\right)k
\end{aligned}$$

$$\pi_{t+1}^{disc} = \lambda k$$

In period t the agents expect π^*. The central bank knows that once it breaks its promise, the choice of the specific value of inflation for period t will not affect its choice later on. It thus focuses on t and minimizes:

$$L_t^{deviate} = \frac{1}{2}\left[\lambda\left(\pi_t - \pi^* - k\right)^2 + \left(\pi_t\right)^2\right]$$

The first order condition is:

$$\begin{aligned}
0 &= \lambda\left(\pi_t - \pi^* - k\right) + \pi_t \\
0 &= \lambda\left(-\pi^* - k\right) + \left(1+\lambda\right)\pi_t \\
\left(1+\lambda\right)\pi_t &= \lambda\left(\pi^* + k\right) \\
\pi_t &= \frac{\lambda}{1+\lambda}\left(\pi^* + k\right)
\end{aligned}$$

Here also, we could have simply used the result of the previous section:

$$\begin{aligned}
\pi_t^{disc} &= \frac{\lambda}{1+\lambda}\pi_t^e + \frac{\lambda}{1+\lambda}k \\
\pi_t^{disc} &= \frac{\lambda}{1+\lambda}\pi^* + \frac{\lambda}{1+\lambda}k \\
\pi_t^{disc} &= \frac{\lambda}{1+\lambda}\left(\pi^* + k\right)
\end{aligned}$$

4.3 Gain and cost from breaking promises

We now consider the choice of the central bank at period t.

If it deviate from $\pi_t = \pi^*$, this gives it a utility gain in period t. This gain is (take this as given):

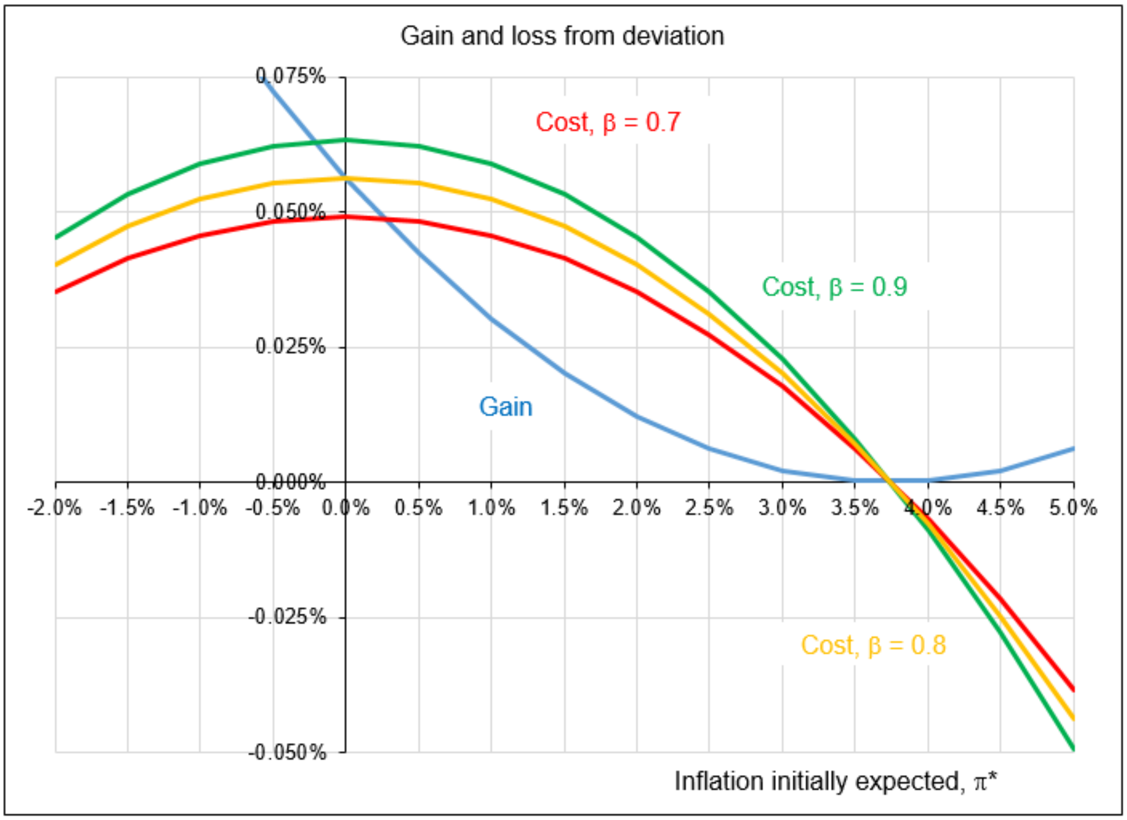
$$Gain\left(\pi^*\right) = \frac{1}{2}\left[\lambda\left(k\right)^2 + \left(\pi^*\right)^2 - \frac{\lambda}{1+\lambda}\left(\pi^* + k\right)^2\right]$$

As the deviation however triggers a change of expectations in period $t+1$, the central bank faces a cost from having broken its promise. This cost is (take this as given):

$$Cost\left(\pi^*\right) = \frac{\beta}{2}\left[\left(\lambda k\right)^2 - \left(\pi^*\right)^2\right]$$

The specific forms of $Gain\left(\pi^*\right)$ and $Cost\left(\pi^*\right)$ are given for reference. To make your work easier, we consider a numerical example. We set $\lambda = 0.25$, $k = 15\%$.

The figure below shows the gain and cost as functions of initial expectations π^* ranging from -2% to 5%. We show three calibrations of the cost: for $\beta = 0.7$ (red line), $\beta = 0.8$ (yellow line) and $\beta = 0.9$ (green line).



Based on this figure:

1. Explain the shape of the cost line in terms of intuition.
2. Explain the shape of the gain line in terms of intuition.
3. Explain what are the values of π^* (high, low) that the bank can credibly deliver.
4. Can we deliver zero inflation?
5. What is the highest inflation that the central bank can credibly deliver? Explain intuitively.
6. Can the central bank always deliver an inflation lower than the level under discretion?

Answer: We start by presenting the derivations.

If the central bank delivers π^* there are never any deviation from expectations ($\pi_t - \pi_t^e = \pi_{t+1} - \pi_{t+1}^e = 0$). The loss in period t (and also in period $t+1$) is:

$$\begin{aligned} L_t^{commit} &= \frac{1}{2}\left[\lambda\left(\pi_t - \pi_t^e - k\right)^2 + \left(\pi_t\right)^2\right] \\ L_t^{commit} &= \frac{1}{2}\left[\lambda\left(\pi^* - \pi^* - k\right)^2 + \left(\pi^*\right)^2\right] \\ L_t^{commit} &= \frac{1}{2}\left[\lambda k^2 + \left(\pi^*\right)^2\right] \end{aligned}$$

If the central bank deviates from its promise, in the future $\pi_{t+1} = \pi^{disc} = \lambda k$. The loss function at period $t+1$ is:

$$
\begin{aligned}
L_{t+1}^{deviate} &= \frac{1}{2}\left[\lambda\left(\pi_{t+1} - \pi_{t+1}^{e} - k\right)^2 + \left(\pi_{t+1}\right)^2\right] \\
L_{t+1}^{deviate} &= \frac{1}{2}\left[\lambda\left(\lambda k - \lambda k - k\right)^2 + \left(\lambda k\right)^2\right] \\
L_{t+1}^{deviate} &= \frac{1}{2}\lambda\left(1+\lambda\right)k^2
\end{aligned}
$$

In period t, the loss function when the central bank deviates is:

$$
\begin{aligned}
L_{t}^{deviate} &= \frac{1}{2}\left[\lambda\left(\pi_{t} - \pi^{*} - k\right)^2 + \left(\pi_{t}\right)^2\right] \\
L_{t}^{deviate} &= \frac{1}{2}\left[\lambda\left(\frac{\lambda}{1+\lambda}\left(\pi^{*}+k\right) - \pi^{*} - k\right)^2 + \left(\frac{\lambda}{1+\lambda}\left(\pi^{*}+k\right)\right)^2\right] \\
L_{t}^{deviate} &= \frac{1}{2}\left[\lambda\left(\frac{-1}{1+\lambda}\left(\pi^{*}+k\right)\right)^2 + \left(\frac{\lambda}{1+\lambda}\left(\pi^{*}+k\right)\right)^2\right] \\
L_{t}^{deviate} &= \frac{1}{2}\left[\lambda\left(\frac{1}{1+\lambda}\right)^2 + \left(\frac{\lambda}{1+\lambda}\right)^2\right]\left(\pi^{*}+k\right)^2 \\
L_{t}^{deviate} &= \frac{1}{2}\left[\lambda + \left(\lambda\right)^2\right]\left(\frac{1}{1+\lambda}\right)^2\left(\pi^{*}+k\right)^2 \\
L_{t}^{deviate} &= \frac{1}{2}\lambda\left(1+\lambda\right)\left(\frac{1}{1+\lambda}\right)^2\left(\pi^{*}+k\right)^2 \\
L_{t}^{deviate} &= \frac{1}{2}\frac{\lambda}{1+\lambda}\left(\pi^{*}+k\right)^2
\end{aligned}
$$

The initial gain from breaking the commitment is:

$$
\begin{aligned}
Gain\left(\pi^{*}\right) &= L_{t}^{commit} - L_{t}^{deviate} \\
Gain\left(\pi^{*}\right) &= \frac{1}{2}\left[\lambda k^2 + \left(\pi^{*}\right)^2\right] - \frac{1}{2}\frac{\lambda}{1+\lambda}\left(\pi^{*}+k\right)^2 \\
Gain\left(\pi^{*}\right) &= \frac{1}{2}\left[\lambda k^2 + \left(\pi^{*}\right)^2 - \frac{\lambda}{1+\lambda}\left(\pi^{*}+k\right)^2\right]
\end{aligned}
$$

The discounted subsequent cost is:

$$
\begin{aligned}
Cost\left(\pi^{*}\right) &= \beta\left[L_{t+1}^{deviate} - L_{t+1}^{commit}\right] \\
Cost\left(\pi^{*}\right) &= \frac{\beta}{2}\lambda\left(1+\lambda\right)k^2 - \frac{\beta}{2}\left[\lambda k^2 + \left(\pi^{*}\right)^2\right] \\
Cost\left(\pi^{*}\right) &= \frac{\beta}{2}\left[\lambda\left(1+\lambda\right)k^2 - \lambda k^2 - \left(\pi^{*}\right)^2\right] \\
Cost\left(\pi^{*}\right) &= \frac{\beta}{2}\left[\lambda^2 k^2 - \left(\pi^{*}\right)^2\right]
\end{aligned}
$$

The figure shows that the gain is a convex function of the inflation expectations, while the cost is a concave function.

Commitment is sustained if the cost exceeds the gain:

$$\begin{aligned}
C(\pi^*) &> G(\pi^*) \\
\beta\frac{1}{2}\left[(\lambda k)^2 - (\pi^*)^2\right] &> \frac{1}{2}\left[\lambda k^2 + (\pi^*)^2\right] - \frac{1}{2}\lambda\frac{(k+\pi^*)^2}{1+\lambda} \\
\beta(\lambda k)^2 - \beta(\pi^*)^2 &> \lambda k^2 + (\pi^*)^2 - \lambda\frac{(k+\pi^*)^2}{1+\lambda} \\
\beta(\lambda k)^2 - \lambda k^2 &> (1+\beta)(\pi^*)^2 - \lambda\frac{k^2 + (\pi^*)^2 + 2k\pi^*}{1+\lambda} \\
\left[\beta(\lambda)^2 + \frac{\lambda}{1+\lambda} - \lambda\right]k^2 &> \left(1+\beta-\frac{\lambda}{1+\lambda}\right)(\pi^*)^2 - \lambda\frac{2k}{1+\lambda}\pi^* \\
\left[(1+\lambda)\beta\lambda + 1 - 1 - \lambda\right]\lambda k^2 &> \left((1+\beta)(1+\lambda) - \lambda\right)(\pi^*)^2 - \lambda 2k\pi^* \\
\left[(1+\lambda)\beta - 1\right]\lambda^2 k^2 &> (1+(1+\lambda)\beta)(\pi^*)^2 - 2\lambda k\pi^* \\
0 &> (1+(1+\lambda)\beta)(\pi^*)^2 - 2\lambda k\pi^* \\
&\quad + \left[1-(1+\lambda)\beta\right](\lambda k)^2 \\
0 &> Pol(\pi^*)
\end{aligned}$$

If π^* goes to plus or minus infinity, $Pol(\pi^*)$ is driven by $(1+(1+\lambda)\beta)(\pi^*)^2$ which is positive and infinite. Thus, commitment is not feasible for very high or very low inflation expectations.

The value of inflation that minimizes $Pol(\pi^*)$ is obtained by setting the derivative to zero:

$$\begin{aligned}
0 &= Pol'\left(\pi^{low}\right) \\
0 &= 2(1+(1+\lambda)\beta)\pi^{low} - 2\lambda k \\
\pi^{low} &= \frac{\lambda k}{1+(1+\lambda)\beta}
\end{aligned}$$

If we evaluate the polynomial at this value, we get that it is negative:

$$\begin{aligned}
Pol\left(\pi^{low}\right) &= (1+(1+\lambda)\beta)\left(\frac{\lambda k}{1+(1+\lambda)\beta}\right)^2 \\
&\quad -2\lambda k\frac{\lambda k}{1+(1+\lambda)\beta} + \left[1-(1+\lambda)\beta\right](\lambda k)^2 \\
Pol\left(\pi^{low}\right) &= \frac{(\lambda k)^2}{1+(1+\lambda)\beta} - 2\frac{(\lambda k)^2}{1+(1+\lambda)\beta} + \left[1-(1+\lambda)\beta\right](\lambda k)^2 \\
Pol\left(\pi^{low}\right) &= \left[1-(1+\lambda)\beta - \frac{1}{1+(1+\lambda)\beta}\right](\lambda k)^2 \\
Pol\left(\pi^{low}\right) &= \frac{(1+(1+\lambda)\beta)(1-(1+\lambda)\beta) - 1}{1+(1+\lambda)\beta}(\lambda k)^2 \\
Pol\left(\pi^{low}\right) &= \frac{\left(1-(1+\lambda)^2\beta^2\right) - 1}{1+(1+\lambda)\beta}(\lambda k)^2 \\
Pol\left(\pi^{low}\right) &= \frac{-(1+\lambda)^2\beta^2}{1+(1+\lambda)\beta}(\lambda k)^2 < 0
\end{aligned}$$

Therefore $Pol(\pi^*)$ is positive for very low values of inflation expectations, reaches a negative minimal value, and is again positive for very high values of inflation expectations. This implies that there are two roots for which $Pol(\pi^*) = 0$.

The roots of the polynomial are given by the usual formula:

$$\begin{aligned}
\pi^* &= \frac{2\lambda k \pm \sqrt[2]{4(\lambda k)^2 - 4(1+(1+\lambda)\beta)(1-(1+\lambda)\beta)(\lambda k)^2}}{2(1+(1+\lambda)\beta)} \\
\pi^* &= \frac{2\lambda k \pm 2\lambda k \sqrt[2]{1 - \left(1-(1+\lambda)^2\beta^2\right)}}{2(1+(1+\lambda)\beta)} \\
\pi^* &= \lambda k \frac{1 \pm \sqrt[2]{(1+\lambda)^2\beta^2}}{1+(1+\lambda)\beta} \\
\pi^* &= \lambda k \frac{1 \pm (1+\lambda)\beta}{1+(1+\lambda)\beta}
\end{aligned}$$

The larger root is simply the inflation under discretion:

$$\begin{aligned}
\pi^{*\max} &= \lambda k \frac{1+(1+\lambda)\beta}{1+(1+\lambda)\beta} \\
\pi^{*\max} &= \lambda k
\end{aligned}$$

Intuitively, as the central bank chooses λk under discretion, this is a promise that it has no problem holding. The smaller root is:

$$\pi^{*\min} = \lambda k \frac{1-(1+\lambda)\beta}{1+(1+\lambda)\beta}$$

The central bank can credibly commit to any inflation level between $\pi^{*\min}$ and $\pi^{\max}$. This range include zero inflation if $\pi^{*\min} < 0$, that is:

$$1 < (1+\lambda)\beta$$

This is the case if the central bank care about the future, i.e. β is close enough to 1. Note that if $\beta \simeq 1$ then $(1+\lambda)\beta \simeq 1+\lambda$ which is clearly larger than 1.

Looking at the figure (or from the above computations), we can answer the various questions.

1. Explain the shape of the cost line in terms of intuition. The cost is zero when inflation expectation is equal to the discretion level $\lambda k = 0.15 * 0.25 = 3.75\%$. If people expect the central bank to choose the inflation level of discretion, then there is no real deviation as the choice under commitment is the same as under a deviation. For lower values of period t expected inflation ($\pi^* < \lambda k$), a deviation will lead agents to expect higher inflation at period $t+1$ ($\pi^e_{t+1} = \lambda k > \pi^*$). The central bank delivers that level in period $t+1$ but by doing so raises the losses. For higher value of period t inflation expectations ($\pi^* > \lambda k$), the cost is negative as a deviation pushes the economy towards lower inflation in period $t+1$, and thus a lower loss.

2. Explain the shape of the gain line in terms of intuition. The gain is zero if agents expect the

level of inflation corresponding to discretion. Again, in that gain the deviation isn't really one. For lower values of period t expected inflation ($\pi^* < \lambda k$), there is a gain as the central bank can temporarily raise output. For higher value of period t inflation expectations ($\pi^* > \lambda k$), deviating means reducing inflation, and thus lowering output. This is unpleasant in terms of output, but it delivers a reduction of the direct cost of inflation from a very high level, and is thus beneficial.

3. Explain what are the values of π^* that the bank can credibly deliver. The central bank cannot credibly deliver very low inflation. This is because the gain from deviating is too high, as a deviation would raise output. Similarly, it cannot credibly deliver very high inflation, as sticking to a promise of high inflation is very costly. It can credibly deliver inflation over a middle range.

4. Can we deliver zero inflation? Not necessarily. If the central bank is very patient (high β), it puts more weight on the future cost following a deviation. It is then less inclined to deviate, and zero inflation is sustainable. If it puts limited weight on the future however (such as $\beta = 0.7$), then zero inflation will not be delivered.

5. What is the higher inflation that the central bank can credibly deliver? Explain intuitively. The top end of the credible range is simply the inflation under discretion. If agents expect the bank to act as it please, this is something that it will definitely do.

6. Can the central bank always deliver an inflation lower than under discretion? Yes. Even if we set a low value of β, the cost line always goes through the discretion inflation, and goes through this point with a negative slope. More generally, the polynomial has two roots and the lowest one is always lower that the discretion inflation..