Macroeconomics A
Problem Set 2

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1 The cake-eating problem

Consider the following problem:

$$\max_{\{c_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \log c_t$$

subject to

$$\sum_{t=0}^{\infty} c_t \leq k_0$$
$$c_t \geq 0 \text{ for } \forall t \geq 0$$

and k_0 is given and fixed. Use the Lagrangian method to derive the optimal consumption plan for this problem, the so-called *cake eating problem*. The name of the problem comes from the first constraint: there is a certain amount of capital that is not productive and can be consumed over time.

Hint: Follow these steps:

1. Set up the Lagrangian.
2. Combine the FOCs to get the Euler equation, which relates consumption in two consecutive periods (period t and $t+1$).
3. Use the complementary slackness constraint (i.e. that the resource constraint is satisfied with equality).
4. How much of the cake is left at time T?
5. Is the transversality condition satisfied?
6. What if the resource constraint is not satisfied?

2 Exponential Utility

Assume that infinite-horizon households maximize their present discounted utility, where period utility $u(c)$ is now given by the exponential form,

$$u(c) = -\frac{1}{\theta} e^{-\theta c}$$

where $\theta > 0$. The behavior of firms is the same as in the Ramsey model we discussed in the lectures.

1. Relate θ to the concavity of the utility function and to the desire to smooth consumption over time. Compute the inter-temporal elasticity of substitution. How does it relate to the level of per capita consumption, c?

2. Find the first-order conditions for a representative household with preferences given by this form of $u(c)$.

3. Combine the first-order conditions for the representative household with those of firms to describe the behavior of c and k over time. (Assume that $k(0)$ is below its steady-state value.)

4. How does the transition depend on the parameter θ?

3 Factor misallocation: Hsieh-Klenow (2009)

This exercise asks you to derive some key results from the misallocation paper by Hsieh and Klenow (2009). You can find the paper on the Moodle page.

Consider a firm that operates in sector s with production function

$$Y_j = A_j K_j^{\alpha} L_j^{1-\alpha}.$$

Firms in this sector are monopolistically competitive, meaning they internalize the impact of their production decision on the price for their product P_j, but not the impact on the sectoral price index P or total household expenditure PY. Firms face isoelastic demand for their products,

$$Y_j = \left(\frac{P_j}{P}\right)^{-\sigma} Y.$$

Firms take factor prices r (for capital) and w (for labor) as given and maximize profits, but make "mistakes" in the sense that firm j behaves as if the price per unit of capital was $r(1+\tau_{K,j})$ and as if the revenue from sales was $(1-\tau_{Y,j})P_j Y_j$.

1. Write down the objective function that the firm maximizes, and calculate the first-order conditions.

2. Show that $\tau_{K,j}$ can be calculated when the total expenditure on labor, wL_j and the total expenditure on capital rK_j are observed. Show that $\tau_{Y,j}$ can be calculated when total sales and total expenditure on labor wL_j are observed.

3. Show that revenue TFP equals

$$P_j A_j = \frac{\sigma}{1-\sigma}\left(\frac{r}{\alpha}\right)^{\alpha}\left(\frac{w}{1-\alpha}\right)^{1-\alpha}\frac{(1+\tau_{K,j})^{\alpha}}{1-\tau_{Y,j}}.$$

   and discuss the intuition for this relationship.

4. Imagine all firms were facing the same distortions: $\tau_{K,j} = \tau_K$, $\tau_{Y,j} = \tau_Y$. How much lower is output in this economy compared to one where there are no distortions, i.e. $\tau_K = 0$, $\tau_Y = 0$?