# Game Theory Analysis in *A Beautiful Mind*

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## 1 Introduction

*A Beautiful Mind* (2001) tells the story of John Forbes Nash, a mathematician who, despite schizophrenia, contributed profoundly to game theory, earning the Nobel Prize in Economics. In the film, there is a notable scene where John Nash and his friends are in a bar and observe a group of women entering: one blonde and four brunettes. The men are particularly attracted to the blonde and consider approaching her. Nash proposes a strategy that deviates from their initial instincts, suggesting that by not all pursuing the blonde, they can achieve a more favorable outcome. This scene is often cited as an illustration of Nash's contributions to game theory, specifically the concept of Nash Equilibrium. However, a closer analysis reveals complexities that challenge the scene's portrayal.
*John Nash: "I don't believe in luck. But I do believe in assigning value to things."*

## 2 Theoretical Model

1. **Formalize this game using the normal-form representation.**

   - Note the buddies choose brunettes be $B$, and choose blonde be $G$, let four buddies be Player 1, 2, 3, 4.
   - Thus the strategy spaces would be $S_1 = S_2 = S_3 = S_4 = \{B, G\}$.
   - Let the payoff be: ending up with the blonde has a payoff of 4, ending up with a brunette has a payoff of 1, and ending up alone is 0, thus the payoff Bi-matrix would be:

Player 2

| | $B$ | $G$ |
|---|---|---|
| $B$ | $(1,1,1,1)$ | $(1,4,1,1)$ |
| $G$ | $(4,1,1,1)$ | $(0,0,0,0)$ |

Player 3

| | $B$ | $G$ |
|---|---|---|
| $B$ | $(1,1,4)$ | $(0,0,0)$ |
| $G$ | $(0,0,0)$ | $(0,0,0)$ |

Player 4

| | $B$ | $G$ |
|---|---|---|
| $B$ | $(0,0,0)$ | $(0,0,0)$ |
| $G$ | $(0,0,0)$ | $(0,0,0)$ |

Table 4: bi-matrix and Nash equilibrium

2. **Is the result in the story a Nash equilibrium?**

   - The result of the story, which is $(B, B, B, B)$, is not a Nash equilibrium.

3. **Find all pure-strategy Nash equilibria for this game.**

   - All the Nash equilibria for this game would be:

$$(B, G, B, B), (G, B, B, B), (B, B, G, B), (G, G, B, B),$$

$$(B, B, B, G), (G, B, G, G), (B, G, G, G), (G, G, G, G).$$

# 3 Conclusion

The analysis reveals that the only pure strategy Nash equilibrium in this scenario is when one man approaches the blonde and the remaining three men each approach a different brunette. This outcome ensures that no individual can unilaterally change their strategy to achieve a better payoff, satisfying the conditions of a Nash equilibrium. The film's suggestion that all men should ignore the blonde to achieve the best collective outcome does not align with the principles of non-cooperative game theory, as individual incentives lead to different strategic choices.