Macroeconomics A: EI056

Problem set 1

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1 The Taylor principle

1.1 Aggregate demand

We start by defining the real interest rate r_t as the difference between the nominal interest rate set by the central bank i_t and subsequent inflation π_{t+1}:

$$r_t = i_t - \pi_{t+1}$$

The central bank sets the real interest rate in line with its long run equilibrium $\overline{r}$, reacting to deviations in inflation today from the target level $\overline{\pi}$:

$$i_t = \overline{r} + \gamma (\pi_t - \overline{\pi}) + \pi_{t+1}$$

The IS relation links output y_t to the deviation of the real interest rate from its long run equilibrium and a demand shock ε_t^{demand} of expected value equal to zero:

$$y_t = -\sigma (r_t - \overline{r}) + \varepsilon_t^{demand}$$

Show that this gives the aggregate demand AD:

$$y_t = -\gamma\sigma (\pi_t - \overline{\pi}) + \varepsilon_t^{demand}$$

How would you describe, in terms of economic intuition, the cases of $\gamma < 0$ and $\gamma > 0$?

1.2 Supply and demand

Aggregate supply AS links output to movements in inflation, and a supply shock ε_t^{supply} of expected value zero:

$$y_t = \frac{1}{\kappa} (\pi_t - \pi_{t-1}) + \varepsilon_t^{supply}$$

Show that inflation is

$$\pi_t = \overline{\pi} + \frac{1}{1+\gamma\kappa\sigma}\left(\pi_{t-1} - \overline{\pi}\right) + \frac{\kappa}{1+\gamma\kappa\sigma}\left(\varepsilon_t^{demand} - \varepsilon_t^{supply}\right)$$

and output is:

$$y_t = -\frac{\gamma\sigma}{1+\gamma\kappa\sigma}\left(\pi_{t-1} - \overline{\pi}\right) + \frac{1}{1+\gamma\kappa\sigma}\varepsilon_t^{demand} + \frac{\gamma\kappa\sigma}{1+\gamma\kappa\sigma}\varepsilon_t^{supply}$$

1.3 Numerical illustration

Set $\kappa = \sigma = 1$ and $\overline{\pi} = \overline{r} = 0$. Consider that at $t-1$ inflation and output are zero.

Using Excel or similar, plot the impact of a positive demand shock at time t ($\varepsilon_t^{demand} = +0.75$ and zero afterwards), and the impact of a the impact of a negative supply shock ($\varepsilon_t^{supply} = -1$ and zero afterwards). Specifically, show the path of output, inflation, and the nominal interest rate. Also, display the evolution of AS and AD

Do so first for $\gamma = 0.5$ and then for $\gamma = -0.25$.

A positive value of γ is called the "Taylor principle". How do you interpret it intuitively? What are its consequences?

2 Commitment and interest rate

2.1 Inflation and interest rate

The economy is summarized by two relation. The first is the IS relation between output y, the real interest rate r, and a real demand shock u of expected value zero:

$$y = -br + u$$

The second is is the aggregate supply that links inflation π to expected inflation π^e, output y, and a real supply shock e of expected value zero:

$$\pi = \pi^e + ay + e$$

The central bank sets a nominal interest rate is the real rate plus inflation expectations: $i = r + \pi^e$. The central bank minimizes a loss function in terms of output deviation from a target k and inflation:

$$L = \frac{1}{2}\left[\lambda (y-k)^2 + (\pi)^2\right]$$

where λ is the weight of output in the loss, relative to the weight of inflation.

The central bank sets the nominal interest rate based on the value of the shocks (one could have the policy based on some forecasts, but for simplicity assume that the shocks are seen by the central bank) u and e. When the public sets its expectations however, it does not see i (expectations are formed before policy acts).

Show that inflation can be written as:

$$\pi = (1+ab)\, Ei - abi + au + e$$

where E is the expectation operator, so Ei is the expectation of the interest rate.

2.2 Policy under commitment

The central bank commits to a policy rule for its interest rate of the form:

$$i = c_0 + c_U u + c_E e$$

The shocks are independent, so $E(eu) = 0$. Show that the ex-ante expected loss (before shocks are realized) is:

$$EL = \frac{1}{2} E\left[\lambda\left(-b\left(c_U u + c_E e\right) + u - k\right)^2 + \left(c_0 - ab\left(c_U u + c_E e\right) + au + e\right)^2\right]$$

The coefficients c_0, c_U and c_E are set optimally to minimize the expected loss EL. Show that:

$$i = \frac{1}{b}\left(u + \frac{a}{\lambda + a^2}e\right)$$

Interpret this result in terms of economic intuition. Explain how the value of b, a and λ affect movements in interest rates.

2.3 Policy under discretion

Under discretion, the central bank sets the interest rate once it has observed the shocks, and takes the expected interest rate i^e as given. The ex-post loss function is:

$$L = \frac{1}{2}\left[\lambda\left(-b\left(i - i^e\right) + u - k\right)^2 + \left(\left(1 + ab\right)i^e - abi + au + e\right)^2\right]$$

Show that (derive the optimal interest rate, then apply expectations):

$$i = \frac{\lambda k}{a} + \frac{1}{b}\left(u + \frac{a}{\lambda + a^2}e\right)$$

How does discretion affect the reaction to shocks?

If you have data on interest rates, how would you assess empirically the degree of credibility of a central bank's commitment to a rule?

3 Design of monetary policy contract

3.1 Setup

Consider the model of commitment versus discretion in the conduct of policy. The social loss function includes deviations of output y around a target y^* and inflation π:

$$L^{social} = \frac{1}{2} E\left[\lambda\left(y - y^*\right)^2 + \pi^2\right]$$

Output is given by aggregate supply, which includes the natural rate of output y_n, unexpected inflation $(\pi - \pi^e)$, and a shock e of expected value zero:

$$y = y_n + \pi - \pi^e + e$$

The central bank aims for a target output that is above the natural rate: $y^* = y_n + k$.

The conduct of policy is delegated to a central bank that has a different loss function:

$$L^{bank} = \frac{1}{2} E\left[(\lambda - \theta)\left(y - y^*\right)^2 + (1 + \theta)\left(\pi - \pi^T\right)^2\right] + t\pi$$

The central bank conducts policy under discretion. θ represents a shock to the central bank's preference. It reflects the uncertainty about the preferences of the central banker between output and inflation. θ is uncorrelated with e and happens once the private sector has set its inflation expectations (therefore $Ee = E\theta = Ee\theta = 0$).

π^T and t are part of the contract between society and the central bank. π^T is the inflation rate that the bank should aim for, and t is a penalty that the central bank pays if it lets inflation become positive.

3.2 Target contract

Consider that the contract between society and the central bank only entails the objective π^T and no penalty ($t = 0$). Show that inflation is:

$$\pi = \pi^T + (\lambda - \theta)\left(k - \frac{1}{1 + \lambda} e\right)$$

Discuss the intuition behind this result.

Using the solution for inflation, we can show (you don't have to, take this as given) that the expected social loss is:

$$L^{social} = \frac{1}{2} E\left[\lambda\left(-(1 + \theta)k + \frac{1 + \theta}{1 + \lambda} e\right)^2 + \left(\pi^T + (\lambda - \theta)k - \frac{\lambda - \theta}{1 + \lambda} e\right)^2\right]$$

Using this expression, show that the optimal inflation target is

$$\pi^T = -\lambda k$$

Provide an interpretation of this result.

3.3 Penalty contract

Consider that the contract between society and the central bank now only entails the penalty t and no objective ($\pi^T = 0$). Show that inflation is:

$$\pi = (\lambda - \theta)\left(k - \frac{1}{1+\lambda}e\right) - \frac{1+\lambda-\theta}{1+\lambda}t$$

Using the solution for inflation, we can show (you don't have to) that the expected social loss is:

$$L^{social} = \frac{1}{2}E\left[\lambda\left(-(1+\theta)k + \frac{1+\theta}{1+\lambda}e + \frac{\theta}{1+\lambda}t\right)^2 + \left((\lambda-\theta)k - \frac{\lambda-\theta}{1+\lambda}e - \frac{1+\lambda-\theta}{1+\lambda}t\right)^2\right]$$

Using this show that the optimal penalty tax is

$$t = \frac{(1+\lambda)\left(\lambda + E\theta^2\right)}{1+\lambda+E\theta^2}k$$

Provide an interpretation of this result.

How does the uncertainty about the bank's preferences $E\theta^2$ affect the penalty?

3.4 Comparing contracts

We now compare the social losses under an optimal inflation target π^T and an optimal penalty t. We can show that the social losses are (this is complex, so you don't need to prove the result):

$$\begin{aligned}
L^{social}_{\text{target}} &= \frac{1}{2}\left[\left[\lambda + (1+\lambda)E\theta^2\right]k^2 + \frac{\lambda+E\theta^2}{1+\lambda}Ee^2\right] \\
L^{social}_{\text{penalty}} &= \frac{1}{2}\left[\frac{E\theta^2}{1+\lambda+E\theta^2}k^2 + \frac{\lambda+E\theta^2}{1+\lambda}Ee^2 + \lambda k^2\right]
\end{aligned}$$

When is a penalty contract preferred to an inflation target contract? Explain the intuition.