Intermediate Microeconomics

Imperfect Competition I: Monopoly

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Perfectly Competitive Market v.s. Imperfect Competition

The "competitive market" gives the first-best (socially efficient) outcome.

- Each participant is a price-taker
- First theorem of welfare economics: Walrasian equilibrium is socially optimal

"Imperfect market" and market failure

- Market power (市场势力): Imperfect competition (不完全竞争): **monopoly (垄断)**, oligopoly (寡头)
  - Industrial organization
- Incomplete information (不完全信息) and strategic behavior (策略性行为)
  - Game theory; Contract theory; Mechanism design
- Externalities (外部性) and public goods (公共物品)
  - Public economics

Outline: Monopoly

- Single pricing
  - monopoly v.s. perfect competition
  - welfare
- Price discrimination
  - first-degree (privacy)
  - second-degree (two-part tariffs)
  - third-degree (separate markets)

Profit Maximization: A Comparison

- Perfect competition:
  - "Many" firms.
  - Each individual firm cannot affect price p
  - Each firm solves $\max_q pq - C(q)$, where p is given as a constant.
  - F.O.C: $p = C'(q) = MC$, i.e., supply curve in a competitive market.
- Monopoly
  - A single firm faces the entire market demand $p(q)$: a downward sloping curve.
  - The monopolist can alter the market price by adjusting quantities.
  - The monopolist solves $\max_q p(q)q - C(q)$, where $p(q)$ is decreasing in q.
  - Will price be higher/lower than, or equal to marginal cost?

Profit Maximization of a Monopolist

Formally, a monopolist solves

$$\max_q p(q)q - C(q)$$

Let $R(q) = p(q)q$ be its revenue (**总收益**). Hence the marginal revenue (**边际收益**) is the amount earned by producing/selling an additional unit of output:

$$MR(q) = R'(q) = p'(q)q + p(q)$$

The first-order condition of profit maximization gives

$$\underbrace{p(q^m) + p'(q^m)q^m}_{=MR(q^m)} - C'(q^m) = 0 \Rightarrow MR(q^m) = MC(q^m)$$

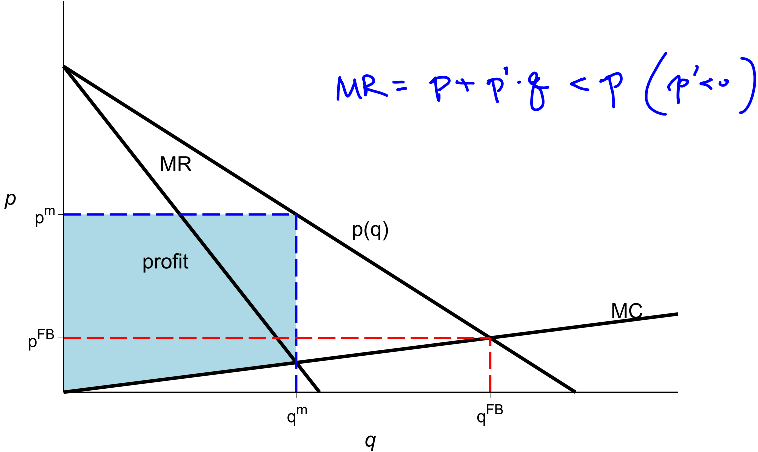
Perfect Competition v.s. Monopoly

- Recall that for the perfectly competitive market, each individual firm solves:
$$\max_q pq - C(q) \Rightarrow p = C'(q^{FB}).$$
- Starting from the equilibrium decision q^{FB} set at the competitive level, now suppose that all suppliers are replaced by a single monopolist. Consider whether the monopolist has an incentive to produce an additional unit of output: the derivative of profit $p(q)q - C(q)$ with respect to q, evaluated at q^{FB}, is
$$\left.\frac{d\pi^m}{dq}\right|_{q=q^{FB}} = p(q^{FB}) + p'(q^{FB})q^{FB} - \underbrace{C'(q^{FB})}_{=p} = p'(q^{FB})q^{FB} < 0.$$
Therefore, the monopolist will not increase but decrease the output.
- The quantity produced by a monopolist is lower than the level determined by a competitive market:

$>$
$$q^{FB} < q^m.$$

Diagram: Monopoly and Perfect Competition

Monopolistic outcome (p^m, q^m) v.s. Perfect competition (p^{FB}, q^{FB})



Decomposition of Marginal Revenue*

- The profit-maximization quantity produced by a monopolist satisfies

$$\underbrace{\underbrace{p(q^m)}_{\text{marginal}}+\underbrace{p'(q^m)q^m}_{\text{infra-marginal}}}_{\text{marginal revenue}}-\underbrace{C'(q^m)}_{\text{marginal cost}}=0\Leftrightarrow MR(q^m)=MC(q^m).$$

为卖出更多产品的降价

- The effect of selling an additional unit:
  - Marginal effect: selling an additional unit brings about $p>0$.
  - Infra-marginal effect: a lower price $\Leftrightarrow$ a greater quantity $\Rightarrow$ since all consumers pay the same price, each unit will be sold at a lower price: $p'(q)q<0$.
  - Marginal cost: producing an additional unit incurs $-C'(q)<0$.
- If the gains outweigh the losses, then the monopoly continue to produce; otherwise, the monopolist reduces output. At optimum, the marginal benefit is equal to the marginal loss, i.e., $MR(q^m)=MC(q^m)$.

FOC and SOC

- The profit-maximization output of a monopolist is derived by the first-order condition (with respect to q):

$$\pi(q) = p(q)q - C(q)$$
$$\frac{d\pi}{dq} = p(q) + p'(q)q - C'(q) = 0$$

- The second-order condition for such a maximization problem requires a negative second-order derivative:

$$\begin{aligned}\frac{d^2\pi}{dq^2} &= p'(q) + p''(q)q + p'(q) - C''(q)\\ &= p''(q)q + 2p'(q) - C''(q) < 0.\end{aligned}$$

Comparative Statics

- Assume constant marginal cost (e.g., CES technology with $\gamma = 1$): $MC = c$. The monopoly's problem is $\max_q p(q)q - cq$.
- The optimal choice of q satisfies (from FOC)

$$p'(q^m)q^m + p(q^m) - c = 0$$

- The second order condition requires that

$$p''(q^m)q^m + 2p'(q^m) < 0.$$

- What is the effect due to a higher cost c? Differentiate the FOC w.r.t. c gives

$$p''(q^m)\frac{dq^m}{dc}q^m + p'(q^m)\frac{dq^m}{dc} + p'(q^m)\frac{dq^m}{dc} - 1 = 0$$
$$\Rightarrow \left[p''(q^m)q^m + 2p'(q^m)\right]\frac{dq^m}{dc} = 1 \Rightarrow \frac{dq^m}{dc} < 0.$$

Practice: compute $\frac{d\pi(q^m)}{dc}$ and $\frac{dp(q^m)}{dc}$.

Measuring Market Power: Lerner's Formula

- Under perfect competition: $p - MC = 0$
- Under monopoly: $p + p'(q)q - MC = 0 \Rightarrow p - MC > 0$
- The degree of market power (市场势力) is captured by $p - MC$, i.e., price markup (价格加成)
- Recall the definition of demand elasticity: $\varepsilon_D = -\frac{dq}{dp}\frac{p}{q}$.
- The monopoly's outcome can be expressed in terms of price markup, and elasticity:

$$p - MC = -p'(q)q \Leftrightarrow \frac{p - MC}{p} = -\frac{\frac{dp}{dq}q}{p} = \frac{1}{-\frac{dq}{dp}\frac{p}{q}} = \frac{1}{\varepsilon_D}$$

- Lerner's formula/index (勒纳指数): $\frac{p-MC}{p} = \frac{1}{\varepsilon_D}$.
  - A higher index $\Leftrightarrow$ a higher markup $\Leftrightarrow$ more market power.
  - A higher markup $\Leftrightarrow$ demand is relatively inelastic

Monopoly and Welfare

- Total surplus, as a measure for social welfare, is the sum of consumer surplus and producer surplus.
- Assume that the equilibrium price is quantity is $p(q^*)$ and q^*, respectively.
- Consumer surplus is the sum of the willingness to pay, minus price:

$$CS = \int_0^{q^*} \left[\underbrace{p(q)}_{\text{demand curve}} - \underbrace{p(q^*)}_{\text{price}} \right] dq.$$

- Producer surplus is the sum of the amount of money received, minus production costs:

$$PS = \int_0^{q^*} \left[\underbrace{p(q^*)}_{\text{price}} - \underbrace{C'(q^*)}_{\text{marginal cost}} \right] dq.$$

- Then, total surplus is

$$W = CS + PS = \int_0^{q^*} \left[p(q) - C'(q) \right] dq.$$

Monopoly and Deadweight Loss (净损失)

- Under monopoly, $p'(q^m)q^m + p(q^m) - C'(q^m) = 0$. Total surplus is

$$W^m = \int_0^{q^m} \left[p(q) - C'(q)\right] dq.$$

- Under perfect competition, $p = C'(q^{FB})$. Total surplus is

$$W^{FB} = \int_0^{q^{FB}} \left[p(q) - C'(q)\right] dq.$$

- Suppose that the monopolist firm is taken over by a benevolent authority, who produces q^{opt} to maximize total surplus W.

$$\max_q \int_0^q \left[p(t) - C'(t)\right] dt$$

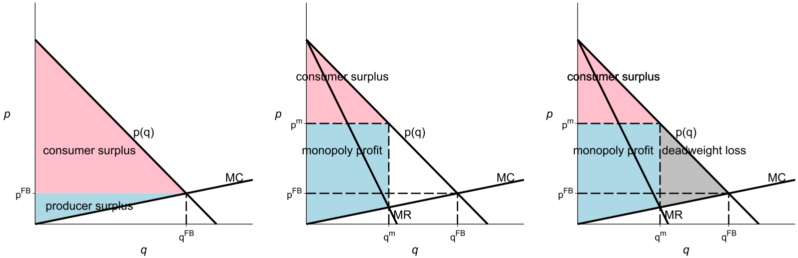
$$\Rightarrow \frac{dW}{dq} = p(q) - C'(q) = 0 \Rightarrow p(q^{opt}) = C'(q^{opt}) \Rightarrow q^{opt} = q^{FB}$$

$$\Rightarrow W^{opt} = W^{FB} = \int_0^{q^{FB}} \left[p(q) - C'(q)\right] dq.$$

- Perfect competition = Socially optimal.
- We have shown that $q^m < q^{FB} = q^{opt}$, hence

$$W^m = \int_0^{q^m} \left[p(q) - C'(q)\right] dq < \int_0^{q^{FB}} \left[p(q) - C'(q)\right] dq = W^{FB}.$$

- Compared with the first-best outcome, the monopoly produces too little, and the price is too high, such that $W^m < W^{FB}$, incurring a deadweight loss (DWL) of $W^{FB} - W^m$.



Two Types of DWL: Monopoly and Tax

Since monopoly generates DWL, consider whether taxing the monopoly can be welfare-improving. The authority imposes *ad valorem* tax of rate τ. The monopolist pays the tax.

- Assume quasi-linear utility: $u_i(x_i)$. For each consumer: $u_i'(x_i^*) = p$. Assume there are n consumers.
- For the monopolist: $\max_q (1-\tau)p(q)q - C(q)$
  - FOC: $(1-\tau)\left[p'(q^m)q + p(q^m)\right] - C'(q^m) = 0$
  - SOC: $(1-\tau)\left[p''(q^m)q^m + 2p'(q^m)\right] - C''(q^m) < 0$
- At equilibrium, $\sum_{i=1}^n x_i^* = q^m$. Note that, x_i^* and q^m are functions of τ.
- $\sum_{i=1}^n x_i^* = q^m \Rightarrow \frac{dq^m}{d\tau} = \frac{d\sum x_i^*}{d\tau}$.
- Differentiate the FOC with respect to τ:
  $-\left[p'(q)q + p(q)\right] + (1-\tau)\left[p''(q)q\frac{dq}{d\tau} + 2p'(q)\frac{dq}{d\tau}\right] = C''(q)\frac{dq}{d\tau}$, hence
  $\frac{dq}{d\tau} = \frac{p'(q)q+p(q)}{(1-\tau)[p''(q)q+2p'(q)]-C''(q)} < 0$.

- Total surplus at equilibrium is

$$W(\tau) = \sum_{i=1}^{n} u_i(x_i^*) - C(q^m)$$

- The first order effect of tax is

$$W'(\tau) = \sum_{i=1}^{n} \underbrace{u_i'(x_i^*)}_{=p} \frac{dx_i^*}{d\tau} - C'(q^m)\frac{dq^m}{d\tau}$$

$$= p \underbrace{\sum_{i=1}^{n} \frac{dx_i^*}{d\tau}}_{=\frac{dq^m}{d\tau}} - \underbrace{C'(q^m)}_{=(1-\tau)(p'q+p)} \frac{dq^m}{d\tau} = \left[p - (1-\tau)\left(p'(q^m)q^m + p\right)\right]\frac{dq^m}{d\tau}$$

$$= \left[-p'(q^m)q^m + \tau \underbrace{\left(p'(q^m)q^m + p\right)}_{=\frac{C'}{1-\tau}}\right]\frac{dq^m}{d\tau} = \left[-p'(q^m)q^m + \frac{\tau}{1-\tau}C'(q^m)\right]\frac{dq}{d\tau} < 0$$

- Monopoly is worse than perfect competition.
- Taxing monopoly is even worse.

Linear Demand

- Some textbooks assume that demand curve is linear, i.e., $p = a - bq$. Where does it come from?
  - A simple way is the "unit-demand" (单位需求) paradigm.
  - A unit mass of consumers. Each consumer has the valuation θ that is drawn from the support $[\underline{\theta}, \overline{\theta}]$ according to $F(\cdot)$.

$$\text{measure of consumers whose } \theta > p = \int_p^{\overline{\theta}} f(\theta)d\theta = 1 - F(p).$$

  - Unit demand (i.e., each buys at most one unit): given price p, buying the product gives $\theta - p$; otherwise, the outside option gives zero.
- If we assume $\underline{\theta} = 0$, $\overline{\theta} = 1$ and uniform distribution $F'(\theta) = f(\theta) = 1$, then the market demand is $q = \Pr(\theta > p) = \int_p^1 1 \cdot d\theta = 1 - p$, or $p = 1 - q$.

Example: Linear Demand and Monopoly Pricing

- The monopolist solves $\max_q p(q)q - cq$, where $p(q) = 1 - q$, and assume $c < p < 1$.
- First-order condition implies $q^m = \frac{1-c}{2} \Rightarrow p^m = \frac{1+c}{2} > c$.
- Consumer surplus can be computed by two ways:
  - by integrating the region below the demand curve and above price:

$$CS = \int_0^{q^m} (p(q) - p^m)\, dq = \frac{(1-c)^2}{8}$$

  - or by integrating the utilities of consumers

$$CS = \int_{p^m}^{1} (\theta - p^m) d\theta = \frac{(1-c)^2}{8}$$

- Producer surplus is $PS = \frac{(1-c)^2}{4}$.
- Total surplus is $W^m = CS + PS = \frac{3(1-c)^2}{8}$.

For market demand $p = 1 - q$, if the market is perfectly competitive:

- $p^{FB} = MC = c$, and hence $q^{FB} = 1 - c$
- Consumer surplus is $\int_0^{1-c} (p(q) - c)\, dq = \frac{(1-c)^2}{2}$.
- Producer surplus is $p^{FB}q - cq = (p - c)q = 0$
- Total surplus is $W^{FB} = CS + PS = CS + 0 = \frac{(1-c)^2}{2}$.
- Compared to the welfare under monopoly, $W^{FB} = \frac{(1-c)^2}{2} > \frac{3(1-c)^2}{8} = W^m$.

In the above examples, we assume that the monopolist charges a single price that applies to all consumers. Because the monopolist has market power, the seller is able to use price discrimination.

Price Discrimination (价格歧视/区别定价)

A monopoly engages in price discrimination if it is able to sell otherwise identical units of output at different prices.

- First-degree or perfect price discrimination: If each buyer can be separately identified by a monopolist, then it may be possible to charge each the maximum price he or she would willingly pay for the good.
- Second-degree price discrimination through two-part tariffs (两部定价):
    - Single two-part tariff: a fixed entry-fee + per-unit price
    - Non-linear two-part tariff: different two-part tariffs (menu: 套餐) designed for different consumers
- Third-degree price discrimination through market separation: the monopoly can separate its buyers into relatively few identifiable markets (such as “rural-urban,” ”domestic-foreign,” or ”prime-time-offprime”) and pursue a separate monopoly pricing policy in each market.

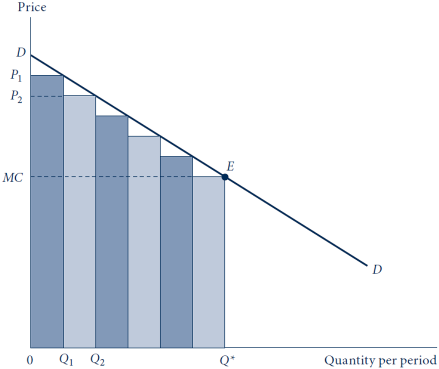
First-Degree: Perfect Discrimination

Claim

Total surplus under first-degree price discrimination is equivalent to the first-best outcome.

- Linear demand example: $p = 1 - q$ and $C(q) = cq$.
  - $p = \theta$ for any $\theta \in [0, 1]$.
  - $CS = 0$ because $\theta - p = 0$.
  - $q = 1 - c$ and $PS = \int_0^{1-c} (p(q) - c)\, dq = \frac{(1-c)^2}{2}$.
- The monopolist exacts all consumer surplus.
- Total surplus is maximized, and is equal to the socially optimal level.
- First-best outcome.

First-Degree Price Discrimination



However, in reality, it's pretty difficult for the firm to acquire the information about "the willingness to pay of a particular consumer."

Using Privacy to Implement First-Degree Price Discrimination (隐私定价)

For simplicity, assume $c = 0$.

- If the monopolist charges a single price for all consumers, the firm solves $\max_p p(q)q - cq$, where $q = \Pr(\theta > p) = \int_p^1 d\theta = 1 - p$. Hence $p^m = \frac{1}{2}$ and $q^m = \frac{1}{2}$.
  - $CS^m = \int_{p^m}^1 (\theta - p^m) d\theta = \frac{1}{8}$; $PS^m = p^m(1 - p^m) = \frac{1}{4}$,
  - Total surplus is $W^m = \frac{3}{8}$.
  - Those whose valuation $\theta < \frac{1}{2}$ are not served.
- Now suppose the firm is able to acquire each consumer's demand information by **voluntary** disclosure: i.e., each consumer **can choose** to or not to "tell" the firm about his/her valuation. And the firm will offer a customized price for each consumer who discloses his/her information.
  - For those who choose not to disclose their private information, the monopolist charges a single price p^m.
  - "History/Behavior Based Price Discrimination (BBPD):" e.g., **大数据杀熟** (browse histories, cookies)

Privacy and First-Degree Price Discrimination

Assume the monopolist announces p^{ND} for those who do not disclose; and charges a customized price $p^D = \theta - \epsilon$ where ϵ is an infinitely small but positive number, for a consumer who discloses his/her valuation/privacy θ.

- Who is willing to disclose his/her privacy?
  - Those whose valuations are below p^{ND} will: buying gives $\theta - p^D = \theta - (\theta - \epsilon) = \epsilon$, which better than not buying.
- What is the profit-maximizing p^{ND}?
  - Given p^{ND}, those whose $\theta > p^{ND}$ will not disclose.
  - The profit from selling a unit to a buyer with $\theta > p^{ND}$ who does not disclose is p^{ND}; The firm can earn more by setting $p^{ND} = \theta - \epsilon$.
  - Similarly, the firm will increase p^{ND} to 1, and all consumers will be served by customized prices.
- Therefore, the monopolist charges customized prices for all consumers. The profit is $\int_0^1 (\theta - \epsilon) d\theta = \frac{1}{2} - \epsilon$. Every consumer will buy with a surplus ϵ, and total consumer surplus is $\int_0^1 \epsilon d\theta = \epsilon$.

- Under privacy pricing: a threat $p^{ND} = 1$ with customized prices $p^D = \theta - \epsilon$
  - $CS = \int_0^1 (\theta - p^D) d\theta = \epsilon$
  - $PS = \int_0^1 (\theta - \epsilon) d\theta = \frac{1}{2} - \epsilon$
  - $W^D = CS + PS = \frac{1}{2}$
- Under single-pricing, $p^m = \frac{1}{2}$, $q^m = \frac{1}{2}$ and
  - $CS^m = \frac{1}{8} > 0$
  - $PS^m = \frac{1}{4} < \frac{1}{2}$
  - $W^m = \frac{3}{8} < \frac{1}{2}$
- Under perfect competition: $p^{FB} = MC = 0$:
  - $CS^{FB} = \int_{p^{FB}}^1 \theta d\theta = \frac{1}{2}$
  - $PS^{FB} = 0$
  - $W^{FB} = \frac{1}{2}$.
- Therefore, total surplus under price discrimination is equivalent to the socially optimal level.
- Reason behind: all consumers are covered with customized pricing; whereby only 50% of consumers are covered without discrimination (incurring deadweight losses).

Third-Degree through Market Separation

Consider two "segmented" markets, A and B.

- What does "segmentation" mean?
  - Consumers in market A cannot "imitate" those who in market B.
- The inverse demand for the two markets is $p_A(q)$ and $p_B(q)$, respectively.
- A and B are isolated $\Rightarrow$ the firm maximizes profits in each market.
- For an additional output to be produced, the firm incurs $C'(q)$.
  - Sell it at market A gives MR_A; sell it at B gives MR_B.
  - If $MR_A > MR_B$, the current unit will be supplied to A; otherwise, supply to B.
  - When an additional unit gives equally profitable profit between A and B, that unit is the last unit to be produced.
- Therefore, $MR_A = MR_B = MC$.

Third-degree price discrimination for n segmented markets:
$MR_A = MR_B = .. = MR_n = MC$

- For market A, the firm solves $\max_q p_A(q)q - C(q) \Rightarrow \frac{p_A - MC}{p_A} = \frac{1}{\varepsilon_A}$;
- For market B, the firm solves $\max_q p_B(q)q - C(q) \Rightarrow \frac{p_B - MC}{p_B} = \frac{1}{\varepsilon_B}$;
- The above two equations give

$$MC = p_A - p_A \frac{1}{\varepsilon_A} = p_B - p_B \frac{1}{\varepsilon_B}$$
$$\Rightarrow \frac{p_A}{p_B} = \frac{1 - \frac{1}{\varepsilon_B}}{1 - \frac{1}{\varepsilon_A}}$$

A higher price for less elastic market (lower $\varepsilon_A \rightarrow$ higher p_A).

Example: Third-Degree Price Discrimination

- Two isolated markets: $q_L = 8 - p_L$ and $q_H = 10 - p_H$. $MC = 2$.
- $p_L = 8 - q_L \Rightarrow MR_L = 8 - 2q_L$; $p_H = 10 - q_B \Rightarrow MR_H = 10 - 2q_H$.
- $MR_L = MR_H = MC = 2 \Rightarrow$
  $8 - 2q_L = 10 - 2q_H = 2 \Rightarrow q_L = 3,\ q_H = 4$.
- $p_L = 5$ and $p_H = 6$.
- Profit is $\pi_L + \pi_H = (p_L q_L - 2q_L) + (p_H q_H - 2q_H) = 25$.
- If the monopolist charges a single price to all markets, then total demand is (horizontal aggregation evaluated at a particular p):
  $Q = q_L + q_H = 18 - 2p$. Inverse demand is $p = 9 - Q/2$.
  $MR = 9 - Q = MC = 2 \Rightarrow Q = 7$, $p = 11/2$ and
  $\pi = pQ - 2Q = 49/2 < 25$.

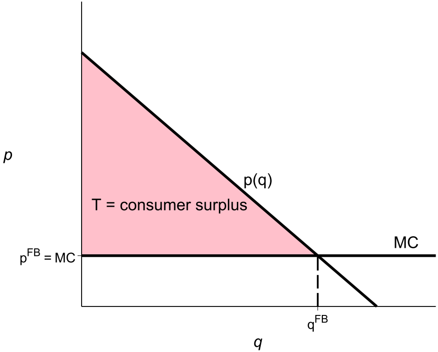
Second-Degree through Price Schedule: Homogeneous (同质的) Consumers

Now, let's focus on one market only. If the firm charges a single price, the consumer surplus is positive. How to extract the surplus through a single price?

- Assume that the demand curve is derived from UMP of a representative consumer with quasi-linear utility, i.e., $u(q)$.
- The monopolist uses two-part tariff:
  1. The consumer has to pay an "entry fee" T to get access to the market.
  2. The per-unit price is p.
- Utility of the consumer: $u(q) - pq - T$. The outside option is normalized to be zero.
  - $\max_q u(q) - pq - T \Rightarrow u'(q) = p$.
  - If $u(q) - pq - T \geq 0$, he/she buys; otherwise, he/she does not buy.
- For the monopolist: $\max_{q,T} T + pq - C(q)$
  - The monopolist can raises T such that each consumer's surplus approaches to zero: $u(q) - pq - T = 0$
  - Plug $T = u(q) - pq$ into the objective: $\max_q u(q) - C(q)$

Two-Part Tariffs: Homogeneous Consumers

- The monopolist solves $\max_q u(q) - C(q) \Rightarrow u'(q) = C'(q)$. Combining $u'(q) = p$, the monopolist should set $u'(q) = p = C'(q)$.
- That is, $p = MC$. The fixed fee is equal to "consumer surplus."
- The first-best outcome is achieved.



Example: Two-Part Tariffs with Homogeneous Consumers

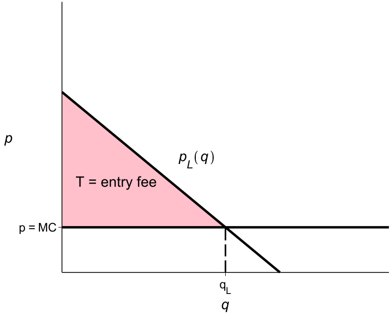
- One consumer with demand curve: $q = 8 - p$
- Marginal cost is $MC = 2$.
- The profit-maximizing two-part tariff:
  - Per-unit price: $p = MC = 2$, hence $q = 8 - 2 = 6$.
  - Entry fee: $T = \int_0^q (8 - q - 2)\, dq = \frac{1}{2}(8 - p)^2 = 18$.
- The profit of the monopoly: $T + pq - 2q = T = 18$.

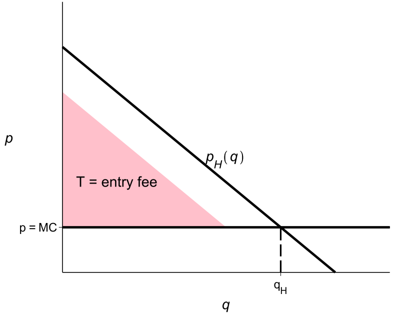
Two-Part Tariffs: Heterogeneous (异质性) Consumers

Now consider two heterogeneous consumers with demand curves $q_L = 8 - p$ and $q_H = 10 - p$.

- Assume that the monopolist uses two-part tariffs (T, p) to serve both consumers, by setting $p = MC = 2$.
- The entry fee is equal to the consumer surplus of the low-demand type: $T = \frac{1}{2}(8 - p)^2 = 18$.
- At $p = MC = 2$, the per-unit profit from each consumer is zero: $pq_L - 2q_L = pq_H - 2q_H = 0$.
- Profit of the monopolist is $2T = 36$.

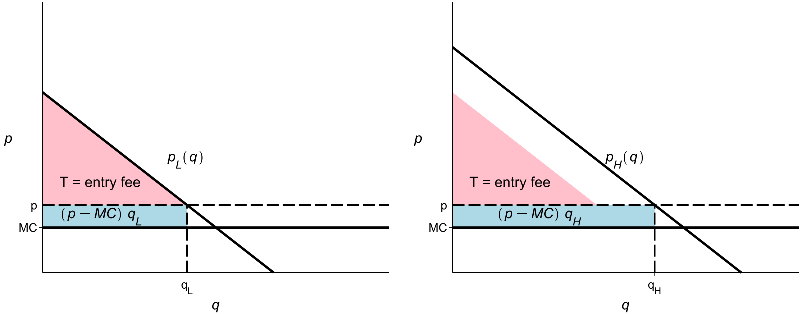
The monopolist uses two-part tariffs (T, p) where $p = MC$ to sell to heterogeneous consumers. The profit is 36.





Can the monopolist do better?

Two-Part Tariffs: Heterogeneous Consumers



In addition to set an entry fee T, the monopolist chooses a per-unit price p that may not necessarily be equal to marginal cost. Then, the entry fee T becomes a function of p.

Two-Part Tariffs: Heterogeneous Consumers

Still, one single fixed fee T and one single price p for all consumers. Now, p is not necessarily equal to MC, but let p be a choice variable.

- The entry fee for the low-type consumer: $T(p) = \int_0^{q_L} (8 - q - p)\, dq$.
- The low-type buys $q_L = 8 - p$ units at price p. Hence $T(p) = \frac{1}{2}(8 - p)^2$.
- The high-type also pays the entry fee $T(p) = \frac{1}{2}(8 - p)^2$.
- The high-type buys $q_H = 10 - p$ units at price p.
- The profit of the monopolist is
  $\pi = 2T(p) + (p - MC)q_L + (p - MC)q_H =$
  $(8 - p)^2 + (p - 2)(8 - p) + (p - 2)(10 - p)$, i.e., a function of p.
- FOC w.r.t. p gives $p = 3$, then $T = 25/2$ and $\pi = 25 + 5 + 7 = 37 > 36$.

Practice: $q_A = A - p$ and $q_B = B - p$, assuming $A > B > c > 0$, where c is constant marginal cost. Provide solutions for first/second/third-degree price schedules. (A challenging issue: for the single two-part tariff, is it possible that type-B will not be served?)

Comparing Two-Part Tariff and Single-Pricing

By using a single price, the monopolist charges p^m such that $MR = MC$. Using two-part tariff, the per-unit price is denoted by p^{STP}. We would like to compare: p^m, p^{STP} and $p^{FB} = c$ (assuming marginal cost is c); and profits generated by the three types of pricing scheme.

- Clearly, for profit, $\pi^{STP} \geq \pi^m \ > 0$. Because compared with single monopoly price p^m, T is the additional choice under two-part tariff: you can choose to set $T = 0$ to make p^{STP} and p^m equivalent.
- Still, consider two different consumers: $\theta_H u(q) - pq - T$ and $\theta_L u(q) - pq - T$. $\theta_H > \theta_L$ and $u'(\cdot) > 0$, $u''(\cdot) < 0$.
- The low-type buys q_L and the high-type buys q_H. The market demand is $Q = q_L + q_H$.
- The optimal single-pricing p^m satisfies $MR = MC \Rightarrow p'(Q)Q + p - c = 0$. Hence $p^m > c = p^{FB}$.
- The per-unit price for two-part tariffs is p^{STP}.

- Under two-part tariffs: for each consumer, $\theta_i u'(q_i^*) = p, \; i = H, L.$
- The entry fee: $T = \theta_L u(q_L^*) - pq_L^*$.
- The profit under two-part tariff:
  $\pi^{STP} = 2T(p) + (p-c)(q_H^*(p) + q_L^*(p)) =$
  $2\left[\theta_L u(q_L^*(p)) - pq_L^*(p)\right] + (p-c)(q_H^*(p) + q_L^*(p))$.
- First-order derivative with respect to p:

$$\frac{d\pi^{STP}}{dp} = 2\underbrace{\theta_L u'(q_L^*)}_{=p}\frac{dq_L^*}{dp} - 2q_L^* - 2p\frac{dq_L^*}{dp} + \underbrace{q_H^* + q_L^*}_{=Q^*} + (p-c)\underbrace{\left(\frac{dq_H^*}{dp} + \frac{dq_L^*}{dp}\right)}_{=\frac{dQ^*}{dp}}$$

$$= -2q_L^* + Q^* + (p-c)\frac{dQ^*}{dp}.$$

- The optimal single-pricing p^m satisfies:
  $p'(Q)Q - p - c = 0 \Rightarrow (p^m - c) = -\frac{dp^m}{dQ}Q^m$.
- If the monopolist replaces the per-unit price under two-part tariff by p^m:
  $\frac{d\pi^{STP}}{dp}\Big|_{p=p^m} = -2q_L < 0$, i.e., the optimal per-unit price p^{STP} under two-part tariff is lower than the monopoly single price p^m: $p^{STP} < p^m$

- Under perfect competition: $p^{FB} = c$.
- Under two-part tariffs, the first-order derivative with respect to p:

$$\frac{d\pi^{STP}}{dp} = 2\underbrace{\theta_L u'(q_L^*)}_{=p}\frac{dq_L^*}{dp} - 2q_L^* - 2p\frac{dq_L^*}{dp} + \underbrace{q_H^* + q_L^*}_{=Q^*} + (p-c)\underbrace{\left(\frac{dq_H^*}{dp} + \frac{dq_L^*}{dp}\right)}_{=\frac{dQ^*}{dp}}$$

- If the monopolist replace the per-unit price by $p^{FB} = c$, then

$$\left.\frac{d\pi^{STP}}{dp}\right|_{p=p^{FB}} = q_H^* - q_L^*$$

  - Because $\theta_H u'(q_H^*) = p^{STP} = \theta_L u'(q_L^*)$ and $u''(\cdot) < 0$, then $q_H^* > q_L^*$.
  - Therefore, $\left.\frac{d\pi^{STP}}{dp}\right|_{p=p^{FB}} > 0$, i.e., the optimal per-unit price p^{STP} under two-part tariff is higher than p^{FB}: $p^{STP} > p^{FB}$
- $p^m > p^{STP} > p^{FB} = c$.

- In this course, we assume that the monopolist uses "single" two-part tariffs, i.e., the same T and p for different consumers.
- In practice, the monopolist frequently designs different menus $(T_1, q_1), ..., (T_n, q_n)$, and let the consumers to choose.
  - The monopolist cannot distinguish the types of different consumers.
  - Type i cannot "imitate" type j: type i will voluntarily choose the menu (T_i, q_i) designed for him/her, instead of another menu (T_j, q_j).
- Non-linear two-part tariffs:
  - Business class v.s. economy class
  - "Standard version" v.s. "premium version"
  - Labor contracts
- Those topics will be covered in the game theory course in the spring semester.