# Macroeconomics A
# Problem Set 4

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## Question 1

Consider an RBC model with a representative household that maximizes utility

$$U_t = E_t \sum_{t=0}^{\infty} \beta^t \left( \log \left( C_t \right) + \nu \left( 1 - L_t \right) \right)$$

where $\nu$ is an increasing and concave function. The production function is

$$Y_t = A_t L_t^{1-\alpha} \left( z_t K_t \right)^{\alpha}$$

with $0 < \alpha < 1$ and $z_t$ denotes a capital utilization choice. For example, low capital utilization may mean running the machines only at half speed or half of the time.

Capital accumulation is described by

$$K_{t+1} = \left( 1 - \bar{\delta} z_t^{\phi} \right) K_t + Y_t - C_t.$$

1. Explain how variable utilitzation of capital is modeled here. Should we assume that $\phi < 1$ or $\phi > 1$? (5 points)

2. Derive and explain the optimality conditions of the social planner's problem. The social planner's problem is to maximize total utility subject to the economy's physical resource constraints (i.e. production, and the law of motion for capital). The social planner can set the levels of consumption, investment, capacity utilization, and labor supply, without having to worry about prices or market clearing. (10 points)

3. Derive an expression for the steady-state capital utilization $z^*$ in terms of the steady-state capital-output ratio. If we want to normalize $z^* = 1$, how should we calibrate $\phi$? (10 points)

4. Compared to a standard RBC model (as in the lecture), do you expect this model to generate a stronger or weaker response of output to a TFP shock? (5 moins)

5. Assuming the model is correct, how is the standard empirical Solow residual $\log Y_t - (1-\alpha)\log L_t - \alpha \log K_t$ related to total factor productivity $\log A_t$? (5 points)

## Question 2

We study a variation of the simplified RBC model from the lecture. Consider an economy populated by a mass of representative households and a mass of representative firms which take output and factor prices as given. The production function is Cobb-Douglas:

$$Y_t = e^{z_t} K_t^{\alpha} L_t^{1-\alpha}$$

where $e^{z_t}$ is total factor productivity, $K$ is capital, and $L$ is labor. Capital fully depreciates in every periods so that

$$K_{t+1} = I_t$$

where $I_t$ is investment.

The representative household supplies labor $L_t$ and consumes $C_t$. It maximizes

$$U = E_0 \left[ \sum_{t=0}^{\infty} \beta^t \left[ \log C_t - e^{\chi_t} L_t \right] \right]$$

where $\chi_t$ is a preference shock. Capital is accumulated by the household and rented to the firm, as usual. Any profits of the firms are rebated to the households. As usual, we have $0 < \alpha < 1$, $0 < \beta < 1$.

1. Write down the budget constraint of the household and the profit function of firms. (5 points)
2. Derive first-order conditions of firms and households. (10 points)
3. Show that when factor markets and final good markets are competitive, firm profits are zero. (5 points)
4. Solve the model and show that the process for output in a competitive equilibrium is

   $$y_t = z_t + \alpha y_{t-1} - (1 - \alpha) \chi_t$$

   (where lowercase letters denote the log of the uppercase letter). (10 points)
5. Assume $y_{-1} = 0$ and $z_t = 0$ for all $t$, and $\chi_t = 0$ for all $t$ except that $\chi_0 = 1\%$. Draw the time path for $\chi_t$, $z_t$, and $y_t$. Explain why $y_t$ is persistent. (5 points)