Macroeconomics A: Review Session IX

Overlapping Generations

Gregory Auclair
allan.auclair@graduateinstitute.ch

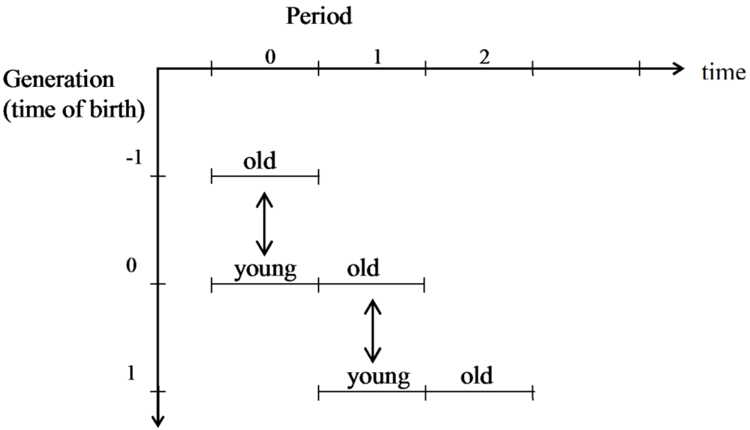
Outline

Table of contents

Why We Use OLG

- With a relative simple setup, get very rich outcomes
  - Generates saving (in levels)
  - Useful to think about long-term demographic or structural changes
  - Analyze Ricardian equivalence, secular stagnation, etc.
- OLG can also include bequests
- Bequests are main source of wealth for most households
  - Low inheritance taxes have increased the persistence of wealth inequality
  - Interesting area of study :)
- With negative interest rates, rational asset bubbles may emerge
  - OLG models can capture some of these dynamics
  - May explain rising property values, etc.

Basic Structure of OLG



Specification

- We can further simplify the two-period model we saw in class
- Log utility is a special case of CRRA utility where $\theta = 1$

$$U(c_t) = \begin{cases} \frac{c_t^{1-\theta}}{1-\theta} & \theta \geq 0; \theta \neq 1 \\ \log(c_t) & \theta = 1 \end{cases}$$

- We can also specify $\beta = \frac{1}{1+\rho}$, therefore the household problem is

$$\max_{c_1, c_2} \quad U_t = log(c_{1,t}) + \beta \log(c_{2,t+1})$$

$$\text{s.t.}$$

$$c_{1,t} + b_t = w_t n_t$$

$$c_{2,t} = (1 + r_t) b_{t-1}$$

Household Optimization Problem

- Let's combine the budget constraints into a single equation

$$c_{1,t} + \frac{c_{2,t+1}}{1+r_{t+1}} = w_t n_t$$

- Now we can write the Lagrangian

$$\mathcal{L}_t = log(c_{1,t}) + \beta \log(c_{2,t+1}) - \lambda_t \left(c_{1,t} + \frac{c_{2,t+1}}{1+r_{t+1}} - w_t n_t \right)$$

- Accordingly the FOCs are

$$\frac{\partial \mathcal{L}_t}{\partial c_{1,t}} = 0 \implies \frac{1}{c_{1,t}} = \lambda_t$$

$$\frac{\partial \mathcal{L}_t}{\partial c_{2,t+1}} = 0 \implies \frac{\beta(1+r_{t+1})}{c_{2,t+1}} = \lambda_t$$

$$\implies c_{2,t+1} = \beta(1+r_{t+1})c_{1,t}$$

Saving and Market Clearing

- Using the solution from the previous slide in the budget constraint

$$c_{1,t} + \frac{\beta(1 + r_{t+1})c_{1,t}}{1 + r_{t+1}} = w_t n_t$$

$$c_{1,t} = \frac{1}{1 + \beta} w_t n_t$$

- Using the original budget constraint

$$b_t = \frac{\beta}{1 + \beta} w_t n_t$$

- Households can only save in capital

$$b_t = k_{t+1}$$

Firm Optimization Problem

- Firms are perfectly competitive and set the wage and interest rate
- Output is Cobb-Douglas and firms pay labor and rent capital,

$$\Pi_t = Y_t - w_t n_t - (r_t + \delta) k_t$$
$$Y_t = A_t k_t^{\alpha} n_t^{1-\alpha}$$

- Profit maximization gives

$$\frac{\partial \Pi_t}{\partial n_t} = 0 \implies w_t n_t = (1-\alpha) Y_t$$
$$\frac{\partial \Pi_t}{\partial k_t} = 0 \implies (r_t + \delta) k_t = \alpha Y_t$$

- We can make hours worked numeraire ($n_t = 1$)

$$w_t = (1-\alpha) y_t$$

$$y_t = A_t k_t^{\alpha}$$

Putting Things Together

- We can now solve for the interest rate using $b_t = k_{t+1}$

$$k_{t+1} = \mathbb{E}_t\left[\frac{\alpha y_{t+1}}{r_{t+1}+\delta}\right] \quad \text{(capital demand)}$$

$$b_t = \frac{\beta}{1+\beta}(1-\alpha)y_t \quad \text{(capital supply)}$$

- The interest rate intermediates capital demand and supply
- Easy to solve for the steady state value of r

$$r_t = \frac{\alpha(1+\beta)}{\beta(1-\alpha)}\frac{y_t}{y_{t-1}} - \delta \quad \Longrightarrow \quad r^* = \frac{\alpha(1+\beta)}{\beta(1-\alpha)} - \delta$$

- Steady state value of A is exogenous, therefore

$$k^* = \left(\frac{\alpha A^*}{r^*+\delta}\right)^{\frac{1}{1-\alpha}}$$

Modeling a Shock

- What happens after a productivity shock?

$$\widetilde{A}_t = \varepsilon_t \quad \text{where} \quad \widetilde{A}_t = \log\left(\frac{A_t}{A^*}\right) \quad \text{and} \quad \epsilon_t \sim N(0, \sigma)$$

- To solve the shock, we can log-linearize the system

$$\widetilde{y}_t = \widetilde{A}_t + \alpha \widetilde{k}_t \tag{1}$$

$$\widetilde{w}_t = \widetilde{y}_t \tag{2}$$

$$\frac{r^*}{\delta + r^*}\widetilde{r}_t = \widetilde{y}_t - \widetilde{y}_{t-1} \tag{3}$$

$$\widetilde{k}_{t+1} = \widetilde{w}_t \tag{4}$$

$$\widetilde{c}_{1,t} = \widetilde{w}_t \tag{5}$$

$$\widetilde{c}_{2,t} = \frac{r^*}{1 + r^*}\widetilde{r}_t + \widetilde{k}_t \tag{6}$$

- Note that $\widetilde{y}_{t-1} = 0$ and $\widetilde{k}_t = 0$ as both were set before the shock

Exercise

For $\{\alpha;\ \beta;\ \delta\} = \{1/3;\ 2/3;\ 0\}$ what is $\widetilde{c}_{2,t+2}$ if $\varepsilon_t = 0.1$?

$$r^* = \frac{\frac{1}{3} \cdot \frac{5}{3}}{\frac{2}{3} \cdot \frac{2}{3}} = \frac{5}{4}$$

$$\widetilde{c}_{2,t+2} = \frac{5}{9}\widetilde{r}_{t+2} + \widetilde{k}_{t+2}$$

$$\widetilde{y}_t = \varepsilon_t = 0.1; \quad \widetilde{k}_{t+1} = 0.1$$

$$\widetilde{y}_{t+1} = \alpha\widetilde{k}_{t+1} = \frac{0.1}{3}; \quad \widetilde{k}_{t+2} = \frac{0.1}{3}$$

$$\widetilde{y}_{t+2} = \alpha\widetilde{k}_{t+2} = \frac{0.1}{9}; \quad \widetilde{r}_{t+2} = \frac{0.1 - 0.3}{9} = -\frac{0.2}{9}$$

$$\widetilde{c}_{2,t+2} = \frac{5}{9} \cdot \frac{-0.2}{9} + \frac{0.1}{3} = -\frac{0.7}{9} \approx -0.08$$

Note: this result is in terms of output per effective labor

Table of contents

Introduction

- Ricardian equivalence: financing government spending out of current taxes or future taxes (and current deficits) will have equivalent effects on the overall economy
- Implies fiscal expansion has small effect
- Higher expected taxes $\rightarrow$ more saving $\rightarrow$ lower aggregate demand
- Fiscal multiplier depends on ‘myopia’ of households or expectation only future generations will be taxed :(
- Heterogeneity matters: constrained, hand-to-mouth households may spend entire fiscal transfer while well-off may save all of it

Taxes in OLG

- Let's add taxes to our model to analyze Ricardian equivalence

$$\max_{c_1, c_2} \quad U_t = log(c_{1,t}) + \beta \log(c_{2,t+1})$$

$$\text{s.t.}$$

$$c_{1,t} + b_t = w_t n_t + \tau_{1,t}$$

$$c_{2,t} = (1 + r_t) b_{t-1} + \tau_{2,t}$$

- Asset market clearing includes government debt d_t

$$b_t = k_{t+1} + d_t$$

$$d_t = \tau_{1,t} + \tau_{2,t}$$

- Government can transfer between generations, incur a net tax and pay down debt, or incur a net transfer and increase debt

Effect of a Transfer between Generations

- Let's say the government makes a transfer between generations

$$\tau_{1,t} = -\tau_{2,t} \qquad \tau_{1,t} > 0$$

- There is no change in debt, no crowding-out of capital
- We can define aggregate demand <u>in the steady state</u> as

$$Y^d = \underbrace{\frac{1}{1+\beta}(wn+\tau_1)}_{c_1} + \underbrace{(1+r)k+\tau_2}_{c_2} + \underbrace{\delta k}_{inv}$$

- Simple to show the tax has no effect on demand in equilibrium

$$b = \frac{\beta}{1+\beta}(wn+\tau_1) \implies r = \frac{\alpha(1+\beta)}{\beta}\frac{Y}{wn+\tau_1} - \delta$$

- Since $b = k$ and $\tau_{1,t} = -\tau_{2,t}$

$$Y^d = \frac{1}{1+\beta}(wn+\tau_1) + \frac{\beta}{1+\beta}(wn+\tau_1) + \alpha Y - \tau_1 = \frac{wn}{1-\alpha}$$

- What about capital? What is the initial effect of the transfer?

Borrowing on Domestic Markets

- Simple to show that crowding-out effect can be strong

$$k_{t+1} = \underbrace{\frac{\beta}{1+\beta}(w_t n_t + \tau_{1,t})}_{b_t} - \underbrace{(\tau_{1,t} + \tau_{2,t})}_{d_t} \qquad r_{t+1} = \frac{\alpha Y_{t+1}}{b_t - d_t} - \delta$$

- This supports idea that government debt should be used judiciously
- Also begs the question why interest rates are low and what to do...
- While borrowing from abroad works, it also lowers consumption in the steady state (we will see this next semester)
- Important to note that OLG captures long-term dynamics, not short-term fluctuations

Thinking about Demographic Structure

- Size of each generation matters and OLG often includes population growth
- World population getting much, much older than in the past

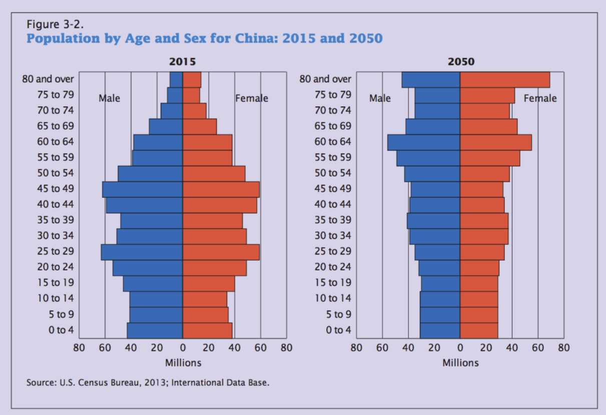


Table of contents

Government Budget Constraint

- We can write the government budget as

$$\underbrace{P_t G_t + (1 + i_t) B_t}_{\text{expenditures}} = \underbrace{P_t T_t + B_{t+1}}_{\text{revenue}}$$

- Here, all bonds are one-period
- We can define the primary deficit as $D_t^p = P_t G_t - P_t T_t$ so that

$$D_t^p + (1 + i_t) B_t = B_{t+1}$$

- Furthermore, let's normalize the variables by GDP Y_t where

$$\frac{B_{t+1}}{Y_t} = \underbrace{\frac{B_{t+1}}{Y_{t+1}}}_{b_{t+1}} \underbrace{\frac{Y_{t+1}}{Y_t}}_{1+g_{t+1}}$$

- Note that g is the growth rate of <u>nominal</u> GDP in this case

Explosive Debt Dynamics

- Using the GDP normalized variables (lower case)

$$\frac{d_t^p}{1+g_{t+1}} + \frac{(1+i_t)b_t}{1+g_{t+1}} = b_{t+1}$$

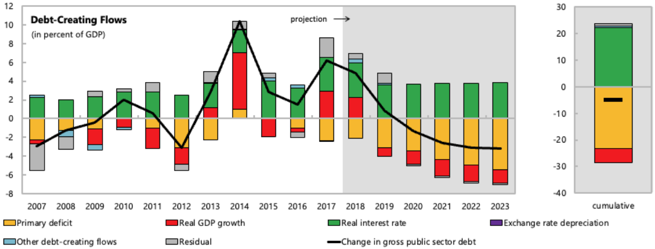
- Set $i_t = i$ and $g_t = g$ for all t and solve recursively with $d^p = 0$

$$\left(\frac{1+i}{1+g}\right)^2 b_t = b_{t+2} \implies \left(\frac{1+i}{1+g}\right)^n b_t = b_{t+n}$$

- When $i > g$, this is explosive despite no primary deficit
- If $i < g$, then debt disappears :)
- Even if the government can lock in low interest rate on debt, future growth is somewhat uncertain
- Also, as government issues more debt, interest rates rise

Assessing Debt Sustainably

- The IMF DSA template is available online
- Deficit, real interest rate, and real GDP growth drive dynamics



- What makes debt risky?
  - Currency composition (commitment to pay in hard currency)
  - Type of creditor (domestic vs. foreign)
  - Maturity mismatches and structure of payments
- Note the real interest rate is a function of inflation $r = i - \pi$