Geneva Graduate Institute (IHEID)

Econometrics II (EI062), Spring 2025

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Problem Set 4

Due: Sunday, 11 May, 23:59

- Prepare concise answers.
- State clearly any additional assumptions, if needed.
- You are encouraged to collaborate in groups but the final write-up should be individual.
- Submit your solutions, along with any code (if applicable), in a **single pdf file** through **Moodle**. If you choose to write your solutions by hand, please make sure your scanned answers are legible.
- Grading scale:

5.5	default grade
6	absolutely no mistakes and particularly appealing write-up (clear and concise answers, decent formatting, etc.)
5	more than a few mistakes, or single mistake and particularly long, wordy answers
4	numerous mistakes, or clear lack of effort (e.g. parts not solved or not really attempted)
1	no submission by due date

Problem 1

This excercise is inspired by McConnell, Margaret and Gabriel Perez-Quiros (2000): "Output Fluctuations in the United States: What has changed since the early 1980's?" *American Economic Review*, 90(5), 1464-76.

Download some aggregate time series from FRED: Real GDP, Real Personal Consumption Expenditure, Real Gross Private Domestic Investment. Use quarterly frequency.

(a) Take logs of GDP, Consumption, and Investment. Plot the three series (you can generate the plots directly in FRED). What are the most striking features?

(b) For each series, estimate a model of the form

$$y_t = \beta_1 + \beta_2 t + u_t$$

using OLS, based on the samples 1965:Q1 to 2006:Q4, 2007:Q1 to 2019:Q4, and 2007:Q1 to 2022:Q2.

(c) According to your estimates, what are the annualized average growth rates (in percent) of GDP, consumption, and investment? Are these series growing, approximately, at the same rate?

(d) For each subsample, compute sample autocorrelation functions for the deviations of output, consumption, and investment (the $\hat{u}_t$'s) from their estimated deterministic trend.

(e) Now compute quarter-on-quarter growth rates of these three series as $\ln y_t - \ln y_{t-1}$, plot the growth rates. What are the most striking features?

(f) Compute the sample means of the growth rates for each of the above subsamples. Compare the growth-rate results to (ii). Also compute the sample standard deviations for the growth rates.

(g) Repeat the analysis in (vi) for the subsamples "before 1984", "between 1984 and 2006". Did the means and the volatility of the series change?

Problem 2

Consider the MA(3) process

$$y_t = (1 - 2.4L + 0.8L^2 - 0.4L^3)u_t \ ,$$

where L denotes the lag operator and

$$\mathbb{E}[u_t u_\tau] = \begin{cases} 1 & \text{if } t = \tau \\ 0 & \text{otherwise} \end{cases} .$$

(a) Define Weak and Strict Stationarity. Is the process y_t weakly stationary? Is it strictly stationary? You may impose additional conditions on the u_t's.

(b) Calculate the autocovariance function of y_t.

(c) Calculate $\mathbb{V}\left[\frac{1}{\sqrt{T}}\sum_{t=1}^{T} y_t\right]$.

(d) Suppose $\{u_t\}$ is i.i.d. with $\mathbb{E}[u_t] = 0$ and $\mathbb{V}[u_t] = \sigma^2$, and define $x_t = u_t u_{t-4}$. Is the process x_t strictly stationary? Is x_t ergodic? Is x_t a White Noise process?

(e) Calculate $\mathbb{E}\left[\frac{1}{T}\sum_{t=1}^{T} x_t\right]$ and $\mathbb{V}\left[\frac{1}{\sqrt{T}}\sum_{t=1}^{T} x_t\right]$.