Macroeconomics A, EI056

Class 9

Overlapping generations model

Cédric Tille

November 21, 2023

What you will get from today class

- Money and **inflation**, the Cagan model.
- Irrelevance of timing of taxes with a representative agent with infinite life (**Ricardian equivalence**).
- Going **beyond the representative agent**: succession of agents with finite lives (the **overlapping generations** (OLG) model).
  - General steps.
  - A simpler version, illustration of how the timing of taxes matters.
- Possible inefficiency of resources allocation in OLG: when **bubbles** can help.
- Combining infinite horizon and OLG models.

A question to start

When the government borrows, investors purchase the debt instead of investing in firms. Government debt is thus to be avoided as it reduces economic activity.

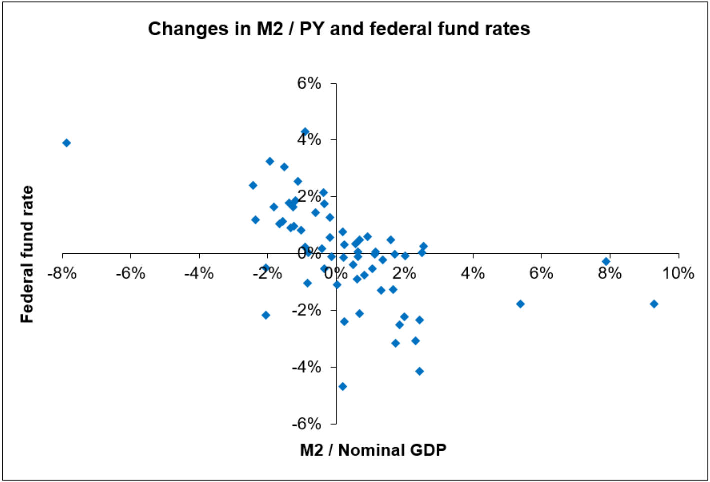
Do you agree? Why or why not?

MONEY AND PRICE

Bringing money in macro models

- **Value of transactions** can be from two angles: price level P * quantity (real GDP) Y; money M * number of times it is used (velocity V): $PY = MV$
- Three broad ways to generate a money demand.
- Real balances in the **utility function** (even with a small weight).
  - Trade off between money (gives utility) and bonds (pay interest).
  - Money demand: real balances linked to consumption and the nominal interest rate ($M/(PY) = 1/V$ inversely related to interest rate).
- **Cash in advance**: one needs to hold real balances to make purchases. Links money and consumption.
- **Shopping technology**: household splits time between work, leisure and shopping.
  - Shopping time is an increasing function of consumption and a decreasing function of money.
  - Generates a money demand relation between real balances, consumption and the interest rate.

- Ratio of money to GDP inversely correlated with the interest rate.



Cagan model of inflation

- Go beyond the $PY = MV$ to be more specific on the demand for real balances.
- Nominal interest rate is equal to the real rate plus inflation expectations: $i_t = r + \pi^e_{t+1}$.
    - Focus on the nominal variables and take the real rate r to be constant (and zero for simplicity).
    - Higher expected **inflation reduces the demand** for money (inflation acts as a tax reducing the real value of cash).
- Money demand is given by:

$$m_t - p_t = -\gamma \pi^e_{t+1} = -\gamma \left(p^e_{t+1} - p_t \right)$$

- Dynamic relation between the current price and future expected prices. Iterating forward, the price reflects **future expected money**:

$$p_t = \frac{1}{1+\gamma} \sum_{s=0}^{\infty} \left(\frac{\gamma}{1+\gamma} \right)^s m^e_{t+s}$$

Constant money growth rate

- Constant growth rate of money: $m_t - m_{t-1} = \mu$. Inflation is constant and equal to μ:

$$\begin{aligned} m_{t+1} - p_{t+1} &= -\gamma\pi \quad ; \quad m_t - p_t = -\gamma\pi \\ (p_{t+1} - p_t) &= (m_{t+1} - m_t) = \mu \end{aligned}$$

- Higher price level when money grows at a fast rate:

$$\begin{aligned} m_t - p_t &= -\gamma\pi \\ p_t &= m_t + \gamma\mu \end{aligned}$$

- Higher growth rate **reduces the demand for real balances**, as money looses its value fast:

$$m_t - p_t = -\gamma\mu$$

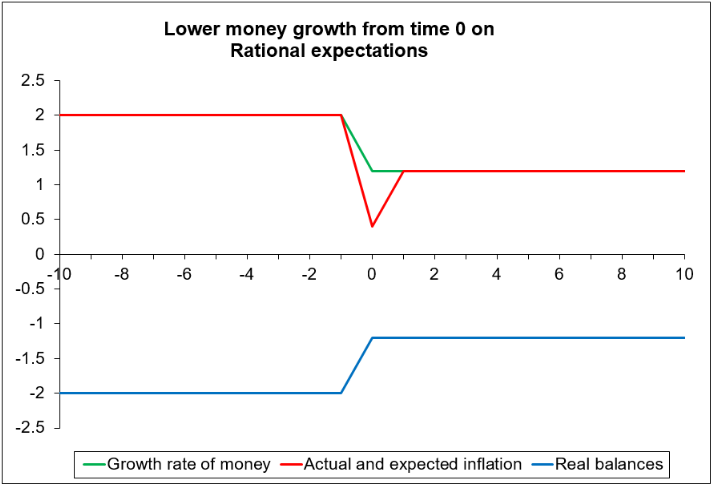
- A lower μ lowers inflation.

Disinflation under rational expectations

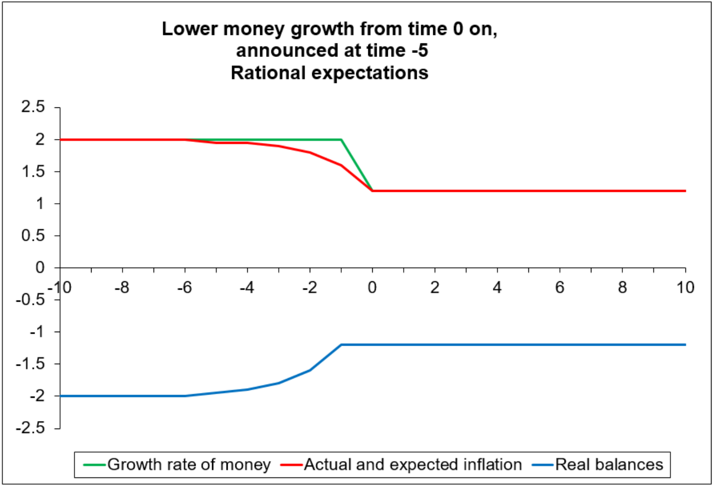
- Agents immediately understand the new regime at time $T+1$. Inflation **drops** from μ_0 to μ_1.
- At the period of transition, inflation **undershoots** for one period.
  - Lower money growth leads to a positive **jump in the demand** for real balances $m-p$.
  - Nominal balances m do not jump up (they only grow at a slower rate). So the **price** p **has to jump** down, a one-shot low inflation
- If the regime is announced before implementation, real balances react right away.
- Transition with adaptive expectations in extra slides. Adaptive expectations

Dynamics under rational expectations

- Lower money growth starting at time 0.



- Lower money growth starting at time 0, announced at time -5.



RICARDIAN EQUIVALENCE

Budget constraints

- Household lives **forever**. Invest in capital k and a government bond b. Both pay the same rate of return.
- **Flow** budget constraint (τ_t denotes taxes):

$$w_t - \tau_t + r_t (k_t + b_t) = c_t + (k_{t+1} + b_{t+1}) - (k_t + b_t)$$

- Government spends g funded by taxes and debt. Flow budget constraint:

$$b_{t+1} = g_t - \tau_t + (1 + r_t) b_t$$

- Add the two constraints. **Taxes do not enter**, only government spending does. Resource use by the government matters, not how it finances it.

$$w_t + r_t k_t = c_t + g_t + k_{t+1} - k_t$$

Intertemporal constraint

- Combine successive flow constraints into an overall **intertemporal** one $\left(R_{t,t+s} = \prod_{i=0}^{s} \left(1/\left(1+r_{t+i}\right)\right)\right)$:

$$
\begin{aligned}
k_t &= \frac{c_t + g_t - w_t}{1+r_t} + \frac{k_{t+1}}{1+r_t} \\
k_t &= \frac{c_t + g_t - w_t}{1+r_t} + \frac{c_{t+1} + g_{t+1} - w_{t+1}}{\left(1+r_t\right)\left(1+r_{t+1}\right)} + \frac{k_{t+2}}{1+r_{t+1}} \\
k_t &= \sum_{s=0}^{\infty} R_{t,t+s} \left[c_{t+s} + g_{t+s} - w_{t+s}\right] + \lim_{s\to\infty} R_{t,t+s} k_{t+s+1}
\end{aligned}
$$

- **Transversality condition**: $\lim_{s\to\infty} R_{t,t+s} k_{t+s+1} = 0$. Capital doesn't grow at a pace higher than the interest rate (no explosive growth).
- **Present value** of private and public consumption = capital + PV wages: Details of intertemporal constraints

$$
\sum_{s=0}^{\infty} R_{t,t+s} \left(c_{t+s} + g_{t+s}\right) = k_t + \sum_{s=0}^{\infty} R_{t,t+s} w_{t+s}
$$

Ricardian equivalence

- Intertemporal **tax switch**: tax cut today funded by debt, higher taxes tomorrow to pay the debt and interest.
- **No impact** on private consumption (no extra government spending).
  - Total tax bill has no changed (lower taxes today vs. higher taxes tomorrow).
  - Household saves to pay for future taxes, buying the very bond that the government issues.
- Key reason: both government and household **view the future identically**, because:
  - Same interest rate.
  - Same time horizon.
- In reality, private agents have shorter horizons. Motivation for overlapping generations (OLG) models.

OVERLAPPING GENERATIONS

Structure of population

- Successions of cohorts of agents with finite lifes. Different cohorts **coexist** at a given time.
- Each agent lives for **two periods**. Agent borne at time t maximizes utility over consumption when young at time t, $c_{1,t}$, and when old at time $t+1$, $c_{2,t+1}$:

$$U_t = \frac{(c_{1,t})^{1-\theta}}{1-\theta} + \frac{1}{1+\rho} \frac{(c_{2,t+1})^{1-\theta}}{1-\theta}$$

  where ρ is the discount rate.
- Young agent supplies one unit of labor paid a wage w_t, invests in bonds and capital for old age earning with rate of return r_{t+1}.

Individual's budget constraint

- Tax $\tau_{1,t}$ when young and $\tau_{2,t+1}$ when old.
- Flow budget constraints when young and old ($s_{1,t}$ denotes savings by the young agent in bonds and capital):

$$\begin{aligned} c_{1,t} + s_{1,t} &= w_t - \tau_{1,t} \\ c_{2,t+1} &= (1 + r_{t+1}) s_{1,t} - \tau_{2,t+1} \end{aligned}$$

- Combine for the **intertemporal** budget constraint:

$$c_{1,t} + \frac{c_{2,t+1}}{1 + r_{t+1}} = w_t - \left(\tau_{1,t} + \frac{\tau_{2,t+1}}{1 + r_{t+1}} \right) = \Omega_t$$

- Ω_t can be interpreted as **wealth** of the young agent, i.e. value of lifetime income.

Euler condition

- Lagrangian solved by the young agent at time t:

$$\begin{aligned}\mathcal{L}_t &= \frac{(c_{1,t})^{1-\theta}}{1-\theta} + \frac{1}{1+\rho}\frac{(c_{2,t+1})^{1-\theta}}{1-\theta} \\ &\quad +\lambda_t\left[\Omega_t - c_{1,t} - \frac{c_{2,t+1}}{1+r_{t+1}}\right]\end{aligned}$$

- First-order conditions for consumption:

$$\begin{aligned}0 &= \frac{\partial \mathcal{L}_t}{\partial c_{1,t}} = (c_{1,t})^{-\theta} - \lambda_t \\ 0 &= \frac{\partial \mathcal{L}_t}{\partial c_{2,t+1}} = \frac{1}{1+\rho}(c_{2,t+1})^{-\theta} - \lambda_t \frac{1}{1+r_{t+1}}\end{aligned}$$

- Combining we get the **Euler equation**:

$$\frac{c_{2,t+1}}{c_{1,t}} = \left(\frac{1+r_{t+1}}{1+\rho}\right)^{\frac{1}{\theta}}$$

Optimal consumption

- Euler condition and budget constraint give consumption when young and old:

$$c_{1,t} = \frac{(1+\rho)^{\frac{1}{\theta}}}{(1+\rho)^{\frac{1}{\theta}} + (1+r_{t+1})^{\frac{1-\theta}{\theta}}}\Omega_t$$

$$c_{2,t+1} = \frac{(1+r_{t+1})^{\frac{1}{\theta}}}{(1+\rho)^{\frac{1}{\theta}} + (1+r_{t+1})^{\frac{1-\theta}{\theta}}}\Omega_t$$

- **Saving** by the young is wage minus consumption: $s_{1,t} = w_t - \tau_{1,t} - c_{1,t}$. Function of taxes, the wage, and the future interest rate.

Channels of interest rate impact

- **3 channels** of impact of interest rate on consumption when young:

$$c_{1,t} = \frac{(1+\rho)^{\frac{1}{\theta}}}{(1+\rho)^{\frac{1}{\theta}} + (1+r_{t+1})^{\frac{1}{\theta}}(1+r_{t+1})^{-1}}\left[w_t - \left(\tau_{1,t} + \frac{\tau_{2,t+1}}{1+r_{t+1}}\right)\right]$$

- **Substitution** effect: higher rate makes savings more attractive, reducing consumption: $(1+r_{t+1})^{\frac{1}{\theta}}$ term.
- **Income** effect: higher rate makes reaching a given value of assets tomorrow easier, lowering savings and raising consumption: $(1+r_{t+1})^{-1}$ term.
- **Wealth** effect: higher interest rate reduces (absolute) present value of future income. With negative future income, lifetime income is higher and so is consumption.
- Log utility ($\theta = 1$): substitution and income effects cancel, consumption is a constant share of wealth.

PRODUCTION AND DYNAMICS

Firm's optimization

- Standard framework of technology with constant productivity ($L = 1$):

$$Y_t = (K_t)^{\alpha} (L)^{1-\alpha} \Rightarrow y_t = (k_t)^{\alpha}$$

  Lower case letters denote scaled by labor.

- **Input demands**: wage and real interest rates (marginal costs) equal to respectively marginal products of labor and capital:

$$w_t = (1-\alpha)(k_t)^{\alpha} \quad ; \quad r_t = \alpha (k_t)^{\alpha-1}$$

Dynamics: assets

- Young agents invest in capital k and government bonds b.
- Assets available at time $t+1$ held by old agents, who bought it using their savings at time t:

$$k_{t+1} + b_{t+1} = s_{1,t}$$

- **Government** budget constraint (abstract from government spending), where b is debt (so $(-b)$ is government's asset):

$$(-b_{t+1}) = \tau_{1,t} + \tau_{2,t} + (1 + r_t)(-b_t)$$

- **Good market clearing**: output equal to private consumption by young and old agents, plus capital accumulation (no depreciation):

$$(k_t)^{\alpha} = c_{1,t} + c_{2,t} + k_{t+1} - k_t$$

Dynamics: capital

- For simplicity: log utility of consumption ($\theta = 1$). Dynamics of capital (recall that $r_{t+1} = \alpha \left(k_{t+1}\right)^{\alpha-1}$): ▸ With general utility

$$
\begin{aligned}
k_{t+1} + b_{t+1} &= s_{1,t} = w_t - \tau_{1,t} - c_{1,t} \\
k_{t+1} &= \tau_{1,t} + \tau_{2,t} - \left(1 + r_t\right) b_t + w_t - \tau_{1,t} - c_{1,t} \\
k_{t+1} &= \frac{1}{2+\rho}\left[\left(1-\alpha\right)\left(k_t\right)^{\alpha} - \tau_{1,t}\right] + \frac{1+\rho}{2+\rho}\frac{\tau_{2,t+1}}{1+\alpha\left(k_{t+1}\right)^{\alpha-1}} \\
&\quad + \left(\tau_{1,t} + \tau_{2,t} - \left(1 + r_t\right) b_t\right)
\end{aligned}
$$

- With no taxes and no government debt, unique **steady state** capital (possibility of multiple steady states for capital when $\theta \neq 1$):

$$
k^* = \left(\frac{1-\alpha}{2+\rho}\right)^{\frac{1}{1-\alpha}}
$$

Contrast with Ramsey model

- In Ramsey, the steady state the Euler (with log utility) is:

$$\frac{c_{t+1}}{c_t} = 1 = \frac{1 + \alpha\left(k^*\right)^{\alpha-1}}{1+\rho} \Rightarrow k^* = \left(\frac{\alpha}{\rho}\right)^{\frac{1}{1-\alpha}}$$

- **Steady states differ** between the Ramsey and OLG economies.
  - Ramsey: Euler pins down capital, and capital dynamics relation gives consumption.
  - OLG: Euler applies **within cohorts** (consumption can change across time) but **not across** cohorts.

Linearization

- Linear approximation around steady state with no taxes (hatted values are log deviations from steady state).
- Consumption of young and old agents:

$$\hat{c}_{1,t} = \hat{w}_t - \hat{\tau}_{1,t} - \frac{1}{1+r^*}\hat{\tau}_{2,t+1} \qquad ; \qquad \hat{c}_{2,t+1} = \hat{r}_{t+1} + \hat{c}_{1,t}$$

- Wage and real interest rate (from firm): $\hat{w}_t = \alpha \hat{k}_t$ and $\hat{r}_t = r^* \left(1+r^*\right)^{-1} (\alpha - 1)\, \hat{k}_t$.
- Dynamics of assets, and government budget constraint:

$$\begin{aligned} \hat{k}_{t+1} + \hat{b}_{t+1} &= \hat{w}_t - \hat{\tau}_{1,t} + \frac{1}{1+r^*}\hat{\tau}_{2,t+1} \\ \hat{b}_{t+1} &= -\left(2+\rho\right)\left(\hat{\tau}_{1,t} + \hat{\tau}_{2,t}\right) + \left(1+r^*\right)\hat{b}_t \end{aligned}$$

- Utility of an agent borne at time t:

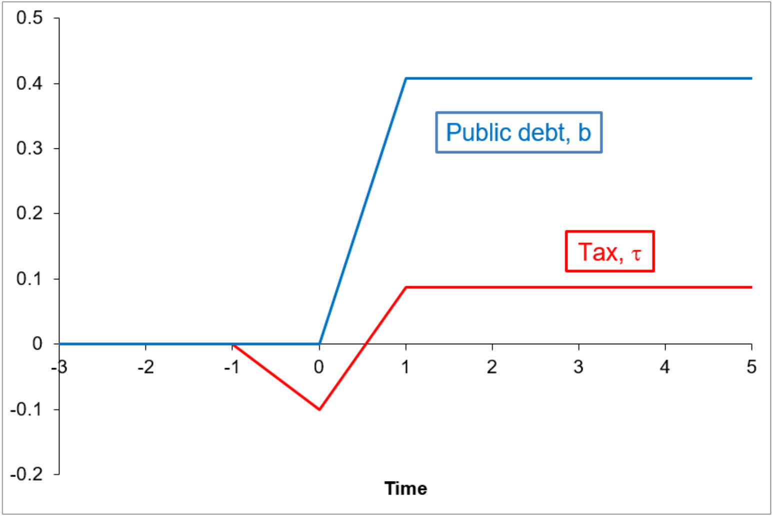
$$\hat{v}_t = \hat{c}_{1,t} + \frac{1}{1+\rho}\hat{c}_{2,t+1}$$

Effect of intertemporal tax shift

- Start at steady state with zero taxes.
- At time t government unexpectedly introduces a tax switch:
  - **Transfer** to both households at time t: $\hat{\tau}_{1,t} = \hat{\tau}_{2,t} = \hat{\tau}_t < 0$.
  - **Tax increase** from $t+1$ on ($\hat{\tau}_{1,t+s} = \hat{\tau}_{2,t+s} = \hat{\tau}_{t+s} > 0$ for $s = 1, 2, ...$), to pay for interest and keep the debt constant.
- Impact at time t:
  - Old agents consume the gift.
  - Young agents see higher lifetime income (future tax hike does not offset initial gift) and increase consumption.
  - Output set in the short run (capital is given), so **higher consumption** lowers investment.
- Impact from time $t+1$ on: agents only face higher taxes and lower wages (capital has gone down), so they suffer.

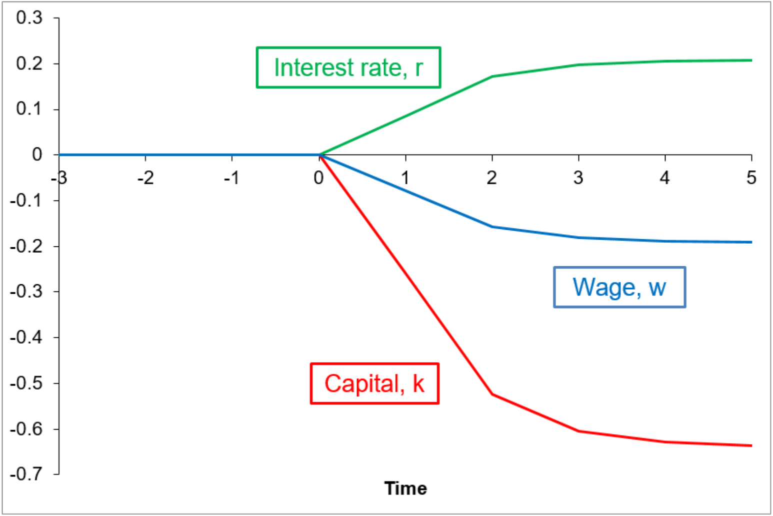
Tax and government debt

- Tax cut, followed by increase. Permanent increase in debt.



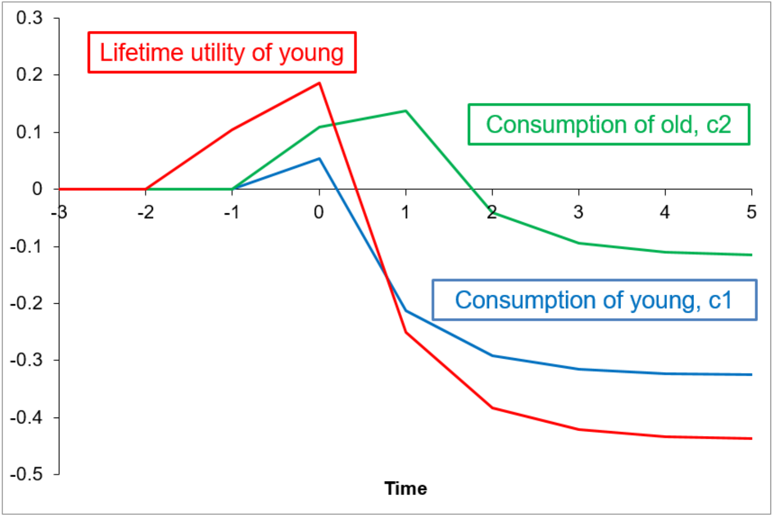
Factor prices and capital

- Crowding out of private capital, raises capital return and reduces wages.



Consumption and utility

- Higher consumption (and utility) of agents alive at the time of the tax cut. Adverse effect for the subsequent generations.



Intuition for the adjustment

- Temporary tax cut alters consumption and output in OLG. It did not do so in representative agent model.
- Agents and the government have **different horizons**.
  - Government has an **infinite** horizon. Switch does not change the net present values of taxes.
  - Agents have **short** horizon. Switch changes their net present value of taxes.
  - Agents at time t see a lower net present value of taxes, future generations see a higher net present value of taxes.

Alternative interpretation: borrowing constraints

- If agents are **prevented from borrowing**, they effectively face an $+\infty$ interest rate, different from the government.
- In Ramsey an agent who would like to borrow but cannot is not on the Euler: marginal utility of current consumption higher than the future marginal utility adjusted by the market interest rate:

$$(c_t)^{-\theta} > \frac{1 + r_{t+1}}{1 + \rho} (c_{t+1})^{-\theta}$$

- Agent faces a **shadow interest rate** (the rate for which the marginal utilities are equalized) higher that the market rate:

$$1 + r_{t+1}^{\text{shadow}} = (1 + \rho) \left(\frac{c_t}{c_{t+1}} \right)^{-\theta} > 1 + r_{t+1}$$

- Tax switch helps as government effectively borrows "on behalf" of the agent. More likely to be effective when agents are constrained, such as in a deep recession.

OLG AND DYNAMIC EFFICIENCY

OLG and saving constraints

- With OLG, "laisser faire" allocation can be highly inefficient, giving a role for policy (Weil 2012).
- Agents live for two periods, population grows at a rate n across cohorts. Output takes the form of **endowments**.
  - Young agents get an endowment e_1.
  - Old agents an endowment e_2.
  - Endowment made of a perishable good that cannot be stored.
- **No intertemporal trade** is possible: specific young and specific old meet only once. No rest-of-the-world to trade with.
- Everyone is forced to consume their allocation in each period.

A patient economy

- Agents care primarily about consumption when old, they would **like to save**.
- Consuming e_1 is inefficient. Shadow interest rate r from the Euler is very low (-100% if young consumption is not enjoyed at all).
- **Pareto improving transfer**: from period t on take $\tau \leq e_1$ from the young and give it to the old.
    - Old at time t are better off.
    - Young at time t, and all future agents, are better off: they consume $e_2 + (1+n)\tau$ when old instead of e_2.
    - Result holds even if there is a storage technology, as long as it delivers a return lower than n.

Transfer mechanism

- Agents want to save, and thus need an asset.
- Role for **public debt** as a way to transfer resources. Agents do not meet each other twice, but they meet the infinitively-lived government twice.
- Another option is "pay as you go" **retirement schemes** where young agents pay for old ones.
- Rational **bubbles** can also help: fundamentally worthless assets that can be bought, held, and sold across generations.

An impatient economy

- Agents care primarily about consumption when young, they would **like to borrow**.
- Interest rate r from the Euler is then very high to make agents want to consume e_2 when old.
- Switch from old to young starting at time t, mirroring the policy for the patient economy?
  - Young agents at time t, and all future agents would be better off.
  - Old agents at time t would be worse off.
  - Switch is **not a Pareto improvement**.
- Policy is thus possible only for an environment of high saving propensity and low interest rates (the patient economy).

CONNECTING OLG

AND INFINITE HORIZON

- **Bequests**: agents live for one period, but care (equally) about future agents:

$$U_t = u(C_t) + \beta U_{t+1} = \sum_{s=t}^{\infty} \beta^{s-t} u(C_s)$$

- Budget constraint is identical to the representative agent model.
- Bequests H implies that agents' planning horizon is infinite, even though life is not:

$$\begin{aligned} C_t + H_{t+1} &= (1+r) H_t + Y_t - T_t \\ \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} C_s &= (1+r) H_t + \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} (Y_s - T_s) \end{aligned}$$

Population growth through new entrants

- Agents have infinite lives. Population grows, but not because current agents grow as in Ramsey model.
- Population grows with **new agents** appearing with **no assets**.
- Each period new agents enter then live forever. Total population grows at a rate n through new agents.
- Distinguish allocation for an **individual** agent from the **per-capita** allocation.
  - Individual agent takes his own future into account, but not the situation of agents that are not yet borne. We are not in a representative agent model.
  - Per-capita variables include growth through the arrival of new agents.
- Tax switch has real effect, as in OLG model. Agents currently alive ignore the tax burden on unborn generations.

Bottom line

- Ricardian equivalence when the public and the government have the same time horizon. In OLG model the public has a finite horizon and the government an infinite horizon.
- Other possibility: borrowing constraints, agents live forever but cannot borrow today.
  - Euler condition does not hold: marginal utility of current consumption exceeds that of future consumption (or Euler condition holds including the shadow value of the constraint).
- Tax cut followed by tax increases is consumed instead of being saved (borrowing constraint implies an effective shorter horizon).
  - Ricardian equivalence then varies across the cycle.

EXTRA SLIDES

Disinflation under adaptive expectations

- Until T money grows at a high rate: $m_T - m_{T-1} = \mu_0$. Starting at $T+1$ money grows at a **slower rate**: $m_{T+1} - m_T = \mu_1 < \mu_0$.
- Start with adaptive inflation expectations:

$$\pi^e_{t+1} - \pi^e_t = \delta\left(\pi_t - \pi^e_t\right)$$

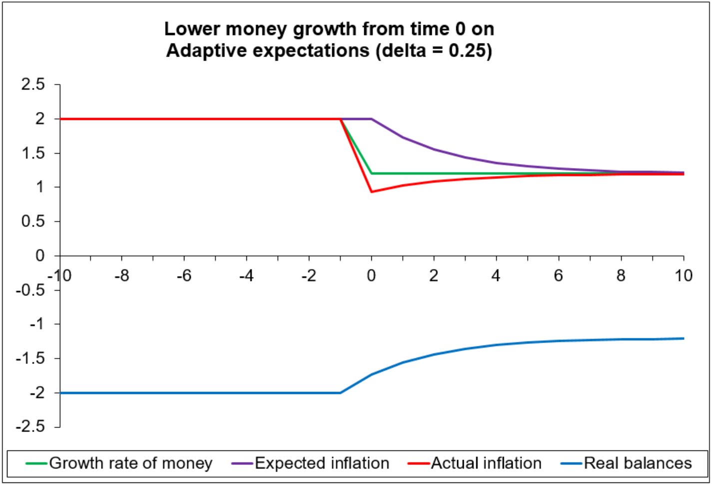
- Inflation expectations come down only gradually:

$$\begin{aligned} \pi^e_{T+s} &= \mu_1 + \left(1 - \frac{\delta}{1-\gamma\delta}\right)^{s-1}(\mu_0 - \mu_1) \\ \pi_{T+s} &= \mu_1 - \frac{\gamma\delta}{1-\gamma\delta}\left(1 - \frac{\delta}{1-\gamma\delta}\right)^{s-1}(\mu_0 - \mu_1) \end{aligned}$$

- Money demand **increases slowly** as agents realize lower inflation is here to stay.

Dynamics under adaptive expectations

- Lower money growth starting at time 0 ($\delta = 0.25$). ◂ Return



Household's intertemporal constraint

- Combine successive flow constraints ($R_{t,t+s} = \prod_{i=0}^{s} (1/(1 + r_{t+i}))$):

$$
\begin{aligned}
(k_t + b_t) &= \sum_{s=0}^{\infty} R_{t,t+s} \left[c_{t+s} - (w_{t+s} - \tau_{t+s})\right] \\
&\quad + \lim_{s\to\infty} R_{t,t+s} (k_{t+s+1} + b_{t+s+1})
\end{aligned}
$$

- Transversality condition: $\lim_{s\to\infty} R_{t,t+s} (k_{t+s+1} + b_{t+s+1}) = 0$.
- The value of debt or assets can grow, but not at a pace higher than the interest rate (no explosive growth).
- Present value of consumption: initial assets + PV after tax income:

$$
\sum_{s=0}^{\infty} R_{t,t+s} c_{t+s} = (k_t + b_t) + \sum_{s=0}^{\infty} R_{t,t+s} (w_{t+s} - \tau_{t+s})
$$

Government's and overall intertemporal constraints

- Combine successive flow constraints:

$$b_t + \sum_{s=0}^{\infty} R_{t,t+s} g_{t+s} = \sum_{s=0}^{\infty} R_{t,t+s} \tau_{t+s}$$

- Present value of spending plus initial debt equal to present value of taxes.
- Combine household and government constraints:

$$\sum_{s=0}^{\infty} R_{t,t+s} c_{t+s} = k_t + \sum_{s=0}^{\infty} R_{t,t+s} \left(w_{t+s} - g_{t+s} \right)$$

- Present value of private consumption equal to the initial capital plus PV of wages net of government spending (alternatively: value of private and public consumption equal to capital plus value of wages). Return

General case of capital dynamics

- Consumptions reflect taxes and wages. Wage and the interest rate are functions of capital.
- Savings and government's budget constraint give **dynamics of capital**.
- Highly **non-linear** relation between k_{t+1} and k_t: ◂ Return

$$
\begin{aligned}
k_{t+1}+b_{t+1} &= s_{1,t}=w_t-\tau_{1,t}-c_{1,t} \\
k_{t+1} &= \tau_{1,t}+\tau_{2,t}-(1+r_t)\,b_t+w_t-\tau_{1,t}-c_{1,t} \\
k_{t+1} &= \frac{(1+r_{t+1})^{\frac{1-\theta}{\theta}}}{(1+\rho)^{\frac{1}{\theta}}+(1+r_{t+1})^{\frac{1-\theta}{\theta}}}\left[(1-\alpha)(k_t)^{\alpha}-\tau_{1,t}\right] \\
&\quad +\frac{(1+\rho)^{\frac{1}{\theta}}}{(1+\rho)^{\frac{1}{\theta}}+(1+r_{t+1})^{\frac{1-\theta}{\theta}}}\frac{\tau_{2,t+1}}{1+r_{t+1}} \\
&\quad +\left(\tau_{1,t}+\tau_{2,t}-(1+r_t)\,b_t\right)
\end{aligned}
$$