Assignment 1.

Problem Set 8
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Monopolistically competitive firm

Consider a monopolistically competitive firm with real unit cost of z_t per unit sold at time t (marginal cost is constant at the firm level). The firm (firm i) sells output at price $p_t(i)$ and faces the following demand function for its output $y_t(i)$:

$$y_t(i) = \left(\frac{p_t(i)}{p_t}\right)^{-\varepsilon_t} y_t$$

where p_t is the general price level, y_t is aggregate output, and ε_t is the price elasticity of demand. The firm's price $p_t(i)$ implies a (gross) markup of $\mu_t(i) = \frac{p_t(i)}{p_t z_t}$ on marginal cost. Profits (in real terms) made by the firm at time t are therefore given by

$$\text{Profits}_{i,t} = z_t^{1-\varepsilon_t} y_t \left(\mu_t(i)^{1-\varepsilon_t} - \mu_t(i)^{-\varepsilon_t}\right)$$

Problem (a). Show that the profit-maximizing markup for a firm with flexible prices is $\mu_t^* = \frac{\varepsilon_t}{\varepsilon_t - 1}$.

Solution.

Take the first order derivative with respect to $\mu_t(i)$ and set it to zero, we have:

$$\begin{aligned}
\frac{\partial \text{Profits}_{i,t}}{\partial \mu_t(i)} &= \frac{\partial}{\partial \mu_t(i)} \left(z_t^{1-\varepsilon_t} y_t \left(\mu_t(i)^{1-\varepsilon_t} - \mu_t(i)^{-\varepsilon_t}\right)\right) = 0 \\
&\Rightarrow z_t^{1-\varepsilon_t} y_t \left((1-\varepsilon_t)\mu_t(i)^{-\varepsilon_t} + \varepsilon_t \mu_t(i)^{-\varepsilon_t - 1}\right) = 0 \\
&\Rightarrow (1-\varepsilon_t)\mu_t(i)^{-\varepsilon_t} + \varepsilon_t \mu_t(i)^{-\varepsilon_t - 1} = 0 \\
&\Rightarrow \mu_t^* = \frac{\varepsilon_t}{\varepsilon_t - 1}
\end{aligned}$$

The firm hires labor in a perfectly competitive market at wage w_t and each unit of labor has productivity a_t. Thus $z_t = w_t/a_t$. The wage is determined by the household labor supply condition

$$w_t = \frac{\nu'(l_t)}{u'(c_t)}$$

where l_t is labor supply and c_t is consumption.

Problem (b). Assume that $u(c_t) = 1 - c_t^{-1}$ and $\nu(l_t) = l_t$. Show that the natural level of output y_t^* is

$$y_t^* = \sqrt{\frac{a_t}{\mu_t^*}}$$

and explain why this means that the efficient level of output is $\hat{y}_t = \sqrt{a_t}$.

Solution.

From the household's labor supply condition:

$$w_t = \frac{\nu'(l_t)}{u'(c_t)} = \frac{1}{c_t^{-2}} = c_t^2.$$

In the flexible price equilibrium, firms set prices to maximize profits, leading to

$$\mu_t(i) = \frac{\varepsilon_t}{\varepsilon_t - 1} = \mu_t^*.$$

Hence the price would also be the same: $p_t(i) = p_t^*$ and the markups become:

$$\mu_t^* = \frac{1}{z_t}.$$

In a perfectly competitive market, the market clearing condition is: $y_t = c_t$. So, we have:

$$z_t = \frac{w_t}{a_t} = \frac{c_t^2}{a_t} = \frac{y_t^2}{a_t} = \frac{1}{\mu_t^*}$$
$$\Rightarrow y_t^* = \sqrt{\frac{a_t}{\mu_t^*}}$$

The efficient level of output occurs when there are no distortions, i.e. when the markup is eliminated: $\mu_t^* = 1$. Thus, $\hat{y}_t = \sqrt{a_t}$. The natural level of output y_t^* is lower than the efficient level $\hat{y}_t$due to the presence of monopolistic competition, which introduces a markup over marginal cost($\mu_t^* > 1$). This markup leads to higher prices and reduced output compared to the efficient (perfect competition) case.

Problem (c). When firms can adjust their prices at random staggered intervals (Calvo pricing), inflation is determined by the New Keynesian Phillips curve:

$$\tilde{\pi}_t = \beta E_t \tilde{\pi}_{t+1} + \kappa \tilde{x}_t + \gamma \tilde{\mu}_t^*$$

where $\tilde{\pi}_t$ is inflation and $\tilde{x}_t$ is the output gap (defined relative to efficient output). The notation $\tilde{x}_t$ indicates the percentage deviation of x_t from its steady-state value.

In this environment, should monetary policy aim to stabilize the output gap $\tilde{x}_t$ completely following an unexpected temporary decrease in a_t? Why or why not?

Solution.

Yes. The monetary policy should aim to stabilize the output gap.

$\tilde{x}_t$ is defined as the difference between actual output y_t and efficient level of output $\hat{y}_t$:

$$\tilde{x}_t = \frac{y_t}{\hat{y}_t}.$$

Recall $y_t^* = \sqrt{\frac{a_t}{\mu_t^*}}$, $w_t = y_t^2$ and $\hat{y}_t = \sqrt{a_t}$, we know that a decrease in a_t will lead to a decrease of both y_t^* and $\hat{y}_t$.

As a_t shock is exogenous, it won't affect the markup term $\tilde{\mu}_t^*$, so we can just ignore it in this case, we have:

$$\tilde{\pi}_t = \beta \mathbb{E}_t \tilde{\pi}_{t+1} + \kappa \tilde{x}_t$$

By targeting the output gap, CB can just change the interest rate t o cancel out the effect of the shock in the IS curve.

If we stabilize the output gap completely in response to a temporary decrease in a_t, we have:

$$\tilde{x}_t = 0 \Rightarrow \tilde{\pi}_{t+1} = 0 \Rightarrow \tilde{\pi}_t = \beta \mathbb{E}_t \tilde{\pi}_{t+1} = 0$$

In this case, output is kept equal to the flexible-price equilibrium level of output. This also guarantees inflation equals zero, thereby eliminating the costly dispersion of relative prices that arises with inflation. When firms do not need to adjust their prices, the fact that prices are sticky is no longer relevant.

Problem (d). Now suppose that markets become more competitive when output is higher (perhaps because of more competition for new customers). In particular, assume that $\varepsilon_t = 1 + \frac{1}{2}y_t$. Price adjustment is staggered according to the Calvo model.

Derive an expression for the desired flexible-price markup μ_t^* in terms of the output gap $x_t \equiv y_t/\hat{y}_t$ and productivity a_t. (Hint: the efficient level of output is still $\hat{y}_t = \sqrt{a_t}$.) Find a log-linear approximation of this equation in terms of $\tilde{\mu}_t^*$, $\tilde{a}_t$, and $\tilde{x}_t$, and interpret it.

Solution.

As given, $\varepsilon_t = 1 + \frac{1}{2}y_t$. The profit-maximizing markup is $\mu_t^* = \frac{\varepsilon_t}{\varepsilon_t - 1}$. So, we have:

$$\mu_t^* = \frac{1 + \frac{1}{2}y_t}{\frac{1}{2}y_t} = 1 + \frac{2}{y_t} = 1 + \frac{2}{x_t\sqrt{a_t}}.$$

We also know that $y_t = x_t\hat{y}_t = x_t\sqrt{a_t}$. Substitute this into the equation, we have:

$$\mu_t^* = 1 + \frac{2}{y_t} \Rightarrow y_t\mu_t^* = y_t + 2.$$

Take the log-linear approximation, we get:

$$\begin{aligned}
\tilde{y}_t &\approx \tilde{x}_t + \frac{1}{2}\tilde{a}_t \\
\tilde{y}_t + \tilde{\mu}_t^* &\approx \frac{\bar{y}}{2+\bar{y}}\tilde{y}_t \\
\Rightarrow \tilde{\mu}_t^* &\approx -\frac{2}{2+\bar{y}}\left(\tilde{x}_t + \frac{1}{2}\tilde{a}_t\right) \\
&= -\frac{2}{2+\sqrt{\bar{a}}}\left(\tilde{x}_t + \frac{1}{2}\tilde{a}_t\right)
\end{aligned}$$

This indicates that the desired markup $\tilde{\mu}_t^*$ decreases when the output gap increases. This reflects the assumption that markets become more competitive when output is higher.

Higher productivity can enhance competition by enabling firms to produce more efficiently, which may encourage them to lower prices to gain market share.

Problem (e). In this environment, should monetary policy aim to stabilize the output gap $\tilde{x}_t$ completely following an unexpected temporary decrease in a_t? Why or why not?

Solution.

No, in this case, the monetary policy should not aim to stabilize the output gap completely following an unexpected temporary decrease in a_t.

As we have a negative coefficient, if we stabilize the output gap completely in response to a temporary decrease in a_t, we have:

$$\tilde{\mu}_t^* = -\frac{1}{2+\sqrt{\bar{a}}}\tilde{a}_t > 0.$$

Then, if we still expect the future inflation to be zero, we have:

$$\tilde{\pi}_t = \kappa\tilde{x}_t + \gamma\tilde{\mu}_t^* > 0.$$

This shows that we have a tradeoff between stabilizing inflation and closing the output gap.

Stabilizing $\tilde{x}_t$ would decrease the inflation pressure, but amplifying the markup, which increase inflation volatility.

So, if CB takes stabilizing inflation as there priority, they should not completely stabilizing $\tilde{x}_t$.

Problem (f). Compare the size of the response of inflation $\tilde{\pi}_t$ to the output gap $\tilde{x}_t$ in the two cases where the price elasticity ε_t is exogenous and where it depends positively on output y_t. Assuming the central bank is minimizing the same standard loss function in both cases, what implications does the dependence of ε_t on y_t have for the optimal balance between stabilizing inflation and stabilizing the output gap (when shocks shift the New Keynesian Phillips curve)? Why?

Solution.

With exonogenous ε_t, we have:

$$\tilde{\pi}_t = \beta \mathbb{E}_t \tilde{\pi}_{t+1} + \kappa \tilde{x}_t.$$

With $\varepsilon_t = 1 + \frac{1}{2} y_t$, we have:

$$\tilde{\pi}_t = \beta \mathbb{E}_t \tilde{\pi}_{t+1} + \kappa \tilde{x}_t - \gamma \frac{2}{2+\sqrt{\bar{a}}}(\tilde{x}_t + \frac{1}{2}\tilde{a}_t) = \beta \mathbb{E}_t \tilde{\pi}_{t+1} + \left(\kappa - \frac{2}{2+\sqrt{\bar{a}}}\gamma\right)\tilde{x}_t - \frac{1}{2+\sqrt{\bar{a}}}\gamma \tilde{a}_t.$$

Thus, the response of inflation to the output gap is smaller when ε_t depends positively on output y_t.

Assuming the loss function is:

$$L_t = \frac{1}{2}\left(\tilde{\pi}_t^2 + \lambda \tilde{x}_t^2\right).$$

The optimal balance problem is:

$$\begin{aligned}
\min_{\pi_t, x_t} \quad & L_t = \frac{1}{2}(\tilde{\pi}_t^2 + a\tilde{x}_t^2) \\
s.t. \quad & \tilde{\pi}_t = \beta \mathbb{E}_t \tilde{\pi}_{t+1} + \left(\kappa - \frac{2}{2+\sqrt{\bar{a}}}\gamma\right)\tilde{x}_t - \frac{1}{2+\sqrt{\bar{a}}}\gamma \tilde{a}_t. \\
& \tilde{\pi}_t = \beta \mathbb{E}_t \tilde{\pi}_{t+1} + \kappa \tilde{x}_t
\end{aligned}$$

Take the first order condition with respect to $\tilde{\pi}_t$ and $\tilde{x}_t$, we have:

$$\begin{aligned}
\tilde{\pi}_t &= -\frac{\lambda}{\kappa}\tilde{x}_t \quad \text{under exogenous } \varepsilon_t; \\
\tilde{\pi}_t &= -\frac{\lambda}{\kappa - \frac{2}{2+\sqrt{\bar{a}}}\gamma}\tilde{x}_t \quad \text{under } \varepsilon_t = 1 + \frac{1}{2}y_t.
\end{aligned}$$

So, the optimal balance between stabilizing inflation and stabilizing the output gap is affected by the dependence of ε_t on y_t.

As $\kappa - \frac{2}{2+\sqrt{\bar{a}}}\gamma < \kappa$, the coefficient of $\tilde{x}_t$ is larger under endogenous ε_t. When ε_t depends positively on output y_t, the inflation rate is more volatile to the output gap. The monetary policy will prioritize inflation control over output gap stabilization.

Short Questions

Problem (2). Explain the channel system of conducting monetary policy, and illustrate its advantages and disadvantages.

Solution.

In a channel system, the central bank sets lower and upper bounds for the interest rate.

1. **Deposit Facility Rate (Floor):** The interest rate the central bank pays on excess reserves deposited by commercial banks.
2. **Lending Facility Rate (Ceiling):** The interest rate at which the central bank lends to commercial banks (typically through overnight loans).

The target policy rate is kept within this corridor, and the central bank uses open market operations to manage liquidity and guide the market rate towards the target.

Advantages:

1. **Interest Rate Control:** The corridor provides clear boundaries for short-term interest rates, enhancing the central bank's ability to control market rates.
2. **Liquidity Management:** By providing standing facilities, banks have predictable access to liquidity, reducing uncertainty in interbank markets.
3. **Stability:** The system can mitigate volatility in interest rates by absorbing shocks through the corridor's floor and ceiling rates.

Disadvantages:

1. **Interest Rate Volatility:** If the corridor is wide, market rates can fluctuate significantly within it, potentially leading to uncertainty.
2. **Ineffective Transmission:** In times of excess reserves (e.g., during quantitative easing), the floor may become the de facto policy rate, weakening the central bank's control over other interest rates.
3. **Dependency on Market Operations:** The system requires active management of liquidity through frequent open market operations, which can be resource-intensive.

Problem (3). Discuss whether (and in which way) optimal monetary policy with commitment is preferable over discretionary optimal monetary policy. Which assumption(s) of the New Keynesian model generate the dynamic externality that drives the differences between discretion and commitment?

Solution.

- **Commitment Policy:**
  - Under commitment, the central bank commits to future policy actions, influencing expectations and economic outcomes today.
  - It can implement time-inconsistent policies that are optimal in the long run but may not be optimal in the short run.
- **Discretionary Policy:**
  - The central bank optimizes policy period by period without committing to future actions.
  - This can lead to suboptimal outcomes due to the inability to influence expectations effectively.

If we suppose a timporary positive cost-push shock at time $t = 0$, then for the commitment policy, the central bank will increase the interest rate to reduce inflation, while for the discretionary policy, x_{t+j} would be negative for a while and the inflation expectations will fall $\Rightarrow \pi_0$ increases by less than under discretion without large decrease in x_0. Hence, outcome leads to higher welfare.

However, commitment policy is worse at future dates. Under the discretionary policy, there are no deviations in output or inflation at any date $t \geq 1$ while, under the commitment policy, output and inflation are too low relative to the targets. This brings us to a time-consistency problem. At date 0, the central bank would like to announce the commitment policy, but it would like to switch to the discretionary policy at date 1. If the private sector anticipates this switch in policy, then the benefits of the commitment policy at date 0 are unattainable because the central bank cannot convince the private sector that π_1 will be negative. The central bank can only achieve the better outcomes if it is able to commit at date 0 to have tight monetary policy at future dates even though it will not want to do that when those future dates arrive.

Assumptions Generating the Dynamic Externality:

- **Future Expectations:**

  - In the New Keynesian model, prices and wages are set based on expectations of future economic conditions.
  - This creates a dynamic externality where current policy affects future expectations and, therefore, current economic outcomes.

- **Sticky Prices and Wages:**
  - Price and wage rigidities cause adjustments to be gradual, making expectations about future policy more impactful.
  - The slow adjustment amplifies the importance of commitment in shaping expectations.

- **Monopolistic Competition and Price Setting:**
  - Firms set prices considering future marginal costs and the anticipated policy path.
  - Commitment allows the central bank to influence these expectations, improving allocation efficiency.