Macroeconomics A; EI056

Short problems

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1 Allocation of consumption through time

1.1 Euler condition

Question: Consider a two-period model without uncertainty, with the following utility of consumption:

$$U_1 = [\ln(C_1) + b\ln(1 - L_1)] + \frac{1}{1+\rho}[\ln(C_2) + b\ln(1 - L_2)]$$

The households earn a wage, rents out capital, and can invest. Initial capital K_1 is given. Capital does not depreciate, and the capital remaining at the end of period 2 is consumed. The budget constraints are:

$$\begin{aligned} C_1 + K_2 &= w_1 L_1 + (1 + r_1) K_1 \\ C_2 &= w_2 L_2 + (1 + r_2) K_2 \end{aligned}$$

Show that the Euler condition is:

$$\frac{1}{C_1} = \frac{1}{C_2}\frac{1 + r_2}{1 + \rho}$$

1.2 Propensity to consume

Question: We can think of the wealth of an agent as the return on her initial capital, K_1, plus the discounted value of her wage earnings.

Combine the Euler and the budget constraint to solve for the level of consumption in period 1. How is the sensitivity of consumption to wealth affected by the interest rate?

2 Labor supply sensitivity

2.1 Representative household

Question: Consider a static model for simplicity. It is inhabited by a representative household with the following utility over consumption and labor:

$$(C, L) = \ln(C) + \frac{1}{1-\gamma}(1 - L)^{1-\gamma}$$

The budget constraint is (w is the real wage and I other income sources, such as capital income):

$$C = wL + I$$

Show that the labor supply is:

$$(1-L)^{\gamma} = \frac{C}{w}$$

How sensitive is the marginal utility of leisure to the quantity of labor? What would you conclude regarding the sensitivity of labor supply to movements in the wage? To answer this, bear in mind that micro studies find a high value for γ.

2.2 Divisible household

Question: Consider now that the representative household is made of a large number of people. We normalize the total size to 1.

Each person either works H hours ($H < 1$) or no hours. The probability that a person works is thus the ratio of total hours worked L to the number of hours worked by each employed person H.

Each person has the same utility function as above. We assume that all the members of the household pool their resources (whether they work or not), so that each consumes the same amount C.

Show that the average utility across the members of the households is:

$$\begin{aligned} U^{avg} &= \ln(C) - \frac{L}{H}\left(\frac{1}{1-\gamma}(1)^{1-\gamma} - \frac{1}{1-\gamma}(1-H)^{1-\gamma}\right) + \frac{1}{1-\gamma}(1)^{1-\gamma} \\ &= \ln(C) - \frac{L}{H}\Omega_1 + \Omega_0 \end{aligned}$$

Treating this average household as a true person, how sensitive is the marginal utility of leisure to the quantity of labor? What would you conclude regarding the sensitivity of labor supply to movements in the wage?

Why can this alternative modelisation make the RBC model more realistic?