Solution Question 1

Temporary shock in Haiti that causes a fall in output.
Suppose Haiti is integrated in the international financial market.
Effect on:

(a) Consumption in the year of the hurricane.
Temporary output shocks only slightly affect consumption even if output declines. The bad shock can be absorbed by borrowing from the rest of the world (i.e., consumption smoothing)

(b) CA in the year of the hurricane.
Haiti runs a CA deficit ($CA < 0$) and borrows from the rest of the world in the year of the hurricane so as to maintain constant consumption levels.

(c) Net investment income in the year after the hurricane.
Net investment income goes down in the year after because Haiti will have to repay interests and debt of the year before. In fact, $\Delta NIIP = CA < 0$ because of borrowing from abroad.

(d) Consumption in future period.
Thanks to financial integration, Haiti can maintain a stable consumption path in future periods.

(e) Current Account Balance in future periods.
Haiti repays the foreign debt in future periods by running a CA surplus.

Suppose now Haiti had no access to international financial market.

(a) Consumption in the year of the hurricane.
Consumption goes down as much as current output falls (i.e., no consumption smoothing)

(b) Current Account in the year of the hurricane.
For a closed economy there is no financial integration and CA remains zero.

(c) Net Investment Income one year after.
For a closed economy there is no financial integration and the Net Investment Income remains zero.

(d) Consumption in future periods.
Consumption follows output and as the bad shock is absorbed, also consumption increases

(e) Current Account balance in future periods.
For a closed economy there is no financial integration and CA remains zero.

Solution Exercise 2

Point a)
The discovery of a new technology can be seen as productivity shock with a positive effect on output. In the case the technology is effective for the future, its discovery results in a positive outcome shock. Note that we assume that there is a lag between the technology discovery and the output increase. Suppose that the discovery will lead output to double in period 2. The economy can start consuming more in period 1 by borrowing from the rest of the world and then it can repay the debt in period 2. This is because we want that level of consumption remains the same across the two periods.

In period 1, the economy consumption increases compared to the initially planned values. The economy borrows from the rest of the world and thus $CA < 0$ and $TB < 0$.

Point b)
In period 2, output is double compared to the situation with no technology shock. Part of this higher output is used to maintain the same level of consumption of period 1 (which is higher than the case with no shock). The other part of output is used to repay the debt accumulated in period 1 ($CA > 0$ and $TB > 0$)

Point c)
Now suppose that the technology turns out to be ineffective. Output does not double in period 2 and consumption drops to the lower level that we would have in the case of no technology shock.

QUESTION 3

Consider a small open economy with capital that has zero foreign assets and liabilities and a balanced current account. Suppose consumption is determined as in the Intertemporal Approach to the Current Account presented by Obstfeld and Rogoff (1996) with a quadratic utility function, so that:

$$C_t = r\,B_{t-1} + \frac{r}{1+r}\sum_{i=0}^{\infty}\frac{1}{(1+r)^i}E_t(Y_{t+i} - I_{t+i}) \qquad (1)$$

where $Y_{t+i} \doteq A_{t+i}F(K_{t-1+i})$, and there is no government.

a) Suppose productivity, A_t, unexpectedly increases in period t. Under which condition does this productivity shock improve the current account (i.e. lead to a surplus)? Assuming that such condition holds, will the current account show a surplus or a deficit or be balanced in the medium run, after productivity and capital will have come back to their initial values A_{t-1} and K_{t-1}? Explain.

$\rho < \bar{\rho}$

The shock must be unexpected and ~~not~~ persistent. Indeed the more persistent the shock, the worst the impact on the CA_t. On the other hand, when the productivity shock is unexpected and temporary, it doesn't lead on an increase in I_t, but only in Y_t, since there is no reason to invest in period t to have more capital in period $t+1$. Thus, because of consumption smoothing according to which C_t will increase but not as much as Y_t, the CA_t will improve. Assuming that such condition holds, in the medium run the CA will return to its initial state, i.e. it will be balanced, since all its component (rB_{t+i-1}; C_{t+i}; Y_{t+i} $etc..$) did not change.

b) Now suppose that, in period t, economic agents expect productivity to increase starting with period $t+1$. How do investment, consumption and the current account change because of these optimistic expectations? How is the net international investment position affected by such changes?.

If economic agents expect this positive (persistent) productivity shock, then at time *t,* they will invest more so that at time *t+1* there is a higher level of productive capital available for production. The consumption at time *t* will start increase because of consumption smoothing, and they will run a trade deficit, also in light of the fact that they expect to be able to repay the debt coming from other countries. Therefore, there will be a CA deficit in *t* due to higher consumption and investments, and trade deficits. This will be reflected in a worsening of the $NIIP_t$.

c) What happens in period $t+1$ if no productivity shock takes place, so that $A_t = A_{t+1}$ and agents realize that their expectations were wrong? How do investment, consumption and the current account change in period $t+1$? In particular, does consumption go back to its initial value, and why? [Hint: to answer the last question consider the consumption function (1) at period $t+2$].

$t+1$

If the agents realize that their expectations were wrong, investment will drop and the current account is going to be negative because of the repayment of the interest on the debt accumulated in the previous period. When agents realize that there is no productivity improvement consumption falls below its initial level because we need a positive trade balance, on top of the fact that there hasn't been an increase in output, to pay for the interests on the debt that is taken up (erroneously) in period t.

EXERCISE 4:

$$C_t = rB_{t-1} + \hat{Y} + \frac{r}{1+r} \sum_{i=0}^{\infty} \frac{1}{(1+r)^i} E_t\left[Y_{t+i} - \bar{Y}\right] \quad [\mathrm{I}]$$

: Permanent Income consumption [sl 44]

H1+H2: $I_t = G_t = 0$.

stochastic process of output:

$$Y_{t+1} - \hat{Y} = \rho(Y_t - \hat{Y}) + \varepsilon_{t+1}, \quad E_t[\varepsilon_{t+i}] = 0 \quad [\mathrm{II}]$$

$$\Rightarrow E_t[Y_{t+i} - \hat{Y}] = \rho^i [Y_t - \hat{Y}]$$

a) Find C_t and CA_t as functions of current shock ε_t:

Replace [II] into [I]:

$$C_t = rB_{t-1} + \hat{Y} + \frac{r}{1+r} \underbrace{\sum_{i=0}^{\infty} \frac{1}{(1+r)^i} \rho^i}_{\frac{1+r}{1+r-\rho}} [Y_t - \hat{Y}]$$

$$= rB_{t-1} + \hat{Y} + \frac{r}{\cancel{1+r}} \cdot \frac{\cancel{r+1}}{1+r-\rho} [Y_t - \hat{Y}] \quad [\mathrm{III}]$$

Rewrite this in terms of the output shock: Eq. [II] on step back and then into Eq. [III]:

$$\underline{Y_t - \hat{Y}} = \rho(Y_{t-1} - \hat{Y}) + \varepsilon_t$$

$$\boxed{C_t = rB_{t-1} + \hat{y} + \frac{r}{1+r-\rho}\left[\rho(Y_{t-1} - \hat{Y})\right] + \frac{r}{1+r-\rho} \varepsilon_t}$$

ubstitute the expression for consumption into CA_t:

$$CA_t = B_t - B_{t-1} = r B_{t-1} + Y_t - C_t \underbrace{- G_t - I_t}_{=0} \quad \text{[sl. 4]}$$

$$\stackrel{!}{=} \cancel{rB_{t-1}} + Y_t - \left[\cancel{rB_{t-1}} + \hat{Y} + \frac{r\rho}{1+r-\rho}\left[\rho(Y_t - \hat{Y})\right] + \frac{r}{1+r-\rho}\varepsilon_t\right]$$

$$\stackrel{!}{=} \underline{(Y_t - \hat{Y})} - \frac{r\rho}{1+r-\rho}\left[\rho(Y_t - \hat{Y})\right] - \frac{r}{1+r-\rho}\varepsilon_t$$

$$\stackrel{!}{=} \rho(Y_{t-1} - \hat{Y}) + \varepsilon_t - \frac{r\rho}{1+r-\rho}(Y_{t-1} - \hat{Y}) - \frac{r}{1+r-\rho}\varepsilon_t$$

$$\Rightarrow \boxed{CA_t = \rho\left(\frac{1-\rho}{1+r-\rho}\right)\left[Y_{t-1} - \hat{Y}\right] + \left(\frac{1-\rho}{1+r-\rho}\right)\varepsilon_t}$$

$$\rho - \frac{r\rho}{1+r-\rho}$$

b) $0<\rho<1$ means that the shock to output ε_t is temporary, i.e the autoregressive process is mean reverting.

There is an improvement in CA_t, in fact the shock is smoothed out by saving since the impact of the shock is not fully transmitted to consumption, b/c of consumption smoothing.

$\neq$

If $\rho = 1$, an unanticipated shock to output $\varepsilon_t > 0$ will not affe[ct] the CA. Intuitively, it means that consumption jumps to a new level to reflect the ever-lasting increase in the output, i.e. the new permanent level of output.

$\Rightarrow$ The more persistent the shock, the more C_t reacts to deviation of the output wrt its mean and therefore the less CA_t chan[ges]

Solution

Exercise 5 – Intertemporal Approach to CA - 30 minutes - 12 points

Consider a small open economy without capital that has no foreign assets and liabilities (NIIP=0), and starts from a balanced current account (CA=0). Consumption is determined as in the Intertemporal Approach by Obstfeld and Rogoff (1996) when the utility function is quadratic:

$$C_t = r\,B_{t-1} + \bar{Y} + \frac{r}{1+r}\sum_{i=0}^{\infty}\frac{1}{(1+r)^i}E_t[Y_{t+i} - \bar{Y}]$$

The stochastic process for output is $Y_{t+1} - \bar{Y} = \rho(Y_t - \bar{Y}) + \varepsilon_{t+1}$ so that $E_t[Y_{t+i} - \bar{Y}] = \rho^i[Y_t - \bar{Y}]$

a) Find consumption C_t and the current account CA_t as functions of the current shock ε_t.

To answer this question just follow the procedure in the slides of Lecture 4

Even if you did not solve point a), answer the following questions by reasoning (no need for calculations).

b) Which type of shocks (temporary or permanent) lead to a significant change in the current account balance? And what shocks do not affect the current account? Explain. [Remember we have no capital]

As there is no capital and thus investment the trade balance is given by output minus consumption. Consumption smoothing implies that consumption reacts very little to temporary shocks and this results in a significant change in the trade balance and current account. By contrast permanent shock do not affect the CA much, because they lead to a significant change in consumption that offsets the increase in output.

c) Now (remember CA=0, NIIP=0), suppose a negative permanent shock, $\varepsilon_t < 0$, hits the economy at time t, but it is erroneously believed to be temporary, i.e. to last only one period. What is the effect of such shock on consumption, the current account and the international investment position at time t? What happens at time t+1 when the agents realize that the shock was permanent? How does consumption change? How is the current account at time t+1? Zero, positive or negative? How is the trade balance?

If the shock is believed to be temporary, consumption would decrease very little and the country would run a current account deficit borrowing from the rest of the world. This implies that the country will enter the following period with a net liability position and a negative investment income.

As agents realize that the shock is permanent, they immediately decrease consumption to the level it can be maintained forever after. Note that consumption must decrease below output because the country has to make payments on the liabilities incurred in period t. This means that the country must run a positive trade balance to make up for the return payments on its liabilities. Hence, the trade surplus covers the payments on the liabilities that remain constant. As a result, the current account is zero a time $t+1$ and expected to remain zero in all following periods.

Question 6

$u(C_t) = \log(C_t)$, $\beta(1+r) \neq 1$

a) HH maximizes:

$$\sum_{i=0}^{\infty} \beta^i \log(C_{t+i})$$

s.t. flow budget constraint:

$$B_t + C_t = (1+r)B_{t-1} + Y_t - G_t - I_t$$

~~The ... is~~

or the IBC:

$$\sum_{i=0}^{\infty} \frac{1}{(1+r)^i} C_{t+i} = (1+r)B_{t-1} + \sum_{i=0}^{\infty} \frac{1}{(1+r)^i} (Y_{t+i} - G_{t+i} - I_{t+i})$$

[Sl. 2]

$$\mathcal{L} = U(C_t) - \lambda \left[\sum_{i=0}^{\infty} \frac{1}{(1+r)^i} C_{t+i} - (1+r)B_{t-1} \dots \right]$$

FOC:

$$\frac{\partial \mathcal{L}}{\partial C_{t+i}} = 0 \rightarrow \beta^i \frac{1}{C_{t+i}} = \lambda \frac{1}{(1+r)^i}$$

But this holds true for all the i's:

$i=0 \rightarrow \frac{1}{C_t} = \lambda$

$i=1 \rightarrow \beta \cdot \frac{1}{C_{t+1}} = \lambda \cdot \frac{1}{(1+r)}$ $\Rightarrow C_{t+1} = \beta(1+r) C_t$

$\Rightarrow C_{t+i} = \beta^i (1+r) C_t$

Interpret:

$\beta(1+r) > 1 \Rightarrow$ Lower consumption at time t yields higher utility in (t+1). More utility to future consumption.

$\beta(1+r) < 1 \Rightarrow$ Consumers prefer consumption today

$\beta(1+r) = 1 \Rightarrow$ Perfect consumption smoothing: indiff b/w today and tomorrow

Replace EULER EQN into IBC to obtain CONSUMPTION FUNCTION:

$$\underbrace{C_t \sum_{i=0}^{\infty} \beta^i \frac{(1+r)^i}{(1+r)^i}}_{\frac{1}{1-\beta}} = \dots$$

$$\Rightarrow \boxed{C_t = \left[(1+r)B_{t-1} + \sum_{i=0}^{\infty} \frac{1}{(1+r)^i}\left(y_{t+i} - G_{t+i} - I_{t+i}\right)\right](1-\beta)}$$

b) Deriva CA_t its deviations from the permanent level:

Def. of permanent level:

$$\sum_{i=0}^{\infty} \frac{1}{(1+r)^i} \bar{X}_t = \sum_{i=0}^{\infty} \frac{1}{(1+r)^i} E_t X_{t+i}$$

$$\bar{X}_t = \frac{r}{(1+r)} \sum_{i=0}^{\infty} \frac{1}{(1+r)^i} E_t X_{t+i}$$

The CA_t definition is:

$$CA_t = B_t - B_{t-1} = rB_{t-1} + Y_t - C_t - G_t - I_t.$$

Re-write consumption function:

$$C_t = \underline{\frac{(1+r) - (1+r)\beta}{(1+r)}} \left[(1+r)B_{t-1} + \sum_{i=0}^{\infty} \frac{1}{(1+r)^i}\left(Y_{t+i} - G_{t+i} - I_{t+i}\right)\right]$$

Replace into CA_t:

$$CA_t = rB_{t-1} + Y_t - G_t - I_t - \frac{(1+r)-(1+r)\beta}{(1+r)}\left[(1+r)B_{t-1} + \sum_{i=0}^{\infty} \frac{1}{(1+r)^i}\left(Y_{t+i} - G_{t+i} - I_{t+i}\right)\right]$$

$$\overset{!}{=} rB_{t-1} + Y_t - G_t - I_t - \frac{r}{1+r}\left[(1+r)B_{t-1} + \sum_{i=0}^{\infty} \dots\right] - \frac{1-(1+r)\beta}{(1+r)}\left[\dots\right]$$

$$\overset{!}{=} (Y - \bar{Y}_t) - (I_t - \bar{I}_t) - (G_t - \bar{G}_t) - \frac{1-(1+r)\beta}{(1+r)}\left[\dots\right]$$