

Macroeconomics B, EI060

Class 2

Intertemporal approach to the current account

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What you will get from today class

- Intertemporal approach of production and consumption: 2 periods.
 - Consumption choice in endowment economy (Harms III.1-3 & my charts, Obstfeld-Rogof (secondary) 1.1).
 - Introducing government (Harms IV.3, OR 1.1.6).
 - Endogenous production (Harms III.4-5, OR 1.2).
- Infinite horizon (Harms III.7, OR 2.1-2.3.2).
- Equilibrium with two countries (Harms III.8, OR 1.3.1-1.3.3).
- External debt solvency (Harms VI.1-2).

A question to start

A country that has a higher income will save more, as it is wealthier.

Do you agree? Why or why not? Think of two cases:

- a) Higher income thanks to additional natural resources, that will be available for a long time.
- b) Higher income thanks to a increase in the value of its GDP, which is temporary

INTERTEMPORAL APPROACH : SIMPLE MODEL

Two periods consumption

- Real model with one good.
- The representative consumer maximizes the intertemporal utility ($\beta < 1$):

$$u(C_1) + \beta u(C_2)$$

- Output is an endowment. The consumer can purchase a bond with real interest rate r . The flow budget constraints are:

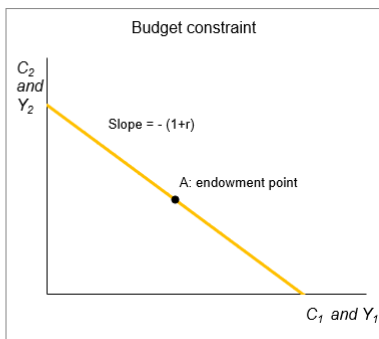
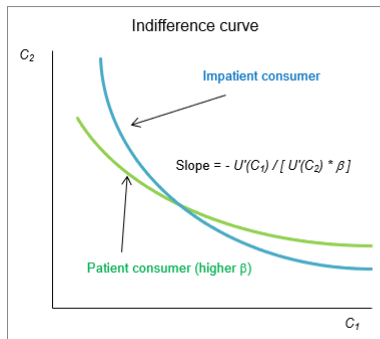
$$C_1 + B_2 = Y_1 \quad ; \quad C_2 = (1 + r) B_2 + Y_2$$

- The intertemporal constraint links the present value of consumption to the present value of income Ω :

$$C_1 + \frac{C_2}{1 + r} = Y_1 + \frac{Y_2}{1 + r} = \Omega$$

Diagram

- Indifference curves, with decreasing marginal utility.
- Budget constraint, going through the endowment point (Y_2, Y_1) .



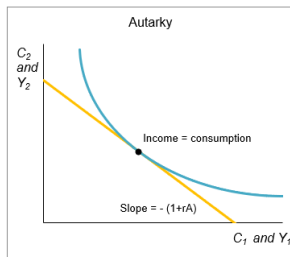
Optimal choice

- Euler condition, with tangent of the indifference curve and the budget constraint:

$$u'(C_1) = \beta(1+r)u'(C_2) \quad \Rightarrow \quad \frac{\beta u'(C_2)}{u'(C_1)} = \frac{1}{1+r}$$

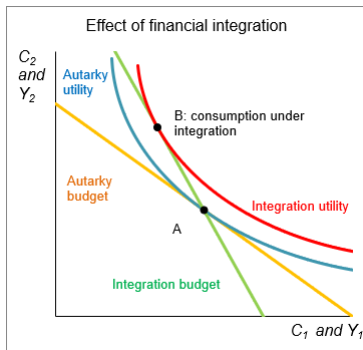
- Marginal utility of giving up a unit today = return $1+r$, the marginal utility of consumption tomorrow, adjusted for the cost of waiting β .
- Autarky: $B_2 = 0$ and $C_1 = Y_1$, $C_2 = Y_2$. Euler gives the interest rate:

$$1+r^A = [u'(Y_1)] / [\beta u'(Y_2)]$$



Effect of financial integration

- Consumption is not connected to output each period.
- A country with an autarky rate $r^A < r$ chooses to save ($C_1 < Y_1$).
- Integration raises the utility (one could always stay in autarky allocation). Not necessarily for all if households are heterogeneous (borrowers are unhappy).



Specific case

- Constant relative risk aversion utility:

$$u(C) = \frac{C^{1-\sigma}}{1-\sigma}$$

- The consumptions and savings are:

$$C_1 = \frac{1}{1 + \beta^{\frac{1}{\sigma}} (1+r)^{\frac{1}{\sigma}-1}} \left[Y_1 + \frac{Y_2}{1+r} \right]$$

$$C_2 = \frac{\beta^{\frac{1}{\sigma}} (1+r)^{\frac{1}{\sigma}}}{1 + \beta^{\frac{1}{\sigma}} (1+r)^{\frac{1}{\sigma}-1}} \left[Y_1 + \frac{Y_2}{1+r} \right]$$

$$B_2 = \frac{1}{1 + \beta^{\frac{1}{\sigma}} (1+r)^{\frac{1}{\sigma}-1}} \left[\beta^{\frac{1}{\sigma}} (1+r)^{\frac{1}{\sigma}-1} Y_1 - \frac{Y_2}{1+r} \right]$$

- If $\sigma = 1$ (log utility), initial consumption is a given share of overall income: $C_1 = \frac{1}{1+\beta} \left[Y_1 + \frac{Y_2}{1+r} \right]$.

Effects of the interest rate

- Three effects on initial consumption:

$$C_1 = \frac{1}{1 + \beta^{\frac{1}{\sigma}} (1 + r)^{\frac{1}{\sigma}} (1 + r)^{-1}} \left[Y_1 + \frac{Y_2}{1 + r} \right]$$

- Substitution: with a higher interest rate, savings is more remunerated. Consume less and save, driven by $(1 + r)^{\frac{1}{\sigma}}$.
- Income: a higher interest rate makes reaching a given value of assets tomorrow easier. Save less and consume, driven by $(1 + r)^{-1}$.
- Wealth: a higher interest rate reduces the net present value of future income, reducing wealth. Consume less, driven by $\frac{Y_2}{1+r}$.

Introducing government spending

- Government purchases G_t and taxes the consumer T_t .
- Consumer's and government's budget constraints:

$$C_1 + B_2^{private} = Y_1 - T_1 \quad ; \quad C_2 = (1 + r) B_2^{private} + Y_2 - T_2$$

$$G_1 + B_2^{public} = T_1 \quad ; \quad G_2 = (1 + r) B_2^{public} + T_2$$

- Adding up, the taxes cancel out. Only government spending matters:

$$C_1 + G_1 + \left(B_2^{private} + B_2^{public} \right) = Y_1$$

$$C_2 + G_2 = (1 + r) \left(B_2^{private} + B_2^{public} \right) + Y_2$$

- Taxes don't matter (Ricardian equivalence) as any change in the path of taxes is undone by the consumer.
 - Assumes they face the same interest rate and planning horizon.

INTERTEMPORAL APPROACH :

ENDOGENOUS OUTPUT

- Produce with capital (depreciates at a rate δ), using decreasing returns to scale:

$$Y_t = A_t F(K_t) \quad ; \quad F' > 0, \quad F'' < 0$$

- The consumer can save in bonds and capital. The budget constraints are:

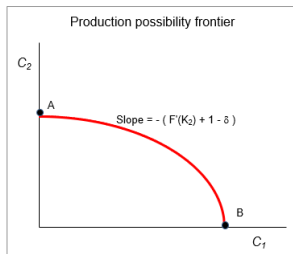
$$\begin{aligned} B_2 &= A_1 F(K_1) + (1 - \delta) K_1 - C_1 - K_2 \\ 0 &= (1 + r) B_2 + A_2 F(K_2) + (1 - \delta) K_2 - C_2 \end{aligned}$$

Production possibility frontier

- In autarky the budget constraint is:

$$\begin{aligned}C_2 &= G(C_1) \\&= A_2 F(A_1 F(K_1) + (1 - \delta) K_1 - C_1) \\&\quad + (1 - \delta) [A_1 F(K_1) + (1 - \delta) K_1 - C_1]\end{aligned}$$

- Concave relation with negative slope between the two consumptions ($G'(C_1) < 0$, $G''(C_1) < 0$).



- Two Euler conditions: one for the bond, and one for capital:

$$u'(C_1) = \beta(1+r)u'(C_2)$$

$$u'(C_1) = \beta(A_2F'(K_2) + (1-\delta))u'(C_2)$$

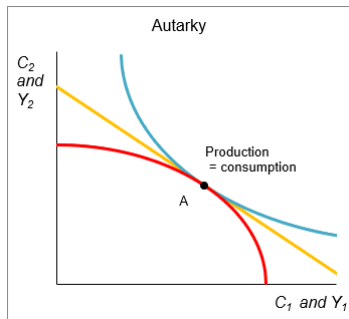
- Arbitrage links capital to the interest rate:

$$A_2F'(K_2) = r + \delta$$

- Capital (investment) demand: a higher rate leads to a lower capital.

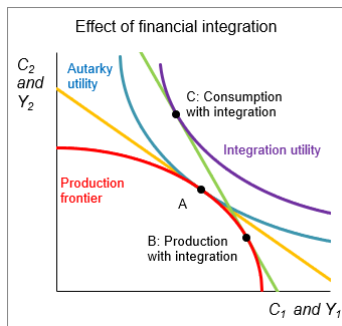
Autarky

- Optimal allocation at the tangent of the production frontier, the utility, and the budget constraint.
- The autarky interest rates ensures that all three lines are tangent.



Effect of financial integration

- Again allows for disconnect of each period consumption - output.
- Integration now also affects the production point on the PPF. A country with an autarky rate $r^A < r$ chooses to save ($C_1 < Y_1$) by a) reducing consumption and b) raising output.



INFINITE HORIZON

Utility and constraint

- Maximizes an intertemporal utility:

$$U_t = \sum_{s=t}^{\infty} \beta^{s-t} u(C_s)$$

- Flow budget constraint (current account $CA_t = B_{t+1} - B_t$):

$$\begin{aligned} C_t + I_t + B_{t+1} &= (1+r)B_t + Y_t \\ CA_t &= rB_t + Y_t - C_t - I_t \end{aligned}$$

- Iterate to get the intertemporal constraint, with transversality $\lim_{T \rightarrow \infty} B_{t+T+1} / (1+r)^{T+1} = 0$:

$$\sum_{s=t}^{\infty} \frac{C_s + I_s}{(1+r)^{s-t}} = (1+r)B_t + \sum_{s=t}^{\infty} \frac{Y_s}{(1+r)^{s-t}}$$

- Maximization again gives the Euler condition
$$u'(C_t) = \beta(1+r)u'(C_{t+1}).$$
- If $\beta(1+r) = 1$ consumption is constant:

$$C_t = rB_t + \frac{r}{1+r} \sum_{s=t}^{\infty} \frac{Y_s - I_s}{(1+r)^{s-t}}$$

- Current account:

$$\begin{aligned} CA_t &= rB_t + Y_t - C_t - I_t \\ CA_t &= Y_t - I_t - \frac{r}{1+r} \sum_{s=t}^{\infty} \frac{Y_s - I_s}{(1+r)^{s-t}} \end{aligned}$$

Permanent vs. temporary levels

- Permanent level of a variable X , denoted by \tilde{X} : constant value that gives the same net present value:

$$\sum_{s=t}^{\infty} \frac{\tilde{X}_t}{(1+r)^{s-t}} = \sum_{s=t}^{\infty} \frac{X_s}{(1+r)^{s-t}}$$

- Current account reflects deviation of output (net of investment) from permanent level. Open economy equivalent to the permanent income hypothesis of consumer's saving:

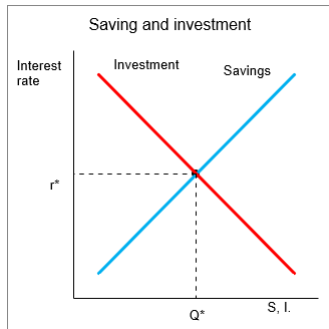
$$CA_t = (Y_t - \tilde{Y}_t) - (I_t - \tilde{I}_t)$$

- Key message: permanent shocks have no effect on the current account, only temporary ones do.

GENERAL EQUILIBRIUM

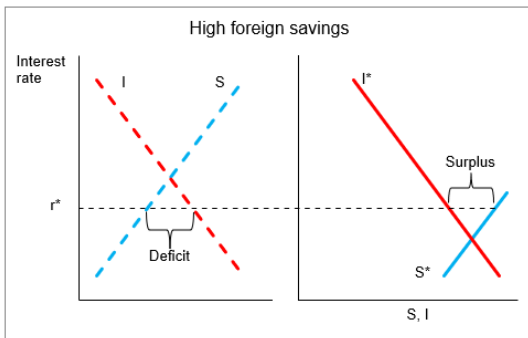
Savings and investment functions

- Savings increases with the interest rate (assume substitution effect dominates): $Savings = S(r)$, $S' > 0$.
- Investment demand decreases with the interest rate:
 $Investment = I(r)$, $I' < 0$.
- Intersection of a country's lines gives the autarky interest rate.



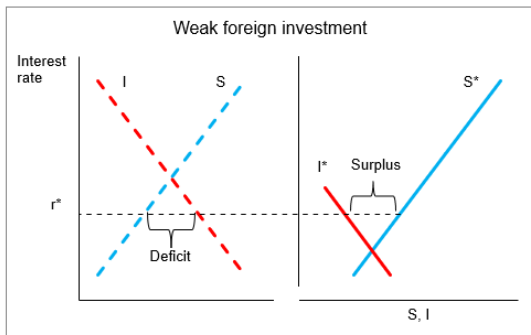
Effect of high savings

- Foreign savings line to the right.
 - Lower interest rate (autarky rate is at the intersection of the dotted lines).
 - Current account deficit at home ($S < I$) and surplus abroad ($S^* > I^*$).
“Savings glut” situation of the 2000’s.



Effect of low investment

- Foreign investment line to the left.
- Lower interest rate (autarky rate is at the intersection of the dotted lines).
- Current account deficit at home ($S < I$) and surplus abroad ($S^* > I^*$).



SOLVENCY

- From the intertemporal budget constraint, a positive net asset position $B_t^{total} > 0$ allows the country to fund a trade deficit ($NX_s < 0$) in net present value terms:

$$\sum_{s=t}^{\infty} \frac{NX_s}{(1+r)^{s-t}} = -(1+r) B_t^{total}$$

- The country is solvent as long as:

$$B_t^{total} > B_t^{total,min} = -\frac{1}{1+r} \sum_{s=t}^{\infty} \frac{NX_s}{(1+r)^{s-t}}$$

Minimal assets

- Investment and government spending are a constant share ϕ of output, which grows at rate g .
- The highest possible trade surplus, which is linked to the lowest possible asset holdings $B_t^{total,min}$, is reached when consumption is as low as possible.
- We take a consumption floor $C_t^{min} = \varphi^{min} Y_t$, with $C_{t+1}^{min} = (1 + g^{Cmin}) C_t^{min}$ where the growth rate is bounded as follows: $0 \leq g^{Cmin} \leq g$.
- The lower bound of asset to GDP is (assuming $g < r$):

$$\frac{B_t^{total,min}}{Y_t} = \frac{\varphi^{min}}{r - g^{Cmin}} - \frac{1 - \phi}{r - g}$$

- As time goes to infinity, the ratios of the various variables to GDP converge, with the values depending on whether $g^{Cmin} < g$ or $g^{Cmin} = g$.

Low floor growth

- If $g^{Cmin} < g$, the ratio C^{min}/Y goes to zero.
- The economy converges to a high debt, and a current account deficit:

$$\frac{B^{total,min}}{Y} \rightarrow -\frac{1-\phi}{r-g}$$
$$\frac{CA}{Y} \rightarrow -g \frac{1-\phi}{r-g} = g \frac{B^{total,min}}{Y}$$

- All output goes towards the net exports needed to sustain the debt:

$$\frac{NX}{Y} \rightarrow 1-\phi$$

High floor growth

- If $g^{Cmin} = g$, the ratio C^{min}/Y is constant at φ^{min} .
- If this is below the available output $1 - \phi$, the country has a debt, and a current account deficit, but needs a trade surplus:

$$\frac{B^{total,min}}{Y} = -\frac{1 - \phi - \varphi^{min}}{r - g}$$
$$\frac{CA}{Y} \rightarrow -g \frac{1 - \phi - \varphi^{min}}{r - g} = g \frac{B^{total,min}}{Y}$$

- It needs a trade surplus to stabilize the debt:

$$\frac{NX}{Y} = 1 - \phi - \varphi^{min}$$

- The higher the growth rate, the larger (more negative) are the debt and current account deficit that are sustainable.