Game Theory

Applications in Oligopoly and Industrial Organization

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Outline

Basics of Oligopolistic (寡头) Models

- Cournot （古诺） Games: choosing quantities
  - Simultaneous game
  - Sequential game: Stackelberg （斯塔克伯格） leader
  - Tacit collusion and grim-trigger: Cartel (卡特尔)
  - Vertical Relationships
- Bertrand (伯川德) Games: choosing prices
  - Simultaneous game
  - Hotelling (霍特林) model
  - Sequential game: Location choice (选址问题) and product Selection
  - *Single-home vs. Multi-home (单/多归属)

Cournot model (古诺模型)

- Assume the inverse demand of a market is $p = 10 - Q$.
- Two sellers (duopoly: 双寡头). Marginal cost of each seller is 2.
- Each seller chooses to produce a **quantity** that maximizes his/her **own** profit.
  - Seller 1 produces q_1; seller 2 produces q_2. Total quantity is $Q = q_1 + q_2$.
- Inverse demand becomes: $p = 10 - (q_1 + q_2)$.
  - Seller 1:
$$\max_{q_1} p(q_1, q_2)q_1 - 2q_1$$
  - Seller 2:
$$\max_{q_2} p(q_1, q_2)q_2 - 2q_2$$

Best Response

- Seller 1: given the rival's choice q_2, seller 1's profit is $\pi_1(q_1, q_2) = pq_1 - 2q_1$. Seller 1 chooses q_1 that is the best response of q_2

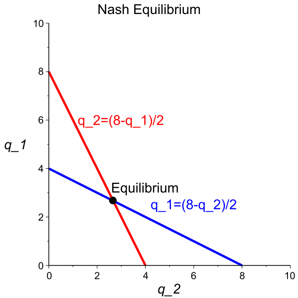
$$\max_{q_1} [10 - q_1 - q_2]\, q_1 - 2q_1$$
$$\Rightarrow q_1^{BR}(q_2) = \frac{8 - q_2}{2}$$

- Seller 2: given the rival's choice q_1, seller 2's profit is $\pi_2(q_1, q_2) = pq_2 - 2q_2$. Seller 2 chooses q_2 that is the best response of q_1

$$\max_{q_2} [10 - q_1 - q_2]\, q_2 - 2q_2$$
$$\Rightarrow q_2^{BR}(q_1) = \frac{8 - q_1}{2}$$

Cournot-Nash Equilibrium

- Strategic Substitutes （策略替代）：downward sloping BR_i



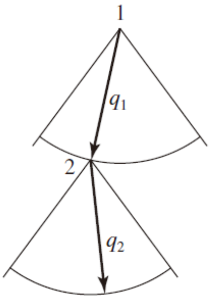
- At the cross-over point, neither party has an incentive to deviate:

$$\begin{cases} q_1 = (8 - q_2)/2 \\ q_2 = (8 - q_1)/2 \end{cases} \Rightarrow \begin{cases} q_1^* = 8/3 \\ q_2^* = 8/3 \end{cases}$$

Stackelberg Model (斯塔克伯格领导者模型)

For the two firms, now suppose that firm 1 sets quantity first. Upon observing firm 1's choice, firm 2 sets quantity.

- Firm 1 moves first: leader firm;
- Firm 2 is the follower.
- $\Rightarrow$ Sequential game.



Market demand: $p = 10 - Q = 10 - q_1 - q_2$. The game lasts two stages. At stage 1, seller 1 (leader) produces q_1. Then at stage 2, seller 2 (follower) produces q_2. Using backward induction:

At stage 2 q_1 is already given, then seller 2 solves

$$\max_{q_2}(10 - q_1 - q_2)q_2 - 2q_2 \Rightarrow q_2(q_1) = \frac{8 - q_1}{2}.$$

At stage 1 seller 1 knows that seller 2 will subsequently respond by setting $q_2 = \frac{8-q_1}{2}$. Hence q_2 in seller 1's profit can be replaced by $q_2 = \frac{8-q_1}{2}$, i.e., seller 1 solves

$$\max_{q_1}(10 - q_1 - q_2(q_1))q_1 - 2q_1$$

$$\Rightarrow \max_{q_1}\left(10 - q_1 - \frac{8 - q_1}{2}\right) q_1 - 2q_1$$

- Seller 1 solves

$$\max_{q_1} \left(10 - q_1 - \frac{8 - q_1}{2}\right) q_1 - 2q_1 \Rightarrow q_1^{\text{leader}} = 4.$$

The profit of seller 1 is $\pi_1^{\text{leader}} = 8 > 7.11$.

- Plug $q_1^L = 4$ into seller 2's best response

$$q_2^{\text{follower}} = \frac{8 - q_1}{2} = 2$$

The profit of seller 2 is
$\pi_2^{\text{follower}} = (10 - 4 - 2) \cdot 2 - 2 \cdot 2 = 4 < 7.11$

- The first-mover advantage for seller 1.

Cartel and Collusion

The market demand is $p = 10 - q_1 - q_2$. The Cournot-Nash outcome is $q_1^* = q_2^* = \frac{8}{3}$. The payoff of each seller is $\pi_1^* = \pi_2^* = \frac{64}{9} \approx 7.11$.

- Suppose the a candidate "collusive" plan: They coordinate the production by forming a Cartel, and split the profit equally. The Cartel solves

$$\max_Q (10 - Q)Q - 2Q \Rightarrow Q^m = 4, \ \pi^m = 16$$

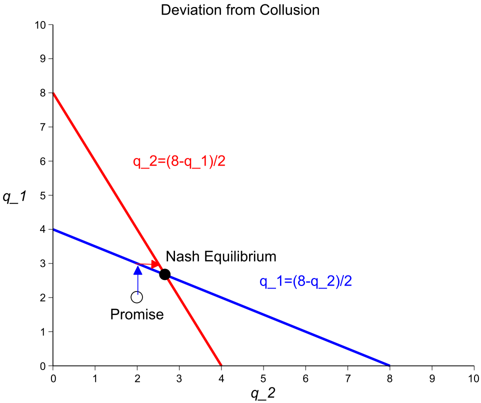
  Each seller produces $q_1^m = q_2^m = 2$, and the payoff of each is $\pi_1^m = \pi_2^m = 8$.
- Suppose the game lasts only one period. Can such collusion stable?

Unilateral Deviation

- Is there any way to make seller 1 better-off?
- If seller 2 keeps his/her promise, by producing $q_2 = 2$, consider seller 1's best response:

$$\max_{q_1}(10-2-q_1)q_1 - 2q_1 \Rightarrow q_1^{dev} = 3 \Rightarrow \pi_1^{dev} = 9 > 8.$$

  The deviation profit is higher than the collusive profit.
- Therefore, seller 1 will deviate by producing $q_1^{dev} = 3$.
- Seller 2 knows that, and will play a best response to $q_1^{dev} = 3$ ...
- Each seller will play the best response to the rival's deviations ... play the Nash equilibrium strategies: $(q_1^*, q_2^*) = (8/3, 8/3)$.



- Sellers have incentives to deviate from the collusive output (incredible promise).
- They will play the Nash equilibrium strategies.

Cooperative Outcome: Grim-trigger Strategy

- If the game lasts for finite *n* periods, by backward induction, they produce the Cournot-Nash outcome at the last stage, and the $n-1$ stage, ...
- The collusive outcome might be achieved if they adopt the grim-trigger strategy and play **infinite** times:

  Grim-trigger "Let's keep playing the collusive strategies $(q_1^m, q_2^m) = (2, 2)$ earning $(\pi_1^m, \pi_2^m) = (8, 8)$, if such collusive outcome is achieved in the previous stage. Otherwise, as long as one of us deviates from such cooperation by playing a best response earning $\pi^{dev} = 9$, then the cooperation ends and I will play the Cournot-Nash strateiges forever, and each will earn $(q_1^*, q_2^*) = \left(\frac{64}{9}, \frac{64}{9}\right)$ in every period in the future."

- The discount factor is δ.

Consider player 1's choice at an arbitrage stage:

- Deviation once gives one-shot payoff $\pi^{dev} = 9$, but receives $\frac{64}{9}$ in each period in the following. The present of "cheating" is

$$\pi^{dev} + \underbrace{\delta\frac{64}{9} + \delta^2\frac{64}{9} + \cdots}_{=\frac{\delta}{1-\delta}\frac{64}{9}} = 9 + \frac{\delta}{1-\delta}\frac{64}{9}$$

- Keeping cooperation gives the present value

$$8 + 8\delta + 8\delta^2 + \cdots = \frac{8}{1-\delta}.$$

Comparing the above two options, the collusive outcome can be achieved provided that

$$\frac{8}{1-\delta} > 9 + \frac{\delta}{1-\delta}\frac{64}{9} \Rightarrow \delta > \frac{9}{17} \approx 0.53$$

Vertical Relations

There are two monopolies. An upstream supplier U and a downstream seller D. Market demand is $p = 10 - q$.

- Two stages:
  - Stage 1: The upstream firm produces inputs and sells to the downstream firm. The wholesale price for per-unit input is w. The marginal cost of the upstream manufacture is 2.
  - Stage 2: The downstream sells the final goods to consumers at price p without costs.
- What is the Nash outcome? If they merge into a single firm, what is the monopolistic outcome?

Using the backward induction:

- At stage 2 Given the whole sale price w, the downstream seller D solves

$$\max_{q}(10-q)q - wq \Rightarrow q^{BR}(w) = \frac{10-w}{2}.$$

- At stage 1 The downstream seller demands q^{BR} units. The upstream seller U solves

$$\max_{w} wq^{BR}(w) - 2q^{BR}(w) \Rightarrow w = 6$$

Plug $w = 6$ into $q^{BR} \Rightarrow q^* = 2$. The market price is $p = 10 - q = 8$. The payoff of the two firms are $\pi^U = 8$, $\pi^D = 4$.

If the two firms get merged, the monopolistic outcome is $q^m = 4$, $p^m = 6$, and the payoff of the entire monopoly is $\pi^m = 16$. Compared with the merged monopoly,

- Consumers bear a higher price (double marginalization: 双重加价)
- A lower industrial profit.

Beyond the above incentives calling for vertical integration (垂直整合), there are some other reasons:

- Coordination & transaction costs (协调与交易成本)
- "Holdup" (敲竹杠)
- Free-riding of the franchisee (加盟商搭便车)
- Risks and output fluctuations (减少波动风险)
- Secret contracts (秘密谈判)

Bertrand Games: Homogeneous Products

Two sellers, A and B, sell homogeneous products. N consumes in the market.

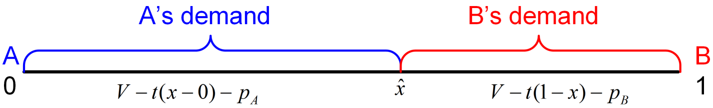
- "Unit-demand": each consumer needs only one and at most one unit of the product, which gives $V >> 0$.
- Two sellers offer prices, p_A and p_B simultaneously.
- The consumer buys from A if $V - p_A > V - p_B \Leftrightarrow p_A < p_B$; Otherwise, buy from B.
- Demand for each seller:

$$D_A = \begin{cases} N & \text{if } p_A < p_B \\ \frac{N}{2} & \text{if } p_A = p_B \\ 0 & \text{if } p_A > p_B \end{cases}, \quad D_B = \begin{cases} N & \text{if } p_B < p_A \\ \frac{N}{2} & \text{if } p_A = p_B \\ 0 & \text{if } p_B > p_A \end{cases}$$

- Assume that sellers have identical marginal cost c.
- Suppose firm B charge p_B, consider the best response of firm A:
    - If $p_A > p_B$, firm A's profit is zero.
    - If $p_A = p_B$, firm A's profit is $p_B \frac{N}{2}$.
    - By decreasing p_A a little bit, say, $p_B - \epsilon$, then A's profit is $(p_B - \epsilon)N$.
    - Because ϵ is infinitely small, then firm A prefers to sets p_A "slightly" below p_B.
- Firm B will think in the same way.
- $p_A^* = p_B^* = c$ is a Bertrand-Nash equilibrium.
- Bertrand Paradox: sellers are not "price-takers" as in the perfect competition, yet price is equal to the marginal cost.

Horizontally Differentiated Duopoly

- In the Bertrand game, we see a "Bertrand paradox" in the sense that each seller earns zero profit, although there are only two sellers.
- In the real world, sellers who compete in prices, are selling differentiated products.
  - Introduce differentiation: horizontal differentiation/Hotelling model/spatial model/linear city model.
- Consider a "linear" beach: $x \in [0, 1]$. One unit of tourists are uniformly distributed along the beach.
- Two ice-cream sellers, A and B, are located at the ends of the beach. A locates at 0 while B locates at 1.
- Each tourist (as a buyer), has the "unit-demand" for ice-cream: each buyer buyers exactly one unit, either from A or from B.



- Both sellers offer prices (p_A, p_B) simultaneously.
- For a buyer located at x, he/she buys one unit either from A or B
  - Utility of buying from A: $V - tx - p_A$
  - Utility of buying from B: $V - t(1-x) - p_B$

    where x and $1-x$ is the distance between the consumer's location and the location of seller A and B, respectively. t is the per-unit "travel cost."
- For a consumer located at x:
  - if $-tx - p_A > -t(1-x) - p_B$, buy from A.
  - if $-tx - p_A < -t(1-x) - p_B$, buy from B
  - if $-tx - p_A = -t(1-x) - p_B$, indifferent.
- The indifferent margin: $\widehat{x} = \frac{1}{2} + \frac{p_B - p_A}{2t}$.

- Seller A's demand: $\Pr(x < \widehat{x}) = \int_0^{\widehat{x}} \frac{1}{1-0} dx = \frac{1}{2} + \frac{p_B - p_A}{2t}$.
- Seller B's demand: $\Pr(x > \widehat{x}) = \int_{\widehat{x}}^{1} \frac{1}{1-0} dx = \frac{1}{2} + \frac{p_A - p_B}{2t}$.
- Seller A solves (given p_B)

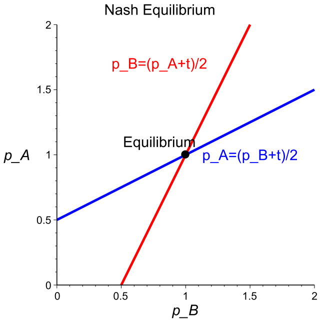
$$\max_{p_A} p_A \left(\frac{1}{2} + \frac{p_B - p_A}{2t} \right) \Rightarrow p_A^{BR}(p_B) = \frac{p_B + t}{2}$$

- Seller B solves (give p_A)

$$\max_{p_B} p_B \left(\frac{1}{2} + \frac{p_A - p_B}{2t} \right) \Rightarrow p_B^{BR}(p_A) = \frac{p_A + t}{2}$$

Bertrand-Nash Equilibrium

Strategic Complements （策略互补）: the Best Price Reply Correspondence is upward sloping



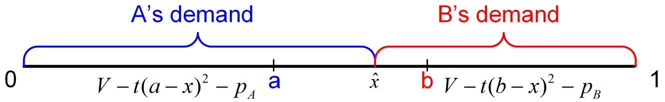
$$\begin{cases} p_A^{BR} = \frac{p_B + t}{2} \\ p_B^{BR} = \frac{p_A + t}{2} \end{cases} \Rightarrow (p_A^*, p_B^*) = (t, t)$$

Product Differentiation and Market Power

- The equilibrium price: $p_A^* = p_B^* = t$. The equilibrium profit: $\pi_A^* = \pi_B^* = \frac{t}{2}$. Both are increasing in t.
- Here, the degree of differentiation is captured by t. A higher t gives rise to a greater level of market power.
- In contrast, if the sellers sell homogeneous products, as is shown by the "Bertrand paradox," the equilibrium price is equal to the marginal cost, i.e., $p_A^* = p_B^* = 0$.

Product Selection (Location Choice)

- The two firms can choose to design their product functionalities or styles.
  - Choose a specific location within $x \in [0, 1]$.
- Consider the two-stage game:
  1. Firms choose locations (one chooses *a* on the left; the other chooses *b* on the right) in stage 1;
  2. Firms offer prices simultaneously in stage 2.



- To avoid calculating "the absolute value between *x* and *a*," let's assume that the "travel cost" is quadratic in distances.
- In order to solve the subgame perfect Nash equilibrium (SPNE), we start to solve firms' pricing strategies in stage 2.

Backward-Induction

- In stage 2, a and b are given. Assume that $0 \leq a \leq b \leq 1$.
  - The travel disutility incurred from buying from point a is $-t(x-a)^2$; If buying from point b, it incurs $-t(x-b)^2$.
  - A marginal consumer who is indifferent between buying from a and b locates at $V - t(x-a)^2 - p_A = V - t(b-x)^2 - p_B \Rightarrow \hat{x} = \frac{a+b}{2} + \frac{p_B - p_A}{2t(b-a)}$
- At stage 2, firms choose prices simultaneously

$$\max_{p_A} p_A \hat{x}, \quad \max_{p_B} p_B (1-\hat{x})$$

- The stage-2-prices are

$$p_A(a,b) = \frac{t}{3}(b^2 - a^2) + \frac{2t(b-a)}{3}$$
$$p_B(a,b) = \frac{t}{3}(a^2 - b^2) + \frac{4t(b-a)}{3}$$

Stage-1 Location

- Plug $p_A(a, b)$ and $p_B(a, b)$ into stage-1's problem:
  - $\pi_A = p_A(a, b)\hat{x}(p_A(a, b), p_B(a, b)) = \frac{t}{18}(a + b + 2)(2b - 2a + b^2 - a^2)$
  - $\pi_B = p_B\left[1 - \hat{x}(p_A(a, b), p_B(a, b))\right] = \frac{t}{18}(4 - a - b)(4b - 4a - b^2 + a^2)$
- At stage 1, each firm chooses a or b:
  - For firm A, $\pi'_A(a) = -\frac{t}{18}(a + b + 2)(2 - b + 3a) < 0$
  - For firm B, $\pi'_B(b) = \frac{t}{18}(4 - a - b)(4 - 3b + a) > 0$
- Therefore, at stage 1, the optimal choices are $a^* = 0$ and $b^* = 1$, i.e., "maximum-differentiation"
- Plug $a^* = 0$ and $b^* = 1$ into stage-2 prices, we have $p_A^* = p_B^* = t$.

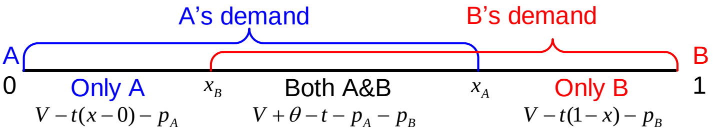
Multi-Homing in Price Competition*

In the benchmark Hotelling's game, each consumer buys from one seller exclusively.

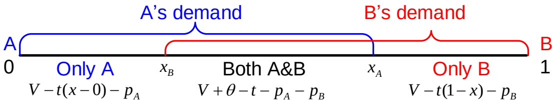
- Firms compete head on for consumers.
- Each firm's best response is upward sloping: price adjusts to the same direction in terms of the rival's.
- In reality, consumer can choose to buy from both sellers: one unit from A and one unit from B (multi-homing).
- Consumer's multi-homing: tx or $t(1-x)$ is the "disutility" incurred from not being able to purchase one's ideal product.
- If one consumer already buys one unit, buying an additional unit from the other sellers gives an incremental utility: θ, i.e., the degree of complementarity (双方产品的互补性).

Buy from Both Sellers

Each consumer has three options: $\{A, B, AB\}$, i.e., buy one unit from A; buy one unit from B; or buy one unit from A and buy one unit from B (Kim and Serfes, 2006).



- Buying from A only gives $V - tx - p_A$; buying from B only gives $V - t(1-x) - p_B$.
- Buying from both sellers gives $V + \theta - tx - t(1-x) - p_A - p_B$;



- B's demand: those who locate to the right side of the one x_B who is indifferent between buying A only and both: $V - tx - p_A = V + \theta - t - p_A - p_B \Leftrightarrow x_B = 1 - \frac{\theta - p_B}{t}$;
- A's demand: those who locate to the left side of the one x_A who is indifferent between buying B only and both: $V - t(1-x) - p_B = V + \theta - t - p_A - p_B \Leftrightarrow x_A = \frac{\theta - p_A}{t}$.
- Each firm's demand is **independent** of the rival's price! Strategic Independence/Local Monopoly (独立策略/局部垄断).

- Note that there are three "indifferent margins"
  - When nobody buys both, the one who is indifferent between A and B locates at $\hat{x} = \frac{1}{2} + \frac{p_B - p_A}{2t}$;
  - When some buyers buy from both, the indifferent margins are $x_A = \frac{\theta - p_A}{t}$ and $x_B = 1 - \frac{\theta - p_B}{t}$.
- $x_A - x_B > 0 \Leftrightarrow p_A + p_B + t < 2\theta$, some consumers buy both; otherwise, nobody buys both. The demand of each firm is

$$D_A = \begin{cases} x_A & \text{if } x_A > x_B \\ \hat{x} & \text{if } x_A < x_B \end{cases}, \quad D_B = \begin{cases} 1 - x_B & \text{if } x_A > x_B \\ 1 - \hat{x} & \text{if } x_A < x_B \end{cases}$$

- The profit of each firm is
  - $\pi_A = \begin{cases} p_A \frac{\theta - p_A}{t} & \text{if } x_A > x_B \\ p_A \left(\frac{1}{2} + \frac{p_B - p_A}{2t} \right) & \text{if } x_A < x_B \end{cases}$
  - $\pi_B = \begin{cases} p_B \frac{\theta - p_B}{t} & \text{if } x_A > x_B \\ p_B \left(\frac{1}{2} + \frac{p_A - p_B}{2t} \right) & \text{if } x_A < x_B \end{cases}$

Firm A's Best Response

A's payoff function depends on the relative position of x_A and x_B:

$$\pi_A = \begin{cases} p_A \frac{\theta - p_A}{t} & \text{if } x_A > x_B \\ p_A \left(\frac{1}{2} + \frac{p_B - p_A}{2t}\right) & \text{if } x_A < x_B \end{cases}$$

- If $x_A > x_B$, firm A and B are independent. A solves

$$\max_{p_A} p_A \frac{\theta - p_A}{t} \Rightarrow p_A = \frac{\theta}{2}, \ \pi_A = \frac{\theta^2}{4t}.$$

- If $x_A < x_B$, firm A and B compete head on for consumers. A solves

$$\max_{p_A} p_A \left(\frac{1}{2} + \frac{p_B - p_A}{2t}\right) \Rightarrow p_A(p_B) = \frac{p_B + t}{2}, \ \pi_A(p_B) = \frac{(p_B + t)^2}{8t}$$

Conditional Reactions (条件最佳回应)

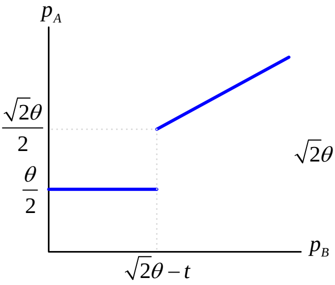
- Choosing to charge $p_A = \frac{\theta}{2}$ gives $\pi_A = \frac{\theta^2}{4t}$;
- Choosing to charge $p_A = \frac{p_B+t}{2}$ gives $\pi_A(p_B) = \frac{(p_B+t)^2}{8t}$.
- The former option is better iff $\frac{\theta^2}{4t} > \frac{(p_B+t)^2}{8t} \Leftrightarrow p_B < \sqrt{2}\theta - t$.
- Therefore, firm A's best-price-correspondence is a piecewise function contingent upon a threshold of the rival's price, i.e.,

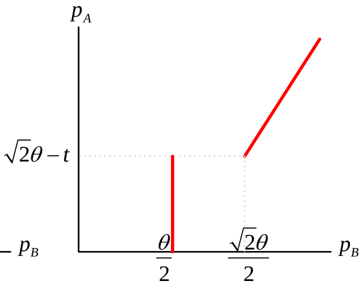
$$p_A^{BR} = \begin{cases} \frac{\theta}{2} & \text{if } p_B < \sqrt{2}\theta - t \\ \frac{p_B+t}{2} & \text{if } p_B > \sqrt{2}\theta - t \end{cases}$$

When p_B is low relative to θ, firm A prices independently (local monopoly); otherwise, firm A and B engage into a price competition.

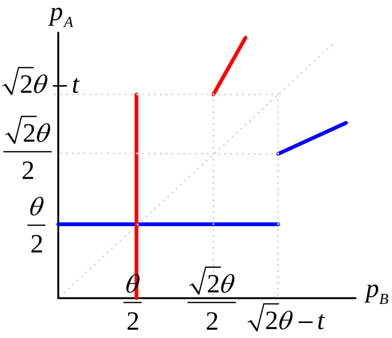
Conditional Reactions

$$p_A^{BR} = \begin{cases} \frac{\theta}{2} & \text{if } p_B < \sqrt{2}\theta - t \\ \frac{p_B+t}{2} & \text{if } p_B > \sqrt{2}\theta - t \end{cases} \quad p_B^{BR} = \begin{cases} \frac{\theta}{2} & \text{if } p_A < \sqrt{2}\theta - t \\ \frac{p_A+t}{2} & \text{if } p_A > \sqrt{2}\theta - t \end{cases}$$

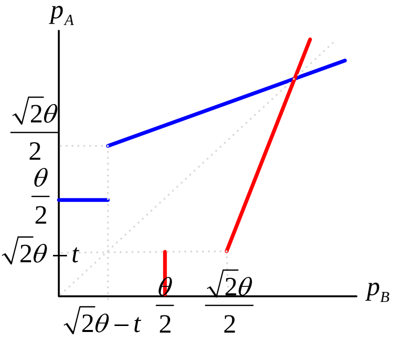




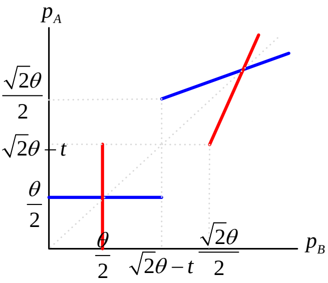
- Firm B's best response can be derived analogously.
- The Bertrand-Nash equilibrium is determined by the intersections between A's and B's best responses.
  - A unique intersection at the horizontal part;
  - A unique intersection at the upward-sloping part;
  - Two intersections: one at the horizontal part and the other at the upward-sloping part.
- The above three states are mutually exclusive, depending on the relative positions of $\sqrt{2}\theta - t$, $\frac{\theta}{2}$ and $\frac{\sqrt{2}\theta}{2}$. Because $\frac{\theta}{2} < \frac{\sqrt{2}\theta}{2}$, there are three possibilities:
  1. $\frac{\theta}{2} < \frac{\sqrt{2}\theta}{2} < \sqrt{2}\theta - t$: a unique intersection at the horizontal part.
  2. $\sqrt{2}\theta - t < \frac{\theta}{2} < \frac{\sqrt{2}\theta}{2}$: a unique intersection at the upward-sloping part.
  3. $\frac{\theta}{2} < \sqrt{2}\theta - t < \frac{\sqrt{2}\theta}{2}$: two intersections.



When $\frac{\sqrt{2}\theta}{2} < \sqrt{2}\theta - t \Leftrightarrow \theta > \sqrt{2}t$, there exists a unique intersection $(p_A^*, p_B^*) = \left(\frac{\theta}{2}, \frac{\theta}{2}\right)$.



When $\sqrt{2}\theta - t < \frac{\theta}{2} \Leftrightarrow \theta < \frac{2t}{2\sqrt{2}-1} \approx 1.094t$, there exists a unique intersection $(p_A^*, p_B^*) = (t, t)$.



When $\frac{\theta}{2} < \sqrt{2}\theta - t < \frac{\sqrt{2}}{2}\theta \Leftrightarrow \frac{2t}{2\sqrt{2}-1} < \theta < \sqrt{2}t$, there are two intersections. The payoff of charging $\left(\frac{\theta}{2}, \frac{\theta}{2}\right)$ is $\frac{\theta^2}{4t}$; The payoff of charging (t, t) is $\frac{t}{2}$.

$$(p_A, p_B) = \begin{cases} (t, t), & \frac{t}{2} > \frac{\theta^2}{4t} \Leftrightarrow \theta < \sqrt{2}t \\ \left(\frac{\theta}{2}, \frac{\theta}{2}\right), & \frac{t}{2} < \frac{\theta^2}{4t} \Leftrightarrow \theta > \sqrt{2}t \end{cases}$$

The Bertrand-Nash equilibrium:

$$(p_A^*, p_B^*) = \begin{cases} \left(\frac{\theta}{2}, \frac{\theta}{2}\right) & \text{if } \theta > \sqrt{2}t \\ (t, t) & \text{if } \theta < \sqrt{2}t \end{cases}$$

Check the necessary conditions:

- when prices are $\frac{\theta}{2}$, $x_A - x_B = \frac{\theta}{t} - 1 > \sqrt{2} - 1 > 0$;
- when prices are t, $x_A - x_B = \frac{2\theta - 3t}{t} < 2\sqrt{2} - 3 \approx -0.172 < 0$.