

Problem Set 4

Due: Sunday, 11 May, 23:59

- Prepare concise answers.
- State clearly any additional assumptions, if needed.
- You are encouraged to collaborate in groups but the final write-up should be individual.
- Submit your solutions, along with any code (if applicable), in a **single pdf file** through **Moodle**. If you choose to write your solutions by hand, please make sure your scanned answers are legible.
- Grading scale:

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| 5.5 | default grade |
| 6 | absolutely no mistakes and particularly appealing write-up (clear and concise answers, decent formatting, etc.) |
| 5 | more than a few mistakes, or single mistake and particularly long, wordy answers |
| 4 | numerous mistakes, or clear lack of effort (e.g. parts not solved or not really attempted) |
| 1 | no submission by due date |

Problem 1

This exercise is inspired by McConnell, Margaret and Gabriel Perez-Quiros (2000): “Output Fluctuations in the United States: What has changed since the early 1980’s?” *American Economic Review*, 90(5), 1464-76.

Download some aggregate time series from FRED: Real GDP, Real Personal Consumption Expenditure, Real Gross Private Domestic Investment. Use quarterly frequency.

- (a) Take logs of GDP, Consumption, and Investment. Plot the three series (you can generate the plots directly in FRED). What are the most striking features?
- (b) For each series, estimate a model of the form

$$y_t = \beta_1 + \beta_2 t + u_t$$

using OLS, based on the samples 1965:Q1 to 2006:Q4, 2007:Q1 to 2019:Q4, and 2007:Q1 to 2022:Q2.

- (c) According to your estimates, what are the annualized average growth rates (in percent) of GDP, consumption, and investment? Are these series growing, approximately, at the same rate?
- (d) For each subsample, compute sample autocorrelation functions for the deviations of output, consumption, and investment (the \hat{u}_t ’s) from their estimated deterministic trend.
- (e) Now compute quarter-on-quarter growth rates of these three series as $\ln y_t - \ln y_{t-1}$, plot the growth rates. What are the most striking features?
- (f) Compute the sample means of the growth rates for each of the above subsamples. Compare the growth-rate results to (ii). Also compute the sample standard deviations for the growth rates.
- (g) Repeat the analysis in (vi) for the subsamples “before 1984”, “between 1984 and 2006”. Did the means and the volatility of the series change?

Problem 2

Consider the MA(3) process

$$y_t = (1 - 2.4L + 0.8L^2 - 0.4L^3)u_t ,$$

where L denotes the lag operator and

$$\mathbb{E}[u_t u_\tau] = \begin{cases} 1 & \text{if } t = \tau \\ 0 & \text{otherwise} \end{cases} .$$

- (a) Define Weak and Strict Stationarity. Is the process y_t weakly stationary? Is it strictly stationary? You may impose additional conditions on the u_t 's.
- (b) Calculate the autocovariance function of y_t .
- (c) Calculate $\mathbb{V} \left[\frac{1}{\sqrt{T}} \sum_{t=1}^T y_t \right]$.
- (d) Suppose $\{u_t\}$ is i.i.d. with $\mathbb{E}[u_t] = 0$ and $\mathbb{V}[u_t] = \sigma^2$, and define $x_t = u_t u_{t-4}$. Is the process x_t strictly stationary? Is x_t ergodic? Is x_t a White Noise process?
- (e) Calculate $\mathbb{E} \left[\frac{1}{T} \sum_{t=1}^T x_t \right]$ and $\mathbb{V} \left[\frac{1}{\sqrt{T}} \sum_{t=1}^T x_t \right]$.