# L10. Imperfect Competition: Workhorse Models

Yuan Zi

EI037: Microeconomics
Geneva Graduate Institute (IHEID)

# Literature

- MWG, Chapter 12

# Introduction

- In perfect competition models, all consumers and producers are assumed to act as price takers, in effect behaving as if the demand and supply functions that they face are infinitely elastic at existing market prices.
- However, this assumption may not be a good one when there are only a few agents on one side of the market, for these agents will possess **market power** – the ability to alter the profitable prices away from competitive levels.
- Markets where agents' market power goes from high to low: monopoly $\to$ oligopoly $\to$ monopolistic competition $\to$ perfect competition.
- In this lecture, we introduce the workhorse models characterizing each of these markets. They (and their variations) constitute 99% of applied models you may confront in later studies.

# Monopoly Pricing

When there is only a single seller in a market, we say it is a monopolist of that good.

- The monopolist's decision constitutes of choosing price $p$ so as to maximize its profits (in terms of the numeraire):

$$\max_{p} \underline{p \cdot q(p)} - \underline{c(q(p))}$$

$x = x(p)$

An equivalent formation is choosing production quantity $q$, as $p = x^{-1}(q)$ anyway:

$$\max_{q} p(q) \cdot q - c(q)$$


- FOC (ignoring a corner solution):

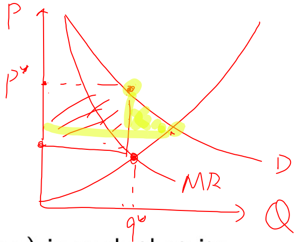
$$\boxed{p'(q^*)q^* + p(q^*) = c'(q^*)}$$

# Monopoly Pricing

When there is only a single seller in a market, we say it is a monopolist of that good.

- FOC (ignoring a corner solution):

$$\underbrace{p'(q^*)q^*}_{-} + p(q^*) = c'(q^*)$$

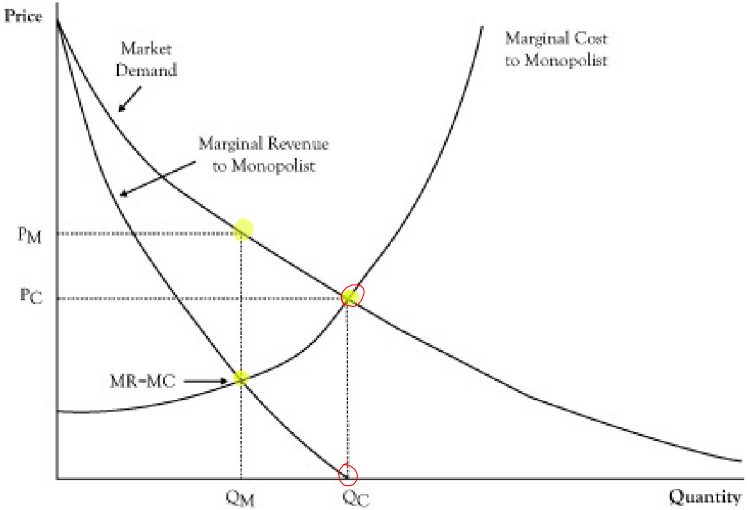



  - $p'(q^*)q^* + p(q^*)$: marginal revenue of production
  - $c'(q^*)$: marginal cost of production

$\Rightarrow$ the monopolist's optimal pricing strategy (or quantity production) is such that its marginal revenue of production equals the marginal cost of production.

$\Rightarrow$ note that $\underbrace{p'(q^*)q^* + p(q^*)}_{MR} < \underbrace{p(q^*)}_{inverse\ \ of\ \ demand}$

# Graphic Illustration



# Oligopoly

When there are more than one, but still not many, firms competing in one market, we say it is a market of oligopolists.

Competition among firms in an oligopolist market is inherently a setting of strategic interaction. The appropriate tool for its analysis is game theory.

- Recall that a game has 4 basic elements. Depending on how we specify these elements (or how the market looks like) we can have many variations.
- We consider 1 environment: static games with perfect information; and 2 models: Bertrand and Cournot.
  - In Bertrand competition firms strategically choose prices.
  - In Cournot competition firms strategically choose quantities.

# The Bertrand Model of Price Competition

We begin with a case where there are two profit-maximizing firms (a duopoly) in a market.

1. Their demand functions are given by $q(p)$, with $q'(p) < 0$ and there exists a $\bar{p} < \infty$ such that $q(p) = 0$ for $p \geq \bar{p}$.
2. Their marginal cost of production is a constant $c$.
3. Two firms simultaneously name their prices $p_1$ and $p_2$. Sales for firm $j$ are then given by

$$q_j(p_j, p_k) = \begin{cases} q(p_j) & \text{if } p_j < p_k \\ \frac{1}{2}q(p_j) & \text{if } p_j = p_k \\ 0 & \text{otherwise} \end{cases}$$

**Proposition 10.1** There is a unique Nash equilibrium $(p_1^*, p_2^*)$ in the Bertrand duopoly model. In this equilibrium, both firms set their prices equal to the marginal cost: $(p_1^* = p_2^* = c)$.

*Can you prove this? What is the intuition?*

*Show that with $J > 2$ number of firms, NE is all firms set their prices equal to $c$.*

**Remarks:**

1. The striking implication of Proposition 10.1 is that with even only two firms we can get the perfect competitive outcome. Thus the Bertrand model predicts that the distortions arising from the exercise of market power are limited to the special case of monopoly.
2. Do you find this model captures (in some cases) the reality? Can you give some examples?
3. Can you think of any reason why this model may be unrealistic in some settings? Can you give some real world examples?

The striking result of Proposition 10.1 requires some strong (although very hidden) assumptions of the model setup.

1. Firms' strategic variable is price, not quantity (once we deviate from the monopoly world, this matters).
2. There is no capacity constraint (one firm can serve the whole economy $q(p)$ as long as the price she charges is lower than her rivals).
3. There are no decreasing returns to scale (if firms' marginal cost of production rises when increasing production, the results will change).
4. There is no production differentiation (consumers only care about price, not who makes the product, as implied by $q(p)$).

# The Cournot Model of Quantity Competition

Two profit-maximization firms (a duopoly) in a market.

1. Demand function is given by $q(p)$ as before.
2. Marginal cost of production is a constant $c$ as before.
3. Two firms simultaneously decide how much to produce, $q_1$ and $q_2$. In equilibrium, the price adjusts to the level that clears the market, $p = q^{-1}(q_1 + q_2)$.

To find a (pure strategy) NE, consider firm $j$'s maximization problem given an output level $\bar{q}_k$ of the other firm

$$\max_{q_j} p(q_j + \bar{q}_k) \cdot q_j - c(q_j)$$

FOC:

$$p'(q_j^* + \bar{q}_k) \cdot q_j^* + p(q_j^* + \bar{q}_k) = c'(q_j^*)$$

$q_j^* = BR(q_k)$

$q_k^* = BR(q_j)$

Therefore in NE:

$$p'(q_j^* + q_k^*) \cdot q_j^* + p^*(q_j^* + q_k^*) = c'(q_j^*); p'(q_j^* + q_k^*) \cdot q_k^* + p^*(q_j^* + q_k^*) = c'(q_k^*)$$

When marginal cost is a constant $c$:

$p'(2q^*)\,q^\circ + p(2q^\circ) = c$

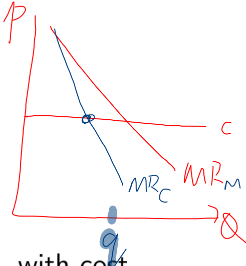
- In Cournot, market equilibrium satisfies

$$p'(q_1^* + q_2^*) \cdot \left(\frac{q_1^* + q_2^*}{2}\right) + p(q_1^* + q_2^*) = c$$

- Compared to monopoly, where the market equilibrium is given by

MR

$$p'(q^*)q^* + p(q^*) = c$$




**Proposition 10.2** In any Nash equilibrium of the Cournot duopoly model, with cost $c > 0$ per unit for the two firms and an inverse demand function $p(.)$ satisfying $p'(q) < 0$, $p(0) > c$, the market price is greater than $c$ (the competitive price) and smaller than the monopoly price.

*Can you prove this (with the help of a graph)? What is the intuition?*

# Graphic Illustration

**Remarks:**

1. The presence of two firms is not sufficient to obtain a competitive outcome in the Cournot model, in contrast with the prediction of the Bertrand Model.
2. The key reason is that in this model, a firm no longer sees itself facing an infinitely elastic demand.
3. In Bertrand, a firm who increases the price a bit at the margin can lose all demand; but in Cournot, a firm who increases quantity a bit will not decrease the price he faces to zero.
4. In Cournot, a firm is more like a monopoly with reduced market power.
   - When the number of firms goes to infinity, equilibrium is the same as that of perfect competition.
   - In the middle ground, we study an environment called monopolistic competition.

# Monopolistic Competition

Monopolistic competition characterizes an industry in which many firms offer products or services that are similar, but are not perfect substitutes.

1. Products are imperfect substitutes hence all firms have market power.
2. But each firm is sufficiently small hence they take the market price as given.

   $\rightarrow$ Monopolistic competitive firms are "hybrids" between monopolists and perfect competitive firms

*Examples*?

We next study one popular model of monopolistic competition – the Dixit–Stiglitz model.

# The Dixit–Stiglitz Model

## Product Differentiation

1. There are $N$ (a large number) firms in the market, where each produces one differentiated good.
2. A representative consumer has $E$ income. His preference over $N$ differentiated goods is given by:

$$U = \left[ \sum_{j=1}^{N} q_j^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \quad ; \; \sigma \geq 1$$

3. Given the utility function, the demand curve that is faced by firm $j$ is given by

$$q_j = q_j(p_j) = \frac{p_j^{-\sigma}}{P^{1-\sigma}} E,$$

where $P$ is the aggregate price index and $P^{1-\sigma} = \sum_{j=1}^{N} p_j^{1-\sigma}$.

**Proof:** The utility maximization problem of the consumer is given by:

$$\max_{q_j} U = \left[\sum_{j=1}^{N} q_j^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}, \quad s.t. \quad \sum_{j=1}^{N} q_j p_j = wL_H \equiv E$$

Write down the Lagrangian:

$$\mathcal{L} = \left[\sum_{j=1}^{N} q_j^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} + \lambda(E - \sum_{j=1}^{N} q_j p_j).$$

Take the F.O.C w.r.t. $q_j$:

$$\left[\sum_{j=1}^{N} q_j^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}-1} q_j^{\frac{\sigma-1}{\sigma}-1} = \lambda p_j, \tag{1}$$

Multiply both sides by $q_j$, and sum over $j = 1, 2, ..., N$:

$$\left[\sum_{j=1}^{N} q_j^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}-1} \sum_{j=1}^{N} q_j^{\frac{\sigma-1}{\sigma}} = \lambda \sum_{j=1}^{N} p_j q_j, \quad \rightarrow \quad U \equiv \left[\sum_{j=1}^{N} q_j^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} = \lambda E, \quad \rightarrow \quad 1/\lambda = E/U,$$

i.e., $1/\lambda$ reflects the shadow price of the consumption bundle, $\left[\sum_{j=1}^{N} q_j^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$. Therefore, we can also let $Q \equiv U$, $1/\lambda \equiv P$. Note $Q \times P = E$

Plug $1/\lambda \equiv P$. back into (1):

$$\left[\sum_{j=1}^{N} q_j^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}-1} q_j^{\frac{\sigma-1}{\sigma}-1} = p_j/P \quad \rightarrow \quad Q^{\frac{1}{\sigma}} q_j^{-\frac{1}{\sigma}} = p_j/P \quad \rightarrow \quad q_j = \frac{p_j^{-\sigma}}{P^{1-\sigma}} PQ = \frac{p_j^{-\sigma}}{P^{1-\sigma}} E$$

Now we are left to show that $P^{1-\sigma} = \sum_{k=1}^{N} p_k^{1-\sigma}$:

Multiply both sides of $q_j = \frac{p_j^{-\sigma}}{P^{1-\sigma}} E$ by $p_j$, then sum over $j = 1, ..., N$ over both sides, we get

$$\sum_{j=1}^{N} p_j q_j = \sum_{j=1}^{N} \frac{p_j^{1-\sigma}}{P^{1-\sigma}} E. \quad \rightarrow \quad 1 = \sum_{j=1}^{N} \frac{p_j^{1-\sigma}}{P^{1-\sigma}},$$

as $\sum_{j=1}^{N} p_j q_j = E$. Note that $P$ does not depend on $\sum_{j=1}^{N}$. Take it out and arrange the terms:

$$P^{1-\sigma} = \sum_{j=1}^{N} p_j^{1-\sigma}.$$

**Remarks:**

1. Note that the elasticity of substitution between two goods $i$ and $j$ is

$$\epsilon_{ij} = -\frac{dln(q_j/q_i)}{dln(p_j/p_i)} = \sigma.$$

- Hence the utility is called constant elasticity of substitution (CES) utility.

2. The elasticity of substitution across varieties is constant and given by $\sigma$:
   - as $\sigma_i \to \infty$, varieties become perfect substitutes.
   - as $\sigma_i \to 1$, we get the Cobb-Douglas case.

3. Notice the "love-of-variety" feature. Under perfect symmetry, $i.e.$, $q_i = q$ for all goods, we have

$$U = N^{\frac{\sigma}{\sigma-1}} q = N^{\frac{1}{\sigma-1}} \times \underbrace{(Nq)}_{\text{``real'' spending}} = N^{\frac{1}{\sigma-1}} E.$$

The more varieties, the greater the consumer utility.

# The Dixit–Stiglitz Model

## Product Differentiation

4. Given the demand curve of firm $j$:

$$q_j = q_j(p_j) = \frac{p_j^{-\sigma}}{P^{1-\sigma}} E,$$

   - firm $j$ has some market power – the demand curve is not flat.
   - but firm $j$ is also sufficiently small – when she makes decisions, she takes aggregate price $P$ as given instead of thinking her action can strategically influence $P$.

5. The profit maximization is given by

$$\max_{p_j} q(p_j)p_j - q(p_j)c.$$

6. Solving firm $j$'s profit maximization problem, we get that the firm charges constant markup over her marginal cost of production (try to solve it yourself, or see notes on Moodle).

$$p_j^* = \frac{\sigma}{\sigma - 1} c.$$

**Remarks:**

i. Monopolistic competition model allows firms to have market power, and at the same avoids to consider strategic interactions that can be very complicated.

ii. It captures some realistic features of incomplete markets, and at the same time maintains analytical solutions and shares some elegance of perfect competition models.

iii. The Dixit–Stiglitz model has a wide application in trade and macroeconomics. It should however be clear that the Dixit–Stiglitz model does *not equal to* but belongs to the set of monopolistic competition models.

iv. Monopolistic competition models nevertheless also have their drawbacks. Can you think of any?

# Perfect Competition

Although somewhat trivial, the points below are helpful to think about the big picture regarding a perfect competitive environment:

1. There are *infinite homogeneous* firms taking market prices as given.
2. All firms charge $p = c$. *(Note that the constant returns to scale assumption is necessary for perfect competition, why?)*
3. Note that the production or demand from "one particular firm" is undetermined under perfect competition, because firms are indifferent from the consumers' perspective.

To some extent, **there is no role for firms** in the perfect competitive environment, the firms' problem is degenerated to **set price equal to marginal cost of production**, and **only aggregate variables matter** (i.e., aggregate demand $q(p)$ and market price $p$)!