Macroeconomics B, EI060

Class 3

Real exchange rate and terms-of-trade

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What you will get from today class

- Comparing price levels: the real exchange rate.
  - Consumption allocation between traded and non-traded goods (Harms IV.4.1-4.4, Obstfeld and Rogoff (secondary) 4.4).
  - Impact on the current account.
- Different traded goods and the terms-of-trade.
  - Consumption allocation (Harms IV.4.5).
  - Impact on current account.
- Link with nominal exchange rate, and special cases (Harms VII.1-3).
- Real exchange rate and productivity (Harms VII.4, OR 4.1-4.2.3).

A question to start

When a country's currency appreciates (even when correcting for different price levels), it looses competitiveness and runs a trade deficit.

Do you agree? Why or why not?

REAL EXCHANGE RATE :

DEFINITION AND PATTERNS

Nominal and real exchange rates

- Nominal exchange rate of Switzerland, E: how many Swiss franc does it take to get a Euro?
  - A higher value is a nominal depreciation.
  - In an analysis always be clear of whether you define the exchange rate that way, or the other way around (both are perfectly fine, but beware of confusion).
- Nominal exchange rate is the (inverse of) international purchasing power of a currency. How does it link to its domestic purchasing power (inverse of price level)?
- Real exchange rate Q. P is the Swiss price level, in Swiss franc, P^* is the European price level, in Euro:

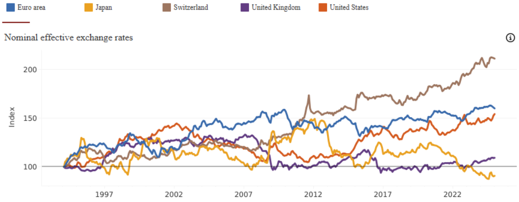
$$Q = \frac{EP^*}{P} = \frac{\frac{CHF}{Euro} * Euro}{CHF}$$

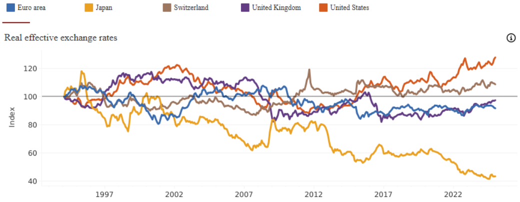
  - A higher value is a real depreciation, so the nominal exchange rate is not just the mirror of domestic purchasing powers.
  - No specific unit of measure (unlike nominal exchange rate), expressed as an index.

Bilateral and effective rates

- Swiss franc - Euro is a bilateral exchange rate.
- A (geometric) weighted-average across a countries trading partners gives an effective (trade weighted exchange rate).
  - Weights can be based on exports, imports, or a mix.
- Database of the Bank for International Settlements: https://data.bis.org/topics/EER go under "tables & dashboards".
  - Can select countries, real or nominal rates.
  - Easy charts and data downloads. Exchange rate as indices, were an increase is an appreciation (inverse of my definition).

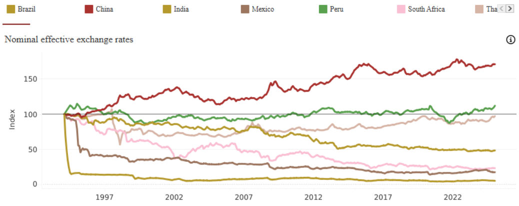
- Trends of nominal appreciations (esp. Switzerland), but less so in real terms. Nominal appreciation reflects low domestic inflation (esp. Japan).

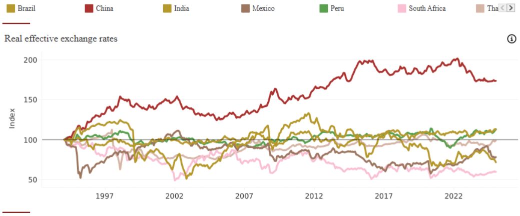




Emerging economies

- Heterogeneous: several nominal depreciations (incl. crises), some steady, some increases (China). Same real pattern, but smaller magnitudes (exc. China).





TRADED VS. NON – TRADED GOODS

Consumption basket

- Small open economy with representative consumer.
- Overall consumption C_t is a (constant elasticity of substitution) basket of a traded good C_t^T and a non-traded good C_t^N:

$$C_t = \left[(\gamma)^{\eta}\left(C_t^T\right)^{1-\eta} + (1-\gamma)^{\eta}\left(C_t^N\right)^{1-\eta}\right]^{\frac{1}{1-\eta}}$$

- The higher η, the less substitutables are the two goods.
- Price of the traded good normalized to 1. Relative price of the non-traded good is P_t^N. Total expenditure:

$$P_t C_t = C_t^T + P_t^N C_t^N$$

Allocation between the two goods

- Minimize the expenditure, subject to the constraints of a given real basket C_t.
- Demands reflects the weight in the basket, and the relative price:

$$C_t^T = \gamma \left[\frac{1}{P_t} \right]^{-\frac{1}{\eta}} C_t \quad ; \quad C_t^N = (1-\gamma) \left[\frac{P_t^N}{P_t} \right]^{-\frac{1}{\eta}} C_t$$

- The higher η, the lower the sensitvity of demand to the price (goods are not so substitutables).
- Price index is the minimum expenditure to get $C_t = 1$:

$$P_t = \left[\gamma + (1-\gamma) \left[P_t^N \right]^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}$$

Real exchange rate

- Real exchange rate is the ratio of the price level abroad to the domestic price level, all denominated in terms of traded good.
- The rest of the world consumes only the traded good:

$$Q_t = \frac{P_t^T}{P_t} = \left[\gamma + (1-\gamma)\left[P_t^N\right]^{\frac{\eta-1}{\eta}}\right]^{-\frac{\eta}{\eta-1}}$$

- Increase is a real depreciation for the domestic country.
- An increase in the price of the non-traded good P_t^N reduces $Q_t$and is a real exchange rate appreciation.

Intertemporal dimension

- Two periods economy, with utility:

$$U_1 = \frac{(C_1)^{1-\sigma}}{1-\sigma} + \beta \frac{(C_2)^{1-\sigma}}{1-\sigma}$$

- Outputs are endowments. Investment in a bonds denominated in traded good units, with return r.
- Budget constraints, in terms of traded goods:

$$\begin{aligned} B_2 + P_1 C_1 &= Y_1^T + P_1^N Y_1^N \\ P_2 C_2 &= Y_2^T + P_2^N Y_2^N + (1+r) B_2 \end{aligned}$$

Dynamic of consumption basket

- Standard Euler condition:

$$
\begin{aligned}
C_2 &= \left[\beta\left(1+r\right)\frac{P_1}{P_2}\right]^{\frac{1}{\sigma}} C_1 \\
C_2 &= \left[\beta\left(1+r^C\right)\right]^{\frac{1}{\sigma}} C_1
\end{aligned}
$$

- The real interest rate is the relative price of current consumption. A higher rate shifts consumption to the future, especially if σ is low (utility is not very concave).
- Two different real interest rates.
  - r in terms of units of traded good. Reflects the price of a traded good today, in terms of a traded good tomorrow.
  - r^C in terms of the consumption basket. Reflects the price of a basket today, in terms of a basket tomorrow. If today the non-traded good is relatively expensive ($P_1^T > P_2^T$) this makes the basket relatively expensive today ($P_1 > P_2$), hence a higher real interest rate.

Dynamics of traded good consumption

- Combine the Euler (dynamics of C) and the static demand for the traded good (link between C and C^T):

$$C_2^T = [\beta (1 + r)]^{\frac{1}{\sigma}} \left[\frac{P_1}{P_2}\right]^{\frac{1}{\sigma} - \frac{1}{\eta}} C_1^T$$

- Usual effect of the real interest rate r.
- Subtle effects of the real exchange rate dynamics (P_1/P_2). If the price of the non-traded good increases through time, the real exchange rate is appreciating between today and tomorrow $P_1 < P_2$.
  - Non-traded good is relatively cheap today.
  - Low consumption real interest rate r^C, shifts consumption basket to the present: higher C_1/C_2, and C_1^T/C_2^T. Driven by intertemporal sensitivity $1/\sigma$.
  - Take advantage of cheaper non-traded good in period 1 by shifting consumption composition towards non-traded today, and traded tomorrow. Reduces C_1^T/C_2^T. Driven by substitutability $1/\eta$.

Traded consumption and current account

- Traded consumption in period 1 is:

$$C_1^T = \frac{1}{1 + (\beta)^{\frac{1}{\sigma}} (1+r)^{\frac{1-\sigma}{\sigma}} \left[\frac{P_2}{P_1}\right]^{\frac{1}{\eta}-\frac{1}{\sigma}}} \left[Y_1^T + \frac{Y_2^T}{1+r}\right]$$

- Consumption and supply of non-traded goods are always equal: $C_t^N = Y_t^N$. Intertemporal constraint then only includes traded goods:

$$C_1^T + \frac{C_2^T}{1+r} \quad = \quad Y_1^T + \frac{Y_2^T}{1+r}$$

- The current account $CA_1 = Y_1^T - C_1^T$ is:

$$CA_1 = Y_1^T - \frac{1}{1 + (\beta)^{\frac{1}{\sigma}} (1+r)^{\frac{1-\sigma}{\sigma}} \left[\frac{P_2}{P_1}\right]^{\frac{1}{\eta}-\frac{1}{\sigma}}} \left[Y_1^T + \frac{Y_2^T}{1+r}\right]$$

- It reflects the dynamics of the real exchange rate, P_2/P_1, not its level.

A simpler case

- We still need to solve for P_2/P_1. Set $\beta(1+r)=1$, and $\eta=1$ (Cobb-Douglas consumption basket)

$$C_t = \left(C_t^T\right)^{\gamma}\left(C_t^N\right)^{1-\gamma}$$

- Consumption allocation shows constant spending shares:

$$\begin{aligned} C_t^T &= \gamma P_t C_t \quad ; \quad P_t^N C_t^N = (1-\gamma) P_t C_t \\ P_t &= \frac{1}{(\gamma)^{\gamma}(1-\gamma)^{1-\gamma}}\left(P_t^N\right)^{1-\gamma} \end{aligned}$$

- Using the fact that $C_t^N = Y_t^N$, the price of the non-traded good increases when its supply shrinks (details in the technical appendix):

$$\frac{P_2^N}{P_1^N} = \left(\frac{Y_1^N}{Y_2^N}\right)^{\frac{\sigma}{1-\gamma+\sigma\gamma}} \quad ; \quad \frac{P_2}{P_1} = \left(\frac{Y_1^N}{Y_2^N}\right)^{\frac{(1-\gamma)\sigma}{1-\gamma+\sigma\gamma}}$$

Current account (1)

- The ratio of current account to traded output is ($1 + g^N = Y_2^N / Y_1^N$ and $1 + g^T = Y_2^T / Y_1^T$):

$$\frac{CA_1}{Y_1^T} = 1 - \frac{1}{1 + \beta \left(1 + g^N\right)^{\frac{(1-\gamma)(1-\sigma)}{1-\gamma+\sigma\gamma}}} \left(1 + \beta \left(1 + g^T\right)\right)$$

- For simplicity starts from a case where all outputs are constant, and the current account is zero.

$$\begin{aligned} \frac{\partial}{\partial g^T} \left(\frac{CA_1}{Y_1^T} \right) &= -\frac{\beta}{1+\beta} < 0 \\ \frac{\partial}{\partial g^N} \left(\frac{CA_1}{Y_1^T} \right) &= (1-\sigma) \frac{\beta}{1+\beta} \frac{(1-\gamma)}{1-\gamma+\sigma\gamma} \end{aligned}$$

- Permanent increases in either output have not effects on the current account or real exchange rate dynamics.

Effect of endowment growth

- Higher g^T leads to a current account deficit, but no change in the real exchange rate dynamics.
  - As in last week, smooth consumption.
- Higher g^N leads to:
  - Real depreciation path $(P_2 < P_1)$
  - Current account deficit, $\sigma > 1$, utility more concave than log (corresponds to $\frac{1}{\sigma} - \frac{1}{\eta} < 0$).
  - Non-traded relative scarce and expensive today, dominant impact is shift of consumption in period 1 from non-traded to traded good, hence deficit.$
- Equally higher g^T and g^N leads to a current account deficit:

$$\frac{\partial}{\partial g^T}\left(\frac{CA_1}{Y_1^T}\right) + \frac{\partial}{\partial g^N}\left(\frac{CA_1}{Y_1^T}\right) = \frac{\beta}{1+\beta}\frac{-\sigma}{1-\gamma+\sigma\gamma} < 0$$

TERMS OF TRADE

Different traded goods

- Two traded goods: one is produced at home (index H) and the other abroad (index F). No non-traded good.
- Consumption of the traded goods is a basket:

$$C_t = \left[(\theta)^{\nu} \left(C_t^H \right)^{1-\nu} + (1-\theta)^{\nu} \left(C_t^F \right)^{1-\nu} \right]^{\frac{1}{1-\nu}}$$

- Normalize the foreign price to 1. The terms-of-trade are the ratio of the home good to the foreign one:

$$Q_t^{tot} = \frac{P_t^H}{P_t^F} = P_t^H$$

Consumption allocation

- Total expenditure is $P_t C_t = Q_t^{tot} C_t^H + C_t^F$. Optimal allocation minimizes it, subject to a target value for C_t, leading to (same steps as for the traded / non-traded analysis):

$$
\begin{aligned}
C_t^H &= \theta \left[\frac{Q_t^{tot}}{P_t} \right]^{-\frac{1}{\nu}} C_t \quad ; \quad C_t^F = (1-\theta) \left[\frac{1}{P_t} \right]^{-\frac{1}{\nu}} C_t \\
P_t &= \left[\theta \left[Q_t^{tot} \right]^{\frac{\nu-1}{\nu}} + (1-\theta) \right]^{\frac{\nu}{1-\nu}}
\end{aligned}
$$

Intertemporal choice

- Maximization of intertemporal log utility ($\sigma = 1$):

$$U_1 = ln\left(C_1\right) + \beta ln\left(C_2\right)$$

- Output of domestic traded good is an endowment. Invest in a bond denominated in the foreign good. The budget constraints are:

$$\begin{aligned} B_2 + P_1 C_1 &= Q_1^{tot} Y_1^H \\ P_2 C_2 &= Q_2^{tot} Y_2^H + (1+r) B_2 \end{aligned}$$

Euler condition

- Allocation given by the Euler condition:

$$
\begin{aligned}
C_2 &= \beta\left(1+r\right)\frac{P_1}{P_2}C_1 \\
C_2 &= \beta\left(1+r^C\right)C_1
\end{aligned}
$$

- The real interest rate in terms of the consumption basket r^C (relative price of the period 1 consumption basket) reflects the dynamics of the terms of trade.
- If the Home good becomes more valuable, $Q_2^{tot} > Q_1^{tot}$, the price of the consumption basket increases through time, $P_2 > P_1$.
- This makes the current basket relatively cheap, i.e. an decrease in the real interest rate.

Current account

- Current account is the difference between the value of the Home good endowment and that of the overall traded consumption basket:

$$CA_1 = Q_1^{tot} Y_1^H - P_1 C_1$$

- The current account reflects the dynamics of the terms of trade:

$$\frac{CA_1}{Q_1^{tot} Y_1^H} = \frac{1}{1+\beta} \left[\beta - \frac{1}{1+r} \frac{Q_2^{tot} Y_2^H}{Q_1^{tot} Y_1^H} \right]$$

- Rising value of the Home endowment $Q_2^{tot} Y_2^H > Q_1^{tot} Y_1^H$ leads to a deficit:
  - Smooth consumption, as endowment tilted to the future.
  - Rising terms-of-trade leads to a decrease in the real interest rate (in terms of the traded basket).
- Solving for the terms-of-trade requires extra steps (Foreign demand, goods market clearing).

COMBINING VARIOUS DIMENSIONS

Nested indices

- The overall consumption basket consists of non-traded and traded goods, with the later being itself a basked of domestic and imported goods (set $\eta = \nu = 1$ for simplicity).

$$C_t = \left(C_t^T\right)^{\gamma}\left(C_t^N\right)^{1-\gamma} \quad ; \quad C_t^T = \left(C_t^H\right)^{\theta}\left(C_t^F\right)^{1-\theta}$$

- Prices are expressed in Home currency, and denoted by P_t^H, P_t^F,P_t^T,P_t^N and P_t.
- Consumption allocation and price indices are:

$$C_t^T = \gamma\frac{P_t C_t}{P_t^T} \quad ; \quad C_t^N = (1-\gamma)\frac{P_t C_t}{P_t^N}$$

$$C_t^H = \theta\frac{P_t^T C_t^T}{P_t^H} \quad ; \quad C_t^F = (1-\theta)\frac{P_t^T C_t^T}{P_t^F}$$

$$P_t = (\gamma)^{-\gamma}(1-\gamma)^{-(1-\gamma)}\left(P_t^T\right)^{\gamma}\left(P_t^N\right)^{1-\gamma}$$

$$P_t^T = (\theta)^{-\theta}(1-\theta)^{-(1-\theta)}\left(P_t^H\right)^{\theta}\left(P_t^F\right)^{1-\theta}$$

Foreign country

- The allocation is similar in the Foreign country, with indices:

$$C_t^* = \left(C_t^{T*}\right)^{\gamma}\left(C_t^{N*}\right)^{1-\gamma} \quad ; \quad C_t^{T*} = \left(C_t^{H*}\right)^{1-\theta}\left(C_t^{F*}\right)^{\theta}$$

- The weight θ applies to the domestic traded good. If $\theta > 0.5$ the basket is tilted towards the domestic traded good.
- Foreign prices prices, expressed in Foreign currency, are denoted by P_t^{H*}, P_t^{F*},P_t^{T*},P_t^{N*} and P_t^*. The indices are:

$$\begin{aligned} P_t^* &= (\gamma)^{-\gamma}(1-\gamma)^{-(1-\gamma)}\left(P_t^{T*}\right)^{\gamma}\left(P_t^{N*}\right)^{1-\gamma} \\ P_t^{T*} &= (\theta)^{-\theta}(1-\theta)^{-(1-\theta)}\left(P_t^{F*}\right)^{\theta}\left(P_t^{H*}\right)^{1-\theta} \end{aligned}$$

Real exchange rate and terms-of-trade

- Nominal exchange rate between the Home and Foreign: E_t (increase is a Home currency nominal depreciation).
- Real exchange rate and terms-of-trade (earnings of an export / cost of an import):

$$Q_t^{rer} = \frac{E_t P_t^*}{P_t} \quad ; \quad Q_t^{tot} = \frac{E_t P_t^{H*}}{P_t^F}$$

- The real exchange rate is:

$$Q_t^{rer} = \left(\frac{\left(E_t P_t^{F*}\right)^{\theta} \left(E_t P_t^{H*}\right)^{1-\theta}}{\left(P_t^H\right)^{\theta} \left(P_t^F\right)^{1-\theta}} \right)^{\gamma} \left(\frac{E_t P_t^{N*}}{P_t^N} \right)^{1-\gamma}$$

- Law of one price: a good sells for the price in all countries:

$$P_t^H = E_t P_t^{H*} \quad ; \quad \frac{P_t^F}{E_t} = P_t^{F*}$$

- Purchasing Power Parity: the consumer price indices are the same in all countries, one converted in the same currency:

$$P_t = E_t P_t^*$$

Some particular cases

- If the basket is symmetric ($\theta = 0.5$), and the law of one price holds, the real exchange rate reflects the price of non-traded goods:

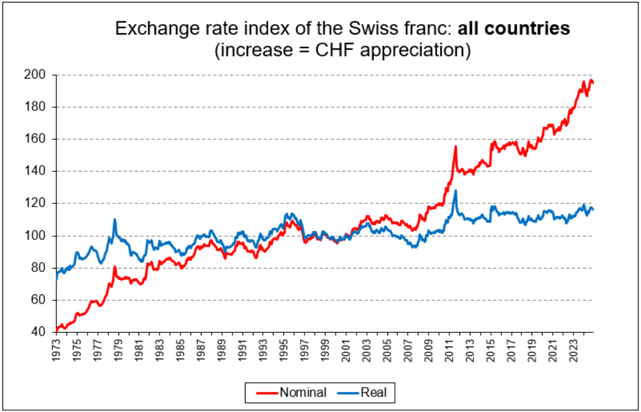
$$Q_t^{rer} = \left(\frac{E_t P_t^{N*}}{P_t^N} \right)^{1-\gamma}$$

- A country with more expensive non-traded goods, for instance due to higher overall income, has a more appreciate (real) currency.
- If the countries have a bias towards domestic goods ($\theta > 0.5$) and the law of one price holds, the real exchange rate is affected by the terms-of-trade:

$$Q_t^{rer} = \left(\left(\frac{1}{Q_t^{tot}} \right)^{2\theta - 1} \right)^{\gamma} \left(\frac{E_t P_t^{N*}}{P_t^N} \right)^{1-\gamma}$$

Does PPP hold?

- Not in absolute terms, reflecting in part different prices of non-traded goods.
- More so in relative terms (first difference): the real exchange rate varies, but is stationary. In other words, prices take time to adjust to the nominal exchange rate.



ENDOGENOUS OUTPUT

Technology

- Small open economy with traded and non-traded good. Price of traded good set a 1, with price of non-traded good equal to P_t^N
  - Higher value raises the consumer price index: a real appreciation.
- Production uses labor and capital, with productivity shifter:

$$Y_t^T = A_t^T \left(K_t^T\right)^{\alpha_T} \left(L_t^T\right)^{1-\alpha_T} \quad ; \quad Y_t^N = A_t^N \left(K_t^N\right)^{\alpha_N} \left(L_t^N\right)^{1-\alpha_N}$$

- Profit maximization implies ($k = K/L$):

$$r = \alpha_T A_t^T \left(k_t^T\right)^{\alpha_T - 1} \quad ; \quad w = (1-\alpha_T) A_t^T \left(k_t^T\right)^{\alpha_T}$$

$$r = P_t^N \alpha_N A_t^N \left(k_t^N\right)^{\alpha_N - 1} \quad ; \quad w = P_t^N (1-\alpha_N) A_t^N \left(k_t^N\right)^{\alpha_N}$$

Solution for price

- Capital mobility gives the world real interest rate r. This gives k_t^T, and in turn w:

$$k_t^T = \left(\frac{\alpha_T A_t^T}{r}\right)^{\frac{1}{1-\alpha_T}} \quad ; \quad w = (1-\alpha_T) A_t^T \left(\frac{\alpha_T A_t^T}{r}\right)^{\frac{\alpha_T}{1-\alpha_T}}$$

- Putting these in the two relation for the non-traded sector gives the price of the non-traded good:

$$P_t^N = \frac{\left(A_t^T\right)^{\frac{1-\alpha_N}{1-\alpha_T}}}{A_t^N} (r)^{\frac{\alpha_N-\alpha_T}{1-\alpha_T}}$$

Impact of productivity

- Increase in productivity in the traded sector A_t^T: Harrod Balassa Samuel
  - Higher capital ratio.
  - Higher wage, thanks to a) direct productivity effect and b) higher capital ratio.
  - Non-traded firms must match the higher wage.
  - Requires them to increase the price, especially if they are more reliant on labor that traded goods producers ($\frac{1-\alpha_N}{1-\alpha_T} > 1$).
- Increase in productivity in the non-traded sector A_t^N:
  - No impact on the traded sector.
  - Wage is unchanged.
  - Non-traded producers fully pass-on the higher productivity into lower prices.
- Dutch disease, if there are several traded good sectors, with a gain in only some of them.
  - Productivity gain leads to a real exchange appreciation.
  - Producers of the traded goods that did not see a gain have to raise wages, but cannot increase their price (set by world markets).

Two countries

- Follow similar steps in the Foreign country.
- The ratio of prices is then:

$$ln\left(\frac{P_t^{N*}}{P_t^N}\right) = \frac{1-\alpha_N}{1-\alpha_T} ln\left(\frac{A_t^{T*}}{A_t^T}\right) - ln\left(\frac{A_t^{N*}}{A_t^N}\right)$$

- The real exchange rate is affected by productivity, but differently depending on which sector sees an increase (Balassa-Samuelson effect).
  - Difference between traded and non-traded sectors.
  - Cross-country difference. A global productivity boom does not impact the real exchange rate. Not surprising, as the real exchange rate is about cross-country differences.