PS 2

Exercise 1

$$\max_{\{c_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \log c_t$$

s.t. $\sum_{t=0}^{\infty} c_t \leq k_0$ we already know that the whole cake is going to be consumed

$c_t \geq 0$, $t \geq 0$

k_0 given

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \ln(c_t) - \lambda\left(\sum_{0}^{\infty} c_t - k_0\right)$$

FOC
- $\frac{d\mathcal{L}}{dc_t} = 0$ $\quad \frac{\beta^t}{c_t} = \lambda$
- $\frac{d\mathcal{L}}{d\lambda} = 0$ $\quad \sum_{t=0}^{\infty} c_t - k_0 = 0$

Eule Equation:

$$\left.\begin{aligned} \lambda &= \frac{\beta^t}{c_t} \\ \lambda &= \frac{\beta^{t+1}}{c_{t+1}} \end{aligned}\right\} \quad \frac{c_{t+1}}{c_t} = \beta$$

2) The complementary slackness implies

$\sum_{t=0}^{\infty} c_t = k_0$ $\quad$ NB: $c_t = \beta^t c_0$

$$\sum_{t=0}^{\infty} \beta^t c_0 = k_0 \quad \Rightarrow \quad \frac{c_0}{1-\beta} = k_0 \qquad c_0 = (1-\beta)k_0$$

and $c_t = \beta^t c_0 = \beta^t (1-\beta) k_0$

• Short focus on the multiplier λ

In an optimization problem with objective function $f(x_1, x_2, \ldots, x_n)$ subject to a constraint $g(x_1, x_2, \ldots, x_n)$, the Lagrange multiplier, tells us the rate of change of the objective function with respect to a change in the constraint.
In other words, λ measures how much the value of the optimal solution would change if the constraint were relaxed (high K_0) or tightened slightly

In our case:
The total amount of the cake is K_0. λ represents the marginal utility of an additional unit of K_0.

from FOC: $\frac{\beta^t}{C_t} = \lambda = \frac{\beta^t}{K_0(1-\beta)\beta^t} = \frac{1}{K_0(1-\beta)} > 0$

If K_0 is small, λ is very large meaning that the agent would gain a lot with an additional unit of K_0
As K_0 increases (relaxing the BC) λ will decrease which is coherent with decreasing marginal utility

$\Rightarrow$ THE AGENT GAINS MARGINALLY LESS FOR EACH UNIT INCREASE IN K_0

3) How much cake left at time T?

$$\sum_{t=0}^{T} c_t = \sum_{t=0}^{T} \beta^t c_0 = \sum_{t=0}^{T} \beta^t (1-\beta) k_0 = \cancel{(1-\beta)} k_0 \frac{(1-\beta^{T+1})}{\cancel{1-\beta}}$$

$$= k_0 (1-\beta^{T+1})$$

$$\text{Cake left} = k_0 - k_0(1-\beta^{T+1}) = \underbrace{k_0 \beta^{T+1}}$$

5) Transversality Condition

$$\lim_{t\to\infty} \beta^t u'(c_t) k_t = \beta^T \frac{1}{\underbrace{c_T}_{c_0\beta^T}} k_T = \lim_{t\to\infty} \beta^T \cdot \frac{1}{\beta^T c_0} k_0 \beta^{T+1} = 0 \checkmark$$

6) If the resource constraint is not satisfied in this problem we would have that

$$\lambda \left(\sum_{t=0}^{\infty} c_t - k_0 \right) \neq 0$$ violating the complementary slackness condition since λ is always positive $\lambda = \frac{\beta^t}{c_t} > 0$

In this case we should adopt different preferences for the individual to make the problem respect the condition to find an optimum

1) Cake-Eating Problem: A more general approach

$$\sum_{t=0}^{t=T} \beta^t \ln(c_t)$$

s.t. $k_{t+1} = k_t - c_t$ => you can rewrite infinitely many budget constraint

$k_0 - c_0 - k_1 \geq 0$ also $k_t \geq 0$ $c_t \geq 0$

$k_1 - c_1 - k_2 \geq 0$

$\vdots$

$k_t - c_t - k_{t+1} \geq 0$

$\vdots$

$k_T - c_T - k_{T+1} \geq 0$ || End of the world, we need a transversality condition

=> This is the same as $c_0 + c_1 + c_2 + \dots c_T = k_0$

$$\mathcal{L} = \left[\sum_{t=0}^{t,T} \beta^t \left[\ln(c_t) + \lambda_t (k_t - c_t - k_{t+1}) \right] + \mu_{T+1} k_{T+1} \right]$$

N.B.: Here we have infinitely many budget constraint and so λ depends on time.

FOCs

1) $\frac{\partial \mathcal{L}}{\partial c_t} = \frac{\beta^t}{c_t} = \beta^t \lambda_t \Rightarrow \frac{1}{c_t} = \lambda_t$

2) $\frac{\partial \mathcal{L}}{\partial k_{t+1}} = -\beta^t \lambda_t + \beta^{t+1} \lambda_{t+1} \Rightarrow \lambda_t = \beta \lambda_{t+1}$

THERE ARE TWO CONSECUTIVE BUDGET CONSTRAINT WHERE k_{t+1} is present

Substitute 2) in 1)

$$\frac{1}{c_t} = \beta \frac{1}{c_{t+1}} \Rightarrow \frac{c_{t+1}}{c_t} = \beta$$ <=> CONSUMPTION GROWTH IS CONSTANT

3) $\frac{\partial \mathcal{L}}{\partial k_{T+1}} = 0 \qquad \mu - \beta^T \lambda_T = 0$

4) • $\sum_{t=0}^{T} \lambda_t (k_t - c_t - k_{t+1}) = 0$ => COMPLEMENTARY SLACKNESS CONDITIONS

• $\mu k_{T+1} = 0$

- $\mu - \beta^T \lambda_T = 0$
- $\mu k_{T+1} = 0$

Combining the two gives

$$\underset{0}{\underset{\parallel}{\mu}} k_{T+1} - \beta^T \lambda_T k_{T+1} = 0$$

$$\boxed{\beta^T \underbrace{\lambda_T}_{>0} k_{T+1} = 0}$$ => This implies $k_{T+1} = 0$

In infinite horizon, a natural extension is the so called transversality condition

$$\lim_{t \to \infty} \beta^t \lambda_t k_{t+1} = 0 = \lim_{t \to \infty} \beta^T u'(c(T)) k_{T+1} = 0$$

Optimal Consumption profile (finite HORIZON T)

$$\frac{c_{t+1}}{c_t} = \beta$$

=> Start in $T+1$, $k_{T+1} = 0$ by slackness.

$$k_{T+1} = k_T - c_T = 0 \Rightarrow k_T = c_T$$

- $\dfrac{c_T}{c_{T-1}} = \beta$
- $c_{T-1} = \dfrac{1}{\beta} \overbrace{k_T}^{bc\ c_T = k_T}$

$$k_T = k_{T-1} - c_{T-1}$$

$$c_{T-1} = \frac{1}{\beta}\left(k_{T-1} - c_{T-1}\right) \Rightarrow c_{T-1} = \frac{1}{1+\beta} k_{T-1}$$

$$c_{T-2} = \frac{1}{\beta} c_{T-1} \Rightarrow c_{T-2} = \frac{1}{\beta} \frac{1}{1+\beta} k_{T-1};$$

$$k_{T-1} = k_{T-2} - c_{T-2}$$

$$C_{T-2} = \frac{1}{\beta+\beta^2}\left(K_{T-2} - C_{T-2}\right) \Rightarrow C_{T-2} = \frac{1}{1+\beta+\beta^2} K_{T-2}$$

$\vdots$

$$C_0 = \frac{1}{1+\beta+\dots\beta^{T+1}} K_0 = \frac{1-\beta}{1-\beta^{T+1}} K_0$$

another way

$$C_T = \beta C_{T-1} = \beta \beta^{T-1} C_0 = \beta^T C_0$$

Now take the total sum

$$\underbrace{\sum_{t=0}^{\infty} C_T}_{K_0} = \sum_{t=0}^{\infty} \beta^T C_0 = \frac{1-\beta^{T+1}}{1-\beta} C_0$$

$$\boxed{K_0 = \frac{1-\beta^{T+1}}{1-\beta} C_0}$$

$\Rightarrow$ Path for C_0

Now let's see it w/ INFINITE HORIZON $(t = \infty)$

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \ln(c_t)$$

$$k_{t+1} = k_t - c_t$$

$$k_{t=0} = k_0$$

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left\{ \ln(c_t) - \lambda_t \left(k_{t+1} - k_t + c_t\right)\right\}$$

FOC $\frac{d\mathcal{L}}{dc_t} = 0 \quad \frac{\beta^t}{c_t} = \lambda_t \beta^t$

$\frac{d\mathcal{L}}{dk_{t+1}} = 0 \quad -\beta^t \lambda_t + \beta^{t+1} \lambda_{t+1} = 0$

$\Rightarrow \frac{c_{t+1}}{c_t} = \beta$

$+ \lim_{t \to \infty} \beta^t \lambda_t k_{t+1} = 0 \Rightarrow$ TRANSVERSALITY

Now I put you two ways of solving the problem, so you see where I need the TVC.

$$k_2 = k_1 - c_1 = k_0 - c_0 - \beta c_0 = k_0 - (1-\beta) c_0$$

$$k_3 = k_2 - c_2 = k_1 - c_1 - \beta c_1 = k_0 - (1-\beta) c_0 - \beta^2 c_0$$

$$\vdots$$

$$k_{T+1} = k_0 - \frac{(1 - \beta^{T+1})}{1 - \beta} c_0$$

$$c_0 = \frac{1-\beta}{1-\beta^{T+1}} k_0 - \frac{1-\beta}{(1-\beta)^{T+1}} k_{T+1}$$

We can't just get rid of k_{T+1}, we need an additional condition

$\Rightarrow$ TRANSVERSALITY!

$$\lim_{T\to\infty} \beta^T u'(c_t) k_{T+1} = 0 \Rightarrow$$

k_{T+1} is the cake left and we computed it before

$$k_{T+1} = k_0 \beta^{T+1}$$

$$k_{T+1} = k_T - c_T = k_0 - \sum_{t=}$$

$$= \lim_{T\to\infty} \beta^T \frac{1}{\beta^T c_0} \cdot \underline{k_0 \beta^{T+1}} = 0 \checkmark$$

Here I proved you that the TVC holds. This is a necessary condition, it should hold EX-ANTE!.

$$c_0 = \underbrace{\frac{(1-\beta)}{1-\beta^{T+1}}}_{\to (1-\beta)^{T+1} \to 1} k_0 + \underbrace{\frac{(1-\beta)}{1-\beta^{T+1}} k_{T+1}}_{\to 0 \text{ by TVC}}$$

$$c_0 = (1-\beta) k_0$$

What about c_t? Use Euler!

$$\frac{c_t}{c_0} = \beta^t = \beta^t c_0 = \beta^t (1-\beta) k_0$$

Simpler version that you can use (bc. TVC holds)

$c_t = \beta^t c_0 \Rightarrow$ Now I take the infinite sum

$$\underbrace{\sum_{t=0}^{\infty} c_t}_{k_0} = \sum_{t=0}^{\infty} \beta^t c_0 \Rightarrow$$

- $c_0 = (1-\beta) k_0$
- $\dfrac{c_t}{c_{t-1}} = \beta$

This two equations describe the whole model

2) Exponential Utility

$$u(c) = -\frac{1}{\theta} e^{-\theta c} \Rightarrow \text{Utility of households over infinite horizon}$$
$\theta > 0$

1)

a) Relate θ to the concavity of utility functions: CHECK 2^{ND} DERIVATIVE TO BE <0

- $\frac{du(c)}{dc} = \frac{d}{dc}\left(-\frac{1}{\theta} e^{-\theta c}\right) = e^{-\theta c} > 0$ ✓
- $\frac{d^2 u(c)}{dc} = \frac{d}{dc}\left(e^{-\theta c}\right) = -\theta e^{-\theta c} < 0$ ✓ for $\theta > 0$

Parameter θ governs the degree of concavity; $\uparrow \theta$ $\uparrow$ concavity

b) Intertemporal Elasticity of substitution

$$IES = \frac{-u'(c)}{c\,u''(c)} = -\frac{e^{-\theta c}}{c \cdot -\theta e^{-\theta c}} = \frac{1}{\theta c}$$

It depends on both θ and c $\Rightarrow$ higher $\theta \Rightarrow \downarrow IES$

higher $c \Rightarrow \downarrow IES$

2) Set up the problem:

The HOUSEHOLD MAXIMIZES:

$$\max \int e^{-\rho t} u(c(t))$$

s.t. $\dot{k}(t) = w(t) + r(t))k(t) - c(t) \Rightarrow$ in per capita terms, assuming population constant $L(t) = L$

and No-Ponzi condition:

$$\lim_{t \to \infty} \left\{ k_t \times \exp\left[-\int_0^t (r(s))\,ds\right] \right\} \geq 0 \Rightarrow \text{Always positive or zero capital in the limit}$$

REMEMBER: 1) The transversality condition (TVC) is about optimal behavior

2) The No-Ponzi condition is a condition to make sure that the flow budget constraints are consistent w/ a lifetime budget constraint (i.e. aggregated across periods)

N.B.

Set up the Hamiltonian:

$$\mathcal{H} = e^{-\rho t}u(c(t)) + \mu(t)\left[w(t) + r(t)k(t) - c(t)\right]$$

1) $H_c(c,k,\mu) = 0 \Rightarrow e^{-\rho t}u'(c(t)) = \mu(t)$

2) $H_k(c,k,\mu) = -\dot{\mu}(t) \Rightarrow \mu(t)\cdot(1+r(t)) = -\dot{\mu}(t)$

$H_\mu(c,k,\mu) = \dot{k}(t)$

$\lim_{t\to\infty} e^{-\rho t}\mu(t)k(t) = 0$ + No ponzi ~~scheme~~ condition and initial condition $k(0) = k_0$

TVC

To compute $\dot{\mu}(t)$ take the time derivative of the first FOC
$\dot{\mu}(t) = \frac{d}{dt}\mu(t) = \frac{d}{dt}e^{-\rho t}u'(c(t))$

Now put 1) + 2) together

$$\cancel{e^{-\rho t}}(1+r(t))u'(c(t)) = \rho \cancel{e^{-\rho t}}u'(c(t)) - \cancel{e^{-\rho t}}u''(c(t))\dot{c}(t)$$

$$\frac{u'(c(t))}{u''(c(t))}\left[r(t) - \rho\right] = -\dot{c}(t) \Rightarrow \frac{\dot{c}(t)}{c} = \sigma(1+r(t)-\rho)$$

$$\text{w/ } \sigma = \left(-\frac{u'(c(t))}{u''(c(t))c}\right)$$

This is the IES

Consumption grows as long as $1+r(t) > \rho$ and how much it grows depends on the IES (θ). To see this Just plug-in the utility function

3) The firm: $\max_{k,L} F(k,L) - w(t)L - R(t)k$

$w(t) = F_L(k,L) = F_L(k,1)$

$R(t) = F_k(k,L) = F_k(k,1)$

- $r(t) = R(t) - \delta \Rightarrow$ Total return of capital

$w(t) = F_L(k,L) = f(k) - f'(k)$

From the capital accumulation

$$\dot{k}(t) = \overbrace{w(t) + r(t))k(t)}^{f(k_t)} - c(t)$$

We know that
$r(t) = F_k(k,L) - \delta$

$\dot{k}(t) = f(k) - \delta k(t) - c(t)$

$$\sigma(c) = -\frac{u'(c)}{u''(c)c}$$

$$\frac{\dot{c}(t)}{c} = (r(t) - \rho)\sigma(c) = (f'(k) - \rho - \delta)\sigma(c)$$

$$\frac{\dot{c}(t)}{c} = [r(t) - \rho]\frac{e^{-\theta c}}{c\theta e^{-\theta c}} = \frac{[r(t) - \rho]}{c\theta}$$

$$\frac{\dot{c}(t)}{c} = \frac{[f'(k) - \rho - \delta]}{\theta c}$$

Steady State

$\dot{k}(t) = 0 \quad \dot{c}(t) = 0$

$f(k^*) - \delta k^* = c^*$ || Net output is consumed in equilibrium

$\dot{c}(t) = 0$

$f'(k) = \rho + \delta$ || Grow until the MP of capital is equal to $\rho + \delta$

5) During the transition towards the SS, the parameter θ governs the speed of consumption growth. A higher θ means lower IES meaning that the individual are more reluctant to adjust consumption in response to changes in the interest rate.
Remember $IES = \frac{1}{CRRA}$ (CRRA = coefficient of risk aversion). So the individual is more risk averse.

• If you want to add population growth the model becomes:

$$\max U = \int_0^{\infty} -\left(\frac{1}{\theta}\right) e^{-\theta c} e^{-(n-\rho)t}\, dt$$

$$\text{s.t.} \quad \dot{k} = w(t) + r k(t) - c(t) - n k(t) \quad \Big| \text{ per capita term}$$

The Hamiltonian

$$\mathcal{H} = -\left(\frac{1}{\theta}\right) e^{-\theta c} e^{-(n-\rho)t} + \mu(t)\left(r k(t) + w(t) - c(t) - n k(t)\right)$$

you'll find the same solution only the evolution of $\dot{k}$ is also influenced by n.

$$\dot{k} = f(k) - c - (n+\delta) k$$

$$\frac{\dot{c}}{c} = \frac{1}{\theta c}\left(f'(k) - \delta - \rho\right)$$

Behaviour of the model.

Exercise 3

Firm: $Y_{SJ} = A_{SJ} K_{SJ}^{\alpha_S} L_{SJ}^{1-\alpha_S}$

Sector: $Y_S = \left(\sum_{j=1}^{M_S} Y_{SJ}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$

Demand for variety J in sector s:

$$Y_J = \left(\frac{P_J}{P}\right)^{-\sigma} Y \qquad P_J = P\left(\frac{Y_J}{Y}\right)^{-\frac{1}{\sigma}} \text{; normalize } P = 1$$

we are considering just one sector

Firm's maximization problem

1) $\max_{K_J, L_J} \; p_J y_J (1-\tau_{J,y}) - w L_J - r(1+\tau_{K,J}) K_J$

s.t. $p_J = \left(\frac{y_J}{y}\right)^{-\frac{1}{\sigma}}$; Substitute inside the objective function

$y_J = A_J K_J^{\alpha} L_J^{1-\alpha}$

$$\max_{K_J, L_J} \; y_J^{1-\frac{1}{\sigma}} y^{\frac{1}{\sigma}} (1-\tau_{J,y}) - w L_J - r(1+\tau_{K,J}) K_J = \left[A_J K_J^{\alpha} L_J^{1-\alpha}\right]^{1-\frac{1}{\sigma}} y^{\frac{1}{\sigma}} - w L_J - r(1+\tau_{K,J}) K_J$$

FOC

$$\frac{d\pi}{dK_J} = (1-\tau_{J,y})\left(1-\frac{1}{\sigma}\right) y_J^{-\frac{1}{\sigma}} y^{\frac{1}{\sigma}} \alpha A_J K_J^{\alpha-1} L_J^{1-\alpha} - r(1+\tau_{K,J}) = 0 \quad 1)$$

$$\frac{d\pi}{dL_J} = (1-\tau_{J,y})\left(1-\frac{1}{\sigma}\right) y_J^{-\frac{1}{\sigma}} y^{\frac{1}{\sigma}} (1-\alpha) A K_J^{\alpha} L_J^{-\alpha} - w = 0 \quad 2)$$

3) $\frac{r(1+\tau_{K,J})}{\alpha} = (1-\tau_{J,y}) y_J^{-\frac{1}{\sigma}} y^{\frac{1}{\sigma}} \frac{A K_J^{\alpha} L_J^{-\alpha} L_J}{K_J}$

4) $\frac{w}{1-\alpha} = (1-\tau_{J,y})\left(1-\frac{1}{\sigma}\right) y_J^{-\frac{1}{\sigma}} y^{\frac{1}{\sigma}} K_J^{\alpha} L_J^{-\alpha} A_J$

$$\frac{r(1+\tau_{K,J})}{\alpha} = \frac{w}{1-\alpha}\frac{L_J}{K_J}$$

$$\boxed{(1+\tau_{K,J}) = \frac{w}{r}\frac{\alpha}{1-\alpha}\frac{L_J}{K_J}}$$

remember: $y_J^{-\frac{1}{\sigma}} y^{\frac{1}{\sigma}} = p_J$

Back to 1)

$$(1-\tau_{J,y})\left(1-\frac{1}{\sigma}\right) p_J \cancel{\alpha} \frac{A_J K_J^{\alpha} L_J^{1-\alpha}}{\cancel{K_J}} = \cancel{r} \cdot \frac{w}{\cancel{r}} \frac{\cancel{\alpha}}{1-\alpha} \frac{L_J}{\cancel{K_J}}$$

$$(1-\tau_{J,y})\left(\frac{\sigma-1}{\sigma}\right)p_J y_J = \frac{1}{1-\alpha} w L_J$$

$$\boxed{(1-\tau_{J,y}) = \left(\frac{\sigma}{1-\sigma}\right)\left(\frac{1}{1-\alpha}\right)\frac{w L_J}{p_J y_J}}$$

3) Revenue Productivity : $P_J A_J$

- Remember from FOC

- $(1-\tau_{J,y})\,\alpha\left(1-\frac{1}{\sigma}\right)p_J y_J = MPK\, k_J$
- $(1-\tau_{J,y})(1-\alpha)\left(1-\frac{1}{\sigma}\right)p_J y_J = MPL\, L_J$

$$\frac{\alpha}{1-\alpha} = \frac{MPK}{MPL}\cdot\frac{k_J}{L_J}$$

$$\frac{k_J}{L_J} = \frac{MPL}{MPK}\frac{\alpha}{1-\alpha}$$

$$(1-\tau_{J,y})\alpha\left(1-\frac{1}{\sigma}\right)p_J y_J + (1-\tau_{J,y})(1-\alpha)\left(1-\frac{1}{\sigma}\right)p_J y_J = MPK\, k_J + MPL\, L_J$$

7) $$(1-\tau_{J,y})\left(1-\frac{1}{\sigma}\right)p_J y_J = MPK\, k_J + MPL\, L_J$$

- Remember P_{30}:

$$y_J = A_J k_J^{\alpha} L_J^{1-\alpha} = A_J\left(\frac{k_J}{L_J}\right)^{\alpha} L_J = A_J\left(\frac{L_J}{k_J}\right)^{1-\alpha} k_J$$

8) $$k_J = \frac{y_J}{A_J}\left(\frac{k_J}{L_J}\right)^{1-\alpha} = \frac{y_J}{A_J}\left(\frac{\alpha}{1-\alpha}\frac{MPL}{MPK}\right)^{1-\alpha}$$; 9) $$L_J = \frac{y_J}{\cancel{A}_J}\left(\frac{\alpha}{1-\alpha}\frac{MPL}{MPK}\right)^{-\alpha}$$

RHS of 7) : $$MPK\frac{y_J}{A_J}\left(\frac{\alpha}{1-\alpha}\frac{MPL}{MPK}\right)^{1-\alpha} + MPL\frac{y_J}{\cancel{A}_J}\left(\frac{\alpha}{1-\alpha}\frac{MPL}{MPK}\right)^{-\alpha}$$

$$= \frac{y_J}{A_J}\left[MPK\left(\frac{\alpha}{1-\alpha}\frac{MPL}{MPK}\right)^{1-\alpha} + MPL\left(\frac{\alpha}{1-\alpha}\frac{MPL}{MPK}\right)^{-\alpha}\right] =$$

$$= \frac{y_J}{A_J}\left[\cancel{MPK}\frac{\alpha}{1-\alpha}\frac{MPL}{\cancel{MPK}}\left(\frac{\alpha}{1-\alpha}\frac{MPL}{MPK}\right)^{-\alpha} + MPL\left(\frac{\alpha}{1-\alpha}\frac{MPL}{MPK}\right)^{-\alpha}\right]$$

$$= \frac{y_J}{A_J}\left[\left(\frac{\alpha}{1-\alpha}\frac{MPL}{MPK}\right)^{-\alpha}\left(\frac{\alpha MPL}{1-\alpha} + MPL\right)\right] = \frac{y_J}{A_J}\left[\left(\frac{\alpha}{1-\alpha}\frac{MPL}{MPK}\right)^{-\alpha}\frac{MPL}{1-}\right]$$

$$= \left(\frac{MPL}{1-\alpha}\right)^{1-\alpha}\left(\frac{MPK}{\alpha}\right)^{\alpha}\frac{y_J}{A_J} = MPK\,k_J + MPL\,l_J$$

Now plug the above expression into 7)

$$MPL = w$$

$$MPK = (1+\tau_{J,k})\, r$$

$$(1-\tau_{J,y})\left(\frac{\sigma-1}{\sigma}\right)p_J \cancel{y_J} = \left(\frac{MPL}{1-\alpha}\right)^{1-\alpha}\left(\frac{MPK}{\alpha}\right)^{\alpha}\frac{\cancel{y_J}}{A_J}$$

$$p_J = \frac{\sigma}{\sigma-1}\left(\frac{w}{1-\alpha}\right)^{1-\alpha}\left(\frac{r}{\alpha}\right)^{\alpha}\frac{(1+\tau_{J,k})^{\alpha}}{1-\tau_{J,y}}\cdot\frac{1}{A_J}$$

4) $\tau_{J,k} = 0$; $\tau_{J,y} = 0$

W/out distortions $\quad p_J A_J = \frac{\sigma}{\sigma-1}\left(\frac{w}{1-\alpha}\right)^{1-\alpha}\left(\frac{r}{\alpha}\right)\frac{1}{1}$ ✓

$$\Rightarrow \frac{(1+\tau_{J,k})^{\alpha}}{1-\tau_{J,y}} \Leftrightarrow$$ Factor of distortion, thus the amount by which output decrease w/ distortion