Chapter IV

Intertemporal Trade: Applications and Extensions

IV.1 Introduction and Overview

In Chapter II, it was shown that the current account reflects the difference between gross national savings and domestic investment, and that it influences the evolution of a country's net international investment position. In Chapter III, we related saving and investment decisions to the intertemporal optimization of consumers and firms. We used the framework of the representative consumer (RC) model to analyze the response of the current account to income fluctuations and to explore how exogenous shocks in a large open economy affect savings and investment in other countries via an adjustment of the world interest rate.

In the current chapter, we want to show that these examples do not exhaust the applicability of the „intertemporal approach" to the current account, and we want to use this approach to analyze a wide set of interesting and relevant questions. To accomplish this task, we will have to extend and modify the basic model. However, we will stick to the fundamental notion that the volume and direction of international capital flows are determined by the interaction of national saving and investment levels, and that these macroeconomic aggregates reflect the intertemporal decisions of rational agents.

In Section IV.2, we will analyze the question how ***demographic change*** – in particular, variations of a country's population growth rate – affects the current account. Starting from the observation that individuals' saving behavior varies across the life cycle, we will show that aggregate savings are determined by the age structure of the population. To achieve this, we will use a theoretical framework that replaces the representative consumer by a sequence of "overlapping generations" whose size may vary over time.

Section IV.3 will focus on the effects of ***fiscal policy***, i.e. the government's spending and taxation decisions. We will thus extend our analysis, which so far focused on *private* saving and investment activities, by considering the possibility of public budget surpluses and deficits. As we will demonstrate, the reaction of the current account to variations in fiscal policy crucially depends on the reaction of private savings and investment to these policy changes.

In Section IV.4, we will drop the assumption that there is a homogeneous, perfectly tradable consumption good, and focus on a more differentiated goods spectrum. We will start by distinguishing between ***tradable*** and ***non-tradable goods*** and explore how this difference affects saving choices. In a further step, we will consider an economy that is specialized in the production of a tradable good, but consumes both domestically produced and imported goods. Finally, we will replace the tradable/non-tradable dichotomy by a model, in which goods are generally tradable, but in which trade is associated with ***trade costs***. As we will show, all these models allow for relative price changes which affect saving and investment behavior – and thus the current account – not only through their influence on the intertemporal budget constraint, but also through their influence on the "effective" (consumption-based) interest rate.

Section IV.5 will deviate from the assumption of perfect foresight to analyze how ***uncertainty*** about future incomes affects individuals' saving behavior. We will start by assuming that the international financial market offers only risk-free bonds as a store of value, and introduce the concept of ***precautionary savings***. In a second step, we will allow agents to purchase state-contingent claims on other countries' output. As we will see, this opens another motive for international capital flows, which complements the objective to smooth consumption and to realize profitable investment opportunities: specifically, the possibility of ***international diversification*** offers the chance to reduce country-specific risk and to implement a more stable consumption path.

IV.2 Demographic Change, International Investment, and the Current Account

IV.2.1 Motivation

The age structure of most industrialized economies will change considerably in the coming decades, and the implications of this ***demographic change*** are the subject of an intensive academic and public debate. According to estimates of the United Nations, the number of persons of working age relative to the number of persons beyond the retirement age in Germany will shrink from three to two between 2010 and 2050. Figure 4.1 illustrates this phenomenon by showing ***old-age dependency ratios*** for various industrialized economies. These ratios are computed by dividing the number of retired persons by the number of persons of working age – usually 15 to 64 years.[1]

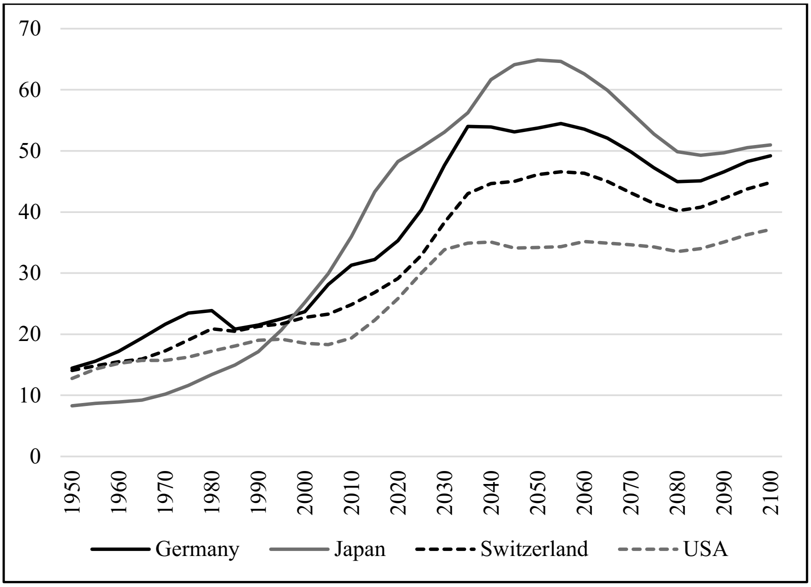

Figure 4.1: Observed and predicted old-age dependency ratios for Germany, Japan, Switzerland and the United States (Source: United Nations, World Population Prospects 2015, High fertility variant).

Several factors are responsible for this evolution: the "baby boom", which started in the USA shortly after the end of the Second World War and in Europe in the mid-1950s, was characterized by high birth rates and resulted in a high supply of workers and employees who started entering labor markets some twenty years later. Around 1965, the baby boom was followed by a rapid decline in fertility. Taking Germany as an example, the average number of children per woman was 2.47 in the years between 1960 and 1965. In the years from 2005 through 2010, this number had dropped to 1.36 (United Nations, 2015). At the same time, technological progress, which results in better nutrition, reduced hazard at the work place, and improved health care, has increased people's life expectancy in many countries. The aging of the "baby boomer generation",

[1] Forecasting population growth in the distant future is, of course, a challenge. For this reason, the United Nations offer different "variants" of their predictions, which are based on alternative assumptions on the evolution of fertility and mortality. In our graph, we are using the so-called "high fertility variant", which assumes rather high birth rates.

combined with increasing longevity and declining birth rates has resulted in the evolution that is described by Figure 4.1.

Population aging directly affects the sustainability of national social security systems. Many of these systems – including the German one – are based on the ***pay-as-you-go*** model, which implies that payments to retirees are financed by contributions of the current labor force. Such a system gets into trouble if the number of retirees relative to workers and employees increases substantially. But even if social security is based on a ***fully-funded system*** – i.e. if old-aged persons live from the savings they have accumulated while working – a changing demographic structure affects aggregate savings, investment, and thus the current account as well as international capital flows. This is due to the fact that individuals' consumption and saving behavior is age-dependent: before entering the work force, savings are usually negative since individuals' consumption is rarely supported by their own income. During working age, individuals have positive savings. Upon retirement, savings decrease and possibly become negative, since individuals run down the assets accumulated during their working age.

As illustrated by Figure 4.2, this ***life cycle theory of consumption*** is, by and large, supported by the evidence: in 2013, the saving rates of households captured by the German Sample Survey of Income and Expenditure obviously depended on the age of the main income earner.[2] Combining the life-cycle hypothesis of consumption with the demographic evolution sketched above yields some hypotheses on the evolution of aggregate savings: as long as a large share of the population is of working age, the national saving rate should be high. As soon as this cohort enters retirement, however, aggregate savings should decrease.

Evoking our model of investment from Chapter III, we can also speculate how demographic change affects aggregate capital formation in an economy: since the marginal productivity of capital positively depends on employment, investment should increase in those periods in which an increase of the labor force is expected for the future. Conversely, if a decrease of the labor force is anticipated, this should result in declining investment.

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[2] When interpreting these data, we have to take into account that they are based on a cross-sectional survey, not on the consideration of the *same* individuals over time. As a consequence, life-cycle effects mix with cohort effects – i.e. the saving rates of the 45-55 year-old may not be high because individuals tend to save a lot at this age, but because *that particular cohort* was exposed to experiences that made it choose a particularly high saving rate. Note also that the values in Figure 4.2 remain positive even after people have entered retirement, and that there is even a slight increase in the age bracket of the very old. This implies that many old people leave a share of their wealth as a bequest to the next generation.

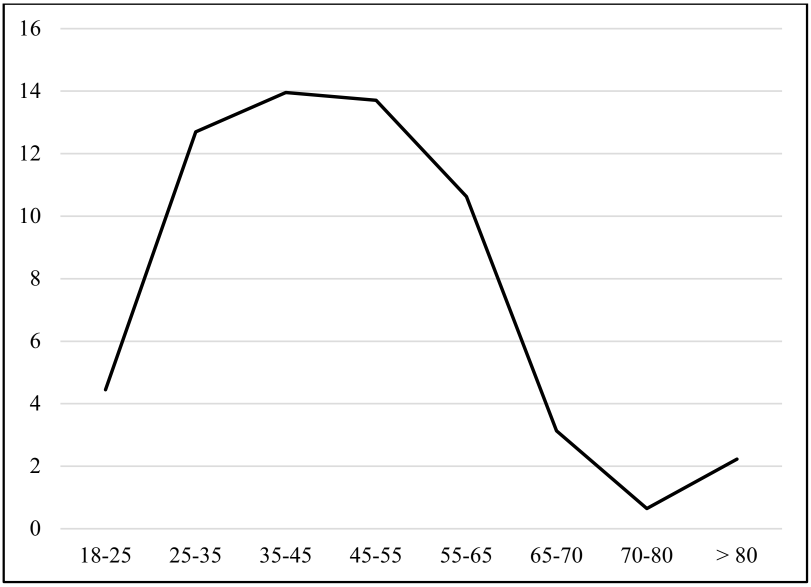

Figure 4.2: Average saving rates of German households depending on the age of the main income earner. Source: Destatis (2015).

On the following pages, we will elaborate and deepen these ideas. To accomplish this task, we will introduce a theoretical framework that resembles the basic model of Chapter III in many respects. However, we will drop the assumption that individuals maximize the utility of their entire "dynasty" when deciding on the time path of consumption. We will then use this framework to explore how a temporary change of the population growth rate affects savings, consumption and the current account.

IV.2.2 A Simple OLG Model

In principle, it is possible to modify the infinite-horizon RC model introduced in Section III.7 to allow for demographic change.[3] However, it is hard to use this framework to model the choices of different generations, to explicitly consider alternative assumptions on individuals' intergenerational altruism, and to analyze how different age groups are affected by policy changes. On the fol-

[3] In two seminal papers, Yaari (1965) and Blanchard (1985) present a "perpetual youth model", in which agents are assumed to have an infinite time horizon and to face a constant probability of death in each period. This modification of the standard model reduces the subjective discount factor and gives rise to a non-degenerate (though constant) age distribution.

lowing pages, we will therefore present a formal approach that allows the explicit modelling of an economy's demographic structure and the combining of the concept of intertemporal optimization with alternative assumptions on individuals' bequest motives. The basic version of this model works as follows:

– Every individual lives for a finite number of periods. By contrast, the economy itself has no terminal period.
– An individual's life can be split into different phases, which differ with respect to his productivity and income sources (labor income vs. capital income). These phases may be of different length.
– Each individual possibly leaves some bequests to his descendants. However the bequest motive differs from the one in the infinite-horizon RC model since the current generation does not account for the utility of subsequent generations, but utility is immediately derived from bequeathing a certain sum to one's offspring.[4]
– In every period, several ***age cohorts*** – i.e. groups of individuals who were born at the same time or in the same time interval – coexist.[5]

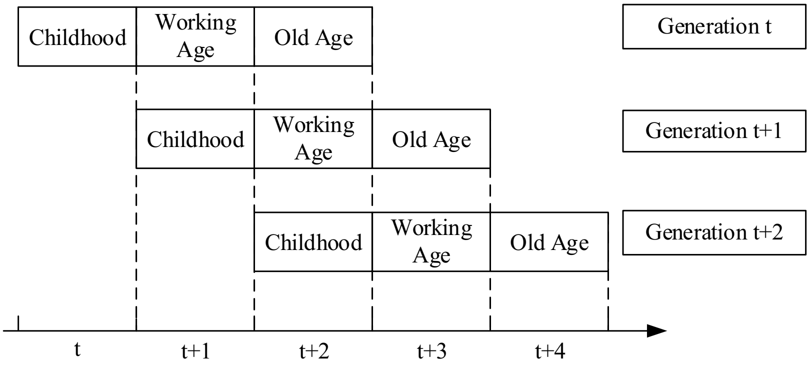

Figure 4.3 : The structure of a simple OLG model

The term ***overlapping generations (OLG) model*** is based on the fact that, in every period, the total population consists of individuals who are in different phases of their lives. In this chapter, we will work with a very simple OLG

[4] The fact that this bequest motive is based on a noble form of selfishness is reflected by the term ***warm-glow preferences*** (Andreoni, 1989).

[5] An age group / cohort may – but does not have to – coincide with a *generation*. In models, in which individual time periods are shorter than 25 years, this distinction is important. However, we will consider a very coarse demographic structure and will therefore use the terms cohort and generation as synonyms.

model, which splits an individual's life into three equally long phases: childhood, working age, retirement. The demographic structure for this special case is depicted in Figure 4.3. We add the following assumptions:

– All individuals have the same preferences represented by a utility function that is characterized by a constant intertemporal elasticity of substitution $(1/\sigma)$.
– In the first part of his life ("childhood") an individual has no income, and his consumption is financed by his parents.
– In the second part of his life ("working age"), every individual gets an exogenous income. This income is identical for all members of a generation.
– All (grown-up) individuals have access to a perfect international capital market. The bonds traded on this market yield an exogenous real interest rate r.
– In the third part of their lives („retirement"), all individuals run down their savings and do not leave any bequests.
– All individuals of a generation have the same number of descendants. The size of a generation born in period $t-1$ is n_{t-1}. The number of descendants per individual in period t is $\mu_t = n_t / n_{t-1}$ [6].

Due to our assumption that a child's consumption is determined by its parents, a representative member of the generation that is born in period $t-1$ chooses his optimal consumption path for the remaining two phases of his life in period t. Specifically, such an individual maximizes the following objective function:

$$U_t = (1+\mu_t)\frac{\left(c_t^y\right)^{1-\sigma}-1}{1-\sigma}+\beta\frac{\left(c_{t+1}^o\right)^{1-\sigma}-1}{1-\sigma} \tag{4.1}$$

In this expression, c_t^y is consumption during working age, and c_{t+1}^o is the individual's consumption during retirement.[7] The fact that period utility at working age is multiplied with the size of the entire household reflects the assumption that parents derive utility from their own and from their descendants' consumption, and that they evenly distribute consumption among family members.

When choosing his optimal consumption path, a member of "generation $t-1$" has to take into account the following constraints:

[6] Taken literally, the model suggests a world in which all parents are single mothers or fathers. However, since we assume that all members of a generation are (economically) identical, and since we abstract from the subtleties of dating and mating, it is straightforward to reconcile the model with more realistic notions of family life.

[7] In this section, lower-case letters denote quantities that refer to single individuals, while upper-case letters represent aggregate magnitudes.

(4.2) $$c_t^y = \frac{y_t - b_{t+1}^o}{1+\mu_t}$$

(4.3) $$c_{t+1}^o = \left(1+r\right) b_{t+1}^o$$

Here, y_t represents the (exogenous) income of a worker in period t, while b_{t+1}^o represents the stock of bonds he will own at the start of period $t+1$. Hence, $y_t - b_{t+1}^o$ is the sum (expressed in goods units) available for consumption to the individual and his offspring in period *t*. Since, by assumption, goods cannot be consumed by several persons, total household consumption $(y_t - b_{t+1}^o)$ has to be divided by the size of the household $(1+\mu_t)$.

Solving the optimization problem of a member of generation $t-1$ yields the following necessary condition for utility maximization:

(4.4) $$\left(c_t^y\right)^{-\sigma} = \beta\left(1+r\right)\left(c_{t+1}^o\right)^{-\sigma}$$

Substituting the constraints (4.2) and (4.3) into (4.4) and solving for b_{t+1}^o allows deriving an individual's optimal savings:

(4.5) $$b_{t+1}^o = \frac{y_t}{1+\left(1+\mu_t\right)\beta^{-\frac{1}{\sigma}}\left(1+r\right)^{\frac{\sigma-1}{\sigma}}}$$

The fact that the variable μ_t lowers savings is due to the effective income reduction associated with higher population growth: for a given income, less resources are available for consumption and saving if consumption has to be spread over a larger number of household members. Conversely, the effect of the interest rate on an individual's savings depends on the intertemporal elasticity of substitution (IES): if the IES is small $(\sigma > 1)$, the income effect of an interest rate increase dominates, and a higher real interest rate results in lower savings. By contrast, if the IES is high ($\sigma < 1$), the substitution and wealth effects dominate, and savings increase in the interest rate.

To arrive at macroeconomic aggregates in general and the current account in particular, we have to multiply generation-specific variables with the respective generation's size. For the evolution of the net international investment position, this implies

(4.6) $$B_{t+1} - B_t = n_{t-1}\, b_{t+1}^o - n_{t-2}\, b_t^o$$

Hence, if the demand for assets of the working-age population in period t exceeds the asset supply of the "dis-saving" old generation – be it, because the generation born in period $t-1$ is large or because this generation has particularly strong incentives to save – the net international investment position increases, i.e. the economy runs a current account surplus. By contrast, if aggregate savings relative to the preceding period are small, this results in a current account deficit.

IV.2.3 "Baby Boom", "Baby Bust", and the Current Account

Equations (4.5) and (4.6) show that demographic change influences aggregate savings both by affecting individual saving decisions and by determining the size of different cohorts. In this subsection, we will use our simple OLG model to analyze the effects of a "***baby boom***" and a subsequent "***baby bust***". Such a sequence of exceptionally high birth rates followed by below-average fertility could be observed in many industrialized countries in the second half of the 20th century, and our model suggests that this demographic change had an important effect on countries' current accounts.

Parameter	Value	Parameter	Value
$..., \mu_1, \mu_2$	1	σ	2
μ_3	1.5	r	0.05
μ_4	0.67	β	0.95
$\mu_5, \mu_6, ...$	1	y_t	1

Table 4.1: Parameters used for the simulation in Figure 4.4.

Using the individual saving function given by (4.5) and the evolution of the net international investment position in (4.6), it is easy to derive the time path of the current account for a given time path of cohort sizes. However, instead of analyzing the resulting difference equation, we will focus on the following numerical example: we assume that, through period $t=2$, the population size is constant – i.e. every individual has exactly one descendant: $\mu_t = 1$ for $t \leq 2$. The further evolution of the population growth rate – a strong increase in period 3, followed by a sharp decrease in period 4 and a return to its long-run value in period 5 – as well as the values chosen for the other model parameters are given by Table 4.1. Figure 4.4 depicts the number of children per worker and shows how the ***age dependency ratio*** – i.e. the number of children and retired persons relative to the labor force – and the current account (relative to aggregate income) evolve over time.

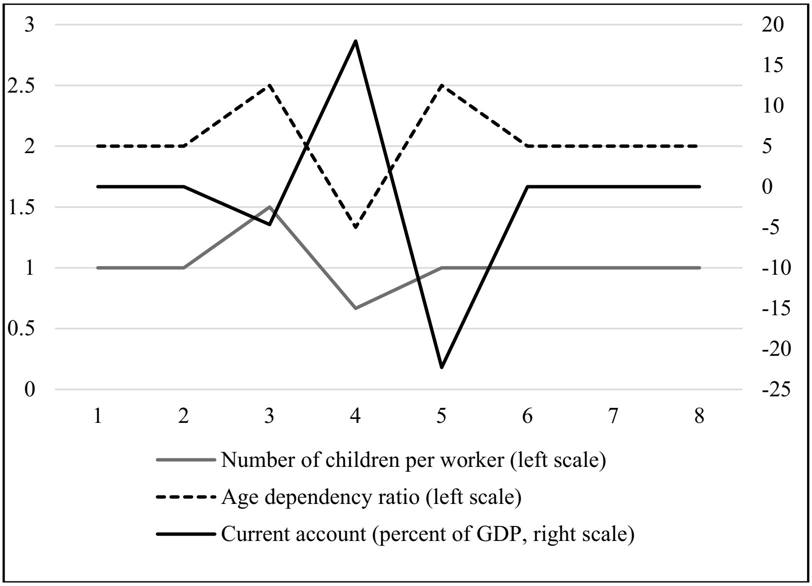

Figure 4.4: Evolution of the birth rate, age dependency ratio and current account (relative to GDP) for a sequence of a "baby boom" and a "baby bust".

The evolution of the current account that is shown in Figure 4.4 can be interpreted as follows: through period 2 and starting with period 6, the current account is balanced. Since the population size is constant in these periods, and since all cohorts have the same preferences and incomes, there is no reason why the net international investment position should vary. In period 3, the economy exhibits a current account deficit, which results from the fact that the large number of children (born in period 3) reduces parents' savings. In period 4, the baby boom generation reaches working age. This raises aggregate savings, generating a current account surplus. Finally, the current account deficit in period 5 reflects the negative savings of the baby boomers who finance consumption in the last part of their lives by running down their assets.

Despite its obvious simplicity, this numerical example conveys an important insight: the age dependency ratio and the current account should be negatively correlated. Figure 4.5 demonstrates that such a correlation can, indeed, be observed for a sample of high- and middle-income countries. Of course, when interpreting this graph we have to take into account that the relationship may be driven by other factors that are related to countries' age dependency ratios. However, the significantly negative effect of that variable is also supported by empirical analyses that explicitly consider the effect of other potential determinants of current account balances.[8]

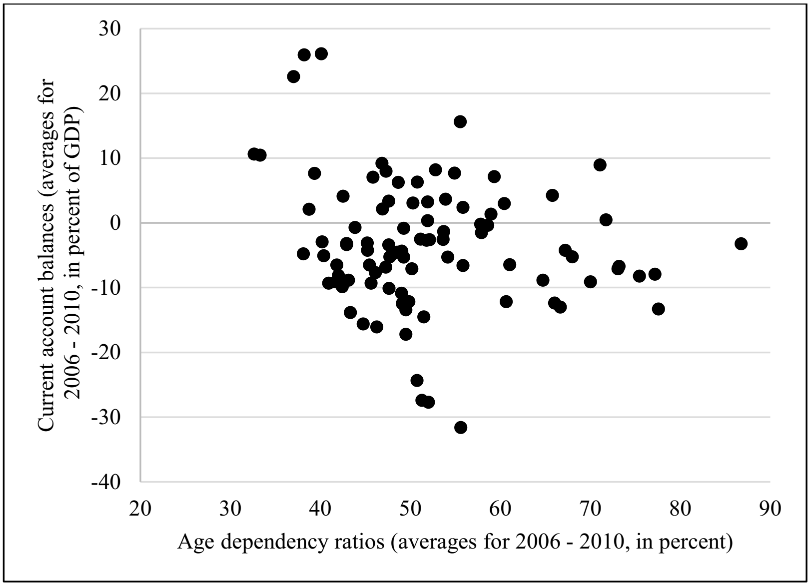

Figure 4.5: Age dependency ratios (averages for 2006 – 2010, in percent) and current account balances as a share of GDP (averages for 2006 – 2010, in percent) for high- and middle-income countries excluding oil-exporting countries. Source: World Bank (World Development Indicators).

IV.2.4 Demographic Change and the Current Account: The Role of Investment

As mentioned in the introduction of this section, a varying population size may affect the current account not only through its effect on national savings, but also by influencing aggregate investment. To further explore this issue, we assume that there is a large number of identical firms, each of whom produces a homogenous good, using the following ***Cobb-Douglas***[9] technology:

$$(4.7) \qquad Y_t = K_t^{\alpha}\, L_t^{1-\alpha}$$

[8] This is shown in the empirical studies by Chinn und Prasad (2003), De Santis and Lührmann (2006), as well as Chinn and Ito (2008).

[9] This specific form of a production function suggested by the mathematician Charles W. Cobb and the economist Paul H. Douglas in 1928 satisfies all the assumptions outlined in Section III.5. It makes, however, specific assumptions on the substitutability of capital and labor.

During their working age, individuals receive a real wage w, which they use to finance their own consumption, their descendants' consumption, and their savings. Savings can be used to purchase internationally traded bonds, but also to buy investment goods, which will be used for production in the next period. For simplicity, we assume that capital completely depreciates in the period in which it is used. The solution to the optimization problem of a representative member of generation $t-1$ looks like the one in (4.5), with the wage in period t replacing the exogenous income Y_t and the left-hand side reflecting the fact that individuals may use both bonds and physical capital as a store of value:

$$b_{t+1}^{o}+k_{t+1}^{o}=\frac{w_t}{1+\left(1+\mu_t\right)\beta^{-\frac{1}{\sigma}}\left(1+r\right)^{\frac{\sigma-1}{\sigma}}}, \tag{4.8}$$

with k_{t+1}^{o} denoting the stock of physical capital carried into period $t+1$, and w_t representing the individual's labor income in period t. When deriving this equation, we have already taken into account that both alternative stores of value have to offer the same return – i.e. in equilibrium, the marginal productivity of capital has to equal the exogenous world interest rate plus the rate of depreciation (assumed to be one). The real wage reflects the marginal product of labor. Using this insight as well as the relationship between the capital-labor ratio and the real interest rate in (4.8), we get:[10]

$$b_{t+1}^{o}=\left(\frac{\alpha}{1+r}\right)^{\frac{1}{1-\alpha}}\left[\frac{1-\alpha}{\alpha}\frac{1+r}{1+\left(1+\mu_t\right)\beta^{\frac{-1}{\sigma}}\left(1+r\right)^{\frac{\sigma-1}{\sigma}}}-\mu_t\right] \tag{4.9}$$

As the occurrence of the population growth rate μ_t in the second part of the squared bracket suggests, there is an additional channel through which population growth affects the current account: this channel reflects the fact that a high birth rate in period t signals a high labor supply in period $t+1$. This, in turn, raises the marginal productivity of capital in period $t+1$, and makes investment in period t more attractive. As a consequence, the stock of bonds carried into the next period decreases and possibly even turns negative.

Hence, adding endogenous investment is likely to reinforce the dynamics of our simple numerical example: during the baby boom, a negative current account balance would not only result from lower savings, but also from higher investment, which would be realized in anticipation of a larger labor force in the future. In period 4, by contrast, savings would increase, while investment

[10] We explicitly derive this result in the appendix to this chapter.

would fall, resulting in an even higher current account surplus. Finally, period 5 would be characterized by negative national savings and a slight increase of investment, generating a current account deficit.

Note that the entire analysis so far has concentrated on the case of a small open economy that faces a constant exogenous interest rate r. By definition, this made it impossible to analyze the effect of demographic change on capital returns. Box 4.1 turns to the question how the future aging of economies may influence asset prices and capital incomes by first looking at a closed economy and then considering the potential effects of international financial integration.

Box 4.1: Will Demographic Change Result in an "Asset Meltdown"?

Recall that in Section III.3, we considered a closed economy, showing that a large supply of assets meeting a low demand for assets results in a high equilibrium real interest rate. Considering the demographic change that is currently under way in many industrialized countries, this gives rise to an alarming hypothesis: as soon as members of the baby boom generation start consuming their savings and selling their wealth, economies will face a dramatic decline of asset prices and a strong reduction of capital returns. This evolution will be reinforced by the fact that a higher capital-labor ratio – due to a falling labor force in combination with a high capital stock – reduces the marginal productivity of capital and thus the autarky interest rate.

However, the prediction of such an "asset meltdown" (Mankiw and Weil, 1989) neglects the fact that the consequences of demographic change on industrialized countries can be mitigated by international capital flows. Figure B4.1 shows that the old-age dependency ratio behaves very differently across world regions: while Europe and North America are in the midst of rapid population aging – albeit with different intensities – this process will start somewhat later in Latin America and Asia, and will not be observed in Africa in the forseeable future. Economic theory predicts much higher returns to capital for the "younger" parts of the world. This, in turn, suggests that the baby boom generation in industrialized countries could prevent the dramatic drop of capital returns that seems inevitable for a closed economy by channeling their savings into countries with a different demographic structure and evolution. Brooks (2000) and Börsch-Supan et al. (2004) present numerical simulations that are based on multi-country/multi-cohort OLG models and that support this conjecture. However, the notion that international investment may prevent the "asset meltdown" in rich economies is based on the assumption of reasonably well-functioning capital markets. This assumption has to be considered with caution if investments in developing countries and emerging markets meet a risky policy environment and uncertain property rights – problems to which we will turn explicitly in Chapter 6.

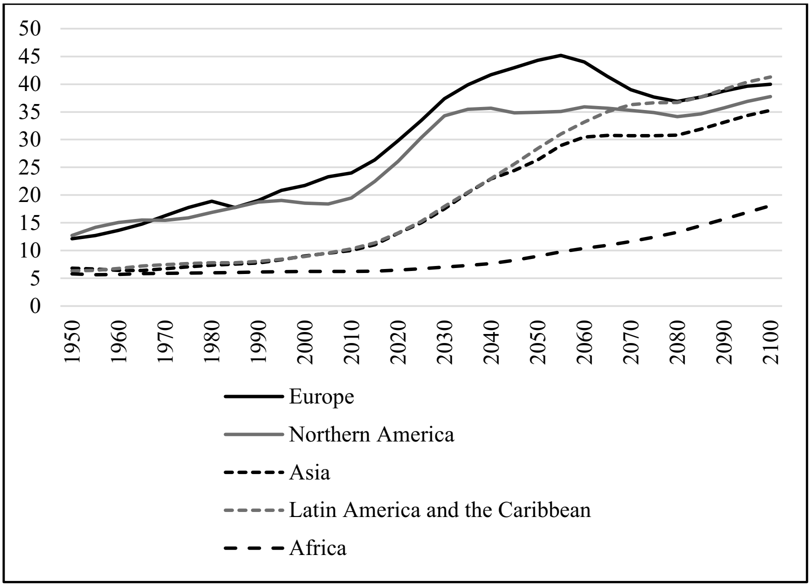

Figure B4.1: Past and predicted age dependency ratios (in percent) in different regions of the world. Source: United Nations (World Population Prospects 2015, high fertility variant).

IV.3 Government Spending, Budget Deficits, and the Current Account

IV.3.1 Motivation

We have shown in Chapter II that national savings have a private and a public component. After exclusively focusing on the determinants and consequences of private saving choices, we will now consider the implications of the government's decisions on spending and taxation. At the core of this analysis will be the question whether and how budget deficits affect a country's current account and net international investment position. Figure 4.6 highlights the importance of these considerations: it depicts the evolution of German public debt relative to GDP and shows that, in 2014, roughly 60 percent of that debt was held by foreign residents and institutions. To make the resulting payments on principal and interest, the government will have to tax domestic residents. By how much this will constrain future consumption possibilities depends on current saving decisions: if private savings increase, higher future taxation will be met by higher capital incomes, and private consumption will have to be lowered to a lesser extent. By contrast, if private savings do not adjust, today's budget deficits imply a reduction of future consumption.

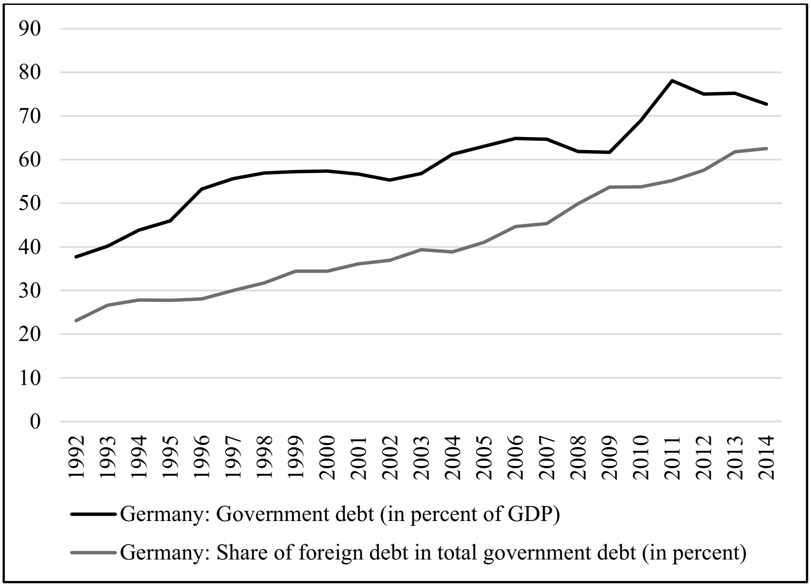

Figure 4.6: Government debt in Germany (relative to GDP, in percent) and the share of government debt held by foreign creditors (in percent). Source: Deutsche Bundesbank and German Ministry of Finance.

The reaction of private savings is also crucial when it comes to understanding the effects of budget deficits on the current account. As we have shown in Chapter II, it is the sum of public and private savings that determines the current account balance for a given level of aggregate investment. If a reduction of *public* savings triggers an increase of *private* savings, the effect of a budget deficit on the current account is muted.

To understand the relationship between budget deficits, private savings, and the current account, we will analyze the decisions of individuals who face a government that partially finances its expenditure by borrowing on the (international) capital market. As we will see, the macroeconomic consequences of a budget deficit crucially depend on whether private individuals understand and account for the government's intertemporal budget constraint when making their own savings decisions.

IV.3.2 Government Spending and Budget Deficits in the Representative-Consumer Model

We start our analysis by returning to the two-period representative consumer (RC) model that was introduced in Section III.2. The country we consider has a government that uses G_t units of the composite good in every period. For the time being, we assume that G_t is exogenous, and that all government spending is used for public consumption. A share of public expenditure is financed by raising taxes. If tax revenues T_t are greater than public spending G_t, we observe a ***primary budget surplus***. If they are smaller we observe a ***primary budget deficit*** – with the word "primary" indicating that interest payments on existing government debt are not included.

We assume that government spending potentially generates utility for the representative consumer. However, the utility function is characterized by additive separability, such that variations in G_t do not affect the marginal utility of private consumption. In period 1, RC maximizes

$$(4.10) \qquad U_1 = u\left(C_1\right) + v\left(G_1\right) + \beta\left[u\left(C_2\right) + v\left(G_2\right)\right]$$

In this expression $v\left(G_t\right)$ with $v' \geq 0$ represents the utility generated by government spending. Optimization takes place subject to the following constraints:

$$(4.11) \qquad B_{t+1}^{priv} = Y_t + \left(1+r\right)B_t^{priv} - T_t - C_t \qquad t = 1, 2$$

$$(4.12) \qquad B_1^{priv} = 0$$

$$(4.13) \qquad B_3^{priv} = 0$$

The subscript *priv* used in these equations characterizes assets and liabilities held by the *private sector*, which we will later distinguish from *public* assets and liabilities. The variable T_t denotes the taxes paid by RC in period *t*. We assume that these taxes are raised in a ***lump-sum*** fashion, i.e. taxation does not affect the incentives which determine individual choices, in particular on consumption and saving.

Combining (4.11) – (4.13) yields RC's intertemporal budget constraint:

(4.14) $$C_1 + \frac{C_2}{1+r} = Y_1 - T_1 + \frac{Y_2 - T_2}{1+r}$$

Hence, the present value of consumption has to equal the present value of income net of taxes. The solution of the optimization problem is characterized by the following intertemporal Euler equation:

(4.15) $$u'(C_1) = \beta(1+r)u'(C_2)$$

The variables that describe the government's decisions on taxation and spending do not enter this equation. This is due to the additive separability of preferences and to our assumption that the government raises lump-sum taxes. If the tax burden in period 2 depended on capital income in that period, this would affect the RC's incentive to save, which would be reflected by the intertemporal Euler equation.

As mentioned above, government spending and tax revenues do not have to be equal in a given period. If the government has access to a capital market, it can incur liabilities to finance budget deficits or purchase assets to store surpluses. We assume that the government faces the same real interest rate as RC. Moreover, we assume that public debt is zero at the start of period 1. Following the same logic as in Chapter III, we also assume that all assets are run down and all liabilities are served at the end of period 2. We therefore have:

(4.16) $$B_{t+1}^{pub} - B_t^{pub} = T_t - G_t + r\, B_t^{pub} \qquad t = 1, 2$$

(4.17) $$B_1^{pub} = 0$$

(4.18) $$B_3^{pub} = 0$$

The difference $B_{t+1}^{pub} - B_t^{pub}$ in equation (4.16) is the government's ***budget balance*** in period t. It is derived by combining the primary budget balance $T_t - G_t$ with the interest payments in period t. These payments are positive (negative) if the government's initial assets are greater (smaller) than its liabilities. If B_t^{pub} is negative, public spending G_t is augmented by interest payments $(r\, B_t^{pub})$. Equations (4.16) – (4.18) can be combined to get

(4.19) $$G_1 + \frac{G_2}{1+r} = T_1 + \frac{T_2}{1+r}$$

Like private agents, the government is subject to an intertemporal budget constraint: the present value of public spending must equal the present value of tax revenues. This condition is obviously satisfied if the government has a balanced budget in every period, i.e. if spending coincides with tax revenues ($G_t = T_t$). However, the intertemporal budget constraint is also compatible with the government running a deficit in period 1: in this case, the tax burden in period 2 is given by

$$(4.20) \qquad T_2 = G_2 + \left(1+r\right)\left(G_1 - T_1\right),$$

i.e. it incorporates the repayment of principal and interest on public debt. If we substitute equation (4.19) into (4.14) we get another version of RC's intertemporal budget constraint:

$$(4.21) \qquad C_1 + \frac{C_2}{1+r} = Y_1 + \frac{Y_2}{1+r} - G_1 - \frac{G_2}{1+r}$$

This expression suggests that the present value of consumption has to coincide with the present value of income minus the present value of government spending. Note that what is relevant is the present value of government expenditure, but not the distribution of taxation across periods. This, in turn, implies that it does not matter for RC's intertemporal budget constraint whether the government completely finances its spending out of current taxation in every period, or incurs a non-zero budget balance in period 1. Since taxation and public spending do not affect the intertemporal Euler equation either, the size of the government's budget deficit in period 1 apparently does not affect RC's optimal consumption path.

This is an important finding: while RC's optimal time path of consumption is influenced by the *present value* of government spending, it does not matter which share of public spending is financed by raising debt in financial markets. The economic intuition behind this principle, which is known as ***Ricardian equivalence***, is straightforward: as shown by equation (4.21), RC *internalizes* the government's intertemporal budget constraint – i.e. he knows that, for a given present value of public spending, lower taxation in the current period is associated with higher taxation in the future. To realize his optimal consumption path, he adjusts his savings. In the simplest constellation, higher private savings exactly compensate for reduced public savings.[11]

[11] The term ***Ricardian Equivalence*** goes back to a comment by Buchanan (1976) on a paper by Barro (1974). It refers to the classical economist David Ricardo (1772-1823), who questioned the notion that there is a difference between tax and credit financing of government expenditure.

How do government spending and budget deficits affect the current account? To answer this question, we use a simple example: we assume that government spending is non-negative in period 1 ($G_1 \geq 0$), but zero in period 2 ($G_2 = 0$). Moreover, we assume that $\beta(1+r)=1$, which implies that RC's optimal consumption path is characterized by a constant level of consumption. Finally, we assume that RC's income (Y_t) does not vary over time. If government spending were zero in period 1 ($G_1 = 0$), RC would obviously consume his income in every period, and the current account balance would be zero. If, however, $G_1 > 0$, we have to make an assumption on the government's financing decision. We assume that the government raises taxes to finance a share α of its spending in period 1, and that the rest is financed by borrowing on the capital market. Combining this assumption with the government's intertemporal budget constraint and our insight on RC's optimal consumption path ($C_1 = C_2$) yields:

$$(4.22) \qquad B_2^{priv} = \frac{1+r-\alpha(2+r)}{2+r} G_1$$

$$(4.23) \qquad B_2^{pub} = -(1-\alpha) G_1$$

$$(4.24) \qquad CA_1 = B_2^{priv} + B_2^{pub} = -\frac{G_1}{2+r}$$

The influence of α on RC's saving decisions is documented by equation (4.22). If this parameter is low, the future government debt and the anticipated tax burden in period 2 are high. This induces RC to accumulate assets in period 1, which will allow him to realize his optimal consumption path despite high future taxation. If, however, $\alpha > (1+r)/(2+r)$, RC incurs liabilities in period 1. Equation (4.24) shows that the effect of α on B_2^{priv} and B_2^{pub} cancels out once we consider *national* savings: while the size of the first-period budget deficit affects the *composition* of a country's external assets and liabilities at the start of period 2, it does not affect the size of the current account balance CA_1. Figure 4.7 illustrates these findings. The graph on the left-hand side sets $\alpha = 1$, implying a balanced budget. The tax acts like a temporary income drop in period 1, shifting the intertemporal budget line to the left. Since RC wants to implement a constant consumption level – recall that $\beta(1+r)=1$ – he borrows on the international capital market. The current account balance is given by the (negative) difference $Y_1 - G_1 - C_1^*$. The graph on the right-hand side sets $\alpha = 0$, i.e. all government spending in period 1 is debt-financed. In this case, the endowment point that is relevant for RC is no longer **A1** (as in the balanced-budget case) but **A2**, which reflects the fact that taxation takes place in period 2. However, this point is on the same intertemporal budget line as **A1** since the present value of RC's income after taxes has not changed. Optimal consumption is now associated with spending $Y_1 - C_1^*$ on assets. In Figure 4.7, this is reflected by the gray bar. To identify the current account balance, we have to combine (positive) private and (negative) public savings, which are represented by the black bar. The sum is $Y_1 - C_1^* - G_1$, i.e. exactly the same current account deficit that we derived for the case of a balanced budget.

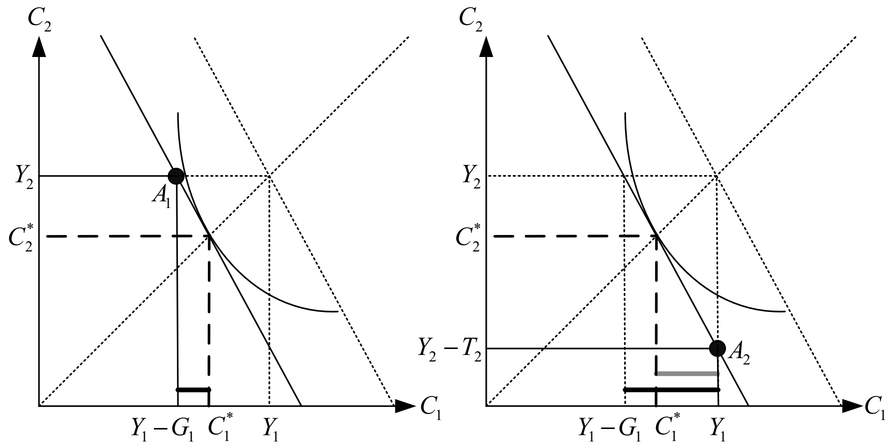

Figure 4.7: Ricardian Equivalence. In the plot on the left-hand side, the government finances first-period spending by raising taxes in that period (i.e. $\alpha = 1$). In the plot on the right-hand side, the government does not raise taxes in period 1 (i.e. $\alpha = 0$), but borrows the necessary amount on the capital market.

The irrelevance of budget deficits for aggregate consumption and the current account is not restricted to the two-period example. It can easily be shown that RC's intertemporal budget constraint in a model with investment and an infinite time horizon looks as follows:

$$(4.25) \qquad \left(1+r\right)B_t + \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}\left(Y_s - I_s - G_s\right) = \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} C_s$$

To derive equation (4.25), we use the laws of motion for private and public assets and liabilities (augmented by investment) and proceed as described in Section III.4. Moreover, we use a transversality condition that has the same interpretation as (3.65). Combining the private and the public intertemporal budget constraints yields (4.25). Private investment is implicitly determined by the condition in (3.39), which – given our assumption of lump-sum taxation – is independent of the government's decisions. Hence, with an infinite time horizon, RC's optimal consumption path and the current account depend on the present value of government spending, but not on its financing decision.

IV.3.3 Ricardian Equivalence: Critique

The theoretical result presented in the previous subsection is an important and prominent benchmark in the analysis of fiscal policy. However, there are numerous arguments that have been brought forward to question its practical relevance. In what follows, we will provide a short, non-exhaustive summary of this discussion:

Critique 1: Private individuals *do not completely internalize the government's budget constraint* when making their saving decisions. If, for example, the private time horizon is shorter than the government's time horizon, a reduced tax burden in the present period may raise some agents' wealth, since a part of the higher tax payments will be borne by future generations. This dampens the reaction of private savings. In the next subsection, we will provide a more detailed analysis of such a scenario.

Critique 2: Due to *frictions in financial markets*, the government may enjoy a higher creditworthiness than private individuals and thus pay a lower interest rate. In this case, the private budget constraint depends on the government's decision to finance its spending through taxes or debt, and it may actually raise social welfare if the government – instead of the private sector – increases its debt burden.

Critique 3: If the government *does not raise lump-sum taxes* but, e.g., a proportional tax on incomes, this induces an adjustment reaction by private individuals. More specifically, a budget deficit in the current period is likely to raise the tax on capital income in the future. This affects incentives to save and invest today, and thus potential output in the future. In this case, the specific combination of tax- and debt-finance affects the time path of production and income.

Apart from these theoretical objections, a first glance at the data raises some doubts about the empirical validity of Ricardian equivalence: Figure 4.8 depicts the relationship between the German budget balance – aggregated over all levels of government – and gross national savings. It would, of course, be naïve to interpret these observations as proof of a causal effect, but the strong correlation between the government's budget balance and national savings is definitely not compatible with the Ricardian idea that increases in the budget deficit are compensated by an increase of private savings, such that gross national savings remain constant.[12]

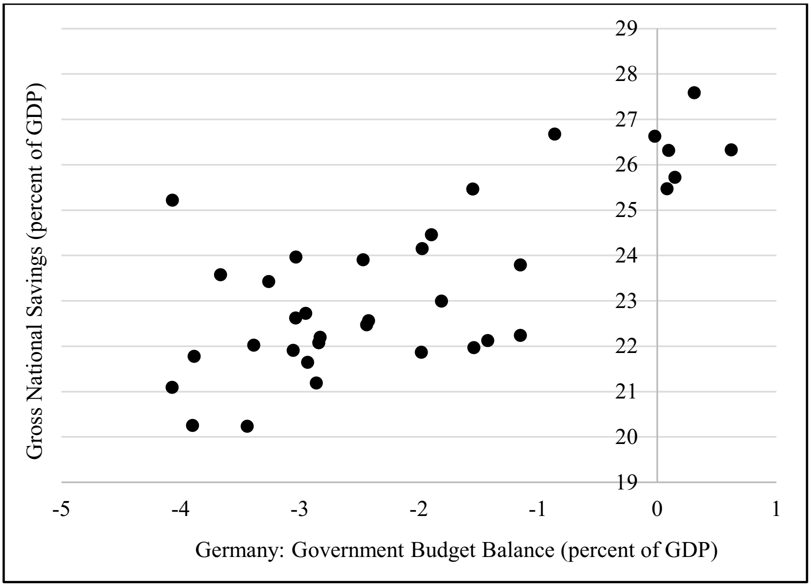

Figure 4.8: General government budget balance and gross national savings in Germany (as a percentage share of GDP), 1980 to 2014. The budget balances in 1995 and 2000 have been adjusted for one-time privatization proceeds and expenses. Source: IMF (World Economic Outlook database) and Sachverständigenrat (2015).

IV.3.4 Budget Deficits in an OLG Model

The idea that the time path of public deficits is irrelevant for the current account crucially depends on the assumption that private individuals have a time horizon which is at least as long as the government's. If this condition is not satisfied, the government's budget constraint is not completely internalized by the private sector, and the present value of taxes that is taken into account by individuals differs from the actual present value of government expenditure.

[12] The following quote suggests that David Ricardo himself seems to have been aware of these objections: "The people who pay taxes … do not manage their private affairs accordingly. We are apt to think that the war is burdensome only in proportion to what we are at the moment called to pay for it in taxes, without reflecting on the probable duration of such taxes." (David Ricardo, 1820, quoted from Elmendorf and Mankiw 1999).

We can use the OLG model introduced in Section IV.2 to analyze the effects of public spending and budget deficits in a framework in which private agents have a finite time horizon and face a government whose lifetime is potentially infinite. We simplify this model and assume that every individual lives for only two periods and that economic activities start in the first period of life. Population size is constant and normalized to one ($n=1$) for simplicity. In the first period of their lives, individuals receive a constant exogenous income y. They have access to an international capital market, which allows them to borrow and lend at an exogenous interest rate r. In every period t, the overall tax burden T_t is evenly split across the generations currently alive, i.e. both the "young" and the "old" individual have to pay taxes of $T_t/2$. Note that T_t is negative if the government pays a transfer to individuals. An individual that is born in period t maximizes the following objective function:

$$(4.26) \qquad U_t = \ln c_t^y + \beta \ln c_{t+1}^o$$

It takes into account the following constraints:

$$(4.27) \qquad c_t^y = y - \frac{T_t}{2} - b_{t+1}^o$$

$$(4.28) \qquad c_{t+1}^o = \left(1+r\right) b_{t+1}^o - \frac{T_{t+1}}{2}$$

Solving this problem yields

$$(4.29) \qquad b_{t+1}^o = \frac{\beta}{1+\beta}\left(y - \frac{T_t}{2} \right) + \frac{1}{\left(1+r\right)\left(1+\beta\right)} \frac{T_{t+1}}{2} = B_{t+1}^{priv}$$

The volume of assets accumulated in period t negatively depends on the tax burden of this period and positively on the taxes expected for the future. This reflects an individual's attempt to compensate for a temporarily higher tax in period t by borrowing on the international capital market. Conversely, a higher tax burden in period t+1 provides an incentive to increase savings to avoid a sharp drop of future consumption. Hence, at the individual level the very mechanisms that characterized the infinite-horizon RC model are still effective[13]. As

[13] The fact that b_{t+1}^o coincides with B_{t+1}^{priv} is due to our assumption that the population size is one.

before, the evolution of the difference between the government's assets and liabilities is described by

$$(4.30) \qquad B_{t+1}^{pub} = T_t - G_t + \left(1+r\right) B_t^{pub}$$

We can compute the current account balance by adding the change of private and public net assets. Using (4.29) and (4.30) as well as the fact that $B_{t+j} = B_{t+j}^{priv} + B_{t+j}^{pub}$ we get

$$(4.31) \qquad B_{t+1} - B_t = \frac{\left(T_{t+1} - T_t\right)}{2\left(1+\beta\right)\left(1+r\right)} - \frac{\beta\left(T_t - T_{t-1}\right)}{2\left(1+\beta\right)} + T_t - G_t + r\, B_t^{pub}$$

This expression illustrates how the time path of taxes and transfers influences consumption and savings of a generation: if a future tax increase is anticipated in period t $(T_{t+1} > T_t)$, the economy as a whole accumulates assets. If the tax burden in period t is greater than in the past $(T_t > T_{t-1})$, the current account balance drops. The reason is that individuals who anticipated higher future taxes in period $t-1$ and accumulated assets to smooth consumption, run down their assets in period *t*.

Using this framework, we now consider the following scenario:[14] public spending G_t is zero in all periods, and through period 2, the government does not collect any taxes from its citizens, nor does it distribute any transfers. Both private and public assets and liabilities are zero at the start of period 2, which implies that $B_2^{priv} = B_2^{pub} = 0$. In period 2, the government credibly announces that every individual will receive a transfer amounting to one unit of the good *in period 3*, i.e. $T_3 = -1$. This „gift“ is financed by raising public debt, which implies that $B_4^{pub} = -1$. In all subsequent periods, the government will not repay the principal on its debt, but raise taxes to make the necessary interest payments, i.e. $T_t = -r\, B_t^{pub}$ with $B_t^{pub} = -1$ for $t \geq 4$. For simplicity, we assume that $\beta\left(1+r\right) = 1$. It is easy to show that such a policy would have no effect on the current account in the *infinite-horizon RC model*: as of period 2, the present value of future transfers and taxes is given by

$$(4.32) \qquad \sum_{s=2}^{\infty} \left(\frac{1}{1+r}\right)^{s-2} T_s = -\frac{1}{1+r} + \left(\frac{1}{1+r}\right)^2 \sum_{s=4}^{\infty} \left(\frac{1}{1+r}\right)^{s-4} r = 0$$

Hence, the intertemporal budget constraint of an infinitely-lived individual would not be affected by the government's policy. As a result, the public budget

[14] This example has been suggested by Obstfeld and Rogoff (1996:138)

deficit in period 3 would be completely compensated by an increase in private savings. National savings would not change, and there would be no reason for fluctuations of the current account.

The reaction of an *OLG economy* to the government's transfer-cum-tax policy are completely different: the fact that the government incurs liabilities in period 3 and raises taxes in all future periods to make the necessary interest payments implies that government debt remains constant ($B_4^{pub} = B_5^{pub} = ... = -1$) and that tax payments amount to $T_4 = T_5 = ... = r$. Substituting these values into (4.31), we arrive at the time path of the current account that is depicted in Table 4.2.

t	T_t	B_t^{pub}	CA_t
2	0	0	$-\frac{1}{2(1+\beta)(1+r)}$
3	-1	0	$-\frac{1}{2}$
4	r	-1	$-\frac{\beta(1+r)}{2(1+\beta)}$
5	r	-1	0

Table 4.2: Taxes, transfers, public debt, and the current account in an OLG model.

The negative current account balance in period 2 may seem surprising at first glance, since the government does not raise any taxes nor does it pay any transfers in that period. However, the mere announcement of future transfer payments affects the young individual's saving decision: in anticipation of this payment, the individual borrows abroad to raise his consumption in period 2. The current account balance in period 3 is negative, too: the old generation repays its debt, and in anticipation of future tax payments the young generation increases its savings. But these private-sector decisions are dominated by the government's borrowing. The current account deficit in period 4, finally, is due to the old generation running down its assets. Starting in period 5, the model economy is in a steady state with all variables remaining at their constant levels.

This example illustrates how a budget deficit that would not affect the current account in the infinite-horizon RC model can result in a sequence of current account deficits in an OLG economy. The economic interpretation is straightforward: since individuals have a finite horizon and no bequest motive, they consume the government's transfer during their lifetime. The generation born in period 2 benefits from the transfer in the second period of its life. The following generation receives a transfer while young, but faces some – albeit smaller – tax payments in old age. All subsequent generations suffer from a higher tax burden. While the present value of the government's transfer-cum-tax policy is zero, this policy asymmetrically affects the intertemporal budget constraints of different generations. This ***intergenerational redistribution*** affects national savings and thus the current account.

IV.4 Limited Tradability, the Terms of Trade, and the Current Account

IV.4.1 Motivation

All the models we have considered so far were based on the assumption that individuals consume and firms produce a single good. Of course, we emphasized in Chapter III that this single good should be interpreted as a "goods basket" that consists of various components. We also stated that the construct of such a "composite" good is legitimate – and, indeed, quite practical – if neither the relative prices nor the expenditure shares of the basket's components vary over time.

However, practical as it may be, the single-good assumption downplays the importance of price fluctuations for individuals' choices. To meet this shortcoming, we will now split the goods bundle into various components. This will allow us to augment the *inter*temporal optimization problem considered so far by an *intra*temporal decision – i.e. we will consider a representative consumer who not only decides on his consumption level at every point in time, but also chooses how to allocate total consumption to different goods. Moreover, we will explore how an anticipated change of relative prices affects consumption, savings and the current account. As we will show, such a price change is relevant since it not only affects RC's intertemporal budget constraint, but also the ***effective interest rate*** that crucially determines saving decisions.

Of course, we will not be able to account for the entire goods spectrum faced by RC. Instead, we will focus on different goods aggregates, which can be distinguished with respect to certain basic characteristics. First, we will make a distinction between ***tradable*** and ***non-tradable goods***. In the second part of this subsection, we will then consider a situation where all goods are tradable, but where countries produce only a share of the entire goods spectrum. This setup will allow us to analyze how a variation of the ***terms of trade*** – i.e. of the relative price of exported goods – affects the current account. In the final part of this section, we will then assume that all goods are tradable, but that trade is associated with ***trade costs***.

IV.4.2 Non-Tradable Goods: Definition and Relevance

So far, we have assumed that all goods could be traded without costs, and that there were no administrative barriers which prevented international exchange. Given these assumptions, the ***law of one price (LOP)*** had to hold universally, i.e. the price of a good – expressed in a common currency – did not depend on the geographical location where it was supplied. Absent trade costs and trade barriers, the LOP can be justified by referring to the immense opportunities for ***international goods arbitrage*** that would result from cross-country price differences: goods could be bought where they are cheaper and sold where the price is higher. However, individuals' arbitrage activities would quickly make such price differences disappear: at the location where prices were initially lower (higher) the additional demand (supply) would result in a price increase (decline), and eventually all price differences would be eliminated.

The LOP may be an appropriate description of reality for some standardized goods that are easily transported. By contrast, for some goods and services the costs of moving the product to the customer (or vice versa) are so high that the arbitrage mechanism sketched above breaks down. While, in the real world, goods and services are probably characterized by different degrees of *tradability*, the model presented on the following pages will distinguish between goods that are *tradable without any costs* and goods that are completely *non-tradable*. Box 4.2 describes an attempt to operationalize this distinction.

What makes non-tradable goods special is that, even in a small open economy, changes in domestic demand and supply can affect relative prices. These price fluctuations, in turn, may bring about reactions that affect both the structure of consumption at every point in time and the optimal consumption path. In the following paragraphs, we will consider these processes in more detail.

Box 4.2: Assessing Tradability

In an important contribution, De Gregorio et al. (1994) distinguish between tradable and non-tradable goods by determining the share of domestic production that is exported. While the authors of that study consider the OECD average for the years 1970 to 1985, Figure B4.3 focuses on Germany and a more recent time span. The figure documents a striking difference between the "tradability" of manufactured goods (such as machinery or chemicals) and the low export share of services.

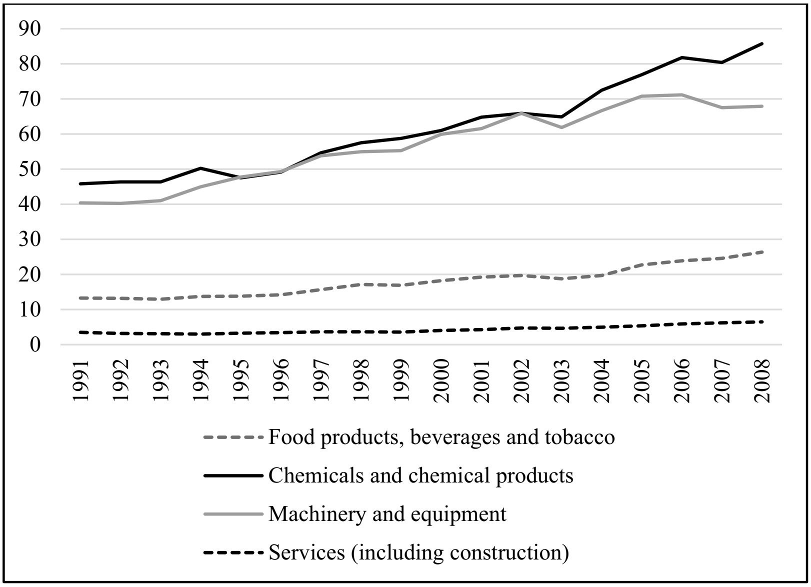

Figure B4.2: Exports relative to total production (in percent) for different industries in Germany. Source: OECD.

De Gregorio et al. (1994) distinguish between *tradable* and *non-tradable* goods by defining a critical export share of 10 percent. By comparing observed values to this threshold, they conclude that services are non-tradable. In fact, although the export share of services has almost doubled between 1991 and 2008, it still fails to cross the 10 percent threshold, and it is much lower than export shares for manufactured goods.

However, we should be aware that the share of exports in production depends as much on firms' and consumers' choices as on the specific properties of goods and services. Once ***trade cost***s start to decrease, the perspective of additional revenue for firms and the additional utility for consumers may dominate the additional costs associated with exporting and importing, and goods/services that used to be sold and purchased only on the domestic market will eventually become traded. For the time being we will stick to the tradables/non-tradables dichotomy. In a later part of this chapter, however, we will, once more, address the difference between (exogenous) "tradability" and (endogenous) "tradedness", and focus on the role of trade costs.

IV.4.3 Non-Tradable Goods in a Small Open Economy

When splitting the consumption bundle into a tradable and a non-tradable component we can, in principle, stick to the two-period framework introduced in Chapter II. However, we have to modify the model to account for the fact that RC not only chooses the optimal time path of consumption, but also decides on how to allocate total consumption to different goods categories.[15]

The utility derived from consuming a combination of the tradable and the non-tradable good in period t is denoted by $u\left(C_t\right)$. We denote the quantity of the tradable good consumed in period t by C_t^T and the quantity of the non-tradable good by C_t^N, and aggregate these quantities by using the following function:

$$(4.33) \qquad C_t = \left[\gamma^{\eta}\left(C_t^T\right)^{1-\eta} + \left(1-\gamma\right)^{\eta}\left(C_t^N\right)^{1-\eta}\right]^{\frac{1}{1-\eta}}$$

with $0<\gamma<1$ and $\eta>0$. The function in (4.33) is characterized by a ***constant elasticity of substitution (CES)*** amounting to $1/\eta$.[16] To demonstrate this and to illustrate the role of the elasticity of substitution, we start by computing the slope of an indifference curve in a diagram with C_t^T on its horizontal axis and C_t^N on its vertical axis. The absolute value of this slope is the ***marginal rate of substitution*** between C_t^N and C_t^T $\left(MRS_{N,T}\right)$:

$$(4.34) \qquad MRS_{N,T} = \left(\frac{\gamma}{1-\gamma}\right)^{\eta}\left(\frac{C_t^N}{C_t^T}\right)^{\eta}$$

Intuitively, $MRS_{N,T}$ indicates how many units of the non-tradable good we can take away from RC if we offer him an additional (marginal) unit of the tradable good and aim at keeping him on the same (period) utility level. Taking logarithms on both sides of equation (4.34), reshuffling terms, and taking a derivative with respect to $\ln MRS_{N,T}$ yields

[15] Of course, we did not deny the existence of such choices in all previous chapters. However, we neglected these static optimization problems since we assumed that neither preferences nor relative goods prices varied over time. The "single" good was therefore a stable bundle of different components.

[16] Note that, while this (***intratemporal***) ***elasticity of substitution*** affects the choice between different goods consumed in one period, the ***intertemporal elasticity of substitution*** determines how the time path of *total* consumption reacts to variations of the interest rate.

$$\frac{d\ln\left(\dfrac{C_t^N}{C_t^T}\right)}{d\ln MRS_{N,T}}=\frac{1}{\eta} \tag{4.35}$$

The left-hand side in (4.35) defines the elasticity of substitution. As we will show below, the consumer's optimal choice among different goods is characterized by an equality of $MRS_{N,T}$ and the relative price of the non-tradable good. Hence, the elasticity of substitution indicates how the structure of consumption reacts to price changes. In case of a CES function, this value is constant, i.e. it does not depend on the volume of overall consumption. If η approaches 0, the elasticity of substitution is infinite, and indifference curves representing these preferences are straight lines in C_t^T / C_t^N space. Conversely, if η becomes infinitely large, the elasticity of substitution approaches zero. In this case, the relationship at which C_t^T and C_t^N are consumed does not react to price changes.

The (intratemporal) elasticity of substitution $1/\eta$ thus plays the same role for the structure of consumption in a given period as the intertemporal elasticity of substitution $1/\sigma$ in the context of dynamic optimization. In fact, both aspects are relevant in a model with two goods and two periods: as we will show, RC reacts to price changes by varying the shares of individual components in his overall consumption bundle *and* by adjusting the total volume of consumption in a given period.

We assume that RC maximizes his lifetime utility by first deciding on how to allocate a given total consumption level in period *t* to different goods. He then chooses the optimal time path of consumption, i.e. C_1 and C_2. The ***intratemporal*** optimization problem can be described as follows: in period *t*, RC faces the (nominal) prices $P_t^{T,m}$ and $P_t^{N,m}$ for tradable and non-tradable goods, respectively, and chooses the quantities C_t^T and C_t^N to minimize the expenditure $P_t^{T,m}C_t^T + P_t^{N,m}C_t^N$ associated with a given overall consumption level C_t, as represented by (4.33).[17] In what follows, we will use the price of the tradable good as ***numéraire***, i.e. we set $P_t^{T,m}=1$ and all prices are expressed in units of the tradable good. As a consequence, $P_t^N \equiv P_t^{N,m} / P_t^{T,m}$ indicates the number of tradable good units that have to be paid to receive one unit of the non-tradable good in period t. In the appendix to this chapter, we show that optimal consumption in period t is characterized by the following condition:

[17] This problem amounts to deriving an ***expenditure function***, with the aggregator C_t assuming the role of the utility level. Hence, the demand functions derived will be ***conditional demand functions***.

(4.36) $$\frac{C_t^N}{C_t^T} = \frac{1-\gamma}{\gamma}\left(\frac{1}{P_t^N}\right)^{\frac{1}{\eta}}$$

Note that this expression is equivalent to $MRS_{N,T} = 1/P_t^N$, where the left-hand side is the marginal rate of substitution from (4.34). At the optimum, $MRS_{N,T}$ thus equals the (inverse) of the relative price of non-tradable goods. Substituting (4.36) into (4.33) and solving for C_t^T and C_t^N yields:[18]

(4.37) $$C_t^T = \left[\gamma^\eta + (1-\gamma)\gamma^{\eta-1}\left(P_t^N\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{1}{\eta-1}} C_t$$

(4.38) $$C_t^N = \left[(1-\gamma)^\eta + \gamma(1-\gamma)^{\eta-1}\left(P_t^N\right)^{\frac{1-\eta}{\eta}}\right]^{\frac{1}{\eta-1}} C_t$$

By multiplying C_t^T and C_t^N with the respective prices (taking into account that $P_t^{T,m} = 1$) and defining the value of total consumption, measured in tradable-goods units, as $P_t\, C_t = C_t^T + P_t^N\, C_t^N$, one arrives at

(4.39) $$P_t\, C_t = \left[\gamma + (1-\gamma)\left(P_t^N\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} C_t$$

The ***aggregate price level*** P_t represents the *minimal* expenditure (in units of the tradable good) that RC has to incur to acquire one unit of the consumption bundle C_t in period t. It follows from (4.39) that

(4.40) $$P_t = \left[\gamma + (1-\gamma)\left(P_t^N\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}$$

Variations of the overall price level P_t thus result from changes in the relative price of the non-tradable good.[19] By substituting the above expression into

[18] The appendix to this chapter presents a detailed derivation of this and subsequent results.

[19] Note the difference between this ***ideal price index*** and the ***consumer price index*** published by statistical authorities. While the former is based on RC's expenditure-minimizing choices, the latter is a weighted mean of individual goods prices, with the weights depending on observed consumption behavior.

(4.37) and (4.38), we arrive at a compact representation of the ***conditional demand*** for the tradable and the non-tradable good:

$$(4.41) \qquad C_t^T = \gamma \left(\frac{1}{P_t} \right)^{-\frac{1}{\eta}} C_t$$

$$(4.42) \qquad C_t^N = (1-\gamma) \left(\frac{P_t^N}{P_t} \right)^{-\frac{1}{\eta}} C_t$$

These equations show that, for a given total volume of consumption C_t, the demand for a good decreases if its price relative to the aggregate price level – $(1/P_t)$ and (P_t^N / P_t) respectively – increases. The strength of this reaction depends on the elasticity of substitution $1/\eta$, which determines the (constant!) ***price elasticity of demand***.

Having solved the *intratemporal* optimization problem, we can now turn to the ***optimal consumption path***. We assume that RC's lifetime utility is given by the following expression:

$$(4.43) \qquad U_1 = \frac{C_1^{1-\sigma} - 1}{1-\sigma} + \beta \frac{C_2^{1-\sigma} - 1}{1-\sigma}$$

RC maximizes this objective function subject to the following constraints:

$$(4.44) \qquad B_{t+1} = Y_t^T + P_t^N Y_t^N + (1+r) B_t - P_t C_t \qquad t = 1, 2$$

$$(4.45) \qquad B_1 = 0$$

$$(4.46) \qquad B_3 = 0$$

The expressions Y_t^T and Y_t^N denote the quantities of the tradable and the non-tradable good produced in period t. The sum $Y_t^T + P_t^N Y_t^N$ thus represents the income (in tradable-good units) that accrues to RC from selling the tradable and the non-tradable good. To facilitate matters, we assume that domestic production of both goods types is exogenous. The expression $P_t C_t$ reflects the value of RC's total consumption expenditures (in tradable-good units). Recall that this expression implies that a given spending level C_t is optimally allocated to the two goods categories.

The payments associated with international borrowing and lending are also defined in terms of the tradable good: an individual who lends B_2 units of the tradable good at the end of period 1 holds a claim to receive $(1+r)B_2$ units of that good at the end of period 2. Combining (4.44) to (4.46), we can derive the following intertemporal budget constraint

$$P_1 C_1 + \frac{P_2 C_2}{1+r} = Y_1^T + P_1^N Y_1^N + \frac{Y_2^T + P_2^N Y_2^N}{1+r}, \tag{4.47}$$

which requires the present value of total consumption (in tradable-good units) to equal the present value of total output. Formulating an appropriate Lagrange function and taking the derivative with respect to C_t yields the necessary condition that characterizes the optimal time path of consumption:

$$C_1^{-\sigma} = \beta(1+r)\left(\frac{P_1}{P_2}\right)C_2^{-\sigma} \tag{4.48}$$

The novel feature of this version of the intertemporal Euler equation is that it equates the intertemporal marginal rate of substitution not only to the world interest rate $(1+r)$, but also to the evolution of the aggregate price level. If RC anticipates that prices will *decrease* $(P_1 / P_2 > 1)$, this *raises* the ***effective (consumption-based) interest rate***: for every consumption unit saved today, RC is not only rewarded by receiving an interest r, but also by the fact that a given income can buy a higher amount of the consumption bundle in period 2 than in period 1. Conversely, the effective interest rate *decreases* if the aggregate price level is expected to *rise*.

Let us briefly take stock: we started by analyzing how much RC spends on the tradable and the non-tradable good, given the level of total consumption in period *t*. In a next step, we determined the optimal consumption path chosen by RC for a given real interest rate and a given evolution of prices. Using these results, we can now derive the current account balance. When doing this, we have to take into account that, by definition, a country's production and consumption of non-tradable goods have to coincide, i.e. $C_t^N = Y_t^N$ for $t = 1, 2$. This implies that we can simplify the intertemporal budget constraint in (4.47) as follows:

$$C_1^T + \frac{C_2^T}{1+r} = Y_1^T + \frac{Y_2^T}{1+r} \tag{4.49}$$

This equation has a straightforward interpretation: since all payments on the international capital market ultimately have to be made in terms of tradable goods, the scope for international borrowing and lending is constrained by the present value of *tradable*-good production.[20] Hence, the current account in period 1 reflects the difference between the production and the consumption of the tradable good:

$$CA_1 = Y_1^T - C_1^T \tag{4.50}$$

Substituting (4.41), (4.48) and (4.49) into (4.50), we get

$$CA_1 = Y_1^T - \frac{Y_1^T + \dfrac{Y_2^T}{1+r}}{1 + \beta^{\frac{1}{\sigma}} \left(1+r\right)^{\frac{1-\sigma}{\sigma}} \left(\dfrac{P_2}{P_1}\right)^{\frac{1}{\eta}-\frac{1}{\sigma}}} \tag{4.51}$$

In what follows, we will focus on the effects of a varying price level, which appears in the denominator of the ratio on the right-hand side, and whose effect apparently depends on the relationship between the *intertemporal* elasticity of substitution $\left(1/\sigma\right)$ and the *intratemporal* elasticity of substitution $\left(1/\eta\right)$. To interpret this finding, we start by analyzing the impact of varying $\left(P_2/P_1\right)$, setting $1/\eta$ equal to zero. By doing this, we exclude all intratemporal adjustment reactions and fix the proportions at which tradable and non-tradable goods are consumed in each period. It follows from (4.51) that, in this special case, consumption of the tradable good unambiguously increases if $\left(P_2/P_1\right)$ increases. A stronger price increase reduces the effective interest rate, and the reaction of consumption is the more pronounced the higher is the intertemporal elasticity of substitution $\left(1/\sigma\right)$: if RC is more willing to accept fluctuations in his consumption level, the decline in the effective interest rate induces a stronger increase of consumption in period 1, which, in turn, lowers the current account balance.

These processes, however, are dampened if the *intra*temporal elasticity of substitution $1/\eta$ is greater than zero. Recall that variations in the aggregate price level are driven by changes of the non-tradable good's price. If $1/\eta$ is strictly positive, RC reacts to an anticipated increase of P_t^N not only by adjusting his consumption path, but also by raising his consumption of the non-tradable good in the current period, thus reducing his demand for the tradable good.

[20] Why didn't we mention this when spelling out RC's intertemporal budget constraint in (4.47)? The reason is that RC does not internalize aggregate market clearing conditions when solving his own optimization problem(s). This may sound inconsistent, given that RC is the only agent whose decisions we consider explicitly. Remember, however, that the artefact of a "representative consumer" just stands in for a large number of individuals who make their decisions in a decentralized fashion, and who consider themselves too small to affect aggregate outcomes.

If the intratemporal elasticity of substitution $1/\eta$ is greater than $1/\sigma$, this effect is strong enough to shift the consumption reaction in period 1 towards the non-tradable good, such that consumption of the tradable good decreases. By illustrating that this has an effect on the current account in period 1, equation (4.51) conveys an important message: if an economy is subject to structural changes – e.g. an exogenous expansion of the sector that provides non-tradable goods – these changes affect the current account even if it seems at first glance as if we were dealing with a purely domestic affair that has no relevance for international transactions. The reason why this naïve view is wrong is that any variation in domestic supply and demand conditions results in price changes that affect the current account through their effects on intratemporal and intertemporal consumption choices.

IV.4.4 Economic Growth, Non-Tradable Goods, and the Current Account: An Example

The analysis of the last subsection has demonstrated that intratemporal price variations affect the effective interest rate, which, in turn, influences RC's consumption choices. However, we treated these variations as exogenous and did not link them to supply and demand conditions in the non-tradable good sector. In this subsection, we want to move one step further and endogenize the evolution of the aggregate price level by relating it to the evolution of goods supply. To keep the model reasonably tractable, we make the simplifying assumption that $\beta(1+r)=1$. As we have demonstrated in Section III.2.4, this implies that RC implements a constant consumption level if the price level does not change over time. Moreover, we assume that the intratemporal elasticity of substitution $1/\eta$ equals one. In this case, the aggregator in (4.33) turns into the following function:[21]

$$(4.52) \qquad C_t = \left(C_t^T\right)^{\gamma} \left(C_t^N\right)^{1-\gamma}$$

It follows from (4.42) that the demand for the non-tradable good is given by

$$(4.53) \qquad C_t^N = \frac{1-\gamma}{P_t^N} P_t C_t \qquad\qquad t = 1, 2$$

Hence, RC devotes a share $(1-\gamma)$ of his total consumption expenditures in period t to purchasing the non-tradable good. By substituting (4.36) for the case of $\eta = 1$ into (4.52) and following the steps that resulted in (4.39), we can show that the aggregate price level for $\eta = 1$ is given by

[21] The specifics of this transformation are shown in the appendix to this chapter.

(4.54) $$P_t = \left(\frac{1}{\gamma}\right)^{\gamma}\left(\frac{1}{1-\gamma}\right)^{1-\gamma}\left(P_t^N\right)^{1-\gamma}$$

By combining (4.53) and (4.54) with the intertemporal Euler equation in (4.48), we get

(4.55) $$\frac{C_2^N}{C_1^N} = \left(\frac{P_1}{P_2}\right)^{\frac{1-\sigma}{\sigma}}\left(\frac{P_1}{P_2}\right)^{\frac{1}{1-\gamma}}$$

To close the model, we use the fact that, in every period, supply and demand of the non-tradable good have to be equal, i.e. $C_t^N = Y_t^N$. By substituting this market equilibrium condition into (4.55), we arrive at the following expression:

(4.56) $$\frac{P_1}{P_2} = \left(\frac{Y_2^N}{Y_1^N}\right)^{\frac{(1-\gamma)\sigma}{1-\gamma+\gamma\sigma}}$$

This equation relates the evolution of the price level to the (exogenous) supply of the non-tradable good. Given the structure of the model, it is not surprising that an increasing supply $\left(Y_2^N > Y_1^N\right)$ results in a declining price index $\left(P_1 > P_2\right)$. By combining (4.51) and (4.56) and by setting $\eta = 1$, we can relate the current account balance in period 1 to the time path of tradable and non-tradable good output:

(4.57) $$CA_1 = Y_1^T - \frac{Y_1^T + \dfrac{Y_2^T}{1+r}}{1 + \left(\dfrac{1}{1+r}\right)\left(\dfrac{Y_2^N}{Y_1^N}\right)^{\frac{(1-\gamma)(1-\sigma)}{1-\gamma+\gamma\sigma}}}$$

We further simplify by assuming that $Y_1^T = Y_1^N = 1$ and $Y_2^T = Y_2^N = (1+g)$ with $g > 0$ – i.e., both sectors grow at an (exogenous) rate g. Substituting this information into (4.57), we derive an expression for the current account relative to the production of the tradable good in period 1:

(4.58) $$\frac{CA_1}{Y_1^T} = 1 - \frac{1 + \left(\dfrac{1+g}{1+r}\right)}{1 + \left(\dfrac{1}{1+r}\right)(1+g)^{\frac{(1-\gamma)(1-\sigma)}{1-\gamma+\gamma\sigma}}}$$

Once more, this expression highlights the role of non-tradable goods for the current account. If the goods bundle exclusively consisted of tradable goods – i.e. if $\gamma = 1$ – a higher growth rate g would unambiguously *reduce* the current account balance: anticipating a higher income in the future and aiming for a constant consumption level, RC would borrow in the first period and use his higher income to repay his debt in the second period.

If, by contrast, γ is smaller than one, we have to take into account that higher growth is associated with a growing supply and thus a declining price of the non-tradable good. The resulting *increase* of the effective interest rate reinforces the effect of economic growth if the intertemporal elasticity of substitution is low $(\sigma > 1)$. In this case, the perspective of receiving a high income in the future dominates, and consumption increases. Conversely, if $\sigma < 1$, the higher effective interest rate reinforces the incentive to save, reduces current consumption, and thus raises the current account balance.

These observations suggest some important conclusions: first, we have learned that gross savings and thus the current account not only depend on growth and interest rates, but also on the sectoral change taking place in an economy. Hence, the current account is also affected by developments that seem to be purely "domestic" – for example, supply and demand on the market for non-tradable goods. This, in turn, implies that the behavior of a small open economy may deviate substantially from the predictions of a model that ignores the existence of non-tradable goods. As the example in this subsection has demonstrated, an economy may exhibit a moderate current account deficit (or even a surplus) even if it experiences rapid economic growth. The important fact to see is that such a behavior need not be driven by constraints on borrowing, but by the anticipated decline of the aggregate price level – which, in turn, may raise aggregate savings.

IV.4.5 The Terms of Trade and the Current Account

While the preceding subsections split the consumption bundle into a tradable and a non-tradable component, we will now return to the assumption that all goods are tradable without costs. Note that, in the model of the previous subsection, this amounts to setting $\gamma = 1$. Instead of assuming that there is one tradable good, however, we consider a tradable goods *bundle* that consists of an *imported* good and an *exported* good. More specifically, we analyze a country that is specialized in the production of one good, but whose residents consume a bundle consisting of the domestic and an imported foreign good.[22] Using the

[22] The assumption that countries are specialized in a certain part of the goods spectrum, leaving the production of other goods to the rest of the world, goes back to Armington (1969)

index H to denote the good produced by the domestic economy and the index F to denote the good produced abroad, but otherwise sticking to the structure of the last subsection, we can write the consumption aggregator as follows:

(4.59) $$C_t = \left[\theta^{\nu}\left(C_t^H\right)^{1-\nu} + \left(1-\theta\right)^{\nu}\left(C_t^F\right)^{1-\nu}\right]^{\frac{1}{1-\nu}}$$

Here, θ is the relative weight of the domestic (traded) good in the domestic consumption aggregator and $1/\nu$ is the elasticity of substitution between domestic and foreign (traded) goods. Using the price of the foreign good as a numéraire, we proceed as in the derivation of (4.40) to compute the aggregate price index

(4.60) $$P_t = \left[\theta\left(\widetilde{Q}_t^{tot}\right)^{\frac{\nu-1}{\nu}} + \left(1-\theta\right)\right]^{\frac{\nu}{\nu-1}}$$

The ratio $\widetilde{Q}_t^{tot} = P_t^H \,/\, P_t^F$ represents the ***terms of trade*** – i.e. the price of the good produced (and exported) by the domestic economy relative to the price of the imported good.[23] The intertemporal budget constraint of the domestic economy is given by

(4.61) $$P_1\, C_1 + \frac{P_2 C_2}{1+r} = \widetilde{Q}_1^{tot}\; Y_1^H + \frac{\widetilde{Q}_2^{tot}\; Y_2^H}{1+r}$$

The consumption possibilities of RC thus depend on the production of the domestic good – which we assume to be exogenous – and the relative price of this production, i.e. the terms of trade. The higher $\widetilde{Q}_t^{tot}$, the higher the value of domestic output measured in units of the foreign good.

We assume that RC's lifetime utility is given by $U_1 = \ln C_1 + \beta \ln C_2$, which implies that the intertemporal elasticity of substitution σ equals one. By combining the intertemporal Euler equation (4.48) for the special case of $\sigma = 1$ with

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and is often referred to as the ***Armington assumption***. Note that this approach is in stark contrast to models of international trade which *endogenize* an economy's production and trade pattern.

[23] Note that we assume that both prices are expressed in a common currency, and that the law of one price (LOP) holds for both goods, i.e. there are no international price differences. For ease of exposition, we follow the standard convention according to which an increase of $\widetilde{Q}_t^{tot}$ represents an "improvement" of the terms of trade. In later chapters, we will deviate from this convention and define the (inverse) terms of trade as $Q_t^{tot} = 1/\widetilde{Q}_t^{tot} = P_t^F / P_t^H$, and we will allow prices of identical goods to differ across countries.

the intertemporal budget constraint in (4.61), we arrive at the following expression:

$$(4.62) \qquad P_1 C_1 = \frac{1}{1+\beta}\left(\widetilde{Q}_1^{tot}\ Y_1^H + \frac{\widetilde{Q}_2^{tot}\ Y_2^H}{1+r} \right)$$

We can write the current account balance in period 1 as the difference between RC's income and his consumption in that period. The current account balance *relative to GDP* – both expressed in units of the foreign good – is thus given by

$$(4.63) \qquad \frac{CA_1}{\widetilde{Q}_1^{tot}\ Y_1^H} = \frac{1}{1+\beta}\left(\beta - \frac{1}{1+r}\frac{\widetilde{Q}_2^{tot}\ Y_2^H}{\widetilde{Q}_1^{tot}\ Y_1^H} \right)$$

This result has a straightforward interpretation: a low real interest rate and a high growth rate of output reduce the current account balance (relative to GDP) in the first period. In this case, RC has an incentive to implement his optimal consumption path by borrowing against future income. In addition, the current account balance in period 1 depends on the evolution of the terms of trade: if the relative price $\widetilde{Q}_t^{tot}$ increases between periods 1 and 2, this raises the value of RC's future income and thus his consumption possibilities. As a consequence, the current account balance decreases. Conversely, if RC anticipates declining terms of trade – i.e. if $\widetilde{Q}_2^{tot} < \widetilde{Q}_1^{tot}$ – this raises the current account balance of the first period.[24]

IV.4.6 Trade Costs, the Effective Interest Rate, and the Current Account

The first sections of this chapter were based on the notion that there is a clear-cut difference between tradable and non-tradable goods. At a closer look, however, this distinction becomes questionable: while there may be some goods for which the LOP holds without reservations and others whose physical properties render international transactions utterly impossible, these are extreme cases which should not mask the fact that, whether a good is traded or not, depends both on exogenous circumstances and agents' optimizing choices. The outcome of these choices may vary as exogenous parameters change, and firms may decide to serve international markets that they have not served before if the additional revenues exceed the additional costs.

[24] Blanchard and Giavazzi (2002), whose contribution served as a basis for this subsection, endogenize the evolution of the terms of trade by relating them to the growth of domestic and foreign output and the resulting interaction of goods demand and supply.

An approach that gives justice to these considerations is to replace the tradables/non-tradables dichotomy by a framework in which all goods are tradable, but trade is associated with ***trade costs***. In an influential contribution, Obstfeld and Rogoff (2000a) explore the macroeconomic implications of such an approach. In what follows, we will adopt their framework to show how accounting for trade costs influences our view on the current account.

As in the last subsection, we assume that the intratemporal elasticity of substitution $(1/\nu)$ equals one. Domestic individuals consume two tradable goods (*H* and *F*), but the domestic economy is specialized in the production of good *H*. Unlike in the previous subsection, we allow good *H* to be produced in other countries as well. The consumption aggregator is given by

(4.64) $$C_t = \left(C_t^H\right)^{\theta} \left(C_t^F\right)^{1-\theta}$$

Unlike in the previous subsection, LOP does not necessarily hold. This is because trade costs possibly drive a wedge between the prices at which identical goods are sold in different locations. We thus define P_t^{HH} as the price at which good *H* trades in country *H*, and P_t^{HF} as the price at which it trades in country *F* – which, for ease of exposition, we assume to be the „rest of the world“. Assuming that good *F* trades at no cost and setting $P_t^{FH} = P_t^{FF} = 1$, we can write the domestic price index as

(4.65) $$P_t = \left(\frac{1}{\theta}\right)^{\theta} \left(\frac{1}{1-\theta}\right)^{1-\theta} \left(P_t^{HH}\right)^{\theta}$$

The price at which good *H* trades abroad, i.e. P_t^{HF}, is given, and we also set it equal to one. The price paid by domestic consumers depends on whether the good is imported or exported. If the good is *exported*, domestic producers have to take into account that there are trade costs which we express as a share τ of the good's original price. Due to these costs, the price which has to be paid by foreign customers increases by a factor $(1+\tau)$.[25] To remain competitive vis-à-vis foreign firms, domestic exporters can only charge a price $1/(1+\tau)$ on every goods unit that they send abroad. Under perfect competition, this is also the price that is charged on domestic consumers, hence

(4.66) $$P_t^{HH} = 1/(1+\tau)$$

[25] Obstfeld and Rogoff model trade costs as ***iceberg costs***, i.e. only a share $(1-\tau)$ of every goods unit sent abroad reaches its destination. While the wedge between domestic and foreign prices looks somewhat different under this specification, the qualitative results are identical.

If, by contrast, consumption of good H exceeds production such that it has to be imported, the domestic price is given by

$$(4.67) \qquad P_t^{HH} = (1+\tau)$$

In this case, domestic producers can charge a higher price on their domestic customers since the price of imports is raised by trade costs.

As documented by the intertemporal Euler equation (4.48), the consumption path of RC depends on the effective interest rate which, in turn, depends on the exogenous world interest rate and on the evolution of the price level. Using (4.65), we get

$$(4.68) \qquad 1+r^{eff} = (1+r)\left(\frac{P_1^{HH}}{P_2^{HH}}\right)^{\theta}$$

To demonstrate the role of trade costs in this context, we consider two scenarios: in the first scenario, domestic production of good H in period 1 is much higher than in period 2, such that it is optimal for RC to export the good in period 1 and to import it in period 2. This implies that P_1^{HH} is given by (4.66) while (4.67) holds for P_2^{HH}. We can thus derive the effective interest rate for the case of a current-account surplus:

$$(4.69) \qquad 1+r_{CA_1>0}^{eff} = (1+r)(1+\tau)^{-2\theta}$$

In this case, the effective interest rate is *below* the world market level. The reason is that trade costs *reduce* the domestic price P_t^{HH} in the first period, while they *raise* it in the second period, and the anticipated price increase enters the effective interest rate. If the intertemporal elasticity of substitution is high, this induces RC to raise total consumption in period 1, such that the current account surplus is lower than in the case of costless trade.

In the second scenario, production of good H increases substantially over time, such that it is *imported* in the first period and *exported* in the second period. In this case of a current-account deficit, the effective interest rate is given by

$$(4.70) \qquad 1+r_{CA_1<0}^{eff} = (1+r)(1+\tau)^{2\theta}$$

which is higher than the world market interest rate. As a consequence, the incentive to save increases, and the current account deficit is dampened.

This is the interesting finding of Obstfeld and Rogoff (2000a): whenever RC tends to run a current account surplus, trade costs *reduce* the effective interest rate, while they *raise* the effective interest rate whenever RC's optimal consumption path would generate a current account deficit. Hence, a positive value of τ reduces both current account surpluses and current account deficits compared to a scenario without trade costs. Based on this observation, Obstfeld and Rogoff argue that the explicit consideration of trade costs can contribute to solving some open questions in international macroeconomics – e.g., the low volume of intertemporal trade, which is at the heart of the "Feldstein-Horioka puzzle" (Box 3.6). Box 4.3 reports the results of two studies that estimate the real-world counterpart of τ.

Box 4.3: How Large are International Trade Costs?

In reality, few goods and services are completely non-tradable, but the assumption of perfectly costless trade seems equally implausible. Instead, the intensity of international goods arbitrage is likely to hinge on the size of trade costs – i.e. the parameter τ introduced above – for individual goods categories. In an important article published in 2004, James Anderson and Eric van Wincoop summarize numerous studies on this topic and present the interesting result that, for industrialized countries, international trade costs amount to 74 percent on average, such that $1+\tau=1.74$ would be a plausible value. As detailed by Anderson and van Wincoop (2004), these costs can be split into pure costs of transportation (21 percent) and costs that are associated with crossing a border (44 percent). The latter, in turn, result from formal trade impediments like tariffs and quantitative restrictions (8 percent), language barriers (7 percent), the use of different currencies (14 percent), information barriers (6 percent), as well as contracting costs (3 percent). Combining these percentages by computing $1.21\cdot 1.08\cdot 1.07\cdot 1.14\cdot 1.06\cdot 1.03=1.21\cdot 1.44=1.74$, we arrive at the 74-percent estimate mentioned above.

While the number presented by Anderson and van Wincoop (2004) gives a cross-country average based on data from various studies and years, it does not reveal how trade costs differ across countries and time periods. In a more recent study, Dennis Novy (2013) provides such information by calculating trade costs on the basis of observed trade flows. Starting from a model that predicts the volume of bilateral exports for a given size of trade costs, he infers these costs by comparing countries' intranational and international trade. While this comprehensive measure of trade costs does not provide any information on the individual components mentioned above (transportation, tariffs, etc.), it offers the advantage of being available for different countries and years. Figure B4.3 illustrates the evolution of country-specific trade costs averaged over the respective countries' trading partners. Apparently, the cross-country differences are considerable. Moreover, the figure documents that trade costs have decreased substantially over the past decades.

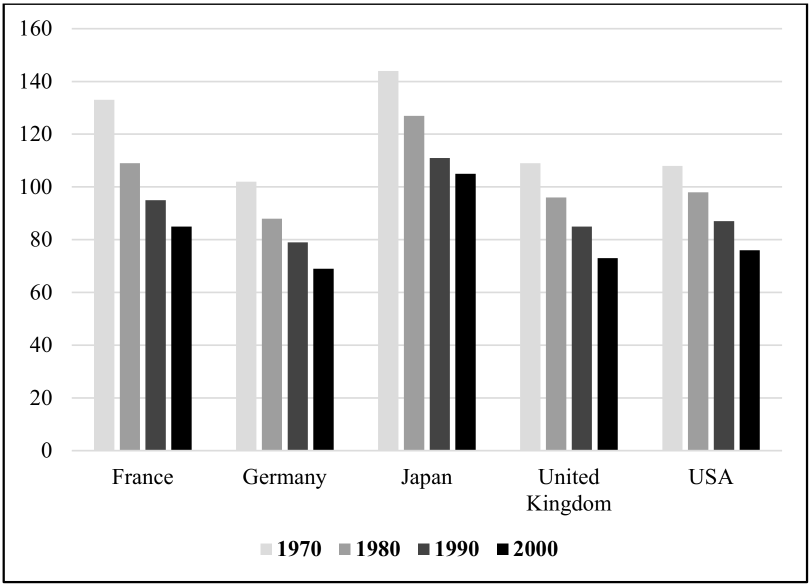

Figure B4.3: Country-specific trade costs τ_i, averaged over countries' trading partners (in percent). Source: Novy (2013).

Some further comments are in place: first, our focus on *international* trade costs should not suggest that *intra*national trade is costless. In fact, Anderson and van Wincoop (2004) provide an estimate of 55 percent for local distribution costs. The numbers provided by Novy (2013) should thus be interpreted as showing by how much the costs of international trade *exceed* the costs of intranational trade. Second, trade costs differ substantially across goods, such that the cross-country differences identified by Novy (2013) are likely to be driven by countries' geographical location and institutions, but also by the specific mix of goods and services these countries produce. Finally, while we have modelled trade costs as variable costs that depend on the volume of exports, we have to be aware that, regardless of the amount of transactions, entering foreign markets is also associated with *fixed costs* that have to be spent on market research, creating a customer network, etc. In an important strand of literature that goes back to the seminal contribution by Marc Melitz (2003), these fixed costs are used to explain why, in most industries, only a small share of firms actually export their products. In another important paper, Fabio Ghironi and Marc Melitz (2005) explore the macroeconomic implications of ***heterogeneous firms*** and trade costs.

IV.5 Uncertainty and International Diversification

IV.5.1 Motivation

All the models that we have presented so far were based on the perfect-foresight assumption – i.e. the notion that individuals and firms, when solving their dynamic optimization problems, could perfectly anticipate the future. There was no uncertainty, either with respect to future income levels or to future interest rates or productivity parameters. Considering the numerous imponderabilia and surprises that individuals face in real life, it goes without saying that such an assumption is not very plausible. This raises the question about how our previous insights change if we explicitly account for the possibility that individuals face uncertainty and that they have to base their decisions on ***expectations*** about the future.

As we will see, the standard model has to be only slightly modified as long as we stick to the assumption that the only security traded on the international capital market is a bond with a *fixed* return. We will show that, in this case, it is quite likely that greater uncertainty has a positive effect on national savings, since individuals react to greater risk by raising their ***precautionary savings***. By contrast, the model changes in a fundamental way if we explicitly allow for securities that are associated with *state-contingent payments*. In this case, there is a novel motive for international financial flows, which augments the two motives emphasized so far (consumption smoothing and the chase for higher returns on investment). For if income fluctuations are not perfectly correlated across countries, there is an incentive to reduce the volatility of consumption *ex-ante* by using financial markets for ***international diversification***. To demonstrate this, we will explicitly model individuals' portfolio decisions, we will determine the prices of the securities traded, and analyze the consequences of optimal portfolio choices for the volatility and international correlation of national consumption levels.

IV.5.2 Decisions under Uncertainty: A Brief Review

We start by using a simple example in order to review some fundamental definitions and concepts, which are relevant for the analysis of decisions under uncertainty: an individual with a time horizon of two periods receives an income Y_1 at the end of the first period and saves an amount S_1. The individual's second-period income is zero, i.e. $Y_2 = 0$, and at the end of that period, he consumes his entire savings, including the returns. The crucial assumption we make is that the return $\tilde{r}$ on savings is uncertain, and that the individual merely knows the probability distribution of $\tilde{r}$. This implies that – for a given volume of savings S_1 – consumption in period 2 is uncertain. Instead of standard utility maximization we thus have to consider the maximization of ***expected utility***:

$$\text{(4.71)} \quad \mathrm{E}(U_1) = u(Y_1 - S_1) + \beta\, \mathrm{E}\{u[(1+\tilde{r})S_1]\}$$

with $\mathrm{E}(x)$ denoting the ***expected value*** of a variable x. For a given saving decision, the individual is facing a ***lottery*** in period 2, whose payoff $(1+\tilde{r})S_1$ is a random variable due to the uncertain return. If the function u is concave – i.e. if $u'' < 0$ – the individual is ***risk-averse***. It follows from ***Jensen's inequality*** that the expected utility generated by such a lottery is smaller than the utility generated by the expected payoff:[26]

$$\text{(4.72)} \qquad \mathrm{E}\{u[(1+\tilde{r})S_1]\} < u\{\mathrm{E}[(1+\tilde{r})S_1]\}$$

If, by contrast, the function u is linear, the individual is ***risk-neutral***, and (4.72) turns into an equation.

We define the ***risk premium*** of a lottery as the sum that a risk-averse individual is willing to pay to avoid the uncertainty associated with that lottery.[27] In our specific example, the *relative* risk premium ϕ is implicitly defined by the following expression:

$$\text{(4.73)} \qquad \mathrm{E}\{u[(1+\tilde{r})S_1]\} = u\{(1-\phi)\mathrm{E}[(1+\tilde{r})S_1]\}$$

[26] Jensen's inequality makes the following statement about a concave function $f(x)$ and two values x_1 and x_2: $\alpha f(x_1) + (1-\alpha) f(x_2) < f[\alpha x_1 + (1-\alpha)x_2]$ with $0 < \alpha < 1$. Since we compute an expected value by adding the realized utility levels, multiplied with the respective probabilities, we can apply this result.

[27] Alternatively, we can define the risk premium as the sum that has to be offered to the individual to make him accept the lottery. Both definitions generate the same result.

In general, ϕ depends on the properties of the function u and on the distribution of $\left(1+\tilde{r}\right)$. Assuming, for simplicity, that $\mathrm{E}\left(1+\tilde{r}\right)=1$, we can derive the following approximation for ϕ:[28]

$$\text{(4.74)} \qquad \phi \approx \frac{-u''\left(S_1\right)S_1}{u'\left(S_1\right)}\frac{Var\left(\tilde{r}\right)}{2}$$

The first ratio on the right-hand side is the ***coefficient of relative risk aversion***.[29] The higher this coefficient, the higher the risk premium an individual would pay to replace the lottery by a certain payoff. The second expression, $Var\left(\tilde{r}\right)$, is the ***variance*** of the return on savings. A high variance reflects a high spread of potential returns and thus a high degree of uncertainty faced by the individual. Obviously, $\phi=0$ if $Var\left(\tilde{r}\right)=0$, i.e. if the return on savings is certain.

Recall the constant intertemporal elasticity of substitution (CIES) utility function introduced in Chapter III:

$$\text{(4.75)} \qquad U_1=\frac{C_1^{1-\sigma}-1}{1-\sigma}+\beta\frac{C_2^{1-\sigma}-1}{1-\sigma},$$

Computing the coefficient of relative risk aversion for this function yields

$$\text{(4.76)} \qquad \frac{-u''\left(S_1\right)S_1}{u'\left(S_1\right)}=\sigma$$

Hence, this function is characterized by a ***constant relative risk aversion, (CRRA)***: the individual's willingness to accept (multiplicative) risks does not depend on his savings. Moreover, the relative risk aversion equals the inverse of the intertemporal elasticity of substitution! This relationship between the IES and the coefficient of relative risk aversion is an important and somewhat problematic property of this class of utility functions. While it is plausible at first glance that risk-averse individuals try to avoid large consumption fluctuations, these are nevertheless two different aspects of preferences, and it may be im-

[28] As detailed by Laffont (1993), deriving this expression involves taking a second-order approximation of $u\left[\left(1+\tilde{r}\right)S_1\right]$ in $\tilde{r}_0=0$ and a first-order approximation of the right-hand side in $\phi_0=0$.

[29] Since $u'>0$ and $u''<0$ this coefficient is always positive.

portant to distinguish the attitude towards consumption volatility from an individual's willingness to accept risk. Box 4.4 presents a utility function that allows for such a distinction.

Box 4.4: Separating the Intertemporal Elasticity of Substitution from Risk Aversion

In order to specify preferences that allow separating the intertemporal elasticity of substitution from the relative risk aversion we can proceed as follows: in a first step, we determine the ***certainty equivalent*** of uncertain future consumption – i.e. the consumption level that generates the same utility as the lottery faced by the individual. Of course, the size of this certainty equivalent depends on the individual's risk aversion. In a second step, the certainty equivalent replaces the (random) second-period consumption in the intertemporal utility function. How an individual allocates his lifetime income to the (certain) consumption in period 1 and the certainty equivalent of consumption in period 2 depends on the intertemporal elasticity of substitution. This principle was initially outlined by Kreps and Porteus (1978) and is reflected by the following utility function suggested by Epstein and Zin (1989):

$$U_1 = \frac{C_1^{1-\sigma}}{1-\sigma} + \beta \frac{\left[(1-\gamma) \mathrm{E}\left(\frac{C_2^{1-\gamma}}{1-\gamma} \right) \right]^{\frac{1-\sigma}{1-\gamma}}}{1-\sigma}$$

In this function, $1/\sigma$ is the intertemporal elasticity of substitution, and γ is the coefficient of relative risk aversion. If $\gamma = \sigma$, this expression coincides with the CIES/CRRA utility function of (4.75) plus a constant. However, the above specification also offers the possibility to choose separate values for the two parameters. Gollier (2002) discusses the properties of this utility function.

IV.5.3 Income Uncertainty and Savings

In the two-period model of a small open RC economy with an exogenous income, there are two parameters which can be uncertain at the start of period 1: the gross return on the net international investment position and the income in period 2. However, if we assume that the only security traded on the international capital market is a bond with a fixed real interest rate, and that debtors always honor their repayment obligations at the end of period 2, the variable r can be predicted with certainty, and the only random variable is the second-period income. In period 1, RC chooses his consumption to maximize the following function:

$$\mathrm{E}(U_1) = u(C_1) + \beta\,\mathrm{E}[u(C_2)] \tag{4.77}$$

RC takes into account the following constraints:

$$B_{t+1} = Y_t + (1+r)B_t - C_t \qquad t = 1, 2\,. \tag{4.78}$$

$$B_1 = 0 \tag{4.79}$$

$$B_3 = 0 \tag{4.80}$$

As in Chapter III, these expressions allow the derivation of the intertemporal budget constraint. Note, however, that (4.78) has to be satisfied under all circumstances – i.e. if RC chooses a negative value of B_2, he has to make sure that he can honor his repayment obligations for all realizations of Y_2 without forcing his consumption to be negative. The solution of the optimization problem is characterized by the following condition:

$$u'(C_1) = \beta(1+r)\,\mathrm{E}[u'(C_2)] \tag{4.81}$$

The crucial difference between this expression and the intertemporal Euler equation in (3.14) is that the right-hand side shows the *expected* marginal utility of consumption in period 2. If the future income is certain, (4.81) coincides with (3.14). By contrast, if Y_2 is uncertain, this potentially affects RC's consumption and saving choices.

In order to further explore the effects of uncertainty on saving decisions, we use the following simple example: we assume that RC's period utility function is logarithmic – i.e. it is characterized by a constant intertemporal elasticity of substitution and relative risk aversion of one. For simplicity, we assume that $\beta(1+r) = 1$, which implies that, under certainty, RC would choose a constant level of consumption. We assume that the first-period income is given by $Y_1 = Y$. The income in period 2 is possibly uncertain and given by

$$Y_2 = \begin{cases} Y + \Delta & \text{with probability } \pi = 0.5 \\ Y - \Delta & \text{with probability } (1-\pi) = 0.5 \end{cases} \tag{4.82}$$

It is easy to show that, by raising Δ, we can model an increasing variance of second-period income (Δ^2) without changing the expected value of Y_2, which is $\mathrm{E}\left(Y_2\right)=Y$. That is, raising Δ is associated with a ***mean-preserving increase in risk***. For this particular example, the intertemporal Euler equation (4.81) is given by:

$$\text{(4.83)} \qquad \frac{1}{Y-B_2}=\frac{0.5}{Y+\Delta+\left(1+r\right)B_2}+\frac{0.5}{Y-\Delta+\left(1+r\right)B_2}$$

The expression on the left-hand side is obviously increasing in B_2 and drawn as the curve LHS in Figure 4.9. The curve RHS $\left(\Delta=0\right)$ reflects the term on the right-hand side for the case without uncertainty. In this case, RC's income does not vary over time, and it is optimal to consume this income in every period – which implies that the current account balance $CA_1=B_2$ is zero. It is easy to show that raising Δ shifts RHS upward. This results in a new point of intersection of the two curves, which is associated with a positive value of B_2. Hence, a mean-preserving increase in income risk raises savings. For a given value Y_1 this generates a current account surplus in period 1.

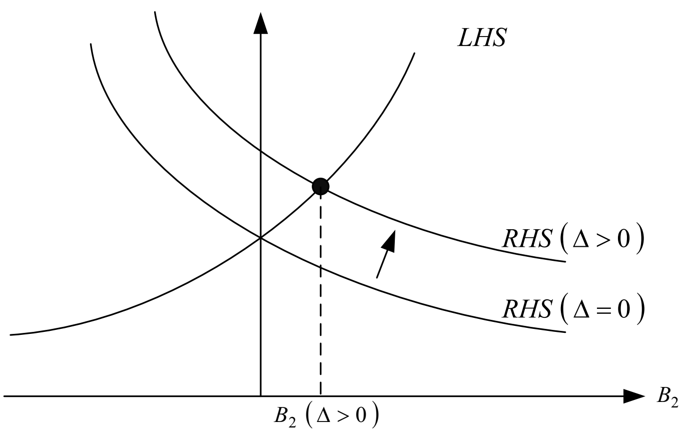

Figure 4.9: The effect of higher income uncertainty on savings and the current account.

The positive effect of uncertainty on ***precautionary savings*** is more general than our specific example. It can be shown that such a relationship holds if marginal utility is convex in consumption, i.e. if $u'''>0$. The role of this condition, which is satisfied for most conventional utility functions, is intuitive: if marginal utility is convex, an increase of uncertainty raises the expression on the right-hand side of the intertemporal Euler equation (4.81). This, in turn, implies that the marginal utility of first-period consumption has to increase as well. For a given income in period 1, this is achieved by reducing consumption and raising savings.[30]

There are several ways to deal with the fact that RC's saving behavior is likely to be affected by uncertainty. The first approach formed the basis of our analysis in Chapter III and the first sections of Chapter IV. The models presented in these sections simply *neglected* the role of uncertainty. Such an approach may be justified if one is willing to assume that the variance of potential shocks is not very large, and that the extent of precautionary savings is rather small. Still, it is hard to explain how the assumption of "perfect foresight" should be compatible with the existence of surprising income changes – as, e.g. in the example we presented in Box 3.10.

The second way to account for the existence of uncertainty is to specify RC's preferences in a fashion that allows dropping the perfect-foresight assumption, but in such a way that uncertainty *per se* does not affect his choices. A functional form that meets this requirement is given by

$$(4.84) \qquad u(C_t) = C_t - \frac{a}{2} C_t^2$$

For such a *quadratic* utility function, the individual is risk-averse, but his *marginal utility* is linear. As a consequence, the right-hand side of the intertemporal Euler equation in (4.81) shows expected future consumption instead of the expected value of a *nonlinear* function of consumption. The variance of uncertain variables (as well as their higher moments) are thus irrelevant, and individuals behave as if they had perfect foresight – with all uncertain future variables being replaced by their expected values. In this case, RC's choices follow the principle of ***certainty equivalence***.

The third – and most convincing – alternative is to use a utility function whose first derivative is not linear in consumption and to acknowledge that consumption and saving choices substantially differ from the perfect-foresight case. If one adopts this approach it is, however, necessary to define exactly the set of securities that are available to individuals. So far, we have assumed that transactions on the international capital market are restricted to buying and selling risk-free bonds. If we add securities that are associated with ***state-contingent payments***, this changes the optimization problem of RC, since the interna-

[30] Kimball (1990) suggests to call the ratio $-C \cdot u'''(C)/u''(C)$ the ***coefficient of relative prudence***.

tional capital market can be used to reduce consumption volatility via international diversification. In the following subsections, we will analyze this possibility in some detail.

IV.5.4 International Diversification: Model Structure

In this subsection, we add a new type of security to the risk-free bond to which we have restricted our attention so far: individuals may now also trade "equity".[31] Recall that we already considered the role of shares in Section III.6 when individuals had to decide about the portion of their wealth that they wanted to hold as fixed-interest-rate bonds and the portion that they wanted to allocate to securities that entitled them to receive firms' dividends. In that context, we demonstrated that the two alternatives are equivalent in a world with perfect foresight, since they have to generate the same (certain) return in equilibrium. In fact, the fundamental difference between bonds and equity only becomes apparent in a model that explicitly accounts for uncertainty: if we abstract from the possibility of default and assume that all securities are held to maturity, the return on a bond is risk-free, while the return on equity is state-contingent. Individuals can exploit this property in order to reduce consumption volatility by means of ***international diversification***.

We use a simple example to illustrate this possibility: in a two-period RC-model with an exogenous income, there are two economies (H and F), which are linked by a perfect international capital market. At the end of period 1, both individuals have to decide how much to consume, and how to allocate their total savings to *fixed-interest bonds* and *equity*. Owning equity establishes a claim to receive a portion of a country's output in the future.[32] The RC in the domestic economy (H) thus maximizes the following function:

$$\text{(4.85)} \qquad \mathrm{E}\left(U_1^H\right) = u\left(C_1^H\right) + \beta\,\mathrm{E}\left[u\left(C_2^H\right)\right]$$

subject to the following constraints:

$$\text{(4.86)} \qquad C_1^H = Y_1^H + V_1^H - B_2^H - x_2^{HH}\,V_1^H - x_2^{HF}\,V_1^F$$

$$\text{(4.87)} \qquad C_2^H = \left(1+r\right)B_2^H + x_2^{HH}\,Y_2^H + x_2^{HF}\,Y_2^F$$

[31] Obstfeld and Rogoff (1996, Chapter 5) consider this constellation as a special case of a model in which individuals trade ***Arrow-Debreu-securities*** that are associated with state-contingent payments.

[32] Note that we somewhat stretch the original definition of equity, which is usually defined as a claim on *firms' profits*. However, if we assume that these profits are a constant share of a country's output – e.g. because both profits and output depend on country-specific technology shocks – the interpretation of equity as claims on countries' output is sound.

In (4.86), V_1^H is the price to be paid in period 1 for a claim on country H's *total* output in period 2 (Y_2^H), and $x_2^{HH} \in [0,1]$ is the portion of his *own* country's future output that the RC of country H purchases in period 1. Conversely, x_2^{HF} is the share of country F's second-period income purchased by the RC of country H in period 1. The right-hand side of (4.86) indicates that RC's consumption and savings in period 1 are financed by his income, but also by selling claims on Y_2^H. Hence, the RC of country H uses the international capital market to offer claims on his economy's *entire* future income, and possibly buys back a part of these claims. The RC of country F maximizes the same objective function and faces identical constraints. When solving the optimization problem of the domestic RC, we arrive at the following necessary conditions for a maximum:

(4.88) $$u'\left(C_1^H\right) = \beta\left(1+r\right)\mathrm{E}\left[u'\left(C_2^H\right)\right]$$

(4.89) $$u'\left(C_1^H\right)V_1^H = \beta\,\mathrm{E}\left[u'\left(C_2^H\right)Y_2^H\right]$$

(4.90) $$u'\left(C_1^H\right)V_1^F = \beta\,\mathrm{E}\left[u'\left(C_2^H\right)Y_2^F\right]$$

IV.5.5 International Diversification: Determining Asset Prices

Equations (4.88) – (4.90) implicitly define the structure of the *portfolio* purchased by the RC of country H in period 1. Moreover, we can derive the prices V_1^H and V_1^F which are paid in equilibrium for the future output of country H and F: since $u'\left(C_1^H\right)$ is not a random variable, we can transform (4.90) into

(4.91) $$V_1^F = \mathrm{E}\left[\frac{\beta\,u'\left(C_2^H\right)}{u'\left(C_1^H\right)}Y_2^F\right]$$

The ratio in squared brackets represents the *inverse* of the (intertemporal) marginal rate of substitution (MRS).[33] The MRS itself usually increases in the ratio C_2^H / C_1^H – i.e. it is higher, the steeper the time path of consumption. To further analyze the expression in (4.91), we can exploit the fact that the expected value of a product of two random variables can be split into the product of the variables' expected values and their ***covariance***. Equation (4.91) can thus be rewritten as

[33] This ratio is sometimes called the ***stochastic discount factor*** or the ***pricing kernel***.

(4.92) $$V_1^F = \mathrm{E}\left[\frac{\beta\, u'\left(C_2^H\right)}{u'\left(C_1^H\right)}\right]\mathrm{E}\left(Y_2^F\right) + Cov\left[\frac{\beta\, u'\left(C_2^H\right)}{u'\left(C_1^H\right)}, Y_2^F\right]$$

The second term on the right-hand side is the covariance between the inverse of the marginal rate of substitution and the output of country F in period 2.[34] By substituting (4.88) into (4.92), we eventually arrive at the price that the international capital market charges for claims on country F's future output:

(4.93) $$V_1^F = \frac{\mathrm{E}\left(Y_2^F\right)}{1+r} + Cov\left[\frac{\beta\, u'\left(C_2^H\right)}{u'\left(C_1^H\right)}, Y_2^F\right]$$

This expression conveys an insight whose importance goes beyond our simple model: in an uncertain world, the value of a security not only depends on its expected return – which, in our case, is the discounted expected output of country F – but also on how much this security contributes to reducing consumption risk. As suggested by equation (4.93), the price V_1^F increases if the covariance between Y_2^F and the inverse of the MRS increases. Such a positive correlation emerges if Y_2^F is exceptionally high (low) whenever the ratio C_2^H / C_1^H is exceptionally low (high). In this case, having a claim on future output in country F reduces the consumption volatility faced by the RC of country H since payments out of this security in period 2 tend to be high when the consumption level is low. For a given *expected* payoff, this raises the attractiveness of the security, and thus its price in equilibrium.

The fact that security prices not only depend on expected payments but also on how these payments affect the risk of individuals' total portfolio is the central insight of the ***Capital Asset Pricing Model (CAPM)***. However, while the original CAPM focuses on the covariance of a security with the "market portfolio", we have derived the ***consumption-based CAPM***, which states that the price of a security depends on how the resulting payments are correlated with individuals' marginal rate of substitution.

IV.5.6 The Optimal Portfolio in Equilibrium

Having derived asset prices in equilibrium, we will now consider individuals' *portfolio choice*, i.e. the decision on how to allocate their total savings to different securities.

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[34] Recall that the covariance of two random variables X and Y is given by $\mathrm{Cov}(X,Y) = \mathrm{E}\left[(X - \mathrm{E}(X))(Y - \mathrm{E}(Y))\right]$. Roughly speaking, the covariance is positive if X and Y tend to move in the same direction and negative if they tend to move in opposite directions.

To simplify the analysis, we assume that individuals in countries H and F have identical preferences, which are characterized by a constant relative risk aversion (CRRA). Using the specification in (4.75), we can thus write (4.89) as

$$(4.94) \qquad V_1^H = \beta \, \mathrm{E}\left[\left(\frac{C_1^H}{C_2^H} \right)^{\sigma} Y_2^H \right]$$

The assumption of identical CRRA preferences substantially simplifies the derivation of equilibrium values for x_2^{HH}, x_2^{HF} etc. since, in this special case, an individual's portfolio decision is independent of his wealth level. As a consequence, agents from H and F do not differ in the share of savings they allocate to country H and country F equity, respectively:

$$(4.95) \qquad \frac{x_2^{HH} \, V_1^H}{V_1^H + Y_1^H - \mu_1^H \, Y_1^W} = \frac{x_2^{FH} \, V_1^H}{V_1^F + Y_1^F - \mu_1^F \, Y_1^W}$$

$$(4.96) \qquad \frac{x_2^{HF} \, V_1^F}{V_1^H + Y_1^H - \mu_1^H \, Y_1^W} = \frac{x_2^{FF} \, V_1^F}{V_1^F + Y_1^F - \mu_1^F \, Y_1^W}$$

In these expressions, μ_1^H denotes country H's share of global consumption in period 1, which, in turn, equals global output $\left(Y_1^W = Y_1^H + Y_1^F\right)$. The left-hand side of equation (4.95) thus shows the amount that country-H residents spend on country H equity in period 1 $\left(x_2^{HH} V_1^H\right)$, relative to their total savings $\left(V_1^H + Y_1^H - \mu_1^H Y_1^W\right)$, with savings reflecting the difference between first-period income – including the proceeds from selling claims on country H's future output – and first-period consumption. The right-hand side does the same for the portion of savings that country F residents spend on country H equity.

The astute reader may wonder why savers' demand for *risk-free bonds* $\left(B_2^H, B_2^F\right)$ does not appear in equations (4.95) and (4.96). The reason is that, due to the symmetry built into this particular model, a strictly positive bond demand or bond supply is not compatible with equilibrium on the international capital market. To see this, suppose that the RC of country H decides to hold a strictly positive amount of risk-free bonds. Due to our assumption on preferences, this would also hold for the RC of country F. As a consequence, the international capital market would be characterized by an excess demand for bonds. If the RC of country H decided to *offer* risk-free bonds instead, the same would hold for country F, and there would be an excess supply of bonds. Hence, the only constellation that entails neither excess demand nor excess supply on the market for risk-free bonds is characterized by $B_2^H = B_2^F = 0$.[35]

Since global consumption equals global output in every period, we must have $\mu_1^F = 1 - \mu_1^H$. Moreover, the claims on a country's future output must add up to one, i.e. $x_2^{FF} = 1 - x_2^{HF}$ and $x_2^{FH} = 1 - x_2^{HH}$. If we substitute these equilibrium conditions into equations (4.95) and (4.96), it is easy to show that $x_2^{HH} = x_2^{HF}$ has to hold in equilibrium. This means that, in period 1, the RC from country *H* purchases a share $x_2^{HH} = x_2^{HF} = x_2^H$ of claims on his own country's and the foreign country's future output. His second-period consumption is thus given by

$$(4.97) \qquad C_2^H = x_2^H \left(Y_2^H + Y_2^F \right) = x_2^H \, Y_2^W$$

This is an important result: while the individual's consumption in period 2 is uncertain *ex-ante*, uncertainty only stems from possible variations of *global* output. By contrast, variations in country *H*'s own output (Y_2^H) only matter through their influence on Y_2^W. International diversification thus allows RC to eliminate all ***idiosyncratic*** – i.e. country-specific – ***risk***. As a result, the variability of consumption is only due to potential fluctuations of world output, i.e. ***aggregate risk***.

Equation (4.97) implies that $\mu_2^H = x_2^H$. To determine the consumption shares μ_1^H and μ_2^H, we can use equation (4.94). Since individuals from both country *H* and country *F* face the same asset prices, it must be the case that

$$(4.98) \qquad \beta \; \mathrm{E}\left(\frac{\mu_1^H \; Y_1^W}{\mu_2^H \; Y_2^W} \, Y_2^H \right)^{\sigma} = V_1^H = \beta \; \mathrm{E}\left(\frac{(1-\mu_1^H)\, Y_1^W}{(1-\mu_2^H)\, Y_2^W} \, Y_2^H \right)^{\sigma}$$

Given that $\mu_2^H = x_2^H$ is determined in period 1, this condition is satisfied if and only if $\mu_1^H = \mu_2^H$ – which implies that the RC of country *H* consumes a constant share $\mu_1^H = \mu_2^H = \mu^H$ of world output in both periods of his life. We can compute the size of this share by substituting $x_2^H = \mu^H$ into condition (4.86):

$$(4.99) \qquad \mu^H = \frac{Y_1^H + V_1^H}{Y_1^W + V_1^W}$$

where we have defined $V_1^W = V_1^H + V_1^F$ as the ex-ante value of world output in period 2. The share of the "global portfolio" that is purchased by the resident

[35] If we dropped the assumption that individuals in both countries have identical preferences, or if individuals' risk aversion depended on their wealth levels, this result would break down, and the volume of risk-free bonds traded on the international capital market could be strictly positive.

of country H is thus determined by his country's (relative) first-period output and the (relative) value of its (uncertain) second-period output. As we have shown in Subsection IV.5.5, the latter value crucially depends on the correlation properties of Y_2^H.

To compute the current account balance of country H in period 1, we subtract the value of consumption $C_1^H = \mu^H Y_1^W$ from the value of its output Y_1^H:

$$CA_1^H = \theta_1^W Y_1^H - \left(1 - \theta_1^W\right) V_1^H \tag{4.100}$$

with $\theta_1^W \equiv V_1^W / (V_1^W + Y_1^W)$. This term has a straightforward interpretation: a higher income Y_1^H drives up savings of country H in period 1, while a higher value of country H's future output V_1^H increases RC's wealth and therefore raises consumption. The tendency to save is reinforced by a high valuation of future global output, which raises θ_1^W. This valuation, in turn, hinges both on individuals' preferences and on the distribution of output growth rates. The important insight to take away from this expression is that the domestic country's current account balance in period 1 may be different from zero even if $Y_1^H = Y_1^F$ and $\mathrm{E}(Y_2^H) = \mathrm{E}(Y_2^F)$. This reflects the fact that consumption also depends on asset prices, and asset prices do not just hinge on expected output, but also on the contribution of a country's output to the riskiness of the "global portfolio".

IV.5.7 International Diversification: Some Evidence

Let us take stock: in the previous subsections, we have considered a simple two-period model, in which a country's (representative) resident faces an uncertain future income. The fact that, apart from risk-free bonds, the international capital market allows to trade claims on state-contingent payments, enables individuals to diversify idiosyncratic – i.e. country-specific – risks. Hence, as long as Y_2^H and Y_2^F are not perfectly correlated, the volatility of consumption can be reduced by receiving a share of Y_2^F whenever Y_2^H is low and vice versa. It is important to see that, unlike in previous chapters, this type of consumption smoothing is due to *ex-ante* diversification, not *ex-post* borrowing and lending.

Does the empirical evidence support these theoretical predictions? To answer this question, we confront individuals' hypothetical portfolio choices with observed data on international investment. Recall that, in the model above, the RC of country H purchases a share x_2^{HH} of his own country's future output and a share x_2^{HF} of country F's future output. The share of "domestic equity" in his overall portfolio, which we denote by s_2^{HH}, is thus given by

$$(4.101) \qquad s_2^{HH} = \frac{x_2^{HH} V_1^H}{x_2^{HH} V_1^H + x_2^{HF} V_1^F}$$

Recall from above that, in a model in which individuals have CRRA-preferences, $x_2^{HH} = x_2^{HF} = x_2^H$. Substituting this into the above expression yields

$$(4.102) \qquad s_2^{HH} = \frac{V_1^H}{V_1^W}$$

Country	Size of Domestic Market in Percent of World Market Capitalization	Observed Portfolio Share of Domestic Equity (in Percent)	Degree of Equity Home Bias
Brazil	1.6	98.6	0.99
South Korea	1.4	88.5	0.88
Japan	8.9	73.5	0.71
USA	32.6	77.2	0.66
United Kingdom	5.1	54.5	0.52
Euro area	13.5	56.7	0.50
Switzerland	2.3	50.9	0.50

Table 4.3: Theoretical and observed shares of domestic equity in countries' total equity wealth and the degree of equity home bias in 2008. Source: Coeurdacier and Rey (2013).

Hence, the share of claims on future domestic output in the portfolio of country-*H* residents should equal the value of country *H*'s future output relative to the value of future global output. While the value of a country's future GDP is not observable, we can take the value of all companies traded on a country's stock market – i.e. country *H*'s ***stock market capitalization*** – as a proxy. Equation (4.102) thus suggests that the share of domestic equity in the portfolio of domestic residents should equal the value of the domestic stock market relative to the "global" stock market's capitalization. Table 4.3 follows Nicolas Coeurdacier and Hélène Rey (2013) by comparing this prediction with recent evidence on international equity investment. The second column of that table presents the ratio on the right-hand side of (4.102) while the third column presents observed data on s_2^{HH} (in percent).

Apparently, the values in the third column are much higher than their theoretical counterparts, which indicates that individuals exhibit a substantial ***home bias*** when deciding on the portion of their wealth that is devoted to domestic equity. The "degree of equity home bias" (EHB^H) is given in the fourth column.

Using the notation introduced above and treating country H as the domestic economy while country F represents the rest of the world, EHB^H can be computed according to the formula

$$EHB^H = 1 - \frac{1 - s_2^{HH,observed}}{V_1^F / V_1^W} = \frac{s_2^{HH,observed} - V_1^H / V_1^W}{1 - V_1^H / V_1^W} \tag{4.103}$$

If equation (4.102) were a correct description of reality, the degree of equity home bias as defined by (4.103) would be zero. By contrast, the greater the discrepancy between the observed share of country H equity in domestic agents' portfolios and the hypothetical share V_1^H / V_1^W, the greater EHB^H. Table 4.3 demonstrates that even in a country that is as firmly integrated into global financial markets as Switzerland, the degree of equity home bias is substantial.

In their study, Coeurdacier and Rey (2013) show that, for most industrialized countries, EHB^H has been decreasing over the past years. Nevertheless the equity home bias still is a puzzle that needs to be explained. Why don't agents exploit the potential for international diversification to a larger extent? The international macroeconomics literature offers various answers to this question. A substantial part of the equity home bias is certainly caused by international capital market imperfections – insecure property rights, asymmetric information etc. – which make it seem advantageous to invest a large share of one's wealth in the home country. Moreover, there are regulatory barriers that make it difficult to fully exploit the scope for international diversification. And finally, we have to take into account that the extent of international diversification may actually be greater than what is suggested by the numbers in Table 4.3: in particular, these numbers do not account for the fact that, by *operating* abroad, domestic multinational firms substantially contribute to international diversification.

Box 4.5: The Consumption Correlation Puzzle

It follows from the model presented in subsection IV.5.6 that

$$\frac{C_2^H}{C_1^H} = \frac{\mu^H Y_2^W}{\mu^H Y_1^W} = \frac{(1-\mu^H) Y_2^W}{(1-\mu^H) Y_1^W} = \frac{C_2^F}{C_1^F}$$

Hence, regardless of the stochastic properties of national GDP levels, the growth rates of consumption in countries H and F should be perfectly correlated. While this strong result is certainly driven by our assumptions on individuals' preferences, the general idea that international diversification reduces the impact of country-specific income variations also applies under less specific assumptions. Hence, consumption fluctuations should predominantly reflect aggregate – i.e. global – shocks and should therefore be highly correlated.

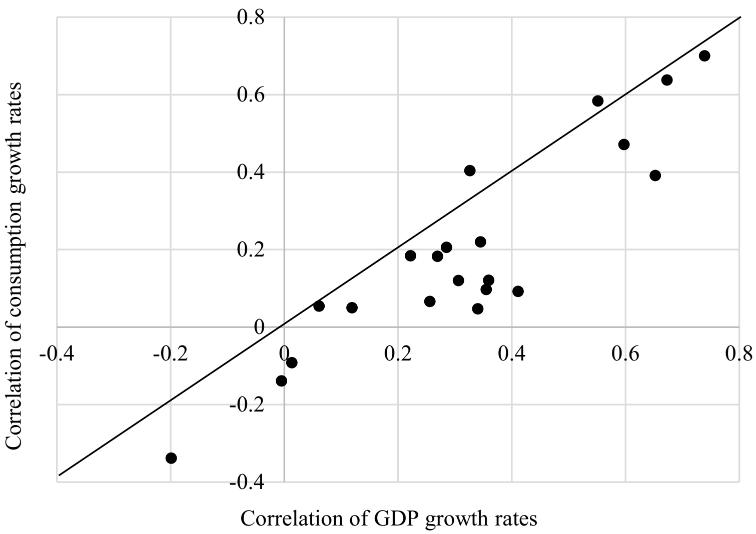

Figure B4.5: Bilateral correlations of GDP and consumption growth rates, 1950-1992. Source: Lewis (1999:574).

This conjecture is not supported by the data: Figure B4.5, which is based on a study by Karen Lewis (1999) and which depicts the correlation of annual growth rates for various industrialized countries, not only shows that income and consumption correlations are almost identical. For most country-pairs, the consumption correlation is actually *lower* than the correlation of income – which is the exact opposite of what is suggested by our theory. Of course, this ***consumption correlation puzzle*** is closely related to the home bias – i.e. agents' apparent reluctance to exploit the scope for international diversification. While the actual degree of countries' home bias is hard to pin down, the scatter plot in Figure B4.6 makes an unambiguous point: apparently, the diversification of country-specific consumption risks falls considerably short of what is suggested by a theory that is based on the notion of frictionless goods markets and a perfect international capital market.

IV.6 Summary and Outlook

The previous sections have extended the basic model of Chapter III by going beyond the perfect-foresight RC framework, thus demonstrating the flexibility and broad applicability of the intertemporal approach. To analyze the consequences of demographic change in Section IV.2, we replaced the construct of a representative consumer by a model of overlapping generations. In Section IV.3, we explicitly considered the effect of (exogenous) government spending, putting a particular emphasis on the question whether and how budget deficits affect national savings and the current account. By allowing for a more sophisticated goods structure in Section IV.4, it was possible to explore the interaction of *inter*temporal and *intra*temporal optimization processes. In this context, we demonstrated that an anticipated change of relative prices influences saving behavior both through its effect on the intertemporal budget constraint and by affecting the effective (consumption-based) interest rate, and that costs of international goods trade may dampen fluctuations of the current account. Finally, Section IV.5 explored the implications of uncertainty for individuals' saving behavior. We showed that a higher variance of future incomes enhances (precautionary) savings. Moreover, we demonstrated that the possibility to trade equity on international financial markets allows to diversify country-specific risks and to reduce the volatility of consumption.

Throughout this chapter, our analysis was restricted to a two-period framework. As a consequence, we could not explore how access to the international capital market affects a country's income *in the long run*. This gap will be closed in the following chapter, which will be devoted to an analysis of the growth effects of international capital flows.

Moreover, we implicitly assumed that individuals always comply with their intertemporal budget constraints when choosing their optimal consumption and investment plans. Hence, our exposition was based on the notion of a *perfect* international capital market, which was – by definition – free from repayment difficulties, debt crises etc. Needless to say that, in reality, capital market imperfections play an important role when it comes to determining the volume, direction and composition of international capital flows. While we already alluded to this fact when discussing the limited extent of international diversification, as reflected by the home bias, Chapter VI will explore the determinants, inherent mechanics, and consequences of financial market imperfections in more detail.

IV.7 Keywords

Age dependency ratio
Budget balance
Capital asset pricing model
Certainty equivalence
Consumption correlation puzzle
Demographic change
Expected utility
Home bias
Ideal price index
Idiosyncratic risk
Intergenerational redistribution
International diversification
Jensen's inequality
Life-cycle theory of consumption
Mean-preserving increase in risk
Non-tradable goods
OLG model
Pay-as-you-go system
Portfolio
Precautionary savings
Primary budget deficit
Relative risk aversion
Ricardian equivalence
State-contingent payments
Stochastic discount factor
Terms of trade
Trade costs
Warm-glow preferences

IV.8 Literature

The second and third sections of this chapter borrow from Chapter 3 in Obstfeld und Rogoff (1996), which offers an in-depth analysis of budget deficits in open economies (with and without demographic heterogeneity). Börsch-Supan (2004) provides a comprehensive discussion of various aspects of demographic change. The OLG model goes back to the seminal contributions by Samuelson (1958) and Diamond (1965), the life-cycle theory of consumption originates with Modigliani und Brumberg (1954). Data on various *demographic* variables can be found on the homepage of the Population Division at the United Nations' Department of Economic and Social Affairs (http://esa.un.org/unpd/wpp/). Elmendorf and Mankiw (1999) offer a survey on the determinants and consequences of government debt. Feyrer and Shambaugh (2012) analyze the effects of US fiscal shocks on other countries' current accounts. The fourth section of this chapter, which considers models with multiple goods, leans on Chapter 4 in Obstfeld and Rogoff (1996). The classical contribution on the role of non-tradable goods in dynamic models goes back to Dornbusch (1983). Obstfeld and Rogoff (2000a) discuss the role of trade costs for intratemporal and intertemporal trade. A recent quantitative assessment of their claim that trade costs go a long way in explaining several puzzles in macroeconomics is offered by Eaton et al. (2015). Francois and Hoekman (2010) provide a recent survey on international services trade. Gollier's (2001) monograph offers an excellent description of behavior under uncertainty. Chapter 5 in Obstfeld and Rogoff

(1996), which has influenced the fifth section of this chapter, applies these concepts to the analysis of saving behavior and portfolio choice in open economies. While we have interpreted claims on a country's GDP as equity, the idea that financial markets should offer such claims was originally proposed by Shiller (1993). More recently, Kamstra and Shiller (2010) suggest to tie payments on government debt to countries' GDP. Lewis (1999) and Coeurdacier and Rey (2013) offer surveys on potential explanations of the equity home bias.

IV.9 Exercises

4.1. Public debt, the current account, and welfare in a model with heterogenous interest rates. In a two-period model of a small open economy with an exogenous income, a country's representative consumer (RC) and government face the following interest rates: while the government can borrow and lend at the interest rate r, RC has to pay the interest rate $r^P > r$ on loans. By contrast, he receives the interest rate r if he lends. We exclude the possibility that the government offers loans to RC.

RC's income in both periods amounts to Y. His utility function is given by

$$U_1 = \ln\left(C_1\right) + \beta \ln\left(C_2\right)$$

In both periods, the government spends the amount G. It can finance these expenditures by raising (lump-sum) taxes or by borrowing on the (international) capital market. However, it must not carry any assets or liabilities beyond the end of period 2. Finally, we assume that

$$G < Y < G + \frac{1}{1+r} G$$

a) Explain intuitively whether Ricardian Equivalence holds in this example.
b) Compute RC's consumption path for the case that the government raises taxes only in the *first* period.
c) Compute RC's consumption path for the case that the government raises taxes only in the *second* period.
d) Show formally which policy (b or c) is associated with higher welfare for RC. Interpret your result.

4.2. Intertemporal elasticity of substitution, intratemporal elasticity of substitution, and the effect of anticipated price changes on consumption. Equation (4.51) indicates that the reaction of the current account to anticipated price changes crucially hinges on the relationship between the *inter*temporal elasticity of substitution $(1/\sigma)$ and the *intra*temporal elasticity of substitution $(1/\eta)$. Corsetti and Pesenti (2001: 438) argue that the tradable and the non-tradable good are complements – i.e. the marginal utility of consuming one good increases in the consumption of the other good – if $(1/\eta)$ is smaller than $(1/\sigma)$. Conversely, the goods are substitutes – i.e the marginal utility of consuming one good decreases in the consumption of the other good – if $(1/\eta)$ is greater than $(1/\sigma)$. Show that these statements are correct and use this finding to interpret equation (4.51).

4.3. Deregulation and the current account. Harms (2005) analyzes a small open economy with a (tradable) manufacturing and a (non-tradable) services sector. Using this framework, he shows that removing barriers to market entry in the services sector *ceteris paribus* results in a current account surplus if the manufacturing sector is relatively capital intensive. Use your intuition to explain the mechanisms that could generate such a result.

4.4. Asset prices in a two-country model. We consider a 2-period model of two large open economies (H and F). Both countries' RCs have identical preferences and maximize their expected utility:

$$U_1^c = \frac{\left(C_1^c\right)^{1-\sigma} - 1}{1-\sigma} + \beta\,\mathrm{E}\left[\frac{\left(C_2^c\right)^{1-\sigma} - 1}{1-\sigma}\right] \qquad c = H, F$$

In period 1, both agents receive an exogenous income Y of one, i.e. $Y_1^H = Y_1^F = 1$. Second-period income is random: In state of nature 1, which occurs with probability $\pi = 0.5$, we observe $Y_{2,1}^H = 2$ and $Y_{2,1}^F = 0$. In state of nature 2, which occurs with probability $(1-\pi) = 0.5$, we observe $Y_{2,2}^H = 1$ and $Y_{2,2}^F = 3$.

In period 1, both RCs can purchase and sell a risk-free bond, which yields an interest rate r. They may also purchase equity, i.e. claims on country H's and F's output at prices V_1^H and V_1^F, respectively.

a) Describe intuitively how the two countries' growth rates are correlated.
b) What is the expected growth rate of world income? What are the expected growth rates of the two countries?
c) We choose the following parameter values: $\sigma = 2$, $\beta = 0.5$. Compute the asset prices V_1^H and V_1^F in equilibrium and interpret your result.

4.5. The current account in a model of international diversification. In the model of international diversification, equation (4.100) has related the current-account balance of country H in period 1 to its income (Y_1^H) and the valuation of its output in period 1 . Suppose that future world output is deterministic, i.e. there is no aggregate risk. Use equations (4.94) and (4.100) to show that, in this case, the home country's first-period current account is given by the following expression:

$$CA_1^H = \frac{\beta\left(1+g^W\right)^{-\sigma} \; Y_1^H}{1+v_1^W}\left[\frac{Y_2^W}{Y_1^W} - \mathrm{E}\left(\frac{Y_2^H}{Y_1^H}\right)\right]$$

where $1+g^W \equiv Y_2^W / Y_1^W$ and $v_1^W \equiv V_1^W / Y_1^W$. Provide an economic interpretation for this expression.

IV.10 Appendix to Chapter IV

IV.10.1 The Evolution of the Net International Investment Position in an OLG Model with Endogenous Production

To derive (4.9) using (4.8), we recall that, in equilibrium, the marginal productivity of capital in period $t+1$ has to equal the sum of the depreciation rate, which we have assumed to be one $\left(\delta=1\right)$, and the world interest rate:

$$\text{(4.A1)} \qquad \alpha\left(\frac{k_{t+1}^o \cdot n_{t-1}}{n_t}\right)^{\alpha-1} = 1+r$$

with n_{t-1} representing the number of capital owners and n_t denoting the number of workers in period $t+1$. We can solve this equation for k_{t+1}^o, using the definition $n_t \big/ n_{t-1} = \mu_t$.

$$\text{(4.A2)} \qquad k_{t+1}^o = \left(\frac{\alpha}{1+r}\right)^{\frac{1}{1-\alpha}} \mu_t$$

The wage in period t is given by

$$\text{(4.A3)} \qquad \left(1-\alpha\right)\left(\frac{k_t^o \; n_{t-2}}{n_{t-1}}\right)^{\alpha} = w_t$$

By moving equation (4.A2) backward by one period and substituting the result into (4.A3) we get

$$w_t = (1-\alpha)\left(\frac{\alpha}{1+r}\right)^{\frac{\alpha}{1-\alpha}}. \tag{4.A4}$$

Substituting (4.A2) and (4.A4) into (4.8) results in (4.9).

IV.10.2 Conditional Demand Functions in a Model with Non-Tradable Goods

To derive the conditional demand for the tradable and the non-tradable good in period t, we write down the following Lagrange function:

$$\begin{aligned} Z = \; & C_t^T + P_t^N\, C_t^N \\ & - \lambda \left\{ \left[\gamma^\eta \left(C_t^T\right)^{1-\eta} + (1-\gamma)^\eta \left(C_t^N\right)^{1-\eta} \right]^{\frac{1}{1-\eta}} - C_t \right\} \end{aligned} \tag{4.A5}$$

RC minimizes the total costs of consumption for a given level of the consumption aggregator C_t. Setting the derivatives with respect to C_t^T and C_t^N equal to zero yields the following necessary conditions for a minimum:

$$\lambda\, C_t^\eta\, \gamma^\eta \left(C_t^T\right)^{-\eta} = 1 \tag{4.A6}$$

$$\lambda\, C_t^\eta\, (1-\gamma)^\eta \left(C_t^N\right)^{-\eta} = P_t^N \tag{4.A7}$$

Combining these expressions generates equation (4.36) in the main text. Substituting this result for C_t^N into (4.33) gives

$$C_t = \left[\gamma^\eta \left(C_t^T\right)^{1-\eta} + (1-\gamma)^\eta \left(\frac{1-\gamma}{\gamma}\right)^{1-\eta} \left(\frac{1}{P_t^N}\right)^{\frac{1-\eta}{\eta}} \left(C_t^T\right)^{1-\eta} \right]^{\frac{1}{1-\eta}} \tag{4.A8}$$

Solving this expression for C_t^T yields (4.37). Substituting (4.37) into (4.36) gives (4.38).

IV.10.3 The Aggregate Price Level in a Model with Non-Tradable Goods

Evaluated in terms of tradable-good units, RC's consumption expenditure in period t amounts to

$$C_t^T + P_t^N C_t^N = P_t C_t \tag{4.A9}$$

Plugging (4.37) and (4.38) into the left-hand side of (4.A9), we get
(4.A10)

$$\left[\gamma^{\eta} + (1-\gamma)\gamma^{\eta-1}\left(P_t^N\right)^{\frac{\eta-1}{\eta}} \right]^{\frac{1}{\eta-1}} C_t$$
$$+\left[(1-\gamma)^{\eta}\left(P_t^N\right)^{\eta-1} + \gamma(1-\gamma)^{\eta-1}\left(P_t^N\right)^{\eta-1}\left(P_t^N\right)^{\frac{1-\eta}{\eta}} \right]^{\frac{1}{\eta-1}} C_t \quad = P_t C_t$$

This expression may be transformed into

$$\begin{aligned} &\gamma\left[\gamma + (1-\gamma)\left(P_t^N\right)^{\frac{\eta-1}{\eta}} \right]^{\frac{1}{\eta-1}} C_t \\ &+(1-\gamma)\left(P_t^N\right)^{\frac{\eta-1}{\eta}}\left[\gamma + (1-\gamma)\left(P_t^N\right)^{\frac{\eta-1}{\eta}} \right]^{\frac{1}{\eta-1}} C_t \quad = P_t C_t \end{aligned} \tag{4.A11}$$

which, in turn, is equivalent to

$$\left[\gamma + (1-\gamma)\left(P_t^N\right)^{\frac{\eta-1}{\eta}} \right]\left[\gamma + (1-\gamma)\left(P_t^N\right)^{\frac{\eta-1}{\eta}} \right]^{\frac{1}{\eta-1}} C_t = P_t C_t \tag{4.A12}$$

The expressions in squared brackets on the left-hand side are identical. They can thus be combined to yield (4.39).

IV.10.4 The Consumption Aggregator for $\eta = 1$

In order to show that the consumption aggregator of (4.33) turns into (4.52) as the elasticity of substitution $(1/\eta)$ approaches one, we start by taking logarithms on both sides of (4.33), taking into account that $x^a = e^{a \ln x}$. This yields

(4.A13) $$\ln C_t = \frac{\ln\left[e^{\eta \ln\gamma}\ e^{(1-\eta)\ln C_t^T} + e^{\eta\ln(1-\gamma)}\ e^{(1-\eta)\ln C_t^N}\right]}{(1-\eta)}$$

Since a ratio is not defined if both the numerator and the denominator are zero, we cannot simply replace η by one. However, we can apply L'Hôpital's rule. To do this, we denote the numerator of (4.A13) by $\ln H$. This yields

(4.A14) $$\ln C_t = \frac{\ln H\left(\eta, \gamma, C_t^T, C_t^N\right)}{1-\eta}$$

In a second step, we take a derivative of $\ln H$ with respect to η:

(4.A15) $$\begin{aligned}\frac{d\ln H}{d\eta} = \ & \Big[\gamma^{\eta}\left(C_t^T\right)^{1-\eta}\left(\ln\gamma - \ln C_t^T\right) \\ & + (1-\gamma)^{\eta}\left(C_t^N\right)^{1-\eta}\left(\ln(1-\gamma) - \ln C_t^N\right)\Big]/H\end{aligned}$$

We can compute the limit of (4.A13) for η approaching one by allowing η to approach one in (4.A15), and by dividing the result by (-1) – the derivative of the denominator of (4.A14) with respect to η. This yields

(4.A16) $$\lim_{\eta\to 1} \ln C_t = \gamma \ln C_t^T + (1-\gamma)\ln C_t^N + \Phi$$

with Φ as a constant. By using the exponential function to turn logarithms into original variables, we arrive at

(4.A17) $$C_t = e^{\Phi}\left(C_t^T\right)^{\gamma}\left(C_t^N\right)^{1-\gamma}$$

The (positive) constant e^{Φ} is irrelevant for individuals' decisions and can therefore be ignored.