Part I: Information, Knowledge and Uncertainty

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Motivation – Why should we care?

- Tackling asymmetric information is a key function of several public policies.
- Many vibrant policy debates (e.g. on the role of universal healthcare or the scope of the welfare state) are related to questions of asymmetric information
- Thus, understanding the mechanisms of Adverse Selection ("Hidden types") and Moral Hazard ("Hidden actions") does not only help to acquire important microeconomic concepts and tools, but provides deep, policy relevant insights.

I.1 Adverse Selection

- We analyze situations of asymmetric information where the agent has some information that the principal does not have about some aspect of product/service quality.
- This is a situation of "hidden types", i.e. Adverse Selection.
- The basic problem is the market for lemons (Akerlof (1970)) where markets collapse.
- The problem can be solved if the informed agent moves first and signals her type (Spence (1973)).
- The problem can be solved if the principal designs a contract to screen the agents (Mirrlees (1971)).

The Market for Lemons - George Akerlof (Nobel 2001)

The Market for Lemons (Cont.)

The Market for Lemons (Cont.)

- The market for lemons shows that markets can collapse because of asymmetric information.
- *Lemons* are used cars of low quality, whereas *peaches* are used cars of good quality.
- In the market for second-hand cars, the seller knows the quality of the car but not the buyer.
- Suppose that quality is $k \in [0, 1]$ uniformly distributed.

The Market for Lemons (Cont.)

- Cars of quality 0 are the worst and cars of quality 1 are the best.
- Sellers of good cars have a reservation price. A seller is willing to sell a car of quality k at $p_0 \cdot k$.
- Buyers value the car at $p_1 \cdot k$ with $p_1 > p_0$.
- Assume $p_1 = \frac{3}{2} p_0$.

Equilibrium for the Market

- If information was symmetric, a car of quality k would be sold at a price p between $p_0 \cdot k$ and $p_1 \cdot k$.

- Given that the quality is not observed, there is a unique price p at which all cars are sold.

- The buyer knows that the only sellers who would be willing to sell their car at p are sellers for whom $p_0 \cdot k \leq p$ or $k \leq \frac{p}{p_0}$. Thus, the expected quality of the car is $E(k) = \frac{1}{2} \cdot \frac{p}{p_0}$ and the expected valuation of the buyer is (recall $p_1 = \frac{3}{2}p_0$)
$$p_1 E(k) = \frac{3}{4}p.$$

- Naturally, *no buyer is willing to spend p to buy a car of value $\frac{3}{4}p$, and no car is sold* in the market.

How Can One Solve the Market for Lemons?

- We need to find a way to distinguish between good and bad cars.
- We can utilize *certification* where a third party is trusted to certify cars.
- We can offer a *warranty* where only sellers of good cars are willing to offer the warranty; that is, the warranty acts as a *signal* that the car is good.
- *Lemon laws* were enacted five years after the publication of this paper. These state laws provide remedies to consumers for automobiles that repeatedly fail to meet certain standards of quality and performance.

Examples of Signaling

- Product warranties signal quality.
- An expensive art work in the lobby of a bank signals financial security.
- A dividend policy signals value of the firm to the shareholders.
- Price signals quality of wine.
- Education signals quality of performance.

Education as a Signal - Michael Spence (Nobel 2001)

Education as a Signal (Cont.)

- Agents are born with a productivity level θ, where $\theta = \theta^H$ with probability p and $\theta = \theta^L$ with probability $1 - p$.
- Agents select their level of education, $e \in \{0, 1\}$.
- High productivity agents suffer a cost c^H for acquiring one unit of education, whereas low productivity agents suffer a cost c^L for acquiring one unit of education.
- We assume that $c^H < c^L$.

The Different Types of Equilibria

- A separating equilibrium where high types choose $e = 1$ and low types choose $e = 0$ (S1).
- A pooling equilibrium where both types choose $e = 0$ (P1).
- A pooling equilibrium where both types choose $e = 1$ (P2).

The Beliefs of Employers

- In a separating equilibrium, **agents getting education result in the employer offering them a wage $w^H = \theta^H$; otherwise,** $w^L = \theta^L$.
- In a pooling equilibrium, along the equilibrium, the employers cannot distinguish between low and high types: the employer will pay a wage corresponding to the expected productivity $p\theta^H + (1-p)\theta^L$.

The Separating Equilibrium S1

- We write down the two *incentive* conditions for the H and L agents.
- H prefers to choose $e = 1$ if
$$\theta^H - c^H \geq \theta^L. \quad [1]$$
- L prefers to choose $e = 0$ if
$$\theta^L \geq \theta^H - c^L. \quad [2]$$
- This equilibrium exists if
$$c^L \geq \Delta\theta \geq c^H.$$

Note: $\theta^H - \theta^L \geq c^H$ [1] & $c^L \geq \theta^H - \theta^L$ [2] where $\Delta\theta = \theta^H - \theta^L$.

The Pooling Equilibrium P1

- Let π be the probability that an agent choosing $e = 1$ is of high productivity ($\rightarrow$ out of equilibrium beliefs). The LHS is the expected productivity.
- Agent H prefers to choose $e = 0$ if
$$p\theta^H + (1-p)\theta^L \geq \pi\theta^H + (1-\pi)\theta^L - c^H.$$
- Agent L prefers to choose $e = 0$ if
$$p\theta^H + (1-p)\theta^L \geq \pi\theta^H + (1-\pi)\theta^L - c^L.$$
- The only binding constraint is the constraint for the H type.
- This equilibrium exists if
$$c^H \geq (\pi - p)\Delta\theta$$
where $\Delta\theta = \theta^H - \theta^L$.

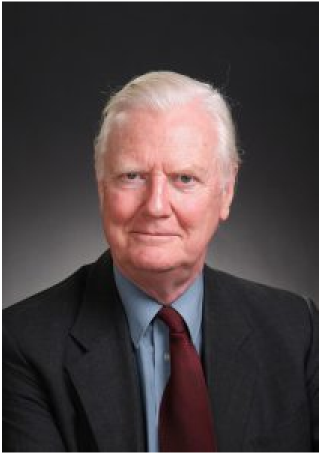
The Pooling Equilibrium P2

- Let π be the probability that an agent choosing $e = 0$ is of high productivity.
- Agent H prefers to choose $e = 1$ if
$$p\theta^H + (1-p)\theta^L - c^H \geq \pi\theta^H + (1-\pi)\theta^L.$$
- Agent L prefers to choose $e = 1$ if
$$p\theta^H + (1-p)\theta^L - c^L \geq \pi\theta^H + (1-\pi)\theta^L.$$
- The only binding constraint is the constraint for the L type.
- This equilibrium exists if
$$c^L \leq (p-\pi)\Delta\theta$$
where $\Delta\theta = \theta^H - \theta^L$.

Education as a Signal

- We have found that there are three equilibria.
- One separating equilibrium, where the high productivity agents get education but not the low ones.
- Two pooling equilibria, where either no type gets education or both types get education.

Screening and Adverse Selection - James Mirrlees (Nobel 1996)



Screening and Adverse Selection

- Suppose now that the sequence of moves is reversed.
- The uninformed agent (P) (mneumonic for *Principal*) moves first, and the informed agent (A) moves second.
- The principal proposes a *screening contract* such that low types and high types choose different alternatives.
- We will study this optimal screening contract in the context of a relation between an employer and an employee.

Types of Agents

- There are two types of employees: the good ones that have a cost of effort $v(e)$, and the bad ones that have a cost of effort $kv(e)$ where $k > 1$.
- The proportion of good types is p whereas the proportion of bad types is $1 - p$.
- $\Pi(e) - w$ is the expected output minus wage; that is, the principal's profit.
- $\underline{U}$ is the reservation utility of the agent whose utility function is given by $u(w)$.

Optimal Contracts with Symmetric Information

- P chooses to maximize $\Pi(e) - w$ subject to the participation constraint $u(w) - (k)v(e) \geq \underline{U}$.

- The contract for the G type is

$$u(w^G) - v(e^G) = \underline{U},$$

- The contract for the B type is

$$u(w^B) - kv(e^B) = \underline{U},$$

Optimal Contracts with Symmetric Information (Cont.)

- We compute the Lagrangian for the good type

$$\begin{aligned} \mathcal{L} &= \Pi(e^G) - w^G \\ &- \lambda(u(w^G) - v(e^G) - \underline{U}). \end{aligned}$$

- Taking the first order conditions, we get:

$$\begin{aligned} \Pi'(e^G) + \lambda v'(e^G) &= 0 \quad \text{or} \quad \Pi'(e^G) = -\lambda v'(e^G) \quad [1] \\ -1 - \lambda u'(w^G) &= 0 \quad \text{or} \quad 1 = -\lambda u'(w^G). \quad [2] \end{aligned}$$

Optimal Contracts with Symmetric Information

- Dividing last slide's first by second condition we get:
- The contract for the G type is

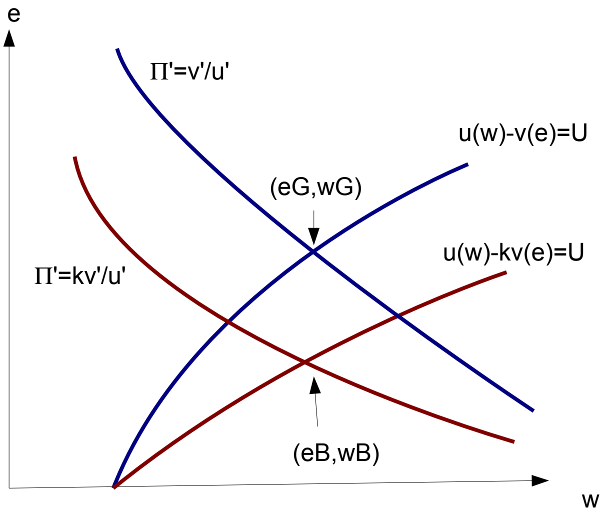
$$\begin{aligned} u(w^G) - v(e^G) &= \underline{U}, \\ \Pi'(e^G) &= \frac{v'(e^G)}{u'(w^G)}. \end{aligned}$$

- The contract for the B type is

$$\begin{aligned} u(w^B) - kv(e^B) &= \underline{U}, \\ \Pi'(e^B) &= \frac{kv'(e^B)}{u'(w^B)}. \end{aligned}$$

- Assume that P's utility concave in effort, A's cost convex in effort, A's utility concave in wage. $\rightarrow$ graph next page.

Optimal Contract



Optimal Contract with Asymmetric Information

- If P does not know the type of the agent, offering the two optimal contracts (w^B, e^B) and (w^G, e^G) will not work.
  - The B type will choose (w^B, e^B).
  - The G type will also choose (w^B, e^B) which gives them higher utility than (w^G, e^G). (by picking "her" G contract she gets U while otherwise more than U, given $k > 1$)
- The optimal contract under asymmetric information must take this into account and introduce an *incentive constraint*, stating that each type prefers the contract that is directed to that type. $\rightarrow$ Basis of the **Revelation Principle**.

The Principal's Problem

- The principal maximizes $p(\Pi(e^G) - w^G) + (1-p)(\Pi(e^B) - w^B)$ subject to participation constraints and incentive compatibility constraints of types G and B:
  - $u(w^G) - v(e^G) \geq \underline{U}$ (PG);
  - $u(w^B) - kv(e^B) \geq \underline{U}$ **(PB)**;
  - $u(w^G) - v(e^G) \geq u(w^B) - v(e^B)$ **(IG)**;
  - $u(w^B) - kv(e^B) \geq u(w^G) - kv(e^G)$ (IB).

The Participation Constraint (PG) Is Not Binding

$$\begin{aligned} u(w^G) - v(e^G) &\geq u(w^B) - v(e^B) \text{ by IG} \\ &> u(w^B) - kv(e^B) \textbf{ because } \text{ k} > 1 \\ &\geq \underline{U} \text{ by PB.} \end{aligned}$$

- This shows that the participation constraint of the good type is never binding and can be ignored.
- PB is binding in equilbirum, as otherwise Principal could deviate and increase benefit by marginally lowering w^B.
- IG also needs to be binding in equilibrium, as otherwise Principle could deviate and increase benefit by marginally lowering w^G (given that PG is not binding)
- IB not binding (remember: w^B pinned down by PB)

Binding Constraints

- In the optimal contract, PB and IG are binding; that is,

$$\begin{aligned} u(w^B) - kv(e^B) &= \underline{U} \\ u(w^G) - v(e^G) &= u(w^B) - v(e^B) > \underline{U}. \end{aligned}$$

- The bad type is at the reservation utility level.
- The good type gets a utility above the reservation utility; that is, an **informational rent**.

Optimal Contract

- We compute the Lagrangian

$$\begin{aligned}\mathcal{L} &= p(\Pi(e^G) - w^G) + (1-p)(\Pi(e^B) - w^B) \\ &+ \lambda(\underline{U} - u(w^B) + kv(e^B)) \\ &+ \mu(u(w^G) - v(e^G) - u(w^B) + v(e^B)).\end{aligned}$$

- Taking the first order conditions, we get:

$$\begin{aligned} p\Pi'(e^G) - \mu v'(e^G) &= 0, \\ -p + \mu u'(w^G) &= 0, \\ (1-p)\Pi'(e^B) + k\lambda v'(e^B) + \mu v'(e^B) &= 0, \\ -(1-p) - \lambda u'(w^B) - \mu u'(w^B) &= 0. \end{aligned}$$

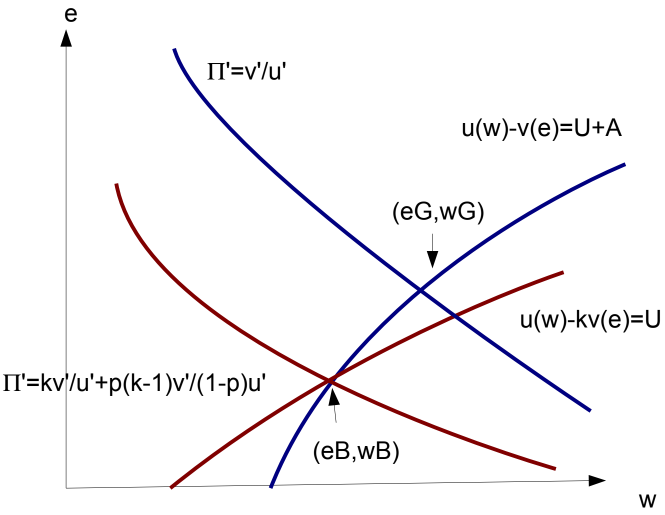
Optimal Contract (Cont.)

- Solving the first order conditions, we obtain:

$$
\begin{aligned}
\Pi'(e^G) &= \frac{v'(e^G)}{u'(w^G)}, \\
\Pi'(e^B) &= \frac{kv'(e^B)}{u'(w^B)} + \frac{p(k-1)v'(e^B)}{(1-p)u'(w^G)}.
\end{aligned}
$$

- *The effort choice of the good type is always efficient.*
- The effort choice of the bad type is inefficient. It is *lower than the efficient effort level*.

Optimal Contract (Cont.)



Summary

- Asymmetric information may result in a market collapse.
- In the market for lemons, the fact that sellers have private information on the quality of the car prevents the market from functioning.
- In a signaling game, the informed agent moves first, and may reveal her type by choosing a different action for each type.
- Signaling games have two types of equilibria: separating equilibria and pooling equilibria.
- In Spence's education model (1973), a plausible equilibrium is the separating equilibrium where only the good type gets education.

Summary (Cont.)

- A screening contract proposes two different effort-wage levels for the two different groups.
- The binding constraints are the participation constraint of the bad type and the incentive constraint of the good type.
- The bad type is pushed to the reservation utility level, whereas the good type collects an informational rent.
- The good type's effort choice is efficient, whereas the bad type's effort choice in the optimal contract is too low.

I.2 Moral Hazard

Moral Hazard (Cont.)

- We consider a principle agent setting of interactions between two agents: a *principal* and an *agent*.
- The principal has the bargaining power and designs a *contract*.
- The agent reacts to the principal's contract and chooses an *action* or a *message*.
- The principal is risk neutral, whereas the agent is risk averse.

Examples

- Landlord (P) vs. Tenant (A)
- Employer (P) vs. Employee (A)
- Insurer (P) vs. Insured (A)

Asymmetric Information

- The agent knows something that the principal cannot observe. As mentioned above, we have:
  - *Hidden Information*: P cannot observe A's type (Adverse Selection). $\rightarrow$ Previous topic.
  - *Hidden Action*: P cannot observe A's action (Moral Hazard).
- Due to asymmetric information, the agent can extract a *rent*.
- Question: How can one minimize the agent's rent? What is the optimal contract chosen by the principal?

Basic Model

- There is a result $x \in X$ (e.g. sales, harvest). The result is always observed by both parties.
- The agent chooses an effort level e.
- The relation between effort and the result is not deterministic but stochastic (if deterministic, the principal could have deduced the effort from the result).
- Results are ordered $x_1 < x_2 < ... < x_n$.
- $\Pr[x = x_i] = p_i(e)$; suppose that $p_i(e) > 0$ for all i, e.

Utilities

- The principal obtains the result x and pays a wage w.
- Her utility is given by $B(x - w)$ with $B' > 0$ and $B'' \leq 0$. *We often assume $B'' = 0$; that is, the principal is risk neutral.*
- The agent's utility is separable in wage and effort:
$$u(w, e) = u(w) - v(e),$$
where $u' > 0, u'' < 0$, $v' > 0, v'' > 0$. (as usual, concave utility function, convex cost function)
- Conflict of interests: the principal cares about the result but not the effort, whereas the agent cares about the effort but not the result.

Reservation Utilities, Participation and Contract

- The time sequence of the model is the following.

1. P offers a contract, specifying a wage for the agent as a function of observed variables.
2. A accepts or rejects the contract. If A rejects, receives a *reservation utility* $\overline{U}$.
3. A chooses effort.
4. Results are realized, and a wage is paid to A.

Contracts with Symmetric Information

- Suppose that information is symmetric; that is, P observes the effort of A.

- A contract specifies both a payment scheme $w(x_i)$ and an effort level e in order to maximize

$$\sum_i p_i(e)B(x_i - w(x_i)),$$

subject to the constraint

$$\sum_i p_i(e)u(w(x_i)) - v(e) \geq \overline{U}.$$

The constraint is called the *participation constraint* of the agent (analogous logic as before).

Optimal Risk Sharing

- The Lagrangian is

$$\mathcal{L} = \sum_i p_i(e)B(x_i - w(x_i)) - \lambda[\overline{U} - (\sum p_i(e)u(w(x_i)) - v(e))].$$

- Fix e. For any x_i, the wage $w(x_i)$ is chosen to maximize the Lagrangian, thus:

$$\frac{\partial \mathcal{L}}{\partial w(x_i)} = -p_i(e)B'(x_i - w(x_i)) + \lambda p_i(e)u'(w(x_i)) = 0.$$

- Hence, for any x_i,

$$\frac{B'(x_i - w(x_i))}{u'(w(x_i))} = \lambda.$$

- *Optimal risk sharing occurs where marginal utilities are equalized for the principal and the agent.*

Principal and Agent

- If P is risk neutral, B' is a constant, so $u'(w(x_i))$ is a constant; that is, $w(x_1) = w(x_2) = ... = w(x_n)$.
- The agent receives a *fixed wage*.

Moral Hazard

- The moral hazard problem arises when the effort of the agent is not observed.
- For one, a farmer's effort is not observed but the harvest is.
- For another, a salesman's effort is not observed but sales are.
- Finally, the behavior of a driver is not observed but accidents are.

Incentive Constraint

- In addition to the participation constraint, there is an *incentive constraint*.

- If P wants to induce effort level e^* it must be that
$$\sum p_i(e^*)u(W(x_i)) - v(e^*) \geq \sum p_i(e)u(W(x_i)) - v(e) \quad \forall e.$$

- If there are only two effort levels, the incentive constraint is simple to write.

- If there are many effort levels, the condition becomes difficult to write!

Two Effort Levels, Two Results

- Suppose that there are two effort levels, e^H and e^L with costs $v(e^H) = c^H > c^L = v(e^L)$.
- There are two possible results x^L, x^H and the probabilities are given by

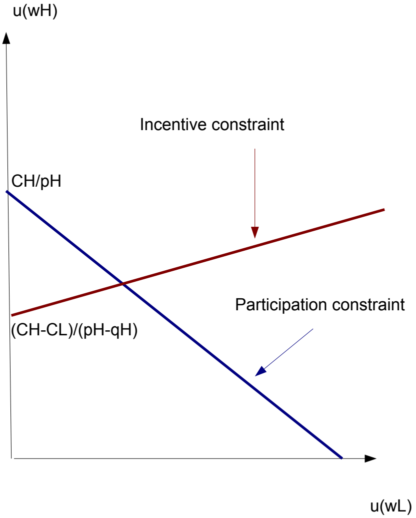
	x^H	x^L
e^H	p^H	p^L
e^L	q^H	q^L

- with $p^H > q^H$, $p^H + p^L = 1$, $q^H + q^L = 1$.

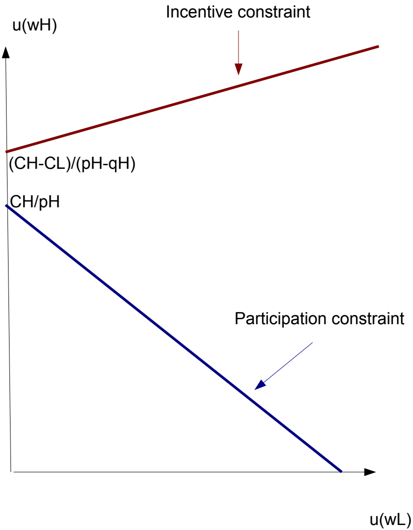
Participation and Incentive Constraints With e^H

- **Suppose that P wants to implement the high effort level.**
- The participation constraint is
$$p^H u(w^H) + p^L u(w^L) - c^H \geq \overline{U}.$$
- The incentive constraint is
$$p^H u(w^H) + p^L u(w^L) - c^H \geq q^H u(w^H) + q^L u(w^L) - c^L$$
or
$$(p^H - q^H) u(w^H) + (p^L - q^L) u(w^L) \geq (c^H - c^L).$$
- In the following graph we will put $\overline{U} = 0$.

Participation and Incentive Constraints (Cont.)



Participation and Incentive Constraints (Cont.)



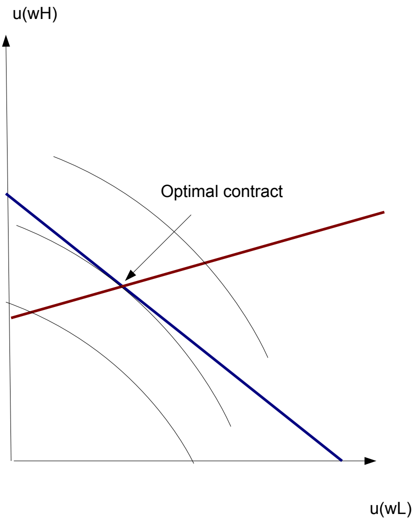
Principal's Profit

- The principal maximizes her profit
$$p^H(x^H - w^H) + p^L(x^L - w^L).$$

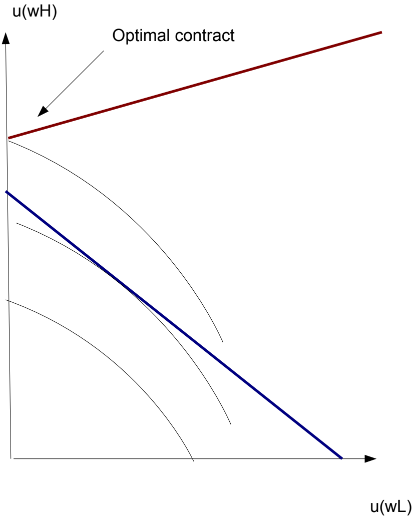
- We consider the isoprofit curves
$$p^H(x^H - w^H) + p^L(x^L - w^L) = \pi.$$

- These curves are decreasing curves in the space $(u(w^H), u(w^L))$ and they increase towards the southwest.

Optimal Contract



Optimal Contract (Cont.)



Interpretation

- The optimal contract always lies on the incentive constraint.
- The agent is sometimes pushed down to her reservation utility, sometimes not.

Mathematical Solution

- The Lagrangian is

$$\begin{aligned}\mathcal{L} &= p^H(x^H - w^H) + p^L(x^L - w^L) \\ &+ \lambda(\overline{U} - (p^H u(w^H) + p^L u(w^L) - c^H) \\ &+ \mu((c^H - c^L) - (p^H - q^H)u(w^H) - (p^L - q^L)u(w^L)).\end{aligned}$$

- This leads to

$$\begin{aligned}-1 - \lambda u'(w^H) - \mu(1 - \frac{q^H}{p^H})u'(w^H) &= 0, \\ -1 - \lambda u'(w^L) - \mu(1 - \frac{q^L}{p^L})u'(w^L) &= 0.\end{aligned}$$

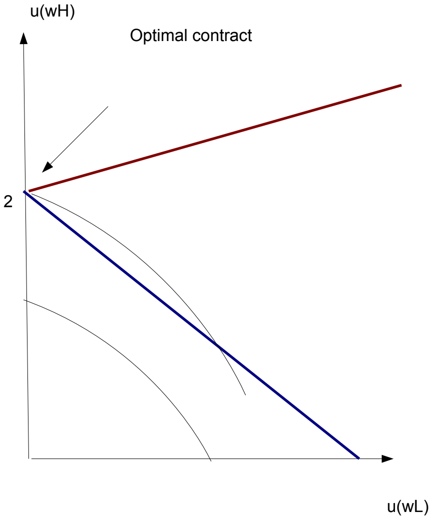
Interpretation of the Solution

- As $\mu \neq 0$, the optimal wages *are not constant*; that is $w^H \neq w^L$.
- In fact, as $\frac{q^H}{p^H} \leq \frac{q^L}{p^L}$, $w^H > w^L$.

Example

- Suppose $x^L = 0$, $x^H = 20$, $p^H = \frac{1}{2}$, $q^H = \frac{1}{4}$, $c^L = 0$, $c^H = \frac{1}{2}$.
- Suppose $u(w) = \sqrt{w}$, $\overline{U} = \frac{1}{2}$.
- The participation constraint is
$$\frac{1}{2}\sqrt{w_H} + \frac{1}{2}\sqrt{w_L} - \frac{1}{2} \geq \frac{1}{2} \rightarrow \sqrt{w_H} + \sqrt{w_L} \geq 2.$$
- The incentive constraint is
$$\frac{1}{4}\sqrt{w_H} - \frac{1}{4}\sqrt{w_L} \geq \frac{1}{2} \rightarrow \sqrt{w_H} = \sqrt{w_L} + 2.$$

Optimal Contract



Optimal Contract (Cont.)

- In the optimal contract, $w^H = 4$, $w^L = 0$.
- The profit of P is $\frac{1}{2}(20 - 4) = 8$.

Participation Constraint With e^L

- **Suppose that P wants to implement the low effort level.**
- The participation constraint is

$$\frac{1}{4}\sqrt{w_H} + \frac{3}{4}\sqrt{w_L} = \frac{1}{2}.$$

- As $w^H = w^L$, the wage is $w = \frac{1}{4}$, and the expected profit

$$\frac{20}{4} - \frac{1}{4} = \frac{19}{4}.$$

Other Models of Moral Hazard

- Severity of punishments.
- Giving bargaining power to the agent.
- Multiple agents: yardstick competition and using information on relative performance.
- Multiple principals: competition between principals and common agency.

Summary

- Consider a contract between two individuals: an agent and a principal.
- The principal proposes the contract, the agent accepts or rejects, and the contract is executed.
- The agent's effort is not observed (hidden action).
- In a symmetric information contract, the principal only faces a participation constraint, and risk is optimally shared between the principal and the agent.
- If the principal is risk neutral, the optimal contract is a fixed wage

Summary (Cont.)

- When the effort of the agent is not observed, the principal offers an incentive contract, where wages depend on the results.
- The principal faces both a participation and an incentive constraint.
- In the optimal contract, the incentive constraint is satisfied with equality; the agent may or may not be driven to her reservation utility.
- In the optimal contract, higher results are rewarded with higher wages.

I.3 Application: Insurance - In a nutshell I

- We will now discuss a particularly salient area where asymmetric information plays a crucial role: Insurance.
- Individuals are generally risk averse
    - This means that uncertainty per se causes disutility and creates a demand for insurance.
    - Private insurance is incomplete or inefficient when there are problems of asymmetric information, i.e. moral hazard and adverse selection, or when there are high administrative costs.
    - For example, the health sector suffers from adverse selection problems.

Insurance - In a nutshell II

- Arguably, the most important source of uncertainty comes from having “good luck” or “bad luck” in life
  - Good and bad luck can be replaced by high and low ability, or high and low income...
  - Insurance against poverty thus emerges as a reason for state existence
  - Taking income from the rich and giving it to the poor (grounded on discussions on social justice)
  - A large part of the public sector is about insurance against poor health (health and disability insurance) or poverty (social assistance)

Risk aversion I

- Risk aversion

Definition

A decision maker is a risk averter (or exhibits **risk aversion**) if for any lottery $F(\cdot)$, the degenerate lottery that yields the amount $\int x dF(x)$ with certainty is at least as good as the lottery $F(\cdot)$ itself. If the decision maker is always [i.e. for any $F(\cdot)$] indifferent between these two lotteries, we say that he is **risk neutral**. Finally, we say that he is **strictly risk averse** if indifference holds only when the two lotteries are the same [i.e. when $F(\cdot)$ is degenerate].

Mas-Collel, Whinston and Green (1995)

Risk aversion II

- For preferences assuming an expected utility representation, it follows from the definition of risk-aversion that the decision maker is risk-averse iff:

$$\int u(x)\, dF(x) \leq u\left(\int x dF(x)\right) \text{ for all } F(\cdot).$$

- This inequality is called *Jensen's inequality*
- It is also the defining property of a concave function
- Therefore, in the context of expected utility theory, strict risk-aversion and strict concavity of $u(\cdot)$ are equivalent

Risk aversion III

- Strict concavity means that the marginal utility of income is decreasing
  - As a result, at any level of wealth x, the utility gain from another dollar is less than (the absolute value of) the utility loss of having a dollar less
  - It follows that, for a risk-averse person, an even chance of getting or losing a dollar is not worth taking

Risk aversion IV

Example

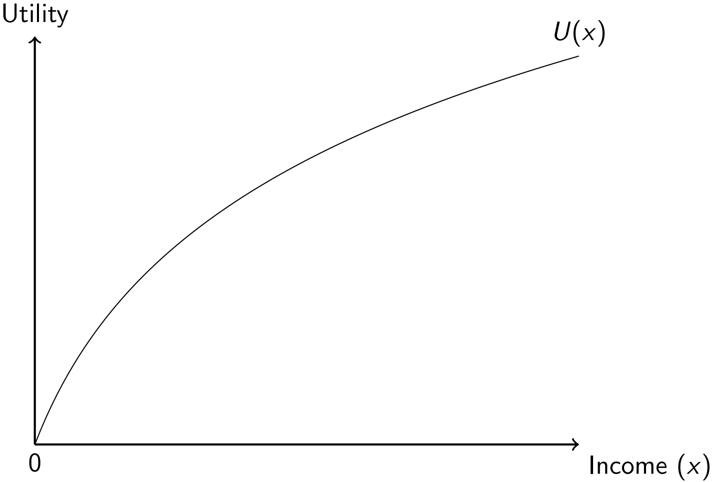
- Bad outcome y_1, good outcome y_2, occurring with probabilities p_1 and p_2 respectively, where $p_2 = 1 - p_1$.
- Expected income: $E(y) = y_a = p_1 y_1 + (1 - p_1) y_2$
- Expected utility: $E(U) = U_a = p_1 U(y_1) + (1 - p_1) U(y_2)$
- A rational risk averse individual would be indifferent between a lottery with an expected income y_a and lower income y^* received with certainty
- Quantity y^* is called the *certainty equivalent* (of this lottery (y_1, y_2, p_1))

Risk aversion V

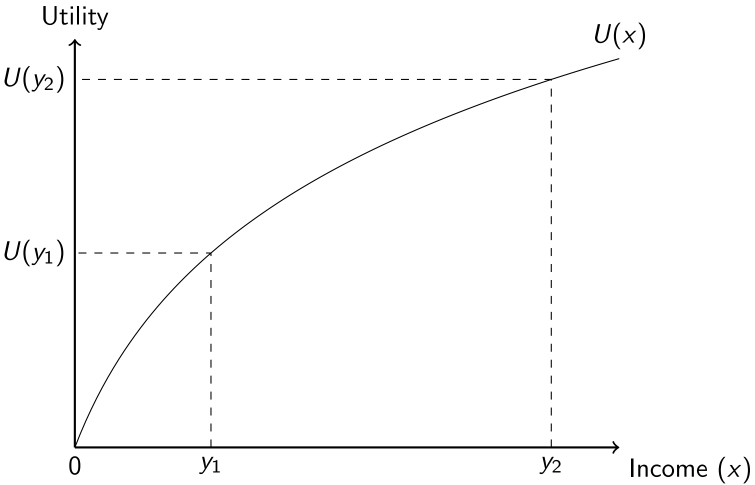
A numerical example:

- Expected income: $E(y) = y_a = 0.5 * 100 + 0.5 * 1000 = 550$
- Expected utility: $E(U) = U_a = 0.5 * U(100) + 0.5 * U(1000)$
- The utility function is concave $\rightarrow$ Ex: $U(x) = ln(x)$
- $U(y_a) = ln(550) = 6.31$
- $U_a = 0.5 * ln(100) + 0.5 * ln(1000) = 5.76 (< 6.31)$
- To find the certainty equivalent y^*: $ln(y^*) = 5.76 \rightarrow y^* = 316$

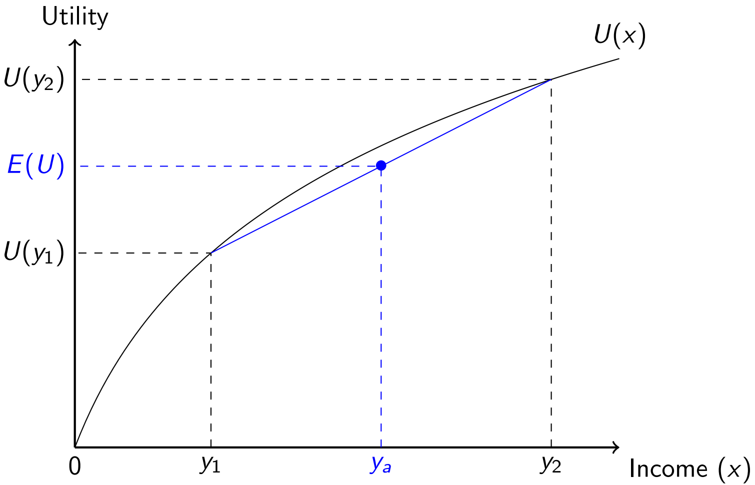
Risk aversion VI



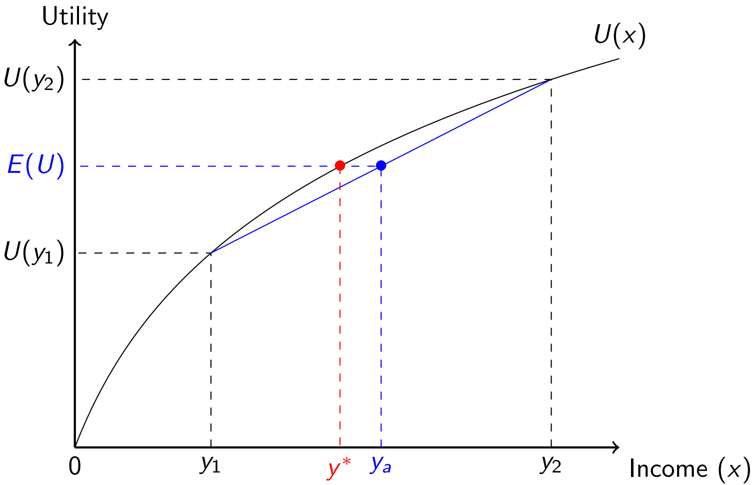
Risk aversion VI



Risk aversion VI



Risk aversion VI



Demand for insurance I

- Let L (loss) be the difference between high and low income:

$$L \equiv Y_2 - Y_1.$$

- Suppose people faced the following contract:
  - One unit of insurance costs q dollars and is paid every period to the insurance company; accident takes place with likelihood p
  - In case of accident, the insured gets 1 dollar back
- For each dollar of insurance sold, expected profits for the insurance company equal

$$\text{Expected Profit } = q - p$$

- When insurance is actuarially fair (meaning that the price of insurance equals its expected cost), then $q = p$

Demand for insurance II

- In such a case, people optimally chose to fully insure and will purchase exactly L units of insurance
- When doing this, people accomplish identical income across the two income states of the world
  - Further, this common income equals the average income of the lottery, y_a
- The law of large numbers makes it possible to exploit "gains from trade" by pooling risk
  - The variance of average income and hence the risk of the individual is zero in the limit when the number of insurance takers N approaches infinity
  - Insurance companies take from those who experienced the good state and give income to those who experienced the bad state

Supply of insurance I

- The actuarial premium: $\pi = (1+\alpha)pL$, α=loading to cover administrative costs and normal profit
- π is the price at which insurance will be supplied in a competitive market
- For private market insurance to be possible and efficient, we need:

1. Independent probabilities
   - If individual probabilities are linked, the insurance fails to provide protection $\rightarrow$ it can only cope with individual risk and not with systemic shocks
2. Probability less than one
   - Otherwise $\pi = (1+\alpha)pL > L \rightarrow$ there will be no demand for this insurance (e.g. chronically ill)

Supply of insurance II

3 Known and estimable probabilities

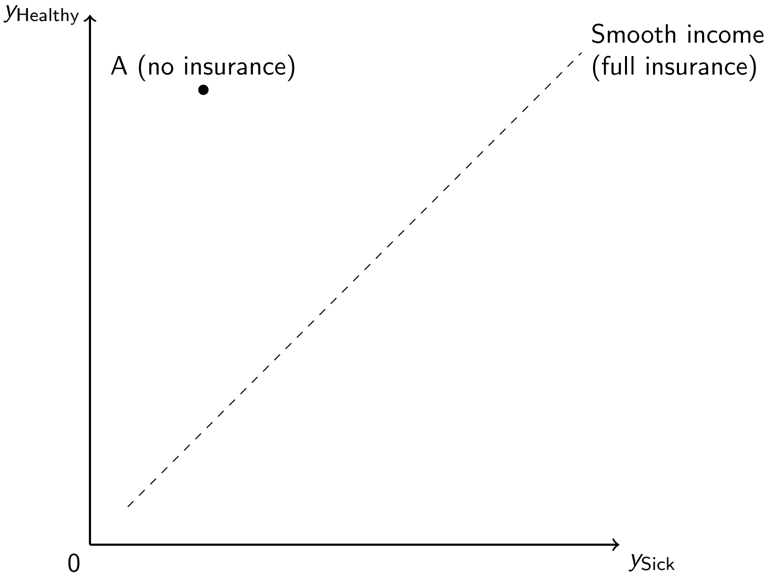
- Insurance addresses risk but cannot cope with uncertainty
- Rare, complex or very variable events cannot be insured
- Ex: Risk of unknown or rare disease, future inflation

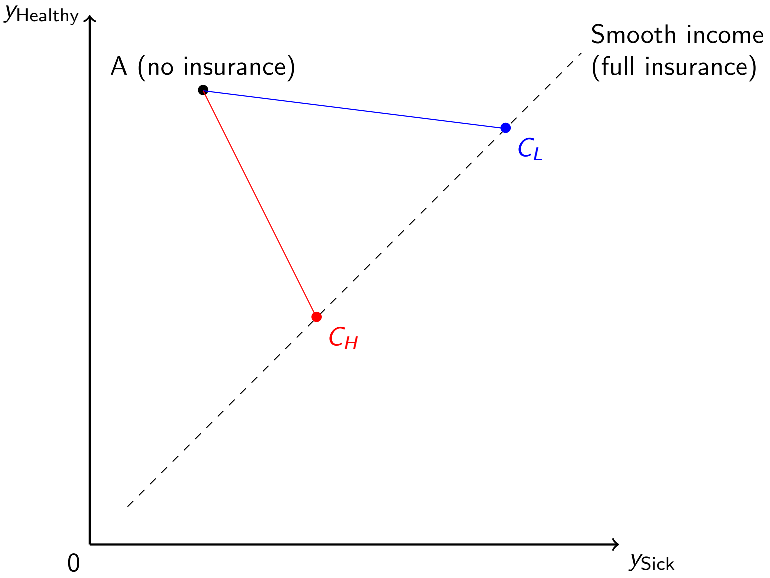
4 No asymmetric information (No adverse selection and moral hazard)

- Cf. next slides

Insurance - Adverse selection I

- As discussed before, a key pioneer is George Akerlof (born 1940; Nobel Prize 2001)
- As in Akerlof's "lemons" problem, an individual that is poor risk will try to hide this information
- First, consider a case where individual probabilities are known
- $\pi_L = (1+\alpha)p_L L \rightarrow$ offered contract C_L (L=low risk)
- $\pi_H = (1+\alpha)p_H L \rightarrow$ offered contract C_H (H=high risk)

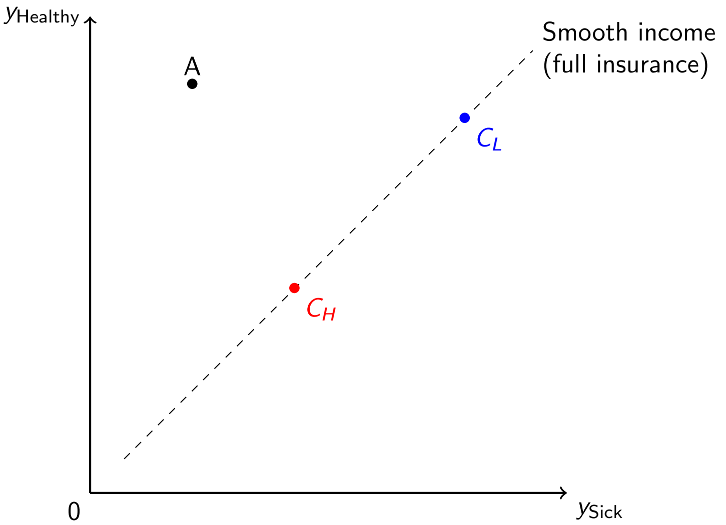

y_{Healthy}
A (no insurance)
Smooth income
(full insurance)
0
y_{Sick}


y_{Healthy}
A (no insurance)
Smooth income
(full insurance)
C_L
C_H
0
y_{Sick}

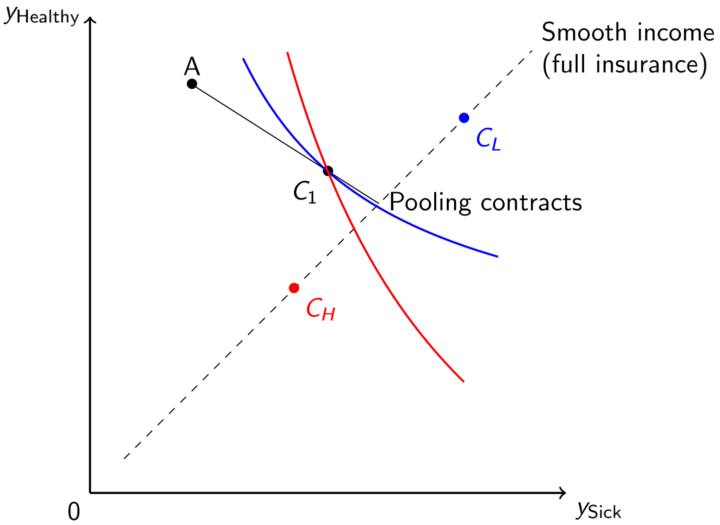
Insurance - Adverse selection III

- However, when an insurer cannot distinguish low and high risk types, different equilibria could take place
- Pooling equilibrium $\rightarrow$ all receive same deal
- Separating equilibrium $\rightarrow$ find ways to separate the two types
- Pooling equilibrium with premium based on average risk $\pi_a = (1+\alpha)(\theta p_H + (1-\theta)p_L)L$, where $\theta =$ proportion high-risk types in society
- Such a pooling equilibrium is socially efficient but unstable

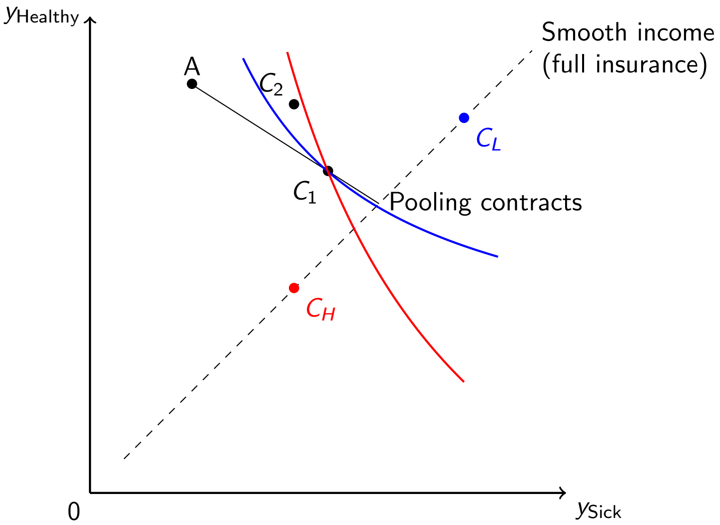
Insurance - Adverse selection IV



Insurance - Adverse selection IV



Insurance - Adverse selection IV



Insurance - Adverse selection V

Implications of Adverse selection:

- By offering C_2 (lower premium and lower insurance), an insurance company would attract only low-risk types.
- "Cream skimming" methods lead to separating equilibria.
- Socially inefficient situation with partial coverage of risks.
- Pooling equilibrium can only be sustained if it is mandatory: state-monitored (social) insurance.

Insurance - Moral hazard

- As seen above, moral hazard is about "hidden actions" $\rightarrow$ the individual can influence the probability p of loss with her actions
- This can induce people to underinvest in prevention
  - Ex: Insurance can lead to less careful driving, parking
- The problem is similar for endogenous L
  - Can result in over-consumption (Ex: repair car lavishly)
- Various measures and mechanisms have been developed to counter these problems
  - Inspection, Frequent claims leading to higher premium, Deductibles, Coinsurance (people pay percentage of claim) etc.
- But none of these imply the full marginal cost for the individual $\rightarrow$ private insurance may not be feasible $\rightarrow$ there can be role for social insurance

Insurance - Summary

- Asymmetric information is at the heart of insurance questions
- Both Adverse Selection and Moral Hazard salient challenges
- These questions are not only important for the very sizable private insurance industry, but also for public policy
- Key policy questions concern the scope for mandatory insurance and the relative size of the welfare state, which is an important political battleground e.g. in the United States