Appendix B

Basic ideas about DSGE

Calibration

For the purposes of the numerical implementation of the model, it is necessary to assign values to the parameters. Generally speaking, the most recommended course of action is to estimate them. On the other hand, calibration is still quite popular among researchers working with DSGE, but to talk in depth about the methods used in DSGE model estimation is not within the realms of this book. So, in short, calibration will be discussed using the RBC model as an example.

The rate of depreciation of capital stock (δ), is usually estimated using a database and an equation representing the movement of capital stock intertemporally, such as the equation (2.26), $I_{ss} = \delta K_{ss}$. For quarterly data, the literature works with values between 0.02 and 0.03. In the case of annual data, it would be between 0.04 and 0.1. In this book, the data are considered to be quarterly, so the equation $\delta = 0.025$ will be assumed.

The parameter β is called the discount factor, representing how the agents assess future utility versus present utility. The literature assumes that this parameter falls between 0.97 for annual data and 0.99 for quarterly data. On the other hand, its value may be obtained from the equation (2.25). Assuming an average quarterly nominal interest rate of four percent ($R_{ss} = 4\%$)[9]:

$$\beta = \frac{1}{R_{ss} + (1 - \delta)} = 0.985$$

[9]As in this model there is no financial bond, the interest rate may be used as a proxy for return on capital.

The proportion of capital used in the production process, α, is obtained from data in the domestic accounts. International literature works with values between 0.3 and 0.4. In this work, it is being adopted $\alpha = 0.35$. The value of the autoregressive parameter, ρ_A, is normally greater than 0.9. However, it is possible to obtain an estimation of the total productivity of factors like the production function residual (known as the Solow Residual), and then estimate this parameter.

There is little consensus over the values for the relative risk aversion coefficient[10], σ. Vereda and Cavalcanti (2010) conducted parameterizations in a quest to determine the parameter limits of a DSGE model for Brazil, and the value found by the authors was between 1 and 3. In this book, the average of this study has been assumed, $\sigma = 2$. The same problem occurs with the marginal disutility of labor[11], φ. The result from the Vereda and Cavalcanti (2010) study is between 0 and 3. The choice for this parameter was also the average of the result of these authors, namely $\varphi = 1.5$.

In summary, to calibrate for DSGE modeling means assigning values to the parameters, in some form. No one form is more correct than the other, but it is always necessary to proceed with caution and common sense.

Blanchard-Kahn (BK) unique solution and stability condition

The model, in linearized form, may be expressed in state-space form as:

$$E\begin{bmatrix} z_t \\ E_t x_{t+1} \end{bmatrix} = A_0 \begin{bmatrix} z_{t-1} \\ x_t \end{bmatrix} + Dr_{n,t} + G\varepsilon_t \tag{121}$$

where z_t is a vector of predetermined variables in time t, x_t is a vector of forward-looking variables. E, A_0, D and G are matrices and ε_t is a shock vector, and

[10]Christiano, Eichenbaum and Evans (2005), Juillard *et al.* (2006) and Rotemberg and Woodford (1997), assign 1, 1.25 and 6.25, respectively.

[11]Christiano, Eichenbaum and Evans (2005) and Juillard *et al.* (2006), define as 1 and 3, respectively.

$$r_{n,t} = K \begin{bmatrix} z_{t-1} \\ x_t \end{bmatrix}$$

where K is defined depending on the model.

So the equation (B.1) is:

$$E \begin{bmatrix} z_t \\ E_t x_{t+1} \end{bmatrix} = [A_0 + DK] \begin{bmatrix} z_{t-1} \\ x_t \end{bmatrix} + G\varepsilon_t$$

or,

$$E \begin{bmatrix} z_t \\ E_t x_{t+1} \end{bmatrix} = A \begin{bmatrix} z_{t-1} \\ x_t \end{bmatrix} + G\varepsilon_t$$

resulting in,

$$\begin{bmatrix} z_t \\ E_t x_{t+1} \end{bmatrix} = \bar{A} \begin{bmatrix} z_{t-1} \\ x_t \end{bmatrix} + \bar{G}\varepsilon_t \tag{122}$$

where $\bar{A} = E^{-1}A$ and $\bar{G} = E^{-1}G$.

Thus rational expectations are formed using a set of information $\{z_s, x_{s+1}, \varepsilon_s\}$, with $s \leq t$.

The condition for unique equilibrium and stability depends on the magnitude of the eigenvalues of the matrix $(A_0 + DK)$. If the number of eigenvalues with an absolute value greater than 1 (unstable root) is equal to the number of forward-looking variables, the system has a unique solution and is also stable on a saddle path (Blanchard and Kahn, 1980). On the other hand, indeterminacy occurs when the number of eigenvalues of the matrix $(A_0 + DK)$ with an absolute value greater than 1 is lower than the number of forward-looking variables (many stable roots). This means that when the shock shifts the economy out of its steady state, many paths exist that lead to equilibrium, in other words, there are multiple solutions to the model. The possibility also exists of the number of unstable roots being greater than the number of forward-looking variables (many unstable roots). In this case, the system has no solution and all paths are explosive (Figure B.1).

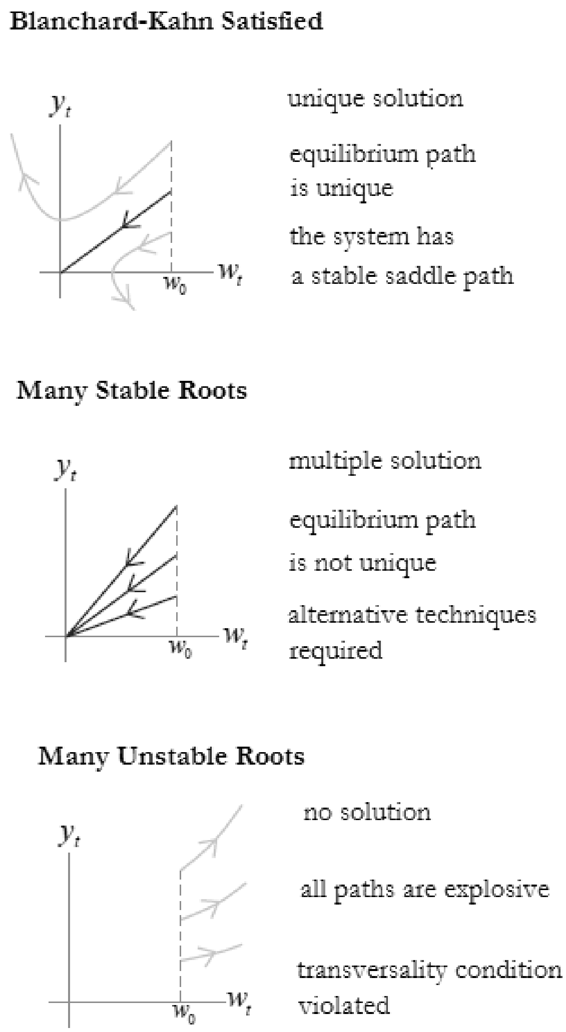

Figure 10: Examples of positive results for the BK analysis.