Demystifying DSGE Models

10. Internationality: The Two-Country Model

- This is the International Economics Section
- But we have so far ignored "internationality"
- This week, we remedy this by considering models which allow for two economies: Home (H) and Foreign (F)
- Typically this is done by considering a "small open economy" (SOE)
- In SOE models, one country is supposed to be small economically, whereas other is large
- ➔ large country is not dependent on small one, but reverse is not true

- To create an SOE model from a closed economy model, we need essentially to:
  - add exports and imports functions
  - create a foreign economy
  - link the two economies together via
    - exchange rate
    - UIP
  - differentiate Home and Foreign goods, for both consumption and investment
  - deal with import and export prices

GSW2012 in OE version

- To begin, let us develop an ***SOE version*** of the ***GSW2012*** model mentioned last week
- Our two economies will be linked by *[NEW]* ***trade*** and by the *[NEW]* ***UIP condition***
- *[NEW]* ***Prices of exports*** will feed into importing country prices
- *[NEW]* ***Exchange rates*** will also be a feature since import/export demand will depend on ***relative prices*** in Home country currency
- ***Final demand*** for ***consumer*** goods will be *[NEW]* a ***weighted average*** of ***Home*** and ***Foreign*** production of such goods

- Similarly, ***[NEW] final demand*** for ***investment*** goods will also be a (different) weighted average of ***Home*** and ***Foreign*** production of such goods
- ***Aggregate output*** in Home will now ***[NEW]*** include ***exports*** and ***imports***:

$$Y = C + I + G + (X - M)$$

where

- X = Home ***exports***; will depend on ***Foreign demand***
- M = Home ***imports***; will be a ***weighted average*** of ***Foreign output*** of consumer and investment goods, again depending on relative prices

- That is basically all that is needed to adapt ***GSW2012*** to open economy version
- However, we must now define carefully the various ***demand functions***
- As seen at beginning of course, demand functions are derived from usual ***Dixit-Stiglitz*** aggregator
- Home consumption is ∴ a CES composite bundle

$$C_t = \left[(1-\alpha) C_{H,t}^{\frac{\eta_c - 1}{\eta_c}} + \alpha C_{F,t}^{\frac{\eta_c - 1}{\eta_c}} \right]^{\frac{\eta_c}{\eta_c - 1}}$$

$\eta_c > 0$ = elasticity of ***substitution*** between domestic (**H**) ***and*** foreign (**F**) consumer goods

- In this CES composite, $C_{H,t}$ and $C_{F,t}$ are themselves usual ***Dixit-Stiglitz aggregates*** of available domestic and foreign produced goods given by

$$C_{H,t} = \left[\int_0^1 C_{H,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \quad \text{and} \quad C_{F,t} = \left[\int_0^1 C_{F,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

- where $\varepsilon > 1$ = (common) elasticity of ***substitution*** between types of ***differentiated*** Home ***or*** Foreign consumer goods

- And same is true for ***investment goods***

$$I_t = \left[(1-\alpha) I_{H,t}^{\frac{\eta_i - 1}{\eta_i}} + \alpha I_{F,t}^{\frac{\eta_i - 1}{\eta_i}} \right]^{\frac{\eta_i}{\eta_i - 1}}$$

  - where $\eta_i > 0$ = elasticity of ***substitution*** between domestic (**H**) ***and*** foreign (**F**) investment goods

- And

$$I_{H,t} = \left[\int_0^1 I_{H,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \quad \text{and} \quad I_{F,t} = \left[\int_0^1 I_{F,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

  - where $\varepsilon > 1$ = (common) elasticity of ***substitution*** between types of ***differentiated*** Home ***or*** Foreign investment goods

- ***Household's optimisation problem*** requires allocation of expenditures across ***both*** types (consumption and investment) of domestic ***and [NEW] foreign*** goods both ***intra***temporally and ***inter***temporally
- Optimal allocation of expenditure across domestic and foreign goods ➔ ***demand functions***

$$C_{H,t} = \left(P_{HC,t}/PC_t\right)^{-\theta_c} C_t \text{ and } C_{F,t} = \left(P_{FC,t}/PC_t\right)^{-\theta_c} C_t$$

- where
  - θ_c = ***elasticity of substitution*** between (differentiated) consumption goods
  - P_{HC} = price in Home market (and Home currency) of ***Home***-produced consumption goods
  - P_{FC} = price in Home market (and Home currency) of ***Foreign***-produced consumption goods
  - PC = overall price level in Home market (and Home currency) of ***composite*** consumption goods [= CPI]

- ➔ ***Demand*** for ***Home***-produced ***consumption*** goods will therefore be
  - ***directly proportional*** to aggregate ***demand*** for consumption goods
  - ***inversely proportional*** to ***relative price*** of such Home-produced goods
- And similarly for ***Foreign***-produced consumption goods
- And ***same*** will be true for ***investment*** goods, whether ***Home*** or ***Foreign***

→

$$I_{H,t} = \left(P_{HI,t}/PI_t\right)^{-\theta_i} I_t \text{ and } I_{F,t} = \left(P_{FI,t}/PI_t\right)^{-\theta_i} I_t$$

- where
  - θ_i = ***elasticity of substitution*** between (differentiated) investment goods
  - P_{HI} = price in Home market (and Home currency) of ***Home***-produced investment goods
  - P_{FI} = price in Home market (and Home currency) of ***Foreign***-produced investment goods
  - PI = overall price level in Home market (and Home currency) of ***composite*** investment goods

- What of ***exports***?
- By a similar logic, Home ***exports*** to Foreign will be
  - ***directly proportional*** to aggregate Foreign ***demand*** for imported goods
  - ***inversely proportional*** to the ***relative price*** (in Foreign) of such Home-produced goods

$$X_{H,t}^{*} = \omega^{*} \left(\frac{P_{H,t}^{*}}{P_{t}^{*}} \right)^{-\eta_{*}} \boxed{Y_{t}^{*}} = \omega^{*} \left(\frac{P_{H,t}}{\boxed{\text{rer}_{t}}} \right)^{-\eta_{*}} Y_{t}^{*}$$

  - where ω^* is ***proportionality*** factor relating ***imports*** to overall ***demand*** ***Y**** in Foreign
  - and ***rer*** = ***real exchange rate***

- ***Nominal exchange rate*** S_t measures units of domestic currency per unit of foreign currency

➔ $S_t P^*_t$ = Foreign price level (P^*) in Home currency

- ***Law of one price*** ➔ $S_t P^*_t = P_{F,t}$

$P_{F,t}$ is price in Home market *and currency* of Foreign good

- ***But*** monopolistically competitive ***pricing power*** of domestic retail firms ➔ ***wedge*** between world market price of foreign goods ($S_t P_t^*$) paid by importing firms and domestic currency price ($P_{F,t}$) of these goods paid by domestic consumers

➔ failure of ***LOOP***

➔

$$\Psi_{F,t} \equiv \frac{S_t P_t^*}{P_{F,t}} \neq 1$$

- which defines what Monacelli (2005) calls "law of one price gap" (***LOOP Gap***)

- ***Real exchange rate rer*** is simply ***nominal*** exchange rate *S*, adjusted by ***ratio*** of Foreign price level *P** to aggregate Home price level (*P = CPI*)

$$rer_t = \frac{S_t P_t^*}{P_t}$$

NB: note that denominator here is P, ***not*** P_F as in defining LOOP Gap

$$\boxed{P_t \equiv PC_t = \left[\alpha P_{H,t}^{1-\eta_c} + (1-\alpha) P_{F,t}^{1-\eta_c}\right]^{\frac{1}{1-\eta_c}}}$$

- A bit of algebra demonstrates that we can write following useful equation linking ***inflation rates***

$$\frac{\text{rer}_t}{\text{rer}_{t-1}} = \frac{\pi_t^S \pi_t^*}{\pi_t}$$

π^S = exchange rate inflation

- In ***GSW2012***, Household j's ***budget constraint*** is

$$C_t(j) + I_t(j) + \frac{\boxed{B_t(j)}}{\varepsilon_t^b R_t P_t} - \boxed{T_t}$$

$$\leq \frac{B_{t-1}(j)}{\boxed{P_t}} + \frac{W_t^h(j) L_t(j)}{P_t} + \frac{R_t^k Z_t(j) K_t(j)}{P_t} - \boxed{a\left(Z_t(j)\right) K_t(j)} + \frac{\boxed{\mathrm{Div}_t}}{P_t}$$

- One-period bond $B_t(j)$ is expressed on a discount basis, where R_t is interest rate
- T_t are lump sum taxes or subsidies
- Div_t are (per-capita) dividends distributed by intermediate goods producers and labour unions
- P_t is overall price level
- Penultimate term represents cost associated with variations in degree of ***capital utilisation***

- Households ↑ their financial assets through ↑ their government nominal bond holdings (B_t), from which they earn an interest rate of R_t
- Return on these bonds is subject to a risk ***shock*** ε_t^b which may be considered as an exogenous ***premium*** in return to bonds, reflecting

$$\frac{B_t(j)}{\varepsilon_t^b R_t P_t}$$

  - inefficiencies in financial sector (leading to some ***premium*** on deposit rate versus risk-free rate set by central bank), or
  - a risk ***premium*** that households require to hold one period bond
- and

$$\ln \varepsilon_t^b = \rho_b \ln \varepsilon_{t-1}^b + \eta_t^b, \eta_t^b \sim N(0, \sigma_b)$$

- In ***open-economy*** context, Home households will ***also*** have access to ***Foreign***'s capital market
- Budget constraint will then ***also*** include revenue from, and expenditure on, ***Foreign bonds***
- Quarter-to-quarter changes in these amounts will obviously depend on changes in (nominal) ***exchange rate*** and ***relative interest rates*** between Home (R_t) and Foreign ($R^F{}_t$)
- Given nominal resource constraint, Home's ***net foreign asset holdings*** (B^F) are ***constrained*** by its ***trade balance*** (measured in Home currency):

$$S_t B^F{}_t = S_t B^F{}_{t-1} + TB_t$$

- And finally (in)famous ***UIP*** condition is derived from ***optimisation*** in SOE context ➔ ***Lagrange Multiplier*** identical for Home and Foreign bond holdings respectively

$$\Lambda_t = \quad R_t \left[\Lambda_{t+1} \frac{P_t}{P_{t+1}} \right] \varepsilon_t^b$$

$$\Lambda_t = \xi_t R_t^F \left[\Lambda_{t+1} \frac{S_{t+1}}{S_t} \frac{P_t}{P_{t+1}} \right] \varepsilon_t^b$$

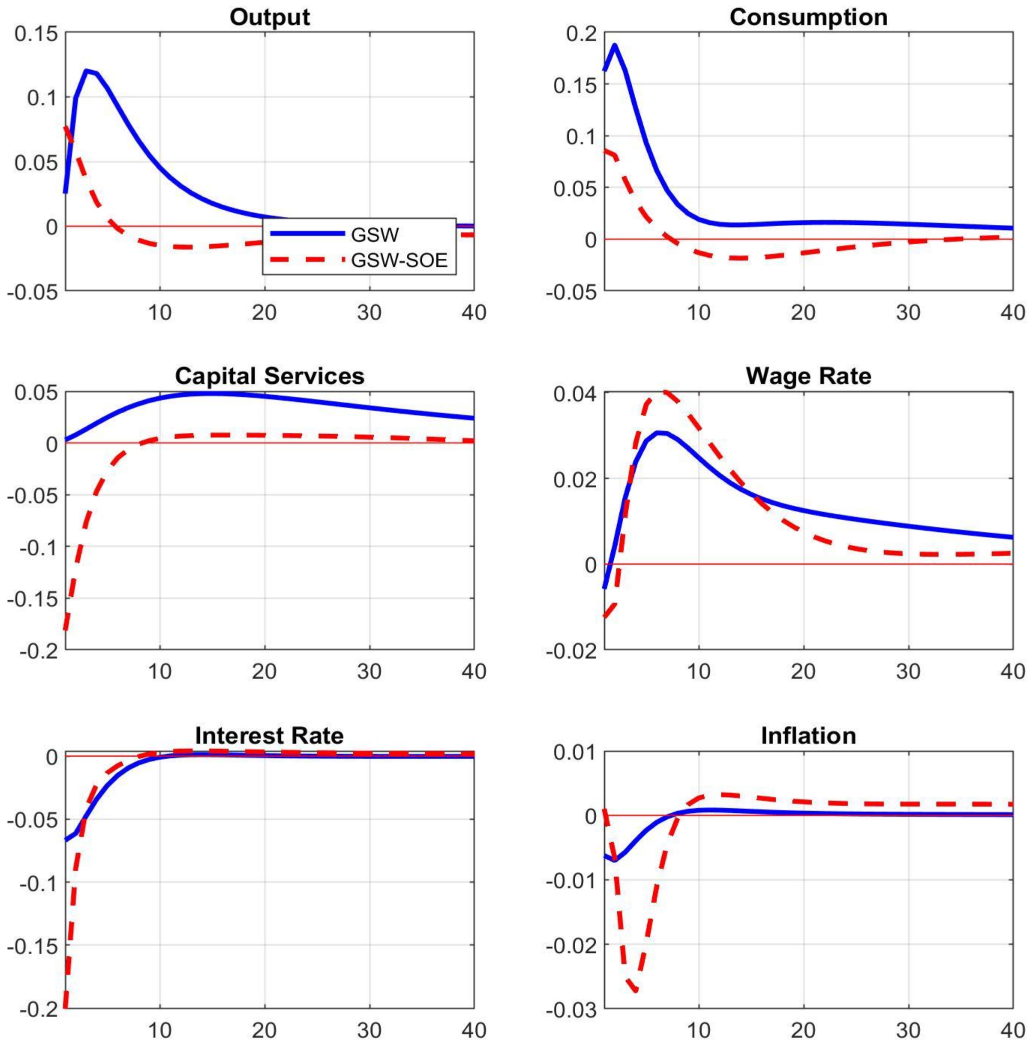
- from which nominal ER (S_t) is pinned down by

$$\frac{S_{t+1}}{S_t} = \frac{R_t}{\xi_t R_t^F}$$

- Assuming ***Foreign economy*** is approximately ***closed*** (ie, influence of SOE [Home] economy on it is ***negligible***), its available ***consumption bundle*** comprises essentially only ***Foreign-produced*** goods
- ***Foreign*** households thus need only decide how to ***allocate expenditures*** across ***these*** goods, within any time period t (***intra***temporally), and also over time (***inter***temporally)
- ***Bonds*** in ***Foreign*** economy are assumed to be in ***zero net supply***
- ***Foreign*** agents are assumed to have ***no access*** to/ interest in Home (SOE) debt markets

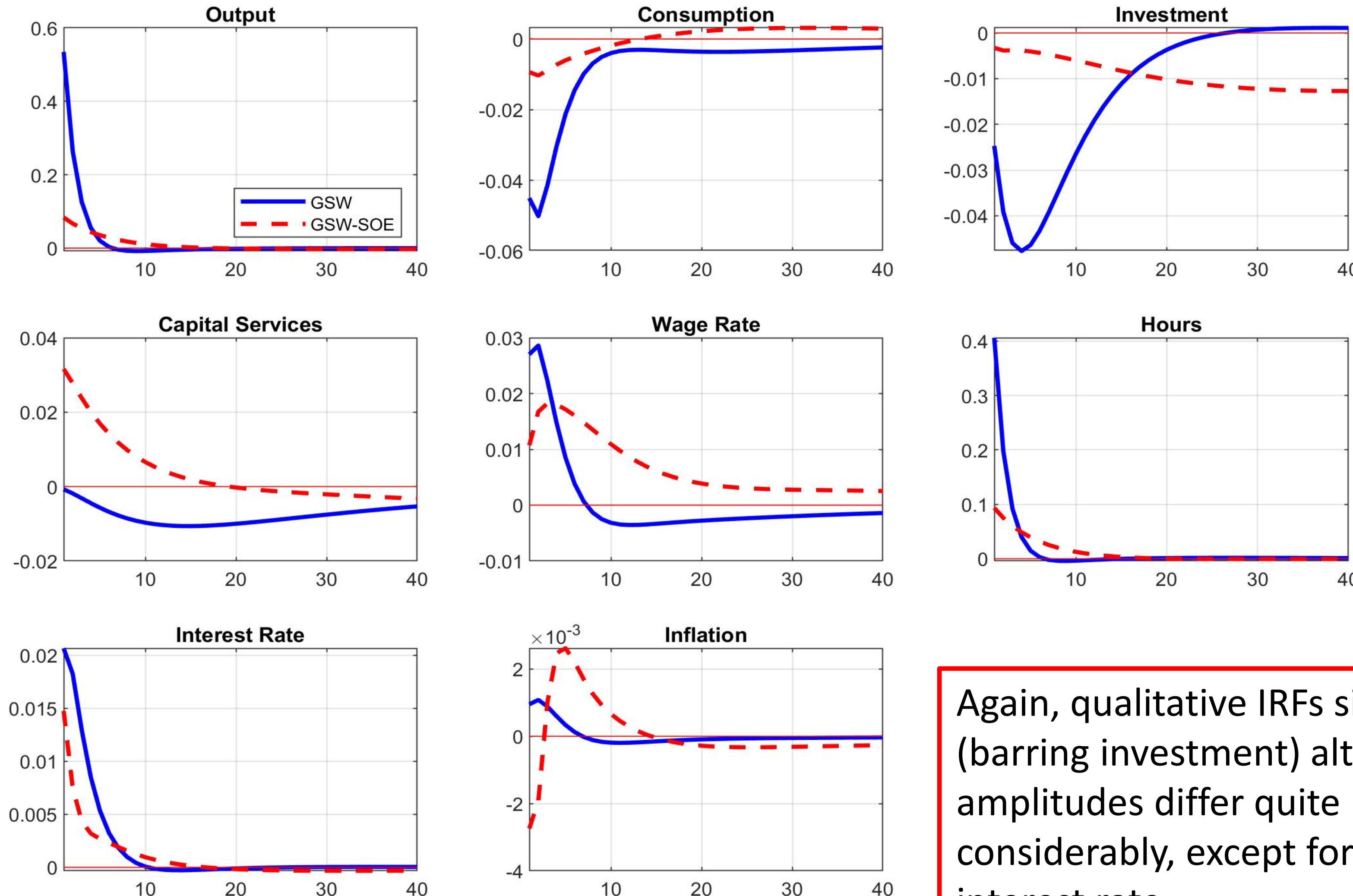
- Thus, all we need on ***Foreign*** economy side are its output, interest rate and inflation rate
- These we construct as simple ***AR(1)*** processes; more sophisticated approaches are of course possible [eg, modelling Foreign economy completely using a DSGE model]
- We now have all necessary ingredients for transforming ***closed-economy*** GSW2012 model into its ***open-economy*** version
- Let us then compare results, using ***Australia*** as our SOE

- First, TFP:



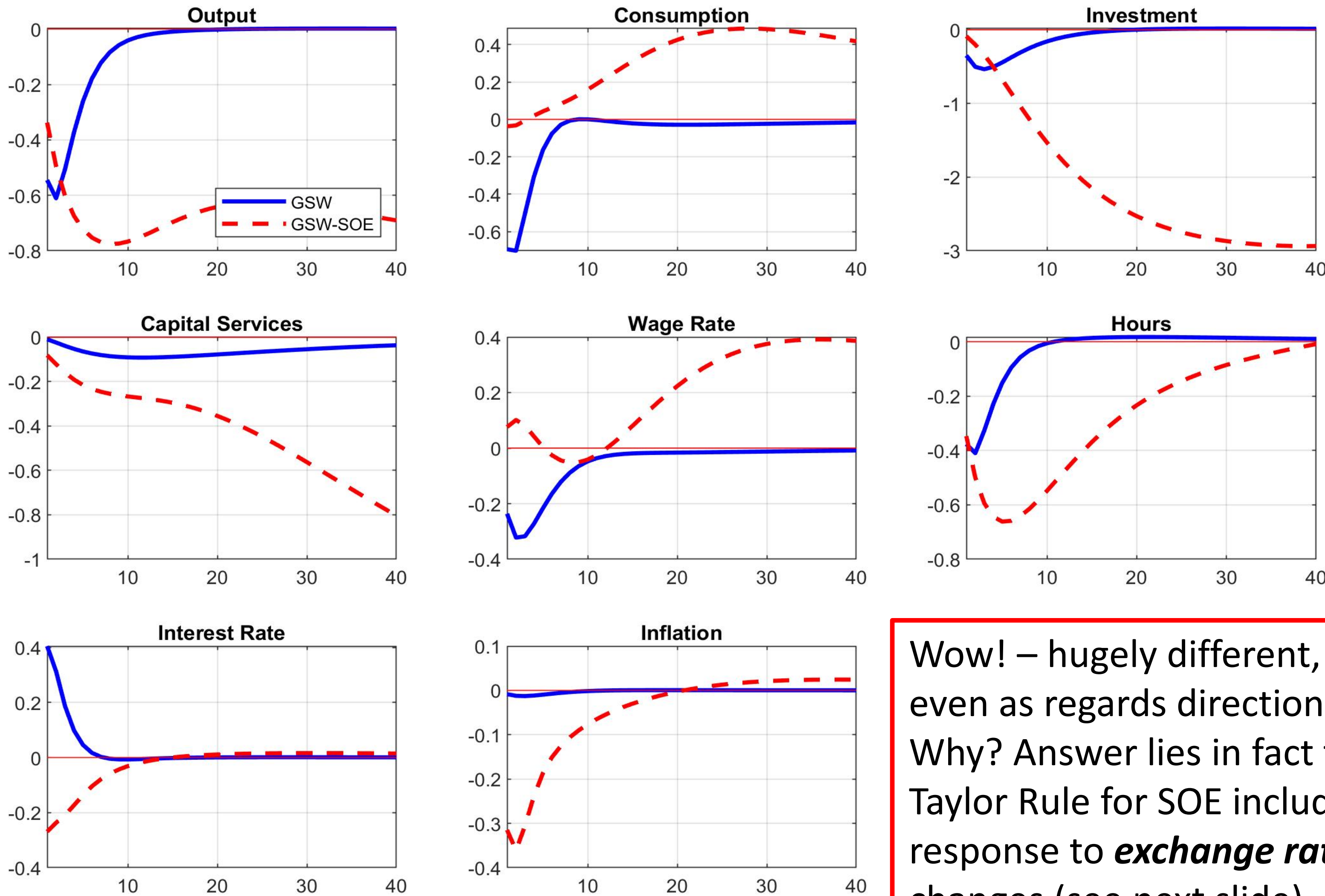
The two models give same qualitative IRFs although amplitudes differ, particularly for investment, interest rate and inflation

- Next, Fiscal Policy:



Again, qualitative IRFs similar (barring investment) although amplitudes differ quite considerably, except for interest rate

- And now MonPol:



Wow! – hugely different, even as regards direction. Why? Answer lies in fact that Taylor Rule for SOE includes response to ***exchange rate*** changes (see next slide)

- ***Monetary policy*** is conducted (as usual) according to a ***Taylor Rule*** in both models; in ***GSW*** it is

$$i_t = \rho_i i_{t-1} + \psi_\pi \pi_t + \psi_y y_t^{gap} + \psi_{\Delta y} \Delta y_t^{gap} + \varepsilon_{M,t}$$

- Policy responds (as usual) to inflation, output gap and output gap growth
- But in ***SOE*** version

$$i_t = \rho_i i_{t-1} + \psi_\pi \pi_t + \psi_y y_t^{gap} + \psi_{\Delta y} \Delta y_t^{gap} + \psi_s \boxed{\Delta S_t} + \varepsilon_{M,t}$$

- So policy also ***[NEW for SOE]*** responds to changes in nominal ***exchange rate***

Rees-Smith-Hall 2016

- Rees-Smith-Hall 2016 is a DSGE model of Australian economy developed by staff at ***Reserve Bank of Australia*** (RBA)
- Based on an SOE and ROW, it contains (as usual) households, firms, a monetary authority and a passive fiscal authority
- Especially interesting: also includes ***five separate sectors***
- ***Households*** derive ***utility*** from consumption and ***disutility*** from labour services, which they supply to firms
- Households' ***saving*** takes form of ***bonds***, denominated in either domestic or foreign currency, and ***capital***, which is specific to each of ***three production sectors***

- *[NEW]* Domestic economy consists of ***five sectors***:
  1. non-tradeables (NT) - intermediate
  2. resources (R) - intermediate
  3. non-resource tradeables (NRT) - intermediate
  4. imports (M) - final
  5. final goods and services sector (F)
- Firms in ***NT, R*** and ***NRT sectors*** produce output using labour, capital ***and resource goods*** as inputs
- ***M sector*** purchases goods from abroad and sells them in domestic economy
- Firms in ***NT, M and NRT sectors*** are ***imperfectly competitive*** → individual firms have pricing power

- In contrast, ***resource commodities (R sector)*** are ***homogeneous*** and price of these goods (in foreign currency) is determined ***entirely abroad***
- ***F sector*** transforms ***domestically sold output*** of NT, M and NRT sectors into ***final goods*** that are then sold to households for use in consumption or investment ***or*** sold to public sector
- Economy ***exports*** resources (R) and non-resource tradeable (NRT) goods
- ***Monetary authorities*** adjust ***nominal interest rate*** to stabilise inflation and aggregate output
- ***Fiscal policy*** is specified as an exogenous government spending process that is funded through lump-sum ***taxation***

- RSH include several ***frictions*** to help model capture empirical regularities in Australian macro data
- ➔ ***price stickiness*** in NT, M and NRT sectors via ***[NEW]*** sector-specific ***quadratic price adjustment costs*** that firms must pay when changing their prices (***Rotemberg vs. Calvo***)
- Price stickiness ➔ (as usual) monetary policy affects ***real activity*** as well as ***prices***
- RSH also include ***quadratic investment adjustment costs*** (à la SW2007) which allow model to match investment volatility seen in Australian data

- Details of RSH model follow now-familiar structure of NK DSGE models, so we shall go quickly
- ***Household j*** maximises lifetime ***utility function***, which is ***separable*** in consumption (C), hours worked (H) and time

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \xi_{c,t} \ln (C_t - hC_{t-1}) - A_L \frac{H_t^{1+\eta}}{1+\eta} \right\}$$

Standard is
$(1-\sigma)^{-1}(C_t - hC_{t-1})^{1-\sigma}$
here σ =1 ➔ logs

- where (as usual)
  - $0 < \beta < 1$ is intertemporal discount rate
  - $\boldsymbol{\eta} > 0$ parametrises responsiveness of hours worked to a change in real wages (Frisch)
  - $\boldsymbol{A_L}$ is a scaling term ➔ model average hours = those in data

- Repeating $$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \xi_{c,t} \ln\left(C_t - hC_{t-1}\right) - A_L \frac{H_t^{1+\eta}}{1+\eta} \right\}$$

- $\xi_{c,t}$ is usual ***preference-shifter shock*** accounting for changes in consumption not explained by other economic features of model

- Households own ***capital stock*** which is ***sector-specific*** and evolves according to law of motion

$$K_{j,t+1} = (1-\delta)K_{j,t} + \Upsilon_t \left[1 - F_t\left(\frac{I_{j,t}}{I_{j,t-1}} \right) \right] I_{j,t}$$

Adjustment costs

- Υ_t is (usual) ***shock*** to ***marginal efficiency of investment*** following a stationary AR process

- ***Hours worked index*** includes hours allocated to ***NT***, ***NRT*** and ***R*** sectors

$$H_t = \left[H_{NT,t}^{1+\sigma} + H_{NRT,t}^{1+\sigma} + H_{R,t}^{1+\sigma} \right]^{\frac{1}{1+\sigma}}$$

- ***σ*** ≥ 0 is (common) elasticity of labour substitution between sectors
- Households ***save*** and thus enter each period with domestic ***bonds*** B_{t-1} and ***foreign*** bonds $S_t B^*_{t-1}$ where S_t is ***nominal exchange rate*** (units of domestic currency per unit of foreign currency)

- Household's ***budget constraint*** is therefore

$$P_tC_t + P_tI_t + \frac{B_t}{R_t} + \frac{S_tB_t^*}{R_t^*\psi_t} \leq \sum_{j=N,NRT,H} \left(W_{j,t}H_{j,t} + R_{j,t}K_{j,t}\right) + R_{L,t}L + B_{t-1} + S_tB_{t-1}^* + \Gamma_t - T_t$$

- where Γ_t is ***profits***
- T_t is net lump sum **transfers *to*** government
- $W_{j,t}$ and $R_{j,t}$ are ***wage rate*** and ***rate of return on capital*** in sector j
- $R_{L,t}$ is ***rate of return on land***
- R_t is ***gross nominal interest rate***
- ψ_t is ***risk premium shock***

Risk premium itself is

$$v_t = \exp\left[-\chi\left(\frac{S_tB_t^*}{P_tY_t}\right) + \psi_t\right]$$

- Now turn to ***productive sectors***
- First, ***NT*** sector (services – finance, construction, etc):
- (standard) ∃ continuum of firms producing ***intermediate goods*** using capital, labour
- ***and [NEW] resources (including land)*** as inputs into
- ***[NEW]*** non-traded sector
- ***NT*** firms sell their output to a ***retailer***, which transforms them into a homogeneous good that it sells to ***final goods*** (F) sector
- Transformation of intermediate goods into ***NT*** sector's composite good follows standard ***Dixit-Stiglitz aggregator***:

- ***Dixit-Stiglitz*** ➔

θ^{NT} = elasticity of substitution across firms

$$Y_{NT,t} = \left[\int_0^1 Y_{NT,t}(i)^{\frac{\theta^{NT}-1}{\theta^n}} di \right]^{\frac{\theta^{NT}}{\theta^{NT}-1}}$$

- ***Demand function*** for each firm's output is

$$Y_{NT,t}(i) = \left(\frac{P_{NT,t}(i)}{P_{NT,t}} \right)^{-\theta^{NT}} Y_{NT,t}$$

a is a technology shock and ***μ*** is a productivity process

- ***Cobb-Douglas production function*** of ***NT*** firm i:

$$Y_{NT,t}(i) \le a_{NT,t} \left(\mu_t H_{NT,t}(i)\right)^{\alpha_{NT}} \left(K_{NT,t}(i)\right)^{\gamma_{NT}} \left(Z_{NT,t}(i)\right)^{1-\alpha_{NT}-\gamma_{NT}}$$

- α_{NT}, γ_{NT} and $(1-\alpha_{NT}-\gamma_{NT})$ are ***input shares*** of labour, capital and resources in NT goods production

- RSH introduce ***price stickiness*** into non-traded sector by assuming that NT firms face ***quadratic cost*** of adjusting their prices, along lines of ***Rotemberg*** (1982) – alternative to ***Calvo***
- Given ***friction***, NT firms choose prices and factor inputs to ***maximise real profits***, given by

$$\Gamma_{NT,t}(i) = \frac{P_{NT,t}(i)Y_{NT,t}(i)}{P_t} - \frac{\boxed{MC_{NT,t}(i)}Y_{NT,t}(i)}{P_t} - \overbrace{\frac{\tau_{\pi}^{NT}}{2}\left[\frac{P_{NT,t}(i)}{\prod_{NT,t-1}^{\chi} \pi *^{1-\chi} P_{NT,t-1}(i)} - 1\right]^2 \frac{P_{NT,t}Y_{NT,t}}{P_t}}^{\text{Rotemberg}}$$

- where ***marginal costs*** are (geometric average)

$$MC_{NT,t}(i) = \frac{\varepsilon_{\pi_{NT},t}}{a_{NT,t}}\overset{\text{labour}}{\left[\frac{W_{NT,t}}{\alpha_{NT}\mu_t}\right]^{\alpha_{NT}}}\overset{\text{capital}}{\left[\frac{R_{NT,t}}{\gamma_{NT}}\right]^{\gamma_{NT}}}\overset{\text{resources}}{\left[\frac{P_{z,t}}{1-\alpha_{NT}-\gamma_{NT}}\right]^{1-\alpha_{NT}-\gamma_{NT}}}$$

- Turning now to non-resource ***tradeable (NRT) sector***:
- It has similar set-up but can ***also*** enter into ***international trade***
- For ***domestic market***, ***NRT*** firms sell their output to a retailer as in NT sector
- But ***[NEW]*** ∃ now ***also*** possibility for firms to sell their outputs to an ***exporter*** which transforms them into a homogeneous good for export to overseas markets
- ***NRT*** firms are permitted to charge ***separate prices*** for goods that they sell ***domestically*** and goods that they ***export*** [*price differentiation* by market]

- Transformation of each firm's intermediate good into ***NRT*** sector's composite good follows usual ***Dixit-Stiglitz aggregator***
- And there are now ***demand functions*** for each firm's output in ***domestic*** (***d***) and ***overseas*** (***x***) markets

θ^{NRT} = elasticity of substitution across firms

$$Y^{d}_{NRT,t}(i) = \left(\frac{P_{NRT,t}(i)}{P_{NRT,t}}\right)^{-\theta^{NRT}} Y^{d}_{NRT,t}$$

$$Y^{x}_{NRT,t}(i) = \left(\frac{P^{*}_{NRT,t}(i)}{P^{*}_{NRT,t}}\right)^{-\theta^{NRT}} Y^{x}_{m,t}$$

Aggregated (no "i") foreign demand for ***NRT*** goods

$$Y^{x}_{NRT,t} = \omega^{*} \left(\frac{P^{*}_{NRT,t}}{P^{*}_{t}}\right)^{\zeta^{*}} Y^{*}_{t}$$

- As in NT sector, each ***NRT*** firm produces using ***Cobb-Douglas production function***

$$Y_{NRT,t}(i) \leq a_{NRT,t} \left(\mu_t H_{NRT,t}(i)\right)^{\alpha_{NRT}} \left(K_{NRT,t}(i)\right)^{\gamma_{NRT}} \left(Z_{NRT,t}(i)\right)^{1-\alpha_{NRT}-\gamma_{NRT}}$$

- And again ***Rotemberg price stickiness*** is introduced ➔ profit function to be maximised

$$\begin{aligned}\Gamma_{NRT,t}(i) = {} & \frac{P_{NRT,t}(i) Y^d_{NRT,t}(i)}{P_t} + \frac{S_t P^*_{NRT,t}(i) Y^x_{NRT,t}(i)}{P_t} - \frac{MC^d_{NRT,t}(i) Y_{NRT,t}(i)}{P_t} \\ & - \frac{MC^x_{NRT,t}(i) Y_{NRT,t}(i)}{P_t} - \frac{\tau_\pi^{NRT}}{2} \left[\frac{P_{NRT,t}(i)}{\prod^{\chi}_{NRT} \pi *^{1-\chi} P_{NRT,t-1}(i)} - 1 \right]^2 \frac{P_{NRT,t} Y^d_{NRT,t}}{P_t} \\ & - \frac{\tau_\pi^{NRT*}}{2} \left[\frac{P^*_{NRT,t}(i)}{\prod^{*\chi}_{NRT,t-1} \pi **^{1-\chi} P^*_{NRT,t-1}(i)} - 1 \right]^2 \frac{S_t P^*_{NRT,t} Y^x_{NRT,t}}{P_t}\end{aligned}$$

Foreign elements

- Next, ***resource (R)*** sector:
- Unlike NT and NRT production sectors, ***R*** sector produces ***homogenous*** output under ***perfect competition*** taking prices as given
- Thus, it ***behaves as though*** it consists of a ***single firm*** producing output via ***Cobb-Douglas production function***

$$Y_{R,t} = a_{R,t} \left(\mu_t h_{R,t}\right)^{\alpha_R} \left(K_{R,t}\right)^{\gamma_R} \left(\mu_t L\right)^{1-\alpha_R-\gamma_R}$$

- ***World markets*** determine prices at which these resources may be sold but it is assumed that although ***LOOP*** holds in LR, it ***fails in SR*** ("LOOP Gap")

- Why a LOOP Gap?
  1. a proportion of Australia's resource exports are sold according to ***pre-determined price contracts***
  2. some resource firms ***hedge*** their overseas currency exposures
- RSH assume that ***half*** of any change in overseas resource prices feeds into domestic resource prices ***in quarter in which price change occurs***, and ∴ around ***95 per cent*** flows through within first ***year***

  0.5 + 0.25 + 0.125 + 0.0625 = 0.9375
- ➔ specific functional form for ***resource price pass-through***:

$$P_{R,t} = \left(S_t P^*_{R,t} \right)^{1/2} \left(P_{R,t-1} \right)^{1/2}$$

- Next, ***M (imports)*** sector:
- Output of imports sector is an aggregate constructed from a continuum of imported varieties according to usual ***Dixit-Stiglitz aggregator***
- with standard ***demand function*** for each variety:

$$Y_{M,t}(i) = \left(\frac{P_{M,t}(i)}{P_{M,t}} \right)^{-\theta^M} Y_{M,t}$$

- and importer ***marginal costs***: $MC_{M,t}(i) = \varepsilon_{\pi_{M},t} \frac{S_t P^*_{M,t}}{P_{M,t}}$

- ***F (final goods)*** sector: it assembles domestically sold output (DFD = domestic final demand) of ***NT***, ***NRT*** and ***M*** sectors via usual ***CES aggregator***

$$DFD_t = \left[\omega_{NT}^{\frac{1}{\zeta}} Y_{NT,t}^{\frac{\zeta-1}{\zeta}} + \omega_{NRT}^{\frac{1}{\zeta}} Y_{NRT,t}^{d\,\frac{\zeta-1}{\zeta}} + \omega_{M}^{\frac{1}{\zeta}} Y_{M,t}^{d\,\frac{\zeta-1}{\zeta}} \right]^{\frac{\zeta}{\zeta-1}}$$

- $\omega_{NT} + \omega_{NRT} + \omega_M = 1$ where ω_{NT}, ω_{NRT} and ω_M are shares of ***NT***, ***NRT*** and ***M*** goods in final domestic good and Y^d_M = tradeable goods sold domestically
- Profit maximisation by final goods producer ➔

$$P_t = \left[\omega_{NT} P_{NT,t}^{1-\zeta} + \omega_{NRT} P_{NRT,t}^{1-\zeta} + \omega_M P_{M,t}^{1-\zeta} \right]^{\frac{1}{1-\zeta}}$$

- ***Government sector*** also purchases goods and services which enter into final demand
- Government ***demand*** G_t is treated as an ***exogenous*** process that evolves according to

$$\ln\left[\frac{G_t}{\mu_t}\right] = (1-\rho_g)\ln(g) + \rho_g \ln\left[\frac{G_{t-1}}{\mu_{t-1}}\right] + \varepsilon_{g,t}$$

- **g** is calibrated so that steady-state share of G_t in GDP matches its observed empirical level
- Government issues ***bonds*** B_t and raises lump-sum ***taxes*** T_t to pay for G_t under budget constraint

$$P_t G_t + B_{t-1} = T_t + \frac{B_t}{R_t}$$

- ***Central Bank*** sets short-term nominal interest rate R_t according to a Taylor-type ***monetary policy rule***

$$\ln\left(\frac{R_t}{R}\right) = \rho_r \ln\left(\frac{R_{t-1}}{R}\right) + (1-\rho_r)\left[\phi_\pi \ln\left(\frac{\Pi_t}{\Pi}\right) + \phi_y \ln\left(\frac{Y_t^{va}}{Y^{va}}\right)\right] + \phi_{\Delta_y} \ln\left(\frac{Y_t^{va}}{Y_{t-1}^{va}}\right) + \phi_q\left(\frac{Q_t}{Q_{t-1}}\right) + \varepsilon_{r,t}$$

- where Π_t is CPI inflation rate and Y^{va}_t is real GDP output gap and letters w/o subscripts = SS values ➔ usual log-deviation variables
  - eg, *ln(R$_t$/R) = ln(R$_t$) – ln(R) = r$_t$*
- R_t thus depends on
  - past nominal **interest rates**
  - current values of **CPI inflation**
  - ***level*** and ***growth rate*** of output
  - change in the ***real exchange rate Q$_t$***

- ***Market clearing***:
- all non-resource tradeable goods (***NRT***) produced must be sold at home (***d***) or abroad (***x***)
- all resources (***R***) produced must be exported (***x***) or used in production of other domestic goods
- GDP = domestic final goods demand + net exports
- Hence

$$Y_{NRT,t} = Y^{d}_{NRT,t} + Y^{x}_{NRT,t}$$

$$Y_{R,t} = Y^{x}_{R,t} + Z_{NT,t} + Z_{NRT,t}$$

$$DFD_t = C_t + I_t + G_t$$

- ***Net exports*** in nominal terms, NX_t, are equal to sum of resource (***R***) and non-resource (***NRT***) export values less import values
- Hence

$$NX_t = P_{R,t} Y^X_{R,t} + S_t P^*_{NRT,t} Y^X_{NRT,t} - S_t P^*_{f,t} Y_{F,t}$$

- ***Current account*** evolution is given by

$$\boxed{\frac{S_t}{R^*_t v_t}} B^*_t = S_t B^*_{t-1} + NX_t$$

$S_t/R_t{}^*v_t$ = price of Foreign bonds B^*_t

- What of ***Foreign economy*** ? RSH follow Gali and Monacelli (2005) in specifying foreign economy as a simplified ***closed economy*** with an IS curve, a Phillips curve and a Taylor Rule:
- Foreign IS curve (log-linearised) is

$$\hat{y}_t^* = E_t\left\{\hat{y}_{t+1}^*\right\} + \left(\hat{r}_t^* - E_t\left\{\hat{\pi}_{t+1}^*\right\}\right) + E_t\left\{\xi_{y^*,t+1}\right\} - \boxed{\xi_{y^*,t}}$$

- where * indicates a foreign-economy variable
- and $\xi_{y^*,t}$ is an AR(1) foreign demand shock

- ***Foreign*** Phillips curve is given by

$$\hat{\pi}_t^* = \beta E_t \{\hat{\pi}_{t+1}^*\} + \frac{\kappa^*}{100}\hat{y}_t^* + \boxed{\varepsilon_{\pi^*,t}}$$

- where $\boldsymbol{\varepsilon_{\pi*,t}}$ is a cost push shock
- And ***Foreign*** Taylor Rule is given by

$$\hat{r}_t^* = \phi_{r^*}\hat{r}_{t-1}^* + (1-\phi_{r^*})\left(\phi_{\phi^*}\hat{\pi}_t^* + \phi_y\hat{y}_t^*\right) + \boxed{\varepsilon_{r^*,t}}$$

- and $\boldsymbol{\varepsilon_{r*,t}}$ is usual monetary shock

- RSH also assume that relative ***price of resources*** in terms of foreign currency is stationary, but subject to transitory deviations according to an AR(1) process
- They allow for two shocks:
  - ***foreign demand*** shocks and
  - ***resource-specific*** price shocks
- so that (log relative) real ***resource*** prices evolve as

$$\hat{p}^*_{R,t} = \left(1 - \rho_{p^*_R}\right)\hat{p}^*_R + \rho_{p^*_R}\hat{p}^*_{R,t-1} + \rho_{Ry,t}\varepsilon_{y^*,t} + \varepsilon_{p^*_R,t}$$

- Finally, RSH model rate of growth of (labour-augmenting) ***technology*** as

$$\Delta\mu_t = \ln(\mu) + \varepsilon_{\mu,t}; \quad \varepsilon_{\mu,t} \sim N\left(0, \sigma_\mu^2\right)$$

- where *ln(μ)* is ***trend rate of productivity growth*** and $\Delta\mu_t = ln(\mu_t/\mu_{t-1})$
- ***Structural shocks*** (consumer preference, marginal efficiency of investment, risk premium and foreign demand , plus sector-specific technology shocks to NT, NRT and R sectors) are assumed to follow an AR(1) process
- Remaining shocks are assumed to be ***white noise***

- Before estimating model, RSH do an ***interesting test***: they check model's steady-state against historical values for "great ratios":

Table 2: Steady-state Properties of the Model

Target	Average 1993–2013	Model
Expenditure (per cent of GDP)		
Household consumption	56.9	55.8
Private investment	21.4	22.7
Public demand	22.5	22.5
Exports	19.5	19.6
Imports	20.6	20.6
Production (per cent of GVA)		
Non-tradeable	64.0	64.0
Other tradeable	26.3	23.1
Mining	9.7	12.9
Trade (per cent of exports)		
Resource exports	40.1	40.1
Other exports	59.9	59.9
Investment demand (per cent of private investment)		
Non-tradeable	58.5	58.4
Other tradeable	27.8	27.7
Mining	13.7	13.9

- This depends not only on model structure, but also on initial calibrations. Still, results are ***good*** !!

- Because estimation of such a large model is hard (even for Bayes !!) RSH estimate model in ***two stages***
- ***First***, they estimate ***large*** economy's parameters
- ***Second***, they estimate ***small*** economy's parameters, ***taking as given*** posterior mean values of ***common*** parameters from first stage
- ***[E:\ReesSmithHall\Estimation_Files\Australian_Economy\multisector_jc3_TeX_binder.pdf and …\multisector_jc3.pdf]***

- As an aside, it is interesting to look at how **Dynare** estimation file is set up to be ***compact***

```
// -- 0. PRELIMINARIES AND DEFINITIONS ----------------------------------
// -- OPTIONS ---
@#define estimate_mod = 1 // Estimate model (1 = yes, 0 = no)
// -- DEFINITIONS OF SECTORS ---
@#define production_sectors =  [ "n", "m", "z" ]
@#define imperfectly_competitive_sectors = [ "n", "m" ]
@#define consumed_sectors = [ "n", "m", "f" ]
// -- 1. MODEL DECLARATIONS AND CALIBRATION ----------------------------
@# include "variable_declarations.mod" // -- DECLARATION OF PARAMETERS AND VARIABLES ---
@# include "static_calibrations.mod" // -- CALIBRATIONS ---
@# include "model_equations_loglinear.mod" // -- MODEL EQUATIONS ---
@# include "dynamic_calibrations_forestimation.mod" // --- Fixed values for remaining parameters ---
@# include "calc_steadystate_nonlinear.mod" // --- STEADY STATE ---
// -- 2. ESTIMATE THE MODEL ----------------------------
@# if estimate_mod == 1
@# include "estimate_model_jc3.mod"
save latest_results.mat oo_ M_
@# endif
// -- 3. CHECK CALIBRATION ----------------------------
@# include "check_calibration_targets.mod" // Calculate and print some implied calibration targets
```

n = NT
m = NRT
z = R
f = M

This is where the previously-estimated foreign parameters are read in

- Look at a part of “variable_declarations.mod”

```
// Looping across the production sectors
@#for j in production_sectors
var  h_@{j},           // Hours worked in sector j
    w_@{j},            // Wage rate
    lambda_@{j},         // Tobin's-q, sorta
    inve_@{j},         // Investment
    k_@{j},            // Capital stock
    rk_@{j},           // Capital rental rate
    a_@{j},            // Sectoral productivity
    p_@{j},            // Sector's output price relative to final demand price index
    z_@{j},            // Sector's demand for commodities
    y_@{j},            // Gross output
    y_va_@{j};           // Value-added

parameters abar_@{j},     // Steady-state relative productivity level
        delta_@{j},    // Depreciation rate
        alpha_h_@{j},  // Labour share in production
        alpha_k_@{j},  // Capital share in production
        alpha_z_@{j};  // Commodity input share in production

parameters rho_a_@{j};   // Persistence of sector-specific technology shocks
varexo     eps_a_@{j};  // Sector-specific technology shocks

@#endfor
```

- Sectoral definitions are incorporated into settings of ***priors*** in *"estimate_model_jc3.mod"*

```
estimated_params;
stderr   eps_g,          2.2,            gamma_pdf,      0.5,     0.4;
stderr   eps_xi_c,       2.0,            gamma_pdf,      0.5,     0.4;
stderr   eps_upsilon,    5.4,            gamma_pdf,      0.5,     0.4;
@#for j in production_sectors
stderr   eps_a_@{j},    3,        gamma_pdf,      0.5,     0.4;
@#endfor

 chi, 0.3, beta_pdf, 0.3, 0.15;

 @#for j in production_sectors
 rho_a_@{j} ,    0.7,         beta_pdf,            0.5,    0.15;
 @#endfor
 phik,      1.78,        gamma_pdf,                4,       1;

 rho_r,             0.86,            beta_pdf,       0.75,    0.1;
 rho_pi,            1.3,             normal_pdf,     1.5,     0.2;
 rho_y,             0.15,            normal_pdf,     0.125,    0.05;
 rho_deltay,        0.05,            normal_pdf,     0.00,     0.025;
 rho_q,             0.00,            normal_pdf,     0.0,     0.05;
 b,                 0.75,            beta_pdf,       0.5,     0.15;

 end;
```

See later in priors table

- They are also used ***elsewhere*** in model, eg:

"model_equations_loglinear.mod"

Notice how this actually constructs the model equation

```
// -- MODEL EQUATIONS -----------------------------------

// -- HOUSEHOLD AND FIRM OPTIMISATION --

// (Eq.7) Utility impact of hours worked depends on a CES aggregate of sectoral hours worked
            (h_ss)^sigma * h =
            @#for j in production_sectors
               + ( h_@{j}_ss^sigma ) * h_@{j}
            @#endfor
           ;

// --- IMPERFECTLY COMPETITIVE SECTORS: set prices optimally given downward-sloping demand ---

@#for j in imperfectly_competitive_sectors

// Capital/labour demand from sector j   ( eq. 10, 11, 15, 16 )
        h_@{j} - k_@{j}(-1) = rk_@{j} - w_@{j} - mu;

// Resources demand from sector j (eq. 11,16 with 10,15,20)
        z_@{j} - k_@{j}(-1) = rk_@{j} - p_z - mu;

// Definition of CES aggregator for CPI (using the total price index P as a numeraire)  (eq.23)
0 =
    @#for j in consumed_sectors
        + omega_@{j}*p_@{j}_ss^(1-zeta)*(1 - zeta)*p_@{j}
    @#endfor
    ;

// Mapping from sectoral inflation rates to relative prices  (eq. 27,28,29)
    @#for j in consumed_sectors
        p_@{j} = p_@{j}(-1) + infl_@{j} - infl;
    @#endfor
```

- As mentioned earlier, RSH estimate model in ***two stages,*** first estimating ***large*** economy's parameters
- Estimation of ***foreign Taylor Rule*** ➔
  - strong response to ***inflation*** and
  - modest response to deviations of ***output*** from its steady-state level
- Also, response of ***resource prices*** to foreign output shocks was found to be small and not significantly different from zero
- And foreign Phillips curve quite ***flat*** ($\kappa^*/100 = 0.016$) in $\hat{\pi}_t^* = \beta E_t\{\hat{\pi}_{t+1}^*\} + \frac{\kappa^*}{100}\hat{y}_t^* + \varepsilon_{\pi^*,t}$

- RSH estimate model using usual Bayesian methodology, which requires priors:

Prior_trunc=0 !

Range

90% CI

$SE_{eps\_e\_m\_star}$	Gamma	0.5000	0.1800	0.4000	0.0000	∞	0.0626	1.2849
rho_g	Beta	0.5000	0.5000	0.1500	0.0000	1.0000	0.2526	0.7474
rho_xi_c	Beta	0.5000	0.5000	0.1500	0.0000	1.0000	0.2526	0.7474
$rho\_upsilon$	Beta	0.5000	0.5000	0.1500	0.0000	1.0000	0.2526	0.7474
rho_psi	Beta	0.5000	0.5000	0.1500	0.0000	1.0000	0.2526	0.7474
$slope\_pi\_n$	Gamma	50.0000	32.0000	30.0000	0.0000	∞	12.7062	107.2975
$slope\_pi\_f$	Gamma	50.0000	32.0000	30.0000	0.0000	∞	12.7062	107.2975
$slope\_pi\_m$	Gamma	50.0000	32.0000	30.0000	0.0000	∞	12.7062	107.2975
$slope\_pi\_m\_star$	Gamma	50.0000	32.0000	30.0000	0.0000	∞	12.7062	107.2975
chi	Beta	0.3000	0.2368	0.1500	0.0000	1.0000	0.0838	0.5734
rho_a_n	Beta	0.5000	0.5000	0.1500	0.0000	1.0000	0.2526	0.7474
rho_a_m	Beta	0.5000	0.5000	0.1500	0.0000	1.0000	0.2526	0.7474
rho_a_z	Beta	0.5000	0.5000	0.1500	0.0000	1.0000	0.2526	0.7474
$phik$	Gamma	4.0000	3.7500	1.0000	0.0000	∞	2.5090	5.7743
rho_r	Beta	0.7500	0.7817	0.1000	0.0000	1.0000	0.5701	0.8971
rho_pi	Gaussian	1.5000	1.5000	0.2000	$-\infty$	∞	1.1710	1.8290
rho_y	Gaussian	0.1250	0.1250	0.0500	$-\infty$	∞	0.0428	0.2072
rho_deltay	Gaussian	0.0000	0.0000	0.0250	$-\infty$	∞	-0.0411	0.0411
rho_q	Gaussian	0.0000	0.0000	0.0500	$-\infty$	∞	-0.0822	0.0822
b	Beta	0.5000	0.5000	0.1500	0.0000	1.0000	0.2526	0.7474

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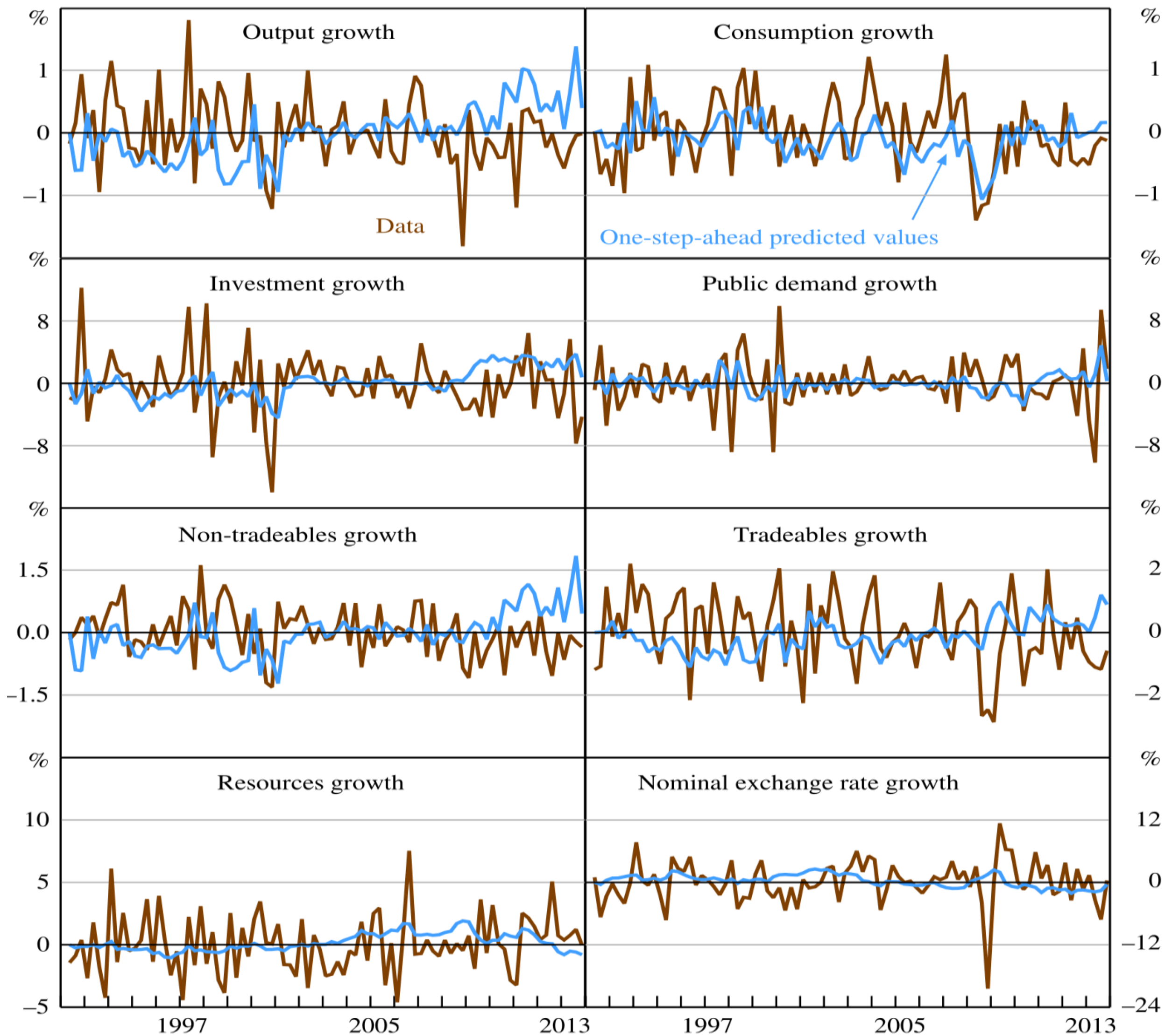
TABLE 4
Model Results: Domestic Block

Parameter	Prior distribution			Posterior distribution			
	Shape	Mean	SD	Mode	Mean	5%	95%
h	beta	0.5	0.15	0.76	0.76	0.67	0.85
$\kappa_{\pi n}$	gamma	50	30	0.20	0.29	0.06	0.55
$\kappa_{\pi f}$	gamma	50	30	0.75	1.06	0.24	1.87
$\kappa_{\pi m}$	gamma	50	30	1.24	1.61	0.33	2.91
$\kappa^*_{\pi m}$	gamma	50	30	1.27	2.10	0.34	4.00
χ	beta	0.3	0.15	0.20	0.22	0.08	0.35
Φ	gamma	4	1	1.68	1.80	1.16	2.37
ρ_r	beta	0.75	0.1	0.87	0.86	0.82	0.90
ϕ_π	normal	1.5	0.2	1.38	1.42	1.07	1.74
ϕ_y	normal	0.125	0.05	0.12	0.12	0.04	0.19
$\phi_{\Delta y}$	normal	0	0.025	0.05	0.05	0.01	0.08
ϕ_q	normal	0	0.05	0.00	0.00	−0.01	0.01
ρ_g	beta	0.5	0.15	0.32	0.34	0.17	0.51
$\rho_{\xi c}$	beta	0.5	0.15	0.71	0.68	0.54	0.84
ρ_Υ	beta	0.5	0.15	0.32	0.34	0.15	0.53
ρ_ψ	beta	0.5	0.15	0.87	0.84	0.77	0.91
ρ_{an}	beta	0.5	0.15	0.78	0.71	0.57	0.86
ρ_{am}	beta	0.5	0.15	0.52	0.47	0.31	0.63
ρ_{az}	beta	0.5	0.15	0.87	0.84	0.73	0.95
Standard deviations (× 100)							
σ_g	gamma	0.5	0.4	2.15	2.21	1.86	2.52
$\sigma_{\xi c}$	gamma	0.5	0.4	2.13	2.26	1.56	2.95
σ_Υ	gamma	0.5	0.4	5.48	5.72	4.02	7.44
σ_ψ	gamma	0.5	0.4	0.50	0.62	0.36	0.85
σ_{an}	gamma	0.5	0.4	2.67	2.65	2.13	3.15
σ_{am}	gamma	0.5	0.4	5.19	5.14	4.35	5.98
σ_{az}	gamma	0.5	0.4	1.77	1.80	1.53	2.03
σ_r	gamma	0.5	0.4	0.11	0.12	0.10	0.13
$\sigma_{\pi n}$	gamma	0.5	0.4	0.22	0.23	0.18	0.28
$\sigma_{\pi f}$	gamma	0.5	0.4	0.43	0.45	0.22	0.68
$\sigma_{\pi m}$	gamma	0.5	0.4	0.79	0.79	0.47	1.11
$\sigma^*_{\pi m}$	gamma	0.5	0.4	3.56	3.74	3.16	4.40

Note: $\kappa_{\pi_n} = 100 \times \frac{\theta^n - 1}{\tau^n_\pi}$, $\kappa_{\pi_f} = 100 \times \frac{\theta^f - 1}{\tau^f_\pi}$, $\kappa_{\pi_m} = 100 \times \frac{\theta^m - 1}{\tau^m_\pi}$, $\kappa_{\pi^*_m} = 100 \times \frac{\theta^n - 1}{\tau^{m^*}_\pi}$.

- Compare model predictions with actual data:

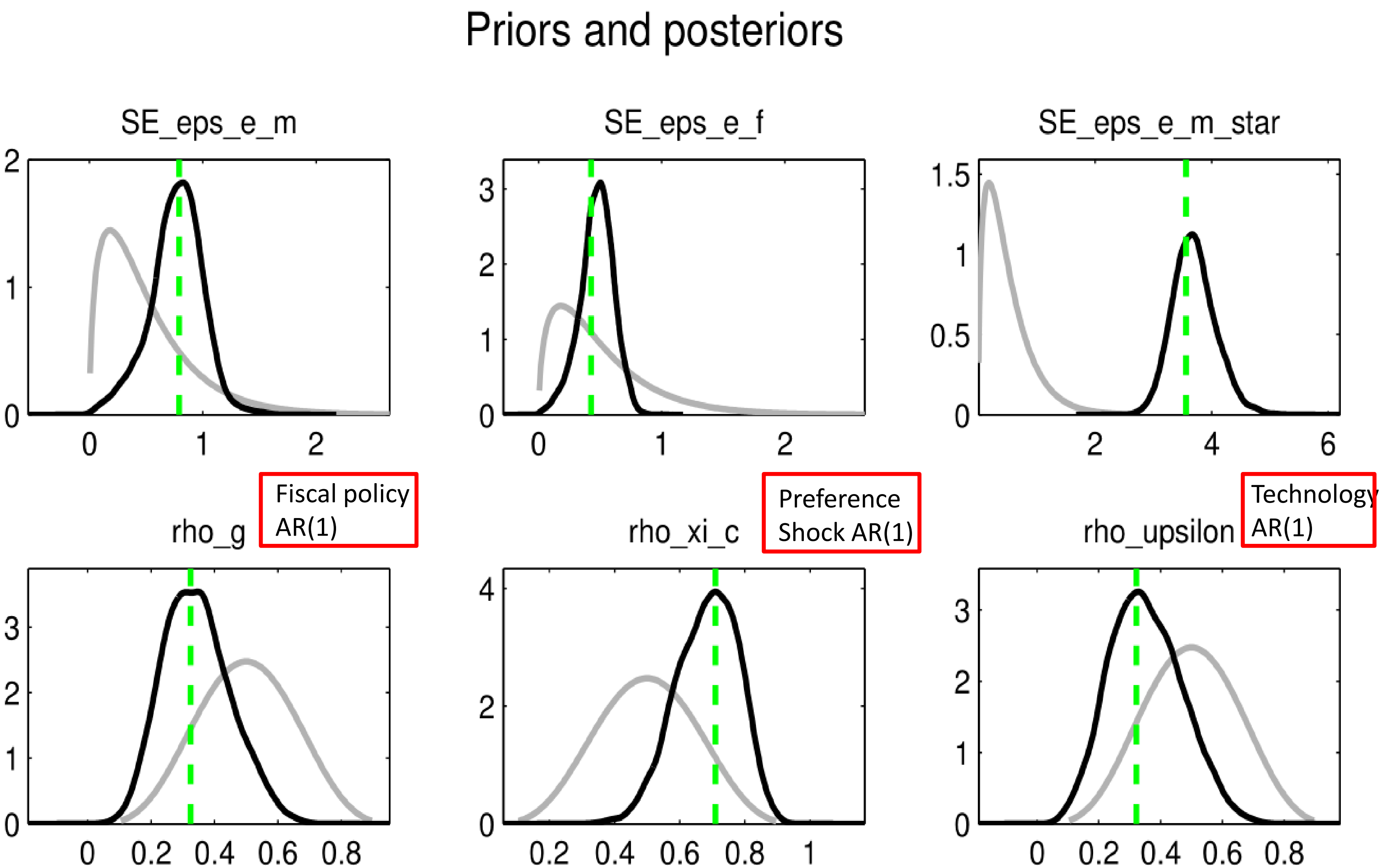


Note: All data series were demeaned prior to estimation.

For most series, model captures ***low-frequency*** variations in data reasonably well, but it struggles to match some ***high-frequency*** movements, particularly for volatile variables such as ***exchange rate*** or ***resource prices***

Model also over-predicts ***GDP growth*** in period after global financial crisis (2008), largely because of large prediction errors for growth rate of ***non-tradeable sector***

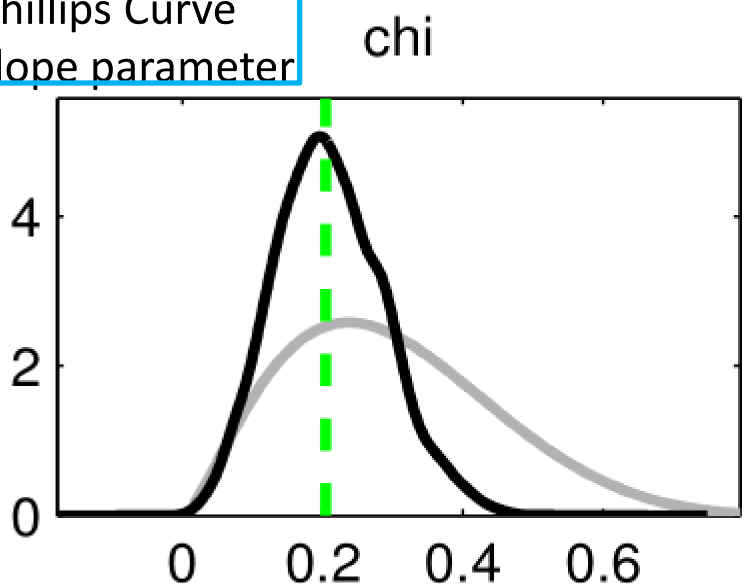
- Some estimated parameter results look like this:



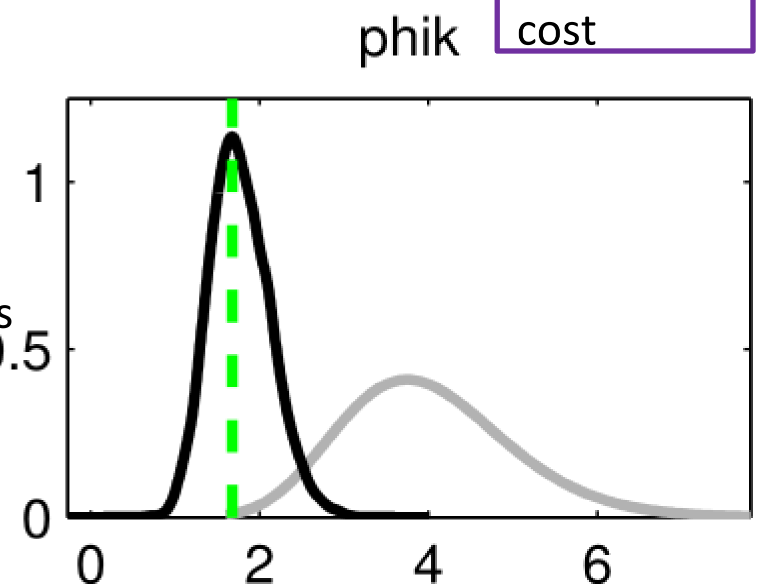
• Continuing with "deep" parameters:

Investment adjustment cost

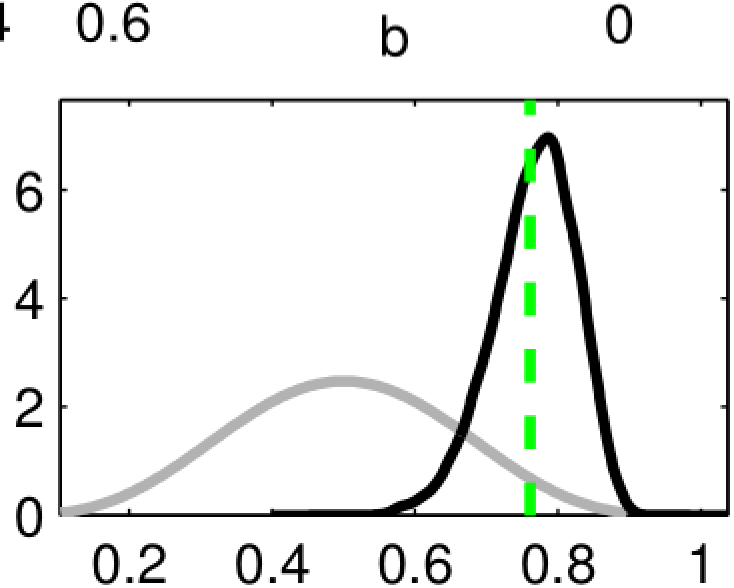
Phillips Curve slope parameter



Parameter governing degree of investment adjustment costs (***phik***) is estimated to be 1.78, ie ***small***



Domestic ***Phillips curves*** for various sectors estimated to be extremely ***flat*** (low value of ***chi***) ➔ high degree of price rigidity: ***NT*** prices ***stickiest***, followed by ***M*** prices, with ***NRT*** sector most ***flexible***



Consumption habits

Habits parameter (***b***) has a posterior mean of 0.77, indicative of considerable ***inertia*** in consumption

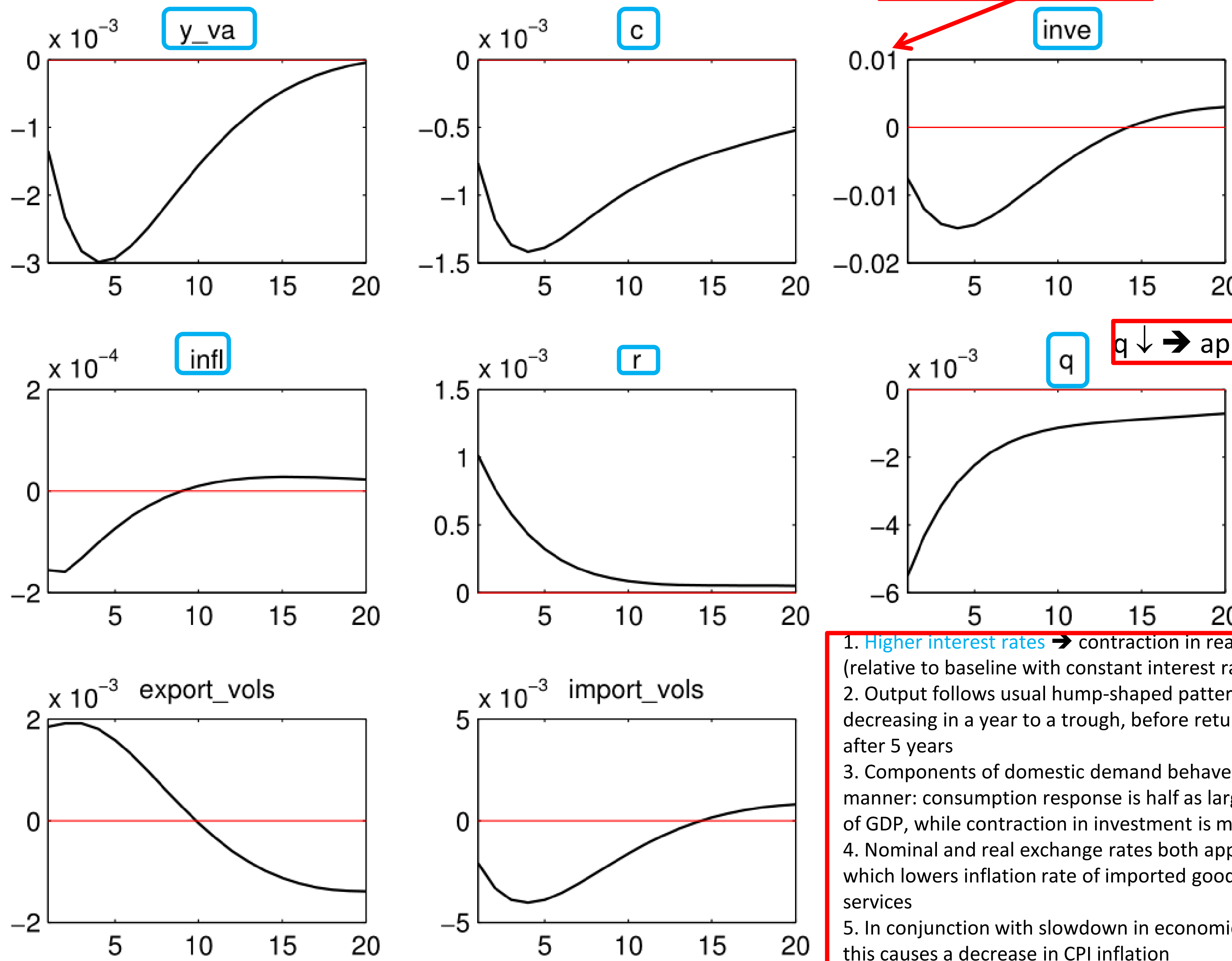
	prior mean	post. mean	90% HPD interval		prior	pstdev
rho_r	0.750	0.8628	0.8224	0.9025	beta	0.1000
rho_pi	1.500	1.4237	1.0675	1.7382	norm	0.2000
rho_y	0.125	0.1200	0.0414	0.1899	norm	0.0500
rho_deltay	0.000	0.0469	0.0126	0.0781	norm	0.0250
rho_q	0.000	-0.0000	-0.0055	0.0058	norm	0.0500

- Estimated coefficients of domestic ***Taylor rule*** → ***strong*** response to ***inflation***
- and ***modest*** response to **output** gap
- But ∃ ***no evidence*** that monetary policy in Australia responds directly to ***exchange rate*** movements (as also found in LS2007)
- Now look at response to a ***monetary policy shock***

Orthogonalized shock to eps_r

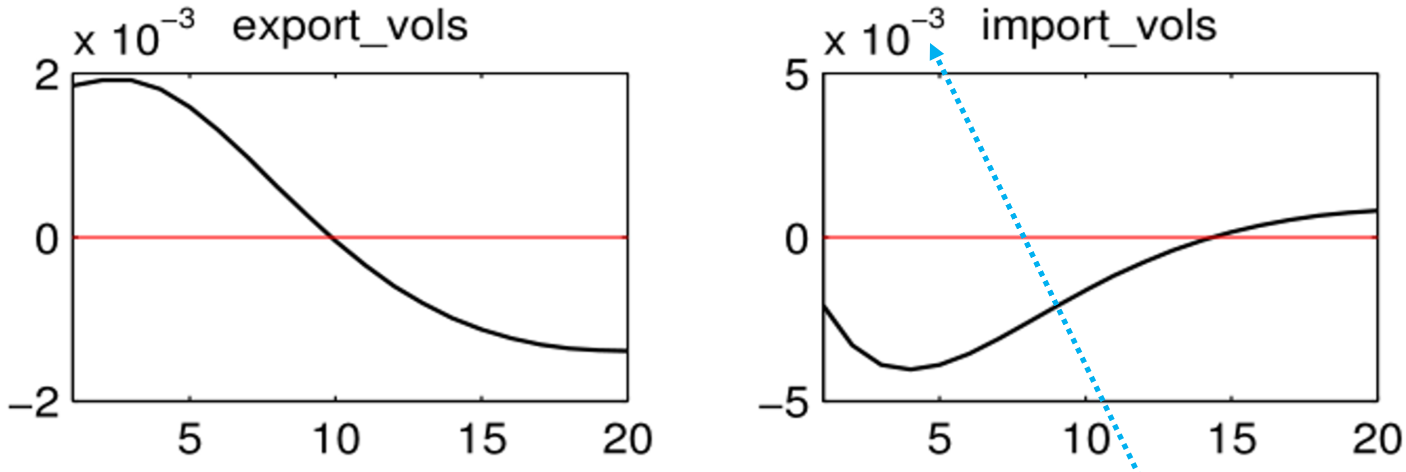
➔ domestic variables respond in a manner consistent with economic theory

Note scale difference



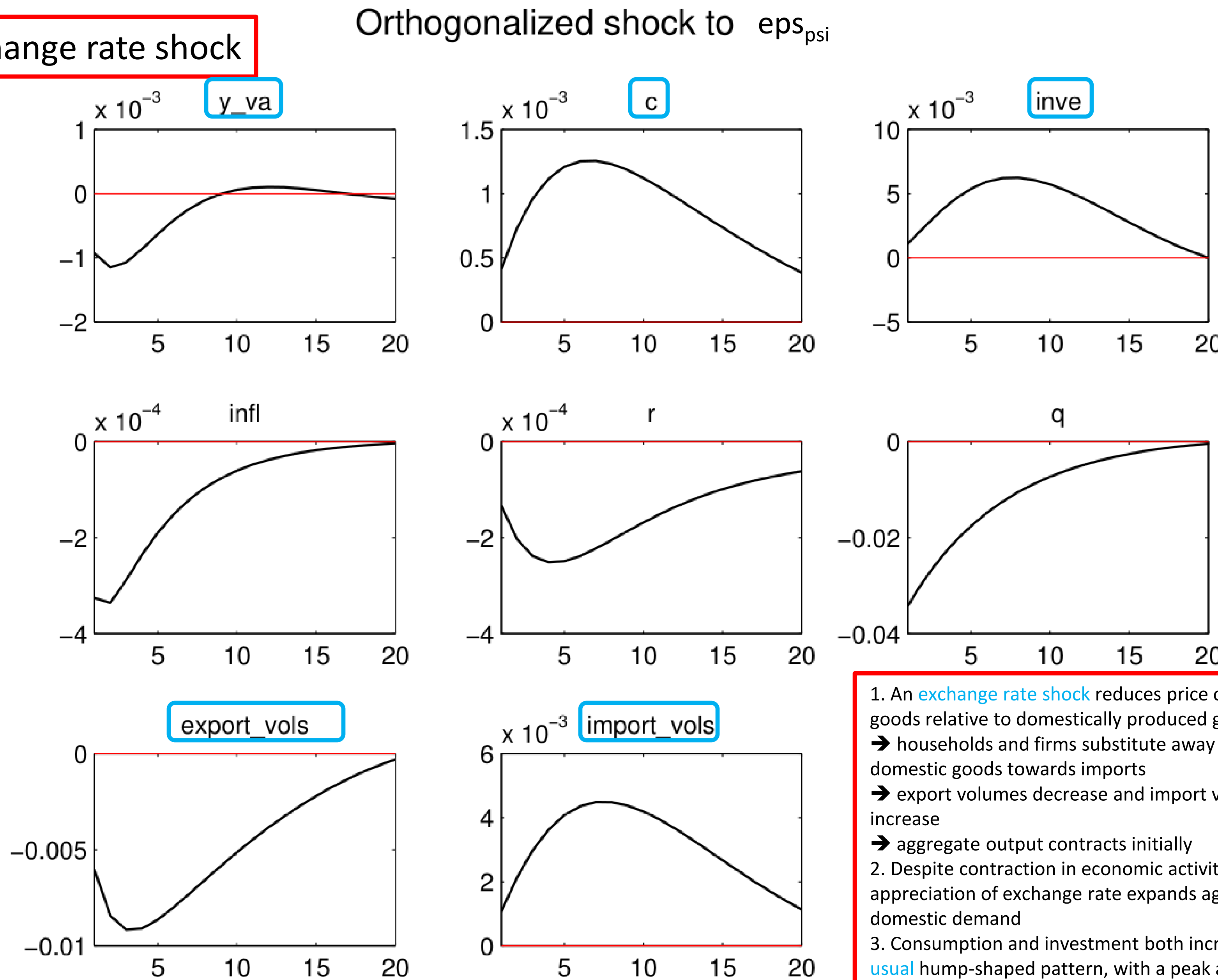
q ↓ ➔ appreciation

1. Higher interest rates ➔ contraction in real economy (relative to baseline with constant interest rates)
2. Output follows usual hump-shaped pattern, decreasing in a year to a trough, before returning to SS after 5 years
3. Components of domestic demand behave in a similar manner: consumption response is half as large as that of GDP, while contraction in investment is much larger
4. Nominal and real exchange rates both appreciate, which lowers inflation rate of imported goods and services
5. In conjunction with slowdown in economic activity, this causes a decrease in CPI inflation



- ***Import volumes*** also contract (though scale is ***very small***) following a positive monetary policy shock, as effects of higher interest rates on domestic demand overwhelm **substitution effects** associated with stronger exchange rate
- Response of ***export volumes***, however, is ***unexpected (but again, very small)***; they ***increase*** on impact, before decreasing below their baseline level after 10 quarters
- ***Initial increase*** in exports – which is common to both resource and non-resource exports – is ***due to lower domestic wages and cheaper capital***, both of which reduce firms' costs

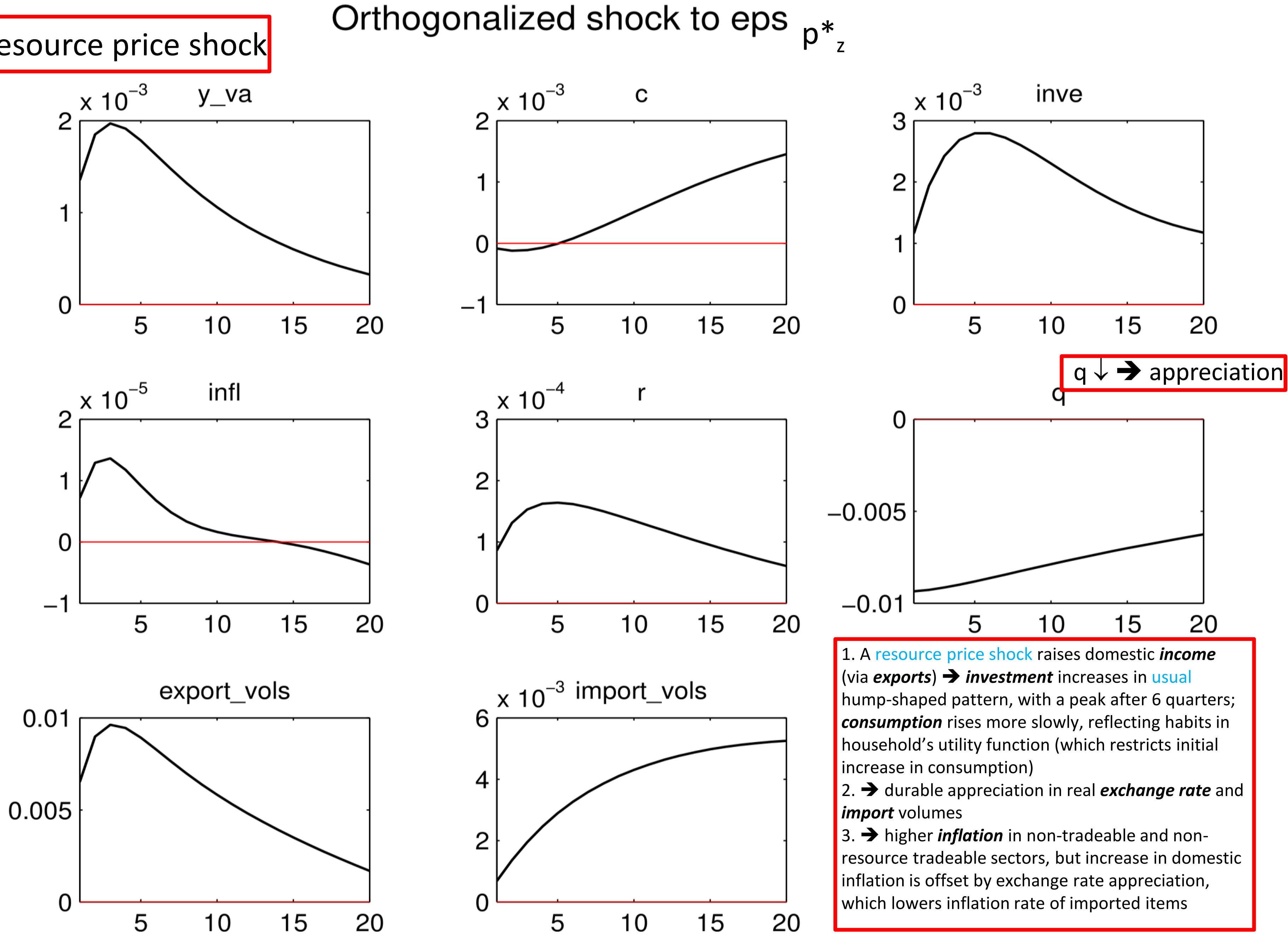
Exchange rate shock



1. An exchange rate shock reduces price of overseas goods relative to domestically produced goods
➔ households and firms substitute away from domestic goods towards imports
➔ export volumes decrease and import volumes increase
➔ aggregate output contracts initially
2. Despite contraction in economic activity, appreciation of exchange rate expands aggregate domestic demand
3. Consumption and investment both increase in usual hump-shaped pattern, with a peak after six to eight quarters

- ***Why*** an expansion in domestic demand?
- Two factors:
- First, **ER** appreciation raises ***purchasing power*** of domestic residents; for a given level of domestic production, they can now afford to consume more ***imports***
- Second, monetary policy responds to shock by lowering ***interest rate***
  – partly due to contraction in real economic activity (y_va)
  – partly reflecting deflationary impacts of exchange rate appreciation, which lowers (***M*** and ***NRT*** but not ***NT***) inflation on impact

Resource price shock



1. A resource price shock raises domestic ***income*** (via ***exports***) ➔ ***investment*** increases in usual hump-shaped pattern, with a peak after 6 quarters; ***consumption*** rises more slowly, reflecting habits in household's utility function (which restricts initial increase in consumption)
2. ➔ durable appreciation in real ***exchange rate*** and ***import*** volumes
3. ➔ higher ***inflation*** in non-tradeable and non-resource tradeable sectors, but increase in domestic inflation is offset by exchange rate appreciation, which lowers inflation rate of imported items

- ***Good idea***: RSH also check their ***steady-state*** for ***compatibility*** with data (see ranges below)

Checking model steady state...

Consumption share of GDP	**ok**	**55.8 vs (50.0,60.0)**
Investment share of GDP	**ok**	**22.7 vs (15.0,25.0)**
Non-tradable share of nominal GVA	**ok**	**64.0 vs (60.0,70.0)**
Other Tradable share of nominal GVA	**ok**	**23.1 vs (20.0,28.0)**
Resources share of nominal GVA	**ok**	**12.9 vs (10.0,15.0)**
Other Tradable share of export values	**ok**	**59.9 vs (50.0,65.0)**
Resources share of export values	**ok**	**40.1 vs (35.0,55.0)**
Nontradable share of investment	**ok**	**58.4 vs (50.0,70.0)**
Resources share of investment	**ok**	**13.9 vs (7.0,15.0)**
Other Tradable share of investment	**ok**	**27.7 vs (15.0,30.0)**
Imports share of GDP	**ok**	**20.6 vs (19.0,30.0)**
Exports share of GDP	**ok**	**19.6 vs (19.0,30.0)**
Public demand share of GDP	**ok**	**22.5 vs (15.0,25.0)**

All tests passed.

Appendix: Justiniano Preston 2010

- Justiniano-Preston 2010 model consists of (a) ***small open economy (SOE)*** and (b) ***rest of the world (ROW)*** and contains (as usual) households, firms, a monetary authority and a passive fiscal authority
- As usual, RH consumes domestically produced ***[NEW] and imported*** goods, supplies labour, and invests in either domestic ***[NEW] or foreign*** one-period bonds
- As usual, firms are divided into domestic producers, retailers and final good producers
- As usual, ***domestic*** firms produce differentiated domestic goods by using labour input and sell these goods domestically ***[NEW] and overseas***

- *[NEW]* Retail firms ***import*** differentiated (***intermediate***) products from ROW and sell them in (Home) SOE's domestic market
- In principle, ∃ also a ***perfectly competitive final goods*** sector that buys domestic and ***import*** varieties and produces a final consumption good
- However, final goods sector is not modelled explicitly since ***perfectly competitive*** firms make zero profit
- ***Note*** that here for simplicity ∃ ***no capital*** (but this could easily be rectified)

- As usual, unlike final goods producers, domestic producers and retailers operate under ***monopolistic competition***
- Recall: under ***imperfect (monopolistic) competition***, output is ***below*** its ***Pareto-optimal*** levels in absence of ***government intervention***, even with perfectly ***flexible prices***
- ➔ ***taxes*** = ***subsidy*** required to eliminate imperfect competition ***distortion***
- Also, ***[NEW]*** ***fiscal policy*** is responsible for ensuring ***zero net debt***

- ***ROW*** is ***large*** compared to ***SOE***
- ➔ all foreign economy variables are taken ***exogenously*** by domestic (SOE) economy
- But ***explicitly modelled*** to ***endogenise*** overall interactions of SOE and ROW

- Details of J-P model follow now-familiar structure of NK DSGE models, so we shall go quickly
- ***Household j*** maximizes lifetime ***utility function***, which is ***separable*** in consumption (C), hours worked (N) and time

$$E_0 \sum_{t=0}^{\infty} \beta^t \tilde{\varepsilon}_{g,t} \left[\frac{(C_t - H_t)^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right]$$

- where (as usual)
  - $0 < \beta < 1$ = ***discount factor***
  - $\sigma > 0$ = inverse of ***elasticity of intertemporal substitution***
  - $\varphi > 0$ = inverse of ***(Frisch) elasticity of labour supply with respect to real wages***

- Repeating $$E_0 \sum_{t=0}^{\infty} \beta^t \tilde{\varepsilon}_{g,t} \left[\frac{(C_t - H_t)^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right]$$
- $\varepsilon_{g,t}$ is a ***preference-shifter shock*** which affects marginal utility of consumption
- $H_t \equiv hC_{t-1}$ is (***external***) habit term, taken as exogenous by individual household
- Consumption C is ***[NEW]*** ***CES composite*** of domestic and import goods

$$C_t = \left[(1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}$$

- where $\eta > 0$ = elasticity of ***substitution*** between domestic ***and*** foreign goods

- In this CES composite, $C_{H,t}$ and $C_{F,t}$ are each usual ***Dixit-Stiglitz aggregates*** of available domestic and foreign produced goods given by

$$C_{H,t} = \left[\int_0^1 C_{H,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \quad \text{and} \quad C_{F,t} = \left[\int_0^1 C_{F,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

- where $\varepsilon > 1$ = (common) elasticity of ***substitution*** between types of ***differentiated*** domestic ***or*** foreign goods

- Available ***assets*** are one-period domestic ***[NEW] and foreign bonds*** → optimisation occurs subject to ***flow budget constraint***

$$P_t C_t + D_t + e_t B_t = D_{t-1}\left(1 + i_{t-1}\right) + e_t B_{t-1}\left(1 + i^*_{t-1}\right) \boxed{\phi_t\left(A_t\right)} + W_t N_t + \Pi_{H,t} + \Pi_{F,t} + T_t$$

- where D_t and B_t denote holdings of one-period domestic and foreign ***bonds*** with ***gross interest rates*** i_t and i_t* and e_t is ***nominal exchange rate***
- Households receive ***wages*** W_t for labour supplied
- $\Pi_{H,t}$ and $\Pi_{F,t}$ denote ***profits*** from equity holdings in domestic and retail firms
- T_t denotes ***taxes and transfers***
- $\phi_t(A_t)$ is a debt-elastic ***interest rate premium*** ensuring stationarity of foreign debt level

- ***Household's optimisation problem*** requires allocation of expenditures across all types of domestic ***[NEW] and foreign*** goods both ***intra***temporally and ***inter***temporally
- Optimal allocation of expenditure across domestic and foreign goods ➔ ***demand functions***

$$C_{H,t}(i) = \left(P_{H,t}(i)/P_{H,t}\right)^{-\theta} C_{H,t} \text{ and } C_{F,t}(i) = \left(P_{F,t}(i)/P_{F,t}\right)^{-\theta} C_{F,t}$$

- where ***CPI*** is $P_t = \left[(1-\alpha)P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta}\right]^{\frac{1}{1-\eta}}$
- and ***Lagrange multiplier*** is

$$\lambda_t = \tilde{\varepsilon}_{g,t} P_t N_t^{\varphi} / W_t$$

$P_{F,t}$ is price in domestic market *and currency* of foreign good

- Assuming ***foreign economy*** is approximately ***closed*** (ie, influence of SOE [domestic] economy on it is ***negligible***), its available ***consumption bundle*** comprises continuum of only ***foreign-produced*** goods $C^*_{F,t}(j)$ for $j \in [0;1]$
- Foreign households thus need only decide how to ***allocate expenditures*** across these goods within any time period t (***intra***temporally) and also over time (***inter***temporally)
- Foreign debt in foreign economy is assumed to be in ***zero net supply***
- Foreign agents are assumed to have ***no access*** to (domestic) SOE debt markets

- ∃ continuum of monopolistically competitive ***domestic firms producing differentiated goods***
- ***Calvo-style price setting*** is assumed, allowing for ***indexation*** to past ***domestic*** goods-price inflation
- ➔ as usual, in any period t, fraction $1 - \theta_H$ of firms set prices ***optimally***, and fraction $0 < \theta_H < 1$ of goods prices are adjusted according to ***Calvo indexation rule*** $\log P_{H,t}(i) = \log P_{H,t-1}(i) + \delta_H \pi_{H,t-1}$
  - where $0 < \delta_H < 1$ measures degree of indexation to previous-period's inflation rate and $\pi_{H,t} = \log(P_{H,t}/P_{H,t-1})$
- ➔prices are re-optimised every $1/(1 - \theta_H)$ periods

- ***[NEW]*** Like domestic ***production*** firms, domestic ***retail*** firms ***also*** face a ***Calvo***-style price-setting problem allowing for ***indexation*** to past inflation
- Fraction $1 - \theta_F$ of firms set prices optimally, while fraction $0 < \theta_F < 1$ of goods prices are adjusted according to ***Calvo indexation*** rule
- ➔ prices are ***re-optimised*** every $1/(1 - \theta_F)$ periods

- In ***foreign*** economy ∃ no analogous optimal ***pricing*** problem
- Because ***imports*** form a ***negligible part*** of foreign consumption bundle, variations in ***import prices*** have ***negligible*** effect on evolution of foreign price index, P^*_t
- ***However,*** optimality conditions for domestic and foreign ***bond*** holdings
- ➔ ***uncovered interest rate parity (UIP) condition***
- restricting relative movements of domestic and foreign ***interest rates***, and changes in nominal ***exchange rate***

$$E_t \lambda_{t+1} P_{t+1} \left[(1 + \iota_t) - (1 + \iota_t^*) \left(e_{t+1} / e_t \right) \phi_{t+1} \right] = 0$$

- ***Nominal exchange rate*** e_t measures units of domestic currency per unit of foreign currency
- ➔ $e_t P^*_t$ = ROW price level in SOE currency
- ***Law of one price* ➔ $e_t P^*_t = P_{F,t}$**

Recall, $P_{F,t}$ is price in domestic market *and currency* of foreign good

- ***But*** monopolistically competitive ***pricing power*** of domestic retail firms
- ➔ failure of ***LOOP***
- ➔

$$\Psi_{F,t} \equiv e_t P^*_t / P_{F,t} \neq 1$$

- which defines what Monacelli (2005) calls "law of one price gap" (***LOOP Gap***)

- ***Monetary policy*** is conducted (as usual) according to a ***Taylor Rule***

$$i_t = \rho_i i_{t-1} + \psi_\pi \pi_t + \psi_y y_t + \psi_{\Delta y} \Delta y_t + \psi_e \Delta e_t + \varepsilon_{M,t}$$

- where $\varepsilon_{M,t}$ is an exogenous disturbance
- Policy responds to ***contemporaneous*** values of inflation, output, output growth and ***[NEW]*** growth rate in nominal ***exchange rate***
- ∃ also usual ***smoothing factor*** ρ
- ***Fiscal policy*** is specified as a ***zero debt*** policy

- These assumptions are for most part very familiar, ➔ usual optimising conditions, comprising an ***Euler equation*** and a ***Phillips curve*** based on ***marginal cost*** and incorporating ***Calvo*** stickiness
- When log-linearised, these lead to following [lower-case ➔ log deviations from SS]:
- ***Euler equation***

$$(1-\alpha)\,c_t = y_t - \alpha\eta\,(2-\alpha)\,s_t - \alpha\eta\psi_{F,t} - \alpha y_t^*$$

- ➔ domestic consumption depends not only on domestic output but also on ***three sources of foreign disturbance***:
  - terms of trade ($s \equiv p_{F,t} - p_{H,t}$),
  - deviations from law of one price (ψ_F),
  - foreign output (y^*)

- ***Phillips Curve and Marginal Cost***

$$\pi_{H,t} - \delta\pi_{H,t-1} = \theta_H^{-1}(1-\theta_H)(1-\theta_H\beta)mc_t + \beta E_t(\pi_{H,t+1} - \delta\pi_{H,t})$$

$$mc_t = \varphi y_t - (1+\varphi)\varepsilon_{a,t} + \alpha s_t + \sigma(1-h)^{-1}(c_t - hc_{t-1})$$

- Domestic price inflation $\pi_{H,t}$ is determined by
  - current ***marginal costs*** (mc),
  - ***expectations*** about inflation in next period
  - most recent observed inflation rate (latter appearing as result of ***price indexation***)
- Note that in **Dynare** implementation, coefficient δ is differentiated between ***domestic*** and ***imported*** inflation δ_H vs δ_F (as it should be)

- ➔ as with domestic consumption, and in ***contrast*** to a ***closed*** economy setting, domestic goods price inflation $\pi_{H,t}$ depends on ***three sources of foreign disturbance***
  - direct and indirect effect of ***terms of trade (s_t)*** on firms' ***marginal costs***

$$mc_t = \varphi y_t - (1+\varphi)\,\varepsilon_{a,t} + \alpha s_t + \sigma\,(1-h)^{-1}\,(c_t - hc_{t-1})$$

  - with latter operating through ToT implications for equilibrium consumption
  - effects of ***foreign output*** (via c_t in Euler equation)

$$(1-\alpha)\,c_t = y_t - \alpha\eta\,(2-\alpha)\,s_t - \alpha\eta\psi_{F,t} - \alpha y_t^*$$

  - ***deviations from law of one price*** ψ_F (also via Euler equation)

- ***Optimality conditions*** for ***retailers'*** pricing problem → ***Phillips curve*** for inflation in domestic currency ***price of imports*** $\pi_{F,t}$

$$\pi_{F,t} - \delta\pi_{F,t-1} = \theta_F^{-1}(1-\theta_F)(1-\theta_F\beta)\psi_{F,t} + \beta E_t\left(\pi_{F,t+1} - \delta\pi_{F,t}\right) + \varepsilon_{cp,t}$$

- $\pi_{F,t}$ is thus determined by LOOP gap given by $\psi_{F,t}$
- and ***expectations*** about next period's inflation rate
- JP2010 also add a ***cost-push shock*** $\varepsilon_{cp,t}$ to capture inefficient variations in mark-ups
- Prices ***indexed*** to past inflation induces ***history dependence*** on most recent observed inflation rate

- Model translated into **Dynare** is quite long and will not be presented here
- Results of estimating model by ***Regularised MLE*** using Canada data supplied by JP2010 (which sets US as "foreign" economy), are shown on next slides
- Estimation took 8min 45sec on a 3.4Ghz IntelCore7 CPU with 32GB RAM (mode_compute=4)....
- We focus on "open economy" specifics

[E:\JustinianoPreston\JP2010a_CaUS_BASE_RegMLE.mod and …_tex_binder.pdf]

These are the estimated values

Table 4: Parameter Values

Parameter	Value	Description
β	0.990	lifetime discount factor
σ	0.291	inverse elast intertemp substn
φ	0.118	inverse elast intertemp labour supply
h	0.527	degree of habit persistence
η	0.377	elast substn betw dom and import goods
α	0.280	share of foreign goods in consumption
θ_H	0.557	domestic producer Calvo parameter
θ_F	0.647	importer Calvo parameter
ρ_g	0.927	domestic preferences persistence
ρ_a	0.875	domestic NFA persistence
ρ_{cpF}	0.053	import cost-push persistence
ρ_{rp}	0.980	risk preference persistence
δ_H	0.062	indexation to domestic inflation
δ_F	0.037	indexation to import inflation

Low!

High!

Low!

- Parameter α, which measures ***share of foreign goods in consumption***, is estimated as 0.280, manifestly much higher than 12% share found in actual Canadian data
- Parameter η, which measures ***elasticity of substitution between domestic and import goods***, is estimated as 0.377, implying rather little such substitution
- ***Indexation*** to ***domestic*** inflation (δ_H) is estimated to be ***double*** that of indexation to ***import*** inflation (δ_F), implying that domestic sector responds ***more sluggishly*** to changing conditions than does import sector, but both are ***very small***

- By contrast, ***Calvo*** parameters (θ_H and θ_F) are reasonably similar
- Calvo parameter for ***domestic*** producers ➔ domestic prices ***re-optimised*** every $1/(1 - \theta_H) \approx 2.25$ quarters, with as noted, a very small degree of indexation (0.06)
- By contrast, for ***importers Calvo*** parameter ➔ prices are re-optimised a bit less frequently – only every $1/(1 - \theta_F) \approx 2.8$ quarters, also with almost no indexation (0.04)

- Results also indicate that intertemporal ***labour supply elasticity*** is quite high (1/.118 = 8.5), more than double that of intertemporal ***goods substitution*** (1/.291 = 3.4)

Remaining parameters

ν_H	0.157	domestic cost channel
ν_F	0.641	import cost channel
ϕ_e	0.408	UIP nominal ER response
ϕ_a	0.100	UIP NFA response
ρ_R	0.662	interest rate persistence
ψ_π	1.928	inflation response
ψ_y	0.041	output response
$\psi_{\Delta y}$	0.943	output gap change response
σ^*	0.572	inverse elast intertemp substn
φ^*	0.828	inverse elast intertemp labour supply
h^*	0.340	degree of habit persistence
δ^*	0.119	indexation to foreign inflation
ν^*	0.129	foreign cost channel
θ^*	0.528	foreign Calvo parameter
ρ_{a*}	0.850	foreign technology persistence
ρ_{R*}	0.730	foreign interest rate persistence
ψ_{π^*}	2.995	foreign inflation response
ψ_{y*}	0.072	foreign output response
$\psi_{\Delta y^*}$	1.000	foreign output chnage response
ρ_{gs}	0.905	foreign preferences persistence

Appendix: Kolasa 2009

- In this Appendix, we consider a two-country DSGE originally developed by Kolasa (2009) at the central bank of Poland
- It is essentially an open-economy version of the familiar SW2007 model
- In the Kolasa model, there are two countries in the world: Home (H) and Foreign (F)
- Each country is inhabited by a continuum of infinite-lived consumers, distributed over the intervals of [0; n] and[n; 1], respectively

- Both countries produce a continuum of differentiated tradable goods,indexed on the interval [0; n] in the Home economy and [n; 1] in the Foreign economy
- Each country produces also an array of nontradable goods, distributed over the same intervals as tradable goods
- Since the general setup of the Foreign country is similar to that for the Home economy, in what follows we focus on the exposition for the latter

- ***Household j*** maximizes the following lifetime ***utility function***, which is separable in consumption, hours worked, and time

$$U_t(j) = E_t \sum_{k=0}^{\infty} \beta^k \left[\frac{\varepsilon_{d,t+k}}{1-\sigma} \left(C_{t+k}(j) - hC_{t-1} \right)^{1-\sigma} - \frac{\varepsilon_{l,t+k}}{1+\varphi} L_{t+k}(j)^{1+\varphi} \right]$$

- where
  - $0 < \beta < 1$ is the ***discount factor***
  - $\sigma > 0$ is the (inverse of the) ***elasticity*** of intertemporal ***substitution***
  - h is the external ***habit persistence*** parameter
  - $\varphi > 0$ is the inverse of the ***Frisch elasticity*** of labour supply with respect to real wages
  - ε_d and ε_l are consumption preference and labour supply ***shocks***

- Household j's ***resource constraint*** is given by

$$P_{C,t}C_t(j) + P_{I,t}I_t(j) + E_t\left\{\Upsilon_{t,t+1}B_{t+1}(j)\right\}$$
$$= B_t(j) + W_t(j)L_t(j) + R_{K,t}K_t(j) + \Pi_{H,t}(j) + \Pi_{N,t}(j) + T_t(j)$$

- where $\Pi_{H,t(j)}$ and $\Pi_{N,t(j)}$ are ***dividends*** from tradable and nontradable goods producers, respectively
- $T_t(j)$ are nominal lump sum government ***transfers*** net of lump sum taxes
- $\Upsilon_{t,t+1}$ is the stochastic discount factor for nominal payoffs, such that $E_t\Upsilon_{t,t+1} = R_t^{-1}$, where R_t is the ***gross return*** on a riskless one-period bond
- B_{t+1} is the ***nominal payoff*** in period t+1 of the portfolio held at the end of period t

- The consumption bundle C_t consists of final ***tradable goods $C_{T,t}$*** and ***nontradable goods $C_{N,t}$***, aggregated according to

$$C_t = \frac{C_{T,t}^{\gamma_c} C_{N,t}^{1-\gamma_c}}{\gamma_c^{\gamma_c}(1-\gamma_c)^{1-\gamma_c}}$$

- where γ_c denotes the share of tradable goods in the total consumption of home households
- Consuming a final tradable good requires ω units of ***nontradable "distribution services" $Y_{D,t}$*** :

$$C_{T,t} = \min\{C_{R,t}; \omega^{-1} Y_{D,t}\}$$

- where $C_{R,t}$ is the bundle of "raw tradable goods"

- The index of ***raw tradable goods*** is defined by

$$C_{R,t} = \frac{C_{H,t}^{\alpha} C_{F,t}^{1-\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}$$

- where
  - $C_{H,t}$ is the bundle of home-made raw tradable goods consumed at home
  - $C_{F,t}$ is the bundle of foreign-made raw tradable goods consumed at home
  - α denotes the share of home goods in the home basket of tradable goods
- The indices of nontradable and both types of tradable goods are in turn given by the following ***Dixit-Stiglitz aggregators*** of individual varieties

$$C_{N,t} = \left[\left(\frac{1}{n} \right)^{\frac{1}{\phi_N}} \int_0^n C_t(z_N)^{\frac{\phi_N - 1}{\phi_N}} dz_N \right]^{\frac{\phi_N}{\phi_N - 1}}$$

$$C_{H,t} = \left[\left(\frac{1}{n} \right)^{\frac{1}{\phi_H}} \int_0^n C_t(z_H)^{\frac{\phi_H - 1}{\phi_H}} dz_H \right]^{\frac{\phi_H}{\phi_H - 1}}$$

$$C_{F,t} = \left[\left(\frac{1}{1-n} \right)^{\frac{1}{\phi_F}} \int_n^1 C_t(z_F)^{\frac{\phi_F - 1}{\phi_F}} dz_F \right]^{\frac{\phi_F}{\phi_F - 1}}$$

- where ϕ_N, ϕ_H and ϕ_F are the ***elasticities of substitution*** across varieties of a given type

- ***Intratemporal*** optimization implies the following ***demand functions*** for each variety of goods

$$C_t(z_N) = \frac{1}{n}(1-\gamma_c)\left(\frac{P_t(z_N)}{P_{N,t}}\right)^{-\phi_N}\left(\frac{P_{N,t}}{P_{C,t}}\right)^{-1} C_t$$

$$C_t(z_H) = \frac{1}{n}\gamma_c\alpha\left(\frac{P_t(z_H)}{P_{H,t}}\right)^{-\phi_H}\left(\frac{P_{H,t}}{P_{R,t}}\right)^{-1}\left(\frac{P_{T,t}}{P_{C,t}}\right)^{-1} C_t$$

$$C_t(z_F) = \frac{1}{1-n}\gamma_c(1-\alpha)\left(\frac{P_t(z_F)}{P_{F,t}}\right)^{-\phi_F}\left(\frac{P_{F,t}}{P_{R,t}}\right)^{-1}\left(\frac{P_{T,t}}{P_{C,t}}\right)^{-1} C_t$$

- where $P_t(z_j)$ is the price of variety z_j, and the others are composite ***price indexes*** of the form

$$P_{H,t} = \left[\frac{1}{n}\int_0^n P_t(z_H)^{1-\phi_H}\, dz_H\right]^{\frac{1}{1-\phi_H}}$$

- And of course the conventional stochastic consumption ***Euler Equation***

$$\beta R_t E_t \left\{ \frac{\varepsilon_{d,t+1}}{\varepsilon_{d,t}} \left(\frac{C_{t+1} - hC_t}{C_t - hC_{t-1}} \right)^{-\sigma} \frac{P_{C,t}}{P_{C,t+1}} \right\} = 1$$

- where as earlier,
- $\sigma > 0$ is the (inverse of the) ***elasticity*** of intertemporal ***substitution***
- R_t is the ***gross return*** on a riskless one-period bond
- $\varepsilon_{d,t}$ is the consumption ***preference shock***

- ***Investment Decisions*** also play a role
- Households spend part of their income on a homogenous investment good, which is transformed into the ***capital stock*** K_{t+1} according to the formula

$$K_{t+1} = (1-\tau)K_t + \varepsilon_{i,t}\left(1 - S(\frac{I_t}{I_{t-1}})\right) I_t$$

- where τ is the depreciation rate
- As in SW, capital accumulation is impacted by investment-specific ***technological progress*** $\varepsilon_{i,t}$, and ***adjustment cost*** represented by the function S

- FOCs for the consumer ➔

$$Q_{T,t} = E_t\left\{\frac{R_{K,t+1}}{P_{C,t+1}}\frac{P_{C,t+1}}{P_{C,t}R_t}\right\} + (1-\tau)E_t\left\{\frac{P_{C,t+1}}{P_{C,t}R_t}Q_{T,t+1}\right\}$$

- which determines the relative price of installed capital (i.e. ***Tobin's Q***), defined as

$$Q_{T,t} \equiv \frac{\lambda_{K,t}}{\lambda_{C,t}P_{C,t}}$$

- where
  - $\lambda_{C,t}$ is the marginal utility of nominal income (which is the Lagrange multiplier on households' budget constraint)
  - $\lambda_{K,t}$ is the Lagrange multiplier on the capital law of motion

- The homogeneous ***investment good*** is produced in a similar fashion as the final consumption good, except that there are ***no distribution costs*** associated with supplying its tradable component
- ∃ ***differences in the tradable-nontradable composition*** between the final consumption basket and the investment basket (i.e. γ_c need not be equal to γ_i)
- But the ***structure*** of the purely tradable component is identical for both types of goods
- This simplifies calibration

- Each household in the ***home country*** supplies monopolistically one distinctive type of ***labour*** L(j), aggregated into a homogenous labour input as usual
- And ***wages*** are subject to the ***Calvo fairy***, leading to the familiar formula for the evolution of the ***aggregate wage index*** (W~ is the optimal wage)

$$W_t = \left[\theta_W \left(W_{t-1}\left(\frac{P_{C,t-1}}{P_{C,t-2}}\right)^{\delta_W}\right)^{1-\phi_W} + (1-\theta_W)\tilde{W}_t^{1-\phi_W}\right]^{\frac{1}{1-\phi_W}}$$

- The wage setting problem faced by ***foreign*** households is ***similar*** and leads to an ***analogous*** first-order condition and an aggregate wage law of motion

Production

- There exist a continuum of identically ***monopolistic competitive firms*** in each of the tradable and nontradable sectors of the domestic economy
- The production technology is homogenous with respect to labour and capital inputs

$$Y_t(z_N) = \varepsilon_{a^N,t} L_t(z_N)^{1-\eta} K_t(z_N)^{\eta}$$
$$Y_t(z_H) = \varepsilon_{a^H,t} L_t(z_H)^{1-\eta} K_t(z_H)^{\eta}$$

- where **η** is output elasticity with respect to capital (common across sectors but not necessarily across countries)
- the **ε** are sector specific productivity parameters

- The output index in each sector is given by the usual ***Dixit-Stiglitz aggregators***

$$Y_{N,t} = \left[\left(\frac{1}{n}\right)^{\frac{1}{\phi_N}} \int_0^n Y_t(z_N)^{\frac{\phi_N - 1}{\phi_N}} dz_N \right]^{\frac{\phi_N}{\phi_N - 1}}$$

$$Y_{H,t} = \left[\left(\frac{1}{n}\right)^{\frac{1}{\phi_H}} \int_0^n Y_t(z_H)^{\frac{\phi_H - 1}{\phi_H}} dz_H \right]^{\frac{\phi_H}{\phi_H - 1}}$$

- Since all firms operate technologies with the ***same relative intensity*** of productive factors and face the ***same prices*** for labour and capital inputs, cost minimization ➔ ***capital-labour ratio*** identical across all domestic firms

$$\frac{W_t L_t}{R_{K,t} K_t} = \frac{1-\eta}{\eta}$$

Prices

- As for wages, firms set their prices according to the usual ***Calvo*** approach
- Here, the prices of firms that do not receive a price signal from the Calvo fairy are ***indexed to the past inflation*** according to the following Calvo rule

$$P_t(z_N) = P_{t-1}(z_N)\left(\frac{P_{N,t-1}}{P_{N,t-2}}\right)^{\delta_N}$$

- where δ_N is the degree of indexation in ***nontradable*** prices

- $MC_{N,t}$ is the ***real marginal cost*** (identical across firms from a given sector since factor markets are homogenous) defined as

$$MC_{N,t} = \frac{1}{P_{N,t}\varepsilon_{a^N,t}} \left(\frac{W_t}{1-\eta}\right)^{1-\eta} \left(\frac{R_{K,t}}{\eta}\right)^{\eta}$$

- As there are no firm-specific shocks in the model, all firms that are allowed to reset their price select the same optimal price ➔ ***home nontradable goods*** price index of familiar form

$$P_{N,t} = \left[\theta_N \left(P_{N,t-1}\left(\frac{P_{N,t-1}}{P_{N,t-2}}\right)^{\delta_N}\right)^{1-\phi_N} + (1-\theta_N)\tilde{P}_{N,t}^{1-\phi_N}\right]^{\frac{1}{1-\phi_N}}$$

- Prices are set in the producer currency and the LOOP holds at the dock for each tradable variety ➔ price of home goods sold abroad and that of foreign goods sold domestically are given by

$$P_t^*(z_H) = ER_t^{-1} P_t(z_H) \quad P_t(z_F) = ER_t P_t^*(z_F)$$

- $\boldsymbol{ER_t}$ is the ***nominal exchange rate*** expressed as units of domestic currency per one unit of foreign currency
- The ***real exchange rate*** $\boldsymbol{Q_t}$ is defined as

$$Q_t = \frac{ER_t P_{C,t}^*}{P_{C,t}}$$

- ***Real exchange rate*** is allowed to deviate from its ***purchasing power parity*** ("LOOP Gap") owing to
  - changes in relative prices of ***tradable vs. nontradable*** goods in both countries (the ***internal exchange rates***)
  - changes in relative ***distribution costs***
  - changes in ***terms-of-trade***, as long as there is some ***home bias*** in preferences (ie, $\alpha \neq \alpha^*$)
- Terms-of-trade S_t are defined as home import prices relative to home export prices

$$S_t = \frac{ER_t P^*_{F,t}}{P_{H,t}}$$

Monetary and fiscal authorities

- ***Monetary*** authorities in both countries respond to the economic conditions through the following simple Taylor-type feedback rule

$$R_t = R_{t-1}^{\rho} \left[\left(\frac{Y_t}{\bar{Y}} \right)^{\phi_y} \left(\frac{P_{C,t}}{(1+\bar{\pi})P_{C,t-1}} \right)^{\phi_\pi} \right]^{1-\rho} \varepsilon_{m,t}$$

- where
  - Y_t is total output produced in the economy
  - $\bar{Y}$ is its steady state level
  - $\bar{\pi}$ is steady state CPI inflation
  - $\varepsilon_{m,t}$ is a monetary policy shock

- ***Fiscal*** authorities are modelled in a very simplistic fashion:
- Government expenditures and transfers to households are fully financed by ***lump sum taxes***, so that the state budget is ***balanced*** each period
- Government spending is fully directed at ***nontradable*** goods and is modelled as a stochastic process $\varepsilon_{g,t}$
- Given the representative agent assumption, Ricardian equivalence holds in the model

- ***Market clearing conditions*** close the model
- Output of each firm producing ***non-tradable goods (N)*** is either consumed ***domestically***, spent on investment or used for distribution services or purchased by the government
- Similarly, all ***tradable goods (H)*** are consumed or invested ***domestically or abroad***
- ➔

$$Y_{N,t} = (1-\gamma_c)\left(\frac{P_{N,t}}{P_{C,t}}\right)^{-1} C_t + \omega\gamma_c\left(\frac{P_{T,t}}{P_{C,t}}\right)^{-1} C_t + (1-\gamma_i)\left(\frac{P_{N,t}}{P_{I,t}}\right)^{-1} I_t + G_t$$

$$\begin{aligned} Y_{H,t} = {} & \alpha\gamma_c\left(\frac{P_{H,t}}{P_{R,t}}\right)^{-1}\left(\frac{P_{T,t}}{P_{C,t}}\right)^{-1} C_t + \frac{1-n}{n}\alpha^*\gamma_c^*\left(\frac{P_{H,t}^*}{P_{R,t}^*}\right)^{-1}\left(\frac{P_{T,t}^*}{P_{C,t}^*}\right)^{-1} C_t^* \\ & + \alpha\gamma_i\left(\frac{P_{H,t}}{P_{R,t}}\right)^{-1}\left(\frac{P_{R,t}}{P_{I,t}}\right)^{-1} I_t + \frac{1-n}{n}\alpha^*\gamma_i^*\left(\frac{P_{H,t}^*}{P_{R,t}^*}\right)^{-1}\left(\frac{P_{R,t}^*}{P_{I,t}^*}\right)^{-1} I_t^* \end{aligned}$$

- Model ***does not*** have a closed-form solution → must be ***log-linearised*** around the non-stochastic steady state
- Model is driven by ***fourteen stochastic shocks***, seven for each country
- Preference, labour supply, government spending, investment efficiency and productivity shocks in the two sectors are assumed to follow ***first-order autoregressive processes***
- Monetary policy shocks are assumed to be ***white noise***
- Monetary policy shocks and the IID innovations to the remaining types of shocks are allowed to be ***correlated across countries***

- For the log-linearised model and output of simulation see ***E:/Kolasa/Kolasa2009_simul_TeX_binder.pdf***