Macroeconomics B, EI060

Class 6

Exchange rate with flexible prices

Cédric Tille

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What you will get from today class

- Nominal exchange rate market.
  - Characteristics (BIS)
- Exchange rate and money market (Harms VIII.1-VIII.4, Obstfeld and Rogoff (secondary) 8.1-8.3.3).
  - Money demand and interest parity.
  - Forward-looking nature of the exchange rate.
- Empirical evidence (Engel and Wu 2024)

A question to start

MONEY MARKET
$M^S \uparrow \Rightarrow$ CHF WEAK

$M^S \rightarrow i \downarrow$
i ARTIFICIAL

The exchange rate being the relative price of currencies, a country's currency depreciates if its money supply is high today.

FROM $t-1$ TO t

RELATIVE
$t-1$ vs t

Do you agree? Why or why not?

EXPECTATIONS
$M_{t+1} \longrightarrow i_{TODAY}$

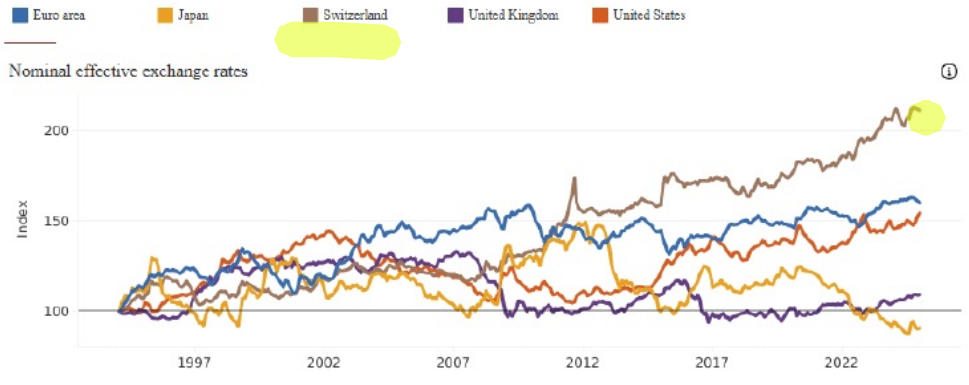
SOME PATTERNS

Nominal exchange rate

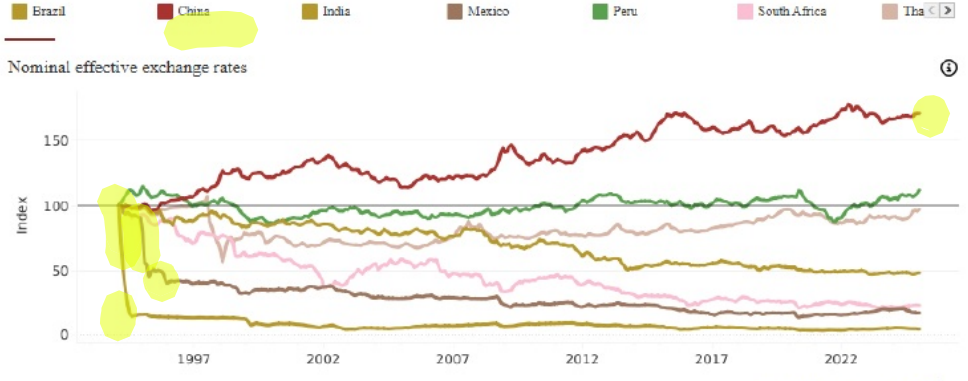
- Price of a currency in terms of another one. Bilateral or effective (index, trade-weighted).
- Trends of nominal appreciations among advanced economies, more varied among emerging ones (https://data.bis.org/topics/EER , under "tables & dashboards".)

ADV

↑ APPRECI. OF CRY'S CURRENCY



EME



Pattern of the exchange rate market

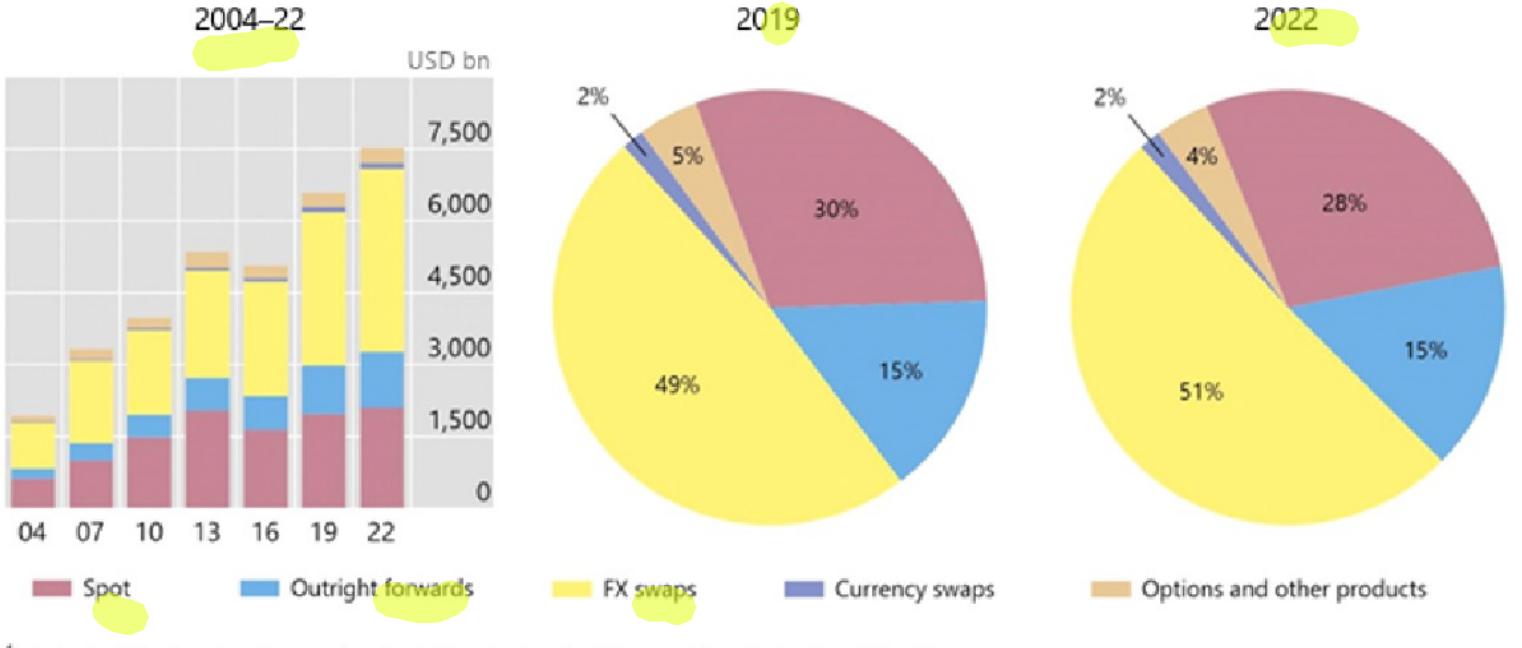
- Every three years the BIS undertakes a survey of the foreign exchange market (last in April 2022).
- Data on volumes, types of instruments, trader, currency pairs.
- Large volume, and growing rapidly: 7.5 trillion $ per day, 6.5% of annual world GDP (115.5 trillion $). 14% growth over 2019.
- FX transaction can be in the form of:
  - Spot: immediate transaction (less than a third of transactions).
  - Forward: price agreed today, with transaction taking place at a later date.
  - Swap: combination of spot and forward (half the transactions), short horizon (less than 7 days).
- The US dollar is present in nearly half the transactions.

Turnover by instruments

- Dominant share of swaps.

Foreign exchange market turnover by instrument[1]

Net-net basis, daily averages in April — Graph 1



[1] Adjusted for local and cross-border inter-dealer double-counting, ie "net-net" basis.

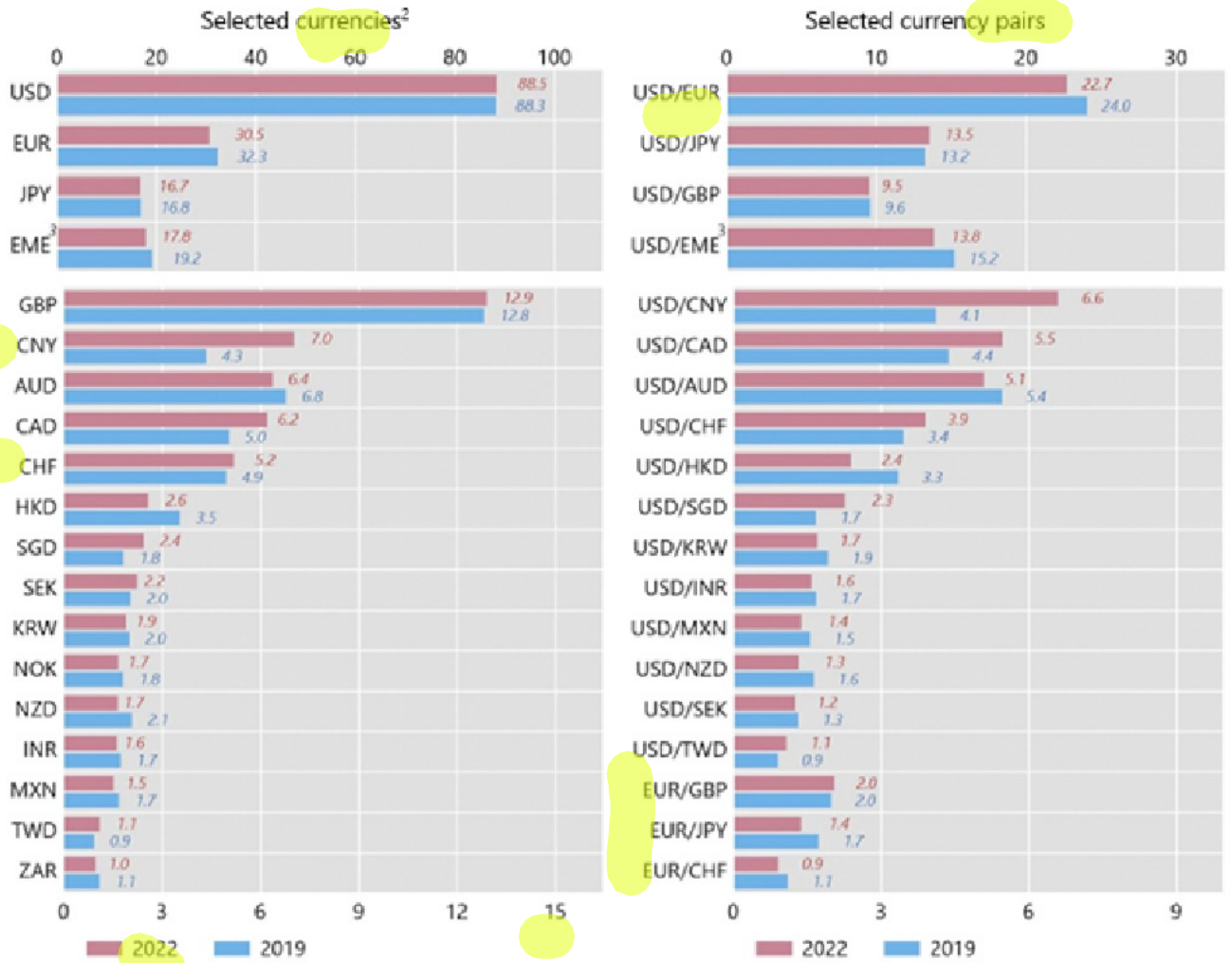
Source: BIS Triennial Central Bank Survey. For additional data by instrument, see Table 1.

Bank for International settlements (2022). "Foreign Exchange Turnover in April 2022", BIS Triennal Central Bank Survey.

- Dominant presence of the US dollar (currency shares add up to 200%).

Foreign exchange market turnover by currency and currency pairs[1]

Net-net basis, daily averages in April, as a percentage of total turnover — Graph 4



[1] Adjusted for local and cross-border inter-dealer double-counting, ie "net-net" basis. [2] As two currencies are involved in each transaction, the sum of shares in individual currencies will total 200%. [3] Emerging market economy currencies excluding the Chinese renminbi and Russian rouble: AED, ARS, BGN, BHD, BRL, CLP, COP, CZK, HKD, HUF, IDR, ILS, INR, KRW, MXN, MYR, PEN, PHP, PLN, RON, SAR, SGD, THB, TRY, TWD and ZAR.

Source: BIS Triennial Central Bank Survey. For additional data by currency and currency pairs, see Tables 4 and 5. See our Statistics Explorer for access to the full set of published data.

Bank for International settlements (2022). "Foreign Exchange Turnover in April 2022", BIS Triennal Central Bank Survey.

EXCHANGE RATE AND MONEY

Investment in different bonds

- Investment at time t in a Home currency bond or a Foreign currency bond.
  - Certain returns at time $t+1$ of i^H_{t+1} and i^F_{t+1} in the respective currencies (slightly different timing notation from book, for consistency with later points).
- Exchange rate: units of Home currency per unit of Foreign currency. An increase is a depreciation of the Home currency.
  - Spot for immediate transaction: E_t. Similarly for the next period E_{t+1}, but uncertain from the point of view of period t.
  - Forward: F_t. Set in period t for transaction in period $t+1$.
- Investment in Foreign bond: one unit of Home currency gives $1/E_t$ units of Foreign currency, which translates for sure into $\left(1+i^F_{t+1}\right)/E_t$ of Foreign currency at time $t+1$.
  - Convert back into Home currency

Interest rate parities

- For investors to be indifferent, expected returns are equalized.
- Covered interest parity using the forward exchange rate (risk free):

$$1 + i^H_{t+1} = \left(1 + i^F_{t+1}\right) \frac{F_t}{E_t}$$

  - Should hold, but has not since 2008 (especially in turmoil times) as arbitrageurs face limits.
- Uncovered parity if only spot transactions. Reflects the expected exchange rate:

$$1 + i^H_{t+1} = \left(1 + i^F_{t+1}\right) \frac{E^{\text{expected}}_{t+1}}{E_t}$$

- Linearized versions around a steady state:

$$i^H_{t+1} = i^F_{t+1} + f_t - e_t \quad ; \quad i^H_{t+1} = i^F_{t+1} + e^{\text{expected}}_{t+1} - e_t$$

Optimizing model of money

- Expected Home agent's utility from consumption, C, and from real money balances, M/P:

$$U_t = \sum_{s=t}^{\infty} \beta^s \left[ln\left(C_s\right) + \chi ln\left(\frac{M_s}{P_s}\right) \right]$$

- Invests in Home money, a bond in Home currency, and a bond in Foreign currency. Flow budget constraint:

$$\begin{aligned} B_{t+1}^H + E_t B_{t+1}^F + M_t = & \left(1 + i_t^H\right) B_t^H + E_t \left(1 + i_t^F\right) B_t^F \\ & + M_{t-1} + P_t \left(Y_t - C_t\right) \end{aligned}$$

- Focus on periods t and $t+1$. Euler condition (expectation denoted by $\mathbb{E}_t$, avoid confusion with the exchange rate E_t):

$$\frac{1}{C_t} = \beta \left(1 + i_{t+1}^H\right) \mathbb{E}_t \left(\frac{P_t}{P_{t+1}} \frac{1}{C_{t+1}}\right)$$

Money demand

- Combine optimal condition with respect to money and the Euler:

$$
\begin{aligned}
0 &= \chi \frac{1}{M_t} - \frac{1}{P_t C_t} + \beta \mathbb{E}_t \left(\frac{1}{P_{t+1} C_{t+1}} \right) \\
0 &= \chi \frac{1}{M_t} - \frac{1}{P_t C_t} + \frac{1}{P_t C_t} \frac{1}{1 + i^H_{t+1}} \\
\frac{M_t}{P_t} &= \chi C_t \frac{1 + i^H_{t+1}}{i^H_{t+1}}
\end{aligned}
$$

- Money demand: real balances increase with consumption and decrease with the interest rate.

Interest parity

- Combine optimality conditions of Home currency and Foreign currency bonds ($\omega_{t+1} = 1/(P_{t+1} C_{t+1})$):

$$\left(1 + i^H_{t+1}\right) \mathbb{E}_t\left(\omega_{t+1}\right) = \left(1 + i^F_{t+1}\right) \mathbb{E}_t\left(\frac{E_{t+1}}{E_t} \omega_{t+1}\right)$$

$$\left(1 + i^H_{t+1}\right) = \left(1 + i^F_{t+1}\right) \mathbb{E}_t\left(\frac{E_{t+1}}{E_t}\right) + \left(1 + i^F_t\right) \frac{\mathbb{CV}_t\left(\left(E_{t+1}/E_t\right), \omega_{t+1}\right)}{\mathbb{E}_t\left(\omega_{t+1}\right)}$$

- Uncovered interest parity conditions, adjusted for covariance.
- Investing in the Foreign currency bond is appealing if the Home currency weakens when the marginal utility is high, $\mathbb{CV}\left(\left(E_{t+1}/E_t\right), \omega_{t+1}\right) > 0$,

Linearized model

- Uncovered interest parity, money demand, and purchasing power parity:

$$\begin{aligned} i_{t+1}^{H} &= i_{t+1}^{F} + \mathbb{E}_t\left(e_{t+1}\right) - e_t \\ m_t - p_t &= c_t - \lambda i_{t+1}^{H} \\ p_t &= e_t + p_t^{F} \end{aligned}$$

- Use PPP to substitute for the price in the money demand:

$$\begin{aligned} m_t - \left(e_t + p_t^{F}\right) &= c_t - \lambda i_{t+1}^{H} \\ i_{t+1}^{H} &= \frac{1}{\lambda}\left[\left(e_t - m_t\right)\right] + \frac{1}{\lambda}\left(c_t + p_t^{F}\right) \end{aligned}$$

Dynamic of the exchange rate

- Use the money demand to substitute for the interest rate in the UIP to get a dynamic relation in the exchange rate:

$$\begin{aligned}\frac{1}{\lambda}\left[\left(e_t - m_t\right)\right] + \frac{1}{\lambda}\left(c_t + p_t^F\right) &= i_{t+1}^F + \mathbb{E}_t\left(e_{t+1}\right) - e_t \\ -m_t + c_t + p_t^F &= \lambda i_{t+1}^F + \lambda \mathbb{E}_t\left(e_{t+1}\right) - (1+\lambda)\, e_t\end{aligned}$$

- Re-arrange:

$$e_t = \frac{\lambda}{1+\lambda}\mathbb{E}_t\left(e_{t+1}\right) + \frac{1}{1+\lambda} m_t + \frac{1}{1+\lambda}\left[\lambda i_{t+1}^F - \left(c_t + p_t^F\right)\right]$$

REAL / FOREIGN

Exchange rate solution

- Set $\lambda i_{t+1}^F - \left(c_t + p_t^F\right) = 0$ and iterate the relation forward, along with the transversality condition $lim_{s\to\infty}\left(\frac{\lambda}{1+\lambda}\right)^s \mathbb{E}_t\left(e_s\right)$:

$$
\begin{aligned}
e_t &= \frac{1}{1+\lambda} m_t + \frac{\lambda}{1+\lambda}\mathbb{E}_t\left(e_{t+1}\right) \\
e_t &= \frac{1}{1+\lambda} m_t + \frac{\lambda}{1+\lambda}\frac{1}{1+\lambda}\mathbb{E}_t\left(m_{t+1}\right) + \left(\frac{\lambda}{1+\lambda}\right)^2 \mathbb{E}_t\left(e_{t+2}\right) \\
e_t &= \frac{1}{1+\lambda}\sum_{s=t}^{\infty}\left[\left(\frac{\lambda}{1+\lambda}\right)^{s-t}\mathbb{E}_t\left(m_s\right)\right]
\end{aligned}
$$

- The exchange rate reflects the money supply.
  - Current value, as well as all future values with a geometrically decreasing weight.

Constant money growth

- Money supply grows at a constant rate μ:

$$m_s = m_t + \mu(s - t)$$

- Money demand in two subsequent periods ($c_t + p_t^F = 0$):

$$m_t - e_t = -\lambda i_{t+1}^H \quad ; \quad m_t + \mu - e_{t+1} = -\lambda i_{t+2}^H$$

- In steady money growth path, the interest rate is constant at i^H. Therefore: $e_{t+1} - e_t = \mu$, and UIP implies ($i^F = 0$): $i^H = \mu$.
- The money demand gives the exchange rate. A higher growth rate of money depreciates the currency:

$$e_t = m_t + \lambda\mu$$

Pre-announced changes

- Money is initially constant at $\underline{m}$, and the exchange rate is $e_t = \underline{m}$.
- At time t the central bank announces that at time $T \geq t$ money will increase permanently to $\overline{m}$. The exchange rate will then be $e_t = \overline{m}$.
- Immediate depreciation of the exchange, with a jump followed by a gradual move:

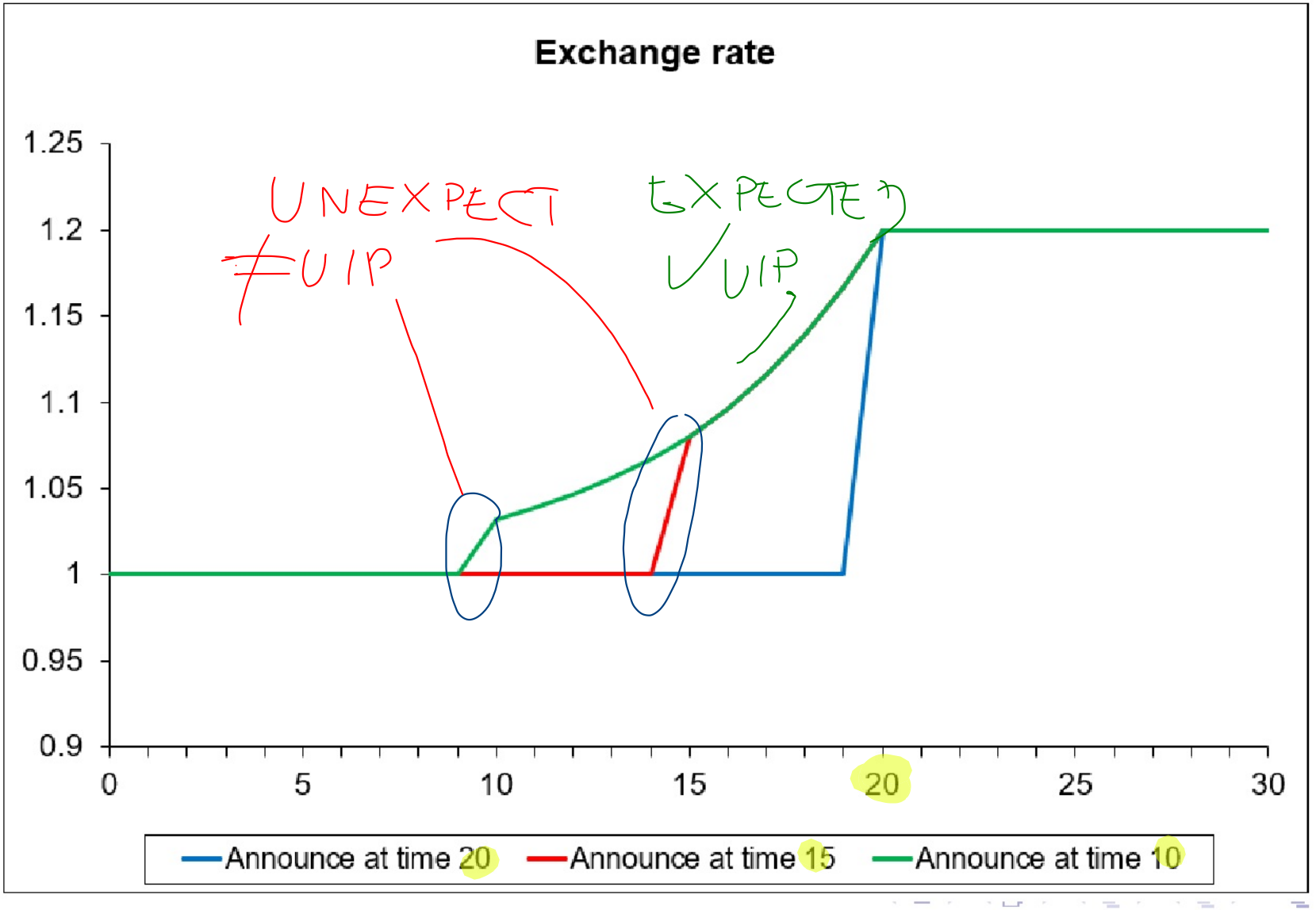
$$e_t = \overline{m} - (\overline{m} - \underline{m}) \left[1 - \left(\frac{\lambda}{1+\lambda} \right)^{T-t} \right]$$

- Initial jump:

$$e_t - \underline{m} = (\overline{m} - \underline{m}) \left(\frac{\lambda}{1+\lambda} \right)^{T-t} > 0$$

Exchange rate dynamics

- Increase of money from 1 to 1.2 at time 20.



Jump vs. subsequent dynamics

- Interest parity is about the expected exchange rate movement from today to tomorrow.
- Shocks can make the interest rate jump unexpectedly today.
- Consider that shift from $\underline{m}$ to $\overline{m}$ happens when announced ($t = T$).
  - Exchange rate was constant, and will be (at $\underline{m}$ and $\overline{m}$ respectively).
  - The interest rate is always equal to the world interest rate.
  - Initial jump for the exchange rate, followed by constant level.

EMPIRICAL ANALYSIS

The challenge of explaining exchange rates

- Long literature testing models of exchange rates, with mixed results (exchange rate "disconnect" from fundamentals).
- Fit has improved over the last 20 years (Engel and Wu 2024).
- Regress the monthly US dollar depreciation (increase is a dollar depreciation) on:
  - Change of real policy interest rates. Tighter US policy should appreciate the dollar.
  - Inflation (expectation): under inflation targeting, higher inflation should appreciate the dollar (expectation of tighter policy).
  - Trade balance: smaller deficit should appreciate the dollar.
  - Higher risk should appreciate the dollar, as US is a safe haven.
  - Convenience (non monetary) return.
- Contrast 1999-2023 with 1973-1998.

Results since 1999

- Signs as expected, and significant. Stronger dollar following tighter US policy (actual or expected), higher risk, smaller deficit.

Table 1: Baseline regression with inflation level, and convenience yield

$$\Delta s_t = \alpha + \beta_1 \Delta r_t + \beta_2 \Delta r_t^* + \beta_3 \pi_t + \beta_4 \pi_t^* + \beta_5 \Delta RISK_t + \beta_6 q_{t-1} + \beta_7 \frac{TB}{GDP}_t + \beta_8 \Delta \eta_t + u_t$$

	AUD	CAD	EUR	GBP	NZD	NOK	SEK	Panel fixed effect	Panel pooled
Δr_t	-1.12***	-1.66***	-2.34***	-1.37***	-0.92***	-1.85***	-1.94***	-1.48***	-1.47***
	(-3.83)	(-6.38)	(-8.11)	(-5.32)	(-2.83)	(-6.37)	(-6.53)	(-7.35)	(-7.30)
Δr_t^*	1.12***	1.16***	2.21***	1.80***	1.06***	0.23	0.80**	0.92***	0.94***
	(4.03)	(4.30)	(5.60)	(4.84)	(2.97)	(1.14)	(2.42)	(5.63)	(5.72)
π_t	-0.25**	-0.21*	-0.69***	-0.33***	-0.45***	-0.21*	-0.58***	-0.34***	-0.33***
	(-2.41)	(-1.66)	(-4.88)	(-2.85)	(-3.50)	(-1.94)	(-4.65)	(-4.32)	(-4.31)
π_t^*	0.03	0.14	0.53***	0.14	0.24*	-0.19	0.24**	0.15**	0.18***
	(0.24)	(0.94)	(4.02)	(1.31)	(1.76)	(-1.48)	(2.57)	(2.13)	(2.83)
$\Delta RISK_t$	-0.03***	-0.02***	-0.01***	-0.01***	-0.02***	-0.02***	-0.02***	-0.02***	-0.02***
	(-10.19)	(-9.60)	(-2.77)	(-4.61)	(-6.67)	(-7.56)	(-5.92)	(-7.70)	(-7.44)
q_{t-1}	-0.01	-0.01	-0.02**	-0.03***	-0.01	-0.03***	-0.01	-0.01**	-0.00
	(-1.34)	(-1.50)	(-2.58)	(-2.72)	(-1.50)	(-2.87)	(-1.33)	(-2.19)	(-0.45)
$\frac{TB}{GDP}_t$	-0.48***	-0.45***	-0.63***	-0.73***	-0.37*	-0.66***	-0.80***	-0.54***	-0.48***
	(-2.76)	(-3.71)	(-3.86)	(-3.49)	(-1.91)	(-3.16)	(-4.00)	(-4.26)	(-3.80)
$\Delta \eta_t$	-1.92**	-2.33***	-0.86	-1.52*	-1.56**	-1.20*	-1.04	-1.38**	-1.45**
	(-2.09)	(-2.93)	(-0.92)	(-1.76)	(-2.11)	(-1.77)	(-1.54)	(-2.24)	(-2.33)
N	296	296	295	296	296	296	296	2071	2071
F	21.80	21.45	13.30	11.56	11.33	16.80	13.12	22.65	21.50
R2	0.38	0.37	0.27	0.24	0.24	0.32	0.27		0.25
R2_adj	0.36	0.36	0.25	0.22	0.22	0.30	0.25		
R2_within								0.25	

Engel, Charles, and Steve Pak Yeung Wu (2024). "Exchange Rate Models are Better than You Think, and Why They Didn't Work in the Old Days", mimeo

Earlier sample

- Only US policy matters, and R2 are lower.

Table 4: Baseline regression with inflation level, sample period Mar 1973-Dec 1998

$$\Delta s_t = \alpha + \beta_1 \Delta r_t + \beta_2 \Delta r_t^* + \beta_3 \pi_t + \beta_4 \pi_t^* + \beta_5 \Delta RISK_t + \beta_6 q_{t-1} + \beta_7 \frac{TB}{GDP_t} + u_t$$

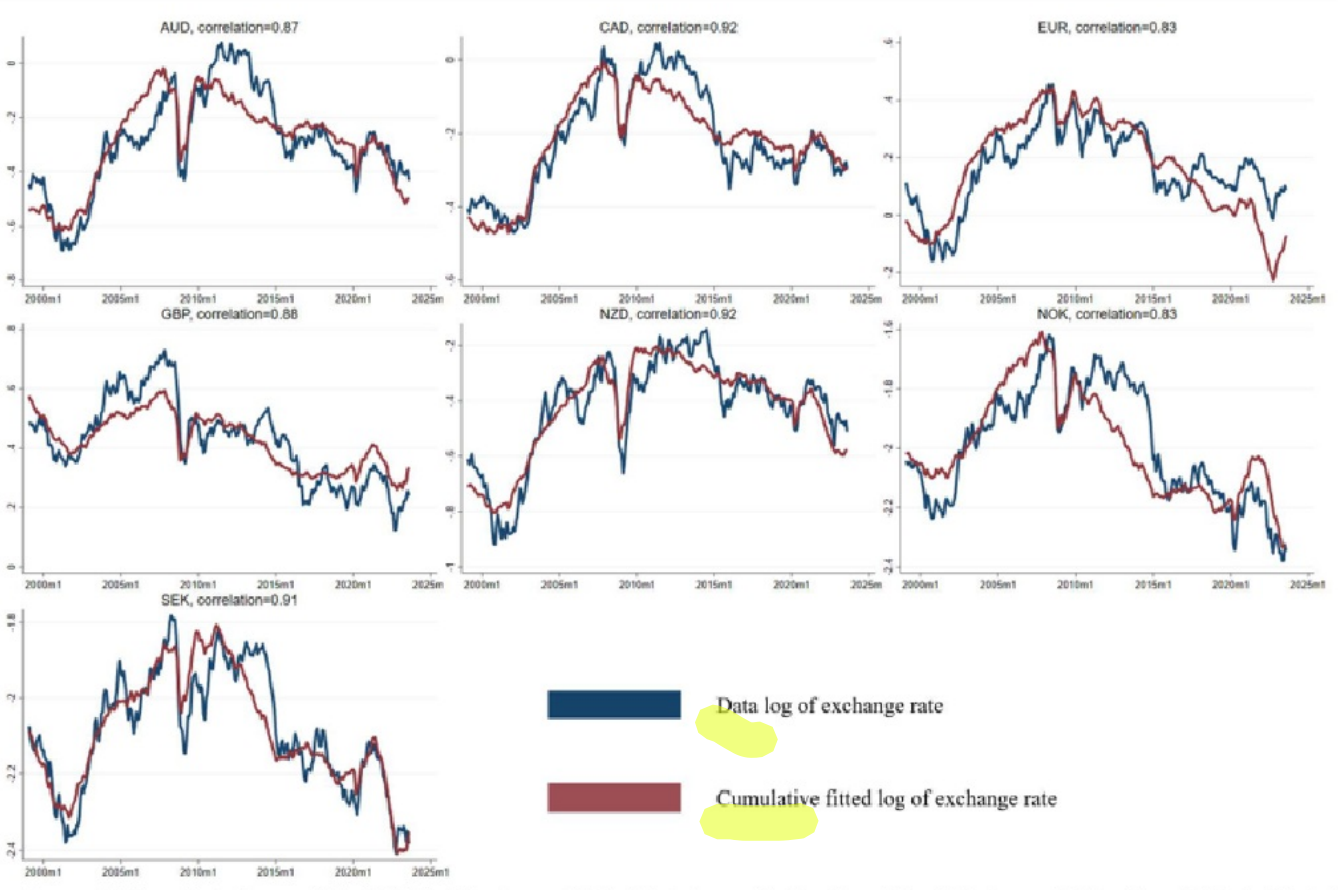
	AUD	CAD	DEM	GBP	NZD	NOK	SEK	Panel fixed effect	Panel pooled
Δr_t	-0.72***	-0.03	-0.77**	-0.72**	-0.18	-0.65**	-0.86***	-0.63***	-0.63***
	(-2.84)	(-0.24)	(-2.48)	(-2.59)	(-0.40)	(-2.49)	(-3.17)	(-3.55)	(-3.56)
Δr_t^*	0.03	-0.51***	-0.22	-0.62***	0.11	0.16	0.35***	0.02	0.02
	(0.20)	(-5.80)	(-0.66)	(-3.57)	(0.74)	(0.84)	(2.99)	(0.28)	(0.28)
π_t	0.08	-0.03	-0.09	0.07	-0.20*	0.07	0.02	0.02	0.02
	(1.21)	(-0.76)	(-1.23)	(0.77)	(-1.86)	(1.03)	(0.27)	(0.42)	(0.44)
π_t^*	0.02	0.05	0.26	-0.02	0.12**	-0.06	-0.01	0.01	0.01
	(0.39)	(1.39)	(1.39)	(-0.33)	(2.18)	(-1.08)	(-0.27)	(0.29)	(0.29)
$\Delta RISK_t$	-0.01***	-0.00	0.00	-0.00	0.00	-0.00	-0.00	-0.00	-0.00
	(-3.61)	(-0.30)	(0.93)	(-0.69)	(0.43)	(-0.09)	(-1.09)	(-1.24)	(-1.23)
q_{t-1}	-0.03*	0.00	0.01	-0.01	0.01	-0.01	0.00	-0.00	-0.00
	(-1.78)	(0.11)	(0.56)	(-0.50)	(1.14)	(-0.98)	(0.46)	(-0.20)	(-0.47)
$\frac{TB}{GDP_t}$	0.19	-0.08	-0.77*	-0.53**	0.01	-0.17	-0.42*	-0.26*	-0.26*
	(0.79)	(-0.91)	(-1.82)	(-2.28)	(0.04)	(-0.86)	(-1.92)	(-1.95)	(-1.92)
N	311	311	311	311	205	311	311	1760	1760
F	2.25	6.91	3.52	4.89	1.36	2.08	3.65	3.34	3.45
R2	0.05	0.14	0.08	0.10	0.05	0.05	0.08		0.03
R2_adj	0.03	0.12	0.05	0.08	0.01	0.02	0.06		
R2_within								0.03	

Engel, Charles, and Steve Pak Yeung Wu (2024). "Exchange Rate Models are Better than You Think, and Why They Didn't Work in the Old Days", mimeo

Dynamics since 1999

- Cumulating regressions fit for monthly exchange rate change gives a good fit of level (challenging test).

Figure 1 Comparing data and model implied exchange rates



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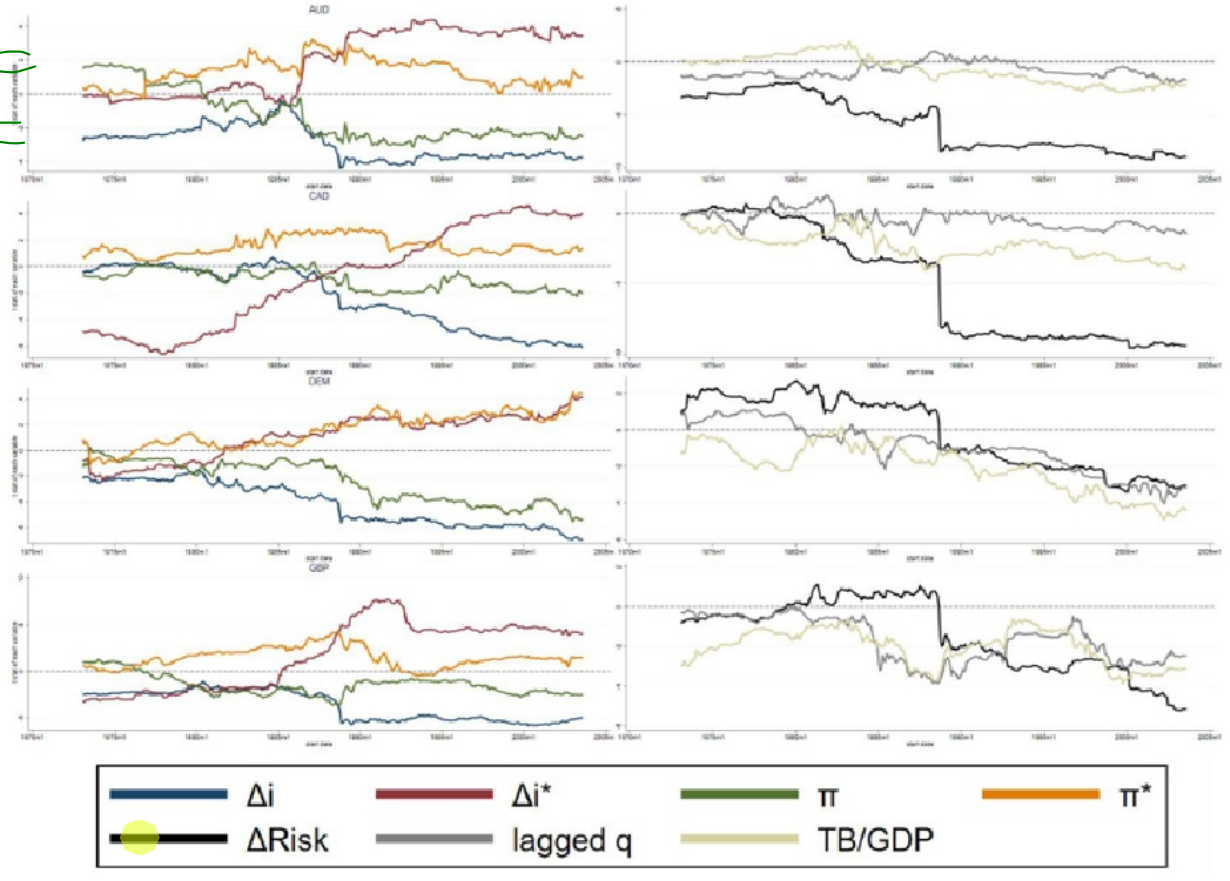
Significance

- T-stats increase in absolute value of the last 20 years.

Figure 2: *t* statistics of 20-year rolling window regressions of equation (1)



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Explaining the pattern

$\frac{\partial i}{\partial \pi} > 1$

- Simple New Keynesian framework, with sticky prices.
  - Uncovered interest rate parity with premium shocks.
  - New Keynesian Phillips curve (cross-country difference), with impact of expected inflation on current inflation.
  - Taylor rule for the interest rate (cross-country difference).
- If the Taylor principle applies, with real interest rate increasing following an inflation surprise, equilibrium is unique.
- Otherwise, possibility of self-fulfilling movements in the exchange rate.
- The improvement of the monetary policy framework since the late 20th century is consistent with the empirical results.

STABILITY