EI035 Econometrics II
The Graduate Institute, Spring 2025, Marko Mlikota

PS3 Solutions

Jingle Fu

Solution (a).

- **Unit (Firm) Fixed Effects:** Each firm i has intercept α_i. When the number of firms N is large but the time period T is fixed, we have to estimate almost $N - T$ parameters. Such problem can lead to inconsistency in the estimation of the common parameters, becaus ethe estimation error in α_i doesn't vanish as $N \to \infty$.
- **Time Fixed Effects:** In contrast, time fixed effects add only T dummies, the number of parameters associated with time dummies is fixed. Hence, their estimation does not create an incidental parameters problem.

In short, while unit fixed effects can cause IPP when T is relative small to N, the addition of time fixed effects does not because the number of time dummies remains fixed and is asymptotically negligible.

Table 1: Fixed Eects Model

	(1) FE b/se
log of employment	0.737***
	(0.063)
log of deflated capital	0.096**
	(0.044)
log of deflated R&D	0.144***
	(0.028)
Observations	2971

```
use GMdata.dta, clear
xtset index yr
xtreg ldsal lemp ldnpt ldrnd i.yr, fe robust
```

Solution (b).

Starting from the original equation for firm i at time t:

$$\Delta ldsal_{it} = \beta_1 \Delta lemp_{it} + \beta_2 \Delta ldnpt_{it} + \beta_3 \Delta ldrnd_{it} + (f_t - f_{t-1}) + \Delta u_{it}$$

Since α_i does not vary over time, it drops out in the differencing.
The time effects appear as differences $f_t - f_{t-1}$. Thus, in the first-differenced equation the levels of the year dummies disappear, but their differences remain.

Solution (c).
As $d357_{it}$ equals 1 for firms in industry 357 and is constant over time, then its effect is absorbed by the firm fixed effect α_i. In the fixed effects (within) estimator, the coefficient on $d357$ is not separately identified. n the first-differenced model, any time-invariant variable will vanish $\Delta d357_{it} = 0 \forall t$.
Including a time-invariant dummy does not worsen the IPP since it adds only one parameter that is either absorbed (or eliminated in first differences).

Solution (d).
As we define $\ddot{y}_{it} = y_{it} - \overline{y}_{it}$ and $\ddot{X}_{it} - \overline{X}_{it}$, we have:

$$\hat{\beta}_{FE-W} = \left(\sum_{i,t} \ddot{X}_{it}' \ddot{X}_{it}\right) \sum_{i,t} \ddot{X}_{it} \ddot{y}_{it}$$

$$\hat{\beta}_{RE} = \left(X'\Omega^{-1}X\right)^{-1} X'\Omega^{-1}y$$

Table 2: Fixed Effects Model

	(1) log of deflated sales b/se
log of employment	0.650***
	(0.031)
log of deflated capital	0.186***
	(0.025)
log of deflated R&D	0.098***
	(0.019)
Observations	856

```
use "GMdata_balanced.dta", clear
xtset index yr
gen d357 = (sic3 == 357)

xtreg ldsal lemp ldnpt ldrnd i.yr##i.d357, fe robust
eststo fe_w
esttab fe_w using d1.tex, replace label booktabs title("FE-W Estimator") ///
    cells("b(fmt(3)) se(fmt(3))")

xtreg ldsal lemp ldnpt ldrnd i.yr##i.d357, re robust
```

Table 3: Random Effects Model

	(1)
	log of deflated sales
	b/se
log of employment	0.582***
	(0.026)
log of deflated capital	0.340***
	(0.019)
log of deflated R&D	0.067***
	(0.016)
Observations	856

```
eststo RE
esttab RE using d2.tex, replace label booktabs title("RE Estimator") ///
    cells("b(fmt(3)) se(fmt(3))")
```

Solution (e).

The Hausman test statistics is:

$$H = \left(\hat{\beta}_{FE} - \hat{\beta}_{RE}\right)' \left[A\mathbb{V}[\hat{\beta}_{FE}] - A\mathbb{V}[\hat{\beta}_{RE}]\right]^{-1} \left(\hat{\beta}_{FE} - \hat{\beta}_{RE}\right)$$

```
hausman fe_w re, sigmamore
```

```
Test of H0: Difference in coefficients not systematic

    chi2(3) = (b-B)'[(V_b-V_B)^(-1)](b-B)
            =  67.24
Prob > chi2 = 0.0000
(V_b-V_B is not positive definite)
```

Hausman test output yields a chi-square statistic of 67.24 (with p-value= 0.0000). The null hypothesis is strongly rejected, indicating that the RE estimator is inconsistent and that the FE estimator is preferred.

Solution (f).

Let$\theta = \beta_1 + \beta_2$. Under the null hypothesis, we have

$$H_0 : \theta = 1.$$

An estimator for θ is

$$\hat{\theta} = \hat{\beta}_1 + \hat{\beta}_2.$$

Suppose the estimated coefficients $\hat{\beta}_1$ and $\hat{\beta}_2$ have the following variance-covariance matrix:

$$V = \begin{pmatrix} \text{Var}(\hat{\beta}_1) & \text{Cov}(\hat{\beta}_1, \hat{\beta}_2) \\ \text{Cov}(\hat{\beta}_1, \hat{\beta}_2) & \text{Var}(\hat{\beta}_2) \end{pmatrix}.$$

Because $\hat{\theta}$ is a sum of $\hat{\beta}_1$ and $\hat{\beta}_2$, by the properties of variance we have:

$$\text{Var}(\hat{\theta}) = \text{Var}(\hat{\beta}_1 + \hat{\beta}_2) = \text{Var}(\hat{\beta}_1) + \text{Var}(\hat{\beta}_2) + 2\ \text{Cov}(\hat{\beta}_1, \hat{\beta}_2).$$

Under $\mathcal{H}_0$, the deviation of $\hat{\theta}$ from its hypothesized value is:

$$\hat{\theta} - 1 = \hat{\beta}_1 + \hat{\beta}_2 - 1.$$

Since, by the Central Limit Theorem, $\hat{\theta}$ is approximately normally distributed in large samples, we can standardize the difference:

$$Z = \frac{\hat{\theta} - 1}{\sqrt{\text{Var}(\hat{\theta})}}.$$

Under H_0, Z is asymptotically standard normal. Squaring this Z statistic gives a chi-square statistic with 1 degree of freedom:

$$\chi^2 = \frac{(\hat{\beta}_1 + \hat{\beta}_2 - 1)^2}{\text{Var}(\hat{\beta}_1) + \text{Var}(\hat{\beta}_2) + 2\ \text{Cov}(\hat{\beta}_1, \hat{\beta}_2)}.$$

This test statistic is compared to the χ^2_1 distribution.

- If χ^2 is large (and the corresponding p-value is small), we reject H_0 and conclude that the sum of β_1 and β_2 is statistically different from 1.
- If χ^2 is not large (and the p-value is large), we do not reject H_0 and there is no evidence against constant returns to scale.

At the conventional 5% level, the p-value (0.0528) is marginally above 0.05, meaning we do not reject $\mathcal{H}_0$.