Problem Set I

EI037 Microeconomics I

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1. Preference-based Approaches

In the preference-based approach, the objectives of the decision maker are summarized by the preference relation $\succeq$. For individual preferences to be "rational", we impose two basic assumptions about $\succeq$: completeness and transitivity.

1.a. Write down the definitions of completeness and transitivity. Can you provide one example of a preference relation $\succeq$ that satisfies completeness but not transitivity?

1.b. Is it possible that a preference relation $\succeq$ satisfies transitivity but not completeness? If you think it is possible, please provide one example. If not, please explain why.

1.c. Do you think the transitivity assumption is reasonable? Can you give a real-world example that violates transitivity, but seems to make common sense?

1.d. From $\succeq$, we can derive another important relation on X, the *strict preference relation* $\succ$:

$$x \succ y \iff x \succeq y \text{ but not } y \succeq x.$$

Show that $\succ$ is transitive but not complete.

2. Choice Rules

In the choice-based approach, the objectives of the decision maker are summarized by the choice structure $(\mathcal{B}, C(\cdot))$. For individual choices to be "reasonable", we impose some restrictions regarding an individual's choice behavior. Apart from the fact that an individual's choice cannot be empty nor out of their budget set, we require that their choice satisfy the weak axiom of revealed preference (or weak axiom in short).

2.a. What is a choice structure $(\mathcal{B}, C(\cdot))$? Provide an example of $(\mathcal{B}, C(\cdot))$.

2.b. What is the weak axiom (WA)? Provide an example of $(\mathcal{B}, C(\cdot))$ that violates WA.

2.c. We defined a revealed preference relation $\succeq^*$ from the observed individual choice behavior in $C(\cdot)$ in class. What is this definition? How does it differ from $\succeq$ defined in the preference-based approach?

2.d. Provide an example of a preference for which an induced choice structure satisfies WA, but the preference itself does not satisfy transitivity nor completeness.

3. Consumer Choice

3.a. Some definitions

In the entire world, there are 10 apples, 10 bananas, 20 bottles of wine, and 10 Gruyere cheese traded in the market economy at the price of 1, 2, 3, and 4 CHF respectively. You as an individual have 60 CHF. Use the notation we studied in Lecture 2.

(i) What is the set of commodities in the economy? What does L equal to?

(ii) Give an example of a commodity vector (i.e., consumption bundle).

(iii) What is the commodity set?

(iv) What is the budget set?

(v) Plot out your budget plane.

3.b. Weak axiom of revealed preference

Consider a consumer who consumes only two goods and satisfies Walras' law. When prices are (2,4), they demand (y,10). When prices are (5,5), they demand (9,3). Nothing else of significance has changed between the two situations.

(i) Suppose that $y = 5$. Do these consumption plans satisfy the weak axiom of revealed preference?

(ii) Suppose that $y = 2$. Do these consumption plans satisfy the weak axiom of revealed preference?

(iii) For which range of y do these consumption plans violate the weak axiom?

4. Classical Demand Theory

4.a. Show that if a consumer has a Cobb-Douglas preference

$$U = x^{\beta} y^{1-\beta}; \text{ with } \beta \in (0, 1),$$

their demand function is homogeneous of degree one w.r.t. wage w, i.e., $x(p, \alpha w) = \alpha x(p, w)$ for all $\alpha > 0$.