International Finance
Lecture IX
Sovereign Debt: Theory
(USG Chapter 13)

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What debt?

- External
  - No difference between net and gross
  - No difference between public and private
- CA is fully financed with debt
  - No equity
  - No remittances or aid
- Deabt is just there to smooth consumption
- Plan
  - Models with state-contingent debt
  - Analytical models without state-contingent debt
  - Quantitative models without state-contingent debt

Default Incentives with State-Contingent Contracts

- We start by analyzing the structure of international debt contracts when agents have access to state-contingent financial instruments but may not be able to commit to repay
- Consider a one-period economy facing the following stochastic endowment

$$y = \bar{y} + \epsilon$$

  $\bar{y} > 0$ is a constant and ϵ is a random variable with $E(\epsilon) = 0$ and density $\pi(\epsilon)$ defined over the interval $[\epsilon^L, \epsilon^H]$
- Thus, $y = \bar{y}$ is the mean of the endowment process and ϵ is an endowment shock satisfying:

$$\int_{\epsilon^L}^{\epsilon^H} \epsilon \pi(\epsilon) d\epsilon = 0$$

- Note $d\epsilon$ is in a different color (bad notation but we need to differentiate from d which we will use for debt)

- Assume that before observing ϵ, the country can buy insurance from foreign investors against the endowment shock
- This insurance takes the form of state-contingent debt $d(\epsilon)$ When $\epsilon < 0$ the country receives payments from the rest of the world, the opposite when $\epsilon > 0$
- Foreign investors are risk neutral, competitive and face an international interest rate equal to zero. With these assumptions the (foreign investors) participation constraint is:

$$\int_{\epsilon^L}^{\epsilon^H} d(\epsilon)\pi(\epsilon)d\epsilon = 0 \tag{13.1}$$

- The country maximizes utility of the representative consumer:

$$\int_{\epsilon^L}^{\epsilon^H} u(c(\epsilon))\pi(\epsilon)d\epsilon \tag{13.2}$$

under the period budget constraint:

$$c(\epsilon) = \bar{y} + \epsilon - d(\epsilon) \tag{13.3}$$

Assume that the country can commit to repay

The country maximizes:

$$\int_{\epsilon^L}^{\epsilon^H} u(c(\epsilon))\pi(\epsilon)d\epsilon \tag{13.2R}$$

under the period budget constraint:

$$c(\epsilon) = \bar{y} + \epsilon - d(\epsilon) \tag{13.3R}$$

and the participation constraint:

$$\int_{\epsilon^L}^{\epsilon^H} d(\epsilon)\pi(\epsilon)d\epsilon = 0 \tag{13.1R}$$

The Lagrangean is (I substituted c using 13.3R):

$$\mathcal{L} = \int_{\epsilon^L}^{\epsilon^H} \left[u(\bar{y} + \epsilon - d(\epsilon)) + \lambda d(\epsilon)\right] \pi(\epsilon)d\epsilon$$

Note that λ (the Lagrange multiplier associated with the participation constraint) is NOT state contingent. The choice variable is d and the first order condition is:

$$u'(c(\epsilon)) = \lambda$$

- Since λ is independent of ϵ, the FOC indicates that consumption is also independent of ϵ
- That is, the optimal contract leads to perfect consumption smoothing.
- Integrate both sides of the period budget constraint (13.3) using $\int_{\epsilon^L}^{\epsilon^H} \pi(\epsilon) d\epsilon$:

$$\int_{\epsilon^L}^{\epsilon^H} c(\epsilon)\pi(\epsilon) d\epsilon = \bar{y} \int_{\epsilon^L}^{\epsilon^H} \pi(\epsilon) d\epsilon + \int_{\epsilon^L}^{\epsilon^H} \epsilon\pi(\epsilon) d\epsilon - \int_{\epsilon^L}^{\epsilon^H} d(\epsilon)\pi(\epsilon) d\epsilon$$

- Note that: $\int_{\epsilon^L}^{\epsilon^H} \pi(\epsilon) d\epsilon = 1$, $\int_{\epsilon^L}^{\epsilon^H} \epsilon\pi(\epsilon) d\epsilon = 0$, and $\int_{\epsilon^L}^{\epsilon^H} d(\epsilon)\pi(\epsilon) d\epsilon = 0$.
- Thus, consumption is constant and equal to average endowment:

$$c(\epsilon) = \bar{y}$$

- Debt payments are equal to the endowment shocks
- All risk is transferred to foreign lenders:

$$d(\epsilon) = \epsilon$$

- Payments move one per one with the endowment:

$$d'(\epsilon) = \frac{\partial d(\epsilon)}{\partial \epsilon} = 1$$

- This derivative tells us how much insurance the contract provides
- A unit slope tells us that we get full insurance

Assume that the country cannot commit

- The country would now default in states of the world in which it needs to make net payments
- To avoid this, we need an incentive compatibility (IC) constraint:

$$d(\epsilon) \leq 0 \tag{13.4}$$

- The representative household's problem becomes

$$\int_{\epsilon^L}^{\epsilon^H} u(c(\epsilon))\pi(\epsilon)d\epsilon \tag{13.2R}$$

Subject to

$$c(\epsilon) = \bar{y} + \epsilon - d(\epsilon) \tag{13.3R}$$

$$\int_{\epsilon^L}^{\epsilon^H} d(\epsilon)\pi(\epsilon)d\epsilon = 0 \tag{13.1R}$$

$$d(\epsilon) \leq 0 \tag{13.4R}$$

- The last constraint is new.

$$\int_{\epsilon^L}^{\epsilon^H} d(\epsilon)\pi(\epsilon)d\epsilon = 0 \tag{13.1R}$$

$$d(\epsilon) \leq 0 \tag{13.4R}$$

- Note that the only contract that satisfies both (13.1) and (13.4) is:

$$d(\epsilon) = 0$$

  for all values of ϵ
- This is a trivial contract that means no debt
- The country is in financial autarky

- With financial autarky, consumption is:

$$c(\epsilon) = \bar{y} + \epsilon$$

- The average consumption is the same as with commitment, but now there is no consumption smoothing at all
- Consumption is as volatile as the endowment
- As preferences are concave, utility under autarky is lower:

$$u\left(\int_{\epsilon^L}^{\epsilon^H} (\bar{y} + \epsilon)\pi(\epsilon)d\epsilon\right) > \int_{\epsilon^L}^{\epsilon^H} u(\bar{y} + \epsilon)\pi(\epsilon)d\epsilon$$

- As $d(\epsilon) = 0$, we have:

$$d'(\epsilon) = 0$$

- The slope is zero.
- No insurance

Direct Sanctions

- Suppose that creditors can seize assets worth k from a defaulting debtor
- The incentive compatibility contract now takes the form:

$$d(\epsilon) \leq k \tag{13.5}$$

- If $k > \epsilon^H$, we can replicate the optimal contract under commitment
- If $k = 0$, we are in the same situation as with no commitment
- The interesting case is the intermediate case

$$k \in (0, \epsilon^H)$$

Direct Sanctions

- The problem of the representative HHS is:

$$\int_{\epsilon^L}^{\epsilon^H} u(c(\epsilon))\pi(\epsilon)d\epsilon \tag{13.2R}$$

Subject to:

$$c(\epsilon) = \bar{y} + \epsilon - d(\epsilon) \tag{13.3R}$$

$$\int_{\epsilon^L}^{\epsilon^H} d(\epsilon)\pi(\epsilon)d\epsilon = 0 \tag{13.1R}$$

$$d(\epsilon) \leq k \tag{13.5R}$$

- (13.1R) is the participation constraint, (13.5R) is the incentive compatibility constraint

- The Lagrangean is:

$$\mathcal{L} = \int_{\epsilon^L}^{\epsilon^H} \left[u(\bar{y} + \epsilon - d(\epsilon)) + \lambda d(\epsilon) + \gamma(\epsilon)[k - d(\epsilon)] \right] \pi(\epsilon) d\epsilon$$

- Note that we have two Lagrange multipliers (λ and $\gamma(\epsilon)$).
- The first does not depend on ϵ; but $\gamma(\epsilon)$ depends on ϵ, and there is a continuum of multiplier for each possible value of ϵ. The FOCs are:

$$u'(c(\epsilon)) = \lambda - \gamma(\epsilon) \tag{13.6}$$

$$\gamma(\epsilon) \geq 0 \tag{13.7}$$

Plus the slackness condition:

$$(k - d(\epsilon))\gamma(\epsilon) = 0 \tag{13.8}$$

- When the incentive compatibility constraint does not bind ($d(\epsilon) < k$), the Lagrange multiplier $\gamma(\epsilon)$ is equal to zero.
- This means that the marginal utility of consumption is λ when $d(\epsilon) < k$ (thus consumption is constant)

- The budget constrain (13.3) must hold ($c(\epsilon) = \bar{y} + \epsilon - d(\epsilon)$). Thus, when the incentive compatibility constraint does not bind (and c is constant), payments to foreigners (debt) must divert from the shock by an endogenous constant $\bar{d}$ ($d(\epsilon) = \bar{y} - \bar{c} + \epsilon \Rightarrow d(\epsilon) = \bar{d} + \epsilon$, with $\bar{d} = \bar{y} - \bar{c}$):

$$d(\epsilon) = \bar{d} + \epsilon$$

- It is possible to show that the incentive compatibility constraint is not binding at relatively low levels of income ($\epsilon < \bar{\epsilon}$) and binding at relatively high levels of income ($\epsilon > \bar{\epsilon}$) (see page 504 for a proof by contradiction)
- Therefore, there is a $\bar{\epsilon}$ such that:

$$d(\epsilon) = \begin{cases} \bar{d} + \epsilon & \text{for } \epsilon < \bar{\epsilon} \\ k & \text{for } \epsilon > \bar{\epsilon} \end{cases} \tag{13.9}$$

- We can show that this expression is continuous in the endowment shock and that $\bar{d} > 0$. That is, we can show that:

$$d(\bar{\epsilon}) = \bar{d} + \bar{\epsilon} = k \tag{13.10}$$

- With direct sanctions, the borrower enjoys some insurance but less than with commitment
- In relatively low endowment states, the borrower will pay $\bar{d} + \epsilon$ which is more than what it would pay with commitment (i.e., ϵ, note that ϵ can be negative, so the borrower might actually receive net payments)
- To see that condition (13.10) holds, substitute 13.9 in the participation constraint 13.1 which says that on average debt payment=0 ($\int_{\epsilon^L}^{\epsilon^H} d(\epsilon)\pi(\epsilon)d\epsilon = 0$)

$$0 = \int_{\epsilon^L}^{\bar{\epsilon}} (\bar{d} + \epsilon)\pi(\epsilon)d\epsilon + \int_{\bar{\epsilon}}^{\epsilon^H} k\pi(\epsilon)d\epsilon$$

- The first integral is average debt payments when $\epsilon < \bar{\epsilon}$ and the second is when $\epsilon > \bar{\epsilon}$.
- In the latter case, repayments are constant at $k = \bar{d} + \bar{\epsilon}$

$$0 = \int_{\epsilon^L}^{\bar{\epsilon}} (\bar{d} + \epsilon)\pi(\epsilon) d\epsilon + \int_{\bar{\epsilon}}^{\epsilon^H} k\pi(\epsilon) d\epsilon$$

Substitute k using 13.10 (this is still a conjecture)

$$0 = \int_{\epsilon^L}^{\bar{\epsilon}} (\bar{d} + \epsilon)\pi(\epsilon) d\epsilon + \int_{\bar{\epsilon}}^{\epsilon^H} (\bar{d} + \bar{\epsilon})\pi(\epsilon) d\epsilon$$

$$0 = \bar{d} + \int_{\epsilon^L}^{\bar{\epsilon}} \epsilon\pi(\epsilon) d\epsilon + \int_{\bar{\epsilon}}^{\epsilon^H} \bar{\epsilon}\pi(\epsilon) d\epsilon$$

$$0 = \bar{d} - \int_{\bar{\epsilon}}^{\epsilon^H} (\epsilon - \bar{\epsilon})\pi(\epsilon) d\epsilon$$

$$\bar{d} = \int_{\bar{\epsilon}}^{\epsilon^H} (\epsilon - \bar{\epsilon})\pi(\epsilon) d\epsilon$$

Since $\bar{\epsilon} < \epsilon^H$ we have

$$\bar{d} > 0$$

- To show that $\bar{d} > 0$, we used the conjecture that the debt contract is continuous in the endowment (ie, $\bar{d} + \bar{\epsilon} = k$). We now show that this is indeed the case
- Use (13.9) to eliminate $d(\epsilon)$ from (13.3) $c(\epsilon) = \bar{y} + \epsilon - d(\epsilon)$ and get

$$c(\epsilon) = \int_{\epsilon^L}^{\bar{\epsilon}} (\bar{y} - \bar{d})\pi(\epsilon) d\epsilon + \int_{\bar{\epsilon}}^{\epsilon^H} (\bar{y} + \epsilon - k)\pi(\epsilon) d\epsilon$$

- Substitute in the objective function and get:

$$\int_{\epsilon^L}^{\bar{\epsilon}} u(\bar{y} - \bar{d})\pi(\epsilon) d\epsilon + \int_{\bar{\epsilon}}^{\epsilon^H} u(\bar{y} + \epsilon - k)\pi(\epsilon) d\epsilon$$

- Use (13.9) to eliminate $d(\epsilon)$ from (13.1) ($\int_{\epsilon^L}^{\epsilon^H} d(\epsilon)\pi(\epsilon) d\epsilon = 0$) and get:

$$\int_{\epsilon^L}^{\bar{\epsilon}} (\bar{d} + \epsilon)\pi(\epsilon) d\epsilon + \int_{\bar{\epsilon}}^{\epsilon^H} k\pi(\epsilon) d\epsilon = \int_{\epsilon^L}^{\bar{\epsilon}} (\bar{d} + \epsilon)\pi(\epsilon) d\epsilon + [1 - F(\epsilon)]k = 0$$

- Where $F(\epsilon) = \int_{\epsilon^L}^{\bar{\epsilon}} \pi(\epsilon) d\epsilon$ is the probability that ϵ is less than $\bar{\epsilon}$

- Using these equations, we find that the optimal contract sets $\bar{d}$ and $\bar{\epsilon}$ to maximize:

$$\int_{\epsilon^L}^{\bar{\epsilon}} u(\bar{y} - \bar{d})\pi(\epsilon)d\epsilon + \int_{\bar{\epsilon}}^{\epsilon^H} u(\bar{y} + \epsilon - k)\pi(\epsilon)d\epsilon \tag{13.11}$$

Subject to :

$$\int_{\epsilon^L}^{\bar{\epsilon}} (\bar{d} + \epsilon)\pi(\epsilon)d\epsilon + [1 - F(\epsilon)]k = 0 \tag{13.12}$$

- Now, differentiate (13.11) and (13.12) with respect to $\bar{d}$ and $\bar{\epsilon}$, set equal to zero and get (after some pain)

$$-u'(\bar{y}-\bar{d})F(\bar{(}\epsilon))d\bar{d}+[(u(\bar{y}-\bar{d})-u(\bar{y}+\bar{\epsilon}-k)]\pi(\bar{\epsilon})d\bar{\epsilon}=0 \quad (13.13)$$

$$[\bar{\epsilon}-k+\bar{d}]\pi(\epsilon)d\epsilon+F(\bar{\epsilon})=0 \quad (13.14)$$

- Combining (13.13) and (13.14), we get:

$$-u'(\bar{y}-\bar{d})[k-\bar{d}-\bar{\epsilon}]+[u(\bar{y}-\bar{d})-u(\bar{y}+\bar{\epsilon}-k)]=0 \quad (13.15)$$

- Conditions (13.12) and (13.15) are a system of two equations in two unknowns $\bar{d}$ and $\bar{\epsilon}$

- Condition (13.15) is satisfied for any pair $(\bar{d}, \bar{\epsilon})$ such that $\bar{d} + \bar{\epsilon} = k$ (ie for continuous contracts as defined in 13.9).
- This is because with $\bar{d} + \bar{\epsilon} = k$, the first term is equal to zero and, setting $\bar{d} = k - \bar{\epsilon}$, we get that the second term is $u(\bar{y} + \bar{\epsilon} - k) - u(\bar{y} + \bar{\epsilon} - k) = 0$
- We now show that there is a continuous contract that satisfies (13.12). To do this, replace $\bar{d}$ in (13.12) by $k + \bar{\epsilon}$ to get

$$k = \int_{\epsilon^L}^{\bar{\epsilon}} (\bar{\epsilon} - \epsilon)\pi(\epsilon) d\epsilon$$

- The function $(\bar{\epsilon} - \epsilon)\pi(\epsilon)$ is non-negative for $\epsilon \leq \bar{\epsilon}$.
- The RHS=0 if $\epsilon^L = \bar{\epsilon}$ and ϵ^H if $\epsilon^H = \bar{\epsilon}$
- Since $k \in (0, \epsilon^H)$, there is at least one value of $\bar{\epsilon}$ that satisfies the above expression
- We have established that there is at least one continuous contract that satisfies (13.12) and (13.15)
- If the density function $\pi(\epsilon)$ is strictly positive for all possible ϵ, there is a unique continuous contract that satisfies both optimality conditions

- With no commitment and direct sanctions, we are in a situation which is between a situation with commitment and a situation with no commitment and no sanctions
- Payment to foreigners increase one per one with shocks when $\epsilon < \bar{\epsilon}$ and are independent of the endowment when $\epsilon > \bar{\epsilon}$

$$d'(\epsilon) = \begin{cases} 1 & \text{for } \epsilon < \bar{\epsilon} \\ 0 & \text{for } \epsilon > \bar{\epsilon} \end{cases}$$

- The larger the punishment k, the better off the borrower is
- (ex-ante, Vito Corleone is your friend, not ex-post)

- The figure in the next slide shows that with commitment consumption is smooth and equal to average endowment
- (the country is risk averse, lenders are risk neutral and there is full transfer of risk)
- With sanctions and no commitment consumption is flat in low endowment states (from ϵ^L to $\bar{\epsilon}$) and increasing in endowment when $\epsilon > \bar{\epsilon}$
- Note that when $\epsilon < \bar{\epsilon}$, consumption is lower that $\bar{y}$. So, there is a safety net, but this safety net is not as good as with commitment
- Finally, with complete markets, default is an out of equilibrium phenomenon (this is guaranteed by the incentive compatibility constraint)
- Discuss what happens in the graph when k increases or decreases

Consumption profiles under full commitment and no commitment with direct sanctions

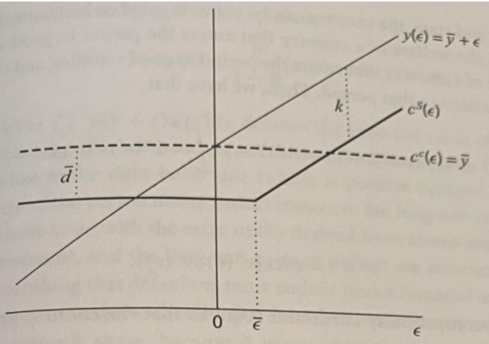


Figure 13.6 Consumption profiles under full commitment and no commitment with direct sanctions.

Note: $c^c(\epsilon)$ and $c^s(\epsilon)$ denote the levels of consumption in state ϵ under commitment and sanctions, respectively, $y(\epsilon) \equiv \bar{y} + \epsilon$ denotes output, and ϵ denotes the endowment shock.

Reputation

- Suppose that creditors don't have access to sanctions but can exclude defaulters from financial markets
- We need a multiperiod model.
- Time is key for reputational models of debt
- Assume that the debtor lives forever and in each period receives an endowment

$$\bar{y} + \epsilon$$

where the density $\pi(\epsilon)$ is time-invariant and the support is as before

- We assume that output is non-storable

- Call $v^b(\epsilon)$, the welfare of a country in bad financial standing (i.e., a country that defaulted and lost access to credit)
- The welfare of a country in bad financial standing is given by (ϵ' is the endowment shock in next period):

$$v^b(\epsilon) \equiv u(\bar{y} + \epsilon) + \beta \int_{\epsilon^L}^{\epsilon^H} v^b(\epsilon')\pi(\epsilon')d\epsilon'$$

  Note that in the current period we consume the full endowment and d has disappeared because we are not repaying and not borrowing again.
- The previous equation can be rewritten as:

$$v^b(\epsilon) = u(\bar{y} + \epsilon) + \frac{\beta}{1-\beta} \int_{\epsilon^L}^{\epsilon^H} u(\bar{y} + \epsilon')\pi(\epsilon')d\epsilon'$$

Consider a debt contract with the following characteristics:

1. Payments are state-contingent but time independent (i.e., in any state ϵ, the country pays $d(\epsilon)$ independently of history)
2. The contract is incentive compatible (i.e., the country prefers to repay under the contract)
3. The contract satisfies the participation constraint (13.1) (i.e., at each date the expected value of payment is equal to zero)

- At any date the country can be in good or bad financial standing.
- $v^g(\epsilon)$ is the value of a country that enters the period in good standing
- $v^c(\epsilon)$ the value of a country that enters the period in good standing and and chooses to honor its obligations in that period

$$v^c(\epsilon) \equiv u(\bar{y} + \epsilon - d(\epsilon)) + \beta \int_{\epsilon^L}^{\epsilon^H} v^g(\epsilon')\pi(\epsilon')d\epsilon' \tag{13.16}$$

and

$$v^g(\epsilon') = max\{v^b(\epsilon'), v^c(\epsilon')\}$$

The incentive compatibility constraint is

$$v^c(\epsilon') \geq v^b(\epsilon') \tag{13.17}$$

The above two expressions imply that:

$$v^g(\epsilon') = v^c(\epsilon')$$

And, in equilibrium, the country never defaults

We can use the above expressions to eliminate $v^g(\epsilon')$ from (13.16)

$$v^c(\epsilon) = u(\bar{y} + \epsilon - d(\epsilon)) + \beta \int_{\epsilon^L}^{\epsilon^H} v^c(\epsilon')\pi(\epsilon')d\epsilon'$$

Iterating this expression forward yields

$$v^c(\epsilon) = u(\bar{y} + \epsilon - d(\epsilon)) + \frac{\beta}{1-\beta} \int_{\epsilon^L}^{\epsilon^H} u(\bar{y} + \epsilon' - d(\epsilon'))\pi(\epsilon')d\epsilon'$$

We can rewrite the incentive compatibility constraint (13.17) as

$$u(\bar{y} + \epsilon - d(\epsilon)) + \frac{\beta}{1-\beta} \int_{\epsilon^L}^{\epsilon^H} u(\bar{y} + \epsilon' - d(\epsilon'))\pi(\epsilon')d\epsilon' \geq$$

$$u(\bar{y} + \epsilon) + \int_{\epsilon^L}^{\epsilon^H} u(\bar{y} + \epsilon')\pi(\epsilon')d\epsilon' \qquad (13.18)$$

This must hold for any state ϵ

If we evaluate (13.18) at first best $(d(\epsilon) = \epsilon)$ and rearrange:

$$u(\bar{y} + \epsilon) - u(\bar{y}) \leq + \frac{\beta}{1-\beta}[u(\bar{y}) - Eu(\bar{y} + \epsilon)]$$

where $Eu(\bar{y} + \epsilon) \equiv \int_{\epsilon^L}^{\epsilon^H} u(\bar{y} + \epsilon')\pi(\epsilon)d\epsilon$ is the expected value of the period utility under autarky.

* The LHS is the short-run gain from defaulting and the RHS is the long-run cost of default (this is positive because concavity of u, remember Jensen's inequality)
* This shows that the first best cannot be implemented on reputational grounds alone because it may violate the the incentive compatibility constraint in certain states
* Incentives to default are stronger when ϵ is big (in general when ϵ is above average, given our assumptio $\epsilon > 0$), and when the representative consumer is impatient (low β)
* Intuition: with big ϵ we keep a lot by defaulting and with low β we don't care much about the future
* The concavity of the utility function also matters (the more concave, the lower the incentive to default because we value more consumption smoothing)
* Summing up: the first best contract is not incentive compatible

Let's now look at the incentive compatible contract. As the contract is time-independent, the problem is stationary and we can focus on the period utility index:

$$\int_{\epsilon^L}^{\epsilon^H} u(\bar{y} + \epsilon - d(\epsilon))\pi(\epsilon)d\epsilon$$

subject to the participation constraint (13.11) and the incentive-compatibility constraint (13.18). The Lagrangean is

$$\mathcal{L} = \int_{\epsilon^L}^{\epsilon^H} u(\bar{y} + \epsilon - d(\epsilon))\pi(\epsilon)d\epsilon + \lambda \int_{\epsilon^L}^{\epsilon^H} d(\epsilon)\pi(\epsilon)d\epsilon +$$

$$+ \int_{\epsilon^L}^{\epsilon^H} \gamma(\epsilon) \left[u(\bar{y} + \epsilon - d(\epsilon)) + \frac{\beta}{1-\beta} \int_{\epsilon^L}^{\epsilon^H} u(\bar{y} + \epsilon' - d(\epsilon'))\pi(\epsilon')d\epsilon' - u(\bar{y} + \epsilon) - \int_{\epsilon^L}^{\epsilon^H} u(\bar{y} + \epsilon')\pi(\epsilon')d\epsilon' \right] \pi(\epsilon)d\epsilon$$

where λ and $\pi(\epsilon)\gamma(\epsilon)$ denote the Lagrange multipliers on the participation constraint (13.1) and the IC constraint (13.18), respectively

The first order conditions related to choosing $d(\epsilon)$ are

$$u'(\bar{y}+\epsilon-d(\epsilon)) = \frac{\lambda}{1+\gamma(\epsilon)+\frac{\beta}{1-\beta}\int_{\epsilon^L}^{\epsilon^H}\gamma(\epsilon')\pi(\epsilon')d\epsilon'} \tag{13.19}$$

$$\gamma(\epsilon) \geq 0$$

and the slackness condition

$$\gamma(\epsilon)\left[u(\bar{y}+\epsilon-d(\epsilon))+\frac{\beta}{1-\beta}\int_{\epsilon^L}^{\epsilon^H}u(\bar{y}+\epsilon'-d(\epsilon'))\pi(\epsilon')d\epsilon'-\frac{\beta}{1-\beta}\int_{\epsilon^L}^{\epsilon^H}u(\bar{y}+\epsilon')\pi(\epsilon')d\epsilon'\right] \tag{13.20}$$

When the incentive compatibility constraints (13.18) is not binding, the slackness condition states that the Lagrange multiplier $\gamma(\epsilon)$ is equal to zero. Hence (13.19) becomes:

$$u'(\bar{y}+\epsilon-d(\epsilon)) = \frac{\lambda}{1+\frac{\beta}{1-\beta}\int_{\epsilon^L}^{\epsilon^H}\gamma(\epsilon')\pi(\epsilon')d\epsilon'} \tag{13.21}$$

$$u'(\bar{y} + \epsilon - d(\epsilon)) = \frac{\lambda}{1 + \frac{\beta}{1-\beta} \int_{\epsilon^L}^{\epsilon^H} \gamma(\epsilon')\pi(\epsilon') d\epsilon'} \tag{13.21}$$

- Note that the RHS is independent of the value of ϵ, we have that the marginal utility of of consumption is constant across states in which the incentive compatibility constraint does not bind
- Hence, consumption is constant in these states and the debt contract takes the form $d(\epsilon) = \bar{d} + \epsilon$, with $\bar{d}$ is a constant
- When the IC constraint is binding, consumption is greater than when the IC is not binding.
- To see this note that because of $\gamma(\epsilon) \geq 0$ for all ϵ, the RHS of (13.19) is smaller or equal to the RHS of (13.21).
- It follows from concavity of the utility function, that consumption is higher in states when the IC constraint is binding

- As before the IC binds in high endowment states (because this is when repayment should be high, creating big incentives to default, for a proof see pages 510-511)
- There is thus a threshold level of the endowment shock $\bar{\epsilon} \leq \epsilon^H$ such that the IC constraint binds for all $\epsilon > \bar{\epsilon}$ and does not bind for $\epsilon < \bar{\epsilon}$. That is:

$$\gamma(\epsilon) = \begin{cases} 1 & \text{for } \epsilon < \bar{\epsilon} \\ 0 & \text{for } \epsilon > \bar{\epsilon} \end{cases}$$

- Consider how the optimal transfer $d(\epsilon)$ varies across states in which the collateral is binding
- Does it increase as one moves from low- to high-endowment states, and by how much?

- To address these questions, let's consider the IC constraint (13.18) holding with equality:

$$u(\bar{y}+\epsilon-d(\epsilon))+\frac{\beta}{1-\beta}\int_{\epsilon^L}^{\epsilon^H} u(\bar{y}+\epsilon'-d(\epsilon'))\pi(\epsilon')d\epsilon' =$$

$$= u(\bar{y}+\epsilon)+\frac{\beta}{1-\beta}\int_{\epsilon^L}^{\epsilon^H} u(\bar{y}+\epsilon')\pi(\epsilon')d\epsilon' \qquad (13.22)$$

- In this expression, the terms $\frac{\beta}{1-\beta}\int_{\epsilon^L}^{\epsilon^H} u(\bar{y}+\epsilon'-d(\epsilon'))\pi(\epsilon')d\epsilon'$ and $\int_{\epsilon^L}^{\epsilon^H} u(\bar{y}+\epsilon'))\pi(\epsilon')d\epsilon'$ are both independent of the current endowment shock ϵ
- Only the first terms on the right- and left-hand sides of (13.22) change with the current level of the endowment

- Differentiating (13.22) wrt the current endowment ϵ yields:

$$d'(\epsilon) = \frac{u'(\bar{y} + \epsilon - d(\epsilon)) - u'(\bar{y} + \epsilon)}{u'(\bar{y} + \epsilon - d(\epsilon))}$$

- As there are no incentives to default when the country receives income from foreigners, the IC constraint only binds when the agents need to make payments ($d(\epsilon) > 0$)
- Since the utility function is concave, it follows that $u'(\bar{y} + \epsilon - d(\epsilon)) > u'(\bar{y} + \epsilon)$ in all states in which the IC constraint binds. So the numerator is positive and smaller than the denominator
- This implies that when the IC constraint binds:

$$0 < d'(\epsilon) < 1$$

- Payments to foreign lenders increase with the level of income but less than one for one

- Is it counterintitutive that, as the current endowment increases, the payment to creditors that can be supported without default also increases? (after all, with high endowment we can keep a lot by defaulting)
- The intuition is that, given a positive level of current payments ($d(\epsilon) > 0$), a small increase in current endowment raises the current-period utility associated with not defaulting $u(\bar{y} + \epsilon - d(\epsilon))$ by more than it raises the utility associated with the alternative of defaulting $u(\bar{y} + \epsilon)$.
- This is because of concavity!
- It follows that in states in which $d(\epsilon) > 0$, the higher the current endowment, the higher the level of of payments to foreign lenders that can be supported without inducing default
- This does not mean that default incentives are weaker when ϵ increases as the discussion above only holds when the IC constraint is binding
- Note the difference with the discussion under sanctions.
- In that case, when the IC constraint binds, payments equal the maximum payment k, which implies that the slope equals zero

Concavity matters: example

- Assume $U(c) = c^{\alpha}$. Then:

$$d'(\epsilon) = \frac{u'(\bar{y} + \epsilon - d(\epsilon)) - u'(\bar{y} + \epsilon)}{u'(\bar{y} + \epsilon - d(\epsilon))}$$

becomes

$$d'(\epsilon) = \frac{\alpha(\bar{y} + \epsilon - d(\epsilon))^{\alpha-1} - \alpha(\bar{y} + \epsilon)^{\alpha-1}}{\alpha(\bar{y} + \epsilon - d(\epsilon))^{\alpha-1}}$$

$$d'(\epsilon) = 1 - \frac{(\bar{y} + \epsilon)^{\alpha-1}}{(\bar{y} + \epsilon - d(\epsilon))^{\alpha-1}} = 1 - \left(\frac{(\bar{y} + \epsilon - d(\epsilon))}{(\bar{y} + \epsilon)}\right)^{1-\alpha}$$

- As α becomes smaller, concavity increases and so does $d'(\epsilon)$
- If $\alpha = 1$ (no concavity)

$$d'(\epsilon) = 1 - \left(\frac{(\bar{y} + \epsilon - d(\epsilon))}{(\bar{y} + \epsilon)}\right)^{0} = 0$$

Consumption profiles under full commitment and no commitment in a reputational model of debt

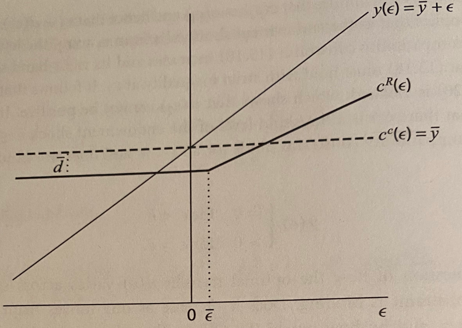


Figure 13.7 Consumption profiles under full commitment and no commitment in a reputational model of debt.

Note: $c^c(\epsilon)$ and $c^R(\epsilon)$ denote the levels of consumption in state ϵ under commitment and no commitment, respectively; $y(\epsilon) \equiv \bar{y} + \epsilon$ denotes output; and ϵ denotes the endowment shock.

Sanctions vs Reputation

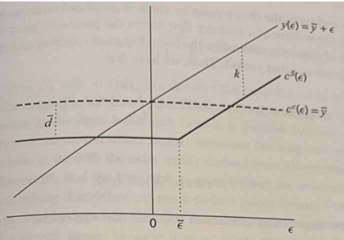


Figure 13.6 Consumption profiles under full commitment and no commitment with direct sanctions.

Note: $c^c(\epsilon)$ and $c^s(\epsilon)$ denote the levels of consumption in state ϵ under commitment and sanctions, respectively, $y(\epsilon) \equiv \bar{y} + \epsilon$ denotes output, and ϵ denotes the endowment shock.

Figure: Sanctions

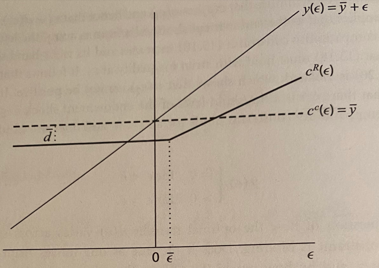


Figure 13.7 Consumption profiles under full commitment and no commitment in a reputational model of debt.

Note: $c^c(\epsilon)$ and $c^R(\epsilon)$ denote the levels of consumption in state ϵ under commitment and no commitment, respectively; $y(\epsilon) \equiv \bar{y} + \epsilon$ denotes output; and ϵ denotes the endowment shock.

Figure: Reputation

Note the slope when $\epsilon > \bar{\epsilon}$. With sanctions it is 1 ($c = \bar{y} + \epsilon - k$). With reputation it is less than 1. There is still some insurance

Non-state-contingent contracts

- With complete financial markets, risk-sharing leads to positive transfers in low-income states and negative transfers in high-income states.
- As a consequence, default incentives are stronger in high-income states (and in equilibrium there is no default)
- This is not in line with the empirical evidence that: (i) defaults happen and (ii) that they tend to happen in bad times
- Also, the more indebted the country is, the higher the default probability
- Let us now assume that there is a single non-state contingent asset

The Eaton-Gersovitz Model

- Based on Eaton and Gersovitz (RES, 1981) and Arellano (AER, 2008).
- Consider a small open economy populated by a large number of identical households with preferences:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$$

- No difference between household, individual, country, government.
- This requires that, besides having identical households and individuals, decisions on international borrowing and repayment are made by a benevolent social planner
- Each period $t \geq 0$, the country get y units of a non-storable consumption good. This endowment is exogenous, stochastic and iid with a distribution:

$$y_t \in Y \equiv [\underline{y}, \bar{y}], \qquad \text{i.i.d.}$$

- One-period non-state-contingent debt, d.

- At the beginning of each period, the country can be in good or bad financial standing
- If the country is in bad financial standing, it cannot borrow or lend in the international financial market
- As a consequence, it needs to consume its endowment:

$$c_t = y_t$$

- If the country is in good financial standing, it decides whether to honor its debt or default
- If it honors its debt, it starts next period in good financial standing
- If it defaults, it goes into bad financial standing immediately

- The budget constraint for a country that starts in GFS and honor its debt is given by (From now on: c current period, c' future)

$$c + d = y + q(d')d' \tag{13.23}$$

  where d is the debt due today, d' is the debt acquired today and due next period, and $q(d')$ is the market price of the country's debt
- One way to think about $q(d')$ is $q(d') = \frac{1}{1+r}$, where r is the interest rate paid by the country.
- In general, $r \geq r^*$; if there is no risk of default $r = r^*$
- Note that q depends on the debt due next period d' and not on the debt due today d.
- Also, q does not depend on y. This comes from the iid assumption (today's realization gives no info about tomorrow's output)
- It would have been different if we had assumed that y is serially correlated

- We assume that BFS is an absorbent state. Once we are there, we stay forever.
- Denote $v^b(y)$ the value of bad financial standing (E is the expectation operator):

$$v^b(y) = u(y) + \beta E v^b(y')$$

- If the country is in good financial standing, the value $v^c(d, y)$ is:

$$v^c(d, y) = \max_{d'} \left[u(y + q(d')d' - d) + \beta E v^g(d', y') \right]$$

subject to

$$d' \leq \bar{d}$$

- where $\bar{d}$ is a debt limit and $v^g(d, y)$ is the value associated with being in good financial standing:

$$v^g(d, y) = \max \left\{ v^b(y), v^c(d, y) \right\}$$

- This economy decides to default if the cost (in terms of foregone consumption) of servicing the debt is larger than the costs of living in autarky forever
- It seems intuitive that default is more likely when d is high and y is low (when y is low the marginal utility of current consumption is very high)

The Default Set

- The default set contains all endowment levels at which the country decides to default for a given level of debt d. Formally:

$$D(d) = \{y \in Y : v^b(y) > v^c(d, y)\}$$

- Clearly, the country never defaults, when it has positive net foreign assets:$D(d) = \emptyset$ if $d \leq 0$
- Using the definition of trade balance $tb \equiv y - c$ and the budget constraint $c + d = y + q(d')d' \Rightarrow y = c + d - q(d')d'$. We get:

$$tb = d - q(d')d'$$

- Then we get **Proposition 13.1:** At debt levels for which the default set is not empty, an economy that chooses not to default will run a trade surplus:

  If $D(d) \neq \emptyset$, then $tb = d - q(d')d' > 0$ for all $d' \leq \bar{d}$.

(proof by contradiction on page 515)

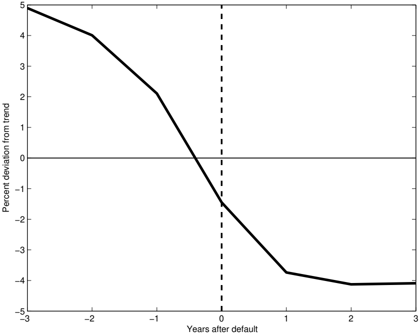
- Note that $q(d')d' - d = c - y$ is the trade balance deficit for a country that decides continuing paying its debt.
- Thus, proposition 13.1 states that a country in which risk of default is non-zero ($D(d) \neq \emptyset$) and that decides to continue to participate in the international financial market will devote part of its endowment to servicing the debt by running a trade balance surplus
- In other words, the country runs trade deficits only when the probability of default is zero ($D(d) = \emptyset$)
- (This results also hold with serially correlated endowments)
- Let us now establish that defaults happen in bad times
- If a country decides to default at a certain level of debt and income, it will also default at lower levels of income
- If the default set is not empty, it is an interval with the lower bound given by $\underline{y}$

Default and economic conditions

- **Proposition 13.2:** defaults happen in bad times:

$$\text{If } y_1 \in D(d) \text{ and } \underline{y} \leq y_2 < y_1, \text{ then } y_2 \in D(d).$$

  Proof on page 516
- This seems to match the data (Figure 13.2)



Default and debt levels

- Defaults happen when debt levels are high. More precisely, the default interval $D(d)$ is larger when debt levels are high
- **Proposition 13.3:** defaults are more likely when debt is high:

  If $D(d) \neq \emptyset$, then $D(d)$ is an interval, $[\underline{y}, y^*(d)]$, where $y^*(d)$ is increasing in d if $y^*(d) < \bar{y}$.

  Proof on pages 516-17
- This is in line with the empirical evidence (Table 13.3)

Default Risk and the Country Premium

- Let us now look at the interest rate faced by the country
- Remember that we assumed that the world interest rate is r^* and that we have risk-neutral and competitive lenders
- It follows that the expected rate of return on the country debt is r^*
- We assume that if the country defaults, lenders get nothing (haircut 100%)
- If the country does not default, lenders get $1/q(d')$ units for each unit lent (this is equivalent to $(1+r)$).
- The participation constraint equates the world interest rate with the expected return:

$$1+r^* = \frac{\text{Prob}\{y' \geq y^*(d')\}}{q(d')} + 0 \times \frac{\text{Prob}\{y' < y^*(d')\}}{q(d')}.$$

or

$$q(d') = \frac{\text{Prob}\{y' \geq y^*(d')\}}{1+r^*} + 0 \times \frac{\text{Prob}\{y' < y^*(d')\}}{1+r^*}.$$

- Note that $\text{Prob}\{y' \geq y^*(d')\}$ is the probability that the country will not default in the next period (and $\text{Prob}\{y' < y^*(d')\}$ is the probability that it will default)
- Define $F(y)$, the cdf of the endowment shock, we get: $\text{Prob}\{y' \geq y^*(d')\} = 1 - F(y^*(d'))$ & $\text{Prob}\{y' < y^*(d')\} = F(y^*(d'))$. So:
$$q(d') = \frac{1 - F(y^*(d'))}{1 + r^*} + 0 \times \frac{F(y^*(d'))}{1 + r^*}$$
- Or:
$$\frac{1}{q(d')(1 + r^*)} \equiv \frac{1 + r}{1 + r^*} = \frac{1}{1 - F(y^*(d'))}$$
- With 100% haircut, the gross country premium equals the inverse of the probability of repayment
- This is approximately equal to one plus the probability of default
- Alternatively, the country spread is approximately equal to the probability of default: $1 - \frac{1+r^*}{1+r} = F(y^*(d'))$ or:
$$\frac{r - r^*}{1 + r} = F(y^*(d'))$$

- Note that if the haircut is not 100%, we get

$$q(d') = \frac{1 - F(y^*(d'))}{1 + r^*} + (1 - h) \times \frac{F(y^*(d'))}{1 + r^*}$$

- and

$$\frac{r - r^*}{1 + r} = hF(y^*(d'))$$

- The spread is less than the probability of default
- With an average haircut of 40%, we expect spreads which are about half the probability of default
- Yet, in the data, country spreads typically exceed the probability of default

Debt and spreads

- Let's go back to our assumption of 100% haircut, use

$$q(d') = \frac{1 - F(y^*(d'))}{1 + r^*}$$

- and compute the derivative of the price of next period's debt with respect to next period's debt:

$$\frac{\partial q(d')}{\partial d'} = \frac{-F'(y^*(d'))\frac{\partial(y^*(d'))}{\partial d'}}{1 + r^*} \leq 0$$

- This is because $F' \geq 0$ (by definition of a cdf) and, by Proposition 13.3, $\frac{\partial(y^*(d'))}{\partial d'} \geq 0$
- As $r = \frac{1}{q(d')} - 1$, we get $\frac{\partial r}{\partial d'} \geq 0$ and we have:
- **Proposition 13.4:** The country spread is nondecreasing in the stock of debt.

Summing up: Three important results

1. Given the stock of debt, default is more likely when output is low
2. Given the state of the economy, default is more likely when debt levels are high
3. Given the state of the economy, spreads are non-decreasing in the level of debt

Saving and the breakdown of reputational lending

- Key assumptions so far are that output is non-storable and that countries in bad financial standing cannot borrow or save in the international financial market
- Why cannot save? $\Rightarrow$ Bulow and Rogoff (1989) showed that if defaulters cannot borrow but can save (i.e., if defaulters can hold positive net foreign assets), then no lending can be supported on reputational grounds
- Intuition: Assume that you get to the maximum debt, then you can simply default, store what you would have used to pay the debt abroad and then use this money saved abroad to mimic what you would have done by accessing the international capital market but without having to borrow and pay interests
- Example that assumes a deterministic economy (it can be shown that this also works with a stochastic economy)

Saving and the breakdown of reputational lending

- Suppose that a deterministic economy where reputational equilibrium supports a path for external debt given by $[d_t]_{t=0}^{\infty}$.
- Where d_t is a level of external debt incurred in period t and due in period $t+1$
- Assume that default is punished with exclusion from borrowing but not saving (this is similar to assuming that output is storable)
- This assumption, together with perfect foresight (deterministic economy), guarantees that any reputational equilibrium featuring positive debt in at least one date must be characterized by no default

- This is because of the participation constraint: if a country defaults in period T, nobody would lend in period $T-1$ because default would happen with certainty next period. Thus $d_{T-1} \leq 0$
- But then, if the country is excluded in period $T-1$, it will have no incentive to honor its debt in that period. As a result, nobody will lend in $T-2$. That is $d_{T-2} \leq 0$
- Keep iterating backwards and you get that $d_t \leq 0$ for all $t \geq 0$
- It follows that in an economy of this type and positive debt, the spread is zero: $r = r^*$ because the probability of default is zero
- The evolution of the equilibrium level of debt is then (becuase $r = r^*$):

$$d_t = (1 + r^*)d_{t-1} - tb_t \tag{13.24}$$

for $t \geq 0$ and where $tb \equiv y_t - c_t$ is the trade balance

- Assume that the endowment path $[y_t]_{t=0}^{\infty}$ is bounded.
- Let $d_T > 0$ be the maximum level of external debt in this equilibrium sequence (ie $d_T \geq d_t$) for all $t \geq -1$
- Does it pay for the country to honor its debt? NO!
- This is because the country could default at $T + 1$ and be excluded from borrowing but able to maintain a level of consumption which at least as high as the level of consumption under repayment
- To see this, let $\tilde{d}_t$ for $t > T$ denote the post-default path of external debt (or external assets if negative). Let the net debt position acquired in the period of default be:

$$\tilde{d}_{T+1} = -tb_{T+1}$$

  where tb_{T+1} is the trade balance prevailing in period $T + 1$ under the original debt sequence $[d_t]$
- By 13.24, we have that $-tb_{T+1} = d_{T+1} - (1 + r^*)d_T$. This implies

$$\tilde{d}_{T+1} = d_{T+1} - (1 + r^*)d_T \tag{13.25}$$

- By assumption $d_T \geq d_{T+1}$ (because d_T is the maximum) and $r^* > 0$, so:

$$\tilde{d}_{T+1} < 0$$

- This tells us that the country can achieve the same trade balance (and consumption) under the default strategy and under no-default strategy without having to borrow internationally ($\tilde{d}_{T+1} < 0$)
- The external debt position in period $T+2$ under default strategy is

$$\tilde{d}_{T+2} = (1+r^*)\tilde{d}_{T+1} - tb_{T+2}$$

  where tb_{T+2} is the trade balance in period $T+2$ under the original (no default) debt sequence $[d_t]$
- Using 13.24 and 13.25 we can rewrite this as:

$$\tilde{d}_{T+2} = d_{T+2} - (1+r^*)^2 d_T < 0$$

  The inequality is because, by assumption, $d_T \geq d_{T+2}$ and $r^* > 0$
- We have now shown that the default strategy can achieve the non-default strategy trade balance at $T+2$ without any international borrowing

- Continuing like this, we can show that the no default sequences of trade balances for $t \geq$ T+1 can be supported by the debt path $\tilde{d}$ satisfying
$$\tilde{d}_t = d_t - (1 + r^*)^{t-T} d_T$$
This is strictly negative for all $t \geq T + 1$
- The fact that the entire post default debt path is negative, implies that the country could also implmenet a post-default path of trade balances $\tilde{tb}_t$ satisfying
$$\tilde{tb}_t \leq tb_t$$
for $t \geq T + 1$ and
$$\tilde{tb}_{t'} < tb_{t'}$$
for at least one $t' \geq T + 1$

- This path would be strictly preferred to the trade balance under no default because it would allow consumption to be strictly higher than in the no-default strategy for at least one period (recall $tb_t = y_t - c_t$)
- Thus, if $\tilde{d}$ can be negative (i.e., post-default the country can save abroad), it is optimal for the country to default after reaching the highest debt level d_T
- But we showed that in this perfect foresight equilibrium, default implies zero debt at all times
- Therefore, no external debt can be supported in Equilibrium on reputational grounds alone if the country can save abroad
- To reiterate: If the country can save abroad, it would find it optimal to default when $d = D_T$ and then accumulate foreign assets ($d < 0$), but with perfect foresight there cannot be reputational equilibrium with positive debt if at any point the country finds it optimal to default. Therefore, if the country can accumulate foreign assets, there cannot be any debt
- Bulow and Rogoff (1989) show that this result also holds with stochastic output

Quantitative Analysis of the The Eaton-Gersovitz Model

- The baseline the analytical example discussed thus far cannot capture the main facts about sovereign debt
- In order to implement a quantitative analysis, we will assume:
  1. Serially correlated endowment process.
  2. Finite period of exclusion (nonzero probability of re-accessing the international capital market)
  3. Output cost of default.
  - We still assume 100% haircut
- With these features, we cannot have an analytical solution but we can characterize the equilibrium dynamics with numerical methods

Serially Correlated Output

- The output process is

$$\ln y_t = \rho \ln y_{t-1} + \sigma_\epsilon \epsilon_t \tag{13.26}$$

  σ_ϵ denotes the standard deviation of the innovation process and ϵ_t is an iid random variable with a standard normal distribution: $\epsilon_t \sim N(0, 1)$
- if $\rho = 0$, y_t has the iid distribution used so far.
- With $\rho > 0$, income is persistent and the probability of default in $t + 1$ depends on y_t (because y_t provides information on y_{t+1}). Thus $q_t = q(d_{t+1}, y_t)$.
- Use data from Argentina (1983:Q1 to 2001:Q4) to define y_t as tradable output measured as the sum of GDP in agriculture, forestry, fishing, mining, and manufacturing.

$$\ln y_t = 0.9317 \ln y_{t-1} + 0.037\, \epsilon_t$$

Finite Exclusion Period

- Exclusion from international capital market does not last forever
- We assume a constant probability of reentry in each period: $\theta \in (0, 1)$.
- This means that the average exclusion period is $1/\theta$. To see this assume that the first period of exclusion is the period of default. Then:
    - Prob. to be excluded exactly 1 quarter: θ
    - Prob. to be excluded exactly 2 quarters: $\theta(1-\theta)$
    - $\vdots$
    - Prob. to be excluded exactly j quarters: $\theta(1-\theta)^{j-1}$
- Average exclusion period: $\theta + \theta(1-\theta) + ...\theta(1-\theta)^{j-1}$

$$= \theta \sum_{j=1}^{\infty} j(1-\theta)^{j-1} = \frac{1}{\theta}$$

$\Rightarrow$ expected length of exclusion is $\frac{1}{\theta}$ quarters.
$\Rightarrow$ The larger θ, the quicker reaccess.

Finite Exclusion Period

- Most calibrated model set θ on the basis of data on average exclusion (table 13.6 shows that typical exclusion period is 4.7-13.7 years).
- Constant probability of re-entry is in line with the exponential distribution of the length of default status (Figure 13.1)
- Earlier we assumed that autarky is an absorbing status.
- This is equivalent to setting $\theta = 0$
- θ is a key parameter that affects:
  - Default frequency
  - Average risk premium
  - Amount of debt that can be sustained in equilibrium
- Note that we assume that after default debt is zero. This is 100% haircut. This is not realistic

The Output Cost of Default

- We will show that exclusion is not sufficient to generate sustainable levels of debt
- Need to add another cost of default: Exogenous output cost
- Upon default, countries lose part of their endowment until they are in bad standing
- We assume that the endowment received by households is not y_t, but $\tilde{y}_t \leq y_t$, where $\tilde{y}_t$ is given by

$$\tilde{y}_t = \begin{cases} y_t & \text{if the country is in good standing} \\ y_t - L(y_t) & \text{if the country is in bad standing} \end{cases},$$

- Where $L(y_t)$ is an output loss function assumed to be positive and nondecreasing.
- This is ad hoc and not based on microfoundations (there have been some attempts to microfound it)

- Assume the following specification for $L(y_t)$

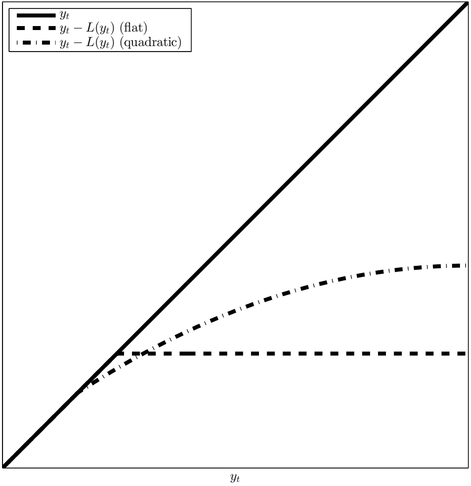
$$L(y_t) = max[0, a_0 + a_1 y_t + a_2 y_t^2] \tag{13.28}$$

- This encompasses several cases.
- Arellano (2008) assumes that when the country is in bad standing it loses any endowment above a certain threshold $\bar{y}$:

$$y_t - L(y_t) = \begin{cases} y_t & \text{if } y_t < \bar{y} \\ \bar{y} & \text{if } y_t \geq \bar{y} \end{cases}, \tag{13.29}$$

- This is 13.28 with $a_0 = \bar{y}$, $a_1 = 1$, and $a_2 = 0$
- The figure in the next slide shows the Arellano endowment net of the output cost (dashed line).
- Chatterjee and Eygungor use a quadratic output loss ($a_0 = 0$; $a_1 < 0$, and $a_2 > 0$), plotted as the dash-dotted line in the figure

Asymmetric Output Cost of Default



The Model

Value of continuing to participate in financial markets

$$v^c(d, y) = \max_{d'} \left\{ u(y + q(d', y)d' - d) + \beta E_y v^g(d', y') \right\}$$

Value of being in bad financial standing

$$v^b(y) = u(y - L(y)) + \beta\theta E_y v^g(0, y') + \beta(1 - \theta) E_y v^b(y')$$

Value of being in good financial standing

$$v^g(d, y) = \max\{v^c(d, y), v^b(y)\}$$

Participation Constraint

$$1 + r^* = \frac{\text{Prob}_y\{v^c(d', y') \geq v^b(y')\}}{q(d', y)} \tag{13.30}$$

Country interest rate: $1 + r \equiv \frac{1}{q(d',y)}$
Country Spread $= r - r^*$

Calibration and Functional Forms

- Preferences:
$$u(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma}.$$
- Output Cost of Default:
$$L(y_t) = \max\{0, a_0 + a_1 y_t + a_2 y_t^2\}.$$
- Time unit is a quarter.
- Parameter values:
  * $\sigma = 2$.
  * $\theta = 0.0385$ probability of reentry.
  * $r^* = 1\%$ per quarter.
  * $\beta = 0.85$.
  * $a_1 = -0.36$; parameter of output loss function.
  * $a_2 = 0.4403$; parameter of output loss function.
  * (β, a_1, a_2) set to match debt to GDP of 60% per quarter; 2.6 defaults per century; output cost of default is 7% per year of autarky.

Computation

- Discretization of State Space
  - $n_y = 200$; Number of output grid points (equally spaced in logs)
  - $n_d = 200$; Number of debt grid points (equally spaced)
  - $[\underline{y}, \overline{y}] = [0.6523, 1.5330]$; 4.2 standard deviations
  - $[\underline{d}, \overline{d}] = [0, 1.5]$;
  - Transition probability matrix computed using Schmitt-Grohé and Uribe (2009) interative procedure, see `tpm.m`7

Quantitative Predictions of the Eaton-Gersovitz Model

Selected First and Second Moments: Data and Model Predictions

	Default frequency	E(d/y)	$E(r-r^*)$	$\sigma(r-r^*)$	$\text{corr}(r-r^*,y)$	$\text{corr}(r-r^*,tb/y)$
Data	2.6	58.0	7.4	2.9	-0.64	0.72
Model	2.6	59.0	3.5	3.2	-0.54	0.81

Note. Data moments are from Argentina over the inter-default period 1994:1 to 2001:3, except for the default frequency, which is calculated over the period 1824 to 2014. The variable d/y denotes the quarterly debt-to-GDP ratio in percent, $r-r^*$ denotes the country premium, in percent per year, y denotes (quarterly detrended) output, and tb/y denotes the trade-balance-to-GDP ratio. The symbols E, σ, and corr denote, respectively, the mean, the standard deviation, and the correlation. In the theoretical model, all moments (other than default frequency) are conditional on the country being in good financial standing. Replication file statistics_model.m in sovereign_default.zip.

- Predicted default frequency while in good standing is 3.2% (which is close to the country spread of 3.5%).
- r is countercyclical; when it rains it pours; this feature should increase consumption volatility relative to output;
- Model explains only half of observed country premium (3.5 versus 7.4 percent).

Spreads and default frequency

By construction, the model can either match the default frequency or the spread but not both, because it implies that the spread is equal to the default probablity:

$$
\begin{aligned}
r - r^* &\approx \ln\left[\frac{1}{q(d', y)(1 + r^*)}\right] \\
&= \ln\left[\frac{1}{\text{Prob}\{\text{repayment in } t+1 \text{ given information in } t\}}\right] \\
&= \ln\left[\frac{1}{1 - \text{Prob}\{\text{default in } t+1 \text{ given information in } t\}}\right] \\
&\approx \text{Prob}\{\text{default in } t+1 \text{ given information in } t\}.
\end{aligned}
$$

Q: Can partial default explain the spread-default frequency differential?

- Partial default:

$$q(d', y) = \frac{\lambda \text{Prob}_y\{v^c(d', y') < v^b(y')\} + \text{Prob}_y\{v^c(d', y') \geq v^b(y')\}}{1 + r^*}.$$

- This implies that

$$r - r^* \approx (1 - \lambda)\text{Prob}\{\text{default in } t+1 \text{ given information in } t\}.$$

- Now spread smaller than default frequency.
- Thus, allowing for partial default only widens the spread-defaul frequency differential.

Q: Does making foreign lenders risk averse close the spread-default frequency differential.

- Lizarazo (2013) shows that this is the case
- However, this result relies on the assumption that default has large negative wealth effects on the foreign lender.
- If defaulting country is small so that wealth of foreign lender is not affected by default, then allowing for risk aversion, does not help much to close the spread-default frequency differential. (Section 13.9 of USG, 2017)

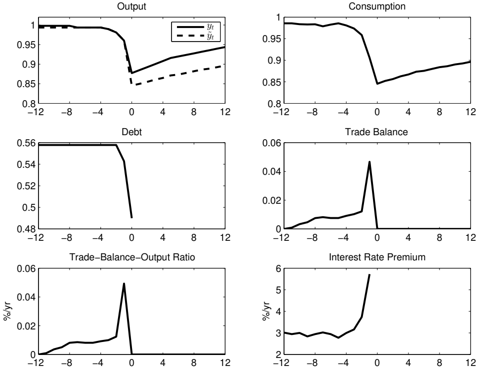
Data and Model Predictions: Standard Business-Cycle Statistics

	$\sigma(c)/\sigma(y)$	$\sigma(tb/y)/\sigma(y)$	$\text{corr}(c, y)$	$\text{corr}(tb/y, y)$
Data				
Emerging Countries	1.23	0.69	0.72	-0.51
Argentina	1.11	0.48	0.75	-0.87
Model	1.22	0.57	0.88	-0.14

Note. Data moments for emerging countries and Argentina are taken from chapter 1, tables 1.6 and 1.9, respectively. The symbols c and y denote the log deviation from trend, tb/y denotes the trade-balance-to-output ratio, and σ and corr denote, respectively, standard deviation and correlation. Replication file for model predictions simu.m in sovereign_default.zip.

- The model captures excess volatility of consumption.
- Why? in good standing interest rate is countercyclical exacerbating consumption adjustment to negative shocks; in bad standing $c = y$. So overall consumption is more volatile than output.
- The model explains sign but not size of $corr(tb/y, y)$. In the absence of default risk this model would predict tb/y to be procyclical. Recall: finance temporary income shocks, and adjust to permanent ones.
- The model predicts countercyclical interest rate, this makes savings, and hence the trade balance itself countercyclical.

Dynamics Around A Typical Default Episode



Notes: Lines display medians of 25-quarter windows centered on default episodes occurring in an artificial time series of 1 million quarters. The default date is normalized to 0. Replication file typical_default_episode.m in sovereign_default.zip.

Observations on the figure

- When does a country default? After a sudden deep contraction in output. From at mean to 1.3 std below mean in just 3 quarters.
- The model can explain that default coincides with end of contraction and beginning of recovery.
- Consumption falls by more than output (no consumption smoothing!).
- Why, because spreads increase from 3 to 6 percent.
- Debt fails to increase prior to default. d/y little changed until period of default. Then it goes down
- But in the data, we see d/y increasing before default and not bnecessarily decreasing after default (puzzle).

Approximating the Eaton-Gersovitz Model: Accuracy Tests

	Grid Points		Default				Correlation	
	n_y	n_d	frequency	E(d/y)	$E(r-r^*)$	$\sigma(r-r^*)$	$(r-r^*, y)$	$(r-r^*, tb/y)$
Data			2.6	58.0	7.4	2.9	-0.64	0.72
Model*	200	200	2.65	59.05	3.47	3.21	-0.54	0.81
Model	25	200	2.30	69.43	3.01	4.20	-0.28	0.44
Model	400	200	2.63	58.64	3.43	3.12	-0.55	0.82
Model	200	400	2.65	59.46	3.44	3.13	-0.55	0.83
Model	400	400	2.65	59.46	3.44	3.13	-0.55	0.83

Note. Data moments are from Argentina over the inter-default period 1994:1 to 2001:3, except for the default frequency, which is calculated over the period 1824 to 2014. The variable d/y denotes the quarterly debt-to-GDP ratio in percent, $r - r^*$ denotes the country premium, in percent per year, y denotes (quarterly detrended) output, and tb/y denotes the trade-balance-to-GDP ratio. The symbols E and σ denote, respectively, the mean and the standard deviation. The symbols n_y and n_d denote the number of grid points for the endowment and debt, respectively. In the theoretical model, all moments are conditional on the country being in good financial standing. Theoretical moments were computed by running the Matlab script statistics_model.m after appropriately adjusting the number of grid points in eg.m.
*Baseline grid specification.

Hatchondo, Martínez, and Sapriza (2010) find that the numerical solution of the Eaton-Gersovitz model deteriorates significantly when the endowment grid is coarsely specified. Correlation of country premium with output falls from -0.54 to -0.28 when n_y is reduced from 200 to 25. Std of premium is 1 percentage point higher under coarser grid.

Reputation or Direct Output Costs? — The Quantitative Irrelevance of Exclusion

- Consider no output cost of default, $L(y) = 0$. Leaving financial autarky as the only cost of default.
- What happens? The debt distribution becomes degenerate.
- For all values of y debt is equal to zero.
- That is, the model cannot support **any** debt in equilibrium.
- This means that the benefit of having access to financial markets must always be smaller than the benefit of defaulting.
- Therefore, the current model is not really a reputational model of default but instead a sanctions model of default.

- But does reputation play any role at all? To answer this question, let's compare the predictions of the model with output costs of default and with and without financial exclusion post default.
- The model then becomes:

$$v^{gc}(d,y) = \max_{d'} \left\{ u(y + q^g(d',y)d' - d) + \beta E_y v^g(d',y') \right\},$$

- The value of defaulting, $v^d(y)$

$$v^d(y) = \max_{d'} \left\{ u(y - L(y) + q^b(d',y)d') + \beta\theta E_y v^g(d',y') + \beta(1-\theta) E_y v^b(d',$$

- The value of being in good financial standing

$$v^g(d,y) = \max\{v^{gc}(d,y), v^d(y)\}.$$

$$q^g(d',y) = \frac{\text{Prob}_y\{v^{gc}(d',y') \geq v^d(y')\}}{1+r^*},$$

- The value of being in bad financial standing and continuing to service the debt, v^{bc},

$$v^{bc}(d,y) = \max_{d'} \left\{ u(y - L(y) + q^b(d',y)d' - d) + \beta\theta E_y v^g(d',y') + \beta(1-\theta)E_y v^b(d',y') \right\}.$$

- The value of being in bad financial standing

$$v^b(d,y) = \max\{v^{bc}(d,y), v^d(y)\}$$

- The reason why $v^b(d,y)$ may not always be equal to $v^d(y)$ is that a country in bad standing may have assets $d < 0$, in which case it will never default.
- The price of debt in periods of bad financial standing, $q^b(d',y)$

$$q^b(d',y) = \frac{\theta\,\text{Prob}_y\{v^{gc}(d',y') \geq v^d(y')\} + (1-\theta)\text{Prob}_y\{v^{bc}(d',y') \geq v^d(y')\}}{1+r^*}$$

The Quantitative Irrelevance of Exclusion

	Default frequency	E(d/y)	$E(r-r^*)$	$\sigma(r-r^*)$	corr$(r-r^*, y)$	corr$(r-r^*, tb/y)$
Data	2.6	58.0	7.4	2.9	-0.64	0.72
Model						
Baseline	2.6	59.0	3.5	3.2	-0.54	0.81
No Exclusion	3.0	53.1	4.1	3.6	-0.61	0.84

* The model in which default is not punished by exclusion from financial markets behaves remarkably similar to the baseline model.
* Specifically, the no-exclusion model can support about the same amount of debt than the model with exclusion.
* The mean debt to output ratio in times of good financial standing is 53 percent per quarter in the no-exclusion model, slightly below the value of 59 percent predicted by the baseline model.
* The average country premium in times of good standing is predicted to be 4.1 percent in the no-exclusion model compared to 3.5 percent in the baseline case. The default frequency conditional on good standing is 3.7 percent. (In this model, up to first order the spread-default frequency differential is zero.)
* The volatility of the country premium and the correlation of the country premium with output and the trade balance to output ratio are also little changed.
* The model predicts that on average the country defaults three times per century compared to a default frequency of 2.6 times per century predicted by the baseline model. $\Rightarrow$ We conclude that exclusion from credit markets plays a negligible role for the quantitative performance of the Eaton-Gersovitz model. The main mechanism supporting debt in equilibrium is the output loss associated with default!

The Role of Discounting: Varying the subjective disount factor, β

- If $\beta \uparrow$, then agents discount the future less. All else constant the cost of default (exclusion as well as output loss) has a higher present discounted value. This should make countries default less often.
- Lower default frequencies then immediately imply lower country premia. And with more ability to repay, debt should incrase. By this argument $\beta \uparrow \Rightarrow d \uparrow$.
- Yet, β does not only affect the present value of the costs of default, it also changes the desired level of debt.
- In a model without default, the more patient agents are the lower is the level of debt, thus, by this argument $\beta \uparrow \Rightarrow d \downarrow$.
- Which force dominates in equilibrium? The next table shows that the first force does.

Varying β

	Default frequency	E(d/y)	$E(r-r^*)$	$\sigma(r-r^*)$	corr$(r-r^*, y)$	corr$(r-r^*, tb/y)$
Data	2.6	58.0	7.4	2.9	-0.64	0.72
Model						
$\beta = 0.85^*$	2.6	59.0	3.5	3.2	-0.54	0.81
$\beta = 0.90$	1.4	71.4	1.6	2.0	-0.52	0.78
$\beta = 0.95$	0.4	87.8	0.5	0.9	-0.51	0.71

* indicates the baseline calibration.

- The larger is β, all else constant, the more debt can be supported, which is the exact opposite of what happens under committment.
- Intuition, higher β increases the present value of the default cost.

Varying the Persistence of the Output Process

Varying ρ holding constant the variance of y

	Default frequency	E(d/y)	$E(r-r^*)$	$\sigma(r-r^*)$	corr$(r-r^*, y)$	corr$(r-r^*, tb/y)$
Data	2.6	58.0	7.4	2.9	-0.64	0.72
Model						
$\rho = 0$	0.1	274.1	0.1	0.1	-0.68	0.65
$\rho = 0.5$	0.2	176.5	0.2	0.2	-0.68	0.67
$\rho = 0.75$	0.7	104.3	0.8	0.6	-0.57	0.66
$\rho = 0.85$	1.5	74.0	1.7	1.4	-0.52	0.73
$\rho = 0.9317^*$	2.6	59.0	3.5	3.2	-0.54	0.81
$\rho = 0.95$	2.8	59.7	3.8	3.5	-0.58	0.85
$\rho = 0.97$	2.8	67.3	3.7	3.7	-0.62	0.85

Note. * = baseline value.

- The higher is ρ the lower is debt and the higher is the default frequency.
- Why? Recall that the output cost kicks in only for high levels of output.
- When $\rho = 0$, it is quite likely that output in the near future is high and hence the country has to pay a high default cost.
- Thus, the country chooses a low default frequency.
- And with infrequent defaults the country can support more debt.

The Welfare Cost of Lack of Commitment

- If agents could commit to repay, then debt would be **100 times larger**, 65.88 versus 0.6133, conditional on good standing, or 0.5091 unconditionally.
- Why? Agents are very impatient, $\beta(1 + r) = 0.8585 < 1$
- Lack of commitment manifests itself as an endogenous borrowing constraint.
- And lack of commitment is **welfare reducing**.
- How much would agents be willing to pay to be able to commit to repay?

- Value function under commitment:

$$v^{\text{com}}(d,y) = \max_{d'} \left\{ u(y + d'(1+r^*)^{-1} - d) + \beta E_y v^{\text{com}}(d', y') \right\}$$

- With d' in hand, we can find equilibrium consumption from

$$c^{\text{com}} = y + d'/(1+r^*) - d$$

- The welfare costs of lack of commitment, $\Lambda(d,y)$:

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{\left(c_t^{\text{com}}\right)^{1-\sigma} - 1}{1-\sigma} = E_0 \sum_{t=0}^{\infty} \beta^t \frac{\left[(1+\Lambda(d_0,y_0))c_t^{\text{nocom}}\right]^{1-\sigma} - 1}{1-\sigma}.$$

- Solving for $\Lambda(d,y)$ yields

$$\Lambda(d,y) = \left[\frac{v^{\text{com}}(d,y)(1-\sigma)(1-\beta)+1}{v^{\text{nocom}}(d,y)(1-\sigma)(1-\beta)+1} \right]^{\frac{1}{1-\sigma}} - 1,$$

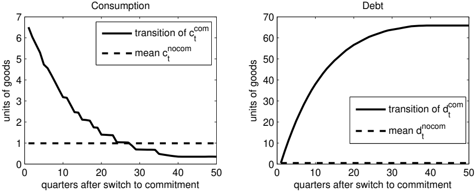
- Note that the welfare cost of lack of commitment are state dependent, that is, Λ is a function of the current state (d,y).

- Compute $v^{\text{nocom}}(d, y)$ and $v^{\text{com}}(d, y)$ and then take unconditional expectations using the ergodic distribution of the state (d, y) associacted with the economy displaying lack of commitment.
- The unconditional mean of the welfare cost of lack of commitment is **273 percent,** that is,

$$100 \times E\Lambda(d, y) = 273$$

- This is an enormous value.
- It means that the consumption stream of an individual living in the economy without commitment must almost **quadruplicate** in order for him to be as well off as living in the economy with commitment to repay debts.
- The totality of this welfare cost is due to the transitional dynamics of switching from no commitment to commitment. Along this transition, debt increases from a mean of 0.50 to a mean of 65.88, and consumption declines from a mean of 0.9873 to a mean of 0.36. Of course, along this transition consumption is temporarily much higher than 0.9873.
- The next figure displays the typical transition from the lack of commitment economy to the commitment economy.

Transition From No Commitment to Commitment



- The typical transition path is the mean of 10,000 transition paths each starting at a pair (d, y) drawn from the ergodic distribution under lack of commitment.
- The transitional dynamics are the key determinant of the welfare gains of commitment.
- Consider a naive approach to welfare evaluation consisting in computing the unconditional welfare in each economy separately.
- Because in the stationary state average consumption in the commitment economy is **one third** as high as consumption in the no-commitment economy, one would erroneously conclude that lack of commitment is welfare improving.

Decentralization Of The Eaton-Gersovitz Model

- We will not do it
- But it is potentially interesting because it allows us to study capital controls
- It also clarifies that capital controls are implicitly present in every EG-style default model
- The decentralized version of the model makes this explicit

Risk Averse Lenders

- We will not do in detail, but it is of interest because
  * Observed country spreads tend to be **larger** than observed default probabilities.
  * Average spread-default-frequency differential greater **200** basis points.
  * By contrast EG model predicts a **negative or zero** spread-default-frequency differential.
  * Possible solution: introduced **risk averse** lenders into the EG model.
- Spread will then be the sum of default probability and compensation for default risk. Why will that help?
  * NO! The assumption of risk averse foreign lenders has quantitatively negligible effects on the predictions of the Eaton-Gersovitz model.
  * The assumption of risk-averse foreign lenders does not change the prediction of a near-zero spread-default-frequency differential
  * Why?
  * Probably because the model economy has highly impatient agents, who can borrow much less than what they would like to borrow under commitment.
  * As a result, the model behaves like one in which the agent is up against a borrowing constraint most of the time.
  * In such a setting, the price of credit is little allocative, and hence variation therein do not affect much consumption or borrowing decisions.
  * This result may change in a setting with default and more patient consumers.