ECON 39: Undergraduate International Trade
Problem Set #3
Professor Treb Allen
Due: Tuesday February 6 at the beginning of class

Preliminaries

This week, we are learning about the Specific Factor model of trade. This problem set is also pretty difficult, but after the last problem set, hopefully this will seem a little easier!

Questions

1. Consider a single country (call it country 1) which produces two products, product A and product B. The country is inhabited by L_1 identical workers and two specific factors, one for each product. Let the quantity of the specific factor in country 1 for product A be A_1 and the quantity for the specific factor in country 1 for product B be B_1. Suppose the production function for the production of product A can be written as:

$$Q_1^A = \left(L_1^A\right)^{\alpha} \left(A_1\right)^{1-\alpha},$$

where Q_1^A is the quantity produced of product A in country 1, L_1^A is the number of workers employed in the production of product A, and $\alpha \in (0,1)$. Similarly, suppose the production function for the production of product B can be written as:

$$Q_1^B = \left(L_1^B\right)^{\beta} \left(B_1\right)^{1-\beta},$$

where Q_1^B is the quantity produced of product B in country 1, L_1^B is the number of workers employed in the production of product B, and $\beta \in (0,1)$.

(a) Suppose that a representative agent for country 1 has preferences $U_1\left(C_1^A, C_1^B\right)$, where C_1^A is the quantity consumed of product A and C_1^B is the quantity consumed of product B.

i. List the exogenous model parameters.
ii. List the endogenous model outcomes.
iii. Define the equilibrium.

(b) Using the labor market clearing condition (i.e. that $L_1 = L_1^A + L_1^B$), write down an equation for the production possibility frontier (i.e. a function for the maximum amount of Q_1^A that can be produced for a given quantity of Q_1^B produced).

(c) Find the marginal product of labor in the production of product A (i.e. $\frac{\partial Q_1^A}{\partial L_1^A}$) and the marginal product of labor in the production of product B (i.e. $\frac{\partial Q_1^B}{\partial L_1^B}$).

(d) Calculate the equilibrium relationship between the relative price $\frac{p_1^A}{p_1^B}$ and the allocation of labor between the two products. (Hint: use the equilibrium condition that workers maximize their income).

(e) Suppose that $U_1\left(C_1^A, C_1^B\right) = \gamma \log C_1^A + (1-\gamma) \log C_1^B$, where $\gamma \in (0,1)$. Calculate the equilibrium relationship between the relative price $\frac{p_1^A}{p_1^B}$ and the quantity consumed of the two products. (Hint: use the equilibrium condition that the representative agent maximizes her utility subject to the income earned in the country).

(f) Combining your previous two answers, write all the endogenous model outcomes as functions only of the exogenous model parameters.

(g) Suppose that workers and owners of each specific factor all have identical preferences to the representative agent. Write down expressions for the *welfare* of workers and owners of each specific factor as a function only of the exogenous model parameters.

2. Now suppose that there exists another country (call it country 2). Suppose that country 2 is identical to country 1 in every way except that $A_2 < A_1$, i.e. country 2 is endowed with less of the specific factor for product A.

   (a) Using you answer to question 1(f), will the autarkic relative price of product A in country 2 (i.e. $\frac{p_2^A}{p_2^B}$) be higher or lower than the autarkic relative price of product A in country 1 (i.e. $\frac{p_1^A}{p_1^B}$)?

   (b) Suppose that country 1 and country 2 now open up to trade with each other.

      i. List the exogenous model parameters.

      ii. List the endogenous model outcomes.

      iii. Define the trade equilibrium.

   (c) Given your answer to question 2(a), which country do you expect will export product A and which country do you expect will export product B?

   (d) Write all the endogenous model outcomes as functions only of the exogenous model parameters. [Note: you should end up with a set of two equations and two unknowns. That's as far as you need to go!]

   (e) How has the relative price changed in country 1 compared to its autarkic price? How has the equilibrium distribution of labor across the two sectors changed? Using your your answer to question 1(g), who gains from trade and who loses?

3. Suppose instead that $U_1\left(C_1^A, C_1^B\right) = \min\left\{C_1^A, C_1^B\right\}$. Show the overall gains from trade for country 1 and country 2 using pictures only.