Macroeconomics A; EI056

Short problems

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1 Adverse selection

1.1 Payoff of borrower

Question: Consider a borrower getting a loan from a lender of an amount L and interest rate r_l.

The borrower invests in a project that delivers $R' + x$ with 50% probability and $R' - x$ with 50% probability.

We assume that $R' - x < (1 + r_l) L$, so in case of bad payoff the borrower just defaults and pays nothing.

What is the expected return for the borrower? How does it depend on the risk x? What is the intuition?

1.2 Payoff of lender

Question: The lender gets the loan paid back, or in case of default gets the return on the project $R' - x$.

What is the expected return for the lender? How does it depend on the risk x? What is the intuition?

1.3 Heterogeneous borrowers

Question: The population is made of 50% of risky borrowers whose projects have $x = x_b$ and 50% of safe borrowers whose projects have $x = x_g < x_b$.

The lender must charge the same interest rate to all borrowers, as he cannot tell who is risky and who is not.

Show that $r_l > r_l^* = \frac{R'+x_g}{L} - 1$ if the safe borrowers do not take a loan.

What is the expected payoff for the risky borrower if $r_l = r_l^*$?

1.4 Expected payoff of lender with heterogeneous borrowers

Question: Show that if all potential borrowers take a loan, the expected payoff of the lender:

Show that if only risky borrowers take a loan, the expected payoff of the lender:

What happens to the lender's expected payoff if the interest rate moves from slightly below r_l^* to slightly above r_l^*?

2 Bank and risk sharing

2.1 Utility under autarky

Question: Consider a two-period model with a unit mass of small agents. Each agent gets 1 unit of a good in period 0. They can consume the unit in period 1, or keep it until period 2 and get $R > 1$ units.

In period 1, each agent learns its type. If impatient, which happens with probability t she get utility only from consumption in period 1. With probability $1 - t$ she get utility only from consumption in period 2. Utility is thus:

$$\frac{1}{1-\sigma}(c_1)^{1-\sigma} \text{with probability } t$$
$$\frac{1}{1-\sigma}(c_2)^{1-\sigma} \text{with probability } 1-t$$

What is the expected utility for an agent who does not transact with anyone else?

2.2 Allocation under pooling

Question: Consider that there is a bank. All agents deposit their unit of endowment.

In period 1, an agent can come to the bank and ask for c_1^* units of consumption. Alternatively, she can come to the bank and ask for c_2^* units of consumption.

The budget constraints of the bank are:

$$tc_1^* + s = 1 \qquad ; \qquad sR = (1-t)\,c_2^*$$

where s is the amount kept from period 1 to 2, and t is the proportion of agents coming to the bank in the first period.

Show that a bank maximizing welfare chooses:

$$c_1^* = \frac{1}{1-(1-t)\left[1-(R)^{\frac{1-\sigma}{\sigma}}\right]} \qquad ; \qquad c_2^* = \frac{1}{1+t\left[(R)^{\frac{\sigma-1}{\sigma}}-1\right]}R$$

How does this compare to the consumption under autarky (assume $\sigma > 1$)?

2.3 Interpretation

Question: What are the values of c_1^* and c_2^* when $R = 1$?

What are the derivatives of c_1^* and c_2^* with respect to R (evaluate them at $R = 1$)?

Show that $c_2^* > c_1^*$. Hint: think about how an increase in R starting at $R = 1$ affects $c_2^* - c_1^*$.

2.4 Interpretation

Question: Intuitively, why is the consumption different under pooling that under autarky?

Would an impatient agent ever lie about who she truly is?

Is it optimal for a patient agent to claim to be impatient?