EI060 Macroeconomics B
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PS1 Solutions

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1 First generation crisis model

1.1 Consumption under a peg

We begin with the Euler equation. Under fixed exchange rate, the exchange depreciation rate $\varepsilon_t = 0$. Thus the interest parity gives that $i = r$, so that $c_t^{-\frac{1}{\sigma}} = \lambda(1 + \theta + \alpha r)$, as we know that $r = \beta$ and θ is the tax rate on consumption, which is constant, consumption will be constant, we can write it as $\tilde{c}$.
We then identify the constant by the intertemporal budget constraint:

$$\begin{aligned}
\alpha_0 + \frac{y}{r} &= \int_0^\infty e^{-rt} \left(\tilde{c}(1+\theta) + rm_t\right) dt \\
&= \int_0^\infty e^{-rt} \left(\tilde{c}(1+\theta) + r\alpha c_t\right) dt \\
&= \int_0^\infty e^{-rt} \tilde{c}(1+\theta+r\alpha) dt \\
&= \tilde{c}(1+\theta+\alpha r) \int_0^\infty e^{-rt} dt \\
&= \tilde{c}(1+\theta+\alpha r) \frac{1}{r}
\end{aligned}$$

Thus we have:

$$\tilde{c} = \frac{\alpha_0 r + y}{1 + \theta + \alpha r}$$

1.2 Unsustainable peg

As we know that the government spending is over a threshold:

$$g > rh_0 + \frac{\alpha_0 r + y}{1 + \theta + \alpha r}$$

From **??**, we know that the consumption is a constant $\tilde{c} = \frac{\alpha_0 r + y}{1+\theta+\alpha r}$, thus the government tax income would be $\theta\tilde{c} = \frac{\theta(\alpha_0 r + y)}{1+\theta+\alpha r}$.
The government tax revenue is:

$$s^p = \theta\tilde{c} - g = \frac{\theta(\alpha_0 r + y)}{1 + \theta + \alpha r} - g < -rh_0 < 0$$

Under a fixed exchange rate, $\varepsilon_t = 0$ and at the steady state, the real balance is constant, $\dot{m}_t = 0$, we know that the foreign reserves cahnge is:

$$\dot{h}_t = rh_t + s_t^p < rh_t - rh_0$$

at $t = 0$, $h_0 < 0$. As time passes, the foreign reserves will be negative and the government will default on its debt. Hence the government cannot continue to run a peg, and the peg is unsustainable.

1.3 Consumption and depreciation pre- and post-break

From the Euler equation, before the abandon of the peg, we have:

$$c_1^{-\frac{1}{\sigma}} = \lambda(1 + \theta + \alpha r)$$

and after the abandon, we have:

$$c_2^{-\frac{1}{\sigma}} = \lambda(1 + \theta + \alpha(r + \varepsilon)).$$

So,

$$\left(\frac{c_1}{c_2}\right) = \frac{1 + \theta + \alpha(r + \varepsilon)}{1 + \theta + \alpha r} > 1$$

as $\varepsilon > 0$, giving that after the abando of the peg, the consumption decrceases.
Now we use that budget constraint and the cash in advance constraint, we have:
With m constant and in steady state of reserves: $\dot{h}_t = 0$,

$$0 = rh_t + (\theta c_2 - g) + \varepsilon m_2 = \theta(c_2 - c_1) + \varepsilon \alpha c_2$$

since $\theta c_1 = g$, and we can solve:

$$\varepsilon = \frac{\theta}{\alpha}\left(\frac{c_1}{c_2} - 1\right)$$

At the equilibrium values, $\frac{c_1}{c_2} > 1$ and thus $\varepsilon > 0$.

1.4 Dynamics of reserves and assets

Before the break, $s_t^p = \theta c_1 - g = 0$, $\dot{m}_t = 0$ and $\varepsilon_t = 0$, so $\dot{h}_t = rh_t$.
For foreign assets, we have:

$$\dot{a}_t = ra_t + y - c_1(1 + \theta) - rm_1 = ra_t + y - c_1(1 + \theta + \alpha r)$$

and the uverall position is:

$$\dot{a}_t + \dot{h}_t = r(a_t + h_t) + y - c_1(1 + \theta + \alpha r) = r(a_t + h_t) + y - \frac{g(1 + \theta + \alpha r)}{\theta}$$

since $c_1 = \frac{g}{\theta}$.
Evaluated at $t = 0$, we have:

$$\dot{h}_0 = rh_0, \quad \dot{a}_0 + \dot{h}_0 = r(a_0 + h_0) + y - \frac{g(1+\theta+\alpha r)}{\theta}$$

which is exactly the current-account balance. Observing $\dot{a}+\dot{h} < 0$ ex-ante would signal an impending crisis.

1.5 Timing of break

Recall that the intertemporal budget constraint is:

$$\frac{g}{r} = h_0 + \int_0^{\infty} e^{-rt} \left[\theta c_t + \dot{m}_t + \varepsilon_t m_t\right] dt + e^{-rT} \left[m_T - m_{T-}\right].$$

Before the break fo the peg($0 \leq t \leq T$):

$$c_t = c_1, \quad \dot{m}_t = 0, \quad \varepsilon_t = 0$$

and after the break, ($t > T$):

$$c_t = c_2, \quad \dot{m}_t = 0, \quad \varepsilon_t = \varepsilon > 0$$

Bringing these conditions into the intertemporal budget constraint, we have:

$$\frac{g}{r} = h_0 + \int_0^{T} e^{-rt}\theta c_1 dt + \int_T^{\infty} e^{-rt} \left[\theta c_2 + \varepsilon m_2\right] dt + e^{-rT} \left[m_T - m_{T-}\right].$$

As $\theta c_1 = g$, we can write:

$$\int_0^T e^{-rt}\theta c_1 dt = g \int_0^T e^{-rt} dt = \frac{g\left(1-e^{-rT}\right)}{r}$$

and that the second term is:

$$\int_T^{\infty} e^{-rt} \left[\theta c_2 + \varepsilon m_2\right] dt = \left[\theta c_2 + \varepsilon m_2\right]\frac{e^{-rT}}{r}$$

Thus,

$$\frac{g}{r} = h_0 + \frac{g\left(1-e^{-rT}\right)}{r} + \frac{[\theta c_2 + \varepsilon m_2]e^{-rT}}{r} + e^{-rT} \left[m_T - m_{T-}\right].$$

As $m_2 = \alpha c_2$, $\varepsilon = \frac{\theta}{\alpha}\left(\frac{c_1}{c_2} - 1\right)$, we can further simplify that:

$$\theta c_2 + \varepsilon m_2 = \theta c_2 + \frac{\theta}{\alpha}\left(\frac{c_1}{c_2} - 1\right)\alpha c_2 = \theta c_2 + \theta(c_1 - c_2) = \theta c_1 = g$$

Thus the budget constraint is reduced to:

$$\begin{aligned}
\frac{g}{r} &= h_0 + \frac{g\left(1-e^{-rT}\right)}{r} + \frac{ge^{-rT}}{r} + e^{-rT}\left[m_T - m_{T-}\right] \\
&= h_0 + \frac{g}{r} + e^{-rT}[m_T - m_{T-}] \\
\Rightarrow h_0 &= -e^{-rT}[m_T - m_{T-}] \\
\Rightarrow T &= \frac{1}{r}\ln\left(\frac{m_{T-} - m_T}{h_0}\right)
\end{aligned}$$

2 Choice of policy regime

2.1 Constant money

Based on the rational expectation and constant money supply assumption, we have:

$$\mathbb{E}_t[e_{t+1}] = m_t = \bar{m}$$

SO, we have $i_{t+1} = \bar{m} - e_t$, and we implement the total output function, we can get:

$$\begin{aligned}
\overline{m} - p_t &= -\eta(\overline{m} - e_t) + \phi y_t + v_t \\
&= -\eta\overline{m} + \eta e_t + \phi\left[\delta(e_t - p_t) + \varepsilon_t\right] + v_t \\
&= -\eta\overline{m} + \eta e_t + \phi\delta(e_t - p_t) + \phi\varepsilon_t + v_t \\
\Rightarrow (\phi\delta - 1)p_t &= (\eta + \phi\delta)e_t + \phi\varepsilon_t + v_t - (1+\eta)\overline{m} \qquad (2.1.1)
\end{aligned}$$

We bring this back to the total supply function, with $\mathbb{E}_{t-1}[p_t] = \overline{m}$(expectation price is equal to the long-term equilibrium), we have:

$$\begin{aligned}
y_t &= \theta(p_t - \overline{m}) \Rightarrow p_t = \frac{y_t}{\theta} + \overline{m} \\
\Rightarrow y_t &= \delta\left(e_t - \overline{m} - \frac{y_t}{\theta}\right) + \varepsilon_t \\
\Rightarrow y_t &= \frac{\theta\left[\delta(e_t - \overline{m}) + \varepsilon_t\right]}{\theta + \delta} \\
\Rightarrow p_t &= \overline{m} + \frac{\delta(e_t - \overline{m}) + \varepsilon_t}{\theta + \delta}
\end{aligned}$$

Bring this back to equation 2.1.1, we have:

$$
\begin{aligned}
(\phi\delta - 1)\left[\overline{m} + \frac{\delta(e_t - \overline{m}) + \varepsilon_t}{\theta + \delta}\right] &= (\eta + \phi\delta)e_t + \phi\varepsilon_t + v_t - (1+\eta)\overline{m} \\
\Rightarrow (\phi\delta + \eta)\overline{m} + \frac{\phi\delta - 1}{\theta + \delta}\left[\delta(e_t - \overline{m}) + \varepsilon_t\right] &= (\eta + \phi\delta)e_t + \phi\varepsilon_t + v_t \\
\Rightarrow \left[\phi\delta + \eta - \frac{(\phi\delta - 1)\delta}{\theta + \delta}\right]\overline{m} + \left[\frac{\phi\delta - 1}{\theta + \delta} - \phi\right]\varepsilon_t &= \left[\eta + \phi\delta - \frac{(\phi\delta - 1)\delta}{\theta + \delta}\right]e_t + v_t \\
\Rightarrow \frac{\phi\delta\theta + \eta(\theta + \delta) + \delta}{\theta + \delta}\overline{m} - \frac{1 + \phi\theta}{\theta + \delta}\varepsilon_t &= \frac{\phi\delta\theta + \eta(\theta + \delta) + \delta}{\theta + \delta}e_t + v_t \\
\Rightarrow \left[(\phi\theta + 1)\delta + \eta(\theta + \delta)\right]\overline{m} - (1 + \phi\theta)\varepsilon_t &= \left[(\phi\theta + 1)\delta + \eta(\theta + \delta)\right]e_t + v_t \\
\Rightarrow e_t = \overline{m} - \frac{(1 + \phi\theta)\varepsilon_t + (\theta + \delta)v_t}{(\phi\theta + 1)\delta + \eta(\theta + \delta)} & \qquad (2.1.2)
\end{aligned}
$$

We bring this result back to p_t, we have:

$$
\begin{aligned}
p_t &= \overline{m} + \frac{\delta(e_t - \overline{m}) + \varepsilon_t}{\theta + \delta} \\
&= \overline{m} + \frac{\varepsilon_t - \frac{\delta(1+\phi\theta)\varepsilon_t + \delta(\theta+\delta)v_t}{(\phi\theta+1)\delta + \eta(\theta+\delta)}}{\theta + \delta} \\
&= \overline{m} + \frac{\eta(\theta + \delta)\varepsilon_t - \delta(\theta + \delta)}{(\theta + \delta)\left[(\phi\theta + 1)\delta + \eta(\theta + \delta)\right]} \\
&= \overline{m} + \frac{\eta\varepsilon_t - \delta v_t}{(1 + \phi\theta)\delta + \eta(\theta + \delta)} \qquad (2.1.3)
\end{aligned}
$$

and that

$$
\begin{aligned}
y_t &= \frac{\theta\left[\delta(e_t - \overline{m}) + \varepsilon_t\right]}{\theta + \delta} \\
&= \frac{\eta\theta\varepsilon_t - \delta\theta v_t}{(\phi\theta + 1)\delta + \eta(\theta + \delta)} \qquad (2.1.4)
\end{aligned}
$$

Then, the variance of output should be:

$$
\mathbb{E}_{t-1}[y_t^2] = \left(\frac{\eta\theta}{(\phi\theta + 1)\delta + \eta(\theta + \delta)}\right)^2 \sigma_\varepsilon^2 + \left(\frac{\delta\theta}{(\phi\theta + 1)\delta + \eta(\theta + \delta)}\right)^2 \sigma_v^2
$$

denote $A = (\phi\theta + 1)\delta + \eta(\theta + \delta)$, we have:

$$
\mathbb{E}_{t-1}[y_t^2] = \frac{\theta^2\eta^2\sigma_\varepsilon^2 + \theta^2\delta^2\sigma_v^2}{A^2}.
$$

2.2 Exchange rate peg

Under a fixed exchange rate, $e_t = \overline{e} = \overline{m}$.
From the total demand function, we have:

$$
y_t = \delta(e_t - p_t) + \varepsilon_t = \delta(\overline{m} - p_t) + \varepsilon_t
$$

From the total supply function, we have:

$$y_t = \theta(p_t - \mathbb{E}_{t-1}[p_t]) = \theta(p_t - \overline{m})$$

Combining these two equations, we have:

$$\begin{aligned}\theta(p_t - \overline{m}) &= \delta(\overline{m} - p_t) + \varepsilon_t \\ \Rightarrow p_t &= \overline{m} + \frac{\varepsilon_t}{\theta + \delta}\end{aligned}$$

Bring p_t back to the total supply function, we have:

$$y_t = \theta(p_t - \overline{m}) = \theta\frac{\varepsilon_t}{\theta + \delta} = \frac{\theta}{\theta + \delta}\varepsilon_t$$

As the exchange rate is pegged, $i_{t+1} = \mathbb{E}_t[e_{t+1}] - e_t = 0$. From the money demand function, we could get:

$$\begin{aligned}m_t - p_t &= \phi y_t + v_t = \phi\frac{\theta}{\theta + \delta}\varepsilon_t + v_t \\ \Rightarrow m_t &= p_t + \phi\frac{\theta}{\theta + \delta}\varepsilon_t + v_t \\ \Rightarrow m_t &= \overline{m} + \frac{\varepsilon_t}{\theta + \delta} + \phi\frac{\theta}{\theta + \delta}\varepsilon_t + v_t \\ &= \overline{m} + \frac{(1 + \phi\theta)\varepsilon_t}{\theta + \delta} + v_t\end{aligned}$$

For the variance:

$$\mathbb{E}_{t-1}[y_t^2] = \left(\frac{\theta}{\theta + \delta}\right)^2 \sigma_\varepsilon^2$$

2.3 Regime choice

The relative volatility is given by the ratio of variance under two policies, given by:

$$\frac{\mathbb{V}[y_t]_{peg}}{\mathbb{V}[y_t]_{money}} = \frac{[\eta(\theta + \delta) + \delta(1 + \phi\theta)]^2}{(\theta + \delta)^2}\frac{\sigma_\varepsilon^2}{\eta^2\sigma_\varepsilon^2 + \delta^2\sigma_v^2}$$

So, if $\sigma_\varepsilon^2/\sigma_v^2 \to 0$, the relative volativity is determined by $\sigma_v^2/\sigma_\varepsilon^2 \to \infty$, thus the ratio is close to 0, thus a peg policy is less volatile than a fixed money supply.
When the main source of volatility is a money demand shock (v_t) rather than a real economy shock (ε_t), a fixed exchange rate regime stabilizes output by allowing the money supply to adjust automatically to offset money demand shocks. In contrast, a fixed money supply regime is unable to adjust in the face of money demand shocks, leading to higher output volatility.

2.4 Optimal rule

Consider the Taylor rule, we can rewrite the money demand equation as:

$$\overline{m} - p_t = -(\eta + \Phi)(e - e_t) + \phi y_t + v_t$$

Compare with section 2.1, we can see that the only difference is the Φ term, which is the Taylor rule coefficient.
THus we only need to replace the η with $(\eta + \Phi)$ in the previous equations, we can get:

$$\begin{aligned}
p_t &= \overline{m} + \frac{(\eta + \Phi)\varepsilon_t - \delta v_t}{(1 + \phi\theta)\delta + (\eta + \Phi)(\theta + \delta)} \\
y_t &= \frac{\theta(\eta + \Phi)\varepsilon_t - \theta\delta v_t}{[(\phi\theta + 1)\delta + (\eta + \Phi)(\theta + \delta)]^2} \\
e_t &= \overline{m} - \frac{(1 + \phi\theta)\varepsilon_t + (\theta + \delta)v_t}{(\phi\theta + 1)\delta + (\eta + \Phi)(\theta + \delta)}
\end{aligned}$$

Then, the variance of output should be:

$$\mathbb{E}_{t-1}[y_t^2] = \frac{\theta^2(\eta + \Phi)^2\sigma_\varepsilon^2 + \theta^2\delta^2 v_t^2}{[(\phi\theta + 1)\delta + (\eta + \Phi)(\theta + \delta)]^2}$$

To solve the optimal value of Φ, we aim to minimizes tha variance of output. First we denote $A = \eta + \Phi$ and $D = A(\theta + \delta) + \delta(1 + \phi\theta)$, then we know that $\frac{dA}{d\Phi} = 1$ and $\frac{dD}{d\Phi} = \theta + \delta$. Take the FOC of $\mathbb{E}_{t-1}[y_t^2]$, we have:

$$\frac{\partial \mathbb{E}_{t-1}[y_t^2]}{\partial \Phi} = \theta^2 \frac{2A\sigma_\varepsilon^2 D^2 - [A^2\sigma_\varepsilon^2 + \delta^2\sigma_v^2]\, 2DD'}{D^4} = 0$$

THen, we have:

$$\begin{aligned}
&A\sigma_\varepsilon^2 D = \left(A^2\sigma_\varepsilon^2 + \delta^2\sigma_v^2\right)(\theta + \delta) \\
\Rightarrow &\left[A^2(\theta + \delta) + A\delta(1 + \phi\theta)\right]\sigma_\varepsilon^2 = A^2(\theta + \delta)\sigma_\varepsilon^2 + (\theta + \delta)\delta^2\sigma_v^2 \\
\Rightarrow &A = \frac{(\theta + \delta)\delta\sigma_v^2}{(1 + \phi\theta)\sigma_\varepsilon^2}
\end{aligned}$$

So, we have:

$$\Phi^* = \frac{(\theta + \delta)\delta\sigma_v^2}{(1 + \phi\theta)\sigma_\varepsilon^2} - \eta.$$

When $\sigma_\varepsilon^2/\sigma_v^2 \to 0$, $\Phi \to \infty$, this implies that the central bank should fix the exchange rate completely. This is consistent with our conclusion in 2.3.
When $\sigma_v^2/\sigma_\varepsilon^2 \to 0$, $\Phi \to -\eta$. This implies that the central bank should implement a fully floating exchange rate regime because at this point $\Phi + \eta \approx 0$, which is equivalent to not reacting to exchange rate fluctuations.
When money demand shocks dominate, it is better to fix the exchange rate to offset these shocks; when real economy shocks dominate, it is better to let the exchange rate float freely to absorb these

shocks.

3 Taxation of debt

3.1 Decentralized and centralized choice