The Euler Equation

Differentiating

$$U(C_t) - V(N_t) + \lambda_t \left(A_t K_{t-1}^{\alpha} N_t^{1-\alpha} - C_t - K_t + (1-\delta) K_{t-1}\right)$$
$$+\beta E_t \left[\lambda_{t+1} \left(A_{t+1} K_t^{\alpha} N_{t+1}^{1-\alpha} + (1-\delta) K_t\right)\right]$$

$$\frac{\partial L}{\partial C_t} \quad : \quad U'(C_t) - \lambda_t = 0$$

$$\frac{\partial L}{\partial K_t} \quad : \quad -\lambda_t + \beta E_t \left[\lambda_{t+1} \left(\alpha \frac{Y_{t+1}}{K_t} + 1 - \delta\right)\right] = 0$$

$$R_{t+1} = \alpha \frac{Y_{t+1}}{K_t} + 1 - \delta$$

Then the FOC for capital can be written as

$$\lambda_t = \beta E_t \left(\lambda_{t+1} R_{t+1}\right)$$

Step 2: combine this with the FOC for consumption

$$\frac{\partial L}{\partial C_t} \quad : \quad U'(C_t) - \lambda_t = 0$$

ie, $\lambda_t = U'(C_t)$ and $\lambda_{t+1} = U'(C_{t+1})$ ➔

$\lambda_t = U'(C_t) = \beta E_t(\lambda_{t+1} R_{t+1}) = \beta E_t(U'(C_{t+1})\ R_{t+1})$ ➔

The "***Euler equation***"

$$U'(C_t) = \beta E_t \left[U'(C_{t+1}) R_{t+1}\right]$$