# Macroeconomics B, EI060

## Class 10

## Optimal policy in stochastic model

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# What you will get from today class

- Stochastic version of the micro-founded model (Corsetti and Pesenti).
  - Beyond the effect of monetary shocks: how should we set monetary policy in response to other shocks (normative view).

- Presenting the concepts: optimal monetary policy in a closed economy.

- Extending to then open economy.
  - How exchange rate pass-through affects the optimal prices.
  - Optimal policy: technical steps, and visual presentation of the key dimension.
  - When cooperation can help.

- Focus on intuition behind optimal policy. Graphical illustration of transmission as extra slides.

*Under sticky prices, the exchange rate transmits shocks across countries. It should therefore be a) kept stable, b) used to stabilize output.*

Do you agree? Why or why not?

# CLOSED ECONOMY MODEL

# Simplifying assumptions

- Stochastic version of the Obstfeld-Rogoff model.
  - Go beyond the ex-post effect of one shock.
  - Consider the ex-ante impact of the possibility of shocks and the associated policy response.
- One-period model, allowing for an analytical solution.
  - Prices are set at the beginning of the period.
  - Shocks then happen, monetary policy reacts, the exchange rate moves, and firms meet the actual demand at preset prices.
- Prices are preset in a forward-looking way. Their level reflects the rules of policy.

# Benchmark: closed economy

- Illustrate key concepts in a closed economy. Representative household with utility of consumption and labor:

$$U = \ln C - \kappa l$$

- CES basked of the various brands $j$:

$$C = \left[ \int_0^1 [C(j)]^{\frac{\theta-1}{\theta}} \, dj \right]^{\frac{\theta}{\theta-1}}$$

- Consumption allocation reflects relative prices:

$$C(j) = \left[ \frac{p(j)}{P} \right]^{-\theta} C \qquad ; \qquad P = \left[ \int_0^1 [p(j)]^{1-\theta} \, dj \right]^{\frac{1}{1-\theta}}$$

- The household gets a wage income and the profits from firms (lump sum).
- Monetary stance $\mu = PC$ (cash-in-advance constraint or a money-in-the-utility approach).

$$\mu = PC$$

- Labor supply, with flexible wages: $W = \kappa PC$ (marginal utility of consumption * real wage = marginal cost of effort).
- Flexible wages reflect the monetary stance:

$$W = \kappa \mu$$

- Each brand $j$ produced by a unique firm. Linear technology using labor with productivity $Z$ common to all firms:

$$Y(j) = Z \times I(j)$$

- Equilibrium: all firms are identical in equilibrium, consumption is equal to output:

$$C = ZI$$

- Each firm sets its price taking account if its market power, with demand:

$$Y(j) = \left[\frac{p(j)}{P}\right]^{-\theta} C$$

- When prices can be set, markup over effective cost (wage adjusted for productivity):

$$P^{\text{flex}} = \frac{\theta}{\theta - 1} \frac{W}{Z} = \frac{\theta \kappa}{\theta - 1} \frac{\mu}{Z}$$

- Leads to constant employment, consumption reflects productivity:

$$l^{\text{flex}} = \frac{\theta - 1}{\theta \kappa} = \bar{l} \qquad ; \qquad C^{\text{flex}} = \frac{\theta - 1}{\theta \kappa} Z$$

- These are the natural rates of employment and output (consumption).

- Flexible price: higher productivity $Z$ reduces firms' costs and prices.
  - Labor unchanged at $\bar{l}$.
  - Higher purchasing power, efficient increase in output and consumption.
- If prices cannot adjust, outcome depends on monetary policy $\mu$.
- With unchanged $\mu$, consumption ($C = \mu/P$) unchanged. Employment is inefficiently low.
- With monetary expansion (higher $\mu$), consumption increases.
  - Higher purchasing power through higher money balances instead of lower prices.
  - Monetary policy can replicate the flexible price allocation.

- Firm maximizes expected profits, discounted by the marginal utility of income (using demand $Y(j) = [p(j)/P]^{-\theta} C$):

$$E \frac{1}{PC} [p(j) Y(j) - WI(j)]$$

- Useful property: if $Z$ is log normal: $\ln(Z) = z \rightsquigarrow N(0, \sigma^2)$, expected level reflects variance:

$$E(Z^a) = E(\exp[az]) = \exp\left[\frac{a^2}{2}\sigma^2\right]$$

- Price is a markup over expected cost. If $\ln \mu$ proportional to $\ln Z$, price "premium":

$$P^{\text{sticky}} = \frac{\theta \kappa}{\theta - 1} E\left(\frac{\mu}{Z}\right) = \frac{\theta \kappa}{\theta - 1} \exp\left[\frac{1}{2} Var(\ln \mu - \ln Z)\right]$$

- Volatile $\mu/Z$ moves the margin of the firm, who "self insures" through a higher price.

# Utility

- Expected effort = natural rate of employment: $E(I) = \bar{I}$.

$$P = \frac{\theta \kappa}{\theta - 1} E\left(\frac{\mu}{Z}\right) = \frac{\theta \kappa}{\theta - 1} E\left(\frac{PC}{Z}\right) = (\bar{I})^{-1} PE(I)$$

- Utility is then expected log consumption ($\pi_k$: probability of state of nature $k$):

$$
\begin{aligned}
E(\ln C) &= E\left(\ln \frac{\mu}{P}\right) \\
E(\ln C) &= E(\ln \mu) - \ln P \\
E(\ln C) &= E(\ln \mu) - \ln E\left(\frac{\mu}{Z}\right) - \ln \frac{\theta \kappa}{\theta - 1} \\
E(\ln C) &= \sum_k \pi_k \ln \mu_k - \ln \sum_k \pi_k \frac{\mu_k}{Z_k} - \ln \frac{\theta \kappa}{\theta - 1}
\end{aligned}
$$

# Optimal policy

- First-order condition with respect to $\mu_k$:

$$0 = \pi_k \frac{1}{\mu_k} - \frac{1}{\sum_k \pi_k \frac{\mu_k}{Z_k}} \pi_k \frac{1}{Z_k}$$

$$\frac{\mu_k}{Z_k} = \sum_k \pi_k \frac{\mu_k}{Z_k}$$

- Hence $\mu_k/Z_k$ is the same across all states, so $\mu_k = Z_k$. Minimizes the preset price as $Var\left(\ln \mu - \ln Z\right) = 0$:

$$P^{\text{sticky}} = \frac{\theta \kappa}{\theta - 1} \exp\left[\frac{1}{2} Var\left(\ln \mu - \ln Z\right)\right] = \frac{\theta \kappa}{\theta - 1}$$

- As $\mu/Z$ is constant, firms don't want to change prices ex-post, even if they can. Price stickiness becomes irrelevant.

# OPEN ECONOMY

# Consumption baskets

- Two countries, Home and Foreign, of equal size. Foreign variables are indexed by * (prices in Foreign currency).

- Nominal exchange rate $\mathcal{E}$, an increase is a depreciation of the Home currency.

- Consumption baskets ($\theta$: elasticity of substitution across brands, $\lambda = 1$: elasticity between Home and Foreign goods):

$$C = [C_H]^{0.5} [C_F]^{0.5}$$

$$C_H = \left[ \int_0^1 [C_{H,j}]^{\frac{\theta-1}{\theta}} \, dj \right]^{\frac{\theta}{\theta-1}} \qquad ; \qquad C_F = \left[ \int_0^1 [C_{F,j}]^{\frac{\theta-1}{\theta}} \, dj \right]^{\frac{\theta}{\theta-1}}$$

- Allocation of consumption reflects relative prices (similar relations in the Foreign country):

$$C_{H,j} = \frac{1}{2} \left[ \frac{P_{H,j}}{P_H} \right]^{-\theta} \frac{PC}{P_H} \qquad ; \qquad C_{F,j} = \frac{1}{2} \left[ \frac{P_{F,j}}{P_F} \right]^{-\theta} \frac{PC}{P_F}$$

$$P = 2 \left[ P_H \right]^{0.5} \left[ P_F \right]^{0.5}$$

- Monetary stances and labor supplies are as in the closed economy: $\mu = PC$, $\mu^* = P^*C^*$ and $W = \kappa\mu$, $W^* = \kappa\mu^*$.
- Technologies are identical to the closed economy: $Y = ZI$ and $Z^*I^*$.
- Home consumption spending is equal to the value of home output $\Rightarrow$ exchange rate is the ratio of monetary stances:

$$\mathcal{E} = \frac{\mu}{\mu^*}$$

- Demand (monetary stance) relations are similar to the closed economy: $\mu = PC$ and $\mu^* = P^*C^*$.
- The natural rates of employment are also similar: $\bar{l} = \bar{l}^*$.
- Good market clearing (balanced) trade reflects world demand:

$$
\begin{aligned}
Y &= \frac{1}{2}\left(\frac{PC}{P_H} + \frac{P^*C^*}{P_H^*}\right) = \frac{\mu}{2}\left(\frac{1}{P_H} + \frac{\mu^*}{\mu P_H^*}\right) = \frac{PC}{2}\left(\frac{1}{P_H} + \frac{1}{\mathcal{E}P_H^*}\right) \\
Y^* &= \frac{1}{2}\left(\frac{PC}{P_F} + \frac{P^*C^*}{P_F^*}\right) = \frac{\mu^*}{2}\left(\frac{\mathcal{E}_t}{P_F} + \frac{1}{P_F^*}\right) = \frac{P^*C^*}{2}\left(\frac{\mathcal{E}}{P_F} + \frac{1}{P_F^*}\right)
\end{aligned}
$$

- Unit elasticities are central in being able to get a closed-form solution.
- Unit elasticity of substitution between Home and Foreign goods $\Rightarrow$ the same fraction of income is spent on imports regardless of the shocks (the quantity-price split of course varies).
- The log-utility is a similar feature in dynamics terms.
- Both imply that even without contingent assets markets are complete. The current account is zero, and the model is effectively static.

- Home firm $j$ sets a home currency price $P(H, j)$ in Home currency for domestic sales.
- It sets a price $\tilde{P}(H, j)$ in a composite currency for exports. Foreign currency price paid by Foreign consumers:

$$P^*(H, j) = \tilde{P}(H, j)(\mathcal{E})^{-\gamma}$$

- $\gamma$: exchange rate pass-through to Foreign consumers. Price set in a basket currency that is $\gamma$ Home currency and $1 - \gamma$ Foreign currency.
    - $\gamma = 1$: producer currency pricing (PCP, full pass-through).
    - $\gamma = 0$: local currency pricing (LCP, no pass-through).
- Revenue, in Home currency, received by the Home firm: $\tilde{P}(H, j)(\mathcal{E})^{1-\gamma}$.
- Similarly the Foreign firm sets a price $\tilde{P}(F, j)$ such that the Home currency price paid by Home consumers is: $P(F, j) = \tilde{P}(F, j)(\mathcal{E})^{\gamma^*}$.

# Optimal domestic prices

- Prices for domestic sales as in the closed economy case:

$$
\begin{aligned}
P\left(H\right) &= \frac{\theta\kappa}{\theta-1}E\left[\frac{\mu}{Z}\right] = \frac{\theta\kappa}{\theta-1}\exp\left[\frac{1}{2}Var\left(\ln\mu-\ln Z\right)\right] \\
P^{*}\left(F\right) &= \frac{\theta\kappa}{\theta-1}E\left[\frac{\mu^{*}}{Z^{*}}\right] = \frac{\theta\kappa}{\theta-1}\exp\left[\frac{1}{2}Var\left(\ln\mu^{*}-\ln Z^{*}\right)\right]
\end{aligned}
$$

# Optimal export prices

- Prices are markups over expected costs, in the basket currency. Home goods sold in the Foreign country:

$$\tilde{P}(H) = \frac{\theta\kappa}{\theta-1}E\left[\frac{(\mu)^\gamma (\mu^*)^{1-\gamma}}{Z}\right]$$

$$\tilde{P}(H) = \frac{\theta\kappa}{\theta-1}\exp\left[\frac{1}{2}Var\left[\gamma\ln\mu + (1-\gamma)\ln\mu^* - \ln Z\right]\right]$$

- Foreign goods sold in the Home country:

$$\tilde{P}(F) = \frac{\theta\kappa}{\theta-1}E\left[\frac{(\mu)^{1-\gamma^*} (\mu^*)^{\gamma^*}}{Z^*}\right]$$

$$\tilde{P}(F) = \frac{\theta\kappa}{\theta-1}\exp\left[\frac{1}{2}Var\left[(1-\gamma^*)\ln\mu + \gamma^*\ln\mu^* - \ln Z^*\right]\right]$$

- Prices reflect the volatility of money supply in the invoicing currency, scaled by the firm's productivity.

# OPTIMAL POLICY :
# TECHNICAL STEPS

# Utility

- We can show $E(I) = E(I^*) = \bar{I}$.

- Welfare is driven by expected log consumption:

$$E(\ln C) = E\left(\ln \frac{\mu}{P}\right)$$

$$E(\ln C) = E(\ln \mu) - E \ln\left(\left[E\frac{\mu}{Z}\right]^{0.5}\left[(\mathcal{E})^{\gamma^*} E\frac{\mu^*}{Z^*}(\mathcal{E})^{1-\gamma^*}\right]^{0.5}\frac{2\theta\kappa}{\theta-1}\right)$$

$$E(\ln C) = \sum_k \pi_k \ln(\mu_k) - \frac{1}{2}\ln\sum_k \pi_k \frac{\mu_k}{Z_k}$$

$$\qquad - \frac{\gamma^*}{2}\sum_k \pi_k\left(\ln(\mu_k) - \ln(\mu_k^*)\right)$$

$$\qquad - \frac{1}{2}\ln\left(\sum_k \pi_k \frac{(\mu_k)^{1-\gamma^*}(\mu_k^*)^{\gamma^*}}{Z_k^*}\right) - \ln\frac{2\theta\kappa}{\theta-1}$$

# Optimality condition

- The first order condition with respect to $\mu_k$ is:

$$
0 \;=\; \frac{\pi_k}{\mu_k} \left[ 1 - \frac{1}{2} \frac{\frac{\mu_k}{Z_k}}{E\frac{\mu}{Z}} - \frac{\gamma^*}{2} - \frac{1-\gamma^*}{2} \frac{\frac{(\mu_k)^{1-\gamma^*}\left(\mu_k^*\right)^{\gamma^*}}{Z_k^*}}{E\frac{(\mu)^{1-\gamma^*}(\mu^*)^{\gamma^*}}{Z^*}} \right]
$$

- We take a log approximation around
  $\mu_k = \mu_k^* = Z_k = Z_k^* = \mu_0 = Z_0 = 1$:

$$
\left[ 1 + (1-\gamma^*)^2 \right] \ln\left(\mu_k\right) \;=\; \ln\left(Z_k\right) + (1-\gamma^*)\left[\ln\left(Z_k^*\right) - \gamma^* \ln\left(\mu_k^*\right)\right]
$$

- Similar optimization in the Foreign country:

$$
\left[ 1 + (1-\gamma)^2 \right] \ln\left(\mu_k^*\right) \;=\; \ln\left(Z_k^*\right) + (1-\gamma)\left[\ln\left(Z_k\right) - \gamma \ln\left(\mu_k\right)\right]
$$

- Symmetric full exchange rate pass-through (PCP, $\gamma = \gamma^* = 1$):

$$\ln(\mu_k) \;=\; \ln(Z_k) \qquad ; \qquad \ln(\mu_k^*) = \ln(Z_k^*)$$

- Symmetric absence of exchange rate pass-through (LCP, $\gamma = \gamma^* = 0$):

$$\ln(\mu_k) \;=\; \ln(\mu_k^*) = \frac{1}{2}\left[\ln(Z_k) + \ln(Z_k^*)\right]$$

- Asymmetric dominance of Home currency (Dominant CP, $\gamma = 1$ and $\gamma^* = 0$):

$$\ln(\mu_k) \;=\; \frac{1}{2}\left[\ln(Z_k) + \ln(Z_k^*)\right] \qquad ; \qquad \ln(\mu_k^*) = \ln(Z_k^*)$$

# OPTIMAL POLICY :
## VISUAL ILLUSTRATION

# Expected utility

- Home utility (deviation from the flexible price allocation) reflects the level at which prices of goods sold in the Home country are set:

$$E\left(\ln C\right) \propto -\frac{1}{2}Var\left[\ln\left(\frac{\mu}{Z}\right)\right] - \frac{1}{2}Var\left[\ln\left(\frac{(\mu)^{1-\gamma^*}(\mu^*)^{\gamma^*}}{Z^*}\right)\right]$$
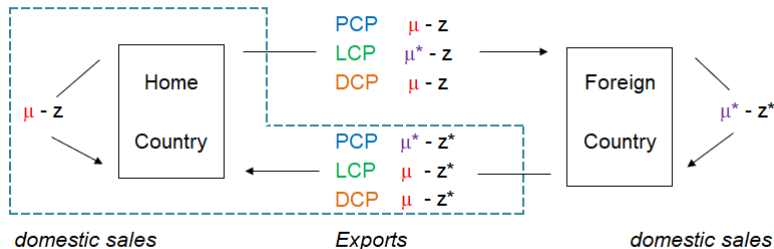
- Similarly for the Foreign utility:

$$E\left(\ln C^*\right) \propto -\frac{1}{2}Var\left[\ln\left(\frac{(\mu)^{\gamma}(\mu^*)^{1-\gamma}}{Z}\right)\right] - \frac{1}{2}Var\left[\ln\left(\frac{\mu^*}{Z^*}\right)\right]$$

- Stabilize the margin of producers *selling* in the country, regardless whether they are local or foreign firms.

- Depends on pass-through and $\gamma$ and $\gamma^*$.

- The Home policy makers cares about the dotted box. Where $\mu$ enters depends on pass-through.
- Only affects imports under LCP or DCP (Home currency used in all trade). Affects exports under PCP and LCP, but not taken into account.



The letters denote the volatility of the money / productivity ratio relevant to the corresponding consumption

| | PCP | $\mu$ - z |
| | LCP | $\mu^*$ - z |
| | DCP | $\mu$ - z |

Home Country

$\mu$ - z

| | PCP | $\mu^*$ - z* |
| | LCP | $\mu$ - z* |
| | DCP | $\mu$ - z* |

Foreign Country

$\mu^*$ - z*

*domestic sales*　　　　*Exports*　　　　*domestic sales*

PCP: $\gamma = \gamma^* = 1$, LCP: $\gamma = \gamma^* = 0$, DCP: $\gamma = 1$, $\gamma* = 0$

# Flexible prices and PCP

- Flexible prices, productivity shocks affect prices, and level and composition of demand.
  - *Level* : 10% increase in home productivity lowers Home good price by 10%. Raises consumption increases by 5% in both countries (Home goods are half the basket).
  - *Composition*: relative price of Home goods falls, demand switches towards them.
- Optimal PCP policy: $\mu = Z$ and $\mu^* = Z^*$. With a 10% increase in Home productivity:
  - Home currency depreciates by 10%, Home import prices increase by 10%, Foreign import prices fall by 10%.
  - Efficient *level*: real balances increase by 5% in Home (10% nominal balance, minus 10% raise import prices) and Foreign (10% decrease import prices).
  - Efficient *composition*: relative price of Home goods falls.

- Optimal LCP policy: $\ln(\mu) = \ln(\mu^*) = 0.5[\ln(Z) + \ln(Z^*)]$. With a 10% increase in Home productivity:
  - Money stance raises by 5% everywhere, no prices change.
  - Efficient *level*: real balances increase by 5% in Home and Foreign.
  - Inefficient *composition*: no change in relative prices.
- Optimal DCP (all invoicing in Home currency):
  $\ln(\mu) = 0.5[\ln(Z) + \ln(Z^*)]$ and $\mu^* = Z^*$. With a 10% increase in Home productivity:
  - Money stance raises by 5% at Home. Only Foreign import price change (reduction by 5%).
  - Inefficient *level*: real balances increase by 5% in Home, but only 2.5% in Foreign.
  - Inefficient *composition*: limited change in relative prices.

# Should we cooperate?

- Cooperation: choose $\mu$ and $\mu^*$ to maximize $0.5 \left( EU + EU^* \right)$, not just $EU$ or $EU^*$.
- Useful only under two conditions:
  - Home policy affects Foreign country, and conversely. Spillover not taken into account with self-centered policy.
  - Spillover effect differs from the domestic impact of policy.
- No need to cooperate under PCP (spillover identical to domestic effect) or LCP (no spillover).
- Useful under DCP: spillover differs from domestic, Home should react more to its shocks:

$$\ln \left( \mu \right) = \frac{2}{3} \ln \left( Z \right) + \frac{1}{3} \ln \left( Z^* \right) \qquad ; \qquad \mu^* = Z^*$$

- With identical consumption baskets in both countries, efficient demand *level* is the same: $C = C^*$.
- Under LCP, policy has no impact on the demand *composition* and focuses on the efficient *level*.
  - $C = C^*$ achieved by $\mu = \mu^*$.
  - No movement in exchange rate: if the exchange rate is useless, don't move it.
- If Home consumption is tilted towards Home goods (domestic bias, or with non-traded goods), $C$ should increase more than $C^*$ when Home productivity increases.
  - Under LCP, requires a larger monetary expansion in Home: $\mu > \mu^*$.
  - Home currency depreciates. The exchange rate has no impact on prices, but should not restrict policy: if the exchange rate is useless, ignore it and don't constrain policy through it.

- Stochastic model to compute optimal policy rules in response to shocks.
- Prices are set, but in a forward-looking way.
  - Price premium: when costs are volatile, firms set higher prices.
- Ideal stabilization: keep costs constant, so firms don't even want to change prices.
- Feasible in closed economy, but not (necessarily) in open economy.
  - Depends on ability to affect prices via the exchange rate (pass-through).
  - Policy care about domestic and foreign firms selling in the country.
- Cooperation can help, but under specific conditions.

EXTRA SLIDES : GRAPHICAL ILLUSTRATION

- Aggregate demand (AD) is the monetary stance:

$$C \ = \ \frac{\mu}{p}$$

- Aggregage supply (AS) is the market clearing + technology:

$$C = Zl$$

- Natural rate (NR) is the flexible price allocation of labor:

$$l^{\text{flex}} = \frac{\theta - 1}{\theta \kappa} = \bar{l}$$

# Graphical relation

- NR gives labor $\bar{l}$, AS gives $C$, and AD gives $P$.



$\mu, Z$ exogenous
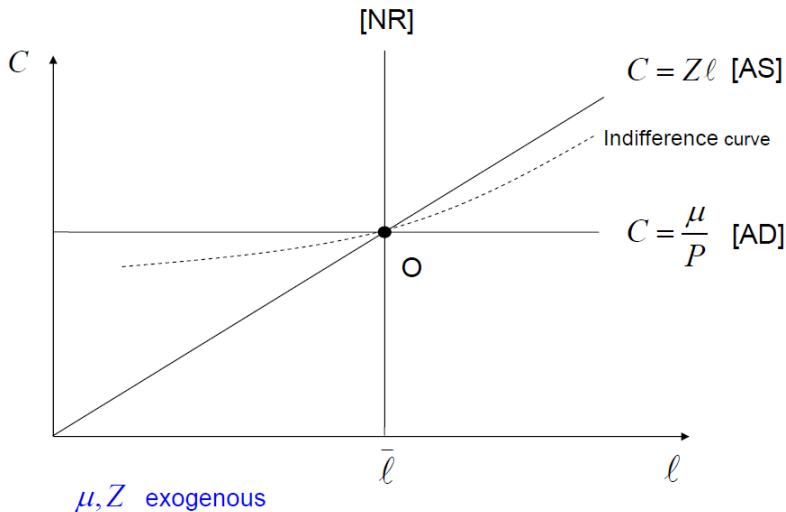
# Impact of productivity shocks

- Higher productivity $Z$ reduces firms' costs.
  - AS rotates up to the left.
- Flexible price: productivity reduces prices.
  - NR: labor unchanged at $\bar{l}$.
  - AS - NR intersection gives higher consumption, AD gives the price level.
- If prices cannot adjust, NR becomes irrelevant (firms are not at the optimal price).
  - If monetary policy does nothing (unchanged $\mu$), AD does not move. Equilibrium at AS - AD intersection, employment is inefficiently low.
  - Increase $\mu$ to move AD up. Equilibrium at AS - AD intersection, higher consumption and unchanged labor.
  - Monetary policy replicates the flexible price allocation.

- Higher productivity rotates AS. Under sticky prices, AD is unchanged.
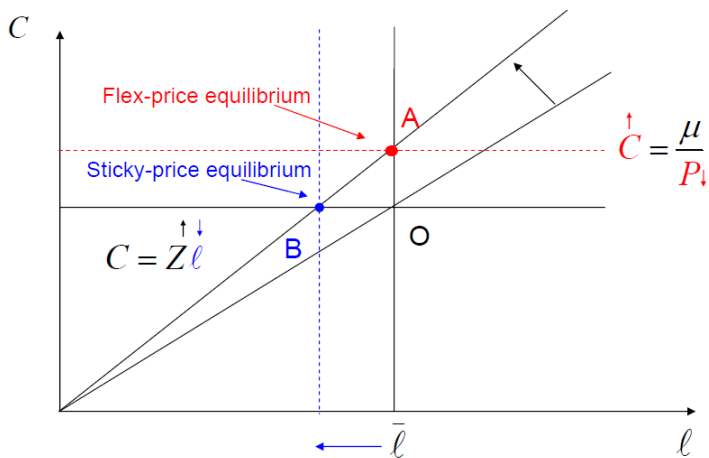
- Monetary expansion increases AD and delivers optimal allocation.
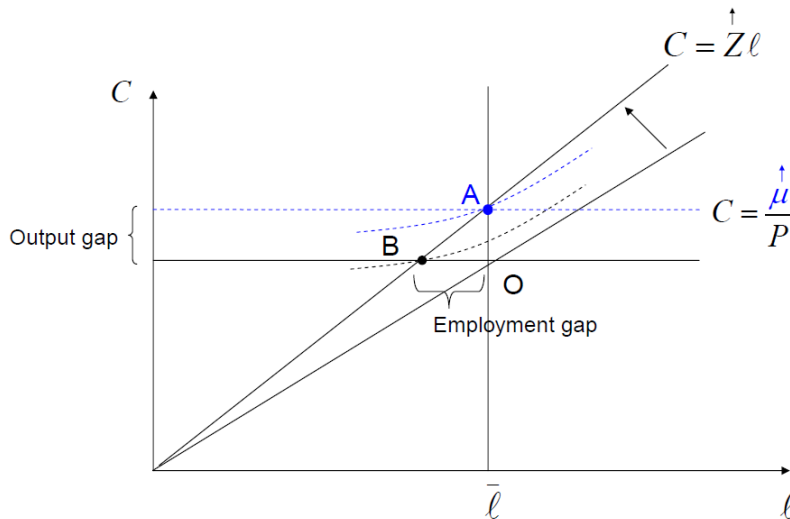
# Open economy figure

- AD and NR relations identical to the closed economy case.
- AS reflects the terms-of-trade. A given amount of home labor gives more home consumption when:
  - Productivity is high
  - Home goods fetch a higher price than foreign goods (terms of trade $\tau$ are favorable):

$$C = Zl\tau \quad ; \quad \tau = \left[\frac{P}{2}\left(\frac{1}{P(H)} + \frac{1}{\mathcal{E}P^*(H)}\right)\right]^{-1}$$

$$C^* = Z^*l^*\tau^* \quad ; \quad \tau^* = \left[\frac{P^*}{2}\left(\frac{\mathcal{E}}{P(F)} + \frac{1}{P^*(F)}\right)\right]^{-1}$$

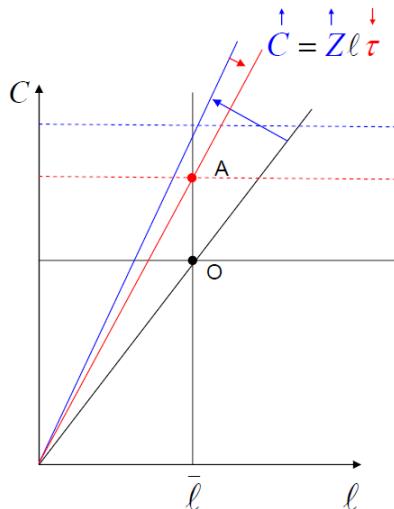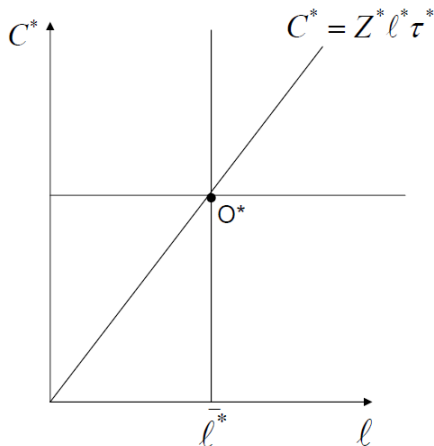# Open economy diagram
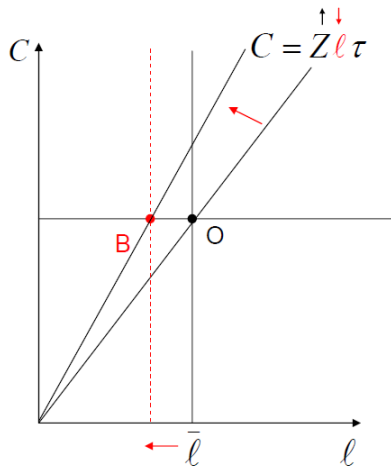
- AS now includes terms-of-trade.
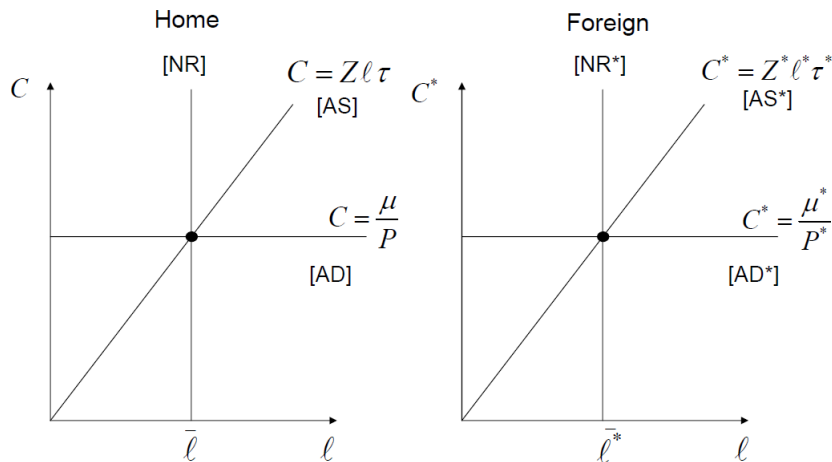
# Productivity under flexible prices

- Increase in Home productivity lowers the price of Home goods. Lower $\tau$ and higher $\tau^*$.



$$C = Z \ell \tau$$

$$C^* = Z^* \ell^* \tau^*$$
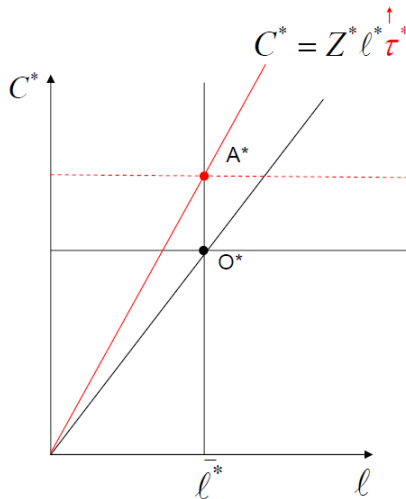
# Productivity under sticky prices

- Only monetary policy can change the terms-of-trade. Higher productivity lowers Home output.

- Monetary expansion lowers $\tau$ and raises $\tau^*$. Replicates the flexible price allocation.