Macroeconomics A; EI056

Short problems

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1 Solow model in discrete time

1.1 Capital dynamics

Question: Consider the model in discrete time. The production function is:

$$Y_t = (K_t)^{\alpha} (A_t L_t)^{1-\alpha} \Rightarrow y_t = (k_t)^{\alpha}$$

where $y_t$and k_t are scaled by effective labor $A_t L_t$. A fraction s_K of output is saved, so the output dynamics are:

$$K_{t+1} - K_t = s_K Y_t - \delta K_t$$

Labor grows at a rate n and productivity at a rate g (that is $L_{t+1} = (1+n) L_t$ and $A_{t+1} = (1+g) A_t$).

Show that the dynamics of scaled capital are:

$$k_{t+1} - k_t = \frac{1}{(1+n)(1+g)} s_K y_t - \frac{(1+n)(1+g) - (1-\delta)}{(1+n)(1+g)} k_t$$

1.2 Steady state

Question: Show that in the steady state:

$$k^* = \left[\frac{s_K}{(1+n)(1+g) - (1-\delta)} \right]^{\frac{1}{1-\alpha}}$$

$$y^* = \left[\frac{s_K}{(1+n)(1+g) - (1-\delta)} \right]^{\frac{\alpha}{1-\alpha}}$$

1.3 Approximation

Question: Show that a linear expansion of the capital dynamics around the steady state implies:

$$\hat{k}_{t+1} - \hat{k}_t = -\frac{(1+n)(1+g)-(1-\delta)}{(1+n)(1+g)}(1-\alpha)\hat{k}_t$$

where $\hat{k}_t = ln(k_t) - ln(k^*)$. You may find useful to use the fact that $k_t = exp(ln(k_t))$.

What can you say about the dynamics of capital? How is the speed of movement affected by α, n, g, δ and s_K?

2 Endogenous growth

2.1 Dynamics of capital and output

Question: Consider the Solow model where labor is constant ($L_t = 1$) and technology is affected by capital:

$$A_t = A_t^{exog} K_t^{\eta}$$

In the standard model we assume $\eta = 0$. Consider that the exogenous component of productivity is constant $A_t^{exog} = 1$. Capital accumulation is as in the previous section:

$$K_{t+1} - K_t = s_K Y_t - \delta K_t$$

Show that the growth rates of capital and output are (α is the weight of capital in the production function, as in the previous question):

$$\begin{aligned} \frac{K_{t+1}}{K_t} &= s_K (K_t)^{(1-\alpha)(\eta-1)} + (1-\delta) \\ \frac{Y_{t+1}}{Y_t} &= \left(\frac{K_{t+1}}{K_t}\right)^{(1-\alpha)(\eta-1)} \end{aligned}$$

2.2 Impact of savings

Question: How does the saving rate s_k affects the growth rate in the long run (along a steady growth path)?

Contrast the general results with the standard Solow model.