# Macroeconomics A: Review Session VII

RBC and NK Models

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# Outline

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# The Role of Investment

- The basic point of RBC is that future capital is chosen (at $t$) before the future state of the economy is revealed ($t + 1$)
- This reflects real world behavior
  - There are delays between investment (say ordering machines or building factories) and production
- Therefore, expectations about future states influence investment decisions today
  - Recall 'business sentiment' as a driver of investment in the IS-TR model
- In the RBC model, capital is a state variable and consumption is a control variable

# The Role of Shocks

- Mathematically, the expectations operator is a weighted average: the value of a given state multiplied by its probability
  - We assume agents solve expected value of future periods
- We generate different states in models through 'shocks'
- Normally, we denote 'shocks' as $\varepsilon_t$
  - To make life easier, we imagine that shocks have a distribution centered at 0
  - The shocks represent shifts away from steady state values
  - Shocks can be persistent, but converge to 0 over time

$$s_t = \rho s_{t-1} + \varepsilon_t \quad \text{where} \quad -1 < \rho < 1$$

Question: Why is it helpful to have $\mathbb{E}[\varepsilon_t] = 0$? '

# Eigenvalues

- $\lambda$ is an eigenvalue of $\mathbf{A}$ and $\mathbf{x}$ is an eigenvector if they satisfy

$$\mathbf{Ax} = \lambda\mathbf{x}$$

- To find the eigenvalues, we can solve

$$(\mathbf{A} - \lambda\mathbf{I})\mathbf{x} = 0$$

- Let's assume that $\mathbf{x}$ is non-zero and solve $\det(\mathbf{A} - \lambda\mathbf{I}) = 0$

$$\text{For} \quad \mathbf{A} = \begin{bmatrix} 2 & 4 \\ 1 & -1 \end{bmatrix} \quad \mathbf{A} - \lambda\mathbf{I} = \begin{bmatrix} 2-\lambda & 4 \\ 1 & -1-\lambda \end{bmatrix}$$

$$\begin{aligned} \det(\mathbf{A} - \lambda\mathbf{I}) = (2-\lambda)(-1-\lambda) - 4 &= 0 \\ \lambda^2 - \lambda - 6 &= 0 \\ (\lambda - 3)(\lambda + 2) &= 0 \end{aligned}$$

- Therefore we have $\lambda_1 = 3$ and $\lambda_2 = -2$ for the eigenvalues

# Eigenvectors

- To solve the eigenvectors, we can now use the eigenvalues

Starting with $\lambda = 3$ $\quad \begin{bmatrix} 2-3 & 4 \\ 1 & -1-3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -1 & 4 \\ 1 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$

$$\begin{aligned} -x_1 + 4x_2 &= 0 \\ x_1 - 4x_2 &= 0 \end{aligned} \qquad \Longrightarrow \qquad \mathbf{v}_1 = a \begin{bmatrix} 4 \\ 1 \end{bmatrix}$$

- Similarly, solving for the case where $\lambda = -2$

$$\begin{aligned} x_1 + \phantom{4}x_2 &= 0 \\ 4x_1 + 4x_2 &= 0 \end{aligned} \qquad \Longrightarrow \qquad \mathbf{v}_2 = a \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

- For now, we will set $a = 1$
- Much of the following is based on Krzysztof Makarski's slides

# Eigendecomposition

- We can take these elements an combine them is a very useful way
- First, let's define three matrices

$$\Lambda = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & -2 \end{bmatrix} \qquad \mathbf{C} = \begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_2 \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ 1 & -1 \end{bmatrix}$$

$$\mathbf{C}^{-1} = \frac{1}{\det(\mathbf{C})} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \begin{bmatrix} 0.2 & 0.2 \\ 0.2 & -0.8 \end{bmatrix}$$

- Some tedious calculations will show you that

$$\mathbf{C}\Lambda\mathbf{C}^{-1} = \mathbf{A}$$

- Notice that $\mathbf{A}^n = \mathbf{C}\Lambda^n\mathbf{C}^{-1}$

# Simple Example

- Let's start with the difference equation

$$x_t = \alpha x_{t-1} + \beta$$

- We can write this recursively as

$$x_t = \alpha^2 x_{t-2} + (1+\alpha)\beta \quad \text{or generally} \quad x_t = \alpha^n x_{t-n} + \sum_{i=0}^{n-1} \alpha^i \beta$$

- Using the identity $\sum_{i=0}^{n-1} a^i = \frac{1-a^n}{1-a}$ and setting $n = t$

$$x_t = \alpha^t x_0 + \beta \frac{1-\alpha^t}{1-\alpha}$$

- The equation is stable when $|\alpha| < 1$, in this case

$$\lim_{t\to\infty} x_t = \frac{\beta}{1-\alpha}$$

- This is equal to the steady state value if you check!!

# System of Difference Equations

- Now we can look at a more complicated example

$$\mathbf{x}_t = \mathbf{A}\mathbf{x}_{t-1} + \mathbf{B} \quad \text{where} \quad \mathbf{x}_t = \begin{bmatrix} x_{1,t} \\ x_{2,t} \\ \cdots \\ x_{n,t} \end{bmatrix}$$

- Here $\mathbf{A}$ is a $n \times n$ matrix and $\mathbf{B}$ is a $n \times 1$ vector
- The steady state is given by $\mathbf{x} = [\mathbf{I} - \mathbf{A}]^{-1}\mathbf{B}$
- If we can decompose $\mathbf{A}$ as $\mathbf{A} = \mathbf{C}\Lambda\mathbf{C}^{-1}$, then

$$\mathbf{x}_t = \mathbf{C}\Lambda\mathbf{C}^{-1}\mathbf{x}_{t-1} + \mathbf{B}$$

$$\mathbf{C}^{-1}\mathbf{x}_t = \Lambda\mathbf{C}^{-1}\mathbf{x}_{t-1} + \mathbf{C}^{-1}\mathbf{B}$$

- Let's define $\bar{\mathbf{x}}_t = \mathbf{C}^{-1}\mathbf{x}_t$ and $\bar{\mathbf{B}} = \mathbf{C}^{-1}\mathbf{B}$ so that

$$\bar{\mathbf{x}}_t = \Lambda\bar{\mathbf{x}}_{t-1} + \bar{\mathbf{B}}$$

# System Stability

- Solving the system recursively (recalling $\Lambda$ is diagonal), we get

$$\bar{\mathbf{x}}_t = \Lambda^n \bar{\mathbf{x}}_{t-n} + \sum_{i=0}^{n-1} \Lambda^i \bar{\mathbf{B}}$$

$$\bar{\mathbf{x}}_t = \Lambda^t \bar{\mathbf{x}}_0 + [\mathbf{I} - \Lambda^t][\mathbf{I} - \Lambda]^{-1} \bar{\mathbf{B}}$$

- If all $|\lambda| < 1$ then the system is **stable**
- If all $|\lambda| > 1$ then the system is **unstable**
- System is **saddle-path stable** if at least one $|\lambda| < 1$
- In many cases, saddle-path solutions are desirable
  - Given initial state variables, the agent can only choose one set of control variables and follows unique path to the steady state

# Blanchard Kahn Method

- Taking what we saw before we can write

$$\mathbf{X}\begin{bmatrix} x_{t+1} \\ \mathbb{E}[y_{t+1}] \end{bmatrix} = \mathbf{Y}\begin{bmatrix} x_t \\ y_t \end{bmatrix} + \mathbf{Z}\varepsilon_t \quad \text{where} \quad \mathbb{E}[\varepsilon_t] = 0 \text{ and } \mathbb{V}[\varepsilon_t] > 0$$

- $x_t$ and $y$ are vectors of $n$ state and $m$ control variables, respectively
- Dividing through by **X** we have

$$\begin{bmatrix} x_{t+1} \\ \mathbb{E}[y_{t+1}] \end{bmatrix} = \mathbf{A}\begin{bmatrix} x_t \\ y_t \end{bmatrix} + \mathbf{B}\varepsilon_t \quad \text{where} \quad \mathbf{A} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$$

- Here $\mathbf{A} = \mathbf{X}^{-1}\mathbf{Y} = \mathbf{C}\Lambda\mathbf{C}^{-1}$ and $\mathbf{B} = \mathbf{X}^{-1}\mathbf{Z}$
- The eigenvalues in the matrix $\Lambda$ can be arranged along the diagonal

$$|\lambda_1| < |\lambda_2| < ... < |\lambda_{n+m}|$$

# Stability

- The model has unique solution if the number of unstable eigenvalues (greater than 1 in absolute value) of the system equals the number ($m$) of forward-looking control variables
- In this case there is one solution, the equilibrium path is unique and the system exhibits saddle-path stability
- Question: How do we determine the initial values of control variables when there is a saddle path?

- Before moving to the next part, lets define

$$\begin{bmatrix} \bar{x}_t \\ \bar{y}_t \end{bmatrix} = \mathbf{C}^{-1} \begin{bmatrix} x_t \\ y_t \end{bmatrix} \quad \text{where} \quad \mathbf{C}^{-1} = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}$$

$$\bar{\mathbf{B}} = \mathbf{C}^{-1}\mathbf{B}$$

# Unstable Eigenvalues

- The submatrix $\Lambda_2$ contains all the unstable eigenvalues
- In our newly defined system, $\bar{y}_t$ is independent of $\bar{x}_t$

$$\begin{bmatrix} \bar{x}_{t+1} \\ \mathbb{E}[\bar{y}_{t+1}] \end{bmatrix} = \begin{bmatrix} \Lambda_1 & 0 \\ 0 & \Lambda_2 \end{bmatrix} \begin{bmatrix} \bar{x}_t \\ \bar{y}_t \end{bmatrix} + \begin{bmatrix} \bar{B}_1 \\ \bar{B}_2 \end{bmatrix} \varepsilon_t \implies \mathbb{E}[\bar{y}_{t+1}] = \Lambda_2 \bar{y}_t + \bar{B}_2 \varepsilon_t$$

- Solving for $\bar{y}_t$

$$\bar{y}_t = \Lambda_2^{-1} \mathbb{E}[\bar{y}_{t+1}] - \Lambda_2^{-1} \bar{B}_2 \varepsilon_t$$

- Iterating forward

$$\bar{y}_t = \Lambda_2^{-2} \mathbb{E}[\bar{y}_{t+2} - \bar{B}_2 \varepsilon_{t+1}] - \Lambda_2^{-1} \bar{B}_2 \varepsilon_t$$

- Recognizing that $\mathbb{E}[\varepsilon_{t+1}] = 0$ and that $\mathbb{E}[\varepsilon_{t+j}] = 0$ in general

$$\bar{y}_t = \Lambda_2^{-j} \mathbb{E}[\bar{y}_{t+j}] - \Lambda_2^{-1} \bar{B}_2 \varepsilon_t$$

# Solving for State and Control Variables

- Recall that $\Lambda_2$ is a diagonal matrix where all $\lambda > 1$, so at the limit

$$\lim_{j\to\infty} \Lambda_2^{-j}\mathbb{E}[\bar{y}_{t+j}] = 0$$

- Using this result, we can solve the control variables

$$\bar{y}_t = -\Lambda_2^{-1}\bar{B}_2\varepsilon_t \quad \text{and} \quad \bar{y}_t = C_{21}x_t + C_{22}y_t$$

$$\implies y_t = -C_{22}^{-1}\Lambda_2^{-1}\bar{B}_2\varepsilon_t - C_{22}^{-1}C_{21}x_t$$

- Question: Would this hold if any $\lambda$ in $\Lambda_2 < 1$?
- Having $y$ in terms of $x$, we can now find the law of motion for $x_t$

$$x_{t+1} = A_{11}x_t + A_{12}y_t + B_1\varepsilon_t$$

$$x_{t+1} = A_{11}x_t + A_{12}(-C_{22}^{-1}\Lambda_2^{-1}\bar{B}_2\varepsilon_t - C_{22}^{-1}C_{21}x_t) + B_1\varepsilon_t$$

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# Incorporating Markups in Macro Models

- Many of the models we have looked at consider perfect competition
- However, it is clear that firms are profitable
- When the costs to consumers is greater than the total cost of production/distribution, firms earn a markup
- In fact, markups vary over time and contribute to economic dynamics
- Not completely straightforward to model

# Estimated Markups in the United States

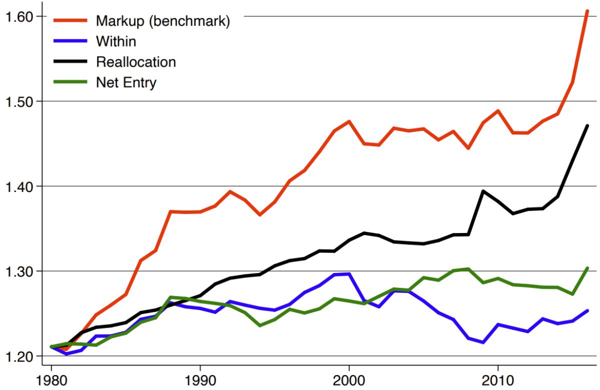


Figure 4: Decomposition of markup growth at the firm level.

Source: De Loecker, Eeckhout, and Unger (2019)

# CES Aggregator

- Often, macro models capture markups using a CES aggregator
- CES stands for ‘constant elasticity of substitution’

$$Q = A\left[\sum_{i}^{N} a_i X_i^{\rho}\right]^{\frac{1}{\rho}} \quad \text{where} \quad \rho = \frac{\sigma - 1}{\sigma} \quad \text{and} \quad \sigma > 1$$

- Here, $Q$ = output; $A$ = aggregate productivity; $a_i$ = input shares; $X_i$ = factors of production; $\sigma$ = elasticity of substitution
- Monopolistic competition: markups are not specific to sectors or products, but economy-wide (identical firms are assumed $a_i = 1/N$)
- Possible to capture dispersion in markups across sectors or products using a nested CES aggregator

# Profit Maximization (No Capital)

- Economy is populated by identical intermediate producers
- Each makes one good and has the same budget constraint

$$y_{it} = \ell_{it} \qquad \text{s.t.} \qquad \Pi_{it} = p_{it} y_{it} - w_t \ell_{it}$$

- A final retailer combines these goods into final output

$$Y_t = \left[ \int_i y_{it}^{\rho} di \right]^{\frac{1}{\rho}}$$

- Final retailer cannot easily substitute goods
- Difficulty changing between varieties ($\rho \to 0$) leads to markup
- Each individual firm owner $j \in i$ maximizes her profits by solving

$$\max_{y_{jt}} \quad \mathcal{L}_{jt} = P_t Y_t - \int_i p_{it} y_{it} di$$

# Finding the Markup

- We can solve for the markup by the firm ($p_{jt}$) as follows

$$\max_{y_{jt}} \quad \mathcal{L}_{jt} = P_t \left[ \int_i y_{it}^{\rho} di \right]^{\frac{1}{\rho}} - \int_i p_{it} y_{it} di$$

$$\frac{\partial \mathcal{L}_{jt}}{\partial y_{jt}} = 0 \implies P_t \left[ \int_i y_{it}^{\rho} di \right]^{\frac{1-\rho}{\rho}} y_{jt}^{\rho - 1} = p_{jt}$$

$$\left( P_t \left[ \int_i y_{it}^{\rho} di \right]^{\frac{1-\rho}{\rho}} y_{jt}^{\rho - 1} \right)^{\frac{1}{1-\rho}} = p_{jt}^{\frac{1}{1-\rho}}$$

$$p_{jt} = P_t \left( \frac{Y_t}{y_{jt}} \right)^{1-\rho}$$

- The problem from the class slides is almost identical

# Markup Over Wages

- Going back to the firm's budget constraint

$$\Pi_{it} = p_{it} y_{it} - w_t \ell_{it}$$

- Let's use the solution from the previous slide

$$\begin{aligned} \Pi_{it} &= P_t \left( \frac{Y_t}{y_{it}} \right)^{1-\rho} y_{it} - w_t \ell_{it} \\ &= P_t Y_t^{1-\rho} y_{it}^{\rho} - w_t \ell_{it} \end{aligned}$$

- The wage is set by the marginal product of labor, for each firm

$$\frac{\partial \Pi_{it}}{\partial \ell_{it}} = 0 \implies \rho P_t Y_t^{1-\rho} y_{it}^{\rho-1} = w_t$$

- Since $y_{it} = \ell_{it}$

$$\rho P_t Y_t^{1-\rho} y_{it}^{\rho} = w_t \ell_{it}$$

# Aggregation

- Now we can aggregate both sides so that

$$\rho P_t Y_t^{1-\rho} \int_i y_{it}^{\rho} di = w_t \int_i \ell_{it} di$$

- The sum of all labor hired by firms equals aggregate labor

$$L_t = \int_i \ell_{it} di \qquad Y_t^{\rho} = \int_i y_{it}^{\rho} di$$

- Solving this, labor gets paid less than under perfect competition :(

$$\rho = \frac{w_t L_t}{P_t Y_t} < 1$$

- Under perfect competition, all output goes to factors of production

# Solving the Price Index

- Rewriting the previous expression gives

$$\left(\frac{p_{it}}{P_t}\right)^{\frac{1}{1-\rho}} y_{it} = Y_t$$

- Raising both sides to $\rho$

$$y_{it}^{\rho} = \left(\frac{p_{it}}{P_t}\right)^{1-\sigma} Y_t^{\rho}$$

- Now integrating both sides

$$Y^{\rho} P_t^{1-\sigma} = Y_t^{\rho} \int_i p_{it}^{1-\sigma} di \quad \Longrightarrow \quad P_t = \left[\int_i p_{it}^{1-\sigma} di\right]^{\frac{1}{1-\sigma}}$$

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# Basic Assumptions of NKPC Model

- Monopolistic competition in the goods market
- Staggered price setting
- Perfectly competitive labor market, flexible wages
- Closed economy
- No capital

# Calvo Pricing

- We define a probability $1-\omega$ a firm can reset its price in any period

$$Pr(t) = \omega^t (1-\omega)$$

- The average price duration is given by $\mathcal{T} = \mathbb{E}[t\, Pr(t)]$

$$\mathcal{T} = (1-\omega) \sum_{t=0}^{\infty} t\, \omega^t = \frac{\omega}{1-\omega}$$

- ECB: average price lasts 13 months in EU, 7 months in US (source)
- With discounting, we can describe price setting behavior as

$$\widetilde{P}_t^* = (1-\beta\omega) \sum_{k=0}^{\infty} (\beta\omega)^k \mathbb{E}_t[\widetilde{P}_{t+k}^*]$$

- Note: this requires log-linearization of firms' optimality condition
- A bit too complicated and tedious, so we skip for now

# Incorporating Markups

- We can define (note that $Y_t = L_t$)

$$\mu_t = \frac{P_t}{\Psi_t} \quad \text{where} \quad \Psi_t = \frac{w_t L_t}{Y_t} = w_t \quad \text{(real marginal cost)}$$

- With no frictions $P_t^* = \mu_t \Psi_t$, with pricing frictions

$$\widetilde{P}_t^* = (1-\beta\omega)\sum_{k=0}^{\infty}(\beta\omega)^k \mathbb{E}_t[\widetilde{\mu}_{t+k} + \widetilde{\psi}_{t+k}]$$

- We can substitute the term

$$\widetilde{P}_{t+1}^* = (1-\beta\omega)\sum_{k=1}^{\infty}(\beta\omega)^k \mathbb{E}_t[\widetilde{\mu}_{t+k} + \widetilde{\psi}_{t+k}]$$

- Into the original equation so that

$$\widetilde{P}_t^* = \beta\omega\mathbb{E}_t[\widetilde{P}_{t+1}^*] + (1-\beta\omega)(\widetilde{\mu}_t + \widetilde{\psi}_t) \tag{1}$$

# Firms Set Prices Above Rate of Inflation

- There is a set of firms $\Omega$ that cannot change their price
- We can write this as

$$\begin{aligned} P_t &= \left[\int_{i\in\Omega} p_{it}^{1-\theta} di + \int_{i\notin\Omega} p_{it}^{1-\theta} di\right]^{\frac{1}{1-\theta}} \\ &= \left[\int_{i\in\Omega} p_{it-1}^{1-\theta} di + \int_{i\notin\Omega} p_{it}^{1-\theta} di\right]^{\frac{1}{1-\theta}} \\ &= \left[\omega P_{t-1}^{1-\theta} + (1-\omega)(P^*)_t^{1-\theta}\right]^{\frac{1}{1-\theta}} \end{aligned}$$

- In the steady state $P_t = P_{t-1} = P_t^*$
- Log-linearization around the steady state gives

$$\widetilde{P}_t = \omega \widetilde{P}_{t-1} + (1-\omega)\widetilde{P}_t^* \quad \Longleftrightarrow \quad \widetilde{P}_t^* = \frac{\widetilde{P}_t - \omega \widetilde{P}_{t-1}}{1-\omega} \qquad (2)$$

- Calvo pricing implies that $\widetilde{P}_t^* > \widetilde{P}_t$ whenever $\widetilde{P}_t > \widetilde{P}_{t-1}$

# The New Keynesian Philips Curve

- Combining results from equations 1 and 2

$$\frac{\widetilde{P}_t - \omega \widetilde{P}_{t-1}}{1-\omega} = \beta\omega \frac{\mathbb{E}_t[\widetilde{P}_{t+1}] - \omega \widetilde{P}_t}{1-\omega} + (1-\beta\omega)(\widetilde{\mu}_t + \widetilde{\psi}_t)$$

- Let's define inflation as $\tilde{\pi}_t = \widetilde{P}_t - \widetilde{P}_{t-1}$
- Rearranging terms gives

$$\tilde{\pi}_t = \beta \mathbb{E}_t[\tilde{\pi}_{t+1}] + \kappa(\widetilde{\mu}_t + \widetilde{\Psi}_t - \widetilde{P}_t) \quad \text{where} \quad \kappa = \frac{(1-\omega)(1-\omega\beta)}{\omega}$$

- Inflation depends positively on real marginal cost $\Psi_t - \widetilde{P}_t$
- We can assume that there is a log-linear relation between changes and the markup and real marginal cost and output

$$\gamma \widetilde{Y}_t = \widetilde{\mu}_t + \widetilde{\Psi}_t - \widetilde{P}_t$$

$$\implies \tilde{\pi}_t = \beta \mathbb{E}_t[\tilde{\pi}_{t+1}] + \kappa\gamma \widetilde{Y}_t \quad \text{(NKPC)}$$

# Household Problem

We now solve for the IS curve, defined as

$$\widetilde{Y}_t = \mathbb{E}_t[\widetilde{Y}_{t+1}] - \frac{1}{\eta}(R_t^n - \mathbb{E}_t[\tilde{\pi}_{t+1}] - \delta)$$

Not hard to do: it is given by the Euler condition for households

$$U = \sum_{t=0}^{\infty} \beta^t \frac{C^{1-\eta}}{1-\eta} \quad \text{s.t.} \quad P_t C_t + B_t = \underbrace{w_t L_t + \Pi_t}_{Y_t} + R_{t-1}^n B_{t-1}$$

Solving the Lagrangian

$$\mathcal{L}_t = \sum_{t=0}^{\infty} \beta^t \left[ \frac{C^{1-\eta}}{1-\eta} - \lambda \left( P_t C_t + B_t = Y_t + R_{t-1}^n B_{t-1} \right) \right]$$

$$\frac{\partial \mathcal{L}_t}{\partial C_t} = 0 \implies C_t^{-\eta} = P_t \lambda_t \qquad \frac{\partial \mathcal{L}_t}{\partial B_t} = 0 \implies \lambda_t = \beta R_t^n \lambda_{t+1}$$

# Solving for the IS Curve

- From the FOCs, we get the Euler equation

$$\left(\frac{C_{t+1}}{C_t}\right)^{\eta} = \beta R_t^n \frac{P_t}{P_{t+1}}$$

- We can log-linearize this

$$\eta \widetilde{C}_{t+1} - \eta \widetilde{C}_t = \log(\beta) + \widetilde{R}_t^n - \mathbb{E}_t[\tilde{\pi}_{t+1}]$$

- Rearranging terms and taking $\widetilde{C}_t = \widetilde{Y}_t$
- We can define $\delta = -log(\beta)$

$$\widetilde{Y}_t = \mathbb{E}_t[\widetilde{Y}_{t+1}] - \frac{1}{\eta}(\widetilde{R}_t^n - \mathbb{E}_t[\tilde{\pi}_{t+1}] - \delta) \quad \text{(IS)}$$

# Taylor Rule

- Central bank pins everything down – the Taylor rule is given by

$$R_t^n = \bar{R}^n \left(\frac{\pi_t}{\pi^*}\right)^{\phi_\pi} \left(\frac{Y_t}{Y^*}\right)^{\phi_y}$$

- When log-linearized this is

$$\widetilde{R}_t^n = \phi_\pi \tilde{\pi}_t + \phi_y \widetilde{Y}_t$$

- If we put this back in the IS equation, it is only a function of $Y$ and $\pi$
- We still have to solve for the timing of the model
- Easiest way is method of undetermined coefficients

$$\widetilde{Y}_t = \Gamma_y \varepsilon_t \qquad \tilde{\pi}_t = \Gamma_\pi \varepsilon_t$$

$$\mathbb{E}_t[\widetilde{Y}_{t+1}] = \rho \Gamma_y \varepsilon_t \qquad \mathbb{E}_t[\tilde{\pi}_{t+1}] = \rho \Gamma_\pi \varepsilon_t$$

- Here $\rho$ is the expected persistence of the shock
- The NKPC and IS equation allow us to solve $\Gamma_\pi$ and $\Gamma_y$