

DSGE models for central bankers

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Central bank policymakers are faced daily with the challenge of interpreting economic data so as to be able to form judgments about the need for and direction of monetary policy changes. Ultimately, such interpretations rely on the implicit personal models which all policymakers carry in their heads. Because it is virtually impossible for the human brain to comprehend and link together appropriately the entire range of factors impinging on macroeconomic outcomes, such personal models are by necessity partial equilibrium in nature, and therefore often internally inconsistent.

Economic modellers since Tinbergen have attempted to reduce the degree of internal inconsistency involved in assessments of economic data. Sophisticated econometric techniques have been developed to improve the statistical qualities of these models, but ultimately, these mainstream simultaneous equations models have remained partial equilibrium in nature. Although applied general equilibrium models – closed systems in which all variables of interest are determined endogenously and therefore internally consistent – have been available since the pioneering efforts of Shoven and Whalley (1972), they have been largely comparative static in nature and therefore limited to such areas as the analysis of taxation or international trade. It is only recently that computing power has been sufficient for properly dynamic general equilibrium models to make their way into the mainstream of economic analysis.

Dynamic Stochastic General Equilibrium (DSGE) models are the current manifestation of this line of analysis. Unlike in the structural equations based on empirical relationships between macroeconomic variables which were the basis of previous models, the behaviour of agents in the standard DSGE models is modeled from the bottom up based on microfoundations (individual economic actors maximise their objective – be it profits or utility – given the constraints that they face). This approach thus focusses on “deep parameters” which are more robust (though not immune) to the Lucas (1976) critique – that rational agents update their expectations and change their behaviour in response to policy changes and thus that many model parameters are not invariant to policy – than those of the older structural equation models. Such models now form part of the suite of models typically used at many central banks to provide background information to monetary policymakers. Importantly in this context, policy variables are treated either as endogenous variables, determined by instrument rules that relate them to other endogenous variables, or as variables whose values are set optimally on the basis of a policy objective function that is part of the model’s specification.

Early DSGE models were strongly criticised for the unreality of their assumptions. But such models have come a long way since Kydland and Prescott (1982) set out simple “real business cycle” (RBC) models assuming perfectly competitive goods, factor and asset markets in which money played no rôle and whose evolution depended solely on exogenous technology shocks, thus eliminating the need for government intervention.

A. Basic structure

The basic structure of most DSGE models in common use today is as follows: The economy features a continuum of households, indexed by $j \in [0,1]$. Each household consumes final goods, supplies a specific type of labour to intermediate goods firms via employment agencies, saves (typically in one-period nominal government bonds), and accumulates physical capital through investment. It transforms physical capital to effective capital by choosing the capital utilisation rate, and then rents the effective capital to intermediate goods firms. Each firm in the intermediate goods sector produces a differentiated intermediate good i using capital and labour inputs according to a production function (typically Cobb-Douglas). Each household is a monopolistic supplier of specialised labour $H_t(j)$, which is combined by perfectly competitive employment

agencies into labour services H_t via an aggregation function (typically Dixit-Stiglitz). This aggregation function contains a measure of substitutability across labour varieties that translates into a (gross) markup of wages over the marginal rate of substitution between consumption and leisure. Once a continuum of intermediate goods $Y_t(i)$ indexed by $i \in [0,1]$ are produced, they are combined into a final consumption good Y_t in a perfectly competitive final-goods producing sector. Finally, the government sets public spending G_t according to some fiscal rule and also sets the nominal interest rate i_t using a monetary policy rule (typically a Taylor Rule). The model is then closed using the resource constraint that output is equal to the sum of consumption, investment, government spending and net exports (if the economy is open).

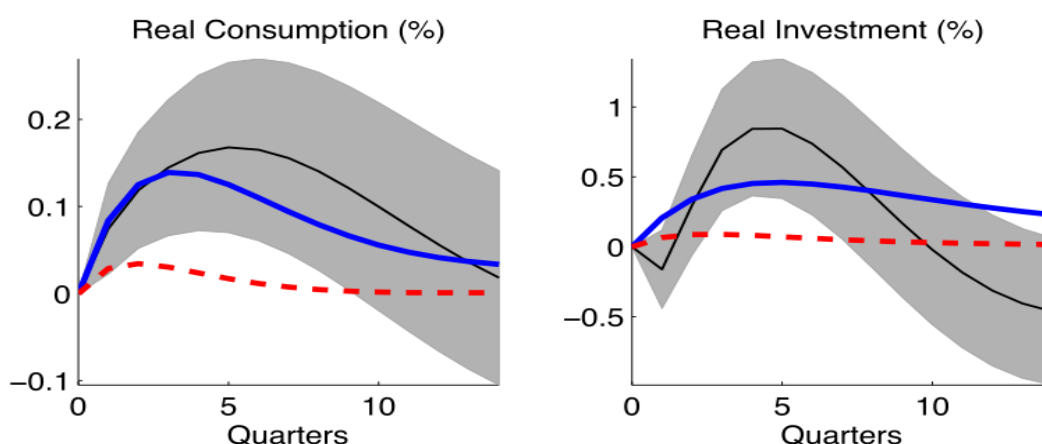
This basic structure is implemented by working out the optimisation conditions for the household and production sectors and from them deriving a model (usually but not necessarily linear in form) which can be simulated to indicate (in both numerical and graphical terms) the behaviour of the modelled economy when confronted with shocks (such as a change in monetary policy). The properties of the simulations evidently are dependent on the values of the parameters characterising the model. In the earliest DSGE models, these were “calibrated” by the researchers on the basis of various studies in the literature. More recently, the parameters have been estimated directly from the data by full information methods – a major computational undertaking given the highly nonlinear nature of the resultant likelihood function.

B. Modifications to the basic structure

Informed by important features of the macro data, DSGE modellers have modified the basic model in many directions to take better into account features of real-world economies not adequately expressed in the early versions of the DSGE model. In what follows, I concentrate on those modifications of most interest to central bank practitioners.

1. Consumption and Investment Frictions

Among the most important of these data features for central bankers is evidently the response of the economy to monetary policy. Considerable accumulated evidence shows that there is a hump-shaped response of consumption and investment (and thence output) to a monetary policy shock (see the solid black line in the graphs below, derived from VAR evidence on the US). The original structure of the DSGE models did not, however, produce such responses (red dotted line).



To produce such responses from the model, the basic model outlined was modified by including “frictions” – in the case of consumption, the addition of habit persistence; in the case of investment, the addition of adjustment costs (producing the solid blue lines in the graphs above):

With habit formation, households choose to raise consumption slowly over time in response to a policy-induced decline in the interest rate, rather than immediately as implied by the basic model, thus producing the observed hump-shaped response. Thus, the utility function which household τ maximises at any point t in time may be written (as in the canonical Smets-Wouters (2007) model)

$$U_t^\tau = \varepsilon_t^B \left(\frac{1}{1-\sigma_c} (C_t^\tau - H_t)^{1-\sigma_c} - \frac{\varepsilon_t^L}{1+\sigma_l} (\ell_t^\tau)^{1+\sigma_l} + \frac{\varepsilon_t^M}{1-\sigma_m} \left(\frac{M_t^\tau}{P_t} \right)^{1-\sigma_m} \right)$$

where H_t is the habit variable, ℓ_t^τ is the labour supplied by household τ , M_t^τ/P_t is the real balances it holds, the ε are various shocks and the σ various “deep” parameters. Habit can be viewed as either “internal” (in which case H_t is a function of C_t^τ) or “external” (in which case H_t is a function of C_t).

Similarly, when it is costly to adjust the rate of investment, households (who are the owners of the economy’s capital goods) raise investment only slowly over time, generating the familiar hump-shaped response-pattern. The introduction of variable capacity utilisation (and costs associated with such variation) reinforces this effect. Again following Smets-Wouters (2007), this gives a law of motion for capital and a budget constraint as

$$K_t(j) = (1 - \delta)_{t-1}(j) + \varepsilon_t^i \left[1 - S \left(\frac{I_t(j)}{I_{t-1}(j)} \right) \right] I_t(j)$$

$$P_t(C_{R,t} + I_t) = W_t L_{R,t} + R_t U_t K_t - P_t K_t \left[\Psi_1 (U_t - 1) + \frac{\Psi_2}{2} (U_t - 1)^2 \right]$$

where $S(\cdot)$ is the adjustment cost function (where $S(1) = 0$, $S'(1) = 0$ and $S''(\cdot) > 0$), U_t is the rate of capital utilisation and the parameters Ψ_1 and Ψ_2 are both > 0 . [The subscript R in the budget constraint relates to Ricardian households as the model allows for both Ricardian and non-Ricardian agents.] Frequently, the adjustment cost function is taken as quadratic:

$$S \left(\frac{I_t}{I_{t-1}} \right) = \frac{\gamma}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2, \quad \gamma > 0$$

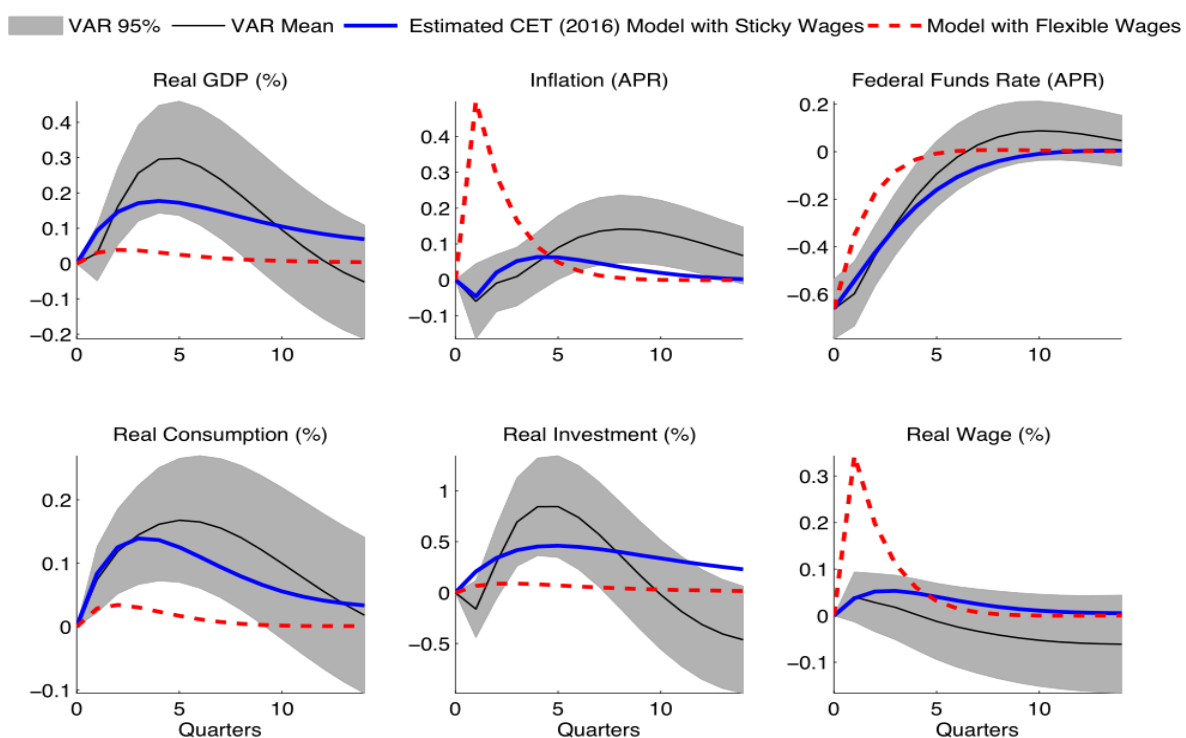
2. Sticky Prices and Wages

Given central banks’ mandates, the response of prices to demand variations is equally of interest. Markets are manifestly not perfect and (so long as inflation is not too high) suppliers typically respond to changes in demand by varying quantities rather than prices, hence real-world prices are “sticky” rather than adjusting immediately as supposed in the basic model. This fact is accommodated in the DSGE model by the device of a “Calvo devil” who plies CEOs with unstinting supplies of liquor until only a given proportion of them is able in each period to stagger to the office to change prices, the remainder drunkenly adopting some sort of indexation to past prices (known as a “Calvo Rule”). However fanciful, this device provides an elegant and

mathematically-tractable way to introduce price friction into the model. The Calvo devil is also assumed to operate in the wage-setting process, ensuring that the model takes into account the real-world practice of staggered labour contracts.

The figure below, which comes from Christiano *et al.* (2018), shows the effect of the sticky wage assumption when the economy is faced with a monetary policy shock (in the form of a reduction in the policy interest rate) in the presence and absence of wage stickiness (prices are sticky in both variants). The model succeeds in accounting for the hump-shaped rise in consumption, investment, and real GDP and the (small) rise in inflation after a policy-induced fall in the policy rate. Also, the wage stickiness introduced leads to real wages being essentially unaffected by the policy shock (solid blue line), whereas in the absence of stickiness (dotted red line), real wages are sharply and rapidly affected. It is also noteworthy that in this case, other results of the model are also adversely affected: wage (and price) stickiness is crucial in obtaining appropriate responses from the model to policy shocks. [The model in question is very similar to that of Smets and Wouters (2007)]

Figure 2: Impulse Responses to a Monetary Policy Shock — VAR vs. Model



3. New-Keynesian Phillips Curve (NKPC)

The introduction of price and wage stickiness gives rise to what is known as the New-Keynesian Phillips Curve, in which current inflation is determined by current economic activities and expectations of future inflation, the latter based on the optimising behavior of individual firms, and the former represented by the output gap between actual and potential output or real marginal cost. This arises because, under Calvo, profit-maximising firms choose prices not only on the basis of current demand and marginal costs, but also on the basis of expected future marginal costs over the (Calvo) period during which they do not expect to be able to change their prices. However, the purely forward-looking inflation model is unable to explain the inflation inertia which is observed in empirical studies, and DSGE modellers have therefore introduced backward-looking agents who set their prices according to the average behaviour of the economy in the previous period, again expressed in the use of a Calvo Rule for non-optimising firms, which produces the “hybrid” NKPC nowadays used in most DSGEs, correspondingly known as “New-Keynesian” or NK DSGEs:

$$\hat{\Pi}_t = \beta_1 E_t \hat{\Pi}_{t+1} + \beta_2 \hat{\Pi}_{t-1} + \kappa \widehat{mc}_t$$

where the $\hat{\cdot}$ indicates (log-) deviation from the steady-state and mc_t is either real marginal cost or the output gap, depending on the modeller's preference.

Although there has been much controversy about the NKPC, Adams and Padula (2011) showed that the above formulation is successful empirically, with the lagged inflation term entering highly significantly in a regression using US data, as shown in the table below (where β is the coefficient on Π_{t+1} and λ the coefficient on either marginal cost or the output gap).

Table 6: The Role of Lagged Variables

	Unit Labor Costs			Output		
β	0.445 (0.124)**	1.153 (0.183)**	0.990 (0.036)**	0.376 (0.094)**	1.089 (0.214)**	1.015 (0.039)**
λ	0.050 (0.019)**	0.097 (0.026)**	0.120 (0.060)*	0.042 (0.015)**	0.053 (0.027)	0.009 (0.042)
Π_{t-1}	0.543 (0.124)**			0.623 (0.096)**		
$E_{t-1}[\Pi_t]$		-0.165 (0.173)			-0.072 (0.198)	
rmc_{t-1}			-0.024 (0.049)			0.053 (0.043)
Observations	125	125	125	125	125	125

An interesting application of the NKPC is made by Casares and Vasquez (2018) who re-estimate a version of the Smets-Wouters (2007) model containing transactions costs for money balances using rolling windows and find that the coefficients (in particular the (slope) coefficient on marginal cost) are not stable, but in ways which are entirely plausible. They find that the estimated posterior mean of the structural parameters give the following NKPC over the first subsample (1961–1981):

$$\pi_t = 0.26\pi_{t-1} + 0.74E_t\pi_{t+1} + 0.0582mc_t + \epsilon_t^p,$$

$$\epsilon_t^p = 0.94\epsilon_{t-1}^p - 0.69\eta_{t-1}^p + \eta_t^p, std(\eta^p) = 0.16\%,$$

whereas for the subsample when the Gibson paradox occurs (1991–2011) the estimated NKPC is:

$$\pi_t = 0.21\pi_{t-1} + 0.79E_t\pi_{t+1} + 0.0064mc_t + \epsilon_t^p,$$

$$\epsilon_t^p = 0.78\epsilon_{t-1}^p - 0.60\eta_{t-1}^p + \eta_t^p, std(\eta^p) = 0.10\%.$$

As is evident, the slope coefficient declines to a ninth of its value in the second period (which they denote the “Gibson paradox” period during which there is a weak correlation of inflation and the nominal interest rate). This result is consistent with the increase in the estimate of the Calvo sticky-price probability (from 0.50 in the 1961–1981 sample period to 0.81 in the Gibson paradox period (1991–2011), implying that the average number of months without optimal pricing increases from 6 to 15.8 months). As argued by Smets and Wouters (2007), low and stable inflation may explain the increase in price stickiness associated with lower menu costs, less price dispersion, and smaller losses of deviating from optimal pricing.

4. Estimation

The improvements detailed above were incorporated into the canonical Smets-Wouters (2007) model, whose simulation properties (using “calibrated” parameters chosen by the researchers on the basis of various studies in the literature) showed that the model responded as expected to the shocks included in it, in particular the shocks to monetary policy. Central banks, however, cannot rely on parameters drawn from the literature and based (almost invariably) on behaviour in other countries (frequently the US). Hence the importance of “taking the model to the data” (the title of a celebrated paper by Ireland (2004) in which he used the full information maximum likelihood (FIML) technique to estimate the model’s parameters).

It is well-known from the econometrics literature that FIML is notoriously sensitive to model structure, and many subsequent attempts to use it to estimate DSGE models foundered on the shoals of flat or rocky likelihood surfaces. In their pioneering paper of 2003, Smets and Wouters (2003) introduced the use of Bayesian methods to estimate the parameters of their model based on seven key macro-economic variables in the Euro area (real GDP, real consumption, real investment, the GDP deflator, real wages, employment and the nominal interest rate). The choice of these variables is driven by the fact that in central banks (for whom both authors work) the model is primarily used for scenario “counterfactual” analysis (examining the implications of different scenarios for key macroeconomic variables), hence it is highly desirable that the dynamics and definitions of the model variables be as similar as possible to those of their real-world counterparts.

The Bayesian approach involves combining the likelihood function with prior distributions for the parameters of the model, to form the posterior density function. This posterior is then optimised with respect to the model parameters either directly or through Monte-Carlo Markov-Chain (MCMC) sampling methods. Because of the extreme difficulty of identifying all the parameters in a high-dimensional and highly nonlinear likelihood function, typically the unidentified parameters are calibrated (these usually include the rate of time discount applied by households, the depreciation rate for physical capital and the share of labour in the production function). The priors are defined by the researcher and draw on both formal (literature) and informal (“expert”) knowledge of the economy’s behaviour.

When Smets and Wouters applied this methodology to the US in their 2007 paper, they observed that the estimated model did a good job of matching the second moments of both growth and inflation:

TABLE 6—ACTUAL, MODEL-BASED, AND COUNTERFACTUAL STANDARD DEVIATIONS OF GDP GROWTH AND INFLATION

	1966:1–2004:4		1966:1–1979:2		1984:1–2004:4		Counterfactual 1984:1–2004:4		
	Actual	Model	Actual	Model	Actual	Model	Shocks	Policy	Structure
Growth	0.86	0.94	1.01	1.13	0.59	0.73	1.21	0.70	0.75
Inflation	0.62	0.57	0.55	0.81	0.25	0.34	1.30	0.39	0.32

Notes: “Actual” refers to the data-based standard deviations over the indicated sample; “model” refers to the standard deviations generated by the DSGE model estimated over the indicated sample. The counterfactual standard deviations for the period 1984:1–2004:4 refer to the standard deviations that would have occurred in this period if the shock processes (“shocks”), the monetary policy rule (“policy”), or the structural parameters (“structure”) would have been the same as the ones estimated in the 1966:1–1979:2 sample.

5. Financial Sector

Following the acute financial crisis beginning in 2008, the early New-Keynesian DSGE models were strongly criticised for their failure to include an explicit financial sector, thus ignoring that rather than originated and propagated primarily by changes in the “real” sector, many business cycles are caused at least in part by disruptions to the flow of resources between different groups of agents when a group of agents defaults on its obligations. In response to this, more recent models have included financial intermediaries, capital producers and entrepreneurs in addition to the intermediate and final goods firms. In these models (for example Del Negro et al. (2015)), banks collect deposits from households and lend to entrepreneurs who use these funds as well as their own wealth to acquire physical capital, which is then rented to intermediate goods producers. Entrepreneurs are subject to idiosyncratic disturbances that affect their ability to manage capital (called a “risk shock” in an influential paper by Christiano et al. (2014), who note that “Even entrepreneurs whom we now think of as sure bets, such as Steve Jobs and Bill Gates, experienced failures as well as the successes for which they are famous”). Retail banks can diversify the idiosyncratic risk of many entrepreneurs and thus can generate a safe return on households’ deposit. In the bond market, there are a continuum of safer entrepreneurs who are assumed not to default and investment banks which have some monopolistic power. The coupon rate of bonds issued is determined as a time-varying markup over the risk-free interest rate, which is a function of the elasticity of the demand for funds in the bond market.

Other modellers have looked at further aspects of the financial sector of special interest to central banks. For example, Verona, Martins and Drumond (2013) study what they call the “shadow banking” sector which in some countries essentially escapes central bank regulation. In their model, the shadow banking system is populated by a continuum of monopolistic competitive investment banks who, seeking to maximise profits, set the coupon rate on bonds as a markup over the risk-free interest rate. Empirically, it is observed that the yield curve (spread between short- and long-term bond yields) is not stable but moves countercyclically, so the authors include both a relation between the elasticity of demand for funds in the bond market and the cyclical state of the economy, which rules the baseline behavior of the spread in bond finance, and a distinction between “normal times” and periods of optimism (associated with financial booms). Doing so brings added realism to the model’s results as total credit (bank loans plus bonds), investment, and the price of capital increase substantially more than without optimism or in alternative models not incorporating the shadow banks.

A further example is Carlstrom et al. (2017) whose model features long-term bond purchases by the central bank, in order to analyse the effect of financial market segmentation and of term-premium targeting on the effectiveness of monetary policy, by incorporating private financial intermediaries within segmented financial markets in which the net worth of financial institutions limits the degree of arbitrage across the term structure. Through portfolio adjustment costs, central bank purchases of long-term bonds have a significant effect on long yields and thereby affect capital investment and the real economy.

6. Housing Sector

Spurred by the rôle played by the housing bust in the late 2000s in generating sharp (in this case, downward) fluctuations in economic activity in such countries as far-flung as the US, Ireland and Spain, economists scrambled to include the housing sector in their DSGE models, noting that the stock of housing wealth accounts for almost half of total household wealth and over 1.5 times annual GDP (flow) in most OECD countries, and that fluctuations in housing investment generally precede corresponding fluctuations in business investment (making housing a “leading indicator”).

In one widely-cited model incorporating the housing sector (Iacoviello (2015)) – which otherwise follows the canonical Smets-Wouters (2007) model incorporating wage and price frictions - the representative household

chooses consumption $C_{H,t}$, housing $H_{H,t}$, and time spent working $N_{H,t}$ to solve the following intertemporal problem:

$$\max E_0 \sum_{t=0}^{\infty} \beta_H^t (\log C_{H,t} + j \log H_{H,t} + \tau \log (1 - N_{H,t}))$$

In this model, sectoral heterogeneity on the supply side allows capturing the different trend and cyclical properties of housing prices and housing investment relative to other prices and to other forms of demand (including the fact that the production of housing is land intensive). In particular, the model is able to explain the observed increase in real housing prices as a consequence of slower technological progress in the housing sector and of the presence of land (a fixed factor) in the production function for new homes. On the demand side, collateral effects of housing prices on borrowing allow for spillovers from the housing market to consumer spending, thus altering the standard propagation mechanism. The presence of credit constrained households reinforces the correlation between movements in consumption and movements in housing wealth. And as found more generally, wage rigidity in the housing sector is crucial to explain important features of the data, in particular the large sensitivity of residential investment to changes in short-term interest rates.

As is seen in the table below, the estimated version of this model does very well in replicating major moments in the data. [In this table, “q” is not Tobin’s q but rather the housing price index and IH is housing investment, IK being non-housing investment.]

TABLE 5—BUSINESS CYCLE PROPERTIES OF THE MODEL

	Model			Data
	Median	2.5%	97.5%	
<i>Panel A. Standard deviation (percent)</i>				
<i>C</i>	1.57	1.20	2.02	1.22
<i>IH</i>	8.19	6.65	10.19	9.97
<i>IK</i>	4.08	3.20	5.23	4.87
<i>q</i>	2.10	1.70	2.62	1.87
π	0.48	0.39	0.58	0.40
<i>R</i>	0.31	0.25	0.39	0.32
<i>GDP</i>	2.20	1.72	2.82	2.17
<i>Panel B. Correlations</i>				
<i>C, GDP</i>	0.87	0.75	0.93	0.88
<i>IH, GDP</i>	0.63	0.43	0.78	0.78
<i>IK, GDP</i>	0.89	0.81	0.94	0.75
<i>q, GDP</i>	0.65	0.43	0.80	0.58
<i>q, C</i>	0.57	0.31	0.75	0.48
<i>q, IH</i>	0.46	0.19	0.67	0.41

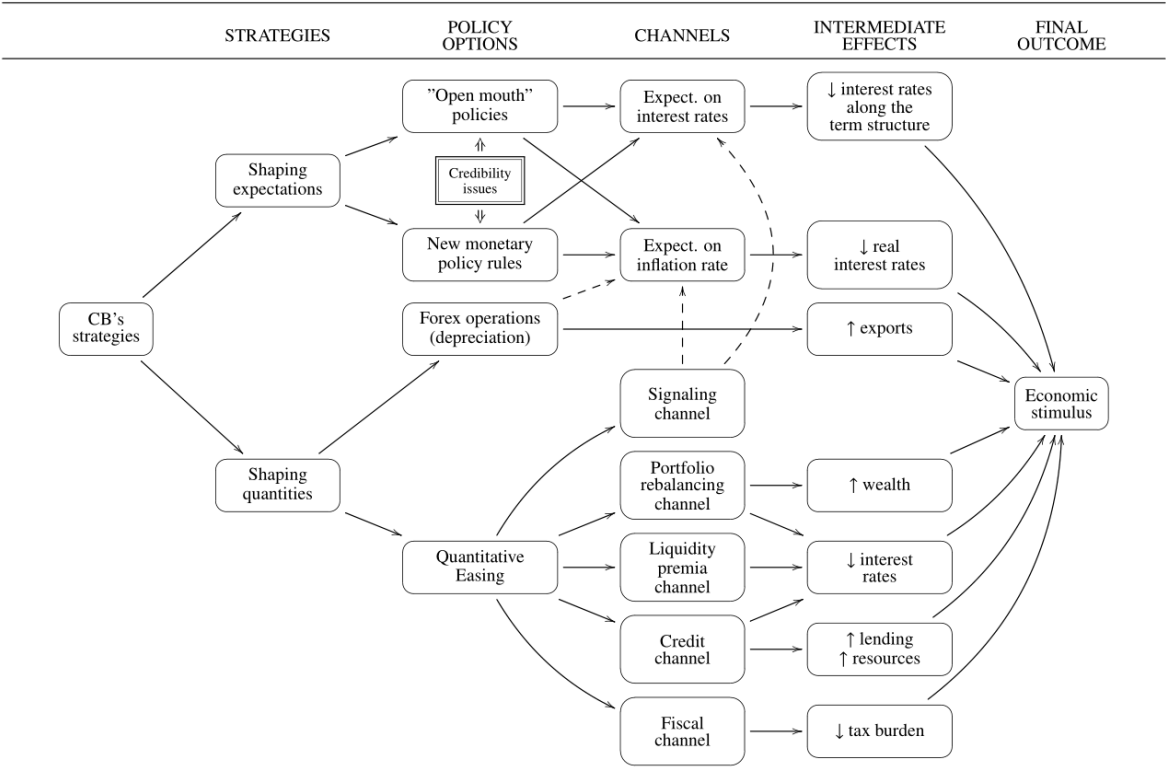
More recent work by central bank researchers (eg, Gibbs, Hambur and Nodari (2018)) has expanded this approach to include housing services explicitly, thus allowing a greater consistency between key macroeconomic variables (such as GDP and CPI inflation, both of which have a large weight placed on housing services – 23% in the Australian CPI), and their model counterparts. Another recent paper (Lambertini *et al.* (2017)) extends the canonical DSGE model so as to replicate the US subprime crisis, analysing how an increase

in risk in the mortgage market raises the default rate and spreads to the rest of the economy, creating a recession.

7. ZLB

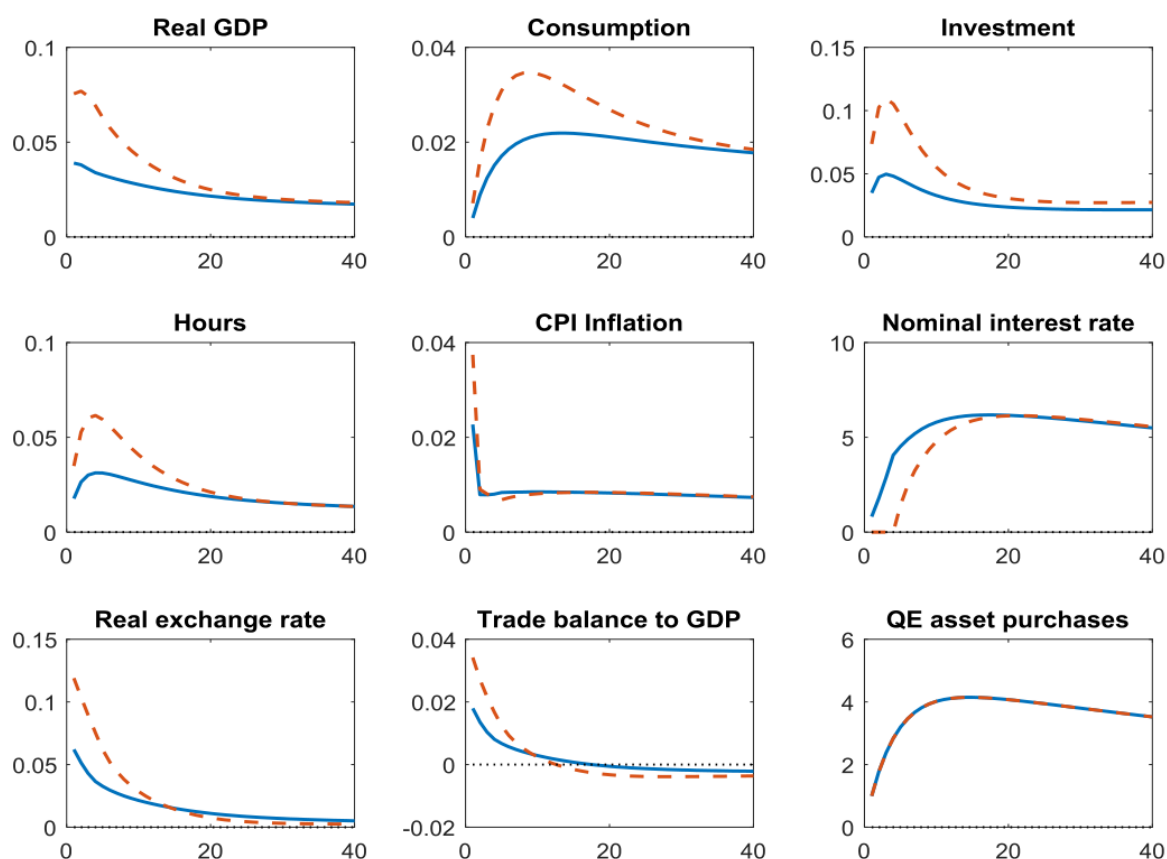
As the global downturn beginning in 2008 unfolded, many advanced economies experienced a serious liquidity shortage combined with an interest rate close to the zero lower bound (ZLB). In the aftermath of the financial crisis, interbank money markets froze up as a result of important bankruptcies (and, more generally, solvency concerns), a consequent widespread lack of confidence, and coordination failures among market participants. As a result, financial markets also broke down with dramatic consequences for the whole economic system. As is well-known, in an effort to spur economic activity and restore financial market functioning, several central banks intervened by reducing the short-term interest rate. The ZLB quickly became a serious concern for monetary institutions since, in such situations, the availability of credit tends to become unresponsive to the quantity of liquidity present in the economic system. Falagiarda (2014) analysed this situation and produced the chart below indicating the various policy options and channels which central banks could adopt.

Figure 1: Facing the ZLB: Strategies, policy options and channels



Falagiarda (2014) then developed a DSGE model for the analysis of QE which was used by Hohberger, Priftis and Vogel (2018) to analyse the effects of quantitative easing (QE) in a model of the Eurozone, based itself on Kollmann *et al.* (2016), where QE is captured by long-term bond purchases by the central bank. They also take into account the ZLB on interest rates which faces most central banks (some, including that of Switzerland, have implemented negative interest rates for deposits at the central bank). In particular, the model allows for examination of the effects of QE both unconstrained and constrained by the ZLB. When unconstrained, the standard monetary policy (Taylor) rule is operational and offsets part of effective QE as it reacts to rising output growth and inflation by tightening short-term interest rates. When by contrast the ZLB is allowed to be occasionally binding (creating a piecewise linear model), an increase in output and inflation through QE or other factors does not lead to tightening of the short-term rate while the constraint is binding. The result is

obviously a stronger impact of QE, as shown in the figure below, in which the solid (blue) line is the response to QE in the unconstrained model, and the dashed (red) line the corresponding response in the ZLB model



The policy recommendation which emerged from this type of analysis is that the interest rate should remain low past the time that the economy starts recovering from the shock that pushed it against the ZLB constraint. This policy prescription has become the theoretical underpinning of so-called forward guidance policies, which are now pursued by several central banks.

It should also be noted that other non-linearities plague the environment within which work central banks. For example, the US Federal Reserve Open Market Committee's (FOMC) guidance about future monetary policy has been highly nonlinear in nature: it involved a régime switch depending on the realization of endogenous variables (e.g. the unemployment rate). Similarly, it is evident that an economy might be more "fragile" (vulnerable to shocks) in certain circumstances, such as when banks or households are more leveraged, than in others, introducing another nonlinearity. DSGE modellers have begun incorporating such non-linearities into their models (eg, Christiano et al. (2016) which allows for just such régime-switching, Gertler and Kiyotaki (2015) which investigates economic fragility, and Aruoba *et al.* (2016) which tackles the formidable complexities of a full nonlinear solution to the ZLB issue).

8. Open Economy Models

In the descriptions above, no mention has been made of a feature important to central banks – the need to operate in a world economy in which exchange rates and trade flows can play an important rôle for monetary policy. The models described above have in the main been of the closed-economy type but much work has gone into extending them to include a foreign sector. The IMF in particular has developed a suite of DSGE models (GIMF) which explicitly take into account the important rôle of the uncovered interest parity (UIP) condition in an open-economy situation (see Anderson *et al.* (2013)). This Global Integrated Monetary and Fiscal model is a multi-region, forward-looking, DSGE model which features both overlapping-generation

households (OLG) that optimise their borrowing and saving decisions over a 20-year planning horizon (using a range of financial assets) and also liquidity-constrained households, who do not save and have no access to credit. All bilateral trade flows (of both intermediate and final goods) are explicitly modeled, as are the relative prices for each region, including exchange rates. These flows are calibrated in the steady state to match the flows observed in the recent data. International linkages are driven by global saving and investment decisions, a by-product of consumers' finite horizons. This leads to uniquely defined current account balances and net foreign asset positions for each region. Since asset markets in these models are incomplete, net foreign asset positions are represented by nominal non-contingent one-period bonds denominated in U.S. dollars. Together with UIP, and long-term movements in the world real interest rate, the magnitude of the international trade linkages is the main determinant of spillover effects from shocks in one region to other regions in the world. When conducting monetary policy, the GIMF's focal-country central bank uses an inflation-forecast-based interest rate rule in which it varies the gap between the actual policy rate and the long-run equilibrium rate to achieve a stable target rate of inflation over time.

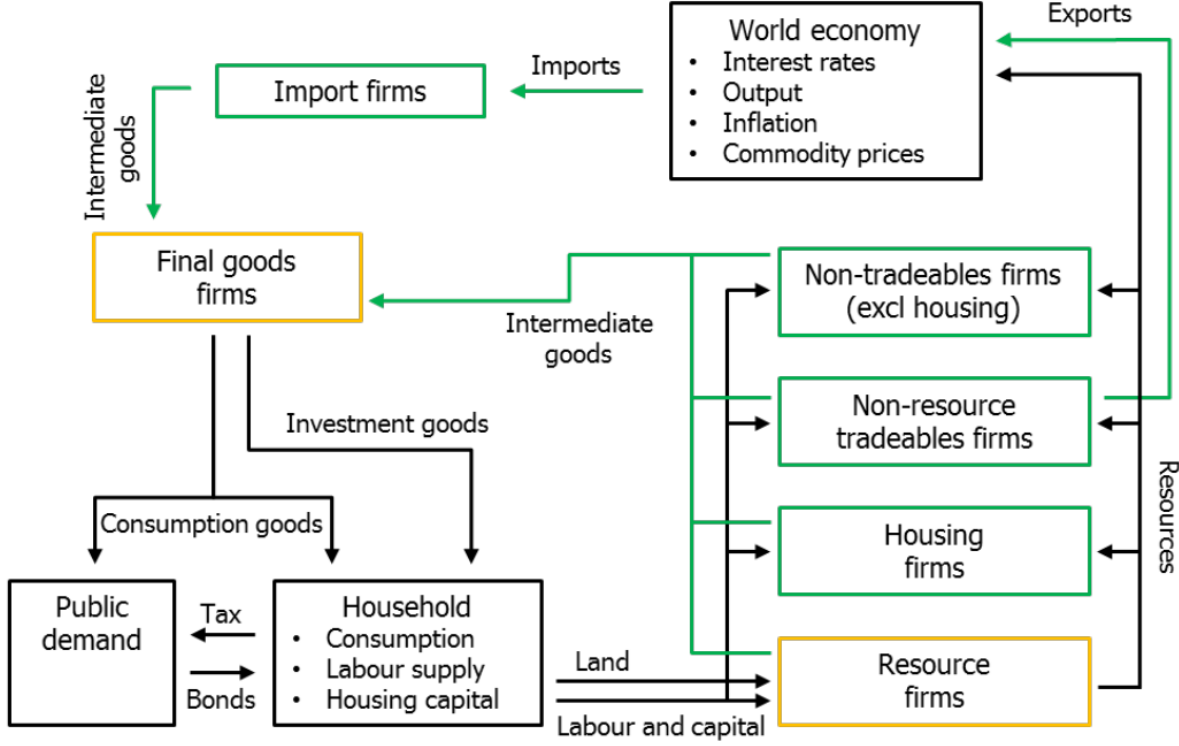
In a version of the model containing the United States, the euro area, Japan, emerging Asia (including China), and, as a single entity, the remaining countries, the response of the real effective exchange rate to a monetary policy shock (in the form of a 100 basis point increase in the U.S. nominal policy interest rate) exacerbates the reduction in demand for U.S. output. Higher real interest rates appreciate the U.S. dollar, reducing import prices for U.S. residents, and raising the price of U.S. exports in the rest of the world. Cheaper imports lead to some expenditure switching away from domestic toward foreign goods in the United States. Additionally, foreign demand declines for the now more expensive U.S. exports. The net impact of all these factors is a slight deterioration in the U.S. current account in the short run.

National central banks have also been at the forefront of research, particularly as regards the features relevant to a "small open economy" (SOE). An early example is that of Adolfson et al. (2007) which used Bayesian techniques to estimate an open economy DSGE model with incomplete exchange-rate pass-through. A very recent example is the already-cited model of the Reserve Bank of Australia (Gibbs, Hambur and Nodari (2018)) whose structure is summarised in the figure below. The SOE – Australia – consists of households and firms that produce and consume in six distinct sectors. There are five intermediate goods and services-producing sectors: resource (mining), non-resource tradeable (manufacturing and agriculture), non-tradeable excluding housing, housing, and imported goods and services. The resource sector is modelled as perfectly competitive and takes the world price of the resource good as given. The remaining sectors are monopolistically competitive and have some power to choose the prices of the goods they sell. Price changes for monopolistically competitive firms are subject to Rotemberg (1982) style price adjustment costs (this being an alternative to the usual Calvo mechanism). Wages in all domestic production sectors face similar adjustment costs. These adjustment costs generate price and wage stickiness in the model. Intermediate goods and services, produced domestically and imported from abroad, are combined into final goods by the perfectly competitive final goods sector, which provides goods for household and government consumption as well as business investment.

In this model, households derive utility from consumption of the composite final good and from the level of the housing stock in the economy, and derive disutility from labour supplied to firms in the four domestic intermediate production sectors. Households earn wages from labour supplied, rents from their ownership of capital, and dividends from ownership of firms. Households may also purchase domestic and foreign nominal bonds. Monetary policy in the model follows a Taylor-type rule that responds more than one-for-one to changes in inflation and positively to changes in real GDP growth. Fiscal policy is assumed passive with lump-sum taxation.

As in most current multi-country DSGE models, the rest-of-world (ROW) economy is significantly less detailed. The ROW is modelled as a two sector closed economy version of the model just described, and also features price stickiness. It purchases resources and tradeable goods from Australia, and sells tradeable goods to Australia. Households in the ROW economy may invest in their own and Australian nominal bonds.

Figure 2: The Model



□ Monopolistically competitive □ Perfectly competitive

For equilibrium to obtain in the model, the UIP condition must hold; in the log-linearised version of the model it is (where q_t is the (log of) the real exchange rate):

$$r_t - r_t^* - \psi_t = (1 - \Phi_\psi) \left[E_t q_{t+1} - q_t + E_t \pi_{t+1} - E_t \pi_{t+1}^* \right] - \Phi_\psi \left[q_t - q_{t-1} + \pi_t - \pi_t^* \right]$$

The real exchange rate also figures in the Taylor Rule followed by the monetary authorities where the short-term nominal interest rate is set according to:

$$\ln \left(\frac{R_t}{R} \right) = \rho_r \ln \left(\frac{R_{t-1}}{R} \right) + (1 - \rho_r) \left[\phi_\pi \ln \left(\frac{\pi_t}{\Pi} \right) + \phi_{dy} \ln \left(\frac{Y_t^{va}}{Y_{t-1}^{va}} \right) + \phi_q \left(\frac{Q_t}{Q_{t-1}} \right) \right] + \varepsilon_{r,t}$$

The parameters R and Π are steady-state interest rates and inflation, respectively and GDP is measured in value-added terms (because the non-tradeable, tradeable, and housing sectors all use resources as inputs, which introduces a wedge between the production and the value added in these sectors).

9. Departures from Rational Expectations

Rational expectations assumes that agents possess complete knowledge about the economic environment (model structure and parameters). Since in practice agents do not know the Rational Expectation Equilibrium

(REE), they are unable to produce model-based predictions of the path of forward-looking variables and have to form their own beliefs about future developments on the basis of the information they observe. In effect, agents gradually learn the “true” parameters of the model by adapting their expectations with the use of a (usually informal and personal) learning algorithm. Some recent papers have formulated versions of such learning algorithms in an attempt to resolve the problem that agents do not have sufficient information about the model and its parameters to formulate the REE-based expectations $E_t Y_{t+1}$ which enter into all DSGE models. Rychalovska (2016) applied this approach to the canonical Smets-Wouters (2007) model (augmented with financial frictions) via an adaptive learning (AL) algorithm which relates the value of the forward variable y_j^f to the model state variables X_j using a reduced-form linear function:

$$y_{j,t}^f = \beta_{j,t-1} X_{j,t-1} + u_{j,t}$$

She finds that replacing RE with AL amplifies the effect of financial frictions and strengthens the propagation of shocks to the real economy by influencing the degree of economic persistence in asset prices and investments. leading to results which are more in line with historical experience (inflation, output and investment growth are all better predicted by the AL model than by the RE model, most pronouncedly during the recession following 2008).

Adam and Merkel (2018) also replace REE expectations in a model which succeeds in jointly replicating the quantitative behaviour of business cycles and stock prices. They consider subjectively Bayesian investors who seek to filter the long-term trend component of capital gains from observed capital gains and as a result extrapolate (to some extent) past capital gains into the future. They justify this by observing that such extrapolative behaviour is in fact consistent with the available survey evidence on investors’ return expectations, whereas REE is strongly rejected by the data. The Bayesian element allows for agents to deviate from rational expectations along some dimensions (eg, future asset prices), but be consistent with REE along other dimensions.

C. What can these DSGE models do (and not do)?

As is evident from the preceding pages, DSGE models have come a very long way from their rather rudimentary beginnings as simple Real Business Cycle models based on perfect competition and instantaneously adjusting goods and factor markets. The canonical NK DSGE introduces nominal variables explicitly: prices, wages, and a nominal interest rate; it departs from the assumption of perfect competition in the goods market, allowing for positive price markups; and it introduces nominal rigidities, both in goods markets and the labour market, generally using the formalism proposed by Calvo (1983). NK DSGEs nowadays also incorporate explicit modelling of vital sectors such as finance and housing and of policies of interest to central banks, such as QE. Methodologies have been developed and applied to enable full-information joint estimation of (nearly all) the model parameters from real-world data and to allow the use of only occasionally binding constraints (such as the ZLB). And the models have been expanded from single-country closed economies to multi-country open economies with cross-country linkages.

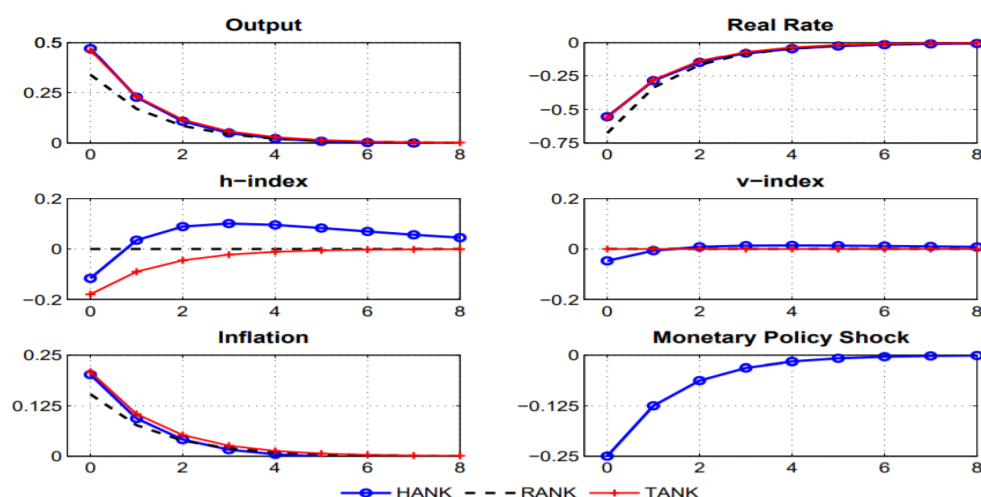
Unlike their predecessors, the (partial equilibrium) simultaneous equations models used for decades, current DSGE models in use (or development) at central banks therefore are capable of providing internally-consistent general equilibrium analysis to enable examination of the implications of different counterfactual scenarios for key macroeconomic variables. The ability to create and examine scenarios in which the central bank shuts off or modifies one or several of the many channels through which monetary policy operates, for example, is a highly valuable feature of modern DSGE models. In addition (although it has not been mentioned above) most current DSGE models also contain fiscal sectors with which the monetary sector interacts in real-world

governments, enabling a much richer consideration of policy tools. And since such DSGE models with nominal rigidities and financial frictions have been shown to provide quantitatively plausible accounts of the financial crisis and the so-called “Great Recession”, they are legitimate frameworks within which to analyze alternative fiscal and monetary policies.

It nevertheless remains that even today’s highly-developed DSGE models are not very useful as forecasting tools. Central banks have for some time been using VAR models (usually in the Bayesian variant and sometimes also with structural constraints - SBVARs) for short-term forecasting purposes and although a few studies (eg, Cai *et al.* (2018)) have indicated that in particular horse races the DSGE model can give a (relatively simple) VAR a run for its money at short-run forecasting, I know of no-one in the central bank forecasting community who would risk her job on the forecasting ability of the bank’s research department DSGE(s). As del Negro and Giannoni (2017) say of the FRB/NY DSGE model: “We’d say we had hits and misses. The biggest success was perhaps forecasting a sluggish recovery since 2010, with output growth around 2% ... [But] it was wrong in 2011 and 2012, when the Arab Spring sent oil prices spiking.” Part of the reason for the ineffectiveness of DSGE models in forecasting has been related by Fair (2019) to excessive aggregation: just as for plant and equipment investment and housing investment (which recent research has separated), services, nondurables, and durable consumption behave quite differently in the macro economy, and much is likely to be lost in aggregating the three.

More fundamentally, most current DSGE models are based on the representative agent (RANK) framework, even if that framework nowadays routinely incorporates (fixed proportions of) two agents (eg, Ricardian and non-Ricardian (or “hand-to-mouth”) consumers – the TANK model). As Gali (2018) points out, the primary channel by which monetary-policy induced interest rate changes affect consumption in the standard NK DSGE model is by causing the representative household to reallocate consumption over time, a requirement for which there is only weak empirical support, in part because many households face binding borrowing constraints. This implies the need for heterogeneous agent (or HANK) models where consumers face idiosyncratic shocks and binding borrowing constraints, thus impacting the effect of monetary policy changes (since interest rate sensitivities may vary in size (and even in the sign of their covariances) across groups). If indeed a large fraction of the population is highly sensitive to labour income shocks (idiosyncratic and aggregate) but not very responsive to interest rate changes, the nature of the monetary policy transmission mechanism in HANK models accounting for this dramatically changes as compared to that in the RANK model. An example of the differences in response to a monetary policy shock is shown below, drawn from Debortoli and Gali (2017) where the “h-index” is a measure of heterogeneity between (liquidity-) constrained and unconstrained agents and the “v-index” is a measure of heterogeneity within unconstrained agents:

Figure 3: The Effects of a Monetary Policy Shock: HANK vs TANK



As for households, so for enterprises in the NK DSGE model – again, the RANK model needs to be replaced by HANK models (eg, as in Ottonello and Winberry (2018), who use a group of heterogeneous production firms which invest in capital using either internal funds or external borrowing and which can default on their debt, leading to an external finance premium). Work is progressing in these areas, but is by no means yet integrated into the work-horse models used at central banks. HANK models are unfortunately very difficult to implement computationally (although Winberry (2018) has made considerable progress in this area), and Gali has suggested approximating them with TANK models (as in the paper cited above) for the near future. Another recent approach is that of Cardaci and Saraceno (2019) who specialise HANK to differentiate households by wage distribution using a Pareto distribution consistent with empirical evidence concerning income inequality, enabling a finer examination consumer debt, default and endogenous credit tightening.

Another fundamental issue is that much of the analysis based on the NK DSGE model has a local nature: specifically, it is carried out using a linear approximation to the (highly nonlinear and intractable) equilibrium conditions around a supposed “steady state” consistent with the inflation target (which is typically small or zero). By construction, therefore, that analysis limits the central banker’s understanding of the economy’s behaviour far from the assumed steady state. This is relevant to the puzzle of why the slow recovery from recession and low inflation in many advanced economies in the wake of their financial crises persisted despite highly expansionary monetary policies with near-zero policy interest rates. Some recent papers have interpreted the crises and subsequent persistent slump as an equilibrium path converging to a “liquidity trap” steady state which co-exists with the locally unique equilibrium around the steady state associated with the targeted inflation rate (eg, Schmitt-Grohé and Uribe (2017)). This issue of a multiplicity of equilibria is clearly of significance for central banks as it implies that two different balanced growth paths may be consistent with equilibrium, one of which (the stagnation trap) is characterized by involuntary unemployment and low growth, while the other may feature high growth and full employment but may not be attainable with standard policy.

As indicated by Williams (2017), another side of this coin is that “Many of the most important issues facing central banks today are related to medium- or long-run developments to both the “supply” and “demand” sides of the economy, including the labour market, productivity, and other structural changes. Because the current crop of DSGE models assumes that all shocks are transitory and that the economy eventually returns to a fixed steady state, these models are not designed to analyse longer-term shifts in demographics, productivity, preferences, or other structural shifts.” Some work has been done in this area (for example in the FRB/US DSGE model – see Laforge (2018)) but more research work is needed to incorporate longer-term trends in the labour market, productivity, and other structural factors into the models. Recent advances in estimating DSGE models using frequency bands may prove helpful in this direction. For example, Gallegati *et al.* (2019) find that when they estimate a DSGE model with financial frictions and banks on subsets of frequency bands corresponding to higher and lower business cycle frequencies, the results vary considerably and the introduction of financial frictions provides a significant improvement at lower business cycle frequencies.

Conclusion

The review above demonstrates the sagacity of the sub-title of Gürkaynak and Tille (2017): “Use as Intended”. DSGE models, even in the most recent of versions, are not forecasting models: for short-term forecasting, nothing will beat a VAR model. However, for counterfactual scenario analysis, nothing will beat a well-constructed recent-vintage NK DSGE model. Economic analysis was once verbal, then comparative-static, then partial-equilibrium, but can now be fully general-equilibrium and dynamic, in nature. Modern NK DSGE models are ideal frameworks within which to test serious economic hypotheses, not only because they are internally consistent, dynamic and general equilibrium, but also because modern computing power has made it possible

to use full-information (maximum likelihood) methods to estimate the parameters of these complex models using both real-world data and the input of sector experts. This is a very major advance compared with anything available in the past, and central banks (as many are) would be well advised to make full use of it.

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