# Intermediate Microeconomics Assignment 4

With Suggested Solution

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1. Consider two consumers, A and B. Their demand functions are given by $q_A = A - p$, and $q_B = B - p$, respectively. A monopoly's marginal cost is a constant $c$. Assume that $A > B > c > 0$. The firm uses the second-degree price discrimination, by offering a fixed fee $T$ and a per-unit price $p$. The menu $(T, p)$ applies to all consumers. Compute the profit-maximizing two-part tariff (Hint: you should discuss under what circumstances, the firm serves both/only type-A consumer(s)).

2. In page 22 of the lecture note (i.e., pirate game), assume that the rule is changed as: Starting from A, each proposes a distribution of coins. If the proposed allocation is approved by a strict majority vote ($> 50\%$ not including $= 50\%$), then it happens. Otherwise, the proposer is thrown overboard and dies, and the next makes a new proposal to begin the system again. Indicate the subgame perfect Nash equilibrium.

3. Consider two firms who compete in quantities (Cournot model). The market demand is given by $P = 10 - Q$. Assume that firm 1's marginal cost is 1, and firm 2's marginal cost is 2.

   (1) If they offer quantities, $q_1$ and $q_2$, simultaneously, solve the equilibrium quantities and profits.

   (2) Assume that firm 1 moves first; upon observing firm 1's choice, firm 2 moves second. Compute the equilibrium quantity choice of each firm.

4. Consider a Hotelling model. A unit mass of consumers are uniformly distributed over $[0, 1]$. Two firms locate at 0 and 1, respectively. For each consumer, buying from firm 0 gives $V - p_0 - tx$; buying from firm 1 gives $V - p_1 - t(1 - x)$, where $V$ denotes the value of buying from one of the two sellers, and $tx$ or $t(1 - x)$ denotes the travel disutility of buying from seller 0 or seller 1, respectively. Firms compete in prices. The marginal cost of each firm is $c_0$, and $c_1$, respectively.

   (1) Assume that each consumer buys at least and at most one unit of the product. Solve the equilibrium price charged by each firm.

   (2) One of the conditions that makes your arguments in (1) valid is that the demand margin of each firm cannot locate outside the $[0, 1]$ interval. What conditions are required to make that happen?

1. Notice that the willingness to pay of consumer A is higher than that of consumer B. Hence, for the single two-part tariffs, there are two cases: (i) consumer B is served; (ii) consumer B is not served.

   (i) When consumer B is served, the per-unit price $p < B$ is offered such that the fixed fee is $T = \frac{1}{2}(B-p)^2$ for both consumers. Evaluated at the per-unit price $p$, consumer A buys $A - p$ units and consumer B buys $B - p$ units. The firm's profit is given by

   $$\max_p 2T + (p-c)(A-p+B-p). \tag{1}$$

   The first-order condition of (1) gives

   $$p_i = \frac{A-B}{2} + c.$$

   Profit is given by

   $$\pi_i = \frac{1}{4}A^2 - \frac{1}{2}AB + \frac{5}{4}B^2 - 2Bc + c^2.$$

   (ii) When $A$ is sufficiently large relative to $B$, it is possible that the fixed fee $T$ is equal to $T = \frac{1}{2}(A-p)^2 > \frac{1}{2}(B-p)^2$ such that consumer B will not buy. Then, the firm solves

   $$\max_p \frac{1}{2}(A-p)^2 + (p-c)(A-p). \tag{2}$$

   The first-order condition of (2) gives $p_{ii}^{STP} = c$, and profit is

   $$\pi_{ii} = \frac{(A-c)^2}{2}.$$

   Comparing $\pi_{ii}$ and $\pi_i$, the derivative of $\pi_{ii} - \pi_i$ with respect to $A$ is $\frac{A+B}{2} - c > 0$, which implies that when $A$ is large enough relative to $B$, the firm will choose case (ii); otherwise, the firm chooses case (i), and the threshold condition is

   $$\pi_{ii} > \pi_i \Rightarrow A > (\sqrt{6}-1)(B-c) + c.$$

2. Using backward induction, we start by analyze the strategies taken by the last two guys, D and E. Because a D can never get one more vote, hence there will be only two possible outcomes for D: be rejected by leaving a positive number of coins for himself, or rejected/not rejected but leaving 0 coin for himself. Nevertheless, E will get 100 coins.

   Now let's move back to check the situation where C, D, and E are left. C knows what will happen if C is rejected. Hence C needs two vote: one from himself (C) and one from D. Therefore, C only need to make D slightly better-off by giving 1 coin to D. That is, C proposes the plan $(99, 1, 0)$ and then C and D will agree.

   Then let's check the situation where B, C, D, and E are making decisions. They know that if B's proposal is rejected, then the outcome will be $(99, 1, 0)$. With 4 people, B needs to get 3 votes: one from himself, and the other two from D and E. Therefore, B need to make D and E better-off compared with $(99, 1, 0)$. Hence, B should give 2 coins to D and 1 coin to E to make that work. That is, B proposes $(97, 0, 2, 1)$. (You might wonder that D will be indifferent between saying yes and saying no if B gives 1 coin to D. But if that is the case, we can "refine" the result by assuming that if D agrees with probability 0.5. Then B's plan will be approved with probability 0.5, with expected payoff: $0.5 \cdot 0 + 0.5 \cdot 98 = 49$. If B gives 2 coin to D instead, he wins definitely and obtains 97.)

Back to A's decision. Knowing that the outcome will be $(97, 0, 2, 1)$ if A's plan is rejected, A needs 3 votes out of 5 to win, hence A needs: one vote from himself, and the other two votes from C and E. Hence A gives 1 coin to C and 2 coins to E to make them better-off compared to the outcome if they enter the next round. Therefore, the subgame perfect Nash equilibrium is that A proposes $(97, 0, 1, 0, 2)$, then A, C, and E agree, then the game ends.

3. (1) Firm 1 solves $\max_{q_1}(9 - q_1 - q_2)q_1$. Firm 2 solves $\max_{q_2}(8 - q_1 - q_2)q_2$. The first-order conditions (best responses) are

$$\begin{cases} 9 - 2q_1^{BR} - q_2 = 0 \\ 8 - q_1 - 2q_2^{BR} = 0 \end{cases}$$

The above two equations imply $q_1^* = \frac{10}{3}$, $q_2^* = \frac{7}{3}$. The equilibrium profit is $\pi_1 = \frac{100}{9}$ and $\pi_2 = \frac{49}{9}$.

(2) In stage 2, firm 2 solves $\max_{q_2}(8 - q_1 - q_2)q_2$ where $q_1$ is given, the first-order condition gives

$$8 - q_1 - 2q_2 = 0 \Rightarrow q_2^{\text{follower}} = 4 - \frac{1}{2}q_1. \tag{3}$$

Back to stage 1, firm 1 solves $\max_{q_1}(9 - q_1 - q_2^{\text{follower}})q_1$, where $q_2$ should be replaced by equation (3), i.e., firm 1 solves $\max_{q_1}\left[9 - q_1 - 4 + \frac{1}{2}q_1\right]q_1$. The first-order condition gives $q_1^{\text{leader}} = 5$. Plug $q_1^{\text{leader}} = 5$ into firm 2's solution, then $q_2^{\text{follower}} = \frac{3}{2}$.

4. (1) A marginal consumer who is indifferent between buying from 0 or buying from 1 locates at

$$V - p_0 - tx = V - p_1 - t(1 - x) \Rightarrow \hat{x} = \frac{1}{2} + \frac{p_1 - p_0}{2t}.$$

Therefore, firm 0's demand is $D_0 = \hat{x}$ and firm 1's demand is $D_1 = 1 - \hat{x}$. Each firm solves

$$\max_{p_0}(p_0 - c_0)\hat{x}; \;\; \max_{p_1}(p_1 - c_1)(1 - \hat{x}).$$

The solution is

$$\begin{cases} p_0^* = t + \frac{2}{3}c_0 + \frac{1}{3}c_1 \\ p_1^* = t + \frac{2}{3}c_1 + \frac{1}{3}c_0 \end{cases} \tag{4}$$

(2) The marginal consumer locates at $\hat{x}$, which is equal to (plug equation 4 into the expression of $\hat{x}$) $\hat{x}(p_0^*, p_1^*) = \frac{1}{2} + \frac{c_1 - c_0}{6t}$. In order to make $\hat{x} \in [0, 1]$, $|c_0 - c_1| \le 3t$ should hold.