

International Finance

Lecture VI: Nominal Rigidities (USG Chapter 9)

Geneva Graduate Institute

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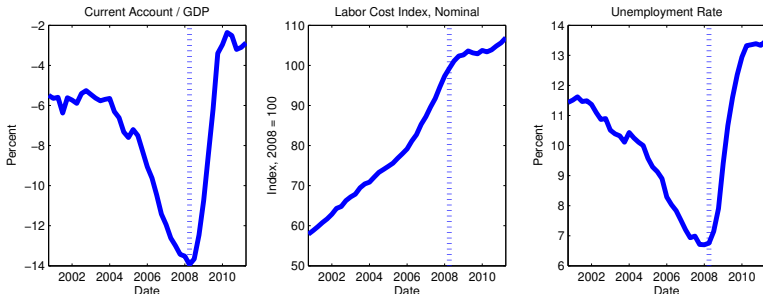
Roadmap

- Chapter 9 develops a theoretical framework in which nominal rigidities result in inefficient adjustments to aggregate disturbances
- The framework can be used in an intuitive graphical manner to demonstrate how nominal rigidities amplify the business cycle in open economies
- The framework can also be used to derive quantitative predictions useful for policy evaluation

Motivation: Peripheral Europe and the Global Crisis of 2008

- Countries in the periphery of the European Union, such as Ireland, Portugal, Greece, and a number of small eastern European countries adopted a fixed exchange rate regime by joining the euro area
- Most of these countries experienced an initial transition into the euro characterized by low inflation, low interest rates, and economic expansion
- The inception of the euro in 1999 was followed by massive capital inflows into the region, possibly driven by expectations of quick convergence of peripheral and core Europe
- Large current account deficits and large increases in nominal hourly wages, with declining rates of unemployment between 2000 and 2008
- When the global crisis of 2008 starts, capital inflows dry up abruptly. Peripheral Europe suffers a severe **sudden stop** (sharp reductions in current account deficits)
- In spite of the collapse in aggregate demand and the lack of a devaluation, nominal hourly wages remain as high as at the peak of the boom
- Massive unemployment affects all countries in the region

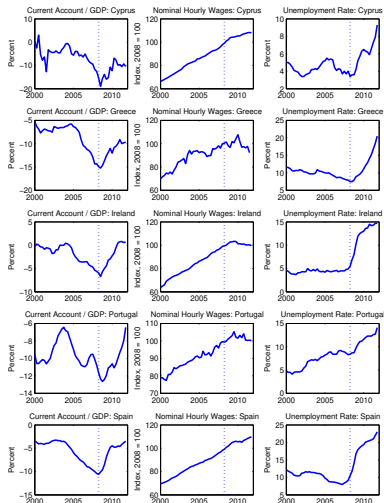
Figure 9.1 Boom-Bust Cycle in Peripheral Europe: 2000-2011



Data Source: Eurostat. Labor Cost Index, Nominal, is the nominal hourly wage rate in manufacturing, construction and services (including the public sector, but for Spain.)

Data represents arithmetic mean of Bulgaria, Cyprus, Estonia, Greece, Ireland, Lithuania, Latvia, Portugal, Spain, Slovenia, and Slovakia.

The Disaggregated Story: Boom-Bust Cycles in Cyprus, Greece, Ireland, Portugal, and Spain.



The challenge with currency pegs

- * Countries adopt currency pegs for a variety of reasons. Often to fight high inflation and to anchor expectations. (Example: Argentina's 1991 convertibility plan.)
- * However, fixed exchange rate arrangements can be difficult to maintain and can have costs.
- * The Achilles' heel of currency pegs is that they hinder the efficient adjustment of the economy to negative external shocks, such as drops in the terms of trade or hikes in the interest-rate.
- * Such shocks produce a contraction in aggregate demand that requires a decrease in the relative price of nontradables, that is, a real depreciation of the domestic currency, in order to bring about an expenditure switch away from tradables and toward nontradables.
- * In turn, the required real depreciation may come about via a nominal devaluation of the domestic currency or via a fall in nominal prices or both.

The challenge with currency pegs

- * The currency peg rules out a devaluation. Thus, the only way the necessary real depreciation can occur is through a decline in the nominal price of nontradables.
- * However, when nominal wages are downwardly rigid, producers of nontradables are reluctant to lower prices, for doing so might render their enterprises no longer profitable.
- * As a result, the necessary real depreciation takes place too slowly, causing recession and unemployment along the way.
- * This narrative goes back at least to Keynes (1925) who argued that Britain's 1925 decision to return to the gold standard at the 1913 parity despite the significant increase in the aggregate price level that took place during World War I would force deflation in nominal wages with deleterious consequences for unemployment and economic activity.
- * Similarly, Friedman's (1953) seminal essay points at downward nominal wage rigidity as the central argument against fixed exchange rates.

To formalize this narrative build an open economy model with:

- Downward nominal wage rigidity
- A traded and a nontraded sector
- Involuntary unemployment

To produce quantitative predictions

- Estimate the key parameters of the model (with particular attention on the parameter governing downward wage rigidity) and estimate the driving forces.
- Characterize response to large negative external shock under a peg and show that the model can explain the observed sudden stop.
- Characterize optimal exchange rate policy.
- Quantify the costs of currency pegs in terms of unemployment and welfare.

The material is based on Schmitt-Grohé and Uribe (JPE, 2016).

9.1 An Open Economy with Downward Nominal Wage Rigidity (DNWR): Key variables and assumptions

- Stochastic and exogenous endowment of tradable goods (this assumption can be relaxed): y_t^T .
- Movements in y_t^T can be interpreted as shocks to the availability of traded goods or shocks to ToT.
- Stochastic and exogenous country interest rate: r_t .
- Nontraded goods, y_t^N , produced with labor, h_t : $y_t^N = F(h_t)$
- Law of one price holds for tradables: $P_t^T = \mathcal{E}_t P_t^*$.
 - * P_t^T , nominal price of tradable goods.
 - * \mathcal{E}_t , nominal exchange rate, domestic-currency price of one unit of foreign currency ($\mathcal{E}_t \uparrow$ depreciation of domestic currency).
 - * P_t^* , foreign currency price of tradable goods.
 - * Assume that $P_t^* = 1$, so that $P_t^T = \mathcal{E}_t$

9.1 An Open Economy with Downward Nominal Wage Rigidity (DNWR): Key variables and assumptions

$$W_t \geq \gamma W_{t-1} \quad (9.6)$$

W_t = nominal wage rate in period t

γ = degree of downward wage rigidity.

$\gamma = 0 \Rightarrow$ fully flexible wages.

Think of γ as being around 1. Empirical evidence suggests $\gamma = 0.99$ at quarterly frequency.

Households

$$\max_{\{c_t^T, c_t^N, d_{t+1}\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t) \quad (9.1)$$

subject to

$$c_t = A(c_t^T, c_t^N) \quad (9.2)$$

$$P_t^T c_t^T + P_t^N c_t^N + \varepsilon_t d_t = P_t^T y_t^T + W_t h_t + \varepsilon_t \frac{d_{t+1}}{1 + r_t} + \Phi_t \quad (9.3)$$

$$d_{t+1} \leq \bar{d} \quad (9.4)$$

$$h_t \leq \bar{h} \quad (9.7)$$

- * (9.2): Consumption is a composite of traded and nontraded goods. $A(\cdot, \cdot)$ increasing, concave, and HD1.
- * (9.3) and (9.4): Households are subject to a debt limit \bar{d} . d_t = one-period debt chosen in t , due in $t + 1$. Debt is denominated in units of foreign currency \rightarrow full liability dollarization. \rightarrow *Original Sin*: In emerging countries almost 100% of external debt issued in foreign currency (Eichengreen, Hausmann, and Panizza, 2005). Country interest rate, r_t , is stochastic.
- * (9.7): Workers supply \bar{h} hours inelastically (labor supply is exogenous), but may not be able to sell them all. They take $h_t \leq \bar{h}$ as given.

Optimality Conditions Associated with the Household Problem

$$\frac{A_2(c_t^T, c_t^N)}{A_1(c_t^T, c_t^N)} = \frac{P_t^N}{P_t^T} \equiv p_t \quad (9.5)$$

remember $P_t^T = \mathcal{E}_t$

$$\lambda_t = U'(A(c_t^T, c_t^N))A_1(c_t^T, c_t^N)$$

$$\lambda_t = \beta(1 + r_t)\mathbb{E}_t\lambda_{t+1}$$

$$P_t^T c_t^T + P_t^N c_t^N + \mathcal{E}_t d_t = P_t^T y_t^T + W_t h_t + \mathcal{E}_t \frac{d_{t+1}}{1 + r_t} + \Phi_t$$

$$h_t \leq \bar{h}$$

The Demand For Nontradables

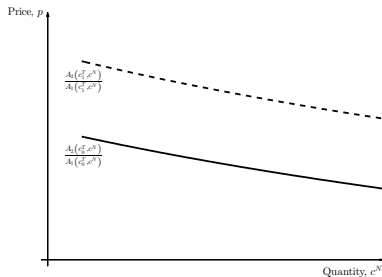
Look again at the optimality condition (9.15)

$$\frac{A_2(c_t^T, c_t^N)}{A_1(c_t^T, c_t^N)} = p_t. \quad (9.15)$$

- * If $A(c^T, c^N)$ is concave ($A_{22} < 0$) and HD1, then given c_t^T , the left-hand side is decreasing in c_t^N .
- * This means that, all other things equal, an increase in p_t reduces the desired demand for nontradables, giving rise to the downward sloping demand schedule shown in the next slide.
- * Note that c_t^T acts as a shifter of the demand schedule for nontradables: given p_t , an increase in c_t^T is associated with an equiproportional desired increase in c_t^N .
- * Of course, this shifter is endogenously determined.

Figure 9.2 The Demand For Nontradables

$$p_t = \frac{A_2(c_t^T, c_t^N)}{A_1(c_t^T, c_t^N)} \quad (9.15)$$



- Here we treat c_t^T as a shifter of the demand schedule.
- An increase in c_t^T from c_0^T to $c_1^T > c_0^T$, shifts the demand schedule up and to the right.
- But what is p_t ?

Firms in the Nontraded Sector (9.1.2)

Nontraded output, $y_t^N = F(h_t)$ (with $F' > 0$ and $F'' < 0$) is produced by perfectly competitive firms which use labor as sole input. Profits, Φ_t are given by:

$$\Phi_t = P_t^N F(h_t) - W_t h_t$$

where P_t^N is the nominal price of nontradables. Firms maximize profits taking as given P_t^N and W_t .

Optimality Condition:

$$P_t^N F'(h_t) = W_t$$

Divide by $P_t^T = \mathcal{E}_t$ and rearrange

$$p_t \equiv \frac{P_t^N}{P_t^T} = \frac{W_t / \mathcal{E}_t}{F'(h_t)}$$

p_t is the relative price of nontradables in terms of tradables.

The Supply Schedule of Nontradables

Let's derive the supply schedule for nontradables in the space (y^N, p) given the real wage, W_t/\mathcal{E}_t .

Note that

$$p_t = \frac{W_t/\mathcal{E}_t}{F'(h_t)}$$

is the usual optimality condition price=marginal cost, we are just using the **real** (i.e., in terms of traded goods) price and marginal cost

Use $h = F^{-1}(y^N)$ to obtain

$$p_t = \frac{W_t/\mathcal{E}_t}{F'(F^{-1}(y_t^N))}$$

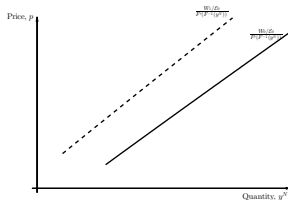
Interpret this relation as a supply schedule of nontradables given the real wage.

Figure 9.3 The Supply Of Nontradables

Supply schedule: $p_t = \frac{W_t/\varepsilon_t}{F'(F^{-1}(y_t^N))}$

Properties:

- upward sloping, the higher the price, the more a firm wishes to produce, given factor prices (for a given level of wages and exchange rate, if p goes up the denominator needs to go down, since F is concave ($F'' < 0$), this requires a higher level of y^N)
- A decrease in nominal wage from W_1 to $W_0 < W_1$ shifts the supply schedule down and to the right.
- A devaluation $\varepsilon_t \uparrow$ (not shown) shifts the supply schedule in the same manner as a nominal wage cut.



Closing of the Labor Market (9.1.3)

Nominal wages are **downwardly** rigid. This is our main friction and it's captured by γ (higher γ more rigid wages)

$$W_t \geq \gamma W_{t-1} \quad (9.6)$$

Labor demand may not exceed supply

$$h_t \leq \bar{h} \quad (9.7)$$

Impose the following slackness condition:

$$(\bar{h} - h_t)(W_t - \gamma W_{t-1}) = 0 \quad (9.8)$$

- This slackness condition says that, if there is involuntary unemployment ($h_t < \bar{h}$), then the lower bound on nominal wages must be binding.
- It also says that if the lower bound on nominal wages is not binding ($W_t > \gamma W_{t-1}$), then the labor market must feature full employment.

Competitive Equilibrium 9.1.4

Market clearing in the Nontraded Sector

$$c_t^N = y_t^N = F(h_t)$$

This must hold at all t . Combining the above condition with the production technology for nontradable, the HHs budget constraint and the definition of firms' profits we obtain the following market clearing condition for traded goods:

$$c_t^T + d_t = y_t^T + \frac{d_{t+1}}{1 + r_t} \quad (9.9)$$

Consumption of traded goods today plus payments of debt contracted yesterday is equal to tradable income today plus the present value of debt owed tomorrow

- Define the (gross) devaluation rate as: $\epsilon_t \equiv \frac{\epsilon_t}{\epsilon_{t-1}}$. If $\epsilon_t > 1$, the domestic currency depreciates (and the other way around)
- Define the real wage in terms of tradable as $w_t \equiv \frac{W_t}{\epsilon_t}$

Competitive Equilibrium

A competitive equilibrium is a set of stochastic processes $\{c_t^T, h_t, w_t, d_{t+1}, p_t, \lambda_t\}_{t=0}^{\infty}$ satisfying:

$$c_t^T + d_t = y_t^T + \frac{d_{t+1}}{1 + r_t} \quad (9.9)$$

$$\lambda_t = U'(A(c_t^T, F(h_t)))A_1(c_t^T, F(h_t)) \quad (9.13)$$

$$\frac{\lambda_t}{1 + r_t} = \beta \mathbb{E}_t \lambda_{t+1} \quad (9.14)$$

$$p_t = \frac{A_2(c_t^T, F(h_t))}{A_1(c_t^T, F(h_t))} \quad (9.15)$$

$$p_t = \frac{w_t}{F'(h_t)} \quad (9.16)$$

$$w_t \geq \gamma \frac{w_{t-1}}{\epsilon_t} \quad (9.17)$$

$$h_t \leq \bar{h} \quad (9.18)$$

$$(\bar{h} - h_t) \left(w_t - \gamma \frac{w_{t-1}}{\epsilon_t} \right) = 0 \quad (9.19)$$

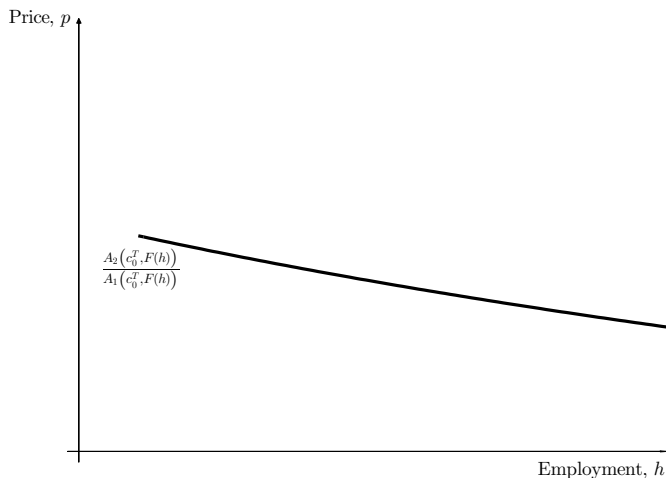
given an exchange rate policy $\{\epsilon_t\}_{t=0}^{\infty}$, initial conditions w_{-1} and d_0 , and exogenous stochastic processes $\{r_t, y_t^T\}_{t=0}^{\infty}$. To characterize the eqm we must specify the exchange-rate regime. We will turn to this next.

A Graphical Representation of (Partial) Equilibrium

- Why partial, because for the moment we take c^T as given.
- In equilibrium, $c^N = y^N = F(h)$. This means that we can draw Figures 9.2 and 9.3 in the employment-relative price of nontradables space, that is, in the space (h, p) (same as before but with h instead of y^N)
- Note that in the previous slide all markets clear except one (labor). Does this violate Walras' law?
- In a sense yes, but Walras' law is not applicable here because we are assuming that there is a price that does not adjust to disequilibrium

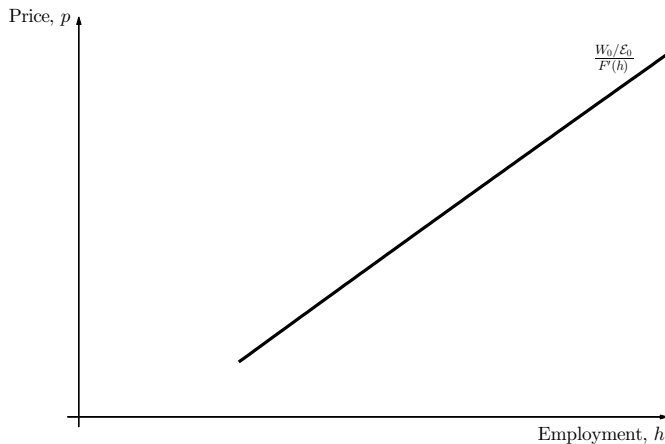
A Graphical Representation of (Partial) Equilibrium

The Demand for Nontradables

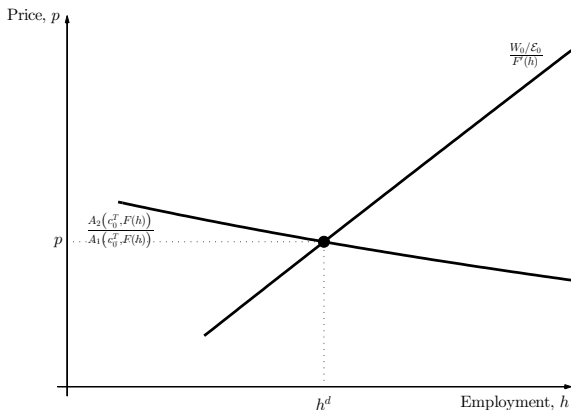


A Graphical Representation of (Partial) Equilibrium

The Supply of Nontradables

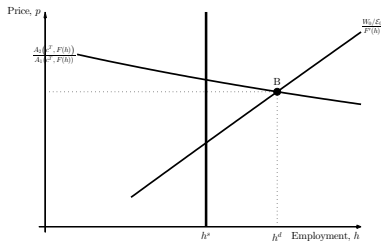


In eqm both (9.15) and (9.16) must hold. We refer to the value of h at which these schedules intersect for given W_0/ε_0 and c_0^T as labor demand, and denote it h^d

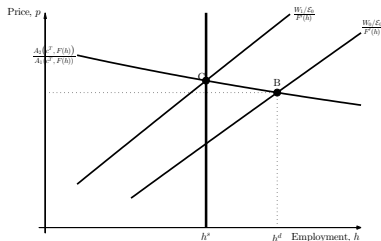


- * Next add the labor supply schedule to the graph. Two generic cases emerge:
- * At given real wages, labor demand exceeds labor supply, or
- * Labor supply exceeds labor demand.
- * Let's consider the first case first.

Suppose given W_0/ε_0 , labor demand exceeds labor supply:

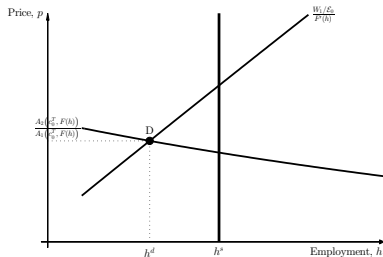


\Rightarrow real wage rises to clear the labor market, $W_0 \uparrow$, until $h^d = h^s$.



In eqm, there is full employment, $h_t = \bar{h}$

Now suppose given W_0/ε_0 , labor supply exceeds labor demand:



\Rightarrow To clear the labor market the real wage must fall, but nominal wages cannot fall due to DNWR (assume $\gamma = 1$). Thus, unless the monetary authority devalues, $\varepsilon \uparrow$, the labor market will fail to clear.

\Rightarrow In eqm, absent a devaluation there is unemployment, $h^d < \bar{h} = h^s$

\Rightarrow In the following sections, we will use the graphical framework (as well as quantitative methods) to analyze boom and bust cycles.

9.2 Currency Pegs

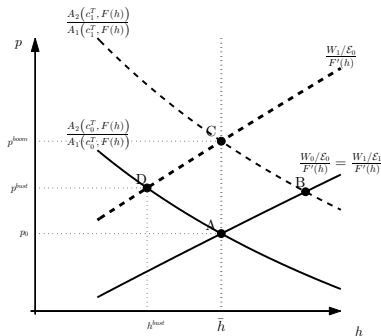
The exchange rate policy is a peg

$$\mathcal{E}_t = \mathcal{E}_0; \quad \forall t \geq 0.$$

Use the graphical apparatus just developed to show that a **boom-bust cycle** leads—as documented in Figure 9.1—to

- nominal wage growth and real appreciation during the boom phase
- involuntary unemployment and insufficient real depreciation during the bust phase

Figure 9.4 Adjustment to a Boom-Bust Cycle under a Currency Peg



Observations on Figure 9.4: Adjustment in a Boom-Bust Episode

- The initial situation is point A. At point A there is full employment, $h = \bar{h}$.
- Now, a boom starts. We capture this by an increase in c^T (perhaps because r falls).
- Given nominal wages the economy moves to point B. But at point B, there is excess demand for labor. Thus nominal wages will rise. By how much? Until the excess demand for labor has disappeared. That will be at point C.
- Thus the boom leads to an increase in nominal wages ($W \uparrow$) and a real appreciation ($p \uparrow$). The economy continues to operate at full employment.
- Next, the boom is over and the bust comes. We capture this by assuming that c^T falls back to its original level, c_0^T .
- This shifts the demand for nontradables back to its original position. The new intersection between supply and demand is at point D. At D, labor supply exceeds labor demand.
- However, because nominal wages are downwardly rigid and the nominal exchange rate is fixed, the supply schedule does not shift, (for simplicity, in the figure, we assume $\gamma = 1$).
- Thus the economy is stuck at point D. At point D, there is involuntary unemployment ($\bar{h} - h^{bust}$) and there is insufficient real depreciation, i.e., p_t does not fall enough (that is, does not fall to p_0) to restore full employment.
- With a peg there are two nominal rigidities: one structural (wages don't adjust downward) and one by policy (the exchange rate cannot move)
- The combination of these two nominal rigidities leads to a **real** rigidity
- The disequilibrium depends by how much past real wage w_{t-1} exceeds the current full employment real wage. If $\gamma < 1$ the wage will eventually go back to equilibrium
- Thus, under a currency peg w_{t-1} becomes a relevant state variable for the economy

The Link between Volatility and Average Unemployment

- The model predicts that aggregate volatility increases the mean level of unemployment.
- The fact that the **second moment has an effect on a first moment gives rise to large welfare benefits of stabilization policy.**
- This prediction is not purely due to the assumption of downward (but not upward, so the economy respond efficiently to positive shocks but not to negative shocks) nominal wage rigidity...
- ...but also due to the assumption that employment is determined by the minimum of labor demand and labor supply. (Note: key difference with Calvo-style sticky wage models in which employment is always demand determined.)
- Note that symmetric wage rigidity is enough for a link between aggregate volatility and mean unemployment but downward (asymmetric) nominal wage rigidity amplifies the connection between aggregate volatility and mean unemployment.

To see this consider the following example:

$$U(A(c_t^T, c_t^N)) = \ln c_t^T + \ln c_t^N$$

$d_t = 0$ (no access to international financial markets)

$$\Rightarrow c_t^T = y_t^T.$$

$$y_t^T = \begin{cases} 1 + \sigma & \text{prob } \frac{1}{2} \\ 1 - \sigma & \text{prob } \frac{1}{2} \end{cases}$$

$$E(y_t^T) = 1 \text{ and } \text{var}(y_t^T) = \sigma^2.$$

$$F(h_t) = h_t^\alpha$$

$$\bar{h} = 1$$

$$\mathcal{E}_t = \mathcal{E} \text{ (currency peg)}$$

$$W_{-1} = \alpha \mathcal{E}$$

The equilibrium conditions associated with this economy are (we list all except wage adjustment, for which we will consider two cases):

$$\frac{c_t^T}{c_t^N} = p_t$$

$$\alpha p_t (h_t)^{\alpha-1} = W_t / \mathcal{E}$$

$$c_t^T = y_t^T$$

$$c_t^N = h_t^\alpha$$

Step 1: Find labor demand: $h_t^d = \frac{\alpha y_t^T}{W_t / \mathcal{E}}$

Step 2: Find equilibrium labor as $h_t = \min\{\bar{h}, h_t^d\}$

Case 1: Assume bi-directional nominal wage rigidity:

$$W_t = W_{t-1} = \alpha \mathcal{E}.$$

Then, if $y_t^T \leq \bar{h} = 1$, $h_t^d = \frac{\alpha y_t^T}{W_t/\mathcal{E}} = y_t^T \leq 1$. (unemployment)

Instead, if $y_t^T > \bar{h} = 1$, $y_t^T \geq h_t^d = 1$ if $y_t^T > \bar{h} = 1$.

The equilibrium level of employment is

$$h_t = \begin{cases} 1 - \sigma & \text{if } y_t^T = 1 - \sigma \\ 1 & \text{if } y_t^T = 1 + \sigma \end{cases}$$

Let $u_t \equiv \bar{h} - h_t$ be the unemployment rate. The equilibrium distribution of u_t is:

$$u_t = \begin{cases} \sigma & \text{with probability } \frac{1}{2} \\ 0 & \text{with probability } \frac{1}{2} \end{cases}.$$

The unconditional mean of the unemployment rate is then given by

$$E(u_t) = \frac{\sigma}{2}.$$

Average level of unemployment increases linearly with the volatility of tradable endowment, in spite of the fact that wage rigidity is symmetric!

Case 2: assume ‘only’ downward nominal wage rigidity, $W_t \geq W_{t-1}$

- When $y_t^T = 1 + \sigma$, we get $\bar{h} = \frac{\alpha(y_t^T)}{W_t/\varepsilon} = \frac{\alpha(1+\sigma)}{W_t/\varepsilon}$, given that $\bar{h} = 1$, we can solve for W_t and get $W_t = \frac{\alpha(1+\sigma)}{\varepsilon}$.
- When $y_t^T = 1 - \sigma$, we would like the salary to go down, but the, **if in the previous period we had $y_t^T = 1 + \sigma$** , salary is stuck at $W_t = \frac{\alpha(1+\sigma)}{\varepsilon}$.
- Substitute y_t^T and W_t into labor demand: $h_t^d = \frac{\alpha y_t^T}{W_t/\varepsilon} = \frac{\alpha(1-\sigma)}{\alpha(1+\sigma)}$. So:

$$h_t = \begin{cases} \frac{1-\sigma}{1+\sigma} & \text{if } y_t^T = 1 - \sigma \\ 1 & \text{if } y_t^T = 1 + \sigma \end{cases}$$

- $E(u_t) = \sigma/(1 + \sigma) > \sigma/2$ (recall that σ must be less than 1).
- Thus uni-directional wage rigidity exacerbates the link between mean unemployment and volatility.

The Peg-Induced Externality

- Under a currency peg and downward nominal wage rigidity, a fall in the country interest rate r_t , can be the prelude to bad things to happen later.
- The reason is that individual agents do not internalize that during the boom nominal wages increase too much, putting the economy in a vulnerable position once the good shock fades away.
- Consider the following environment:

$$U(A(c_t^T, c_t^N)) = \ln c_t^T + \ln c_t^N$$

$$F(h_t) = h_t^\alpha; \quad 0 < \alpha < 1$$

$$\bar{h} = 1; \quad y_t^T = y^T > 0; \quad W_t \geq W_{t-1};$$

$$\beta(1+r) = 1; \quad d_0 = 0; \quad w_{-1} = \alpha y^T$$

$$r_t = \begin{cases} r & t < 0 \\ \underline{r} < r & t = 0 \\ r & t > 0 \end{cases}$$

The Peg-Induced Externality (Continued)

- Assumptions: (i) log utility function; (ii) constant tradable endowment; (iii) full downward wage rigidity ($\gamma = 1$); (iv) r is a parameter; and (v) at period 0 the economy is at full employment with zero debt
- We'll discuss the solution of the equilibrium in algebraic and graphical form.
- To help the interpretation, let's discuss intuitively what is going on:
 - i The fall in the interest rate in period 0 induces an expansion in the desired demand for consumption goods, of all types, tradables and nontradable.
 - ii The increased demand for tradables causes a trade balance deficit, a deficit in the current account, and an increase in external debt in period 0.
 - iii The increased demand for nontradables cause a rise in wages and a rise in the relative price of nontradables (i.e., an appreciation of the real exchange rate).

The Peg-Induced Externality (Continued)

- In period 1, the interest rate goes back up to its permanent value r , causing a contraction in the demand for consumption goods (both tradables and nontradables), and a reversal in the trade balance and the current account.
- The contraction in the demand for nontradables causes a derived contraction in the demand for labor.
- However, because nominal wages are downwardly rigid and the nominal exchange rate is fixed, the real wage doesn't fall, causing involuntary unemployment.
- Involuntary unemployment is highly persistent. (In fact, in this example, because $\gamma = 1$, it never disappears)

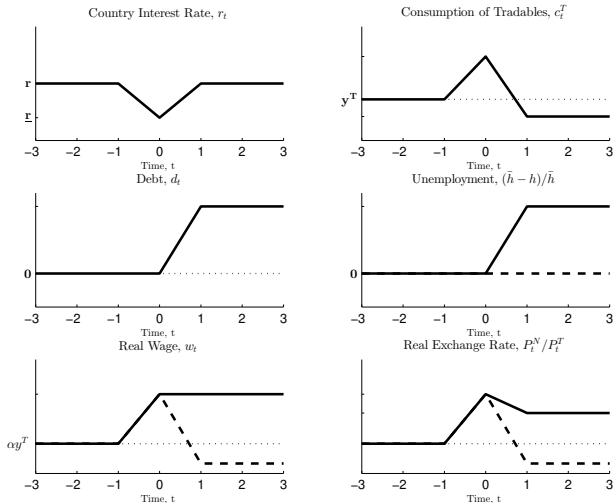
The Peg-Induced Externality (Continued)

It is possible to plug these assumptions in the model described earlier and show that the equilibrium has the following closed-form solution (we need to conjecture and then show that the debt limit is not binding, see page 303 for details):

$$\begin{aligned}c_0^T &= y^T \left[\frac{1}{1 + \underline{r}} + \frac{r}{1 + r} \right] > y^T \\c_t^T &= y^T \left[\frac{1}{1 + r} + \frac{r}{1 + r} \frac{1 + \underline{r}}{1 + r} \right] < y^T, \\d_t &= y^T \left[1 - \frac{1 + \underline{r}}{1 + r} \right] > 0, \\h_0 &= 1; \\h_1 &= h_2 \cdots = \frac{1 + \underline{r}}{1 + r} < 1\end{aligned}$$

This is the equilibrium level of unemployment. The larger the decline in r (i.e, the smaller \underline{r} is) the larger unemployment in the long run

A Temporary Decline in the Country Interest Rate: Graphical representation



———— currency peg

- - - flexible wage economy or optimal exchange rate economy

Remarks

- Note that with flexible wages the response of tradable consumption and debt is the same.
- This is because in this example preferences for tradable and nontadables are additive and separable
- However, the other variables are different.
- We are always at full employment and the real exchange rate and real wages depreciate
- The real depreciation leads consumers to switch from tradable to nontradables and to run the trade surplus necessary to service the higher external debt

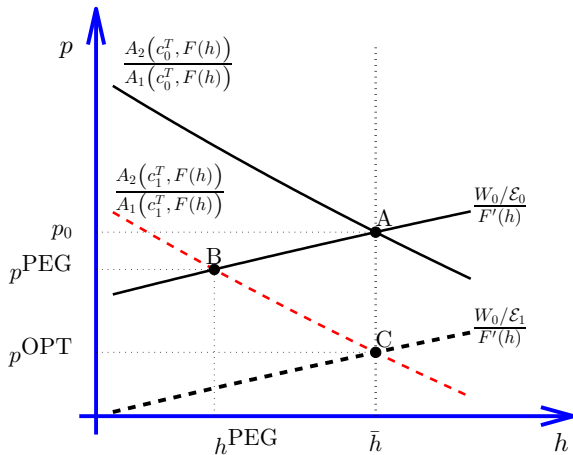
Section 9.3: Motivation

- We have just seen that under an exchange rate peg a negative external shock may lead to involuntary unemployment. How would optimal exchange rate policy look like?
- In this section we show that under optimal policy
 - i there is full employment
 - ii negative external shocks call for devaluations.
- We begin with a graphical explanation.

The graph on the next slide illustrates how the optimal exchange rate policy works

- The initial situation is point A . Suppose a negative shock (possibly an increase in the country interest rate), shifts the demand schedule down and to the left. Without government intervention, the supply schedule does not move, because W is downwardly rigid. The equilibrium would then be point B , with involuntary unemployment equal to $\bar{h} - h^{PEG}$.
- Suppose now that the government devalues the domestic currency from E_0 to $E_1 > E_0$. The devaluation lowers the real wage from W_0/E_0 to W_0/E_1 , causing the supply schedule to shift down and to the right.
- If the devaluation is just right (the dashed lines display the equilibrium under optimal exchange-rate policy.), the new supply schedule will cross the new demand schedule at point C , preserving full employment (we say preserving and not restoring full employment because the economy jumps from A to C , without visiting B).
- Note: the fall in labor cost caused by the drop in the real wage allows firms to cut prices from p_0 to p^{OPT} and induces households to switch expenditure away from tradables and toward nontradables.

Optimal Exchange-Rate Policy (again, assume $\gamma = 1$)



$c_1^T < c_0^T$ (negative shock, possibly $r_t \uparrow$)
 $\varepsilon_1 > \varepsilon_0$ (optimal devaluation)

9.3 Optimal Exchange Rate Policy

Combine optimality conditions (9.15) and (9.16) and rearrange to get

$$w_t = \frac{A_2(c_t^T, F(h_t))}{A_1(c_t^T, F(h_t))} F'(h_t)$$

Set $h_t = \bar{h}$ and get an important reference point: the **full-employment real wage**, denoted $\omega(c_t^T)$. This is defined as the real wage that clears the labor market,

$$\omega(c_t^T) \equiv \frac{A_2(c_t^T, F(\bar{h}))}{A_1(c_t^T, F(\bar{h}))} F'(\bar{h})$$

The full employment exchange rate policy ensures that $w_t = \omega(c_t^T)$. Note that the property of A assure that the function $\omega()$ is strictly increasing in c^T : $\omega'(c_t^T) > 0$

Now plug $w_t = \omega(c_t^T)$ into the downward wage rigidity condition (9.17) and set the (gross) devaluation rate, $\epsilon_t = \mathcal{E}_t / \mathcal{E}_{t-1}$, to eliminate unemployment:

$$\epsilon_t \geq \frac{\gamma W_{t-1} / \mathcal{E}_{t-1}}{\omega(c_t^T)}$$

Note: There is a whole family of optimal exchange-rate policies. Under any member of this policy, $h_t = \bar{h}$ and $w_t = \omega(c_t^T)$ for all t .

9.3 Optimal Exchange Rate Policy

- To see that $\epsilon_t \geq \frac{\gamma W_{t-1}/\epsilon_{t-1}}{\omega(c_t^T)}$ gives full employment, assume that this policy allows for $h_t < \bar{h}$ for some t .
- Then, the slackness condition (9.19: $(\bar{h} - h_t) \left(w_t - \gamma \frac{w_{t-1}}{\epsilon_t} \right) = 0$) would tell use that $w_t = \gamma w_{t-1}/\epsilon_t$.
- Solve this expression for ϵ_t and plug this value into $\epsilon_t \geq \frac{\gamma W_{t-1}/\epsilon_{t-1}}{\omega(c_t^T)}$ to obtain $w_t \leq \omega(c_t^T)$
- Use (9.15) and (9.16) to eliminate w_t and $\omega(c_t^T) \equiv \frac{A_2(c_t^T, F(\bar{h}))}{A_1(c_t^T, F(\bar{h}))} F'(\bar{h})$ to eliminate $\omega(c_t^T)$.
- This yields the inequality:

$$\frac{A_2(c_t^T, F(h_t))}{A_1(c_t^T, F(h_t))} F'(h_t) \leq \frac{A_2(c_t^T, F(\bar{h}))}{A_1(c_t^T, F(\bar{h}))} F'(\bar{h})$$

- But the left hand side is strictly decreasing in h . So, the only value that satisfies this inequality (and it satisfies it as equality) is \bar{h} .
- This negates the original assumption that $h_t < \bar{h}$

Remark on Optimal Exchange Rate Policy

Optimal exchange rate policy is:

$$\epsilon_t \geq \frac{\gamma W_{t-1} / \mathcal{E}_{t-1}}{\omega(c_t^T)}$$

- Because $\omega'(c_t^T) > 0$, optimal devaluations occur in periods of contraction of aggregate demand.
- It follows that **contractions are devaluatory as opposed to devaluations being contractionary**
- What happens if $\gamma = 0$? We get:

$$\epsilon_t \equiv \frac{\mathcal{E}_t}{\mathcal{E}_{t-1}} \geq \frac{\gamma W_{t-1} / \mathcal{E}_{t-1}}{\omega(c_t^T)}$$
$$\mathcal{E}_t \geq 0$$

Which always holds

The Ramsey Optimal Exchange-Rate Policy

- We saw that it is possible to choose an exchange rate policy that keeps unemployment at zero. But is this policy optimal? After all (in our model) people do not care about employment, they care about consumption.
- A way to check is to find the social planner's optimal exchange rate policy. It is possible to show that a policy that keeps unemployment at zero is Pareto Optimal.
- The social planner solves the problem:

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(A(c_t^T, F(h_t)))$$

- subject to (9.9) and (9.13)-(9.19).
- This is complicated. Strategy: solve a less constrained problem and then show that its solution satisfies (9.9) and (9.13)-(9.19).

Consider the less restricted problem of choosing $\{c_t^T, h_t, d_{t+1}\}_{t=0}^{\infty}$ to

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(A(c_t^T, F(h_t)))$$

subject to the following subset of the equilibrium conditions:

$$c_t^T + d_t = y_t^T + \frac{d_{t+1}}{1 + r_t} \quad (9.9)$$

$$h_t \leq \bar{h} \quad (9.18)$$

- * Clearly, a good guess for the solution for labor is $h_t = \bar{h}$ for all t or full employment at all times.
- * Intuition: One nominal friction and one instrument that can fully offset it. Hence possible to obtain the flexible wage allocation (which here coincides with the first best).
- * But to show that this is indeed the allocation under the optimal exchange-rate policy, we must show that the solution to the above social planner's problem satisfies the all of the competitive equilibrium conditions, that is, conditions (9.9) and (9.13)-(9.19).
- * To see this, proceed by construction: set λ_t to satisfy (9.13), p_t to satisfy (9.15), w_t to satisfy (9.16), ϵ_t to satisfy (9.17), (9.18) is a constraint of the social planner's problem, and so is (9.9). Because $h_t = \bar{h}$, (9.19) holds. That (9.14) holds follows from the definition of λ_t and the first-order condition of the social planner.
- * See pages 307-309

9.4 Empirical Evidence On Downward Nominal Wage Rigidity

- Downward nominal wage rigidity is the central friction in the present model \Rightarrow natural to ask if it is empirically relevant.
- Downward nominal wage rigidity has been studied empirically from a number of perspectives:
 - Evidence from micro and macro data.
 - Studies focusing on rich, emerging, and poor countries.
 - Studies focusing on formal and informal labor markets.
- **By product:** Will obtain an estimate of the parameter γ governing wage stickiness in the model (useful for quantitative analysis).

Downward Nominal Wage Rigidity: (A) Evidence From Micro Data from Developed Countries

- ① United States, 1986-1993, SIPP panel data
- ② United States, 1996-1999, SIPP panel data
- ③ United States, 1997-2016, CPS panel data
- ④ Other Developed Countries

United States, 1986-1993, SIPP panel data

Probability of Decline, Increase, or No Change in Wages

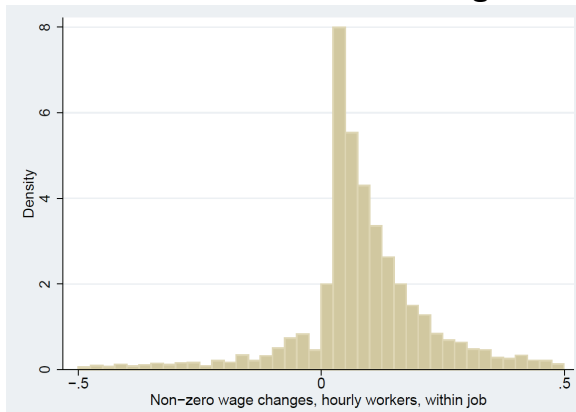
	Interviews One Year apart	
	Males	Females
Decline	5.1%	4.3%
Constant	53.7%	49.2%
Increase	41.2%	46.5%

Source: Gottschalk (2005). Note: Male and female hourly workers not in school, 18 to 55 at some point during the panel. All nominal-wage changes are within-job wage changes, defined as changes while working for the same employer. SIPP panel data.

- Large mass at 'Constant' suggests nominal wage rigidity.
- Small mass at 'Decline' suggests downward nominal wage rigidity.

United States 1996-1999, SIPP panel data

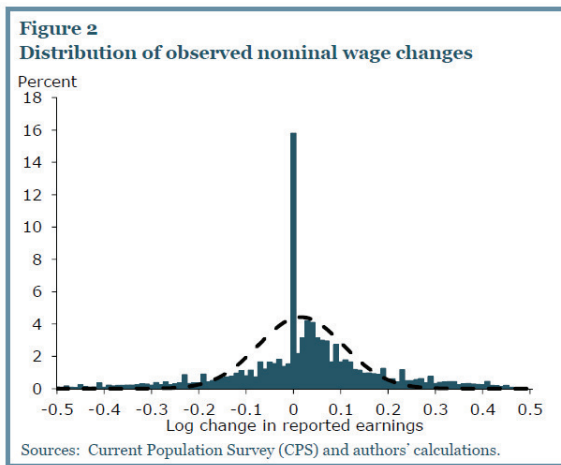
Distribution of Non-Zero Nominal Wage Changes



Source: Barattieri, Basu, and Gottschalk (2012). SIPP panel data.

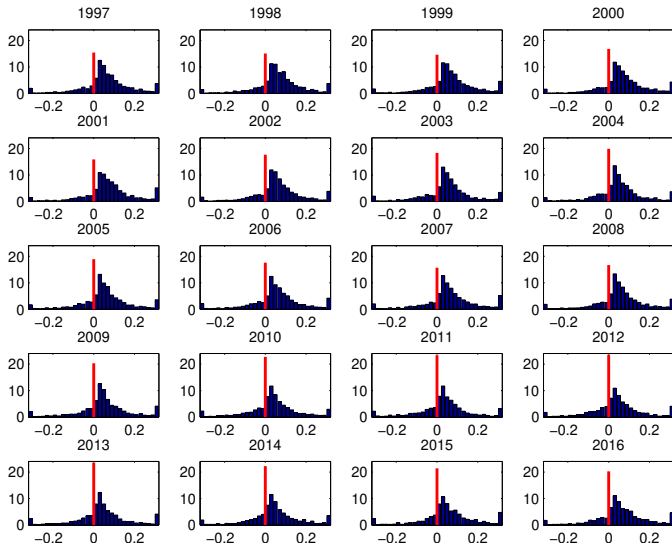
3. United States 2011, CPS data.

Distribution of Nominal Wage Changes, U.S. 2011



Source: Daly, Hobijn, and Lucking (2012).

- The next slide shows the distributions of Nominal Wage Changes in the United States for each year from 1997 to 2016, from CPS panel data
- **Nominal wage change:** Year-over-year log changes in nominal hourly wages of hourly-paid job stayers.
- In each panel, the horizontal axis shows bins of year-over-year percent change in the nominal hourly wage of an hourly-paid jobstayer. The bin size is two percent, with the exception of a wage-freeze bin, which is defined as an exact zero change.
- The vertical axis measures the share of workers in each bin.
- Each wage change distribution is based on about 5,000 workers.



Source: Jo, Schmitt-Grohé, and Uribe (2017).

Observations on the figure

- Large spike at zero wage changes.
- Many more wage increases than wage cuts.
- Fraction of wage freezes is cyclical, rises from 15 percent in 2007 to 20 percent in 2009.
- Much smaller cyclical increase in wage cuts.

4. Micro Evidence On Downward Nominal Wage Rigidity From Other Developed Countries

- Canada: Fortin (1996).
- Japan: Kuroda and Yamamoto (2003).
- Switzerland: Fehr and Goette (2005).
- Industry-Level Data: Holden and Wulfsberg (2008), 19 OECD countries from 1973 to 1999.

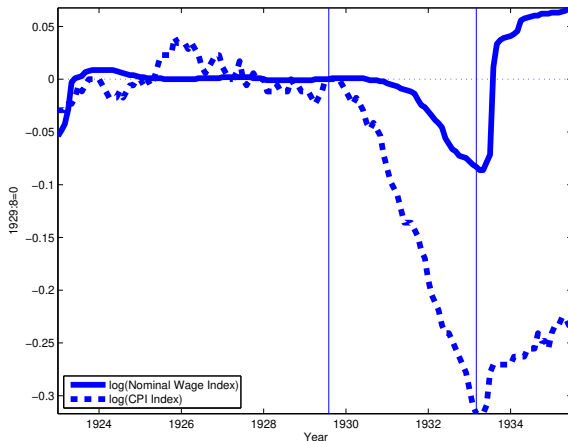
Evidence From Informal Labor Markets

- Are nominal wages downwardly flexible in informal labor markets, where labor unions, wage legislation, or regulation play, if any, a small role?
- Kaur (2012) addresses this issue by examining the behavior of nominal wages, employment, and rainfall in casual daily agricultural labor markets in rural India (500 districts from 1956 to 2008).
- Finds asymmetric nominal wage adjustment:
 - W_t increases in response to positive rainfall shocks
 - W_t failure to fall, labor rationing, and unemployment are observed in response to negative rain shocks.
- Inflation (uncorrelated with local rain shocks) tends to moderate rationing and unemployment during negative rain shocks, **suggesting downward rigidity in nominal rather than real wages.**

Evidence From the Great Depression in the U.S.

- How do nominal wages behave during extraordinary contractions?
- The next slide shows the nominal wage rate and the consumer price index in the United States from 1923:1-1935:7.
- Between 1929 and 1931 the U.S. economy experienced an enormous contraction in employment of 31%.
- Nonetheless, during this period nominal hourly wages fell by 0.6% per year, while consumer prices fell by 6.6% per year. See the figure on the next slide.
- **Massive increase in real wages in the middle of a recession**
- A similar pattern is observed during the second half of the Depression. By 1933, real wages were 26% higher than in 1929, in spite of a highly distressed labor market.

Figure 9.8 Nominal Wage Rate and Consumer Prices, United States 1923:1-1935:7

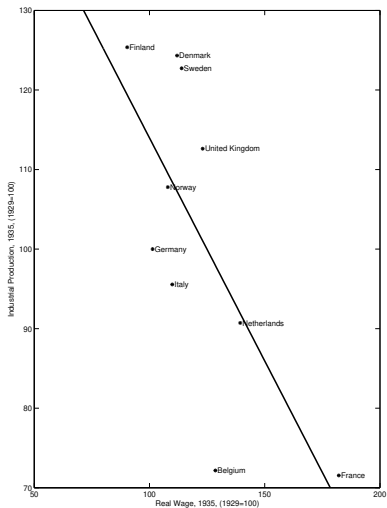


Solid line: natural logarithm of an index of manufacturing money wage rates. Broken line: logarithm of the consumer price index.

Evidence From the Great Depression In Europe

- Countries that left the gold standard earlier recovered faster than countries that remained on gold.
 - Left Gold Early (sterling bloc): United Kingdom, Sweden, Finland, Norway, and Denmark.
 - **Countries That Stuck To Gold (gold bloc):** France, Belgium, the Netherlands, and Italy.
- The gold standard is akin to a currency peg. A peg not to another currency, but to gold.
- When the sterling-bloc left gold, they effectively devalued, as their currencies lost value against gold.
- The figure on the next slide shows that between 1929 and 1935 the sterling-bloc experienced less real wage growth and a larger increase in industrial production than the gold bloc.

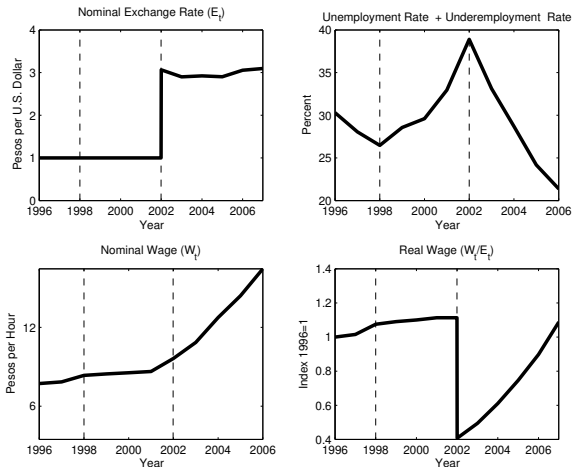
Changes In Real Wages and Industrial Production, 1929-1935



Evidence From Emerging Countries

- Argentina pegged the peso at a 1-to-1 rate to the dollar between 1991 and 2001.
- Starting in 1998, the economy was buffeted by a number of large negative shocks (weak commodity prices, large devaluation in Brazil, large increase in country premium).
- Not surprisingly, between 1998 and 2001, unemployment rose sharply. See the figure on the next slide.
- Nonetheless, nominal wages remained remarkably flat.
- This evidence is consistent with downward nominal wage rigidity, and suggests that γ , the parameter governing downward wage rigidity in the model, is about 1.
- Why $\gamma \approx 1$? The slackness condition $(\bar{h} - h_t)(W_t - \gamma W_{t-1}) = 0$ (recall $\epsilon_t = 1$ between 1991 and 2001) can be written as $(U_t)(\frac{W_t}{W_{t-1}} - \gamma) = 0$, implies that if unemployment is high, then wages must grow at the gross rate γ .
- Argentine wages were flat $\Rightarrow \gamma \approx 1$.

Argentina 1996-2006



Implied Value of γ : Around unity.

Evidence From Peripheral Europe (2008-2011)

- The next slide shows the unemployment rate and nominal wage growth between 2008:Q1 and 2011:Q2 in 12 European countries that were either in the eurozone or pegging to the euro.
- Between 2008 and 2011, all countries in the periphery of Europe experienced increases in unemployment; Some very large increases.
- In spite of extreme duress in the labor market, nominal hourly wages experienced increases in most countries and modest declines in only a few.
- The slide following the table explains how to use the information in the table to infer a range for γ .

Table 9.2 Unemployment, Nominal Wages, and γ Evidence from the Eurozone

Country	Unemployment Rate		Wage Growth	Implied Value of γ
	2008Q1 (in percent)	2011Q2 (in percent)	$\frac{W_{2011Q2}}{W_{2008Q1}}$ (in percent)	
Bulgaria	6.1	11.3	43.3	1.028
Cyprus	3.8	6.9	10.7	1.008
Estonia	4.1	12.8	2.5	1.002
Greece	7.8	16.7	-2.3	0.9982
Ireland	4.9	14.3	0.5	1.0004
Italy	6.4	8.2	10.0	1.007
Lithuania	4.1	15.6	-5.1	0.996
Latvia	6.1	16.2	-0.6	0.9995
Portugal	8.3	12.5	1.91	1.001
Spain	9.2	20.8	8.0	1.006
Slovenia	4.7	7.9	12.5	1.009
Slovakia	10.2	13.3	13.4	1.010

Note. W is an index of nominal average hourly labor cost in manufacturing, construction, and services, including the public sector (except for Spain). Source: Schmitt-Grohé and Uribe (JPE, 2016)

How To Infer γ From European Data

As explained before, the slackness condition of the model, $(W_t - \gamma W_{t-1})(\bar{h} - h_t) = 0$, implies that if unemployment is positive, then nominal wages must be growing at the rate γ : $\frac{W_t}{W_{t-1}} = \gamma$.

How to calculate γ :

$$\gamma = \left(\frac{W_{2011:Q2}}{W_{2008:Q1}} \right)^{\frac{1}{13}}$$

Subtract 0.6% per quarter to adjust for foreign inflation and long-run growth (because they are not explicitly incorporated in the model) to obtain the estimate:

$$\gamma \in [0.99, 1.022]$$

Quantitative Analysis

(Sections 9.5-9.8 and 9.10)

(We will not do in detail but discuss some of the results)

Functional Forms

Assume a CRRA form for preferences, a CES form for the aggregator of tradables and nontradables, and an isoelastic form for the production function of nontradables:

$$U(c) = \frac{c^{1-\sigma} - 1}{1-\sigma}$$

$$A(c^T, c^N) = \left[a(c^T)^{1-\frac{1}{\xi}} + (1-a)(c^N)^{1-\frac{1}{\xi}} \right]^{\frac{1}{1-\frac{1}{\xi}}}$$

$$F(h) = h^\alpha,$$

with $\sigma, \xi, a, \alpha > 0$.

Calibration

Parameter	Value	Description
γ	0.99	Degree of downward nominal wage rigidity
σ	2	Inverse Intertemp. elast. of subst.
y^T	1	Steady-state tradable output
\bar{h}	1	Labor endowment
a	0.26	Share of tradables
ξ	0.5	Intratemp. elast. of subst.
α	0.75	Labor share in nontraded sector
β	0.9635	Quarterly subjective discount factor

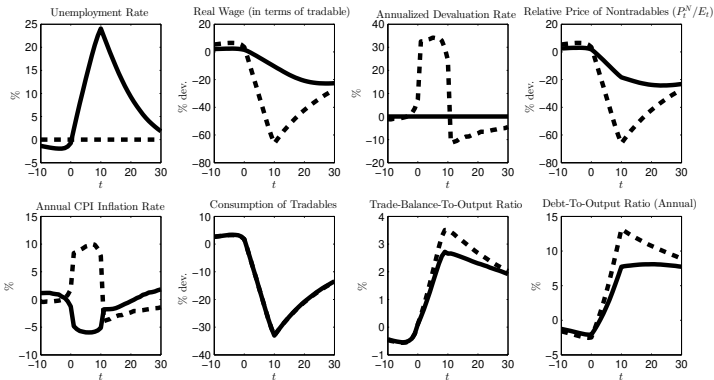
Note:

$\sigma = 2$ is widely used in business cycle analysis, and $\xi = 0.5$ is within the range of values estimated for emerging countries (see the survey by Akinici, 2011). Consequently, the restriction $\xi = 1/\sigma$ is quite compelling on empirical and computational grounds.

Crisis Dynamics Under A Currency Peg and Under Optimal Exchange-Rate Policy

- We are interested in characterizing quantitatively the response of the model economy to large contractions like the ones observed in Argentina in 2001 and in the periphery of Europe in 2008. In Argentina, for instance, traded output fell by 2 standard deviations in a period of two and a half years (10 quarters). Accordingly, we use the following operational definition of an external crisis.
- **Definition of an External Crisis.** A crisis is a situation in which in period t tradable output, y_t^T , is at or above average, and 10 quarters later, in period $t + 10$, it is at least two standard deviations below trend.
- **The Typical External Crisis:** Simulate the model for 20 million periods. Extract all windows of time in which y_t^T conforms to the definition of a crisis. For each variable of interest, average all windows and subtract its unconditional mean (i.e., the mean taken over the 20 million observations).

Pegs Amplify Negative External Shocks



———— Currency Peg

- - - Optimal Exchange-Rate Policy

Observations

- Large contraction in c^T , driven primarily by the hike in the country interest rate. The trade balance, $y_t^T - c_t^T$, actually improves in spite of the fact that y^T falls sharply. The response of c^T is independent of exchange-rate policy, because $\xi = 1/\sigma$.
- **Currency Pegs:** large increase in unemployment (25%), because the real wage does not fall sufficiently (stays 40% above the full-employment real wage). Firms don't cut prices because labor cost remains high. As a result, consumers don't switch spending away from tradables and toward nontradables.
- **Optimal Exchange-Rate Policy:** It cannot avoid the contraction in the tradable sector. But it can prevent the external crisis to spread to the nontradable sector. In fact, it preserves full employment throughout the crisis (this result was also established analytically earlier in this chapter). Large devaluations of around 30% per year for 2.5 years. Consistent with devaluations post Convertibility in Argentina. Devaluations bring the real wage down ($\varepsilon \uparrow \Rightarrow w = W/\varepsilon \downarrow$), fostering employment and allowing the real exchange to depreciate ($p = P^N/\varepsilon \downarrow$). Real depreciation facilitates expenditure switch toward nontradables.

The Welfare Costs of Currency Pegs

Find the compensation, measured as percent increase in the stream of consumption in the peg economy, denoted $\Lambda(s_t)$, that makes agents indifferent between living under a peg or under the optimal exchange-rate policy, given the current state $s_t = (y_t^T, r_t, d_t, w_{t-1})$. This compensation is implicitly given by

$$\mathbb{E} \left\{ \sum_{j=0}^{\infty} \beta^j U \left(c_{t+j}^{\text{PEG}} (1 + \Lambda(s_t)) / 100 \right) \middle| s_t \right\} = \mathbb{E} \left\{ \sum_{j=0}^{\infty} \beta^j U \left(c_{t+j}^{\text{OPT}} \right) \middle| s_t \right\},$$

Solve for $\Lambda(s_t)$ to obtain

$$\Lambda(s_t) = 100 \left\{ \left[\frac{v^{\text{OPT}}(y_t^T, r_t, d_t)(1 - \sigma) + (1 - \beta)^{-1}}{v^{\text{PEG}}(y_t^T, r_t, d_t, w_{t-1})(1 - \sigma) + (1 - \beta)^{-1}} \right]^{1/(1-\sigma)} - 1 \right\},$$

where $v^{\text{OPT}}(y_t^T, r_t, d_t) \equiv \mathbb{E} \left\{ \sum_{j=0}^{\infty} \beta^j U \left(c_{t+j}^{\text{OPT}} \right) \middle| s_t \right\}$

and $v^{\text{PEG}}(y_t^T, r_t, d_t, w_{t-1}) \equiv \mathbb{E} \left\{ \sum_{j=0}^{\infty} \beta^j U \left(c_{t+j}^{\text{PEG}} \right) \middle| s_t \right\},$

with the expectation taken over the distribution of s_t in the peg economy. The welfare cost of a peg, $\Lambda(s_t)$, is a random variable as it is a function of the state in period t , s_t . When $\sigma = 1/\xi$ only state variable that is policy dependent is w_{t-1} and c_t^T is policy independent. Thus, when $\sigma = 1/\xi$ the only source of welfare loss of suboptimal exchange rate policy stems from the dynamics of c_t^N .

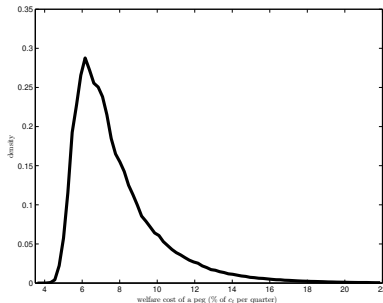
The Welfare Costs of Currency Pegs

Model	Welfare Cost		Unemployment
	Mean	Median	Mean Rate
Baseline ($\gamma = 0.99$)	7.8	7.2	11.7

Note. The welfare cost of a currency peg is expressed in percent of consumption. Welfare costs are computed over the distribution of the state $(y_t^T, r_t, d_t, w_{t-1})$ induced by the peg economy.

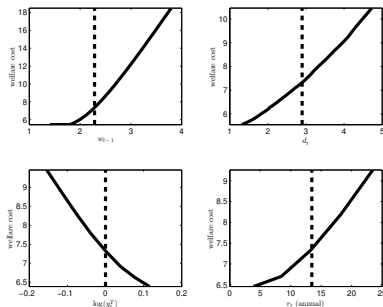
Observation: Large welfare costs of currency pegs. All of the cost is explained by lost consumption of nontradables due to unemployment in that sector.

Probability Density Function of the Welfare Cost of Currency Pegs



Observation: The distribution of welfare costs of pegs is highly skewed to the right, suggesting the existence of initial states, $(y_t^T, r_t, d_t, w_{t-1})$, in which pegs are highly costly in terms of unemployment. The next slide identifies such states.

Welfare Cost of Currency Pegs and the Initial State



Note. In each plot, all states except the one shown on the horizontal axis are fixed at their unconditional mean values. The dashed vertical lines indicate the unconditional mean of the state displayed on the horizontal axis (under a currency peg if the state is endogenous).

Observation: Currency pegs are more costly the higher the initial past wage, the higher the initial stock of external debt, the lower the initial endowment of tradables, and the higher the initial country interest rate.

Alternative Parameterizations and Model Specifications

- 9.10 Varying the Degree of Wage Rigidity
- 9.11 Symmetric Wage Rigidity
- 9.13 Endogenous Labor Supply
- 9.14 Production in the Traded Sector
- 9.15 Product Price Rigidity

Varying the Degree of Downward Wage Rigidity

Model	Welfare Cost		Unempl. Rate
	Mean	Median	
Baseline ($\gamma = 0.99$)	7.8	7.2	11.7
Lower Downward Wage Rigidity			
$\gamma = 0.98$	5.7	5.3	8.9
$\gamma = 0.97$	3.5	3.3	5.6
$\gamma = 0.96$	2.8	2.7	4.6
Higher Downward Wage Rigidity			
$\gamma = 0.995$	14.3	13.0	19.5

Observation: Sizable welfare costs and unemployment even for highly flexible wages, e.g., $\gamma = 0.96$. Recall, $\gamma = 0.96$ means that wages can fall frictionlessly by 16% per year.

Symmetric Wage Rigidity

- Is more wage flexibility always welfare increasing?
- We have just seen that the welfare costs of a currency peg increase as the degree of downward wage rigidity, γ , increases. So the answer with downward rigidity only is Yes.
- But in general, not always.
- We now consider a different way of increasing wage rigidity, namely, bi-directional wage rigidity:

$$\frac{1}{\gamma} \geq \frac{W_t}{W_{t-1}} \geq \gamma$$

- We will see that this increase in wage rigidity is welfare enhancing.

The Welfare Costs of Pegs: Symmetric Wage Rigidity ($\gamma = 0.99$)

	Welfare Cost Mean	Unempl. Rate
Downward only: $\frac{W_t}{W_{t-1}} \geq \gamma$	7.8	11.7
Upward and downward: $\frac{1}{\gamma} \geq \frac{W_t}{W_{t-1}} \geq \gamma$	3.3	5.2

- Welfare costs under symmetric rigidity, while still large, are half that under downward wage rigidity. Thus greater wage flexibility is welfare decreasing. Why? Symmetric wage rigidity alleviates the peg-induced externality (we saw this theoretically).
- To the extent that downward wage rigidity is the case of greatest empirical relevance, this result suggests that models with upward and downward wage rigidity underestimate the welfare costs of currency pegs.

Product Price Rigidity

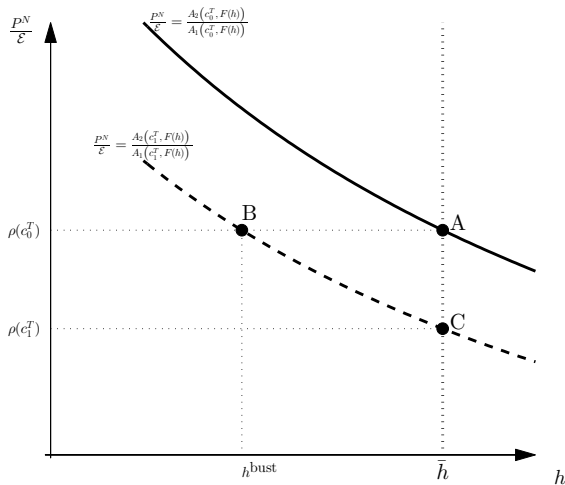
Assume now that nominal wages are fully flexible and that instead prices (of non-tradable) are sticky. Consider downward nominal price rigidity and symmetric price rigidity.

downward price rigidity:
$$\frac{P_t^N}{P_{t-1}^N} \geq \gamma_p$$

symmetric price rigidity:
$$\frac{1}{\gamma_p} \geq \frac{P_t^N}{P_{t-1}^N} \geq \gamma_p$$

Calibrate models as before, but set $\gamma = 0$ and $\gamma_p = 0.99$.

Adjustment to a Negative External Shock with Downward Price Rigidity Under A Currency Peg



$$c_1^T < c_0^T; \quad \gamma_p = 1$$

Observations:

- When c^T falls from c_0^T to c_1^T , full employment would occur if prices could decline to $\rho(c_1^T)$, point C in the figure. But because of downward nominal price rigidity, P_t^N/\mathcal{E} remains at $\rho(c_0^T)$. At this price households only demand $F(h^{bust})$ units of nontradables and the economy suffers of unemployment due to weak demand.
- Optimal policy calls for a devaluation that lowers P_t^N/\mathcal{E}_t down to $\rho(c_1^T)$ and restores full employment. Hence contractions continue to be devaluatory!
- The economy continues to suffer from a peg induced externality. Increases in P_t^N during booms should be limited to avoid unemployment during the recession phase of the cycle.

Price Rigidity And The Welfare Costs of Currency Pegs

Parameterization	Welf Cost	Unempl Rate
Baseline (wage rigidity, $\gamma = 0.99$ and $\gamma_p = 0$.)	7.8	11.7
Nominal Price Rigidity ($\gamma = 0$, $\gamma_p = 0.99$)		
Downward Price Rigidity, $P_t^N / P_{t-1}^N \geq \gamma_p$	9.9	14.1
Symmetric Price Rigidity, $1/\gamma_p \geq P_t^N / P_{t-1}^N \geq \gamma_p$	4.4	6.6
Calvo Price Rigidity, $\theta = 0.7$	3.6	N/A

Note. The welfare cost of a currency peg is expressed in percent of consumption per quarter. Unemployment rates are expressed in percent.

Observations.

- The welfare costs of pegs under downward price rigidity are as large as under downward wage rigidity. The welfare costs under symmetric price rigidity are only about half as large as under downward price rigidity. It follows that adding upward price rigidity ameliorates the peg induced externality, just like adding upward wage rigidity ameliorates it in the model model with wage rigidity.
- The last line, labeled Calvo pricing, pertains to an economy with Calvo-type price rigidity in the nontraded sector. The probability of not being able to change the nominal price is $\theta = 0.7$ per period. The calibration of the shocks is the same as in the baseline model.
- The Calvo model is one of symmetric price rigidity and so it is not surprising that the welfare costs of pegs are most similar to those associated with the economy with the bi-directional price rigidity studied earlier.

Summary of Theoretical results:

● Analytical results

- * The combination of a currency peg and downward nominal wage rigidity creates an externality that amplifies the severity of contractions.
- * The model predicts that the average rate of unemployment is increasing in the degree of aggregate volatility.
- * The DNWR model predicts significant amplification of booms-bust cycles under suboptimal monetary policy (and in particular under currency pegs.).

Summary of Theoretical results:

● Quantitative Results:

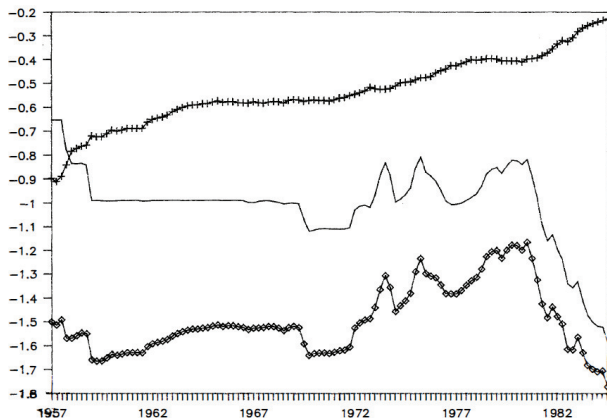
- * The costs of currency pegs due to downward nominal wage rigidity are large:
 - * in terms of welfare, 4 to 10% of consumption per period.
 - * and in terms of unemployment, 10 to 30%.
- * These results are robust to a variety of changes parameter values and model specifications, including endogenous labor supply, bidirectional nominal rigidity, and product price rigidity.
- * The welfare costs of currency pegs are higher the higher the past real wage, the higher the level of external debt, the lower the level of tradable output, and the higher the country interest rate.

The Mussa Puzzle: Section 9.12

- Mussa (1986) provides evidence against the “nominal exchange rate regime neutrality” hypothesis.
- Methodology: compare variations in nominal and real exchange rates in quarterly data from 13 industrialized countries during the fixed exchange rate period (1957-1970) with variations in the floating exchange rate period (1973-1984)
- Observables:
 \mathcal{E}_t = nominal exchange rate vis-à-vis the U.S. dollar
 P_t^* = U.S. Consumer Price Index
 P_t = Domestic Consumer Price Index
 $RER_t = \frac{\mathcal{E}_t P_t^*}{P_t}$
- Take a look at the next figure. It shows 3 series: the natural logarithms of the dollar French franc (i) nominal and (ii) real exchange rates as well as the (iii) log of the relative CPIs.

FIGURE 1

France v. U.S



Source: This is figure 1 of Mussa 1986. Solid line: nominal exchange rate, \mathcal{E}_t ; plus-line: ratio of CPIs, P_t^*/P_t ; diamond line: real exchange rate, $RER_t = \mathcal{E}_t P_t^*/P_t$; variables are shown in natural logarithms.

Comments on the figure:

- Quarterly data, 1957:Q2 to 1984:Q3.
- Fixed exchange rate: Bretton Woods, 1957:Q2-1970:Q4.
- French franc was pegged to the U.S. dollar.
- Floating exchange rate: Post-Bretton Woods, 1973:Q1-1984:Q3.
- Selection of sample period: Why a gap in the sample period? Mussa argues that 1970Q4 to 1973Q1 is a transition period. The float starts in March 1973.
- Why does Mussa start his sample in 1957, this was after all not the beginning of the Bretton Woods agreement?
- Mussa justifies the start date by saying that this is when the IFS **tapes** start having the raw data.
- Why does Mussa end the sample 1983Q3? This must have been the most recent observation when he wrote the paper.

- The graph shows that during the Bretton Woods period (1957-1970) the nominal exchange rate was basically constant.
- In the post-Bretton Woods period (1973-1984) it fluctuates quite a bit.
- This confirms treating these two periods as **having a different nominal exchange rate regime**.
- The striking feature of the graph is that the real exchange rate mimics the behavior of the nominal exchange rate throughout.
- The flipside of this observation is that the times series properties of relative prices, P_t^*/P_t , remained remarkable constant across the two periods.
- Mussa argues that under the 'nominal exchange rate regime neutrality' hypothesis one should not observe such a high correlation between nominal and real exchange rates.

Mussa documents three facts:

- 1 The standard deviation of the real depreciation rate is smaller during the peg era.
- 2 The standard deviations and serial correlations of the nominal and real depreciation rates are similar to each other in each exchange rate regime.
- 3 The volatility of national CPI inflation is about the same during the fixed and the floating regime sample period.

Mussa documents three facts:

Let

$$\epsilon_t^{RER} \equiv \ln RER_t - \ln RER_{t-1}$$

$$\epsilon_t \equiv \ln \mathcal{E}_t - \ln \mathcal{E}_{t-1}$$

$$\epsilon_t^P \equiv \ln P_t^*/P_t - \ln P_{t-1}^*/P_{t-1}$$

Illustrate the 3 facts for France

France	1957Q2-1970Q4	1973Q1-1984Q3
$\text{var}(\epsilon_t^{RER})$	5.258	23.590
$\text{var}(\epsilon_t)$	8.197	24.275
$\text{corr}(\epsilon_t^{RER}, \epsilon_{t-1}^{RER})$	0.1150	0.3376
$\text{corr}(\epsilon_t, \epsilon_{t-1})$	0.1776	0.4317
$\text{var}(\epsilon_t^P)$	1.543	0.540

Taken from Table 1.4 of Mussa (1986)

Theoretical Implications of the Mussa Puzzle

- The Mussa facts are referred to as a puzzle because they suggest that relative prices depend on the behavior of nominal prices. This is inconsistent with flexible-price models in the RBC tradition, possibly augmented with a demand for money, which represented the predominant paradigm around the time of Mussa's writing (e.g., Stockman, 1988).
- The Mussa puzzle suggests a role for nominal rigidities, which gained renewed popularity since the mid 1990s.
- Here, we wish to address two questions:
 - a Are the predictions of the DNWR model consistent with the Mussa facts.
 - b When seen through the lens of the DNWR model, do the Mussa facts tell us anything about the optimality or not of the monetary and exchange-rate regime post Bretton Woods?

Real and Nominal Exchange Rates Under Fixed And Floating Exchange-Rate Regimes as Predicted by the DNWR Model

	Peg	Float	
		Optimal	Anti optimal
$\text{std}(\epsilon_t^{RER})$	12.0	32.5	5.2
$\text{std}(\epsilon_t)$	0	45.2	44.0
$\text{corr}(\epsilon_t^{RER}, \epsilon_{t-1}^{RER})$	0.18	-0.04	-0.04
$\text{corr}(\epsilon_t, \epsilon_{t-1})$	—	-0.04	0.95
$\text{corr}(\epsilon_t^{RER}, \epsilon_t)$	—	0.99	-0.15
$\text{std}(\pi_t)$	13.2	13.2	44.3

Note. Standard deviations are expressed in percent per year. The optimal floating exchange-rate policy is given by $\epsilon_t = w_{t-1}/\omega(c_t^T)$, and the suboptimal floating exchange-rate policy is given by $\epsilon_t = \omega(c_t^T)/w_{t-1}$.

Observations on the Table

- In line with Mussa's first fact, the predicted standard deviation of the real depreciation rate is much larger under the optimal floating exchange rate policy than under the peg (first line of the table).
- In line with Mussa's second fact, under the optimal flexible exchange rate regime, the nominal and real exchange rates have similar standard deviation and first-order serial correlations, and are highly positively contemporaneously correlated (lines 2-5 of the table).
- In line with Mussa's third fact, the predicted volatility of CPI inflation is the same under the peg and the optimal floating regime (last line of the table).

Was Post-Bretton-Woods Exchange-Rate Policy Optimal?

- Often, empirical studies classify exchange-rate regimes into fixed or floating, and then derive stylized facts associated with each regime. This practice is problematic because in reality there is an infinite family of floating exchange-rate regimes, not just one, which can induce different dynamics of nominal and real variables.
- As an illustration, consider the 'anti optimal' exchange rate policy:

$$\epsilon_t = \frac{\omega(c_t^T)}{w_{t-1}}.$$

which is the inverse of the optimal policy studied in the quantitative analysis.

- The last column of the previous table shows that under the anti optimal floating exchange-rate policy, the model fails to capture all three of the Mussa facts.
- Seen through the lens of the DNWR model, it follows that Mussa's facts can be interpreted as suggesting that during the early post-Bretton-Woods period, overall, the dynamics of inflation and nominal and real exchange rates were consistent with the optimal exchange rate policy.

Staggered Price Setting: The Calvo Model

- Is *the* canonical models used in monetary economics. Proposed by Calvo (1983) and later refined by Woodford (1996) and Yun (1996)
- Assumes staggered price setting
- Price rigidity is bidirectional, that is, the upward and downward adjustment of nominal prices is sluggish
- Differences wrt the models of nominal rigidities studied earlier in the chapter:
 - 1 firms are forced to satisfy demand even if the price is below marginal cost. An implication of this difference is that in the Calvo model the labor supply must be wage elastic for price stickiness to have first-order effects
 - 2 Calvo model assumes imperfect competition in product markets. This assumption allows firms be price setters and to have nonzero finite demand even when their prices differ from those of their closest competitors.
 - 3 Wages are flexible and there is no involuntary unemployment.
- **Jump to slide 122**

Households

$$\max E_0 \sum_{t=0}^{\infty} \beta^t [U(c_t) - V(h_t)] \quad (9.41)$$

As before, we assume that the consumption good is a composite of tradable and nontradable consumption goods with the aggregation technology

$$c_t = A(c_t^T, c_t^N) \quad (9.42)$$

The constraints are:

$$P_t^T c_t^T + P_t^N c_t^N + \varepsilon_t d_t = P_t^T y_t^T + W_t h_t + \Phi_t + T_t + \frac{\varepsilon_t d_{t+1}}{1 + r_t} \quad (9.43)$$

$$d_{t+1} \leq \bar{d} \quad (9.44)$$

Assume law of one price holds for tradables and foreign price of tradables is constant and equal to 1

$$P_t^T = \mathcal{E}_t$$

Optimality conditions of the household's problem under (9.42)-(9.44) are:

$$\frac{A_2(c_t^T, c_t^N)}{A_1(c_t^T, c_t^N)} = p_t$$

$$\lambda_t = U'(c_t)A_1(c_t^T, c_t^N)$$

$$\frac{\lambda_t}{1 + r_t} = \beta E_t \lambda_{t+1} + \mu_t$$

$$\mu_t \geq 0$$

$$\mu_t(d_{t+1} - \bar{d}) = 0$$

$$V'(h_t) = \lambda_t \frac{W_t}{P_t^T}$$

The last condition is the labor supply schedule and it states that the number of hours supplied is increasing the real wage (W_t/P_t^T) and the marginal utility of wealth λ_t

Firms Producing Final Nontraded Goods

Production technology: Dixit Stiglitz production function in which the nontradable good is produced with a continuum variety of intermediate nontraded inputs

$$y_t^N = \left[\int_0^1 \left(a_{it}^N \right)^{1-\frac{1}{\mu}} di \right]^{\frac{1}{1-\frac{1}{\mu}}}, \quad (9.45)$$

y_t^N = output of the final nontraded good

a_{it}^N = quantity of intermediate goods of type $i \in [0, 1]$ used in the production of the final nontraded good

$\mu > 1$ elasticity of substitution across varieties

Environment: perfect competition

Firm profits:

$$P_t^N y_t^N - \int_0^1 P_{it}^N a_{it}^N di,$$

Profit maximization implies intermediate input demand of the form

$$a_{it}^N = y_t^N \left(\frac{P_{it}^N}{P_t^N} \right)^{-\mu} \quad (9.46)$$

Demand for the intermediate good of variety i is increasing in the level of final output and decreasing in the relative price of the variety in terms of the final good (P_{it}^N), with a price elasticity of $-\mu$.

Using this expression to eliminate a_{it}^N from the Dixit-Stiglitz aggregator (9.45) yields

$$P_t^N = \left[\int_0^1 (P_{it}^N)^{1-\mu} di \right]^{\frac{1}{1-\mu}} \quad (9.47)$$

Price of the final nontraded good is increasing and homogeneous of degree one in the price of the intermediate nontraded goods

Firms Producing Nontraded Intermediate Goods

Production technology for variety i

$$y_{it}^N = h_{it}^\alpha; \quad \alpha \in (0, 1] \quad (9.48)$$

Environment: Monopolistic competition, firms are price setters

Production is demand determined. This means that, given the posted price P_{it}^N , firms must set production to ensure that all customers are served, that is,

$$y_{it}^N = a_{it}^N \quad (9.49)$$

Profits

$$\Phi_{it} = P_{it}^N a_{it}^N - (1 - \tau) W_t h_{it}$$

τ = a labor subsidy to offset distortions introduced by imperfect competition. Its presence facilitates the characterization of optimal monetary policy, as it results in a model with a single distortion, namely the one stemming from price rigidity. That is, monetary policy here will not be called up to correct distortions stemming from imperfect competition.

Price setting problem of producer of variety i

Use (9.46), (9.48), and (9.49) to eliminate a_{it}^N , h_{it} , and y_{it}^N , respectively, from the expression for profits in period t :

$$P_{it}^N y_t^N \left(\frac{P_{it}^N}{P_t^N} \right)^{-\mu} - (1 - \tau) W_t y_t^{N \frac{1}{\alpha}} \left(\frac{P_{it}^N}{P_t^N} \right)^{-\frac{\mu}{\alpha}} .$$

Under price flexibility, the optimal pricing decision consists in choosing P_{it}^N to maximize the above expression.

Under price stickiness, the pricing problem is different because firms, by assumption, cannot reoptimize prices every period.

With probability $\theta \in (0, 1)$ a firm cannot reset its price in period t and must charge the same price as in the previous period, and with probability $1 - \theta$ it can adjust the price freely. Consider the pricing decision of a firm that can reoptimize its price in period t .

- Let \tilde{P}_{it}^N denote the price chosen in t . Then, with probability θ , the price will continue to be \tilde{P}_{it}^N in period $t + 1$. With probability θ^2 , the price will continue to be \tilde{P}_{it}^N in period $t + 2$, and so on. In general, with probability θ^s the price will continue to be \tilde{P}_{it}^N in period $t + s$.
- The original Calvo (1983) formulation assumes that \tilde{P}_{it}^N is set following an ad hoc rule of thumb. The innovation introduced by Yun (1996) is to assume that the firm picks \tilde{P}_{it}^N in a profit-maximizing fashion.
- As in much of the modern new-Keynesian literature (e.g., Woodford, 2003), we adopt Yun's approach. Specifically, the present discounted value of profits associated with \tilde{P}_{it}^N is given by

$$E_t \sum_{s=0}^{\infty} Q_{t,t+s} \theta^s \left[\tilde{P}_{it}^N y_{t+s}^N \left(\frac{\tilde{P}_{it}^N}{P_{t+s}^N} \right)^{-\mu} - (1 - \tau) W_{t+s} y_{t+s}^N \frac{1}{\alpha} \left(\frac{\tilde{P}_{it}^N}{P_{t+s}^N} \right)^{-\frac{\mu}{\alpha}} \right],$$

$Q_{t,t+s}$ = state-contingent nominal discount factor that converts nominal payments in period $t + s$ into a nominal payment in period t .

The firm picks \tilde{P}_{it}^N to maximize the pdv of profits. The FOC is:

$$E_t \sum_{s=0}^{\infty} Q_{t,t+s} \theta^s y_{t+s}^N \left(\frac{\tilde{P}_{it}^N}{P_{t+s}^N} \right)^{-\mu} \left\{ \frac{\mu-1}{\mu} \tilde{P}_{it}^N - \frac{1}{\alpha} (1-\tau) W_{t+s} \left[y_{t+s}^N \left(\frac{\tilde{P}_{it}^N}{P_{t+s}^N} \right)^{-\mu} \right]^{\frac{1-\alpha}{\alpha}} \right\} = 0. \quad (9.50)$$

$\frac{\mu-1}{\mu} \tilde{P}_{it}^N$ = marginal revenue in period $t+s$

$\frac{1}{\alpha} (1-\tau) W_{t+s} \left[y_{t+s}^N \left(\frac{\tilde{P}_{it}^N}{P_{t+s}^N} \right)^{-\mu} \right]^{\frac{1-\alpha}{\alpha}} = \frac{(1-\tau) W_{t+s}}{\alpha (h_{it+s}^N)^{\alpha-1}}$ = marginal cost in period $t+s$.

$y_{t+s}^N \left(\frac{\tilde{P}_{it}^N}{P_{t+s}^N} \right)^{-\mu}$ = quantity sold in period $t+s$

- Price set in period t equates the present discounted values of marginal revenues and marginal costs weighted by the level of production
- Notice that the first-order condition (9.50) has only one firm specific variable, namely, \tilde{P}_{it}^N . It follows that any firm i regardless of its history will charge the same price in period t if it gets the change to reoptimize the price. This feature of the Calvo-style model greatly facilitates aggregations.

Aggregation and Equilibrium: Aggregating inflation in the non-traded sector (9.16.4)

Use the fact that every firm that can change the price in period t will choose the same price in the definition of the price index for nontradables, equation (9.47) becomes

$$\begin{aligned}(P_t^N)^{1-\mu} &= \int_0^1 (P_{it}^N)^{1-\mu} di \\ &= \theta P_{t-1}^N{}^{1-\mu} + (1-\theta) (\tilde{P}_t^N)^{1-\mu}.\end{aligned}$$

Dividing both sides of the above expression by $(P_t^N)^{1-\mu}$ yields

$$1 = \theta(\pi_t^N)^{\mu-1} + (1-\theta)(\tilde{p}_t^N)^{1-\mu},$$

where $\pi_t^N \equiv P_t^N / P_{t-1}^N$ denotes the gross rate of inflation of nontradables, and $\tilde{p}_t^N \equiv \tilde{P}_t^N / P_t^N$ denotes the relative price of reoptimized prices in terms of final nontraded goods.

Aggregate output in the nontraded sector and total hours worked

Total hours worked, denoted h_t , is the sum of hours worked across all intermediate goods producing firms, that is,

$$h_t = \int_0^1 h_{it} di.$$

To obtain a relationship between total hours worked and aggregate output of nontradables, use (9.48) to eliminate h_{it} from the above expression. Then we have that $h_t = \int_0^1 y_{it}^{1/\alpha} di = \int_0^1 \left[y_t^N \left(\frac{P_{it}^N}{P_t^N} \right)^{-\mu} \right]^{1/\alpha} di = y_t^{N^{1/\alpha}} \int_0^1 \left(\frac{P_{it}^N}{P_t^N} \right)^{-\mu/\alpha} di$. Let

$$s_t \equiv \int_0^1 \left(\frac{P_{it}^N}{P_t^N} \right)^{-\mu/\alpha} di.$$

The variable s_t measures price dispersion. If all i varieties sell for the same price, then $s_t = 1$, otherwise, as will become clear shortly, $s_t \geq 1$. Using the definition of s_t in the above expression yields

$$y_t^N = s_t^{-\alpha} h_t^\alpha.$$

This expression shows that price dispersion acts as a negative productivity shock. The higher is price dispersion, the lower will be the aggregate level of output associated with a given level of employment. When prices are fully flexible, all firms charge the same price ($P_{it}^N = P_t^N$), so $s_t = 1$ and $y_t^N = h_t^\alpha$. It follows that s_t represents a measure of output loss due to the presence of price rigidity.

Competitive Equilibrium

A competitive equilibrium is a set of processes c_t^T , π_t^N , p_t , h_t , λ_t , w_t , \tilde{p}_t^N , y_t^N , s_t , d_{t+1} , μ_t , $pvmc_t$, and $pvmr_t$ satisfying

$$c_t^T + d_t = y_t^T + \frac{d_{t+1}}{1 + r_t}, \quad (1)$$

$$\lambda_t = U'(A(c_t^T, y_t^N))A_1(c_t^T, y_t^N), \quad (2)$$

$$\frac{\lambda_t}{1 + r_t} = \beta E_t \lambda_{t+1} + \mu_t, \quad (3)$$

$$d_{t+1} \leq \bar{d}, \quad (4)$$

$$\mu_t \geq 0,$$

$$\mu_t(d_{t+1} - \bar{d}) = 0,$$

$$\frac{A_2(c_t^T, y_t^N)}{A_1(c_t^T, y_t^N)} = p_t, \quad (5)$$

$$V'(h_t) = \lambda_t w_t, \quad (6)$$

$$y_t^N = s_t^{-\alpha} h_t^\alpha, \quad (7)$$

$$s_t = \theta s_{t-1} (\pi_t^N)^{\mu/\alpha} + (1 - \theta) (\tilde{p}_t^N)^{-\mu/\alpha}, \quad (8)$$

$$1 = \theta (\pi_t^N)^{\mu-1} + (1 - \theta) (\tilde{p}_t^N)^{1-\mu}, \quad (9)$$

$$pvmr_t = pvmc_t, \quad (10)$$

$$pvmc_t = \frac{1 - \tau}{\alpha} (y_t^N)^{\frac{1}{\alpha}} w_t (\tilde{p}_t^N)^{-\frac{\mu}{\alpha}} + \beta \theta E_t \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{\tilde{p}_t^N}{\tilde{p}_{t+1}^N} \frac{1}{\pi_{t+1}^N} \right)^{-\frac{\mu}{\alpha}} pvmc_{t+1}, \quad (11)$$

$$pvmr_t = \frac{\mu - 1}{\mu} y_t^N p_t (\tilde{p}_t^N)^{1-\mu} + \beta \theta E_t \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{\tilde{p}_t^N}{\tilde{p}_{t+1}^N} \frac{1}{\pi_{t+1}^N} \right)^{1-\mu} pvmr_{t+1}, \quad (12)$$

$$p_t = p_{t-1} \frac{\pi_t^N}{\epsilon_t}, \quad (13)$$

given exogenous processes y_t^T and r_t , initial conditions d_0 and s_{-1} , an exchange-rate policy ϵ_t , and a subsidy policy τ .

Following the standard in the literature, we assume that the government sets the labor subsidy so as to offset the distortion in the labor market created by the presence of imperfect competition in product markets, that is,

$$\tau = \frac{1}{\mu}. \quad (14)$$

9.16.6 Optimal Monetary Policy

With this subsidy policy in place, and if there is no initial price dispersion, $s_{\hat{a}1} = 1$ the policy of full price stabilization in the nontraded sector

$$\pi_t^N = 1$$

is consistent with the solution to the social planner's problem of the flexible price economy. Hence optimal monetary policy fully stabilizes prices in the sector with nominal rigidities.

What exchange rate supports the optimal monetary policy? Use the equilibrium condition (15) to obtain

$$\epsilon_t = \frac{p_{t-1}}{p_t}$$

It follows that any shock that causes a real exchange rate depreciation (i.e., causes p_t to fall) in the flexible-price equilibrium—such as a contraction in tradable output y_t^T or an increase in the world interest rate R_t^* , must be accompanied by a devaluation. (For example, this says that during the global financial crisis when borrowing conditions on world capital market worsened, countries should have devalued.) Thus, the present model shares two important predictions with the models of downward nominal wage rigidity or downward nominal price rigidity studied earlier in this chapter:

- stabilization of the nominal price of nontradables is optimal.
- contractions are devaluatory.

9.16.8 Crisis Dynamics in the Calvo Model

Numerical Solution technique: use perturbation instead of global methods. They are applicable here as we do not have occasionally binding constraints (no downward nominal wage rigidity). But perturbation methods force us to induce stationarity in a different way than earlier in the chapter. Instead of borrowing limit plus uncertainty, following Schmitt-Grohé and Uribe (2003), we assume that

$$r_t = r_t^* + \psi \left[e^{d_{t+1} - \bar{d}} - 1 \right], \quad (15)$$

where r_t is now an endogenous variable and r_t^* is an exogenous variable.

Functional Forms and Calibration

As in DNWR model, assume

$$U(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma},$$

$$A(c^T, c^N) = \left[a(c^T)^{1-\frac{1}{\xi}} + (1-a)(c^N)^{1-\frac{1}{\xi}} \right]^{\frac{1}{1-\frac{1}{\xi}}},$$

$$F(h) = h^\alpha,$$

$$V'(h) = \varphi(\bar{h} - h)^{-\chi},$$

Calibration of the Calvo Model

Parameter	Value	Description
θ	0.7	Probability of no price change in nontraded sector
μ	6	Elasticity of subst. across intermediate nontradables
σ	2	Inverse of intertemporal elasticity of consumption
β	1.0316^{-1}	Quarterly subjective discount factor
φ	1.11	Preference parameter
χ	1	Preference parameter
\bar{h}	3	Labor endowment
a	0.26	Share of tradables
ξ	0.5	Elasticity of substitution between tradables and nontradables
α	0.75	Labor share in nontraded sector
ψ	0.0000335	Parameter of debt-elastic interest rate
\bar{d}	2.9014	Parameter of debt-elastic interest rate
y^T	1	Steady-state tradable output
r^*	0.0316	Steady-state interest rate (quarterly)

Note. The time unit is a quarter.

Observations on the Calibration:

- The values assigned to σ , a , ξ , α , y^T , and r^* are same as in DNWR baseline model.
- The values assigned to φ , χ , and \bar{h} are same as in DNWR model with endogenous labor supply.

This will make the predictions comparable ... hopefully.

2 new parameters relative to DNWR model:

- θ This is the probability of not being able to change the price in a given period
- There is almost no empirical evidence on frequency of nominal price changes of non-tradables in emerging countries? Maybe because for much of the postwar period emerging countries experienced relatively high levels of inflation.
- An exception to the dearth of evidence is Gagnon (2009). He uses Mexican micro consumer-price data and reports the monthly frequency of nominal price changes in nonregulated services in Mexico slightly below 10 percent over two low-inflation periods, one preceding and the other following the 1994 “Tequila” crisis. This evidence suggests a quarterly probability of no price change of nontradables of 70 percent.
- Accordingly, we set θ equal to 0.7.

2 new parameters relative to DNWR model:

- ψ this is the parameter governing the debt elasticity of the interest rate.
- We calibrate ψ to match the unconditional standard deviation of (percentage deviations from trend of) tradable consumption of 18.5 percent implied by the models studied earlier in this chapter.
- Those models do not incorporate a debt-elastic interest rate. Instead, stationarity is induced by setting $\beta(1+r) < 1$ and approximating the equilibrium with global methods.
- The implied value of ψ is

$$\psi = 0.000335$$

Driving Forces:

$$\begin{bmatrix} \ln y_t^T \\ \ln \frac{1+r_t^*}{1+r^*} \end{bmatrix} = \begin{bmatrix} 0.79 & -1.36 \\ -0.01 & 0.86 \end{bmatrix} \begin{bmatrix} \ln y_{t-1}^T \\ \ln \frac{1+r_{t-1}^*}{1+r^*} \end{bmatrix} + \Gamma \epsilon_t, \quad (9.31)$$

with

$$\Sigma_\epsilon = I; \quad \Gamma \Gamma' = \begin{bmatrix} 0.00123 & -0.00008 \\ -0.00008 & 0.00004 \end{bmatrix}; \quad r^* = 0.0316,$$

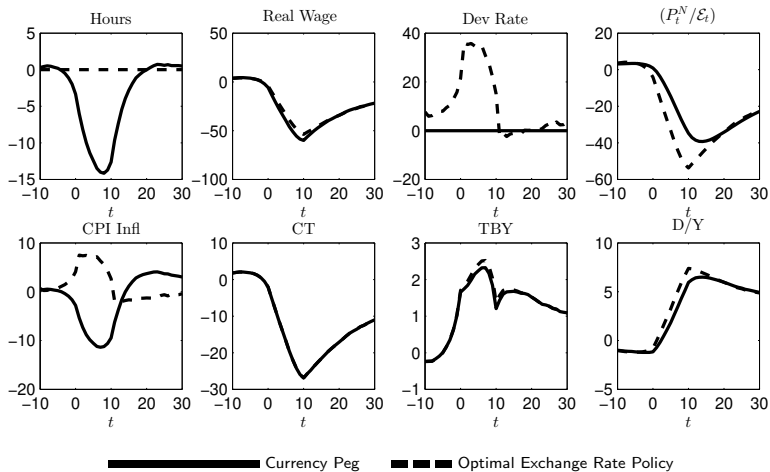
This is the same process as the one used in the DNWR model.

As for the DNWR model, we wish to characterize dynamics in a during a large external crisis

- under a currency peg, and
- under optimal monetary policy

Recall the definition of large external crisis: $\hat{y}_0^T \geq 0$ in quarter 0 and $\hat{y}_{10}^T \leq 2\sigma_y^T$

Crisis Dynamics in the Calvo Model: The Role Of Exchange-Rate Policy



Observations on the figure

- * Under optimal monetary policy employment is unaffected by the large external shocks. This prediction of the Calvo model is akin to the full-employment result obtained under optimal exchange-rate policy in the DNWR model.
- * There are large devaluations throughout the crisis in the order of 30 to 40 percent per year. The devaluations allow the price of nontradables, which is rigid in nominal terms, to decline in real terms, that is, they allow the real exchange rate to depreciate.
- * The real depreciation induces an expenditure switch away from tradables and toward nontradables.
- * The primary role of the optimal policy is to prevent the external crisis from spreading to the nontraded sector. By contrast, under the currency peg, employment falls by about 15 percent. Why, price stickiness prevents the real exchange rate to depreciate.
- * Because preferences are separable in tradable and nontradable consumption ($\sigma = 1/\xi$), the response of c_t^T is identical (down 25%) under the peg and the optimal monetary policy.
- * The dynamic responses predicted by the Calvo and DNWR models are largely similar with **one important exception**:
- * Under a peg, the real wage declines in the Calvo model but not in the DNWR model. This is so because nominal wages are flexible and firms, rationed by weak demand, cut their labor demand, which drives the market clearing wage down.
- * This prediction is counterfactual. During crises in fixed-exchange-rate economies or in monetary unions (see, for instance, the cases of Argentina 1998-2001 and the periphery of Europe 2008-2011) private sector real wages failed to fall.