Chapter III

The Basic Model of Intertemporal Trade

III.1 Introduction and Overview

The review of balance of payments relationships presented in Chapter II gave us the following important insights:

– The current account balance mirrors international capital flows. A country that runs a current account surplus acquires claims on future payments from abroad, a country that runs a current account deficit raises its liabilities towards the rest of the world.
– The current account balance reflects the difference between national savings and national investment – with these magnitudes resulting from both public and private decisions. If national savings exceed national investment, domestic residents acquire foreign assets, and this is mirrored by a current account surplus.

Knowing the rules and mechanics of balance of payments accounting does not provide an *explanation* of current account fluctuations and international capital flows. Nor can we use the results of the previous chapter to *evaluate* the welfare implications or the long-run sustainability of current account balances. However, these insights indicate which direction any further analysis has to take: since the current account balance reflects the difference between aggregate savings and investment, we have to understand the fundamental determinants of saving and investment decisions. These decisions are necessarily dynamic because both savings and investments are chosen with regard to future periods – in the first case because households want to enjoy a certain consumption level in the future, in the second because firms want to sustain or expand productive capacity. These considerations are at the heart of the ***intertemporal approach to the current account*** that will be introduced in this chapter. On the following pages we will present a model in which saving and investment decisions are explicitly based on the intertemporal optimization of rational agents. This model represents a key building block of modern open-economy macroeconomics.

We will start by introducing this formal framework in a very stylized way – by presenting the model of a representative consumer who lives for two periods, who is able to perfectly predict the future, and who receives an exogenous income in both periods of his life. In the following sections and chapters we will then gradually relax these restrictive assumptions. However, we will see that the key relationships and trade-offs that characterize the simple two-period model are still present in more complex versions of this model. A thorough grasp of the basic framework thus crucially facilitates the understanding of subsequent extensions and applications.

In addition to emphasizing the didactic value of the representative-consumer model, we will also show that, under certain conditions, a market economy, in which many consumers and firms are independently realizing their plans, behaves *as if* it were governed by the decisions of a representative consumer. The basic framework thus offers a practical and theoretically sound approach to model the behavior of macroeconomic variables in a parsimonious way. Conversely, we will be careful to highlight the limits of this approach: in particular when we want to understand the implications of international capital flows on the welfare of *heterogeneous* individuals we have to abandon the concept of the representative consumer.

The major part of this chapter will refer to a small open economy whose saving and investment decisions do not influence prices and interest rates on the international capital market. In the last section of this chapter, we will then drop this assumption and derive the equilibrium interest rate that results from the interaction of two large economies. This framework will enable us to analyze how exogenous changes in one country affect the "world interest rate" and thus saving and investment decisions in the other country.

III.2 Consumption, Savings, and the Current Account in a Small Open Economy with an Exogenous Income

III.2.1 Model Structure and Assumptions

We consider an economy that is populated by a single ***„representative" consumer (RC)***.[1] RC faces the following situation:

[1] Other expositions refer to a "representative household" or a „representative agent". Our decision to introduce the "representative consumer" is motivated by the fact that his initials remind us of Daniel Defoe's Robinson Crusoe – an allusion that succinctly illustrates the strengths and limitations of the simple model. Note that, unlike in the case of Defoe's famous islander, the gender of RC is not clearly defined. In what follows, we will refer to him as a male individual, bearing in mind that, of course, all our insights also apply to his female incarnations.

– He lives for two periods and has no offspring.
– At the beginning of the first period, RC has neither assets nor liabilities.
– In both periods, he consumes a single good which should be interpreted as a fixed *bundle* of goods. This good cannot be stored.
– In both periods, he receives an exogenous amount of this good as „income“.
– RC does not face any uncertainty, i.e. he has ***perfect foresight*** and can therefore perfectly predict his income path.
– RC has access to an international capital market that allows buying and selling assets which yield a fixed real interest rate of *r*.
– RC lives in a ***small open economy***. Hence, his behavior does not affect the real interest rate prevailing on the international capital market.

Given these constraints, RC maximizes the following objective function:

$$U_1 = u(C_1) + \beta u(C_2) \tag{3.1}$$

The ***lifetime utility*** of RC is given by U_1 and depends on first- and second-period consumption C_1 and C_2. The specification of the utility function in (3.1) is characterized by additive separability, with $u(C_t)$ reflecting ***instantaneous utility*** in period *t*. The function u is continuous, differentiable, increasing in C_t, and strictly concave, i.e. $u' > 0$, $u'' < 0$. The concavity assumption implies that instantaneous utility is increasing in consumption, but marginal utility is decreasing, i.e. the additional utility generated by an additional unit of consumption becomes smaller as the consumption level increases.[2]

The parameter $\beta \in [0,1]$ in (3.1) represents RC's ***subjective discount factor*** and reflects his ***time preference***: the lower β, the lower the utility generated by a given level of consumption in period 2 relative to the same level of consumption in period 1, i.e. the more "impatient" RC.[3]

When choosing the optimal ***consumption path***, i.e. levels of C_1 and C_2 that maximize his utility, RC has to take into account the following constraints:

$$B_{t+1} = (1+r)B_t + Y_t - C_t \quad \text{with } t = 1, 2 \tag{3.2}$$

$$B_1 = 0 \tag{3.3}$$

[2] The assumption of additive separability implies that the marginal utility of consumption in one period is not affected by the consumption level in another period. This considerably simplifies the analysis, and the assumption of additive separability is therefore widely used in macroeconomic models.

[3] The subjective discount factor β is related to the subjective ***discount rate*** b through $\beta = 1/(1+b)$.

$$(3.4) \qquad B_3 = 0$$

Equation (3.2) characterizes the evolution of the ***net international investment position*** of the country inhabited by RC. This equation can be derived by combining (2.10) with (2.13), setting the compensation of employees as well as the balances on the secondary income account and the capital account equal to zero. Moreover, we abstract from investment I_t. This is due to our assumption of an exogenous income: there is just no reason to change the capital stock in this version of the model, and we can safely set investment equal to zero. Finally, there is no government in the model for the time being, and all consumption decisions are taken by a (representative) private individual.

The variable B_t stands for the net international investment position of the country inhabited by RC *at the beginning* of period t, with t adopting the values 1 and 2. As outlined in Chapter 2, B_t can be positive, negative, or zero. If B_t is positive, RC enters period t with positive net claims towards the rest of the world. If this variable is negative, his liabilities exceed his assets. Since we assume that bonds with a fixed interest rate are the only type of asset traded on the international capital market, we interpret a negative value of B_t as "external debt". Due to the assumption of perfect foresight this simplification is innocuous. Nevertheless, we emphasize once more that, in reality, a country's net international investment position does not just consist of bonds and credit contracts, but also of equity-type assets and liabilities like company stock and direct firm ownership.

The variable r is the ***real interest rate*** that has to be paid on all loans taken up on the international capital market. The fact that r is not affected by RC's borrowing and lending behavior is due to our ***small open economy*** assumption. Moreover, this fact reflects the assumption of ***perfect capital mobility***: on the international capital market, RC meets other agents who allow him to borrow and lend any amount at the given interest rate as long as he complies with the constraints (3.2) to (3.4).[4] Note, finally, that r does not have a time subscript, i.e. we are assuming that the interest rate is constant over time. As we will see, this assumption is innocuous in a two-period model in which only the return on assets between the start and the end of period 2 is relevant.

For the case of a positive net international investment position $\left(B_t > 0\right)$, equation (3.2) should be read as follows: at the start of period t, the representative consumer holds B_t bonds, each of which entitles him to receive $\left(1+r\right)$ units of the consumption good at the end of the same period. At the end of that period, RC realizes this claim, devoting a portion of the resulting payment

[4] For the time being, we thus ignore the possibility that RC or anyone else in the model denies repayment of outstanding liabilities. Such a decision would amount to a ***default***, the determinants and consequences of which will be analyzed in Chapter VI.

(measured in goods units!) to the purchase of new bonds $\left(B_{t+1}\right)$, which he carries into the following period. At first glance, this interpretation may seem exotic, since real-world bonds usually define claims on future *money payments*, not *goods deliveries*. In fact, we could have expressed the law of motion (3.2) in terms of monetary units. By suggesting the above interpretation, we do not deny the existence of money as a unit of account and a medium of exchange, but we emphasize the fact that every monetary payment ultimately defines a claim on goods units. Box 3.1 further elaborates on this idea.

Box 3.1: Nominal and Real Interest Rates

To better understand the relationship between nominal and real magnitudes we consider a simple example: at the end of period t, an individual receives a nominal income of $P_t\,Y_t$ monetary units (say, Euros), with Y_t representing his real income in goods units and P_t the (money) price of a clearly-defined goods basket. Out of this income, the individual spends the sum $P_t\,C_t$ on consumption. The rest is spent on the purchase of bonds, each of which costs one monetary unit. At the end of period $t+1$ the individual receives a ***nominal interest*** i_{t+1} on each of these bonds, sells these bonds at one monetary unit, and uses the resulting sum to finance his consumption $P_{t+1}\,C_{t+1}$. The value of consumption in period $t+1$ is thus given by the following expression:

$$P_{t+1}\,C_{t+1}=\left(1+i_{t+1}\right)\left(P_t\,Y_t-P_t\,C_t\right)$$

Factoring out P_t on the right-hand side and dividing both sides by P_{t+1} yields

$$C_{t+1}=\left(1+i_{t+1}\right)\frac{P_t}{P_{t+1}}\left(Y_t-C_t\right)$$

This can be written as

$$C_{t+1}=\frac{\left(1+i_{t+1}\right)}{\left(1+\pi_{t+1}\right)}\left(Y_t-C_t\right)$$

with $\pi_{t+1}=\left(P_{t+1}-P_t\right)/P_t$ as the ***inflation rate***. The left-hand side of this expression gives consumption in period $t+1$, measured in goods units. On the right-hand side, the difference between *real* income and *real* consumption in period t is multiplied by the ratio of $\left(1+i_{t+1}\right)$ and $\left(1+\pi_{t+1}\right)$, i.e. the ***(gross) real interest rate*** $\left(1+r_{t+1}\right)$. The term "gross" reflects the fact that both principal and interest payments are included, and the term „real" indicates that the nominal interest rate is corrected for possible changes in the price level, such that all payments are defined in goods units.

Throughout this book, we will make the important assumption that the real interest rate determines agents' saving decisions, and that the nominal interest rate is linked to the real interest rate by taking into account the ***expected inflation rate*** π_{t+1}^{e}. In a first approximation, this principle, which goes back to the economist Irving Fisher (1867-1947), can be represented by the so-called ***Fisher-equation*** $i_{t+1}=r_{t+1}+\pi_{t+1}^{e}$. Under our assumption of perfect foresight, the *expected* inflation rate π_{t+1}^{e} coincides with the *realized* inflation rate π_{t+1}. Hence, price level changes are perfectly anticipated, and result in an adjustment of the nominal interest rate without affecting the real interest rate. As a consequence, we can ignore nominal magnitudes and define all payments in terms of goods units – even if we are aware that, in reality, individuals use money as a unit of account and medium of exchange, and that most financial contracts are defined in nominal terms.

The „initial condition" in (3.3) reflects our assumption that RC has neither assets nor liabilities at the start of his life, and is chosen to simplify the exposition. By contrast, the "terminal condition" in (3.4) is more than a mere assumption: given the positive marginal utility of consumption, RC has no incentive to carry claims into the third period, since this period is no longer relevant for him. Conversely, every creditor would insist on the repayment of remaining liabilities at the end of period 2. Hence, only $B_3=0$ is compatible with agents' rational behavior.

III.2.2 The Intertemporal Budget Constraint

It is instructive to first approach RC's intertemporal optimization problem by using a graphical exposition. Point **A** in Figure 3.1 defines the endowment point, i.e. the exogenous combination of Y_1 and Y_2 faced by RC. If he didn't have access to the international capital market, RC's only option would be to consume his (given) income in both periods.[5]

[5] Of course, this statement hinges on our assumption that the consumption good cannot be stored.

The possibility to borrow and lend allows RC to consume an amount that exceeds or falls short of his income – i.e. to realize a combination of C_1 and C_2 that does not necessarily coincide with the endowment point. However, when choosing his optimal consumption path, he faces an ***intertemporal budget constraint***. This constraint is derived by combining equations (3.2) – (3.4) and can be written as follows:

(3.5) $$C_2 = Y_2 + (1+r)(Y_1 - C_1)$$

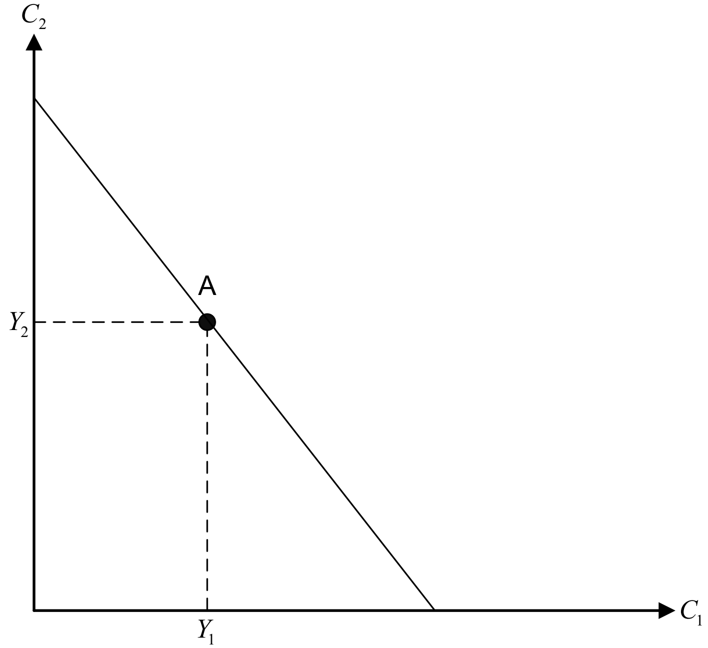


Figure 3.1: Endowment point and intertemporal budget constraint

In Figure 3.1, the intertemporal budget constraint is depicted as a straight line. It crosses the endowment point **A** since substituting $C_1 = Y_1$ in (3.5) implies $C_2 = Y_2$. The slope of the straight line is $-(1+r)$: a higher interest rate on the international capital market makes it steeper. A reshuffling of (3.5) yields the following expression:

(3.6) $$C_1 + \frac{C_2}{1+r} = Y_1 + \frac{Y_2}{1+r}$$

This representation of the intertemporal budget constraint has a straightforward interpretation: RC can choose any combination of C_1 and C_2 as long as the ***present value*** of his consumption equals the present value of his lifetime income. This constraint, which is also called the ***intertemporal solvency condition***, is much less restrictive than the requirement that consumption and income coincide in both periods. The possibility of borrowing and lending thus substantially widens RC's scope when it comes to choosing his optimal consumption path. Of course, he doesn't have to exploit this scope, since the endowment point is also part of the intertemporal budget constraint. Hence, despite having access to the international capital market, RC may deliberately choose the consumption path that he would have been forced to realize under financial autarky.

III.2.3 The Objective Function

In the preceding subsection we introduced the intertemporal budget constraint faced by RC when he decided on his optimal consumption path. We now consider his intertemporal optimization problem.

RC's goal is to maximize his lifetime utility subject to the intertemporal budget constraint, i.e. to reach the highest level of the utility function U_1, as defined by (3.1). Given the assumptions we have made on $u(C_t)$ (continuity, differentiability, concavity), a certain level of utility can be represented by an ***indifference curve*** as in Figure 3.2. Due to our assumption that u is monotonically increasing in consumption, "higher" indifference curves that combine a given value of C_1 with a higher value of C_2 yield a higher level of utility – i.e. RC prefers point $\widetilde{C}$ on the indifference curve $\widetilde{U}_1$ to point C on U_1. Conversely, a given level of utility is sustained if RC is compensated for a reduction in C_1 by an increase of C_2 – e.g. by moving from point C to point C' in Figure 3.2.

RC's willingness to trade off consumption in different periods against each other is mirrored by the slope of the indifference curve in a given point. By totally differentiating (3.1) and rearranging terms – taking into account that, by definition, utility remains constant on an indifference curve – we can easily derive this slope[6]:

$$\frac{dC_2}{dC_1} = -\frac{u'(C_1)}{\beta\, u'(C_2)} \tag{3.7}$$

[6] Totally differentiating (3.1) yields $dU_1 = u'(C_1)dC_1 + \beta u'(C_2)dC_2$. Setting $dU_1 = 0$ and rearranging terms yields the expression in (3.7).

It is intuitive that the slope of the indifference curve is negative: if C_1 increases, C_2 has to be reduced to keep the level of utility constant. The absolute value of dC_2 / dC_1 is called the ***(intertemporal) marginal rate of substitution (MRS)***. It represents the volume of period-2 consumption that RC would be willing to sacrifice in order to marginally increase his period-1 consumption, maintaining a constant utility level. Note that, for most specifications of $u(C_t)$, MRS changes as we move along the indifference curve.

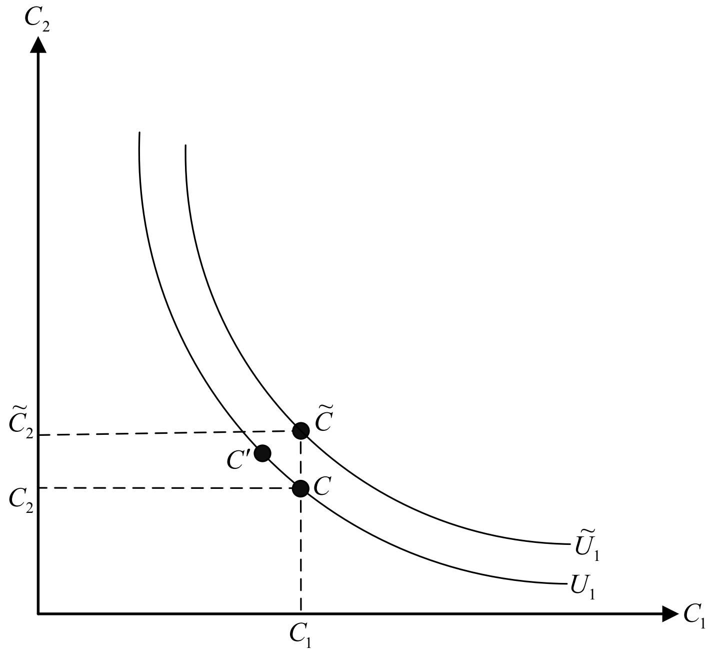


Figure 3.2: Indifference curves of the representative consumer

III.2.4 The Optimal Consumption Path

The problem faced by RC consists in maximizing lifetime utility U_1 subject to the constraint (3.6). In graphical terms, this amounts to reaching the highest indifference curve in Figure 3.2 without violating the intertemporal budget constraint depicted in Figure 3.1. If we assume that RC's preferences are represented by indifference curves that look like those in Figure 3.2, this condition is satisfied at a point where the indifference curve is tangential to the intertemporal budget constraint.[7] At such a point, the indifference curve and the intertemporal budget constraint have the same slope. The optimal consumption path is thus characterized by the (intertemporal) marginal rate of substitution coinciding with the (gross) real interest rate.

To derive the solution of RC's problem analytically, we start by introducing some additional notation and define

$$\Omega_1 \equiv Y_1 + \frac{Y_2}{1+r} \tag{3.8}$$

In what follows, we will call Ω_1 the ***wealth*** of RC as of period 1. Wealth does not just consist of financial and real assets at the beginning of period 1 (which we have assumed to be zero), but is interpreted as the present value of the – perfectly anticipated – lifetime income. Using this notation, we can write down the following ***Lagrange function***:

$$Z = u(C_1) + \beta\, u(C_2) + \lambda\left(\Omega_1 - C_1 - \frac{C_2}{1+r}\right) \tag{3.9}$$

The necessary conditions for an optimum are found by taking derivatives of (3.9) with respect to C_1, C_2, and λ:

$$u'(C_1) - \lambda = 0 \tag{3.10}$$

$$\beta\, u'(C_2) - \frac{\lambda}{1+r} = 0 \tag{3.11}$$

$$\Omega_1 - C_1 - \frac{C_2}{1+r} = 0 \tag{3.12}$$

By solving (3.11) for λ, substituting the result into (3.10), and rearranging terms, we arrive at the following expression:

$$\frac{u'(C_1)}{\beta\, u'(C_2)} = 1 + r \tag{3.13}$$

[7] To exclude the possibility that the indifference curves intersect an axis such that the optimal consumption may be a ***boundary solution*** with zero-consumption in one of the periods we usually assume that $u'(0)$ is infinitely large.

This equation describes the tangential solution mentioned above: RC chooses a combination of C_1 and C_2 such that the intertemporal marginal rate of substitution coincides with the (gross) real interest rate. We can rewrite the expression in (3.13) as follows:

(3.14) $$u'(C_1) = (1+r)\beta u'(C_2)$$

This representation of the optimality condition is known as the ***intertemporal Euler equation***, named after the Swiss mathematician Leonhard Euler (1707 – 1783). It succinctly illustrates the trade-off faced by RC when he decides on his optimal consumption path: a marginal increase in savings in period 1 and the resulting reduction of consumption generates a marginal utility loss, which is represented by the left-hand side of equation (3.14). Conversely, an increase in period 1 savings expands consumption possibilities in period 2 and thus generates additional utility in the future. This is reflected by the term $u'(C_2)$ on the right-hand side. Moreover, every goods unit saved in period 1 raises consumption in period 2 by $(1+r)$ units. However, from the perspective of period 1, future consumption gets a lower weight, which is captured by the discount factor β on the right-hand side. Thus, the intertemporal Euler equation requires that the marginal utility loss of additional savings coincides with the marginal utility gain. If this condition weren't satisfied, RC could increase his utility by varying his savings and thus change his consumption plan.

Without making any further assumptions on the function $u(C_t)$, we can use the intertemporal Euler equation to characterize the evolution of RC's consumption over time. Figure 3.3 depicts the ratio $u'(C_1)/u'(C_2)$ as a function of the ratio C_1/C_2. The negative slope of this curve is due to the concavity of $u(C_t)$, which implies that $u'(C_t)$ decreases as C_t increases. Obviously, $u'(C_1)/u'(C_2)=1$ if $C_1/C_2=1$. If we denote the optimal consumption level in period t by C_t^*, the Euler equation requires that $u'(C_1^*)/u'(C_2^*)=\beta(1+r)$. Hence, $C_1^*/C_2^*<1$ if $\beta(1+r)>1$, as shown in Figure 3.3. Conversely, optimal consumption is *falling* over time, i.e. $C_1^*/C_2^*>1$, if $\beta(1+r)<1$. Finally, RC implements a constant consumption level over time if $\beta(1+r)=1$. The role of the expression $\beta(1+r)$ for the slope of the optimal consumption path is easily explained. If $\beta(1+r)=1$, the interest rate exactly compensates RC for his time preference.[8] This makes it optimal to consume the same amount of goods in

[8] Recall that the ***discount rate*** b and the ***discount factor*** β are linked by $\beta = 1/(1+b)$. Hence $\beta(1+r)=1$ implies $r=b$.

both periods. Conversely, if $\beta\left(1+r\right)>1$ the market offers a particularly attractive reward for increased savings, and RC has an incentive to realize a consumption path that is increasing over time.

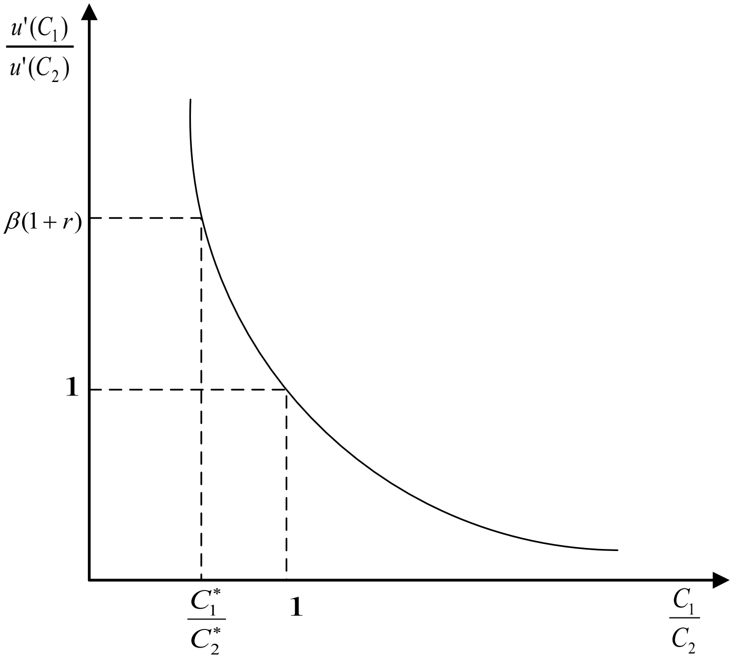


Figure 3.3: The relationship between C_1^* and C_2^* if $\beta(1+r)>1$.

So far, we have paid little attention to the Lagrange parameter λ. It is, however, worthwhile to shed light on the economic interpretation of this variable. To do this, we substitute the optimal values C_1^* and C_2^* into the Lagrange function (3.9). This gives the ***indirect utility function***

$$(3.15) \qquad Z^* = u\left(C_1^*\right) + \beta\, u\left(C_2^*\right) + \lambda^*\left(\Omega_1 - C_1^* - \frac{C_2^*}{1+r}\right)$$

A variation of Ω_1 obviously affects the maximal utility Z^* that RC can attain. In principle, the total effect of such a change consists of both a direct and an indirect effect, with the latter working through an adjustment of C_1^* and C_2^*:

$$(3.16) \qquad \frac{dZ^*}{d\Omega_1} = \lambda^* + \left[u'\left(C_1^*\right) - \lambda^*\right]\frac{\partial C_1^*}{\partial \Omega_1} + \left[\beta\, u'\left(C_2^*\right) - \frac{\lambda}{1+r}\right]\frac{\partial C_2^*}{\partial \Omega_1}$$

However, if we take into account the necessary conditions in (3.10) and (3.11) the expression in (3.16) boils down to

$$\text{(3.17)} \qquad \frac{dZ^*}{d\Omega_1} = \lambda^*$$

This means that we can ignore the indirect effects of a variation of Ω_1, and the Lagrange parameter λ^* reflects the change in RC's maximally attainable utility that results from a variation of Ω_1.[9] For this reason λ^* is sometimes called the ***marginal utility of wealth***.

III.2.5 The Current Account

Let us briefly summarize what we have found out so far: we have considered the decisions of a representative consumer (RC), i.e. an individual whom we have treated as the sole inhabitant of an entire country. For a given time path of income, RC chooses his consumption path to maximize utility. Since he has access to an international capital market, he may borrow and lend, such that consumption and income do not have to coincide at every point in time. We can now analyze how RC's decisions affect the current account balance of his country.

Absent capital account transactions and valuation effects, the current account reflects the change of a country's net international investment position during a period, i.e.

$$\text{(3.18)} \qquad CA_t = B_{t+1} - B_t$$

Using the assumptions of our two-period model, we thus get

$$\text{(3.19)} \qquad CA_1 = B_2$$

$$\text{(3.20)} \qquad CA_2 = -B_2$$

This is intuitive: since RC has neither assets nor liabilities at the beginning of period 1 $(B_1 = 0)$, the first-period current account equals the net international investment position at the end of period 1. At the end of period 2, the net international investment position has to return to zero. That is, if RC borrows in

[9] This is an application of the so-called ***envelope theorem.*** A detailed exposition of this theorem can be found, e.g., in Mas-Colell et al. (1995:964).

period 1 he has to repay his debt at the end of period 2 and to make the associated interest payments. The opposite constellation applies if RC lends to foreigners in period 1.

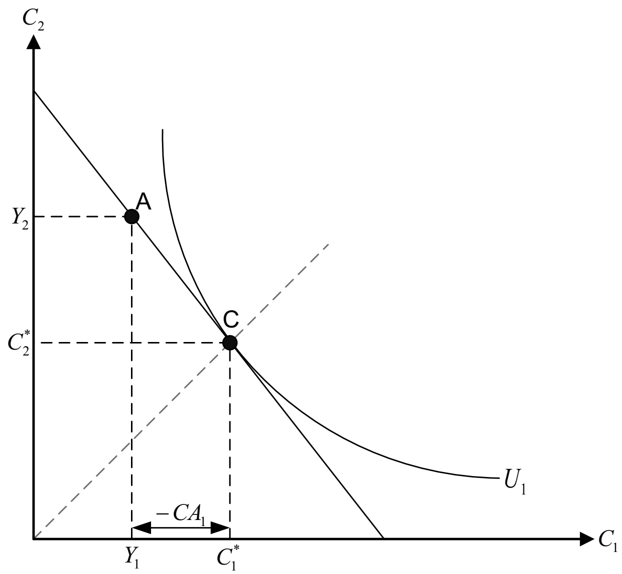


Figure 3.4: The optimal consumption path and the current account

Figure 3.4 combines all these elements for the case that $Y_2 > Y_1$ and $\beta(1+r)=1$: RC faces an increasing income path (endowment point **A**). However, his optimal consumption path is characterized by a constant level of consumption, i.e. it is located on the dotted 45°-line (point **C**). To implement the optimal consumption path, RC borrows in period 1, which results in a negative net international investment position at the beginning of period 2 $(B_2 < 0)$. Since, by assumption, $B_1 = 0$, this is reflected by a negative current account balance in period 1. In Figure 3.4, the absolute value of this current account deficit (CA_1) is reflected by the difference between Y_1 and C_1. In period 2, the current account balance is given by the difference between GNI and consumption, i.e. $CA_2 = Y_2 + rB_2 - C_2$.

III.2.6 Comparative-Static Analysis

How are the optimal consumption path and the current account affected by a changing time path of income or variations in the world interest rate? As we will show, the ***comparative-static properties*** of the model – i.e. the reaction of the ***endogenous*** variables to changes of an ***exogenous*** variable – crucially depend on the properties of the utility function.

An increase of the real interest rate r not only affects the relative price of current consumption in terms of future consumption, but also the wealth of RC, i.e. the present value of his (current and future) income. We therefore have to consider three effects: a ***substitution effect*** which reflects the fact that an increasing interest rate makes shifting consumption into the future more appealing, an ***income effect***, which is due to the fact that, for a given wealth level, a higher interest rate raises RC's consumption possibilities, and finally a ***wealth effect***, which captures the negative influence of a higher interest rate on the present value of future income.

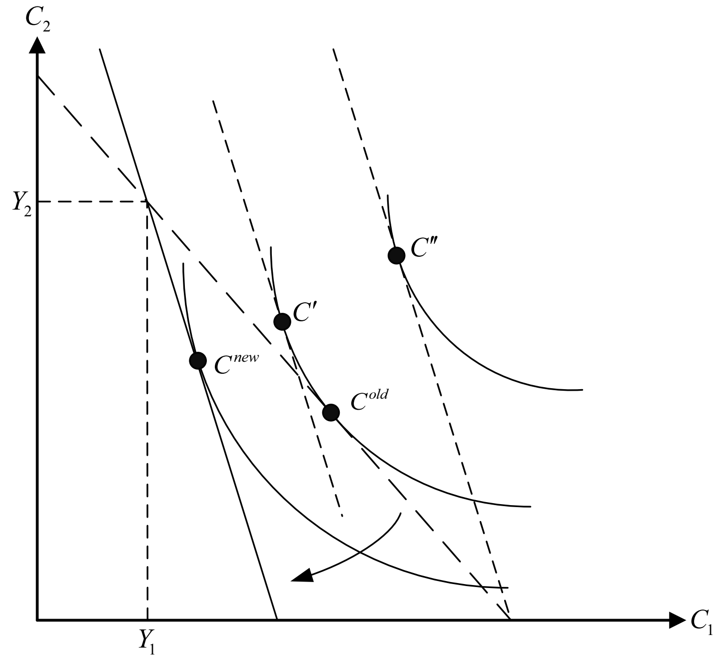


Figure 3.5: Substitution effect, income effect and wealth effect of an increasing interest rate

These effects are depicted in Figure 3.5: a rising interest rate makes the intertemporal budget constraint, which is anchored in the endowment point, steeper. The old consumption path (point C^{old}) is no longer optimal. The adjustment to the new optimum C^{new} can be split into two steps: the (***Hicksian***) ***substitution effect*** identifies a consumption path that RC would choose if he faced the new interest rate, but wanted to maintain the old utility level. This is point C', which is located on the old indifference curve. However, this consumption path is not feasible, since it is off the new budget constraint. The movement from point C' to point C^{new} can be split into an income effect and a wealth effect: the *income effect* is driven by a rightward shift of the intertemporal budget constraint, reflecting the fact that, for a given wealth level, the relative price of future consumption has decreased. This expands RC's consumption possibilities, and it is not surprising that C'' is characterized by a higher consumption level in *both* periods. At the same time, however, the increasing interest rate reduces RC's wealth, since his income in period 2 is discounted more strongly. This shifts the intertemporal budget constraint to the left. The *wealth effect* is reflected by the transition from C'' to C^{new}.

The relative strength of the substitution, income and wealth effects crucially hinge on the ***intertemporal elasticity of substitution (IES)*** that characterizes RC's preferences. With a low intertemporal elasticity of substitution, RC has a strong interest in implementing a "smooth" consumption path. He therefore tries to avoid stark differences between C_1 and C_2, such that a rise of the real interest rate generates a rather weak substitution effect. Conversely, an individual whose preferences feature a high elasticity of substitution is willing to accept a volatile consumption path. In this case the substitution effect is likely to be strong and possibly dominates the total reaction to a rising interest rate. Box 3.2 further elaborates on this concept.

Box 3.2: The Intertemporal Elasticity of Substitution (IES)

The ***intertemporal elasticity of substitution (IES)*** is defined as the percentage change of the ratio $\left(C_2 / C_1\right)$ that results from a one-percent change of the (intertemporal) marginal rate of substitution (MRS):

$$IES = \frac{d\ln\left(C_2 / C_1\right)}{d\ln MRS} = \frac{d\left(C_2 / C_1\right)}{d\,MRS}\frac{MRS}{\left(C_2 / C_1\right)}$$

In this expression, $\ln X$ is the (natural) logarithm of variable X. Note that an individual's optimal consumption path is characterized by the equality of the intertemporal MRS and the gross real interest rate. The IES thus indicates by how much the *optimal* ratio $\left(C_2^* / C_1^*\right)$ reacts to a change of the interest rate. The smaller the IES, the stronger the curvature of indifference curves in $\left(C_1, C_2\right)$-space, and the weaker the adjustment of the time path of consumption to a marginal change of the real interest rate. Modern macroeconomic theory amply uses variants of the following utility function:

$$U_1 = \frac{C_1^{1-\sigma} - 1}{1-\sigma} + \beta \frac{C_2^{1-\sigma} - 1}{1-\sigma}$$

This function is characterized by a ***constant IES*** of $(1/\sigma)$. A constant IES implies that the strength of the substitution effect does not depend on an individual's wealth. Moreover, we can use L'Hôpital's rule to show that instantaneous utility is logarithmic if $\sigma = 1$.[10]

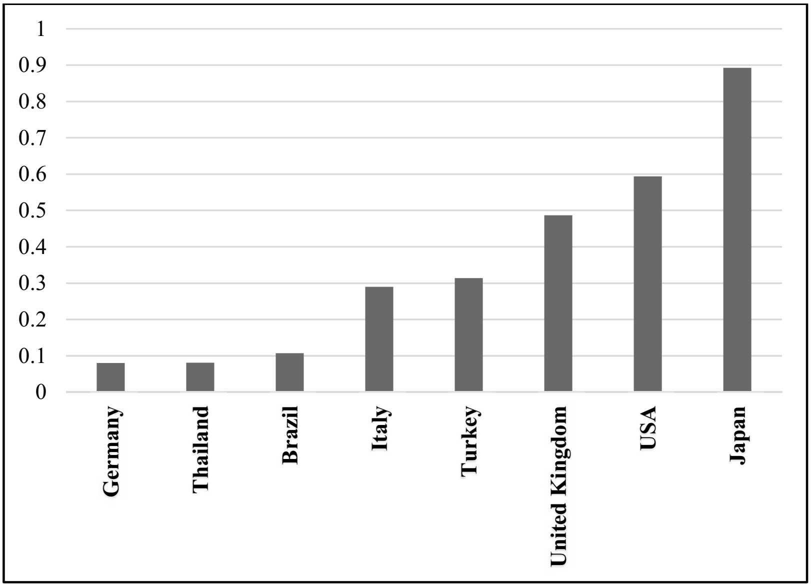


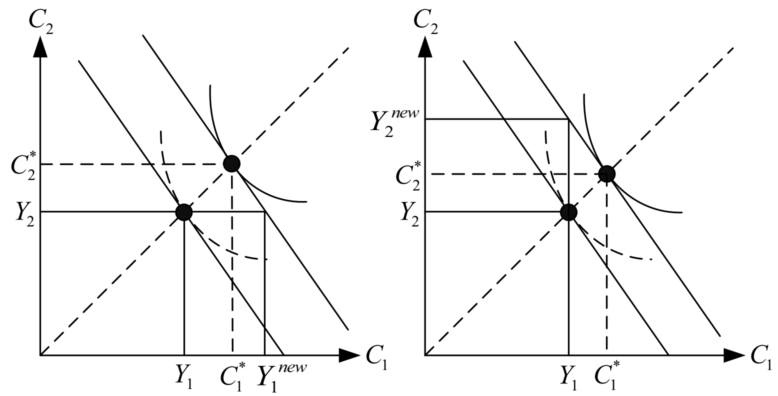
Figure B3.2: Mean estimates of the intertemporal elasticity of substitution for selected countries. Source: Havranek et al. (2015).

Given the importance of the IES for individuals' saving decisions, it is not surprising that there are numerous studies which estimate this parameter, using either microeconomic (household) data or macroeconomic time series, and differing both with respect to the countries and time spans considered and with respect to the econometric methods applied. In a meta-analysis published in 2015, Tomas Havranek, Roman Horvath, Zuzana Irsova and Marek Rusnak summarize the results of 169 empirical studies on the IES. They find that the average estimate of the IES is 0.5, but that estimates differ substantially across countries. Figure B3.2 shows the mean estimates for some countries, as reported by Havranek et al. (2015). The

[10] We demonstrate this in the appendix to this chapter.

study also finds that, controlling for other potential determinants, estimates of the IES tend to be higher in countries with a higher per-capita income and greater asset market participation – with Germany being an interesting exception. The paper's result is not surprising, since variations in consumption are harder to tolerate in poor economies, in which a large share of income is devoted to bare necessities, and harder to implement in countries in which financial markets are less developed. This, however, suggests that assuming an IES that is constant across countries and wealth levels is far from trivial.

Of course, the exact reaction of RC to *income* changes also depends on his preferences. However, our preceding analysis offers some guidance on the direction of these effects: it is quite likely that an increase of first-period income (Y_1) *ceteris paribus* – i.e. keeping Y_2 constant – results in higher savings. Unless the IES is infinite, RC transfers at least some share of the additional income into the future. Compared to the initial equilibrium, this raises the current account balance in the first period (see Figure 3.6a). The opposite effect can be observed if Y_2 increases while Y_1 stays constant: in this case, RC realizes an optimal consumption path by borrowing on the international capital market, which amounts to consuming a portion of the (future!) income rise in the present period. As a consequence, the current account balance decreases, possibly resulting in a current account deficit as in Figure 3.6b.



a) A higher income in period 1 **b)** A higher income in period 2

Figure 3.6: Income fluctuations and the current account for $\beta(1+r)=1$

Both examples illustrated by Figure 3.6 convey an important message: the current account reflects the direction and size of *transitory* income fluctuations. A temporary reduction of income is likely to result in a current account deficit, while a temporary increase of income is likely to generate a current account surplus. In both cases, RC exploits his access to the international capital market and offsets the income fluctuations via borrowing or lending. Fluctuations of the current account balance thus result from income fluctuations, combined with RC's ***consumption smoothing*** objective.

III.2.7 Gains from Intertemporal Trade

In many ways, our simple model is analogous to the optimization problem of an individual who is endowed with given quantities of goods, faces exogenous prices, and who has to decide on the structure of his consumption bundle.

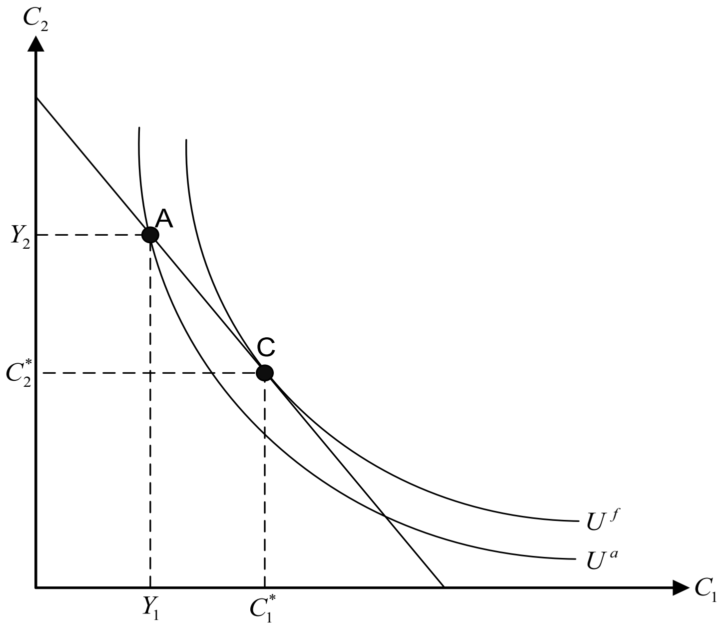


Figure 3.7: Gains from intertemporal trade

However, instead of focusing on the choice of different goods in one period, our model considers *one* (composite) good that may be consumed at two different points in time. In this sense, transactions on the international capital market reflect ***intertemporal trade***: individuals reduce their current consumption in exchange for financial claims that will enable them to enjoy higher consumption in the future. Or they raise their present consumption by promising future payments that will then reduce their consumption.

We have already mentioned in Subsection III.2.2 that having access to the international capital market is unlikely to make RC worse off: the option of intertemporal trade does not prevent him from choosing the consumption path that he would have chosen in autarky. In fact, we have all reason to believe that intertemporal trade *raises* RC's utility, since the option to borrow and lend allows him to realize a consumption path that would not have been feasible in autarky. In Figure 3.7, this is reflected by the fact that RC reaches a higher indifference curve once he is able to move away from the autarky point **A**. The difference between the autarky utility level U^a and the utility in case of financial integration U^f represents the ***gains from intertemporal trade***. These gains are higher, the greater the difference between the consumption path chosen in the open economy and the consumption path chosen in autarky.

Note, however, that this result refers to the welfare of a *representative consumer*. If all inhabitants of our model economy were equal, the notion that there are non-negative gains from intertemporal trade could, of course, be easily transferred to a framework with multiple individuals. But does this important normative concept also apply to a world that is populated by heterogeneous individuals who independently pursue their personal objectives? To answer this question, we have to abandon the concept of the representative consumer and to consider a "market economy" that consists of a large number of heterogeneous individuals with (possibly) different income paths and distributional interests.

III.3 The Current Account in a Market Economy with Exogenous Incomes

III.3.1 Motivation

The model we introduced in the last sections featured many simplifying assumptions, including the notion that the behavior of macroeconomic variables could be explained by referring to a single (representative) individual's optimizing decisions. Given the size and complexity of real-world economies, this simplification seems, of course, rather outlandish, and it is desirable to explain the evolution of aggregate consumption, the net international investment position, and the current account as reflecting the interaction of many different agents. This leaves us with the question of whether the results of the preceding subsections are still valid if we replace the representative consumer with a ***market economy*** – i.e. if we assume that the model economy consists of many different individuals whose consumption and savings choices are guided by their respective incomes and by the prices they observe, and that there is no central institution which coordinates their behavior.

To approach this question we start by deriving the interest rate that equates the demand and supply on the capital market of a *(financially) closed* economy. In this context, we will show that the behavior of market participants can be aggregated in a simple way if certain assumptions about preferences and market structure are satisfied: the RC model will turn out to be a useful and legitimate shortcut that we can use to model the aggregate behavior of a market economy.

In addition to making the model more realistic, moving away from the concept of the representative consumer allows consideration of the distributive consequences of opening an economy to the international capital market. As we will show, financial integration does not necessarily increase the utility of *every* individual. Hence, there is a possible tension between the *aggregate* gains from intertemporal trade that we have identified in the previous section and the welfare effects of financial integration at the *individual* level.

III.3.2 The Autarky Interest Rate

In what follows, we adopt the framework introduced at the start of section III.2, and add the following assumptions:

- The economy consists of N individuals, denoted by an index i. These individuals take prices as given – i.e., they do not recognize the influence of their choices on market outcomes.
- In financial autarky, agents do *not* have access to the international capital market. However, there is a national capital market on which agents can borrow and lend. Borrowing is realized by selling securities that promise a fixed (real) interest rate, lending is realized by purchasing these securities.

Given these assumptions, the objective function and the intertemporal budget constraint of agent i read as follows:

(3.21) $$U_1^i = u\left(C_1^i\right) + \beta\, u\left(C_2^i\right)$$

(3.22) $$C_1^i + \frac{C_2^i}{1+r} = Y_1^i + \frac{Y_2^i}{1+r}$$

The only difference between this optimization problem and the RC model of the preceding section model consists in the index i which allows for different income and consumption levels across individuals. However, we assume that agents' preferences are characterized by the same utility function – i.e. both the function u and the discount factor β are identical within the population. Since individuals face the same interest rate, the optimal consumption path is characterized by the following intertemporal Euler equation:

(3.23) $$u'\left(C_1^i\right) = \beta\left(1+r\right)u'\left(C_2^i\right)$$

For a given time path of incomes, equation (3.23) implies a certain demand for assets B_2^i. If this demand is positive, the individual wants to *lend* at the given interest rate. If B_2^i is negative he wants to *borrow*. We can thus write the asset demand of agent i as a function of his income path and the interest rate:

(3.24) $$B_2^i = f\left(Y_1^i, Y_2^i, 1+r\right)$$

In a ***market equilibrium***, aggregate net demand for assets is zero: all agents realize their optimal plans, and total demand for assets equals total supply. The equilibrium price that emerges in such a situation is the (gross) real interest rate $1+r^a$, with the index a indicating that this is the ***autarky interest rate*** which prevails in a financially closed economy. In order to compute this autarky interest rate, one has to add up the asset demands (3.24) of all N individuals and to set this sum equal to zero. Hence, the autarky interest rate is implicitly defined by the following equation:

(3.25) $$\sum_{i=1}^{N} f\left(Y_1^i, Y_2^i, 1+r^a\right) = 0$$

While it is possible to discuss the existence and uniqueness of an equilibrium interest rate under fairly general conditions, computing a specific value of this interest rate requires to make an assumption about individuals‘ preferences. In what follows, we assume that their utility function is characterized by a constant intertemporal elasticity of substitution:

$$u\left(C_t^i\right) = \frac{\left(C_t^i\right)^{1-\sigma} - 1}{1-\sigma}$$

This implies that the intertemporal Euler equation is given by

(3.26) $$\left(C_1^i\right)^{-\sigma} = \beta\left(1+r\right)\left(C_2^i\right)^{-\sigma}.$$

By solving the intertemporal budget constraint (3.22) for C_2^i, substituting the result into (3.26), and taking into account that $B_2^i = Y_1^i - C_1^i$, we can derive agent i's demand for assets:

$$(3.27) \qquad B_2^i = \frac{\left[\beta(1+r)\right]^{\frac{1}{\sigma}} Y_1^i - Y_2^i}{(1+r) + \left[\beta(1+r)\right]^{\frac{1}{\sigma}}}$$

Apparently, B_2^i can be positive or negative: in the former case ($B_2^i > 0$), agent i *lends* in the first period, in the latter case ($B_2^i < 0$) he *borrows*. The numerator of the right-hand side in equation (3.27) indicates that, for a given interest rate and given preferences, borrowing is more likely if $Y_2^i > Y_1^i$. In equilibrium, demand and supply on the capital market have to add up to zero. Substituting (3.27) into (3.25) and defining the economy's per capita income in both periods to be $Y_1 = \sum_{i=1}^N Y_1^i / N$ and $Y_2 = \sum_{i=1}^N Y_2^i / N$ respectively, we arrive at the following expression for the autarky interest rate:

$$(3.28) \qquad 1 + r^a = \frac{1}{\beta}\left(\frac{Y_2}{Y_1}\right)^{\sigma}$$

Apparently, the autarky interest rate is the higher, the higher the growth rate of the economy's per capita income. This is intuitive: if Y_2 is higher than Y_1, many agents have an incentive to borrow in order to effectively shift a part of their high future income into the present.[11] This raises the aggregate demand for credit and thus the equilibrium interest rate. A higher value of σ enhances this effect. Individuals whose preferences are characterized by a low intertemporal elasticity of substitution are willing to pay a high interest rate on loans that allow them to reduce the volatility of their consumption. The third variable that determines the autarky interest rate is the discount factor β: if this parameter is low – i.e. if agents are "impatient" – potential lenders have to be compensated with a higher interest rate for reducing their consumption in period 1.

Notice that the autarky interest rate in (3.28) depends on preference parameters (β, σ) and the evolution of per capita income (Y_2/Y_1), but not on the *distribution of income* in the two periods. This means that it is of no importance whether incomes are distributed equally or concentrated in the hands of a few individuals. In fact, the right-hand side of equation (3.28) can also be interpreted as follows: it reflects the (intertemporal) marginal rate of substitution of

[11] By substituting (3.28) into (3.27) we can show that $B_2^i < 0$ if $Y_1^i / Y_1 < Y_2^i / Y_2$.

an individual who receives the economy's entire income in both periods – i.e. the MRS of the representative consumer in the endowment point! This striking result is due to a specific property of the utility function that we have used: a look at the intertemporal Euler equation in (3.26) reveals that the *growth rate* of an individual's consumption does not depend on the *level* of his consumption in both periods. For a given interest rate, it is the same for rich and for poor individuals. This is the key implication of ***homothetic preferences***, of which the utility function we have used is a specific example. As detailed in Box 3.3, the marginal rate of substitution for homothetic preferences depends on the ratio C_2 / C_1, but not on the *level* of consumption that an individual can afford. Since all agents face the same interest rate and since they all choose a point where this interest rate coincides with the marginal rate of substitution, the assumption of homothetic preferences implies that poor and rich individuals choose the same growth rate of consumption. This, in turn, implies that the equilibrium interest rate in autarky only depends on aggregate magnitudes. The distribution of incomes is irrelevant, and an economy that consists of many individuals who optimize in a decentralized and non-coordinated way, behaves *as if* it were driven by the decisions of a single ("representative") consumer.

This result explains the immense popularity of the representative consumer in modern macroeconomics: under some – admittedly restrictive – assumptions, the behavior of a complex market economy can be modelled in an extremely parsimonious, yet correct fashion.

Box 3.3: Homothetic Preferences and Aggregation

It is the key property of homothetic preferences that the (intertemporal) marginal rate of substitution (MRS) of individual i depends on the ratio C_2^i / C_1^i, but not on absolute consumption levels in the two periods:

$$MRS^i = h\left(\frac{C_2^i}{C_1^i}\right)$$

where h is a function whose properties depend on the specifics of individuals' utility function. Moving along a ray through the origin where C_2^i / C_1^i is constant, all indifference curves thus have the same slope. Combining this property with the assumption that all individuals adjust their behavior to the same interest rate, and considering that, on the optimal consumption path, the MRS coincides with the (gross) interest rate, we get

$$1+r=h\left(\frac{C_2^i}{C_1^i}\right)$$

Since $(1+r)$ and the function h are assumed to be the same for all individuals, the preceding equation implies that $C_2^i / C_1^i = C_2 / C_1$, i.e. the slope of the time path of consumption depends on the interest rate and on parameters of the utility function, but not on an individual's wealth. This is illustrated by Figure B3.3: while individuals A and B enjoy very different consumption *levels* in both periods, they choose the same ratio C_2 / C_1. This implies that the growth rate of aggregate consumption does not depend on how aggregate wealth is distributed between A and B, and that we can replace these individuals by a "representative" consumer who has the economy's total wealth at his disposal.

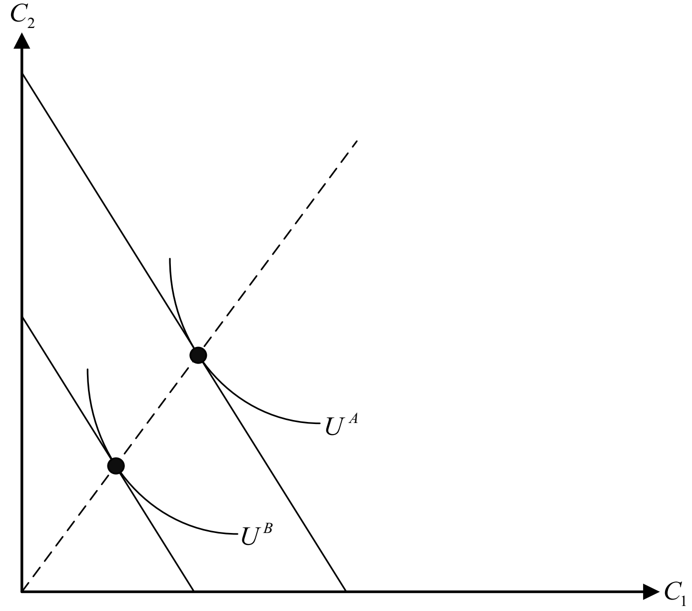


Figure B3.3: Homothetic preferences

It is easy to show that the *isoelastic* utility function we introduced in Section III.2.6 is homothetic. Note, however, that the assumption of homothetic preferences is sufficient, but not necessary for justifying the representative-consumer approach. Adopting this approach is also possible if individuals' preferences are described by a ***quasilinear*** utility function, which is given by $U_1 = C_1 + \beta u(C_2)$.

III.3.3 Gains from Intertemporal Trade?

The popularity of the RC-model should not mask the fact that this framework makes it impossible to address a whole range of interesting questions. In particular, it does not allow assessment of how the transition from financial autarky to financial integration affects the welfare of *individual agents*. With free access to the international capital market, optimal consumption is still characterized by the intertemporal Euler equation (3.26). However, the relevant interest rate is no longer r^a, but the world market interest rate r. Unless r happens to coincide with r^a, agents choose another time path of consumption and reach another utility level than in autarky.

We use a simple example to illustrate the possibility that an agent can actually experience a welfare loss as a result of financial openness. The individual we consider has a strictly positive income in period 1, but *no* income in period 2. Moreover, we assume that his preferences are characterized by a constant intertemporal elasticity of substitution, and for simplicity we set $\sigma = 1$, which implies $u(C_t^i) = \ln C_t^i$. Deriving the agent's optimal consumption path and substituting the result into the utility function gives his lifetime utility

$$(3.29) \qquad U_1^i = \ln\left(\frac{Y_1^i}{1+\beta}\right) + \beta \ln\left(\frac{\beta(1+r)Y_1^i}{1+\beta}\right)$$

If the transition from financial autarky to financial integration results in a *decreasing* interest rate, this lowers the agent's lifetime utility. The explanation for this finding is simple: since the agent has a positive first-period income, but no second-period income, he necessarily is a saver, i.e. he purchases assets in the first period and consumes principal and interest in the second period. A lower interest rate reduces the agent's capital income for given savings, and even if he optimally adjusts to the new situation, he reaches a lower utility level than in autarky.

This insight is not really surprising. In the same way that international goods trade creates "winners" and "losers" at the national level, *intertemporal trade*, which is reflected by asset trade on the international capital market, has nontrivial distributional consequences. At the same time, it holds true that the economy as a whole, personified by the representative consumer, realizes positive gains from intertemporal trade. The solution to this putative paradox rests in the fact that the sum of welfare gains exceeds the sum of welfare losses. In a world that is characterized by perfect competition and free of distortions like external effects or informational asymmetries, access to the international capital market possibly results in a situation which – compared to (financial) autarky – leaves no agent worse off and some agents better off. This important result is a variant of the ***first theorem of welfare economics***, which claims that equilibria on frictionless markets are ***pareto-efficient***.[12] However, reaching such a situation requires the existence of a ***transfer mechanism*** through which winners compensate the losers. Hence, financial integration ***potentially pareto-dominates*** financial autarky. Box 3.4 illustrates this by means of a simple example. If there is no transfer mechanism, it is not surprising that financial integration does not meet the unconditional support of the entire population. In this case, aggregate welfare gains coexist with welfare losses of individual agents.

Box 3.4: Gains and Losses from Intertemporal Trade

We consider an economy that lasts for two periods and that is populated by two individuals with identical preferences. For simplicity, we assume that instantaneous utility is logarithmic. Individual A (the „creditor“) receives two units of the consumption good in the first period, and nothing in the second period. By contrast, individual B (the „debtor“) receives zero units in period 1 and two units in period 2. Hence, the economy's aggregate income is constant, and it follows from (3.28) that the autarky interest rate is

$$1+r^a=\frac{1}{\beta}.$$

Solving the intertemporal optimization problem for individual i and substituting the optimal consumption path into the utility function, we get indirect utility for an arbitrary time path of income and an arbitrary interest rate:

$$U_1^i=\ln\left(\frac{1}{1+\beta}\left(Y_1^i+\frac{Y_2^i}{1+r}\right)\right)+\beta\ln\left(\frac{\beta\left(1+r\right)}{1+\beta}\left(Y_1^i+\frac{Y_2^i}{1+r}\right)\right).$$

Combining this result with the above assumptions on the income of individuals A and B as well as the autarky interest rate, we can derive their welfare levels in autarky:

[12] This is a general result, which also holds if the assumptions on preferences underlying the representative-consumer model are not satisfied.

$$U_1^{A,a} = (1+\beta)\ln\left(\frac{2}{1+\beta}\right), \quad U_1^{B,a} = (1+\beta)\ln\left(\frac{2\beta}{1+\beta}\right)$$

Once both agents have access to the international capital market, the world market interest rate, which we denote by $(1+r^*)$, becomes relevant. For ease of notation, we define $\phi \equiv (1+r^*)\beta$. If ϕ is greater (smaller) than one, the world market interest rate is above (below) the autarky interest rate. It is easy to show that access to the international capital market never lowers the welfare of a "representative consumer" who receives one goods unit in both periods. To determine RC's utility in autarky $(U_1^{RC,a})$, we substitute the autarky interest rate derived above (r^a) into the indirect utility function. This yields $U_1^{RC,a} = 0$. To demonstrate that access to the international capital market cannot make RC's utility under financial integration $(U_1^{RC,f})$ negative, we show that $U_1^{RC,f}$ is a function that reaches its minimum $U_1^{RC,f} = 0$ for $\phi = 1$, and that it is strictly decreasing (increasing) for $\phi < 1$ $(\phi > 1)$. Hence, if the world interest rate happens to coincide with the autarky interest rate, RC's utility does not change. If there is a difference between r^a and r^*, RC's utility increases as a result of financial integration.

Comparing the utility levels of agents A and B in autarky and with financial integration, we see that the creditor (debtor) loses if ϕ is smaller (greater) than one. However, in both cases we can determine a transfer scheme that completely compensates the loser and still leaves the winner strictly better off. To show this, we introduce the ***compensating variation*** Δ^i with $i = A, B$ – a payment in period 1 that leaves the loser from financial integration as well off as under financial autarky. Replacing $\beta(1+r)$ in the indirect utility function by ϕ, and Y_1^i and Y_2^i by the assumed values for the two individuals, we arrive at two equations that implicitly define Δ^A and Δ^B:

$$\ln\left(\frac{2+\Delta^A}{1+\beta}\right) + \beta\ln\left(\frac{\phi(2+\Delta^A)}{1+\beta}\right) = U_1^{A,a},$$

$$\ln\left(\frac{\Delta^B + 2/(1+r^*)}{1+\beta}\right) + \beta\ln\left(\frac{\phi\left(\Delta^B + 2/(1+r^*)\right)}{1+\beta}\right) = U_1^{B,a}$$

where the utility levels under autarky $U_1^{A,a}$ and $U_1^{B,a}$, to which the left-hand sides are equated, have been derived above. Solving both equations for Δ^A and Δ^B, we get

$$\Delta^A = 2\,\phi^{\frac{-\beta}{1+\beta}} - 2, \quad \Delta^B = 2\,\beta\,\phi^{\frac{-\beta}{1+\beta}} - 2\,\beta\,\phi^{-1}$$

Δ^A is positive and Δ^B is negative if ϕ is smaller than one. The opposite holds if $\phi > 1$. This means that individual A (the „creditor“) has to be compensated if the world market interest rate is lower than the autarky interest rate, while individual *B* (the “debtor”) would be willing to share a part of his income. Conversely, the compensating variation would be negative for individual *A* and positive for individual *B* if the interest rate increased as a result of financial integration. Finally, we can show that Δ^A is smaller than Δ^B in absolute value if the interest rate decreases, and that the opposite holds if the interest rate increases: the winner from financial integration would be able to completely compensate the loser and still realize a strictly positive welfare gain. This implies that, if there is an appropriate compensation mechanism, financial integration pareto-dominates financial autarky.

III.4 Intertemporal Trade with Production

So far, we have assumed that RC faces an *exogenous* time path of income. The simplest interpretation – that Y_1 and Y_2 represent a supply of goods that accrues to RC like „manna from heaven“ – can be easily replaced by a more sophisticated reading. Suppose that RC produces the composite consumption good using the following ***production function***:

(3.30) $$Y_t = A_t\, F\left(K_t, L_t\right)$$

In (3.30), K_t is the stock of ***physical capital*** at the start of period t – i.e. the total volume of all machines, factory plants etc. that are used in production – and L_t is ***employment*** in that period. Finally, the parameter A_t represents ***total factor productivity (TFP)***. A higher value of TFP raises the output Y_t that is produced for given inputs K_t and L_t. For the time being, we simply assume that the function F is increasing in both arguments. If both the capital stock and employment are constant, variations in Y_t are solely generated by fluctuations of A_t – so-called ***technology shocks*** that determine RC's productivity in a given period. This example illustrates a simple way to add a “supply side” to our model: setting $F\left(K_t, L_t\right)$ equal to one for simplicity, we could simply replace Y_t by A_t in all the expressions derived above, and we could adopt all qualitative results.

However, this example also illustrates the shortcomings of such a simplistic approach and indicates which aspects should be added to move our framework closer to reality: first, we should allow for variations in employment, and we should also account for the fact that the capital stock K_t varies over time. The second extension is particularly important since, as we have seen above, national investment plays a crucial role in determining the current account balance. The next section will therefore focus on the economic mechanisms that determine the volume of investment.

III.5 Savings, Investment, and the Current Account

We now extend the formal framework that we have introduced in Section III.2 and assume that RC produces the income he receives in every period by combining capital and labor according to the production function (3.30). In addition, we allow both the capital stock K_t and employment L_t to change over time. While the evolution of the capital stock will be at the center of our attention, we will assume that employment is exogenous, i.e. we will not consider RC's labor supply decision. We start by recalling his objective function:

(3.1) $$U_1 = u\left(C_1\right) + \beta\, u\left(C_2\right)$$

When maximizing his lifetime utility, RC has to consider the following constraints:

(3.3) $$B_1 = 0$$

(3.4) $$B_3 = 0$$

(3.30) $$Y_t = A_t\, F\left(K_t, L_t\right) \qquad t = 1, 2\,.$$

Concerning the production function in (3.30), we make the following assumptions on the first and second partial derivatives and the cross derivatives: $F_K > 0, F_L > 0, \quad F_{KK} < 0, F_{LL} < 0, F_{KL} = F_{LK} > 0$. This means that raising the amount of capital used while keeping employment of labor constant raises production, but the additional output decreases as we increase the use of capital – i.e. the production function is characterized by a ***diminishing marginal productivity of capital***. The same applies to the other factor of production: the ***marginal productivity of labor*** is strictly positive, but diminishing. The positive cross derivatives indicate that the marginal productivity of capital increases in employment and vice versa.

Further constraints to consider are:

(3.31) $B_{t+1} = (1+r)B_t + Y_t - C_t - I_t \qquad t = 1, 2.$

(3.32) $K_{t+1} = (1-\delta)K_t + I_t \qquad t = 1, 2.$

(3.33) $K_1 > 0$

(3.34) $K_3 = 0$

Equation (3.31) is the law of motion of the net international investment position familiar from (3.2), but this time accounting for gross investment I_t. Due to our focus on a small open economy, we keep assuming that the world interest rate $(1+r)$ is exogenous. Equation (3.32) indicates that the capital stock at the beginning of period $t+1$ (K_{t+1}) is the sum of gross investment I_t and of the capital stock available at the start of period t (K_t), reduced by physical depreciation δK_t. The ***rate of depreciation*** δ, with $\delta \in [0, 1]$, indicates that a share of the capital stock wears out in every period. The expression in (3.32) is based on the assumption that any investment in period t cannot be used productively before period $t+1$. Whether this assumption is realistic depends on the length of the periods we consider.

The new law of motion for the net international investment position in (3.31) accounts for the fact that RC can alternatively accumulate internationally traded bonds or domestic physical capital. The fact that gross investment shows up in (3.31) is based on the assumption that the consumption good we consider can also be used as a factor of production. This idea might be acceptable for some goods – say, computers – but usually the transformation of consumption goods into investment goods is costly. We detail in Box 3.5 that this can be accounted for by adding explicit ***costs of adjustment*** to the model.

As reflected by (3.33), we assume that RC's capital stock is strictly positive at the beginning of period 1. The assumption that the capital stock at the start of period 3 is zero (equation (3.34)) follows the same logic that led us to set $B_3 = 0$: since RC does not plan beyond period 2 it is optimal to completely dismantle and consume the remaining capital stock at the end of that period.

Substituting (3.30) and (3.32) into (3.31) yields the following law of motion for the net international investment position:

(3.35) $B_{t+1} = (1+r)B_t + A_t F(K_t, L_t) - C_t - K_{t+1} + (1-\delta)K_t$

Combining this with the boundary conditions (3.3) – (3.4) and (3.33) – (3.34) and solving for C_1 and C_2, we can write RC's objective function as follows:[13]

$$(3.36) \qquad \begin{aligned} U_1 = {} & u\left[A_1\, F\left(K_1, L_1\right) - B_2 + \left(1-\delta\right)K_1 - K_2\right] \\ & + \beta\, u\left[A_2\, F\left(K_2, L_2\right) + \left(1+r\right)B_2 + \left(1-\delta\right)K_2\right] \end{aligned}$$

RC chooses the values of B_2 and K_2 to maximize his lifetime utility. Taking derivatives of (3.36) with respect to B_2 and K_2 we get the following first-order conditions for an optimum:

$$(3.37) \qquad u'\left(C_1\right) = u'\left(C_2\right)\beta\left(1+r\right)$$

$$(3.38) \qquad u'\left(C_1\right) = u'\left(C_2\right)\beta\left[A_2\, F_K\left(K_2, L_2\right) + 1 - \delta\right]$$

The condition in (3.37) is the intertemporal Euler equation that we know from Section III.2 and that characterizes optimal consumption. Equation (3.38) has the same structure: the left-hand side gives the marginal utility of consumption in period 1, the right hand side gives the discounted marginal utility of consumption in period 2, multiplied by a term that reflects the marginal return of physical capital in that period plus the share of the capital stock that is left after accounting for depreciation. Combining (3.37) and (3.38) yields

$$(3.39) \qquad A_2\, F_K\left(K_2, L_2\right) - \delta = r$$

Equation (3.39) implicitly defines the optimal capital stock in period 2 and therefore optimal investment in period 1. This optimum is reached if the marginal return on an additional unit of physical capital equals the exogenous world interest rate, i.e. if the yields of the two alternative "stores of value" that are available to RC coincide. The marginal return on physical capital investment is given by the marginal productivity of capital – i.e. the additional output that can be generated by a marginal increase of the capital stock in period 2 – minus the rate of depreciation, which captures the depletion of physical capital over time.

Box 3.5: Adjustment Costs

Equation (3.39) is based on the assumption that the (composite) consumption good can simply be transformed into a (composite) investment good,

[13] Of course, we could also have solved this problem by using a Lagrangian function as in Subsection III.2.4. Substituting the constraints into the objective function is an alternative approach that saves on notation, but sacrifices information on the marginal utility of wealth.

which can be used in production in the following period. This assumption is not very realistic – in particular if the index t refers to rather brief time spans. To capture the notion that changes in the capital stock are associated with costs that exceed the mere price of investment goods, our model can be augmented by introducing ***adjustment costs***. In this case, the law of motion of the net international investment position (3.31) reads

$$B_{t+1} = (1+r)B_t + A_t\,F(K_t, L_t) - C_t - \Phi(I_t, K_t)$$

Here, $\Phi(I_t, K_t)$ captures the total costs of investment. We follow Hayashi's (1982) assumption that the pure adjustment costs are a quadratic function, i.e.

$$\Phi(I_t, K_t) = I_t + \frac{\phi}{2}\left(\frac{I_t}{K_t}\right)^2$$

This specification implies that both increasing and reducing the capital stock is costly, and that the size of adjustment costs depends on the volume of investment relative to the existing capital stock.

To derive optimal consumption and investment in this model, we use the simplifying assumption that the depreciation rate δ is zero. To proceed, we set up a Lagrangian function that is based on (3.36), but accounts for adjustment costs of investment. Moreover, we introduce three constraints that reflect the evolution of the capital stock as well as the fact that the capital stock at the end of period 2 must not be negative, i.e. $K_3 \geq 0$. Hence, RC maximizes

$$\begin{aligned} Z \;=\; & u\left[A_1\,F(K_1, L_1) - I_1 - \frac{\phi}{2}\left(\frac{I_1}{K_1}\right)^2 - B_2\right] \\ & + \beta\,u\left[A_2\,F(K_2, L_2) + (1+r)B_2 - I_2 - \frac{\phi}{2}\left(\frac{I_2}{K_2}\right)^2\right] \\ & + \mu_1(I_1 + K_1 - K_2) + \mu_2(I_2 + K_2 - K_3) + \mu_3 K_3 \end{aligned}$$

with μ_i representing the Lagrange parameter on constraint *i*, more specifically, the marginal utility (or "shadow price") of additional investment and capital in period *i*. Taking derivatives of Z with respect to B_2, I_1, K_2, I_2 and K_3 yields the following conditions that characterize optimal consumption and investment:

$$u'(C_1) = \beta(1+r)u'(C_2)$$

$$\left(1+\phi\frac{I_1}{K_1^{\,2}}\right)u'(C_1) = \mu_1$$

$$\beta\left[A_2\,F_K(K_2,L_2)+\frac{\phi}{K_2}\left(\frac{I_2}{K_2}\right)^2\right]u'(C_2)+\mu_2 = \mu_1$$

$$\beta\left(1+\phi\frac{I_2}{K_2^2}\right)u'(C_2) = \mu_2$$

$$\mu_2 = \mu_3$$

The first condition is the well-known intertemporal Euler equation. The second condition equates the marginal cost of investment and the marginal utility of a higher capital stock in period 2. The latter is expressed by the „shadow price“ μ_1. Defining $q_1 \equiv \frac{\mu_1}{u'(C_1)}$, we can transform this condition into:

$$I_1 = \frac{q_1-1}{\phi}K_1^{\,2}$$

This implies that investment in period 1 should be positive (negative) if q_1 is greater (smaller) than one, i.e. if the marginal utility of investment in period 1 (μ_1) is greater than the marginal disutility from reducing period-1 consumption. This condition is reminiscent of the famous result that firms should raise their investment as long as ***Tobin's q*** – the effect of additional capital on firm value – exceeds the price of investment goods (which, in this model, equals one).[14] Unlike Tobin, however, we do not focus on the market value of a firm, but on a comparison of marginal utilities.

The third optimality condition relates the marginal utility of first-period investment (μ_1) to three effects: first, additional capital enables RC to produce higher output in period 2. Moreover, it reduces the adjustment costs associated with future investment. And finally, investment in period 1 generates a capital stock that is evaluated at the shadow price μ_2 in period 2.

[14] This insight goes back to Brainard and Tobin (1968) and Tobin (1969).

The fourth optimality condition, which results from taking the derivative with respect to I_2, has the same interpretation as the second one. However, we now have to take into account that – unlike in the model without adjustment costs – it is not necessarily optimal for RC to dismantle the entire capital stock at the end of period 2. If high costs of negative investment make it preferable to carry a positive capital stock into the third period, the non-negativity constraint for K_3 is not binding, and it holds true that $\mu_3 = \mu_2 = 0$. Alternatively, optimal second-period investment is $I_2 = -K_2$ and we have $\mu_3 = \mu_2 > 0$. In this case, the value of μ_2 is given by the fourth optimality condition. By combining this result with the other three optimality conditions we arrive at an equation that implicitly defines the optimal capital stock in period 2:

$$1 + r = \frac{1 + A_2\, F_K\left(K_2, L_2\right)}{1 + \phi\, I_1 \,/ K_1^2}$$

If $\phi = 0$, i.e. if there are no adjustment costs, this expression coincides with (3.39) for the special case of $\delta = 0$. If ϕ is strictly positive the optimal capital stock differs from the one defined by (3.39). If optimal first-period investment is strictly positive $\left(I_1 > 0\right)$ the denominator of the ratio on the right-hand side is greater than one. This implies that the marginal productivity of capital has to be greater than the real interest rate. By contrast, if the optimal investment decision amounts to reducing the capital stock $\left(I_1 < 0\right)$, the marginal productivity of capital has to be lower than the real interest rate. Due to our assumption of a diminishing marginal productivity of capital, this implies that the existence of adjustment costs dampens both expansions and contractions of the capital stock.

The general principle that investment reacts less strongly to exogenous shocks if changes in the capital stock are associated with adjustment costs does not just apply to our two-period model, but also holds in more general settings.

Notice that none of the parameters that characterize the utility function appears in equation (3.39): apparently, RC's preferences are irrelevant when it comes to determining optimal investment. To better understand this result, it is worthwhile to derive the intertemporal budget constraint using equations (3.3) – (3.4) and (3.31). This yields

$$C_1 + \frac{C_2}{1+r} = Y_1 - I_1 + \frac{Y_2 - I_2}{1+r} \tag{3.40}$$

RC maximizes his utility, given the constraint that the present value of consumption has to equal the present value of income less investment. Substituting the production function and the law of motion for the capital stock into (3.40) gives

$$\begin{aligned} C_1 + \frac{C_2}{1+r} &= A_1\, F\left(K_1, L_1\right) + \left(1-\delta\right)K_1 - K_2 \\ &\quad + \frac{A_2\, F\left(K_2, L_2\right) + \left(1-\delta\right)K_2}{1+r} \end{aligned} \tag{3.41}$$

Regardless of his optimal consumption path, it must be in the interest of RC to maximize the expression on the right-hand side of (3.41). It is easy to show that this goal is achieved if the condition in (3.39) is satisfied.

For a given interest rate, we can thus split RC's optimization problem into two parts: in a first step, he chooses investment to maximize the present value of his lifetime income. In a second step, he takes the present value of lifetime income as given and chooses an optimal consumption path that is characterized by the intertemporal Euler equation.

As mentioned above, RC has two options to realize his optimal consumption path: by borrowing and lending on the international capital market, and by investing into physical capital. Let us assume for the time being that the first alternative is not available, i.e. that RC lives in a financially closed economy, and that he chooses the optimal level of consumption and investment in period 1. When doing so, he takes the initial capital stock K_1 as given.

Points O_1 and O_2 in Figure 3.8 represent two extreme alternatives: At O_1 , RC consumes all his income as well as the capital stock left over after production in period 1. This implies that consumption in the second period is zero. At point O_2, RC invests all his first-period income and the remaining capital stock, which raises consumption in period 2, but pushes consumption in period 1 down to zero. The points O_1 and O_2 are linked by the ***(intertemporal) transformation curve***, which represents the feasible combinations of C_1 and C_2 that can be implemented by an appropriate choice of investment in period 1. It is easy to show that the concave shape of this curve results from the negative second derivative of the production function with respect to the capital stock. In autarky, RC would choose the point on the transformation curve that maximizes his utility. In Figure 3.8 this is point **A**, where the (intertemporal) marginal rate of substitution – i.e. the slope of the indifference curve – coincides with the (intertemporal) ***marginal rate of transformation*** – i.e. the slope of the transformation curve. This point defines consumption in both periods and thus the difference between income and investment $\left(Y_t^a - I_t^a\right)$ for $t = 1, 2$ in autarky.

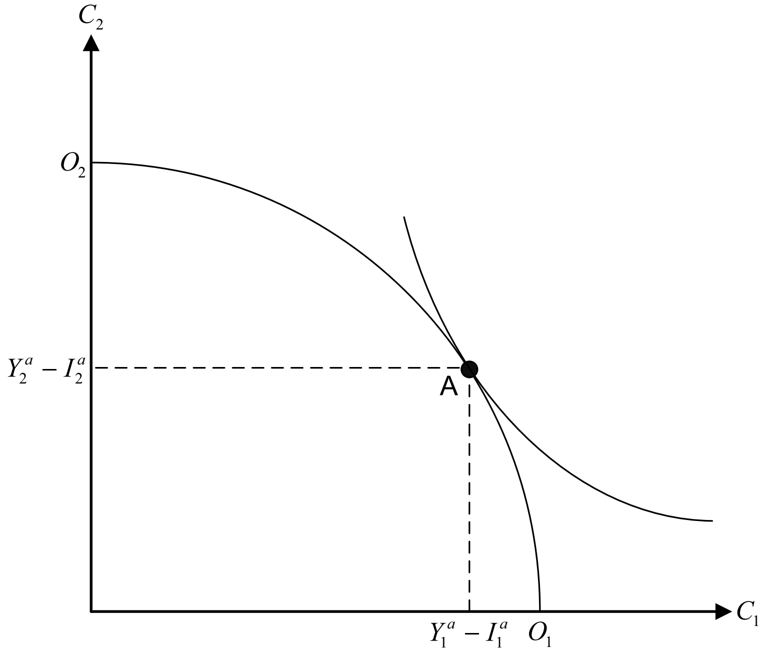


Figure 3.8: The intertemporal transformation curve and the optimal consumption path in autarky

If RC has access to the international capital market, point **A** in Figure 3.9 is still feasible. But when deciding on his optimal consumption, RC is also free to choose any other point on the dotted line that runs through point **A** and that represents the intertemporal budget constraint (3.40) for a given time path of investment. Given the relatively low interest rate that gives rise to the flat intertemporal budget constraint, RC can obviously expand his consumption possibilities by increasing period 1 investment and moving to point **B**. The reason is straightforward: in point **A**, the left-hand side of (3.39) exceeds the world interest rate. This means that the marginal return on investment is greater than the costs of borrowing. The possibility to finance capital accumulation by tapping the international capital market induces RC to choose a level of investment that is higher than what would have been optimal in autarky.

Point **B** defines the difference $Y_1^* - I_1^*$ in an open economy. In this point, the marginal productivity of capital (less the rate of depreciation) in period 2 is equal to the (exogenous) world interest rate. What is relevant for consumption is the linear budget constraint that is tangential to the transformation curve in point **B**. On this straight line, RC chooses the combination of C_1 and C_2 that maximizes his utility. The optimum is given by point **C** where the (intertemporal) marginal rate of substitution equals the real interest rate. The difference between $Y_1^* - I_1^*$ and C_1^* represents the current account balance in period 1. This difference is negative in Figure 3.9, which implies that the economy populated by RC exhibits a current account deficit in period 1. It goes without saying that this outcome applies to this particular example, and that there are parameter constellations that may result in a positive difference $Y_1^* - I_1^* - C_1^*$, i.e. a current account surplus in period 1. However, this does not change the logic underlying Figure 3.9.

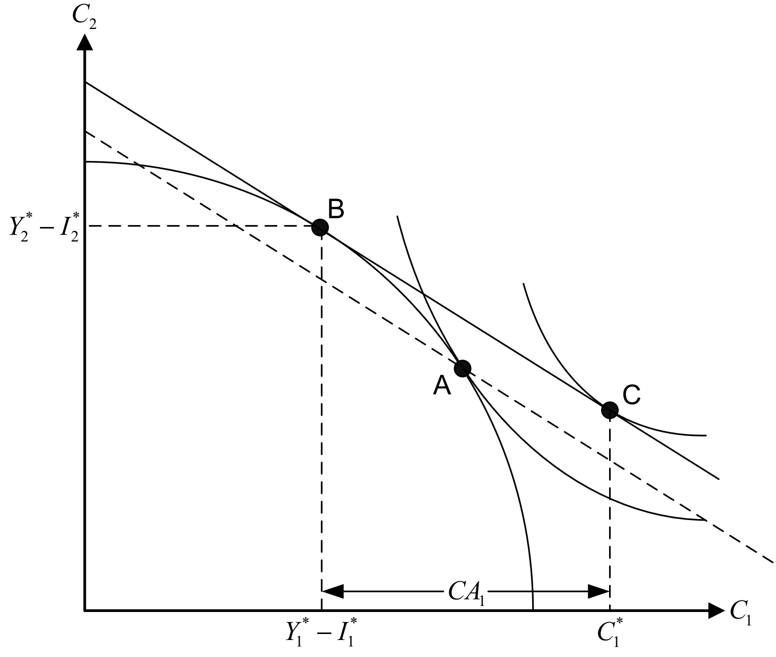


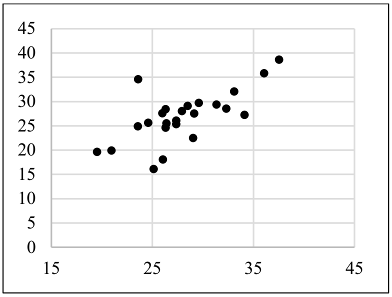
Figure 3.9: The current account and the gains from intertemporal trade in a small open economy with investment

In addition to illustrating a possible combination of investment and consumption chosen by RC in a financially open economy, Figure 3.9 documents that having access to the international capital market raises RC's utility. This welfare gain can be split into two parts: for a given time path of income and investment, the possibility to engage in intertemporal trade allows RC to realize a consumption path that is possibly superior to the consumption path in autarky. These are the “gains from intertemporal trade” that we already introduced in Subsection III.2.7. The additional gains that we identified in the current subsection result from the fact that the international capital market enables RC to *increase* the present value of his lifetime income. This is because, by borrowing abroad, he can realize a volume of investment that, in autarky, would have been associated with an enormous reduction of consumption and utility.

Box 3.6: The Rise and Fall of the „Feldstein-Horioka-Puzzle”

In an influential paper that was published in 1980, Martin Feldstein and Charles Horioka presented the results of a simple empirical analysis: for a set of OECD countries, they compared national savings and aggregate investment (as a share of GDP). The theory presented in this chapter suggests that these two variables should have little to do with each other – at least in a homogeneous group of developed economies. The international capital market should enable countries to realize a volume of investment that is much bigger than their savings. At the same time, there should be countries whose savings exceed their investment. Given this conjecture, the results of Feldstein and Horioka were rather surprising: they found that, in their sample, national savings and national investment were almost perfectly correlated. Apparently, international financial integration had proceeded more slowly than expected. Otherwise, current account balances – both surpluses and deficits – would have been much larger.

After the publication of the „Feldstein-Horioka puzzle“, an enormous amount of studies tried to explain the high correlation between national savings and investment, emphasizing, for example, the role of other variables that were driving both savings and investment or the importance of trade costs that reduced current account balances. As time went by, however, the puzzle seemed to become obsolete. In fact, both observed current account surpluses and observed current account deficits started to increase on a massive scale around the turn of the millennium. The two figures below document this evolution by showing saving rates (on the horizontal axis) and investment rates (on the vertical axis) for a subset of rich OECD countries. The left-hand side graph shows averages for the years 1970 to 1974, the right-hand side graph refers to the years 2000 to 2004. Obviously, the correlation between savings and investment was much lower in the more recent period. This indidates that the rapid integration of international capital markets eventually severed the link between savings and investment, and that the Feldstein-Horioka puzzle had quietly lost its relevance.



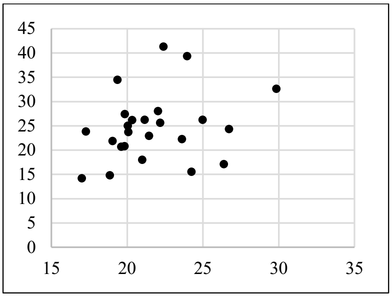


Figure B3.6: Average saving rates (horizontal axis) and investment rates (vertical axis) for high-income OECD countries (in percent). Left-hand side: averages for 1970 to 1974. Right-hand side: averages for 2000 to 2004. Source: World Bank (World Development Indicators).

As we have already discussed in Section III.3.2, the important finding that intertemporal trade raises the welfare of RC should not mask the fact that having access to the international capital market is not necessarily associated with utility gains for *all* agents in an economy. In the above example the possibility of utility losses was linked to movements in the interest rate, which differently affected the welfare of agents with different income paths. Once we add the possibility that financial integration influences the time path of capital accumulation in an economy, there are further effects on the income distribution, which result from a changing production structure and changing factor prices. To better understand these changes, it is necessary to adopt a less aggregate perspective and to consider an economy that is not populated by RC, but by a large number of agents who pursue their goals in a non-coordinated way.

III.6 Savings, Investment, and the Current Account in a Market Economy

III.6.1 Motivation

In Section III.3, we considered a version of the model in which a large number of agents chose their optimal consumption paths for a given interest rate and a given, individual-specific, time path of income. In this context we showed that the assumption of homothetic preferences allows the modelling of the behavior of macroeconomic aggregates –savings, consumption and the current account – *as if* they reflected the optimizing choices of a single "representative" individual. The RC model could thus be interpreted as a concise representation of a market economy.

In this section, we will go one step further and adopt the formal framework of Section III.3, but abandon the assumption that agents' incomes are exogenously given. To model the supply side of this economy, we will use the production function that we introduced in Section III.5. The representative consumer-producer will, however, be replaced by a large number of firms and consumers, and instead of modeling RC's optimization problem, we will explicitly analyze labor markets and financial markets as well as the decisions that drive firms' investment decisions and labor demand, as well as consumers' demand for assets.

III.6.2 Consumers' Saving Decision and Portfolio Choices

We continue to assume that the economy we are considering lasts for only two periods. As in Section III.3, lifetime utility of individual i, with $i \in \{1,2,...,N\}$, is thus given by:

$$U_1^i = u\left(C_1^i\right) + \beta\, u\left(C_2^i\right) \tag{3.21}$$

Agent i is maximizing U_1^i subject to the following constraints:

$$B_{t+1}^i + \sum_j x_{t+1}^{ij} V_t^j = w_t L_t^i + \left(1+r\right) B_t^i + \sum_j x_t^{ij}\left(d_t^j + V_t^j\right) - C_t^i \quad t = 1,2\,. \tag{3.42}$$

$$B_1^i = 0 \tag{3.43}$$

$$B_3^i = 0 \tag{3.44}$$

$$x_3^{ij} = 0 \tag{3.45}$$

The assumptions and arguments behind the constraints (3.43) and (3.44) are well-known by now. What's new about the law of motion in (3.42) is that agent i can devote his savings to two different purposes: while B_t^i represents the net stock of internationally tradable bonds that are held by agent i at the beginning of period t, and that yield an exogenous interest rate r over that period, the variable V_t^j stands for the market value of firm j at the end of period t, with $j \in \{1,2,...,J\}$. The share in firm j held by agent i at the beginning of period $t+1$ is given by the variable $x_{t+1}^{ij} \in [0,1]$. An individual can thus buy and sell internationally traded bonds that promise a fixed interest payment, and he can purchase shares of firms – i.e. ***equity*** – which establish a claim on future dividends $\left(d_{t+1}^{j}\right)$, and which can later be sold at a price V_{t+1}^{j}.[15] Note that B_{t+1}^{i} can also be negative. In this case, agent i borrows at the end of period t and has to repay his debt in the future.[16] The terminal conditions (3.44) and (3.45) reflect the fact that agents who have a finite time horizon don't find it optimal to carry a positive stock of securities into the third period and will not be allowed to leave period 2 with a negative stock of securities.

The right-hand side of (3.42) reflects the individual's savings, i.e. the difference between income and consumption. Income consists of labor income, which depends on the agent's labor supply L_t^i and the real wage w_t. Note that we allow for the possibility that labor supply differs across individuals and time periods. However, we do not *endogenize* individuals' labor supply, i.e. we take L_t^i as given.

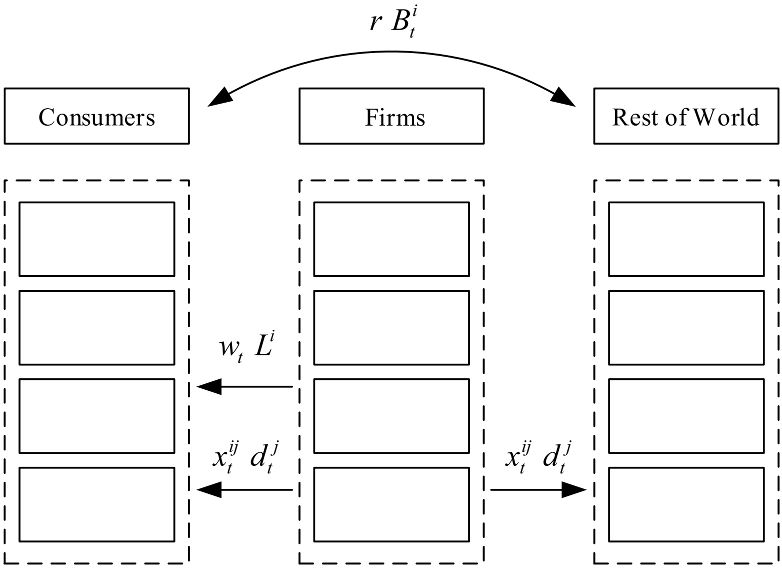


Figure 3.10: Payment flows in a market economy

Apart from his labor income, agent i receives a financial income: this income consists partly of interest and principal payments on the bonds he held at the beginning of period t – provided, of course, that the stock of bonds is positive. If $B_t^i < 0$, i.e. if the agent went into debt at the end of the previous period, these

[15] To keep the analysis simple, we exclude the possibility of domestic residents purchasing shares of *foreign* firms.

[16] We assume that x_{t+1}^{ij} is non-negative, thus excluding that individuals "short-sell" equity.

payments are negative. The other part of an agent's financial income goes back to the equity he holds in various firms: depending on the share of firm j he owns, he receives dividend payments $x_t^{ij}\, d_t^j$ at the end of period t. Moreover, he can sell his shares at a price V_t^j to domestic or foreign buyers. For the first time, we thus explicitly consider the possibility that agents trade both ***bonds*** and ***equity*** on the international capital market. The payments that take place between consumers, firms and the rest of the world are illustrated in Figure 3.10.

For given initial endowments and given prices, the problem of individual i amounts to choosing B_2^i and x_2^{ij} to maximize lifetime utility U_1^i. It is straightforward to derive the necessary conditions for an optimum:

$$u'\left(C_1^i\right) = \beta\left(1+r\right)u'\left(C_2^i\right) \tag{3.46}$$

$$u'\left(C_1^i\right) = \beta\left(\frac{d_2^j + V_2^j}{V_1^j}\right)u'\left(C_2^i\right) \tag{3.47}$$

Equation (3.46) is the well-known intertemporal Euler equation. The expression in (3.47) has the same structure, but the gross interest rate $\left(1+r\right)$ is replaced by the return on holding a share of enterprise j. This return is given by the sum of dividend payments and the resale value at the end of period 2, divided by the price at which the firm was bought in period 1. By combining (3.46) and (3.47) we arrive at

$$\left(1+r\right) = \frac{d_2^j + V_2^j}{V_1^j} \tag{3.48}$$

Equation (3.48) shows that, in equilibrium, the return on equity has to coincide with the (exogenous) real interest rate. Of course, this requirement is an implication of our assumption that agents have perfect foresight. If we allowed for uncertainty about future dividend payments and market valuations, the ***no-arbitrage condition*** in (3.48) would have to be replaced by an equation that accounted for the different risk associated with bonds and equity.

A further simplification is due to our two-period setup: since firm j no longer exists in period 3 its resale value at the end of period 2 is zero, i.e. $V_2^j = 0$. Using this and re-arranging (3.48) gives

$$V_1^j = \frac{d_2^j}{1+r} \tag{3.49}$$

The value of firm j at the end of period 1 thus reflects the present value of its total dividend in period 2.

III.6.3 Firms' Labor Demand and Investment

How high are firms' dividends in periods 1 and 2? This depends on their investment and production decisions. We assume that, in period 1, the management of firm j maximizes the present value of current and future dividends:

(3.50) $$\Pi_1^j = d_1^j + \frac{d_2^j}{1+r}$$

These dividends reflect the difference between revenues and labor costs as well as expenses for investment. For periods 1 and 2 this implies:

(3.51) $$d_1^j = A_1\, F\left(K_1^j, L_1^j\right) - w_1\, L_1^j - \left[K_2^j - \left(1-\delta\right) K_1^j\right]$$

(3.52) $$d_2^j = A_2\, F\left(K_2^j, L_2^j\right) - w_2\, L_2^j + \left(1-\delta\right) K_2^j$$

The fact that the production function F in (3.51) and (3.52) does not carry the subscript j reflects the important assumption that firms may differ in terms of their capital stock and employment, but not with respect to the technology they use. Hence, the production function is identical for all firms. Moreover, equation (3.52) documents that the entire capital stock is liquidated at the end of period 2, and that the proceeds are paid out as dividends. Concerning the technology used by firms, we assume that it is characterized by ***constant returns to scale***, i.e. the production function F is ***linearly homogeneous***. This implies that a firm's output is multiplied by λ if all factor inputs – here: labor and capital – are multiplied by λ. Box 3.7 explains this concept in more detail.

Box 3.7: Homogeneous Functions and Returns to Scale

A production function F whose arguments are the factors of production K and L is ***homogeneous*** if, for an arbitrary $\lambda > 0$, we have

$$F\left(\lambda K, \lambda L\right) = \lambda^x F\left(K, L\right)$$

In this expression, x represents the ***degree*** of homogeneity: If $x = 1$, the function F is ***homogeneous of degree one***, i.e. ***linearly homogeneous***. Linear homogeneity implies that a firm's output is twice (three times, four times etc.) as large if the input of *all* factors of production is doubled (tripled, quadrupled etc.). In this case, the technology is characterized by ***constant returns to scale***. It is easy to show that a firm with a linearly homogeneous production function has ***constant average costs***: since a doubling of output is achieved by a doubling of all factor inputs, costs are a linear function of output, and average costs do not depend on the amount produced. By contrast, ***increasing returns to scale*** are associated with average costs that are decreasing in the amount produced. In the above example, this is the case if x is greater than one. Finally, ***decreasing returns to scale*** are characterized by increasing average costs.

The management of firm j chooses three variables: employment in periods 1 and 2 (L_1^j, L_2^j) and the capital stock in period 2 (K_2^j). When making this decision, managers face ***perfect competition*** on all markets, i.e. they take the goods price, the interest rate and the real wage in both periods as given. An optimal decision is characterized by the following conditions:

$$A_t\, F_L\left(K_t^j, L_t^j\right) = w_t \qquad t = 1, 2. \tag{3.53}$$

$$A_2\, F_K\left(K_2^j, L_2^j\right) - \delta = r \tag{3.54}$$

Equation (3.53) states that the firm's optimal labor demand equates the marginal productivity of labor and the real wage in period t. The condition in (3.54) requires the marginal productivity of capital less the depreciation rate to equal the interest rate. We have already seen this equation in Section III.5. Unlike (3.39), however, equation (3.54) does not refer to the investment decision of a representative consumer, but to the choice of a single firm.

III.6.4 Equilibrium

In equilibrium, firms' aggregate labor demand equals individuals' total labor supply. As we will show, we can combine this equilibrium condition with the optimality conditions (3.53) – (3.54) to derive the real wage in both periods as well as the capital stock in period 2.

Because firms' production functions are characterized by constant returns to scale, we can exploit the fact that partial derivatives of linearly homogeneous functions are *homogeneous of degree zero* – i.e. they do not change if all arguments are multiplied by the same number. The conditions (3.53) – (3.54) are thus equivalent to

(3.55) $$A_t\, F_L\left(\frac{K_t^j}{L_t^j},1\right)=w_t \qquad t=1,2\,.$$

(3.56) $$A_2\, F_K\left(\frac{K_2^j}{L_2^j},1\right)-\delta=r$$

These conditions determine the ***capital-labor ratio*** chosen by firm j in both periods. Since all firms face the same market wage, they choose identical capital-labor ratios, which, in turn, have to coincide with the capital-labor ratio at the economy-wide level, i.e.

(3.57) $$\frac{K_t^j}{L_t^j}=\frac{K_t}{L_t}, \qquad t=1,2\,.$$

with K_1 being given and L_1 as well as L_2 reflecting the (exogenous) aggregate labor supply in the two periods. Concerning K_2, the conditions in (3.56) and (3.57) imply

(3.58) $$A_2\, F_K\left(\frac{K_2}{L_2},1\right)-\delta=r$$

Hence, for a given labor supply L_2 and a given real interest rate r, the aggregate capital stock is unambiguously defined by equation (3.58). This equation is well-known to us: it is the condition that characterized the optimal investment decision of RC in Section III.5. Hence, firms in a market economy that use identical constant-returns-to-scale technologies behave like one big firm that exploits all productive capacity. The output of this ***representative firm*** is described by summing up the output of all individual firms, i.e.

(3.59) $$\sum_{j=1}^{J} A_t\, F\left(K_t^j, L_t^j\right)=A_t\, F\left(K_t, L_t\right)$$

This result is due to our constant-returns-to-scale assumption, i.e. the notion that it does not matter for aggregate production whether the total capital stock and the total labor supply is used by a single firm or spread over multiple firms. Moreover, our assumption of frictionless markets is crucial: if individual firms enjoyed ***monopoly power***, the equilibrium in a decentralized economy would differ substantially from the equilibrium in a representative-consumer model.

Such a difference would also arise in the presence of ***externalities***, i.e. if some economically relevant interactions did not take place on markets.[17]

The notion of the "representative firm" considerably simplifies our analysis since it allows analyzing aggregate production and investment without explicitly modeling the interaction of heterogeneous firms on goods and factor markets. This simplification carries over to consumers' portfolio decision: the only decision individual i has to take is which portion of his savings he devotes to shares of the representative firm whose total value is given by V_t, and which portion to internationally traded bonds. Summing up the simplified version (3.42) over all N individuals we get

$$\sum_{i=1}^{N} B_{t+1}^i + \sum_{i=1}^{N} x_{t+1}^i V_t = w_t \sum_{i=1}^{N} L_t^i + (1+r)\sum_{i=1}^{N} B_t^i + \sum_{i=1}^{N} x_t^i \left(d_t + V_t\right) - \sum_{i=1}^{N} C_t^i \tag{3.60}$$

In what follows, we define the aggregate magnitudes $B_t \equiv \sum_{i=1}^N B_t^i, C_t \equiv \sum_{i=1}^N C_t^i$ etc. Moreover, we assume that $\sum_{i=1}^N x_1^i = 1$. This implies that, at the beginning of period 1, the representative firm is completely owned by domestic residents. However we allow for the possibility that shares of this firm are sold to foreign residents at the end of period 1, such that $\sum_{i=1}^N x_2^i = \theta$ with $\theta \in [0,1]$. In our two-period model, this results in the intertemporal budget constraint

$$C_1 + \frac{C_2}{1+r} = Y_1 + (1-\theta)V_1 - I_1 + \frac{\theta\, d_2 + w_2\, L_2}{1+r} \tag{3.61}$$

where we have already taken into account that, as described by equation (3.51), first-period dividends of the representative firm are given by $d_1 = Y_1 - w_1 L_1 - I_1$.

If domestic individuals sell a portion $(1-\theta)$ of their domestic firm shares to foreigners at the end of period 1, this reduces their claims on future dividends. Combining the above equation with the no-arbitrage condition (3.49) and the definition of second-period dividends in (3.52), we can turn it into

$$C_1 + \frac{C_2}{1+r} = Y_1 - I_1 + \frac{Y_2 - I_2}{1+r} \tag{3.62}$$

This is another well-known expression: it is the intertemporal budget constraint which the representative consumer had to take into account when deciding on

[17] A detailed analysis of the normative properties of market equilibria is given by Boadway and Bruce (1984). German-speaking readers will find Breyer and Kolmar (2005) very useful.

his optimal consumption path in Section III.5. To characterize this decision, we can use the results from Section III.3: if agents have identical homothetic preferences, the behavior of macroeconomic aggregates, which results from a multitude of individual, uncoordinated choices, can be modelled *as if* it reflected the decisions of a single representative agent.

The importance of this finding can hardly be exaggerated, since the representative consumer is omnipresent in modern macroeconomic research. While it may seem abstruse at first glance to treat an entire economy as if it consisted of only one single producer/consumer, the last paragraphs have provided a justification for this approach. Under the assumption that all agents have identical homothetic preferences, that firms' technologies are characterized by constant returns to scale, and that markets work perfectly, the behavior of a market economy is identical to the behavior of an RC economy.[18]

The concept of the representative consumer is popular, but not uncontested in economics. Note, however, that a critique of the RC model does not invalidate the general approach to base the explanation of macroeconomic phenomena on an analysis of consumers' and firms' dynamic optimization problems. But if one questions the assumptions on which the RC model is based, aggregation works differently: instead of postulating the existence of a representative consumer/producer from the outset, one starts by considering the decisions of single firms and agents. The interaction of those individual entities results in market equilibria that ultimately determine the behavior of macroeconomic aggregates. This approach is more challenging than the use of a representative consumer, but the choices of individual agents are determined by the very objectives, constraints and trade-offs that we know from the RC model.

III.6.5 Gains from Intertemporal Trade?

The use of the RC framework for the analysis of macroeconomic aggregates is tied to a set of assumptions whose plausibility may be contestable. But even when accepting these assumptions we have to be aware that the representative-consumer model is not suitable to analyze the distributional consequences of international financial integration. In Section III.3, we used an exchange economy to discuss the possibility that the "aggregate gains from intertemporal trade" may contrast with welfare losses at the individual level. Once we endogenize production and investment, we have to take into account that the

[18] Once we move from a perfect-foresight environment to a framework that allows for uncertainty, the applicability of the RC model depends on how individuals cope with risk: if they can trade securities that allow them to insure against all idiosyncratic – i.e. individual-specific – shocks, markets are said to be complete, and the RC model is appropriate. In Chapter IV, we will consider how uncertainty affects individual decisions.

access to the international capital market possibly changes factor prices and thus agents' current and future incomes.

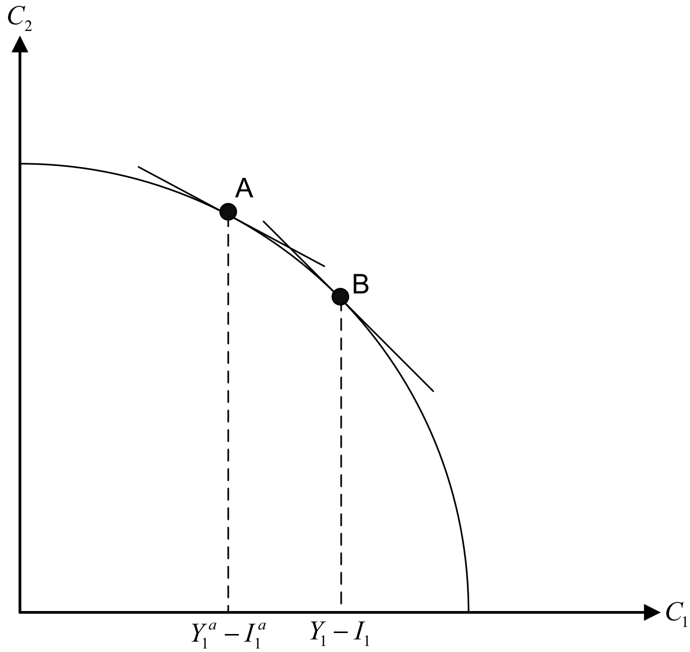


Figure 3.11: Financial integration and optimal investment

To illustrate this, we use a simple example: Figure 3.11 shows the intertemporal transformation curve of an economy that chooses point **A** in autarky – be it for technological reasons, be it due to its citizens' preferences. This point reflects a high investment level in period 1 and thus a high capital stock and high output in period 2. The autarky interest rate $1 + r^a$ is given by the slope of the tangent in point **A**. Once we move from financial autarky to financial integration, the option to borrow and lend on the international capital market possibly affects investment decisions in period 1.

In Figure 3.11 we assume that the world interest rate is higher than the autarky interest rate, such that first-period investment decreases (point **B**): the return offered by the international capital market is just higher than the marginal productivity of capital in point **A**. Hence, it is more attractive to "export capital" instead of building up the domestic capital stock. The drop of domestic investment lowers the capital stock in period 2, and this is associated with a lower real wage w_2.[19] In this example, the future wage level thus decreases as a result of financial integration, and it is possible that, for some individuals, the present value of income decreases relative to the autarky situation. The lower wage has to be weighed against the welfare gains from implementing a more favorable consumption path, though. Moreover, returns to capital are higher in period 2, which boosts individuals' future incomes. Nevertheless, it is easy to come up with constellations where the transition from autarky to financial integration generates welfare losses at the individual level – in particular if some agents do not have access to the international capital market and therefore completely rely on their labor income. However, this does not disqualify the important result that – under the ideal conditions of frictionless markets and perfect competition – an economy as a whole benefits from having access to the international capital market. If there exists a transfer mechanism that allows compensating the losers, policymakers can target a situation in which individual welfare gains from financial integration are never negative. The fact that ***financial globalization*** rarely meets unlimited support is due to the fact that these transfer mechanisms are often underdeveloped in reality, and that there is a limited arsenal of policy tools that can be used to dampen the distributional effects of financial globalization.

III.7 An Infinite Time Horizon

III.7.1 Motivation

So far, we have considered an economy whose life span was limited to two periods. Whereas such an assumption may be justified with respect to individual agents, it is hard to explain why entire *countries* should face some terminal period in which the total capital stock and the net international investment position are liquidated and consumed. Moreover, the short time span we considered made it impossible to analyze adjustment processes that last for more than one period. Given these drawbacks, this section will demonstrate how the RC model with an exogenous income can be extended to an infinite horizon. As we will show, many of the insights we have gained in the preceding sections still apply in an infinite-horizon framework.

[19] This follows from the assumption that the cross derivative of the production function F_{LK} is strictly positive.

III.7.2 Optimization with an Infinite Time Horizon

We consider a representative consumer (RC) who maximizes the following objective function in period t:

$$(3.63) \qquad U_t = \sum_{s=t}^{\infty} \beta^{s-t}\, u\left(C_s\right)$$

The sign ∞ indicates that the optimization now refers to an infinite time horizon. Given that life is finite for most human beings, this seems like another hair-raising assumption. However, as detailed in Box 3.8, an infinite horizon can be justified by the assumption that agents take into account the welfare of all subsequent generations. They are thus maximizing the utility of an entire dynasty, and implement the optimal consumption path by choosing an appropriate time path of bequests.[20]

Box 3.8: The "Dynastic" Utility Function

Suppose that every individual lives for one period and has a single offspring. The offspring's utility is taken into account when making optimal choices, i.e. an individual living in period t maximizes

$$U_t = u\left(C_t\right) + \beta\, U_{t+1}$$

Note that the second term on the right-hand side does not just depend on the offspring's *consumption* in period $t+1$. Instead, since the offspring's utility depends on the well-being of his descendant, RC implicitly also takes into account the utility of the subsequent generation. Hence,

$$U_t = u\left(C_t\right) + \beta\, u\left(C_{t+1}\right) + \beta^2\, U_{t+2}$$

Repeating this iteration *ad infinitum* generates the objective function in (3.63). In every period, RC decides on the optimal consumption path for himself and all subsequent generations. This consumption path is implemented by choosing an appropriate time path of bequests. Note that these

[20] Be sure to understand the conceptual difference between the time subscript t, which denotes the period in which optimal plans are designed, and the subscript s, which denotes the periods these plans refer to. We have to introduce this distinction since, unlike in a two-period model, intertemporal optimization does not just take place in one period. Instead, RC moves through time (from period t to t+1 to t+2 etc.), constantly revising his plans and possibly adjusting them to newly acquired information.

bequests can be positive as well as negative, i.e. subsequent generations may benefit from inherited *assets*, but they are also responsible for inherited *liabilities*.

The specification of the objective function makes sure that RC sticks to his optimal consumption path unless he receives new information.[21] Hence, his behavior is ***time consistent***, and he has no incentive to revise his plans at a later point in time. We should be aware that this is a very restrictive description of human behavior: as we all know, everyday life is characterized by failed plans, revised resolutions, and broken promises. In Box 3.9, we introduce the concept of "***hyperbolic discounting***", which can be used to integrate *time-inconsistent* behavior into dynamic models.

We assume that RC faces an exogenous time path of income, such that there is no investment. When maximizing his objective function, RC has to take into account the well-known law of motion for the net international investment position:

$$(3.64) \qquad B_{s+1} = \left(1+r\right) B_s + Y_s - C_s \qquad s = t, t+1, t+2, ...$$

Note, however, that this equation now refers to an *infinite sequence* of periods. Concerning the initial condition, it is possible to set B_t equal to zero if we consider t to be the "first period". In all subsequent periods $\left(t+1, t+2, ...\right)$, however, the net international investment position depends on past consumption decisions.

Box 3.9: Hyperbolic Discounting

It is an important implication of the utility function in (3.63) that the intertemporal marginal rate of substitution (MRS) between two arbitrary periods s and $s+1$ does not depend on how far these periods are away from the current planning period t. The concept of ***hyperbolic discounting*** which is described, e.g., in Angeletos et al. (2001) abandons this concept and replaces (3.63) by the following utility function:

[21] In principle, our perfect foresight-assumption excludes the arrival of new information by definition. This contradiction is solved by assuming that there are "non-anticipated shocks" whose potential occurrence is not taken into account by the representative consumer when he chooses his optimal consumption path.

$$U_t = \sum_{s=t}^{\infty} \left(\frac{1}{1+\alpha(s-t)} \right)^{\frac{\gamma}{\alpha}} u(C_s)$$

This specification implies that the intertemporal MRS *decreases* as periods move into the more distant future – a property that also applies to preferences which are characterized by ***quasi-hyperbolic discounting***:

$$U_t = u(C_t) + \delta \sum_{s=t+1}^{\infty} \beta^{s-t} u(C_s) \quad \text{with } 0 < \delta < 1$$

From the perspective of period t the intertemporal MRS between $t+1$ and $t+2$ is given by

$$MRS^{t}_{t+1,t+2} = u'(C_{t+1}) / \left[\beta u'(C_{t+2}) \right].$$

However, once we move to period $t+1$, we have

$$MRS^{t+1}_{t+1,t+2} = u'(C_{t+1}) / \left[\delta \beta u'(C_{t+2}) \right]$$

This means that a reduction of consumption in period $t+1$ is easier to realize if $t+1$ is in the distant future. A twenty-year old person easily plans to reduce his consumption at the age of 35 in order to save for retirement. However, once this person approaches the 35th year of his life, renouncing consumption turns out to be much harder, and there is a temptation to renege on the original plan, i.e. to save much less than originally intended.

The empirical plausibility of hyperbolic and quasi-hyperbolic discounting – as opposed to the ***exponential discounting*** of the standard model – is supported by the observation that a lot of people are willing to lock their savings into very illiquid retirement plans. Apparently, they are aware of the danger of time-inconsistent behavior and prefer commitment over flexibility.

With an infinite time horizon, the terminal condition of (3.4) becomes obsolete: if there is no final period, there is no point in time at which the net international investment position has to equal zero. In fact, the expression in (3.4) is replaced by the following ***transversality condition***:

(3.65) $$\lim_{\tau \to \infty} \left(\frac{1}{1+r} \right)^{\tau} B_{t+\tau+1} = 0$$

This condition requires the absolute value of the net international investment position to grow at a rate that is lower than the real interest rate r. It would be violated if RC financed the entire repayment – i.e. principal and interest – that is due in a given period by issuing new debt. This practice was at the heart of a snow-ball system set in motion in the early 20th century by the Italo-American trickster Charles Ponzi. Ponzi (and most of his later imitators) ended up in jail, which suggests that "***Ponzi schemes***" may be successful in the short run, but are rarely sustainable in the long run. This observation, in turn, provides a justification for imposing the above transversality condition.

To derive the intertemporal budget constraint faced by RC at the beginning of period t, we start by re-arranging the expression in (3.64):

(3.66) $$B_t = \left(\frac{1}{1+r} \right) \left(C_t - Y_t + B_{t+1} \right)$$

The same condition holds for period $t+1$, hence

(3.67) $$B_t = \left(\frac{1}{1+r} \right) \left(C_t - Y_t \right) + \left(\frac{1}{1+r} \right)^2 \left(C_{t+1} - Y_{t+1} + B_{t+2} \right)$$

Continuing this iteration, we eventually arrive at the following expression:

(3.68) $$\left(1+r\right) B_t = \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} \left(C_s - Y_s \right) + \lim_{\tau \to \infty} \left(\frac{1}{1+r} \right)^{\tau} B_{t+\tau+1}$$

The transversality condition introduced above allows us to eliminate the second term on the right-hand side. This yields

(3.69) $$\sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} C_s = \left(1+r\right) B_t + \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} Y_s$$

The infinite-time-horizon version of the intertemporal budget constraint has the same interpretation as (3.6) in the two-period model: the present value of consumption must not be greater than the present value of income, plus principal and interest on the initial net international investment position. Since utility is monotonically increasing in consumption, RC makes sure that his consumption does not fall short of the maximally feasible level, such that the equation in (3.69) holds exactly. As long as RC complies with this constraint he can use the international capital market to spread his consumption over different time periods.

The optimal consumption path is determined by maximizing the following Lagrange function:

$$Z_t = \sum_{s=t}^{\infty} \beta^{s-t} u(C_s) + \lambda_t \left[(1+r) B_t + \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} Y_s - \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} C_s \right] \tag{3.70}$$

with λ_t representing the marginal utility of wealth in period t, as in Subsection III.2.4. Maximizing (3.70) over C_s yields the first-order condition, which implicitly characterizes the optimal consumption path

$$u'(C_s) = \beta (1+r) u'(C_{s+1}) \qquad s = t, t+1, t+2, ... \tag{3.71}$$

Combined with the law of motion for the net international investment position, this *infinite sequence of Euler equations* determines the consumption path from the present period t until infinity.

In the special case of $\beta(1+r)=1$, we can characterize this consumption path without even knowing RC's utility function. As we have demonstrated in Section III.2, this case is characterized by $C_t = C_{t+1} = C_{t+2} = ...$ etc. Factoring out C_t in (3.69) we find that[22]

$$C_t = r B_t + \left(\frac{r}{1+r} \right) \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} Y_s \tag{3.72}$$

The expression in (3.72) once more highlights the role of financial integration: whereas his income Y_s may be subject to strong fluctuations, the ability to borrow and lend on the international capital market allows RC to realize a flat consumption path. This constant consumption level depends on his initial net international investment position and on the present value of his lifetime income. If

[22] When deriving this expression, we use the result that $\sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} = \frac{1+r}{r}$ for $|1/(1+r)| < 1$.

$\beta(1+r)\neq 1$, optimal consumption follows an increasing or a falling trend. But in this case the access to the international capital market also allows to iron out temporary income fluctuations. The simple example presented in Box 3.10 uses this model to describe the reaction of consumption and the current account to a temporary income shock.

If we augment the infinite-horizon model as in Section III.5 by introducing a production function and investment, we have to consider an additional set of optimality conditions starting in period *t*. RC maximizes lifetime utility by choosing the optimal capital stock in all following periods. The resulting first-order conditions are given by

$$(3.73) \qquad A_{s+1}\, F_K\left(K_{s+1}, L_{s+1}\right) - \delta = r \qquad s = t, t+1, t+2, ...$$

Accounting for investment, the intertemporal budget constraint is given by

$$(3.74) \qquad \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} C_s = (1+r)B_t + \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}\left(Y_s - I_s\right),$$

with period-*s* investment being implicitly defined by the initial capital stock in every period and the condition in (3.73), which characterizes the optimal future capital stock.

Box 3.10: A Temporary Income Shock and the Current Account

We assume that $\beta(1+r)=1$ and that the net international investment position of RC is zero at the beginning of period t. Until period t, RC receives a constant income Y. In period t, RC faces a non-anticipated shock that reduces his income by Δ goods units. Subsequent periods are characterized by

$$Y_s = Y - \rho^{s-t}\Delta \qquad \text{for } s = t, t+1, t+2, ...$$

and $0 \leq \rho \leq 1$.

If ρ is smaller than one, the income reduction disappears over time. Using (3.72), we can show that consumption in period t immediately drops to the following level:

$$C_t = Y - \frac{r}{1+r-\rho}\Delta$$

Notice that, for $0 \le \rho < 1$, the reduction of consumption is less pronounced than the reduction of income. The reason is that RC uses his access to the international capital market to iron out the temporary income shock. The current account balance in period t is given by

$$CA_t = -\frac{(1-\rho)}{1+r-\rho}\Delta$$

The role of the parameter ρ for the level of consumption and the current account has a straightforward interpretation: the higher ρ – i.e. the *more persistent* the income shock – the stronger the reduction of consumption, and the lower the current account deficit in the periods after the shock. In the extreme case of a *permanent* income reduction ($\rho = 1$) the current account remains balanced since the income path does not offer any scope for consumption smoothing.

III.7.3 A Variable Interest Rate

So far we have assumed that the exogenous world market interest rate does not change over time. In a model with an infinite time horizon, this assumption is not really plausible. If we allow for changes of r in a model with an exogenous income, the law of motion of the net international investment position reads

$$B_{s+1} = Y_s + (1+r_s)B_s - C_s \qquad s = t, t+1, t+2, ... \tag{3.75}$$

with r_s representing the interest rate prevailing in period s. By iterating forward and using a modified version of the transversality condition in (3.65), we arrive at the following intertemporal budget constraint:[23]

$$(1+r_t)B_t + \sum_{s=t}^{\infty}\prod_{j=t}^{s}\left(\frac{1}{1+r_j}\right)Y_s = \sum_{s=t}^{\infty}\prod_{j=t}^{s}\left(\frac{1}{1+r_j}\right)C_s \tag{3.76}$$

The optimal time path of consumption is characterized by the intertemporal Euler equation

$$u'(C_s) = \beta(1+r_{s+1})u'(C_{s+1}) \qquad s = t, t+1, t+2, ... \tag{3.77}$$

[23] Note that $\prod_{j=t}^{s}\left(\frac{1}{1+r_j}\right) = \left(\frac{1}{1+r_t}\right)\left(\frac{1}{1+r_{t+1}}\right)\cdots\left(\frac{1}{1+r_s}\right)$

This expression differs from (3.71) only by allowing for a time-variant interest rate.

III.8 Endogenizing the World Interest Rate

III.8.1 Motivation

One of the crucial elements of our analysis so far has been the assumption that the world interest rate is not affected by saving and investment decisions of the country under consideration. We thus analyzed a ***small open economy*** whose representative consumer took the real interest rate r as an exogenously given parameter.

In this section we want to understand the forces that determine the interest rate which emerges as an equilibrium price on the international capital market. Our analysis will be based on the assumption that the world consists of two large countries ("home" H and "foreign" F). Both countries are large enough for changes in their national saving and investment behavior to affect the world interest rate. However, individuals in H and F take the interest rate as given. This assumption makes sure that neither consumers nor firms behave strategically, using their saving and investment choices to influence the world market interest rate.

III.8.2 The World Interest Rate in Equilibrium

As in Section III.5, we restrict our attention to a two-period model, and we assume that both countries have a net international investment position of zero at the beginning of period 1. Hence,

$$(3.78) \qquad CA_1^i = S_1^i - I_1^i = B_2^i \qquad\qquad i = H, F$$

Equilibrium on the global capital market is characterized by

$$(3.79) \qquad B_2^H + B_2^F = 0$$

This has a straightforward interpretation: since our model world consists of two countries, net international investment positions have to add up to zero. If country H's net demand for assets in period 1 is positive ($B_2^H > 0$), it has to meet a supply of equal size from country F ($B_2^F = -B_2^H < 0$). The price that sustains such an equilibrium is the world interest rate. As indicated by equation (3.78), the net demand for assets equals the current account balance in period 1 which, in turn, reflects the difference between domestic savings and domestic investment.

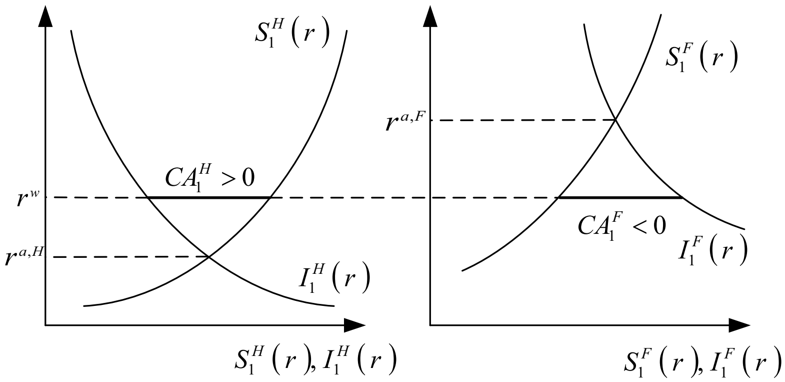


Figure 3.12: The world interest rate in equilibrium

We have seen in previous sections that both investment and savings depend on the interest rate: for a given initial capital stock, investment is decreasing in r. This is due to our assumption of diminishing marginal returns to capital. As stated by (3.39), a lower future capital stock – and hence lower investment – is thus optimal if the interest rate increases, and we can draw the domestic and the foreign investment function in Figure 3.12 as curves that are falling in r. Concerning the saving functions in H and F, we draw them as curves that are increasing in the interest rate. We thus implicitly assume that the substitution and the wealth effect of an interest rate change dominate the income effect.

The autarky interest rates $r^{a,H}$ and $r^{a,F}$ are given by the intersection of saving and investment functions in both countries. This follows from the fact that the current account must be zero under (financial) autarky, such that all domestic investment is financed out of domestic savings. In Figure 3.12, country H saves more and invests less for a given interest rate than country F. This constellation results in the autarky interest rate being lower in H than in F. If we allow inhabitants of the two countries to trade assets on the international capital market, the "world interest rate" r^w establishes an equilibrium. With this interest rate, the global net-demand for assets is zero: country H exhibits a current account surplus which, in absolute value, coincides with the current account deficit of country F. In country H, the rising interest rate raises savings and lowers investment compared to financial autarky. This reflects the fact that it becomes more attractive for residents of country H to purchase foreign assets instead of investing domestically. The consequences of financial integration in country F mirror those in country H: national savings *decrease*, and the additional investment is financed by tapping the international capital market. As a result, the current account exhibits a deficit.

III.8.3 Comparative-Static Analysis: Changes in the World Interest Rate

How do changes in one country affect the global interest rate and thus saving and investment behavior around the world? As an example, we consider an exogenous reduction of aggregate savings in country H. Such a change could be caused by individuals in H expecting a higher income in period 2 and thus reducing their savings in period 1.

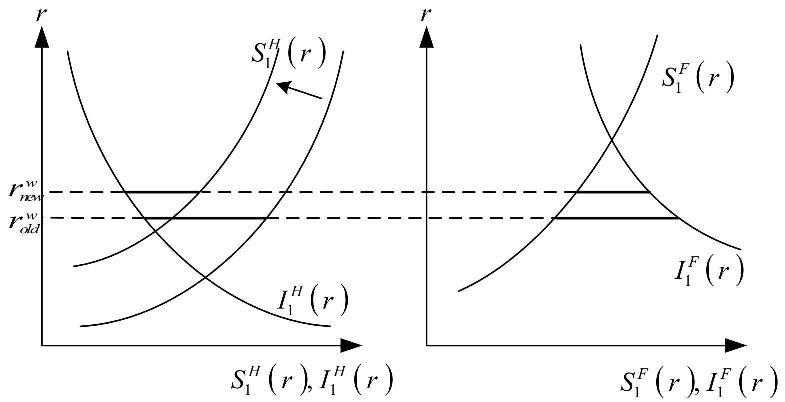


Figure 3.13: Declining savings in country H and the world interest rate.

In Figure 3.13, this exogenous variation is represented by a shift of country H's saving function to the left. With such a change, the "old" interest rate r_{old}^w no longer sustains an equilibrium since the current account surplus in country H falls short of financing the current account deficit of country F. Instead, r_{new}^w emerges as a new equilibrium interest rate. By moving from the old to the new equilibrium, the difference between savings and investment decreases in country H. Via the higher interest rate, this also affects saving and investment behavior in country F: savings increase while investment drops.

This example highlights an important aspect of financial globalization: while having access to the international capital market sustains consumption smoothing via borrowing and lending, it is also associated with a greater exposure to changes of international financial conditions – for better or worse: if savings in another part of the world increase, this potentially enhances domestic investment. Conversely, a sudden increase of the world interest rate, possibly driven by declining savings in another country, may result in a reduction of domestic investment, and this effect is only partly buffered by higher domestic savings.

III.9 Summary and Outlook

The intertemporal model introduced in this chapter is at the core of modern macroeconomic theories that base the analysis of aggregate variables on the consideration of microeconomic decisions, and that focus on the dynamic optimization problems behind individuals' choices of consumption, savings and investment. As we have hopefully managed to demonstrate, this approach provides us with a useful framework to understand the trade-offs shaping individuals' and firms' behavior, and it enables us to analyze the effects of exogenous parameter changes on the current account as well as the international transmission of macroeconomic shocks.

Moreover, the basic model represents a starting point for various extensions, which move it closer to reality and allow us to analyze a host of further interesting and relevant issues. In the following chapter, we will first abandon the notion that subsequent generations are linked by „dynastic preferences" and use the resulting model to analyze the consequences of demographic change. In a next step, we will explore how explicitly accounting for government spending and taxation affects aggregate consumption, investment, and the current account. Moreover, we will consider how the existence of non-tradable goods and variations in relative prices affects agents' behavior. And finally, we will drop the perfect foresight-assumption and analyze the influence of uncertainty on agents' saving behavior and international capital flows. All these extensions, however, will largely use elements of the basic model that we introduced in this chapter.

III.10 Keywords

Aggregation
Adjustment costs
Constant returns to scale
Consumption smoothing
Diminishing marginal productivity of capital
Dynastic utility function
Envelope theorem
Equity
Exponential discounting
Feldstein-Horioka puzzle
Financial globalization
Fisher equation
Gross investment
Homothetic preferences
Hyperbolic discounting
Inflation rate
Instantaneous utility
Intertemporal approach
Intertemporal budget constraint
Intertemporal elasticity of substitution
Intertemporal Euler equation
Intertemporal trade
Lifetime utility
Market equilibrium

No-arbitrage condition	Representative consumer
Pareto efficiency	Representative firm
Perfect competition	Subjective discount factor
Perfect foresight	Technology shock
Physical capital	Time consistency
Ponzi scheme	Time preference
Present value	Tobin's q
Production function	Total factor productivity
Rate of depreciation	Transversality condition
Real interest rate	

III.11 Literature

The intertemporal approach to the current account that we introduced in the previous pages and that will shape the following chapters, goes back to the seminal contributions by Buiter (1981), Sachs (1981), Obstfeld (1982), Dornbusch (1983), as well as Svensson and Razin (1983). Obstfeld and Rogoff (1995a) offer a survey article, while Obstfeld und Rogoff's (1996) masterly graduate textbook keeps shaping modern open-economy macroeconomics – including large parts of this chapter. A recent discussion of the issues touched upon in this chapter is provided by Gourinchas and Rey (2014). Blanchard and Giavazzi (2002) were among the first to recognize the disappearance of the Feldstein-Horioka puzzle and to provide an interpretation. Kirman (1992) and Carroll (2000) offer a well-founded critique of the representative-consumer model.

III.12 Exercises

3.1. The current account in a small open economy. We consider a small open economy with a time horizon of two periods. The representative consumer's lifetime utility is given by

$$U_1 = \frac{C_1^{1-\sigma} - 1}{1-\sigma} + \beta \frac{C_2^{1-\sigma} - 1}{1-\sigma}$$

RC produces a homogeneous good using the following technology:

$$Y_t = A_t K_t^{\alpha} \qquad t = 1, 2$$

We assume that the rate of depreciation δ equals one. The physical capital stock thus evolves according to the following equation:

$$K_{t+1} = I_t$$

RC has access to the international capital market where he can borrow and lend at the exogenous interest rate r. The evolution of the net international investment position is given by

$$B_{t+1} = (1+r)B_t + Y_t - C_t - I_t$$

We assume that $B_1 = 0$ and that $K_1 > 0$. Moreover, the usual boundary conditions $(B_3 = 0, K_3 = 0)$ hold.

a) Start with an educated guess on how the exogenous variables K_1 and A_2 affect the first-period current account CA_1. Explain through which channels these variables influence the current account.
b) Compute the economy's current account balance in the first period.
c) Show formally how an increase of K_1 and A_2 affects the first-period current account. Provide an economic interpretation for your results.

3.2. The German current account. As shown in Chapter II, Germany exhibited current account deficits for most of the 1990s. Use the basic intertemporal model and your knowledge of recent German history to explain this observation.

3.3. Current account balances and the interest rate in a two-country model with exogenous incomes. We consider a two-period model of two large open economies that are linked by a perfect international capital market. Lifetime utility of the representative consumer in country c $(c = H, F)$ is given by

$$U_1^c = \ln C_1^c + \beta \ln C_2^c$$

Both countries face an exogenous time path of incomes. We assume that

$$Y_1^H = Y_1^F, \qquad Y_2^F = \phi Y_2^H \text{ with } \phi \geq 1.$$

The evolution of the net international investment position is given by

$$B_{t+1}^c = (1+r)B_t^c + Y_t^c - C_t^c$$

with r representing the real world interest rate. We assume that $B_1^c = 0$ and $B_3^c = 0$ for $c = H, F$.

a) Explain why, in such a setting, the interest rate r is not an exogenous variable.
b) Start with an educated guess on how the parameter ϕ influences the current accounts of countries H and F as well as the world interest rate.
c) Compute the net international investment position for country H and F at the start of period 2 – i.e. B_2^H and B_2^F – as a function of the interest rate r.
d) Derive the world interest rate in equilibrium.
e) Derive the current account balance of country H in period 1. Do these results confirm your conjecture from part b)?

3.4. Current account balances and the interest rate in a two-country model with investment. We consider a two-period model of two large open economies that are linked by a perfect international capital market. Lifetime utility of the representative consumer in country c $(c = H, F)$ is given by

$$U_1^c = \ln C_1^c + \beta \ln C_2^c$$

The production function in country c $(c = H, F)$ reads

$$Y_t^c = A_t^c \left(K_t^c\right)^\alpha$$

From problem 1, we adopt the laws of motion for the physical capital stock and the net international investment position as well as the assumption on the depreciation rate and the boundary conditions

$$\delta = 1, B_1^c = 0, K_1^c > 0, B_3^c = 0, K_3^c = 0 \text{ for } c = H, F.$$

a) Compute the net international investment position for country H and F at the start of period 2 – i.e. B_2^H and B_2^F – as a function of the interest rate r.
b) Derive the world interest rate in equilibrium.
c) Compute the current account balance of country H in period 1.
d) How do the variables A_2^H and K_1^H affect the world interest rate and the current account balance of country H? Provide an economic interpretation for your results.

3.5. International investment and factor prices. As in Problem 3.4, we consider a two-period model of two large open economies. The lifetime utility of consumer i in country c $\left(c = H, F\right)$ is given by

$$U_1^{i,c} = \ln\left(C_1^{i,c}\right) + \beta \ln\left(C_2^{i,c}\right)$$

Firm j in country c $\left(c = H, F\right)$ uses the following production function:

$$Y_t^{j,c} = \left(K_t^{j,c}\right)^{\alpha} \left(L_t^{j,c}\right)^{1-\alpha} \qquad \text{with } c = H, F$$

In both countries, all individuals offer the same (exogenous) amount of labor, and aggregate labor supply is normalized to one, i.e. $L_t^c = 1$ for $c = H, F$ and $t = 1, 2$. At the beginning of period 1, capital stocks differ across countries, i.e. $K_1^F = \phi\, K_1^H$ with $0 < \phi < 1$. Moreover, we assume that the rate of depreciation δ equals one.

a) Compute the capital stock in countries H and F in period 2, assuming that no country has access to the international capital market (financial autarky).
b) Compute the capital stock in countries H and F in period 2 assuming that in both countries agents have the possibility to buy foreign assets and to sell domestic assets (financial integration).
c) How does financial integration affect the GDP of countries H and F in period 2? How does it affect the countries' GNI?
d) How does financial integration affect real wages in countries H and F in period 2?
e) Discuss under which conditions financial integration lowers the welfare of an individual in country H.
f) Do such welfare losses at the individual level contradict the notion that an economy reaps welfare gains when engaging in intertemporal trade?

3.6. Time consistent behavior. We consider a small open economy with a time horizon of three periods. Lifetime utility of the representative consumer is given by

$$U_1 = \ln C_1 + \beta \ln C_2 + \beta^2 \ln C_3$$

We assume that $\beta\left(1 + r\right) = 1$. The income of RC is exogenous. Specifically, we assume that $Y_1 = 1, Y_2 = 4, Y_3 = 1$. Moreover, we assume that $B_1 = 0$ and $B_4 = 0$.

a) Start with an educated guess on the evolution of RC's consumption and of the country's current account.
b) Derive the consumption levels and current account balances that RC plans for periods 1, 2 and 3.
c) Show that RC has no incentive to change his plans if he re-optimizes at the beginning of period 2.
d) What does this imply for the time consistency of RC's behavior? Under which conditions does he have an incentive to deviate from his original plans in period 2?

III.13 Appendix to Chapter III

In Box 3.2 we introduced a utility function that is characterized by a constant intertemporal elasticity of substitution. In this appendix we want to show that

$$\text{(3.A1)} \qquad \lim_{\sigma \to 1} \frac{C_1^{1-\sigma} - 1}{1-\sigma} = \ln C_1$$

We start from the observation that it is not possible to simply set σ in (3.A1) equal to one: in this case, both the numerator and the denominator of this ratio would be zero. However, we can use ***L'Hôpital's rule***. This rule requires taking the derivative of the numerator and the denominator with respect to σ. For the numerator this yields

$$\text{(3.A2)} \qquad \frac{d\,C_1^{1-\sigma}}{d\,\sigma} = \frac{d\,e^{(1-\sigma)\ln C_1}}{d\,\sigma} = \left(-\ln C_1\right) C_1^{1-\sigma}$$

Taking the derivative of the denominator in (3.A1) with respect to σ, combining both terms, and setting σ equal to one in the resulting expression produces the result in (3.A1).