Macroeconomics A
Problem Set 7

Johannes Boehm

Geneva Graduate Institute — Fall 2024

1 The zero lower bound: discretion and commitment

Suppose the central bank has the following loss function: $L_t = \frac{1}{2}x_t^2$, where x_t is the output gap (defined relative to the efficient level of output). The output gap is determined by the IS equation

$$x_t = \mathbb{E}_t x_{t+1} - \sigma(i_t - \mathbb{E}_t \pi_{t+1} - \hat{r}_t)$$

where i_t is the nominal interest rate (controlled by the central bank), π_t is the inflation rate, and $\hat{r}_t$ is the efficient interest rate. Inflation is determined by the Phillips curve

$$\pi_t = \mathbb{E}_t \pi_{t+1} + \kappa x_t$$

(assuming the discount factor $\beta = 1$, and ignoring cost-push shocks).

Nominal interest rates are subject to a zero lower bound. Since all variables, including i_t, are given as percentage deviations from steady-state levels, and since the steady-state nominal interest rate is likely to be positive, the lower bound on it can be stated as $i_t \geq -b$ for some $b > 0$. (Note: i_t is the percentage deviation of the *gross* nominal interest rate from the steady-state level. The gross nominal interest rate is equal to one at the "zero lower bound"!)

1. What factors would determine the size of b?

2. Now suppose at time t there is a temporary negative shock to the efficient interest rate $\hat{r}_t$, so that $\hat{r}_t = \hat{r} < 0$ and $\hat{r}_{t+1} = \hat{r}_{t+2} = \ldots = 0$. Find the optimal interest rate i_t when the central bank acts with discretion, and the resulting value of the output gap x_t. Distinguish between the cases $\hat{r} \geq -b$ and $\hat{r} < -b$ in your answer.

3. Suppose that $\hat{r} < -b$, and that the shock now lasts for two periods, hence $\hat{r}_t = \hat{r}_{t+1} = \hat{r}$ and $\hat{r}_{t+2} = \hat{r}_{t+3} = \ldots = 0$. Find the level of the output gap x_t when the central bank follows the optimal policy with discretion. [Hint: work backwards, starting from period $t + 2$.] Compare your answer to part (b) (assuming $\hat{r}$ is the same in both cases) and explain the intuition.

4. Suppose again that the shock is only temporary ($\hat{r}_{t+1} = \hat{r}_{t+2} = \ldots = 0$), and that $\hat{r}_t = \hat{r} < -b$. Assume the central bank is now able to commit to an interest-rate policy at time t for two periods, that is, it can choose both i_t and i_{t+1} at time t (but not inflence outcomes further in the future). Find the optimal policy with commitment (minimizing the sum $L_t + L_{t+1}$, and assuming the shock is such that the zero lower bound will not bind at time $t + 1$). Compare the total loss $L_t + L_{t+1}$ under discretion and commitment, and comment on the behaviour of long-term interest rates in the two cases.

5. Describe the time-inconsistency problem inherent in the optimal commitment found in part (d). Explain whether there is an analogy with the inflation bias problem.