Problem Set 2

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Question 1: tariffs and quotas

1: Small Country without monopoly

Consider a small open economy with domestic demand and supply functions:

$$D(P) = 200 - 10P$$
$$S(P) = 50 + 5P$$

(a) Autarky Equilibrium

Equating demand and supply:

$$200 - 10P = 50 + 5P$$
$$150 = 15P \quad \Longrightarrow \quad P = 10$$
$$Q = D(10) = 200 - 10 \cdot 10 = 100$$

Result: Autarky price is $P = 10$, quantity is $Q = 100$.

(b) Free Trade Equilibrium (World Price = 5)

With a world price $P_W = 5$:

$$Q_D = 200 - 10 \cdot 5 = 150$$
$$Q_S = 50 + 5 \cdot 5 = 75$$
$$\text{Imports} = Q_D - Q_S = 75$$

Result: Under free trade, price is $P = 5$, domestic supply $Q_S = 75$, demand $Q_D = 150$, imports $M = 75$.

(c) Import Quota: $M = 30$

Quota limits imports to 30 units:

$$(200 - 10P) - (50 + 5P) = 30$$
$$150 - 15P = 30$$
$$P = \frac{150 - 30}{15} = 8$$

$$Q_D = 200 - 10 \cdot 8 = 120$$
$$Q_S = 50 + 5 \cdot 8 = 90$$

Result: Price rises to $P = 8$, supply $Q_S = 90$, demand $Q_D = 120$, imports $M = 30$.

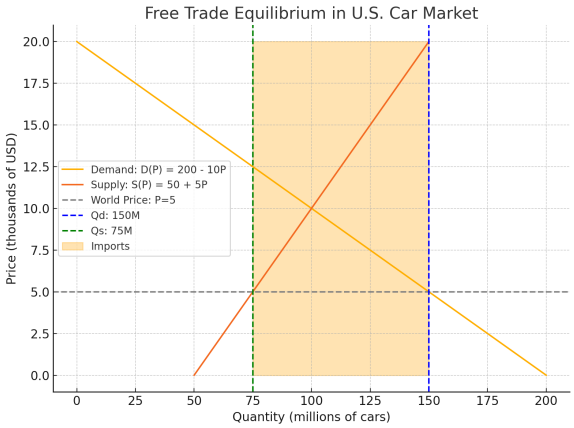

Figure 1: U.S. under free trade

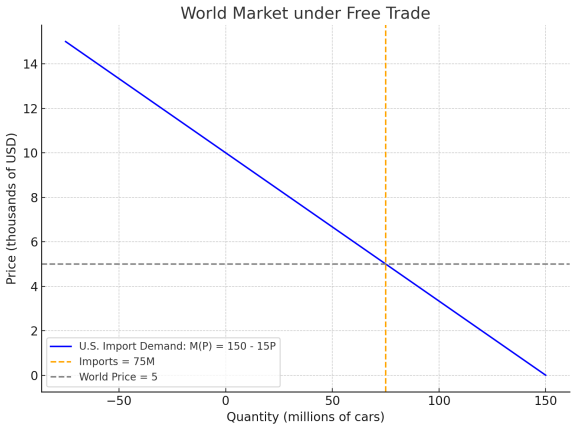

Figure 2: World market under free trade

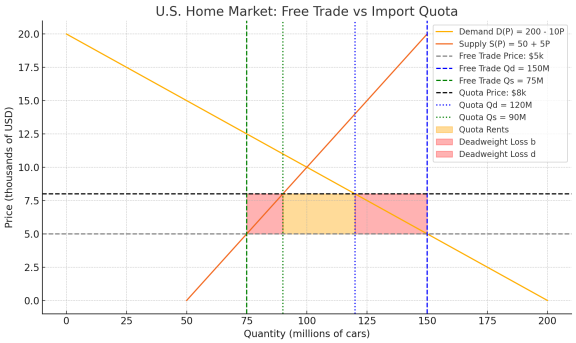

Figure 3: U.S. with quota

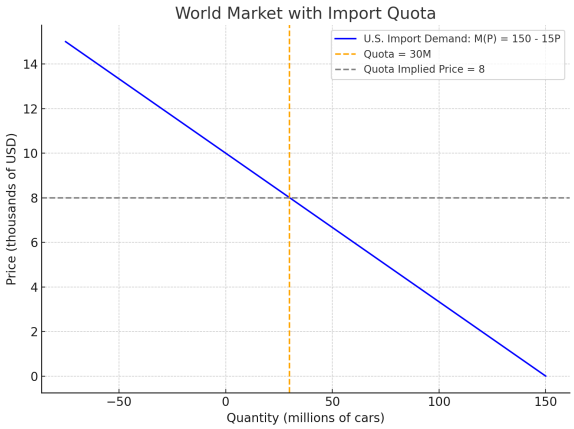

Figure 4: World market with quota

(d) Welfare Effects of the Quota

Quota Rents:

$$\text{Quota Rent} = (8 - 5) \cdot 30 = 90$$

Assuming rents are fully dissipated through rent-seeking.

Deadweight Loss:

$$\text{Production DWL} = \frac{1}{2} \cdot (8 - 5) \cdot (90 - 75) = 22.5$$

$$\text{Consumption DWL} = \frac{1}{2} \cdot (8 - 5) \cdot (150 - 120) = 45$$

$$\text{Total DWL} = 22.5 + 45 = 67.5$$

Total Welfare Loss:

$$\text{Total Loss} = \text{Quota Rents} + \text{DWL} = 90 + 67.5 = 157.5$$

Advice for the President: The import quota leads to a total welfare loss of 157.5. Although it increases domestic production by 15 units, it does so at the expense of higher

consumer prices and inefficient resource allocation. The entire quota rent of 90 is wasted due to rent-seeking behavior. Therefore, imposing an import quota is not advisable as it harms consumers, reduces national welfare, and benefits only a small group of insiders.

(e) Equivalent Import Tariff Comparison

But unlike the quota, tariff revenues go to the government rather than being wasted.

Advice for the President: If protection is politically necessary, a tariff is significantly better than an import quota. It avoids wasteful rent-seeking and generates public revenue. The tariff provides the same degree of import limitation while mitigating welfare losses.

2: Monopoly and Quotas

Consider the U.S. car market with the following demand and supply functions:

$$D(P) = 200 - 10P$$
$$MC(Q) = \frac{Q}{5} - 10$$

Imports are perfectly elastic at world price $P_W = 5$ unless restricted by quotas.

(a) Import Quota: $M = 30$ with Monopoly

Under monopoly, the firm maximizes profit by equating marginal revenue (MR) with marginal cost (MC). The residual demand is:

$$Q + 30 = 200 - 10P \quad \Longrightarrow \quad P = \frac{170 - Q}{10}$$

Total Revenue (TR):

$$TR = P \cdot Q = \left(\frac{170 - Q}{10}\right) Q = 17Q - \frac{Q^2}{10}$$

Marginal Revenue (MR):

$$MR = \frac{dTR}{dQ} = 17 - \frac{Q}{5}$$

Equating MR and MC:

$$17 - \frac{Q}{5} = \frac{Q}{5} - 10 \quad \Longrightarrow \quad 27 = \frac{2Q}{5} \quad \Longrightarrow \quad Q = 67.5$$

$$Q_D = Q + M = 67.5 + 30 = 97.5$$
$$P = \frac{170 - 67.5}{10} = 10.25$$

Result: The monopolist reduces domestic production to $Q = 67.5$, raises the market price to $P = 10.25$, imports remain fixed at $M = 30$.

(b) graph for monopoly equilibrium with quota

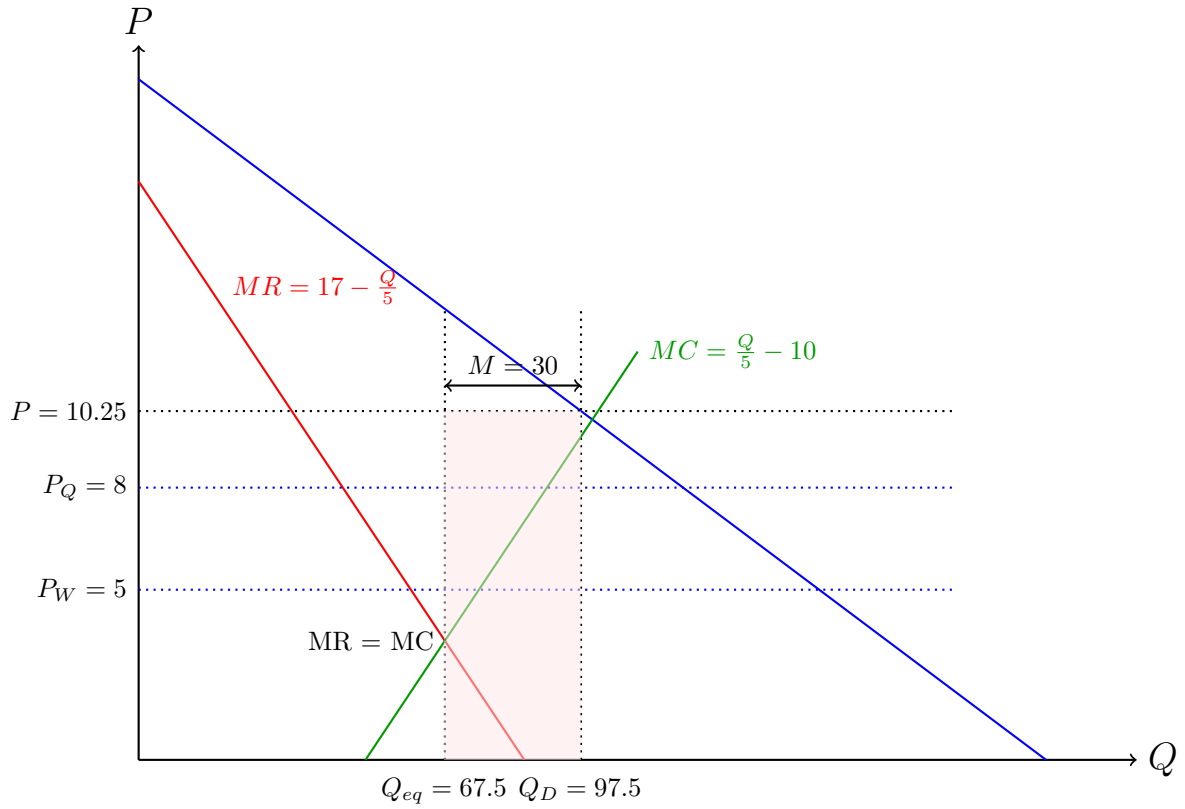

Figure 5: U.S. market with monopoly and quota

(c) effect with quota

Under free trade, the U.S. market is competitive, and domestic firms produce where price equals marginal cost. When an import quota is imposed and the domestic industry is monopolized, the firm maximizes profit by restricting output to $Q = 67.5$. This is because the quota limits foreign competition, allowing the monopolist to face a steeper residual demand curve. By producing less and raising prices, the monopolist increases profits at the expense of consumers.

Impact on Employment: Lower domestic production directly translates into fewer jobs in the U.S. automotive industry. Ironically, while the quota is often justified as a means to protect domestic employment, under monopoly conditions it incentivizes the firm to cut output and employment in order to maintain higher prices and profits.

Conclusion: In the presence of a monopolist, an import quota can have the opposite of its intended effect: reducing domestic employment rather than protecting it.

(d)policy comparison

An import quota and an equivalent tariff can both limit foreign competition and raise domestic prices. However, their welfare implications differ significantly in the presence of a domestic monopolist.

Quota:

- Grants the monopolist a fixed market share of residual demand.
- Encourages the monopolist to restrict output further to raise prices.
- Leads to a larger reduction in domestic production, higher prices, and greater deadweight loss.
- Creates quota rents, which may be dissipated through rent-seeking.

Tariff:

- Raises the cost of imports but allows the quantity of imports to adjust based on market price.
- Limits the monopolist's ability to restrict output since higher prices incentivize more imports.
- Generates government revenue, reducing overall welfare loss.

Conclusion: In terms of U.S. welfare, a tariff is preferable to an equivalent import quota when a domestic monopolist is present. The tariff retains competitive pressure through the elasticity of imports and transforms potential rents into government revenue, resulting in lower welfare loss compared to a quota.

This illustrates the principle that **price-based instruments** (like tariffs) are generally less distortionary than **quantity-based restrictions** (like quotas) in the presence of market power.

Question 2: Subsidies

Consider the Swiss market for widgets with the following functions:

$$D(P) = 600 - 10P$$
$$S(P) = 100 + 10P$$

The world price is $P_W = 10$.

1. Autarky Equilibrium

In autarky, equilibrium is found by equating supply and demand:

$$600 - 10P = 100 + 10P$$
$$500 = 20P \quad \Longrightarrow \quad P = 25$$

$$Q = D(25) = 600 - 10 \cdot 25 = 350$$

Result: Autarky equilibrium price is $P = 25$, quantity is $Q = 350$.

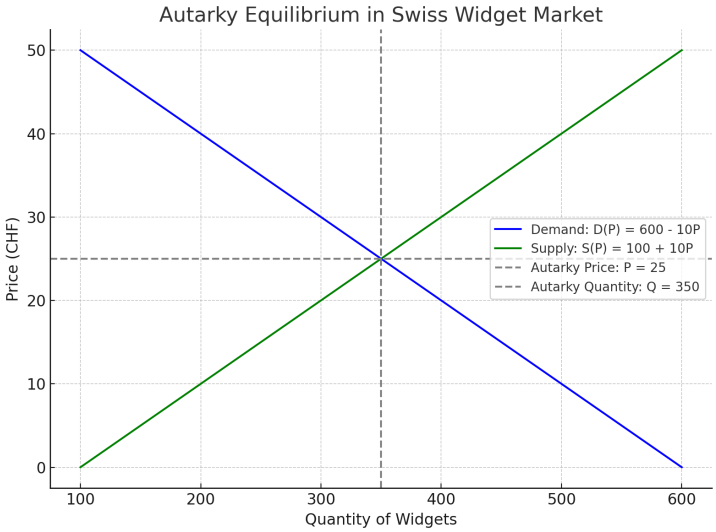

Figure 6: Autarky

2. Free Trade

(a) Quantities under Free Trade:

$$Q_S = S(10) = 100 + 10 \cdot 10 = 200$$
$$Q_D = D(10) = 600 - 10 \cdot 10 = 500$$
$$M = Q_D - Q_S = 300$$

Gievn that the demand is more than the supply domestically, Switzerland is the importer of widgets at the given world price. The import volume is 300.

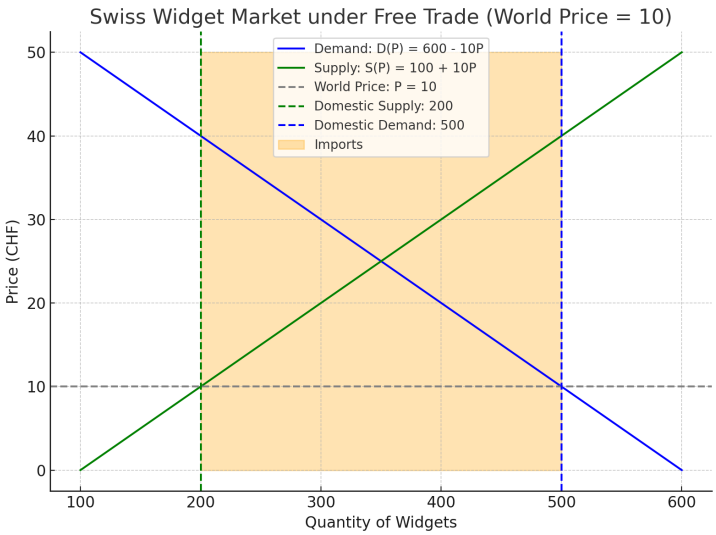

Figure 7: Free trade

(b) Welfare Calculations:

- Consumer Surplus (CS):

$$CS = \frac{1}{2} \cdot 500 \cdot (60 - 10) = 12500$$

- Producer Surplus (PS):

$$PS = \frac{1}{2} \cdot (200 + 100) \cdot 10 = 1500$$

- Government Revenue (R):

$$R = 0$$

- Welfare (W):

$$W = CS + PS = 14000$$

- Government Objective (V):

$$V = W + 0.2 \cdot PS = 14300$$

3. Import Tariff Analysis

(a) Market Outcome with Tariff $t = 5$:

$$\begin{aligned} P_T &= 10 + 5 = 15 \\ Q_S &= S(15) = 100 + 10 \cdot 15 = 250 \\ Q_D &= D(15) = 600 - 10 \cdot 15 = 450 \\ M &= Q_D - Q_S = 200 \end{aligned}$$

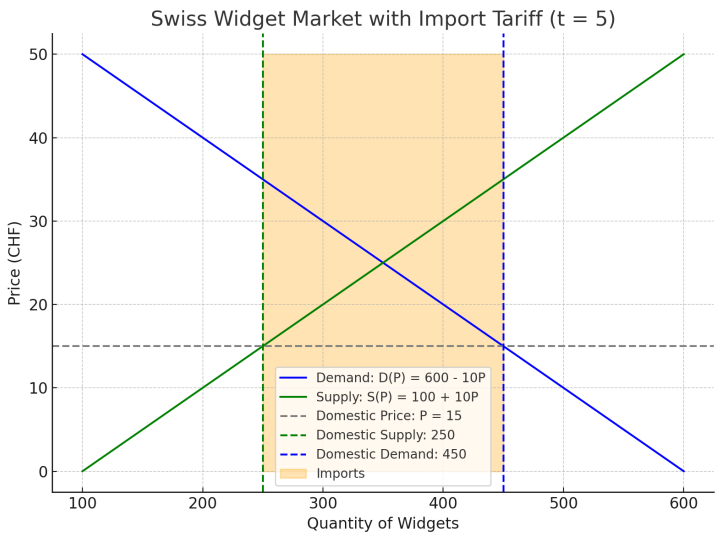

Figure 8: Import tariff

(b) Welfare Effects:

- Consumer Surplus:
$$CS = \frac{1}{2} \cdot 450 \cdot (60 - 15) = 10125$$
- Producer Surplus:
$$PS = \frac{1}{2} \cdot (250 + 100) \cdot 15 = 2625$$
- Tariff Revenue:
$$R = 5 \cdot 200 = 1000$$
- Welfare:
$$W = CS + PS + R = 13750$$
- Government Objective:
$$V = W + 0.2 \cdot PS = 14275$$

(c) Policy Implication:

- As a welfare optimizer, tariff is not a good idea. Because welfare decreases under the tariff compared to free trade ($W = 13750 < 14000$).
- Tariff is also not a good idea for optimizing V as government objective V also decreases compared to free trade ($V = 14275 < 14300$).
- Intuition: Tariff is not recommended in this case.

3.4 Production Subsidy Analysis

A unit production subsidy s lowers producers' marginal cost by s francs. Graphicallythis equals to shift the supply curve to the right, and the price remains as $P_W = 10$, but the marginal cost of the producer is $P_W - s$.

(a) Quantities under $s = 5$

$$Q_S = 100 + 10(P_W + s) = 100 + 10(10 + 5) = 250, \quad Q_D = D(10) = 600 - 10 \cdot 10 = 500,$$
$$\text{Imports} = Q_D - Q_S = 250 \quad \text{(down from 300)}.$$

(b) Welfare Components

$$\begin{aligned}
CS &= \tfrac{1}{2}\,(60 - 10)\,Q_D = \tfrac{1}{2} \cdot 50 \cdot 500 = 12500, \\
PS &= \int_0^{Q_S} \left[P_W - \left(\tfrac{Q-100}{10} - s\right)\right] dQ = 5s^2 + 200s + 2000 \overset{s=5}{=} 3125, \\
R &= -s\,Q_S = -5 \cdot 250 = -1250, \\
W &= CS + PS + R = 14500 - 5s^2 \overset{s=5}{=} 1475, \\
V &= W + 0.2\,PS = 14900 + 40s - 4s^2 \overset{s=5}{=} 15000.
\end{aligned}$$

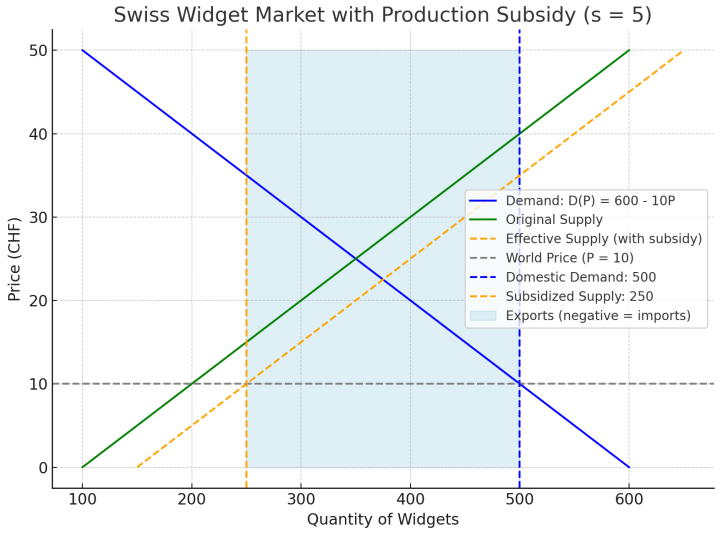

Figure 9: Import subsidy

(c) Comparison with Free Trade

	Free Trade	Subsidy ($s = 5$)
W	14000	14375
V	14300	15000

- **If the government maximizes total welfare W:** The subsidy leaves W only slightly higher than free trade because CS is unchanged while PS increases and fiscal cost partly offsets the gain. The subsidy is only marginally better than free trade from a pure W-maximization perspective.
- **If the government maximizes the weighted objective $V = W + 0.2\,PS$:** Under free trade, $V_{\text{FT}} = 14300$; under the subsidy, $V_{\text{sub}} = 15000$. Because $V_{\text{sub}} > V_{\text{FT}}$, the subsidy is unambiguously *desirable* if the government places extra weight on producer surplus.
- **Intuition:** The production subsidy leaves consumer surplus unchanged (price stays at \$10) and transfers resources to domestic producers, raising PS. When W is the sole objective, the subsidy's fiscal cost partly offsets the gain in PS, so the net welfare gain is limited. When PS is weighted more heavily, the subsidy's boost to PS dominates, making it optimal under the government's objective.

(d) Comparison to an Equivalent Import Tariff

- Under an import tariff $t = 5$, we found

$$(CS, PS, W, V)_{\text{tariff}} = (10125,\ 2625,\ 13750,\ 14275).$$

  Under the production subsidy $s = 5$, we have

$$(CS, PS, W, V)_{\text{subsidy}} = (12500,\ 3125,\ 14375,\ 15000).$$

- **Illustrated Principle:** This comparison demonstrates the *targeting principle*: a policy that directly targets the intended beneficiary (domestic producers) via a

subsidy outperforms an indirect policy (tariff) that distorts both production and consumption. In other words, price-based instruments targeted at the locus of desired support are less distortionary than quantity-based or border-tax instruments.

5. Optimal Production Subsidy

(a) CS, PS, and R as Functions of s

$$Q_S(s) = 200 + 10s, \qquad Q_D = 500.$$

$$CS(s) = 12500, \quad PS(s) = 5s^2 + 200s + 2000, \quad R(s) = -s\,Q_S(s) = -s\,(200 + 10s).$$

(b) Government Objective

$$V(s) = CS(s) + PS(s) + R(s) + 0.2\,PS(s) = 14900 - 4s(s - 10). \tag{8}$$

(c) Optimal Subsidy

$$V'(s) = 40 - 8s = 0 \quad \Longrightarrow \quad s^* = 5.$$

(d) Recommendation

Because the government maximizes V, it should implement a **production subsidy of 5 CHF per widget**. The policy yields the highest attainable government surplus ($V = 15000$). Nonetheless, the fiscal cost warrants:

- sunset provisions to curb long-term budget pressure;
- periodic reviews tied to productivity and export performance;
- consideration of alternative, less distortionary supports (e.g. targeted R&D incentives).