

1. Consider a country (the U.S.) and two goods (footballs and soccer balls). Suppose that the country is inhabited by  $L_{US}$  workers,  $F_{US}$  units of a specific factor used in the production of footballs and  $S_{US}$  units of a specific factor used in the production of soccer balls. Suppose that the total quantity of soccer balls that can be produced is determined by the production function  $Q_{US}^{SB} = (S_{US})^{\frac{1}{2}} (L_{US}^{SB})^{\frac{1}{2}}$ , where  $L_{US}^{SB}$  is the labor allocated to the production of soccer balls. Similarly, suppose that the total quantity of footballs that can be produced is determined by the production function  $Q_{US}^{FB} = (F_{US})^{\frac{1}{2}} (L_{US}^{FB})^{\frac{1}{2}}$ , where  $L_{US}^{FB}$  is the labor allocated to the production of footballs.
  - (a) Find an equation that depends only on exogenous model parameters for the Production Possibility Frontier in the U.S.
  - (b) Suppose that the representative agent in the U.S. has preferences  $U = \min \{C_{US}^{SB}, C_{US}^{FB}\}$ , where  $C_{US}^{SB}$  and  $C_{US}^{FB}$  are the quantity consumed of soccer balls and footballs, respectively. Find an equation for the equilibrium autarkic relative price of footballs to soccer balls (i.e.  $\frac{p_{US}^{FB}}{p_{US}^{SB}}$ ) that depends only on exogenous model parameters.
  - (c) Suppose now that there is another country (Mexico), where we assume that  $\frac{S_{MEX}}{F_{MEX}} > \frac{S_{US}}{F_{US}}$ . If the two countries began trading, how would the labor allocation across the sectors change compared to the autarkic equilibrium? Show your answer using figures. Who would lose from trade in the U.S.?
  - (d) Suppose that in anticipation of losing from trade, those that would lose from trade in the U.S. (that you identified in part (c)) lobby the government to impose a domestic tax (or subsidy) on footballs so that the equilibrium autarkic relative price with the subsidy is  $\frac{\tilde{p}_{US}^{FB}}{\tilde{p}_{US}^{SB}} = \tau \frac{p_{US}^{FB}}{p_{US}^{SB}}$ , where  $\tau > 1$  indicates that the government has imposed a tax on footballs that has made footballs relatively more expensive and  $\tau < 1$  indicates that the government has imposed a subsidy on footballs that makes the footballs relatively less expensive. What is the smallest tax or subsidy (i.e the  $\tau$  closest to one) such that the losers identified in part (c) do not lose from trade?
2. Consider a world with two countries and two goods. As in class, let us call the countries U.S. and Mexico and the goods footballs and soccer balls. In each country  $i$ , suppose that footballs are produced combining labor  $L_i^{FB}$  and a football specific factor  $F_i$ , soccer balls are produced combining labor  $L_i^{SB}$  and a soccer ball specific factor  $S_i$ . Assume that the representative agent in country  $i$  has preferences  $U_i = \min \{C_i^{FB}, C_i^{SB}\}$ , where  $C_i^{FB}$  is the consumption of footballs and  $C_i^{SB}$  is the consumption of soccer balls. In each country  $i$ , assume that the production function for footballs is  $Q_i^{FB} = (L_i^{FB})^{\frac{1}{2}} (F_i)^{\frac{1}{2}}$  and the production function for soccer balls is  $Q_i^{SB} = (L_i^{SB})^{\frac{1}{2}} (S_i)^{\frac{1}{2}}$ . Finally, assume that both countries have identical populations (i.e.  $L_{US} = L_{MEX}$ ), identical football specific factors (i.e.  $F_{US} = F_{MEX}$ ) and the only difference between the two countries is that Mexico has more of the soccer ball specific factor (i.e.  $S_{US} < S_{MEX}$ ).
  - (a) Draw the production possibility frontiers of both countries and show the equilibrium production and consumption of both countries in free trade. How can we see from your figure that the markets clear?
  - (b) Suppose that Mexico's endowment of the soccer ball specific factor increases. How would the world price change? Who in the U.S. would benefit and who would lose from this change? Explain your answer using figures.
  - (c) Suppose now that we have the following actual numbers:  $L_{US} = L_{MEX} = 2$ ,  $F_{US} = F_{MEX} = 1$ ,  $S_{US} = 1$  and  $S_{MEX} = 2$ . What is the equilibrium quantity produced and consumed in each location as a function of the world price? What is the equation that (implicitly) defines the world price?

3. Consider a world with two countries and two goods. As in class, let us call the countries U.S. and Mexico and the goods footballs and soccer balls. Suppose that the U.S. is endowed with 16 units of labor ( $L_{US}$ ), 16 units of the specific factor used in the production of footballs ( $F_{US}$ ) and 16 units of the specific factor used in the production of soccer balls ( $S_{US}$ ). Suppose that Mexico is endowed with 16 units of labor ( $L_{MEX}$ ), 4 units of the specific factor used in the production of footballs ( $F_{MEX}$ ) and 16 units of the specific factor used in the production of soccer balls ( $S_{MEX}$ ). The representative agent in each country  $i \in \{US, MEX\}$  has preferences  $U_i = \min \{C_i^{FB}, C_i^{SB}\}$  where  $C_i^{FB}$  is the consumption in country  $i$  of footballs and  $C_i^{SB}$  is the consumption in country  $i$  of soccer balls. In each country  $i \in \{US, MEX\}$ , the production function of footballs is  $Q_i^{FB} = (L_i^{FB})^{\frac{1}{2}} (F_i)^{\frac{1}{2}}$ , where  $L_i^{FB}$  is the labor allocated to the production of footballs in country  $i$ . Finally, in each country  $i \in \{US, MEX\}$ , the production function of soccer balls is  $Q_i^{SB} = (L_i^{SB})^{\frac{1}{2}} (S_i)^{\frac{1}{2}}$ , where  $L_i^{SB}$  is the labor allocated to the production of soccer balls in country  $i$ .
- Suppose that both the U.S. and Mexico are initially in autarky. Draw the production possibilities frontier for both countries (with footballs on the x-axis and soccer balls on the y-axis). Indicate the equilibrium autarkic production and consumption of both goods in both countries (no need to solve for the actual numbers). In which country is the autarky relative price of footballs to soccer balls higher? What is the economic intuition for this difference in prices?
  - Now suppose that the two countries open to trade. Draw a new figure showing the equilibrium production and consumption of both goods in both countries (again, no need to solve for the actual numbers). Which country exports what? How do we see on the figure that markets clear?
  - Use a (new) figure to show how trade changed the income of each of the three factors in the U.S. (workers, football specific factor, soccer ball specific factor). Carefully label on the figure the following six things: (i) the original income of workers; (ii) the original income of the soccer ball specific factor; (iii) the original income of the football specific factor; (iv) the new income of the soccer ball specific factor; (v) the new income of the football specific factor; and (vi) the new income of workers.
  - Use math to show how trade changes how much soccer balls and footballs U.S. workers are able to consume. What is the economic intuition for these changes?