Macroeconomics A; EI060

Technical appendix: Mundell-Fleming

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1 Mundell-Fleming (Harms IX.2)

1.1 Structure

1.1.1 Market for money, and interest parity

The demand for money (linearized) links the real balance, $m-p$, to output, y, the domestic nominal interest rate, i^H, and a shock ζ of expected value zero.

$$m-p = \phi y - \lambda i^H + \zeta$$

The uncovered interest parity links the domestic interest rate to the foreign one, i^F, and the expected depreciation, $e^e - e$, where an increase of e represents a depreciation of the domestic currency.

$$i^H = i^F + e^e - e$$

We consider that $e^e - e = 0$, either because the country has an exchange rate peg, or because any exchange rate adjustment happens fully at time t, with the exchange rate then being constant.

1.1.2 Market for goods

Output reflects consumption, investment, government spending and net exports:

$$y = c + inv + g + nx$$

Consumption increases with income (GDP) and a wealth, or confidence, parameter Ω. Investment reflects the interest rate and firms' confidence μ. Government spending is exogenous. Net exports increase with the exchange rate (with set prices, a depreciation raises competitiveness), increase with foreign demand y^F, and decrease as output y increases because this draws in imports.

$$\begin{aligned} c &= \gamma y + \gamma_\Omega \Omega \\ inv &= -\sigma i^H + \sigma_\mu \mu \\ nx &= \delta e - \rho y + \varphi y^F \end{aligned}$$

where $0 < \gamma < 1$. Combining these, we write the output as follows, with ξ reflecting a demand shock of expected value zero:

$$\begin{aligned} y &= c + inv + g + nx \\ y &= \gamma y + \gamma_\Omega \Omega - \sigma i^H + \sigma_\mu \mu + g + \delta e - \rho y + \varphi y^F + \xi \\ (1 - \gamma + \rho)\, y &= \gamma_\Omega \Omega - \sigma i^H + \sigma_\mu \mu + g + \delta e + \varphi y^F + \xi \\ y &= \frac{-\sigma i^H + \delta e + g + \gamma_\Omega \Omega + \sigma_\mu \mu + \varphi y^F + \xi}{1 - \gamma + \rho} \end{aligned}$$

As prices are fixed, we set $p = 0$ for simplicity.

1.2 Solution of the model

1.2.1 The closed economy

Recall that in a closed economy, there is no exchange rate or net exports. The system is:

$$\begin{aligned} m &= \phi y - \lambda i^H + \zeta \\ y &= \frac{-\sigma i^H + g + \gamma_\Omega \Omega + \sigma_\mu \mu + \xi}{1 - \gamma} \end{aligned}$$

Use the money market to substitute for the interest rate:

$$\begin{aligned} m &= \phi y - \lambda i^H + \zeta \\ y &= -\frac{\sigma}{1-\gamma} i^H + \frac{g + \gamma_\Omega \Omega + \sigma_\mu \mu + \xi}{1-\gamma} \\ y &= -\frac{\sigma}{1-\gamma}\frac{1}{\lambda}\left[\phi y - m + \zeta\right] + \frac{g + \gamma_\Omega \Omega + \sigma_\mu \mu + \xi}{1-\gamma} \\ \left[\lambda(1-\gamma) + \sigma\phi\right] y &= \sigma m - \sigma\zeta + \lambda\left[g + \gamma_\Omega \Omega + \sigma_\mu \mu + \xi\right] \\ y &= \frac{1}{\lambda(1-\gamma) + \sigma\phi}(\sigma m + \lambda g) + \frac{\lambda(\gamma_\Omega \Omega + \sigma_\mu \mu)}{\lambda(1-\gamma) + \sigma\phi} + \frac{\lambda\xi - \sigma\zeta}{\lambda(1-\gamma) + \sigma\phi} \end{aligned}$$

The interest rate is given by the money market equilibrium:

$$\begin{aligned} i^H &= \frac{1}{\lambda}\left[\phi y - m + \zeta\right] \\ i^H &= \frac{\phi}{\lambda} y - \frac{1}{\lambda} m + \frac{1}{\lambda}\zeta \\ i^H &= \frac{\phi}{\lambda}\frac{1}{\lambda(1-\gamma) + \sigma\phi}(\sigma m + \lambda g) + \frac{\phi}{\lambda}\frac{\lambda\left[\gamma_\Omega \Omega + \sigma_\mu \mu\right]}{\lambda(1-\gamma) + \sigma\phi} + \frac{\phi}{\lambda}\frac{\lambda\xi - \sigma\zeta}{\lambda(1-\gamma) + \sigma\phi} - \frac{1}{\lambda} m + \frac{1}{\lambda}\zeta \end{aligned}$$

$$
\begin{aligned}
i^H &= \left(\frac{\sigma\phi}{\lambda(1-\gamma)+\sigma\phi}-1\right)\frac{1}{\lambda}m+\frac{\phi}{\lambda(1-\gamma)+\sigma\phi}g+\frac{\phi\left[\gamma_\Omega\Omega+\sigma_\mu\mu\right]}{\lambda(1-\gamma)+\sigma\phi}+\frac{\phi\xi}{\lambda(1-\gamma)+\sigma\phi}+\frac{1}{\lambda}\left(\frac{-\sigma\phi}{\lambda(1-\gamma)+\sigma\phi}+1\right)\zeta \\
i^H &= -\frac{(1-\gamma)}{\lambda(1-\gamma)+\sigma\phi}m+\frac{\phi}{\lambda(1-\gamma)+\sigma\phi}g+\frac{\phi(\gamma_\Omega\Omega+\sigma_\mu\mu)}{\lambda(1-\gamma)+\sigma\phi}+\frac{\phi\xi+(1-\gamma)\zeta}{\lambda(1-\gamma)+\sigma\phi}
\end{aligned}
$$

1.2.2 Exchange rate float

Under an exchange rate float, e is endogenous. We use the interest parity condition to substitute for the interest rate.

The output follows from the money market equilibrium:

$$
\begin{aligned}
m &= \phi y-\lambda i^F+\zeta \\
y &= \frac{m}{\phi}+\frac{\lambda i^F-\zeta}{\phi}
\end{aligned}
$$

The exchange rate follows from the equilibrium of the market for goods:

$$
\begin{aligned}
y &= \frac{-\sigma i^F+g+\gamma_\Omega\Omega+\sigma_\mu\mu+\varphi y^F+\xi}{1-\gamma+\rho}+\frac{\delta}{1-\gamma+\rho}e \\
\frac{1-\gamma+\rho}{\phi}m+\frac{1-\gamma+\rho}{\phi}\left(\lambda i^F-\zeta\right) &= -\sigma i^F+g+\gamma_\Omega\Omega+\sigma_\mu\mu+\varphi y^F+\xi+\delta e \\
e &= \frac{1-\gamma+\rho}{\phi\delta}m-\frac{g}{\delta}-\frac{\gamma_\Omega\Omega+\sigma_\mu\mu+\varphi y^F}{\delta} \\
&\quad+\left(\frac{1-\gamma+\rho}{\phi\delta}\lambda+\frac{\sigma}{\delta}\right)i^F-\left(\frac{1-\gamma+\rho}{\phi\delta}\zeta+\frac{\xi}{\delta}\right)
\end{aligned}
$$

1.2.3 Exchange rate peg

Under an exchange rate peg, e is set and m is endogenous. We use the interest parity condition to substitute for the interest rate.

The output follows from the equilibrium of the market for goods:

$$
y = \frac{-\sigma i^F+\delta e+g+\gamma_\Omega\Omega+\sigma_\mu\mu+\varphi y^F+\xi}{1-\gamma+\rho}
$$

The money supply is given by the money market equilibrium:

$$
\begin{aligned}
m &= \phi y-\lambda i^F+\zeta \\
m &= \phi\frac{\delta e+g+\gamma_\Omega\Omega+\sigma_\mu\mu+\varphi y^F}{1-\gamma+\rho}-\left(\lambda+\frac{\sigma\phi}{1-\gamma+\rho}\right)i^F+\frac{\xi}{1-\gamma+\rho}+\zeta
\end{aligned}
$$

2 The overshooting model (Obstfeld and Rogoff 9.2.1-9.2.3)

2.1 Structure, and adjustment process

We consider a perfect foresight model (dropping the expectations). The real exchange rate is $q_t=e_t+p^*-p_t$, with an increase indicating a depreciation of the country's currency.

The dynamics of the exchange rate is related to the interest rate through the uncovered interest parity condition, where i^* is the constant foreign interest rate:

$$i_{t+1} \quad = \quad i^* + e_{t+1} - e_t$$

The money demand is positively linked to output, and inversely related to the interest rate:

$$m_t - p_t \quad = \quad -\eta i_{t+1} + \phi y_t$$

In the long run, output is equal to $\bar{y}$ and the real exchange rate is $\bar{q}$. The short run output deviates from $\bar{y}$ when the real exchange rate is weaker than the long run value, i.e. $q_t > \bar{q}$:

$$y_t - \bar{y} \quad = \quad \delta\left(q_t - \bar{q}\right)$$

The price level adjust gradually to its long run value. $\tilde{p}_t$ represents the flexible price level, which is the price consistent with the real exchange rate being equal to $\bar{q}$:

$$e_t + p^* - \tilde{p}_t \quad = \quad \bar{q}$$

The price increases when output exceeds its long run value, and when the flexible price level increases:

$$\begin{aligned} p_{t+1} - p_t &= \psi\left(y_t - \bar{y}\right) + \left(\tilde{p}_{t+1} - \tilde{p}_t\right) \\ p_{t+1} - p_t &= \psi\left(y_t - \bar{y}\right) + \left(e_{t+1} - e_t\right) \qquad (1) \end{aligned}$$

2.2 Exchange rate dynamics

The model is characterized by the dynamics of the nominal and real exchange rates. We start with the real exchange rate. From the price dynamics (1), we get:

$$\begin{aligned} p_{t+1} - p_t &= \psi\left(y_t - \bar{y}\right) + \left(e_{t+1} - e_t\right) \\ 0 &= \psi\left(y_t - \bar{y}\right) + \left(e_{t+1} - p_{t+1}\right) - \left(e_t - p_t\right) \\ 0 &= \psi\left(y_t - \bar{y}\right) + \left(p^* + e_{t+1} - p_{t+1}\right) - \left(p^* + e_t - p_t\right) \\ 0 &= \psi\left(y_t - \bar{y}\right) + q_{t+1} - q_t \\ q_{t+1} - q_t &= -\psi\delta\left(q_t - \bar{q}\right) \qquad (2) \end{aligned}$$

We assume that $\psi\delta < 1$ so the real exchange rate converges. The real exchange rate increases when $q_t < \bar{q}$.

Now turn to the nominal exchange rate. Using the interest rate parity, the money demand is:

$$\begin{aligned} m_t - p_t &= -\eta i_{t+1} + \phi y_t \\ m_t - p_t &= -\eta i^* - \eta\left(e_{t+1} - e_t\right) + \phi y_t \\ m_t - p_t &= -\eta\left(e_{t+1} - e_t\right) + \phi\delta\left(q_t - \bar{q}\right) + \phi\bar{y} - \eta i^* \end{aligned}$$

$$\begin{aligned}
m_t + q_t - e_t - p^* &= -\eta\left(e_{t+1} - e_t\right) + \phi\delta\left(q_t - \bar{q}\right) + \phi\bar{y} - \eta i^* \\
m_t + q_t - e_t &= -\eta\left(e_{t+1} - e_t\right) + \phi\delta\left(q_t - \bar{q}\right) + \left[\phi\bar{y} - \eta i^* + p^*\right] \\
m_t + q_t - e_t &= -\eta\left(e_{t+1} - e_t\right) + \phi\delta\left(q_t - \bar{q}\right)
\end{aligned}$$

where we assume $\bar{y} = i^* = p^* = 0$ for simplicity. The exchange rate dynamics are then:

$$\begin{aligned}
m_t + q_t - e_t &= -\eta\left(e_{t+1} - e_t\right) + \phi\delta\left(q_t - \bar{q}\right) \\
\eta\left(e_{t+1} - e_t\right) &= -m_t - q_t + e_t + \phi\delta\left(q_t - \bar{q}\right) \\
e_{t+1} - e_t &= \frac{e_t}{\eta} - \frac{1-\phi\delta}{\eta} q_t - \frac{\phi\delta\bar{q} + m_t}{\eta} \qquad (3)
\end{aligned}$$

From the nominal exchange rate dynamics (3), a constant nominal exchange rate implies a positive relation between e_t and q_t, assuming $1 > \phi\delta$.

2.3 Exchange rate overshooting: analytical solution

2.3.1 Dynamic relations

Consider that the economy starts at a steady state where $m_t = \bar{m}$. The dynamics of the nominal exchange rate (3) imply:

$$\begin{aligned}
\bar{e} - \bar{e} &= \frac{\bar{e}}{\eta} - \frac{1-\phi\delta}{\eta}\bar{q} - \frac{\phi\delta\bar{q} + \bar{m}}{\eta} \\
0 &= \bar{e} - \left(1 - \phi\delta\right)\bar{q} - \phi\delta\bar{q} - \bar{m} \\
0 &= \bar{e} - \bar{q} - \bar{m} \\
0 &= \bar{e} - \bar{e} - p^* + \bar{p} - \bar{m}
\end{aligned}$$

As $p^* = 0$ this implies that the price level simply reflects the money supply: $\bar{p} = \bar{m}$.

We write the dynamics of the real exchange rate (2) as:

$$\begin{aligned}
q_{t+1} - q_t &= -\psi\delta\left(q_t - \bar{q}\right) \\
q_{t+1} - \bar{q} - q_t + \bar{q} &= -\psi\delta\left(q_t - \bar{q}\right) \\
q_{t+1} - \bar{q} &= \left(1 - \psi\delta\right)\left(q_t - \bar{q}\right) \\
q_s - \bar{q} &= \left(1 - \psi\delta\right)^{s-t}\left(q_t - \bar{q}\right) \qquad (4)
\end{aligned}$$

Now turn to the dynamics of nominal exchange rate (3) :

$$\begin{aligned}
e_{t+1} - e_t &= \frac{e_t}{\eta} - \frac{1-\phi\delta}{\eta} q_t - \frac{\phi\delta\bar{q} + m_t}{\eta} \\
\left(e_{t+1} - \bar{q}\right) - \left(e_t - \bar{q}\right) &= \frac{e_t - \bar{q}}{\eta} + \frac{\bar{q}}{\eta} - \frac{1-\phi\delta}{\eta} q_t - \frac{\phi\delta\bar{q} + m_t}{\eta} \\
\left(e_{t+1} - \bar{q}\right) - \frac{1+\eta}{\eta}\left(e_t - \bar{q}\right) &= -\frac{1-\phi\delta}{\eta}\left(q_t - \bar{q}\right) - \frac{m_t}{\eta} \\
e_t - \bar{q} &= \frac{\eta}{1+\eta}\left(e_{t+1} - \bar{q}\right) + \frac{1-\phi\delta}{1+\eta}\left(q_t - \bar{q}\right) + \frac{m_t}{1+\eta}
\end{aligned}$$

This gives a dynamics relation in $e_t - \bar{q}$, which we iterate forward (using the transversality condition):

$$\begin{aligned}
e_t - \bar{q} &= \frac{1-\phi\delta}{1+\eta}(q_t - \bar{q}) + \frac{m_t}{1+\eta} + \frac{\eta}{1+\eta}\left[\frac{\eta}{1+\eta}(e_{t+2} - \bar{q}) + \frac{1-\phi\delta}{1+\eta}(q_{t+1} - \bar{q}) + \frac{m_{t+1}}{1+\eta}\right] \\
e_t - \bar{q} &= \frac{1-\phi\delta}{1+\eta}\sum_{s=t}^{\infty}\left(\frac{\eta}{1+\eta}\right)^{s-t}(q_s - \bar{q}) + \frac{1}{1+\eta}\sum_{s=t}^{\infty}\left(\frac{\eta}{1+\eta}\right)^{s-t} m_s \\
e_t - \bar{q} &= \frac{1-\phi\delta}{1+\eta}\sum_{s=t}^{\infty}\left(\frac{\eta(1-\psi\delta)}{1+\eta}\right)^{s-t}(q_t - \bar{q}) + \frac{1}{1+\eta}\sum_{s=t}^{\infty}\left(\frac{\eta}{1+\eta}\right)^{s-t} m_s \\
e_t - \bar{q} &= \frac{1-\phi\delta}{1+\eta}\frac{1}{1-\frac{\eta(1-\psi\delta)}{1+\eta}}(q_t - \bar{q}) + \frac{1}{1+\eta}\sum_{s=t}^{\infty}\left(\frac{\eta}{1+\eta}\right)^{s-t} m_s \\
e_t - \bar{q} &= \frac{1-\phi\delta}{1+\eta\psi\delta}(q_t - \bar{q}) + \frac{1}{1+\eta}\sum_{s=t}^{\infty}\left(\frac{\eta}{1+\eta}\right)^{s-t} m_s
\end{aligned}$$

2.3.2 Impact of a permanent money supply shock

Initially m_s is constant at $\bar{m}$, and $q_t = \bar{q}$, implying that the nominal exchange rate is simply the real exchange rate and the monetary stance:

$$\bar{e} = \bar{q} + \frac{1}{1+\eta}\frac{1}{1-\frac{\eta}{1+\eta}}\bar{m} = \bar{q} + \bar{m}$$

Consider that at time 0 the money supply increases permanently to $\bar{m}'$. The price level is a set state variable and is given by $p_0 = \bar{m}$. The nominal exchange rate is given by the forward looking dynamic relation:

$$\begin{aligned}
e_t - \bar{q} &= \frac{1-\phi\delta}{1+\eta\psi\delta}(q_t - \bar{q}) + \frac{1}{1+\eta}\sum_{s=t}^{\infty}\left(\frac{\eta}{1+\eta}\right)^{s-t}\bar{m}' \\
e_t - \bar{q} &= \frac{1-\phi\delta}{1+\eta\psi\delta}(q_t - \bar{q}) + \frac{1}{1+\eta}\frac{1}{1-\frac{\eta}{1+\eta}}\bar{m}' \\
e_t &= \bar{q} + \frac{1-\phi\delta}{1+\eta\psi\delta}(q_t - \bar{q}) + \bar{m}' \qquad (5)
\end{aligned}$$

Before the shock, the real exchange rate is: $\bar{q} = \bar{m} - \bar{e}$. After the shock, the real exchange rate q_0 combines the nominal exchange rate ($e_0 = q_0 + p_0$) above for $t = 0$ and p_0:

$$\begin{aligned}
e_0 &= \bar{q} + \frac{1-\phi\delta}{1+\eta\psi\delta}(q_0 - \bar{q}) + \bar{m}' \\
q_0 + p_0 &= \bar{q} + \frac{1-\phi\delta}{1+\eta\psi\delta}(q_0 - \bar{q}) + \bar{m}' \\
(q_0 - \bar{q}) &= \frac{1-\phi\delta}{1+\eta\psi\delta}(q_0 - \bar{q}) + \bar{m}' - p_0 \\
\frac{\eta\psi\delta + \phi\delta}{1+\eta\psi\delta}(q_0 - \bar{q}) &= \bar{m}' - \bar{m} \\
q_0 &= \bar{q} + \frac{1+\eta\psi\delta}{\phi\delta + \eta\psi\delta}(\bar{m}' - \bar{m}) \qquad (6)
\end{aligned}$$

The monetary expansion brings q_0 above $\bar{q}$. The real exchange rate goes back to $\bar{q}$ according to (2) as shown above.

The nominal exchange rate right after the shock is:

$$\begin{aligned}
e_0 &= p_0 + q_0 \\
e_0 &= \bar{m} + \left[\bar{q} + \frac{1+\eta\psi\delta}{\phi\delta+\eta\psi\delta}\left(\bar{m}' - \bar{m}\right)\right] \\
e_0 &= \bar{m}' - \left(\bar{m}' - \bar{m}\right) + \bar{q} + \frac{1+\eta\psi\delta}{\phi\delta+\eta\psi\delta}\left(\bar{m}' - \bar{m}\right) \\
e_0 &= \bar{q} + \bar{m}' + \frac{1-\phi\delta}{\phi\delta+\eta\psi\delta}\left(\bar{m}' - \bar{m}\right)
\end{aligned}$$

In the long run $\bar{e}' = \bar{q} + \bar{m}'$, therefore:

$$e_0 - \bar{e}' = \frac{1-\phi\delta}{\phi\delta+\eta\psi\delta}\left(\bar{m}' - \bar{m}\right) \tag{7}$$

which is positive when $1 > \phi\delta$, so the immediate nominal exchange rate is depreciated compared to the long run value.

The path of the exchange rate for $t \geq 0$ is as follows. We take the forward looking nominal exchange rate (5) and the dynamics of the real exchange rate (4):

$$\begin{aligned}
e_t &= \bar{q} + \frac{1-\phi\delta}{1+\eta\psi\delta}\left(q_t - \bar{q}\right) + \bar{m}' \\
e_t &= \bar{q} + \bar{m}' + \frac{1-\phi\delta}{1+\eta\psi\delta}\left(1-\psi\delta\right)^t\left(q_0 - \bar{q}\right) \\
e_t &= \bar{q} + \bar{m}' + \frac{1-\phi\delta}{1+\eta\psi\delta}\left(1-\psi\delta\right)^t\frac{1+\eta\psi\delta}{\phi\delta+\eta\psi\delta}\left(\bar{m}' - \bar{m}\right) \\
e_t &= \bar{q} + \bar{m}' + \left(1-\psi\delta\right)^t\frac{1-\phi\delta}{\phi\delta+\eta\psi\delta}\left(\bar{m}' - \bar{m}\right)
\end{aligned}$$

After the initial exchange rate jump, we use this result to write the exchange rate dynamics as:

$$\begin{aligned}
e_{t+1} - e_t &= \left[\bar{q} + \bar{m}' + \left(1-\psi\delta\right)^{t+1}\frac{1-\phi\delta}{\phi\delta+\eta\psi\delta}\left(\bar{m}' - \bar{m}\right)\right] \\
&\quad - \left[\bar{q} + \bar{m}' + \left(1-\psi\delta\right)^{t}\frac{1-\phi\delta}{\phi\delta+\eta\psi\delta}\left(\bar{m}' - \bar{m}\right)\right] \\
e_{t+1} - e_t &= \left[\left(1-\psi\delta\right)^{t+t} - \left(1-\psi\delta\right)^{t}\right]\frac{1-\phi\delta}{\phi\delta+\eta\psi\delta}\left(\bar{m}' - \bar{m}\right) \\
e_{t+1} - e_t &= \left[1-\psi\delta-1\right]\left(1-\psi\delta\right)^{t}\frac{1-\phi\delta}{\phi\delta+\eta\psi\delta}\left(\bar{m}' - \bar{m}\right) \\
e_{t+1} - e_t &= -\psi\delta\left(1-\psi\delta\right)^{t}\frac{1-\phi\delta}{\phi\delta+\eta\psi\delta}\left(\bar{m}' - \bar{m}\right) \\
e_{t+1} - e_t &= -\left(1-\psi\delta\right)^{t}\psi\delta\left(e_0 - \bar{e}'\right)
\end{aligned}$$

The overshooting depreciation is followed by a steady appreciation.

The interest parity condition implies that domestic interest rate remains low:

$$\begin{aligned} i_{t+1} &= i^* + e_{t+1} - e_t \\ i_{t+1} &= i^* - (1 - \psi\delta)^t \, \psi\delta \, (e_0 - \bar{e}') \end{aligned}$$