The Graduate Institute | Geneva
International Finance B – Fall 2021
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Problem Set 6
First Generation Model & Second Generation Model

First generation Model

Exercise 1

Consider the Flood and Garber (1984) model presented in Lecture 8 with the Uncovered Interest Parity condition (UIP) modified to account for an exchange-rate risk premium, q_t

$$i_t = i^* + \frac{E_t S_{t+1} - S_t}{S_t} + q_t \qquad \text{UIP}$$

$$P_t = S_t \qquad \text{PPP}$$

$$M_t = R_t^F + D_t \qquad \text{Money Supply}$$

$$M_t^d = P_t(Y - \alpha i_t) \qquad \text{Money Demand}$$

The exchange-rate risk premium increases with expected depreciation: $q_t = \bar{q} + \gamma \dfrac{E_t S_{t+1} - S_t}{S_t}$ with $\bar{q} > 0$ and $\gamma > 0$

The central banks pegs the exchange rate at $\bar{S}$ and finance the government deficit so that CB domestic credit increases at the deterministic rate μ; i.e. $D_t - D_{t-1} = \mu$ (Certainty case).

a) Find the shadow exchange rate, $\hat{S}_t$, that would prevail in a floating exchange-rate regime (fundamental solution) in the presence of the exchange-rate risk premium.

b) Derive the time of the attack T when foreign reserves are depleted and the central bank must let the exchange rate float. Also find the amount of remaining reserves that are taken away all at once when the run takes places at T.

c) What implications does the existence of an exchange rate premium have on the timing of the run on reserves and on the amount of reserves taken away.

d) Now suppose that $\gamma = 0$, while $\bar{q}$ depends on investor's risk aversion or risk perception. Suppose the exchange rate is fixed, and the shadow exchange rate is below the fixed parity, $\bar{S}$. Discuss what may happen in the case of a sudden increase in the exchange premium $\bar{q}$ due to a change in risk perception, e.g. because of a wake-up call originating from another country's crisis.

Exercise 2 - The Bubble

Suppose the shadow exchange rate includes a Bubble, A, growing at a rate β/α, so that it is equal to

$$\hat{S}_t^+ = \frac{\alpha}{\beta^2}\mu + \frac{1}{\beta}D_t + A\left(1+\frac{\beta}{\alpha}\right)^{t-T}$$

a) Derive the time of the attack T when foreign reserves are depleted and the central bank must let the exchange rate float.

b) Find the amount of remaining reserves that are taken away all at once when the run takes places at T.

Second generation Model

Exercise 3

Consider the following Loss function for the Central Bank:

$$L = (S^* - S_t)^2 + 3(E_t\Delta S_{t+1})^2 + C \qquad (1)$$

Where S_t is the exchange rate chosen by the Central Bank; S^* is the optimal equilibrium exchange rate, $E_t\Delta S_{t+1}$ is the expected rate of devaluation, and C is the political cost of devaluation ($C = 0$ if the peg is maintained).

If the CB maintains the exchange rate fixed, then $S_t = \bar{S}$, while, if the CB devalues, it sets $S_t = S^*$. If investors expect the CB to devalue, then $E_t\Delta S_{t+1} = S^* - \bar{S}$; otherwise $E_t\Delta S_{t+1} = 0$. (Recall that after a devaluation $E_t\Delta S_{t+1} = 0$). Assume $S^* = 3$ and $\bar{S} = 1$.

a) For which values of C a fixed exchange rate is a rational expectations equilibrium? For which values of C a devaluation is a rational expectations equilibrium?

b) Suppose that the political cost of a devaluation, C, decreases with the number of countries that, after joining the fixed exchange regime, decide to devalue. Specifically, take $C = 0.5(N-n)^2$, where $N = 7$ is the initial number of countries joining the fixed exchange regime and n is the number of countries that, eventually, devalue or abandon the regime.

Consider again the same country as in part a) that has a Loss function as in equation (1) and $S^* = 3$ and $\bar{S} = 1$. How many countries, n, have to devalue before a speculative attack against this country can be self-fulfilling? How many countries, n, have to devalue before a devaluation becomes the only possible equilibrium for the country under consideration?

c) If the countries belonging the fixed-exchange area differ regarding the misalignment of their exchange rate, $S^* - \bar{S}$, what do the results of part b) suggest about the timing and the propagatiion of currency crises?

Exercise 4

The Greek government must decide whether to raise taxes (and cut spending) or to default in order to stabilize the public debt at the level $\widehat{D}$. Hence, the government must generate a surplus, T, equal to:

$$T = (1+\rho)R\,sD(1-x) + R(1-s)D(1-x) - \widehat{D}$$

where R is the gross return on debt in the absence of a default premium, ρ is the default premium and s (for (with $0 < s < 1$) is the fraction of the debt to be rolled over in the current period (s stands for short-maturity). Finally, x is the fraction of the debt that is not repaid (i.e. the haircut) [Note: there is no risk premium on the fraction of debt, $(1-s)D$, that was issued in the past and is not going to be rolled over].

If the government decides to honor its obligations, then $x = 0$, while, if it defaults, $x = \bar{x} < 1$.
Expectations matter as they determine ρ. To this end, note that in a rational expectations (perfect foresight) equilibrium $(1+\rho)(1-Ex) = 1$, where investors' expectations of x can either be $Ex = 0$ or $Ex = \bar{x}$. In turn, this implies that either $(1+\rho) = 1$ or $(1+\rho) = (1-\bar{x})^{-1}$.

The government loss function is:

$$L = \alpha T^2 + C$$

where C is the cost of default, say, due to disruption of the financial sector, reputation loss, etc. To simplify the algebra assume $\widehat{D} = 0$, with a little loss of generality.

a) Characterize the possible equilibria, in terms of s, $\bar{x}$, C, etc.

b) Are there values for these variables/parameters for which multiple equilibria exist? Discuss conditions for multiple equilibria.

c) Consider the effect of the availability of a credit line, L, from other European countries so that:

$$T = R(1+\rho)sD(1-x) + R(1-s)D(1-x) - L$$

d) (where it is assumed $\widehat{D} = 0$). How generous must be the loan, L, from other countries to rule out the default equilibrium? Find the minimum L to avoid default in terms of the variables/parameters of the model.

Exercise 5

Consider the following Loss function for the Central Bank:

$$L = 2(S^* - S_t)^2 + (E_t \Delta S_{t+1})^2 + C$$

Where S_t is the exchange rate chosen by the Central Bank; S^* is the optimal equilibrium exchange rate, $E_t \Delta S_{t+1}$ is the expected rate of devaluation, and C is the political cost of devaluation ($C = 0$ if the peg is maintained). If the CB maintains the exchange rate fixed, then $S_t = \bar{S}$, while, if the CB devalues, it sets $S_t = S^*$. If investors expect the CB to devalue, then $E_t \Delta S_{t+1} = S^* - \bar{S}$; otherwise $E_t \Delta S_{t+1} = 0$. (Recall that after a devaluation $E_t \Delta S_{t+1} = 0$).

a) Suppose $S^* = 3$, $\bar{S} = 1$ and $C = 10$. Which rational expectations equilibrium(a) is(are) possible? What determines a devaluation?

b) Continue to assume $S^* = 3$ and $C = 10$, but now suppose that the exchange rate is pegged at $\bar{S} = 2$. Which rational expectations equilibrium(a) is(are) possible? What determines a devaluation?

c) What does this Exercise suggest regarding the role/importance of fundamentals in Second Generation Models of currency crises?