EI035 Econometrics II

The Graduate Institute, Spring 2025, Marko Mlikota

PS1 Solutions

Jingle Fu

Solution (a).

```r
set.seed(2025)
n <- 100
u_i <- rnorm(n, mean = 0, sd = sqrt(5))
g_i <- rgamma(n, shape = 2, scale = 2)
r_i <- rbinom(n, size = 1, prob = 0.5)
x_star_i <- numeric(n)

for (i in 1:n) {
if (r_i[i] == 1) {
x_star_i[i] <- rgamma(1, shape = 3, scale = 1)
  } else {
    x_star_i[i] <- rgamma(1, shape = 7, scale = 1)
  }
}

beta_0 <- 400
beta_1 <- 5
beta_2 <- 200
beta_3 <- 10

y_i <- beta_0 + beta_1 * x_star_i + beta_2 * r_i + beta_3 * g_i + u_i

n1_i <- rnorm(n, mean = 10, sd = sqrt(3))
n2_i <- rnorm(n, mean = 5 + sqrt(x_star_i), sd = sqrt(3))

data <- data.frame(
    y = y_i,
    x_star = x_star_i,
    r = r_i,
    g = g_i,
    n1 = n1_i,
    n2 = n2_i
  )
```

Solution (b).

We consider the true model:

$$y_i = \beta_0 + \beta_1 x_i^* + \beta_2 r_i + \beta_3 g_i + u_i,$$

with the following Data Generating Process (DGP):

- $u_i \sim N(0, 5)$.
- $g_i \sim \Gamma(2, 2)$, so that

$$\mathbb{E}[g_i] = \frac{2}{2} = 1, \quad \mathbb{V}[g_i] = \frac{2}{2^2} = \frac{2}{4} = 0.5.$$

- $r_i \in \{0, 1\}$ with $P(r_i = 1) = 0.5$.
- Conditionally on r_i, the fertilizer variable x_i^* is distributed as:
  - If $r_i = 1$: $x_i^* \sim \Gamma(3, 1)$, so that

$$\mathbb{E}[x_i^* \mid r_i = 1] = 3, \quad \mathbb{V}[x_i^* \mid r_i = 1] = 3.$$

  - If $r_i = 0$: $x_i^* \sim \Gamma(7, 1)$, so that

$$\mathbb{E}[x_i^* \mid r_i = 0] = 7, \quad \mathbb{V}[x_i^* \mid r_i = 0] = 7.$$

By the law of total expectation, we have:

$$\mathbb{E}[x_i^*] = 0.5 \cdot 3 + 0.5 \cdot 7 = 5.$$

Similarly, by the law of total variance:

$$\mathbb{V}[x_i^*] = \mathbb{E}\left[\mathbb{V}[x_i^* \mid r_i]\right] + \mathbb{V}\left(\mathbb{E}[x_i^* \mid r_i]\right) = 0.5 \cdot 3 + 0.5 \cdot 7 + 0.5\Big[(3-5)^2 + (7-5)^2\Big] = 5 + 4 = 9.$$

Calculation of $\mathrm{Cov}(x_i^*, g_i)$

We have

$$\mathrm{Cov}(x_i^*, g_i) = \mathbb{E}[x_i^* g_i] - \mathbb{E}[x_i^*]\,\mathbb{E}[g_i].$$

Since $\mathbb{E}[x_i^*] = 5$ and $\mathbb{E}[g_i] = 1$, it remains to compute $\mathbb{E}[x_i^* g_i]$. Conditioning on r_i,

$$\mathbb{E}[x_i^* g_i] = E\left[\mathbb{E}[x_i^* g_i \mid r_i]\right] = 0.5\,\mathbb{E}[x_i^* g_i \mid r_i = 1] + 0.5\,\mathbb{E}[x_i^* g_i \mid r_i = 0].$$

For each group we write:

$$\mathbb{E}[x_i^* g_i \mid r_i = i] = \mathbb{E}[x_i^* \mid r_i = i]\mathbb{E}[g_i \mid r_i = i] + \mathrm{Cov}(x_i^*, g_i \mid r_i = i), \quad i = 0, 1.$$

Since the DGP does not explicitly state a dependence between x_i^* and g_i, we denote their conditional covariance by

$$\mathrm{Cov}(x_i^*, g_i \mid r_i = i) = \rho_i \sqrt{\mathbb{V}[x_i^* \mid r_i = i]\mathbb{V}[g_i]}, \quad i = 0, 1,$$

with ρ_i being the (conditional) correlation. Thus, for $r_i = 1$:

$$\mathbb{E}[x_i^* g_i \mid r_i = 1] = 3 \cdot 1 + \rho_1 \sqrt{3 \cdot 0.5} = 3 + \rho_1 \sqrt{1.5},$$

and for $r_i = 0$:

$$\mathbb{E}[x_i^* g_i \mid r_i = 0] = 7 \cdot 1 + \rho_0 \sqrt{7 \cdot 0.5} = 7 + \rho_0 \sqrt{3.5}.$$

Averaging, we obtain:

$$\mathbb{E}[x_i^* g_i] = 5 + \frac{1}{2}\left(\rho_1 \sqrt{1.5} + \rho_0 \sqrt{3.5}\right).$$

Therefore,

$$\mathrm{Cov}(x_i^*, g_i) = \frac{1}{2}\left(\rho_1 \sqrt{1.5} + \rho_0 \sqrt{3.5}\right).$$

Regression 1: $y_i = \beta_0 + \beta_1 x_i^* + \mathrm{error}_i$
The working regression is

$$y_i = \beta_0 + \beta_1 x_i^* + \varepsilon_i, \quad \text{with} \quad \varepsilon_i = \beta_2 r_i + \beta_3 g_i + u_i.$$

The OLS estimator for β_1 is given by

$$\hat{\beta}_1 = \beta_1 + \frac{\mathrm{Cov}\left(x_i^*,\, \beta_2 r_i + \beta_3 g_i\right)}{\mathbb{V}[x_i^*]}.$$

By linearity,

$$\mathrm{Cov}\left(x_i^*,\, \beta_2 r_i + \beta_3 g_i\right) = \beta_2\ \mathrm{Cov}(x_i^*, r_i) + \beta_3\ \mathrm{Cov}(x_i^*, g_i).$$

Calculation of $\mathrm{Cov}(x_i^*, r_i)$:

$$\mathrm{Cov}(x_i^*, r_i) = \mathbb{E}[x_i^* r_i] - \mathbb{E}[x_i^*]\,\mathbb{E}[r_i].$$

Since

$$\mathbb{E}[x_i^* r_i] = P(r_i = 1)\mathbb{E}[x_i^* \mid r_i = 1] + P(r_i = 0) \cdot 0 = 0.5 \cdot 3 = 1.5,$$

and $\mathbb{E}[r_i] = 0.5$, it follows that

$$\mathrm{Cov}(x_i^*, r_i) = 1.5 - 5 \cdot 0.5 = 1.5 - 2.5 = -1.$$

Thus, the probability limit of $\hat{\beta}_1$ is:

$$\operatorname{plim} \hat{\beta}_1 = \beta_1 + \frac{\beta_2(-1) + \beta_3\left\{\frac{1}{2}\left(\rho_1 \sqrt{1.5} + \rho_0 \sqrt{3.5}\right)\right\}}{9}.$$

That is, the **omitted variable bias** is:

$$\text{Bias}^{(1)} = \frac{-\beta_2 + \frac{1}{2}\beta_3\left(\rho_1\sqrt{1.5} + \rho_0\sqrt{3.5}\right)}{9}.$$

Regression 2: $y_i = \beta_0 + \beta_1 x_i^* + \beta_2 r_i + \text{error}_i$
Now the regression is:

$$y_i = \beta_0 + \beta_1 x_i^* + \beta_2 r_i + \varepsilon_i, \quad \text{with} \quad \varepsilon_i = \beta_3 g_i + u_i.$$

By the Frisch–Waugh–Lovell theorem, we partial out r_i from x_i^*. Define the residual:

$$\tilde{x}_i = x_i^* - \mathbb{E}[x_i^* \mid r_i],$$

with

$$\mathbb{E}[x_i^* \mid r_i] = \begin{cases} 3, & r_i = 1, \\ 7, & r_i = 0. \end{cases}$$

Then the OLS estimator becomes:

$$\hat{\beta}_1 = \beta_1 + \frac{\text{Cov}\left(\tilde{x}_i, \beta_3 g_i\right)}{\mathbb{V}[\tilde{x}_i]} = \beta_1 + \beta_3\,\frac{\text{Cov}(\tilde{x}_i, g_i)}{\mathbb{V}[\tilde{x}_i]}.$$

Assuming that $\mathbb{E}[g_i \mid r_i] = \mathbb{E}[g_i] = 1$, note that

$$\text{Cov}(\tilde{x}_i, g_i) = \text{Cov}(x_i^*, g_i) - \text{Cov}(\mathbb{E}[x_i^* \mid r_i], g_i).$$

A brief calculation shows:

$$\text{Cov}(\mathbb{E}[x_i^* \mid r_i], g_i) = 0.5\,[3 \cdot 1 + 7 \cdot 1] - \mathbb{E}[x_i^*]\mathbb{E}[g_i] = 5 - 5 = 0.$$

Also,

$$\mathbb{V}[\tilde{x}_i] = E\left[\mathbb{V}[x_i^* \mid r_i]\right] = 0.5 \cdot 3 + 0.5 \cdot 7 = 5.$$

Thus, the probability limit is:

$$\text{plim}\,\hat{\beta}_1 = \beta_1 + \frac{\beta_3\ \text{Cov}(x_i^*, g_i)}{5} = \beta_1 + \frac{\beta_3\,\frac{1}{2}\left(\rho_1\sqrt{1.5} + \rho_0\sqrt{3.5}\right)}{5}.$$

That is, the bias in Regression 2 is:

$$\text{Bias}^{(2)} = \frac{\beta_3\ \left(\rho_1\sqrt{1.5} + \rho_0\sqrt{3.5}\right)}{10}.$$

The asymptotic variance is:

$$\mathbb{V}[\hat{\beta}_1] \approx \frac{1}{n}\,\frac{\mathbb{V}[\beta_3 g_i + u_i]}{5}.$$

Regression 3: $y_i = \beta_0 + \beta_1 x_i^* + \beta_2 r_i + \beta_3 g_i + \text{error}_i$
Here the regression exactly matches the true DGP:

$$y_i = \beta_0 + \beta_1 x_i^* + \beta_2 r_i + \beta_3 g_i + u_i.$$

Under the exogeneity assumption $\mathbb{E}[u_i \mid x_i^*, r_i, g_i] = 0$, the OLS estimator for β_1 is **consistent**:

$$\operatorname{plim} \hat{\beta}_1 = \beta_1.$$

Its finite-sample variance is given by the standard OLS formula:

$$\mathbb{V}[\hat{\beta}_1] = \sigma_u^2 \left[(X'X)^{-1}\right]_{11},$$

where the design matrix X includes the moments and cross-moments of x_i^*, r_i, and g_i.

Regression 4: $y_i = \beta_0 + \beta_1 x_i^* + \beta_2 r_i + \beta_3 g_i + \beta_4 n_i^1 + \text{error}_i$
Since $n_i^1 \sim N(10, 3)$ is generated independently of x_i^*, r_i, g_i, and u_i, it is an *irrelevant regressor*. Under the standard OLS assumptions, the inclusion of an irrelevant regressor does not cause bias in the estimated coefficient of x_i^*:

$$\operatorname{plim} \hat{\beta}_1 = \beta_1.$$

However, its inclusion may increase the finite-sample variance of $\hat{\beta}_1$. In particular, the variance formula now becomes

$$\mathbb{V}[\hat{\beta}_1] = \sigma_u^2 \left[(X'X)^{-1}\right]_{11},$$

where the design matrix X now includes the column corresponding to n_i^1. If n_i^1 is only weakly correlated with x_i^*, then the increase in variance is modest. In summary, **Regression 4** yields a consistent estimator for β_1, with no additional bias but possibly a slight inflation in variance.

Regression 5: $y_i = \beta_0 + \beta_1 x_i^* + \beta_2 r_i + \beta_3 g_i + \beta_4 n_i^2 + \text{error}_i$
Here,

$$n_i^2 \sim N\left(5 + \sqrt{x_i^*},\, 3\right),$$

so that n_i^2 is a non-linear function of x_i^*. This implies that n_i^2 is correlated with x_i^*. In particular, since

$$\mathbb{E}[n_i^2 \mid x_i^*] = 5 + \sqrt{x_i^*},$$

we have

$$\operatorname{Cov}(x_i^*, n_i^2) \neq 0.$$

The inclusion of n_i^2 does not cause endogeneity provided that

$$\mathbb{E}[u_i \mid x_i^*, r_i, g_i, n_i^2] = 0.$$

Thus, by the Frisch–Waugh–Lovell theorem, the coefficient β_1 is still identified and

$$\operatorname{plim} \hat{\beta}_1 = \beta_1.$$

However, the strong correlation between x_i^* and n_i^2 increases multicollinearity. To see this more formally, consider the variance of $\hat{\beta}_1$ in a multiple regression:

$$\mathbb{V}[\hat{\beta}_1] = \sigma_u^2 \left[(X'X)^{-1}\right]_{11}.$$

When x_i^* is highly collinear with n_i^2, the effective variation in x_i^* (after partialling out the effect of n_i^2 along with r_i and g_i) is reduced. Denote by R^2_{x,n^2} the coefficient of determination from regressing x_i^* on the other regressors (including n_i^2). Then, the variance inflation factor (VIF) for $\hat{\beta}_1$ is given by

$$\text{VIF} = \frac{1}{1 - R^2_{x,n^2}}.$$

Thus, the asymptotic variance becomes

$$\mathbb{V}[\hat{\beta}_1] \approx \frac{\sigma_u^2}{n\,\mathbb{V}[x_i^*]} \cdot \frac{1}{1 - R^2_{x,n^2}},$$

which is larger than that in Regression 3 (which does not include n_i^2). In summary, while **Regression 5** still provides a consistent estimate of β_1, the estimator‘s variance is inflated due to the high collinearity between x_i^* and n_i^2.

```r
reg1 <- lm(y ~ x_star, data = data)
summary_reg1 <- summary(reg1)

reg2 <- lm(y ~ x_star + r, data = data)
summary_reg2 <- summary(reg2)

reg3 <- lm(y ~ x_star + r + g, data = data)
summary_reg3 <- summary(reg3)

reg4 <- lm(y ~ x_star + r + g + n1, data = data)
summary_reg4 <- summary(reg4)

reg5 <- lm(y ~ x_star + r + g + n1 + n2, data = data)
summary_reg5 <- summary(reg5)

extract_results <- function(reg_summary, reg_name) {
    beta1_estimate <- reg_summary$coefficients["x_star", "Estimate"]
```

```
    beta1_se <- reg_summary$coefficients["x_star", "Std. Error"]
    beta1_true <- 5

    cat(paste0("\n", reg_name, ":\n"))
    cat(paste0("Estimated : ", round(beta1_estimate, 4), "\n"))
    cat(paste0("True : ", beta1_true, "\n"))
    cat(paste0("Standard Error: ", round(beta1_se, 4), "\n"))
    cat(paste0("Difference from true value: ", round(beta1_estimate - beta1_true, 4), "\n"))
    cat(paste0("Adjusted Rš: ", round(reg_summary$adj.r.squared, 4), "\n"))
}

extract_results(summary_reg1, "Regression 1 (y ~ x_star)")
extract_results(summary_reg2, "Regression 2 (y ~ x_star + r)")
extract_results(summary_reg3, "Regression 3 (y ~ x_star + r + g)")
extract_results(summary_reg4, "Regression 4 (y ~ x_star + r + g + n1)")
extract_results(summary_reg5, "Regression 5 (y ~ x_star + r + g + n1 + n2)")
```

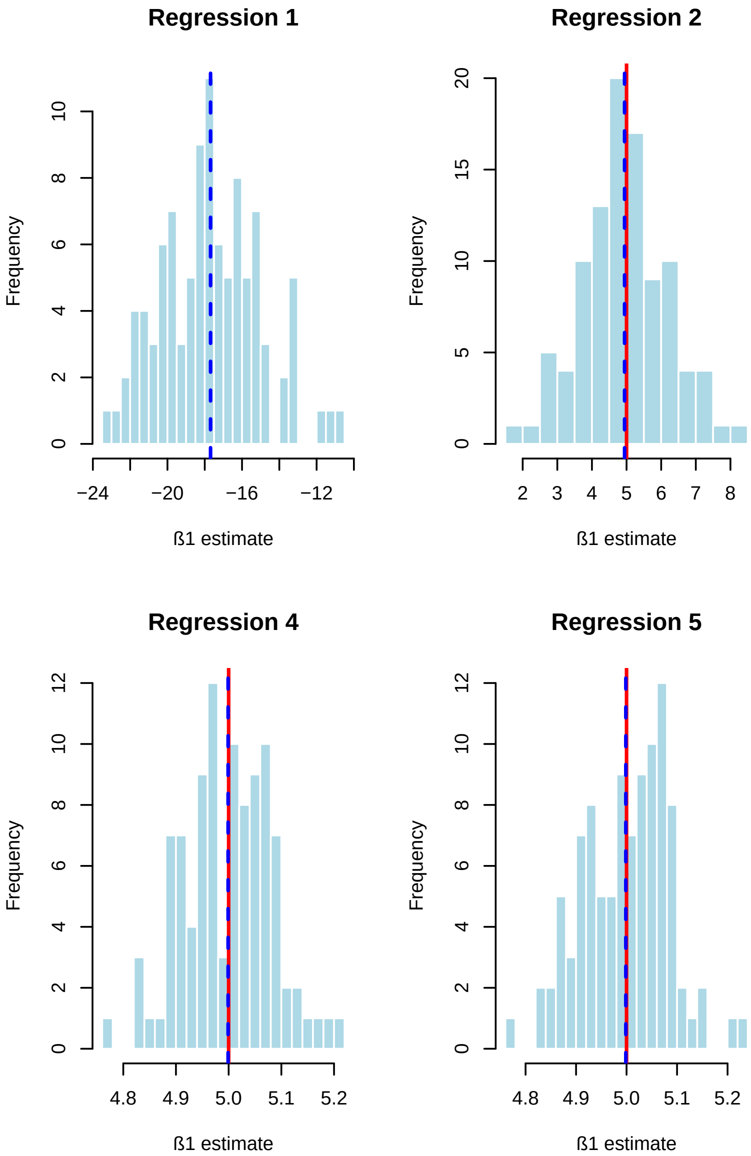
Table 1: Regression Results for Question (b)

Regression	Estimated β_1	True β_1	SE	Difference	Adjusted R^2
$y \sim x^*$	-16.9809	5	2.5386	-21.9809	0.3065
$y \sim x^* + r$	4.3648	5	1.2873	-0.6352	0.9029
$y \sim x^* + r + g$	5.0395	5	0.0957	0.0395	0.9995
$y \sim x^* + r + g + n1$	5.0675	5	0.0962	0.0675	0.9995
$y \sim x^* + r + g + n1 + n2$	5.0628	5	0.0976	0.0628	0.9995

Solution (c).

From a theoretical perspective, the Monte Carlo simulation allows us to empirically approximate the sampling distribution of each estimator. The histograms should confirm our theoretical derivations:

1. $\hat{\beta}_1^{(1)}$ should show a systematic bias from the true value of 5.
2. $\hat{\beta}_1^{(2)}$ might show a smaller bias if there's still correlation between g_i and x_i^* after controlling for r_i.
3. $\hat{\beta}_1^{(3)}$, $\hat{\beta}_1^{(4)}$, and $\hat{\beta}_1^{(5)}$ should be centered around 5, but with increasing variance.




```
set.seed(2025)
M <- 100
n <- 100

beta1_estimates <- matrix(NA, nrow = M, ncol = 5)
colnames(beta1_estimates) <- c("Reg1", "Reg2", "Reg3", "Reg4", "Reg5")

for (m in 1:M) {
  u_i <- rnorm(n, mean = 0, sd = sqrt(5))
  g_i <- rgamma(n, shape = 2, scale = 2)
  r_i <- rbinom(n, size = 1, prob = 0.5)

  x_star_i <- numeric(n)
  for (i in 1:n) {
    if (r_i[i] == 1) {
```

```
      x_star_i[i] <- rgamma(1, shape = 3, scale = 1)
    } else {
      x_star_i[i] <- rgamma(1, shape = 7, scale = 1)
    }
  }

  beta_0 <- 400
  beta_1 <- 5
  beta_2 <- 200
  beta_3 <- 10

  y_i <- beta_0 + beta_1 * x_star_i + beta_2 * r_i + beta_3 * g_i + u_i

  n1_i <- rnorm(n, mean = 10, sd = sqrt(3))
  n2_i <- rnorm(n, mean = 5 + sqrt(x_star_i), sd = sqrt(3))

  data <- data.frame(
    y = y_i,
    x_star = x_star_i,
    r = r_i,
    g = g_i,
    n1 = n1_i,
    n2 = n2_i
  )

  reg1 <- lm(y ~ x_star, data = data)
  reg2 <- lm(y ~ x_star + r, data = data)
  reg3 <- lm(y ~ x_star + r + g, data = data)
  reg4 <- lm(y ~ x_star + r + g + n1, data = data)
  reg5 <- lm(y ~ x_star + r + g + n1 + n2, data = data)

    # Store  estimates
  beta1_estimates[m, 1] <- coef(reg1)["x_star"]
  beta1_estimates[m, 2] <- coef(reg2)["x_star"]
  beta1_estimates[m, 3] <- coef(reg3)["x_star"]
  beta1_estimates[m, 4] <- coef(reg4)["x_star"]
  beta1_estimates[m, 5] <- coef(reg5)["x_star"]
}

beta1_df <- data.frame(
  Estimate = c(beta1_estimates),
  Regression = rep(colnames(beta1_estimates), each = M)
)

beta1_summary <- data.frame(
    Regression = colnames(beta1_estimates),
    Mean = colMeans(beta1_estimates),
    SD = apply(beta1_estimates, 2, sd),
```

```
    Bias = colMeans(beta1_estimates) - 5
)

par(mfrow = c(2, 3))
for (i in 1:5) {
  hist(beta1_estimates[, i],
       main = past\mathbb{E}["Regression", i],
       xlab = " estimate",
       breaks = 20,
       col = "lightblue",
       border = "white")
  ablin\mathbb{E}[v = 5, col = "red", lwd = 2]  # True value
  ablin\mathbb{E}[v = mean(beta1_estimates[, i]], col = "blue", lty = 2, lwd = 2)  # Mean estimate
}
```

Solution (d).

When $x_i^* \mid (r_i = 1) = x_i^* \mid (r_i = 0) \sim \Gamma(5, 1)$

If we set

$$x_i^* \mid (r_i = 1) = x_i^* \mid (r_i = 0) \sim \Gamma(5, 1),$$

then

$$\mathbb{E}[x_i^* \mid r_i] = 5 \quad \text{for both } r_i = 0, 1,$$

and hence

$$\mathbb{E}[x_i^*] = 5 \quad \text{and} \quad \mathbb{V}[x_i^*] = 5.$$

In this case,

$$\mathrm{Cov}(x_i^*, r_i) = \mathbb{E}[x_i^* r_i] - \mathbb{E}[x_i^*]\mathbb{E}[r_i] = 0.5 \cdot 5 - 5 \cdot 0.5 = 0.$$

Thus, in Regression 1 the omitted variable bias reduces to:

$$\mathrm{Bias}^{(1)} = \frac{0 + \beta_3 \ \mathrm{Cov}(x_i^*, g_i)}{5}.$$

When $\beta_2 = 0$

Then the bias in Regression 1 simplifies to:

$$\mathrm{Bias}^{(1)} = \frac{\beta_3 \ \mathrm{Cov}(x_i^*, g_i)}{\mathbb{V}[x_i^*]}.$$

A similar simplification applies for Regression 2.

When $r_i = 1$ with probability 0.1

Then,

$$\mathbb{E}[r_i] = 0.1, \quad \mathbb{E}[x_i^*] = 0.1 \cdot 3 + 0.9 \cdot 7 = 6.6,$$

and

$$\mathbb{E}[x_i^* r_i] = 0.1 \cdot 3 = 0.3.$$

Thus,

$$\text{Cov}(x_i^*, r_i) = 0.3 - 6.6 \cdot 0.1 = 0.3 - 0.66 = -0.36.$$

Accordingly, the bias in Regression 1 becomes:

$$\text{Bias}^{(1)} = \frac{\beta_2(-0.36) + \beta_3 \, \text{Cov}(x_i^*, g_i)}{\mathbb{V}[x_i^*]},$$

with $\mathbb{V}[x_i^*]$ recalculated under the new mixture proportions.

When $\beta_3 = 50$

Then, for Regression 1, the bias is:

$$\text{Bias}^{(1)} = \frac{-\beta_2 + 50 \cdot \frac{1}{2}\left(\rho_1\sqrt{1.5} + \rho_0\sqrt{3.5}\right)}{\mathbb{V}[x_i^*]}.$$

For the original DGP with $\mathbb{V}[x_i^*] = 9$, the bias becomes substantially larger due to the amplified effect of β_3.

Table 2: Simulation Summary Statistics for Question (d)

	Scenario	Regression	Mean	SD	Bias
Reg11	$x_i \vert r_1 = 1$ $x_i \vert r_i = 0$	Reg1	5.530295	4.3563060	0.5302951
Reg21	$x_i \vert r_1 = 1$ $x_i \vert r_i = 0$	Reg2	4.994777	1.3270007	-0.0052234
Reg31	$x_i \vert r_1 = 1$ $x_i \vert r_i = 0$	Reg3	4.991522	0.1105130	-0.0084783
Reg12	$\beta_2 = 0$	Reg1	5.211875	0.9578191	0.2118745
Reg22	$\beta_2 = 0$	Reg2	5.282412	1.2969897	0.2824116
Reg32	$\beta_2 = 0$	Reg3	5.005810	0.0950364	0.0058101
Reg13	$p(r_i = 1) = 0.1$	Reg1	-4.152274	2.6530512	-9.1522744
Reg23	$p(r_i = 1) = 0.1$	Reg2	4.941895	1.1553780	-0.0581046
Reg33	$p(r_i = 1) = 0.1$	Reg3	4.993826	0.0973356	-0.0061737
Reg14	$\beta_3 = 50$	Reg1	-18.371055	6.5915167	-23.3710551
Reg24	$\beta_3 = 50$	Reg2	4.394012	6.1969404	-0.6059882
Reg34	$\beta_3 = 50$	Reg3	4.996611	0.1112587	-0.0033893

```r
run_monte_carlo <- function(x_star_equal = FALSE, beta2_zero = FALSE, r_
    prob = 0.5, beta3_value = 10) {
  M <- 100
  n <- 100

  beta1_estimates <- matrix(NA, nrow = M, ncol = 3)
  colnames(beta1_estimates) <- c("Reg1", "Reg2", "Reg3")

  for (m in 1:M) {
    # Generate data according to modified DGP
    u_i <- rnorm(n, mean = 0, sd = sqrt(5))
    g_i <- rgamma(n, shape = 2, scale = 2)
    r_i <- rbinom(n, size = 1, prob = r_prob)
```

```

    x_star_i <- numeric(n)
    if (x_star_equal) {
      x_star_i <- rgamma(n, shape = 5, scale = 1)
    } else {
      for (i in 1:n) {
        if (r_i[i] == 1) {
          x_star_i[i] <- rgamma(1, shape = 3, scale = 1)
        } else {
          x_star_i[i] <- rgamma(1, shape = 7, scale = 1)
        }
      }
    }

    beta_0 <- 400
    beta_1 <- 5
    beta_2 <- ifels\mathbb{E}[beta2_zero, 0, 200]
    beta_3 <- beta3_value

    y_i <- beta_0 + beta_1 * x_star_i + beta_2 * r_i + beta_3 * g_i + u_i

    data <- data.frame(
      y = y_i,
      x_star = x_star_i,
      r = r_i,
      g = g_i
    )

    reg1 <- lm(y ~ x_star, data = data)
    reg2 <- lm(y ~ x_star + r, data = data)
    reg3 <- lm(y ~ x_star + r + g, data = data)

    beta1_estimates[m, 1] <- coef(reg1)["x_star"]
    beta1_estimates[m, 2] <- coef(reg2)["x_star"]
    beta1_estimates[m, 3] <- coef(reg3)["x_star"]
  }

  return(beta1_estimates)
}

results_original <- run_monte_carlo()
results_xstar_equal <- run_monte_carlo(x_star_equal = TRUE)
results_beta2_zero <- run_monte_carlo(beta2_zero = TRUE)
results_r_prob_0.1 <- run_monte_carlo(r_prob = 0.1)
results_beta3_50 <- run_monte_carlo(beta3_value = 50)

calc_summary <- function(results, scenario_name) {
```

```
  summary_df <- data.frame(
    Scenario = rep(scenario_name, 3),
    Regression = c("Reg1", "Reg2", "Reg3"),
    Mean = colMeans(results),
    SD = apply(results, 2, sd),
    Bias = colMeans(results) - 5
  )
  return(summary_df)
}

summary_original <- calc_summary(results_original, "Original DGP")
summary_xstar_equal <- calc_summary(results_xstar_equal, "x_star equal")
summary_beta2_zero <- calc_summary(results_beta2_zero, " = 0")
summary_r_prob_0.1 <- calc_summary(results_r_prob_0.1, "r_prob = 0.1")
summary_beta3_50 <- calc_summary(results_beta3_50, " = 50")

all_summaries <- rbind(
  summary_original,
  summary_xstar_equal,
  summary_beta2_zero,
  summary_r_prob_0.1,
  summary_beta3_50
)

latex_table_d <- kable(all_summaries, format = "latex", booktabs = TRUE,
                       caption = "Simulation Summary Statistics for
   Question (d)")

output_file_d <- "d.tex"
cat(latex_table_d, file = output_file_d)
cat("\n% Table saved to ", output_file_d, "\n", sep = "", file = output
   _file_d, append = TRUE)


plot_scenario_comparison <- function(original, modified, title) {
  par(mfrow = c(2, 3))

  for (i in 1:3) {
    hist(original[, i],
         main = past\mathbb{E}["Reg", i, "- Original"],
         xlab = " estimate",
         breaks = 15,
         col = "lightblue",
         border = "white",
         xlim = rang\mathbb{E}[c(original[, i], modified[, i]]))
    ablin\mathbb{E}[v = 5, col = "red", lwd = 2]
    ablin\mathbb{E}[v = mean(original[, i]], col = "blue", lty = 2, lwd
   = 2)
```

```

    hist(modified[, i],
         main = past\mathbb{E}["Reg", i, "- Modified"],
         xlab = " estimate",
         breaks = 15,
         col = "lightgreen",
         border = "white",
         xlim = rang\mathbb{E}[c(original[, i], modified[, i]]))
    ablin\mathbb{E}[v = 5, col = "red", lwd = 2]
    ablin\mathbb{E}[v = mean(modified[, i]], col = "blue", lty = 2, lwd
   = 2)
  }
  mtext(title, side = 3, line = -1.5, outer = TRUE)
}

par(mfrow = c(1, 1))
```