PS 2

QUESTION 1 : BENEFITS OF FINANCIAL INTEGRATION:

- FOR AN INVESTOR :
  - ↳ CONSUMPTION SMOOTHING
  - ↳ MINIMIZE THE RISK, i.e INSURE INCOME AGAINST RISK

  } She can buy assets w/ higher expected returns oversea + more diversified portfolio

- FOR A FIRM :
  - ↳ production growth ⇐ I, Access to foreign credit markets thus increasing I, otherwise not possible b/c of saving constraints. This also reduces the cost of capital and the competition with more efficient firms can induce an improvement in the production process.
  - ↳ Better institutional environment
  - ↳ New management technology

b/c ca

Q 2

W = Share of wealth(1) in R_D

$r_D, r_F \Rightarrow r_D = r_F \Rightarrow w, (1-w) \mid$ Var is minimized.

$R_p = r_D w + r_F(1-w)$

$$Var(R_p) = Var(r_D)w^2 + Var(r_F)(1-w)^2 + 2cov(r_D, r_F)w(1-w)$$

a) $Var(r_D) = 2 \quad Var(r_F) = 8 \quad Cov(r_D, r_F) = 0 \quad ?w$

$$Var(R_p) = 2w^2 + 8(1-w)^2 + 0$$
$$\overset{!}{=} 2w^2 + 8(1 + w^2 - 2w)$$
$$\overset{!}{=} 10w^2 + 8 - 16w$$

$$\frac{\partial Var(R_p)}{\partial w} = 0 \Rightarrow 20w - 16 = 0 \Rightarrow w = 0.8$$

$$\underbrace{(1-w) = 0.2}$$

This is due to the fact that the covariance b/w r_D and r_F is zero and therefore for diversification purposes it allows to hedge against the risk arising from domestic assets. In fact, portfolio diversification reduces the risk for any given expected return.

b) $Cov(r_D, r_F) = -4.$

$$Var(r_p) = 2w^2 + 8(1-w)^2 - 8w(1-w)$$
$$\overset{!}{=} 2w^2 + 8 + \cancel{8w^2} - 16w - 8w + \cancel{8w^2}$$
$$\overset{!}{=} 18w^2 - 24w + 8$$

$$\frac{\partial Var(r_p)}{\partial w} = 0 \Rightarrow 36w - 24 = 0 \Rightarrow w \approx 0.67$$

With $cov(r_D, r_F) < 0$ the gross returns will go in opposite direction. The distribution across assets more evenly distributed b/c of higher possibility to hedge the risk. $\Rightarrow$

Q 3

Ponzi - Game condition:

↳ country finances its liabilities and related income payments by issuing new liabilities. ≡ A country pays interests by borrowing more.

$$\lim_{T\to\infty} \frac{1}{(1+r)^T} \underbrace{NFL_T}_{-B_T} \leq 0$$

The debt (NFL) is supposed to grow at a lower rate than ⓡ. If it was to run at a rate ⓡ, it would mean that the liabilities and the interests on them are paid by issuing more debt.

Debt is growing over time b/c I'm paying interests on my initial debt, and no Trade surpluses are generated.

$$NFL_T = (1+r)^T NFL_0$$

b) Transversality condition:

$$\lim_{T\to\infty} \frac{1}{(1+r)^T} B_T \geq 0$$: I don't want other countries to play Ponzi game against me.

$$\Rightarrow \boxed{\lim_{T\to\infty} \frac{1}{(1+r)^T} NFL_T = 0}$$

Flow budget constraint: [Accumulation of net foreign assets]

$$NFL_1 = (1+r) NFL_0 - TB_1$$

→ Derive IBC:

$$NFL_0 = \frac{NFL_1}{(1+r)} + \frac{TB_1}{(1+r)}$$

Shift by one period:

$$NFL_1 = \frac{NFL_2}{(1+r)} + \frac{TB_2}{(1+r)}$$

Replace:

$$NFL_0 = \frac{NFL_2}{(1+r)^2} + \frac{TB_2}{(1+r)} + \frac{TB_1}{(1+r)}$$

Iterate N times:

$$NFL_0 = \frac{TB_1}{(1+r)} + \frac{TB_2}{(1+r)^2} + \dots + \frac{TB_T}{(1+r)^T} + \frac{NFL_T}{(1+r)^T}$$

Write the last term in terms of B_T:

$$NFL_0 = \frac{TB_1}{(1+r)} + \frac{TB_2}{(1+r)^2} + \dots + \frac{TB_T}{(1+r)^T} - \underbrace{\frac{B_T}{(1+r)^T}}$$

Take the limit and apply transversality condition: $\lim_{T\to\infty} \frac{B_T}{(1+r)^T} = 0$

$$NFL_0 = \frac{TB_1}{(1+r)} + \frac{TB_2}{(1+r)^2} + \dots \lim_{T\to\infty} \frac{TB_T}{(1+r)^T}$$

To be sustainable, NFL_0:

- Run a trade surplus at some point in the future, i.e. Not possible to have perpetual trade deficits.
- However, a perpetual (limited) current account deficit is possible even if $NFL_0 > 0$ ($\neq$ 2 periods). why?
  - you use trade surpluses to pay a fraction (income payment/ interests) so that the liabilities will grow but less than ①.

[sl. 16] In math:

$$TB_t = \varphi r NFL_{t-1} > 0 \quad \left[\begin{array}{l}\text{I use trade surpluses}\\ \text{to pay income payments}\end{array}\right]$$

and CA_t remains in deficit:

$$CA_t = \underbrace{TB_t - rNFL_{t-1}}_{\substack{\text{By definition}\\ CA_t = TB_t + iB_{t-1}}} = \varphi r NFL_{t-1} - rNFL_{t-1}$$

$$= -r(1-\varphi) NFL_{t-1} < 0$$

$r(1-\varphi) < r$: No Ponzi.

This is possible b/c NFL evolves as:

$$NFL_t = NFL_{t-1} - CA_t = (1 + r - \varphi r) NFL_t$$

$$\Delta NFL_t = \underbrace{r(1-\varphi)}_{} NFL_t$$

$\triangleright$ rate of growth debt and TB.

c) $NFL = 60 bln$ $r = 5\%$? $\overline{TB}$ | IBC satisfied. [See pg. 15]

$$60 \times 10^9 = \frac{TB}{1{,}05} + \frac{TB}{(1{,}05)^2} + \dots + \frac{TB}{(1{,}05)^\infty}$$

$$= TB \underbrace{\left[\frac{1}{1{,}05} + \dots \frac{1}{(1{,}05)^\infty}\right]}_{20}$$

$$\Rightarrow TB = \frac{60 \times 10^9}{20} \Rightarrow TB = 3 \times 10^9$$

$r = 8\%$

$$60 \times 10^9 = TB \underbrace{\left[\frac{1}{1{,}08} + \frac{1}{(1{,}08)^2} + \dots + \frac{1}{(1{,}08)^\infty}\right]}_{12.5}$$

$$\Rightarrow TB = \frac{60 \times 10^9}{12.5} = 48 \times 10^8$$

EXERCISE 4

$NFL_0 = -B_0 = 60$ $r = 0.10$. $T = 2$

a) $CA_1 = 24$ (surplus)

? TB_1, NFL_1, TB_2 | $B_2 = 0$

$\therefore B_1 - B_0 = CA_1 \rightarrow B_1 + 60 = 24 \Rightarrow NIIP_1 = +36$

Beginning of 2 Beginning of 1

$NFL_1 = +36$ [End period 1].

Positive CA only means NIIP has improved: in fact $\Delta NIIP$ is positive.

$B_2 = TB_2 + (1+r)B_1$: Formula on accumulation of assets in real terms. (7)

End period 2 End period 1 cf. 7

It's a debt position

$0 = TB_2 - (1,10)36 \Rightarrow TB_2 = 39{,}6$

$B_1 = TB_1 + (1+r)B_0$

$-36 = TB_1 - (1,10)60 \Rightarrow TB_1 = 30$ (6)

Check (11):

$$\frac{TB_1}{1+r} + \frac{TB_2}{1+r} = NFL_0 \rightarrow 60 = \frac{30}{1{,}10} + \frac{39{,}6}{(1{,}10)^2}$$

check:

$$CA_1 = TB_1 - rNFL_0$$

$$24 = -0.10 * 60 + TB_1 \Rightarrow TB_1 = 30$$

b) $r_2 = 0.20$. Find new CA_2 and TB_2

(7) $$0 = TB_2 + (1+r)B_1$$

$$0 = TB_2 - (1{,}20)36 \Rightarrow TB_2 = 43.2$$

Check: $$-66 = -30 - \frac{TB_2}{1{,}20}$$

$$CA_2 = TB_2 - rNFL_1 = 43.2 - 0.20 \cdot 36 = 36.$$

c) $CA_1 = -6$ (deficit)

? $TB_1, NFL_1, TB_2 \mid B_2 = 0$

$$CA_1 = TB_1 - rNFL_0 \rightarrow TB_1 = CA_1 + rNFL_0 \overset{!}{=} -6 + 0.10 \cdot 60 = 0$$

$$B_1 - B_0 = CA_1 \rightarrow B_1 + 60 = -6 \Rightarrow B_1 = -66 \Rightarrow NIIP_1 = 66$$

$$0 = TB_2 + 1.1(-66) \Rightarrow 72.6$$

B_2 has to be zero

Q5

$NFL_0 = -B = 45 \quad TB_1 = TB_2 = \dots = TB_{10} = -2 \quad r = 0.01$

a) From $t=11$, ? $\overline{TB}$ | IBC satisfied

$$NFL_0 = \frac{TB^1}{1,01} + \frac{TB^1}{(1,01)^2} + \dots + \frac{TB^1}{(1,01)^{10}} +$$

$$+ \frac{TB^{11}}{(1,01)^{11}} + \dots + \frac{TB^{11}}{(1,01)^{\infty}}$$

$$45 = -2 \Big[\overbrace{\frac{1}{1,01} + \frac{1}{(1,01)^2} + \dots \frac{1}{(1,01)^{10}}}^{9.47} \Big] +$$

$$TB^{11} \Big[\underbrace{\frac{1}{(1,01)^{11}} + \dots \frac{1}{(1,01)^{\infty}}}_{90.53} \Big]$$

$\Rightarrow TB^{11} = 0.706$

b) $t = 16$

$$45 = -2 \Big[\underbrace{\frac{1}{1,01} + \dots \frac{1}{(1,01)^{15}}}_{13..87} \Big] + TB^{16} \Big[\underbrace{\frac{1}{(1,01)^{16}} + \dots}_{86.13} \Big]$$

$\Rightarrow TB^{16} = 0.84$

c) ? $\overline{TB}$ | $NFL_{60} = 0$

$$B_{60} = TB_{60} + (1+r)\,TB_{59} + \dots (1+r)^{59}\,TB_1 + (1+r)^{60} B_0$$

$$B_{60} = TB \left[1 + (1+r) + \dots (1+r)^{59} \right] + (1+r)^{60} B_0$$

$$0 = TB * 81.67 - 45 * 1.82 \Rightarrow TB = \frac{45 * 1.82}{81,67} = 0.77$$

d) $r = 0.02$

$$45 = -2 \Big[\underbrace{\frac{1}{1.02} + \dots \frac{1}{(1.02)^{10}}}_{8.98} \Big] + TB \Big[\underbrace{\frac{1}{(1.02)^{11}} + \dots}_{41} \Big]$$

$$\Rightarrow TB = \frac{45 + 17.96}{41} = 1.53$$

The lower the interest rate, the easier to respect the Ponzi condition and therefore to lower the trade surplus to win.

A negative return on net foreign liabilities

- Consider the interest/income payments on net foreign liabilities $NFL_t = L_t - A_t$ where L are liabilities and A assets:

Return on net liabilities or net investment income (paid)

- Return $= i_L L_t - i_A A_t$ (1) net investment income paid
- Return $= i_L L_t - i_L A_t + i_L A_t - i_A A_t$ (2)
- Return $= i_L(L_t - A_t) - (i_A - i_L)A_t$ (3)

- The usual way to compute the average nominal rate of return on net liabilities is to divide the return by net foreign iabilities

Rate of return

- $i^{US} = \dfrac{\textbf{Return}}{L_t - A_t} = i_L - (i_A - i_L)\dfrac{A_t}{L_t - A_t}$ (4)

- The rate of return in eq. (4) is negative for

- $i_L - (i_A - i_L)\dfrac{A_t}{L_t - A_t} < 0$ (5)

A negative return on net foreign liabilities

- $i_L(1+\frac{A_t}{L_t - A_t}) < i_A \frac{A_t}{L_t - A_t}$ (6) $i_L \frac{L_t}{L_t - A_t} < i_A \frac{A_t}{L_t - A_t}$ (7)
- $i_A > i_L \frac{L_t}{A_t}$ (8) **Solution a)** Condition for $i^{US} < 0$

- Note that this condition can be obtained by the imposing a negative return in the equation shown in the Problem: Return $= i_L L_t - i_A A_t < 0$

Solution b)

- At the end of 2020 we had A_t =154% of GDP and $L_t - A_t$ = 67% and thus L_t = 154+67=220%.
- As a result the nominal rate i^{US} is negative for
  - $i_A > i_L \frac{221}{154} = i_L$ 1.435
- The rate of return on asset must be 43.5% higher than on liabilities

A negative return on net foreign liabilities

Solution b) Second Part

- A positive investment income equal to 1% of GDP with $i_L = 0.01$ obtains if
  - $i_A A_t - i_L L_t = i_A 154 - 0.01 \text{x} 221 = 1$
  - $i_A = \frac{3.21}{154}$

Solution c)

- If US financial integration increases, so that both A_t and L_t increase by 40%, then Investment Income increases by 40% as well:
  - Invest income(t+1)= $i_A A_t(1.4) - i_L L_t(1.4)$ =Invest income (t) x 1.4
- Note that the Liability position also increases by 40%, e.g. it increases from 221% of GDP to 309.4 of GDP

Stabilizing Trade Balance

In terms of GDP the Net Liability Position evolves as follows (see equation 3 in slide 10 of Lecture 27)

- $Nfl_t = (1 + i_t^{us} - g_t^N)Nfl_{t-1} - Tb_t - x_t - V_t$ (1)
- The trade balance, Tb_t, that stabilizes Nfl_t so that $\Delta Nfl_t = 0$ solves
  $\Delta Nfl_t = (i_t^{us} - g_t^N)Nfl_{t-1} - Tb_t - x_t - V_t = 0$ (2)
- Assuming zero valuation changes, $V_t = 0$, and net transfers plus labor income equal to $x_t = -0.7\%$ of GDP and Nfl_{t-1} =67% in 2020
- $Tb_t = (i_t^{us} - g_t^N)Nfl_{t-1} + 0.7 = i_t^{us}Nfl_{t-1} - g_t^N Nfl_{t-1} + 0.7$ (3)
- With a negative return on the liability position of $i_t^{us}Nfl_{t-1} = -1\%$ and a nominal growth $g_t^N = 0.044$ and Nfl_{t-1} =67% we have
- $Tb_t = -1 - 2.9 + 0.7 = -3.2$ % of GDP
- Absent valuation changes **a stable liability position would require the trade deficit not to exceed 3.2%**.

Stabilizing Trade Balance

Note that this result obtains because of the high nominal growth rate of GDP expected for 2021 as the economy bounces back from the Covid crisis

If GDP grew only by only 2% because of a slower recovery from the crisis, then $g_t^N Nfl_{t-1}$ would be $g_t^N Nfl_{t-1}$ = 0.02x67=1.34 and the trade balance needed for foreign liability stabilization would be

- $Tb_t = -1 - 1.3 + 0.7 = -1.6$ % of GDP

- For a stable liability position the trade deficit should not exceed 1.6%.
- It is worth noting that in 2020 the Trade deficit has reached 3.3%.
  If such a deficit remained in 2021 the liability position would grow even under favorable expectations of a nominal GDP growth of 4.4%

Data Appendix

US Balance of Payment and NIIP 2018-2020

	2019	2019	2020	2020	2019	2020
Exports goods and services	2 528 262		2 127 254		%GDP	%GDP
Imports goods and services	3 105 127		2 808 954			
Trade Balance - million		**-576 865**		**-681 700**	**-2.7**	**-3.3**
Investment income received	1 128 966		952 148			
Investment income paid	880 562		761 297			
Investment income net		**248 404**		**190 851**	**1.2**	**0.9**
Labor income received	6 725		6 166			
Labor income paid	18 785		15 443			
Labor income net		**-12 060**		**-9 277**	**-0.1**	**0.0**
Current transfer receipts	141 984		142 040			
Current transfer payments	281 689		289 124			
Current transfer net		**-139 705**		**-147 084**	**-0.7**	**-0.7**
Current Account Balance		**-480 226**		**-647 210**	**-2.2**	**-3.1**
	2018	**2019**	**2020**	**18 %GDP**	**19 %GDP**	**20 %GDP**
Foreign Assets - billion	25233.8	29152.8	32156.0	122.4	136.0	153.6
Foreign Liabilities - billion	34908.2	40203.3	46248.1	169.4	187.6	220.9
Net Investment Position	**-9 674**	**-11 051**	**-14 092**	**-47.0**	**-51.6**	**-67.3**
GDP billion	**2018**	20612	**2019**	21433	**2020**	20937