Lecture Notes:
International Trade I

Based on lectures by **Monika Mrázová(UNIGE)** in Spring semester, 2025

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These lecture notes were taken in the course *International Trade I* taught by **Monika Mrázová(UNIGE)** at Graduate Institute of International and Developoment Studies, Geneva as part of the International Economics program (Semester II, 2025).

Currently, these are just drafts of the lecture notes. There can be typos and mistakes anywhere. So, if you find anything that needs to be corrected or improved, please inform at `jingle.fu@graduateinstitute.ch`.

Contents

Lecture 1.

Comparative Advantage and Gains from Trade

1.1 Internatioanl trade: Standard Assumptions

What distinguished trade theory from general-equilibrium analysis is the exiatence of a **hierarchical market structure**.

- 'International' good markets
- 'Domestic' factor markets.

Question.

- How does the integration of good markets affect good prices?
- How do changes in good prices, in turn, affect factor prices, factor allocation, production and welfare?

While these assumptions are less fundamental, we will also often assume that:

Assumption 1.1.1.

- Consumers have identical homothetic preferences in each country (representative agent)
- Model is static.

1.2 Neoclassical trade

"Neoclassical trade theory" is characterized by three key assumptions:

1. **Perfect competition** in all markets
2. **Constant returns to scale** in production
3. **No distortions**

Note.
Increasing returns to scale (IRS) are a much more severe issue addressed by "New" trade theory

Let's first stick to the general case and show how simple revealed preference arguments can be used to establish two important results:

1. *Gains from trade*(Samuelson 1939)
2. *Law of comparative advantage*(Deardorff 1980)

1.2.1 Basic environment

Consider a world economy with $n = 1, 2, \cdots, N$ countries, each populated by $h = 1, \cdots, H$ households. There are $g = 1, \cdots, G$ goods:

- $y^n \equiv (y_1^n, \cdots, y_G^n) \equiv$ Output vector in country n
- $c^{nh} \equiv (c_1^{nh}, \cdots, c_G^{nh}) \equiv$ Consumption vector of household h in country n
- $p^n \equiv (p_1^n, \cdots, p_G^n) \equiv$ Price vector in country n

There are $f = 1, \cdots, F$ factors:

- $v^n \equiv (v_1^n, \cdots, v_F^n) \equiv$ Factor endowment vector in country n
- $w^n \equiv (w_1^n, \cdots, w_F^n) \equiv$ Factor price vector in country n

Supply side

We denote by Ω^n the set of combinations (y, v) feasible in country n, our assumption of constant returns to scale implies that Ω^n is a convex set.

Definition 1.2.1 (Revenue function).

The **revenue function** in country n of a firm producing output y using factors v is a function $r^n(y, v)$ such that:

$$r^n(y, v) \equiv \max_y \{py | (y, v) \in \Omega^n\} \tag{1.1}$$

Note (see Dixit-Norman pp. 31-36 for details).

- Revenue function summarizes all relevant properties of technology;
- Under perfect competition, y^n maximizes the value of output in country n:

$$r^n(p^n, v^n) = p^n y^n.$$

Demand side

We denote by u^{nh} the utility function of household h in country n.

Definition 1.2.2 (Expenditure function).

The **expenditure function** of household h in country n is a function $e^{nh}(p^n, u^{nh})$ such that:

$$e^{nh}(p, u) \equiv \min_c \{pc | u^{nh}(c) \geq u\} \tag{1.2}$$

Note (see Dixit-Norman pp. 59-64 for details).

- Here factor endowments are in fixed supply, but easy to generalize to case where households choose factor supply optimally
- Holding p fixed, $e^{nh}(p, u)$ is increasing in u.

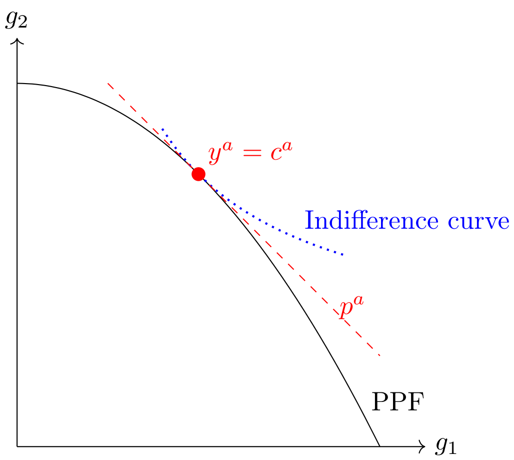
- Household's optimization implies:

$$e^{nh}(p^n, u^{nh}) = p^n c^{nh}$$

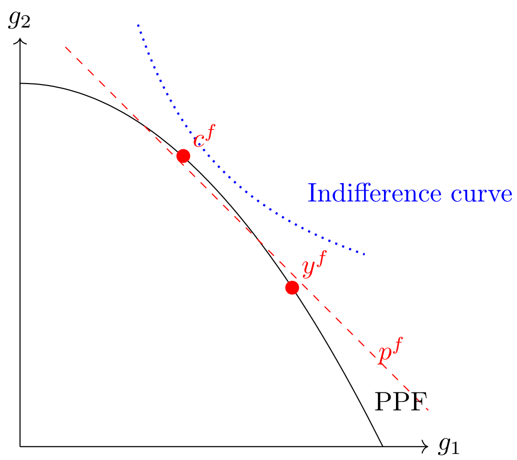
where c^{nh} and u^{nh} are the consumption and utility level of the household h in country n in equilibrium, respectively.

1.2.2 Gains from Trade

In the next propositions, when we say "in a neoclassical trade model", we mean in a model where equations (1.1) and (1.2) hold in any equilibrium.



(a) Autarky Equilibrium



(b) Free Trade Equilibrium

Figure 1.1: Equilibria for a Small Country

Formula of Gains from Trade

Arkolakis, Costinot, Rodriguez-Clare (AER, 2012)

Assumption 1.2.1.

- CES utility function(Dixit-Stiglitz)

$$U = \left[\int q(\omega)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$

- One factor production(labor) and constant RS
- 'Iceberg' trade costs
- Import demand system is CES: artial equilibrium (given wages) elasticity of aggregate bilateral trade flow relative to domestic demand is ε w.r.t. trade costs τ_{ij} fo any i, j.

Define a "foreign shock" as any change in (foreign) endowments and trade costs that do not affect a country's endowment or its ability to serve its own market. Define $\hat{W} = \frac{W'}{W}$ and $\hat{\lambda}_{jj} = \frac{\lambda'_{jj}}{\lambda_{jj}}$

Proposition 1.2.1. The change in country j's real income associated with any foreign shock can be computed as $\hat{W}_j = \hat{\lambda}_{jj}^{\frac{1}{\varepsilon}}$, where λ_{ij} is the share of country j's spending on country i's goods.

Corollary 1.2.1. Gains from TRade relative to autarky can be computed as $\hat{W}_j = \lambda_{jj}^{-\frac{1}{\varepsilon}}$.

One household per country

Consider first the case where there is just one household per country, $H = 1$. Without risk of confusion, we drop h and n from all variables.

We denote by:

- (y^a, c^a, p^a) the vector of output, consumption and good prices under autarky;
- (y^f, c^f, p^f) the vector of output, consumption and good prices under free trade.
- u^a and u^f the utility levels under autarky and free trade.

Proposition 1.2.2. *In a neoclassical trade model with one household per country, free trade makes all households (weakly) better off.*

Proof.

Under free trade, households can consume at prices p^f. By definition of the expenditure function, we have:

$$\begin{aligned} e(p^f, u^a) &\leq p^f c^a \\ &= p^f y^a \\ &\leq r(p^f, v^f) \\ &= e(p^f, u^f) \end{aligned}$$

Since $e(p, \cdot)$ is increasing, we get $u^f \geq u^a$. □

Note.

- Two inequalities in the previous proof correspond to consumption and production gains from trade.
- Previous inequalities are weak. Equality if kinks in IC or PPF.
- Previous proposition only establishes that households always prefer "free trade" to "autarky." It **does not** say anything about the comparisons of trade equilibria.

Multiple households per country: domestic lump-sum transfers

With multiple households per country, moving away from autarky is likely to create winners and losers. In order to establish the Pareto-superiority of trade, we will need to allow for **policy instruments**. We start with *domestic lump-sum transfers* and then *commodity taxes*.

We now reintroduce the index h and denote by:

- c^{ah} and c^{fh} the consumption vectors of household h under autarky and free trade;
- v^{ah} and v^{fh} the endowment vectors of country h under autarky and free trade;
- u^{ah} and u^{fh} the utility levels of household h under autarky and free trade;
- τ^h the lump-sum transfer from the government to household h under free trade. [1]

Proposition 1.2.3.

In a neoclassical trade model with multiple households per country, there exist domestic lump-sum transfers such that free trade is (weakly) Pareto superiority than autarky in all countries.

Proof.

For any h, set the lump-sum transfer τ^h such that:

$$\tau^h = (p^f - p^a)c^{ah} - (w^f - w^a)v^{fh}.$$

Budget constraint unde rautarky implies that: $p^a c^{ah} \leq w^a v^{fh}$. Therefore, we have:

$$p^f c^{ah} \leq w^f v^{fh} + \tau^h.$$

Thus c^{ah} is still int he budget set of household h under free trade.

By definition, the government revenue is given by:

$$\begin{aligned}
-\sum \tau^h &= (p^a - p^f)\sum c^{ah} - (w^a - w^f)\sum v^{fh} \\
&= (p^a - p^f)y^a - (w^a - w^f)v^f \\
&= -p^f y^a + w^f v^f \\
&\geq -r(p^f, v^f) + w^f v^f \\
&= -(p^f y^f - w^f v^f) = 0.
\end{aligned}$$

□

So, each household can buy its autarky consumption bundle at free trade prices and still have some money left. But, the government must know individual preferences to implement the transfers.

If it does not, households can manipulate mechanism by altering their announcements or autarky behavior. In other words, lump-sum transfers typically are not incentive compatible.

Multiple households per country: commodity taxes

We now restrict the set of instruments to commodity taxes/subsidies.

Suppose that the government can affect the porices faced by households under free trade by setting τ^{good} and τ^{factor}:

$$\begin{aligned}
p^h &= p^f + \tau^{good} \\
w^h &= w^f + \tau^{factor}.
\end{aligned}$$

[1] $\tau^h \leq 0 \Leftrightarrow$ lump-sum tax and $\tau^h \geq 0 \Leftrightarrow$ lump-sum subsidy.

Proposition 1.2.4.

In a neoclassical trade model with multiple households per country, there exist commodity taxes/subsidies such that free trade is (weakly) Pareto superior to autarky in all countries.

Proof.

Consider two following taxes:

- $\tau^{good} = p^a - p^f$
- $\tau^{factor} = w^a - w^f$

By construction, household is indifferent between autarky and free trade. Now consider the government revenue:

$$\begin{aligned}
-\sum \tau^h &= \sum \tau^{good} c^{ah} - \sum \tau^{factor} v^{fh} \\
&= (p^a - p^f)\sum c^{ah} - (w^a - w^f)\sum v^{fh} \\
&= (p^a - p^f)y^a - (w^a - w^f)v^f \\
&= -p^f y^a + w^f v^f \\
&\geq -r(p^f, v^f) + w^f v^f \\
&= -(p^f y^f - w^f v^f) = 0.
\end{aligned}$$

□

Tax revenue is non-negative. If all households are on the same side of the market for at least for at least one good ot factor, government can cut a tax or raise a subsidy to generate Pareto improvement.

This scheme sacrifices the consumer gains from trade, but preserves the gains from reorganizing production.

Note.

The previous proof only relies on existence of *production gains* from trade.

- It's closely related to Diamod and Mirrlees (1971) result on the production efficiency
- When only commodity taxes are available, DM sho that production should remain efficient at a social optimum
- Thus, trade, acting as an expansion of PPF, should remain free(ignore issues of market power)

But, if there's a kink in the PPF, there are no production gains. [a]

Factor taxation still informationally intensive: need to know endowments in efficiency units, may lead to different business taxes

[a]Similar problem with "moving costs", see Feenstra p. 185

1.2.3 Law of Comparative Advantage

The previous results have focused on normative predictions. Let's take a look at how the same revealed preference argument can be used to make positive predictions about patterns of trade.

Theorem 1.2.1 (Law of Comparative Advantage).

Countries tend to export goods in which they have a CA, i.e. lower relative autarky prices compared to other countries

Lecture 2.

Ricardian Model

2.1 Introduction

2.1.1 Reasons of trade

- **Countries' differences: comparative advantage**
  - Productivity: Ricardo
  - Endowments: Heckscher-Ohlin
- **Countries' similarities: economies of scale**

2.1.2 Comparative advantage

Note.

Stanislaw Ulam's challenge to Paul Samuelson: "name me one proposition in all of the social sciences which is both true and non-trivial".

Samuelson's answer: Comparative advantage. "That it is logically true need not be argued before a mathematician; that is is not trivial is attested by the thousands of important and intelligent men who have never been able to grasp the doctrine for themselves or to believe it after it was explained to them.".[a]

[a]P.A. Samuelson (1969), "The Way of an Economist," in P.A. Samuelson, ed., International Economic Relations: Proceedings of the Third Congress of the International Economic Association, Macmillan: London, pp. 1-11.

In the simplest and earliest complete model of production and trade, the reason for trade is **comparative advantage**. And the source of comparative advantage is **differences in production technologies**.

These are the differnces in production functions, not the differences in labor productivities due to different endowments of capital which is the type of Heckscher-Ohlin model.

2.1.3 Basic assumptions

- Labor is the only factor of production
- Constant returns to scale
- Perfect competition
- Full employment
- Endowments given, confined to country but intersectorally mobile within each country
- Two countries with different technologies (production functions)
- Number of goods: $n \geq 2$

2.1.4 Technology

Question. How is efficiency measured?

With only one factor, we measure by the usage of the factor: **labbor requirement per unit output**.

We define the quantity of labour (e.g. number of hours) necessary to produce one unit of good i as a_i. If $a_i = 2$, it means that 2 hours of labor are needed to produce one unit of good i.

So, the labor productivity is the inverse of the labor requirement a_i, the higher the labor requirement, the lower the productivity.

2.1.5 Production Possibility Frontier: Refresher

PPF is the maximum possible production level for a given technology and factor endowment.

- Constraint: points outside the PPF are not feasible;
- Efficiency: points inside the PPF are inefficient, those on the PPF are efficient;
- Opportunity cost: the slope of the PPF is the opportunity cost of producing one more unit of good i in terms of good j.
- Concavity: the PPF is usually concave to the origin due to the law of diminishing returns.

Technology progress shifts the PPF outwards.

2.2 The two-sector Model

2.2.1 A Simple Numerical Example

Let's begin with a simple example: US and India producing corn and auto.

	US	India
Labor force	$L = 200$	$L^* = 800$
Labor per unit corn	$a_c = 8$	$a_C^* = 50$
Labor per unit auto	$a_A = 10$	$a_A^* = 40$

Specific assumption of this example : US is more efficient/productive at producing both goods, meaning US has an absolute advantage in both goods.

Production Possibility Frontier

US

The full-employment condition is:

$$L = a_c Q_c + a_A Q_A$$

The PPF is a straight line connecting these two points:

$$Q_A = \frac{L}{a_A} - \frac{a_C}{a_A} Q_C$$

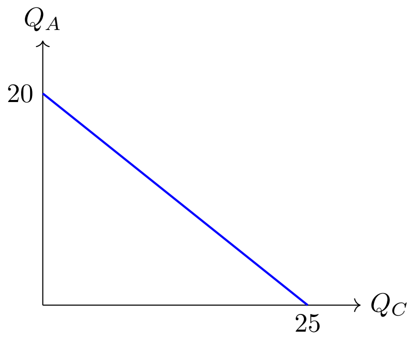


Figure 2.1: US PPF

India

The full-employment condition is:

$$L^* = a_C^* Q_C + a_A^* Q_A$$

The PPF is a straight line connecting these two points:

$$Q_A^* = \frac{L^*}{a_A^*} - \frac{a_C^*}{a_A^*} Q_C$$

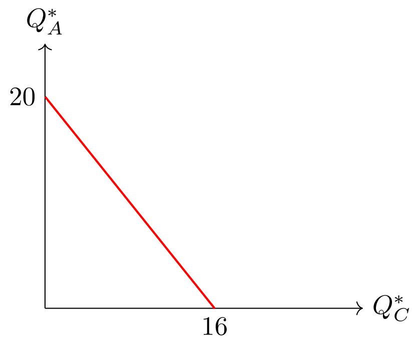


Figure 2.2: India PPF

Relative price and Technology

> **Notation.**
> Relative price of corn: $P = \frac{P_C}{P_A}$;
> Relative quantity of corn: $Q = \frac{Q_C}{Q_A}$;
> L and L^* are the endowments of labor(in efficient units) in the two countries;
> w and w^* are the wage(in efficiency units) in the two countries.

Workers are paid the value of their marginal products: $w_C = \frac{P_C}{a_C}$ and $w_A = \frac{P_A}{a_A}$.

Perfectly mobile labor: If both goods are produced, the wage in both sectors must be the same.

$$w_C = w_A \Leftrightarrow P = \frac{a_C}{a_A}.$$

Production and Supply in the US

If $P = \frac{a_C}{a_A} = 0.8$, the international price(relative price) is equal to the domestic opportunity cost of producing corn in terms of auto. Trade cannot bring extra gains. Hence the production set doesn't affect the consumption set, and the production set can be anywhere on the PPF. $1 \leq \frac{Q_c}{Q_A} < \infty$.

If $P < \frac{a_C}{a_A} = 0.8$, international price of C is lower than the domestic opportunity cost of producing C in terms of A. US will produce only A and import C. $Q_C = 0$, $Q_A = 20$, and $\frac{Q_C}{Q_A} = 0$.

If $P > \frac{a_C}{a_A} = 0.8$, international price of C is higher than the domestic opportunity cost of producing C in terms of A. US will produce only C and export A. $Q_A = 0$, $Q_C = 25$, and $\frac{Q_C}{Q_A} = \infty$.

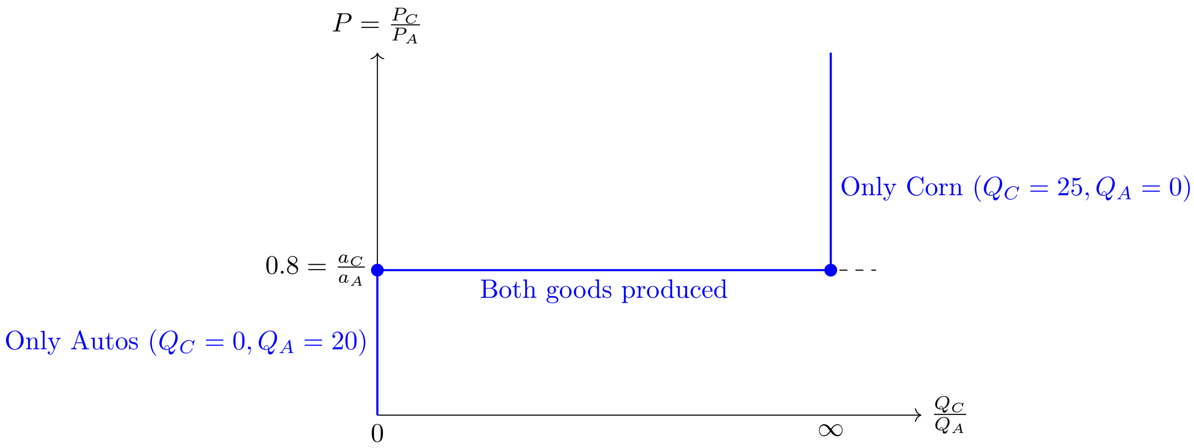


Figure 2.3: US Relative supply

Autarky equilibria in the two countries

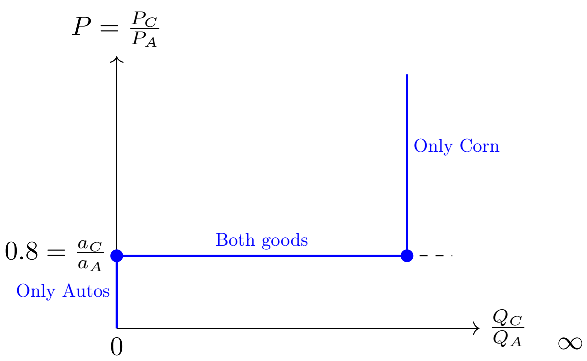


Figure 2.4: US Relative Supply

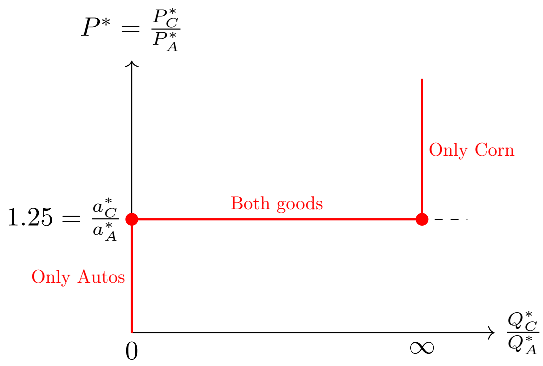


Figure 2.5: India Relative Supply

Trade: Relative supply

When we combine the two countries to form the world market, we get the following relative supply curve:

- When $P < 0.8$, both countries produce only autos, world ratio $R = 0$.

- When $P = 0.8$, US can vary production between 20 autos, no corn (world $R = 0$), and 0 autos, 25 corn (world $R = 1.25$) while India produces only autos, so world $R \in [0, 1.25]$.

- When $0.8 < P < 1.25$, US produces only corn, India produces only autos, so $R = \frac{25}{20} = 1.25$.

- When $P = 1.25$, India can vary production while US produces only corn, so world $R \in [1.25, \infty)$.

- When $P > 1.25$, both countries produce only corn, so world $R = \infty$.

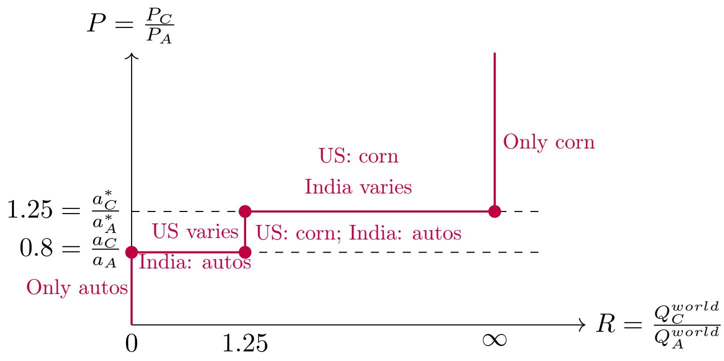


Figure 2.6: World Relative Supply Curve

Trading Equilibrium

Depending on position of relative demand, the trading equilibrium can be one of three types:

A. $P = 0.8$, US produces both goods, India produces only autos;

B. $0.8 < P < 1.25$, US produces only corn, India produces only autos;

C. $P = 1.25$, US produces only corn, India produces both goods.

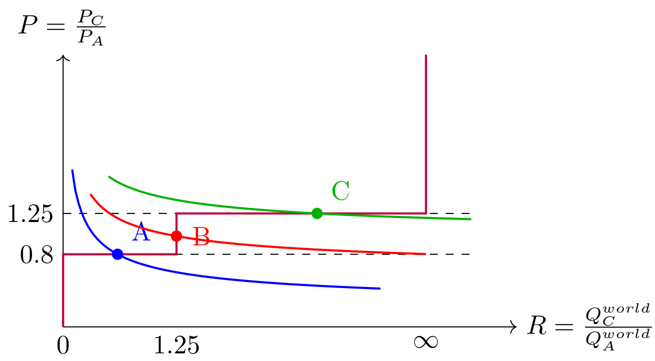


Figure 2.7: Trading Equilibrium

Efficient production and world PPF

Suppose initially all labor produc autos in both: 40 in all. If any corn is produced, it's better to do so by switching sone labor in the US: because wach auto not produced releases 10 labor which can then produce 1.25 corn, while in India, each less auto yields only 0.8 more corn.

Only when all US labor has been diverted to producing corn should any Indian labor be switched.

Conversely, starting with all corn: 41 units, to produce any autos, Indian labor should be switched.

This despite the US producing autos more efficiently than India: only 10 units of labor against 40. The reason: the US produces corn even more efficiently: only 8 units of labor against 50. What matters is the ratio (opportunity cost): $\frac{10}{8} > \frac{40}{50}$, or $\frac{10}{40} > \frac{8}{50}$.

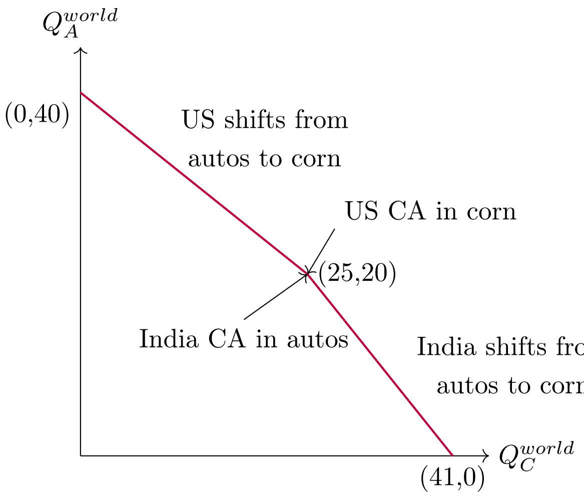


Figure 2.8: World Production Possibility Frontier

TRading equilibrium and world PPF

If preferences are identical and homthetic everywhere, we can draw the world indifference curves. Depending on their shape, there are three types of outcomes:

A. US produces both goods, India produces only autos;

B. US produces only corn, India produces only autos;

C. US produces only corn, India produces both goods.

Relative price of corn = slope of PPF: 0.8 in A, 1.25 in C, between 0.8 and 1.25 in B.

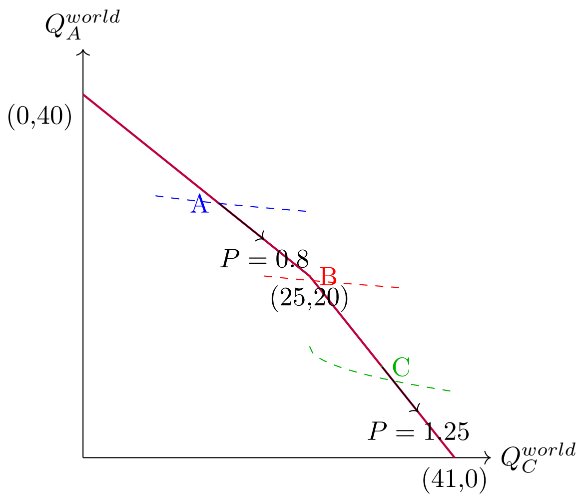


Figure 2.9: Trading Equilibrium on the World PPF

2.3 Dornbusch, Fischer, and Samuelson 1977: Continuum of Goods

Consider a world economy with only two countries: Home and Foreign. Use asterisks to denote foreign variables.

Note.

Ricardian models differ from other neoclassical trade models in that there is only **one factor of production** in the model.

We denote by:

- L and L^* as the labor endowments in Home and Foreign respectively.
- w and w^* as the wages(in efficiency units) in Home and Foreign respectively.

There's a continuum of goods indexed by $z \in [0, 1]$. Since there are CRS, we can define the constant unit labot requirements in both countries: $a(z)$ and $a^*(z)$.

Without loss of generaliry, we order goods such that $A(z) \equiv \frac{a^*(z)}{a(z)}$ is decreasing.

- Hence Home has a comparative advantage in goods with low-z goods.
- For simplicity, we'll assume strict monotonicity.

Previous supply-side assumptions are all we need to make qualitative predictions about pattern of trade.

Let $p(z)$ denote the price of good z in Home under free trade.

The profit-maximization condition for a firm producing good z is:

$$p(z) - w\,a(z) \leq 0, \text{with equality if z produced at home.}$$
$$p(z) - w^*\,a^*(z) \leq 0, \text{with equality if z produced abroad.}$$

Proposition 2.3.1.

There exists $\tilde{z} \in [0, 1]$, s.t. Home produces all goods with $z < \tilde{z}$ and Foreign produces all goods with $z > \tilde{z}$.

Proof.

By contradiction. Suppose that there exists $z' < z$, s.t. z produced at Home and z' is produced abroad. Then:

$$\begin{aligned} p(z) - w\,a(z) &= 0, \\ p(z') - w\,a(z') &\leq 0, \\ p(z') - w^*\,a^*(z') &= 0, \\ p(z) - w^*\,a^*(z) &\leq 0. \end{aligned}$$

This implies that:

$$w\,a(z)w^*\,a^*(z) = p(z)\,p(z') \leq w\,a(z')\,w^*\,a^*(z),$$

which can be rearranged as:

$$\frac{a^*(z')}{a(z')} \leq \frac{a^*(z)}{a(z)}$$

which contradicts that $A(z)$ is strictly decreasing. □

This proposition(2.3.1) simply shows tha tHome should produce and specialize in the goods in which it has a CA.

Note.

- Proposition(2.3.1) doesn't rely on continuum of goods.
- Continuum of goods + continuity of $A(z)$ is important to derive the relative wage:

$$A(\tilde{z}) = \frac{w}{w^*} \equiv \omega. \tag{2.1}$$

Equation(2.1) is the key result of the Dornbusch, Fischer and Samuelson(1997) model. It gives us that: conditional on wages, goods should be produced in the country where it's cheaper to do so.

Then, we take a look at the demand side to pin down the relative wage ω.

2.3.1 Demand side

Assume that consumers have identical Cobb-Douglas preferences around the world.

We denote by $b(z) \in (0,1)$ the share of expenditure spent on good z:

$$b(z) = \frac{p(z)c(z)}{wL} = \frac{p(z)c^*(z)}{w^*L^*}$$

where $c(z)$ and $c^*(z)$ are the consumption of good z in Home and Foreign respectively.

By definition, share of expenditure satisfies: $\int_0^1 b(z)dz = 1$.

Then, we denote by $\theta(\tilde{z}) = \int_0^{\tilde{z}} b(z)dz$ the fraction of income spent on goods produced at Home(spent in both countries).

The trade balance requires:

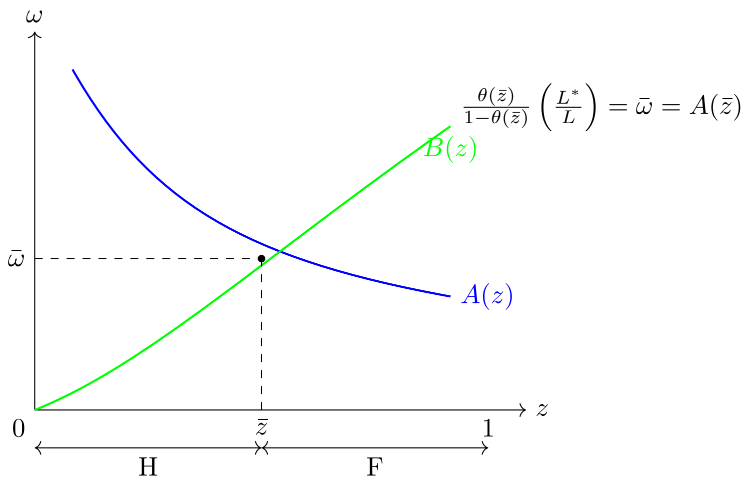
$$\theta(\tilde{z})w^*L^* = [1 - \theta(\tilde{z})]wL$$

where LHS od the Home exports an d RHE is the Home imports.

WE can then rearrange the equation(2.1) to get:

$$\omega = \frac{\theta(\tilde{z})}{1-\theta(\tilde{z})}\left(\frac{L^*}{L}\right) \equiv B(\tilde{z}). \tag{2.2}$$

Note that $B' > 0$ and: an increase in $\tilde{z}$ leads to a trade surplus at Home, which must be compensated by an increase in Home's relative wage ω.



- Efficient international specialization, Equation (3), and trade balance, (4), jointly determine $(\bar{z}, \bar{\omega})$

Since Ricardian model is a neoclassical model, general results derived in previous lecture hold. However, we can directly show the existence of gains from trade in this environment.

- Set $w = 1$ under autarky and free trade.
- Indirect utility[1] of Home representative houssehold depends on $p(\cdot)$.
- For goods z produced at Home under free trade: no change sompared to autarky.
- For goods z produced Foreign under free trade:

$$p(z) = w^* a^*(z) < a(z)$$

- Since all prices go down, indirect utility must go up.

2.3.2 Consequences of (Relative) Country Growth

Suppose that L^*/L increases, then the relative wage ω must increase and $\tilde{z}$ goes down. At initial wages, an increase in L^*/L leads to a trade deficit in Foreign, which must be compensated by an increase in ω. An increase in L^*/L raises indirect utility, i.e. real wage, of representative household at Home and lowers it in Foreign.

- We set $w = 1$ before and after the change in L^*/L.
- For goods z whose production remains at Home, there's no change in $p(z)$.
- For goods z whose production remains in Foreign, we have: $w \uparrow \Rightarrow w^* \downarrow \Rightarrow p(z) = w^* a^*(z) \downarrow$.
- For goods z whose production changes from Home to Foreign, we have: $p(z) = w^* a^*(z) \leq a(z) \Rightarrow p(z) \downarrow$.
- So Home gains. Similar logic implies Foreign welfare loss.

> **Note.**
>
> In spite of CRS at the industry level, everything is as if we had DRS at the country-level.
>
> As foreing's size increases, it specialized in sectors in which it is relatively less productive(compared to home), which worsens it's terms of trade, so lowers real GDP per capita.
>
> The flatter the A schedule, the smaller this effect.

If there's an improvement in Home technology, then unit of labor requirement in Home $a(z)$ will be reducing, hence $A(z)$ is increasing - shifting upward. As we have shown in 2.1, this shows **an increase in home relative wage** i.e. higher income, and **an increase in home's share of world production and trade.**[2]

[1] The maximum utility under budget constraint.

[2] As shown previously, $\omega = \frac{\theta(\tilde{z})}{1-\theta(\tilde{z})}\left(\frac{L^*}{L}\right)$, if ω increases, and we fix $\frac{L^*}{L}$, then $\theta(\tilde{z})$ increases.

2.4 J. Eaton and Kortum 2002: Many Countries

The model of Dornbusch, Fischer, and Samuelson 1977 is based on the Ricardian model where trade and specialization patterns are determined by different productivities.

In the Dornbusch-Fischer-Samuelson model, there were no restrictions on the productivity distributions, Eaton and Kortum were able to extent the model to a case with many countries by sacrificing this generality.

The Eaton-Kortum model is based on the following assumptions:

- Perfect Competition
- N countries, insteat of 2 in DFS
- Continuum of goods ($u \in [0, 1]$)
- CRS technology: labor only
- CES preferences with elasticity of substitution $\sigma > 0$

2.4.1 Model Setup

The world

In this model, there a finite number of countries $i \in S \equiv \{1, \cdots, N\}$; (unlike previous models, there are technical difficulties in extending the model to a continuum of countries). There are a continuum of goods Ω.

However, unlike Krugman 1980 and M. J. Melitz 2003, models which follow(in Lecture 4 and 5), *every country is able to produce every good.* Countries, however, varies(exogeneously) in their productivity of each good $z_i(\omega)$ - varying by country i and good $\omega \in \Omega$.

The J. Eaton and Kortum 2002 has no concept of a firm. Instead, it is assumed that all goods ω are are produced using the same bundle of inputs with a constant returns to scale technology. Let the cost of a bundle of inputs in country i be c_i, so that the cost of producing one unit of $\omega \in \Omega$ in country i is $\frac{c_i}{z_i(\omega)}$.

Firstly, we introduce the **trade costs**: transport costs and tariffs. This is not an innovation of Eaton and Kortum, the original Dornbush-Fischer-Samuelson model also had trade costs, but for simplification, we left them out.

Trade costs are modelled as iceberg costs, a popular device invented by SamuelsonSamuelson 1954. That is, we assume that for delivering one unit of a good from country i to country j, it's necessary to ship $d_{ij} \geq 1$ units of good: the rest "melts away" during transit.

Supply

Each good is assumed to be sold in perfectly competitive markets, so that the price a consumer in country j would pay for good ω produced in country i is given by:

$$p_{ij}(\omega) = \frac{c_i}{z_i(\omega)} d_{ij}.$$

However, consumers in country $j \in S$ are assumed to only purchase good $\omega \in \Omega$ from the country who can provide it at the lowest price, so the price a consumer in $j \in N$ actually pays for good ω is:

$$p_j(\omega) = \min_{i \in S} p_{ij}(\omega) = \min_{i \in S} \left\{ \frac{c_i}{z_i(\omega)} d_{ij} \right\}$$

Here, the basic idea of Eaton and Kortum is already present in equation above: a country $j \in S$ is more likely to purchase a good $\omega \in \Omega$ from country $i \in S$ if

(1) it has a lower cost c_i;

(2) it has a higher good productivity $z_i(\omega)$;

(3) it has a lower trade cost d_{ij}.

The main innovation of Eaton and Kortum, however, is to assume that the productivities $z_i(\omega)$ are drawn from a Fréchet distribution[3]. This means that for each $i \in S$ and $\forall \omega \in \Omega$, the cumulative distribution function F_i is:

$$F_i(z) = \Pr\left\{ z_i(\omega) \leq z \right\} = \exp\left\{ -T_i z^{-\theta} \right\}$$

where $T_i > 0$ is a measure of the aggregate productivity of country i(country i's state of technology), and $\theta > 1$, which measures teh intra-industry heterogrneity, idiosyncratic variation in know-how across goods $\rightarrow$ drive intra-industry trade(comparative advantage).[4].

- High θ means less variability and less intra-industry trade;
- It's important to assume that θ is common across industries;
- θ guides the impact of changes in fundamental productivity on aggregate trade flows.

Note (Why Fréchet?).

Why make this particular distributional assumption for productivities?

Kortum 1997 showed that if the technology of producing goods is determined by the best "idea" of how to produce, then the limiting distribution is indeed Fréchet, where T_i reflects the country's stock of ideas. More generally, consider the random variable:

$$M_n = \max\{X_1, \cdots, X_n\}$$

where X_n are i.i.d.

The Fisher-Tippett-Gnedenko theorem states that the only (normalized) distribution of M_n as $n \rightarrow \infty$ is an extreme value distribution, of which Fréchet is one of three types (Type II). Note that a conditional logit model assumes that the error term is Gumbel (Type I) extreme value distributed.

If random variable x is Gumbel distributed, then $\ln x$ is Fréchet distributed. Hence, loosely speaking, the Fréchet distribution works better for models that are log linear (like the gravity equation), whereas the Gumbel distribution works better for models that are linear.

[3] To find more properties of the Fréchet distribution, see the appendix 5.1.3.

[4] A larger value of T_i decreases F_i fro any $z \geq 0$. It increases the probability of larger values of z and $\theta > 1$, which is assumed to be constant across countries, governs the distribution of productivities across goods within countries. As θ increases, the heterogeneity of productivity across goods declines.

Demand

As in previous models, consumers have CES preferences so that the representative agent in country j has utility:

$$U_j = \Big(\int_\Omega q_j(\omega)^{\frac{\sigma-1}{\sigma}} d\omega\Big)^{\frac{\sigma}{\sigma-1}},$$

where $q_j(\omega)$ is the quantity that country j consumes of good ω.

Note that unlike the Krugman (1980)Krugman 1980 model, not every good produced in every country will be sold to country j.

Indeed, good $\omega \in \Omega$ will be produced by all countries but country j will only purchase it from one country. The CES preferences will yield a Dixit-Stiglitz price index:

$$P_j \equiv \Big(\int_\Omega p_j(\omega)^{1-\sigma}\Big)^{\frac{1}{1-\sigma}}.$$

2.4.2 Equilibrium

We now consider the equilibrium of the model. Instead of relying on the CES demand equation as in the previous models, we use a probabilistic formulation in order to solve the model.

Prices

In perfect competition only the lowest cost producer of a good will supply that particular good. Thus, we want to derive the distribution of the minimum price over a set of prices offered by producers in different countries:

$$p_j = \min\{p_{ij}, \cdots, p_{Nj}\}.$$

In order to solve the model, we take advantage of the properties of the Fréchet distribution.

First, we consider the probability that country $i \in S$ is able to offer country j good ω for a price less than p. Because the technology is i.i.d. across goods, we can define:

$$G_{ij}(p) \equiv \Pr\Big\{p_{ij}(\omega) \le p\Big\}$$

Using the perfect competition price equation(2.4.1), we can write:

$$\begin{aligned} G_{ij}(p) &= \Pr\Big\{\frac{c_i}{z_i(\omega)} d_{ij} \le p\Big\} \\ &= 1 - \Pr\Big\{z_i(\omega) \ge \frac{c_i d_{ij}}{p}\Big\} \\ &= 1 - F_i\Big(\frac{c_i d_{ij}}{p}\Big) \\ &= 1 - \exp\Big\{-T_i\Big(\frac{c_i d_{ij}}{p}\Big)^{-\theta}\Big\} \end{aligned}$$

Consider now the probability that country j pays a price less than p for good ω.

Again, because the technology is i.i.d. across goods, the probability will be tha same fo all goods $\omega \in \Omega$. Define:

$$G_j(p) \equiv \Pr\Big\{p_j(\omega) \le p\Big\} \tag{2.3}$$

Because country j buys from the least cost provider, using equation 2.4.1, we can write:

$$\begin{aligned}
G_j(p) &= \Pr\Big\{\min_{i\in S} p_{ij}(\omega) \le p\Big\} \\
&= 1 - \Pr\Big\{\min_{i\in S} p_{ij}(\omega) \ge p\Big\} \\
&= 1 - \Pr\Big\{\cap_{i\in S}\Big(p_{ij}(\omega) \ge p\Big)\Big\} \\
&= 1 - \prod_{i\in S}\Pr\Big\{p_{ij}(\omega) \ge p\Big\} \\
&= 1 - \prod_{i\in S}\Big(1 - G_{ij}(p)\Big)
\end{aligned}$$

Substituting G_{ij} into this equation, we have:

$$\begin{aligned}
G_j(p) &= 1 - \prod_{i\in S}\Big(1 - \Big(1 - \exp\Big\{-T_i\Big(\frac{c_i d_{ij}}{p}\Big)^{-\theta}\Big\}\Big)\Big) \\
&= 1 - \prod_{i\in S}\exp\Big\{-T_i\Big(\frac{c_i d_{ij}}{p}\Big)^{-\theta}\Big\} \\
&= 1 - \exp\Big\{-p^\theta \sum_{i\in S} T_i\Big(c_i d_{ij}\Big)^{-\theta}\Big\} \\
&= 1 - \exp\Big\{-p^\theta \Phi_j\Big\}
\end{aligned}$$

where $\Phi_j = \sum_{i\in S} T_i(c_i d_{ij})^{-\theta}$.

This tells us the distribution of prices across goods in country j. This will further allow us to calculate the price index P_j for country j.

$$\begin{aligned}
P_j &= \Big(\int_\Omega p_j(\omega)^{1-\sigma}\Big)^{\frac{1}{1-\sigma}} \\
\Leftrightarrow P_j^{1-\sigma} &= \int_0^\infty p^{1-\sigma} dG_j(p) \\
&= \int_0^\infty p^{1-\sigma}\Phi_j \exp\Big\{-p^\theta \Phi_j\Big\}dp \\
&= \theta\Phi_j \int_0^\infty p^{\theta-\sigma}\exp\Big\{-p^\theta\Phi_j\Big\}dp.
\end{aligned}$$

Define $x = p^\theta \Phi_j$, so $dx = \theta\Phi_j p^{\theta-1}dp$,with a change of variable, we have:

$$\begin{aligned}
P_j^{1-\sigma} &= \int_0^\infty \Big(\frac{x}{\Phi_j}\Big)^{\frac{1-\sigma}{\theta}} \exp\{-x\}dx \\
&= \Phi_j^{-\frac{1-\sigma}{\theta}} \int_0^\infty x^{\frac{1-\sigma}{\theta}} \exp\{-x\}dx \\
&= \Phi_j^{-\frac{1-\sigma}{\theta}} \Gamma\Big(\frac{1-\sigma}{\theta}+1\Big) \\
\Leftrightarrow P_j &= \Phi_j^{-\frac{1}{\theta}} \Gamma\Big(\frac{\theta+1-\sigma}{\theta}\Big)^{\frac{1}{1-\sigma}}
\end{aligned}$$

where $\Gamma(x) = \int_0^\infty x^{t-1}e^{-x}dx$ is the gamma function.

Hence, the equilibrium price index for a CES utililty with elasticity of substitution $\sigma < 1+\theta$ in country $j \in N$ can be written as:

$$P_j = \gamma\Big(\sum_{i\in S} T_i(c_i d_{ij})^{-\theta}\Big)^{-\frac{1}{\theta}}$$

where $\gamma \equiv \Gamma\left(\frac{\theta+1-\sigma}{\theta}\right)^{\frac{1}{1-\sigma}}$ is a constant.

2.4.3 Gravity

What is the probability that a country i actually buys a good ω from another country j?

Because all goods receive i.i.d. draws and there are a continuum of varieties, by the law of large numbers, this probability will be equal to the fraction of goods i sells to j.

$$\begin{aligned}
\pi_{ij} &= \Pr\left\{p_{ij}(\omega) \le \min_{k\in S\setminus j} p_{kj}(\omega)\right\} \\
&= \int_0^\infty \Pr\left\{p \le \min_{k\in S\setminus j} p_{kj}(\omega)\right\} dG_{ij}(p) \\
&= \int_0^\infty \Pr\left\{\cap_{k\in S\setminus j}(p_{kj}(\omega) \ge p)\right\} dG_{ij}(p) \\
&= \int_0^\infty \prod_{k\in S\setminus j}\Big(1 - G_{kj}(p)\Big) dG_{ij}(p)
\end{aligned}$$

Substituting the distribution of price yields:

$$\begin{aligned}
\pi_{ij} &= \int_0^\infty \prod_{k\in S\setminus j}\left(1 - G_{kj}(p)\right) dG_{ij}(p) \\
&= \int_0^\infty \prod_{k\in S\setminus j}\left(1 - \exp\{-p^\theta \Phi_k\}\right) dG_{ij}(p) \\
&= \int_0^\infty \prod_{k\in S\setminus j}\left(\exp\left\{-T_k\left(\frac{c_k d_{kj}}{p}\right)^{-\theta}\right\}\right)\left(\frac{\partial}{\partial p}\left(1 - \exp\left\{-T_i\left(\frac{c_i d_{ij}}{p}\right)^{-\theta}\right\}\right)\right) dp \\
&= T_i(c_i d_{ij})^{-\theta}\int_0^\infty \theta p^{\theta-1}\exp\left\{\sum_k -T_k\left(\frac{c_k d_{kj}}{p}\right)^{-\theta}\right\} dp \\
&= T_i(c_i d_{ij})^{-\theta}\int_0^\infty \theta p^{\theta-1}\exp\left\{-p^{\theta\Phi_j}\right\} dp \\
&= \frac{T_i(c_i d_{ij})^{-\theta}}{\Phi_j}\int_0^\infty \theta p^{\theta-1}\Phi_j\exp\left\{-p^\theta\Phi_j\right\} dp \\
&= \frac{T_i(c_i d_{ij})^{-\theta}}{\Phi_j}\cdot - \exp\left\{-p^\theta\Phi_j\right\}\Big|_{p=0}^{\infty} \\
&= \frac{T_i(c_i d_{ij})^{-\theta}}{\Phi_j}
\end{aligned}$$

Hence, the fraction of goods exported form i to j just depends on i's share on j's Φ_j.

Intuition.

Intuitively, π_{ij} is increasing in T_j: if country j is on average more productive, then country i will import a lot from j; but decreasing in c_i and d_{ij}: if country j is more expensive in wages or trade costs are higher, then it's a less attractive source of imprts to country i.

Note (More about the Fréchet Distribution).

π_{ij} is the fraction of goods that j purchases from i, it may not be the fraction of j's income that is spent on goods from country i. However, it turns out that with Fréchet distribution, *the distribution* *of prices of goods that country j actually purchases from any country $i \in S$ will be the same.*

Intuitively, what is happening is that origins with better comparative advantage (lower trade costs, better productivity, etc.) in selling to j will exploit its advantage by selling a greater number of goods to j exactly up to the point where the distribution of prices it offers to j is the same as j's overall price distribution.

While the result depends heavily on the Fréchet distribution, the distribution of prices offered to an importing country j is independent from the origin, country j's average expenditure per good doesn't depend on the source of the good.

As a result, the fraction of goods purchased from a particular origin i, denoted by π_{ij}, is equal to the fraction of j's income spent on goods form i:

$$\lambda_{ij} \equiv \frac{X_{ij}}{Y_j} = \frac{T_i(c_i d_{ij})^{-\theta}}{\Phi_j} \tag{2.4}$$

This implies that the total expenditure of j on goods from country i is:

$$X_{ij} = \pi_{ij} E_j \Rightarrow X_{ij} = \frac{T_i(c_i d_{ij})^{-\theta}}{\Phi_j} E_j.$$

Suppose that $c_i = w_i$ and substitute into the price index, yields:

$$X_{ij} = \gamma^{-\theta} d_{ij}^{-\theta} w_i^{-\theta} T_i E_j P_j^{\theta}.$$

As in previous models, we can also push the gravity equation a little bit further. Note that in general equilibrium, the total income of a country will equal the amount it sells to all other countries:

$$Y_i = \sum_{j \in S} X_{ij}$$

Substitute X_{ij} into Y_i, yields:

$$\begin{aligned} Y_i &= \sum_{j \in S} \gamma^{-\theta} d_{ij}^{-\theta} w_i^{-\theta} T_i E_j P_j^{\theta} \\ &= \gamma^{-\theta} w_i^{-\theta} T_i \sum_{j \in S} d_{ij}^{-\theta} E_j P_j^{\theta} \\ \Leftrightarrow \gamma^{-\theta} w_i^{-\theta} T_i &= \frac{Y_i}{\sum_{j \in S} d_{ij}^{-\theta} E_j P_j^{\theta}} \end{aligned}$$

Thus, we can write the gravity equation in a more general form:

$$\begin{aligned} X_{ij} &= \gamma^{-\theta} d_{ij}^{-\theta} w_i^{-\theta} T_i E_j P_j^{\theta} \\ &= d_{ij}^{-\theta} \times \frac{Y_i}{\Omega^{-\theta}} \times \frac{E_j}{P_j^{-\theta}} \end{aligned}$$

where $\Omega_i \equiv \left(\sum_{k \in S} d_{ik}^{-\theta} \frac{E_k}{P_k^{-\theta}} \right)^{-\frac{1}{\theta}}$. We will see soon that if trade costs are symmetric, $\Omega_i = P_i$.

Note (Notation in Lecture slide).

Denote total spending in country i on goods from country j as X_{ij}; country i's total spending on all goods as $X_i \equiv \sum_j X_{ij}$.

We know that $\frac{X_{ij}}{X_i} = \pi_{ij}$, so

$$X_{ij} = \frac{T_j(c_j d_{ij})^{-\theta}}{\Phi_i} X_i$$

Let $Y_j = \sum_i X_{ij}$ be country j's total sales, then we can write:

$$Y_j = \sum_i \frac{T_j(c_j d_{ij})^{-\theta} X_i}{\Phi_i} = T_j c_j^{-\theta} \Omega_j^{-\theta}$$

where $\Omega_j = \sum_i d_{ij}^{-\theta} \frac{X_i}{P_i^{-\theta}}$ is the average price index of country i.

Solving $T_j c_j^{-\theta}$ from $Y_j = T_j c_j^{-\theta} \Omega_j^{-\theta}$ and plug into X_{ij},

$$X_{ij} = \frac{X_i Y_j d_{ij}^{-\theta}}{\Omega}{}^{\theta}_{i} \Phi_i.$$

Ug $p_i = \gamma \Phi_i^{\frac{1}{\theta}}$, we get:

$$X_{ij} = \gamma^{-\theta} X_i Y_j d_{ij}^{-\theta} (p_i \Omega_j)^{\theta}.$$

Remark.

In this model, changes in trade costs affect the extensive margin, i.e. which goods an origin country trades with a destination country. As the bilateral trade costs rise, the origin country is the least cost provider in fewer goods; the greater the θ, the less heterogeneity in a country's productivity across different goods, so there are a greater number of goods for which it is no longer the least cost provider.

Hence, the J. Eaton and Kortum 2002 model is similar to the M. J. Melitz 2003 model (which we'll discuss in later chapters) in that the elasticity of trade to trade costs ultimately depends on the density of producers/firms that are indifferent between exporting and not exporting: the greater the heterogeneity in productivity, the lower the density of these marginal producers.

2.4.4 Welfare

From the CES preferences, the welfare of a worker in country i is given by:

$$W_i = \frac{w_i}{P_i} \tag{2.5}$$

Recall from above that $\lambda_{ij} = \frac{X_{ij}}{E_j}$ is the fraction of j's expenditure spent on i. From the Gravity equation , we have:

$$\lambda_{ij} = \gamma^{-\theta} d_{ij}^{-\theta} c_i^{-\theta} T_i P_j^{\theta}$$

which, given $d_{ii} = 1$, implies:

$$\lambda_{ii} = \gamma^{-\theta} W_i^{-\theta} T_i \Leftrightarrow W_i = C^{-1} \lambda_{ii}^{-1} T_i^{\frac{1}{\theta}}.$$

Hence, as in the Krugman (1980)Krugman 1980 model, welfare can be expressed as a function of technology and the openness of a country.

2.4.5 Ratio Methods

Starting with Head and Ries 1999, several researchers have developed so-called ratio methods. The idea behind these is to eliminate multilateral resistance terms by taking ratios, so that there is no longer a need for fixed effects.

To illustrate ratio methods, we consider here the approach due to Romalis 2007. Consider the Eaton-Kortum gravity equation given by EK gravity equation 2.4.4. Then for country of origin i and country of destination j, we can write:

$$S_{ij} \equiv \frac{X_{ij}/X_i}{X_{jj}/X_j} = \frac{\Phi_j}{\Phi_i} d_{ij}^{-\theta} = \left(\frac{p_j d_{ij}}{p_i}\right)^{-\theta}$$

which shows country j's share in country i's expenditure nromalized by its own share.

This method shows the importance of trade costs and comparative advantage in determining trade volumes. If there are no trade barriers, then $S_{ij} = 1$.

2.4.6 Gains from Trade

What are the gain from trade in the Eaton-Kortum model? That is, what is the difference in consumer welfare between a world with trade and a world in which every country exists in autarky?

We know that the share of national income spent on a country's own goods is:

$$\pi_{ii} = \frac{X_{ii}}{Y_i} = \frac{T_i w_i^{-\theta}}{\Phi_i}.$$

We also know that $P_i = \gamma \Phi_i^{-\frac{1}{\theta}}$, so

$$\omega_i = \frac{w_i}{P_i} = \gamma^{-1} T_i^{\frac{1}{\theta}} \pi_{ii}^{-\frac{1}{\theta}}.$$

Under autarky, we have: $\omega_i^A = \gamma^{-1} T_i^{\frac{1}{\theta}}$, hence the gain from trade is:

$$GT_i = \frac{\omega_i}{\omega_i^A} = \pi_{ii}^{-\frac{1}{\theta}} \tag{2.6}$$

Trade elasticity θ and share of expenditure on domestic goods π_{ii} are sufficient statistics to compute GT.

> **Intuition.**
>
> As $\pi_{ii} \leq 1$, all countries gain from opening up to trade (as long as this leads to them doing some actual trade).
>
> Note also that if the parameter θ is very large (meaning productivity dispersion is low, higher heterogeneity), gains from trade are almost zero.
>
> Indeed, low productivity dispersion within countries means that a country produces essentially all goods with its average productivity T_j. Then, there are no differences in opportunity costs across countries, hence no comparative advantage and no international specialization.

2.5 Weak points of Ricardian Model

In Ricardian model, we simply assume the labor differences (labor input coefficients), but how do they persist when technology is a recipe that everyone can use?

Some other skills are required in using the recipe. Another possible answer is: labor productivity differences are due to differences in other factors, especially capital.

Also, with just one factor mobile across sectors, there is only one wage, giving no diributive conflict. But such conflict is an important aspect of reality.

Lecture 3.

The Specific Factors Model

3.1 Introduction

Like the simple Ricardian model, it assumes an economy that produces two goods and that can allocate its labor supply between the two sectors. Unlike the Ricardian model, however, the specific factors model allows for the existence of factors of production besides labor. Whereas labor is a mobile factor that can move between sectors, these other factors are assumed to be specific.

3.1.1 Factor Proportion Theory

The law of comparative advantage establishes the relationship between relative autarky prices and trade flows.

- Countries differ in terms of factor abundance [i.e relative factor supply]
- Goods differ in terms of factor intensity [i.e relative factor demand]

Interaction between the two terms will determine differences in relative autarky prices, and in turn, the pattern of trade.

3.2 Model Setup

Let's consider an economy with two countries: $g = 1, 2$, and three factors of production: L, K_1, and K_2.

- L is the mobile factor (labor) that can move between sectors.
- K_1 and K_2 are "immobile" factors, can only be employed in one.

We denote by:

- p_1 and p_2 the prices of the two goods.
- w, r_1 and r_2 the prices of the three factors.

Output of good g is given by:

$$y_g = f^g(L_g, K_g)$$

where

- L_g is the amount of labor used in sector g.
- K_g is the amount of capital used in sector g.
- f^g is the production function of good g.
  - Assume that f^g is positive, increasing and concave in both arguments.

- Assume that f^g is homogeneous of degree one in (L_g, K_g), i.e. CRS.

By asusming that f^g is concave, we have a decreasing marginal product $f^g_{LL} < 0$ and $f_{KK} < 0$. The model is isomorphic to DRS model(Dornbusch, Fischer, and Samuelson 1977): $y_g = f^g(L_g)$ with $f^g_{LL} < 0$. Payments to specific factors under CRS is the same as profits under DRS.

The model shares the same assumpmtions as the Ricardian model below:

- Perfect competition in both sectors.
- Full employment
- Endowments given, immobile across countries
- Constant returns to scale in each sector

Different assumpmtions are:

- Labour L is mobile between sectors;
- Capital K_g are immobile between sectors(sector specific).

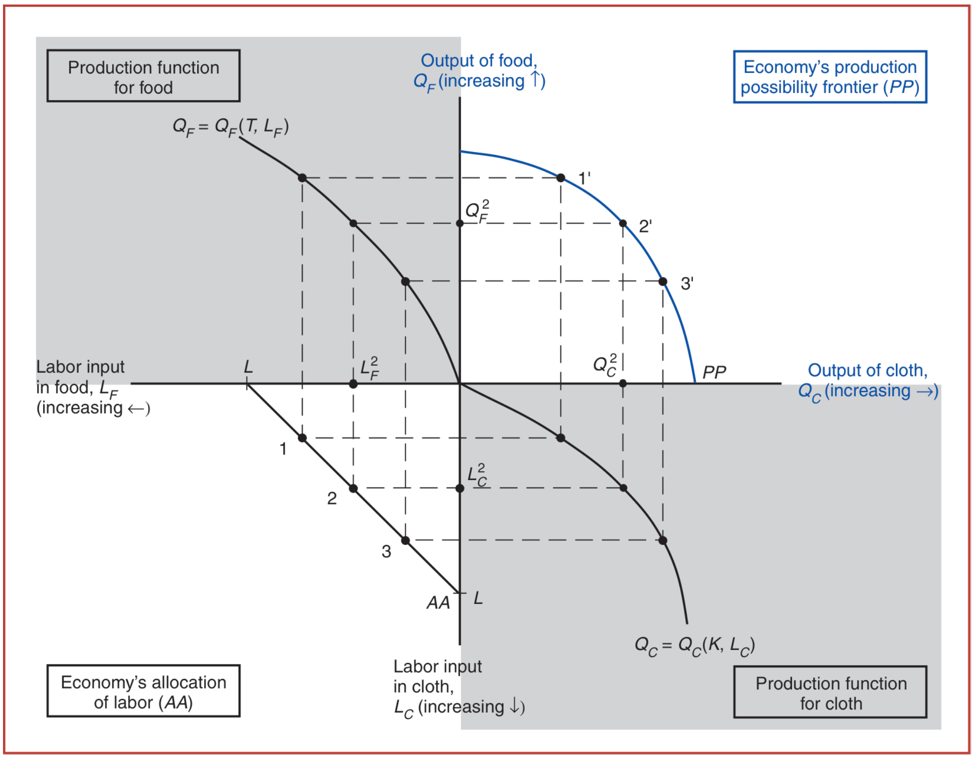


Figure 3.1: Production Possibility Frontier

Production of Good 1 and 2 is determined by the allocation of labor. In the lower-left quadrant, the allocation between sectors can be illustrated by a point on line AA, which represents all combinations of labor inupt of good 1 and 2 that sum up to total labor supplt L.

The curves in the lower-right and upper-left quadrants represent the production functions for both goods, these allow determination of output (Q^2_C, Q^2_F) given output.

Lecture 4.

Monopolistic Competition and Trade

Lecture 5.

Trade Policy

5.1 Introduction

5.1.1 Motivation and Plan

Trade Policy Literature

1. Economic Emvironment:
2. Political environment
3. Constraints on the set of feasible contracts

5.1.2 Introduction to Graphical Analysis

5.1.3 Social Welfare effects of a tariff

Appendix

Fréchet Distribution

The type II extreme value distribution, also called the Fréchet distribution, is one of three distributions that can arise as the limiting distribution of the maximum of a sequence of independent random variables. The distribution function for the Fréchet distribution is

$$F(x) = \exp\left\{ -\left(\frac{x-\mu}{\sigma}\right)^{-\theta} \right\},$$

for $x > \mu$,where $\theta > 0$ is a shape parameter, $\sigma > 0$ is a scale parameter and $\mu \in \mathbb{R}$ is a location parameter. The density of the Fréchet distribution is

$$f(x) = \frac{\theta}{\sigma}\left(\frac{x-\mu}{\sigma}\right)^{-\theta-1} \exp\left\{ -\left(\frac{x-\mu}{\sigma}\right)^{-\theta} \right\},$$

for $x > \mu$. If X is a Fréchet-distributed random variable then

$$\begin{aligned}
E(X) &= \int_{\mu}^{\infty} x\frac{\theta}{\sigma}\left(\frac{x-\mu}{\sigma}\right)^{-\theta-1} \exp\left\{ -\left(\frac{x-\mu}{\sigma}\right)^{-\theta} \right\} dx \\
&= \sigma \int_0^{\infty} y^{-\frac{1}{\theta}} e^{-y} dy + \mu \int_0^{\infty} e^{-y} dy \\
&= \sigma\Gamma\left(\frac{\theta-1}{\theta}\right) + \mu,
\end{aligned}$$

where $y := \left(\frac{x-\mu}{\sigma}\right)^{-\theta}$and

$$\Gamma(z) = \int_0^{\infty} t^{z-1} e^{-t} dt$$

is the Gamma function.

Now assume that $\mu = 0$,take $T := \sigma^{\theta}$ and rewrite the distribution function as

$$F(x) = e^{-Ax^{-\theta}},$$

so that the Fréchet distribution is now parameterized by θ, A.

Recommended Resources

Books

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