

Lecture Notes: Econometrics II

Based on lectures by [Marko Mlikota](#) in Spring semester, 2025

Draft updated on February 17, 2025

These lecture notes were taken in the course *Econometrics II* taught by [Marko Mlikota](#) at Graduate of International and Development Studies, Geneva as part of the International Economics program (Semester II, 2024).

Currently, these are just drafts of the lecture notes. There can be typos and mistakes anywhere. So, if you find anything that needs to be corrected or improved, please inform at jingle.fu@graduateinstitute.ch.

<h2>Contents</h2>

1.	Review of Econometrics I	1
1.1.	Basic assumptions	1
1.2.	Endogeneity	1
	Recommended Resources	4

Lecture 1.

Review of Econometrics I

1.1 Basic assumptions

As we know,

$$\hat{\beta} = (X'X)^{-1}X'y \xrightarrow{P} \beta$$

if

1. Model is correctly specified: $y_i = x_i'\beta + u_i$
2. X is full rank
3. $\mathbb{E}[x_i u_i] = 0$: x_i is exogenous.
4. Unbiased CIA: $\mathbb{E}[u_i | x_i] = 0$

Theorem 1.1.1 (Frisch-Waugh-Lovell (FWL) theorem).

Recall: $\hat{Y} = X\hat{\beta} = X(X'X)^{-1}X'Y = P_X Y$, $Y = \hat{Y} + \hat{U} \rightarrow \hat{U} = (I - P_X)Y = M_X Y$.

Take $Y = X_1\beta_1 + X_2\beta_2 + U = X\beta' + U$, let $P_1 = X_1(X_1'X_1)^{-1}X_1'$, $M_1 = I - P_1$.

And write $M_1 Y = M_1 X_2 \beta_2 + M_1 U$, then

$$\beta_{2, \hat{O}LS} = \hat{b}.$$

1.2 Endogeneity

Three reasons for endogeneity:

1. Measurement error: x_i is measured with error.

Assume the true Regression is: $y_i = x_i^{*'}\beta + \varepsilon_i$, $\mathbb{E}[x_i^* \varepsilon_i] = 0$, we run: $y_i = x_i'\beta + u_i$, $x_i = x_i^* + v_i$, $u_i = \varepsilon_i - v_i'\beta$.

$$\begin{aligned} \mathbb{E}[x_i u_i] &= \underbrace{\mathbb{E}[x_i \varepsilon_i]}_0 - \mathbb{E}[x_i v_i']\beta \\ &= -\mathbb{E}[(x_i^* + v_i)v_i']\beta \\ &= \underbrace{-\mathbb{E}[x_i^* v_i']}_0 \beta - \mathbb{E}[v_i v_i']\beta \\ &= -\mathbb{E}[v_i v_i']\beta \end{aligned}$$

2. Simultaneity(Reverse causality): x_i is endogenous.

$$y_i = x_i'\beta + u_i = x_{i1}^*\beta_1 + x_{i2}\beta_2 + u_i, \quad x_i = z_i'\gamma + y_i\delta + v_i.$$

3. Omitted variables: x_i is correlated with u_i .

The true regression is: $y_i = x_i'\beta + w_i'\delta + \varepsilon_i$, $\mathbb{E}[x_i \varepsilon_i] \neq 0$, $\mathbb{E}[w_i \varepsilon_i] = 0$.

We run: $y_i = x_i'\beta + u_i$, then

$$\begin{aligned}\mathbb{E}[x_i u_i] &= \mathbb{E}[x_i(w_i'\delta + \varepsilon_i)] \\ &= \mathbb{E}[x_i w_i']\delta + \underbrace{\mathbb{E}[x_i \varepsilon_i]}_0\end{aligned}$$

Appendix

Recommended Resources

Books

- [1] James H. Stock and Mark W. Watson. *Introduction to Econometrics*. 4th ed. New York: Pearson, 2003
- [2] Jeffrey M. Wooldridge. *Introductory Econometrics: A Modern Approach*. 7th ed. Cengage Learning, 2020
- [3] Bruce E. Hansen. *Econometrics*. Princeton, New Jersey: Princeton University Press, 2022
- [4] Fumio Hayashi. *Econometrics*. Princeton, New Jersey: Princeton University Press, 2000
- [5] Jeffrey M. Wooldridge. *Econometric Analysis of Cross Section and Panel Data*. 2nd ed. Cambridge, Massachusetts: The MIT Press, 2010
- [6] Joshua Chan et al. *Bayesian Econometric Methods*. 2nd ed. Cambridge, United Kingdom: Cambridge University Press, 2019
- [7] Badi H. Baltagi. *Econometric Analysis of Panel Data*. 6th ed. Cham, Switzerland: Springer, 2021
- [8] James D. Hamilton. *Time Series Analysis*. Princeton, New Jersey: Princeton University Press, 1994. ISBN: 9780691042893
- [9] Takeshi Amemiya. *Advanced Econometrics*. Cambridge, MA: Harvard University Press, 1985

Others

- [10] Roger Bowden. “The Theory of Parametric Identification”. In: *Econometrica* 41.6 (1973), pp. 1069–1074. DOI: [10.2307/1914036](https://doi.org/10.2307/1914036)
- [11] Robert I. Jennrich. “Asymptotic Properties of Non-linear Least Squares Estimators”. In: *The Annals of Mathematical Statistics* 40.2 (1969), pp. 633–643. DOI: [10.1214/aoms/1177697731](https://doi.org/10.1214/aoms/1177697731)
- [12] Michael P. Keane. “A Note on Identification in the Multinomial Probit Model”. In: *Journal of Business & Economic Statistics* 10.2 (1992), pp. 193–200. DOI: [10.1080/07350015.1992.10509906](https://doi.org/10.1080/07350015.1992.10509906)
- [13] Thomas J. Rothenberg. “Identification in Parametric Models”. In: *Econometrica* 39.3 (1971), pp. 577–591. DOI: [10.2307/1913267](https://doi.org/10.2307/1913267)
- [14] George Tauchen. “Diagnostic Testing and Evaluation of Maximum Likelihood Models”. In: *Journal of Econometrics* 30 (1985), pp. 415–443. DOI: [10.1016/0304-4076\(85\)90149-6](https://doi.org/10.1016/0304-4076(85)90149-6)
- [15] Abraham Wald. “Note on the Consistency of the Maximum Likelihood Estimate”. In: *The Annals of Mathematical Statistics* 20.4 (1949), pp. 595–601. DOI: [10.1214/aoms/1177729952](https://doi.org/10.1214/aoms/1177729952)
- [16] Halbert White. “Maximum Likelihood Estimation of Misspecified Models”. In: *Econometrica* 50.1 (1982), pp. 1–25. DOI: [10.2307/1912526](https://doi.org/10.2307/1912526)