Macroeconomics A

Lecture 9 - Idiosyncratic Risk and Incomplete Markets Equilibrium

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Last week

- Two benchmarks: autarky and complete markets. Both rejected by the data: either too little or too much risk-sharing.
- Non-state contingent bonds: the PIH and certainty equivalence. Unfortunately, borrowing constraints and risk seems to matter.
- Precautionary savings: due to prudence and borrowing constraints
- Solving the model in partial equilibrium: given a process for income, and r fixed, how does the consumption function look like?

This week

Look at models that feature a non-trivial endogenous distribution of wealth

Questions:

- How much of the observed wealth inequality is due to uninsurable earnings variation across agents?
- How much of aggregate savings is due to the precautionary motive?
- How do various policies affect the distribution? Effects on inequality and welfare?

  (Note that policy affects also equilibrium prices $\Leftarrow$ endogenous!)
- Is the implied consumption behavior consistent with facts about asset prices? ($\Rightarrow$ field of consumption-based asset pricing)
- How large are the welfare losses of a rise in labor market risk?

Aiyagari (1994)

Key elements:

1. Consumption choice under the fluctuation of income, with borrowing constraints and noncontingent bonds (a la Bewley models, see last week)
2. Aggregate neoclassical production function
3. Asset market equilibrium

$\Rightarrow$ look for stationary equilibria

1. the income fluctuation problem
   - idiosyncratic risk
   - only non-state contingent asset (risk-free bond) and an exogenous borrowing constraint
   - $\Rightarrow$ two reasons to save: inter-temporal substitution and pre-cautionary motive (shield against future negative income shocks)
   - agents who have a long sequence of bad shocks will have low wealth and will be close to the borrowing constraint
   - agents who have a long sequence of good shocks will have high wealth
   - $\Rightarrow$ endogenous distribution of wealth
   - integrating wealth over all agents $\rightarrow$ **aggregate supply of capital**

2. aggregate production function
   - competitive representative firm
   - profit maximization
   - CRS production technology
   - $\Rightarrow$ aggregate demand for capital

3. equilibrium on the asset market
   - aggregate supply of capital = aggregate demand for capital
   - $\Rightarrow$ equilibrium interest rate

<u>note:</u> in an AD equilibrium, the economy is observationally equivalent to a representative agent model with a stationary amount of savings, and the steady state is described by
$\beta(1+r)=1$
here the agents have more reasons to save $\Rightarrow$ more capital $\Rightarrow$ lower interest rate $\Rightarrow$ $\beta(1+r)<1$

Consumers - workers

- continuum (measure 1) of infinitely lived agents
- time-separable utility function: $E_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$
  $u' > 0$, $u'' < 0$, $\beta \in (0, 1)$
- the budget constraint

$$c_t + a_{t+1} = (1 + r_t)a_t + w_t l_t$$

  * l_t is the efficiency units of labor the agent supplies
  * a_{t+1} is the amount of risk free bond the agent buys, the price is one; it pays $1 + r_{t+1}$ in the next period
  * the interest rate is independent of the individual state

- the exogenous liquidity constraint

$$a_{t+1} \geq -\phi$$

- inelastic labor supply, with an exogenous process for labor productivity:
  l_t drawn from density $g(l)$ with support $[l_{min}, l_{max}]$
- the shocks are iid across individuals and over time
- $\Rightarrow$ the aggregate supply of efficiency units of labor is:

$$L_t = \int_{l_{min}}^{l_{max}} lg(l)dl \ \text{ for all } \ t \geq 0$$

- $\Rightarrow$ **no aggregate uncertainty**, L_t exogenous

Financial position of households

- at the beginning of period t, an individual takes assets, a_t and labor income, $w_t l_t$ as given
- as the productivity process is iid, future draws, l_τ are unpredictable from current draw l_t
- assume for now that the real wage, w and the real interest rate, r are expected to remain constant
- let $\widehat{a}_t$ denote accumulated assets plus the borrowing limit: $\widehat{a}_t \equiv a_t + \phi$
- the borrowing constraint is then equivalent to: $\widehat{a}_{t+1} \geq 0$
- let z_t denote the maximum amount of consumption that can be obtained in period t: $z_t = wl_t + (1+r)a_t + \phi = wl_t + (1+r)\widehat{a}_t - r\phi$

Consumer's problem in a recursive form

$$
\begin{aligned}
v(z; w, r) &= \max_{c, \widehat{a}'} \left(u(c) + \beta \int_{l_{min}}^{l_{max}} v(z'; w, r) g(l) dl \right) \\
\text{s.t. } \; c + \widehat{a}' &= z \\
z' &= wl' + (1 + r)\widehat{a}' - r\phi \\
\widehat{a}' &\geq 0
\end{aligned}
$$

- where r, w are exogenous to the individual, but endogenous to the economy
- z is the only state variable
- we assumed that w, r are constant, which is true in a stationary environment r, w

<u>solution</u>: policy function $A(z; w, r)$ which determines the choice variable $\widehat{a}'$

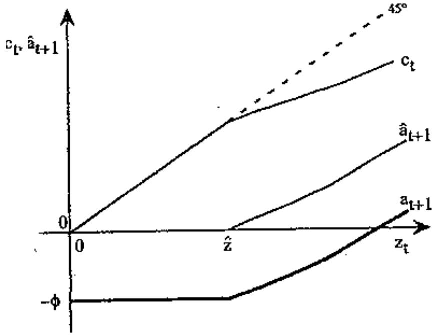

FIGURE Ia
Consumption and Assets as Functions of Total Resources

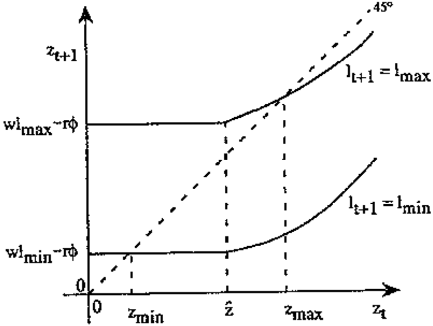

FIGURE Ib
Evolution of Total Resources

- if individual resources are very low in the current period, the individual will consume everything, i.e. $A(z; r, w) = 0$
- higher levels of z induce some saving, so resources are divided between consumption and saving
- the policy function, $A(z; r, w)$, and the random draw of l determine the evolution of each individuals's available resources over time:

$$z_{t+1} = wl_{t+1} + (1 + r)A(z_t; w, r) - r\phi$$

the economy is described by the distribution of z across individuals

the law of motion for this is:

$$\lambda_{t+1}(z) = \int_0^\infty g\left(\frac{z - (1+r)A(\widetilde{z}; w, r) + r\phi}{w}\right)\lambda_t(\widetilde{z})d\widetilde{z}$$

under some conditions there exists a unique, stable, stationary distribution for z: λ^*

the long-run total amount of assets in the economy is then:

$$\int_0^\infty (A(z; r, w) - \phi)\lambda^*(z)dz$$

Technology, markets, feasibility

- representative firm has a CRS technology: $Y_t = F(K_t, L_t)$
- the depreciation rate of capital is $\delta \in (0, 1)$
- all markets (final good, labor, capital) are competitive
- the feasibility constraint of the economy is

$$F(K_t, L_t) = C_t + I_t = C_t + K_{t+1} - (1 - \delta)K_t$$

Stationary Recursive Competitive Equilibrium

consists of a value function $v : z \to \mathbb{R}$, policy functions for the household $A : z \to \mathbb{R}^+$, a stationary probability distribution λ^*, real numbers (K, L, w, r) given $g(\cdot)$ and ϕ such that

- the policy function A solves the household's problem and v is the associated value function, given w and r
- prices satisfy $r + \delta = F_K(K, L)$ and $w = F_N(K, L)$
- the labor market clears, $\int_{l_{min}}^{l_{max}} lg(l)dl = L$
- the capital market clears, $\int_0^{\infty}(A(z; r, w) - \phi)\lambda^*(z)dz = K$
- given A and g, λ^* satisfies

$$\lambda^*(z) = \int_0^{\infty} g\left(\frac{z - (1 + r)A(\widetilde{z}; w, r) + r\phi}{w}\right)\lambda^*(\widetilde{z})d\widetilde{z}$$

Existence and uniqueness of equilibrium

- equilibrium in the labor market exists, and is unique
- demand for capital: $K(r) = F_K^{-1}(r+\delta)$, continuous, decreasing
- supply of capital: $A(r) = \int_0^\infty (A(z; r, w) - \phi)\lambda_r^*(z)dz$
  * if we could show that this crosses the demand, then we prove existence
  $\rightarrow$ possible, but complicated, we skip it here
  * if in addition we could show that $A(r)$ is increasing, then we would also have uniqueness
  $\rightarrow$ no results on monotonicity
  income and substitution effects of r (could rule out turning by specific utility function)
  hard to assess how r affects the distribution of assets

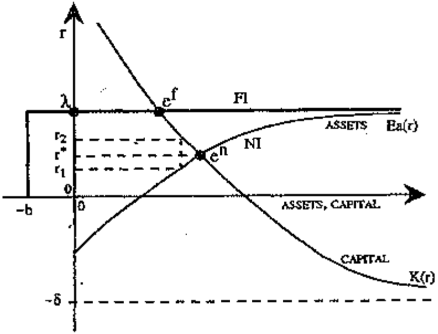

FIGURE IIb
Steady-State Determination

Full insurance

Full insurance (or equivalently no uncertainty)

- If $r < 1/\beta - 1$ then, if the borrowing constraint is not binding, $c_{t+1} > c_t$ (from the Euler eqn) and at some point the borrowing constraint becomes binding. Hence supply of capital is $-\phi$
- If $r = 1/\beta - 1$ then, regardless of whether BC is binding, the supply of capital will remain whatever it was in the beginning $(c_t = c_{t+1})$
- If $r > 1/\beta - 1$ then $c_{t+1} < c_t$ and his asset holdings/supply of capital will diverge to ∞.

Computation of the equilibrium

1. make an initial guess, $r_0 \in (-\delta, 1 - \frac{1}{\beta})$
2. given r_0, obtain the wage rate $w(r_0)$
3. given r_0 and $w(r_0)$, solve the agent's problem ($\Rightarrow$ Value function iteration, Bewley problem) and obtain $A(z; r_0, w_0)$
4. given these and $g(l)$ by iteration we can obtain the stationary distribution $\lambda(r_0)$
   - simulate a large number of households (say 10,000): initialize each individual in the sample with a pair (a_0, l_0) and hence z_0, using the decision rule $A(z_0; r_0, w(r_0))$ and a random number generator that replicates $g(l)$, update the households' individual state (z) in every period t
   - for every t, compute a set of cross-sectional moments J_t which summarize the distribution of assets (mean, variance, various percentiles); stop when J_t and J_{t-1} are close enough $\rightarrow$ the cross-sectional distribution has converged
     (we know that for any given r, a unique invariant distribution will be reached for sure)

5. compute the supply of capital

$$A(r_0) = \int_0^{z_{max}} (A(z; r_0, w(r_0)) - \phi)\lambda_{r_0}(z) dz$$

this can be done by using the model generated data in step 4

6. compare $K(r_0)$ and $A(r_0)$
* if $K(r_0) > A(r_0)$, then the new guess for r has to be higher
* if $K(r_0) < A(r_0)$, then the new guess for r has to be lower
for example by bi-section:

$$r_1 = \frac{1}{2}(r_0 + F_K(A(r_0), L) - \delta)$$

better: Brent's method (= matlab's `fzero()`)

7. update your guess to r_1 and go back to step 1; keep iterating until reaching convergence of the interest rate

Chapter 4 by Rios-Rull in Cooley (1995) describes how to compute equilibria in these types of models

Calibration

this is already needed for the numerical solution method described before

- technology: Cobb-Douglas, capital share $\alpha = 1/3$
- depreciation: $\delta = 0.06$
- utility function: CRRA, the coefficient of relative risk aversion $\gamma \in [1, 5]$, typically 1 or 2 is most commonly used
- β could be chosen to match the aggregate wealth-income ratio for the US (around 3 without residential capital)
  this requires internal parameter calibration, which is quite hard
  an alternative is to match the wealth-income ratio of 3 under complete markets, implies to achieve 3 in the calibrated incomplete markets model we need a β slightly below 0.951
- g to match the distribution of labor income
- ϕ - natural debt limit, or match the fraction of people in the data that hold negative wealth (15%) - internal calib

can use the model to predict the amount of aggregate precautionary savings in the US:

- log utility and iid shocks $\Rightarrow r^* \approx 1/\beta - 1$, no precautionary savings
  intuition: low risk aversion and low persistence shocks $\rightarrow$ low self-insurance motive
- CRRA utility, $\gamma = 5$, 0.9 autocorrelation of income shocks $\Rightarrow$ precautionary savings is 14% of aggregate output
  intuition: high risk aversion $\rightarrow$ consumption fluctuations are very costly, high persistence $\rightarrow$ income can stay low for a long time
- these are the two extremes, a more realistically calibrated model ($\gamma \approx 2$) predicts that the precautionary saving rate would be around 5% of GDP, about 25% of total savings

<u>note</u>: equilibrium considerations put some discipline on the model - given demand for capital only one interest rate works
(amount of savings if r gets closer to $1/\beta - 1$ behaves differently in the equilibrium model)

Comparative statics

- borrowing limit: suppose we increase ϕ, i.e. make the borrowing limit more generous $\Rightarrow$ $A(r)$ shifts up, the equilibrium amount of capital decreases and the interest rate increases
- risk aversion: increase risk aversion $\Rightarrow$ individuals are more concerned about consumption smoothing, they accumulate more buffer-stock savings, for any r $A(r)$ is larger $\Rightarrow$ $A(r)$ shifts down, more capital, lower interest rate
- income process: increase the variance of income $\Rightarrow$ again, people accumulate more, $A(r)$ shifts down
  increasing the persistence has a similar, but quantitatively bigger effect

Efficiency and constrained efficiency

- is the competitive equilibrium efficient, does is achieve the first best?
- answer: it clearly is not, as the first best allocation is the one achieved under complete markets, where $\beta R = 1$
  where agents fully insure themselves against the idiosyncratic labor risk
- in the Aiyigari model there is an **over-accumulation of capital** compared to the first-best
- first best can be achieved by public insurance: tax away all of the income, and redistribute it equally across agents
  note: this would not work if the households also had a labor-leisure choice $\rightarrow$ this would introduce a trade-off between insurance and efficiency

- a different question: do the markets perform efficiently relative to the set of allocations achievable with the same structure? $\rightarrow$ **constrained efficiency**
- the planner tells each agent how much to consume and how much to save
- facing the same technology and asset structure
- there is an externality in the competitive equilibrium: each agent's decision has an impact on prices, which he does not take into account when making his choices
- $\Rightarrow$ the competitive equilibrium is constrained inefficient
- the planner takes this effect into account when telling the agents how much to consume and save

the planner maximizes

$$\Omega(\lambda) = \max_{f(a,l)} \int_{\mathcal{A}\times\mathcal{L}} u(aR(\lambda) + lw(\lambda) - f(a,l))d\lambda + \beta\Omega(\lambda')$$

* subject to what? * where $a' = f(a,l)$ is the amount of assets that the planner asks an individual with assets a and labor productivity l to have next period
* when the planner assigns $f(a,l)$ to an individual, he changes the total amount of saving (capital) next period, thus influencing the total amount of resources of everyone in the next period
* the Euler equation for the optimal saving of an agent with state a, l is:

$$u_c \geq \beta R(\lambda') \int_{l_{min}}^{l_{max}} u'_c g(l')dl + \beta \int_{\mathcal{A}\times\mathcal{L}} (a' F'_{KK} + l' F'_{LK}) u'_c d\lambda'$$

the extra term compared to the competitive EE is:

$$\beta \int_{\mathcal{A}\times\mathcal{L}} (a' F'_{KK} + l' F'_{LK}) u'_c d\lambda'$$

$\rightarrow$ the planner internalizes the effects of individual savings on prices

- the blue term captures the effect of an extra unit of saving on next period capital income and labor income
- more savings $\Rightarrow$ more capital $\Rightarrow$ more labor income and less capital income
- this effect is averaged over all agents, weighted by the marginal utility from consumption
- term can be positive or negative, as $F_{KK} < 0$ and $F_{KL} > 0$ why isn't there such a term in case of complete markets?

- if the income of the poor is labor-intensive, then the expression is positive
- $\Rightarrow$ the planner wants the agents to save more
- the competitive equilibrium features **under-accumulation of capital**
- <u>intuition</u>: the planner wants to redistribute from the rich to the poor
- if the poor have mostly labor income, then the way to redistribute is to increase equilibrium wages by inducing agents to save more than in equilibrium
  larger individual savings increase the aggregate capital stock and increase wages
- Davila, Hong, Krusell and Rios-Rull (2012) calibrate this model to the US economy and find that the constrained efficient capital stock is 3.5 (!!) times higher than the competitive equilibrium capital stock

Wealth inequality

- in this model agents are ex-ante identical, they only differ due to the variation in their income realizations
  $\Rightarrow$ differences are only driven by luck
- the path of shocks lead to an endogenous consumption and wealth distribution
- natural questions: if idiosyncratic earnings shocks are the only source of heterogeneity how much can the model explain of the observed wealth inequality?

	mean	mean/median	Gini	share of top 1%
earnings	21.1	1.57	0.61	7.5%
wealth	47.4	4.03	0.80	31%

Source: Budria, Diaz-Gimenez,Quadrini, Rios-Rull (2002) data: both wealth and earnings are skewed, but wealth much more so, the top 1% of the wealth distribution holds more than 30% of total wealth

model generates too much asset holding at the bottom and not enough at the top, the Gini generated by the model is 0.4 as opposed to 0.8 in the data

1. reduce the incentives for the poor to save for self-insurance
   * modeling the welfare state properly helps a lot $\sim$ public insurance schemes
   see for example Hubbard, Skinner and Zeldes (1995)
2. increase the incentives for the rich to accumulate capital
   * entrepeneurship (Quadrini (2000), Kitao (2008))
   * heterogeneity in the discount factor (Krusell, Smith (1997))
   * bequests (De Nardi (2003))
   * very high income realizations with very low probability (Castaneda, Diaz-Gimenez, Rios-Rull (2003))

Aggregate uncertainty

so far we assumed that there is no aggregate uncertainty $\Rightarrow$ cannot study the effects of incomplete markets on macro dynamics

Krusell and Smith (1998) introduce aggregate uncertainty, where aggregate shocks have two effects on the economy:

1. change total factor productivity
2. change employment

extra difficulty in solving these models: the entire distribution of wealth becomes a state variable (this is infinite dimensional object), which we have to keep track of as it is not stationary $\Rightarrow$ **impossible to solve explicitly** for the equilibrium allocations

good news: we can approximate the exact equilibrium

Krusell-Smith: sketch

- Consumers

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \log(c_t)$$

s.t.

$$a_{t+1} = (1 + r_t - \delta) a_t + w_t \epsilon_t + h(1 - \epsilon_t) - c_t$$

and

$$a_{t+1} \geq \underline{a}$$

- Firms:

$$\max z_t K_t^{\alpha} L_t^{1-\alpha} - r_t K_t - w_t L_t$$

- Aggregate state: $z_t \in \{g, b\}$ with transition probabilities $\phi_{zz'}$
- Idiosyncratic state $\epsilon_t \in \{0, 1\}$
- As before, slight abuse of notation: should index individual-specific variables by i, i.e. a_t^i, ϵ_t^i, etc. But we're lazy.

State variables

What do we need to make our decisions?

- a_t, ϵ_t, and z_t (and conditional expectations of future z and ϵ – but here it's a one-period Markov chain, so no need)
- We need to predict $w_{t+1}, r_{t+1}, w_{t+2}, r_{t+2}, \ldots$. These are going to depend on the distribution of individuals' state variables. Call this Γ_t.
- Then the state space is $(a_t, \epsilon_t, z_t, \Gamma_t)$. Can show that this is enough.

Then the Bellman eqns are

$$V(a, \epsilon, z, \Gamma) = \max_{c,a'} \log(c) + \beta E \left(V(a', \epsilon', z\, \Gamma') | \epsilon, z \right)$$

subject to the budget constraint, borrowing constraint, and

$$\Gamma' = H(\Gamma, z, z')$$

H is the law of motion for the distribution Γ.

Equilibrium

- Equilibrium: Consumers optimize, firms optimize, markets clear

$$K_t = \int a_t d\Gamma_t(a_t, \epsilon_t), \quad L_t = \int \epsilon_t d\Gamma_t(a_t, \epsilon_t)$$

  and consistency of the law of motion (Γ' is generated by $a'(a, \epsilon, z, \Gamma)$ and ϵ' with Γ and z'.
- Conceptually not hard. But big problems for how to solve it.
- What do we do with Γ, H?
- Idea of Krusell and Smith: perhaps not the full distribution of capital holdings are important, but some moments are sufficient?
- Also: Den Haan (1996), Rios-Rull (1997).

General K-S algorithm

- Assume the following approximate law of motion for the aggregate state:
$$m_{t+1} = \tilde{H}(m_t, z_t; \eta)$$
where m_t is a vector of moments of Γ_t, and $\tilde{H}$ is a parametric function with coefficients $\eta = \eta(z)$.
- Assume that the agents are boundedly rational and believe that $\tilde{H}$ is the actual law of motion of the aggregate variables. Solve the utility maximization of the households under these beliefs.
- Then simulate many agents for many time periods, and use regression analysis to update the parameters of $\tilde{H}$.
- In case the fit remains bad, use more moments or higher order polynomials for $\tilde{H}$.

Specifically for Krusell-Smith's model

1. Start with the guess

$$\log(K') = \begin{cases} a_0 + b_0 \log(K) & \text{if } z = g \\ a_1 + b_1 \log(K) & \text{if } z = b \end{cases}$$

2. Solve the households' problem:

$$V(a, \epsilon, z, K) = \max_{c, a'} \log(c) + E\left(V(a', \epsilon', z, K')\right)$$

   s.t. the BC, borrowing constraint, and the guess for the law of motion for K. Note that r and w can be calculated from the state variables.

3. Simulate the economy using the policy functions obtained in the previous step. Obtain the simulated series for K and z.
4. Use OLS to estimate the new coefficients from the time series for K and z.
5. If the fit is sufficiently good, stop. (Original K-S: use R^2. Not good: see Den Haan's handbook chapter)

Results

- Consumption and saving resemble those of a representative agent, unless a is very low
- Little is lost by considering only the mean of the asset distribution (i.e. you don't need higher moments here)
- $\Leftrightarrow$ we also do not gain much from looking at the heterogeneous agent economy when thinking about business cycles

Results

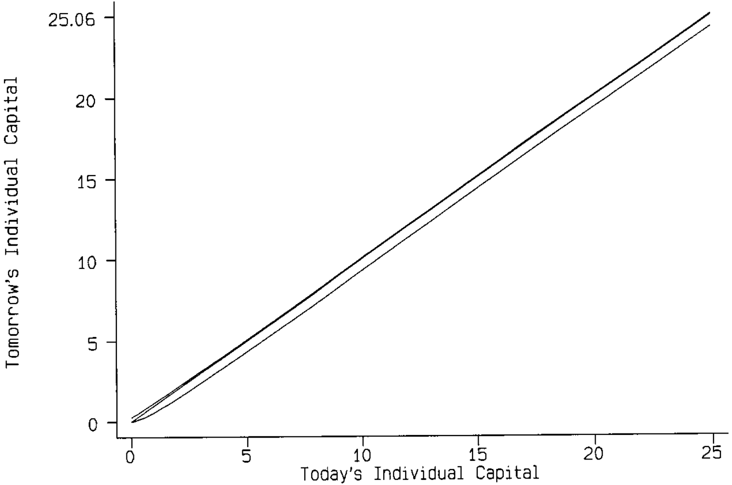

FIG. 2.—An individual agent's decision rules (benchmark model, aggregate capital = 11.7, good aggregate state).

Why?

Intuition:

- under complete markets there is a linear relationship between saving and wealth
- this is not true in the Aiyigari and in the Krusell and Smith model: precautionary saving should be higher for agents who hold fewer assets $\Rightarrow$ non-linear saving function
- for agents who are rich enough, the policy function is roughly linear

  they achieve good self-insurance with a small amount of saving, most of their saving is due to the inter-temporal motive rather than the insurance motive
- non-linearity is created by those who cannot self-insure by saving and borrowing $\rightarrow$ those who are very poor and are close to the borrowing constraint
- but these agents are a relatively unimportant part of the overall distribution of assets

Application of K-S algorithm

Pretty much anywhere where you need GE and a distribution of agents.

- Cost of business cycles (K-S 1999 RED), Asset pricing (Gomes and Michaelides and many others), Tax reform (Nishiyama and Smetters 2005 JPE)
- Monetary policy/state-dependent pricing: Golosov and Lucas 2007 JPE
- Monetary policy and redistribution: Doepke and Schneider 2006
- IO sectoral models (Weintraub Benkard Van Roy 2007, somewhat related)
- many many others

Welfare and Policy

Model is good candidate for studying optimal policy

- Incomplete insurance (taxes can be used for risk-sharing)
- General equilibrium (gov has budget constraint)
- Can study effect on wealth/income distribution

How to do this?

- Set a welfare criterion (weighted sum of utilities, min of utilities, . . .)
- Parameterize policy space: what kind of taxes can be used?
- Solve the model for each set of policies, simulate, and calculate welfare
- Iterate over policies to find "optimal policy"

Welfare and Policy

This stuff in practice

- Advantage: you are fairly independent on functional forms, can put in a lot of different stuff
- Disadvantage: you can't put in stuff that means that value functions/policy functions become discontinuous. Discrete choices are hard to do (labor/IO people who do this usually go to partial equilibrium)
- Results are often dependent on how exactly the policy space looks
- ...and on calibrations
- Robustness is easy to do for calibrations, harder for policy space
- Models are not very "transparent"; Intuition is hard