Macroeconomics A
Problem Set 6

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Question 1: The CES consumption aggregator

Consider the CES consumption aggregator $c = \left(\int c(i)^{\frac{\epsilon-1}{\epsilon}} di\right)^{\frac{\epsilon}{\epsilon-1}}$.

1. Find the demand functions that minimize the expenditure $\int p(i)c(i)di$ needed to obtain a particular level of consumption c of the basket of goods.

2. Show that the minimum expenditure required is $\int p(i)c(i)di = pc$ where $p = (\int p(i)^{1-\epsilon}di)^{\frac{1}{1-\epsilon}}$ is the price index.

Question 2: Deriving the IS curve

Consider the New Keynesian model. Define the output gap x_t as

$$x_t = \frac{y_t}{\hat{y}_t} \tag{1}$$

where $\hat{y}_t$ denotes the efficient level of output. Recall the Euler equation for consumption (with $u'(c) = c_t^{-1/\sigma}$ and market clearing, $y_t = c_t$)

$$y_t^{-\frac{1}{\sigma}} = \mathbb{E}_t \left(\frac{1+r_t}{1+\bar{r}} y_{t+1}^{-\frac{1}{\sigma}} \right)$$

where $\bar{r}$ is the real interest rate in steady state, $\beta = 1/(1+\bar{r})$.

1. Log-linearize the Euler equation around the steady state.

2. Noting that the approximated Euler equation should also hold in an efficient world (i.e. for the efficient level of output and the efficient real interest rate), use approximations for the Fisher equation

$$r_t = i_t - \pi_{t+1}$$

and for the definition of the output gap (1) to show that

$$\widetilde{x}_t = \mathbb{E}_t \left(\widetilde{x}_{t+1} - \sigma(\widetilde{i}_t - \widetilde{\pi}_{t+1}) \right) + \sigma \widetilde{\hat{r}}_t$$

Question 3: The three-equation New Keynesian model

Consider the New Keynesian model, which consists of the New Keynesian Phillips curve,

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa x_t + e_t,$$

the IS curve

$$x_t = \mathbb{E}_t \left(x_{t+1} - \sigma(i_t - \pi_{t+1}) \right) + v_t$$

where $v_t = \sigma \widetilde{\hat{r}}_t$ is a shock to the efficient real interest rate, and a monetary policy rule of the form

$$i_t = \alpha_1 \pi_t + u_t$$

where u_t is a shock (with the usual properties: mean zero, uncorrelated with all other variables). All variables in the three equations, except the shocks, are in log deviations from the steady state (i.e. I dropped the $\widetilde{\ }$).

1. Interpret the monetary policy rule. Which sign would you expect α to have?
2. State what effect the following have on e_t, v_t, and u_t:
   (a) A temporary rise in productivity (TFP) a_t
   (b) A permanent rise in productivity productivity at (i.e. the percentage increase in a_t and a_{t+1} is the same)
   (c) A temporary rise in the competitiveness of markets as measured by the price elasticity ϵ_t
   (d) A shift in monetary policy caused by new personnel on the monetary policy committee
3. Consider a temporary shock to e_t and assume that this has no effect on expectations of the future, so that $\mathbb{E}_t \pi_{t+1} = 0$ and $\mathbb{E}_t x_{t+1} = 0$. No other shock occurs at the same time, thus $v_t = 0$ and $u_t = 0$. Find expressions for π_t and x_t in terms of e_t and interpret your results.
4. Suppose that the monetary policy rule is modified so that $i_t = \hat{r}_t + \alpha \pi_t + u_t$. If this interest rate rule is used then what are the effects of a temporary shock to the natural interest rate (and hence v_t) on inflation and the output gap?
5. Is it possible to redesign the monetary policy rule to achieve the same effect found in part (d) but now in response to a shock to e_t? Explain why or why not.