ECON 39: Undergraduate International Trade
Problem Set #2
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This problem set is not for credit, but is good practice for midterm #1

Preliminaries

This week, we are learning about two modern extensions of comparative advantage theory: one to many goods (and two countries, possibly with trade costs), and one to many goods, many countries, and arbitrary trade costs. This problem set, while short, is going to be tricky, but do your best!

Questions

1. [DFS '77] Consider a world with two countries $i \in \{1, 2\}$, each inhabited by L_i consumers. Suppose that each country has the technology to produce a continuum $g \in [0, 1]$ of goods. Let $\alpha_i(g)$ be the unit labor cost of producing good $g \in [0, 1]$ in country $i \in \{1, 2\}$. Normalize the wage in country 2 to one and let w denote the relative wage in country 1.
   (a) Suppose that $\alpha_1(g) = g^2$ and $\alpha_2(g) = g$. If the equilibrium wage is $w = 2$, which country would produce which good?
   (b) Suppose consumers have Cobb-Douglas preferences with equal demand shifters for all goods, i.e.:
   $$U_i = \int_0^1 \log C_i(g)\, dg$$
   Solve for the equilibrium quantity consumed of good g by a consumer in i as a function of the good's price $p_i(g)$ and the total income in country i, Y_i.
   (c) Now let us focus on the equilibrium:
      i. List the exogenous parameters of the model.
      ii. List the endogenous outcomes of the model.
      iii. State the equilibrium conditions.
   (d) Suppose $L_1 = 1$ and $L_2 = 2$, unit labor costs are as in 1(a) and preferences are as in 1(b). Find the equilibrium relative wage, incomes in the two countries and pattern of specialization.
   (e) Suppose the population of country 1 doubled to $L_1 = 2$. Show using both math and a figure how equilibrium wages would change. Would the equilibrium pattern of specialization change?
2. [EK '02]. Consider a world with N countries with iceberg trade costs from i to j $\tau_{ij} \geq 1$. Let A_i be the aggregate productivity in country i and let w_i be its wage. Recall that the value of trade flows from i to j can be written as:
   $$X_{ij} = \tau_{ij}^{-\theta} w_i^{-\theta} A_i^{\theta} P_j^{\theta} w_j L_j,$$
   where $P_j \equiv \left(\sum_{k=1}^{K} \tau_{kj}^{-\theta} w_k^{-\theta} A_k^{\theta}\right)^{-\frac{1}{\theta}}$ is the price index. Recall too that the welfare of a worker in location j can be written as $U_j = \frac{w_j}{P_j}$. Suppose that the wages are exogenous. [Note: This assumption is sometimes made by assuming there is an "outside sector" that is costlessly traded that all countries produce. I am not a fan of this assumption.] Derive the elasticity of welfare in country j to a change in the productivity in country i, i.e. $\frac{\partial \ln U_j}{\partial \ln A_i}$. Is this variable observed in the data? What is the intuition for this result?