Macroeconomics A

Lecture 5 - Optimal Monetary Policy

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Optimal Monetary Policy

What should monetary policy aim for?

- Average inflation rate? (= zero?)
- Stability of inflation?

Several considerations:

1. Menu costs: keep inflation at zero to minimise menu costs
2. Incomplete indexation of taxes: when households have to pay taxes on nominal (labor, capital) income, a increase in inflation might push them into a higher tax bracket $\Rightarrow$ distorts incentives $\Rightarrow$ should target zero inflation
3. Zero lower bound for nominal interest rates: targeting positive inflation rates leaves more space to decrease i during a recession
4. Measured inflation may overstate actual inflation due to incomplete adjustment for quality improvements $\Rightarrow$ target positive measured inflation
5. Nominal wages do not adjust downwards: target positive average inflation to allow real wages to fall in times of low TFP

Optimal Monetary Policy

Output/Unemployment stabilization

- Here: think of unemployment as inversely moving with output gap
- More careful analysis of unemployment would require modeling the labor market (search & matching models)

Trying to bring output up to the efficient level $\Leftrightarrow$ closing the output gap

- But can we push output above the steady-state (= natural) level?
- Note: natural level $y_t^* <$ efficient level $\hat{y}_t$

Note: in the following I'll ignore the ˜. Everything is in percentage deviation from the non-stochastic steady state.

Optimal Monetary Policy: Loss function

Obvious choice for CB's optimal MP problem would be to maximize welfare, i.e. the NPV of expected utility of households, subject to the optimal responses and market clearing conditions of the model.

- Woodford (2003) shows that (to a second order approximation) this is equivalent to minimizing a *loss function* in the economy: time t loss function is

$$L_t = \frac{1}{2}\left((\pi_t - \pi^*)^2 + a(x_t - x^*)^2\right)$$

- Notation as in NKPC: everything in pct deviations from steady state
- π^* is an *inflation target*
- x^* is an *output gap target*
- a governs the CB's relative importance of hitting the output gap target

Optimal MP problem

Now: assume that CB cannot commit to a policy, i.e. chooses current period monetary policy only. Then, optimal MP problem is

$$\min_{i_t} L_t = \frac{1}{2}\left((\pi_t - \pi^*)^2 + a(x_t - x^*)^2\right)$$

s.t. the NKPC and IS curves,

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa x_t + e_t \tag{1}$$

$$x_t = \mathbb{E}_t\left(x_{t+1} + \sigma \pi_{t+1}\right) - \sigma i_t + v_t \tag{2}$$

Interest rate i_t only features in IS curve, hence this problem is equivalent to picking an output gap x_t and omitting IS curve (interest rate policy i_t can then be backed out using the IS curve):

$$\min_{\pi_t, x_t} L_t = \frac{1}{2}\left((\pi_t - \pi^*)^2 + a(x_t - x^*)^2\right) \tag{3}$$

$$\text{s.t.} \quad \pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa x_t + e_t \tag{4}$$

$$\tag{5}$$

Optimal MP problem

Straightforward to solve for optimal x_t, π_t. Combine FOC

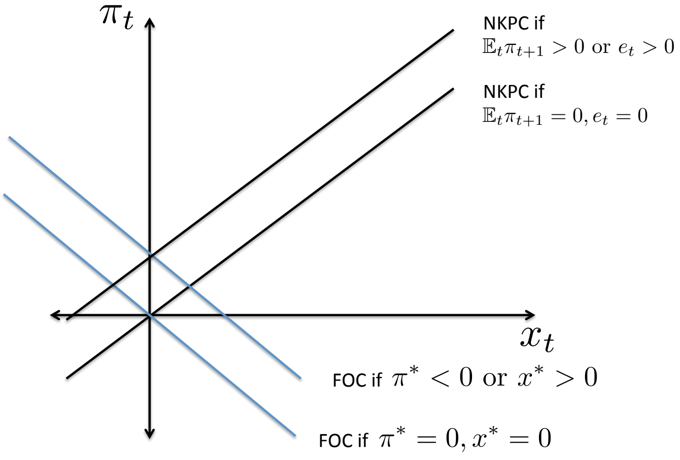
$$\kappa(\pi_t - \pi^*) + a(x_t - x^*) = 0$$

with NKPC to find x_t and π_t (and then IS to find i_t).

Conclusions:

1. When the economy is hit by **shocks to the efficient real interest rate** (such as TFP shocks), the CB should just move nominal interest rate accordingly so that the shock cancels out in the IS curve
   - No tradeoff for CB. Can stabilize both inflation and output gap.
2. When the economy is hit by **cost-push shocks** (i.e. shocks to markups), the CB faces a tradeoff that is given by the NKPC.
   - Tradeoff becomes worse when inflation expectations $\mathbb{E}_t\pi_{t+1}$ are high or cost-push shock e_t is high (NKPC shifts up)
   - Optimally chosen point depends on CB preferences: π^*, x^*, a.

Graphically



Central bank's preferences

Parameter a governs importance of output gap stabilization.

- $a \to \infty$ means that CB will only care about output gap; $Var(\pi)$ large, $Var(x) = 0$. Doveish CB.
- $a = 0$ means that CB will only care about inflation; $Var(x)$ large, $Var(\pi) = 0$. Hawkish CB.

Note that $Var(X) = \mathbb{E}X^2$ if $\mathbb{E}X = 0$ (cf. central bank's loss function).

So far our solution takes expectations about future variables as given. In order to be able to solve for x_t, π_t, i_t as a function of parameters and shocks only, you need to know (a) whether shocks are persistent or not (i.e. $\mathbb{E}_t e_{t+1}$ etc), (b) what the central bank is expected to do in the future.

If the CB minimizes the loss function L_t in every period and implements the solution, this is called *optimal monetary policy under discretion*.

Inflation bias

Suppose that CB chooses $\pi^* = 0$ but $x^* > 0$ in every period. Assume no shocks.

$$\min_{i_t} L_t = \frac{1}{2}\left(\pi_t^2 + a(x_t - x^*)^2\right)$$

- FOC is
$$x_t = x^* - \frac{\kappa}{a}\pi_t$$
- NKPC is
$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa x_t$$
- Plug in NKPC_{t+1} (recursively) to get
$$\pi_t = \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \kappa x_{t+j}$$

  and now the FOC to get
$$\pi_t = \frac{\kappa}{1-\beta} x^* - \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \frac{\kappa^2}{a} \pi_{t+j}$$

Inflation bias

- Need to solve for expectations of future variables. Note that in the next period, the CB's problem will look exactly like in this period. Hence optimal solution must be the same in all periods $\pi_t = \bar{\pi}$ for all t. Because of rational expectations:

  $$\pi_t = \mathbb{E}_t \pi_{t+1} = \bar{\pi}$$

  Hence, solving for π_t above we get

  $$\pi_t = \bar{\pi} = \frac{\kappa}{1 - \beta + \frac{\kappa^2}{a}} x^*$$

  and hence

  $$x_t = x^* - \frac{\kappa}{a} \frac{\kappa}{1 - \beta + \frac{\kappa^2}{a}} x^*$$

Inflation bias

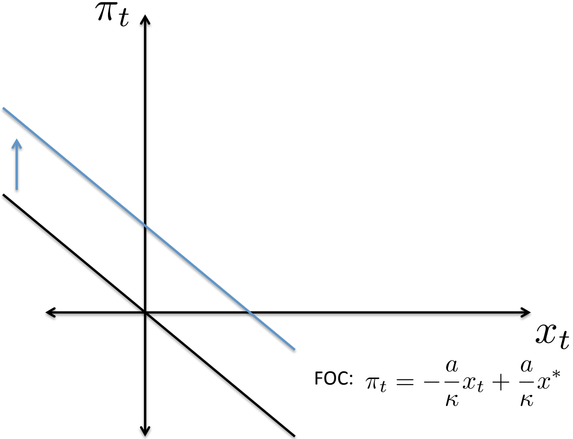
- How large are these magnitudes? Remember $\beta \approx 0.97$. For $\beta \to 1$,

$$\lim_{\beta \to 1} x_t = 0, \quad \lim_{\beta \to 1} \pi_t = \frac{a}{\kappa} x^*$$

- Hence x_t is actually pretty close to 0, and π_t is positive, which means we're far away from the target! (we wanted to achieve $x_t \to x^*$ and $\pi_t \to 0$!)
- What happened?

Output gap target x^* shifts FOC up

Period t

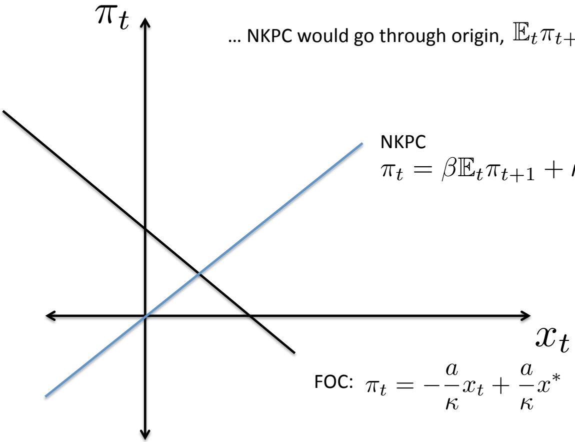


FOC: $\pi_t = -\frac{a}{\kappa}x_t + \frac{a}{\kappa}x^*$

If inflation expectations were zero…

Period t

… NKPC would go through origin, $\mathbb{E}_t \pi_{t+1} = 0$

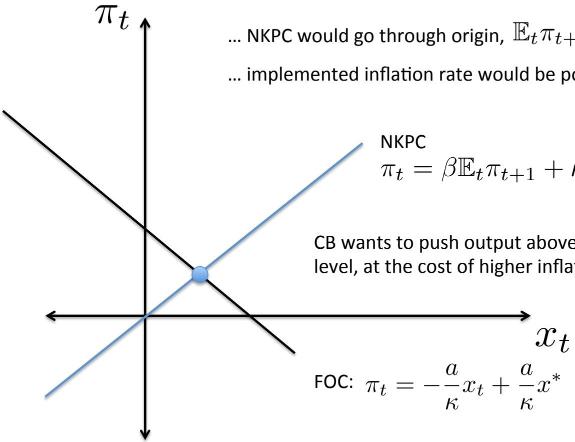


NKPC
$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa x_t$

FOC: $\pi_t = -\frac{a}{\kappa} x_t + \frac{a}{\kappa} x^*$

If inflation expectations were zero…

Period t



… NKPC would go through origin, $\mathbb{E}_t \pi_{t+1} = 0$

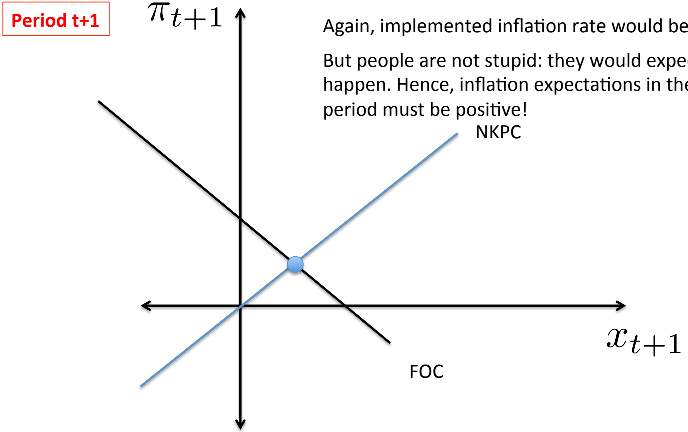
… implemented inflation rate would be positive.

NKPC
$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa x_t$$

CB wants to push output above its long-term level, at the cost of higher inflation.

FOC: $$\pi_t = -\frac{a}{\kappa} x_t + \frac{a}{\kappa} x^*$$

People expect the same thing to happen in the next period!

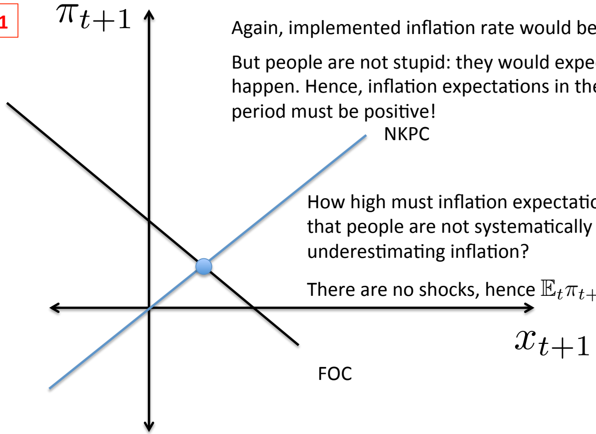


Again, implemented inflation rate would be positive!

But people are not stupid: they would expect this to happen. Hence, inflation expectations in the first period must be positive!

People expect the same thing to happen in the next period!

Period t+1



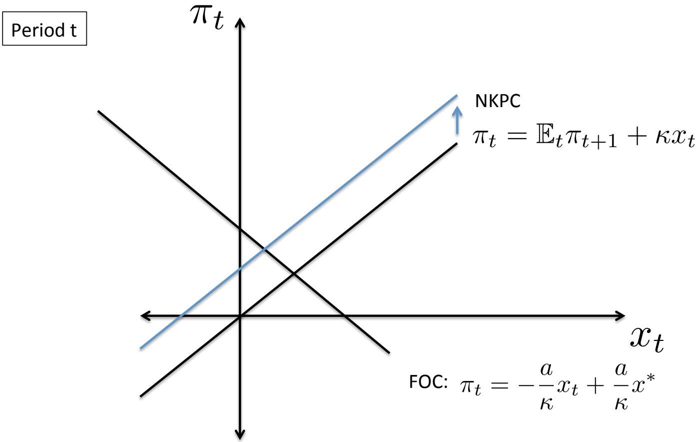
Again, implemented inflation rate would be positive!

But people are not stupid: they would expect this to happen. Hence, inflation expectations in the first period must be positive!

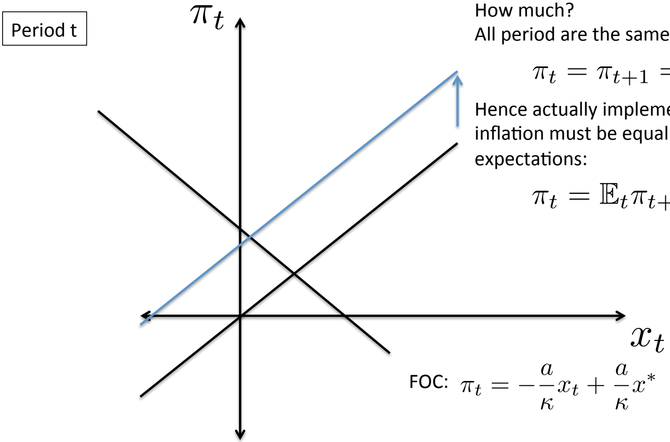
How high must inflation expectations be so that people are not systematically underestimating inflation?

There are no shocks, hence $\mathbb{E}_t \pi_{t+1} = \pi_{t+1}$!

Higher inflation expectations shift period t NKPC up



Higher inflation expectations shift period t NKPC up



How much?

All period are the same (no shocks)

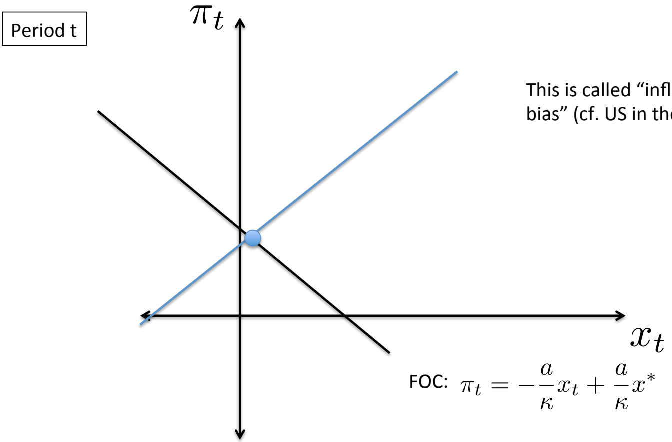
$$\pi_t = \pi_{t+1} = \ldots$$

Hence actually implemented inflation must be equal to inflation expectations:

$$\pi_t = \mathbb{E}_t \pi_{t+1}$$

FOC: $\pi_t = -\frac{a}{\kappa} x_t + \frac{a}{\kappa} x^*$

Final outcome, high inflation, low output gain



This is called "inflation bias" (cf. US in the 1970's)

FOC: $\pi_t = -\frac{a}{\kappa}x_t + \frac{a}{\kappa}x^*$

Optimal monetary policy with commitment

Instead of re-optimizing every period, the CB could plan a MP rule (a set of $\{i_t\}_t$ contingent on shocks). This would generally lead to a preferable outcome, and is called *optimal monetary policy with commitment*.

- However, this only works when the people believe the central bank's announcement, i.e. the central bank can "set" expectations of future variables.
- Simple scenario: $x^* = \pi^* = 0$, cost-push shock positive $e_t > 0$ but short-lived: $\mathbb{E}_t e_{t+j} = 0$ for $j > 0$. Assume we're at time $t = 0$.
- Intertemporal loss function:

$$\min_{\{\pi_t\}_t, \{x_t\}_t} L = \frac{1}{2}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\pi_t^2 + a x_t^2\right)$$

  subject to the NKPC in every period: for $t = 0, 1, 2, \ldots,$

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa x_t + e_t$$

Solving this

- Lagrangian:

$$\mathcal{L}_t = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{1}{2}(\pi_t^2 + ax_t^2) + \lambda_t(\pi_t - \beta\pi_{t+1} - \kappa x_t - e_t) \right)$$

- FOCs:

$$\pi_t + \lambda_t - \lambda_{t-1} = 0 \quad \text{for } t \geq 1 \tag{6}$$

$$ax_t - \kappa\lambda_t = 0 \quad \text{for } t \geq 0 \tag{7}$$

- Note that the NKPC at time -1 is not a restriction, hence FOC for π_0 is

$$\pi_0 + \lambda_0 = 0$$

- Eliminate λ_t to get

$$x_t - x_{t-1} = -\frac{\kappa}{a}\pi_t \tag{8}$$

$$x_0 = -\frac{\kappa}{a}\pi_0 \tag{9}$$

Could solve for solution, but really ugly...

Time inconsistency of the policy

- The optimal monetary policy with commitment features *history dependence*, i.e. policy decision for period t depends on what we want to implement in period $t-1$.
- Exception: optimal policy for period 0 does not depend on past (why? economy is only forward-looking)
- Imagine at time 1 we'd be solving the optimal policy problem with commitment again. We would then implement

$$x_1 = -\frac{\kappa}{a}\pi_1$$

  instead of what we would choose if we used the optimal problem from time zero:

$$x_1 - x_0 = -\frac{\kappa}{a}\pi_1$$

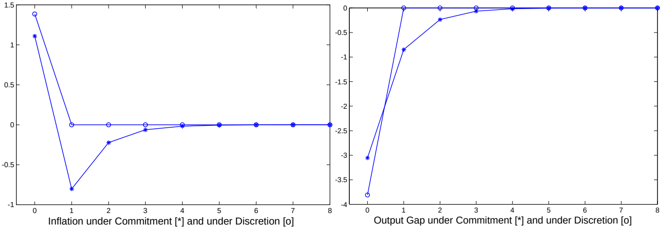
- This directly shows that the optimal policy with commitment is *time inconsistent*: at time 0, what we want to implement for period 1 is not what we *will want* to implement for period 1 at time 1.

Time inconsistency of the policy

- Hence, the CB has an incentive to deviate from its past announcements (and that would even be entirely in the interest of the households)
- So perhaps the people should not believe the announcement? Then we can (and should) only do discretionary monetary policy over a one-period horizon.
- Hence: intertemporally optimal policy can only implemented when the CB can *commit* to a rule (rule = what happens to x and π in the present and future). Hence the name "optimal policy under commitment"
- In practice: importance of CB's mandate and communication strategy

Commitment vs. discretion

Numerical solution: impulse responses to a temporary positive cost-push shock at time $t = 0$:



Under commitment:

- x_{t+j} is negative for a while $\Rightarrow$ inflation expectations fall (NKPC lower) $\Rightarrow$ π_0 increases by less than under discretion without large decrease in x_0.
- Optimal policy under commitment internalizes the dynamic externality of inflation expectations via the NKPC.
- Hence, outcome leads to higher welfare (intuitively, and also formally).

What if the CB cannot observe everything?

In reality, the CB faces uncertainty:

- Shock uncertainty (e.g. e_t not directly observed)
- Parameter uncertainty (e.g. slope of NKPC κ not observed)

Pretty straightforward to study optimal policy problem with these extensions in the linear-quadratic framework. Basic outcomes:

- If there is uncertainty about something that enters *additively* (like shocks), the CB should ignore the uncertainty and act based on the expectation.
- If there is uncertainty about something that enters *multiplicatively* (like parameters in front of the variables), then the CB should do MP more cautiously (i.e. muted responses).

But if there is parameter uncertainty, the CB should be able to learn the correct parameters over time...
Now: what if the private agents cannot observe everything?

Lucas Island Model (1972, 1973)

- Assume there is a continuum of islands on the unit interval, indexed by i.
- The (log deviation of the) price level on island i is partly determined by aggregate shocks (to the average price level), and partly by island-specific idiosyncratic shocks z_t^i,

$$p_t^i = p_t + z_t^i$$

  We assume that the z_t^i are i.i.d. with mean zero, variance σ_z, gaussian, and orthogonal to p_t.
- The supply curve of goods on island i would in principle depend on the difference between island i price and aggregate price:

$$\left(\quad y_t^i = b(p_t^i - p_t) \quad \right)$$

  Unfortunately firms on island i dont observe the average price level p_t, and need to infer it from (a) overall information set I_t (containing e.g. past values), (b) island-specific price p_t^i:

$$y_t^i = b(p_t^i - E(p_t | I_t, p_t^i))$$

- Firms form model-consistent expectations to infer the aggregate component of the price level:

$$p_t = E(p_t | I_t, p_t^i) + \epsilon_t$$

  Since E is an orthogonal projection, the prediction error ϵ_t is orthogonal to the prediction and has conditional variance σ_p^2.
- Projection theorem for normal random variables:

$$E(Y|X) = E(Y) + \frac{Cov(Y, X)}{Var(X)}(X - E(X))$$

- Hence

$$E(p_t | I_t, p_t^i) = E(p_t | I_t) + \frac{\sigma_p^2}{\sigma_p^2 + \sigma_z^2}(p_t^i - E(p_t^i | I_t))$$

  Intuition: if σ_z^2 is small, then $p_t^i \approx p_t$ and p_t^i is a good predictor for p_t.
- $\theta \equiv \sigma_p^2 / (\sigma_p^2 + \sigma_z^2)$ is called the *signal-to-noise ratio*.

- Note that because z_t^i has mean zero (well, conditional on I_t), we have that $E(p_t^i|I_t) = E(p_t|I_t)$.
- The supply curve becomes

$$y_t^i = b(1-\theta)(p_t^i - E(p_t|I_t))$$

and on aggregate

$$y_t = b(1-\theta)(p_t - E(p_t|I_t))$$

hence $E(y_t|I_t) = 0$.
- Write down demand curve

$$m_t = p_t + y_t - v_t$$

where v_t is a money demand shock, m_t is a money supply shock (orthogonal to I_t). Then,

$$m_t - E(m_t) = p_t - E(p_t|I_t) + y_t - (v_t - E(v_t))$$

- Plug in the AS curve,

$$m_t - E(m_t) = (1 + b(1-\theta))(p_t - E(p_t|I_t)) - v_t + E(v_t)$$

- Hence

$$\sigma_p^2 = \frac{\sigma_m^2 + \sigma_v^2}{(1 + b(1-\theta))^2}$$

Results:

- Only unanticipated shocks matter: if $v_t = E(v_t)$ and $m_t = E(m_t)$, then $p_t = E(p_t)$ and $y_t = 0$.
- This is despite agents being nowhere close to full information (but they're right on average!)
- Rational Expectations is key here!
- Additional assumptions: no nominal rigidities, and policymaker has same information set as private agents

Lucas-Sargent-Wallace *Policy Ineffectiveness Proposition*

Problems with the Lucas Islands model

- Effects of MP should be very short-lived
- More general: empirical evidence rejects the policy ineffectiveness hypothesis
- Aggregate information is readily available. Why don't the agents look them up? ($\Rightarrow$ Rational inattention literature)
- Islands? Wtf!?

Mankiw and Reis (2002): Sticky information

(see also Walsh, Chapter 5.2.3)

- Instead of having a probability $1-\phi$ of being allowed to reset prices (Calvo/NK), assume that firms can always update their prices but receive new information only with probability $1-\phi$. Hence a firm that updated information j periods ago sets prices

$$p_t(j) = \mathbb{E}_{t-i} p_t^*$$

- Assume that optimal prices are (in logs)

$$p_t^* = p_t + \alpha x_t$$

  where x_t is again the output gap (which contains real marginal cost)
- If all firms would set $p_t(i) = p_t^*$, then $p_t^* = p_t$ and $x_t = 0$.
- With frictions: all firms who set their price i periods ago are the same, denote by $p_t^i = E_{t-i} p_t^*$.

- Analogously to the Calvo calculations:

$$p_t = (1-\phi)\sum_{i=0}^{\infty}\phi^i E_{t-i} p_t^* = (1-\phi)\sum_{i=0}^{\infty}\phi^i E_{t-i}(p_t + \alpha x_t) \quad (10)$$

and, taking differences,

$$\begin{aligned} p_t - p_{t-1} = (1-\phi)(p_t + \alpha x_t) + (1-\phi)\sum_{i=1}^{\infty}\phi^i E_{t-i}(\Delta p_t + \alpha \Delta x_t) \\ - (1-\phi)^2 \sum_{i=0}^{\infty}\phi^i E_{t-i-1}(p_t + \alpha x_t) \end{aligned}$$

- Use (10) again to rewrite the last sum as $(\phi p_t - (1-\phi)\alpha x_t)/((1-\phi)\phi)$ and get

$$\pi_t = \frac{1-\phi}{\phi}\alpha x_t + (1-\phi)\sum_{i=1}^{\infty}\phi^i E_{t-i}(\pi_t + \alpha \Delta x_t)$$

- This is the Sticky Information Phillips Curve.

$$\pi_t = \frac{1-\phi}{\phi}\alpha x_t + (1-\phi)\sum_{i=1}^{\infty}\phi^i E_{t-i}(\pi_t + \alpha\Delta x_t)$$

- Now *past* expectations of *current* inflation rates determine the current inflation rate.
- Hence inflation will respond sluggishly to shocks.
- Close model by assuming quantity theory demand curve, as above:

  $$m_t = p_t + x_t - v_t$$

  and in differences,

  $$\Delta m_t = \pi_t + \Delta x_t - \Delta v_t$$

  Assume money supply process is

  $$\Delta m_t = \rho\Delta m_{t-1} + u_t$$

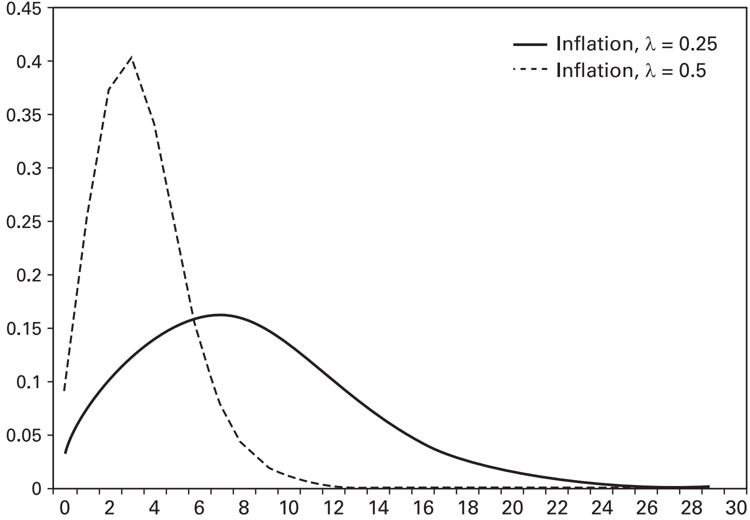


Figure 5.1
Response of inflation to a unit innovation in money growth in the sticky information model.

(Source: Walsh Ch. 5. Note that $\lambda \equiv 1 - \phi$)

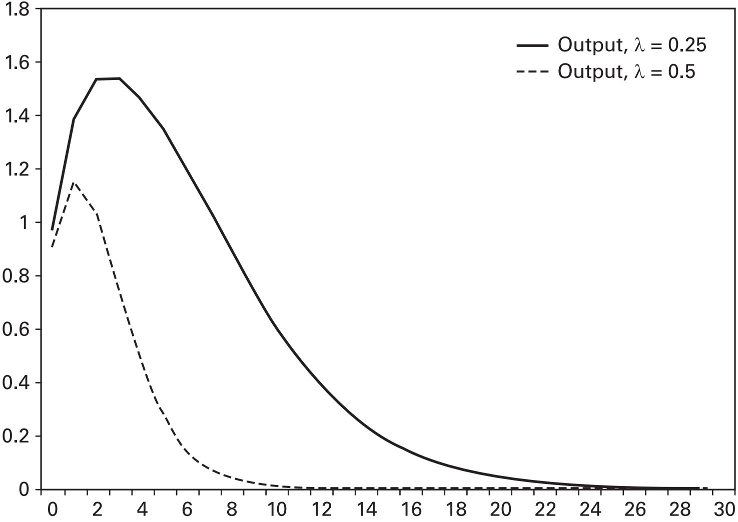


Figure 5.2
Response of output to a unit innovation in money growth in the sticky information model.

(Source: Walsh Ch. 5. Note that $\lambda \equiv 1 - \phi$)

Criticism

- Microfoundations? :/
- How should we identify SI from SP? Shape of hump different, but there could be a lot of things that determine the shape of the hump...

Some recent research areas

- Higher-order expectations: Keynesian beauty contest (trying to guess the guess of the others). Can amplify things (e.g. Woodford, 2001, Kris Nimark, Marios Angeletos)
- Learning. If you learn fast enough, information frictions should not matter. Generally it's hard to make agents learn slowly enough so that we can match the persistence of shocks in VARs. (Sargent, Marcet, Adam, Evans)
- Rational Inattention. Argues that people have limited information processing capabilities, pick the information that is most valuable to them. (Sims, Mackowiak, Wiederholt, Woodford)
  (*But do they really know what information is available?)*
- Robustness: What if agents don't know what the right model is? Can we still find good policies? (Lars Hansen, Tom Sargent)

Big research problem: information flows are usually not observed.