Macroeconomics A

Lecture 4 - Money and the New Keynesian Model

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'Old' Monetary Economics (up to late 70s)

Does an unanticipated shock to money affect real side of the economy (quantities)?

- Classical view: no. If prices are perfectly flexible, any unanticipated shock to money supply proportionally changes wages and prices, leaves quantities unchanged (money neutrality result) Think: Edgeworth box (undergrad micro)
  - That does not mean that gov printing money and spending it has no effect:
  - Redistributionary channel: gov prints money and spends it $\rightarrow$ prices increase $\rightarrow$ nominal assets lose in value $\rightarrow$ negative wealth effect
  - Money supply increase acts like a tax on nominal asset holdings $\rightarrow$ "inflation tax"

'Old' Monetary Economics (up to late 70s)

Does an unanticipated shock to money affect real side of the economy (quantities)?

- Keynesian view: yes. If prices are rigid, an unanticipated increase in the money supply will act like a positive wealth effect (higher purchasing power).
- Again: this assumes that new money is distributed proportionally to existing money holdings

'Old' Monetary Economics (up to late 70s)

How should we conduct monetary policy?

- Monetarists: steady money supply growth, in line with money demand growth (i.e. in line with real output growth)
  - Not too tight: Great Depression was partly caused by too tight MP
  - But having loose MP to stimulate the economy will only lead to inflation (neoclassical argument).
- Keynesians: use expansionary MP to stimulate the economy in a recession (except liquidity trap)
- Inflation of 70's and subsequent successful tight MP (Volcker) led to Monetarists becoming very influential

Some problems with 'old' models

- IS-LM and Classical models are inherently static. Inflation is something dynamic. Hard to talk about dynamic effects in a static model
- Old models are subject to the Lucas critique (policy changes may affect people's response to the policies)
- Hard to take seriously from a quantitative perspective

RBC models already try to address all three concerns, so people have started putting money into these models.

- Outcome from this program is the "New Keynesian Synthesis": combine RBC models with price rigidities.
- Result: Money is non-neutral in the short run (Keynesian element), and neutral in the long run (classical element).
- But how do we know that this is the right outcome?

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Most widely accepted econometric methodology: Vector Autoregressions (VARs)

- In micro: try to get causal inference by regressing y on x, with assumptions on exogeneity

Empirical evidence on the effects of money supply shocks

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Most widely accepted econometric methodology: Vector Autoregressions (VARs)

- In micro: try to get causal inference by regressing y on x, with assumptions on exogeneity
- Here, similar idea. Example with detrended money stock M and detrended real output Y:

$$Y_t = \phi_{YY} Y_{t-1} + \phi_{YM} M_t + u_t^Y$$
$$M_t = \phi_{MY} Y_t + \phi_{MM} M_{t-1} + u_t^M$$

where u^M are money supply shocks and u^Y are output supply (e.g. TFP) shocks (independent from other time t variables and from each other).

Simple example

$$Y_t = \phi_1 Y_{t-1} + \phi_2 M_t + \phi_3 M_{t-1} + u_t^Y$$
$$M_t = \phi_4 Y_t + \phi_5 M_{t-1} + \phi_6 Y_{t-1} + u_t^M$$

- Problem: OLS will provide inconsistent estimates of the ϕ's
- RHS variables are endogenous
- We could do it if we had direct measures of the shocks, or an instrument (we don't)

Rearrange to get:

$$Y_t = \frac{1}{1-\phi_2\phi_4}\left((\phi_1 + \phi_2\phi_6)Y_{t-1} + (\phi_3 + \phi_2\phi_5)M_{t-1} + u_t^Y + \phi_2 u_t^M\right)$$
$$M_t = \frac{1}{1-\phi_2\phi_4}\left((\phi_1\phi_4 + \phi_6)Y_{t-1} + (\phi_3\phi_4 + \phi_5)M_{t-1} + \phi_4 u_t^Y + u_t^M\right)$$

(Continued from last slide)

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- But now we can run OLS! That gives us consistent estimates of the coefficients (under the assumption that the u are $\perp$ to variables at $t-1$)

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- But now we can run OLS! That gives us consistent estimates of the coefficients (under the assumption that the u are $\perp$ to variables at $t-1$)
- Still, we only estimate four coefficients, and have six unknown parameters.
- The residuals that we get are

$$\varepsilon_t^Y = \frac{1}{1 - \phi_2\phi_4}\left(u_t^Y + \phi_2 u_t^M\right)$$
$$\varepsilon_t^M = \frac{1}{1 - \phi_2\phi_4}\left(\phi_4 u_t^Y + u_t^M\right)$$

Need to make additional identifying assumptions. Two alternatives:

1. $\phi_4 = 0$, i.e. monetary policy does not react to the contemporaneous level of output (because of data limitations).
   - Then:
   $$\varepsilon_t^Y = \phi_2 \varepsilon_t^M + \frac{1}{1 - \phi_2 \phi_4} u_t^Y$$
   - Can estimate ϕ_2 from regression of ε_t^Y on ε_t^M.
   - Then get the other four ϕ's from the estimated coefficients in the OLS regression before.
2. $\phi_2 = 0$, i.e. output does not react to a change in monetary policy (because of decision lags)
   - Then:
   $$\varepsilon_t^M = \phi_4 \varepsilon_t^Y + \frac{1}{1 - \phi_2 \phi_4} u_t^M$$
   - Can estimate ϕ_4 from regression of ε_t^M on ε_t^Y.
   - Again, get the other four ϕ's from the estimated coefficients in the OLS regression before.

Identification

Either assumption identifies all the parameters of the VAR.

- Once we have all the ϕ parameters, we know the economy's response to a money shock u_t^M or to a real shock u_t^Y. Equations from before:

$$Y_t = \frac{1}{1-\phi_2\phi_4}\Big((\phi_1+\phi_2\phi_6)Y_{t-1} + (\phi_3+\phi_2\phi_5)M_{t-1} + u_t^Y + \phi_2 u_t^M\Big)$$
$$M_t = \frac{1}{1-\phi_2\phi_4}\Big((\phi_1\phi_4+\phi_6)Y_{t-1} + (\phi_3\phi_4+\phi_5)M_{t-1} + \phi_4 u_t^Y + u_t^M\Big)$$

- Hence we have an estimated impulse response that does not rely on a theoretical model! (but still relies on linear functional form & identification assumptions...)
- In principle you can do this for any type of shock and any variables (as long as you have good identification assumptions!)
- Started by Chris Sims (1980). Nobel Prize 2011. Many other variations of identification restrictions (e.g. Blanchard-Quah 1989)

Christiano, Eichenbaum, Evans (1999)

Study effect of a monetary shock (as defined through the VAR)

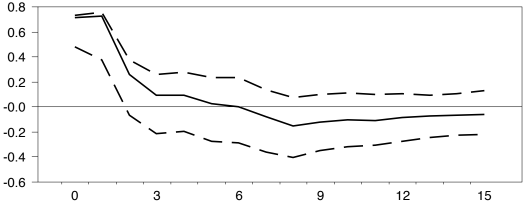
- US quarterly data, 1965–1995
- Includes GDP, price level, federal funds rate, money supply, and some other variables. More lags as well.
- Several identification assumptions.

Main findings:

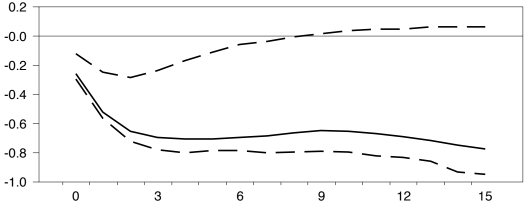
- Slow adjustment of output and prices (prices more sluggish than output)
- Short-run non-neutrality but long-run neutrality of money.

Response to a contractionary monetary shock

Interest rate (FFR) rises:

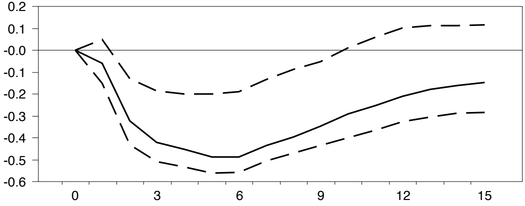


Money supply (M2) falls:

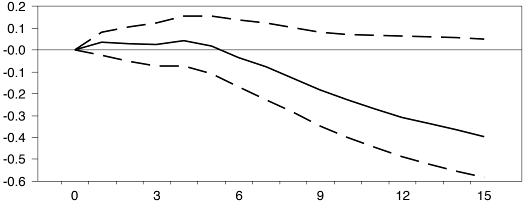


Response to a contractionary monetary shock

Aggregate output falls:



Prices fall (relative to trend)



The Monetary Transmission Mechanism

- Ultimately, we need a model that generates impulse responses similar to the ones by Christiano, Eichenbaum, Evans (1999).
- Monetary policy should be a demand shock: loose MP should stimulate demand through increased consumption and investment.
- We'll look at several building blocks first, and will then assemble some of them to form the New Keynesian model.
- 1. What determines demand (consumption/investment)?
- 2. What is monetary policy exactly?
- 3. The role of nominal rigidities

Households and nominal assets

Let's extend the household's problem by an option to hold nominal assets.

- Here: nominal assets are bonds B_t which pay interest at nominal rate i_t
- Ignore labor-leisure choice here ($L = 1$). Households maximize

$$\max E_t \sum_{i=0}^{\infty} U(C_{t+i})$$

subject to budget constraints

$$\begin{aligned} P_{t+i}C_{t+i} + P_{t+i}K_{t+i+1} + B_{t+i+1} \\ = W_{t+i}L + P_{t+i}(1 + r_{t+i})K_{t+i} + (1 + i_{t+i})B_{t+i} \end{aligned}$$

where W is the nominal wage, P is the price of consumption (=capital) goods, and r_{t+i} is the rental rate of capital net of depreciation.

First-order conditions

(E_t of:)

- For C_{t+i}:
$$U'(C_{t+i}) = P_{t+i}\lambda_{t+i}$$
- For K_{t+i+1}:
$$P_{t+i}\lambda_{t+i} = \lambda_{t+i+1}P_{t+i+1}(1 + r_{t+i+1})$$
- For B_{t+i+1}:
$$\lambda_{t+i} = \lambda_{t+i+1}(1 + i_{t+i+1})$$
- Combining the first two we get the standard Euler equation:
$$U'(C_t) = E_t\left((1 + r_{t+1})U'(C_{t+1}\right)$$
- Combining the first and third,
$$U'(C_t) = E_t\left(\frac{1 + i_{t+1}}{1 + \pi_{t+1}}U'(C_{t+1})\right)$$

where inflation $\pi_{t+1} = (P_{t+1} - P_t)/P_t$.

Lessons

$$U'(C_t) = E_t\left((1 + r_{t+1})U'(C_{t+1})\right)$$
$$U'(C_t) = E_t\left(\frac{1 + i_{t+1}}{1 + \pi_{t+1}}U'(C_{t+1})\right)$$

1. What matters for the intertemporal consumption decision is the *real* interest rate, not the nominal interest rate.
2. The FOC are necessary conditions for having both positive B and positive K. Hence, in order for this to be the case, we need to have that in expectation
$$1 + r_{t+1} = \frac{1 + i_{t+1}}{1 + \pi_{t+1}}$$
3. This motivates the following definition: the real return (interest rate) of a nominal asset with nominal return $1 + i_{t+1}$ is $\frac{1+i_{t+1}}{1+\pi_{t+1}}$. This is generally not known at time t (because inflation is not known).
4. This relationship is called the *Fisher equation*.

What is the Fisher equation?

$$1 + r_{t+1} = \frac{1 + i_{t+1}}{1 + \pi_{t+1}}$$

1. The Fisher equation can be a *definition* of the *real* return on a nominal asset (e.g "nominal return on a 1y bond is 4%, inflation is 2%, so real return is approx. 2%")
2. In general, it's the relationship between asset returns that has to hold so that the household is willing to hold positive amounts of these assets in their portfolio (or: if you allow negative holdings, to clear asset markets)
   2.1 When households are risk-neutral (i.e. linear utility) the FOCs become
   $$E_t(1 + r_{t+1}) = E_t\left(\frac{1 + i_{t+1}}{1 + \pi_{t+1}}\right)$$
   2.2 In a deterministic world,
   $$1 + r_{t+1} = \frac{1 + i_{t+1}}{1 + \pi_{t+1}}$$
   which is often approximated as $i_{t+1} = i_{t+1} - \pi_{t+1}$.

Firms

Okay, for household consumption it is the real interest rates that matter. What about firms?

- Profit maximization, with perfect competition and frictionless factor markets:

$$MPK = r$$

- Again, *real* interest rate matters.
- Conclusion: if monetary policy should affect demand (consumption and investment), then it needs to affect the real interest rate.

Next: the tools of monetary policy.

The tools of monetary policy

Central bank wants to stimulate demand

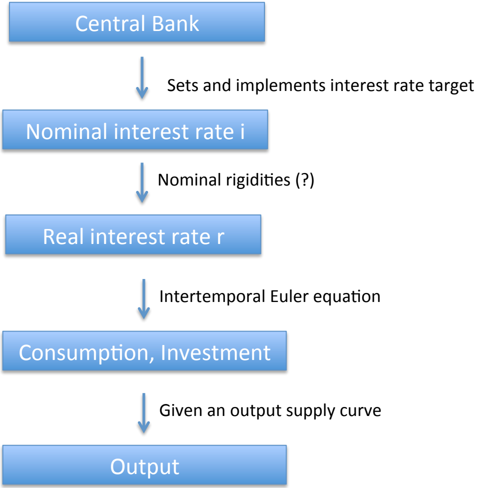
- Need to be able to change the real interest rate
- How do we do that?
- Weapon of choice is the nominal interest rate
  - We could also change the money supply. Together with the money demand curve, this would implement a particular nominal interest rate
  - Better to target nominal interest rate directly.

Agenda:

- First, a model of central bank's tools and the interbank market (similar: Walsh 3rd Ed, Chapter 11.4.3)
- Then, back to macro: New Keynesian model (Walsh Ch. 8, Gali 2008 Ch. 3)

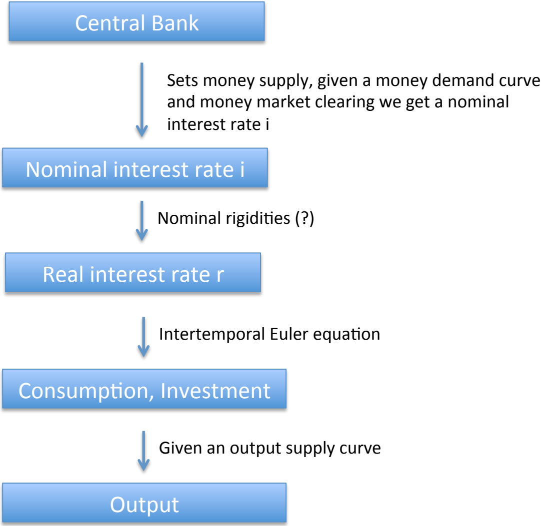
Overview

In a cashless economy:

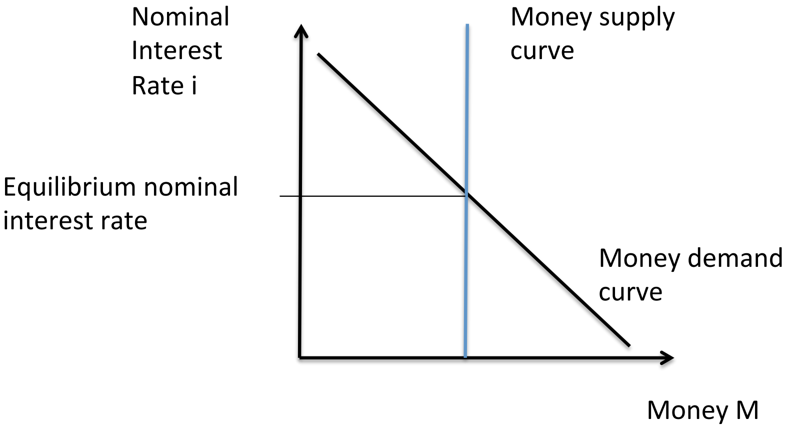


Overview

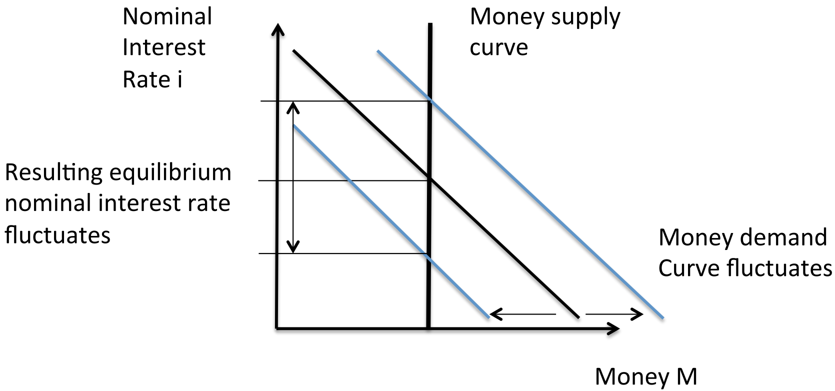
In an economy with money



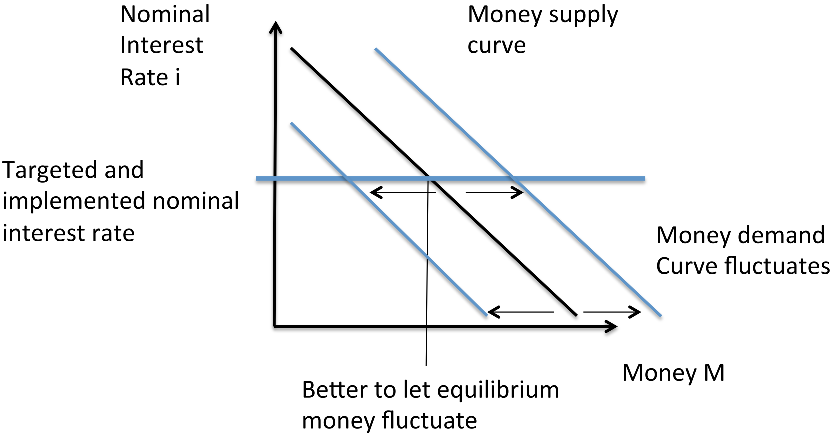
Money Market equilibrium



Changes in money demand cause i fluctuations



Better to target and implement a particular interest rate



A model of the interbank market

- Market participants: n banks
- Banks can hold **reserves** (deposits at the central bank)
- Reserves are used for settling transfers between banks arising from the payments system (e.g. a customer of one bank writes a cheque payable to the customer of another)
- Banks are unsure of the transfers they wil need to make
- Banks can borrow reserves from one another in the interbank market, but do not know how much reserves they will require at the point they participate in the interbank market
- Banks can also acquire reserves through the repo (repurchase agreement) market. The central bank can be a participant in this market.

The model

Notation

- B_j = beginning-of-period holding of reserves by bank j (after previous debts settled)
- I_j = net borrowing in the interbank market (negative for lending) by bank j
- P_j = net bond repos (sale and repurchases) by bank j
- T_j = net payment bank j must make to other banks (uncertain)
- R_j = balance of bank j's account at end of time period (uncertain because T_j is not known in advance)

Accounting:

$$R_j = B_j + I_j + P_j - T_j$$

Interbank market and repo market

Interbank market:

- Uncollateralized lending between banks (potentially risky)
- Interest rate$=i$

Repo market:

- Sell bond (e.g. Treasury bond) now and agree to repurchase (at a higher price, reflecting the repo interest rate) later
- Effectively a collateralized loan, so risk-free if collateral is good.

Let us assume that banks are not thought likely to default on interbank loans, so these are perceived to be risk-free. Hence: interbank interest rate should be the same as repo rate.

Central-bank standing facilities

- **Deposit facility:** central bank offers an interest rate of i_d on positive balances in banks' accounts at the end of the time period
- **Borrowing facility:** central bank charges an interest rate of i_b on negative balances in banks' accounts at the end of the time period
  - positive spread over deposit rate: $i_b > i_d$
  - equivalent to an offer to make loans at the end of period, with the load credited to the bank's account
  - in practice, collateral required — we shall assume banks have enough collateral to use it

These are referred to as **standing facilities** because banks know the facilities will be available to them to use as needed at known interest rates (and without any restriction on the magnitude of their use).

Balance on account next period

Reserve balance of bank j at the beginning of next period (after interbank loans repaid, repos mature, and standing facilitiy rates paid/levied):

$$B_j' = R_j - (1+i)(I_j + P_j) + \begin{cases} i_d R_j & \text{if } R_j \geq 0 \\ i_b R_j & \text{if } R_j < 0 \end{cases}$$

Since $R_j = B_j + I_j + P_j - T_j$:

$$B_j' = B_j - i(I_j + P_j) - T_j + \begin{cases} i_d(B_j + I_j + P_P - T_j) & \text{if } T_j \leq B_j + I_j + P_j \\ i_b(B_j + I_j + P_P - T_j) & \text{if } T_j > B_j + I_j + P_j \end{cases}$$

Transfer T_j is unknown until end of period, so B_j' is uncertain when banks choose I_j and P_j. Density function of T_j is $f(T_j)$ and distribution function is $F(T_j)$. Assume $\mathbb{E}(T_j)$=0.

Bank objective function

Assume risk-neutral banks. Aim to maximize expected next period reserve balance $\mathbb{E}(B_j')$. Choice variables: interbank transaction $= I_j$, repo market transaction $= P_j$.

$$\mathbb{E}(B_j') = B_j - i(I_j + P_j) + i_d \int_{-\infty}^{B_j+I_j+P_j} (B_j + I_j + P_j - T_j) f(T_j) dT_j + i_b \int_{B_j+I_j+P_j}^{\infty} (B_j + I_j + P_j - T_j) f(T_j) dT_j$$

First-order conditions:

$$\frac{\partial \mathbb{E}(B_j')}{\partial I_j} = 0 \quad \text{and} \quad \frac{\partial \mathbb{E}(B_j')}{\partial P_j} = 0$$

These lead to the same equation (liquidity choice is only thing that matters), so we only need to consider one.

First-order condition

To derive this, use the Leibiz rule for the derivative of an integral:

$$\frac{\partial}{\partial I_j}\int_{-\infty}^{B_j+I_j+P_j}(B_j+I_j+P_j-T_j)f(T_j)dT_j=\int_{-\infty}^{B_j+I_j+P_j}f(T_j)dT_j \tag{1}$$

$$+((B_j+I_j+P_j)-(B_j+I_j+P_j))f(B_j+I_j+P_j)=F(B_j+I_j+P_j) \tag{2}$$

Similarly:

$$\frac{\partial}{\partial I_j}\int_{B_j+I_j+P_j}^{\infty}(B_j+I_j+P_j-T_j)f(T_j)dT_j=1-F(B_j+I_j+P_j)$$

Hence, the first-order condition reduces to

$$-i+i_dF(B_j+I_j+P_j)+i_b(1-F(B_j+I_j+P_j))=0$$

Equivalently (at margin, expected benefit = expected loss):

$$(i-i_d)F(B_j+I_j+P_j)=(i_b-i)(1-F(B_j+I_j+P_j))$$

Optimal use of interbank and repo markets

The optimal demand for interbank and repo lending is characterized by the equation:

$$F(B_j + I_j + P_j) = \frac{i_b - i}{i_b - i_d}$$

- Since $0 \leq F(B_j + I_j + P_j) \leq 1$, the FOC implies $i_d \leq i \leq i_b$, so the interest rate i must lie in the channel bounded by i_b and i_d in equilibrium. Intuition: No one would lend at an interest rate that is lower than what you get from the central bank; no one would borrow at an interest rate that is higher than what you'd have to pay to the central bank.
- Since F is increasing, there is a negative relationship between i and $I_j + P_j$.
- Initial balance B_j is predetermined, so equation implies banks with high B_j will have low or negative values of $I_j + P_j$, i.e. will be lenders.

Aggregation and market clearing

- Aggregate beginning-of-period balances: $B = \sum_{j=1}^{n} B_j$
- Aggregate end-of-period balances $R = \sum_{j=1}^{n} R_j$
- Payments between banks cancel out: $\sum_{j=1}^{n} T_j = 0$
- Repo market transactions net out to central bank's open-market operation: $\sum_{j=1}^{n} P_j = P$, where P is (reverse) repos by central bank
- Since $R_j = B_j - T_j + I_j + P_j$, it follows that $R = B + P$ in the aggregate, so P is net injection of new reserves.
- Since $B_j + I_j + P_j$ is the same for all banks: $B_j + I_j + P_j = \frac{R}{n}$

Equilibrium interbank interest rate

Equilibrium i is determined by the demand curve for reserves (aggregate FOC)

$$F(\frac{R}{n}) - \frac{i_b - i}{i_b - i_d}$$

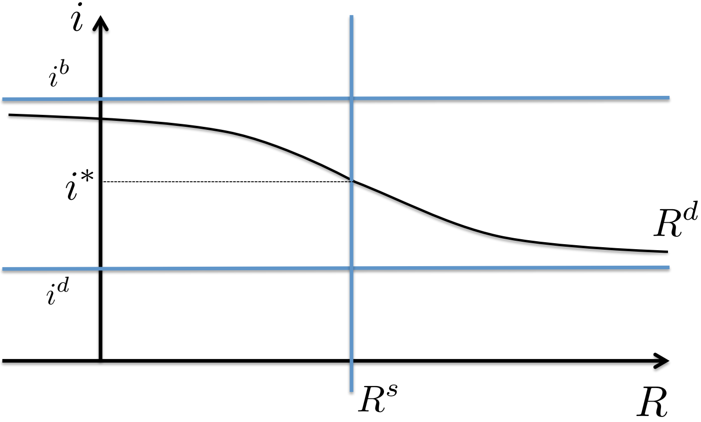
The central bank directly controls

- the deposit rate i_d
- the borrowing rate i_b
- open market operations P

These indirectly determine

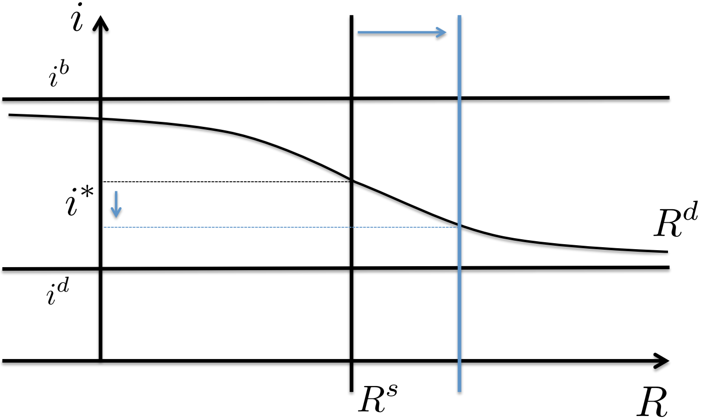
- Total end-of-period reserves $R = B + P$
- Interbank and repo rate i

Interbank market with channel system

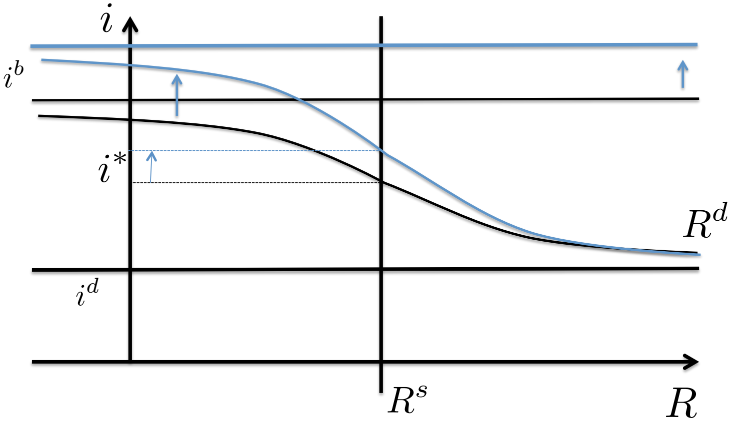


Demand: The equation $F(\frac{R}{n}) - \frac{i_b - i}{i_b - i_d}$, which implies a negative relationship between i and R.
Supply: The equation $R = B + P$ (inelastic — assuming CB can use open market operations)

Increase in Open Market Operations decreases equilibrium interbank interest rate



Increase in borrowing rate stretches demand curve, increases equilibrium interbank interest rate



Comparative statics

Holding constant other policy instruments:

- An increase in i_d “squashes” the demand curve from below: i rises
- An increase in i_b “streches the demand curve from above: i rises
- An increase in total reserves R shifts the supply curve to the right: i falls

A simultaneous increase of i_d and i_b by the same amount implies a parallel upward shift of the demand curve
If T_j has a symmetric distribution, then setting $R = 0$ implies i will be exactly halfway between i_d and i_b.

Changing interest rates

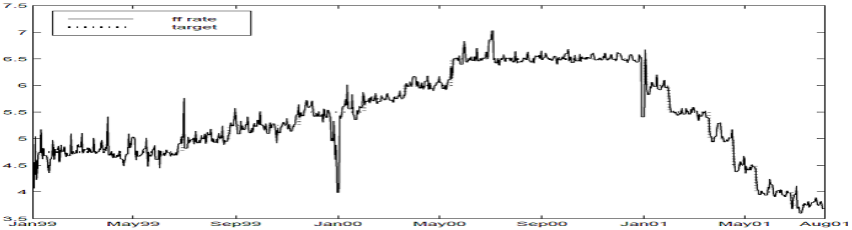
- Normally we think that changing interest rates requires a carefully calibrated open-market operation (need to know money demand function)
- This is much easier with the channel system: given R, i_d, and i_b, suppose the interest rate is initially i, which satisfies $F(R/n) = (i_b - i)/(i_b - i_d)$.
- Now suppose central bank would like to increase i by x, i.e. achieve $i' = i + x$. Set new deposit and borrowing rates $i'_d = i_d + x$ and $i'_b = i_b + x$. Note that

$$\frac{i'_b - i'}{i'_b - i'_d} = \frac{(i_b + x) - (i + x)}{(i_b + x) - (i_d + x)} = F\left(\frac{R}{n}\right)$$

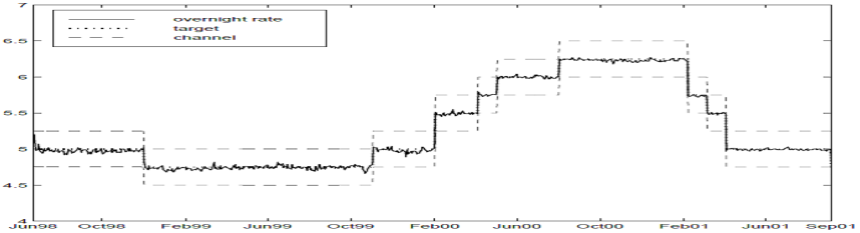
  so no new open market operation ($R' = R$) is required.
- Intuition: reserve demand depends on interbank rate **relative** to standing facility rates in channel system.

Success of the channel-based system

US (traditional system) — greater volatility around target than



interest rate in Australia (channel system)



How wide should the channel be?

A narrower channel has advantages:

- Guarantee that interest rate fluctuations are smaller
- Less need for "fine-tuning" open market operations

But there can be disadvantages. Think, e.g. $i_d = i_b$

- Trading in interbank market dries up
- CB becomes the intermediary for all borrowing and lending between banks. Hence, the CB incurs all the costs from having to monitor the credit-worthiness of banks (that's not the role of central banks!)

Interest rate maturity

The model above shows how the central bank can “set” short-term interbank interest rates (overnight, one week)

But how does this affect medium- and long-term interest rates?

More generally, how are interest rates of different maturities related?

- Imagine an investor wants to invest over two years. He has the choice of
  - buying a two-year bond, and keeping it until it matures

$$\text{return} = 1 + i_t^{(2y)}$$

  - buying a one-year bond, and, after one year, buy another one-year bond

$$\text{return} = (1 + i_t^{(1y)})(1 + i_{t+1}^{(1y)})$$

Expectations hypothesis

- The **Expectations Hypothesis** says that in expectations these two returns should be the same

$$1 + i_t^{(2y)} = \mathbb{E}_t(1 + i_t^{(1y)})(1 + i_{t+1}^{(1y)})$$

- cf. portfolio choice model from last week
- This should hold exactly if agents are risk-neutral. With risk aversion, one would adjust the expected returns for riskiness
- Still: current long-term rates are determined from current short-term rates and expectations of future short term rates.
- Can use announcements of future policies to affect current long-term rates (forward guidance)

New Keynesian Model

Benchmark macroeconomic model since the early 2000's.

- Start off with baseline RBC model, add a nominal asset (one-period bond)
- Price stickiness only makes sense when firms can set prices; hence, need some degree of market power. Special case of Rotemberg & Woodford.
- Nominal rigidities: Calvo pricing: firm-specific shock that tells you whether you're allowed to reset your price.

Again, will need to log-linearize to keep dynamics tractable.

Households

Representative household, maximizes utility

$$\max \sum_{i=0}^{\infty} \beta^i \left(u(c_{t+1}) - \nu(l_{t+i}) \right)$$

subject to budget constraints (for all i)

$$p_t c_t + b_t = p_t w_t l_t + p_t d_t + (1 + i_{t-1}) b_{t-1}$$

- c_t is quantity of baskets of goods consumed (baskets to be defined later), p_t is the price of each basket in terms of money
- w_t is the real wage
- d_t are dividends (= firms' profits) received from owning firms
- i_t nominal interest rate
- b_t holding of nominal bonds

For simplicity: labor is the only factor of production.

Consumption and Labor Supply

Intertemporal optimality condition for consumption as always

$$u'(c_t) = \beta \mathbb{E}_t (1 + r_t) u'(c_{t+1})$$

where r_t is the real interest rate implied by the Fisher equation:

$$1 + r_t = \frac{1 + i_t}{1 + \pi_{t+1}}$$

and $\pi_{t+1} = (p_{t+1} - p_t)/p_t$ is expected inflation defined in terms of the price index p_t for the consumption basket.
Labor supply: intratemporal FOC

$$MRS_{l,c} = \frac{v'(l_t)}{u'(c_t)} = w_t$$

Consumption aggregator

- Introduce the basket of imperfectly substitutable goods through a consumption aggregator, a description of consumer preferences over the whole set of goods (indexed by i).
- Formula linking consumption $c(i)$ of each good $i \in [0, 1]$ to an "amount" of total baskets c enjoyed by the household.
- Most common aggregator formula is the CES (constant elasticity of substitution; also called Dixit-Stiglitz)

$$c = \left(\int c(i)^{\frac{\epsilon - 1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon - 1}}$$

(ϵ is the elasticity of substitution)

Expenditure minimization

- Suppose that the household faces prices $p(i)$. Since utility depends on the $c(i)$ only through the basket c, we can solve the utility maximization problem in two steps
  - Given prices $p(i)$, solve for the cheapest way to assemble c baskets: cost minimization problem
  - Then, maximize utility over quantity c of baskets (and labor supply) ¡- we've already done that
- Expenditure minimization:

$$\min \int p(i)c(i)di \tag{3}$$

$$s.t. \quad \left(\int c(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}} = c \tag{4}$$

(To solve this, you can treat the integral like a sum.)

Solving this

- Lagrangian:

$$\mathcal{L} = \int p(i)c(i)di - \lambda \left(\left(\int c(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}} - c \right)$$

- FOC for $c(i)$:

$$p(i) = \lambda c(i)^{-\frac{1}{\epsilon}} \left(\int c(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}-1}$$

Plug in constraint to get

$$p(i) = \lambda c(i)^{-\frac{1}{\epsilon}} c^{\frac{1}{\epsilon}}$$

and rearrange to get

$$c(i) = \left(\frac{p(i)}{\lambda} \right)^{-\epsilon} c$$

Solving this

- Need to find Lagrange multiplier. Plug in the expression for λ into the constraint to get

$$\left(\int \left(\frac{p(i)}{\lambda} \right)^{1-\epsilon} c^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}} = c$$

- Take c and λ out of the integral and cancel to get

$$\lambda = \left(\int p(i)^{1-\epsilon} di \right)^{\frac{1}{1-\epsilon}}$$

- Interpretation of λ as the shadow price: "If you tighten the constraint by one unit, the objective function increases by λ"
- i.e. "If you have to buy one unit of the basket more, you have to pay λ more". Hence: λ is the price of one basket,

$$p_t = \lambda.$$

Demand function

- Hence, we get the **demand function** for variety i:

$$c(i) = \left(\frac{p(i)}{p}\right)^{-\epsilon} c$$

where

$$p_t = \left(\int p(i)^{1-\epsilon} di\right)^{\frac{1}{1-\epsilon}}$$

is the overall price index.

- Interpretation: demand depends on the relative price $p(i)/p$
- Price elasticity of demand is now constant ϵ

Production technology and market clearing

Output $y(i)$ is governed by the production function

$$y_t(i) = a_t h_t(i)$$

- a_t = productivity (TFP), assumed to be the same for all goods
- $h_t(i)$ = hours of labor employed in producting good i

Goods market clearing: (assuming no investment of gov spending)

$$c_t(i) = y_t(i) \quad \text{and} \quad c_t = y_t$$

Labor market clearing: (interpret l_t as average labor supplied per good:

$$\int h_t(i) di = l_t$$

Demand, revenues, and costs

Firm i faces demand function:

$$y_t(i) = \left(\frac{p(i)}{p}\right)^{-\epsilon} y_t \quad \text{with} \quad p_t = \left(\int p(i)^{1-\epsilon} di\right)^{\frac{1}{1-\epsilon}}$$

Firm i has the power to set price $p_t(i)$, but it is too small to affect the general price level p_t or aggregate demand y_t (monopolistic competition).

Firm can hire labor in a perfectly competitive market. Let z_t denote the real cost of production per unit of output. With linear production function for $y(i)$, z_t is also the (real) marginal cost of producting a unit of output:

$$z_t = \frac{w_t}{a_t}$$

- Total real revenue $= (p_t(i)/p_t)y_t(i)$
- Total real cost of production $= z_t y_t(i)$

Profits

Profits (in real terms) made by firm i at time t are paid out as dividends, denoted by $d_t(i)$:

$$d_t(i) = \left(\frac{p_t(i)}{p_t} - z_t\right) y_t(i) = \left(\frac{p_t(i)}{p_t} - z_t\right)\left(\frac{p_t(i)}{p_t}\right)^{-\epsilon} y_t$$

Let $\mu_t(i)$ be firm i's (gross) markup of price over marginal cost:

$$\mu_t(i) \equiv \frac{p_t(i)}{p_t z_t}$$

The expression for profits/dividends can be rewritten in terms of the markup $\mu_t(i)$:

$$d_t(i) = z_t^{1-\epsilon} y_t (\mu_t(i)^{1-\epsilon} - \mu_t(i)^{-\epsilon})$$

With flexible prices

Consider first a world with flexible prices.
Price is set to maximize profits. First order condition:

$$\frac{\partial d_t(i)}{\partial \mu_t(i)} = z_t^{1-\epsilon} y_t((1-\epsilon)\mu_t(i)^{-\epsilon} + \epsilon \mu_t(i)^{-\epsilon-1}) = 0$$

Since z_t, y_t, $\mu_t(i)$ are > 0, you can cancel to get:

$$\mu_t^* = \frac{\epsilon}{\epsilon - 1}$$

A higher price elasticity ϵ results in a lower markup (perfect competition is limiting case $\epsilon \to \infty$; minimum price elasticity is one: $\epsilon > 1$)

General equilibrium with flexible prices

We now find equilibrium aggregate output y_t assuming firms have flexible prices.

Make some simplifying assumptions: work with a particular utility function:

$$u(c) = \frac{c^{1-\frac{1}{\sigma}} - 1}{1 - \frac{1}{\sigma}} \quad \text{and} \quad \nu(l) = l$$

hence the labor supply condition $\nu'(l_t)/u'(c_t) = w_t$ becomes

$$y_t^{\frac{1}{\sigma}} = w_t$$

(market clearing condition $c_t = y_t$ imposed)

Labor demand: Firm's profit maximization implies that wage equals the **marginal revenue product of capital**:

$$w_t = \frac{a_t}{\mu_t^*}$$

Flexible-price world: natural level of output

With flexible prices, all firms set the same markup: $\mu_t(i) = \mu_t^*$. Hence, the same price: $p(i) = p_t^*$. Hence, the markups become (see before)

$$\mu_t^* = \frac{1}{z_t}$$

The **natural level of output** is defined as the level of aggregate output y_t that prevails when prices are fully flexible.
Can find this by equating labor supply and labor demand with flexible prices:

$$y_t^{\frac{1}{\sigma}} = w_t = \frac{a_t}{\mu_t^*}$$

hence the natural level of output y_t^* is

$$y_t^* = \left(\frac{a_t}{\mu_t^*}\right)^{\sigma}$$

- Natural output is higher when productivity a_t is higher
- Natural output is lower when market power (as measured by μ_t^*) is higher

Natural real interest rate

The **natural (real) interest rate** is the real interest rate consistent with output being always at its natural level.

This requires $c_t = y_t = y_t^*$, so with the Euler equation,

$$u'(y_t^*) = \beta(1 + r_t^*)u'(y_{t+1}^*)$$

with r_t^* denoting the natural interest rate. Using $u'(c) = c^{-\frac{1}{\sigma}}$:

$$(y_t^*)^{-\frac{1}{\sigma}} = \beta(1 + r_t^*)(y_{t+1}^*)^{-\frac{1}{\sigma}} \text{ and formula for } y_t^*,$$

$$\frac{\mu_t^*}{a_t} = \beta(1 + r_t^*)\frac{\mu_{t+1}^*}{a_{t+1}}$$

Hence, the natural interest rate is given by:

$$1 + r_t^* = (1 + \bar{r})\frac{a_{t+1}}{a_t}\frac{\mu_t^*}{\mu_{t+1}^*}$$

where $\bar{r}$ is defined as the average interest rate consistent with the discount factor β, i.e. $\beta = 1/(1 + \bar{r})$.

Efficiency of output

Imperfect competition introduces a distortion, so market allocation of output is not Pareto efficient.

Perfect competition is consistent with efficiency, so we can ask what level of output is efficient by finding the equilibrium in the special case of perfect competition:

- perfect competition: perfectly elastic demand curve because of perfect substitutability (no market power): $\epsilon \to \infty$
- then, zero markup: $\mu_t^* \to 1$

Efficient output $\hat{y}_t$ found using formula for natural output, and setting $\mu_t^* = 1$:

$$\hat{y}_t = a_t^{\sigma}$$

Efficient interest rate $\hat{r}_t$ is the real interest rate consistent with output at its efficient level:

$$1 + \hat{r}_t = (1 + \bar{r})\frac{a_{t+1}}{a_t}$$

Nominal Rigidities

We're now going to introduce nominal rigidities in the form of sticky prices. Tricky to model, so we're going to make some simplifications.

Assume **Calvo-Yun pricing** assumption:

- In each time period, a randomly selected fraction $1 - \phi$ of firms receive an opportunity to set a new price
- All other firms must continue to use the same price as before.

The parameter ϕ represents the importance of the pricing friction. Calvo pricing is a simple idea that stands in for a much more complicated approach to modelling price adjustment:

- firms face a fixed physical cost of adjusting prices
- would then need to calculate the points in time when gains from price adjustment exceed the menu cost
- Very hard to do (deal with heterogeneity, re-setting thresholds, etc.). Example: Gertler and Leahy (2008)

Setting a new price

Suppose firm i sets a new price at time t. Newly set price $= s_t$.

- In each subsequent period: probability ϕ that price will not be changed
- Hence probability that price is still used at time $T > t$ is ϕ^{T-t}.

Firm maximized expected present discounted value of profits/dividends associated with using price s_t:

$$d_t(i) + \phi \frac{d_{t+1}(i)}{(1+r_t)} + \phi^2 \frac{d_{t+2}(i)}{(1+r_t)(1+r_{t+1})} + \cdots$$

where dividends (=profits) $d_t(i), d_{t+1}(i), \ldots$ are calculated assuming the price remains "sticky" at s_t:
$p_t(i) = p_{t+1}(i) = \cdots = s_t$.
(Note: if firm is allowed to re-set price (probability $= 1 - \phi$), it solves a new optimization problem, hence we don't need to take that into account here)

Setting a new price

From the expression for firms' profits (see before)

$$d_t(i) = \left(\frac{p_t(i)}{p_t} - z_t\right)\left(\frac{p_t(i)}{p_t}\right)^{-\epsilon} y_t$$

we get the objective function to maximize:

$$\max \mathbb{E}_t \sum_{i=0}^{\infty} \left(\prod_{j=1}^{i} \frac{1}{1 + r_{t+j-1}}\right) \phi^i \left(\frac{p_t(i)}{p_{t+i}} - z_{t+i}\right)\left(\frac{p_t(i)}{p_{t+i}}\right)^{-\epsilon} y_{t+i}$$

The first-order condition is that

$$\mathbb{E}_t \sum_{i=0}^{\infty} \left(\prod_{j=1}^{i} \frac{1}{1 + r_{t+j-1}}\right) \phi^i \left((1-\epsilon)\frac{p_t(i)}{p_{t+i}} + \epsilon z_{t+i}\right) \frac{1}{p_t(i)} \left(\frac{p_t(i)}{p_{t+i}}\right)^{-\epsilon} y_{t+i}$$

equals zero.

Approximating the FOC

Now plug in the Euler equation to get rid of the interest rates:

$$\mathbb{E}_t \sum_{i=0}^{\infty} y_{t+i}^{1+1/\sigma} \beta^i \phi^i \left((1-\epsilon) \frac{p_t(i)}{p_{t+i}} + \epsilon z_{t+i} \right) \frac{1}{p_t(i)} \left(\frac{p_t(i)}{p_{t+i}} \right)^{-\epsilon} = 0$$

Note that the optimal reset price $p_t(i)$ does not depend on the firm (everything forward-looking), denote by s_t for simplicity. Rearrange:

$$\mathbb{E}_t \sum_{i=0}^{\infty} (\beta\phi)^i y_{t+i}^{1+1/\sigma} \left(\frac{s_t}{p_{t+i}} \right)^{1-\epsilon} = \frac{\epsilon}{\epsilon - 1} \mathbb{E}_t \sum_{i=0}^{\infty} (\beta\phi)^i y_{t+i}^{1+1/\sigma} z_{t+i} \left(\frac{s_t}{p_{t+i}} \right)^{-\epsilon}$$

Approximating the FOC

Now loglinearize around nonstochastic steady state and cancel terms to get

$$\widetilde{s}_t = (1 - \beta\phi)\mathbb{E}_t \left(\sum_{i=0}^{\infty} (\beta\phi)^i (\widetilde{p}_{t+i} + \widetilde{z}_{t+i}) \right)$$

Interpretation: Firm sets price in accordance with current and expected future price level and cost deviations.
Write the equation for time $t+1$ and subtract from this equation to get:

$$\widetilde{s}_t - \beta\phi\mathbb{E}_t\widetilde{s}_{t+1} = (1 - \beta\phi)(\widetilde{p}_t + \widetilde{z}_t)$$

Overall price level

Remember formula for overall price level:

$$p_t = \left(\int p(i)^{1-\epsilon} di \right)^{\frac{1}{1-\epsilon}}$$

Loglinearize this to get: (it helps to write the integral as the limit of the sum)

$$\widetilde{p}_t = \int \widetilde{p}(i) di$$

How old are the prices of the firms?

- $(1-\phi)$ reset their prices at time t, use s_t
- $(1-\phi)\phi$ reset their prices at time $t-1$, use s_{t-1}
- $(1-\phi)\phi^2$ reset their prices at time $t-2$, use s_{t-2}, etc.

Hence:

$$\widetilde{p}_t = (1-\phi)\widetilde{s}_t + (1-\phi)\phi\widetilde{s}_{t-1} + (1-\phi)\phi^2\widetilde{s}_{t-2} + \cdots$$

Overall price level

Write for $t-1$ and subtract to get:

$$\widetilde{p}_t - \phi\widetilde{p}_{t-1} = (1-\phi)\widetilde{s}_t$$

Interpretation: Price level today is a weighted average of past price level $\widetilde{p}_{t-1}$ and the newly set price $\widetilde{s}_t$.
Inflation is defined as $\pi_t = (p_t - p_{t-1})/p_{t-1}$, take loglinearization:

$$\widetilde{\pi}_t = \widetilde{p}_t - \widetilde{p}_{t-1}$$

Plug in the above to get

$$\widetilde{\pi}_t = (1-\phi)(\widetilde{s}_t - \widetilde{p}_{t-1})$$

and subtract $(1-\phi)\widetilde{p}_t$ from both sides, rearrange,

$$\phi\widetilde{\pi}_t = (1-\phi)(\widetilde{s}_t - \widetilde{p}_t)$$

Deriving the New Keynesian Phillips curve

Take the firms' FOC

$$\widetilde{s}_t = (1-\beta\phi)(\widetilde{p}_t + \widetilde{z}_t) + \beta\phi\mathbb{E}_t\widetilde{s}_{t+1},$$

now subtract $\widetilde{p}_t$ from both sides and multiply by $(1-\phi)$,

$$(1-\phi)(\widetilde{s}_t - \widetilde{p}_t) = (1-\beta\phi)(1-\phi)\widetilde{z}_t + \beta\phi(1-\phi)(\mathbb{E}_t\widetilde{s}_{t+1} - \widetilde{p}_t),$$

substitute in the last two equations from the previous slide (the penultimate for period $t+1$),

$$\phi\widetilde{\pi}_t = \beta\phi\mathbb{E}_t\widetilde{\pi}_{t+1} + (1-\beta\phi)(1-\phi)\widetilde{z}_t,$$

and divide by ϕ to get

$$\widetilde{\pi}_t = \beta\mathbb{E}_t\widetilde{\pi}_{t+1} + \underbrace{\frac{(1-\beta\phi)(1-\phi)}{\phi}}_{=:\gamma}\widetilde{z}_t,$$

The output gap

Define the **output gap** as the (log) difference between the actual output y_t and the efficient level of output $\hat{y}_y$:

$$x_t \equiv \frac{y_t}{\hat{y}_t}$$

(Note that even in the nonstochastic steady state, $y_t < \hat{y}_t$. Why?) Recall equations for

- real marginal cost $z_t = w_t / a_t$,
- labor supply/equilibrium: $w_t = y_t^{1/\sigma}$,
- efficient level of output $\hat{y}_t = a_t^\sigma$

combine to get

$$z_t = \left(\frac{y_t}{\hat{y}_t}\right)^{\frac{1}{\sigma}} = x_t^{\frac{1}{\sigma}}$$

Or in log deviations, $\widetilde{z}_t = \frac{1}{\sigma}\widetilde{x}_t$

The New Keynesian Phillips curve

Plug the last equation into inflation equation before to get

$$\widetilde{\pi}_t = \beta \mathbb{E}_t \widetilde{\pi}_{t+1} + \underbrace{\frac{\gamma}{\sigma}}_{=:\kappa} \widetilde{x}_t$$

In our derivation we assumed that ϵ is constant. If instead you make it stochastic and allow shocks to it (cf. Rotemberg-Woodford), you would have shocks to markups even in a frictionless world:

$$\mu_t^* = \frac{\epsilon_t}{\epsilon_t - 1}$$

Derivation as above, with $\widetilde{z}_t + \widetilde{\mu}_t^*$ replacing $\widetilde{z}_t$. Hence

$$\widetilde{\pi}_t = \beta \mathbb{E}_t \widetilde{\pi}_{t+1} + \kappa \widetilde{x}_t + \underbrace{\gamma \widetilde{\mu}_t^*}_{=:e_t}$$

This is the **New Keynesian Phillips curve**.

The New Keynesian Phillips curve

$$\widetilde{\pi}_t = \beta \mathbb{E}_t \widetilde{\pi}_{t+1} + \kappa \widetilde{x}_t + e_t$$

Inflation (or: log-deviation of inflation from steady-state (=0)) depends on

- expected future inflation $\beta \mathbb{E}_t \widetilde{\pi}_{t+1}$: firms may not be allowed to reset their prices in the future, but want to set their price in line with other firms ("strategic complementarity") to maximize profits. Result: firms try to anticipate future inflation and increase current price in line with expected future inflation.
- current output gap $\kappa \widetilde{x}_t$: high real marginal cost $\rightarrow$ high prices. κ is the "slope" of the PC: $\phi = 0$ means $\kappa \rightarrow \infty$ and PC becomes vertical (classical world). $\phi = 1$ means constant prices and $\kappa = 0$, horizontal PC.
- "cost-push shock" e_t: shock to magnitude of markups over real marginal cost. Do we really need this (why not just TFP shocks?)? Answer in homework.

The IS equation

The NKPC explains inflation $\widetilde{\pi}_t$ in terms of the output gap $\widetilde{x}_t$.
What determines the output gap?
Take consumption Euler equation with market clearing ($c_t = y_t$):

$$y_t^{-\frac{1}{\sigma}} = \beta \mathbb{E}_t \left((1 + r_t) y_{t+1}^{-\frac{1}{\sigma}} \right)$$

$$y_t^{-\frac{1}{\sigma}} = \mathbb{E}_t \left(\frac{1 + r_t}{1 + \bar{r}} y_{t+1}^{-\frac{1}{\sigma}} \right)$$

where $\beta = 1/(1 + \bar{r})$, and log-linearize:

$$\widetilde{y}_t = \mathbb{E}_t(\widetilde{y}_{t+1} - \sigma \widetilde{r}_t)$$

Plug in approximate Fisher equation $\widetilde{r}_t = \widetilde{i}_t - \widetilde{\pi}_{t+1}$ to get

$$\widetilde{y}_t = \mathbb{E}_t \left(\widetilde{y}_{t+1} - \sigma(\widetilde{i}_t - \widetilde{\pi}_{t+1}) \right)$$

The IS equation

Write approximated Euler equation for the efficient level of output (approximation exactly as before):

$$\widetilde{\hat{y}}_t = \mathbb{E}_t(\widetilde{\hat{y}}_{t+1} - \sigma \widetilde{\hat{r}}_t)$$

Subtract this from the last eqn on the previous slide and use the definition of the output gap: $\widetilde{x}_t = \widetilde{y}_t - \widetilde{\hat{y}}_t$,

$$\widetilde{x}_t = \mathbb{E}_t\left(\widetilde{x}_{t+1} - \sigma(\widetilde{i}_t - \widetilde{\pi}_{t+1})\right) + \underbrace{\sigma \widetilde{\hat{r}}_t}_{=:v_t}$$

is the **IS equation** of the NK model. v_t is a shock to the efficient real interest rate.

The New Keynesian Model

To summarize, we can write the approximated dynamics of the model as

$$\text{(NKPC)} \qquad \widetilde{\pi}_t = \beta \mathbb{E}_t \widetilde{\pi}_{t+1} + \kappa \widetilde{x}_t + e_t$$

$$\text{(IS)} \qquad \widetilde{x}_t = \mathbb{E}_t \left(\widetilde{x}_{t+1} - \sigma \big(\widetilde{i}_t - \widetilde{\pi}_{t+1} \big) \right) + v_t$$

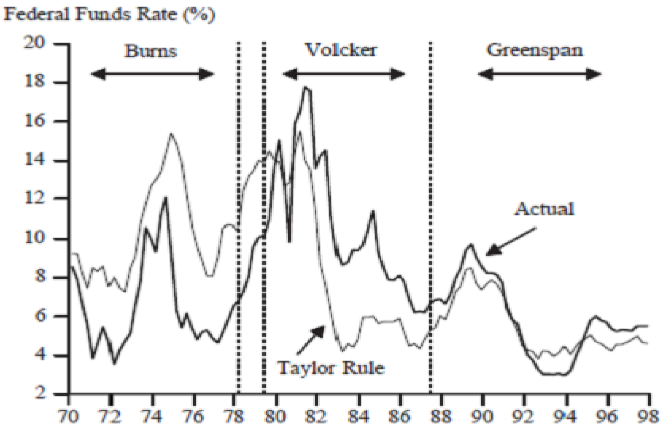
Sometimes researches specify that monetary policy (through the interest rate) follows a *Taylor rule*, i.e. interest rates respond to inflation and output gap deviations, subject to a shock u_t,

$$\text{(TR)} \qquad \widetilde{i}_t = \alpha_1 \widetilde{\pi}_t + \alpha_2 \widetilde{x}_t + u_t$$

which generally delivers a pretty good description of how MP is done.

Can show that $\alpha_1 > 1$ is required to have a unique equilibrium ("Taylor principle").

Comparing the Taylor rule and actual policy decisions



Clarida, Gali, Gertler (2000) estimate Taylor rules for different Fed chairmen and blame an estimated $\alpha_1 < 1$ under Burns & Miller for high inflation.

Calibrating and Simulating the NK model

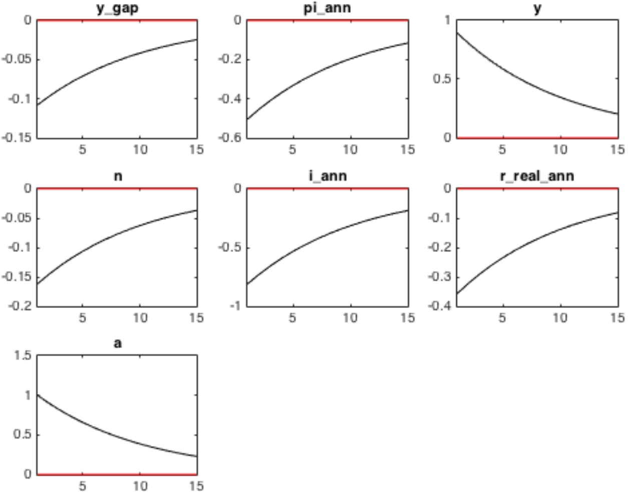
Calibrate model to quarterly frequency

- β: real return $\approx 4\%$, set $\beta = 0.99$
- Log period utility $u(c_t) = \log c_t$
- Frisch elasticity of labor supply equal to 1 (see RBC)
- $\theta = 2/3$ (avg price duration of three quarters)
- Taylor rule coefficients $\alpha_1 = 1.5$ and $\alpha_2 = 0.5/4$ from Taylor (1999), which captures Greenspan era MP.
- Persistence of monetary shock $= 0.5$.

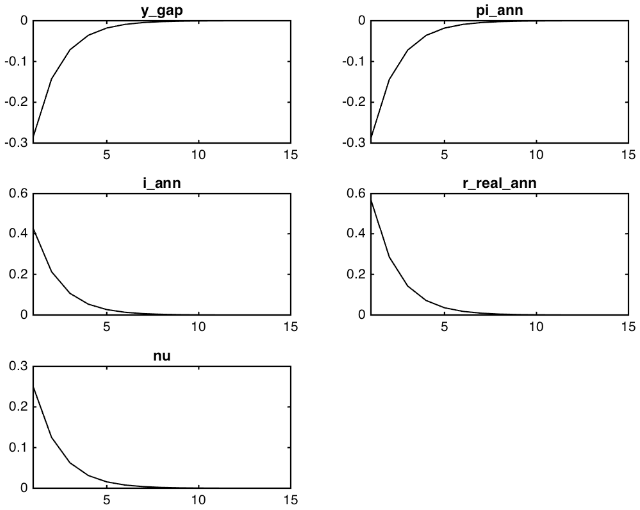
Details of calibration see Gali (2008) Chapter 3.

Response to a one SD negative TFP shock

(i, r, π are annualized)



Response to a one SD contractionary monetary shock



(note: here "nu" corresponds to u_t)

Success of the NK model?

- Original model due to Smets and Wouters (2003), though most people cite Christiano, Eichenbaum, Evans (2005). Yun (1996) was the first one to put Calvo prices into a DSGE model (but few people noticed).
- CEE manage to tweak to model to match the VAR impulse responses to a monetary shock.

Biggest problem: the Calvo pricing assumption ("Calvo fairy") does not really make sense.

- Ideally price setting should be coming from firm's optimal responses to policy (think: Lucas critique)
- Large literature (even before NK model) where price rigidities are coming from adjustment costs to prices (menu costs)
- Alternative approach: imperfect information

State-dependent pricing

Early insight: Stickiness at the micro level does not necessarily translate into stickiness at the macro level

- Study a simple model that shows this: Caplin and Spulber (1987, QJE)
- No idiosyncratic shocks
- Unit mass of firms, indexed by $i \in [0, 1]$. Continuous time.
- Let p_{it} be the log of firm i's price at time t, and p^*_{it} the (log) profit-maximizing (optimal) price.
- Price level is average price of all firms,

$$p_t = \int_0^1 p_{it} di$$

- Assume that

$$p^*_{it} = p^*_t = p_t + \xi y_t$$

  with $0 < \xi < 1$ capturing market structure/market power.

- Let aggregate demand be equal to real money balances in the economy
$$y_t = m_t - p_t$$
where money supply is assumed to be exogenous and growing (at possibly varying growth rates) but continuous
- Use AD and profit max to get
$$p_t^* = (1-\xi)p_t + \xi m_t$$
- If there are no frictions in the economy, then $p_{it} = p_{it}^*$ and hence $p_t = m_t$ and money is neutral.
- Let $g_{it} \equiv p_{it} - p_t^*$ be firm i's gap between actual and desired price
- Assume that the firm follow a *one-sided* (s, S) pricing rule:
    - When g_{it} falls below s, the firm resets its price so that $g_{it} = S$
    - Otherwise, the firm does not change its price
- Need to specify an initial distribution of price gaps: assume g_{it} is uniformly distributed between s and S.
- Then the distribution is invariant over time.

- Suppose Δt time passes, and money supply grows by Δm.
- From above,

$$\Delta p^* = (1-\xi)\Delta p + \xi \Delta m$$

- If firms take no actions, all price gaps fall by Δp^*
- Since the gaps are uniformly distributed, a fraction $\Delta p^*/(S-s)$ falls under the s threshold and raise their prices by $S-s$
- Hence, the overall price level changes by

$$\Delta p = \left(\frac{\Delta p^*}{S-s}\right) \times (S-s) + \left(1 - \frac{\Delta p^*}{S-s}\right) \times 0 = \Delta p^*$$

- Substitute this into the above to get

$$\Delta p^* = (1-\xi)\Delta p^* + \xi \Delta m$$

  and hence $\Delta p = \Delta m$ and $\Delta y = 0$.
- Money is neutral.

On the firm level prices are sticky, but on the aggregate they are not. Why?

- Selection effects: only the ones with lowest prices raise price
- Magnitude of price increase is such that average moves one-to-one with target

Yes, the result hinges on some very particular assumptions:

- Uniform distribution of price gaps
- Exogenous one-sided (s, S) pricing rule (which may or may not be optimal)
- Continuity

But the point of the paper is to show that micro-stickiness per se is not enough: it matters who adjusts, when, and by how much.

Subsequent work

- Caplin-Leahy (1991) allow money supply to fall: two-sided (s, S) rule. Money not neutral. Danziger (1999) puts the Caplin-Spulber selection story into RBC. Money neutral.
- Dotsey-King-Wolman (1999): firm-specific menu cost. Everything more tractable.
- Bils and Klenow (2004), Klenow and Kryvtsov (2008), Klenow and Malin (2011), Nakamura and Steinsson (2008) collect micro-evidence on price setting.
- Golosov and Lucas (2007, JPE) argue that if you put menu costs into DSGE and match Bils-Klenow micro evidence on prices, you get a very weak and short response to money shocks.
- Gertler and Leahy (2008, JPE) match Nakamura-Steinsson and get substantially bigger responses (see also Midrigan (Ecta 2011))
- Then the financial crisis hit, and we had other things in mind.