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Monetary Transmission Channels in DSGE Models: Decomposition of Impulse Response Functions Approach

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Abstract The paper presents decomposition of impulse response functions (IRFs) as a new diagnostic tool for dynamic stochastic general equilibrium (DSGE) models. This method works with any DSGE model of arbitrary complexity or theoretical background. It is also applicable to any policy transmission channels. We illustrate it with monetary transmission mechanisms in two New Keynesian general equilibrium models: QUEST_III model of the European Commission and Smets–Wouters model of the USA economy. For that purpose, we use DYNARE platform for solving the models and provide a MATLAB file for IRFs decomposition. The underlying software can handle decomposition of IRFs using both the first-order and the second-order approximation of Taylor series to equilibrium relations. An IRF aggregates partial contributions of all state variables to impulse responses of a model's variable to a stochastic shock. The IRF decomposition identifies individual contributions of state variables and marks each particular channel that a policy shock uses to propagate throughout the model. We show in two illustrated cases that monetary transmission channels might be quite distinct even if DSGE models employ the same (Taylor) policy rule and reveal similar IRFs. More specifically, IRFs initiated by a monetary shock might misrepresent the pure interest rate impact on some variables. Decomposition of monetary IRFs casts more light on flexibility needed in an economy to contain negative impact of a monetary shock.

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1 Introduction

Impulse response functions (IRFs) are a standard tool to explore how a dynamic stochastic general equilibrium (DSGE) model reacts to small stochastic disturbances on its steady state. It is assumed that the model can restore its steady state after such a shock, and the main task of a researcher is to explain how this is done. Since economic policy measures can be modeled as external shocks to the economy, IRFs are a useful means of studying the potential effects of alternative macroeconomic scenarios within a DSGE framework.

However, IRFs can have unexpected shapes with surprising adjustment paths for some variables. To explain this, researchers usually refer to the structural equations of the model and try to trace the paths of particular endogenous variables. This approach may have results in simple DSGE models, but in medium or large models, it is less accurate. Additionally, IRFs are not based on the structural equations of the model, but on its reduced form equations. What drives the path of IRFs cannot be attributed to the whole set of endogenous variables, but only to the subset of state variables. This is a consequence of relying on numerical solution for DSGE models.

IRFs are regularly reported in a closed form. In this paper we will open them to reveal the effects of each state variable and its contribution to the final value of IRFs. This is what we mean by the term IRF decomposition, or alternatively called, partial IRFs. In large models, for practical purposes, these effects should be reduced to no more than ten key variables. We believe this approach will facilitate better understanding of how adjustment mechanisms work in complex DSGE models.

The paper is organized in the following way. In the second section, we briefly discuss the origin and computation of IRFs in DSGE models. In the third section, we explain how IRFs can be decomposed into partial IRFs. In the fourth section, we apply this method to the QUEST_III model of the European Commission. The fifth section shows new insights into monetary transmission channels and therefore addresses rigidity impacts. In the sixth section, we use the same method in the Smets–Wouters model of the USA economy and compare alternative transmission channels. In the seventh section we explain how to use a MATLAB function for doing decomposition. Finally, we briefly conclude.

2 Impulse Response Functions

Christopher Sims' seminal paper on vector autoregressive (VAR) models (1980) has offered a new analytical tool for policy analysis in a dynamic macroeconomics framework. In order to assess effects of alternative policy scenarios, one can simply study the impulse response functions (IRFs) from the VAR model. By using IRFs, policymakers can track the responses of a model's variables to innovations of the model's shocks, and evaluate performance of the underlying adjustment mechanism in the economy under consideration. This technique immediately became a key part of many empirical dynamic models and a standard procedure in user-friendly econometrics packages (EViews and RATS for instance). Advanced textbook discussion of this technique can be found in Hamilton (1994).

Although this analytical tool captures dynamics in multiple time series framework, it was quickly adopted by the alternative optimization-based dynamic macroeconomic models—dynamic stochastic general equilibrium (DSGE) models. The Real Business Cycle (RBC) literature must be credited for inventing, and contributing to early development of, these type of models. The literature began with Kydland and Prescott (1982), but gained widespread attention after Hansen presented two models (1985). It is interesting to notice that IRFs were not present in either of these seminal papers. IRFs were integrated into the new framework by Long and Plosser (1983), and broadly disseminated by King et al. (1988) and (2002). A clear textbook exposition of RBC models, with particular attention to IRFs, can be found in McCandless (2008).

RBC models used IRFs to study business cycle fluctuations caused by real, not monetary, shocks in an environment with flexible prices. On the other hand, Rotemberg and Woodford (1997) applied the same concept of IRFs, within the similar class of optimization models, but for a completely opposite purpose. Rotemberg and Woodford developed a structural econometric model for the sack of stimulating effects of the optimal monetary policy, and calibrated the twin DSGE model in such a way as to mimic VAR's IRFs. They changed the basic RBC's assumption that prices freely adjust to external shocks, and assumed that nominal rigidities prevail in goods and labor markets with appropriate cost adjustment. The outcome was far-reaching. With this switch in assumptions, an alternative approach to DSGE modelling has emerged: the New Keynesian (NK) school of DSGE models. Early survey of this literature can be found in Clarida et al. (1999), and an advanced textbook interpretation in Woodford (2003).

For the purpose of our analysis, it does not so much matter whether DSGE models belong to RBC or NK tradition. What matters more is how solutions are obtained. DSGE models usually do not have close analytical solutions and the underlying non-linear system of difference equations needs to be numerically solved. To do so, one option is to use value function iteration in a recursive macroeconomic framework developed by Stokey et al. (1989), and Ljungqvist and Sargent (2000).

An alternative approach is based on linear approximation of non-linear systems. As the name of the method indicates, one has to initially (log) linearize the model's non-linear equations around the steady state, and then solve the resulting linear system of equations by using state-space representation of the problem. Under certain conditions, this method provides a good approximation of non-linear models with reasonably low costs in terms of computing resources and accuracy. The key reference in the literature is still Blanchard and Kahn (1980). For more information on solution algorithms, the singularity issue and unclear distinction between state and control variables see King and Watson (2002), King et al. (2002), Klein (2000), Sims (2002) and Uhlig (2001).

We also follow the linear approximation method in this paper, but our software allows the second-order approximation as well. We start with the perturbation method, Judd (1998) and Judd and Jin (2002) which allows for k-order Taylor expansion of the policy function around the steady state, and afterwards restrict the approximation of the policy function to the first-order Taylor expansion. This first-order perturbation method exactly corresponds to the solution obtained by standard linearization of first-order conditions, according to Collard and Juillard (2001).[1] The second-order approximation should be used in models where variance has an impact on decision rules.

There are many different algorithms publicly available for solving DSGE models, so one does not have to write his/her own code for this purpose (Adjemian et al. 2011; Anderson 2008; Klein 2000; Sims 2002; Uhlig 2001). Among them, we refer to DYNARE that works as preprocessor to the MATLAB mathematical software. DYNARE is a platform for the solution, simulation and estimation of DSGE models developed by Michel Juillard and his collaborators; see Adjemian et al. (2011). We therefore rely on DYNARE code for solving DSGE models and write our own code programmed in MATLAB only for decomposing IRFs.

We will now briefly explain the process of computing IRFs, following Adjemian et al. (2011), before moving to the IRFs decomposition method that will provide the partial IRFs. A DSGE model of rational expectations can be represented in general form by a set of first order and equilibrium conditions:

$$
\begin{aligned}
\mathbb{E}_t\{f(\boldsymbol{y}_{t+1}, \boldsymbol{y}_t, \boldsymbol{y}_{t-1}, \boldsymbol{\varepsilon}_t)\} &= 0 \\
\mathbb{E}(\boldsymbol{\varepsilon}_t) &= 0 \\
\mathbb{E}(\boldsymbol{\varepsilon}_t \cdot \boldsymbol{\varepsilon}_t') &= \Sigma_\varepsilon
\end{aligned}
\tag{1}
$$

where $\mathbb{E}_t$ is expectation operator, f are structural equations, $\boldsymbol{y}_t$ is a vector of endogenous variables, and $\boldsymbol{\varepsilon}_t$ is a vector of stochastic shocks. The system of equations (1) comprises linear and non-linear first-order difference equations, with leads and lags, which have no explicit algebraic solution. The solution has to be numerically computed in the form of policy functions which relate all endogenous variables in the current period to the endogenous variables of the previous period, and current shocks. To be more precise, endogenous variables in the current period are to be expressed as a function of only state variables in the previous period and current shocks:

$$
\boldsymbol{y}_t = g(\boldsymbol{y}_{t-1}, \boldsymbol{\varepsilon}_t) \tag{2}
$$

The policy functions g are computed by linearizing the system (1) around the steady state ($\bar{\boldsymbol{y}}$) using the first-order Taylor expansion and the certainty equivalence principle:

$$
\boldsymbol{y}_t = \bar{\boldsymbol{y}} + \boldsymbol{g}_y \cdot (\boldsymbol{y}_{t-1} - \bar{\boldsymbol{y}}) + \boldsymbol{g}_u \cdot \boldsymbol{\varepsilon}_t \tag{3}
$$

Endogenous variables in Eq. (3) can be split into state $\boldsymbol{s}_t$ and control variables $\boldsymbol{c}_t$, $\boldsymbol{y}_t = \boldsymbol{s}_t \cup \boldsymbol{c}_t$, and transform into deviations from the steady states $\hat{\boldsymbol{s}}_t = \boldsymbol{s}_t - \bar{\boldsymbol{s}}$ and $\hat{\boldsymbol{c}}_t = \boldsymbol{c}_t - \bar{\boldsymbol{c}}$. Then, evolution of the system (3) can be rearranged as follows:

[1] For the later discussion, it is important to notice that the policy function provides the optimal values of the control variables as a function of the state variables. The *state variables* are those variables whose values are already determined by agents' actions in period t − 1 or set in period t by an exogenous process. The *control variables* in period t are those variables whose values rational agents choose in that period in order to maximize an objective function under constraints.

$$\begin{bmatrix} \hat{\boldsymbol{s}}_t \\ \hat{\boldsymbol{c}}_t \\ \hat{\boldsymbol{\varepsilon}}_t \end{bmatrix} = \begin{bmatrix} \boldsymbol{g}_s^s & 0 & \boldsymbol{g}_s^\varepsilon \\ \boldsymbol{g}_c^s & 0 & \boldsymbol{g}_c^\varepsilon \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \hat{\boldsymbol{s}}_{t-1} \\ \hat{\boldsymbol{c}}_{t-1} \\ \hat{\boldsymbol{\varepsilon}}_t \end{bmatrix} \tag{4}$$

Submatrix g_s^s denotes responses of $\hat{s}_t$ to movements in $\hat{s}_{t-1}$, while submatrix g_s^ε denotes responses of $\hat{s}_t$ to movements in the exogenous shock terms ε_t. Submatrices $\boldsymbol{g}_c^s$ and $\boldsymbol{g}_c^\varepsilon$ capture responses of the control variables to the movement of state variables and exogenous shocks, respectively. From Eq. (4) it is obvious that only the state variables and the exogenous shocks drive the model.

IRFs are directly calculated from the policy functions (4). One has to start from the initial value of variables given by the steady state and draw the innovation $\bar{\varepsilon}$ to the exogenous shock of interest, and iterate on as many times as the number of future periods that have been chosen. The results are IRFs. Note that there might be $l = 1, \ldots k, \ldots, \bar{l}$ exogenous shocks, and each of them creates its own IRF. The column vectors of all innovations $\hat{\boldsymbol{\varepsilon}}_t$ are defined as follows: $\varepsilon_{l,t|l=k,t=1} = \bar{\varepsilon}$, $\varepsilon_{l,t|l=k,t>1} = 0$, $\varepsilon_{l,t|l\neq k,t\geq 1} = 0$, where $\bar{\varepsilon}$ is the initial shock of the kth innovation. DYNARE automatically provides IRFs, and stores a simplified representation of matrices $\boldsymbol{g}_y$ and $\boldsymbol{g}_u$ in the workspace. However, submatrices g_s^s, g_s^u, g_c^s and g_c^u should be created outside DYNARE.

Evolution of the system (4) is a recursive process that has two different steps, for t = 1 and t > 1, as shown in Eq. (4′):

$$\begin{bmatrix} \hat{\boldsymbol{s}}_{t|t=1} \\ \hat{\boldsymbol{c}}_{t|t=1} \\ \hat{\boldsymbol{\varepsilon}}_{t|t=1} \\ \hat{\boldsymbol{s}}_{t|t>1} \\ \hat{\boldsymbol{c}}_{t|t>1} \\ \hat{\boldsymbol{\varepsilon}}_{t|t>1} \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \boldsymbol{g}_s^\varepsilon & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \boldsymbol{g}_c^\varepsilon & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \boldsymbol{g}_s^s & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \boldsymbol{g}_c^s & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \hat{\boldsymbol{\varepsilon}}_t \\ \hat{\boldsymbol{s}}_{t-1} \\ \hat{\boldsymbol{c}}_{t-1} \\ \mathbf{0} \end{bmatrix} \tag{4′}$$

In the first period exogenous shocks are the only driving factors. They induce deviations of all variables from the steady state, and after that disappear. From the second period on, the state variables continue to drive the evolution of IRFs. Equation (4) clarifies the role of state and control variables, while the modified Eq. (4′) prepares the ground for calculation individual impacts of each state variable in the period $t|t > 1$, which is the core of IRFs decomposition process.

The assumption of the certainty equivalence can be removed with the second-order Taylor approximation to the steady state. In fact, the second-order approximation is a generalization of the first-order approximation. It has the following form:

$$\begin{aligned} \hat{\boldsymbol{y}}_t = \boldsymbol{g}_y \hat{\boldsymbol{y}}_{t-1} + \boldsymbol{g}_u \boldsymbol{\varepsilon}_t + 0.5 \cdot \Big(\boldsymbol{g}_{\sigma\sigma} \sigma^2 + \boldsymbol{g}_{yy} \cdot (\hat{\boldsymbol{y}}_{t-1} \otimes \hat{\boldsymbol{y}}_{t-1}) + \boldsymbol{g}_{\varepsilon\varepsilon} (\boldsymbol{\varepsilon}_t \otimes \boldsymbol{\varepsilon}_t) \Big) \\ + \boldsymbol{g}_{y\varepsilon} (\hat{\boldsymbol{y}}_{t-1} \otimes \boldsymbol{\varepsilon}_t) \end{aligned} \tag{5}$$

where σ^2 is the variance and $\otimes$ is the Kronecker tensor product. The columns of matrix $\boldsymbol{g}_{yy}$ correspond to the Kronecker product of the vector of state variables, the matrix columns of $\boldsymbol{g}_{\varepsilon\varepsilon}$ correspond to the Kronecker product of exogenous shocks and the matrix columns of $\boldsymbol{g}_{y\varepsilon}$ correspond to the Kronecker product of the vector of state variables by the vector of exogenous shocks. It is obvious from (5) that the second-order approximation depends on the outcome of first-order approximation to the steady state. In equations (5) the first two terms are identical with those from Eq. (3), while the remaining parts are specific for extension of the solution to a second-order approximation.

The systems of Eqs. (4) and (5) can be written in a more compact way.

$$\hat{\boldsymbol{y}}_t = \boldsymbol{G} \cdot \hat{\boldsymbol{x}}_t \tag{6}$$

$$\hat{\boldsymbol{y}}_t = \boldsymbol{G} \cdot \hat{\boldsymbol{x}}_t + \boldsymbol{\psi}^{\mathbf{2}} + \boldsymbol{\Gamma} \cdot \hat{\boldsymbol{x}}_t \otimes \hat{\boldsymbol{x}}_t \tag{7}$$

where

$$\boldsymbol{G} = \begin{bmatrix} \boldsymbol{g}_s^s & 0 & \boldsymbol{g}_s^\varepsilon \\ \boldsymbol{g}_c^s & 0 & \boldsymbol{g}_c^\varepsilon \\ 0 & 0 & 1 \end{bmatrix}, \quad \boldsymbol{\psi}^{\mathbf{2}} = 0.5\sigma^2 \begin{bmatrix} g_{\sigma\sigma}^s \\ g_{\sigma\sigma}^c \\ 0 \end{bmatrix}, \quad \hat{\boldsymbol{x}}_t \otimes \hat{\boldsymbol{x}}_t = \begin{bmatrix} \hat{\boldsymbol{s}}_{t-1} \otimes \hat{\boldsymbol{s}}_{t-1} \\ \hat{\boldsymbol{s}}_{t-1} \otimes \hat{\boldsymbol{c}}_{t-1} \\ \hat{\boldsymbol{s}}_{t-1} \otimes \hat{\boldsymbol{\varepsilon}}_t \\ \hat{\boldsymbol{c}}_{t-1} \otimes \hat{\boldsymbol{s}}_{t-1} \\ \hat{\boldsymbol{c}}_{t-1} \otimes \hat{\boldsymbol{c}}_{t-1} \\ \hat{\boldsymbol{c}}_{t-1} \otimes \hat{\boldsymbol{\varepsilon}}_t \\ \hat{\boldsymbol{\varepsilon}}_t \otimes \hat{\boldsymbol{s}}_{t-1} \\ \hat{\boldsymbol{\varepsilon}}_t \otimes \hat{\boldsymbol{c}}_{t-1} \\ \hat{\boldsymbol{\varepsilon}}_t \otimes \hat{\boldsymbol{\varepsilon}}_t \end{bmatrix}$$

$$\boldsymbol{\Gamma} = \begin{bmatrix} 0.5\boldsymbol{g}_s^{s\otimes s} & \mathbf{0}\otimes\mathbf{0} & \boldsymbol{g}_s^{s\otimes\varepsilon} & \mathbf{0}\otimes\mathbf{0} & \mathbf{0}\otimes\mathbf{0} & \mathbf{0}\otimes\mathbf{0} & \mathbf{0}\otimes\mathbf{0} & \mathbf{0}\otimes\mathbf{0} & 0.5\boldsymbol{g}_s^{\varepsilon\otimes\varepsilon} \\ 0.5\boldsymbol{g}_c^{s\otimes s} & \mathbf{0}\otimes\mathbf{0} & \boldsymbol{g}_c^{s\otimes\varepsilon} & \mathbf{0}\otimes\mathbf{0} & \mathbf{0}\otimes\mathbf{0} & \mathbf{0}\otimes\mathbf{0} & \mathbf{0}\otimes\mathbf{0} & \mathbf{0}\otimes\mathbf{0} & 0.5\boldsymbol{g}_c^{\varepsilon\otimes\varepsilon} \\ \mathbf{0}\otimes\mathbf{0} & \mathbf{0}\otimes\mathbf{0} & \mathbf{0}\otimes\mathbf{0} & \mathbf{0}\otimes\mathbf{0} & \mathbf{0}\otimes\mathbf{0} & \mathbf{0}\otimes\mathbf{0} & \mathbf{0}\otimes\mathbf{0} & \mathbf{0}\otimes\mathbf{0} & \mathbf{1} \end{bmatrix}$$

and $\hat{\boldsymbol{y}}_t = \left[\hat{\boldsymbol{s}}_t, \hat{\boldsymbol{c}}_t, \hat{\boldsymbol{\varepsilon}}_t\right]'$, $\hat{\boldsymbol{x}}_t = \left[\hat{\boldsymbol{s}}_{t-1}, \hat{\boldsymbol{c}}_{t-1}, \hat{\boldsymbol{\varepsilon}}_t\right]'$. Matrix $\boldsymbol{\Gamma}$ contains submatrices with different dimensions.[2] For example, submatrix $\boldsymbol{g}_s^{s\otimes s}$ has dimension $\bar{i}x\bar{i}^2$, e.g. $\bar{i}$ rows and $\bar{i}^2$ columns. Sparse submatrices $\mathbf{0}\otimes\mathbf{0}$ also have different dimensions depending on the place where they are located. All those submatrices should match corresponding dimensions in the column vector of the Kronecker product $\hat{\boldsymbol{x}}_t \otimes \hat{\boldsymbol{x}}_t$. For example, vector $\hat{\boldsymbol{s}}_{t-1} \otimes \hat{\boldsymbol{s}}_{t-1}$ has dimension $\bar{i}x\bar{i}$. It should be noticed that the Kronecker product of innovations $\hat{\boldsymbol{\varepsilon}}_t \otimes \hat{\boldsymbol{\varepsilon}}_t$ has dimension $\bar{l}x\bar{l}$.

[2] For indices $i = 1, \ldots, \bar{i}$, $j = 1, \ldots, \bar{j}$, and $l = 1, \ldots, \bar{l}$ that represent state and control variables, and exogenous shocks, respectively, matrix **G** has the following dimension $\begin{bmatrix} \bar{i}x\bar{i} & \bar{i}x\bar{j} & \bar{i}x\bar{l} \\ \bar{j}x\bar{i} & \bar{j}x\bar{j} & \bar{j}x\bar{l} \\ \bar{l}x\bar{i} & \bar{l}x\bar{j} & \bar{l}x\bar{l} \end{bmatrix}$, while matrix **Γ** is much more structured $\begin{bmatrix} \bar{i}x\bar{i}^2 & \bar{i}x\bar{i}\cdot\bar{j} & \bar{i}x\bar{i}\cdot\bar{l} & \bar{i}x\bar{i}\cdot\bar{i} & \bar{i}x\bar{j}^2 & \bar{i}x\bar{j}\cdot\bar{l} & \bar{i}x\bar{l}\cdot\bar{i} & \bar{i}x\bar{l}\cdot\bar{j} & \bar{i}x\bar{l}^2 \\ \bar{j}x\bar{i}^2 & \bar{j}x\bar{i}\cdot\bar{j} & \bar{j}x\bar{i}\cdot\bar{l} & \bar{j}x\bar{j}\cdot\bar{i} & \bar{j}x\bar{j}^2 & \bar{j}x\bar{j}\cdot\bar{l} & \bar{j}x\bar{l}\cdot\bar{i} & \bar{j}x\bar{l}\cdot\bar{j} & \bar{j}x\bar{l}^2 \\ \bar{l}x\bar{i}^2 & \bar{l}x\bar{i}\cdot\bar{j} & \bar{l}x\bar{i}\cdot\bar{l} & \bar{l}x\bar{i}\cdot\bar{i} & \bar{l}x\bar{j}^2 & \bar{l}x\bar{j}\cdot\bar{l} & \bar{l}x\bar{l}\cdot\bar{i} & \bar{l}x\bar{l}\cdot\bar{j} & \bar{l}x\bar{l}^2 \end{bmatrix}$ and $\hat{x}_t \otimes \hat{x}_t = \left[\bar{i}x\bar{i},\ \bar{i}x\bar{j},\ \bar{i}x\bar{l},\ \bar{j}x\bar{i},\ \bar{j}x\bar{j},\ \bar{j}x\bar{l},\ \bar{l}xi,\ \bar{l}x\bar{j},\ \bar{l}x\bar{l}\right]'$.

3 Decomposition of IRFs

In order to explain decomposition of an IRF, let us start with the first-order approximation to the non-linear system of Eq. (1). As we already stated, calculating an IRF from Eq. (4) is a straightforward task: draw ε_t and calculate (4). Thus, the state and control variables move away from their steady states and we can iterate the calculation with $\forall t > 1\ \varepsilon_t = 0$ up to the length of an IRF. Let us rewrite Eq. (4) as:

$$\hat{\boldsymbol{y}}_t = \boldsymbol{G} \cdot \hat{\boldsymbol{s}}_{t-1} = \begin{bmatrix} \boldsymbol{g}_s \\ \boldsymbol{g}_c \\ 0 \end{bmatrix} \cdot \hat{s}_{t-1} \tag{4''}$$

where $\boldsymbol{g}_s$ represents the submatrix of rows in matrix $\mathbf{G}$ of the state variables, and $\boldsymbol{g}_c$ the corresponding submatrix of rows of the control variables. Then, impulse response functions of both the state and control variables are for $t|t > 1$:

$$IRF_t^s = \boldsymbol{g}_s \cdot \hat{\boldsymbol{s}}_{t-1} \tag{8}$$

$$IRF_t^c = \boldsymbol{g}_c \cdot \hat{\boldsymbol{s}}_{t-1} \tag{8'}$$

From this point on, decomposition of IRFs is computationally a straightforward process. It is given by calculating element-wise product of the row vector of the coefficient matrix $\mathbf{G}$ and the column vector $\hat{\boldsymbol{s}}_{t-1}$. Instead of an array multiplication of two vectors, we now have the corresponding element-wise multiplication in the form:

$$\widetilde{IRF}^s_{t,i} = \{\boldsymbol{g}_s \cdot \hat{\boldsymbol{s}}_{t-1} \odot \boldsymbol{I}_i\}(i), \qquad \forall i = 1, \ldots, \bar{i} \tag{9}$$

$$\widetilde{IRF}^c_{t,i} = \{\boldsymbol{g}_c \cdot \hat{\boldsymbol{s}}_{t-1} \odot \boldsymbol{I}_i\}(i), \qquad \forall i = 1, \ldots, \bar{i} \tag{9'}$$

where symbol $\odot$ denotes element-wise multiplication of two arrays, $\boldsymbol{I}_i$ is the i^{th} column vector of the identity matrix $\mathbf{I}$, and { }(i) denotes the i^{th} element of the vector in parentheses { }. It should be reiterated that an IRF decomposition extracts contributions of each state variables to the impulse response function of an endogenous variable for $t|t > 1$. During the iteration process, the policy functions (8) and (8′) sum up individual impacts of state variables and report the aggregate outcome. If one keeps track of this process and extracts individual contributions of each state variable to the IRF value for each period of iteration, the results are partial IRFs. Therefore the sum of individual contributions must be equal to the value of IRFs for each period of iteration $IRF_t^s = \sum_{i=1}^{\bar{i}} \widetilde{IRF}^s_{t,i}$ and $IRF_t^c = \sum_{i=1}^{\bar{i}} \widetilde{IRF}^c_{t,i}$.

Calculation of *IRF*s and their decomposition $\widetilde{IRF}$, when solution of the model is based on the second-order approximation, are technically more demanding but follows the same line of reasoning:

$$IRF_t^s = \boldsymbol{g}_s \cdot \hat{\boldsymbol{s}}_{t-1} + \boldsymbol{\psi}_s^2 + \boldsymbol{\Gamma}_{ss} \cdot (\hat{\boldsymbol{s}}_{t-1} \otimes \hat{\boldsymbol{s}}_{t-1}) \tag{10}$$

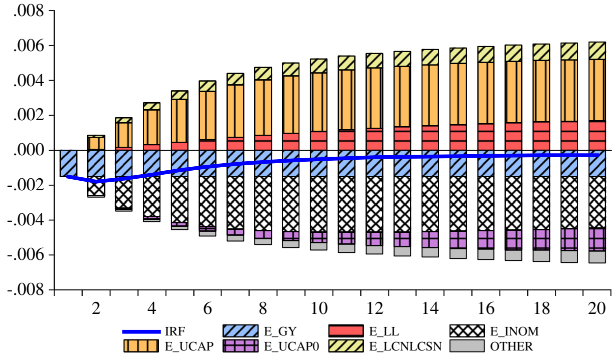

Fig. 1 Cumulative impulse responses of output to a monetary shock (*line* refers to aggregate IRF and *bars* to decomposed effects of state variables)

$$IRF_t^c = \boldsymbol{g}_c \cdot \hat{\boldsymbol{s}}_{t-1} + \boldsymbol{\psi}_c^2 + \boldsymbol{\Gamma}_{sc} \cdot (\hat{\boldsymbol{s}}_{t-1} \otimes \hat{\boldsymbol{s}}_{t-1}) \tag{10'}$$

$$\widetilde{IRF}^s_{t,i} = \{\boldsymbol{g}_s \cdot \hat{\boldsymbol{s}}_{t-1} \odot \boldsymbol{I}_i\}(i) + \boldsymbol{\psi}_s^2 + \boldsymbol{\Gamma}_{ss} \cdot (\hat{\boldsymbol{s}}_{t-1} \odot \boldsymbol{I}_i) \otimes (\hat{\boldsymbol{s}}_{t-1} \odot \boldsymbol{I}_i),\ \forall i = 1, \ldots, \bar{i} \tag{11}$$

$$\widetilde{IRF}^c_{t,i} = \{\boldsymbol{g}_c \cdot \hat{\boldsymbol{s}}_{t-1} \odot \boldsymbol{I}_i\}(i) + \boldsymbol{\psi}_c^2 + \boldsymbol{\Gamma}_{sc} \cdot (\hat{\boldsymbol{s}}_{t-1} \odot \boldsymbol{I}_i) \otimes (\hat{\boldsymbol{s}}_{t-1} \odot \boldsymbol{I}_i),\ \forall i = 1, \ldots, \bar{i} \tag{11'}$$

where $\boldsymbol{\Gamma}_{ss}$ and $\boldsymbol{\Gamma}_{sc}$ are the first and second row of matrix $\boldsymbol{\Gamma}$, respectively, and $\boldsymbol{\psi}_s^2$ and $\boldsymbol{\psi}_c^2$ partition the variance vector for state and control variables.

As we have already mentioned, the impulse response functions are a tool for analyzing the inter-relation between the model variables. They show how the equilibrium is restored after an exogenous shock, but their decomposition reveals the underlying impact of key driving factors in this process. We will illustrate the concept of IRF decomposition in two New Keynesian general equilibrium models—QUEST_III model of the European Commission (Ratto et al. 2009)[3] and Smets–Wouters model of the USA economy (Smets and Wouters 2007).[4] Both models are linearized or log-linearized versions of the initial non-linear models, so our decomposition software will use only the first-order approximation option. In a general non-linear model that is solved by using the second-order approximation to the equilibrium relations, the set of Eqs. (11)–(11′) will drive the IRFs decomposition (Fig. 1).

4 Monetary Policy Channels in QUEST_III Model

Ratto *et al.* (2009) investigate broad impacts of fiscal and monetary policy measures in the EU area. On the fiscal side, they trace responses of output and other variables to

[3] To download the QUEST III source code an email request should be sent to Marco Ratto, https://ec.europa.eu/jrc/en/econometric-statistical-software. We downloaded it on 20 July 2013.

[4] We obtained DYNARE source codes for this model from the DYNARE forum at the web site address: http://www.dynare.org/phpBB3/viewtopic.php?f=1&t=3750.

government transfer payment, consumption and investment shocks. On the monetary side, they study effects of a monetary shock on real variables. Additionally, they analyse impacts of world demand and technology shocks. We will focus in this paper only on monetary transmission channels.[5] For that purpose let us start with the impact of a monetary shock on real output (GDP). Three key equations are the Cobb–Douglas function, the output gap equation and the Taylor rule of monetary policy.

$$g_t^Y = (1-\alpha)\cdot(g_t^K + g_t^{UCAP}) + \alpha\cdot(g_t^L + g_t^{TFP}) + (1-\alpha^G)\cdot g_t^{KG} \tag{12}$$

$$\log\tilde{y}_t = (1-\alpha)\cdot\left(logUCAP_t - log\overline{UCAP}_t\right) + \alpha\cdot\left(L_t - \bar{L}_t\right) \tag{13}$$

$$\begin{aligned} i_t &= \gamma\cdot i_{t-1} + (1-\gamma)\cdot\left[\left(\frac{1}{\beta}-1\right) + \bar{\pi}_t + \rho\cdot\left(\pi_t^C - \bar{\pi}_t\right) + \rho_1\cdot\log\tilde{y}_{t-1}\right] \\ &\quad + \rho_2\left(\log\tilde{y}_t - \log\tilde{y}_{t-1}\right) + \xi_t^M \end{aligned} \tag{14}$$

Equation (12) is the Cobb–Douglas production function in terms of growth rates, where g_t^Y, g_t^K, g_t^L and g_t^{KG} are growth rates of output, private capital, labor and government capital (mostly infrastructure). Growth rates of capital utilization (g_t^{UCAP}) and total factor productivity (g_t^{TFP}) augment capital and labor inputs. Equation (13) defines the output gap ($\tilde{y}_t$) as deviation of capital and labour utilization from their long run trends ($\overline{UCAP}_t$ and $\bar{L}_t$ respectively). Equation (14) is the Taylor rule, where β is the rate of time preferences, and $\bar{\pi}_t$ is the target path of inflation.

From these equations one might infer that a one-off increase of monetary shock (ξ_t^M) initially changes interest rate (i_t), inflation (π_t^C) and output gap ($\tilde{y}_t$). Output gap further changes capacity utilization rate, which, in turn, generates impacts on growth rates of output, capital and labor. From those three channels the monetary impact spreads throughout the QUEST_III model. However, the monetary shock propagation is not so simple or straightforward. It will later be clear that, for example, inflation does not play any significant role in this process. To demonstrate that point, we report decomposed IRFs in Fig. 1, where their decomposition is presented with hatched stack bars. Isolated effects of the interest rate or its partial IRFs are always hatched with a cross pattern on the white background.

Ratto et al. (2009) report in Fig. 5 of their paper impulse responses of the eighteen model's variables to a monetary shock. We reproduce all these IRFs in Fig. 6 in the Appendix as solid lines and augment them with their decomposition. The first graph in Fig. 6 exactly corresponds to the graph in Fig. 1. It is evident that tightening of monetary policy produces a long-run negative effect on the output level, but with diminishing marginal impacts.

Looking at the line of IRF in Fig. 1 we cannot say anything about three important things: (i) whether all of the mentioned variables are the key drivers of output adjustment to a change in monetary policy; (ii) which of those variables have positive or negative effects on output; and finally (iii) what the size is of impact of the key drivers in this adjustment mechanism. Knowing those three things we will be able to better assess the propagation path of a monetary shock throughout the model and to properly

[5] Ratto et al. (2009) report corresponding IRFs in several graphs from Figs. 2, 3, 4, 5, 6 and 7. For us, Figure 5 contain the reference IRF graphs, since they directly refer to monetary transmission mechanisms.

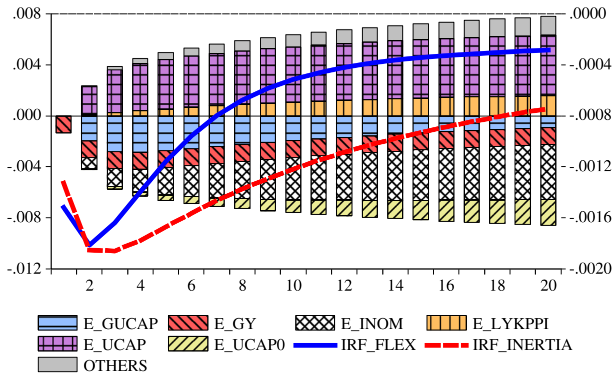

Fig. 2 Rigidities in capacity utilization (right scale for *lines* of aggregate IRFs and left scale for *bars* of decomposed IRFs)

analyse monetary policy in a real world situation if the model correctly represents the actual economy.

To obtain the answers to those questions, we do IRFs decomposition. IRFs decomposition, however, calls for a subtle distinction between an impact of the monetary shock on a variable and that variable's feedback to all other variables that are induced by the monetary shock. This corresponds to the distinction that we make between aggregate IRFs and partial IRFs. The impact of a monetary shock is measured by the aggregate IRF. It takes into account decomposed effects of all state variables. The feedback of the interest rate is a decomposed IRF that is put into motion by the monetary shock. This impact is isolated from all other state variables' effects. It is a partial impact and does not coincide with the aggregate impact of a monetary shock to the interest rate.

This subtle distinction between aggregate and partial impacts of the interest rate change is not recognized in the paper by Ratto et al. (2009) or in the literature. The authors write that "the [monetary] shock leads to a rise in the (annualized) nominal short-term interest rate of 0.4% points on impact…The monetary policy shock is not very persistent and nominal interest rates return quickly to base" (p.28). This conclusion is based on the inspection of Fig. 5 in their paper, more specifically the sixteenth graph in this figure. We have replicated and modified their Fig. 5 with our *irf_decomposition* function and presented it in this paper as Fig. 6 in the Appendix. Aggregate IRFs are printed as solid lines for which the right scale applies. On that account it is possible to easily compare our IRFs with Ratto et al.'s. This is the part of replication. The part of modification refers to IRFs decomposition. Decomposed IRFs are presented as stacked bars for which the left scale applies.[6]

We recognize the lack of persistence of a monetary shock to the interest rate. The adjustment path of the interest rate (E_INOM) is almost exclusively driven by inertia (see graph sixteen in Fig. 6). Other state variables play very limited roles. Relative

[6] Of course, the sum of decomposed IRFs must be equal to the aggregate IRF. That is not visually obvious due to different graph scales.

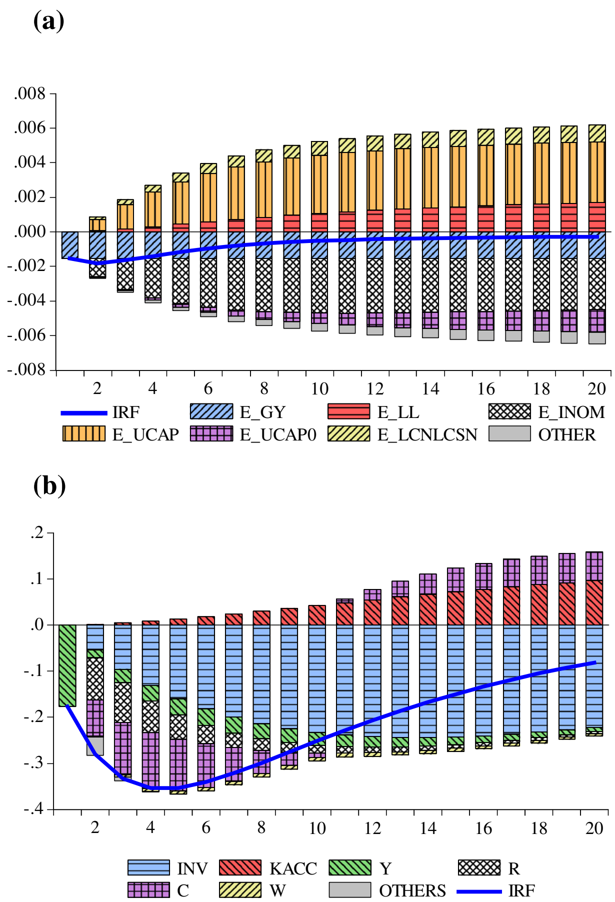

Fig. 3 IRFs of output to a monetary shock in two alternative models. **a** Impulse responses of output to a monetary shock in Ratto et al. model. **b** Impulse responses of output to a monetary shock in Smets–Wouters model

import prices (E_LPMP) and output gap (E_LYGAP) push up slightly the nominal interest rate, but improvements in the capacity utilisation (E_UCAP) forced the interest rate back to the steady state. Overall, the interest rate practically returns to the steady state in ten periods. However, the individualized impact of the interest rate on the output level lasts permanently (see first graph in Fig. 6 or 1). After twenty periods this impact is neutralized by other state variables, and IRF of the output to a monetary shock broadly returns to the steady state. Among other state variables, capacity utilization (E_UCAP), employment (E_LL) and share of the Ricardian consumption in GDP (E_LCNLCSN) offset negative effects of the interest rate rise.

The individual effect of a rise in the interest rate on the output level following a monetary shock is much stronger than the aggregate effect of the monetary shock. The aggregate effect in this case veils the particular impact of the interest rate increase. The interest rate adjustment permanently and significantly depresses the output level,

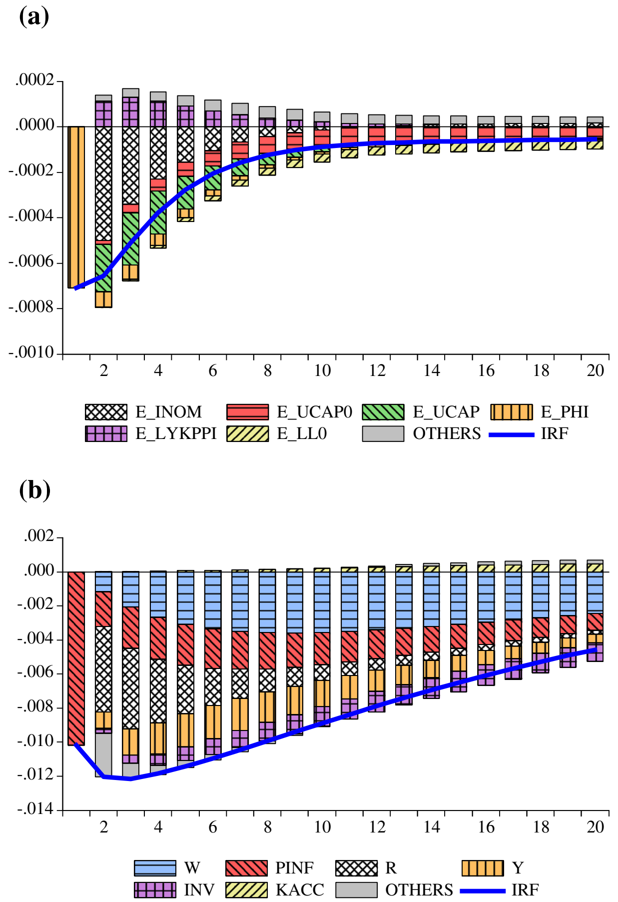

Fig. 4 IRFs of inflation to a monetary shock in two alternative models. **a** Impulse responses of inflation to a monetary shock in Ratto et al. model. **b** Impuls responses of inflation to a monetary shock in Smets–Wouters model

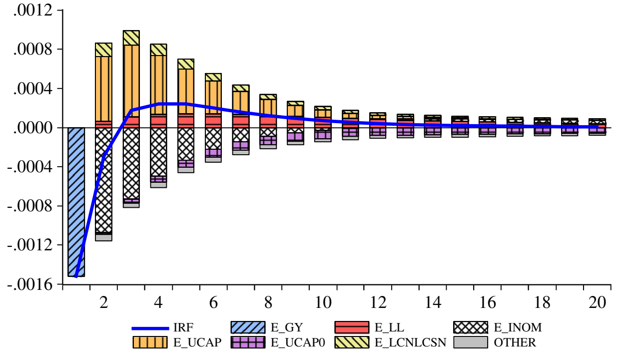

Fig. 5 Period impulse responses of output to a monetary shock (*line* refers to aggregate IRF and *bars* to decomposed effects of state variables)

but this negative effect is eventually overcome by positive effects of other state variables.

As we mentioned, Fig. 6 in the Appendix focuses on the eighteen variables from the QUEST_III model. Within this subset of variables, the interest rate significantly influences output, investment, trade balance, real wage rate, output gap, inflation and CPI, real interest rate and real exchange rate. Quite the opposite, its influence on consumption, both Ricardian and liquidity constrained consumptions, government consumption and investment, transfer payments and fiscal deficit is limited if not negligible. This finding is strikingly falsified by the aggregate IRFs.

We summarize impacts of the interest rate adjustment after a monetary shock on selected variables from the QUEST_III model in Table 1. It is interesting to note that even if prices play a key role in the Taylor rule, and the expected inflation is the major concern for the monetary policy of any central bank, the QUEST_III model does not assign such a strong weight to them as state variables. In the subset of the eighteen variables, only in two cases (E_PHI and E_PHIC) does inflation take place among the group of key state variables. The relative export price (E_LPXP) also significantly drives the trade balance (E_TBYN). Impulse reactions to a monetary shock of output, consumption and investment (both private and government), employment, transfer payments and fiscal balance are more sensitive to real variables, than to price changes. That challenges a widespread opinion in the literature that monetary policy primarily works through nominal, not real channels.

5 Rigidities

Decomposition of IRFs provides important information for conducting monetary policy. The above findings suggest that the reaction of output to a monetary shock crucially depends on flexible adjustment of capacity utilization. If such adjustment is subject to some delays or obstacles caused by technology or institutional rigidity, cost of monetary tightening in terms of forgiven output might be much higher. That is what right now intuition tells us. Ratto et al. (2009) have anticipated some rigidity in terms of incurred costs. They write that, "Firms also face technological and regulatory constraints which restrict their […] capacity utilization decisions […] Costs associated with the utilization of capital can result from higher maintenance costs associated with a more intensive use of capital […Firms] jointly determine the optimal […] capacity utilization by equating […] the marginal product of capital services to the marginal cost of increasing capacity" (p. 4).

That means firms are aware that higher maintenance costs might be associated with a more intensive use of capital, and can make optimal decisions about capacity utilization taking into account those costs. Technically speaking, firms take FOC from the Lagrangian function of the profit maximization problem with respect to capacity utilization, which incorporates adjustment costs of capacity utilization. The resulting equation is:

$$(1-\alpha)\cdot\eta_t\cdot\frac{Y_t\cdot P_t}{K_t\cdot PI_t}=UCAP_t\cdot\left[\rho_1^{ucap}+\rho_2^{ucap}\left(UCAP_t-\overline{UCAP_t}\right)\right] \tag{15}$$

Table 1 Direct and indirect impacts of the interest rate after a monetary shock on the selected variables

Selected variables	Type of IRFs	Direction of impact	Intensity	Counterbalancing by
Output	Cumulative	Negative	Strong	Capacity utilisation
Consumption	Cumulative	Negative	Moderate	Capacity utilisation
Investment	Cumulative	Negative	Strong	Investment expenditure to physical capital ratio
Non-Ricardian consumption	Cumulative	Negative	Strong	Capacity utilisation and consumption inertia
Ricardian consumption	Cumulative	Negative	Moderate	Capacity utilisation
Trade deficit	Discrete	Negative	Strong	Foreign output
Government consumption	Cumulative	Positive	Weak	Government consumption inertia
Government investment	Cumulative	Negative	Moderate	Output gap
Transfer payments	Cumulative	Negative	Moderate	Employment
Employment	Cumulative	Negative	Strong	Employment inertia
Real wages	Cumulative	Negative	Strong	Capacity utilization
Fiscal deficit	Discrete	Positive	Weak	Debt inertia
Output gap	Discrete	Negative	Strong	Steady-state output gap
Inflation	Discrete	Negative	Strong	Output-capital ratio
CPI	Discrete	Negative	Strong	Output-capital ratio
Interest rate	Discrete	Positive	Strong	Capacity utilization
Real interest rate	Discrete	Positive	Strong	Output-capital ratio
Nominal exchange rate	Cumulative	Negative	Strong	Inertia in real exchange rate

The steady-state capacity utilization ($\overline{UCAP_t}$) is normalized to 1 for the initial period and gradually evolves according to the following moving average process:

$$\overline{UCAP}_t = \rho^{ucap} \cdot \overline{UCAP}_{t-1} + (1 - \rho^{ucap}) \cdot UCAP_t \tag{16}$$

With η_t, Y_t, K_t, P_t and PI_t are denoted monopolistic price margin, output and capital levels, price of final goods and price of investment goods, respectively. Parameter α represents the capital share in output, while parameters ρ_1^{ucap} and ρ_2^{ucap} are from the quadratic equation of cost adjustment of capacity utilization. On the other hand, the capacity utilization rate ($UCAP_t$) is to adjust as to equate the marginal product of capital services to the marginal cost of increasing capacity. The growth rate of capacity utilization is therefore flexible and defined as:

$$g_t^{ucap} = \ln UCAP_t - \ln UCAP_{t-1} \tag{17}$$

Let us now consider the opposite case when a higher interest rate decreases capacity utilization due to lower final demand. Less intensive use of capital would imply lower maintenance costs in terms of the model equations. However, this is not enough to prevent further decrease in capacity utilization. A change in relative prices, because of the lower maintenance costs, is not present in the group of the leading six drivers for capacity utilization. As Fig. 8 in the Appendix shows, the initial interest rate increase is permanently depressing capacity utilization. When this negative trend sets off, the process of inertia is additionally contributing to an even lower rate of the capacity utilization. Adjustments in employment only partially compensate for decreasing capacity utilization.

The interesting question is what happen if the adjustment of capacity utilization was not a symmetric process in which expansion and contraction phases are not equally flexible. One can imagine that a higher interest rate will not immediately reduce capacity since firms might continue production with building up inventories in order to avoid unpleasant firing of workers. In such a case, it is reasonable to add a sluggish adjustment of the capacity utilization to the basic set up of the model.

We simply model this by modifying Equ.80 in the following way[7]:

$$g_t^{ucap} = \rho_3^{ucap} \cdot g_{t-1}^{ucap} + \left(1 - \rho_3^{ucap}\right) \cdot [\ln UCAP_t - \ln UCAP_{t-1}] \tag{17b}$$

Under this setting, the cost of tightening monetary policy in terms of forgiven output is much higher. The dotted line in Fig. 2 represents the corresponding IRF. Alternatively, the solid line reproduces impulse responses to a monetary shock under the assumption of flexible capacity utilization. It is evident that a sluggish adjustment in capacity utilization generates higher output drop than a flexible adjustment. One can measure the effect of a sluggish capacity adjustment by comparing two related IRFs. Additionally, decomposed IRFs provide information about how key state variables drive responses of the output to a monetary shock under the framework of frictions. This information can improve our understanding of the transmission mechanism of the monetary policy in the real world, which is full of frictions, delayed reactions and sluggish adjustments.

6 Monetary Policy Channels in Smets–Wouters Model

Monetary policy analysis uses DSGE models based on micro decisions of economic agents in a business cycle framework. They add the Taylor rule, to that optimization framework, as a function that describes monetary policy reaction of central banks under their goal to keep inflation low and output high as much as possible. Alternative versions of this function give rise to a general discussion about which monetary policy rules are optimal. We will do here a different exercise. We are not going to compare IRFs of two alternative monetary policy rules in the same model, but IRFs from two models with the same Taylor rule. Our point is that the monetary transmission mechanism not only depends on the Taylor rule, but also on the structure of the model.

[7] We set $\rho_3^{ucap} = 0.90$.

Additionally, separate effects of monetary policy conduct are veiled with general equilibrium adjustment to a monetary shock by other state variables and design of equations that describe agents' behavior in the model. We will illustrate this point by comparing another well-known DSGE model developed for the USA economy by Smets and Wouters (2007), to the QUESY_III model of the European economic area.

For that purpose, we slightly modified Equ. (14) of Smets–Wouters model and set the Taylor rule in the form that is fully compatible with Ratto et al. (2009) Equ. (14):

$$r_t = \gamma \cdot r_{t-1} + (1-\gamma) \cdot \left\{ \left[\frac{1}{\beta} - 1 + (1-\rho_p) \cdot \bar{\pi}_t \right] + \rho_p \cdot \pi_t + \rho_y \cdot (y_t - \bar{y}_t) + \rho_{\Delta y} \left[(y_t - \bar{y}_t) - (y_{t-1} - \bar{y}_{t-1}) \right] \right\} + \varepsilon_t^r \tag{18}$$

where r_t, π_t, y_t are interest rate, inflation and output, respectively, while their steady-state counterparts are $\bar{\pi}_t$ and $\bar{y}_t$. Modification refers to the first square bracket in Equ. (14), which introduces the steady-state real interest rate (β is the rate of time preference). Since Ratto et al. (2009) did not assume that the monetary policy shock follows a first-order autoregressive process, but that it instead is one-off innovation, we also modified the monetary shock equation by imposing a restriction on the inertia parameter $\rho_R = 0$.

Both models now work with the same Taylor rule, but description of the underlying economy differs in some important points. Therefore, monetary transmission channels are different even if both models repeat similar responses of output and inflation to a rise in the policy interest rate. We summarize differences between the two models in Table 2. Decomposed impulse responses of an extensive set of variables to a monetary shock in Smets–Wouters model are presented in Fig. 7 in the Appendix.

Let us first compare aggregate and decomposed impulse responses of output to a monetary shock from two models in Fig. 3. The aggregate effect of monetary tightening is persistent and evidently negative in both models (solid lines). Drop acceleration is rather short in Ratto et al. (2009) model, where the absolute minimum is achieved in the second period, while it takes a little bit longer in Smets–Wouters model, where the same minimum occurs at the fifth period. Further similarities, except output inertia, are lacking. In Ratto et al. (2009) model an individualized impact of the interest rate plays a strong negative role all the time. Its negative result accumulates up to the twelfth period and only slightly weaken afterwards. On the other hand, similar impacts of the interest rate on output are much weaker in Smets–Wouters model and are constantly dying out. They practically disappear after the twelfth period. Negative impacts of a rise in the interest rate are partially offset by comovements of capacity utilization and employments in Ratto et al. (2009) model. Quite the opposite, capacity utilization plays a minor role in Smets–Wouters model. Instead, the process of capital accumulation is much more important. Capital accumulation is constantly relaxing the output drop. Since consumption recovered in the second half of the periods under consideration, consumption contributed to a lower fall in output

Table 2 Alternative drivers in monetary channels of the two models

Variables	Ratto et al.		Smets–Wouters	
	The main drivers	Supplement drivers	The main drivers	Supplement drivers
Output	Capacity utilization and interest rate	Labor, output inertia and consumption	Investment	Capital accumulation, consumption and interest rate
Consumption	Capacity utilization	Labor, consumption inertia and interest rate	Inertia and investment	Interest rate, output and capital accumulation
Investment	Investment-capital ratio	Interest rate	Investment inertia	Interest rate
Labor	Employment inertia	Capacity utilization and interest rate	Investment	Consumption, capital accumulation and interest rate
Wage rate	Capacity utilization	Labor and interest rate	Wage rate inertia	Interest rate and output
Inflation	Interest rate	Capacity utilization	Wage rate	Inflationary inertia, interest rate and output
Interest rate	Inertia	Capacity utilization	Inertia	Output and investment

in that stage. The overall impact of a rise in the interest rate on output is amazingly similar in both models, but the underlying adjustment mechanisms are completely different.

Monetary policy has proved to be a useful tool for containing inflationary expectation. It is evident from Fig. 4 that an increase in the interest rate brings down inflation in both models. However, adjustment mechanisms are far from being comparable. Inflation inertia in Smets–Wouters model is strong, while it is weak in Ratto et al.'s model. In the latter case, inflation inertia disappeared rather quickly, while in the former case it is present up to the end. The interest rate weakens inflationary expectations all the time in Smets–Wouters model. It, however, evaporates in Ratto et al.'s model in ten periods. In addition, other factors discipline inflation rate. Lower wage costs and shrinking demand due to decreasing investments additionally push down inflation rate in Smets–Wouters model. Similar stabilizing effects are present in Ratto et al.'s model but are due to a different driver. Capacity utilization adjustment—current and steady state—reduces inflation rate. Once again, the overall impact of a rise in the interest rate on inflation is amazingly similar in both models, but the underlying adjustment mechanisms are completely different.

7 Decomposition Function

We need first to solve a DSGE model by using DYNARE and then to run MATLAB function *irf_decomposition* in order to get IRFs decomposition.[8] This function has a general form as follows:

irf_decomposition(endo_var_name, shock_name, shock_value, max_num_for_ plotting, periods, irf_type, order).

Endogenous variables (*endo_var_name)* and stochastic shocks (*shock_name*) must be declared under single quotation marks. The function applies at a time for only one endogenous variable and one shock. The size of the shock (*shock_value*) is up to a researcher's choice. Usually, it is either one percent increase in the shock or a standard deviation of it. Both models use a standard deviation of shocks. Since standard deviations of the monetary shock were different across the models, we normalize monetary shock to 0.01 in both cases.

DYNARE reports IRFs in the same format as the underlying variables enter the model. For example, the output variable in the QUEST_III model is expressed in terms of growth rates. There is no output level as the model's variable. Hence, the reported IRF is a discrete or period IRF, which takes into account only the rate of change of the output response materialized in a particular period following an initial monetary shock. To get the effect of the same shock on the output level, one has to accumulate IRFs. DYNARE does not have an option to distinguish between period and cumulative IRFs. Therefore, we put an option (*irf_type*) in the *irf_decomposition* function to compute cumulative IRFs of any variable to any shock. When doing decomposition of IRFs one has to specify what type of IRFs is decomposing: discrete or cumulative. The *irf_type* is a numeric cell, which takes alternative values "0" or "1". Value "0" triggers computation and decomposition of discrete IRFs, while value "1" triggers computation and decomposition of cumulative IRFs.

In our example of the output responses to a monetary shock from the QUEST_III model in Fig. 1, we presented cumulative IRFs. The reason for this was that Ratto et al. (2009) did the same and we wanted to replicate their results. However, if one runs the QUEST_III model under DYNARE command, she/he will not get Fig. 1. DYNARE saves data on IRFs in a structural file under the label oo_.*irfs.NameVariable_NameShock*. If the command *stoch_simul* does not suppress display of graphs, then DYNARE prints graphs of IRFs in the form of a line as in Fig. 5.

This graph shows a period impulse response of the output growth rate to a monetary shock and its decomposition. The growth rate of output initially falls due to a higher interest rate, but quickly rebounded and turned to a positive value. In the first period of adjustment, output growth rate (E_GY in the model notation) deeply drops as a direct consequence of the initial interest rate increase. In the first period there is present only the direct effect of a monetary shock. Indirect effects or propagation of the monetary shock starts to work from the second period on. In that period two

[8] The source code of this function is available on request by email to miroljub.labus@belox.rs.

remarkably opposite impacts happened. The interest rate (E_INOM) still pulled down output growth rate, but the capacity utilisation (E_UCAP) started to neutralize its negative effect. Since the third period, other state variables had more influence. On the negative side there are two impacts: movement in the long-run steady state capacity utilisation (E_UCAP0) and the net effect of all other non-displayed state variables (OTHER). On the positive side the rate of employment (E_LL in logarithms) and the share of Ricardian consumption in GDP (E_LCNLCSN in logarithms) contributed the most. Since the fourth period discrete positive effects outperformed negative effects, and discrete IRF becomes positive. Changes to the impacts of the interest rate and capacity utilization practically died out after ten periods, but their cumulative effects continue to shape the output level.

One has to carefully select the value of *irf_type* cell. The rule of thumb is that if a model works with growth rates, it must be “1” for IRFs of level variables, and “0” for growth rates. That was the case for the QUEST_III model. Therefore, we indicated in Table 1 in the second row what type of IRFs we were working with. In the case of Smets–Wouters model, the suggested value for *irf_type* is “0”.

The QUEST_III model has 107 endogenous variables, out of which 68 are state variables. Each state variable contributes to IRFs, some of them significantly, the others negligibly. It would be impractical and unproductive to graph all of those impacts. Therefore we introduce in the function *irf_decomposition* an argument related to the number of the most significant state variables, impacts of which would be displayed. Researchers can manipulate this number until they get the most informative graph. This is accomplished with parameter max_num_for_plotting.

We reported six key state variables from the QUEST_III model in Fig. 1 that are related to monetary transmission channels. For the illustration, we now report the ten state variables in Fig. 9 in the Appendix. By increasing the number of displayed variables, one might reveal effects of additional state variables which have a less important role. For example, we need to increase the number of displayed state variables to ten in order to highlight the relative import price as one of the key drivers. Further increase of this number to fifteen-displayed state variables will additionally highlight the relative export price. To put the relative price of consumer goods into the key drivers group, we need to increase this group to twenty state variables. It is evident that partial contributions of the relative prices or inflation rate to the response of aggregate IRF of output to a monetary shock are rather minor. This is what one hardly expects if he/she believes in the Taylor rule and its impact on output.

It is also worth noticing that the Net Foreign Assets (NFA or E_BGYN) also has a rather limited role as the state variable. It appears in the QUEST_III model in the interest rate parity equation (19):

$$i_t = i_t^W + g_{t+1}^E + \varphi \frac{NFA_t}{y_t} + \xi_t^{\varphi} \tag{19}$$

where the foreign interest rate is i_t^W, an expected change of the nominal exchange rate is g_{t+1}^E, and the share of real NFA in GDP is expressed as $\frac{NFA_t}{y_t}$. NFA is composed of foreign bonds that are subject to an external financial intermediation premium risk (ξ_t^{φ}). Its role as a state variable is not visible, unless the number of displayed state variables is considerably increased.

Finally, the remaining cell in the function *irf_decomposition* refers to the number of periods for which IRFs are computed (*periods)*. It is also a numeric cell. We used the number "20" and shorted the length of IRFs' display to 20 points in order to emphasis initial responses and visually inspect their paths in a clear-cut way. Ratto et al. (2009) reported IRFs for 40 periods even if the adjustment process took a much shorter time to restore the steady-state. Smets and Wouters also work with 20 periods of adjustment for IRFs.[9]

Finally, the function *irf_decomposition* has the last option *order*. This option selects the order of approximation for IRFs decomposition. If the solution to the non-linear model is obtained by linearization or log-linearization based on the first-order approximation, the value must be "1". If the solution is based on the second-order approximation, the value is "2". Ratto et al. (2009) model is log-linearized, while Smets and Wouters is linearized, hence, the order option must be "1".

8 Conclusion

An IRF is obtained in a DSGE model using the reduced form equations, which approximate the numerical solutions to non-linear DSGE models. IRFs are computed in an iterative way by adding up the individual contributions of the state variables to each variable of the model following the initial one-period shock of the control variable. This process is closely followed in this paper and the individual contributions to the aggregate IRF are extracted for each period of iteration. The outcome is, as we call it, IRF decomposition that generates partial IRFs.

We illustrate application of this method in the QUEST_III model of the European Commission and Smets–Wouters model of the USA economy. The same method can be applied in any DSGE model of arbitrary complexity. The underlying software can handle decomposition of IRFs using both the first-order and the second-order approximation of Taylor series to equilibrium relations.

Using IRFs decomposition we cast new light on the monetary policy transmission mechanism in DSGE models. The interest rate influence is either veiled or assigned to aggregate impacts of other state variables. With IRF decomposition we can better understand this mechanism. For example, in the QUEST_III framework the negative effect of a rise in the interest rate is offset by flexible adjustment of the capacity utilization rate. If such flexibility is absent, the negative impact of an interest rate shock on output will be much higher.

Acknowledgements We thank to Marco Ratto for useful comments and research guidance. Of course, we are responsible for computation and findings.

[9] In our example from Fig. 1, we use the following *irf_decomposition('E_GY','E_EPS_M',0.01,6,20,1)*.

Appendix

See Figs. 6, 7, 8 and 9.

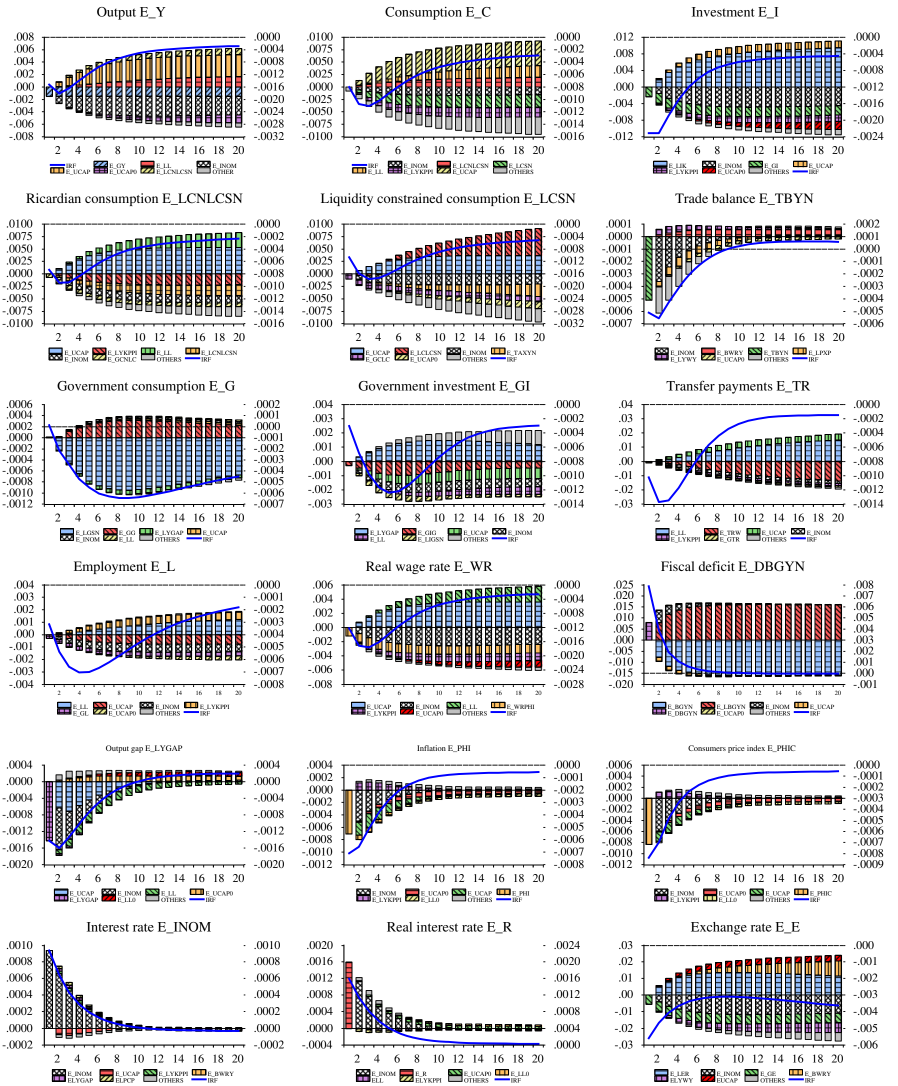

Fig. 6 Decomposition of IRFs to a monetary shock (*right scale* for *lines* of aggregate IRFs and *left scale* for *bars* of decomposed IRFs)

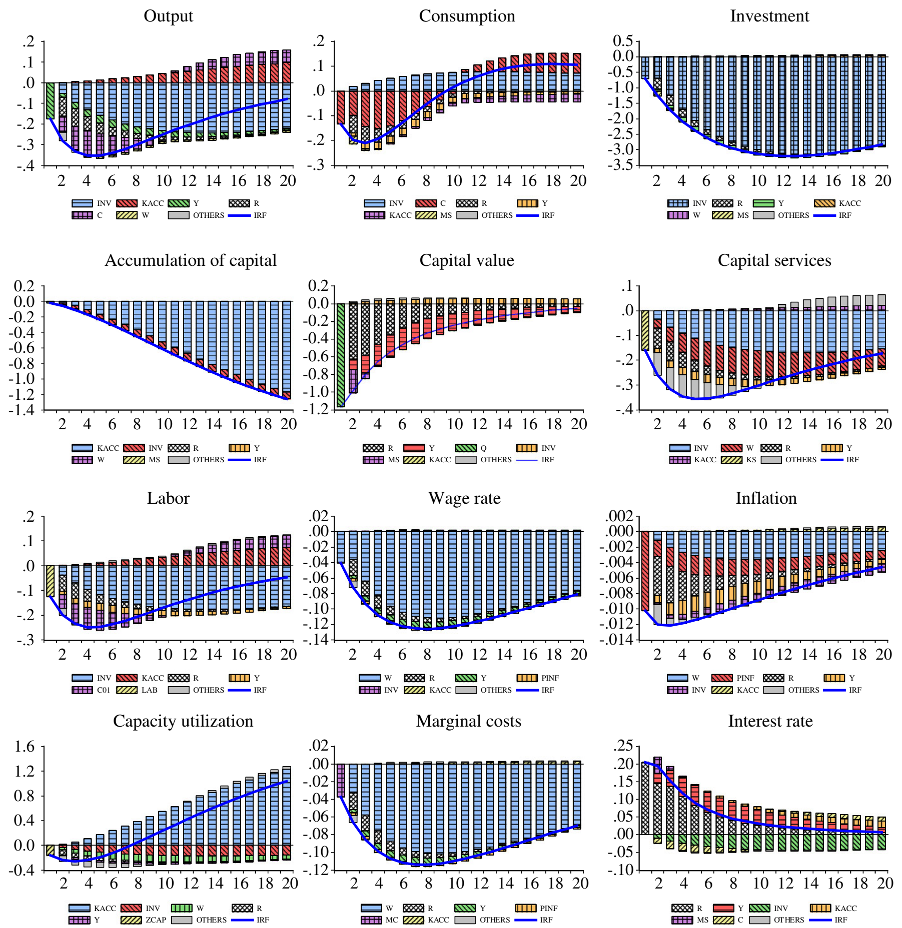

Fig. 7 Decomposition of IRFs to a monetary shock in Smets–Wouters model

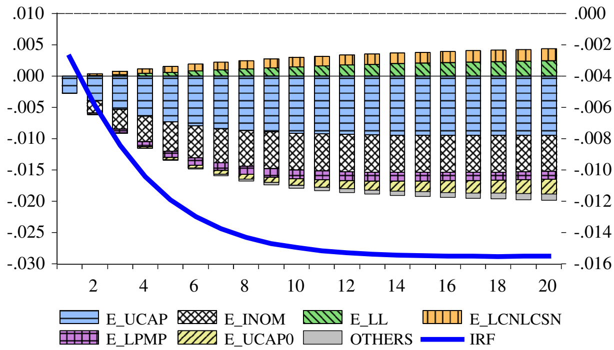

Fig. 8 Impulse responses of capacity utilization to a monetary shock (*right scale* for *lines* of aggregate IRFs and *left scale* for *bars* of decomposed IRFs)

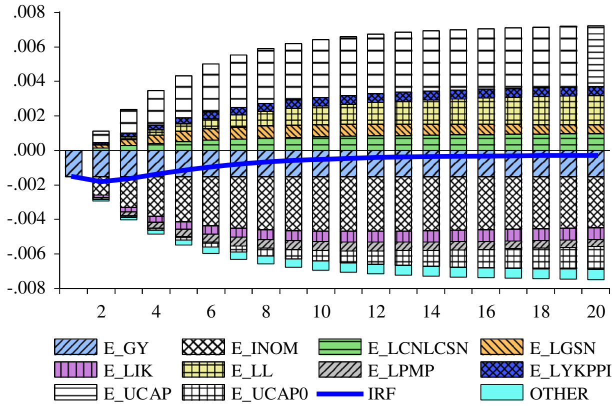

Fig. 9 Impulse responses of output to a monetary shock—decomposed effects due to ten state variables

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