L9 Example - Find WPBE of a General Game

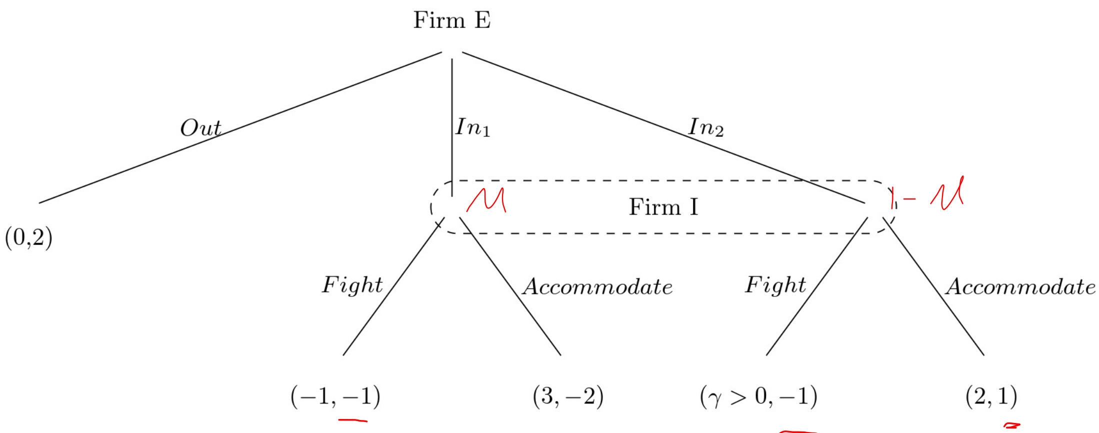
Yuan Zi

EI037 Microeconomics

General solution strategy:

- Find out the best responses (i.e., optimal strategies) that are consistent with a given belief
- Verify that the belief is consistent with rivals' strategy (i.e. belief is correct)
- Verify the strategies are the best response to each other, given the correct belief
- Sometimes you need to do this back and forth a couple of times to find the WPBE that satisfies all requirements or to rule out strategies or beliefs that cannot be part of a WPBE.

The game we studied in class:



Answer:

Step 1. Introducing some notations to facilitate our analysis

- Let $\sigma_1, \sigma_2, \sigma_3$ be the probability that Firm E plays Out, In_1, In_2, respectively.
- Let μ be Firm I's belief that he is at In_1 and $1-\mu$ that he is at In_2, when he is called to move.

Step 2. What is firm I's best response given his belief?

Firm I will find it optimal to choose *Fight*, if the expected payoff of playing *Fight* is higher than *Accommodate*. That is:

$$\underbrace{-1*\mu+(-1)*(1-\mu)}_{F} > \underbrace{-2*\mu+(1)*(1-\mu)}_{A}.^{1} \quad \Rightarrow \quad \mu > \frac{2}{3}$$

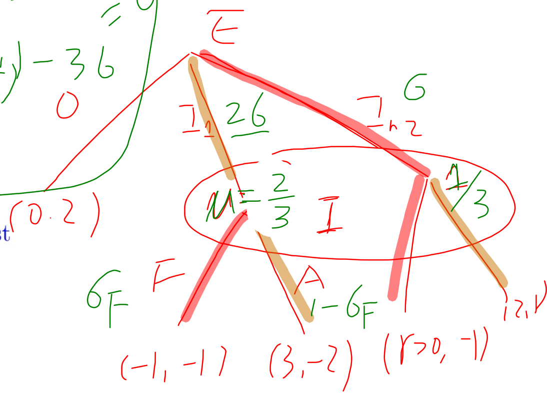
[1]The right-hand side (RHS) of this expression gives Firm I's expected payoff of playing *Fight*, the left-hand side (LHS) is Firm I's expected payoff of playing *Accommodate*, given the belief

$\sigma_E = (0, \frac{2}{3}, \frac{1}{3})$

$\sigma_I = (\frac{1}{\gamma+2}, 1-\frac{1}{\cdot}) - 3\sigma$

$\mu = \frac{2}{3}$

$\Rightarrow \sigma = \frac{1}{3}$



Therefore, Firm I will find it optimal to play:

- Fight if $\mu > \frac{2}{3}$
- Accommodate if $\mu < \frac{2}{3}$
- Indifferent if $\mu = \frac{2}{3}$

Step 4. Verify if Firm I's best response and belief are consistent with Firm E's best response.

- Suppose $\mu > \frac{2}{3}$ is part of a WPBE.
  - From step 2, we know that when $\mu > \frac{2}{3}$, Firm I will choose *Fight*.
  - Given Firm I chooses *Fight*, it is optimal for Firm E to choose In_2.
  - However, to be consistent with Firm E's strategy, firm I's belief has to be $\mu = 0$, which contradicts $\mu > \frac{2}{3}$.
  - So $\mu > \frac{2}{3}$ cannot be part of a WPBE.
- We can prove in the same way that $\mu < \frac{2}{3}$ cannot be part of a WPBE.
- So $\mu = \frac{2}{3}$ is the only possible belief left. In what follows, we will try to find strategies that are both optimal responses and consistent with this belief. If we find them, then they constitute WPBE.

Step 5. Find out I's mixed strategy.

- We verified in step 4 that Firm I's pure strategies and beliefs supporting pure strategies cannot be part of the WPBE.
  - So if WPBE exists, in equilibrium, Firm I can only play mixed strategy with belief $\mu = \frac{2}{3}$.
  - To be consistent with I's belief, Firm E must also play a mixed strategy, with $\sigma_2 = 2 * \sigma_3$
- However, firm E will randomize between In_1 and In_2 **only if** there is no one pure strategy dominating the other pure strategy.
  - This is the case only if, given the I's strategy, firm E's expected payoff from playing In_1 is the same as playing In_2.
  - Let Firm I play *Fight* with probability σ_F. This implies:

$$\underbrace{(-1) * \sigma_F + 3 * (1 - \sigma_F)}_{E\ In_1} = \underbrace{\gamma * \sigma_F + 2 * (1 - \sigma_F)}_{In_2} \quad \Rightarrow \quad \sigma_F = \frac{1}{\gamma + 2}$$

· Exp(Out)

· Exp(In_1) = In_2

Step 6. Check if "Out" is part of Firm E's mixed strategy.

- When $\sigma_F = \frac{1}{\gamma+2}$, the expected payoff of playing In_1 or In_2 for Firm E is

$$(-1) * \frac{1}{\gamma+2} + 3 * (1 - \frac{1}{\gamma+2}) \quad \Rightarrow \quad \frac{3\gamma+2}{\gamma+2}.$$

  That is, the expected payoff of playing "In_1" and "In_2" (payoff: $\frac{3\gamma+2}{\gamma+2}$) is higher than "Out" (payoff: 0) . So "Out" is not part of Firm E's mixed strategy. So in equilibrium, $\sigma_1 = 0, \sigma_2 = 2/3, \sigma_3 = 1/3$.

Step 7. Put everything together, and double check.

Given our analysis above, the WPBE of the game is given by:

- Belief: $\mu = 2/3$
- Strategy of E: $(\sigma_1, \sigma_2, \sigma_3) = (1, 2/3, 1/3)$
- Strategy of I: $(\sigma_F, 1 - \sigma_F) = (\frac{1}{\gamma+2}, 1 - \frac{1}{\gamma+2})$

Now that you have found the WPBE, you can now check your answers again: 1. whether the strategies are mutually best responses, 2. whether the strategies are consistent with the beliefs, 3. whether the beliefs are consistent with the strategies. If the answers are all "yes", you can be sure you got the exercise right. :)