

EI135: International Trade I
Spring 2025
Problem Set 1

Due Date: Monday, March 31, 17h00 (by email)

Note: You may work alone or in pairs. If the latter, each team should submit a single response. Typewritten answers are preferred. If you choose to write by hand, please make sure it is neat and legible.

1. Gains from Trade

Initially, Australia (A) trades freely with Britain (B). Both are large countries. The vector of equilibrium prices in the initial free-trade equilibrium is \mathbf{p}_1 . Now it becomes possible for Australia to trade (freely) with China (C). The vector of equilibrium prices in the three-country world is \mathbf{p}_2 . China runs a trade deficit with Britain balanced by a surplus with Australia. Either (i) prove that the representative consumer in Australia always gains from the entry of China into the trading system, or (ii) provide a sufficient condition for the representative Australian to gain.

2. Ricardian Trade and Technological Progress

Consider a Ricardian model of trade with two countries, Home and Foreign, and two goods, Clothing and Food. The labour requirements per unit of output are summarized in the table below:

	Home	Foreign
Labour force	$L = 1000$	$L^* = 1000$
Labour per unit of Clothing	$a_C = 1/z$	$a_C^* = 1$
Labour per unit of Food	$a_F = 1/z$	$a_F^* = 1/4$

In other words, each Home worker can produce z units of either food or clothing per work hour, where $z > 0$ is an index of Home worker productivity. Each Foreign worker can produce either 1 unit of clothing or 4 units of food per work hour. Each country has an endowment of 1000 work hours.

All workers in both countries have the same preferences for clothing and food, represented by the utility function $U(C, F) = CF$ in obvious notation. All prices in both countries are measured in a common unit of account.

1. Which country has absolute advantage in which good? How does your answer depend on the value of z ?

2. Which country has comparative advantage in which good? How does your answer depend on the value of z ?
3. First assume that home productivity is given by $z = 2$.
 - (a) Draw the world relative supply of clothing.
Hint: Place the relative quantity of clothing $\frac{C}{F}$ on the horizontal axis and the relative price $\frac{P_C}{P_F}$ at the vertical axis.
 - (b) Add the relative demand into your previous graph of relative supply to determine the free trade equilibrium. What is the specialization pattern in this equilibrium? What is the equilibrium relative price of clothing $\frac{P_C}{P_F}$ under free trade?
 - (c) Let w and w^* represent the wages paid to Home and Foreign workers per hour; calculate the ratio $\frac{w}{w^*}$ under free trade.
4. Now assume that Home's productivity z increases, but that both countries remain completely specialized in the new free trade equilibrium.
 - (a) Calculate expressions for $\frac{P_C}{P_F}$ and $\frac{w}{w^*}$ as functions of z .
 - (b) How does the increase in z change the welfare of each Home and Foreign worker? Explain the intuitive explanation for your finding.

3. Two-by-Two-by-Two with Fixed Coefficients

For the purpose of this exercise, the world consists of two countries: Austria (A) and Germany (G); two goods: Buns (B) and Sausages (S); and two factors of production: Labor (L) and Capital (K). All consumers in both countries consume the goods in bundles of fixed proportions (Leontief indifference curves), consisting of one Bun and one Sausage; each such bundle is called a Hotdog. The factor requirements per unit of output of the two goods are also fixed, and are the same across countries. They are shown in the following table:

	Bun	Sausage
Labor	$a_{LB} = 1$	$a_{LS} = 2$
Capital	$a_{KB} = 3$	$a_{KS} = 1$

The factors are immobile across countries, but mobile across sectors within each country. The factor endowments in the two countries are fixed, and are shown in the following table:

	Austria	Germany
Labor	$L_A = 420$	$L_G = 600$
Capital	$K_A = 460$	$K_G = 900$

You will be asked to draw various diagrams, and the parts of the question are steps in the construction of parts of the diagrams. You should only submit the final diagrams which show all of the parts of each, labeling all the points and lines appropriately. The diagrams do not need to be drawn very accurately to scale; a sketch that shows the relative positions of all the points and the relative slopes of all the lines correctly will suffice.

1. **Preliminaries and intuition:** Which country is relatively capital abundant, which country is relatively labor abundant? Which good is relatively capital intensive, which is relatively labor intensive? Which country has a comparative advantage in which good? Do you expect the autarkic relative price of Buns $P = P_B/P_S$ to be higher in Austria or Germany? If the two countries trade, what kind of trade pattern do you expect to see?
2. **Production possibility frontiers:** Write B for the output of Buns and S for the output of Sausages.
 - (a) Write down the equations that represent the full employment condition for Labor and Capital in Austria. In a diagram with Buns on the horizontal axis and Sausages on the vertical axis, show the lines that represent these full employment conditions. Show the production possibility frontier (PPF). At which point are both factors fully employed? What is the output of the two goods at this full-employment point?
 - (b) Repeat these steps for Germany.
3. **Autarkic equilibria:**
 - (a) Add the line that represents the consumption proportions (the corners of the Leontief indifference curves) into the PPF graphs for the two countries. What is the autarkic equilibrium production and consumption of Hotdogs in both countries? What is the autarkic relative price of Buns in both countries? Is this in line with your answer to (a)? Which theorem does this confirm?
 - (b) Are both factors fully employed in both countries? If not, which one suffers some unemployment and where? Write W for the wage rate and R for the return to capital. Write down the equations that represent the zero pure profit (price equals unit cost) conditions for the two goods. Use this to find the autarkic factor prices in both countries. How many Hotdogs can each worker and each owner of a unit of capital buy in Austria and Germany?
Hint: Remember that in microeconomics, anything that is in excess supply in equilibrium must have a zero price.
4. **Trading equilibrium:**
 - (a) Suppose the relative price of Buns in terms of Sausages is $P = P_B/P_S = 2$. Inspect the PPFs to find the production and consumption choices of the two countries at this relative price. Is this an equilibrium? Why, or why not?
 - (b) Find the free-trade factor prices in both countries, again in units of Hotdogs. Which theorem did you confirm with this calculation?
 - (c) Compare the free-trade factor prices to the situation in autarky. Who gains and who loses from trade? Which other theorem did you confirm with this calculation? What can you say about the effects of trade in aggregate? What does your answer imply for the distributional conflict?

4. Two-by-Two-by-Two

Consider the following world economy. There are two countries, H and F , and two production factors, capital K and labor L . Country H is endowed with $K_H = 3/4$ and $L_H = 1/4$, while F is endowed with $K_F = 1/4$ and $L_F = 3/4$. Let w_J and r_J be the wage rate and the rental rate in country J , $J = H, F$.

There are two goods, $i = 1, 2$, which yield utility to all households according to $u(c_1, c_2) = c_1^{1/2} c_2^{1/2}$, where c_i is consumption of good i . The countries have access to common technologies, which are given by $x_1 = L_1$ and $x_2 = K_2$. All markets are perfectly competitive.

1. Write down the conditions for autarky equilibrium in each country. Derive an expression for the equilibrium relative factor prices in each country as a function of exogenous variables and parameters.
2. Solve for the world integrated equilibrium. Which variables determine factor prices now?
3. Assume free trade in goods and no international factor mobility. Is the trade equilibrium characterized by factor price equalization (FPE)? Find the patterns of production and trade in the trade equilibrium. Describe the factor content of trade.
4. Now assume that the technologies for producing good 1 are different in the two countries: $x_{1H} = 9L_{1H}$ and $x_{1F} = L_{1F}$. The technologies for producing good 2 are common and as described above. Factor endowments are the same as above. Discuss whether the equilibrium is characterized by FPE.
5. Find the wage-rental ratios in the trade equilibrium. Find the equilibrium patterns of production and trade. Describe the factor content of trade.
6. Now assume that production functions are Cobb-Douglas, with $x_1 = A_1 K_1^{1/3} L_1^{2/3}$ and $x_2 = A_2 K_2^{2/3} L_2^{1/3}$, where $A_1 = A_2 = \left(\frac{2}{3}\right)^{-2/3} \left(\frac{1}{3}\right)^{-1/3}$. Factor endowments are as above. Compute the integrated equilibrium. Draw the FPE set. Discuss whether the equilibrium is characterized by FPE.
7. Find the wage-rental ratios in the trade equilibrium. Describe the pattern of trade.

5. Gravity with Multilateral Resistance

There are N countries, each of which produces a unique good. In every country, one unit of labor produces one unit of the local output. The labor supply in country i is L_i . Labor and product markets are perfectly competitive. There are iceberg trading costs between every pair of countries: τ_{ij} units of the good produced in country i must be shipped in order that one unit arrives in country j .

Consumers in country j have symmetric preferences with an elasticity of substitution of σ between any pair of goods. More specifically, utility of the representative consumer in country j is given by

$$U_j = \left[\sum_{k=1}^N \alpha_{kj}^{1/\sigma} x_{kj}^{(\sigma-1)/\sigma} \right]^{\sigma/(\sigma-1)}$$

where x_{kj} is consumption of the good produced in country k and α_{kj} is a preference parameter.

1. Let λ_{ij} be the share of total spending in country j that is devoted to goods imported from country i . Prove that

$$\lambda_{ij} = \frac{\alpha_{ij} w_i^{1-\sigma} \tau_{ij}^{1-\sigma}}{\sum_k \alpha_{kj} w_k^{1-\sigma} \tau_{kj}^{1-\sigma}} \quad (1)$$

2. Let $X_i = w_i L_i$ be GDP in country i , let p_{ij} be the local price in country j of goods imported from country i , and let $X_{ij} = p_{ij} x_{ij}$ be the value of imports in j from country i . Use the consumer budget constraint and trade balance to prove that

$$X_{ij} = X_i X_j \frac{\alpha_{ij}}{\sum_k \alpha_{ik} \left(\frac{\tau_{ik}}{P_k} \right)^{1-\sigma}} \left(\frac{\tau_{ij}}{P_j} \right)^{1-\sigma} X_k \quad (2)$$

where

$$P_j = \left[\sum_{k=1}^N \alpha_{kj} (p_{kj})^{1-\sigma} \right]^{1/(1-\sigma)}$$

is the price index in country j . Interpret (2) as a gravity equation subject to multi-lateral resistance.

3. Now prove that welfare in country j is given by

$$W_j = \alpha_{jj}^{1/(\sigma-1)} \lambda_{jj}^{-1/(\sigma-1)}$$