DSGE 2024 EXAM

Three hours allowed

November 12, 2024

This exam is based upon the celebrated model set out in Erceg, Henderson and Levin (2000), "Optimal Monetary Policy with Staggered Wage and Price Contracts", *Journal of Monetary Economics*, 46 (March), pp. 281-313, which introduced Sticky Wages into the DSGE world. Below is Table 1 of the above paper, which sets out the main (log-linearised) equations of the model

Table 1: Key Equations

$g_t = g_{t+1\|t} - \frac{1}{\sigma\bar{\ell}_C}\left(i_t - \pi_{t+1\|t} - r_t^*\right)$	(goods demand)
$mpl_t = \zeta_t^* - \frac{\alpha}{1-\alpha} g_t$	(marginal product of labor)
$mrs_t = \zeta_t^* + \left(\frac{\chi\bar{\ell}_L}{1-\alpha} + \sigma\bar{\ell}_C\right) g_t$	(marginal rate of substitution)
$\pi_t = \beta\pi_{t+1\|t} + \kappa_p\left(\zeta_t - mpl_t\right)$	(price setting)
where $\kappa_p = \frac{(1-\xi_p\beta)(1-\xi_p)}{\xi_p}$	
$\omega_t = \beta\omega_{t+1\|t} + \kappa_w\left(mrs_t - \zeta_t\right)$	(wage setting)
where $\kappa_w = \frac{(1-\xi_w\beta)(1-\xi_w)}{\xi_w\left(1+\chi\bar{\ell}_L\left(\frac{1+\theta_w}{\theta_w}\right)\right)}$	
$\zeta_t = \zeta_{t-1} + \omega_t - \pi_t$	(real wage change)

Most symbols are self-explanatory and familiar, with the exception of ζ*, which represents the flex-price real wage; r* is the flex-price interest rate (definitions are given below).

The (log-linearised) flex-price equations are given by equations 20 and 21 in the paper, as below:

$$
\begin{aligned}
y_t^* &= \frac{(1-\alpha)\sigma\bar{\ell}_U}{\Lambda} u_t - \frac{(1-\alpha)\chi\bar{\ell}_Z}{\Lambda} z_t + \left(1 + \frac{\chi\bar{\ell}_L}{\Lambda}\right) x_t \\
\zeta_t^* &= \frac{-\alpha\sigma\bar{\ell}_U}{\Lambda} u_t + \frac{\alpha\chi\bar{\ell}_L}{\Lambda} z_t + \frac{\chi\ell_L + \alpha\bar{\ell}_C}{\Lambda} x_t \\
r_t^* &= \sigma\bar{\ell}_C\left(y_{t+1|t}^* - y_t^*\right) + \sigma\bar{\ell}_U\left(u_{t+1|t} - u_t\right) \\
\Lambda &= \alpha + \chi\bar{\ell}_{\mathrm{L}} + (1-\alpha)\sigma\bar{\ell}_C
\end{aligned}
\tag{1}
$$

$$
\bar{\ell}_C = \frac{\bar{C}}{(\bar{C}-\bar{U})}, \quad \bar{\ell}_U = \frac{\bar{U}}{(\bar{C}-\bar{U})}, \quad \bar{\ell}_{\mathrm{L}} = \frac{\bar{L}}{(1-\bar{L}-\bar{Z})}, \quad \bar{\ell}_Z = \frac{\bar{Z}}{(1-\bar{L}-\bar{Z})}.
$$

In EHL2000, X is productivity, U is a "consumption shock" and Z is a "leisure shock"; the lower-case variables above are their log-linear counterparts, which are all three assumed to follow a simple AR(1) process. The elements with over-bars ($\overline{C}$, etc) are the steady-state (SS) values of the corresponding model variables. EHL derive SS values as follows: $Lss = 0.27$; Uss $= 0.3163$; Zss $= 0.03$; $Yss = 10*$ Uss ; Css $=$ Yss.

Question 1

Based on the model set out above, construct a Dynare programme to simulate the EHL economy using as a basis the attached partial mod-file **EHL.mod**, which you may use directly for exam purposes. To do so, open the zip file **Exam.zip** and extract the contents to a clean **exam** directory on your computer. To complete the model, you will need to introduce a **Taylor Rule** for the interest rate; include interest rate smoothing ρ_r and reaction parameters φ_π, φ_y for inflation and output (here "g") and a simple IID MonPol shock. You will therefore have a total of 14 equations. Note that as "*" means multiplication in Dynare, you should use notation like rn, yn and rwn for the flex-price variables r*, y* and ζ*. You will also find it convenient to use the composite parameters I have included in the partial mod-file **EHL.mod**.

Question 2

Using the model constructed in Q1 above, and the parameter values set out in the partial mod-file **EHL.mod**, simulate the EHL economy over 24 quarters and comment on your results, with special emphasis on the effects of the MonPol shock. To do so, you may find it useful to calculate the values of the composite parameters.

Question 3

Open Matlab and point it to the exam directory you created in Q1 above. The data provided in the file **Data4Exam.xls** is a subset of the data used by Smets and Wouters, updated to 2018Q4. The columns contain observations drawn from FRED on (i) real GDP, (ii) the GDP deflator, (iii) Population (in the form of an index with a base of 1992Q3 = 1), (iv) the US Federal Funds rate, (v) hourly earnings, respectively, from 1947Q1 to 2018Q4 inclusive. Using this data, construct appropriate **observed variables** to input into the model, using the methodology of SW (ie, first differences of log real per capita output, etc.). However, rather than inserting trends into the **measurement equations** you will use to map the input data to the model variables (as did SW), simply map your constructed variables directly (eg, yobs = g, etc) and then use "prefilter=1" in your estimation command to get rid of any non-zero mean.

Question 4

Now **estimate** (via **RegMLE** with mode_compute $= 1$ and - of course - mh_replic $= 0$) the parameters of this model **except for $\boldsymbol{\beta}$**. Thus, you will be estimating twelve structural parameters - $\alpha, \sigma, \theta_w, \chi, \xi_p, \xi_w, \varphi_\pi, \varphi_y, \rho_a, \rho_x, \rho_u, \rho_r$ - plus four standard errors (for e^x, e^z, e^u and e^{ms}). You may use the parameter values used in Q2 above as the prior means for your estimation. Restrict the time period over which you estimate to 1971Q1 - 2015Q4 inclusive. You will have four possible observed variables to use for estimation purposes (as there are four shocks); however, using the inflation variable causes the estimation to fail, so do not use it in your estimation. Comment on your results.