

Demystifying DSGE Models

2. Adding Bells and Whistles – The Calvo Fairy or Price and Wage Stickiness

Outline

- I. Under the Hood – Nuts and Bolts of a DSGE Model
- II. Adding Bells and Whistles – The NK DSGE in all its Glory
- III. Case Study – The Canonical Smets-Wouters DSGE
- IV. Bringing DSGE Models to the Data – Beyond Calibration and Simulation
- V. Extensions to the NK DSGE: The Financial and Housing Sectors
- VI. Smörgåsbord – Unemployment and Environment
- VII. Internationality – The Open Economy

Demystifying DSGE Models

Summary: The Basic RBC

- Last week we looked at a simple RBC model:
- Utility function [CRRA – there are many others]:

$$U(C_t) - V(L_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{L_t^{1+\varphi}}{1+\varphi}$$

- Production function [Cobb-Douglas]:

$$Y_t = A_t K_t^\alpha L_t^{1-\alpha}$$

- Law of motion of capital:

$$K_{t+1} = (1 - \delta)K_t + I_t$$

- TFP process:

$$\log A_t = \rho \log A_{t-1} + \epsilon_t$$

- Optimising, model became:

$$C_t^\sigma L_t^\varphi = \frac{W_t}{P_t} \quad (\text{Labour supply})$$

$$\left(\frac{E_t C_{t+1}}{C_t} \right)^\sigma = \beta \left[(1 - \delta) + E_t \left(\frac{R_{t+1}}{P_{t+1}} \right) \right] \quad (\text{Euler Equation})$$

$$K_{t+1} = (1 - \delta)K_t + I_t \quad (\text{Law of motion of capital})$$

$$Y_t = A_t K_t^\alpha L_t^{1-\alpha} \quad (\text{Production function})$$

$$K_t = \alpha M C_t \frac{Y_t}{R_t} \quad (\text{Demand for capital})$$

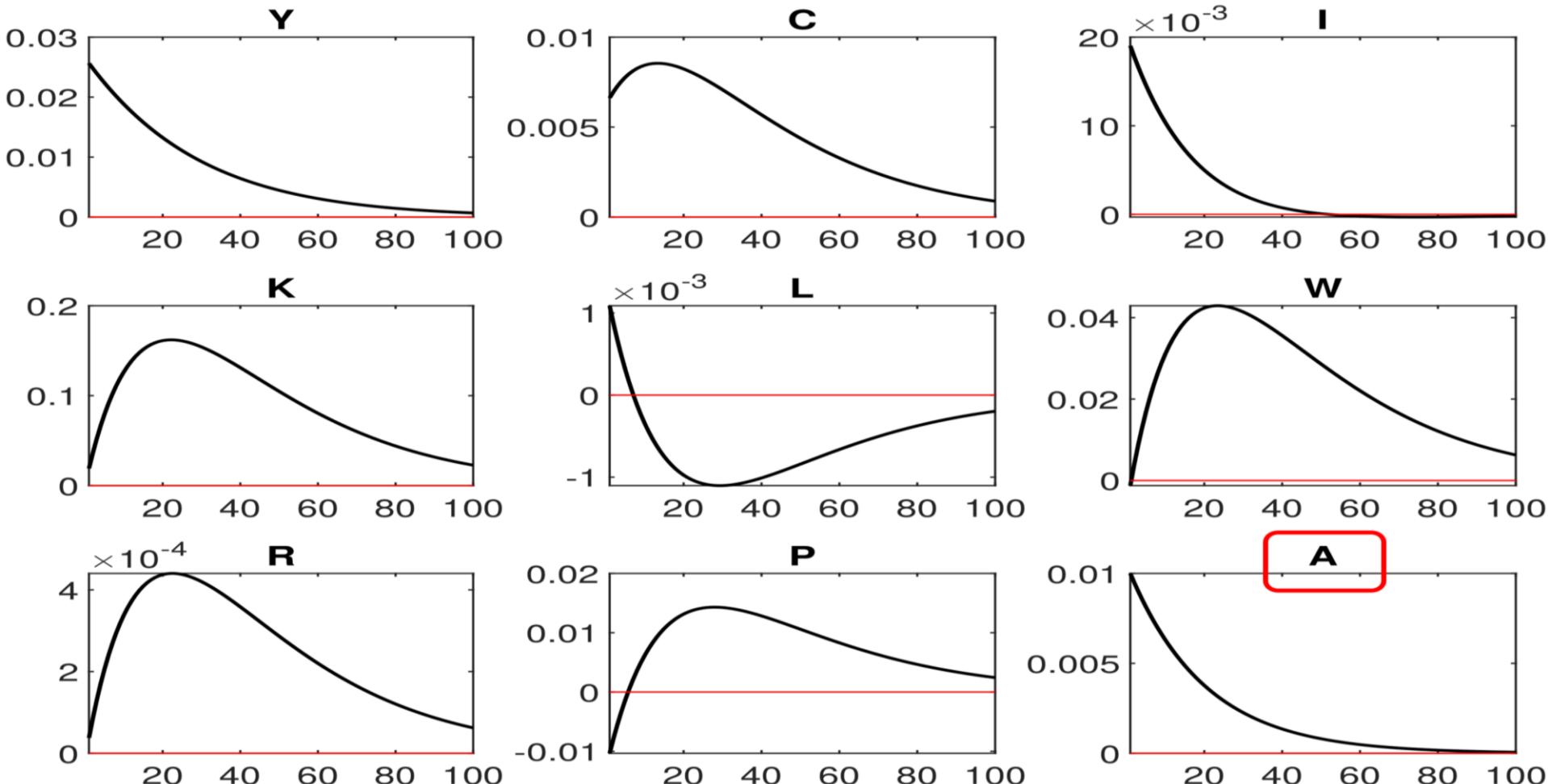
$$L_t = (1 - \alpha) M C_t \frac{Y_t}{W_t} \quad (\text{Demand for labour})$$

$$P_t = \frac{1}{A_t} \left(\frac{W_t}{1-\alpha} \right)^{1-\alpha} \left(\frac{R_t}{\alpha} \right)^\alpha \quad (\text{Price level})$$

$$I_t = Y_t - C_t \quad (\text{Equilibrium condition})$$

$$\log A_t = \rho_A \log A_{t-1} + \epsilon_t \quad (\text{Productivity shock})$$

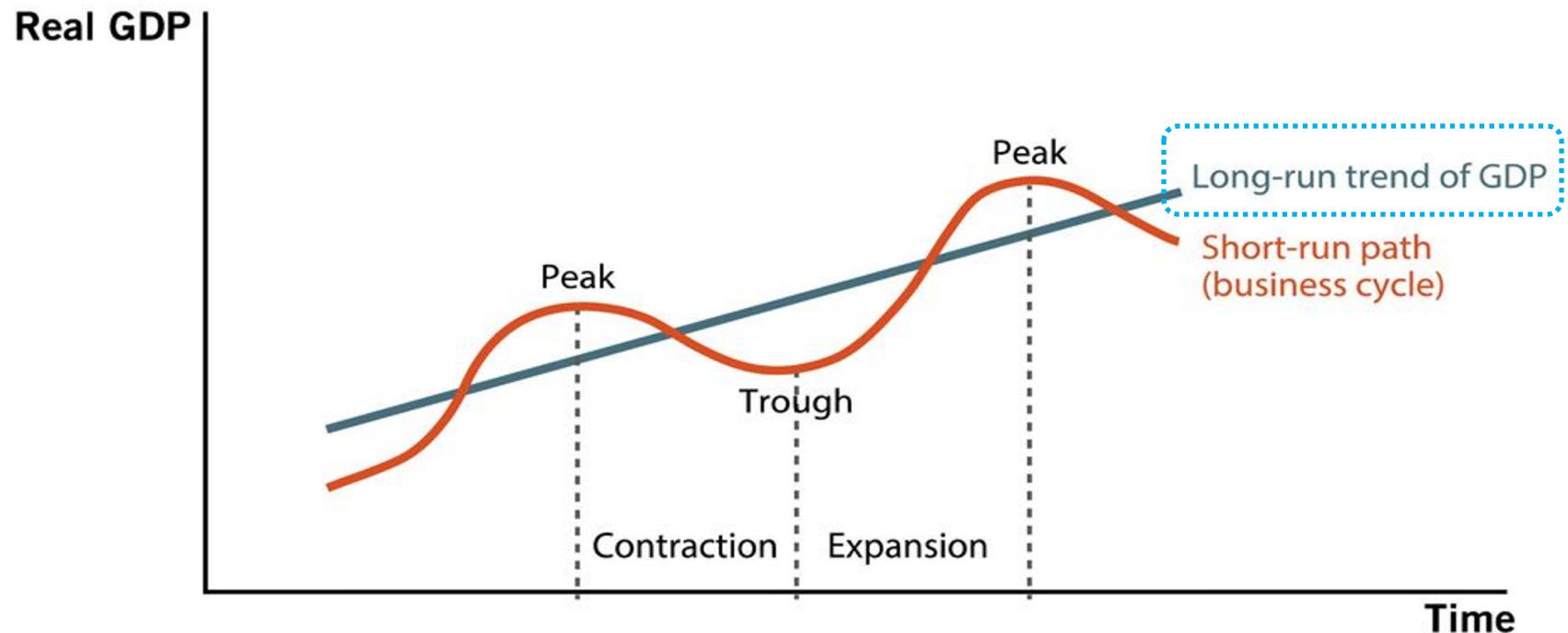
- We then simulated the behaviour of this economy, tracing out the way in which the macro variables responded to a shock in TFP



- We might well have stopped there
- But we were ambitious!
- Rather than just tracing out the behaviour of highly *trending* macro variables, we wanted to look at the *fluctuations* of the economy around these trends
- This led us to look at data like the **green** line in the slide after next

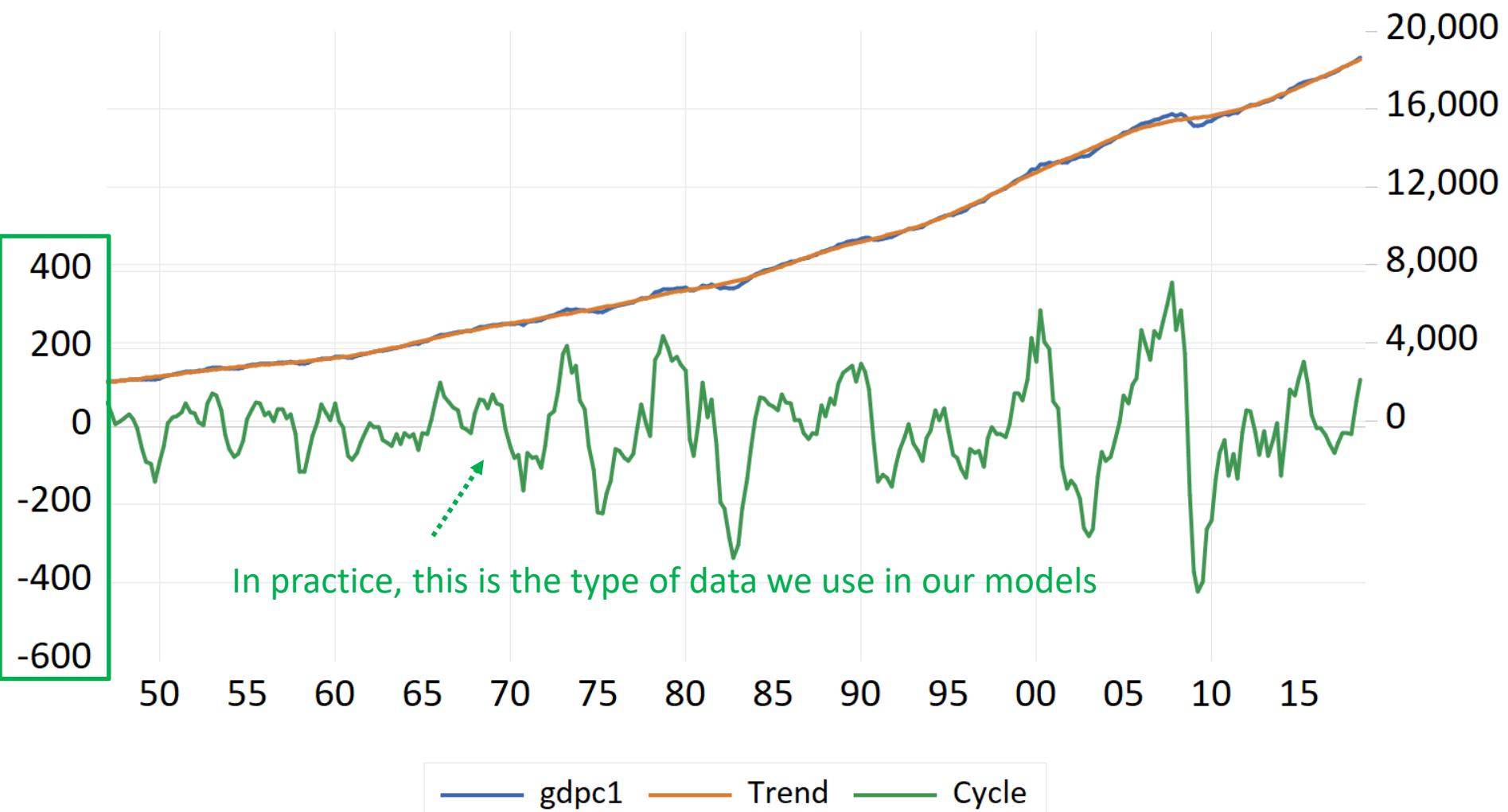
- “*Long-run trend of GDP*” = “Balanced Growth Path”, aka “Steady State”

The Business Cycle

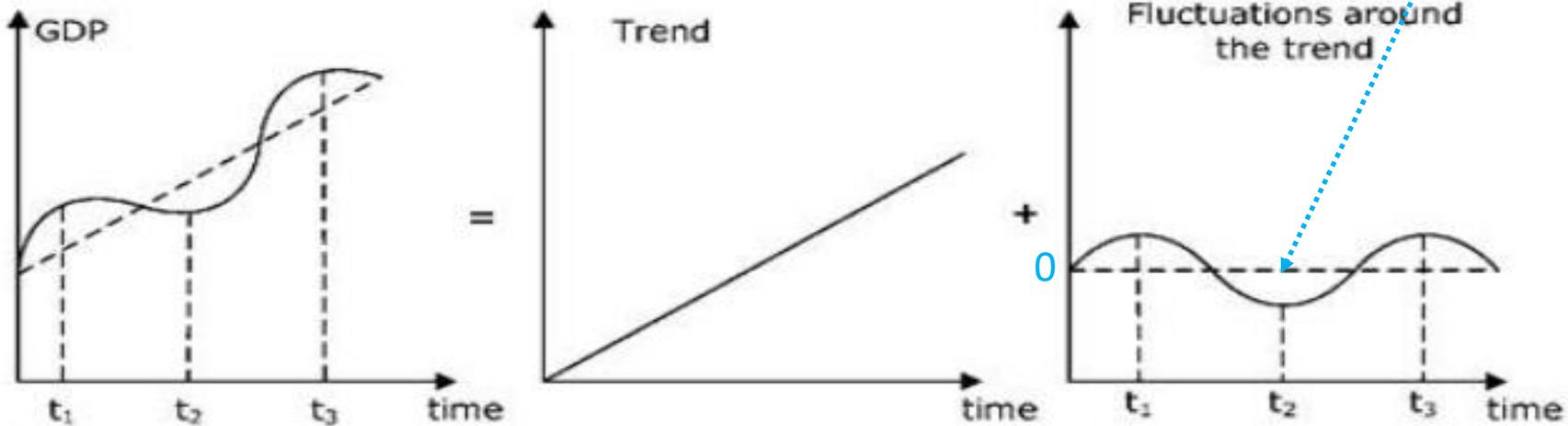


This shows *real GDP per capita* for the US over the period 1947Q1 – 2018Q3 (blue), the HP filter trend (brown) and the difference between the two (the “cycle”) in green

Hodrick-Prescott Filter (lambda=1600)



- And we noted that ***on average*** [ie, “**steady state**”], these fluctuations had a value of 0



-
- To put our model into a form based on these ***fluctuations***, we ***log-linearised*** → growth rates [but never forget that this is just an ***approximation*** to an underlying ***nonlinear*** model!]

- Result of doing so was *simple linear equations* in the *transformed* variables (ie, log deviations from steady state = growth rates)
- *Example:* production function

$$y_t = a_t + \alpha k_t + (1 - \alpha) l_t$$

- This states that *dlogs* (= growth rates) of output are a simple *weighted average* of
 - *dlogs* of *capital* services k (assumed identical to stock)
 - *dlogs* of *labour* hours l
 - modified by a *TFP* productivity process a

- *Summarising our log-linearised model:*

$$\sigma c_t + \varphi l_t = w_t - p_t \quad (\text{Labour supply})$$

$$\frac{\sigma}{\beta} (c_{t+1} - c_t) = \frac{R_{ss}}{P_{ss}} (r_{t+1} - p_{t+1}) \quad (\text{Euler equation})$$

$$k_{t+1} = (1 - \delta) k_t + \delta i_t \quad (\text{Law of motion of capital})$$

$$y_t = a_t + \alpha k_t + (1 - \alpha) l_t \quad (\text{Production function})$$

$$k_t = y_t - (r_t - p_t) \quad (\text{Demand for capital})$$

$$l_t = y_t - (w_t - p_t) \quad (\text{Demand for labour})$$

$$p_t = (1 - \alpha) w_t + \alpha r_t - a_t \quad (\text{Price level})$$

$$y_t = \frac{C_{ss}}{Y_{ss}} c_t + \frac{I_{ss}}{Y_{ss}} i_t \quad (\text{Equilibrium condition})$$

$$a_t = \rho_A a_{t-1} + e_t \quad (\text{Productivity shock})$$

The New-Keynesian Model

- **NK model** introduces into basic RBC more realistic specifications:
 - *imperfect competition* ⇒ Today
 - *frictions* in *prices* and in *wages* ⇒ Today
 - *habit formation* in consumption ⇒ Today
 - *non-Ricardian* agents ⇒ Today
 - *adjustment costs* in investment ⇒ Today
 - *capacity* (non-)utilisation *costs* ⇒ Today
 - *government* sector (*monetary* and *fiscal*) ⇒ Thursday
 - *financial* sector ⇒ Week 5
 - *housing* sector ⇒ Week 5
 - *unemployment* ⇒ Week 6
 - *environment* ⇒ Week 6
 - *foreign trade* ⇒ Week 7

Imperfect Competition and Price Stickiness

- Price and wage “*frictions*”
- Published evidence about frequency of changes in prices and wages indicates that:
 - Prices and wages are *temporarily rigid*
 - Prices and wages are *adjusted* [“*reset*”] on average two or three times a year Thus, In any given quarter, some prices and wages will be reset, but most will have been set some time in the past
 - Prices and wages are *not* reset simultaneously
 - Changes in prices of *tradable* goods are *more frequent* than those of *non-tradable* goods
- Hence incorporating price and wage “*stickiness*” into model makes eminent sense

- How to deal with this wage and price stickiness?
- “*Calvo fairy*” (or “Calvo devil” as she is sometimes called)
- Calvo fairy visits a certain *proportion* of (wholesale) *firms* and a (different) proportion of *households* in *each period*
- and *permits* those firms or households to *change* their prices or wages to their *utility-maximisation* levels
- *Remaining* firms and households are obliged to follow a “*Calvo rule*” for prices and wages

- “Calvo rule” can have many forms:
 - Keeping *same* price or wage level as in previous period
 - *Updating* prices or wages using *steady state* gross inflation rate (“*indexation*”)
 - Updating prices or wages using *previous period’s* gross inflation rate
 - Something else entirely
- Incorporating this in model is straightforward:
- Since *nothing changes* as regards optimising behaviour of *households*, we may simply *copy over* utility maximising conditions of basic RBC model

- \rightarrow repeat Household's *labour supply* function as

σ is the inverse of the intertemporal rate of substitution

$$C_{j,t}^{\sigma} L_{j,t}^{\varphi} = \frac{W_t}{P_t}$$

φ is the inverse of the Frisch elasticity of labour supply \rightarrow more convex is utility function for leisure (higher is φ), more *inelastic* is labour supply \rightarrow higher real wage

- and *Euler Equation* as

$$\left(\frac{E_t C_{j,t+1}}{C_{j,t}} \right)^{\sigma} = \beta \left[(1 - \delta) + E_t \left(\frac{R_{t+1}}{P_{t+1}} \right) \right]$$

- and *law of motion of capital (capital supply)* as

$$K_{j,t+1} = (1 - \delta)K_{j,t} + I_{j,t}$$

- Now add *imperfect competition* to NK model
→ now have both **wholesale** and **retail** sector
- **Retail** sector is *lazy*: it just *aggregates intermediate* goods produced by **wholesale** sector (using *Dixit-Stiglitz* aggregator – there are others!)
- → problem of representative **retail** firm: maximise its *profit function* taking into account *input costs* of the “j” varieties of intermediate goods:

$$\max_{Y_{j,t}} \left[P_t Y_t - \int_0^1 P_{j,t} Y_{j,t} dj \right]$$


eg, aggregate “sales” of a retail shop

- **Dixit-Stiglitz** aggregator (CES) → $\max_{Y_{j,t}} \left[P_t Y_t - \int_0^1 P_{j,t} Y_{j,t} dj \right]$

$$\max_{Y_{j,t}} P_t \left(\int_0^1 Y_{j,t}^{\frac{\psi-1}{\psi}} dj \right)^{\frac{\psi}{\psi-1}} - P_{j,t} \int_0^1 Y_{j,t} dj$$

Y_t ↗

- FOCs (using **Chain Rule** for differentiation)

$$\frac{\psi}{\psi-1} P_t \left(\int_0^1 Y_{j,t}^{\frac{\psi-1}{\psi}} dj \right)^{\frac{\psi}{\psi-1}-1} \frac{\psi-1}{\psi} Y_{j,t}^{\frac{\psi-1}{\psi}-1} - P_{j,t} = 0$$

- → **pricing rule for retail goods**

$$P_t = \left[\int_0^1 P_{j,t}^{1-\psi} dj \right]^{\frac{1}{1-\psi}}$$

Elasticity of substitution
 between different varieties
 is ψ . When variety j is 1%
 more expensive than
 average variety, then
 demand for this particular
 good will be $\psi\%$ lower

- **Wholesale** firms *not lazy* → **work** to produce, then sell their *differentiated products* to retail firms
- Wholesale firm solves its problem in *two stages*
- *First*, takes prices of *factors of production* (return on capital and wages) as given and determines amounts of capital and labour which will minimise *total production cost*

$$\min_{L_{j,t}, K_{j,t}} [W_t L_{j,t} + R_t K_{j,t}]$$

- *subject to* use of a given *technology* (typically *assumed to be Cobb-Douglas*) and law of motion of productivity (typically *autoregressive*)

- → (copying from first week's RBC model):

$$Y_{j,t} = A_t K_{j,t}^\alpha L_{j,t}^{1-\alpha}$$

$$\log A_t = \rho_A \log A_{t-1} + \epsilon_t$$

- Using Lagrangian and solving FOCs → **demand** of wholesale firm ***for labour and capital*** (as first week):

$$L_{j,t} = (1 - \alpha) MC_{j,t} \frac{Y_{j,t}}{W_t}$$

$$K_{j,t} = \alpha MC_{j,t} \frac{Y_{j,t}}{R_t}$$

- *Production technology* here *same* as RBC → *reuse first week's* RBC equation for $MC_{j,t}$

$$MC_{j,t} = \frac{1}{A_t} \left(\frac{W_t}{1 - \alpha} \right)^{1-\alpha} \left(\frac{R_t}{\alpha} \right)^\alpha$$

- “*Price stickiness*” distinguishes NK from RBC and will require a *mark-up* over perfectly-flexible-price version of MC defined above [ie, $P \neq MC$!!]
- → “*Calvo fairy*” and *second stage* of wholesale firm’s problem

- → objective of wholesale firm permitted by Calvo fairy to ***optimally reset price (P^*)*** of its good is

$$\max_{P_{j,t}^*} E_t \sum_{i=0}^{\infty} (\beta\theta)^i \left(P_{j,t}^* Y_{j,t+i} - TC_{j,t+i} \right)$$

- Maximisation → FOCs which → ***optimal price level for price-adjusting firm*** in each period

$$P_{j,t}^* = \left(\frac{\psi}{\psi - 1} \right) E_t \sum_{i=0}^{\infty} (\beta\theta)^i MC_{j,t+i}$$

θ = *share of firms* following Calvo rule for *prices*

ψ = *elasticity of substitution* between wholesale *goods*

- \rightarrow aggregate price level each period

share able to reset price = $1-\theta$

share using Calvo Rule = θ

$$P_t^{1-\psi} = \int_0^{\theta} P_{t-1,j}^{1-\psi} dj + \int_{\theta}^1 P_{t,j}^{*1-\psi} dj$$

- \rightarrow

Assumed Calvo Rule: could be *different*

$$P_t = \left[\theta P_{t-1}^{1-\psi} + (1 - \theta) P_t^{*1-\psi} \right]^{\frac{1}{1-\psi}}$$

- Add ***equilibrium condition*** $Y_t = C_t + I_t \rightarrow$ model is complete: ***copy*** much from **first week's RBC**:

$$C_t^\sigma L_t^\varphi = \frac{W_t}{P_t} \quad (\text{Labour supply})$$

$$\left(\frac{E_t C_{t+1}}{C_t} \right)^\sigma = \beta \left[(1 - \delta) + E_t \left(\frac{R_{t+1}}{P_{t+1}} \right) \right] \quad (\text{Euler Equation})$$

$$K_{t+1} = (1 - \delta)K_t + I_t \quad (\text{Law of motion of capital})$$

$$Y_t = A_t K_t^\alpha L_t^{1-\alpha} \quad (\text{Production function})$$

$$K_t = \alpha M C_t \frac{Y_t}{R_t} \quad (\text{Demand for capital})$$

$$L_t = (1 - \alpha) M C_t \frac{Y_t}{W_t} \quad (\text{Demand for labour})$$

- ***Completing*** initial NK model:

$$MC_t = \frac{1}{A_t} \left(\frac{W_t}{1-\alpha} \right)^{1-\alpha} \left(\frac{R_t}{\alpha} \right)^\alpha \quad (\text{Marginal cost})$$

$$P_t^* = \left(\frac{\psi}{\psi-1} \right) E_t \sum_{i=0}^{\infty} (\beta\theta)^i MC_{t+i} \quad (\text{Optimal price level})$$

$$P_t = \left[\theta P_{t-1}^{1-\psi} + (1 - \theta) P_t^{*1-\psi} \right]^{\frac{1}{1-\psi}} \quad (\text{General price level})$$

$$\pi_t = \frac{P_t}{P_{t-1}} \quad (\text{Gross inflation rate})$$

$$I_t = Y_t - C_t \quad (\text{Equilibrium condition})$$

$$\log A_t = \rho_A \log A_{t-1} + \epsilon_t \quad (\text{Productivity shock})$$

- Equation for *optimal price level [aka, reset price]* for price-adjusting firm in each period

$$P_{j,t}^* = \left(\frac{\psi}{\psi - 1} \right) E_t \sum_{i=0}^{\infty} (\beta \theta)^i MC_{j,t+i}$$

Mark-up over MC

- *Formidable* equation → *infinite sum* of marginal costs
- → *impossible* to write this (non-linear) model directly in Dynare (without some *tricks* ...)
- Luckily, we have just such a *trick*:

- A trick to get rid of *infinite sums*
- Suppose we have
- $\sum_{i=0}^{\infty} (\lambda^i A_{t+i}) = A_t + \lambda A_{t+1} + \lambda^2 A_{t+2} + \lambda^3 A_{t+3} + \lambda^4 A_{t+4} + \dots$
- Call this (infinite) sum B_t
- Then $B_{t+1} = A_{t+1} + \lambda A_{t+2} + \lambda^2 A_{t+3} + \lambda^3 A_{t+4} + \lambda^4 A_{t+5} + \dots$
- $\rightarrow \lambda B_{t+1} = \lambda A_{t+1} + \lambda^2 A_{t+2} + \lambda^3 A_{t+3} + \lambda^4 A_{t+4} + \dots$
- So
- $B_t - \lambda B_{t+1} = A_t$
- **Neat!!**

$$B_t = \sum_{i=0}^{\infty} (\lambda^i A_{t+i})$$

- *Optimal* price under a *Calvo* approach (abstracting from “j”) is

$$P_t^* = \left(\frac{\psi}{\psi - 1} \right) E_t \sum_{i=0}^{\infty} (\beta\theta)^i MC_{t+i}$$

- Recall, $B_t = \sum_{i=0}^{\infty} (\lambda^i A_{t+i})$
- In B, set $A = [\psi/(\psi-1)]MC$ and $\lambda = \beta\theta$,
- $\rightarrow P_t^* = B_t$
- **But** $B_t - \lambda B_{t+1} = A_t \rightarrow P_t^* - (\beta\theta)P_{t+1}^* = A_t$
- Hence $P_t^* = \beta\theta P_{t+1}^* + \underbrace{[\psi/(\psi-1)]MC_t}_{\text{Mark-up over marginal cost}} \leftarrow \text{Neat!!}$

- We may now write down new non-linear **NK model** directly in **Dynare**
- **predetermined_variables K;**
- **model;**
- $C^{\sigma} L^{\varphi} = \frac{W_t}{P_t}$ $\left(\frac{E_t C_{j,t+1}}{C_{j,t}}\right)^{\sigma} = \beta \left[(1 - \delta) + E_t \left(\frac{R_{t+1}}{P_{t+1}}\right) \right]$
- $C^{\sigma} L^{\varphi} = W/P;$
- $(C(+1)/C)^{\sigma} = \beta * ((1 - \delta) + R(+1)/P(+1));$
- $K(+1) = (1 - \delta) * K + I;$
- $Y = A * (K^{\alpha}) * (L^{1-\alpha});$ $A_t K_{j,t}^{\alpha} L_{j,t}^{1-\alpha}$
- $K = \alpha * MC * Y / R;$
- $L = (1 - \alpha) * MC * Y / W;$
- $MC = (1/A) * (W/(1-\alpha))^{(1-\alpha)} * (R/\alpha)^{\alpha};$
- $I = Y - C;$
- $Pstar = \beta * \theta * Pstar(+1) + (\psi / (\psi - 1)) * MC;$
- $P = (\theta * (P(-1)^{(1-\psi)}) + (1 - \theta) * ((Pstar^{(1-\psi)})^{(1/(1-\psi))}));$
- $\pi = P / P(-1);$
- $\log(A) = \rho_A * \log(A(-1)) + e;$
- **end;**
- ***Note new equations in orange box, including definition of (gross) inflation rate***

- We may now proceed to ***simulate*** model
- But since model is ***nonlinear*** [we have not log-linearised here] **Dynare** needs ***steady-state*** values to check for fulfilment of infamous ***Blanchard-Kahn*** conditions
- If no ***analytical*** steady-state solution exists, we may ask **Dynare** to attempt to solve model itself for steady-state values
- But this requires that we provide **Dynare** with ***initial guesses*** at those steady-state values
- And if they are not very good ...

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HOME PLOTS APPS

FILE VARIABLE CODE SIMULINK ENVIRONMENT RESOURCES

Current Folder

Command Window

Workspace

Command History

Start Google Photos File Explorer MATLAB Help Add-ons

John Cuddy - The Graduate Institute, Geneva 22 September 2025 34

The screenshot shows the MATLAB R2013b interface. The Command Window is active, showing the prompt `f>>`. The Workspace browser on the right lists various variables and their properties. The Command History window shows a series of commands run in the session, including multiple `dynare` calls for different model files. The Current Folder browser on the left shows a directory structure with several files and subfolders related to RBC models. The bottom taskbar includes icons for Start, Google Chrome, Photos, File Explorer, MATLAB, Help, and Add-ons.

Now start from a better set of initial values

The screenshot shows the MATLAB R2013b interface. The top menu bar includes HOME, PLOTS, APPS, FILE, VARIABLE, CODE, SIMULINK, ENVIRONMENT, and RESOURCES. The left sidebar has sections for Current Folder, Command Window, and Details. The Command Window is active with the prompt 'fx >> |'. The Workspace browser on the right lists variables like ASS, M_, alpha, ans, bayestopt_, betta, dataset_, dataset_info, delta, emptydateso..., emptydseries..., estim_params_, estimation_info, and ex0_. The Command History window at the bottom shows a list of commands run, including dynare and clc.

Current Folder

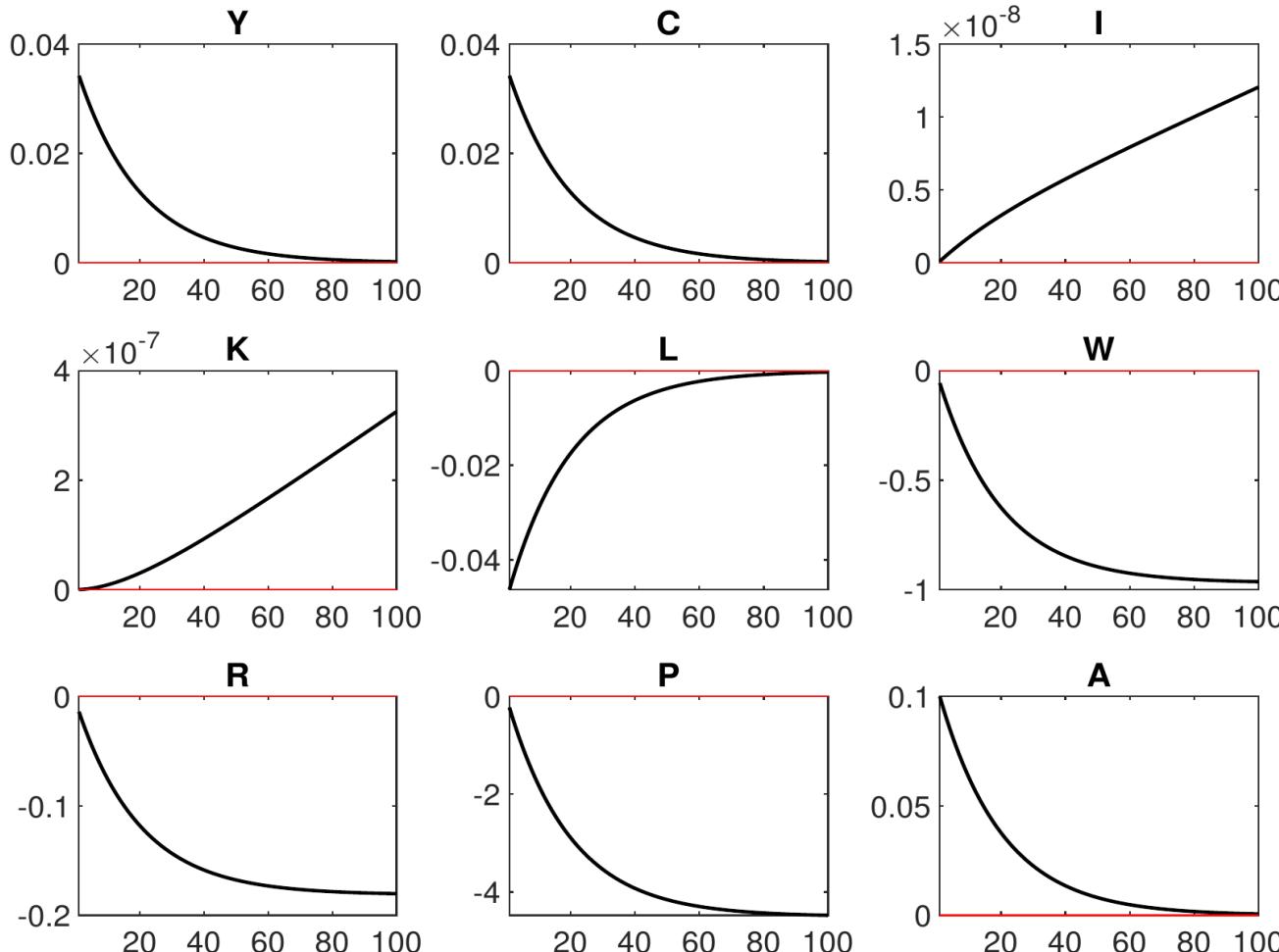
Command Window

Workspace

Command History

- As just seen, starting from a **better set** of initial values → following IRFs

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Note: these are **levels**, not log-differences (ie, growth rates) so shapes are quite different from those found previously

Figure 1: Impulse response functions (orthogonalized shock to e).

- *If* an *analytical* solution exists, then *instead* of setting Initial Values, we may write down *steady-state model* directly
- Fortunately for this model, *Costa* does solution for us [see pp. 75-79 for details]
- Results are *identical*

// Steady-state values

steady_state_model;

A = 1; % ==> log(A)=0 ==> no shock until subsequently imposed

P = 1; % Normalisation from Walras' Law (k goods ==> k-1 independent equations in equilibrium ==> may set aggregate price level to 1)

R = P*((1/betta)-(1-delta));

MC = ((psi-1)/psi)*(1-betta*theta)*P;

W = (1-alpha)*(MC^(1/(1-alpha)))*((alpha/R)^(alpha/(1-alpha)));

Y = ((R/(R-delta*alpha*MC))^(sigma/(sigma+phi)))*(((W/P)*((W/((1-alpha)*MC))^phi))^(1/(sigma+phi)));

I = delta*alpha*MC*Y/R;

C = (1/(Y^(phi/sigma)))*(((W/P)*(W/((1-alpha)*MC))^phi)^(1/sigma));

L = (1-alpha)*MC*Y/W;

K = alpha*MC*Y/R;

Pstar = P;

pi = 1;

end;

- In many cases, especially with more complex models, either *analytical* solution is *unavailable*, or it is *impossible* to find initial values which satisfy infamous *BK conditions*
- In these cases, *only* solution is to resort to *approximating* non-linear model, e.g. via *log-linearisation* technique [there are others]
- Although it is not necessary for this simple model, it will be very useful to have done so when model becomes more complex
- Let us begin:

- For *eight* equations, there is *no change* from first week:
 - labour supply
 - capital supply (law of motion of capital)
 - Euler equation
 - production function
 - demand for capital
 - demand for labour
 - equilibrium condition
 - law of motion of productivity shock
- all *remain identical* since *unaffected* by price frictions

- most of *linearised* model just *copied* over from first week:

$$\sigma c_t + \varphi l_t = w_t - p_t \quad (\text{Labour supply})$$

$$\frac{\sigma}{\beta} (c_{t+1} - c_t) = \frac{R_{ss}}{P_{ss}} (r_{t+1} - p_{t+1}) \quad (\text{Euler Equation})$$

$$k_{t+1} = (1 - \delta) k_t + \delta i_t \quad (\text{Law of motion of capital})$$

$$y_t = a_t + \alpha k_t + (1 - \alpha) l_t \quad (\text{Production function})$$

$$k_t = mc_t + y_t - r_t \quad (\text{Demand for capital})$$

$$l_t = mc_t + y_t - w_t \quad (\text{Demand for labour})$$

First week we used p
because $p = mc$ in RBC.
But not here!

$$y_t = \frac{C^*}{Y^*} c_t + \frac{I^*}{Y^*} i_t \quad (\text{Equilibrium condition})$$

$$a_t = \rho_A a_{t-1} + e_t \quad (\text{Productivity shock})$$

- Now for something **New**: Log-linearising Marginal Cost equation

$$MC_t = \frac{1}{A_t} \left(\frac{W_t}{1-\alpha} \right)^{1-\alpha} \left(\frac{R_t}{\alpha} \right)^\alpha$$

- New, but yet again using trick

recall that *lower-case* →
dlog of *Upper-case*

$$Z = \zeta W^b X^c Y^d \Rightarrow z = bw + cx + dy$$

- $mc_t = (1 - \alpha) w_t + \alpha r_t - a_t$

Here, $\zeta = (1-\alpha)^{-(1-\alpha)} \alpha^{-\alpha}$

- Also **New** : log-linearising **Phillips Curve** [see Costa pp. 80-82 for details]

$$P_t^* = \left(\frac{\psi}{\psi-1} \right) E_t \sum_{i=0}^{\infty} (\beta \theta)^i MC_{t+i}$$

$$P_t = \left[\theta P_{t-1}^{1-\psi} + (1 - \theta) P_t^{*1-\psi} \right]^{\frac{1}{1-\psi}}$$

$$\boxed{\pi_t} = \frac{P_t}{P_{t-1}}$$

Gross inflation rate !!

Simple forward-looking
Phillips Curve

- $\rightarrow \boxed{pi_t} = \beta pi_{t+1} + \frac{(1 - \beta \theta)(1 - \theta)}{\theta} \underbrace{(mc_t - p_t)}$

Net inflation rate

Mark-up over marginal cost

- *Dynare* simulation model log-linearised equations :

//1-Labour Supply

$\sigma*c + \phi*l = w - p;$

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//2-Euler Equation

$(\sigma/\beta)*(c(+1)-c) = (Rss/Pss)*(r(+1)-p(+1));$

//3-Law of motion of Capital

$k(+1) = (1-\delta)*k + \delta*i;$

//4-Production Function

$y = a + \alpha*k + (1-\alpha)*l;$

//5-Demand for Capital

$k = y - (r - p);$

//6-Demand for Labour

$l = y - (w - p);$

//7-Marginal Cost

$mc = (1-\alpha)*w + \alpha*r - a;$

//8-Phillips Curve

$\pi = \beta*\pi(+1) + ((1-\theta)*(1-\beta*\theta)/\theta)*(mc-p);$

New !!

//9- Inflation Rate

$\pi = p - p(-1);$

//10-Goods Market Equilibrium Condition

$Yss*y = Css*c + Iss*i;$

//11-Productivity Shock

$a = rho_a*a(-1) + e;$

MATLAB R2013b

HOME PLOTS APPS

FILE VARIABLE CODE SIMULINK ENVIRONMENT RESOURCES

New Script New Open Compare Import Data Save Clear Workspace Analyze Code Run and Time Simulink Library Layout Set Path Preferences Community Request Support Help Add-Ons

G: MyCourseDSGEs2020 Class2020 DynareTests

Current Folder

Name

- +BasicRBC_nonlin_v2
- BasicNKDSGE_nl_v1
- BasicNKDSGE_nl_v1a
- BasicRBC_lin
- BasicRBC_lin_4PPWk2
- BasicRBC_lin_v2
- BasicRBC_nl_v1
- BasicRBC_nl_v2
- BasicRBC_nonlin
- BasicRBC_nonlin_v2
- Ch3_4PPWk2_lc
- Whelan_RBC_w_ss
- Whelan_RBC_w_ss_v2
- Whelan_RBC_w_ss_v3
- Whelan_RBC_w_ss_v3a
- BasicNKDSGE_nl_v1.log
- BasicNKDSGE_nl_v1.m
- BasicNKDSGE_nl_v1.mod
- BasicNKDSGE_nl_v1_dynamic.m
- BasicNKDSGE_nl_v1_dynamic.tex
- BasicNKDSGE_nl_v1_dynamic_content.tex
- BasicNKDSGE_nl_v1_set_auxiliary_variabl...

Select a file to view details

Command Window

f> >> ↻

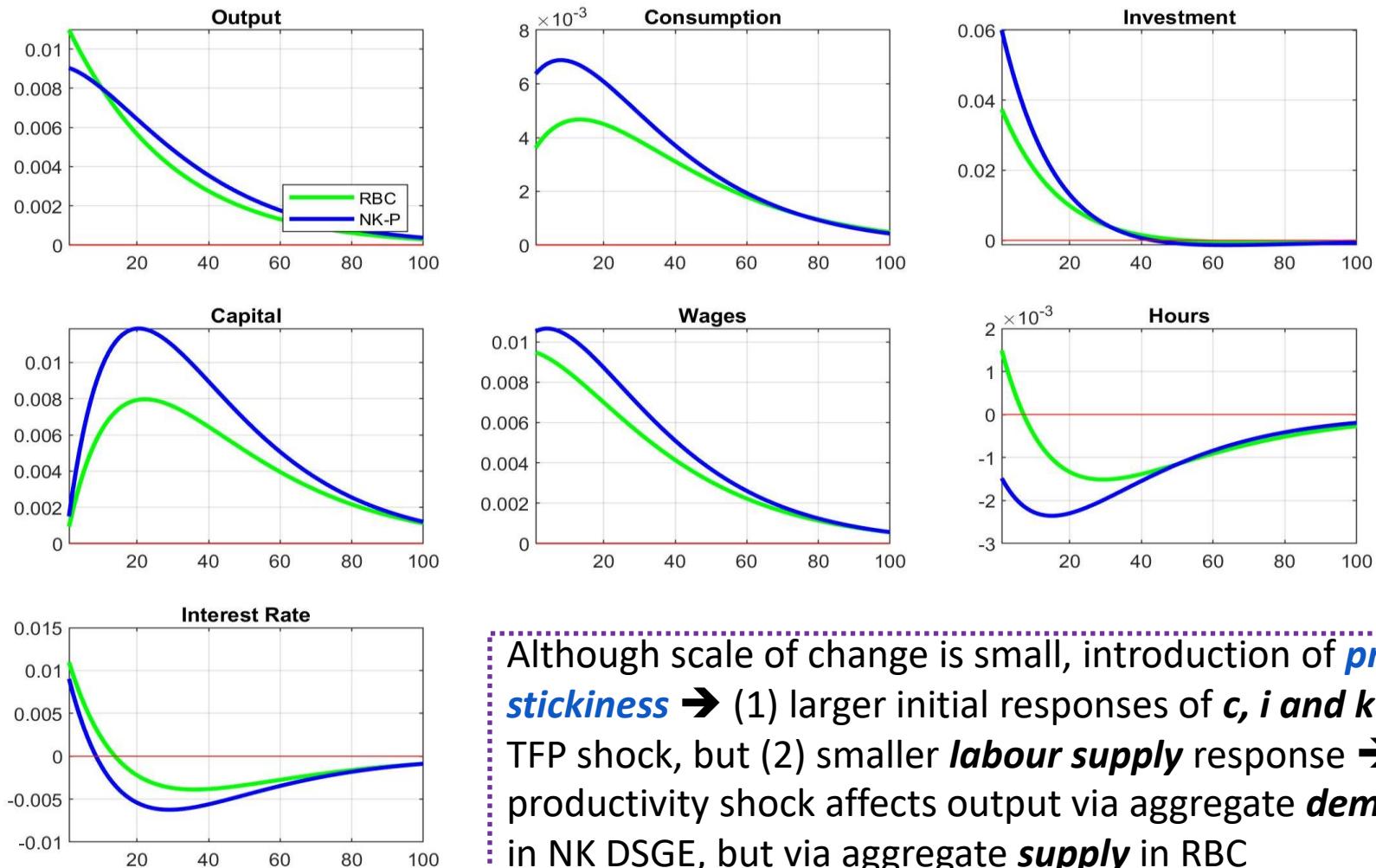
Workspace

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ans	11x1 double	0 0		
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c	2000x1 double	-0.... 1..		
c_e	100x1 double	4.2... 68		
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emptydseries...	0x0 dseries			

Command History

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dynare BasicRBC_nl_v2.mod
clc
dynare BasicRBC_nl_v2.mod
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clc
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dynare BasicNKDSGE_nl_v1.mod
clc
dynare BasicNKDSGE_nl_v1a.mod
clc
dynare Ch3_4PPWk2_lc.mod
clc
dynare Ch3_4PPWk2_lc.mod
clc
```

• Comparing IRFs of NK DSGE and RBC:



Although scale of change is small, introduction of **price stickiness** → (1) larger initial responses of ***c, i and k*** to TFP shock, but (2) smaller ***labour supply*** response → productivity shock affects output via aggregate ***demand*** in NK DSGE, but via aggregate ***supply*** in RBC

Wage Stickiness

- **Recall:** Monopolistic competition → an aggregating retail firm which uses **Dixit-Stiglitz** aggregator
- Here **[NEW]** assume that households supply their ***differentiated labour*** to a representative union which ***aggregates that labour*** ($L_{j,t}$) into a single labour input (L_t) via **Dixit-Stiglitz** aggregator
- →

$$L_t = \left(\int_0^1 L_{j,t}^{\frac{\psi_w - 1}{\psi_w}} dj \right)^{\frac{\psi_w}{\psi_w - 1}}$$

- *Labour aggregating firm* maximises its profit function

$$\max_{L_{j,t}} \left[W_t L_t - \int_0^1 W_{j,t} L_{j,t} dj \right]$$

- Solving for FOCs and doing a bit of algebra [*see Costa pp. 94-97 for details*] → demand function for *differentiated labour*

$$L_{j,t} = L_t \left(\frac{W_t}{W_{j,t}} \right)^{\psi_w}$$

- A bit more algebra → *aggregate wage level*

$$W_t = \left(\int_0^1 W_{j,t}^{1-\psi_w} dj \right)^{\frac{1}{1-\psi_w}}$$

- But now, Calvo fairy [*she's back again!*] introduces **wage stickiness** into this economy
- → aggregate wage level composed of ***two components***
 - wages for households permitted by ***Calvo fairy*** to select an “optimal” wage
 - wages for those having to follow a ***Calvo Rule***

- Specifically, each household is given
 - probability $1-\theta_w$ of optimally defining its wage
 - probability θ_w of “Calvo Rule” in determining its wage in next period (wage “stickiness”)
- Typically, ***Calvo Rule*** implies one of three possibilities
 - maintain previous period’s wage
 - update wage using steady-state gross inflation rate (π_{ss})
 - update wage using previous period’s gross inflation rate (π_{t-1})
- Here ***assume*** wage remains at previous period’s level when applying Calvo Rule

- **RBC**: Household Lagrangian:

$$\mathcal{L} = E_t \sum_{t=0}^{\infty} \beta^t \left\{ \left[\frac{C_{j,t}^{1-\sigma}}{1-\sigma} - \frac{L_{j,t}^{1+\varphi}}{1+\varphi} \right] - \lambda_{j,t} [P_t C_{j,t} + P_t K_{j,t+1} - P_t(1-\delta)K_{j,t} - W_t L_{j,t} - R_t K_{j,t}] \right\}$$



This is just $P_t I_t$

- But this ***does not take into account*** that only a proportion θ_w of households may now set their wage optimally
- Omitting parts of Lagrangian not related to Labour and Wages, → Household problem becomes ...

$$\max_{W_{j,t}^*} E_t \sum_{i=0}^{\infty} (\beta \theta_w)^i \left\{ -\frac{L_{j,t+i}^{1+\varphi}}{1+\varphi} - \lambda_{t+i} \left[-W_{j,t}^* L_{j,t+i} \right] \right\}$$

- Maximisation \rightarrow FOCs \rightarrow *optimal wage* level for wage-adjusting household in each period

$$W_{j,t}^* = \left(\frac{\psi_w}{\psi_w - 1} \right) E_t \sum_{i=0}^{\infty} (\beta \theta_W)^i C_{j,t+i}^\sigma L_{j,t+i}^\varphi P_{t+i}$$

- All households which Calvo fairy allows to define their optimal wages choose ***same wages*** W^* (as they are subject to ***same constraints***)
- → ***overall nominal wage*** level each period

$$W_t^{1-\psi_w} = \int_0^{\theta_w} W_{t-1,j}^{1-\psi_w} dj + \int_{\theta_w}^1 W_{t,j}^{*1-\psi_w} dj$$

- → complete wage model (*does this look familiar?*)

$$W_t = \left[\theta_w W_{t-1}^{1-\psi_w} + (1 - \theta_w) W_t^{*1-\psi_w} \right]^{\frac{1}{1-\psi_w}}$$

So complete model becomes:

$$W_{j,t}^* = \left(\frac{\psi_w}{\psi_w - 1} \right) E_t \sum_{i=0}^{\infty} (\beta \theta_W)^i C_{j,t+i}^\sigma L_{j,t+i}^\varphi P_{t+i} \text{ (Definition of optimal wages)}$$

$$W_t^{1-\psi_w} = \int_{j=0}^{\theta_w} W_{t-1}^{1-\psi_w} dj + \int_{j=\theta_w}^1 W_t^{*1-\psi_w} dj \text{ (Level of aggregate wages)}$$

$$\pi_{W,t} = W_t / W_{t-1} \text{ (Gross wage inflation rate)}$$

$$\left(\frac{E_t C_{t+1}}{C_t} \right)^\sigma = \beta \left[(1 - \delta) + \left(\frac{E_t R_{t+1}}{E_t P_{t+1}} \right) \right] \text{ (Euler equation)}$$

As before:

$$K_{t+1} = (1 - \delta)K_t + I_t \quad (\text{Law of motion of capital})$$

$$Y_t = A_t K_t^\alpha L_t^{1-\alpha} \quad (\text{Production function})$$

$$K_t = \alpha MC_t \frac{Y_t}{R_t} \quad (\text{Demand for capital})$$

$$L_t = (1 - \alpha) MC_t \frac{Y_t}{W_t} \quad (\text{Demand for labour})$$

$$MC_t = \frac{1}{A_t} \left(\frac{W_t}{1-\alpha} \right)^{1-\alpha} \left(\frac{R_t}{\alpha} \right)^\alpha \quad (\text{Marginal cost})$$

And again, as before:

$$P_t^* = \left(\frac{\psi}{\psi-1} \right) E_t \sum_{i=0}^{\infty} (\beta\theta)^i MC_{t+i}$$

(Optimal price level)

$$P_t = \left[\theta P_{t-1}^{1-\psi} + (1 - \theta) P_t^* - \psi \right]^{\frac{1}{1-\psi}}$$

(General price level)

$$\pi_t = \frac{P_t}{P_{t-1}}$$

(Gross inflation rate)

$$Y_t = C_t + I_t$$

(Equilibrium condition)

$$\log A_t = (1 - \rho_A) \log A_{ss} + \rho_A \log A_{t-1} + \epsilon_t$$

(Productivity shock)

- Using *same infinite sum trick* as before, we may transform

$$W_{j,t}^* = \left(\frac{\psi_w}{\psi_w - 1} \right) E_t \sum_{i=0}^{\infty} (\beta \theta_W)^i C_{j,t+i}^\sigma L_{j,t+i}^\varphi P_{t+i}$$

- into

$$W_t^* = \beta \theta W_{t+1}^* + [\underbrace{\psi_w / (\psi_w - 1)}_{\text{Wage Mark-up over marginal cost}}] C_t^\sigma L_t^\varphi P_t$$

Wage Mark-up over marginal cost

- And proceed with new non-linear model
- However, it will be easier later if we continue to *log-linearise* this model

- *Only difference* between Dynare models with and without wage stickiness is *first two equations*, which replace single labour supply equation

//1-Phillips Curve for Wages

$\pi_w = \beta * \pi_w + ((1 - \theta_w) * (1 - \beta * \theta_w) / \theta_w) * (\sigma * c + \phi * l - (w - p));$

//2- Wage Inflation Rate

$\pi_w = w - w(-1);$

- “*Phillips curve for wages*” is result of log-linearising equation for optimal wage of households allowed by Calvo fairy to set wages optimally
- “*Wage Inflation Rate*” is simply a *definition*

- **→ Model**

//1-Phillips Curve for Wages

```
piw = beta*piw(+1)+((1-thetaw)*(1-beta*thetaw)/thetaw)*(sigma*c+phi*l-(w-p));
```

//2- Wage Inflation Rate

```
piw = w - w(-1);
```

//3-Euler Equation

```
(sigma/beta)*(c(+1)-c)=(Rss/Pss)*(r(+1)-p(+1));
```

//4-Law of Motion of Capital

```
k(+1) = (1-delta)*k + delta*i;
```

//5-Production Function

```
y = a + alpha*k + (1-alpha)*l;
```

//6-Demand for Capital

```
k = y - (r - p);
```

//7-Demand for Labour

```
l = y - (w - p);
```

//8-Marginal Cost

```
mc = ((1-alpha)*w + alpha*r - a);
```

//9-Phillips Curve

```
pi = beta*pi(+1)+((1-theta)*(1-beta*theta)/theta)*(mc-p);
```

//10 Inflation Rate

```
pi = p - p(-1);
```

//11-Goods Market Equilibrium Condition

```
Yss*y = Css*c + Iss*i;
```

//12-Productivity Shock

```
a = rhoa*a(-1) + e;
```

New !!

MATLAB R2013b

HOME PLOTS APPS

New Script New Open Find Files Import Data Open Variable Analyze Code Run and Time Preferences

New New Open Compare Import Data Workspace Clear Commands Simulink Library Layout Set Path Parallel Add-Ons

FILE VARIABLE CODE SIMULINK ENVIRONMENT RESOURCES

G: MyCourseDSGEs2020 Class2020 DynareTests

Current Folder

+ BasicRBC_nonlin_v2

BasicNKDSGE_nl_v1

BasicNKDSGE_nl_v1a

BasicRBC_lin

BasicRBC_lin_4PPWk2

BasicRBC_lin_v2

BasicRBC_nl_v1

BasicRBC_nl_v2

BasicRBC_nonlin

BasicRBC_nonlin_v2

Ch3_4PPWk2_lc

Whelan_RBC_w_ss

Whelan_RBC_w_ss_v2

Whelan_RBC_w_ss_v3

Whelan_RBC_w_ss_v3a

BasicNKDSGE_nl_v1.log

BasicNKDSGE_nl_v1.m

BasicNKDSGE_nl_v1.mod

BasicNKDSGE_nl_v1_dynamic.m

BasicNKDSGE_nl_v1_dynamic.tex

BasicNKDSGE_nl_v1_dynamic_content.tex

BasicNKDSGE_nl_v1_set_auxiliary_variabl...

Select a file to view details

Command Window

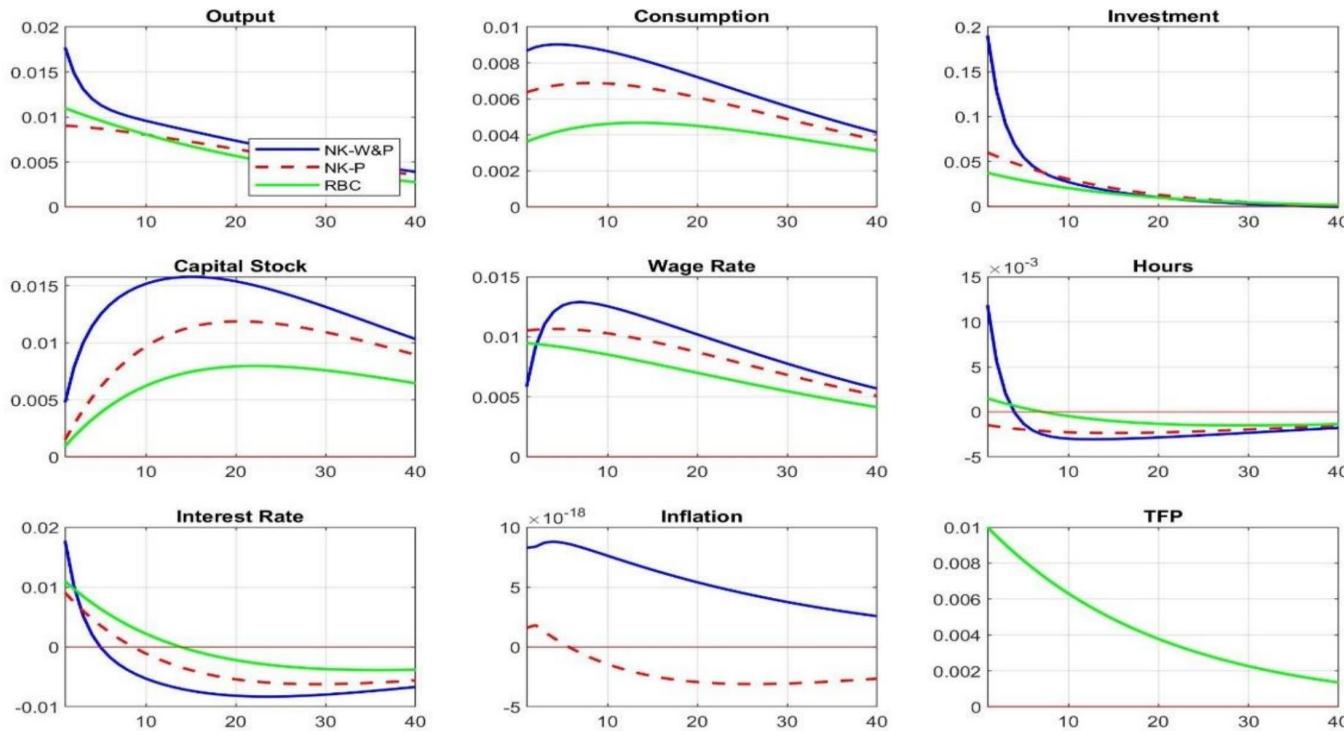
fx >> | I

Workspace

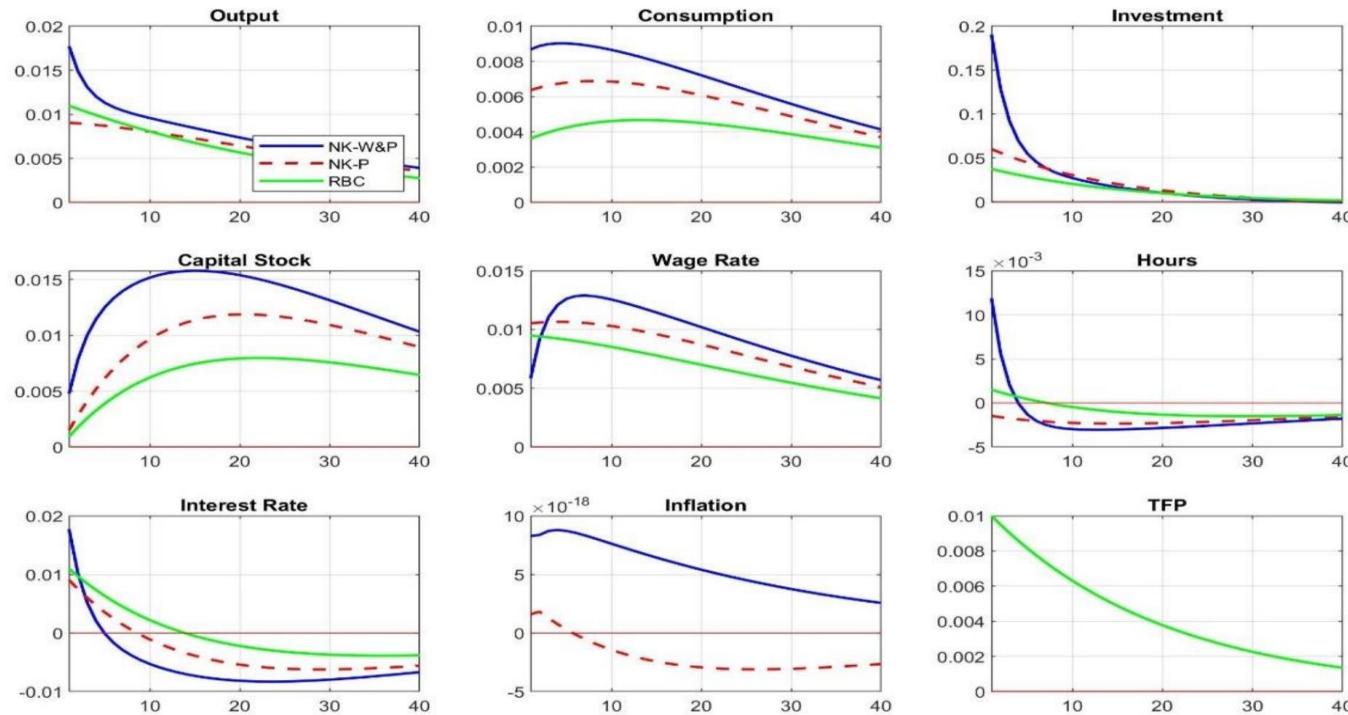
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a	2000x1 double	-1.... 1.		
a_e	100x1 double	0.6... 10		
alpha	0.3500	0.3... 0.		
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bayestopt_	[]			
beta	0.9850	0.9... 0.		
c	2000x1 double	-1.... 1.		
c_e	100x1 double	4.2... 68		
dataset_	[]			
dataset_info	[]			
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Command History

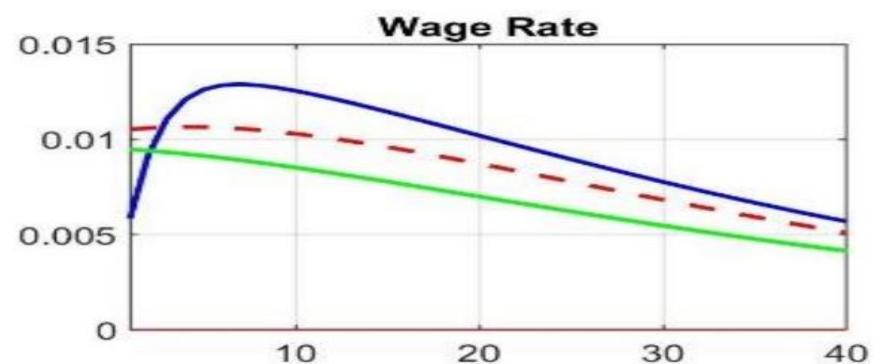
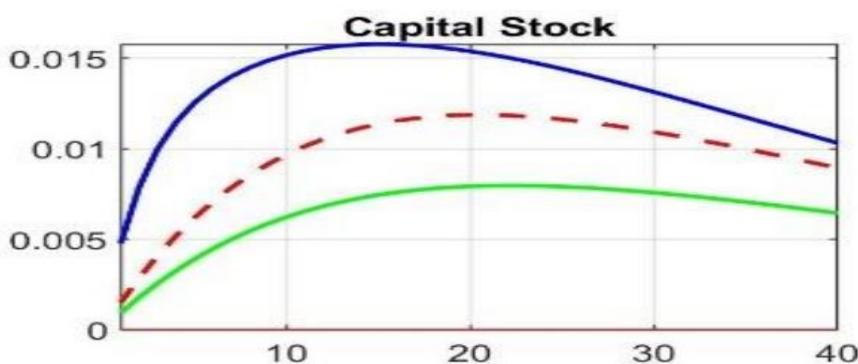
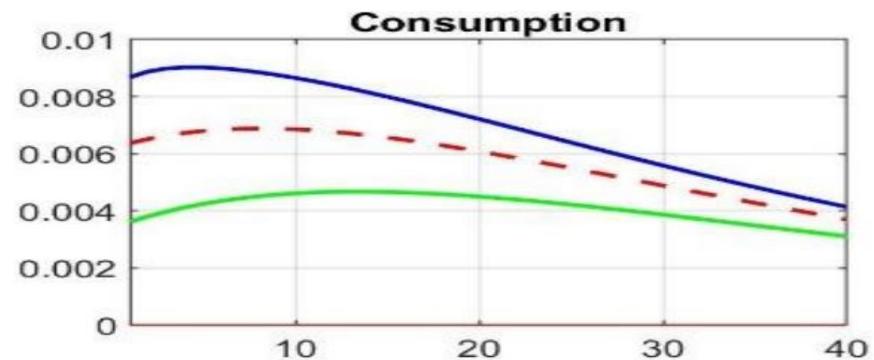
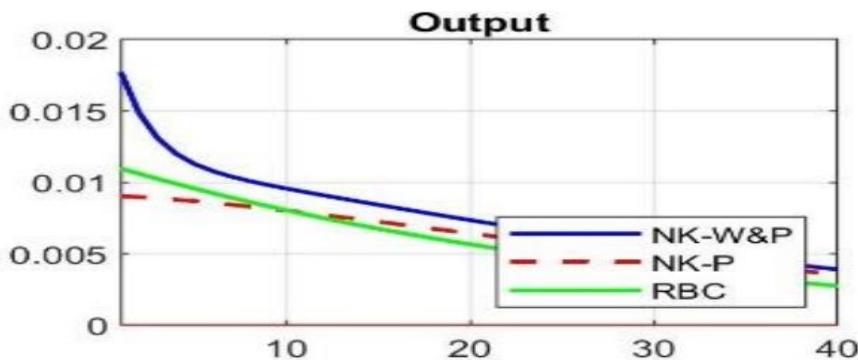
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clc
dynare BasicRBC_lin_4PPWk2.mod
clc
dynare BasicNKDSGE_nl_v1
clc
dynare BasicNKDSGE_nl_v1.mod
clc
dynare BasicNKDSGE_nl_v1a.mod
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clc
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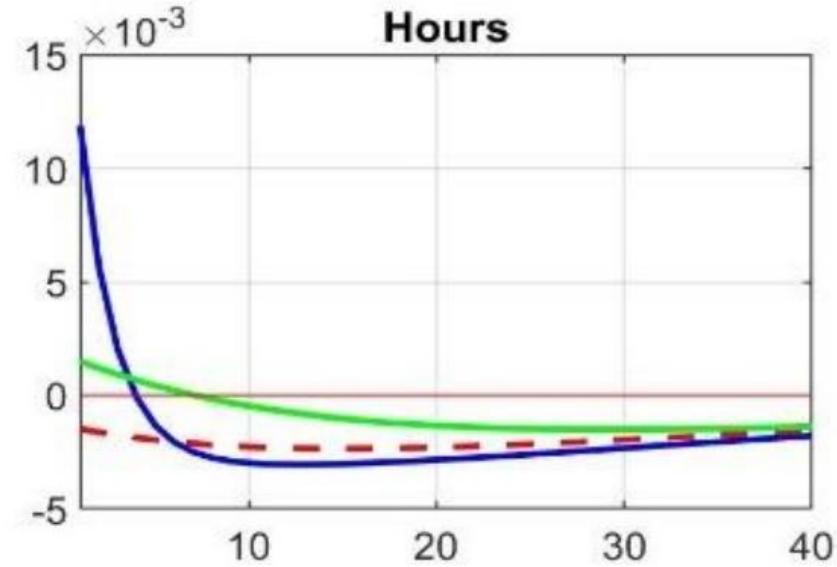
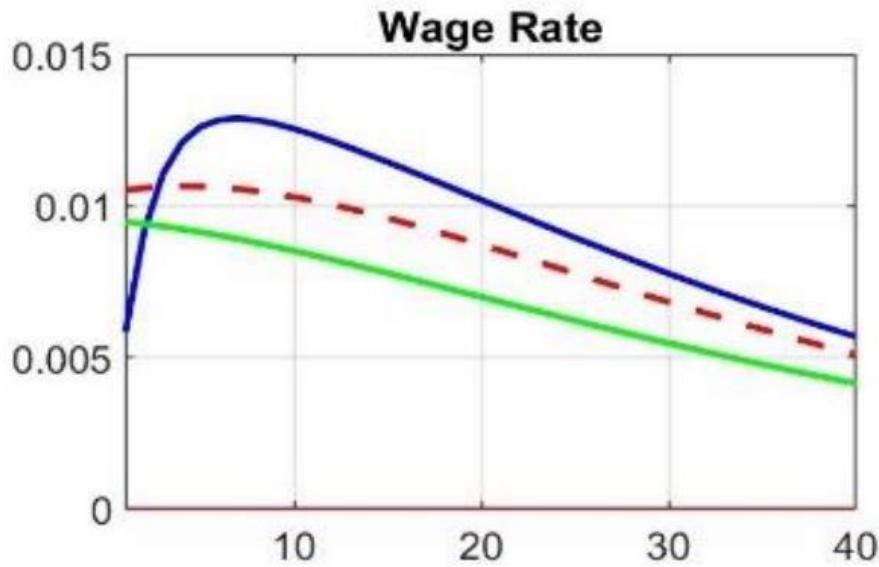
- Productivity shock (*identical* in all three cases – hence only single colour [green] shown for TFP)
 - first impacts *investment*, which in all three models rises (relative to SS),
 - → rise (although much smaller – observe carefully the scales!) in *output* and *consumption* through aggregate spending effect
 - and also (with a lag) spurs growth of capital



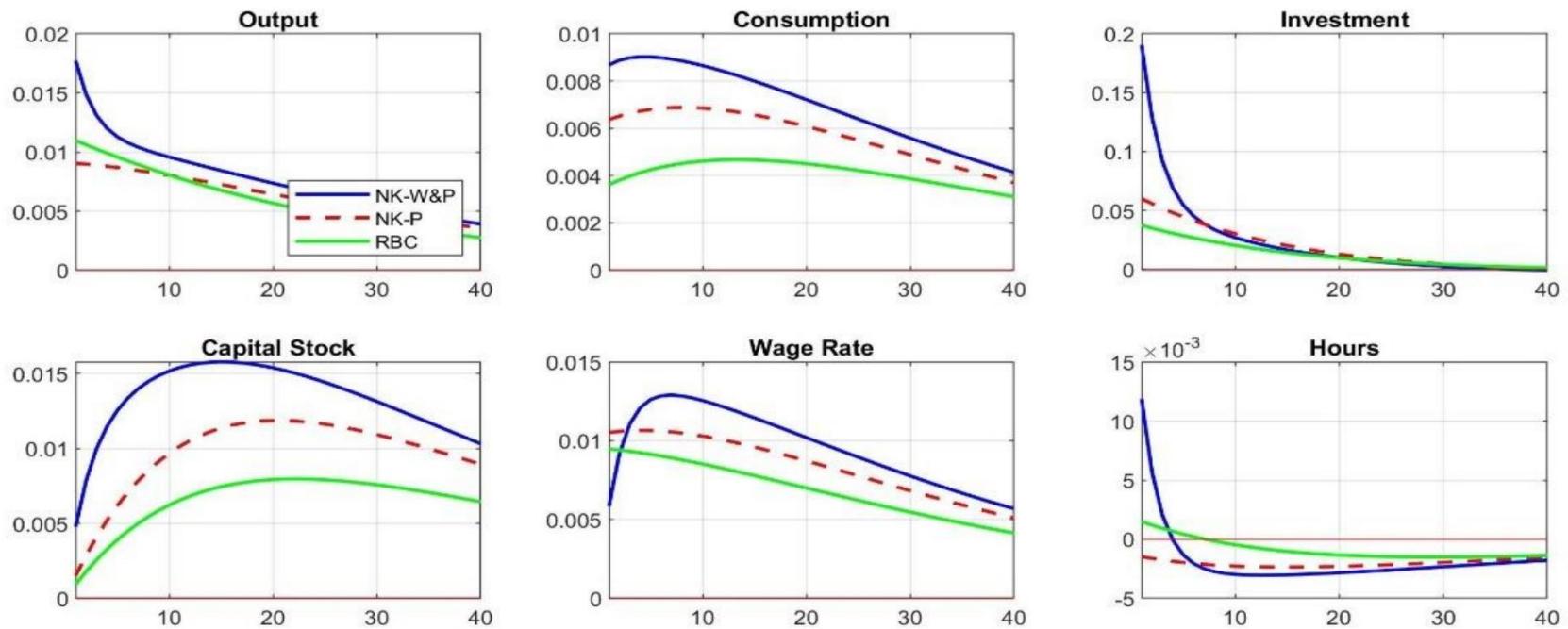
- In these models, there is not yet a government sector, so no Taylor Rule is in operation, and interest rate shown is not a **policy** rate, but **return on capital**, which initially rises with improvement in productivity, before falling back as that shock dies away
- There is no **inflation** variable in our RBC model, so there is no green line in that graph, but in any case, values are essentially rounding errors (again, look at scale)



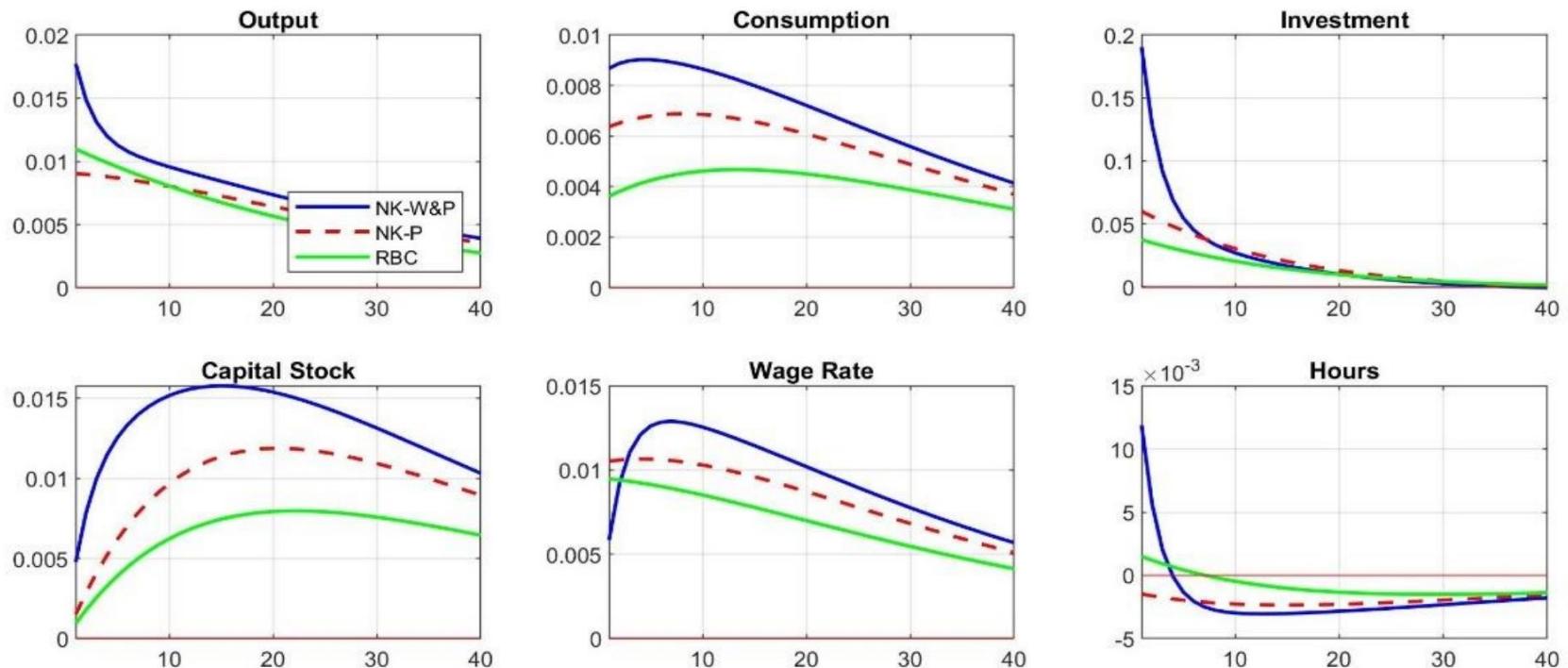
- Next, consider **differences** among three models: First, as frictions are incorporated, aggregate variables (**output**, **consumption** and **capital**) become more **persistent** (ie, their tails stay higher for longer)
- Hence, RBC (green) line invariably is **flatter** and returns more rapidly to zero (ie, its steady state value, as these are log-differences vis-à-vis steady state) than is case for either NK model



- Now, **hours worked** and **wage rate**: In NK-P (dotted red) model, where prices are sticky, but not wages, initial increase in level of wages causes a **substitution** of **leisure** for work, and consequently dotted red (NK-P) line for **hours** is both **below** green (RBC) line, and even **negative**
- By contrast, in NK-W&P (blue line) model, **stickiness** of wages causes **wage rate** to adjust more **slowly** to improved marginal productivity of labour induced by TFP shock, thus **limiting** (relative to NK-P and RBC models) increase in household **income**, and also inducing an actual initial (small – see scale) **increase** in **hours worked** (blue line), although this rapidly dies out as (lagged) wage rate increase subsequently filters into household income

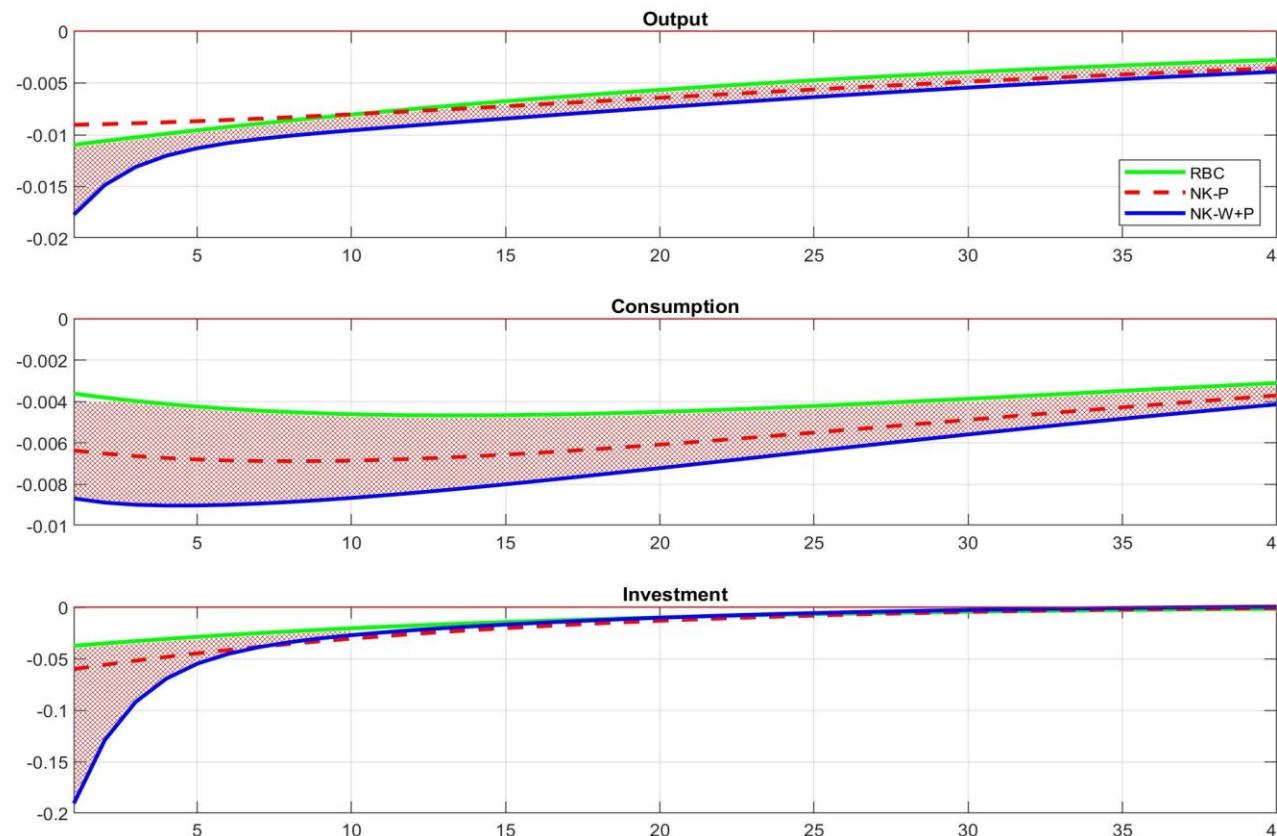


- But why is *initial impact* on *output* so much higher in NK-W&P (blue line) model versus RBC (green) and NK-P (dotted red)?
- Answer comes from a consideration of *costs* involved in producing this output: they are sum of *capital rental* and *labour rental* costs
- And in NK-W&P model, increase in cost of labour rental (ie, *wage rate*) initially does not rise as much as it “should” because of *wage stickiness*
- Hence, investment initially rises much *more* in NK-W&P (blue line) model than in the others, causing both *output (simultaneously)* and *capital* (with a *lag*) also to rise much more, comparatively



- Finally, why is initial impact on output ***smaller*** (although only marginally) in NK-P (dotted red) model than in RBC (green) model?
- Answer was essentially given in a previous slide: ***hours worked*** in NK-P model initially turn ***negative***, and fall below those in RBC model
- Whereas ***investment*** effect is only marginally higher in NK-P (dotted red) model than in RBC (green) model
- Result: ***output*** produced by these smaller working hours also falls short

- What would happen if you were employed by your **central bank** and mistakenly used an **RBC** (green) instead of an **NK DSGE** with wage and price **frictions** (blue) to analyse a **negative** productivity shock such as from Covid?
- You would be **fired**, because you would **underestimate** the shock's effects and thus propose a **weaker** countercyclical policy than necessary:



- **NK model** introduces into basic RBC more realistic specifications:
 - *imperfect* competition
 - *frictions* in *prices* and in *wages*
 - *habit* formation in consumption
 - *non-Ricardian* agents
 - *adjustment costs* in investment
 - *capacity* (non-)utilisation *costs*
 - *government* sector (*monetary* and *fiscal*)
 - *financial* sector ⇒ Week 5
 - *housing* sector ⇒ Week 5
 - *unemployment* ⇒ Week 6
 - *environment* ⇒ Week 6
 - *foreign* trade ⇒ Week 7

Non-Ricardian Agents and Habit Formation

- Now enter realm of *heterogeneous agents* (“HANK”)
- “*Ricardian*” households:
 - forward looking
 - base spending not only on current income, but also on *expected future* income (“Ricardian equivalence”)
- RBC: all households assumed to have *free access to financial markets*
- for carrying forward present income to future (via *saving*) and
- for bringing future income to present (via *borrowing*)
- → “Complete markets and Arrow-Debreu securities” assumption

- But many agents face *liquidity constraints*:
 - willing to go into *debt* to increase present consumption levels
 - but *do not have access to credit*
- → cannot maximise their intertemporal utility
- → “*Non-Ricardian*” or “*rule-of-thumb*” or “*hand-to-mouth*” or “*LAMP – Limited Asset Market Participation*” agents
- Many empirical studies both macro- and micro- → a *significant proportion* of a country’s population is subject to liquidity constraints

- **NK**: allow *some* households to be *non-Ricardian*
- Liquidity restrictions affecting non-Ricardian households → *no intertemporal maximisation*
- → *consumption* of non-Ricardian agents *equals income* in each period (*assumption* – ∃ others)
- → Budget constraint of *Ricardian* households is

$$P_t (C_{R,t} + I_t) = W_t L_{R,t} + R_t K_t$$

- But that of *non-Ricardian* households will be
- $$P_t C_{NR,t} = W_t L_{NR,t}$$
- because *no saving, no investment* (→ no $R_t K_t$ hence no I_t) for non-Ricardians

- Assume *fraction* ω_R of households has *access to financial markets* (and therefore act as *Ricardian* agents)
- but remaining fraction ($1 - \omega_R$) do *not* have access to these markets
- → simply consume all current available household income (“*hand-to-mouth*” or “*LAMP*” agents)
- Since only two types of agents → “*TANK*” model
- [If >2 types of agent → “*HANK*” model – “heterogeneous agent NK” model]

- Next, introduce *habit formation* into model
- *RBC*: utility of households depends only on *current* consumption and not on *previous* consumption
- But empirically, behaviour of households follows a fairly *regular pattern*
- If household is used to a certain level of consumption and suddenly a *shock* [eg, Covid] alters its income, it does not immediately change its pattern of consumption, but uses its *savings* to *mitigate* this alteration
- → “*habit formation*”

- **NK**: utility function *intertemporally additively separable*
- **New**: maximise utility in terms of *additional utility* of consumption today (C_t) over that from “habitual” level (defined as $\phi_c C_{t-1}$)
→ *quasi-difference* in consumption: $U(C_t - \phi_c C_{t-1})$
 - where $1 > \phi_c > 0$ = *persistence coefficient* of habit formation
- If *persistence coefficient* ϕ_c is large (*near 1*) then there will be *very little additional utility* (since quasi-difference $C_t - \phi_c C_{t-1}$ is then close to 0)

- Whereas if persistence coefficient ϕ_c is small (*near 0*) → ***very little habit formation***, then additional utility will be very substantial (since $C_t - \phi_c C_{t-1}$ is then close to C_t)
- Why bother with “***habit formation***”?
- Fuhrer (2000) demonstrates ability of habit-formation model to produce ***empirically-observed hump-shaped impulse response*** of consumption to transitory but persistent income shock (which model without habit formation ***fails to do*** adequately)

- Two types of habit formation: *external and internal*
- *External* consumption habits do *not* depend on agents' past decisions wrt *individual* consumption
- *instead* depend on economy's *aggregate* consumption ("keeping up with the Joneses")
- $\rightarrow U(C_{j,t} - \Phi_c C_{t-1})$
- *Internal* habit formation: individual's habits determined in terms of *own past* consumption
- $\rightarrow U(C_{j,t} - \Phi_c C_{j,t-1})$

Remember, we use *real per capita* consumption in our model
- *If* $C_{j,t-1} \approx C_{t-1}$ \rightarrow no practical difference

- Now introduce (*internal*) ***habit formation*** to NK model → utility function now includes also (*own*) past consumption
- → ***Ricardian*** households ($j = R$) maximise utility function similar to RBC :
 additional utility

$$\max_{C_{R,t}, K_{t+1}} E_t \sum_{t=0}^{\infty} \beta^t \left[\frac{(C_{R,t} - \phi_c C_{R,t-1})^{1-\sigma}}{1-\sigma} - \frac{L_{R,t}^{1+\varphi}}{1+\varphi} \right]$$

- with budget constraint

$$P_t (C_{R,t} + I_t) = W_t L_{R,t} + R_t K_t$$

- Lagrangian is set up and maximised as last week:

$$\mathcal{L} = E_t \sum_{t=0}^{\infty} \beta^t \left\{ \left[\frac{(C_{R,t} - \phi_c C_{R,t-1})^{1-\sigma}}{1-\sigma} - \frac{L_{R,t}^{1+\varphi}}{1+\varphi} \right] - \lambda_{R,t} [P_t C_{R,t} + P_t K_{t+1} - P_t(1-\delta)K_t - W_t L_{R,t} - R_t K_t] \right\}$$

P_t I_t recall

- NB: ***only Ricardians save*** → K has no “ R ” subscript
- Because of presence of habit formation, FOCs are now rather more complex than previously, where we had the simple formulation

$$\frac{\partial \mathcal{L}}{\partial C_{j,t}} = C_{j,t}^{-\sigma} - \lambda_{j,t} P_t = 0$$

$$\lambda_{j,t} = \frac{C_{j,t}^{-\sigma}}{P_t}$$

Lagrange Multiplier = “shadow price” of (real) consumption

- Now we have

See derivation pp.124-125 of Costa

$$\frac{\partial \mathcal{L}}{\partial C_{R,t}} = \left(\frac{1}{1-\sigma} \right) \frac{\partial \left[(C_{R,t} - \phi_c C_{R,t-1})^{1-\sigma} \right]}{\partial C_{R,t}}$$

$$-\lambda_{R,t} P_t + \left(\frac{1}{1-\sigma} \right) \beta \frac{\partial \left[(E_t C_{R,t+1} - \phi_t C_{R,t})^{1-\sigma} \right]}{\partial C_{R,t}} = 0$$

$$\boxed{\lambda_{j,t} = \frac{C_{j,t}^{-\sigma}}{P_t}}$$

- which can be simplified to

$$(C_{R,t} - \phi_c C_{R,t-1})^{-\sigma} - \lambda_{R,t} P_t - \phi_c \beta (E_t C_{R,t+1} - \phi_c C_{R,t})^{-\sigma} = 0$$

- Rewriting this \rightarrow **Lagrange Multiplier** explicitly

$$\lambda_{R,t} = \frac{(C_{R,t} - \phi_c C_{R,t-1})^{-\sigma}}{P_t} - \phi_c \beta \frac{(E_t C_{R,t+1} - \phi_c C_{R,t})^{-\sigma}}{P_t}$$

- So much for Ricardians
- For *non-Ricardians*, budget constraint is *simpler* (K and I drop out, remember):

$$P_t C_{NR,t} = W_t L_{NR,t}$$

- However, Utility function for (hand-to-mouth) *non-Ricardians* ($j = NR$) identical to that for Ricardians
- → *Lagrange Multiplier for non-Ricardians*

$$\lambda_{NR,t} = \frac{(C_{NR,t} - \phi_c C_{NR,t-1})^{-\sigma}}{P_t} - \phi_c \beta \frac{(E_t C_{NR,t+1} - \phi_c C_{NR,t})^{-\sigma}}{P_t}$$

- *Aggregating* consumption and labour as

$$C_t = \omega_R C_{R,t} + (1 - \omega_R) C_{NR,t}$$

$$L_t = \omega_R L_{R,t} + (1 - \omega_R) L_{NR,t}$$

- → everything needed since *all other equations remain identical*
- Log-linearisation → Dynare model shown in Appendix
- J:\MyCourseDSGEs2025\Powerpoints\Wk2\BasicNK2025_Habit_plus_Ricardian.mod

MATLAB R2013b

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FILE VARIABLE CODE SIMULINK ENVIRONMENT RESOURCES

Find Files New Variable Import Data Open Variable Clear Commands Analyze Code Run and Time Set Path Preferences Community Request Support Add-Ons

Current Folder G: MyCourseDSGEs2020 Class2020 DynareTests

Command Window

fx >> I

Workspace

Name	Type	Value	Min	Max
M_	1x1 struct			
a	2000x1 double	-0.... 0.		
a_e	20x1 double	0.0... 0.1		
alpha	0.3500	0.3... 0.3		
ans	0x0 dseries			
bayestopt_	[]			
beta	0.9850	0.9... 0.!		
c	2000x1 double	-0.... 0.		
c_e	20x1 double	0.0... 0.1		
cnr	2000x1 double	-0.... 0.		
cnr_e	20x1 double	0.0... 0.1		
cr	2000x1 double	-0.... 0.		
cr_e	20x1 double	0.0... 0.1		
dataset_	[]			

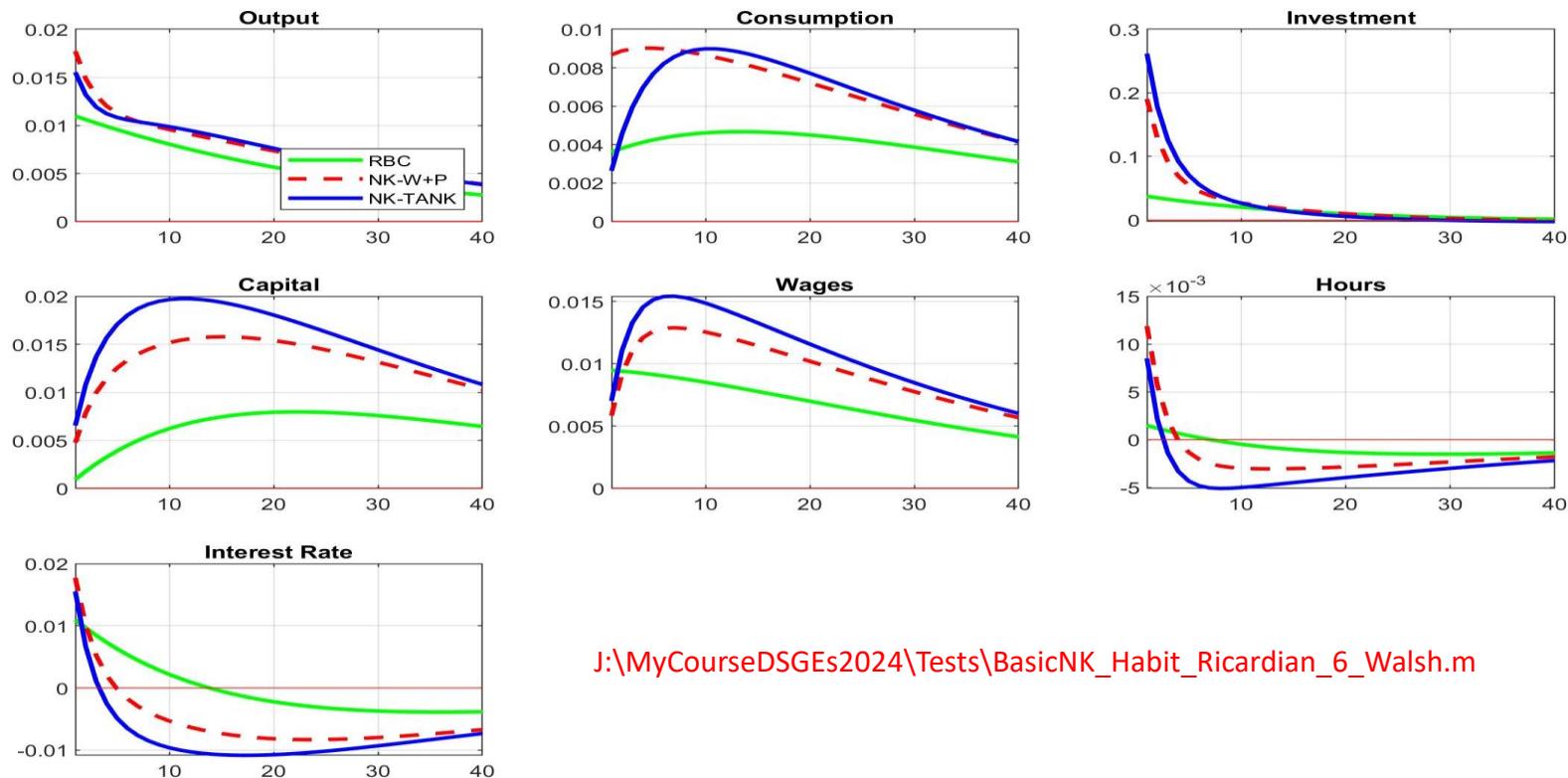
Command History

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clc
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clc
dynare BasicNKDSGE_nl_v1.mod
clc
dynare BasicNKDSGE_nl_v1a.mod
clc
dynare Ch3_4PPWk2_lc.mod
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dynare Ch3_4PPWk2_lc.mod
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dynare Ch3_4PPWk2_lc.mod
clc
dynare Ch4_4PPWk2_lc.mod
clc
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clc
dynare Ch5_4PPWk2_lc.mod
clc
```

Start Chrome File Home MATLAB Help

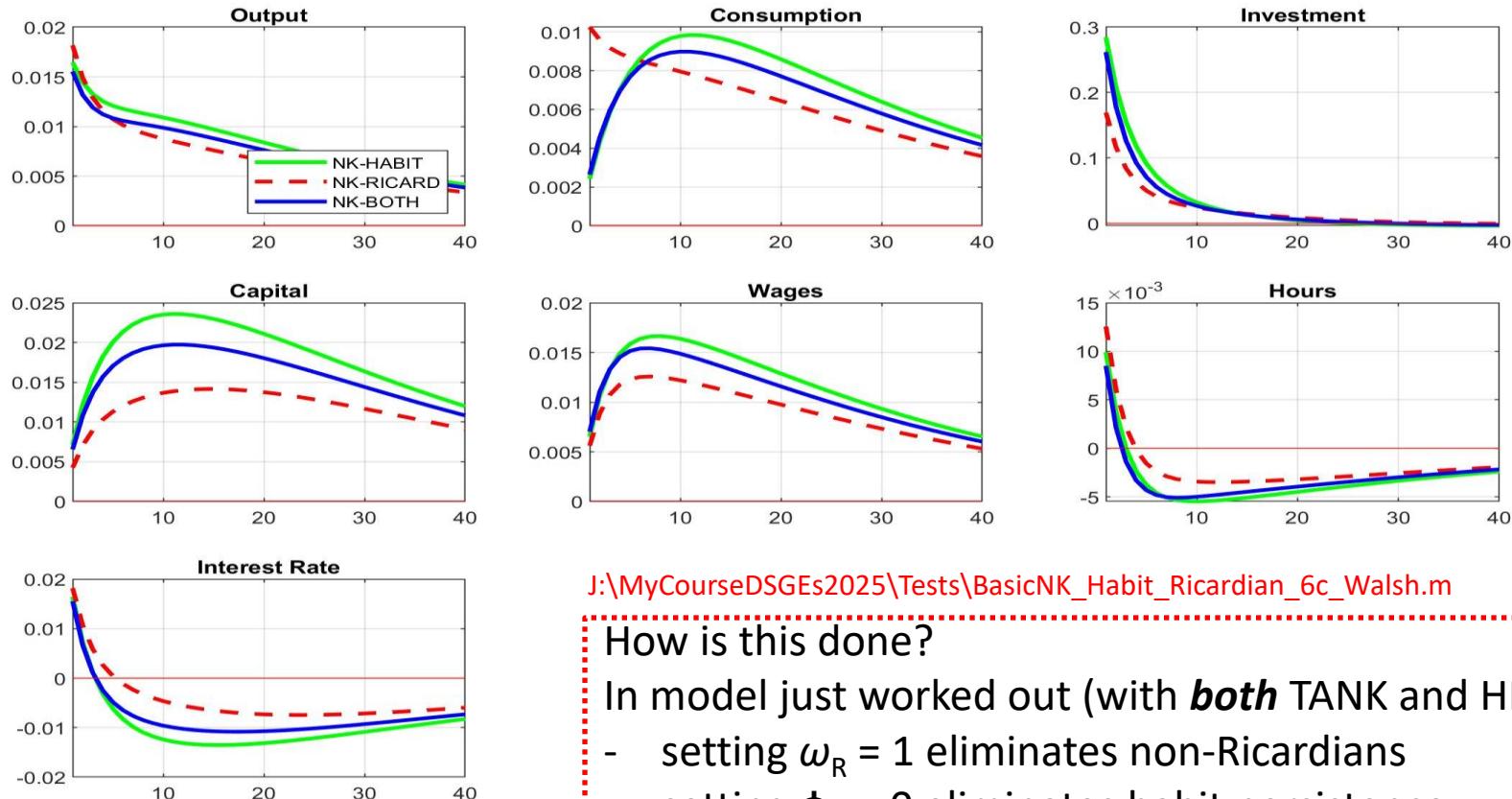
ENG 4:47 PM
SF 7/28/2020

- Comparing results of this model with previous NK (which included imperfect competition and wage and price stickiness) → *small impact* of **TANK** and **HP** assumptions except on consumption

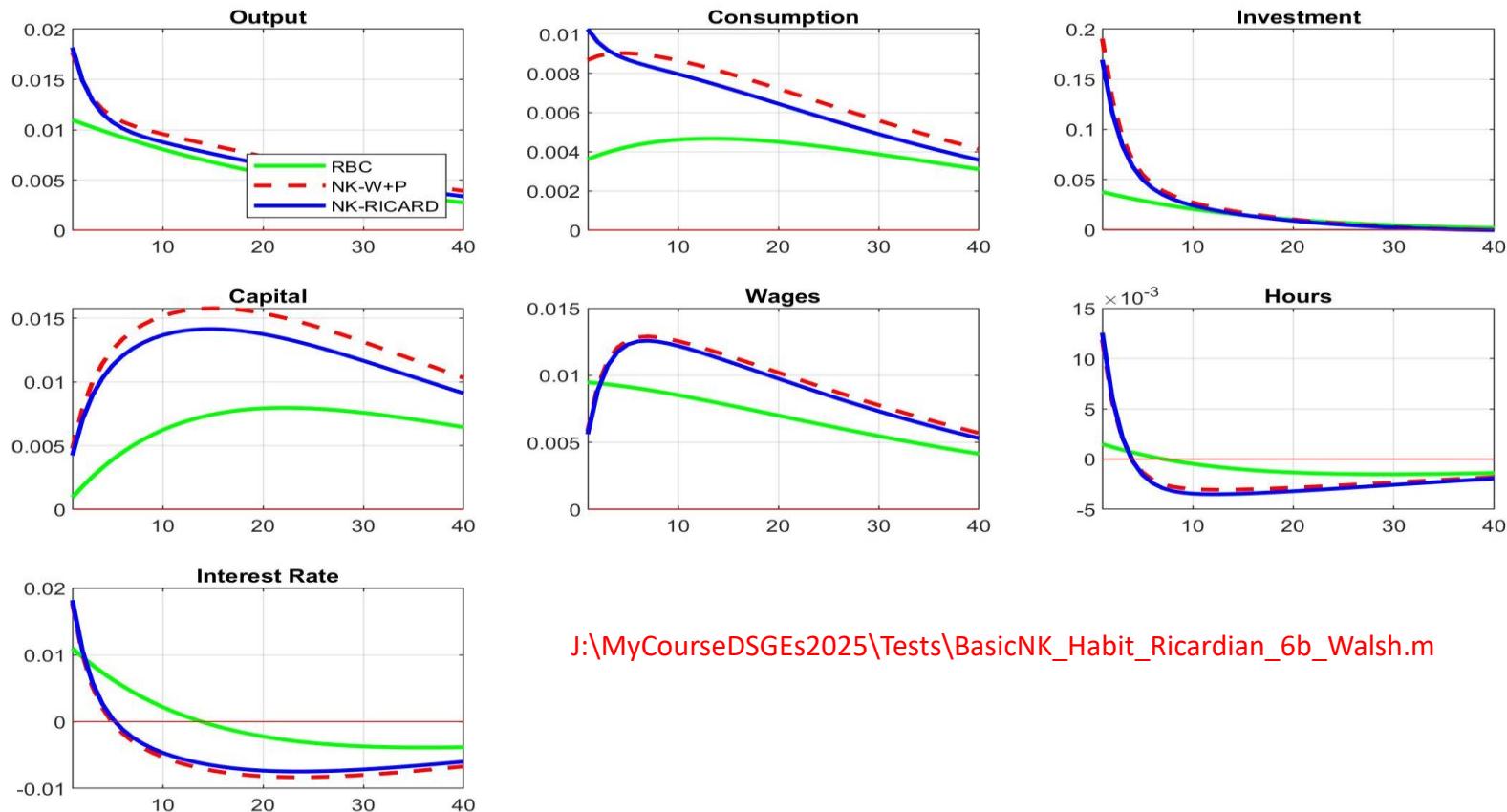


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- Separating out **TANK** and **HP** assumptions shows that impact of inclusion of ***non-Ricardians*** dominates that of ***habit*** persistence

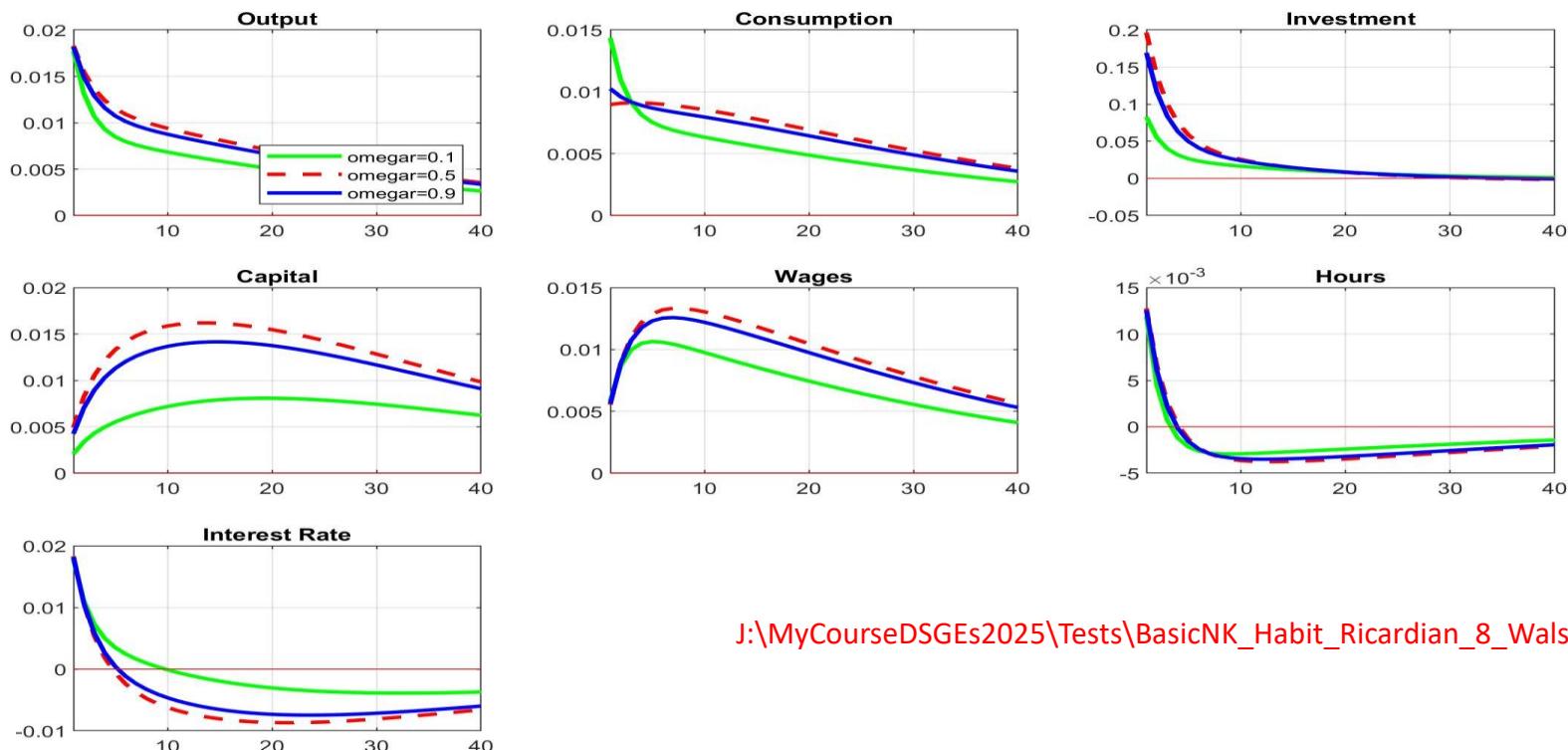


- Compared with difference between **RBC** and **NK-W&P**, inclusion of ***non-Ricardians*** is quite minor, except (as before) for consumption



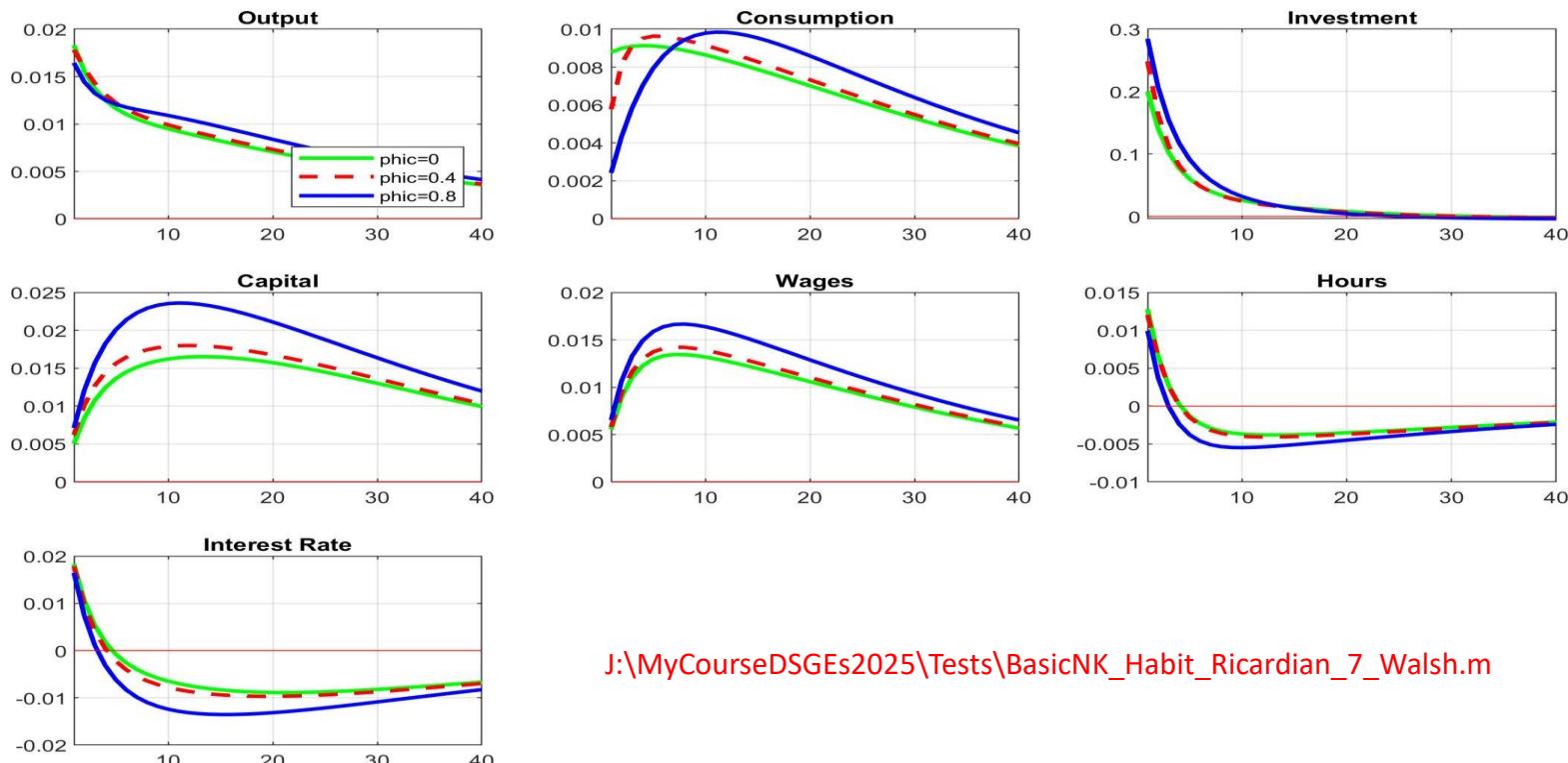
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- However, as share of *non-Ricardians* increases (ω_R decreases), supply of *capital* is compromised as NRs do not have access to *financial markets* → lower *i* and subsequently lower *c* and *y*



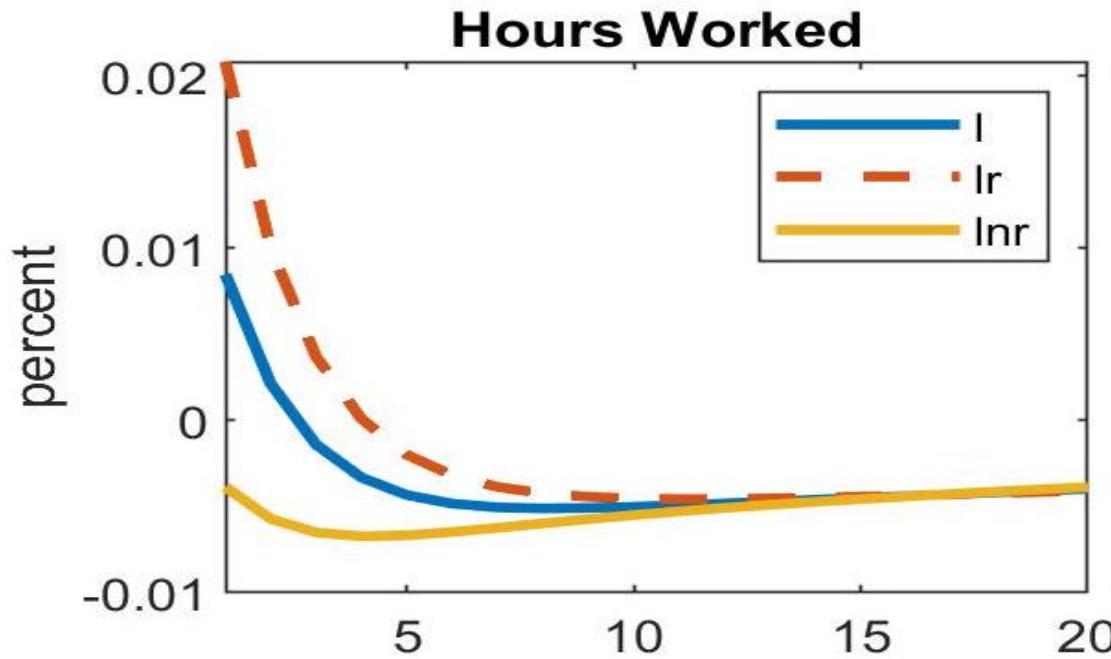
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- *Degree of habit* persistence ϕ_c is evidently important for *consumption*, but also alters *income effect* of productivity rise → sharper fall in labour supply, and sharper rises in wages and capital



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- Behaviour of *two types of labour*



- *Non-Ricardian* (“hand-to-mouth”) households vary their supply of labour *much less* than do “optimisers” (*Ricardians*) in response to TFP shock

- **NK model** introduces into basic RBC more realistic specifications:
 - *imperfect* competition
 - *frictions* in *prices* and in *wages*
 - *habit* formation in consumption
 - *non-Ricardian* agents
 - *adjustment costs* in investment
 - *capacity* (non-)utilisation *costs*
 - *government* sector (*monetary* and *fiscal*)
 - *financial* sector ⇒ Week 5
 - *housing* sector ⇒ Week 5
 - *unemployment* ⇒ Week 6
 - *environment* ⇒ Week 6
 - *foreign* trade ⇒ Week 7

Adjustment Costs in Investment and Capacity (non-)Utilisation Costs

- RBC: capital stock changes *smoothly* from period to period → no constraint on investment process
- But capital stock in *real world* = factories, warehouses, machinery, equipment, etc
- *Cannot* be instantaneously built and installed
- → capital investment for individual firm “*lumpy*”
- Difficult to model
- Could introduce *Calvo fairy* here to deal with this!
- But instead ...

- Include an *adjustment cost* in investment process
→ changes “law of motion of capital”
- So instead of just I_t , we have [\exists other versions!]

$$K_{t+1} = (1 - \delta)K_t + I_t \left[1 - \frac{\chi}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 \right]$$

- This strange (quadratic) formulation → adjustment cost = 0 when objective is simply to maintain a *constant* level of capital ($I_t = I_{t-1}$ → “steady state”)

- What about *capacity utilisation*?
- RBC: installed capital is always used to *full capacity* (utilisation rate = 100%)
- But *empirical* evidence → capacity utilisation related to firm's position in business cycle
- Firm might have built a plant, but because of weakening economy, it is functioning < maximum capacity
- → opportunity cost

- Let U_t represent rate of ***capacity utilisation*** → Cobb-Douglas production function amended to

$$Y_{j,t} = A_t (U_t K_{j,t})^\alpha L_{j,t}^{1-\alpha}$$

This changes capital from a ***stock*** concept to a ***flow*** concept

- changes also to Lagrangian associated with Ricardian households' utility function and to ***budget constraint***

“Effective flow of services” from capital stock

$$P_t (C_{R,t} + I_t) = W_t L_{R,t} + R_t U_t K_t - P_t K_t \left[\Psi_1 (U_t - 1) + \frac{\Psi_2}{2} (U_t - 1)^2 \right]$$

Adjustment cost function – SW quadratic

- Amended Lagrangian for *Ricardians*

$$\begin{aligned} \mathcal{L} = E_t \sum_{t=0}^{\infty} \beta^t & \left\{ \left[\frac{(C_{R,t} - \phi_c C_{R,t-1})^{1-\sigma}}{1-\sigma} - \frac{L_{R,t}^{1+\varphi}}{1+\varphi} \right] \right. \\ & - \lambda_{R,t} [P_t (C_{R,t} + I_t) - W_t L_{R,t} - R_t U_t K_t + \\ & P_t K_t \left[\Psi_1 (U_t - 1) + \frac{\Psi_2}{2} (U_t - 1)^2 \right]] \\ & \left. - Q_t \left[K_{t+1} - (1 - \delta) K_t - I_t \left[1 - \frac{\chi}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 \right] \right] \right\} \end{aligned}$$

↑

- Q_t is “shadow price” [ie, marginal value] of capital, aka **Tobin’s Q** [recall, \equiv ratio of firm market value to replacement value of firm’s assets]

- FOCs \rightarrow **Tobin's Q** [shadow price of capital]

See Costa pp. 157-160 for details

$$Q_t = \beta E_t \left\{ (1 - \delta) Q_{t+1} + \lambda_{R,t+1} R_{t+1} U_{t+1} \right. \\ \left. - \lambda_{R,t+1} P_{t+1} \left[\Psi_1 (U_{t+1} - 1) + \frac{\Psi_2}{2} (U_{t+1} - 1)^2 \right] \right\}$$

- FOCs also \rightarrow equation for **demand for investment assets**

$$\lambda_{R,t} P_t - Q_t \left[1 - \frac{\chi}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 - \chi \left(\frac{I_t}{I_{t-1}} \right) \left(\frac{I_t}{I_{t-1}} - 1 \right) \right] \\ = \chi \beta E_t \left[Q_{t+1} \left(\frac{I_{t+1}}{I_t} \right)^2 \left(\frac{I_{t+1}}{I_t} - 1 \right) \right]$$

- and for **installed capacity** $\frac{R_t}{P_t} = \Psi_1 + \Psi_2 (U_{t-1})$

Remember, R here is price of capital, **not** a policy rate

Only change from previous log-linearised model
is in *investment-related* equations; rest *identical*
→ see details in Appendix

//4-Tobin's Q

$$(Qss/\beta)^* \boxed{q} = (1-\delta)*Qss*q(+1) + \text{LAMBDA}Rss*Rss*Uss * (\lambda + r(+1) + u(+1)) - \text{LAMBDA}Rss*Pss*\Psi1*Uss*u(+1);$$

//5-Demand for Installed Capacity

$$(Rss/Pss)*(r-p) = \Psi2*Uss*u;$$

//6-Investment Demand

$$\text{LAMBDA}Rss*Pss*(\lambda + p) - Qss*q + \chi*Qss*(i - i(-1)) = \chi*\beta*Qss*(i(+1) - i);$$

//13-Production Function

$$y = a + \alpha*(\boxed{u+k}) + (1-\alpha)*l;$$

//14-Demand for Capital

$$\boxed{u+k} = y - (r-p);$$

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Current Folder

Command Window

Workspace

Command History

Start Google MATLAB Python R Java Video

John Cuddy - The Graduate Institute, Geneva 22 September 2025 99

The screenshot shows a MATLAB R2013b interface. The Command Window is active, showing the prompt 'fx >> I'. The Workspace browser on the right lists various variables and their properties. The Command History window shows a series of commands run in sequence. The desktop taskbar at the bottom includes icons for Start, Google, MATLAB, Python, R, Java, and Video.

Current Folder

- +BasicRBC_nonlin_v2
- BasicNKDSGE_nl_v1
- BasicNKDSGE_nl_v1a
- BasicRBC_lin
- BasicRBC_lin_4PPWk2
- BasicRBC_lin_v2
- BasicRBC_nl_v1
- BasicRBC_nl_v2
- BasicRBC_nonlin
- BasicRBC_nonlin_v2
- Ch3_4PPWk2_lc
- Ch4_4PPWk2_lc
- Ch5_4PPWk2_lc
- Whelan_RBC_w_ss
- Whelan_RBC_w_ss_v2
- Whelan_RBC_w_ss_v3
- Whelan_RBC_w_ss_v3a
- BasicNKDSGE_nl_v1.log
- BasicNKDSGE_nl_v1.m
- BasicNKDSGE_nl_v1.mod
- BasicNKDSGE_nl_v1_dynamic.m
- BasicNKDSGE_nl_v1_dynamic.tex

Select a file to view details

Command Window

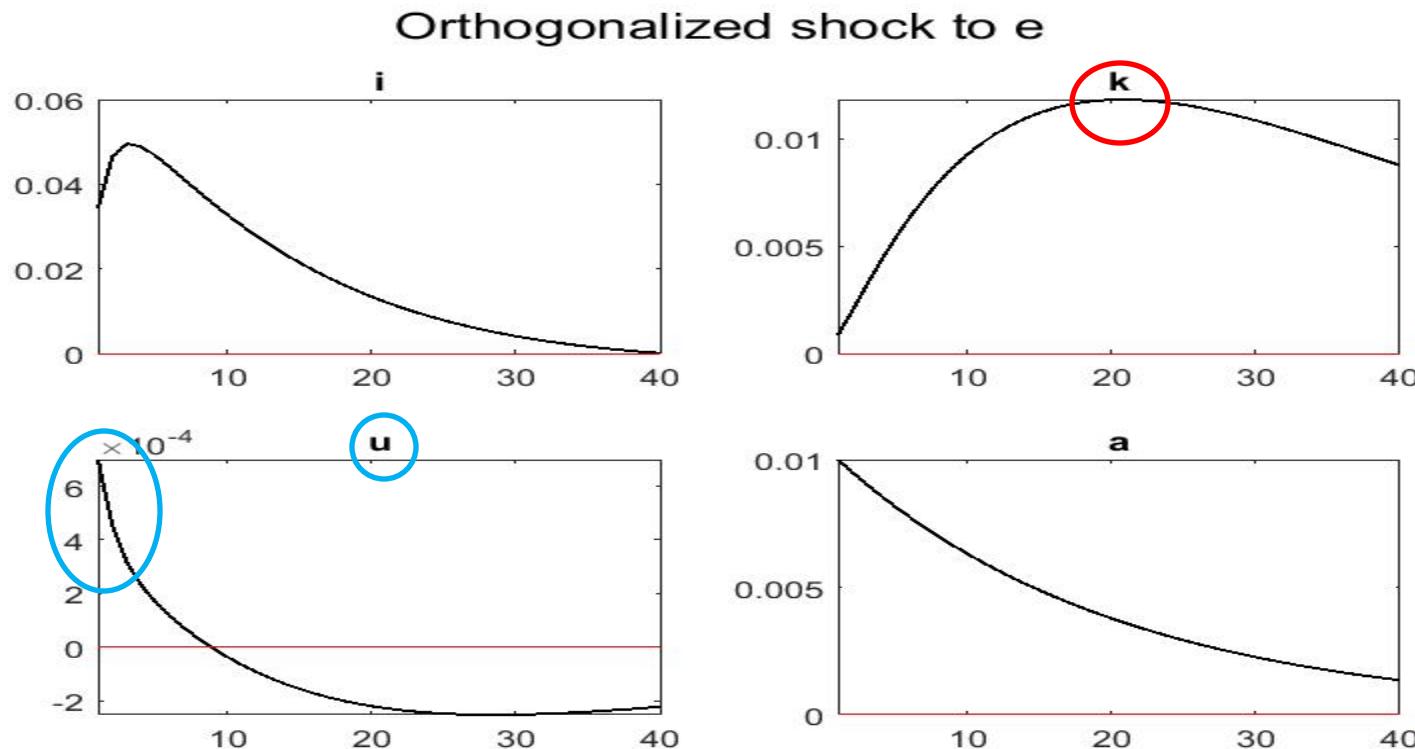
Workspace

Command History

Name	Type	Value	Min	Max
M_	1x1 struct			
a	2000x1 double	-0.... 0..		
a_e	20x1 double	0.0... 0.1		
alpha	0.3500	0.3... 0.3		
ans	0x0 dseries			
bayestopt_	[]			
beta	0.9850	0.9... 0.1		
c	2000x1 double	-0.... 0..		
c_e	20x1 double	0.0... 0.1		
cnr	2000x1 double	-0.... 0..		
cnr_e	20x1 double	0.0... 0.1		
cr	2000x1 double	-0.... 0..		
cr_e	20x1 double	0.0... 0.1		
dataset_	[]			

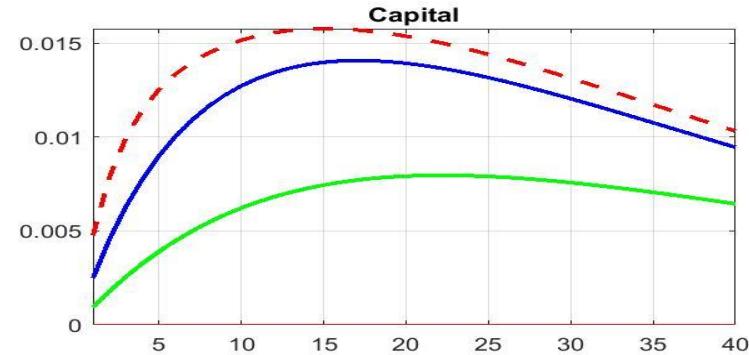
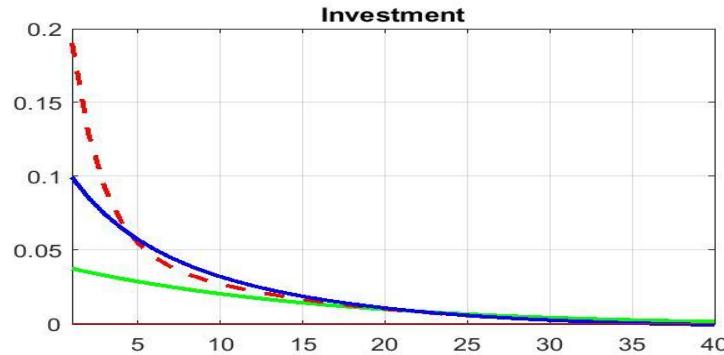
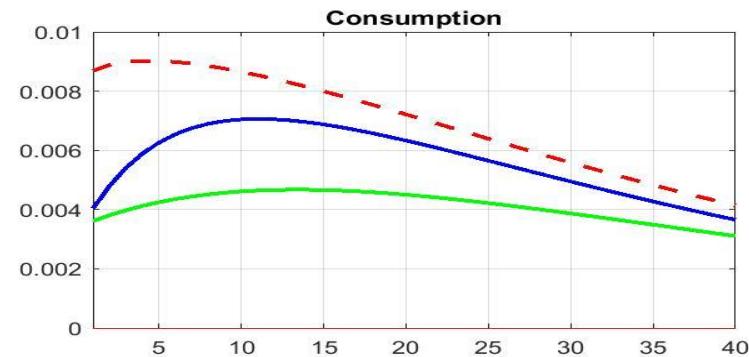
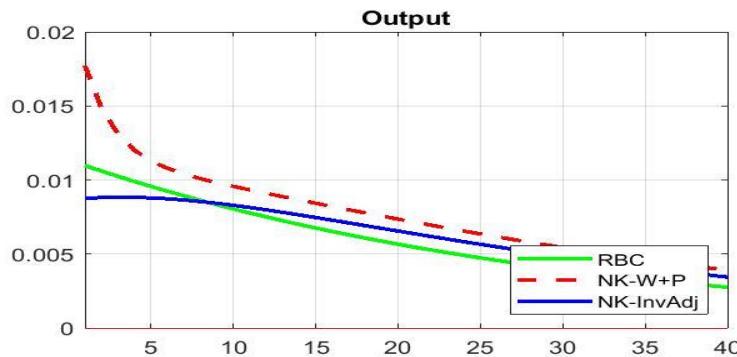
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dynare BasicNKDSGE_nl_v1.mod
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dynare Ch3_4PPWk2_lc.mod
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dynare Ch5_4PPWk2_lc.mod
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dynare Ch3_4PPWk2_lc.mod
clc
dynare Ch4_4PPWk2_lc.mod
clc
dynare Ch5_4PPWk2_lc.mod
clc

- Investment ***adjustment costs*** introduced here → supply of ***capital*** stock ***responds*** only ***slowly*** to higher demand caused by productivity increase [it takes 5 years = 20 quarters to reach peak]

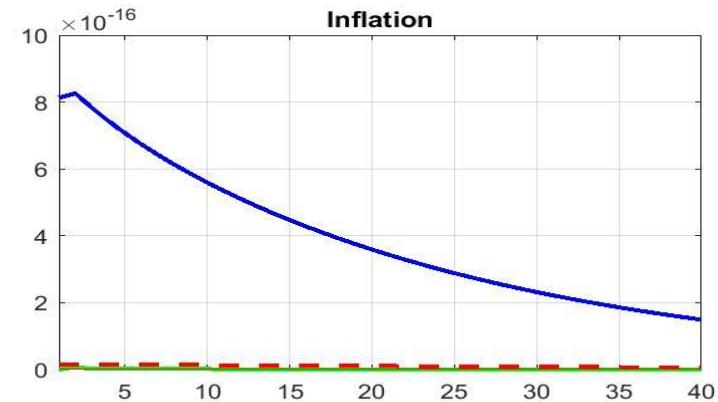
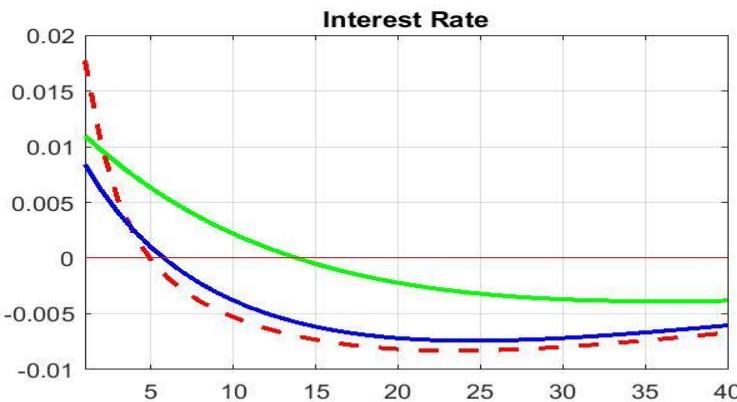
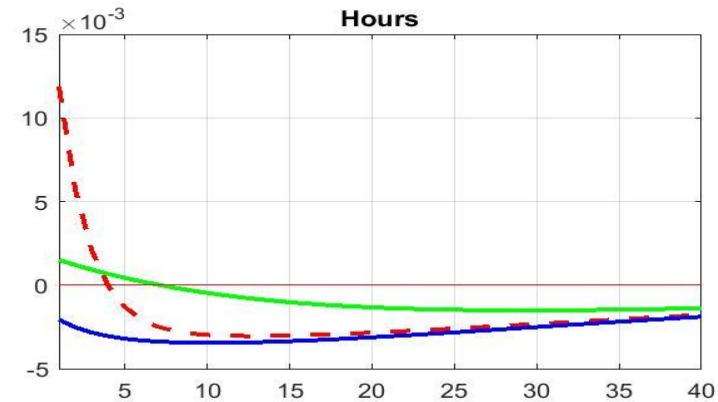
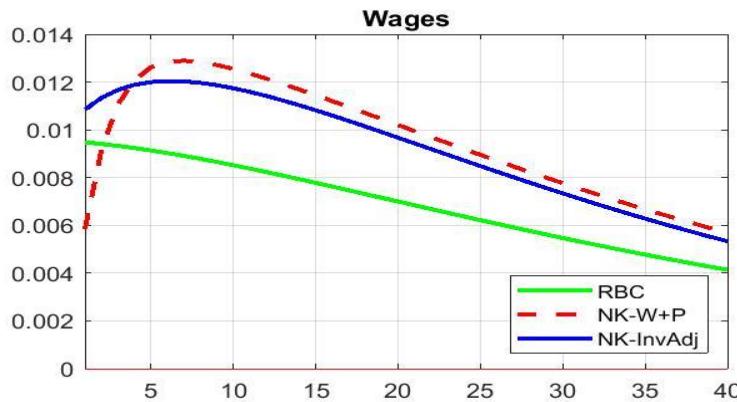


- any short-run ***output*** growth is made possible by an ***increase in utilisation*** of installed capacity ***u*** until capital supply responds

- Compared with basic NK model (which includes price and wage stickiness) we see an ***initial sharp downward impact*** of investment and capacity utilisation rigidities on both investment and capital



- ***Wage*** rates and ***Hours*** worked also show a ***significant impact*** of addition of these new rigidities: “*smoother*” behaviour than in NK-W+P



- **NK model** introduces into basic RBC more realistic specifications:
 - *imperfect* competition
 - *frictions* in *prices* and in *wages*
 - *habit* formation in consumption
 - *non-Ricardian* agents
 - *adjustment costs* in investment
 - *capacity* (non-)utilisation *costs*
 - *government* sector (*monetary* and *fiscal*)
 - *financial* sector ⇒ Week 5
 - *housing* sector ⇒ Week 5
 - *unemployment* ⇒ Week 6
 - *environment* ⇒ Week 6
 - *foreign* trade ⇒ Week 7

Government sector (monetary and fiscal)

- RBC: no government sector
- But in practice \exists :
- *Monetary* authority (Central Bank) focuses on *stabilising prices* (using Taylor rule in DSGE models)
- *Fiscal* authority (Treasury) decides on *scale and composition of government spending*, among
 - supplying public goods and services to private agents
 - income transfers
 - public capital investment
- But expenditure must be paid for! →
 - *taxation*
 - issuing public *debt*
 - issuing *currency*
 - differing costs and benefits to society depending on source

Fiscal Authority

- Taxes affect ***both*** Ricardian and non-Ricardian ***household*** decisions
- Assume following:
 - tax on acquisition of consumer goods and investment assets, τ_c
 - tax on income from labour, τ_l
 - tax on income from capital, τ_k [$\tau_k \ll \tau_l$ – Zucman!]
- Government ***also*** makes ***lump-sum transfer*** of net income ($TRANS_t$) to households, observing the ***proportion*** of Ricardian and non-Ricardian agents

- **Ricardian** households **save** by acquiring (1-period) **bonds**, discounted at basic rate of interest, **issued by government** in each period (B/R^B) R^B is **gross** interest rate
- → Ricardian problem

$$\max_{C_{R,t}, K_{t+1}^P, U_t, I_t^P, B_{t+1}} E_t \sum_{t=0}^{\infty} \beta^t \left[\frac{(C_{R,t} - \phi_c C_{R,t-1})^{1-\sigma}}{1-\sigma} - \frac{L_{R,t}^{1+\varphi}}{1+\varphi} \right]$$

- subject to

+ because tax **adds to** cost of consumption and investment goods (VAT)

$$P_t (1 + \tau_t^c) (C_{R,t} + I_t^P) + \frac{B_{t+1}}{R_t^B} = W_t L_{R,t} (1 - \tau_t^l) + R_t U_t K_t^P (1 - \tau_t^k) \\ - P_t K_t^P \left[\Psi_1 (U_t - 1) + \frac{\Psi_2}{2} (U_t - 1)^2 \right] + B_t + \omega_R P_t TRANS_t$$

Net cost of bonds purchased this quarter, and maturing **next** quarter

Funds received from bonds maturing **this** quarter

- Law of motion of *private* capital

$$K_{t+1}^P = (1 - \delta)K_t^P + I_t^P \left[1 - \frac{\chi}{2} \left(\frac{I_t^P}{I_{t-1}^P} - 1 \right)^2 \right]$$

- → Lagrangian for Ricardian households

$$\begin{aligned} \mathcal{L} = E_t \sum_{t=0}^{\infty} \beta^t & \left\{ \left[\frac{(C_{R,t} - \phi_c C_{R,t-1})^{1-\sigma}}{1-\sigma} - \frac{L_{R,t}^{1+\varphi}}{1+\varphi} \right] \right. \\ & - \lambda_{R,t} \left[P_t (1 + \tau_t^c) (C_{R,t} + I_t^P) + \frac{B_{t+1}}{R_t^B} - W_t L_{R,t} (1 - \tau_t^l) \right. \\ & - R_t U_t K_t^P (1 - \tau_t^k) + P_t K_t^P \left[\Psi_1 (U_t - 1) + \frac{\Psi_2}{2} (U_t - 1)^2 \right] \\ & \left. - B_t - \omega_R P_t [TRANS_t] \right] \\ & \left. - Q_t \left[K_{t+1}^P - (1 - \delta)K_t^P - I_t^P \left[1 - \frac{\chi}{2} \left(\frac{I_t^P}{I_{t-1}^P} - 1 \right)^2 \right] \right] \right\} \end{aligned}$$

- Solving FOCs → For derivation, see Costa pp. 193-199

1. Lagrange multiplier for Ricardian households:

$$\lambda_{R,t} = \frac{(C_{R,t} - \phi_c C_{R,t-1})^{-\sigma}}{P_t (1 + \tau_t^c)} - \phi_c \beta \frac{(E_t C_{R,t+1} - \phi_c C_{R,t})^{-\sigma}}{P_t (1 + \tau_t^c)}$$

2. Tobin's Q:

$$Q_t = \beta E_t \left\{ (1 - \delta) Q_{t+1} + \lambda_{R,t+1} R_{t+1} U_{t+1} \frac{(1 - \tau_{t+1}^k)}{} \right. \\ \left. - \lambda_{R,t+1} P_{t+1} \left[\Psi_1 (U_{t+1} - 1) + \frac{\Psi_2}{2} (U_{t+1} - 1)^2 \right] \right\}$$

3. Equation for demand for installed capacity:

$$\frac{R_t}{P_t} = \frac{1}{(1 - \tau_t^k)} [\Psi_1 + \Psi_2 (U_{t-1})]$$

4. Equation for (private) investment demand:

$$\begin{aligned}\lambda_{R,t} P_t (1 + \tau_t^c) - Q_t & \left[1 - \frac{\chi}{2} \left(\frac{I_t^P}{I_{t-1}^P} - 1 \right)^2 - \chi \left(\frac{I_t^P}{I_{t-1}^P} \right) \left(\frac{I_t^P}{I_{t-1}^P} - 1 \right) \right] \\ & = \chi \beta E_t \left[Q_{t+1} \left(\frac{I_{t+1}^P}{I_t^P} \right)^2 \left(\frac{I_{t+1}^P}{I_t^P} - 1 \right) \right]\end{aligned}$$

5. Euler equation for public *bonds*:

$$\frac{\lambda_{R,t}}{R_t^B} = \beta E_t \lambda_{R,t+1}$$

Intertemporal optimisation

- ***Non-Ricardian*** households
- As before, but now ***taking into account taxes***:

$$\max_{C_{NR,t}} E_t \sum_{t=0}^{\infty} \beta^t \left[\frac{(C_{NR,t} - \phi_c C_{NR,t-1})^{1-\sigma}}{1-\sigma} - \frac{L_{NR,t}^{1+\varphi}}{1+\varphi} \right]$$

- subject to

$$P_t (1 + \tau_t^c) C_{NR,t} = W_t L_{NR,t} (1 - \tau_t^l) + (1 - \omega_R) P_t TRANS_t$$

- Solving FOCs → Lagrange multiplier for non-Ricardian households

$$\lambda_{NR,t} = \frac{(C_{NR,t} - \phi_c C_{NR,t-1})^{-\sigma}}{P_t (1 + \tau_t^c)} - \phi_c \beta \frac{(E_t C_{NR,t+1} - \phi_c C_{NR,t})^{-\sigma}}{P_t (1 + \tau_t^c)}$$

- Wages determined as in previous models, but now taking into account taxes and transfers
- → optimal wage, where “j” represents either Ricardian (R) or non-Ricardian (NR) households, since ***there is no qualitative difference*** in labour which they provide

$$W_{j,t}^* = \left(\frac{\psi_w}{\psi_w - 1} \right) E_t \sum_{i=0}^{\infty} (\beta \theta_w)^i \left[\frac{L_{j,t+i}^\psi}{\lambda_{j,t+i} (1 - \tau_t^l)} \right]$$

- thus ***aggregate wage level*** is (as before)

$$W_t = \left[\theta_w W_{t-1}^{1-\psi_w} + (1 - \theta_w) W_t^{*1-\psi_w} \right]^{\frac{1}{1-\psi_w}}$$

Firms

- ***Public capital*** (superscript “G”) now also in production function for intermediate goods

$$Y_{j,t} = A_t K_{j,t}^P \alpha_1 L_{j,t}^{\alpha_2} K_{j,t}^G \alpha_3$$

- → marginal cost

$$MC_{j,t} = \frac{1}{A_t K_{j,t}^G \alpha_3} \left(\frac{W_t}{\alpha_2} \right)^{\alpha_2} \left(\frac{R_t}{\alpha_1} \right)^{\alpha_1}$$

- ***Does not affect*** optimal pricing decision, which remains

$$P_{j,t}^* = \left(\frac{\psi}{\psi - 1} \right) E_t \sum_{i=0}^{\infty} (\beta \theta)^i MC_{j,t+i}$$

Government

Net income from bonds sold *this* quarter, but maturing *next* quarter

- No *seignorage* → government's *budget constraint*

$$\frac{B_{t+1}}{R_t^B} - B_t + T_t = P_t G_t + P_t I_t^G + P_t \text{TRANS}_t$$

Cost of repaying bonds maturing *this* quarter

- and *tax revenue* is

$$T_t = \tau_t^c P_t (C_t + I_t^P) + \tau_t^l W_t L_t + \tau_t^k (R_t - \delta) K_t^P$$

- Also assume that each fiscal instrument evolves following a “*fiscal policy rule*”
- Many forms possible
- Assume a *common* rule of the form

$$\frac{Z_t}{Z_{ss}} = \left(\frac{Z_{t-1}}{Z_{ss}} \right)^{\gamma_Z} \left(\frac{B_t}{Y_{t-1}P_{t-1}} \quad \frac{Y_{ss}P_{ss}}{B_{ss}} \right)^{(1-\gamma_Z)\phi_Z} S_t^Z$$

- where Z represents each of the fiscal policy instruments in turn (τ^c , τ^l , τ^k)
- Why this form?
- Take logs → LHS: $\log(Z_t) - \log(Z_{ss})$... *looks familiar?*

- This rule also ensures that ***as issued debt rises*** (B_t increases) ***so too do tax rates*** to maintain balance
- A “fiscal shock” S^Z is attached to each rule

$$\log S_t^Z = \rho_Z \log S_{t-1}^Z + \varepsilon_{Z,t}$$

- Law of motion of ***public capital***

$$K_{t+1}^G = (1 - \delta_G) K_t^G + I_t^G$$

- So much for *Treasury*; what about *Central Bank*?
- Central Bank follows a standard *Taylor Rule* with a twofold objective: price stability and economic growth

$$\frac{R_t^B}{R_{SS}^B} = \left(\frac{R_{t-1}^B}{R_{SS}^B} \right)^{\gamma_R} \left[\left(\frac{\pi_t}{\pi_{SS}} \right)^{\gamma_\pi} \left(\frac{Y_t}{Y_{SS}} \right)^{\gamma_Y} \right]^{(1-\gamma_R)} S_t^m$$
 - where γ_Y and γ_π = sensitivities of basic interest rate in relation to output and inflation rate
 - γ_R = interest rate smoothing parameter
 - R^B = *policy rate* (= interest rate on government bonds)
- Again, this form of Taylor Rule is used because taking logs → LHS: $\log(R_t^B) - \log(R_{SS}^B)$

- Here too, I have included a monetary shock, S^m which evolves according to
$$\log S_t^m = \rho_m \log S_{t-1}^m + \varepsilon_{m,t}$$
- Other equations may be ***carried over*** from previous model
- Main ***changes*** are those shown on following slides

$P_t(1 + \tau_t^c)(C_{R,t} + I_t^P) + \frac{B_{t+1}}{R_t^B} = W_t L_{R,t}(1 - \tau_t^l) + R_t U_t K_t^P(1 - \tau_t^k)$	
$-P_t K_t^P \left[\Psi_1(U_t - 1) + \frac{\Psi_2}{2} (U_t - 1)^2 \right] + B_t + \omega_R P_t TRANS_t$	(Ricardian budget)
$\frac{\lambda_R t}{R_t^s} = \beta E_t \lambda_{R,t+1}$	(Public bond Euler)
$\frac{B_{t-1}}{R_t^B} - B_t + T_t = P_t G_t + P_t I_t^G + P_t TRANS_t$	(Gov't budget)
$T_t = \tau_t^c P_t (C_t + I_t^P) + \tau_t^l W_t L_t + \tau_t^k (R_t - \delta) K_t^P$	(Gov't revenue)
$\frac{Z_t}{Z_{ss}} = \left(\frac{Z_{t-1}}{Z_{ss}} \right)^{\gamma_Z} \left(\frac{B_t}{Y_{t-1} P_{t-1}} \quad \frac{Y_{ss} P_{ss}}{B_{ss}} \right)^{(1-\gamma_Z)\phi_Z} S_t^Z$	(Fiscal policy rule)
$K_{t+1}^G = (1 - \delta_G) K_t^G + I_t^G$	(Public capital)
$\frac{R_t^B}{R_{Bs}^B} = \left(\frac{R_{t-1}^B - 1}{R_{ss}^B} \right)^{\gamma_R} \left[\left(\frac{\pi_t}{\pi_{ss}} \right)^{\gamma_\pi} \left(\frac{Y_t}{Y_{ss}} \right)^{\gamma_s} \right]^{(1-\gamma_R)} S_t^m$	(Taylor Rule)
$Y_t = C_t + I_t^P + I_t^G + G_t$	(Equilibrium)
$\log A_t = \rho_A \log A_{t-1} + \varepsilon_t$	(Productivity shock)
$\log S_t^Z = \rho_Z \log S_{t-1}^Z + \varepsilon_{Z,t}$	(Fiscal shock)
$\log S_t^m = \rho_m \log S_{t-1}^m + \varepsilon_{m,t}$	(Monetary shock)

I have also added several “shocks” to model, so that it is possible to observe *effects* of *fiscal* and *monetary policy* [only *structural* shock is – as previously – to *productivity*]

//33 - Income Transfer Shock

$\text{STRANS} = \text{rhoTRANS} * \text{STRANS}(-1) + e_TRANS;$

Changing this from 0 permits us to see how altering VAT affects economy – note *negative* sign

//34 - Consumption Tax Shock (Negative => Reduction in Tax Rate!)

$\text{Stau_c} = \text{rhotauc} * \text{Stau_c}(-1) - e\tau_c;$

//35 - Labour Income Tax Shock (Negative => Reduction in Tax Rate!)

$\text{Stau_l} = \text{rhotaul} * \text{Stau_l}(-1) - e\tau_l;$

//36 - Capital Income Tax Shock (Negative => Reduction in Tax Rate!)

$\text{Stau_k} = \text{rhotauk} * \text{Stau_k}(-1) - e\tau_k;$

//37- Monetary Shock (Negative => Reduction in Interest Rate!)

$\text{Sm} = \text{rhom} * \text{Sm}(-1) - e_m;$

- Model uses following parameters

must sum to unity	α_1	0.300	private capital share
	α_2	0.650	labour share
	α_3	0.050	public capital share
	β	0.985	discount factor
ρ_a		0.900	autocorrelation technology shock
σ		2.000	CRRA coefficient
ϕ		1.500	Frisch elasticity
ψ		8.000	elasticity of substitution - intermediate goods
ψ_w		21.000	elasticity of substitution - labour
δ		0.025	depreciation rate - private
δ_g		0.025	depreciation rate - public
θ		0.750	Calvo parameter - prices
θ_w		0.750	Calvo parameter - wages

- Model uses following parameters

γ_r	0.800	Taylor Rule - interest rate sensitivity
γ_p	1.500	Taylor Rule - inflation sensitivity
γ_y	0.500	Taylor Rule - output sensitivity
τ_c	0.160	Tax rate on consumption SS
τ_l	0.170	Tax rate on labour SS
τ_k	0.080	Tax rate on capital SS
ϕ_c	0.800	Habit persistence
ω_r	0.500	Proportion of Ricardians
ψ_1	0.047	Capacity utilisation parameter 1
ψ_2	1.000	Capacity utilisation parameter 2
χ	1.000	Investment adjustment cost parameter

HOME PLOTS APPS

FILE VARIABLE CODE SIMULINK ENVIRONMENT RESOURCES

Find Files New Variable Import Data Open Variable Clear Commands Analyze Code Run and Time Set Path Preferences Community Request Support Add-Ons

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Current Folder Name +BasicRBC_nonlin_v2 BasicNKDSGE_nl_v1 BasicNKDSGE_nl_v1a BasicRBC_lin BasicRBC_lin_4PPWk2 BasicRBC_lin_v2 BasicRBC_nl_v1 BasicRBC_nl_v2 BasicRBC_nonlin BasicRBC_nonlin_v2 Ch3_4PPWk2_lc Ch4_4PPWk2_lc Ch5_4PPWk2_lc Ch6_4PPHW2_lc Whelan_RBC_w_ss Whelan_RBC_w_ss_v2 Whelan_RBC_w_ss_v3 Whelan_RBC_w_ss_v3a BasicNKDSGE_nl_v1.log BasicNKDSGE_nl_v1.m BasicNKDSGE_nl_v1.mod BasicNKDSGE_nl_v1_dynamic.m

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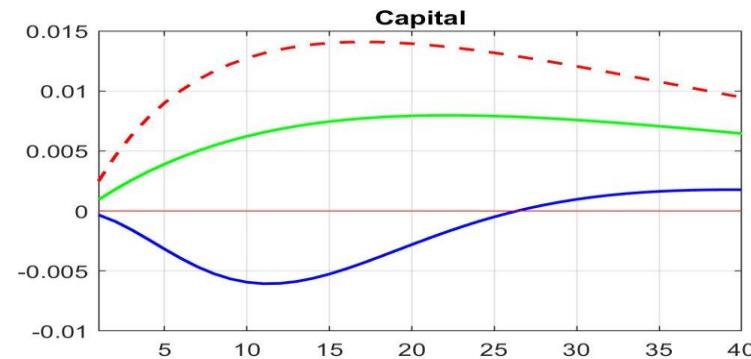
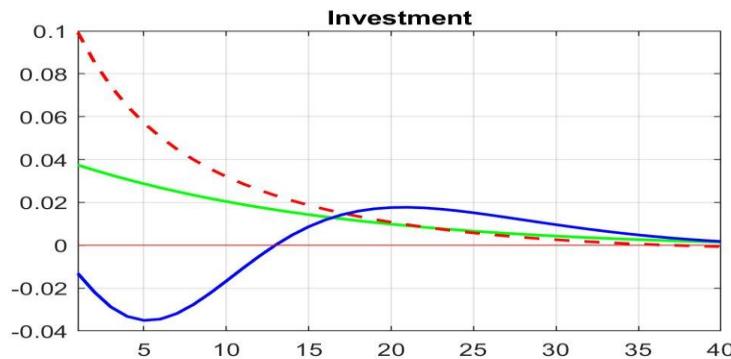
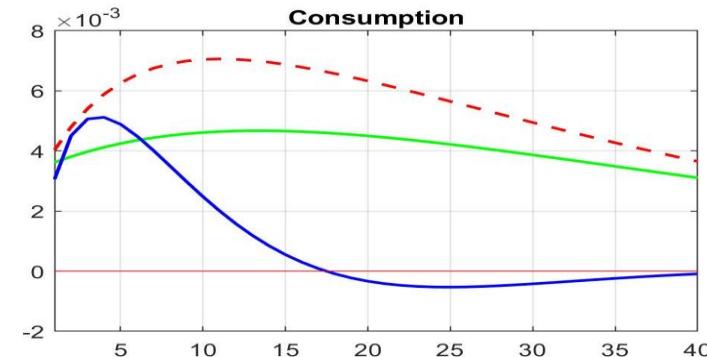
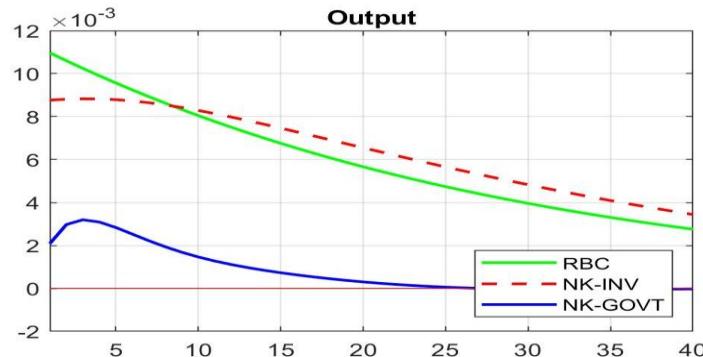
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Psi2	1	1	1	
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a_e	100x1 double	6.2...	0.1	
alpha	0.3500	0.3...	0.1	
ans	0x0 dseries			
bayestopt_	[]			
beta	0.9850	0.9...	0.1	
c	2000x1 double	-0....	0.	
c_e	100x1 double	3.7...	0.1	
chi	1	1	1	
cnr	2000x1 double	-0....	0.	
cnr_e	100x1 double	3.5...	0.1	

Command History

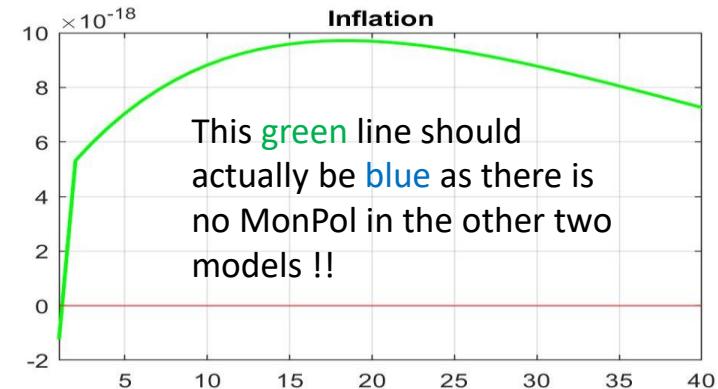
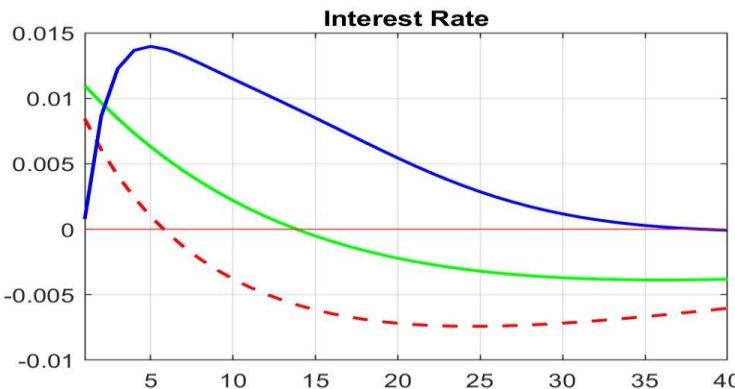
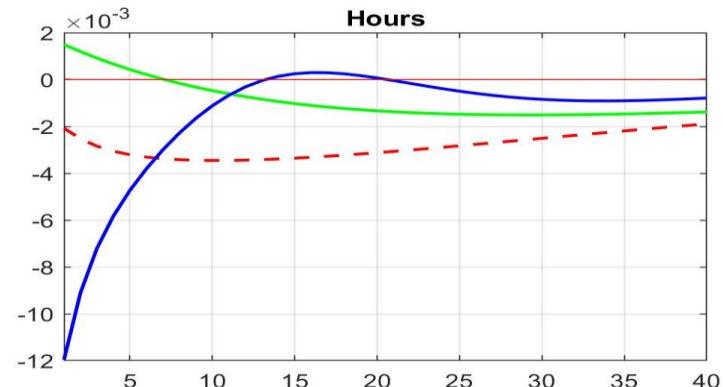
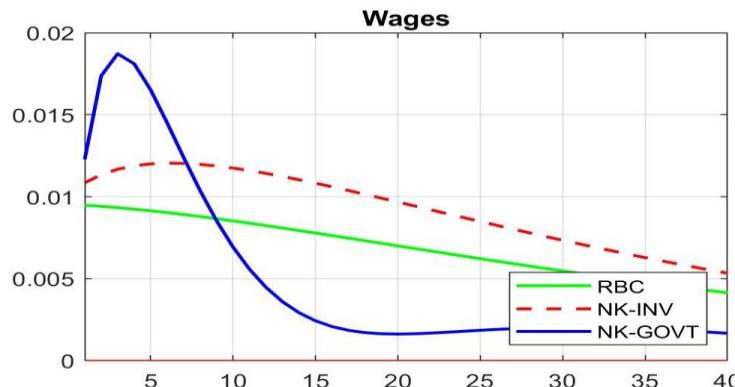
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dynare Ch5_4PPWk2_lc.mod
clc
dynare Ch6_4PPHW2_lc.mod
clc
```

- Compare effects of ***productivity shock*** to that of previous (INV) model

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- → inclusion of taxes has a ***sharp effect*** on consumption and investment → income and capital



- Evidently monetary policy also weighs sharply on interest rate

Appendix 1: More Details

Non-Ricardian Agents and Habit Formation

New equations related to heterogeneous agents, $j \in \{R, NR\}$

//1-Lagrangian for Ricardian Households

```
lambdar = (sigma/((1-phic)*(1-phic*beta)))*(phic*beta*(cr(+1)-phic*cr)-(cr-phic*cr(-1)))-p;
```

//2-Phillips Curve for Wages for Ricardian Households

```
piw = beta*piw(+1)+((1-thetaw)*(1-beta*thetaw)/thetaw)*(lr-lambdar-w);
```

//3-Gross Wage Inflation Rate

```
piw = w - w(-1);
```

See derivation pp.136-139 of Costa

//4-Euler Equation

```
(Pss/beta)*(lambdar+p-lambdar(+1))=(1-delta)*Pss*p(+1) + Rss*r(+1);
```

//5-Law of Motion of Capital

$$k = (1-\delta) * k(-1) + \delta * i;$$

//6-Lagrangian for Non-Ricardian Households

$$\lambda_{\text{LNR}} = (\sigma / ((1-\phi_{\text{IC}}) * (1-\phi_{\text{IC}} * \beta))) * (\phi_{\text{IC}} * \beta * (c_{\text{NR}}(1) - \phi_{\text{IC}} * c_{\text{NR}}) - (c_{\text{NR}} - \phi_{\text{IC}} * c_{\text{NR}}(-1))) - p;$$

//7-Phillips Curve for Wages for Non-Ricardian Households

$$\pi_w = \beta * \pi_w(1) + ((1 - \theta_w) * (1 - \beta * \theta_w) / \theta_w) * (l_{\text{NR}} - \lambda_{\text{LNR}} - w);$$

//8-Budget Constraint for Non-Ricardian Households

$$p + c_{\text{NR}} = w + l_{\text{NR}};$$

//9-Aggregate Consumption

$$C_{\text{SS}} * c = \omega * C_{\text{RSS}} * c_r + (1 - \omega) * C_{\text{NRR}} * c_{\text{NR}};$$

//10-Aggregate Labour Supply

$$L_{\text{SS}} * l = \omega * L_{\text{RSS}} * l_r + (1 - \omega) * L_{\text{NRR}} * l_{\text{NR}};$$

//11-Production Function

$y = a + \alpha*k + (1-\alpha)*l;$

//12-Demand for Capital

$k = y - (r - p);$

//13-Demand for Labour

$l = y - (w - p);$

11-18 are just copied over
from previous model

//14-Marginal Cost

$mc = ((1-\alpha)*w + \alpha*r - a);$

//15-Phillips Curve

$\pi_t = \beta_t * \pi_{t-1} + ((1-\theta_t) * (1-\beta_t * \theta_t) / \theta_t) * (mc - p_t);$

//16-Gross Inflation Rate

$\pi_t = p_t - p_{t-1};$

//17-Goods Market Equilibrium Condition

$Y_{ss} * y_t = C_{ss} * c_t + I_{ss} * i_t;$

//18-Productivity Shock

$a_t = \rho_a * a_{t-1} + e_t;$

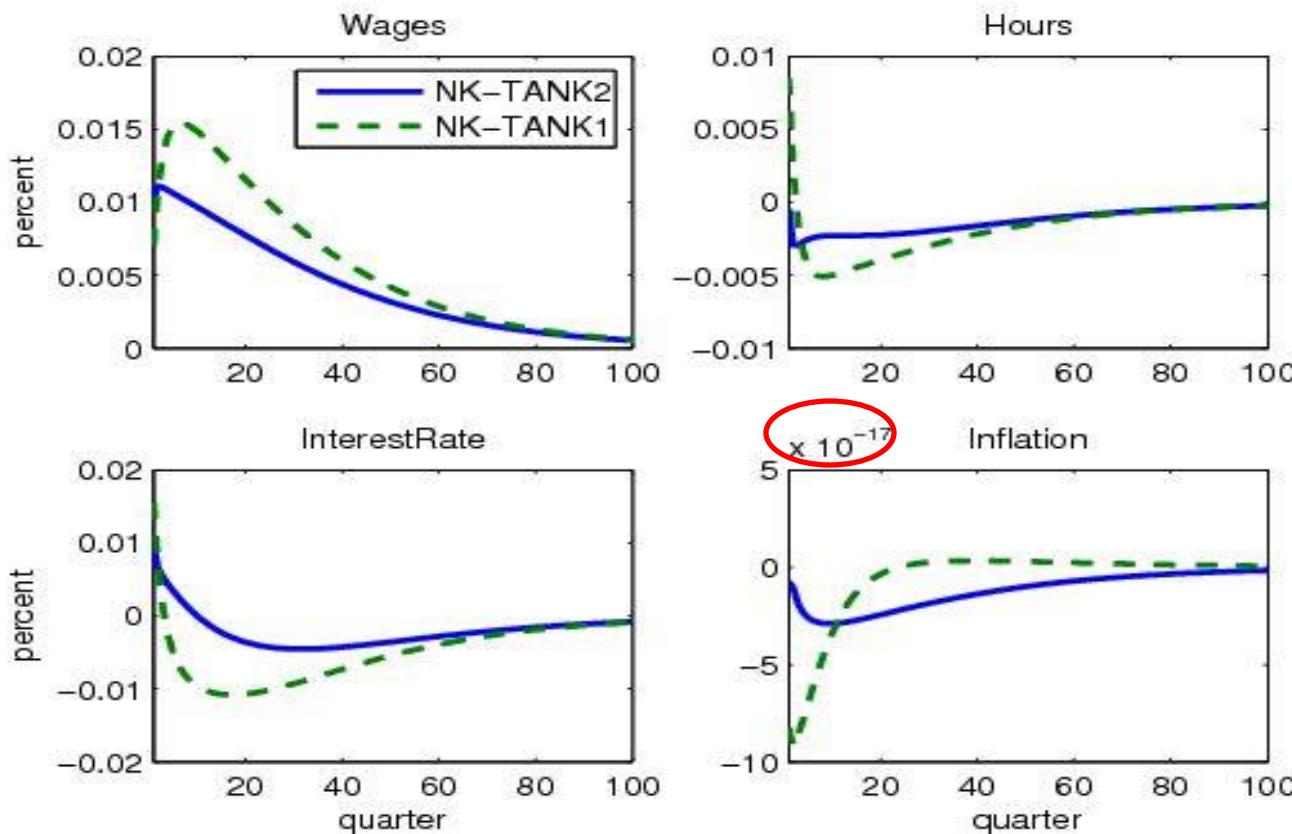
- The results cited were derived from the following

Parameter		Calibrated value
σ	Relative risk aversion coefficient	2
ϕ	Marginal disutility with respect to labor supply	1.5
α	Elasticity of output with respect to capital	0.35
β	Discount factor	0.985
δ	Depreciation rate	0.025
ρ_A	Autoregressive parameter of productivity	0.95
σ_A	Standard deviation of productivity	0.01
θ	Price stickiness parameter	0.75
ψ	Elasticity of substitution between intermediate goods	8
θ_w	Wage stickiness parameter	0.75
ψ_w	Elasticity of substitution between differentiated labour	21
φ_c	Habit persistence	0.8
ω_R	Participation of Ricardian agents in consumption and labor	0.5

- What if we reduce (or increase) degree of price and wage **stickiness**?

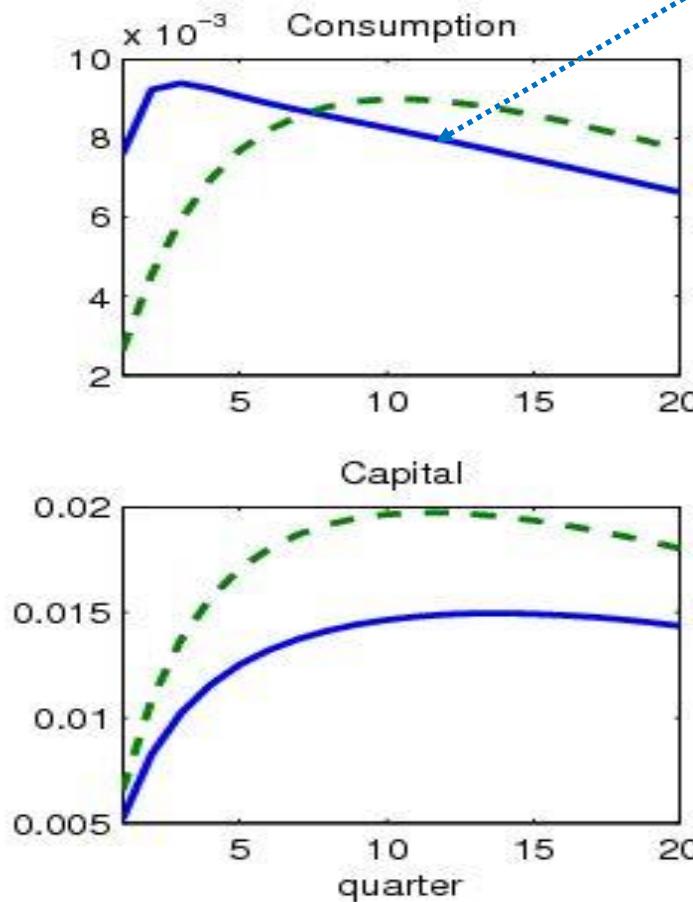
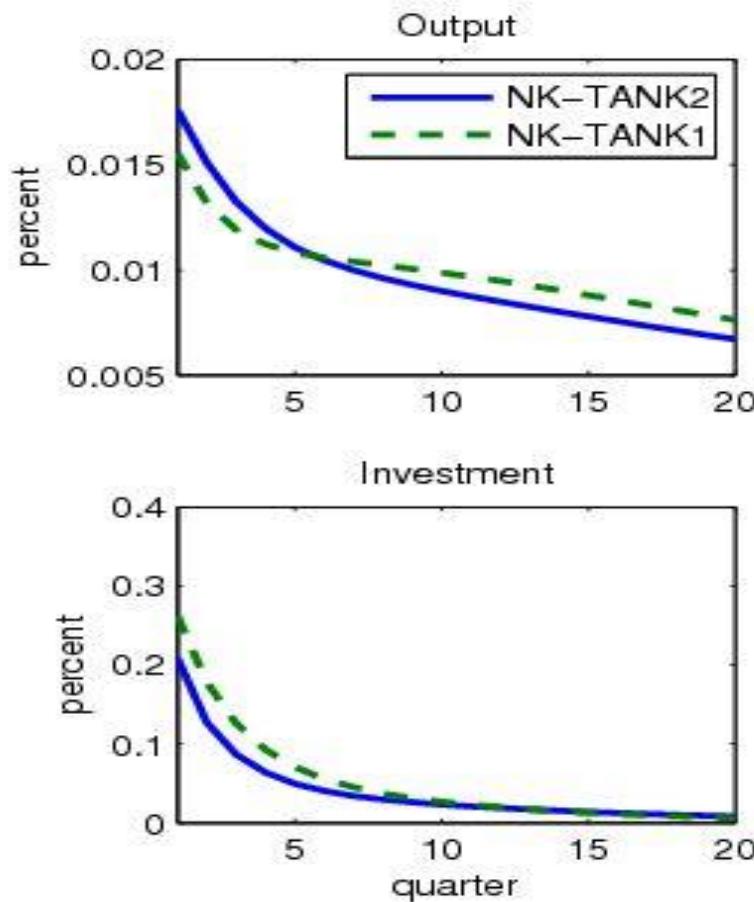
- Fraction of firms and households obliged by Calvo
fairy to follow a Calvo Rule was set at **three-quarters**; what if this is reduced to **one-third**?

Since a greater share can already optimally adjust, they respond less to a shock



- Habit persistence parameter was set at four-fifths; what if this is reduced to one-third?

This means that the additional utility of consumption is anyway much greater, so it responds less to a shock



Adjustment Costs in Investment and Capacity (non-)Utilisation Costs

-  Model

$$\begin{aligned} & \lambda_{R,t} P_t - Q_t \left\{ 1 - \frac{\chi}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 - \chi \left(\frac{I_t}{I_{t-1}} \right) \left(\frac{I_t}{I_{t-1}} - 1 \right) \right\} \\ &= \chi \beta E_t Q_{t+1} \left(\frac{E_t I_{t+1}}{I_t} \right)^2 \left(\frac{E_t I_{t+1}}{I_t} - 1 \right) \end{aligned} \quad (\text{Demand for investment assets})$$

$$K_{t+1} = (1 - \delta) K_t + I_t \left[1 - \frac{\chi}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 \right] \quad (\text{Law of motion of capital})$$

$$\begin{aligned} Q_t &= \beta E_t \left\{ (1 - \delta) Q_{t+1} + \lambda_{R,t+1} R_{t+1} U_{t+1} \right. \\ &\quad \left. - \lambda_{R,t+1} P_{t+1} \left[\Psi_1 (U_{t+1} - 1) + \frac{\Psi_2}{2} (U_{t+1} - 1)^2 \right] \right\} \\ \frac{R_t}{P_t} &= \Psi_1 + \Psi_2 (U_t - 1) \end{aligned}$$

(Tobin's Q)

(Demand for installed capacity)

- ***Remainder* of model *familiar*:**

$W_{j,t}^* = \left(\frac{\psi_W}{\psi_W - 1} \right) E_t \sum_{i=0}^{\infty} (\beta \theta_W)^i \frac{L_{NR,j,t+i}}{\lambda_{NR,t+i}}^{\psi}$	(NR optimal wage)
$P_t C_{NR,t} = W_t L_{NR,t}$	(NR budget)
$C_t = \omega_R C_{R,t} + (1 - \omega_R) C_{NR,t}$	(Aggregate consumption)
$L_t = \omega_R L_{R,t} + (1 - \omega_R) L_{NR,t}$	(Aggregate Labour)
$Y_t = A_t [U_t K_{j,t}]^{\alpha} L_t^{1-\alpha}$	(Production function)
$U_t K_t = \alpha M C_t \frac{Y_t}{R_t}$	(Demand for capital) New
$L_t = (1 - \alpha) M C_t \frac{Y_t}{W_t}$	(Demand for labour)
$M C_t = \frac{1}{A_t} \left(\frac{W_t}{1-\alpha} \right)^{1-\alpha} \left(\frac{R_t}{\alpha} \right)^{\alpha}$	(Marginal cost)
$P_t^* = \left(\frac{\psi}{\psi - 1} \right) E_t \sum_{i=0}^{\infty} (\beta \theta)^i M C_{t+i}$	(Optimal price level)
$P_t = \left[\theta P_{t-1}^{1-\psi} + (1 - \theta) P_t^* \right]^i M C_{t+i}$	(General price level)
$\pi_t = P_t / P_{t-1}$	(Equilibrium)
$\log A_t = \rho_A \log A_{t-1} + \epsilon_t$	(Productivity shock)

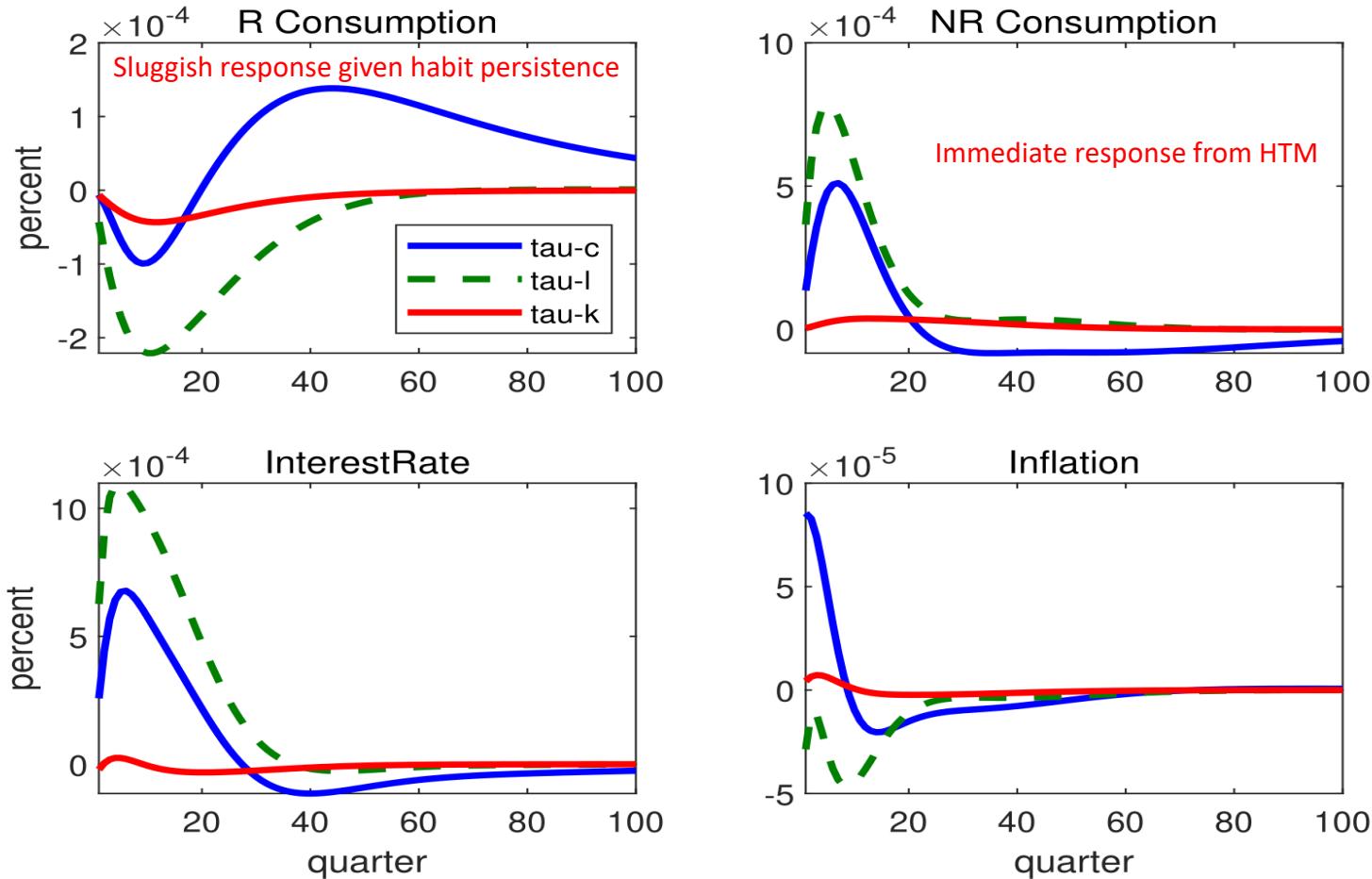
Government sector (monetary and fiscal)

- Comparing effects of various *tax rates* (on consumption, labour and capital)

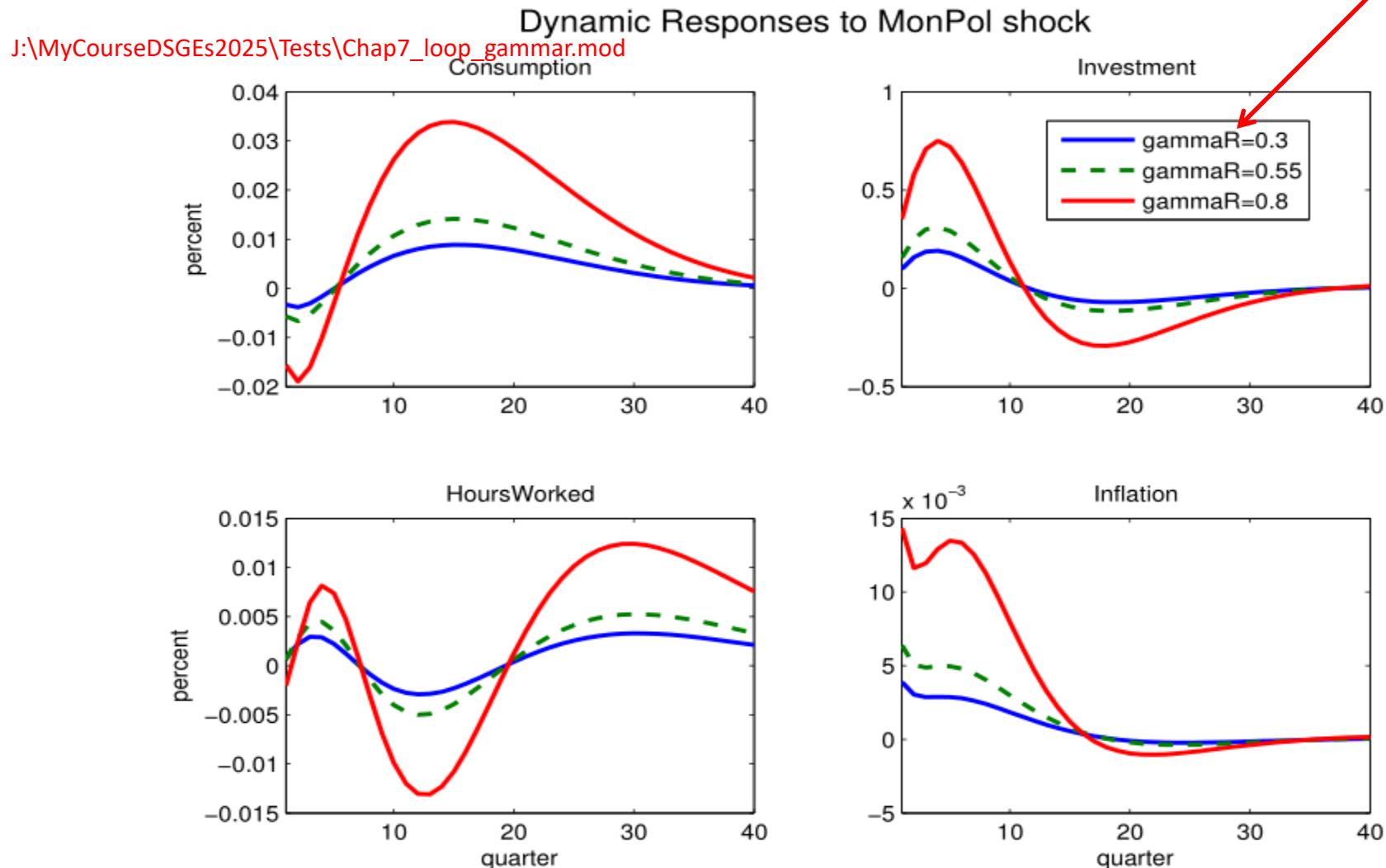
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Remember, these shocks are negative!!

Dynamic Responses to Tax shocks

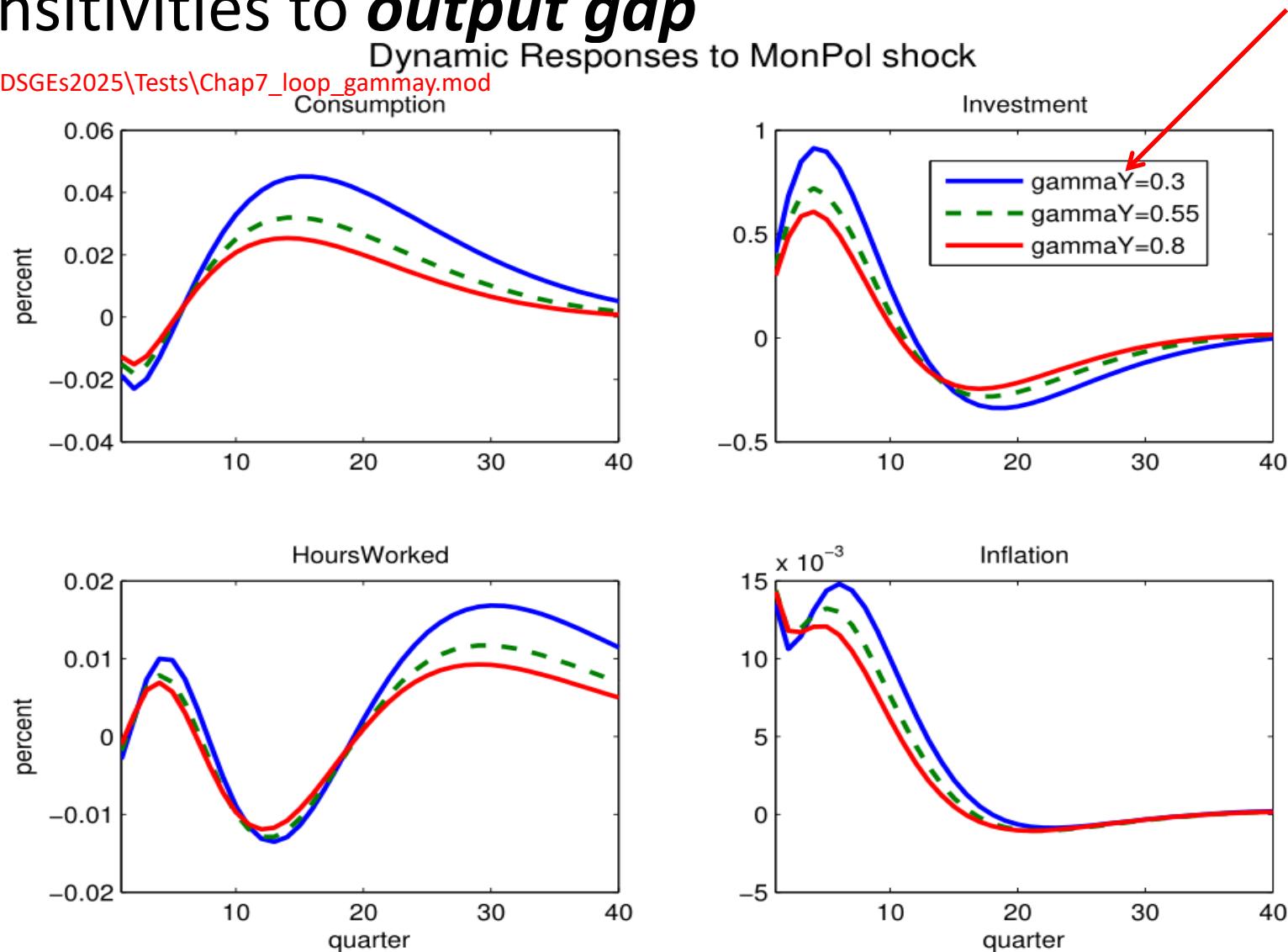


- Comparing effects of different Taylor Rule sensitivities to *interest rate*



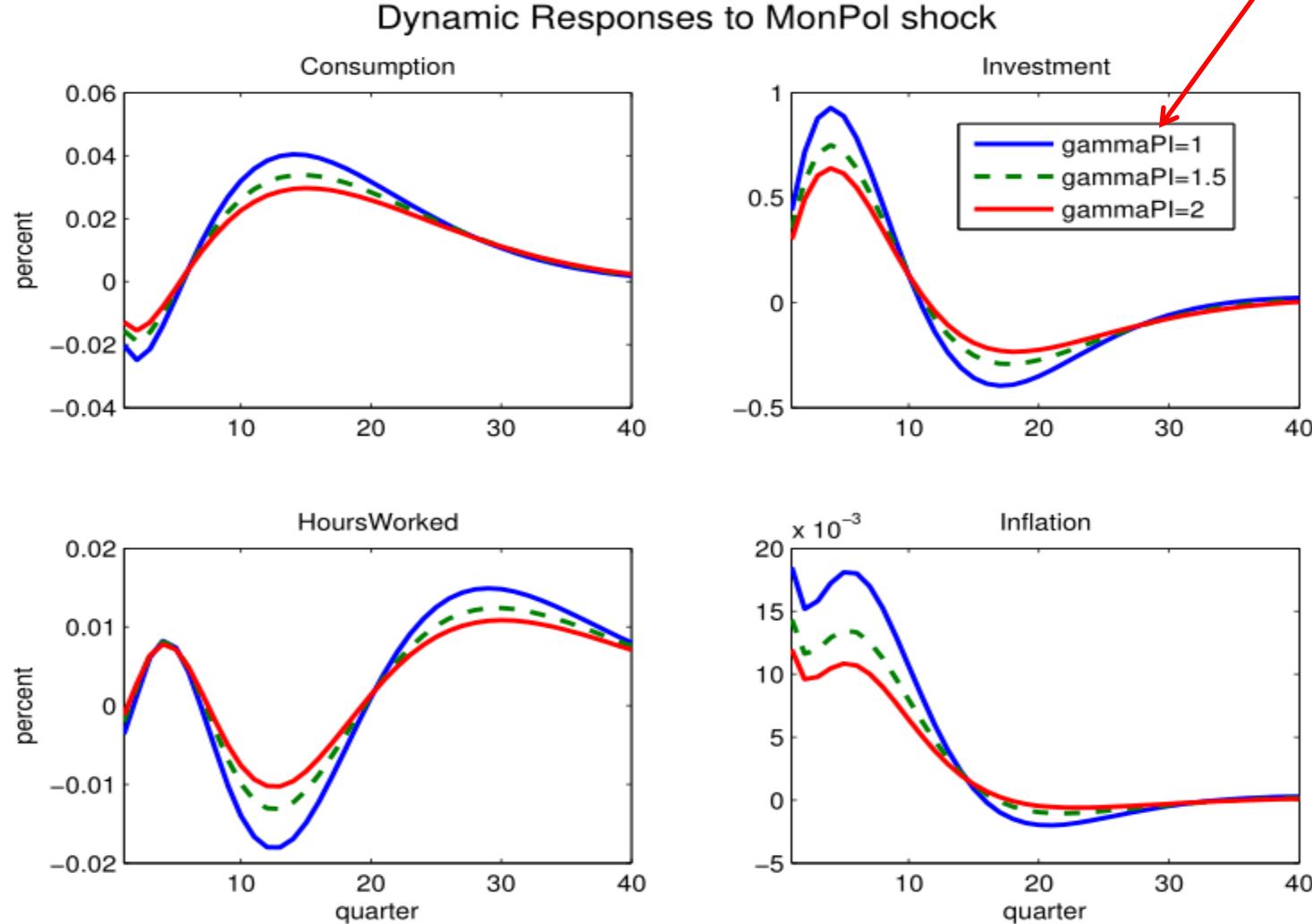
- Comparing effects of different Taylor Rule sensitivities to *output gap*

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Dynamic Responses to MonPol shock



- Comparing effects of different Taylor Rule sensitivities to *inflation rate*

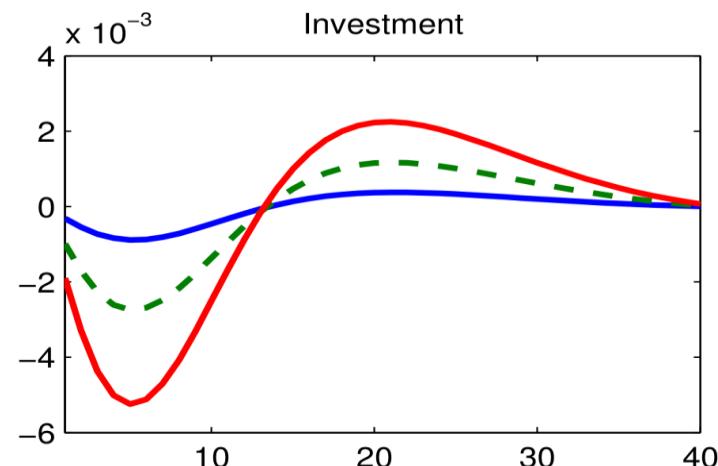
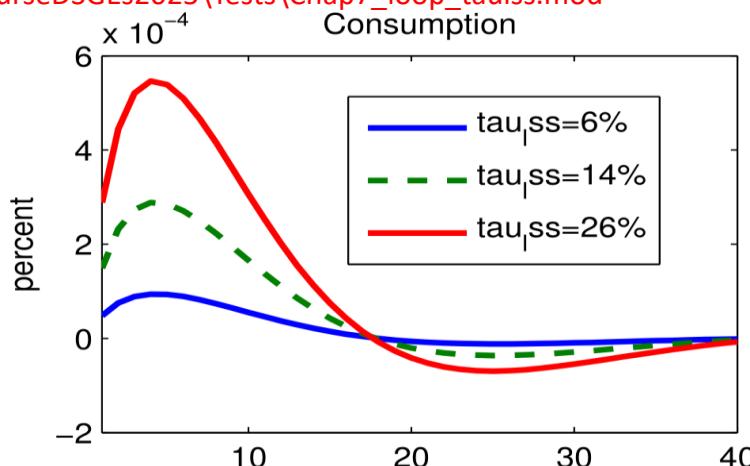
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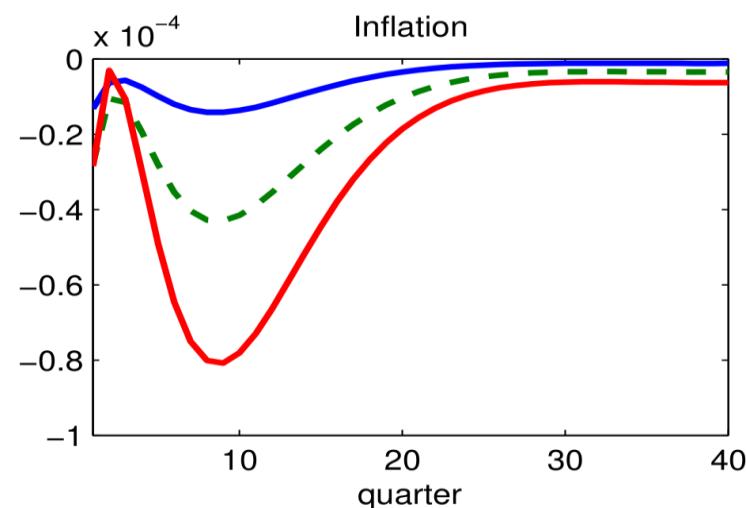
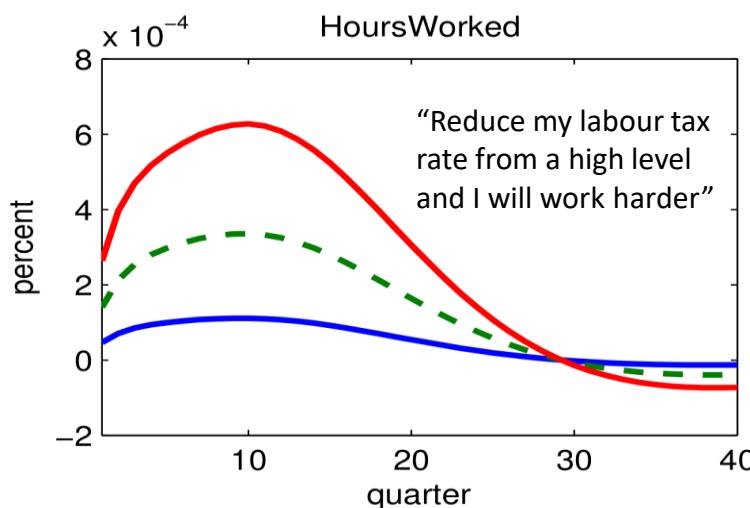
• Comparing effects of different tax rates - *Labour*

Dynamic Responses to Labour Tax Rate

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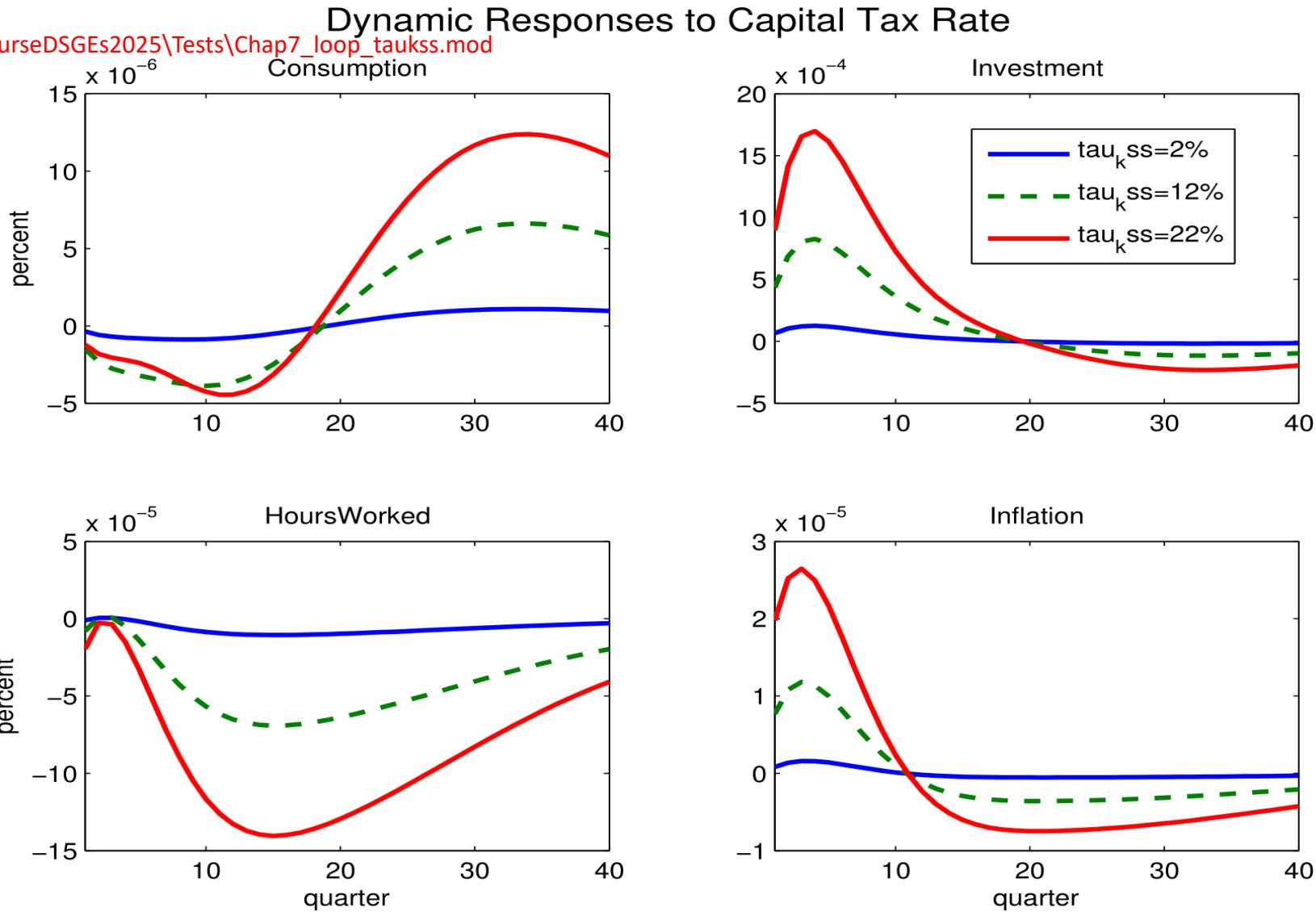


Remember, these shocks are negative!!



- Comparing effects of different tax rates - *Capital*

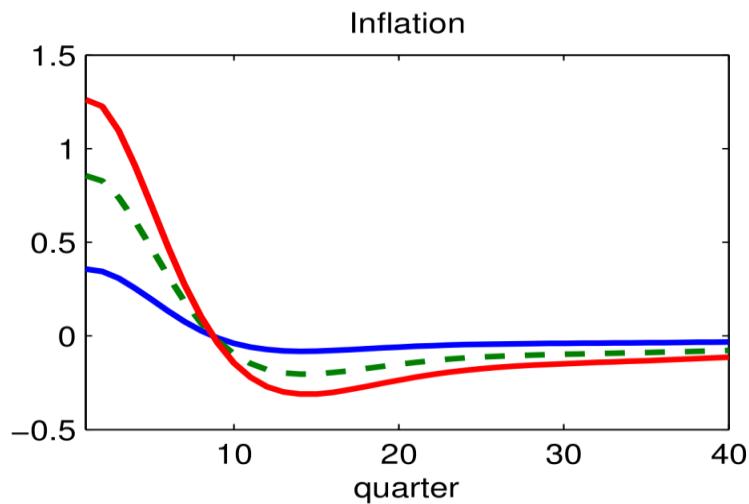
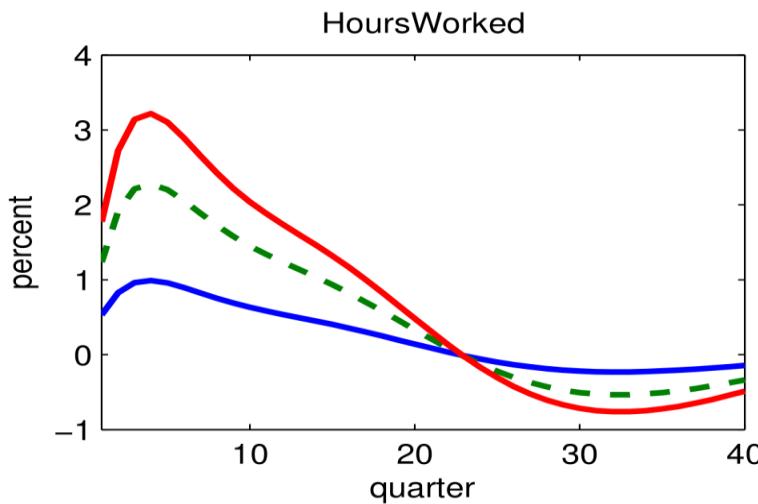
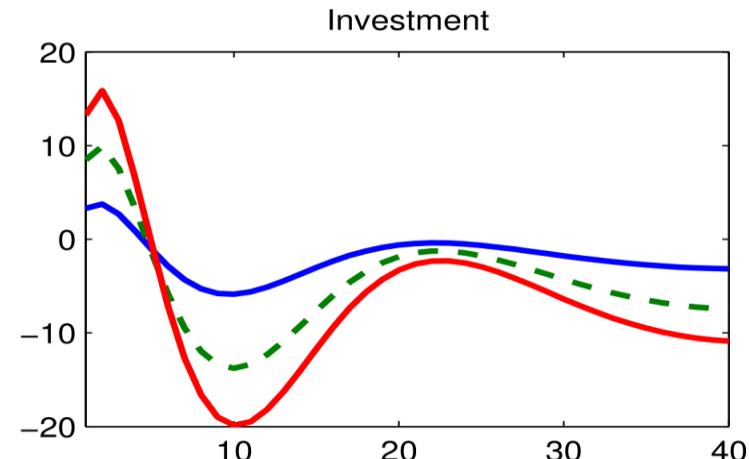
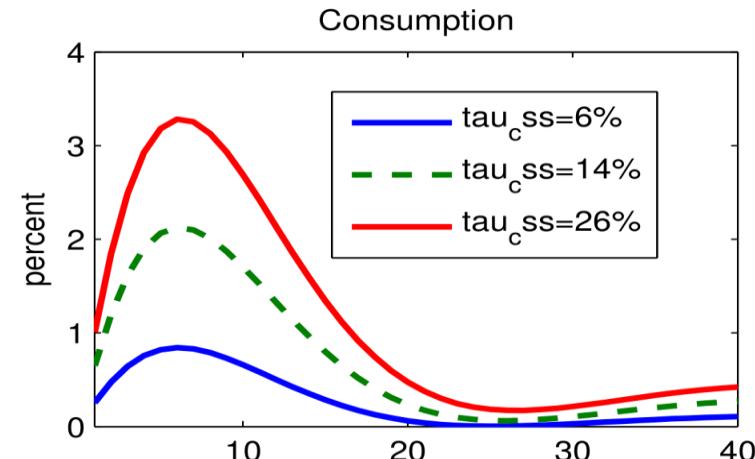
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- Comparing effects of different tax rates - ***Consumption***

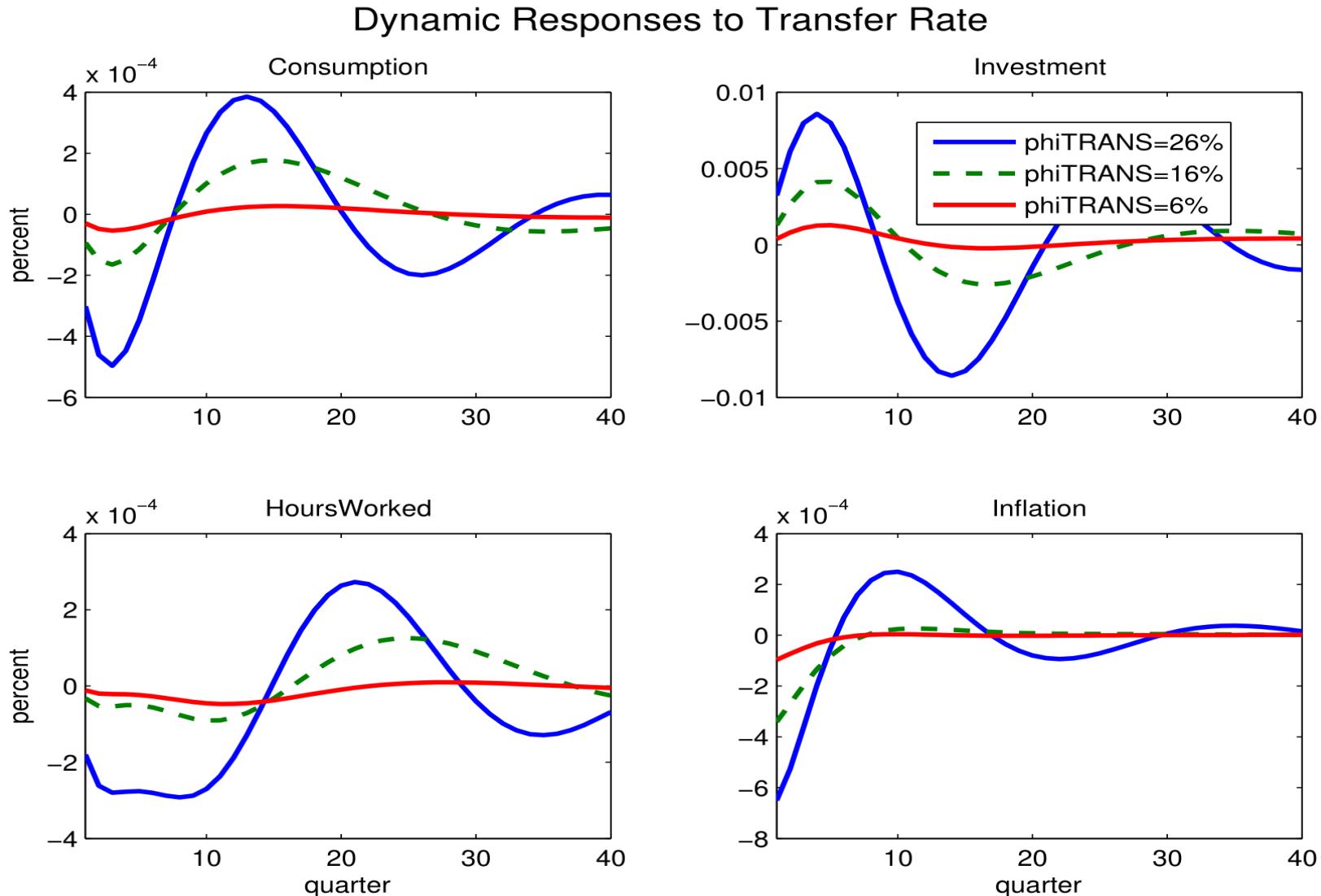
Dynamic Responses to Consumption Tax Rate

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- Comparing effects of different *Transfer* rates

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Appendix 2: Bounded Rationality

- In a very important new paper, economists from the central banks of Canada and the Netherlands introduce methodology for dealing with the awkwardness of “rational expectations” (RE):
- Hommes *et al* (2023), “Behavioral learning equilibria in New Keynesian models”, *Quantitative Economics*, 14, 1401–1445.
- The RE paradigm [which we have been using] is a model-consistent approach where, by construction, agents’ *expectations* are *on average* confirmed by the *realisations* of the model

- Agents in RE models are *assumed* to know a large number of state variables, shocks, and parameters to form their expectations
- But in medium- and large-scale DSGE models, such assumptions lead to *information sets* supposed to be used by agents which are *implausibly large*
- Moreover, to *match* the *persistence* of macroeconomic variables, RE models typically need to be augmented by highly *persistent exogenous shocks* or other sources of persistence, such as consumption *habits* [which we have used] and *indexation* in prices and wages [ditto]

- Hommes *et al* therefore introduce a form of “***bounded rationality***” in which agents are ***not*** perfectly rational (in the above sense) in their expectations, but instead base them on more ***limited*** information sets
- There are many ways to construct such “boundedly rational” expectations
- Among the simplest (and therefore most appealing for everyday use by real human beings) ways is to use a ***parsimonious*** forecasting scheme, such as simple AR(1), AR(2) or VAR(1) models

- Hommes *et al* use what they call a “Behavioral Learning Equilibrium (BLE)” approach
- Along a BLE, agents ***forecast*** the states of the economy by simple, but optimal, univariate AR(1) rules
- The AR(1) rules are optimal in the sense that it is ***assumed*** that the ***mean*** and the ***first-order autocorrelation*** of all forecasts coincide with the ***actual*** mean and the first-order autocorrelation of the ***realisations***

- In the linearized DSGE framework, RE and BLE are both linear equilibrium models but they satisfy different fixed-point conditions:
- In RE, agents are *assumed* to have *perfect* structural knowledge of the model
- By contrast, in BLE, agents do *not* know the *cross-correlations* among the variables and do *not* observe the *shocks*
- but instead use parsimonious univariate AR(1) rules and are *assumed* to know the *correct mean* and *first-order autocorrelation* coefficients

- Hommes *et al* show that in the *Smets-Wouters* model which we study next week, most structural parameters have similar values when estimated under RE or BLE assumptions
- But a few have quite *different* values, and these are sufficient to make the model behave much better in terms of *persistence-tracking*
- In addition, *model fit* (as measured by the log-likelihood at the mode) is *much better* for the BLE model
- Further, when they conduct a pseudo out-of-sample *forecasting* exercise, the BLE model beats the RE model for virtually all variables up to 8 quarters ahead

- What of the *learning* aspect in BLE?
- Under BLE, agents *learn* the two parameters of each AR(1) rule consistent with the *observable sample* averages and first-order autocorrelations of the model's state variables
- This is a *one-time event* given the data set
- However, it is possible to go beyond this:
- A natural *learning process* of a BLE model would be for agents to use a univariate AR(1) rule for every variable and *update* their beliefs about the mean and persistence in every period as *new* observations become available

- This process is known as “*sample autocorrelation (SAC) learning*”
- Hommes *et al* show that when *SAC learning* is used for estimating the parameters of the Smet-Wouters model, the results are *even better* than for BLE, in terms both of *model fit* and *forecasting*, while changing only little the estimated parameters
- On the other hand, replacing RE with simple AR(1) beliefs (RE vs. BLE) improves the model fit *more* than introducing *time variation* in AR(1) beliefs (BLE vs. SAC)