EXERCISE

1) Find $\hat{S}_t$ when you have a premium.

shadow exchange that would prevail if peg is abandoned and start floating

$$S_t = \frac{\alpha}{\beta^2}\mu + \frac{1}{\beta}D_t \quad \text{when} \quad i_t = i_t^* + \frac{E_t S_{t+1} - S_t}{S_t}$$

INTERESTRATE ON DOMESTICALLY DENOMINATED BOND = INTERESTRATE ON FOREIGN DOMINATED BONDS CORRECTED BY EXCHANGE RATE

THIS MUST HOLD, SO IF ANY OF THE COMPONENTS ON THE RHS CHANGES (NO $\bar{S}$), i_t HAS TO CHANGE. HOW? BY CHANGING DEMAND/SUPPLY OF MONEY $\Rightarrow$ MONEY SUPPLY BECOMES ENDOGENOUS (= NO INDEPENDENCE)

ex. Increase in demand of foreign assets puts pressure on exchange rate S_t. why? S_t = Rate that equates Demand/supply of a currency vs another currency $\to$ French people have to buy UK goods in \$ and UK people have to buy Fr goods in €. So in Fr an increase in X would increase demand for \$ $\Rightarrow$ The price of it $(S_t)\uparrow$

But here we have $i_t = i_t^* + \frac{E_t S_{t+1} - S_t}{S_t} + q_t$

where $q_t = \bar{q} + \gamma \frac{E_t S_{t+1} - S_t}{S_t}$: Exchange rate risk premium.

Equating $\underbrace{M_t = M_t^d}_{\text{one moment before the attack e'ne in eq}}$ and knowing $S_t = P_t$ [sl. 24]

* There is one single good in the economy whose domestic currency price is P_t and foreign currency price is 1. PPP $P_t = S_t P_t^*$

! WHEN CB RUNS OUT OF RF, IT IS FORCED TO DEVALUE

Equ at the moment of the attack

$$D_t = \hat{S}_t\left(y_t - \alpha\left(i^* + \frac{E_t S_{t+1} - S_t}{S_t} + \bar{q} + \gamma \frac{E_t S_{t+1} - S_t}{S_t}\right)\right)$$

$$D_t = \hat{S}_t(Y_t - \alpha i^*) - \hat{S}_t\left(\alpha\left(\frac{E_t S_{t+1} - S_t}{S_t}\right) + \alpha\bar{q} + \alpha\gamma\left(\frac{E_t S_{t+1} - S_t}{S_t}\right)\right)$$

At the moment of the attack: $\hat{S}_t = S_t$:

$$D_t = \hat{S}_t(Y_t - \alpha i^*) - \alpha(E_t\hat{S}_{t+1} - \hat{S}_t) - \alpha\bar{q}\hat{S}_t - \alpha\gamma(E_t\hat{S}_{t+1} - \hat{S}_t)$$

$$\overset{!}{=} \hat{S}_t(y_t - \alpha i^*) - \alpha\left[(1+\gamma)(E_t\hat{S}_{t+1} - \hat{S}_t) + \bar{q}\hat{S}_t\right]$$

$$\overset{!}{=} \hat{S}_t(y_t - \alpha i^*) - \alpha(1+\gamma)(E_t\hat{S}_{t+1}) + \alpha(1+\gamma)\hat{S}_t - \alpha\bar{q}\hat{S}_t$$

$$\overset{!}{=} \hat{S}_t\left[\underbrace{(y_t - \alpha i^*)}_{\beta} + \alpha(1+\gamma) - \alpha\bar{q}\right] - \alpha(1+\gamma)(E_t\hat{S}_{t+1})$$

$$\Rightarrow \hat{S}_t = \frac{D_t}{[\beta + \alpha(1+\gamma) - \alpha\bar{q}]} + \frac{\alpha(1+\gamma)}{(\beta + \alpha(1+\gamma) - \alpha\bar{q})} E_t\hat{S}_{t+1}$$

Guess the solution:

$$\hat{S}_t = \lambda_0 + \lambda D_t$$

$$\lambda_0 + \lambda_1 D_t = \frac{D_t}{[\beta + \alpha(1+\gamma) - \alpha\bar{q}]} + \frac{\alpha(1+\gamma)}{[\beta + \alpha(1+\gamma) - \alpha\bar{q}]} E_t\left[\lambda_0 + \lambda_1 \overbrace{D_{t+1}}^{D_t + \mu}\right]$$

$$\lambda_0 + \lambda_1 D_t = \frac{1}{(\beta+\alpha(1+\gamma)-\alpha\bar{q})} D_t + \frac{\alpha(1+\gamma)}{(\beta+\alpha(1+\gamma)-\alpha\bar{q})} \lambda_1 D_t +$$

$$+ \frac{\alpha(1+\gamma)}{(\beta+\alpha(1+\gamma)-\alpha\bar{q})} (\lambda_0 + \lambda_1 \mu)$$

$$\lambda_1 \cancel{D_t} = \frac{1}{(\beta+\alpha(1+\gamma)-\alpha\bar{q})} \cancel{D_t} + \frac{\alpha(1+\gamma)}{(\beta+\alpha(1+\gamma)-\alpha\bar{q})} \lambda_1 \cancel{D_t}$$

$$\lambda_1 \left(1 - \frac{\alpha(1+\gamma)}{\beta+\alpha(1+\gamma)-\alpha\bar{q}}\right) = \frac{1}{(\beta+\alpha(1+\gamma)-\alpha\bar{q})}$$

$$\Rightarrow \lambda_1 = \frac{1}{\beta-\alpha\bar{q}}$$

$$\lambda_0 = \frac{\alpha(1+\gamma)}{(\beta+\alpha(1+\gamma)-\alpha\bar{q})} \lambda_0 + \frac{\alpha(1+\gamma)}{(\beta+\alpha(1+\gamma)-\alpha\bar{q})} \cdot \frac{1}{(\beta-\alpha\bar{q})} \cdot \mu$$

$$\lambda_0 \left(\frac{\beta+\alpha(1+\gamma)-\alpha\bar{q}-\alpha(1+\gamma)}{\beta+\alpha(1+\gamma)-\alpha\bar{q}}\right) = \frac{\alpha(1+\gamma)}{\beta+\alpha(1+\gamma)-\alpha\bar{q}} \cdot \frac{1}{\beta-\alpha\bar{q}} \cdot \mu$$

$$\Rightarrow \lambda_0 = \frac{\alpha(1+\gamma)\mu}{(\beta-\alpha\bar{q})^2}$$

$$\Rightarrow \hat{S}_t = \frac{\alpha(1+\gamma)\mu}{(\beta-\alpha\bar{q})^2} + \frac{1}{(\beta-\alpha\bar{q})} D_t.$$

b) Derive T when $\bar{S} = \hat{S}_t$ [Time of attack $\bar{S} = \hat{S}_t$ (or S_T)]

$$\bar{S} = \frac{\alpha(1+\gamma)\mu}{(\beta-\alpha\bar{q})^2} + \frac{1}{(\beta-\alpha\bar{q})} D_T$$

Noting that $D_T = D_0 + \mu T$ [sl. 25] and knowing that when the peg is maintained: $i_t = i^* + \bar{q}$, we also know that: $\underbrace{R_0 + D_0}_{M^s} = \bar{S}(y - \alpha i^* - \alpha q)$

b/c $M^s = M^d$

$\hookrightarrow S_t(Y - \alpha i_t)$

$$\Rightarrow \bar{S} = \frac{(R_0 + D_0)}{\beta - \alpha\bar{q}}.$$

and $\beta = y - \alpha i^*$

equate the two:

$$\frac{R_0+D_0}{(\beta-\alpha\bar{q})} = \frac{\alpha(1+\gamma)\mu}{(\beta-\alpha\bar{q})^2} + \frac{1}{(\beta-\alpha\bar{q})}(D_0+\mu T)$$

$$R_0 = \frac{\alpha(1+\gamma)\mu}{(\beta-\alpha\bar{q})} + \mu T \Rightarrow T = \frac{R_0}{\mu} - \frac{\alpha(1+\gamma)\mu}{(\beta-\alpha\bar{q})}$$

c) w/o risk premium: VS w/ risk premium:

$$T = \frac{R_0}{\mu} - \frac{\alpha}{\beta} \qquad T' = \frac{R_0}{\mu} - \frac{\alpha(1+\gamma)\mu}{(\beta-\alpha\bar{q})}$$

when is $T' > T$?

$$-\frac{\alpha(1+\gamma)}{(\beta-\alpha\bar{q})} > -\frac{\alpha}{\beta} \rightarrow \frac{\alpha}{\beta} > \frac{\alpha(1+\gamma)}{(\beta-\alpha\bar{q})} \Rightarrow \beta\gamma+\alpha\bar{q}<0$$

which cannot be true, so the attack happens earlier.

d) $\gamma = 0$ and $\hat{S}_t < \bar{S}$:

If $q_t \uparrow$ b/c $\bar{q}\uparrow$, then suddenly $\hat{S}_t \uparrow$ (see expression above), precipitating the crisis. Now in fact, the foreign reserves would be acquired at a higher price.

EXERCISE 3 : SGM

$$L = \underbrace{(S^* - S_t)^2}_{\text{cost of misalignment}} + \overbrace{3(\underbrace{E_t\,\Delta S_{t+1}}_{\text{Exp. deval. by investors}})^2}^{\text{cost of devaluation}} + C$$

[$S_t\uparrow$: Devaluation]

! $S_t = S^*$: FUNDAMENTALS ARE PERFECT, IF GAP IS LARGE THEY ARE WRONG.

S_t : exchange rate chosen by CB

S^* : optimal exchange $=3$

$\bar{S} = 1$ (pegged)

$C = 0$ (No political cost if peg is maintained)

$E_t\,\Delta S_{t+1} = 0$: Maintaining exch. rate (always $\bar{S}$ at t and $t+1$)

$E_t\,\Delta S_{t+1} = S^* - \bar{S}$: Devaluation and exchange rate set equal to optimal are reflecting the fundamentals.

a) ? c/ maintaining is an equ

$L_{F|F} = (S^* - \bar{S})^2 = (3-1)^2 = 4 \qquad L_{D|F} = C$

Fixed exchange rate is an equ iff:

$L_{F|F} < L_{D|F} \Rightarrow (S^* - \bar{S})^2 < C \Rightarrow C > 4$

valuating is an equ:

$L_{F|D} = (s^* - \bar{s})^2 + \beta (s^* - \bar{s})^2 = 4(s^* - \bar{s})^2 = 4(3-1)^2 = 16$

$L_{D|D} = C$

$L_{D|D} < L_{F|D} \Rightarrow C < 16$

$C = 0.5(N-n)^2$ N=7: initial nr of countries joining fixed regime
n: nr of countries abandoning

Countries with the weakest fundamentals devaluate first. Then the crisis propagates to the others since the political cost decreases with countries devaluating.

$n = 2 \Rightarrow C = 0.5(7-2)^2 = 12.5$: Both equilibria possible

$n = 3 \Rightarrow C = 0.5(7-3)^2 = 8$: Both

$n = 4 \Rightarrow C = 0.5(7-4^2) = 4.5$: Both

$n = 5 \Rightarrow$ Devaluation is the only equ. In fact, suppose $n=1 \Rightarrow C = 18$ and fixed exchange is the only equ. $C_5 = 2$ and devaluation is the only equ.

Expectations play a crucial role when there are multiple equilibria:

$\alpha (s^* - \bar{s})^2 < C < (\alpha + \beta)(s^* - s)^2 \Rightarrow \boxed{4 < C < 16}$

c) If $(s^* - \bar{s})$ is large, then a devaluation is more likely to happen and it is going to happen faster. In fact, raising the i to maintain the fixed exchange rate becomes too costly. Devaluation expectations are self-fulfilling.

EXERCISE : SGM

$$T = (1+\rho) R s D(1-x) + R(1-s) D(1-x) - \hat{D}$$ ← gov't surplus

$\hat{D}$: level of debt to stabilize

R = Return on the debt

ρ = default premium

s = fraction of debt to be rolled over

x = (haircut). Fraction of debt not repaid: $x = 0$: gov't repays; $x = \bar{x} < 1$: default

Loss function: $L = \alpha T^2 + c$

a) Possible equilibria:

$$L = \alpha[R(1+\rho)sD(1-x) + R(1-s)D(1-x)]^2 + c$$

•) Gov't doesn't expect default and neither does the mkt: $E_x = \rho = c = 0$

$$L_1 = \alpha[RsD + R(1-s)D]^2 = \alpha(RD)^2$$

•) Gov't defaults $[x = \bar{x}]$ but $\rho = 0$ b/c mkt doesn't expect this:

$$L_2 = \alpha[RsD(1-\bar{x}) + R(1-s)D(1-\bar{x})]^2 + c$$
$$\stackrel{!}{=} \alpha[RD(1-\bar{x})]^2 + c$$

=) Gov't doesn't default iff $L_1 < L_2$:

$$\alpha(RD)^2 < \alpha[RD(1-\bar{x})]^2 + c \;\Rightarrow\; \alpha(RD)^2[1-(1-\bar{x})^2] < c$$

•) mkt expects default $(1+\rho) = (1-\bar{x})^{-1}$ and gov't repays $x = 0$

$$L_3 = \alpha[RsD(1-\bar{x})^{-1} + R(1-s)D]^2$$
$$\stackrel{!}{=} \alpha[RD(s(1-\bar{x})^{-1} + (1-s))]^2$$
$$\stackrel{!}{=} \alpha\left[RD\left(\frac{1-\bar{x}+s\bar{x}}{1-\bar{x}}\right)\right]^2$$

•) mkt expects default and gov't defaults $x = \bar{x}$:

$$L_4 = \alpha[R(1+\rho)sD(1-\bar{x}) + R(1-s)D(1-\bar{x})]^2 + c$$
$$\stackrel{!}{=} \alpha[RD(s + (1-s)(1-\bar{x}))]^2 + c$$
$$\stackrel{!}{=} \alpha[RD(1-\bar{x}+s\bar{x})]^2 + c$$

=) Default is an equ under this condition:

$$L_4 < L_3$$

$$\alpha[RD(1-\bar{x}+s\bar{x})]^2 + c < \alpha\left[RD\left(\frac{(1-\bar{x}+s\bar{x})}{(1-\bar{x})}\right)\right]^2$$

$$c < \alpha(RD)^2[1-\bar{x}+s\bar{x}]\frac{[1-\bar{x}+s\bar{x}]^2}{(1-\bar{x})^2}$$

Combining the two, we obtain the range of multiple equilibria:

$$\alpha(RD)^2\left[1-(1-\bar{x})^2\right] < c < \alpha(RD)^2\left[1-\bar{x}+s\bar{x}\right]\frac{\left[1-\bar{x}+s\bar{x}\right]^2}{(1-\bar{x})^2} \quad [1]$$

- Low D => Good fundamentals. Eliminate default equ
- High D => Contrary: bad fundamentals.
- Intermediate D => multiple equilibria are possible if (s) is large enough. This is because a self-fulfilling equ can be driven by expectations of default ($\rho>0$), only if the share of the debt to roll over is high enough that the higher int. rate has an impact on gov't loss and decision to default

Structure of the debt matters

(s) is small => Expectations have no role in driving up the cost of rolling over the debt

c) Availability of credit line:

$$T = R(1+\rho)sD(1-x) + R(1-s)D(1-x) - \hat{D} - L, \quad \hat{D}=0$$

New loss function:

$$L = \alpha\left[R(1+\rho)sD(1-x) + R(1-s)D(1-x) - L\right]^2 + c$$

If expectations of default are positive: $(1+\rho)=(1-\bar{x})^{-1}$ and gov't repays the debt:

$$L_5 = \alpha\left[RD\frac{1-\bar{x}+s\bar{x}}{1-\bar{x}} - L\right]^2$$

" " and gov't does default:

$$L_6 = \alpha\left[RDs + RD(1-s)(1-\bar{x}) - L\right]^2 + c$$

To rule out self-fulfilling equ when there is possibility of a loan from other countries:

$$L_5 < L_6$$

$$\alpha\left[RD\frac{1-\bar{x}+s\bar{x}}{1-\bar{x}} - L\right]^2 < \alpha\left[RD(1-\bar{x}+s\bar{x}) - L\right]^2 + c$$

=> L has to be generous enough to avoid default b/c LHS decreases at a faster rate w/ L than RHS.

The condition above is satisfied when

[2]

$$L \Rightarrow \frac{1}{2}\left[RD(1-\bar{x}+s\bar{x})\frac{(2-\bar{x})}{(1-\bar{x})} - \frac{c}{\alpha}\frac{(1-\bar{x})}{RD(1-\bar{x}+s\bar{x})\bar{x}}\right]$$

which is a decreasing function of the short-term debt.

Comparing [1] and [2] by re-writing the latter:

$$\alpha (RD)^2\left[1-(1-\bar{x})^2\right]\frac{\left[1-\bar{x}+s\bar{x}\right]^2}{(1-\bar{x})^2} - 2 < \frac{\alpha RD(1-\bar{x}+s\bar{x})\bar{x}}{(1-\bar{x})} \leq c$$

$\Rightarrow$ There exists a ___ such that both are satisfied.

EXERCISE

$$L = 2(S^* - S_t)^2 + (E_t \Delta S_{t+1}) + c$$

maintain $S_t = \bar{S}$
devalue $S_t = S^*$

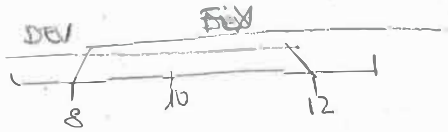
a) $S^* = 3$, $\bar{S} = 1$, $c = 10$

Both equilibria are possible (devaluation + fixed exchange) since $c=10$

For $c < 2(S^*-\bar{S})^2 + (S^*-\bar{S}) = 12$: CB devalues.

$c > 2(S^*-\bar{S})^2 = 8$: CB fixed exchange

$\Rightarrow 8 < c < 12$: For intermediate values, multiple equilibria are possible and therefore expectations play a role. w/ $c=10$, if they expect a devaluation the equ is going to be a devaluation and the same for the fixed exchange rate.



b) $S^* = 3$, $\bar{S} = 2$, $c = 10$

Fixed exchange : $2(S^*-\bar{S})^2 < c \rightarrow 2(1)^2 < 10$: ✓

Devaluation : $c < 3(S^*-\bar{S})^2 \rightarrow c < 3 \rightarrow 10 < 3$: ✗

c) Whether the good equ or a crisis realizes, depends on investors' expecta[tions] when these are on CB devaluating, they are self-fulfilling since $E_t S_{t+1}$ raises c making the peg more costly.

$\bar{S} \leq 2$: D

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b/c

, whereas for

EXERCISE 2 - The Bubble

$$S_t^+ = \underbrace{\frac{\alpha}{\beta^2}\mu + \frac{1}{\beta}D_t}_{\text{Fundamental}} + \underbrace{A\left(1 + \frac{\beta}{\alpha}\right)^{t-T}}_{\text{Bubble}}$$

$$\bar{S} = \frac{\alpha}{\beta^2}\mu + \frac{1}{\beta}\underbrace{(D_0 + \mu T)}_{D_T} + A\left(1 + \beta/\alpha\right)^{T-T} \quad \text{as } t = T.$$

At the time of the attack we have demand = supply : $\bar{S}\beta = R_0 + D_0$

$$\frac{R_0 + D_0}{\beta} = \frac{\alpha}{\beta^2}\mu + \frac{1}{\beta}(D_0 + \mu T) + A \rightarrow R_0 + \cancel{D_0} = \frac{\alpha}{\beta}\mu + \cancel{D_0} + \mu T + \beta A$$

$$\rightarrow T = \frac{R_0}{\mu} + \frac{\alpha}{\beta} - \frac{\beta}{\mu}A$$

$$\rightarrow R_T^F = \alpha(F_T S_{T+1} - \underset{S_T}{\bar{S}}) = \alpha\left(\frac{\alpha}{\beta^2}\mu + \frac{1}{\beta}D_{T+1} + A\left(1 + \beta/\alpha\right) - \frac{\alpha}{\beta^2}\mu + \frac{1}{\beta}D_T - A\right)$$

$$= \alpha\left(\frac{\mu}{\beta} + \frac{\beta}{\alpha} \cdot A\right) = \frac{\alpha\mu}{\beta} + \beta A$$

PS 6

Question 1

b: ~~and~~ before the attack T-

$$R^f_{T-} + D_{T-} = \bar{S}(Y - \alpha(i^* + \bar{q}))$$

At the time of the attack (T):

$$\cancel{D_T} = \bar{S}\left(Y - \alpha\left(i^* + \frac{E_T S_{T+1} - \bar{S}}{\bar{S}} + \bar{q} + \gamma \frac{E_T S_{T+1} - \bar{S}}{\bar{S}}\right)\right) =$$

$$\underbrace{\bar{S}(Y - \alpha(i^* + \bar{q}))}_{\cancel{D_{T-}} + R^f_{T-}} - \alpha(1+\gamma)(E_T S_{T+1} - \bar{S})$$

$$\Longrightarrow R^f_{T-} = \alpha(1+\gamma)(E_T S_{T+1} - \bar{S})$$

based on what we found in part a:

$$R^f_T = \alpha(1+\gamma)\left[\frac{\alpha(1+\cancel{\gamma})\mu}{(\cancel{\beta} - \alpha\bar{q})^2} + \frac{1}{(\beta - \alpha\bar{q})} \underbrace{D_{T+1}}^{D_T + \mu} - \frac{\alpha(\cancel{1}+\gamma)\mu}{(\cancel{\beta} - \alpha\bar{q})^2} - \frac{1}{(\beta - \alpha\bar{q})} D_T\right] =$$

$$\frac{\alpha(1+\gamma)}{\beta - \alpha\bar{q}} \cdot \mu \rightarrow$$ the amount of remaining reserves that are taken away at time T

without risk premium : $R^f_T = \frac{\alpha\mu}{\beta}$ } calculated in the lecture slides

$$\frac{\alpha(1+\gamma)}{\beta - \alpha\bar{q}} \cdot \mu > \frac{\cancel{\alpha}\mu}{\beta} \Longrightarrow \underbrace{\frac{\overbrace{1+\gamma}^{\text{positive}}}{\underbrace{\beta - \alpha\bar{q}}_{\text{positive}}} > \frac{1}{\beta}} \checkmark$$