Micro II
Problem Set 3
SOLUTIONS

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A simple model of probabilistic voting and ideological bias

There is a mass 1 of voters. Assume that three factors affect voter i's voting strategy: (1) the economic policy implemented q, (2) her individual ideological bias σ^i toward candidate B, and (3) the popularity δ of politician B. We assume that σ^i is uniformly distributed on $[-\frac{1}{2\phi}, \frac{1}{2\phi}]$. Moreover, δ is the same for all voters and is drawn from the uniform distribution on $[-\frac{1}{2\psi}, \frac{1}{2\psi}]$. The distributions are common knowledge, but only agent i observes her own parameter σ^i. Then, i's preferences over the policy implemented by A are summarized by $W(q^A; \alpha^i)$, whereas the preferences over the policy implemented by politician B take the final form

$$W(q^B; \alpha^i) + \sigma^i + \delta \tag{1}$$

The timing is as follows: first, each voter observes σ^i, and politicians simultaneously and noncooperatively announce platforms q^A and q^B. Second, δ is realized. Third, elections take place, and last, the announced policy is implemented.

a) Give an interpretation of σ^i. Characterize the agent who is indifferent between voting for politician A and voting for politician B for given policies q^A and q^B. Suppose that $\alpha^i = \alpha$. Compute candidate A's vote share as well as her probability of winning.

Interpretation of σ^i: voter i's ideological bias for candidate B, i.e. how this voter i personally likes the politician, which comes from some private knowledge, relationship or taste (e.g. religion, looks, personal experiences). Indifferent agent ("swing voter", denoted by a tilde):

$$W(q^A, \alpha^i) = W(q^B, \alpha^i) + \delta + \tilde{\sigma}^i$$

With homogenous agents, $\alpha^i = \alpha$, $\tilde{\sigma}^i = \tilde{\sigma}$:

$$\tilde{\sigma} = W(q^A, \alpha) - W(q^B, \alpha) - \delta$$

For certain δ, the vote share for candidate A = percentage of the voters who prefer candidate A[1]*:*

$$\begin{aligned}
\pi_A(\delta) &= Prob(\sigma \leq \tilde{\sigma}) \\
&= F_\sigma(\tilde{\sigma}) \\
&= \int_{-\frac{1}{2\phi}}^{\tilde{\sigma}} f_\sigma(\sigma)\, d\sigma \\
&= \phi\sigma\Big|_{-\frac{1}{2\phi}}^{\tilde{\sigma}} \\
&= \phi\left(\tilde{\sigma} - \left(-\frac{1}{2\phi}\right)\right) \\
&= \tilde{\sigma}\phi + \frac{1}{2} \\
&= \phi\left[W(q^A, \alpha) - W(q^B, \alpha) - \delta\right] + \frac{1}{2}
\end{aligned}$$

[1]The density of a uniformly distributed variable is found as follows: $f(\sigma) = \frac{1}{b-a}$ where a is the lower bound and b is the upper bound of the interval on which the variable is distributed. Therefore in our case $f(\sigma) = \frac{1}{\frac{1}{2\phi} - (-\frac{1}{2\phi})} = \phi$. The distribution function can be found by integrating up to the desired number: $F(\sigma \leq \tilde{\sigma}) = \int_a^{\tilde{\sigma}} \phi d\sigma = \Big|\sigma\phi\Big|_{-\frac{1}{2\phi}}^{\tilde{\sigma}} = \phi\tilde{\sigma} + \frac{1}{2}$

The probability of winning of candidate A:

$$\begin{aligned}
p_A &= Prob(\pi_A(\delta) \geq \frac{1}{2}) = Prob(\tilde{\sigma}\phi \geq 0) = Prob\big((W(q^A,\alpha) - W(q^B,\alpha) - \delta)\phi \geq 0\big) \\
&= Prob\big(\delta \leq W(q^A,\alpha) - W(q^B,\alpha)\big) = \int_{-\frac{1}{2\psi}}^{(W^A - W^B)} \psi d\delta = \psi\delta\Big|_{-\frac{1}{2\psi}}^{W^A - W^B} \\
&= \psi[W(q^A,\alpha) - W((q^B,\alpha)] + \frac{1}{2}
\end{aligned}$$

b) Which platforms do the politicians select? Which one is implemented? Discuss.

Homogenous agents with the same α: Politician A chooses policy q^A to maximize the probability of winning, given B's policy q^B

$$\begin{aligned}
& \max_{q^A} p_A \\
= \quad & \max_{q^A} \psi[W(q^A,\alpha) - W((q^B,\alpha)] + \frac{1}{2} \\
\Longleftrightarrow \quad & \max_{q^A} W\left(q^A,\alpha\right)
\end{aligned}$$

The analogous holds true for politician B: B wants to maximize $p_B = Prob(\pi_B(\delta) \geq \frac{1}{2}) = Prob(\pi_A(\delta) < \frac{1}{2}) = 1 - Prob(\pi_A(\delta) \geq \frac{1}{2}) = 1 - p_A$. So her problem is

$$\begin{aligned}
& \max_{q^B}(1 - p_A) \\
= \quad & \max_{q^B}\{1 - \psi[W(q^A,\alpha) - W((q^B,\alpha)] - \frac{1}{2}\} \\
\Longleftrightarrow \quad & \max_{q^B} W\left(q^B,\alpha\right)
\end{aligned}$$

Hence, $q^A = q^B = q^*$ (Utilitarian social optimum). The same as under Downsian Competition (because $\alpha^M = E(\alpha) = \alpha$).

c) Suppose that agents are heterogeneous. What does this imply for the equilibrium?

Heterogeneous agents with different α^i: For a voter i with taste α^i for public goods, the probability for her to vote for politician A, or say, the probability for politician A to gain voter i's vote is:

$$\Pr\left[\sigma^i < \tilde{\sigma}\left(\alpha^i\right)\right] = \tilde{\sigma}\left(\alpha^i\right)\phi + \frac{1}{2}$$

For certain δ, the vote share for candidate A is:

$$\begin{aligned}
\pi_A(\delta) &= \int_{\alpha^i} \Pr[\sigma^i < \tilde{\sigma}\left(\alpha^i\right)]dF(\alpha^i) \\
&= \int_{\alpha^i} [\tilde{\sigma}\left(\alpha^i\right)\phi + \frac{1}{2}]dF(\alpha^i) \qquad (2)\\
&= \int_{\alpha^i} [\tilde{\sigma}\left(\alpha^i\right)\phi]dF(\alpha^i) + \frac{1}{2} \qquad (3)\\
&= \int_{\alpha^i} [\left(W(q^A, \alpha^i) - W(q^B, \alpha^i) - \delta\right)\phi]dF(\alpha^i) + \frac{1}{2} \\
&= \phi \int_{\alpha^i} [W(q^A, \alpha^i) - W(q^B, \alpha^i)]dF(\alpha^i) - \phi\delta + \frac{1}{2} \qquad (4)
\end{aligned}$$

The probability of winning of candidate A:

$$\begin{aligned}
p_A &= Prob(\pi_A(\delta) \geq \frac{1}{2}) = Prob(\int_{\alpha^i} [\tilde{\sigma}\left(\alpha^i\right)\phi]dF(\alpha^i) \geq 0) \qquad (5)\\
&= Prob\big((\phi \int_{\alpha^i} [W(q^A, \alpha^i) - W(q^B, \alpha^i)]dF(\alpha^i) - \phi\delta \geq 0\big) \\
&= Prob\big(\delta \leq \int_{\alpha^i} [W(q^A, \alpha^i) - W(q^B, \alpha^i)]dF(\alpha^i)\big)
\end{aligned}$$

The candidate A chooses q^A to maximize the probability of winning p_A

$$
\begin{aligned}
& \max_{q_A} p_A \\
= \quad & \max_{q_A} \int_{\alpha^i} [W(q^A, \alpha^i) - W(q^B, \alpha^i)] dF(\alpha^i) \\
\iff \quad & \max_{q_A} \int_{\alpha^i} W(q^A, \alpha^i) dF(\alpha^i)
\end{aligned}
$$

which is the same objective function as the Utilitarian social planner. It is analogous for candidate B.

d) Discuss your results and compare them with the results obtained under Downsian competition.

Here the candidate will choose the Utilitarian social optimum. Why? Because each candidate weights voters of different α^i with the same probability distribution as the Utilitarian social planer would do, i.e. the average is decisive. Since the individual ideological bias σ^i is distributed uniformly and identically across i, this doesn't distort the weighting. In contrast, in Downsian competition the Utilitarian social optimum is only obtained when all voters have the same $\alpha^i = \alpha$, while otherwise both parties will converge to the platform preferred by the median voter who has α^M.

e) What alternative assumptions could make the equilibrium platform differ from the social optimum?

For example, the following two extensions could generate this: 1) different Swing voter densities for different groups of voters, 2) assumption that some groups of voters are organized and some are not in a setting where lobbying can influence popularity.

f) What alternative assumption could lead to divergence of the platforms of the two parties?

The assumption that parties are unable to credibly commit to a platform ex ante.

g) How realistic is the probabilistic voting setting? Are there other aspects of real-world politics that are ignored in this framework?

For example, there may be different types of candidates with high or low competence and voters can use past performance to screen out good from bad types (adverse selection). There could also be extensions where incumbents can influence the salience of particular issues and manipulate beliefs on performance to endogenously influence the σ *and* δ *(moral hazard).*