ECON 39: International Trade
Midterm #1 Practice Problems
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Practice Problems

1. True / False / Uncertain. For each of the following statements, tell me if the statement is true, false or uncertain and explain why using words, figures, or both. Note: the explanation is more important than the answer!

   (a) "In a Ricardian trade model, the representative agent of a country is always made better off by trade if the equilibrium relative world price is different than the equilibrium autarkic price."

   False. While this is true for any non-corner solution, it could be the case that a country produces at their PPF corner both before and after trade liberalization, which leaves the representative agent no better off.

   (b) "Iceberg trade costs are so-called because there is a probability that a shipment will fail to arrive because the ship carrying it runs into an iceberg."

   False. Iceberg trade costs are so-called because they assume that a fraction of a good disappears in transit, much like how an iceberg would "melt" as you shipped it from one location to another.

   (c) "In a Ricardian model, if workers in country A have higher wages than workers in country B in autarky, then opening up trade between the two countries will cause the wage of workers in country A to fall."

   - *False. In autarky (under the assumption of incomplete specialization), the wage in each sector will be equalized:*

   $$w_1 = \frac{p_1}{\alpha_1} = \frac{p_2}{\alpha_2} = w_2 = w \iff \frac{p_2}{p_1} = \frac{\alpha_2}{\alpha_1}.$$

   If we normalize the price of good 1 to 1, this implies that the autarkic price of good 2 is $\frac{\alpha_2}{\alpha_1}$ and the autarkic wage is equal to $\frac{1}{\alpha_1}$. Suppose the equilibrium relative world price p^* is greater than $\frac{\alpha_2}{\alpha_1}$. Then we have:

   $$w_1 = \frac{1}{\alpha_1} \text{ and } w_2 = \frac{p^*}{\alpha_2} > \frac{\alpha_2}{\alpha_1} \times \alpha_2 = \frac{1}{\alpha_1}.$$

   Hence, workers would specialize in the production of good 2 and receive a nominal wage greater than their autarkic wage. Conversely, suppose that $p^* \leq \frac{\alpha_2}{\alpha_1}$. Then $w_1 = \frac{1}{\alpha_1}$ and $w_2 = \frac{p^*}{\alpha_2} < \frac{\alpha_2}{\alpha_1} \times \alpha_2 = \frac{1}{\alpha_1}$. In this case, workers would specialize in the production of good 1 and their nominal wage would be $\frac{1}{\alpha_1}$, just as in autarky. In either case, the wage in the country would not fall.

2. Consider a world with two countries and two goods. As in class, let us call the countries U.S. and Mexico and the goods footballs and soccer balls. Suppose that there are 10 workers in the U.S. and 6 workers in Mexico. Each worker in the U.S. *can make* either 10 soccer balls or 15 footballs, whereas each worker in Mexico *can make* either 10 soccer balls or 5 footballs. Let the utility of *each worker* in each country be $U = \left(c^{SB}\right)^{\frac{1}{2}} \left(c^{FB}\right)^{\frac{1}{2}}$, where c^{SB} is the quantity that the worker consumes of soccer balls and c^{FB} is the quantity that the worker consumes of footballs.

   (a) In autarky, what is the equilibrium utility for a U.S. worker?

   From the preferences, we know that each worker wants to spend half of her income on soccer balls and half on footballs. To produce both goods then, the relative price of footballs to soccer balls has to make workers indifferent between the production of both goods. This implies that:

   $$10 \times p^{SB}_{US} = 15 \times p^{FB}_{US} \iff \frac{p^{FB}_{US}}{p^{SB}_{US}} = \frac{2}{3}.$$

   If we normalize the price of soccer balls to one. Notice that this implies the income of a worker in the U.S. is $10 \times p^{SB}_{US} = 1$. With an income of 10, a worker can consume $c^{SB}_{US} = \frac{1}{2}\frac{w_{US}}{p^{SB}_{US}} = 5$

soccer balls and $c_{US}^{FB} = \frac{1}{2}\frac{w_{US}}{p_{US}^{FB}} = \frac{1}{2}\frac{10}{\frac{2}{3}} = 7.5$. *This means that the consumer has utility* $U = \left(c_{US}^{SB}\right)^{\frac{1}{2}}\left(c_{US}^{FB}\right)^{\frac{1}{2}} = (37.5)^{\frac{1}{2}}$.

(b) Define the world equilibrium.

"For any population of workers in the U.S. (L_{US}) *and Mexico* (L_{MEX})*, productivities in the production of footballs and soccer balls* $\left\{\alpha_{US}^{FB}, \alpha_{MEX}^{FB}, \alpha_{US}^{SB}, \alpha_{MEX}^{SB}\right\}$*, and preferences of the representative agent* $U_{MEX}(\cdot)$ *and* $U_{US}(\cdot)$*, equilibrium is defined as the quantity of footballs and soccer balls produced in each country* $\left\{Q_{US}^{FB}, Q_{MEX}^{FB}, Q_{US}^{SB}, Q_{MEX}^{SB}\right\}$*, the quantity of footballs and soccer balls consumed in each country* $\left\{C_{US}^{FB}, C_{MEX}^{FB}, C_{US}^{SB}, C_{MEX}^{SB}\right\}$*, and the relative price of footballs to soccer balls* $\frac{p^{FB}}{p^{SB}}$ *such that: (1) given relative prices, workers in each country maximize their income; (2) given relative prices and the country income, the representative agent maximizes its utility; and (3) for both soccer balls and footballs, the total world production equals the total world consumption."*

(c) Solve the world equilibrium. With free trade, what is the equilibrium utility for a U.S. worker?

The U.S. clearly has a comparative advantage in footballs. I will guess that the U.S. produces both footballs and soccer balls, while Mexico specializes in soccer balls. If Mexico specializes in soccer balls, we know that $Q_{MEX}^{SB} = 6 \times 10 = 60$ *and* $Q_{MEX}^{FB} = 0$. *Since the U.S. produces both footballs and soccer balls, from the first equilibrium condition, it must be that U.S. workers are indifferent between the two sectors so that:*

$$10 \times p^{SB} = 15 \times p^{FB} \iff \frac{p^{FB}}{p^{SB}} = \frac{2}{3},$$

i.e. the world price is equal to the U.S. autarkic price. Normalize the price of soccer balls to one, i.e. $p^{SB} = 1$. *This implies that the income in the U.S. of a worker is:*

$$w_{US} = p^{SB} \times 10 = 10$$

and the total income in Mexico is:

$$w_{MEX} = p^{SB} \times 10 = 10$$

From the second equilibrium condition, the representative agent in both countries wants to splits its income evenly between footballs and soccer balls. This means that:

$$\begin{aligned}
C_{MEX}^{FB} &= \frac{1}{2}\frac{w_{MEX} \times L_{MEX}}{p^{FB}} = \frac{1}{2}\frac{60}{2/3} = 45;\\
C_{MEX}^{SB} &= \frac{1}{2}\frac{w_{MEX} \times L_{MEX}}{p^{SB}} = \frac{60}{2} = 30;\\
C_{US}^{FB} &= \frac{1}{2}\frac{w_{US} \times L_{US}}{p^{FB}} = \frac{1}{2}\frac{100}{2/3} = 75;\\
C_{US}^{SB} &= \frac{1}{2}\frac{w_{US} \times L_{US}}{p^{SB}} = \frac{100}{2} = 50;
\end{aligned}$$

From the third equilibrium condition, total production must equal total consumption in each good. Since Mexico is producing 60 *soccer balls, this means that the U.S. has to make* 20 *soccer balls (i.e.* $Q_{US}^{SB} = 20$) *to sate the world demand for 80 soccer balls (i.e.* $Q_{US}^{FB} = 80$*). Similarly, the U.S. has to produce* 120 *footballs. The total labor requirement for this is:*

$$\frac{20}{10} + \frac{120}{15} = 2 + 8 = 10,$$

which is equal to the number of workers in the U.S., so we are in an equilibrium. Since $c_{US}^{FB} = C_{US}^{SB}/L_{US} = 7.5$ *and* $c_{US}^{SB} = C_{US}^{SB}/L_{US} = 5$*, the welfare of a worker in the U.S. is the same as before:*

$$U = \left(c_{US}^{SB}\right)^{\frac{1}{2}}\left(c_{US}^{FB}\right)^{\frac{1}{2}} = (37.5)^{\frac{1}{2}}$$

(d) Suppose there was a zombie attack in the U.S., from which only 4 workers in the U.S. survived. Calculate the welfare in the post-zombie equilibrium for these workers in U.S. How has the welfare in the U.S. changed? Why did this change occur?

If there are only four workers in the United States, then the above equilibrium cannot hold, since at the previous prices we have:

$$C_{MEX}^{FB} = \frac{1}{2}\frac{w_{MEX} \times L_{MEX}}{p^{FB}} = \frac{1}{2}\frac{60}{2/3} = 45;$$
$$C_{MEX}^{SB} = \frac{1}{2}\frac{w_{MEX} \times L_{MEX}}{p^{SB}} = \frac{60}{2} = 30;$$
$$C_{US}^{FB} = \frac{1}{2}\frac{w_{US} \times L_{US}}{p^{FB}} = \frac{1}{2}\frac{40}{2/3} = 30;$$
$$C_{US}^{SB} = \frac{1}{2}\frac{w_{US} \times L_{US}}{p^{SB}} = \frac{40}{2} = 20;$$

so that there is only a demand for 50 *soccer balls, but Mexico is producing* 60 *soccer balls. Hence we have to find a different equilibrium. Let us suppose that both countries fully specialize. Then* $Q_{US}^{FB} = 4 \times 15 = 60$, $Q_{US}^{SB} = 0$, $Q_{MEX}^{FB} = 0$ *and* $Q_{MEX}^{SB} = 60$. *Again, normalize the price of soccer balls to 1 so that we only have to solve for the price of footballs. The wage of a worker in the U.S. is* $w_{US} = 15 \times p^{FB}$ *and the wage of a worker in Mexico is* $w_{MEX} = 10 \times p^{SB} = 10$. *From the second equilibrium condition we have:*

$$C_{US}^{FB} = \frac{1}{2}\frac{w_{US} \times L_{US}}{p^{FB}} = \frac{1}{2}(15 \times 4) = 30$$
$$C_{US}^{SB} = \frac{1}{2}\frac{w_{US} \times L_{US}}{p^{SB}} = \frac{1}{2}\left(15p^{FB} \times 4\right) = 30p^{FB}$$
$$C_{MEX}^{FB} = \frac{1}{2}\frac{w_{MEX} \times L_{MEX}}{p^{FB}} = \frac{1}{2}\left(\frac{10 \times 6}{p^{FB}}\right) = \frac{30}{p^{FB}}$$
$$C_{MEX}^{SB} = \frac{1}{2}\frac{w_{MEX} \times L_{MEX}}{p^{SB}} = \frac{1}{2}(10 \times 6) = 30$$

From the third equilibrium condition to hold, the total production of footballs has to equal the total consumption of footballs, i.e:

$$60 = 30 + \frac{30}{p^{FB}},$$

so that $p^{FB} = 1$. *It is straightforward to show that* $p^{FB} = 1$ *also ensures that the total production of soccer balls is equal to the total consumption of soccer balls, i.e.:*

$$60 = 30p^{FB} + 30.$$

With $p^{FB} = 1$, *a worker in the U.S. consumes* $\frac{30}{4}$ *footballs and* $\frac{30}{4}$ *soccer balls, so her utility is:*

$$U = \left(\frac{30}{4}\right)^{\frac{1}{2}}\left(\frac{30}{4}\right)^{\frac{1}{2}} = 7.5,$$

which is greater than before. Intuitively, what is happening is that with fewer workers, the world price is now different than the autarkic price, allowing the U.S. to specialize, and extending the CPF past the PPF.

3. Consider a world with two countries and two goods. As in class, let us call the countries U.S. and Mexico and the goods footballs and soccer balls. Suppose that there are 10 workers in the U.S. and 8 workers in Mexico. Each worker in the U.S. *can make* either 1 soccer balls or 2 footballs, whereas each worker in Mexico *can make* either 1 soccer ball or 1 football. Let the utility of *each worker* in each country be $U = \left(c^{SB}\right)^{\frac{1}{2}}\left(c^{FB}\right)^{\frac{1}{2}}$, where c^{SB} is the quantity that the worker consumes of soccer balls and c^{FB} is the quantity that the worker consumes of footballs.

(a) Which country has a comparative advantage in footballs? Which country has a comparative advantage in soccer balls?

- *From the question, we have* $\alpha_{US}^{FB} = \frac{1}{2}$, $\alpha_{US}^{SB} = 1$, $\alpha_{MEX}^{FB} = 1$ *and* $\alpha_{MEX}^{SB} = 1$. *Hence we have:*

$$\frac{\alpha_{US}^{FB}}{\alpha_{US}^{SB}} = \frac{1}{2} < 1 = \frac{\alpha_{MEX}^{FB}}{\alpha_{MEX}^{SB}},$$

so that the U.S. has a comparative advantage in the production of footballs and Mexico has the comparative advantage in the production of soccer balls.

(b) Define the free trade equilibrium.

- *"For the set of productivities* $\{\alpha_{US}^{FB}, \alpha_{US}^{SB}, \alpha_{MEX}^{FB}, \alpha_{MEX}^{SB}\}$, *labor endowments* $\{L_{US}, L_{MEX}\}$, *and utility functions* U_{US}, U_{MEX} *(which in this case are the same), the free trade equilibrium is defined as a set of quantities produced* $\{Q_{US}^{FB}, Q_{US}^{SB}, Q_{MEX}^{FB}, Q_{MEX}^{SB}\}$, *quantities consumed* $\{C_{US}^{FB}, C_{US}^{SB}, C_{MEX}^{FB}, C_{MEX}^{SB}\}$, *and relative price* $\frac{p^{FB}}{p^{SB}}$ *[optional: and labor allocated to each sector] such that: (1) given prices, workers choose the sector to work in to maximize their income; (2) given prices and incomes, consumers maximize their utility, and (3) the total quantity produced of each good is equal to the total quantity consumed."*

(c) Guess that the free trade equilibrium is characterized by complete specialization of each country. Is this an equilibrium? Why or why not?

- *This is not an equilibrium. From above, we know that the US would completely specialize in the production of footballs, yielding 20 footballs. Similarly, Mexico would completely specialize in the production of soccer balls, yielding 8 soccer balls. We now need to solve for the price. Let* $p_{SB} = 1$ *so we only need to find* p_{FB}. *The income of the US and Mexico is equal to:*

$$Y_{US} = p^{FB} \times 20$$

$$Y_{MEX} = 8$$

Given the Cobb-Douglas preferences, we know that the total consumption of footballs is:

$$C_{US}^{FB} + C_{MEX}^{FB} = \frac{1}{2}\left(\frac{Y_{US} + Y_{MEX}}{p^{FB}}\right) = \frac{1}{2}\left(\frac{p^{FB} \times 20 + 8}{p^{FB}}\right) = 10 + \frac{4}{p^{FB}}.$$

From market clearing, we know that $C_{US}^{FB} + C_{MEX}^{FB} = 20$. *Hence we have:*

$$20 = 10 + \frac{4}{p^{FB}} \iff p^{FB} = \frac{2}{5}.$$

However, this cannot be an equilibrium because if the price of footballs was $\frac{2}{5}$, *then workers in the U.S. would prefer to specialize in the production of soccer balls (since* $w_{US}^{SB} = 1$ *and* $w_{US}^{FB} = 2p^{FB} = \frac{4}{5}$*).*

(d) Guess that the free trade equilibrium is characterized by incomplete specialization of the U.S. Is this an equilibrium? Why or why not?

- *Yes, this is an equilibrium. To see this, we note that incomplete specialization on the part of the U.S. pins down the world price:*

$$2p^{FB} = p^{SB} \iff \frac{p^{FB}}{p^{SB}} = \frac{1}{2}.$$

Given those prices, the income of each country is:

$$Y_{US} = 10 \text{ and } Y_{MEX} = 8.$$

Given prices, incomes, and preferences, the quantity demanded by consumers of footballs and soccer balls is hence:

$$C_{US}^{FB} + C_{MEX}^{FB} = \frac{1}{2}\frac{Y_{US}}{p^{FB}} + \frac{1}{2}\frac{Y_{MEX}}{p^{FB}} = 18$$

$$C_{US}^{SB} + C_{MEX}^{SB} = \frac{1}{2}Y_{US} + \frac{1}{2}Y_{MEX} = 9.$$

It remains to see if markets clear. We know that Mexico completely specializes in the production of soccer balls, yielding 8 soccer balls. Hence, we need that the US can produce 1 soccer balls and 18 footballs. The total labor requirement of doing that is:

$$\alpha_{US}^{SB}Q_{US}^{SB} + \alpha_{US}^{FB}Q_{US}^{FB} = 1 \times 1 + \frac{1}{2} \times 18 = 10,$$

which is the population of the US, so markets clear.

(e) What is the equilibrium utility of a worker in the U.S.?

- *From above, we have* $w_{US} = \frac{Y_{US}}{L_{US}} = 1$ *and* $p^{FB} = \frac{1}{2}$ *so that* $\frac{C_{US}^{FB}}{L_{US}} = \frac{1}{2} \times \frac{1}{\frac{1}{2}} = 1$ *and* $\frac{C_{US}^{SB}}{L_{US}} = \frac{1}{2} \times 1 = \frac{1}{2}$*. Hence a worker in the US has utility:*

$$U = (1)^{\frac{1}{2}} \left(\frac{1}{2}\right)^{\frac{1}{2}} = \left(\frac{1}{2}\right)^{\frac{1}{2}}.$$

(f) Suppose that the population of Mexico increases from 8 to 20. What is the new equilibrium utility of a worker in the U.S.? What is the intuition for this change?

- *To answer this, we have to recalculate the equilibrium. I am going to guess that both countries completely specialize. In this case, we have:*

$$Q_{US}^{FB} = 20 \text{ and } Q_{MEX}^{SB} = 20.$$

The income of the US and Mexico is equal to:

$$Y_{US} = p^{FB} \times 20$$

$$Y_{MEX} = 20$$

Given the Cobb-Douglas preferences, we know that the total consumption of footballs is:

$$C_{US}^{FB} + C_{MEX}^{FB} = \frac{1}{2}\left(\frac{Y_{US} + Y_{MEX}}{p^{FB}}\right) = \frac{1}{2}\left(\frac{p^{FB} \times 20 + 20}{p^{FB}}\right) = 10 + \frac{10}{p^{FB}}.$$

From market clearing, we know that $C_{US}^{FB} + C_{MEX}^{FB} = 20$*. Hence we have:*

$$20 = 10 + \frac{10}{p^{FB}} \iff p^{FB} = 1.$$

This is an equilibrium, because U.S. workers are happy to specialize in footballs and Mexican workers are indifferent between producing footballs and soccer balls. Finally, to calculate the utility of a US worker, we proceed as above: $w_{US} = \frac{Y_{US}}{L_{US}} = 2$ *and* $p^{FB} = 1$ *so that* $\frac{C_{US}^{FB}}{L_{US}} = \frac{1}{2} \times 2 = 1$ *and* $\frac{C_{US}^{SB}}{L_{US}} = \frac{1}{2} \times 2 = 1$*. Hence a worker in the US has utility:*

$$U = (1)^{\frac{1}{2}} (1)^{\frac{1}{2}} = 1.$$

Since $1 > \left(\frac{1}{2}\right)^{\frac{1}{2}}$*, this means that the utility of a US worker actually increased! Intuitively, as Mexico gets larger, the world price shifts toward the Mexican autarkic price, which means that the relative price of the good the US had a comparative advantage in increased, thereby increasing the US gains from trade. (This is known as an improvement in the terms of trade for the US).*