Macroeconomics A

Lecture 7 - Labor Market Search & Matching Models

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Geneva Graduate Institute, Fall 2024

The neoclassical model of the labor market

Central question for macro and labor: what determines the level of employment and unemployment in the economy?

The neoclassical model

- theoretically cannot deal with unemployment – there is no involuntary unemployment
  - Given prices, some agents optimally choose to work zero hours
  - There is supply and demand only; demand determined by technology or by demand for output; supply driven by inter- or intratemporal substitution
  - There can be under-employment due to wage stickiness, but there is no unemployment in equilibrium
  - $\Rightarrow$ does not fit the statistical definition of unemployment
- empirically cannot explain fluctuations in employment
  - Employment and wage movement do not jointly fit the data

Some facts about the labor market

- unemployment is a persistent phenomenon
  $\rightarrow$ can wage/price stickiness be the reason?
- large flows of workers between employment, unemployment and non-participation states
- $$\Delta u = \text{inflow} - \text{outflow}$$

  inflow: due to job loss or new entry from non-participation
  outflow: due to job finding or exit into non-participation (retirement, school, inactivity)
- employed workers often change jobs – with a wage gain or wage reduction

How should labor market frictions be modeled?

- incentive problems, efficiency wages
- wage rigidities, bargaining, non-market clearing prices
- search frictions

search and matching: costly process for workers (firms) to find the right jobs (workers)

- very similar to
  - searching for a flat
  - searching for a spouse
  - searching for the best loans on offer
- many applications of the search model

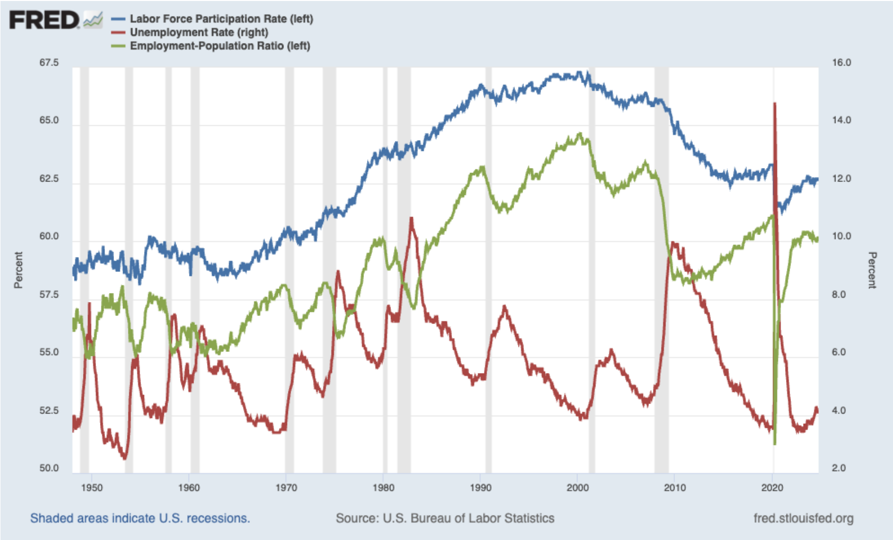
Facts about job flows

- job creation is mildly pro-cyclical
- job destruction is strongly counter-cyclical
- job destruction leads job creation
  it is the driving force of the business cycle – especially in economies with flexible labor markets
- job creation seems to be the main cause of long-run changes in unemployment

Facts about worker flows

- worker turnover about three times as large as job turnover
- worker quits are strongly pro-cyclical
  $\leftrightarrow$ offset by the counter-cyclical job destruction rate
  recession $\rightarrow$ job destruction increases
  $\rightarrow$ voluntary quitting decreases
  $\Rightarrow$ inflow to unemployment increases, but less
- unemployment changes driven mainly by the outflow from unemployment
- in monthly data: employment $\leftrightarrow$ non-participation flows $\approx$
  employment $\leftrightarrow$ unemployment flows

US employment/unemployment statistics



Shimer's exercise

the change in the unemployment rate is

$$u_{t+1} - u_t = s_t(1 - u_t) - f_t u_t$$

- u_t – unemployment rate
- s_t – separation rate
- f_t – job finding rate
- ignore exit from the labor force, and entry from out of labor force

denote average rates by:

$$\bar{s} = \sum_{t=1}^{T} \frac{s_t}{T} \quad \text{and} \quad \bar{f} = \sum_{t=1}^{T} \frac{f_t}{T}$$

Shimer's exercise

Compare the actual unemployment rate with

1. a hypothetical unemployment rate constructed using the average (a constant) separation rate:

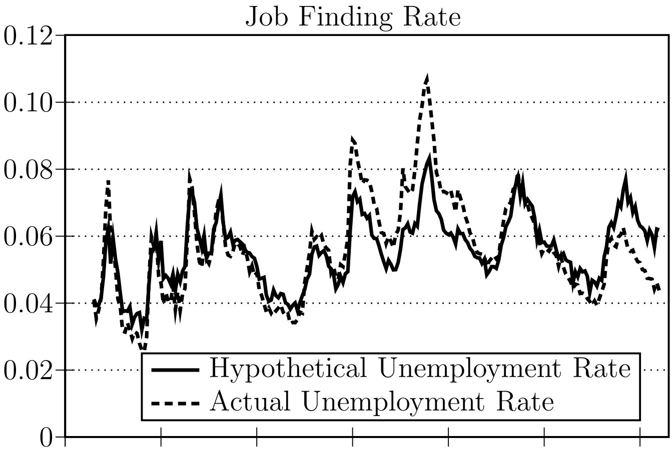
$$u_{t+1} - u_t = \bar{s}(1 - u_t) - f_t u_t$$

2. a hypothetical unemployment rate constructed using the average (a constant) job finding rate:

$$u_{t+1} - u_t = s_t(1 - u_t) - \bar{f} u_t$$

The role of the job finding rate

holding the separation rate constant at $\bar{s}$



Source: Shimer (2005)

The role of the separation rate

holding the job finding rate constant at $\overline{f}$

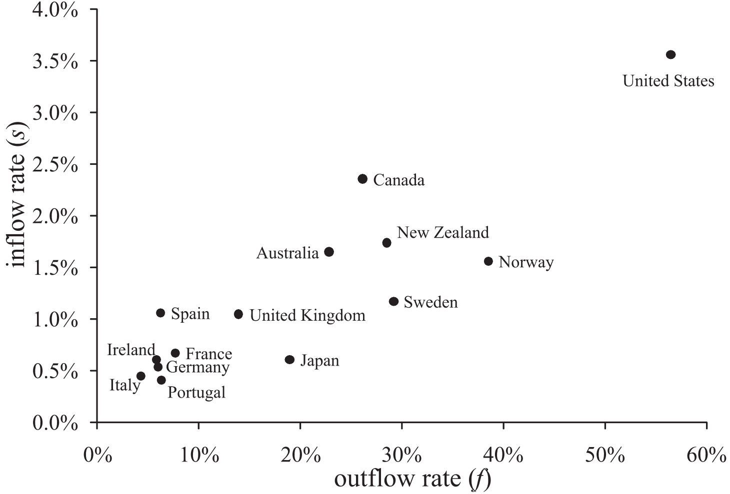
Source: Shimer (2005)

Lessons from Shimer's exercise

- separation rate not so important in the evolution of unemployment
- job finding rate is a more important determinant of unemployment
- why?
  separation rates do increase during recessions
  BUT the job finding rate is high in the US, even during recessions
  even if more workers get laid off, they find a job quickly $\rightarrow$ job separation rate not so important

But countries are different

FIGURE 1.—AVERAGE INFLOW AND OUTFLOW RATES ACROSS COUNTRIES



Source: Elsby, Hobijn, Sahin (2013), Figure 1.

Shimer's exercise for other countries

changes in unemployment are due to

- UK: 71% inflow rate, 29% outflow rate (Elsby, Smith, Wadsworth (2010))
- Spain: 57% inflow rate, 43% outflow rate (Petrongolo and Pissarides (2009))

First generation search models (PE, McCall)

- Start right away in continuous time, as we'll be going there anyway
- Infinite horizon
- Workers start out unemployed, receive unemployment benefits $b \cdot \delta t$ in every infinitesimal time interval δt
- Job offers arrive according to a Poisson process with rate $a > 0$, i.e. the in each period of length δt a worker receives $n = 0, 1, 2, \ldots$ offers with probability $a(n, \delta t)$, where

  $$a(n, \delta t) = \frac{e^{a\delta t}(a\delta t)^n}{n!}$$

  and the number of offers is independent across time
- Then, $\{N(t), t \geq 0\}$ is a Poisson pricess with rate $a > 0$, and

  $$P(N(\delta t) = 1) = e^{a\delta t} a\delta t + O(\delta t)$$

  $$P(N(\delta t) \geq 2) = O(\delta t)$$

  Reminder: a function f is $O(\delta t)$ if $\lim_{\delta t \to 0} \frac{f(\delta t)}{\delta t} = 0$

Value function of an unemployed worker

- Jobs are all the same, pay a flow of w, perpetually (employment is an absorbing state)
- Constant discount rate r
- Unemployed workers decide whether to take job offers

Value function of an unemployed worker, U, satisfies the Bellman equation

$$U_t = \frac{b \cdot \delta t}{1 + r\delta t} + e^{a\delta t} a\delta t \frac{\max(W_{t+\delta t}, U_{t+\delta t})}{1 + r\delta t} + (1 - e^{a\delta t} a\delta t)\frac{U_{t+\delta t}}{1 + r\delta t}$$

- Very similar to the value of holding an asset:
- cash flow
- Option arrives $\rightarrow$ expectation of the capital gain
- option does not arrive $\rightarrow$ continuation value

The value of an unemployed worker

rearranging terms and dividing by δt

$$rU_t = b + a\left(\max(W_{t+\delta t}, U_{t+\delta t}) - U_{t+\delta t}\right) + \frac{U_{t+\delta t} - U_t}{\delta t}$$

taking the limit as $\delta t \to 0$ and omitting subscripts for convenience yields:

$$rU = b + a\left(\max(W, U) - U\right) + \dot{U}$$

this is an arbitrage equation for the valuation of human capital:

- investing the value at a safe return
- if you leave the asset in the labor market
  - flow return
  - expected capital gain from change of state
  - capital gains from changes in evaluation – in the steady state, b, a, r are all constant + infinite horizon $\Rightarrow$ stationary solutions to the valuation equations, i.e. $\dot{U} = 0$

Closing the model

Assume that

- employment is an absorbing state
- during employment, the worker earns forever the (flow) wage w

Then the value function of an employed worker is

$$W = \int_0^\infty e^{-rt} w dt = \frac{w}{r}$$

and the stationary solution to U satisfies

$$rU = b + a\left(\max\left(\frac{w}{r}, U\right) - U\right)$$

When does the unemployed take a job?

$$W \geq U$$
$$\frac{w}{r} \geq U$$
$$w \geq rU$$

$\Rightarrow$ optimal stopping rule: *reservation wage*, the minimum acceptable wage

$$\xi \equiv rU$$

if you stay unemployed, you can never be worse off than rU, so unless someone pays at least this, you won't start working

For a known wage offer distribution

assume that offers arrive from a known wage distribution, $F(w)$

$$rU = b + a\left(\int_0^A \max\left(\frac{w}{r}, U\right) dF(w) - U\right)$$

wages under the reservation wage, $\xi = rU$ are rejected:

$$rU = b + a\int_\xi^A \left(\frac{w}{r} - U\right) dF(w)$$

collecting terms and solving for the reservation wage

$$\xi = \frac{r}{r + a(1 - F(\xi))} b + \frac{a}{r + a(1 - F(\xi))} \int_\xi^A w dF(w)$$

$a(1 - F(\xi))$ is the transition from unemployment to employment

Second-generation models: Search & Matching

(Mortensen and Pissarides, 1994)

Two-sided matching

to close the one-sided search model, to derive a proper equilibrium of the economy, we need to

- make the arrival rate, a, endogenous
  $\rightarrow$ specify decision of firms whether or not to offer jobs
- ensure that employment is not an absorbing state
  $\rightarrow$ exogenous job destruction, at rate λ
  interpretation: negative shocks arrive to existing matches, that destroy the match
  $\rightarrow$ the worker becomes unemployed, the job is destroyed
- the wage is the result of bargaining between the matched worker and firm
  assume for now: single wage offer, which satisfies $w \geq \xi$
  $\rightarrow$ all job offers are accepted, outflow rate from unemployment is a

Value of employed and unemployed

- the value of an unemployed worker is (as before)

$$rU = b + a(W - U)$$

- using that all jobs pay the same wage rate, w
- $\rightarrow$ nobody has an incentive to quit or to search for another job while employed
- $\Rightarrow$ the value of an employed worker is

$$rW = w - \lambda(W - U)$$

Combining the above two and rearranging:

$$W - U = \frac{w - b}{r + a + \lambda}$$

The matching function

Key assumption: the aggregate flow depends on an aggregate matching function

- Black box – similar to a production function: gives number of matches as a function of the inputs into the search process
- $m = m(u, v) \rightarrow$ determines # matches, where v is the mass of vacancies
- Assumptions on $m(\cdot, \cdot)$
  - Continuous and differentiabe
  - Positive first partial derivatives, negative second partial derivatives
  - CRS
- The empirical literature (Petrongolo & Pissarides 2001) found that a Cobb-Douglas function matches the data well:

$$m = Au^{\eta}v^{1-\eta}$$

  where usually $\eta = 0.5$.

The matching function

job matching is pairwise $\Rightarrow$

$$m = au = qv$$

- arrival rate of workers to vacant jobs

$$q = \frac{m(u,v)}{v} = m\left(\frac{u}{v}, 1\right) \equiv m(\theta^{-1}, 1) \equiv q(\theta)$$

  - q is a decreasing function of θ: $q'(\theta) < 0$
  - the elasticity of q wrt θ is $\frac{\partial q}{\partial \theta}\frac{\theta}{q} \equiv -\eta \in (-1, 0)$
- arrival rate of jobs to workers

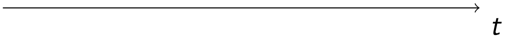
$$a = \frac{m(u,v)}{u} = m\left(1, \frac{v}{u}\right) = a(\theta) = \theta q(\theta)$$

  - a is an increasing function of θ: $a'(\theta) > 0$
  - the elasticity of a wrt θ is $\frac{\partial a}{\partial \theta}\frac{\theta}{a} \in (0, 1)$

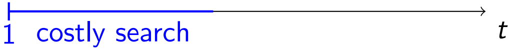
Market tightness

- $\theta = \frac{v}{u}$ is a measure of **market tightness**
- u is a state
  v is the firm's control $\rightarrow$ this drives unemployment
- however, probably both firms and workers ignore their effect on θ when they make their search choices
  $\rightarrow$ **search externalities**

Timing of decisions



Timing of decisions



1. firm posts vacancy
   - the firms post vacancies until there are no rents to be made
   - the value of a vacancy in equilibrium has to be zero $\Leftrightarrow V = 0$
   - new jobs produce with the best technology

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2. worker arrives
   - wage bargaining takes place
     w acceptable to both parties $\Leftrightarrow W \geq U$ and $J \geq 0$
   - **job creation** takes place if there is an agreement
   - production begins

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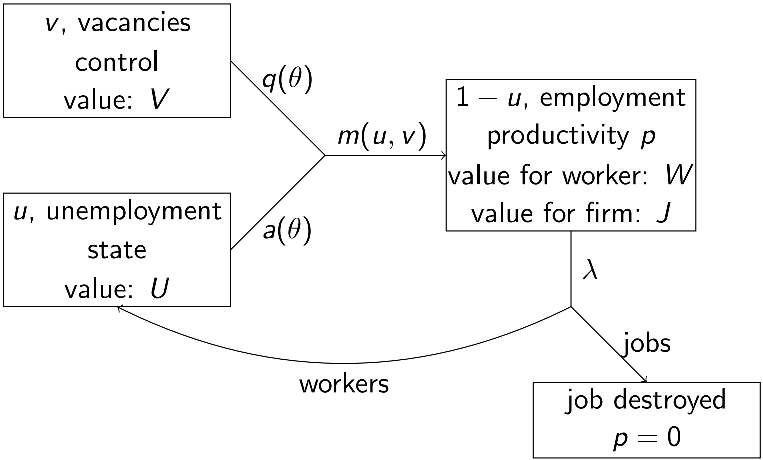
   - production begins
3. idiosyncratic productivity shock arrives
   - investment is irreversible
     if the shock reduces the net value of the job below zero $\Leftrightarrow$
     $J + W < U$
   - **job destruction**

Flow of people and jobs



The value of vacancies and jobs

- V – value of a vacant job
- J – value of an occupied job
- a job is an asset owned by the firm, and its value is determined by arbitrage equations

$$rV = -pc + q(\theta)(J - V)$$
$$rJ = p - w - \lambda J$$

- r – discount rate
- p – value of product, productivity $\rightarrow$ higher value for better workers
- pc – cost of maintaining vacancy $\rightarrow$ it is more costly to find people to fill skilled vacancies
- w – wage rate

Job creation

firm creates a job vacancy when there are gains from entering the market and there is free entry
$\Rightarrow$ zero profits, $V = 0$

$$rV = -pc + q(\theta)(J - V)$$
$$V = 0 \Leftrightarrow J = \frac{pc}{q(\theta)}$$

- J – the value of having a worker = the PV(expected profits)
- $\frac{1}{q(\theta)}$ – the expected duration of a vacancy
- $\frac{pc}{q(\theta)}$ – the expected total cost of finding a worker

Job creation

the value of a filled job

$$rJ = p - w - \lambda J$$
$$J = \frac{p - w}{r + \lambda}$$

<u>note</u>: for the firm to accept a wage $w \Leftrightarrow p \geq w$

combining the two equations on the value of filled jobs:

$$J = \frac{pc}{q(\theta)} \quad \& \quad J = \frac{p - w}{r + \lambda}$$

$$\underbrace{p - w}_{\text{profit flow}} - \underbrace{\frac{(r + \lambda)pc}{q(\theta)}}_{\text{expected cost of finding a worker}} = 0$$

job creation condition: generalization of the labor demand condition, downward sloping in the θ - w space

Wage determination

Value functions of workers and unemployed:

$$rU = z + \theta q(\theta)(W - U)$$
$$rW = w + \lambda(U - W)$$

Wages are determined through a process of **bargaining** between firm and worker

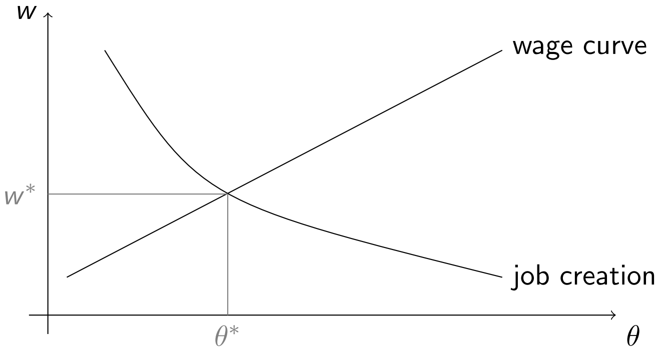
- Each party can leave the negotiations $\rightarrow$ wage is between worker outside option and marginal product of worker
- Using Nash bargaining, each party gets outside option plus a fraction (β for worker, $1 - \beta$ for firm) of the difference between total surplus and outside options

This process yields a wage equation:

$$w = (1 - \beta)z + \beta p(1 + c\theta)$$

- increasing in worker outside option z
- increasing in β (because $p > z$)
- increasing in worker marginal product p
- increasing in market tightness θ (outside option)

Equilibrium wages and market tightness



wage curve – bargained wage as a function of θ

$$w = (1-\beta)z + \beta p(1 + c\theta)$$

job creation curve – zero profit from vacancies, optimal θ as a function of w

$$p - w - \frac{(r+\lambda)pc}{q(\theta)} = 0$$

Equilibrium

in this economy the equilibrium is a path for

the controls:

- wages – from the wage equation: $J \geq 0, W \geq U$
- job vacancies (or tightness) – from job creation $V = 0$

the state:

- employment – **condition for the evolution of unemployment**

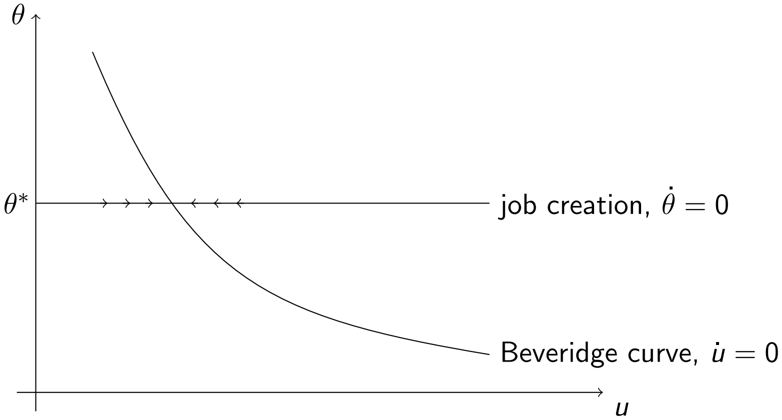
$$\dot{u} = \lambda(1-u) - m(u,v) = \lambda - (\lambda + \theta q(\theta))u$$

given θ^*, this has a unique stable solution, **the natural rate of unemployment**

$$u = \frac{\lambda}{\lambda + \theta^* q(\theta^*)}$$

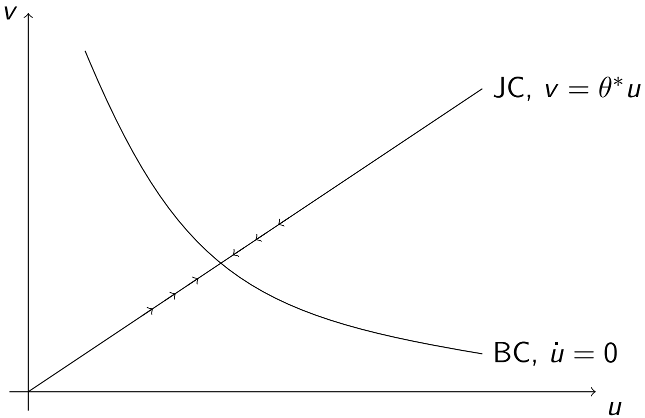
the above is the **Beveridge curve**

Equilibrium tightness and unemployment



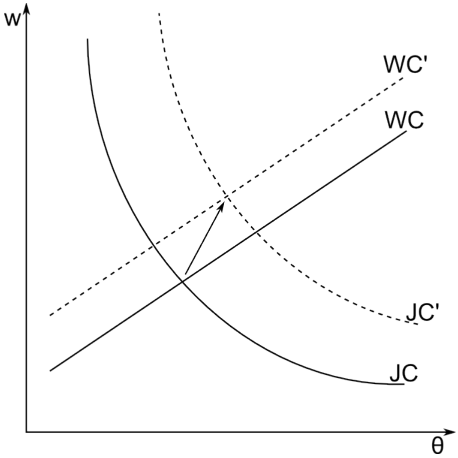
- θ is a control variable, its equilibrium is independent of u
- u is a state variable, and it is stable
- the $\dot{\theta} = 0$ line is the saddle path
- θ jumps to its equilibrium value, and the economy moves along the saddle path to the steady state

Equilibrium vacancies and unemployment

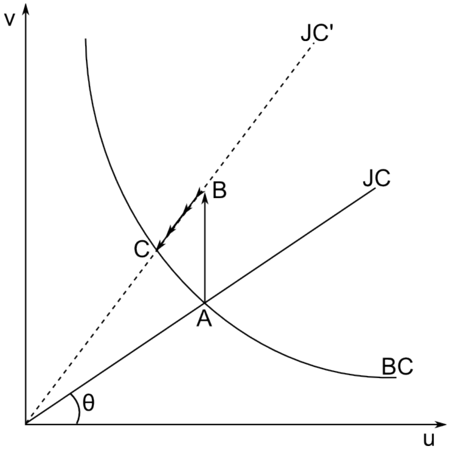


- v is a control variable
- u is a state variable, and it is stable
- the $v = \theta^* u$ line is the saddle path
- v jumps on the saddle path, and the economy moves along it to the steady state

Permanent increase in A (MPL)



Permanent increase in A (MPL)



Temporary positive TFP shock

- Over time, JC curve will rotate back to the original position
- response of u will be U-shaped (bad pun, i know)
- if unemployment flow utility b is high enough, elasticity of wage is low
- Can it explain the business cycle moments of unemployment and vacancies?

The unemployment volatility puzzle

Shimer (2005) argues that (reasonable) empirical implementations of this model

- can match contemporaneous correlations between unemployment and vacancies
- *cannot* match the volatility of unemployment, vacancies, tightness, job finding rate (no amplification)
- *cannot* match the correlations with labor productivity

Where's the problem?

What happens after a TFP shock?

- Wage increases, but not as much as the MPL
- Hence firms want to increase employment. More vacancies get posted.
- Over time, this reduces the unemployment rate.

Hence:

- Since the intensive margin of labor supply is fixed, higher wages do not increase labor supply
- Any effect on u must go through vacancy posting
- But wage is too elastic: $MPL - w$ increases by too little, few vacancies are being posted

Possible solutions

- Sticky wages (within the participation constraints): Hall (2005)
- Make unemployment not a big deal: $\beta = 0.05$, $b = 1$, low vacancy posting costs. (Hagedorn and Manovskii, 2008).
- Fixed costs for vacancy posting (Pissarides, 2009).

Additional slides: derivation of the wage equation

Wage bargaining

- J is value of the filled job to the firm (depends on w)
- V is the value of the unfilled vacancy to the firm
- W is the value of the job to the worker
- U is the value of being unemployed (outside option)

Assumption: wage is set through *generalized Nash bargaining*, i.e. to maximize the Nash product

$$(w - U)^{\beta}(J - V)^{1-\beta} \text{where } \beta \in (9, 1)$$

Solution is such that

$$W - U = \beta(J - V + W - U)$$

β is the bargaining power of the worker, and mathematically the share of the surplus that is allocated to the worker.

Some tedious math

Express $W - U$ from the worker's Bellman eqns:

$$W - U = \frac{w - z}{r + \lambda + \theta q(\theta)}$$

Express $J - V$ from the firm's Bellman eqn for J, and $V = 0$:

$$J - V = \frac{p - w}{r + \lambda}$$

Plug both into the Nash solution equation

$$(1 - \beta)\frac{w - z}{r + \lambda + \theta q(\theta)} = \beta \frac{p - w}{r + \lambda}$$

expand, cancel, and put w on one side to get:

$$w = (1 - \beta)z + \beta \frac{(p - w)\theta q(\theta)}{r + \lambda} + \beta p$$

$$w = (1-\beta)z + \beta\frac{(p-w)\theta q(\theta)}{r+\lambda} + \beta p$$

Notice that $J - V = (p - w)/(r + \lambda)$ and because of free entry into vacancy posting ($V = 0$), $J - V = pc/q(\theta)$, hence

$$w = (1-\beta)z + \beta pc\theta + \beta p$$

and the finished wage equation is

$$w = (1-\beta)z + \beta p(1 + c\theta)$$

Wage is a weighted average of the worker's outside income flow z, and its marginal product p, adjusted for how costly it would be for the firm to look for a replacement ($c\theta$).