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An estimated New-Keynesian model with unemployment as excess supply of labor ☆

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ABSTRACT

Wage stickiness is incorporated to a New-Keynesian model with variable capital to drive endogenous unemployment fluctuations defined as the log difference between aggregate labor supply and aggregate labor demand. We estimated such model using Bayesian econometric techniques and quarterly US data. The second-moment statistics of the unemployment rate in the model provide a reasonable fit to those observed in US data. Our results also show that mainly wage-push shocks together with demand shifts and monetary policy shocks are the major determinants of unemployment fluctuations. Compared to an estimated New-Keynesian model without unemployment (Smets and Wouters 2007): wage stickiness is higher, labor supply elasticity is lower, the slope of the New-Keynesian Phillips curve is flatter, and the importance of technology innovations on output variability increases.

1. Introduction

The New-Keynesian macro model has been extended in recent years to incorporate the endogenous determination of unemployment fluctuations in the labor market.[1] Taking the search frictions approach, Walsh (2005) and Trigari (2009) introduced unemployment as the gap between job creation and destruction that results in a labor market with real rigidities à *la* Mortensen and Pissarides (1994). Alternatively, Casares (2007, 2010), Galí (2011), Casares et al., 2012, and Galí et al. (2012) assume nominal rigidities on wage setting to produce mismatches between labor supply and labor demand that deliver unemployment fluctuations. In another recent paper, Michaillat (2012) explores the interactions between search frictions, job rationing and wage rigidity and finds asymmetric patterns in business cycle fluctuations of unemployment.

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[1] Referential New-Keynesian models without unemployment are Christiano et al. (2005), Smets and Wouters (2003, 2007) and all the model variants collected in Woodford (2003). They belong to the family of dynamic stochastic general equilibrium (DSGE) models.

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This paper presents novel theoretical and empirical contributions. On the theoretical side, our model simultaneously accommodates unemployment fluctuations due to sticky wages, with variable capital, that brings labor-capital reallocations at the firm level. In contrast to the vast majority of the related literature, our model generates unemployment fluctuations without resorting to search frictions in the labor market. Hence, the model combines most of the nominal and real rigidities of full-fledged New-Keynesian models – Calvo-type price stickiness, consumption habits, investment adjustment costs, variable capital utilization, etc. –, with a labor market which generalizes that of Casares (2010) to include natural-rate unemployment and a rental market for capital. In that regard, we replace the standard way of introducing wage rigidities on labor contracts set by households (which follows the seminal paper by Erceg et al. (2000)) for a labor market structure in which excess-labor-supply unemployment stems from sticky wages. As a result, wage dynamics depend inversely upon fluctuations of the rate of unemployment. We also discuss the implications on inflation dynamics: the New-Keynesian Phillips curve turns flatter because of the negative effect of relative prices over relative nominal wages at the firm level.

On the empirical front, this paper includes the observed rate of unemployment in the estimation and provides a comparison between our proposed model and the Smets and Wouters (2007) New-Keynesian model, which is a well-known reference model in the DSGE literature. We follow a Bayesian econometric strategy to estimate the models using US quarterly data during the Great Moderation period (1984:1–2007:4).[2] In the comparison across models, we find similar estimates of many structural model parameters, though three remarkable differences. First, wage stickiness is significantly higher in the model with unemployment while price stickiness is nearly the same across models. As a consequence, the introduction of unemployment as excess supply of labor raises the average length of labor contracts (3.23 quarters with unemployment and 2.13 quarters without unemployment). Second, the labor supply curve is significantly more inelastic in the model with unemployment. Finally, the elasticity of capital adjustment costs is lower in the model with unemployment.

Our estimated New-Keynesian model reproduces the US business cycle features at least as well as DSGE models without unemployment and, crucially, provides a good characterization of US unemployment volatility and persistence. Furthermore, the impulse-response functions provide reasonable reactions of unemployment to technology innovations, demand shocks, monetary shocks and cost-push shocks. In the variance-decomposition analysis, model results indicate that the driving forces of unemployment fluctuations are mainly wage inflation shocks together with risk premium (demand-side) shocks, and monetary shocks playing a secondary role. Importantly, the model provides a good matching of the lead-lag comovement between the unemployment rate and output growth.

This paper is related to Galí et al. (2012), though developed independently. There are five important differences across models. First, and most importantly, in Galí et al. (2012) -following Galí (2011) - unemployment is perfectly correlated with the average wage mark-up and therefore moves in tandem with workers' market power. In contrast to assuming market power of households, this paper includes an intertemporal equation to set wages that match labor supply and labor demand. In turn, unemployment is introduced at a decentralized level that interacts with the price setting behavior. Second, marginal costs are firm specific only in our model due to firm-specific nominal wage setting. Third, both models feature a similar number of shocks but Galí et al. (2012) resort to wage mark-up shocks, whereas we introduce push shocks on indexed wages. Fourth, our model provides a reasonable characterization of the business cycle during the Great Moderation without having to assume away short-run wealth effects on labor. Fifth, wage-push shocks are here less important in explaining fluctuations across all aggregate variables than the wage markup shocks in Galí et al. (2012).

There is also a straightforward comparison of this paper to Casares et al. (2012), which ignores endogenous capital accumulation and consumption habits, and uses linear detrending to obtain the observable series for estimation. In turn, nominal rigidities on both price and wage setting are estimated to be significantly lower here, reducing the role of demand-side shocks for both output and unemployment variability.

The remainder of the paper proceeds as follows. Section 2 describes the model with sticky wages, unemployment as excess supply of labor and variable capital. Section 3 introduces the estimation procedure and discusses the estimation results. Section 4 presents the empirical fit of the two models along three important dimensions (second-moment statistics, variance decomposition and impulse-response functions) and compares some of the model-implied dynamic cross-correlations with those in the data. This section also provides a robustness analysis along with two important dimensions. First, the sample period is extended to include the most recent recession. Second, the two models are estimated without any labor measure as observable variable. Finally, Section 5 concludes with a summary of the main results.

2. A New-Keynesian model with unemployment

This section introduces unemployment in a New-Keynesian model with sticky wages and endogenous capital accumulation. Thus, we borrow most of the elements of the New-Keynesian model described in Smets and Wouters (2007) except for the labor market and wage setting behavior. On that dimension, we extend Casares (2010) and Casares et al. (2012), with the addition of variable capital accumulation and consumption habits. In contrast to Smets and Wouters (2007), employment

[2] Following Galí et al. (2012), we focus our analysis on the Great Moderation period in order to avoid issues associated with parameter instability and potential parameter estimate distortion due to the presence of non-linearities showing up during the Great Recession induced by (i) the zero lower bound on the Fed funds rate, and (ii) the binding downward nominal wage rigidities discussed in Galí et al. (2012). Moreover, the focus on the Great Moderation will allow us to compare our estimation results with those reported in Galí et al. (2012). Below, in a robustness section, we will also estimate the two models extending the sample period including data from the most recent recession period.

variability is determined only by the extensive margin of labor (number of employees), assuming that the number of hours per worker is inelastically supplied as in Hansen (1985).[3] Hence, there is a representative household that supplies a variable number of workers for all differentiated types of labor while each firm demands one specific kind.[4] Let us denote $L_t^d(i)$ as the labor demand for jobs in type-i firm and $L_t^s(i)$ as the labor supply of workers in period t for that i firm, so that the rate of unemployment at the i firm in period t is

$$u_t(i) = 1 - \frac{L_t^d(i)}{L_t^s(i)}. \tag{1}$$

In the model, wage rigidity causes unemployment fluctuations around the (constant) natural rate, u^n. Thus, if wages were flexible they would adjust to make the current rate of unemployment equal to u^n. Following the wage-rigidity assumption of Bénassy (1995), and, more recently, of Casares (2007), Casares (2010), labor contracts can be revised to equate labor supply and demand only when firms and households receive a market signal to set such natural-rate wage. Otherwise, the wage will be automatically adjusted by applying an *ad-hoc* indexation rule. Introducing wage stickiness à *la* Calvo (1983), the market signal for wage setting arrives with a fixed probability. If possible, the natural-rate wage for labor contracts in firm i is set at the value that results from the intertemporal condition:

$$E_t^{\xi_w} \sum_{j=0}^{\infty} \overline{\beta}^j \xi_w^j u_{t+j}(i) = \frac{u^n}{1 - \overline{\beta}\xi_w}, \tag{2}$$

where $\overline{\beta}$ is the discount rate that incorporates detrending from long-run growth, ξ_w is the Calvo (1983)-type constant probability of not being able to make a natural-rate wage revision, and $E_t^{\xi_w}$ is the rational expectations operator conditional on the lack of revisions in the future. Plugging (1) and the corresponding expressions for future periods in (2) and taking a loglinear approximation give the following expression

$$E_t^{\xi_w} \sum_{j=0}^{\infty} \overline{\beta}^j \xi_w^j \left(l_{t+j}^s(i) - l_{t+j}^d(i) \right) = 0, \tag{3}$$

where $l_{t+j}^s(i)$ and $l_{t+j}^d(i)$ represent the log deviations, in any $t+j$ period, from their respective steady-state levels of the labor supply of workers and the labor demand for jobs of type-i labor, $L_{t+j}^s(i)$ and $L_{t+j}^d(i)$.[5] In the absence of wage stickiness ($\xi_w = 0.0$), there would be a perfect matching between fluctuations of labor supply and labor demand at firm level: $l_t^s(i) = l_t^d(i)$ in (3) and $u_t(i) = u^n$ in (2). Put differently, wage rigidities bring about gaps between the amounts of supply of labor (workers provided by the household) and the demand for labor (jobs demanded by the firm) that make the effective rate of unemployment deviate from a constant natural rate of unemployment.

Hence, the nominal wage adjusts depending on the fluctuations of either labor supply or labor demand entering (3). Adapting the household optimizing program of Smets and Wouters (2007), the first order condition on type-i labor supply implies a positive reaction of the relative labor supply to both the firm-specific wage and relative unemployment rate as follows[6]

$$l_t^s(i) - l_t^s = \frac{1}{\sigma_l} \left(\widetilde{W}_t(i) - \frac{1}{1-u^n}(u_t(i) - u_t) \right), \tag{4}$$

where $l_t^s = \int_0^1 l_t^s(i)di$ is the log deviation of aggregate labor supply from the steady-state level, σ_l is the elasticity parameter of labor disutility, $\widetilde{W}_t(i) = \log W_t(i) - \log W_t$ is the relative nominal wage, and u_t is the aggregate rate of unemployment defined as 1.0 minus the ratio between aggregate labor demand (effective employment, L_t)[7] and aggregate labor supply (labor force, L_t^s)

$$u_t = 1 - \frac{L_t}{L_t^s}.$$

In semi-loglinear terms, firm-level and aggregate rates of unemployment become

$$u_t(i) = u^n + (1-u^n)\left(l_t^s(i) - l_t^d(i) \right), \tag{5a}$$

$$u_t = u^n + (1-u^n)\left(l_t^s - l_t \right), \tag{5b}$$

where $l_t = \int_0^1 l_t^d(i)di$ is the log deviation of effective employment from its steady-state level. Substituting both (5a) and (5b) in (4) yields

[3] This assumption relies on the generally accepted view that most variability of total hours worked in modern economies is explained by changes in the number of employed people whereas fluctuations of the number of hours at work have significantly less influence (Cho and Cooley, 1994; Mulligan, 2001).

[4] Woodford (2003, chapter 3), uses this labor market scenario for fluctuations of the intensive margin of labor (hours), claiming that the existence of heterogeneous labor services is more adequate for sticky-price models than the common assumption of an homogeneous labor market.

[5] Throughout the paper, lower-case variables denote log deviations with respect to steady-state levels.

[6] See the technical appendix – Section A – for the optimizing program of the representative household and the derivation of the first order conditions.

[7] The effective amount of employment is demand determined, as usually assumed in Keynesian models and many macroeconomics textbooks (e.g., Abel et al., 2011; Blanchard, 2011).

$$l_t^s(i) - l_t^s = \frac{1}{\sigma_l}\left(\widetilde{W}_t(i) - \left(l_t^s(i) - l_t^d(i) - l_t^s + l_t\right)\right). \tag{6}$$

Regarding firm-level labor demand, we also borrow the production technology and capital rental market used in Smets and Wouters (2007), which result in the relative labor demand equation[8]

$$l_t^d(i) - l_t = -\theta\widetilde{P}_t(i) - \alpha\widetilde{W}_t(i), \tag{7}$$

which introduces the Dixit–Stiglitz demand elasticity θ, the capital share parameter from a Cobb-Douglas technology, α, and the relative price $\widetilde{P}_t(i) = \log P_t(i) - \log P_t$. For non-revised labor contracts, the nominal wage is automatically adjusted by applying an indexation rule that combines a weight $0 < \iota_w < 1$ for lagged inflation, π_{t-1}, and the complementary weight $1 - \iota_w$ for the steady-state inflation rate plus a stochastic wage-push shock, $\pi + \varepsilon_t^w$. If firm j cannot revise the labor contract, it will apply the following nominal wage adjustment

$$W_t(j) = W_{t-1}(j)\left[(1+\pi_{t-1})^{\iota_w}(1+\pi+\varepsilon_t^w)^{1-\iota_w}\right]. \tag{8}$$

This wage indexation rule is very similar to the one assumed in Smets and Wouters (2007), with the only difference that we include a cost-push shock ε_t^w to replace the wage mark-up shock of their model. As shown in the technical appendix (Section B), the average relative wage set in period t, $\widetilde{W}_t^* = \int_0^1 \log W_t^*(i)di - \log W_t$, is

$$\widetilde{W}_t^* = -\frac{(1-\overline{\beta}\xi_w)(1+\sigma_l^{-1})}{(\sigma_l^{-1}+\alpha)(1-u^n)(1+\Lambda)}E_t\sum_{j=0}^{\infty}\overline{\beta}^j\xi_w^j(u_{t+j}-u^n) + E_t\sum_{j=1}^{\infty}\overline{\beta}^j\xi_w^j\left(\pi_{t+j}^w - \iota_w\pi_{t+j-1} - (1-\iota_w)\varepsilon_{t+j}^w\right), \tag{9}$$

where $\int_0^1 \log W_t^*(i)di$ provides the average log of the nominal wage across types that received the Calvo signal to make the intertemporal clearing of labor supply and demand as in 3. There is a rich model structure embedded in (9) as the coefficient Λ is given by $\frac{\tau_1\overline{\beta}\xi_w\theta}{\sigma_l^{-1}+\alpha}\left(1 - \frac{\xi_p(1-\overline{\beta}\xi_w)}{1-\overline{\beta}\xi_w\xi_p}\right)$, where τ_1 is the elasticity of relative prices with respect to lagged relative wages from the relationship $\widetilde{P}_t^*(i) = \widetilde{P}_t^* + \tau_1\widetilde{W}_{t-1}(i)$.[9] Meanwhile, the wage indexation rule (8) implies a proportional relationship between $\widetilde{W}_t^*$ and the rate of wage inflation, π_t^w, adjusted by the indexation factors

$$\widetilde{W}_t^* = \frac{\xi_w}{1-\xi_w}\left(\pi_t^w - \iota_w\pi_{t-1} - (1-\iota_w)\varepsilon_t^w\right). \tag{10}$$

Combining (9) and (10) results in the wage inflation equation

$$\pi_t^w = \overline{\beta}E_t\pi_{t+1}^w + \iota_w\pi_{t-1} - \overline{\beta}\iota_w\pi_t - \frac{(1-\overline{\beta}\xi_w)(1-\xi_w)(1+\sigma_l^{-1})}{\xi_w(\sigma_l^{-1}+\alpha)(1-u^n)(1+\Lambda)}(u_t-u^n) + (1-\iota_w)(\varepsilon_t^w - \overline{\beta}E_t\varepsilon_{t+1}^w). \tag{11}$$

Thus, wage inflation dynamics are inversely related to the rate of unemployment like in the old-fashion empirical Phillips curve.[10] For the real wage equation, we can take the log difference to its definition, $w_t = \log\left(\frac{W_t}{P_t}\right)$, to obtain

$$w_t - w_{t-1} = \pi_t^w - \pi_t. \tag{12}$$

Using (11) in (12) and solving out for the log of the real wage leads to

$$w_t = w_1 w_{t-1} + (1-w_1)(E_t w_{t+1} + E_t\pi_{t+1}) - w_2\pi_t + w_3\pi_{t-1} - w_4(u_t-u^n) + w_5(\varepsilon_t^w - \overline{\beta}E_t\varepsilon_{t+1}^w), \tag{13}$$

where $w_1 = \frac{1}{1+\overline{\beta}}$, $w_2 = \frac{1+\overline{\beta}\iota_w}{1+\overline{\beta}}$, $w_3 = \frac{\iota_w}{1+\overline{\beta}}$, $w_4 = \frac{1}{1+\overline{\beta}}\frac{(1-\overline{\beta}\xi_w)(1-\xi_w)(1+\sigma_l^{-1})}{\xi_w(\sigma_l^{-1}+\alpha)(1-u^n)(1+\Lambda)}$, and $w_5 = \frac{1-\iota_w}{1+\overline{\beta}}$.

Let us turn now to derive the price inflation equation, the so-called New-Keynesian Phillips curve. The presence of unemployment as excess supply of labor is going to influence inflation dynamics through the effect of firm-specific wage setting on firm-specific real marginal costs. For its derivation, we start from the loglinearized equation for the optimal price in Smets and Wouters (2007)[11]:

$$p_t^*(i) = (1-\overline{\beta}\xi_p)E_t^{\xi_p}\sum_{j=0}^{\infty}\overline{\beta}^j\xi_p^j\left(A\left(mc_{t+j}(i) + \lambda_{t+j}^p\right) + p_{t+j} - \iota_p\sum_{k=1}^{j}\pi_{t+k-1}\right), \tag{14}$$

where $p_t^*(i) = \log P_t^*(i)$ is the log of the optimal price set by firm i, $A > 0$ is a constant parameter that depends upon the Kimball (1995) goods market aggregator and the steady-state price mark-up,[12] and $E_t^{\xi_p}$ is the rational expectations operator conditional on the lack of optimal pricing after period t. The log of the optimal price depends on the expectation of three factors: the

[9] Details available in the technical appendix, Section B.

[10] Remarkably, the slope coefficient in the wage inflation Eq. (11) is different from the one found in Casares (2010) and Casares et al. (2012) due to the presence of variable capital and a competitive rental market together with a constant natural rate of unemployment.

[11] This result is provided in the technical appendix of Smets and Wouters (2007), available at http://www.aeaweb.org/aer/data/june07/20041254_app.pdf.

[12] Concretely, $A = 1/((\Phi-1)\varepsilon_p + 1)$ where ε_p is the curvature of the Kimball aggregator and Φ is the steady-state price mark-up.

log of the real marginal costs, $mc_{t+j}(i)$, exogenous price mark-up variations, λ^p_{t+j}, and the log of the aggregate price level adjusted by the indexation rule, $p_{t+j} - \iota_p \sum_{k=1}^{j} \pi_{t+k-1}$. Since $p_{t+j} = p_t + \sum_{k=1}^{j} \pi^p_{t+k}$, the following optimal relative price ($\widetilde{P}^*_t(i) = \log P^*_t(i) - \log P_t$) is obtained:

$$\widetilde{P}^*_t(i) = A(1-\overline{\beta}\xi_p)E_t^{\xi_p}\sum_{j=0}^{\infty}\overline{\beta}^j\xi_p^j\left(mc_{t+j}(i) + \lambda^p_{t+j}\right) + E_t\sum_{j=1}^{\infty}\overline{\beta}^j\xi_p^j\left(\pi_{t+j} - \iota_p\pi_{t+j-1}\right). \tag{15}$$

Unlike Smets and Wouters (2007), the real marginal cost is firm-specific in our model as a consequence of firm-specific nominal wages. Taking logs in the definition of the firm-specific real marginal cost gives[13]

$$mc_t(i) = (1-\alpha)(\log W_t(i) - p_t) + \alpha \log r^k_t - z_t, \tag{16}$$

where z_t is the log of capital utilization. Summing up across all firms and subtracting the result from (16) leads to

$$mc_t(i) = mc_t + (1-\alpha)\widetilde{W}_t(i). \tag{17}$$

Generalizing (17) for $t+j$ periods and inserting the resulting expressions in (15) yields

$$\widetilde{P}^*_t(i) = A(1-\overline{\beta}\xi_p)E_t^{\xi_p}\sum_{j=0}^{\infty}\overline{\beta}^j\xi_p^j\left(mc_{t+j} + (1-\alpha)\widetilde{W}_{t+j}(i) + \lambda^p_{t+j}\right) + AE_t\sum_{j=1}^{\infty}\overline{\beta}^j\xi_p^j\left(\pi_{t+j} - \iota_p\pi^p_{t+j-1}\right), \tag{18}$$

which makes that average relative prices, $\widetilde{P}^*_t = \int_0^1 \log P^*_t(i)di - \log P_t$, across all firms that in period t are able to optimally set prices, evolve as follows[14]

$$\widetilde{P}^*_t = \frac{A(1-\overline{\beta}\xi_p)}{1+\Theta}E_t\sum_{j=0}^{\infty}\overline{\beta}^j\xi_p^j\left(mc_{t+j} + \lambda^p_{t+j}\right) + E_t\sum_{j=1}^{\infty}\overline{\beta}^j\xi_p^j\left(\pi_{t+j} - \iota_p\pi_{t+j-1}\right), \tag{19}$$

where $\Theta = \tau_2 A(1-\alpha)\left(1 - \frac{(1-\overline{\beta}\xi_p)\xi_w}{1-\overline{\beta}\xi_p\xi_w}\right)$ and τ_2 is the (negative) elasticity of relative wages with respect to relative prices in $\widetilde{W}^*_t(i) = \widetilde{W}^*_t - \tau_2\widetilde{P}_t(i)$. Recalling the price indexation rule of Smets and Wouters (2007) brings, after loglinearization, that relative optimal prices and the rate of inflation are related as follows

$$\widetilde{P}^*_t = \frac{\xi_p}{1-\xi_p}\left(\pi_t - \iota_p\pi_{t-1}\right),$$

which can be substituted into (19) to obtain

$$\pi_t - \overline{\beta}\xi_p E_t\pi_{t+1} = \iota_p\pi_{t-1} - \overline{\beta}\xi_p\iota_p\pi^p_t + \frac{A(1-\overline{\beta}\xi_p)(1-\xi_p)}{\xi_p(1+\Theta)}\left(mc_t + \lambda^p_t\right) + \frac{1-\xi_p}{\xi_p}\overline{\beta}\xi_p E_t\left(\pi_{t+1} - \iota_p\pi_t\right).$$

Finally, we can put together terms on current and expected next period's inflation, re-scale the mark-up shock at $\varepsilon^p_t = \pi_3\lambda^p_t$ and -following the Smets and Wouters (2007) convention- introduce μ^p_t as the log deviation of the price mark-up ($mc_t = -\mu^p_t$), so that the inflation equation becomes

$$\pi_t = \pi_1\pi_{t-1} + \pi_2 E_t\pi_{t+1} - \pi_3\mu^p_t + \varepsilon^p_t, \tag{20}$$

where $\pi_1 = \frac{\iota_p}{1+\overline{\beta}\iota_p}$, $\pi_2 = \frac{\overline{\beta}}{1+\overline{\beta}\iota_p}$, and $\pi_3 = \frac{1}{1+\overline{\beta}\iota_p}\frac{(1-\overline{\beta}\xi_p)(1-\xi_p)}{\xi_p((\phi_p-1)\varepsilon_p+1)(1+\Theta)}$. Interestingly, the inflation Eq. (20) is a hybrid New-Keynesian Phillips curve where the slope coefficient is affected by the presence of nominal rigidities on both the goods and the labor market.[15] Thus, the slope of (20) depends on the value of the sticky-wage probability, ξ_w , that is contained in Θ, reflecting the complementarities between pricing and wage setting assumed here that are absent in standard DSGE models ($\widetilde{P}^*_t(i) = \widetilde{P}^*_t + \tau_1\widetilde{W}_{t-1}(i)$ and $\widetilde{W}^*_t(i) = \widetilde{W}^*_t - \tau_2\widetilde{P}_t(i)$). In turn, the New-Keynesian Phillips curve (20) is flatter than the one derived in Smets and Wouters (2007) which had a slope coefficient $\pi_3^{SW} = \frac{1}{1+\overline{\beta}\iota_p}\frac{(1-\overline{\beta}\xi_p)(1-\xi_p)}{\xi_p((\phi_p-1)\varepsilon_p+1)} = \pi_3(1+\Theta)$. Intuitively, the response of inflation to an increase in the real marginal cost is weaker in our model because relative wages will move downwards due to higher relative prices. Therefore, a more moderate initial increase in prices set by firms will be enough to maximize intertemporal profit as they anticipate lower marginal costs when nominal wages are reset.

Hence, Eqs. (13) and (20) collect the effects of unemployment as excess supply of labor in the dynamics of both the real wage and inflation. The demand-side equations, the monetary policy rule and all the stochastic elements of the model (except for the wage indexation shock) are borrowed from Smets and Wouters (2007) since all these equations can be reached with no influence of the wage setting behavior and unemployment fluctuations. We also take from their paper the following

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[13] The real marginal cost of firm i in period t is $MC_t(i) = \frac{\left(\frac{W_t(i)}{P_t}\right)^{1-\alpha}(R^k_t)^{\alpha}}{Z_t\alpha^{\alpha}(1-\alpha)^{1-\alpha}}$.

[14] The algebra involved is provided in the technical appendix, Section C.

[15] The slope coefficient of the New-Keynesian Phillips curve (20) is also analytically different from the one obtained in the model of Casares (2010) and Casares et al. (2012), which feature unemployment as excess supply of labor and constant capital.

shock process structures: the AR(1) technology shock $\varepsilon_t^a = \rho_a \varepsilon_{t-1}^a + \eta_t^a$, the AR(1) risk premium disturbance that shifts the demand for purchases of both consumption and investment goods $\varepsilon_t^b = \rho_b \varepsilon_{t-1}^b + \eta_t^b$, the exogenous spending (fiscal, net exports) shock driven by an AR(1) process with an extra term capturing the potential influence of technology innovations on exogenous spending $\varepsilon_t^g = \rho_g \varepsilon_{t-1}^g + \eta_t^g + \rho_{ga}\eta_t^a$, the AR(1) investment shock $\varepsilon_t^i = \rho_i \varepsilon_{t-1}^i + \eta_t^i$, the AR(1) monetary policy shock: $\varepsilon_t^R = \rho_R \varepsilon_{t-1}^R + \eta_t^R$, the ARMA(1,1) price mark-up shock: $\varepsilon_t^p = \rho_p \varepsilon_{t-1}^p + \eta_t^p - \mu_p \eta_{t-1}^p$, and the ARMA(1,1) wage push shock $\varepsilon_t^w = \rho_w \varepsilon_{t-1}^w + \eta_t^w - \mu_w \eta_{t-1}^w$. The technical appendix – Sections D and E – displays the complete set of equations of the model and discusses the equation-to-equation comparison across models. From now on, we will refer to the model with unemployment as the CMV model while the model of Smets and Wouters (2007) will be the SW model, taking the initial letters of the authors' last names.

3. Estimation

We estimate both models with US data during the Great Moderation period, from the first quarter of 1984 to the last quarter of 2007. This period is characterized by mild fluctuations of aggregate variables (see Stock and Watson, 2002, among others). Thus, the estimation exercises do not suffer from some potential miss-specification sources, such as parameter instability in both the private sector – for instance, Calvo probabilities (Moreno, 2004) – and the monetary policy reactions to inflation or output. Indeed, some authors argue that a sound monetary policy implementation is the main factor behind the low business cycle volatility in this period (Clarida et al., 1999). In our main estimation we exclude the years of the recent credit crisis, where monetary policy is more accurately characterized through unconventional measures and the zero interest rate lower bound. As a robustness exercise we report results with a sample including these years.

Regarding the data set, we take as observable variables quarterly time series of the inflation rate, the Federal funds rate, civilian employment and the log differences of the real Gross Domestic Product (GDP), real consumption, real investment, and the real wage.[16] Thus, variables displaying a long-run trend enter the estimation procedure in log differences to extract their stationary business cycle component.[17] In the estimation of the CMV model, we add the quarterly unemployment rate as another observable variable and ignore the log of employment in order to consider the same (number of) shocks in the two models. The data were retrieved from the Federal Reserve of St. Louis (FRED2) database.

The estimation procedure also follows Smets and Wouters (2007). Thus, we employ a two-step Bayesian procedure. In the first step, the log posterior function is maximized in a way that combines the prior information of the parameters with the empirical likelihood of the data. In a second step, we perform the Metropolis-Hastings algorithm to compute the posterior distribution of the parameter set.[18] It should be noted that in the estimation of the CMV model, the slope coefficients in the inflation and real wage equations were introduced as implicit functions of the undetermined coefficients τ_1 and τ_2. These coefficients can be analytically solved through a non-linear two-equation system. We choose the positive values associated with these solutions, as implied by theory.

In terms of the priors, we select the same prior distributions as Smets and Wouters (2007) for the estimation of the two models (see the first three columns in Tables 1A and 1B), and we also borrow their notation for the structural parameters. In the CMV model we have two additional parameters: the Dixit–Stiglitz elasticity of substitution across goods, θ, and the steady-state unemployment rate, u. The prior mean of these two parameters is set at 6.0, in line with previous studies.

Tables 1A and 1B show the estimation results of both the SW and CMV models and report the posterior mean estimates together with the 5% and 95% intervals of the posterior distribution.

As the last three columns of Tables 1A and 1B show, our version of the SW model – with a three-year longer sample period and considering the extensive margin of labor – confirms their estimates of the structural parameters. Across models, there are three main differences. First, the labor supply is less elastic in the CMV model, where σ_l is 4.57 , while it is 2.01 in the SW model. The internal reallocations between labor and capital at firm-specific wages (only in the CMV model) cuts the aggregate response of labor supply to changes in the real wage.[19] Second, the Calvo probability of wage stickiness is higher in the CMV model, where ξ_w is 0.69, while it is 0.53 in the SW model.[20] Therefore, the introduction of unemployment as excess supply of labor increases the estimated average length of labor contracts from $(1-0.53)^{-1} = 2.13$ quarters to $(1-0.69)^{-1} = 3.23$ quarters. There are trade-offs between real wage and unemployment fluctuations in Eq. (13) governed by the slope coefficient at

[16] The rate of inflation is obtained as the first difference of (the log of) the implicit GDP deflator, whereas the real wage is computed as the ratio between nominal compensation per hour and the GDP price deflator. Smets and Wouters (2007) estimate their model with (the log of) hours. As an alternative, and in order to facilitate a comparison with the estimation results of the CMV model, this paper estimates a version of the SW model where employment variability is determined only by the extensive margin of labor. Thus, we estimate the SW model using (the log of) civilian employment.

[17] In this way, we avoid the well-known measurement error implied by standard filtering treatments.

[18] All estimation exercises are performed with DYNARE free routine software, which can be downloaded from http://www.dynare.org. A sample of 250,000 draws was used (ignoring the first 20% of draws). A step size of 0.3 resulted in an average acceptation rate of roughly 30% across the five Metropolis-Hastings blocks used.

[20] This result somewhat challenges the findings of Champagne and Kurmann (2013), who argue that there was an increase in wage flexibility over the Great Moderation in the US. Indeed, our estimate at $\xi_w = 0.69$ is very close to the standard calibration of Calvo probabilities at 0.75 to render a one-time wage change per year. Moreover, our estimated value of ξ_w is in line with the estimated values reported by Smets and Wouters (2007) for the Great Inflation (1961:1–1979:2) and the Great Moderation (1984:1–2004:2) periods (0.65 and 0.74, respectively), showing no evidence of higher wage flexibility during the Great Moderation period.

Table 1A
Priors and estimated posteriors of the structural parameters (sample: 1984-2007).

	Priors			Posteriors					
				CMV model			SW model		
	Distr.	Mean	Std. D.	Mean	5%	95%	Mean	5%	95%
φ: Cost of adjusting capital	Normal	4.00	1.50	4.69	2.77	6.50	6.36	4.49	8.38
h: Habit formation	Beta	0.70	0.10	0.46	0.33	0.60	0.60	0.47	0.74
σ_c: Risk aversion	Normal	1.50	0.37	0.99	0.67	1.29	1.26	0.93	1.59
σ_l: Inverse Frisch elasticity	Normal	2.00	0.75	4.57	3.63	5.36	2.01	1.02	2.97
ξ_p: Price Calvo probability	Beta	0.50	0.10	0.75	0.66	0.84	0.72	0.64	0.81
ξ_w: Wage Calvo probability	Beta	0.50	0.10	0.69	0.56	0.79	0.53	0.40	0.67
ι_w: Wage indexation	Beta	0.50	0.15	0.34	0.14	0.55	0.47	0.22	0.71
ι_p: Price indexation	Beta	0.50	0.15	0.29	0.10	0.44	0.37	0.16	0.57
ψ: Capital utilizat. adjust. cost	Beta	0.50	0.15	0.81	0.69	0.93	0.75	0.61	0.89
Φ: Steady-state price mark-up	Normal	1.25	0.12	1.61	1.44	1.77	1.52	1.37	1.67
r_π: Inflation (policy rule)	Normal	1.50	0.25	2.00	1.64	2.38	2.04	1.69	2.35
ρ: Inertia (policy rule)	Beta	0.75	0.10	0.84	0.81	0.88	0.82	0.77	0.86
r_y: Output gap (policy rule)	Normal	0.12	0.05	0.14	0.06	0.23	0.03	0.00	0.07
$r_{\Delta y}$: Output growth (policy rule)	Normal	0.12	0.05	0.23	0.17	0.29	0.16	0.10	0.20
π: Steady-state inflation	Gamma	0.62	0.10	0.66	0.55	0.76	0.74	0.59	0.89
$100(\beta^{-1}-1)$: Discount rate	Gamma	0.25	0.10	0.25	0.11	0.38	0.19	0.08	0.29
$\bar{l}$: Steady-state log labor	Normal	0.00	2.00	–	–	–	5.24	2.58	7.77
γ: Steady-state output growth	Normal	0.40	0.10	0.43	0.36	0.51	0.41	0.35	0.46
α: Capital share	Normal	0.30	0.05	0.17	0.13	0.21	0.21	0.17	0.25
θ: Demand elasticity	Normal	6.00	1.50	6.51	4.17	8.94	–	–	–
u: Steady-state unemployment rate	Normal	6.00	2.00	5.92	5.40	6.52	–	–	–

Table 1B
Priors and estimated posteriors of the shock processes (sample: 1984–2007).

	Priors			Posteriors					
				CMV model			SW model		
	Distr.	Mean	Std. D.	Mean	5%	95%	Mean	5%	95%
σ_a: Std of productivity innovation	Invgamma	0.10	2.00	0.42	0.34	0.50	0.37	0.32	0.41
σ_b: Std of risk premium innov.	Invgamma	0.10	2.00	0.07	0.05	0.08	0.13	0.06	0.20
σ_g: Std of spending innov.	Invgamma	0.10	2.00	0.39	0.34	0.44	0.39	0.35	0.44
σ_i: Std of investment innov.	Invgamma	0.10	2.00	0.31	0.24	0.38	0.33	0.25	0.40
σ_R: Std of monetary innov.	Invgamma	0.10	2.00	0.12	0.11	0.14	0.12	0.10	0.14
σ_p: Std of price index. innov.	Invgamma	0.10	2.00	0.10	0.08	0.12	0.10	0.07	0.12
σ_w: Std of wage index. innov.	Invgamma	0.10	2.00	3.71	1.45	7.12	0.34	0.27	0.41
ρ_a: Persistence of prod. shock	Beta	0.50	0.20	0.98	0.97	0.997	0.94	0.90	0.98
ρ_b: Persistence of risk prem. shock	Beta	0.50	0.20	0.88	0.82	0.94	0.51	0.10	0.88
ρ_g: Persistence of spending shock	Beta	0.50	0.20	0.96	0.93	0.98	0.96	0.94	0.99
ρ_i: Persistence of investment shock	Beta	0.50	0.20	0.71	0.60	0.83	0.69	0.57	0.81
ρ_R: Persistence of monetary shock	Beta	0.50	0.20	0.25	0.13	0.39	0.36	0.21	0.53
ρ_p: Persistence of price shock	Beta	0.50	0.20	0.70	0.54	0.89	0.83	0.71	0.96
ρ_w: Persistence of wage shock	Beta	0.50	0.20	0.89	0.84	0.95	0.96	0.93	0.99
μ_p: Moving-average of price shock	Beta	0.50	0.20	0.55	0.34	0.79	0.62	0.42	0.81
μ_w: Moving-average of wage shock	Beta	0.50	0.20	0.21	0.04	0.35	0.67	0.49	0.85
ρ_{ga}: Correlation of prod. & spend shocks	Beta	0.50	0.20	0.35	0.17	0.52	0.41	0.24	0.59

$w_4 = 0.32$, which shape the short-run effects of stabilizing policies.[21] Third, the elasticity of capital adjustment costs is higher in the SW model, where φ is 6.36, while it is 4.69 in the CMV model. Since the estimated nominal rigidity is greater in the labor market of the CMW model, capital accumulation is more sensitive to changes in either the marginal product of capital or the interest rates.

In both models, the inverse of the elasticity of intertemporal substitution (σ_c) is quite low, near the log utility consumption preference, and the presence of habit persistence is moderate, as h is in the vicinity of 0.5 in the CMV model and 0.6 in the SW model. The elasticity of capital utilization adjustment cost ψ is 0.81 in the CMV model and just below that at 0.75 in the SW model. The Calvo probability of price stickiness (ξ_p) is high, around 0.75, and very similar across models. Both wage and price indexation parameters (ι_w and ι_p, respectively) are between 0.25 and 0.50, and somewhat higher in the SW model. The New-Keynesian Phillips curve is flatter in the CMV model (see discussion in Section 2). Concretely, the estimated slopes

[21] However, in line with the results of Justiniano et al. (2013), the volatility of the output gap relative to current output fluctuations is low in the estimated model (34%), which may indicate that most output variability has been driven by productivity shocks with a strong co-movement between natural-rate and effective output (94% in the CMV estimated model).

are $\pi_3 = 0.0051$ in the CMV model, lower than $\pi_3^{SW} = 0.0127$ in the SW model. The reason behind this result is that the CMV model incorporates more rigidities in the connection between firm-level pricing and wage setting. Such frictions are collected in the term Θ of the slope coefficient that takes a estimated value at $\Theta = 0.72$ in the CMV model whereas it is absent in the SW model.

Monetary policy parameters are similar across models, with a stabilizing interest reaction of inflation, r_π, close to 2, a response to output gap growth, $r_{\Delta y}$, near 0.20, and a high policy rule persistence parameter, ρ, between 0.82 and 0.84. The only noticeable difference is that the response to the output gap, r_y, is not significantly different from zero in the SW model, whereas it is small, 0.14, but significant in the CMV model. The estimate of one plus the fixed-cost share, Φ, is slightly higher in the CMV model. The estimates of the steady-state parameter that determines the long-run rate of growth, γ, the real interest rate, $100(\beta^{-1} - 1)$, and the rate of inflation, π, are similar in the two models, as well as a low estimate of the capital share in the production function, α. Finally, the elasticity of substitution across goods, θ, and the steady state rate of unemployment, u, are only estimated in the CMV model, and are in line with the values chosen as priors.

Table 1B shows the standard deviations and autocorrelations of the seven structural shocks. The estimates of the standard deviations of the innovations look similar in both models. The only difference lies in the volatility of the wage-push innovation, which is significantly higher in the CMV model. As shown in the technical appendix, this is due to the fact that the wage inflation equation differs across models and the wage-push shock also has a different interpretation; it is a wage indexation shock in the CMV model while it is a wage mark-up shock in the SW model.[22] Again, the estimates of persistence and moving-average parameters are similar across models. The only difference lies in the persistence parameter associated with the wage-push shock, which is lower in the CMV model. Moreover, the monetary policy shock is the one exhibiting the lowest first-order autocorrelation – around 0.30 – in both models. Technology, risk premium, exogenous spending and wage-push innovations are highly persistent across models. The two models feature moderately persistent investment and price-push innovations.

4. Empirical fit

This section compares the performance of the SW and CMV models along several dimensions in the first three subsections. First, we analyze the ability of the two models to reproduce second-moment statistics found in US quarterly data. Second, we study the contribution of each structural shock in explaining the total variance decomposition of macroeconomic variables. Third, we carry out an impulse-response analysis. The fourth subsection analyzes the ability of the CMV model to replicate US lead-lag comovements between the unemployment rate and the output growth rate, and between the rate of inflation and the output growth rate. Finally, the fifth subsection shows parameter estimates for alternative subsamples and observable variables.

4.1. Second-moment statistics

Table 2 shows second-moment statistics obtained from actual data, and the ones found in the estimated CMV and SW models.

In general, the two models do a good job in reproducing the cyclical features of the data. Thus, both models approximate quite closely the standard deviations in the US economy of output growth, consumption growth, investment growth, the real wage growth, the log of civilian employment, price inflation, and the nominal interest rate during the Great Moderation. The CMV model matches all these volatilities better except for the nominal interest rate and the change in the real wage. Importantly, the introduction of unemployment as excess supply of labor in the estimated CMV model reproduces the unemployment rate volatility reasonably well. In contrast, results show a clear labor volatility mismatch in the SW model due to an excessive standard deviation whereas the CMV falls short in this dimension.

The contemporaneous correlations between each variable and the output growth rate are also reported in Table 2 as a measure of their procyclical or countercyclical behavior. In general, cyclical correlations obtained in model simulations are close to their data counterparts; most numbers are in a similar range except for the ones of the nominal interest rate (mildly negative in the models and mildly positive in the data) and the rate of unemployment (mildly procyclical at 0.14 in US data and mildly countercyclical at -0.23 in the CMV model).[23] Finally, the two models do a reasonable job in replicating the first-order autocorrelation of all variables, with the exception of an excessive inflation persistence in the SW model. Interestingly, the CMV model does a good job in reproducing the second-moment statistics of the US Great Moderation period without resorting to the device of considering an arbitrarily small short-term wealth effect introduced in Galí et al. (2012).

4.2. Variance decomposition

Table 3 shows the total variance decomposition analysis for the CMV and SW models. In the CMV model, technology innovations η^a explain around 40% of the fluctuations in output and consumption growth. Meanwhile, demand (risk-premium) shocks, η^b, drive 65% of the variability of the nominal interest rate, 18% of output growth, 22% of employment and 15% of the rate of unemployment. The influence of exogenous spending (fiscal/net exports) shocks, η^g, is moderately low as it deter-

[22] Moreover, the wage-push shock in the real wage equation of the CMV model appears multiplied by the coefficient w_5, which is estimated to be close to 1/3 while it is a unit coefficient in the real wage equation of the SW model. As a result, the effective size of the wage-push shock is similar across models.

[23] Further discussion on this issue comes below in Section 4.4.

Table 2
Second-moment statistics (sample: 1984–2007).

	Δy	Δc	Δi	Δw	l	u	R	π
US data								
Std. deviation, %	0.54	0.50	1.47	0.72	2.00	1.04	0.59	0.23
Corr. withΔy	1.0	0.56	0.61	0.00	0.04	0.14	0.09	−0.14
Autocorrelation	0.23	0.10	0.52	0.13	0.97	0.98	0.98	0.61
Estimated CMV model								
Std. deviation, %	0.67	0.62	1.73	0.89	1.15	0.88	0.40	0.28
	(0.60,0.73)	(0.55,0.67)	(1.47,1.93)	(0.78,1.01)	(0.92,1.35)	(0.67,1.05)	(0.31,0.47)	(0.22,0.33)
Corr. with Δy	1.0	0.64	0.52	0.39	0.08	−0.23	−0.09	−0.09
		(0.55,0.75)	(0.44,0.60)	(0.28,0.50)	(0.01,0.13)	(−0.28,−0.19)	(−0.14,−0.04)	(−0.18,−0.02)
Autocorrelation	0.27	0.37	0.62	0.38	0.93	0.92	0.94	0.72
	(0.21,0.32)	(0.30,0.44)	(0.55,0.70)	(0.27,0.48)	(0.91,0.95)	(0.90,0.95)	(0.92,0.96)	(0.62,0.80)
Estimated SW model								
Std. deviation, %	0.77	0.63	1.78	0.82	3.05	–	0.42	0.40
	(0.69,0.84)	(0.54,0.71)	(1.50,1.99)	(0.71,0.92)	(1.98,3.75)		(0.33,0.49)	(0.31,0.45)
Corr. with Δy	1.0	0.68	0.68	0.31	0.12	–	−0.18	−0.24
		(0.59,0.76)	(0.61,0.75)	(0.18,0.43)	(0.08,0.17)		(−0.26,−0.09)	(−0.38,−0.11)
Autocorrelation	0.37	0.46	0.61	0.32	0.98	–	0.94	0.83
	(0.29,0.44)	(0.37,0.56)	(0.50,0.69)	(0.17,0.43)	(0.96,0.99)		(0.92,0.96)	(0.78,0.89)

Table 3
Long-run variance decomposition (sample: 1984–2007).

Estimated CMV model								
Innovations	Δy	Δc	Δi	Δw	l	u	R	π
Technology, η^a	0.41	0.39	0.04	0.03	0.19	0.01	0.06	0.03
Risk premium, η^b	0.18	0.25	0.08	0.13	0.22	0.15	0.65	0.17
Fiscal/net exports, η^g	0.18	0.11	0.01	0.01	0.12	0.01	0.03	0.01
Investment, η^i	0.07	0.04	0.77	0.03	0.11	0.02	0.10	0.02
Interest-rate, η^R	0.12	0.17	0.06	0.09	0.14	0.11	0.06	0.07
Wage-push, η^w	0.01	0.02	0.01	0.62	0.14	0.69	0.06	0.20
Price-push, η^p	0.03	0.02	0.04	0.10	0.08	0.01	0.04	0.50
Estimated SW model								
Innovations	Δy	Δc	Δi	Δw	l	u	R	π
Technology, η^a	0.11	0.05	0.04	0.01	0.02	–	0.09	0.03
Risk premium, η^b	0.19	0.45	0.03	0.06	0.03	–	0.20	0.06
Fiscal/net exports, η^g	0.23	0.03	0.01	0.01	0.06	–	0.05	0.02
Investment, η^i	0.18	0.02	0.77	0.04	0.07	–	0.26	0.09
Interest-rate, η^R	0.09	0.15	0.03	0.05	0.03	–	0.09	0.11
Wage-push, η^w	0.12	0.24	0.06	0.66	0.71	–	0.24	0.40
Price-push, η^p	0.07	0.07	0.06	0.17	0.08	–	0.07	0.30

mines 18% of output growth and even lower shares of the other variables.[24] Innovations in the investment adjustment costs, η^i, only have a substantial impact on investment fluctuations (77%) whereas monetary policy shocks, η^R, explain between 11% and 17% of fluctuations of output growth, consumption growth, employment and the rate of unemployment. Meanwhile, inflation (price-push) shocks, η^p, are the main determinant of inflation variability (50%) but account for not more than 10% of the variance share of the other variables. The wage-push (indexation) shock, η^w, exerts a strong influence on the rate of unemployment (69%) and the real wage growth (62%) and a more moderate influence on inflation (20%) and employment (14%), while having a weak impact on the rest of the variables.

Therefore, the introduction of unemployment as excess supply of labor in an estimated New-Keynesian model implies that the main driving forces behind US unemployment fluctuations are shifts driven by wage-push shocks (69%), risk-premium variations (15%), and also monetary policy shocks (11%). Innovations in technology barely explain 1% of unemployment variability. By contrast, the dynamics of output growth are substantially influenced by technology shocks (41%), as well as a mix of demand-side perturbations (risk-premium shocks, exogenous spending innovations and monetary policy shocks that jointly take 48% in its variance decomposition).

In the estimated SW model, technology innovations, η^a, are less influential on business cycle fluctuations than in the CMV model, affecting only 11% of output changes and lower percentages of the rest of the variables. The absence of unemployment implies that all labor force are employed and technology shocks lose significance on production variability (in comparison to the CMV model with unemployment). The risk-premium shocks, η^b, account for 19% of the variability of

[24] As in Smets and Wouters (2007), the role of the exogenous spending shock is to bring demand-determined changes in output that are not collected by either private consumption or private investment, such as fiscal shocks or exports/imports variations.

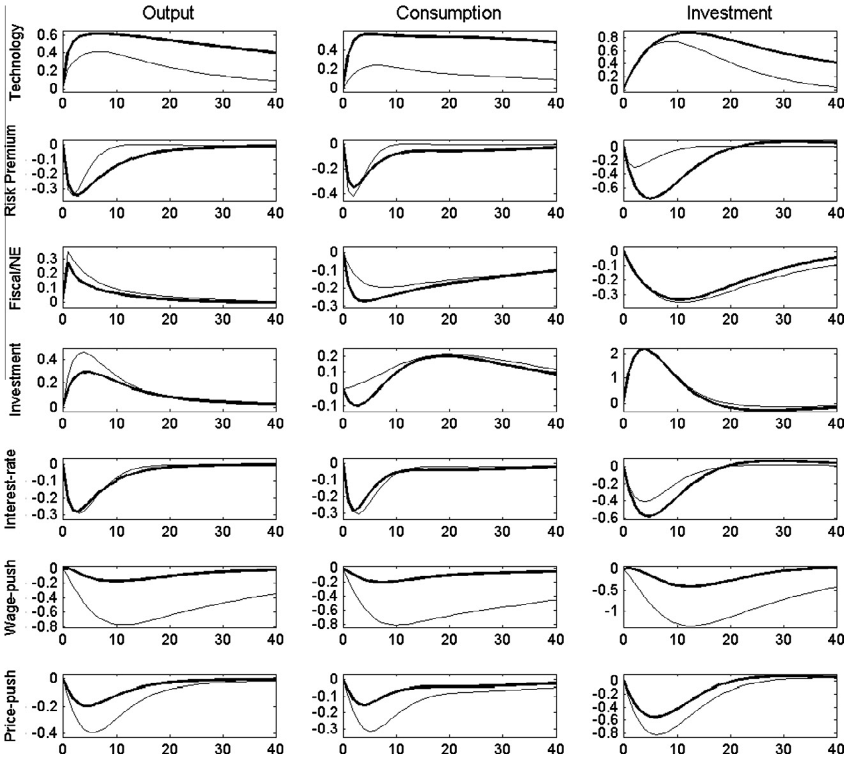

Fig. 1. Impulse response functions of output, consumption, and investment. CMV model (thick line) and SW model (thin line).

output growth and 45% of consumption growth, while their influence on the movements of the nominal interest rate and inflation are much lower than in the CMV model at only 20% and 6%, respectively. The influence of exogenous spending shocks, η^g, is moderate with 23% of fluctuations of output growth, while the investment shock, η^i, mainly affects private investment (77% of its variability) and has a moderate influence on the nominal interest rate (26%) and output growth (18%). Monetary policy shocks, η^R, account for 9% and 15% of changes in output and consumption, respectively; whereas inflation shocks, η^p, only explain significant fractions of variability of inflation (30%) and the changes in the real wage (17%). Wage-push shocks are more influential in the SW model than in the CMV model as they explain 66% of variations in the real wage growth, but in addition they also determine a high share of employment fluctuations (71%), 40% of inflation variability, 24% of consumption growth and nominal interest rate fluctuations, and 12% of variability of output growth.[25]

It is worthwhile highlighting that the results for long-run variance decompositions obtained from the estimated CMV model present some relevant differences with respect to those reported in Galí et al. (2012). Hence, the CMV model gives a relatively large importance to demand (risk premium, monetary policy and fiscal/net export) shocks in explaining most cyclical fluctuations, whereas wage markup shocks emerge as a key driving force behind many variables in Galí et al. (2012).[26] More concretely, they find most variability due to wage markup shocks (80%), whereas in our case demand shocks account for nearly one third of the variance. Despite these differences, both models confer a large importance to productivity shocks in explaining output fluctuations.

4.3. Impulse-response functions

Figs. 1–3 plot the impulse response functions obtained in the SW and CMV estimated models to the seven -one standard deviation- structural shocks. Across figures, we observe that responses are quite similar for both models in terms of sign and dynamics. However, the CMV model shows greater responses to technology shocks and weaker to wage-push and price-push shocks. In particular, Fig. 1 shows that the technology shock increases output, consumption and investment, with the effects being higher and more persistent in the CMV model. The risk premium shock results in similar declines of these three variables, whereas the investment shock increases output and investment. The fiscal-net exports (exogenous spending) shock

[25] This result will be reflected in the relatively large reactions of these variables to the estimated wage-push shock displayed in the impulse-response analysis conducted below.

[26] Demand shocks are even more important for both output and unemployment fluctuations in the constant-capital estimated model of Casares et al. (2012).

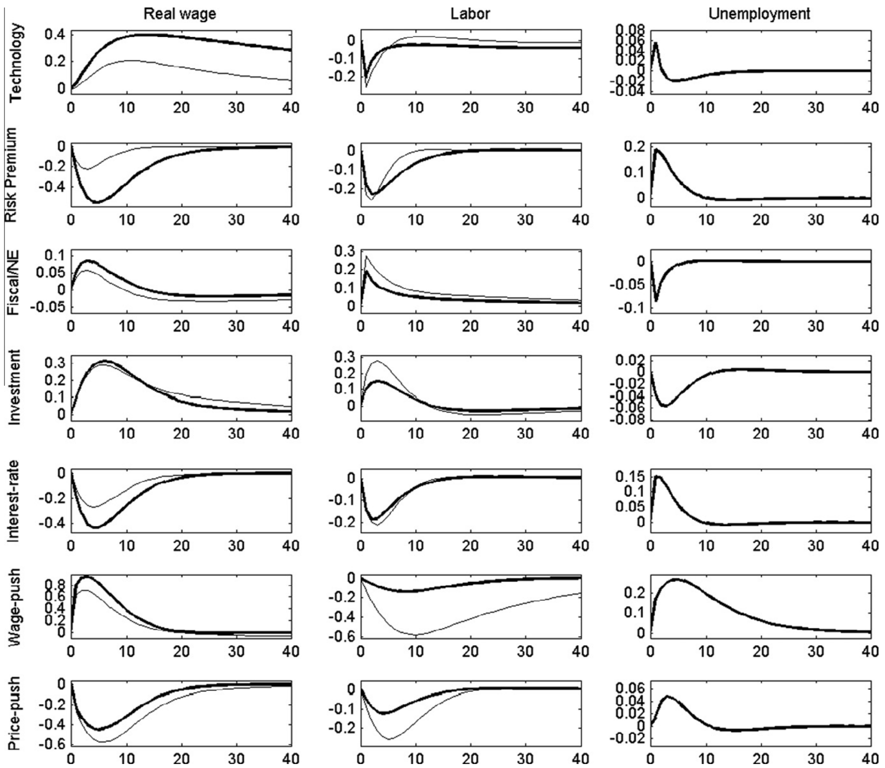

Fig. 2. Impulse responses of the real wage, labor, and the rate of unemployment. CMV model (thick line) and SW model (thin line).

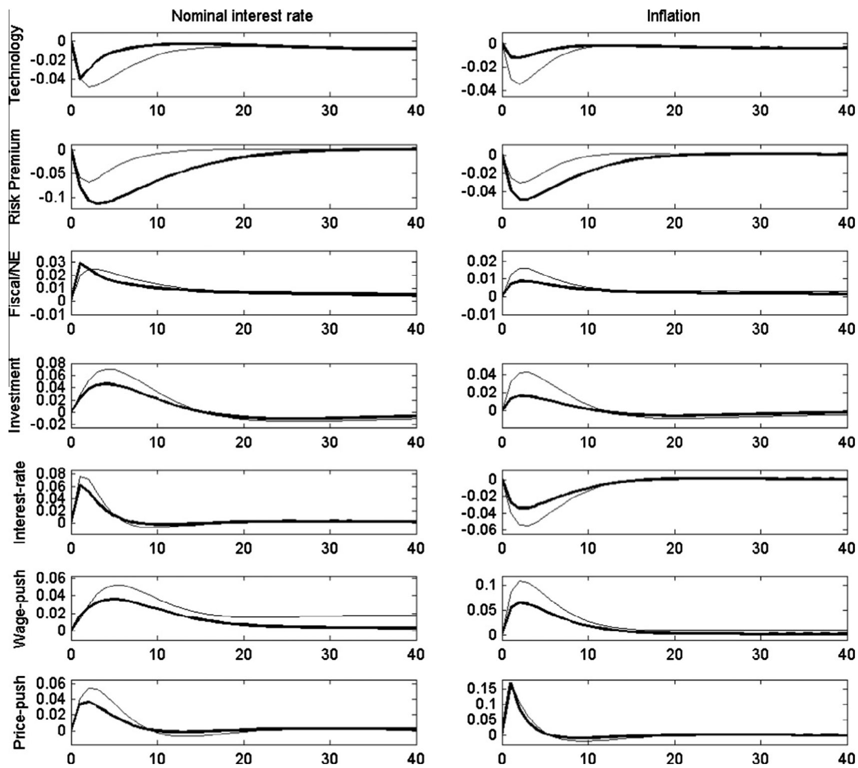

Fig. 3. Impulse response functions of the nominal interest rate and the rate of inflation. CMV model (thick line) and SW model (thin line).

increases output but crowds out investment and consumption in both models due to the consequent monetary policy tightening (see Fig. 3 below). The interest-rate shock has a negative impact on these three variables, as is typical from sticky-price models. Models disagree substantially in the effects of the wage-push shocks. These different effects do not come as a surprise since the interpretation of this shock in the two models, as explained above, is different. Thus, the CMV model provides a slight decrease in output while in the SW model we find a much deeper and more persistent fall in output. Price mark-up shocks are also more contractionary on output, consumption and investment in the SW model, through the implied increase in interest rates in response to higher inflation.

Fig. 2 shows the responses of the labor market variables to the structural shocks. While both the SW and CMV models include log fluctuations of the real wage and labor (employment), the CMV model captures the response of the unemployment rate as well, in contrast to the SW model. The real wage rises after technology, investment, fiscal-net exports and wage-push shocks, whereas it decreases after risk premium, monetary policy and price-push shocks. Meanwhile, employment falls countercyclically in both models when there is a positive technology shock. This is a characteristic response in New-Keynesian models with sticky prices, as discussed in Galí (1999). By contrast, procyclical reactions of employment are always reported after demand-side disturbances such as risk-premium shocks, investment shocks, and fiscal-net exports shocks. Both models imply declines of employment in reaction to price and wage cost-push shocks. However, the fall of employment after a wage-push shock is much deeper and persistent in the SW model than in the CMV model (see Fig. 2), which is consistent with the variance decomposition analysis conducted above.[27]

The reactions of unemployment – only reported in the CMV model – are closely and inversely related to those of employment, as the influence of labor supply variability is small due to the low estimated labor supply elasticity.[28] A positive technology shock increases unemployment only during the quarter of the shock. Demand shocks (risk premium, investment, fiscal-net exports and monetary policy) bring procylical reactions of unemployment, that display quite persistent dynamic patterns. The unemployment rate also raises after both wage and price cost-push shocks, since monetary policy reacts through higher interest rates to these supply-side disturbances.

Fig. 3 shows the responses of the nominal interest rate and inflation. The plots are rather similar across models, although the reactions in the SW model show more amplitude after wage-push, technology, and investment shocks, and less after risk premium shocks. Technology innovations bring a countercyclical decrease of inflation and the nominal interest rate whereas three of the demand shocks (risk premium, investment, and fiscal-net exports) result in procyclical responses of inflation and interest rates. The interest rate shock represents an unexpected monetary policy tightening that brings a realistic U-shaped decline in inflation (as observed in Romer and Romer (2004)). Finally, both wage and price push shocks increase inflation and, as a result, trigger a gradual and persistent increase in the nominal interest rate.

4.4. Dynamic cross-correlation functions

This section studies the ability of the CMV model to reproduce two important comovement patterns observed in US business cycles. First, we examine the dynamic correlations between the rate of unemployment and the output growth rate in a model-to-data comparison (Fig. 4). Then, we assess the capacity of the model to replicate the dynamic cross correlations between inflation and output growth rates (Fig. 5). These figures compare the lead-lag correlation functions in the data with those implied by the CMV model.[29] They also show the $\pm$ two-standard deviation confidence interval bands derived from simulated data obtained from 5000 independent draws for the seven innovations of the CMV model.

Fig. 4 shows the lead and lag correlation structure between output growth and the rate of unemployment observed in actual US data and the corresponding comovement implied by the CMV model within the model-implied statistical confidence interval. The estimated model does not reproduce the positive contemporaneous correlation between the US rates of unemployment and output growth. However, the model replicates the positive correlation between lagged rates of unemployment and current output growth rate (i.e., cases with $j < 0$ in Fig. 4) and the negative correlation between future unemployment rates and current output growth rate (i.e., cases with $j > 0$ in Fig. 4). These two different dynamic patterns between output growth and unemployment observed in US data capture two distinct stages of the business cycle. Thus, when the economy starts to recover from a recession unemployment is high, so high unemployment rates in the early stages of economic recovery anticipates an economic boots. By contrast, when an economy is slowing down, a low output growth rate now is followed by an unemployment rate rise in the near future.

[27] This higher sensitivity to wage-push shocks in the SW model is the result of its particular labor market assumptions. Households must attend firm-specific relative labor demand as constraints in their optimizing programs. In turn, those households that apply the indexation rule with the positive wage shock will suffer from a significant employment cut. This implies contractionary effects on consumption due to the non-separability between labor and consumption in the utility function, which justifies the difference in the response of output growth across models (see Fig. 1).

[28] In log fluctuations from steady-state, labor supply (labor force) can be obtained as the sum of the response of labor plus the response of the rate of unemployment.

[29] Comparing model and data dynamic cross-correlations presents advantages with respect to the comparison between VAR and model impulse responses. In particular, no assumptions are needed about shock identification, while in the VAR impulse response analysis these assumptions may be at odds with the model. Despite these difficulties of mapping between the structural model and the VAR shocks, we estimated a simple VAR and compared the implied impulse-response functions with those implied by our structural model. Results are available upon request.

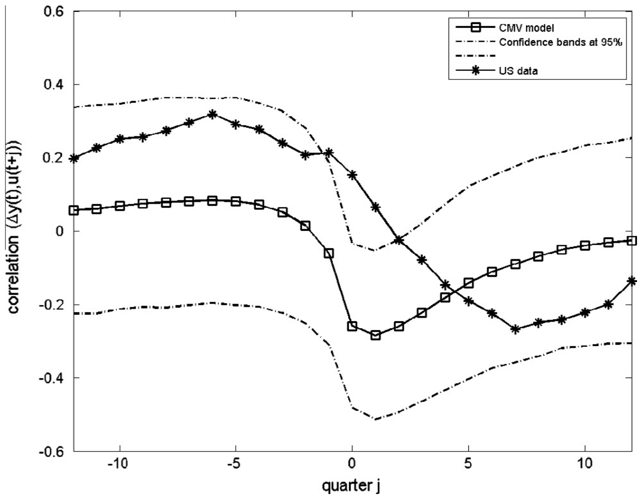

Fig. 4. Dynamic cross-correlation between output growth, Δy_t, and the unemployment rate, u_{t+j}.

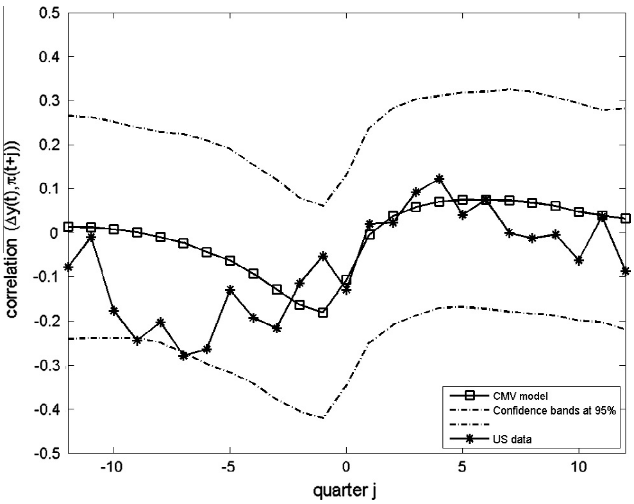

Fig. 5. Dynamic cross-correlation between output growth, Δy_t, and inflation, π_{t+j}.

Similarly, Fig. 5 compares the dynamic comovement patterns between inflation and output growth. Once again the estimated CMV model does a good job in reproducing the lead-lag pattern shape displayed by actual US data.[30] Thus, the model reproduces two stylized facts. First, low lagged inflation anticipates high current output growth (i.e. for $j < 0$). Second, high current output growth anticipates high future inflation 1-4 quarters ahead.

4.5. Robustness analysis

This subsection develops a robustness analysis along two important dimensions. First, we examine whether extending the sample period to include data from the most recent recession affects our main conclusions. Second, we re-estimate the CMV and SW models removing the series of unemployment and employment as observables. In this way, we assess whether the different estimation results found in the two models are mainly due to different model dynamics or are simply the result of using a different labor market observables in estimation. For the sake of brevity, the next two subsections only highlight the main conclusions obtained from this robustness analysis.[31]

[30] Smets and Wouters (2007) also report a good matching to dynamic cross correlations between inflation and Hodrick–Prescott filtered output in their model without unemployment. However, Galí et al. (2012) do not report dynamic correlations between output growth and unemployment.
[31] The full set of estimation results from this robustness analysis are available from the authors upon request.

4.5.1. Extended sample analysis

We re-estimate the two models using the sample period 1984:1–2013:2. The estimation results with the extended sample are rather similar to those found using only data from the Great Moderation with a few exceptions. First, the estimate of the inverse of the Frisch elasticity, $\sigma_l = 5.20$, is larger in the estimated CMV model for the extended sample. The opposite occurs for the SW model ($\sigma_l = 1.47$). Therefore, the difference between the estimates of σ_l across models is larger in the extended sample. Second, the estimate of price stickiness, ξ_p, is similar (0.81) in the two models when considering the extended sample. Thus, the estimated average length of optimal price changes increases from $(1-0.75)^{-1} = 4.0$ quarters during the Great Moderation to $(1-0.81)^{-1} = 5.27$ quarters when data from the Great Recession is also considered. Similarly, the estimated average length of labor contracts increases to $(1-0.84)^{-1} = 6.52$ quarters in the estimated CMV model with the extended sample. Finally, the estimates of wage-push shock parameters in the CMV model change. In particular, the size and persistence of wage-push innovations are lower ($\sigma_w = 1.85, \rho_w = 0.76$) in the extended sample period than in the Great Moderation ($\sigma_w = 3.71, \rho_w = 0.89$). In contrast, the estimate of the moving average parameter, μ_w, is larger in the extended sample than in the Great Moderation-only period (0.53 vs. 0.21) indicating that the recent recession features high-frequency movements of wage shocks.

Regarding the empirical fit of the second-moment statistics in the extended sample, estimation results are similar to those found using the Great Moderation sample. Thus, the CMV model still matches most second-moment statistics better than the SW model. Moreover, the CMV model is able to capture the weak, negative correlation between unemployment and output growth characterizing the extended sample. Furthermore, the variance decomposition analysis of the estimated CMV model shows that the importance of wage-push shocks in explaining unemployment rate fluctuations sharply decreases from 69% in the Great Moderation sample to 35% in the extended sample. By the same token, the importance of demand shocks, such as risk premium and interest rate shocks, increases in the extended sample (to take 33% and 20%, respectively).

4.5.2. Estimation analysis without a labor measure as observable

This sub-section discusses whether the better fit of the CMV model compared to the SW model is a result of the improved model structure or the different data set used (i.e., the estimated CMV uses the unemployment rate as an observable variable whereas the estimated SW considers the level of employment). For this purpose, we re-estimate both models without any labor measure as observable for the Great Moderation sample. We found that a few parameter estimates were sensitive to ignoring a labor measure as observable. The estimate of wage stickiness in the CMV model, $\xi_w = 0.84$, is larger when ignoring the unemployment rate as an observable variable. This value is also larger than the estimated value in the SW model ($\xi_w = 0.62$). The opposite occurs for the size of wage innovation shock, $\sigma_w = 2.04$, which is lower when the unemployment rate is not considered as an observable in the estimation of the CMV model. Nevertheless, this estimated value is still much larger than the one obtained estimating the SW model. Additionally, the volatility of technology innovations, $\sigma_a = 0.08$, and the persistence of technology and risk premium shocks ($\rho_a = 0.86$ and $\rho_b = 0.77$) are much lower when ignoring the rate of unemployment as observable in the estimated CMV model. Finally, the estimated CMV model parameter values of the cost of adjusting capital, $\varphi = 6.03$, capital adjustment costs, $\psi = 0.68$, the inverse of Frisch elasticity, $\sigma_l = 2.51$, and the steady-state price mark-up, $\Phi = 1.39$, were similar to the estimates found for the SW model either using employment as observable or ignoring it.

Regarding the empirical fit of second-moment statistics when not including any labor measure in the estimation of the two models, the CMV still performs better than the SW model for most statistics, but the overall fit-improvement provided by the CMV model in this case is rather small. Three noticeable exceptions are the observed volatility of employment, the contemporaneous correlation between inflation and output growth, and the autocorrelation of the first-difference of the real wage which are better fitted by the CMV model.

From this analysis, we conclude that the two models provide different characterizations of labor market dynamics, but these differences are stressed when a labor measure is included in the set of observable variables used in the estimation procedure. Using observables from the labor market in the estimation helps identify the parameters describing the labor market such the inverse of Frisch elasticity, wage stickiness, and wage-push shock parameters together with those parameters describing capital adjustments costs (φ and ψ), since these are linked to labor demand decisions in models with endogenous capital.

5. Conclusions

This paper introduces a model that combines elements of Smets and Wouters (2007) and Casares (2010) in a way that incorporates unemployment as excess supply of labor in a medium-scale New-Keynesian model. The alternative labor market assumptions have implications for the real wage equation (where the real wage is inversely related to the rate of unemployment) and also for the New Keynesian Phillips curve (where the slope coefficient is lower and it depends upon the level of wage stickiness).

The structural model parameters were estimated with Bayesian techniques and then compared to the estimates of the benchmark New-Keynesian model of Smets and Wouters (2007). Most parameter estimates are quite similar across models. The only substantial differences are that in the model with unemployment wages are stickier (longer average length of labor contracts), the labor supply curve is less elastic, and the elasticity of capital adjustment costs is lower. The empirical comparison also shows that both models do a similar job in reproducing many of the features characterizing recent US business cycles. Importantly, our model with unemployment is able to replicate volatility and persistence of the US unemployment rate, and its lead-lag patterns with output growth fluctuations. The impulse-response functions show that the rate of unemployment reacts in a countercyclical way to demand shocks and price-push shocks, whereas the response is initially procyclical and later countercyclical after productivity innovations and mildly countercyclical after wage-push innovations.

Our results also indicate that fluctuations in the unemployment rate are mainly driven by wage inflation shocks, with risk premium shocks and monetary shocks playing a secondary role. Meanwhile, technology shocks have almost no role. Regarding output variability, changes in output are driven by technology innovations and various demand shocks (risk premium, fiscal/net exports, investment and monetary shocks to jointly account for half of total variability). The model without unemployment gives less influence to technology shocks and more to cost-push shocks.

Appendix A. Labor supply and labor demand of type *i*

A.1. Labor supply

Households maximize intertemporal utility subject to a budget constraint. The representative household provides all types of labor services and receives instantaneous utility given by the following function

$$\left[\frac{1}{1-\sigma_c}\left(C_t - hC^A_{t-1}\right)^{1-\sigma_c}\right] \exp\left(\frac{\sigma_c-1}{1+\sigma_l}\int_0^1 \left(L^s_t(i)\right)^{1+\sigma_l} di\right),$$

where $\sigma_c, \sigma_l > 0, C_t$ is current consumption of the household, C^A_{t-1} is lagged aggregate consumption and $L^s_t(i)$ is the supply of labor of the type employed in the i-th firm. The household budget constraint is

$$\int_0^1 \frac{(1-u_t(i))W_t(i)L^s_t(i)}{P_t} di + \frac{R^k_t Z_t K_{t-1}}{P_t} - a(Z_t)K_{t-1} + \frac{B_{t-1}}{P_{t-1}(1+\pi_t)} + \frac{Div_t}{P_t} = C_t + I_t + \frac{B_t}{\exp(\varepsilon^b_t)(1+R_t)P_t} - T_t.$$

The first order conditions for consumption and labor supply of type i that result from the household optimizing program are

$$\left(C_t - hC^A_{t-1}\right)^{-\sigma_c} \exp\left(\frac{\sigma_c-1}{1+\sigma_l}\int_0^1 \left(L^s_t(i)\right)^{1+\sigma_l} di\right) - \Xi_t = 0, \qquad C^{foc}_t$$

$$-L_t(i)^{\sigma_l}\left[\left(C_t - hC^A_{t-1}\right)^{1-\sigma_c}\right] \exp\left(\frac{\sigma_c-1}{1+\sigma_l}\int_0^1 \left(L^s_t(i)\right)^{1+\sigma_l} di\right) + \Xi_t \frac{(1-u_t(i))W_t(i)}{P_t} = 0, \qquad (L^{foc}_t(i))$$

where Ξ_t is the Lagrange multiplier of the budget constraint in period t. Inserting (C^{foc}_t) in $(L^{foc}_t(i))$ and rearranging terms leads to the optimal supply of i-type labor

$$L^s_t(i) = \left(\frac{(1-u_t(i))\frac{W_t(i)}{P_t}}{C_t - hC^A_{t-1}}\right)^{1/\sigma_l},$$

which in log-linear terms is

$$l^s_t(i) = \frac{1}{\sigma_l}\left(\log W_t(i) - p_t - \frac{1}{1-u^n}(u_t(i) - u^n) - \frac{1}{(1-h/\gamma)}(c_t - (h/\gamma)c_{t-1})\right).$$

Loglinearizing and aggregating across all types of labor services yields the relative labor supply equation

$$l^s_t(i) - l^s_t = \frac{1}{\sigma_l}\left(\widetilde{W}_t(i) - \frac{1}{1-u^n}(u_t(i) - u_t)\right).$$

A.2. Labor demand

From the optimizing behavior, the labor demand of firm i is obtained from the loglinear version of the condition that equates the ratio of marginal products of labor and capital to the relative input prices, $\frac{1-\alpha}{\alpha}\frac{K^d_t(i)}{L^d_t(i)} = \frac{W_t(i)/P_t}{r^k_t}$,

$$l^d_t(i) = k^d_t(i) - \log W_t(i) + p_t + \log r^k_t.$$

As in Smets and Wouters (2007), the loglinearized production function, with technology shocks ε^a_t, is $y_t(i) = (1-\alpha)l^d_t(i) + \alpha k^d_t(i) + \varepsilon^a_t$, which determines the log of firm-specific capital demand

$$k_t^d(i) = \frac{1}{\alpha}\left(y_t(i) - (1-\alpha)l_t^d(i) - \varepsilon_t^a\right).$$

Substituting the value of $k_t^d(i)$ from the last expression in the labor demand equation and rearranging terms results in

$$l_t^d(i) = y_t(i) - \alpha(\log W_t(i) - p_t) + \alpha \log r_t^k - \varepsilon_t^a.$$

As discussed in Woodford (2003, page 168), the Kimball (1995) scheme for the aggregation of goods – also used in the Smets and Wouters (2007)'s model –, yields a log approximation of demand-determined relative output that is inversely related to the relative price,

$$y_t(i) = y_t - \theta\widetilde{P}_t(i),$$

where $\theta > 0$ defines the elasticity of demand and the relative price is $\widetilde{P}_t(i) = \log P_t(i) - \log P_t = \log P_t(i) - \int_0^1 \log P_t(i)di$. Inserting $y_t(i) = y_t - \theta\widetilde{P}_t(i)$ in the labor demand equation gives

$$l_t^d(i) = y_t - \theta\widetilde{P}_t(i) - \alpha(\log W_t(i) - p_t) + \alpha \log r_t^k - \varepsilon_t^a.$$

Summing up across all firms and taking the difference between firm-specific and aggregate values results in a firm-specific labor demand equation

$$l_t^d(i) = -\theta\widetilde{P}_t(i) - \alpha\widetilde{W}_t(i) + l_t,$$

which introduces l_t as the log deviation from steady state of demand-determined employment obtained from the aggregation of log deviations on firm-specific labor demand $l_t = \int_0^1 l_t^d(i)di$.

Appendix B. Derivation of Eq. (9): The dynamics of relative wages

Eq. (3) in the text governs wage setting with the intertemporal targeting of $l_{t+j}^s(i) - l_{t+j}^d(i)$. Recalling labor supply and labor demand schedules (Eqs. (6) and (8) in the main text, respectively) for any $t+j$ period,

$$l_{t+j}^s(i) - l_{t+j}^d(i) = \frac{1}{\sigma_l}\left(\widetilde{W}_{t+j}(i) - \left(l_{t+j}^s(i) - l_{t+j}^d(i) - l_{t+j}^s + l_{t+j}\right)\right) + l_{t+j}^s - l_{t+j} + \theta\widetilde{P}_{t+j}(i) + \alpha\widetilde{W}_{t+j}(i), \tag{A1}$$

which can be simplified as follows

$$l_{t+j}^s(i) - l_{t+j}^d(i) = \frac{\sigma_l^{-1} + \alpha}{1+\sigma_l^{-1}}\widetilde{W}_{t+j}(i) + \frac{\theta}{1+\sigma_l^{-1}}\widetilde{P}_{t+j}(i) + \left(l_{t+j}^s - l_{t+j}\right). \tag{A2}$$

Substituting both (A2) and the definition $u_t = u^n + (1-u^n)(l_t^s - l_t)$, in (A1) yields

$$E_t^{\xi_w}\sum_{j=0}^{\infty}\overline{\beta}^j\xi_w^j\left[\frac{\sigma_l^{-1}+\alpha}{(1+1/\sigma_l)}\widetilde{W}_{t+j}(i) + (1-u^n)^{-1}(u_{t+j} - u^n) + \frac{\theta}{1+1/\sigma_l}\widetilde{P}_{t+j}(i)\right] = 0. \tag{A3}$$

Using a log-linear approximation to the wage indexation rule (8), the conditional expectation of future relative wages that cannot be revised to accommodate labor supply/demand changes becomes

$$E_t^{\xi_w}\widetilde{W}_{t+j}(i) = \widetilde{W}_t^*(i) + E_t\sum_{k=1}^{j}\left(\iota_w\pi_{t+k-1} + (1-\iota_w)\varepsilon_{t+k}^w - \pi_{t+k}^w\right), \tag{A4}$$

where $\widetilde{W}_t^*(i) = \log W_t^*(i) - \log W_t$ is the log difference between the natural-rate wage set in the firm $i, W_t^*(i)$, and the aggregate wage, W_t, and $\pi_{t+j}^w = \log W_{t+j} - \log W_{t+j-1}$ is wage inflation in period $t+j$. The relative wage consistent with (A3) and (A4) is

$$\widetilde{W}_t^*(i) = -\frac{(1-\overline{\beta}\xi_w)}{\sigma_l^{-1}+\alpha}E_t^{\xi_w}\sum_{j=0}^{\infty}\overline{\beta}^j\xi_w^j\left(\frac{1+1/\sigma_l}{1-u^n}(u_{t+j}-u^n) + \theta\widetilde{P}_{t+j}(i)\right) - E_t\sum_{j=1}^{\infty}\overline{\beta}^j\xi_w^j\left(\iota_w\pi_{t+j-1} + (1-\iota_w)\varepsilon_{t+k}^w - \pi_{t+j}^w\right), \tag{A5}$$

which implies that the relative wage depends negatively on the stream of the economy-wide rate of unemployment and also negatively on the stream of relative prices. As in Casares (2010), let us introduce the following guess: relative optimal pricing and relative wage setting are related as follows

$$\widetilde{P}_t^*(i) = \widetilde{P}_t^* + \tau_1\widetilde{W}_{t-1}(i),$$
$$\widetilde{W}_t^*(i) = \widetilde{W}_t^* - \tau_2\widetilde{P}_t(i),$$

where $\widetilde{P}_t^*(i) = \log P_t^*(i) - \log P_t$ is the firm-specific relative optimal price, $\widetilde{P}_t^* = \int_0^1 \log P_t^*(i)di - \log P_t$ is the aggregate relative optimal price, $\widetilde{W}_t^* = \int_0^1 \log W_t^*(i)di - \log W_t$ is the aggregate relative wage, and τ_1 and τ_2 are coefficients to be determined by equilibrium conditions. We want to express the expected stream of relative prices, $E_t^{\xi_w}\sum_{j=0}^{\infty}\overline{\beta}^j\xi_w^j\widetilde{P}_{t+j}(i)$, as a function of the current relative prices in order to have a log-linear relation of the type $\widetilde{W}_t^*(i) = \widetilde{W}_t^* - \tau_2 \widetilde{P}_t(i)$. Beginning with $\widetilde{P}_{t+1}(i)$, the Calvo aggregation scheme implies

$$E_t^{\xi_w}\widetilde{P}_{t+1}(i) = \xi_p\left(\widetilde{P}_t(i) + \iota_p\pi_t - E_t\pi_{t+1}\right) + (1-\xi_p)E_t^{\xi_w}\widetilde{P}^*_{t+1}(i), \tag{A6}$$

where the second term is $E_t^{\xi_w}\widetilde{P}^*_{t+1}(i) = E_t\widetilde{P}^*_{t+1} + \tau_1\widetilde{W}^*_t(i)$ using our guess on relative prices for $t+1$ conditional on having a natural-rate wage contract set in t. Using that information in (A6) yields

$$E_t^{\xi_w}\widetilde{P}_{t+1}(i) = \xi_p\left(\widetilde{P}_t(i) + \iota_p\pi_t - E_t\pi_{t+1}\right) + (1-\xi_p)\left(E_t^{\xi_w}\widetilde{P}^*_{t+1} + \tau_1\widetilde{W}^*_t(i)\right). \tag{A7}$$

The Calvo aggregation scheme with price indexation implies $\widetilde{P}^*_{t+1} = \frac{\xi_p}{1-\xi_p}(\pi_{t+1} - \iota_p\pi_t)$. Taking rational expectations and substituting in (A7) give

$$E_t^{\xi_w}\widetilde{P}_{t+1}(i) = \xi_p\widetilde{P}_t(i) + \tau_1(1-\xi_p)\widetilde{W}^*_t(i). \tag{A8}$$

Analogously to (A6), $E_t^{\xi_w}\widetilde{P}_{t+2}(i)$ is a linear combination of non-adjusted relative prices and optimal relative prices:

$$E_t^{\xi_w}\widetilde{P}_{t+2}(i) = \xi_p\left(E_t^{\xi_w}\widetilde{P}_{t+1}(i) + \iota_pE_t\pi_{t+1} - E_t\pi_{t+2}\right) + (1-\xi_p)E_t^{\xi_w}\widetilde{P}^*_{t+2}(i),$$

where using (A8) for $E_t^{\xi_w}\widetilde{P}_{t+1}(i)$ leads to

$$E_t^{\xi_w}\widetilde{P}_{t+2}(i) = \xi_p\left(\xi_p\widetilde{P}_t(i) + \tau_1(1-\xi_p)\widetilde{W}^*_t(i) + \iota_pE_t\pi_{t+1} - E_t\pi_{t+2}\right) + (1-\xi_p)E_t^{\xi_w}\widetilde{P}^*_{t+2}(i). \tag{A9}$$

The relationship between relative prices and wages in period $t+2$ conditional on the lack of wage resetting is $E_t^{\xi_w}\widetilde{P}^*_{t+2}(i) = E_t\widetilde{P}^*_{t+2} + \tau_1E_t^{\xi_w}\widetilde{W}^*_{t+1}(i) = E_t\widetilde{P}^*_{t+2} + \tau_1\left(\widetilde{W}^*_t(i) + \iota_w\pi_t + (1-\iota_w)E_t\varepsilon^w_{t+1} - E_t\pi^w_{t+1}\right)$, that inserted in (A9)

$$E_t^{\xi_w}\widetilde{P}_{t+2}(i) = \xi_p\left(\xi_p\widetilde{P}_t(i) + \tau_1(1-\xi_p)\widetilde{W}^*_t(i) + \iota_pE_t\pi_{t+1} - E_t\pi_{t+2}\right)$$
$$+ (1-\xi_p)\left(E_t\widetilde{P}^*_{t+2} + \tau_1\left(\widetilde{W}^*_t(i) + \iota_w\pi_t + (1-\iota_w)E_t\varepsilon^w_{t+1} - E_t\pi^w_{t+1}\right)\right),$$

where using $\widetilde{P}^*_{t+2} = \frac{\xi_p}{1-\xi_p}(\pi_{t+2} - \iota_p\pi_{t+1})$ simplifies to

$$E_t^{\xi_w}\widetilde{P}_{t+2}(i) = \xi_p^2\widetilde{P}_t(i) + \tau_1\left(1-\xi_p^2\right)\widetilde{W}^*_t(i) - \tau_1(1-\xi_p)\left(E_t\pi^w_{t+1} - \iota_w\pi_t - (1-\iota_w)E_t\varepsilon^w_{t+1}\right). \tag{A10}$$

A generalization of (A8) and (A10) results in the following rule:

$$E_t^{\xi_w}\widetilde{P}_{t+j}(i) = \xi_p^j\widetilde{P}_t(i) + \tau_1\left(1-\xi_p^j\right)\widetilde{W}^*_t(i) - \tau_1E_t\sum_{k=1}^{j-1}\left(1-\xi_p^{j-k}\right)\left(\pi^w_{t+k} - \iota_w\pi_{t+k-1} - (1-\iota_w)E_t\varepsilon^w_{t+k}\right),$$

implying the following expected sum of discounted relative prices:

$$E_t^{\xi_w}\sum_{j=0}^{\infty}\overline{\beta}^j\xi_w^j\widetilde{P}_{t+j}(i) = \frac{1}{1-\overline{\beta}\xi_w\xi_p}\widetilde{P}_t(i)$$
$$+ \tau_1\left(\frac{\overline{\beta}\xi_w}{1-\overline{\beta}\xi_w} - \frac{\overline{\beta}\xi_w\xi_p}{1-\overline{\beta}\xi_w\xi_p}\right)\left(\widetilde{W}^*_t(i) - E_t\sum_{j=1}^{\infty}\overline{\beta}^j\xi_w^j\left(\pi^w_{t+j} - \iota_w\pi_{t+j-1} - (1-\iota_w)E_t\varepsilon^w_{t+j}\right)\right). \tag{A11}$$

Substituting (A11) in the relative wage equation (A5), we obtain:

$$(1+\Lambda)\widetilde{W}^*_t(i) = -\frac{\theta(1-\overline{\beta}\xi_w)}{(\sigma_l^{-1}+\alpha)(1-\overline{\beta}\xi_w\xi_p)}\widetilde{P}_t(i) - \frac{(1-\overline{\beta}\xi_w)(1+1/\sigma_l)}{(\sigma_l^{-1}+\alpha)(1-u^n)}E_t\sum_{j=0}^{\infty}\overline{\beta}^j\xi_w^j(u_{t+j} - u^n)$$
$$+ (1+\Lambda)E_t\sum_{j=1}^{\infty}\overline{\beta}^j\xi_w^j\left(\pi^w_{t+j} - \iota_w\pi_{t+j-1} - (1-\iota_w)\varepsilon^w_{t+j}\right), \tag{A12}$$

with $\Lambda = \frac{\tau_1\overline{\beta}\xi_w\theta}{\sigma_l^{-1}+\alpha}\left(1 - \frac{\xi_p(1-\overline{\beta}\xi_w)}{1-\overline{\beta}\xi_w\xi_p}\right)$. Equation (A12) proves right the proposed linear relation $\widetilde{W}^*_t(i) = \widetilde{W}^*_t - \tau_2\widetilde{P}_t(i)$, with the following analytical solution for τ_2

$$\tau_2 = \frac{\theta(1-\overline{\beta}\xi_w)}{(\sigma_l^{-1}+\alpha)(1-\overline{\beta}\xi_w\xi_p)(1+\Lambda)},$$

and the following expression for the aggregate relative wage set in period t (Eq. (9) in the main text)

$$\widetilde{W}^*_t = -\frac{(1-\overline{\beta}\xi_w)(1+\sigma_l^{-1})}{(\sigma_l^{-1}+\alpha)(1-u^n)(1+\Lambda)}E_t\sum_{j=0}^{\infty}\overline{\beta}^j\xi_w^j(u_{t+j} - u^n) + E_t\sum_{j=1}^{\infty}\overline{\beta}^j\xi_w^j\left(\pi^w_{t+j} - \iota_w\pi_{t+j-1} - (1-\iota_w)\varepsilon^w_{t+j}\right).$$

Appendix C. Derivation of Eq. (19) on the dynamics of average relative prices

As described in the main text indicates, the loglinearized relative price $\widetilde{P}_t^*(i)$ depends upon terms on relative wages $\widetilde{W}_{t+j}(i)$ as follows

$$\widetilde{P}_t^*(i) = A(1-\overline{\beta}\xi_p)E_t^{\xi_p}\sum_{j=0}^{\infty}\overline{\beta}^j\xi_p^j\left(mc_{t+j} + (1-\alpha)\widetilde{W}_{t+j}(i) + \lambda_{t+j}^p\right) + E_t\sum_{j=1}^{\infty}\overline{\beta}^j\xi_p^j\left(\pi_{t+j} - \iota_p\pi_{t+j-1}^p\right). \tag{A13}$$

To be consistent with the value of the undetermined coefficient τ_1 implied by $\widetilde{P}_t^*(i) = \widetilde{P}_t^* + \tau_1\widetilde{W}_{t-1}(i)$, we must relate $E_t^{\xi_p}\sum_{j=0}^{\infty}\overline{\beta}^j\xi_p^j\widetilde{W}_{t+j}(i)$ to $\widetilde{W}_{t-1}(i)$. The Calvo scheme applied for wage setting in period t results in

$$\widetilde{W}_t(i) = \xi_w\left(\widetilde{W}_{t-1}(i) + \iota_w\pi_{t-1} + (1-\iota_w)\varepsilon_t^w - \pi_t^w\right) + (1-\xi_w)\widetilde{W}_t^*(i).$$

Using the proposed conjecture, $\widetilde{W}_t^*(i) = \widetilde{W}_t^* - \tau_2\widetilde{P}_t(i)$, conditional on optimal pricing in period t allows us to write $\widetilde{W}_t^*(i)$ depending upon the aggregate relative value of new labor contracts, $\widetilde{W}_t^*$, and also upon the relative optimal price, $\widetilde{P}_t^*(i)$

$$\widetilde{W}_t^*(i) = \widetilde{W}_t^* - \tau_2\widetilde{P}_t^*(i),$$

which can be inserted in the previous expression to reach

$$\widetilde{W}_t(i) = \xi_w\left(\widetilde{W}_{t-1}(i) + \iota_w\pi_{t-1} + (1-\iota_w)\varepsilon_t^w - \pi_t^w\right) + (1-\xi_w)\left(\widetilde{W}_t^* - \tau_2\widetilde{P}_t^*(i)\right).$$

Recalling that $\widetilde{W}_t^* = \frac{\xi_w}{1-\xi_w}(\pi_t^w - \iota_w\pi_{t-1} - (1-\iota_w)\varepsilon_t^w)$ from Calvo-type sticky wages, and cancelling terms, we transform the previous expression into

$$\widetilde{W}_t(i) = \xi_w\widetilde{W}_{t-1}(i) - \tau_2(1-\xi_w)\widetilde{P}_t^*(i). \tag{A14}$$

Repeating the procedure one period ahead for $E_t^{\xi_p}\widetilde{W}_{t+1}(i)$, we have

$$E_t^{\xi_p}\widetilde{W}_{t+1}(i) = \xi_w\left(\widetilde{W}_t(i) + \iota_w\pi_t + (1-\iota_w)E_t\varepsilon_{t+1}^w - E_t\pi_{t+1}^w\right) + (1-\xi_w)E_t^{\xi_p}\widetilde{W}_{t+1}^*(i). \tag{A15}$$

Using $\widetilde{W}_{t+1}^*(i) = \widetilde{W}_{t+1}^* - \tau_2\widetilde{P}_{t+1}(i)$ conditional on no-optimal pricing in $t+1$ yields

$$E_t^{\xi_p}\widetilde{W}_{t+1}^*(i) = \widetilde{W}_{t+1}^* - \tau_2\left(\widetilde{P}_t^*(i) + \iota_p\pi_t - E_t\pi_{t+1}\right),$$

which can be inserted in (A15) together with (A14) and also $\widetilde{W}_{t+1}^* = \frac{\xi_w}{1-\xi_w}(\pi_{t+1}^w - \iota_w\pi_t - (1-\iota_w)\varepsilon_{t+1}^w)$ to obtain (after dropping terms that cancel out)

$$E_t^{\xi_p}\widetilde{W}_{t+1}(i) = \xi_w^2\widetilde{W}_t(i) - \tau_2(1-\xi_w^2)\widetilde{P}_t^*(i) + \tau_2(1-\xi_w)(E_t\pi_{t+1} - \iota_p\pi_t). \tag{A16}$$

A generalization of (A10) and (A16) for a $t+j$ future period gives the following expression

$$E_t^{\xi_p}\widetilde{W}_{t+j}(i) = \xi_w^{j+1}\widetilde{W}_{t-1}(i) - \tau_2\left(1-\xi_w^{j+1}\right)\widetilde{P}_t^*(i) + \tau_2\left(1-\xi_w^{j-k+1}\right)E_t\sum_{k=1}^{j}(\pi_{t+k} - \iota_p\pi_{t+k-1}). \tag{A17}$$

Using (A17), the expected sum of the stream of conditional relative wages becomes

$$E_t^{\xi_p}\sum_{j=0}^{\infty}\overline{\beta}^j\xi_p^j\widetilde{W}_{t+j}(i) = \frac{\xi_w}{1-\overline{\beta}\xi_w\xi_p}\widetilde{W}_{t-1}(i) - \tau_2\left(\frac{1}{1-\overline{\beta}\xi_p} - \frac{\xi_w}{1-\overline{\beta}\xi_w\xi_p}\right)\widetilde{P}_t^*(i)$$
$$+ \tau_2\left(\frac{1}{1-\overline{\beta}\xi_p} - \frac{\xi_w}{1-\overline{\beta}\xi_w\xi_p}\right)E_t\sum_{j=1}^{\infty}\overline{\beta}^j\xi_p^j(\pi_{t+j} - \iota_p\pi_{t+j-1}). \tag{A18}$$

Substituting (A18) in (A13) yields

$$(1+\Theta)\widetilde{P}_t^*(i) = \frac{A(1-\alpha)(1-\overline{\beta}\xi_p)\xi_w}{1-\overline{\beta}\xi_w\xi_p}\widetilde{W}_{t-1}(i) + A(1-\overline{\beta}\xi_p)E_t\sum_{j=0}^{\infty}\overline{\beta}^j\xi_p^j\left(mc_{t+j} + \lambda_{t+j}^p\right) + (1+\Theta)E_t\sum_{j=1}^{\infty}\overline{\beta}^j\xi_p^j(\pi_{t+j} - \iota_p\pi_{t+j-1}),$$

where $\Theta = \tau_2A(1-\alpha)\left(1 - \frac{(1-\overline{\beta}\xi_p)\xi_w}{1-\overline{\beta}\xi_p\xi_w}\right)$. The last result validates $\widetilde{P}_t^*(i) = \widetilde{P}_t^* + \tau_1\widetilde{W}_{t-1}(i)$ with τ_1 given by

$$\tau_1 = \frac{A(1-\alpha)(1-\overline{\beta}\xi_p)\xi_w}{(1-\overline{\beta}\xi_p\xi_w)(1+\Theta)},$$

and implies the following dynamic equation for average optimal prices (Eq. (19) in the main text)

$$\widetilde{P}_t^* = \frac{A(1-\overline{\beta}\xi_p)}{1+\Theta}E_t\sum_{j=0}^{\infty}\overline{\beta}^j\xi_p^j\left(mc_{t+j} + \lambda_{t+j}^p\right) + E_t\sum_{j=1}^{\infty}\overline{\beta}^j\xi_p^j(\pi_{t+j} - \iota_p\pi_{t+j-1}).$$

Appendix D. New-Keynesian model with unemployment as excess supply of labor and variable capital

Set of log-linearized dynamic equations:

- Aggregate resource constraint:

$$y_t = c_y c_t + i_y i_t + z_y z_t + \varepsilon_t^g, \tag{A24}$$

where $c_y = \frac{C}{Y} = 1 - g_y - i_y, i_y = \frac{I}{Y} = (\gamma - 1 + \delta)\frac{K}{Y}$, and $z_y = r^k \frac{K}{Y}$ are steady-state ratios. As in Smets and Wouters (2007), the depreciation rate and the exogenous spending-GDP ratio are fixed in the estimation procedure at $\delta = 0.025$ and $g_y = 0.18$.

- Consumption equation:

$$c_t = c_1 c_{t-1} + (1 - c_1)E_t c_{t+1} + c_2(l_t - E_t l_{t+1}) - c_3(R_t - E_t \pi_{t+1}) + \varepsilon_t^b, \tag{A25}$$

where $c_1 = \frac{h/\gamma}{1+h/\gamma}, c_2 = \frac{[(\sigma_c-1)wL/C]}{\sigma_c(1+h/\gamma)}$ and $c_3 = \frac{(1-h/\gamma)}{\sigma_c(1+h/\gamma)}$.

- Investment equation:

$$i_t = i_1 i_{t-1} + (1 - i_1)E_t i_{t+1} + i_2 q_t + \varepsilon_t^i, \tag{A26}$$

where $i_1 = \frac{1}{1+\overline{\beta}}$, and $i_2 = \frac{1}{(1+\overline{\beta})\gamma^2\varphi}$ with $\overline{\beta} = \beta\gamma^{(1-\sigma_c)}$.

- Arbitrage condition (value of capital, q_t):

$$q_t = q_1 E_t q_{t+1} + (1 - q_1)E_t r_{t+1}^k - \left(R_t - E_t\pi_{t+1} - \left(\frac{1 - h/\gamma}{\sigma_c(1 + h/\gamma)}\right)^{-1} \varepsilon_t^b\right), \tag{A27}$$

where $q_1 = \overline{\beta}\gamma^{-1}(1 - \delta) = \frac{(1-\delta)}{(r^k+1-\delta)}$.

- Log-linearized aggregate production function:

$$y_t = \phi_p(\alpha k_t^s + (1 - \alpha)l_t + \varepsilon_t^a), \tag{A28}$$

where $\phi_p = 1 + \frac{\phi}{Y} = 1 + \frac{\text{Steady state fixed cost}}{Y}$ and α is the capital-share in the production function.[32]

- Effective capital (with one period time-to-build):

$$k_t^s = k_{t-1} + z_t. \tag{A29}$$

- Capital utilization:

$$z_t = z_1 \log r_t^k, \tag{A30}$$

where $z_1 = \frac{1-\psi}{\psi}$.

- Capital accumulation equation:

$$k_t = k_1 k_{t-1} + (1 - k_1)i_t + k_2 \varepsilon_t^i, \tag{A31}$$

where $k_1 = \frac{1-\delta}{\gamma}$ and $k_2 = \left(1 - \frac{1-\delta}{\gamma}\right)(1 + \overline{\beta})\gamma^2\varphi$.

- Price mark-up (negative of the log of the real marginal cost):

$$\mu_t^p = mpl_t - w_t = \alpha(k_t^s - l_t) + \varepsilon_t^a - w_t. \tag{A32}$$

- New-Keynesian Phillips curve (price inflation dynamics):

$$\pi_t = \pi_1 \pi_{t-1} + \pi_2 E_t \pi_{t+1} - \pi_3 \mu_t^p + \varepsilon_t^p, \tag{A33}$$

where $\pi_1 = \frac{\iota_p}{1+\overline{\beta}\iota_p}, \pi_2 = \frac{\overline{\beta}}{1+\overline{\beta}\iota_p}$, and $\pi_3 = \frac{1}{1+\overline{\beta}\iota_p}\left[\frac{(1-\overline{\beta}\xi_p)(1-\xi_p)}{\xi_p((\phi_p-1)\varepsilon_p+1)(1+\Theta)}\right]$ with $\Theta = \tau_2\left(1 - \frac{(1-\overline{\beta}\xi_p)\xi_w}{1-\overline{\beta}\xi_p\xi_w}\right)$. The coefficient of the curvature of the Kimball goods market aggregator is fixed in the estimation procedure at $\varepsilon_p = 10$ as in Smets and Wouters (2007).[33]

- Optimal demand for capital by firms

$$-(k_t^s - l_t) + w_t = \log r_t^k. \tag{A34}$$

- Unemployment rate equation, using $u_t = u^n + (1 - u^n)(l_t^s - l_t)$ and the aggregation of log supply of labor at firm level, $L_t^s(i) = \left(\frac{(1-u_t(i))\frac{W_t(i)}{P_t}}{C_t - hC_{t-1}^A}\right)^{1/\sigma_l}$,

[32] From the zero profit condition in steady-state, it should be noticed that ϕ_p also represents the value of the steady-state price mark-up.

[33] Using Dixit–Stiglitz output aggregators as in Smets and Wouters (2003) or Christiano et al. (2005), the slope coefficient on the price mark-up changes to $\pi_3 = \frac{1}{1+\beta\gamma^{(1-\sigma_c)}\iota_p}\left[\frac{(1-\overline{\beta}\xi_p)(1-\xi_p)}{\xi_p(1+\Theta)}\right]$.

$$\left(1+\frac{1}{\sigma_l}\right)(u_t-u^n)=\frac{(1-u^n)}{\sigma_l}w_t-\frac{(1-u^n)}{\sigma_l(1-h/\gamma)}(c_t-(h/\gamma)c_{t-1})-(1-u^n)l_t, \tag{A35}$$

where l_t^s denotes log-fluctuations of the aggregate labor supply and l_t denotes log-fluctuations of demand-determined aggregate labor.

- Real wage dynamic equation:

$$w_t=w_1w_{t-1}+(1-w_1)(E_tw_{t+1}+E_t\pi_{t+1})-w_2\pi_t+w_3\pi_{t-1}-w_4(u_t-u^n)+w_5\left(\varepsilon_t^w-\overline{\beta}E_t\varepsilon_{t+1}^w\right), \tag{A36}$$

where $w_1=\frac{1}{1+\overline{\beta}}, w_2=\frac{1+\overline{\beta}\iota_w}{1+\overline{\beta}}, w_3=\frac{\iota_w}{1+\overline{\beta}}$,

$w_4=\frac{1}{1+\overline{\beta}}\frac{\left(1-\overline{\beta}\xi_w\right)(1-\xi_w)\left(1+\sigma_l^{-1}\right)}{\xi_w\left(\sigma_l^{-1}+\alpha\right)(1-u^n)(1+\Lambda)}$ with $\Lambda=\frac{\tau_1\overline{\beta}\xi_w\theta}{\sigma_l^{-1}+\alpha}\left(1-\frac{\xi_p\left(1-\overline{\beta}\xi_w\right)}{1-\overline{\beta}\xi_w\xi_p}\right)$, and $w_5=\frac{1-\iota_w}{1+\overline{\beta}}$.

- Monetary policy rule, a Taylor-type rule for nominal interest rate management:

$$R_t=\rho R_{t-1}+(1-\rho)[r_\pi\pi_t+r_Y(y_t-y_t^p)]+r_{\triangle y}\left[(y_t-y_t^p)-\left(y_{t-1}-y_{t-1}^p\right)\right]+\varepsilon_t^R. \tag{A37}$$

Potential (natural-rate) variables are obtained assuming flexible prices, flexible wages and shutting down price mark-up and wage indexation shocks. They are denoted by adding a superscript "p" .

- Flexible-price condition (no price mark-up fluctuations, $\mu_t^p=mpl_t-w_t=0$):

$$\alpha\left(k_t^{s,p}-l_t^p\right)+\varepsilon_t^a=w_t^p. \tag{A38}$$

- Flexible-wage condition (no wage cost-push shock fluctuations, $(1-u^n)\frac{W_t}{P_t}=mrs_t$ and $l_t^{s,p}=l_t^p$):

$$w_t^p=\sigma_l l_t^p+\frac{1}{1-h/\gamma}\left(c_t^p-h/\gamma c_{t-1}^p\right). \tag{A39}$$

- Potential aggregate resources constraint:

$$y_t^p=c_yc_t^p+i_yi_t^p+z_yz_t^p+\varepsilon_t^g. \tag{A40}$$

- Potential consumption equation:

$$c_t^p=c_1c_{t-1}^p+(1-c_1)E_tc_{t+1}^p+c_2\left(l_t^p-E_tl_{t+1}^p\right)-c_3\left(R_t^p-E_t\pi_{t+1}^p\right)+\varepsilon_t^b. \tag{A41}$$

- Potential investment equation:

$$i_t^p=i_1i_{t-1}^p+(1-i_1)E_ti_{t+1}^p+i_2q_t^p+\varepsilon_t^i. \tag{A42}$$

- Arbitrage condition (value of potential capital, q_t^p):

$$q_t^p=q_1E_tq_{t+1}^p+(1-q_1)E_tr_{t+1}^{k,p}-\left(R_t^p-E_t\pi_{t+1}^p-\left(\frac{1-h/\gamma}{\sigma_c(1+h/\gamma)}\right)^{-1}\varepsilon_t^b\right). \tag{A43}$$

- Log-linearized potential aggregate production function:

$$y_t^p=\phi_p\left(\alpha k_t^{s,p}+(1-\alpha)l_t^p+\varepsilon_t^a\right). \tag{A44}$$

- Potential capital (with one period time-to-build):

$$k_t^{s,p}=k_{t-1}^p+z_t^p. \tag{A45}$$

- Potential capital utilization:

$$z_t^p=z_1\log r_t^{k,p}. \tag{A46}$$

- Potential capital accumulation equation:

$$k_t^p=k_1k_{t-1}^p+(1-k_1)i_t^p+k_2\varepsilon_t^i. \tag{A47}$$

- Potential demand for capital by firms ($\log r_t^{k,p}$ is the log of the potential rental rate of capital):

$$-\left(k_t^{s,p}-l_t^p\right)+w_t^p=\log r_t^{k,p}. \tag{A48}$$

- Monetary policy rule (under flexible prices and flexible wages):

$$R_t^p=\rho R_{t-1}^p+(1-\rho)[r_\pi\pi_t^p]+\varepsilon_t^R. \tag{A49}$$

Equations-and-variables summary

– Set of equations:
Eqs (A24)–(A49) determine solution paths for 26 endogenous variables. The subset (A38)–(A49) is introduced to solve the potential (natural-rate) block of the model.
– Set of variables:
Endogenous variables (26): $y_t, c_t, i_t, z_t, l_t, R_t, \pi_t, q_t, \log r_t^k, k_t^s, k_t, u_t - u^n, \mu_t^p, w_t, y_t^p, c_t^p, i_t^p, z_t^p, l_t^p, R_t^p, \pi_t^p, q_t^p, \log r_t^{k,p}, k_t^{s,p}, k_t^p$, and w_t^p.

Predetermined variables (12): $c_{t-1}, i_{t-1}, k_{t-1}, \pi_{t-1}, w_{t-1}, R_{t-1}, y_{t-1}, y_{t-1}^p, c_{t-1}^p, i_{t-1}^p$, k_{t-1}^p, and R_{t-1}^p.

Exogenous variables (7): AR(1) technology shock $\varepsilon_t^a = \rho_a \varepsilon_{t-1}^a + \eta_t^a$, AR(1) risk premium shock $\varepsilon_t^b = \rho_b \varepsilon_{t-1}^b + \eta_t^b$, AR(1) exogenous spending shock cross-correlated to technology innovations $\varepsilon_t^g = \rho_g \varepsilon_{t-1}^g + \eta_t^g + \rho_{ga}\eta_t^a$, AR(1) investment shock $\varepsilon_t^i = \rho_i \varepsilon_{t-1}^i + \eta_t^i$, AR(1) monetary policy shock $\varepsilon_t^R = \rho_R \varepsilon_{t-1}^R + \eta_t^R$, ARMA(1,1) price mark-up shock $\varepsilon_t^p = \rho_p \varepsilon_{t-1}^p + \eta_t^p - \mu_p \eta_{t-1}^p$, and ARMA(1,1) wage push shock $\varepsilon_t^w = \rho_w \varepsilon_{t-1}^w + \eta_t^w - \mu_w \eta_{t-1}^w$.

Appendix E. Model comparison

In order to recover the Smets and Wouters (2007) model, we must introduce the following modifications in Eqs. (A33), (A35) and (A36):

(i) New-Keynesian Phillips Curve (price inflation dynamics):

$$\pi_t = \pi_1 \pi_{t-1} + \pi_2 E_t \pi_{t+1} - \pi_3^{SW} \mu_t^p + \varepsilon_t^p, \tag{A33$'$}$$

where $\pi_1 = \frac{\iota_p}{1+\bar\beta\iota_p}$, $\pi_2 = \frac{\bar\beta}{1+\bar\beta\iota_p}$, and $\pi_3 = \frac{1}{1+\bar\beta\iota_p}\left[\frac{(1-\bar\beta\xi_p)(1-\xi_p)}{\xi_p((\phi_p-1)\varepsilon_p+1)}\right]$. Notice the changes in the slope coefficient $\pi_3^{SW} > \pi_3$.

(ii) Wage mark-up (log difference between the marginal rate of substitution between working and consuming and the real wage) which replaces the rate of unemployment present in the CMV model:

$$\mu_t^w = w_t - mrs_t = w_t - \left(\sigma_l l_t + \frac{1}{1-h/\gamma}(c_t - h/\gamma c_{t-1})\right). \tag{A35$'$}$$

(iii) Real wage dynamic equation:

$$w_t = w_1 w_{t-1} + (1-w_1)(E_t w_{t+1} + E_t \pi_{t+1}) - w_2 \pi_t + w_3 \pi_{t-1} - w_4^{SW} \mu_t^w + \varepsilon_t^w, \tag{A36$'$}$$

where $w_1 = \frac{1}{1+\bar\beta}$, $w_2 = \frac{1+\bar\beta\iota_w}{1+\bar\beta}$, $w_3 = \frac{\iota_w}{1+\bar\beta}$, and $w_4^{SW} = \frac{1}{1+\bar\beta}\left[\frac{(1-\bar\beta\xi_w)(1-\xi_w)}{\xi_w((\phi_w-1)\varepsilon_w+1)}\right]$ with the curvature of the Kimball labor aggregator fixed at $\varepsilon_w = 10.0$ and a steady-state wage mark-up fixed at $\phi_w = 1.5$ as in Smets and Wouters (2007). Notice the new slope coefficient w_4^{SW}.

In addition, the coefficient c_2 from equations (A25) and (A41) suffers a slight change to accommodate the fact that there is a wage mark-up in the SW model. In turn, the coefficient to be used in the SW model is $c_2 = \frac{[(\sigma_c-1)wL/(\phi_w C)]}{\sigma_c(1+h/\gamma)}$.

The set of variables is identical in the two models except for the replacement of the wage mark-up μ_t^w in the SW model for the deviation of the rate of unemployment, $u_t - u^n$, in the CMV model. In turn there are 26 equations and 26 endogenous variables in both models.

Appendix F. Supplementary material

Supplementary data associated with this article can be found, in the online version, at http://dx.doi.org/10.1016/j.jmacro.2014.01.010.

References

Abel, A., Bernanke, B., Croushore, D., 2011. Macroeconomics, seventh ed. Pearson Education.
Bénassy, J.P., 1995. Money and wage contracts in an optimizing model of the business cycle. J. Monetary Econ. 35, 303–315.
Blanchard, O., 2011. Macroeconomics, fifth ed. Pearson Education.
Calvo, G.A., 1983. Staggered pricing in a utility-maximizing framework. J. Monetary Econ. 12, 383–396.
Casares, M., 2007. Firm-specific or household-specific sticky wages in the new-Keynesian model? Int. J. Central Banking 3, 181–240.
Casares, M., 2010. Unemployment as excess supply of labor. J. Monetary Econ. 57, 233–243.
Casares, M., Moreno, A., Vázquez, J., 2012. Wage stickiness and unemployment fluctuations: an alternative approach. SERIEs – J. Spanish Econ. Assoc. 3, 395–422.
Champagne, J., Kurmann, A., 2013. The great increase in relative wage volatility in the United States. J. Monetary Econ. 60, 166–183.
Cho, J.O., Cooley, T.F., 1994. Employment and hours over the business cycle. J. Econ. Dynam Control 18, 411–432.
Christiano, L.J., Eichenbaum, M., Evans, C.L., 2005. Nominal rigidities and the dynamic effects of a shock to monetary policy. J. Political Econ. 113, 1–45.
Clarida, R., Galí, J., Gertler, M., 1999. The science of monetary policy: a new-Keynesian perspective. J. Econ. Lit. 37, 1661–1707.
Erceg, C.J., Henderson, D.W., Levin, A.T., 2000. Optimal monetary policy with staggered wage and price contracts. J. Monetary Econ. 46, 281–313.

Galí, J., 1999. Technology, employment, and the business cycle: do technology shocks explain aggregate fluctuations? Am. Econ. Rev. 89, 249–271.
Galí, J., 2011. The return of the wage Phillips curve. J. Eur. Econ. Assoc. 9, 436–461.
Galí, J., Smets, F.R., Wouters, R., 2012. Unemployment in an estimated new Keynesian model. In: Acemoglu, D., Woodford, M. (Eds.), NBER Macroeconomics Annual, vol. 26. MIT Press, Cambridge, pp. 329–360.
Hansen, G.D., 1985. Indivisible labor and the business cycle. J. Monetary Econ. 16, 309–327.
Justiniano, A., Primiceri, G.E., Tambalotti, A., 2013. Is there a trade-off between inflation and output stabilization? Am. Econ. J. Macroecon. 5, 1–31.
Kimball, M.S., 1995. The quantitative analytics of the basic neomonetarist model. J. Money Credit Banking 27, 1241–1277.
Michaillat, P., 2012. Do matching frictions explain unemployment? Not in bad times. Am. Econ. Rev. 102, 1721–1750.
Moreno, A., 2004. Reaching inflation stability. J. Money Credit Banking 36, 801–825.
Mortensen, D.T., Pissarides, C.A., 1994. Job creation and job destruction in the theory of unemployment. Rev. Econ. Studies 61, 397–415.
Mulligan, C., 2001. Aggregate implications of indivisible labor. Advances in Macroeconomics, vol. 1. Berkeley Electronic Press (article 4).
Romer, C.D., Romer, D.H., 2004. A new measure of monetary shocks: derivation and implications. Am. Econ. Rev. 94, 1055–1084.
Smets, F.R., Wouters, R., 2003. An estimated dynamic stochastic general equilibrium model of the Euro area. J. Eur. Econ. Assoc. 1, 1123–1175.
Smets, F.R., Wouters, R., 2007. Shocks and frictions in US business cycles: a Bayesian DSGE approach. Am. Econ. Rev. 97, 586–606.
Stock, J.H., Watson, M.W., 2002. Has the business cycle changed and why? In: Gertler, M., Rogoff, K. (Eds.), NBER Macroeconomics Annual, vol. 17. MIT Press, Cambridge, pp. 159–218.
Trigari, A., 2009. Equilibrium unemployment, job flows, and inflation dynamics. J. Money Credit Banking 41, 1–33.
Walsh, C.E., 2005. Labor market search, sticky prices, and interest rate policies. Rev. Econ. Dynam. 8, 829–849.
Woodford, M., 2003. Interest and Prices: Foundations of a Theory of Monetary Policy. Princeton University Press, Princeton.