Solution to PS 8

1. (a)-(c) see slides, standard derivation

   (d)
$$\hat{y}_t = \sqrt{a_t}$$
$$\mu_t^* = \frac{\epsilon_t}{\epsilon_t - 1} = \frac{2 + y_t}{y_t} = \frac{2 + x_t \hat{y}_t}{x_t \hat{y}_t} = \frac{2 + x_t\sqrt{a_t}}{x_t\sqrt{a_t}} = \frac{2}{x_t\sqrt{a_t}} + 1$$
   Log-linearization:
$$\begin{aligned} \left(\mu_t^* - 1\right) x_t \sqrt{a_t} &= 2 \\ \left(\bar{\mu}_t^* e^{\tilde{\mu}_t^*} - 1\right) \bar{x}_t e^{\tilde{x}_t} \sqrt{\bar{a}_t e^{\tilde{a}_t}} &= 2 \end{aligned}$$
   Approximation:
$$\left(\bar{\mu}_t^* \left(1 + \tilde{\mu}_t^*\right) - 1\right) \bar{x}_t \left(1 + \tilde{x}_t\right) \sqrt{\bar{a}_t} \left(1 + \frac{1}{2}\tilde{a}_t\right) = 2$$
   Taylor approximation of first order:
$$\left(\bar{\mu}_t^* - 1\right) \bar{x}_t \sqrt{\bar{a}_t} + \bar{x}_t \bar{\mu}_t^* \sqrt{\bar{a}_t} \tilde{\mu}_t^* + \left(\bar{\mu}_t^* - 1\right) \bar{x}_t \sqrt{\bar{a}_t} \tilde{x}_t + \frac{1}{2}\left(\bar{\mu}_t^* - 1\right) \bar{x}_t \sqrt{\bar{a}_t} \tilde{a}_t = 2$$
   and since in steady state
$$\left(\bar{\mu}_t^* - 1\right) \bar{x}_t \sqrt{\bar{a}_t} = 2$$
   this simplifies to
$$\frac{\bar{\mu}_t^*}{\bar{\mu}_t^* - 1} \tilde{\mu}_t^* + \tilde{x}_t + \frac{1}{2}\tilde{a}_t = 0.$$
   Interpretation: by assumption, demand elasticity is increasing in level of output, so the markup in a flexible-price world decreases with output. Note that $\tilde{y}_t = \tilde{x}_t + \frac{1}{2}\tilde{a}_t$, so $\tilde{\mu}_t^*$ is decreasing in both $\tilde{x}_t$ and $\tilde{a}_t$.

   (e) No. The fact that now $\tilde{\mu}_t^*$ depends on $\tilde{x}_t$ and $\tilde{a}_t$ means that both will appear in the Phillips curve, so as $\tilde{a}_t$ decreases the PC will shift up, which will increase inflation. It is no longer the case that both output gap and inflation can be fully stabilized in the wake of a technology shock ("divine coincidence", see discussion in the lecture). The central bank faces a trade-off between stabilizing inflation and stabilizing the output gap, and the intersection between FOC curve and PC gives the optimal (discretionary) response of the central bank (draw graph).

   (f) The standard NKPC is
$$\tilde{\pi}_t = \beta E_t \tilde{\pi}_{t+1} + \kappa \tilde{x}_t + \gamma \tilde{\mu}_t^*$$
   whereas when competition depends on the level of output, we also have
$$\begin{aligned} \tilde{\pi}_t &= \beta E_t \tilde{\pi}_{t+1} + \kappa \tilde{x}_t + \gamma \frac{\bar{\mu}_t^* - 1}{\bar{\mu}_t^*} \left(-\tilde{x}_t - \frac{1}{2}\tilde{a}_t\right) \\ &= \beta E_t \tilde{\pi}_{t+1} + \left(\kappa - \gamma \frac{\bar{\mu}_t^* - 1}{\bar{\mu}_t^*}\right) \tilde{x}_t - \gamma \frac{\bar{\mu}_t^* - 1}{\bar{\mu}_t^*} \frac{1}{2}\tilde{a}_t \end{aligned}$$

which means that the Phillips curve is flatter. When there is a positive (i.e. inflationary) cost-push shock, output goes down because of increased market power of firms, but now that reduction in output increases markups even further (and therefore output etc). For the same upward shift of the Phillips curve (regardless of whether it's due to a shock to a or to ϵ) the CB now faces a tougher situation, in the sense that the same implemented level of inflation would lead to a lower x (show graph). Note that the fact that the PC is flatter also enters the FOC (it's making it steeper: the CB has an additional incentive to stabilize output, so that inflation does not get pushed up even more). Assuming $\pi^* = x^* = 0$ and transitory shocks, we have that the FOC of the CB's optimal discretionary MP problem is

$$\pi_t = -\frac{a}{\left(\kappa - \gamma \frac{\bar{\mu}_t^* - 1}{\bar{\mu}_t^*}\right)} x_t$$

Combining this with the new NKPC (where e_t is the upward shift)

$$\tilde{\pi}_t = \left(\kappa - \gamma \frac{\bar{\mu}_t^* - 1}{\bar{\mu}_t^*}\right) \tilde{x}_t + e_t$$

leads to

$$\tilde{x}_t = -\frac{\left(\kappa - \gamma \frac{\bar{\mu}_t^* - 1}{\bar{\mu}_t^*}\right)}{\left(\kappa - \gamma \frac{\bar{\mu}_t^* - 1}{\bar{\mu}_t^*}\right)^2 + a} e_t$$

which can either rise or fall with $\gamma \frac{\bar{\mu}_t^* - 1}{\bar{\mu}_t^*}$ depending on how large a is.

$$\frac{\partial}{\partial \kappa} \frac{\kappa}{\kappa^2 + a} = \frac{\left(\kappa^2 + a\right) - 2\kappa^2}{\left(\kappa^2 + a\right)^2} = \frac{a - \kappa^2}{\left(\kappa^2 + a\right)^2}$$

That's because the changed incentives of the CB push against the flattening of the PC curve.

2. See lecture slides

3. See lecture slides. Discussion of dynamic externality (firms have strategic complementarity in price setting, and therefore firms want to increase prices when expecting inflation because they may not be able to increase prices in the future). CB uses announcement of lower-than-normal inflation in the future to shift downwards the NKPC in the present. Discussion of the fact that commitment requires credibility, and is time inconsistent.