Macroeconomics A; EI060

Short problems

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1 Exchange rate and money

Question: The model consists of the uncovered interest parity, money demand, and purchasing power parity. In linearized terms, omitting variables that are not relevant, we have:

$$\begin{aligned} i_{t+1}^{H} &= \mathbb{E}_t\left(e_{t+1}\right) - e_t \\ m_t - p_t &= -\lambda i_{t+1}^{H} \\ p_t &= e_t \end{aligned}$$

Show that:

$$e_t = \frac{\lambda}{1+\lambda}\mathbb{E}_t\left(e_{t+1}\right) + \frac{1}{1+\lambda}m_t$$

Then show that:

$$e_t = \frac{1}{1+\lambda}\sum_{s=t}^{\infty}\left[\left(\frac{\lambda}{1+\lambda}\right)^{s-t}\mathbb{E}_t\left(m_s\right)\right]$$

2 Constant money growth rate: useful expressions

Question: The money supply grows at a constant rate μ:

$$m_s = m_t + \mu\left(s-t\right)$$

We first derive some useful properties. Recall that:

$$\sum_{s=0}^{\infty}\left[\left(\frac{\lambda}{1+\lambda}\right)^{s}\right] = \frac{1}{1-\frac{\lambda}{1+\lambda}} = 1+\lambda$$

The expression $\left(\frac{\lambda}{1+\lambda}\right)^{s-1}s$ is the derivative of $\left(\frac{\lambda}{1+\lambda}\right)^{s}$ with respect to $\frac{\lambda}{1+\lambda}$. As the sum is a linear function, the sum of a derivative is the derivative of the sum. Show that:

$$\sum_{s=0}^{\infty}\left[\left(\frac{\lambda}{1+\lambda}\right)^{s-1}s\right] = \frac{\partial\left(1+\lambda\right)}{\partial\left(\frac{\lambda}{1+\lambda}\right)}$$

The recall the chain rule $\frac{\partial f(\lambda)}{\partial g(\lambda)} = \frac{\partial f(\lambda)}{\partial \lambda} / \left[\frac{\partial g(\lambda)}{\partial \lambda}\right]$ and show:

$$\sum_{s=0}^{\infty} \left[\left(\frac{\lambda}{1+\lambda} \right)^{s-1} s \right] = (1+\lambda)^2$$

3 Constant money growth rate: useful expressions

Question: Using the results above, show that:

$$e_t = m_t + \lambda \mu$$

4 Autoregressive money process

Question: In terms of movements around a steady state, money follows an autoregressive process:

$$m_{t+1} = \rho m_t + \epsilon_{t+1}$$

Show that

$$\mathbb{E}_t (m_s) = \rho^{s-t} m_t$$

And show that the exchange rate is:

$$e_t = m_t \frac{1}{1 + \lambda (1-\rho)}$$