Problem Set II

Yuan Zi

EI037 Microeconomics I

1. Consumption, Production, and Competitive Equilibrium

Consider an economy of L identical consumers, where each consumer has a Cobb-Douglas preference over two goods x and y

$$U = x^{\frac{1}{2}} y^{\frac{1}{2}}.$$

The total number of good x in the economy is fixed and equals to X. Each consumer owns a share $\frac{1}{L}$ of X. Good y is produced by firms using labor and good x. The firm's technology is Cobb-Douglas

$$y = x^{\frac{1}{2}} \ell^{\frac{1}{2}}.$$

where each consumer supplies one unit of labor ℓ. The wage is normalized to be equal to 1. Denote the price of x and y by p_x and p_y, respectively.

1.a. Solve for the consumer's demand of x and y as functions of prices p_x and p_y, and income. (Hint: first write down the consumer's income as a function of wage, prices and the return from owning units of x.)

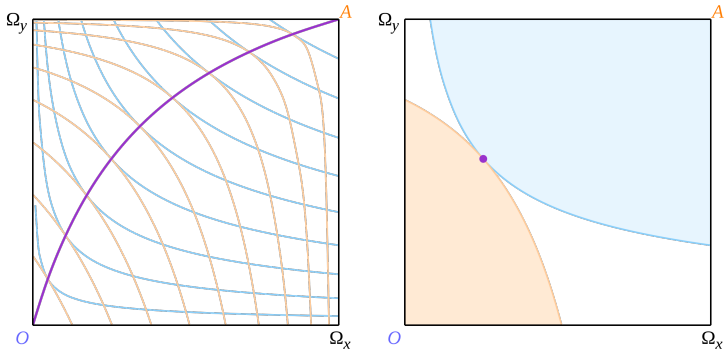
1.b. Solve for the firms' demand of x and supply of y as functions of p_x, p_y, and wage.

1.c. Write down the market clearing condition(s) and find the equilibrium values of p_x and p_y.

2. Fundamental Welfare Theorems

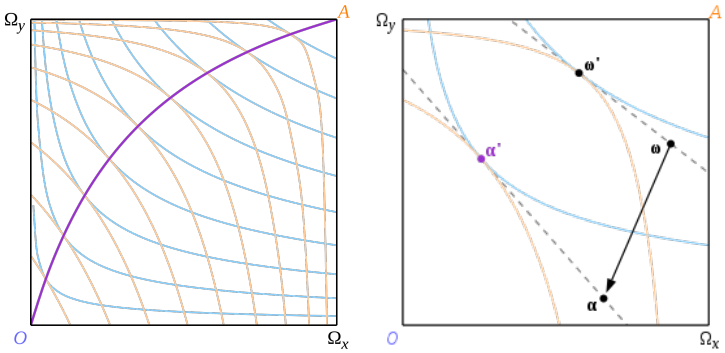
2.a. Explain the First Fundamental Welfare Theorem in words.

2.b. Explain the First Fundamental Welfare Theorem through the following Edgeworth boxes.



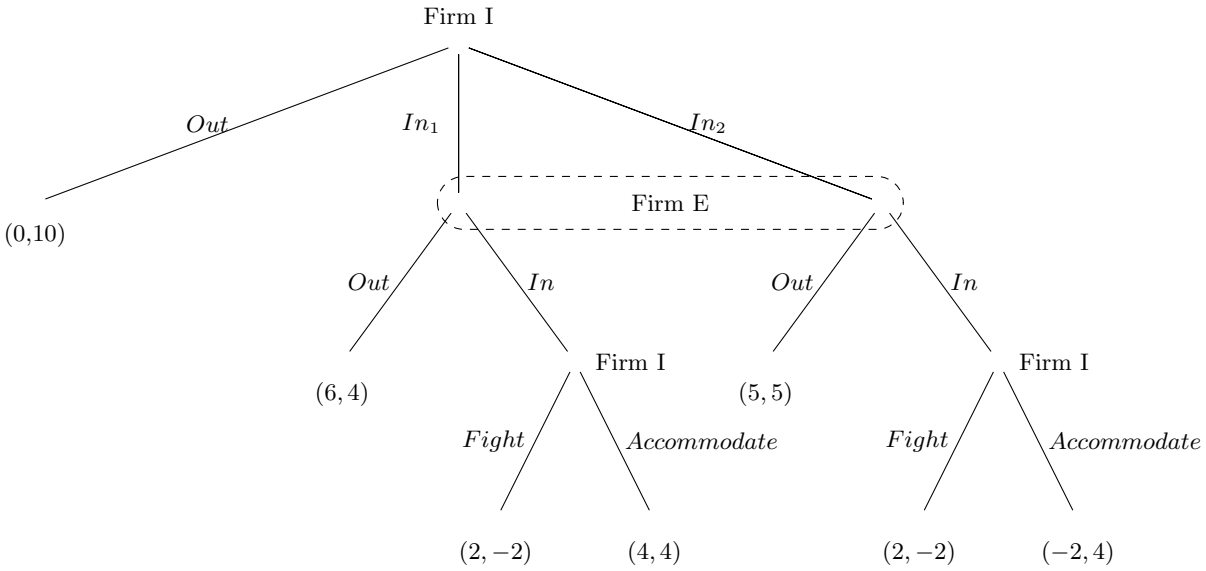
2.c. Explain the Second Fundamental Welfare Theorem in words.

2.d. Explain the Second Fundamental Welfare Theorem through the following Edgeworth boxes.



3. Strategic Interactions

Consider the following two-player game with imperfect information.



3.a. What are Firm I's possible strategies? And what are Firm E's possible strategies?

3.b. With the help of strategies, write down the normal form representation of the game.

3.c. Write down the representation of the two firms' mixed strategy.

3.d. Show that for any behavior strategy that Firm I might play, there is a realization equivalent mixed strategy – that is, a mixed strategy that generates the same probability distribution over the terminal nodes for any mixed strategy choice by Firm E.

3.e. Does the game have any pure strategy Nash Equilibrium(s)? If so, find them.

3.f. Does the game have any mixed strategy Nash Equilibrium(s)? If so, find them.

3.e. Find all the subgames of this game.

3.h. Does the game have any pure strategy Subgame Perfect Nash Equilibrium(s)? If so, find them.

3.i. Can you add an information set to the game, so that it no longer satisfies perfect recall? If possible, provide one example. If not possible, justify why.

3.j. Can you introduce Nature and modify the game tree so that it represents a game with incomplete information? There are many ways of designing this, providing one example would be sufficient.