

Demystifying DSGE Models

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PS6 Solutions

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Q4.

Persistence and price stickiness. Posteriors point to very flexible domestic prices ($\text{Calvo } \theta_H = 0.0414$) alongside sticky import prices ($\theta_F = 0.7703$). This asymmetry—common in emerging small open economies—helps rationalize incomplete (and gradual) pass-through from the external sector into CPI. Moderate indexation ($\delta_H = 0.3604$, $\delta_F = 0.5510$) supports inflation persistence without tipping the Phillips curves into over-backwardness.

Monetary policy rule. The estimated Taylor-rule coefficient on inflation ($\psi_\pi = 1.9047 > 1$) exceeds unity, confirming that the Central Bank responds aggressively to inflation deviations, thereby ensuring determinacy. The coefficient on output ($\psi_y = 0.2077$) is small, implying that stabilization of real activity plays a secondary role. The direct exchange-rate term is small but non-zero ($\psi_{de} = 0.1489$), suggesting limited—but present—leaning against exchange-rate movements in the policy rule. Interest rate smoothing ($\psi_r = 0.8283$) indicates considerable policy inertia, aligning with empirical evidence of gradual monetary adjustments.

Nominal and real rigidities. The degree of indexation to past inflation ($\delta_H = 0.3604$, $\delta_F = 0.5510$) points to moderate backward-looking behavior, helping the model reproduce observed inflation persistence. The habit parameter ($h = 0.6415$) suggests significant consumption smoothing, which dampens the volatility of output and consumption in response to shocks.

External transmission and shock processes. Persistence in foreign variables ($b_1 \approx 0.9398$ for output, $c_1 \approx 0.9339$ for interest rates) ensures that international conditions transmit slowly to the domestic economy. The high autoregressive coefficients for domestic technology ($\rho_a = 0.7606$) and exchange-rate shocks ($\rho_q = 0.8836$) highlight the persistent nature of supply and financial disturbances.

Shock volatility and variance decomposition. Among estimated standard deviations, the most volatile drivers are the terms-of-trade and foreign-output shocks ($\sigma_{es} = 0.46$, $\sigma_{ey^*} = 0.55$), underlining the exposure of small open economy to external commodity price movements and foreign demand fluctuations. Domestic technology shocks ($\sigma_{ea} = 0.68$) also play a notable role, while monetary and UIP shocks are relatively moderate. domestic technology shocks also play a notable role, while monetary

Dynamic properties. Autocorrelations in key observables—such as output (0.89 at lag 1) and interest rate (0.96)—show that the model captures the persistence found in actual Brazilian data. The correlation matrix indicates strong positive comovement between consumption and output, and a negative correlation between terms of trade and real exchange rate, both consistent with open-economy theory.

Overall, the posterior modes reflect a coherent structure: a credible and inflation-focused monetary policy, moderate nominal rigidities, and high exposure to external shocks—typical of emerging-market open economies.

Table 1: Estimated Priors and Posterior Modes (Q4 Run)

Parameter	Description	Dynare Name	Prior Mean	Posterior Mode
<i>Preferences and Technology</i>				
h	Habit persistence	h	0.5	0.6415
σ	Inverse IES	sigma	1.5	0.1798
φ	Inverse Frisch elasticity	varphi	1.5	0.1971
<i>Open-Economy Structure</i>				
α	Share of foreign goods in CPI	alpha	0.5	0.8678
η	Trade elasticity / demand curvature	eta	1.5	0.9156
<i>Price Setting and Indexation</i>				
θ_H	Calvo parameter (home prices)	thetaH	0.5	0.0414
θ_F	Calvo parameter (import prices)	thetaF	0.5	0.7703
δ_H	Indexation to home inflation	deltaH	0.5	0.3604
δ_F	Indexation to import inflation	deltaF	0.5	0.5510
<i>Monetary Policy Rule</i>				
ψ_r	Interest-rate smoothing	psir	0.5	0.8283
ψ_π	Response to inflation	psi_pi	1.5	1.9047
ψ_y	Response to output (gap/growth, per file setup)	psiy	0.25	0.2077
ψ_{de}	Response to nominal exchange-rate change	pside	0.2	0.1489
<i>Shock Persistence (AR(1)) and Foreign Block</i>				
ρ_a	Productivity persistence (home)	rhoa	0.5	0.7606
ρ_s	Terms-of-trade persistence	rhos	0.5	0.5059
ρ_q	Exchange-rate (risk-premium) persistence	rhoq	0.5	0.8836
ρ_r	Monetary shock persistence	rhor	0.5	0.3551
a_1	Foreign inflation persistence	a1	0.5	0.2467
b_1	Foreign output persistence	b1	0.5	0.9398
c_1	Foreign interest-rate persistence	c1	0.5	0.9339

Notes: THe table reports the posterior mean of the parameters. Priors are as specified in the model file.

Table 2: Variance Decomposition by Shock
(Q4 Run, Percent)

	e_q	e_r	e_a	e_s	e_{y^*}	e_{π^*}	e_{r^*}
y	10.44	13.34	33.44	0.04	39.16	0.01	3.56
c	37.84	48.52	0.10	0.18	0.40	0.02	12.93
s	30.96	39.41	17.84	0.15	1.29	0.05	10.30
q	37.86	47.87	0.09	0.15	1.44	0.07	12.52
r	52.19	7.76	0.22	0.05	1.37	0.03	38.38
π	35.54	37.35	3.30	0.19	3.96	0.04	19.62
π_H	33.67	39.96	10.91	3.10	0.79	0.13	11.43
π_F	35.78	35.49	1.85	0.21	4.82	0.01	21.84

Notes: Entries are posterior-mode variance shares from Dynare.

Q5.

Monetary policy shock

Q4 Benchmark A contractionary MonPol shock produces: r up on impact; (c, y) down (hump-shaped), (π, π_H, π_F) down; real and nominal exchange rates appreciate; terms of trade improve. This is the standard NK–SOE mechanism: tighter intertemporal margin (Euler), lower marginal costs (Phillips), and a UIP channel that transmits through the exchange rate into import prices.

Q5a Impact responses of consumption and output are sharper and peak earlier than in Q4. Removing external habit raises the effective intertemporal elasticity, so the Euler equation delivers stronger immediate expenditure switching when r jumps. Exchange-rate appreciation is also slightly larger on impact, and the real side returns to steady state faster (less internal propagation without habit).

Q5b Inflation disinflates more on impact and is less persistent than in **Q4** (both π_H and π_F panels flatten faster). With weaker backward-looking pressure, the price Phillips curves transmit the policy shock more cleanly to prices. As a result, the **required real-side adjustment is milder and shorter-lived**: output and consumption troughs are a touch smaller and closer to impact; the exchange-rate path is similar in level but **returns more quickly**, reflecting faster closure of the price gap that drives pass-through.

Q5c Dropping the Δe term removes the direct **lean-against-the-wind** channel. In the IRFs this shows up as **larger and more persistent exchange-rate appreciation**, and a **somewhat bigger disinflation** via import-price pass-through. Real activity bears **slightly more of the adjustment** (net-export channel amplified by the stronger appreciation). Note this variant also changes the model’s block structure (more recursive blocks), but determinacy is preserved.

Under a monetary policy shock, across Q4→Q5a→Q5b→Q5c, we move from (i) **demand-side amplification** (no habit → larger, earlier real responses), to (ii) **nominal smoothing** (no indexation → faster, less persistent inflation and faster ER reversion), to (iii) **open-economy amplification** (no ER term → bigger, longer ER movements and more pass-through). The **baseline** strikes a middle ground: sufficient inertia to match persistence, but with disciplined anti-inflation policy.

Foreign Output shock

Q4 Benchmark A favorable external demand shock raises y and c ; the **exchange rate appreciates** (both real and nominal), **terms of trade deteriorate**, and **inflation falls modestly**; the policy rate **declines** (the rule accommodates the disinflation/appreciation). This aligns with an export-demand channel plus pass-through: stronger foreign demand improves domestic activity but, via appreciation and cheaper imports, **pulls down CPI inflation**.

Q5a Consumption rise more on impact and peak earlier, because intertemporal smoothing is weaker. And the output jump immediately with a gradual decrease to benchmark. With weaker real responses, the appreciation is **slightly smaller on impact**, and the policy rate **falls a bit less** initially (inflation drops a touch less via faster pass-through to prices). The net effect is a **front-loaded expansion**, with **faster normalization** afterward.

Q5b Removing indexation **dampens inflation persistence** further: π , π_H , and π_F decline a bit more on impact and **revert sooner**. With price gaps closing faster, **policy lowers the rate by less and for a shorter period**, and the appreciation path is **less persistent** than in Q4. Real-side responses are **similar in amplitude but shorter-lived**, reflecting the reduced nominal propagation.

Q5c Without the Δe term, policy **does not mechanically accommodate an appreciation**. Relative to Q4, the **policy rate falls by less**, so the **currency appreciates more/for longer** to clear the UIP condition. The bigger appreciation implies **stronger (more negative) inflation responses** but **dampens the real expansion** through the net-export channel; output and consumption **peak slightly lower and normalize more slowly** than under Q4. This is precisely the open-economy trade-off the ER term was designed to mitigate.

Under a foreign output shock, no-habit **amplifies and front-loads** the real expansion; no-indexation **speeds up nominal adjustment** and shortens the cycle; removing the ER term **strengthens appreciation and pass-through, dampening the real gains and deepening the disinflation**.

Relative to the estimated baseline, **No-Habit** (Q5a) strengthens and accelerates real responses (bigger impact, earlier peaks), **No-Indexation** (Q5b) cleans up nominal inertia (larger on-impact inflation move, faster reversion, smaller/prompt real adjustments), and **No-ER-term** (Q5c) hands more control to **UIP** and **pass-through**, yielding **bigger and more persistent exchange-rate movements** that **intensify disinflation** and **trim real gains** after external shocks. The combined evidence across MonPol and Foreign Output disturbances paints a consistent SOE picture: **policy design** (habit/indexation/ER term) redistributes the adjustment **between prices, the exchange rate, and quantities** rather than overturning the core transmission.

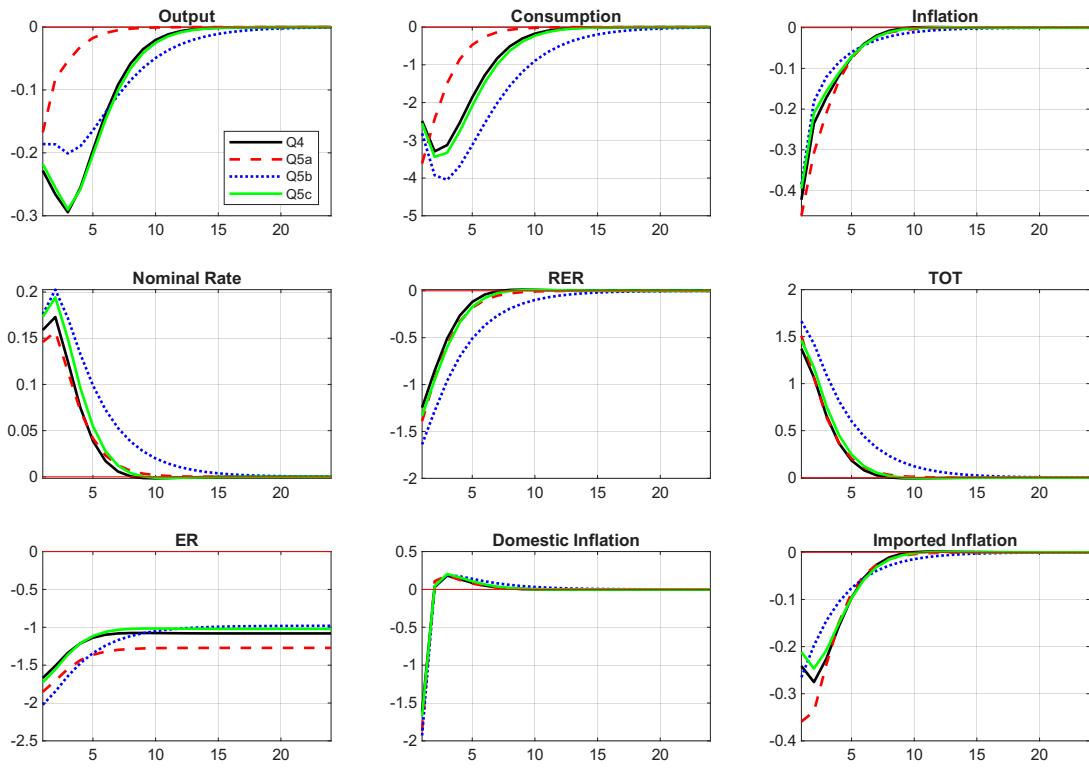
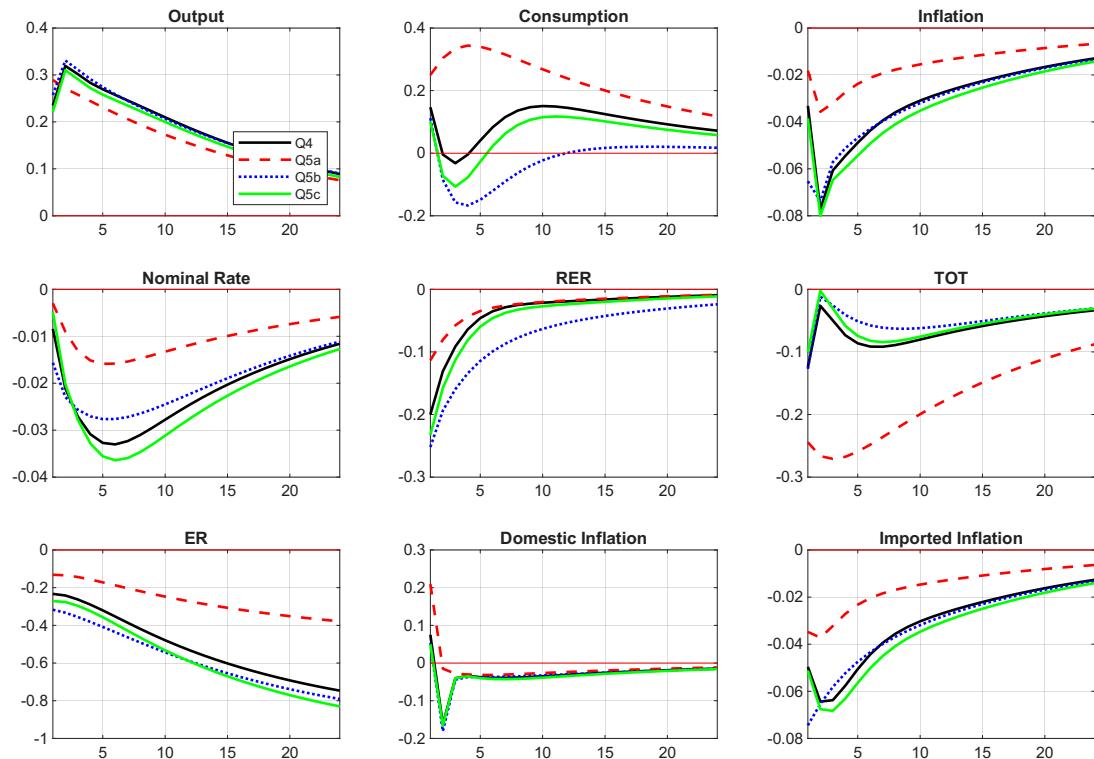


Figure 1: IRFs — Monetary policy shock (ε_r)

Notes: This figure presents the IRF of the benchmark model (solid blue), the no-habit model (dashed red), the no-indexation model (dotted blue), and the no-exchange-rate-term model (dashed green) under a monetary policy shock.

Figure 2: IRFs — Foreign Output shock (ε_{y^*})

Notes: This graphs gives the IRF of the benchmark model (solid blue), the no-habit model (dashed red), the no-indexation model (dotted blue), and the no-exchange-rate-term model (dashed green) under a foreign output shock.