

Macroeconomics B, EI060

Class 10

Optimal policy in stochastic model

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What you will get from today class

- **Stochastic** version of the micro-founded model (Corsetti and Pesenti).
 - Beyond the **effect of monetary shocks**: how **should we set monetary policy** in response to other shocks (**normative** view). **RULE**
- Presenting **the concepts**: optimal monetary policy in a **closed** economy.
- Extending to then **open economy**.
 - **How exchange rate pass-through** affects the optimal prices.
 - Optimal policy: technical steps, and **visual presentation** of the key dimension.
 - When **cooperation** can help.
- Focus on **intuition** behind **optimal policy**. Graphical illustration of transmission as extra slides.

A question to start

SR: STABLE γ w C.P

INSURANCE \hookrightarrow RISK SHARING $\frac{U(C)}{U(C^*)} = \frac{E^P}{P}$

Under sticky prices, the exchange rate transmits shocks across countries. It should therefore be a) kept stable, b) used to stabilize output. GAP $Y = Y^{\text{EFFICIENT}}$

b RISK ABSORBER ✓ IF $FX \rightarrow P$

\hookrightarrow INDEPENDENCE

Do you agree? Why or why not?

CLOSED ECONOMY MODEL

Simplifying assumptions

- **Stochastic** version of the Obstfeld-Rogoff model.
 - Go **beyond** the **ex-post** effect of one shock.
 - Consider the **ex-ante** impact of the possibility of shocks and the associated policy response.
- One-period model, allowing for an analytical solution.
 - **Prices are set** at the beginning of the period. FORWARD LOOKING
 - Shocks then happen, **monetary policy** reacts, the **exchange rate** moves, and **firms** meet the actual demand at preset prices.
- Prices are preset in a **forward-looking** way. Their level reflects the rules of policy.

Benchmark: closed economy

- Illustrate key concepts in a closed economy. Representative household with utility of consumption and labor:

$$U = \ln C - \kappa l$$

- CES basket of the various brands j :

$$C = \left[\int_0^1 [C(j)]^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}}$$

- Consumption allocation reflects relative prices:

$$C(j) = \left[\frac{p(j)}{P} \right]^{-\theta} C \quad ; \quad P = \left[\int_0^1 [p(j)]^{1-\theta} dj \right]^{\frac{1}{1-\theta}}$$

Labor supply and aggregate demand

- The household gets a **wage income** and the profits from firms (lump sum).

$$M \frac{w}{P} = C$$

- Monetary stance** $\mu = PC$ (cash-in-advance constraint or a money-in-the-utility approach).

$$POLICY(\mu) = PC$$

- Labor supply, with flexible wages: $W = \kappa PC$ (marginal utility of consumption * real wage = marginal cost of effort).
- Flexible wages reflect the monetary stance:

$$M \rightarrow W$$

$$W = \kappa \mu$$

- Each brand j produced by a unique firm. Linear technology using labor with productivity Z common to all firms:

SHOCKS

$$Y(j) = Z \times I(j)$$

- Equilibrium: all firms are identical in equilibrium, consumption is equal to output:

$$C = ZI$$

- Each firm sets its price taking account of its market power, with demand:

$$Y(j) = \left[\frac{p(j)}{P} \right]^{-\theta} C$$

Allocation under flexible prices

- When prices can be set, markup over effective cost (wage adjusted for productivity):

$$P^{\text{flex}} = \frac{\theta}{\theta - 1} \frac{W}{Z} = \frac{\theta \kappa}{\theta - 1} \frac{\mu}{Z}$$

- Leads to constant employment, consumption reflects productivity:

$$I^{\text{flex}} = \frac{\theta - 1}{\theta \kappa} = \bar{I} \quad ; \quad C^{\text{flex}} = \frac{\theta - 1}{\theta \kappa} Z$$

- These are the natural rates of employment and output (consumption).

Impact of productivity shocks

- Flexible price: higher productivity Z reduces firms' costs and prices.
 - Labor unchanged at \bar{l} .
 - Higher purchasing power, efficient increase in output and consumption.
- If prices cannot adjust, outcome depends on monetary policy μ .
- With unchanged μ , consumption ($C = \mu/P$) unchanged. Employment is inefficiently low.
- With monetary expansion (higher μ), consumption increases.
 - Higher purchasing power through higher money balances instead of lower prices.
 - Monetary policy can replicate the flexible price allocation.

Endogenous sticky price

- Firm maximizes expected profits, discounted by the marginal utility of income (using demand $Y(j) = [p(j)/P]^{-\theta} C$):

$$E \frac{1}{PC} [p(j) Y(j) - WI(j)]$$

- Useful property: if Z is log normal: $\ln(Z) = z \sim N(0, \sigma^2)$, expected level reflects variance:

$$E(Z^a) = E(\exp[az]) = \exp\left[\frac{a^2}{2}\sigma^2\right]$$

- Price is a markup over expected cost. If $\ln \mu$ proportional to $\ln Z$, price "premium":

$$P^{\text{sticky}} = \frac{\theta\kappa}{\theta-1} E\left(\frac{\mu}{Z}\right) = \frac{\theta\kappa}{\theta-1} \exp\left[\frac{1}{2} \text{Var}(\ln \mu - \ln Z)\right]$$

- Volatile μ/Z moves the margin of the firm, who "self insures" through a higher price.

- Expected effort = natural rate of employment: $E(I) = \bar{I}$.

$$P = \frac{\theta\kappa}{\theta-1} E\left(\frac{\mu}{Z}\right) = \frac{\theta\kappa}{\theta-1} E\left(\frac{PC}{Z}\right) = (\bar{I})^{-1} PE(I)$$

- Utility is then **expected log consumption** (π_k : probability of state of nature k):

$$E(\ln C) = E\left(\ln \frac{\mu}{P}\right)$$

$$E(\ln C) = E(\ln \mu) - \ln P$$

$$E(\ln C) = E(\ln \mu) - \ln E\left(\frac{\mu}{Z}\right) - \ln \frac{\theta\kappa}{\theta-1}$$

$$E(\ln C) = \sum_k \pi_k \ln \mu_k - \ln \sum_k \pi_k \frac{\mu_k}{Z_k} - \ln \frac{\theta\kappa}{\theta-1}$$

Optimal policy

- First-order condition with respect to μ_k :

$$0 = \pi_k \frac{1}{\mu_k} - \frac{1}{\sum_k \pi_k \frac{\mu_k}{Z_k}} \pi_k \frac{1}{Z_k}$$

$$\frac{\mu_k}{Z_k} = \sum_k \pi_k \frac{\mu_k}{Z_k} = \cancel{\mu} \cancel{Z}$$

- Hence μ_k/Z_k is the same across all states, so $\mu_k = Z_k$. Minimizes the preset price as $\text{Var}(\ln \mu - \ln Z) = 0$:

$$P^{\text{sticky}} = \frac{\theta \kappa}{\theta - 1} \exp \left[\frac{1}{2} \text{Var}(\ln \mu - \ln Z) \right] = \frac{\theta \kappa}{\theta - 1}$$

$C \approx C^{\text{flex}}$

- As μ/Z is constant, firms don't want to change prices ex-post, even if they can. Price stickiness becomes irrelevant.

OPEN ECONOMY

Consumption baskets

- Two countries, **Home** and **Foreign**, of equal size. Foreign variables are indexed by $*$ (prices in Foreign currency).
- **Nominal exchange rate \mathcal{E}** , an increase is a depreciation of the Home currency.
- Consumption baskets (θ : elasticity of substitution across brands, $\lambda = 1$: elasticity between Home and Foreign goods):

LAST WEEK $\lambda = 1$ $n = 1/\theta$ $C = [C_H]^{0.5} [C_F]^{0.5}$

$$C_H = \left[\int_0^1 [C_{H,j}]^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \quad ; \quad C_F = \left[\int_0^1 [C_{F,j}]^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}}$$

Consumption allocation

- Allocation of consumption reflects relative prices (similar relations in the Foreign country):

$$C_{H,j} = \frac{1}{2} \left[\frac{P_{H,j}}{P_H} \right]^{-\theta} \frac{PC}{P_H} \quad ; \quad C_{F,j} = \frac{1}{2} \left[\frac{P_{F,j}}{P_F} \right]^{-\theta} \frac{PC}{P_F}$$

$$P = 2 [P_H]^{0.5} [P_F]^{0.5}$$

- Monetary stances and labor supplies are as in the closed economy:
 $\mu = PC$, $\mu^* = P^* C^*$ and $W = \kappa \mu$, $W^* = \kappa \mu^*$.
- Technologies are identical to the closed economy: $Y = ZI$ and $Z^* I^* \leftarrow Y$
- Home consumption spending is equal to the value of home output \Rightarrow exchange rate is the ratio of monetary stances:

$$\mathcal{E} = \frac{\mu}{\mu^*}$$

Demand and market clearing

- Demand (monetary stance) relations are similar to the closed economy: $\mu = PC$ and $\mu^* = P^* C^*$.
- The natural rates of employment are also similar: $\bar{I} = \bar{I}^*$.
- Good market clearing (balanced) trade reflects world demand: TOT

$$Y = \frac{1}{2} \left(\frac{PC}{P_H} + \frac{P^* C^*}{P_H^*} \right) = \frac{\mu}{2} \left(\frac{1}{P_H} + \frac{\mu^*}{\mu P_H^*} \right) = \frac{PC}{2} \left(\frac{1}{P_H} + \frac{1}{\mathcal{E} P_H^*} \right)$$
$$Y^* = \frac{1}{2} \left(\frac{PC}{P_F} + \frac{P^* C^*}{P_F^*} \right) = \frac{\mu^*}{2} \left(\frac{\mathcal{E}}{P_F} + \frac{1}{P_F^*} \right) = \frac{P^* C^*}{2} \left(\frac{\mathcal{E}}{P_F} + \frac{1}{P_F^*} \right)$$

Key assumptions

$$\lambda = 1 \quad P_H C_H = \frac{1}{2} P C$$

$$P_F C_F = 1$$

$$\text{LASIW} > \frac{b}{1-n} = \underbrace{(x-1)}_{\text{full PT}} e$$

- Unit elasticities are central in being able to get a closed-form solution.
- Unit elasticity of substitution between Home and Foreign goods \Rightarrow the same fraction of income is spent on imports regardless of the shocks (the quantity-price split of course varies).
- The log-utility is a similar feature in dynamics terms.
- Both imply that even without contingent assets markets are complete. The current account is zero, and the model is effectively static.

$$\frac{\ln C_H + \beta \ln C_F}{1-\sigma} + \bar{U} = 1$$

$$\frac{C_H}{C_F} = \frac{1}{2} \left\{ \ln C_H + \lambda \ln C_F \right\}$$

Exchange rate pass-through

- Home firm j sets a **home currency price** $P(H, j)$ in Home currency for **domestic sales**. \simeq **CLOSED**
- It sets a price $\tilde{P}(H, j)$ in a **composite currency** for **exports**. Foreign currency **price paid by Foreign consumers**: **BASKET**

$$P^*(H, j) = \tilde{P}(H, j) (\mathcal{E})^{-\gamma}$$

- γ : **exchange rate pass-through** to Foreign consumers. Price set in a basket currency that is γ Home currency and $1 - \gamma$ Foreign currency.
 - $\gamma = 1$: **producer currency pricing** (PCP, **full pass-through**).
 - $\gamma = 0$: **local currency pricing** (LCP, **no pass-through**).
- **Revenue, in Home currency**, received by the Home firm:
 $\tilde{P}(H, j) (\mathcal{E})^{1-\gamma}$.
- Similarly the Foreign firm sets a price $\tilde{P}(F, j)$ such that the Home currency price paid by **Home consumers** is: $P(F, j) = \tilde{P}(F, j) (\mathcal{E})^{\gamma^*}$.

Optimal domestic prices

$$\text{MAX } E \frac{1}{PC} \overline{T} \text{ P&F HS}$$

- Prices for **domestic sales** as in the **closed** economy case:

$$P(H) = \frac{\theta\kappa}{\theta-1} E \left[\frac{\mu}{Z} \right] = \frac{\theta\kappa}{\theta-1} \exp \left[\frac{1}{2} \text{Var}(\ln \mu - \ln Z) \right]$$

$$P^*(F) = \frac{\theta\kappa}{\theta-1} E \left[\frac{\mu^*}{Z^*} \right] = \frac{\theta\kappa}{\theta-1} \exp \left[\frac{1}{2} \text{Var}(\ln \mu^* - \ln Z^*) \right]$$

- Prices are **markups** over **expected costs**, in the basket currency. Home goods sold in the Foreign country:

$$\tilde{P}(H) = \frac{\theta\kappa}{\theta-1} E \left[\frac{(\mu)^\gamma (\mu^*)^{1-\gamma}}{Z} \right] - \mu^{\text{BASKET}}$$

$$\tilde{P}(H) = \frac{\theta\kappa}{\theta-1} \exp \left[\frac{1}{2} \text{Var} [\gamma \ln \mu + (1-\gamma) \ln \mu^* - \ln Z] \right]$$

- Foreign goods sold in the Home country:

$$\tilde{P}(F) = \frac{\theta\kappa}{\theta-1} E \left[\frac{(\mu)^{1-\gamma^*} (\mu^*)^{\gamma^*}}{Z^*} \right]$$

$$\tilde{P}(F) = \frac{\theta\kappa}{\theta-1} \exp \left[\frac{1}{2} \text{Var} [(1-\gamma^*) \ln \mu + \gamma^* \ln \mu^* - \ln Z^*] \right]$$

- Prices reflect the volatility of money supply in the invoicing currency, scaled by the firm's productivity.

OPTIMAL POLICY : TECHNICAL STEPS

Utility

- We can show $E(I) = E(I^*) = \bar{I}$.
- Welfare is driven by expected log consumption:

$$E(\ln C) = E\left(\ln \frac{\mu}{P}\right)$$

$$E(\ln C) = E(\ln \mu) - E \ln \left(\left[E \frac{\mu}{Z} \right]^{0.5} \left[(\mathcal{E})^{\gamma^*} E \frac{\mu^*}{Z^*} (\mathcal{E})^{1-\gamma^*} \right]^{0.5} \frac{2\theta\kappa}{\theta-1} \right)$$

$$E(\ln C) = \sum_k \pi_k \ln(\mu_k) - \frac{1}{2} \ln \sum_k \pi_k \frac{\mu_k}{Z_k}$$

$$\bar{P}\bar{I} \leftarrow -\frac{\gamma^*}{2} \sum_k \pi_k (\ln(\mu_k) - \ln(\mu_k^*))$$

$$\text{PRESENT} = -\frac{1}{2} \ln \left(\sum_k \pi_k \frac{(\mu_k)^{1-\gamma^*} (\mu_k^*)^{\gamma^*}}{Z_k^*} \right) - \ln \frac{2\theta\kappa}{\theta-1}$$

Optimality condition

- The **first order** condition with respect to μ_k is:

$$0 = \frac{\pi_k}{\mu_k} \left[1 - \frac{1}{2} \frac{\mu_k}{E \frac{\mu}{Z}} \underbrace{\left(\frac{\gamma^*}{2} - \frac{1 - \gamma^*}{2} \frac{\frac{(\mu_k)^{1 - \gamma^*} (\mu_k)^{\gamma^*}}{Z_k^*}}{E \frac{(\mu)^{1 - \gamma^*} (\mu^*)^{\gamma^*}}{Z^*}} \right)}_{\text{PT ex post}} \right]$$

PRESET P IMP

CLOSED

- We take a **log approximation** around

$$\mu_k = \mu_k^* = Z_k = Z_k^* = \mu_0 = Z_0 = 1:$$

HOME REACTION FUNCTION

$$\left[1 + (1 - \gamma^*)^2 \right] \ln(\mu_k) = \ln(Z_k) + (1 - \gamma^*) [\ln(Z_k^*) - \gamma^* \ln(\mu_k^*)]$$

- Similar optimization in the Foreign country:

FOR

$$\left[1 + (1 - \gamma)^2 \right] \ln(\mu_k^*) = \ln(Z_k^*) + (1 - \gamma) [\ln(Z_k) - \gamma \ln(\mu_k)]$$

Particular cases

- Symmetric full exchange rate pass-through (PCP, $\gamma = \gamma^* = 1$):

~~CLOSED~~ $\underline{\ln(\mu_k)} = \ln(Z_k)$; $\ln(\mu_k^*) = \ln(Z_k^*)$

~~FULL STAR~~

- Symmetric absence of exchange rate pass-through (LCP, $\gamma = \gamma^* = 0$):

~~FX PEGGED~~ $\cancel{\times}_{\text{FULL STAR}} \underline{\ln(\mu_k)} = \underline{\ln(\mu_k^*)} = \frac{1}{2} [\ln(Z_k) + \ln(Z_k^*)]$

- Asymmetric dominance of Home currency (Dominant CP, $\gamma = 1$ and $\gamma^* = 0$):

$$\ln(\mu_k) = \frac{1}{2} [\ln(Z_k) + \ln(Z_k^*)] ; \ln(\mu_k^*) = \ln(Z_k^*)$$

OPTIMAL POLICY : VISUAL ILLUSTRATION

Expected utility

- Home utility (deviation from the flexible price allocation) reflects the level at which prices of goods sold in the Home country are set:

$$E(\ln C) \propto -\frac{1}{2} \text{Var} \left[\ln \left(\frac{\mu}{Z} \right) \right] - \frac{1}{2} \text{Var} \left[\ln \left(\frac{(\mu)^{1-\gamma^*} (\mu^*)^{\gamma^*}}{Z^*} \right) \right]$$

H SELLING f1 *F r eign SELLING f1*

- Similarly for the Foreign utility:

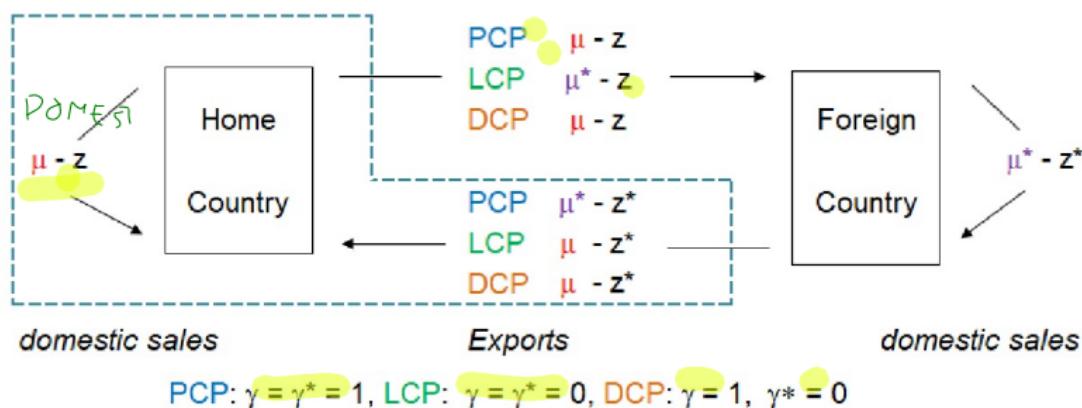
$$E(\ln C^*) \propto -\frac{1}{2} \text{Var} \left[\ln \left(\frac{(\mu)^{\gamma} (\mu^*)^{1-\gamma}}{Z} \right) \right] - \frac{1}{2} \text{Var} \left[\ln \left(\frac{\mu^*}{Z^*} \right) \right]$$

- Stabilize the margin of producers selling in the country, regardless whether they are local or foreign firms.
- Depends on pass-through and γ and γ^* .

Where domestic policy enters

- The **Home policy makers** cares about the **dotted box**. Where μ enters depends on pass-through.
- Only affects imports under LCP or DCP (Home currency used in all trade). Affects exports under PCP and LCP, but not taken into account.

The letters denote the volatility of the money / productivity ratio
relevant to the corresponding consumption



Flexible prices and PCP

- Flexible prices, productivity shocks affect prices, and level and composition of demand.
 - Level: 10% increase in home productivity lowers Home good price by 10%. Raises consumption increases by 5% in both countries (Home goods are half the basket).
 - Composition: relative price of Home goods falls, demand switches towards them.
- Optimal PCP policy: $\mu = Z$ and $\mu^* = Z^*$. With a 10% increase in Home productivity:
 - Home currency depreciates by 10%, Home import prices increase by 10%, Foreign import prices fall by 10%.
 - Efficient level: real balances increase by 5% in Home (10% nominal balance, minus 10% raise import prices) and Foreign (10% decrease import prices).
 - Efficient composition: relative price of Home goods falls.

- Optimal LCP policy: $\ln(\mu) = \ln(\mu^*) = 0.5 [\ln(Z) + \ln(Z^*)]$. With a 10% increase in Home productivity:
 - Money stance raises by 5% everywhere, no prices change.
 - Efficient level: real balances increase by 5% in Home and Foreign.
 - Inefficient composition: no change in relative prices.
- Optimal DCP (all invoicing in Home currency):
 $\ln(\mu) = 0.5 [\ln(Z) + \ln(Z^*)]$ and $\mu^* = Z^*$. With a 10% increase in Home productivity:
 - Money stance raises by 5% at Home. Only Foreign import price change (reduction by 5%).
 - Inefficient level: real balances increase by 5% in Home, but only 2.5% in Foreign.
 - Inefficient composition: limited change in relative prices.

Should we cooperate?

- Cooperation: choose μ and μ^* to maximize $0.5(EU + EU^*)$, not just EU or EU^* .
- Useful only under two conditions:
 - Home policy affects Foreign country, and conversely. Spillover not taken into account with self-centered policy.
 - Spillover effect differs from the domestic impact of policy.
- No need to cooperate under PCP (spillover identical to domestic effect) or LCP (no spillover).
- Useful under DCP: spillover differs from domestic, Home should react more to its shocks:

$$\ln(\mu) = \frac{2}{3} \ln(Z) + \frac{1}{3} \ln(Z^*) ; \quad \mu^* = Z^*$$

Extension: asymmetric baskets

- With identical consumption baskets in both countries, efficient demand level is the same: $C = C^*$.
- Under LCP, policy has no impact on the demand composition and focuses on the efficient level.
 - $C = C^*$ achieved by $\mu = \mu^*$.
 - No movement in exchange rate: if the exchange rate is useless, don't move it.
- If Home consumption is tilted towards Home goods (domestic bias, or with non-traded goods), C should increase more than C^* when Home productivity increases.
 - Under LCP, requires a larger monetary expansion in Home: $\mu > \mu^*$.
 - Home currency depreciates. The exchange rate has no impact on prices, but should not restrict policy: if the exchange rate is useless, ignore it and don't constrain policy through it.

C
 C^*

Main takeaways

- Stochastic model to compute optimal policy rules in response to shocks.
- Prices are set, but in a forward-looking way.
 - Price premium: when costs are volatile, firms set higher prices.
- Ideal stabilization: keep costs constant, so firms don't even want to change prices.
 $w = z$
- Feasible in closed economy, but not (necessarily) in open economy.
 - Depends on ability to affect prices via the exchange rate (pass-through).
 - Policy care about domestic and foreign firms selling in the country.
- Cooperation can help, but under specific conditions.

EXTRA SLIDES : GRAPHICAL ILLUSTRATION

- Aggregate demand (AD) is the monetary stance:

$$C = \frac{\mu}{p}$$

- Aggregate supply (AS) is the market clearing + technology:

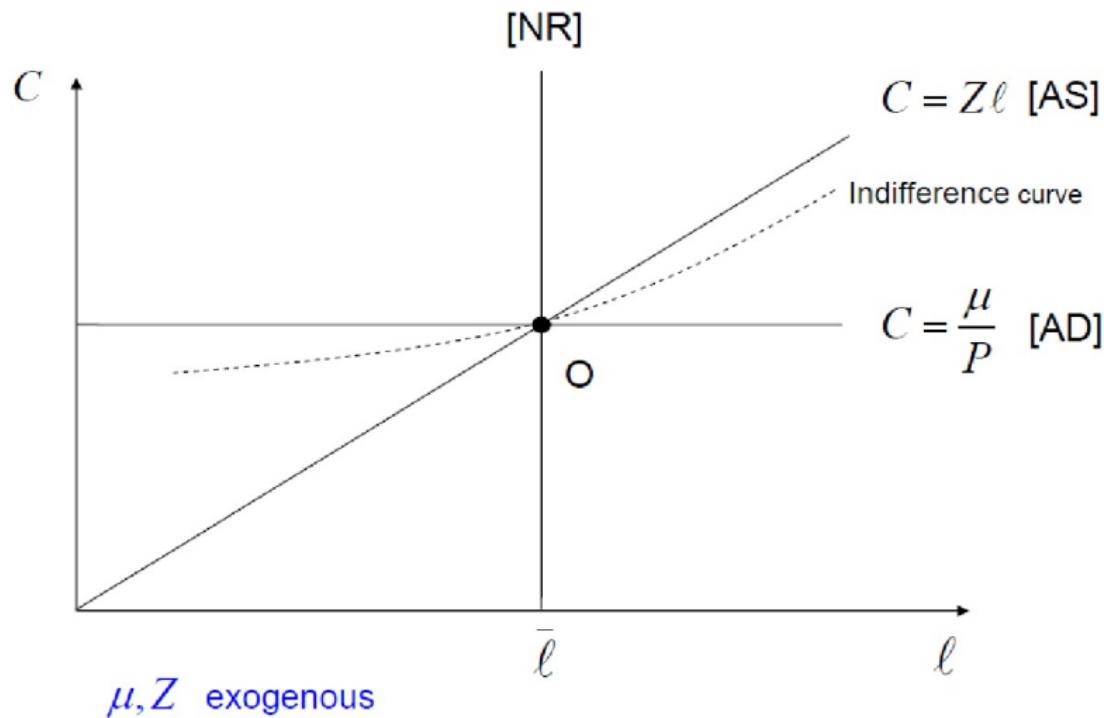
$$C = ZI$$

- Natural rate (NR) is the flexible price allocation of labor:

$$f^{\text{flex}} = \frac{\theta - 1}{\theta \kappa} = \bar{I}$$

Graphical relation

- NR gives labor $\bar{\ell}$, AS gives C , and AD gives P .

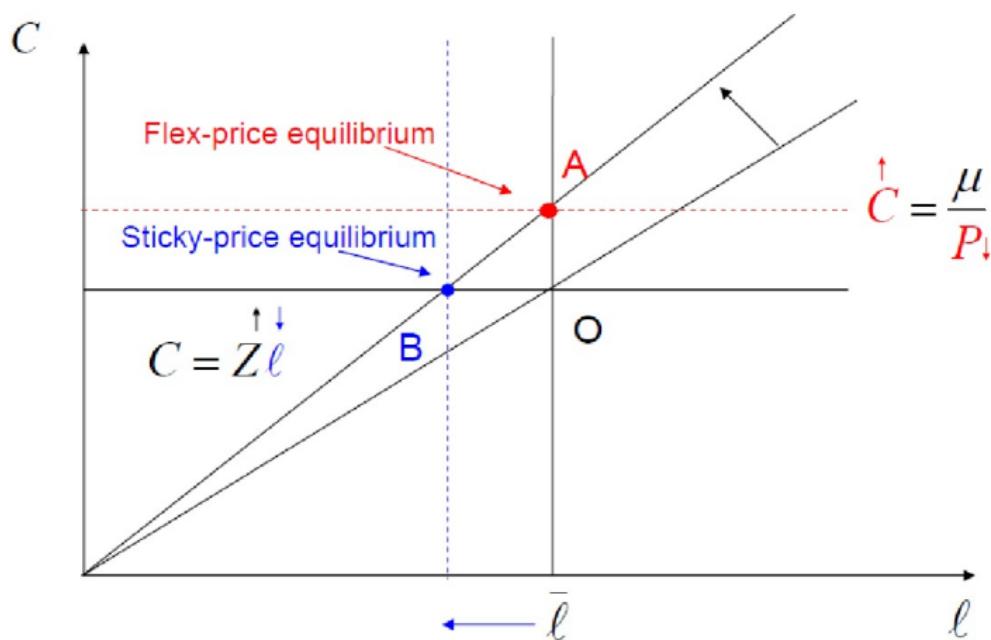


Impact of productivity shocks

- Higher productivity Z reduces firms' costs.
 - AS rotates up to the left.
- Flexible price: productivity reduces prices.
 - NR: labor unchanged at \bar{I} .
 - AS - NR intersection gives higher consumption, AD gives the price level.
- If prices cannot adjust, NR becomes irrelevant (firms are not at the optimal price).
 - If monetary policy does nothing (unchanged μ), AD does not move. Equilibrium at AS - AD intersection, employment is inefficiently low.
 - Increase μ to move AD up. Equilibrium at AS - AD intersection, higher consumption and unchanged labor.
 - Monetary policy replicates the flexible price allocation.

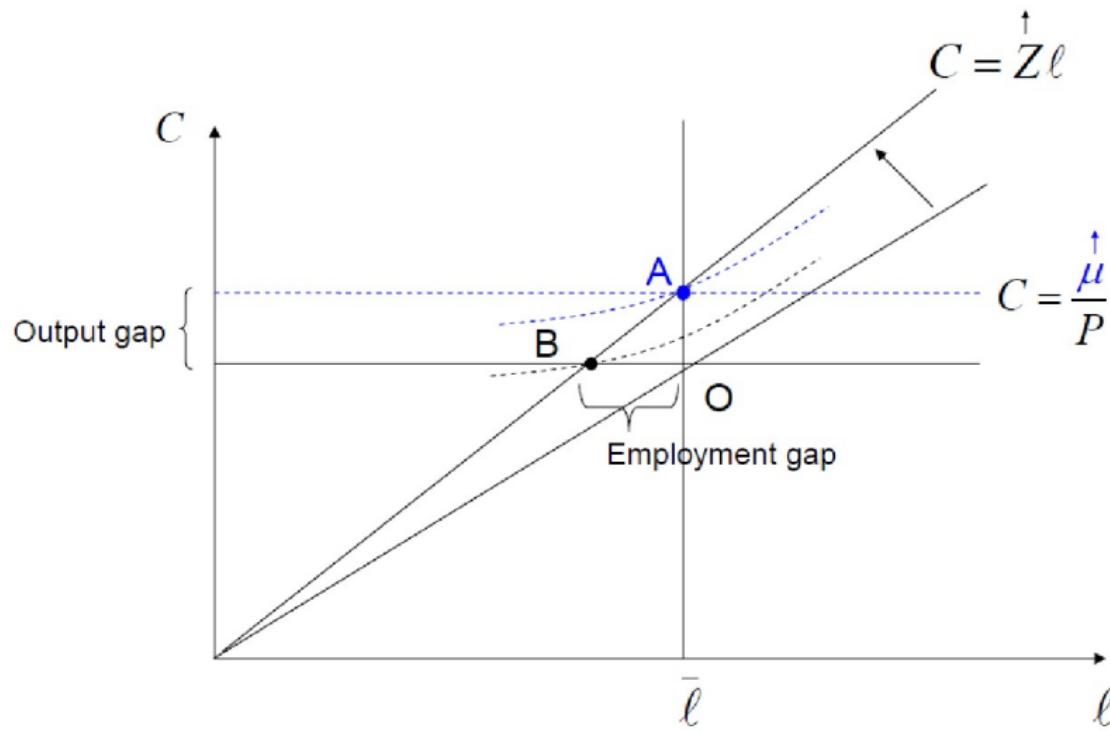
Illustration without policy

- Higher productivity rotates AS. Under sticky prices, AD is unchanged.



Optimal policy

- Monetary expansion increases AD and delivers optimal allocation.



Open economy figure

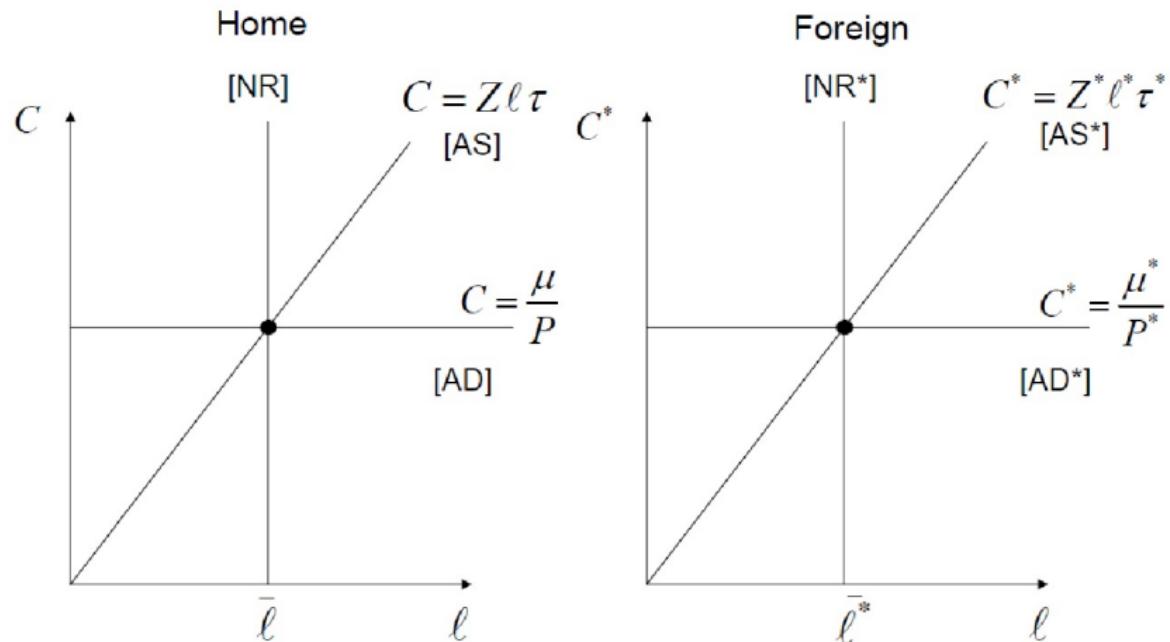
- AD and NR relations identical to the closed economy case.
- AS reflects the terms-of-trade. A given amount of home labor gives more home consumption when:
 - Productivity is high
 - Home goods fetch a higher price than foreign goods (terms of trade τ are favorable):

$$C = Zl\tau \quad ; \quad \tau = \left[\frac{P}{2} \left(\frac{1}{P(H)} + \frac{1}{\mathcal{E}P^*(H)} \right) \right]^{-1}$$

$$C^* = Z^*l^*\tau^* \quad ; \quad \tau^* = \left[\frac{P^*}{2} \left(\frac{\mathcal{E}}{P(F)} + \frac{1}{P^*(F)} \right) \right]^{-1}$$

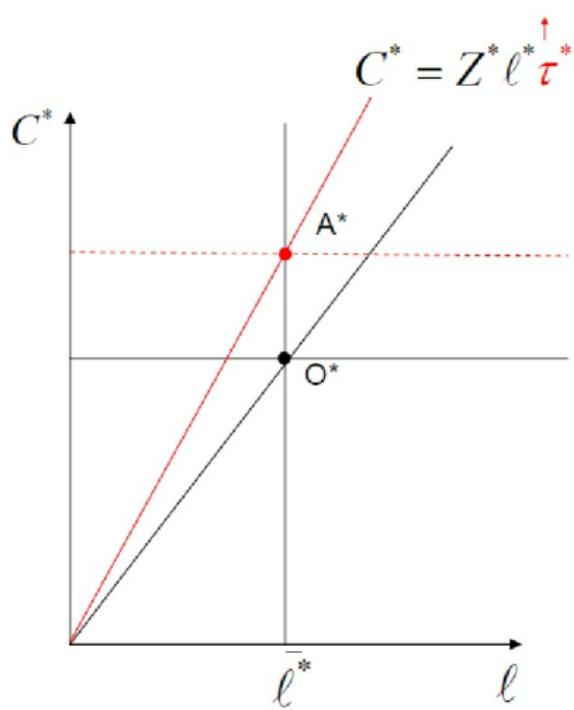
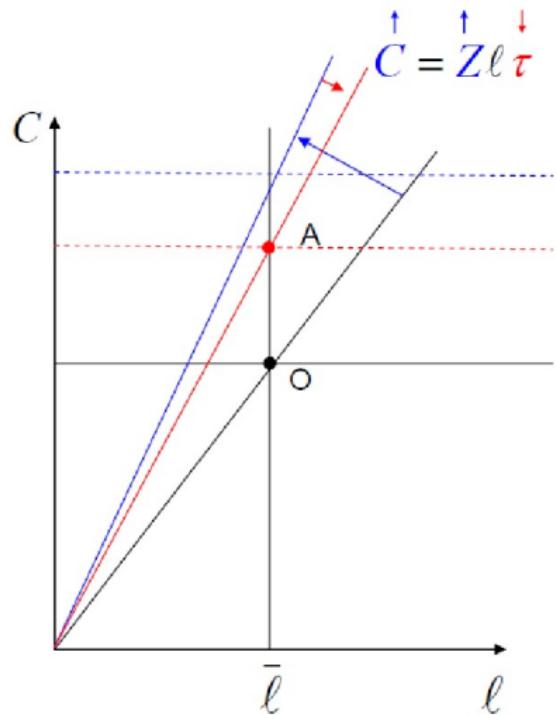
Open economy diagram

- AS now includes terms-of trade.



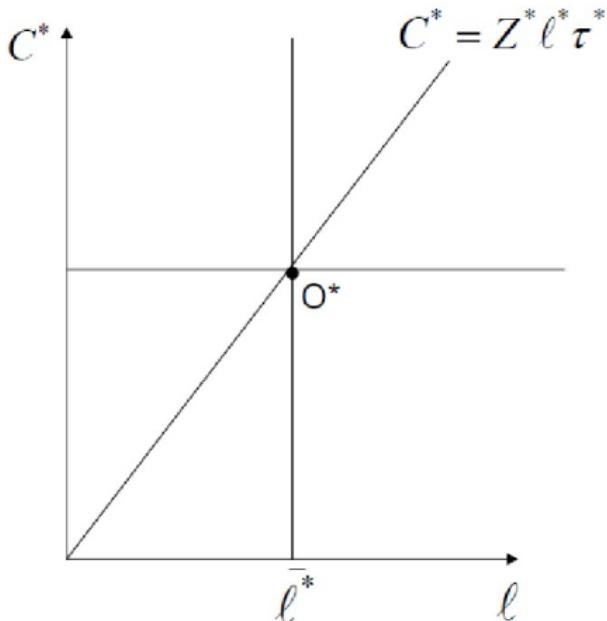
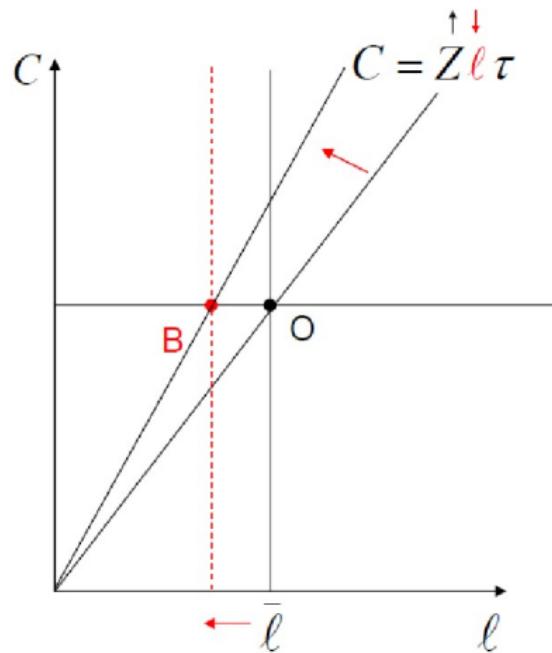
Productivity under flexible prices

- Increase in Home productivity lowers the price of Home goods. Lower τ and higher τ^* .



Productivity under sticky prices

- Only monetary policy can change the terms-of-trade. Higher productivity lowers Home output.



Optimal policy under PCP

- Monetary expansion lowers τ and raises τ^* . Replicates the flexible price allocation.

