Macroeconomics A; EI060

Short problems

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1 Infinite horizon optimization

Question: Consider that the consumer maximizes an intertemporal log utility:

$$U_t = \sum_{s=t}^{\infty} \beta^{s-t} ln\left(C_s\right)$$

Output is an endowment growing at a rate g, and the agent can save in a bond:

$$\begin{aligned} C_t + B_{t+1} &= (1+r) B_t + Y_t \\ CA_t &= rB_t + Y_t - C_t \end{aligned}$$

Show that:

$$C_{t+1} = \beta (1+r) C_t$$

2 Consumption solution

Question: Assume that $g < r$. Show that consumption at time t is:

$$C_t = (1-\beta)\left[(1+r) B_t + \sum_{s=t}^{\infty} \frac{Y_s}{(1+r)^{s-t}}\right]$$

3 Net exports and the current account

Question: Show that the trade balance and the current account are:

$$\begin{aligned} NX_t &= -(1-\beta)(1+r) B_t + \frac{\beta(1+r)-(1+g)}{r-g} Y_t \\ CA_t &= [\beta(1+r)-1] B_t + \frac{\beta(1+r)-(1+g)}{r-g} Y_t \end{aligned}$$

Show that the debt dynamics are:

$$B_{t+1} = \beta (1+r) B_t + \frac{\beta(1+r)-(1+g)}{r-g} Y_t$$

Is the ratio of debt to GDP $\frac{B_t}{Y_t}$ constant? How about the ratio of other variables to GDP?

4 Constant consumption

Question: Consider that $\beta(1+r)=1$.

What are the values of consumption C_t, net exports NX_t, the current account CA_t, in periods t and $t+1$?

What are the debt dynamics $B_{t+1}-B_t$?

Are variables constant, in terms of ratio to GDP?

5 Autarky

Question: If the economy is in autarky, consumption is always equal to output.

Show that the real interest rate is:

$$1+r^A=\frac{1+g}{\beta}$$

6 Open economy, autarky interest rate

Question: Consider that the economy is open, and the world interest rate is equal to the autarky interest rate $1+r^A$.

What are the values of consumption C_t, net exports NX_t, the current account CA_t, in periods t and $t+1$?

What are the debt dynamics $B_{t+1}-B_t$?

Are variables constant, in terms of ratio to GDP?