International Trade I: Theory

The Heckscher-Ohlin Model (2)[1]

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[1] These lecture notes are based on materials from A. Costinot, A. Dixit, R. Feenstra, Feenstra&Taylor, and J. P. Neary.

Outline of the Lecture

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1. **Two-by-two-by-two Heckscher-Ohlin model**
   - Integrated equilibrium
   - Heckscher-Ohlin Theorem
2. **High-Dimensional Predictions**

Basic environment

- Previous results hold for small open economies
  - relative good prices were taken as exogenously given
- We now turn world economy with two countries, North and South
- We maintain the two-by-two HO assumptions:
  - there are two goods, $g = 1, 2$, and two factors, K and L
  - identical technology around the world, $y_g = f_g(K_g, L_g)$
  - identical homothetic preferences around the world, $d_g^c = \alpha_g(p)L^c$
- What is the pattern of trade in this environment?

Strategy

- Start from **Integrated Equilibrium** $\equiv$ competitive equilibrium that would prevail if *both* goods and factors were freely traded
- Consider **Free Trade Equilibrium** $\equiv$ competitive equilibrium that prevails if goods are freely traded, but factors are not
- Ask: Can free trade equilibrium reproduce integrated equilibrium?
  - If factor prices are equalized through trade, the answer is yes
- In this situation, one can then use homotheticity to go from differences in factor endowments to pattern of trade

Integrated equilibrium

- **Integrated equilibrium** corresponds to (p, ω, y) such that:

$$(ZP): \quad p = A'(\omega)\omega \tag{1}$$

$$(GM): \quad y = \alpha(p)(\omega' v) \tag{2}$$

$$(FM): \quad v = A(\omega)y \tag{3}$$

  where

  - $p \equiv (p_1, p_2)$, $\omega \equiv (w, r)$, $A(\omega) \equiv [a_{fg}(\omega)]$, $y \equiv (y_1, y_2)$, $v \equiv (L, K)$, $\alpha(p) \equiv [\alpha_1(p), \alpha_2(p)]$
  - $A(\omega)$ derives from cost-minimization
  - $\alpha(p)$ derives from utility-maximization

Free-trade equilibrium

- **Free-trade equilibrium** corresponds to $(p^t, \omega^n, \omega^s, y^n, y^s)$ such that:

$$(ZP): \quad p^t \leq A'(\omega^c)\omega^c \tag{4}$$

$$(GM): \quad y^n + y^s = \alpha(p^t)(\omega^{n\prime}v^n + \omega^{s\prime}v^s) \tag{5}$$

$$(FM): \quad v^c = A(\omega^c)y^c \text{ for } c = n, s \tag{6}$$

  where (4) holds with equality if good is produced in country c

- **Definition:** Free-trade equilibrium replicates integrated equilibrium if $\exists(y^n, y^s) \geq 0$ such that $(p, \omega, \omega, y^n, y^s)$ satisfy conditions (4)-(6).

Factor-Price-Equalization (FPE) Set

- **Definition:** (v^n, v^s) are in the FPE set if $\exists(y^n, y^s) \geq 0$ such that condition (16) holds for $\omega^n = \omega^s = \omega$.

- **Lemma:** If (v^n, v^s) are in the FPE set, then free-trade equilibrium replicates integrated equilibrium.

- **Proof:** By definition of the FPE set, $\exists(y^n, y^s) \geq 0$ such that

$$v^c = A(\omega)y^c$$

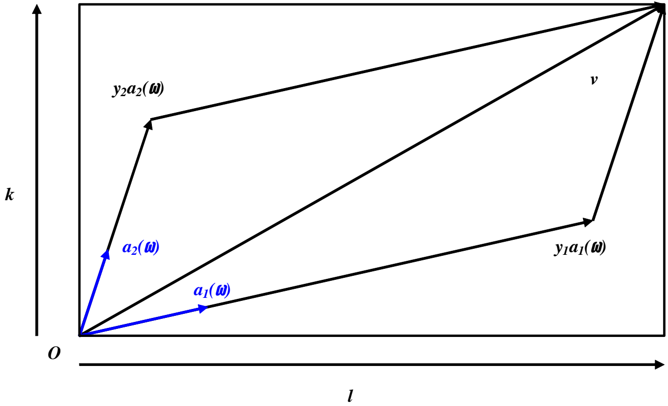
So condition (6) holds. Since $v = v^n + v^s$, this implies

$$v = A(\omega)(y^n + y^s)$$

Combining this expression with condition (3), we obtain $y^n + y^s = y$. Since $\omega^{n\prime}v^n + \omega^{s\prime}v^s = \omega' v$, condition (5) holds as well. Finally, condition (1) directly implies (4) holds.

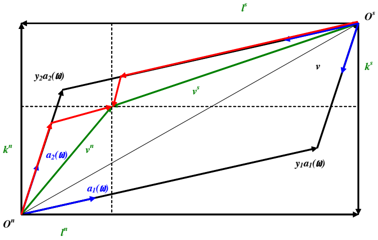
Integrated equilibrium: graphical analysis

- Factor market clearing in the integrated equilibrium:



The "Parallelogram"

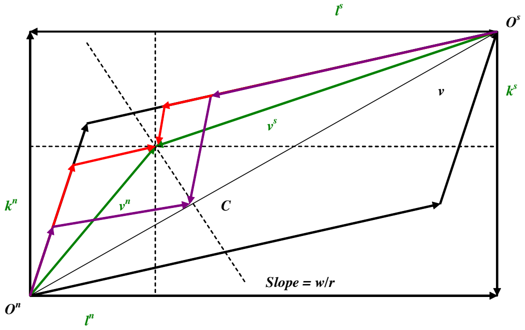
- **FPE set** $\equiv (v^n, v^s)$ inside the parallelogram



- When v^n and v^s are inside the parallelogram, we say that they below to the same **diversification cone**
- This is a very different way of approaching FPE than FPE Theorem
  - Here, we have shown that there can be FPE iff factor endowments are not too dissimilar, whether or not there are no FIR
  - Instead of taking prices as given - whether or not they are consistent with integrated equilibrium - we take factor endowments as primitives

Heckscher-Ohlin Theorem: graphical analysis

- Suppose that (v^n, v^s) is in the FPE set
- **HO Theorem:** In the free trade equilibrium, each country will export the good that uses its abundant factor intensively



- Outside the FPE set, additional technological and demand considerations matter (e.g. FIR or no FIR)

Heckscher-Ohlin Theorem: alternative proof

- HO Theorem can also be derived using Rybczynski effect:
  1. Rybczynski theorem $\Rightarrow y_2^n/y_1^n > y_2^s/y_1^s$ for any p
  2. Homotheticity $\Rightarrow c_2^n/c_1^n = c_2^s/c_1^s$ for any p
  3. This implies $p_2^n/p_1^n < p_2^s/p_1^s$ under autarky
  4. Law of comparative advantage $\Rightarrow$ HO Theorem

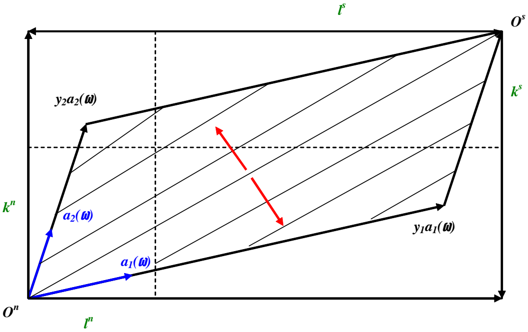
Trade and inequality

- Predictions of HO and SS Theorems are often combined:
  - HO theorem $\Rightarrow p_2^n/p_1^n < p_2/p_1 < p_2^s/p_1^s$
  - SS Theorem $\Rightarrow$ Moving from autarky to free trade, real return of abundant factor increases, whereas real return of scarce factor decreases
  - If North is skill-abundant relative to South, inequality increases in the North and decreases in the South

- So why may we observe a rise in inequality in the South in practice?
  - Southern countries are not moving from autarky to free trade
  - Technology is not identical around the world
  - Preferences are not homothetic and identical around the world
  - There are more than two goods and two countries in the world

Trade volumes

- Let us define trade volumes as the sum of exports plus imports
- Inside FPE set, iso-volume lines are parallel to diagonal (HKa p.23)
  - the further away from the diagonal, the larger the trade volumes
  - factor abundance rather than country size determines trade volume

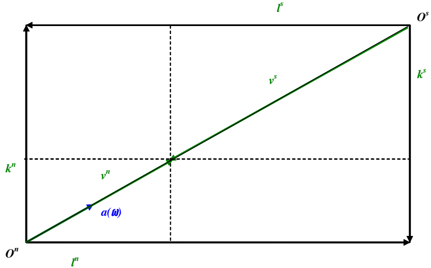


Outline of the Lecture

1. Two-by-two-by-two Heckscher-Ohlin model
2. High-Dimensional Predictions

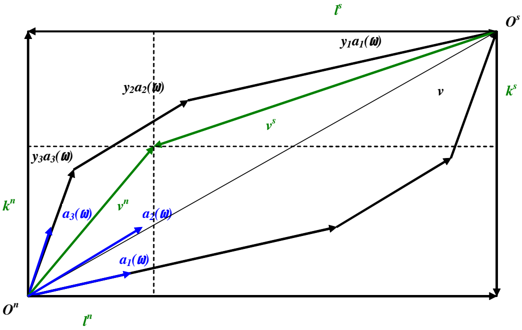
FPE (I): More factors than goods

- Suppose now that there are F factors and G goods
- By definition, (v^n, v^s) is in the FPE set if $\exists(y^n, y^s) \geq 0$ s.t. $v^c = A(\omega)y^c$ for $c = n, s$
- If $F = G$ ("even case"), the situation is qualitatively similar
- If $F > G$, the FPE set will be "measure zero": $\{v | v = A(\omega)y^c \text{ for } y^c \geq 0\}$ is a G-dimensional cone in F-dimensional space
- Example: "Macro" model with 1 good and 2 factors



FPE (II): More goods than factors

- If $F < G$, there will be indeterminacies in production, (y^n, y^s), and so, trade patterns, but FPE set will still have positive measure
- Example: 3 goods and 2 factors



Stolper-Samuelson-type result: “Friends and Enemies”

- SS Theorem was derived by differentiating zero-profit condition
- With an arbitrary number of goods and factors, we still have

$$\hat{p}_g = \sum_f \theta_{fg} \hat{w}_f \tag{7}$$

where w_f is the price of factor f and $\theta_{fg} \equiv w_f a_{fg}(\omega)/c_g(\omega)$

- Now suppose that $\hat{p}_{g_0} > 0$ and $\hat{p}_g = 0$ for all $g \neq g_0$
- Equation (7) immediately implies the existence of f_1 and f_2 s.t.

$$\begin{aligned} \hat{w}_{f_1} \geq & \hat{p}_{g_0} > \hat{p}_g = 0 \text{ for all } g \neq g_0 \\ \hat{w}_{f_2} < & \hat{p}_g = 0 < \hat{p}_{g_0} \text{ for all } g \neq g_0 \end{aligned}$$

- So every good is “friend” to some factor and “enemy” to some other (Jones and Scheinkman 1977)

Rybczynski-type results

- Rybczynski Theorem was derived by differentiating the factor market clearing condition
- If $G = F > 2$, same logic implies that increase in endowment of one factor decreases output of one good and increases output of another (Jones and Scheinkman 1977)
- If $G < F$, increase in endowment of one factor may increase output of all goods (Ricardo-Viner)
- If $G > F$, indeterminacies in production imply that we cannot predict changes in output vectors

Heckscher-Ohlin-type results

- Since HO Theorem derives from Rybczynski effect + homotheticity, problems of generalization in the case $G < F$ and $F > G$ carry over to the Heckscher-Ohlin Theorem
- If $G = F > 2$, we can invert the factor market clearing condition

$$y^c = A^{-1}(\omega)v^c$$

- By homotheticity, the vector of consumption in country c satisfies

$$d^c = s^c d$$

  where $s^c \equiv c$'s share of world income, and $d \equiv$ world consumption
- Good and factor market clearing requires

$$d = y = A^{-1}(\omega)v$$

- Combining the previous expressions, we get net exports

$$t^c \equiv y^c - d^c = A^{-1}(\omega)(v^c - s^c v)$$

Heckscher-Ohlin-Vanek Theorem

- Without assuming that $G = F$, we can still derive sharp predictions if we focus on the factor content of trade rather than commodity trade
- We define the net exports of factor f by country c as

$$\tau_f^c = \sum_g a_{fg}(\omega) t_g^c$$

- In matrix terms, this can be rearranged as

$$\tau^c = A(\omega) t^c$$

- HOV Theorem: In any country c, net exports of factors satisfy

$$\tau^c = v^c - s^c v$$

- So countries should export the factors in which they are abundant compared to the world: $v_f^c > s^c v_f$
- Assumptions of HOV Theorem are extremely strong: identical technology, FPE, homotheticity
  - One shouldn't be too surprised if it performs miserably in practice