Macroeconomics A: EI056

Quizz

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Class of October 10, 2023

1 Steady state in Solow model

Question: How is the steady-state capital determined in the Solow model?

Answer: The steady state capital reflects the balance between two aspects. The first is capital accumulation, which is a constant saving share of output. This accumulation increases as the capital increases (because more capital means more output), but to an ever lower extent as we have decreasing returns to scale. The second aspect is the drag on capital coming from its depreciation, as well as the exogenous growth of population and productivity (because these directly lower the ratio between capital and effective labor, and we focus on this ratio). This drag is proportional to capital.

For a low value of capital, the drag is limited but output is high (capital is very productive), so the first aspect dominates and capital increases. At very high values of capital, the first aspect is relatively limited (the marginal unit of capital does not give much output) but the drag is large. The second aspect then dominates and capital decreases. The steady state is the point where the two aspects balance each other out.

2 Drivers of growth

Question: What drives growth in the Solow model?

Answer: First, it is important to distinguish between growth at the steady state, and growth along a convergence towards the steady state.

At the steady state, the ratio between capital (physical or human) and labor / productivity is constant. Capital then grows at the same rate as population and technology ($n + g$ in the class notation). As human capital, physical capital, and labor / productivity all grow at the same rate, output also grows at this rate. Output per capita (Y/L) grows at the rate of productivity, g.

During a convergence to the steady state – say from a low level of capital – growth is driven by growth in labor / productivity (as above), but also by the accumulation of human and physical

capital towards their steady state value. Thus, k and h increase (there are constant in the steady state). The exact value of growth depends on the savings rate. If the economy saves a lot, capital accumulates quickly and growth is fast.

3 Impact of savings

Question: Is saving a good thing in the Solow model?

Answer: It depends on what variable we consider. Focus on the steady state. A higher saving rate clearly implies a higher capital and more output. However, it does not necessarily imply higher consumption. This is because while there is more output, a bigger fraction of it is saved. When the saving rate is low (meaning lower that the weight on capital in the production function), then higher saving primarily boost output and thus consumption. When the saving rate is very high however, adding capital does not boost output much, and thus we have a higher saving rate on a marginally higher output, implying a lower consumption.

4 Golden rule

Question: What is the Golden Rule? What is dynamic inefficiency?

Answer: The steady state capital and output in the Solow model depend on the savings rate. If the savings is very low, most output is consumed and little is left to invest in capital. Capital is then low, and so is output. If the savings rate is very high, a lot of output is invested. The capital stock is then high. However, maintaining the capital stock at that level requires a substantial effort to offset depreciation, and little is left for consumption. The Golden Rule states that the savings rate should be equal to the weight of capital in the production (α in the class notation), where the capital is high enough to produce substantial consumption, but not so high as to put a drain on resources to offset depreciation.

The concept of dynamic inefficiency is related to the Golden Rule. Consider an economy that starts with a capital below the Golden rule, and decides to reduce its savings rate permanently. This allows for a short run increase in consumption, but is paid by a long run reduction in the capital stock and consumption. If the economy starts with a capital above the Golden Rule however, the reduction of savings frees resources for consumption in the long run, as the burden of offsetting depreciation is reduced. Consumption then increases in the short and the long run, a free lunch.

5 Human capital

Question: How does introducing human capital improve the model?

Answer: The share of physical capital, α, is about $1/3$. Without human capital, this means that two-thirds of the factors of production consist of labor which cannot be accumulated. If capital plays such a small role, it is relatively easy to quickly accumulate capital from the (relatively) large

output. The economy then converges very quickly to the steady, in fact it does so at a speed that is not realistic.

In addition, if capital is a small share of output, generating a given ratio of per capita GDP between two countries (say 10) requires a large discrepancy of capital per capita (say 1'000) as capital affects output only through a small exponent. The model can then match the observed output inequality across countries only by assuming an unrealistically large difference in the capital stocks.

Introducing human capital implies that the labor input is not just hours worked, but also includes skills, which is a factor that can be accumulated. We can then have an economy that is more capital (human and physical) intensive. With the two capitals being a large share of output, it takes time to build up the capital stocks, and convergence is then slow. In addition, the two capital affect output through a larger exponent, so we can generate the observed output inequality across countries by assuming a realistic difference in the capital stocks (say 30).

6 Endogenous growth

Question: In the "AK" model ($Y_t = A_t K_t$), what is the impact of the savings rate? Why does it differ from Solow?

Answer: In the Solow model, the saving rate affects the steady state *level* of (scaled) GDP, but not the long run *growth rate*. In the AK model, output is linear in capital, which is itself linear in output ($\dot{K}_t = sY_t - \delta K_t$). This implies that the growth rate of capital (and of output) is affected by savings ($\dot{K}_t / K_t = sA_t - \delta$).

The reason why savings affect the growth rate is that the AK model does not have the built-in "brake" mechanism in the Solow model, namely the decreasing returns to scale. In Solow, capital accumulation become less effective as we accumulate capital, which leads to a point where more capital is self-defeating as depreciation and dilution (growth of population and exogenous productivity) dominates. This pins down a capital level. No suh