Geneva Graduate Institute (IHEID)

Econometrics II (EI062), Spring 2025

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Problem Set 5

Due: Sunday, 18 May, 23:59

- Prepare concise answers.
- State clearly any additional assumptions, if needed.
- You are encouraged to collaborate in groups but the final write-up should be individual.
- Submit your solutions, along with any code (if applicable), in a **single pdf file** through **Moodle**. If you choose to write your solutions by hand, please make sure your scanned answers are legible.
- Grading scale:

5.5	default grade
6	absolutely no mistakes and particularly appealing write-up (clear and concise answers, decent formatting, etc.)
5	more than a few mistakes, or single mistake and particularly long, wordy answers
4	numerous mistakes, or clear lack of effort (e.g. parts not solved or not really attempted)
1	no submission by due date

Problem 1

Consider the following bivariate VAR(1):

$$y_t = \Phi_1 y_{t-1} + u_t \ , \quad u_t = \Phi_\varepsilon \varepsilon_t \ , \quad \varepsilon_t \overset{i.i.d.}{\sim} N(0, I).$$

where $y_t = (w_t, h_t)'$ is composed of log wages w_t and log hours h_t. The vector of structural shocks $\varepsilon_t = (\varepsilon_{a,t}, \varepsilon_{b,t})'$ is composed of a labor demand shock (technology shok) $\varepsilon_{a,t}$ and a labor supply shock (preference shock) $\varepsilon_{b,t}$.

(a) What condition does Φ_1 have to satisfy so that y_t is stationary?

(b) Suppose y_t is stationary, derive the autocovariances of order zero and one, denoted by $\Gamma_{yy}(0)$ and $\Gamma_{yy}(1)$.

(c) Derive the impulse response function of y_{t+h}, $h = 0, 1, \ldots$ with respect to the vector of structural shocks ε_t. How do log wages react to a labor supply shock that occurred 3 periods before?

(d) Describe the identification problem in the context of this VAR.

(e) Supose we are willing to assume that, contemporaneously, hours worked are only affected by preferences, not technology. What restrictions does this assumption impose on Φ_ε? Is this enough to uniquely identify Φ_ε?

(f) Alternatively, suppose that we assume that a labor supply shock $\varepsilon_{b,t}$ moves wages and hours in opposite directions upon impact, whereas a demand shock $\varepsilon_{a,t}$ moves wages and hours in the same direction. What restrictions does this assumption impose on Φ_ε? Is this enough to uniquely identify Φ_ε?

We can think of wages and hours being determined by an interplay of labor supply and labor demand. Let $h_t = \varphi^D(w_t, y_{t-1}; \varepsilon_{a,t}, \varepsilon_{b,t})$ be the demand and $h_t = \varphi^S(w_t, y_{t-1}; \varepsilon_{a,t}, \varepsilon_{b,t})$ the supply function. They show the relationship between hours and wages (i.e. quantity and price in the labor market), whereby these functions (think of labor/supply curves) depend on current technology and preference shocks as well as past hours and wages.[1]

(h) Suppose we assume that the labor demand is only affected by the technology shock, not the preference shock, whereas labor supply is affected by both shocks:

$$h_t = \varphi^D(w_t, y_{t-1}; \varepsilon_{a,t}) \quad \text{and} \quad h_t = \varphi^S(w_t, y_{t-1}; \varepsilon_{a,t}, \varepsilon_{b,t}) \ .$$

[1] In a structural economic model for the labor market, we would typically assume exogenous preference and technology processes (not i.i.d., but persistent, e.g. AR(1)s), which then, combined with the households' (workers') and firms' utility and profit maximization problems, lead to such demand and supply functions. Owing to the persistence preference and technology processes, the demand and supply functions will depend on past preference and technology shocks, which are summarized by past hours and wages in y_{t-1}.

Is this enough to uniquely identify Φ_ε?

Hint: remember that demand must equal supply at all times.

(i) Does your answer change if, on top of the above assumption, we assume that

$$\frac{\partial w_t}{\partial \varepsilon_{b,t}} = (\alpha - 1)\frac{\partial h_t}{\partial \varepsilon_{b,t}}$$

for some given α; conditional on α, is it possible to uniquely identify the elements of Φ_ε? If yes, show how you can solve for Φ_ε based on α and the reduced-form VAR parameters.