L6. Competitive Market Economies

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Literature

- MWG (1995), Chapters 10 A-C

This and the Next Lecture

Fundamental issue of Economics: the organization of production and the allocation of the resulting commodities among consumers

Positive (or descriptive) perspective:

What are the market equilibrium production and consumption? What factors (or institutional mechanisms) determine them and how?

Normative (or prescriptive) perspective:

What constitutes a *socially optimal* plan of production and consumption? (ofc we need to be more specific about what "socially optimal" means.)

"What is" vs. "What should be"

Today's Lecture

Competitive (or perfectly competitive) market economies

Two key concepts

i. Competitive equilibrium (or Walrasian equilibrium)

ii. Pareto optimality (or Pareto efficiency)

- The concept of a competitive equilibrium provides us with an appropriate notion of market equilibrium for competitive market economies.
- The concept of Pareto optimality offers a minimal and uncontroversial test that any socially optimal economic outcome should pass.

The relationship between these two concepts

- In the special context of the **partial equilibrium model**.

Before We Start

- This is the first time we consider an entire economy in which consumers and firms interact through markets

- Remember the tools/concepts we will introduce are for studying this interaction

- What we learn here is very much a first pass, but these materials are "0 to 1" to almost all realistic, fun, and complex micro problems that you may encounter later.

Basic Environment

$w = [w_1 \ \cdots \ w_I]$

$\begin{bmatrix} \vdots \\ w_{l1} \\ \vdots \end{bmatrix}$

w_i

$w_l =$

- [Dimension] Consider an economy consisting of
  - I consumers ($i = 1, ..., I$)
  - J firms ($j = 1, ..., J$)
  - L goods ($l = 1, ..., L$)

- [Preference] Consumer i's preferences over consumption bundles $x_i = (x_{1i}, ..., x_{Li})$ in his consumption set $X_i \subset \mathbb{R}^L$ are represented by the utility function $u_i(.)$.

- [Endowment] The total amount of each good initially available in the economy is called *endowment*, $w = (w_i, ..., w_l)$.

- [Production] Each element of Y_j is a production vector $y_j = (y_{1j}, ..., y_{Lj}) \in \mathbb{R}^L$. Total net amount of good l available to the economy $w_l + \sum_j y_{lj}$ (recall negative entries in a production vector denote input usage).

Feasible Allocation

Definition 6.1 (Feasible Allocation)
All economic allocations $(x_1, ..., x_I, y_1, ..., y_J)$ are a specification of a consumer vector for each consumer i and a production vector for each firm j. The allocation $(x_1, ..., x_I, y_1, ..., y_J)$ is feasible if

$$\sum_{i=1}^{I} x_{li} \leq w_l + \sum_{j=1}^{J} y_{lj}, \text{ for } l = 1, ..., L.$$

An economic allocation is feasible $\Rightarrow$ What consumers consume in total should be no greater than what is available in the economy.

Pareto Optimality

It is often interesting to ask whether an economic system is producing "optimal" economic outcomes.

How to define "optimality"?

Definition 6.2 (Pareto Optimality)
A feasible allocation $(x_1, ..., x_I, y_1, ..., y_J)$ is Pareto optimal (or Pareto efficient) if there is no other feasible allocation $(x'_1, ..., x'_I, y'_1, ..., y'_J)$ such that $u_i(x'_i) \geq u_i(x_i)$ for all $i = 1, ..., I$ and $u_i(x'_i) > u_i(x_i)$ for some i.

A feasible allocation is Pareto optimal $\Rightarrow$ There is no alternative feasible way to organize production and distribution of goods that makes some consumers better off without making others worse off.

Pareto Optimality

- Can you give some examples of Pareto improvement?
  - “use-up” feasibility
  - adj allocation
  - adj production

- Can you think of any limitations of the criterion of Pareto optimality?

Pareto Optimality

- Can you think of any limitations of the criterion of Pareto optimality?

  - Incomplete "comparison" concept
  - It does not ensure that an allocation is in any sense equitable
  - Is a Pareto allocation always "better" than a non-Pareto allocation? Not really!

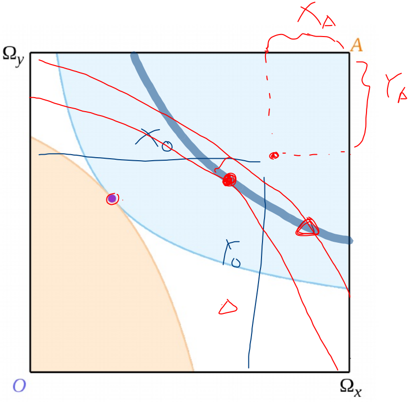
- Limited concept, but serves as in important "minimal" test for the desirability of an allocation.

- It does, at the very least, say that there is no waste in the allocation of resources in society $\Rightarrow$ A non-Pareto allocation always has room for Pareto improvement.

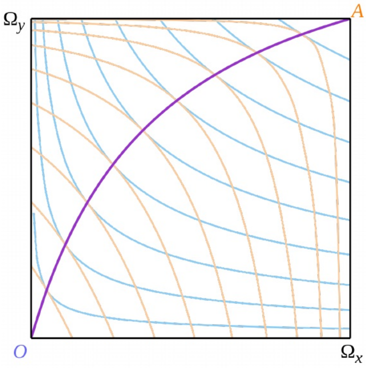
Edgeworth Box and Pareto Set

Wikipedia "Edgeworth box"

- Two commodities, X and Y.
- Two consumers, Octavio and Abby.



(a) Tangible indifference curves



(b) Pareto set (purple curve)

Competitive Equilibria

In addition, we suppose that consumer i owns a share θ_{ij} of firm j (where $\sum_i \theta_{ij} = 1$), giving him a claim of fraction θ_{ij} of firm j's profits.

Definition 6.3 (Competitive (or Walrasian) Equilibirum) The allocation $(x_1^*, ..., x_I^*, y_1^*, ..., y_J^*)$ and price vector $p^* \in \mathbb{R}^L$ constitute a competitive (or Walrasian) equilibrium if the following conditions are satisfied:

i) **Profit maximization:** for each firm j, y_j^* solves

$$max_{y_j \in Y_j} \; p^* \cdot y_j$$

ii) **Utility maximization:** for each consumer i, x_i^* solves

$$max_{x_i \in X_i} \; u_i(x_i) \quad s.t. \; p^* \cdot x_i \leq p^* \cdot w_i + \sum_{j=1}^{J} \theta_{ij}(p^* \cdot y_j^*)$$

iii) **Market clearing:** for each good $l = 1, ..., L$,

$$\sum_{i=1}^{I} x_{li} = w_l + \sum_{j=1}^{J} y_{lj}$$

Competitive Equilibria

Discussion

- Conditions i) and ii) reflect the underlying assumption which is common to nearly all economic models – agents seek to do what is best for themselves.

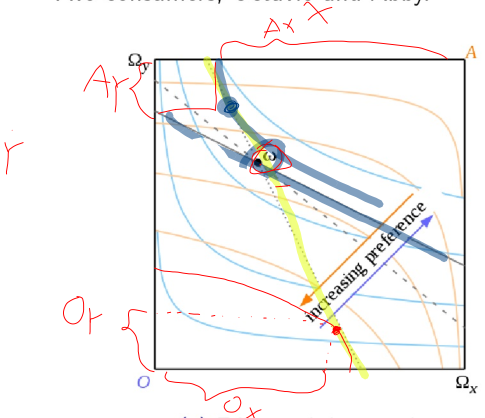
- Condition iii) requires that, at the equilibrium price, the designed production and consumption decisions identified by i) and ii) are mutually compatible – even though agents only do what best for themselves, market signals (here price) will coordinate their behavior to reach iii).

Key insight of iii) is not 'no excess resources $\geq$' (which is also implied by i) and ii), but mutual compatibility $=$ (i.e.,'market clear').

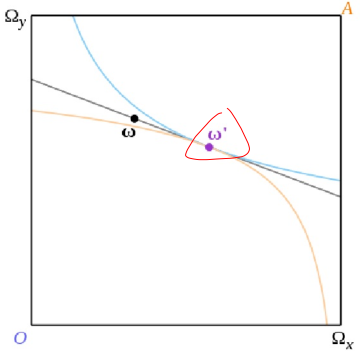
Edgeworth Box and Competitive Equilibrium

Pure exchange economy

- Two commodities, X and Y.
- Two consumers, Octavio and Abby.



(a) Edgeworth box market



(b) Equilibrium in an Edgeworth box

Competitive Equilibria

Note 1

- Note that if $(x_1^*, ..., x_I^*, y_1^*, ..., y_J^*)$, p^* constitute a competitive equilibrium, then so does $(x_1^*, ..., x_I^*, y_1^*, ..., y_J^*)$, $\alpha \cdot p^*$ for any scalar $\alpha > 0$.

- So we can normalize one good (or one factor)'s price without loss of generality.

- You will see many papers normalizing wage, or one good's price to 1 for this reason.

Competitive Equilibria

Note 2

Add up all budget constraint

$\sum_i pX_i = \sum_i pw_i + \sum_i \sum_j \theta_{ij} p Y_j$

$\Rightarrow pX = pw + \sum_j pY_j = pY$

$\Leftrightarrow P_1X_1 + P_2X_2 + \cdots P_LX_L = P_1W_1 + \cdots + P_LW_L + P_1Y_1 + \cdots + P_LY_L$

E1

Lemma 6.1 If the allocation $(x_1, ..., x_I, y_1, ..., y_J)$ and price vector $p >> 0$ satisfy the market clearing condition for all goods $l \neq k$, and if every consumers' budget constraint is satisfied with equality, so that $p \cdot x_i = p \cdot w_i + \sum_j \theta_{ij} p \cdot y_j$ for all i, then the market for good k also clears.

- **Proof:** adding up all consumers' budget constraint gives an additional equation (E1); straightforward to show "E1+ (L-1) market clearing conditions" ensures the last market clearing condition must hold.

- You will see many papers leave one market clearing condition when solving for competitive equilibrium for this reason.

l's market clearing condition

$X_l = W_l + Y_l \Rightarrow P_lX_l = P_lW_l + P_lY_l$

we have L-1 of those. Call them Eq1 ... Eq L-1

$\Rightarrow E1 - Eq1 - \cdots Eq_{L-1} \Rightarrow P_LX_L = P_LW_L + P_LY_L \Rightarrow X_L = W_L + Y_L$; Q.E.D.

Partial Equilibrium Competitive Analysis

Justification of the Partial Equilibrium Analysis

Marshallian partial equilibrium analysis envisions the market for one good that constitutes a small part of the overall economy.

The small size of the market facilitates two important simplifications for the analysis of market equilibrium:

1. When expenditure on the good under study is a small portion of a consumer's total expenditure, only a small fraction of any additional dollar of wealth will be spent on this good; consequently, we can expect wealth effects for it to be small.

2. With similarly dispersed substitution effects, the small size of the market under study should lead the prices of other goods to be approximately unaffected by changes in this market.

$\rightarrow$ Because of the fixity of prices, we are justified in treating the expenditure of other goods as a single composite commodity, which we call the numeraire.

Two-good Quasilinear Model

Basic Environment

- [Dimension] Consider an economy consisting of
  - I consumers ($i = 1, ..., I$)
  - J firms ($j = 1, ..., J$)
  - 2 goods, (x and the numeraire m); quantity consumption also as x, m, resp.

- [Preference] Consumer i's preferences over consumption bundles (m_i, x_i) takes a quasilinear form: $u_i(m_i, x_i) = m_i + \phi_i(x_i)$.

  Assp: $\phi_i(.)$ is bounded above, $\phi_i'(.) > 0, \phi_i''(.) < 0$, normalize $\phi_i(0) = 0$

- [Endowment] The total amount of m initially available in the economy is given by $w = (w_{m1}, ..., w_{mI})$; no initial endowment of x.

- [Production] Each firm j is able to produce good x from good m. The production technology (or cost function) is $z_j = c_j(q_j)$, i.e., to produce q_j amount of I, firm j needs to use $z_j = c_j(q_j)$ amount of m.

  Assp: $c_i'(.) > 0, c_i''(.) > 0$.

Two-good Quasilinear Model

Identify Equilibria

$P_m = 1$

$L = m + \phi(x) - \lambda(m + px - E)$

FOC:

$1 = \lambda$

$\phi'(x) = p$

i) **Profit maximization:** for each firm j solve

$$max_{q_j \geq 0} \; p^* q_j - 1 c_j(q_j) \;\Rightarrow\; p^* \leq c_j'(q_j), \text{with equality if } q_j^* \neq 0.$$

ii) **Utility maximization:** for each consumer i solve

$$max_{x_i \in R_+, m_i \in R} \; m_i + \phi_i(x_i)$$

$$s.t. \; 1 m_i + p^* x_i \leq 1 w_{mi} + \sum_{j=1}^{J} \theta_{ij}(p^* q_j^* - c_j(q_j^*))$$

$$\Rightarrow \phi_i'(x_i) \leq p^*, \text{ with equality if } x_i^* > 0.$$

iii) **Market clearing:** for each good $l = x$ (recall only need to check L-1 markets clear, so no need to write down the market clearing condition for m),

$$\sum_{i=1}^{I} x_i^* = \sum_{j=1}^{J} q_j^*$$

Two-good Quasilinear Model

Identify Equilibria

Hence, we conclude that the allocation $(x_1^*, ..., x_I^*, q_1^*, ..., q_J^*)$ and the price p^* constitute a competitive equilibrium if and only if

$$C1).\ p^* \leq c_j'(q_j), \text{with equality if } q_j^* = 0\ j = 1, ..., J$$

J

$$C2).\ \phi_i'(x_i) \leq p^*, \text{with equality if } x_i^* > 0\ i = 1, ..., I$$

I

$$C3).\ \sum_{i=1}^{I} x_i^* = \sum_{j=1}^{J} q_j^*$$

1

- For any interior solution, condition C1 says that firm j's marginal benefit from selling good l equals its marginal cost $c_j'(q_j)$.
- Condition C2 says that consumer i's marginal benefit from consuming good x equals its marginal cost p^* (due to quasi-linear funct., only x's own price matter).
- Condition C3 is the market clearing condition.

$\Rightarrow$ I+J+1 equations exactly identify I+J+1 values $\{(x_1^*, ..., x_I^*, q_1^*, ..., q_J^*), p^*\}$.

Graphic Analysis

C1 - C3 have a very important property: they do not involve, in any manner, the endowments or ownership shares of consumers. As a result, we see that the equilibrium allocation and price are independent of the distribution of endowments and ownership shares. This important simplification arises from the quasilinear form of consumer preferences.

We can therefore present the competitive equilibrium of this model nicely using the traditional Marshallian graph $\rightarrow$ the equilibrium price can be identified as the point of intersection of aggregate demand and aggregate supply curves.

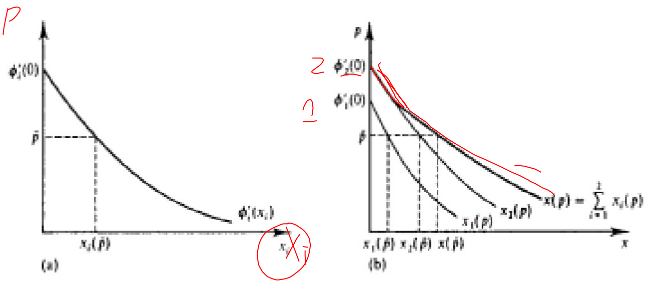


Figure 10.C.1
Construction of the aggregate demand function.
(a) Determination of consumer i's demand.
(b) Construction of the aggregate demand function ($I = 2$).

Graphic Analysis

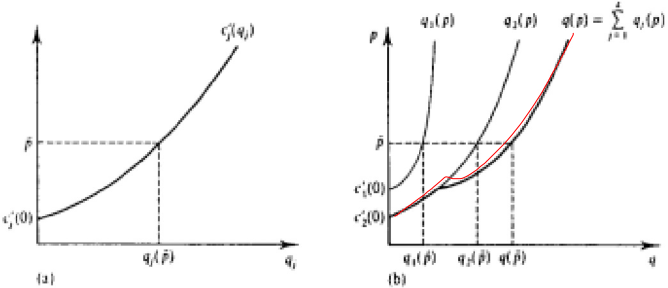


Figure 10.C.2
Construction of the aggregate supply function.
(a) Determination of firm j's supply.
(b) Construction of the aggregate supply function ($J = 2$).

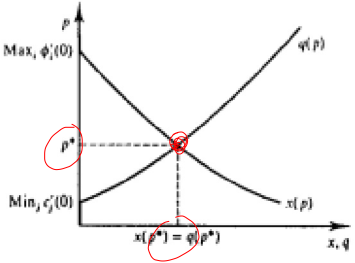


Figure 10.C.3
The equilibrium price equates demand and supply.

Comparative Statics

How changes in underlying market conditions affect the equilibrium outcome of a competitive market?

Sufficient to know how to do so with specific models & graphic analysis.

Example

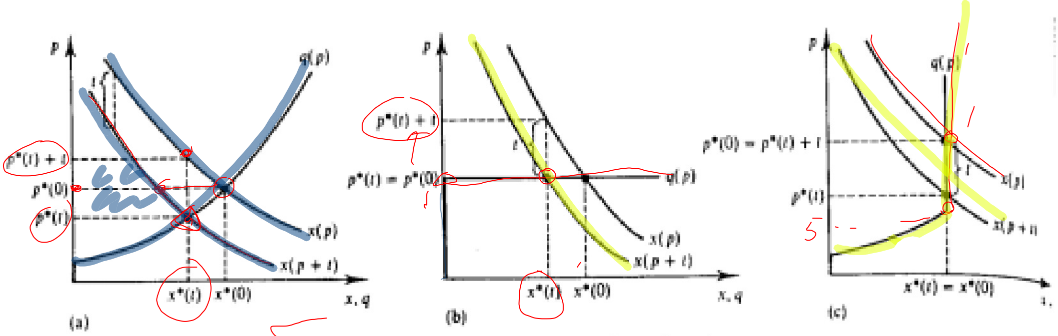


Figure 10.C.7 Comparative statics effects of a sales tax.