Econ 39: International Trade

The Ricardian Model

Treb Allen

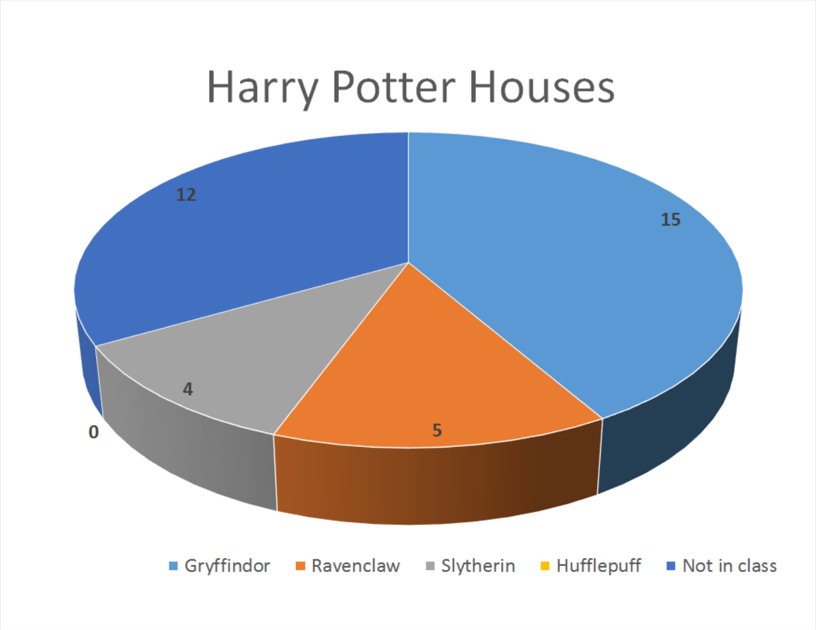
Winter 2018

Plan for the week

- Our first trade model: The Ricardian Trade Model.
- One of the most important results in international trade: the principal of comparative advantage.
- The simplest model that demonstrates how countries can (almost) always gain from trade.
- Today: go through the basics of the model without trade.
- Next class: Introduce trade.
- But first, a broad motivation and an introduction to modeling.

Today's Teams

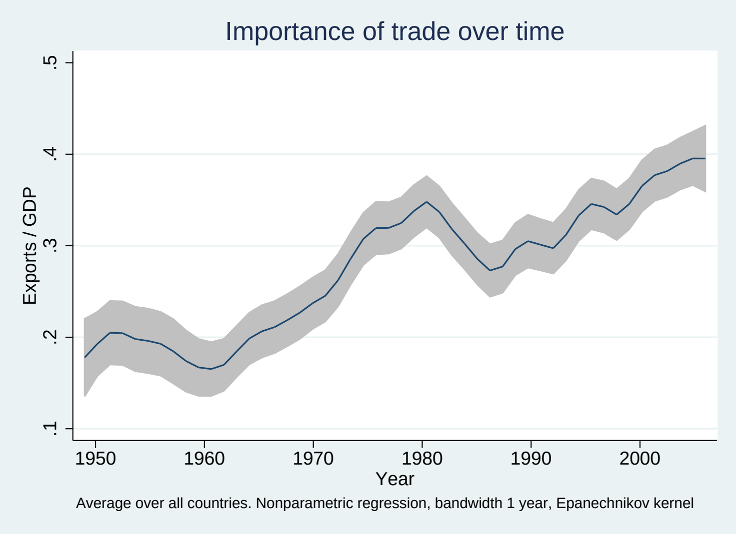
Today's Teams



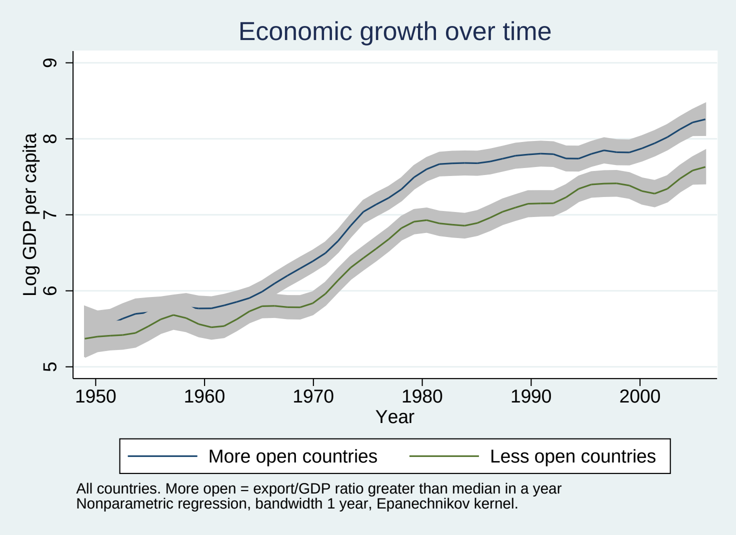
Why study trade?

- Reason #1: Trade is an increasingly important part of the world economy.
- Reason #2: Trade seems closely linked with economic growth.

Reason #1: Trade over time



Reason #2: GDP per capita over time



Why study trade (ctd)?

- Central message of class: In almost all cases, trade makes all countries better off.
- We call this the *gains from trade*.
- Despite the potential gains, much resistance to freer trade.
- Reason #3: How can we understand this resistance?


DEATH
BY CHINA
ONE LOST JOB
AT A TIME
MADE IN CHINA

TRUMP APPOINTS "DEATH BY CHINA" FILMMAKER TO LEAD NEW TRADE COUNCIL

Peter Navarro, a fierce anti-China critic, will serve as an advisor to the incoming president.

BY ABIGAIL TRACY
DECEMBER 22, 2016 10:09 AM

Everett: MCDDEBY EC005

An introduction to economic models

- Trade models are *general equilibrium* models.
- What does this mean? Roughly speaking, there are a lot of moving pieces.
- To avoid confusion, it is very important to be organized while modeling.
- Three parts of any trade model:
  - Exogenous model parameters.
  - Endogenous model outcomes.
  - An equilibrium concept.

Exogenous model parameters

- The exogenous model parameters are the elements of the model that are taken "as given."
- You as the economist choose their values.
- They do not respond to other elements of the model.
- But these are the things that you as an economist can change to see how the outcome of the model can change.

Endogenous model outcomes

- The endogenous model outcomes are the elements of the model that "move around."
- You as the economist do not get to choose their values.
- Instead, they respond to the other elements of the model (including both the exogenous model parameters and other endogenous outcomes).
- These are the parts of the model that you are interested in watching change when you change the exogenous model parameters.

An equilibrium concept

- An equilibrium concept is the set of rules that tells you what the endogenous model outcomes should be for a given set of exogenous model parameters.
- An equilibrium may be one or more conditions.
- Every model should have the following phrase:

  > *"Given a [insert set of exogenous model parameters here], equilibrium is defined by the [insert endogenous model outcomes here] such that [list equilibrium conditions here]."*

Example from one of my recent papers

2.1.3 Equilibrium

The CES assumption implies that the welfare of living in a particular location can be written as an indirect function of the real wage and the overall amenity value:

$$W(i) = \frac{w(i)}{P(i)} u(i). \tag{5}$$

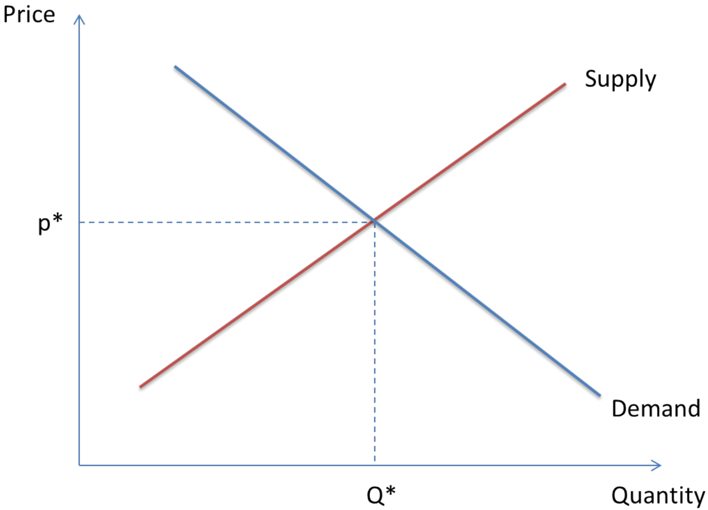
Welfare is said to be *equalized* if for all $i \in S$ there exists a $W > 0$ such that $W(i) \leq W$, with equality if $L(i) > 0$. That is, welfare is equalized if the welfare of living in every inhabited location is the same and the welfare of living in every uninhabited location is no greater than the welfare of the inhabited locations.

Markets are said to *clear* if the income is equal to the value of goods sold in all locations, i.e. for all $i \in S$:

$$w(i) L(i) = \int_S X(i,s)\, ds. \tag{6}$$

Given a regular geography with parameters σ, α, and β, we define a *spatial equilibrium* as a distribution of economic activity such that (i) markets clear; (ii) welfare is equalized; and (iii) the aggregate labor market clears:

Example: Supply and Demand



Example: Supply and Demand

- What are the three parts of that model?

- Exogenous model parameters:
  - Supply curve and demand curve.
- Endogenous model outcomes:
  - Price and quantity.
- Equilibrium concept:
  - "For a given supply curve and demand curve, equilibrium is defined by a price and quantity such that demand equals supply."

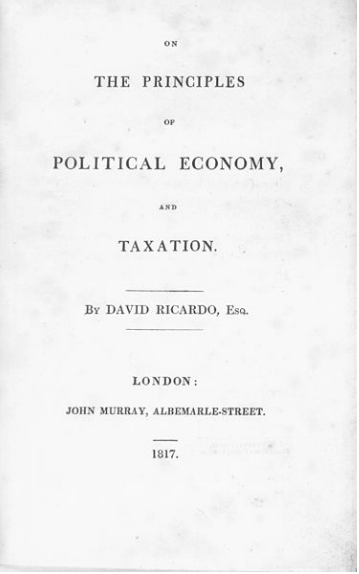
A precautionary note

- Benefit of economic models: clearly state assumptions, follow model to logical conclusion.
- A good economic model abstracts from the unimportant to focus on a particular mechanism.
- However, models are (at best) a stylized notion of reality.
- In particular, the exogenous parameters of one model may very well be the endogenous outcomes of another.
- Economics in general requires careful thought about:
    - How the model assumptions affect the model outcomes ("thinking within the model").
    - What other aspects are missing from the model that may affect the outcomes in reality ("thinking outside the model").

David Ricardo (1772-1823)

- Third of seventeen children.
- Eloped at the age of 21.
- Became rich by purposefully misleading the public about the outcome of the Battle of Waterloo and buying British securities at a steep discount.
- Died from an ear infection at 51.

The Ricardian Model of Trade



Ricardo wrote his first economics article at the age of 37.

Model assumptions

- Two countries
- Two goods
- One factor of production
- Country-specific production technology

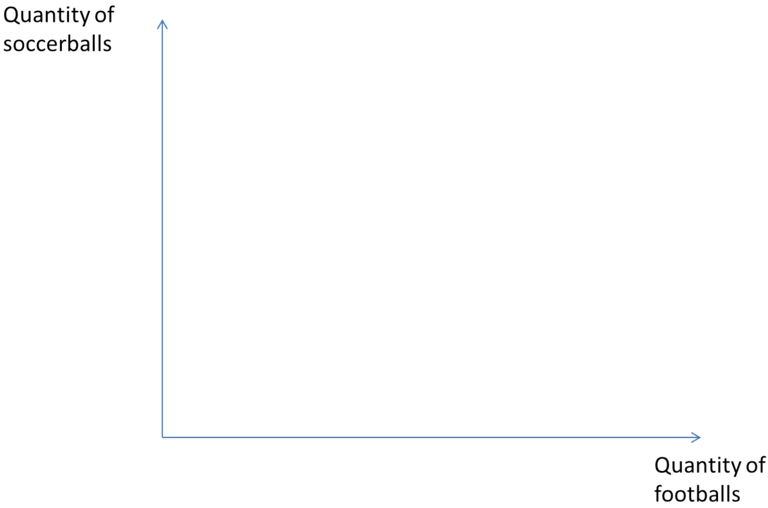
Example

- Two countries: USA and Mexico
- Two goods: Footballs and soccer balls
- One factor of production: Labor
  - USA: 100 people.
  - Mexico: 50 people.
- Country-specific production technology:
  - USA: Each worker can produce *1* football or *1* soccer ball.
  - Mexico: Each worker can produce *1* football or *2* soccer balls.

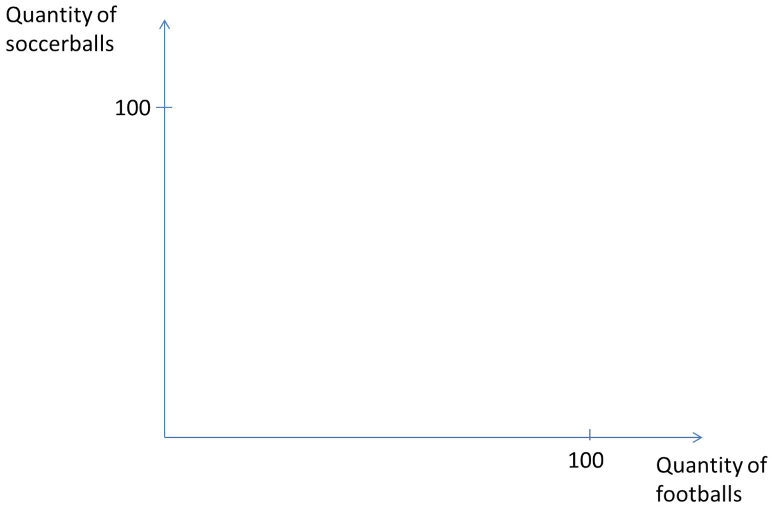
The production possibility frontier

- Consider the U.S. in autarky. ("Autarky" means in the absence of trade).
- We first characterize the **production possibilities** of the United States:
  - The **set of production possibilities** of the United States is all the different combinations of footballs and soccer balls that the U.S. can produce.
  - The **production possibility frontier** (PPF) is the most soccer balls the U.S. can produce *for each number* of footballs produced.

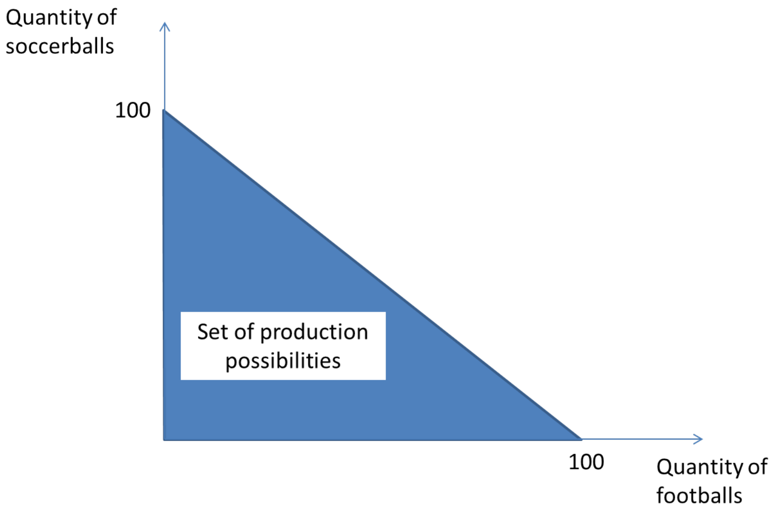
PPF for the Example



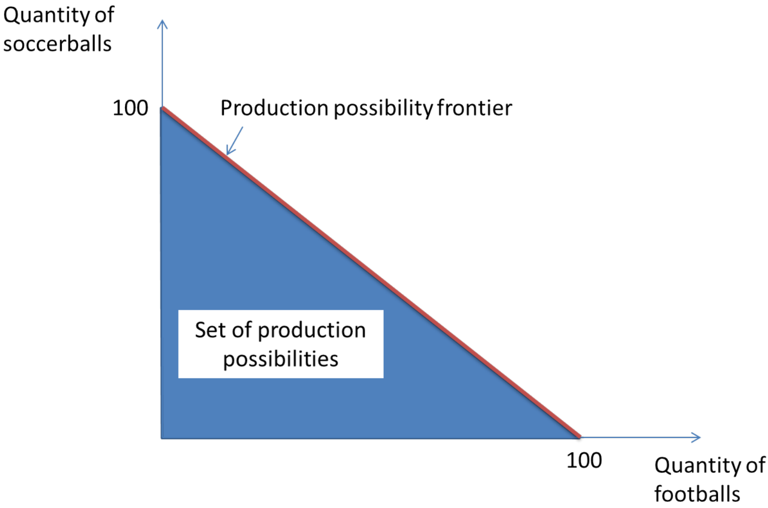
PPF for the Example



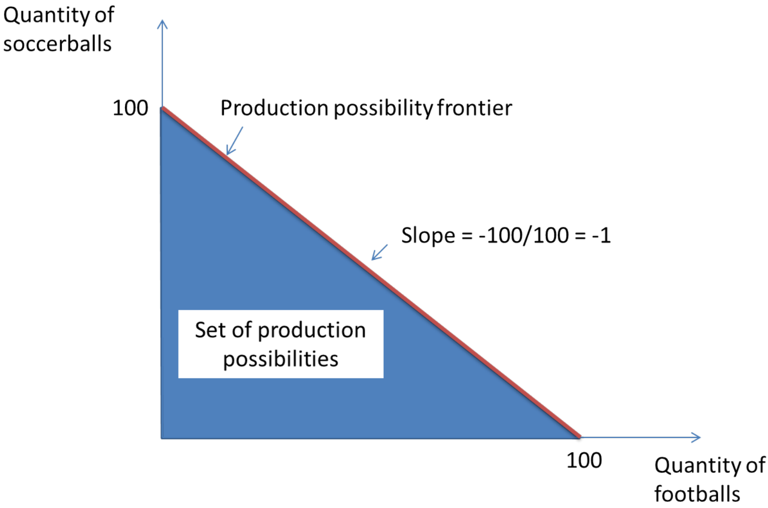
PPF for the Example



PPF for the Example



PPF for the Example



Warning: Math ahead

- I am now going to define things formally using mathematics.
  - (This is a general plan for the course: first provide the intuition, then formalize it).
- Why use math at all?
  - Allows us to derive general insights (does not depend on particular examples).
  - Sometimes, models get too complicated to give full insight in a picture.
- What do you need to know?
  - Definitely understand the intuition.
  - Work through the math on the problem sets (they are meant to be hard).
  - Math is fair game for exams.
- **Ask questions and slow me down if I go too fast!**

The production possibility frontier generalized

- Let Q_{US}^{FB} denote the number of footballs produced and Q_{US}^{SB} denote the number of soccer balls produced by the U.S.

- Let α_{US}^{FB} and α_{US}^{SB} denote how much labor is required to produce a single football or soccer ball by a U.S. worker, respectively. We call this the **unit labor cost**;
  - Note that the unit labor cost is the inverse of worker productivity.
- Let L_{US} be the number of workers in the U.S.

The production possibility frontier generalized

- Then the **set of production possibilities** is:

$$\{Q_{US}^{FB}, Q_{US}^{SB} \mid \alpha_{US}^{FB} Q_{US}^{FB} + \alpha_{US}^{SB} Q_{US}^{SB} \leq L_{US}\}$$

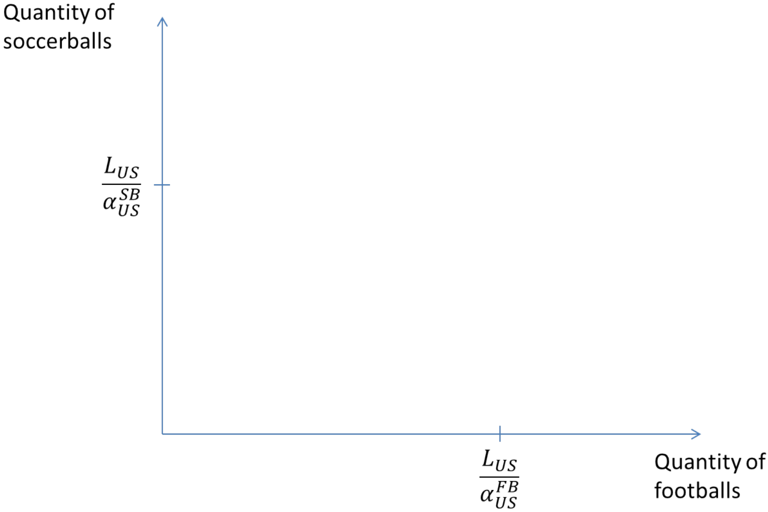
- And the **production possibility frontier** is:

$$Q_{US}^{SB}\left(Q_{US}^{FB}\right) \equiv \max_{Q>0} Q \text{ s.t. } \alpha_{US}^{SB} Q + \alpha_{US}^{FB} Q_{US}^{FB} \leq L_{US}$$

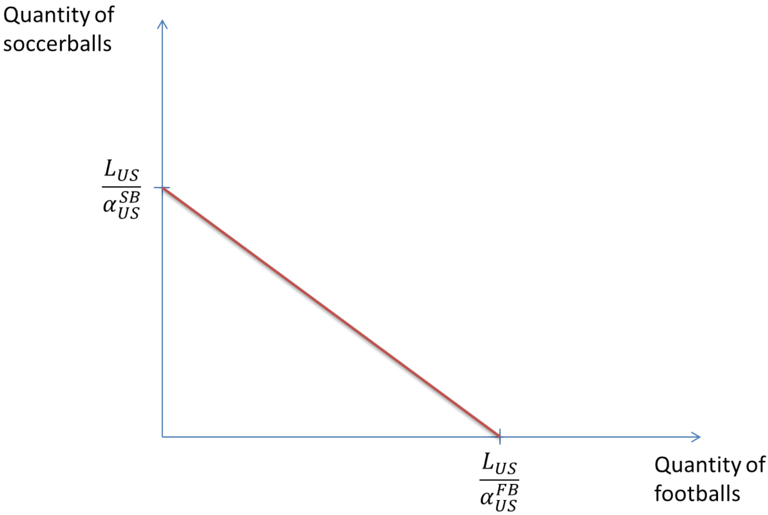
- [Class question]: what is the solution to above equation?
  - Answer:

$$Q_{US}^{SB}\left(Q_{US}^{FB}\right) = \frac{L_{US}}{\alpha_{US}^{SB}} - \frac{\alpha_{US}^{FB}}{\alpha_{US}^{SB}} Q_{US}^{FB}$$

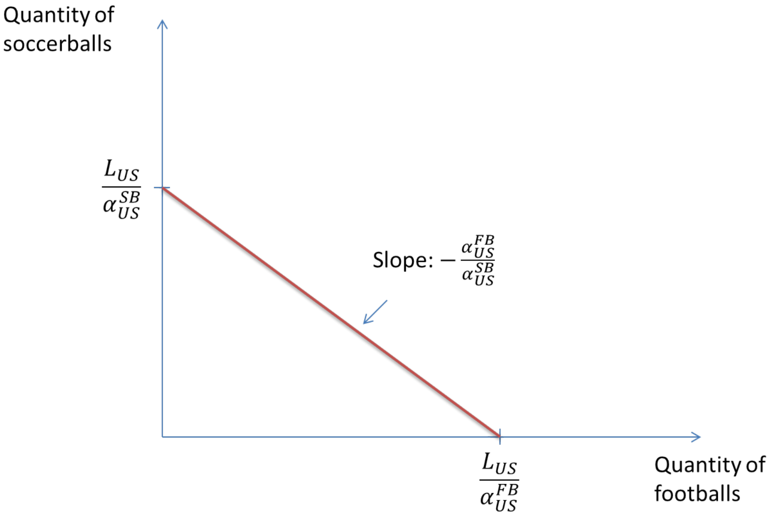
More general PPF



More general PPF



More general PPF



Opportunity cost

- The slope of the production possibility frontier is $-\frac{\alpha_{US}^{FB}}{\alpha_{US}^{SB}}$.
- [Class question]: What is the economic interpretation of this slope?
- To see this:
  - A worker can make $1/\alpha_{US}^{SB}$ soccer balls or $1/\alpha_{US}^{FB}$ footballs.
  - Equivalently, it takes α_{US}^{SB} workers to make a soccer ball and α_{US}^{FB} workers to make a football.
  - Hence, for each football made, we could have made $\frac{\alpha_{US}^{FB}}{\alpha_{US}^{SB}}$ soccer balls.
- We call this the **opportunity cost** of producing a football.

Prices and specialization

- Now suppose the price of footballs and soccer balls in the U.S. are p_{US}^{FB} and p_{US}^{SB}.

- Take the perspective of a worker.

- [Class question]: Suppose (as in the example) that $\alpha_{US}^{FB} = \alpha_{US}^{SB} = 1$. Suppose that $p_{US}^{FB} = 2$ and $p_{US}^{SB} = 1$. Then what would happen?

Prices and specialization (ctd.)

- [Class question]: Now suppose that $p_{US}^{FB} = 4$ and $p_{US}^{SB} = 2$. Then what would happen?
  - Note: The units of the prices don't actually matter. What matters is the relative price $\frac{p_{US}^{FB}}{p_{US}^{SB}}$.
  - This is true throughout the course (and in economics more generally; it doesn't matter if something is measured in cents or dollars)

Prices and specialization (ctd.)

- [Class question]: What is the relative price $\frac{p_{US}^{FB}}{p_{US}^{SB}}$ at which workers are indifferent between producing soccer balls and footballs?
  - Answer: when $\frac{p_{US}^{FB}}{p_{US}^{SB}} = 1$ workers earn the same amount regardless if they produce soccer balls or footballs.

Prices and specialization (ctd.)

- [Class question]: For a general technology α_{US}^{FB} and α_{US}^{SB}, at what relative price are workers indifferent between producing soccer balls and footballs?
  - Answer: Worker revenue from producing footballs is $\frac{p_{US}^{FB}}{\alpha_{US}^{FB}}$ and is $\frac{p_{US}^{SB}}{\alpha_{US}^{SB}}$, so they are indifferent when:

$$\frac{p_{US}^{FB}}{\alpha_{US}^{FB}} = \frac{p_{US}^{SB}}{\alpha_{US}^{SB}} \iff$$
$$\frac{p_{US}^{FB}}{p_{US}^{SB}} = \frac{\alpha_{US}^{FB}}{\alpha_{US}^{SB}}$$

Preferences

- To determine equilibrium, we need to specify the preferences of workers.
- For simplicity, suppose there is a "representative agent" in the economy who receives utility:

$$W = U\left(C_{US}^{SB}, C_{US}^{FB}\right),$$

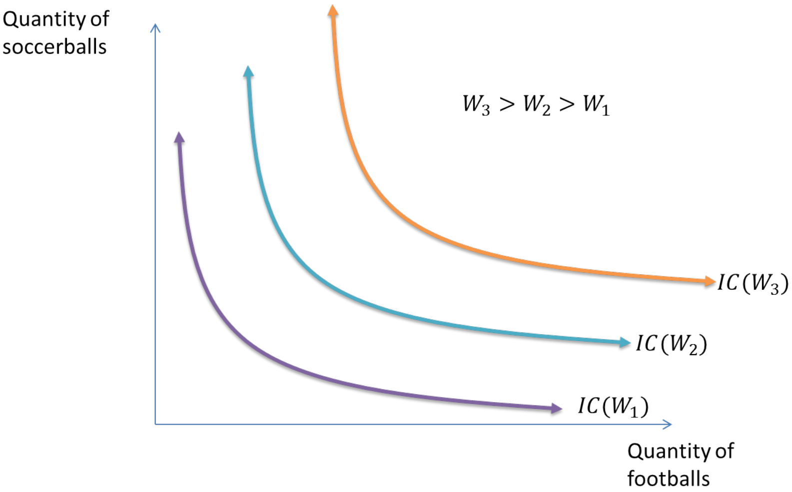
  where:
  - C_{US}^{SB} is the quantity of soccer balls consumed in the United States.
  - C_{US}^{FB} is the quantity of footballs consumed in the United States.
  - U is some (given) function
  - W is a number that tells you the total utility of the representative agent.
- I will assume that $\frac{\partial U\left(C_{US}^{SB}, C_{US}^{FB}\right)}{\partial C_{US}^{SB}} > 0$ and $\frac{\partial U\left(C_{US}^{SB}, C_{US}^{FB}\right)}{\partial C_{US}^{FB}} > 0$.

Indifference curves

- We can use the preferences $W = U\left(C_{US}^{SB}, C_{US}^{FB}\right)$ to rank any consumption combination.
- That is, if $U\left(C_{US}^{SB}, C_{US}^{FB}\right) > U\left(\tilde{C}_{US}^{SB}, \tilde{C}_{US}^{FB}\right)$, then we know that the representative agent would prefer to consume $\left\{C_{US}^{SB}, C_{US}^{FB}\right\}$ rather than $\left\{\tilde{C}_{US}^{SB}, \tilde{C}_{US}^{FB}\right\}$.
- The typical way to show this on a diagram is to draw indifference curves.
- An **indifference curve** is a set of all consumption bundles that yield the same utility. Formally the indifference curve corresponding to utility W_1 is:

$$IC\left(W_1\right) = \left\{C_{US}^{SB}, C_{US}^{FB} \mid U\left(C_{US}^{SB}, C_{US}^{FB}\right) = W_1\right\}$$

Indifference curves



- [Class question]: Why are the indifference curves curved like they are?

Types of indifference curves

- [Class question] Suppose that:

$$U\left(C_{US}^{SB}, C_{US}^{FB}\right) = C_{US}^{SB} + \beta C_{US}^{FB}$$

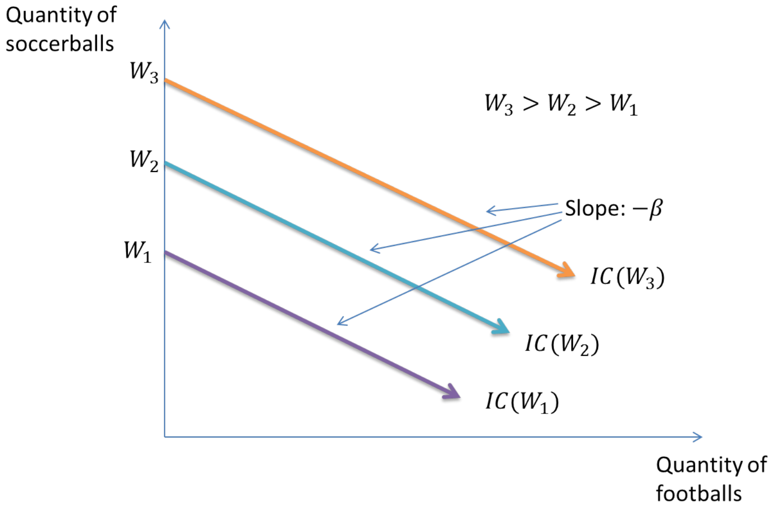
What will the indifference curves look like?

  - Answer:

$$W = C_{US}^{SB} + \beta C_{US}^{FB} \iff$$
$$C_{US}^{SB} = W - \beta C_{US}^{FB}$$

so that the indifference curves will be straight lines with intercept W and slope $-\beta$.

Types of indifference curves



Types of indifference curves

- [Class question] Suppose that:

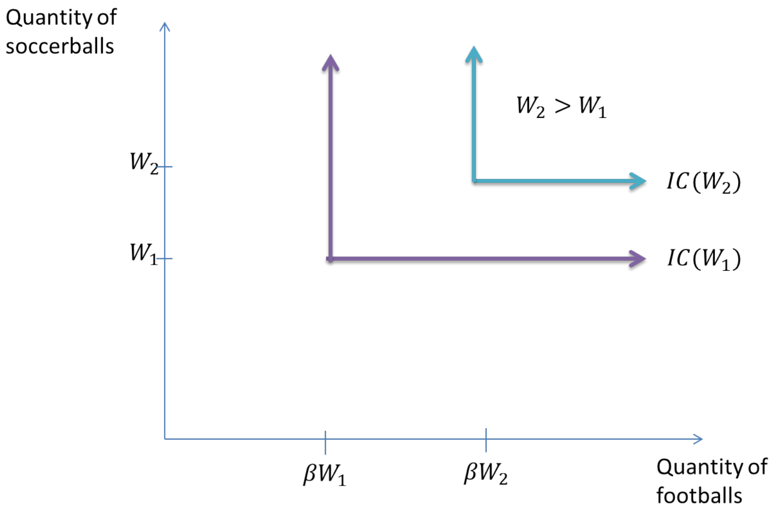
$$U\left(C_{US}^{SB}, C_{US}^{FB}\right) = \min\left\{C_{US}^{SB}, \beta C_{US}^{FB}\right\}$$

What will the indifference curves look like?

  - Answer: The key thing to note if you consume C_{US}^{SB}, then the utility will be the same if you consume βC_{US}^{FB} or $\beta C_{US}^{FB} + x$ for any $x > 0$.

  - Hence these preferences (known as "Leontief" preferences) will have a "kink" at $\{x, \beta x\}$.

Types of indifference curves



Autarkic equilibrium

- We can now (finally!) define the autarkic equilibrium.
- [Class question]: What are the exogenous model parameters?
  - Answer: Productivities α_{US}^{SB} and α_{US}^{FB}, population L_{US}, and preferences $U(\cdot,\cdot)$ (and the same for Mexico).
- [Class question]: What are the endogenous model outcomes?
  - Answer: Production Q_{US}^{SB} and Q_{US}^{FB}, consumption C_{US}^{SB} and C_{US}^{FB}, and relative prices $\frac{p_{US}^{FB}}{p_{US}^{SB}}$ (and the same for Mexico).

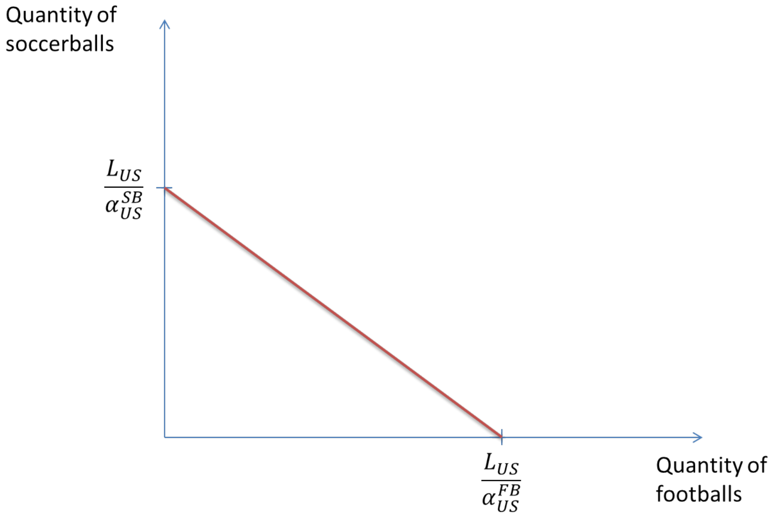
Autarkic Equilibrium

- Defining the equilibrium:

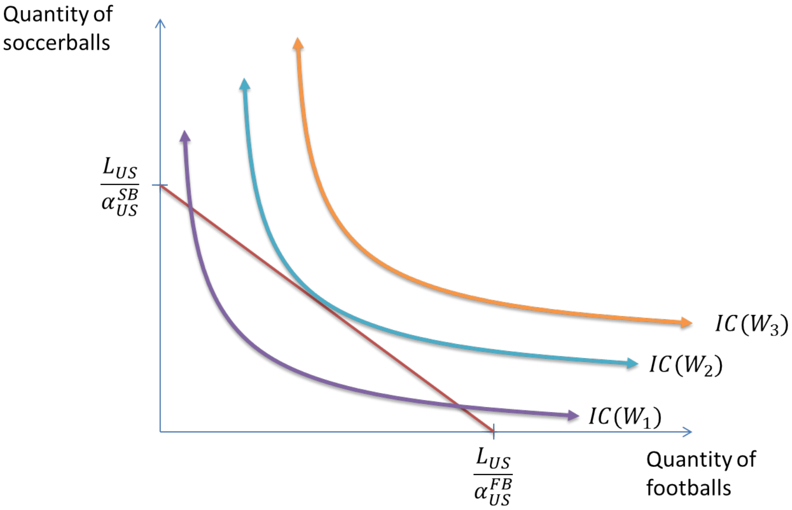
  "For any set of productivities α_{US}^{SB} and α_{US}^{FB}, population L_{US}, and preferences $U(\cdot,\cdot)$, equilibrium is defined as a set of Production Q_{US}^{SB} and Q_{US}^{FB}, consumption C_{US}^{SB} and C_{US}^{FB}, and relative prices $\frac{p_{US}^{FB}}{p_{US}^{SB}}$ such that..."

- [Class question]: Any guesses as to what the equilibrium conditions are?
  1. The utility of the representative agent is maximized.
  2. Workers maximize their revenue.
  3. Consumption is equal to production.
- [Class question]: Which of the three equilibrium conditions will change when we introduce trade?
  - Answer: the last one.

Autarkic Equilibrium



Autarkic Equilibrium



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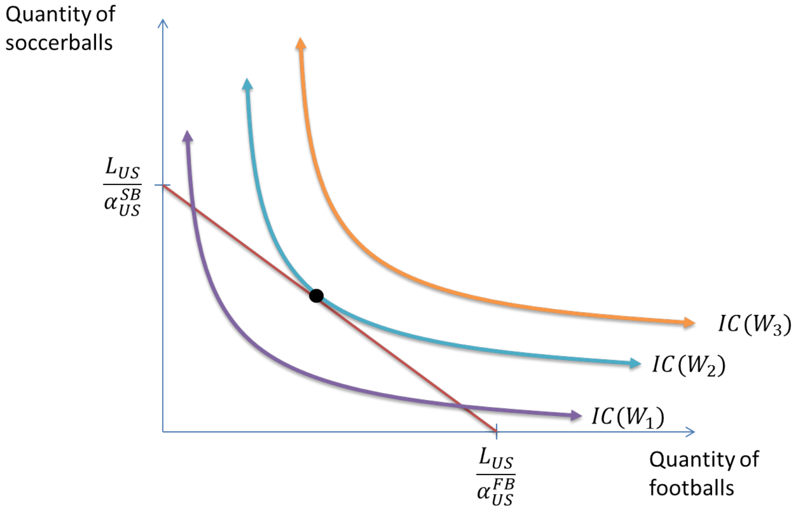
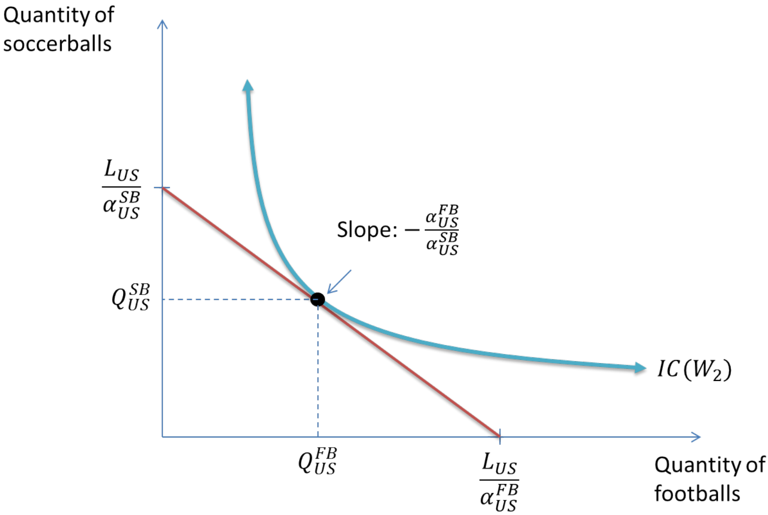


Autarkic Equilibrium



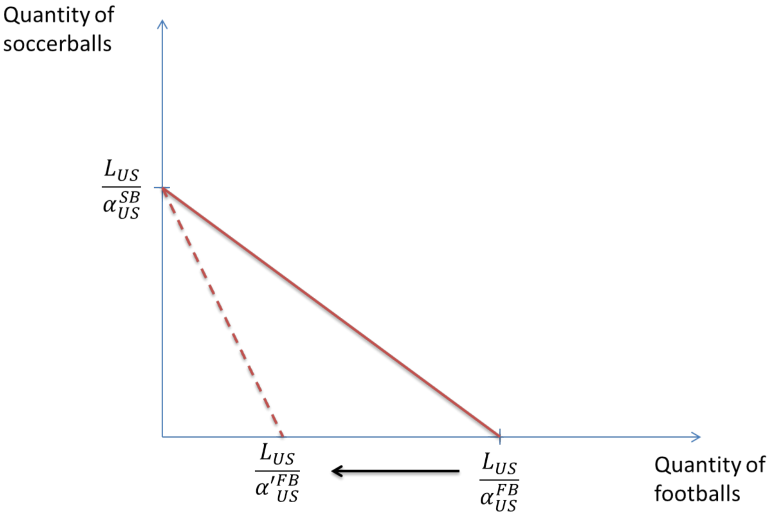
Autarkic Equilibrium Recap

- Equilibrium prices are “pinned-down” by the production technology:
$$\frac{p_{US}^{FB}}{p_{US}^{SB}} = \frac{\alpha_{US}^{FB}}{\alpha_{US}^{SB}}$$
  - [Class question]: Will this always be the case?
  - Answer: No, but if it is not the case, the country will completely specialize in the production of one good.
- Total quantity produced is determined by the point where the indifference curve lies tangent to the production possibility frontier.
  - This depends on preferences.
- Consumption is simply equal to production.

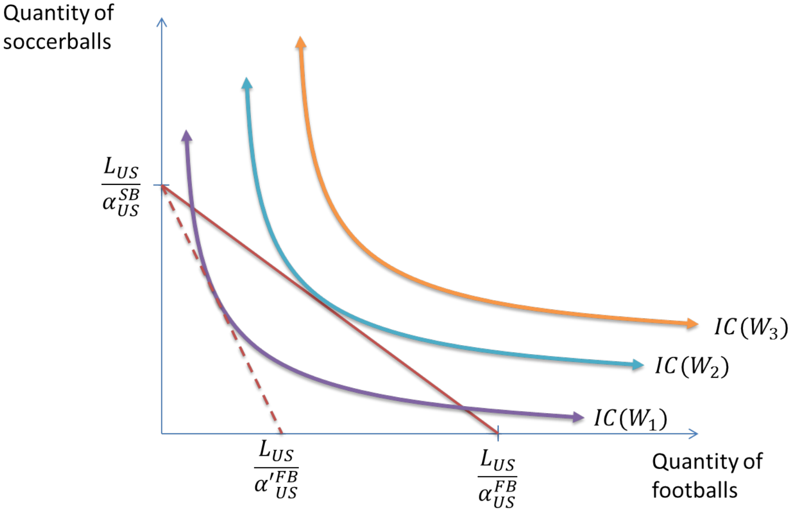
“Comparative static” example

- Economists love to derive “comparative statics”
- A **comparative static** tells us how an equilibrium object changes as we change model fundamentals.
- These make good exam questions.
- For example: if we decrease the productivity of football production, what happens to relative prices and the equilibrium production/consumption of footballs and soccer balls?

"Comparative static" example



“Comparative static” example



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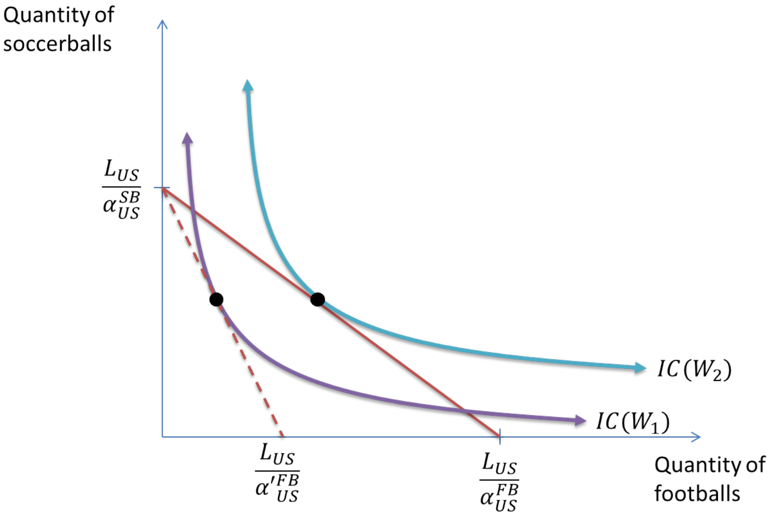

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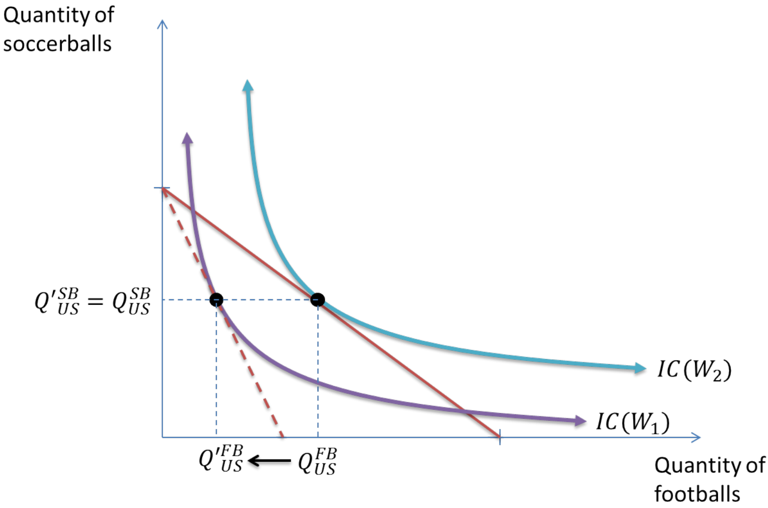

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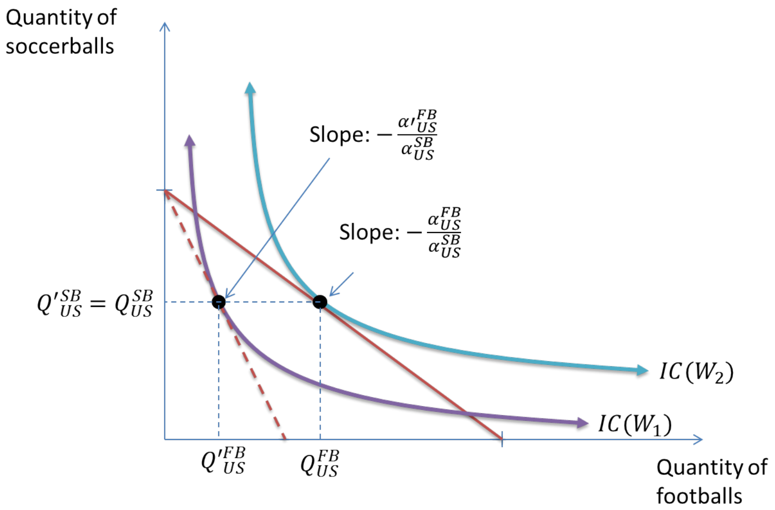


"Comparative static" example

- So reducing the productivity of footballs:
  - Reduces the equilibrium consumption and production of footballs.
  - Increases the relative price of footballs to soccer balls.
  - Has no affect on the equilibrium consumption and production of soccer balls.
    - More generally, the effect could go either way depending on the strength of the income and substitution effects.

Mathematical example

- As above:
  - US has population of workers L_{US}.
  - Workers produce soccer balls and footballs with unit labor costs α_{US}^{SB} and α_{US}^{FB}, respectively.
- Suppose workers have preferences:

$$U\left(C_{US}^{FB}, C_{US}^{SB}\right) = \left(C_{US}^{FB}\right)^{\beta}\left(C_{US}^{SB}\right)^{1-\beta},$$

  - where $\beta \in (0,1)$.
  - These preferences are known as **Cobb-Douglas** preferences.
  - One of my go-to preferences for exams (other go-to: Leontief).
- Question: What is the equilibrium quantity of footballs and soccer balls consumed per worker?

Mathematical example: Production

- Step #1(a): We calculate the PPF. From above, recall that labor can be used either to produce footballs or soccer balls:

$$Q_{US}^{FB}\alpha_{US}^{FB} + Q_{US}^{SB}\alpha_{US}^{SB} = L^{US}$$

  - Can then write the quantity produced of soccer balls as a function of footballs:

$$Q_{US}^{SB} = \frac{L^{US}}{\alpha_{US}^{SB}} - \frac{\alpha_{US}^{FB}}{\alpha_{US}^{SB}}\left(Q_{US}^{FB}\right)$$

Mathematical example: Production (ctd.)

- Step #1(b): We calculate the relative price. In autarky, the relative price of footballs to soccer balls is equal to the (negative) of the slope of the PPF:

$$\frac{p_{US}^{FB}}{p_{US}^{SB}} = -\frac{\partial Q_{US}^{SB}}{\partial Q_{US}^{FB}}$$

  - [Class question: What is the intuition? Is this true with trade?]
  - In this linear case, we then have (as we found above) that:

$$\frac{p_{US}^{FB}}{p_{US}^{SB}} = \frac{\alpha_{US}^{FB}}{\alpha_{US}^{SB}}$$

Mathematical example: Consumption

- Step #2(a): We calculate the wage of a worker.
  - Assume the price of soccer balls is 1 [Why is this okay?].
  - A worker can produce $\frac{1}{\alpha_{US}^{SB}}$ soccer balls. Hence her wage is:
    $$w_{US} = p_{US}^{SB} \times \frac{1}{\alpha_{US}^{SB}} = \frac{1}{\alpha_{US}^{SB}}$$
    .
  - [Class question: what would happen if I had calculated wages using her football production?].

Mathematical example: Consumption (ctd)

- Step #2(b): We calculate the equilibrium consumption of a worker.
  - Maximize utility subject to the worker's budget constraint:

$$\max_{C_{US}^{FB}, C_{US}^{SB}} \left(C_{US}^{FB}\right)^{\beta} \left(C_{US}^{SB}\right)^{1-\beta} \text{ s.t. } p_{US}^{FB} C_{US}^{FB} + C_{US}^{SB} \leq w_{US}$$

  - The Lagrangian is:

$$\mathcal{L} : \left(C_{US}^{FB}\right)^{\beta} \left(C_{US}^{SB}\right)^{1-\beta} - \lambda \left(p_{US}^{FB} C_{US}^{FB} + C_{US}^{SB} - w_{US}\right)$$

  - Yielding first order conditions:

$$\beta \left(\frac{C_{US}^{SB}}{C_{US}^{FB}}\right)^{1-\beta} = \lambda p_{US}^{FB} \text{ and } (1-\beta) \left(\frac{C_{US}^{FB}}{C_{US}^{SB}}\right)^{\beta} = \lambda$$

Mathematical example: Consumption (ctd)

- Recall first order conditions:

$$\beta\left(\frac{C_{US}^{SB}}{C_{US}^{FB}}\right)^{1-\beta}=\lambda p_{US}^{FB} \text{ and } (1-\beta)\left(\frac{C_{US}^{FB}}{C_{US}^{SB}}\right)^{\beta}=\lambda$$

- Combining both equations to get rid of λ yields:

$$\beta C_{US}^{SB}=(1-\beta)\,p_{US}^{FB}C_{US}^{FB}$$

- Using the budget constraint, this implies:

$$C_{US}^{SB}=(1-\beta)\,w_{US} \text{ and } p_{US}^{FB}C_{US}^{FB}=\beta w_{US}$$

- Implication: With Cobb-Douglas preferences, always spend a constant fraction of income on each good, where fraction pinned down by exponent!

Mathematical example: Equilibrium

- Step #3: Combine production and consumption equilibrium relationships:
  - Prices: $p_{US}^{FB} = \frac{\alpha_{US}^{FB}}{\alpha_{US}^{SB}}$ and $p_{US}^{SB} = 1$
  - Wages: $w_{US} = \frac{1}{\alpha_{US}^{SB}}$
  - Consumption: $C_{US}^{SB} = (1-\beta)\, w_{US}$ and $C_{US}^{FB} = \frac{\beta w_{US}}{p_{US}^{FB}}$
- Answer to the question:

$$C_{US}^{SB} = \frac{1-\beta}{\alpha_{US}^{SB}} \text{ and } C_{US}^{FB} = \frac{\beta}{\alpha_{US}^{FB}}$$

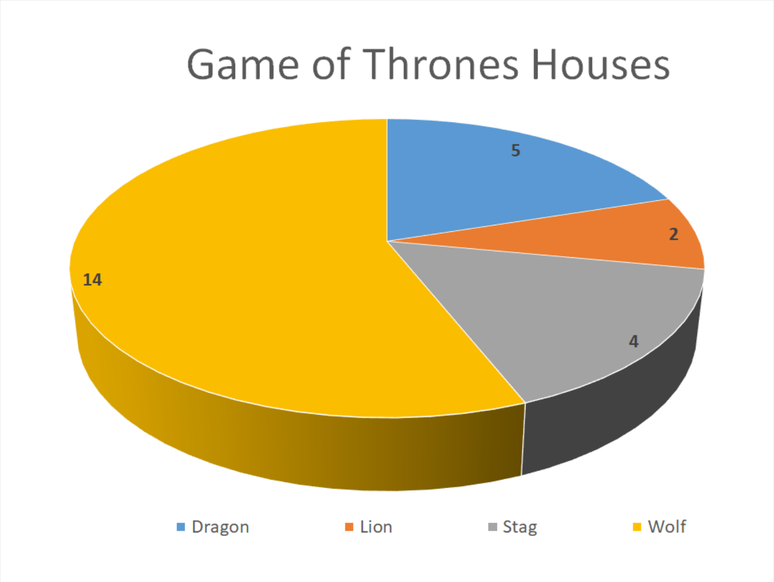
- [Class questions: whats the intuition for β and the unit costs? Why doesn't the labor supply affect the production decision?]

Plan for the day

- Introduce trade into the Ricardian Model...
- But first go through practice problem in autarky.
- Show where the gains from trade come from.
- Find the equilibrium.
- Note: this is a lot of important material:
  - We may take more than one day.
  - Ask lots of questions - I want to make sure everything is clear!

Today's Teams

Today's Teams



Mathematical example

- As above:
  - US has population of workers L_{US}.
  - Workers produce soccer balls and footballs with unit labor costs α_{US}^{SB} and α_{US}^{FB}, respectively.
- Suppose workers have preferences:

$$U\left(C_{US}^{FB}, C_{US}^{SB}\right) = \left(C_{US}^{FB}\right)^{\beta}\left(C_{US}^{SB}\right)^{1-\beta},$$

  - where $\beta \in (0,1)$.
  - These preferences are known as **Cobb-Douglas** preferences.
  - One of my go-to preferences for exams (other go-to: Leontief).
- Question: What is the equilibrium quantity of footballs and soccer balls consumed per worker?

Mathematical example: Production

- Step #1(a): We calculate the PPF. From above, recall that labor can be used either to produce footballs or soccer balls:

$$Q_{US}^{FB}\alpha_{US}^{FB} + Q_{US}^{SB}\alpha_{US}^{SB} = L^{US}$$

  - Can then write the quantity produced of soccer balls as a function of footballs:

$$Q_{US}^{SB} = \frac{L^{US}}{\alpha_{US}^{SB}} - \frac{\alpha_{US}^{FB}}{\alpha_{US}^{SB}}\left(Q_{US}^{FB}\right)$$

Mathematical example: Production (ctd.)

- Step #1(b): We calculate the relative price. In autarky, the relative price of footballs to soccer balls is equal to the (negative) of the slope of the PPF:

$$\frac{p_{US}^{FB}}{p_{US}^{SB}} = -\frac{\partial Q_{US}^{SB}}{\partial Q_{US}^{FB}}$$

  - [Class question: What is the intuition? Is this true with trade?]
  - In this linear case, we then have (as we found above) that:

$$\frac{p_{US}^{FB}}{p_{US}^{SB}} = \frac{\alpha_{US}^{FB}}{\alpha_{US}^{SB}}$$

Mathematical example: Consumption

- Step #2(a): We calculate the wage of a worker.
  - Assume the price of soccer balls is 1 [Why is this okay?].
  - A worker can produce $\frac{1}{\alpha_{US}^{SB}}$ soccer balls. Hence her wage is:
    $$w_{US} = p_{US}^{SB} \times \frac{1}{\alpha_{US}^{SB}} = \frac{1}{\alpha_{US}^{SB}}$$
    .
  - [Class question: what would happen if I had calculated wages using her football production?].

Mathematical example: Consumption (ctd)

- Step #2(b): We calculate the equilibrium consumption of a worker.
  - Maximize utility subject to the worker's budget constraint:

$$\max_{C_{US}^{FB}, C_{US}^{SB}} \left(C_{US}^{FB}\right)^{\beta} \left(C_{US}^{SB}\right)^{1-\beta} \text{ s.t. } p_{US}^{FB} C_{US}^{FB} + C_{US}^{SB} \leq w_{US}$$

  - The Lagrangian is:

$$\mathcal{L} : \left(C_{US}^{FB}\right)^{\beta} \left(C_{US}^{SB}\right)^{1-\beta} - \lambda \left(p_{US}^{FB} C_{US}^{FB} + C_{US}^{SB} - w_{US}\right)$$

  - Yielding first order conditions:

$$\beta \left(\frac{C_{US}^{SB}}{C_{US}^{FB}}\right)^{1-\beta} = \lambda p_{US}^{FB} \text{ and } (1-\beta) \left(\frac{C_{US}^{FB}}{C_{US}^{SB}}\right)^{\beta} = \lambda$$

Mathematical example: Consumption (ctd)

- Recall first order conditions:

$$\beta\left(\frac{C_{US}^{SB}}{C_{US}^{FB}}\right)^{1-\beta}=\lambda p_{US}^{FB} \text{ and } (1-\beta)\left(\frac{C_{US}^{FB}}{C_{US}^{SB}}\right)^{\beta}=\lambda$$

- Combining both equations to get rid of λ yields:

$$\beta C_{US}^{SB}=(1-\beta)\, p_{US}^{FB} C_{US}^{FB}$$

- Using the budget constraint, this implies:

$$C_{US}^{SB}=(1-\beta)\, w_{US} \text{ and } p_{US}^{FB} C_{US}^{FB}=\beta w_{US}$$

- Implication: With Cobb-Douglas preferences, always spend a constant fraction of income on each good, where fraction pinned down by exponent!

Mathematical example: Equilibrium

- Step #3: Combine production and consumption equilibrium relationships:
  - Prices: $p_{US}^{FB} = \frac{\alpha_{US}^{FB}}{\alpha_{US}^{SB}}$ and $p_{US}^{SB} = 1$
  - Wages: $w_{US} = \frac{1}{\alpha_{US}^{SB}}$
  - Consumption: $C_{US}^{SB} = (1-\beta)\, w_{US}$ and $C_{US}^{FB} = \frac{\beta w_{US}}{p_{US}^{FB}}$
- Answer to the question:

$$C_{US}^{SB} = \frac{1-\beta}{\alpha_{US}^{SB}} \text{ and } C_{US}^{FB} = \frac{\beta}{\alpha_{US}^{FB}}$$

- [Class questions: whats the intuition for β and the unit costs? Why doesn't the labor supply affect the production decision?]

The Famous Challenge

- Stan Ulam (1909-1984)
  - Mathematician
  - Worked on the Manhattan project
  - Namesake of the "Teller-Ulam" thermonuclear weapons
- Paul Samuelson (1915-2009)
  - Economist
  - First American to win the Nobel Prize in Economics
  - "Foremost academic economist of the 20th century" (NY Times)
- Ulam's challenge to Samuelson:

  "Name one proposition in the social sciences that is both true and non-trivial."
- Samuelson's (immediate) response: comparative advantage!

The Game

- Which team can produce the most "ice cream cones"?
- Two players from each team, judges from the other two teams.
- Practice:
  - How many "cones" can each team make in 30 seconds?
  - How many "scoops" can each team make in 30 seconds?
- Contest #1:
  - How many completed "ice cream cones" can each team make in 1 minutes?
  - One point per cone.
- Contest #2:
  - Now, let the teams trade cones and scoops.
- How many completed "ice cream cones" can each team make with trade in 1 minute?

The Game (ctd).

- [Class questions]:
  - Which team was better at producing "cones"? What about "scoops"?
  - How did what the teams produce change when they traded? Why?
  - Which team (or teams) did better with trade compared to autarky?

Model Setup

- Two countries: U.S. and Mexico.
- Two goods: footballs and soccer balls.
- One factor of production: labor.

Exogenous Model Parameters

- Population:
  - Population in U.S.: L_{US}
  - Population in Mexico: L_{MEX}.
- Productivity:
  - One worker can produce $\frac{1}{\alpha_{US}^{FB}}$ footballs or $\frac{1}{\alpha_{US}^{SB}}$ soccer balls in U.S.
  - One worker can produce $\frac{1}{\alpha_{MEX}^{FB}}$ footballs or $\frac{1}{\alpha_{MEX}^{SB}}$ soccer balls in Mexico.
- Preferences:
  - "Representative agent" in each country that receives utility:
$$W_i = U_i\left(C_i^{SB}, C_i^{FB}\right),$$
    for $i \in \{US, MEX\}$.
  - Assume that $\frac{\partial U_i\left(C_i^{SB}, C_i^{FB}\right)}{\partial C_i^{SB}} > 0$ and $\frac{\partial U_i\left(C_i^{SB}, C_i^{FB}\right)}{\partial C_i^{FB}} > 0$ for $i \in \{US, MEX\}$.

Endogenous model outcomes

- Quantity of soccer balls and footballs produced in each country (Q^{SB}_{US}, Q^{SB}_{MEX}, Q^{FB}_{US}, Q^{FB}_{MEX}).

- Quantity of soccer balls and footballs consumed in each country (C^{SB}_{US}, C^{SB}_{MEX}, C^{FB}_{US}, C^{FB}_{MEX}).

- World relative price of footballs to soccer balls: p^{FB}/p^{SB}.

Equilibrium Conditions

1. **Optimal production:** Given relative prices, workers choose what to produce in order to maximize their income.

2. **Optimal consumption:** Given relative prices and the income earned from production, the representative agents choose what to consume in order to maximize their utility

3. **Market clearing:** Total world consumption of each good equals total world production.

- [Class question: which of these conditions has changed from autarky?]

Comparative versus Absolute Advantage

- We say that Mexico has an **absolute advantage** in the production of soccer balls if:

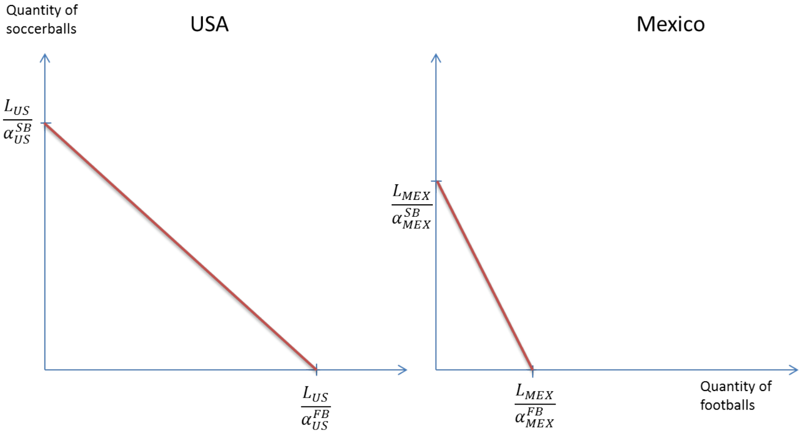
$$\alpha_{MEX}^{SB} < \alpha_{USA}^{SB}$$

  - [Class question: what does this equation mean?]
  - [Class question: will a country always have an absolute advantage in at least one good?]
- We say that Mexico has a **comparative advantage** in the production of soccer balls if:

$$\frac{\alpha_{MEX}^{SB}}{\alpha_{MEX}^{FB}} < \frac{\alpha_{US}^{SB}}{\alpha_{US}^{FB}}$$

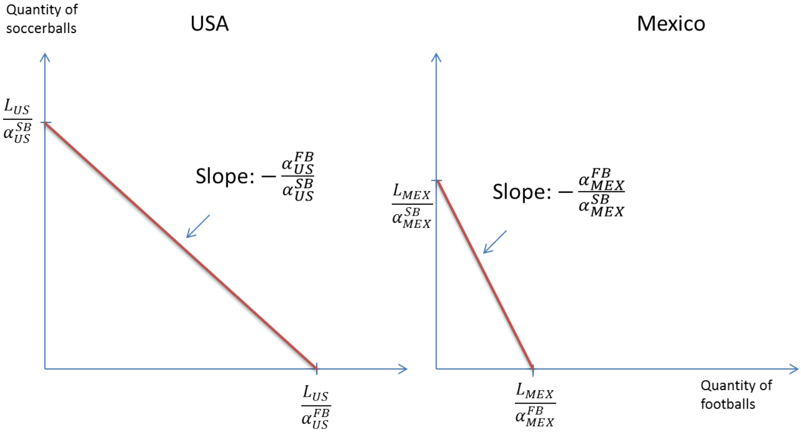
  - [Class question: what does this equation mean?]
  - [Class (trick) question: will a country always have a comparative advantage in at least one good?]

Production Possibilities Frontiers



- [Class question: Which country has the absolute advantage in footballs? soccer balls?]
- [Class question: Which country has the comparative advantage in footballs? soccer balls?]

Production Possibilities Frontiers



- Note:

$$\frac{\alpha_{MEX}^{FB}}{\alpha_{MEX}^{SB}} > \frac{\alpha_{US}^{FB}}{\alpha_{US}^{SB}} \iff \frac{\alpha_{MEX}^{SB}}{\alpha_{MEX}^{FB}} < \frac{\alpha_{US}^{SB}}{\alpha_{US}^{FB}}$$

Relative prices and specialization

- With trade, the relative price of footballs to soccer balls is the same in both countries.
  - [Class question: why?]
  - [Class question: can you think of reasons that this may not be true?]
- How does the world relative price affect the production choices of workers?

Relative prices and specialization (ctd.)

- Consider the decision of workers in the United States of what to produce:
  - Earn $\frac{p^{FB}}{\alpha^{FB}_{US}}$ for producing footballs and $\frac{p^{SB}}{\alpha^{SB}_{US}}$ for producing soccer balls.
  - Will produce footballs if $\frac{p^{FB}}{\alpha^{FB}_{US}} > \frac{p^{SB}}{\alpha^{SB}_{US}} \iff \frac{p^{FB}}{p^{SB}} > \frac{\alpha^{FB}_{US}}{\alpha^{SB}_{US}}$.
  - Will produce soccer balls if $\frac{p^{FB}}{\alpha^{FB}_{US}} < \frac{p^{SB}}{\alpha^{SB}_{US}} \iff \frac{p^{FB}}{p^{SB}} < \frac{\alpha^{FB}_{US}}{\alpha^{SB}_{US}}$.
  - Will be indifferent if $\frac{p^{FB}}{\alpha^{FB}_{US}} = \frac{p^{SB}}{\alpha^{SB}_{US}} \iff \frac{p^{FB}}{p^{SB}} = \frac{\alpha^{FB}_{US}}{\alpha^{SB}_{US}}$.
- Similarly, workers in Mexico will produce footballs if $\frac{p^{FB}}{p^{SB}} > \frac{\alpha^{FB}_{MEX}}{\alpha^{SB}_{MEX}}$, soccer balls if $\frac{p^{FB}}{p^{SB}} < \frac{\alpha^{FB}_{MEX}}{\alpha^{SB}_{MEX}}$ and will be indifferent if $\frac{p^{FB}}{p^{SB}} = \frac{\alpha^{FB}_{MEX}}{\alpha^{SB}_{MEX}}$.

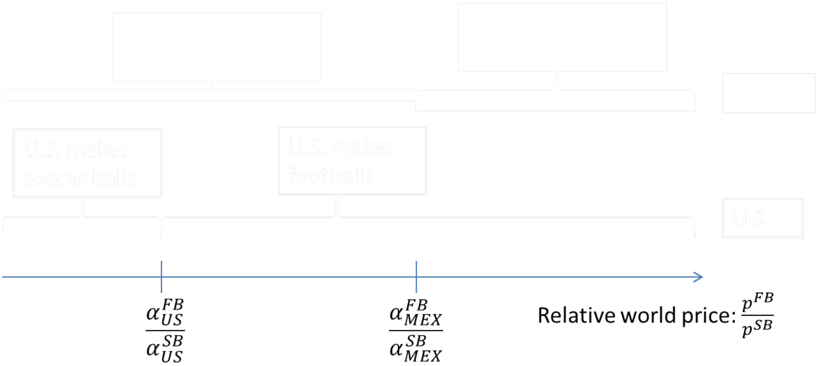
Relative prices and specialization (ctd.)

- Recall that we assumed:

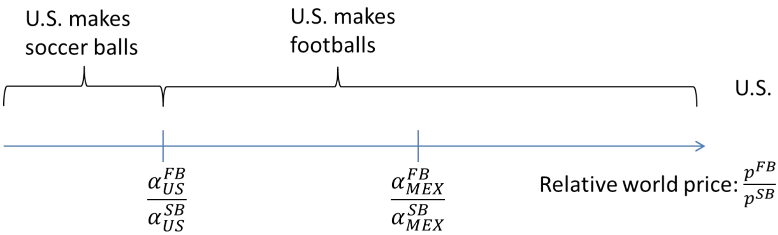
$$\frac{\alpha^{FB}_{MEX}}{\alpha^{SB}_{MEX}} > \frac{\alpha^{FB}_{US}}{\alpha^{SB}_{US}}$$

- Hence we can figure out what each country produces for any world relative price.

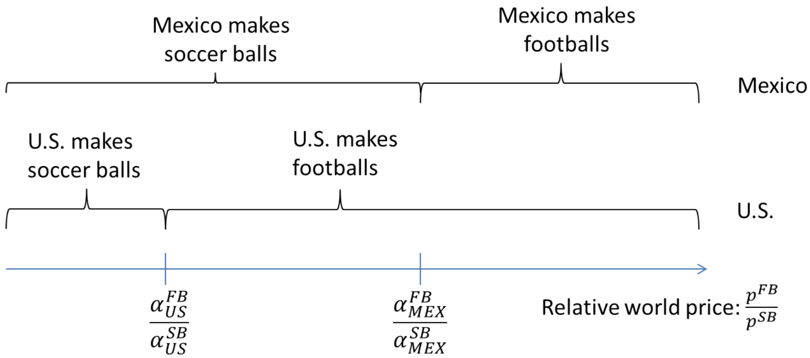
Relative prices and specialization (ctd.)



Relative prices and specialization (ctd.)



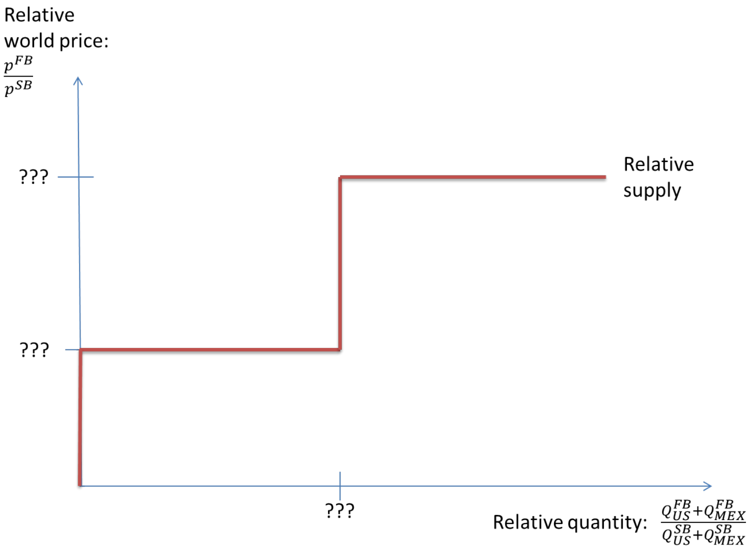
Relative prices and specialization (ctd.)



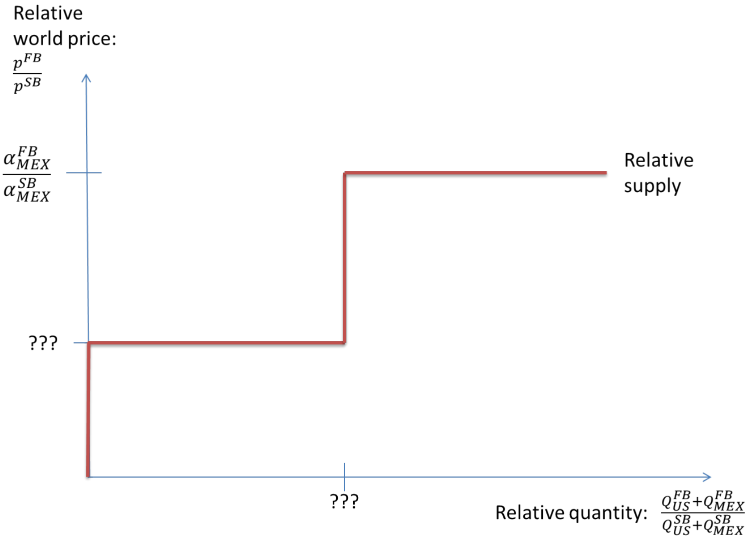
Relative Supply Curve

- With this specialization in mind, we can construct a "relative" supply curve.
- $\frac{Q_{US}^{FB}+Q_{MEX}^{FB}}{Q_{US}^{SB}+Q_{MEX}^{SB}}$ is the quantity of footballs produced worldwide relative to the quantity of soccer balls produced worldwide.
- How does $\frac{Q_{US}^{FB}+Q_{MEX}^{FB}}{Q_{US}^{SB}+Q_{MEX}^{SB}}$ depend on $\frac{p^{FB}}{p^{SB}}$?
- Aside: this is why trade economists hate traditional supply demand curves - they're flipped the wrong way!

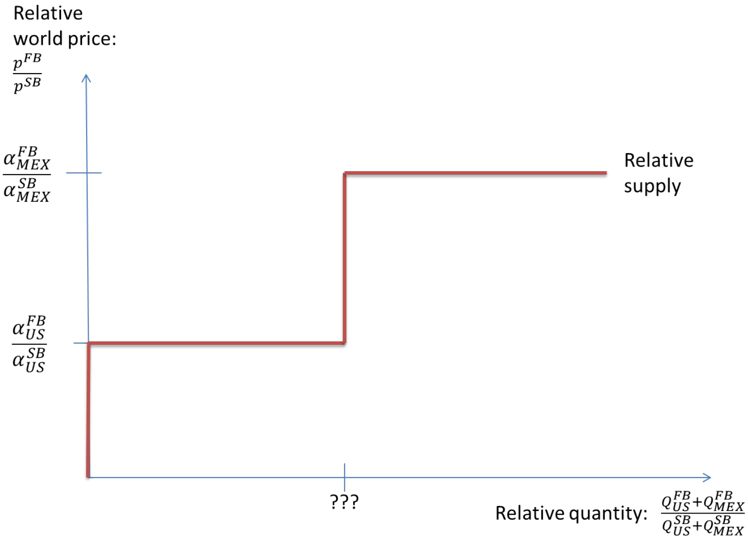
Relative Supply Curve



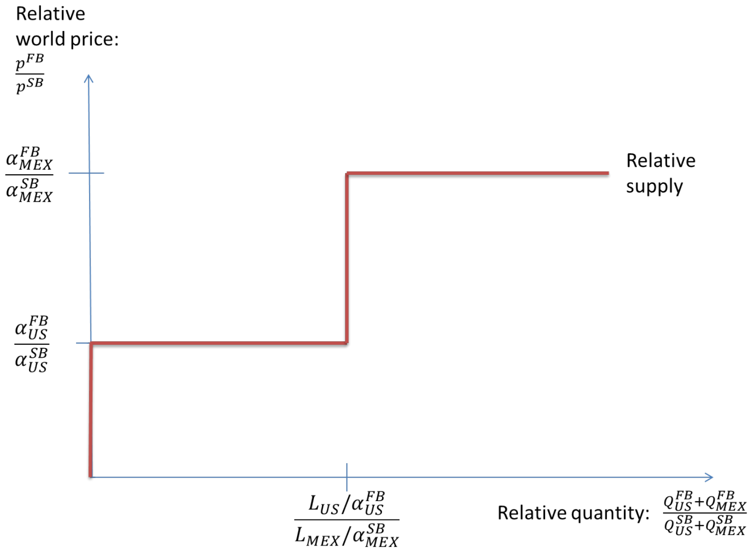
Relative Supply Curve



Relative Supply Curve



Relative Supply Curve



Consumption possibility frontier

- The **set of consumption possibilities** is the set of consumption bundles that are feasible for workers in a country to consume.
- The **consumption possibility frontier (CPF)** is the most soccer balls that can be consumed in a country for a given consumption of footballs.
- [Class question: why have we only considered the production possibility frontier so far?].
- Key insight: with trade, the CPF need not be the same as the PPF.
- Why? Because a country can import more of a good than it produces (in exchange for exporting more of another good than it consumes).
- The CPF depends on the world relative price $\frac{p^{FB}}{p^{SB}}$.

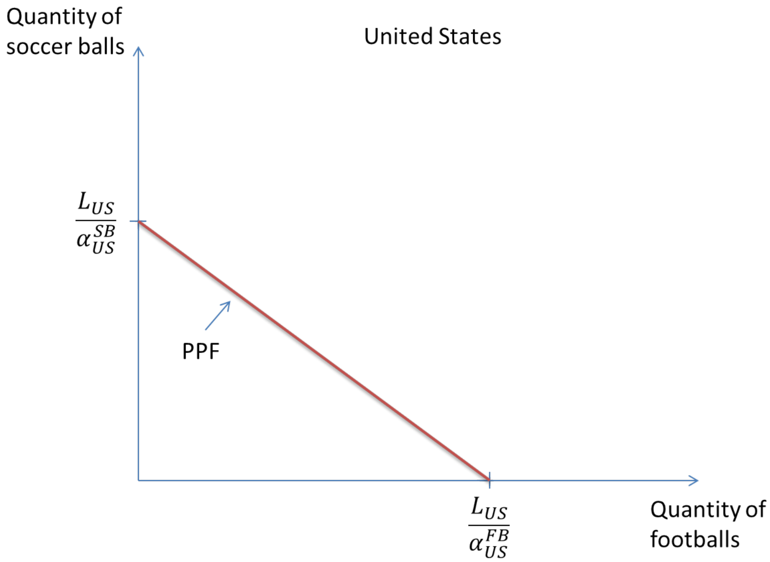
Consumption possibility frontier

- Consider the United States.
- Suppose that $\frac{p^{FB}}{p^{SB}} > \frac{\alpha_{US}^{FB}}{\alpha_{US}^{SB}}$, so that U.S. specializes in footballs.
- This earns the U.S. an income of $\frac{L_{US}}{\alpha_{US}^{FB}} \times p^{FB}$.
- Then the U.S. could purchase up to $\frac{L_{US}}{\alpha_{US}^{FB}} \times \frac{p^{FB}}{p^{SB}}$ soccer balls (income divided by price of soccer balls).
- Note that:

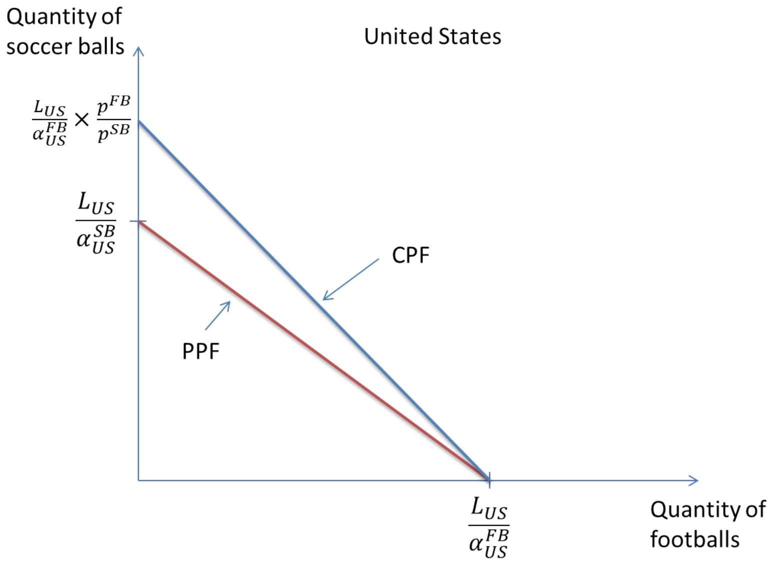
$$\frac{L_{US}}{\alpha_{US}^{FB}} \times \frac{p^{FB}}{p^{SB}} > \frac{L_{US}}{\alpha_{US}^{FB}} \times \frac{\alpha_{US}^{FB}}{\alpha_{US}^{SB}} = \frac{L_{US}}{\alpha_{US}^{SB}}$$

- Hence, the U.S. can now consume more soccer balls than it can produce!

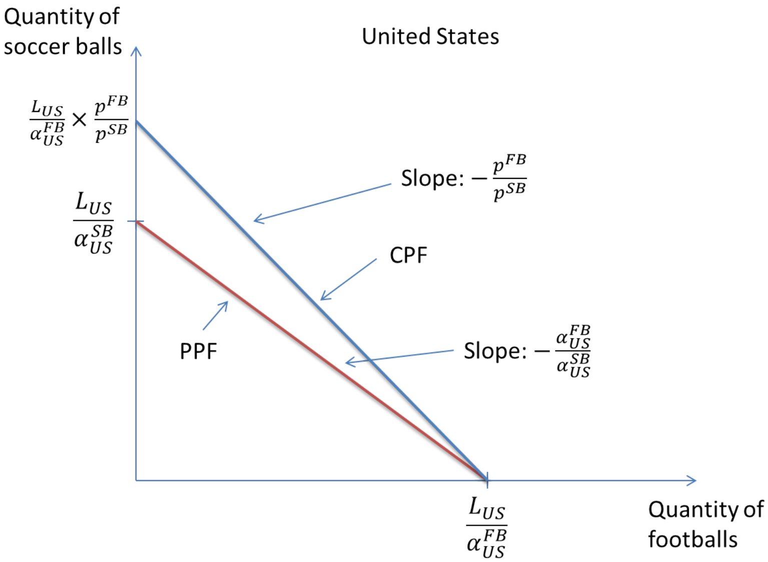
U.S. consumption possibility frontier



U.S. consumption possibility frontier



U.S. consumption possibility frontier



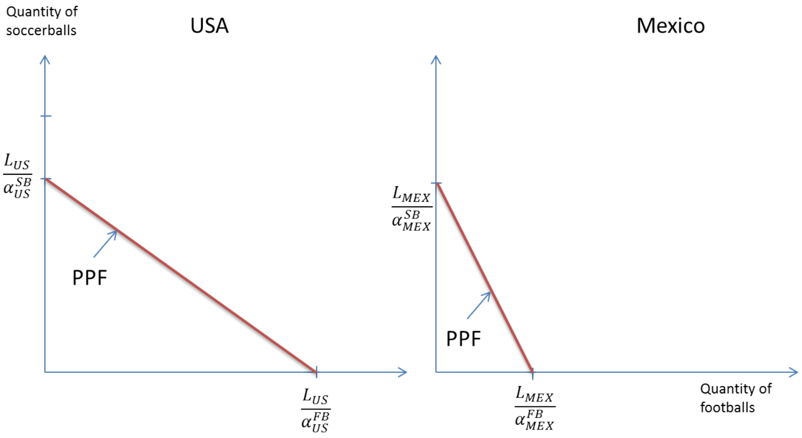
Mexico consumption possibility frontier

- Now consider Mexico.
- Suppose that $\frac{p^{FB}}{p^{SB}} < \frac{\alpha^{FB}_{MEX}}{\alpha^{SB}_{MEX}}$, so that Mexico specializes in soccer balls.
- This earns the Mexico an income of $\frac{L_{MEX}}{\alpha^{SB}_{MEX}} \times p^{SB}$.
- Then Mexico could purchase up to $\frac{L_{MEX}}{\alpha^{SB}_{MEX}} \times \frac{p^{SB}}{p^{FB}}$ footballs.
- Note that:

$$\frac{L_{MEX}}{\alpha^{SB}_{MEX}} \times \frac{p^{SB}}{p^{FB}} > \frac{L_{MEX}}{\alpha^{SB}_{MEX}} \times \frac{\alpha^{SB}_{MEX}}{\alpha^{FB}_{MEX}} = \frac{L_{MEX}}{\alpha^{FB}_{MEX}}$$

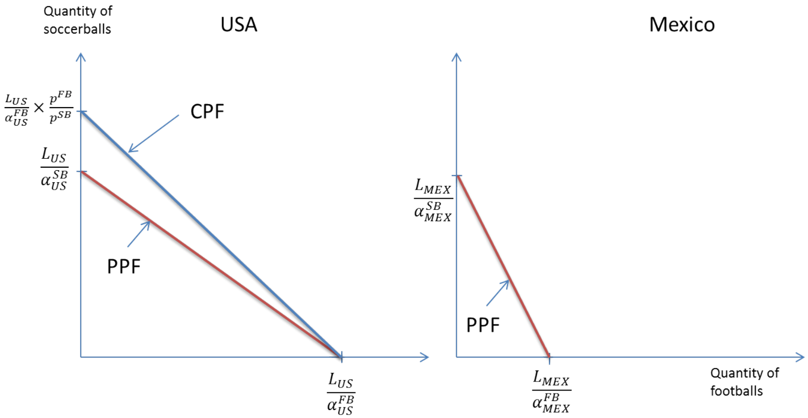
- Hence, Mexico can now consume more footballs than it can produce!
- If $\frac{\alpha^{FB}_{US}}{\alpha^{SB}_{US}} < \frac{p^{FB}}{p^{SB}} < \frac{\alpha^{FB}_{MEX}}{\alpha^{SB}_{MEX}}$, both the U.S. and Mexico have CPF that are outside their PPF!

World CPF



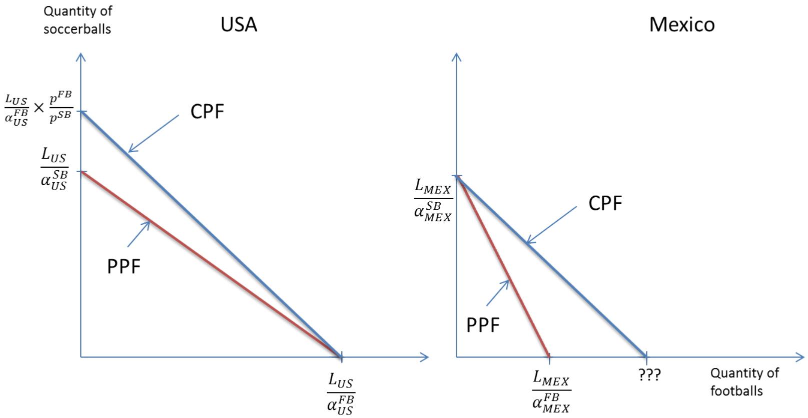
Quantity of soccerballs
USA
Mexico
$\frac{L_{US}}{\alpha_{US}^{SB}}$
PPF
$\frac{L_{US}}{\alpha_{US}^{FB}}$
$\frac{L_{MEX}}{\alpha_{MEX}^{SB}}$
PPF
$\frac{L_{MEX}}{\alpha_{MEX}^{FB}}$
Quantity of footballs

World CPF



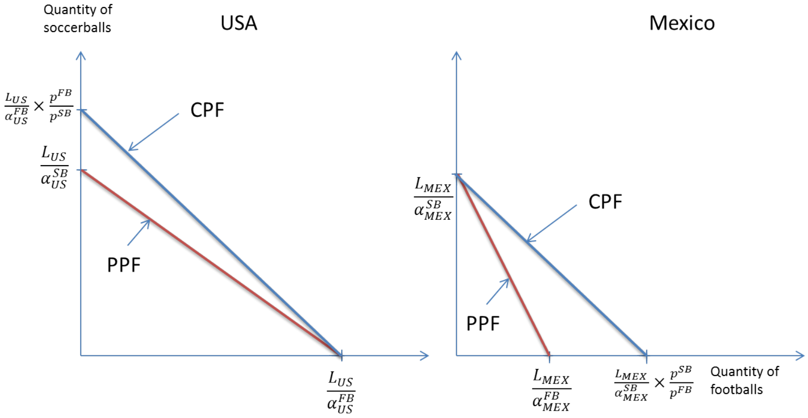
Quantity of soccerballs
USA
Mexico
$\frac{L_{US}}{\alpha_{US}^{FB}} \times \frac{p^{FB}}{p^{SB}}$
CPF
$\frac{L_{US}}{\alpha_{US}^{SB}}$
PPF
$\frac{L_{MEX}}{\alpha_{MEX}^{SB}}$
PPF
$\frac{L_{US}}{\alpha_{US}^{FB}}$
$\frac{L_{MEX}}{\alpha_{MEX}^{FB}}$
Quantity of footballs

World CPF



Quantity of soccerballs
USA
Mexico
$\frac{L_{US}}{\alpha_{US}^{FB}} \times \frac{p^{FB}}{p^{SB}}$
CPF
$\frac{L_{US}}{\alpha_{US}^{SB}}$
PPF
$\frac{L_{US}}{\alpha_{US}^{FB}}$
$\frac{L_{MEX}}{\alpha_{MEX}^{SB}}$
CPF
PPF
$\frac{L_{MEX}}{\alpha_{MEX}^{FB}}$
???
Quantity of footballs

World CPF

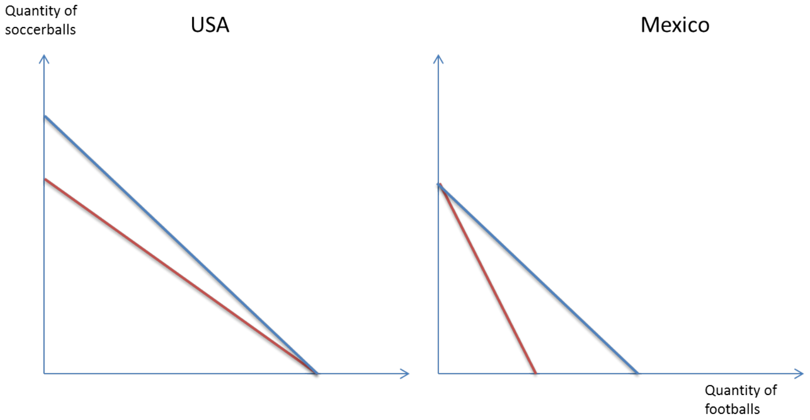


- [Class question: what happens if $\frac{p^{FB}}{p^{SB}} = \frac{\alpha_{US}^{FB}}{\alpha_{US}^{SB}}$? If $\frac{p^{FB}}{p^{SB}} = \frac{\alpha_{MEX}^{FB}}{\alpha_{MEX}^{SB}}$?]

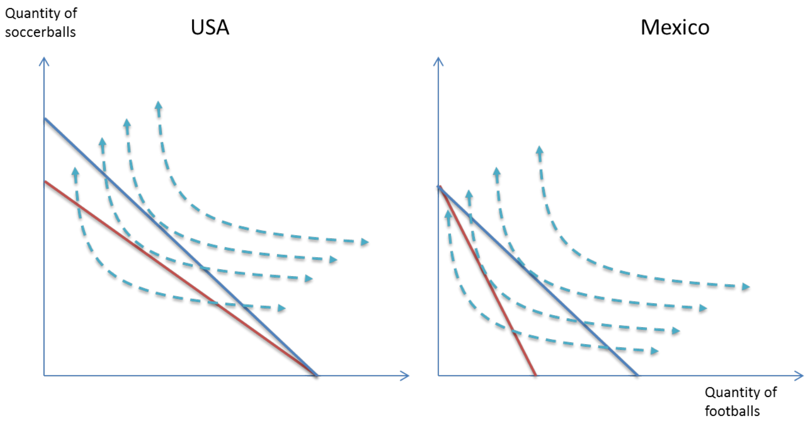
Gains from Trade

- Given preferences of the "representative agents", we can determine the utility of any consumption bundle.
- Recall $\frac{\partial U_i\left(C_i^{SB}, C_i^{FB}\right)}{\partial C_i^{SB}} > 0$ and $\frac{\partial U_i\left(C_i^{SB}, C_i^{FB}\right)}{\partial C_i^{FB}} > 0$ for $i \in \{US, MEX\}$.
- Then can use the indifference curves to determine the welfare gains from trade.

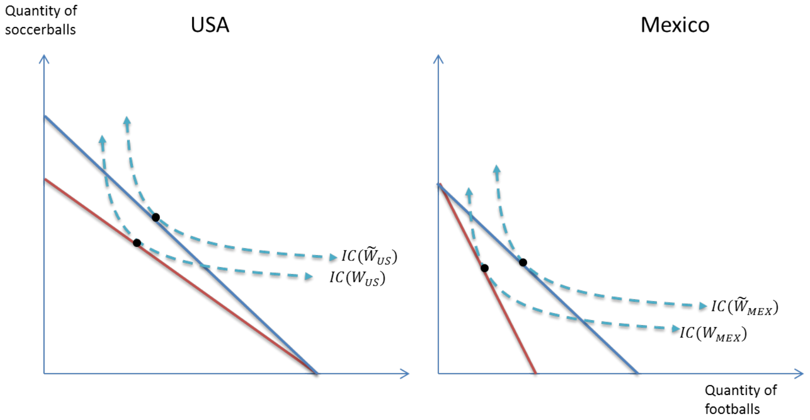
Gains from Trade



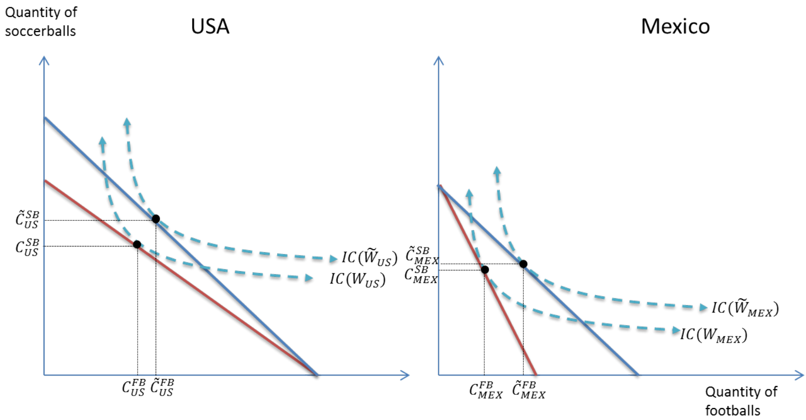
Gains from Trade



Gains from Trade



Gains from Trade

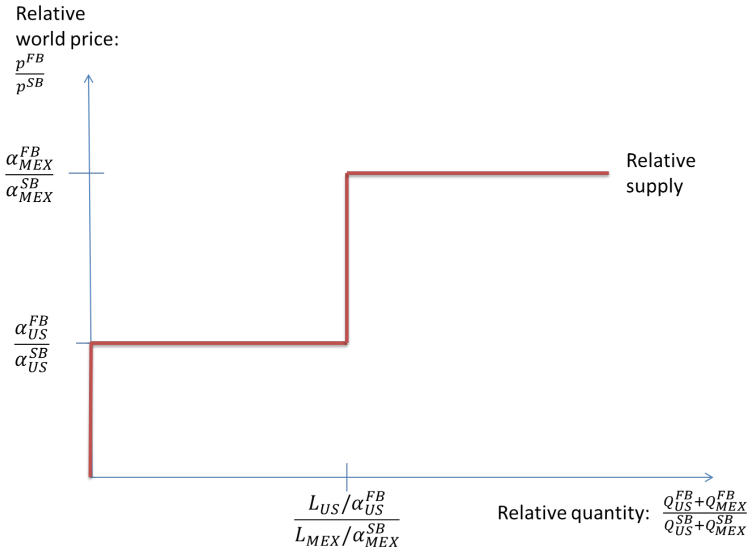


- [Class question: who gains from trade if $\frac{p^{FB}}{p^{SB}} = \frac{\alpha^{FB}_{US}}{\alpha^{SB}_{US}}$?]
- [Class question: is there a way for a country to lose from trade? To become no better off?]

Closing the model

- For a given relative price, we have now seen:
  - What workers in each country will produce.
  - What the representative agent in each country will consume.
- All that is left to do is determine the equilibrium relative price. To do so, we combine:
  - The relative supply curve (which we have already seen).
  - The relative demand curve (which we will now construct).

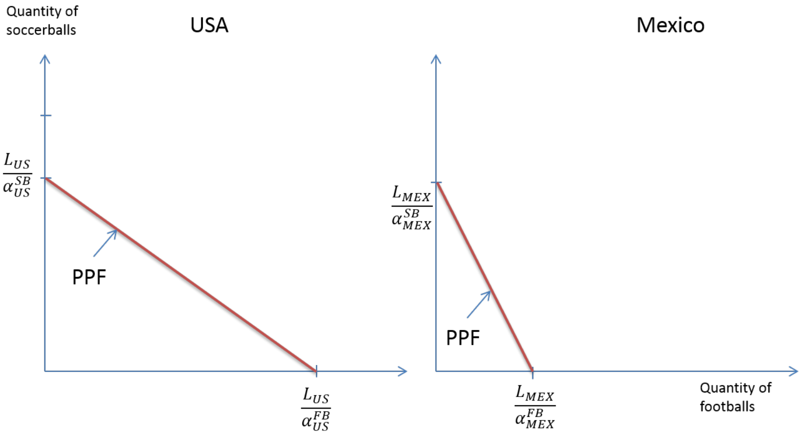
Recall: The relative supply curve



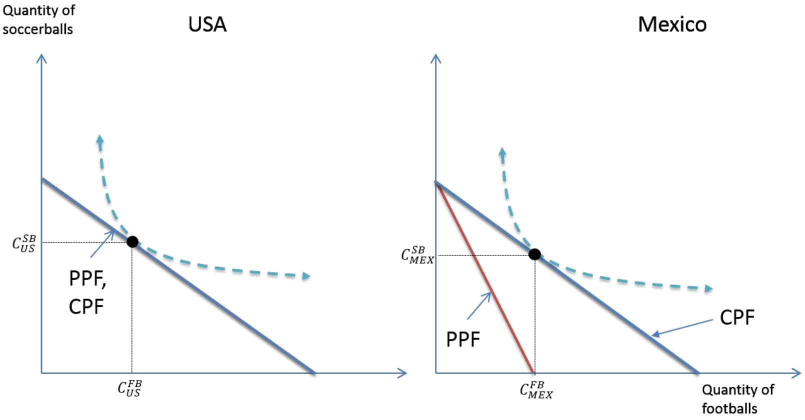
Finding the equilibrium world price

- [Class question: What would happen if the world price $\frac{p^{FB}}{p^{SB}} > \frac{\alpha^{FB}_{MEX}}{\alpha^{SB}_{MEX}}$?]
- [Class question: What would happen if the world price $\frac{p^{FB}}{p^{SB}} < \frac{\alpha^{FB}_{US}}{\alpha^{SB}_{US}}$?]
- Unless the preferences are such that the representative agents would be willing to only consume one good, this means we can focus on potential equilibrium prices $\frac{p^{FB}}{p^{SB}} \in \left[\frac{\alpha^{FB}_{US}}{\alpha^{SB}_{US}}, \frac{\alpha^{FB}_{MEX}}{\alpha^{SB}_{MEX}}\right]$.

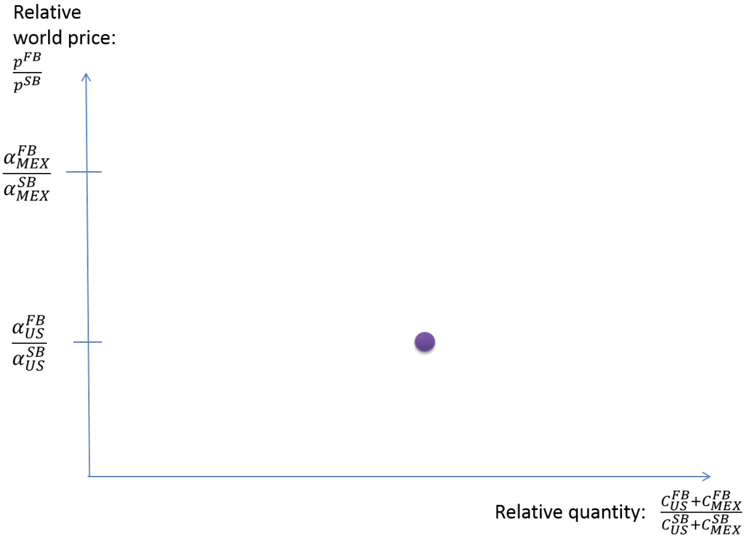
PPF if world price is $\frac{\alpha^{FB}_{US}}{\alpha^{SB}_{US}}$



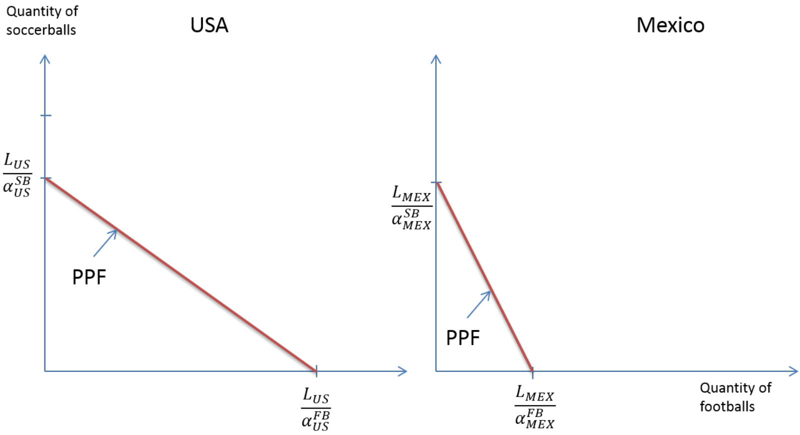
CPF if world price is $\frac{\alpha_{US}^{FB}}{\alpha_{US}^{SB}}$



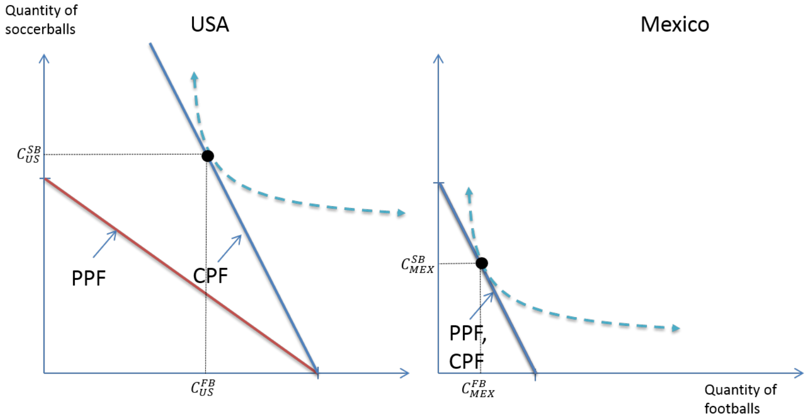
Relative demand if world price is $\frac{\alpha_{US}^{FB}}{\alpha_{US}^{SB}}$



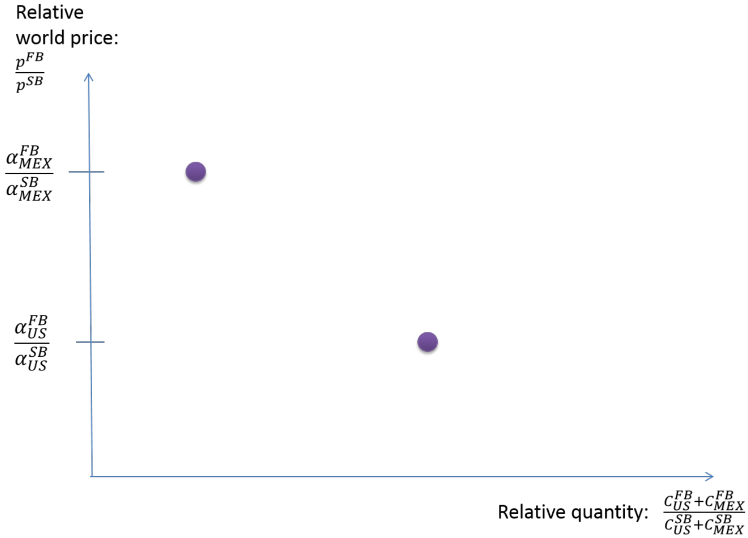
PPF if world price is $\frac{\alpha^{FB}_{MEX}}{\alpha^{SB}_{MEX}}$



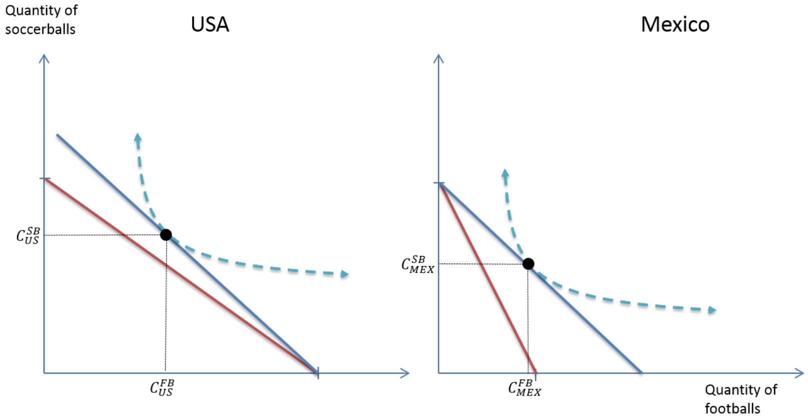
CPF if world price is $\frac{\alpha^{FB}_{MEX}}{\alpha^{SB}_{MEX}}$



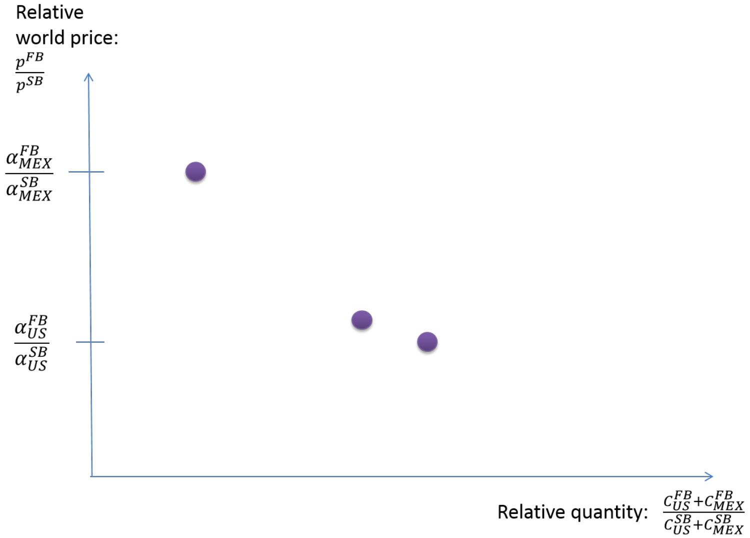
Relative demand if world price is $\frac{\alpha^{FB}_{MEX}}{\alpha^{SB}_{MEX}}$



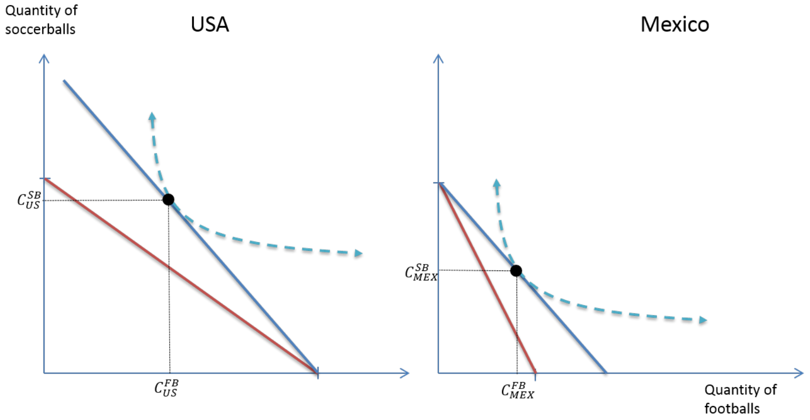
CPF if $\frac{\alpha^{FB}_{US}}{\alpha^{SB}_{US}} < \frac{p^{FB}}{p^{SB}} < \frac{\alpha^{FB}_{MEX}}{\alpha^{SB}_{MEX}}$



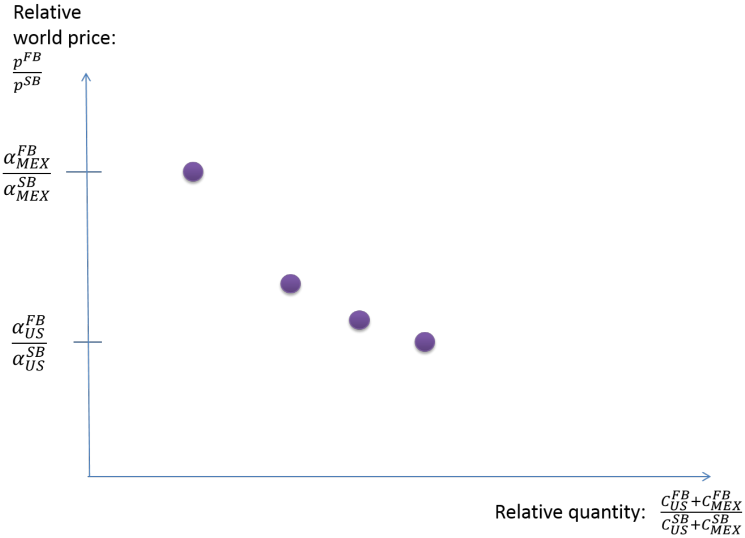
Relative demand if $\frac{\alpha_{US}^{FB}}{\alpha_{US}^{SB}} < \frac{p^{FB}}{p^{SB}} < \frac{\alpha_{MEX}^{FB}}{\alpha_{MEX}^{SB}}$



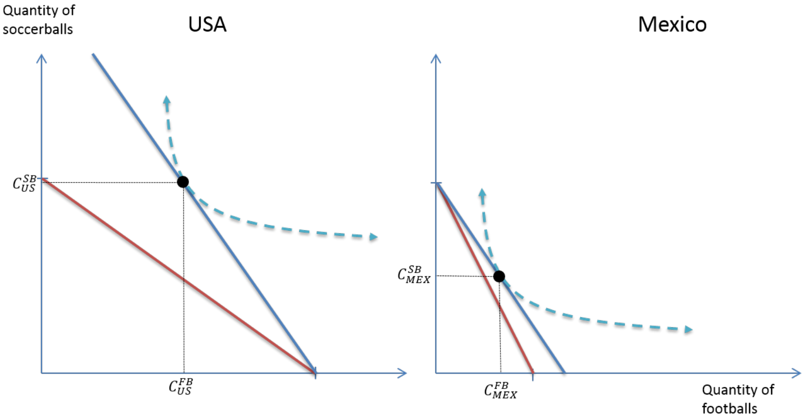
CPF if $\frac{\alpha^{FB}_{US}}{\alpha^{SB}_{US}} < \frac{p^{FB}}{p^{SB}} < \frac{\alpha^{FB}_{MEX}}{\alpha^{SB}_{MEX}}$



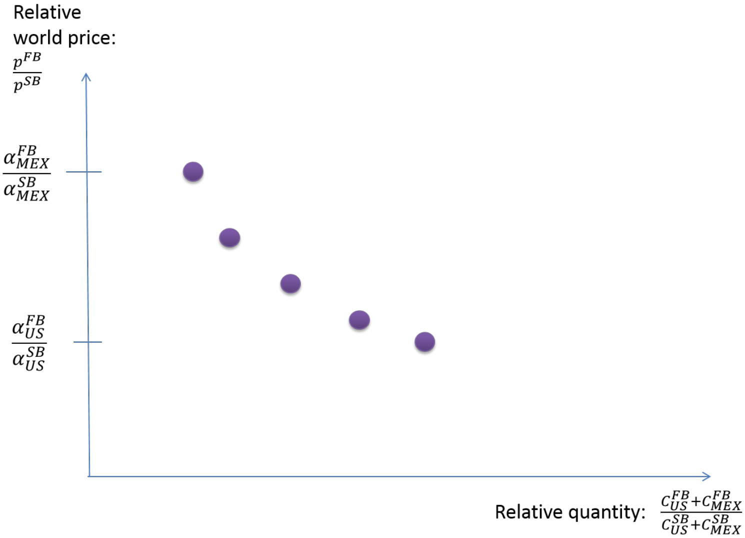
Relative demand if $\frac{\alpha_{US}^{FB}}{\alpha_{US}^{SB}} < \frac{p^{FB}}{p^{SB}} < \frac{\alpha_{MEX}^{FB}}{\alpha_{MEX}^{SB}}$



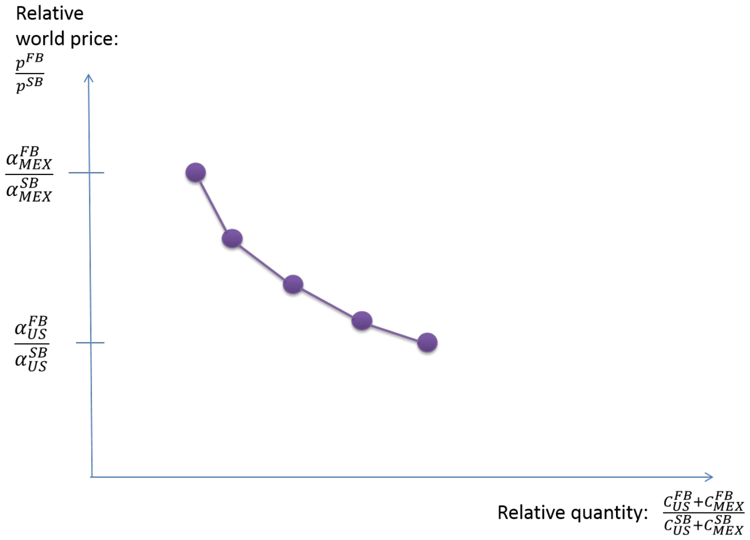
$$\text{CPF if } \frac{\alpha^{FB}_{US}}{\alpha^{SB}_{US}} < \frac{p^{FB}}{p^{SB}} < \frac{\alpha^{FB}_{MEX}}{\alpha^{SB}_{MEX}}$$



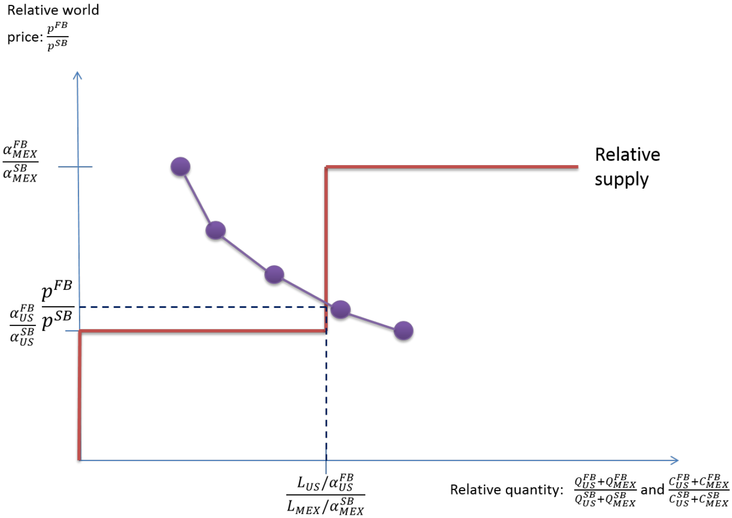
Relative demand if $\frac{\alpha_{US}^{FB}}{\alpha_{US}^{SB}} < \frac{p^{FB}}{p^{SB}} < \frac{\alpha_{MEX}^{FB}}{\alpha_{MEX}^{SB}}$



Relative demand function



Equilibrium world price



Finding the equilibrium world price

- Generally, the equilibrium is one of two types:
  - Type 1:
    - One country completely specializes in the production of comparative advantage good.
    - Other country produces both goods.
    - Equilibrium price is determined by relative productivity of country producing both goods (easy).
    - Equilibrium quantities produced in country producing both goods ensures its income equals its spending (hard).
  - Type 2:
    - Both countries completely specialize in production of their respective comparative advantage good.
    - Equilibrium quantities determined by corner of PPF (easy).
    - Equilibrium price is determined by ensuring total demand equals total supply (hard).

Example: Setup

- Suppose that:
  - $L_{US} = 10$, $L_{MEX} = 5$
  - $\alpha_{US}^{FB} = 1$, $\alpha_{US}^{SB} = 1$, $\alpha_{MEX}^{FB} = 2$, $\alpha_{MEX}^{SB} = 1$
  - $U\left(C_i^{FB}, C_i^{SB}\right) = \left(C_i^{FB}\right)^{0.5}\left(C_i^{SB}\right)^{0.5}$ for $i \in \{US, MEX\}$
- [Class questions:]
  - What is the equilibrium world price?
  - What is the equilibrium consumption of workers in the U.S. and Mexico?

Example: Production

- Step #1(a): Guess the pattern of specialization
  - [Class question: Who has the comparative advantage in what?]
  - Answer: US has comparative advantage in footballs, Mexico has comparative advantage in soccer balls.
  - This tells us that there are three possible equilibria:
    1. US produces both footballs and soccer balls, Mexico only produces soccer balls.
    2. US produces only footballs, Mexico only produces soccer balls.
    3. US produced only footballs, Mexico produces footballs and soccer balls.
  - Rule of thumb: first guess the equilibrium where the larger / more productive country produces both.

Example: Production (ctd)

- Step #1(b): Guess US produces both footballs and soccer balls, calculate world price.
  - Let $p^{SB} = 1$.
  - [Class question: What is the equilibrium world price?]
  - U.S. workers have to be willing to produce both products:
$$p^{FB} = 1.$$

Example: Production (ctd)

- Step #1(c): Calculate production in both countries:
  - Mexico is easy: $Q^{SB}_{MEX} = \frac{L_{MEX}}{\alpha^{SB}_{MEX}} = 5.$
  - US is harder:

$$\begin{aligned} L_{US} &= \alpha^{FB}_{US} Q^{FB}_{US} + \alpha^{SB}_{US} Q^{SB}_{US} \iff \\ Q^{SB}_{US} &= \frac{1}{\alpha^{SB}_{US}} L_{US} - \frac{\alpha^{FB}_{US}}{\alpha^{SB}_{US}} Q^{FB}_{US} \iff \\ Q^{SB}_{US} &= 10 - Q^{FB}_{US}. \end{aligned}$$

  - In total:

$$Q^{SB}_{World} = 15 - Q^{FB}_{US}$$

$$Q^{FB}_{World} = Q^{FB}_{US}$$

Example: Consumption

- Step #2(a): Calculate worker income in both countries:
  - U.S. workers: $w_{US} = \frac{p^{FB}}{\alpha_{US}^{FB}} = \frac{p^{SB}}{\alpha_{US}^{SB}} = 1.$
  - Mexico workers: $w_{MEX} = \frac{p^{SB}}{\alpha_{MEX}^{SB}} = 1.$
- Step #2(b): Calculate total consumption given income and preferences in both countries:
  - Cobb-Douglas preferences imply that workers spend $\beta = 0.5$ of their money on each good:
  - $C_{US}^{FB} = \frac{\beta \times w_{US} \times L_{US}}{p^{FB}} = 5.$ $C_{US}^{SB} = \frac{(1-\beta) \times w_{US} \times L_{US}}{p^{SB}} = 5.$
  - $C_{MEX}^{FB} = \frac{\beta \times w_{MEX} \times L_{MEX}}{p^{FB}} = 2.5.$
    $C_{MEX}^{SB} = \frac{(1-\beta) \times w_{MEX} \times L_{MEX}}{p^{SB}} = 2.5.$
  - $C_{World}^{FB} = 7.5.$ $C_{World}^{SB} = 7.5.$

Example: Market clearing

- Step #3: Check to see if production can satisfy consumption for both goods (i.e. do markets clear?).
  - Market clearing implies $C_{World}^{FB} = Q_{World}^{FB}$. Recall that $Q_{World}^{FB} = Q_{US}^{FB}$ and $C_{World}^{FB} = 7.5$ so that $Q_{US}^{FB} = 7.5$.
  - If $Q_{US}^{FB} = 7.5$, then $Q_{US}^{SB} = 2.5$ and $Q_{World}^{SB} = 7.5$. Since $C_{World}^{SB} = 7.5$, both goods markets clear and we found the equilibrium!
- [Class question: suppose we guessed that both countries completely specialize; what would have happened?]

Conclusion and next steps

- Lessons from today:
  - Trade allows CPF to exceed PPF, which creates gains from trade.
  - Gains from trade come from differences in comparative advantage (not absolute advantage).
  - Finding the equilibrium can be difficult (but is certainly fair game for the exam!)
- Next week:
  - Extend the model to incorporate many goods (1977) and countries (2002).
  - Despite the added complication, the same basic forces are at work.