Midterm Exam

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EI037 Microeconomics

This is a closed book exam. You need to solve this exam alone and independently. Your answers should be legible, clear, and concise. In order to get full credit you have to give complete answers, including how answers are derived. Partial answers will lead to partial credit. Wrong additional statements (i.e., guessing) might reduce the given credit.

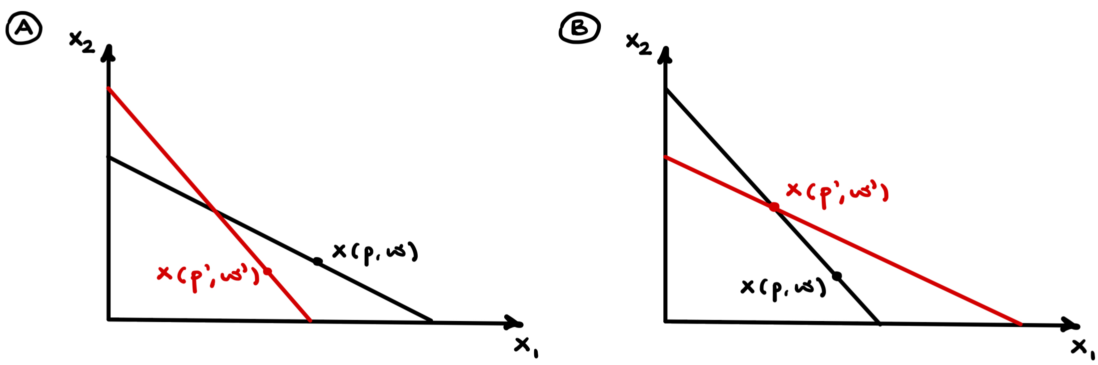
You have 2 hours to complete this exam. Each sub-question is worth 0.5 points, adding up to a total of 6 points. Allocate your time wisely across questions. Good luck!

1. Consumer Choice and Preferences

1.a. Explain in words or graphs the concept of the *weak axiom of revealed preference* (WA). Provide an example of when the WA is violated.

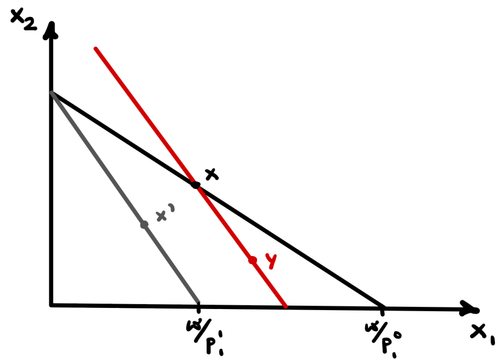
Answer: *The weak axiom of revealed preference (WA) explains that when there are two bundles $x(p,w)$ and $x(p',w')$, if $px(p',w') \leq w$ and $x(p,w) \neq x(p',w')$, then it must be that $p'x(p,w) > w'$.*

Graph A below shows that, at (p,w) both bundles $x(p,w)$ and $x(p',w')$ are affordable, but $x(p,w)$ is chosen and preferred over $x(p',w')$. At (p',w'), $x(p',w')$ is chosen because $x(p,w)$ is not affordable. The graph therefore shows us that the WA is satisfied. In graph B, both $x(p,w)$ and $x(p',w')$ are affordable in both states. However, the consumer makes different (and inconsistent) choices in each period, which violates the WA.



1.b. Explain in graphs the concept of the *compensated law of demand* (CLD). Provide an example of when the CLD is violated.

Answer: *The compensated law of demand means that $(p'-p)\cdot(x(p',w')-x(p,w)) \leq 0$. This is shown in the graph below, where it also holds that $\Delta w = \Delta p \cdot x(p,w)$. As shown in the graph below, as the price of x_1 increases from p_1^0 to p_1^1, the consumer chooses bundle X' instead of bundle X. The CLD compensates the consumer, given the new prices, such that X would still* *be affordable. Logically, a consumer would then choose a new bundle on the compensated budget plane that is higher than X. Bundle Y shown in the graph below violates the CLD.*



1.c. The preference relation $\succeq$ defined on the consumption set $X = \mathbf{R}_+^L$ is said to be *monotone* if $x >> y$ implies that $x \succ y$; and *strongly monotone* if $x > y$ implies that $x \succ y$. Show that if $\succeq$ is strongly monotone, then it is monotone.

Answer: *Strongly monotone denotes that $\forall x \in X$, if $x > y$, then $x \succ y$. Then, $x \neq y$ and $x > y$ also implies $x \succ y$. Hence, $x >> y$ implies $x \succ y$, which is the definition of monotone. Hence, if $\succeq$ is strongly monotone, then it is monotone.*

1.d. The preference relation $\succeq$ defined on the consumption set $X = \mathbf{R}_+^L$ is said to be *weakly monotone* if and only if $x \geq y$ implies that $x \succeq y$. Show that if $\succeq$ is transitive, locally non-satiated, and weakly monotone, then it is monotone.

Answer: *Suppose that $x >> y$. To prove $x \succ y$, first note that: since $x >> y$ implies $x \geq y$, and given that $\succeq$ is weakly monotone, we have $x \succeq y$.*

If we define $\varepsilon = \min\{x_1 - y_1, \ldots, x_L - y_L\} > 0$ then for every $z \in X$, if $||y - z|| < \varepsilon$, then $x >> z$. By local non-satiation, there exists $z^* \in X$ such that $||y - z^*|| < \varepsilon$ and $z^* \succ y$. By $x >> z^*$ and weak monotonicity, $x \succeq z^*$. Given $x \succeq z^*$, $z^* \succ y$, by transitivity or proposition 1.B.1(iii) in MWG, $x \succ y$. Thus, $\succeq$ is monotone and we conclude the proof.

2. Classical Demand Theory

Consider the following utility function

$$u(x) = x_1 + 2x_2^{\frac{1}{2}}$$

2.a. Find the Walrasian demand function for goods 1 and 2 as a function of prices and wealth.

Answer: *We begin by solving for the consumer's utility maximization problem, in order to obtain their optimal demand of each good. We have*

$$\max_{x_1, x_2 \geq 0} \quad x_1 + 2x_2^{\frac{1}{2}} \quad s.t. \ p_1x_1 + p_2x_2 \leq w$$

The problem's Lagrangian is therefore

$$\mathcal{L} = x_1 + 2x_2^{\frac{1}{2}} - \lambda(p_1x_1 + p_2x_2 - w)$$

Take the first order derivative w.r.t x_1, x_2, respectively:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial x_1} &= 1 - \lambda p_1 = 0 \\ \frac{\partial \mathcal{L}}{\partial x_2} &= x_2^{-\frac{1}{2}} - \lambda p_2 = 0 \end{aligned} \quad \Rightarrow \quad \frac{p_1}{p_2} = \frac{1}{x_2^{-\frac{1}{2}}} = x_2^{\frac{1}{2}} \quad \Rightarrow \quad x_2 = \frac{p_1^2}{p_2^2}$$

Substituting x_2 into the consumer's budget constraint, we have

$$p_1x_1 + p_2\frac{p_1^2}{p_2^2} = w \quad \Rightarrow \quad p_1x_1 = w - \frac{p_1^2}{p_2} \quad \Rightarrow \quad x_1^*(p,w) = \frac{w}{p_1} - \frac{p_1}{p_2}$$

$$\text{and} \quad x_2^*(p,w) = \frac{p_1^2}{p_2^2}$$

2.b. Find the Hicksian (compensated) demand function $h(p,u)$ for goods 1 and 2.

Answer: *We solve for the consumer's expenditure minimization problem (EMP)*

$$\min_{x_1,x_2 \geq 0} \quad p_1x_1 + p_2x_2 \quad s.t. \quad x_1 + 2x_2^{\frac{1}{2}} \geq \bar{u}$$

The consumer's Lagrangian is set up as

$$\mathcal{L} = p_1x_1 + p_2x_2 - \lambda(x_1 + 2x_2^{\frac{1}{2}} - \bar{u})$$

Take the first order derivative w.r.t x_1, x_2, respectively:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial x_1} &= p_1 - \lambda = 0 \\ \frac{\partial \mathcal{L}}{\partial x_2} &= p_2 - \lambda x_2^{-\frac{1}{2}} = 0 \end{aligned} \quad \Rightarrow \quad \frac{p_1}{p_2} = \frac{1}{x_2^{-\frac{1}{2}}} = x_2^{\frac{1}{2}} \quad \Rightarrow \quad x_2 = \frac{p_1^2}{p_2^2}$$

Substituting x_2 into the consumer's utility constraint, we have

$$x_1 + 2\frac{p_1}{p_2} = \bar{u} \quad \Rightarrow \quad x_1 = \bar{u} - 2\frac{p_1}{p_2} \quad \Rightarrow \quad h_1^*(p,u) = \bar{u} - 2\frac{p_1}{p_2}$$

$$\text{and} \quad h_2^*(p,u) = \frac{p_1^2}{p_2^2}$$

2.c. Find the expenditure function $e(p,u)$, and verify Shephard's lemma.

Answer: *The consumer's expenditure function, $e(p,u)$ is given by*

$$\begin{aligned} e(p,u) &= p_1h_1(p,u) + p_2h_2(p,u) \\ &= p_1\left(\bar{u} - 2\frac{p_1}{p_2}\right) + p_2\frac{p_1^2}{p_2^2} = p_1\bar{u} - 2\frac{p_1^2}{p_2} + \frac{p_1^2}{p_2} \\ e(p,u) &= p_1\bar{u} - \frac{p_1^2}{p_2} \end{aligned}$$

We are asked to verify Shephard's lemma, that is, $h(p,u) = \nabla_p e(p,u)$.

$$\begin{aligned}
\frac{\partial e(p,u)}{\partial p_1} &= \bar{u} - \frac{2p_1}{p_2} = h_1^*(p,u) \\
\frac{\partial e(p,u)}{\partial p_2} &= (-1)\cdot(-1)\cdot p_1^2 p_2^{-2} = \frac{p_1^2}{p_2^2} = h_2^*(p,u)
\end{aligned}$$

2.d. Find the indirect utility function $v(p,w)$, and verify Roy's identity.

Answer: *To obtain the consumer's indirect utility function $v(p,w)$, we plug the optimal Marshallian demands we found in item (a) into the consumer's utility function $u(x_1,x_2)$*

$$\begin{aligned}
v(p,w) &= u(x_1^*, x_2^*) \\
&= x_1^* + 2{x_2^*}^{\frac{1}{2}} = \left(\frac{w}{p_1} - \frac{p_1}{p_2}\right) + 2\left(\frac{p_1^2}{p_2^2}\right)^{\frac{1}{2}} = \frac{w}{p_1} - \frac{p_1}{p_2} + 2\frac{p_1}{p_2} \\
&= \frac{w}{p_1} + \frac{p_1}{p_2}
\end{aligned}$$

To verify Roy's identity, we first derive each of it's elements separately

$$\begin{aligned}
\frac{\partial v(p,w)}{\partial p_1} &= -wp_1^{-2} + \frac{1}{p_2} \\
\frac{\partial v(p,w)}{\partial p_2} &= -\frac{p_1}{p_2^2} \\
\frac{\partial v(p,w)}{\partial w} &= \frac{1}{p_1}
\end{aligned}$$

Then

$$\begin{aligned}
-\frac{\frac{\partial v(p,w)}{\partial p_1}}{\frac{\partial v(p,w)}{\partial w}} &= -\frac{-wp_1^{-2} + \frac{1}{p_2}}{\frac{1}{p_1}} = \frac{-\frac{1}{p_2} + \frac{w}{p_1^2}}{\frac{1}{p_1}} = \frac{w}{p_1} - \frac{p_1}{p_2} = x_1^*(p,w) \\
-\frac{\frac{\partial v(p,w)}{\partial p_2}}{\frac{\partial v(p,w)}{\partial w}} &= -\frac{-\frac{p_1}{p_2^2}}{\frac{1}{p_1}} = \frac{p_1^2}{p_2^2} = x_2^*(p,w)
\end{aligned}$$

3. Welfare Economics

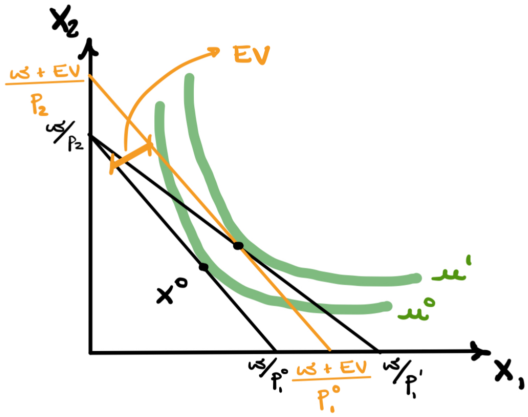
Consider an economy with two goods, 1 and 2. Consider a price change from the initial price vector p^0 to a new price vector $p^1 \leq p^0$.

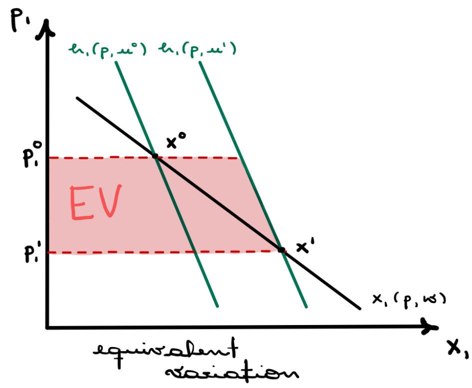
3.a. Write the mathematical definition of the *equivalent variation* (EV). Explain the concept using both words and graphs.

Answer: *Mathematically, there are two ways to denote the equivalent variation (EV).*

$$\begin{aligned}
&(1) \qquad EV = \int_{p_0}^{p_1} h(p,u^1)dp \\
&(2) \qquad v(p^0, w + EV) = v(p^1, w)
\end{aligned}$$

EV refers to the amount of money the consumer should be given in the current state so that they will be indifferent between staying at old prices or changing to new prices, whenever prices are proposed to change.
Graphically,





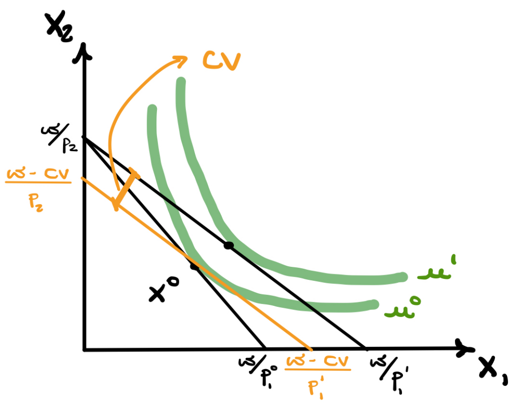
3.b. Write the mathematical definition of the *compensating variation* (CV). Explain the concept using both words and graphs.

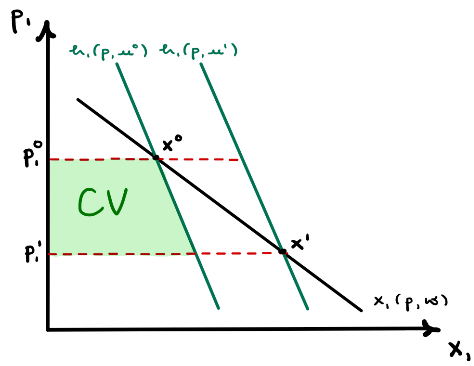
Answer: *Mathematically, there are two ways to denote the compensating variation (CV).*

$$(1) \qquad CV = \int_{p_0}^{p_1} h(p, u^0) dp$$

$$(2) \qquad v(p^0, w) = v(p^1, w - CV)$$

CV refers to the amount of money that should be taken away from the consumer in the future state such that they will be indifferent between staying at old prices or changing to new prices, whenever prices have already changed.
Graphically,





3.c. Show that if $u(x) = x_1 + 2x_2^{\frac{1}{2}}$, then $CV(p^0, p^1, w) = EV(p^0, p^1, w)$ for any (p^0, p^1, w). (*Hint:*

you can choose p_1 as the numéraire – i.e., fix $p_1 = 1$.)

Answer: *Following from the results obtained in question 2, we know that the consumer's indirect utility function is $v(p,w) = \frac{w}{p_1} + \frac{p_1}{p_2}$. Letting p_1 be the numéraire, we have $v(p,w) = w + \frac{1}{p_2}$. We assume that the price of good 2 changes from p_2^0 to p_2^1.*

We therefore have that

$$v(p^0, w + EV) = w + EV + \frac{1}{p_2^0} \quad \textit{and} \quad v(p^1, w) = w + \frac{1}{p_2^1}$$
$$v(p^0, w) = w + \frac{1}{p_2^0} \quad \textit{and} \quad v(p^1, w - CV) = w - CV + \frac{1}{p_2^1}$$

Solving for EV and CV, we therefore have

$$w + EV + \frac{1}{p_2^0} = w + \frac{1}{p_2^1} \quad \Rightarrow \quad EV = \frac{1}{p_2^1} - \frac{1}{p_2^0}$$
$$w + \frac{1}{p_2^0} = w - CV + \frac{1}{p_2^1} \quad \Rightarrow \quad CV = \frac{1}{p_2^1} - \frac{1}{p_2^0} = EV$$

3.d. Explain the intuition behind the results of 3.c. In this case, what is the shadow price of money?

Answer: *From the consumer's UMP in question 2, we have that*

$$\mathcal{L} = x_1 + 2x_2^{\frac{1}{2}} - \lambda(p_1 x_1 + p_2 x_2 - w)$$

From the FOC with respect to x_1, we have that $\lambda p_1 = 1$. Having set p_1 as the numéraire, this translates to $\lambda = 1$, where λ is the shadow price of money. This result shows that the shadow price for the given utility function, $u(x_1, x_2) = x_1 + 2x_2^{\frac{1}{2}}$, is only relevant with respect to the price of good x_1. The marginal utility of increasing 1 unit of wealth is fixed if the price of good x_1 is unchanged. Therefore, regardless of whether staying at old prices p_2^0 or moving to new prices p_2^1, the consumer's marginal utility of increasing 1 unit of wealth will be the same. A consumer being given EV will thus be equally well off as if paying CV, since $EV = CV$.