

Econ 39: International Trade

Week #5: The Specific Factors Model

Treb Allen

Winter 2018

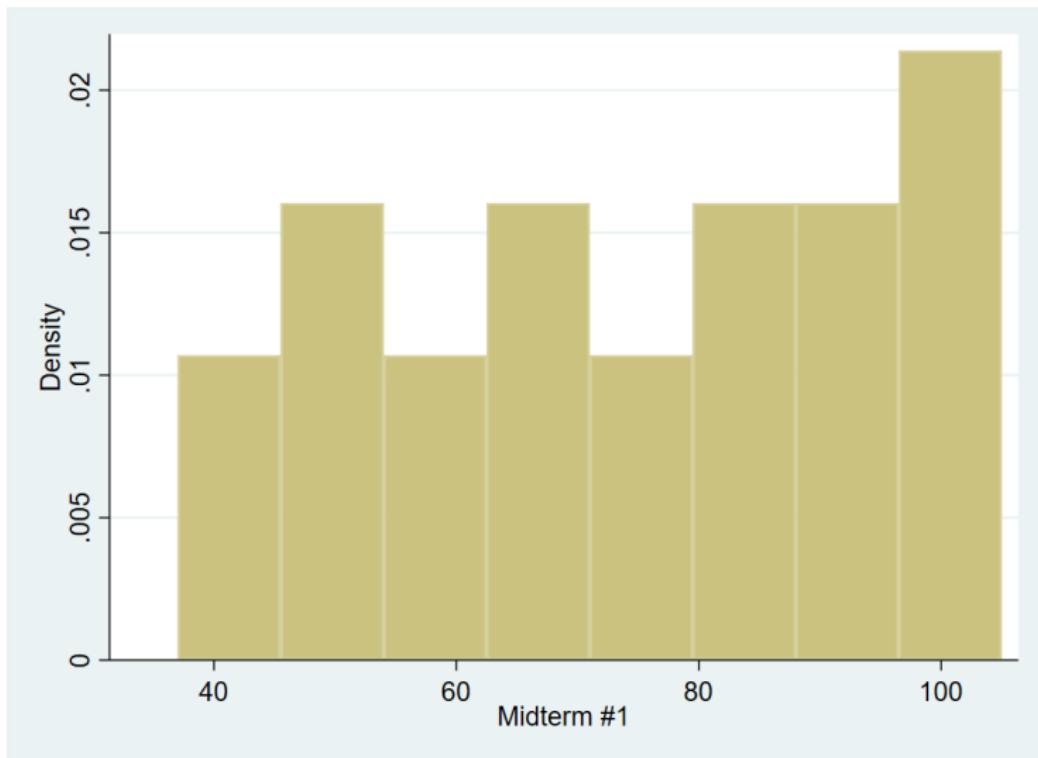
Plan for the day

- ▶ Begin discussion of first model where there are winners and losers from trade
- ▶ Logistical announcements:
 - ▶ “Testing the Ricardo Model” lecture slides are optional reading
 - ▶ Problem set #3 assigned, due next Tuesday (February 6).

Class evaluation

- ▶ Now that I've evaluated you, it's your turn to evaluate me!
- ▶ [Complete class evaluation]

Midterm: Grade distribution



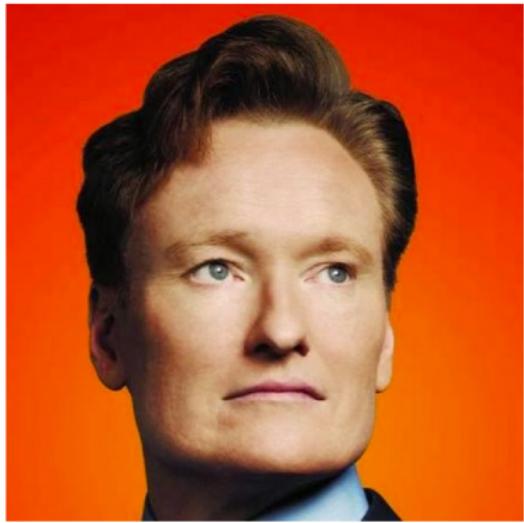
Midterm: Approximate Grades

A+: 100<=	C: 55-59
A: 93-99	C-: 50-54
A-: 90-92	D+: 47-49
B+: 80-89	D: 43-46
B: 70-79	D-: 40-42
B-: 65-69	F: 40>
C+: 60-64	

Grade appeal policy (reminder)

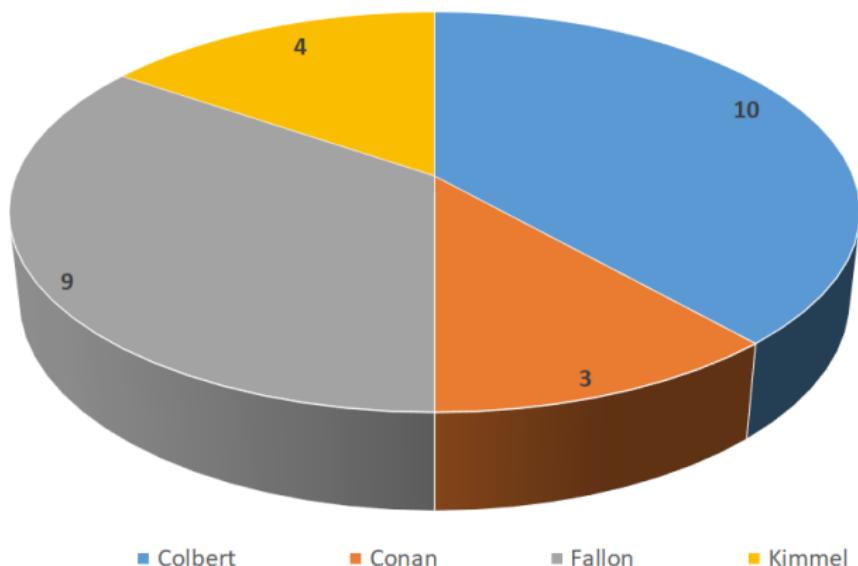
- ▶ Because grades are relative, I am very strict about the grade appeal process. Two types of appeals:
- ▶ Arithmetic error appeal
 - ▶ If I added up the total score wrong.
 - ▶ Write “arithmetic appeal” on the top of the first page of the exam, sign, and return.
 - ▶ I will add up all the scores again.
- ▶ Full exam appeal
 - ▶ If you feel there were any grading mistakes.
 - ▶ Write “full appeal” on the top of the first page of the exam, sign, and return to me.
 - ▶ I will regrade the entire exam.
- ▶ For both types of appeals:
 - ▶ I will not discuss grading prior to an appeal.
 - ▶ Appeals are final. I am happy to discuss grading after an appeal during office hours, but will not change the grade.
 - ▶ The grade after an appeal may be lower than the original

Today's Teams



Today's Teams

Late Night Hosts



The Gains from Trade in the Ricardian Model

- ▶ In the Ricardian model, we saw that no country can be made worse off by openness to trade.
- ▶ [Class question: Where did the gains of trade come from?]
- ▶ However, in the real world, there is currently *substantial* political resistance to trade.

"I know 'em all"

≡ SECTIONS HOME SEARCH

Travelers Stranded and Protests Swell Over Trump Order Internal Memo at State Department Opposes Trump's Travel Ban Trump Vows to 'Do a Big Num' Dodd-Frank Regulations

POLITICS

Trump Abandons Trans-Pacific Partnership, Obama's Signature Trade Deal

By PETER BAKER JAN. 23, 2017



President Trump pulled out of the Trans-Pacific Partnership on Monday and proposed massive cuts in taxes and regulations for businesses. By THE ASSOCIATED PRESS. Photo by Doug Mills/The New York Times. Watch in Times Video »

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WASHINGTON — President Trump upended America's traditional, bipartisan trade policy on Monday as he formally abandoned the ambitious, 12-nation Trans-Pacific Partnership brokered by his predecessor and declared an end to the era of multinational trade agreements that defined global economics for decades.

► Movie

What is missing from the Ricardian model?

- ▶ In the Ricardian model, it is not the wages in each country that matter but instead the real wages (i.e. how much of each good that a worker can consume).
- ▶ Hence, openness to trade will always (weakly) increase the consumption possibility frontier:
 - ▶ Workers specialize in their comparative advantage, sell it on the world market.
 - ▶ Use their income to import other goods at lower relative prices than in autarky.
- ▶ [Class question: What is not realistic in this model?]

What is missing from the Ricardian model?

- ▶ The week we will discuss the **specific factors** model.
 - ▶ Basic idea: it is (infinitely) costly for some factors of production to switch what is being produced.
 - ▶ Increased specialization can make the owners of these factors worse off.
 - ▶ Not everyone gains from trade.
 - ▶ But countries still overall gain from trade.

Model Setup

- ▶ Two countries: U.S. and Mexico.
- ▶ Two goods: footballs and soccer balls.
- ▶ *Three factors of production:*
 - ▶ Labor
 - ▶ Freely mobile across the football and soccer ball sectors
 - ▶ Football **specific factor**
 - ▶ Only used in the production of footballs
 - ▶ Soccer **specific factor**
 - ▶ Only used in the production of soccer balls
- ▶ [Class question: What are examples of specific factors?]
- ▶ [Class question: What in the real world is the concept of a specific factor supposed to capture?]

Terminology

- ▶ For now, let us consider the United States in autarky.
- ▶ Terminology:
 - ▶ Let S_{US} be the quantity of the soccer ball specific factor in the United States.
 - ▶ Let F_{US} be the quantity of the football specific factor in the United States.
 - ▶ Let L_{US} be the number of workers in the United States.
 - ▶ Let L_{US}^{SB} be the number of workers employed in the production of soccer balls.
 - ▶ Let L_{US}^{FB} be the number of workers employed in the production of footballs.

Production

- In the specific factors model, we assume that the total quantity of footballs produced Q_{US}^{FB} depends on the amount of labor and the amount of the football specific factor used:

$$Q_{US}^{FB} = Q^{FB}(F_{US}, L_{US}^{FB}) \quad (1)$$

- Equation (1) is known as a **production function**.
- Similar equation for the production of soccer balls:

$$Q_{US}^{SB} = Q^{SB}(S_{US}, L_{US}^{SB})$$

Assumptions

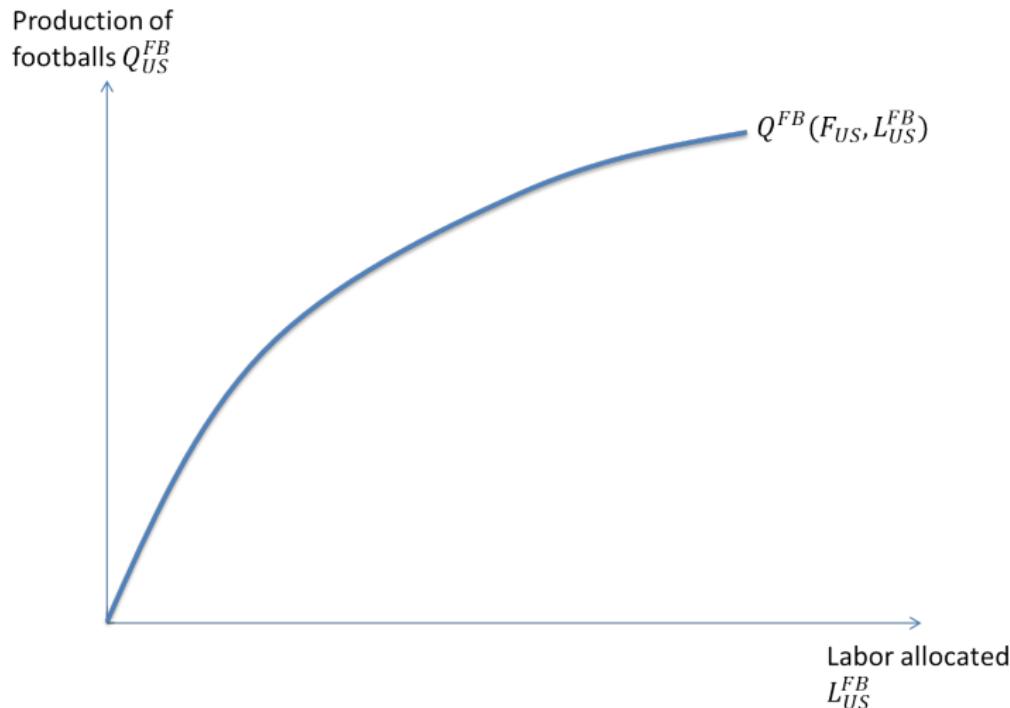
For all $i \in \{FB, SB\}$:

- ▶ $\frac{\partial Q^i}{\partial L_{US}^i} > 0$, $\frac{\partial Q^{FB}}{\partial F} > 0$, $\frac{\partial Q^{SB}}{\partial S} > 0$.
 - ▶ [Class question: what do these equations mean?]
- ▶ $\frac{\partial^2 Q^{FB}}{(\partial F)^2} < 0$, $\frac{\partial^2 Q^{SB}}{(\partial S)^2} < 0$, $\frac{\partial^2 Q^i}{(\partial L_{US}^i)^2} < 0$
 - ▶ [Class question: what do these equations mean?]
- ▶ For all $\alpha > 0$, $Q^i(\alpha F_i, \alpha L_{US}^i) = \alpha Q^i(F_i, L_{US}^i)$
 - ▶ [Class question: what does this equation mean?]

Important Concepts

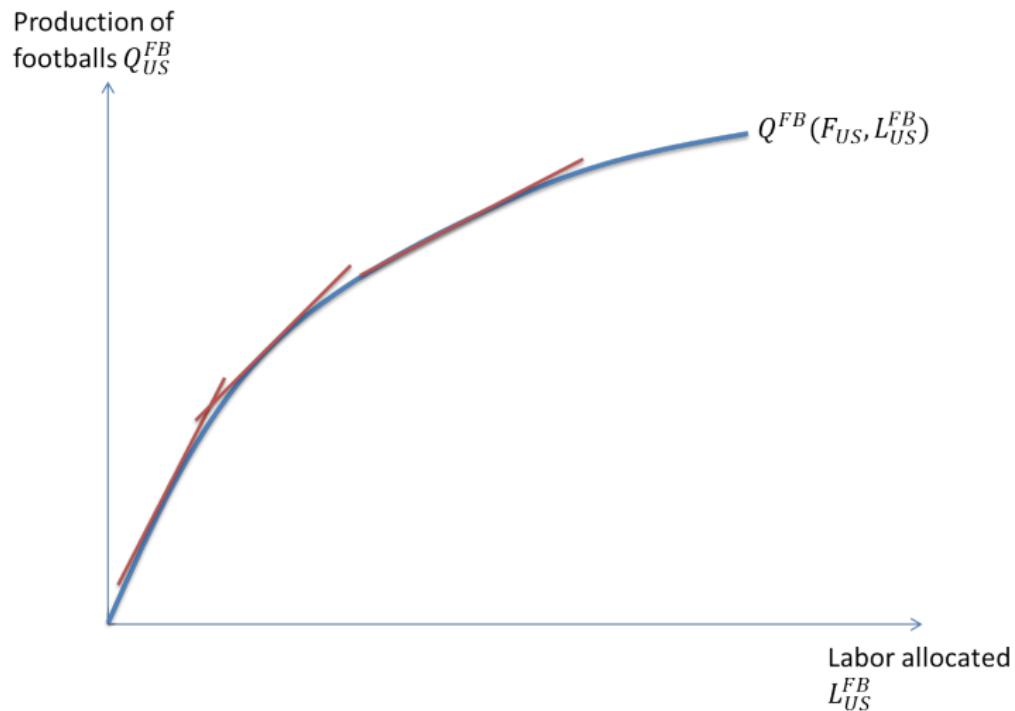
- ▶ The **marginal product of labor** is how much an additional worker increases the total production of a good.
 - ▶ [Class question: If we added another worker to the production of footballs in the U.S., how much would the production of footballs increase?]
 - ▶ The marginal product of labor in the production of footballs is $\frac{\partial Q^{FB}(F_{US}, L_{US}^{FB})}{\partial L_{US}^{FB}}$.
 - ▶ The marginal product of labor in the production of soccer balls is $\frac{\partial Q^{SB}(S_{US}, L_{US}^{SB})}{\partial L_{US}^{SB}}$.
- ▶ The production of a good is subject to **diminishing marginal returns** (of labor) if the marginal product of labor is decreasing as additional workers are added.
- ▶ [Class question: why would we expect diminishing marginal returns in the specific factor model?]

Production Function for Footballs

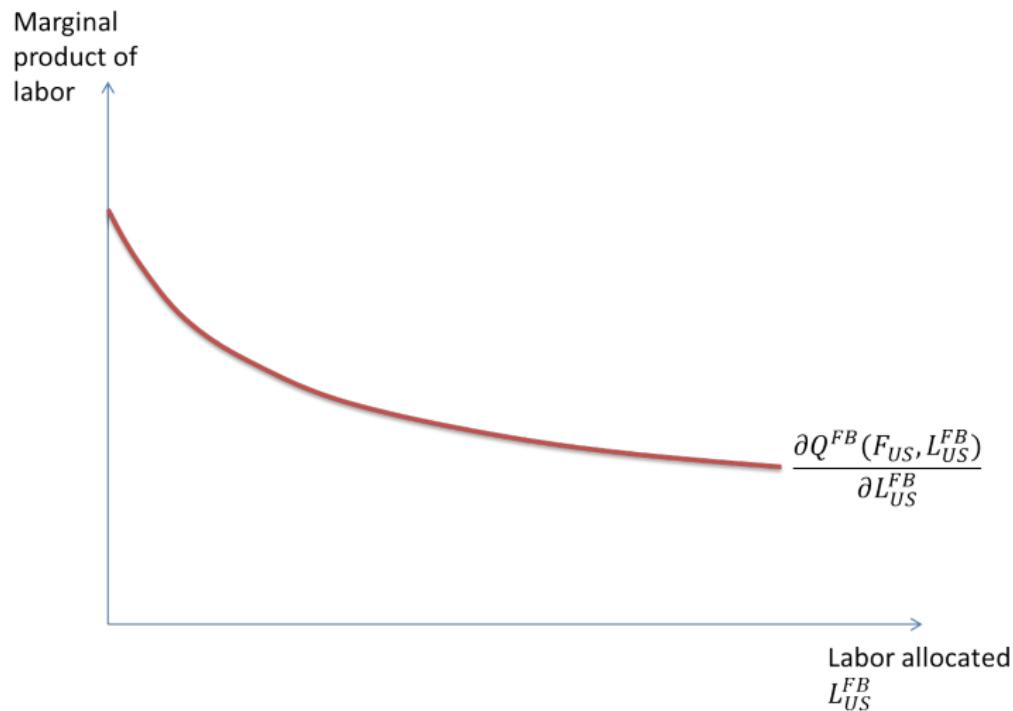


- ▶ [Class question: How can we find the marginal product of labor from this figure?]

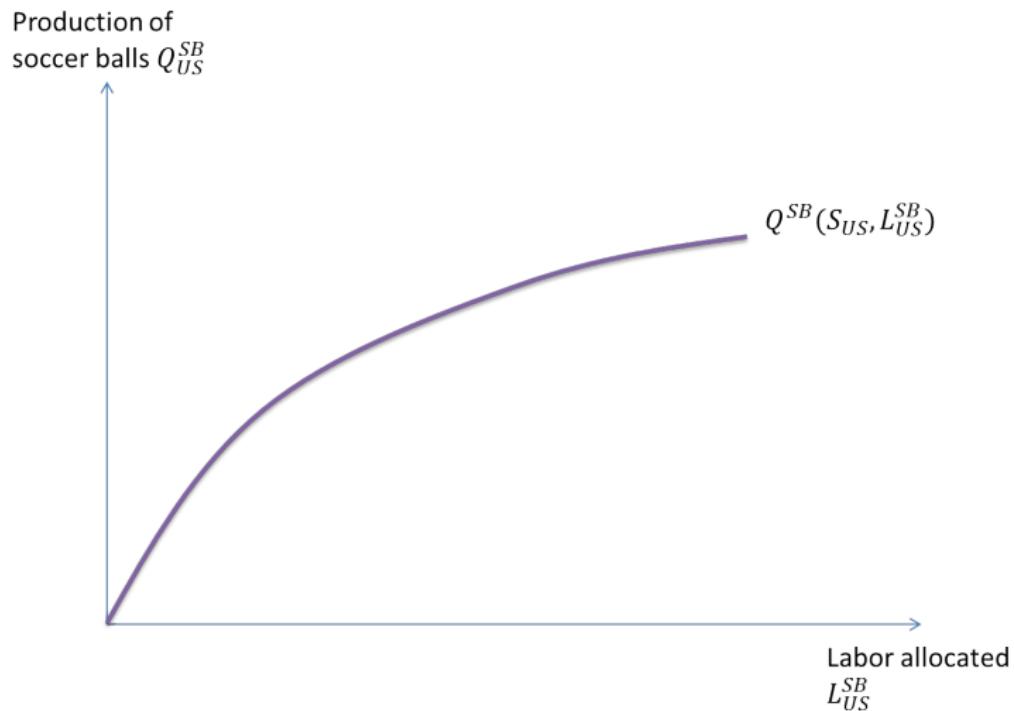
Production Function for Footballs



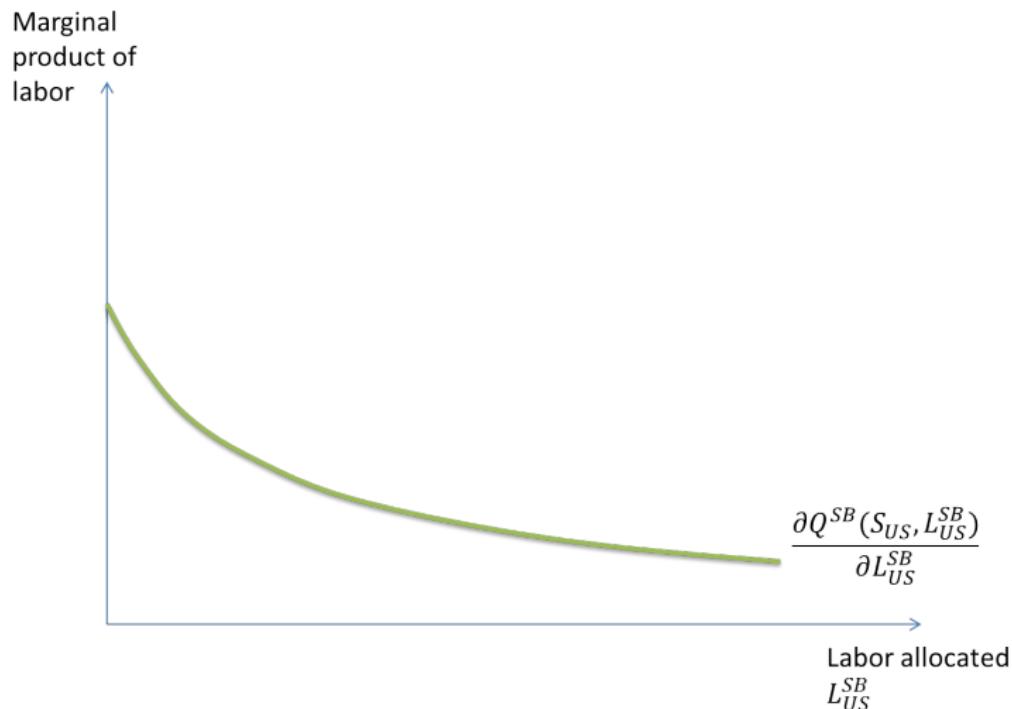
Marginal Product of Labor for Footballs



Production Function of Soccer Balls



Marginal Product of Labor for Soccer Balls



Constructing the Production Possibility Frontier

- ▶ As with the Ricardian model, an important part of determining the equilibrium is the construction of the production possibility frontier (PPF).
- ▶ Basic idea remains the same: What is the most soccer balls can be produced for a given number of footballs?
- ▶ Trick is to use all the labor not used for footballs in the production of soccer balls:

$$L_{US} = L_{US}^{FB} + L_{US}^{SB}$$

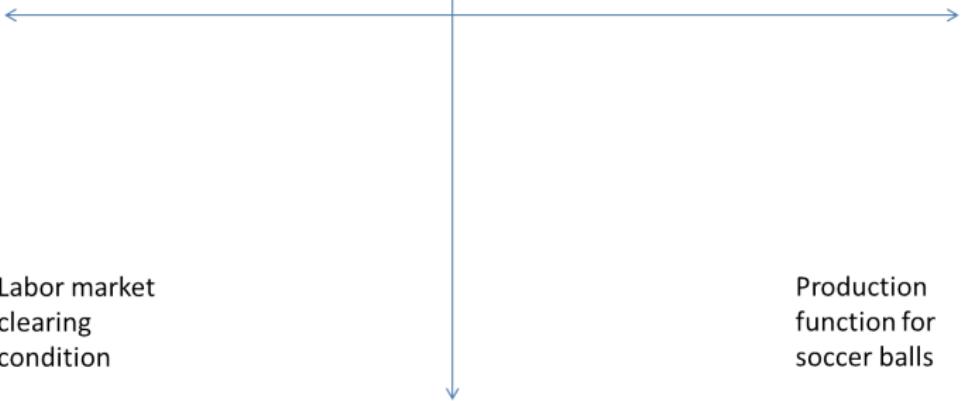
We call this equation the **labor market clearing condition.**

Constructing the Production Possibility Frontier

- ▶ In the Ricardian model, this was easy because the marginal product of labor was constant (so we could just draw a straight line).
- ▶ Now we have to account for the diminishing marginal returns due to the specific factors.
- ▶ To do so, we draw one of my favorite figures!

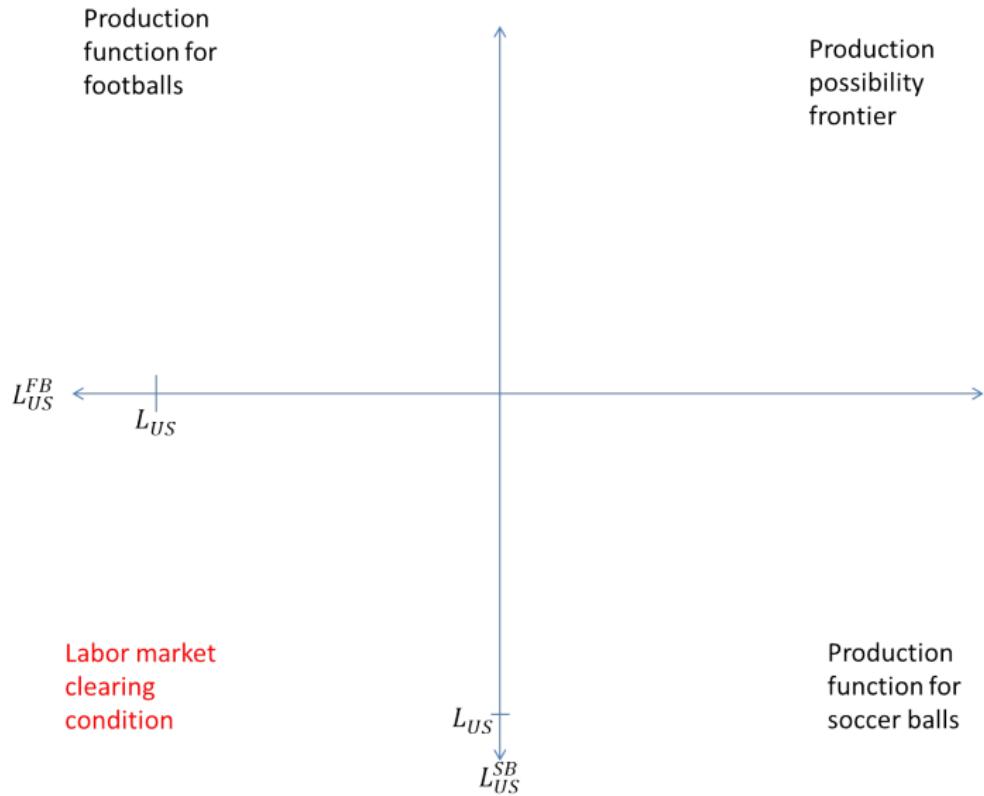
Production
function for
footballs

Production
possibility
frontier



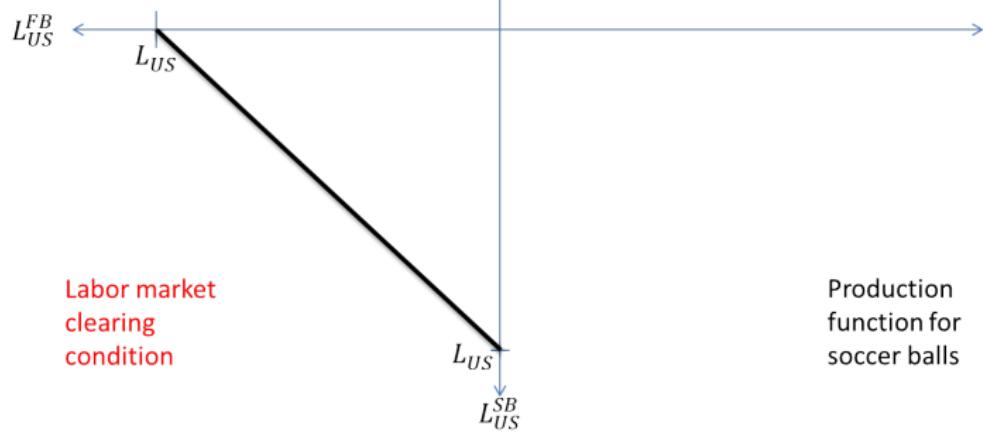
Labor market
clearing
condition

Production
function for
soccer balls



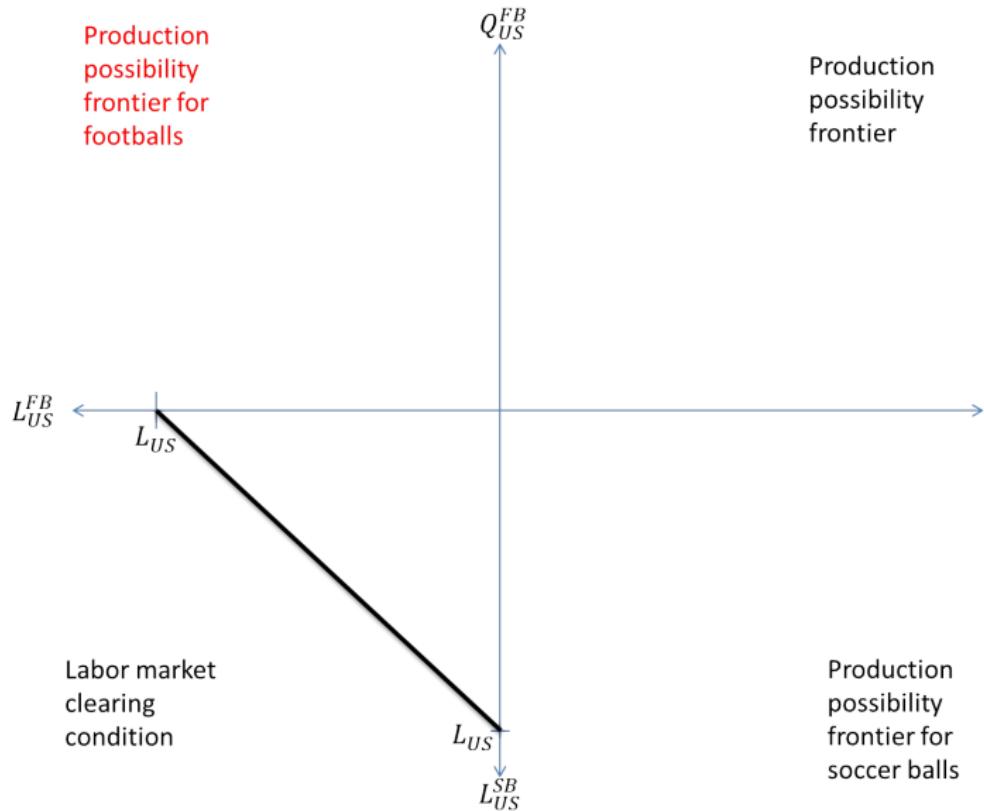
Production
function for
footballs

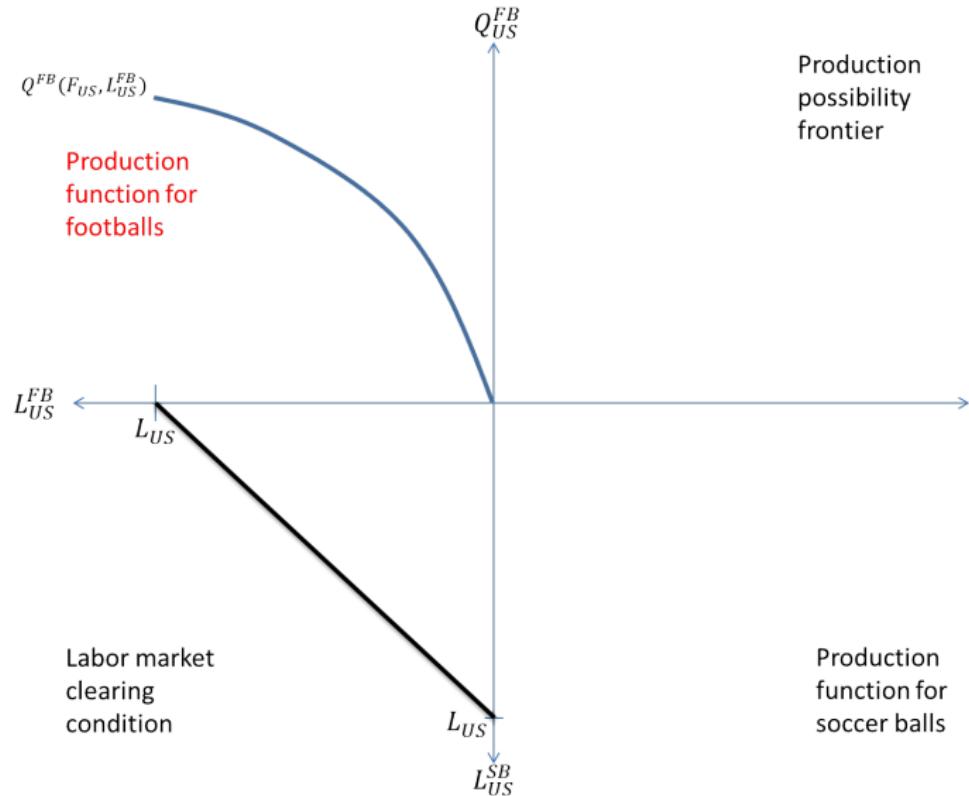
Production
possibility
frontier

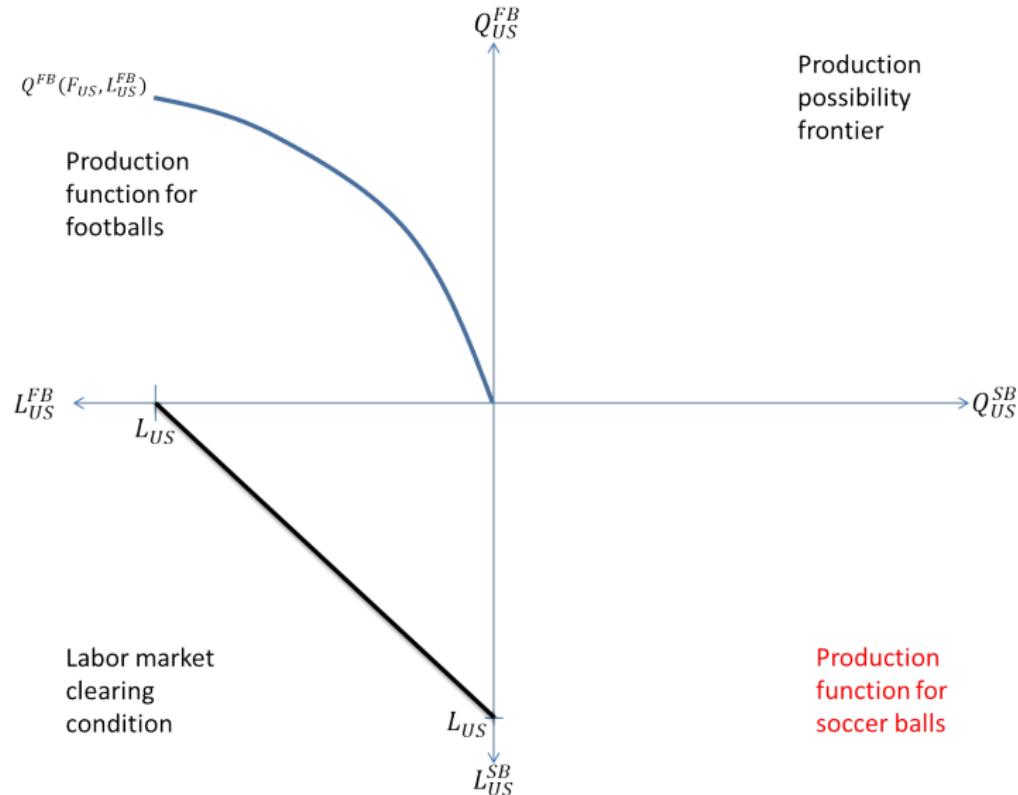


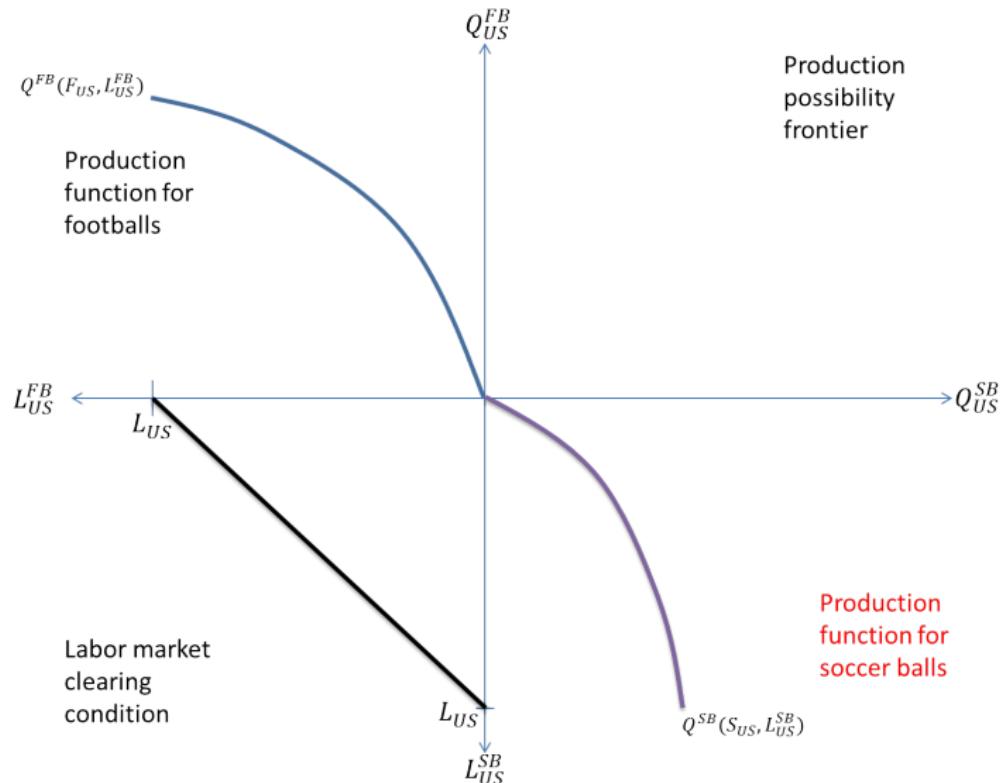
Labor market
clearing
condition

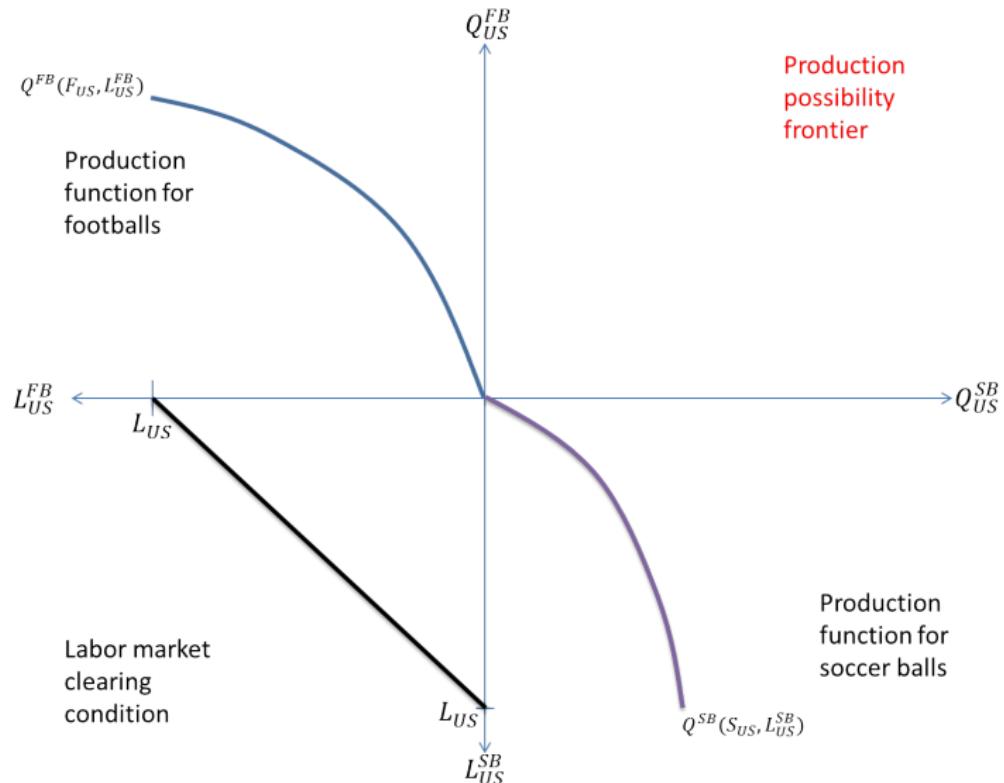
Production
function for
soccer balls

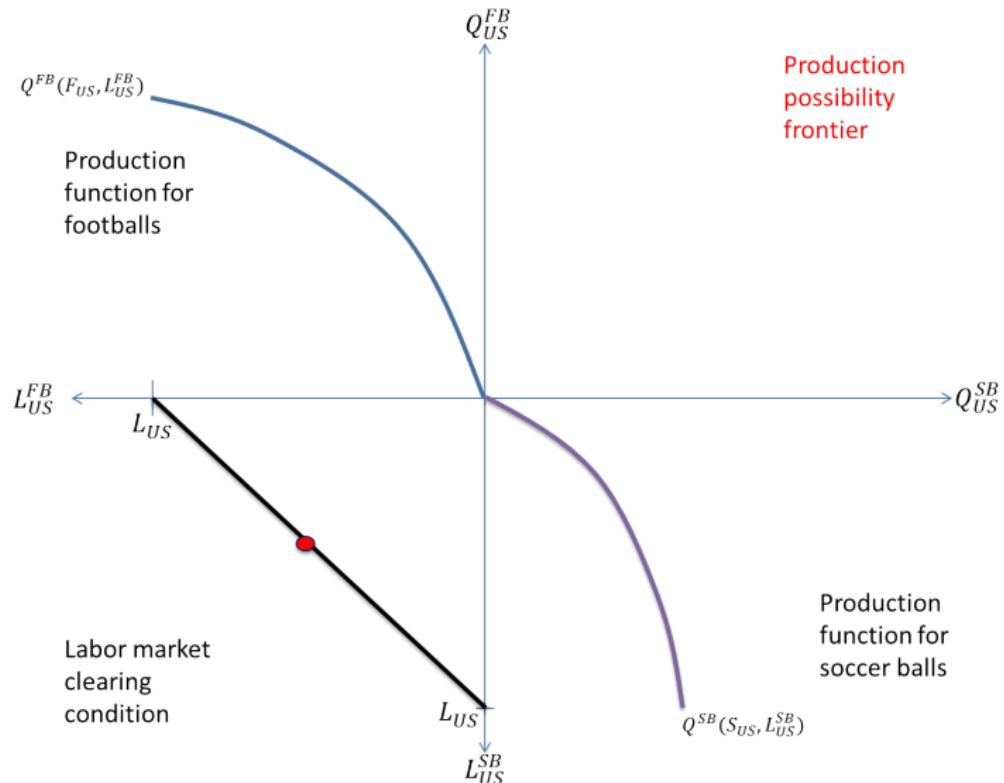


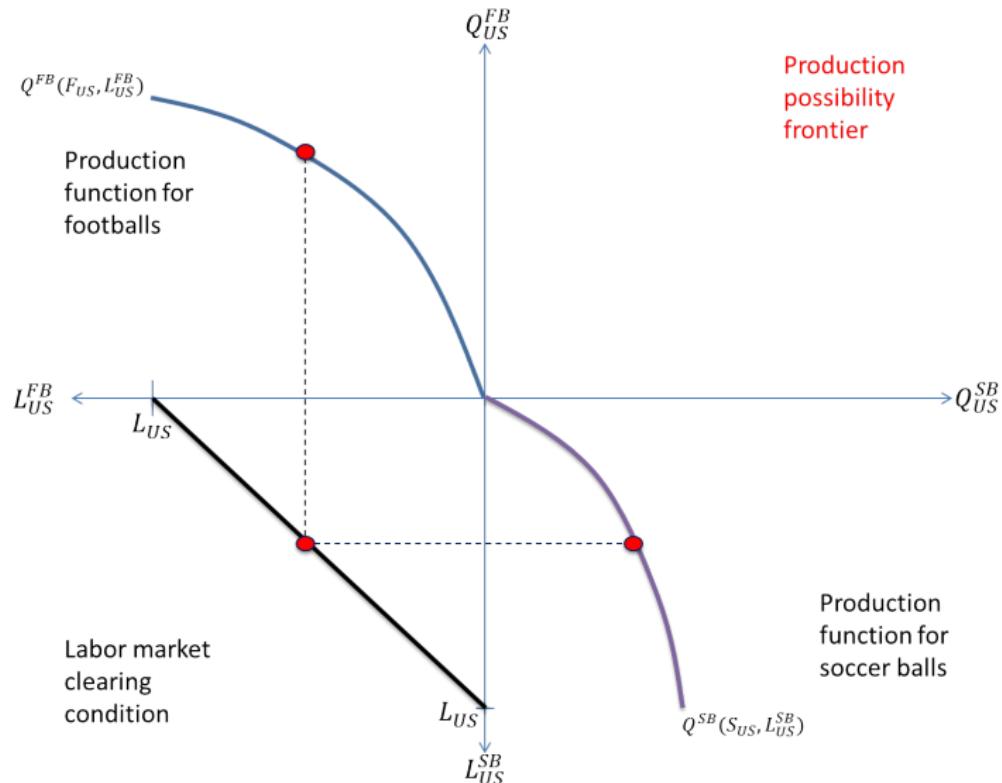


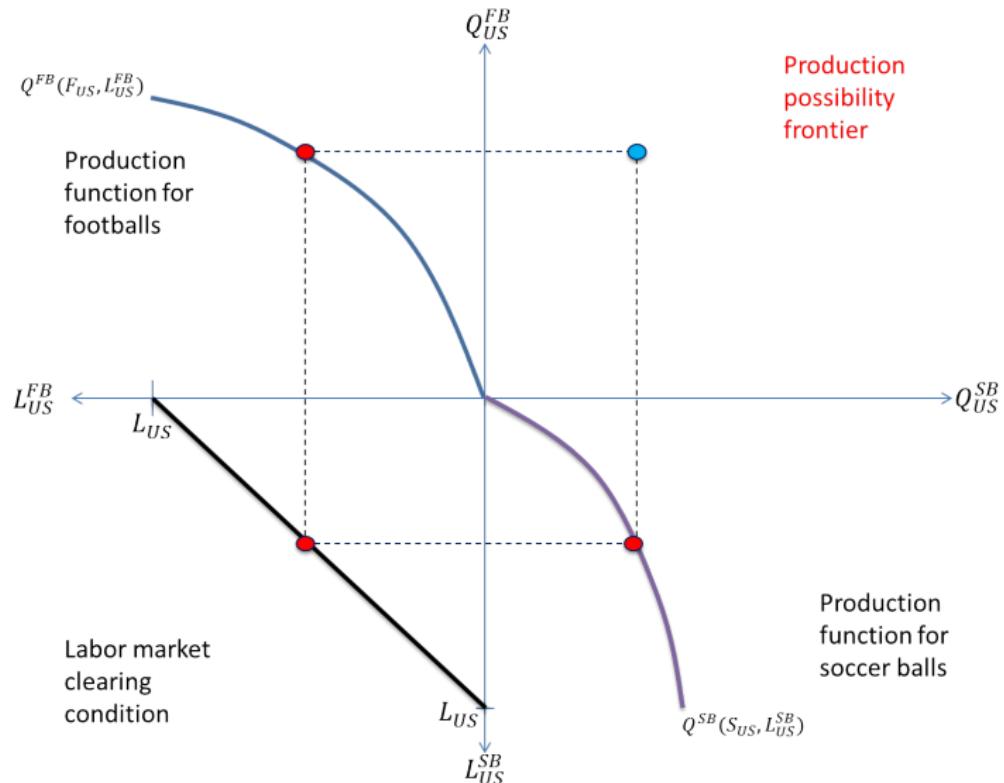


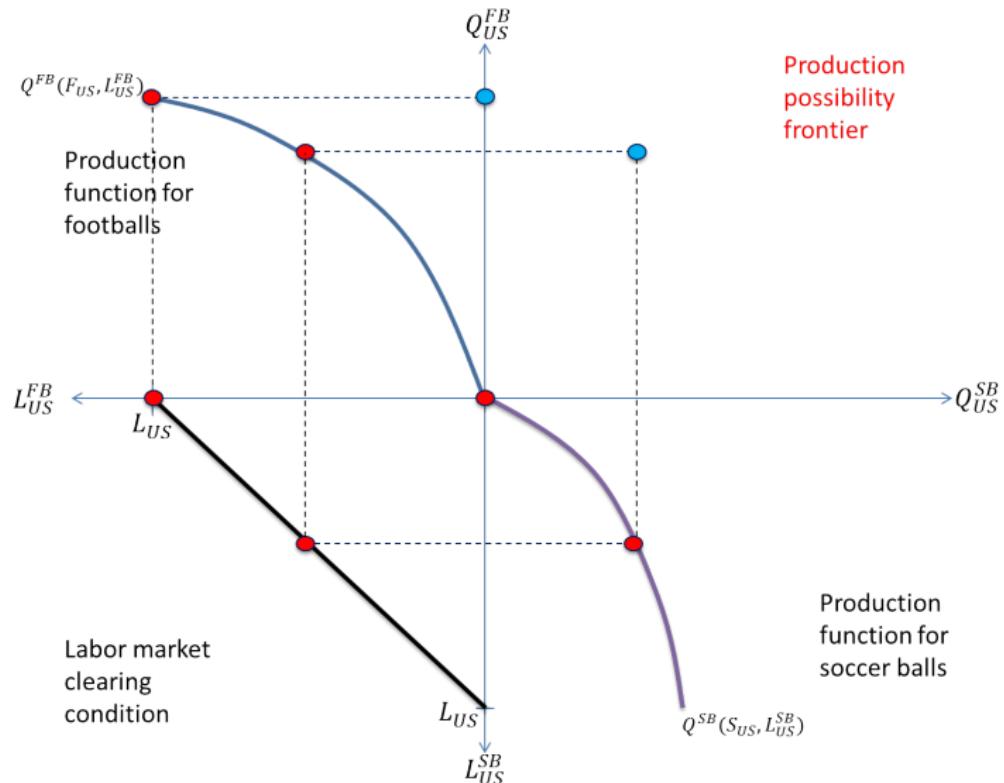


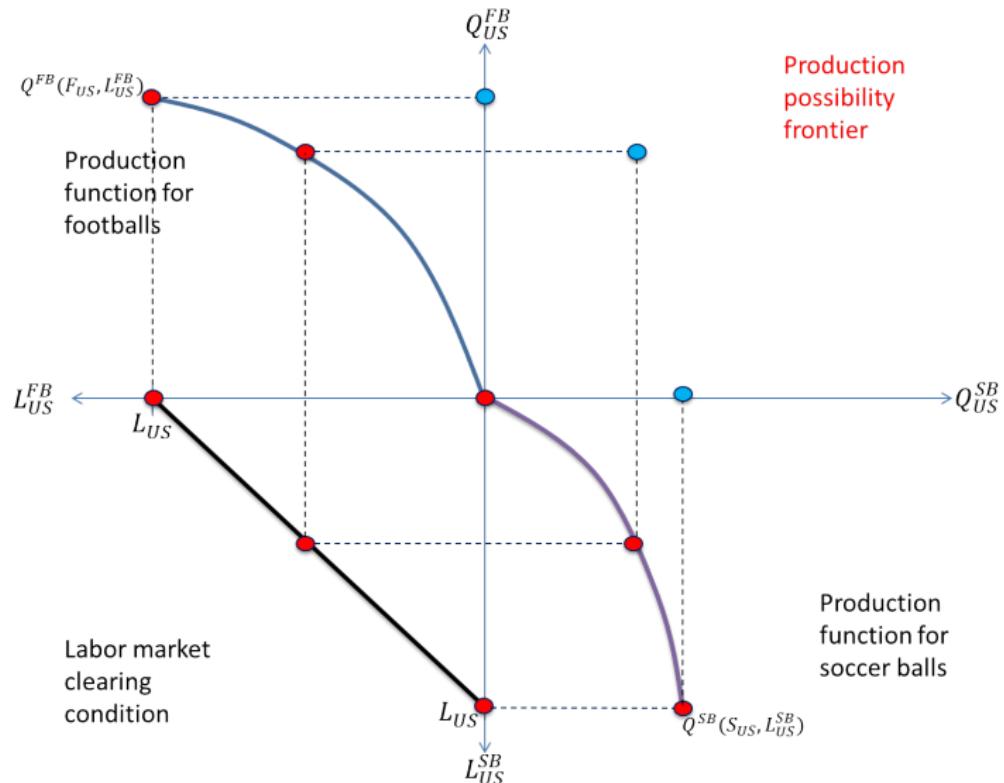


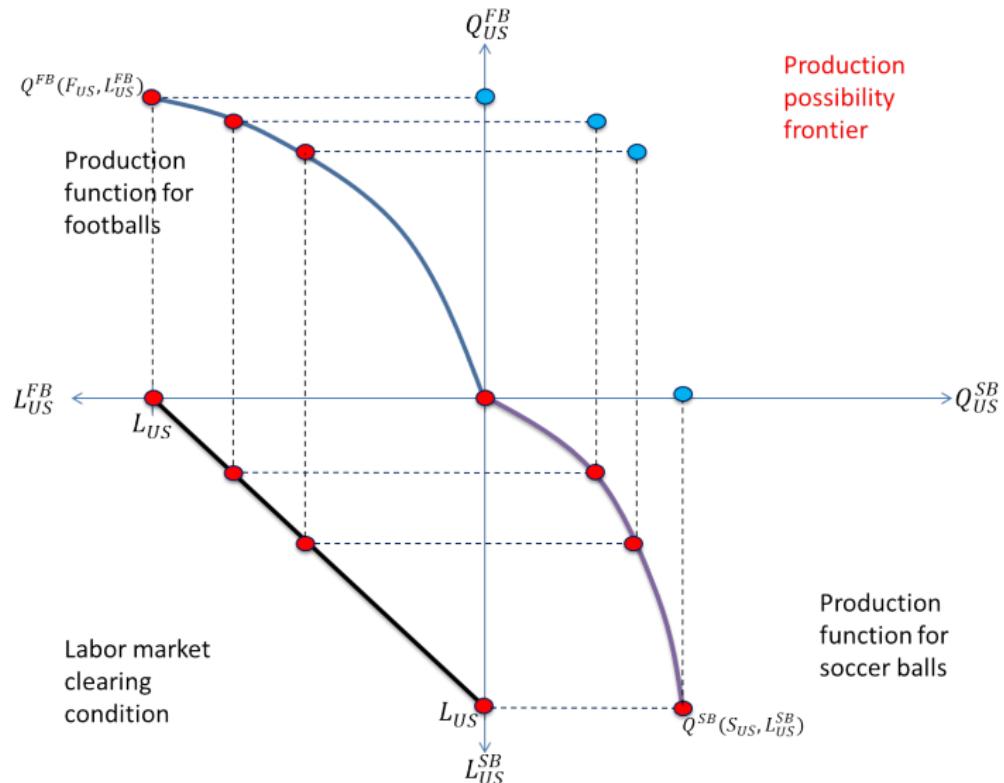


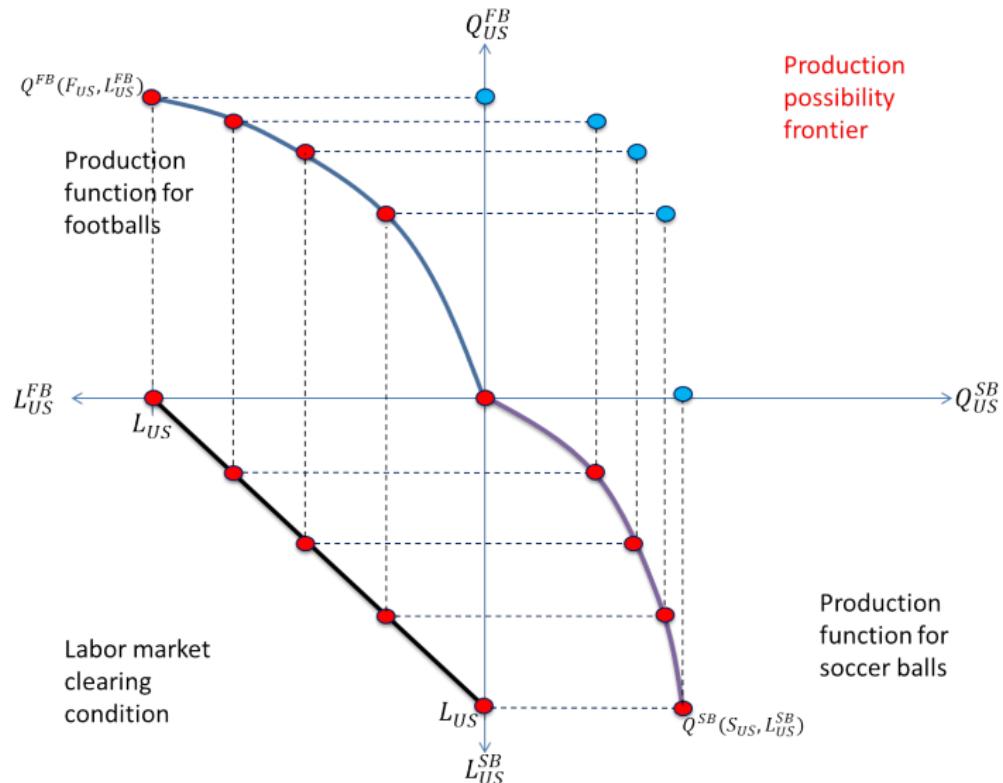


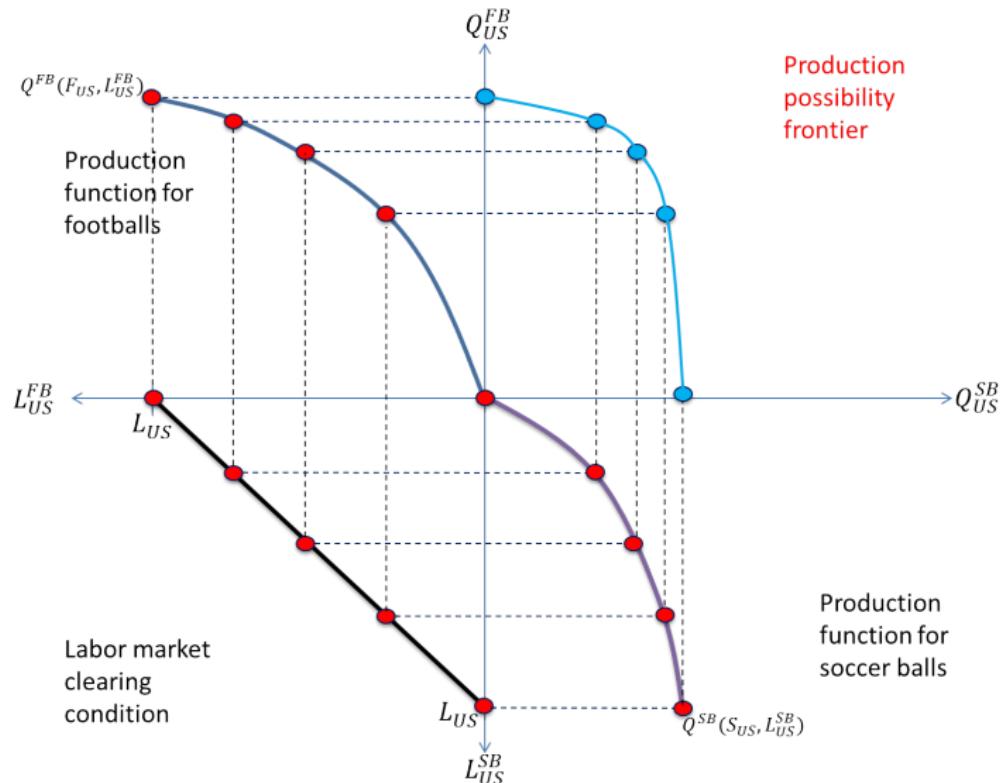


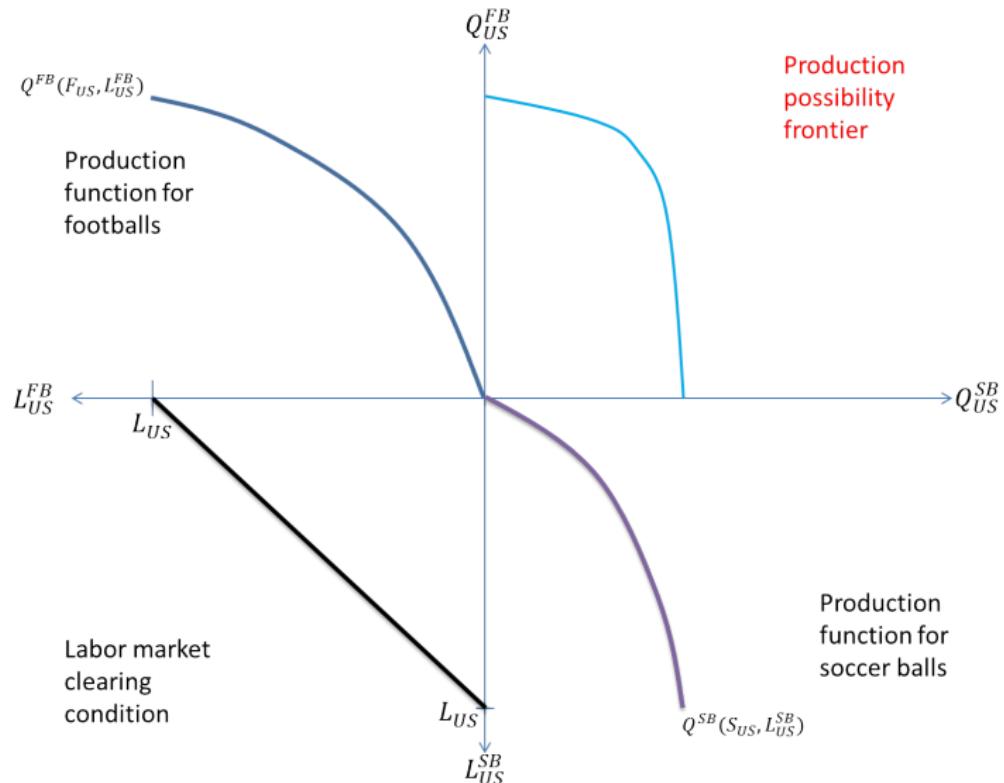




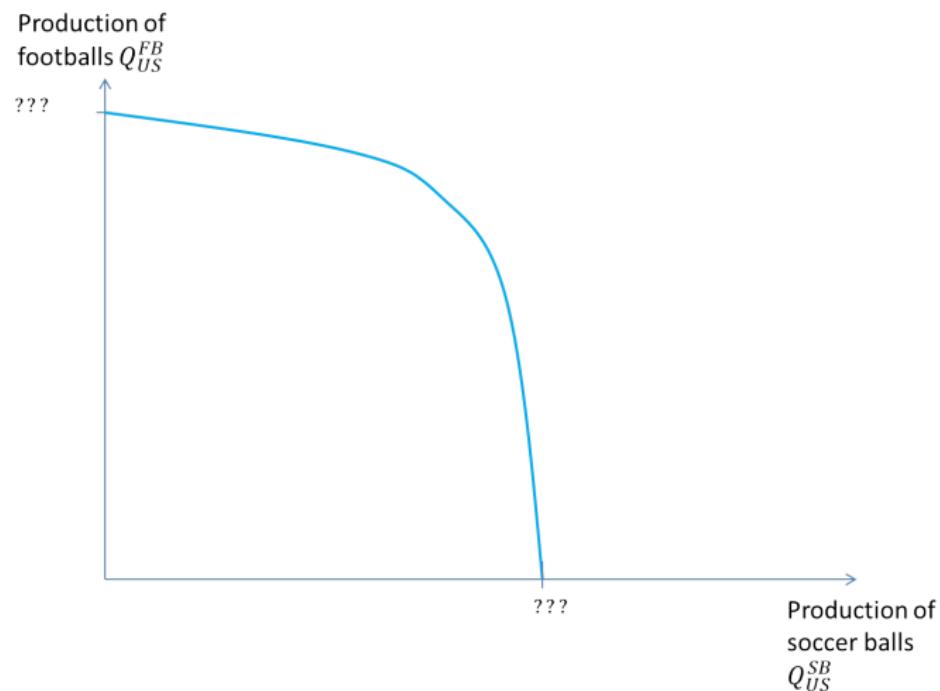




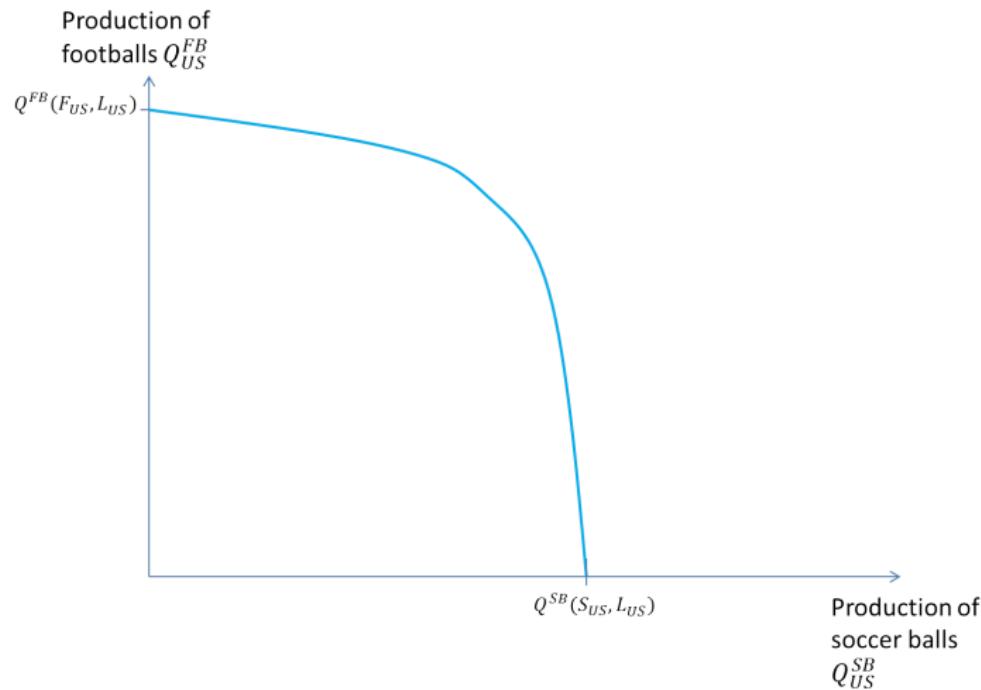




Production Possibility Frontier



Production Possibility Frontier



- ▶ [Class question: What is the slope of the PPF?]

The Slope of the PPF

- ▶ Recall from the Ricardian model that the slope of the PPF reflects the **opportunity cost** of increasing the production of one good in terms of the lost production of the other good.
- ▶ As written we have:

$$\text{Slope of PPF} = \frac{\text{Additional production of footballs}}{\text{Foregone production of soccer balls}}$$

- ▶ [Class question: what is the increased production of footballs from one additional unit of labor?]
- ▶ [Class question: what is the loss of production of soccer balls from one fewer units of labor?]

The Slope of the PPF

- ▶ Define:

$$MPL_{US}^{FB} \equiv \frac{\partial Q^{FB}(F_{US}, L_{US}^{FB})}{\partial L_{US}^{FB}}$$

$$MPL_{US}^{SB} \equiv \frac{\partial Q^{SB}(S_{US}, L_{US}^{SB})}{\partial L_{US}^{SB}}$$

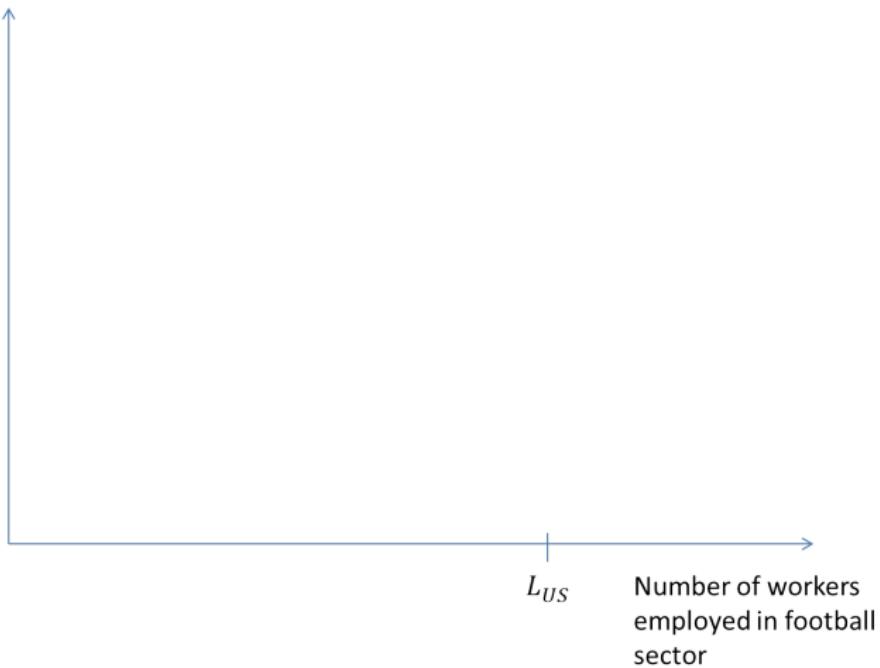
- ▶ Note: MPL_{US}^{FB} depends on L_{US}^{FB} (this is why I prefer the calculus notation).
- ▶ Then we have:

$$\text{Slope of PPF} = -\frac{MPL_{US}^{FB}}{MPL_{US}^{SB}}$$

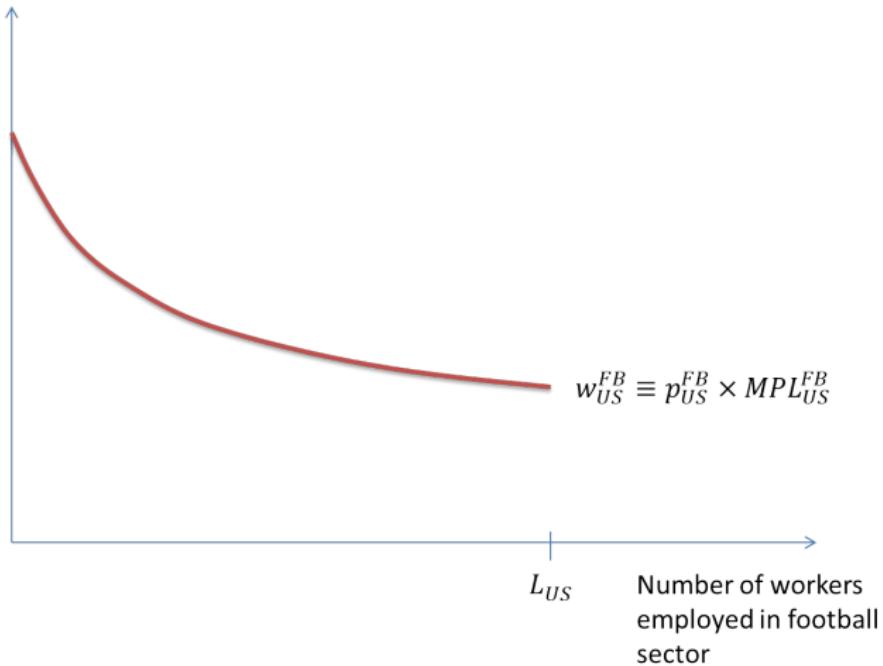
Income of Workers

- ▶ As with the Ricardian model, let us now consider the production decision of a particular U.S. worker.
- ▶ Each worker takes as given the prices of both goods *and the production decisions of all other workers*.
- ▶ [Class question: what is the income of a worker producing soccer balls?]
 - ▶ Number of soccer balls that a (marginal) worker can produce given the production decisions of all other workers: MPL_{US}^{SB}
 - ▶ Therefore the income of a worker is
 $w_{US}^{SB} \equiv p_{US}^{SB} \times MPL_{US}^{SB}$
- ▶ [Class question: what is the income of a worker producing footballs?]
 - ▶ The income of a worker is $w_{US}^{FB} \equiv p_{US}^{FB} \times MPL_{US}^{FB}$

Income earned by a
worker in each sector

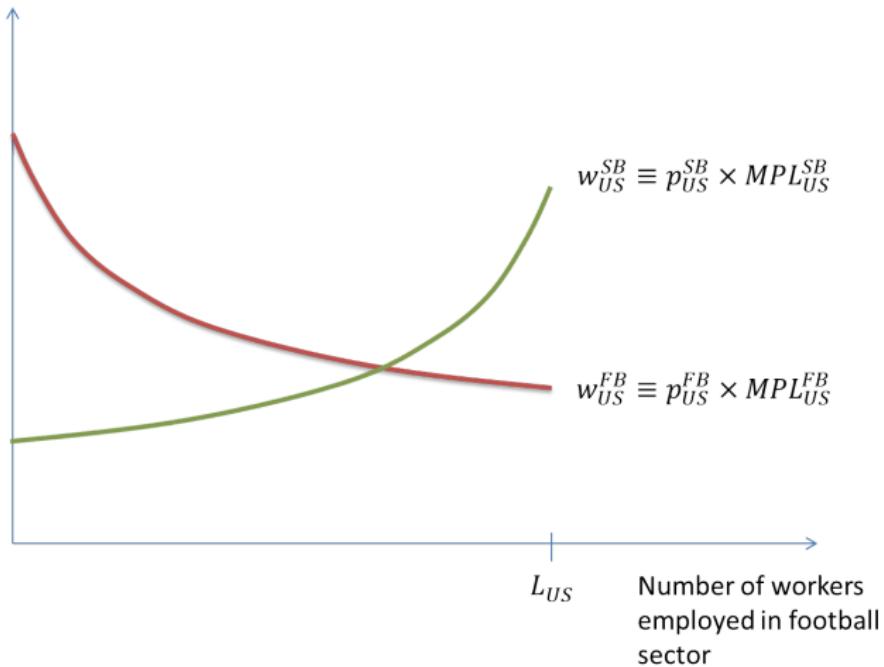


Income earned by a worker in each sector



- ▶ [Class question: why are the wages in the football sector decreasing with the number of workers employed in football?]

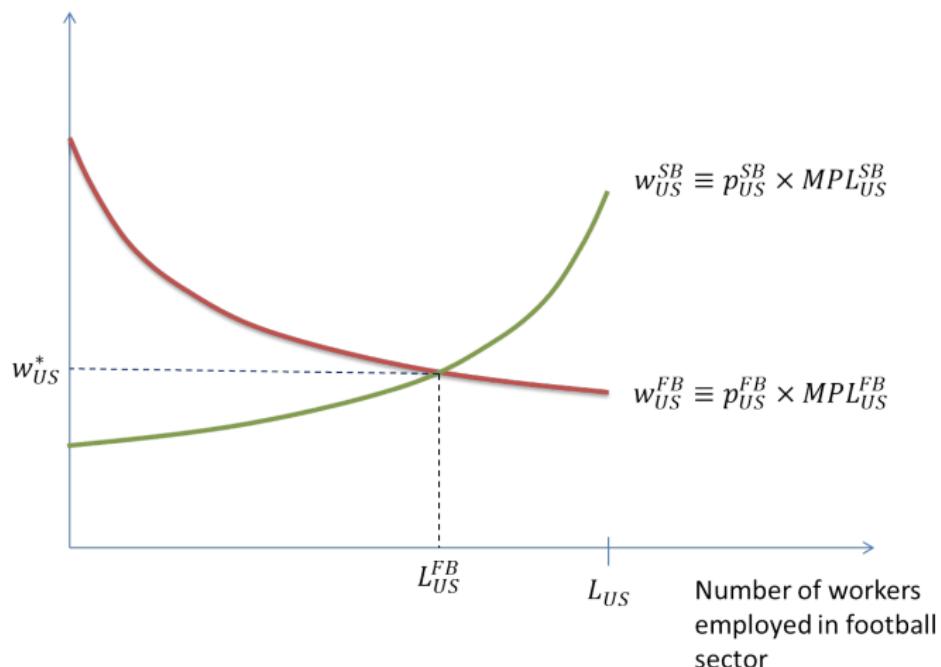
Income earned by a worker in each sector



- ▶ [Class question: why are the wages in the soccer ball sector increasing with the number of workers employed in football?]

Equilibrium wages

Income earned by a worker in each sector



- [Class question: What is the equilibrium number of people employed in the soccer ball sector (i.e., L^{SB})?]

Equilibrium wages

- In equilibrium, workers must be indifferent between producing soccer balls and footballs, so that:

$$w_{US}^{SB} = w_{US}^{FB} \equiv w_{US}^*$$

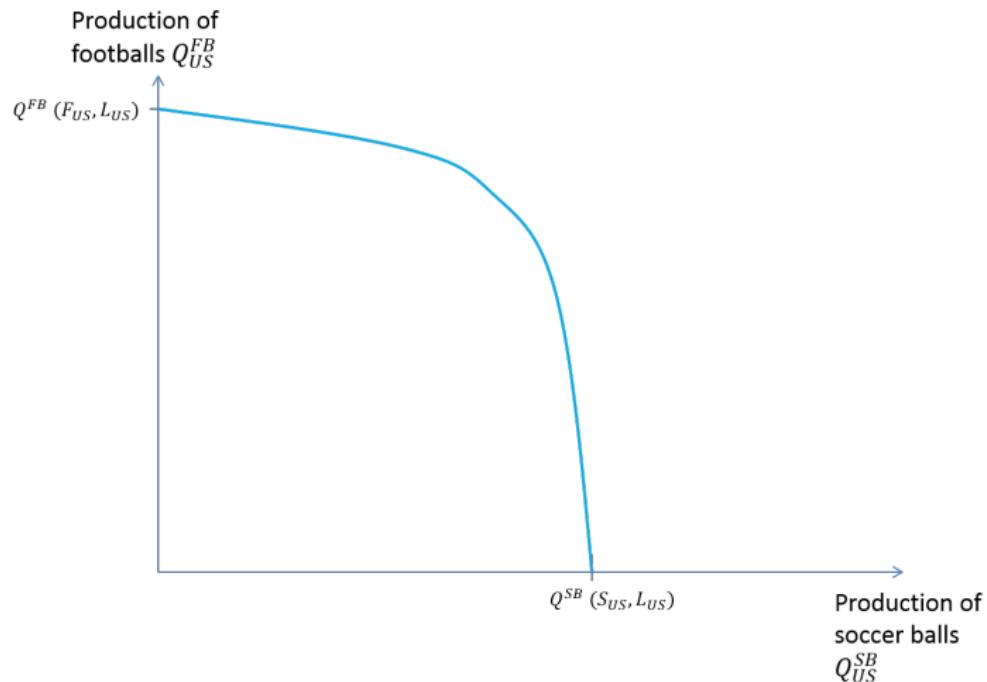
- Since $w_{US}^{SB} \equiv p_{US}^{SB} \times MPL_{US}^{SB}$ and $w_{US}^{FB} \equiv p_{US}^{FB} \times MPL_{US}^{FB}$ this means:

$$p_{US}^{SB} \times MPL_{US}^{SB} = p_{US}^{FB} \times MPL_{US}^{FB} \iff$$

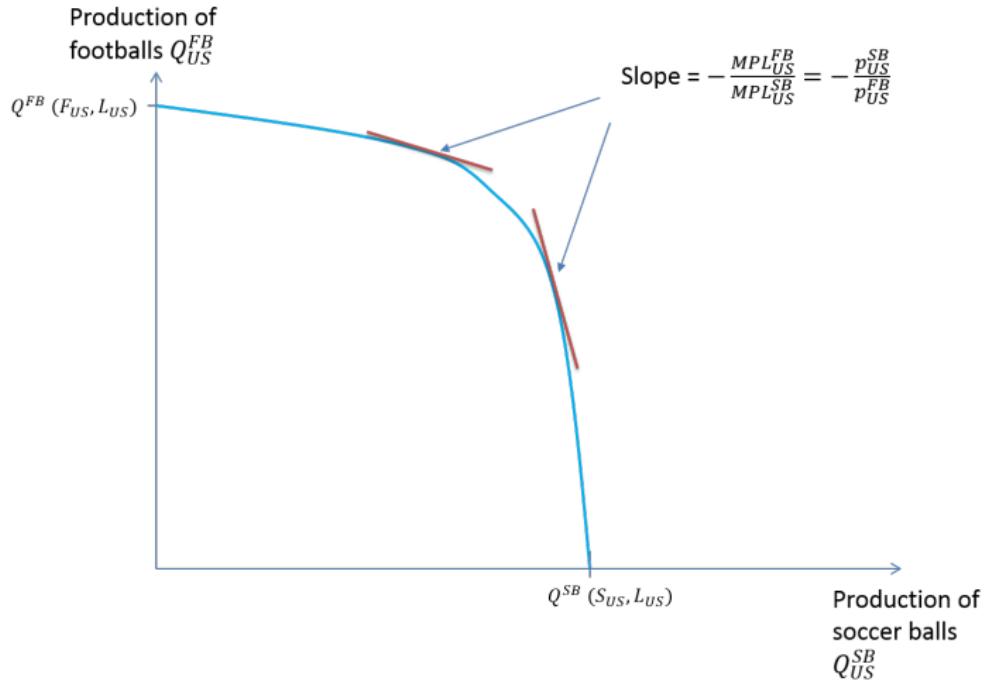
$$\frac{MPL_{US}^{SB}}{MPL_{US}^{FB}} = \frac{p_{US}^{FB}}{p_{US}^{SB}}$$

- This means that for a given relative price, labor moves between the production of footballs and soccer balls until the ratio of marginal products of labor for both goods is equal to the relative price.
- [Class question: how is this different from how the allocation of labor is affected by prices in the Ricardian model?]

Prices and the PPF



Prices and the PPF



- ▶ [Class question: As $\frac{p_{US}^{SB}}{p_{US}^{FB}}$ increases, what happens to $\frac{Q_{US}^{SB}}{Q_{US}^{FB}}$?]

Plan for the day

- ▶ Specific factors (ctd).

Midterm evaluations: Concerns

- ▶ Major concerns with class thus far:
 1. Slides posted too close to class.
 2. Too few examples of how to solve questions.
 3. Lectures too fast paced / math confusing.
 4. More real world examples.

Midterm evaluations: Proposed class changes

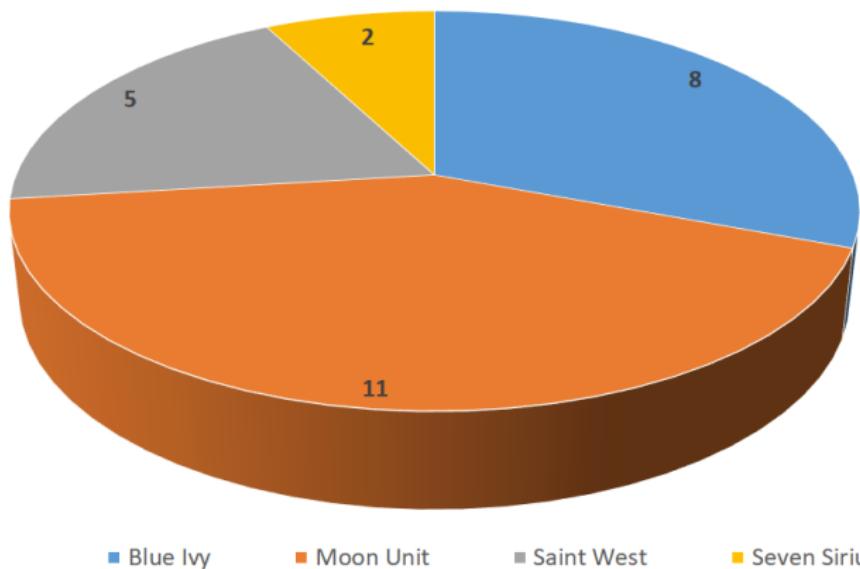
1. Slides posted too close to class:
 - ▶ All lecture slides are now available online.
2. Too few examples of how to solve questions:
 - ▶ We will spend 15 minutes at the end of each section of lecture working as a class through an example.
 - ▶ Additional practice problems (and their solutions) on class website.
3. Lectures too fast paced / math confusing:
 - ▶ We will slow down the pace of lectures (e.g. specific factors / H-O may take three weeks instead of two).
4. More real world examples:
 - ▶ Will focus last few weeks of course on applying our theories to the real world.

Today's Teams



Today's Teams

Celebrity Baby Names



Preferences

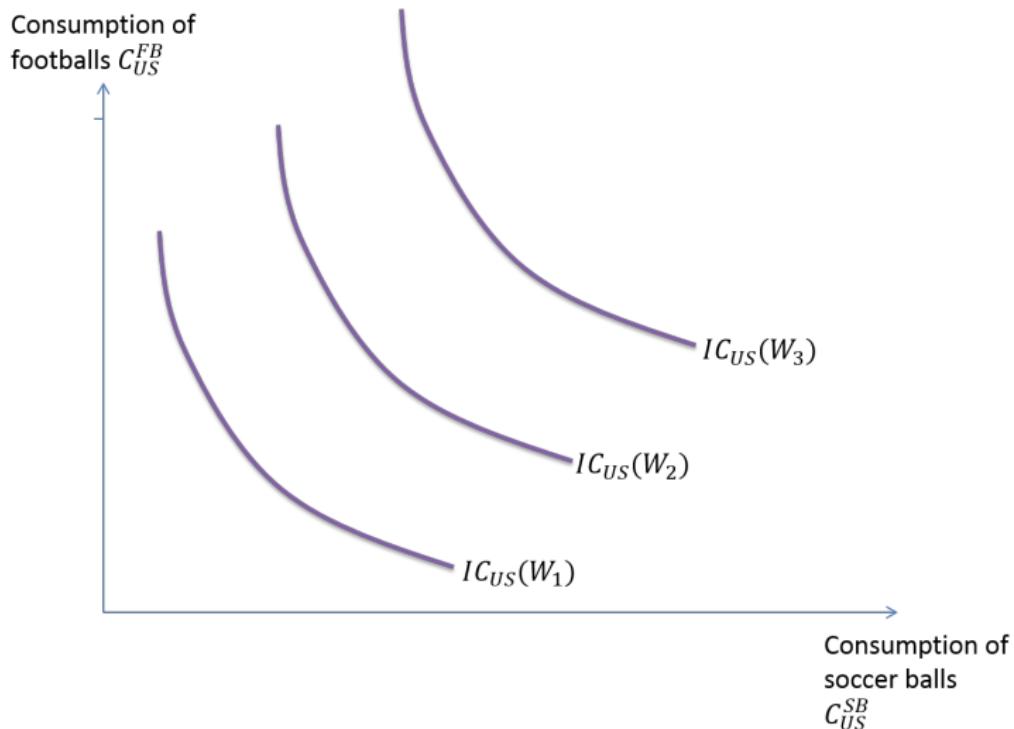
- ▶ As in the Ricardian model, assume that there is a representative agent in each country with preferences:

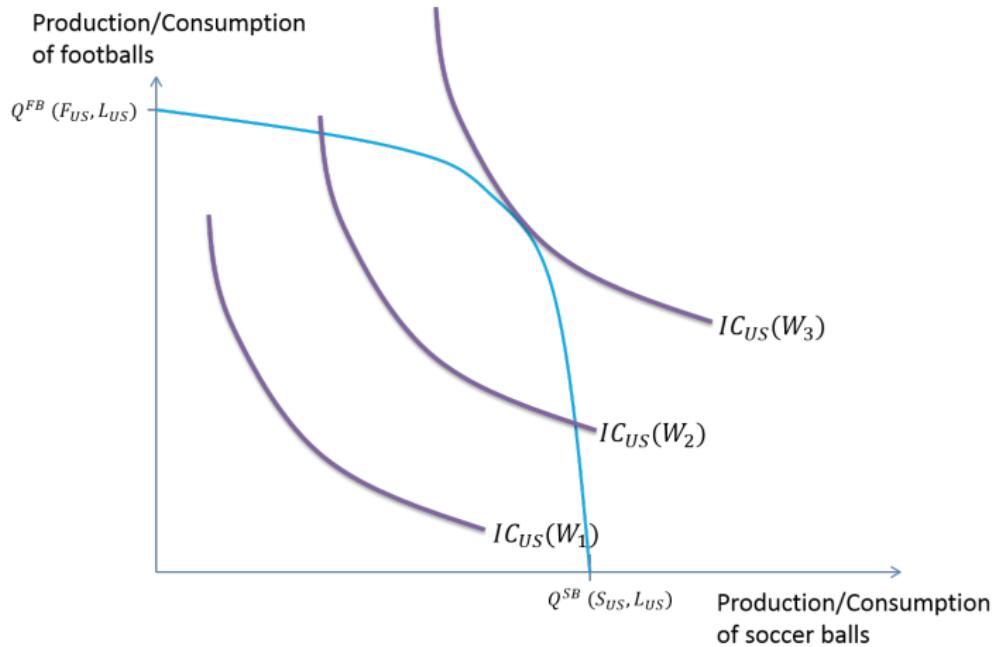
$$W = U(C_{US}^{FB}, C_{US}^{SB}),$$

where $\frac{\partial U}{\partial C_{US}^{FB}} > 0$ and $\frac{\partial U}{\partial C_{US}^{SB}} > 0$

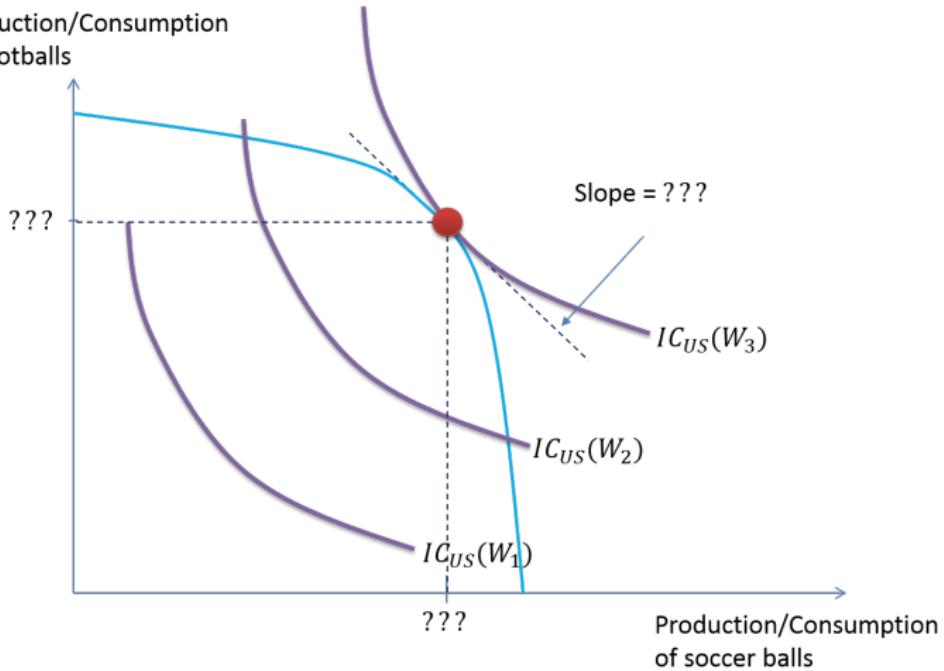
- ▶ This allows us to overlay indifference curves on the production possibility frontier figure.
- ▶ Recall:

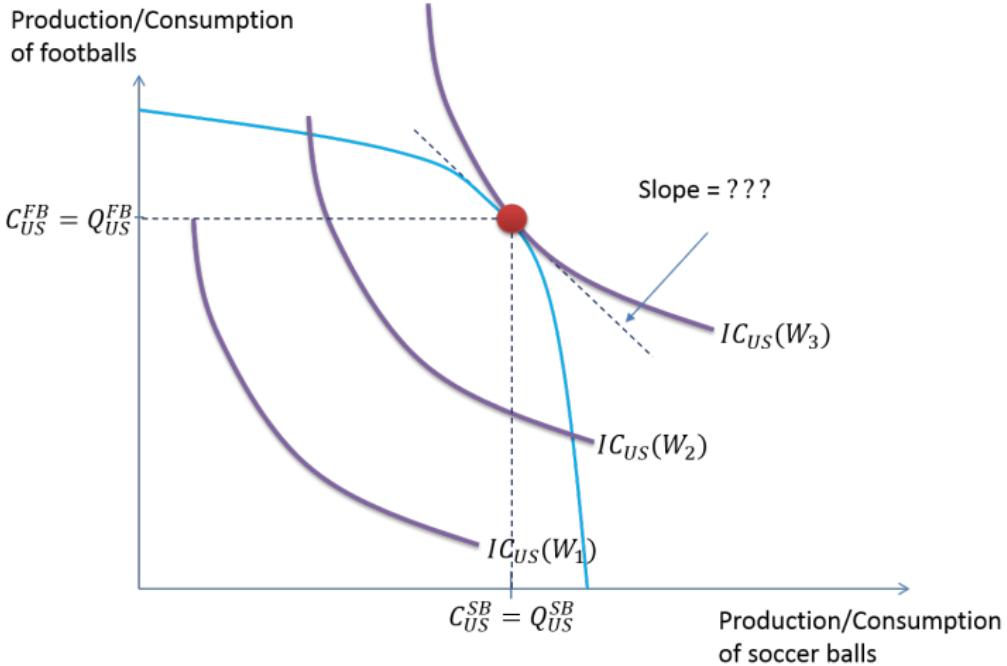
$$IC_{US}(W) \equiv \{C_{US}^{FB}, C_{US}^{SB} \mid U(C_{US}^{FB}, C_{US}^{SB}) = W\}$$

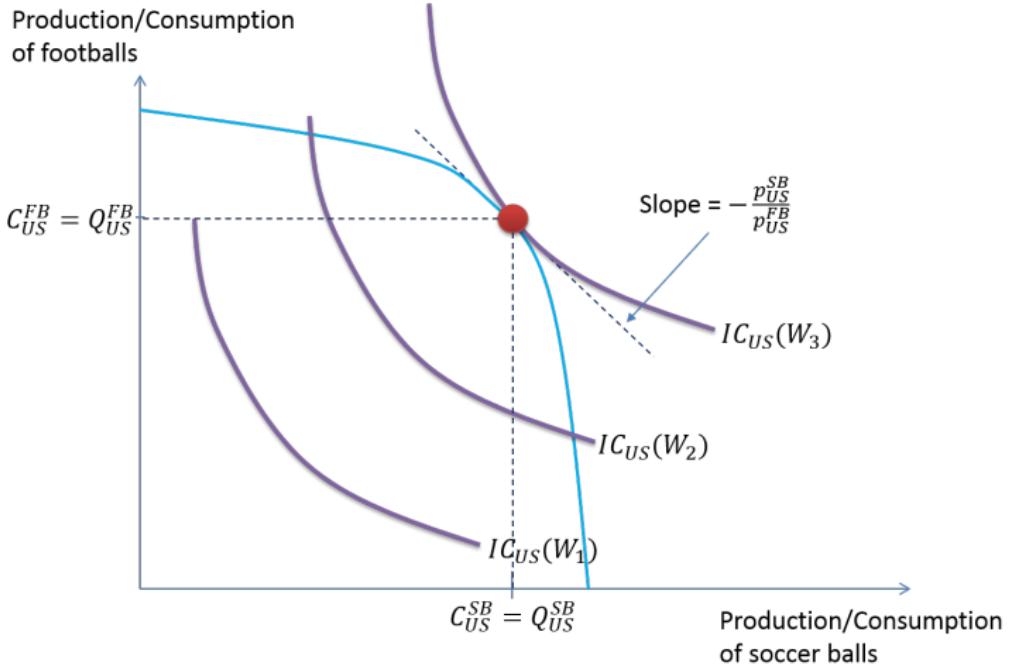




Production/Consumption
of footballs







Equilibrium

- ▶ Now we can (finally!) define an equilibrium
- ▶ [Class question: what are the exogenous model parameters for the U.S.?]
 - ▶ Production functions $Q^{FB}(\cdot, \cdot)$, $Q^{SB}(\cdot, \cdot)$
 - ▶ Preferences $U(\cdot)$
 - ▶ Factor endowments L_{US} , F_{US} , and S_{US}
- ▶ [Class question: what are the endogenous model outcomes for the U.S.?]
 - ▶ Quantity produced Q_{US}^{FB} and Q_{US}^{SB}
 - ▶ Quantity consumed C_{US}^{FB} and C_{US}^{SB}
 - ▶ Relative prices $\frac{p_{US}^{SB}}{p_{US}^{FB}}$

Equilibrium

- ▶ “For any production functions $Q^{FB}(\cdot, \cdot)$, $Q^{SB}(\cdot, \cdot)$, preferences $U(\cdot)$, and factor endowments L_{US} , F_{US} , and S_{US} , equilibrium is the quantity produced Q_{US}^{FB} and Q_{US}^{SB} , quantity consumed C_{US}^{FB} and C_{US}^{SB} , and relative prices $\frac{p_{US}^{SB}}{p_{US}^{FB}}$ such that....”
 1. Given prices and the production choice of all other workers, each worker maximizes her income.
 2. Given prices and total country income, the representative consumer chooses what to consume to maximize her utility.
 3. For each good, the total quantity consumed is equal to the total quantity produced.

Returns to each factor of production

- ▶ How much income does each factor of production get?
- ▶ Labor
 - ▶ From above, the income of a worker producing soccer balls is $p_{US}^{SB} \times MPL_{US}^{SB}$ and the income of a worker producing footballs is $p_{US}^{FB} \times MPL_{US}^{FB}$
 - ▶ From equilibrium condition #1, we have:

$$w_{US} = p_{US}^{SB} \times MPL_{US}^{SB} = p_{US}^{FB} \times MPL_{US}^{FB}$$

Returns to each factor of production

- ▶ Specific factor S_{US}
 - ▶ The total amount spent on soccer balls is $p_{US}^{SB} \times C_{US}^{SB}$.
 - ▶ The total amount going to workers producing soccer balls is $w_{US} \times L_{US}^{SB}$.
 - ▶ The remainder $p_{US}^{SB} \times C_{US}^{SB} - w_{US} \times L_{US}^{SB}$ goes to the specific factor S_{US} .
 - ▶ The amount earned per unit of specific factor S_{US} is:

$$\frac{1}{S_{US}} (p_{US}^{SB} \times C_{US}^{SB} - w_{US} \times L_{US}^{SB})$$

- ▶ Similarly, the amount earned per unit of specific factor F_{US} is:

$$\frac{1}{F_{US}} (p_{US}^{FB} \times C_{US}^{FB} - w_{US} \times L_{US}^{FB})$$

Easier way of seeing the returns to the specific factor

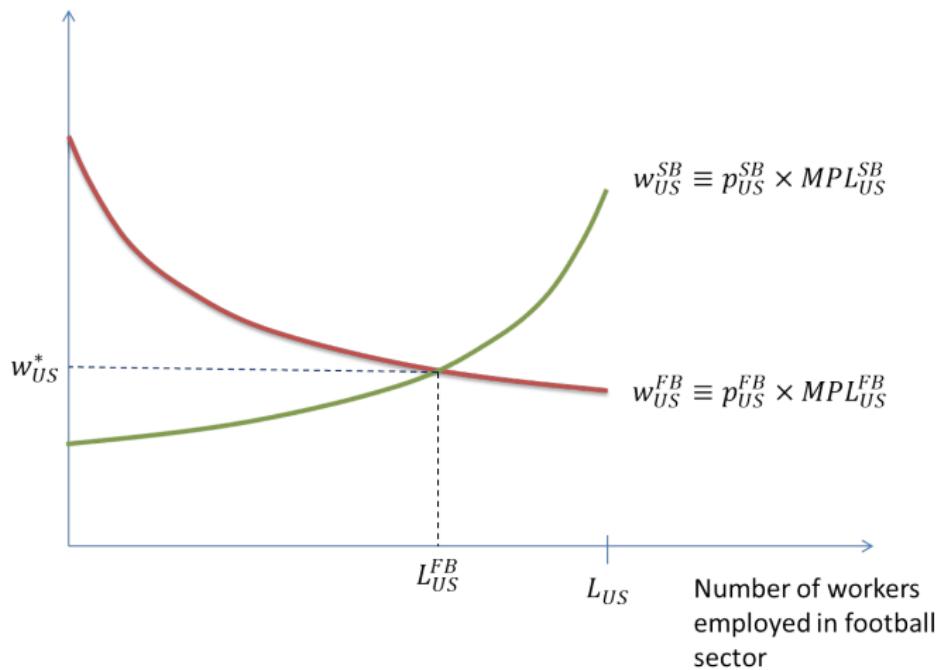
- ▶ In equilibrium, $C_{US}^{FB} = Q_{US}^{FB}$, so that total income earned from the production of footballs is $p_{US}^{FB} \times Q_{US}^{FB}$.
- ▶ Recall from production function that $Q_{US}^{FB} = Q^{FB}(F_{US}, L_{US}^{FB})$.
- ▶ Use the fundamental theorem of calculus:

$$\begin{aligned} p_{US}^{FB} \times Q_{US}^{FB} &= p_{US}^{FB} \times \int_0^{L_{US}^{FB}} \frac{\partial Q^{FB}(F_{US}, l)}{\partial L_{US}^{SFB}} dl \iff \\ &= \int_0^{L_{US}^{FB}} p_{US}^{FB} \times MPL_{US}^{FB}(l) dl \end{aligned}$$

- ▶ This leads to the following simple picture.

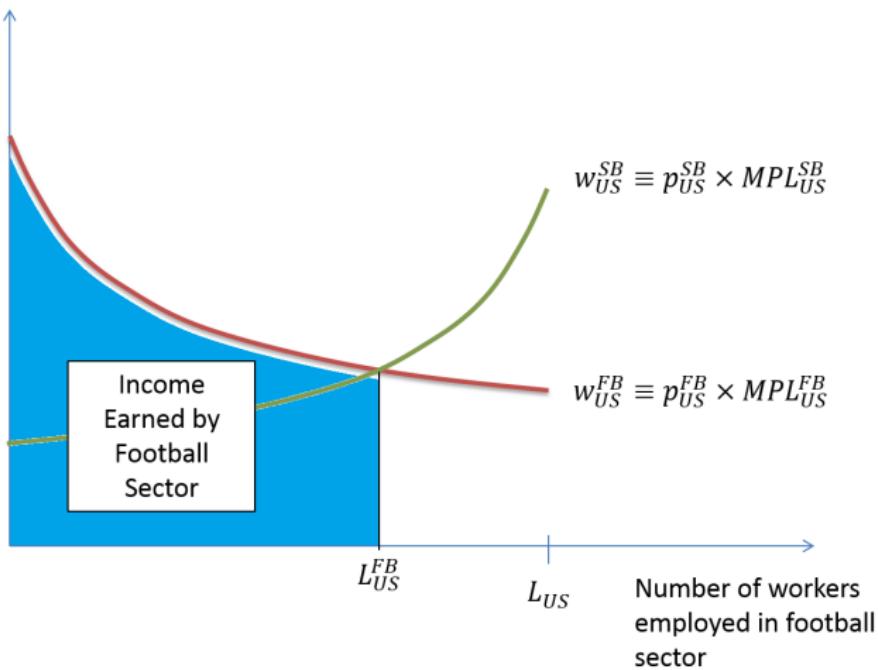
Returns to Specific Factors

Income earned by a worker in each sector



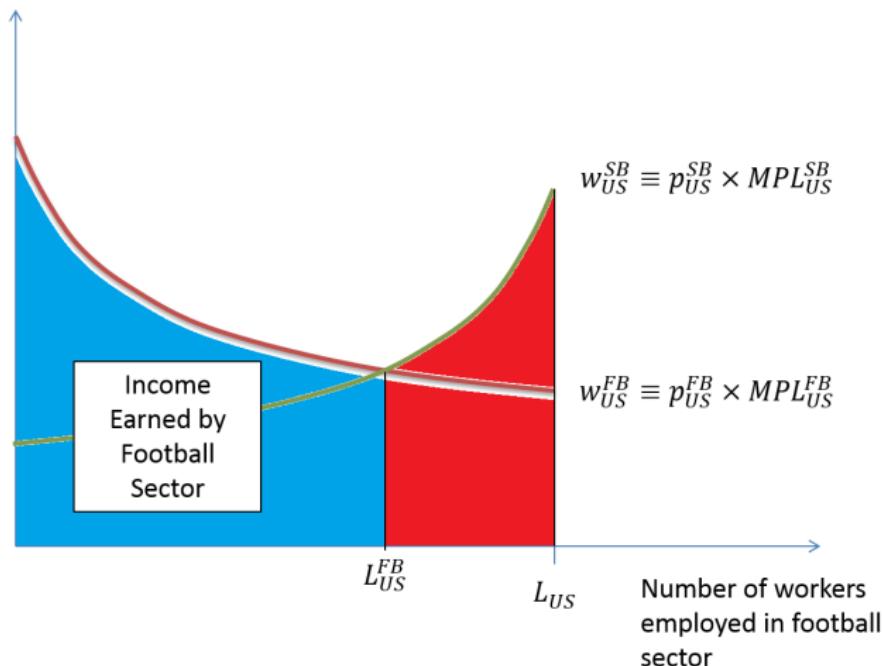
Returns to Specific Factors

Income earned by a worker in each sector



Returns to Specific Factors

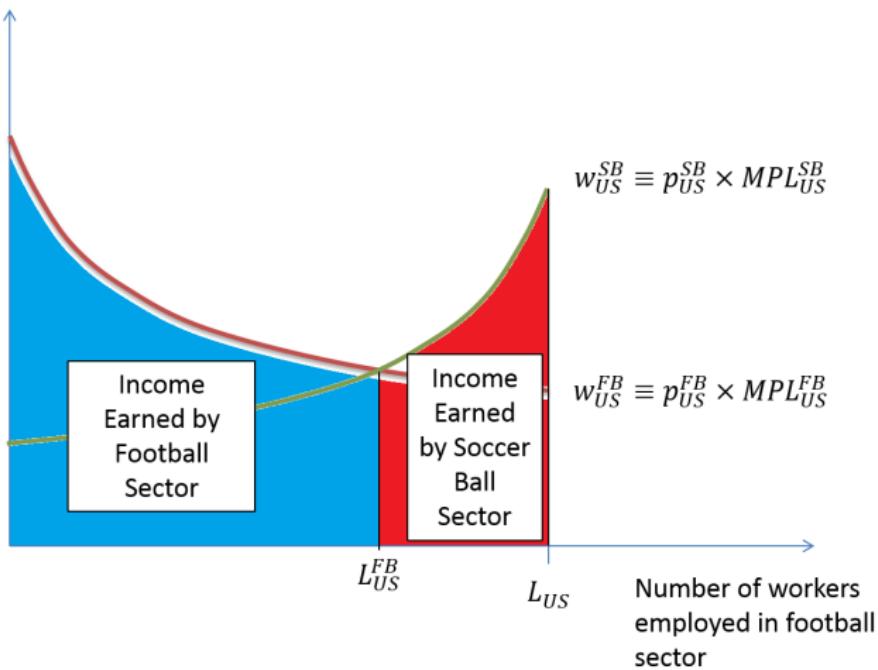
Income earned by a worker in each sector



- ▶ [Class question: what does the red area represent?]

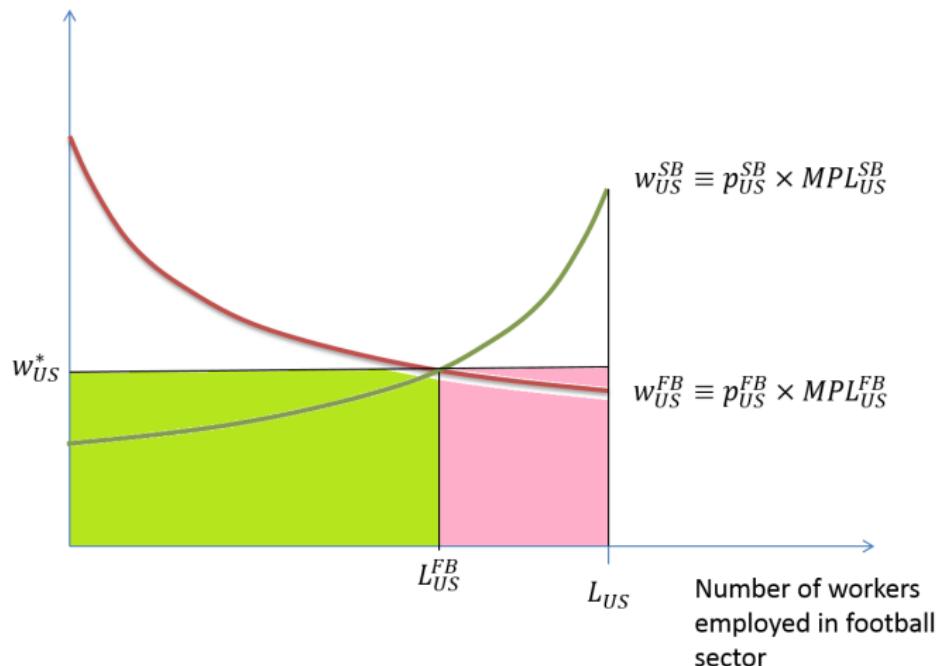
Returns to Specific Factors

Income earned by a worker in each sector



Returns to Specific Factors

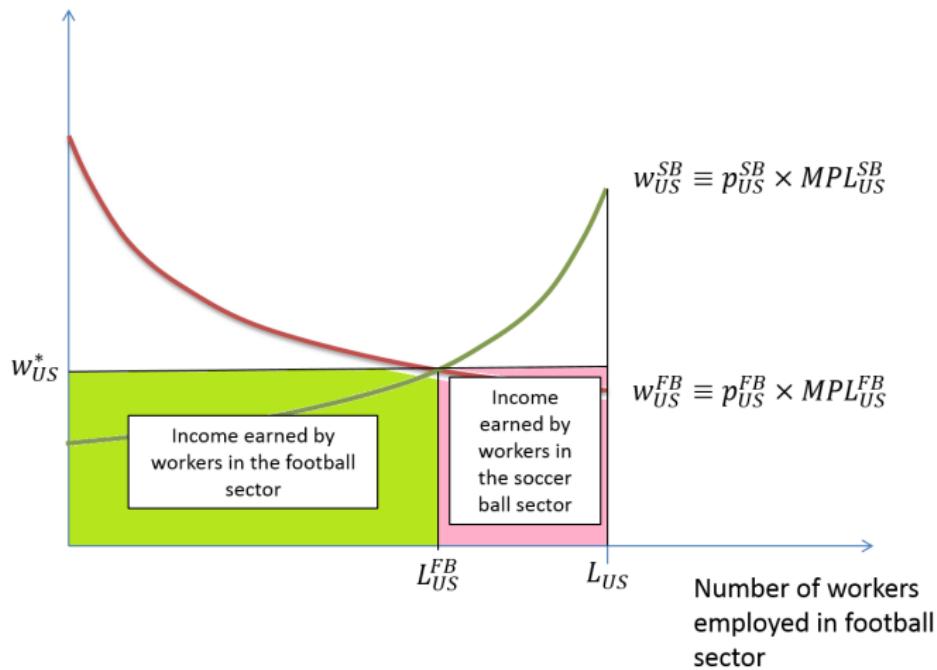
Income earned by a worker in each sector



- ▶ [Class question: what are the pink & green areas?]

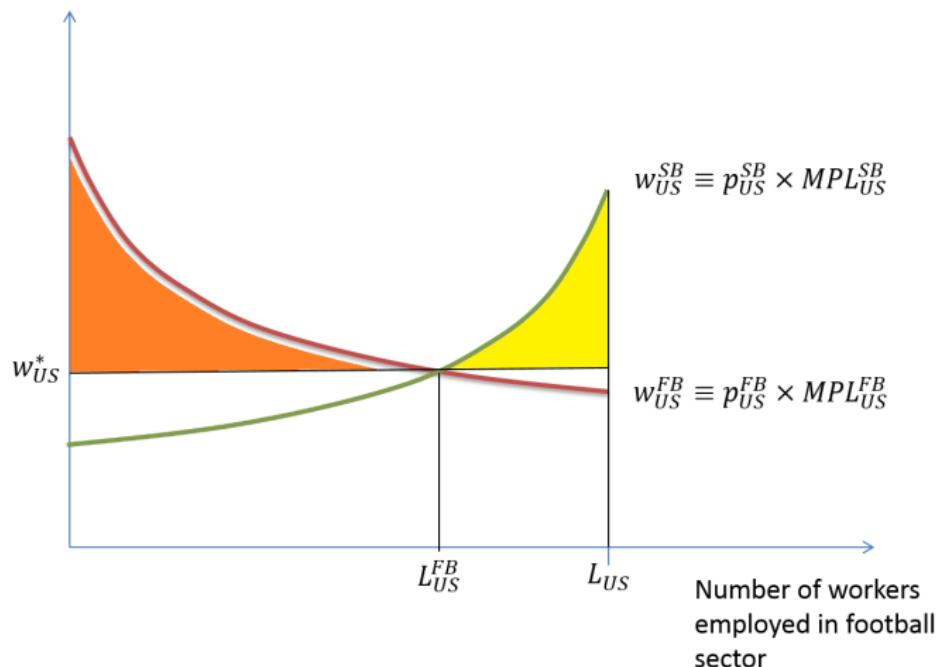
Returns to Specific Factors

Income earned by a worker in each sector



Returns to Specific Factors

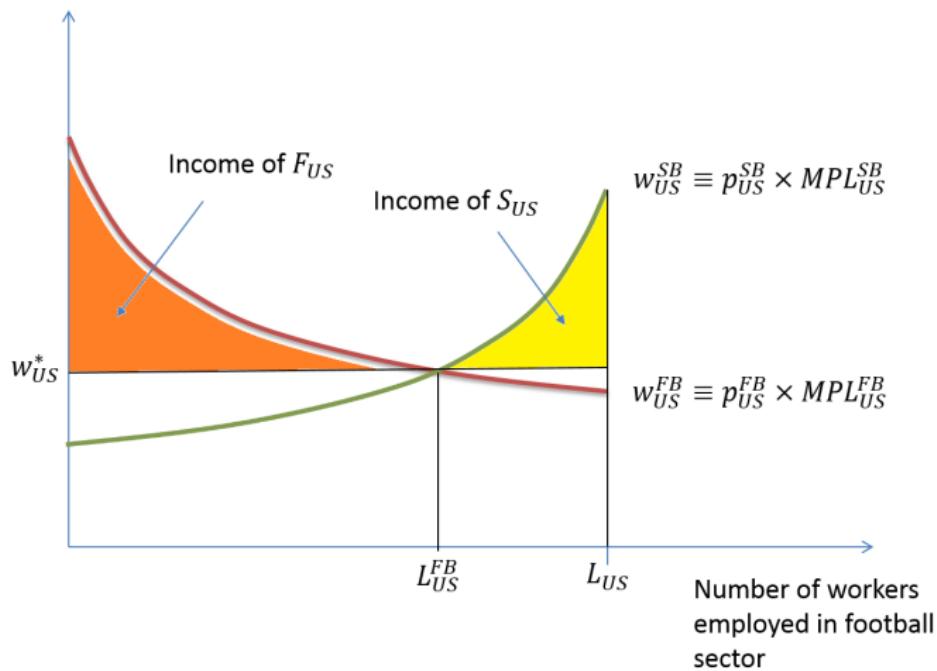
Income earned by a worker in each sector



- ▶ [Class question: what are the orange & yellow areas?]

Returns to Specific Factors

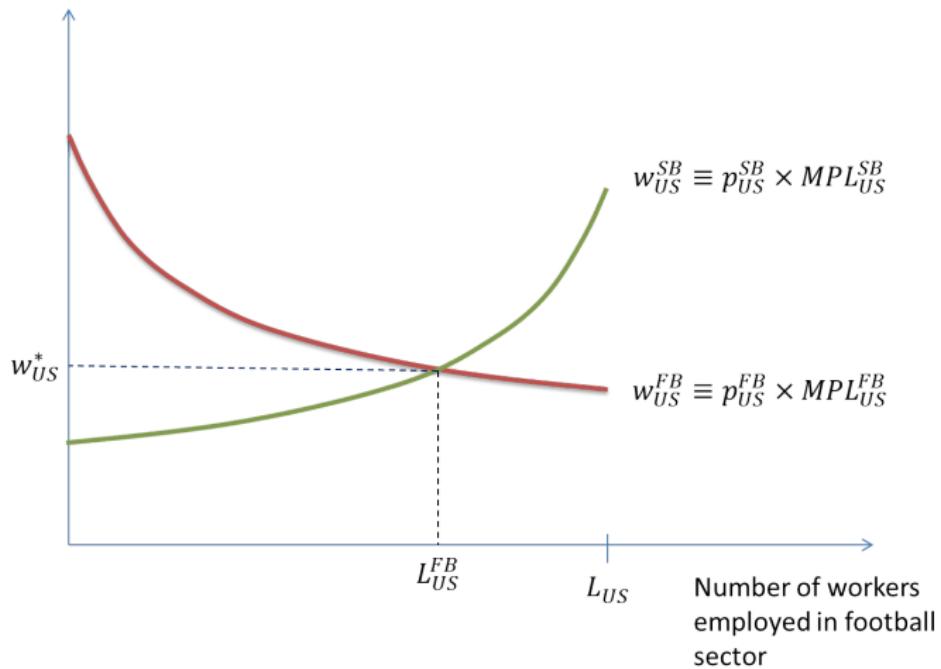
Income earned by a worker in each sector



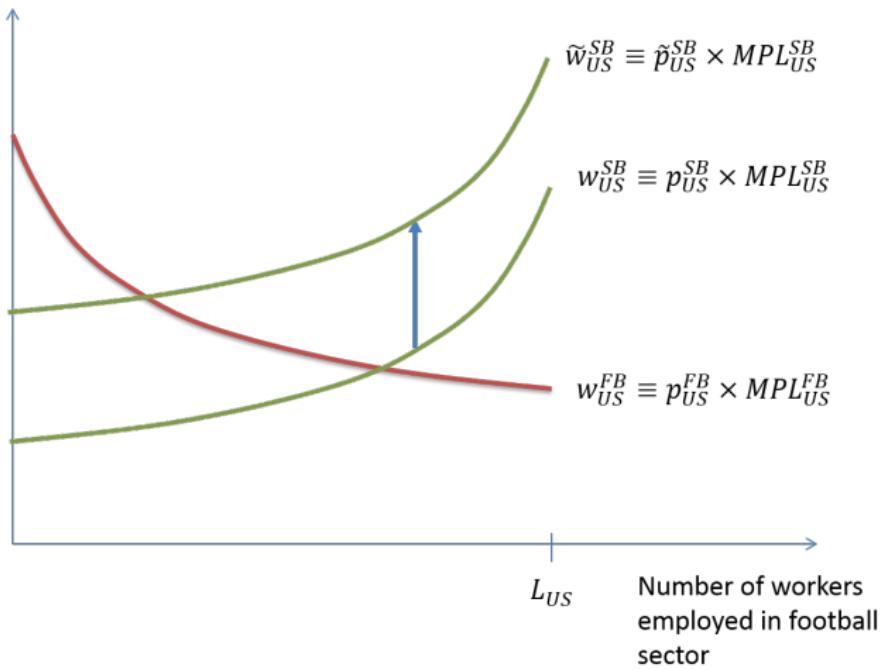
Effect of a Relative Price Change

- ▶ Suppose that the relative price of soccer balls $\frac{p_{US}^{SB}}{p_{US}^{FB}}$ increases.
- ▶ How does this affect the returns to each factor of production?
- ▶ Consider labor first.

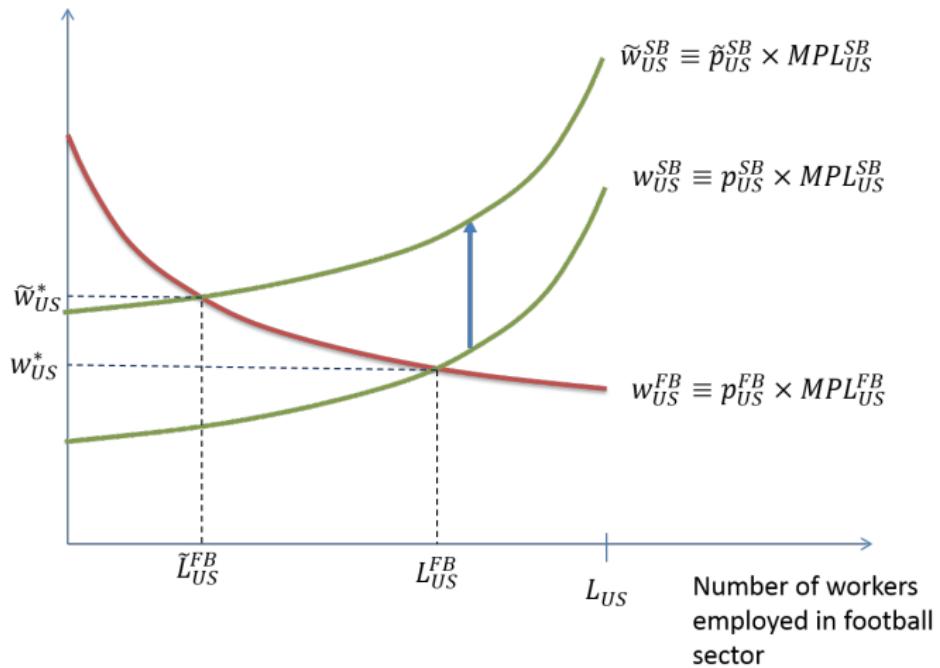
Income earned by a
worker in each sector



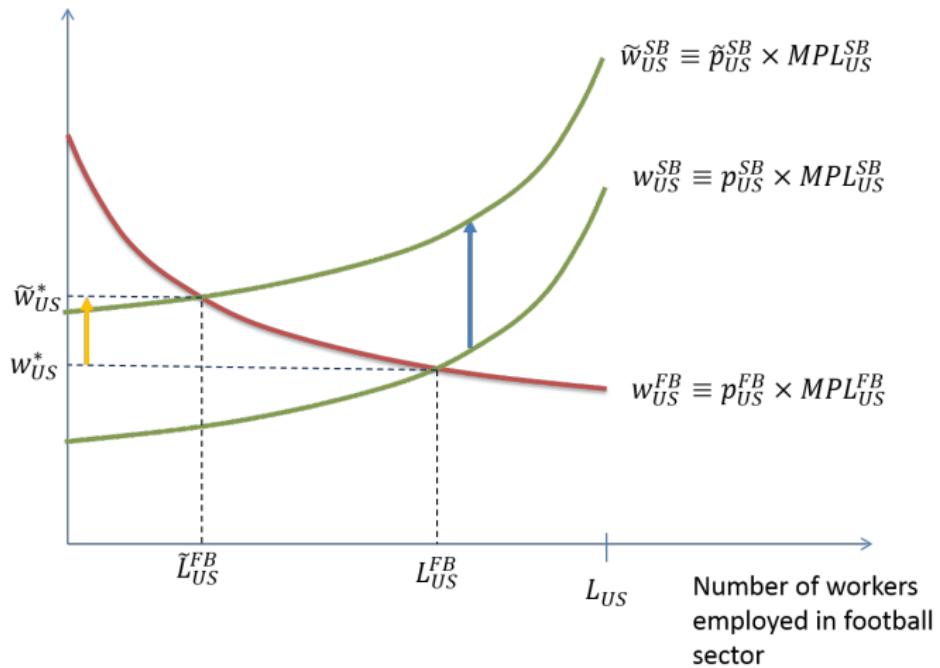
Income earned by a
worker in each sector



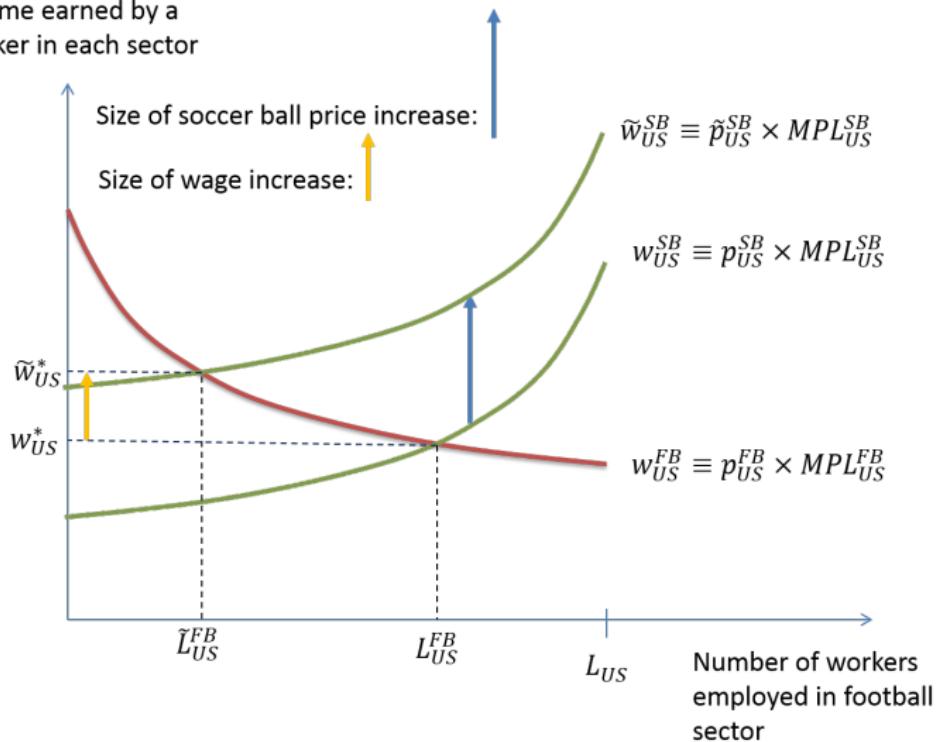
Income earned by a
worker in each sector



Income earned by a
worker in each sector



Income earned by a worker in each sector



- ▶ [Class question: Why does the wage not increase as much as the price?]

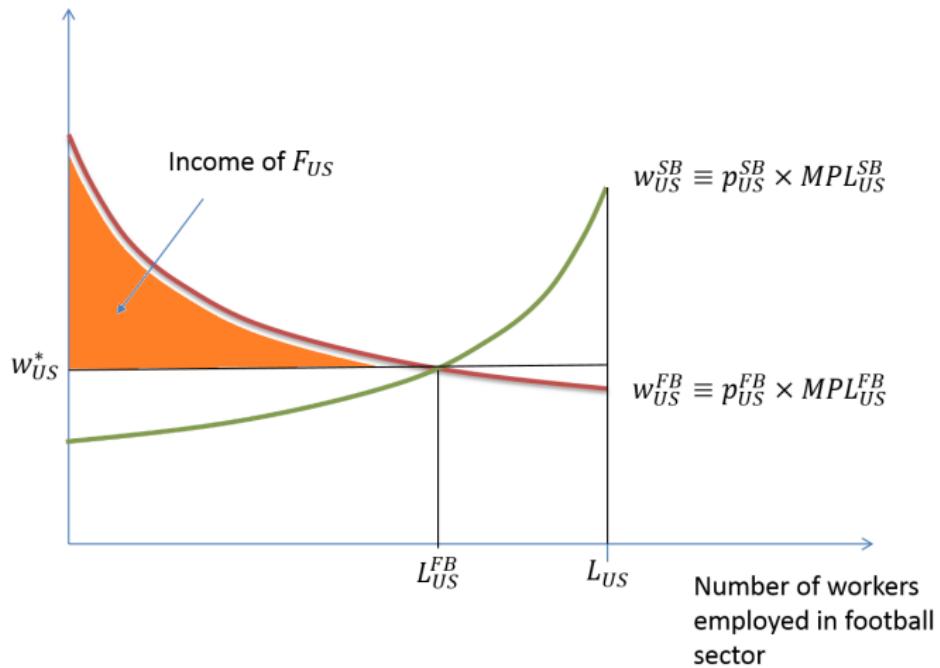
Effect of a Relative Price Change on Labor

- ▶ Conclusion: an increase in the price of soccer balls causes wages to increase, but by a smaller amount.
- ▶ How does this affect the purchasing power of workers?
 - ▶ [Class question: can workers now purchase more or less footballs?]
 - ▶ [Class question: can workers now purchase more or less soccer balls?]
 - ▶ [Class question: are workers better off or worse off?]
- ▶ Hence, the effect of a change in prices on workers welfare is ambiguous!

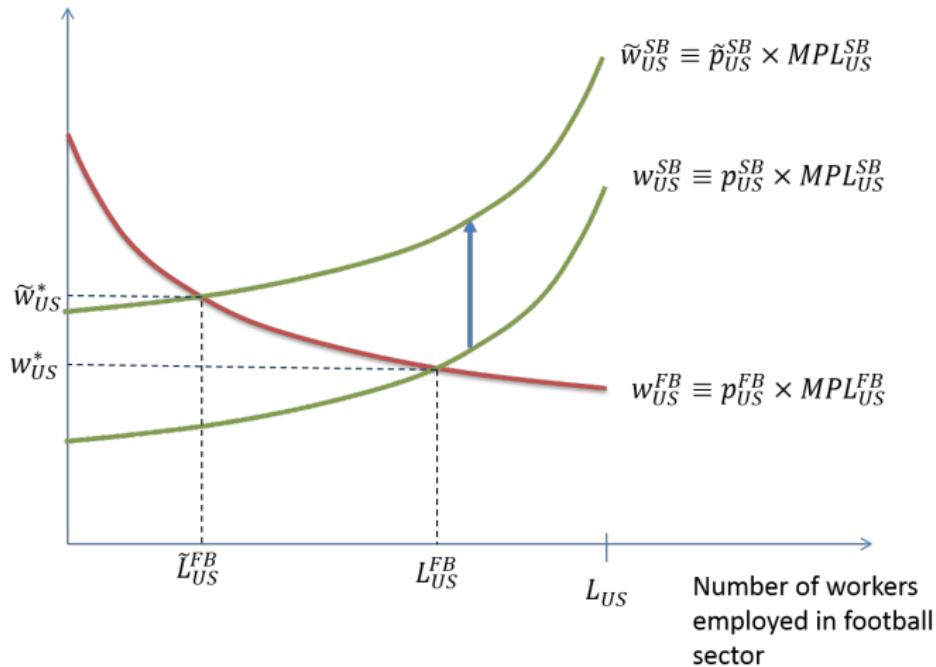
Effect of a Relative Price Change on Specific Factors

- ▶ Suppose that the relative price of soccer balls $\frac{P_{US}^{SB}}{P_{US}^{FB}}$ increases.
- ▶ How does this affect the returns to each factor of production?
- ▶ Consider ~~labor first~~.
- ▶ Consider now the returns to the specific factor F_{US} .

Income earned by a
worker in each sector

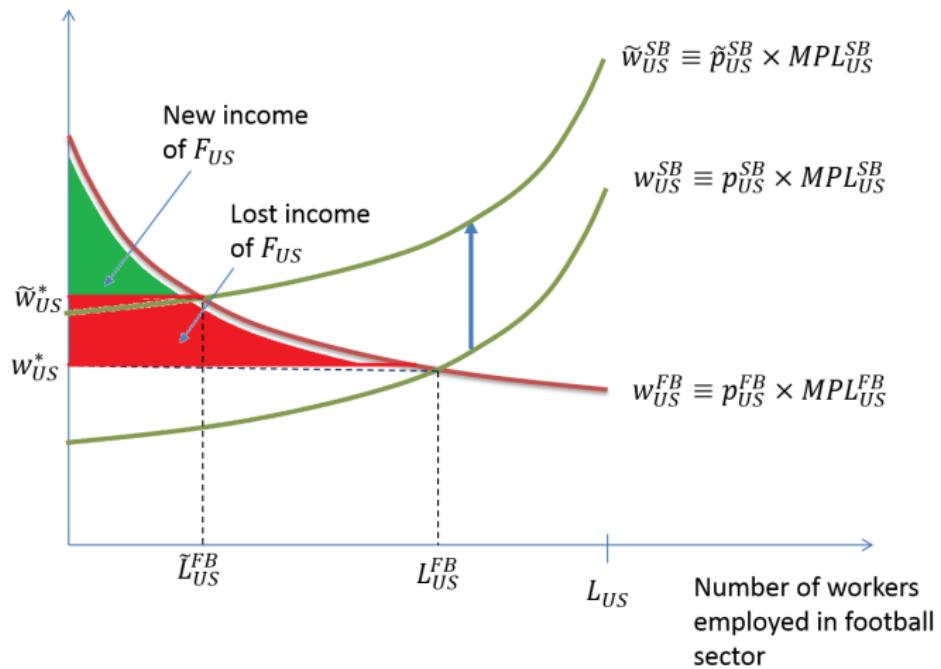


Income earned by a worker in each sector



- ▶ [Class question: What is the new income of F_{US} ?]

Income earned by a worker in each sector



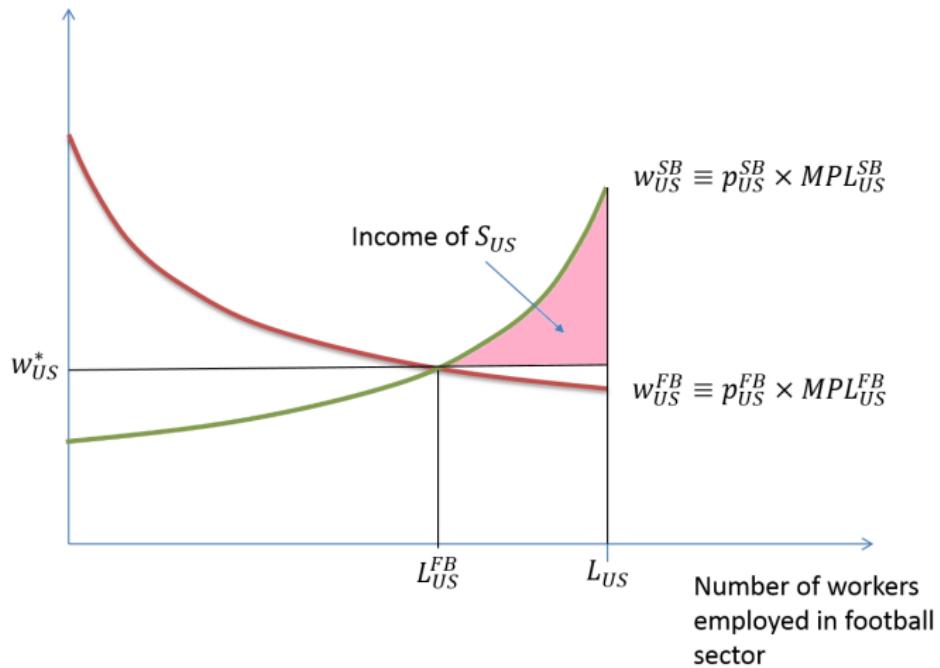
Effect of a Relative Price Change on F_{US}

- ▶ Conclusion: an increase in the price of soccer balls causes the returns to the football specific factor to go down.
- ▶ How does this affect the purchasing power of the owner of F_{US} ?
 - ▶ [Class question: can the owners of F_{US} now purchase more or less footballs?]
 - ▶ [Class question: can the owners of F_{US} now purchase more or less soccer balls?]
 - ▶ [Class question: are the owners of F_{US} better off or worse off?]
- ▶ Hence, the effect of an increase in the price of soccer balls is unambiguously bad for the owner of the football specific factor!

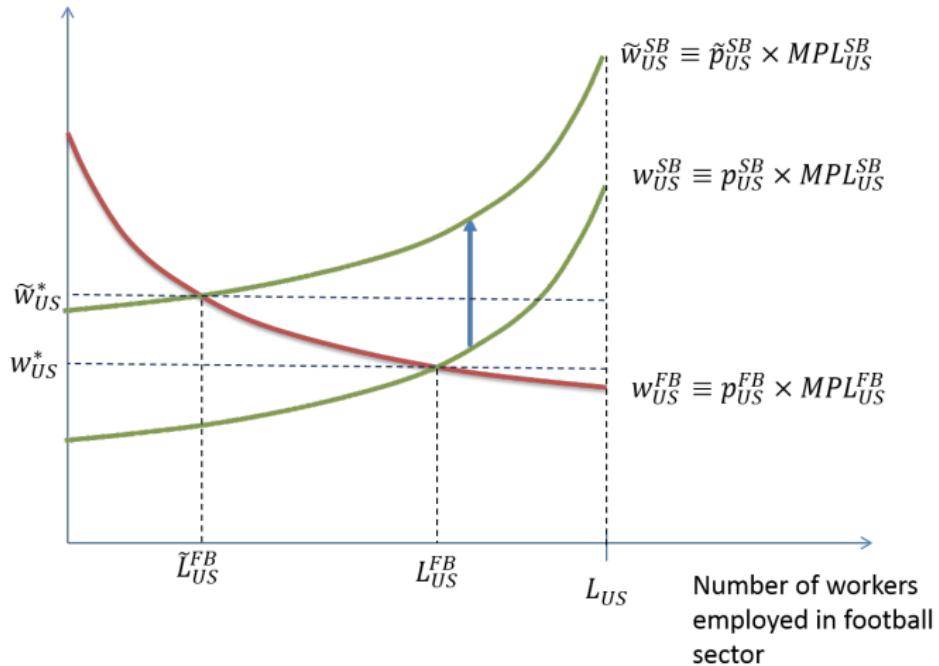
Effect of a Relative Price Change on Specific Factors

- ▶ Suppose that the relative price of soccer balls $\frac{p_{US}^{SB}}{p_{US}^{FB}}$ increases.
- ▶ How does this affect the returns to each factor of production?
- ▶ Consider labor first.
- ▶ Consider now the returns to the specific factor F_{US} .
- ▶ Consider now the returns to the specific factor S_{US} .

Income earned by a
worker in each sector

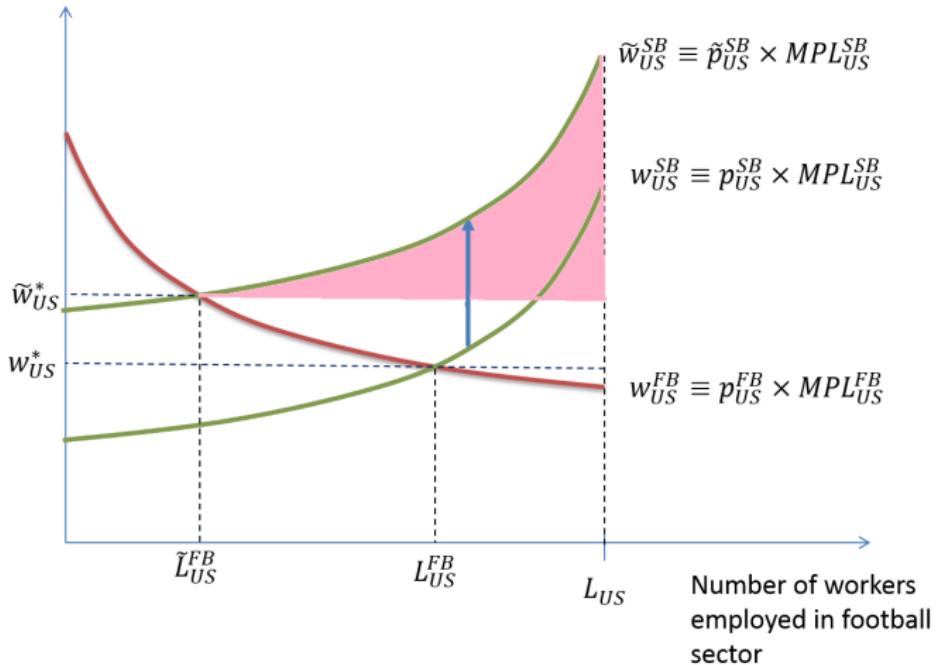


Income earned by a worker in each sector



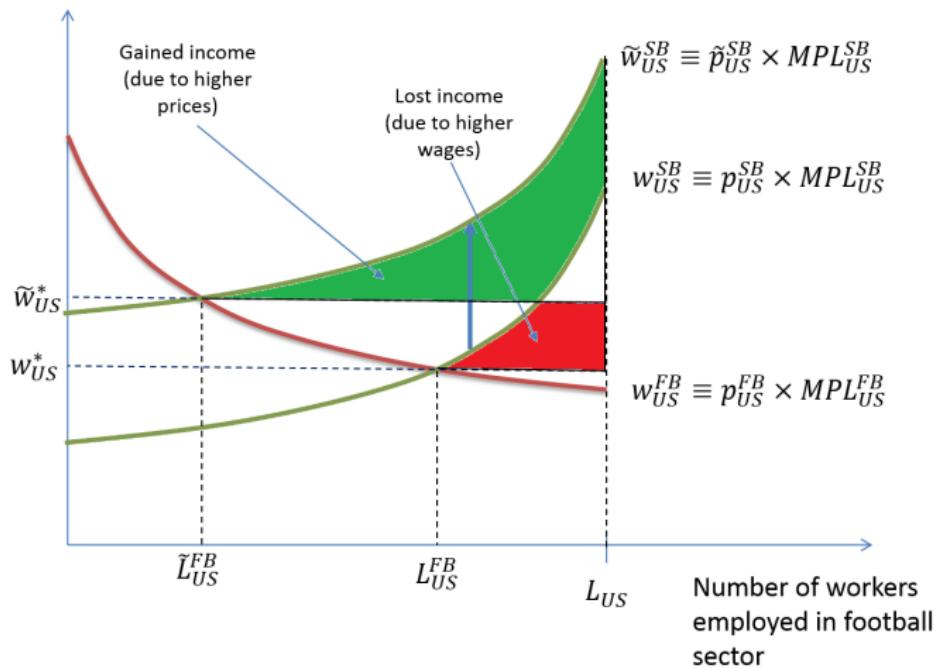
- ▶ [Class question: What is the new income of S_{US} ?]

Income earned by a worker in each sector



- ▶ [Class question: Is the new income of S_{US} greater or less than the old income?]

Income earned by a worker in each sector



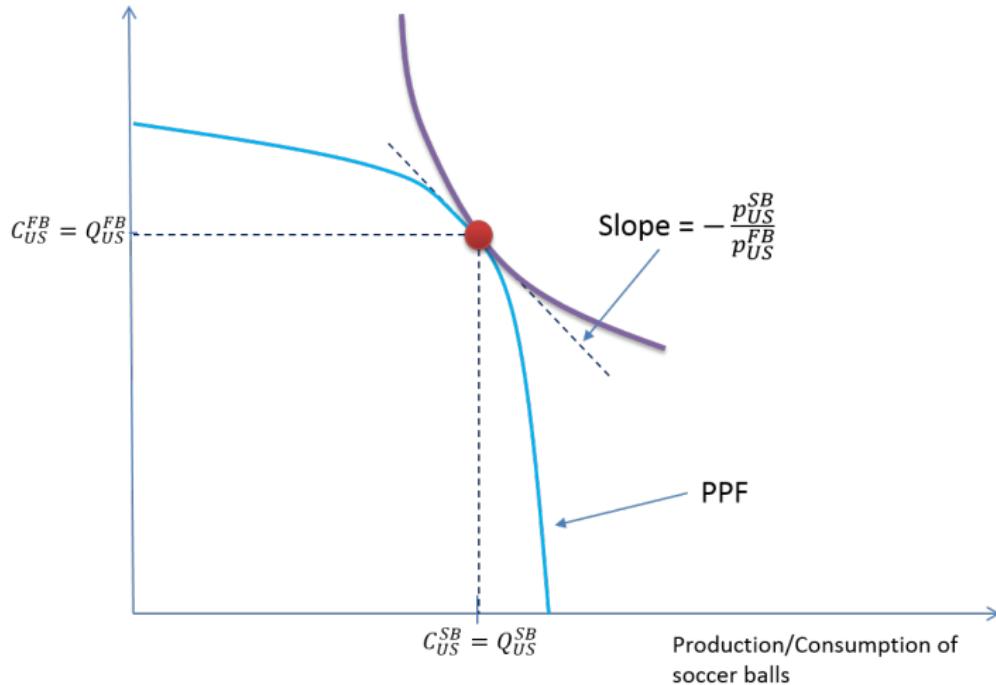
Effect of a Relative Price Change on S_{US}

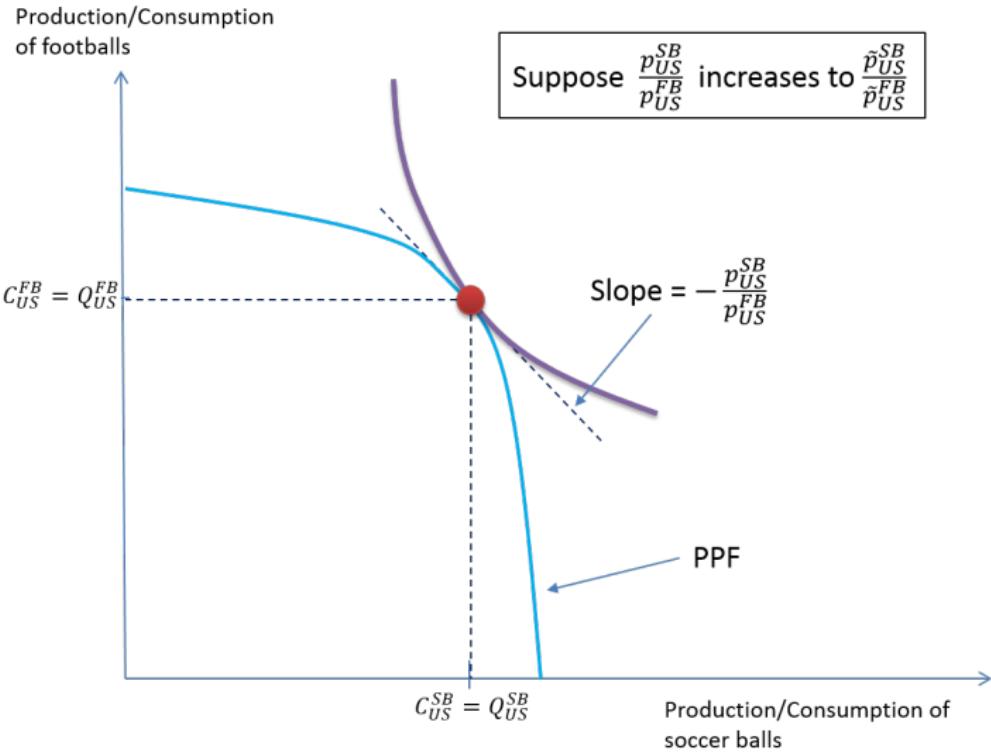
- ▶ Conclusion: an increase in the price of soccer balls causes the returns to the soccer ball specific factor to go up.
- ▶ How does this affect the purchasing power of the owner of S_{US} ?
 - ▶ [Class question: can the owners of S_{US} now purchase more or less footballs?]
 - ▶ [Class question: can the owners of S_{US} now purchase more or less soccer balls?]
 - ▶ [Class question: are the owners of S_{US} better off or worse off?]
- ▶ Hence, the effect of an increase in the price of soccer balls is unambiguously good for the owner of the soccer ball specific factor!

Summary of the distributional effects of a price change

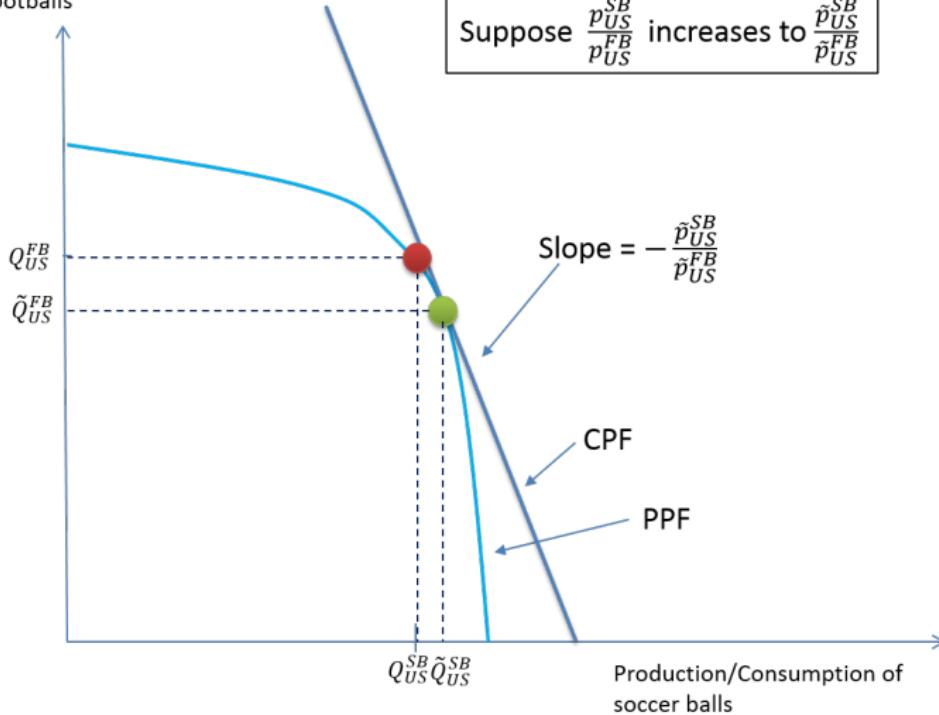
- ▶ If the relative price of a good increases:
 - ▶ The welfare of the owners of the specific factor of that good will increase.
 - ▶ The welfare of the owners of the specific factor of the other good will decrease.
 - ▶ Workers may be made better off or worse off.
- ▶ What about the overall welfare of the country?

Production/Consumption
of footballs

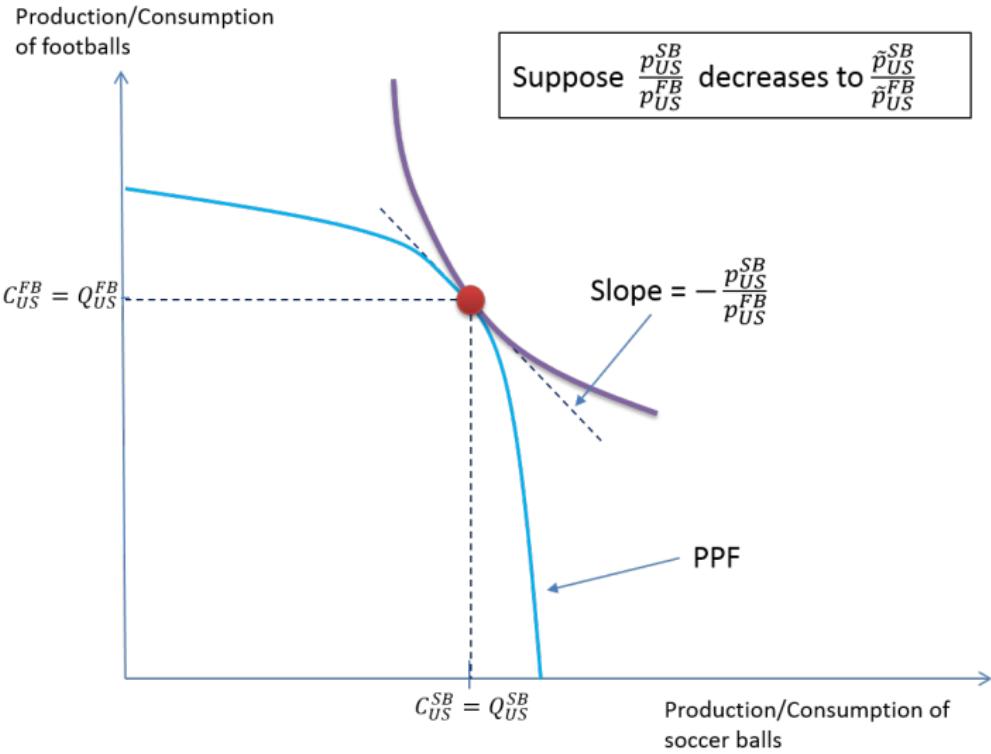




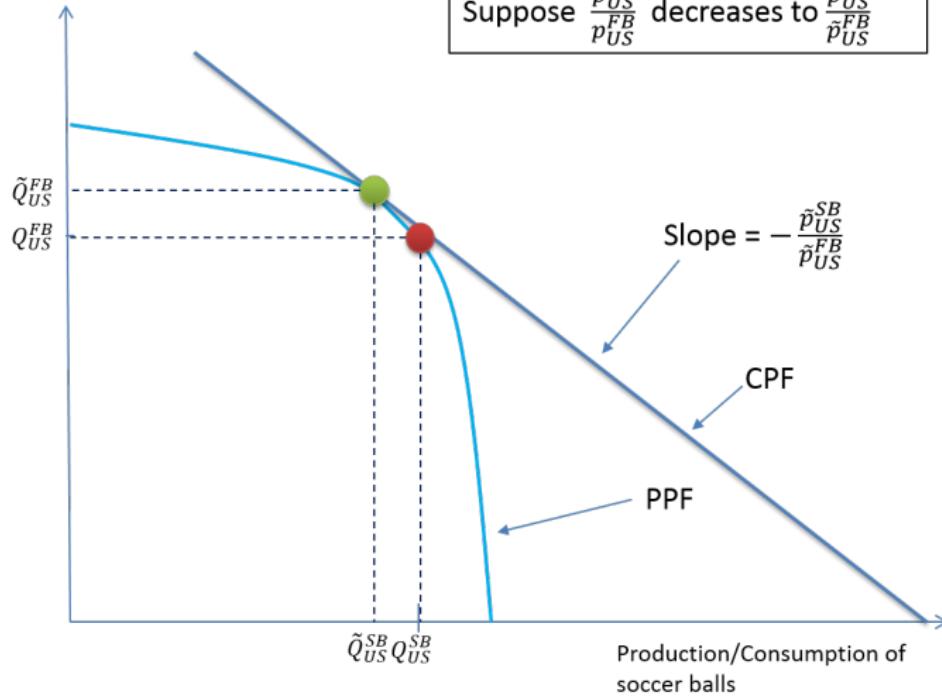
Production/Consumption
of footballs



- ▶ [Class question: Where is the new consumption bundle?]
- ▶ [Class question: Is the entire country better off?]



Production/Consumption
of footballs



- ▶ [Class question: Did the change in prices make the entire country better off?]

Summary of the distributional effects of a price change

- ▶ If the relative price of a good increases:
 - ▶ The welfare of the owners of the specific factor of that good will increase.
 - ▶ The welfare of the owners of the specific factor of the other good will decrease.
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- ▶ What about the overall welfare of the country?
 - ▶ So while some factors lose out, the country as a whole is never worse off from a price change.

From last time: Distributional effects of a price change

- ▶ If the relative price of a good increases:
 - ▶ The welfare of the owners of the specific factor of that good will increase.
 - ▶ The welfare of the owners of the specific factor of the other good will decrease.
 - ▶ Workers may be made better off or worse off.
- ▶ What about the overall welfare of the country?
 - ▶ So while some factors lose out, the country as a whole is never worse off from a price change.

Trade Equilibrium

- ▶ Consider a world with two countries, U.S. and Mexico.
- ▶ [Class question: what are the exogenous model parameters?]
 - ▶ Production functions $Q_{US}^{FB}(\cdot, \cdot)$, $Q_{US}^{SB}(\cdot, \cdot)$, $Q_{MEX}^{FB}(\cdot, \cdot)$, $Q_{MEX}^{SB}(\cdot, \cdot)$
 - ▶ Preferences $U_{US}(\cdot)$, $U_{MEX}(\cdot)$
 - ▶ Factor endowments L_{US} , F_{US} , S_{US} and L_{MEX} , F_{MEX} , S_{MEX}
- ▶ [Class question: what are the endogenous model outcomes?]
 - ▶ Quantity produced Q_{US}^{FB} , Q_{US}^{SB} and Q_{MEX}^{FB} , Q_{MEX}^{SB}
 - ▶ Quantity consumed C_{US}^{FB} , C_{US}^{SB} and C_{MEX}^{FB} , C_{MEX}^{SB}
 - ▶ Relative prices $\frac{p^{SB}}{p^{FB}}$

Trade Equilibrium

"For any production functions $Q_{US}^{FB}(\cdot, \cdot)$, $Q_{US}^{SB}(\cdot, \cdot)$, $Q_{MEX}^{FB}(\cdot, \cdot)$, $Q_{MEX}^{SB}(\cdot, \cdot)$, preferences $U_{US}(\cdot)$, $U_{MEX}(\cdot)$, and factor endowments L_{US} , F_{US} , S_{US} and L_{MEX} , F_{MEX} , S_{MEX} , equilibrium is the quantity produced Q_{US}^{FB} , Q_{US}^{SB} and Q_{MEX}^{FB} , Q_{MEX}^{SB} , quantity consumed C_{US}^{FB} , C_{US}^{SB} and C_{MEX}^{FB} , C_{MEX}^{SB} and relative prices $\frac{p^{SB}}{p^{FB}}$ such that...."

1. *In each country, given prices and the production choice of all other workers, each worker maximizes her income.*
2. *Given prices and total country income, the representative consumer in each country chooses what to consume to maximize her utility.*
3. *For each good, the total quantity consumed in the world is equal to the total quantity produced in the world.*

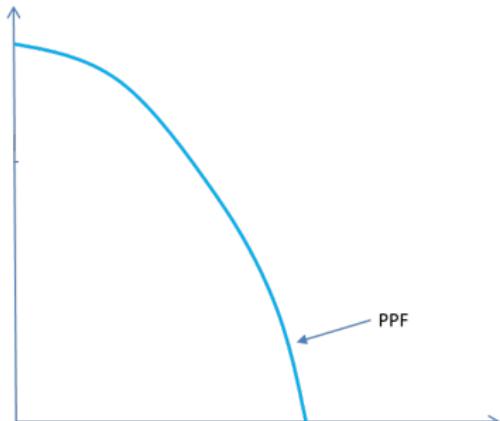
Determining the Trade Equilibrium

- ▶ Introducing trade allows the relative price to differ from the domestic relative marginal product of labor.
- ▶ This difference decouples the consumption possibility frontier from the production possibility frontier.
- ▶ Each country specializes (incompletely) in their comparative advantage good.
- ▶ Equilibrium world relative price is “pinned down” by ensuring that total production equals total consumption.

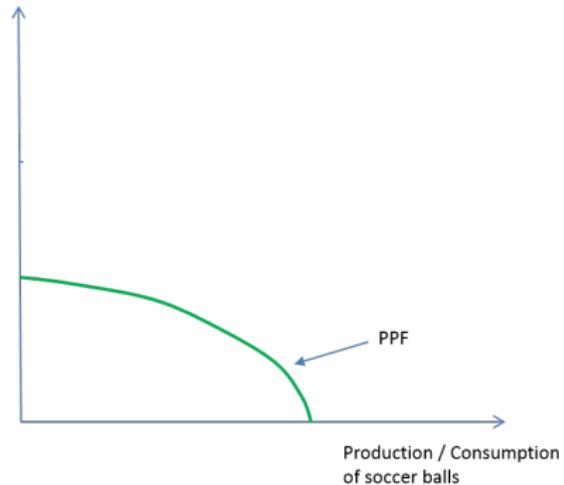
Production Possibility Frontiers

U.S.

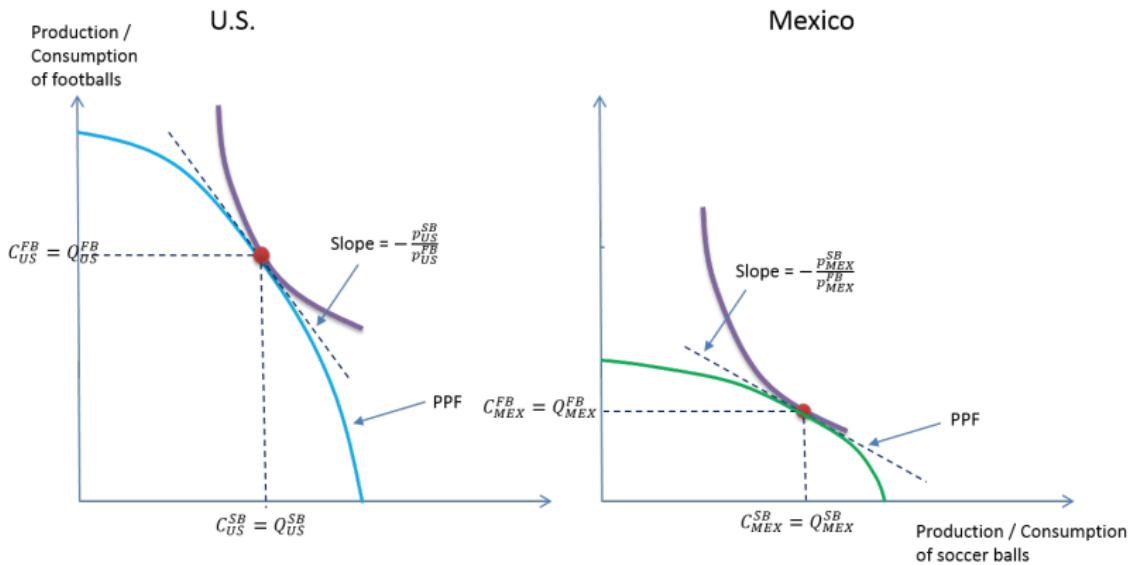
Production /
Consumption
of footballs



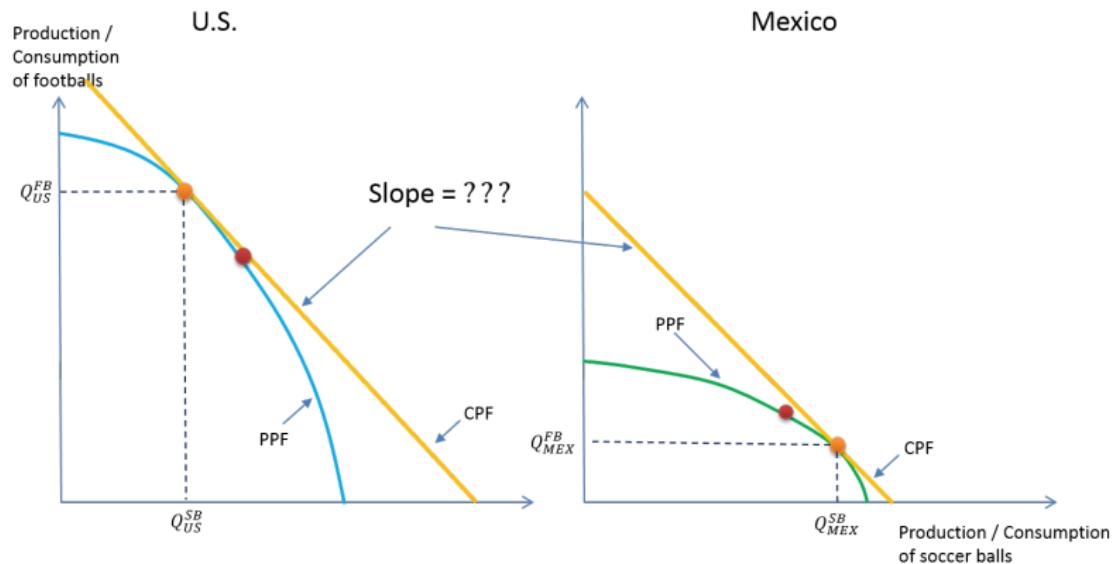
Mexico



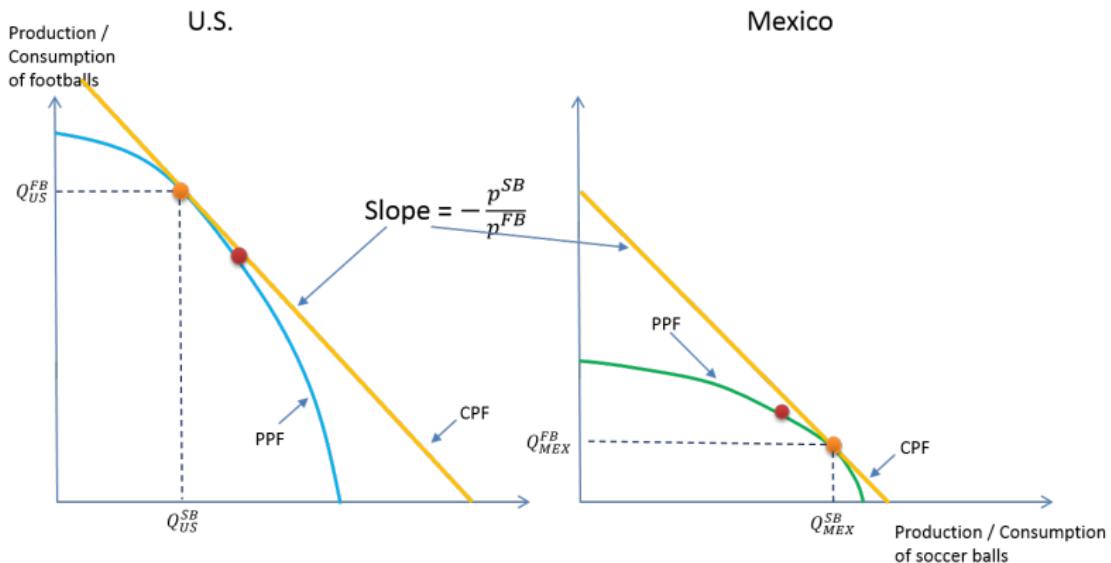
Autarkic equilibrium



Trade equilibrium

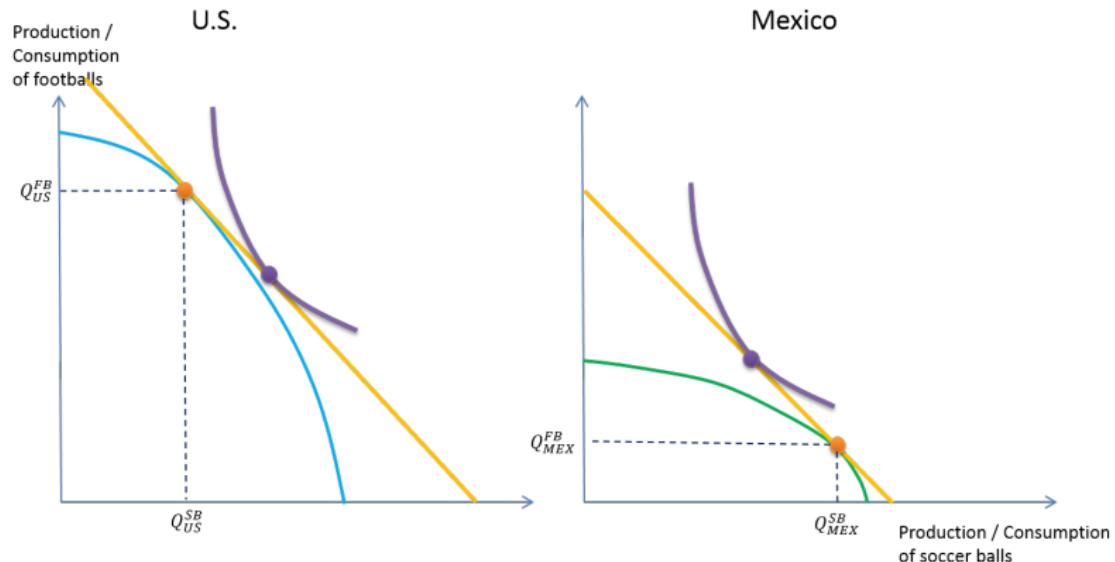


Trade equilibrium



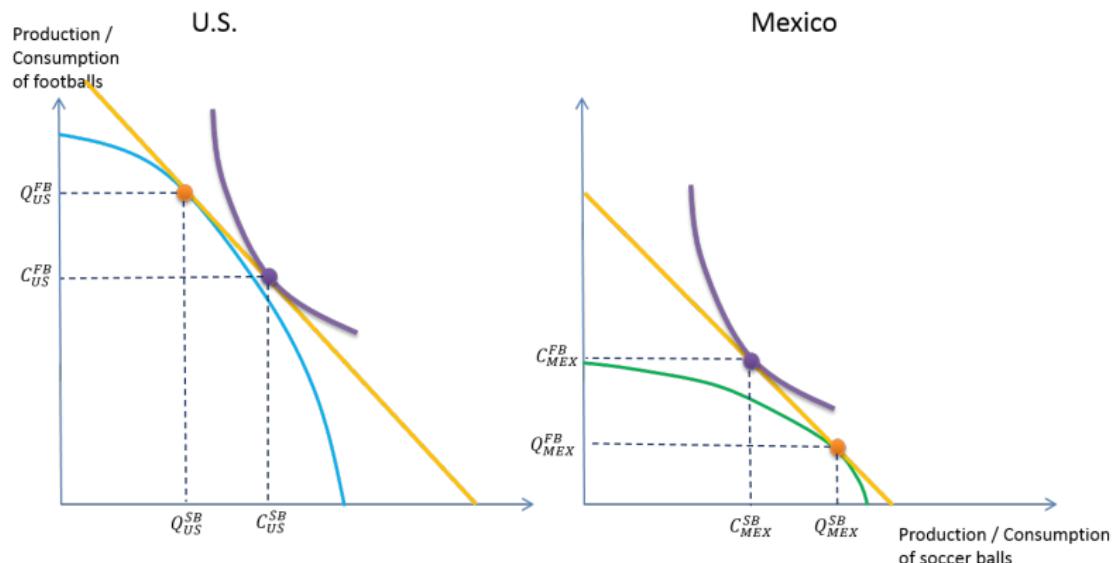
- ▶ [Class question: How did trade affect the pattern of specialization?]

Trade equilibrium



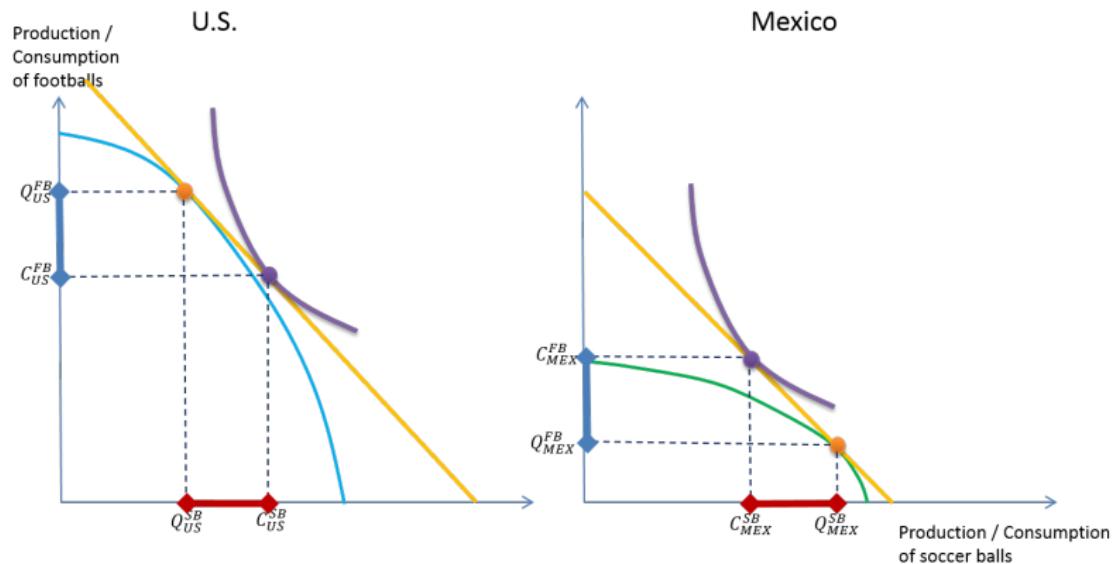
- ▶ [Class question: Who exports what?]
- ▶ [Class question: who did trade make better off?]

Trade equilibrium



- ▶ [Class question: How do we know this is an equilibrium?]

Trade equilibrium



Recap of the Trade Equilibrium

- ▶ Unlike the Ricardian model, there is incomplete specialization.
- ▶ This makes things much easier, since it implies that the equilibrium world relative price is in between the two country's autarkic prices.
- ▶ Whichever country has the higher autarkic relative price for soccer balls will specialize and export in footballs (and vice versa).
 - ▶ [Class question: What is the intuition for this?]
- ▶ This implies that *trade will raise the relative price of the good that a country exports.*

Winners and Losers from Trade

- ▶ Recall that we saw that a relative price increase in soccer balls will:
 - ▶ Benefit the owners of the soccer ball specific factor.
 - ▶ Hurt the owners of the football specific factor.
 - ▶ Have an ambiguous effect on the mobile factor (labor).
- ▶ Since trade raises the relative price of the export good, this implies that trade will:
 - ▶ Benefit the owners of the factor specific to the export good.
 - ▶ Hurt the owners of the factor specific to the import good.
 - ▶ Have an ambiguous effect on the mobile factor.
- ▶ [Class question: How may this result help explain the political resistance to free trade?]

Example: Exogenous parameters

- ▶ Factors of production
 - ▶ Labor: $L_{US} = 10, L_{MEX} = 5$
 - ▶ Specific factors:
 - ▶ $S_{US} = 1, F_{US} = 4, S_{MEX} = 1, F_{MEX} = 1$
- ▶ Production technology:
 - ▶ $Q_i^{FB} = (L_i^{FB})^{0.5} (F_i)^{0.5}, Q_i^{SB} = (L_i^{SB})^{0.5} (S_i)^{0.5},$
 $i \in \{US, MEX\}$
- ▶ Preferences:
 - ▶ $U_i = \min \{ C_i^{FB}, 2C_i^{SB} \}, i \in \{US, MEX\}$
- ▶ Question: what is the equilibrium world price, consumption, and production?

Step #1: Worker maximization

- ▶ Normalize $p^{SB} = 1$. Consider US first.
- ▶ Goal: write down the quantity produced of both footballs and soccer balls as a function of the price of footballs.
- ▶ Step #1(a): Calculate wages in both sectors, equalize the returns to both sectors:
 - ▶ Marginal product of labor in the production of footballs:

$$MPL_{US}^{FB} = \frac{\partial Q_{US}^{FB} (L_{US}^{FB}, F_{US})}{\partial L_{US}^{FB}} = \frac{\partial (L_{US}^{FB})^{0.5} (4)^{0.5}}{\partial L_{US}^{FB}} \iff$$

$$MPL_{US}^{FB} = (L_{US}^{FB})^{-0.5}$$

- ▶ Marginal product of labor in the production of soccer balls:

$$MPL_{US}^{SB} = \frac{\partial Q_{US}^{SB} (L_{US}^{SB}, S_{US})}{\partial L_{US}^{SB}} = \frac{\partial (L_{US}^{SB})^{0.5} (1)^{0.5}}{\partial L_{US}^{SB}} \iff$$

$$MPL_{US}^{SB} = 0.5 \times (L_{US}^{SB})^{-0.5}$$

Step #1: Worker maximization (ctd.)

- Under the assumption that US produces both goods in equilibrium, we have:

$$\begin{aligned} p^{FB} \times MPL_{US}^{FB} &= MPL_{US}^{SB} \iff \\ p^{FB} \times (L_{US}^{FB})^{-0.5} &= 0.5 \times (L_{US}^{SB})^{-0.5} \iff \\ 4(p^{FB})^2 L_{US}^{SB} &= L_{US}^{FB} \end{aligned}$$

- Step #1(b): Impose labor market clearing (i.e. $L_{US}^{SB} + L_{US}^{FB} = L_{US}$), solve for labor allocation to both sectors:

$$\begin{aligned} 4(p^{FB})^2 L_{US}^{SB} &= 10 - L_{US}^{SB} \iff \\ L_{US}^{SB} &= \frac{10}{(4(p^{FB})^2 + 1)} \text{ and } L_{US}^{FB} = \frac{40(p^{FB})^2}{4(p^{FB})^2 + 1} \end{aligned}$$

Step #1: Worker maximization (ctd.)

- ▶ Step #1(c): Calculate total quantity produced using production function:

$$Q_{US}^{FB} = 4p^{FB} \left(\frac{10}{4(p^{FB})^2 + 1} \right)^{0.5}$$

$$Q_{US}^{SB} = \left(\frac{10}{(4(p^{FB})^2 + 1)} \right)^{0.5}$$

Step #1: Worker maximization (ctd.)

- ▶ Step #1(d): Do the same thing for Mexico:

- ▶ Marginal product of labor in soccer ball production:

$$MPL_{MEX}^{FB} = 0.5 (L_{MEX}^{FB})^{-0.5} \text{ and } MPL_{MEX}^{SB} = 0.5 (L_{MEX}^{SB})^{-0.5}$$

- ▶ Equalizing wages implies:

$$\begin{aligned} p^{FB} (L_{MEX}^{FB})^{-0.5} &= (L_{MEX}^{SB})^{-0.5} \iff \\ L_{MEX}^{FB} &= (p^{FB})^2 L_{MEX}^{SB} \end{aligned}$$

- ▶ Labor market clearing implies:

$$L_{MEX}^{SB} = \frac{5}{(p^{FB})^2 + 1} \text{ and } L_{MEX}^{FB} = \frac{5 (p^{FB})^2}{(p^{FB})^2 + 1}$$

- ▶ Production in Mexico:

$$Q_{MEX}^{FB} = p^{FB} \left(\frac{5}{(p^{FB})^2 + 1} \right)^{0.5} \quad \& \quad Q_{MEX}^{SB} = \left(\frac{5}{(p^{SB})^2 + 1} \right)^{0.5}$$

Step #2: Consumer maximization

- ▶ Step 2(a): Calculate income. Consider US first:

$$Y_{US} = Q_{US}^{SB} + p^{FB} Q_{US}^{FB} \iff$$

$$Y_{US} = \left(\frac{10}{(4(p^{FB})^2 + 1)} \right)^{0.5} + 4(p^{FB})^2 \left(\frac{10}{4(p^{FB})^2 + 1} \right)^{0.5} \iff$$

$$Y_{US} = (1 + 4(p^{FB})^2) \left(\frac{10}{(4(p^{FB})^2 + 1)} \right)^{0.5} \iff$$

$$Y_{US} = \left(10 (1 + 4(p^{FB})^2) \right)^{0.5}$$

Step #2: Consumer maximization

- ▶ Step 2(a): Calculate income for Mexico:

$$Y_{MEX} = Q_{MEX}^{SB} + p^{FB} Q_{MEX}^{FB} \iff$$

$$Y_{MEX} = \left(\frac{5}{(p^{FB})^2 + 1} \right)^{0.5} + (p^{FB})^2 \left(\frac{5}{(p^{FB})^2 + 1} \right)^{0.5} \iff$$

$$Y_{MEX} = \left(5 \left(1 + (p^{FB})^2 \right) \right)^{0.5}$$

Step #2: Consumer maximization (ctd.)

- ▶ Step 2(b): Given preferences, maximize utility subject to prices and income:

$$\max U(C_i^{FB}, C_i^{SB}) \text{ s.t } C_i^{SB} + p^{FB} C_i^{FB} \leq Y_i$$

- ▶ Recall $U(C_i^{FB}, C_i^{SB}) = \min \{C_i^{FB}, 2C_i^{SB}\}$.
- ▶ Trick:

- ▶ Equate two elements of minimization:

$$C_i^{FB} = 2C_i^{SB}$$

- ▶ Substitute into budget constraint:

$$C_i^{SB} + p^{FB} C_i^{FB} = Y_i \iff$$

$$C_i^{SB} + 2p^{FB} C_i^{SB} = Y_i \iff$$

$$C_i^{SB} = \frac{Y_i}{1 + 2p^{FB}} \text{ and } C_i^{FB} = \frac{2Y_i}{1 + 2p^{FB}}$$

Step #2: Consumer maximization (ctd.)

- ▶ Step 2(c): Substitute actual incomes in to get consumption as a function of prices:

$$C_{US}^{SB} = \frac{\left(10 \left(1 + 4 (p^{FB})^2\right)\right)^{0.5}}{1 + 2p^{FB}}$$

$$C_{US}^{FB} = \frac{2 \left(10 \left(1 + 4 (p^{FB})^2\right)\right)^{0.5}}{1 + 2p^{FB}}$$

$$C_{MEX}^{SB} = \frac{\left(5 \left(1 + (p^{FB})^2\right)\right)^{0.5}}{1 + 2p^{FB}}$$

$$C_{MEX}^{FB} = \frac{2 \left(5 \left(1 + (p^{FB})^2\right)\right)^{0.5}}{1 + 2p^{FB}}$$

Step #3: Market clearing

- ▶ Step 3(a): Choose one sector (e.g. footballs). Calculate total world demand and total world supply as a function of price:
- ▶ World demand:

$$C_{World}^{FB} = C_{US}^{FB} + C_{MEX}^{FB} \iff$$

$$C_{World}^{FB} = \frac{2 \left(10 \left(1 + 4 (p^{FB})^2 \right) \right)^{0.5}}{1 + 2p^{FB}} + \frac{2 \left(5 \left(1 + (p^{FB})^2 \right) \right)^{0.5}}{1 + 2p^{FB}}$$

$$C_{World}^{FB} = 2 \frac{\left(10 \left(1 + 4 (p^{FB})^2 \right) \right)^{0.5} + \left(5 \left(1 + (p^{FB})^2 \right) \right)^{0.5}}{1 + 2p^{FB}}$$

Step #3: Market clearing

- ▶ World supply:

$$Q_{World}^{FB} = Q_{US}^{FB} + Q_{MEX}^{FB} \iff$$

$$Q_{World}^{FB} = 4p^{FB} \left(\frac{10}{4(p^{FB})^2 + 1} \right)^{0.5} + p^{FB} \left(\frac{5}{(p^{FB})^2 + 1} \right)^{0.5} \iff$$

$$Q_{World}^{FB} = p^{FB} \left(4 \left(\frac{10}{4(p^{FB})^2 + 1} \right)^{0.5} + \left(\frac{5}{(p^{FB})^2 + 1} \right)^{0.5} \right) \iff$$

Step #3: Market clearing (ctd.)

- ▶ Step 3(b): Equalize world demand and world supply for footballs, solve for p^{FB} :

$$2 \frac{\left(10\left(1 + 4(p^{FB})^2\right)\right)^{0.5} + \left(5\left(1 + (p^{FB})^2\right)\right)^{0.5}}{1 + 2p^{FB}} = \\ p^{FB} \left(4 \left(\frac{10}{4(p^{FB})^2 + 1}\right)^{0.5} + \left(\frac{5}{(p^{FB})^2 + 1}\right)^{0.5}\right)$$

- ▶ Notes:
 - ▶ This function can be solved for p^{FB} using a computer, but (at least by me) not by hand.
 - ▶ If I cannot do it, I will not expect you to be able to do it on a midterm.
 - ▶ However, you should feel comfortable understanding how the math in this example is related to the pictures above.

Next steps

- ▶ Specific factors model provided more realism in the production function and uneven gains from trade across factors.
- ▶ We will now consider the “long-run” version of the specific factors model, where the specific factors can move across sectors (i.e. they are no longer specific).
- ▶ This is the famous Heckscher-Ohlin model (which dominated the trade literature for almost fifty years).