International Finance
Lecture I: Endowment Economy

Graduate Institute of International and Development Studies

September 23, 2024

The course

- Textbook
  - Martín Uribe and Stephanie Schmitt-Grohé (2017) *Open Economy Macroeconomics*, Princeton University Press (henceforth US-G)
- Grading
  - Final Exam on December 19
- Today
  - Stylized Facts and Endowment Economy: Chapters 1 and 2 of US-G

10 Business-Cycle Facts Around the World

Fact 1: [High Global Volatility] The cross-country average standard deviation of output is about twice as large as its U.S. counterpart.
Fact 2: [Excess Consumption Volatility] On average across countries, private consumption including durables is more volatile than output.
Fact 3: [Global Ranking of Volatilities] The ranking of cross-country average standard deviations from top to bottom is: imports, investment, exports, government spending, consumption, and output.
Fact 4: [Procyclicality of the Components of Aggregate Demand] On average, across countries, consumption, investment, exports, and imports are positively correlated with output.
Fact 5: [Countercyclicality of the Trade Balance and the Current Account] On average, across countries, the trade balance, trade-balance-to-output ratio, current account, and current-account-to-output ratio are negatively correlated with output.
Fact 6: [Acyclicality of the Share of Government Consumption in GDP] On average across countries, the share of government consumption in output is roughly uncorrelated with output.
Fact 7: [Persistence] The components of aggregate supply (output and imports) and aggregate demand (consumption, government spending, investment, and exports) are all positively serially correlated.
Fact 8: [Excess Volatility of Poor and Emerging Countries] Business cycles in emerging or poor countries are about twice as volatile as business cycles in rich countries.
Fact 9: [Excess Consumption Volatility in Poor and Emerging Countries] The relative consumption volatility is higher in poor and emerging countries than in rich countries.
Fact 10: [The Countercyclicality of Government Spending Increases with Income] The share of government consumption is countercyclical in rich countries, but acyclical in emerging and poor countries.

High Global Volatility

Business-Cycle Statistic	World	U.S.
σ_y	6.2%	2.9%

Fact 1: The cross-country average standard deviation of output is twice as large as its U.S. counterpart.

This is mostly driven by EMDEs.
Advanced economies are similar to the US

Excess Consumption Volatility

Business-Cycle Statistic	World Average
$\frac{\sigma_c}{\sigma_y}$	1.05

Fact 2: On average, across countries, private consumption (including durables) is more volatile than output.

Ranking of Volatilities

Business-Cycle Statistic	World Average
$\frac{\sigma_m}{\sigma_y}$	3.23
$\frac{\sigma_i}{\sigma_y}$	3.14
$\frac{\sigma_x}{\sigma_y}$	3.07
$\frac{\sigma_g}{\sigma_y}$	2.26
$\frac{\sigma_c}{\sigma_y}$	1.05

Fact 3: The ranking of cross-country average standard deviations from top to bottom is: imports, investment, exports, government spending, consumption, and output.

Cyclicality

Business-Cycle Statistic	World Average
corr($cons, y$)	0.69
corr(inv, y)	0.66
corr(exp, y)	0.19
corr(imp, y)	0.24
corr(trb, y)	-0.18
corr(cab, y)	-0.28
corr($gc/y, y$)	-0.02

Fact 4: Consumption, investment, exports, and imports are procyclical.
Fact 5: The trade balance and the current account are countercyclical.
Fact 6: The share of government consumption in output is acyclical.

Persistence

Business-Cycle Statistic	World Average
corr(y_t, y_{t-1})	0.71
corr(c_t, c_{t-1})	0.66
corr(g_t, g_{t-1})	0.76
corr(i_t, i_{t-1})	0.56
corr(x_t, x_{t-1})	0.68
corr(m_t, m_{t-1})	0.61

Fact 7: All components of demand ($cons, gov, inv, exp$) and supply (y, m) are positively serially correlated.

Excess Volatility of EMDEs

Business-Cycle Statistic	Poor	Emerging	Rich
σ_y	6.1%	8.7%	3.3%

Fact 8: Business Cycles in poor and emerging countries are about **twice** as volatile as business cycles in rich countries.

Less Consumption Smoothing in EMDEs

Business-Cycle Statistic	Poor	Emerging	Rich
σ_c/σ_y	1.12	0.98	0.87

Fact 9: The relative consumption volatility is higher in poor and emerging countries than in rich countries.

The Countercyclicality of Government Spending Increases With Income

Business-Cycle Statistic	Poor	Emerging	Rich
corr$(g/y, y)$	0.08	-0.08	-0.39

Fact 10: The share of government consumption is countercyclical in rich countries, but acyclical in emerging and poor countries.

US-G Chapter 2

- The purpose of this chapter is to build a canonical dynamic, general equilibrium model of the open economy and contrast its predictions with some of the empirical regularities documented above
- The model economy developed in this chapter is simple enough to allow for a full characterization of its equilibrium dynamics using pen and paper.
- The economy is inhabited by households that receive an exogenous but stochastic endowment of perishable goods each period.
- In addition, households have access to an internationally traded bond, which they use to smooth consumption in response to random income disturbances.
- In turn, consumption smoothing gives rise to equilibrium movements in the trade balance and the current account.
- The framework is called the *intertemporal approach to the current account*.

The Model

The representative household chooses processes for consumption (c_t) and and debt (d_t), to maximize:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t) \tag{2.1}$$

subject to the sequential budget constraint (2 sources of fund):

$$c_t + (1+r)d_{t-1} = y_t + d_t, \tag{2.2}$$

and to the no-Ponzi-game constraint:

$$\lim_{j \to \infty} E_t \frac{d_{t+j}}{(1+r)^j} \leq 0, \tag{2.3}$$

given d_{-1}, where y_t is an exogenous but stochastic endowment of perishable goods (this is the sole source of uncertainty) and r is a constant interest rate on an internationally traded bond.

At the optimal allocation, the no-Ponzi-game constraint must hold with equality (see p. 27-28 for a proof). When the NPG holds with equality, it's called *transversality condition.*

Comment: In the closed economy context, this is the permanent income model.

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t)$$

- U period utility function which is continuously differentiable and strictly increasing and concave in c
- β is the subjective discount factor
- E_t is the mathematical expectation operator based on information available at time t
- We are assuming perfect capital markets, individuals can borrow and lend at the risk-free rate r
- The exogenous stochastic endowment process y_t is the *sole source of uncertainty*
- The HH has two sources of funds: y_t and new debt but needs to pay old debt with interest

Optimality Conditions

The Lagrangian associated with the household's problem is

$$\mathcal{L}_0 = E_0 \sum_{t=0}^{\infty} \beta^t \{U(c_t) + \lambda_t[d_t + y_t - (1+r)d_{t-1} - c_t]\},$$

where λ_t is a Lagrange multiplier. The optimality conditions with respect to c_t and d_t are:

$$U'(c_t) = \lambda_t,$$

and

$$\lambda_t = \beta(1+r)E_t\lambda_{t+1}.$$

Combining these two expression, yields the *Euler Equation*

$$U'(c_t) = \beta(1+r)E_t U'(c_{t+1}). \tag{2.4}$$

Interpretation: At the margin, the household is indifferent between consuming a unit of good today or saving it and consuming it the next period along with the interest.

As all HHs are the same, c_t and d_t can be interpreted as the per capita consumption and net foreign liabilities

A rational expectations equilibrium can be defined as a pair of processes $[c_t, d_t]_t^\infty$, satisfying the period budget constraint and the NPG, given the initial condition d_{t-1} and the exogenous process $[y_t]_t^\infty$

The Intertemporal Resource Constraint

Writing the sequential budget constraint (2.2) for period $t+j$, dividing by $(1+r)^j$, and taking expected values conditional on information available in period t yields:

$$\frac{E_t c_{t+j}}{(1+r)^j} + \frac{E_t d_{t-1+j}}{(1+r)^{j-1}} = \frac{E_t y_{t+j}}{(1+r)^j} + \frac{E_t d_{t+j}}{(1+r)^j}.$$

Then sum from $j=0$ to $j=J$:

$$\sum_{j=0}^{J} \frac{E_t c_{t+j}}{(1+r)^j} + (1+r)d_{t-1} = \sum_{j=0}^{J} \frac{E_t y_{t+j}}{(1+r)^j} + \frac{E_t d_{t+J}}{(1+r)^J}.$$

Taking limit for $J \to \infty$ and using the transversality condition (equation (2.3) holding with equality) yields the intertemporal resource constraint

$$(1+r)d_{t-1} = \sum_{j=0}^{\infty} \frac{E_t(y_{t+j} - c_{t+j})}{(1+r)^j}. \tag{2.5}$$

Interpretation: at every point in time, the economy must be expected to put aside a stream of resources, $\{E_t y_{t+j} - E_t c_{t+j}\}_{j=0}^{\infty}$, large enough in present discounted value to cover the outstanding external debt.

Can An Economy Run A Perpetual Trade Deficit?

Because in this economy there is only one good, the trade balance (the difference between exports and imports), denoted tb_t, is given by the difference between output and consumption, that is:

$$tb_t \equiv y_t - c_t. \tag{2.6}$$

Using this definition, we can rewrite the intertemporal resource constraint (2.5) as

$$(1+r)d_{t-1} = \sum_{j=0}^{\infty} \frac{E_t tb_{t+j}}{(1+r)^j}.$$

- The answer to the above question depends on the country's initial debt position:
  * If the country is a net debtor ($d_{t-1} > 0$), then it must generate an expected trade surplus in at least one period, $E_t tb_{t+j} > 0$ for some j.
  * That is, if the country starts out as a net debtor, then it cannot run perpetual trade deficits. However, if $d_{t-1} < 0$ the country could run a permanent trade defict

Two Simplifying Assumptions

The following two assumptions are handy because they allow for a closed-form solution of the model:
(1) The subjective and market discount rates are equal

$$\beta = \frac{1}{1+r}.$$

(2) The period utility function is quadratic

$$U(c) = -\frac{1}{2}(c - \overline{c})^2, \tag{2.7}$$

where $\overline{c}$ is a satiation point large enough so that $c < \overline{c}$ in equilibrium. Under these assumptions, the Euler equation (2.4) becomes

$$c_t = E_t c_{t+1}, \tag{2.8}$$

that is, consumption follows a **random walk**. Households expect to maintain their current level of consumption forever (non long-run growth in consumption)

$$E_t c_{t+j} = c_t$$

for all $j > 0$.

Two useful variables

Before presenting the closed form solution to the model, we introduce two variables that will be convenient in their characterization and interpretation

— nonfinancial permanent income, y_t^p

— difference between current income and permanent income, $y_t - y_t^p$

Nonfinancial permanent income, y_t^p

Define nonfinancial permanent income, denoted y_t^p, as the **constant** level of income that, if received in all future periods $t+j$ for $j \geq 0$, is equal to the expected present discounted value of the stochastic income process as of time t. That is, define y_t^p as the solution to

$$y_t^p \sum_{j=0}^{\infty} \frac{1}{(1+r)^j} = \sum_{j=0}^{\infty} \frac{E_t y_{t+j}}{(1+r)^j}$$

Solving for y_t^p we obtain

$$y_t^p = \frac{r}{1+r} \sum_{j=0}^{\infty} \frac{E_t y_{t+j}}{(1+r)^j}. \tag{2.10}$$

This equation says that y_t^p is a weighted average of expected future income levels, with weights adding up to one: $1 = \frac{r}{1+r} \sum_{j=0}^{\infty} \frac{1}{(1+r)^j}$.

The difference between current and permanent income, $y_t - y_t^p$

$$
\begin{aligned}
y_t - y_t^p &= y_t + \frac{1}{r} y_t^p - \left(\frac{1+r}{r}\right) y_t^p \\
&= y_t + \sum_{j=0}^{\infty} \frac{E_t y_{t+j}}{(1+r)^{j+1}} - \sum_{j=0}^{\infty} \frac{E_t y_{t+j}}{(1+r)^{j}} \\
&= \sum_{j=1}^{\infty} \frac{E_t y_{t+j-1}}{(1+r)^{j}} - \sum_{j=1}^{\infty} \frac{E_t y_{t+j}}{(1+r)^{j}} \\
&= -\sum_{j=1}^{\infty} \frac{E_t y_{t+j} - E_t y_{t+j-1}}{(1+r)^{j}}
\end{aligned}
$$

$\Rightarrow$

$$
y_t - y_t^p = -\sum_{j=1}^{\infty} \frac{E_t \Delta y_{t+j}}{(1+r)^{j}} \tag{2.27}
$$

Interpretation: Permanent income exceeds current income when income is expected to grow in the future.

The Closed-Form Equilibrium Solution

Use the definition of y_p and the fact that $E_t c_{t+j} = c_t$ to eliminate $E_t c_{t+j}$ from the intertemporal resource constraint (2.5):

$$c_t = y_t^p - rd_{t-1}, \tag{2.11}$$

Because d_{t-1} is predetermined in t and y_t^p is exogenous, equation (2.11) represents the closed-form solution for c_t.

Interpretation: in equilibrium consumption is equal to nonfinancial permanent income minus interest on existing initial debt.

In every period, the optimal plan is to spend the permanent income in consumption and interest payments: $y_t^p = c_t + rd_{t-1}$,

The Equilibrium Behavior of the Trade Balance

Using the equilibrium level of consumption given in (2.11), we obtain the closed-form solution for the trade balance, $tb_t = y_t - c_t$, as

$$tb_t = y_t - y_t^p + rd_{t-1} \tag{2.16}$$

The trade balance responds countercyclically to changes in current income if permanent income increases by more than current income in response to increases in current income:

$$\frac{\partial tb}{\partial y_t} = 1 - \frac{\partial y_p}{\partial y_t}$$

The Equilibrium Behavior of the Trade Balance

The trade balance responds countercyclically to changes in current income if permanent income increases by more than current income in response to increases in current income. $\frac{\partial tb}{\partial y_t} = 1 - \frac{\partial y_p}{\partial y_t}$

⇒ This is an important result, because Fact 5 above says that the trade balance is countercyclical ($\frac{\partial tb}{\partial y_t} < 0$)
⇒ To capture this fact in the context of the present model, the endowment process must be such that permanent income increases by more than one for one with current income ($\frac{\partial y_p}{\partial y_t} > 1$).
⇒ What type of endowment process satisfies this requirement? We will return to this point soon.

The Equilibrium Behavior of External Debt

Use (2.11) again to eliminate c_t from the sequential budget constraint (2.2) to get

$$d_t - d_{t-1} = y_t^p - y_t. \tag{2.12}$$

which says that the economy borrows to cover deviations of current income from permanent income.

Like consumption, external debt has a unit root.

The Equilibrium Behavior of the Current Account

The current account is defined as the trade balance minus interest payments on the external debt (rd_{t-1} net investment income on the net foreign assets position. No dark matter here)

$$ca_t \equiv tb_t - rd_{t-1}. \tag{2.13}$$

Using the closed-form solution for tb_t given in (2.16), we get

$$ca_t = y_t - y_t^p. \tag{2.15}$$

The country runs a CA deficit when current income is lower than permanent income. Fact 5 says that the current account is countercyclical.
In order to explain this empirical regularity through this model, as in the case of the trade balance, permanent income must increase more than one for one with current income.
Comparing the above expression with (2.12) , we have that

$$ca_t = -(d_t - d_{t-1}). \tag{2.14}$$

This is known as the **fundamental balance of payments identity**: the current account equals the change in the country's net asset position.

Let's collect the results obtained thus far:

When $\beta(1+r) = 1$ and $U(c) = -1/2(c - \bar{c})^2$, then the model has the following closed-form solution:

$$c_t = y_t^p - rd_{t-1} \tag{2.11}$$

$$d_t = d_{t-1} + y_t^p - y_t \tag{2.12}$$

$$ca_t = y_t - y_t^p \tag{2.15}$$

$$tb_t = y_t - y_t^p + rd_{t-1} \tag{2.16}$$

where

$$y_t^p \equiv \frac{r}{1+r} \sum_{j=0}^{\infty} \frac{E_t y_{t+j}}{(1+r)^j} \tag{2.10}$$

and d_{-1} and the stochastic process for y_t are exogenously given.
Memo item:

$$ca_t = y_t - y_t^p = -\sum_{j=1}^{\infty} \frac{E_t \Delta y_{t+j}}{(1+r)^j} \tag{2.27}$$

A General Principle I

Take another look at the expression $ca_t = y_t - y_t^p$.
$\Rightarrow$ It says that the current account is used whenever:

- The economy experiences temporary deviations of output from permanent income, that is, whenever $y_t - y_t^p \neq 0$.
- By contrast, **permanent** output shocks, that is movements in y_t that leave $y_t - y_t^p$ unchanged do not produce movements in the current account.

Thus, the following principle emerges:

> *Finance temporary shocks (by running current account surpluses or deficits) with little change in consumption and adjust to permanent shocks by changing consumption but not the current account.*

A General Principle II

We can use what we derived so far to obtain the equilibrium trade balance as:

$$tb_t = y_t - y_t^p - rd_{t-1}$$

- The trade balance responds countercyclically to changes in current income if permanent income increases by more than current income (and the other way around)
- As expectations of future income are so important in driving the trade balance (and the current account), the model is known as the intertemporal approach to the balance of payments
- The behavior of the CA depends crucially on the interaction between current and permanent income
- In turn, this is driven by the stochastic process that drives the endowment

2.2 An AR(1) Income Process

Suppose y_t follows the law of motion

$$y_t - \bar{y} = \rho(y_{t-1} - \bar{y}) + \epsilon_t,$$

where ϵ_t is a white noise with mean zero and variance σ_ϵ^2 and $\bar{y}$ is a positive constant. The parameter $\rho \in (-1, 1)$ measures persistence. The j-step-ahead forecast of y_t as of period t is

$$E_t y_{t+j} = \bar{y} + \rho^j (y_t - \bar{y}).$$

Thus, if $\rho > 0$, which is the case of greatest empirical interest, the forecast converges monotonically to the unconditional mean of the process, namely $\bar{y}$ (note that we are assuming $\rho < 1$). The more persistent the process (the higher ρ) is, the slower the speed of convergence of the forecast to the unconditional mean will be.

Permanent Income with AR(1) Endowment

Use $E_t y_{t+j} - \bar{y} = \rho^j (y_t - \bar{y})$ to eliminate $E_t y_{t+j}$ from (2.10) to obtain

$$y_t^p - \bar{y} = \frac{r}{1 + r - \rho}(y_t - \bar{y}). \tag{2.17}$$

This expression is quite intuitive:
• if the endowment process is temporary ($\rho \to 0$), then only a small fraction, $r/(1 + r)$ of movements in the endowment are incorporated into permanent income.
• if the endowment process is highly persistent ($\rho \to 1$), most of movements in the current endowment are reflected in movements in permanent income (permanent income is close to the endowment itself; with $\rho = 1$ $y_t^p = y_t$).

Permanent Income with AR(1) Endowment

A key variable is $y_t - y_t^p$, which under the AR(1) process is given by

$$y_t - y_t^p = \frac{1-\rho}{1+r-\rho}(y_t - \bar{y}). \qquad (*)$$

The coefficient

$$\frac{1-\rho}{1+r-\rho}$$

is positive for all possible values of ρ (in other words $\frac{\partial y_p}{\partial y_t} < 1$), implying that $y_t - y_t^p$ is procyclical.

Therefore the trade balance and the current account are predicted to be procyclical — which is not in line with the empirical evidence

Consumption Adjustment with AR(1) Income

By (2.11) and (2.17)

$$c_t = y_t^p - rd_{t-1} = \bar{y} + \frac{r}{1+r-\rho}(y_t - \bar{y}) - rd_{t-1} \qquad (2.18)$$

• if $\rho = 0$ consumption increases less than one for one with current income. Why? Because current income is higher than permanent income.
• if $\rho \approx 1$, consumption adjusts one-for-one with current output, as current income is equal to permanent income.

External Debt Adjustment with AR(1) Income

By (2.12) and (*)

$$d_t = d_{t-1} + (y_t^p - y_t) = d_{t-1} - \frac{1-\rho}{1+r-\rho}(y_t - \bar{y}) \tag{2.21}$$

• if $\rho = 0$ debt decreases almost one-for-one with income. Why? The country saves most of the temporary income increase to be able to consume not only more in the current period but also in all future periods. (finance temporary shocks)

• if $\rho \approx 1$, debt is unchanged. Why? Because income will be also higher in the future and hence there is no need to save. (adjust to permanent shocks)

• Note that the above equation shows that debt follows a random walk (non-stationary process). A temporary positive income shock leads to a permanent decline in net foreign liabilities

• As the behavior of the TB is driven by the dynamics of external debt. Temporary positive income shocks lead to a long-run deterioration of the trade balance

Current Account Adjustment with AR(1) Income

By (2.15) and (*)

$$ca_t = y_t - y_t^p = \frac{1-\rho}{1+r-\rho}(y_t - \bar{y}) \tag{2.20}$$

• if $\rho = 0$ the change in the current account is close to the change in current income. Why? The country saves most of the temporary income increase to be able to consume not only more in the current period but also in all future periods.
• if $\rho \approx 1$, the current account is unchanged (and equal to zero). Why? Because income will be also higher in the future and hence there is no need to save.

Trade Balance Adjustment with AR(1) Income

By (2.16) and (*)

$$tb_t = y_t - y_t^p + rd_{t-1} \tag{1}$$

• if $\rho = 0$, then most of the change in current income is exported resulting in a trade balance improvement.

• if $\rho \approx 1$, then none of the change in current income is exported and the trade balance is unchanged.

• The more temporary the income shock is, the more volatile the CA and TB will be.

• When $\rho = 0$ the standard deviation of the CA equals $\frac{\sigma_y}{(1+r)}$ which is close to σ_y.

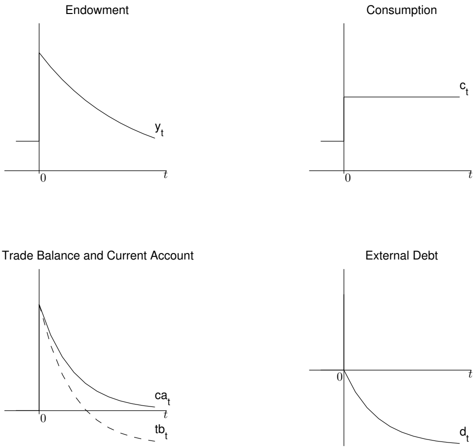
• When $\rho \rightarrow 0$, the standard deviation of the CA goes to zero

Response to a positive and persistent endowment shock: AR(1) process

The next graph displays the impulse response of y_t, c_t, d_t, tb_t, and ca_t to a unit increase in y_t assuming zero initial debt, $d_{t-1} = 0$ for a value of $\rho \in (0, 1)$.

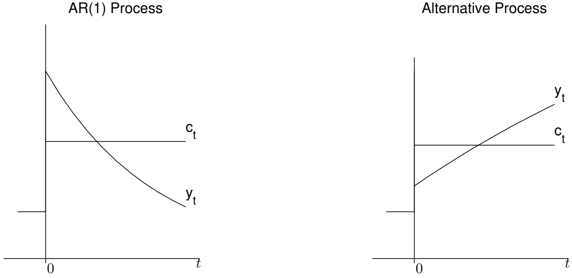
- Consumption increase permanently but by less than income
- The trade balance and the CA improve and then converge to preshock level (CA) and below preshock level (TB). Hence short run improvement and long run deterioration in the TB.
- Net foreign assets (liabilities) increase (decrease)

Response to a positive and persistent endowment shock: AR(1) process



Counterfactual Predictions with AR(1) Endowment

* The figure on the previous slide illustrates that a positive endowment shock causes an improvement in the trade balance and the current account.
* This is counterfactual as both of these variables are countercyclical.
* The intuition behind the model's prediction is provided by the left panel of the following figure



Counterfactual Predictions with AR(1) Endowment

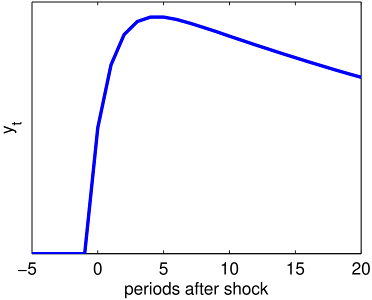
- Under the assumed AR(1) process, an increase in output in the current period is expected to die out over time. So future output is expected to be lower than current output.
- As a result, consumption smoothing households save part of the current increase in output for future consumption.
- Now suppose we could come up with an output process like the one shown on the right panel of the figure.
- There, an increase in output today creates expectations of even higher output in the future. As a result, consumption today increases by more than output, as consumption-smoothing households borrow against future income.
- In this environment, an output shock would tend to worsen the trade balance and the current account, which is more in line with the data.
- What type of endowment processes can give rise to an upward sloping impulse response for output?

2.3 An AR(2) Income Process

$$y_t = \bar{y} + \rho_1(y_{t-1} - \bar{y}) + \rho_2(y_{t-2} - \bar{y}) + \epsilon_t \tag{2.22}$$

Impulse Response of Endowment

AR(2) with $\rho_1 = 1.5$ and $\rho_2 = -0.51$.



* impulse response is hump-shaped, that is, the peak output response occurs several periods after the shock occurs.
* current level of output **may** rise by less than permanent income, that is, the change in y_t may be less than the change in y_t^p.
* If so, the trade balance and the current account will deteriorate in response to an increase in output, bringing the model closer to the data.

Restrictions on ρ_1 and ρ_2 to ensure stationarity

The income process should be mean reverting (stationary), i.e., $E_t y_{t+j}$ exists for all $j \geq 0$ and

$$\lim_{j\to\infty} E_t y_{t+j} = \bar{y} \quad \forall t \geq 0$$

Let $Y_t = \begin{bmatrix} y_t - \bar{y} \\ y_{t-1} - \bar{y} \end{bmatrix}$ Then write (2.22) as:

$$Y_t = R\, Y_{t-1} + \begin{bmatrix} \epsilon_t \\ 0 \end{bmatrix} \quad \text{with } R = \begin{bmatrix} \rho_1 & \rho_2 \\ 1 & 0 \end{bmatrix}$$

This implies that

$$E_t Y_{t+j} = R^j Y_t \tag{2.23}$$

Thus $\lim_{j\to\infty} E_t y_{t+j} = \bar{y}$ holds for any initial conditions y_t, y_{t-1}, if and only if both eigenvalues of **R** lie inside the unit circle.

Restrictions on ρ_1 and ρ_2 to ensure stationarity

The eigenvalues of any 2x2 matrix **M** lie inside the unit circle if and only if

$$|det(M)| < 1$$
$$|tr(M)| < 1 + det(M)$$

In our application we have,

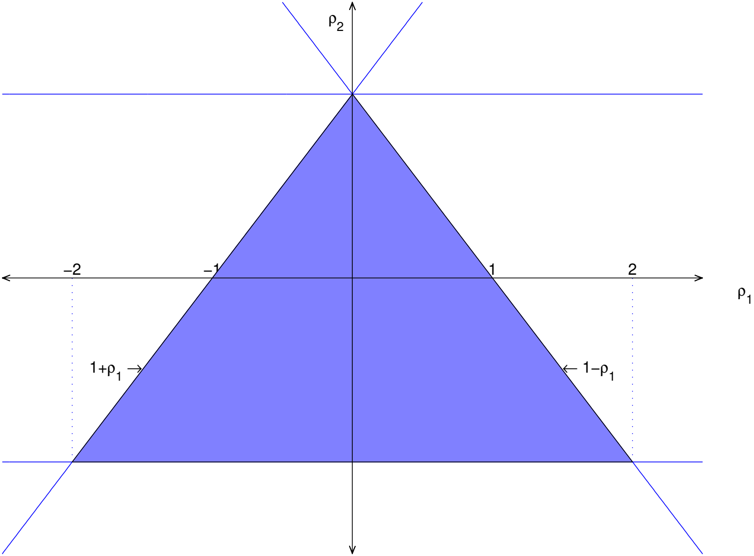
$$det(R) = -\rho_2 \quad \text{and} \quad tr(M) = \rho_1$$

It follows that the AR(2) process is mean reverting iff

$$\rho_2 < 1 - \rho_1$$
$$\rho_2 < 1 + \rho_1$$
$$\rho_2 > -1$$

Take a look at the graph on the next slide. The set of allowable pairs (ρ_1, ρ_2) are those inside the triangle.

Restrictions on ρ_1 and ρ_2 to ensure stationarity



Restrictions on ρ_1 and ρ_2 to ensure countercyclical impact response of trade balance

- But stationarity is not enough, we also want that permanent income increase more than current income
- Recall that the key variable determining the response of the trade balance and the current account is: $y_t - y_t^p$
- This is the difference between current income and permanent income.
- We want to know for which pairs (ρ_1, ρ_2) current income increases by less than permanent income in response to a positive income shock in the AR(2) case.

Permanent Income with AR(2) Endowment

Recall the definition of permanent income, y_t^p, given in (2.10):

$$y_t^p - \bar{y} = \frac{r}{1+r} \sum_{j=0}^{\infty} (1+r)^{-j} E_t(y_{t+j} - \bar{y})$$

Use (2.23)

$$\begin{aligned}
E_t Y_{t+j} &= \mathbf{R}^j Y_t \\
(1+r)^{-j} E_t Y_{t+j} &= (\mathbf{R}/(1+r))^j Y_t \\
\sum_{j=0}^{\infty} (1+r)^{-j} E_t Y_{t+j} &= \sum_{j=0}^{\infty} (\mathbf{R}/(1+r))^j Y_t \\
\frac{r}{1+r} \sum_{j=0}^{\infty} (1+r)^{-j} E_t Y_{t+j} &= \frac{r}{1+r} [\mathbf{I} - \mathbf{R}/(1+r)]^{-1} Y_t \\
&= \frac{r}{(1+r)(1+r-\rho_1) - \rho_2} \begin{bmatrix} 1+r & \rho_2 \\ 1 & 1+r-\rho_1 \end{bmatrix} Y_t
\end{aligned}$$

From here we obtain

$$y_t^p - \bar{y} = \frac{r}{(1+r)(1+r-\rho_1) - \rho_2} \left[(1+r)(y_t - \bar{y}) + \rho_2 (y_{t-1} - \bar{y}) \right] \tag{2.24}$$

$\Rightarrow$ The impact response of permanent income, y_t^p, to an increase in y_t is always positive. (Can you show this?)
What happens if $\rho_2 = 0$?

The key variable $y_t - y_t^p$ with AR(2) Endowment

From (2.24) it follows that

$$y_t - y_t^p = \gamma_0(y_t - \bar{y}) + \gamma_1(y_{t-1} - \bar{y})$$

with

$$\gamma_0 = \frac{(1 - \rho_1 - \rho_2) + r(1 - \rho_1)}{(1 - \rho_1 - \rho_2) + r(1 - \rho_1) + r(1 + r)}$$

The model therefore predicts a counterycyclical response on impact of $y_t - y_t^p$ iff

$$\gamma_0 < 0.$$

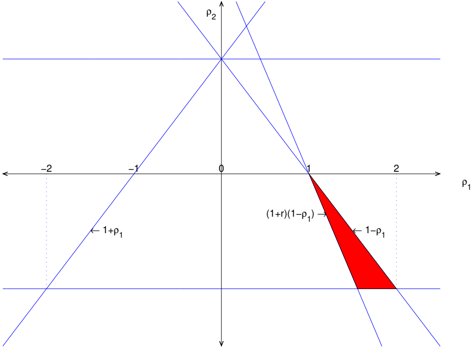
This requires that the numerator and the denominator of γ_0 have different signs, which we can ensure by adding the requirement

$$\rho_2 > (1 + r)(1 - \rho_1) \tag{**}$$

to the requirements of stationarity.

The graph on the next slide shows there exist combinations of (ρ_1, ρ_2) that satisfy this criterion and the stationarity requirements. Thus in principle the model with an AR(2) endowment process can predict a countercyclical response of the trade balance and the current account. It is then an empirical question whether output conforms to those restrictions.

Restrictions on ρ_1 and ρ_2 to ensure countercyclical impact response of trade balance with AR(2) income



We need $\rho_1 > 1$. Why? To have future income higher than current income. And we need $\rho_2 < 0$ for stationarity, but not too negative to avoid that the initial increase in output is followed by a future decline. If ρ_2 becomes too negative (close to -1) the impulse response of the endowment starts oscillating instead of being hump shaped, in this case a positive shock to income will first drive y up

Summary of adjustment with AR(2) income

In response to a positive income shock in period t:

- y_t^p increases
- By (2.11): $c_t = y_t^p - rd_{t-1}$, hence c_t procyclical on impact.
- and, provided $\rho_2 > (1+r)(1-\rho_1)$
  - $y_t^p - y_t$ increases,
  - By (2.12): $d_t = d_{t-1} + y_t^p - y_t$, hence d_t increases.
  - By (2.15): $ca_t = y_t - y_t^p$, hence ca_t countercyclical
  - By (2.16): $tb_t = y_t - y_t^p + rd_{t-1}$, hence tb_t countercyclical
- We have shown that it is possible for the model to predict a counter cyclical trade balance adjustment even under a stationary income process.
- Thus non-stationarity is not a necessary assumption for a countercyclical adjustment.
- Next, we show that a non-stationary income process can give rise to a countercyclical trade balance adjustment as well.

2.4 A Nonstationary Income Process

Assume that it income levels are non-stationary but the change in income is stationary:

$$\Delta y_t \equiv y_t - y_{t-1} \tag{2.25}$$

$$\Delta y_t = \rho \Delta y_{t-1} + \epsilon_t \tag{2.26}$$

Note that with this process a positive output shock $\epsilon_t > 0$ leads to a permanent increase in income. Suppose $\Delta y_{t-1} = 0$ and $\epsilon_t > 0$.
The path of output conditional on expectations at time t is given by:

$$y_t = y_{t-1} + \epsilon_t$$

$$E_t(y_{t+1} - y_t) = \rho(y_t - y_{t-1})$$
$$E_t y_{t+1} = (1+\rho)y_t - \rho y_{t-1} = (1+\rho)(y_{t-1} + \epsilon_t) - \rho y_{t-1} = y_{t-1} + (1+\rho)\epsilon_t$$
$$E_t y_{t+2} = y_{t-1} + (1 + \rho + \rho^2)\epsilon_t$$

...

$$E_t y_{t+\infty} = y_{t-1} + \frac{1}{1-\rho}\epsilon_t$$

It follows that

$$y_t < E_t y_{t+1} < E_t y_{t+1} ... < E_t y_{t+\infty} \tag{2}$$

The key variable $y_t - y_t^p$ with Nonstationary Income

In general (i.e., not using any particular stochastic process for the income process)

$$y_t - y_t^p = -\sum_{j=1}^{\infty} \frac{E_t \Delta y_{t+j}}{(1+r)^j} \tag{2.27}$$

When income follows (2.26), then

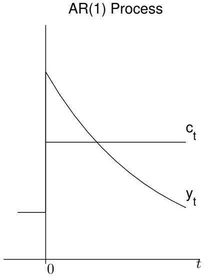
$$E_t \Delta y_{t+j} = \rho^j \Delta y_t$$

and hence (2.27) can be expressed as

$$y_t - y_t^p = -\frac{\rho}{1+r-\rho} \Delta y_t \qquad (***)$$

* This equation says that permanent income increases by more than current income if $\boldsymbol{\rho > 0}$. So, in this case, the growth rate shock is persistent.
* Hence, the level of output is expected to rise in the future. Faced with an increasing income profile HHs want to borrow against future income (thus, countercyclical response to CA)

The figure on the left shows the AR(1) level specification and the figure on the right the AR(1) change specification



Current Account Adjustment with Nonstationary Income

By (2.16) ($tb_t = y_t - y^p + rd_{t-1}$) and (***)

$$tb_t = y_t - y_t^p + rd_{t-1} = -\frac{\rho}{1+r-\rho}\Delta y_t + rd_{t-1}$$

and by (2.15) and (***)

$$ca_t = y_t - y_t^p = -\frac{\rho}{1+r-\rho}\Delta y_t$$

Hence, as long as, $\boldsymbol{\rho > 0}$, the economy with nonstationary income predicts a countercyclical impact response of the trade balance and of the current account.

Why? Agents increase consumption more than income in anticipation of future income increases.

Excess Consumption Volatility with Nonstationary Income

Can the model account for the empirical regularity that consumption changes are more volatile than output changes?
Let

$$\Delta c_t \equiv c_t - c_{t-1}$$

and $\sigma_{\Delta c}$ and $\sigma_{\Delta y}$ denote the standard deviation of consumption and output changes, respectively.
To solve for Δc_t use:

$$ca_t = y_t - c_t - rd_{t-1}$$

Take differences

$$ca_t - ca_{t-1} = \Delta y_t - \Delta c_t - r(d_{t-1} - d_{t-2}).$$

Excess Consumption Volatility with Nonstationary Income

Noting that $d_{t-1} - d_{t-2} = -ca_{t-1}$ and solving for Δc_t, we obtain:

$$\begin{aligned}
\Delta c_t &= \Delta y_t - ca_t + (1+r)ca_{t-1} \\
&= \Delta y_t + \frac{\rho}{1+r-\rho}\Delta y_t - \frac{\rho(1+r)}{1+r-\rho}\Delta y_{t-1} \\
&= \frac{1+r}{1+r-\rho}\Delta y_t - \frac{\rho(1+r)}{1+r-\rho}\Delta y_{t-1} \\
&= \frac{1+r}{1+r-\rho}\ \epsilon_t. \qquad (2.29)
\end{aligned}$$

The change in consumption is a white noise. Why?
By the Euler equation (2.8).

Excess Consumption Volatility with Nonstationary Income

By (2.29)

$$\sigma_{\Delta c} = \frac{1+r}{1+r-\rho}\sigma_{\epsilon}.$$

and by equation (2.26) (and the Wold decomposition)

$$\sigma_{\Delta y} = \frac{\sigma_{\epsilon}}{\sqrt{1-\rho^2}}$$

so that

$$\frac{\sigma_{\Delta c}}{\sigma_{\Delta y}} = \left[\frac{1+r}{1+r-\rho}\right]\sqrt{1-\rho^2} \tag{2.30}$$

- When $\rho = 0$ (i.e., income is a RW), consumption and output changes are equally volatile.
- When $\rho > 0$, consumption changes can become more volatile than output changes.
- To see this: RHS of (2.30) is increasing in ρ at $\rho = 0$.
- Since consumption and output changes are equally volatile at $\rho = 0$, it follows that there are values of ρ in the interval $(0, 1)$ for which the volatility Δc is higher than that of Δy.

Excess Consumption Volatility with Nonstationary Income

- This property ceases to hold as Δy_t becomes highly persistent.
- This is because as $\rho \to 1$, the variance of Δy_t becomes infinitely large as changes in income become a random walk
- Whereas, as expression (2.29) shows, Δc_t follows an i.i.d. process with finite variance for all values of $\rho \in [0, 1]$.

Just in case. Do you remember the Wold decomposition?

If I have $\Delta y_t = \rho \Delta y_{t-1} + \epsilon_t$. As long as $\rho < 1$, Δy is stationary.

The Wold decomposition allows me to transform my AR process into an infinite MA.

I can write: $\Delta y_t = \epsilon_t + \rho \epsilon_{t-1} + \rho^2 \epsilon_{t-2} + \rho^3 \epsilon_{t-3} + ... + \rho^t \epsilon_0$

The variance is $Var(\Delta y) = E[\Delta y_t - E(\Delta y_t)]^2$. But $E(\Delta y_t) = 0$,

So, $Var(\Delta y) = E[(\epsilon_t + \rho \epsilon_{t-1} + \rho^2 \epsilon_{t-2} + ..)(\epsilon_t + \rho \epsilon_{t-1} + \rho^2 \epsilon_{t-2})]$.

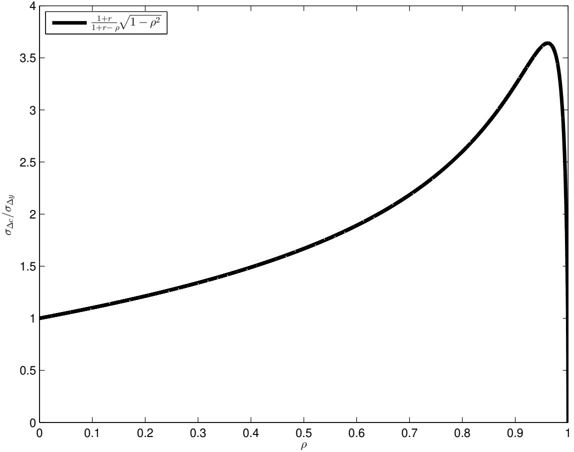
$Var(\Delta y) = E[\epsilon_t^2 + \rho^2 \epsilon_{t-1}^2 + \rho^4 \epsilon_{t-2}^2 + \rho^6 \epsilon_{t-3}^2 + ...]$

$Var(\Delta y) = \sigma_\epsilon^2 [1 + \rho^2 + \rho^4 + \rho^6 + ...]$

$Var(\Delta y) = \frac{\sigma_\epsilon^2}{1-\rho^2} \Rightarrow \sigma_{\Delta y} = \frac{\sigma_\epsilon}{\sqrt{1-\rho^2}}$

Excess Volatility of Consumption Changes and the Persistence of Output Changes

Plot of equation (2.30) as a function of ρ for an interest rate of 4% ($r = 0.04$).



Excess Volatility of Consumption Changes and the Persistence of Output Changes

- When $\rho = 0$ current income is equal to permanent income and the current account is zero at all time (all adjustments happen instantaneously) and consumption moves with income (same volatility)
- For positive values of ρ consumption can become more volatile than the income process

2.5 Testing the Intertemporal Approach to the Current Account (will not do this)

- Hall (1978) initiates a large empirical literature testing the random walk hypothesis for consumption implied by the permanent income hypothesis.
- Campbell (1987) tests the predictions of the permanent income model for savings.
- Nason and Rogers (2006) test the predictions regarding the current account.

In the present model savings and the current accounts are equal (because we are abstracting from investment).

2.5 Testing the Intertemporal Approach to the Current Account

Combining (2.15) and (2.27) yields

$$ca_t = y_t - y_t^p = -\sum_{j=1}^{\infty}(1+r)^{-j}E_t\Delta y_{t+j} \tag{2.28}$$

- This equation says that a country runs a current account deficit (i.e., borrows from the rest of the world), when the present discounted value of future income changes is positive.
- And the country runs a current account surplus (i.e., lends to the rest of the world), when current income exceeds permanent income.
- It also says that current account data should forecast income changes.

Testable Restrictions of the Intertemporal Approach to the Current Account

The testable restriction we focus on is:

$$ca_t = -\sum_{j=1}^{\infty} \frac{E_t \Delta y_{t+j}}{(1+r)^j} \qquad (2.28\ \text{R})$$

- We have observations on the LHS of this equality because we have current account data.
- But we do not have data on the RHS. How can we get an estimate of the RHS?
- The idea is to use a vector autoregression (VAR) model in output changes (Δy_t) and the current account (ca_t) to construct an estimate of the RHS given a calibrated value of r.
- Given the estimate of the RHS, we can test the hypothesis that the LHS is equal to the RHS.

Testable Restrictions of the Intertemporal Approach to the Current Account

To construct an estimate of the RHS, proceed as follows.
Estimate the VAR model:

$$x_t = Dx_{t-1} + \epsilon_t \tag{2.31}$$

where

$$x_t = \begin{bmatrix} \Delta y_t \\ ca_t \end{bmatrix}$$

and D is a 2x2 matrix of coefficients and ϵ_t is a 2x1 mean-zero i.i.d. process.

$H_t =$ information contained in the vector x_t. Forecast of x_{t+j} given H_t:

$$E_t[x_{t+j}|H_t] = D^j x_t$$

Sum the forecasts:

$$\sum_{j=1}^{\infty}(1+r)^{-j}E_t[x_{t+j}|H_t] = \left[I - \frac{D}{1+r}\right]^{-1}\frac{D}{1+r}\begin{bmatrix}\Delta y_t\\ ca_t\end{bmatrix}.$$

The LHS is a 2×1 vector whose first element is $\sum_{j=1}^{\infty}(1+r)^{-j}E_t[\Delta y_{t+j}|H_t]$.
Thus, pre-multiplying this expression by $\begin{bmatrix}1 & 0\end{bmatrix}$ we obtain:

$$\sum_{j=1}^{\infty}(1+r)^{-j}E_t[\Delta y_{t+j}|H_t] = \begin{bmatrix}1 & 0\end{bmatrix}\left[I - \frac{D}{1+r}\right]^{-1}\frac{D}{1+r}\begin{bmatrix}\Delta y_t\\ ca_t\end{bmatrix}.$$

Let

$$F \equiv -\begin{bmatrix}1 & 0\end{bmatrix}\left[I - \frac{D}{1+r}\right]^{-1}\frac{D}{1+r}.$$

Testable Restrictions of the Intertemporal Approach to the Current Account

Use this expression as the estimate of the RHS of (2.28)

$$\widehat{RHS} = F \begin{bmatrix} \Delta y_t \\ ca_t \end{bmatrix}$$

The left-hand-side of (2.28) can be written as:

$$LHS = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} \Delta y_t \\ ca_t \end{bmatrix}$$

The Null hypothesis we wish to test is:

$$H_0 : F = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

* The null hypothesis thus consists of 2 restrictions imposed on (a function of) the coefficients of the matrix D.
* Nason and Rogers (2006) estimate a VAR with 4 lags using Canadian data on the current account and GDP net of investment and government spending. (The above VAR assumes 1 lag; in the next slide we show how to modify the test to 4 lags.) The sample period is 1963:Q1-1997:Q4. They assume that $r = 0.037$ per year.
* Their data strongly **reject the null hypothesis**. They find that the Wald statistic, which reflects the distance between F and $\begin{bmatrix} 0 & 1 \end{bmatrix}$, is 16.1 with an asymptotic p-value of 0.04. This p-value means that if H_0 were true, then the Wald statistic would take a value of 16.1 or higher only 4 out of 100 times.

Testable Restrictions of the Intertemporal Approach to the Current Account

The VAR of Nason and Rogers has 4 lags, but our example above assumed a VAR with one lag. Here we show how to perform the test for a VAR with 4 lags.
Estimate the following VAR(4):

$$\Delta y_t = \sum_{j=1}^{4} a_j \Delta y_{t-j} + \sum_{j=1}^{4} b_j \Delta ca_{t-j} + \epsilon_t^1$$

and

$$ca_t = \sum_{j=1}^{4} c_j \Delta y_{t-j} + \sum_{j=1}^{4} d_j \Delta ca_{t-j} + \epsilon_t^2$$

Testable Restrictions of the Intertemporal Approach to the Current Account

Then let:

$$x_t = \begin{bmatrix} \Delta y_t & \Delta y_{t-1} & \Delta y_{t-2} & \Delta y_{t-3} & ca_t & ca_{t-1} & ca_{t-2} & ca_{t-3} \end{bmatrix}'$$

The vector x_t evolves over time as

$$x_t = Dx_{t-1} + u_t$$

where D is a 8x8 matrix of coefficients

$$D = \begin{bmatrix} a_1 & a_2 & a_3 & a_4 & b_1 & b_2 & b_3 & b_4 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ c_1 & c_2 & c_3 & c_4 & d_1 & d_2 & d_3 & d_4 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

and u_t is the 8x1 vector :

$$u_t = [\epsilon_t^1 \quad 0 \quad 0 \quad 0 \quad \epsilon_t^2 \quad 0 \quad 0 \quad 0]'$$

Testable Restrictions of the Intertemporal Approach to the Current Account

Following the same steps as in the AR(1) case, we obtain.

$$\sum_{j=1}^{\infty}(1+r)^{-j}E_t[\Delta y_{t+j}|H_t] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}\left[I - \frac{D}{1+r}\right]^{-1}\frac{D}{1+r}x_t$$

Let:

$$F \equiv -\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}\left[I - \frac{D}{1+r}\right]^{-1}\frac{D}{1+r}.$$

Then the RHS of 2.28 can be expressed as

$$RHS = Fx_t$$

The LHS of (2.28) can be written as:

$$LHS = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} x_t$$

For the VAR(4) the Null hypothesis becomes:

$$H_0 : F = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

A second testable restriction.

Start again with the following implication of the theoretical model

$$ca_t = -\sum_{j=1}^{\infty} \frac{E_t \Delta y_{t+j}}{(1+r)^j} \tag{2.28 R}$$

and express it as

$$ca_t = \frac{-E_t \Delta y_{t+1}}{1+r} - \sum_{j=2}^{\infty} \frac{E_t \Delta y_{t+j}}{(1+r)^j} = \frac{-E_t \Delta y_{t+1}}{1+r} + \frac{1}{1+r} E_t ca_{t+1}$$

Rearranging yields:

$$E_t \Delta y_{t+1} - E_t ca_{t+1} + (1+r) ca_t = 0$$

Let

$$Z_{t+1} \equiv \Delta y_{t+1} - ca_{t+1} + (1+r) ca_t$$

Note that Z_{t+1} is observable. Regress Z_{t+1} on x_t (or past x_t , the information contained in H_t). If the theoretical model is true, all coefficients on x_t should be zero, that is, is should be true that

$$\left[\begin{bmatrix} 1 & -1 \end{bmatrix} D + (1+r) \begin{bmatrix} 0 & 1 \end{bmatrix} \right] = \begin{bmatrix} 0 & 0 \end{bmatrix}$$

Nason and Rogers find that this hypothesis is rejected with a p-value of 0.06.

Chapter 2: Conclusions

- The propagation mechanism invoked by canonical intertemporal model of the current account does not provide a satisfactory account of the observed current account dynamics.
- For AR(1) output specifications the model predicts the current account to be procyclical, whereas it is countercyclical in the data.
- AR(2) or nonstationary specifications for the income process can in principle imply a countercyclical adjustment of the current account.
- Yet, empirical tests rejects the basic mechanism of the intertemporal model of the current account namely that current accounts are equal to future expected income changes.
- To bring observed and predicted behavior of the current account closer together, in the following chapters we will enrich both the model's sources of fluctuations and its propagation mechanism.