Game Theory

Dynamic Games with Complete Information

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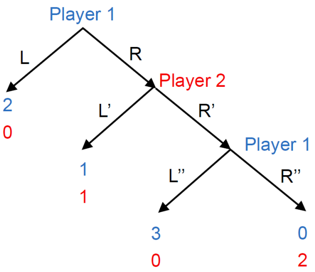
Outline

- Sequential Rationality and Backward Induction
  序贯理性与逆向归纳法
- Subgame Perfect Nash Equilibrium
  子博弈完美纳什均衡
- Extensive v.s. Normal Representation
  博弈的拓展型与标准型
- Multi-stage Game
  多阶段博弈

An Example Using "Backward Induction"

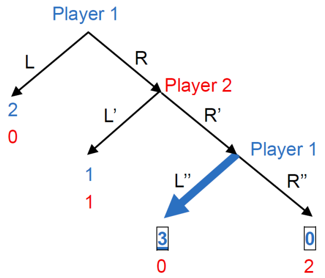
- 5 rational pirates: A, B, C, D, E. They decide to how to distribute 100 coins. Starting from A, each proposes a plan of distribution. If the proposed plan is approved by a majority or tie vote ($\geq 50\%$), then it happens. Otherwise, the proposer is thrown overboard and dies, and the next makes a new proposal to the next round.
- The thought experiment: because all players are completely "rational," they will first infer the actions taken by the player who moves last.
  1. Starting with D and E (since a "tie vote" is sufficient): D proposes $(100, 0)$
  2. C, D, and E: C proposes $(99, 0, 1)$
  3. B, C, D, and E: B proposes $(99, 0, 1, 0)$
  4. A, B, C, D, and E: A proposes $(98, 0, 1, 0, 1)$

Game Tree and Order of Moves

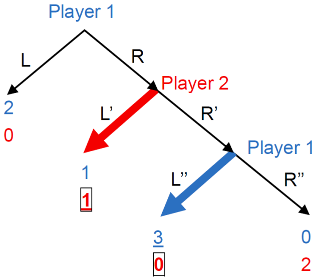


1. Player 1 chooses L or R, where L ends the game with payoffs of 2 to player 1 and 0 to player 2.
2. Player 2 observes 1's choice. If 1 chose R then 2 chooses L' or R', where L' ends the game with payoffs of 1 to both.
3. Player 1 observes 2' choice (and recalls his or her own choice in the first stage). If the earlier choices were R and R' then 1 chooses L'' or R'', both of which end the game, L'' with payoffs of 3 to player 1 and 0 to player 2 and R'' with analogous payoffs of 0 and 2.

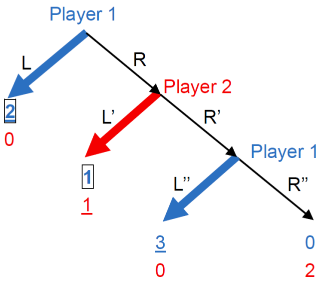
The Backward Induction Outcome



At stage 3 by comparing L'' and R'', player 1 chooses L'' getting payoff 3 (better than R'' which gives 0)



At stage 2 by comparing L' and R', player 2 chooses L' (because $1 > 0$)



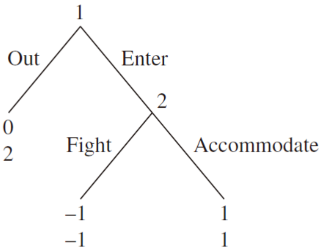
At stage 1 by comparing L and R, player 1 chooses L getting payoff 2 (because $2 > 1$).

- The final outcome is: Player 1 chooses L and the game ends.
- The "three boldfaced arrows" constitutes the "subgame perfect Nash equilibrium."

Extensive-Form Game (博弈的拓展型)

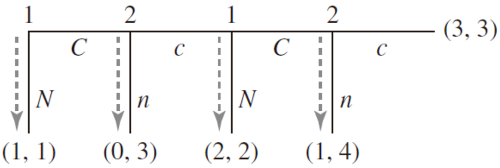
- Set of players
- Players' payoffs as a function of outcomes
- Order of moves
- Actions of players when they can move
- The knowledge that players have when they can move
- Probability distributions over exogenous events
- The structure of the extensive-form game represented above (including this sentence) is common knowledge among all players

Example: Entry Game



- The potential entrant firm (player 1), decides whether or not to enter the market
- The incumbent firm (player 2), decides how to respond to an entry by either fighting or accommodating.
- SPNE: 1 enters and 2 accommodate
- "Fight" is an incredible threat

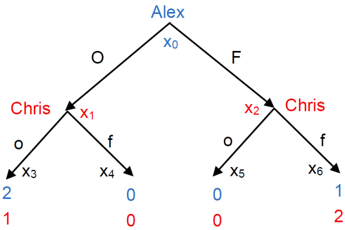
Example: Centipede Game



- Player 1 can terminate the game immediately by choosing N in his first node or can continue by choosing C. Player 2 faces the same choice, nad if player 2 chooses to continue then the ball is back in player 1's court, who again can terminate and continue...
- SPNE: player 1 chooses N at first stage
- "Curse of Rationality"

Example: Battle of Sexes with First-Mover Advantage

		Chris o	Chris f
Alex	O	2,1	0,0
	F	0,0	1,2

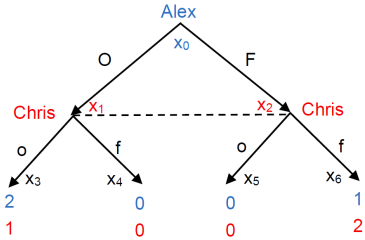


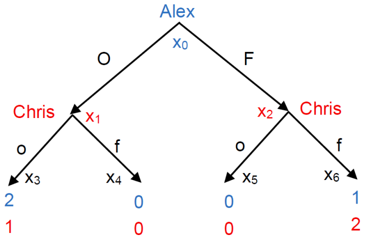
Left-side table: Alex and Chris move simultaneously. Then two pure-strategy Nash equilibrium.

Right-side game tree: Alex moves first

- The backward-induction outcome for Alex is better than the outcome in simultaneous-move game.
- The first-mover advantage (先行者优势) comes from restricting Alex's choices (" 先斩后奏").

Extensive-Form Representation of a Static Game





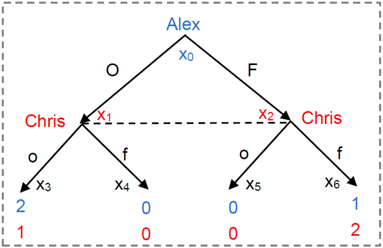
Left-side: The simultaneous-move game can be represented by an extensive-form

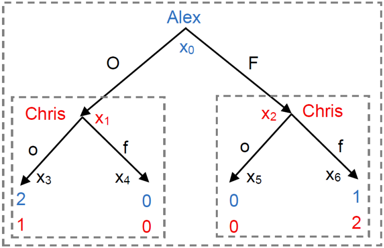
- Information set （信息集）: $\{x_1, x_2\}$, i.e., x_1 and x_2 can not be distinguished

Right-side: Player 1 moves first

- Information set: $\{x_1\}, \{x_2\}$, i.e., player 2 knows exactly where he/she stands at x_1 or at x_2.

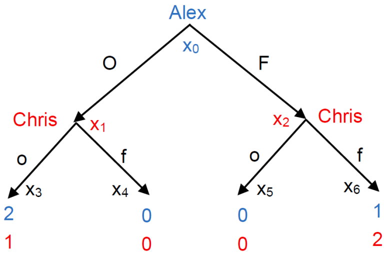
Subgames (子博弈)





- Nodes: x_0, x_1,..., x_6
- Proper subgame:
  - One proper subgame (left graph): starting from x_0
  - Three proper subgames (right graph): the whole game starting from x_0; and two proper subgames starting from x_1 and x_2, respectively.

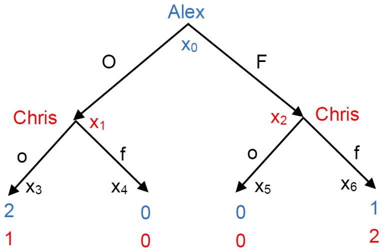
Pure Strategies in Extensive-Form Games



A pure strategy for player i is a **complete plan of play** that describes which pure action player i will choose at each of his/her information set (node).

- Player 1's choices at node x_0: $\{O, F\}$
- Player 2 makes a "plan" by listing all possible combinations of actions chosen at node x_1 and x_2, respectively: $\{oo, of, fo, ff\}$, where the first (resp., second) letter denotes player 2's action at node x_1 (resp., x_2).

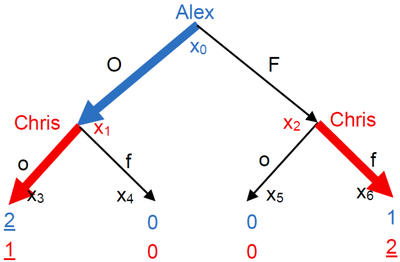
Normal-Form Representation of Extensive-Form Games



		Chris $(x_1 x_2)$			
		oo	of	fo	ff
Alex	O	2,1	2,1	0,0	0,0
(x_0)	F	0,0	1,2	0,0	1,2

- Transferring extensive-form into the normal form seems to miss the dynamic feature
- The concept of Nash equilibrium is static in nature
  - Players take the strategies of others as given, and in turn they play a best response.

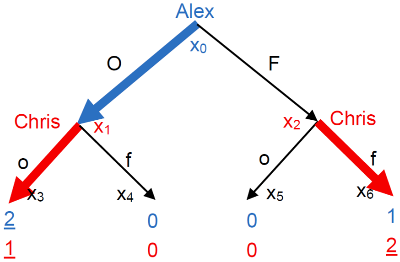
Nash Equilibrium



		Chris (x_1x_2)			
		oo	of	fo	ff
Alex	O	$\underline{2},\underline{1}$	$\underline{2},\underline{1}$	$\underline{0},0$	0,0
(x_0)	F	0,0	$1,\underline{2}$	$\underline{0},0$	$\underline{1},\underline{2}$

- Three pure-strategy Nash equilibrium: (O, oo), (O, of) and (F, ff)
  - By definition: all the three are best responses.
  - (O, oo): Chris "can choose to" to play o at node x_2 while she knows that x_2 will not be reached. Check: Alex will not deviate.
  - (O, of): Chris "plans" to play f as long as x_2 is reached (although x_2 is not actually reached). Check: Alex will not deviate.
  - (F, ff): Chris "can choose to" play f at node x_1 while x_2 is actually reached.
- Complete plan of actions, on and off the equilibrium paths.

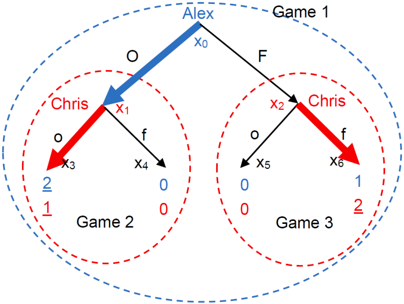
Sequential Rationality and Backward Induction



		Chris (x_1x_2)			
		oo	of	fo	ff
Alex	O	$\underline{2},\underline{1}$	$\mathbf{\underline{2},\underline{1}}$	$\underline{0},0$	0,0
(x_0)	F	0,0	1,$\underline{2}$	$\underline{0},0$	$\underline{1},\underline{2}$

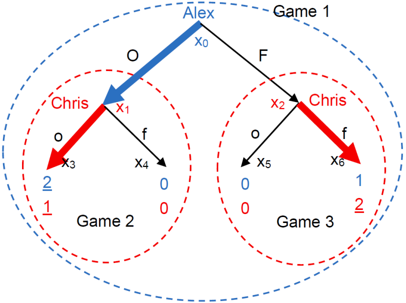
- Are (O, oo) and (F, ff) sequentially rational?
- We know the backward induction outcome should be (O, of)
- We need a new solution concept with respect to dynamic games—Subgame Perfect Nash Equilibrium (SPNE)—a refinement of Nash equilibrium that survives backward induction (逆向归纳) and is sequentially rational (序贯理性).

Subgame Perfect Nash Equilibrium (SPNE)



		Chris ($x_1 x_2$)			
		oo	of	fo	ff
Alex (x_0)	O	2,1	**2,1**	0,0	0,0
	F	0,0	1,2	0,0	1,2

- Concept: Nash equilibrium in **every** proper subgame, i.e., best responses
  - not only on the equilibrium path
  - but also off the equilibrium path (including those subgames that are not reached in equilibrium)
- Among the 3 Nash equilibrium, only (O, of) is SPNE.
  - SPNE **refines** NE. 精炼

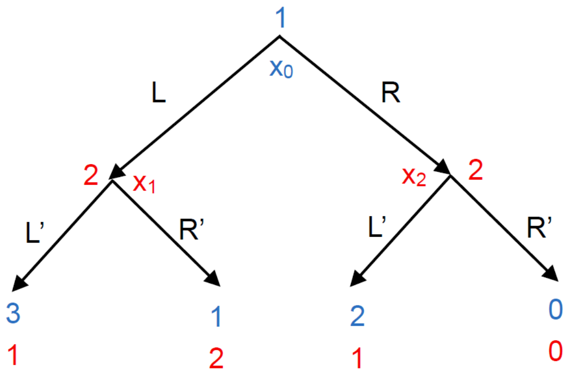


Chris (x_1x_2)

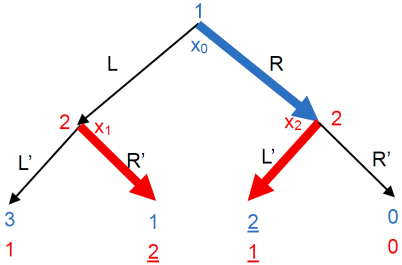
		oo	of	fo	ff
Alex	O	2,1	**2,1**	0,0	0,0
(x_0)	F	0,0	1,2	0,0	1,2

- (O, oo): the second *o* in *oo* is not a best response off the equilibrium path (game 3).
- (F, ff): the first *f* in *ff* is not a best response in game 2 (when Alex chooses *F*, the equilibrium path becomes game 1 $\rightarrow$ game 3; game 2 is off the equilibrium path).
- Only (O, of) are the best responses in game 1, 2 and 3, and coincides with the set of NE that survive backward induction. Therefore, (O, of) is a SPNE.

Example

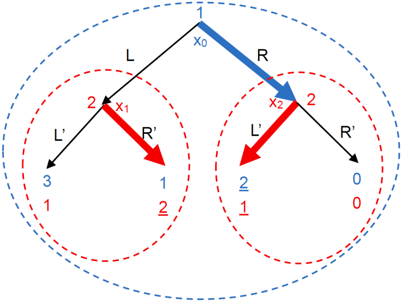


1. 1 chooses an action from $\{L, R\}$
2. 2 observes 1's action and then chooses from $\{L', R'\}$



		2			
		L'L'	L'R'	R'L'	R'R'
1	L	3,1	3,1	1,2	1,2
	R	2,1	0,0	2,1	0,0

- A complete plan of play:
  - Player 1's plan node x_0: L or R
  - Player 2's plan of play at node x_1 and x_2:
    - $(L'L')$: play L' at x_1 and L' at x_2;
    - $(L'R')$: play L' at x_1 and R' at x_2;
    - $(R'L')$: play R' at x_1 and L' at x_2;
    - $(R'R')$: play R' at x_1 and R' at x_2.
- Two pure-strategy Nash equilibrium (player 1, player 2)
  - $(L, R'R')$
  - $(R, R'L')$

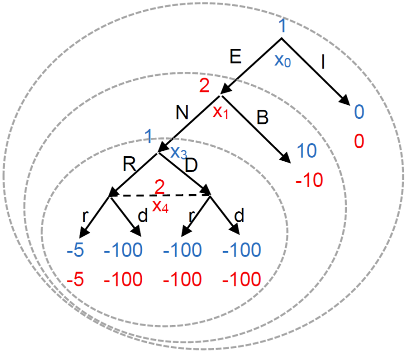


		2			
		L'L'	L'R'	R'L'	R'R'
1	L	$\underline{3},1$	$\underline{3},1$	$1,\underline{2}$	$\underline{1},\underline{2}$
	R	$2,\underline{1}$	0,0	$\mathbf{\underline{2},\underline{1}}$	0,0

- SPNE: $(R, R'L')$.
  - Bold path: including $R \to L'$ on the equilibrium path; and $L \to R'$ that is not reached at equilibrium (off equilibrium path)
  - $(R, R'L')$ is an Nash equilibrium of every proper subgame
- NE but not SPNE: $(L, R'R')$
  - Indeed a best response
  - Not an equilibrium in each proper subgame

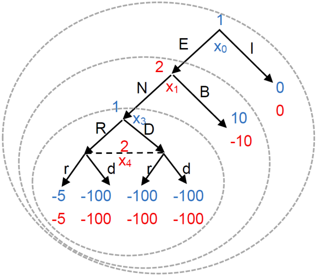
Example: Mutually Assured Destruction

- Cuban missile crisis of 1962:
  - U.S. found Soviet nuclear missiles in Cuba. U.S. escalated the crisis by quarantining Cuba. The USSR then backed down, agreeing to remove its missiles from Cuba.
- Could the suggest “if you don’t back off we both pay dearly” be a credible threat?



- Player 1: U.S.; Player 2: USSR
  1. 1 chooses to ignore I (with 0 each); or escalate the situation E
  2. 2 can back down B (losing face -10) or proceed to a nuclear confrontation N
  3. War stage (simultaneous-move): retreat (r and R) gives -5; or choose Doomsday (D or d) that gives -100.

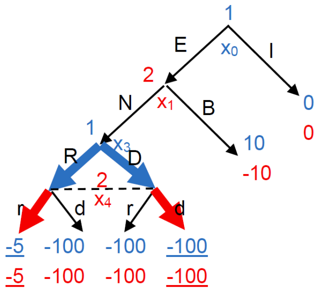
Normal-Form Representation:



		Player 2 (x_1x_4)			
		Br	Bd	Nr	Nd
Player 1 (x_0x_3)	IR	0, 0	0, 0	**0, 0**	0, 0
	ID	0, 0	0, 0	0, 0	0, 0
	ER	10, -10	10, -10	-5, -5	-100,-100
	ED	10, -10	**10, -10**	-100,-100	-100,-100

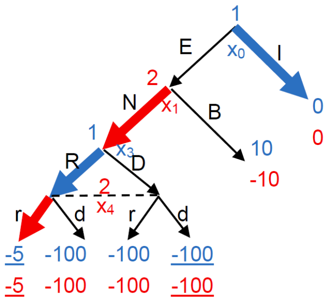
- 1's strategy set: $\{IR, ID, ER, ED\}$, where the first letter denotes the action taken at the beginning step; the second letter denotes the action taken when the "war-stage" is reached.
- 2's strategy set: $\{Br, Bd, Nr, Nd\}$, where the first letter denotes the action taken at the second step; the second letter denotes the action taken at the "war-stage."
- Six pure-strategy Nash equilibrium
- Next: solve SPNE

The War Stage



		Player 2 (x_4)	
		r	d
Player 1 (x_3)	R	**-5, -5**	-100, -100
	D	-100, -100	**-100, -100**

- There are three proper subgames: (1) the whole game; (2) starting from player 2 chooses B or N; (3) war stage
- The war stage: simultaneous-move. Two pure-strategy Nash equilibria
  1. NE1: (R, r) both retreat
  2. NE2: (D, d) Doomsday
- To proceed, consider two possible NEs as two cases.



- Case 1: for NE1 at war stage, (IR, Nr)
- Case 2: for NE2 at war stage, (ED, Bd)
- Two SPNE: (1) 1 chooses *I* (0) because 1 believes that if not then 2 will choose *N* and both *r* (-5); (2) 1 chooses *E* because 1 believes that 2 will treat this as a signal that 1 is willing to "go all the way"—both play the Doomsday—and 2 who shares the same beliefs, will back off.
- In both cases, the war game is *off the equilibrium path*. Nonetheless it is the expected behavior in the last and final stage that dictates how players will play.
- E.g., second-strike from survival forces (ballistic missile submarines)

Sequential Bargaining

- Two players 1 and 2 are bargaining over one dollar. They alternate in making offers: first player 1 makes a proposal that player 2 can accept or reject; if player 2 rejects then 2 makes a proposal that 1 can accept or reject; and so on.
- Each offer takes one period. Players are impatient. They discount payoffs received in later periods by the factor δ per period.
- Assume 3 periods:
  1. $t = 1$: 1 proposes to take a share s_1; $1 - s_1$ for 2; if 2 rejects, then
  2. $t = 2$: 2 proposes to give s_2 to player 1 (hence $1 - s_2$ to himself/herself); if 1 rejects, then
  3. $t = 3$: 1 receives a share s_3, leaving $1 - s_3$ for 2.

Backward induction:

1. At $t = 3$, player 1 gets $s_3 = 1$.
2. At $t = 2$, stage-3's payoff s_3 worths δs_3 at stage 2. 1 will accept s_2 if and only if $s_2 \geq \delta s_3$. Player 2 gets $1 - \delta s_3$
3. At $t = 1$, 1 knows that 2 can receive $1 - s_2$ in the next round by rejecting 1's offer now. Receiving $1 - s_2$ worths $\delta(1 - s_2)$ now. Providing 2 with $\delta(1 - s_2)$ will end the game at stage 1. Player 1 gets $s_1 = 1 - \delta(1 - s_2)$.

- $s_3^* = 1 \Rightarrow s_2^* = \delta s_3^* = \delta \Rightarrow s_1^* = 1 - \delta(1 - s_2^*) = 1 - \delta(1 - \delta)$
- 1 proposes $(s_1^*, 1 - s_1^*)$ at stage 1 and 2 will accept.
- If the game lasts for infinite rounds, by induction, observe that

$$s_1^* = 1 - \delta + \delta^2 - \delta^3 + \ldots = \frac{1}{1 + \delta}$$

Example: Wages and Employment in a Unionized Firm*[1]

- A union and a firm. The union prefers high wage w and high employment L. The union's utility $U(w, L)$ is increasing and concave in both arguments.
- Given w, the firm chooses to hire labor L to maximize profit $\pi(w, L) = R(L) - wL$, where $R(L)$ is the revenue function that is increasing and concave.
- First, the union makes a wage demand; Second, observing w, the firm makes hiring decisions L.
- Using backward induction, at stage 2, the firm solves

$$\max_{L} R(L) - wL \Rightarrow R'(L^{BR}) = w$$

$L^{BR}(w)$ is a best response of w.

[1] Not required

- Back to the stage 1: the union chooses w^* subjected to $L^{BR}(w)$ such that the indifferent curve (w, L) reaches the highest possible level.
- SPNE: $(w^*, L^{BR}(w^*))$ and $R'(L^{BR}) = w^*$.
- Is SPNE socially efficient? Consider a benevolent planner who tries to maximize the sum of the payoffs of the two parties: $W = R(L) - wL + U(w, L)$

$$\max_{w,L} R(L) - wL + U(w, L)$$

If L increases a little bit, then $\frac{\partial W}{\partial L} = R'(L) - w + U'_L$.
- However, if we plug the Nash outcome $(w^*, L^{BR}(w^*))$ into the first-order derivative:

$$\left.\frac{\partial W}{\partial L}\right|_{R'(L^{BR})=w^*} = \underbrace{R'(L^{BR}) - w^*}_{=0} + U'_L(w^*, L^{BR}) > 0,$$

which implies that, starting from the SPNE, a further increase of L will be welfare-improving.

Two-Stage Repeated Game (重复博弈)

- Recall the Prisoner's dilemma, where M and F are replaced by R and L. And payoffs are replaced by positive numbers.

		Player 2	
		L_2	R_2
Player 1	L_1	$\underline{1},\underline{1}$	5,0
	R_1	0,5	4,4

- For one-shot game, the unique Nash equilibrium is (L_1, L_2). However, (R_1, R_2) is the socially efficient outcome.
- Suppose the game is played twice. Using backward induction, it is clear that (L_1, L_2) is the second-stage equilibrium is the first-stage equilibrium.

	L_2	R_2
L_1	$\underline{1}$,$\underline{1}$	5,0
R_1	0,5	4,4

The game at stage two: Nash equilibrium (L_1, L_2)

	L_2	R_2
L_1	$\underline{1+1}$,$\underline{1+1}$	$\underline{5+1}$,0+1
R_1	0+1,$\underline{5+1}$	4+1,4+1

The "entire game" from the view of the first stage.

- When the game is played twice, we use the right-side graph to represent the "entire game."
- Because the second-stage outcome is (L_1, L_2) which gives $(1, 1)$, hence $(1, 1)$ is added to each payoff pair.
- The unique SPNE is (L_1, L_2) in both stages. And cooperation (R_1, R_2) can not be achieved.
- If the stage game G has a **unique** Nash equilibrium, then for **finite** periods T, the repeated game $G(T)$ has a unique SPNE: the Nash equilibrium of G is played in every stage.

Multiple Equilibria Case

- Consider the following game with multiple Nash equilibria:

	L_2	M_2	R_2
L_1	$\underline{1},\underline{1}$	$\underline{5},0$	0,0
M_1	$0,\underline{5}$	4,4	0,0
R_1	0,0	0,0	$\underline{3},\underline{3}$

  - Two NE: (L_1, L_2) and (R_1, R_2)
- Suppose the game is played twice: The second-stage NE will be (L_1, L_2) or (R_1, R_2), then (M_1, M_2) could be a SPNE played at stage 1.
  - Observe that (M_1, M_2) is better than (L_1, L_2) and (R_1, R_2).
  - In order to achieve (M_1, M_2) at stage 1, consider whether the following "threat" is credible:
    "If (M_1, M_2) is played at stage 1, then we play (R_1, R_2) at stage 2; Otherwise, we play (L_1, L_2) at stage 2 if any of the other 8 outcomes occurs at stage 1."

	L_2	M_2	R_2
L_1	1, 1	5, 0	0, 0
M_1	0, 5	4, 4	0, 0
R_1	0, 0	0, 0	3, 3

Second-stage game

	L_2	M_2	R_2
L_1	1+1,1+1	5+1,0+1	0+1,0+1
M_1	0+1,5+1	4+3,4+3	0+1,0+1
R_1	0+1,0+1	0+1,0+1	3+1,3+1

The entire game from the view of stage 1

- For the anticipation: play R after M; play L if the stage-1 outcome is not M, then the payoff of the entire game can be represented by the right side graph.
- Three Nash equilibrium in the right side (hence three SPNE of the entire game):
  - (L_1, L_2): corresponds to SPNE $((L_1, L_2), (L_1, L_2))$
  - (R_1, R_2): corresponds to SPNE $((R_1, R_2), (L_1, L_2))$
  - (M_1, M_2) is a qualitatively different result: SPNE $((M_1, M_2), (R_1, R_2))$
- Renegotiation is not considered here.

Finitely vs. Infinitely Repeated Game (有限/无限次重复博弈)

		Player 2	
		L_2	R_2
Player 1	L_1	$\underline{1},\underline{1}$	$\underline{5},0$
	R_1	$0,\underline{5}$	$4,4$

- For **finite** periods, (L_1, L_2) is NE for each period.
    - At stage n, both choose (L_1, L_2).
    - At stage $n-1$, both choose (L_1, L_2).
    - ...
    - Play (L_1, L_2) in **every** period.
- Consider whether the "cooperative outcome" (R_1, R_2) can be achieved if the game is played for **infinite** times.
  Define δ the discount factor.
- They adopt "grim-trigger" strategy （触发策略）: history dependent.

Grim-Trigger Strategy (触发策略)

- We introduce two components in the infinitely repeated game.
  - Preference for "future:" define $\delta \in [0, 1)$ the discount factor. \$10 obtained in the next period is valued by $\delta \cdot 10$ currently. \$10 obtained in the next next period is valued by $\delta \cdot 10$ in the next period, and hence is valued by $\delta \cdot (\delta \cdot 10) = \delta^2 \cdot 10$ currently.
  - History-dependent strategy: "grim-trigger." What I will play depends on the what we have done in the previous rounds.

		Player 2	
		L_2	R_2
Player 1	L_1	$\underline{1},\underline{1}$	$\underline{5}$,0
	R_1	0,$\underline{5}$	4,4

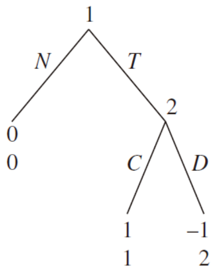
- If one player believes that the opponents' behavior is independent of history, then there can be no role for considering how current play affects future play $\Rightarrow$ unconditionally repeat (L_1, L_2).
- Grim-trigger: Play R if R is achieved in the previous round; otherwise, play L forever.

		Player 2	
		L_2	R_2
Player 1	L_1	$\underline{1},\underline{1}$	$\underline{5},0$
	R_1	$0,\underline{5}$	$4,4$

- Grim-trigger: Play R if R was played in the previous round; otherwise, play L forever.
  - Keep playing R gives 4 each round;
  - One-shot deviation gives 5, but gets only 1 in each round in future.
- From an arbitrage stage:
  - Stick with R gives a stream of future value: $4 + 4\delta + 4\delta^2 + \cdots = \frac{4}{1-\delta}$
  - One-shot deviation gives $5 + 1 \cdot \delta + 1 \cdot \delta^2 + \cdots = 5 + \frac{\delta}{1-\delta}$
- Cooperative outcome (R_1, R_2) is achieved if

$$\underbrace{\frac{4}{1-\delta}}_{\text{cooperation}} \geq \underbrace{5}_{\text{one-shot cheating}} + \underbrace{\frac{\delta}{1-\delta}}_{\text{never cooperate}} \Rightarrow \delta \geq \frac{1}{4}$$

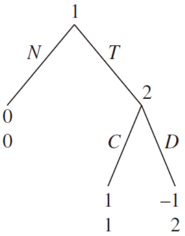
Example: Trust Game and Reputation



1. At stage 1, player 1 chooses to trust (T) or not trust (N) player 2. If 1 chooses N, the game ends.
2. If 1 trusts 2, 2 chooses to cooperate (C) or defect (D).

- For one-shot game, the SPNE is 1 chooses N, which is not "socially efficient"

Suppose the game repeats infinitely times



- Grim-trigger:
  - 1 trusts 2, and if there's no deviations from (T, C), then trust him/her again; otherwise never trust him/her: trust forever implies $\frac{1}{1-\delta} \geq 0$
  - 2 cooperate, and if there's no deviations from (T, C), then cooperate again; otherwise never cooperate: cooperate forever implies $\frac{1}{1-\delta} \geq 2 + 0 \cdot \frac{\delta}{1-\delta} \Rightarrow \delta \geq \frac{1}{2}$.
- The cooperative outcome is achieved if $\delta \geq \frac{1}{2}$ (both players confirm that the two inequalities hold simultaneously).

Application: Time-Consistent Monetary Policy*[2]

- Effectiveness of monetary policy in macroeconomics:
  - Keynesian: fiscal policy
  - Neoclassical: rational expectations
  - Nominal rigidity and unexpected inflation
- The "surprise inflation" helps (rules v.s. discretion)
  1. At stage 1, the employers form an expectation of inflation π^e
  2. At stage 2, the monetary authority chooses inflation level π
- Payoff of employer: $-(\pi - \pi^e)^2$ (correctly estimates inflation to maintain zero profit)
- Payoff of the monetary authority: $-c\pi^2 - (y - y^*)^2$
  - prefer low inflation
  - "efficient" output y^*
  - The actual output is a function of target output y^* and surprise inflation: $y = by^* + d(\pi - \pi^e)$, where $b < 1$ and $d > 0$.

[2]Not required

- At stage 2, the monetary authority solves

$$\max_{\pi} W(\pi, \pi^e) = -c\pi^2 - [(b-1)y^* + d(\pi - \pi^e)]^2$$

$$\frac{dW}{d\pi} = 0 \Rightarrow \pi^{BR}(\pi^e) = \frac{d^2}{c+d^2}\pi^e + \frac{d(1-b)}{c+d^2}y^*$$

- At stage 1, the employers maximize $-(\pi^*(\pi^e) - \pi^e)^2$, which gives $\pi^{BR}(\pi^e) = \pi^e$. Hence, the inflation for a finite stage game is

$$\pi^e = \frac{d(1-b)}{c}y^* \equiv \pi_s$$

- Therefore, the Nash outcome for a finite game is $\pi = \pi^e = \pi_s$, and the payoff of the authority is $W(\pi_s, \pi_s) = -c\pi_s^2 - (b-1)^2 y^{*2}$. The payoff for the employer is 0.
- However, the authority can be better off if $\pi = \pi^e = 0$, which gives $W(0,0) = -(b-1)^2 y^{*2} > W(\pi_s, \pi_s)$.
- The prisoners' dilemma.

- Consider the infinitely repeated game with discount factor δ:
  - Employers hold expectations $\pi^e = 0$ if $\pi = 0$ in all previous rounds
  - If the monetary authority deviates to $\pi^{BR}(\pi^e) = \pi^{BR}(0)$ once, then it lead to $\pi^e = \pi_s$ forever.
- Keep promise (rule): $W(0,0) \cdot \frac{1}{1-\delta}$
- One-shot deviation (discretion): $W(\pi^*(0), 0) + W(\pi_s, \pi_s)\frac{\delta}{1-\delta}$
- Keeping the rule instead of discretion provided that

$$\underbrace{W(0,0) \cdot \frac{1}{1-\delta}}_{\text{rules}} \geq \underbrace{W(\pi^*(0), 0)}_{\text{surprise inflation}} + \underbrace{W(\pi_s, \pi_s)\frac{\delta}{1-\delta}}_{\text{ineffective monetary policy}}$$

  - $\Rightarrow \delta \geq \frac{c}{2c+d^2}$

What can you learn from this lecture?

- 多次博弈，从最后一个行动的人的角度考虑问题
- “日久”见人心：合作与否取决于长期还是短期
    - 要考虑当前的事态是否是“一锤子买卖”；还是以后“抬头不见低头见”
- “狼来了的故事”：只有“老实人”才能骗人
    - 维持信誉靠长期，毁于一旦