Lecture 2: Trade with Firms

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1 Introduction

In the previous lecture, we saw how we could justify the gravity relationship in trade using the ad-hoc assumption that every country produces a unique good as well as the assumption that consumers have a "love of variety" (i.e. they want to consume at least a little bit of every one of the goods). In this lecture, we will dispense of the first assumption by introducing firms into the model. However, we will continue to rely heavily on the second assumption by assuming that each firm produces a unique variety and consumers would like to consume at least a little bit of every variety.

The model considered today was introduced by Krugman (1980) and was an important part of the reason he won a Nobel prize. As we will see next week, the model can be generalized even further to consider heterogeneous firms. This was the insight of the incredibly influential paper by Melitz (2003), which, according to Google Scholar, has since publication averaged two citations a day!

2 Model Set-up

Let us now turn to the set-up of the model.

2.1 The world

As in the Armington model, suppose there is a compact set S of countries, where I will keep the notation that i is an origin country and a j is a destination country. Also as in the Armington model, each country $i \in S$ will be populated by an exogenous measure L_i of workers/consumers, where as before, each worker supplies her unit of labor inelastically. As in the Armington model, suppose that labor is the only factor of production.

2.2 Supply

Unlike the Armington model, we no longer assume that each country produces a unique variety. Instead, suppose that there is a continuum Ω of possible varieties that the world can produce, and suppose that every firm in the world produces a distinct variety $\omega \in \Omega$. Because there is a one to one mapping between firms and varieties, we can label each firm by the variety it produces. Let the set of varieties produced by firms located in country $i \in S$ be denoted by $\Omega_i \subset \Omega$.

A key feature of the Krugman (1980) model is that there are **increasing returns to scale**, i.e. the average cost of production is lower the more that is produced. All else equal, this will lead to gains from trade, since by taking advantage of demand from multiple countries, firms can lower their average costs. To succinctly model the increasing returns from scale, we suppose that a firm has to incur a fixed entry cost f_i^e in order to produce. (The e might seem like unnecessary notation; however, we keep it here because in future models there will be both an entry cost and a fixed cost of serving a particular destination). We assume the fixed cost of entry (like the marginal cost) is paid to domestic workers so that f_i^e is the number of workers employed in the entry sector (think of them as the workers who build the firm).

After the firm has payed the fixed cost, we assume it is subject to a constant marginal cost of production for every unit produced. Let the productivity of all firms producing in country i be homogeneous and denoted by A_i, i.e. it costs $\frac{1}{Ai}$ units of labor to produce one unit of the differentiated variety. (Next week, we will see what happens when the productivity differs across firms within a country).

Finally, as in the Armington model, we suppose that firms are subject to iceberg trade costs $\{\tau_{ij}\}_{i,j \in S}$.

2.3 Demand

As in the Armington model, we assume that consumers have CES preferences over varieties. Hence a representative consumer in country $j \in S$ gets utility U_j from the consumption of goods shipped by all other firms in all other countries, where:

$$U_j = \left(\sum_{i \in S} \int_{\Omega_i} q_{ij} \left(\omega\right)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}}, \tag{1}$$

where $q_j(\omega)$ is the quantity consumed in country j of variety ω. Note that for simplicity, I no longer include a preference shifter (although one could easily be incorporated) so that consumers treat all firms in all countries equally.

3 Gravity

We now show how such a model can yield a gravity equation.

3.1 Optimal demand

The consumer's utility maximization problem is very similar to the Armington model (which shouldn't be particularly surprising, given preferences are virtually the same). In particular, a consumer in country $j \in S$ optimal quantity demanded of good $\omega \in \Omega$ is:

$$q_{ij}(\omega) = p_{ij}(\omega)^{-\sigma} Y_j P_j^{\sigma-1},$$

where:

$$P_j \equiv \left(\sum_{i \in S} \int_{\Omega_i} p_{ij}(\omega)^{1-\sigma} d\omega \right)^{\frac{1}{1-\sigma}} \tag{2}$$

is the Dixit-Stiglitz price index.

The amount spent on variety ω is simply the product of the quantity and the price:

$$x_{ij}(\omega) = p_{ij}(\omega)^{1-\sigma} Y_j P_j^{\sigma-1}. \tag{3}$$

Note that we derived a very similar expression in the Armington model, from which the gravity equation followed almost immediately. In this model however, this is the amount spent on the goods from a particular firm, so we now need to aggregate across all firms in country i to determine bilateral trade flows between i and j, i.e.:

$$X_{ij} \equiv \int_{\Omega_i} x_{ij}(\omega) d\omega = Y_j P_j^{\sigma-1} \int_{\Omega_i} p_{ij}(\omega)^{1-\sigma} d\omega. \tag{4}$$

3.2 Optimal supply

We now determine the equilibrium prices that firms set. This is nearly identical to the derivation for the monopolistic competition structure of the Armington model. Let $c_i \equiv \frac{w_i}{A_i}$ denote the marginal cost of production by a firm.The optimization problem faced by firm ω is:

$$\max_{\{q_{ij}(\omega)\}_{j \in S}} \sum_{j \in S} \left(p_{ij}(\omega) q_{ij}(\omega) - c_i \tau_{ij} q_{ij}(\omega) \right) - w_i f_i^e \text{ s.t. } q_{ij}(\omega) = p_{ij}(\omega)^{-\sigma} Y_j P_j^{\sigma-1} \ \forall j \in S$$

so that, conditional on positive production (more on that below), the first order conditions imply:

$$p_{ij}(\omega) = \frac{\sigma}{\sigma-1} c_i \tau_{ij} \tag{5}$$

Because every firm is charging the same price [Class question: would this be true if firms had different productivities?], we can substitute the price equation (5) into the gravity equation (4) to yield:

$$X_{ij} = Y_j P_j^{\sigma-1} \int_{\Omega_i} \left(\frac{\sigma}{\sigma-1} c_i \tau_{ij} \right)^{1-\sigma} d\omega \iff$$

$$X_{ij} = \left(\frac{\sigma}{\sigma-1} \right)^{1-\sigma} \tau_{ij}^{1-\sigma} \left(\frac{w_i}{A_i} \right)^{1-\sigma} M_i Y_j P_j^{\sigma-1} \tag{6}$$

where $M_i \equiv \int_{\Omega_i} d\omega$ is the measure of firms producing in country i. Comparing this equation to the one derived for the Armington model with monopolistic, we see that the two expressions are nearly identical - the only difference here is that we have to keep track of the mass of firms M_i.

3.3 Free entry

The final thing we have to do is determine the equilibrium number of firms that enter. In this model the mass of firms M_i is determined by the **free entry condition** which states that the profits of all firms must be equal to zero. The justification for this condition is that there is a large mass of potential firms (or equivalently, other differentiated products that could be produced), who choose not to enter. [Class question: why do they not enter?][1]

Hence, to determine the equilibrium mass of firms, we need to calculate the profits of any particular firm. Firms profits are:

$$\pi_i(\omega) \equiv \sum_j \left(p_{ij}(\omega) - c_i \tau_{ij}\right) q_{ij}(\omega) - w_i f_i^e \tag{7}$$

Substituting the consumer demand expression (1) and the price expression (5) into equation (7) yields:

$$\begin{aligned}
\pi_i(\omega) &= \sum_j \left(\frac{\sigma}{\sigma-1} c_i \tau_{ij} - c_i \tau_{ij}\right) \left(\frac{\sigma}{\sigma-1} c_i \tau_{ij}\right)^{-\sigma} Y_j P_j^{\sigma-1} - w_i f_i^e \iff \\
\pi_i(\omega) &= \sum_j \left(\left(\frac{\sigma}{\sigma-1} - 1\right)\right) \left(\frac{\sigma}{\sigma-1}\right)^{-\sigma} \left(c_i \tau_{ij}\right)^{1-\sigma} Y_j P_j^{\sigma-1} - w_i f_i^e \iff \\
\pi_i(\omega) &= \sum_j \left(\frac{1}{\sigma-1}\right) \left(\frac{\sigma}{\sigma-1}\right)^{-\sigma} \left(c_i \tau_{ij}\right)^{1-\sigma} Y_j P_j^{\sigma-1} - w_i f_i^e \iff \\
\pi_i(\omega) &= \frac{1}{\sigma} \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} \sum_j \left(c_i \tau_{ij}\right)^{1-\sigma} Y_j P_j^{\sigma-1} - w_i f_i^e
\end{aligned}$$

It turns out that in this framework, the profits of a firm have a simple relationship to the quantity the firm produces, which greatly simplifies the equilibrium. To see this, we first relate the profits a firm to its revenues. Note that from equation (3) and the price expression (5) that the revenue a producer receives is:

$$r_i(\omega) \equiv \sum_{j \in S} p_{ij}(\omega) q_{ij}(\omega) = \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} \sum_{j \in S} \left(c_i \tau_{ij}\right)^{1-\sigma} Y_j P_j^{\sigma-1}$$

[1] To see that the entry of additional firms pushes down the profits of any particular firm, note that combining expression (2) for the Dixit-Stiglitz price index with the price expression (5) from the producers optimization problem yields:

$$P_j \equiv \frac{\sigma}{\sigma-1} \left(\sum_{i \in S} M_i \left(c_i \tau_{ij}\right)^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$$

Hence, an increase in the number of firms producing in any country reduces the price index, thereby decreasing profits. [Class question: what is the intuition for why more firms lowers the price index?]

so that variable profits are simply equal to revenue divided by the elasticity of substitution, i.e.:

$$\pi_i(\omega) + w_i f_i^e = \frac{1}{\sigma} r_i(\omega). \tag{8}$$

[Class question: what is the intuition of this result?].

From the price equation (5), if we assume that $\tau_{ii} = 1$, we can decompose the total revenue produced by a firm into the total quantity it produces and the price:

$$r_i(\omega) = p_i(\omega) q_i(\omega) = \frac{\sigma}{\sigma - 1}\left(\frac{w_i}{A_i}\right) q_i(\omega), \tag{9}$$

where I imposed the fact that the marginal cost $c_i = \frac{w_i}{A_i}$.

From the free entry condition, total profits of a firm are zero, i.e. $\pi_i(\omega) = 0$. Applying the free entry condition to equation (8) yields:

$$w_i f_i^e = \frac{1}{\sigma} r_i(\omega) \tag{10}$$

Substituting equation (9) into (10) then yields:

$$\begin{aligned} w_i f_i^e &= \frac{1}{\sigma - 1}\left(\frac{w_i}{A_i}\right) q_i(\omega) \iff \\ (\sigma - 1) f_i^e &= \frac{q_i(\omega)}{A_i} \end{aligned}$$

i.e. in equilibrium, the fixed cost of entry will be proportional $\frac{q_i(\omega)}{A_i}$, which is the amount of labor used in production. The last step is to note that the total labor used by all firms (for both entry and for production) has to equal the total number of workers in the country, L_i:

$$\begin{aligned} M_i\left(f_i^e + \frac{q_i(\omega)}{A_i}\right) &= L_i \iff \\ M_i\left(f_i^e + (\sigma - 1) f_i^e\right) &= L_i \iff \\ M_i &= \frac{1}{\sigma}\frac{L_i}{f_i^e}. \end{aligned} \tag{11}$$

In equilibrium, the number of firms is proportional to the population of a country and inversely proportional to the entry costs and the elasticity of substitution. [Class question: what is the intuition for each of these comparative statics?]. Finally, we can the equilibrium number of firms given by expression (11) into the gravity equation given by (6) to yield:

$$X_{ij} = \frac{1}{\sigma}\left(\frac{\sigma}{\sigma - 1}\right)^{1-\sigma} \tau_{ij}^{1-\sigma}\left(\frac{w_i}{A_i}\right)^{1-\sigma} \frac{L_i}{f_i^e} Y_j P_j^{\sigma-1} \tag{12}$$

Because the equilibrium number of firms is pinned down by exogenous model parameters, the Krugman (1980) gravity equation (12) can be formally shown to be isomorphic to Armington model discussed in the previous class. This means that with an appropriate transformation of model fundamentals both models will yield identical predictions for the equilibrium outcomes of the model. Hence, in a sense, making the Armington model more realistic by replacing the Armington assumption with a trade model with firms did not end up changing anything.

4 Trade with firms

We now consider the implications of a model where firms are trading.

4.1 Patterns of trade

First, notice that because all firms are identical and there are no fixed costs of exporting, all firms will sell to all countries (we will see what happens when we introduce firm heterogeneity and fixed costs of exporting next class). Second, note that the elasticity of trade flows to the variable trade cost is:

$$-\frac{\partial \ln X_{ij}}{\partial \ln \tau_{ij}} = \sigma - 1$$

In this model, the trade elasticity is increasing with the elasticity of substitution because the greater the elasticity of substitution, the greater the competition amongst firms. As a result, any cost differential between supplying firms due to trade costs results in exporting firms having a much smaller market share.

4.2 Welfare

Recall that CES preferences are homothetic so that we can write the welfare of a country in any CES model as:

$$U_i = \frac{Y_i}{P_i}.$$

If income in a country accrues entirely to its workers (so that $Y_i = w_i L_i$) then the per capita welfare W_i can be written as:

$$W_i \equiv \frac{U_i}{L_i} = \frac{Y_i}{L_i P_i} = \frac{w_i}{P_i}. \tag{13}$$

Define $\lambda_{ij} \equiv \frac{X_{ij}}{Y_j}$ to be the fraction of country j's expenditure that is spent on goods from i. Then from the gravity equation (12) above we have:

$$\lambda_{ij} \equiv \frac{X_{ij}}{Y_j} = \frac{1}{\sigma}\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} \tau_{ij}^{1-\sigma} \left(\frac{w_i}{A_i}\right)^{1-\sigma} \frac{L_i}{f_i^e} P_j^{\sigma-1} \tag{14}$$

If we consider how much i consumes of its own produce - the inverse of its trade openness, under the assumption that $\tau_{ii} = 1$ equation (14) becomes:

$$\begin{aligned} \lambda_{ii} &= \frac{1}{\sigma}\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} \left(\frac{w_i}{P_i}\right)^{1-\sigma} A_i^{\sigma-1} \frac{L_i}{f_i^e} \iff \\ \left(\frac{w_i}{P_i}\right)^{1-\sigma} &= \lambda_{ii}\sigma \left(\frac{\sigma}{\sigma-1}\right)^{\sigma-1} A_i^{1-\sigma} \frac{f_i^e}{L_i} \iff \\ W_i &= \lambda_{ii}^{\frac{1}{1-\sigma}} \sigma^{\frac{1}{1-\sigma}} \left(\frac{\sigma-1}{\sigma}\right) A_i \left(\frac{L_i}{f_i^e}\right)^{\frac{1}{\sigma-1}}. \end{aligned} \tag{15}$$

Hence, the welfare of a country is increasing in its trade openness, its productivity, and its population, while it is decreasing in the fixed cost of entry. As we will see, expression (15) is common to many different trade models.

5 Next steps

In the next two classes we will add even more realism to the microfoundations of the gravity model by adding heterogeneity. On Monday, we will extend the Krugman (1980) by assuming that firms vary in their productivity using the Melitz (2003) model. On Wednesday, we will examine the Eaton and Kortum (2002) model, which like the Armington model abstracts from firms, but recovers the concept of comparative advantage in gravity models by assuming that countries vary in their productivities across a continuum of varieties.

As you will show in your problem set, all of these different models will yield gravity equations that are isomorphic to (12). As a result, in the following weeks we will examine the general equilibrium properties that all of these models share in common.

References

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