Lecture 3: Trade with Heterogeneous Firms

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1 Introduction

The Krugman (1980) model, while micro-founding the gravity equation based on a story of equilibrium firm entry, made the simplifying assumption that all firms were ex-ante identical. With the advent of digitized data on firm-level trading partners, however, it became clear that there existed an enormous heterogeneity in firm's exporting behavior Bernard, Jensen, Redding, and Schott (2007) provides an excellent overview of the empirical patterns concerning firms in international trade, of which we mention just a few. First, the vast majority of firms do not export; in the U.S. in 2000, only 4% of firms were exporters. Second, amongst those 4% of firms that did export, 96% of the value of exports came from just 10% of exporters. Third, comparing the firms that export to those that do not, the exporting firms tend to be larger, more productive, more skill- and capital-intensive, and to pay higher wages. These differences are apparent even before exporting begins, suggesting that more productive firms choose to export (rather than the act of exporting increasing the productivity of firms).

In response to these new empirical findings, Melitz (2003) developed an extension of the Krugman (1980) where firms varied (exogenously) in their productivities and self selected into exporting. This model has proven enormously successful for a number of reasons: first, it is able to capture many (but not all) of the empirical facts mentioned above, most notably that larger firms will be more likely to export; second, the model has proven incredibly flexible, generating a huge number of "extensions" to capture additional empirical patterns; and third, the model generates a new (potential) source for gains from trade: if falling trade costs leads higher productivity firms to grow and lower productivity firms to shrink, this reallocation of factors of production will increase the average productivity of a country. While there is some debate about whether this is actually an additional gain from trade (as we will see in a few weeks), the idea that greater trade can make a country more productive by increasing competition has made the (rare) leap from academic to political discourse; for example the U.S. trade representative web page lists as one of its major "benefits of trade" the fact that "trade expansion benefits families and businesses by supporting more productive, higher paying jobs in our export sectors."

2 Model Set-up

Let us now turn to the set-up of the model.

2.1 The world

As in the previous models, there is a compact set S of countries, where I will keep the notation that i is an origin country and a j is a destination country. Each country $i \in S$ is be populated by an exogenous measure L_i of workers/consumers where each worker supplies her unit of labor inelastically. Suppose that labor is the only factor of production.

2.2 Supply

As in the Krugman (1980) model, suppose that there is a continuum Ω of possible varieties that the world can produce, and suppose that every firm in the world produces a distinct variety $\omega \in \Omega$. Let the set of varieties produced by firms located in country $i \in S$ be denoted by $\Omega_i \subset \Omega$. (Note that Ω_i is an equilibrium object, as it will depend on the number of firms that are actively producing).

Instead of the fixed entry cost in Krugman (1980) model, suppose that there is a mass M_i of firms from country $i \in S$ and that firms must incur a fixed cost $f_{ij} > 0$ to export to each destination $j \in S$.[1]

The major innovation of the Melitz (2003) model is that firms are heterogeneous. To model this, we suppose that each firm in $i \in S$ has a productivity φ drawn from some cumulative distribution function $G_i(\varphi)$, i.e. it costs a firm with productivity φ exactly $\frac{1}{\varphi}$ units of labor to produce a single unit of its differentiated variety. In what follows, we will sometimes identify each firm by its productivity (since all firms with the same productivity within a particular country will act the same way) and sometimes identify each firm by its variety ω (since every firm produces a unique variety).

Finally, as in previous models, we suppose that all firms within a country are subject to iceberg trade costs $\{\tau_{ij}\}_{i,j \in S}$.

2.3 Demand

As in the Krugman (1980) model, we assume that consumers have CES preferences over varieties. Hence a representative consumer in country $j \in S$ gets utility U_j from the consumption of goods shipped by all other firms in all other countries, where:

$$U_j = \left(\sum_{i \in S} \int_{\Omega_{ij}} \left(q_{ij}(\omega) \right)^{\frac{\sigma}{\sigma-1}} d\omega \right)^{\frac{\sigma-1}{\sigma}}, \tag{1}$$

where $q_{ij}(\omega)$ is the quantity consumed in country j of variety ω.

[1] In Melitz (2003), it was assumed that there was an additional entry cost f_i^e that determined the equilibrium mass of firms M_i. In the Chaney (2008) version of the model, M_i was assumed (for simplicity) to be proportional to the income in the origin. The Chaney (2008) version of the model has become more widely used because it allows for arbitrary bilateral trade costs (the original Melitz (2003) model imposed symmetry).

3 Equilibrium

We now consider the equilibrium of the model.

3.1 Optimal demand

The consumer's utility maximization problem is identical to that of Krugman (1980): A consumer in country $j \in S$ optimal quantity demanded of good $\omega \in \Omega$ is:

$$q_{ij}(\omega) = p_{ij}(\omega)^{-\sigma} Y_j P_j^{\sigma-1}, \tag{2}$$

where:

$$P_j \equiv \left(\sum_{i \in S} \int_{\Omega_i} p_{ij}(\omega)^{1-\sigma} \, d\omega \right)^{\frac{1}{1-\sigma}} \tag{3}$$

is the Dixit-Stiglitz price index.

The amount spent on variety ω is simply the product of the quantity and the price:

$$x_{ij}(\omega) = p_{ij}(\omega)^{1-\sigma} Y_j P_j^{\sigma-1}. \tag{4}$$

To determine total trade flows, we need to aggregate across all firms in country i:

$$X_{ij} \equiv \int_{\Omega_i} x_{ij}(\omega) \, d\omega = Y_j P_j^{\sigma-1} \int_{\Omega_i} p_{ij}(\omega)^{1-\sigma} \, d\omega. \tag{5}$$

Unlike the Krugman (1980) model, firms with different productivities will charge different prices, so the integral in equation (5) becomes more complicated.

3.2 Optimal supply

We now determine the equilibrium prices that a firm with productivity φ sets (where we now identify firms by their productivity). The optimization problem is:

$$\max_{\{q_{ij}(\varphi)\}_{j \in S}} \sum_{j \in S} \left(p_{ij}(\varphi) \, q_{ij}(\varphi) - \frac{w_i}{\varphi} \tau_{ij} q_{ij}(\varphi) - f_{ij} \right) \text{ s.t. } q_{ij}(\varphi) = p_{ij}(\varphi)^{-\sigma} Y_j P_j^{\sigma-1}.$$

Substituting the constraint into the maximand yields:

$$\max_{\{q_{ij}(\varphi)\}_{j \in S}} \sum_{j \in S} \left(p_{ij}(\varphi)^{1-\sigma} Y_j P_j^{\sigma-1} - \frac{w_i}{\varphi} \tau_{ij} p_{ij}(\varphi)^{-\sigma} Y_j P_j^{\sigma-1} - f_{ij} \right)$$

The first order condition implies that a firm from $i \in S$ with productivity φ, conditional on selling to destination j, will charge a price:

$$p_{ij}(\varphi) = \frac{\sigma}{\sigma - 1} \frac{w_i}{\varphi} \tau_{ij} \tag{6}$$

Combining the optimal price (equation (6)) and the optimal demand (equation (2)) gives the total revenue of a firm (conditional on exporting) to be:

$$x_{ij}(\varphi) \equiv p_{ij}(\varphi) q_{ij}(\varphi) = \left(\frac{\sigma}{\sigma-1}\frac{w_i}{\varphi}\tau_{ij}\right)^{1-\sigma} Y_j P_j^{\sigma-1} \tag{7}$$

and variable profits conditional on entering (note that we now define $\pi_{ij}(\varphi)$ as profits without the fixed costs):

$$\begin{aligned}
\pi_{ij}(\varphi) &\equiv \left(p_{ij}(\varphi) - \frac{w_i}{\varphi}\tau_{ij}\right) q_{ij}(\varphi) \\
&= \left(\frac{\sigma}{\sigma-1}\frac{w_i}{\varphi}\tau_{ij} - \frac{w_i}{\varphi}\tau_{ij}\right)\left(\frac{\sigma}{\sigma-1}\frac{w_i}{\varphi}\tau_{ij}\right)^{-\sigma} Y_j P_j^{\sigma-1} \\
&= \frac{1}{\sigma}\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\left(\frac{w_i}{\varphi}\tau_{ij}\right)^{1-\sigma} Y_j P_j^{\sigma-1} \\
&= \frac{1}{\sigma} x_{ij}(\varphi)
\end{aligned} \tag{8}$$

Note that both revenue and profits are increasing in a firm's productivity. [Class question: Why is this?]

3.3 Aggregation

We now discuss how to use the optimal behavior on the part of each firm to construct the aggregate variables necessary to generate a gravity equation. Let $\mu_{ij}(\varphi)$ be the (equilibrium) probability density function of the productivities of firms from country i that sell to country j and let M_{ij} be the (equilibrium) measure of firms exporting from i to j.

Then we can write the average prices charged by all firms in $i \in S$ selling to $j \in S$ as:

$$\begin{aligned}
\int_{\Omega_i} p_{ij}(\omega)^{1-\sigma} d\omega &= \int_0^\infty M_{ij}\left(\frac{\sigma}{\sigma-1}\frac{w_i}{\varphi}\tau_{ij}\right)^{1-\sigma}\mu_{ij}(\varphi)\, d\varphi \iff \\
&= \left(\frac{\sigma}{\sigma-1} w_i \tau_{ij}\right)^{1-\sigma} M_{ij}\int_0^\infty \varphi^{\sigma-1}\mu_{ij}(\varphi)\, d\varphi \iff \\
&= M_{ij}\left(\frac{\sigma}{\sigma-1} w_i\tau_{ij}\right)^{1-\sigma}(\tilde{\varphi}_{ij})^{\sigma-1},
\end{aligned}$$

where $\tilde{\varphi}_{ij} \equiv \left(\int_0^\infty \varphi^{\sigma-1}\mu_{ij}(\varphi)\, d\varphi\right)^{\frac{1}{\sigma-1}}$ captures the "average" productivity of producers from i selling to j. This allows us to write the gravity equation (5) as:

$$X_{ij} = \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} \tau_{ij}^{1-\sigma} w_i^{1-\sigma} M_{ij} (\tilde{\varphi}_{ij})^{\sigma-1} Y_j P_j^{\sigma-1}. \tag{9}$$

Equation (9) resembles the gravity equation from Krugman (1980), except that we now have to keep track of both the number of firms selling to j (M_{ij}) and their average productivity $\tilde{\varphi}_{ij}$. Note that as the average productivity of entrants increases, the trade flows increase. [Class question: what is the intuition for this?].

3.4 Selection into exporting

In order to determine the equilibrium number of entrants M_{ij} and the average productivity of entrants $\tilde{\varphi}_{ij}$, we have to consider the export decisions of firms. A firm from country $i \in S$ with productivity φ conditional on producing will export to j if and only if:

$$\pi_{ij}(\varphi) \geq f_{ij}$$

From equations (7) and (8) we can write this as:

$$\frac{1}{\sigma}\left(\frac{\sigma}{\sigma-1}\frac{w_i}{\varphi}\tau_{ij}\right)^{1-\sigma} Y_j P_j^{\sigma-1} \geq f_{ij} \iff$$
$$\varphi \geq \varphi_{ij}^* \equiv \left(\frac{\sigma f_{ij}\left(\frac{\sigma}{\sigma-1}w_i\tau_{ij}\right)^{\sigma-1}}{Y_j P_j^{\sigma-1}}\right)^{\frac{1}{\sigma-1}}. \tag{10}$$

Hence, only firms that are sufficiently productive will find it profitable to incur the fixed cost of exporting to destination j. This means that the model matches the empirical fact that larger and more productive firms select into exporting.

Together, equations (10) and (18) allow us to determine the "average" productivity of producers selling from i to j:

$$\tilde{\varphi}_{ij} = \left(\frac{1}{1-G_i\left(\varphi_{ij}^*\right)}\int_{\varphi_{ij}^*}^{\infty}\varphi^{\sigma-1}dG_i(\varphi)\right)^{\frac{1}{\sigma-1}}. \tag{11}$$

and the density of firms selling from i to j:

$$M_{ij} = \left(1-G_i\left(\varphi_{ij}^*\right)\right)M_i, \tag{12}$$

so that the gravity equation (9) becomes:

$$X_{ij} = \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\tau_{ij}^{1-\sigma}w_i^{1-\sigma}M_i\left(\int_{\varphi_{ij}^*}^{\infty}\varphi^{\sigma-1}dG_i(\varphi)\right)Y_j P_j^{\sigma-1}. \tag{13}$$

3.5 The Pareto Distribution

In this section, we show that when the distribution of firm productivities is a Pareto distribution, the model above simplifies nicely. This insight is due to Chaney (2008). Suppose that $\varphi \in [1,\infty)$ and:

$$G_i(\varphi) = 1 - \varphi^{-\theta_i}, \tag{14}$$

where θ_i is the *shape parameter* of the distribution. We assume that $\theta_i > \sigma - 1$ (this parametric assumption is necessary in order for trade flows to be finite). Note that as θ_i increases, the probability that the productivity is below any given φ increases, i.e. the heterogeneity of producers is decreasing in θ_i.

If the productivities are Pareto distributed, then we can write:

$$\int_{\varphi_{ij}^*}^{\infty} \varphi^{\sigma-1} dG_i(\varphi) = \int_{\varphi_{ij}^*}^{\infty} \varphi^{\sigma-1} \left(\frac{d\left(1-\varphi^{-\theta_i}\right)}{d\varphi} \right) d\varphi \iff$$
$$= \theta_i \int_{\varphi_{ij}^*}^{\infty} \varphi^{\sigma-\theta_i-2} d\varphi \iff$$
$$= \frac{\theta_i}{\theta_i+1-\sigma} \left(\varphi_{ij}^*\right)^{\sigma-\theta_i-1}$$

Recall from equation (10) above that we can write the export threshold φ_{ij}^* as a function of the fixed cost of export so that:

$$\int_{\varphi_{ij}^*}^{\infty} \varphi^{\sigma-1} dG_i(\varphi) = \frac{\theta_i}{\theta_i+1-\sigma} \left(\varphi_{ij}^*\right)^{\sigma-\theta_i-1} \iff$$
$$= \frac{\theta_i}{\theta_i+1-\sigma} \left(\frac{\sigma f_{ij} \left(\frac{\sigma}{\sigma-1} w_i \tau_{ij}\right)^{\sigma-1}}{Y_j P_j^{\sigma-1}} \right)^{\frac{\sigma-\theta_i-1}{\sigma-1}} \tag{15}$$

Substituting expression (15) into the gravity equation (13) above then yields:

$$X_{ij} = \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} \tau_{ij}^{1-\sigma} w_i^{1-\sigma} M_i \left(\int_{\varphi_{ij}^*}^{\infty} \varphi^{\sigma-1} dG_i(\varphi) \right) Y_j P_j^{\sigma-1} \iff$$
$$X_{ij} = \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} \tau_{ij}^{1-\sigma} w_i^{1-\sigma} M_i \left(\frac{\theta_i}{\theta_i+1-\sigma} \left(\frac{\sigma f_{ij} \left(\frac{\sigma}{\sigma-1} w_i \tau_{ij}\right)^{\sigma-1}}{Y_j P_j^{\sigma-1}} \right)^{\frac{\sigma-\theta_i-1}{\sigma-1}} \right) Y_j P_j^{\sigma-1} \iff$$
$$X_{ij} = C_1 \left(\tau_{ij} w_i\right)^{-\theta_i} f_{ij}^{\frac{\sigma-\theta_i-1}{\sigma-1}} M_i \left(Y_j P_j^{\sigma-1}\right)^{\frac{\theta_i}{\sigma-1}} \tag{16}$$

where $C_1 \equiv \sigma^{\frac{\sigma-\theta_i-1}{\sigma-1}} \left(\frac{\sigma}{\sigma-1}\right)^{-\theta_i} \left(\frac{\theta_i}{\theta_i+1-\sigma}\right)$.

4 Trade with firm heterogeneity

Armed with the gravity equation (16) we have calculated, we now turn to the implications of a trade model with heterogeneous firms.

4.1 Extensive and intensive margins of trade

Equation (16) bears a resemblance to the gravity equation derived by Krugman (1980), but the elasticity of trade flows with respect to variable trade costs is related to the Pareto shape parameter instead of the elasticity of substitution! Since we have assumed that $\theta_i > \sigma - 1$, this means that trade flows have become *more* responsive to changes in trade costs than in the Krugman (1980) model.

What gives? Intuitively, as trade costs fall two things happen: first, the firms already producing will export more (this is known as the **intensive margin**); second, smaller firms who were not exporting previously will begin to export (this is known as the **extensive margin**). Both of these effects will tend to increase trade; since the Krugman (1980) model only had the first effect, the model with heterogeneous firms will predict larger responses of trade flows to changes in trade costs.

We can actually determine the elasticity of both margins of trade separately to see the relative importance of both effects. (This was the central point of Chaney (2008)To do so, recall the Leibnez rule:

$$\frac{\partial}{\partial z}\int_{a(z)}^{b(z)} f(x,z)\,dx = \int_{a(z)}^{b(z)} \frac{\partial f(x,z)}{\partial z} dx + f(b(z),z)\frac{\partial b(z)}{\partial z} - f(a(z),z)\frac{\partial a(z)}{\partial z}$$

Combining the original gravity equation (5) with the threshold exporting decision (10) we have:

$$X_{ij} = M_i \int_{\varphi_{ij}^*}^{\infty} x_{ij}(\varphi)\,dG_i(\varphi)$$

Hence we can write the elasticity of trade flows with respect to variable trade costs as:

$$-\frac{\partial \ln X_{ij}}{\partial \ln \tau_{ij}} = -\frac{\partial X_{ij}}{\partial \tau_{ij}}\frac{\tau_{ij}}{X_{ij}} = -\frac{\int_{\varphi_{ij}^*}^{\infty} \frac{\partial}{\partial \tau_{ij}} x_{ij}(\varphi)\,\tau_{ij} dG_i(\varphi)}{\int_{\varphi_{ij}^*}^{\infty} x_{ij}(\varphi)\,dG_i(\varphi)} + \frac{x_{ij}\left(\varphi_{ij}^*\right)\tau_{ij}\frac{\partial \varphi_{ij}^*}{\partial \tau_{ij}} dG_i\left(\varphi_{ij}^*\right)}{\int_{\varphi_{ij}^*}^{\infty} x_{ij}(\varphi)\,dG_i(\varphi)},$$

where the first term reflects the effect of a change in trade costs on the intensive margin and the second term reflects the change on the extensive margin. From the revenue equation (7) we have:

$$\frac{\partial}{\partial \tau_{ij}} x_{ij}(\varphi) = \frac{\partial}{\partial \tau_{ij}}\left(\frac{\sigma}{\sigma-1}\frac{w_i}{\varphi}\tau_{ij}\right)^{1-\sigma} Y_j P_j^{\sigma-1} = (1-\sigma)\frac{x_{ij}(\varphi)}{\tau_{ij}}.$$

so that:

$$-\frac{\int_{\varphi_{ij}^*}^{\infty} \frac{\partial}{\partial \tau_{ij}} x_{ij}(\varphi)\,\tau_{ij} dG_i(\varphi)}{\int_{\varphi_{ij}^*}^{\infty} x_{ij}(\varphi)\,dG_i(\varphi)} = \sigma - 1,$$

i.e. a decline in trade costs will cause all firms currently producing to increase their production with an elasticity of $\sigma - 1$ (this is the original Krugman (1980) effect). [Class question: Why is the intensive margin increasing with σ?]

From equation (10) governing the threshold productivity:

$$\frac{\partial \varphi_{ij}^*}{\partial \tau_{ij}} = \frac{\partial}{\partial \tau_{ij}}\left(\frac{\sigma f_{ij}\left(\frac{\sigma}{\sigma-1} w_i \tau_{ij}\right)^{\sigma-1}}{Y_j P_j^{\sigma-1}}\right)^{\frac{1}{\sigma-1}} = \frac{\varphi_{ij}^*}{\tau_{ij}}$$

so that:

$$\begin{aligned}
\frac{x_{ij}\left(\varphi_{ij}^*\right)\tau_{ij}\frac{\partial\varphi_{ij}^*}{\partial\tau_{ij}}dG_i\left(\varphi\right)}{\int_{\varphi_{ij}^*}^{\infty}x_{ij}\left(\varphi\right)dG_i\left(\varphi\right)} &= \frac{x_{ij}\left(\varphi_{ij}^*\right)\varphi_{ij}^*dG_i\left(\varphi_{ij}^*\right)}{\int_{\varphi_{ij}^*}^{\infty}x_{ij}\left(\varphi\right)dG_i\left(\varphi\right)} \iff \\
&= \frac{\left(\frac{\sigma}{\sigma-1}w_i\tau_{ij}\right)^{1-\sigma}Y_jP_j^{\sigma-1}\left(\varphi_{ij}^*\right)^{\sigma-1-\theta_i}}{\left(\frac{\sigma}{\sigma-1}w_i\tau_{ij}\right)^{1-\sigma}Y_jP_j^{\sigma-1}\int_{\varphi_{ij}^*}^{\infty}\varphi^{\sigma-2-\theta_i}d\varphi} \iff \\
&= \frac{\left(\varphi_{ij}^*\right)^{\sigma-1-\theta_i}}{\frac{1}{\theta_i-\sigma+1}\left(\varphi_{ij}^*\right)^{\sigma-1-\theta_i}} \iff \\
&= \theta_i-\sigma+1,
\end{aligned}$$

i.e. on the extensive margin, a decline in trade costs will induce less productive firms to enter the market. When the elasticity of substitution is low (i.e. σ is low), even the less productive firms will be able to capture relatively large market share, so that the difference in size between the entering firms and the existing firms is small, meaning that the effect on the extensive margin will be larger. With a Pareto distribution, the extensive margin dominates the intensive margin.

4.2 Free entry and the allocation of factors across firms

Up until now, we have taken the mass of producing firms M_i to be exogenous. We now consider what would happen if it was endogenously determined by a free entry condition (much as in Krugman (1980)). Suppose now that firms have to incur an entry cost $f_i^e > 0$ *prior* to learning their productivity. Then the free entry condition will require that the expected profits are equal to the entry cost:

$$f_i^e = E_\varphi\left[\sum_{j\in S}\max\left\{\pi_{ij}\left(\varphi\right)-f_{ij},0\right\}\right] \tag{17}$$

Since more productive firms are (weakly) more profitable in every market, this implies there will be an equilibrium productivity threshold φ_i^* where firms, upon drawing a productivity, will choose to produce only if their productivity exceeds φ_i^*. This implies that we can write re-write equation (17) as:

$$\begin{aligned}
f_i^e &= \int_{\varphi^*}^{\infty}\sum_{j\in S}\max\left\{\pi_{ij}\left(\varphi\right)-f_{ij},0\right\}dG_i\left(\varphi\right) \iff \\
f_i^e &= \sum_{j\in S}\int_{\max\left\{\varphi_i^*,\varphi_{ij}^*\right\}}^{\infty}\left(\pi_{ij}\left(\varphi\right)-f_{ij}\right)dG_i\left(\varphi\right),
\end{aligned} \tag{18}$$

i.e. the fixed entry cost is simply equal to the sum across all destinations of the profits in those destinations for firms who are sufficiently productive to both pay the fixed entry and export costs.

What would happen to the profits of firms of different productivities if we were to lower the variable trade cost τ_{ij} for some $j \in S$? First, consider a firm whose productivity is greater than the threshold productivity necessary to export to j, i.e. $\varphi \geq \varphi_{ij}^*$. From equation (8), its profits are:

$$\pi_{ij}(\varphi) = \frac{1}{\sigma}\left(\frac{\sigma}{\sigma-1}\frac{w_i}{\varphi}\tau_{ij}\right)^{1-\sigma} Y_j P_j^{\sigma-1} - f_{ij},$$

so that holding all else equal, its total profits must increase. Furthermore, the more productive this firm is, the greater the increase in its profits since $-\frac{\partial^2 \pi_{ij}(\varphi)}{\partial \tau_{ij} \partial \varphi} > 0$. If the total profits for all firms $\varphi \geq \varphi_{ij}^*$ are increasing, then the expected profits of entering the market will also increase; because of the free entry condition, this will induce a greater number of firms to enter the market, increasing the demand for local labor and driving up wages. As a result, the profits of firms with $\varphi < \varphi_{ij}^*$ will go down (as will the firms with productivities $\varphi < \varphi_{ij}^* + \varepsilon$), so that in equilibrium, only the most productive firms productivities will increase. In addition, as the wages increase, the minimum productivity required to produce anything at all (i.e. φ_i^*) will increase, forcing the least productive firms in the model to exit. Hence, the model implies that greater openness to trade will increase the average productivity of producing firms and will allocate labor toward the more productive firms.

5 Next steps

The Melitz (2003) model provides the backbone for many (most?) of the major trade papers written in the past ten years. While we will not have time to discuss its many extensions in detail, we should note a few. Helpman, Melitz, and Yeaple (2004) endogenizes a firm's decision whether to export or to pursue foreign direct investment (FDI). Melitz and Ottaviano (2008) derive a version of the model with linear demand (instead of CES) to analyze how mark-ups endogenously respond to trade liberalizations. Helpman, Melitz, and Rubinstein (2008) discuss how the model (with a bounded distribution of productivity) can be used to explain the zero trade flows observed in bilateral trade data and what it suggests for the estimation of empirical gravity models. Helpman, Itskhoki, and Redding (2010) incorporate labor market frictions into a Melitz (2003) framework. Arkolakis (2010) extends the Melitz (2003) framework to incorporate market penetration costs. Eaton, Kortum, and Kramarz (2011) use the Melitz (2003) framework to structurally estimate the exporting behavior of French firms.

In the next class, we will be examining the other major trade model of the past fifteen years: Eaton and Kortum (2002). While it has not proven as amenable as the Melitz (2003) model to various extensions, it provides an incredibly elegant way of introducing the classical trade force of comparative advantage into a many country world with arbitrary trade costs.

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