Macroeconomics B, EI060

Class 2

Intertemporal approach to the current account

Cédric Tille

February 26, 2025

What you will get from today class

SM OE

- Intertemporal approach of production and consumption: 2 periods.
  - Consumption choice in endowment economy (Harms III.1-3 & my charts, Obstfeld-Rogof (secondary) 1.1).
  - Introducing government (Harms IV.3, OR 1.1.6).
  - Endogenous production (Harms III.4-5, OR 1.2).
- Infinite horizon (Harms III.7, OR 2.1-2.3.2).
- Equilibrium with two countries (Harms III.8, OR 1.3.1-1.3.3).
- External debt solvency (Harms VI.1-2).

A question to start

LIFE CYCLE

A country that has a higher income will save more, as it is wealthier.

$\frac{S}{Y} \rightarrow \quad S \nearrow \quad Y \nearrow$

Do you agree? Why or why not? Think of two cases:

ENDOW vs $Y = F(K)$

a) Higher income thanks to additional natural resources, that will be available for a long time.

PERMANENT

$\Rightarrow S \quad I \nearrow$

b) Higher income thanks to a increase in the value of its GDP, which is temporary

$S \leftarrow$ temporary

INTERTEMPORAL APPROACH : SIMPLE MODEL

Two periods consumption

- Real model with one good. 2 PERIOD $G_{t=1}$ GOOD $t=2$
- The representative consumer maximizes the intertemporal utility ($\beta < 1$):

PATIENT

$$u(C_1) + \beta u(C_2)$$

- Output is an endowment. The consumer can purchase a bond with real interest rate r. The flow budget constraints are:

PRICES –

$$C_1 + B_2 = Y_1 \qquad ; \qquad C_2 = (1 + r) B_2 + Y_2$$

- The intertemporal constraint links the present value of consumption is to the present value of income Ω:

UNITS GOOD (
PERIOD 1

WEALTH

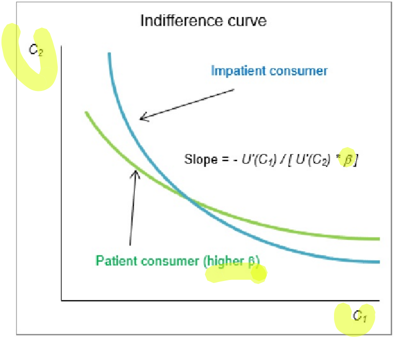
NPV

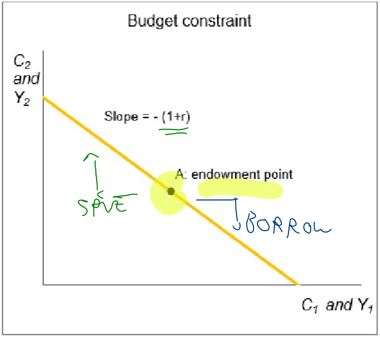
$$\boxed{C_1 + \frac{C_2}{1+r}} = Y_1 + \frac{Y_2}{1+r} = \Omega$$

$\frac{1}{1+r}$ "P_2"

Diagram

- Indifference curves, with decreasing marginal utility.
- Budget constraint, going through the endowment point (Y_2, Y_1).





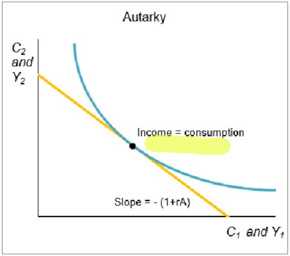
Optimal choice

- Euler condition, with tangent of the indifference curve and the budget constraint:

$$u'(C_1) = \beta(1+r)\,u'(C_2) \qquad \Rightarrow \qquad \frac{\beta u'(C_2)}{u'(C_1)} = \frac{1}{1+r}$$

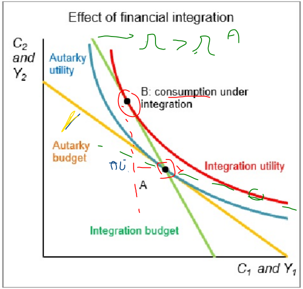
- Marginal utility of giving up a unit today = return $1+r$, the marginal utility of consumption tomorrow, adjusted for the cost of waiting β.
- Autarky: $B_2 = 0$ and $C_1 = Y_1$, $C_2 = Y_2$. Euler gives the interest rate:

$$1 + r^A \;=\; \left[u'(Y_1)\right] / \left[\beta u'(Y_2)\right]$$



Effect of financial integration

- Consumption is not connected to output each period.
- A country with an autarky rate $r^A < r$ chooses to save $(C_1 < Y_1)$.
  low
- Integration raises the utility (one could always stay in autarky allocation). Not necessarily for all if households are heterogeneous (borrowers are unhappy).



Specific case

- Constant relative risk aversion utility:

CRRA

$$u(C) = \frac{C^{1-\sigma}}{1-\sigma}$$

- The consumptions and savings are:

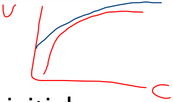
$$\begin{aligned}
C_1 &= \frac{1}{1+\beta^{\frac{1}{\sigma}}(1+r)^{\frac{1}{\sigma}-1}}\left[Y_1 + \frac{Y_2}{1+r}\right] \\
C_2 &= \frac{\beta^{\frac{1}{\sigma}}(1+r)^{\frac{1}{\sigma}}}{1+\beta^{\frac{1}{\sigma}}(1+r)^{\frac{1}{\sigma}-1}}\left[Y_1 + \frac{Y_2}{1+r}\right] \\
B_2 &= \frac{1}{1+\beta^{\frac{1}{\sigma}}(1+r)^{\frac{1}{\sigma}-1}}\left[\beta^{\frac{1}{\sigma}}(1+r)^{\frac{1}{\sigma}-1}Y_1 - \frac{Y_2}{1+r}\right]
\end{aligned}$$

- If $\sigma = 1$ (log utility), initial consumption is a given share of overall income: $C_1 = \frac{1}{1+\beta}\left[Y_1 + \frac{Y_2}{1+r}\right]$.

Effects of the interest rate

σ HIGH

σ LOW



$\frac{1}{\sigma}$ low

- Three effects on initial consumption:

$$C_1 = \frac{1}{1 + \beta^{\frac{1}{\sigma}} (1+r)^{\frac{1}{\sigma}} (1+r)^{-1}} \left[Y_1 + \frac{Y_2}{1+r} \right]$$

- Substitution: with a higher interest rate, savings is more remunerated. Consume less and save, driven by $(1+r)^{\frac{1}{\sigma}}$.
- Income: a higher interest rate makes reaching a given value of assets tomorrow easier. Save less and consume, driven by $(1+r)^{-1}$.
- Wealth: a higher interest rate reduces the net present value of future income, reducing wealth. Consume less, driven by $\frac{Y_2}{1+r}$.

Introducing government spending

LUMP SUM

- Government purchases G_t and taxes the consumer T_t.
- Consumer's and government's budget constraints:

$$C_1 + B_2^{private} = Y_1 - T_1 \quad ; \quad C_2 = (1+r)\, B_2^{private} + Y_2 - T_2$$

$$G_1 + B_2^{public} = T_1 \quad ; \quad G_2 = (1+r)\, B_2^{public} + T_2$$

- Adding up, the taxes cancel our. Only government spending matters:

$$\underbrace{C_1 + G_1}_{\tilde{C}_1} + \underbrace{\left(B_2^{private} + B_2^{public} \right)}_{\tilde{B}_2} = Y_1$$

$$\underbrace{C_2 + G_2}_{\tilde{C}_2} = (1+r) \left(B_2^{private} + B_2^{public} \right) + Y_2$$

$$C_1 + G_1 + \frac{C_2 + G_2}{1+r} = Y_1 + \frac{Y_2}{1+r}$$

- Taxes don't matter (Ricardian equivalence) as any change in the path of taxes is undone by the consumer.
  - Assumes they face the same interest rate and planning horizon.

INTERTEMPORAL APPROACH :

ENDOGENOUS OUTPUT

Technology and production frontier

- Produce with capital (depreciates at a rate δ), using decreasing returns to scale:

$$Y_t = A_t F(K_t) \quad ; \quad F' > 0, \;\; F'' < 0$$

- The consumer can save in bonds and capital. The budget constraints are:

Y_1 + LEFTOVER

$K_2 +$

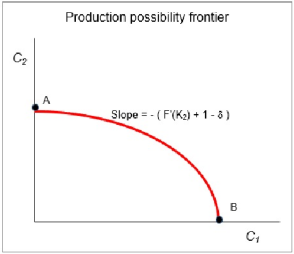
$$\begin{aligned} B_2 &= A_1 F(K_1) + (1-\delta) K_1 - C_1 - K_2 \\ 0 &= (1+r) B_2 + A_2 F(K_2) + (1-\delta) K_2 - C_2 \end{aligned}$$

Production possibility frontier

- In autarky the budget constraint is:

$$
\begin{aligned}
C_2 &= G\left(C_1\right) \\
&= A_2 F\left(\overbrace{A_1 F\left(K_1\right) + (1-\delta) K_1 - C_1}^{K_2}\right) \\
&\quad + (1-\delta)\left[\underbrace{A_1 F\left(K_1\right) + (1-\delta) K_1 - C_1}_{K_2}\right]
\end{aligned}
$$

- Concave relation with negative slope between the two consumptions ($G'\left(C_1\right) < 0$, $G''\left(C_1\right) < 0$).



Optimal allocation

- Two Euler conditions: one for the bond, and one for capital:

$$
\begin{aligned}
u'(C_1) &= \beta(1+r)\,u'(C_2) \\
u'(C_1) &= \beta\left(A_2F'(K_2)+(1-\delta)\right)u'(C_2)
\end{aligned}
$$

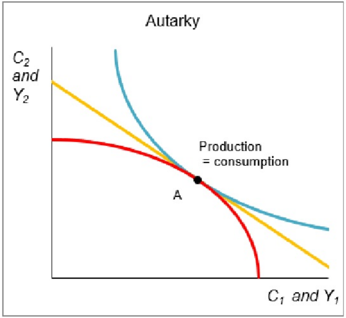
- Arbitrage links capital to the interest rate:

$$A_2F'(K_2) = r+\delta$$

- Capital (investment) demand: a higher rate leads to a lower capital.

Autarky

- Optimal allocation at the tangent of the production frontier, the utility, and the budget constraint.
- The ~~autarky interest rates~~ ensures that all three lines are tangent.



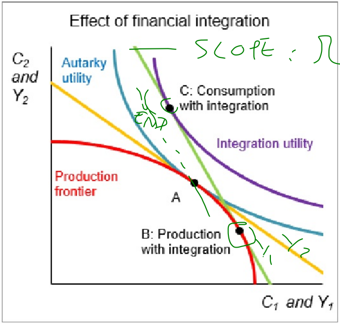
Effect of financial integration

- Again allows for disconnect of each period consumption - output.
- Integration now also affects the production point on the PPF. A country with an autarky rate $r^A < r$ chooses to save $(C_1 < Y_1)$ by a) reducing consumption and b) raising output.

S

I

PPF



SLOPE : R WORLD

INFINITE HORIZON

Utility and constraint

- Maximizes an intertemporal utility:

$I_t = K_{t+1} - K_t(1-\delta)$

$$U_t = \sum_{s=t}^{\infty} \beta^{s-t} u(C_s)$$

- Flow budget constraint (current account $CA_t = B_{t+1} - B_t$):

$B_t(1+r) = \dots B_{T+n}$

$B_{t+1}(1+r) = \dots B_{t+T}$

$$\begin{aligned} C_t + I_t + B_{t+1} &= (1+r) B_t + Y_t \\ CA_t &= \underbrace{rB_t}_{PI} + \underbrace{Y_t - C_t - I_t}_{TB} \end{aligned}$$

- Iterate to get the intertemporal constraint, with transversality $\lim_{T\to\infty} B_{t+T+1} / (1+r)^{T+1} = 0$:

$$\sum_{s=t}^{\infty} \frac{C_s + I_s}{(1+r)^{s-t}} = (1+r) B_t + \sum_{s=t}^{\infty} \frac{Y_s}{(1+r)^{s-t}}$$

Optimization

$$\sum_{s=t}\left(\frac{1}{1+r}\right)^{s-t} = \frac{1}{1-\frac{1}{1+r}}$$

- Maximization again gives the Euler condition $u'(C_t) = \beta(1+r)\,u'(C_{t+1})$.
- If $\beta(1+r) = 1$ consumption is constant:

$\Rightarrow C_t = C_{t+1}$

$$C_t = rB_t + \frac{r}{1+r}\sum_{s=t}^{\infty}\frac{Y_s - I_s}{(1+r)^{s-t}}$$

- Current account:

$$\begin{aligned} CA_t &= rB_t + Y_t - C_t - I_t \\ CA_t &= Y_t - I_t - \frac{r}{1+r}\sum_{s=t}^{\infty}\frac{Y_s - I_s}{(1+r)^{s-t}} \end{aligned}$$

"AVG"

Permanent vs. temporary levels

"AVG"

- Permanent level of a variable X, denoted by $\tilde{X}$: constant value that gives the same net present value:

$$\sum_{s=t}^{\infty} \frac{\tilde{X}_t}{(1+r)^{s-t}} = \sum_{s=t}^{\infty} \frac{X_s}{(1+r)^{s-t}}$$

- Current account reflects deviation of output (net of investment) from permanent level. Open economy equivalent to the permanent income hypothesis of consumer's saving:

$$(Y_t - \bar{I}_t) - (\tilde{Y}_t - \tilde{\bar{I}}_t)$$

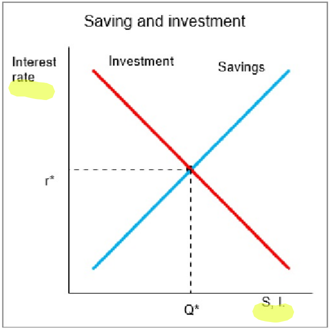
$$CA_t = \left(Y_t - \tilde{Y}_t\right) - \left(I_t - \tilde{I}_t\right)$$

- Key message: permanent shocks have no effect on the current account, only temporary ones do.

GENERAL EQUILIBRIUM

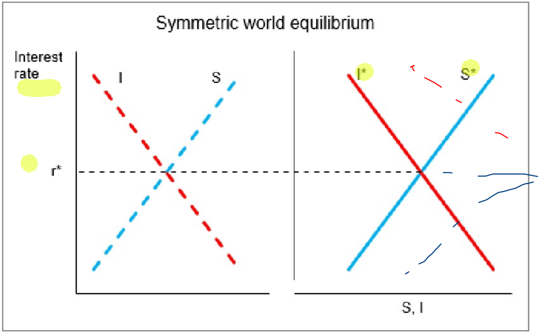
Savings and investment functions

- Savings increases with the interest rate (assume substitution effect dominates): $Savings = S(r)$, $S' > 0$.
- Investment demand decreases with the interest rate: $Investment = I(r)$, $I' \overset{<}{\cancel{>}} 0$.
- Intersection of a country's lines gives the autarky interest rate.



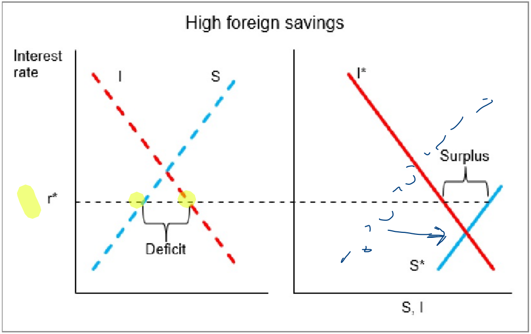
Open economy

- The world interest rate is given by the sum of domestic and foreign savings, and the sum of domestic and foreign investment.
  - Horizontal sums of lines below.
- In a symmetric world, the interest rate is the same as in autarky and $S = I$ in each country (zero current account).



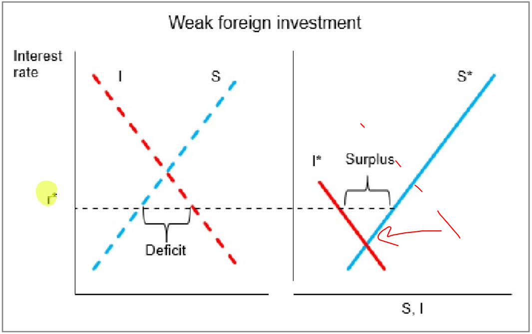
Effect of high savings

- Foreign savings line to the right.
  - Lower interest rate (autarky rate is at the intersection of the dotted lines).
  - Current account deficit at home ($S < I$) and surplus abroad ($S^* > I^*$). "Savings glut" situation of the 2000's.



Effect of low investment

- Foreign investment line to the left.
  - Lower interest rate (autarky rate is at the intersection of the dotted lines).
  - Current account deficit at home ($S < I$) and surplus abroad ($S^* > I^*$).



SOLVENCY

Next exports and debt

NO TRON SV .

- From the intertemporal budget constraint, a positive net asset position $B_t^{total} > 0$ allows the country to fund a trade deficit ($NX_s < 0$) in net present value terms:

$$\sum_{s=t}^{\infty} \frac{NX_s}{(1+r)^{s-t}} = -(1+r)\, B_t^{total}$$

- The country is solvent as long as:

$$B_t^{total} > B_t^{total,min} = -\frac{1}{1+r} \sum_{s=t}^{\infty} \frac{NX_s}{(1+r)^{s-t}}$$

DEPT : B < 0

Minimal assets

- Investment and government spending are a constant share ϕ of output, which grows at rate g. $Y_t^{AV} : (1-\phi) Y_t$
- The highest possible trade surplus, which is linked to the lowest possible asset holdings $B_t^{total,min}$, is reached when consumption is as low as possible.
- We take a consumption floor $C_t^{min} = \varphi_t^{min} Y_t$, with $C_{t+1}^{min} = \left(1 + g^{Cmin}\right) C_t^{min}$ where the growth rate is bounded as follows: $0 \leq g^{Cmin} \leq g$.
- The lower bound of asset to GDP is (assuming $g < r$):

$$\frac{B_t^{total,min}}{Y_t} = \frac{\varphi^{min}}{r - g^{Cmin}} - \frac{1-\phi}{r-g}$$

- As time goes to infinity, the ratios of the various variables to GDP converge, with the values depending on whether $g^{Cmin} < g$ or $g^{Cmin} = g$.

Low floor growth

- If $g^{Cmin} < g$, the ratio C^{min}/Y goes to zero.
- The economy converges to a high debt, and a current account deficit:

CA < 0

NX > 0

$$\frac{B^{total,min}}{Y} \to -\frac{1-\phi}{r-g}$$

$$\frac{CA}{Y} \to -g\frac{1-\phi}{r-g} = g\frac{B^{total,min}}{Y}$$

- All output goes towards the net exports needed to sustain the debt:

$$\frac{NX}{Y} \to \to 1-\phi$$

High floor growth

- If $g^{Cmin} = g$, the ratio C^{min}/Y is constant at φ^{min}.
- If this is below the available output $1-\phi$, the country has a debt, and a current account deficit, but needs a trade surplus:

$$\frac{B^{total,min}}{Y} = -\frac{1-\phi-\varphi^{min}}{r-g}$$

$$\frac{CA}{Y} \rightarrow -g\frac{1-\phi-\varphi^{min}}{r-g} = g\frac{B^{total,min}}{Y}$$

- It needs a trade surplus to stabilize the debt:

$$\frac{NX}{Y} = 1-\phi-\varphi^{min}$$

- The higher the growth rate, the larger (more negative) are the debt and current account deficit that are sustainable.