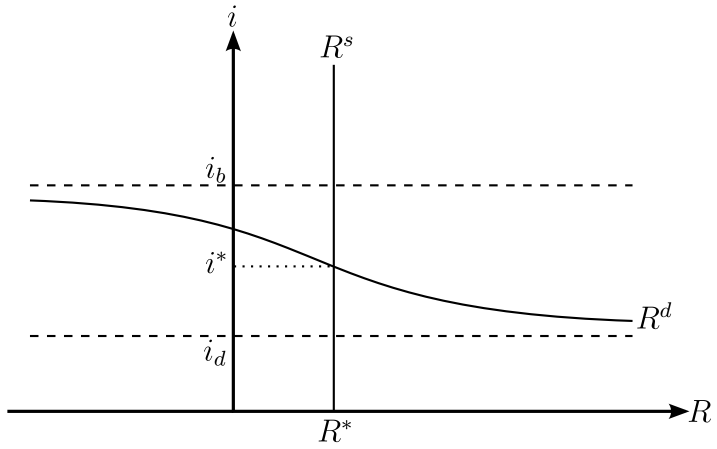
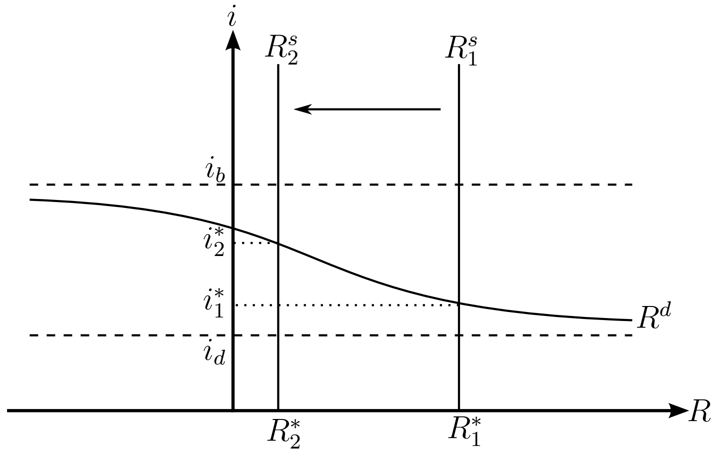
2. The interbank market

(a) Suppose the bank increases its holdings of reserves by borrowing in the interbank market. This borrowing costs i. There is a probability $F(R/n)$ that the bank will have a positive balance on its account, in which case these extra reserves will earn interest rate i_d. The expected cost of holding the reserves is therefore $(i - i_d)F(R/n)$. The expected benefit is reducing the cost of borrowing from the central bank's borrowing facility, for which there is a probability $1 - F(R/n)$ that the bank will need to use it. The expected benefit is therefore $(i_b - i)(1 - F(R/n))$. A bank optimizes its holdings of reserves by equating the expected marginal benefit to the expected marginal cost.

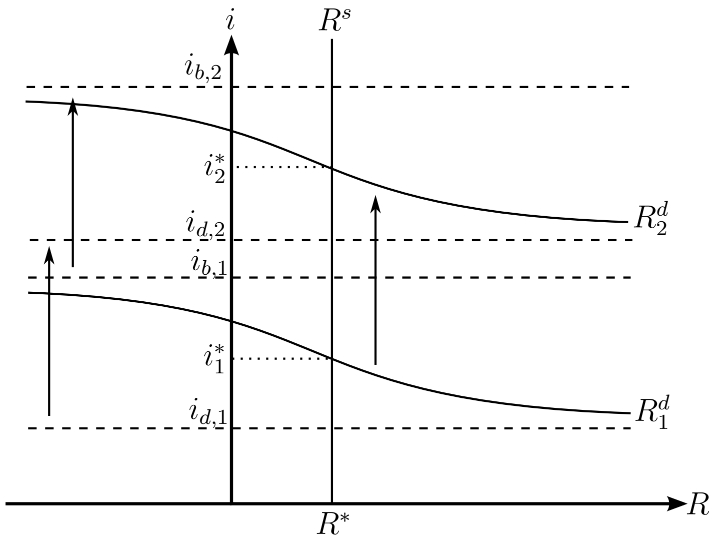
(b) The demand for reserves is determined by the banks' optimality condition. If $i < i_d$ then banks would want to borrow unlimited amounts of reserves to put on deposit at the central bank. If $i > i_b$ then banks would want to lend out unlimited reserves to others, by lending their existing reserves and borrowing more from the central bank. It follows that reserve demand is bounded between i_b and i_d. Inside this interval, since $F(\cdot)$ is a cumulative distribution function, the reserve demand curve has the same shape, but reversed. The net supply of reserves to the market is the initial supply plus or minus the change resulting from central-bank open-market operations. Taking the level of these operations as given, reserve supply is vertical. The equilibrium interbank interest rate is determined by the intersection of reserve supply and reserve demand.



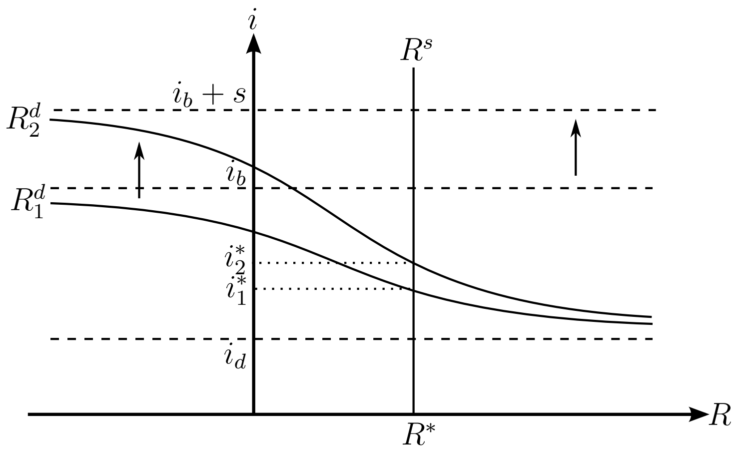
(c) i. A contractionary open-market operation decreases the total supply of reserves in the market. Reserve supply shifts to the left, so the equilibrium interbank rate is higher given that i_d and i_b, and hence the position of the reserve demand curve, are unchanged.



ii. If both i_b and i_d are increased by the same amount as the change in the target interest rate then the interval between i_d and i_b, and the reserve demand curve within it, make a parallel shift upwards. The equilibrium interbank rate rises by as much as the channel shifts up by. No open-market operation is needed.



(d) Let s denote the cost of the "stigma" as a fraction of the amount borrowed (assume it is proportional to borrowing for simplicity). The perceived cost of borrowing from the central bank is now $i_b + s$. Borrowing in the interbank market (importantly assumed not be subject to the same stigma, perhaps because it is assumed that a private bank would not provide a loan unless it was confident of being repaid) costs i as usual. The optimality condition becomes $(i - i_d)F(R/n) = (i_b + s - i)(1 - F(R/n))$. This implies the upper end of the interval $[i_b, i_d]$ shifts upwards by s, stretching the reserve demand curve within the interval. The interbank rate rises as a result.



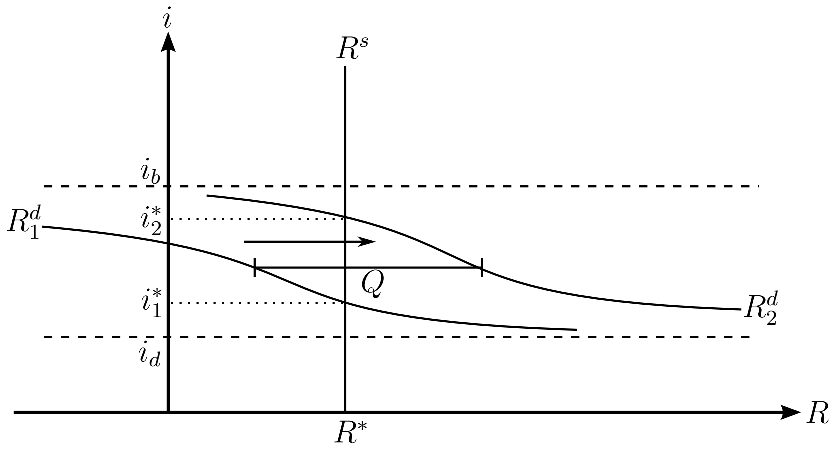
Note that it is now possible to have a case where i is greater than i_b, so the central bank cannot guarantee market rates remain within the channel defined by its standing facility rates (this case is not shown in the diagram).

(e) Suppose reserve requirements of Q are imposed, as described in the question. Suppose a bank was holding reserves R/n after interbank and repo trades, but before it must make its (unknown) net transfer T. After the transfer is made, it has probability $F(R/n)$ of having a positive reserve balance. However, with reserve requirements, the threshold for receiving the deposit interest rate i_d is now Q, and the bank has probability $F(R/n - Q)$ of having a balance above Q. When its balance is below Q after the net transfer, it is charged the (penalty) borrowing rate i_b on the difference between Q and its balance (even if the balance is positive). The probability of this is $1 - F(R/n - Q)$.

The bank's optimality condition becomes

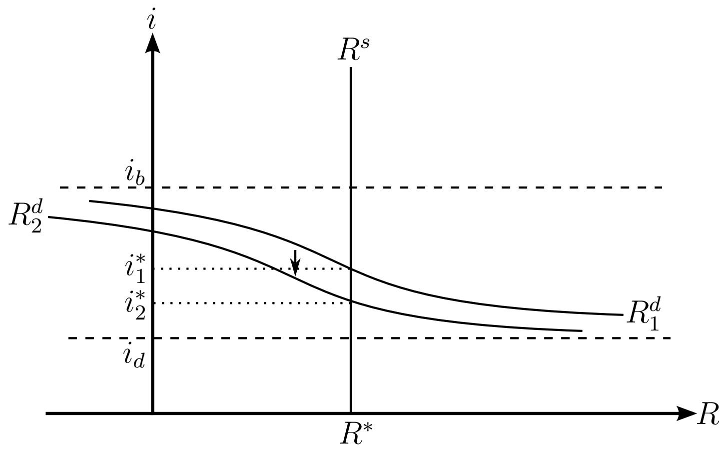
$$(i - i_b)F(R/n - Q) = (i_b - i)(1 - F(R/n - Q))$$

This implies that the channel for i remains between i_d and i_b, but that the reserve demand shifts horizontally to the right by Q (note that the diagram is set up so that the variable on the horizontal axis is reserves per bank). The equilibrium interbank rate rises as a result, unless the supply of reserves is also increased.



(f) **Channel system**:

- Implementing new interest rate decisions requires only changing both i_b and i_d by the change in the target rate (see part (c)ii.). Assuming the initial interest rate was equal to the central bank's initial target rate, this change suffices to make the new market rate equal to the new target. No additional open-market operation is needed.
- However, reserve demand may not remain constant (uncertainty about payments may change for banks, which can be modelled as a change in the cumulative distribution function). When reserve demand shifts, market rates will change, so the central bank would need to perform fine-tuning open-market operations to keep the market rate exactly at the centre of the channel.



Floor system:

- The repo market is flooded with surplus reserves so that interbank rate drops to i_d. At this point there is no opportunity cost for banks of holding on to these reserves, so they are willingly held.
- A change to the target interest rate requires simply that the deposit rate i_d be changed by the same amount. This shifts the floor (and the reserve demand curve) upwards. The intersection with reserve supply will be at the new floor.
- Since all reserve demand functions eventually become flat at the extremes, the market rate should not be sensitive to shifts in reserve demand. Thus, in principle, no fine-tuning open-market operations are required (which would be needed under a channel system if the aim is to keep the market rate exactly in the middle of the channel).