Final exam

Mathematics and Statistics for Economists

September 9, 2022

The exam will be graded over 100 points.

Exercise 1 (5 points) Given the matrix

$$\mathbf{M} = \begin{bmatrix} 1 & 0 & 5 \\ 2 & 1 & 6 \\ 3 & 4 & 0 \end{bmatrix}$$

Use Gauss-Jordan elimination to compute the inverse of $\mathbf{M}$.

Exercise 2 (15 points) Let $\mathbf{A} = \begin{bmatrix} 3 & 5 & -2 \\ 0 & 2 & 0 \\ 0 & 2 & 1 \end{bmatrix}$

(a) Find the characteristic polynomial of $\mathbf{A}$.

(b) Find the eigenvalues of $\mathbf{A}$.

(c) Determine the definiteness of $\mathbf{A}$.

(d) Find the eigenvectors of $\mathbf{A}$.

Exercise 3 (10 points) Let

$$z = x^2 - 2xy + y^2, \quad x = r + \theta, \quad y = r - \theta$$

Using chain rule, compute the derivatives

$$\frac{dz}{dr} \quad \text{and} \quad \frac{dz}{d\theta}$$

Express these derivatives in function of r and θ.

Exercise 4 (5 points) Find the gradient vector and the Hessian matrix for

$$f(x, y) = \frac{1}{x} + xe^{-y}$$

Exercise 5 (10 points) Discuss the concavity and convexity of the function

$$f(x, y) = -6x^2 + (2a + 4)xy - y^2 + 4ay$$

according to the values of a.

Exercise 6 (10 points) Maximize the function $f(x, y) = x + y^2$ subject to $x^2 + y^2 = 1$.

Exercise 7 (10 points) In the famous Gali and Monacelli (2005) paper, the optimal consumption allocation to domestic and foreign goods are given by

$$C_{H,t} = (1 - \alpha)\Big(\frac{P_{H,t}}{P_t}\Big)^{-\eta} C_t \tag{1}$$

$$C_{F,t} = \alpha\Big(\frac{P_{F,t}}{P_t}\Big)^{-\eta} C_t \tag{2}$$

where $C_{H,t}, C_{F,t}, C_t, P_{F,t}, P_{H,t}$ and P_t are variables. η and α are parameters.

Log-linearize equation 1. Deduct the log-linearized version of equation 2.

Exercise 8 (10 points) Suppose that A and B are events defined on a sample space S, where $P(A) = 0.4$, $P(B) = 0.5$ and $P(A \cap B) = 0.1$. Answer the following:

(a) Are A and B independent?

(b) Find $P(A \cup B)$.

(c) Find $P(A \cap B^c)$ (*Hint*: use a Venn diagram).

(d) Find $P(A|B)$.

(e) Find $P(A|B^c)$.

Exercise 9 (15 points) The discrete random variables X and Y have their joint distribution displayed below.

Probabilities	$X = 1$	$X = 2$	$X = 3$
$Y = 1$	0.1	???	0
$Y = 2$	???	0.2	0.1

Additionally, it is known that $P(X = 2) = 0.6$.

(a) Find $P(X = Y)$.

(b) Find $P(X = 2 \cap Y = 1)$.

(c) Find $P(Y = 2)$.

(d) Are X and Y independent?

(e) Find the expected $E(X)$ and $E(X^2)$.

(f) Find the standard deviation $\sigma(X)$.

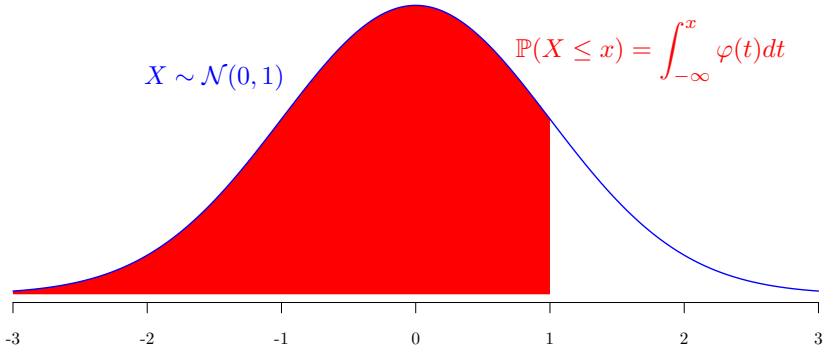
Exercise 10 (10 points) A production line manufactures 1000-ohm (Ω) resistors that have 10 percent tolerance. Let X denote the resistance of a resister. Assuming that X is a normal r.v. with mean 1000 and variance 2500, find the probability that a resistor picked at random will be rejected.

Bonus exercise (10 points) A normal distribution with unknown mean μ and known standard deviation 10 is sampled 5 times. Then the null hypothesis $H_0 : \mu = 0$ is tested against the alternative $H_a : \mu > 0$ with significance level 5%.

(a) If the five sample values are -3, 7, 2, 10 and 7, find 90% and 95% confidence intervals for the mean μ.

(b) If the five sample values are -3, 7, 2, 10 and 7, calculate the p-value and five the outcome of the hypothesis test.

(c) Fill the blank: the null hypothesis will be rejected when the sample mean is greater than _____.



x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990