Macroeconomics A; EI056

Technical appendix: Efficiency wages, matching, and the interaction between goods and labor markets

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1 Efficiency wages

1.1 Basic idea

The basic idea of the efficiency wage model is that wages are not allocative, but serve as incentive devices. The firm maximizes profits which are the value of output (price normalized to one times output Y) minus the wage bill (wage w times labor L). Output is produced using labor through a production function $F(eL)$.

In the usual model e is a parameter that the firm takes as given. The firm also takes the real wage w as a given. The maximization of profits then sets the real wage equal to the marginal product of labor:

$$\begin{aligned} 0 &= \frac{\partial}{\partial L}\left[F(eL) - wL\right] \\ w &= eF'(eL) \end{aligned}$$

Once we combine this with a labor supply the wage clears the labor market and there is no involuntary unemployment (anybody willing to work at the equilibrium wage can work).

With efficiency wages, e is effort. Workers work harder when paid more, so $e = e(w)$ with $e' > 0$. The firm now realizes that the wage can be used to induce effort, and is thus not taking the wage as given. It maximizes expected profits by setting L and w. The first-order condition with respect to labor is as before:

$$\begin{aligned} 0 &= \frac{\partial}{\partial L}\left[F(e(w)L) - wL\right] \\ w &= e(w)F'(e(w)L) \end{aligned}$$

The first-order condition with respect to the wage is:

$$
\begin{aligned}
0 &= \frac{\partial}{\partial w}\left[F\left(e\left(w\right)L\right) - wL\right] \\
0 &= e'\left(w\right)LF'\left(e\left(w\right)L\right) - L \\
1 &= e'\left(w\right)F'\left(e\left(w\right)L\right)
\end{aligned}
$$

Using the first order condition with respect to labor this is rewritten as:

$$
\begin{aligned}
1 &= e'\left(w\right)F'\left(e\left(w\right)L\right) \\
1 &= e'\left(w\right)\frac{w}{e\left(w\right)}
\end{aligned}
$$

This determines the wage. The first order condition with respect to labor then determines employment. However, nothing ensures that this wage - employment combination also satisfies the labor supply, thus we can have involuntary unemployment.

1.2 Introducing multiple firms

A variant of the basic idea is that the effort by workers in a firm is increasing in the wage that the firm pays w, decreasing in the wage that other firms pay w_a, and increasing in the unemployment rate u as workers do not want to loose their job:

$$
e = e\left(w, w_a, u\right) \qquad ; \qquad e_1 > 0 \;\; ; \;\; e_2 < 0 \;\; ; \;\; e_3 > 0
$$

where e_i is the derivative of the function e with respect to its i'th argument.

The first-order conditions with respect to the firm's employment and own wage are as above:

$$
\begin{aligned}
w &= e\left(w, w_a, u\right)F'\left(e\left(w, w_a, u\right)L\right) \\
1 &= e_1\left(w, w_a, u\right)\frac{w}{e\left(w, w_a, u\right)}
\end{aligned}
$$

We pick a more specific parametrization of the function e. There is effort only if the wage w exceeds a reservation value x, which is increasing in the other firms' wage and decreasing in the unemployment rate:

$$
e = \left(\frac{w - x}{x}\right)^{\beta} = \left(\frac{w - \left(1 - bu\right)w_a}{\left(1 - bu\right)w_a}\right)^{\beta} \qquad \text{if } w > x
$$

The optimal condition with respect to effort is then:

$$\begin{aligned}
1 &= e_1(w, w_a, u)\frac{w}{e(w, w_a, u)} \\
1 &= \beta\left(\frac{w-(1-bu)w_a}{(1-bu)w_a}\right)^{\beta-1}\frac{1}{(1-bu)w_a}\frac{w}{\left(\frac{w-(1-bu)w_a}{(1-bu)w_a}\right)^{\beta}} \\
1 &= \beta\left(\frac{w-(1-bu)w_a}{(1-bu)w_a}\right)^{\beta}\frac{1}{w-(1-bu)w_a}\frac{w}{\left(\frac{w-(1-bu)w_a}{(1-bu)w_a}\right)^{\beta}} \\
1 &= \beta\frac{w}{w-(1-bu)w_a} \\
w-(1-bu)w_a &= \beta w \\
w(1-\beta) &= (1-bu)w_a
\end{aligned}$$

In equilibrium all firms must set the same wage. Thus we must have:

$$\begin{aligned}
1-\beta &= 1-bu \\
\beta &= bu \\
u &= \frac{\beta}{b}
\end{aligned}$$

The equilibrium unemployment rate thus only depends on the parameters of the effort function $e(w, w_a, u)$.

1.3 The Shapiro-Stiglitz model of efficiency wages

There are $\bar{L}$ workers. Workers maximize an intertemporal utility:

$$U = \int_{t=0}^{\infty} e^{-\rho t}u_t dt$$

u depends on the employment status. If unemployed it is normalized at zero. If employed, the worker gets a wage w_t and exerts an effort e_t that can be zero or $\bar{e}$. Zero effort is a situation of "shirking". The utilities are then:

$$\begin{array}{lll}
u = & 0 & \text{when unemployed} \\
u = & w & \text{when employed and shirking} \\
u = & w-\bar{e} & \text{when employed and working}
\end{array}$$

Workers produce only when exerting an effort. The firm profits are:

$$\pi_t = F(\bar{e}L_t) - w_t(L_t + S_t)$$

where L is the numbers of employee who exert an effort and S is the number who do not.

Jobs are destroyed at an exogenous rate b. In addition, if a worker exerts no effort he is caught with probability q and fired. Finally, unemployed workers find a job at a rate a. We focus on a steady state and denote the values of being a worker exerting effort, a worker exerting no effort, and an unemployed, as V_E, V_S and V_U. The values are driven by the following equations:

$$
\begin{aligned}
\rho V_E &= w - \bar{e} + b\left(V_U - V_E\right) \\
\rho V_S &= w + (b+q)\left(V_U - V_S\right) \\
\rho V_U &= a\left(V_E - V_U\right)
\end{aligned}
$$

These are to asset return. Consider the relation for V_E. V_E is the value of the "asset" of being employed and working. The rate of return is ρ. The return ρV_E consists of a wage stream (net of effort) $w - \bar{e}$ which is similar to a dividend, and a capital loss $V_U - V_E$ in case of being fired, which happens with probability b.

Express the last relation as:

$$
V_U = \frac{a}{\rho + a} V_E \tag{1}
$$

Substitute this in the first relation:

$$
\begin{aligned}
\rho V_E &= w - \bar{e} + b\left(V_U - V_E\right) \\
\rho V_E &= w - \bar{e} + b\left(\frac{a}{\rho + a} V_E - V_E\right) \\
\rho V_E &= w - \bar{e} - \frac{b\rho}{\rho + a} V_E \\
V_E &= \frac{\rho + a}{\rho + a + b} \frac{w - \bar{e}}{\rho} \qquad (2)
\end{aligned}
$$

(1) then becomes:

$$
V_U = \frac{a}{\rho + a} V_E = \frac{a}{\rho + a + b} \frac{w - \bar{e}}{\rho}
$$

The value for V_S is then computed as:

$$
\begin{aligned}
\rho V_S &= w + (b+q)\left(V_U - V_S\right) \\
(\rho + b + q) V_S &= w + (b+q) V_U \\
V_S &= \frac{1}{\rho + b + q} w + \frac{a(b+q)}{\rho + a + b} \frac{1}{\rho + b + q} \frac{w - \bar{e}}{\rho} \qquad (3)
\end{aligned}
$$

Firms pay a wage that just ensures that workers exert effort: $V_E = V_S$. Using (2) and (3) this implies:

$$
\begin{aligned}
\frac{\rho+a}{\rho+a+b}\frac{w-\bar{e}}{\rho} &= \frac{1}{\rho+b+q}w+\frac{a\left(b+q\right)}{\rho+a+b}\frac{1}{\rho+b+q}\frac{w-\bar{e}}{\rho} \\
0 &= \frac{1}{\rho+b+q}w+\left(\frac{a\left(b+q\right)}{\rho+a+b}\frac{1}{\rho+b+q}-\frac{\rho+a}{\rho+a+b}\right)\frac{w-\bar{e}}{\rho} \\
0 &= w+\frac{1}{\rho+a+b}\left(a\left(b+q\right)-\left(\rho+a\right)\left(\rho+b+q\right)\right)\frac{w-\bar{e}}{\rho} \\
0 &= w+\frac{1}{\rho+a+b}\left(-a\rho-\rho\left(\rho+b+q\right)\right)\frac{w-\bar{e}}{\rho} \\
0 &= w-\frac{a+\rho+b+q}{\rho+a+b}\left(w-\bar{e}\right) \\
w &= \bar{e}+\frac{a+\rho+b}{q}\bar{e} \qquad (4)
\end{aligned}
$$

(1)-(3) then imply:

$$V_U=\frac{a}{q}\frac{\bar{e}}{\rho} \qquad ; \qquad V_E=V_S=\frac{\rho+a}{q}\frac{\bar{e}}{\rho}$$

All employee exert effort. Flows in and out of unemployment cancel out in a steady state:

$$bL=a\left(\bar{L}-L\right)\Rightarrow a=\frac{bL}{\bar{L}-L}\Rightarrow a+b=\frac{\bar{L}}{\bar{L}-L}b$$

where L is employment and $\bar{L}$ is the labor force (the number of firms is set to one). (4) then becomes:

$$w=\bar{e}+\frac{a+\rho+b}{q}\bar{e}=\bar{e}+\left(\rho+\frac{\bar{L}}{\bar{L}-L}b\right)\frac{\bar{e}}{q}=\bar{e}+\left(\rho+\frac{1}{u}b\right)\frac{\bar{e}}{q}$$

where u is the unemployment rate.

The firm profits are:

$$\pi=F\left(\bar{e}L\right)-wL$$

Which is maximized with respect to L at:

$$\bar{e}F'\left(\bar{e}L\right)=w$$

2 An insiders - outsiders model

Workers are either insiders or outsiders, with only insiders bargaining over wages. Insiders choose a wage and let the firm hire as many people as it wants at that wage. Insiders are the workers that were employed last period:

$$N_{It}=L_{t-1}$$

The firm's profits are:

$$\pi_t=A_t\left(L_t\right)^{\alpha}-w_tL_t$$

Conditional on the wage, the labor demand is:

$$\begin{aligned}
\alpha A_t (L_t)^{\alpha-1} &= w_t \\
L_t &= (\alpha A_t)^{\frac{1}{1-\alpha}} \left(\frac{1}{w_t}\right)^{\frac{1}{1-\alpha}} \\
L_t &= C_t (w_t)^{-\beta} \qquad (5)
\end{aligned}$$

where $C_t = (\alpha A_t)^{\frac{1}{1-\alpha}}$ and $\beta = \frac{1}{1-\alpha}$. Productivity is subjected to shocks, so we split C into an expected component and a shock:

$$C_t = C_t^E \varepsilon_t \qquad (6)$$

The utility for an insider when employed is proportional to the wage, and given by w_t^b. If employment drops below the number of insiders, the utility for all insiders is:

$$\frac{L_t}{N_{It}} w_t^b$$

If employment is equal to the number of insiders, the utility is w_t^b. The utility remains at this level if employment is larger as insiders do not care about outsiders. The choice of wage then maximizes (using (5) and (6)):

$$\begin{aligned}
u_t &= E \min\left[\frac{L_t}{N_{It}}, 1\right] w_t^b \\
u_t &= E \min\left[\frac{C_t (w_t)^{-\beta}}{N_{It}}, 1\right] w_t^b \\
u_t &= E \min\left[\frac{C_t^E \varepsilon_t (w_t)^{-\beta}}{N_{It}}, 1\right] w_t^b
\end{aligned}$$

C_t^E, N_{It} and w_t are known before the shock. Define:

$$\chi_t = \frac{C_t^E (w_t)^{-\beta}}{N_{It}} \Rightarrow w_t = \left(\frac{C_t^E}{\chi_t N_{It}}\right)^{\frac{1}{\beta}} \qquad (7)$$

We then write:

$$\begin{aligned}
u_t &= E \min\left[\frac{C_t^E (w_t)^{-\beta}}{N_{It}} \varepsilon_t, 1\right] w_t^b \\
u_t &= E \min\left[\chi_t \varepsilon_t, 1\right] \left(\frac{C_t^E}{\chi_t N_{It}}\right)^{\frac{b}{\beta}} \\
u_t &= \left(\frac{C_t^E}{N_{It}}\right)^{\frac{b}{\beta}} E \min\left[\chi_t \varepsilon_t, 1\right] (\chi_t)^{-\frac{b}{\beta}} \qquad (8)
\end{aligned}$$

For a given χ_t, C_t^E and N_{It} do not alter the maximum value of (8). Therefore they do not affect the choice of χ_t. As they are the only parameters that vary through time in the wage setting, this implies that $\chi_t = \chi^*$. (5) and (7) then imply:

$$\begin{aligned} w_t &= \left(\frac{C_t^E}{\chi_t N_{It}}\right)^{\frac{1}{\beta}} = \left(\frac{C_t^E}{\chi^* N_{It}}\right)^{\frac{1}{\beta}} \\ L_t &= C_t \left(w_t\right)^{-\beta} = C_t^E \varepsilon_t \left(\frac{C_t^E}{\chi^* N_{It}}\right)^{-1} = \varepsilon_t \chi^* N_{It} \end{aligned}$$

3 A search and matching model

3.1 Matching technology and values

Workers and firms are matched to fill positions. A worker gets a wage w if employed and benefits b if not. A filled position produces an output y. In addition to the wage, maintaining a position costs c, whether it's filled or not. We assume that $y > b + c$ The streams of payoff for workers and firms are thus:

employed	:	w
unemployed	:	b
filled position	:	$y - w - c$
unfilled position	:	$-c$

The total number of worker is 1. E are employed and $U = 1 - E$ are unemployed. Open positions are advertised through vacancies V.

Jobs are destroyed at an exogenous rate λ. Workers and positions are matched through a technology:

$$M = kU^{1-\gamma} V^{\gamma} = kU \left(\theta\right)^{\gamma}$$

where $\theta = V/U$ is the tightness of the labor market.

The probability for a worker to find a position is:

$$a = \frac{M}{U} = k \left(\theta\right)^{\gamma}$$

The probability for a vacancy to be filled is:

$$\alpha = \frac{M}{V} = \frac{U}{V}\frac{M}{U} = k \left(\theta\right)^{\gamma-1} = k \left(\left(\frac{a}{k}\right)^{\frac{1}{\gamma}}\right)^{\gamma-1} = \left(k\right)^{\frac{1}{\gamma}} \left(\frac{1}{a}\right)^{\frac{1-\gamma}{\gamma}}$$

Focus on the steady state. The worker's value of being employed is:

$$\rho V_E = w + \lambda \left(V_U - V_E\right)$$

The value of being unemployed is:

$$\rho V_U = b + a \left(V_E - V_U\right)$$

Taking the difference, we get:

$$V_E - V_U = \frac{w-b}{\lambda+\rho+a} \tag{9}$$

The value of a filled position for the firm is:

$$\rho V_F = y - w - c + \lambda\left(V_V - V_F\right)$$

The value of a vacant position is:

$$\rho V_V = -c + \alpha\left(V_F - V_V\right)$$

Taking the difference, we get:

$$V_F - V_V = \frac{y-w}{\lambda+\rho+\alpha} \tag{10}$$

3.2 Wage and employment

We assume that worker get a share ϕ of the surplus generated by filling a vacant position:

$$\begin{aligned} V_E - V_U &= \phi\left[\left(V_E - V_U\right) + \left(V_F - V_V\right)\right] \\ V_E - V_U &= \frac{\phi}{1-\phi}\left(V_F - V_V\right) \end{aligned}$$

Combining this with (9)-(10) we write:

$$\begin{aligned} \frac{w-b}{\lambda+\rho+a} &= \frac{\phi}{1-\phi}\frac{y-w}{\lambda+\rho+\alpha} \\ \left(1-\phi+\frac{\lambda+\rho+a}{\lambda+\rho+\alpha}\phi\right)w-(1-\phi)\,b &= \frac{\lambda+\rho+a}{\lambda+\rho+\alpha}\phi y \\ \left(1-\phi+\frac{\lambda+\rho+a}{\lambda+\rho+\alpha}\phi\right)w &= \frac{\lambda+\rho+a}{\lambda+\rho+\alpha}\phi\left(y-b\right)+\left(1-\phi+\frac{\lambda+\rho+a}{\lambda+\rho+\alpha}\phi\right)b \\ w &= \frac{1}{1-\phi+\frac{\lambda+\rho+a}{\lambda+\rho+\alpha}\phi}\frac{\lambda+\rho+a}{\lambda+\rho+\alpha}\phi\left(y-b\right)+b \\ w &= b+\Phi\phi\left(y-b\right) \end{aligned} \tag{11}$$

where:

$$\Phi = \frac{\lambda+\rho+a}{\lambda+\rho+a+(\alpha-a)(1-\phi)}$$

If $\alpha = a$ we have $\Phi = 1$. If $\alpha > a$ the odds of success are larger for a firm than a worker and $\Phi < 1$.

To determine employment, we first consider the value of a vacant position. Using (10) and (11):

$$\begin{aligned}
\rho V_V &= -c + \alpha \left(V_F - V_V\right) \\
\rho V_V &= -c + \alpha \frac{y - w}{\lambda + \rho + \alpha} \\
\rho V_V &= -c + \alpha \frac{1}{\lambda + \rho + \alpha} \left(1 - \Phi\phi\right)\left(y - b\right) \\
\rho V_V &= -c + \frac{\alpha \left(1 - \phi\right)}{\lambda + \rho + \alpha \left(1 - \phi\right) + a\phi} \left(y - b\right)
\end{aligned}$$

a and α are endogenous and need to be determined. In the steady state, the matches offset the job destruction to leave employment constant:

$$\begin{aligned}
M &= \lambda E \\
kU^{1-\gamma}V^{\gamma} &= \lambda E \\
Ua &= \lambda E \\
\left(1 - E\right) a &= \lambda E \\
a &= \frac{\lambda E}{1 - E}
\end{aligned}$$

hence:

$$\alpha = \left(k\right)^{\frac{1}{\gamma}} \left(\frac{1}{a}\right)^{\frac{1-\gamma}{\gamma}} = \left(k\right)^{\frac{1}{\gamma}} \left(\frac{1 - E}{\lambda E}\right)^{\frac{1-\gamma}{\gamma}}$$

a is increasing in E and α is decreasing in E. Note that:

$$\begin{aligned}
\frac{\partial}{\partial \alpha}\left[\frac{\alpha \left(1 - \phi\right)}{\lambda + \rho + \alpha \left(1 - \phi\right) + a\phi}\right] &= \frac{\left(1 - \phi\right)\left(\lambda + \rho + a\phi\right)}{\left[\lambda + \rho + \alpha \left(1 - \phi\right) + a\phi\right]^2} > 0 \\
\frac{\partial}{\partial a}\left[\frac{\alpha \left(1 - \phi\right)}{\lambda + \rho + \alpha \left(1 - \phi\right) + a\phi}\right] &= \frac{-\alpha \left(1 - \phi\right)\phi}{\left[\lambda + \rho + \alpha \left(1 - \phi\right) + a\phi\right]^2} < 0
\end{aligned}$$

Therefore ρV_V is a decreasing function of E. If E goes to zero, $a = 0$, $\alpha \to \infty$ and $\rho V_V = -c + y - b > 0$. If E goes to 1, $a \to \infty$, $\alpha = 0$ and $\rho V_V = -c < 0$.

Vacancies are costless to setup, and thus their value must be zero.

$$\begin{aligned}
0 &= V_V \\
0 &= -c + \frac{\alpha \left(1 - \phi\right)}{\lambda + \rho + \alpha \left(1 - \phi\right) + a\phi} \left(y - b\right)
\end{aligned}$$

which gives a value of E between 0 and 1.

4 Shocks and institutions

This section is based three lectures by Olivier Blanchard *The Economics of Unemployment: Shocks, Institutions, and Interactions*. It is not exam material and is provided in case you want to deepen the topic.

4.1 The role of shocks in unemployment

4.1.1 The model

This segment is based on lecture 1. The aggregate production function is given by:

$$y = (a(1-u))^{\alpha}(k)^{1-\alpha} \tag{12}$$

where y is output, a productivity, u the unemployment rate implying that employment is $1-u$, and k capital. a grows at a rate g_a. Wages and the cost of capital are denoted by w and c. The firm maximizes:

$$(an)^{\alpha}(k)^{1-\alpha} - w(1-u) - ck$$

In the short run capital is fixed. The labor demand is then the first-order condition with respect to employment:

$$\frac{w}{a} = \left(\frac{k}{a(1-u)}\right)^{1-\alpha} \Rightarrow (1-u) = \frac{k}{a}\left(\frac{w}{a}\right)^{\frac{-1}{1-\alpha}} = \frac{k}{a}f\left(\frac{w}{a}\right) \qquad ; \qquad f' < 0 \tag{13}$$

In the long run capital can adjust and we get:

$$\begin{aligned} c &= \left(\frac{a(1-u)}{k}\right)^{\alpha} = \left(\frac{a}{k}(1-u)\right)^{\alpha} = \left(\frac{a}{k}\frac{k}{a}\left(\frac{w}{a}\right)^{\frac{-1}{1-\alpha}}\right)^{\alpha} \\ &= \left(\frac{w}{a}\right)^{\frac{-\alpha}{1-\alpha}} = g\left(\frac{w}{a}\right) \end{aligned} \tag{14}$$

Take a wage setting relation that is a negative relation between the real wage and the unemployment rate:

$$\frac{w}{a} = zh(u) \qquad ; \qquad h' < 0 \tag{15}$$

4.1.2 Shocks

Consider a productivity slowdown that g_a permanently decreases. Is this is understood right away, the pace of wage increases also slows down to keep w/a constant. If instead wage-setters think that productivity is given by a^*, (15) implies:

$$\frac{w}{a^*} = zh(u) \Rightarrow \frac{w}{a} = \frac{a^*}{a}zh(u) = z'h(u)$$

where $z' > z$ when $a^* > a$.

Another shock is an increase in the real interest rate that raises the cost of capital c.

4.2 The role of institutions in unemployment

4.2.1 The model

This segment is based on the second lecture.

Consumption consists of a basket of imperfectly substitutable brands. The firm producing a brand i faces the following demand:

$$Y_i = \frac{Y}{n} D\left(\frac{P_i}{P}\right) \qquad ; \qquad D' < 0 \quad ; \quad D(1) = 1$$

where n is the number of firms. The elasticity of demand with respect to prices depends on the extent of competition:

$$\sigma = \bar{\sigma} g(n) \qquad ; \qquad g' > 0 \quad ; \quad g(\infty) = \infty$$

Output is produced using a linear technology:

$$Y_i = N_i$$

The demand is written as a function of price to output:

$$Y_i = \frac{Y}{n}\left(\frac{P_i}{P}\right)^{-\sigma} \Rightarrow \frac{P_i}{P} = \left(\frac{Y}{n}\frac{1}{Y_i}\right)^{\frac{1}{\sigma}} \tag{16}$$

The marginal revenue for a firms is given by:

$$\begin{aligned} MRP_i &= \frac{\partial}{\partial Y_i}\left(\frac{P_i}{P} Y_i\right) = \frac{\partial}{\partial Y_i}\left(\left(\frac{Y}{n}\right)^{\frac{1}{\sigma}} (Y_i)^{\frac{\sigma-1}{\sigma}}\right) \\ &= \frac{\sigma - 1}{\sigma}\left(\frac{Y}{n}\right)^{\frac{1}{\sigma}} (Y_i)^{\frac{-1}{\sigma}} \\ &= \frac{\sigma - 1}{\sigma}\left(\frac{Y}{n}\frac{1}{Y_i}\right)^{\frac{1}{\sigma}} \end{aligned} \tag{17}$$

The reservation wage of workers is inversely linked to unemployment:

$$\left(\frac{W}{P}\right)_R = bk(u) \qquad ; \qquad k' < 0 \tag{18}$$

Wages are determined by bargaining. The surplus for workers is:

$$N_i\left[\frac{W_i}{P} - bk(u)\right] \tag{19}$$

were W_i is the wage offered by firm i. The firm cares about profits:

$$\frac{P_i}{P} Y_i - \frac{W_i}{P} N_i \tag{20}$$

The bargaining power of workers is β and that of firms is $1 - \beta$.

4.2.2 Short-run equilibrium

In the short run the numbers of firms is given. The solution proceeds in two steps. First, maximize the value of the total surplus from employment. Second, set the wage to split this surplus between the firm and workers.

The total value of the surplus is the sum of (19) and (20). Using (16) it is written as:

$$\begin{aligned}
& \frac{P_i}{P}Y_i - \frac{W_i}{P}N_i + N_i\left[\frac{W_i}{P} - bk(u)\right] \\
= & \frac{P_i}{P}Y_i - N_i bk(u) \\
= & \frac{P_i}{P}Y_i - Y_i bk(u) \\
= & \frac{Y}{n}\left(\frac{P_i}{P}\right)^{1-\sigma} - \frac{Y}{n}\left(\frac{P_i}{P}\right)^{-\sigma} bk(u)
\end{aligned}$$

The price P_i maximizes this surplus:

$$\begin{aligned}
0 &= (1-\sigma)\frac{Y}{n}\left(\frac{P_i}{P}\right)^{-\sigma}\frac{1}{P} + \sigma\frac{Y}{n}\left(\frac{P_i}{P}\right)^{-\sigma-1}\frac{1}{P}bk(u) \\
0 &= (1-\sigma)\frac{Y}{n}\frac{P_i}{P}\frac{1}{P} + \sigma\frac{Y}{n}\frac{1}{P}bk(u) \\
0 &= (1-\sigma)\frac{P_i}{P} + \sigma bk(u) \\
\frac{P_i}{P} &= \frac{\sigma}{\sigma-1}bk(u) \qquad (21)
\end{aligned}$$

Note that (21) is such that the marginal revenue (17) is equal to the reservation wage (18):

$$\frac{\sigma-1}{\sigma}\left(\frac{Y}{n}\frac{1}{Y_i}\right)^{\frac{1}{\sigma}} = \frac{\sigma-1}{\sigma}\frac{P_i}{P} = bk(u)$$

The surplus is split using the wage. The method is the Nash rule, which maximizes a weighted product of the firm's and workers' surpluses, with the weight reflecting the bargaining power:

$$0 = \frac{\partial}{\partial(W_i/P)}\left(\frac{P_i}{P}Y_i - \frac{W_i}{P}N_i\right)^{1-\beta}\left(N_i\left[\frac{W_i}{P} - bk(u)\right]\right)^{\beta}$$

Using (16)-(21) this becomes:

$$
\begin{aligned}
0 &= \frac{\partial}{\partial\left(W_i/P\right)}\left(\frac{Y}{n}\left(\frac{P_i}{P}\right)^{1-\sigma}-\frac{W_i}{P}\frac{Y}{n}\left(\frac{P_i}{P}\right)^{-\sigma}\right)^{1-\beta}\left(\frac{Y}{n}\left(\frac{P_i}{P}\right)^{-\sigma}\left[\frac{W_i}{P}-bk\left(u\right)\right]\right)^{\beta}\\
0 &= \frac{\partial}{\partial\left(W_i/P\right)}\left[\begin{array}{c}\left(\frac{Y}{n}\left(\frac{\sigma}{\sigma-1}bk\left(u\right)\right)^{1-\sigma}-\frac{W_i}{P}\frac{Y}{n}\left(\frac{\sigma}{\sigma-1}bk\left(u\right)\right)^{-\sigma}\right)^{1-\beta}\\ \times\left(\frac{Y}{n}\left(\frac{\sigma}{\sigma-1}bk\left(u\right)\right)^{-\sigma}\left[\frac{W_i}{P}-bk\left(u\right)\right]\right)^{\beta}\end{array}\right]\\
0 &= \frac{Y}{n}\left(\frac{\sigma}{\sigma-1}bk\left(u\right)\right)^{-\sigma}\frac{\partial}{\partial\left(W_i/P\right)}\left(\frac{\sigma}{\sigma-1}bk\left(u\right)-\frac{W_i}{P}\right)^{1-\beta}\left(\frac{W_i}{P}-bk\left(u\right)\right)^{\beta}\\
0 &= \frac{\partial}{\partial\left(W_i/P\right)}\left(\frac{\sigma}{\sigma-1}bk\left(u\right)-\frac{W_i}{P}\right)^{1-\beta}\left(\frac{W_i}{P}-bk\left(u\right)\right)^{\beta}\\
0 &= -\left(1-\beta\right)\left(\frac{\sigma}{\sigma-1}bk\left(u\right)-\frac{W_i}{P}\right)^{-\beta}\left(\frac{W_i}{P}-bk\left(u\right)\right)^{\beta}\\
&\quad+\beta\left(\frac{\sigma}{\sigma-1}bk\left(u\right)-\frac{W_i}{P}\right)^{1-\beta}\left(\frac{W_i}{P}-bk\left(u\right)\right)^{\beta-1}\\
0 &= -\left(1-\beta\right)\left(\frac{W_i}{P}-bk\left(u\right)\right)+\beta\left(\frac{\sigma}{\sigma-1}bk\left(u\right)-\frac{W_i}{P}\right)\\
0 &= -\frac{W_i}{P}+\left[1-\beta+\beta\frac{\sigma}{\sigma-1}\right]bk\left(u\right)\\
\frac{W_i}{P} &= \frac{\sigma-1+\beta}{\sigma-1}bk\left(u\right) \qquad (22)
\end{aligned}
$$

The expressions so far are for an individual firm. Overall all firms are identical and $P_i = P$. (21) implies:

$$\frac{\sigma}{\sigma-1}bk\left(u\right)=1 \qquad (23)$$

which implies that unemployment is high when the elasticity substitution is low:

$$
\begin{aligned}
0 &= \frac{\sigma}{\sigma-1}bk'\left(u\right)du-\frac{1}{\left(\sigma-1\right)^2}bk\left(u\right)d\sigma\\
0 &= \sigma\left(\sigma-1\right)k'\left(u\right)du-k\left(u\right)d\sigma\\
\frac{du}{d\sigma} &= \frac{k\left(u\right)}{\sigma\left(\sigma-1\right)k'\left(u\right)}<0
\end{aligned}
$$

(23) sets the aggregate employment:

$$
\begin{aligned}
1 &= \frac{\sigma}{\sigma-1}bk\left(u\right)\\
1 &= \frac{\sigma}{\sigma-1}bk\left(\frac{L-N}{L}\right)
\end{aligned}
$$

Using (23), (22) implies:

$$\frac{W_i}{P}=\frac{\sigma-1+\beta}{\sigma-1}bk\left(u\right)=\frac{\sigma-1+\beta}{\sigma-1}\frac{\sigma-1}{\sigma}=\frac{\sigma-1+\beta}{\sigma} \qquad (24)$$

4.2.3 Long-run equilibrium

In the long run the numbers of firms is endogenous, and adjusts such that the profits of firms are equal to a real entry cost cY_i (we assume $c < 1 - \beta$).

Using the results in the previous section, profits are:

$$
\begin{aligned}
& \frac{P_i}{P}Y_i - \frac{W_i}{P}N_i - cY_i \\
= & \left[\frac{P_i}{P} - \frac{W_i}{P} - c\right] Y_i \\
= & \left[1 - \frac{\sigma - 1 + \beta}{\sigma} - c\right] Y_i \\
= & \left[\frac{1-\beta}{\sigma} - c\right] Y_i
\end{aligned}
$$

Profits are zero when:

$$
\begin{aligned}
c\sigma &= 1 - \beta \\
\bar{\sigma} g(n) &= \frac{1-\beta}{c}
\end{aligned}
$$

which pins down the number of firms. It implies:

$$
\begin{aligned}
c\bar{\sigma} g'(n)\, dn &= -d\beta - dc\bar{\sigma} g(n) - cg(n)\, d\bar{\sigma} \\
dn &= \underbrace{-\frac{1}{g'(n)}}_{<0} \left[\frac{1}{c\bar{\sigma}} d\beta + \frac{g(n)}{c} dc + \frac{g(n)}{\bar{\sigma}} d\bar{\sigma}\right]
\end{aligned}
$$

Combining with (23) we write:

$$
\begin{aligned}
1 &= \frac{\sigma}{\sigma - 1} bk(u) \\
1 &= \frac{\frac{1-\beta}{c}}{\frac{1-\beta}{c} - 1} bk(u) \\
1 &= \frac{1-\beta}{1-\beta-c} bk(u)
\end{aligned}
$$

which implies that long run unemployment is high when b, c, or β are high:

$$
\begin{aligned}
(1-\beta)\, bk(u) &= 1 - \beta - c \\
(1-\beta)\, bk'(u)\, du &= -d\beta - dc + bk(u)\, d\beta - (1-\beta)\, k(u)\, db \\
(1-\beta)\, bk'(u)\, du &= -(1 - bk(u))\, d\beta - dc - (1-\beta)\, k(u)\, db \\
du &= \underbrace{-\frac{1}{k'(u)}}_{>0} \left[\frac{1}{\sigma}\frac{1}{(1-\beta)\, b} d\beta + \frac{1}{(1-\beta)\, b} dc + \frac{k(u)}{b} db\right]
\end{aligned}
$$

The real wage is given by (24):

$$
\begin{aligned}
\frac{W_i}{P} &= \frac{\sigma - 1 + \beta}{\sigma} \\
&= \frac{\frac{1-\beta}{c} - 1 + \beta}{\frac{1-\beta}{c}} \\
&= \frac{\frac{1}{c} - 1}{\frac{1}{c}} \\
&= 1 - c
\end{aligned}
$$

4.3 Employment dynamics and protection

This section is based on lecture 3 by Blanchard.

Creating new jobs requires a matching of workers and firms. Firms post vacancies, v, and there are u unemployed workers looking for work. Hires h are generated through a matching process:

$$h = \sqrt{zuv}$$

where z is the productivity of the matching process. It is the product of exit rates:

$$z = x_u x_v \qquad ; \qquad x_u = \frac{h}{u} \qquad ; \qquad x_v = \frac{h}{v}$$

Job are destroyed at a rate λ which is a decreasing function of a firing cost f. The feasible wage (i.e. the wage that leaves no profits for firms) is also a decreasing function of the firing cost:

$$w = \phi\,(f, ...) \qquad ; \qquad \phi_f < 0$$

b is the level of unemployment benefits. The wage is then written as:

$$
\begin{aligned}
w &= b + g\left(\frac{x_u}{x_v}, f, ...\right) = b + g\left(\frac{v}{u}, f, ...\right) \\
g_{v/u} &> 0 \qquad ; \qquad g_f > 0
\end{aligned}
$$

The bargained wage is equal to the feasible wage:

$$
\begin{aligned}
\phi\,(f, ...) &= b + g\left(\frac{x_u}{x_v}, f, ...\right) \\
\phi\,(f, ...) &= b + g\left(\frac{(x_u)^2}{z}, f, ...\right)
\end{aligned}
$$

This implies:

$$\begin{aligned}
\phi_f df &= db + 2g_{v/u}\frac{x_u}{z}dx_u - g_{v/u}\left(\frac{x_u}{z}\right)^2 dz + g_f df \\
2g_{v/u}\frac{x_u}{z}dx_u &= \left(\phi_f - g_f\right) df - db + g_{v/u}\left(\frac{x_u}{z}\right)^2 dz \\
dx_u &= \underbrace{\frac{\phi_f - g_f}{2g_{v/u}\frac{x_u}{z}}}_{<0} df + \underbrace{-\frac{1}{2g_{v/u}\frac{x_u}{z}}}_{<0} db + \underbrace{\frac{1}{2}\frac{x_u}{z}}_{>0} dz
\end{aligned}$$

In a steady states flows in and out of unemployment cancel out:

$$\lambda\left(1-u\right) = x_u u \Rightarrow u = \frac{\lambda}{\lambda + x_u}$$

This implies:

$$\begin{aligned}
du &= \frac{1}{\lambda + x_u}\lambda_f df - \frac{\lambda}{\left(\lambda + x_u\right)^2}\lambda_f df - \frac{\lambda}{\left(\lambda + x_u\right)^2} dx_u \\
du &= \frac{x_u}{\left(\lambda + x_u\right)^2}\lambda_f df - \frac{\lambda}{\left(\lambda + x_u\right)^2}\left[\frac{\phi_f - g_f}{2g_{v/u}\frac{x_u}{z}} df - \frac{1}{2g_{v/u}\frac{x_u}{z}} db + \frac{1}{2}\frac{x_u}{z} dz\right] \\
du &= \frac{1}{\left(\lambda + x_u\right)^2}\left[\underbrace{x_u\lambda_f}_{<0} + \underbrace{-\lambda\frac{\phi_f - g_f}{2g_{v/u}\frac{x_u}{z}}}_{>0}\right] df \\
&\quad - \frac{\lambda}{\left(\lambda + x_u\right)^2}\left[-\frac{1}{2g_{v/u}\frac{x_u}{z}} db + \frac{1}{2}\frac{x_u}{z} dz\right]
\end{aligned}$$