Macroeconomics A; EI060

Short problems

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1 Optimal prices

Question: A Home firm sets a price $P(z,h)$ for domestic sales and $P^*(z,h)$ (in Foreign currency) for export sales.

The demand it faces is:

$$Y(z,h) = n\left[\frac{P(z,h)}{P(h)}\right]^{-\theta}\left[\frac{P(h)}{P}\right]^{-\lambda} C + (1-n)\left[\frac{P^*(z,h)}{P^*(h)}\right]^{-\theta}\left[\frac{P^*(h)}{P^*}\right]^{-\lambda} C^*$$

Each unit of output is produced using A units of labor paid W.

Show that the optimal prices are:

$$P(z,h) = \mathcal{E}P^*(z,h) = \frac{\theta}{\theta-1}\frac{W}{A}$$

Answer: It takes $1/A$ unit of labor for each unit of output. The profits are:

$$\begin{aligned}
\Pi(z,h) = & \left(P(z,h) - \frac{W}{A}\right) n\left[\frac{P(z,h)}{P(h)}\right]^{-\theta}\left[\frac{P(h)}{P}\right]^{-\lambda} C \\
& + \left(\mathcal{E}P^*(z,h) - \frac{W}{A}\right)(1-n)\left[\frac{P^*(z,h)}{P^*(h)}\right]^{-\theta}\left[\frac{P^*(h)}{P^*}\right]^{-\lambda} C^*
\end{aligned}$$

The first-order condition with respect to $P(z,h)$ is:

$$\begin{aligned}
0 &= \frac{\partial \Pi(z,h)}{\partial P(z,h)} \\
0 &= n\left[\frac{P(z,h)}{P(h)}\right]^{-\theta}\left[\frac{P(h)}{P}\right]^{-\lambda} C \\
&\quad -\theta\left(P(z,h) - \frac{W}{A}\right) n\left[\frac{P(z,h)}{P(h)}\right]^{-\theta-1}\frac{1}{P(h)}\left[\frac{P(h)}{P}\right]^{-\lambda} C \\
0 &= n\frac{P(z,h)}{P(h)} - \theta\left(P(z,h) - \frac{W}{A}\right) n\frac{1}{P(h)} \\
0 &= P(z,h) - \theta\left(P(z,h) - \frac{W}{A}\right) \\
P(z,h) &= \frac{\theta}{\theta-1}\frac{W}{A}
\end{aligned}$$

The first-order condition with respect to $P^*(z,h)$ is:

$$
\begin{aligned}
0 &= \frac{\partial \Pi(z,h)}{\partial P^*(z,h)} \\
0 &= \mathcal{E}(1-n)\left[\frac{P^*(z,h)}{P^*(h)}\right]^{-\theta}\left[\frac{P^*(h)}{P^*}\right]^{-\lambda} C^* \\
&\quad -\theta\left(\mathcal{E}P^*(z,h)-\frac{W}{A}\right)(1-n)\left[\frac{P^*(z,h)}{P^*(h)}\right]^{-\theta-1}\frac{1}{P^*(h)}\left[\frac{P^*(h)}{P^*}\right]^{-\lambda} C^* \\
0 &= \mathcal{E}(1-n)\frac{P^*(z,h)}{P^*(h)} - \theta\left(\mathcal{E}P^*(z,h)-\frac{W}{A}\right)(1-n)\frac{1}{P^*(h)} \\
0 &= \mathcal{E}P^*(z,h) - \theta\left(\mathcal{E}P^*(z,h)-\frac{W}{A}\right) \\
\mathcal{E}P^*(z,h) &= \frac{\theta}{\theta-1}\frac{W}{A}
\end{aligned}
$$

2 GG line

Question: The linear approximation shows that in the long run relative consumption is linked to the current account as:

$$\bar{\mathsf{c}} - \bar{\mathsf{c}}^* = \frac{1+\lambda}{2\lambda}\frac{1-\beta}{\beta}\frac{\bar{\mathsf{b}}}{1-n}$$

The Euler condition is:

$$\bar{\mathsf{c}} - \bar{\mathsf{c}}^* = \mathsf{c} - \mathsf{c}^*$$

In the short run, output and the current accounts are:

$$\mathsf{y} - \mathsf{y}^* = \lambda \mathsf{e}$$

The current account is (recalling that the initial b is zero):

$$\frac{\bar{\mathsf{b}}}{1-n} + (\mathsf{c} - \mathsf{c}^*) = -\mathsf{e} + \mathsf{y} - \mathsf{y}^*$$

Show that we get the GG line:

$$\mathsf{e} = \left[1 + \frac{\beta}{1-\beta}\frac{2\lambda}{1+\lambda}\right]\frac{1}{\lambda-1}(\mathsf{c} - \mathsf{c}^*)$$

Answer: Combine the current account and the output difference:

$$
\begin{aligned}
\frac{\bar{\mathsf{b}}}{1-n} + (\mathsf{c} - \mathsf{c}^*) &= -\mathsf{e} + \mathsf{y} - \mathsf{y}^* \\
\frac{\bar{\mathsf{b}}}{1-n} + (\mathsf{c} - \mathsf{c}^*) &= \frac{\lambda-1}{\lambda}(\mathsf{y} - \mathsf{y}^*) \\
\frac{\bar{\mathsf{b}}}{1-n} + (\mathsf{c} - \mathsf{c}^*) &= \frac{\lambda-1}{\lambda}\lambda\mathsf{e} \\
\frac{\bar{\mathsf{b}}}{1-n} + (\mathsf{c} - \mathsf{c}^*) &= (\lambda-1)\mathsf{e}
\end{aligned}
$$

We solve $\frac{\bar{\mathsf{b}}}{1-n}$ as a function of $\mathsf{c}-\mathsf{c}^*$ starting from the long run relation:

$$\begin{aligned}
\bar{\mathsf{c}}-\bar{\mathsf{c}}^* &= \frac{1+\lambda}{2\lambda}\frac{1-\beta}{\beta}\frac{\bar{\mathsf{b}}}{1-n} \\
\frac{1-\beta}{\beta}\frac{\bar{\mathsf{b}}}{1-n} &= \frac{2\lambda}{1+\lambda}\left(\bar{\mathsf{c}}-\bar{\mathsf{c}}^*\right) \\
\frac{\bar{\mathsf{b}}}{1-n} &= \frac{\beta}{1-\beta}\frac{2\lambda}{1+\lambda}\left(\bar{\mathsf{c}}-\bar{\mathsf{c}}^*\right)
\end{aligned}$$

The Euler condition implies that $\bar{\mathsf{c}}-\bar{\mathsf{c}}^*=\mathsf{c}-\mathsf{c}^*$.

Using all this, the current account becomes:

$$\begin{aligned}
\frac{\bar{\mathsf{b}}}{1-n}+\left(\mathsf{c}-\mathsf{c}^*\right) &= (\lambda-1)\,\mathsf{e} \\
\frac{\beta}{1-\beta}\frac{2\lambda}{1+\lambda}\left(\mathsf{c}-\mathsf{c}^*\right)+\left(\mathsf{c}-\mathsf{c}^*\right) &= (\lambda-1)\,\mathsf{e} \\
\left[1+\frac{\beta}{1-\beta}\frac{2\lambda}{1+\lambda}\right]\left(\mathsf{c}-\mathsf{c}^*\right) &= (\lambda-1)\,\mathsf{e} \\
\left[1+\frac{\beta}{1-\beta}\frac{2\lambda}{1+\lambda}\right]\frac{1}{\lambda-1}\left(\mathsf{c}-\mathsf{c}^*\right) &= \mathsf{e}
\end{aligned}$$

3 Relative welfare with complete pass-through

Question: The linearized welfare expressions are:

$$\begin{aligned}
\mathsf{u}_t &= \mathsf{c}-\frac{\theta-1}{\theta}\mathsf{y}+\frac{\beta}{1-\beta}\left[\bar{\mathsf{c}}-\frac{\theta-1}{\theta}\bar{\mathsf{y}}\right] \\
\mathsf{u}_t^* &= \mathsf{c}^*-\frac{\theta-1}{\theta}\mathsf{y}^*+\frac{\beta}{1-\beta}\left[\bar{\mathsf{c}}^*-\frac{\theta-1}{\theta}\bar{\mathsf{y}}^*\right]
\end{aligned}$$

The long run and short run current account expressions are:

$$\begin{aligned}
\frac{1}{1-n}\bar{\mathsf{b}}+\left(\bar{\mathsf{c}}-\bar{\mathsf{c}}^*\right) &= \frac{1}{\beta}\frac{1}{1-n}\bar{\mathsf{b}}+\left[\bar{\mathsf{p}}\left(h\right)-\bar{\mathsf{p}}^*\left(f\right)-\bar{\mathsf{e}}\right]+\left(\bar{\mathsf{y}}-\bar{\mathsf{y}}^*\right) \\
\frac{\bar{\mathsf{b}}}{1-n}+\left(\mathsf{c}-\mathsf{c}^*\right) &= -\mathsf{e}+\mathsf{y}-\mathsf{y}^*
\end{aligned}$$

and the long run and short run outputs are:

$$\begin{aligned}
\bar{\mathsf{y}}-\bar{\mathsf{y}}^* &= -\lambda\left[\bar{\mathsf{p}}\left(h\right)-\bar{\mathsf{p}}^*\left(f\right)-\bar{\mathsf{e}}\right] \\
\mathsf{y}-\mathsf{y}^* &= \lambda\mathsf{e}
\end{aligned}$$

Show that the relative welfare is:

$$\mathsf{u}_t-\mathsf{u}_t^* = \frac{\lambda-\theta}{\lambda\theta}\left[\left(\mathsf{y}-\mathsf{y}^*\right)+\frac{1-\beta}{\beta}\left(\bar{\mathsf{y}}-\bar{\mathsf{y}}^*\right)\right]$$

Answer: Wese the output demands to substitute for exchange rate and prices in the current accounts:

$$\frac{1}{1-n}\bar{\mathsf{b}}+\left(\bar{\mathsf{c}}-\bar{\mathsf{c}}^*\right) = \frac{1}{\beta}\frac{1}{1-n}\bar{\mathsf{b}}+\left[\bar{\mathsf{p}}\left(h\right)-\bar{\mathsf{p}}^*\left(f\right)-\bar{\mathsf{e}}\right]+\left(\bar{\mathsf{y}}-\bar{\mathsf{y}}^*\right)$$

$$\begin{aligned}
(\bar{\mathsf{c}} - \bar{\mathsf{c}}^*) &= \frac{1-\beta}{\beta}\frac{1}{1-n}\bar{\mathsf{b}} + [\bar{\mathsf{p}}(h) - \bar{\mathsf{p}}^*(f) - \bar{\mathsf{e}}] + (\bar{\mathsf{y}} - \bar{\mathsf{y}}^*) \\
(\bar{\mathsf{c}} - \bar{\mathsf{c}}^*) &= \frac{1-\beta}{\beta}\frac{1}{1-n}\bar{\mathsf{b}} - \frac{1}{\lambda}(\bar{\mathsf{y}} - \bar{\mathsf{y}}^*) + (\bar{\mathsf{y}} - \bar{\mathsf{y}}^*) \\
(\bar{\mathsf{c}} - \bar{\mathsf{c}}^*) &= \frac{1-\beta}{\beta}\frac{1}{1-n}\bar{\mathsf{b}} + \frac{\lambda-1}{\lambda}(\bar{\mathsf{y}} - \bar{\mathsf{y}}^*)
\end{aligned}$$

and:

$$\begin{aligned}
\frac{\bar{\mathsf{b}}}{1-n} + (\mathsf{c} - \mathsf{c}^*) &= -\mathsf{e} + \mathsf{y} - \mathsf{y}^* \\
\frac{\bar{\mathsf{b}}}{1-n} + (\mathsf{c} - \mathsf{c}^*) &= -\frac{1}{\lambda}(\mathsf{y} - \mathsf{y}^*) + (\bar{\mathsf{y}} - \bar{\mathsf{y}}^*) \\
\frac{\bar{\mathsf{b}}}{1-n} + (\mathsf{c} - \mathsf{c}^*) &= \frac{\lambda-1}{\lambda}(\mathsf{y} - \mathsf{y}^*)
\end{aligned}$$

We write the welfare difference as:

$$\mathsf{u}_t - \mathsf{u}_t^* = (\mathsf{c} - \mathsf{c}^*) - \frac{\theta-1}{\theta}(\mathsf{y} - \mathsf{y}^*) + \frac{\beta}{1-\beta}\left[(\bar{\mathsf{c}} - \bar{\mathsf{c}}^*) - \frac{\theta-1}{\theta}(\bar{\mathsf{y}} - \bar{\mathsf{y}}^*)\right]$$

We use the current accounts to substitute for consumption:

$$\begin{aligned}
\mathsf{u}_t - \mathsf{u}_t^* &= \left(\frac{\lambda-1}{\lambda}(\mathsf{y} - \mathsf{y}^*) - \frac{\bar{\mathsf{b}}}{1-n}\right) - \frac{\theta-1}{\theta}(\mathsf{y} - \mathsf{y}^*) \\
&\quad + \frac{\beta}{1-\beta}\left[\frac{1-\beta}{\beta}\frac{1}{1-n}\bar{\mathsf{b}} + \frac{\lambda-1}{\lambda}(\bar{\mathsf{y}} - \bar{\mathsf{y}}^*) - \frac{\theta-1}{\theta}(\bar{\mathsf{y}} - \bar{\mathsf{y}}^*)\right] \\
\mathsf{u}_t - \mathsf{u}_t^* &= -\frac{\bar{\mathsf{b}}}{1-n} + \frac{\beta}{1-\beta}\frac{1-\beta}{\beta}\frac{1}{1-n}\bar{\mathsf{b}} \\
&\quad + \left(\frac{\lambda-1}{\lambda} - \frac{\theta-1}{\theta}\right)(\mathsf{y} - \mathsf{y}^*) + \frac{\beta}{1-\beta}\left(\frac{\lambda-1}{\lambda} - \frac{\theta-1}{\theta}\right)(\bar{\mathsf{y}} - \bar{\mathsf{y}}^*) \\
\mathsf{u}_t - \mathsf{u}_t^* &= \left(\frac{\lambda\theta-\theta}{\lambda\theta} - \frac{\lambda\theta-\lambda}{\lambda\theta}\right)\left[(\mathsf{y} - \mathsf{y}^*) + \frac{\beta}{1-\beta}(\bar{\mathsf{y}} - \bar{\mathsf{y}}^*)\right] \\
\mathsf{u}_t - \mathsf{u}_t^* &= \frac{\lambda-\theta}{\lambda\theta}\left[(\mathsf{y} - \mathsf{y}^*) + \frac{\beta}{1-\beta}(\bar{\mathsf{y}} - \bar{\mathsf{y}}^*)\right]
\end{aligned}$$