ECON 39: Undergraduate International Trade
Practice Problems, Week #2
Winter 2017, Professor Treb Allen

Setting

Consider a world with a single country (say, the United States) which is inhabited with L workers. Suppose the workers each have one unit of time, with which they can either produce good 1 or good 2. Suppose it takes α_1 units of time to produce good 1 and α_2 units of time to produce good 2. Finally, suppose the workers have preferences expressed by the following utility function:

$$U = \min\{\beta_1 c_1, \beta_2 c_2\},$$

where c_1 and c_2 are the quantities that each worker consumes of good 1 and good 2, respectively, and $\beta_1 > 0$ and $\beta_2 > 0$. (These preferences are known as *Leontief* preferences). Let q_1 and q_2 denote the quantity produced by each worker of good 1 and 2, respectively. Finally, let p_1 and p_2 be the price of goods 1 and 2, respectively.

Questions

1. What are the exogenous model parameters in this setting?
2. What are the endogenous model outcomes in this setting?
3. Let us first consider the problem of the worker *producing* things.
   (a) What does a worker try to maximize in her production decision?
   (b) What endogenous outcomes does the worker choose in her production decision?
   (c) What endogenous outcomes does the worker take as given in her production decision?
   (d) What constraints does the worker face in her production decision?
   (e) Write down the mathematical expression of the worker's production decision problem:
   (f) In equilibrium, will the worker ever spend all of her time producing just one good? Why or why not?
   (g) What are the first order conditions of the mathematical expression in 3(d)? What is their intuition?
   (h) Can we determine how much of each good is produced from the worker's production decision? Why or why not?
   (i) Can we determine what a workers income is?
4. Now let us consider the problem of the worker *consuming* things.
   (a) What does a worker try to maximize in her consumption decision?
   (b) What endogenous outcomes does the worker choose in her consumption decision?
   (c) What endogenous outcomes does the worker take as given in her consumption decision?
   (d) What constraints does the worker face in her consumption decision?
   (e) Write down the mathematical expression of the worker's consumption decision problem:
   (f) Solve for the equilibrium consumption as a function of β_1, β_2, p_1, p_2, and y.
5. Let us now calculate the market equilibrium.
   (a) Define the market equilibrium:
   (b) Solve for the equilibrium quantity produced / consumed of each good solely as a function of exogenous model parameters.

(c) Does L appear in your expressions? Why or why not?

(d) If we multiplied β_1 and β_2 by the same positive number, would it change the equilibrium consumption and production? Why or why not?

(e) Draw a picture of the equilibrium (i.e. the production possibilities frontier and the indifference curve). Carefully label any intersections of curves and the axis.

6. Finally, let us do a counterfactual: what happens to the consumption of good 2 if workers become more efficient at producing good 1? Show the answer on the board and derive $-\frac{\partial c_2}{\partial \alpha_1}$.