

Lecture Notes: International Trade I

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These lecture notes were taken in the course *International Trade I* taught by **Monika Mrázova** at Graduate of International and Development Studies, Geneva as part of the International Economics program (Semester II, 2024).

Currently, these are just drafts of the lecture notes. There can be typos and mistakes anywhere. So, if you find anything that needs to be corrected or improved, please inform at jingle.fu@graduateinstitute.ch.

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Lecture 1.

Comparative Advantage and Gains from Trade

1.1 Internatioanl trade: Standard Assumptions

What distinguished trade theory from general-equilibrium analysis is the exiatence of a **hierarchical market structure**.

- ‘International’ good markets
- ‘Domestic’ factor markets.

Question.

- How does the integration of good markets affect good prices?
- How do changes in good prices, in turn, affect factor prices, factor allocation, production and welfare?

While these assumptions are less fundamental, we will also often assume that:

Assumption 1.1.1.

- Consumers have identical homothetic preferences in each country (representative agent)
- Model is static.

1.2 Neoclassical trade

“Neoclassical trade theory” is characterized by three key assumptions:

1. **Perfect competition** in all markets
2. **Constant returns to scale** in production
3. **No distortions**

Note.

Increasing returns to scale (IRS) are a much more severe issue addressed by “New” trade theory

Let’s first stick to the general case and show how simple revealed preference arguments can be used to establish two important results:

1. *Gains from trade*(Samuelson 1939)
2. *Law of comparative advantage*(Deardorff 1980)

1.2.1 Basic environment

Consider a world economy with $n = 1, 2, \dots, N$ countries, each populated by $h = 1, \dots, H$ households.

There are $g = 1, \dots, G$ goods:

- $y^n \equiv (y_1^n, \dots, y_G^n) \equiv$ Output vector in country n
- $c^{nh} \equiv (c_1^{nh}, \dots, c_G^{nh}) \equiv$ Consumption vector of household h in country n
- $p^n \equiv (p_1^n, \dots, p_G^n) \equiv$ Price vector in country n

There are $f = 1, \dots, F$ factors:

- $v^n \equiv (v_1^n, \dots, v_F^n) \equiv$ Factor endowment vector in country n
- $w^n \equiv (w_1^n, \dots, w_F^n) \equiv$ Factor price vector in country n

Supply side

We denote by Ω^n the set of combinations (y, v) feasible in country n , our assumption of constant returns to scale implies that Ω^n is a convex set.

Definition 1.2.1 (Revenue function).

The **revenue function** in country n of a firm producing output y using factors v is a function $r^n(y, v)$ such that:

$$r^n(y, v) \equiv \max_y \{py \mid (y, v) \in \Omega^n\} \quad (1.1)$$

Note (see Dixit-Norman pp. 31-36 for details).

- Revenue function summarizes all relevant properties of technology;
- Under perfect competition, y^n maximizes the value of output in country n :

$$r^n(p^n, v^n) = p^n y^n.$$

Demand side

We denote by u^{nh} the utility function of household h in country n .

Definition 1.2.2 (Expenditure function).

The **expenditure function** of household h in country n is a function $e^{nh}(p^n, u^{nh})$ such that:

$$e^{nh}(p, u) \equiv \min_c \{pc \mid u^{nh}(c) \geq u\} \quad (1.2)$$

Note (see Dixit-Norman pp. 59-64 for details).

- Here factor endowments are in fixed supply, but easy to generalize to case where households choose factor supply optimally
- Holding p fixed, $e^{nh}(p, u)$ is increasing in u .

- Household's optimization implies:

$$e^{nh}(p^n, u^{nh}) = p^n c^{nh}$$

where c^{nh} and u^{nh} are the consumption and utility level of the household h in country n in equilibrium, respectively.

1.2.2 Gains from Trade

In the next propositions, when we say “in a neoclassical trade model”, we mean in a model where equations (1.1) and (1.2) hold in any equilibrium.

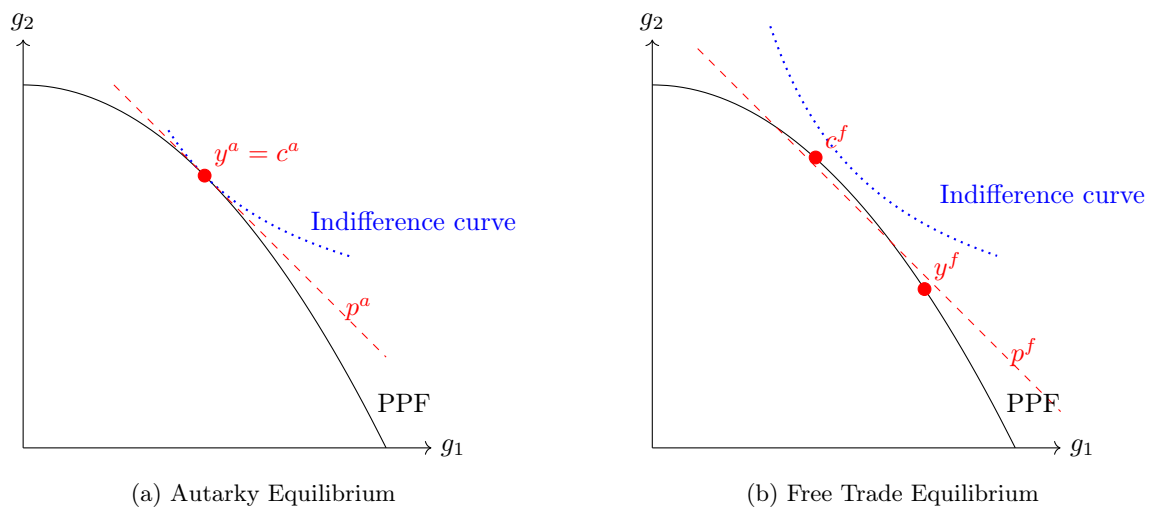


Figure 1.1: Equilibria for a Small Country

Formula of Gains from Trade

Arkoulakis, Costinot, Rodriguez-Clare (AER, 2012)

Assumption 1.2.1.

- CES utility function (Dixit-Stiglitz)

$$U = \left[\int q(\omega)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$

- One factor production (labor) and constant RS
- ‘Iceberg’ trade costs
- Import demand system is CES: partial equilibrium (given wages) elasticity of aggregate bilateral trade flow relative to domestic demand is ε w.r.t. trade costs τ_{ij} for any i, j .

Define a “foreign shock” as any change in (foreign) endowments and trade costs that do not affect a country’s endowment or its ability to serve its own market. Define $\hat{W} = \frac{W'}{W}$ and $\hat{\lambda}_{jj} = \frac{\lambda'_{jj}}{\lambda_{jj}}$

Proposition 1.2.1. The change in country j 's real income associated with any foreign shock can be computed as $\hat{W}_j = \hat{\lambda}_{jj}^{-\frac{1}{\epsilon}}$, where λ_{ij} is the share of country j 's spending on country i 's goods.

Corollary 1.2.1. Gains from TRade relative to autarky can be computed as $\hat{W}_j = \lambda_{jj}^{-\frac{1}{\epsilon}}$.

One household per country

Consider first the case where there is just one household per country, $H = 1$. Without risk of confusion, we drop h and n from all variables.

We denote by:

- (y^a, c^a, p^a) the vector of output, consumption and good prices under autarky;
- (y^f, c^f, p^f) the vector of output, consumption and good prices under free trade.
- u^a and u^f the utility levels under autarky and free trade.

Proposition 1.2.2. In a neoclassical trade model with one household per country, free trade makes all households (weakly) better off.

Proof.

Under free trade, households can consume at prices p^f . By definition of the expenditure function, we have:

$$\begin{aligned} e(p^f, u^a) &\leq p^f c^a \\ &= p^f y^a \\ &\leq r(p^f, v^f) \\ &= e(p^f, u^f) \end{aligned}$$

Since $e(p, \cdot)$ is increasing, we get $u^f \geq u^a$. □

Note.

- Two inequalities in the previous proof correspond to consumption and production gains from trade.
- Previous inequalities are weak. Equality if kinks in IC or PPF.
- Previous proposition only establishes that households always prefer “free trade” to “autarky.” It **does not** say anything about the comparisons of trade equilibria.

Multiple households per country: domestic lump-sum transfers

With multiple households per country, moving away from autarky is likely to create winners and losers. In order to establish the Pareto-superiority of trade, we will need to allow for **policy instruments**. We start with *domestic lump-sum transfers* and then *commodity taxes*.

We now reintroduce the index h and denote by:

- c^{ah} and c^{fh} the consumption vectors of household h under autarky and free trade;
- v^{ah} and v^{fh} the endowment vectors of country h under autarky and free trade;
- u^{ah} and u^{fh} the utility levels of household h under autarky and free trade;
- τ^h the lump-sum transfer from the government to household h under free trade.¹

Proposition 1.2.3.

In a neoclassical trade model with multiple households per country, there exist domestic lump-sum transfers such that free trade is (weakly) Pareto superiority than autarky in all countries.

Proof.

For any h , set the lump-sum transfer τ^h such that:

$$\tau^h = (p^f - p^a)c^{ah} - (w^f - w^a)v^{fh}.$$

Budget constraint under autarky implies that: $p^a c^{ah} \leq w^a v^{fh}$. Therefore, we have:

$$p^f c^{ah} \leq w^f v^{fh} + \tau^h.$$

Thus c^{ah} is still in the budget set of household h under free trade.

By definition, the government revenue is given by:

$$\begin{aligned} -\sum \tau^h &= (p^a - p^f) \sum c^{ah} - (w^a - w^f) \sum v^{fh} \\ &= (p^a - p^f)y^a - (w^a - w^f)v^f \\ &= -p^f y^a + w^f v^f \\ &\geq -r(p^f, v^f) + w^f v^f \\ &= -(p^f y^f - w^f v^f) = 0. \end{aligned}$$

□

So, each household can buy its autarky consumption bundle at free trade prices and still have some money left. But, the government must know individual preferences to implement the transfers.

If it does not, households can manipulate mechanism by altering their announcements or autarky behavior. In other words, lump-sum transfers typically are not **incentive compatible**.

Multiple households per country: commodity taxes

We now restrict the set of instruments to commodity taxes/subsidies.

Suppose that the government can affect the prices faced by households under free trade by setting τ^{good} and τ^{factor} :

$$\begin{aligned} p^h &= p^f + \tau^{good} \\ w^h &= w^f + \tau^{factor}. \end{aligned}$$

¹ $\tau^h \leq 0 \Leftrightarrow$ lump-sum tax and $\tau^h \geq 0 \Leftrightarrow$ lump-sum subsidy.

Proposition 1.2.4.

In a neoclassical trade model with multiple households per country, there exist commodity taxes/subsidies such that free trade is (weakly) Pareto superior to autarky in all countries.

Proof.

Consider two following taxes:

- $\tau^{good} = p^a - p^f$
- $\tau^{factor} = w^a - w^f$

By construction, household is indifferent between autarky and free trade. Now consider the government revenue:

$$\begin{aligned}
 -\sum \tau^h &= \sum \tau^{good} c^{ah} - \sum \tau^{factor} v^{fh} \\
 &= (p^a - p^f) \sum c^{ah} - (w^a - w^f) \sum v^{fh} \\
 &= (p^a - p^f) y^a - (w^a - w^f) v^f \\
 &= -p^f y^a + w^f v^f \\
 &\geq -r(p^f, v^f) + w^f v^f \\
 &= -(p^f y^f - w^f v^f) = 0.
 \end{aligned}$$

□

Tax revenue is non-negative. If all households are on the same side of the market for at least for at least one good or factor, government can cut a tax or raise a subsidy to generate Pareto improvement.

This scheme sacrifices the consumer gains from trade, but preserves the gains from reorganizing production.

Note.

The previous proof only relies on existence of *production gains* from trade.

- It's closely related to Diamond and Mirrlees (1971) result on the production efficiency
- When only commodity taxes are available, DM show that production should remain efficient at a social optimum
- Thus, trade, acting as an expansion of PPF, should remain free (ignore issues of market power)

But, if there's a kink in the PPF, there are no production gains. ^a

Factor taxation still informationally intensive: need to know endowments in efficiency units, may lead to different business taxes

^aSimilar problem with "moving costs", see Feenstra p. 185

1.2.3 Law of Comparative Advantage

The previous results have focused on normative predictions. Let's take a look at how the same revealed preference argument can be used to make positive predictions about patterns of trade.

Theorem 1.2.1 (Law of Comparative Advantage).

Countries tend to export goods in which they have a CA, i.e. lower relative autarky prices compared to other countries

Ricardian Model

2.1 Introduction

2.1.1 Reasons of trade

- **Countries' differences: comparative advantage**
 - Productivity: Ricardo
 - Endowments: Heckscher-Ohlin
- **Countries' similarities: economies of scale**

2.1.2 Comparative advantage

Note.

Stanislaw Ulam's challenge to Paul Samuelson: "name me one proposition in all of the social sciences which is both true and non-trivial".

Samuelson's answer: Comparative advantage. "That it is logically true need not be argued before a mathematician; that it is not trivial is attested by the thousands of important and intelligent men who have never been able to grasp the doctrine for themselves or to believe it after it was explained to them."^a

^aP.A. Samuelson (1969), "The Way of an Economist," in P.A. Samuelson, ed., International Economic Relations: Proceedings of the Third Congress of the International Economic Association, Macmillan: London, pp. 1-11.

In the simplest and earliest complete model of production and trade, the reason for trade is **comparative advantage**. And the source of comparative advantage is **differences in production technologies**.

These are the differences in production functions, not the differences in labor productivities due to different endowments of capital which is the type of Heckscher-Ohlin model.

2.1.3 Basic assumptions

- Labor is the only factor of production
- Constant returns to scale
- Perfect competition
- Full employment
- Endowments given, confined to country but intersectorally mobile within each country
- Two countries with different technologies (production functions)
- Number of goods: $n \geq 2$

2.1.4 Technology

Question. How is efficiency measured?

With only one factor, we measure by the usage of the factor: **labor requirement per unit output**.

We define the quantity of labour (e.g. number of hours) necessary to produce one unit of good i as a_i . If $a_i = 2$, it means that 2 hours of labor are needed to produce one unit of good i .

So, the labor productivity is the inverse of the labor requirement a_i , the higher the labor requirement, the lower the productivity.

2.1.5 Production Possibility Frontier: Refresher

PPF is the maximum possible production level for a given technology and factor endowment.

- Constraint: points outside the PPF are not feasible;
- Efficiency: points inside the PPF are inefficient, those on the PPF are efficient;
- Opportunity cost: the slope of the PPF is the opportunity cost of producing one more unit of good i in terms of good j .
- Concavity: the PPF is usually concave to the origin due to the law of diminishing returns.

Technology progress shifts the PPF outwards.

2.2 The two-sector Model

2.2.1 A Simple Numerical Example

Let's begin with a simple example: US and India producing corn and auto.

	US	India
Labor force	$L = 200$	$L^* = 800$
Labor per unit corn	$a_c = 8$	$a_C^* = 50$
Labor per unit auto	$a_A = 10$	$a_A^* = 40$

Specific assumption of this example : US is more efficient/productive at producing both goods, meaning US has an absolute advantage in both goods.

Production Possibility Frontier

US

The full-employment condition is:

$$L = a_c Q_c + a_A Q_A$$

The PPF is a straight line connecting these two points:

$$Q_A = \frac{L}{a_A} - \frac{a_C}{a_A} Q_C$$

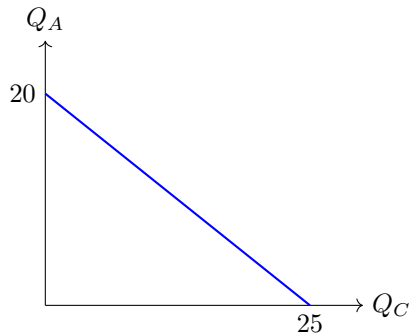


Figure 2.1: US PPF

India

The full-employment condition is:

$$L^* = a_C^* Q_C + a_A^* Q_A$$

The PPF is a straight line connecting these two points:

$$Q_A^* = \frac{L^*}{a_A^*} - \frac{a_C^*}{a_A^*} Q_C$$

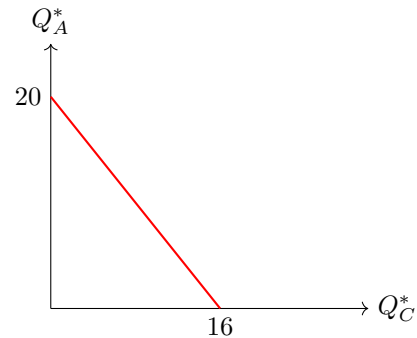


Figure 2.2: India PPF

Relative price and Technology

Notation.

Relative price of corn: $P = \frac{P_C}{P_A}$;

Relative quantity of corn: $Q = \frac{Q_C}{Q_A}$;

L and L^* are the endowments of labor(in efficient units) in the two countries;

w and w^* are the wage(in efficiency units) in the two countries.

Workers are paid the value of their marginal products: $w_C = \frac{P_C}{a_C}$ and $w_A = \frac{P_A}{a_A}$.

Perfectly mobile labor: If both goods are produced, the wage in both sectors must be the same.

$$w_C = w_A \Leftrightarrow P = \frac{a_C}{a_A}.$$

Production and Supply in the US

If $P = \frac{a_C}{a_A} = 0.8$, the international price(relative price) is equal to the domestic opportunity cost of producing corn in terms of auto. Trade cannot bring extra gains. Hence the production set doesn't affect the consumption set, and the production set can be anywhere on the PPF. $1 \leq \frac{Q_C}{Q_A} < \infty$.

If $P < \frac{a_C}{a_A} = 0.8$, international price of C is lower than the domestic opportunity cost of producing C in terms of A. US will produce only A and import C. $Q_C = 0$, $Q_A = 20$, and $\frac{Q_C}{Q_A} = 0$.

If $P > \frac{a_C}{a_A} = 0.8$, international price of C is higher than the domestic opportunity cost of producing C in terms of A. US will produce only C and export A. $Q_A = 0$, $Q_C = 25$, and $\frac{Q_C}{Q_A} = \infty$.

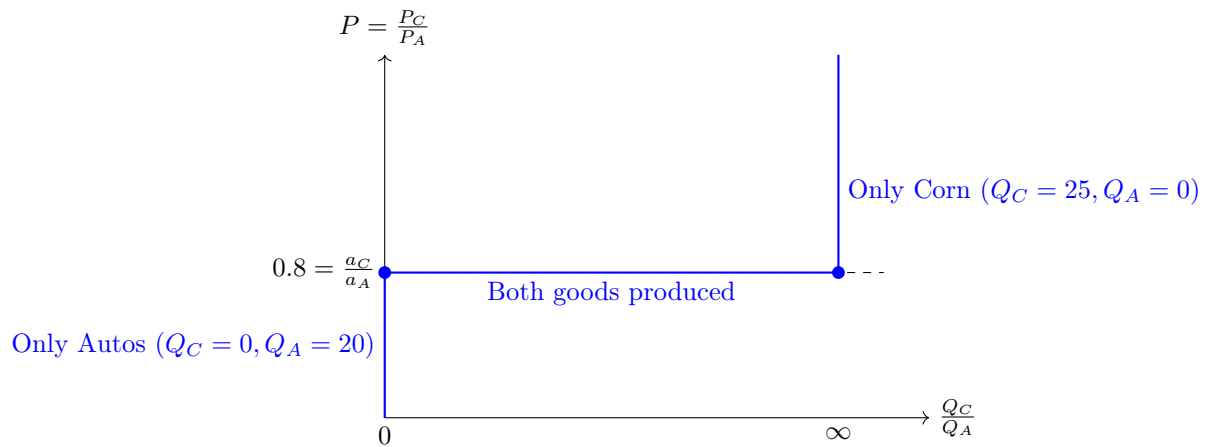


Figure 2.3: US Relative supply

Autarky equilibria in the two countries

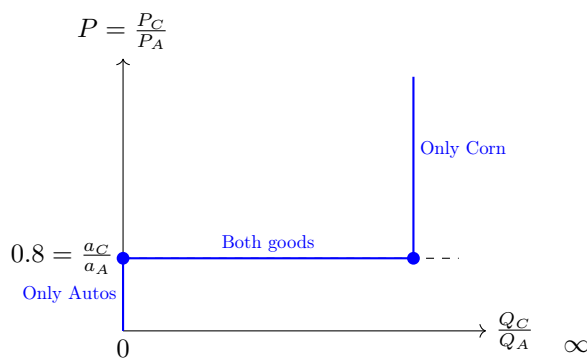


Figure 2.4: US Relative Supply

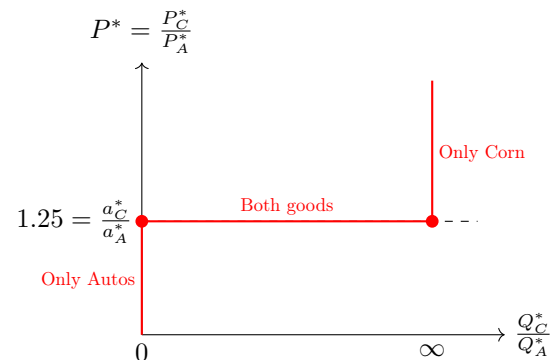


Figure 2.5: India Relative Supply

Trade: Relative supply

When we combine the two countries to form the world market, we get the following relative supply curve:

- When $P < 0.8$, both countries produce only autos, world ratio $R = 0$.
- When $P = 0.8$, US can vary production between 20 autos, no corn (world $R = 0$), and 0 autos, 25 corn (world $R = 1.25$) while India produces only autos, so world $R \in [0, 1.25]$.
- When $0.8 < P < 1.25$, US produces only corn, India produces only autos, so $R = \frac{25}{20} = 1.25$.
- When $P = 1.25$, India can vary production while US produces only corn, so world $R \in [1.25, \infty)$.
- When $P > 1.25$, both countries produce only corn, so world $R = \infty$.

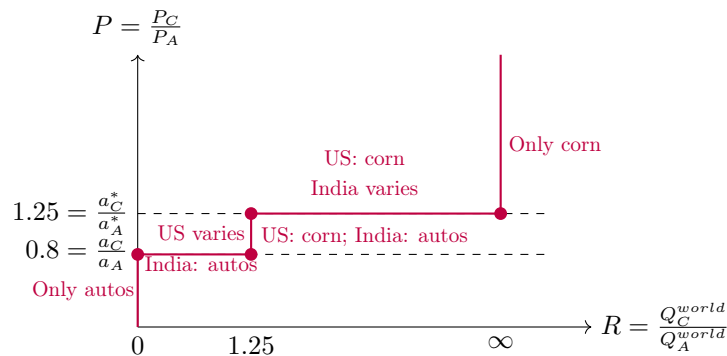


Figure 2.6: World Relative Supply Curve

Trading Equilibrium

Depending on position of relative demand, the trading equilibrium can be one of three types:

- A. $P = 0.8$, US produces both goods, India produces only autos;
- B. $0.8 < P < 1.25$, US produces only corn, India produces only autos;
- C. $P = 1.25$, US produces only corn, India produces both goods.

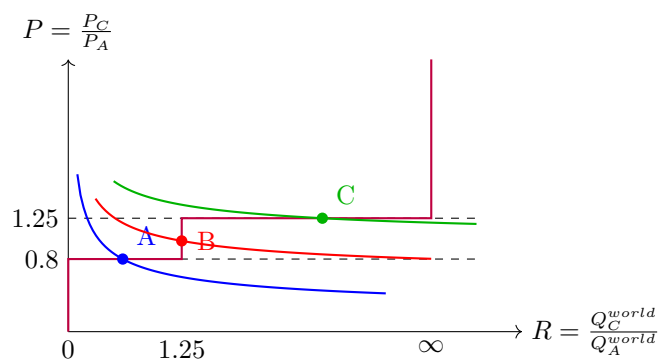


Figure 2.7: Trading Equilibrium

Efficient production and world PPF

Suppose initially all labor produce autos in both: 40 in all. If any corn is produced, it's better to do so by switching some labor in the US: because each auto not produced releases 10 labor which can then produce 1.25 corn, while in India, each less auto yields only 0.8 more corn.

Only when all US labor has been diverted to producing corn should any Indian labor be switched.

Conversely, starting with all corn: 41 units, to produce any autos, Indian labor should be switched.

This despite the US producing autos more efficiently than India: only 10 units of labor against 40. The reason: the US produces corn even more efficiently: only 8 units of labor against 50. What matters is the ratio (opportunity cost): $\frac{10}{8} > \frac{40}{50}$, or $\frac{10}{40} > \frac{8}{50}$.

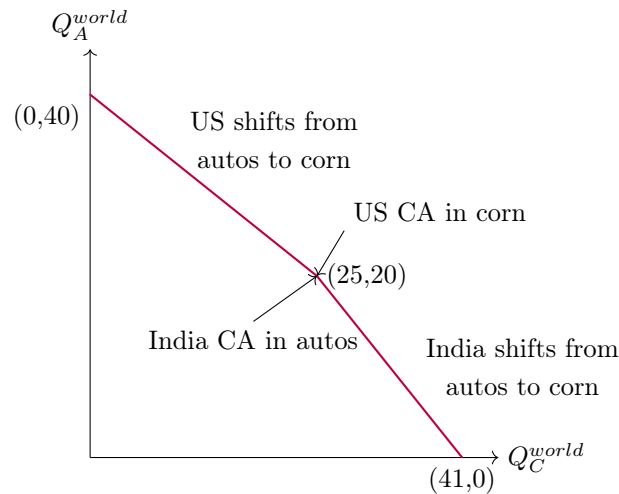


Figure 2.8: World Production Possibility Frontier

Trading equilibrium and world PPF

If preferences are identical and homothetic everywhere, we can draw the world indifference curves. Depending on their shape, there are three types of outcomes:

- A. US produces both goods, India produces only autos;
- B. US produces only corn, India produces only autos;
- C. US produces only corn, India produces both goods.

Relative price of corn = slope of PPF: 0.8 in A, 1.25 in C, between 0.8 and 1.25 in B.

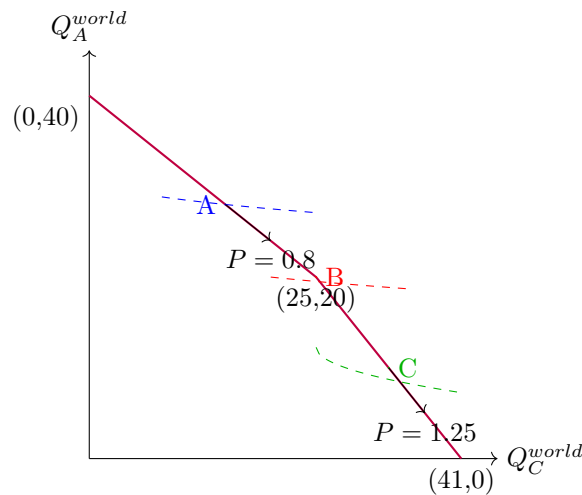


Figure 2.9: Trading Equilibrium on the World PPF

2.3 Dornbusch, Fischer and Samuelson(1997): Continuum of Goods[5]

Consider a world economy with only two countries: Home and Foreign. Use asterisks to denote foreign variables.

Note.

Ricardian models differ from other neoclassical trade models in that there is only **one factor of production** in the model.

We denote by:

- L and L^* as the labor endowments in Home and Foreign respectively.
- w and w^* as the wages (in efficiency units) in Home and Foreign respectively.

There's a continuum of goods indexed by $z \in [0, 1]$. Since there are CRS, we can define the constant unit labor requirements in both countries: $a(z)$ and $a^*(z)$.

Without loss of generality, we order goods such that $A(z) \equiv \frac{a^*(z)}{a(z)}$ is decreasing.

- Hence Home has a comparative advantage in goods with low- z goods.
- For simplicity, we'll assume strict monotonicity.

Previous supply-side assumptions are all we need to make qualitative predictions about pattern of trade.

Let $p(z)$ denote the price of good z in Home under free trade.

The profit-maximization condition for a firm producing good z is:

$$\begin{aligned} p(z) - w a(z) &\leq 0, \text{ with equality if } z \text{ produced at home.} \\ p(z) - w^* a^*(z) &\leq 0, \text{ with equality if } z \text{ produced abroad.} \end{aligned}$$

Proposition 2.3.1.

There exists $\tilde{z} \in [0, 1]$, s.t. Home produces all goods with $z < \tilde{z}$ and Foreign produces all goods with $z > \tilde{z}$.

Proof.

By contradiction. Suppose that there exists $z' < z$, s.t. z produced at Home and z' is produced abroad. Then:

$$\begin{aligned} p(z) - w a(z) &= 0, \\ p(z') - w a(z') &\leq 0, \\ p(z') - w^* a^*(z') &= 0, \\ p(z) - w^* a^*(z) &\leq 0. \end{aligned}$$

This implies that:

$$w a(z) w^* a^*(z) = p(z) p(z') \leq w a(z') w^* a^*(z),$$

which can be rearranged as:

$$\frac{a^*(z')}{a(z')} \leq \frac{a^*(z)}{a(z)}$$

which contradicts that $A(z)$ is strictly decreasing. □

This proposition (2.3.1) simply shows that Home should produce and specialize in the goods in which it has a CA.

Note.

- Proposition(2.3.1) doesn't rely on continuum of goods.
- Continuum of goods + continuity of $A(z)$ is important to derive the relative wage:

$$A(\tilde{z}) = \frac{w}{w^*} \equiv \omega. \quad (2.1)$$

Equation(2.1) is the key result of the Dornbusch, Fischer and Samuelson(1997) model. It gives us that: conditional on wages, goods should be produced in the country where it's cheaper to do so.

Then, we take a look at the demand side to pin down the relative wage ω .

2.3.1 Demand side

Assume that consumers have identical Cobb-Douglas preferences around the world.

We denote by $b(z) \in (0, 1)$ the share of expenditure spent on good z :

$$b(z) = \frac{p(z)c(z)}{wL} = \frac{p(z)c^*(z)}{w^*L^*}$$

where $c(z)$ and $c^*(z)$ are the consumption of good z in Home and Foreign respectively.

By definition, share of expenditure satisfies: $\int_0^1 b(z)dz = 1$.

Then, we denote by $\theta(\tilde{z}) = \int_0^{\tilde{z}} b(z)dz$ the fraction of income spent on goods produced at Home(spent in both countries).

The trade balance requires:

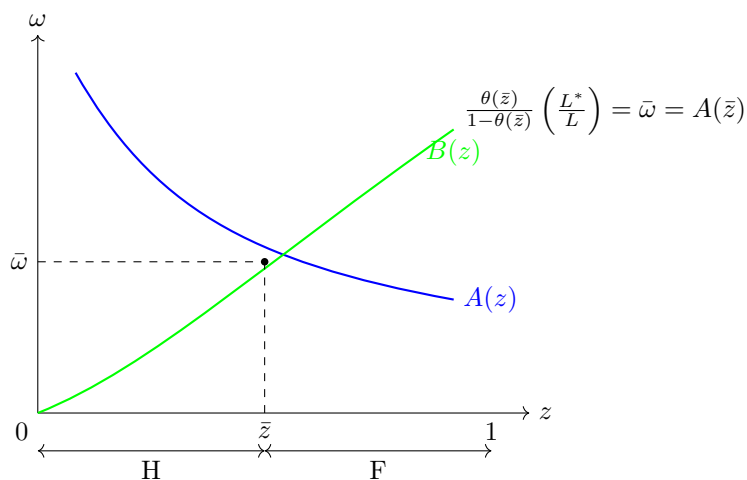
$$\theta(\tilde{z})w^*L^* = [1 - \theta(\tilde{z})]wL$$

where LHS od the Home exports an d RHE is the Home imports.

WE can then rearrange the equation(2.1) to get:

$$\omega = \frac{\theta(\tilde{z})}{1 - \theta(\tilde{z})} \left(\frac{L^*}{L} \right) \equiv B(\tilde{z}). \quad (2.2)$$

Note that $B' > 0$ and: an increase in \tilde{z} leads to a trade surplus at Home, which must be compensated by an increase in Home's relative wage ω .



- Efficient international specialization, Equation (3), and trade balance, (4), jointly determine $(\bar{z}, \bar{\omega})$

Since Ricardian model is a neoclassical model, general results derived in previous lecture hold. However, we can directly show the existence of gains from trade in this environment.

- Set $w = 1$ under autarky and free trade.
- Indirect utility¹ of Home representative household depends on $p(\cdot)$.
- For goods z produced at Home under free trade: no change compared to autarky.
- For goods z produced Foreign under free trade:

$$p(z) = w^* a^*(z) < a(z)$$

- Since all prices go down, indirect utility must go up.

2.3.2 Consequences of (Relative) Country Growth

Suppose that L^*/L increases, then the relative wage ω must increase and \tilde{z} goes down. At initial wages, an increase in L^*/L leads to a trade deficit in Foreign, which must be compensated by an increase in ω . An increase in L^*/L raises indirect utility, i.e. real wage, of representative household at Home and lowers it in Foreign.

- We set $w = 1$ before and after the change in L^*/L .
- For goods z whose production remains at Home, there's no change in $p(z)$.
- For goods z whose production remains in Foreign, we have: $w \uparrow \Rightarrow w^* \downarrow \Rightarrow p(z) = w^* a^*(z) \downarrow$.
- For goods z whose production changes from Home to Foreign, we have: $p(z) = w^* a^*(z) \leq a(z) \Rightarrow p(z) \downarrow$.
- So Home gains. Similar logic implies Foreign welfare loss.

Note.

In spite of CRS at the industry level, everything is as if we had DRS at the country-level.

As foreign's size increases, it specializes in sectors in which it is relatively less productive (compared to home), which worsens its terms of trade, so lowers real GDP per capita.

The flatter the A schedule, the smaller this effect.

If there's an improvement in Home technology, then unit of labor requirement in Home $a(z)$ will be reducing, hence $A(z)$ is increasing - shifting upward. As we have shown in 2.1, this shows **an increase in home relative wage** i.e. higher income, and **an increase in home's share of world production and trade**.²

¹The maximum utility under budget constraint.

²As shown previously, $\omega = \frac{\theta(\tilde{z})}{1-\theta(\tilde{z})} \left(\frac{L^*}{L} \right)$, if ω increases, and we fix $\frac{L^*}{L}$, then $\theta(\tilde{z})$ increases.

2.4 The Eaton-Kortum model (2002): Many Countries[6]

The model of Dornbusch, Fischer, and Samuelson (1977)[5] is based on the Ricardian model where trade and specialization patterns are determined by different productivities.

In the Dornbusch-Fischer-Samuelson model, there were no restrictions on the productivity distributions, Eaton and Kortum were able to extend the model to a case with many countries by sacrificing this generality.

The Eaton-Kortum model is based on the following assumptions:

- Perfect Competition
- N countries, instead of 2 in DFS
- Continuum of goods ($u \in [0, 1]$)
- CRS technology: labor only
- CES preferences with elasticity of substitution $\sigma > 0$

2.4.1 Model Setup

The world

In this model, there is a finite number of countries $i \in S \equiv \{1, \dots, N\}$; (unlike previous models, there are technical difficulties in extending the model to a continuum of countries). There is a continuum of goods Ω .

However, unlike Krugman (1980)[7] and Melitz (2003)[8], models which follow (in Lecture 4 and 5), *every country is able to produce every good*. Countries, however, vary (exogeneously) in their productivity of each good $z_i(\omega)$ - varying by country i and good $\omega \in \Omega$.

The Eaton and Kortum (2002)[6] has no concept of a firm. Instead, it is assumed that all goods ω are produced using the same bundle of inputs with a constant returns to scale technology. Let the cost of a bundle of inputs in country i be c_i , so that the cost of producing one unit of $\omega \in \Omega$ in country i is $\frac{c_i}{z_i(\omega)}$.

Firstly, we introduce the **trade costs**: transport costs and tariffs. This is not an innovation of Eaton and Kortum, the original Dornbusch-Fischer-Samuelson model also had trade costs, but for simplification, we left them out.

Trade costs are modelled as iceberg costs, a popular device invented by Samuelson[9]. That is, we assume that for delivering one unit of a good from country i to country j , it's necessary to ship $d_{ij} \geq 1$ units of good: the rest "melts away" during transit.

Supply

Each good is assumed to be sold in perfectly competitive markets, so that the price a consumer in country j would pay for good ω produced in country i is given by:

$$p_{ij}(\omega) = \frac{c_i}{z_i(\omega)} d_{ij}.$$

However, consumers in country $j \in S$ are assumed to only purchase good $\omega \in \Omega$ from the country who can provide it at the lowest price, so the price a consumer in $j \in N$ actually pays for good ω is:

$$p_j(\omega) = \min_{i \in S} p_{ij}(\omega) = \min_{i \in S} \left\{ \frac{c_i}{z_i(\omega)} d_{ij} \right\}$$

Here, the basic idea of Eaton and Kortum is already present in equation above: a country $j \in S$ is more likely to purchase a good $\omega \in \Omega$ from country $i \in S$ if

- (1) it has a lower cost c_i ;
- (2) it has a higher good productivity $z_i(\omega)$;
- (3) it has a lower trade cost d_{ij} .

The main innovation of Eaton and Kortum, however, is to assume that the productivities $z_i(\omega)$ are drawn from a Fréchet distribution³. This means that for each $i \in S$ and $\forall \omega \in \Omega$, the cumulative distribution function F_i is:

$$F_i(z) = \Pr\{z_i(\omega) \leq z\} = \exp\{-T_i z^{-\theta}\}$$

where $T_i > 0$ is a measure of the aggregate productivity of country i (country i 's state of technology)⁴.

Note (Why Fréchet?).

Why make this particular distributional assumption for productivities?

Kortum (1997)[10] showed that if the technology of producing goods is determined by the best “idea” of how to produce, then the limiting distribution is indeed Fréchet, where T_i reflects the country's stock of ideas. More generally, consider the random variable:

$$M_n = \max\{X_1, \dots, X_n\}$$

where X_n are i.i.d.

The Fisher-Tippett-Gnedenko theorem states that the only (normalized) distribution of M_n as $n \rightarrow \infty$ is an extreme value distribution, of which Fréchet is one of three types (Type II). Note that a conditional logit model assumes that the error term is Gumbel (Type I) extreme value distributed.

If random variable x is Gumbel distributed, then $\ln x$ is Fréchet distributed. Hence, loosely speaking, the Fréchet distribution works better for models that are log linear (like the gravity equation), whereas the Gumbel distribution works better for models that are linear.

Demand

As in previous models, consumers have CES preferences so that the representative agent in country j has utility:

$$U_j = \left(\int_{\Omega} q_j(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}},$$

where $q_j(\omega)$ is the quantity that country j consumes of good ω .

Note that unlike the Krugman (1980)[7] model, not every good produced in every country will be sold to country j .

³To find more properties of the Fréchet distribution, see the appendix 2.4.2.

⁴A larger value of T_i decreases F_i for any $z \geq 0$. It increases the probability of larger values of z and $\theta > 1$, which is assumed to be constant across countries, governs the distribution of productivities across goods within countries. As θ increases, the heterogeneity of productivity across goods declines.

Indeed, good $\omega \in \Omega$ will be produced by all countries but country j will only purchase it from one country. The CES preferences will yield a Dixit-Stiglitz price index:

$$P_j \equiv \left(\int_{\Omega} p_j(\omega)^{1-\sigma} \right)^{\frac{1}{1-\sigma}}.$$

2.4.2 Equilibrium

We now consider the equilibrium of the model. Instead of relying on the CES demand equation as in the previous models, we use a probabilistic formulation in order to solve the model.

Prices

In perfect competition only the lowest cost producer of a good will supply that particular good. Thus, we want to derive the distribution of the minimum price over a set of prices offered by producers in different countries:

$$p_j = \min\{p_{ij}, \dots, p_{Nj}\}.$$

In order to solve the model, we take advantage of the properties of the Fréchet distribution.

First, we consider the probability that country $i \in S$ is able to offer country j good ω for a price less than p . Because the technology is i.i.d. across goods, we can define:

$$G_{ij}(p) \equiv \Pr\{p_{ij}(\omega) \leq p\}$$

Using the perfect competition price equation(2.4.1), we can write:

$$\begin{aligned} G_{ij}(p) &= \Pr\left\{\frac{c_i}{z_i(\omega)} d_{ij} \leq p\right\} \\ &= 1 - \Pr\left\{z_i(\omega) \geq \frac{c_i d_{ij}}{p}\right\} \\ &= 1 - F_i\left(\frac{c_i d_{ij}}{p}\right) \\ &= 1 - \exp\left\{-T_i\left(\frac{c_i d_{ij}}{p}\right)^{-\theta}\right\} \end{aligned}$$

Consider now the probability that country j pays a price less than p for good ω .

Again, because the technology is i.i.d. across goods, the probability will be the same for all goods $\omega \in \Omega$. Define:

$$G_j(p) \equiv \Pr\{p_j(\omega) \leq p\} \tag{2.3}$$

Because country j buys from the least cost provider, using equation 2.4.1, we can write:

$$\begin{aligned} G_j(p) &= \Pr\left\{\min_{i \in S} p_{ij}(\omega) \leq p\right\} \\ &= 1 - \Pr\left\{\min_{i \in S} p_{ij}(\omega) \geq p\right\} \\ &= 1 - \Pr\left\{\cap_{i \in S} (p_{ij}(\omega) \geq p)\right\} \\ &= 1 - \prod_{i \in S} \Pr\{p_{ij}(\omega) \geq p\} \\ &= 1 - \prod_{i \in S} (1 - G_{ij}(p)) \end{aligned}$$

Substituting G_{ij} into this equation, we have:

$$\begin{aligned}
 G_j(p) &= 1 - \prod_{i \in S} \left(1 - \left(1 - \exp \left\{ -T_i \left(\frac{c_i d_{ij}}{p} \right)^{-\theta} \right\} \right) \right) \\
 &= 1 - \prod_{i \in S} \exp \left\{ -T_i \left(\frac{c_i d_{ij}}{p} \right)^{-\theta} \right\} \\
 &= 1 - \exp \left\{ -p^\theta \sum_{i \in S} T_i (c_i d_{ij})^{-\theta} \right\} \\
 &= 1 - \exp \left\{ -p^\theta \Phi_j \right\}
 \end{aligned}$$

where $\Phi_j = \sum_{i \in S} T_i (c_i d_{ij})^{-\theta}$.

Appendix

Fréchet Distribution

The type II extreme value distribution, also called the Fréchet distribution, is one of three distributions that can arise as the limiting distribution of the maximum of a sequence of independent random variables. The distribution function for the Fréchet distribution is

$$F(x) = \exp \left\{ - \left(\frac{x - \mu}{\sigma} \right)^{-\theta} \right\},$$

for $x > \mu$, where $\theta > 0$ is a shape parameter, $\sigma > 0$ is a scale parameter and $\mu \in \mathbb{R}$ is a location parameter. The density of the Fréchet distribution is

$$f(x) = \frac{\theta}{\sigma} \left(\frac{x - \mu}{\sigma} \right)^{-\theta-1} \exp \left\{ - \left(\frac{x - \mu}{\sigma} \right)^{-\theta} \right\},$$

for $x > \mu$. If X is a Fréchet-distributed random variable then

$$\begin{aligned} E(X) &= \int_{\mu}^{\infty} x \frac{\theta}{\sigma} \left(\frac{x - \mu}{\sigma} \right)^{-\theta-1} \exp \left\{ - \left(\frac{x - \mu}{\sigma} \right)^{-\theta} \right\} dx \\ &= \sigma \int_0^{\infty} y^{-\frac{1}{\theta}} e^{-y} dy + \mu \int_0^{\infty} e^{-y} dy \\ &= \sigma \Gamma \left(\frac{\theta - 1}{\theta} \right) + \mu, \end{aligned}$$

where $y := \left(\frac{x - \mu}{\sigma} \right)^{-\theta}$ and

$$\Gamma(z) = \int_0^{\infty} t^{z-1} e^{-t} dt$$

is the Gamma function.

Now assume that $\mu = 0$, take $T := \sigma^{\theta}$ and rewrite the distribution function as

$$F(x) = e^{-Ax^{-\theta}},$$

so that the Fréchet distribution is now parameterized by θ, A .

Recommended Resources

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