

Macroeconomics A, EI056

Class 9

Overlapping generations model

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# What you will get from today class

- Money and **inflation**, the **Cagan model**.
- Irrelevance of **timing of taxes** with a **representative agent** with infinite life (**Ricardian equivalence**).
- Going **beyond the representative agent**: **succession of agents** with **finite lives** (the **overlapping generations** (OLG) model).
  - General steps.
  - A **simpler version**, illustration of how the **timing of taxes** matters.
- Possible **inefficiency** of resources allocation in OLG: when **bubbles** can help.
- Combining infinite horizon and OLG models.

# A question to start

## REASONS G-DEBT SAFE

CROWDING OUT When the government **borrowed**, investors purchase the debt instead of **investing** in firms. Government debt is thus to be avoided as it reduces economic activity.

MULTIPLIER  $G \uparrow \rightarrow Y \uparrow$  DEBT  $\Rightarrow K^{\text{PUBLIC}}$   
 $G \rightarrow K^{\text{PRIVATE}}$  CROWDING IN

$$T_b \downarrow T_{t+1} \uparrow$$

Do you agree? Why or why not?

$\rightarrow \infty$  HORIZON  $\rightarrow X$  EFFECT

$\rightarrow OLG \rightarrow \checkmark$  CROWDING OUT

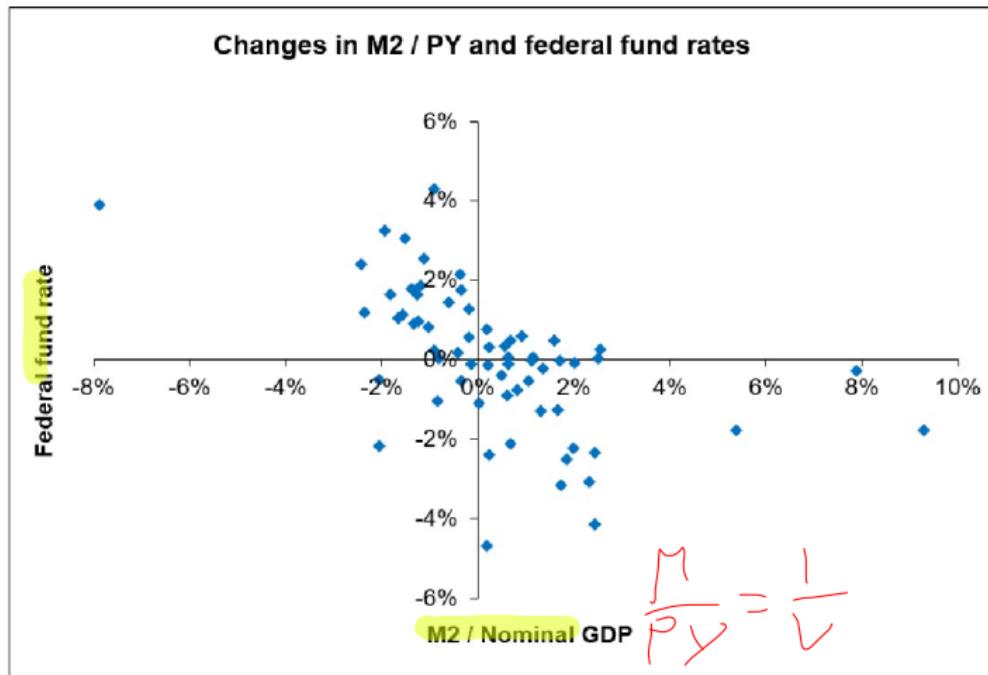
# MONEY AND PRICE

# Bringing money in macro models

- **Value of transactions** can be from two angles: price level  $P$  \* quantity (real GDP)  $Y$ ; money  $M$  \* number of times it is used (velocity  $V$ ):  $PY = MV$  <sub>GIVEN</sub> <sub>GIVEN</sub>  $\rightarrow M = P$
- Three broad ways to generate a money demand.
- Real balances in the utility function (even with a small weight).  
 $M/P$ 
  - Trade off between money (gives utility) and bonds (pay interest).
  - Money demand: real balances linked to consumption and the nominal interest rate ( $M/(PY) = 1/V$  inversely related to interest rate).
- Cash in advance: one needs to hold real balances to make purchases.  
Links money and consumption.  $PC = M$
- Shopping technology: household splits time between work, leisure and shopping.
  - Shopping time is an increasing function of consumption and a decreasing function of money.
  - Generates a money demand relation between real balances, consumption and the interest rate.

# Money and the interest rate

- Ratio of money to GDP inversely correlated with the interest rate.



# Cagan model of inflation

- Go beyond the  $PY = MV$  to be more specific on the demand for real balances.
- Nominal interest rate is equal to the real rate plus inflation expectations:  $i_t = r + \pi_{t+1}^e$ .
  - Focus on the nominal variables and take the real rate  $r$  to be constant (and zero for simplicity).  $\rightarrow r = 0$
  - Higher expected inflation reduces the demand for money (inflation acts as a tax reducing the real value of cash).
- Money demand is given by:  $\ln(M_x/P_t) = m_x$   
$$\frac{M_x}{P_t} \leftarrow \text{CON LINEAR}$$
$$m_t - p_t = -\gamma \pi_{t+1}^e = -\gamma (p_{t+1}^e - p_t)$$
- Dynamic relation between the current price and future expected prices. Iterating forward, the price reflects **future expected money**:

FWD LOOK

$$p_t = \frac{1}{1+\gamma} \sum_{s=0}^{\infty} \left( \frac{\gamma}{1+\gamma} \right)^s m_{t+s}^e$$

# Constant money growth rate

- Constant growth rate of money:  $m_t - m_{t-1} = \mu$ . Inflation is constant and equal to  $\mu$ :

$$m_{t+1} - p_{t+1} = -\gamma\pi ; \quad m_t - p_t = -\gamma\pi$$
$$\cancel{(p_{t+1} - p_t)} = (m_{t+1} - m_t) = \mu$$

- Higher price level when money grows at a fast rate:

$$m_t - p_t = -\gamma\mu$$
$$p_t = m_t + \gamma\mu$$

- Higher growth rate reduces the demand for real balances, as money loses its value fast:

$$m_t - p_t = -\gamma\mu$$

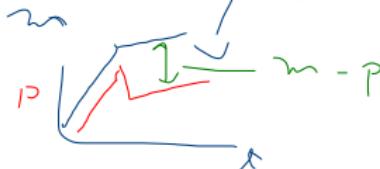
- A lower  $\mu$  lowers inflation.

# Disinflation under rational expectations

1) INTIAL SS. 2) CONGRIN 3) TRANS. M  
POLICY:  $\mu$  ↓ FOREVER

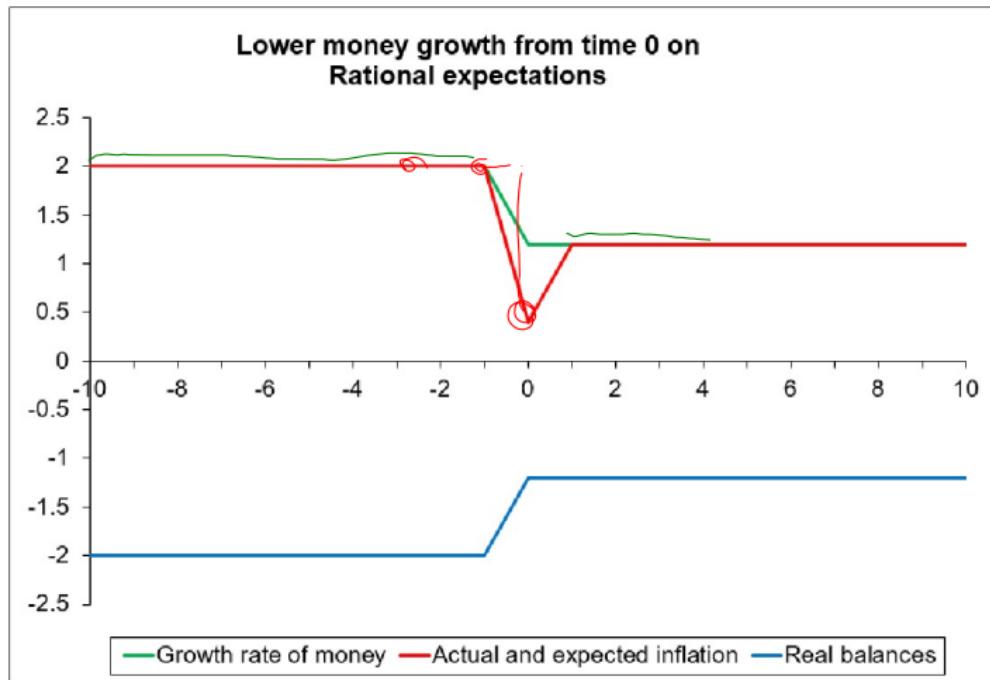


- Agents immediately understand the new regime at time  $T + 1$ . Inflation drops from  $\mu_0$  to  $\mu_1$ .
- At the period of transition, inflation undershoots for one period.
  - Lower money growth leads to a positive jump in the demand for real balances  $m - p$ .
  - Nominal balances  $m$  do not jump up (they only grow at a slower rate). So the price  $p$  has to jump down, a one-shot low inflation
- If the regime is announced before implementation, real balances react right away.
- Transition with adaptive expectations in extra slides. [► Adaptive expectations](#)



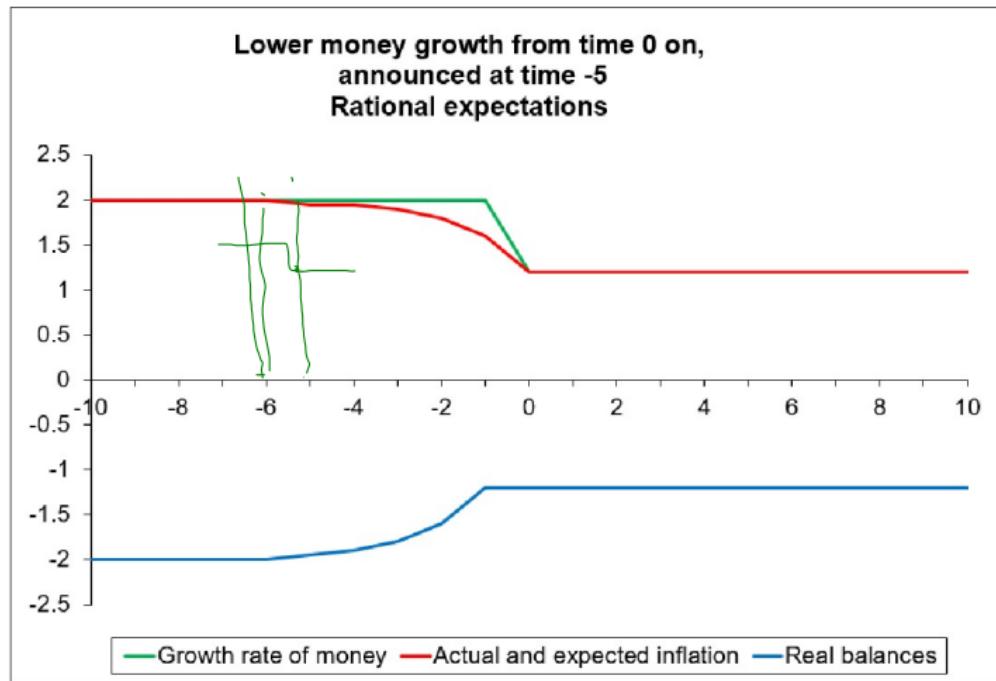
# Dynamics under rational expectations

- Lower money growth starting at time 0.



# Dynamics with news under rational expectations

- Lower money growth starting at time 0, announced at time -5.



# RICARDIAN EQUIVALENCE

# Budget constraints

- Household **lives forever**. Invest in **capital  $k$**  and a government bond  **$b$** . Both pay the same rate of return.
- **Flow budget constraint** ( $\tau_t$  denotes **taxes**): *SAVINGS*

$$w_t - \tau_t + r_t(k_t + b_t) = c_t + \overbrace{(k_{t+1} + b_{t+1})}^{\text{SAVINGS}} - (k_t + b_t)$$

- Government **spends  $g$**  funded by taxes and debt. Flow budget constraint:

$$b_{t+1} - b_t = g_t - \tau_t + r_t k_t$$

$$b_{t+1} = g_t - \tau_t + (1 + r_t) b_t$$

- Add the two constraints. **Taxes do not enter**, only government spending does. Resource use by the government matters, not how it finances it.

$$\cancel{\tau_t} \quad \cancel{g_t} \quad w_t + r_t k_t = c_t + g_t + k_{t+1} - k_t$$

# Intertemporal constraint

- Combine successive flow constraints into an overall **intertemporal** one

$$(R_{t,t+s} = \prod_{i=0}^s (1/(1+r_{t+i}))):$$

$$\begin{aligned} k_t &= \frac{c_t + g_t - w_t}{1+r_t} + \frac{k_{t+1}}{1+r_t} \\ k_t &= \frac{c_t + g_t - w_t}{1+r_t} + \frac{c_{t+1} + g_{t+1} - w_{t+1}}{(1+r_t)(1+r_{t+1})} + \frac{k_{t+2}}{1+r_{t+1}} \\ k_t &= \sum_{s=0}^{\infty} R_{t,t+s} [c_{t+s} + g_{t+s} - w_{t+s}] + \lim_{s \rightarrow \infty} R_{t,t+s} k_{t+s+1} \end{aligned}$$

- Transversality condition:**  $\lim_{s \rightarrow \infty} R_{t,t+s} k_{t+s+1} = 0$ . Capital doesn't grow at a pace higher than the interest rate (no explosive growth).
- Present value** of private and public consumption = capital + PV wages: [Details of intertemporal constraints](#)

$$\sum_{s=0}^{\infty} R_{t,t+s} (c_{t+s} + g_{t+s}) = k_t + \sum_{s=0}^{\infty} R_{t,t+s} w_{t+s}$$

## C UNAFFECTED

- Intertemporal tax switch: tax cut today funded by debt, higher taxes tomorrow to pay the debt and interest.
- No impact on private consumption (no extra government spending).
  - Total tax bill has no changed (lower taxes today vs. higher taxes tomorrow).
  - Household saves to pay for future taxes, buying the very bond that the government issues.
- Key reason: both government and household view the future identically, because:
  - Same interest rate.
  - Same time horizon.
- In reality, private agents have shorter horizons. Motivation for overlapping generations (OLG) models.

# OVERLAPPING GENERATIONS

# Structure of population

2 PERIODS

- Successions of cohorts of agents with finite lives. Different cohorts coexist at a given time.
- Each agent lives for two periods. Agent borne at time  $t$  maximizes utility over consumption when young at time  $t$ ,  $c_{1,t}$ , and when old at time  $t+1$ ,  $c_{2,t+1}$ :

$$U_t = \frac{(c_{1,t})^{1-\theta}}{1-\theta} + \frac{1}{1+\rho} \frac{(c_{2,t+1})^{1-\theta}}{1-\theta}$$

where  $\rho$  is the discount rate.

- Young agent supplies one unit of labor paid a wage  $w_t$ , invests in bonds and capital for old age earning with rate of return  $r_{t+1}$ .

## Individual's budget constraint

- Tax  $\tau_{1,t}$  when young and  $\tau_{2,t+1}$  when old.
- Flow budget constraints when young and old ( $s_{1,t}$  denotes savings by the young agent in bonds and capital):

$$c_{1,t} + s_{1,t} = w_t - \tau_{1,t}$$

$$c_{2,t+1} = (1 + r_{t+1}) s_{1,t} - \tau_{2,t+1}$$

- Combine for the **intertemporal** budget constraint:

*NPV* (

$$c_{1,t} + \frac{c_{2,t+1}}{1 + r_{t+1}} = w_t - \left( \tau_{1,t} + \frac{\tau_{2,t+1}}{1 + r_{t+1}} \right) = \Omega_t$$

- $\Omega_t$  can be interpreted as **wealth** of the young agent, i.e. value of lifetime income.

# Euler condition

- Lagrangian solved by the young agent at time  $t$ :

$$\begin{aligned}\mathcal{L}_t = & \frac{(c_{1,t})^{1-\theta}}{1-\theta} + \frac{1}{1+\rho} \frac{(c_{2,t+1})^{1-\theta}}{1-\theta} \\ & + \lambda_t \left[ \Omega_t - c_{1,t} - \frac{c_{2,t+1}}{1+r_{t+1}} \right]\end{aligned}$$

- First-order conditions for consumption:

$$0 = \frac{\partial \mathcal{L}_t}{\partial c_{1,t}} = (c_{1,t})^{-\theta} - \lambda_t$$

$$0 = \frac{\partial \mathcal{L}_t}{\partial c_{2,t+1}} = \frac{1}{1+\rho} (c_{2,t+1})^{-\theta} - \lambda_t \frac{1}{1+r_{t+1}}$$

- Combining we get the **Euler equation**:

$$\frac{c_{2,t+1}}{c_{1,t}} = \left( \frac{1+r_{t+1}}{1+\rho} \right)^{\frac{1}{\theta}}$$

# Optimal consumption

- Euler condition and budget constraint give consumption when young and old:

$$\begin{aligned} c_{1,t} &= \frac{(1+\rho)^{\frac{1}{\theta}}}{(1+\rho)^{\frac{1}{\theta}} + (1+r_{t+1})^{\frac{1-\theta}{\theta}}} \Omega_t \\ c_{2,t+1} &= \frac{(1+r_{t+1})^{\frac{1}{\theta}}}{(1+\rho)^{\frac{1}{\theta}} + (1+r_{t+1})^{\frac{1-\theta}{\theta}}} \Omega_t \end{aligned}$$

- Saving by the young is wage minus consumption:

$s_{1,t} = w_t - \tau_{1,t} - c_{1,t}$ . Function of taxes, the wage, and the future interest rate.

# Channels of interest rate impact

- 3 channels of impact of interest rate on consumption when young:

$$c_{1,t} = \frac{(1 + \rho)^{\frac{1}{\theta}}}{(1 + \rho)^{\frac{1}{\theta}} + (1 + r_{t+1})^{\frac{1}{\theta}}} \left[ w_t - \left( \tau_{1,t} + \frac{\tau_{2,t+1}}{1 + r_{t+1}} \right) \right]$$

- **Substitution effect:** higher rate makes savings more attractive, reducing consumption:  $(1 + r_{t+1})^{-\frac{1}{\theta}}$  term. *FULLER*
- **Income effect:** higher rate makes reaching a given value of assets tomorrow easier, lowering savings and raising consumption:  
 $(1 + r_{t+1})^{-1}$  term.
- **Wealth effect:** higher interest rate reduces (absolute) present value of future income. With negative future income, lifetime income is higher and so is consumption.
- **Log utility ( $\theta = 1$ ):** substitution and income effects cancel, consumption is a constant share of wealth.

# PRODUCTION AND DYNAMICS

## Firm's optimization

- Standard framework of technology with constant productivity ( $L = 1$ ):

$$Y_t = (K_t)^\alpha (L)^{1-\alpha} \Rightarrow y_t = (k_t)^\alpha$$

Lower case letters denote scaled by labor.

- **Input demands:** wage and real interest rates (marginal costs) equal to respectively marginal products of labor and capital:

$$w_t = (1 - \alpha)(k_t)^\alpha ; \quad r_t = \alpha(k_t)^{\alpha-1}$$

$\text{MP}_L \qquad \qquad \qquad \text{MP}_K$

## Dynamics: assets

- Young agents invest in capital  $k$  and government bonds  $b$ .
- Assets available at time  $t + 1$  held by old agents, who bought it using their savings at time  $t$ :

$$k_{t+1} + b_{t+1} = s_{1,t}$$

- Government budget constraint (abstract from government spending), where  $b$  is debt (so  $(-b)$  is government's asset):

$$(-b_{t+1}) = \tau_{1,t} + \tau_{2,t} + (1 + r_t)(-b_t)$$

- Good market clearing: output equal to private consumption by young and old agents, plus capital accumulation (no depreciation):

$$(k_t)^\alpha = c_{1,t} + c_{2,t} + k_{t+1} - k_t$$

## Dynamics: capital

- For simplicity: **log utility** of consumption ( $\theta = 1$ ). Dynamics of capital (recall that  $r_{t+1} = \alpha (k_{t+1})^{\alpha-1}$ ): With general utility

$k_{t+1} + b_{t+1} = s_{1,t} = w_t - \tau_{1,t} - c_{1,t}$

$\xleftarrow[\text{exR}]{\text{QoT}} k_{t+1} = \tau_{1,t} + \tau_{2,t} - (1 + r_t) b_t + w_t - \tau_{1,t} - c_{1,t}$

$$k_{t+1} = \frac{1}{2 + \rho} [(1 - \alpha) (k_t)^\alpha - \tau_{1,t}] + \frac{1 + \rho}{2 + \rho} \frac{\tau_{2,t+1}}{1 + \alpha (k_{t+1})^{\alpha-1}} + (\tau_{1,t} + \tau_{2,t} - (1 + r_t) b_t)$$

- With **no taxes** and **no government debt**, unique **steady state** capital (possibility of multiple steady states for capital when  $\theta \neq 1$ ):

$$k^* = \left( \frac{1 - \alpha}{2 + \rho} \right)^{\frac{1}{1-\alpha}}$$

## Contrast with Ramsey model

- In Ramsey, the steady state the Euler (with log utility) is:

$$\frac{c_{t+1}}{c_t} = 1 = \frac{1 + \alpha (k^*)^{\alpha-1}}{1 + \rho} \Rightarrow k^* = \left(\frac{\alpha}{\rho}\right)^{\frac{1}{1-\alpha}}$$

- Steady states differ between the Ramsey and OLG economies.
  - Ramsey: Euler pins down capital, and capital dynamics relation gives consumption.
  - OLG: Euler applies within cohorts (consumption can change across time) but not across cohorts.

# Linearization

- Linear approximation around steady state with no taxes (hatted values are log deviations from steady state).
- Consumption of young and old agents:

$$\hat{c}_{1,t} = \hat{w}_t - \hat{\tau}_{1,t} - \frac{1}{1+r^*} \hat{\tau}_{2,t+1} ; \quad \hat{c}_{2,t+1} = \hat{r}_{t+1} + \hat{c}_{1,t}$$

- Wage and real interest rate (from firm):  $\hat{w}_t = \alpha \hat{k}_t$  and  $\hat{r}_t = r^* (1+r^*)^{-1} (\alpha-1) \hat{k}_t$ .
- Dynamics of assets, and government budget constraint:

$$\begin{aligned}\hat{k}_{t+1} + \hat{b}_{t+1} &= \hat{w}_t - \hat{\tau}_{1,t} + \frac{1}{1+r^*} \hat{\tau}_{2,t+1} \\ \hat{b}_{t+1} &= -(2+\rho)(\hat{\tau}_{1,t} + \hat{\tau}_{2,t}) + (1+r^*) \hat{b}_t\end{aligned}$$

- Utility of an agent borne at time  $t$ :

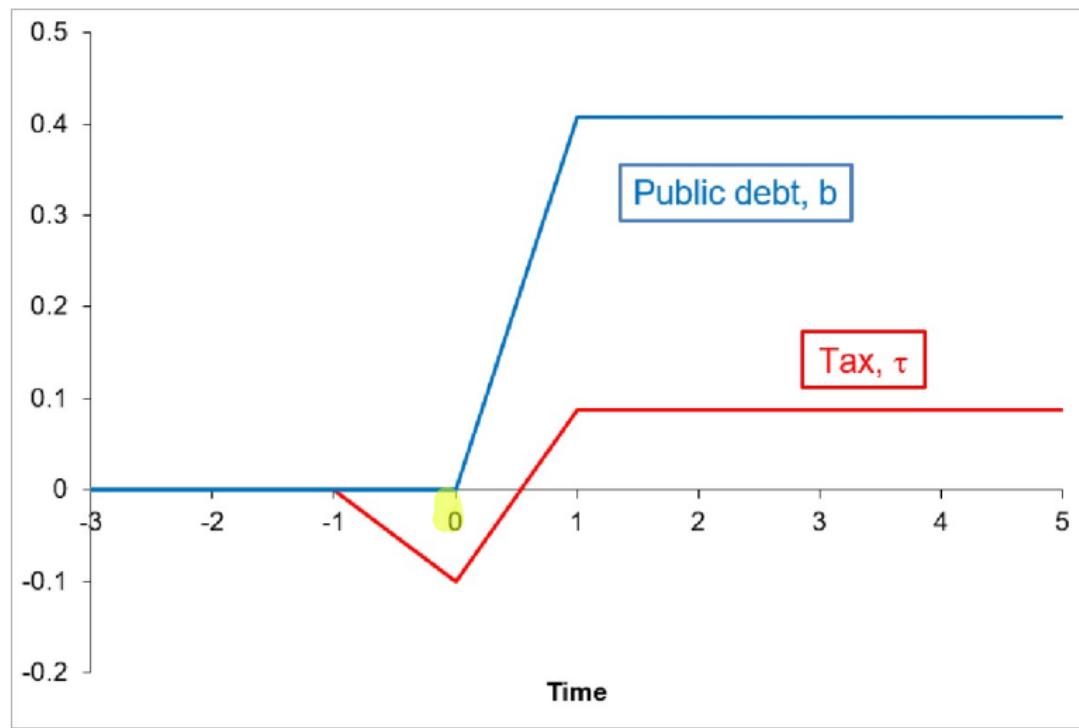
$$\hat{v}_t = \hat{c}_{1,t} + \frac{1}{1+\rho} \hat{c}_{2,t+1}$$

# Effect of intertemporal tax shift

- Start at steady state with zero taxes.
- At time  $t$  government unexpectedly introduces a tax switch:
  - Transfer to both households at time  $t$ :  $\hat{\tau}_{1,t} = \hat{\tau}_{2,t} = \hat{\tau}_t < 0$ .
  - Tax increase from  $t+1$  on ( $\hat{\tau}_{1,t+s} = \hat{\tau}_{2,t+s} = \hat{\tau}_{t+s} > 0$  for  $s = 1, 2, \dots$ ), to pay for interest and keep the debt constant.
- Impact at time  $t$ :
  - Old agents consume the gift.
  - Young agents see higher lifetime income (future tax hike does not offset initial gift) and increase consumption.  
*(Row 1)*
  - Output set in the short run (capital is given), so higher consumption lowers investment.  
*Out*
- Impact from time  $t+1$  on: agents only face higher taxes and lower wages (capital has gone down), so they suffer.

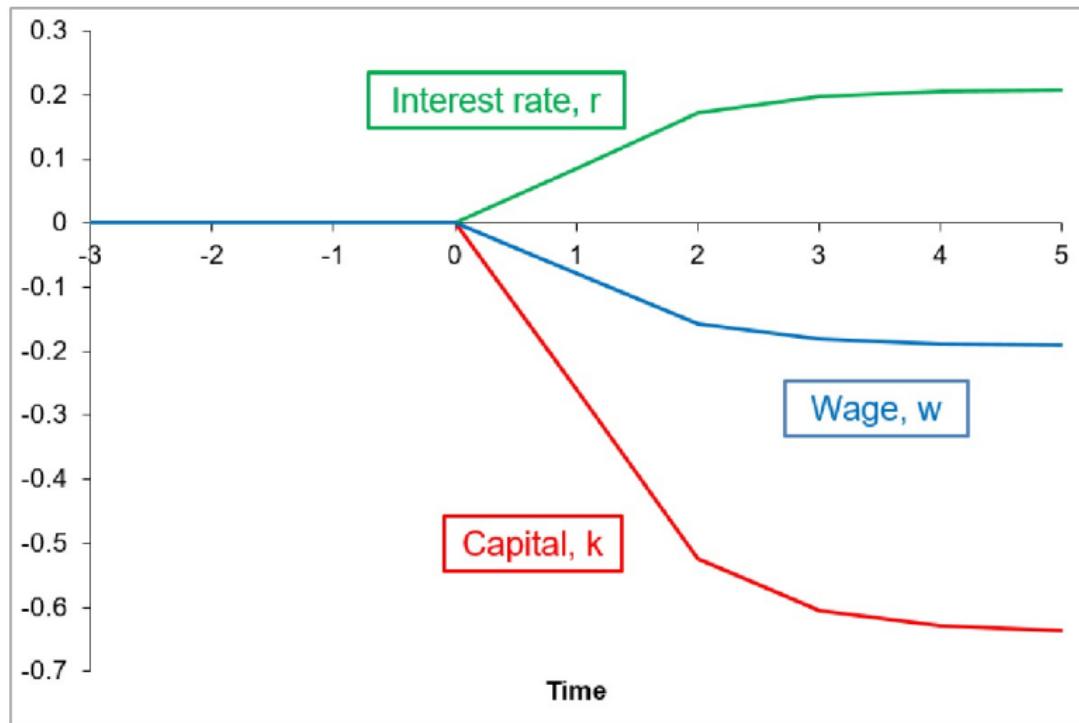
# Tax and government debt

- Tax cut, followed by increase. Permanent increase in debt.



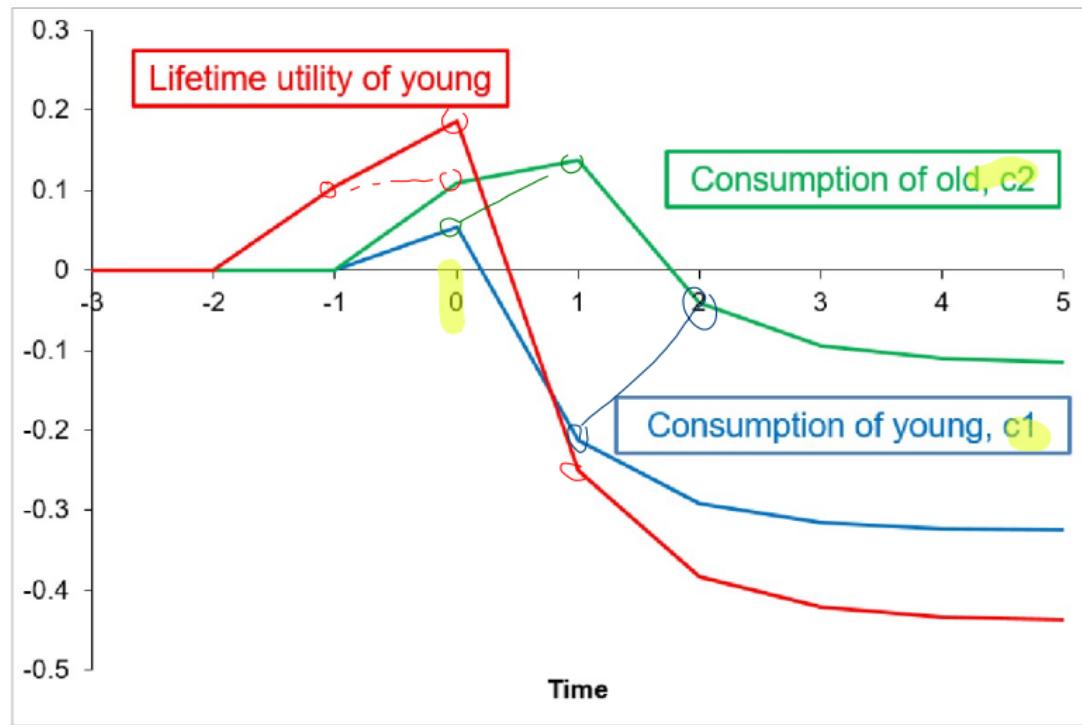
# Factor prices and capital

- Crowding out of private capital, raises capital return and reduces wages.



# Consumption and utility

- Higher consumption (and utility) of agents alive at the time of the tax cut. Adverse effect for the subsequent generations.



# Intuition for the adjustment

- Temporary tax cut alters consumption and output in OLG. It did not do so in representative agent model.
- Agents and the government have **different horizons**.
  - Government has an **infinite** horizon. Switch does not change the net present values of taxes.
  - **Agents have short horizon**. Switch changes their net present value of taxes.
  - Agents at time  $t$  see a lower net present value of taxes, future generations see a higher net present value of taxes.

## Alternative interpretation: borrowing constraints

- If agents are **prevented from borrowing**, they effectively face an  $+\infty$  interest rate, different from the government.
- In **Ramsey** an agent who would like to borrow but cannot is **not on the Euler**: marginal utility of current consumption higher than the future marginal utility adjusted by the market interest rate:

$$\text{USUAL} \quad (c_t)^{-\theta} > \frac{1 + r_{t+1}^{\text{BOND}}} {1 + \rho} (c_{t+1})^{-\theta}$$

- Agent faces a **shadow interest rate** (the rate for which the marginal utilities are equalized) higher than the market rate:

$$1 + r_{t+1}^{\text{shadow}} = (1 + \rho) \left( \frac{c_t}{c_{t+1}} \right)^{-\theta} > 1 + r_{t+1}$$

- **Tax switch** helps as government effectively borrows "on behalf" of the agent. More likely to be effective when agents are constrained, such as in a deep recession.

# OLG AND DYNAMIC EFFICIENCY

# OLG and saving constraints

- With OLG, “laissez faire” allocation can be highly inefficient, giving a role for policy (Weil 2012).
- Agents live for two periods, population grows at a rate  $n$  across cohorts. Output takes the form of **endowments**.
  - Young agents get an endowment  $e_1$ .
  - Old agents an endowment  $e_2$ .
  - Endowment made of a perishable good that cannot be stored.
- No intertemporal trade** is possible: specific young and specific old meet only once. No rest-of-the-world to trade with.
- Everyone is forced to consume their allocation in each period.

# A patient economy

- Agents care primarily about consumption when old, they would **like to save**.
- Consuming  $e_1$  is inefficient. Shadow interest rate  $r$  from the Euler is very low ( $-100\%$  if young consumption is not enjoyed at all).
- **Pareto improving transfer:** from period  $t$  on take  $\tau \leq e_1$  from the young and give it to the old.
  - Old at time  $t$  are better off.
  - Young at time  $t$ , and all future agents, are better off: they consume  $e_2 + (1 + n)\tau$  when old instead of  $e_2$ .
  - Result holds even if there is a storage technology, as long as it delivers a return lower than  $n$ .

# Transfer mechanism

- Agents want to save, and thus need an asset.
- Role for **public debt** as a way to transfer resources. Agents do not meet each other twice, but they meet the infinitively-lived government twice.
- Another option is “pay as you go” **retirement schemes** where young agents pay for old ones.
- Rational **bubbles** can also help: fundamentally worthless assets that can be bought, held, and sold across generations.

# An impatient economy

- Agents care primarily about consumption when young, they would **like to borrow**.
- Interest rate  $r$  from the Euler is then very high to make agents want to consume  $e_2$  when old.
- Switch from old to young starting at time  $t$ , mirroring the policy for the patient economy?
  - Young agents at time  $t$ , and all future agents would be better off.
  - Old agents at time  $t$  would be worse off.
  - Switch is **not a Pareto improvement**.
- Policy is thus possible only for an environment of high saving propensity and low interest rates (the patient economy).

# CONNECTING OLG AND INFINITE HORIZON

# Infinite-life models with and OLG features

- **Bequests:** agents live for one period, but care (equally) about future agents:

$$U_t = u(C_t) + \beta U_{t+1} = \sum_{s=t}^{\infty} \beta^{s-t} u(C_s)$$

- Budget constraint is identical to the representative agent model.
- Bequests  $H$  implies that agents' planning horizon is infinite, even though life is not:

$$\begin{aligned} C_t + H_{t+1} &= (1+r)H_t + Y_t - T_t \\ \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} C_s &= (1+r)H_t + \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} (Y_s - T_s) \end{aligned}$$

# Population growth through new entrants

- Agents have infinite lives. Population grows, but not because current agents grow as in Ramsey model.
- Population grows with **new agents** appearing with **no assets**.
- Each period new agents enter then live forever. Total population grows at a rate  $n$  through new agents.
- Distinguish allocation for an **individual** agent from the **per-capita** allocation.
  - Individual agent takes his own future into account, but not the situation of agents that are not yet borne. We are not in a representative agent model.
  - Per-capita variables include growth through the arrival of new agents.
- Tax switch has real effect, as in OLG model. Agents currently alive ignore the tax burden on unborn generations.

- Ricardian equivalence when the public and the government have the same time horizon. In OLG model the public has a finite horizon and the government an infinite horizon.
- Other possibility: borrowing constraints, agents live forever but cannot borrow today.
  - Euler condition does not hold: marginal utility of current consumption exceeds that of future consumption (or Euler condition holds including the shadow value of the constraint).
- Tax cut followed by tax increases is consumed instead of being saved (borrowing constraint implies an effective shorter horizon).
  - Ricardian equivalence then varies across the cycle.

# EXTRA SLIDES

# Disinflation under adaptive expectations

- Until  $T$  money grows at a high rate:  $m_T - m_{T-1} = \mu_0$ . Starting at  $T + 1$  money grows at a **slower rate**:  $m_{T+1} - m_T = \mu_1 < \mu_0$ .
- Start with adaptive inflation expectations:

$$\pi_{t+1}^e - \pi_t^e = \delta (\pi_t - \pi_t^e)$$

- Inflation expectations come down only gradually:

$$\pi_{T+s}^e = \mu_1 + \left(1 - \frac{\delta}{1 - \gamma\delta}\right)^{s-1} (\mu_0 - \mu_1)$$

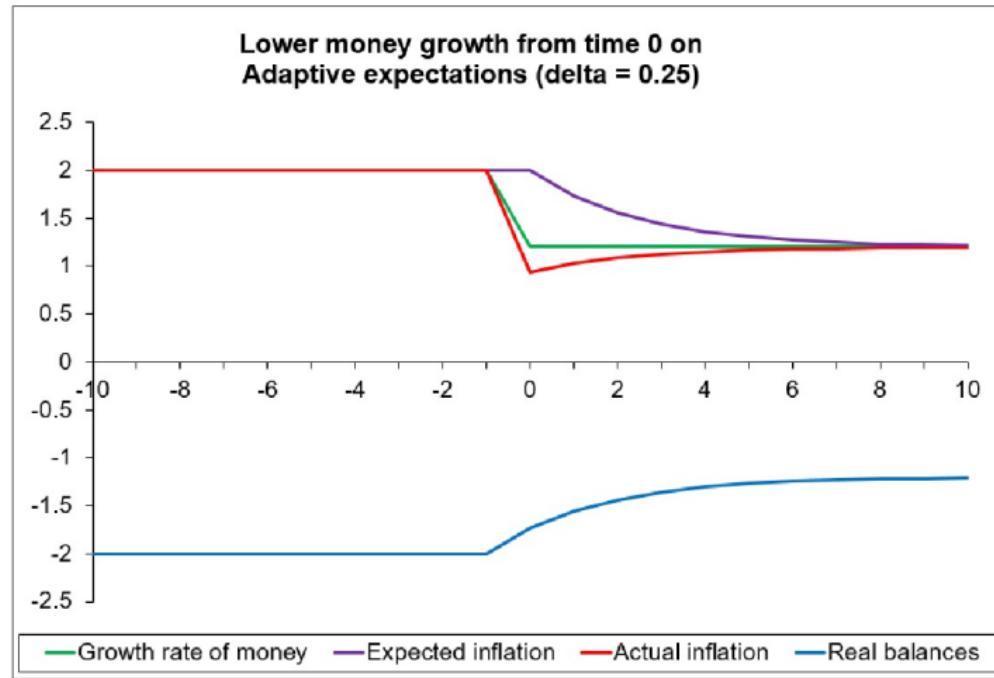
$$\pi_{T+s} = \mu_1 - \frac{\gamma\delta}{1 - \gamma\delta} \left(1 - \frac{\delta}{1 - \gamma\delta}\right)^{s-1} (\mu_0 - \mu_1)$$

- Money demand **increases slowly** as agents realize lower inflation is here to stay.

# Dynamics under adaptive expectations

- Lower money growth starting at time 0 ( $\delta = 0.25$ ).

[◀ Return](#)



## Household's intertemporal constraint

- Combine successive flow constraints ( $R_{t,t+s} = \prod_{i=0}^s (1/(1+r_{t+i}))$ ):

$$(k_t + b_t) = \sum_{s=0}^{\infty} R_{t,t+s} [c_{t+s} - (w_{t+s} - \tau_{t+s})] \\ + \lim_{s \rightarrow \infty} R_{t,t+s} (k_{t+s+1} + b_{t+s+1})$$

- Transversality condition:  $\lim_{s \rightarrow \infty} R_{t,t+s} (k_{t+s+1} + b_{t+s+1}) = 0$ .
- The value of debt or assets can grow, but not at a pace higher than the interest rate (no explosive growth).
- Present value of consumption: initial assets + PV after tax income:

$$\sum_{s=0}^{\infty} R_{t,t+s} c_{t+s} = (k_t + b_t) + \sum_{s=0}^{\infty} R_{t,t+s} (w_{t+s} - \tau_{t+s})$$

# Government's and overall intertemporal constraints

- Combine successive flow constraints:

$$b_t + \sum_{s=0}^{\infty} R_{t,t+s} g_{t+s} = \sum_{s=0}^{\infty} R_{t,t+s} \tau_{t+s}$$

- Present value of spending plus initial debt equal to present value of taxes.
- Combine household and government constraints:

$$\sum_{s=0}^{\infty} R_{t,t+s} c_{t+s} = k_t + \sum_{s=0}^{\infty} R_{t,t+s} (w_{t+s} - g_{t+s})$$

- Present value of private consumption equal to the initial capital plus PV of wages net of government spending (alternatively: value of private and public consumption equal to capital plus value of wages).

[◀ Return](#)

# General case of capital dynamics

- Consumptions reflect taxes and wages. Wage and the interest rate are functions of capital.
- Savings and government's budget constraint give **dynamics of capital**.
- Highly **non-linear** relation between  $k_{t+1}$  and  $k_t$ : [◀ Return](#)

$$k_{t+1} + b_{t+1} = s_{1,t} = w_t - \tau_{1,t} - c_{1,t}$$

$$k_{t+1} = \tau_{1,t} + \tau_{2,t} - (1 + r_t) b_t + w_t - \tau_{1,t} - c_{1,t}$$

$$\begin{aligned} k_{t+1} &= \frac{(1 + r_{t+1})^{\frac{1-\theta}{\theta}}}{(1 + \rho)^{\frac{1}{\theta}} + (1 + r_{t+1})^{\frac{1-\theta}{\theta}}} [(1 - \alpha)(k_t)^\alpha - \tau_{1,t}] \\ &\quad + \frac{(1 + \rho)^{\frac{1}{\theta}}}{(1 + \rho)^{\frac{1}{\theta}} + (1 + r_{t+1})^{\frac{1-\theta}{\theta}}} \frac{\tau_{2,t+1}}{1 + r_{t+1}} \\ &\quad + (\tau_{1,t} + \tau_{2,t} - (1 + r_t) b_t) \end{aligned}$$