Macroeconomics A; EI056

Short problems

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1 Balance sheet of the Swiss National Bank

1.1 Current structure

Question: The SNB published its latest results on October 31:
https://www.snb.ch/en/publications/communication/press-releases/2023/pre_20231031_1
What are the main components of the balance sheet at end September 2023?
What is the main difference with the Fed?

Answer: The figures are on pages 4 and 5.

The balance sheet totals 821 billions Swiss franc. This consists of a fair amount of gold (57 billions), but mostly of foreign assets (703 billions) such as bonds of the US and European governments, and holdings in foreign stock markets.

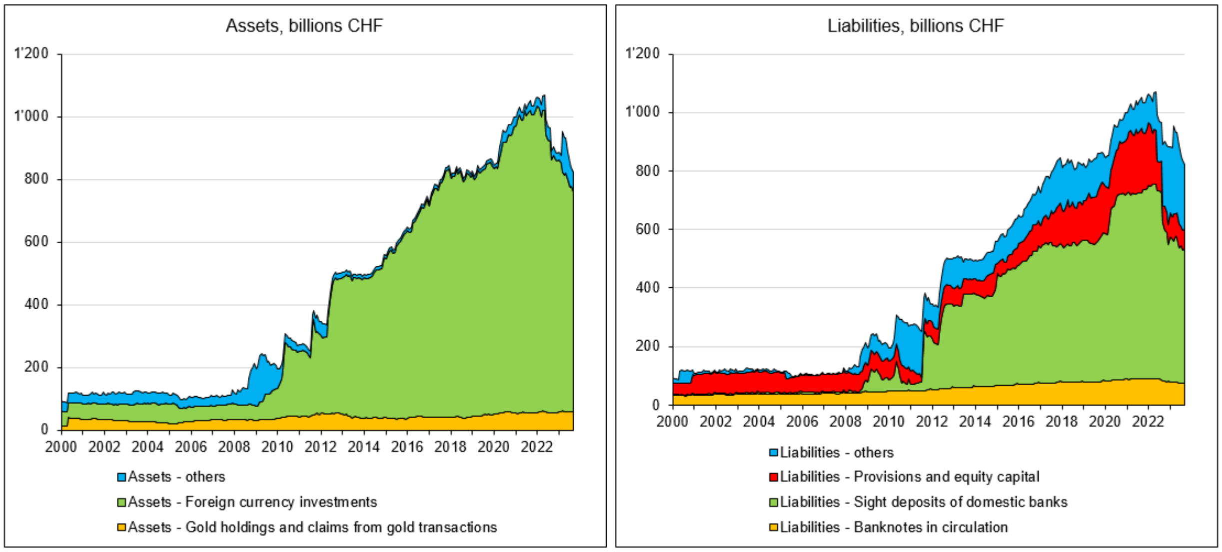
The difference from the Fed is that the Fed holds US domestic assets, while the SNB holds foreign currency assets. This is for two reasons. First, the Swiss government debt is very small, so the SNB couldn't do quantitative easing with only Swiss assets. Second, as the Swiss franc is a safe haven currency, when investors (Swiss and foreign) want to buy Swiss franc, the SNB can either let the exchange rate appreciate, or produce the needed Swiss franc and sell them against the foreign currency assets that private investors want to get rid of.

On the liability side, the biggest components are cash (74 billion) and accounts of banks at the SNB (460 billion). This is similar to the Fed. A difference is that the SNB has a larger equity. This is because it is a publicly traded company, and as such must value its assets at market prices. The strong returns in previous years have led to a build up of equity, which was substantially dented in 2022 and 2023 when asset prices fell and the Swiss franc appreciated (a strong Swiss franc reduces the CHF value of assets held in foreign currencies).

1.2 Changes through time

Question: The SNB publishes its monthly balance sheet on its website:
https://data.snb.ch/en/topics/snb/cube/snbbipo
Since 2000, how has the size and composition of the balance sheet evolved (focus on the main components). Has the evolution been smooth?

Answer: The attached spreadsheet provides the figures. The charts below show the main categories.



The balance sheet was very steady until 2008, and then started increasing. The increase has been essentially in the form of buying foreign currency assets (the other assets in 2008-2012 include the fund holding and selling the assets acquired from UBS during the 2008 SNB intervention in support of the bank). On the liability side, the increase has been in the form of deposits by banks. In other words, the SNB paid for the asset it purchased by selling Swiss francs to banks.

The increase has gone through different phases. Step increase in 2010, 2011 and 2012 (times of tensions in the euro area), then stability from 2012 (when Mario Draghi made clear that the ECB would do “whatever it takes” to save the euro) to 2015 (when the SNB abandoned the peg against the euro). There is then a time of steady increases until 2017. The balance sheet remain flat after that, until the Covid increase when the SNB intervenes a lot, like other central banks. Since early 2022, the balance sheet has decreased. This is due to losses on assets (which reduce equity capital) and recently the tighter monetary policy (reduction of deposits by banks). The takeover of Credit Suisse in 2023 led to an increase in the “other” assets, specifically in the form of liquidity support by the SNB.

On the liability side we see an increase of the equity capital from 2015 to late 2021. This reflects the good return on the foreign assets (including stocks) held by the SNB. Since 2022, losses from lower asset prices and the stronger Swiss franc have reduced the equity capital,

2 Cagan model

2.1 Inflation under constant money supply

Question: The Cagan model is based on the money demand:

$$m_t - p_t = -\gamma \pi^e_{t+1} = -\gamma \left(p^e_{t+1} - p_t \right)$$

Iterating forward, this gives the price level:

$$p_t = \frac{1}{1+\gamma} \sum_{s=0}^{\infty} \left(\frac{\gamma}{1+\gamma} \right)^s m^e_{t+s}$$

Consider that the central bank keeps the money supply constant at $m_t = m^A$.
What is inflation?
What is the price level?

Answer: Inflation is equal to zero. To see that, take the money demand in two successive periods:

$$\begin{aligned} m_t - p_t &= -\gamma\left(p_{t+1} - p_t\right) \\ m_{t+1} - p_{t+1} &= -\gamma\left(p_{t+2} - p_{t+2}\right) \end{aligned}$$

As the money is constant:

$$\begin{aligned} m^A - p_t &= -\gamma\left(p_{t+1} - p_t\right) \\ m^A - p_{t+1} &= -\gamma\left(p_{t+2} - p_{t+1}\right) \end{aligned}$$

Take the difference:

$$\begin{aligned} -p_t + p_{t+1} &= -\gamma\left(p_{t+1} - p_{t+2} - p_t + p_{t+1}\right) \\ \pi_{t+1} &= -\gamma\left(-\pi_{t+2} + \pi_{t+1}\right) \\ (1+\gamma)\,\pi_{t+1} &= \gamma\pi_{t+2} \end{aligned}$$

As inflation must be constant in a steady state, the only possible value is $\pi_{t+1} = \pi_{t+2} = 0$.

This can also be seen from the price formula:

$$\begin{aligned} p_t &= \frac{1}{1+\gamma}\sum_{s=0}^{\infty}\left(\frac{\gamma}{1+\gamma}\right)^s m^e_{t+s} \\ p_t &= \frac{1}{1+\gamma}\sum_{s=0}^{\infty}\left(\frac{\gamma}{1+\gamma}\right)^s m^A \\ p_t &= m^A\frac{1}{1+\gamma}\sum_{s=0}^{\infty}\left(\frac{\gamma}{1+\gamma}\right)^s \\ p_t &= m^A\frac{1}{1+\gamma}\frac{1}{1-\frac{\gamma}{1+\gamma}} \\ p_t &= m^A\frac{1}{1+\gamma}\frac{1+\gamma}{1+\gamma-\gamma} \\ p_t &= m^A \end{aligned}$$

We can do the same computation for p_{t+1}, which implies zero inflation.

2.2 Change in level, rational expectations

Question: Consider that we are initially in a situation where money has been constant at m^A and is expected to remain so.

At period t we are told that starting at period $t+3$ money will be constant at $m^B = m^A + e$.

What will inflation and the price level be at period $t+3$?

Answer: Starting from period $t+3$ we are in the same situation as initially, simply at a different level of the money supply.

We can thus apply the same formula:

$$p_{t+3} = m^B = m^A + e = p_{t-1} + e$$

Inflation is again zero.

2.3 Adjustment dynamics

Question: Now consider the path of adjustment. Show that:

$$\begin{aligned}
\pi_t &= \left(\frac{\gamma}{1+\gamma}\right)^3 e \\
\pi_{t+1} &= \frac{1}{1+\gamma}\left(\frac{\gamma}{1+\gamma}\right)^2 e \\
\pi_{t+2} &= \frac{1}{1+\gamma}\frac{\gamma}{1+\gamma} e \\
\pi_{t+3} &= \frac{1}{1+\gamma} e
\end{aligned}$$

How does inflation behave along the adjustment path?

Answer: Until period $t-1$ included, the price is $p_{t-1} = m^A$. The money demands at the various periods are as follows.

$$\begin{aligned}
m_t - p_t &= -\gamma\left(p_{t+1} - p_t\right) \\
m_{t+1} - p_{t+1} &= -\gamma\left(p_{t+2} - p_{t+1}\right) \\
m_{t+2} - p_{t+2} &= -\gamma\left(p_{t+3} - p_{t+2}\right) \\
m_{t+3} - p_{t+3} &= -\gamma\left(p_{t+4} - p_{t+3}\right)
\end{aligned}$$

Using our results, the last one simply implies $p_{t+3} = p_{t+4} = m^B = m^A + e$. The first three are written as:

$$\begin{aligned}
m^A - p_t &= -\gamma\left(p_{t+1} - p_t\right) \\
m^A - p_{t+1} &= -\gamma\left(p_{t+2} - p_{t+1}\right) \\
m^A - p_{t+2} &= -\gamma\left(m^B - p_{t+2}\right)
\end{aligned}$$

The last implies:

$$\begin{aligned}
m^A - p_{t+2} &= -\gamma\left(m^B - p_{t+2}\right) \\
m^A + \gamma m^B &= (1+\gamma)\, p_{t+2} \\
(1+\gamma)\, m^A + \gamma e &= (1+\gamma)\, p_{t+2} \\
p_{t+2} &= m^A + \frac{\gamma}{1+\gamma} e
\end{aligned}$$

Note that we can get this results from the formula of the price level:

$$p_{t+2} = \frac{1}{1+\gamma}\sum_{s=0}^{\infty}\left(\frac{\gamma}{1+\gamma}\right)^s m_{t+2+s}$$

$$
\begin{aligned}
p_{t+2} &= \frac{1}{1+\gamma} m_{t+2} + \frac{1}{1+\gamma} \sum_{s=1}^{\infty} \left(\frac{\gamma}{1+\gamma} \right)^s m_{t+2+s} \\
p_{t+2} &= \frac{1}{1+\gamma} m^A + \frac{1}{1+\gamma} \sum_{s=1}^{\infty} \left(\frac{\gamma}{1+\gamma} \right)^s m^B \\
p_{t+2} &= \frac{1}{1+\gamma} m^A + m^B \frac{1}{1+\gamma} \frac{\gamma}{1+\gamma} \sum_{s=0}^{\infty} \left(\frac{\gamma}{1+\gamma} \right)^s \\
p_{t+2} &= \frac{1}{1+\gamma} m^A + m^B \frac{1}{1+\gamma} \frac{\gamma}{1+\gamma} \frac{1}{1-\frac{\gamma}{1+\gamma}} \\
p_{t+2} &= \frac{1}{1+\gamma} m^A + \frac{\gamma}{1+\gamma} m^B \\
p_{t+2} &= m^A + \frac{\gamma}{1+\gamma} e
\end{aligned}
$$

Using this we write the money demand of time $t+1$:

$$
\begin{aligned}
m^A - p_{t+1} &= -\gamma \left(p_{t+2} - p_{t+1} \right) \\
m^A &= -\gamma \left(m^A + \frac{\gamma}{1+\gamma} e \right) + (1+\gamma) p_{t+1} \\
(1+\gamma) m^A + \frac{\gamma^2}{1+\gamma} e &= (1+\gamma) p_{t+1} \\
p_{t+1} &= m^A + \left(\frac{\gamma}{1+\gamma} \right)^2 e
\end{aligned}
$$

Here also, we can get this results from the formula of the price level:

$$
\begin{aligned}
p_{t+1} &= \frac{1}{1+\gamma} \sum_{s=0}^{\infty} \left(\frac{\gamma}{1+\gamma} \right)^s m_{t+1+s} \\
p_{t+1} &= \frac{1}{1+\gamma} m_{t+1} + \frac{1}{1+\gamma} \frac{\gamma}{1+\gamma} m_{t+2} + \frac{1}{1+\gamma} \sum_{s=2}^{\infty} \left(\frac{\gamma}{1+\gamma} \right)^s m_{t+1+s} \\
p_{t+1} &= \frac{1}{1+\gamma} \frac{1+2\gamma}{1+\gamma} m^A + \frac{1}{1+\gamma} \sum_{s=2}^{\infty} \left(\frac{\gamma}{1+\gamma} \right)^s m^B \\
p_{t+1} &= \frac{1}{1+\gamma} \frac{1+2\gamma}{1+\gamma} m^A + m^B \frac{1}{1+\gamma} \left(\frac{\gamma}{1+\gamma} \right)^2 \sum_{s=0}^{\infty} \left(\frac{\gamma}{1+\gamma} \right)^s \\
p_{t+1} &= \frac{1}{1+\gamma} \frac{1+2\gamma}{1+\gamma} m^A + m^B \frac{1}{1+\gamma} \left(\frac{\gamma}{1+\gamma} \right)^2 \frac{1}{1-\frac{\gamma}{1+\gamma}} \\
p_{t+1} &= \frac{1}{1+\gamma} \frac{1+2\gamma}{1+\gamma} m^A + \left(\frac{\gamma}{1+\gamma} \right)^2 m^B \\
p_{t+1} &= \left(\frac{1}{1+\gamma} \frac{1+2\gamma}{1+\gamma} + \left(\frac{\gamma}{1+\gamma} \right)^2 \right) m^A + \left(\frac{\gamma}{1+\gamma} \right)^2 e \\
p_{t+1} &= \left(1+2\gamma+\gamma^2 \right) \left(\frac{1}{1+\gamma} \right)^2 m^A + \left(\frac{\gamma}{1+\gamma} \right)^2 e \\
p_{t+1} &= m^A + \left(\frac{\gamma}{1+\gamma} \right)^2 e
\end{aligned}
$$

Finally, the money demand at time t is:

$$\begin{aligned}
m^A - p_t &= -\gamma\left(p_{t+1} - p_t\right) \\
m^A &= -\gamma\left(m^A + \left(\frac{\gamma}{1+\gamma}\right)^2 e\right) + (1+\gamma)\, p_t \\
(1+\gamma)\, m^A + \gamma\left(\frac{\gamma}{1+\gamma}\right)^2 e &= (1+\gamma)\, p_t \\
p_t &= m^A + \left(\frac{\gamma}{1+\gamma}\right)^3 e
\end{aligned}$$

Here also, we can get this results from the formula of the price level:

$$\begin{aligned}
p_t &= \frac{1}{1+\gamma}\sum_{s=0}^{\infty}\left(\frac{\gamma}{1+\gamma}\right)^s m_{t+s} \\
p_t &= \frac{1}{1+\gamma} m_t + \frac{1}{1+\gamma}\frac{\gamma}{1+\gamma} m_{t+1} + \frac{1}{1+\gamma}\left(\frac{\gamma}{1+\gamma}\right)^2 m_{t+2} + \frac{1}{1+\gamma}\sum_{s=3}^{\infty}\left(\frac{\gamma}{1+\gamma}\right)^s m_{t+s} \\
p_t &= \frac{1}{1+\gamma}\left(1 + \frac{\gamma}{1+\gamma} + \left(\frac{\gamma}{1+\gamma}\right)^2\right) m^A + \frac{1}{1+\gamma}\sum_{s=3}^{\infty}\left(\frac{\gamma}{1+\gamma}\right)^s m^B \\
p_t &= \frac{1}{1+\gamma}\left(1 + \frac{\gamma}{1+\gamma} + \left(\frac{\gamma}{1+\gamma}\right)^2\right) m^A + m^B\frac{1}{1+\gamma}\left(\frac{\gamma}{1+\gamma}\right)^3 \sum_{s=0}^{\infty}\left(\frac{\gamma}{1+\gamma}\right)^s \\
p_t &= \frac{1}{1+\gamma}\left(1 + \frac{\gamma}{1+\gamma} + \left(\frac{\gamma}{1+\gamma}\right)^2\right) m^A + m^B\frac{1}{1+\gamma}\left(\frac{\gamma}{1+\gamma}\right)^3 \frac{1}{1-\frac{\gamma}{1+\gamma}} \\
p_t &= \frac{1}{1+\gamma}\left(1 + \frac{\gamma}{1+\gamma} + \left(\frac{\gamma}{1+\gamma}\right)^2\right) m^A + \left(\frac{\gamma}{1+\gamma}\right)^3 m^B \\
p_t &= \frac{1}{1+\gamma}\left(1 + \frac{\gamma}{1+\gamma} + \left(\frac{\gamma}{1+\gamma}\right)^2 + \gamma\left(\frac{\gamma}{1+\gamma}\right)^2\right) m^A + \left(\frac{\gamma}{1+\gamma}\right)^3 e \\
p_t &= \frac{1}{1+\gamma}\left(1 + \frac{\gamma}{1+\gamma} + \gamma\frac{\gamma}{1+\gamma}\right) m^A + \left(\frac{\gamma}{1+\gamma}\right)^3 e \\
p_t &= m^A + \left(\frac{\gamma}{1+\gamma}\right)^3 e
\end{aligned}$$

Using these results, the inflation rates are as follows. For period t:

$$\pi_t = \left(\frac{\gamma}{1+\gamma}\right)^3 e$$

For period $t+1$:

$$\begin{aligned}
\pi_{t+1} &= \left(\frac{\gamma}{1+\gamma}\right)^2 e - \left(\frac{\gamma}{1+\gamma}\right)^3 e \\
\pi_{t+1} &= \left(1 - \frac{\gamma}{1+\gamma}\right)\left(\frac{\gamma}{1+\gamma}\right)^2 e \\
\pi_{t+1} &= \frac{1}{1+\gamma}\left(\frac{\gamma}{1+\gamma}\right)^2 e
\end{aligned}$$

For period $t+2$:

$$\begin{aligned}
\pi_{t+2} &= \frac{\gamma}{1+\gamma}e - \left(\frac{\gamma}{1+\gamma}\right)^2 e \\
\pi_{t+2} &= \left(1 - \frac{\gamma}{1+\gamma}\right)\frac{\gamma}{1+\gamma}e \\
\pi_{t+2} &= \frac{1}{1+\gamma}\frac{\gamma}{1+\gamma}e
\end{aligned}$$

For period $t+3$:

$$\begin{aligned}
\pi_{t+3} &= e - \frac{\gamma}{1+\gamma}e \\
\pi_{t+3} &= \left(1 - \frac{\gamma}{1+\gamma}\right)e \\
\pi_{t+3} &= \frac{1}{1+\gamma}e
\end{aligned}$$

The inflation adjustment takes place through an initial jump in period t,with a subsequent convergence.

From period $t+1$ on inflation gradually accelerate:

$$\frac{\pi_{t+2}}{\pi_{t+1}} = \frac{1+\gamma}{\gamma} > 1$$

In period t inflation increases. It may however be smaller or larger than the subsequent inflation, depending on the value of γ:

$$\frac{\pi_{t+1}}{\pi_t} = \frac{1}{\gamma}$$