International Trade I

The Heckscher-Ohlin Model[1]

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Spring 2025

[1] These lecture notes are based on materials from A. Costinot, A. Dixit, R. Feenstra, Feenstra&Taylor, and J. P. Neary.

Outline of the Lecture

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Reading

Heckscher-Ohlin Model

- *F pp. 31-41, 64-71, 83-93
- Jones, R., and P. Neary. “The Positive Theory of International Trade.” pp. 14-21
- Jones, .W. (1965), “The Structure of Simple General Equilibrium Model,” Journal of Political Economy, 73, 557-572
- *Introductory level:* FT Chs. 4 and 5 or KOM Ch. 5

Outline of the Lecture

Basic environment

- Consider an economy with:
  - Two goods, $g = 1, 2$
  - Two factors with endowments L and K
    - ★ both factors are "mobile", can be employed in both sectors

- Output of good g is given by

$$y_g = f^g(L_g, K_g)$$

where:
  - L_g and K_g are the (endogenous) amounts of labor and capital in sector g
  - f^g is the production function in sector g:
    - ★ positive, increasing, concave
    - ★ homogenous of degree 1 in (L_g, K_g), i.e. CRS

Dual approach

- $c_g(w,r) \equiv$ unit cost function in sector g

$$c_g(w,r) = \min_{L,K}\{wL + rK | f^g(L,K) \geq 1\}$$

  where w and r the price of labor and capital

- $a_{fg}(w,r) \equiv$ unit demand for factor f in the production of good g
- Using the Envelope Theorem, it is easy to check that:

$$a_{Lg}(w,r) = \frac{dc_g(w,r)}{dw} \quad \text{and} \quad a_{Kg}(w,r) = \frac{dc_g(w,r)}{dr}$$

- $A(w,r) \equiv [a_{fg}(w,r)]$: matrix of total factor requirements

Equilibrium conditions: SOE

- Like in RV model, we first look at the case of a SOE
  - So no need to look at good market clearing

- **Profit-maximization:**

$$p_g \leq w a_{Lg}(w,r) + r a_{Kg}(w,r) \text{ for all } g = 1,2 \tag{1}$$

$$p_g = w a_{Lg}(w,r) + r a_{Kg}(w,r) \text{ if } g \text{ is produced in equilibrium} \tag{2}$$

- **Factor-market clearing:**

$$L = y_1 a_{L1}(w,r) + y_2 a_{L2}(w,r) \tag{3}$$

$$K = y_1 a_{K1}(w,r) + y_2 a_{K2}(w,r) \tag{4}$$

Outline of the Lecture

Factor Price Equalization

- **Question:** *Can trade in goods be a (perfect) substitute for trade in factors?*
  - First classical result from the HO literature answers: YES

- To establish this result formally, we'll need the following definition:

- **Definition:** Factor Intensity Reversal (FIR) does not occur if:
  (i) $a_{L1}(w,r)/a_{K1}(w,r) > a_{L2}(w,r)/a_{K2}(w,r)$ for all (w,r); or
  (ii) $a_{L1}(w,r)/a_{K1}(w,r) < a_{L2}(w,r)/a_{K2}(w,r)$ for all (w,r).

Factor Price Insensitivity (FPI)

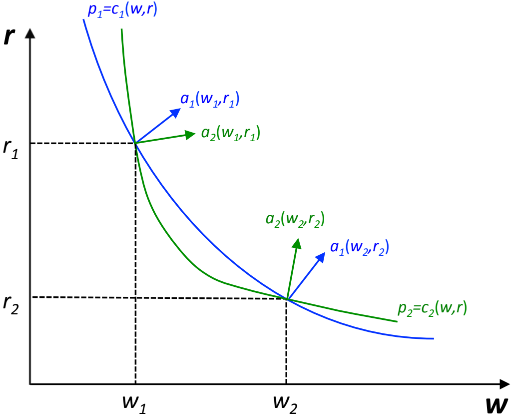
- **Lemma:** If both goods are produced in equilibrium and FIR does not occur, then factor prices $\omega \equiv (w, r)$ are uniquely determined by good prices $p \equiv (p_1, p_2)$.

- **Proof:** If both goods are produced in equilibrium, then $p = A'(\omega)\omega$. By Gale and Nikaido (1965), this equation admits a unique solution if $a_{fg}(\omega) > 0$ for all f, g and $\det[A(\omega)] \neq 0$ for all ω, which is guaranteed by no FIR.

- **Comments:**
  - Good prices rather than factor endowments determine factor prices
  - In a closed economy, good prices and factor endowments are, of course, related, but not for a small open economy
  - Proof already suggests that "dimensionality" will be an issue for FIR

Factor Price Insensitivity (FPI): graphical analysis

- Link between no FIR and FPI can be seen graphically:



- If iso-cost curves cross more than once, then FIR must occur

Factor Price Equalization (FPE) Theorem

- The previous lemma directly implies (Samuelson 1949) that:
- **FPE Theorem:** If two countries produce both goods under free trade with the same technology and FIR does not occur, then they must have the same factor prices.
- **Comments:**
  - Trade in goods can be a "perfect substitute" for trade in factors
  - Countries with different factor endowments can sustain same factor prices through different allocation of factors across sectors
  - Assumptions for FPE are stronger than for FPI: we need free trade and same technology in the two countries...
  - For next results, we'll maintain assumption that both goods are produced in equilibrium, but won't need free trade and same technology

Outline of the Lecture

Stolper-Samuelson (1941) Theorem

- **Stolper-Samuelson Theorem:** An increase in the relative price of a good will increase the real return to the factor used intensively in that good, and reduce the real return to the other factor.

- **Proof:** W.l.o.g. suppose that (i) $a_{L1}(\omega)/a_{K1}(\omega) > a_{L2}(\omega)/a_{K2}(\omega)$ and (ii) $\hat{p}_2 > \hat{p}_1$. Differentiating the zero-profit condition (2), we get

$$\hat{p}_g = \theta_{Lg}\hat{w} + (1 - \theta_{Lg})\hat{r} \tag{5}$$

where $\theta_{Lg} \equiv w a_{Lg}(\omega)/c_g(\omega)$. Equation (5) implies

$$\hat{w} \geq \hat{p}_1, \hat{p}_2 \geq \hat{r} \quad \text{or} \quad \hat{r} \geq \hat{p}_1, \hat{p}_2 \geq \hat{w}$$

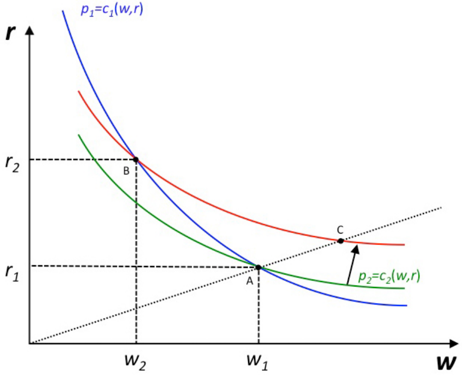
By (i), $\theta_{L2} < \theta_{L1}$. So (ii) requires $\hat{r} > \hat{w}$. Combining the previous inequalities, we get

$$\hat{r} > \hat{p}_2 > \hat{p}_1 > \hat{w}$$

Stolper-Samuelson (1941) Theorem

- **Comments:**
  - The chain of inequalities $\hat{r} > \hat{p}_2 > \hat{p}_1 > \hat{w}$ is referred as a "magnification effect"
  - SS predict both winners and losers from change in relative prices
  - Like FPI and FPE, SS entirely comes from zero-profit condition ($+$ no joint production)
  - Like FPI and FPE, sharpness of the result hinges on "dimensionality"
  - In the empirical literature, people often talk about "Stolper-Samuelson effects" whenever looking at changes in relative factor prices (though changes in relative good prices are rarely observed)

Stolper-Samuelson (1941) Theorem: graphical analysis



- Like for FPI and FPE, all economic intuition could be gained by looking at the simpler Leontieff case:
  - In the general case, iso-cost curves are not straight lines, but under no FIR, same logic applies

Outline of the Lecture

Rybczynski (1941) Theorem

- Previous results have focused on the implication of zero profit condition, Equation (2), for factor prices
- We now turn our attention to the implication of factor market clearing, Equations (3) and (4), for factor allocation
- **Rybczynski Theorem:** An increase in factor endowment will increase the output of the industry using it intensively, and decrease the output of the other industry

Rybczynski (1941) Theorem: proof

- **Proof:** W.l.o.g. suppose that (i) $a_{L1}(\omega)/a_{K1}(\omega) > a_{L2}(\omega)/a_{K2}(\omega)$ and (ii) $\hat{K} > \hat{L}$. Differentiating factor-market-clearing conditions (3) and (4), we get

$$\hat{L} = \lambda_{L1}\hat{y}_1 + (1 - \lambda_{L1})\hat{y}_2 \tag{6}$$

$$\hat{K} = \lambda_{K1}\hat{y}_1 + (1 - \lambda_{K1})\hat{y}_2 \tag{7}$$

where $\lambda_{L1} \equiv a_{L1}(\omega)y_1/L$ and $\lambda_{K1} \equiv a_{K1}(\omega)y_1/K$. Equations (6) and (7) imply

$$\hat{y}_1 \geq \hat{L}, \hat{K} \geq \hat{y}_2 \quad \text{or} \quad \hat{y}_2 \geq \hat{L}, \hat{K} \geq \hat{y}_1$$

By (i), $\lambda_{K1} < \lambda_{L1}$. So (ii) requires $\hat{y}_2 > \hat{y}_1$. Combining the previous inequalities, we get

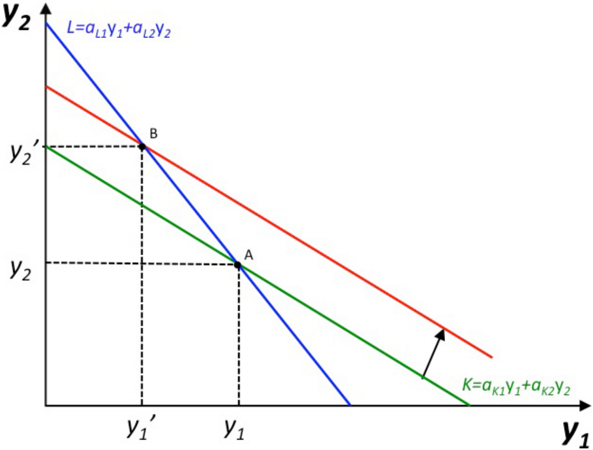
$$\hat{y}_2 > \hat{K} > \hat{L} > \hat{y}_1$$

Rybczynski (1941) Theorem: comments

- Like for FPI and FPE Theorems:
  - (p_1, p_2) is exogenously given $\Rightarrow$ factor prices and factor requirements are not affected by changes in factor endowments
  - Empirically, Rybczynski Theorem suggests that impact of immigration may be very different in closed vs. open economy
- Like for SS Theorem, we have a "magnification effect"
- Like for FPI, FPE, and SS Theorems, sharpness of the result hinges on "dimensionality"

Rybczynski (1941) Theorem: graphical analysis 1

- Since good prices are fixed, it is as if we were in Leontieff case

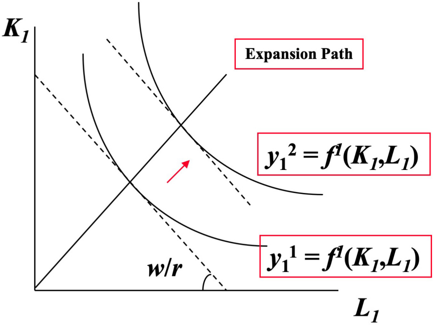


Constructing the Edgeworth Box Diagram

- Isoquant diagram illustrates technology in one sector
- If factor prices are fixed, then least-cost point on any particular isoquant is determined:

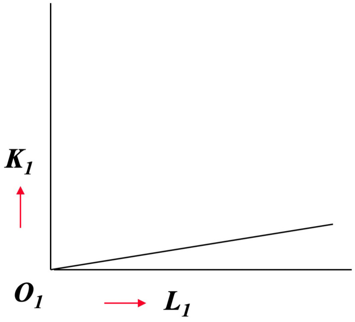
$$slope = -MRS = -\frac{W}{R}$$

- Assume that factor prices remain fixed
- With CRS, locus of least-cost points on different isoquants is a straight line from origin (*Expansion Path*)

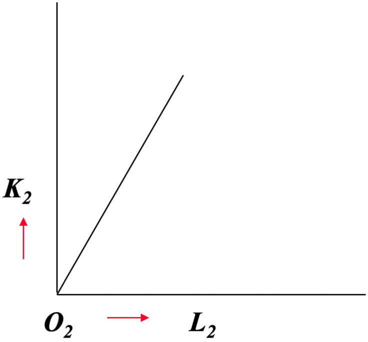


Constructing the Edgeworth Box Diagram 2

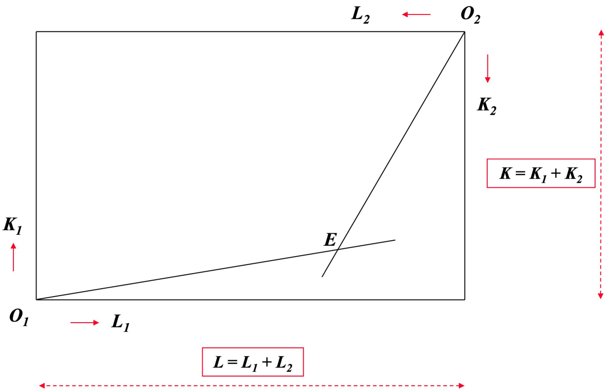
Expansion path for sector 1



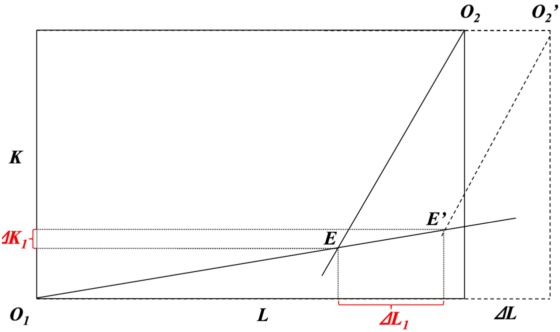
Expansion path for sector 2



Factor Allocation in the Edgeworth Box



Increase in Home Labor



- Additional labor in the economy is fully employed
- Capital-labor ratio in each industry is unchanged
- Sector 1 (labor-intensive) expands, sector 2 contracts

Rybczynski Theorem - Economic mechanism

- Increase in L puts downward pressure on wage W
- This encourages expansion of L-intensive sector 1
- L-intensive sector draws capital *and* labor from other sector
- Since factor prices settle at their original level, both sectors end up with the same factor proportions as initially
- Expanding sector grows by more than the economy average

Consequence of Factor Price Insensitivity

If goods prices do not change and a country continues to produce both goods, endowment changes do not affect factor prices

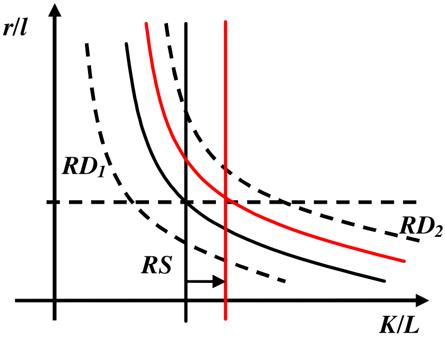
- Factor prices do not change, because factor proportions in both industries stay the same
- The economy can absorb the extra amount of a factor by increasing the output of the industry using that factor intensively and reducing the output of the other industry

Real-world examples

- Black Death in 13th century Europe
- Great Famine in Ireland, 1846-49
- Russian emigration to Israel in 1990’s
- Mariel boat lift

Rybczynski (1941) Theorem: graphical analysis 2

- Rybczynski effect can also be illustrated using relative factor supply and relative factor demand:



- Cross-sectoral reallocations are at the core of HO predictions:
  - For relative factor prices to remain constant, aggregate relative demand must go up, which requires expansion capital intensive sector

Outline of the Lecture

Basic environment

- Previous results hold for small open economies
  - relative good prices were taken as exogenously given
- We now turn world economy with two countries, North and South
- We maintain the two-by-two HO assumptions:
  - there are two goods, $g = 1, 2$, and two factors, K and L
  - identical technology around the world, $y_g = f_g(K_g, L_g)$
  - identical homothetic preferences around the world, $d_g^c = \alpha_g(p)L^c$
- What is the pattern of trade in this environment?

Strategy

- Start from **Integrated Equilibrium** $\equiv$ competitive equilibrium that would prevail if *both* goods and factors were freely traded
- Consider **Free Trade Equilibrium** $\equiv$ competitive equilibrium that prevails if goods are freely traded, but factors are not
- Ask: Can free trade equilibrium reproduce integrated equilibrium?
  - If factor prices are equalized through trade, the answer is yes
- In this situation, one can then use homotheticity to go from differences in factor endowments to pattern of trade

Integrated equilibrium

- **Integrated equilibrium** corresponds to (p, ω, y) such that:

$$(ZP): \quad p = A'(\omega)\omega \tag{8}$$

$$(GM): \quad y = \alpha(p)(\omega' v) \tag{9}$$

$$(FM): \quad v = A(\omega)y \tag{10}$$

  where
  - $p \equiv (p_1, p_2)$, $\omega \equiv (w, r)$, $A(\omega) \equiv [a_{fg}(\omega)]$, $y \equiv (y_1, y_2)$, $v \equiv (L, K)$, $\alpha(p) \equiv [\alpha_1(p), \alpha_2(p)]$
  - $A(\omega)$ derives from cost-minimization
  - $\alpha(p)$ derives from utility-maximization

Free-trade equilibrium

- **Free-trade equilibrium** corresponds to $(p^t, \omega^n, \omega^s, y^n, y^s)$ such that:

$$(ZP): \quad p^t \leq A'(\omega^c)\omega^c \tag{11}$$

$$(GM): \quad y^n + y^s = \alpha(p^t)(\omega^{n\prime}v^n + \omega^{s\prime}v^s) \tag{12}$$

$$(FM): \quad v^c = A(\omega^c)y^c \text{ for } c = n, s \tag{13}$$

  where (11) holds with equality if good is produced in country c

- **Definition:** Free-trade equilibrium replicates integrated equilibrium if $\exists(y^n, y^s) \geq 0$ such that $(p, \omega, \omega, y^n, y^s)$ satisfy conditions (11)-(13).

Factor-Price-Equalization (FPE) Set

- **Definition:** (v^n, v^s) are in the FPE set if $\exists(y^n, y^s) \geq 0$ such that condition (13) holds for $\omega^n = \omega^s = \omega$.

- **Lemma:** If (v^n, v^s) are in the FPE set, then free-trade equilibrium replicates integrated equilibrium.

- **Proof:** By definition of the FPE set, $\exists(y^n, y^s) \geq 0$ such that

$$v^c = A(\omega)y^c$$

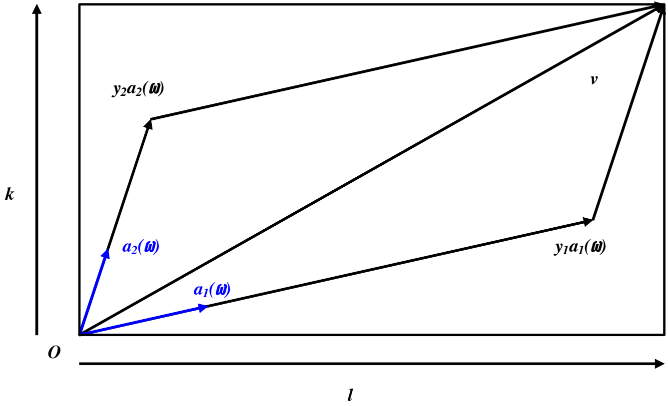
So condition (13) holds. Since $v = v^n + v^s$, this implies

$$v = A(\omega)(y^n + y^s)$$

Combining this expression with condition (10), we obtain $y^n + y^s = y$. Since $\omega^{n\prime} v^n + \omega^{s\prime} v^s = \omega' v$, condition (12) holds as well. Finally, condition (8) directly implies (11) holds.

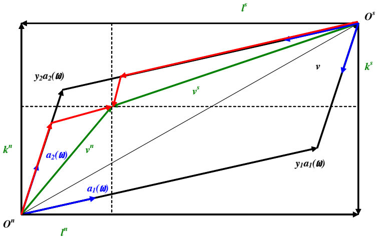
Integrated equilibrium: graphical analysis

- Factor market clearing in the integrated equilibrium:



The “Parallelogram”

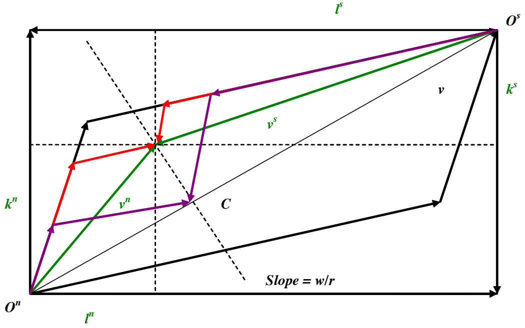
- **FPE set** $\equiv (v^n, v^s)$ inside the parallelogram



- When v^n and v^s are inside the parallelogram, we say that they below to the same **diversification cone**
- This is a very different way of approaching FPE than FPE Theorem
  - Here, we have shown that there can be FPE iff factor endowments are not too dissimilar, whether or not there are no FIR
  - Instead of taking prices as given - whether or not they are consistent with integrated equilibrium - we take factor endowments as primitives

Heckscher-Ohlin Theorem: graphical analysis

- Suppose that (v^n, v^s) is in the FPE set
- **HO Theorem:** In the free trade equilibrium, each country will export the good that uses its abundant factor intensively



- Outside the FPE set, additional technological and demand considerations matter (e.g. FIR or no FIR)

Heckscher-Ohlin Theorem: alternative proof

- HO Theorem can also be derived using Rybczynski effect:
  1. Rybczynski theorem $\Rightarrow y_2^n/y_1^n > y_2^s/y_1^s$ for any p
  2. Homotheticity $\Rightarrow c_2^n/c_1^n = c_2^s/c_1^s$ for any p
  3. This implies $p_2^n/p_1^n < p_2^s/p_1^s$ under autarky
  4. Law of comparative advantage $\Rightarrow$ HO Theorem

Trade and inequality

- Predictions of HO and SS Theorems are often combined:
  - HO theorem $\Rightarrow p_2^n/p_1^n < p_2/p_1 < p_2^s/p_1^s$
  - SS Theorem $\Rightarrow$ Moving from autarky to free trade, real return of abundant factor increases, whereas real return of scarce factor decreases
  - If North is skill-abundant relative to South, inequality increases in the North and decreases in the South

- So why may we observe a rise in inequality in the South in practice?
  - Southern countries are not moving from autarky to free trade
  - Technology is not identical around the world
  - Preferences are not homothetic and identical around the world
  - There are more than two goods and two countries in the world

Trade volumes

- Let us define trade volumes as the sum of exports plus imports
- Inside FPE set, iso-volume lines are parallel to diagonal (HKa p.23)
  - the further away from the diagonal, the larger the trade volumes
  - factor abundance rather than country size determines trade volume