Final Exam

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EI037 Microeconomics

This is a closed-book exam. You need to solve this exam alone and independently. Your answers should be legible, clear, and concise. In order to get full credit, you have to give complete answers, including how the answers are derived. Partial answers will lead to partial credit. Wrong additional statements (i.e., guessing) might reduce the given credit. The exam is 2 hours. Good luck!

1. Consumption, Production, and Competitive Equilibrium (1.5pt)

Consider an economy where there are 150 consumers, where each has a Cobb-Douglas preference over two goods x and y:

$$U = x^{\frac{1}{2}} y^{\frac{1}{2}}.$$

Each consumer supplies one unit of labor l and owns $\frac{1}{3}$ unit of capital k. Good x is produced using only labor l, with a linear production function:

$$x = 2 \times l.$$

Good y is produced using capital and labor, with a Leontief production function:

$$y = \min\{l, k\}.$$

Denote the prices of labor and capital by w, r, respectively. Denote the prices of x and y by p_x and p_y, respectively. The wage is normalized to equal to 1.

1.a. Solve for the consumers' aggregate demand of x and y as functions of prices p_x, p_y, and income. (Hint: first write down the consumer's income as a function of wage, capital price r, and other known variables.)

Answer: *Given the Cobb-Douglas preferences, consumers allocates half of their income to x and the other half to y. Thus:*

$$D_x = \frac{150w + 50r}{2p_x} \quad ; \quad D_y = \frac{150w + 50r}{2p_y}$$

1.b. Solve for the firms' supply of x and y as a function of p_x, p_y, w, and capital price r. (Hint: the Leontief production function essentially implies that the factors of production are used in fixed proportions.)

Answer: *Given the Leontief production function for y, labor and capital are used in fixed proportions (1:1). By cost minimization, the cost of producing one unit of y is fixed at $w + r$. In a competitive market, producers are indifferent to production levels if $p_y = w + r$. If $p_y < w + r$, production ceases; if $p_y > w + r$, production becomes infinite.*

Similarly, one unit of x is produced using $1/2$ unit of labor, leading to a fixed cost of $w/2$. If $p_x < w/2$, no x will be supplied, and if $p_x > w/2$, supply becomes infinite.
Thus, the supply of x and y can be summarized as follows:

$$S_x = \infty \quad if \quad p_x \geq \frac{w}{2}; \quad S_x = 0 \quad if \quad p_x < \frac{w}{2},$$
$$S_y = \infty \quad if \quad p_y \geq w + r; \quad S_y = 0 \quad if \quad p_y < w + r.$$

1.c. Write down the market clearing condition(s), then find the equilibrium values of p_x, p_y, and r.

Answer: *From 1.a. and 1.b we know that, in equilibrium, we must have that $p_x = \frac{w}{2}$ and $p_y = w + r$. Then market clearing conditions imply*

$$D_x = \frac{150w + 50r}{2p_x} = \frac{150w + 50r}{w} \Rightarrow L_x = 2 * D_x = 2\left(\frac{150w + 50r}{w}\right); \; K_x = 0.$$

where L_x and K_x are the labor demand and capital demand for producing x, respectively. Similarly, we have

$$D_y = \frac{150w + 50r}{2p_y} = \frac{150w + 50r}{2(w + r)} \Rightarrow L_y = D_y = \frac{150w + 50r}{2(w + r)}; \; K_y = D_y = \frac{150w + 50r}{2(w + r)}.$$

Labor and capital market clearing imply that

$$L_x + L_y = 150; \quad K_x + K_y = 50$$

Plugging in the expressions derived above and $w = 1$:

$$K_y = 50 \Rightarrow \frac{150 + 50r}{2(1 + r)} = 50 \Rightarrow \frac{3 + r}{2(1 + r)} = 1 \Rightarrow r = 1$$

The competitive equilibrium is therefore given by $p_x = \frac{w}{2} = \frac{1}{2}$, $p_y = w + r = 2$, $w = 1$ and $r = 1$.

2. Monopoly and Oligopoly Pricing (1.5pt)

Consider one monopolist firm in a market. Its marginal cost of production is 1. The demand curve they face is $q = 10 - p$.

2.a. Find the price, quantity, and profit of the firm in equilibrium.

Answer: *We start with the firm's profit function:*

$$\pi = p(10 - p) - (10 - p)$$

Taking the first-order condition (FOC) of the profit function, we solve for the equilibrium price:

$$FOC: 10 - 2p + 1 = 0 \quad \Rightarrow \quad p = 5.5$$

Substituting this price into the demand curve, we find the equilibrium quantity:

$$q = 10 - p \quad \Rightarrow \quad q = 4.5$$

Finally, substituting the equilibrium price and quantity into the profit function, we get the equilibrium profit:

$$\pi = p(10 - p) - (10 - p) \quad \Rightarrow \quad \pi = 20.25$$

Now assume that there are two firms instead of one firm in the market. They simultaneously and independently select non-negative prices. Let p_1 be firm 1's price and p_2 be firm 2's price. The firms' products are differentiated. After prices are set, consumers demand $5 - p_1 + p_2$ units of the good produced by firm 1 and $5 - p_2 + p_1$ units of the good produced by firm 2. Each firm produces at a marginal cost of production of 1.

2.b. Find the pure-strategy Nash equilibrium of this game.

Answer: *Similar to part 2.a, we begin with the profit function for firm 1:*

$$\pi_1 = p_1(5 - p_1 + p_2) - (5 - p_1 + p_2)$$

Taking the first-order condition (FOC) of the profit function, we solve for firm 1's equilibrium price as a function of firm 2's price. This gives us firm 1's best response function:

$$FOC: 5 - 2p_1 + p_2 + 1 = 0 \quad \Rightarrow \quad p_1 = \frac{6 + p_2}{2}$$

Since this is a symmetric game, in equilibrium $p_1 = p_2$. Substituting $p_2 = p_1$ into firm 1's best response function, we get:

$$p_1 = \frac{6 + p_1}{2} \quad \Rightarrow \quad p_1 = p_2 = 6$$

Thus, the pure strategy Nash equilibrium of the game is $(p_1, p_2) = (6, 6)$. One can then solve for the equilibrium quantity and profit for each firm in a manner similar to 2.a.

2.c. Compare the equilibrium production quantities of one firm in question (a) vs. in question (b). Which one is higher? What is the intuition for this result?

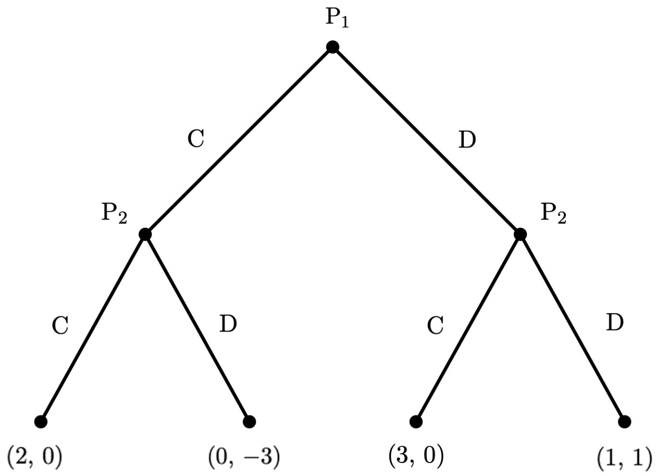
Answer: *Both the equilibrium price and quantity are higher in 2.b compared to 2.a. This is due to a unique feature of the consumer demand: the presence of (expensive) good 2 generates a positive externality for good 1. Specifically, as seen from the demand function, as p_2 increases, consumers want to buy more of good 1. In essence, the presence of good 2 boosts the demand for good 1 and increases its market power and vice versa, leading to higher equilibrium prices and quantities for both goods.*

This type of demand is interesting and not unrealistic. For example, luxury brands like Chanel and Gucci may face similar demand curves for their bags. In case 2.b, consumers also spend more on both goods in equilibrium, assuming goods 1 and 2 make up a small fraction of the consumer's overall consumption bundle.

3. Strategic Interactions (3pt)

Consider the following dynamic game with perfect information (which is a variation of the Prisoners' Dilemma). Two players, P_1 and P_2, sequentially choose to play cooperate (C), or defect (D). In all subsequent questions, unless stated otherwise, you only need to consider pure strategy equilibriums.

Figure 1:



3.a. Find all pure-strategy Nash equilibrium(s) of this game.

Answer:
Player 1 has 2 strategies. They are:

- *C: Play 1 plays C*
- *D: Play 1 plays D*

Player 2 has 4 strategies. They are:

- *CC: Play C if Player 1 plays C, play C if Player 1 plays D*
- *CD: Play C if Player 1 plays C, play D if Player 1 plays D*
- *DC: Play D if Player 1 plays C, play C if Player 1 plays D*
- *DD: Play D if Player 1 plays C, play D if Player 1 plays D*

We can therefore express the game in its normal form as follows:

	Player 2			
Player 1	*CC*	*CD*	*DC*	*DD*
C	$(2,0)$	$(2,0)$	$(0,-3)$	$(0,-3)$
D	$(3,0)$	$(1,1)$	$(3,0)$	$(1,1)$

where each cell contains the payoffs for both players in the format (Player 1's payoff, Player 2's payoff).

Given the actions of the other players, each player's best response can be determined. A Nash Equilibrium (NE) occurs when player 1's and player 2's actions are mutual best responses to each other. Given this definition, we can find out the following two NE:

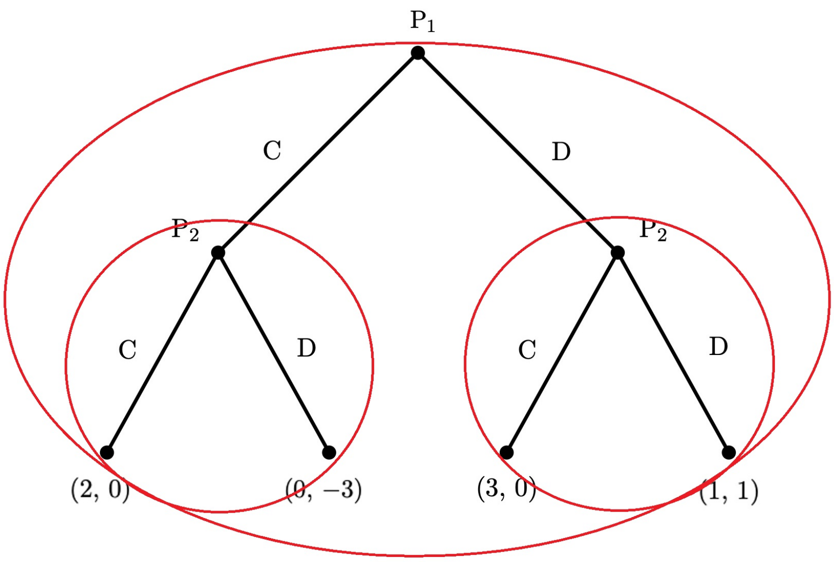
	Player 2			
Player 1	*CC*	*CD*	*DC*	*DD*
C	$(2, \underline{0})$	$(\underline{2}, \underline{0})$	$(0, -3)$	$(0, -3)$
D	$(\underline{3}, 0)$	$(1, \underline{1})$	$(\underline{3}, 0)$	$(\underline{1}, \underline{1})$

- *p1: s=C. p2: play C if p1 play C, play D if p1 play D.* *(or (C, CD))*
- *p1: s=D. p2: play D if p1 play C, play D if p1 play D.* *(or (D, DD))*

3.b. How many subgames does this game have?

Answer: *There are 3 subgames, as circled below*

Figure 2:



3.c. Find all subgame perfect Nash equilibrium(s) of this game.

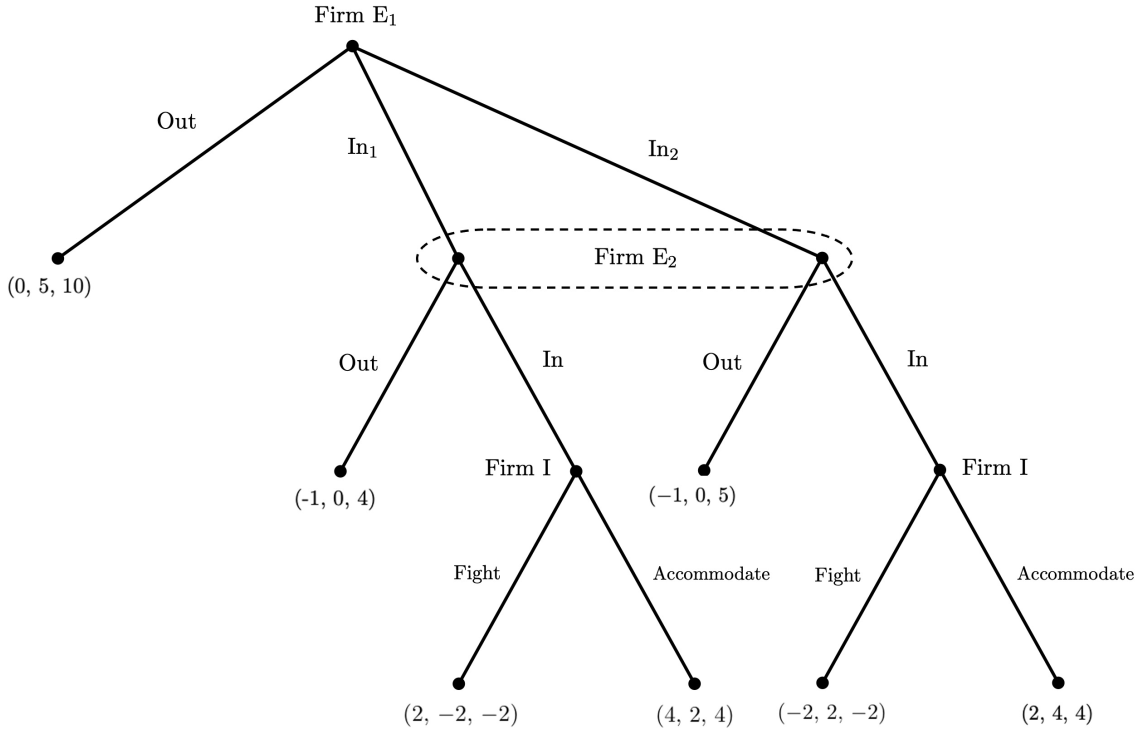
Answer: *To find all SPNE for this game, we can use backward induction.This will yield one SPNE:*

p1: s=C. p2: play C if p1 play C, play D if p1 play D. *(or (C, CD))*

(Note: A strategy is a complete contingent plan. Therefore, it is NOT correct to answer "$p1 : s = C; p2 : s = C$" or "Play 2's strategy to play C if 1 plays C."

Consider the following dynamic game with *imperfect* information.

Figure 3:



3.d. Find the pure-strategy Nash equilibrium(s) of this game.

Answer:

Player 1 (Firm E_1) has 3 strategies. They are:

- *Out, In_1, In_2*

*Player 2 (Firm E_2) cannot distinguish In_1 and In_2 in this game. Therefore, there is only one information set player 2 is called to move. At this information set, she has two possible actions, therefore altogether she has 1*2=2 strategies. They are:*

- *O: Play Out if Player E_1 plays In*
- *I: Play In if Player E_1 plays In*

Player 3 (Firm I) has 2 strategies. They are:

- *Fight, Accomendate*

To find out NE, we need to express the game in its normal form. Since we have 3 players, we can present the game in two matrices.

Firm I plays Fight

	Firm E_2	
Firm E_1	*O*	*I*
Out	$(\underline{0}, \underline{5}, 10)$	$(0, \underline{5}, 10)$
In_1	$(-1, \underline{0}, 4)$	$(\underline{2}, -2, 2)$
In_2	$(-1, 0, 5)$	$(-2, \underline{2}, -2)$

Firm I plays Accomendate

	Firm E_2	
Firm E_1	*O*	*I*
Out	$(\underline{0}, \underline{5}, \mathit{10})$	$(0, \underline{5}, \mathit{10})$
In_1	$(-1, 0, \mathit{4})$	$(\underline{4}, \underline{2}, \mathit{4})$
In_2	$(-1, 0, \mathit{5})$	$(2, \underline{4}, \mathit{4})$

where each cell contains the payoffs for both players in the format (Player 1's payoff, Player 2's payoff, Player 3's payoff).

A Nash Equilibrium (NE) occurs when all three players' actions are mutual best responses to each other. Given this definition, highlight all best reactions in the above matrices, we can find out the following 3 NEs:

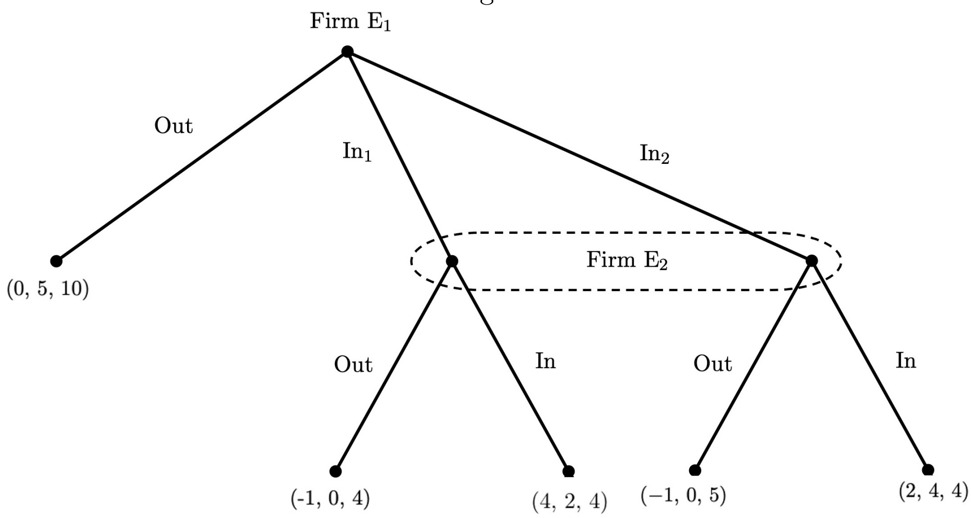
- *(Out, Out, Fight) (Out, Out, Accommodate)*
- *(In_1, In, Accommodate)*

(Notes: First, determine the best responses for Firms E_1 and E_2 when Firm I plays Fight and Accommodate, respectively, as shown with underlines in the tables above. Then, based on the strategies of Firms E_1 and E_2, identify whether Firm I's best response is to play Fight or Accommodate, as highlighted in the tables.)

3.e. Find the subgame perfect Nash equilibrium(s) of this game. How many subgames does this game have?

Answer: *This game has 3 subgames. The full game, and two subgames starting from "Firm I".*
Solve SPNE by backward induction. We can reduce the game to

Figure 4:



Since firm I will always choose "Accommodate" in the last two subgames, the reduced game has only one relevant subgame: itself. Thus, we can use the normal form described in 3.d (i.e., the second matrix) to analyze the game. In that matrix, there is one NE:

- (In_1, In, Accommodate)

These represent the Subgame Perfect Nash Equilibria (SPNE) of the game.

3.f. Find the weak perfect Bayesian equilibrium(s) of this game.

Answer: *From 3.e, we eliminate SPNE that is inconsistent with beliefs. Clearly, if E_2 ever reaches her information set, In is the dominant strategy. So we can eliminate (In_1, IO, Accommodate).*

The Weak Perfect Bayesian Equilibrium (WPBE) also requires that beliefs be consistent with the strategies of players on the equilibrium path. Off the equilibrium path, beliefs can be incorrect, they must align with the strategy profiles.

Note that regardless of E_2's belief about her information set, choosing In is the dominant strategy. Therefore,

- $(In_1, In, Accommodate)$ *with E_2's belief being* $(1, 0)$.

is a WPBE of the game.