Solutions to Problem Set 6

November 13, 2024

Problem 3: New Keynesian Model Analysis

(a)

The three equations system of the NK model is the following:

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa x_t + e_t, \tag{1}$$
$$x_t = \mathbb{E}_t \left(x_{t+1} - \sigma \left(i_t - \pi_{t+1} \right) \right) + v_t, \tag{2}$$
$$i_t = \alpha_1 \pi_t + u_t \tag{3}$$

The shocks e_t and v_t in the Phillips curve and IS equation of the New Keynesian model can be explained in terms of changes to the exogenous variables: Total Factor Productivity (TFP) a_t and the price elasticity of demand ϵ_t. These relationships are given by:

$$e_t = \gamma \mu_t^* \quad \text{and} \quad v_t = \sigma \tilde{r}_t$$

where μ_t^* is the profit-maximizing markup of price on marginal cost, and $\tilde{r}_t$ is the efficient interest rate. These terms are defined as:

$$\mu_t^* = \frac{\epsilon_t}{\epsilon_t - 1} \quad \text{and} \quad \tilde{r}_t = (1 + \bar{r}) \frac{a_{t+1}}{a_t} - 1$$

- **Temporary rise in productivity:** A temporary increase in a_t with no change in a_{t+1} leads to a decrease in $\tilde{r}_t$, resulting in a negative shock to v_t. This affect the output gap by lowering it, since the efficient level of output raises by more than what the actual level of output does. The latter happens because of price stickiness not all the firms can reset their price (down) to exploit this increase in productivity and so output increases just for the firms that are able to reset prices.

- **Permanent rise in productivity:** An increase in both a_t and a_{t+1} leaves $\frac{a_{t+1}}{a_t}$ unchanged, so v_t is unaffected. This implies that also x is unaffected. All else equal a permanent shock to productivity does not affect the output gap (in log-deviations) but it does affect the level of output.

- **Increase in competitiveness:** An increase in ϵ_t lowers μ_t^*, creating a negative shock to e_t.
- **Unexpected interest-rate decision:** An exogenous change in monetary policy constitutes a shock to u_t.

(b)

In the absence of expected future shifts in exogenous variables, assume $x_{t+1} = 0$ and $\pi_{t+1} = 0$ (these refer to expectations of the two variables). Considering only a cost-push shock e_t (set $v_t = 0$), the model reduces to:

$$\pi_t = \kappa x_t + e_t, \quad x_t = -\sigma i_t, \quad i_t = \alpha \pi_t$$

Substituting, we find:

$$x_t = -\alpha\sigma\pi_t, \quad \pi_t = \frac{1}{1+\alpha\kappa\sigma} e_t$$

and the nominal interest rate from the Taylor rule:

$$i_t = \frac{\alpha}{1+\alpha\kappa\sigma} e_t$$

and the output gap:

$$x_t = -\frac{\alpha\sigma}{1+\alpha\kappa\sigma} e_t$$

(c)

We have a new MP rule $i_t = \hat{r}_t + \alpha\pi_t + v_t$. Since we are focusing on the v_t shock in this part, we set $e_t = 0$ and u_t. We know that $v_t = \sigma\hat{r}_t$, where $\hat{r}_t$ is the efficient interest rate (strictly, this is an approximation). The modified Taylor rule is $i_t = \hat{r}_t + \alpha\pi_t$, which can be substituted into the IS equation:

$$x_t = -\sigma(\hat{r}_t + \alpha\pi_t) + \sigma\hat{r}_t = -\sigma\alpha\pi_t$$

Therefore, inflation π_t and the output gap are jointly determined by:

$$\pi_t = \kappa x_t \quad \text{and} \quad x_t = -\alpha\sigma\pi_t$$

The efficient interest rate $\hat{r}_t$ (and the associated shock v_t) do not appear in these equations. It is clear that the solution is $x_t = 0$ and $\pi_t = 0$ for all t even when shocks to the efficient interest rate $\hat{r}_t$ occur. Hence, by following the modified Taylor rule, the central bank can insulate both inflation and the output gap from shocks to the efficient interest rate (such as TFP shocks). Of course, it needs to know the value of $\hat{r}_t$ to put this policy into practice.

(d)

It is not possible to achieve the same result for the cost-push shock e_t. The reason is that this shock occurs in the Phillips curve equation, whereas the efficient interest rate shock v_t is found in the IS equation. The central bank has a policy instrument (the nominal interest rate) which directly affects the IS equation, while it can only affect the Phillips curve indirectly through the effect of interest rates on the output gap x_t. Consequently, the central bank cannot insulate both inflation and the output gap from an e_t shock. The effects of this shock must fall on one or the other (or both) of these variables, so there are trade-offs for the central bank to make depending on its preferences for stabilizing π_t or x_t (more on this later in the course).