Macroeconomics A

Lecture 8 - Consumption

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Introduction and motivation

- consumption is the largest part of GDP $\rightarrow$ around 2/3 for most developed economies
- probably utility and welfare depend the most heavily on consumption
- consumption is a part of a dynamic decision problem $\Rightarrow$ cannot be analyzed in isolation

What we will do

- Analyze economies with **heterogeneous agents**
- Useful for questions pertaining to distribution of resources, inequality, and the effects of policies. Macro models of the last 10 years.

Benchmark consumption model without uncertainty

Assume perfect foresight and no uncertainty. Consumers can save in a risk-free one period bond ⇒ the period t budget constraint is:

$$c_t + b_{t+1} \leq (1 + r_t)b_t + y_t$$

Imagine that they can also "buy" period t consumption in period 0 ⇒ the price of period-t consumption relative to period-0 consumption, for a given price of period 0 consumption $q_0 > 0$, is

$$q_t = \frac{q_0}{(1 + r_1)(1 + r_2) + ...(1 + r_t)}$$

Using this we can then aggregate the per period BCs until T:

$$\sum_{t=0}^{T} q_t c_t + q_T b_{T+1} \leq q_0(1 + r_0)b_0 + \sum_{t=1}^{T} q_t y_t$$

Note: this inter-temporal BC is equivalent to the stream of per-period BCs

Independent of whether

- the agent dies at T
  in which case it is optimal not to leave bequests: $b_{T+1} = 0$
- or the agent lives forever
  in which case we impose that as $T \to \infty$, $q_T b_{T+1} \to 0$

The inter-temporal budget constraint is given by:

$$\sum_{t=0}^{T} q_t c_t \leq q_0(1 + r_0)b_0 + \sum_{t=1}^{T} q_t y_t = q_0((1 + r_0)b_0 + h_0) = q_0 x_0$$

- where $(1 + r_0)b_0$ is the **financial wealth** of the agent at time 0
- $h_0 = \sum_{t=1}^{T} \frac{q_t}{q_0} y_t$ is the **human wealth** of the agent at time 0
- x_0 is the agent's **effective wealth** in period 0

The agent's problem can then be re-written as:

$$\max \sum_{t=0}^{T} \beta^t U(c_t)$$

$$\text{s.t.} \sum_{t=0}^{T} q_t c_t \leq q_0 x_0$$

Note:

- equivalent to the cake-eating problem, as $q_o x_o$ is given at 0
  $\rightarrow$ this is due to the fact that we are not looking at general equilibrium, i.e. the consumption-saving choice does not affect the interest rates
- this can be viewed as a static consumption problem
  $\rightarrow$ we can interpret c_t as different consumption goods, and q_t as the price of these goods
  nevertheless the FOC is the standard Euler equation

Introducing uncertainty

What is the optimal path of consumption if there is uncertainty about the realization of income, y_t?
$\rightarrow$ introduce **idiosyncratic shocks**

Some notation:

- $s_t \in S_t$ is the current state of the economy
- $s^t = \{s_0, s_1, s_2, ...s_t\}$ is the history of the economy up to time t, where $s^t \in S^t \equiv S_0 \times S_1 \times ... \times S_t$
- $\pi(s^t)$ is the probability of history s^t occurring
- $y_t^i(s^t)$ is the income of individual i in period t if history s^t occurred, and $\sum_{i \in I} y_t^i(s^t) = Y_t(s^t)$ is the aggregate income in the economy upon the realization of s^t

Benchmark 1 - Autarky

- endowment economy
- no insurance markets, i.e. you cannot trade across states, $s_t \in S_t$, at a given point in time, t
- no storage technology to transfer resources across time (in our previous notation: $b_t = 0$)

in such a setup it is optimal for the agent to consume all his income in each period:

$$c_t^i(s^t) = y_t^i(s^t)$$

Benchmark 2 - Complete markets

- endowment economy
- full set of insurance markets: agents can trade assets that pay off one unit of consumption if a particular history s^t occured $\Leftarrow$ "Arrow-Debreu securities"
- with S states in each period, T periods, there are $1 + S + S^2 + S^3 + ... + S^T$ markets
- let $p_t(s^t)$ be the price in terms of period-0 consumption of a unit of income (or consumption, i.e. the price of the AD-security) in period t after history s^t

for every individual $i \in I$, the agent's budget constraint at time 0 is:

$$\sum_{t=o}^{T} \sum_{s^t \in S^t} p_t(s^t)(c_t^i(s^t) - y_t^i(s^t)) = 0$$

every transfer of income across **states** and **time** is possible, as long as the expected discounted consumption expenditures equal (or do not exceed) the expected discounted income for each individual

The AD competitive equilibrium

- is an allocation of consumption: $\{c_t^i(s^t)\}_{t=0}^{\infty}$ for all s^t, i
- and a set of prices: $p_t(s^t)$ for all s^t

such that

- given $p_t(s^t)$ for every i $\{c_t^i(s^t)\}_{t=0}^{\infty}$ solves

$$\max \sum_{t=0}^{\infty} \beta^t \sum_{s^t} \pi(s^t) U(c_t^i(s^t))$$

$$\text{s.t.} \quad \sum_{t=o}^{T} \sum_{s^t \in S^t} p_t(s^t)(c_t^i(s^t) - y_t^i(s^t)) = 0$$

- markets clear

$$\sum_i c_t^i(s^t) = Y_t(s^t) \quad \text{for all } s^t \in S^t, t \geq 0$$

Competitive equilibrium

First order conditions: $\forall i, s^t$

$$\beta^t \pi(s^t) U'(c_t^i(s^t)) = p_t(s^t) \lambda^i$$

- **Result 1:** Perfect risk sharing:

$$\frac{U'(c_t^i(s^t))}{U'(c_t^j(s^t))} = \frac{\lambda^i}{\lambda^j}$$

The relative marginal utility does not depend on state/history.

  - Given that

$$\sum_i (U')^{-1}\left(\frac{\lambda^i}{\lambda^j} U'(c_t^j(s^t))\right) = \sum_i c_t^i(s^t) = \sum_i y_t^i(s^t) = Y_t(s^t)$$

    only *aggregate* income matters for *individual* consumption
  - Hence, agents fully insure against idiosyncratic risk (but cannot insure against aggregate risk!)
  - History independence

Competitive equilibrium

First order conditions: $\forall i, s^t$

$$\beta^t \pi(s^t) U'(c_t^i(s^t)) = p_t(s^t)\lambda^i$$

- **Result 2:** Consumption smoothing

$$\beta^{t-\tau} \frac{\pi(s^t) U'(c_t^i(s^t))}{\pi(s^\tau) U'(c_t^i(s^\tau))} = \frac{p_t(s^t)}{p_\tau(s^\tau)}$$

  - Desire to smooth consumption across time and states attenuated by prices and probabilities
  - Marginal rate of substitution = Marginal rate of transformation
  - Can think of state s^t and state s^τ unit of consumption as two different goods, as in micro
  - Hence, outcome is pareto-efficient

Competitive equilibrium = Efficient allocation

- Conditions of the First Welfare Theorem hold $\Rightarrow$ Competitive equilibrium is pareto efficient
- Hence, can be modeled as outcome to a social planner's problem

$$\max_{c_t^i(s^t)} \sum_{t=0}^{T} \sum_{s^t \in S^t} \beta^t \pi(s^t) \sum_{i \in I} \alpha^i U(c_t^i(s^t))$$

$$\text{s.t.} \sum_{i \in I} c_t^i(s^t) = Y_t(s^t) \text{ for all } t \geq 0, s^t \in S^t$$

yields identical outcome to decentralized Arrow-Debreu model when $\alpha^i = \left(\lambda^i\right)^{-1}$.

Empirical implications

both autarky and complete markets benchmarks have empirically testable implications

$\rightarrow$ estimate the following from microdata

$$\Delta \log c_t^i = \beta_1 \Delta \log C_t + \beta_2 \Delta \log y_t^i + \varepsilon_t$$

- autarky implies: $\beta_1 = 0, \beta_2 = 1$
- Arrow-Debreu with CRRA implies: $\beta_1 = 1, \beta_2 = 0$

both hypotheses are rejected

$\Rightarrow$ have to look for a model of partial consumption insurance

The permanent income hypothesis

Assume instead of complete markets that the household can only trade in a **non state-contingent asset**, i.e. they can only buy one-period bonds (which have return r)

$\Rightarrow$ the consumer has an incentive to hold some bonds in order to smooth consumption. BC:

$$b_t + y_t = c_t + \frac{b_{t+1}}{1+r}$$

This is the "standard" setup in the RBC models.
Euler equation:

$$u'(c_t) = \beta(1+r)E_t\left(u'(c_{t+1})\right)$$

Make two additional assumptions ($\Rightarrow$ strict version of the PIH)

1. quadratic preferences: $U(c) = a_1 c - \frac{1}{2}a_2 c^2$
2. the interest rate on the bond equals the inverse of the discount rate: $\beta(1+r) = 1$

Then the Euler equation becomes

$$a_1 - a_2 c_t = \beta(1+r)E_t(a_1 - a_2 c_{t+1}) \;\;\Rightarrow\;\; E_t c_{t+1} = c_t$$

$\Rightarrow$ consumption is a martingale
Law of iterated expectations yields $E_t c_{t+j} = c_t$ for all $j \geq 0$

using the budget constraint:

$$c_t = y_t + b_t - \frac{1}{1+r} E_t b_{t+1} = y_t + b_t - \frac{1}{1+r} E_t \left(c_{t+1} - y_{t+1} + \frac{b_{t+2}}{1+r} \right)$$

re-arranging:

$$c_t + \frac{E_t c_{t+1}}{1+r} = b_t + y_t + \frac{E_t y_{t+1}}{1+r} - \frac{E_t b_{t+2}}{(1+r)^2}$$

keep substituting out the latest asset holding to get:

$$\sum_{j=0}^{\infty} \left(\frac{1}{1+r} \right)^j E_t c_{t+j} = b_t + \underbrace{\sum_{j=0}^{\infty} \left(\frac{1}{1+r} \right)^j E_t y_{t+j}}_{\equiv h_t}$$

$$c_t = \frac{r}{1+r} \left(b_t + h_t \right)$$

consumption today only depends on the annuity value of income, on the **permanent income**

Consumption dynamics

innovations in consumption:

$$
\begin{aligned}
\Delta c_t &\equiv c_t - c_{t-1} = c_t - E_{t-1} c_t \\
&= \frac{r}{1+r} \left(b_t - E_{t-1} b_t + \sum_{j=0}^{\infty} \left(\frac{1}{1+r} \right)^j \left(E_t y_{t+j} - E_{t-1}(E_t y_{t+j}) \right) \right) \\
&= \frac{r}{1+r} \sum_{j=0}^{\infty} \left(\frac{1}{1+r} \right)^j \left(E_t y_{t+j} - E_{t-1} y_{t+j} \right)
\end{aligned}
$$

the change in consumption between period $t-1$ and t is due to the **revision of permanent income**, and it is proportional to the revision of expected earnings due to information that arrived in the same period

Implications so far

- consumption is a random walk

$$E_{t-1}\Delta c_t = 0$$

- there is no insurance, just consumption smoothing $\Rightarrow$ minimize Δc
- the marginal propensity to consume from wealth is

$$\frac{r}{1+r}$$

- the marginal propensity to consume from an innovation to current income depends on the persistence of the income process
  $\rightarrow$ lower persistence $\Rightarrow$ smaller reaction in consumption to current income changes

Testing the PIH using macro data

- Hall (1978): If

$$c_t = E_t c_{t+1}$$

  then no current information should be able to predict consumption growth (at least, if this information is in the agents' information set)
  - Rejects the PIH: you can predict consumption growth
- Wilcox (1989): increases in Social Security benefits pre-announced
  - PIH: consumption should spike when benefits increase is announced, not when implemented
  - Rejects PIH.
  - Possible reason: liquidity constraints: households can save in anticipation of shocks, but can't borrow against them

Testing the PIH using micro data

- Zeldes (1989) tests for liquidity constraints using PSID data
  - PIH seems to hold better for "rich" households, but not for "poor" households
  - Verifies the PIH with liquidity constraint modification?
  - Perhaps – or maybe poor households more myopic
- Shea (1995) matches PSID data to union contracts
  - Looks at cases where decrease in wages are pre-announced
  - Households do not show increase in savings in anticipation of consumption decline
  - $\Rightarrow$ PIH violation not due to liquidity constraints

With more general utility function, certainty equivalence breaks (prudence: $u''' > 0$ means increases in variance of income increases saving)

With more general utility function

break certainty equivalence: savings react to changes in income uncertainty under certain conditions

Two-period prudence model
Consider the following problem

$$\max_{c_0, c_1, b_1} u(c_0) + \beta E(u(c_1))$$

$$\text{s.t. } c_0 + b_1 = y_0$$

$$c_1 = Rb_1 + \widetilde{y}_1$$

- y_0, income in initial period is given
- $\widetilde{y}_1$, income in the next period, is exogenous, stochastic
- assume: $\beta R = 1$

the Euler equation is:

$$u'(y_o - b_1) = E(u'(Rb_1 + \widetilde{y}_1))$$

- b_1 is the only unknown
- under the usual concavity assumption, $u'' < 0$
  $\Rightarrow$ lhs is increasing in b_1
  while the rhs is decreasing
- $\Rightarrow$ b_1^* uniquely determined
- $c_0^* = y_0 - b_1^*$, current consumption is decreasing in savings

What is the effect of a mean-preserving spread?

assume that uncertainty over next period income increases, but the mean stays the same

$$\widetilde{y}_1 = \overline{y}_1 + \varepsilon$$

where $\overline{y}_1$ is the mean, and ε is stochastic: $E(\varepsilon) = 0, Var(\varepsilon) = \sigma_\varepsilon$
the EE becomes:

$$u'(y_o - b_1) = E(u'(Rb_1 + \overline{y}_1 + \varepsilon))$$

a mean preserving spread of ε

- if u' is convex $\Rightarrow b_1^*$ increases, and c_0^* falls
- if u' linear $\Rightarrow b_1^*$, c_0^* does not change
- if u' is concave $\Rightarrow b_1^*$ decreases, and c_0^* increases

- introspection: b_1^* increases $\Rightarrow u'$ is convex, $u''' > 0$
- convex u' unavoidable if we consider that $u'(c) > 0$, $u''(c) < 0$ and $c \geq 0 \Rightarrow u'(c)$ strictly convex near 0 and ∞
- $\Leftrightarrow$ the agent is said to be **prudent**

prudence is an additional motive for saving, to shield from possible negative realizations of the income shock next period $\rightarrow$ precautionary savings or self-insurance

Saving motives

1. inter-temporal motive present if $\beta R > 1 \Rightarrow$ pushes the individual to postpone consumption
2. smoothing motive present even if $\beta R = 1$ and quadratic utility (i.e. uncertainty not important) $\Rightarrow$ individual wants to smooth consumption (cf. autarky!)
3. precautionary or self-insurance motive present shield against future income shocks
4. life-cycle motive present if earnings vary over the life cycle, i.e. saving for retirement

Borrowing constraints in a stochastic model

Consider the following problem

$$\max_{\{c_t, b_{t+1}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t U(c_t)$$

$$\text{s.t.} \quad c_t + b_{t+1} = (1+r)b_t + y_t$$

$$\text{and} \quad b_{t+1} \geq 0$$

using multiplier $\beta^t \lambda_t \geq 0$ for the borrowing constraint of time t the FOC is:

$$u'(c_t) = \beta(1+r)E_t u'(c_{t+1}) + \lambda_t \Rightarrow u'(c_t) \geq \beta(1+r)E_t u'(c_{t+1})$$

How to solve this in practice

We're coming (once again) to the point where we cannot solve our models analytically. This means we need to go to the computer...

- $+$ Perhaps the most useful thing you'll learn in this course (if you end up doing macro/IO/structural labor/anything where you really care about quantitative results)
- $-$ we need to program; this time there's no Dynare around to do the work for us

Many different solution techniques. We're going to focus on one: value function iteration

Value function iteration

Start with a simple example (that we could solve analytically – good to check solution)

- Partial equilibrium, $R = R_t = 1.05$
- $\beta = 0.9$, $u(c) = \log(c)$
- No risk: $y_t = 1$, saving through one-period bond b_t, no borrowing constraint.

Problem is

$$\max \sum_{t=0}^{\infty} \beta^t \log(c_t)$$

subject to

$$c_t + b_t = y_t + Rb_{t-1}$$

in every period.

Value function iteration

Idea of the algorithm is to write down the Bellman equation and solve for the value function:

$$V(w_t) = \max_{c_t} \left(u(c_t) + \beta V(R(w_t - c_t) + y_{t+1}) \right)$$

where $w_t \equiv y_t + Rb_{t-1}$.

- Here, $V(\cdot)$ is given recursively (it depends on itself)
- If we knew $V(\cdot)$, we could simply solve the maximization problem for the optimal consumption level c_t (for every wealth level w_t). Unfortunately, we don't know $V(\cdot)$
- But we can start with a guess for the value function, solve for the optimal c_t for every wealth level, and use the Bellman equation to update the guess for the value function:

Value function iteration: the algorithm

Algorithm:

1. Set $\tilde{V}_0(\cdot)$ to some reasonable starting guess (i.e. increasing, concave)
2. Update:

$$\tilde{V}_j(w_t) \equiv \max_{c_t} \Big(u(c_t) + \beta \tilde{V}_{j-1}(R(w_t - c_t) + y_{t+1}) \Big)$$

3. Stop when $\tilde{V}_j$ and $\tilde{V}_{j-1}$ are very similar:

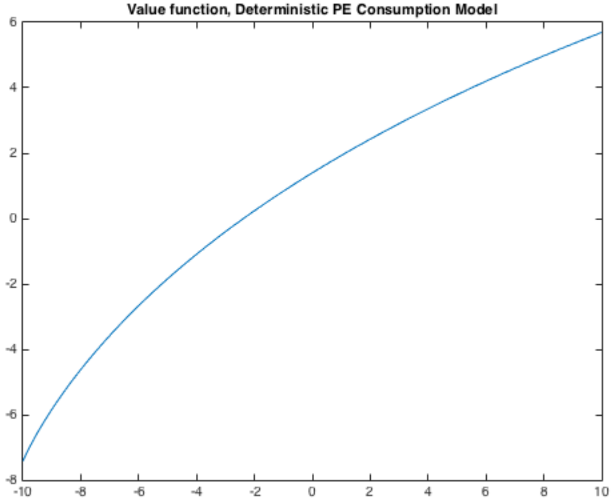
$$||\tilde{V}_j - \tilde{V}_{j-1}|| < \epsilon$$

If some assumptions are satisfied (e.g. the updating procedure is a contraction mapping in the Banach space of bounded continuously differentiable functions on a convex set) then this procedure will converge to a unique solution, which is the actual value function.

Mathematical details: Stokey, Lucas, Prescott book

In Practice

- In order to implement this on the computer, you need to parameterize the current guess for the Value function. Typically, this is done by calculating the function value for w on a grid/lattice, and interpolating it in between the grid points.
- You also need to say what happens outside the grid. Since this will generally be a bad approximation, the solution will probably be more accurate in the "interior" of the grid, than on the "edges".
- In each updating step, you solve, for every value w_t on the grid, the maximization problem in c
- Starting guess for value function is important.


Value function, Deterministic PE Consumption Model
6
4
2
0
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-4
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-8
-10
-8
-6
-4
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2
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6
8
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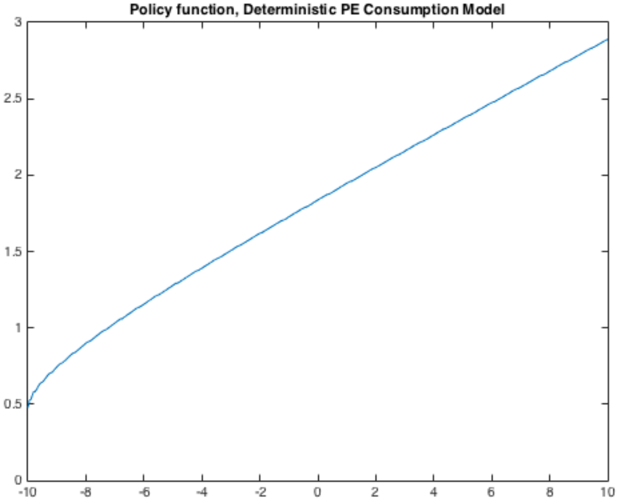

Policy function, Deterministic PE Consumption Model
3
2.5
2
1.5
1
0.5
0
-10
-8
-6
-4
-2
0
2
4
6
8
10

Making this slightly more complicated

So far, everything has been deterministic (we had $y_t = 1$ in every period). Now let's make it stochastic by assuming that y_t is stochastic and i.i.d over time (with distribution with bounded support).

Bellman equation becomes

$$V(w_t) = \max_{c_t} \left(u(c_t) + \beta E_t V(R(w_t - c_t) + y_{t+1}) \right)$$

- Value function iteration works as before. Only complication is that we have the expectation operator in there. Remember that

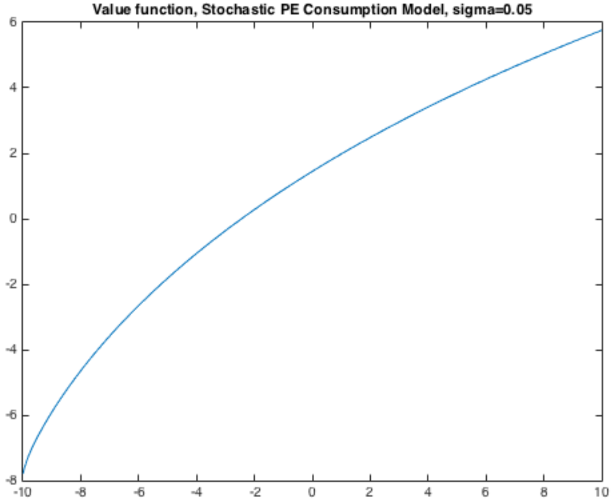
  $$E(g(X)) = \int g(X) dX$$

  hence

$$E_t V(R(w_t - c_t) + y_{t+1}) = \int V(R(w_t - c_t) + y) f(y) dy$$

In Practice

- Most software packages for numerical analysis have libraries for numerical integration ("quadrature")
  - `quad()`, `quadl()`, etc in MATLAB
  - `quadgk()` in Julia
  - QUADPACK/netlib from Fortran, or if you really need high performance
- All numerical integration algorithms *approximate* the integral (think: Riemann sums). Errors may get compounded in the value function iteration procedure; need to set error tolerance levels sufficiently low.
- If you want to make the program run really fast, you can use the grid structure for V to calculate the integral yourself
- If you want to make it more precise, you can pick the grid points optimally ("chebyshev nodes")


Value function, Stochastic PE Consumption Model, sigma=0.05
6
4
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-10
-8
-6
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-2
0
2
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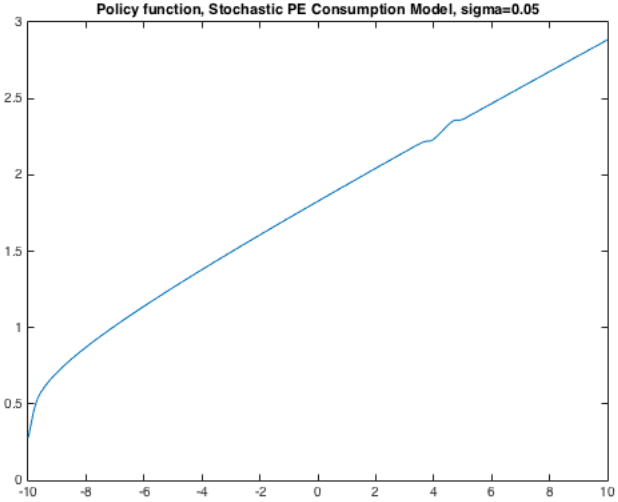
Policy function, Stochastic PE Consumption Model, sigma=0.05
3
2.5
2
1.5
1
0.5
0
-10
-8
-6
-4
-2
0
2
4
6
8
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Buffer-stock saving model

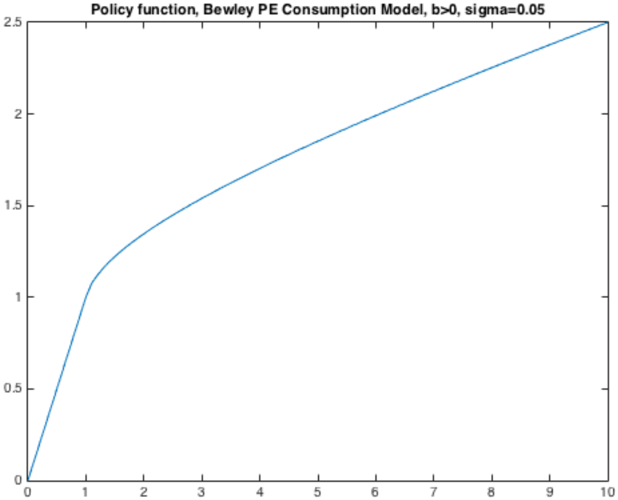
From here it's easy to go to a model with the borrowing constraint. Take the model with stochastic income, and add a borrowing constraint:

$$b_t \geq 0 \qquad \forall t.$$

Then the Bellman equation becomes

$$V(w_t) = \max_{c_t \leq w_t} \left(u(c_t) + \beta E_t V(R(w_t - c_t) + y_{t+1}) \right)$$

- Easy. The only difference is a bound constraint on the numerical optimization for c.
- We're also eliminating the need to consider $w_t < 0$, can adjust the grid for w accordingly.


Policy function, Bewley PE Consumption Model, b>0, sigma=0.05
0
0.5
1
1.5
2
2.5
0
1
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3
4
5
6
7
8
9
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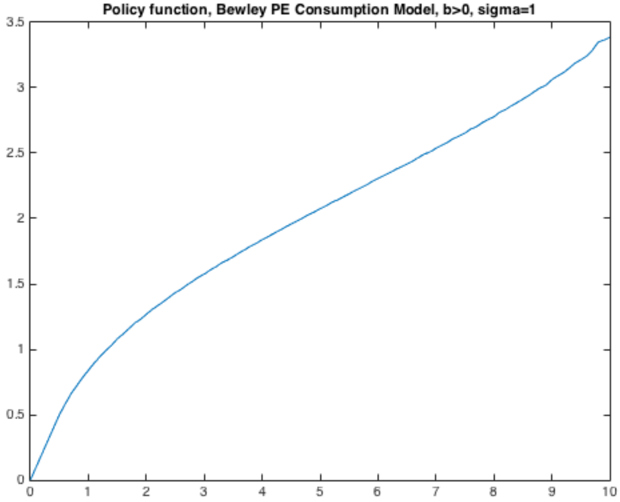
Policy function, Bewley PE Consumption Model, b>0, sigma=1
3.5
3
2.5
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1.5
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0.5
0
0
1
2
3
4
5
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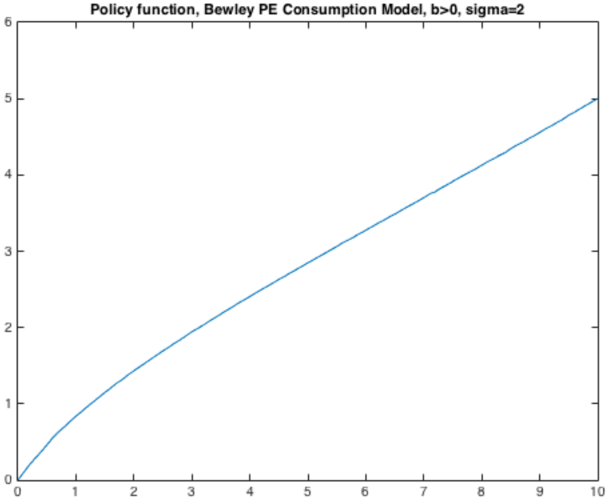
Policy function, Bewley PE Consumption Model, b>0, sigma=2
0
1
2
3
4
5
6
0
1
2
3
4
5
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7
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Lessons

With borrowing constraints:

- If agents' wealth is low, they're consuming most of what they have
- However, when they're rich, they're saving more for bad times
- Buffer-stock saving behavior

Consumption response to an income shock: experimental evidence

All these models have a prediction for the consumption response (marginal propensity to consume, MPC) to an unexpected one-off income shock.

The size of this response matters for the ability of fiscal policy to stimulate demand.

Sizable literature (in particular by Jonathan Parker and co-authors) to estimate average MPC following stimulus checks in the US. Some econometric problems (see Borusyak et al., 2023).

Boehm, Fize, Jaravel (2025) estimate the consumption response to a one-off unexpected transfer of 300 eur among French households.

- "Clean" estimates, can estimate distribution of MPC

BFJ (2025): Setup

Collaboration with French retail bank Credit Mutuel Alliance Federale

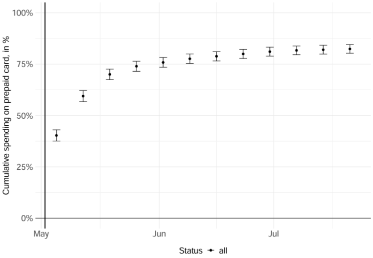
- Access to (anonymized) panel of household transactions
- Restrict to those that only bank with CM (about 90k hh's)
- $\sim$ 1,000 households receive transfer of 300 eur

Three treatment groups:

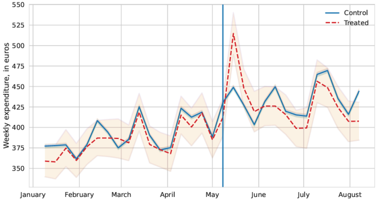
- Group 1: 300 eur on pre-paid Mastercard, no restrictions
- Group 2: Expiry date after three weeks
- Group 3: 10% negative interest rate every week on unspent balances

Compare trajectory of consumption expenditures to control group (no transfer). Study how consumption response depends on household characteristics.

A. Cumulative Spending on Prepaid Card

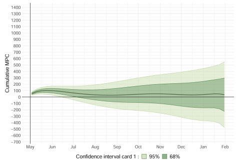


B. Average Total Spending in the Raw Data, Weekly

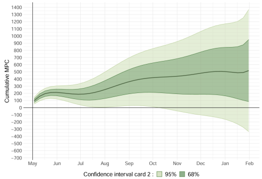


- Significantly higher consumption expenditure (treatment card + all other cards/cash) following transfer for (pooled) treatment group compared to control group.
- You give people money, people spend money.
- But did the expiry/negative interest rate do anything?

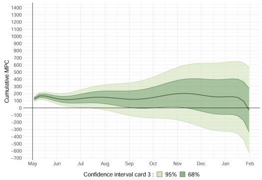
A. Group 1, no restrictions on treatment card



B. Group 2, expiration after three weeks



C. Group 3, negative rates every week



- One-month MPC: G1: 23% of 300 eur, G2: 61% of 300, G3: 36% of 300
- Inconsistent with rationality: almost every household has consumption expenditures of $>$ 300 eur over three weeks. Expiry date of G2 should not bind.
- G1 Consumption response much higher than what you'd get from standard PIH consumers
- Consumption response slightly higher for hh's with low liquid assets ($\rightarrow$ constrained in borrowing?), but still very high even for hh's with plenty of liquid assets.
- Lots of challenges for theory.