International Finance
Lecture III: Open Economy RBC

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Motivation

- In the previous chapter, we built a model of the open economy driven by productivity shocks and argued that it can capture the observed countercyclicality of the trade balance.
- We also established that two features of the model are important for making this prediction possible.
  - First, productivity shocks must be sufficiently persistent.
  - Second, capital adjustment costs must not be too strong.
- Here, we ask more questions about the ability of that model to explain observed business cycles.
- In particular, we ask whether it can explain the sign and magnitude of business-cycle indicators, such as the standard deviation, serial correlation, and correlation with output of output, consumption, investment, the trade balance, and the current account.

The Small Open Economy RBC Model

To make the models studied in chapters 2 and 3 more empirically realistic and to give them a better chance to account for observed business-cycle regularities add:

1. Endogenous labor supply and demand
2. Uncertainty in the technology shock process
3. Capital depreciation

The resulting theoretical framework is known as the Small Open Economy Real-Business-Cycle model, or, succinctly, the SOE-RBC model.

We build on Enrique Mendoza's seminal paper: "Real Business Cycles in a Small Open Economy," AER 1991

Utility Function

Infinite number of identical HHs with the following preferences:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, h_t) \tag{4.1}$$

c is consumption and h hours worked and $U_c > 0$ and $U_h < 0$

Period Budget Constraint

$$c_t + i_t + \Phi(k_{t+1} - k_t) + (1 + r_{t-1})d_{t-1} = y_t + d_t \tag{4.2}$$

i_t is gross investment (now, we will have depreciation)

$\Phi(k_{t+1} - k_t)$ are capital adjustment cost, $\Phi(0) = \Phi'(0) = 0$; $\Phi''(0) > 0$

The restrictions $\Phi(0) = \Phi'(0)$ ensure that in steady state adjustment costs are nil and that the relative price of capital goods in terms of consumption goods is unity

Production, Law of Motion of Capital, & NPG

Linearly homogeneous production function that takes capital and labor services as inputs:

$$y_t = A_t F(k_t, h_t) \tag{4.3}$$

A_t is an exogenous and stochastic productivity shock. This is the only source of aggregate fluctuations in the model

The stock of capital evolves according to:

$$k_{t+1} = (1-\delta)k_t + i_t \tag{4.4}$$

δ is depreciation of physical capital

We also have the usual NPG constraint:

$$\lim_{j\to\infty} E_t \frac{d_{t+j}}{\prod_{s=0}^{j}(1+r_s)} \leq 0 \tag{4.5}$$

The Household's Maximization Problem

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, h_t) \tag{4.1}$$

subject to

$$c_t + i_t + \Phi(k_{t+1} - k_t) + (1 + r_{t-1})d_{t-1} = y_t + d_t \tag{4.2}$$

$$y_t = A_t F(k_t, h_t) \tag{4.3}$$

$$k_{t+1} = (1 - \delta)k_t + i_t \tag{4.4}$$

$$\lim_{j \to \infty} E_t \frac{d_{t+j}}{\prod_{s=0}^{j}(1 + r_s)} \leq 0 \tag{4.5}$$

Additions/differences to the model analyzed in Chapter 3

- Endogenous labor supply, $U(c_t, h_t)$
- Endogenous labor demand, $F(k_t, h_t)$
- Uncertainty, A_t is stochastic
- The interest rate is no longer constant, $r_t \neq r$
- Depreciation, δ no longer 0

Maximization Problem

Use the production function (4.3) and the law of motion of capital (4.4) to eliminate y_t and i_t from the period budget constraint and obtain

$$d_t = (1 + r_{t-1})d_{t-1} - A_t F(k_t, h_t) + c_t + k_{t+1} - (1 - \delta)k_t + \Phi(k_{t+1} - k_t)$$

Write the Lagrangian:

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \{U(c_t, h_t) +$$

$$+ \lambda_t [A_t F(k_t, h_t) + (1 - \delta)k_t + d_t - c_t - (1 + r_{t-1})d_{t-1} - k_{t+1} - \Phi(k_{t+1} - k_t)]\}.$$

Household's Optimality Conditions

The first order conditions wrt λ_t, d_t, c_t, h_t, and k_t are:

$$c_t + k_{t+1} - (1-\delta)k_t + \Phi(k_{t+1} - k_t) + (1 + r_{t-1})d_{t-1} = A_t F(k_t, h_t) + d_t \tag{4.6}$$

$$\lambda_t = \beta(1 + r_t)E_t\lambda_{t+1} \tag{4.7}$$

$$U_c(c_t, h_t) = \lambda_t \tag{4.8}$$

$$-U_h(c_t, h_t) = \lambda_t A_t F_h(k_t, h_t) \tag{4.9}$$

$$1 + \Phi'(k_{t+1} - k_t) = \beta E_t \frac{\lambda_{t+1}}{\lambda_t}\left[A_{t+1}F_k(k_{t+1}, h_{t+1}) + 1 - \delta + \Phi'(k_{t+2} - k_{t+1})\right] \tag{4.10}$$

Labor demand and labor supply

Optimality conditions (4.6), (4.7), (4.8) and (4.10) are similar to what we had before

Condition (4.9) equates the demand of labor with the supply of labor.

Take Equation (4.9) and divide by (4.8) to eliminate λ_t and you get

$$-\frac{U_h(c_t, h_t)}{U_c(c_t, h_t)} = A_t F_h(k_t, h_t) \tag{4.11}$$

- The LHS of this expression is the household's labor supply schedule (it's the marginal rate of substitution between leisure and consumption, this is increasing in hours worked holding the level of consumption constant)
- The RHS is the marginal product of labor. This is decreasing in labor, holding capital constant

Productivity

The law of motion of the productivity shock is assumed to be a first order AR process

$$\ln A_{t+1} = \rho \ln A_t + \widetilde{\eta}\epsilon_{t+1} \tag{4.12}$$

Where ϵ is white noise i.i.d with mean zero and unit standard deviation, the parameter $\tilde{\eta}$ is the standard deviation of innovations to productivity and the parameter $\rho \in (-1, 1)$ dictates the serial correlation of the productivity shock.

With these assumptions, the expected value of productivity at time $t + 1$ conditional on information available at time t is given by:

$$E_t \ln A_{t+1} = \rho \ln A_t$$

and

$$E_t \ln A_{t+j} = \rho^j \ln A_t$$

Hence, $\ln A_t$ converges to zero at rate ρ

Inducing Stationarity: External debt-Elastic Interest Rate (EDEIR)

* In the previous models we saw that equilibrium with one internationally traded bond and $(1+r)\beta = 1$ gives us a random walk for d_t, c_t, and the trade balance.
* Under prefect foresight, the model predicts that the SS levels of d_t, c_t, and the trade balance depend on initial conditions such as the initial level of debt itself.
* This does not mean that the SS is indeterminate, but it means that the SS is history dependent
* The fact that these variables are non-stationary complicates the task of approximating equilibrium dynamics.
* Available numerical approximation techniques requires stationarity of the state variables
* USG induce stationarity by making the interest rate debt elastic

Inducing Stationarity: External debt-Elastic Interest Rate (EDEIR)

$$r_t = r^* + p(\widetilde{d}_t) \tag{4.14}$$

r^* = constant world interest rate (assumed to be constant for simplicity)
$\tilde{d}_t$ = cross-sectional average of debt (HHs take it as exogenous)
$p(\tilde{d}_t)$ = country interest-rate premium (spread).
$p(\cdot)$ is a strictly increasing function: the interest rate is increasing in average debt

* This condition induces stationarity because, as debt levels increase, so does interest rate, this leads to an increase in savings that curbs debt growth (substitution effect dominates income effect).
* This assumption is in line with what we observe in EM where interest rates tend to increase with the level of external debt.
* From a theoretical point of view, this is driven by the presence of financial frictions

Since HHs are identical, equilibrium average debt equals individual debt:

$$\widetilde{d}_t = d_t \tag{4.15}$$

Equilibrium Conditions

Using Equations (4.8), (4.14) and (4.15) to eliminate λ_t, r_t, and $\widetilde{d}_t$ from (4.5)-(4.7) and (4.10), we get

$$d_t = c_t + k_{t+1} - (1-\delta)k_t + \Phi(k_{t+1} - k_t) + [1 + r^* + p(d_{t-1})]d_{t-1} - A_t F(k_t, h_t) \tag{4.16}$$

$$U_c(c_t, h_t) = \beta(1 + r^* + p(d_t))E_t U_c(c_{t+1}, h_{t+1}) \tag{4.17}$$

$$1 = \beta E_t \left\{ U_c(c_{t+1}, h_{t+1}) \frac{[A_{t+1} F_k(k_{t+1}, h_{t+1}) + 1 - \delta + \Phi'(k_{t+2} - k_{t+1})]}{U_c(c_t, h_t)[1 + \Phi'(k_{t+1} - k_t)]} \right\} \tag{4.18}$$

An equilibrium is a set of processes d_t; c_t; h_t; k_{t+1}; A_t satisfying (4.11), (4.12) and (4.16)-(4.19), given A_0, d_{-1}, k_0 and the process $[\epsilon_t]_{t=0}^{\infty}$

For capital, we have a second-order difference equation as it features k_t, k_{t+1} and k_{t+2}. We would like to have a system of first-order difference equations. To this end, introduce the auxiliary variable k_t^f and impose

$$k_t^f = k_{t+1}$$

Note that k_t^f is in the information set of period t.

Recall that the trade balance and the current account are defined as:

The Trade Balance

$$tb_t \equiv y_t - c_t - i_t - \Phi(k_{t+1} - k_t) \tag{4.20}$$

The Current Account

$$ca_t \equiv tb_t - r_{t-1}d_{t-1} \tag{4.21}$$

We could also write the CA as the change in net foreign assets

$$ca_t = d_{t-1} - d_t$$

Equilibrium Conditions

- $k_t^f = k_{t+1}$ together with (4.11) and (4.16)-(4.18) form a system of non-linear stochastic first-order difference equations in the 5 unknowns: c_t, h_t, d_{t-1}, k_t, and k_t^f.
- This system of equations does not have a closed form solution.
- One way to proceed is to use numerical techniques to find a first-order accurate approximate solution around the nonstochastic steady state.
- This is a local approximation
- Another possibility is to use a global solution method.
- We will not go into the details of these techniques (sections 4.6 and 4.13 of the book)

Decentralized Economy

- So far we assumed that the household decides everything
- We could build an alternative decentralized economy in which these activities are performed in the marketplace:
  - HHs consume, supply labor, and own capital which they rent to firms.
  - HHs takes wages (w_t), profits (π_t), and share (s_t) prices (p_t) as exogenously given (because they are atomistic)
  - In this economy, there are two types of firms: firms that produce consumption goods and firms that produce investment goods
  - The decentralized economy is useful because it provides predictions for the equilibrium behavior of relative prices like the real wage, the rental rate of capital, and the value of the stock market.
  - It yields the usual results that wage=MPL and rental rate of capital=MPK
- The decentralized economy is described in Section 4.2 of the book.
- It yields the same equilibrium behavior of the centralized economy

→ First fundamental theorem of welfare economics: in economic equilibrium, a set of complete markets, with complete information, and perfect competition, will be Pareto optimal

- We now need functional forms for the utility function, production function, capital adjustment cost function, and country interest rate premium

Functional Forms

Period utility function is Greenwood, Hercowitz, and Huffman (GHH) with Constant Relative Risk Aversion (CRRA)

$$U(c,h) = \frac{\left(c - \omega^{-1} h^{\omega}\right)^{1-\sigma} - 1}{1-\sigma}; \quad \omega > 1; \sigma > 0$$

Debt-elastic interest rate (country premium is an increasing and convex function of average external debt)

$$p(d) = \psi \left(e^{d-\bar{d}} - 1\right); \quad \psi > 0$$

Production function is Cobb Douglas

$$F(k,h) = k^{\alpha} h^{1-\alpha}; \quad \alpha \in (0,1)$$

Adjustment cost function is quadratic

$$\Phi(x) = \frac{\phi}{2} x^2; \quad \phi > 0$$

6 structural parameters: σ, ω, ψ, $\bar{d}$, α, ϕ

Period utility function

$$U(c,h) = \frac{\left(c - \omega^{-1} h^{\omega}\right)^{1-\sigma} - 1}{1-\sigma}; \quad \omega > 1; \sigma > 0$$

Another way to write this is

$$U(c,h) = \frac{G(c,h)^{1-\sigma} - 1}{1-\sigma}$$

with $G(c,h) = c - \frac{h^{\omega}}{\omega}$

- The subindex $G(c,h)$ is called the Greenwood, Hercowitz, and Huffman (GHH) utility function.
- GHH preferences imply that labor supply (ie the marginal rate of substitution between leisure and consumption) is independent of the level of consumption.
- Under GHH preferences: $h_t^{\omega-1} = w_t$, The labor supply has a wage elasticity of $\frac{1}{\omega-1}$ and is independent of c_t
- First introduced by Mendoza (1991), GHH preferences are useful because they allow employment to be procyclical (if not wealth effect may lead persistent productivity shocks would to cause a decrease in employment)
- $U(c,h)$ display constant relative risk aversion (CRRA) over the sub-utility index $G(c,h)$. The parameter σ measures the degree of relative risk aversion and $\frac{1}{\sigma}$ is the inter-temporal elasticity of substitution

CRRA: Reminder

The **Relative Risk Aversion (RRA)** is a measure of how risk-averse an individual is, *relative* to their wealth or consumption level. It is defined as:

$$RRA(c) = -\frac{c \cdot U''(c)}{U'(c)}$$

Where:

- $U'(c)$ is the **first derivative** of the utility function, representing the **marginal utility** of consumption or wealth,
- $U''(c)$ is the **second derivative** of the utility function, representing how marginal utility changes as consumption or wealth changes,
- c is the level of consumption or wealth.
  - The first derivative of the utility function with respect to consumption represents the additional utility (satisfaction) derived from a small increase in wealth or consumption.
  - The second derivative of the utility function with respect to consumption represents how marginal utility changes as wealth or consumption changes.
  - For a risk-averse individual, marginal utility decreases as consumption increases, implying $U''(c) < 0$.

The formula has several key components:

- **The ratio** $\frac{U''(c)}{U'(c)}$: This measures how fast the marginal utility is decreasing due to the concavity of the utility function. A faster drop in marginal utility implies higher risk aversion, as the individual values additional wealth less and is more inclined to avoid potential losses.
- **Multiplying by wealth** c: This adjusts the measure to the individual's current level of wealth. It scales the risk aversion according to how wealthy the person is, providing a **relative** measure of risk aversion.
- **Negative sign**: Since $U''(c)$ is negative for risk-averse individuals, the negative sign ensures that the measure of risk aversion is positive, reflecting the degree of risk aversion correctly.

Interpretation of RRA

- $RRA(c) = 0$: This indicates a person is **risk-neutral**. They are indifferent to risk and will always choose the option with the highest expected return, regardless of the risk.
- $RRA(c) > 0$: This indicates a person is **risk-averse**.
- $RRA(c) < 0$: This indicates a person is **risk-seeking**.

Example of CRRA:

With CRRA, a person's relative risk aversion remains constant regardless of changes in their wealth level.
The standard form of the Constant Relative Risk Aversion (CRRA) utility function is given by:

$$U(c) = \frac{c^{1-\rho}}{1-\rho}, \quad \rho \neq 1$$

where:

- c is the level of consumption (or wealth),
- ρ is the coefficient of relative risk aversion.

For the special case when $\rho \Rightarrow 1$, the utility function takes the logarithmic form:

$$U(c) = \log(c)$$

dividing by zero!!!!!!! (remember l'Hôpital's Rule)

Example of CRRA:

$$RRA(c) = -\frac{c \cdot U''(c)}{U'(c)}$$

We will now calculate $U'(c)$ and $U''(c)$ for the CRRA utility function $U(c) = \frac{c^{1-\rho}}{1-\rho}$.
The first derivative is:

$$U'(c) = \frac{d}{dc}\left(\frac{c^{1-\rho}}{1-\rho}\right) = c^{-\rho}$$

The second derivative is:

$$U''(c) = \frac{d}{dc}\left(c^{-\rho}\right) = -\rho c^{-\rho-1}$$

Using the definition of RRA, we get:

$$RRA(c) = -\frac{c \cdot \left(-\rho c^{-\rho-1}\right)}{c^{-\rho}}$$

Simplifying:

$$RRA(c) = -\left(-\rho\right) \cdot \frac{c^{-\rho-1} \cdot c}{c^{-\rho}} = \rho$$

Relative versus Absolute Risk Aversion

Absolute Risk Aversion (ARA) measures how an individual's risk aversion changes in absolute terms, that is, how their risk tolerance varies for a fixed amount of risk, regardless of their wealth. The formula for ARA is:

$$ARA(c) = -\frac{U''(c)}{U'(c)}$$

ARA focuses on the absolute amount of wealth the individual is willing to put at risk. It describes how much an individual would risk regardless of how wealthy they are.
A common utility function used for constant absolute risk aversion is:

$$U(c) = -e^{-\alpha c}$$

Where α is the coefficient of absolute risk aversion. In this case, the **ARA** remains constant and does not depend on the level of wealth c.

- If $\alpha > 0$, the individual is **risk-averse**.
- If $\alpha = 0$, the individual is **risk-neutral**.
- If $\alpha < 0$, the individual is **risk-seeking**.

With $U(c) = \frac{c^{1-\rho}}{1-\rho}$ ARA is blue not constant: $ARA = \frac{\rho}{c}$

Production Function

- The Cobb-Douglas production function $F(k,h) = k^{\alpha}h^{1-\alpha}$ implies unitary elasticity of substitution between labor and capital.
- A 1% increase in the wage-to-rental ratio $\frac{w_t}{u_t}$ induces firms to increase the capital-labor ratio by 1%. (u_t is the rental rate of capital)
- To see this, use the fact that wages = MPL ($F_h = w$) and the rental rate of capital=MPK ($F_k = u$).
- With Cobb-Douglas: $F_h = (1-\alpha)(k/h)^{\alpha}$ and $F_k = (\alpha)(k/h)^{\alpha-1}$
- Hence, $\frac{w}{u} = \frac{1-\alpha}{\alpha}\frac{k}{h}$.
- In equilibrium the capital to labor ratio is proportional to the wage-rental ratio

Adjustment cost and interest rate premium

The quadratic adjustment cost function (with parameter $\phi > 0$):

$$\Phi(x) = \frac{\phi}{2}x^2$$

implies that that net investment (either positive or negative) generates quadratic resource costs

The country-interest rate premium (with parameters $\psi_1 > 0$ and $\bar{d}$):

$$p(d) = \psi_1 \left(e^{d-\bar{d}} - 1\right)$$

implies that the country premium is an increasing and concave function of net external debt

Characterizing the Deterministic Steady State

- Assume that the variance of the innovation to productivity η is zero.
- We refer to such an environment as a deterministic economy.
- The deterministic steady state is an equilibrium of the deterministic economy in which all endogenous variables are constant over time.
- This steady state is the quadruple (d, k, c, h) (we removed the time subscript) satisfying:

$$-\frac{U_h(c,h)}{U_c(c,h)} = AF_h(k,h) \tag{4.11'}$$

As $F_h(k,h) = (1-\alpha)\left(\frac{k}{h}\right)^{\alpha}$, we get: $-\frac{U_h(c,h)}{U_c(c,h)} = (1-\alpha)\left(\frac{k}{h}\right)^{\alpha}$

$$c + \delta k + (r^* + p(d))d = AF(k,h) \tag{4.16'}$$

$$1 = \beta(1 + r^* + p(d)) \tag{4.17'}$$

$$1 = \beta\left[AF_k(k,h) + 1 - \delta\right] \tag{4.18'}$$

Characterizing the Deterministic Steady State

We care about the deterministic steady state because:

1. To a first approximation, the deterministic SS coincides with the average position of the model economy.
   * Hence, matching average values of the observed counterparts (for instance matching predicted and observed values of labor shares, consumption shares, trade balance) can help us identifying structural parameters that can be used to calibrate the model
2. The deterministic SS is usually the point around which the equilibrium conditions of the stochastic process are approximated

Characterizing the Deterministic Steady State

Let's remove time subscripts and evaluate (4.17) at equilibrium. We get:

$$1 = \beta(1 + r^* + \psi(e^{d-\bar{d}} - 1)) \tag{4.17''}$$

As usual, let's assume that $(1 + r^*)\beta = 1$ (note that it is r^*, not r). Thus, the last term $(\psi(e^{d-\bar{d}} - 1))$ needs to be zero, which guarantees that $d = \bar{d}$

The SS version of (4.18), yields:

$$1 = \beta \left[A\alpha(k/h)^{\alpha-1} + 1 - \delta\right] \tag{4.18''}$$

this expression gives us the SS value of the capital to labor ratio κ

$$\kappa \equiv \frac{k}{h} = \left(\frac{\beta^{-1} - 1 + \delta}{\alpha}\right)^{\frac{1}{\alpha-1}}$$

We can use κ to eliminate the capital-labor ratio from Equation (4.11) and get an expression for the SS level of hours

$$h = [(1-\alpha)\kappa^{\alpha}]^{\frac{1}{\omega-1}}$$

Given the SS values of labor and capital to labor, we get the SS level of capital as $k = \kappa h$. Finally, we can get the SS level of consumption by evaluating (4.16) at equilibrium. This gives us

$$c = r^* \bar{d} + \kappa^{\alpha} h - \delta k$$

Characterizing the Deterministic Steady State

Rearranging things, with some pain, we get the following steady-state equations

$$h^{\omega-1} = A(1-\alpha)(k/h)^{\alpha} \tag{4.11''}$$

$$c + \delta k + (r^* + \psi(e^{d-\bar{d}}))d = A(k/h)^{\alpha} h \tag{4.16''}$$

$$1 = \beta(1 + r^* + \psi(e^{d-\bar{d}} - 1)) \tag{4.17''}$$

$$1 = \beta\left[A\alpha(k/h)^{\alpha-1} + 1 - \delta\right] \tag{4.18''}$$

- This is a system of 4 equations in 4 unknown endogenous variables, (c, d, h, k) and 7 unknown parameters, $\omega, \alpha, \delta, r^*, \psi, \bar{d}, \beta$.
- (From (4.12), we know that in steady state $A = 1$).
- The model has 4 additional structural parameters, σ, ϕ, ρ, $\tilde{\eta}$, which do not enter in the steady state but which also need to be assigned values to.
- In sum, there are 11 strctural parameters to be calibrated.
- They are:

$$\begin{bmatrix} \omega & \alpha & \delta & r^* & \beta & \sigma & \phi & \rho & \tilde{\eta} & \bar{d} & \psi \end{bmatrix}$$

Calibration

- An important step in computing the quantitative predictions of a business cycle model is to assign values to the structural parameters.
- There are two ways of doing this:
  - i Econometric estimation (GMM, IRF matching, Maximum likelihood, bayesian methods);
  - ii Calibration.
- Here we focus on calibration
- Calibration assigns values to the parameters in three different ways:
  - a By using sources unrelated to the macro data that the model wants to explain
  - b By matching first moments of the data that the model aims to explain
  - c By matching second moments of the data that the model aims to explain

Calibration

* USG assume that the time unit is one year and calibrate the model to the Canadian economy.
* USG adopt (almost) the same calibration as Mendoza (1991).
* Note that Mendoza uses a different stationarity inducing device (the internal discount factor (IDF) model, discussed in section 4.10.4 of the book) and hence that calibration does not assign a value to ψ.
* USG set ψ to ensure that the EDEIR model predicts the same volatility of the current-account-to-output ratio as the IDF model. The value that achieves that is $\psi = 0.000742$ (the table below is Table 4.1 in the book)

σ	$1 + r^* = 1/\beta$	δ	α	ω	ϕ	ρ	σ_ϵ	$\bar{d}$
2	1.04	0.1	0.32	1.455	0.028	0.42	0.0129	0.7442

Given values for the structural parameters, the steady state can be computed using the Matlab program available on the book's website. This yields:

c	d	h	k
1.1170	0.7442	1.0074	3.3977

The Calibration Strategy

* To obtain these values of the structural parameters (Table 4.1 in the book), three types of restrictions were imposed:

  Category a: restrictions using sources unrelated to the data that the model aims to explain, **4** parameters: $\sigma = 2$ (intertemporal elestaticity of substitution), $\delta = 0.1$ (depreciation), $r^* = 0.04$ (world interest rate), $\beta = 1/(1 + r^*) = 0.962$.

  Category b: restrictions to match first moments of the data that the model aims to explain, **2** parameters: α (capital elasticity of the production function), $\overline{d}$ (driver of the interest rate premium)

$$\text{Average labor share in Canada} = 0.68$$

$$\text{Average trade-balance-to-output ratio in Canada} = 0.02$$

  Category c: restrictions to match second moments of the data that the model aims to explain, **5** parameters: ω, ϕ, ψ, ρ, $\tilde{\eta}$. The second moments to be matched are: σ_y, σ_h, σ_i, $\sigma_{tb/y}$, $\text{corr}(\ln y_t, \ln y_{t-1})$

The Calibration Strategy

* How to implement this calibration strategy?
* The restrictions in category **a** translate immediately into values for structural parameters.
* To go from the restrictions in categories **b** and **c** to the values of the structural parameters shown in Table 4.1, one proceeds as follows:

  The labor share, s_h, is defined as

$$s_h = \frac{wh}{y}$$

In the decentralized economy we have: $A_t F_h(k_t, h_t) = w_t$

Thus, in the steady state:

$$s_h = \frac{AF_h(k,h)h}{AF(k,h)}$$

Using the assumed functional form for $F(\cdot)$ yields: $s_h = (1-\alpha)$ (we should know this from basic micro, when we studies the properties of a Cobb-Douglas with CRS)

Hence, we have that $\alpha = 1 - s_h = 1 - 0.68$, that is,

$$\alpha = 0.32$$

The Calibration Strategy

- The parameter $\bar{d}$ is set to match the observed average trade balance-to-output ratio in Canada which is 2%
- Combining the resource constraint of Equation (4.16) with the definition of trade balance of Equation (4.20), we get that in steady state

$$tb = r^* \bar{d}$$

- This condition says that in SS, the country needs to generate a trade surplus sufficiently large to service its external debt.
- Dividing both sides by SS output y and solving for $\bar{d}$ yields

$$\bar{d} = \frac{tb/y}{r^*} y$$

- We know that $tb/y = 2\%$ and $r^* = 4\%$. If we can get a value for y (SS output) we have $\bar{d}$.
- However, we know that

$$y = \left[(1-\alpha)\kappa^{\omega}\right]^{\frac{1}{\omega-1}}$$

- We also know that $\kappa = [\alpha/(r^* + \delta)]^{\frac{1}{1-\alpha}}$. The only thing that we don't have is ω (which governs the wage elasticity). For this, we need to start by guessing.

The Calibration Strategy

* Now we have parameters for σ, δ, r^*, β, and α, and we still need to assign numerical values to a vector of parameters θ with

$$\theta \equiv \left[\begin{array}{cccccc} \omega & \bar{d} & \phi & \psi & \rho & \tilde{\eta} \end{array} \right]$$

- ω is the driver of wage elasticity of labor supply
- ϕ drives the magnitude of the capital adjustment cost
- ψ drives the debt sensitivity of the interest rate
- ρ drives the persistence of the technological shock
- $\tilde{\eta}$ drives the volatility of the technological shock

The Calibration Strategy

* Now we have parameters for σ, δ, r^*, β, and α, and we still need to assign numerical values to a vector of parameters θ with

$$\theta \equiv \left[\begin{array}{cccccc} \omega & \bar{d} & \phi & \psi & \rho & \tilde{\eta} \end{array} \right]$$

* We calibrate them with the following steps:

Step 1: Guess a value for each element of θ (except $\bar{d}$ because this is determined by the other parameters)

Step 2: Given the guess for ω, find h using (4.18")

$$\frac{k}{h} = \left(\frac{r^* + \delta}{A\alpha} \right)^{\frac{1}{\alpha - 1}}$$

in (4.11")

$$h = ((1-\alpha)A(k/h)^{\alpha})^{1/(\omega-1)}$$

With h in hand, find k and y, as $k = (k/h)h$ and $y = A(k/h)^{\alpha}h$, respectively.

The Calibration Strategy

Step 3: Let s_{tb} denote the trade-balance-to-output ratio. In the steady state,

$$s_{tb} = \frac{r^* d}{y}$$

Solve this expression for d

$$d = \frac{s_{tb} y}{r^*}$$

Then use (4.17") and the restriction that $\beta(1 + r^*) = 1$ to obtain

$$\bar{d} = d$$

Step 4: Find c from (4.16")

$$c = y - \delta k - r^* d$$

Step 5: With the steady state values of (c, k, d, h) and all structural parameters in hand approximate the equilibrium dynamic and compute the model's predictions for

$$x(\theta) \equiv \begin{bmatrix} \sigma_y & \sigma_h & \sigma_i & \sigma_{tb/y} & corr(\ln y_t, \ln y_{t-1}) \end{bmatrix}$$

Step 6: Find the distance

$$D = |x(\theta) - x^*|$$

where x^* denotes the vector of targeted moments observed in Canadian data. These are: standard deviation of output 2.81%; standard deviation of hours 2.02%, standard deviation of investment 9.82%; standard deviation of trade balance/output 1.87 p.p; and serial correlation of output 0.62

Step 7: Keep adjusting θ until D is less than some threshold D^*.

* Comment: In general there does not exist a θ that makes the distance D exactly equal to zero. Hence one has to pick some threshold for the distance, D^*.
* Comment: We want to focus on the distance of the parameters that we are not matching directly.

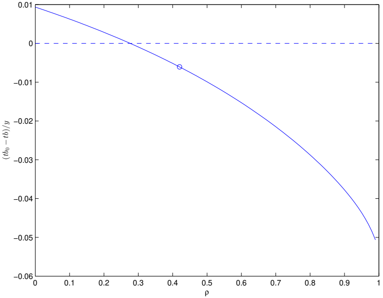
How to proceed:

- If the volatility of output predicted by the model is too low, increase the volatility of the innovation of the technology shock $\tilde{\eta}$ (and the other way around)
- If the predicted volatility of investment is too high increase ϕ
- In general, there is no guarantee that we can match all what we want to match. If we can't, the model does not work well
- But some distance should be tolerated
- Also note that we could use different parameters in the different categories (for instance we could put ω in category **a** and try to assign it a value based on micro estimate of the elasticity of labor supply)

How are we doing?

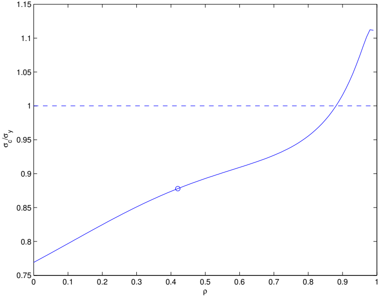
* Before analyzing to which extend the SOE-RBC model can account for the observed Canadian business cycle, let's first study the predictions of this model regarding the predictions for which we build intuition in Chapters 2 and 3.
* In particular, there we showed that:
    1. The more persistent productivity shocks are, the more likely an initial deterioration of the trade balance will be.
    2. The more persistent the technology shock is, the higher the volatility of consumption relative to output will be.
    3. The more pronounced are capital adjustment costs, the smaller will be the initial trade balance deterioration in response to a positive and persistent productivity shock.
* The next two figures show that these analytical results do indeed hold in the fully-fledged stochastic dynamic open economy RBC model. (We look at adjustment costs later)

Impact response of the trade balance as a function of the persistence of the technology shock



* The figure shows the impact response of the trade balance to a one percent positive innovation in productivity predicted by the model
* The response of the trade balance is measured in units of steady-state output.
* All parameters other than ρ take the values shown in Table 4.1.
* The open circle indicates the baseline value of ρ (0.42).
* The figure shows that the more persistent the productivity shock is the more negative the impact response of the trade balance will be.
* For $\rho > 0.3$, the response of the trade balance is negative, confirming the analytical results of chapters 2 and 3.

Relative volatility of consumption as a function of the persistence of the stationary technology shock



* The relative standard deviation shown is that implied by the model
* All parameters other than ρ take the values shown in Table 4.1.
* The open circle indicates the baseline value of ρ (0.42).
* The figure shows that the more persistent stationary productivity shocks are, the higher the standard deviation of consumption relative to the standard deviation of output will be, just as derived analytically in the permanent income model of Chapter 2.
* For the Canadian economy, consumption is less volatile than income. But Canada is an advanced economy.

We now follow Mendoza (1991) and compare the predictions of the SOE-RBC model with Canadian data.

Why Canada? Because it is a small open economy and it is the economy studied in Mendoza (1991).

Some Empirical Regularities of the Canadian Economy

Variable	Canadian Data		
	σ_{x_t}	$\rho_{x_t,x_{t-1}}$	ρ_{x_t,GDP_t}
y	2.8	0.62	1
c	2.5	0.7	0.59
i	9.8	0.31	0.64
h	2	0.54	0.8
$\frac{tb}{y}$	1.9	0.66	-0.13

Source: Mendoza AER, 1991. Annual data (1946-85). Log-quadratically detrended.

* Volatility ranking: $\sigma_{tb/y} < \sigma_c < \sigma_y < \sigma_i$.
* Consumption, investment, and hours are procyclical.
* The trade-balance-to-output ratios is countercyclical.
* All variables considered are positively serially correlated.
* Similar stylized facts emerge from other small developed countries (see, e.g., chapter 1).

Empirical and Theoretical Second Moments

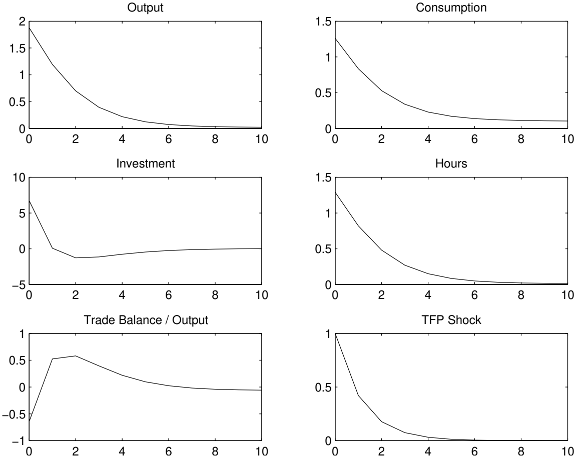
	Canadian Data						Model		
	1946 to 1985			1960 to 2011					
	σ_{x_t}	$\rho_{x_t,x_{t-1}}$	ρ_{x_t,y_t}	σ_{x_t}	$\rho_{x_t,x_{t-1}}$	ρ_{x_t,y_t}	σ_{x_t}	$\rho_{x_t,x_{t-1}}$	ρ_{x_t,y_t}
y	2.8	0.6	1	3.7	0.9	1	3.1	0.6	1
c	2.5	0.7	0.6	2.2	0.7	0.6	2.7	0.8	0.8
i	9.8	0.3	0.6	10.3	0.7	0.8	9.0	0.1	0.7
h	2.0	0.5	0.8	3.7	0.7	0.8	2.1	0.6	1
$\frac{tb}{y}$	1.9	0.7	-0.1	1.7	0.8	0.1	1.8	0.5	-0.04
$\frac{ca}{y}$							1.4	0.3	0.05

* σ_h, σ_i, σ_y, $\sigma_{tb/y}$, and $\rho_{y_t,y_{t-1}}$ were targeted by calibration, so no real test here.
* Model correctly places σ_c below σ_y and σ_i and σ_c above σ_h and $\sigma_{tb/y}$.
* Model correctly makes tb/y countercyclical (but note what happens for the expanded dataset).
* Model overestimates the correlations of hours and consumption with output (it predicts a correlations of 1 and 0.8, while the actual correlations are 0.8 and 0.6).
* The fact that the predicted correlation of hours with output is 1 is due to the GHH utility index (See condition (4.11) that can be written as $h_t^{\omega} = (1-\alpha)y_t$. The log linearized form is $\omega\hat{h}_t = \hat{y}_t$, which implies that $\hat{h}_t$ and $\hat{y}_t$ are perfectly correlated.)

Response to a Positive Technology Shock

- Let's look at the IRFs to a technology shock of size 1 in period 0.
- Increase in y, c, i, and h
- Deterioration of the trade balance both in levels (not shown) and as a share of GDP (shown)
- The fact that the trade balance deteriorates in levels indicates that on impact the increase in domestic absorption $(i_0 + c_0)$ is larger than the increase in output
- Also note that the initial response of consumption is smaller than the increase in y while the response of investment is about 4 times larger than the increase in y
- Investment plays a key role in creating the counter-cyclical response of the trade balance

Response to a Positive Technology Shock



Source: Schmitt-Grohé and Uribe (JIE, 2003)

Persistence and Adjustment costs

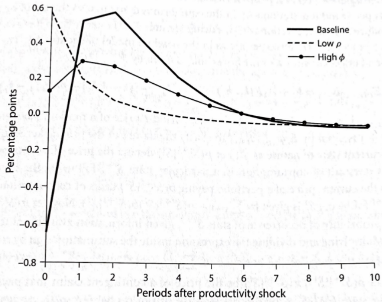


Figure 4.2 Response of the trade-balance-to-output ratio to a positive technology shock.

* Chapter 3 showed that the negative response of the trade balance to a productivity shock requires that adjustment costs are not too large and that the productivity shock is sufficiently persistent
* The figure shows the response of the trade balance to a a technology shock of size 1 in period 0. The solid line shows the benchmark model
* The dashed line shows a model where persistence is half that of benchmark model ($\rho = 0.21$). In this case, since the shock will die out quickly, the response of investment is weak and the trade balance is pro-cyclical. Moreover, since the shock is temporary, HHs will save most of the additional income.
* The bulleted line shows the case of high capital adjustment costs ($\phi = 0.084$, this is three times what we used in the benchmark model). High adjustment costs discourage firms from investing. As a result, the response of aggregate demand is weak and the trade balance goes into surplus.

4.9 The Complete Asset Markets (CAM) Model

* So far, we assumed that there is only one traded bond and that this bond is not contingent
* It would be possible to build a model with a complete set of Arrow-Debreu securities. This is the Complete Asset Market Model (Section 4.9 of the book)
* This allows agents to smooth consumption under all states of nature. Marginal utility of consumption is constant
* Complete asset market induce stationary (this is because in the model the marginal utility of consumption is exogenous and no longer follows a random walk), so no need to include ad hoc features to induce stationarity
* Deriving the CA balance is a bit more complicated because payments depend on the state-of the world
* Quantitative results almost identical to those of the imperfect market model (except that CAM predicts a strongly countercyclical CA, while incomplete market model predicts an cyclical CA)

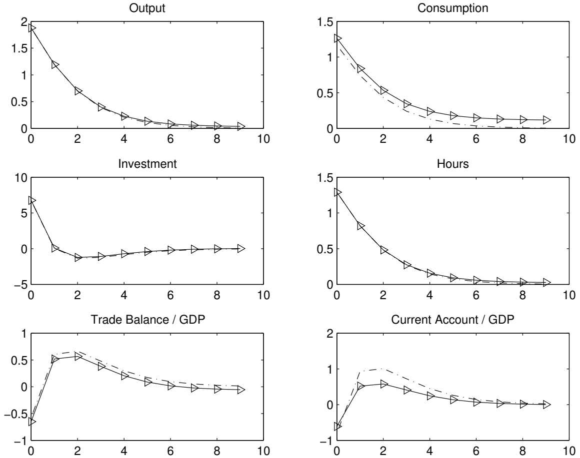
The SOE-RBC Model With Complete Asset Markets: Predicted Second Moments

	σ_{x_t}		$\rho_{x_t,x_{t-1}}$		ρ_{x_t,GDP_t}	
variable	CAM	1 Bond	CAM	1 Bond	CAM	1 Bond
y	3.1	3.1	0.61	0.62	1.00	1.00
c	1.9	2.71	0.61	0.78	1.00	0.84
i	9.1	9.0	0.07	0.07	0.66	0.67
h	2.1	2.1	0.61	0.62	1.00	1.00
$\frac{tb}{y}$	1.6	1.78	0.39	0.51	0.13	-0.04
$\frac{ca}{y}$	3.1	1.45	-0.07	0.32	-0.49	0.05

Note. Standard deviations are measured in percentage points. The columns labeled CAM are produced with the Matlab program $\texttt{cam}_r un.m!availableat$

`http://www.columbia.edu/ mu2166/closing.htm!.`

Impulse Response to a Unit Technology Shock One-Bond Versus Complete Asset Market Models



Dash-diamond, 1 Bond model. Dash-dotted, complete-asset-market model.

Alternative Ways to Induce Stationarity (will not do)

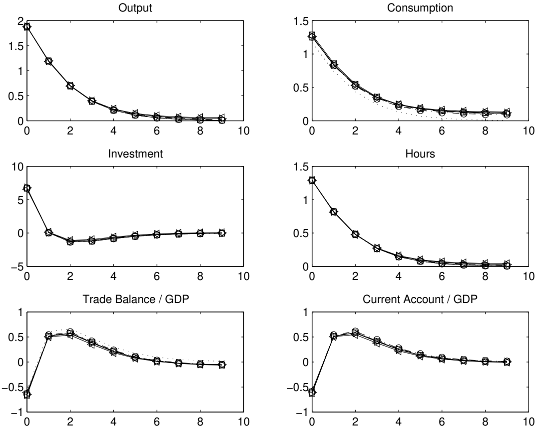
- The Internal Debt-Elastic Interest Rate (IDEIR) Model
  - It assumes that interest rate responds to individual debt (instead of average debt) $r_t = r^* = p(d_t)$
- The Portfolio Adjustment Cost (PAC) Model
  - There is a cost related to adjusting portfolio. The budget constraint includes this cost $\Psi(d_t)$
- The External Discount Factor (EDF) Model
  - The subjective discount factor depends on the average values of endogenous variables $\theta_{t+1} = \beta(\tilde{c}_t, \tilde{h}_t)\theta_t$
- The Internal Discount Factor (IDF) Model
  - The subjective discount factor depends on the individual values of endogenous variables $\theta_{t+1} = \beta(c_t, h_t)\theta_t$

Alternative Ways to Induce Stationarity (will not do)

- The Perpetual-Youth (PY) Model
  - * People do not live forever and they have a constant probability of dying (Blanchard, 1985, has the first formulation in continuous time)
  - * When people die a fraction of debt disappears. This induces stationarity
- Global Solutions
  - * We are now assuming that $\beta(1 + r^*) < 1$
  - * No longer RW on debt and consumption because people want to anticipate consumption.
  - * So, consumption keeps decreasing.
  - * When there is no uncertainty consumption approaches asymptotically its lowest possible level and debt converges to a well-defined value.
  - * In the limit people will be infinitely unhappy
  - * When uncertainty is introduced there is the risk of being infinitely unhappy earlier
  - * People will thus have precautionary savings
  - * Precautionary savings cannot be captured with a first-order approximation. A higher order approximation is needed
  - * Solution algorithm based on a recursive (Bellman) equation (Section 4.13)

4.12 Inducing Stationarity: Quantitative Comparison of Alternative Methods

Impulse Response to a Unit Technology Shock in Models 1 Through 5



Source: Schmitt-Grohé and Uribe (JIE, 2003). Note. Solid line, endogenous discount factor. Squares, endogenous discount factor without internalization. Dashed line, Debt-elastic interest rate. Dash-dotted line, Portfolio adjustment cost. Dotted line, complete asset markets. Circles, No stationarity inducing elements.