# **International Trade I Monopolistic Competition**

Monika Mrázová

Spring 2025

## **Outline of the Lecture**

- Introduction
- 2 Monopolistic competition
- Monopolistic competition with heterogeneous firms: Melitz (2003)
- Monopolistic competition with variable elasticity demands

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- 1 Introduction
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#### **Motivation**

- Problem 1: previous models explain why different countries trade, but a large proportion of world trade is between similar countries
- So far, all models have assumed:
  - Perfect competition
  - Constant returns to scale
- In reality, many industries exhibit some degree of imperfect competition and increasing returns to scale
- This introduces many new features:
  - New reasons and gains from trade:
    - Trade increases market size and so allows the exploitation of economies of scale
    - ★ Trade increases the variety of products available
    - ★ Trade increases competition
  - The pattern of trade may not be efficient

#### **Motivation**

- Problem 2: Nineties have seen a boom in the availability of micro-level data. Previous theories are at odds with (or cannot account for) many micro-level facts:
  - Within a given industry, there is firm-level heterogeneity
  - Fixed costs matter in export related decisions
  - Within a given industry, more productive firms are more likely to export
  - Trade liberalization leads to intra-industry reallocation across firms
  - These reallocations are correlated with productivity and export status
- Melitz (2003) will develop a model featuring facts 1 and 2 that can explain facts 3, 4, and 5
  - ► This is the most influential trade paper in the last 20 years

## Today's plan

- Monopolistic competition
  - Krugman (1979)
  - An important special case: CES utility
- Heterogenous firms: Melitz (2003)
  - Krugman (1980) meets Hopenhayn (1992)
  - Selection into exports and the impact of trade
- Monopolistic competition with heterogeneous firms and variable elasticity demands

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Demand Manifold: Mrázová and Neary (2017)

## Reading

- \*F pp. 137-141 and 163-169
- \*Krugman, P. (1979), "Increasing Returns Monopolistic Competition, and International Trade," Journal of International Economics, 9, 469-479.
- \*Melitz, M. (2003), "The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity", Econometrica, 71:6, 1695-1725.
- Krugman, P. (1980), "Scale economies, product differentiation, and the pattern of trade," American Economic Review, 70, 950-959
- Melitz, M. J., and S. J. Redding (2015), "Heterogeneous Firms and Trade." Handbook of International Economics, 4th ed, 4: 1-54. Elsevier, 4, 1-54,
- Introductory level: KOM chs. 7 and 8, and FT ch. 6
- Paul Krugman, Nobel Prize Lecture (2008): The Increasing Returns Revolution in Trade and Geography, available at: http://www.nobelprize.org/nobel\_prizes/

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- Introduction
- Monopolistic competition
  - Monopolistic pricing: recap
  - Krugman (1979)
  - An important special case: CES utility
- Monopolistic competition with heterogeneous firms: Melitz (2003)
- 4 Monopolistic competition with variable elasticity demands

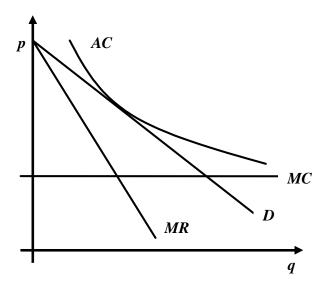
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#### Basic idea

- Monopoly pricing: Each firm faces a downward-sloping demand curve
  - ▶ Profit maximization  $\Rightarrow MR = MC$
- No strategic interaction:
  - Each demand curve depends on the prices charged by other firms
  - But since the number of firms is large, each firm ignores its impact on the demand faced by other firms
- Free entry: Firms enter the industry until profits are driven to zero for all firms
  - Free entry  $\Rightarrow p = AR = AC$

# **Graphical analysis**



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## **Endowments, preferences and technology**

- Endowments: All agents are endowed with 1 unit of labor
- Preferences: All agents have the same utility function given by

$$U = \int_0^n u(c_i)di$$

#### where

- u(0) = 0, u' > 0, and u'' < 0 (love of variety)
- ▶ Elasticity of demand:  $\sigma(c) \equiv -\frac{u'}{cu''} > 0$  is such that  $\sigma' \leq 0$  (why?)
- IRS Technology: Labor used in the production of each "variety" is

$$l_i = f + q_i/\varphi$$

where  $\varphi \equiv common$  productivity parameter

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## **Equilibrium conditions**

Consumer maximization:

$$p_i = \lambda^{-1} u'(c_i)$$

• Profit maximization (MR = MC):

$$p_i = \left[\frac{\sigma(c_i)}{\sigma(c_i) - 1}\right] \cdot \left(\frac{w}{\varphi}\right)$$

• Free entry (P = AC):

$$\left(p_i - \frac{w}{\varphi}\right)q_i = wf$$

Good and labor market clearing:

$$q_i = Lc_i$$

$$L = nf + \int_0^n \frac{q_i}{\varphi} di$$

## **Equilibrium conditions rearraged**

- Symmetry  $\Rightarrow p_i = p, q_i = q$ , and  $c_i = c$  for all  $i \in [0, n]$
- ullet c and p/w are simultaneously characterized by

$$(PP): \quad \frac{p}{w} = \left[\frac{\sigma(c)}{\sigma(c) - 1}\right] \frac{1}{\varphi}$$

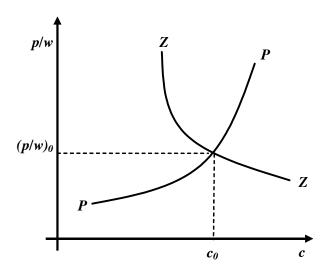
$$(ZZ): \quad \frac{p}{w} = \frac{f}{q} + \frac{1}{\varphi} = \frac{f}{Lc} + \frac{1}{\varphi}$$

• *n* can then be computed using market clearing conditions

$$n = \frac{1}{f/L + c/\varphi}$$

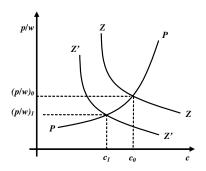
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# **Graphical analysis**



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## Gains from trade revisited



- Suppose that two identical countries open up to trade
  - ► This is equivalent to a doubling of country size (which would have no effect in a neoclassical trade model)
- Because of IRS, opening up to trade now leads to:
  - ▶ Increased product variety:  $c_1 < c_0 \Rightarrow \frac{1}{f/2L + c_1/\varphi} > \frac{1}{f/L + c_0/\varphi}$ ▶ Pro-competitive/efficiency effects:  $(p/w)_1 < (p/w)_0 \Rightarrow q_1 > q_0$

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## Trade economists' most preferred demand system

 Constant Elasticity of Substitution (CES) utility corresponds to the case where:

$$U = \int_0^n (c_i)^{\frac{\sigma - 1}{\sigma}} di$$

where  $\sigma > 1$  is the elasticity of substitution between pair of varieties

- This is the case considered in Krugman (1980)
- What is there to like about CES utility?
  - ► Homotheticity  $(u(c) \equiv (c)^{\frac{\sigma-1}{\sigma}}$  is actually the *only* functional form such that U is homothetic)
  - ► Can be derived from discrete choice model with i.i.d extreme value shocks (see Feenstra Appendix B)
- Is it empirically reasonable?

## Special properties of the equilibrium

■ Because of monopoly pricing, CES ⇒ constant markups:

$$\frac{p}{w} = \left[\frac{\sigma}{\sigma - 1}\right] \frac{1}{\varphi}$$

 Because of zero profit, constant markups ⇒ constant output per firm:

$$\frac{p}{w} = \frac{f}{q} + \frac{1}{\varphi}$$

 Because of market clearing, constant output per firm ⇒ constant number of varieties per country:

$$n = \frac{L}{f + q/\varphi}$$

- So gains from trade only come from access to Foreign varieties
  - ▶ IRS provide an intuitive reason why Foreign varieties are different
  - But consequences of trade would now be the same if we had maintained CRS with different countries producing different goods

## Special properties of the equilibrium

- Decentralized equilibrium is efficient
- Decentralized equilibrium solves:

$$\max_{q_i,n} \int_0^n p_i(q_i) q_i di \quad \text{s.t.} \quad nf + \int_0^n \frac{q_i}{\varphi} di \le L$$

A central planner would solve:

$$\max_{q_i,n} \int_0^n (q_i)^{\frac{\sigma-1}{\sigma}} di \quad \text{ s.t. } \quad nf + \int_0^n \frac{q_i}{\varphi} di \leq L$$

- Under CES,  $p_i(q_i)q_i \propto (q_i)^{\frac{\sigma-1}{\sigma}} \Rightarrow$  two solutions coincide
  - ► This is unique to CES (in general, entry is distorted)
  - ► This implies that many properties of perfectly competitive models carry over to this environment

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  - The model
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#### Introduction

- Two building blocs:
  - Krugman (1980): CES, IRS technology, monopolistic competition
  - Propension (1992): equilibrium model of entry and exit
- CES preferences allow a clever comparison with Krugman (1980): existence of a representative firm
- We study here a simplified (static) version of Melitz (2003) with Pareto distribution of productivities

#### **Demand**

 Like in Krugman (1980), representative agent has CES preferences:

$$U = \left( \int_{\omega \in \Omega} q(\omega)^{\frac{\sigma - 1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma - 1}}$$

where  $\sigma > 1$  is the elasticity of substitution

• Consumption and expenditures for each variety are given by

$$q(\omega) = Q \left[ \frac{p(\omega)}{P} \right]^{-\sigma} \equiv Ap(\omega)^{-\sigma}$$
 (1)

$$r(\omega) = R \left[ \frac{p(\omega)}{P} \right]^{1-\sigma}$$
 (2)

where  $P \equiv \left[\int_{\omega \in \Omega} p(\omega)^{1-\sigma} d\omega\right]^{\frac{1}{1-\sigma}}$ ,  $R \equiv \int_{\omega \in \Omega} r(\omega) d\omega$ , and  $Q \equiv R/P$ 

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## **Production: assumptions**

- Like in Krugman (1980), labor is the only factor of production
  - $L \equiv \text{total endowment}, w = 1 \equiv \text{wage}$
- ullet Firms differ in their productivity  $\varphi$
- Like in Krugman (1980), there are IRS in production

$$l(\varphi) = f + q/\varphi \tag{3}$$

## **Production: implications**

 Like in Krugman (1980), monopolistic competition implies that prices are a constant mark-up over marginal cost

$$\frac{p(\varphi)}{w} = \left[\frac{\sigma}{\sigma - 1}\right] \frac{1}{\varphi} \tag{4}$$

CES preferences with monopoly pricing, (2) and (4), imply

$$r(\varphi) = R \left[ P \frac{\sigma - 1}{\sigma} \varphi \right]^{\sigma - 1} \tag{5}$$

 The two assumptions, (3) and (4), further imply that profits are a constant fraction of revenue minus the fixed production cost

$$\pi(\varphi) \equiv r(\varphi) - l(\varphi) = \frac{r(\varphi)}{\sigma} - wf$$

where  $r(\varphi) = p(\varphi)q(\varphi)$ 

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## **Production: comments**

• Higher productivity  $\varphi$  in the model implies higher *measured* productivity

$$\frac{r(\varphi)}{l(\varphi)} = w \frac{\sigma}{\sigma - 1} \left[ 1 - \frac{f}{l(\varphi)} \right]$$

More productive firms produce more and earn higher revenues

$$\frac{q(\varphi_1)}{q(\varphi_2)} = \left(\frac{\varphi_1}{\varphi_2}\right)^{\sigma} \quad \text{amd} \quad \frac{r(\varphi_1)}{r(\varphi_2)} = \left(\frac{\varphi_1}{\varphi_2}\right)^{\sigma-1}$$

ullet can also be interpreted in terms of quality. This is isomorphic to a change in units of account, which would affect prices, but nothing else

## Aggregation

By definition, the CES price index is given by

$$P = \left[ \int_{\omega \in \Omega} p(\omega)^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}}$$

• Since all firms with productivity  $\varphi$  charge the same price  $p(\varphi)$ , we can rearrange CES price index as

$$P = \left[ \int_0^{+\infty} p(\varphi)^{1-\sigma} M\mu(\varphi) d\varphi \right]^{\frac{1}{1-\sigma}}$$

#### where

- $ightharpoonup M \equiv$  mass of (surviving) firms in equilibrium
- $\mu(\varphi) \equiv$  (conditional) pdf of firm-productivity levels in equilibrium

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## **Aggregation**

Combining the previous expression with monopoly pricing (4), we get

$$P = M^{\frac{1}{1-\sigma}} w \left[ \frac{\sigma}{\sigma - 1} \right] \frac{1}{\tilde{\varphi}} = M^{\frac{1}{1-\sigma}} p(\tilde{\varphi})$$

where

$$\tilde{\varphi} \equiv \left[ \int_0^{+\infty} \varphi^{\sigma - 1} \mu(\varphi) d\varphi \right]^{\frac{1}{\sigma - 1}}$$

One can do the same for all aggregate variables

$$R = Mr(\tilde{\varphi}), \quad \Pi = M\pi(\tilde{\varphi}), \quad Q = M^{\frac{\sigma}{\sigma-1}}q(\tilde{\varphi})$$

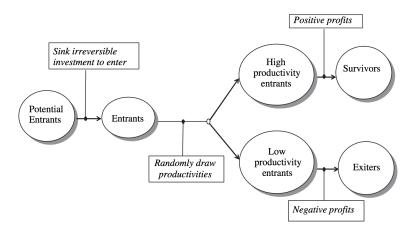
#### Comments:

- ▶ These are the same aggregate variables we would get in a Krugman (1980) model with a mass M of identical firms with productivity  $\tilde{\varphi}$
- ▶ But productivity  $\tilde{\varphi}$  now is an endogenous variable which may respond to changes in trade cost, leading to aggregate productivity changes

## **Entry and exit**

- In order to determine how  $\mu(\varphi)$  and  $\tilde{\varphi}$  get determined in equilibrium, one needs to specify the entry and exit of firms
- Timing is similar to Hopenhayn (1992):
  - There is a large pool of identical potential entrants deciding whether to become active or not
  - ② Firms deciding to become active pay a (sunk) fixed cost of entry  $f_E>0$  and get a productivity draw  $\varphi$  from a cdf G (with density  $g(\varphi)$ )
    - ★ To decide, they compute their expected profits  $\mathbb{E}[\pi]$
  - After observing their productivity draws, firms decide whether to remain active or not
    - ★ Firms are deciding whether to pay or not a fixed cost f in order to start producing
    - ★ To do this, they check whether  $\pi(\varphi) \leq 0$

## Timing and entry



Source: Greenaway Kneller (2007)

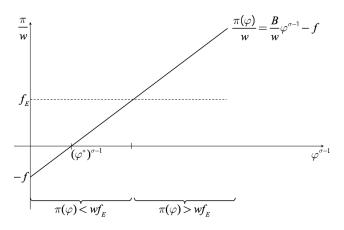
## **Closed economy**

- Solve backwards
- A firm  $\varphi$  produces if it is productive enough i.e.

$$\pi(\varphi) \ge 0$$

- There is a cutoff firm  $\varphi^*$  such that  $\pi(\varphi^*) = 0$
- Among firms who have paid the sunk cost, those with a productivity lower than  $\varphi^*$  choose not to produce (and therefore do not pay f)

# **Closed economy: sorting**



Source: Melitz & Redding (2012).  $B = \frac{(\sigma-1)^{\sigma-1}}{\sigma^{\sigma}} A$ 

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## Aggregate productivity

• Once we know  $\varphi^*$ , we can compute the pdf of firm-productivity levels

$$\mu(\varphi) = \begin{cases} \frac{g(\varphi)}{1 - G(\varphi^*)} & \quad \text{if} \quad \varphi \geq \varphi^* \\ 0 & \quad \text{if} \quad \varphi < \varphi^* \end{cases}$$

Accordingly, the measure of aggregate productivity is given by

$$\tilde{\varphi}(\varphi^*) = \left[\frac{1}{1 - G(\varphi^*)} \int_{\varphi^*}^{+\infty} \varphi^{\sigma - 1} g(\varphi) d\varphi\right]^{\frac{1}{\sigma - 1}}$$

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## Free entry condition

- ullet Let  $ar{\pi}\equiv\Pi/M$  denote average profits per period for surviving firms
- Free entry requires the total expected value of profits to be equal to the fixed cost of entry

$$\mathbb{E}[\pi] = \int_0^\infty \pi(\varphi) g(\varphi) d\varphi$$
$$= 0 \times G(\varphi^*) + \bar{\pi} \times [1 - G(\varphi^*)] = f_E w$$

• Free Entry Condition (FE):

$$\bar{\pi} = \frac{f_E w}{1 - G(\varphi^*)} \tag{6}$$

 Holding constant the fixed costs of entry, if firms are less likely to survive, they need to be compensated by higher average profits

## Zero cutoff profit condition

- Definition of  $\varphi^*$  can be rearranged to obtain a second relationship between  $\varphi^*$  and  $\bar{\pi}$
- By definition of  $\bar{\pi}$ , we know that

$$\bar{\pi} = \Pi/M = \pi[\tilde{\varphi}(\varphi^*)] \Leftrightarrow \bar{\pi} = fw \left[ \frac{r[\tilde{\varphi}(\varphi^*)]}{\sigma fw} - 1 \right]$$

• By definition of  $\varphi^*$ , we know that

$$\pi(\varphi^*) = 0 \Leftrightarrow r(\varphi^*) = \sigma f w$$

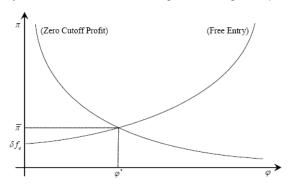
Two previous expressions imply ZCP condition:

$$\bar{\pi} = fw \left[ \frac{r[\tilde{\varphi}(\varphi^*)]}{r(\varphi^*)} - 1 \right] = fw \left[ \left( \frac{\tilde{\varphi}(\varphi^*)}{\varphi^*} \right)^{\sigma - 1} - 1 \right]$$
 (7)

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# Closed economy equilibrium

- FE and ZCP, (6) and (7), determine a unique  $(\bar{\pi}, \varphi^*)$ , and therefore  $\tilde{\varphi}$ , independently of country size L
  - the only variable left to compute is M, which can be done using free entry and labor market clearing as in Krugman (1980)



Source: Melitz (2003)

## **Closed-form using a Pareto**

- Why Pareto?
  - Pareto distribution of productivities + CES preferences = Pareto size distribution of firms
  - ► Found (more or less) in the data (Axtell, 2011) but left tail looks more log-normal...
  - Pareto has good properties: a truncated Pareto is Pareto
- Two-parameter distribution  $(\varphi_{\min}, k)$ :  $G(\varphi) = 1 \left(\frac{\varphi_{\min}}{\varphi}\right)^k$
- Then the cutoff is simply

$$(\varphi^*)^k = \frac{\sigma - 1}{k - (\sigma - 1)} \varphi_{\min}^k \frac{f}{f_E}$$

#### Remark

$$\varphi^* = \varphi_{\min} \left( \frac{\sigma - 1}{k - (\sigma - 1)} \frac{f}{f_E} \right)^{\frac{1}{k}}$$

- $\varphi^*$  does not depend on market size L
- An increase in market size does not change the cutoff
  - Will be restored when moving away from CES preferences.
- Growth in country size and costless trade will therefore have the same impact as in Krugman (1980):
  - ▶ welfare ↑ because of ↑ in total number of varieties in each country

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## **Open-economy model**

- In the absence of trade costs, we have seen trade integration does not lead to any intra-industry reallocation
- In order to move away from such (counterfactual) predictions,
   Melitz (2003) introduces two types of trade costs:
  - Iceberg trade costs: in order to sell 1 unit abroad, firms need to ship  $\tau > 1$  units
  - **2 Fixed exporting costs:** in order to export abroad, firms must incur an additional fixed cost  $f_X$  (information, distribution, or regulation costs) after learning their productivity  $\varphi$
- In addition, Melitz (2003) assumes that  $c=1,\ldots,n$  countries are symmetric so that  $w_c=1$  in all countries

#### **Production**

Monopoly pricing now implies

$$p_D(\varphi) = \frac{1}{\rho \varphi}$$
 and  $p_X(\varphi) = \frac{\tau}{\rho \varphi}$ 

where  $\rho = (\sigma - 1)/\sigma$ 

Revenues in the domestic and export markets are

$$r_D(\varphi) = R_D[P_D\rho\varphi]^{\sigma-1}$$
 and  $r_X(\varphi) = \tau^{1-\sigma}R_X[P_X\rho\varphi]^{\sigma-1}$ 

- Note that by symmetry, we must have  $P_D = P_X = P$  and  $R_D = R_X = R$
- Profits in the domestic and export markets are

$$\pi_D(\varphi) = rac{r_d(\varphi)}{\sigma} - fw$$
 and  $\pi_X(\varphi) = rac{r_X(\varphi)}{\sigma} - f_X w$ 

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## **Productivity cutoffs**

- Like in the closed economy, we let  $\varphi^*$  be the cutoff to enter the domestic market
- In addition, let  $\varphi_X^*$  be the export cutoff
- In order to have both exporters and non-exporters in equilibrium,  $\varphi_X^* > \varphi^*$ , we assume that  $\tau^{\sigma-1} f_X > f$

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## **Selection into exports**

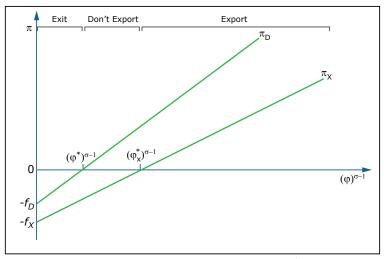


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# Profits and productivity in the open economy

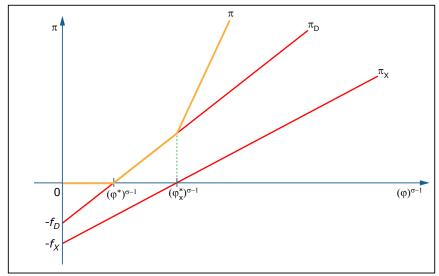


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## The impact of trade

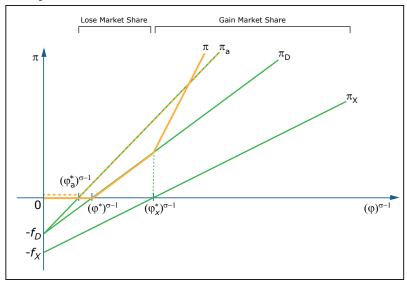


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## The impact of trade

• In line with empirical evidence, exposure to trade forces the least productive firms to exit:  $\varphi^*>\varphi^*_a$ 

#### Intuition:

- For exporters: Profits ↑ due to export opportunities, but ↓ due to the entry of foreign firms in the domestic market (P ↓)
- ► For non-exporters: only the negative second effect is active

## Other comparative static exercises

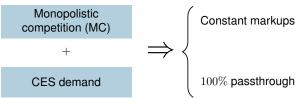
- Starting from autarky and moving to trade is theoretically standard, but not empirically appealing
- Melitz (2003) also considers:
  - ▶ Increase in the number of trading partners *n*
  - Decrease in iceberg trade costs τ
  - ▶ Decrease in fixed exporting costs  $f_X$
- Same qualitative insights in all scenarios:
  - Exit of least efficient firms
  - Reallocation of market shares from less from more productive firms
  - Welfare gains

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#### Question

Workhorse model in int. economics: Melitz (2003)



- Proliferation of alternatives to CES with variable elasticity:
  - Quadratic preferences: Melitz-Ottaviano (2008)
  - Stone-Geary LES: Simonovska (2010)
  - Translog: Feenstra-Weinstein (2010)
  - ► CARA: Behrens-Murata (2007)
  - ► Bulow-Pfleiderer: Atkin-Donaldson (2012)
- How do assumptions about demand affect comparative statics questions?
  - → The Demand Manifold: representation of perceived demand in elasticity-curvature space

## Introducing the Demand manifold: A Core Model

- Monopoly firm facing an inverse demand function p(x), p' < 0
- Consistent with monopolistic competition
  - ► Firm's-Eye View of Demand
  - Firm takes the demand function as given: "perceived"
  - In general/industry equilibrium, the demand function has extra arguments ... see later
- Fixed marginal cost c

## **Profit maximisation**

First-order condition:

$$p(x) + xp'(x) = c$$

Second-order condition:

$$2p'(x) + xp''(x) < 0$$

which can be re-written as

$$\frac{p(x)}{c} = \frac{\varepsilon(x)}{\varepsilon(x) - 1}$$

$$\rho(x) < 2$$

#### where

- $\varepsilon(x) \equiv -\frac{p(x)}{xp'(x)}$  is the demand elasticity/slope
- $\bullet \ \rho(x) \equiv -\frac{xp''(x)}{p'(x)}$  is the demand convexity/curvature

## Two key demand parameters

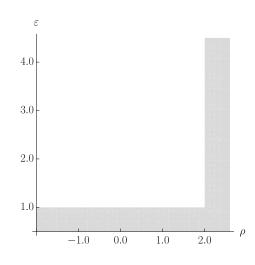
**1** Elasticity  $\varepsilon$ :

$$p(x) + xp'(x) = c \ge 0$$
  
 $\Rightarrow \boxed{\varepsilon(x) \ge 1}$ 

**2** Convexity  $\rho$ :

$$2p'(x) + xp''(x) < 0$$

$$\Rightarrow \boxed{\rho(x) < 2}$$



#### The Demand Manifold

= visualisation of the demand function p(x) in the  $(\varepsilon, \rho)$  space

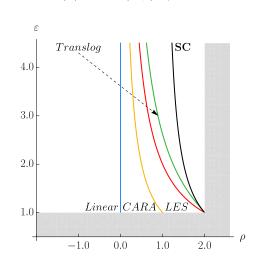
• In general, both  $\varepsilon$  and  $\rho$  vary with sales  $\rightarrow$  a curve

• Exception: CES/iso-elastic case:  $p(x) = \beta x^{-1/\sigma}$ 

$$\Rightarrow \quad \varepsilon = \sigma, \ \ \rho = \frac{\sigma + 1}{\sigma}$$

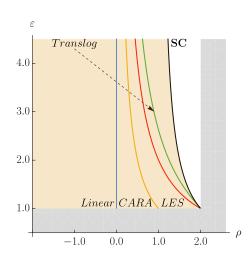
 $\Rightarrow$  Point manifold:

$$\rho = \frac{\varepsilon + 1}{\varepsilon}$$



# **Subconvexity**

- p(x) is subconvex at  $x^0$  IFF:
  - p(x) is less convex than a CES demand function with the same elasticity:  $\rho < \frac{\varepsilon+1}{\varepsilon}$
  - ▶  $\log p(x)$  is concave in  $\log x$
  - $\begin{array}{c} \varepsilon \text{ is decreasing in sales:} \\ \varepsilon_x = \frac{\varepsilon}{x} \left[ \rho \frac{\varepsilon + 1}{\varepsilon} \right] < 0 \end{array}$
- Subconvexity = "Marshall's  $2^{nd}$  Law of Demand"
  - confirmed empirically
  - theoretically plausible (Dixit-Stiglitz, Krugman, etc.)



## Manifold: Definition and Existence

Given a demand function  $p = p_0(x)$  defined over a range  $X(p_0) \subseteq R_{\geq 0}$ :

$$\Omega(p_0) \equiv \left[ (\varepsilon, \rho) : \varepsilon = -\frac{p_0(x)}{x p_0'(x)}, \ \rho = -\frac{x p_0''(x)}{p_0'(x)}, \ \forall x \in X(p_0) \right]$$

#### **Proposition**

For every continuous, three-times differentiable, strictly-decreasing demand function,  $p_0(x)$ , other than the CES, the set  $\Omega(p_0)$  corresponds to a smooth curve in  $\{\varepsilon, \rho\}$  space.

Exception: CES manifold is a point

## **Manifold Invariance**

• When is the demand manifold invariant to shocks?

▶ Graph

- ► Linear: p(x) = a bx► CARA:  $x(p) = a + b \log p$ ► LES:  $x(p) = a + bp^{-1}$ ► Translog:  $x(p) = (a + b \log p)/p$ Manifold is invariant to a and b
  - ► CES:  $p(x) = \beta x^{-1/\sigma}$  → Manifold is invariant to  $\beta$ , not to  $\sigma$

#### **Proposition**

Assume that  $\rho_x$  is non-zero. Then, the demand manifold is invariant with respect to a vector parameter  $\phi$  if and only if both  $\varepsilon$  and  $\rho$  depend on x and  $\phi$  through a common sub-function of either (a) x and  $\phi$ ; or (b) p and  $\phi$ ; i.e.:

$$\varepsilon(x,\phi)=\tilde{\varepsilon}[F(x,\phi)] \quad \text{and} \quad \rho(x,\phi)=\tilde{\rho}[F(x,\phi)]; \quad \text{or} \qquad \qquad \textbf{(8)}$$

$$\varepsilon(p,\phi) = \tilde{\varepsilon}[G(p,\phi)]$$
 and  $\rho(p,\phi) = \tilde{\rho}[G(p,\phi)]$  (9)

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# Competition Effects of a Globalization Shock: Setup

- Krugman (1979) → Melitz (2003) and beyond
- L identical consumers in a given country, k identical countries
- Globalization: increase in k
- ullet Firms sell in many markets, they sell x to an individual consumer, total output: y=kLx
- Additive separability  $\Rightarrow$  demand function for a typical good  $u'(x) = \lambda p$
- Ex-post profits of a typical firm:

$$\pi(\underbrace{c}_{-},\underbrace{\lambda}_{+},\underbrace{k}_{+}) \equiv \max_{y} \Big(p(y,\lambda,k) - c\Big)y$$

where  $p(y, \lambda, k) = \lambda^{-1} u'(y/kL)$ 

## **Free-entry Condition**

• Expected value of firm profits  $\bar{v}(\lambda, k) = \text{sunk entry cost } f_e$ :

$$\bar{v}(\underline{\lambda}, \underline{k}) \equiv \int_{\underline{c}}^{\overline{c}} v(c, \lambda, k) g(c) dc = f_e$$

with

$$v(c, \lambda, k) \equiv \max(0, \pi(c, \lambda, k) - f)$$

where g(c) is the distribution of firm marginal costs

#### Effects of a Globalization on Profits

Effects of globalization on every firm's profits:

$$\frac{d \log \pi(c, \lambda, k)}{d \log k} = \underbrace{\frac{\partial \log \pi(c, \lambda, k)}{\partial \log k}}_{(M)} + \underbrace{\frac{\partial \log \pi(c, \lambda, k)}{\partial \log \lambda}}_{(C)} \underbrace{\frac{d \log \lambda}{d \log k}}_{(C)}$$

where the change in the level of competition is determined in turn by:

$$\frac{d \log \lambda}{d \log k} = -\frac{\partial \log \bar{v}(\lambda, k)}{\partial \log k} / \frac{\partial \log \bar{v}(\lambda, k)}{\partial \log \lambda}$$

## Globalization as a Two-Edged Sword

Effects of globalization on every firm's profits:

- Direct effect: Market Expansion
  - Raises its profits
- Indirect effect: Competition
  - ► Raises all firms' profits ⇒ Encourages entry
    - ⇒ Increases competition
      - ⇒ Reduces each firm's profits
- Net effect ambiguous in general...

## Globalization as a Two-Edged Sword

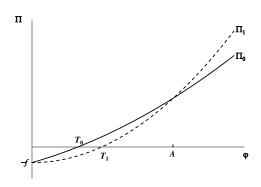
With additive separability:

$$\frac{d\log\pi(c,\lambda,k)}{d\log k} = \underbrace{1}_{(M)} - \underbrace{\frac{\varepsilon(c,\lambda,k)}{\bar{\varepsilon}(\lambda,k)}}_{(C)}$$

where  $\bar{\varepsilon}(\lambda,k)\equiv\int_{\underline{c}}^{\overline{c}} \frac{v(c,\lambda,k)}{\bar{v}(\lambda,k)} \varepsilon(c,\lambda,k) g(c)\,\mathrm{d}c$  is the firm-value-weighted average elasticity of demand across all firms

⇒ Globalization raises profits of larger firms and reduces those of smaller firms IFF demands are subconvex. ("Mathew Effect")

#### The Matthew Effect of Globalization



#### With subconvex demand:

- Large firms expand, small firms contract, some exit
- On average, exporters become more productive