

Macroeconomics B, EI060

Class 5

Frictions in financial markets

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What you will get from today class

- **Defaults** from borrowers.
 - Analysis with a **non-contingent bond** (Harms VI.3.1-3.3).
 - **Risk premia** (Vegh 2.4.3)
 - Analysis with **contingent assets** (Obstfeld and Rogoff 6.1.1.1-6.1.1.5).
- **Moral hazard** in international investment (OR 6.4.1).

A question to start

WILLINGN vs ABILITY

COST PAY vs. BENEFIT DEF

The threat of future complete exclusion from international financial markets can lead a country to repay its debts most of the time.

TILED / UNITED

Do you agree? Why or why not?

DEFAULT WITH NON – CONTINGENT ASSETS

Beyond full enforceability

- In previous classes, we assume that when a country borrow, it will repay the amount agreed upon in the future.
- Disputable assumption with countries, as harder to enforce payments than for households.
- We first consider borrowing in bonds.
 - Enforce repayment through threat of exclusion from markets.
 - Enforce through domestic cost of default.
 - Endogenous default probability and risk premium.

UNCER

Infinite horizon

- Small open economy with an infinite horizon and endowment.
- One good, with a bond denominated in the good with interest rate r .
Flow budget constraint (B is asset, so $(-B)$ is debt):

$$B_{t+1} + C_t = Y_t + (1 + r) B_t$$

- Iterate (with transversality condition) to get the intertemporal constraint:

$$\sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} C_s = (1+r) B_t + \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} Y_s$$

- Take a log utility of consumption, and the usual assumption of $\beta(1+r)=1$ to get perfect smoothing:

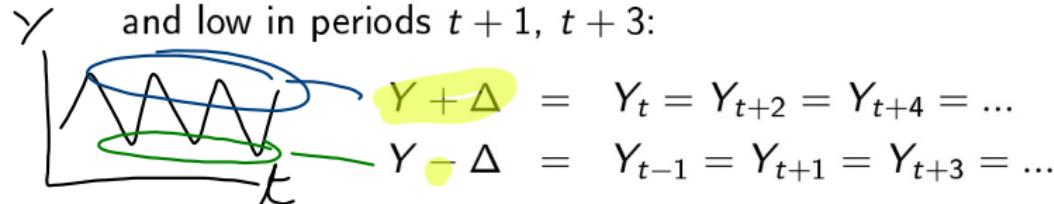
$B < 0 \rightarrow DEBT$

$$C_t = rB_t + \frac{r}{1+r} \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} Y_s$$

- This assumes that required payments are indeed made.

Fluctuating income

- Income has an "up - down" pattern, being high in periods $t, t+2, t+4$ and low in periods $t+1, t+3$:



- With commitments to payments (non default), consumption is:

$$C_t^{ND} = rB_t + Y + \frac{r}{2+r}\Delta$$

- This implies the following path for assets, with the agent saving in high income states and dissaving in low income states:

$$B_t = B_{t+4} = B_{t+2} = \dots$$

$$\frac{2}{2+r}\Delta + B_t = B_{t+1} = B_{t+3} = \dots$$

- This is not a problem if $B_t > 0$ as the agent never becomes a debtor.
But what if $B_t < 0$?

Consumption with default

$$\begin{aligned} B_K < 0 &\quad \text{IF PAY} \\ B_K = 0 &\quad \text{IF DEF} \end{aligned}$$

- Default sets the negative B_t to zero.
- As a punishment, the country is excluded from financial markets, even as a saver.
- Subsequent consumption is equal to endowment.

$$\begin{aligned} Y + \Delta &= C_t^D = C_{t+2}^D = C_{t+4}^D = \dots \\ Y - \Delta &= C_{t-1}^D = C_{t+1}^D = C_{t+3}^D = \dots \end{aligned}$$

Utilities

- With log utility, the utility in the absence of default is:

$$\begin{aligned} U_t^{ND} &= \sum_{s=t}^{\infty} (\beta)^{s-t} \ln(C_s^{ND}) \\ &= \frac{1+r}{r} \ln \left(rB_t + Y + \frac{r}{2+r} \Delta \right) \end{aligned}$$

- With default, the utility does not depend on debt:

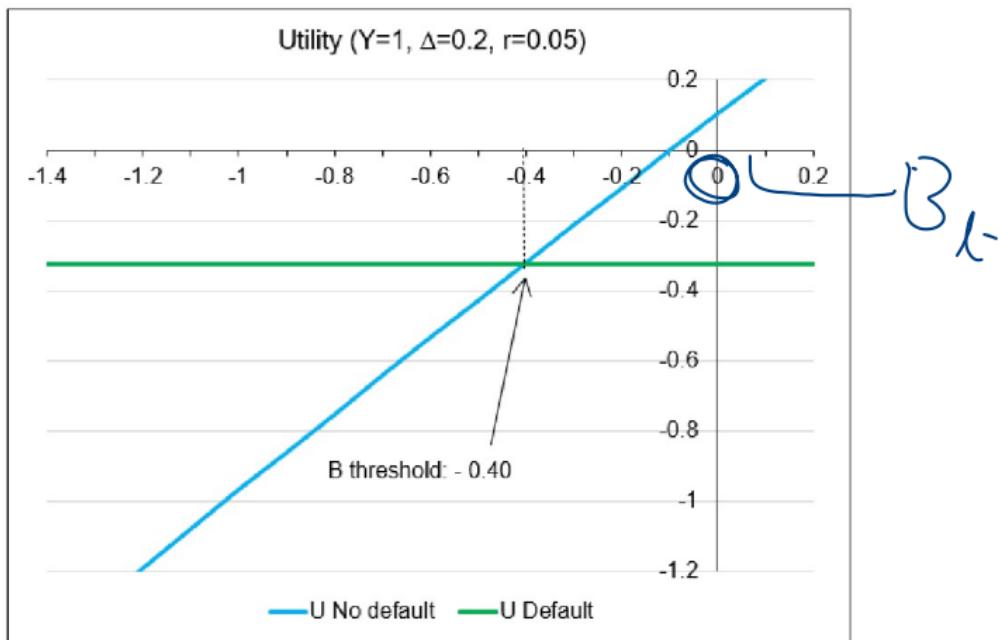
$$\begin{aligned} U_t^D &= \sum_{s=t}^{\infty} (\beta)^{s-t} \ln(Y_s) \\ &= \left(\ln(Y + \Delta) + \ln(Y - \Delta) \frac{1}{1+r} \right) \frac{(1+r)^2}{r(2+r)} \end{aligned}$$

B_t ~~\neq~~
 $U^D > U^{ND}$

- Default is optimal if the country is highly indebted, B_t is low ($B_t < 0$ if $\Delta = 0$).

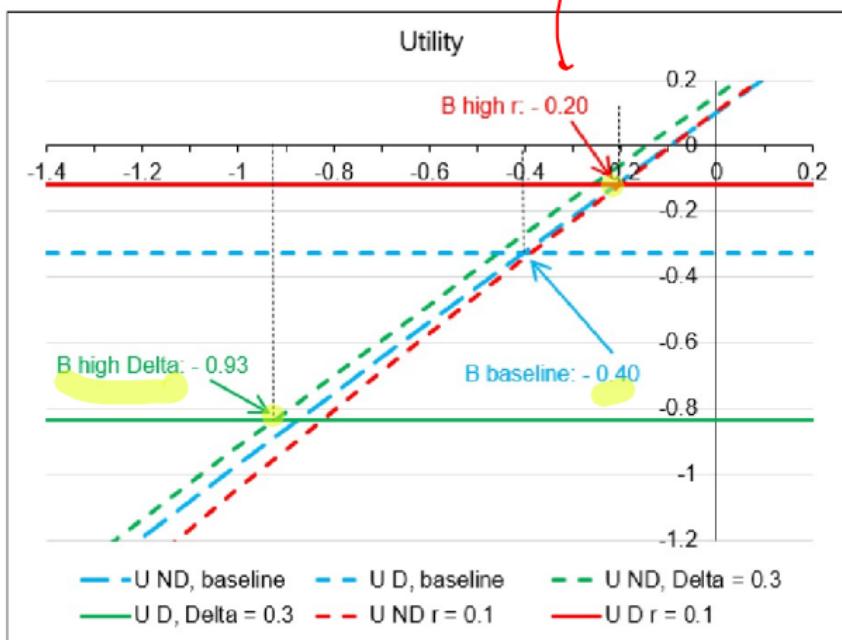
Baseline

- Default under high debt (left hand side) as $U_t^{ND} < U_t^D$.



Alternatives

- More debt is sustainable (low threshold) if Δ is high: big need to smooth.
- Less debt is sustainable (high threshold) if r is high: little weight put on the future cost of exclusion.



Some caveats

- Repayment is enforced by the threat of exclusion.
- This applies to all interaction: the country will not be able to borrow, but also not be able to save.
 - Exclusion from savings is questionable.
- Empirically, exclusion is not seen much. Countries that default can then re-access the market.

Domestic cost of default



- Sovereign default often disrupt credit access to private firms, making investment and output lower.
- The same model as above, without fluctuations ($\Delta = 0$).
- Default leaves output unchanged at time of default, but reduces all future outputs to γY , where $\gamma < 1$.
- Without default, consumption is (log utility, $\beta(1+r) = 1$):

$$C_t^{ND} = rB_t + Y$$

Allocation under default

- If the country defaults, consumption is

$$C_t^D = \frac{r}{1+r} \left(Y + \sum_{s=t+1}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} Y \right)$$
$$C_t^D = \frac{\gamma + r}{1+r} Y < Y$$

- Default is chosen if the debt $(-B_t)$ is high enough. More likely if r is high (little weight put on low future consumption) and γ is high (output is resilient):

$$\ln \left(\frac{\gamma + r}{1+r} Y \right) > \ln (rB_t + Y)$$
$$\frac{-B_t}{Y} > \frac{1 - \gamma}{r(1+r)}$$

$\gamma \uparrow \quad \frac{1-\gamma}{r(1+r)} \downarrow$

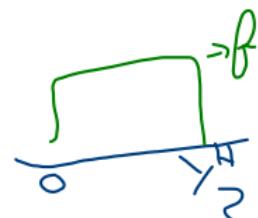
RISK PREMIUM

Debt and premium

- Debt level, probability of default, and risk premium are all related.
- Illustration through 2 period model (some of the algebra complex, focus on intuition).
- Lending through bonds, with interest rate $1 + r^s$ (in the absence of default). Default with probability π , in which case the lender gets $z(1 + r^s)$, where $1 - z$ is the haircut. The lender requires expected return $1 + r$.

TARGET

$$1 + r = (1 - \pi)(1 + r^s) + \pi z(1 + r^s)$$
$$1 + r = (1 - \pi(1 - z))(1 + r^s)$$



- From now on set $z = 0$.
- Period 2 output is uniformly distributed: $y_2 \in [0, y_2^H]$.

Default choice

- Without default, repays the debt with interest: $(1 + r^s) d_1$.
- With default, there is a true resource cost ϕy_2 born by the borrower.
Default is optimal if output is low:

$$y_2 < \frac{(1 + r^s) d_1}{\phi} < \quad (1 + r^s) d_1 > \phi y_2 \Rightarrow D \checkmark$$

- Probability π of default is the probability that output is lower than $(1 + r^s) d_1 / \phi$:

$$\pi = \frac{(1 + r^s) d_1}{\phi y_2^H} \leftarrow P_{\text{UNIF DIS}}$$

- Lender arbitrage then requires:

$$1 + r = \left(1 - \frac{(1 + r^s) d_1}{\phi y_2^H}\right) (1 + r^s)$$

- Quadratic expression in $1 + r^s$. Two equilibria, one with high debt cost and default risk (but unstable), one with low.

Intertemporal utility

RISK NEUTRAL

- Borrowing household maximizes a linear utility: $U = C_1 + \frac{1}{1+\delta} E C_2$.
Assume $r < \delta$ so she wants to borrow.
- Budget constraints (d_1 is debt), depending on default or not in the second period:

$$\beta = \frac{1}{1+\delta}$$

$$\begin{aligned}C_1 &= y_1 + d_1 \\C_2^{ND} &= y_2 - (1 + r^s) d_1 \\C_2^D &= (1 - \phi) y_2\end{aligned}$$

- Utility is raised by debt and reduced by the risk of losing some output under default (cost of default is ultimately borne by the borrower):

$$U = Y_1 + \frac{\delta - r}{1 + \delta} d_1 + \frac{1}{1 + \delta} E(y_2) - \frac{1}{1 + \delta} \pi \phi E(y_2 | D)$$

Optimal allocation

- Using the uniform distribution of output, utility is:

$$U = y_1 + \frac{\delta - r}{1 + \delta} d_1 + \frac{1}{1 + \delta} \frac{y_2^H}{2} (1 - \phi \pi^2) \quad \text{with } \pi = f(\delta)$$

- The first-order condition for debt is:

$$\delta - r = \phi \pi y_2^H \frac{\partial \pi}{\partial d_1}$$

- Borrowing has a benefit, as the agent is impatient ($\delta - r > 0$), but a cost as it raises the risk of costly default.
- Equation system consists of this optimality, as well as the probability of default and the lender's arbitrage:

$$\pi = \frac{(1 + r^s) d_1}{\phi y_2^H} ; \quad 1 + r = (1 - \pi)(1 + r^s)$$

Differentiating these gives an expression for $\frac{\partial \pi}{\partial d_1}$ (some algebra steps).

Solution

- The solution for the default probability, risk premium, and debt is:

PAY IF ND
LENDER OUTSIDE

$$\begin{aligned}\pi &= \frac{\delta - r}{1 + 2\delta - r} \\ \frac{1 + \rho s_{\text{risk}}}{1 + r} &= 1 + \frac{\delta - r}{1 + \delta} \\ \frac{d_1}{\phi y_2^H} &= \frac{(1 + \delta)(\delta - r)}{(1 + r)(1 + 2\delta - r)^2}\end{aligned}$$

- Recovery rate ϕ only affects the debt level, but not the probability of default and the risk premium.
- Higher impatience (~~higher~~) raises the interest rate, the probability of default, and the amount of debt.

DEFAULT WITH CONTINGENT ASSETS

- Two period small economy. Agent consumes only period 2, but trades financial assets in period 1 to maximize expected utility $U = Eu(C_2)$.
- Uncertain endowment in period 2: $Y_2 = Y + \epsilon$, where ϵ is a shock with uniform distribution over $[\epsilon_-, \epsilon_+]$.
- Contract with a risk neutral foreign insurer. Small economy pays a state contingent amount $P(\epsilon)$ in period 2 (negative amount is a payment from the insurer). $\xrightarrow{\text{STATE SPECIFIC}}$
 - Consumption is $C_2(\epsilon) = Y + \epsilon - P(\epsilon)$.
- Without default, $P(\epsilon)$ maximizes expected utility subject to the constraint that the insurer makes zero expected profits $0 = EP(\epsilon)$.
 - Full insurance: $P(\epsilon) = \epsilon$ and $C_2(\epsilon) = Y$. Risk efficiently moved from the risk averse agent to the risk neutral one.
- Default risk: country may decide not to pay when contract requires $P(\epsilon) > 0$ Insurer seizes a share η of output: $\eta(Y + \epsilon)$.

Contract with default

- Maximizes **expected utility**, subject to $0 = EP(\epsilon)$ (multiplier μ), and the **constraint** that payment cannot exceed what can be seized (inequality constraint, multiplier $\lambda(\epsilon)$):

INCENTIVE COMPATIBILITY $P(\epsilon) \leq \eta(Y + \epsilon) \Rightarrow \times$ DEFAULT

- Optimality conditions ($\pi(\epsilon)$ is the probability):

$$u'(Y + \epsilon - P(\epsilon)) = \mu - \frac{\lambda(\epsilon)}{\pi(\epsilon)} \quad \begin{matrix} > 0 \\ \text{IF} \\ \partial \cap \partial \end{matrix}$$
$$0 = \lambda(\epsilon) [P(\epsilon) - \eta(Y + \epsilon)] \quad \begin{matrix} \text{CST} \\ \text{IF} \end{matrix} \quad \begin{matrix} \text{BINDING} \\ \text{PINDING} \end{matrix}$$

States with high vs. low income

- With **low income**, ϵ below a threshold e , there is **no default** and full insurance (e and P_0 to be determined):

$$P(\epsilon) = P_0 + \epsilon ; C(\epsilon) = Y - P_0$$

- With **high income**, the country has an incentive to default, and the constraint is binding. There is **partial insurance** ($\eta < 1$):

$$P(\epsilon) = \eta(Y + \epsilon) ; C(\epsilon) = (1 - \eta)(Y + \epsilon)$$

- Combining the two sets of equations when $\epsilon_i = e$ gives:

$$P_0 = \eta(Y + e) - e, \text{ so when the constraint is not binding:}$$

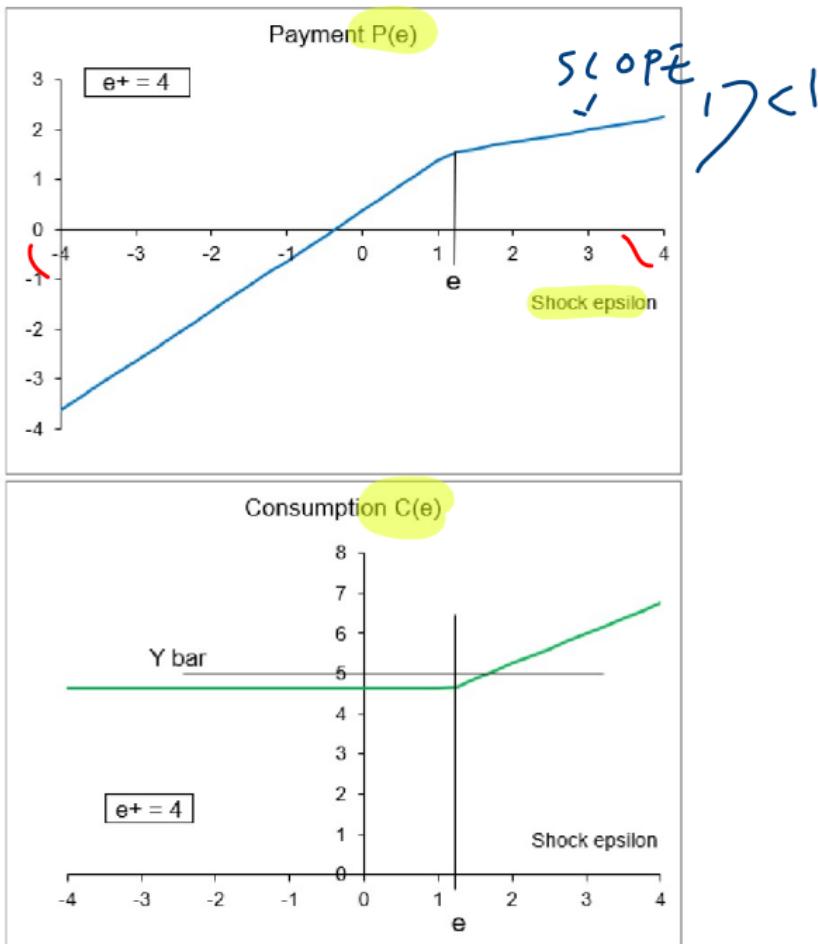
$$C(\epsilon) = Y - P_0 = (1 - \eta)(Y + e) \text{ THRESHOLD}$$

- e is obtained from the condition $0 = EP(\epsilon)$. It is increasing in η (insurance over a broad range). No insurance if $\eta = 0$ as $e = -\epsilon_+$:

$$e = -\epsilon_+ + 2\sqrt{\frac{\eta}{1-\eta} Y \epsilon_+}$$

$$C \neq f(\epsilon)$$

- To the left (low income) there is full insurance, but at a consumption level below the one under no frictions.
- To the right, the need to avoid default limits the extent of insurance.



- Insurance can be sustained over a larger range of shocks when the country can accumulate foreign assets which can be seized by the lender.
COLLATERAL
- Collateral allows for insurance even when no output can be seized ($\eta = 0$).
- In a repeated game, repayment can be sustained by the threat of exclusion from insurance in the future in case of default.
 - If enough weight is put on the future, full insurance can be sustained.
 - Requires infinite horizon, as with finite horizon the threat from exclusion loses power.

MORAL HAZARD

Unobservable actions

- So far all actions were **fully observable**. Consider now that the borrower can take some actions that the lender cannot see, and which may not be in the lender's favor.
- **Small open economy with two periods.** Endowment Y_1 in period 1. Consumption **only takes place in period 2**, with a **linear utility** $U = C_2$.
- The initial endowment can be invested in two ways.
 - **Safe investment** abroad with interest rate r .
 - **Risky technology** where **investment I** delivers Z with probability $\pi(I)$ and zero otherwise. **Investment raises the probability of success**, with decreasing returns ($\pi' > 0$, $\pi'' < 0$, $Z\pi'(0) > 1 + r$).

FOT OUTP: Z wth PROB $\tilde{\pi}(I)$

Frictionless case

- In period 1, the country borrows D (cost discussed below), invests I and lends L at the rate r . The investment / borrowing satisfies:

$$L + I = Y_1 + D$$

- If the investment is successful, the lender gets P . Otherwise, he gets nothing. The payment is such that the expected return corresponds to the one on the bond:

$$P\pi(I) = (1 + r)(I - Y_1)$$

- Expected consumption is:

$$EC_2 = (1 + r)(Y_1 - I) + \pi(I)Z$$

- Maximizing consumption equalizes the marginal product and cost of investment, and lending in bonds is pointless ($L = 0$):

$$Y_1 \cancel{\times} Z\pi'(\tilde{I}) = 1 + r$$

- Investment only reflects fundamentals $Z/(1 + r)$.

Asymmetric information

- The lender observes outputs Y_1 and Z , and the debt D . She cannot tell where the money is invested (I or L).
- The borrower chooses I and L , once D and P are set (no repayment in case of failure). There would be no problem if P can be indexed to I .
- Consumption of the borrower ($L = Y_1 + D - I$):

$$C_2 = \begin{cases} Z - P + (1 + r) \underbrace{(Y_1 + D - I)}_L & \text{if successful} \\ (1 + r)(Y_1 + D - I) & \text{if not} \end{cases}$$

- Expected consumption:

$$EC_2 = \pi(I)(Z - P) + (1 + r)(Y_1 + D - I)$$

Optimal investment

- Expected consumption is maximized by:

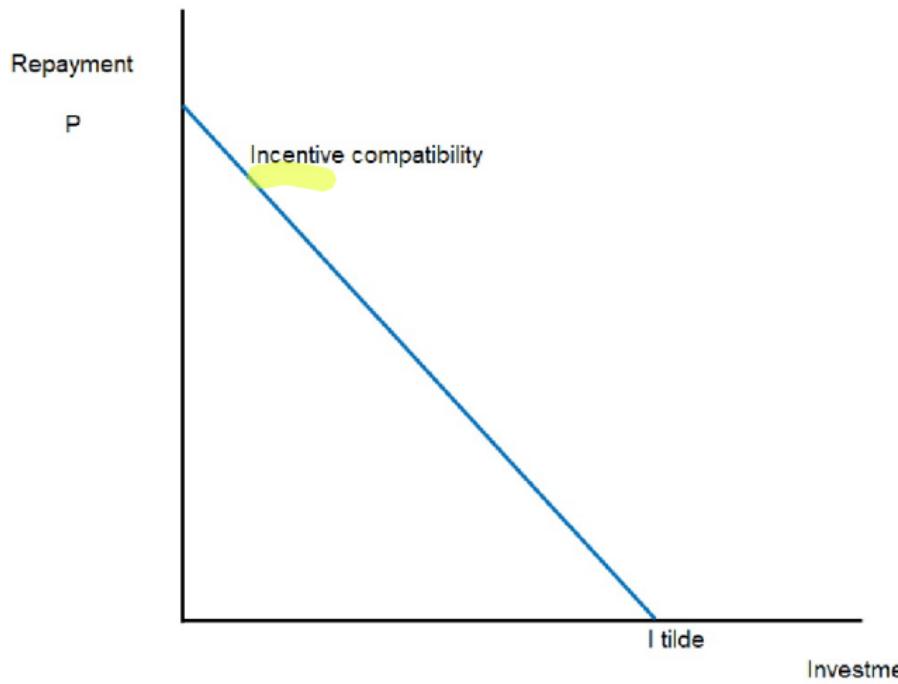
$$\pi'(I)(Z - P) = 1 + r$$

- As in the first best $Z\pi'(\tilde{I}) = 1 + r$, investment is lower under moral hazard: $\pi'(I) > \pi'(\tilde{I})$ hence $I < \tilde{I}$.
- The country invests some money on the world markets (L is not seen by the lender), which she can keep if things go wrong.
- Incentive compatibility condition: P is a decreasing function of I ($P = 0$ when $I = \tilde{I}$).

$$\begin{aligned}P &= Z - \frac{1+r}{\pi'(I)} \\ \frac{\partial P}{\partial I} &= \frac{1+r}{[\pi'(I)]^2} \pi''(I) < 0\end{aligned}$$

Incentive compatibility

- Higher repayment in case of success leads to higher “hiding” in bonds and lower investment.



Lender arbitrage

- Lender requires same expected return as the bond:

$$P\pi(I) = (1+r)(I - Y_1).$$

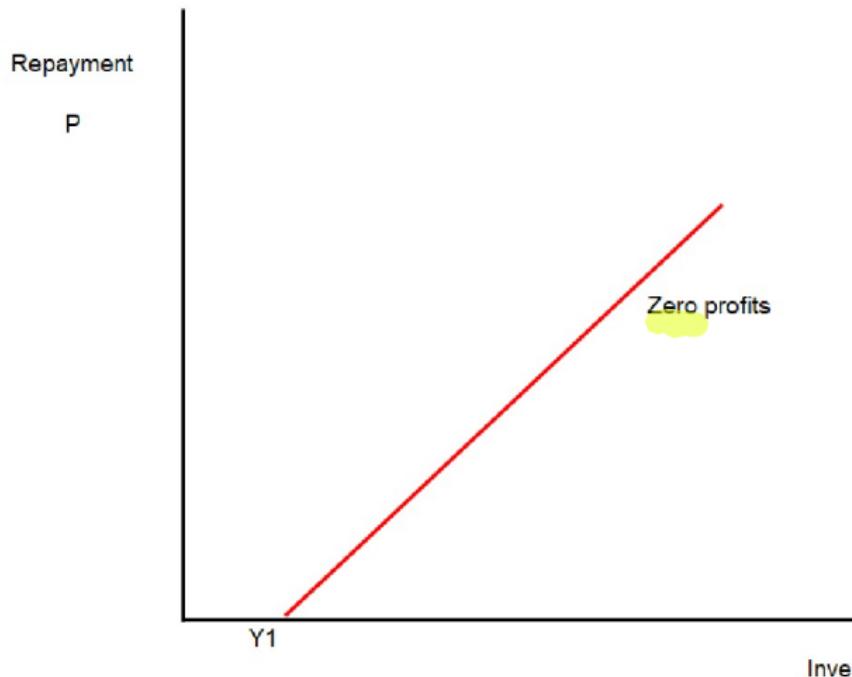
- Increasing relation between P and I , with $P = 0$ when $I = Y_1 < \tilde{I}$:

$$\frac{dP}{dI} = (1+r) \frac{\pi(I) - (I - Y_1)\pi'(I)}{[\pi(I)]^2} > 0$$

- Higher Y_1 lowers P for a given I .
- In equilibrium we need have $L = 0$. Otherwise the borrower would have an extra cost (in the end all inefficiencies are paid by the borrower).
- As $\pi'(I)(Z - P) = 1 + r$ (borrower's incentive). we have $\pi'(I)Z > 1 + r$. Marginal expected return of physical investment exceeds the risk free rate.

Arbitrage

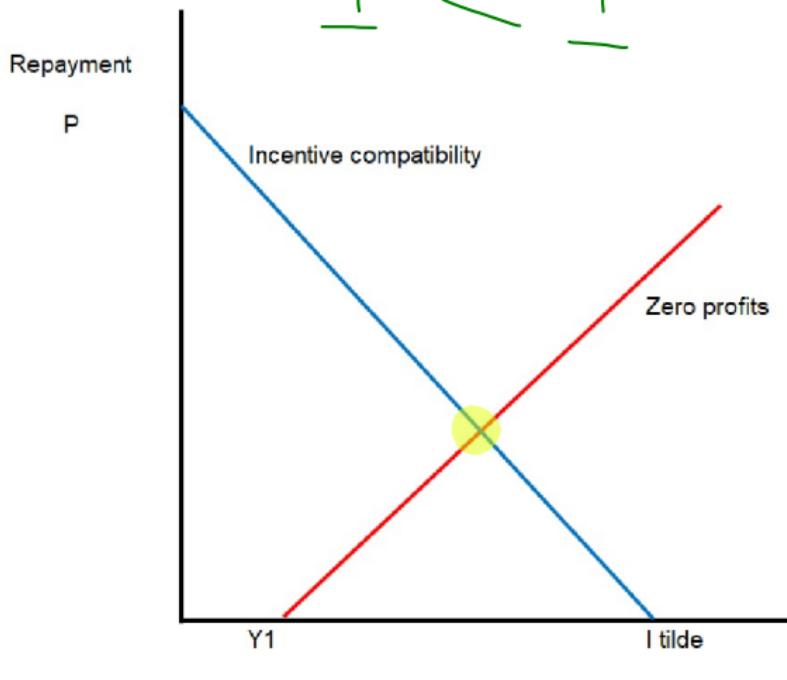
- Higher investment raises loan, hence expected repayment. Not fully undone by higher probability of success.



Equilibrium

- Both line intersect, with investment lower than under the efficient allocation.

$$\underline{I} < \underline{I}^F \beta$$



Higher income

- Higher Y_1 reduces the need to borrow, and lowers P in the lender's zero profits. Red line shifts to the right, with higher investment. Y_1 (net worth) matters in addition to $Z / (1 + r)$.

