ECON 39: Undergraduate International Trade
Problem Set #1
Winter 2018, Professor Treb Allen
Due: Tuesday January 16 at the beginning of class

Preliminaries

This week, we are learning about the Ricardian model of trade. A word of warning: the following problem set is hard. I am purposely keeping things abstract so that when it comes time for the first midterm, you will be more than adequately prepared!

Questions

1. Suppose there is a single country (call it country 1) inhabited by L_1 identical workers. Workers have one unit of time with which to produce and can choose to produce either product A or product B (or split her time between the two). A worker can make $\frac{1}{\alpha_1^A}$ units of product A in a unit of time or $\frac{1}{\alpha_1^B}$ units of product B in a unit of time.

   (a) Define the production possibility set and the production possibility frontier for country 1.

   (b) Suppose that each worker has preferences $U_1\left(C_1^A, C_1^B\right)$, where C_1^A is the quantity consumed of product A and C_1^B is the quantity consumed of product B. Let Q_1^A and Q_1^B denote the quantity produced of products A and B, respectively.

      i. List the exogenous model parameters.
      ii. List the endogenous model outcomes.
      iii. Define the equilibrium.

   (c) Suppose that $U_1\left(C_1^A, C_1^B\right) = \beta_1 \ln\left(C_1^A\right) + \beta_2 \ln\left(C_1^B\right)$, where $\beta_1 > 0$ and $\beta_2 > 0$. Find the equilibrium.

2. Now suppose that there is an additional country (call it country 2) inhabited by L_2 identical workers. As in country 1, workers have one unit of time with which to produce and can choose to produce either product A or product B (or split her time between the two). However, the productivity of workers in country 2 is different: a country 2 worker can make $\frac{1}{\alpha_2^A}$ units of product A in a unit of time or $\frac{1}{\alpha_2^B}$ units of product B in a unit of time.

   (a) Suppose that workers in both countries have identical preferences $U\left(C_i^A, C_i^B\right)$, where $i \in \{1, 2\}$ (that is, i is the name of the country).

      i. List the exogenous model parameters.
      ii. List the endogenous model outcomes.
      iii. Define the equilibrium.

   (b) Suppose that $\alpha_1^A > \alpha_2^A$ and $\alpha_1^B > \alpha_2^B$ (i.e. that country 2 is more efficient at producing both goods). In equilibrium, will it be the case that $C_1^A > Q_1^A$, $C_1^A < Q_1^A$, $C_1^A = Q_1^A$, or not enough information?

   (c) Suppose that $\frac{\alpha_1^A}{\alpha_1^B} > \frac{\alpha_2^A}{\alpha_2^B}$. In equilibrium, will it be the case that $C_1^A > Q_1^A$, $C_1^A < Q_1^A$, $C_1^A = Q_1^A$, or not enough information?

   (d) Continue to assume that $\frac{\alpha_1^A}{\alpha_1^B} > \frac{\alpha_B^A}{\alpha_B^B}$. In addition, suppose that $U\left(C_i^A, C_i^B\right) = \beta_1 \ln\left(C_i^A\right) + \beta_2 \ln\left(C_i^B\right)$, where $\beta_1 > 0$ and $\beta_2 > 0$ and $i \in \{1, 2\}$.

      i. Suppose that in equilibrium, one country completely specializes in the production of a good, while the other produces some of each good. Which country will be specializing in what good? What will the equilibrium relative price be? Find the equilibrium consumption and production of both goods in both countries.

ii. Suppose that in equilibrium, both countries completely specialize in the production of a single good. How much of each good will be produced in the world? Find the equilibrium relative price and the equilibrium consumption of both goods in both countries.