Econ 39: International Trade

Week #3: Modern Comparative Advantage

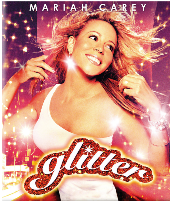
Treb Allen

Winter 2018

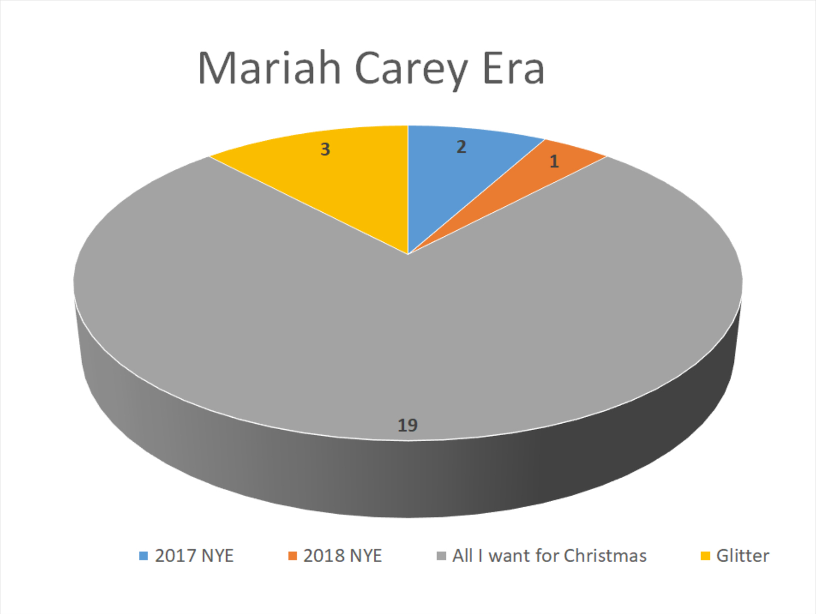
Plan for the week

- Finish the Ricardian trade model.
- Extend the principal of comparative advantage to a more realistic setting
  - Tuesday: Multiple goods (Dornbusch, Fischer and Samuelson '77)
  - Thursday: Multiple goods + multiple countries (Eaton and Kortum '02)
- For an excellent overview of both, read Eaton and Kortum "Putting Ricardo to Work" '12

Today's Teams




Today's Teams



Rudi Dornbusch

- MIT Economist.
- 1942-2002.
- Advised two of your book's authors (Krugman and Obstfeld)

Stanley Fischer

- MIT Economist (1977-1988)
- Advised your (probably) Economics 1 book author (Mankiw) and Ben Bernacke.
- Former Chief Economist of the World Bank
- Current Vice Chairman of the Federal Reserve

Paul Samuelson

- Nobel Prize in 1970.
- NYT top economist of the 20th century.
- His nephew is Larry Summers.
- Ph.D. advisors were Leontief and Schumpeter. After taking his oral examinations, Leontief turned to Schumpeter and asked, "Well, did we pass?"

Recap: Setup of the Ricardian model

- 2 countries
- 2 goods
- 1 factor of production
- Technology:
  - Unit labor cost: α_c^g for all $g \in \{FB, SB\}$ and $c \in \{US, MEX\}$
  - [Class question: what does α_c^g mean?]
- Suppose:

$$\frac{\alpha_{US}^{SB}}{\alpha_{US}^{FB}} > \frac{\alpha_{MEX}^{SB}}{\alpha_{MEX}^{FB}}$$

  - [Class question: which country has a comparative advantage in what?]
  - [Class question: which country will specialize in what?]

Ricardo's original example

- In 1817, Ricardo posited the following example of the number of workers required to make two goods in two countries:
# of workers to produce:	Footballs	Soccer balls
United States	100	120
Mexico	90	80
  - Paul Samuelson called these the "four magic numbers."
  - Note: Mexico requires less labor to produce both goods.
- But trade benefits both countries! To see this:
  - Suppose the world equilibrium free trade price is one.
  - U.S. can use 100 workers to produce 1 football, then buy 1 soccer ball, so soccer balls only costs England 100 workers (instead of 120 in autarky).
  - Mexico can use 80 workers to produce 1 soccer ball, then buy 1 football, so footballs only costs Portugal 80 workers (instead of 90 in autarky).

Prices of goods versus prices of labor

- Suppose the wage of workers is 1 in Mexico and w in the US.
  - [Class question: why is okay to assume the wage in Mexico is 1?]
- [Class question: if a worker in US made footballs, what would its price be?]
  - Answer: $p^{FB} = 100w$
- In general, the price of a good g being produced by a worker in country c is:

$$p^g = \alpha_c^g w_c,$$

  - where g is the good, c is the country, and α_c^g is the unit labor cost.
  - Tight link between the price of goods and the price of labor.

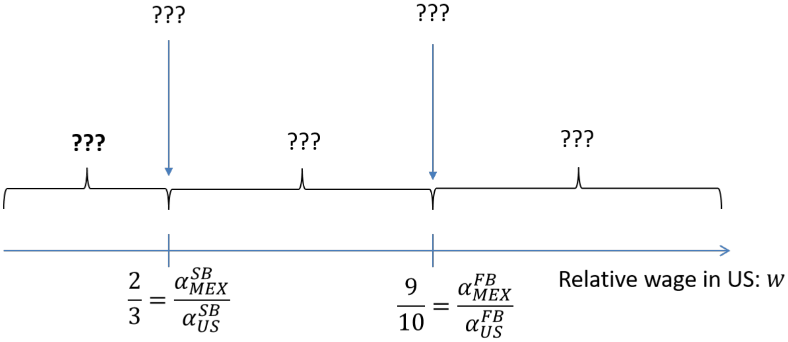
Ricardo's original example, revisited

- Now let us think about labor in terms of prices. The "four magic numbers" become:

Cost required to produce:	Footballs	Soccerballs
United States	$100w$	$120w$
Mexico	90	80

- Crucial insight: can figure out who produces what based on consumers choosing the cheapest source of a good:
  - [Class question: Suppose that $120w < 80 \iff w < \frac{2}{3}$: what country supplies what goods?]
    - Implication: if wage in US is less than 2/3 of Mexico, consumers in the world only buy from the U.S.
  - Can use this insight to figure out what country produces what goods as a function of wages.
    - [Class question: how is this related to how we split up the production based on world prices last class?]

Who produces what cheapest?



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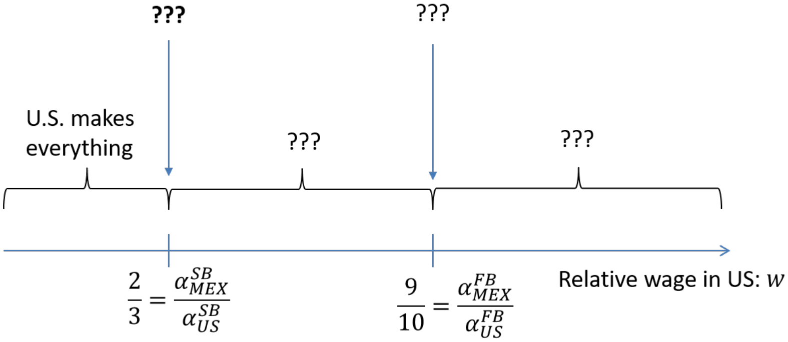

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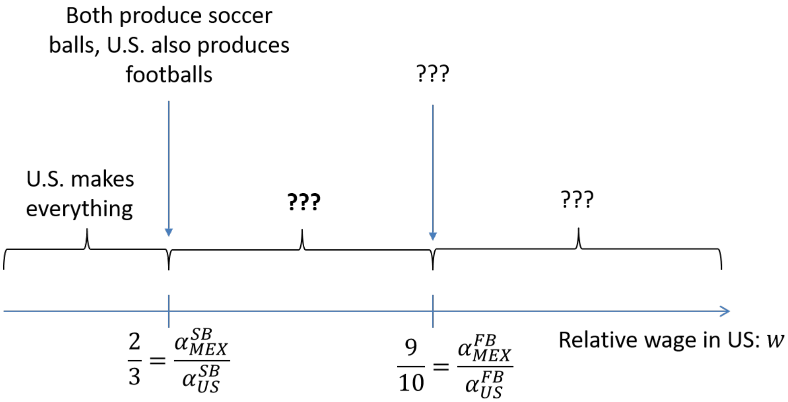

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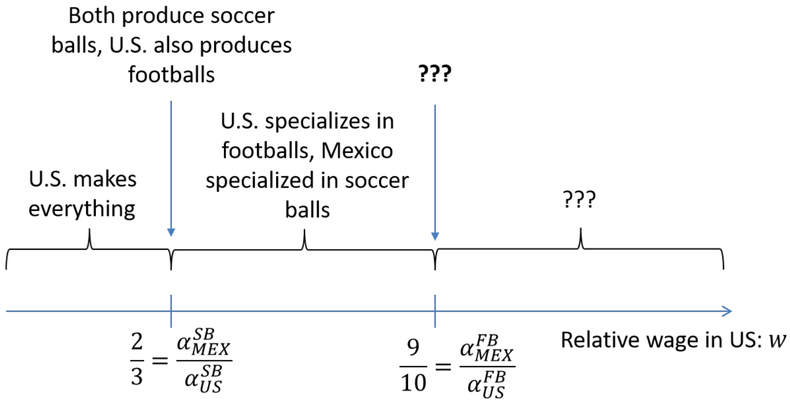

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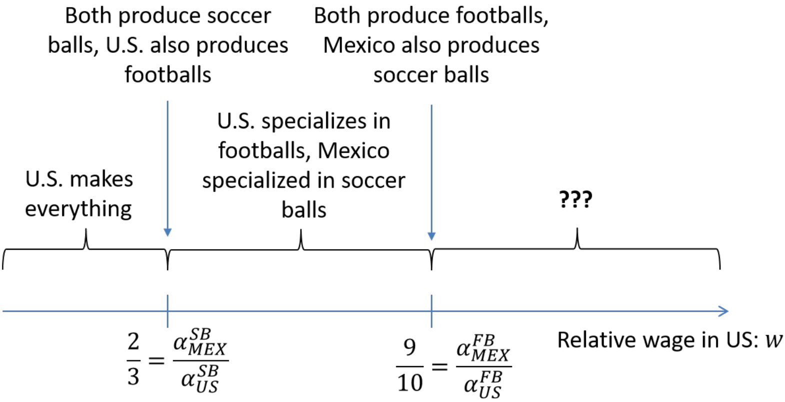

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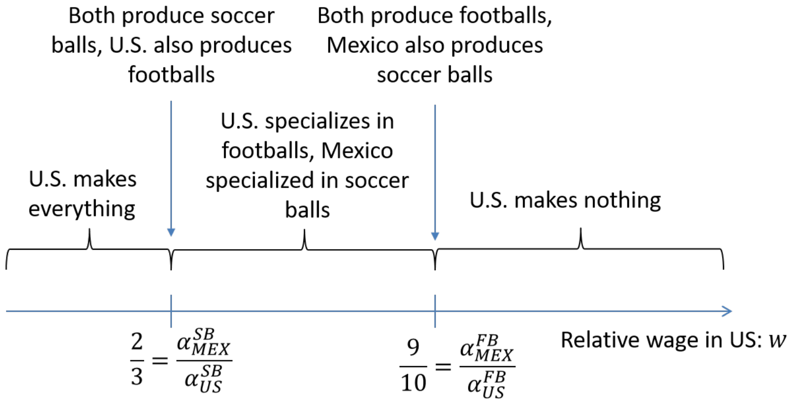
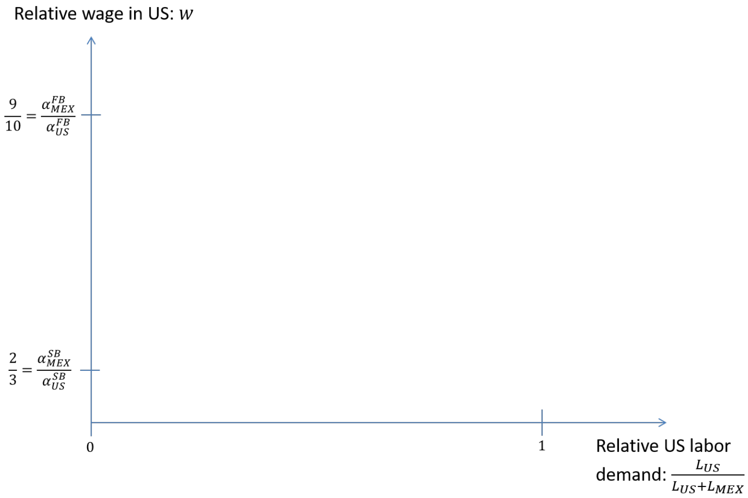


Constructing a labor demand curve

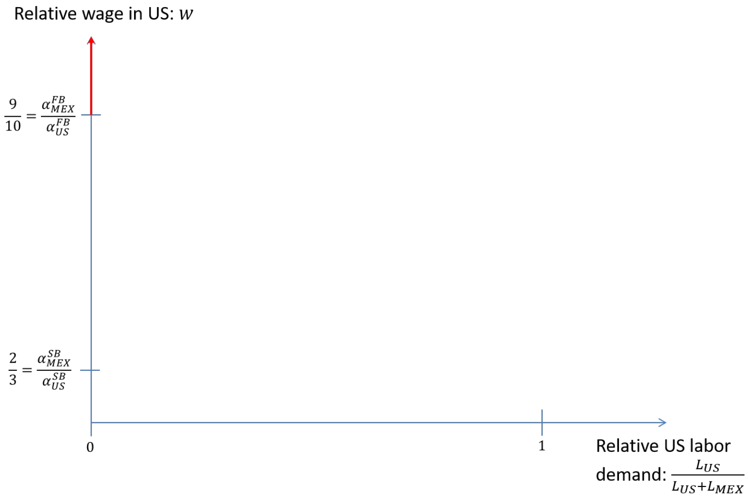
- (Technical assumption): Suppose preferences are homothetic and the same for both countries (so world demand depends only on world income and the relative prices).
- Then we can construct a world (relative) demand curve for labor from the U.S.
  - As the relative wage in the U.S. falls, the world buys more from the U.S., increasing the relative demand for U.S. labor.
  - Two margins at which demand expands:
    - *Extensive* margin: The U.S. produces more goods.
    - *Intensive* margin: The U.S. produces more units of a particular good.

The world labor demand curve



- [Class question: If $w > \frac{9}{10}$, how much U.S. and Mexico worker time will be demanded by the world?]

The world labor demand curve



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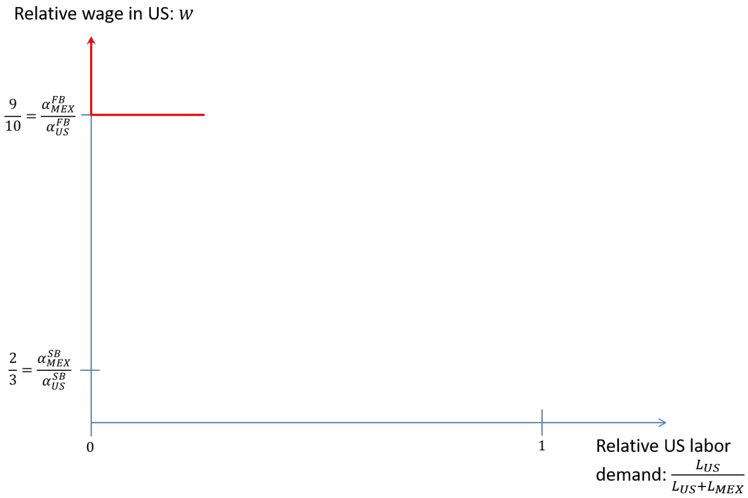
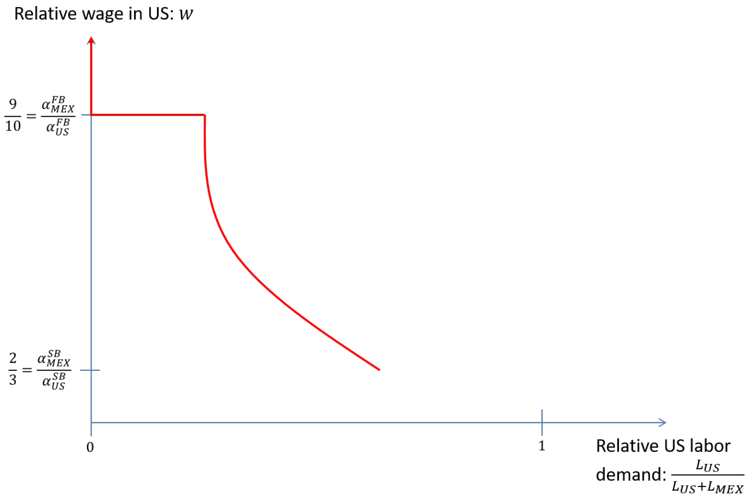


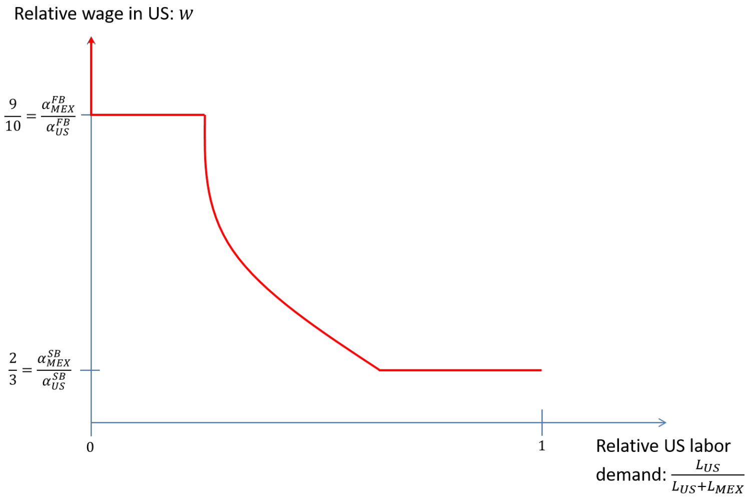
- [Class question: If $w \in \left(\frac{2}{3}, \frac{9}{10}\right)$, how much U.S. and Mexico worker time will be demanded by the world?]

The world labor demand curve



- [Class question: If $w = \frac{2}{3}$, how much U.S. and Mexico worker time will be demanded by the world?]

The world labor demand curve



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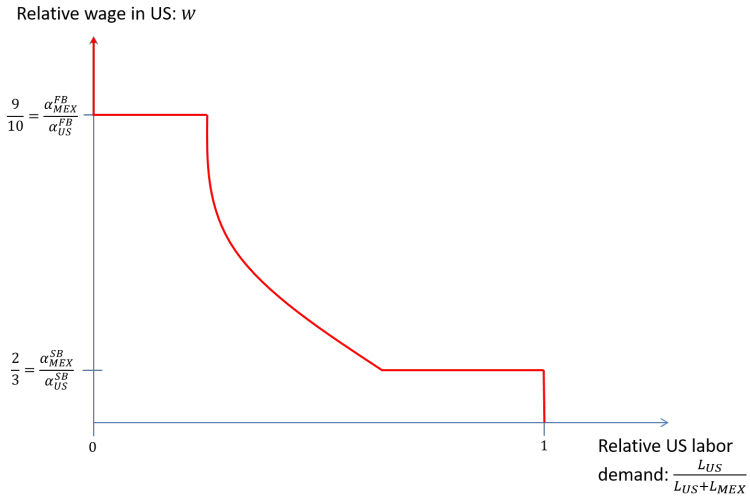
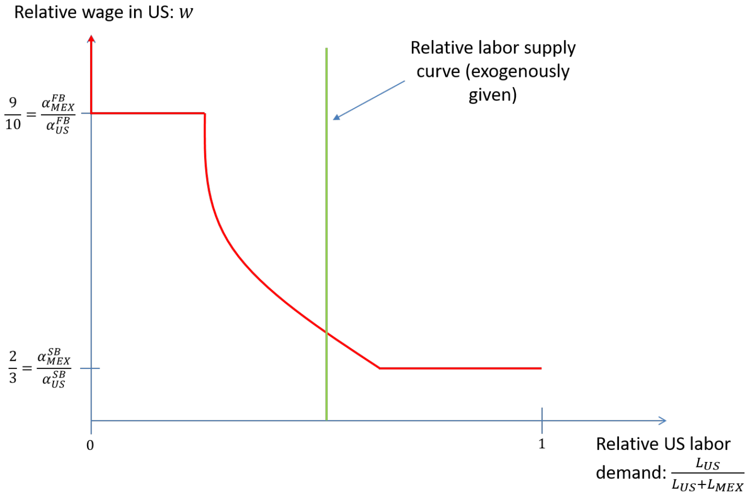


- [Class question: How is the equilibrium relative wage determined?]

Model equilibrium

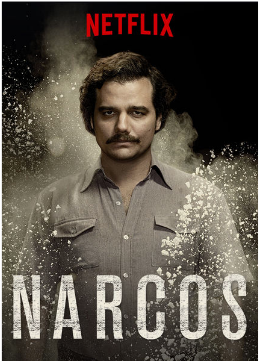


- [Class question: Where does a fall in wages have an extensive margin effect and where does it have an intensive margin effect?]

Plan for the day

- Finish the DFS '77 Ricardian framework with many goods
- Then extend to incorporate more than two countries.
- This was thought to be impossible, for reasons that will become clear.
- Eaton and Kortum '02 discovered an elegant probabilistic technique that makes the model surprisingly tractable.
- Payoff is huge: we can now analyze real world trade data using a Ricardian model.
- Warning: some of the math is tricky. Ask questions when things are not clear!

Today's Teams



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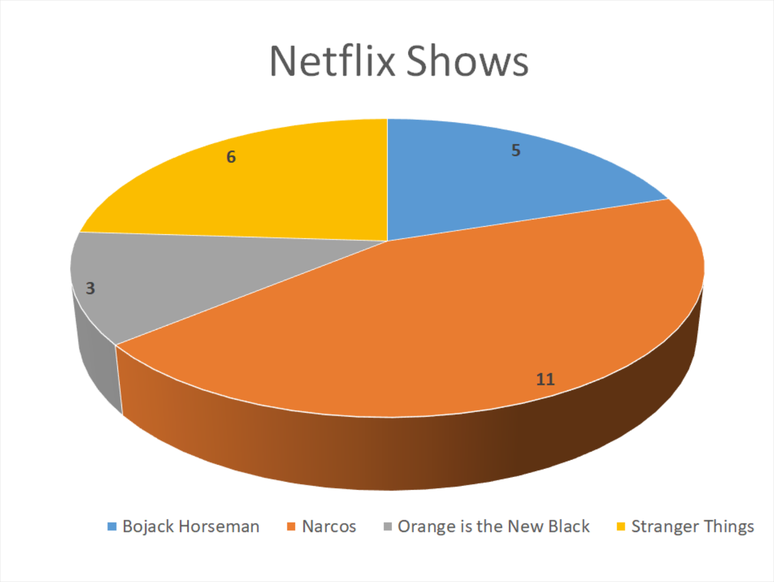
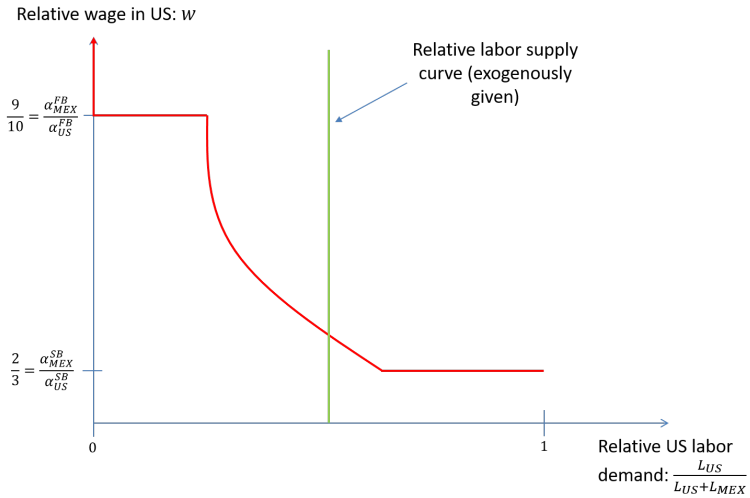


Recap: Basic Ricardian Model



Three goods

- Now let us see what happens if we introduce another good: baseballs.
- Suppose it takes 100 US workers or 75 Mexican workers to produce a baseball, i.e. $\alpha_{US}^{BB} = 100$ and $\alpha_{MEX}^{BB} = 75$.
- Note that $\frac{\alpha_{MEX}^{BB}}{\alpha_{US}^{BB}} = \frac{3}{4}$.
- We can construct a **chain of comparative advantage** across all three goods:

$$\frac{\alpha_{MEX}^{FB}}{\alpha_{US}^{FB}} = \frac{9}{10} > \frac{\alpha_{MEX}^{BB}}{\alpha_{US}^{BB}} = \frac{3}{4} > \frac{\alpha_{MEX}^{SB}}{\alpha_{US}^{SB}} = \frac{2}{3}$$

  - [Class question: can you always construct a chain of comparative advantage with two countries and more than two goods?]
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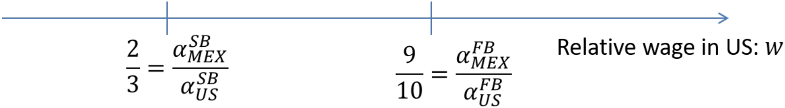
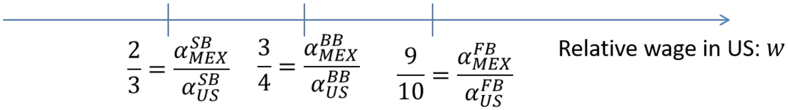
Three goods: Who produces what cheapest?



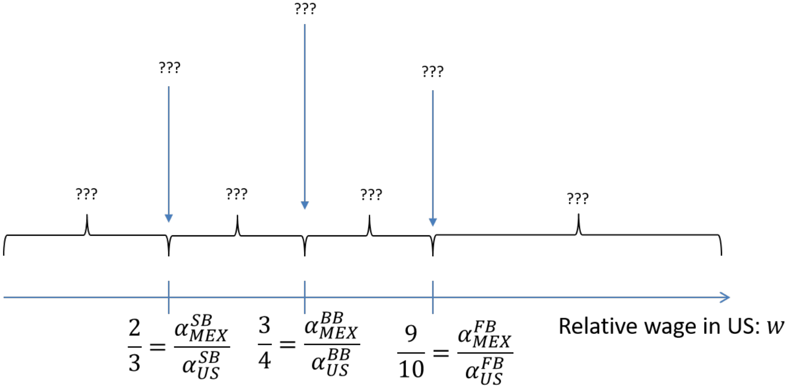
- [Class question: where do we add baseballs to this figure?]

Three goods: Who produces what cheapest?



- [Class question: Who produces what when $w < \frac{2}{3}$?]

Three goods: Who produces what cheapest?



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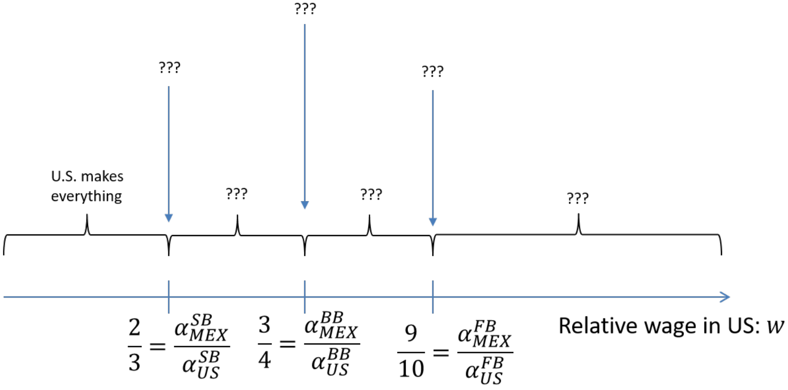

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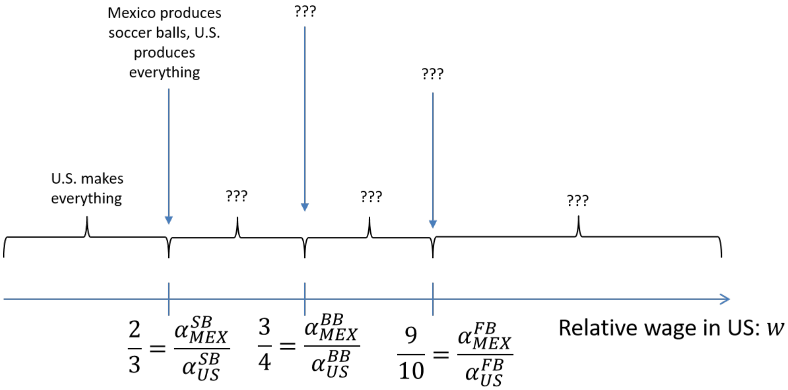
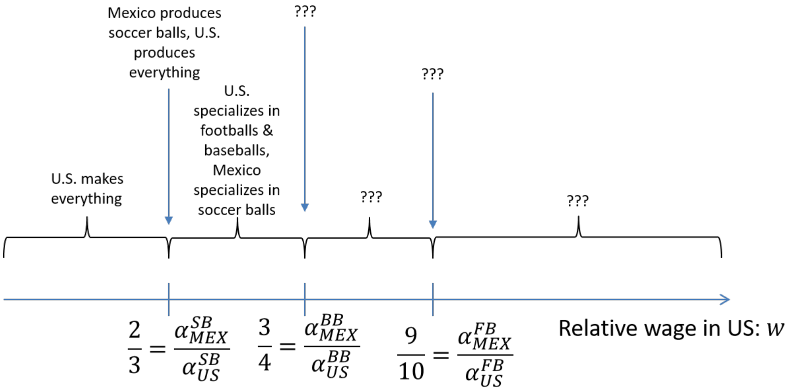


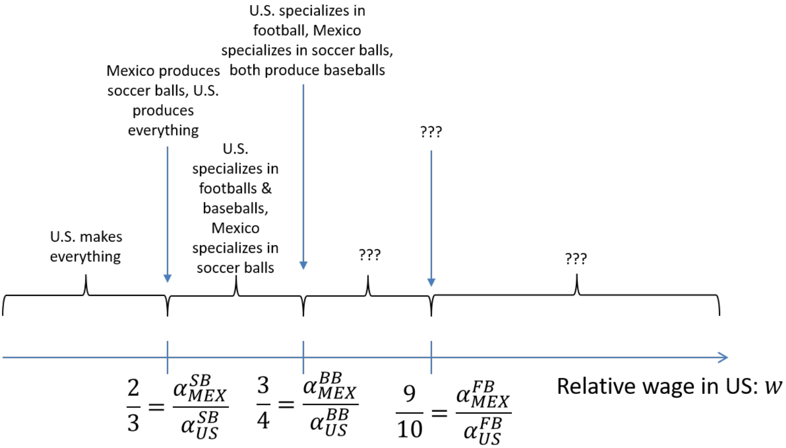
- [Class question: Who produces what when $w \in \left(\frac{2}{3}, \frac{3}{4}\right)$?]

Three goods: Who produces what cheapest?



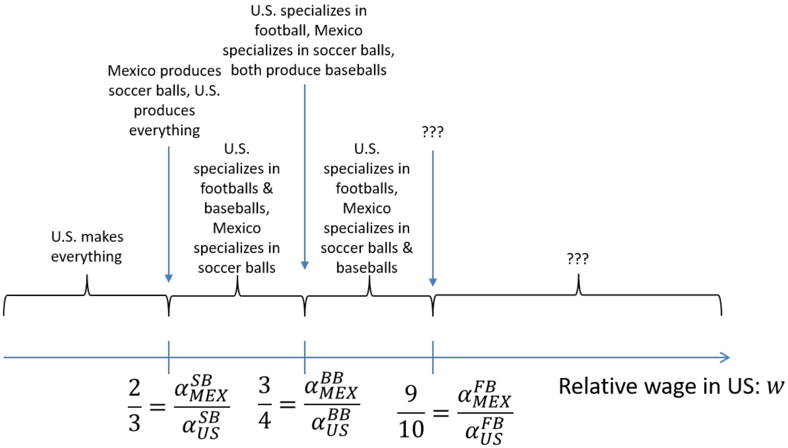
- [Class question: Who produces what when $w = \frac{3}{4}$?]

Three goods: Who produces what cheapest?



- [Class question: Who produces what when $w \in \left(\frac{3}{4}, \frac{9}{10}\right)$?]

Three goods: Who produces what cheapest?



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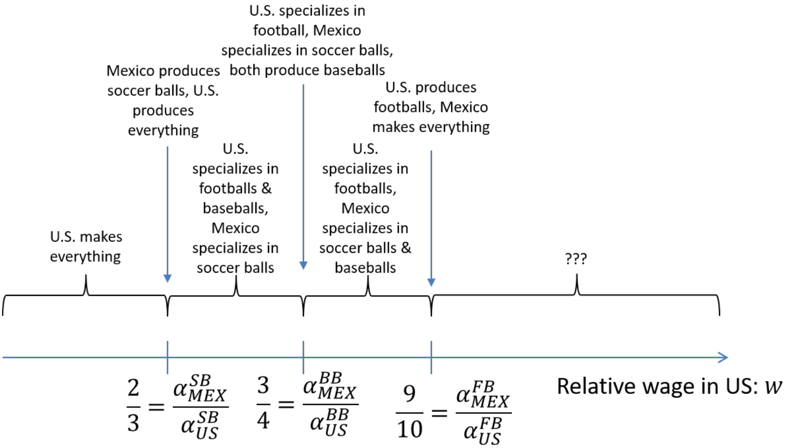

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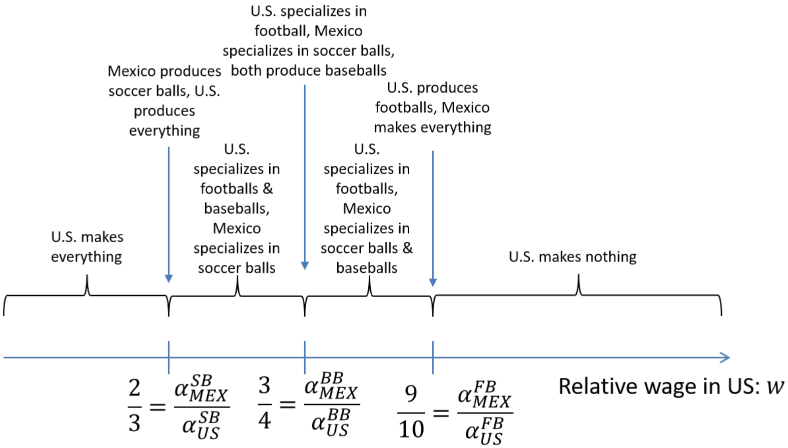
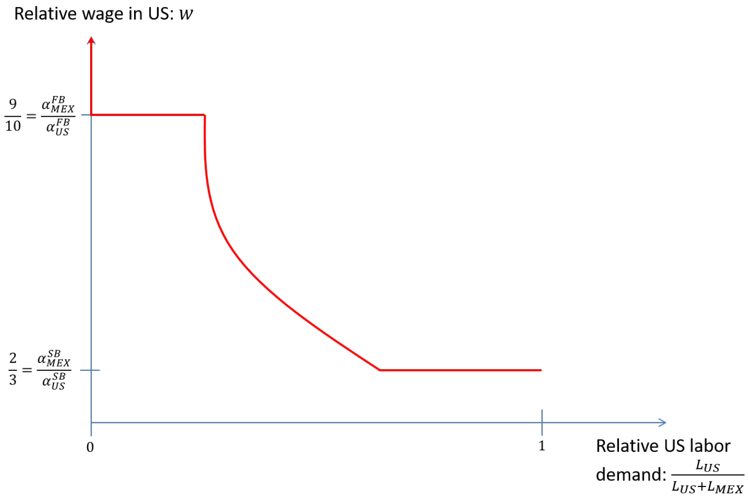
Three goods: Who produces what cheapest?



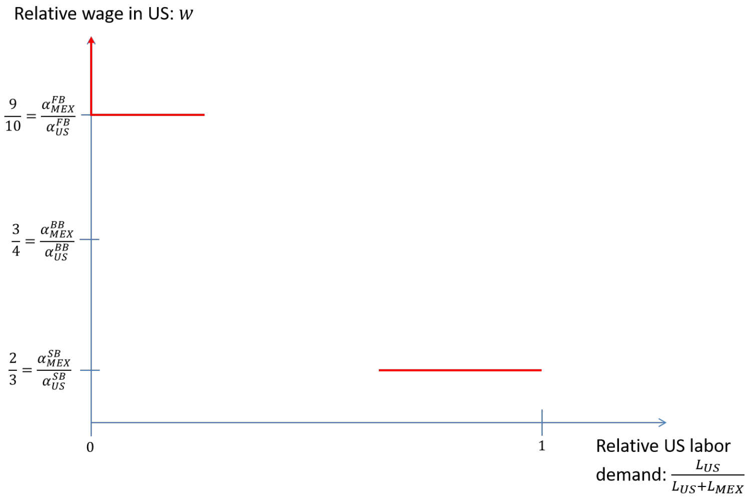
- [Class question: What does the relative demand for labor look like?]

Three goods: Relative labor demand



- [Class question: How does the relative demand curve change from the two good case?]

Three goods: Relative labor demand



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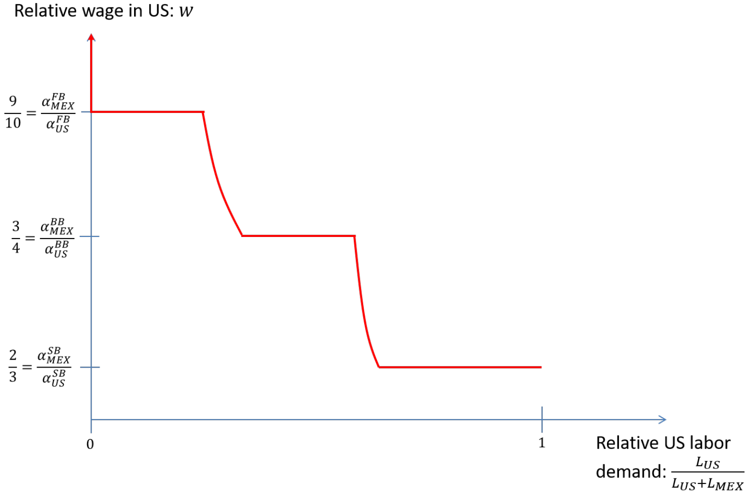
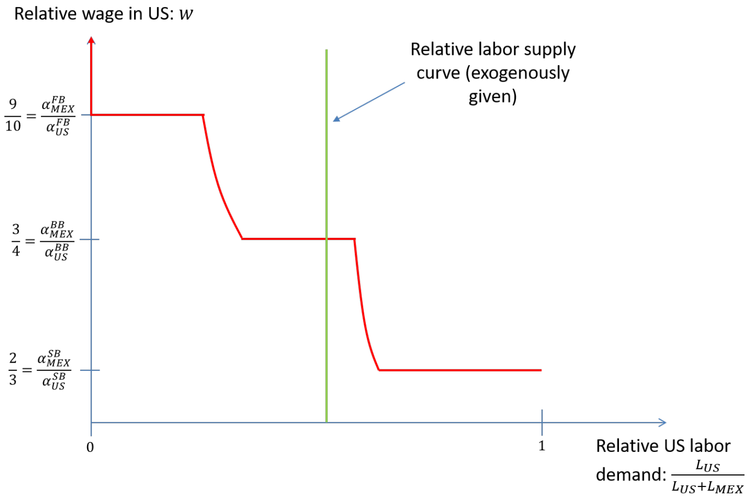
Three goods: Relative labor demand



- [Class question: How do we find the equilibrium?]

Three goods: Equilibrium

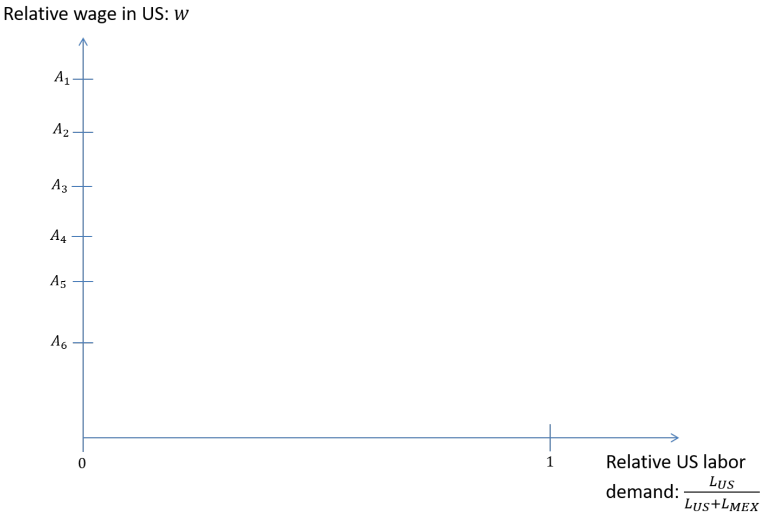


- [Class question: How many different types of equilibria are possible?]

G goods

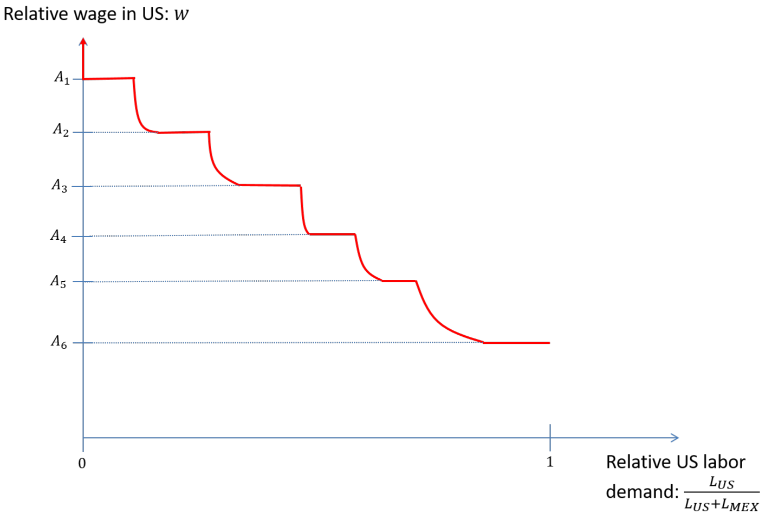
- Now suppose there are $G > 2$ different goods.
- Define $A_g \equiv \frac{\alpha^g_{MEX}}{\alpha^g_{US}}$ to be the efficiency of production of good $g \in \{1, ..., G\}$ in US relative to Mexico.
- Without loss of generality, assume that if $g' > g$, then $A_{g'} \leq A_g$ for all $g, g' \in \{1, ..., G\}$.
  - This implies that we have $A_1 \geq A_2 \geq ... \geq A_G$
  - [Class question: Why can this be done without loss of generality?]

G goods: Relative labor demand curve



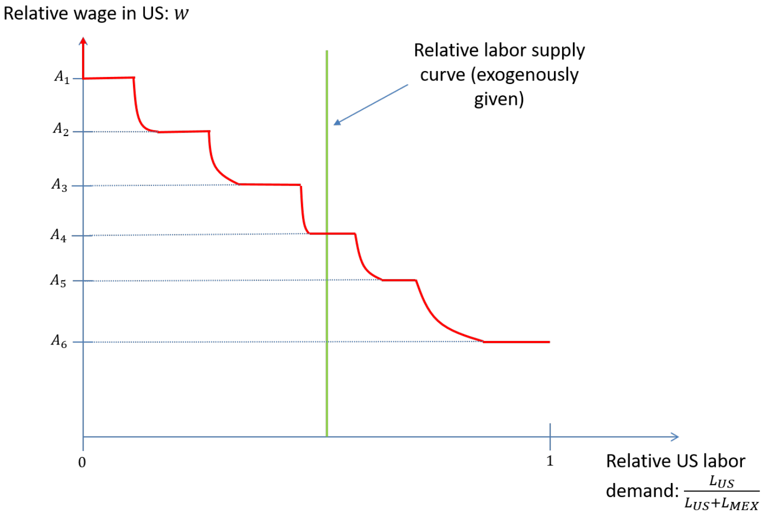
- [Class question: What will the relative demand look like?]

G goods: Relative labor demand curve



- [Class question: How do we determine the equilibrium?]

G goods: Relative labor demand curve

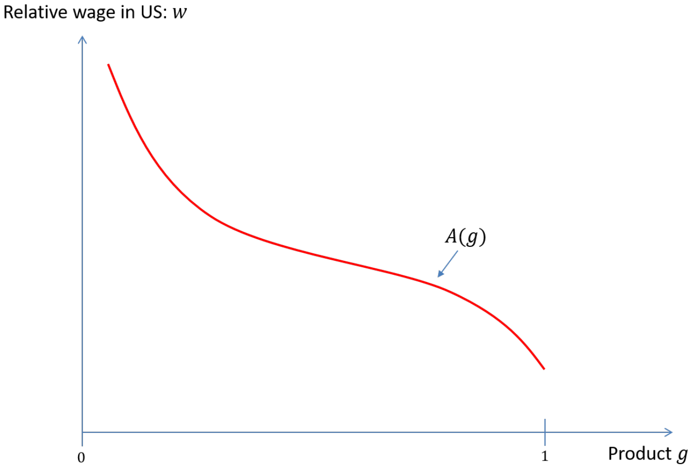


- [Class question: What is the equilibrium pattern of specialization?]

Dornbusch, Fischer, and Samuelson (AER 1977)

- As we saw above, introducing many goods into the two-country Ricardian model, while possible, makes it a pain to solve:
  - With G goods, there are $2G-1$ possible patterns of specialization to check.
- Dornbusch, Fischer, and Samuelson (American Economic Review, 1977) key insight: A continuum of goods makes things much easier!
  - Suppose there are $g \in [0,1]$ goods.
  - As above, let $A(g) \equiv \frac{\alpha(g)_{MEX}}{\alpha(g)_{US}}$ be the productivity of US producing good g relative to Mexico.
  - As above, organize goods so that if $g' > g$, then $A(g) \geq A(g')$.
  - That is, we can treat $A(g)$ as a downward sloping function of g.

Comparative advantage schedule



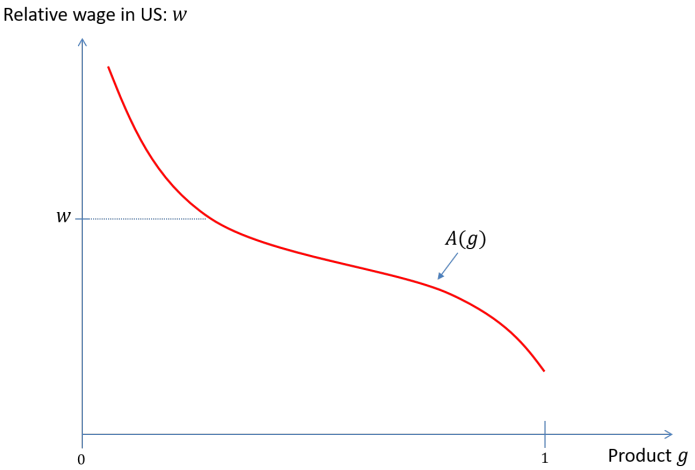
Patterns of production

- Suppose the equilibrium relative wage in the U.S. is w.
- Then there will exist some good $g^* \in [0,1]$ such that:

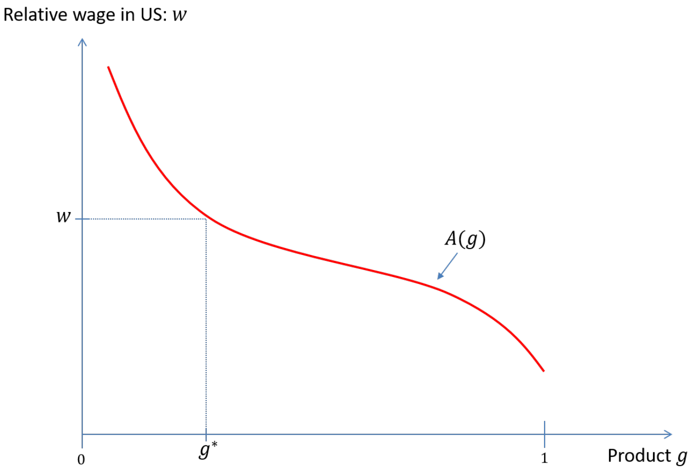
$$A(g^*) = w$$

  - [Class question: who will produce goods $g \in [0, g^*)$?
  - [Class question: who will produce goods $g \in (g^*, 1]$?
  - [Class question: who will produce good g^*?
- Because good g^*comprises a tiny fraction (formally, measure zero) of the aggregate economy, we can ignore it.
  - This is why the continuum makes things easier: it allows us to only look at equilibria where each good is only produced by one country!

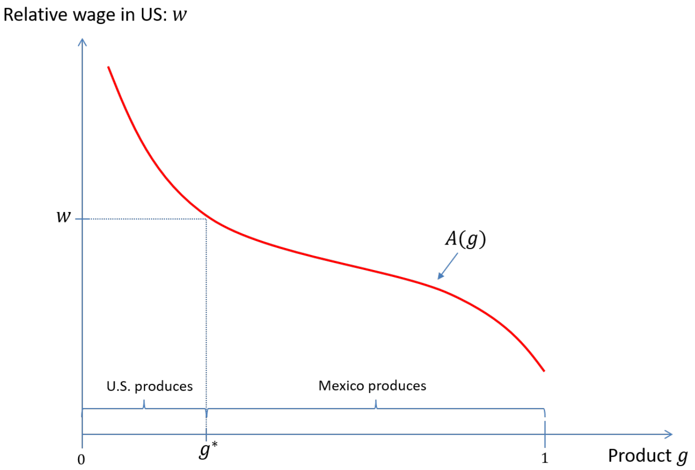
Patterns of production



Patterns of production



Patterns of production



Constructing the supply curve

- Note that the x-axis has changed from the previous demand curve: instead of relative labor demand/supplied, it is indexed by the goods.
  - To find the equilibrium, we need to calculate the amount of goods the U.S. supplies as a factor of its wage.
- Suppose preferences in each country $c \in \{US, MEX\}$ are:

$$U_c = \int_0^1 \log C_c(g)\, dg$$

  - [Class question: What do we know about the solution of the utility maximization problem from these preferences?]

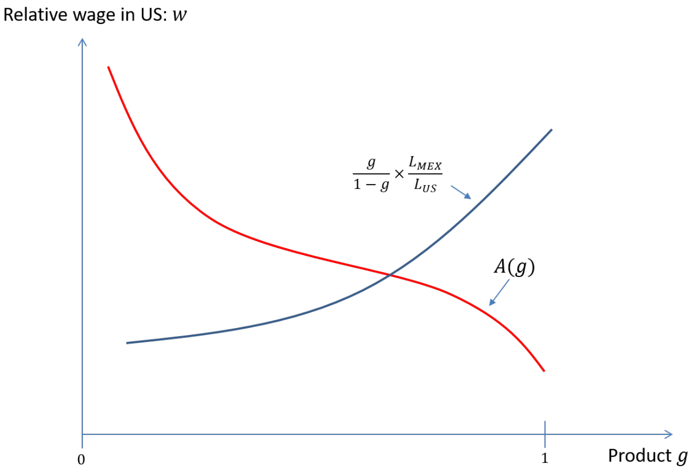
Constructing the supply curve (ctd.)

- Note:
  - Payments to U.S. workers: wL_{US}.
  - Total income in world: $Y^{world} = wL_{US} + L_{MEX}$
  - At threshold g, total expenditure on U.S. products (from Cobb-Douglas preferences): gY^{world}.
- For markets to clear, it must be the case that the amount of money spent on goods produced in the U.S. are equal to the payments of U.S. workers:

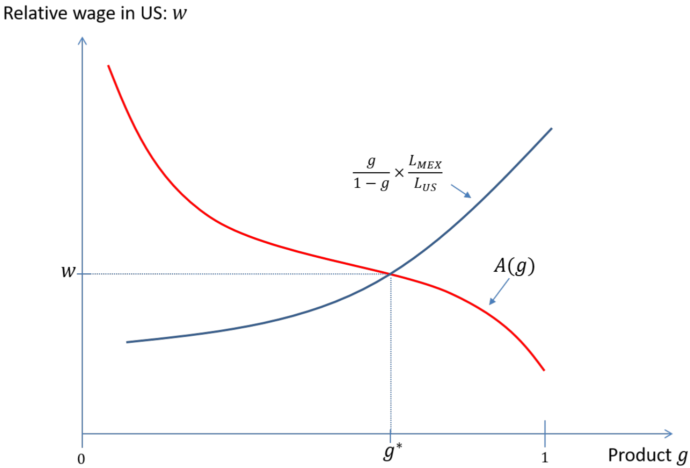
$$wL_{US} = gY^{world} \iff$$
$$wL_{US} = g\left(wL_{US} + L_{MEX}\right) \iff$$
$$w = \frac{g}{1-g} \times \frac{L_{MEX}}{L_{US}},$$

which gives us our supply curve as a function of g!

Equilibrium



Equilibrium

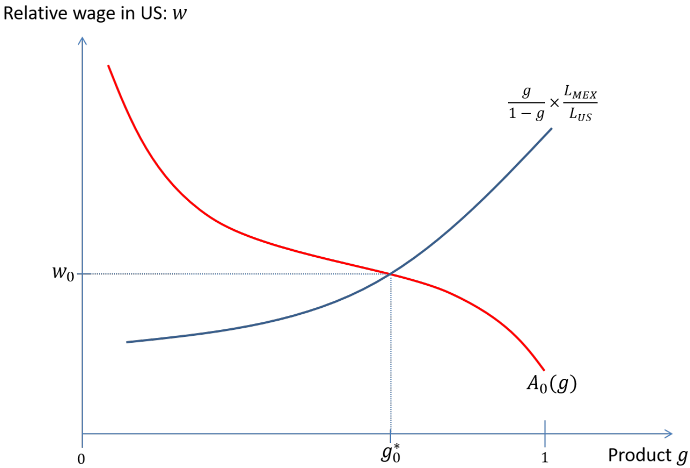


- [Class question: Define the equilibrium.]

Counterfactuals in DFS

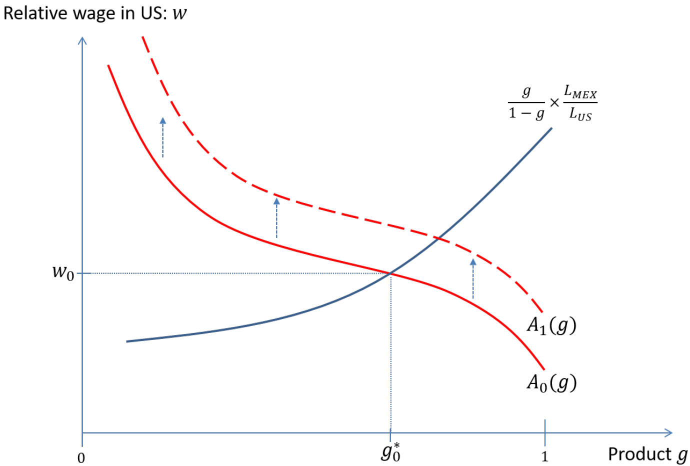
- Suppose that the U.S. gets more productive at producing all goods.
  - How will this affect U.S. relative wages?
  - How will this affect which goods the U.S. produces?

Equilibrium



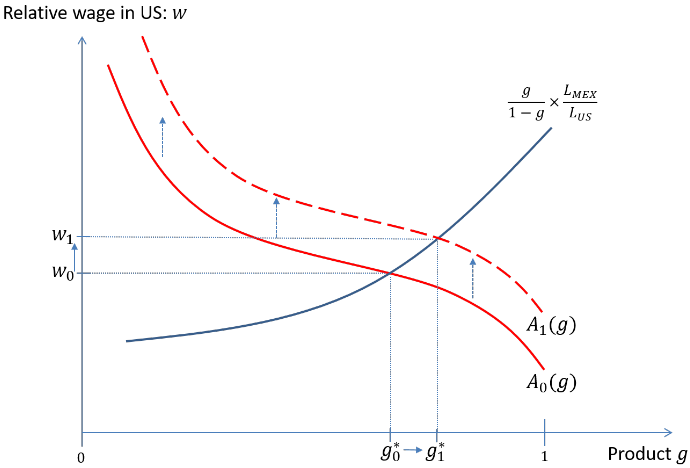
- [Class question: How will this figure change?]

Equilibrium



- [Class question: What happens to wages and patterns of trade?]

Equilibrium



- [Class question: What is the intuition for the change in wages?]

Introducing trade costs

- Thus far in the course, we have assumed the two countries are perfectly integrated when trade occurs.

- Now suppose there are trade costs.
  - We will model trade costs as **iceberg trade costs**: In order for one unit of a good to arrive in the destination, $\tau \geq 1$ units of the good must be shipped.

  - [Class question: Why do you think these costs called "iceberg" costs?]

Consumer sourcing decision

- If a consumer in the US wants to import one unit of good g from Mexico, she must buy τ units, so her cost is $\tau p_{MEX}(g)$.
- A consumer in the US will then choose to purchase locally if:

$$\begin{aligned}
p_{US}(g) \leq \tau p_{MEX}(g) &\iff \\
\alpha_{US}(g)\, w \leq \tau \alpha_{MEX}(g) &\iff \\
w \leq \tau \times \frac{\alpha_{MEX}(g)}{\alpha_{US}(g)} &\iff \\
w \leq \tau \times A(g)
\end{aligned}$$

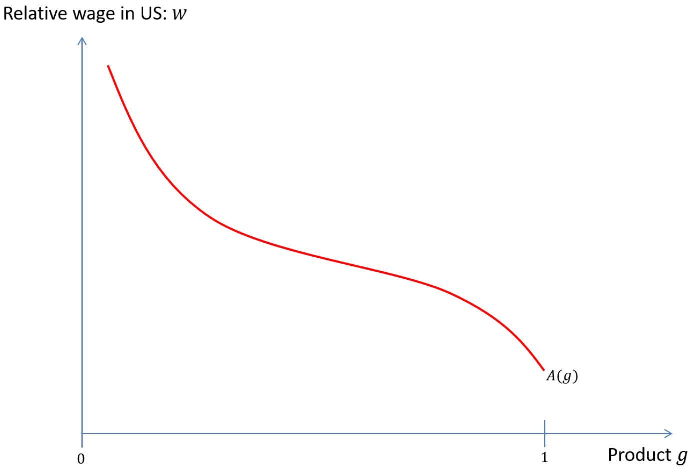
Consumer sourcing decision (ctd.)

- [Class question: what about a consumer in Mexico deciding whether or not to import g?]
  - Derivation:

$$\begin{aligned} p_{MEX}(g) \leq \tau p_{US}(g) &\iff \\ \alpha_{MEX}(g) \leq \tau \alpha_{US}(g) w &\iff \\ w \geq \frac{1}{\tau} \times \frac{\alpha_{MEX}(g)}{\alpha_{US}(g)} &\iff \\ w \geq \frac{1}{\tau} \times A(g) & \end{aligned}$$

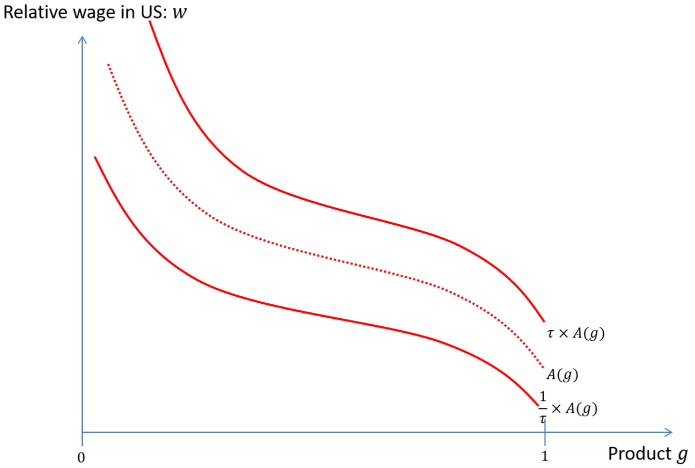
- Hence, if $w \in \left(\frac{1}{\tau} A(g), \tau A(g)\right)$, both countries will source locally and no trade will occur.

Consumer sourcing decision

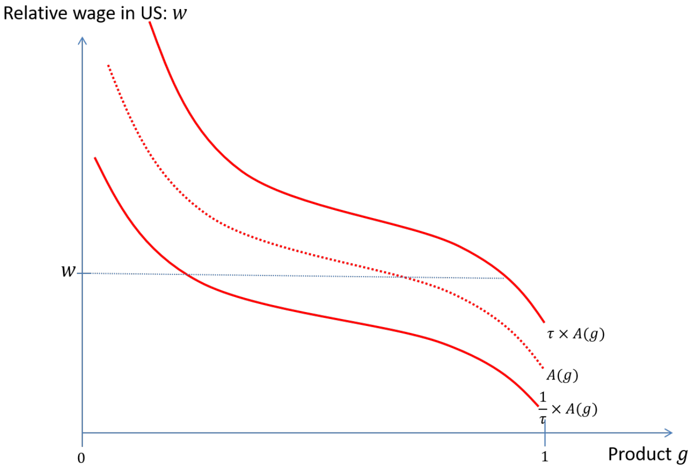


- [Class question: How do trade costs affect this figure?]

Consumer sourcing decision

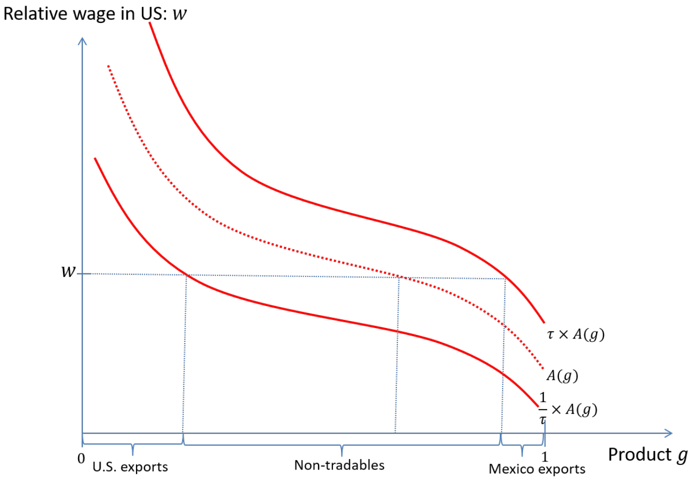


Consumer sourcing decision



- [Class question: Who produces what given this particular wage?]

Consumer sourcing decision



Conclusion and next class

- Key take-aways:
  - Extending the model to multiple goods increases the number of possible equilibria types to check...
  - But extending the model to a continuum of goods makes the problem simpler by focusing only on the extensive margin of trade.
  - With trade costs, certain goods (where countries have similar relative productivities) are produced by both countries
- Next step:
  - We will extend the model to incorporate multiple countries.
  - Requires a somewhat tricky new technical technique.
  - But brings us close to the frontier of trade!

Jonathan Eaton

- Professor at Princeton, Yale, Virginia, Boston University, NYU, Brown, and (now) Penn State.
- Co-editor at the *Journal of International Economics* for 10 years (Professor Staiger is the editor).
- Won the Frisch Medal from *Econometrica* (awarded only every two years).

Samuel Kortum

- Professor at Boston University (where he met Jonathan), University of Minnesota, University of Chicago, and now Yale University.
- Advised my advisor while at University of Minnesota.
- Incredibly thoughtful and generous with his time.

Maurice René Fréchet

- French mathematician.
- Tutored by Jacques Hadamard in high school.
- Two and half years in the front lines of World War I, still found time to write several articles.

Example: Three countries and three goods

- Suppose there are three countries and three goods with the following labor requirements:
-

	Canada	U.S.	Mexico
Baseballs	10	10	10
Footballs	5	7	3
Soccer balls	4	3	2

- "Guess" an equilibrium #1: U.S. produces baseballs, Canada produces footballs, Mexico produces soccer balls.
  - Can check that comparing any two goods satisfies comparative advantage requirements.
  - Example: U.S./baseballs, Canada/footballs makes sense because U.S. has comparative advantage in baseballs:

$$\frac{10}{7} < \frac{10}{5}$$

Example: Three countries and three goods (ctd)

- Suppose there are three countries and three goods with the following labor requirements:

	Canada	U.S.	Mexico
Baseballs	10	10	10
Footballs	5	7	3
Soccer balls	4	3	2

- But that is not the unique equilibrium! Another one is U.S./soccer balls, Canada/baseballs, and Mexico/footballs, which also satisfies pair-wise comparative advantage.
  - Example: U.S./soccer balls and Canada/baseballs:

$$\frac{3}{10} < \frac{4}{10}$$

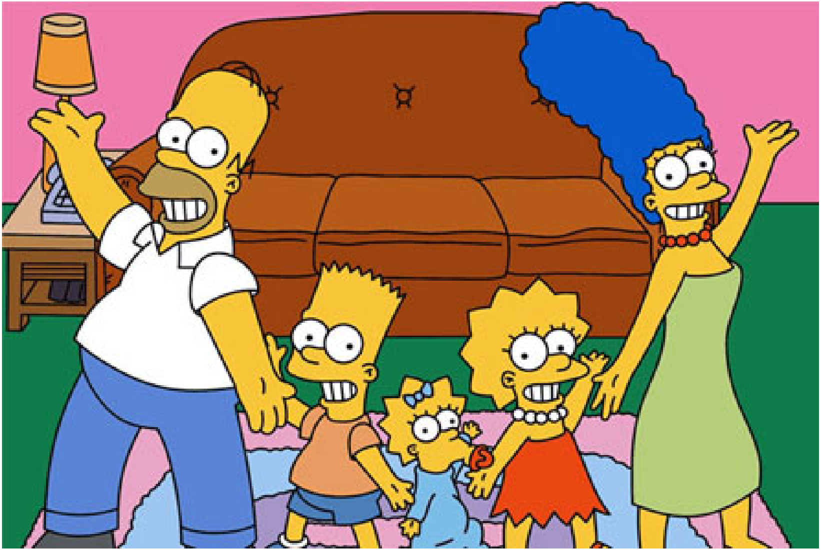
The challenge of many countries and many goods

- Many more types of equilibria that can occur (and would have to check)
- Pair-wise comparative advantage does not uniquely determine pattern of specialization.
- [Class question: Any ideas how to make the problem easier to solve?]
- Note: a continuum of goods $g \in [0, 1]$ by itself will not work, as we just defined the pair-wise comparative advantage $A(g)$.

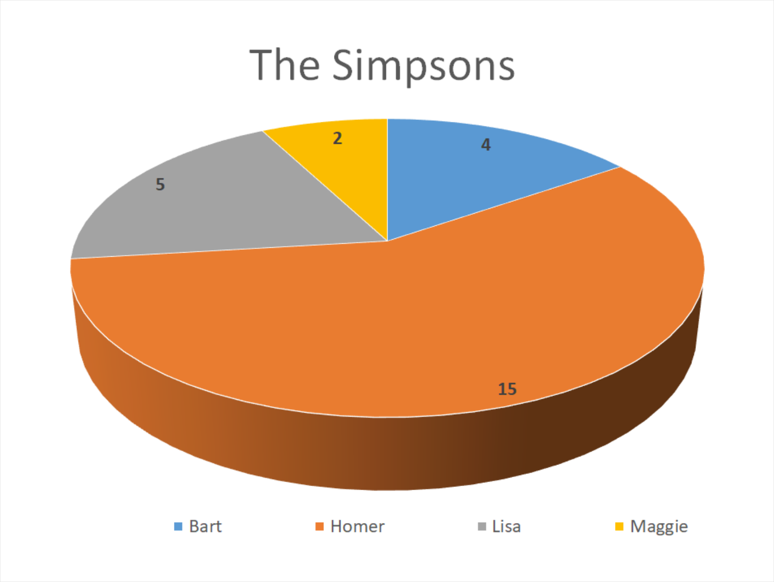
Plan for the day

- Finish Eaton Kortum trade model.
- If time: practice problem.
- Reminder: Midterm #1 on Thursday (show up early)!

Today's Teams



Today's Teams



Geography

- N different countries.
  - In what follows, I will call i the origin country and j the destination country, where $i, j \in \{1, ..., N\}$
  - Note: when $N > 2$, we're beyond figures!
- Geography: trade from i to j subject to an iceberg trade cost $\tau_{ij} \geq 1$.
  - [Class question: What does an iceberg trade cost mean?].

Production

- Each country i is endowed with a fixed amount of labor L_i.
- Like DFS '77, there are a continuum of goods $g \in [0, 1]$.
- Technology: Let $\alpha_i(g)$ be the unit labor cost of producing good g in country i.
- Let w_i be the (endogenous) wage of a worker in location i.
- [Class question: what is the price of good g from i in location j?]
    - Answer:
    $$p_{ij}(g) = \tau_{ij} w_i \alpha_i(g)$$

The key insight

- Suppose each country differs in its unit labor cost of producing a good g, $\alpha_i(g)$.
- Rather than specifying a particular function of $\alpha_i(g)$ for each country, let us treat α_i as a random variable.
  - That is, there is some function F_i (known as a **cumulative distribution function**) such that:

$$\Pr\{\alpha_i(g) \le x\} = F_i(x)$$

  - Intuition: we do not specify which goods a country is good at, just the probability that the country is good at producing a good.
- Since there are a continuum of goods $g \in [0, 1]$:
  - The **law of large numbers** says the $\Pr\{\alpha_i(g) \le x\}$ is equal to the fraction of goods that require less than x units of labor to produce.

But what distribution should we choose?

- Goals in choosing an appropriate distribution:
  - It should be tractable (i.e. it should play well with the idea of consumers choosing the least cost source of each product).
  - It should make sense (i.e. have a believable microfoundation about where the distribution comes from).
- Eaton Kortum '02 solution: The Fréchet distribution![1]
  - CDF of unit labor costs:

$$\Pr\left\{\alpha_i\left(g\right) \leq x\right\} = 1 - e^{-\left(A_i x\right)^{\theta}}$$

[1] In EK '02, they assume productivity is Fréchet distributed. Since productivity is the inverse of the unit labor cost, it turns out the unit labor cost is Weibull distributed.

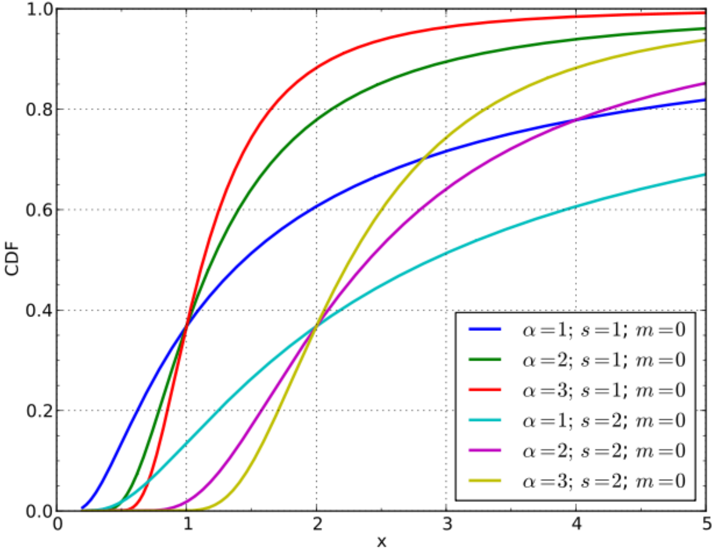
The Fréchet distribution

- CDF of unit labor costs:

$$\Pr\left\{\alpha_i\left(g\right) \leq x\right\} = 1 - e^{-\left(A_i x\right)^{\theta}}$$

- [Class question: As A_i increases, what happens to the probability that a good requires less than x units of labor?]
    - **Scale parameter** A_i governs country i's absolute advantage.
- [Class question: As θ increases, what happens to how the probability changes with x?
    - **Shape parameter** θ (constant across countries) governs how different goods are (higher θ indicates less variance).

The Fréchet distribution (for productivities)



Why the Fréchet distribution?

- The Fréchet (and Weibull) distributions are both "extreme value distributions"
  - The distributions arise when you take the minimum (or maximum) from a number of random variables.
  - In earlier work, Eaton and Kortum developed models where countries become more productive by holding on to good ideas and discarding bad ideas.
  - Over time, the distribution of technologies would then progress to an extreme value distribution.
- The Fréchet (and Weibull) distributions are also "closed" under maximization and minimization
  - The distribution of the maximum (or minimum) of several random variables that are each Fréchet distributed is also Fréchet distributed.
  - This helps immensely with calculating the equilibrium patterns of trade.

Preferences

- As for DFS '77, let us assume that consumers in each country have Cobb-Douglas preferences over all goods:

$$U_j = \int_0^1 \log\left(C_j\left(g\right)\right) dg$$

- This implies that consumers spend an equal amount of their income on all goods.
- [Class question: What does this imply about the relationship between the fraction of goods a country i exports to j and the value of trade from i to j?]
- Note: EK '02 actually have more general "constant elasticity of substitution" preferences, but we will not worry about those until Week 8.

Buying a good g

- As in DFS '77, we can figure out which country produces what by asking, from the perspective of a consumer in country j, which is the cheapest source of good g?
- Recall that the price of a good g from country i consumed in country j is:

$$p_{ij}(g) = \tau_{ij} w_i \alpha_i(g)$$

- The actual price a consumer in country j will actually pay for good g will be the minimum across all possible sources (including sourcing locally):

$$p_j(g) = \min_{i \in \{1,\ldots,N\}} \{p_{ij}(g)\}$$

Price distribution

- Calculate probability that a good costs less than x in 7 steps:
  1. $\Pr\{p_j(g) \leq x\} = \Pr\left\{\min_{i\in\{1,\ldots,N\}}\{p_{ij}(g)\} \leq x\right\} \iff$
  2. $\Pr\{p_j(g) \leq x\} = \Pr\left\{\min_{i\in\{1,\ldots,N\}}\{\tau_{ij}w_i\alpha_i(g)\} \leq x\right\} \iff$
  3. $\Pr\{p_j(g) \leq x\} = 1 - \cap_{i\in\{1,\ldots,N\}}\Pr\{\tau_{ij}w_i\alpha_i(g) \geq x\} \iff$
  4. $\Pr\{p_j(g) \leq x\} = 1 - \cap_{i\in\{1,\ldots,N\}}\Pr\left\{\alpha_i(g) \geq \frac{x}{w_i\tau_{ij}}\right\} \iff$
  5. $\Pr\{p_j(g) \leq x\} = 1 - \prod_{i=1}^{N}\left(1 - F_i\left(\frac{x}{w_i\tau_{ij}}\right)\right) \iff$
  6. $\Pr\{p_j(g) \leq x\} = 1 - \prod_{i=1}^{N}\left(e^{-\left(A_i\frac{x}{w_i\tau_{ij}}\right)^{\theta}}\right) \iff$
  7. $\Pr\{p_j(g) \leq x\} = 1 - e^{-\sum_{i=1}^{N}\left(\frac{A_i}{w_i\tau_{ij}}\right)^{\theta}x^{\theta}}$
- Key take-away: distribution of prices in j is also Weibull distributed with scale parameter $\left(\sum_{i=1}^{N}\left(\frac{A_i}{w_i\tau_{ij}}\right)^{\theta}\right)^{\frac{1}{\theta}}$.
  - [Class question: what factors allow consumers in country j to purchase goods for lower prices?]

Trade flows

- We can use a similar technique to calculate the probability that a consumer in country j will purchase a good from country i. Call this probability π_{ij}.
- Skipping the math (see Lecture #4 of my Ph.D. notes if you're interested), we have:

$$\pi_{ij} = \Pr\left\{p_{ij}(g) = \min_{i \in \{1,\ldots,N\}} \{p_{ij}(g)\}\right\} \implies$$

$$\pi_{ij} = \frac{A_i^\theta \tau_{ij}^{-\theta} w_i^{-\theta}}{\sum_{k=1}^N A_k^\theta \tau_{kj}^{-\theta} w_k^{-\theta}}$$

  - [Class question: what factors cause country j to be more likely to import from i?]
  - [Class question: what is the fraction of goods that j imports from i?]
  - [Class question: what is the value of trade flows going from i to j?]

Trade flows (ctd).

- Given Law of Large Numbers, π_{ij} is also the fraction of goods that j imports from i.
- Cobb Douglas preferences imply that π_{ij} is also the fraction of income that consumers in j spend on i.
- Let Y_j be income in j and let X_{ij} be value of trade from i to j.
- As in DFS '77, market clearing requires income is equal to payments to labor $Y_j = w_j L_j$.
- Then we have:

$$\begin{aligned} X_{ij} &= \pi_{ij} Y_j \iff \\ X_{ij} &= \pi_{ij} w_j L_j \\ X_{ij} &= \frac{A_i^{\theta} \tau_{ij}^{-\theta} w_i^{-\theta}}{\sum_{k=1}^{N} A_k^{\theta} \tau_{kj}^{-\theta} w_k^{-\theta}} w_j L_j \qquad (1) \end{aligned}$$

- Equation (1) gives trade flows between all countries as a function of geography, productivities, and wages!

Equilibrium

- [Class question: what are the exogenous model parameters?]
  - Trade costs $\{\tau_{ij}\}$
  - Absolute advantage parameters $\{A_i\}$ ("productivity")
  - Strength of comparative advantage θ
  - Populations $\{L_i\}$
  - Preferences.
- [Class question: what are the endogenous model outcomes?]
  - Prices: wage in each country $\{w_i\}$
  - Quantities: bilateral trade flows $\{X_{ij}\}$

Equilibrium (ctd).

- [Class question: what is the equilibrium statement?]
  - "Given a set of trade costs, productivities, populations, preferences, and the parameter θ, equilibrium is defined by a set of bilateral trade flows $\{X_{ij}\}$ and wages $\{w_i\}$ such that in all countries $i \in \{1, ..., N\}$:
    - Producers are maximizing profits given prices (i.e. they are indifferent across all goods they are producing);
    - Given prices and incomes, consumers are maximizing their utility (i.e. they buy each good from the cheapest source and spend an equal amount of their income on all goods); and
    - Markets clear (i.e. income in a country is equal to the payments to labor and also equal to its total sales)."
- [Class question: what happened to the price normalization?]

Market clearing

- Income in a country is equal to the payments to labor:

$$Y_i = w_i L_i$$

- Income in a country is also equal to its total sales:

$$Y_i = \sum_{j=1}^{N} X_{ij}.$$

- Combining the two equations and substituting in the expression for bilateral trade flows yields:

$$w_i L_i = \sum_{j=1}^{N} \frac{A_i^{\theta} \tau_{ij}^{-\theta} w_i^{-\theta}}{\sum_{k=1}^{N} A_k^{\theta} \tau_{kj}^{-\theta} w_k^{-\theta}} w_j L_j$$

  - The last equation holds for all N countries and depends only on the N wages.
  - It turns out you can solve it very quickly on the computer for the unique equilibrium wages (up to scale).

The indirect utility function

- Return briefly to the two good model with a Cobb-Douglas utility function:

$$U = C_1^{\beta} C_2^{1-\beta}$$

- As we saw last week, the optimal consumption is:

$$C_1 = \beta \frac{Y}{p_1};\ C_2 = (1-\beta)\frac{Y}{p_2}$$

- We can then figure out how much utility a consumer gets from income Y and prices $\{p_1, p_2\}$:

$$\begin{aligned}
U &= C_1^{\beta} C_2^{1-\beta} \iff \\
U &= \left(\beta \frac{Y}{p_1}\right)^{\beta} \left((1-\beta)\frac{Y}{p_2}\right)^{1-\beta} \iff \\
U &= \left(\beta^{\beta}(1-\beta)^{1-\beta}\right) \frac{Y}{p_1^{\beta} p_2^{1-\beta}} \qquad (2)
\end{aligned}$$

- Equation (2) is known as an **indirect utility function**.

The indirect utility function (ctd.)

- Now consider the Cobb-Douglas utility function with a continuum of goods (and equal preference weights):

$$U_j = \int_0^1 \log C_j(g)\, dg$$

- It turns out the indirect utility function (of an individual worker) is:

$$U_j = \frac{w_j}{P_j},$$

  where $P_j \equiv \prod_0^1 p_j(g)^{dg}$ is the **price index**.
  - [Class question: What is the interpretation of the price index?]
  - [Class question: Why is it okay to ignore the constant?]

The indirect utility function (ctd.)

- Recall that we know the probability distribution of $p_j(g)$:

$$\Pr\{p_j(g) \leq x\} = 1 - e^{-\sum_{i=1}^{N}\left(\frac{A_i}{w_i \tau_{ij}}\right)^{\theta} x^{\theta}}$$

  - Weibull distribution with scale parameter $\left(\sum_{k=1}^{N} A_k^{\theta} \tau_{kj}^{-\theta} w_k^{-\theta}\right)^{\frac{1}{\theta}}$.
- Without going through the math, it turns out the price index is:

$$P_j = \left(\sum_{k=1}^{N} A_k^{\theta} \tau_{kj}^{-\theta} w_k^{-\theta}\right)^{-\frac{1}{\theta}}$$

The trade equation

- From the last slide, the price index is:

$$P_j = \left(\sum_{k=1}^{N} A_k^{\theta} \tau_{kj}^{-\theta} w_k^{-\theta} \right)^{-\frac{1}{\theta}}$$

- Recall from equation (1) that trade flows are :

$$X_{ij} = \frac{A_i^{\theta} \tau_{ij}^{-\theta} w_i^{-\theta}}{\sum_{k=1}^{N} A_k^{\theta} \tau_{kj}^{-\theta} w_k^{-\theta}} w_j L_j$$

- Combining the two yields:

$$X_{ij} = \tau_{ij}^{-\theta} A_i^{\theta} w_i^{-\theta} w_j L_j P_j^{\theta}$$

Own trade shares

- From the last slide:

$$X_{ij} = \tau_{ij}^{-\theta} A_i^{\theta} w_i^{-\theta} w_j L_j P_j^{\theta}$$

- Let us focus on how much consumers in i consume of goods made in i (X_{ii}).
  - Assume $\tau_{ii} = 1$. [Class question: Is this reasonable?]
- Then we can write:

$$X_{ii} = A_i^{\theta} w_i^{-\theta} w_i L_i P_i^{\theta} \iff$$

$$\left(\frac{X_{ii}}{w_i L_i}\right) = A_i^{\theta} \left(\frac{w_i}{P_i}\right)^{-\theta} \iff$$

$$\frac{w_i}{P_i} = \left(\frac{X_{ii}}{w_i L_i}\right)^{-\frac{1}{\theta}} A_i.$$

  - [Class question: How do we interpret the left hand side of the expression?]
  - [Class question: How do we interpret $\frac{X_{ii}}{w_i L_i}$?]

The gains from trade

- From the last slide, welfare is proportional to overall productivity and inversely proportional to trade openness:

$$U_i = \pi_{ii}^{-\frac{1}{\theta}} A_i.$$

- From this equation, we can immediately estimate the welfare loss of county i moving from current level of openness back to autarky.
  - [Class question: what does π_{ii} equal in autarky?]
  - Welfare in autarky:

$$U_i^{autarky} = A_i$$

  - Welfare in trade:

$$U_i^{trade} = \pi_{ii}^{-\frac{1}{\theta}} A_i$$

  - Welfare loss of moving from trade to autarky:

$$U_{ii}^{loss} = \frac{U_i^{autarky}}{U_i^{trade}} = \pi_{ii}^{\frac{1}{\theta}}$$

The gains from trade: Examples

- The shape parameter θ is thought to equal about 4.
- The U.S. spends approximately 75% of its income on goods produced domestically.
- Hence, if the U.S. shut down to trade, it's welfare would be about $(0.75)^{\frac{1}{4}} \approx 0.93$ as high as it is currently, or a 7% loss.
- In contrast, $\pi_{MEX,MEX} \approx 0.5$, so loss to Mexico from shutting its borders is $(0.5)^{\frac{1}{4}} \approx 0.84$, or a 16% loss.
- [Class question: Do these numbers seems large or small to you?]
- [Class question: Is there anything missing in this model that you can think of?]

Conclusion

- In two (and a half) weeks, we covered 200 years of comparative advantage!
- We ended up with a model that incorporates many goods, many countries, and a rich geography of trade costs.
- But we have only seen the tip of the iceberg:
  - Can extend the EK '02 framework to incorporate multiple sectors, multiple factors of production, factor mobility, etc.
  - Can use the framework to inform a number of important policy debates (trade agreements, immigration policy, infrastructure development, etc).
  - We will return to some of these issues in weeks #9 and #10...
  - ... but if you're interested, they would also make great senior thesis projects!