Micro II
Problem Set 4
SOLUTIONS

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Conflict with different "prizes"

1. In the lecture it was assumed that the "prize" (i.e. "stake") was the same for both groups. This is not necessarily the case, and in this problem set we will relax this assumption, while keeping the rest of the game as simple as possible.

   There are two groups, i and j. Each group chooses the amount of fighting resources, D_i resp. D_j, that maximizes the expected payoff as a response to the fighting effort contributed by the opposing group. Conflict has two possible outcomes (v, d), v corresponds to a victory of group i and d corresponds to a defeat of group i (and hence a victory of group j). The payoffs for each of the groups in the event of the two possible outcomes can be labelled as (U_{vi}, U_{di}) and (U_{dj}, U_{vj}), where $U_{vi} > U_{di}$ and $U_{dj} > U_{vj}$. The payoff function of groups i and j are:

$$U_i = qU_{vi} + (1-q)U_{di} - D_i \tag{1}$$

$$U_j = (1-q)U_{dj} + qU_{vj} - D_j \tag{2}$$

   with the probability of group i winning given by the following contest success function:

$$q = \frac{D_i}{D_i + D_j} \tag{3}$$

   We assume that the per unit cost of fighting is unitary (which could be interpreted again as opportunity cost of production).

   (a) What can be the reasons that groups face different gains from winning a contest?

(b) Derive the reaction functions. Display this graphically.

(c) Derive the Nash equilibrium. Display this graphically.

(d) What is the equilibrium probability of group i winning? What is the total cost from conflict?

(e) Fighting effort

  (i) If group i has more to gain in case of victory ceteris paribus, i.e. $U_{vi} > U_{dj}$, $U_{di} = U_{vj}$, which group will fight harder in equilibrium?

  (ii) What if $U_{vi} = U_{dj} - k$, $U_{di} = U_{vj} - k$, where k is some constant?

  (iii) Set $U_{di} = U_{vj} = 0$. Compare country A where $U_{vi} = 3$, $U_{dj} = 1$ with country B where $U_{vi} = 2$, $U_{dj} = 2$. In which country will total fighting effort be larger? Is this a general feature?

  (iv) Do you know real world examples that fit well the predictions of this framework?

Solution:

(a) There are several situations in which a given group has more to win or lose from victory than the opponent: For example, i) if there are asymmetric incentives for post-conflict atrocities, ii) in the case of intrinsically motivated separatist movements, iii) if winning groups i or j are treated differently by the international community (e.g. embargoes, boycotts).

(b) The expected payoffs of groups i and j are

$$U_i = \frac{D_i}{D_i + D_j} U_{vi} + (1 - \frac{D_i}{D_i + D_j}) U_{di} - D_i \tag{4}$$

$$U_j = \frac{D_j}{D_i + D_j} U_{dj} + (1 - \frac{D_j}{D_i + D_j}) U_{vj} - D_j. \tag{5}$$

The payoff U_i is strictly concave in D_i. Differentiating U_i with respect to D_i in (4) we obtain

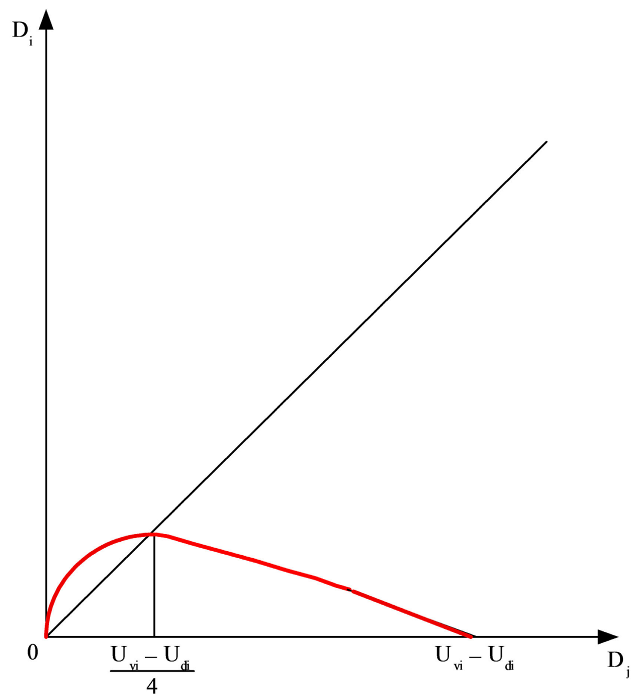
$$\frac{\partial U_i}{\partial D_i} = \frac{D_j}{(D_i + D_j)^2} \Big(U_{vi} - U_{di} \Big) - 1 \stackrel{!}{=} 0. \tag{6}$$

Denoting by Δ_i the utility differential $U_{vi} - U_{di}$, (6) can be solved for D_i to obtain the reaction function:

$$D_i = \sqrt{D_j} \sqrt{\Delta_i} - D_j. \tag{7}$$

This is the reaction function of group i; for group j it is analogous. The best reply function is represented in Figure 1, with D_j on the

Figure 1: Reaction function



x-axis, and D_i on the y-axis. It is concave, starts at the origin with infinite steepness, reaches its maximum when it crosses the 45 degree line, at $D_i = D_j = \frac{\Delta_i}{4}$, and intersects the axis at $D_j = \Delta_i$.

(c) Using,

$$D_i = \sqrt{D_j}\sqrt{\Delta_i} - D_j \tag{8}$$

$$D_j = \sqrt{D_i}\sqrt{\Delta_j} - D_i \tag{9}$$

We can obtain the equilibrium fighting efforts by either substituting one reaction function into the other and then solve for the fighting effort. Or we could work with the first order conditions: set them equal (because they are both equal to zero in the optimum) and then solve for the fighting effort. The resulting equilibrium fighting efforts are:

$$D_i^* = \Delta_j\Big(\frac{\Delta_i}{\Delta_j + \Delta_i}\Big)^2 \tag{10}$$

and

$$D_j^* = \Delta_i\Big(\frac{\Delta_j}{\Delta_j + \Delta_i}\Big)^2 \tag{11}$$

We diagrammatically represent an equilibrium in Figure 2. The equilibrium strategies correspond to the point of intersection of the two best reply curves.

(d) The equilibrium winning probability for group i, q^*, can be easily computed to be

$$q^* = \frac{\Delta_i}{\Delta_i + \Delta_j}. \tag{12}$$

The total warfare cost is:

$$D^* = D_j^* + D_i^* = \frac{\Delta_i \Delta_j}{\Delta_j + \Delta_i}. \tag{13}$$

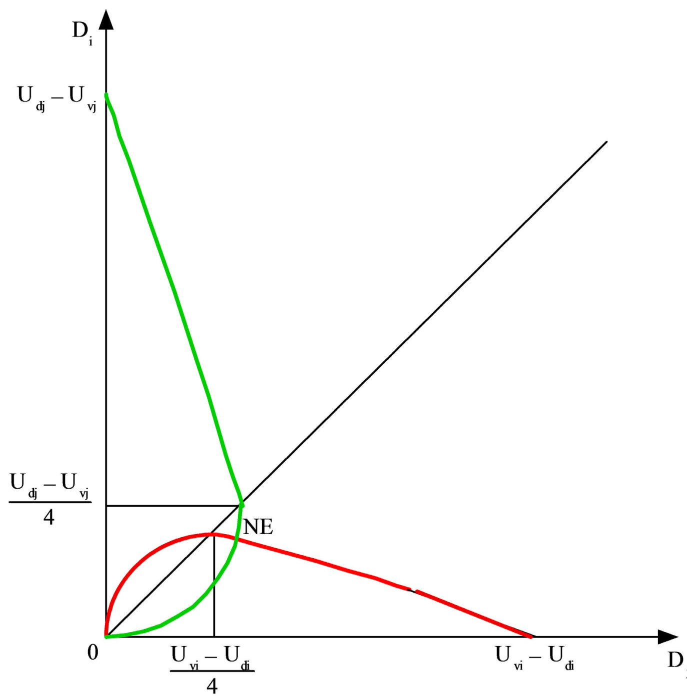
(e) Fighting effort

(i) In this case $\Delta_i > \Delta_j$. Clearly, $D_i^* > D_j^*$, the more motivated group i fights harder and has a higher probability of victory, i.e. $q^* > 1/2$.

(ii) In this case $\Delta_i = \Delta_j$, and everything is symmetrical (both groups make the same fighting effort and $q^* = 1/2$). Only the payoff *difference* between winning and losing matters.

(iii) Using the formula $D^* = D_j^* + D_i^* = \frac{\Delta_i \Delta_j}{\Delta_j + \Delta_i}$, we obtain $D_A^* = \frac{3}{4} < D_B^* = 1$. Yes, this is a general feature. Symmetry maximizes the total fighting efforts.

Figure 2: Nash equilibrium



(iv) There are plenty of examples of small groups who fought hard and successfully and in the case of defeat would have risked execution, such as e.g. Tutsi Rebels in Rwanda 1994.