


Exercise 2

a) Write the VAR(p) system

$$1) \quad U_t = \phi_1 U_{t-1} + \phi_2 U_t + \phi_3 M_{t-1} + \eta_t^U$$

\downarrow unemployment

interest rate shock
(MP)

$$2) \quad M_t = \phi_4 U_t + \phi_5 M_{t-1} + \phi_6 i_{t-1} + \eta_t^M$$

$$\Rightarrow \eta_t^U, \eta_t^M \perp M_t, i_t \quad \forall t = 1, \dots, T-1$$

b) Can we use OLS? No! ENDGENEITY DUE TO REVERSE CAUSALITY

Substitute 2) in 1)

$$U_t = \phi_1 U_{t-1} + \phi_2 (\phi_4 U_t + \phi_5 M_{t-1} + \phi_6 i_{t-1} + \eta_t^M) + \phi_3 M_{t-1} + \eta_t^U$$

$\Rightarrow \text{cov}(U_t, \eta_t^M) \neq 0 \Rightarrow \text{EXOGENEITY ASSUMPTION VIOLATED}$, so OLS estimator is not consistent \Rightarrow remember $E(M_t | X_t) = 0$

c) Which identification strategy?

First Option: find the PF of the system:

$$U_t = \frac{1}{1 - \phi_2 \phi_4} \left[(\phi_1 + \phi_2 \phi_6) U_{t-1} + (\phi_3 + \phi_2 \phi_5) M_{t-1} + \phi_4 \eta_t^M + \eta_t^U \right]$$

$$M_t = \frac{1}{1 - \phi_2 \phi_4} \left[(\phi_1 \phi_4 + \phi_6) U_{t-1} + (\phi_3 \phi_4 + \phi_5) M_{t-1} + \phi_4 \eta_t^U + \eta_t^M \right]$$

We can rule out ϕ_4 , but cannot identify all the coefficients
 \Rightarrow 6 coefficients, 4 regressors:

$$\text{Residuals} = \begin{cases} \varepsilon_t^L = \frac{1}{1-\phi_2\phi_3} [\eta_t^L + \phi_2 \eta_t^M] & 3) \\ \varepsilon_t^M = \frac{1}{1-\phi_2\phi_3} [\eta_t^M + \phi_3 \eta_t^L] & 4) \end{cases}$$

AE residuals are a function of the structural shock

- Assume $\phi_4 = 0$; i.e. unemployment does not react to interest rate movements instantly (adjustment lag)

From DDF estimation I can find ϕ_2 by regressing one residual on the other:

$$\text{Eq 4): } \varepsilon_t^M = \eta_t^N$$

$$\text{Eq 3): } \boxed{\varepsilon_t^L = \phi_2 \varepsilon_t^M}$$

+ Estimate this, obtain ϕ_2

- $\phi_4 = 0$
- $\phi_1, \phi_3, \phi_5, \phi_6$ can be easily obtained.
- You can also say that $\phi_2 = 0 \Rightarrow$ CB does not react to immediate news from the real economy.

Other strategies:

Find instruments for η_t^L , that is a proxy of a MP shock.

\hookrightarrow Narrative Approach [Romes and Raves (2001)]

Exercise 2

1) From FOC seen in class:

$$\frac{(i^* - i_d) F(R/n)}{1} = \frac{(i_b - i)(1 - F(R/n))}{1}$$

- Expected cost of borrowing in the interbank mkt out of the earnings from the deposited reserves at the end of the period weighted by $F(R/n)$
- Expected benefit of reducing the cost of borrowing from the CB's standing facility weighted by the probability that the bank needs to use it

A Bank equate $i^* C = MB$ to decide the optimal amount of C and R to choose at the beginning of the period

- 2) The demand for reserves is determined by the banks' FOC.
 If $i < i_d \Rightarrow$ bank borrows unlimited amount from CBs
 If $i > i_b \Rightarrow$ bank would want to lend unlimited amount to others and borrowing from CB at i_b .

\Rightarrow IT FOLLOWS THAT $\Rightarrow i_d \leq i \leq i_b$

Shape of the demand for reserves \Rightarrow It is a CDF

$$\frac{(b - c)}{(b - i)}$$

