Macroeconomics B, EI060

Class 5

Frictions in financial markets

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What you will get from today class

- Defaults from borrowers.
  - Analysis with a non-contingent bond (Harms VI.3.1-3.3).
  - Risk premia (Vegh 2.4.3)
  - Analysis with contingent assets (Obstfeld and Rogoff 6.1.1.1-6.1.1.5).
- Moral hazard in international investment (OR 6.4.1).

A question to start

The threat of future complete exclusion from international financial markets can lead a country to repay its debts most of the time.

Do you agree? Why or why not?

DEFAULT WITH NON – CONTINGENT ASSETS

Beyond full enforceability

- In previous classes, we assume that when a country borrow, it will repay the amount agreed upon in the future.
- Disputable assumption with countries, as harder to enforce payments than for households.
- We first consider borrowing in bonds.
    - Enforce repayment through threat of exclusion from markets.
    - Enforce through domestic cost of default.
    - Endogenous default probability and risk premium.

Infinite horizon

- Small open economy with an infinite horizon and endowment.
- One good, with a bond denominated in the good with interest rate r. Flow budget constraint (B is asset, so $(-B)$ is debt):

$$B_{t+1} + C_t = Y_t + (1+r) B_t$$

- Iterate (with transversality condition) to get the intertemporal constraint:

$$\sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} C_s = (1+r) B_t + \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} Y_s$$

- Take a log utility of consumption, and the usual assumption of $\beta (1+r) = 1$ to get perfect smoothing:

$$C_t = r B_t + \frac{r}{1+r} \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} Y_s$$

- This assumes that required payments are indeed made.

Fluctuating income

- Income has a up - down" pattern, being high in periods t, $t+2$, $t+4$ and low in periods $t+1$, $t+3$:

$$\begin{aligned} Y + \Delta &= Y_t = Y_{t+2} = Y_{t+4} = ... \\ Y - \Delta &= Y_{t-1} = Y_{t+1} = Y_{t+3} = ... \end{aligned}$$

- With commitments to payments (non default), consumption is:

$$C_t^{ND} = rB_t + Y + \frac{r}{2+r}\Delta$$

- This implies the following path for assets, with the agent saving in high income states and dissaving in low income states:

$$\begin{aligned} B_t &= B_{t+4} = B_{t+2} = ... \\ \frac{2}{2+r}\Delta + B_t &= B_{t+1} = B_{t+3} = ... \end{aligned}$$

- This is not a problem if $B_t > 0$ as the agent never becomes a debtor. But what if $B_t < 0$?

Consumption with default

- Default sets the negative B_t to zero.
- As a punishment, the country is excluded from financial markets, even as a saver.
- Subsequent consumption is equal to endowment.

$$
\begin{aligned}
Y + \Delta &= C_t^D = C_{t+2}^D = C_{t+4}^D = ... \\
Y - \Delta &= C_{t-1}^D = C_{t+1}^D = C_{t+3}^D = ...
\end{aligned}
$$

Utilities

- With log utility, the utility in the absence of default is:

$$
\begin{aligned}
U_t^{ND} &= \sum_{s=t}^{\infty} (\beta)^{s-t} \, ln\left(C_s^{ND}\right) \\
&= \frac{1+r}{r} ln\left(rB_t + Y + \frac{r}{2+r}\Delta\right)
\end{aligned}
$$

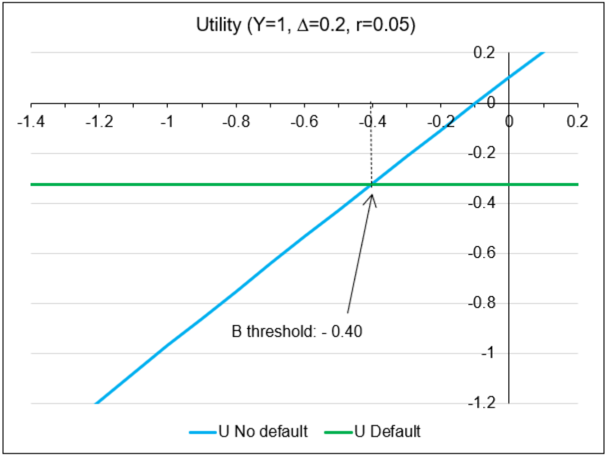
- With default, the utility does not depend on debt:

$$
\begin{aligned}
U_t^{D} &= \sum_{s=t}^{\infty} (\beta)^{s-t} \, ln\left(Y_s\right) \\
&= \left(ln\left(Y+\Delta\right) + ln\left(Y-\Delta\right)\frac{1}{1+r}\right)\frac{\left(1+r\right)^2}{r\left(2+r\right)}
\end{aligned}
$$

- Default is optimal if the country is highly indebted, B_t is low ($B_t < 0$ if $\Delta = 0$).

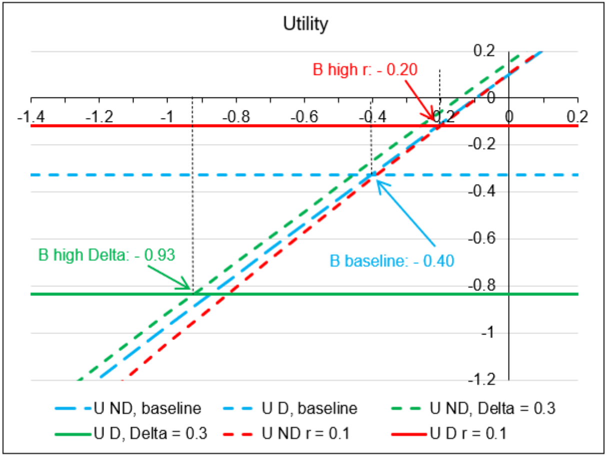
Baseline

- Default under high debt (left hand side) as $U_t^{ND} < U_t^D$.



Alternatives

- More debt is sustainable (low threshold) if Δ is high: big need to smooth.
- Less debt is sustainable (high threshold) if r is high: little weight put on the future cost of exclusion.



Some caveats

- Repayment is enforced by the threat of exclusion.
- This applies to all interaction: the country will not be able to borrow, but also not be able to save.
    - Exclusion from savings is questionable.
- Empirically, exclusion is not seen much. Countries that default can then re-access the market.

Domestic cost of default

- Sovereign default often disrupt credit access to private firms, making investment and output lower.
- The same model as above, without fluctuations ($\Delta = 0$).
- Default leaves output unchanged at time of default, but reduces all future outputs to γY, where $\gamma < 1$.
- Without default, consumption is (log utility, $\beta (1 + r) = 1$):

$$C_t^{ND} = rB_t + Y$$

Allocation under default

- If the country defaults, consumption is

$$C_t^D = \frac{r}{1+r}\left(Y + \sum_{s=t+1}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}\gamma Y\right)$$

$$C_t^D = \frac{\gamma + r}{1+r} Y < Y$$

- Default is chosen if the debt $(-B_t)$ is high enough. More likely if r is high (little weight put on low future consumption) and γ is high (output is resilient):

$$ln\left(\frac{\gamma + r}{1+r} Y\right) > ln\left(rB_t + Y\right)$$

$$\frac{-B_t}{Y} > \frac{1-\gamma}{r\left(1+r\right)}$$

RISK PREMIUM

Debt and premium

- Debt level, probability of default, and risk premium are all related.
- Illustration through 2 period model (some of the algebra complex, focus on intuition).
- Lending through bonds, with interest rate $1 + r^s$(in the absence of default). Default with probability π, in which case the lender gets $z\left(1 + r^s\right)$, where $1 - z$ is the haircut. The lender requires expected return $1 + r$:

$$\begin{aligned} 1 + r &= (1-\pi)\left(1 + r^s\right) + \pi z\left(1 + r^s\right) \\ 1 + r &= \left(1 - \pi\left(1 - z\right)\right)\left(1 + r^s\right) \end{aligned}$$

- From now on set $z = 0$.
- Period 2 output is uniformly distributed:$y_2 \in \left[0, y_2^H\right]$.

Default choice

- Without default, repays the debt with interest:$(1 + r^s)\, d_1$.
- With defaults, there is a true resource cost ϕy_2 born by the borrower. Default is optimal if output is low:

$$(1 + r^s)\, d_1 \quad > \quad \phi y_2$$

- Probability π of default is the probability that output is lower than $(1 + r^s)\, d_1 / \phi$:

$$\pi \quad = \quad \frac{(1 + r^s)\, d_1}{\phi y_2^H}$$

- Lender arbitrage then requires:

$$1 + r \quad = \quad \left(1 - \frac{(1 + r^s)\, d_1}{\phi y_2^H}\right)(1 + r^s)$$

- Quadratic expression in $1 + r^s$. Two equilibria, one with high debt cost and default risk (but unstable), one with low.

Intertemporal utility

- Borrowing household maximizes a linear utility: $U = C_1 + \frac{1}{1+\delta} EC_2$. Assume $r < \delta$ so she wants to borrow.
- Budget constraints (d_1 is debt), depending on default or not in the second period:

$$\begin{aligned} c_1 &= y_1 + d_1 \\ c_2^{ND} &= y_2 - (1 + r^s)\, d_1 \\ c_2^{D} &= (1 - \phi)\, y_2 \end{aligned}$$

- Utility is raised by debt and reduced by the risk of losing some output under default (cost of default is ultimately borne by the borrower):

$$U = Y_1 + \frac{\delta - r}{1+\delta} d_1 + \frac{1}{1+\delta} E\left(y_2\right) - \frac{1}{1+\delta} \pi \phi E\left(y_2 \mid D\right)$$

Optimal allocation

- Using the uniform distribution of output, utility is:

$$U = y_1 + \frac{\delta - r}{1+\delta} d_1 + \frac{1}{1+\delta} \frac{y_2^H}{2} \left(1 - \phi \pi^2\right)$$

- The first-order condition for debt is:

$$\delta - r = \phi \pi y_2^H \frac{\partial \pi}{\partial d_1}$$

- Borrowing has a benefit, as the agent is impatient ($\delta - r > 0$), but a cost as it raises the risk of costly default.
- Equation system consists of this optimality, as well as the probability of default and the lender's arbitrage:

$$\pi = \frac{\left(1 + r^s\right) d_1}{\phi y_2^H} \quad ; \quad 1 + r = \left(1 - \pi\right)\left(1 + r^s\right)$$

Differentiating these gives an expression for $\frac{\partial \pi}{\partial d_1}$ (some algebra steps).

Solution

- The solution for the default probability, risk premium, and debt is:

$$\begin{aligned}
\pi &= \frac{\delta - r}{1 + 2\delta - r} \\
\frac{1 + r^s}{1 + r} &= 1 + \frac{\delta - r}{1 + \delta} \\
\frac{d_1}{\phi y_2^H} &= \frac{(1 + \delta)(\delta - r)}{(1 + r)(1 + 2\delta - r)^2}
\end{aligned}$$

- Recovery rate ϕ only affects the debt level, but not the probability of default and the risk premium.
- Higher impatience (higher ρ) raises the interest rate, the probability of default, and the amount of debt.

DEFAULT WITH CONTINGENT ASSETS

Income risk and insurance

- Two period small economy. Agent consumes only period 2, but trades financial assets in period 1 to maximize expected utility $U = Eu(C_2)$.
- Uncertain endowment in period 2: $Y_2 = Y + \epsilon$, where ϵ is a shock with uniform distribution over $[\epsilon_-, \epsilon_+]$.
- Contract with a risk neutral foreign insurer. Small economy pays a state contingent amount $P(\epsilon)$ in period 2 (negative amount is a payment from the insurer).
  - Consumption is $C_2(\epsilon) = Y + \epsilon - P(\epsilon)$.
- Without default, $P(\epsilon)$ maximizes expected utility subject to the constraint that the insurer makes zero expected profits $0 = EP(\epsilon)$.
  - Full insurance: $P(\epsilon) = \epsilon$ and $C_2(\epsilon) = Y$. Risk efficiently moved from the risk averse agent to the risk neutral one.
- Default risk: country may decide not to pay when contract requires $P(\epsilon) > 0$ Insurer seizes a share η of output: $\eta(Y + \epsilon)$.

Contract with default

- Maximizes expected utility, subject to $0 = EP(\epsilon)$ (multiplier μ), and the constraint that payment cannot exceed what can be seized (inequality constraint, multiplier $\lambda(\epsilon)$):

$$P(\epsilon) \leq \eta (Y + \epsilon)$$

- Optimality conditions ($\pi(\epsilon)$ is the probability):

$$\begin{aligned} u'(Y + \epsilon - P(\epsilon)) &= \mu - \frac{\lambda(\epsilon)}{\pi(\epsilon)} \\ 0 &= \lambda(\epsilon) [P(\epsilon) - \eta (Y + \epsilon)] \end{aligned}$$

States with high vs. low income

- With low income, ϵ below a threshold e, there is no default and full insurance (e and P_0 to be determined):

$$P(\epsilon) = P_0 + \epsilon \quad ; \quad C(\epsilon) = Y - P_0$$

- With high income, the country has an incentive to default, and the constraint is binding. There is partial insurance ($\eta < 1$):

$$P(\epsilon) = \eta (Y + \epsilon) \quad ; \quad C(\epsilon) = (1 - \eta)(Y + \epsilon)$$

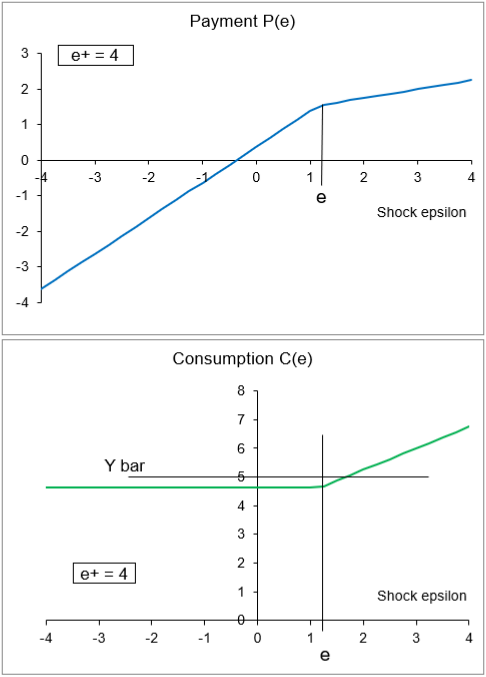
- Combining the two sets of equations when $\epsilon_i = e$ gives: $P_0 = \eta (Y + e) - e$, so when the constraint is not binding:

$$C(\epsilon) = Y - P_0 = (1 - \eta)(Y + e)$$

- e is obtained from the condition $0 = EP(\epsilon)$. It is increasing in η (insurance over a broad range). No insurance if $\eta = 0$ as $e = -\epsilon_+$:

$$e = -\epsilon_+ + 2\sqrt{\frac{\eta}{1 - \eta} Y \epsilon_+}$$

- To the left (low income) there is full insurance, but at a consumption level below the one under no frictions.
- To the right, the need to avoid default limits the extent of insurance.



Extensions

- Insurance can be sustained over a larger range of shocks when the country can accumulate foreign assets which can be seized by the lender.
- Collateral allows for insurance even when no output can be seized ($\eta = 0$).
- In a repeated game, repayment can be sustained by the threat of exclusion from insurance in the future in case of default.
    - If enough weight is put on the future, full insurance can be sustained.
    - Requires infinite horizon, as with finite horizon the threat from exclusion loses power.

MORAL HAZARD

Unobservable actions

- So far all actions were fully observable. Consider now that the borrower can take some actions that the lender cannot see, and which may not be in the lender's favor.
- Small open economy with two periods. Endowment Y_1 in period 1. Consumption only takes place in period 2, with a linear utility $U = C_2$.
- The initial endowment can be invested in two way.
  - Safe investment abroad with interest rate r.
  - Risky technology where investment I delivers Z with probability $\pi(I)$ and zero otherwise. Investment raises the probability of success, with decreasing returns ($\pi' > 0$, $\pi'' < 0$, $Z\pi'(0) > 1 + r$).

Frictionless case

- In period 1, the country borrows D (cost discussed below), invests I and lends L at the rate r. The investment / borrowing satisfies:

$$L + I = Y_1 + D$$

- If the investment is successful, the lender gets P. Other wise, he gets nothing. The payment is such that the expected return correspond to the one on the bond:

$$P\pi(I) = (1+r)(I - Y_1)$$

- Expected consumption is:

$$EC_2 = (1+r)(Y_1 - I) + \pi(I) Z$$

- Maximizing consumption equalizes the marginal product and cost of investment, and lending in bonds is pointless ($L = 0$):

$$Z\pi'\left(\tilde{I}\right) = 1 + r$$

- Investment only reflects fundamentals $Z/(1+r)$.

Asymmetric information

- The lender observes outputs Y_1 and Z, and the debt D. She cannot tell where the money is invested (I or L).
- The borrower chooses I and L, once D and P are set (no repayment in case of failure). There would be no problem if P can be indexed to I.
- Consumption of the borrower ($L = Y_1 + D - I$):

$$\begin{aligned} C_2 &= Z - P + (1+r)(Y_1 + D - I) && \text{if successful} \\ C_2 &= (1+r)(Y_1 + D - I) && \text{if not} \end{aligned}$$

- Expected consumption:

$$EC_2 = \pi(I)(Z - P) + (1+r)(Y_1 + D - I)$$

- Expected consumption is maximized by:

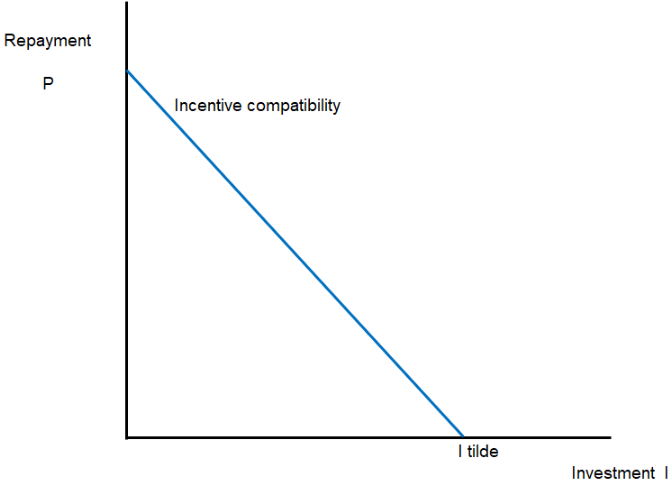
$$\pi'(I)(Z-P) = 1+r$$

- As in the first best $Z\pi'\left(\tilde{I}\right) = 1+r$, investment is lower under moral hazard: $\pi'(I) > \pi'\left(\tilde{I}\right)$ hence $I < \tilde{I}$.
- The country invests some money on the world markets (L is not seen by the lender), which she can keep if things go wrong.
- Incentive compatibility condition: P is a decreasing function of I ($P = 0$ when $I = \tilde{I}$).

$$\begin{aligned} P &= Z - \frac{1+r}{\pi'(I)} \\ \frac{\partial P}{\partial I} &= \frac{1+r}{\left[\pi'(I)\right]^2}\pi''(I) < 0 \end{aligned}$$

Incentive compatibility

- Higher repayment in case of success leads to higher “hiding” in bonds and lower investment.



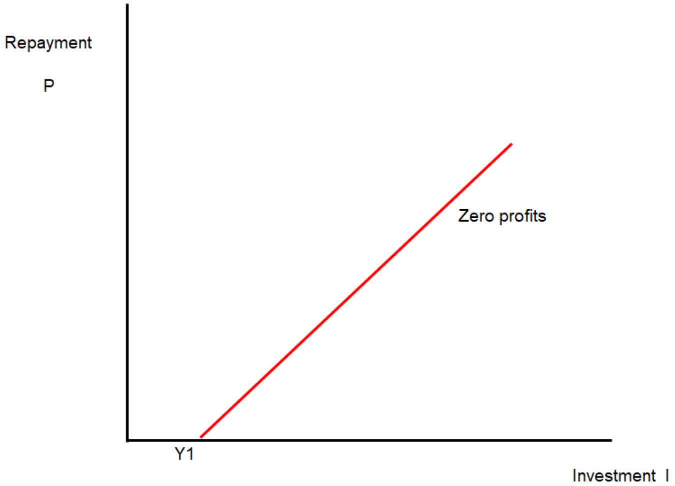
Lender arbitrage

- Lender requires same expected return as the bond: $P\pi(I) = (1+r)(I - Y_1)$.
- Increasing relation between P and I, with $P = 0$ when $I = Y_1 < \tilde{I}$:

$$\frac{dP}{dI} = (1+r)\frac{\pi(I) - (I - Y_1)\pi'(I)}{[\pi(I)]^2} > 0$$

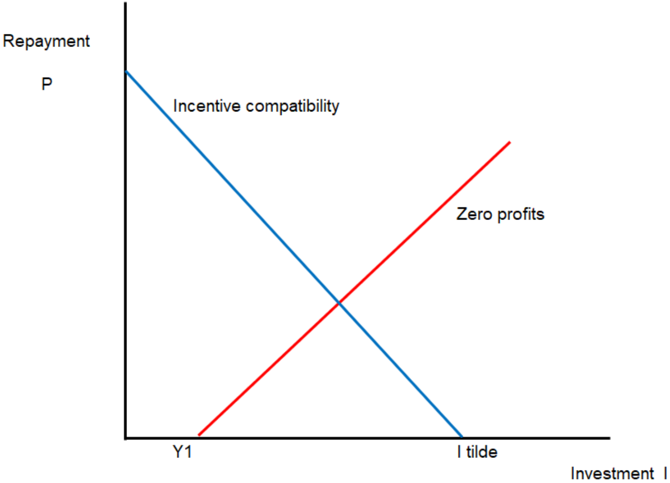
- Higher Y_1 lowers P for a given I.
- In equilibrium we need have $L = 0$. Otherwise the borrower would have an extra cost (in the end all inefficiencies are paid by the borrower).
- As $\pi'(I)(Z - P) = 1 + r$ (borrower's incentive). we have $\pi'(I)Z > 1 + r$. Marginal expected return of physical investment exceeds the risk free rate.

- Higher investment raises loan, hence expected repayment. Not fully undone by higher probability of success.



Equilibrium

- Both line intersect, with investment lower than under the efficient allocation.



Higher income

- Higher Y_1 reduces the need to borrow, and lowers P in the lender's zero profits. Red line shifts to the right, with higher investment. Y_1 (net worth) matters in addition to $Z/(1+r)$.