Lecture 11: Estimating Gravity Trade Models

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1 Introduction

In the previous few classes, we have seen how to calibrate gravity models to coincide with observed trade data and perform counterfactuals for any change in bilateral frictions. One lesson from these procedures is that the gravity constants play an enormously important role in determining the general equilibrium forces of the model. A natural follow-up question is how to estimate these gravity constants. This is the question we turn to in this lecture.

2 Universal Gravity Framework (reminded)

First, let us briefly recall the framework we are considering.

For a given set of bilateral frictions $\{K_{ij}\}$, income shifters $\{B_i\}$ and gravity constants α and β, the universal gravity framework satisfies the following equilibrium conditions:

1. The value of trade flows between any two locations satisfy the **gravity equation**:

$$X_{ij} = K_{ij}\gamma_i\delta_j, \tag{1}$$

   where $\mathbf{K} \equiv \{K_{ij}\}$ is assumed to be exogenous (i.e. it is a model parameter), while $\{\gamma_i\}$ and $\{\delta_j\}$ are endogenous.

2. In all locations, the **goods market clears**:

$$Y_i = \sum_{j \in S} X_{ij}; \tag{2}$$

3. In all locations, **trade is balanced**:

$$Y_i = \sum_{i \in S} X_{ji}; \tag{3}$$

4. In all locations, the **generalized labor market clearing condition** holds:

$$Y_i = B_i \gamma_i^{\alpha} \delta_i^{\beta}, \tag{4}$$

where α and β are the **gravity constants**.

Combining conditions 1-4, we have the following two equilibrium equations:

$$B_i \gamma_i^{\alpha} \delta_i^{\beta} = \sum_j K_{ij} \gamma_i \delta_j \tag{5}$$

$$B_i \gamma_i^{\alpha} \delta_i^{\beta} = \sum_j K_{ji} \gamma_j \delta_i \tag{6}$$

3 Gravity constants and the trade elasticity

An important subset of the models that fit into the universal framework are those for which $\beta = 0$. This is because in many of these models the gravity constant α is related to the partial elasticity of trade flows to iceberg trade costs. For example, in the Armington model, we have the gravity equation:

$$X_{ij} = \tau_{ij}^{1-\sigma} w_i^{1-\sigma} A_i^{\sigma-1} P_j^{\sigma-1} E_j$$

and the labor market clearing condition

$$Y_i = E_i = w_i L_i$$

Because the origin fixed effect $\gamma_i \equiv \left(\frac{A_i}{w_i}\right)^{\sigma-1}$, we can write the labor market clearing condition:

$$Y_i = A_i L_i \gamma_i^{\frac{1}{1-\sigma}},$$

so that the gravity constant $\alpha \equiv \frac{1}{1-\sigma}$. Combining this with the gravity equation, we have that the elasticity of trade flows to iceberg trade costs (which we will refer to as the **trade elasticity**) is the inverse of the gravity constant:

$$\frac{\partial \ln X_{ij}}{\partial \ln \tau_{ij}} = 1 - \sigma = \frac{1}{\alpha}.$$

Hence, for these models, identifying the gravity constant is equivalent to identifying the trade elasticity. As a result, we will turn our focus to procedures for identifying the trade elasticity, which first requires an examination of traditional reduced form gravity equations.

4 Gravity estimators

Let us first do a little algebra using conditions 1-3 of the universal gravity framework in order to contrast some traditional estimators with more structural approaches. By combining the gravity equation with the balanced trade condition, we can write the destination fixed effect δ_j as a function of its income Y_j and the origin fixed effects in all other countries:

$$\begin{aligned}
Y_j &= \sum_{i \in S} X_{ij} \iff \\
Y_j &= \sum_{i \in S} K_{ij} \gamma_i \delta_j \iff \\
\delta_j &= \frac{Y_j}{\sum_{i \in S} K_{ij} \gamma_i}
\end{aligned}$$

Substituting this expression back into the gravity equation yields an expression for bilateral trade flows that depends only on the origin fixed effect:

$$\begin{aligned}
X_{ij} &= K_{ij} \gamma_i \delta_j \iff \\
X_{ij} &= \frac{K_{ij} \gamma_i}{\sum_{k \in S} K_{kj} \gamma_k} Y_j. \qquad (7)
\end{aligned}$$

Substituting equation (7) into the goods market clearing condition allows us to write the origin fixed effect γ_i as a function of the origin income Y_i and the origin fixed effects in all other countries:

$$\begin{aligned}
Y_i &= \sum_{j \in S} X_{ij} \iff \\
Y_i &= \sum_{j \in S} \frac{K_{ij} \gamma_i}{\sum_{j \in S} K_{kj} \gamma_k} Y_j \iff \\
\gamma_i &= \frac{Y_i}{\sum_{j \in S} \frac{K_{ij}}{\sum_{k \in S} K_{kj} \gamma_k} Y_j}.
\end{aligned}$$

Finally, substituting this expression back into the gravity equation (7) allows us to write bilateral trade flows as a function of the (exogenous) bilateral frictions K_{ij}, the income in the origin and destination, and measures of the "bilateral resistance":

$$\begin{aligned}
X_{ij} &= \frac{K_{ij} \gamma_i}{\sum_{k \in S} K_{kj} \gamma_k} Y_j \iff \\
X_{ij} &= K_{ij} \times \frac{Y_i}{\left(\sum_{j \in S} \frac{K_{ij}}{\sum_{k \in S} K_{kj} \gamma_k} Y_j \right)} \times \frac{Y_j}{\left(\sum_{k \in S} K_{kj} \gamma_k \right)} \iff \\
X_{ij} &= K_{ij} \times \frac{Y_i}{\left(\sum_{k \in S} K_{ik} \frac{Y_k}{\Pi_k} \right)} \times \frac{Y_j}{\Pi_j}, \qquad (8)
\end{aligned}$$

where we define $\Pi_j \equiv \sum_{k \in S} K_{kj} \gamma_k$. Let us call equation (8) the **structural gravity equation**.

4.1 The traditional gravity estimator

Until about a decade ago, almost all estimation procedures based on the gravity equation assumed that trade frictions K_{ij} were a linear function of observed bilateral covariates (e.g. distance, common language, shared border, etc.) $\mathbf{T}_{ij}$, i.e.:

$$K_{ij} = \mathbf{T}_{ij}\beta + \varepsilon_{ij}$$

and estimated β by running the following regression:

$$\ln X_{ij} = \mathbf{T}_{ij}\beta + \ln Y_i + \ln Y_j. \tag{9}$$

Call equation (9) the **traditional gravity estimator**. Comparing the traditional estimating gravity equation to the structural gravity equation (8), it is immediately obvious that the traditional estimating gravity equation is missing (i.e. not controlling for) Π_i or $\sum_{k\in S} K_{ik}\frac{Y_k}{\Pi_k}$. Since we can write the structural gravity equation as:

$$X_{ij} = \frac{K_{ij}\frac{Y_j}{\Pi_j}}{\sum_{k\in S} K_{ik}\frac{Y_k}{\Pi_k}} \times Y_i,$$

we can see that the structural gravity equation implies that the share of trade flows from i to j depends on how large $K_{ij}\frac{Y_j}{\Pi_j}$ is to $K_{ik}\frac{Y_k}{\Pi_k}$ in all other countries. Since $\Pi_j \equiv \sum_{k\in S} K_{kj}\gamma_k$, destinations that are more economically remote (in terms of having lower K_{kj} than average) will tend to have lower Π_j, which will cause country $i \in S$ to export a greater share of its total trade to those destinations. Intuitively, this is because more remote countries will have higher price indices, and hence will be willing to pay more for any given good. This is what Anderson and Van Wincoop (2003) refer to as "multilateral resistance."

Because Π_j will (generically) varies across destinations, the traditional estimating gravity equation will suffer from omitted variable bias. Furthermore, because Π_j depends on the average trade friction between j and the rest of the world, it will be correlated with K_{ij}, which will result in biased estimates of β. This means that you should *never use the traditional estimating gravity equation to estimate trade costs*. Indeed, Baldwin and Taglioni (2006) award papers doing this with the "gold medal error" of estimating gravity equations.

4.2 The fixed effects gravity estimator

An alternative to the traditional estimator is to take logs of the gravity equation (1):

$$\ln X_{ij} = \ln K_{ij} + \ln \gamma_i + \ln \delta_j.$$

If we assume that $\ln K_{ij} = \mathbf{T}_{ij}\beta + \varepsilon_{ij}$, this equation becomes:

$$\ln X_{ij} = \mathbf{T}_{ij}\beta + \ln \gamma_i + \ln \delta_j + \varepsilon_{ij}. \tag{10}$$

Call equation (10) the **fixed effects gravity estimator**. Since $\mathbf{T}_{ij}$ are observed and $\ln \gamma_i$ and $\ln \delta_j$ can be estimated by including dummy variables for each origin country and each destination country (note: there are two dummy variables for country), β can be estimated consistently by applying the fixed effects estimator to equation (10) and γ_i and δ_j can be consistently estimated (to scale) from the coefficients on the dummy variables.

Given the estimates from equation (10), the fixed effects gravity estimator allows us to recover the multilateral resistance terms (to scale) in the structural gravity equation as follows. By taking exponents of the estimates, we can back out predicted values (up to scale) of the origin fixed effect and the bilateral trade costs:

$$\hat{\gamma}_i \equiv \exp\left(\widehat{\ln \gamma_i}\right)$$
$$\hat{K}_{ij} \equiv \exp\left(\mathbf{T}_{ij}\hat{\beta}\right)$$

Since $\Pi_j \equiv \sum_{k \in S} K_{kj}\gamma_k$, we then can construct an estimate of the destination multilateral resistance term:

$$\hat{\Pi}_j \equiv \sum_{k \in S} \hat{K}_{kj}\hat{\gamma}_k = \sum_{k \in S} \exp\left(\mathbf{T}_{kj}\hat{\beta} + \widehat{\ln \gamma_k}\right)$$

which then allows you to construct the origin multilateral resistance term $\sum_{k \in S} \hat{K}_{ik}\frac{Y_k}{\hat{\Pi}_k}$. Note that nowhere in these derivations did we use the estimated destination fixed effect, which suggests that the destination fixed effects are "nuisance parameters," i.e. they are unnecessary (given the equilibrium conditions) to fully derive the gravity equation. This is because, as we saw above, balanced trade implies that the destination fixed effect is pinned down by the origin fixed effects:

$$\delta_j = \frac{Y_j}{\sum_{i \in S} K_{ij}\gamma_i},$$

a restriction that is not made in equation (9). This is the major drawback of this estimation procedure: by relying just on the gravity structure of the model, the fixed effects estimator imposes no equilibrium conditions on the estimation, and as such, the resulting estimates will (generically) not ensure that the goods market clears or that trade is balanced. The other major drawback of the fixed effect estimation procedure is that there may be computational difficulties to including so many dummy variables, especially if one is interested in estimating γ_i and δ_j. (However, if one is simply interested in estimating β, there exist new ways of doing so without having to invert the large dependent variable matrix, see Guimaraes and Portugal (2010)).

That being said, the fixed effects gravity estimator is probably the most common estimator of gravity equations today, as it is simple to implement and model consistent (with the caveat above). It is also straightforward to extend the fixed effects gravity estimator to include multiple years (by adding origin-country-year and destination-country-year dummies), multiple industries (by adding origin-country-industry and destination-country-industry dummies), etc.

4.3 The ratio gravity estimator

An alternative approach is to consider as the dependent variable the (log) trade *shares* rather than the (log) trade *levels*. From gravity equation (1) we have:

$$\frac{X_{ij}}{X_{jj}} = \frac{K_{ij}\gamma_i\delta_j}{K_{jj}\gamma_j\delta_j} = \frac{K_{ij}\gamma_i}{K_{jj}\gamma_j} \implies$$
$$\ln\left(\frac{X_{ij}}{X_{jj}}\right) = \ln\left(\frac{K_{ij}}{K_{jj}}\right) + \ln\gamma_i - \ln\gamma_j.$$

If we assume that $\ln\left(\frac{K_{ij}}{K_{jj}}\right) = \mathbf{T}_{ij}\beta + \varepsilon_{ij}$, then this equation becomes:

$$\ln\left(\frac{X_{ij}}{X_{jj}}\right) = \mathbf{T}_{ij}\beta + \ln\gamma_i - \ln\gamma_j + \varepsilon_{ij}. \tag{11}$$

Following Head and Mayer (2013), we call the (11) the **ratio gravity estimator**. Because the destination fixed effect is constrained to be the negative of the origin fixed effect, the ratio gravity estimator no longer has any nuisance parameters in the estimation, which makes the estimation easier to implement using dummy variables. However, while the ratio gravity estimator has N (where N is the number of countries) fewer parameters to estimate than the fixed effect estimator, it also has N fewer observations since any time $i = j$ equation (11) simplifies to the trivial equation $0 = \varepsilon_{ii}$; hence the degrees of freedom remains unchanged. In addition, as with the fixed effect estimator, the ratio gravity estimator is based only on the gravity equation (1), so it too does not impose that the general equilibrium conditions hold.

Furthermore, since unlike the fixed effects estimator, the ratio gravity estimator only identifies the relative trade frictions rather than the absolute trade frictions. That is, taking exponents of $\ln\left(\frac{K_{ij}}{K_{jj}}\right) = \mathbf{T}_{ij}\beta + \varepsilon_{ij}$ yields:

$$\hat{K}_{ij} = \exp\left(\mathbf{T}_{ij}\hat{\beta}\right)\hat{K}_{jj}.$$

This means that we are unable to recover the multilateral resistance term Π_j since:

$$\sum_{k\in S}\exp\left(\mathbf{T}_{kj}\hat{\beta} + \hat{\ln}\gamma_k\right) = \frac{1}{\hat{K}_{jj}}\sum_{k\in S}\hat{K}_{kj}\hat{\gamma}_k = \frac{\hat{\Pi}_j}{\hat{K}_{jj}}.$$

In order to call this expression the multilateral resistance, one would have to assume (as is often done) that $K_{jj} = 1$, i.e. internal trade is costless.

5 Recovering the trade elasticity from gravity estimates

In the gravity regressions above, we estimated the bilateral trade friction matrix K_{ij}. In models where $K_{ij} = \tau_{ij}^{\frac{1}{\alpha}}$, if we observed the iceberg trade costs and had estimates for K_{ij}, we could estimate the gravity constant with a simple log-linear regression:

$$\ln\tau_{ij} = \alpha\ln K_{ij} + \varepsilon_{ij}.$$

Unfortunately, because trade flows alone will only allow us to identify the total bilateral trade frictions $\{K_{ij}\}$, so we need to rely additional data to recover the iceberg trade costs. The most popular method of doing so is to rely on price data and the no-arbitrage condition.

6 The no arbitrage condition

If prices are proportional to marginal costs and there are iceberg trade costs then for any product $\omega \in \Omega$ produced in any origin $i \in S$ and for any destinations $j \in S$ and $k \in S$:

$$\frac{p_{ij}(\omega)}{p_{ik}(\omega)} = \frac{\tau_{ij}}{\tau_{ik}}.$$

If we assume that $\tau_{ii} = 1$ for all $i \in S$ then setting $k = i$ yields the **no-arbitrage condition**:

$$\frac{p_{ij}(\omega)}{p_{ii}(\omega)} = \tau_{ij}. \tag{12}$$

The no-arbitrage condition provides an exceedingly simply and surprisingly powerful way of identifying the iceberg trade costs. The simplicity of the identification is self-evident: if an origin sells a good to itself and sells it to another destination, then the iceberg trade costs is simply equal to the ratio of the destination price to the origin price.

Why is the result surprisingly powerful? It is because no-arbitrage condition should hold regardless of the model. To see this, suppose that the no-arbitrage condition did not hold and instead that $\frac{p_{ij}(\omega)}{p_{ii}(\omega)} > \tau_{ij}$. This should not be an equilibrium, because any self-interested arbitrageur could purchase the good in $i \in S$, resell the good in $j \in S$, and make a profit. Conversely, suppose that $\frac{p_{ij}(\omega)}{p_{ii}(\omega)} < \tau_{ij}$. This implies that whoever was selling the good from i to j ought to have just sold locally. In the first case, money was being left on the table, while in the latter case, money was being thrown away, both of which tend to make us economists nervous.[1]

Despite the simplicity and power of using the no-arbitrage condition to identify the iceberg trade costs, there are three major difficulties in empirically implementing the estimation strategy:

1. The observed prices in both the origin and destination have to be for the same good ω. Observing prices of identical goods is especially difficult for differentiated varieties; for example, prices of t-shirts across locations may vary because of the quality of the t-shirts rather than because of trade costs.

2. Even if the goods are identical, we need to know that the good was produced in $i \in S$ and sold to $j \in S$. If $j \in S$ purchased a good from another location (or produced it locally), there is no reason that the no-arbitrage equation must hold with location i. Note the inherent tension between the first difficulty and this difficulty: if one is able to find a good that is truly identical across locations (e.g. a commodity), there is a high likelihood that it is produced in many locations.

[1] In my job market paper, I argue that it can be the case that $\frac{p_{ij}}{p_{ii}} > \tau_{ij}$ if it is costly for arbitrageurs to discover what the price is in other locations. I believe that such "information frictions" are quantitatively important in real world markets.

3. The no-arbitrage condition only holds if the price of the good is proportional to the marginal cost of production. This assumption would be violated, for example, if producers had market power and were able to charge variable mark-ups in different destinations. Note in the case of CES, producers do not charge variable mark-ups, but this result is particular to the CES case (and likely unrealistic).

Let us now discuss some of the approaches taken in the trade literature that have attempted (more or less successfully) to navigate these three difficulties.

7 Estimating the no-arbitrage conditions

We now consider four methods used to estimate the no-arbitrage conditions.

7.1 The Eaton and Kortum (2002) approach

Eaton and Kortum (2002) observe 50 manufactured products across the 19 countries in their data set. They note that if a product $\omega \in \Omega$ is not traded between the two countries, then it must be the case that producers of ω found it more profitable to sell domestically, i.e.:

$$p_i(\omega) > \frac{p_j(\omega)}{\tau_{ij}} \iff \tau_{ij} > \frac{p_j(\omega)}{p_i(\omega)},$$

i.e. the iceberg trade costs exceed the price gap. Conversely, if a product is traded, then the no arbitrage equation holds with equality. These two facts imply that the price ratio of all products is bounded above by the iceberg trade cost.

Since they do not observe which of the 50 manufactured products are actually traded between any pair of countries, they employ a "brute force" method of estimating the trade cost by taking the maximum price ratio observed across all products as their measure of the bilateral iceberg trade costs:

$$\hat{\tau}_{ij}^{EK} \equiv \max_{\omega\in\Omega} \frac{p_j(\omega)}{p_i(\omega)}. \tag{13}$$

Equation (13) is a valid estimator of the true iceberg trade costs if at least one of the observed products is traded, prices are measured perfectly, the products observed are identical, and prices are proportional to marginal cost.[2]

Using this estimator, Eaton and Kortum (2002) find a trade elasticity (i.e. $\frac{1}{\alpha}$) of roughly eight; i.e. a 10 percent increase in trade costs is associated with an 80% decline in trade flows. More recently, Simonovska and Waugh (2014) have argued that because it is possible for none of the observed products to have actually been traded, an estimator based on equation (13) will be biased downwards. Because observed trade flows can be rationalized equally well with a higher trade elasticity and lower trade costs or a lower trade elasticity and higher trade costs, if the estimated trade costs are biased downwards, the implied elasticity of trade will be biased upwards. They develop a simulated method of moments estimator that corrects this error, and find an elasticity of trade of approximately four, which currently is the standard in the trade literature.

[2] Recognizing that prices likely are measured with error, Eaton and Kortum (2002) actually use the second highest observed price ratio as their preferred estimator of the iceberg trade cost.

7.2 The Donaldson (forthcoming) approach

In Donaldson (forthcoming) (which we will see in detail in a few lectures), the author had a clever solution to the three difficulties mentioned above of estimating the no-arbitrage condition. He found a homogeneous good where the unique location of production was known: salt! As he writes:

> "Throughout Northern India, several different types of salt were consumed, each of which was regarded as homogenous and each of which was only capable of being made at one unique location."

In the simplest case, having such a good would allow one to construct bilateral trade costs immediately from the no-arbitrage equation, as $\tau_{ij} = \frac{p_{ij}(\omega)}{p_{ii}(\omega)}$. However, even with "perfect" good for which to apply the no-arbitrage condition, Donaldson (forthcoming) faced two additional difficulties. First, it turned out that he did not observe the price of a variety $\omega \in \Omega$ of salt at the origin. Second, since not every location $i \in S$ produced its own unique variety of salt, at best, he could only apply the no-arbitrage condition to find a subset of the bilateral iceberg trade costs.

To solve both problems, Donaldson (forthcoming) made a parametric assumption that $\ln \tau_{ij} = \mathbf{T}_{ij}\beta + \varepsilon_{ij}$. With this assumption, the no arbitrage condition becomes:

$$\begin{aligned}
\tau_{ij} &= \frac{p_{ij}(\omega)}{p_{ii}(\omega)} \iff \\
\ln p_{ij}(\omega) &= \ln p_{ii}(\omega) + \ln \tau_{ij} \iff \\
\ln p_{ij}(\omega) &= \ln p_{ii}(\omega) + \mathbf{T}_{ij}\beta + \varepsilon_{ij}. \qquad (14)
\end{aligned}$$

By including a salt-variety ω fixed effect, β can be estimated using just the observed variation in prices of a particular variety across destinations. Once β is estimated, the trade costs between any origin and destination can be imputed from the parametric assumption. Furthermore, α can be estimated by regressing bilateral trade flows on $\mathbf{T}_{ij}\hat{\beta}$ using the gravity regression in equation (1). Donaldson (forthcoming) estimates the elasticity of trade flows for each commodity in his data set separately and finds a mean of roughly four, consistent with Simonovska and Waugh (2014).

The Donaldson (forthcoming) approach assumes that salt is traded in perfectly competitive markets, which, because salt is a commodity, seems reasonable.

7.3 The Allen (2014) approach

In Allen (2014), I use the spatial dispersion in prices of agricultural commodities (which, unlike Donaldson (forthcoming), were produced in many regions) in order to infer the size of trade costs.

The insight of the approach in this paper is to note that even when two countries do not trade, the no arbitrage condition provides information about the size of the trade costs. The intuition is the same as in the Eaton and Kortum (2002) above: if a particular commodity is not observed to be traded between two locations, then this must mean that the trade cost exceeded the price gap between the two locations, i.e. $X_{ij}(\omega) = 0 \implies \tau_{ij} > \frac{p_j(\omega)}{p_i(\omega)}$, whereas when trade does occur between the two locations, then this must mean that the no-arbitrage equation holds, i.e. $X_{ij}(\omega) > 0 \implies \tau_{ij} = \frac{p_j(\omega)}{p_i(\omega)}$.

Suppose we observe the price of a particular commodity $\omega \in \Omega$ in each location $i \in S$ in each period $t \in \{1, ..., T\}$, i.e. $p_{it}(\omega)$. Furthermore, suppose for any pair of origin $i \in S$ and destination $j \in S$, we observe whether or not trade flows occur in each period $t \in \{1, ..., T\}$, i.e. $\mathbf{1}\{X_{ijt}(\omega) > 0\}$. Finally, suppose that the bilateral trade cost of commodity ω in time t depend on a time invariant bilateral trade cost and an idiosyncratic error that is i.i.d. across time periods:

$$\ln \tau_{ijt}(\omega) = \ln \tau_{ij}(\omega) + \varepsilon_{ijt}(\omega),$$

where $\varepsilon_{ijt}(\omega) \sim N(0, \sigma^2)$.

We can then estimate $\ln \tau_{ij}(\omega)$ using a maximum likelihood routine. The log likelihood function can be written as:

$$\begin{aligned} l(\ln \tau_{ij}, \sigma) = \sum_{t=1}^{T} (&\mathbf{1}\{X_{ijt}(\omega) > 0\} \ln \phi\left(\frac{1}{\sigma}\left(\ln \frac{p_{jt}(\omega)}{p_{it}(\omega)} - \ln \tau_{ij}\right)\right) +, \\ &\mathbf{1}\{X_{ijt}(\omega) = 0\} \ln\left(1 - \Phi\left(\frac{1}{\sigma}\left(\ln \frac{p_{jt}(\omega)}{p_{it}(\omega)} - \ln \tau_{ij}\right)\right)\right)) \end{aligned}$$

which bears a very close resemblance to a Tobit estimator. Using this estimator to identify trade costs and then regressing trade flows on these trade costs to identify the trade elasticity yields an elasticity of a little bit more than two in the context of agricultural trade flows between islands in the Philippines.

In the presence of information frictions where positive trade flows merely indicate that the price ratio exceeds the bilateral trade costs (i.e. $X_{ij}(\omega) > 0 \implies \tau_{ij} < \frac{p_j(\omega)}{p_i(\omega)}$), the the log likelihood function can be written as:

$$\begin{aligned} l(\ln \tau_{ij}, \sigma) = \sum_{t=1}^{T} (&\mathbf{1}\{X_{ijt}(\omega) > 0\} \ln \Phi\left(\frac{1}{\sigma}\left(\ln \frac{p_{jt}(\omega)}{p_{it}(\omega)} - \ln \tau_{ij}\right)\right) +, \\ &\mathbf{1}\{X_{ijt}(\omega) = 0\} \ln\left(1 - \Phi\left(\frac{1}{\sigma}\left(\ln \frac{p_{jt}(\omega)}{p_{it}(\omega)} - \ln \tau_{ij}\right)\right)\right)). \end{aligned}$$

In this case, the log likelihood function is identical to the following Probit regression:

$$\mathbf{1}\{X_{ijt}(\omega) > 0\} = \beta \ln \frac{p_{jt}(\omega)}{p_{it}(\omega)} + \alpha_{ij},$$

where $\beta = \frac{1}{\sigma}$ and $\alpha_{ij} = -\frac{1}{\sigma} \ln \tau_{ij}(\omega)$. Hence, identifying the iceberg trade cost is straightforward: you regress (using a Probit) whether or not trade flows occurred on the observed log price ratio and a constant. The coefficient on the price ratio identifies the variance of the distribution of measurement error in the trade costs and the variance, combined with the constant, identifies the iceberg trade cost. Intuitively, as the measurement error goes to zero, any increase of the log price ratio above the threshold α_{ij} will induce trade with probability one, so that β will approach infinity.

The advantage of this approach relative to Eaton and Kortum (2002) is that it explicitly allows for measurement error in trade costs; the advantage of this approach relative to

Donaldson (forthcoming) is that one does not need to observe exactly where a product was produced. The disadvantage relative to both approaches is that requires knowing whether or not trade flows occurred at the product level.

Like in the Donaldson (forthcoming) approach, an assumption in the Allen (2014) approach is that prices are proportional to marginal costs, which because the focus is on agricultural commodities, seems reasonable.

8 Gravity trade models where $\beta \neq 0$

All the proceeding analysis constrained itself to gravity trade models where the trade elasticity provided sufficient information to recover the gravity constants, i.e. β was assumed to be zero. We now turn to the general case.

From our discussion of identification, we know that trade data alone is insufficient to estimate the gravity constants; however, if both trade data and information about trade costs are observed, then the gravity constants can be recovered. Suppose, for example, that (the change in) trade costs is a function of a vector of observables $\hat{T}_{ij}$, i.e. $\ln \hat{K}_{ij} = \hat{T}'_{ij}\mu$, where the prime denotes a transpose. Then the gravity constants can be recovered in a two-stage estimation process. First, one estimates the (log) change in exporter and importer shifters using the observed (log) change in trade flows, $\ln \hat{X}^o_{ij}$:

$$\ln \hat{X}^o_{ij} = \hat{T}'_{ij}\mu + \ln \hat{\gamma}_i + \ln \hat{\delta}_j + \varepsilon_{ij},$$

where we interpret the residual ε_{ij} as classical measurement error. Second, one estimates the gravity constants by projecting the observed (log) change in income, $\ln \hat{Y}^o_i$, on the estimated change in exporter and importer shifters $\left\{\ln \hat{\gamma}^E_i\right\}$ and $\left\{\ln \hat{\delta}^E_j\right\}$:

$$\ln \hat{Y}^o_i = \alpha \ln \hat{\gamma}^E_i + \beta \ln \hat{\delta}^E_i + \nu_i.$$

While theoretically straightforward, this procedure is practically difficult, as the model predicts that the residual ν_i – unless it is pure measurement error – will be correlated with both $\ln \hat{\gamma}^E_i$ and $\ln \hat{\delta}^E_i$. This omitted variable bias arises because any unobserved change in the income shifter B_i (which causes the income of a location to be higher than observables would imply) will enter the residual and increase both the location's exports (through goods market clearing) and imports (through balanced trade). As a result, estimates of α and β will be biased upwards.

An alternative procedure is to rely on the general equilibrium structure of the model. By incorporating the general equilibrium effects within the estimator, there is no need for a two stage estimation procedure. In particular, we use the structure of the model – which incorporates both the gravity structure (corresponding to the first stage above) and the generalized labor market clearing condition (corresponding to the second stage above) – to calculate the change in the exporter and importer shifters directly. Formally, we can estimate the gravity constants α and β and the trade cost parameter μ by minimizing the squared errors of the observed change in trade costs and the predicted change in trade costs:

$$(\alpha^*, \beta^*, \mu^*) \equiv \arg \min_{\alpha,\beta \in \mathbb{R}, \mu \in \mathbb{R}^S} \sum_i \sum_j \left(\ln \hat{X}_{ij}^o - \hat{T}_{ij}' \mu - \ln \hat{\gamma}_i \left(\hat{\mathbf{T}}\mu; \alpha, \beta \right) - \ln \hat{\delta}_j \left(\hat{\mathbf{T}}\mu; \alpha, \beta \right) \right)^2, \tag{15}$$

where we emphasize that the change in the origin and destination shifters will be determined in general equilibrium and depend on both the gravity constants and the trade cost parameter.

It turns out that equation (15) is best solved by first estimating the μ given a set of gravity constants α and β and then solving for the α and β. Denote $\mu(\alpha, \beta)$ as the trade cost parameter which minimizes the squared error for a given α and β, i.e.:

$$\mu(\alpha, \beta) = \arg \min_{\mu \in \mathbb{R}^S} \sum_i \sum_j \left(\ln \hat{X}_{ij}^o - \hat{T}_{ij}' \mu - \ln \hat{\gamma}_i \left(\hat{\mathbf{T}}\mu; \alpha, \beta \right) - \ln \hat{\delta}_j \left(\hat{\mathbf{T}}\mu; \alpha, \beta \right) \right)^2$$

First order conditions are:

$$-2 \sum_i \sum_j \left(\hat{T}_{ij} + \left(\sum_k \sum_l \hat{T}_{kl} \left(\frac{\partial \ln \hat{\gamma}_i}{\partial \ln K_{kl}} + \frac{\partial \ln \hat{\delta}_j}{\partial \ln K_{kl}} \right) \right) \right) \left(\ln \hat{X}_{ij} - \hat{T}_{ij}' \mu^* - \ln \hat{\gamma}_i \left(\hat{T}\mu^* \right) - \ln \hat{\delta}_j \left(\hat{T}\mu^* \right) \right) = 0$$

Consider the following first order approximations of the log change in the exporter and importer shifters:

$$\ln \hat{\gamma}_i \left(\hat{\mathbf{T}}\mu \right) \approx \sum_k \sum_l \frac{\partial \ln \gamma_i}{\partial \ln K_{kl}} \hat{T}_{kl}' \mu \text{ and } \ln \hat{\delta}_j \left(\hat{\mathbf{T}}\mu \right) \approx \sum_k \sum_l \frac{\partial \ln \delta_j}{\partial \ln K_{kl}} \hat{T}_{kl}' \mu. \tag{16}$$

Substituting these approximations into the first order conditions yields:

$$\begin{aligned} 0 = &\sum_i \sum_j \left(\hat{T}_{ij} + \left(\sum_k \sum_l \hat{T}_{kl} \left(\frac{\partial \ln \hat{\gamma}_i}{\partial \ln K_{kl}} + \frac{\partial \ln \hat{\delta}_j}{\partial \ln K_{kl}} \right) \right) \right) \times \\ &\left(\ln \hat{X}_{ij} - \left(\hat{T}_{ij}' + \sum_k \sum_l \frac{\partial \ln \gamma_i}{\partial \ln K_{kl}} \hat{T}_{kl}' + \sum_k \sum_l \frac{\partial \ln \delta_j}{\partial \ln K_{kl}} \hat{T}_{kl}' \right) \mu^* \right), \end{aligned}$$

or equivalently:

$$\mu^* = \left(\sum_i \sum_j \sum_k \sum_l \frac{\partial \ln \hat{X}_{ij}}{\partial \ln K_{kl}} \hat{T}_{kl} \hat{T}_{ij}' \right)^{-1} \sum_i \sum_j \sum_k \sum_l \frac{\partial \ln \hat{X}_{ij}}{\partial \ln K_{kl}} \hat{T}_{kl} \hat{T}_{ij}' \ln \hat{X}_{ij}$$

It turns out that this expression has an intuitive interpretation. To see this, let us first write it in matrix form. Let $\hat{\mathbf{T}}$ denote the $N^2 \times S$ vector whose $\langle i + j(N-1) \rangle$ row is the $1 \times S$ vector $\hat{T}_{ij}'$, $\mathbf{D}(\alpha, \beta)$ is the $N^2 \times N^2$ matrix whose $\langle i + j(N-1), k + l(N-1) \rangle$ element is $\frac{\partial \ln X_{ij}}{\partial \ln K_{kl}}$ (which we know from previous lecture is a function only of the gravity constants and observed trade flows), and $\hat{\mathbf{y}}$ denote the $N^2 \times 1$ vector whose $\langle i + j(N-1) \rangle$ row is $\ln \hat{X}_{ij}^o$. Then the general equilibrium gravity estimator is:

$$\mu(a,\beta) = \left(\left(\mathbf{D}(\alpha,\beta)\,\hat{\mathbf{T}}\right)'\left(\mathbf{D}(\alpha,\beta)\,\hat{\mathbf{T}}\right)\right)^{-1}\left(\mathbf{D}(\alpha,\beta)\,\hat{\mathbf{T}}\right)'\hat{\mathbf{y}}. \tag{17}$$

Equation (17) says that, to a first order, the general equilibrium estimator is the coefficient one gets from of an ordinary squares regression of the observed hatted variables on a "general equilibrium transformed" explanatory variable $\hat{T}_{ij}^{GE}$:

$$\ln \hat{X}_{ij}^{o} = \left(\hat{T}_{ij}^{GE}\right)'\mu + \varepsilon_{ij},$$

where:

$$\hat{T}_{ij}^{GE} \equiv \sum_k \sum_l \frac{\partial \ln \hat{X}_{ij}}{\partial \ln \hat{K}_{kl}} \hat{T}_{kl}.$$

Intuitively, the general equilibrium transformed regressors capture the effect of the entire set of explanatory variables on any particular observed bilateral trade flow. As a result, $\mu(\alpha,\beta)$ directly accounts for all (first-order) general equilibrium effects arising from the network structure of trade flows.

We can then find the gravity constants α and β which minimize the total squared error. From equation (17) (and the fact that a projection matrix is idempotent), the estimation of the gravity constants can be written as:

$$(\alpha^*,\beta^*) = \arg\min_{\alpha,\beta\in\mathbb{R}} \hat{\mathbf{y}}'\left(\mathbf{I} - \hat{\mathbf{T}}\left(\left(\mathbf{D}(\alpha,\beta)\,\hat{\mathbf{T}}\right)'\left(\mathbf{D}(\alpha,\beta)\,\hat{\mathbf{T}}\right)\right)^{-1}\left(\mathbf{D}(\alpha,\beta)\,\hat{\mathbf{T}}\right)'\right)\hat{\mathbf{y}}. \tag{18}$$

Equation (18) can be estimated using traditional optimization procedures. Using the trade cost shock of joining the WTO, Allen, Arkolakis, and Takahashi (2014) find gravity constants are approximately -30, consistent with a trade elasticity of fourteen and a labor share in the production function of 0.08.

9 Conclusion and next steps

At this point, you have seen all the tools necessary to bring a structural gravity model to the data. In the next few classes, I will present papers which I consider to be "gold standard" examples of how to combine structural general equilibrium gravity models with careful empirical work: Donaldson (forthcoming) (who studies the effect of the construction of railroads in India) and Ahlfeldt, Redding, Sturm, and Wolf (forthcoming) (who study the effect of the construction and collapse of the Berlin Wall).

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