we decided to use data only for the inflation targeting period which unfortunately it is not a long sample.

3 A simple small open economy model

The stylized economy is analogous to the open economy model presented in Kam et al. (2009). We consider the case of a small open economy where households are assumed to maximize a utility function over an infinite life horizon. The household derives utility from leisure and consumption relative to an external habit parameter. There are two types of firms in the economy: domestic goods firms and importing firms. The continuum of monopolistically competitive domestic goods firms produce differentiated goods and operate a linear production technology. The continuum of import retail firms add markups to goods imported at world prices.

Sticky price is introduced to the model following Calvo, where some of the firms choose their prices optimally and the other part chooses according to past inflation. Furthermore, in determining the domestic currency price of the imported good, retail firms are assumed to be monopolistically competitive giving rise to deviations from the law of one price. Finally, monetary policy is assumed to be conducted according to a Taylor-type rule and fiscal policy is specified as a zero debt policy.

The foreign economy is exogenous to the domestic economy and for simplicity it is assumed that output, inflation and real interest rate of the foreign economy are given by uncorrelated AR(1) processes.

3.1. Household sector

The economy is inhabited by a representative household who maximizes an intertemporal utility function given by:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{(C_t - H_t)^{1-\sigma}}{1-\sigma} - \frac{N_t^{1-\varphi}}{1-\varphi} \right), (1)$$

where β^t is the discount factor, σ and φ are the inverse elasticities of intertemporal substitution and labor supply, respectively. N_t is the labor supply and $H_t = hC_{t-1}$ is an external habit which is assumed to be proportional to aggregate past consumption. C_t is a composite consumption index given by a constant elasticity of substitution (CES) function:

$$C_t = \left[(1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \quad (2)$$

where α is the share of foreign goods in the domestic consumption bundle. $C_{H,t}$ and $C_{F,t}$ are the usual Dixit–Stiglitz aggregates of the domestic and foreign produced goods:

$$C_{H,t} = \left[\int_0^1 C_{H,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}} \text{ and } C_{F,t} = \left[\int_0^1 C_{F,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}}. \quad (3)$$

Optimal allocation of the household expenditure across each good type gives rise to the demand functions:

$$C_{H,t}(i) = \left(P_{H,t}(i)/P_{H,t}\right)^{-\varepsilon} C_{H,t} \text{ and } C_{F,t}(i) = \left(P_{F,t}(i)/P_{F,t}\right)^{-\varepsilon} C_{F,t} \quad (4)$$

for all $i \in [0, 1]$ where the aggregate price levels are defined as:

$$P_{H,t} = \left[\int_0^1 P_{H,t}(i)^{1-\varepsilon} di\right]^{\frac{1}{1-\varepsilon}} \text{ and } P_{F,t} = \left[\int_0^1 P_{F,t}(i)^{1-\varepsilon} di\right]^{\frac{1}{1-\varepsilon}}. \quad (5)$$

The optimal consumption demand of home and foreign goods can be derived as:

$$C_{H,t} = (1-\alpha)\left(P_{H,t}/P_t\right)^{-\eta} C_t \text{ and } C_{F,t} = \alpha\left(P_{F,t}/P_t\right)^{-\eta} C_t \quad (6)$$

where P_t is the domestic price consumer price index (CPI):

$$P_t = \left[(1-\alpha)P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta}\right]^{\frac{1}{1-\eta}}. \quad (7)$$

The total consumption expenditures by domestic households are given by $P_{H,t}C_{H,t} + P_{F,t}C_{F,t} = P_tC_t$. Hence, the representative household intertemporal budget constraint, for all $t > 0$, is:

$$P_tC_t + \mathbb{E}_t\{Q_{t,t+1}D_{t+1}\} \leq D_t + W_tN_t + T_t \quad (8)$$

where D_{t+1} is the nominal pay-off in period $t + 1$ of the portfolio held at the end of period t. $Q_{t,t+s}$ is the stochastic discount factor. The stochastic discount factor will be inversely related to the gross return on a nominal risk-less one-period bond $\mathbb{E}_tQ_{t,t+1} = R^{-1}$. W_t are wages earned on labor supplied N_t. Finally, T_t denotes government lump-sum taxes and transfers.

The intratemporal condition relating labor supply to the real wage must satisfy:

$$(C_t - H_t)^{\sigma} N_t^{\varphi} = \frac{W_t}{P_t}. \quad (9)$$

The intertemporal optimality for the household decision problem is:

$$\boldsymbol{\beta}\left(\frac{C_{t+1}-H_{t+1}}{C_t-H_t}\right)^{-\sigma}\left(\frac{P_t}{P_{t+1}}\right)=\boldsymbol{Q}_{t,t+1} \qquad (10)$$

and taking expectations yields the stochastic Euler equation:

$$\boldsymbol{\beta R_t}\mathbb{E}_t\left\{\left(\frac{C_{t+1}-H_{t+1}}{C_t-H_t}\right)^{-\sigma}\left(\frac{P_t}{P_{t+1}}\right)\right\}=\mathbf{1} \quad (11)$$

3.2. Domestic goods firms

There is a continuum of monopolistically competitive domestic firms $i \in [0, 1]$ producing differentiated goods. Calvo-style price setting is assumed which allows inflation to be partly a jump variable and also partially backward looking (Kam et al., 2009).

Goods are produced by firms using a linear production technology $Y_{H,t}(i) = \epsilon_{a,t}N_t(i)$ where $\epsilon_{a,t}$ is exogenous domestic technology shock. In each period t, a fraction $1 - \theta_H$ of firms set prices optimally and the remaining fraction of firms ($0 < \theta_H < 1$) partially index their price according to:

$$\boldsymbol{P_{H,t}(i)} \ = \ \boldsymbol{P_{H,t-1}(i)}\left(\frac{P_{H,t-1}}{P_{H,t-2}}\right)^{\delta_H} \qquad (12)$$

where $\delta_H \in [0,1]$ measures the degree of inflation indexation. All firms that set prices optimally in period t face the same decision problem and consequently they will set the same price $P_{H,t}^{new}$. Given Calvo price setting, the evolution of the aggregate domestic goods price index is given by:

$$\boldsymbol{P_{H,t}}=\left\{(\mathbf{1}-\boldsymbol{\theta_H})(\boldsymbol{P_{H,t}^{new}})^{\mathbf{1}-\boldsymbol{\varepsilon}}+\boldsymbol{\theta_H}\left[\boldsymbol{P_{H,t-1}}\left(\frac{P_{H,t-1}}{P_{H,t-2}}\right)^{\delta_H}\right]^{\mathbf{1}-\boldsymbol{\varepsilon}}\right\}^{\frac{1}{1-\varepsilon}}. \qquad (13)$$

Firms setting prices in period t face a demand curve given by the demand constraint:

$$\boldsymbol{Y_{H,t+s}(i)}=\left[\frac{P_{H,t}(i)}{P_{H,t+s}}\left(\frac{P_{H,t+s-1}}{P_{H,t-1}}\right)^{\delta_H}\right]^{-\varepsilon}\left(\boldsymbol{C_{H,t+s}}+\boldsymbol{C_{H,t+s}^{*}}\right) \qquad (14)$$

where * denotes parameters and variables of the rest of the world.

The firm's price-setting problem in period t is to maximize the expected present discounted value of profits:

$$\max_{P_{H,t}(i)} \mathbb{E}_t \sum_{s=0}^{\infty} \boldsymbol{Q_{t,t+s}\theta_H^s Y_{H,t+s}(i)} \left[\boldsymbol{P_{H,t}(i)} \left(\frac{\boldsymbol{P_{H,t+s-1}}}{\boldsymbol{P_{H,t-1}}}\right)^{\boldsymbol{\delta_H}} - \boldsymbol{P_{H,t+s}MC_{H,t+s})}\right], \quad (15)$$

$$s.t. (14)\ for\ t,s\ \in\ \mathbb{N}$$

where the real marginal cost is:

$$\boldsymbol{MC_{H,t+s}} = \frac{\boldsymbol{W_{t+s}}}{\boldsymbol{\epsilon_{a,t+s}P_{H,t+s}}}. \quad (16)$$

The factor θ_H^s in the equation (15) is the probability that the firm will not allowed to adjust its price in the next *s* periods.

The firm's optimization problem implies the first-order condition:

$$\mathbb{E}_t \sum_{s=0}^{\infty} \boldsymbol{Q_{t,t+s}\theta_H^s Y_{H,t+s}(i)} \left[\boldsymbol{P_{H,t}(i)} \left(\frac{\boldsymbol{P_{H,t+s-1}}}{\boldsymbol{P_{H,t-1}}}\right)^{\boldsymbol{\delta_H}} - \left(\frac{\varepsilon}{\varepsilon-1}\right) \boldsymbol{P_{H,t+s}(i)MC_{H,t+s}}\right] = \mathbf{0}.$$

(17)

Let the home goods inflation rate be $\pi_{H,t} \coloneqq ln(P_{H,t}/P_{H,t-1})$ and $y_t \coloneqq ln(Y_t/Y_{ss})$ be the percentage deviation of home output from steady state. The log-linear approximation of the optimal pricing decision rule gives the following Phillips curve for domestic goods inflation:

$$\boldsymbol{\pi_{H,t} - \delta_H\pi_{H,t-1} = \beta\mathbb{E}_t(\pi_{H,t+1} - \delta_H\pi_{H,t}) + \lambda_H mc_{H,t}} \quad (18)$$

where $\lambda_H = (1-\beta\theta_H)(1-\theta_H)\theta_H^{-1}$ and

$$\boldsymbol{mc_{H,t} = \varphi y_t - (1+\varphi)\epsilon_{a,t} + \alpha s_t + \frac{\sigma}{1-h}(y_t^* - hy_{t-1}^*) + q_t} \quad (19)$$

3.3. Importing firms

Importing firms are assumed to buy imported goods at competitive prices. However, in determining the domestic price of the imported good, firms are assumed to be monopolistically competitive. This small degree of pricing power leads to a violation of the law of one price in the short run (Justiniano and Preston, 2010b). The law of one price (LOP) gap, that is, the difference between the price of imported goods in domestic currency terms and the domestic retail price of imported goods, in log-linear terms is defined as:

$$\boldsymbol{\psi_{F,t} = e_t + p_t^* - p_{F,t}} \quad (20)$$

where e_t is the percentage deviation of nominal exchange rate from its steady state.

Importing firms also face a Calvo-style price setting problem. In any period *t*, an fraction $1 - \theta_F$ of firms set prices optimally while the other fraction of firms adjusts goods price according to an indexation rule as described in the last section. Hence, the evolution of the imports price index is given by:

$$\boldsymbol{P_{F,t}} = \left\{(\mathbf{1} - \boldsymbol{\theta_F})(\boldsymbol{P_{F,t}^{new}})^{\mathbf{1}-\boldsymbol{\varepsilon}} + \boldsymbol{\theta_F}\left[\boldsymbol{P_{F,t-1}}\left(\frac{\boldsymbol{P_{F,t-1}}}{\boldsymbol{P_{F,t-2}}}\right)^{\boldsymbol{\delta_F}}\right]^{\mathbf{1}-\boldsymbol{\varepsilon}}\right\}^{\frac{\mathbf{1}}{\mathbf{1}-\boldsymbol{\varepsilon}}}. \qquad (21)$$

Firms setting prices in period *t* face a demand curve given by the demand constraint:

$$\boldsymbol{Y_{F,t+s}}(\boldsymbol{i}) = \left[\frac{\boldsymbol{P_{F,t}}(\boldsymbol{i})}{\boldsymbol{P_{F,t+s}}}\left(\frac{\boldsymbol{P_{F,t+s-1}}}{\boldsymbol{P_{F,t-1}}}\right)^{\boldsymbol{\delta_F}}\right]^{-\boldsymbol{\varepsilon}}\boldsymbol{C_{F,t+s}}. \qquad (22)$$

The firm's price-setting problem in period *t* is to maximize the expected present discounted value of profits:

$$\max_{\boldsymbol{P_{F,t}}(\boldsymbol{i})} \mathbb{E}_{\boldsymbol{t}}\sum_{\boldsymbol{s}=\mathbf{0}}^{\infty}\boldsymbol{Q_{t,t+s}}\boldsymbol{\theta_F^s}\boldsymbol{Y_{F,t+s}}(\boldsymbol{i})\left[\boldsymbol{P_{F,t}}(\boldsymbol{i})\left(\frac{\boldsymbol{P_{F,t+s-1}}}{\boldsymbol{P_{F,t-1}}}\right)^{\boldsymbol{\delta_F}} - \tilde{\boldsymbol{e}}_{\boldsymbol{t+s}}\boldsymbol{P_{F,t+s}^{*}}(\boldsymbol{i})\right], \qquad (23)$$

$$s.t.(22)\ for\ t,s\ \in\ \mathbb{N}$$

The firm's optimization problem implies the first order condition:

$$\mathbb{E}_{\boldsymbol{t}}\sum_{\boldsymbol{s}=\mathbf{0}}^{\infty}\boldsymbol{Q_{t,t+s}}\boldsymbol{\theta_F^s}\boldsymbol{Y_{F,t+s}}(\boldsymbol{i})\left[\boldsymbol{P_{F,t}}(\boldsymbol{i})\left(\frac{\boldsymbol{P_{F,t+s-1}}}{\boldsymbol{P_{F,t-1}}}\right)^{\boldsymbol{\delta_F}} - \left(\frac{\boldsymbol{\varepsilon}}{\boldsymbol{\varepsilon}-\mathbf{1}}\right)\tilde{\boldsymbol{e}}_{\boldsymbol{t+s}}\boldsymbol{P_{F,t+s}^{*}}(\boldsymbol{i})\right] = \mathbf{0}. \qquad (24)$$

The log-linearization of this condition gives:

$$\boldsymbol{\pi_{F,t}} = \boldsymbol{\beta}\mathbb{E}_{\boldsymbol{t}}(\boldsymbol{\pi_{F,t+1}} - \boldsymbol{\delta_F}\boldsymbol{\pi_{F,t}}) + \boldsymbol{\delta_F}\boldsymbol{\pi_{F,t-1}} + \boldsymbol{\lambda_F}\boldsymbol{\psi_{F,t}} \qquad (25)$$

where $\lambda_F = (1 - \beta\theta_F)(1 - \theta_F)\theta_F^{-1}$.

3.4. Monetary policy

As in Kam et al. (2009) it is assumed that the monetary policy is conducted according to the simple Taylor type rule in log-linear terms:

$$\boldsymbol{r_t} = \boldsymbol{\rho_r}\boldsymbol{r_{t-1}} + (\mathbf{1} - \boldsymbol{\rho_r})(\boldsymbol{\psi_\pi}\boldsymbol{\pi_t} + \boldsymbol{\psi_y}\boldsymbol{y_t} + \boldsymbol{\psi_{\Delta e}}\boldsymbol{\Delta e_t}) + \boldsymbol{\varepsilon_{r,t}} \qquad (26)$$

where $\rho_r, \psi_\pi, \psi_y, \psi_{\Delta e}$ are the policy responses to the lag of the nominal interest rate, inflation, output growth and the change in the nominal exchange rate, respectively.

$\varepsilon_{r,t} \sim i.i.d.(0, \sigma_r^2)$ is an exogenous monetary policy shock or implementation error in the conduct of policy.

The change in the nominal exchange rate was included in the equation despite the low evidence that central banks in many countries explicitly respond to it (Justiniano and Preston, 2010b, Palma and Portugal, 2014 and Lubik and Schorfheide, 2007).

3.5. International risk sharing, terms of trade and equilibrium

The rest of the world solves a similar problem to the small open economy. Therefore, similar first-order conditions for optimal labor supply and consumption also hold. From equation (10) it is possible to derive the uncovered interest parity condition or the no-arbitrage condition for exchange rates:

$$\boldsymbol{R_t - R_t^* \frac{\tilde{e}_t}{\tilde{e}_{t+1}} = 0.} \qquad (27)$$

A log-linear approximation of this equation, and taking expectations with respect to the time t, yields the nominal interest parity condition:

$$\mathbb{E}_t \boldsymbol{e_{t+1} - e_t = r_t - r_t^*}, \qquad (28)$$

where $e_t = ln(\tilde{e_t}/e_{ss})$ and domestic and foreign interest rates are $r_t = R_t - 1$ and $r^*_t = R^*_t - 1$, respectively.

The real exchange rate is defined as:

$$\boldsymbol{Q_t = \tilde{e}_t \frac{P_t^*}{P_t}}. \qquad (29)$$

Log-linearizing around the deterministic steady state yields:

$$\boldsymbol{q_t = e_t + p_t^* - p_t}. \qquad (30)$$

The terms of trade (ToT) is the ratio of the foreign goods price index to the home goods price index. In log-linear terms it is:

$$s_t = p_{F,t} + p_{H,t}. \qquad (31)$$

Goods market clearing condition in the domestic economy requires for all t that domestic output equals total domestic and foreign demand for home produced goods:

$$\boldsymbol{Y_{H,t} = C_{H,t} + C_{H,t}^*}. \qquad (32)$$

The demand for home and foreign consumption goods can be written in log-linear form as:

$$c_{H,t} = (1-\alpha)[\alpha\eta s_t + c_t] \text{ and } c^*_{H,t} = \alpha[\eta(s_t + \psi_{F,t}) + y^*_t].$$

Therefore, (32) can be written as:

$$\boldsymbol{y_t = (2-\alpha)\alpha\eta s_t + (1-\alpha)c_t + \alpha\eta\psi_{F,t} + \alpha y^*_t} \quad (33)$$

3.6. The log-linearized model

A summary of the log-linear approximations of the model´s first order conditions around a non-stochastic steady state are presented below. Let $c_t \coloneqq ln(C_t/C_{ss})$, $y_t \coloneqq ln(Y_t/Y_{ss})$, $q_t \coloneqq ln(Q_t/Q_{ss})$ denote the percentage deviation of home consumption, output and real exchange rate from their respective steady states, where X_{ss} is the deterministic steady state value of a variable X_t, and $\pi_t \coloneqq ln(P_t/P_{t-1})$ is the inflation rate.

The consumption Euler equation is obtained by log-linearizing (10) and taking the expectations on the time *t*:

$$\boldsymbol{c_t - hc_{t-1} = \mathbb{E}_t(c_{t+1} + hc_t) - \frac{1-h}{\sigma}(r_t - \mathbb{E}_t\pi_{t+1})}. \quad (34)$$

Domestic goods inflation is given by (18):

$$\pi_{H,t} = \beta\mathbb{E}_t(\pi_{H,t+1} - \delta_H\pi_{H,t}) + \delta_H\pi_{H,t-1} + \lambda_H\left[\varphi y_t - (1+\varphi)\epsilon_{a,t} + \alpha s_t + \frac{\sigma}{1-h}(c_t - hc_{t-1})\right]. \quad (35)$$

Imports inflation is given by (25):

$$\pi_{F,t} = \beta\mathbb{E}_t(\pi_{F,t+1} - \delta_F\pi_{F,t}) + \delta_F\pi_{F,t-1} + \lambda_F[q_t - (1-\alpha)s_t]. \quad (36)$$

Combining (30) and (28) yields the real interest parity condition:

$$\mathbb{E}_t(q_{t+1} - q_t) = (r_t - \mathbb{E}_t\pi_{t+1}) - (r^*_t - \mathbb{E}_t\pi^*_{t+1}) + \epsilon_{q,t} \quad (37)$$

where $\epsilon_{q,t}$ is a real interest parity shock. This shock is introduced to capture deviations from the uncovered interest parity condition resulting from risk premium shocks (Matheson, 2010).

First-differencing the terms of trade equation (31):

$$s_t - s_{t-1} = \pi_{F,t} - \pi_{H,t} + \epsilon_{s,t} \quad (38)$$

where $\epsilon_{s,t}$ is terms of trade (ToT) shock.

Goods market clearing condition (32) in combination with the LOP gap (20) yields:

$$y_t = (1-\alpha)c_t + \alpha\eta q_t + \alpha\eta s_t + \alpha y_t^*. \qquad (39)$$

The first-difference of CPI definition gives CPI inflation:

$$\pi_t = (1-\alpha)\pi_{H,t} + \alpha\pi_{F,t}. \quad (40)$$

Exogenous stochastic processes for ToT, technology, and real-interest parity shocks are given by:

$$\epsilon_{j,t} = \rho_j \epsilon_{j,t-1} + \varepsilon_{j,t};\ \varepsilon_j \sim i.i.d.(0, \sigma_j^2)$$

for j = s, a, q. Finally, as in Kam et al. (2009), it is assumed for simplicity that the foreign processes $\{\pi^*, y^*, r^*\}$ are given by uncorrelated AR(1) processes:

$$\pi_t^* = a_1\pi_{t-1}^* + \varepsilon_{\pi^*,t}\ ,$$

$$y_t^* = b_1 y_{t-1}^* + \varepsilon_{y^*,t}\ ,$$

$$r_t^* = c_1 r_{t-1}^* + \varepsilon_{r^*,t}\ .$$

where $\varepsilon_{i,t} \sim N(0, \sigma_i^2)$, for $i = \pi_t^*$, y_t^* and r_t^*.

The model is solved using the set of equations (34) - (40) that describe the domestic economy and monetary policy rule (26) in the variables $\{c_t, y_t, r_t, q_t, s_t, \pi_t, \pi_{H,t}, \pi_{F,t}\}$, combined with the processes for the exogenous disturbances $\{\epsilon_{s,t}, \epsilon_{a,t}, \epsilon_{q,t}\}$ and the foreign economy $\{\pi_t^*, y_t^*, r_t^*\}$.

4 Data

In estimation, we use seven observable series to match the same number of structural shocks in the model. We collect quarterly time series of the variables $\{q_t, y_t, \pi_t, r_t, \pi_t^*, y_t^*, r_t^*\}$, for the period of inflation targeting regime (2000Q1–2014Q4), totaling 60 observations[1]. We use the index of real exchange rate, q_t; Home output (real GDP per capita), y_t; Home CPI inflation (*IPCA* index), π_t; Home nominal interest rate (*Selic* rate), r_t; U.S. inflation (CPI), π_t^*; U.S. output (real GDP per capita), y_t^*; and U.S. interest rates (FED fund rates), r_t^*.

[1] Inflation targeting was adopted in June 1999 but I dropped the first observations.