Macroeconomics A; EI056

Short problems

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1 Utility functions

1.1 Derivatives

Question: We consider two standard utility functions in macroeconomics
The first is the so called CRRA (constant relative risk aversion):

$$U_{CRRA}\left(c\right)=\frac{c^{1-\theta}}{1-\theta}$$

The second is the CARA (constant absolute risk aversion), more standard in finance:

$$U_{CARA}\left(c\right)=-\exp\left[-\chi c\right]$$

Compute the first and second derivatives.

1.2 Elasticity

Question: The absolute risk aversion of a function is $-U''/U'$, while the relative risk aversion is $-cU''/U'$.
Compute these for CARA and CRRA.
Can you see why macroeconomists prefer CRRA, while finance economists like CARA?

2 Dynamics of consumption and capital

2.1 Steady state

Question: Consider the Ramsey model seen in class. The production function is a function of capital (scaled by labor) as in the Solow model: $y_t=\left(k_t\right)^{\alpha}$
The overall model boils down to the Euler condition and the budget constraint (capital dynamics):

$$\begin{aligned}\frac{c_{t+1}}{c_t} &= \left(\frac{1+\alpha\left(k_{t+1}\right)^{\alpha-1}-\delta}{1+\rho}\right)^{\frac{1}{\theta}} \\ \left(1+n\right)\left(k_{t+1}-k_t\right) &= \left(k_t\right)^{\alpha}-\left(n+\delta\right)k_t-c_t\end{aligned}$$

Show that the steady state is:

$$k^* = c^* \left(\frac{\alpha}{\rho+\delta}\right)^{\frac{1}{1-\alpha}} \qquad ; \qquad c^* = \left[\frac{\rho+\delta}{\alpha} - (n+\delta)\right]\left(\frac{\alpha}{\rho+\delta}\right)^{\frac{1}{1-\alpha}}$$

Show that the real interest rate is $r^* = \rho + \delta$.

2.2 Linearization of the Euler

Question: Show that the Euler condition is linearized as (where $\hat{x}_t = (x_t - x^*)/x^*$):

$$\hat{c}_{t+1} - \hat{c}_t = -(1-\alpha)\frac{1}{\theta}\frac{r^*}{1+r^*-\delta}\hat{k}_{t+1}$$

2.3 Linearization of the budget constraint

Question: Show that the budget constraint is linearized as:

$$\hat{k}_{t+1} = \frac{1+r^*-\delta}{1+n}\hat{k}_t - \frac{1}{\alpha}\frac{r^*-\alpha(n+\delta)}{1+n}\hat{c}_t$$

2.4 Undetermined coefficients: consumption

Question: The form of the linearized model is that current consumption and future capital are both linear function of current capital:

$$\hat{c}_t = \eta_{ck}\hat{k}_t \qquad ; \qquad \hat{k}_{t+1} = \eta_{kk}\hat{k}_t$$

where η_{ck} and η_{kk} are coefficients to compute.

Using the budget constraint, shows that:

$$\eta_{ck} = \frac{\alpha(1+r^*-\delta)}{r^*-\alpha(n+\delta)} - \frac{\alpha(1+n)\eta_{kk}}{r^*-\alpha(n+\delta)}$$

2.5 Undetermined coefficients: capital

Question: Using the result for η_{ck} and the Euler condition, shows that:

$$\begin{aligned} 0 = & -\alpha(1+r^*-\delta)(1+n)(\eta_{kk})^2 \\ & + \left[\begin{array}{c} \alpha(1+r^*-\delta)(1+n) + \alpha(1+r^*-\delta)^2 \\ +(r^*-\alpha(n+\delta))\frac{1}{\theta}(1-\alpha)r^* \end{array}\right]\eta_{kk} \\ & -\alpha(1+r^*-\delta)^2 \end{aligned}$$

What are the values of the two solutions for η_{kk}? Don't compute them, but instead think of whether they are positive, negative, larger or smaller than 1 in absolute value.

Which solution makes economic sense?