Macroeconomics A; EI060

Short problems

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Class of March 5, 2025

1 Consumption allocation

Question: Consider a two period model where the consumers maximizes a log utility:

$$U_1 = ln\left(C_1\right) + \beta ln\left(C_2\right)$$

The consumption basket is given by:

$$\begin{aligned} C_t &= \left(C_t^T\right)^{\gamma}\left(C_t^N\right)^{1-\gamma} \\ C_t^T &= \left(C_t^H\right)^{\theta}\left(C_t^F\right)^{1-\theta} \end{aligned}$$

Take the Foreign traded good to have a price of 1. The price of the Home traded good is P_t^H and that of the non-traded good P_t^N .

Show that:

$$\begin{aligned} P_t^H C_t^H &= \gamma\theta P_t C_t \\ C_t^F &= \gamma\left(1-\theta\right) P_t C_t \\ P_t^N C_t^N &= \left(1-\gamma\right) P_t C_t \end{aligned}$$

and:

$$P_t = \frac{1}{\left(\gamma\theta\right)^{\gamma\theta}\left(\gamma\left(1-\theta\right)\right)^{\gamma(1-\theta)}\left(1-\gamma\right)^{1-\gamma}}\left(P_t^H\right)^{\gamma\theta}\left(P_t^N\right)^{1-\gamma}$$

2 Intertemporal allocation

Question: Outputs are endowments, and the consumer can save in a bond denominated in the Foreign traded good, with a return r. We asssume $\beta\left(1+r\right)=1$.

Show that:

$$\begin{aligned} C_2 &= \left(1+r^C\right) C_1 \\ 1+r^C &= \left(\frac{P_1^H}{P_2^H}\right)^{\gamma\theta}\left(\frac{P_1^N}{P_2^N}\right)^{1-\gamma} \end{aligned}$$

3 Real exchange rate

Question: The consumption of non-traded good is equal to its endowment each period. Show that:

$$\frac{P_2}{P_1} = \left(\frac{Y_1^N}{Y_2^N}\right)^{1-\gamma}\left(\frac{P_1^H}{P_2^H}\right)^{\gamma\theta}$$

4 Current account

Question: The value of the spending on traded goods in a period is: $P_1^H C_1^H + C_1^F$. Show that:

$$P_1^H C_1^H + C_1^F + \beta\left(P_2^H C_2^H + C_2^F\right) = P_1^H Y_1^H + \beta P_2^H Y_2^H$$

Using the allocation of consumption, and the Euler condition, show that:

$$P_1^H C_1^H + C_1^F = \frac{1}{1+\beta}\left[P_1^H Y_1^H + \beta P_2^H Y_2^H\right]$$

Show that the current account is:

$$\frac{CA_1}{P_1^H Y_1^H} = \frac{\beta}{1+\beta}\left(1 - \frac{P_2^H Y_2^H}{P_1^H Y_1^H}\right)$$

What is the impact of the dynamics of the non-traded endowment?