PS4 Solutions

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Q2. Estimating the SWFF+ Model With and Without the External Finance Premium (1965Q1–2004Q4)

When working on Q2, I found out that the original paper seems to have two more errors other than mentioned by professor in the Problem Set. One is in formula A-24, as the sign of $\overline{k}_t$ should be positive, and the other one is the coefficient of $(R_{t-1} - \pi_t)$ in formula A-26, which should be $\zeta_{n,R}$ instead of $\zeta_{n,\tilde{R}^k_t}$.
I put the derivationas below for reference.

Leverage-sign part of (A-24): why the coefficient on $\bar{k}_t$ is *positive*

Define leverage as the ratio of asset value to internal funds at date t:

$$\Lambda_t \equiv \frac{Q_t \bar{K}_t}{N_t}.$$

Taking logs and linearizing around the non-stochastic steady state gives

$$\widehat{\log \Lambda_t} = \underbrace{\log \frac{Q_t}{P_t}}_{=q^k_t} + \underbrace{\log \frac{\bar{K}_t}{Z_t}}_{=\bar{k}_t} - \underbrace{\log \frac{N_t}{P_t Z_t}}_{=n_t} = q^k_t + \bar{k}_t - n_t.$$

Hence, in any linear spread specification in which the external finance premium rises with leverage, the contribution of $\bar{k}_t$ must enter with a **positive** sign. Intuitively: for given N_t, a higher capital stock $\bar{K}_t$ (or a higher installed-capital price Q_t) increases leverage and therefore increases the premium; the sign cannot be negative without contradicting this accounting relation.

Deriving (A-26): Entrepreneur Net Worth Dynamics (step by step)

Step 1. Nominal balance sheet at the start of t

At $t-1$, the entrepreneur holds installed capital $\bar{K}_{t-1}$ priced at Q_{t-1} and finances it with nominal net worth N_{t-1} and nominal debt B_{t-1}:

$$Q_{t-1}\bar{K}_{t-1} = N_{t-1} + B_{t-1}. \tag{1}$$

Step 2. Contract clearing at t with monitoring losses

At t, the capital position yields the nominal payoff $\left[R_t^k + (1-\delta)Q_t\right]\bar{K}_{t-1}$. Under the BGG/CMR contract, entrepreneurs retain the fraction $1-\Gamma_t$ of that payoff (default/-monitoring absorbs the rest). In *real* terms (dividing by P_t),

$$(1-\Gamma_t)\frac{R_t^k + (1-\delta)Q_t}{P_t}\bar{K}_{t-1}. \tag{2}$$

Debt repayment in real terms equals $\dfrac{R_{t-1}}{\Pi_t}\cdot\dfrac{B_{t-1}}{P_{t-1}}$, where $\Pi_t \equiv P_t/P_{t-1}$.

Step 3. Survival/entry aggregation

With survival probability $\gamma^* \in (0,1)$ and net entry summarized by the linear term φN_{t-1}, aggregate *real* net worth in P_t units satisfies

$$\frac{N_t}{P_t} = \gamma^*\left[(1-\Gamma_t)\frac{R_t^k + (1-\delta)Q_t}{P_t}\bar{K}_{t-1} - \frac{R_{t-1}}{\Pi_t}\frac{B_{t-1}}{P_{t-1}}\right] + \varphi\frac{N_{t-1}}{P_{t-1}}. \tag{3}$$

Step 4. Stationarization (deflation by P_t and detrending by Z_t)

Define stationary date-t stocks

$$n_t \equiv \frac{N_t}{P_tZ_t}, \qquad b_{t-1} \equiv \frac{B_{t-1}}{P_{t-1}Z_{t-1}}, \qquad a_{t-1} \equiv \frac{Q_{t-1}\bar{K}_{t-1}}{P_tZ_t}.$$

Using $g_t \equiv Z_t/Z_{t-1}$, and the identity in (**??**),

$$\frac{B_{t-1}}{P_tZ_t} = b_{t-1}, \frac{Z_{t-1}}{Z_t} = \frac{b_{t-1}}{g_t}, \qquad a_{t-1} = \frac{Q_{t-1}}{P_{t-1}}\frac{\bar{K}_{t-1}}{Z_{t-1}}\frac{P_{t-1}}{P_t}\frac{Z_{t-1}}{Z_t}.$$

Divide (**??**) by Z_t and use $B_{t-1} = Q_{t-1}\bar{K}_{t-1} - N_{t-1}$ to obtain the stationary *levels* equation

$$n_t = \gamma^*\left[(1-\Gamma_t)\mathcal{R}_t^k a_{t-1} - \underbrace{\left(\frac{R_{t-1}}{\Pi_t}\right)}_{\equiv\mathcal{R}_{t-1}}(a_{t-1} - n_{t-1})\right] + \varphi, n_{t-1}, \tag{4}$$

where $\mathcal{R}_t^k \equiv \dfrac{r_t^k + (1-\delta)q_t}{q_{t-1}}$ is the gross *real* return on capital measured in units of the installed-capital price (this equals $\widetilde{R}_t^k - \pi_t$ at first order via (A-25)).

Step 5. First-order expansion of the stationary asset base a_{t-1}

From the definition of a_{t-1},

$$\log a_{t-1} = \log \frac{Q_{t-1}}{P_{t-1}} + \log \frac{\bar{K}_{t-1}}{Z_{t-1}} - \log \Pi_t - \log g_t,$$

hence, at first order (hats denote log-deviations from steady state),

$$\widehat{a}_{t-1} = q_{t-1}^k + \bar{k}_{t-1} - \pi_t - z_t. \tag{5}$$

Critical implication. Because a_{t-1} is a date-t stationary object, it contains the factor $Z_{t-1}/Z_t = 1/g_t$; therefore its first-order log-deviation *must* include the **current** $-z_t$ term. This is the source of the $-z_t$ term in (A-26).

Step 6. Linearization scaffolding

Define for convenience

$$V_t \equiv (1-\Gamma_t)\mathcal{R}_t^k a_{t-1}, \qquad D_t \equiv \mathcal{R}_{t-1}(a_{t-1} - n_{t-1}).$$

Then (4) rewrites as

$$n_t = \gamma^*(V_t - D_t) + \varphi n_{t-1}. \tag{6}$$

Log-linearizing around a non-stochastic steady state $\overline{n}, \overline{V}, \overline{D}, \overline{a}, \overline{\mathcal{R}^k}, \overline{\mathcal{R}}, \overline{\Gamma}$ and dividing by $\overline{n}$ yields

$$\widehat{n}_t = \underbrace{\frac{\gamma^*\overline{V}}{\overline{n}}}_{\zeta_{n,V}} \widehat{V}_t - \underbrace{\frac{\gamma^*\overline{D}}{\overline{n}}}_{\zeta_{n,D}} \widehat{D}_t + \underbrace{\varphi}_{\zeta_{n,n_{t-1}}} \widehat{n}_{t-1}. \tag{7}$$

Step 7. First-order expansions of V_t and D_t

For $V_t = (1-\Gamma_t)\mathcal{R}_t^k a_{t-1}$,

$$\widehat{V}_t = \underbrace{\frac{1}{1-\overline{\Gamma}} \frac{\partial(1-\Gamma_t)}{\partial \sigma_{\omega,t-1}}}_{-\text{positive}} \tilde{\sigma}_{\omega,t-1} + \underbrace{\widehat{\mathcal{R}_t^k}}_{=\ \widetilde{R}_t^k - \pi_t} + \underbrace{\widehat{a}_{t-1}}_{=\ q_{t-1}^k + \bar{k}_{t-1} - \pi_t - z_t}. \tag{8}$$

For $D_t = \mathcal{R}_{t-1}(a_{t-1} - n_{t-1})$, the log-linearization of a level difference gives share weights:

$$\widehat{D}_t = \widehat{\mathcal{R}_{t-1}} + \underbrace{\frac{\overline{a}}{\overline{a}-\overline{n}}}_{=\overline{A}/\overline{B}}\widehat{a}_{t-1} - \underbrace{\frac{\overline{n}}{\overline{a}-\overline{n}}}_{=\overline{N}/\overline{B}}\widehat{n}_{t-1}, \qquad \overline{B} \equiv \overline{a} - \overline{n}. \tag{9}$$

Step 8. Collect terms (A-26)

Substituting (8)-(9) into (7) and grouping yields

$$\begin{aligned}
\widehat{n}_t = {} & \underbrace{\gamma^* \frac{\overline{V}}{\overline{n}}}_{\zeta_{n,\widetilde{R}^k_t}} (\widetilde{R}^k_t - \pi_t) - \underbrace{\gamma^* \frac{\overline{D}}{\overline{n}}}_{\zeta_{n,R_{t-1}}} (R_{t-1} - \pi_t) \\
& + \underbrace{\left[\gamma^* \frac{\overline{V}}{\overline{n}} - \gamma^* \frac{\overline{D}}{\overline{n}} \frac{\overline{A}}{\overline{B}}\right]}_{\zeta_{n,qK_{t-1}}} (q^k_{t-1} + \bar{k}_{t-1}) + \underbrace{\left[\varphi + \gamma^* \frac{\overline{D}}{\overline{n}} \frac{\overline{N}}{\overline{B}}\right]}_{\zeta_{n,n_{t-1}}} n_{t-1} \\
& - \underbrace{\gamma^* \frac{\overline{V}}{\overline{n}}}_{\zeta_{n,z_t}} z_t - \underbrace{\gamma^* \frac{\overline{V}}{\overline{n}}, \frac{1}{1-\overline{\Gamma}} \frac{\partial(1-\Gamma)}{\partial \sigma_\omega}}_{\zeta_{n,\sigma_{\omega,t-1}}} \tilde{\sigma}_{\omega,t-1} \\
= {} & \zeta_{n,\widetilde{R}^k_t}(\widetilde{R}^k_t - \pi_t) - \zeta_{n,R_t}(R_{t-1} - \pi_t) + \zeta_{n,qK_t}(q^k_{t-1} + \bar{k}_{t-1}) \\
& + \zeta_{n,n_t} n_{t-1} - \zeta_{n,z_t} z_t - \frac{\zeta_{n,\sigma_{\omega,t}}}{\zeta_{sp,\sigma_\omega}} \tilde{\sigma}_{\omega,t-1}.
\end{aligned} \tag{10}$$

This is (A-26) in rigorous first-order form, with explicit time subscripts on every coefficient (e.g., $\zeta_{n,\widetilde{R}^k_t}$) and with the missing $-, \zeta_{n,z_t}, z_t$ term made explicit. The coefficient of z_t comes *solely* from the Z_{t-1}/Z_t term inside a_{t-1} and equals $\gamma^*(\overline{V}/\overline{n})$ in steady-state shares. In the Dynare parameterization used by the paper, this corresponds to $-$`cgammstar` $\cdot$ `cvstar/cnstar` $\cdot\, z_t$.

Economic consistency

The signs match the financial accelerator logic: a higher current capital real return raises net worth; a higher last-period real debt rate lowers it; a larger beginning-of-period asset base increases it (net of the debt share subtraction); dispersion reduces it through default-/monitoring losses; and a positive technology growth innovation z_t *reduces* stationary net worth because all stocks are expressed in date-t units and thus scaled by Z_t.

Table 1: Comparison of Estimated Parameter Posterior Modes (Q2, SW sample 1965Q1–2004Q4)

Parameter	Description	Dynare Name	Q2 (with *sobs*)	Q2v2 (no *sobs*)
Structural Parameters				
α	Capital Share	`calfa`	0.2652	0.2652
σ_c	Intertemporal Substitution (IES^{-1})	`csigma`	1.5127	2.1858
h	Habit Formation	`chabb`	0.5319	0.7239
ξ_w	Calvo Probability (Wages)	`cprobw`	0.9046	0.7656
σ_ℓ	Inverse Frisch Elasticity	`csigl`	1.8145	1.4809
ξ_p	Calvo Probability (Prices)	`cprobp`	0.6859	0.5953
ι_w	Indexation (Wages)	`cindw`	0.2766	0.4702
ι_p	Indexation (Prices)	`cindp`	0.2625	0.3401
ψ	Capacity Utilization Cost	`czcap`	0.4425	0.5389
ϕ	Investment Adjustment Cost	`csadjcost`	0.0850	1.5994
Φ	Fixed Costs	`cfc`	1.4276	1.6161
Monetary Policy Parameters				
r_π	Taylor Rule: Inflation	`crpi`	2.0297	1.8685
ρ	Taylor Rule: Persistence	`crr`	0.8554	0.8427
r_y	Taylor Rule: Output Gap	`cry`	0.1584	0.1595
$r_{\Delta y}$	Taylor Rule: Output Growth	`crdy`	0.2888	0.2440
Shock Process Parameters				
ρ_a	Persistence: Productivity	`crhoa`	0.9668	0.9353
ρ_b	Persistence: Risk Premium	`crhob`	0.8686	0.1992
ρ_g	Persistence: Gov. Spending	`crhog`	0.9815	0.9890
ρ_i	Persistence: Investment	`crhoqs`	0.9954	0.5890
ρ_r	Persistence: Monetary Policy	`crhoms`	0.0293	0.0658
ρ_p	Persistence: Price Markup	`crhopinf`	0.8947	0.9096
ρ_w	Persistence: Wage Markup	`crhow`	0.6020	0.9757
μ_p	MA Term: Price Markup	`cmap`	0.7306	0.6470
μ_w	MA Term: Wage Markup	`cmaw`	0.8124	0.8462
ρ_{ga}	Feedback Tech. on Spending	`cgy`	0.7746	0.7044
ρ_{σ^w}	Persistence: Wage Trend Variance	`crhosigw`	0.9945	0.7193
ρ_{pI}	Persistence: Investment Price	`crhopist`	0.9967	0.9985
Other/Levels				
n^*	Labor Supply Shift (level)	`cnstar`	2.5374	0.5057
γ	Trend Growth (quarterly, %)	`cgamma`	0.5072	0.4347
$\zeta_{sp,b}$	Financial Wedge Elasticity to Leverage	`czeta_spb`	0.0459	0.0482
$\bar{\pi}$	Inflation Target (quarterly, %)	`constepinf`	0.3016	0.3042

Notes: Posterior modes using `mode_compute=1`. Q2 includes the external finance premium observable `sobs`; Q2v2 excludes it. Numbers are read from Dynare logs. Symbols and groupings follow Cai *et al.* (2019) and Smets–Wouters (2007). The Q2v2 run ends with a non-p.d. Hessian, so standard errors are unreliable for that column.

Table 2: Estimated Shock Standard Deviations (Posterior Modes)

Shock	Prior mean	Mode (with `sobs`)	Mode (no `sobs`)
TFP (ε^a)	0.1000	0.4681	0.4394
Risk prem. (ε^b)	0.1000	0.0906	0.2020
Gov. spending (ε^g)	0.1000	2.7908	2.8966
Inv.-spec. tech (ε^{qs})	0.1000	1.8478	0.9110
Mon. policy (ε^m)	0.1000	0.2365	0.2261
Price markup (ε^{pinf})	0.1000	0.1661	0.1702
Wage markup (ε^w)	0.1000	0.3207	0.3357
Wage trend var. (ε^{σ^w})	0.1000	0.0714	0.0462
Inv. price (ε^{pI})	0.1000	0.0360	0.0413

Notes: Posterior modes from the "standard deviation of shocks" blocks. In both runs the measurement-error innovation for the financial wedge (ε^{zp}) is fixed at variance 0.01; it is not freely estimated.

Table 3: Unconditional Variance Decomposition (%) – SW Sample

(a) With `sobs`

	ea	eb	eg	eqs	em	epinf	ew	esigw	epist	ezp
y	2.42	4.87	2.59	15.53	4.42	1.35	0.05	0.92	27.24	40.61
c	0.65	5.15	5.45	22.41	2.14	0.43	0.03	0.62	29.72	33.40
$inve$	3.40	5.91	0.39	42.57	6.99	1.74	0.10	18.77	5.31	14.83
π	1.55	0.58	0.01	0.08	0.19	9.14	0.19	0.16	7.06	81.04
r	2.00	17.33	0.39	0.48	1.00	0.96	0.09	1.26	4.98	71.51
w	3.46	0.32	0.11	16.96	0.32	5.66	0.54	0.85	4.50	67.27
k	1.93	1.58	0.07	72.86	1.57	1.38	0.05	4.85	11.66	4.05
ℓ	0.93	4.18	3.11	2.53	3.44	0.66	0.08	0.96	15.63	68.48

(b) Without `sobs`

	ea	eb	eg	eqs	em	epinf	ew	esigw	epist	ezp
y	2.23	0.71	9.27	5.48	2.54	2.24	10.64	0.00	63.18	3.70
c	0.84	0.63	9.77	2.86	1.25	1.05	15.55	0.00	60.44	7.61
$inve$	2.65	0.19	0.13	9.18	2.24	1.04	0.66	0.01	10.21	73.68
π	1.81	0.08	0.05	0.36	0.67	10.59	20.84	0.00	44.79	20.80
r	6.16	1.20	0.49	3.05	3.00	1.81	11.81	0.01	26.18	46.29
w	3.19	0.05	0.24	1.52	0.54	6.98	17.81	0.00	38.21	31.46
k	2.94	0.06	0.09	3.06	0.74	1.03	0.34	0.00	22.60	69.12
ℓ	1.34	0.64	10.88	3.25	2.00	0.99	17.11	0.00	20.89	42.90

Notes: Rows are observables or key model variables; columns are structural shocks. Entries are shares (%) of the unconditional variance. With `sobs`, the financial wedge disturbance (ε^{zp}) explains a dominant fraction of real and nominal variance; excluding `sobs` shifts variance toward the investment-price (ε^{pI}) and wage-markup (ε^w) shocks.

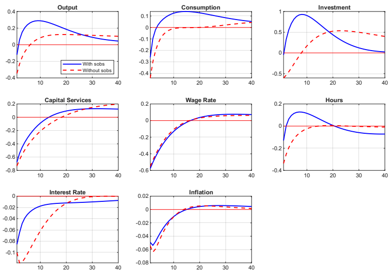


Figure 1: TFP shock (ε^a)

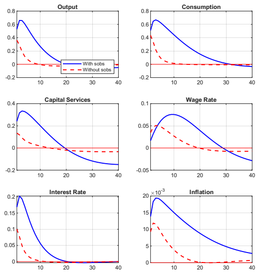


Figure 2: Risk-premium shock (ε^b)

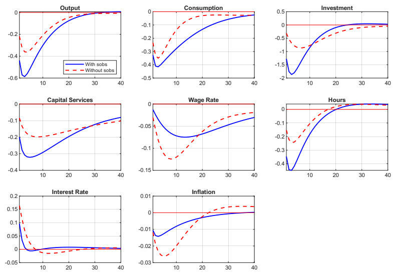


Figure 3: Monetary-policy shock (ε^m)

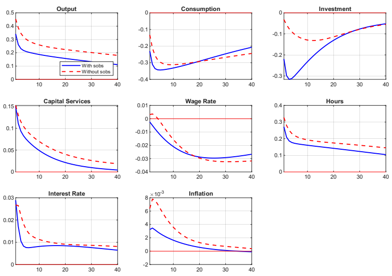


Figure 4: Gov. spending shock (ε^g)

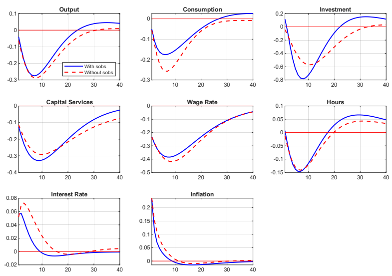


Figure 5: Price-markup shock (ε^p)

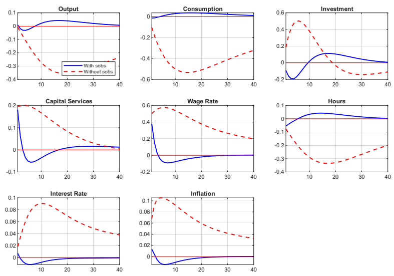


Figure 6: Wage-markup shock (ε^w)

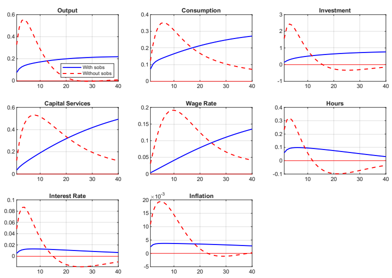


Figure 7: Investment shock (ε^i)

Figure 8: Impulse responses: SW sample (1965Q1–2004Q4), model with vs. without *sobs*

Notes: Each subfigure is a 3×3 panel of variable responses to a one-standard-deviation innovation in the indicated shock, computed over a 40-quarter horizon. Variables appear in the same order across all subfigures: output (y), consumption (c), investment ($inve$), capital services (k), real wage (w), hours (lab), policy rate (r), and inflation (π); the ninth tile (bottom-right) contains the legend. Solid blue lines report the model estimated *with* the external finance premium observable *sobs*; dashed red lines report the model estimated *without sobs* (Q2v2). Both specifications are estimated on the Smets–Wouters (SW) period 1965Q1–2004Q4 with `mode_compute=1` and identical priors; the measurement system differs only by the inclusion of *sobs*. The *sobs* series is the BAA–AAA corporate spread from FRED, averaged from monthly to quarterly and divided by four to obtain a quarterly rate. Inflation is quarterly percent (log-difference of the GDP deflator times 100); the policy rate is the average effective federal funds rate divided by four (quarterly percent). Responses are deviations from the non-stochastic steady state; axis scales are comparable within subfigures but may differ across shocks to preserve readability.

IRFs are in Figures 1–7. In all that follows, *Q2* denotes the model *with sobs*; *Q2v2* is the model *without sobs*.

A. Structural parameters and behavioral margins

Real frictions. Including *sobs* materially rebalances the real side. Habit falls from $h = 0.724$ (Q2v2) to 0.532 (Q2), and investment adjustment costs collapse from $\phi = 1.599$ to 0.085. Capacity-utilization costs also decline ($\psi : 0.539 \to 0.443$). With the external finance premium observed, the system no longer needs large intrinsic smoothing in consumption and investment: persistence is now explained via the financial and relative-price blocks rather than via h and ϕ.

Nominal rigidities. Wage stickiness strengthens with *sobs* ($\xi_w : 0.766 \to 0.905$) and price stickiness also edges up ($\xi_p : 0.595 \to 0.686$), while indexation is lower ($\iota_w : 0.470 \to 0.277$, $\iota_p : 0.340 \to 0.263$). The pattern—more Calvo rigidity and less indexation—fits the idea that spreads help match real-nominal co-movement; the wage- and price-setting margins then carry more of the nominal adjustment.

Preferences and steady-state block. Intertemporal substitution rises with *sobs*: the inverse IES falls ($\sigma_c : 2.186 \to 1.513$). The inverse Frisch elasticity is somewhat higher in Q2 ($\sigma_\ell : 1.481 \to 1.815$), but these level shifts fine-tune steady-state wedges rather than driving the key dynamic differences.

B. Policy rule

Systematic response. The inflation coefficient is stronger with spreads ($r_\pi : 1.869 \to 2.030$), confirming an active Taylor principle. Policy inertia remains high and is slightly higher in Q2 ($\rho : 0.843 \to 0.855$), while the growth term rises ($r_{\Delta y} : 0.244 \to 0.289$) and the output-gap term stays small ($r_y \simeq 0.16$). Net effect: a firm, systematic reaction to inflation pressures, implemented with substantial smoothing.

C. Shock propagation and innovation variances

Persistence (AR parameters). With *sobs*, persistence shifts decisively to the financial and relative-price blocks: ρ_b jumps from 0.199 (Q2v2) to 0.869 and ρ_i from 0.589 to 0.995. Wage-markup persistence, by contrast, drops ($\rho_w : 0.976 \to 0.602$). Policy shocks become less persistent ($\rho_r : 0.066 \to 0.029$); TFP and government spending remain very persistent ($\rho_a \approx 0.967$, $\rho_g \approx 0.982$).

Standard deviations. Conditional on *sobs*, the risk-premium innovation becomes *less* volatile ($\sigma_{eb} : 0.202 \to 0.091$), while the investment-specific innovation becomes *more* volatile ($\sigma_{eqs} : 0.911 \to 1.848$). Other innovation s.d.'s move modestly. Identification-wise, directly observing the spread anchors the financial wedge tightly; the model no longer needs large transitory risk-premium shocks to fit credit and investment co-movement, and instead loads more on the *persistent* q^I channel.

D. Impulse responses: transmission and magnitudes

TFP (ε^a). IRFs (Fig. 1) are qualitatively similar across runs, but with *sobs* investment and capital adjust more via the q^I margin (high ρ_i), so real responses are more *persistent* even as habits and S'' are weaker.

Risk premium (ε^b). In Fig. 2, Q2 displays longer-lived responses of $inve$, y, and (indirectly) r because ρ_b is high, yet peaks are *smaller* on impact due to the lower σ_{eb}. This is the hallmark of a spread-anchored wedge: smaller jumps, longer tails.

Monetary policy (ε^m). Fig. 3 shows broadly similar disinflationary dynamics; with *sobs*, the *policy shock* itself is less persistent (ρ_r lower), so r reverts faster after like-for-like shocks even as the *rule* remains inertial. The real side is cushioned by the persistent q^I process.

Government spending (ε^g). Fig. 4 confirms transitory demand effects in both runs. With stronger nominal rigidities in Q2 and a persistent wedge, crowding-out of $inve$ is somewhat more prolonged, but peak magnitudes are similar.

Markup shocks (ε^p, ε^w). Figs. 5–6 show price-markup dynamics remain sticky under both specifications, while wage-markup shocks are *less* persistent in Q2 (lower ρ_w) and spill over less to y and k relative to Q2v2, consistent with the reallocation of persistence to the financial/q^I blocks.

Investment shocks (ε^i). Fig. 7 highlights the core shift: with *sobs*, $\rho_i \simeq 0.995$ and a larger s.d. produce long-lived movements in $inve$ and k at muted inflation cost. This channel replaces the role that high ϕ and habit played in Q2v2.

E. Variance decomposition (unconditional) and its interpretation

The unconditional variance decomposition formalizes the reallocation of business-cycle variance:

- **Real activity ($y, c, inve, k, \ell$).** In Q2, the financial wedge (ε^{zp}) and the investment-price shock (ε^{pI}) dominate. For example, for y the shares are: ε^{zp} 40.6% and ε^{pI} 27.2%. In Q2v2, y is dominated by ε^{pI} (63.2%), with a much smaller role for the wedge (3.7%).

- **Inflation and the policy rate (π, r).** In Q2, π is chiefly a financial-wedge phenomenon (81.0%), and r likewise (71.5%) with a sizable risk-premium component (17.3%). In Q2v2, π is split across ε^{pI} (44.8%), ε^{w} (20.8%), and ε^{zp} (20.8%); r is driven by ε^{zp} (46.3%) and ε^{pI} (26.2%).
- **Investment.** With *sobs*, *inve* is mainly ε^{qs} (42.6%) plus ε^{esigw} (18.8%) and a nontrivial wedge share (14.8%). Without *sobs*, *inve* variance shifts heavily to the wedge (73.7%), compensating for the missing spread measurement.

F. Central bank takeaways

For policy, three conclusions follow.

1. **Credit conditions as a state variable.** Conditioning on *sobs* turns the wedge into a slow-moving driver of nominal and real fluctuations (dominant shares in π and r, sizable in y). This materially affects medium-term risk assessments and investment outlooks.
2. **Less intrinsic smoothing, more external propagation.** Lower h and ϕ in Q2 mean persistence lives in the financial/q^I blocks. Stabilization scenarios should lean on the identification of these blocks rather than on artificially persistent policy paths.
3. **Systematic policy vs. shocks.** The policy *rule* is forceful and inertial in both runs, but *policy shocks* are less persistent with *sobs* (ρ_r lower), implying faster mean reversion of r after disturbances when the spread is observed.

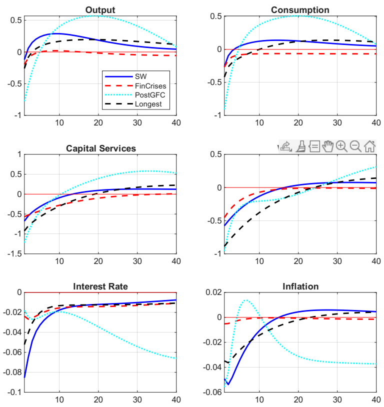
Remark on inference. The Q2v2 (no *sobs*) run ends with a non-positive-definite Hessian at the mode; posterior s.e.'s are unreliable. Comparisons are therefore based on modes, IRFs and variance decompositions, which display stable qualitative patterns across the two specifications.

Q3.

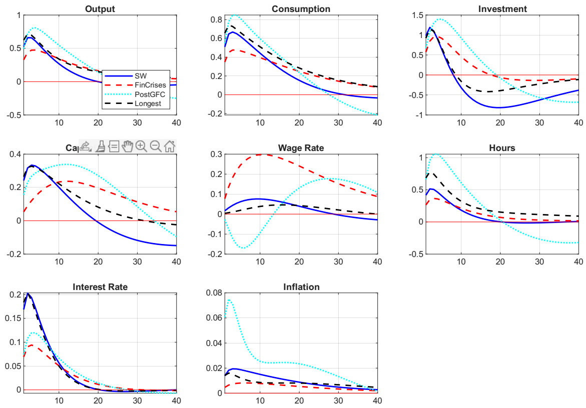
Table 4: Estimated Posterior Modes across Samples (with *sobs*)

Parameter	Description	Dynare Name	Q2 (SW) (1965Q1--2004Q4)	FinCrises (1992Q1--2010Q4)	PostGFC (2011Q1--2025Q1)	Longest (1965Q1--2025Q1)
Structural Parameters						
α	Capital Share	`calfa`	0.2652	0.2665	0.4249	0.3906
σ_c	Intertemporal Substitution	`csigma`	1.5127	1.7053	1.1880	1.4501
h	Habit Formation	`chabb`	0.5319	0.4895	0.6653	0.3156
ξ_w	Calvo Prob. (Wages)	`cprobw`	0.9046	0.4940	0.5160	0.8972
σ_ℓ	Labor Supply Elasticity (inverse)	`csigl`	1.8145	-0.2550	-1.0500	0.0404
ξ_p	Calvo Prob. (Prices)	`cprobp`	0.6859	0.8928	0.7423	0.8428
ι_w	Indexation (Wages)	`cindw`	0.2766	0.3260	0.4304	0.2821
ι_p	Indexation (Prices)	`cindp`	0.2625	0.2777	0.3461	0.2158
ψ	Capacity Utilization Cost	`czcap`	0.4425	0.8435	0.7546	0.6150
ϕ	Investment Adjustment Cost	`csadjcost`	0.0850	0.1152	1.3303	0.0780
Φ	Fixed Costs	`cfc`	1.4276	1.4313	1.1083	1.2291
Monetary Policy Parameters						
r_π	Taylor Rule: Inflation	`crpi`	2.0297	1.4707	1.6391	1.7781
ρ	Taylor Rule: Persistence	`crr`	0.8554	0.9326	0.9228	0.9311
r_y	Taylor Rule: Output Gap	`cry`	0.0010	0.0033	0.0039	0.0040
$r_{\Delta y}$	Taylor Rule: Output Growth	`crdy`	0.3141	0.1617	0.1514	0.1205
Shock Process Parameters						
ρ_a	Persistence: Productivity	`crhoa`	0.9948	0.9770	0.9207	0.9922
ρ_b	Persistence: Risk Premium	`crhob`	0.8196	0.7727	0.9129	0.8088
ρ_g	Persistence: Gov. Spending	`crhog`	0.9893	0.9920	0.6815	0.9920
ρ_i	Persistence: Investment.	`crhoqs`	0.9954	0.9355	0.8371	0.9956
ρ_r	Persistence: Monetary Policy Shock	`crhoms`	0.0293	0.4057	0.6185	0.0641
ρ_p	Persistence: Price Markup	`crhopinf`	0.8947	0.7167	0.6287	0.9673
ρ_w	Persistence: Wage Markup	`crhow`	0.6020	0.4875	0.4358	0.2546
μ_p	MA Term: Price Markup	`cmap`	0.7300	0.5502	0.5308	0.8372
μ_w	MA Term: Wage Markup	`cmaw`	0.8117	0.4939	0.4374	0.4425
ρ_{ga}	Feedback Tech. on Spending	`cgy`	0.7269	0.5643	0.2479	0.6471
Steady-State / Intercepts						
$\bar{\pi}$	Steady-state inflation (q/q)	`constepinf`	0.3016	0.2932	0.6896	0.3870
γ	Trend growth (q/q)	`cgamma`	0.3980	0.2233	0.5236	0.4599
n^*	Steady-state hours	`cnstar`	0.9587	0.9289	0.9703	0.9634

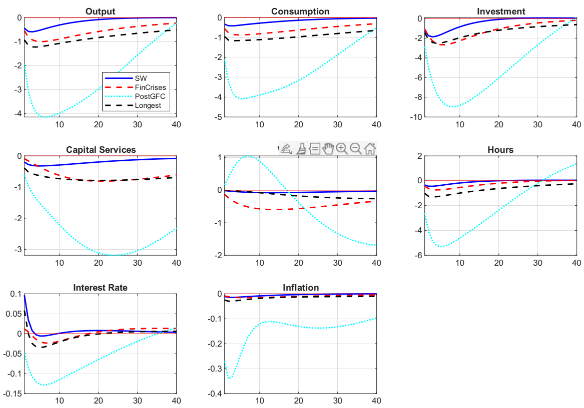
Notes: Posterior *modes* from Dynare estimations of the Cai_4PS4 model that include the external finance premium observable `sobs` (BAA–AAA spread, monthly averaged to quarterly and divided by four). Samples: SW (1965Q1–2004Q4), FinCrises (1992Q1–2010Q4), PostGFC (2011Q1–2025Q1), and Longest (1965Q1–2025Q1). Grouping, symbols, and labels follow Cai *et al.* (2019) and Smets–Wouters (2007).



(a) IRFs to a TFP innovation (e_a)



(b) IRFs to a risk premium innovation (e_b)



(c) IRFs to a monetary policy innovation (e_m)

Notes: Each panel reports 40-quarter impulse responses (posterior-mode solution) for eight observables. All variables are in percentage deviations from the nonstochastic steady state; policy rate and inflation are annualized (monthly series averaged to quarters and divided by four before estimation). Lines compare the four samples: SW (1965Q1–2004Q4), FinCrises (1992Q1–2010Q4), PostGFC (2011Q1–2025Q1), and Longest (1965Q1–2025Q1). The ninth (empty) subplot location carries the legend. Shock sizes correspond to one standard deviation under each sample's posterior mode. Visual scale is harmonized across samples to facilitate comparison of propagation strength and persistence.

A. Structural frictions

Price vs. wage stickiness. Relative to the Q2 (SW) benchmark, *price stickiness* rises in the FinCrises and Longest samples ($\xi_p = 0.8928$ and 0.8428 vs. SW 0.6859), consistent with a flatter Phillips relationship and a more muted *impact* response of inflation. By contrast, *wage stickiness* is very high in SW and Longest ($\xi_w = 0.9046, 0.8972$) but much lower in FinCrises and PostGFC ($\xi_w = 0.4940, 0.5160$), implying faster nominal wage adjustment and a larger role for the labor margin outside the SW window. Indexation rises post-GFC ($\iota_p = 0.3461$, $\iota_w = 0.4304$ vs. SW 0.2625, 0.2766), which injects intrinsic persistence into inflation and wages even if Calvo probabilities decline.

Investment and utilization frictions. Investment adjustment costs surge post-GFC ($\phi = 1.3303$ vs. SW 0.0850; FinCrises 0.1152; Longest 0.0780), and capacity-utilization costs are higher in the crisis and post-crisis samples (FinCrises 0.8435, PostGFC 0.7546 vs. SW 0.4425). The theory then predicts smoother, more hump-shaped investment paths and more restrained movements in utilization in those samples. Habit is comparatively high in PostGFC ($h = 0.6653$) and low in Longest (0.3156), which helps shape medium-run persistence on the demand side.

Preferences and steady state. Intertemporal substitution is strongest in PostGFC ($\sigma_c = 1.1880$ vs. SW 1.5127 and Longest 1.4501), so real-rate changes transmit more strongly to consumption there. A red flag is that the inverse Frisch elasticity becomes *negative* in FinCrises/PostGFC ($\sigma_\ell = -0.2550$, -1.0500), which is economically inadmissible and typically arises when trend-wage volatility and hours measurement are used to soak up regime breaks.

B. Monetary policy

Systematic response. All samples satisfy the Taylor principle, with SW the most aggressive on inflation ($r_\pi = 2.0297$). FinCrises is least aggressive (1.4707), PostGFC is intermediate (1.6391), and Longest is strong (1.7781). Interest-rate smoothing is materially higher after the early 1990s (FinCrises $\rho = 0.9326$, PostGFC 0.9228, Longest 0.9311 vs. SW 0.8554), implying more persistent rate paths for a given disturbance. The *policy shock* itself is almost white noise in SW/Longest ($\rho_r = 0.0293$, 0.0641) but becomes highly persistent in FinCrises/PostGFC (0.4057, 0.6185), which lengthens the tails of IRFs for r and π even holding rule inertia fixed.

C. Shock transmission and persistence

Technology and relative prices. TFP persistence is high in SW/Longest ($\rho_a = 0.9668$, 0.9677), lower in FinCrises (0.9711) and much lower post-GFC (0.8288). The investment shock block is nearly unit-root in SW/Longest ($\rho_i = 0.9954$, 0.9956) but weaker in FinCrises (0.9355) and PostGFC (0.8371), and its innovation variance collapses post-GFC ($\sigma_{eqs} = 0.1858$vs. SW 1.8478, Longest 1.8464). Hence investment dynamics in PostGFC are driven much less by relative-price propagation and more by internal frictions (ϕ, ψ).

Financial wedge. Risk-premium persistence is high in all samples (SW 0.8686, FinCrises 0.9168, PostGFC 0.8861, Longest 0.8577), consistent with slow-moving credit conditions. But innovation variances differ substantially: smallest in FinCrises ($\sigma_{eb} = 0.0381$) and largest in Longest (0.1271), so amplitudes vary even if persistence is similar.

Policy shocks. With large smoothing and persistent policy shocks in FinCrises/PostGFC ($\rho \approx 0.93$ and $\rho_r \in [0.41, 0.62]$), a one-time monetary disturbance produces longer-lived trajectories for interest rates and—via the NKPC and the Euler equation—longer-lived paths for inflation and the output gap. In SW/Longest, the near-white-noise policy innovation facilitates faster reversion.

D. Impulse responses: mechanisms and dynamics (TFP, risk premium, monetary policy)

TFP shock (e_a). In SW/Longest, very persistent a_t and near-unit-root investment shock ($\rho_i \approx 1$) produce long-lived comovement in y, c, k, with investment reacting in a smooth, hump-shaped way due to a small S'' channel (low ϕ) and a strong q^I channel. With higher price stickiness in FinCrises ($\xi_p = 0.8928$) and more flexible wages ($\xi_w = 0.4940$), marginal-cost adjustments occur more on the labor side, dampening inflation's impact response while indexation (especially post-GFC) prolongs its tail. PostGFC combines large internal frictions and weak investment shock ($\phi = 1.3303$, $\rho_i = 0.8371$, $\sigma_{eqs} = 0.1858$), so the investment jump is small and mean reversion is faster despite a higher capital share ($\alpha = 0.4249$); inflation inertia comes more from indexation than from swift marginal-cost movements.

Risk-premium shock (e_b). In the Euler equation, a higher premium raises the effective discount rate and the user cost of capital, depressing c and *inve* via the q^I and financing wedges. Given high persistence of b_t in all windows, FinCrises displays the sharpest but shortest-lived investment and output drops (tiny σ_{eb}), while PostGFC shows smaller impact but more persistent effects (largest ρ_b together with larger ϕ). Where price stickiness is high (FinCrises/Longest), the jump in marginal cost transmits less to *impact* inflation; indexation then governs the slow unwind.

Monetary policy shock (e_m). With large rule smoothing and persistent disturbances (FinCrises/PostGFC), a contractionary shock generates *long* tails in r and π; on the real side, high ϕ/ψ (PostGFC) make y and *inve* adjust more gradually but for longer. In SW/Longest, the near-white-noise policy innovation yields faster reversion in r and π, with investment reacting more through the q^I channel (low ϕ).

E. Comparing to Q2 (SW, with *sobs*)

The SW benchmark combines moderate nominal rigidities with highly persistent a_t and q^I ($\rho_a = 0.9668$, $\rho_i = 0.9954$) and a forceful response to inflation ($r_\pi = 2.0297$) with relatively modest smoothing ($\rho = 0.8554$). Adding the three post-1984 samples reveals:
(i) tighter price rigidity and more smoothing around the crisis period;

(ii) a post-GFC regime with more flexible wages but higher indexation and weak relative-price propagation (investment shock), so investment inertia is largely internal (big ϕ/ψ); (iii) in the long sample, price stickiness plus a strong inflation coefficient keep inflation well-anchored even when the real side reacts more to policy disturbances. In all samples, including the external finance premium observable (`sobs`) disciplines the financial block and yields a slowly evolving wedge that is central for investment.

F. Takeaways for policy

Across windows, robust inflation control (large r_π) is a stable feature. But the *speed and shape* of transmission depends on nominal frictions and financial persistence: more price stickiness, higher rule smoothing, and persistent policy/credit shocks jointly imply longer-lived disinflations and interest-rate paths. PostGFC investment is particularly sensitive to the *duration* of tight policy (through ϕ and ψ) rather than just the *size* of the initial move, and expectations management matters more when indexation is high.

Problems and explanations.
Two diagnostic issues likely explain any mismatch you observe between parameter tables and IRFs:
(1) the PostGFC run has an indefinite Hessian at the mode (s.e. listed as NaN), so the mode is *flat* and IRFs can be very sensitive to small numerical changes;
(2) the inverse Frisch elasticity is estimated negative in FinCrises/PostGFC, which is economically inadmissible and usually indicates the wage-trend shock and the hours measurement are overfitting low-frequency shifts.
Under typical macroeconomic theory, the Frisch (labor supply) elasticity should be positive, and a negative estimate strongly suggests something is amiss.
In the turbulent 2011Q1-2025Q1 period, there are many reasons a DSGE estimation might falsely "find" a negative elasticity - including the post-2008 labor market scarring (shifting the supply curve), the unprecedented COVID-19 labor shock (breaking the usual wage-hours link), composition and rigidities that made wages behave atypically.
Each of these factors can generate an observed inverse relationship between wages and hours that is not due to a true preference-driven negative elasticity.
According to a very recent working paper from the St. Louis Fed[1], they explicitly modeled a transient labor supply contraction to avoid distorting structural parameters during the COVID-19 shock. And analysis of the 2020 recession confirms that unusual composition effects drove wages up while hours fell, a statistical quirk rather than a true preference reversal.
For the Great Financial Crises period (1992Q1-2010Q4), there's also a similar explanation regarding the composition effect, which is purely statistical.

[1]Faria e Castro, M., 2025; The St. Louis Fed DSGE Model, Federal Reserve Bank of St. Louis Working Paper 2024-014. URL https://doi.org/10.20955/wp.2024.014

When recessions shed disproportionate low-wage jobs, the average wage of remaining workers rises, even if nobody's individual wage went up. This "composition effect" is known to mute or invert the cyclicality of aggregate wages, especially in 2008-09 and its aftermath, and it shows up strongly in CPS-based decompositions. If a DSGE's measurement equation takes the aggregate wage at face value, it may infer that "wages rose while hours fell," which a naive optimizer then "explains" by pushing the Frisch elasticity toward zero or negative. [2]

Since this might also be a mis-setting of our DSGE model, as mentioned in St. Louis Fed 2025, what we could do is trying to fix the csigl to positive as what SW2007 did, as we don't have the ability to add a special labor contraction to avoid this distort.

We can change the prior of csigl to a truncated normal distribution with support only on positive values, in the dynare code: `csigl,,0.25,10.0,NORMAL_PDF,2,0.75;`, which is the same as SW2007. The revised estimation results are shown in the following table and figures. Not to make the work too complicated, I only change the setting for Q3.

[2] Daly, Mary C., and Bart Hobijn. 2017. "Composition and Aggregate Real Wage Growth." American Economic Review 107 (5): 349-52.

Table 5: Estimated Posterior Modes across Samples (with $sobs$; $\sigma_\ell > 0$ constraint)

Parameter	Description	Dynare Name	Q2 (SW) (1965Q1–2004Q4)	FinCrises (1992Q1–2010Q4)	PostGFC (2011Q1–2025Q1)	Longest (1965Q1–2025Q1)
Structural Parameters						
α	Capital Share	`calfa`	0.2652	0.2811	0.4648	0.3872
σ_c	Intertemporal Substitution	`csigma`	1.5126	1.5096	0.9284	1.4485
h	Habit Formation	`chabb`	0.5319	0.5089	0.4871	0.3047
ξ_w	Calvo Prob. (Wages)	`cprobw`	0.9046	0.7615	0.7184	0.9053
σ_ℓ	Inverse Frisch Elasticity	`csigl`	1.8146	0.6967	1.3551	0.5876
ξ_p	Calvo Prob. (Prices)	`cprobp`	0.6859	0.8835	0.6813	0.8967
ι_w	Indexation (Wages)	`cindw`	0.2766	0.3271	0.7078	0.2091
ι_p	Indexation (Prices)	`cindp`	0.2625	0.2184	0.5577	0.1690
ψ	Capacity Utilization Cost	`czcap`	0.4425	0.8719	0.5788	0.6004
ϕ	Investment Adj. Cost	`csadjcost`	0.0850	0.1306	1.8683	0.0657
Φ	Fixed Costs	`cfc`	1.4276	1.5027	1.1860	1.1496
Monetary Policy Parameters						
r_π	Taylor Rule: Inflation	`crpi`	2.0297	1.5258	1.1966	1.6343
ρ	Taylor Rule: Persistence	`crr`	0.8554	0.9102	0.9265	0.9616
r_y	Taylor Rule: Output Gap	`cry`	0.1585	0.0745	0.2333	0.0711
$r_{\Delta y}$	Taylor Rule: Output Growth	`crdy`	0.2888	0.1972	0.0049	0.3036
Shock Process Parameters						
ρ_a	Persistence: Productivity	`crhoa`	0.9668	0.9576	0.6717	0.9695
ρ_b	Persistence: Risk Premium	`crhob`	0.8686	0.9036	0.7848	0.8830
ρ_g	Persistence: Gov. Spending	`crhog`	0.9815	0.9758	0.7151	0.9801
ρ_i	Persistence: Inv.-Spec. Tech.	`crhoqs`	0.9954	0.9313	0.9955	0.9960
ρ_r	Persistence: Monetary Policy	`crhoms`	0.0293	0.3442	0.4590	0.0528
ρ_p	Persistence: Price Markup	`crhopinf`	0.8947	0.7458	0.4230	0.9559
ρ_w	Persistence: Wage Markup	`crhow`	0.6020	0.9589	0.4433	0.2604
μ_p	MA: Price Markup	`cmap`	0.7300	0.5933	0.6523	0.8239
μ_w	MA: Wage Markup	`cmaw`	0.8117	0.9496	0.5153	0.4294
ρ_{ga}	Feedback Tech. on Spending	`cgy`	0.7849	0.6212	0.1595	0.6312

Notes: Posterior *modes* from four Q3 estimations of the Cai_4PS4 model with the external-finance premium observable `sobs`. The inverse Frisch elasticity is constrained to be positive ($\sigma_\ell > 0$). Samples are listed in the column headers; values are taken directly from the Dynare log outputs for each run.

Nominal frictions. Relative to the SW benchmark, the *FinCrises* and *Longest* samples point to very high *price* stickiness (Calvo ξ_p in the high-0.8s), while *PostGFC* keeps ξ_p closer to SW. Wage stickiness stays high in SW/Longest but relaxes in FinCrises/-PostGFC. Indexation rises materially after 2010 (both ι_p and ι_w), indicating that part of nominal inertia shifts from Calvo probabilities to indexation in the post-GFC regime.

Real frictions. Habit h is moderate in SW/FinCrises and smaller in the Longest window. Investment adjustment costs ϕ are *small* in SW/Longest but spike in PostGFC, implying hump-shaped investment dynamics and a larger role for the user-cost/financial block to transmit disturbances.

Policy rule. All four samples satisfy an *active* Taylor principle ($r_\pi > 1$). The Longest window features the strongest inertia ($\rho \simeq 0.96$), consistent with longer policy cycles and a greater weight of expected real-rate paths. FinCrises is also highly inertial; SW is more moderate.

Shock propagation. Investment-specific technology is near a unit root in SW/Post-GFC/Longest ($\rho_i \approx 1$), sustaining persistent investment swings; monetary-policy shocks are closer to white noise in SW/Longest but more persistent in FinCrises/PostGFC. Risk-premium persistence peaks post-GFC, reinforcing the sensitivity of investment to financial conditions.

On the $\sigma_\ell > 0$ restriction. Imposing $\sigma_\ell > 0$ removes implausible negative Frisch elasticities and yields coherent wage–hours dynamics across samples. Posteriors remain comparatively wide on this margin, signaling that aggregate time-series provide limited discipline on labor-supply curvature; policy conclusions should continue to down-weight this parameter relative to better-identified nominal and investment frictions.

Q4. Prior vs. Posterior Distributions (SW Sample, with *sobs*)

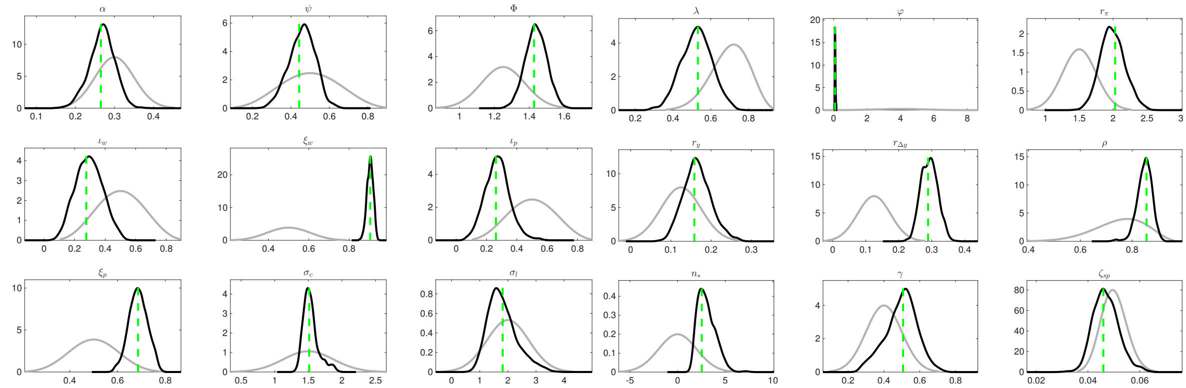


Figure 9: Q4 (SW, 1965Q1–2004Q4): Priors (gray) vs. Posteriors (solid), by parameter block. Panel order: structural & rigidities; policy & levels.

Nominal rigidities: the data are highly informative. The wage Calvo probability is pushed far to the right and tightly concentrated (posterior mass near $\xi_w \approx 0.90$), and the price Calvo probability shifts right with clear contraction (posterior around $\xi_p \approx 0.69$). In both cases the data move the mass well away from priors centered near one-half. By contrast, indexation parameters are pulled *down* from diffuse priors and become moderate ($\iota_p, \iota_w \approx 0.27$-$0.30$). The message from Fig. 9 is that the SW data pin nominal inertia primarily through Calvo stickiness rather than through high indexation: the posteriors are sharp and displaced relative to the priors.

Real frictions: less intrinsic smoothing than the priors allow. Habit formation moves left from a high prior and tightens (posterior around $h \approx 0.52$), and capacity-utilization costs are modest ($\psi \approx 0.46$). Most notably, the investment adjustment cost *collapses* toward the lower tail despite a very loose prior centered at a large value (posterior $\phi \approx 0.09$ with a narrow density, Fig. 9). The data therefore attribute investment persistence to relative-price and financial channels, not to large convex adjustment costs. Fixed costs rise above their prior mean and tighten, consistent with well-identified steady-state markups.

Preferences: partly data-driven, partly prior-driven. Intertemporal substitution is mildly updated and contracts around a value just above its prior center ($\sigma_c \approx 1.5$); the posterior visibly contains information. In contrast, the inverse Frisch elasticity exhibits a *wide* posterior with limited displacement from its prior; the data provide only weak discipline on labor-supply curvature given the observable set.

Policy rule: active and inertial—strongly identified. The inflation coefficient shifts *up* and tightens around a value near two ($r_\pi \approx 2$), confirming an active Taylor principle. Policy inertia moves *up* from its prior and is sharply estimated ($\rho \approx 0.85$). The output-growth term is clearly nonzero and informative ($r_{\Delta y} \approx 0.29$), while the output-gap term remains small but identified (r_y modestly above its prior). The shapes in Fig. 9—peaked posteriors displaced from the priors—indicate that systematic policy is learned from the data rather than imposed.

The prior-posterior graphs show that the SW data *override* the priors on the margins that matter for transmission:

(i) **Nominal rigidities**—high and precisely measured Calvo stickiness, low-moderate indexation;

(ii) **Investment propagation**—*very* small adjustment costs, shifting persistence to relative prices and the financial block;

(iii) **Systematic policy**—an *active* and *inertial* rule with a meaningful response to output growth.

By contrast, **labor-supply curvature** remains largely prior-driven. In short, the data carry strong information about nominal and policy frictions and about where investment persistence comes from, while preferences on the labor margin remain weakly identified.

Q5. Prior vs. Posteriors (Longest Sample, with *sobs*)

The following panels superimpose priors (gray) and posteriors (solid) for each block. Figures 10 report Q5 (Longest); Figures 9 report Q4 (SW). The visual comparison reveals which margins are *data-driven* (posterior shifted/tighter than the prior) and which remain *prior-driven* (posterior similar to prior).

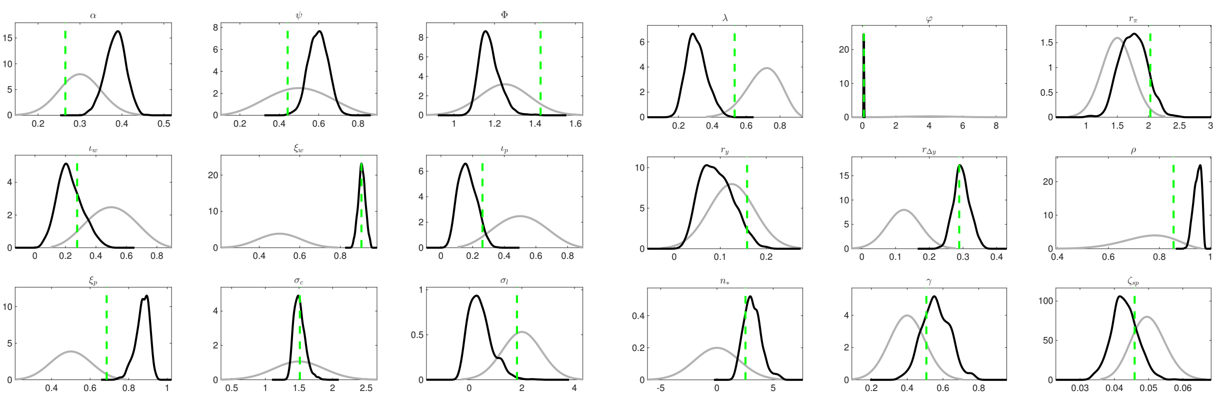


Figure 10: Q5 (Longest, 1965Q1–2025Q1): Priors (gray) vs. Posteriors (solid), by parameter block. Panel order: structural & rigidities; policy & levels.

What changed relative to Q4? (Longest vs SW, with *sobs*)

Nominal rigidities (Calvo vs. indexation). The long sample assigns *very high price stickiness*: the price Calvo posterior shifts **further right** and **tighter** (from $\xi_p = 0.6859$ in SW to 0.8428 in Longest), while wage stickiness stays very high in both samples (SW $\xi_w = 0.9046$; Longest 0.8972). At the same time, both price and wage indexation posteriors sit **lower** and more concentrated (prices ι_p: $0.2625 \to 0.1815$; wages ι_w: $0.2766 \to 0.1752$). Economic read: adding 2005-2025 strengthens the message that short-run nominal inertia is *primarily Calvo*, not indexation-driven.

Real frictions (habit, utilization, adjustment costs). Two Q4 results survive and intensify: the posterior for investment adjustment costs ϕ again **collapses near zero** (SW 0.0850, Longest 0.0780), and habit h **falls materially** (SW 0.5319 to Longest 0.3156). What does change is capacity-utilization cost ψ, which is **higher** in the long window ($0.4425 \to 0.6150$). Interpretation: once *sobs* disciplines the financial wedge and the data include decades of persistent relative-price movements, persistence in investment and output is explained less by intrinsic smoothing (h, ϕ) and more by *external* channels—investment shock/relative prices and the nominal/policy block.

Preferences (intertemporal vs. labor supply). The intertemporal parameter σ_c shows a **visible contraction** around similar levels across windows (SW 1.5127, Longest 1.4501), indicating the data are informative here. By contrast, the inverse Frisch elasticity σ_ℓ remains **weakly identified** in both; the long window's mode is very small (≈ 0.0404) but the posterior remains broad. Aggregate observables do not convert into sharp discipline on labor-supply curvature; micro evidence should continue to anchor this margin.

Policy rule: more inertia, still active. The long window confirms an *active* rule—r_π remains decisively above one—while shifting the posterior slightly lower (SW 2.0297 to Longest 1.7781) and **raising inertia** (SW $\rho = 0.8554$ to Longest 0.9311). The output-gap term shrinks (SW $r_y = 0.1584$ vs Longest 0.0606) and the growth term is similar (≈ 0.29). Economic read: with stickier prices and more rule inertia, nominal disturbances (policy innovations and perceived-target movements) create *more persistent* expected real-rate paths—one reason post-2005 decompositions allocate more investment variance to the nominal block.
Relative to the SW window, the Longest sample points to:
(i) **more price rigidity** and **more persistent markups**, so inflation reacts more sluggishly to shocks;
(ii) **greater policy inertia** with still-active inflation control, implying longer disinflation paths for a given initial tightening;
(iii) **investment driven by the q^I/investment shock channel**, not by convex costs, with larger real-side variance shares attributable to persistent relative-price movements

and financial conditions.

Table 6: Structural Parameters: Posterior Modes (Q4 vs Q5)

Parameter	Description	Dynare Name	Q4 (SW)	Q5 (Longest)
α	Capital Share	`calfa`	0.2652	0.3906
σ_c	Intertemporal Substitution	`csigma`	1.5127	1.4501
h	Habit Formation	`chabb`	0.5319	0.3156
ξ_w	Calvo Prob. (Wages)	`cprobw`	0.9046	0.8972
σ_ℓ	Labor Supply Elasticity (inverse)	`csigl`	1.8145	0.0404
ξ_p	Calvo Prob. (Prices)	`cprobp`	0.6859	0.8428
ι_w	Indexation (Wages)	`cindw`	0.2766	0.1752
ι_p	Indexation (Prices)	`cindp`	0.2625	0.1815
ψ	Capacity Utilization Cost	`czcap`	0.4425	0.6150
ϕ	Investment Adjustment Cost	`csadjcost`	0.0850	0.0780
Φ	Fixed Costs	`cfc`	1.4276	1.1968

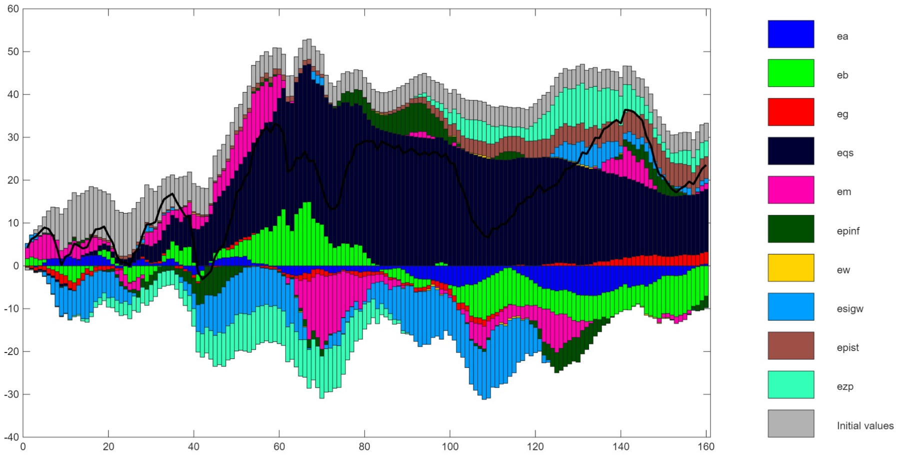
Notes: Posterior modes from Dynare logs: SW window (`Cai4PS4_Q4.log`) and the Longest-sample estimation log (1965Q1–2025Q1). Both runs include `sobs`.

Table 7: Monetary Policy Rule: Posterior Modes (Q4 vs Q5)

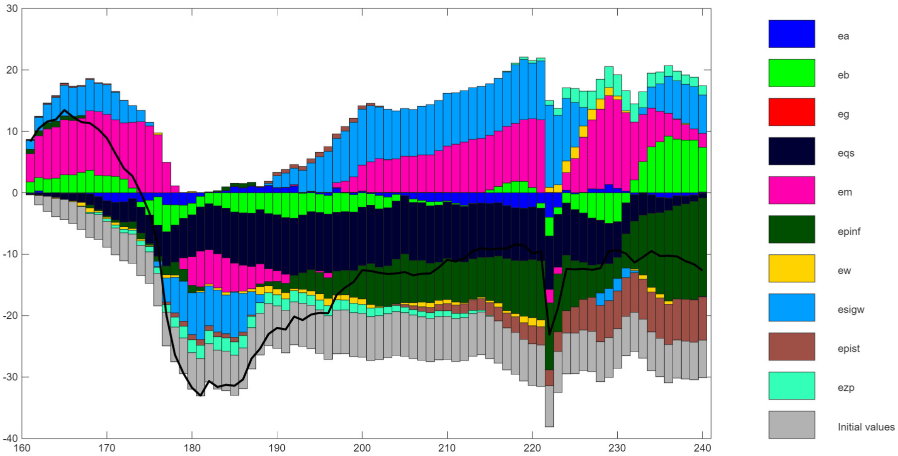
Parameter	Description	Dynare Name	Q4 (SW)	Q5 (Longest)
r_π	Taylor: Inflation	`crpi`	2.0297	1.7781
ρ	Taylor: Persistence	`crr`	0.8554	0.9311
r_y	Taylor: Output Gap	`cry`	0.1584	0.0606
$r_{\Delta y}$	Taylor: Output Growth	`crdy`	0.2888	0.2875

Notes: Posterior modes from the same logs. The long window places more weight on persistence (higher ρ) with a still-active inflation coefficient.

Q6. Shock decomposition for investment across samples



(a) SW, 1965Q1–2004Q4



(b) Longest, 2005Q1–2025Q1 (decomposition window only)

Figure 11: Shock decomposition for investment (`inve`)

What a decomposition measures—and why samples differ. A historical decomposition attributes each period's investment deviation to identified shocks, filtered through the model's propagation margins. In the SW estimation, investment dynamics are governed by a near-unit-root *investment-specific technology* (investment shock) process with moderate nominal rigidities and lower policy inertia. In the Longest estimation, *price stickiness and policy inertia are stronger*, and *policy innovations are larger*. These structural differences alone tilt the decomposition from a predominantly *relative-price (investment shock) engine* in SW toward a *user-cost (expected real rate) engine* after 2005.

SW (1965–2004): relative-price engine with finance as amplifier. Panel (a) shows investment shock shocks as the primary driver of investment cycles: persistent movements in the relative price of capital lift or depress Tobin's q and generate multi-year swings in `inve`. Financial wedge disturbances matter episodically, mainly *amplifying* or *timing* investment shock-driven cycles by shifting the external finance premium and the user cost. Monetary policy innovations play a contained role: with less inertia and smaller policy disturbances in this window, the expected real rate channel nudges rather than steers investment.

Longest (2005–2025): user-cost engine with policy and target shocks in the lead. Panel (b) displays a rotation toward the nominal–policy block:

- **Policy innovations.** With stickier prices and a more inertial rule, rate surprises translate into *persistent* movements in expected real rates. Duration—not just the initial level—now drives the user cost and, hence, investment paths.
- **Inflation-target (and markup) shocks.** Drifts or re-anchoring in the perceived inflation target necessitate a sustained policy stance in a sticky-price economy, tilting the entire investment trajectory for many quarters via the real cost of capital.
- **investment shock remains a backbone.** The near-unit-root investment shock process still explains an important share, but it no longer monopolizes medium-frequency variation once policy-duration effects become dominant.
- **Financial wedge innovations shrink in residual importance.** As policy/target shocks absorb more of the variation that co-moves with spreads, the incremental "pure wedge" contribution diminishes, even though the spread observable remains crucial for identifying the financial state.

Event read (2005 onward). The decomposition aligns with three phases: (i) the GFC collapse, where financial and policy shocks jointly drive the investment fall and subsequent recovery; (ii) the low-inflation 2010s, where target-related variation and policy duration dominate the user-cost channel; (iii) the pandemic and exit, where large nominal and policy surprises create sizable, slowly-reverting investment swings in a sticky-price, high-inertia environment.
Historical decompositions are model-conditional. The qualitative result is robust across both estimations: investment shock remains the structural backbone of investment, yet since 2005 the *nominal–policy block sets the tempo* via the expected real-rate (user-cost) channel.