

PS4 2025 Questions (Week 5)

Due before class Week 6 (21 October 2025)

This Problem Set is based on the 2019 paper of Cai *et al*, “DSGE forecasts of the lost recovery”, *International Journal of Forecasting* (**attached**), in which they discuss several models in use at the NY Fed. Most of that discussion concerns the model they dub as “SWFF”, but here we shall concern ourselves with a “descendant” model they call “SWFF+” in which Cai *et al* essentially add to the SW2007 model equations for:

- gross nominal return on capital for entrepreneurs
- entrepreneurial equity [net wealth]
- changes in cross-sectional dispersion of managerial ability across entrepreneurs
- spread between expected return on capital and riskless rate

These equations feed back into the investment equation and from there feed forward into investment and (subsequently) output and consumption and thence the other variables in the model. Moreover, the Cai *et al* model also deviates from the original Smets-Wouters specification by detrending the non-stationary model variables by a **stochastic** rather than a **deterministic** trend. For details, see equations A-1 and A-2 of their *Online Appendix* (also **attached**). See also, for SWFF+, their discussion in section A.4 of that document, where the technology process of section A.1 is augmented with a long-run component.

Cai *et al* also modify the Taylor Rule to include a time-varying **inflation target** π^* , so that the equation now reads

$$r = \text{crpi}*(1-\text{crr})*(\text{pinf}-\text{pist}) + \text{cry}*(1-\text{crr})*(y-yf) + \text{crdy}*(y-yf-y(-1)+yf(-1)) + \text{crr}*r(-1) + \text{ms};$$

$$\text{pist} = \text{rho_pist}*\text{pist}(-1)+\text{psi_pist};$$

Cai *et al* further introduce a new variable to measure the spread between the return on capital for entrepreneurs and the nominal interest rate so that there is an additional measurement equation

$$\text{sobs} = 100*(\text{Rktil} - r) + \text{sp_ss};$$

where sp_ss is the steady-state spread, which they calibrate at .02 (2 percent).

QUESTIONS

Q1. Starting from the (attached) model ***Cai_SkeletonPS4.mod***, make the necessary adjustments to complete the model. Use the (obvious) notation $Rktil$ and n for expected return to entrepreneurial capital (equations A-24 and A-25 in the Online Appendix) and net worth (A-26), respectively, and $rkstar$ for the steady-state return to capital.

Note that equation A-24 contains an **error**: there is a coefficient missing in front of b : $(c\sigma*(1+chabb*\exp(-c\gamma)))/(1-chabb*\exp(-c\gamma))$, which I have denoted in the skeleton model as `ff3`

Equation A-26 **also** contains an **error**: a term in z is missing: your equation for net worth should therefore end with $-c\gamma_{mm}star*cvstar/cnstar*z$, which I have denoted in the skeleton model as `ff4*z`

where $c\gamma_{mm}star$, $cvstar$ and $cnstar$ are financial friction parameters specified in another paper by the same authors, which also contains the values of the various other financial parameters, as below:

%Financial Frictions Parameters

```
czeta_spb = 0.0443;  
cgamma_star = 0.9900;  
cvstar = 2.4708;  
cnstar = 2.4492;  
czeta_nRk = 1.6938;  
czeta_nR = 0.6930;  
czeta_nsigw = 0.0043;  
czeta_spsigw = 0.0276;  
czeta_nqk = 0.0021;  
czeta_nn = 0.9987;
```

Since Cai *et al* do not provide the flex-price equations which they used, I have included them in the attached skeleton model ***Cai_SkeletonPS4.mod***. The skeleton also contains the priors to be used for estimation purposes. Note that – by contrast with SW – Cai *et al* do not **estimate** the various steady-state

related elements but instead assume them to be **fixed**. They therefore lie **outside** the model and I have included the corresponding values in the skeleton. Name your adjusted model **Cai_4PS4.mod**.

Q2. The SW DataSet you constructed last week contains all the variables needed except for data on the external finance premium – defined here as the BAA-AAA corporate bond interest rate spread (*sobs*). You will need to go back to the FRED website and download this data. As it is available only at a monthly frequency, you will have to do the same averaging process as last week. Note also that since these are *annual* rates, you will need to divide your results by 4 to obtain quarterly versions. Using this data, estimate (using `mode_compute=1`) the **Cai_4PS4.mod** model over the (SW) period 1965Q1-2004Q4 both with and without the use of the *sobs* variable. Compare and contrast your results, using as a basis the attached Matlab file **SWFFplus_Walsh.m**. Include in your comparison the variance decomposition.

Q3. Now re-estimate the **Cai_4PS4.mod** model **including sobs** over the periods 1992Q1 - 2010Q4 [“FinCrises”], 2011Q1 - 2025Q1 [“PostGFC”] and 1965Q1 – 2025Q1 [“Longest”]. Note that it may be necessary to use “`mode_compute = 5`” in estimating the “Longest” model, depending on your processor. Again, compare and contrast your results, including with those from Q2 (using *sobs*) in particular by looking at the deep parameters and the IRFs for TFP, risk premium and monetary policy.

Q4. Now re-run the **Cai_4PS4.mod** model **including sobs** for the SW data period but using (1) the `mode_file` from your estimation in Q2 (with `mode_compute=0`) and (2) RWMH with `mh_replic=20000`. What do the graphs showing **priors and posteriors** tell you about the information contained in the data vis-à-vis the priors? Focus your discussion on the important structural parameters.

Q5. Repeat Q4 but now using the “Longest” period 1965Q1 - 2025Q1. What differences do you find as compared to your results in Q4?

Q6. Now use the models of Q4 and Q5 to create shock decompositions for investment. For the SW period, use the command

`"shock_decomposition(init_state =0, first_obs=1,nobs=160) inve;"`

For the Longest period, use the command

`"shock_decomposition(init_state =0,first_obs=161,nobs=80) inve;"`

in order to create a decomposition starting from 2005Q1. Compare your results and comment.