Macroeconomics A; EI056

Short problems

Cédric Tille

Class of November 21, 2023

1 Intertemporal budget constraint

1.1 Household's flow constraint (a)

Question: At time t the household consumes c_t, saves in capital, $k_{t+1} - k_t$ (saving is the change in an asset holding between the beginning of a period and the end of the period (also beginning of next period)) and in government bonds $b_{t+1} - b_t$. The income is the wage w_t, net of taxes τ_t, plus the return r_t on assets (capital and bonds):

$$w_t - \tau_t + r_t (k_t + b_t) = c_t + (k_{t+1} + b_{t+1}) - (k_t + b_t)$$

Show that:

$$(k_t + b_t) = R_{t,t} [c_t - (w_t - \tau_t)] + R_{t,t+1} [c_{t+1} - (w_{t+1} - \tau_{t+1})] + R_{t,t+1} (k_{t+2} + b_{t+2})$$

where:

$$R_{t,t} = \frac{1}{1 + r_t} \quad , \quad R_{t,t+1} = \frac{1}{1 + r_t} \frac{1}{1 + r_{t+1}}$$

1.2 Household's flow constraint (b)

Question: Show that:

$$(k_t + b_t) = \sum_{s=0}^{\infty} R_{t,t+s} [c_{t+s} - (w_{t+s} - \tau_{t+s})] + \lim_{k \to \infty} R_{t,t+k} (k_{t+k+1} + b_{t+k+1})$$

where $R_{t,t+s} = \frac{1}{1+r_t} \frac{1}{1+r_{t+1}} \cdots \frac{1}{1+r_{t+s}} = \prod_{i=0}^{s} \frac{1}{1+r_{t+i}}$.

We assume that the transversality condition holds: $\lim_{k \to \infty} R_{t,t+k} (k_{t+k+1} + b_{t+k+1}) = 0$.

Do changes in tax matter for consumption? Does government debt b_t matter for consumption?

1.3 Government's constraint

Question: In period t the government spends g_t and pays the interest on its debt. This is financed by taxes and borrowing:

$$\tau_t + b_{t+1} - b_t = g_t + r_t b_t$$

Show that:

$$b_t = \sum_{s=0}^{\infty} R_{t,t+s}\left(\tau_{t+s} - g_{t+s}\right)$$

1.4 Aggregate constraint

Question: Combining the steps so far, do taxes matter for consumption? Does government debt b_t matter for consumption?

2 From individual to average measures

2.1 Individual choice

Question: Take the model where each agent lives for two periods. Agent borne at time t maximizes utility over consumption when young at time t, $c_{1,t}$, and when old at time $t+1$, $c_{2,t+1}$:

$$U_t = ln\left(c_{1,t}\right) + \frac{1}{1+\rho} ln\left(c_{2,t+1}\right)$$

Consider that there is an endowment y_t when young, and that the agent can store goods with an exogenous return r. For simplicity, we abstract from taxes.

Show that:

$$c_{2,t+1} = \frac{1+r}{1+\rho} c_{1,t}$$

Show that optimal consumption is:

$$\begin{aligned} c_{1,t} &= \frac{1+\rho}{2+\rho} y_t \\ c_{2,t+1} &= \frac{1+r}{2+\rho} y_t \\ s_{1,t} &= \frac{1}{2+\rho} y_t \end{aligned}$$

2.2 Per capita consumption

Question: Consider size of successive generations grows at a rate $1+n$.

Show that per capita consumption at time t is:

$$c_t^{\text{capita}} = \frac{1+n}{2+n} c_{1,t} + \frac{1}{2+n} c_{2,t}$$

2.3 Consumption and saving ratio

Question: We assume that endowment grows at a rate g: $y_t = (1+g)\, y_{t-1}$. Show that the aggregate consumption to output ratio is:

$$\frac{c_t^{\text{capita}}}{y_t^{\text{capita}}} = \frac{1+\rho}{2+\rho} + \frac{1}{(1+n)\,(1+g)} \frac{1+r}{2+\rho}$$

and that the ratio of net saving to output is:

$$\frac{s_t^{\text{capita}}}{y_t^{\text{capita}}} = \frac{(1+n)\,(1+g)-1}{(1+n)\,(1+g)} \frac{1}{2+\rho}$$

where s_t^{capita} denotes net savings, $N_t s_{1,t} - N_{t-1} s_{1,t-1}$, scaled by endowment per capita y_t^{capita}.

How do the per-capit consomption/output and saving/output compare to the corresponding ratios for a young agent?