Lecture 4: Comparative advantage with many countries

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1 Introduction

The Krugman (1980) model and especially its Melitz (2003) extension did an excellent job in providing a tractable framework while matching a number of empirical patterns observed in the data. However, because both models relied on economies of scale and imperfect competition as the main forces behind the gains from trade, they were part of a "new trade theory" that bore very little resemblance to the "classical trade theory" that emphasized comparative advantage and specialization as the forces behind the gains from trade. As Krugman writes in the first sentences of Krugman (1980):

> "For some time now there has been considerable skepticism about the ability of comparative cost theory to explain the actual pattern of international trade. Neither the extensive trade among industrial countries, nor the prevalence in this trade of two-way exchanges of differentiated products, make much sense in terms of standard theory. As a result, many people have concluded that a new framework for analyzing trade is needed."

The other factor inhibiting the use of "classical trade theory" in empirical gravity models was the perceived intractability of such a problem. The traditional Ricardian comparative advantage framework relied on two countries and two goods; many attempts to generalize the framework quickly led to a nightmare of corner solutions. While Dornbusch, Fischer, and Samuelson (1977) provided a tractable framework for a continuum of varieties with two countries, it was thought to be impossible to extend the framework to many countries and arbitrary trade costs (both which are necessary to deliver a gravity-like equation). (The closest generalization to many countries was the local comparative static analysis of Wilson (1980)).

This is where Eaton and Kortum (2002) enters. Using a model that bears a resemblance to a discrete-choice framework (a la McFadden (1973)), they show how to derive gravity expressions for trade flows in a world with many countries, arbitrary trade costs (i.e. arbitrary geography), where trade is only driven by technological differences across countries (i.e.

comparative advantage). The Eaton and Kortum (2002) framework not only shattered the age-old belief that there couldn't be a Ricardian gravity model, the model developed turns out to be remarkably elegant. It is also surprising that despite looking very different from the models we have considered thus far, the Eaton and Kortum (2002) trade expression remains formally isomorphic to those models.

2 Model Set-up

Let us now turn to the set-up of the model.

2.1 The world

In this model, there a finite number of countries $i \in S \equiv \{1, ..., N\}$; (unlike previous models, there are technical difficulties in extending the model to a continuum of countries). There are a continuum of goods Ω. However, unlike in Krugman (1980) and Melitz (2003), *every country is able to produce every good.* Countries, however, vary (exogenously) in their productivity of each good; in particular, let $z_i(\omega)$ denote country i's efficiency at producing good $\omega \in \Omega$.

The Eaton and Kortum (2002) has no concept of a firm. Instead, it is assumed that all goods $\omega \in \Omega$ are produced using the same bundle of inputs with a constant returns to scale technology. Let the cost of a bundle of inputs in country $i \in S$ be c_i so that the cost of producing one unit of $\omega \in \Omega$ in country $i \in S$ is $\frac{c_i}{z_i(\omega)}$.

Finally, like the previous models we considered, suppose there is an iceberg trade cost $\tau_{ij} \geq 1$ of trading a good from $i \in S$ to $j \in S$.

2.2 Supply

Each good is assumed to be sold in perfectly competitive markets, so that the price a consumer in country $j \in S$ would pay *if* she were to purchase good $\omega \in \Omega$ from country $i \in S$ is:

$$p_{ij}(\omega) = \frac{c_i}{z_i(\omega)} \tau_{ij}. \tag{1}$$

However, consumers in country $j \in S$ are assumed to only purchase good $\omega \in \Omega$ from the country who can provide it at the lowest price, so the price a consumer in $j \in N$ actually pays for good $\omega \in \Omega$ is:

$$p_j(\omega) \equiv \min_{i \in S} p_{ij}(\omega) = \min_{i \in S} \frac{c_i}{z_i(\omega)} \tau_{ij}. \tag{2}$$

The basic idea behind the Eaton and Kortum (2002) is already present in equation (2): a country $j \in S$ will be more likely to purchase good $\omega \in \Omega$ from country $i \in S$ if (1) it has a lower unit cost c_i; (2) it has a higher good productivity $z_i(\omega)$; and/or (3) it has a lower trade cost τ_{ij}.

One of the major innovations of the Eaton and Kortum (2002) model is that the productivity $z_i(\omega)$ is treated as a random variable drawn independently and identically for each

$\omega \in \Omega$. Define F_i to be the cumulative distribution function of the productivity in country $i \in S$. That is, for each $i \in S$, for all $\omega \in \Omega$:

$$F_i(z) \equiv \Pr\{z_i(\omega) \leq z\}$$

Eaton and Kortum (2002) assume that $F_i(z)$ is **Fréchet distributed** so that for all $z \geq 0$:

$$F_i(z) = \exp\left\{-T_i z^{-\theta}\right\}, \tag{3}$$

where $T_i > 0$ is a measure of the aggregate productivity of country i (note that a larger value of T_i decreases $F_i(z)$ for any $z \geq 0$, i.e. it increases the probability of larger values of z and $\theta > 1$ (which is assumed to be constant across countries) governs the distribution of productivities across goods within countries (as θ increases, the heterogeneity of productivity across goods declines).

Why make this particular distributional assumption? In Kortum (1997), one of the authors shows that if the technology of producing goods is determined by the best "idea" of how to produce, then the limiting distribution is indeed Fréchet, where T_i reflects the country's stock of ideas. More generally, consider the random variable:

$$M_n = \max\{X_1, ..., X_n\},$$

where X_i are i.i.d. The Fisher–Tippett–Gnedenko theorem states that the only (normalized) distribution of M_n as $n \to \infty$ is an extreme value distribution, of which Fréchet is one of three types (Type II) [Class question: why normalized?]. Note that a conditional logit model assumes that the error term is Gumbel (Type I) extreme value distributed. If random variable x is Gumbel distributed, $\ln x$ is distributed Fréchet; hence, loosely speaking, the Fréchet distribution works better for models that are log linear (like the gravity equation), whereas the Gumbel distribution works better for models that are linear.

2.3 Demand

As in previous models, consumers have CES preferences so that the representative agent in country j has utility:

$$U_j = \left(\int_\Omega q_j(\omega)^{\frac{\sigma-1}{\sigma}} d\omega\right)^{\frac{\sigma}{\sigma-1}},$$

where $q_j(\omega)$ is the quantity that country j consumes of good ω. Note that unlike the Krugman (1980) model, not every good produced in every country will be sold to country j. Indeed, good $\omega \in \Omega$ will be produced by all countries but county j will only purchase it from one country. However, like the previous models considered, the CES preferences will yield a Dixit-Stiglitz price index:

$$P_j \equiv \left(\int_\Omega p_j(\omega)^{1-\sigma} d\omega\right)^{\frac{1}{1-\sigma}} \tag{4}$$

3 Equilibrium

We now consider the equilibrium of the model. Instead of relying on the CES demand equation as in the previous models, we use a probabilistic formulation in order to solve the model.

3.1 Prices

First, let us consider the probability that country $i \in S$ is able to *offer* country $j \in S$ good $\omega \in \Omega$ for a price less than p. Because the technology is i.i.d across goods, this probability will be the same for all goods $\omega \in \Omega$. Define:

$$G_{ij}(p) \equiv \Pr\{p_{ij}(\omega) \leq p\}$$

Using the perfect competition price equation (1) and the functional form of the Fréchet distribution (3), we have:

$$\begin{aligned}
G_{ij}(p) &\equiv \Pr\{p_{ij}(\omega) \leq p\} \iff \\
G_{ij}(p) &= \Pr\left\{\frac{c_i}{z_i(\omega)}\tau_{ij} \leq p\right\} \iff \\
G_{ij}(p) &= \Pr\left\{\frac{c_i}{p}\tau_{ij} \leq z_i(\omega)\right\} \iff \\
G_{ij}(p) &= 1 - \Pr\left\{z_i(\omega) \leq \frac{c_i}{p}\tau_{ij}\right\} \iff \\
G_{ij}(p) &= 1 - F_i\left(\frac{c_i}{p}\tau_{ij}\right) \iff \\
G_{ij}(p) &= 1 - \exp\left\{-T_i\left(\frac{c_i}{p}\tau_{ij}\right)^{-\theta}\right\}
\end{aligned} \tag{5}$$

Consider now the probability that country $j \in S$ *pays* a price less than p for good $\omega \in \Omega$. Again, because the technology is i.i.d across goods, this probability will be the same for all goods $\omega \in \Omega$. Define:

$$G_j(p) \equiv \Pr\{p_j(\omega) \leq p\}$$

Because country $j \in S$ buys from the least cost provider, using equation (2) and some basic tools of probability, we can write:

$$\begin{aligned}
G_j(p) &= \Pr\left\{\min_{i \in S} p_{ij}(\omega) \leq p\right\} \iff \\
&= 1 - \Pr\left\{\min_{i \in S} p_{ij}(\omega) \geq p\right\} \iff \\
&= 1 - \Pr\left\{\cap_{i \in S}\left(p_{ij}(\omega) \geq p\right)\right\} \iff \\
&= 1 - \prod_{i \in S}\left(1 - G_{ij}(p)\right)
\end{aligned} \tag{6}$$

Substituting equation (5) into equation (6) yields:

$$\begin{aligned}
G_j(p) &= 1 - \prod_{i \in S} (1 - G_{ij}(p)) \iff \\
&= 1 - \prod_{i \in S} \exp\left\{ -T_i \left(\frac{c_i}{p} \tau_{ij} \right)^{-\theta} \right\} \iff \\
&= 1 - \exp\left\{ -p^{\theta} \sum_{i \in S} T_i (c_i \tau_{ij})^{-\theta} \right\} \iff \\
&= 1 - \exp\left\{ -p^{\theta} \Phi_j \right\}, \qquad (7)
\end{aligned}$$

where $\Phi_j \equiv \sum_{i \in S} T_i (c_i \tau_{ij})^{-\theta}$. Equation (7) tells us what the distribution of prices will be across goods for country j. This, in turn, will allow us to calculate the price index in country j, P_j. Starting with the definition of the price index from equation (5), we have:

$$\begin{aligned}
P_j &\equiv \left(\int_{\Omega} p_j(\omega)^{1-\sigma} d\omega \right)^{\frac{1}{1-\sigma}} \iff \\
P_j^{1-\sigma} &= \int_0^{\infty} p^{1-\sigma} dG_j(p) \iff \\
P_j^{1-\sigma} &= \int_0^{\infty} p^{1-\sigma} \left(\frac{d}{dp} \left(1 - \exp\left\{ -p^{\theta} \Phi_j \right\}\right) \right) dp \iff \\
P_j^{1-\sigma} &= \theta \Phi_j \int_0^{\infty} p^{\theta - \sigma} \exp\left\{ -p^{\theta} \Phi_j \right\} dp.
\end{aligned}$$

Define $x \equiv p^{\theta} \Phi_j$ so that with a change of variables we have:

$$\begin{aligned}
P_j^{1-\sigma} &= \int_0^{\infty} \left(\frac{x}{\Phi_j} \right)^{\frac{1-\sigma}{\theta}} \exp\{-x\} dx \iff \\
P_j^{1-\sigma} &= \Phi_j^{-\frac{1-\sigma}{\theta}} \int_0^{\infty} x^{\frac{1-\sigma}{\theta}} \exp\{-x\} dx \iff \\
P_j^{1-\sigma} &= \Phi_j^{-\frac{1-\sigma}{\theta}} \Gamma\left(\frac{\theta + 1 - \sigma}{\theta} \right) \iff \\
P_j &= \Phi_j^{-\frac{1}{\theta}} \Gamma\left(\frac{\theta + 1 - \sigma}{\theta} \right)^{\frac{1}{1-\sigma}},
\end{aligned}$$

where $\Gamma(t) \equiv \int_0^{\infty} x^{t-1} e^{-x} dx$ is the Gamma function.

Hence, the equilibrium price index in country $j \in N$ can be written as:

$$P_j = C \left(\sum_{i \in S} T_i (c_i \tau_{ij})^{-\theta} \right)^{-\frac{1}{\theta}}, \qquad (8)$$

where $C \equiv \Gamma\left(\frac{\theta+1-\sigma}{\theta} \right)^{\frac{1}{1-\sigma}}$. [Class questions: What does this mean if trade is costless? When trade is infinitely costly?]

3.2 Trade

Now suppose we are interested in determining the probability that $i \in S$ is the least cost provider of good $\omega \in \Omega$ to destination $j \in S$. Because all goods receive i.i.d. draws and there are a continuum of varieties, by the law of large numbers, this probability will be equal to the fraction of goods i sells to j. Define:

$$
\begin{aligned}
\pi_{ij} &\equiv \Pr\left\{p_{ij}(\omega) \leq \min_{k \in S \setminus j} p_{kj}(\omega)\right\} \iff \\
&= \int_0^\infty \Pr\left\{\min_{k \in S \setminus j} p_{kj}(\omega) \geq p\right\} dG_{ij}(p) \iff \\
&= \int_0^\infty \Pr\left\{\cap_{k \in S \setminus j} \left(p_{kj}(\omega) \geq p\right)\right\} dG_{ij}(p) \iff \\
&= \int_0^\infty \prod_{k \in S \setminus j} \left(1 - G_{kj}(p)\right) dG_{ij}(p) \qquad (9)
\end{aligned}
$$

Substituting the distribution of price offers from equation (5) into equation (9) yields:

$$
\begin{aligned}
\pi_{ij} &= \int_0^\infty \prod_{k \in S \setminus j} \left(1 - G_{kj}(p)\right) dG_{ij}(p) \iff \\
&= \int_0^\infty \prod_{k \in S \setminus j} \left(\exp\left\{-T_k \left(\frac{c_k}{p}\tau_{kj}\right)^{-\theta}\right\}\right)\left(\frac{d}{dp}\left(1 - \exp\left\{-T_i\left(\frac{c_i}{p}\tau_{ij}\right)^{-\theta}\right\}\right)\right) dp \iff \\
&= T_i \left(c_i \tau_{ij}\right)^{-\theta} \int_0^\infty \theta p^{\theta - 1}\left(\exp\left\{-p^\theta \Phi_j\right\}\right) dp \iff \\
&= \frac{T_i \left(c_i \tau_{ij}\right)^{-\theta}}{\Phi_j}\left(-\exp\left\{-p^\theta \Phi_j\right\}|_0^\infty\right) \\
&= \frac{T_i \left(c_i \tau_{ij}\right)^{-\theta}}{\Phi_j} \qquad (10)
\end{aligned}
$$

Hence, the fraction of goods exported from i to j just depends on $i's$ share in $j's$ Φ_j. Note that more productive countries, countries with lower unit costs, and countries with lower bilateral trade costs (all relative to other countries) will comprise a larger fraction of the goods sold to j.

Note that π_{ij} is the fraction of *goods* that $j \in S$ purchases from $i \in S$; it may not be the fraction of $j's$ income that is spent on goods from country i. However, it turns out that with the Fréchet distribution, *the distribution of prices of goods that country j actually purchases from any country $i \in S$ will be the same.* To see this, note that the probability country $i \in S$ is able to offer good $\omega \in \Omega$ for a price lower than $\tilde{p}$ conditional on i having the lowest price is simply the product of inverse of the probability that i has the lowest cost good and the probability that j receives a price offer lower that $\tilde{p}$:

$$\begin{aligned}
\Pr\left\{p_{ij}(\omega) \leq \tilde{p} | p_{ij}(\omega) \leq \min_{k \in S \setminus i} p_{kj}(\omega)\right\} &= \frac{1}{\pi_{ij}} \int_0^{\tilde{p}} \Pr\left\{\min_{k \in S \setminus i} p_{kj}(\omega) \geq p\right\} dG_{ij}(p) \iff \\
&= \frac{1}{\pi_{ij}} \int_0^{\tilde{p}} \prod_{k \in S \setminus i} (1 - G_{kj}(p))\, dG_{ij}(p) \iff \\
&= \frac{1}{\pi_{ij}} \frac{T_i (c_i \tau_{ij})^{-\theta}}{\Phi_j} \left(-\exp\left\{-p^{\theta} \Phi_j\right\}|_0^{\tilde{p}}\right) \\
&= \frac{1}{\pi_{ij}} \frac{T_i (c_i \tau_{ij})^{-\theta}}{\Phi_j} \left(1 - \exp\left\{-\tilde{p}^{\theta} \Phi_j\right\}\right) \\
&= G_j(\tilde{p}).
\end{aligned}$$

Intuitively, what is happening is that origins with better comparative advantage (lower trade costs, better productivity, etc.) in selling to j will exploit its advantage by selling a greater number of goods to j exactly up to the point where the distribution of prices it offers to j is the same as j's overall price distribution.

While this result depends heavily on the Fréchet distribution, it greatly simplifies the process of determining trade flows. Since the distribution of prices offered to an importing country $j \in S$ is *independent of the origin*, country $j's$ average expenditure per good does not depend on the source of the good. As a result, the fraction of goods purchased from a particular origin (π_{ij}) is equal to the fraction of $j's$ income spent on goods from country i, $\lambda_{ij} \equiv \frac{X_{ij}}{Y_j}$. This implies that the total expenditure of j on goods from country i is:

$$X_{ij} = \pi_{ij} E_j,$$

where from equation (10) we have:

$$X_{ij} = \frac{T_i (c_i \tau_{ij})^{-\theta}}{\Phi_j} E_j \tag{11}$$

Supposing that $c_i = w_i$ and substituting in equation (8) for the price index yields:

$$X_{ij} = C^{-\theta} \tau_{ij}^{-\theta} w_i^{-\theta} T_i E_j P_j^{\theta}. \tag{12}$$

Hence, the Eaton and Kortum (2002) model yields a nearly identical gravity equation to the Armington model of Anderson (1979), except that the relevant elasticity is θ instead of $\sigma - 1$.

Why is the trade elasticity different in this model? Recall that in the Armington and Krugman (1980) models, how responsive trade flows were to trade costs depended on how demand for a good was affected by the good's price, which was determined by consumer's elasticity of substitution. In this model, however, changes in trade costs affect the *extensive margin*, i.e. which goods an origin country trades with a destination country. As the bilateral trade costs rise, the origin country is the least cost provider in fewer goods; the greater the θ, the less heterogeneity in a country's productivity across different goods, so there are a greater number of goods for which it is no longer the least cost provider. Hence, the Eaton and Kortum (2002) model is similar to the Melitz (2003) model in that the elasticity of trade to trade costs ultimately depends on the density of producers/firms that are indifferent between exporting and not exporting: the greater the heterogeneity in productivity, the lower the density of these marginal producers.

4 Implications

We now use the Eaton and Kortum (2002) model as an example to discuss two implications that hold for gravity models more generally.

4.1 Welfare

From the CES preferences, the welfare of a worker in country $i \in S$ can be written as:

$$W_i \equiv \frac{w_i}{P_i}. \tag{13}$$

Recall from above that $\lambda_{ij} \equiv \frac{X_{ij}}{E_j}$ is the fraction of $j's$ expenditure spent on i. From gravity equation (12) we then have that:

$$\lambda_{ij} = C^{-\theta} \tau_{ij}^{-\theta} w_i^{-\theta} T_i P_j^{\theta},$$

which, given $\tau_{ii} = 1$, implies:

$$\begin{aligned} \lambda_{ii} &= C^{-\theta} W_i^{-\theta} T_i \iff \\ W_i &= C \lambda_{ii}^{-\frac{1}{\theta}} T_i^{\frac{1}{\theta}}. \end{aligned} \tag{14}$$

Hence, as in the Krugman (1980) model, welfare can be expressed as a function of technology and the openness of a country. Indeed, as far as I am aware, Eaton and Kortum (2002) were the first to derive this expression, although the expression is usually known as the "ACR" equation after Arkolakis, Costinot, and Rodríguez-Clare (2012), who derived the conditions under which it holds more generally (we will see more of this in several weeks).

4.2 Gravity

As in previous models, we can also push the gravity equation a little bit further. Note that in general equilibrium, the total income of a country will equal the amount it sells to all other countries:

$$Y_i = \sum_{j \in S} X_{ij} \tag{15}$$

Substituting gravity equation (12) into equation (15) yields:

$$Y_i = \sum_{j \in S} C^{-\theta} \tau_{ij}^{-\theta} w_i^{-\theta} T_i E_j P_j^{\theta} \iff$$

$$Y_i = C^{-\theta} w_i^{-\theta} T_i \sum_{j \in S} \tau_{ij}^{-\theta} E_j P_j^{\theta} \iff$$

$$C^{-\theta} w_i^{-\theta} T_i = \frac{Y_i}{\sum_{j \in S} \tau_{ij}^{-\theta} E_j P_j^{\theta}} \tag{16}$$

Now substituting equation (16) back into the gravity equation (12) yields:

$$X_{ij} = C^{-\theta} \tau_{ij}^{-\theta} w_i^{-\theta} T_i E_j P_j^{\theta} \iff$$

$$X_{ij} = \tau_{ij}^{-\theta} \times \frac{Y_i}{\Pi_i^{-\theta}} \times \frac{E_j}{P_j^{-\theta}}, \tag{17}$$

where

$$\Pi_i \equiv \left(\sum_{k \in S} \tau_{ik}^{-\theta} \frac{E_k}{P_k^{-\theta}} \right)^{-\frac{1}{\theta}}.$$

We will see soon that if trade costs are symmetric, $P_i = \Pi_i$.

5 Next steps

The Eaton and Kortum (2002) model remains the primary framework for the study of trade in perfectly competitive markets (especially agriculture). However, because of a lack of a real concept of a firm, it has proven less popular than the Melitz (2003) model for the study of other sectors of the economy. In Bernard, Eaton, Jensen, and Kortum (2003), the authors did extend the framework to one where there is Bertrand competition between firms. The basic idea is straightforward: the price charged by the single firm that exports a variety is the marginal cost of the second best firm. While this allows for endogenous (non-constant) markups, the extension required somewhat more complicated probability tools and has turned out to be slightly less tractable than Melitz (2003) extensions such as Melitz and Ottaviano (2008), where there are endogenous markups due to non-CES preferences.

This class concludes the examination of the micro-foundations of the gravity equation. For the next two weeks, we will be discussing the general equilibrium properties of gravity models. This will include the properties of the equilibrium, how to actually solve the model, and particularly elegant solutions (although I am biased!) for when labor is mobile.

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