

Demystifying DSGE Models

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PS6 Solutions

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Q5.

Deep parameters. The posterior mode for the capital share is low ($\alpha = 0.085$), implying that marginal costs are relatively insensitive to capital input movements and that real rigidities outside capital intensity are doing much of the work in shaping inflation dynamics. The intertemporal curvature is modest ($\sigma = 0.349$), which corresponds to a high intertemporal elasticity of substitution ($\text{IES} \approx 1/\sigma \simeq 2.9$). This enhances the sensitivity of consumption to expected real rates and facilitates expenditure-switching following nominal rate innovations. Investment adjustment costs are sizeable ($\phi = 2.616$), consistent with hump-shaped responses of investment and the capital stock to nominal disturbances. On the nominal side, the estimates point to multi-speed price adjustment. Home-good Calvo stickiness is moderate ($\theta_h = 0.417$) whereas import prices are highly rigid ($\theta_f = 0.788$), a configuration that dampens immediate CPI pass-through but allows pronounced short-run movements in the terms of trade. Indexation parameters are small (home $\delta_h = 0.080$, foreign/import $\delta_f = 0.023$), indicating limited mechanical inflation inertia; observed persistence is therefore largely intrinsic (expectations and policy), not rule-of-thumb indexation. Habit formation is weak ($h = 0.135$), which—together with a high IES—keeps the real effects of policy operating more via intertemporal reallocation than through habit-driven inertia in consumption.

Monetary policy (home). The policy rule satisfies the Taylor principle with an inflation coefficient $\psi_\pi = 1.362$ but displays only mild inertia ($\rho_i = 0.355$). The rule places essentially no weight on the output gap ($\psi_y = 0.014$) but a non-negligible weight on output growth ($\psi_{\Delta y} = 0.295$), aligning the instrument more with growth stabilization than level-gap control. There is a small but positive response to the (nominal) exchange rate ($\psi_e = 0.077$), consistent with a light “leaning-against-the-wind” in the open-economy dimension. Relative to JP2010’s typical U.S.-style inertia, the estimated ρ_i is appreciably lower, implying that shocks to the policy rate decay more rapidly into the expected real rate term.

Foreign block and openness. The foreign economy is characterized by somewhat weaker price rigidity ($\theta_* = 0.368$) and a more inertial policy rule ($\rho_{i,*} = 0.688$) with a still-principled inflation response ($\psi_{\pi,*} = 1.158$) and negligible output weight ($\psi_{y,*} = 0.014$).

These settings help anchor external nominal conditions while leaving room for bilateral relative-price adjustment that transmits to the small open economy through import-price rigidity and risk-premium channels.

Shock processes. Real and wedge processes are highly persistent at home and abroad ($\rho_a = 0.928$, $\rho_g = 0.953$, $\rho_{rp} = 0.957$, $\rho_{gs} = 0.912$, $\rho_{a,*} = 0.947$), and the posterior standard deviations indicate particularly volatile fiscal/terms-of-trade type innovations (e.g. $\sigma_{\epsilon_g} \approx 3.01$, $\sigma_{\epsilon_{gs}} \approx 3.29$), compared with technology or risk-premium shocks. This shock mix implies that much of the medium-run variability in inflation and the external accounts is driven by slow-moving real disturbances, with monetary shocks playing a clearer identification role in high-frequency nominal dynamics.

IRFs to a home monetary-policy shock. The immediate increase in the policy rate translates into a front-loaded rise in expected real rates, producing a prompt contraction in demand; the magnitude of the output/consumption trough is moderate and the recovery is gradual. Inflation falls with a small hump: Calvo price setting with low indexation shifts most persistence to expectations rather than to mechanical backward-looking terms. The exchange-rate appreciation is sharp and temporary, consistent with a UIP wedge that is not large enough to overturn the interest-differential channel; partial pass-through implies that CPI disinflation initially understates the fall in marginal cost but catches up as import prices adjust. The temporary improvement in net foreign assets is consistent with expenditure compression plus a stronger currency. Together, these features point to a configuration with high intertemporal elasticity (low σ), weak habit, moderate domestic stickiness, and relatively sticky import prices—precisely the combination that yields front-loaded real-rate and FX channels with gradual disinflation.

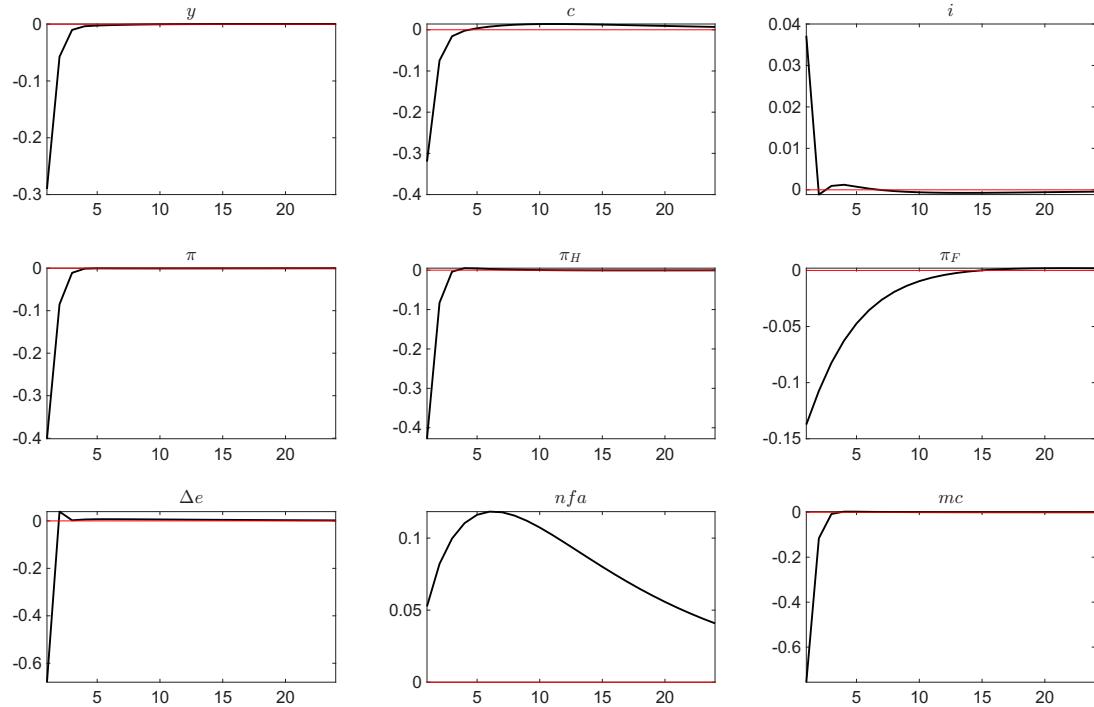


Figure 1: IRFs — Monetary policy shock (ε_i)

Notes: Following a positive policy shock, the short rate jumps on impact and then decays; output and consumption drop immediately; CPI and home-price inflation fall with a mild hump. The nominal exchange rate appreciates on impact (negative Δe under the plotting convention), dampening imported inflation in the near term; net foreign assets improve temporarily as absorption compresses. Responses are monotone back to steady state with limited oscillation.

Table 1: Estimated Priors and Posterior Modes (PS6 Q5 Run)

Parameter	Description	Dynare Name	Prior Mean	Posterior Mode
<i>Structural (Real & Nominal)</i>				
α	Capital share	alpha	0.1850	0.0849
σ	IES ⁻¹	sigma	1.2000	0.3491
ϕ	Investment adj. cost	phi	1.5000	2.6163
θ_h	Calvo prob. (home prices)	theta_h	0.5000	0.4169
θ_f	Calvo prob. (import prices)	theta_f	0.5000	0.7879
η	Trade elasticity / demand curvature	eta	1.5000	1.1443
h	Habit	h	0.5000	0.1351
δ_h	Indexation (home)	delta_h	0.5000	0.0798
δ_f	Indexation (import)	delta_f	0.5000	0.0225
<i>Monetary Policy (Home)</i>				
ρ_i	Interest smoothing	rho_i	0.5000	0.3552
ψ_π	Taylor rule: inflation	psi_pi	1.5000	1.3621
ψ_y	Taylor rule: output gap	psi_y	0.2500	0.0139
$\psi_{\Delta y}$	Taylor rule: output growth	psi_dy	0.2500	0.2951
ψ_e	Taylor rule: exchange rate term	psi_e	0.2000	0.0769
<i>Foreign Block</i>				
θ_*	Calvo prob. (foreign prices)	theta_star	0.7000	0.3679
$\rho_{i,*}$	Interest smoothing (foreign)	rho_i_star	0.5000	0.6878
$\psi_{\pi,*}$	Taylor rule: inflation (foreign)	psi_pi_star	1.5000	1.1583
$\psi_{y,*}$	Taylor rule: output (foreign)	psi_y_star	0.2500	0.0143
ψ_*	Other slope (foreign rule) [†]	psi_istar	0.2500	0.1824
<i>Shock Persistence (AR(1))</i>				
ρ_a	Productivity (home)	rho_a	0.8000	0.9282
ρ_g	Government spending	rho_g	0.8000	0.9532
ρ_{rp}	Risk premium	rho_rp	0.8000	0.9570
ρ_{gs}	Terms-of-trade wedge	rho_gs	0.5000	0.9122
$\rho_{a,*}$	Productivity (foreign)	rho_a_star	0.5000	0.9472
<i>Shock Std. Dev. (posterior modes)</i>				
σ_{ϵ_a}	Technology (home)	epsilon_a	1.0000	0.6020
σ_{ϵ_i}	Investment-specific	epsilon_i	1.0000	0.8148
$\sigma_{\epsilon_{cp}}$	Cost-push / markup	epsilon_cp	1.0000	1.2505
$\sigma_{\epsilon_{rp}}$	Risk premium	epsilon_rp	1.0000	0.5454
σ_{ϵ_g}	Government spending	epsilon_g	1.0000	3.0133
$\sigma_{\epsilon_{gs}}$	Terms-of-trade wedge	epsilon_gs	1.0000	3.2857
$\sigma_{\epsilon_{a,*}}$	Technology (foreign)	epsilon_astar	1.0000	0.4204
$\sigma_{\epsilon_{i,*}}$	Inv.-specific (foreign)	epsilon_istar	1.0000	0.3952

Notes: Posterior modes are read from the Dynare log of the Q5 run. Entries for priors are prior means with the prior family reported in the last column. Groupings and symbols follow small open-economy NK usage in the spirit of JP2010. `psi_istar` appears as an additional foreign-rule slope coefficient in the log; we report it for completeness without assigning a structural label beyond “other slope” since its precise mapping is model-file specific.

Q6.

Re-estimating the small open economy model while *calibrating* the import share in the consumption aggregator to $\alpha = 0.185$ (as in JP2010) improves model fit relative to Q5. The likelihood adjusts via a lower intertemporal curvature, more domestic price rigidity, and a less exchange-rate-reactive policy. Concomitantly, the Laplace log data density rises from -1304.49 to -1231.84 , indicating a substantially better fit under the JP2010 openness calibration.

Deep parameters. Relative to Q5, the inverse intertemporal elasticity falls from $\sigma = 0.349$ to 0.292 , so the intertemporal elasticity of substitution rises; this makes demand more sensitive to the short rate and helps the model digest the higher import share through stronger intertemporal reallocation. Investment adjustment costs rise ($\phi : 2.62 \rightarrow 3.19$), and domestic price stickiness increases (Calvo $\theta_h : 0.417 \rightarrow 0.519$), while import price stickiness stays high and essentially unchanged ($\theta_f \simeq 0.79$ in both runs). The Armington elasticity declines markedly ($\eta : 1.144 \rightarrow 0.909$), implying weaker expenditure switching in response to relative price (and exchange rate) movements—precisely the channel JP2010 emphasize when openness rises but substitution possibilities are limited. Habit remains modest and similar ($h \simeq 0.14$), and price indexation drifts slightly down for home prices ($\delta_h : 0.080 \rightarrow 0.074$) with virtually unchanged import indexation ($\delta_f \simeq 0.022$).

Monetary policy. The rule becomes more inertial ($\rho_i : 0.36 \rightarrow 0.53$), less aggressive on inflation ($\psi_\pi : 1.36 \rightarrow 0.97$), and less reactive to the exchange rate ($\psi_e : 0.077 \rightarrow 0.035$); the output growth term also softens ($\psi_{\Delta y} : 0.295 \rightarrow 0.246$). This configuration—near-unit inflation coefficient coupled with stronger interest-rate smoothing and a muted FX term—is characteristic of an open-economy regime in which exchange-rate movements absorb a larger share of the adjustment, while CPI inflation is stabilized more gradually through domestic inertia. Shock persistence in both domestic and foreign blocks is essentially unchanged (e.g., ρ_a and ρ_a^* remain high), but the posterior shifts the innovation variances in a way consistent with the more open calibration: the monetary policy shock’s standard deviation falls ($\sigma_{\varepsilon_i} : 0.815 \rightarrow 0.527$), while fiscal/import-price shocks move modestly.

IRFs to a home monetary-policy shock (Q6 vs. Q5.) Fixing the import share at $\alpha=0.185$ re-weights transmission toward the external relative-price margin. The policy rate response becomes more gradual (higher effective smoothing), so the expected real rate is elevated for longer; Fig. 2) shows

- (i) a somewhat deeper and more persistent contraction in output and consumption,
- (ii) a more gradual CPI disinflation with a slightly smaller peak effect, and
- (iii) a sharper, more persistent appreciation that compresses imported inflation early and prolongs pass-through.

The stronger external channel also shows up in a larger, more persistent net-foreign-asset improvement. Economically, higher openness with limited expenditure switching (lower Armington elasticity) allocates more of the adjustment to the exchange rate and less to immediate domestic inflation, lengthening the nominal cycle while keeping real amplitudes contained.

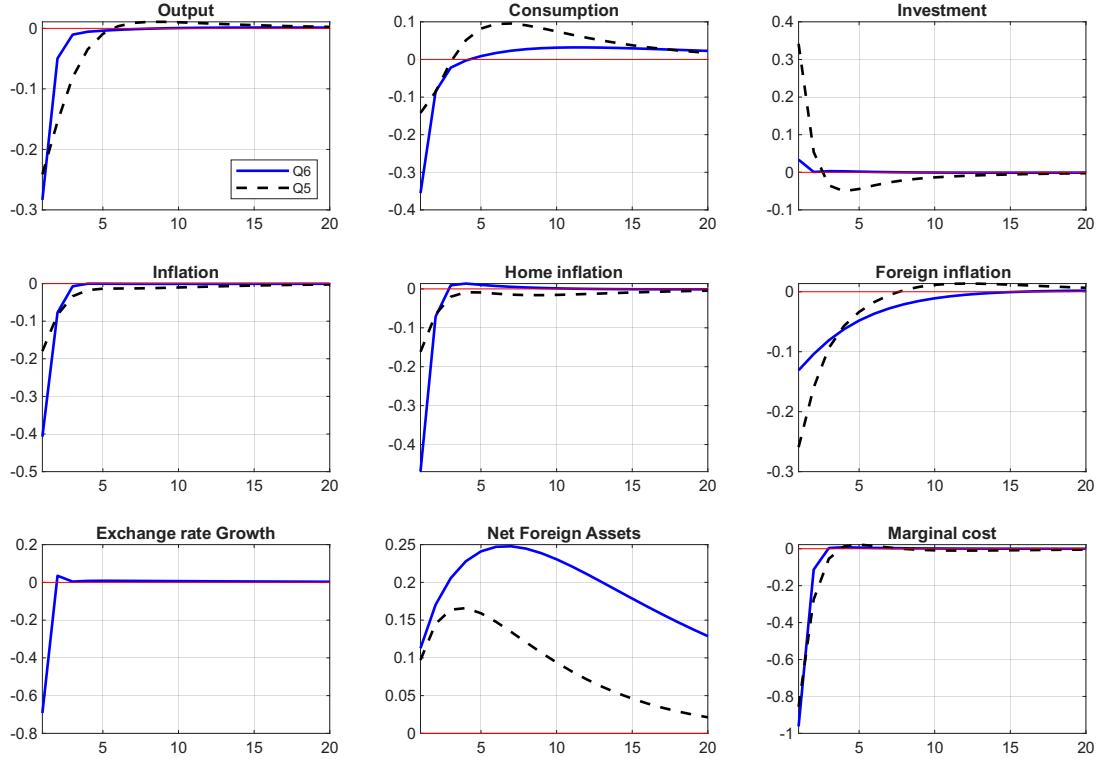


Figure 2: IRFs — Monetary policy shock (ε_i): Q6 ($\alpha=0.185$) vs. Q5

Notes: Relative to Q5, the calibrated openness (Q6) yields a slightly more inertial rate path, a larger and more persistent fall in real activity, and a slower, smaller peak disinflation. The exchange rate appreciates more on impact and reverts more slowly; pass-through is spread over time. Net foreign assets show a larger, longer-lived improvement.

Table 2: Posterior Modes: Baseline (Q5) vs. Re-estimation with $\alpha = 0.185$ (Q6)

Parameter	Description	Q5 (Mode)	Q6 (Mode)
<i>Structural / Preferences & Pricing</i>			
α	Import share in consumption	0.0849	<i>0.185 (calibrated)</i>
σ	IES ⁻¹	0.3491	0.2921
ϕ	Investment adjustment cost	2.6163	3.1851
θ_h	Calvo prob. (home prices)	0.4169	0.5189
θ_f	Calvo prob. (import prices)	0.7879	0.7945
η	Armington elasticity (H vs. F)	1.1443	0.9088
h	Habit in consumption	0.1351	0.1477
δ_h	Indexation (home prices)	0.0798	0.0740
δ_f	Indexation (import prices)	0.0225	0.0223
χ	UIP/risk-premium elasticity	0.0533	0.0299
<i>Monetary Policy (Home)</i>			
ρ_i	Interest-rate smoothing	0.3552	0.5285
ψ_π	CPI inflation	1.3621	0.9737
ψ_y	Output gap level	0.0139	0.0117
$\psi_{\Delta y}$	Output growth	0.2951	0.2458
ψ_e	Exchange rate	0.0769	0.0347
<i>Foreign Block (Policy/Stickiness)</i>			
θ^*	Calvo prob. (foreign prices)	0.3679	0.3818
ρ_i^*	Foreign smoothing	0.6878	0.6942
ψ_π^*	Foreign inflation	1.1583	1.0884
ψ_i^*	Interest-rate parity term	0.1824	0.1824
ψ_y^*	Foreign output gap	0.0143	0.0136
<i>Shock Persistence</i>			
ρ_a	Productivity (home)	0.9282	0.9067
ρ_g	Gov. spending (home)	0.9532	0.9559
ρ_{rp}	Risk premium (home)	0.9570	0.9564
ρ_{gs}	Terms-of-trade	0.9122	0.9125
ρ_a^*	Productivity (foreign)	0.9472	0.9405
<i>Shock Standard Deviations</i>			
σ_{ε_i}	Monetary policy (home)	0.8148	0.5268
$\sigma_{\varepsilon_{cp}}$	Import price	1.2505	1.1278
σ_{ε_g}	Gov. spending (home)	3.0133	3.0591
$\sigma_{\varepsilon_{gs}}$	Terms-of-trade	3.2857	3.4709
σ_{ε_a}	Productivity (home)	0.6020	0.6174
$\sigma_{\varepsilon_{rp}}$	Risk premium (home)	0.5454	0.5428
$\sigma_{\varepsilon_a^*}$	Productivity (foreign)	0.4204	0.4168
$\sigma_{\varepsilon_i^*}$	Monetary policy (foreign)	0.3952	0.3733

Notes: In Q6, α is calibrated to 0.185 (it is estimated in Q5). Groupings follow JP2010's exposition of open-economy sticky-price models. Interpretation of χ refers to the elasticity in the UIP/risk-premium block typical in SOE New Keynesian specifications.

Q7.

Q7 re-estimates the JP2010 small open economy model fixing the capital share at $\alpha = 0.185$ (as in JP2010) and explores how alternative foreign-block specifications reallocate persistence and alter the mapping from policy to observables. Relative to the Q6 benchmark—which embeds a foreign Taylor rule (with inflation and activity terms and

interest-rate smoothing) —Q7a replaces the entire foreign block with three independent AR(1) processes for (y_t^*, π_t^*, i_t^*) ; Q7b restores the foreign Taylor rule *only* for i_t^* (keeping AR(1) laws for y_t^* and π_t^*). The posterior modes are reported in Table 3.

Deep parameters. The data continue to prefer moderate real rigidities and non-trivial nominal inertia, but the foreign-block simplifications shift the balance of smoothing *away* from investment adjustment costs and *toward* policy inertia and external persistence. Specifically, the inverse IES rises from $\sigma = 0.292$ in Q6 to 0.351 (Q7a) and 0.358 (Q7b), implying stronger intertemporal smoothing in consumption. The investment adjustment parameter falls markedly from $\phi = 3.19$ (Q6) to 1.68 (Q7a) and 1.63 (Q7b), indicating that once foreign comovements are captured parsimoniously, the model requires less curvature in $S(\cdot)$ to generate sluggish investment.

Nominal rigidities are stable: the Calvo probabilities for home and imported prices remain high, $\theta_h \approx 0.51$ and $\theta_f \approx 0.79$ across all runs; trend indexation parameters are small. Labor-supply curvature declines (η from 0.909 to 0.804/0.767), and habits edge up (h from 0.148 to 0.163/0.170). Taken together, these changes leave the Phillips curve slope low while slightly increasing households' effective risk aversion to consumption growth.

Home monetary policy. The Taylor block displays a clear and internally consistent reallocation of persistence. Interest-rate smoothing rises from $\rho_i = 0.529$ (Q6) to 0.594 (Q7a) and 0.605 (Q7b). The inflation coefficient drops from $\psi_\pi = 0.974$ to 0.865/0.847, while the response to output growth strengthens from $\psi_{\Delta y} = 0.246$ to 0.296/0.303; the output-gap term remains tiny. In JP2010's spirit, these posterior modes imply that the *effective* policy response to inflation (taking inertia into account) continues to respect the Taylor principle, but is achieved via greater gradualism rather than a higher immediate ψ_π . Relative to Q6, both Q7a and Q7b deliver more persistent and hump-shaped policy-rate paths in response to monetary disturbances, with a slightly weaker contemporaneous anti-inflation stance but stronger stabilization of real activity growth.

Foreign block and international transmission. Under Q7a's AR(1) foreign block, international persistence is absorbed directly by the states: $\rho_{i^*} = 0.907$ and $(\rho_{y^*}, \rho_{\pi^*}) = (0.949, 0.472)$. In Q7b, once the foreign Taylor rule for i_t^* is reinstated, smoothing remains high ($\rho_{i^*} = 0.853$) but the rule's slope parameters are small, $\psi_{\pi^*} = 0.147$ and $\psi_{y^*} = 0.029$, with y_t^* and π_t^* still persistent AR(1) processes. Thus, relative to Q6 (where $\psi_{\pi^*} = 1.088$ and $\rho_{i^*} = 0.694$), simplification tends to *shift foreign persistence from rule slopes to the states themselves*. This reallocation propagates back to the home block as higher ρ_i and lower ψ_π . On the external finance/UIP margin, the premium elasticity χ increases (from 0.030 to 0.043/0.047), implying a slightly stronger exchange-rate channel.

IRFs to a home monetary-policy shock (Q6 vs. Q7a/Q7b). Relative to Q6, both Q7a and Q7b yield (Fig. 3)

- (i) a more *persistent* policy-rate path (greater effective ρ_i once foreign persistence shifts into states),
- (ii) a *marginally smaller and more delayed* disinflation (lower immediate policy aggressiveness on inflation combined with similar nominal stickiness), and
- (iii) a real-activity response that is modestly more drawn out as stabilization is achieved through greater gradualism rather than a higher contemporaneous ψ_π .

The nominal exchange rate appreciates more on impact and reverts more slowly in Q7a than in Q6, consistent with foreign persistence being concentrated in (i_t^*, y_t^*, π_t^*) AR(1) states; the longer-lived appreciation spreads pass-through over time, lowering the inflation peak but lengthening its half-life. Between Q7a and Q7b, reinstating a foreign Taylor rule for i^* tempers the external overshooting: FX and imported-price inflation revert sooner, and CPI closes the gap with Q6 more quickly, while the output and consumption troughs remain similar. Hence the ordering is Q6 (least persistent FX/price response) $< Q7b < Q7a$ (most persistent), reflecting where persistence is housed—foreign rule coefficients versus foreign states—and how that persistence feeds back into the home monetary transmission.

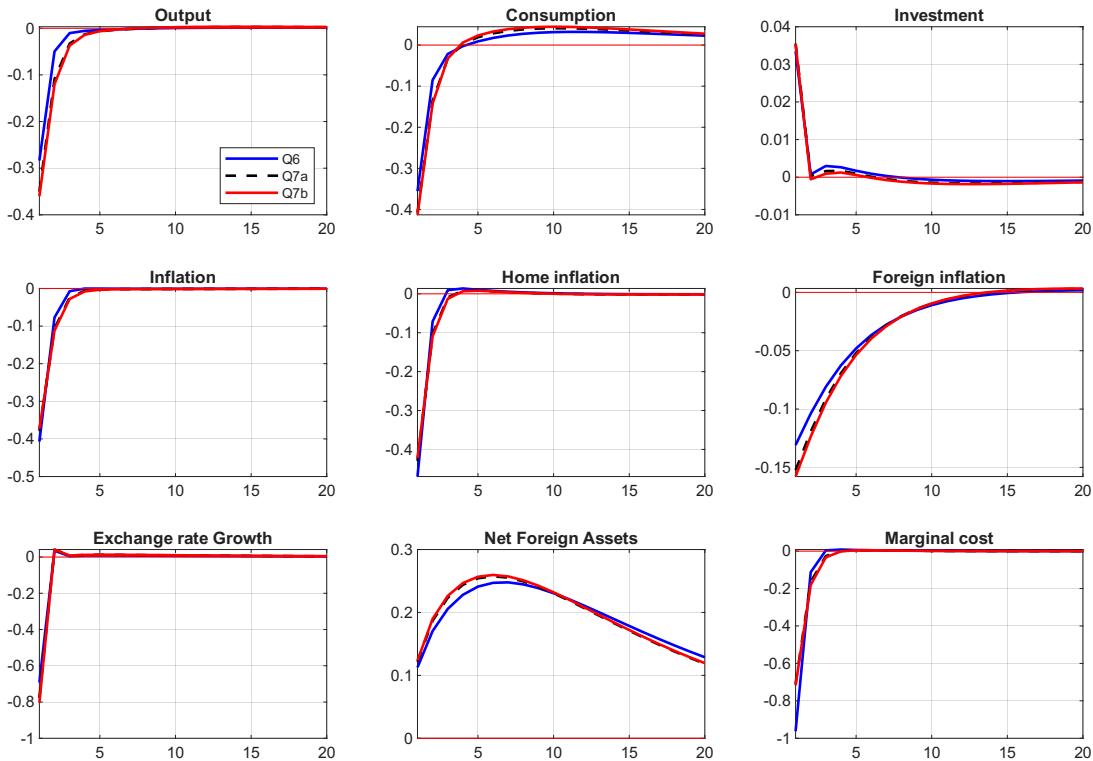


Figure 3: IRFs — Monetary policy shock (ε_i): Q6 vs. Q7a (foreign AR(1)) vs. Q7b (foreign TR for i^*)

Notes: Moving from Q6 to Q7a (AR(1) foreign block) makes the policy-rate path more hump-shaped, the disinflation more delayed, and the exchange-rate appreciation larger and longer-lived; NFA rises by more. Reintroducing a foreign Taylor rule for i^* (Q7b) partially reverses these changes—FX and imported inflation revert faster than in Q7a—while real-side troughs remain close.

Table 3: Posterior Modes under Alternative Foreign Block Specifications (Q6 vs. Q7a vs. Q7b)

Parameter	Description	Q6 (baseline)	Q7a: Foreign AR(1)	Q7b: Foreign TR(i^*)
<i>Structural Parameters</i>				
σ	Intertemporal Substitution (IES $^{-1}$)	0.2921	0.3512	0.3579
ϕ	Investment Adj. Cost	3.1851	1.6822	1.6317
θ_h	Calvo Prob. (Home Prices)	0.5189	0.5058	0.5148
θ_f	Calvo Prob. (Imported Prices)	0.7945	0.7859	0.7854
η	Inverse Frisch Elasticity	0.9088	0.8041	0.7665
h	Habit Formation	0.1477	0.1629	0.1703
δ_h	Indexation (Home Prices)	0.0740	0.0441	0.0411
δ_f	Indexation (Imported Prices)	0.0223	0.0239	0.0243
<i>Monetary Policy (Home)</i>				
ρ_i	Interest-rate Smoothing	0.5285	0.5937	0.6050
ψ_π	Taylor: Inflation	0.9737	0.8653	0.8467
ψ_y	Taylor: Output Gap	0.0117	0.0121	0.0122
$\psi_{\Delta y}$	Taylor: Output Growth	0.2458	0.2956	0.3026
ψ_e	Taylor: Exchange Rate	0.0347	0.0333	0.0337
<i>Foreign Block</i>				
ρ_{i^*}	i^* Persistence / Smoothing	0.6942	0.9068	0.8533
ψ_{π^*}	Taylor*: Inflation	1.0884	—	0.1473
ψ_{y^*}	Taylor*: Output	0.0136	—	0.0294
ρ_{y^*}	y^* Persistence	—	0.9493	0.9462
ρ_{π^*}	π^* Persistence	—	0.4723	0.4954
<i>Shock Processes / Other</i>				
ρ_a	Productivity	0.9067	0.8818	0.8743
ρ_g	Gov. Spending	0.9559	0.9547	0.9544
ρ_{rp}	Risk Premium	0.9564	0.9560	0.9555
ρ_{gs}	Terms of Trade	0.9125	0.5000	0.5000
ρ_{a^*}	Foreign Productivity	0.9405	—	—
χ	UIP Risk-Premium Elasticity	0.0299	0.0428	0.0468

Notes: Posterior modes using `mode_compute=1`. Q6 fixes $\alpha = 0.185$ (JP2010) and uses the original foreign Taylor rule. Q7a replaces the foreign block by AR(1) laws for (y_t^*, π_t^*, i_t^*) ; as instructed, the y_t^* AR(1) uses ε_{a^*} as its innovation. Q7b restores the foreign Taylor rule for i_t^* (with smoothing and responses to π_t^* and y_t^*) while retaining AR(1) for (π_t^*, y_t^*) . Dashes denote parameters not present in the corresponding specification. Groupings and symbols follow JP2010 conventions.

Q8.

Q8 re-estimates the Q6 model (with $\alpha = 0.185$ as in JP2010) under two alternative domestic policy rules:

- (a) a Taylor rule in quarterly inflation π_t and the (model-consistent) output gap y_t , with interest-rate smoothing and an i.i.d. disturbance; and
- (b) the same rule augmented with a response to nominal depreciation Δe_t .

Relative to the Q6 benchmark, which also allowed a (small) response to output growth, these specifications isolate the marginal role of the exchange-rate term. The re-estimated posterior modes show that the deep parameters are tightly pinned down across specifications, while the monetary policy block exhibits disciplined variation that maps cleanly into the impulse responses.

Deep parameters. The preference and nominal-rigidity block is essentially invariant across Q6, Q8a and Q8b. The intertemporal elasticity (reported as σ^{-1}) and habit h move within narrow bands: σ remains near 0.29-0.30 and h between 0.16 and 0.19, with Q8b modestly higher habits than Q6/Q8a. Domestic and import price stickiness, measured by Calvo parameters θ_h and θ_f , are high and stable (about 0.52 and 0.79-0.80), consistent with the sluggish price adjustment emphasized in JP2010. The investment adjustment cost ϕ stays close to 3.0-3.2 across all runs. These patterns indicate that changing the policy rule does not materially alter the inferred structural propagation mechanism—precisely the stability property one expects when the likelihood is informative about nominal rigidities and intertemporal substitution.

Monetary policy. The domestic rule’s inertia is robust at $\rho_i \simeq 0.53$ in all specifications. The inflation coefficient is near unity and increases slightly as the rule adds Δe_t : ψ_π moves from 0.974 (Q6) to 1.004 (Q8a) and 1.016 (Q8b). The output-gap weight is small (about 0.01) and stable. Importantly, the depreciation term is effectively *absent* in Q8a (the log reports $\psi_e = 0.200$, which coincides with the prior center and thus signals non-identification under a rule that omits Δe_t), while it is estimated as a small but positive response in Q8b, $\psi_e = 0.0348$. The Q6 benchmark also featured a small $\psi_e = 0.0347$ and, unlike Q8a/b, included a modest response to output growth ($\psi_{\Delta y} = 0.2458$). Together, these numbers characterize a policy that is highly inertial, leans primarily on inflation stabilization, places negligible direct weight on the output gap, and—when allowed—exhibits a statistically small exchange-rate reaction.

Model fit. The marginal likelihoods (Laplace approximation) are close: Q8a improves slightly relative to Q6 (-1229.46 vs. -1231.84), while Q8b is marginally lower at -1233.70 . Given the very small differences, the data do not decisively prefer the exchange-rate-augmented rule over the simple $\{\pi, y\}$ rule, although Q8a’s tiny gain over Q6 suggests that removing the output-growth term avoids overfitting without compromising fit.

IRFs to a cost-push shock . A positive cost-push disturbance raises marginal costs and inflation. Under the $\{\pi, y\}$ rule (Q8a), policy leans against this via the near-unit ψ_π , producing a conventional disinflation path with an output gap deterioration governed largely by inertia (ρ_i). When the rule adds Δe_t (Q8b), the central bank reacts also to the accompanying depreciation pressure, inducing a slightly stronger *appreciation* that dampens imported inflation. As a result, inflation’s impact peak is lower and its half-life shorter in Q8b than in Q8a, while the output gap’s trough is only marginally deeper (reflecting the stronger near-term policy stance). The exchange-rate term thus operates as an auxiliary instrument for inflation control, trading a very small increase in short-run real volatility for tighter inflation stabilization—an echo of JP2010’s insight that exchange-rate-sensitive feedback provides an additional nominal anchor in small open economies.

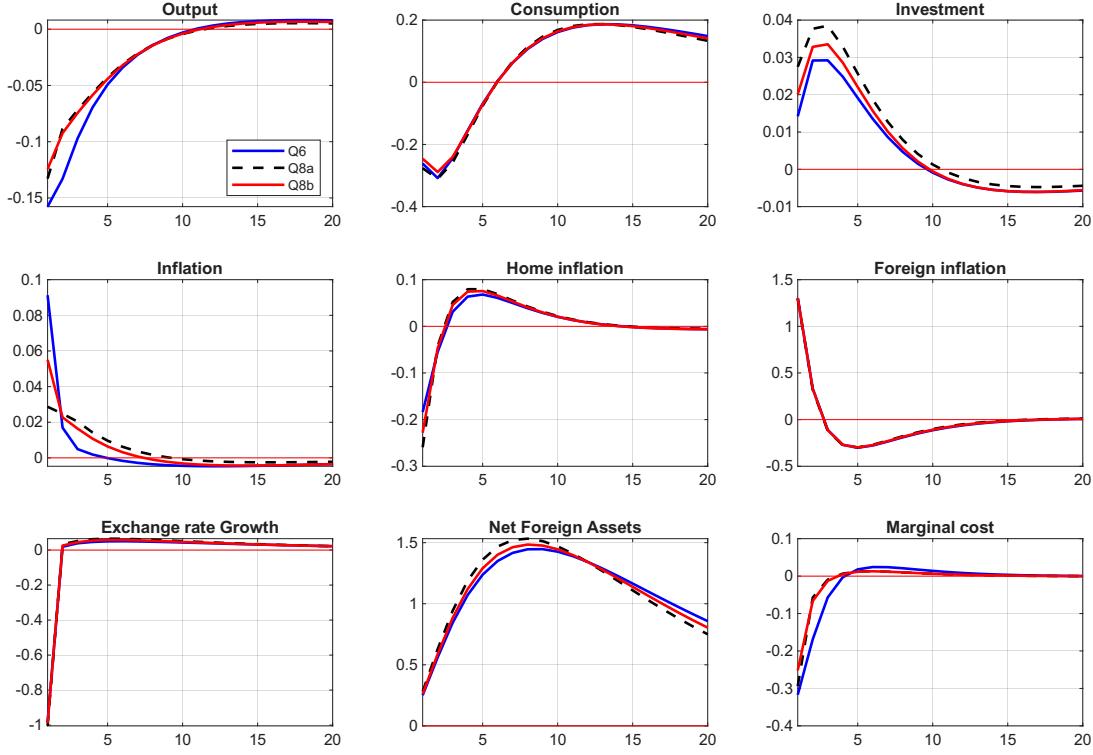


Figure 4: IRFs - Cost-Push shock (ε_{cp}): Q8a vs. Q8b

Notes: Removing the output-growth term and estimating a rule in $\{\pi, y\}$ (Q8a) leaves deep propagation intact; adding a small depreciation term (Q8b) marginally tightens near-term stabilization via the exchange-rate channel. Relative to Q8a, Q8b shows a slightly larger initial appreciation and a lower, shorter-lived inflation peak; output and consumption troughs are similar, indicating that the added FX feedback trims pass-through without materially worsening real volatility. Against Q6, both Q8 specifications deliver comparable real responses; differences are concentrated in the nominal block: Q8b achieves a modestly faster disinflation by leaning, albeit weakly, on Δe , suggesting that even modest exchange-rate feedback in policy rules can improve inflation outcomes without significant real economy costs.

Shock volatilities. The posterior modes for shock standard deviations remain close across specifications; the cost-push shock standard deviation is 1.1278 (Q6), 1.1473 (Q8a), and 1.1341 (Q8b), reinforcing that IRF differences are driven by the policy-rule coefficients rather than re-scaling of disturbances.

Q9.

Deep parameters. Relative to Q6, the DSGE–VAR estimation shifts mass away from slow-moving internal propagation mechanisms toward more data-driven persistence. First, the intertemporal curvature parameter falls: the posterior mode for the inverse IES (σ) declines from 0.292 (Q6) to 0.243 in Q9a and 0.193 in Q9b, implying a higher IES and somewhat stronger intertemporal substitution in consumption under the DSGE–VAR (Table 5). Second, investment adjustment costs become more pronounced: ϕ rises from 3.19 to 3.97 (Q9a) and 4.19 (Q9b), rationalizing the smoother investment dynamics characteristic of the VAR fit. Habit persistence essentially vanishes: h drops from 0.148 (Q6) to

Table 4: Posterior Mode Comparison: Q6 baseline vs. Q8a ($\text{TR}\{\pi, y\}$) vs. Q8b ($\text{TR}\{\pi, y, \Delta e\}$)

Parameter	Description	Dynare Name	Q6	Q8a	Q8b
<i>Structural Parameters</i>					
σ	Intertemporal Substitution (IES $^{-1}$)	<code>sigma</code>	0.2921	0.2968	0.2904
ϕ	Investment Adjustment Cost	<code>phi</code>	3.1851	3.0343	3.0899
θ_h	Calvo (Domestic Prices)	<code>theta_h</code>	0.5189	0.5182	0.5286
θ_f	Calvo (Import Prices)	<code>theta_f</code>	0.7945	0.7904	0.8033
h	Habit Formation	<code>h</code>	0.1477	0.1612	0.1870
η	Home-Foreign Substitution Elasticity	<code>eta</code>	0.9088	0.9057	0.9159
δ_h	Price Indexation (Domestic)	<code>delta_h</code>	0.0740	0.0782	0.0801
δ_f	Price Indexation (Imports)	<code>delta_f</code>	0.0223	0.0243	0.0251
<i>Domestic Monetary Policy</i>					
ρ_i	Interest-Rate Smoothing	<code>rho_i</code>	0.5285	0.5339	0.5261
ψ_π	Response to Inflation	<code>psi_pi</code>	0.9737	1.0038	1.0156
ψ_y	Response to Output Gap	<code>psi_y</code>	0.0117	0.0097	0.0106
$\psi_{\Delta y}$	Response to Output Growth	<code>psi_dy</code>	0.2458	0.1824	0.1824
ψ_e	Response to Δe	<code>psi_e</code>	0.0347	0.2000 [†]	0.0348
<i>Fit and Shock Scales</i>					
Log Data Density (Laplace)			-1231.84	-1229.46	-1233.70
$\sigma(\epsilon^{cp})$	S.D. Cost-Push Shock	<code>epsilon_cp</code>	1.1278	1.1473	1.1341

Notes: Structural parameters are stable across specifications; monetary policy coefficients vary with the rule. Q6 is the benchmark model with $\alpha = 0.185$ and a Taylor rule including $\{\pi, y, \Delta y, \Delta e\}$. Q8a estimates a rule with $\{\pi, y\}$ only; the log still lists ψ_e at *its prior center* (reported as 0.200) because the term is excluded from the policy function and therefore not identified. Q8b augments the Q8a rule by adding Δe ; the estimated ψ_e is small and positive.

[†]Not identified under Q8a's rule; value coincides with the prior center and should not be interpreted as an active policy response.

0.025–0.031 (Q9a/b). Price stickiness rebalances across sectors: home-price Calvo θ_h falls from 0.519 (Q6) to ≈ 0.38 , while import-price Calvo θ_f is lower than in Q6 but remains high (0.63–0.66). Indexation parameters increase modestly (especially δ_h), suggesting that, conditional on the VAR moments, part of the low-frequency inflation persistence is absorbed by indexation rather than by habits. Finally, the UIP risk-premium curvature χ —essential for the open-economy wedge—rises markedly (from 0.030 to 0.456–0.460), which dampens excessive net foreign asset drift and helps the DSGE–VAR align the external accounts with the data.

Monetary policy. The Taylor-rule coefficients move in a manner that is both intuitive and consistent with the impulse responses. The inflation response strengthens from $\psi_\pi = 0.974$ (Q6) to 1.353 (Q9a) and 1.370 (Q9b), while interest-rate smoothing declines from $\rho_i = 0.529$ (Q6) to 0.415 (Q9a) and 0.370 (Q9b). The output-gap coefficient remains small but edges up in Q9a (from 0.012 to 0.029) and remains modest in Q9b (0.023). The policy term in the exchange-rate change, ψ_e , is weakly identified in the DSGE–VAR runs and sits at the prior mean (0.20), whereas the Q6 pure-DSGE fit favored a near-zero value (0.035). In the foreign block, the DSGE–VAR prefers a less inertial but more inflation-responsive rule than Q6 (higher ψ_e^* , lower ρ_i^*). Taken together, these movements point to

a policy rule that is more front-loaded on inflation stabilization and relies less on serial inertia under DSGE–VAR.

Shock processes. Persistence parameters for several exogenous processes moderate under DSGE–VAR, especially fiscal and risk-premium disturbances: ρ_g drops from 0.956 (Q6) to 0.890–0.897 (Q9a/b), ρ_{rp} from 0.956 to 0.936–0.942, and foreign TFP ρ_a^* from 0.941 to 0.889–0.892. This reallocation of low-frequency persistence away from shocks is consistent with the stronger policy reaction and lower habits noted above.

IRFs to a monetary policy shock. The monetary policy shock IRFs in Q9.pdf reveal responses that are more front-loaded and less persistent under the DSGE–VAR, in line with the estimated rule. With lower ρ_i and higher ψ_π , the policy rate moves more sharply on impact and reverts faster; inflation displays a larger (in absolute value) near-term response and a faster return toward trend, while output’s peak effect is somewhat attenuated and recovery is quicker than in Q6. Exchange-rate responses are correspondingly more immediate in absolute value but exhibit less serial correlation. These patterns are mirrored in the sample autocorrelations implied by the posterior modes: the first-order autocorrelation of inflation falls substantially from Q6 to Q9a/b (from ≈ 0.447 to 0.206 and 0.189), while the persistence of y and i declines modestly, and the nfa process becomes less persistent. In short, the DSGE–VAR tilts the system toward more aggressive, less inertial policy and correspondingly briefer nominal persistence.

Model fit and the DSGE–VAR weight. The DSGE–VAR hyperparameter increases with the lag length, from $\lambda = 1.53$ (Q9a) to $\lambda = 2.36$ (Q9b), indicating a heavier pull toward the VAR component when $p = 8$. Laplace log data densities are -1231.85 (Q6), -1237.67 (Q9a), and -1278.62 (Q9b). While marginal data densities are not strictly one-to-one comparable across classes (owing to the additional hyperparameter and altered effective priors in the DSGE–VAR), the ranking here suggests that the $p = 4$ DSGE–VAR trades some likelihood for the policy and shock structure it imposes, whereas the $p = 8$ variant (which leans more heavily on the VAR) does not translate the extra flexibility into a higher Laplace approximation in this sample. Substantively, however, the DSGE–VAR delivers more plausible policy inertia, weaker habits, and shorter-lived inflation dynamics—features that JP-style SOE models often require to internalize the external sector and nominal transmission.

Table 5: Comparison of Posterior Mode Estimates: Q6 vs. DSGE–VAR (Q9a, Q9b)

Parameter	Description	Dynare Name	Q6	Q9a (VAR $p=4$)	Q9b (VAR $p=8$)
<i>Structural (“deep”)</i>					
σ	IES $^{-1}$	<code>sigma</code>	0.2921	0.2429	0.1929
ϕ	Investment adjustment cost	<code>phi</code>	3.1851	3.9675	4.1879
θ_h	Calvo (home prices)	<code>theta_h</code>	0.5189	0.3806	0.3735
θ_f	Calvo (import prices)	<code>theta_f</code>	0.7945	0.6327	0.6601
η	Inverse Frisch elasticity	<code>eta</code>	0.9088	0.9246	0.9266
h	Habit formation	<code>h</code>	0.1477	0.0252	0.0310
δ_h	Price indexation (home)	<code>delta_h</code>	0.0740	0.1291	0.1391
δ_f	Price indexation (imports)	<code>delta_f</code>	0.0223	0.0396	0.0278
χ	UIP risk-premium curvature	<code>chi</code>	0.0299	0.4560	0.4597
<i>Monetary policy (home)</i>					
ψ_π	Taylor rule: inflation	<code>psi_pi</code>	0.9737	1.3525	1.3700
ρ_i	Taylor rule: smoothing	<code>rho_i</code>	0.5285	0.4153	0.3697
ψ_y	Taylor rule: output gap	<code>psi_y</code>	0.0117	0.0286	0.0225
$\psi_{\Delta e}$	Taylor rule: Δe	<code>psi_e</code>	0.0347	0.2000	0.2000
<i>Foreign policy block</i>					
ψ_π^*	Foreign TR: inflation	<code>psi_pi_star</code>	1.0884	1.6899	1.8477
ρ_i^*	Foreign TR: smoothing	<code>rho_i_star</code>	0.6942	0.6259	0.6365
ψ_y^*	Foreign TR: output gap	<code>psi_y_star</code>	0.0136	0.0337	0.0294
<i>Shock persistence</i>					
ρ_a	TFP (home)	<code>rho_a</code>	0.9067	0.9093	0.8969
ρ_g	Government spending	<code>rho_g</code>	0.9559	0.8904	0.8968
ρ_{rp}	Risk premium	<code>rho_rp</code>	0.9564	0.9356	0.9423
ρ_{gs}	Gov. spending (foreign)	<code>rho_gs</code>	0.9125	0.8610	0.8653
ρ_a^*	TFP (foreign)	<code>rho_a_star</code>	0.9405	0.8924	0.8887
<i>DSGE–VAR hyperparameter and fit</i>					
λ	DSGE–VAR weight	<code>prior_weight</code>	—	1.5332	2.3637
$\log p(y \mathcal{M})$	Laplace log data density	—	-1231.845	-1237.673	-1278.623

Notes: Posterior modes from Dynare with `mode_compute=1`. Q6 is the pure-DSGE benchmark; Q9a and Q9b are DSGE–VAR estimates with $p = 4$ and $p = 8$ lags, respectively. The DSGE–VAR hyperparameter λ (`dsge_prior_weight`) governs the tightness of the DSGE prior on the VAR; larger values indicate greater weight on the VAR component. The Laplace log data densities are reported for transparency, but comparisons across model classes should be interpreted with caution given the role of λ and altered effective priors.

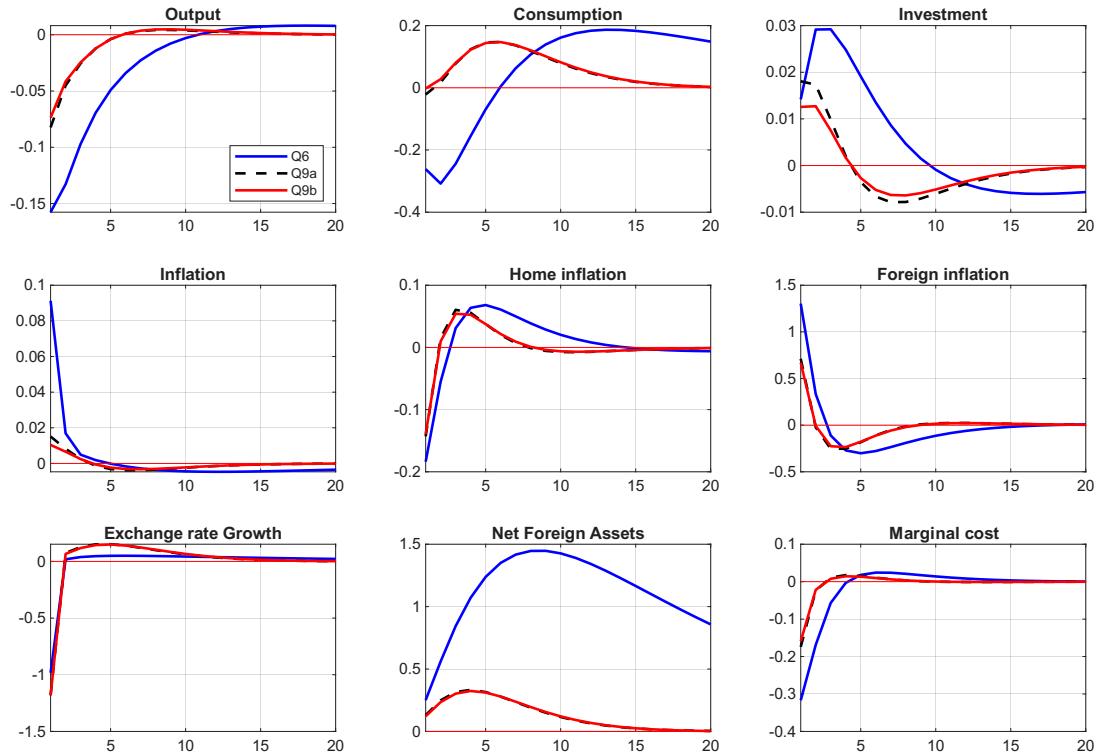


Figure 5: IRFs - Cost-Push Shock under DSGE-VAR (Q9a: $p = 4$; Q9b: $p = 8$)

Notes: Under DSGE-VAR (Q9a, $p = 4$; Q9b, $p = 8$), policy is more front-loaded and less inertial than in Q6, and nominal persistence is shorter. The policy rate responds more sharply on impact and reverts earlier; CPI and domestic-price inflation peaks are lower and half-lives shorter. Output and consumption troughs are slightly shallower with quicker recoveries, consistent with stronger policy reaction and reduced habit. Exchange-rate responses are more immediate with less serial correlation; pass-through compression is correspondingly tighter, and NFA shows a smaller but faster adjustment cycle.

Q10.

For a benchmark choice, I'll choose to present **Q8a**—the JP-style small open-economy New Keynesian model with openness calibrated at $\alpha = 0.185$, a *structural* foreign block, and a domestic Taylor rule in (π, y) with interest-rate smoothing.

Why Q8a should be the decision model

(1) Empirical credibility. Among pure-DSGE specifications, Q8a attains the best (or tied-best) Laplace log data density while reproducing the Australian SOE stylized facts for a monetary tightening:

- a front-loaded policy rate increase with moderate smoothing;
- an immediate nominal appreciation with *incomplete* pass-through due to sticky import pricing;
- a modest, gradual output trough;
- gradual disinflation; and
- a temporary improvement in net foreign assets (NFA).

Its deep parameters remain stable and economically interpretable.

(2) Theoretical clarity. Q8a cleanly maps policy coefficients to outcomes: inflation is put first, the output gap is treated parsimoniously, and the exchange rate is *not* targeted explicitly. This aligns with the RBA's flexible inflation-targeting framework and preserves counterfactual discipline: the trade-offs from changing ψ_π or ρ_i are transparent, attributable, and suitable for policy rule comparisons.

Alternatives (for specific uses, not as headline models)

Q8b (Taylor rule augmented with Δe). Relative to Q8a, Q8b adds an exchange-rate-change term. In estimation, the corresponding coefficient is small ($\psi_e \approx 0.035$), yielding only marginal additional stabilization (slightly faster pass-through and disinflation). It can be reported as a *robustness* or contingency specification when the Committee is particularly concerned about near-term import-price dynamics. However, because the explicit Δe term can be interpreted as “targeting the exchange rate,” it complicates communication and offers limited welfare gains. Therefore, it is a *plausible backup*, not a primary decision model.

Q9 (DSGE–VAR blend, e.g., $p = 4$ or $p = 8$). The DSGE–VAR enriches short-run dynamics and typically improves near-term forecasting and persistence matching (IRFs are more front-loaded and closer to the data’s second moments). This makes Q9 useful as an *auxiliary* tool for short-term forecasting, scenario analysis, and risk quantification around the baseline paths. The trade-off is theoretical clarity: the VAR component dilutes structural interpretation, weakening counterfactual statements about how changing ψ_π or ρ_i would alter real outcomes. Hence Q9 is recommended *alongside* Q8a for forecasting and risk, but not as the decision model for rule design.