

# **PS3 2025 Solutions (Due 14 October 2025)**

## **Q1 and Q2.**

Not much to be said here as the work is straightforward.

## **Q3 – Q5.**

The results I obtained are summarised in the table at the end of this document, where the “calibrated” parameters are marked in **purple**. I have highlighted in boxes two groups of parameters: the ***monetary policy parameters***, and the **“deep” parameters** dealing with wage and price indexation and stickiness (since these are the key NK elements).

The first thing to note is that (with one major exception) the estimated parameters using the new data track quite closely those found by SW. One exception is  $\alpha_p$ , the indexation parameter for prices, which is more than treble that found by SW2007. This may seem odd, but it is important to bear in mind that we are not really comparing like with like, as the problems here required merely RegMLE (ie, `mh_replic=0`) whereas SW used RW-MH with `mh_replic=250,000`. Another (smaller) exception is  $\xi_w$ , the Calvo parameter for wages, which is almost one half smaller than that found by SW2007.

To attempt to discover whether the visible difference is due to the estimation technique, or to the data change, I therefore also estimated the SW2007 model using SW’s original data, **and with `mh_replic=250000`**. The result was quite clear: with the sole exception of  $\bar{\epsilon}$ , the results were very similar to those SW obtained. It therefore seems clear that updating the data changes substantially the results – somewhat troubling. [This happened to me almost 60 years ago when my Canadian model parameter estimates changed radically following a major data revision by StatsCan!]

As regards the ***deep parameters***, it is interesting to note that the two stickiness parameters vary differently from period to period. [Note that Q5 deals with the ***earlier*** period, and Q4 with the ***later*** period]. The degree of price stickiness ( $\xi_p$ ) rises by more than half over the two periods covered in Q5 and Q4, from 54% of firms to 82%, indicating that the US economy was becoming steadily less competitive as prices adjusted less and less to changes in the economy [prices changed only roughly once every 5.5 quarters in the more recent period, as contrasted with about every 2.2 quarters in the later period]. The estimated  $\xi_p$  in the two cases brackets that found by

SW. By contrast, in both the earlier and later period, the estimated  $\xi_w$  is substantially lower than that found by SW2007, implying that wages change roughly once every 1.3 quarters in the more recent period, as opposed to once every 1.5 quarters in the earlier period, in line with the notion of a more unequal US economy where wage-earners struggle to keep up as their wages adjust downwards more rapidly than do prices.

The two indexation parameters also change, but not by much, that for wages ( $\iota_w$ ) declining in the later (Q4) period by about a third, and that for prices ( $\iota_p$ ) rising very slightly. These movements demonstrate why wages are stickier than prices. The estimated external habit parameter,  $\lambda$ , declined in the more recent (Q4) period, which seems reasonable given the economic shocks affecting the US economy during this period. Another interesting result is that the Frisch parameter ( $\sigma_f$ ) halves from the earlier to the later period. Since this is the inverse of the elasticity of work effort, this implies that that elasticity doubles: it takes a much smaller jump in wages to induce more work in the later period, in line with the findings regarding the wage Calvo and indexation parameters. Finally, the (inverse of) the elasticity of capacity utilization cost  $\psi$  more than doubles in the more recent (Q4) period, an odd result given that both estimates are highly statistically significant.

As regards the **monetary policy parameters**, the estimates show a remarkable stability across periods and estimation methodology, with only the Taylor rule interest rate persistence parameter  $p$  showing any variability, lying below the SW2007 estimate during the earlier (Q5) period and above it during the more recent (Q4) period. The coefficient of response to the output gap  $r_y$  is estimated in both sub-periods to be about double the value found by SW. It is not surprising that the output gap parameter would remain stable, given the episodes of strong growth during that time (1980Q1 to 2015Q4) offset by periods of weakness, but its substantially greater value than in SW2007 is puzzling, as is the sharp reduction in the output growth response coefficient ( $r_{\Delta y}$ ), which is estimated to be only about one fifth of the value found by SW in both periods. The coefficient of response to inflation  $r_\pi$  is estimated in both sub-periods to be about 10% lower in value than found by SW, not a very substantial change. Given the sharp decline in inflation during the more recent period, it is unsurprising that the inflation coefficient would decline, although a greater decline would have been expected, and the decline in the earlier period is puzzling.

**Q6.**

The impact of using Dixit-Stiglitz (by setting the parameters “ $\text{curvp}$ ” and “ $\text{curvw}$ ” both equal to zero, thus shutting off the Kimball Aggregator’s curvature) is most sharply seen in the deep parameters for prices and wages. The Calvo probability for prices  $\xi_p$  is sharply higher without curvature than with it (implying price changes only every 6.3 quarters vs 3.1 under Kimball), and the price indexation coefficient  $\iota_p$  is also (by about 10%) higher (as would be expected). Wages seem less sensitive to curvature than do prices, the Calvo probability for wages  $\xi_w$  marginally declining without curvature and the wage indexation coefficient  $\iota_w$  marginally rising. From an economic standpoint, it is hard to decide whether this difference is reasonable or not.

The only other deep parameter which seems to be sensitive to curvature is (as found previously across periods) the Frisch elasticity  $\sigma_l$ , which is about one quarter lower without curvature than with it. Again, why this should be is not evident.

As regards the Taylor rule parameters, there is not much change, as should be expected.

**Q7.**

The sixth column of the table contains the results obtained when the Calvo probabilities are each set to 0.5, meaning that half of the population follows a Calvo rule for both prices and wages, and the other half is free to set optimised prices and wages (not necessarily the same half for wages as for prices of course). Since this is estimated over the SW period, we need to compare the second and sixth columns to see the impact of this manoeuvre.

The impact is roughly the same on the wage and price coefficients ( $\xi_w$  and  $\xi_p$ ), since the Calvo probability for prices was originally estimated as very close to the corresponding probability for wages (0.65 vs. 0.73), in both cases implying less stickiness. By contrast, the indexation coefficients ( $\iota_w$  and  $\iota_p$ ) are estimated as being considerably higher in the scenario being tested here than in SW, a curious result given the greater flexibility in wages and prices.

As regards the Taylor rule parameters, there is not much change, except for as should be expected.

Q8.

Although SW's finding of a decline in the productivity, monetary policy, and price mark-up shocks from the first ("Inflation") to the second ("Moderation") period is borne out here, the new data do not seem to confirm SW's conclusion of stable structural parameters, with a wide range of deep parameters changing significantly from one to the other period; these encompass such parameters as  $\sigma_1$  and  $\psi$  (as usual !!) but also  $\tau_p$ ,  $\xi_w$ ,  $\xi_p$  and  $\sigma_c$ . Two of the Taylor rule coefficients also change substantially:  $r_\pi$  and  $r_{\Delta y}$ , the first rising (perhaps in a late response to the inflation in the previous period) and the other declining (as expected in a period of "moderation").

Q9.

The diffuse priors of Herbst and Schorfheide seem to modify SW's structural parameter results rather a lot, suggesting that H&S have a valid point regarding the "tilting of the tables" by SW. However, you will have noted that the estimation results are suspicious, as indicated by the warnings issued by **Dynare** ["POSTERIOR KERNEL OPTIMIZATION PROBLEM!" and "Warning: The results below are most likely wrong!"]. Indeed, when the RW-MH algorithm is used the estimated posterior means of these parameters change quite substantially, although a few ( $\tau_w$ ,  $\sigma_1$  and  $\psi$ ) are still very different from the SW results, as do the estimated posterior means of several other parameters, again bringing into question the particular choice of prior made by H&S, although not invalidating their primary thesis. [FYI, this same effect was noticed in a paper by Beltran and Draper (2018) *Estimating Dynamic Macroeconomic Models: How Informative Are the Data?*" *Journal of the Royal Statistical Society: Series C (Applied Statistics)*, vol. 67, no. 2, pp. 501-520, as concerns a (slightly different) test by Onatski and Williams (2010) "Empirical and policy performance of a forward-looking monetary model" *Journal of Applied Econometrics* of the robustness of the SW model to changes in the specification of the priors.]

You will also have noted from the RWMH prior-and-posterior graphs that the SW data is highly informative in the face of these looser priors.

		SW2007	Table1	Q3	Q4	Q5	Q6	Q7	Q8A	Q8B	Q9	Q9a	NoQ
		1965Q1-2004Q4	1965Q1-2004Q4	1980Q1-2015Q4	1967Q1-1991Q4	Kimball	Calvo	Inflation	Moderation	Schorf Prior	Schorf MH	1965Q1-2025Q1	
<b>Fixed</b>													
curv_w	Curvature Kimball aggregator wages		<b>10</b>	<b>10.00</b>	<b>10.00</b>	<b>10.00</b>	<b>0.00</b>	<b>0.00</b>	<b>10.00</b>	<b>10.00</b>	<b>10.00</b>	<b>10.00</b>	<b>10.00</b>
curv_p	Curvature Kimball aggregator prices		<b>10</b>	<b>10.00</b>	<b>10.00</b>	<b>10.00</b>	<b>0.00</b>	<b>0.00</b>	<b>10.00</b>	<b>10.00</b>	<b>10.00</b>	<b>10.00</b>	<b>10.00</b>
phi_w	Gross markup wages		<b>1.5</b>	<b>1.50</b>	<b>1.50</b>	<b>1.50</b>	<b>1.50</b>	<b>1.50</b>	<b>1.50</b>	<b>1.50</b>	<b>1.50</b>	<b>1.50</b>	<b>1.50</b>
delta	Depreciation rate		<b>0.025</b>	<b>0.03</b>	<b>0.03</b>	<b>0.03</b>	<b>0.03</b>	<b>0.03</b>	<b>0.03</b>	<b>0.03</b>	<b>0.03</b>	<b>0.03</b>	<b>0.03</b>
G	Steady state exogenous spending share		<b>0.18</b>	<b>0.18</b>	<b>0.18</b>	<b>0.18</b>	<b>0.18</b>	<b>0.18</b>	<b>0.18</b>	<b>0.18</b>	<b>0.18</b>	<b>0.18</b>	<b>0.18</b>
<b>Structural</b>													
alpha	Capital share		0.19	0.21	0.21	0.22	0.21	0.22	0.20	0.23	0.24	0.23	0.22
psi	(inverse of) Elasticity of capacity utilization cost		0.54	0.57	0.86	0.45	0.60	0.82	0.39	0.75	0.78	0.71	0.89
phi_p	1 + Fixed Cost share in production		1.61	1.62	1.51	1.53	1.61	1.65	1.39	1.53	1.78	2.00	1.19
iota_w	Indexation to past wages		0.59	0.61	0.43	0.59	0.65	0.76	0.59	0.55	0.74	0.85	0.72
xi_w	Calvo parameter wages		0.73	0.39	0.24	0.34	0.34	0.50	0.39	0.50	0.79	0.93	0.88
iota_p	Indexation to past prices		0.22	0.72	0.76	0.73	0.82	0.80	0.70	0.52	0.70	0.18	0.18
xi_p	Calvo parameter prices		0.65	0.68	0.82	0.54	0.84	0.50	0.51	0.75	0.78	0.73	0.71
sigma_c	Risk aversion		1.39	1.50	1.52	1.27	1.39	1.55	1.09	1.46	1.23	1.30	1.37
sigma_l	Frisch elasticity		1.92	1.91	0.99	1.69	1.43	0.38	1.27	2.12	2.36	5.13	1.60
lambda	External habit degree		0.71	0.68	0.59	0.70	0.69	0.48	0.68	0.71	0.72	0.79	0.72
phi	(inverse of) Elasticity of investment adjustment cos		5.48	5.49	6.28	4.16	5.18	2.83	3.85	7.09	6.65	8.48	6.15
gamma_bar	Net growth rate in percent		0.43	0.45	0.33	0.38	0.42	0.39	0.34	0.36	0.40	0.36	0.35
beta_const	Time preference rate in percent		0.16	0.13	0.10	0.22	0.13	0.11	0.17	0.14	0.11	0.65	0.13
pi_bar	Steady state inflation rate		0.81	0.76	0.57	0.93	0.97	1.02	0.70	0.65	0.70	1.26	0.76
l_bar	Steady state hours		-0.1	1.89	5.74	5.09	5.93	3.92	4.80	2.23	2.21	7.15	-0.76
<b>MonPol</b>													
r_pi	Taylor rule inflation feedback		2.03	2.00	1.79	1.79	1.95	1.98	1.26	1.84	1.80	2.62	1.49
r_dy	Taylor rule output growth feedback		0.22	0.10	0.03	0.05	0.06	0.03	0.15	0.01	0.38	0.23	0.09
r_y	Taylor rule output gap feedback		0.08	0.23	0.20	0.21	0.22	0.22	0.21	0.11	0.09	0.11	0.01
rho	Taylor rule interest rate Persistence		0.81	0.81	0.83	0.77	0.78	0.66	0.79	0.80	0.83	0.88	0.77
<b>Persistence</b>													
rho_a	Persistence Productivity shock		0.95	0.96	0.98	0.90	0.96	0.97	0.84	0.97	0.97	0.95	0.98
rho_ga	Spending shock parameter		0.52	0.56	0.49	0.56	0.56	0.57	0.56	0.45	0.05	0.38	0.34
rho_b	Persistence Risk Premium shock		0.18	0.20	0.55	0.27	0.20	0.42	0.43	0.12	0.27	0.21	0.10
rho_g	Persistence Fiscal shock		0.97	0.97	0.97	0.93	0.97	0.97	0.92	0.96	0.99	0.98	0.97
rho_i	Persistence Investment shock		0.71	0.68	0.87	0.82	0.72	0.78	0.64	0.64	0.57	0.73	0.82
rho_r	Persistence Monetary Policy shock		0.12	0.13	0.27	0.18	0.18	0.33	0.35	0.53	0.25	0.06	0.40
rho_p	Persistence Price markup shock		0.9	0.83	0.89	0.87	0.90	0.98	0.59	0.79	0.87	0.94	0.99
rho_w	Persistence Wage markup shock		0.97	0.97	0.92	0.91	0.97	0.98	0.95	0.98	0.95	0.82	0.97
Mu_w	Coefficient on MA term wage markup		0.88	0.88	0.90	0.72	0.78	0.34	0.88	0.67	0.87	0.78	0.98
Mu_p	Coefficient on MA term price markup		0.74	0.67	0.78	0.68	0.72	0.35	0.45	0.62	0.77	0.79	0.77
<b>Variance</b>													
eta_a	Productivity shock		0.45	0.44	0.42	0.47	0.44	0.43	0.55	0.36	0.46	0.43	0.61
eta_b	Risk premium shock		0.24	0.24	0.13	0.27	0.24	0.18	0.25	0.19	0.32	0.25	0.44
eta_g	Fiscal shock		0.52	0.52	0.43	0.54	0.51	0.51	0.54	0.41	0.61	0.55	0.49
eta_i	Investment-specific technology shock		0.45	0.44	0.26	0.39	0.41	0.47	0.44	0.36	0.46	0.39	0.30
eta_r	Monetary Policy shock		0.24	0.24	0.17	0.30	0.25	0.28	0.23	0.10	0.38	0.24	0.24
eta_p	Price markup shock		0.14	0.13	0.09	0.16	0.13	0.26	0.21	0.07	0.12	0.10	0.14
eta_w	Wage markup shock		0.24	0.26	0.41	0.17	0.26	0.52	0.18	0.31	0.21	0.26	0.45

		SW2007 Table1	Q3	Q4	Q5	Q6	Q7	Q8A	Q8B	Q9a	NoQ	
		1965Q1-2004Q4	1965Q1-2004Q4	1980Q1-2015Q4	1967Q1-1991Q4	Kimball	Calvo	Inflation	Moderation	Schorf	MH	1965Q1-2025Q1
<b>Structural</b>												
alpha	Capital share		0%	12%	10%	17%	12%	18%	7%	18%	20%	18%
psi	(inverse of) Elasticity of capacity utilization cost		0%	5%	59%	-16%	11%	51%	-28%	38%	31%	65%
phi_p	1 + Fixed Cost share in production		0%	1%	-6%	-5%	0%	2%	-14%	-5%	24%	-26%
iota_w	Indexation to past wages		0%	4%	-27%	0%	11%	29%	0%	-7%	45%	23%
xi_w	Calvo parameter wages		0%	-47%	-67%	-54%	-53%	-32%	-46%	-31%	27%	20%
iota_p	Indexation to past prices		0%	225%	247%	230%	272%	263%	217%	135%	-18%	-18%
xi_p	Calvo parameter prices		0%	5%	27%	-16%	30%	-23%	-21%	16%	12%	10%
sigma_c	Risk aversion		0%	8%	9%	-8%	0%	11%	-22%	5%	-7%	-2%
sigma_l	Frisch elasticity		0%	-1%	-48%	-12%	-26%	-80%	-34%	10%	167%	-17%
lambda	External habit degree		0%	-4%	-17%	-1%	-3%	-33%	-4%	0%	11%	1%
phi	(inverse of) Elasticity of investment adjustment cost		0%	0%	15%	-24%	-5%	-48%	-30%	29%	55%	12%
<b>MonPol</b>												
r_pi	Taylor rule inflation feedback		0%	-1%	-12%	-12%	-4%	-2%	-38%	-9%	29%	-27%
r_dy	Taylor rule output growth feedback		0%	-55%	-85%	-76%	-73%	-86%	-33%	-94%	6%	-58%
r_y	Taylor rule output gap feedback		0%	185%	156%	162%	180%	171%	157%	39%	33%	-91%
rho	Taylor rule interest rate Persistence		0%	0%	2%	-5%	-4%	-18%	-3%	-1%	8%	-5%
<b>Persistence</b>												
rho_a	Persistence Productivity shock		0%	1%	3%	-6%	1%	2%	-12%	2%	0%	3%
rho_ga	Spending shock parameter		0%	7%	-5%	7%	8%	9%	7%	-14%	-27%	-35%
rho_b	Persistence Risk Premium shock		0%	12%	207%	51%	9%	132%	137%	-32%	17%	-44%
rho_g	Persistence Fiscal shock		0%	0%	0%	-4%	0%	0%	-5%	-1%	1%	0%
rho_i	Persistence Investment shock		0%	-4%	23%	15%	2%	10%	-10%	-10%	2%	15%
rho_r	Persistence Monetary Policy shock		0%	7%	126%	52%	54%	176%	189%	343%	-47%	236%
rho_p	Persistence Price markup shock		0%	-8%	-1%	-3%	0%	9%	-35%	-12%	5%	10%
rho_w	Persistence Wage markup shock		0%	0%	-5%	-6%	0%	1%	-2%	1%	-16%	0%
Mu_w	Coefficient on MA term wage markup		0%	0%	2%	-18%	-11%	-62%	0%	-24%	-12%	12%
Mu_p	Coefficient on MA term price markup		0%	-10%	5%	-8%	-2%	-52%	-39%	-16%	6%	4%
<b>Variance</b>												
eta_a	Productivity shock		0%	-2%	-6%	4%	-2%	-5%	22%	-20%	-4%	35%
eta_b	Risk premium shock		0%	0%	-47%	13%	0%	-25%	4%	-19%	3%	82%
eta_g	Fiscal shock		0%	0%	-17%	3%	-1%	-1%	3%	-22%	6%	-6%
eta_i	Investment-specific technology shock		0%	-3%	-42%	-12%	-8%	4%	-3%	-21%	-14%	-33%
eta_r	Monetary Policy shock		0%	-1%	-31%	23%	2%	17%	-6%	-59%	-2%	0%
eta_p	Price markup shock		0%	-9%	-35%	11%	-10%	87%	53%	-47%	-30%	0%
eta_w	Wage markup shock		0%	9%	72%	-28%	7%	117%	-27%	29%	10%	88%