Lecture 10: Optimal Trade Policy

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Spring 2015

Northwestern ECON 460: Graduate International Trade

1 Introduction

In the previous class, we showed how to calculate the elasticity of welfare in any location to a change in trade costs between any two locations solely as a function of observed trade flows and the "gravity constants" α and β. This result turns out to be quite useful when examining optimal trade policy, i.e. how policy makers can alter their trade costs (or tariffs) in order to maximize welfare.

As an aside, there is a long tradition of examining both optimal trade policy and the political economy of trade policy in the trade literature (see e.g. Johnson (1953), Grossman and Helpman (1994), and Staiger and Bagwell (1999)). However, there has been much less work done examining the effect of trade policy or calculating optimal trade policy in many-country quantitative general equilibrium gravity models like the ones we have been focusing on in this class. One notable exception is the work done by Ralph Ossa at University of Chicago - Booth, who in Ossa (2011) quantifies optimal tariffs in a single sector Krugman model with fixed wages and in Ossa (2014) quantifies optimal tariffs in a multi-sector model with and endogenous wages (and political economy considerations). Below, we build off of these papers, along with Allen, Arkolakis, and Takahashi (2014).

2 Results from previous lectures

Let us briefly review some results from previous lectures that will prove useful in what follows.

2.1 The Universal Gravity framework

First, let us briefly recall the framework we are considering.

For a given set of bilateral frictions $\{K_{ij}\}$, income shifters $\{B_i\}$ and gravity constants α and β, the universal gravity framework satisfies the following equilibrium conditions:

1. The value of trade flows between any two locations satisfy the **gravity equation**:

$$X_{ij} = K_{ij}\gamma_i\delta_j, \tag{1}$$

where $\mathbf{K} \equiv \{K_{ij}\}$ is assumed to be exogenous (i.e. it is a model parameter), while $\{\gamma_i\}$ and $\{\delta_j\}$ are endogenous.

2. In all locations, the **goods market clears**:

$$Y_i = \sum_{j \in S} X_{ij}; \tag{2}$$

3. In all locations, **trade is balanced**:

$$Y_i = \sum_{i \in S} X_{ji}; \tag{3}$$

4. In all locations, the **generalized labor market clearing condition** holds:

$$Y_i = B_i \gamma_i^{\alpha} \delta_i^{\beta}. \tag{4}$$

5. Welfare in location i is inversely related to trade openness, as in Arkolakis, Costinot, and Rodríguez-Clare (2012):

$$W_i = \tilde{C}_i \lambda_{ii}^{-\rho} = C_i \gamma_i^{\rho(\alpha-1)} \delta_i^{\rho(\beta-1)},$$

where $C_i \equiv \tilde{C}_i \left(\frac{B_i}{K_{ii}}\right)^{\rho}$ and ρ are assumed to be known.

In what follows, let us suppose we observe a set of (balanced) trade flows $\{X_{ij}\}$ and know the gravity constants α and β.

2.2 World welfare elasticities

Recall from Proposition 1 of Lecture #6 ("Welfare in Gravity Trade Models") that there exists a set of Pareto weights $\{\omega_i\}$ such that the equilibrium of the universal gravity framework has the following dual interpretation:

$$\max \sum_i \omega_i W_i \text{ s.t. } \sum_{i \in S} \sum_{j \in S} K_{ij}\gamma_i\delta_j = \sum_{i \in S} B_i \gamma_i^{\alpha} \delta_i^{\beta} \tag{5}$$

Define $W \equiv \sum_{i \in S} \omega_i W_i$ to be world welfare. By applying the envelope theorem to the dual interpretation (5), we can calculate the elasticity of world welfare to a change in any bilateral trade frictions:

$$\frac{\partial \ln W}{\partial \ln K_{ij}} = \frac{1}{\rho} \frac{X_{ij}}{Y^W}, \tag{6}$$

i.e. the elasticity of world welfare to a change in bilateral trade frictions between two countries is proportional to the value of trade between those countries.

2.3 Location-specific welfare elasticies

Recall from Section 4.2 of Lecture #9 ("Calibration and Counterfactuals of Gravity Trade Models") that a log linearization of the equilibrium conditions at the equilibrium implies that the elasticity of the equilibrium origin and destination fixed effects in every country to a change in any bilateral trade frictions can be written as:

$$\frac{\partial \ln \gamma_l}{\partial \ln K_{ij}} = X_{ij} \times \left(A^+_{l,i} + A^+_{N+l,j} - c \right) \tag{7}$$

$$\frac{\partial \ln \delta_l}{\partial \ln K_{ij}} = X_{ij} \times \left(A^+_{N+l,i} + A^+_{l,j} - c \right), \tag{8}$$

where $A^+_{i,j}$ is the $\langle i, j \rangle$ element of the $2N \times 2N$ matrix $\mathbf{A}^+$:

$$\mathbf{A}^+ = \begin{pmatrix} (\alpha - 1)\,\mathbf{Y} & \beta \mathbf{Y} - \mathbf{X} \\ \alpha \mathbf{Y} - \mathbf{X}^T & (\beta - 1)\,\mathbf{Y} \end{pmatrix}^+, \tag{9}$$

and c is a constant that ensures the normalization holds. (For example, if we normalize world income to be one, i.e. $Y^W \equiv \sum_{i \in S} Y_i = 1$, then $c \equiv \frac{1}{(\alpha+\beta)} \sum_l Y_l \left(\alpha \left(A_{l,i} + A_{N+l,j} \right) + \beta \left(A_{N+l,i} + A_{l,j} \right) \right)$. From Condition 5, local welfare can be written as a function of the local origin and destination fixed effect, i.e. $W_i = C_i \gamma_i^{\rho(\alpha-1)} \delta_i^{\rho(\beta-1)}$, so that we can use equations (7) and (8) to calculate the elasticity of welfare in any location to any change in bilateral trade frictions:

$$\frac{\partial \ln W_l}{\partial \ln K_{ij}} = \rho \left((\alpha - 1) \frac{\partial \ln \gamma_l}{\partial \ln K_{ij}} + (\beta - 1) \frac{\partial \ln \delta_l}{\partial \ln K_{ij}} - \frac{\partial \ln K_{ll}}{\partial \ln K_{ij}} \right). \tag{10}$$

Note that while equation (10) is certainly more complicated than (6), it is still an analytical expression that depends only on observed trade flows and the gravity constants (which are assumed to be known).

3 Optimal Trade Policy: Trade Frictions

We now characterize the optimal trade policy decision in its most general form before turning to two particular examples. Formally, consider a planner who seeks to choose a set of changes in trade frictions in order to maximize a weighted average of the (first-order) change in welfare across all locations:

$$\max_{\{z_{ij}\}} \sum_{l \in S} \sum_{i \in S} \sum_{j \in S} \omega_l \frac{\partial \ln W_l}{\partial \ln K_{ij}} z_{ij} \text{ s.t. } G(\mathbf{z}) = 0, \tag{11}$$

where ω_l is the weight the planner places on the change in welfare in location l, z_{ij} is the percentage change in the bilateral trade friction K_{ij}, $\mathbf{z}$ is the $N \times N$ matrix whose $\langle i, j \rangle$ element is z_{ij} and $G(\mathbf{z})$ is a function which specifies the constraint under which the planner operates. For example, $G(\mathbf{z}) = \sum_{i \in S} \sum_{j \in S} p_{ij} z_{ij} - B$ would reflect the fact that the total expenditure on bilateral trade friction reductions has a budget of B (where the price of a percentage increase in K_{ij} is p_{ij}). In general, $G(\cdot)$ captures whatever factors prevent the planner from simply reducing trade frictions (increasing K_{ij}) as much as possible, thereby making the problem economically interesting. Note that while our framework offers no guidance on what form those constraints take, our procedure is equally valid for any (differentiable) $G(\cdot)$.

From equation (10), once the matrix $\mathbf{A}^{+}$ has been calculated from observed trade flows and the gravity constants, the elasticity of welfare in any location $l \in S$ with respect to the change in trade costs between any two countries $i \in S$ and $j \in S$, i.e. $\frac{\partial \ln W_l}{\partial \ln K_{ij}}$, can be immediately determined from a linear combination of elements of the matrix. (Note that while it is possible to calculate the set of welfare elasticities $\left\{\frac{\partial \ln W_l}{\partial \ln K_{ij}}\right\}_{i,j}$ without the use of equation (10), doing so would require changing each bilateral trade cost separately by a small amount and recalculating the model equilibrium; since there are N^2 bilateral trade costs, such a process would be onerous).

Once the complete set of welfare elasticities is calculated equation (10), equation (11) is simply a constrained optimization problem with the familiar first order necessary conditions for all $i \in S$ and $j \in S$:

$$\omega_l \frac{\partial \ln W_l}{\partial \ln K_{ij}} = \lambda \frac{\partial G(\mathbf{z})}{\partial z_{ij}}. \tag{12}$$

The N^2 equations in equation (12) along with the constraint $G(\mathbf{z}) = 0$ can then be jointly solved to determine the welfare-maximizing trade cost changes $\mathbf{z}^*$ and the Lagrange multiplier λ.

3.1 Example: "WTO"

Consider first a planner who attempts to maximize a weighted average of the world-wide increase in welfare subject to a "non-discrimination" constraint where a location must equally reduce its trade frictions with all its exporting and importing partners. We assume that the weights attached to each location are those so that the welfare maximization problem corresponds to the competitive equilibrium. We assume for simplicity that the trade friction reduction costs are convex. Then equation (11) becomes:

$$\max_{\{z_i\}_{i\in S}} \sum_{i\in S} \sum_{j\neq i} \frac{\partial \ln W}{\partial \ln K_{ij}} z_i z_j \text{ s.t. } \sum_{i \in S} z_i^2 = 1.$$

(We exclude the $i = j$ terms so that domestic trade costs remain unchanged). The first order conditions (12) now imply that the welfare-maximizing multilateral trade friction reductions $\{z_i^*\}$ solve the following system of equations:

$$\begin{aligned} 2\lambda z_i^* &= \sum_{j\neq i} \left(\frac{\partial \ln W}{\partial \ln K_{ij}} + \frac{\partial \ln W}{\partial \ln K_{ji}} \right) z_j^* \iff \\ \tilde{\lambda} z_i^* &= \sum_{j \neq i} \left(X_{ij} + X_{ji}\right) z_j^*, \end{aligned} \tag{13}$$

where λ is the Lagrange multiplier, $\tilde{\lambda} \equiv 2\rho^{-1}\lambda Y^W$, and the second line relied on the relationship between the world welfare elasticity and observed trade flows given in equation (6) $\frac{\partial \ln W}{\partial \ln K_{ij}} = \frac{1}{\rho} \frac{X_{ij}}{Y^W}$. Equation (13) shows that *the reduction in bilateral trade frictions which* *maximizes world welfare is simply the eigenvector of the observed trade flows corresponding to the largest eigenvalue* (when the trade matrix is added to its transpose and has zeros along the diagonal). Furthermore, that largest eigenvalue is proportional to the elasticity of world welfare to increasing the extent of the trade friction reductions in a welfare-maximizing way.

3.2 Example: "Infrastructure improvement"

Suppose a planner ("government official") in a single country i is trying to decide how to allocate a fixed amount of resources B across L different infrastructure projects, which are assumed to only affect export trade costs. Let the value of investment in port $l \in \{1, ..., L\}$ be denoted by I_l. The maximization problem them becomes:

$$\max_{\{I_l\}_l} \ln W_i \left(\{I_l\}\right) \text{ s.t. } \sum_{l=1}^{L} I_l \leq B$$

First order conditions imply:

$$\frac{\partial \ln W_i}{\partial I_l} = \frac{\partial \ln W_i}{\partial I_k}$$

for $l, k \in \{1, .., L\}$ such that $I_l > 0$ and $I_k > 0$. Note that we can rewrite the first order conditions as:

$$\sum_j \frac{\partial \ln W_i}{\partial \ln K_{ij}} \times \frac{\partial \ln K_{ij}}{\partial \ln I_l} = \sum_j \frac{\partial \ln W_i}{\partial \ln K_{ij}} \times \frac{\partial \ln K_{ij}}{\partial \ln I_k}, \tag{14}$$

i.e. if we know how each infrastructure project affects all the export frictions, we can use equation (14) to allocate the resources across the different infrastructure projects. However, given that $\frac{\partial \ln W_i}{\partial \ln K_{ij}}$ are elasticities (i.e. they ignore higher order effects), equation (14) is better suited to evaluate the efficiency of observed spending across current infrastructure projects.

4 Optimal Trade Policy: Tariffs

The above analysis has focused on optimal policy in terms of changing (technological) trade costs, which is appropriate for the study of physical trade costs (e.g. infrastructure improvement) or non-tariff barriers. However, the analysis above is inappropriate for examining the effect of tariffs on welfare, as it does not incorporate the fact that tariffs are a source of revenue. In what follows, we show that when there are not intermediates (i.e. the gravity constant $\beta = 0$), we can incorporate tariffs (and the revenue they generate) into our universal gravity framework.

4.1 The model

To be explicit, let us consider a simple Armington model with both technological iceberg trade costs $\tau_{ij} \geq 1$ and ad-valorem tariffs $\tilde{t}_{ij}$. Define $t_{ij} \equiv 1 + \tilde{t}_{ij}$ to be one plus the ad valorem tariff. Then as in lecture #1, we can write the value of trade flows from i to j that is *sent from i prior to tariffs being levied* as:

$$X_{ij} = \tau_{ij}^{1-\sigma} t_{ij}^{-\sigma} A_i^{\sigma-1} w_i^{1-\sigma} P_j^{\sigma-1} E_j, \tag{15}$$

where A_i is the productivity in location $i \in S$, w_i is the wage, P_j is the ideal Dixit-Stiglitz price index, E_j is expenditure, and σ is the elasticity of substitution. [Class question: why is the exponent on t_{ij} $-\sigma$ instead of $1-\sigma$?]. As in the basic Armington model, income in country i from trade is equal to its total sales:

$$Y_i = \sum_{j \in S} X_{ij}. \tag{16}$$

Unlike the basic Armington model, however, total income (and hence expenditure) also includes the revenue earned from tariffs T_i:

$$E_i = Y_i + T_i, \tag{17}$$

where tariff revenue is equal to the bilateral tariff charged on all trade being sent[1]:

$$T_i = \sum_{j \in S} \tilde{t}_{ji} X_{ji}. \tag{18}$$

The total expenditure by consumers in country i is also equal to its total imports plus the tariffs incurred:

$$E_i = \sum_{j \in S} \left(1 + \tilde{t}_{ji}\right) X_{ji}. \tag{19}$$

Combining equations (17), (18), and (19), we can show that the inclusion of tariff revenue does not change the fact that trade is balanced:

$$\begin{aligned} Y_i + T_i = \sum_j \left(1 + \tilde{t}_{ji}\right) X_{ji} &\iff \\ Y_i + \sum_j \tilde{t}_{ji} X_{ji} = \sum_j \left(1 + \tilde{t}_{ji}\right) X_{ji} &\iff \\ Y_i = \sum_{j \in S} X_{ji}. & \end{aligned} \tag{20}$$

Intuitively, because tariffs are levied on domestic consumers and then are redistributed back to those same consumers, while tariffs distort prices, they do not affect trade balance (when trade flows are measured at the dock of the origin country). [Class question: what is the difference between changing tariff t_{ij} and changing iceberg trade cost τ_{ij}?]

Finally, the income from sales is redistributed back to workers:

$$Y_i = w_i L_i.$$

Because we can re-write equation (15) as:

$$X_{ij} = K_{ij} \gamma_i \delta_j, \tag{21}$$

[1] If we had instead supposed that tariffs are only levied on goods that actually arrive, we would have $T_i = \sum_j \frac{\tilde{t}_{ji}}{\tau_{ji}} X_{ji}$, which does not change the following analysis in any substantive way.

where $\gamma_i \equiv A_i^{\sigma-1} w_i^{1-\sigma}$, $\delta_j \equiv P_j^{\sigma-1} E_j$, and $K_{ij} \equiv \tau_{ij}^{1-\sigma} t_{ij}^{-\sigma}$, we can re-write the labor market clearing condition as:

$$\begin{aligned} Y_i &= A_i L_i \gamma_i^{\frac{1}{1-\sigma}} \iff \\ Y_i &= B_i \gamma_i^{\alpha}, \end{aligned} \tag{22}$$

where $B_i \equiv A_i L_i$ and $\alpha \equiv \frac{1}{1-\sigma}$.

Note that equations (21), (16), (20), and (22) satisfy conditions 1, 2, 3, and 4 of the Universal Gravity framework, respectively. Hence, we can use equations (7) and (8) to calculate the elasticity of the origin and destination fixed effects to any change in tariffs since:

$$\frac{\partial \ln \gamma_i}{\partial \ln t_{kl}} = \frac{\partial \ln \gamma_i}{\partial \ln K_{kl}} \times \frac{\partial \ln K_{kl}}{\partial \ln t_{kl}} = -\sigma \times X_{ij} \times \left(A^+_{l,i} + A^+_{N+l,j} - c\right) \tag{23}$$

$$\frac{\partial \ln \delta_i}{\partial \ln t_{kl}} = \frac{\partial \ln \delta_i}{\partial \ln K_{kl}} \times \frac{\partial \ln K_{kl}}{\partial \ln t_{kl}} = -\sigma \times X_{ij} \times \left(A^+_{N+l,i} + A^+_{l,j} - c\right). \tag{24}$$

4.2 Optimal tariffs

Unfortunately, in the presence of tariffs, the simple expression for welfare given in Condition 5 above does not hold, which makes deriving the elasticity of welfare in a particular location to any tariff change, i.e. $\frac{\partial \ln W_i}{\partial \ln t_{kl}}$, a little more involved.

Recall that welfare in a location is equal to the real expenditure, i.e. $W_i = \frac{E_i}{P_i}$. (Note that we are now considering the total welfare of a country; we could divide by L_i to get the welfare per person). Recall from the reformulation of trade flows into the universal framework (equation (21)), we have:

$$\delta_i \equiv P_i^{\sigma-1} \left(Y_i + T_i\right)$$

and, combined with equation (18), we have:

$$T_i = \sum_j \tilde{t}_{ji} K_{ji} \gamma_j \delta_i$$

Combining these two equations allows us to solve for the price index as a function of the origin and destination fixed effects:

$$\begin{aligned} \delta_i &= P_i^{\sigma-1} \left(\gamma_i^{\frac{1}{1-\sigma}} A_i L_i + \sum_j \tilde{t}_{ji} K_{ji} \gamma_j \delta_i\right) \iff \\ P_i &= \left(\frac{\left(\gamma_i^{\frac{1}{1-\sigma}} A_i L_i + \sum_j \tilde{t}_{ji} K_{ji} \gamma_j \delta_i\right)}{\delta_i}\right)^{\frac{1}{1-\sigma}} \iff \\ P_i &= \left(\gamma_i^{\frac{1}{1-\sigma}} A_i L_i \delta_i^{-1} + \sum_j \tilde{t}_{ji} K_{ji} \gamma_j\right)^{\frac{1}{1-\sigma}}. \end{aligned} \tag{25}$$

Combining equation (25) and the fact that $Y_i + T_i = \delta P_i^{1-\sigma}$, we can write welfare solely as a function of the origin and destination fixed effects:

$$
\begin{aligned}
W_i &= \frac{(Y_i + T_i)}{P_i} \iff \\
W_i &= \frac{\delta_i P_i^{1-\sigma}}{P_i} \iff \\
W_i &= \delta_i P_i^{-\sigma} \iff \\
W_i &= \delta_i \left(\frac{\left(\gamma_i^{\frac{1}{1-\sigma}} A_i L_i + \sum_j \tilde{t}_{ji} K_{ji} \gamma_j \delta_i \right)}{\delta_i} \right)^{\frac{\sigma}{\sigma-1}} \iff \\
W_i &= \left(\delta_i^{-\frac{1}{\sigma}} \gamma_i^{\frac{1}{1-\sigma}} A_i L_i + \delta_i^{\frac{\sigma-1}{\sigma}} \sum_j \tilde{t}_{ji} K_{ji} \gamma_j \right)^{\frac{\sigma}{\sigma-1}} \qquad (26)
\end{aligned}
$$

We can then take logs and differentiate with respect to some bilateral trade friction K_{kl}:

$$
\begin{aligned}
\frac{\partial \ln W_i}{\partial \ln K_{kl}} &= \frac{\sigma}{\sigma-1} \frac{\partial \ln \left(\delta_i^{-\frac{1}{\sigma}} \gamma_i^{\frac{1}{1-\sigma}} A_i L_i + \delta_i^{\frac{\sigma-1}{\sigma}} \sum_j \tilde{t}_{ji} K_{ji} \gamma_j \right)}{\partial \ln K_{kl}} \iff \\
\frac{\partial \ln W_i}{\partial \ln K_{kl}} &= \frac{\sigma}{\sigma-1} \frac{\partial \ln \left(\gamma_i^{\frac{1}{1-\sigma}} A_i L_i + \sum_j \tilde{t}_{ji} K_{ji} \gamma_j \delta_i \right)}{\partial \ln K_{kl}} - \frac{1}{\sigma-1} \frac{\partial \ln \delta_i}{\partial \ln K_{kl}} \iff \\
\frac{\partial \ln W_i}{\partial \ln K_{kl}} &= \frac{\sigma}{\sigma-1} \frac{\frac{\partial \gamma_i^{\frac{1}{1-\sigma}} A_i L_i}{\partial \ln K_{kl}} + \frac{\partial \sum_j \tilde{t}_{ji} K_{ji} \gamma_j \delta_i}{\partial \ln K_{kl}}}{Y_i + \sum_j \tau_{ji} X_{ji}} - \frac{1}{\sigma-1} \frac{\partial \ln \delta_i}{\partial \ln K_{kl}} \iff \\
\frac{\partial \ln W_i}{\partial \ln K_{kl}} &= \frac{\sigma}{\sigma-1} \frac{\frac{1}{1-\sigma} \frac{\partial \ln \gamma_i}{\partial \ln K_{kl}} \times Y_i + \frac{\partial \sum_j \tilde{t}_{ji} K_{ji} \gamma_j \delta_i}{\partial \ln K_{kl}}}{Y_i + \sum_j \tilde{t}_{ji} X_{ji}} - \frac{1}{\sigma-1} \frac{\partial \ln \delta_i}{\partial \ln K_{kl}} \iff \\
\frac{\partial \ln W_i}{\partial \ln K_{kl}} &= -\frac{\sigma}{(\sigma-1)^2} \frac{\partial \ln \gamma_i}{\partial \ln K_{kl}} \times \frac{Y_i}{E_i} + \frac{\sigma}{\sigma-1} \frac{\frac{\partial \sum_j \tilde{t}_{ji} K_{ji} \gamma_j \delta_i}{\partial \ln K_{kl}}}{E_i} - \frac{1}{\sigma-1} \frac{\partial \ln \delta_i}{\partial \ln K_{kl}} \iff \\
\frac{\partial \ln W_i}{\partial \ln K_{kl}} &= -\frac{\sigma}{(\sigma-1)^2} \left(\frac{Y_i}{E_i} \right) \frac{\partial \ln \gamma_i}{\partial \ln K_{kl}} - \frac{1}{\sigma-1} \frac{\partial \ln \delta_i}{\partial \ln K_{kl}} + \frac{\sigma}{\sigma-1} \sum_j \frac{\tilde{t}_{ji} X_{ji}}{E_i} \left(\frac{\partial \ln \tilde{t}_{ji}}{\partial \ln K_{kl}} + \frac{\partial \ln K_{ji}}{\partial \ln K_{kl}} + \frac{\partial \ln \gamma_j}{\partial \ln K_{kl}} + \right. \\
\frac{\partial \ln W_i}{\partial \ln t_{kl}} &= \left(\frac{\sigma}{\sigma-1} \right)^2 \left(\frac{Y_i}{E_i} \right) \frac{\partial \ln \gamma_i}{\partial \ln K_{kl}} + \frac{\sigma}{\sigma-1} \frac{\partial \ln \delta_i}{\partial \ln K_{kl}} - \frac{\sigma^2}{\sigma-1} \sum_j \frac{\tilde{t}_{ji} X_{ji}}{E_i} \left(\frac{\partial \ln \tilde{t}_{ji}}{\partial \ln K_{kl}} + \frac{\partial \ln K_{ji}}{\partial \ln K_{kl}} + \frac{\partial \ln \gamma_j}{\partial \ln K_{kl}} + \right. \\
\frac{\partial \ln W_i}{\partial \ln t_{kl}} &= \left(\frac{\sigma}{\sigma-1} \right)^2 \left(\frac{Y_i}{E_i} \right) \frac{\partial \ln \gamma_i}{\partial \ln K_{kl}} + \frac{\sigma}{\sigma-1} \frac{\partial \ln \delta_i}{\partial \ln K_{kl}} - \frac{\sigma^2}{\sigma-1} \sum_j \lambda_{ji} \left(\frac{\partial \ln \tilde{t}_{ji}}{\partial \ln K_{kl}} + \frac{\partial \ln X_{ji}}{\partial \ln K_{kl}} \right) \tilde{t}_{ji}, \qquad (27)
\end{aligned}
$$

where in the last line we used $\frac{\partial \ln W_i}{\partial \ln t_{kl}} = \frac{\partial \ln W_i}{\partial \ln K_{kl}} \times \frac{\partial \ln K_{kl}}{\partial \ln t_{kl}} = -\sigma \frac{\partial \ln W_i}{\partial \ln K_{kl}}$. Because we know

$\left\{\frac{\partial \ln \gamma_i}{\partial \ln K_{kl}}\right\}$ and $\left\{\frac{\partial \ln \delta_i}{\partial \ln K_{kl}}\right\}$ from equations (23) and (24), equation (27) provides an analytical expression for the effect of any bilateral tariff change on the welfare in any country.

As a result, we can finally turn to the question of optimal tariff policy. Country i will choose the set of (log) import tariffs $\{\ln t_{ji}\}_j$ in order to maximize its (log) welfare:

$$\max_{\{\ln t_{ki}\}_k} \ln W_i\left(\{\ln t_{ki}\}_k\right)$$

First order conditions are simply for all $k \in S$:

$$\begin{aligned}
\frac{\partial \ln W_i}{\partial \ln t_{ki}} &= 0 \iff \\
\frac{\sigma}{\sigma-1}\left(\frac{Y_i}{E_i}\right)\frac{\partial \ln \gamma_i}{\partial \ln K_{ki}} + \frac{\partial \ln \delta_i}{\partial \ln K_{ki}} &= \sigma \sum_j \lambda_{ji}\left(\frac{\partial \ln \tilde{t}_{ji}}{\partial \ln K_{ki}} + \frac{\partial \ln X_{ji}}{\partial \ln K_{ki}}\right)\tilde{t}_{ji} \iff \\
\frac{\sigma}{\sigma-1}\left(\frac{Y_i}{E_i}\right)\frac{\partial \ln \gamma_i}{\partial \ln K_{ki}} + \frac{\partial \ln \delta_i}{\partial \ln K_{ki}} &= \sigma \sum_j \lambda_{ji}\frac{\partial \ln X_{ji}}{\partial \ln K_{ki}}\tilde{t}_{ji} + \sigma\lambda_{ki}\left(-\frac{1}{\sigma}\frac{1+\tilde{t}_{ki}}{\tilde{t}_{ki}}\right)\tilde{t}_{ki} \iff \\
\frac{\sigma}{\sigma-1}\left(\frac{Y_i}{E_i}\right)\frac{\partial \ln \gamma_i}{\partial \ln K_{ki}} + \frac{\partial \ln \delta_i}{\partial \ln K_{ki}} &= \sigma \sum_j \lambda_{ji}\frac{\partial \ln X_{ji}}{\partial \ln K_{ki}}\tilde{t}_{ji} - \lambda_{ki}t_{ki} \iff \\
-\frac{1}{\sigma-1}\left(\frac{Y_i}{E_i}\right)\frac{\partial \ln \gamma_i}{\partial \ln t_{ki}} - \frac{1}{\sigma}\frac{\partial \ln \delta_i}{\partial \ln t_{ki}} &= -\sum_j \lambda_{ji}\frac{\partial \ln X_{ji}}{\partial \ln t_{ki}}\tilde{t}_{ji} - \lambda_{ki}t_{ki} \iff \\
\lambda_{ki}t_{ki} + \sum_j \lambda_{ji}\frac{\partial \ln X_{ji}}{\partial \ln t_{ki}}\tilde{t}_{ji} &= \frac{1}{\sigma}\left(\frac{\sigma}{\sigma-1}\left(\frac{Y_i}{E_i}\right)\frac{\partial \ln \gamma_i}{\partial \ln t_{ki}} + \frac{\partial \ln \delta_i}{\partial \ln t_{ki}}\right) \iff \\
\lambda_{ki} + \sum_j \lambda_{ji}\frac{\partial \ln X_{ji}}{\partial \tilde{t}_{ki}}\tilde{t}_{ji} &= \frac{1}{\sigma}\left(\frac{\sigma}{\sigma-1}\left(\frac{Y_i}{E_i}\right)\frac{\partial \ln \gamma_i}{\partial \tilde{t}_{ki}} + \frac{\partial \ln \delta_i}{\partial \tilde{t}_{ki}}\right) \iff \\
\lambda_{ki} + \sum_j \lambda_{ji}\frac{\partial \ln X_{ji}}{\partial \tilde{t}_{ki}}\tilde{t}_{ji} &= \left(\left(\frac{1-\sigma}{\sigma}\right)\frac{\partial \ln W_i^0}{\partial \tilde{t}_{ki}} - \frac{1}{\sigma}\frac{\partial \ln\left(\frac{Y_i}{E_i}\right)}{\partial \tilde{t}_{ki}} - \frac{1}{\sigma-1}\left(\frac{T_i}{E_i}\right)\frac{\partial \ln \gamma_i}{\partial \tilde{t}_{ki}}\right)
\end{aligned} \tag{28}$$

where we used the fact that:

$$
\begin{aligned}
\frac{\partial \ln \lambda_{ii}}{\partial \ln K_{ki}} &= \frac{\partial}{\partial \ln K_{ki}} \ln \left(\frac{X_{ii}}{Y_i} \frac{Y_i}{E_i} \right) \iff \\
\frac{\partial \ln \lambda_{ii}}{\partial \ln K_{ki}} &= \frac{\partial}{\partial \ln K_{ki}} \ln \left(\frac{K_{ii}\gamma_i\delta_i}{B_i\gamma_i^{\alpha}} \right) + \frac{\partial \ln \left(\frac{Y_i}{E_i}\right)}{\partial \ln K_{ki}} \iff \\
\frac{\partial \ln \lambda_{ii}}{\partial \ln K_{ki}} &= \frac{\partial \ln \delta_i}{\partial \ln K_{ki}} + (1-\alpha) \frac{\partial \ln \gamma_i}{\partial \ln K_{ki}} + \frac{\partial \ln \left(\frac{Y_i}{E_i}\right)}{\partial \ln K_{ki}} \iff \\
\frac{\partial \ln \lambda_{ii}}{\partial \ln K_{ki}} &= \frac{\partial \ln \delta_i}{\partial \ln K_{ki}} + \left(\frac{\sigma}{\sigma - 1} \right) \frac{\partial \ln \gamma_i}{\partial \ln K_{ki}} + \frac{\partial \ln \left(\frac{Y_i}{E_i}\right)}{\partial \ln K_{ki}} \iff \\
(1-\sigma) \frac{\partial \ln W_i^0}{\partial \ln K_{ki}} &= \frac{\partial \ln \delta_i}{\partial \ln K_{ki}} + \left(\frac{\sigma}{\sigma - 1} \right) \frac{\partial \ln \gamma_i}{\partial \ln K_{ki}} + \frac{\partial \ln \left(\frac{Y_i}{E_i}\right)}{\partial \ln K_{ki}} \iff \\
(1-\sigma) \frac{\partial \ln W_i^0}{\partial \tilde{t}_{ki}} - \frac{\partial \ln \left(\frac{Y_i}{E_i}\right)}{\partial \tilde{t}_{ki}} &= \frac{\partial \ln \delta_i}{\partial \tilde{t}_{ki}} + \left(\frac{\sigma}{\sigma - 1} \right) \frac{\partial \ln \gamma_i}{\partial \tilde{t}_{ki}}
\end{aligned}
$$

The first term on the left hand side of equation (28) captures the increase in revenue associated with the tariff increase (holding trade flows constant). The second term on the left hand size captures the decrease in revenue caused by the decline in trade flows because of the higher trade costs. The right hand side captures the standard welfare effects of changing bilateral trade flows.

Note that we can solve for the optimal tariffs $\left\{\tilde{t}_{ji}\right\}_j$ by the simple inversion:

$$
\tilde{t}_i = \left[\lambda_{ji} \frac{\partial \ln X_{ji}}{\partial \ln \tilde{t}_{ki}} \right]_{kj}^{-1} \left[\left(\frac{1-\sigma}{\sigma} \right) \frac{\partial \ln W_i^0}{\partial \tilde{t}_{ki}} - \frac{1}{\sigma} \frac{\partial \ln \left(\frac{Y_i}{E_i}\right)}{\partial \tilde{t}_{ki}} - \frac{1}{\sigma - 1} \left(\frac{T_i}{E_i} \right) \frac{\partial \ln \gamma_i}{\partial \tilde{t}_{ki}} - \lambda_{ki} \right]_k ,
$$

When we consider a local deviation of tariffs around the no tariff case (so that $Y_i = E_i$), we have:

$$
\tilde{t}_i = - \left[\lambda_{ji} \frac{\partial \ln X_{ji}}{\partial \tilde{t}_{ki}} \right]_{kj}^{-1} \left[\left(\frac{\sigma - 1}{\sigma} \right) \frac{\partial \ln W_i^0}{\partial \tilde{t}_{ki}} + \lambda_{ki} \right]_k . \tag{29}
$$

Loosely speaking, equation (29) says that the optimal tariff (starting from no tariffs) trades off the standard welfare (loss) of increasing trade costs with revenue gain from higher tariffs. A country will increase its tariffs until the welfare cost is equal to the increase in revenue.

5 Conclusion and next steps

This lecture highlights the power of combining observed trade flows with the theoretical results we have developed thus far. We should emphasize that the analysis of trade policy using these quantitative frameworks is still young, and we expect that there are many advances to come.

Note that thus far, we have assumed we know the value of the gravity constants α and β. These constants play important roles in determining the general equilibrium forces of the model. Up next, we consider how to estimate those constants.

References

Allen, T., C. Arkolakis, and Y. Takahashi (2014): "Universal gravity," *NBER Working Paper*, (w20787).

Arkolakis, C., A. Costinot, and A. Rodríguez-Clare (2012): "New Trade Models, Same Old Gains?," *American Economic Review*, 102(1), 94–130.

Grossman, G. M., and E. Helpman (1994): "Protection for Sale," *The American Economic Review*, pp. 833–850.

Johnson, H. G. (1953): "Optimum tariffs and retaliation," *The Review of Economic Studies*, pp. 142–153.

Ossa, R. (2011): "A "New Trade" Theory of GATT/WTO Negotiations," *Journal of Political Economy*, 119(1), 122–152.

——— (2014): "Trade Wars and Trade Talks with Data," *American Economic Review*, 104(12), 4104–46.

Staiger, R. W., and K. Bagwell (1999): "An Economic Theory of GATT," *American Economic Review*, 89(1), 215–248.