Plan for Today's Lecture

1. Allen and Arkolakis (2014)
   1. Introduction
   2. Model set-up
   3. Equilibrium characterization
   4. Estimation
   5. Counterfactuals

Allen and Arkolakis (QJE, 2014)

- General spatial economic model
  - Combines gravity structure with labor mobility.
  - Any continuous bilateral trade costs ("geographic location").
  - Any continuous topography of amenities and productivities ("local characteristics").
- Flexible productivity and amenity spillovers
  - Special cases are isomorphic to seminal economic geography models.
- Tractable general equilibrium structure
  - Show solutions are special cases of well-understood mathematical systems.
  - Characterize the conditions for existence, uniqueness, and stability.
  - Derive simple equations governing relationship between equilibrium economic activity and geography.

Geography

Compact set S of locations inhabited by $\bar{L}$ workers.

Location $i \in S$ is endowed with:
Differentiated variety (Armington assumption).
Productivity $\bar{A}(i)$.
Amenity $\bar{u}(i)$.

For all $i, j \in S$, let the iceberg bilateral trade cost be $T(i, j)$.

Terminology
$\bar{A}$ and $\bar{u}$ are the **local characteristics**.
T determines **geographic location**.
Together, $\bar{A}$, $\bar{u}$, and T comprise the **geography** of S.

A geography is **regular** if $\bar{A}$, $\bar{u}$ and T are continuous and bounded above and below by strictly positive numbers.

Workers

Endowed with identical CES preferences over differentiated varieties with elasticity of substitution $\sigma > 1$.

Can choose to live/work in any location $i \in S$.

Receive wage $w(i)$ for their inelastically supplied unit of labor.

Welfare in location i is:

$$W(i) = \left(\int_{s \in S} q(s,i)^{\frac{\sigma-1}{\sigma}} ds \right)^{\frac{\sigma}{\sigma-1}} u(i)$$

where $q(s,i)$ is the per capita quantity consumed in location i of the good produced in location s and $u(i)$ is the local amenity.

Production

Labor is the only factor of production, $L(i)$ is the density of workers.

Productivity of worker in location i is $A(i)$.

Perfect competition implies price of good from i is $\frac{w(i)}{A(i)} T(i,j)$ in location j.

Functions w and L comprise the **distribution of economic activity**.

Productivity and amenity spillovers

Productivity is potentially subject to externalities:

$$A(i) = \bar{A}(i) L(i)^{\alpha}$$

Amenities are potentially subject to externalities:

$$u(i) = \bar{u}(i) L(i)^{\beta}$$

Isomorphisms:

Monopolistic competition with free entry: $\alpha = \frac{1}{\sigma - 1}$.

Cobb-Douglas preferences over non-tradable sector: $\beta = -\frac{1-\gamma}{\gamma}$.

Heterogeneous (extreme-value) worker preferences: $\beta = -\frac{1}{\theta}$.

Terminology

Markets are said to **clear** if for all $i \in S$:

$$w(i) L(i) = \int_S X(i, s) \, ds,$$

where $X(i, j)$ is the value of trade flows from $i \in S$ to $j \in S$.

Welfare is said to be **equalized** if there exists $W \in \mathbb{R}_{++}$ such that for all $i \in S$, $W(i) \leq W$, with the equality strict if $L(i) > 0$.

Equilibrium

A **spatial equilibrium** is a distribution of economic activity such that:

Markets clear,

Welfare is equalized,

The aggregate labor market clears, i.e. $\int_S L(s)\, ds = \bar{L}$. Characterization

A spatial equilibrium is **regular** if L and w are strictly positive and continuous.

A spatial equilibrium is **point-wise locally stable** if $\frac{dW(i)}{dL(i)} < 0$ for all $i \in S$.

Equilibrium without spillovers

Suppose $\alpha = \beta = 0$ so that $A(i) = \bar{A}(i)$ and $u(i) = \bar{u}(i)$.

From welfare equalization:

$$w(i)^{1-\sigma} = W^{1-\sigma} \int_S T(s,i)^{1-\sigma} u(i)^{\sigma-1} A(s)^{\sigma-1} w(s)^{1-\sigma} ds$$

From balanced trade:

$$L(i) w(i)^{\sigma} = W^{1-\sigma} \int_S T(i,s)^{1-\sigma} A(i)^{\sigma-1} u(s)^{\sigma-1} L(s) w(s)^{\sigma} ds$$

These two equations are eigenfunctions of $w(i)^{1-\sigma}$ and $L(i) w(i)^{\sigma}$, respectively.

Equilibrium without spillovers: Theorem

Theorem

For any regular geography with exogenous productivity and amenities:

1. *There exists a unique equilibrium.*
2. *The equilibrium is regular and point-wise locally stable.*
3. *Equilibrium can be determined using an iterative procedure.*

Proof.

Application of Jentzsch's theorem (generalization of the Perron-Frobenius theorem) and Fredholm's Theorems. □

Equilibrium with spillovers

Can rewrite balanced trade and utility equalization as:

$$L(i)^{1-\alpha(\sigma-1)} w(i)^{\sigma} = W^{1-\sigma} \int_S T(i,s)^{1-\sigma} \bar{A}(i)^{\sigma-1} \bar{u}(s)^{\sigma-1} L(s)^{1+\beta(\sigma-1)} w(s)^{\sigma} ds$$

$$w(i)^{1-\sigma} L(i)^{\beta(1-\sigma)} = W^{1-\sigma} \int_S T(s,i)^{1-\sigma} \bar{A}(s)^{\sigma-1} \bar{u}(i)^{\sigma-1} w(s)^{1-\sigma} L(s)^{\alpha(\sigma-1)} ds$$

If $T(i,s) = T(s,i)$ for all $i, s \in S$ then the solution can be written as:

$$A(i)^{\sigma-1} w(i)^{1-\sigma} = \phi L(i) w(i)^{\sigma} u(i)^{\sigma-1}$$

$$L(i)^{\tilde{\sigma}\gamma_1} = K_1(i) W^{1-\sigma} \int_S T(s,i)^{1-\sigma} K_2(s) \left(L(s)^{\tilde{\sigma}\gamma_1}\right)^{\frac{\gamma_2}{\gamma_1}} ds,$$

where $K_1(i)$ and $K_2(i)$ are functions of $\bar{A}(i)$ and $\bar{u}(i)$, γ_1, γ_2, and $\tilde{\sigma}$ are functions of α, β, and σ.

The last equation is a Hammerstein non-linear integral equation.

Equilibrium with spillovers: Theorem

Theorem

Consider any regular geography with endogenous productivity and amenities with T symmetric. Define $\gamma_1 \equiv 1 - \alpha(\sigma - 1) - \beta\sigma$ and $\gamma_2 \equiv 1 + \alpha\sigma + (\sigma - 1)\beta$. If $\gamma_1 \neq 0$, then:

1. *There exists a regular equilibrium.*
2. *If $\gamma_1 < 0$, no regular equilibria are point-wise locally stable.*
3. *If $\gamma_1 > 0$, all equilibria are regular and point-wise locally stable.*
4. *If $\frac{\gamma_2}{\gamma_1} \in [-1, 1]$, the equilibrium is unique.*
5. *If $\frac{\gamma_2}{\gamma_1} \in (-1, 1]$, the equilibrium can be determined using an iterative procedure.*

Result 5 implies this can be very easy to do...

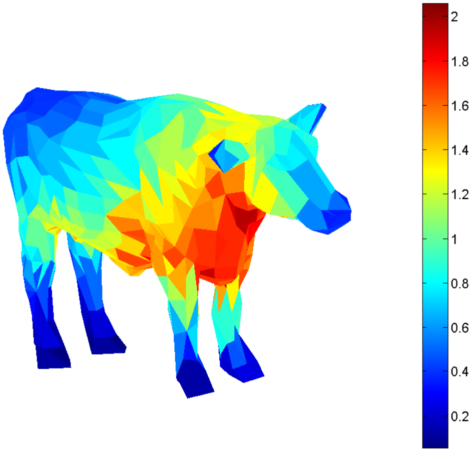
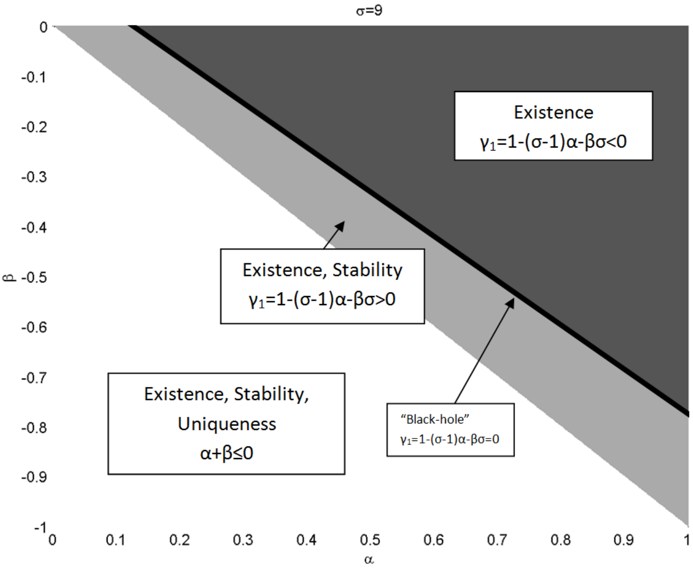


Figure: Equilibria with endogenous amenities and productivity



Geography and the equilibrium distribution of labor

When trade costs are symmetric, equilibrium distribution of labor can be written as a log-linear function of the underlying geography:

$$\gamma_1 \ln L(i) = C_L + (\sigma - 1) \ln \bar{A}(i) + \sigma \ln \bar{u}(i) + (1 - 2\sigma) \ln P(i)$$

Implications:

When equilibrium is point-wise locally stable, population is increasing in $\bar{A}$ and $\bar{u}$.

Price index is a sufficient statistic for geographic location.

Conditional on price index, productivity and amenity spillovers only affect elasticity of $L(i)$ to geography.

Geographic trade costs

Suppose S is a compact surface (e.g. a line, plane, or sphere).

Let $\tau : S \to R_+$ be a continuous function, where $\tau(i)$ is the instantaneous trade cost of traveling over location $i \in S$.

Define the **geographic trade cost** $T(i,j) = f(t(i,j))$, $f' > 0$, $f(0) = 1$ to be the total iceberg trade cost incurred traveling along the least cost route from i to j, i.e.

$$t(i,j) = \min_{\gamma \in \Gamma(i,j)} \int_0^1 \tau(\gamma(t)) \, || \frac{d\gamma(t)}{dt} || dt \qquad (1)$$

where $\gamma : [0,1] \to S$ is a path and $\Gamma(i,j) \equiv \{\gamma \in C^1 | \gamma(0) = i, \gamma(1) = j\}$ is the set of all paths.

$f(t) = \exp(t)$ natural choice since $\prod_0^1 (1 + \tau(x) dx) = \exp\left(\int_0^1 \tau(x) dx\right)$, but can show T satisfied triangle inequality $\iff$ f is log subadditive.

Determining the optimal path

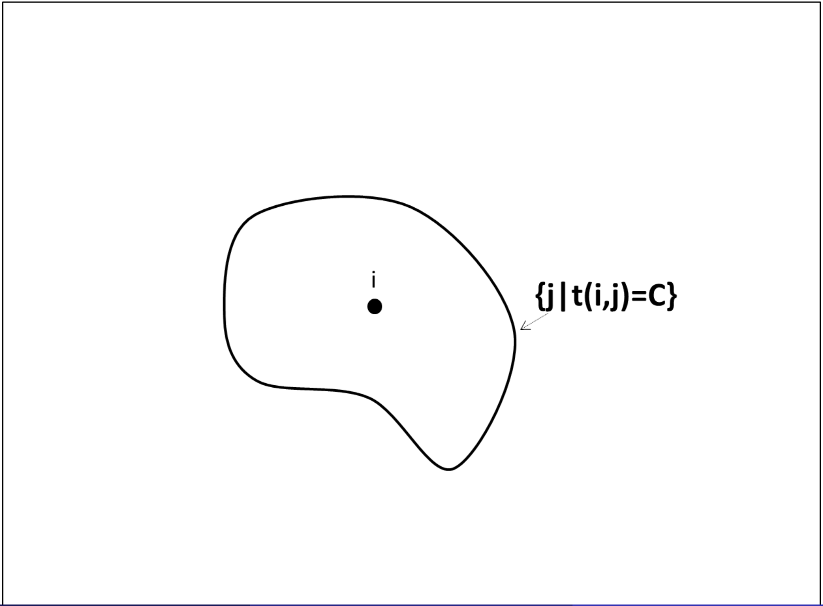
Equation (1) appears in a number of branches of physics. A necessary condition for its solution is the following *eikonal* equation:

$$||\nabla t\,(i,j)\,|| = \tau\,(j) \tag{2}$$

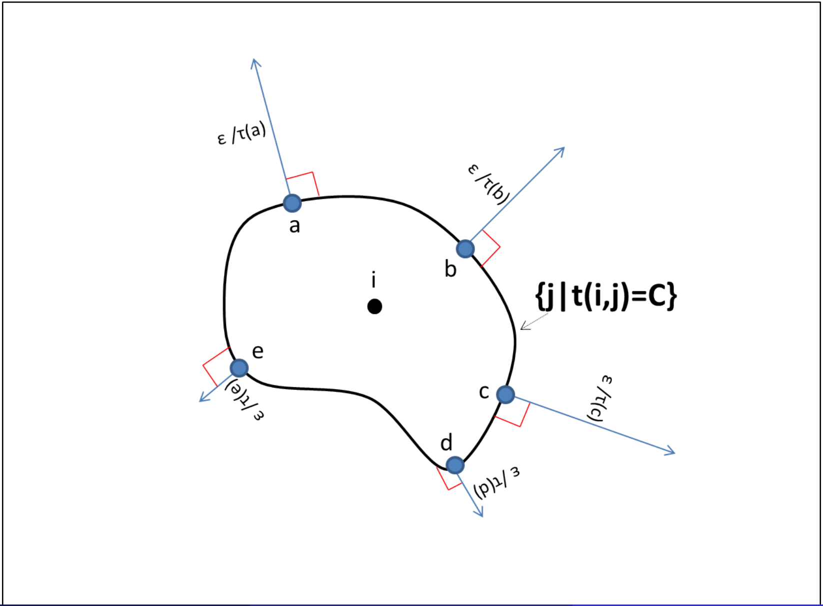
where the gradient is taken with respect to j.

Simple geometric interpretation: the trade cost contour expands outward in the direction orthogonal to the contour at a rate inversely proportional to the instantaneous trade cost.

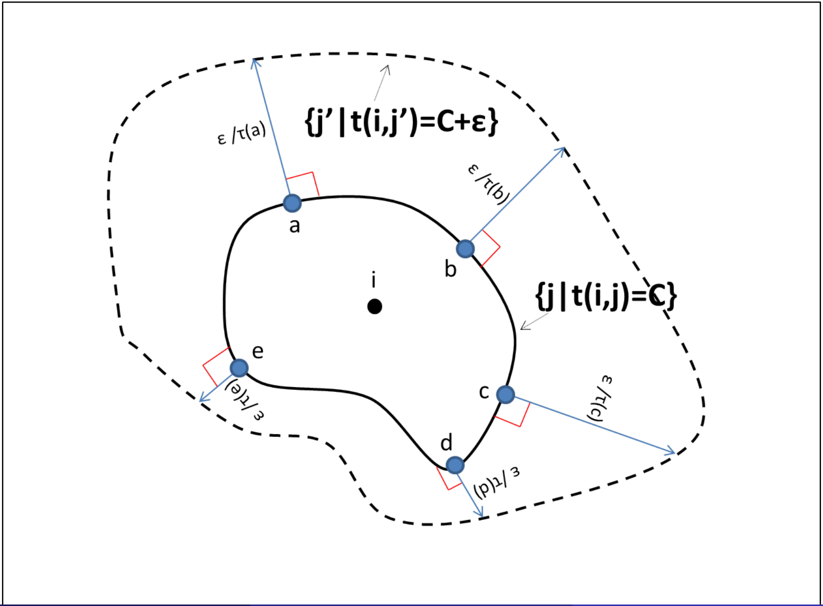
Eikonal Equation Illustrated



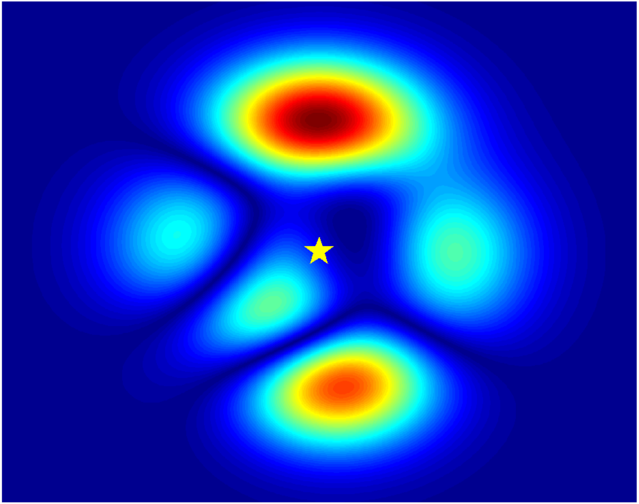
Eikonal Equation Illustrated



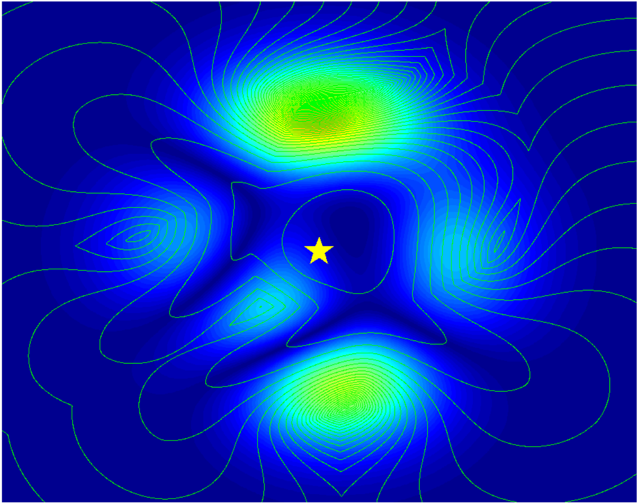
Eikonal Equation Illustrated



Eikonal Equation Illustrated



Eikonal Equation Illustrated



Trade and the topography of the spatial economy: Overview

Estimate the geography of the United States.

Estimate bilateral trade costs

Given trade costs, identify (composite) productivities and amenities

Quantify the importance of geographic location.

Perform counterfactual exercise: remove the Interstate Highway System.
Note: Cannot identify σ, α or β; they do analysis for a large variety of α and β, assume $\sigma = 9$.

Estimating bilateral trade costs

Goal: Find trade costs that best rationalize the bilateral trade flows observed in 2007 Commodity Flow Survey (CFS).

Three step process: Using **Fast Marching Method** (which operationalizes the Eikonal equation) and observed **transportation network**, calculate the (normalized) distance between every CFS area for each major mode of travel (road, rail, air, and water).

Using a **discrete choice** framework and observed **mode-specific bilateral trade shares**, estimate the relative cost of each mode of travel.

Using a **gravity** model and observed **total bilateral trade flows**, pin down normalization (and incorporate non-geographic trade costs).

Estimating trade costs

For any $i, j \in S$, suppose $\exists$ traders $t \in T$ choosing mode $m \in \{1, ..., M\}$ of transit where cost is:

$$\exp\left(\tau_m d_m(i,j) + f_m + \nu_{tm}\right)$$

Then mode-specific bilateral trade shares are:

$$\pi_m(i,j) = \frac{\exp\left(-a_m d_m(i,j) - b_m\right)}{\sum_k \left(\exp\left(-a_k d_k(i,j) - b_k\right)\right)},$$

where $a_m \equiv \theta \tau_m$ and $b_m \equiv \theta f_m$.
Combined with model, yields gravity equation:

$$\ln X_{ij} = \frac{\sigma - 1}{\theta} \ln \sum_m \left(\exp\left(-a_m d_{mij} - b_m\right)\right) + (1 - \sigma)\, \beta' \ln \mathbf{C}_{ij} + \delta_i + \delta_j$$

Estimate a_m and b_m using bilateral trade shares, θ using gravity equation.
Notes:
No mode switching.
Assume $f_{road} = 0$ to pin down scale.

Estimating A and u

Can we identify a topography of productivities A and amenities u consistent with the estimated T and observed distribution of economic activity (w and L)?

Yes (see Theorem 3 in the paper).

Intuition: consider locations a and b with identical bilateral trade costs, i.e. for all $s \in S$, $T(a,s) = T(b,s)$. Then:

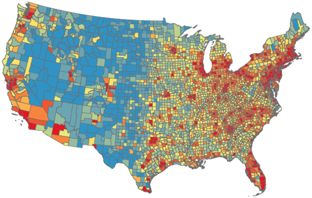
Utility equalization implies $\frac{u(b)}{u(a)} = \frac{w(a)}{w(b)}$.

Balanced trade implies $\frac{A(a)}{A(b)} = \left(\frac{L(a)w(a)^{\sigma}}{L(b)w(b)^{\sigma}}\right)^{\frac{1}{\sigma-1}}$.

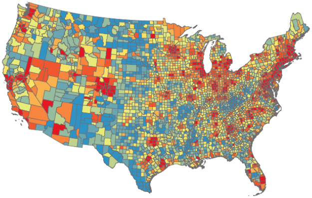
Note: $\bar{A}$ and $\bar{u}$ cannot be identified without knowledge of α and β.

US Application

Figure 12: United States population density and wages in 2000



Population density

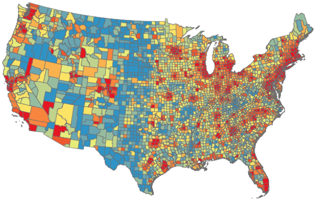


Wages

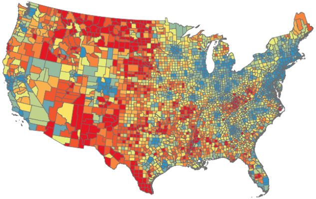
Notes: This figure shows the relative population density (top) and wages (bottom) within the United States in the year 2000 by decile. The data are reported at the county level; red (blue) indicate higher (lower) deciles. (Source: MPC (2011a)).

US Application

Figure 13: Estimated composite productivity and amenity



Composite productivity



Composite amenity

Notes: This figure shows the estimated composite productivity (top) and amenity (bottom) by decile. The data are reported at the county level; red (blue) indicate higher (lower) deciles.

Importance of geographic location

What explains the difference in economic activity across space?

Model yields following equilibrium relationship:

$$\ln Y(i) = C + \gamma_1 \ln \bar{A}(i) + \gamma_2 \ln \bar{u}(i) + \gamma_3 \ln P(i),$$

where $Y(i) \equiv w(i) L(i)$.

Apply a Shapley decomposition to determine what fraction of the variation in $\ln Y(i)$ is due to local characteristics (i.e. $\ln \bar{A}(i)$ and $\ln \bar{u}(i)$) and geographic location (i.e. $\ln P(i)$).

Do for all $\alpha \in [0,1]$, $\beta \in [-1,0]$ for robustness.

Removing the IHS: Cost-benefit analysis

Estimated annual cost of the IHS (interstate highway system): $\approx$ \$100 billion

Annualized cost of construction: $\approx$ \$30 billion (\$560 billion @5%/year) (CBO, 1982)

Maintenance: $\approx$ \$70 billion (FHA, 2008)

Estimated annual gain of the IHS: $\approx \$150 - 200$ billion

Welfare gain of IHS: $1.1 - 1.4\%$.

Given homothetic preferences and holding prices fixed, can multiply welfare gain by U.S. GDP.

Suggests gains from IHS substantially greater than costs.