Game Theory Assignment 1

Due on 14:00PM, March 31 (THU, 5th week), 2022

*(Questions with * are optional)*

Name Student ID

1. Consider a reduced-form of the "number game" we played in class: Two players, denoted by A and B. Each chooses one number from the strategy set $S_i = \{0, 1, 2\}, i = A, B$ simultaneously. The one who chooses the number that is closer to the 2/3 of average of the two numbers win the game. The payoff of wining and losing is 1 and 0, respectively. If they choose the same number, each gets 0.5.

   (1) Write down all the strategy profiles and the associated payoffs of each player, e.g., $v_i(s_A, s_B) = \cdots$ means when player A chooses s_A and B chooses s_B, then the playoff of player i is $\cdots$.

   (2) Plot the matrix representation of the game.

   (3) Is the (pure-strategy) Nash equilibrium of the game can be found by iterated elimination of strictly dominated pure strategies (IESDS)? If so, using IESDS to find the pure-strategy equilibrium. Please draw a separate matrix for each time when you eliminate one column/row.

   (4) Using the "best-response" approach to find the (pure-strategy) Nash equilibrium. E.g., underlining the payoff of player i for his/her best responses.

2. Recall the "battle of sexes" game described in class. Find the mixed-strategy Nash equilibrium of the game (of course when doing so, you will find that the pure-strategy equilibrium, if any, is (are) special case(s)).

3. Three firms are considering entering a new market. The payoff for each firm that enters is $\frac{150}{n}$, where n is the number of firms that enter. The cost of entering is 62. Not entering gives 0.

   (1) Find all the pure-strategy Nash equilibria.

   (2) *Find the symmetric mixed-strategy equilibrium in which all three players enter with the same probability.

4. Two roommates each need to choose to clean their apartment, and each can choose an amount of time $t_i \geq 0$ to clean. If their choices are t_i and t_j, then player i's payoff is given by $(10 - t_j)t_i - t_i^2$.

   (1) What is the best response correspondence of each player?

   (2) What is the Nash equilibrium (pure-strategy) of the game?

5. Consider the "tragedy of the commons" example discussed in class. Now assume there are n players, and the payoff of player i is $\ln(k_i) + \ln(K - \sum_{i=1}^{n} k_i)$.

   (1) Solve the pure-strategy Nash equilibrium.

   (2) How does the Nash outcome compare to the socially efficient outcome as n approaches to infinity?

6. * Imaging there are two politicians, each caring only about being elected. There are one mass of citizens and their political preference are represented by the $[0, 1]$ interval: e.g., the point 0 can be interpreted as a "left" leaning citizen and point 1 can be interpreted as a "right" leaning citizen. Assume that all citizens are uniformly distributed among the $[0, 1]$ line. For the two politicians, each candidate proposes a policy, denoted by a and b, and $a, b \in [0, 1]$. Each citizen votes for the candidate who proposes a policy that is closer to his/her own political preference—for example, if $0 < a < b < 1$, then $\left[0, \frac{a+b}{2}\right]$ will vote for a and the remaining ones will vote for b (if $a = b$, then each citizen flips a coin to decide). The two candidate proposes their policies simultaneously, and the outcome is determined by the majority rule. Show that the unique pure-strategy Nash equilibrium is $a = b = \frac{1}{2}$. (Assume that when each candidate gets half of the votes, then each wins with half probability)