Public Finance: preference: $w^i = c^i + H(g)$ ← by consumption & gov spending

$H' \geq 0 \quad H'' \leq 0$

spending by taxing income, τ: $c^i = (1-\tau)y^i$

$g^i = \tau y^i$. no investment

$E[y^i] = y$. $F(y^M) = \frac{1}{2}$ y^M = median of y^i.

$$W^i(g) = c^i + H(g) = (1-\tau)y^i + H(g) = \frac{y-g}{y} \cdot y^i + H(g)$$

$$\frac{\partial W^i(g)}{\partial g} = 0 \Rightarrow \frac{y^i}{y} = H'(g) \rightarrow g^i = H_g^{-1}\left(\frac{y^i}{y}\right) \rightarrow \frac{\partial g^i}{\partial y^i} < 0. \quad g^i \text{ decrease in } y^i.$$

Total welfare $w = \int W^i(g)\, dF = \int \left[(y-g)\frac{y^i}{y} + H(g)\right] dF$

$$= (y-g) + H(g) = W(g)$$

Probabilistic Voting: other dimension of candidates 非g (policy) 方面.

Rich, Middle, Poor. R, m, P Groups: personality, ethnic, etc.

$\underset{J}{=}$

$y^R > y^M > y^P$. population share α^J. $\sum_J \alpha^J = 1$, $\sum_J \alpha^J y^J = y$ ← avg income

Vote base on policy & ideology: g & σ.

$i \in J$ vote for A if $W^J(g_A) > W^J(g_B) + \sigma^{iJ} + \delta$.

σ^{iJ}: i's biase to B other than policy, $\sigma^{iJ} > 0$: prefer B. in personality etc.

$E[\sigma^{iJ}] = 0$, uniform distributed on $\left[-\frac{1}{2\phi^J}, \frac{1}{2\phi^J}\right]$

density ϕ^J. δ: avg relative popularity of B. unif d. $\left[-\frac{1}{2\psi}, \frac{1}{2\psi}\right]$.

density ψ.

Events: (1). Candidates announce g_A, g_B independently, and they know voter's preference, σ^{iJ} and δ's distribution, but not yet the realization

(2) real value of δ is realized.

(3) Elections

Swing Voter: $\sigma^J = W^J(g_A) - W^J(g_B) - \delta$

all $i \in J$ with $\sigma^{iJ} \geq \sigma^J$ prefer B

indifferent

Vote share for A: $\pi_A^J = \frac{1}{2} + \sigma^J \phi^J = \left[\sigma^J - \left(-\frac{1}{2\phi^J}\right)\right]\phi^J$

As σ^J depends on future realization of δ. π_A^J is RV for both A & B.

$\pi_B^J = \frac{1}{2} - \sigma^J \phi^J = \frac{1}{2} + \phi^J \cdot \left[W^J(g_B) - W^J(g_A) + \delta\right]$

Total vote share is weighted avg of vote shares across all J.

$$\pi_B = \sum_J \alpha^J \pi_B^J = \sum_J \alpha^J \cdot \left\{\frac{1}{2} + \phi^J\left[W^J(g_B) - W^J(g_A) + \delta\right]\right\}.$$

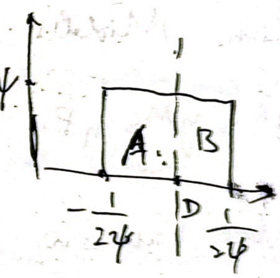
$$= \frac{1}{2} + \delta \cdot \sum_J \alpha^J \phi^J + \sum_J \alpha^J \phi^J \cdot \left[W^J(g_B) - W^J(g_A)\right].$$

Prob of B winning is $\mathbb{P}[\pi_B \geq \frac{1}{2}]$

$$P_B = \underset{\delta}{P}\left\{\delta \sum_J \alpha^J \phi^J \geq \sum_J \alpha^J \phi^J \left[W^J(g_A) - W^J(g_B)\right]\right\}$$

Denote $\phi = \sum_J \alpha^J \phi^J$ the weighted avg density across groups.

$$\Rightarrow P_B = \underset{\delta}{P}\left\{\delta \geq \underbrace{\frac{1}{\phi}\sum_J \alpha^J \phi^J \left[W^J(g_A) - W^J(g_B)\right]}_{D}\right\}$$



$$P_B = \frac{1}{2} - \psi D = \frac{1}{2} + \frac{\psi}{\phi}\sum_J \alpha^J \phi^J \left[W^J(g_B) - W^J(g_A)\right]$$

$P_B = \left(-D + \frac{1}{2\psi}\right) \cdot \psi$
$= \frac{1}{2} - \psi D$

$$P_A = \frac{1}{2} + \frac{\psi}{\phi}\sum_J \alpha^J \phi^J \left[W^J(g_A) - W^J(g_B)\right]$$

→ Prob of winning is smooth function of distance between electoral policies

→ competition becomes less stiff

Independently, A & B wants to maximize prob of winning.

$$g_B = \max_g \sum_J \alpha^J \phi^J W^J(g) = g_A$$ — Nash Equilibrium, only one.

Because they have same optimization problem, same concave preferences & same tech for taxing ⟹ Same policy

In Eq, they max. a weighted Welfare function

P2

$\phi^J \alpha^J$ — is the weight where $\begin{cases} \alpha^J & \text{~~utilitarian optimum~~, also group size} \\ \phi^J & \text{group density — summarize how responsive are voters in each group} \end{cases}$

$$\max_g \sum_J \alpha^J \phi^J W^J(g)$$

$$= \max_g \sum_J \alpha^J \phi^J \left[(y-g)\frac{y^J}{y} + H(g) \right]$$

FOC w.r.t. g: $H'(g)\sum \alpha^J \phi^J = \frac{1}{y} \sum_J \alpha^J \phi^J y^J$, as $\phi = \sum_J \alpha^J \phi^J$

$\Rightarrow g^S = H_g^{-1}\left(\frac{\tilde{y}}{y}\right)$; $\tilde{y} = \frac{\sum_J \alpha^J \phi^J y^J}{\phi}$, is the swing-voting equilibrium

Can be socially optimal is ϕ^J is the same in all groups

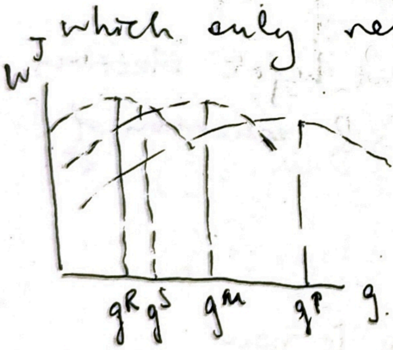
$\Rightarrow \phi^J = \phi$. $\tilde{y} = y$. $g^S = g^*$, if ϕ^J is high $\rightarrow$ group homogenous to ideology $\rightarrow$ more swing voters.

Eg. $\phi^R > \phi^M > \phi^P$. poor have fewer swing voters

$\Rightarrow \tilde{y} > y \longrightarrow g^S < g^*$. Smaller size of gov. than utilitarian optimum.

$\longrightarrow$ Opposite median-Voter Model. which says larger inequality $\rightarrow$ larger gov.

which only restore is poor have strongest political clout.



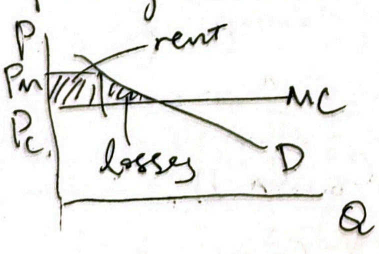
$\rightarrow$ Neutral groups with many swing voters becom attractive target.

$\rightarrow$ please more mobile voters than median voters

Rent-Seeking $\rightarrow$ invest energy, resources for obtaining rents & benefits related to the state.

Monopoly rent $\Rightarrow$ TV licenses. etc.

Primary seekers: interest groups (Lobbies) — unions. farmers' association etc.



3 types of 寻租 expenditure. socially wasteful.

① Efforts & expend. of potential recipients of monopoly

② eff of gov. to obtain/react to expend of ①

③ 3-party distortion.

Olson: Why some ... more prone to lobbying than others?

Groups more likely to be stable & avoid free-riding if

- ↓ membership; private goods to member; homogeneous

low organization costs; potential payoff in case of success large for certain

lobbying 3.1 Prob Voting Model.

groups derive power 100% from attributes to voters.

well-defined interest groups may influence through other forms.

⟶ 8.x campaign contributions by interest groups (Baron 1994)

Assume all groups have same density $\phi^J = \phi$. → socially optimal.

Groups may be organized in a lobby $O^J = \begin{cases} 1 & \text{group J organized} \\ 0 & \text{not organized} \end{cases}$

Organized can contribute to campaign of A or B.

C_P^J is contribution per member of group J to candidate $P = A$ or B.

Total contribution to Candidate P: $C_P = \sum_J O^J \alpha^J C_P^J$

Contributions paid after g^A & g_B announced but before elections & realization of δ.

↳ finance campaign spending

affect relative popularity.

Avg relative popularity of party B (previously δ), is now

$$\delta = \tilde{\delta} + h \sum_J O^J \alpha^J (C_B^J - C_A^J)$$

$\tilde{\delta}$: distributed uniformly with density ψ. $\left[-\frac{1}{2\psi}, \frac{1}{2\psi}\right]$

h: effectiveness of campaign spending.

The Swing voter in Group J becomes:

$$\sigma^J = W^J(g_A) - W^J(g_B) - \left[\tilde{\delta} + h \sum_J \alpha^J O^J (C_B^J - C_A^J)\right]$$

For prob of winning $\phi^J = \phi$. $\delta \to \tilde{\delta} + h \sum_J O^J \alpha^J (C_B^J - C_A^J)$

$$P_B = \mathbb{P}_{\delta}\left\{ \delta + h\sum_J O^J\alpha^J(C_B^J - C_A^J) \geq \frac{1}{\phi}\sum_J \alpha^J \phi\left[W^J(g_A) - W^J(g_B)\right] \right\}$$

$$= \frac{1}{2} + \psi\left\{ \sum_J \alpha^J\left[W^J(g_B) - W^J(g_A)\right] + h\sum_J O^J\alpha^J(C_B^J - C_A^J) \right\}.$$

$$P_A = \frac{1}{2} + \psi\left\{ \sum_J \alpha^J\left[W^J(g_A) - W^J(g_B)\right] + h\sum_J O^J\alpha^J(C_A^J - C_B^J) \right\}$$

Then. Backward induction:

Stage 2: how lobbies choose campaign contributions

Stage 1: how candidates choose policy g_A. g_B

Stage 2. Group J $\longrightarrow$ max. $\mathbb{E}[\text{Utility}]$ − cost

$$\max \; P_A W^J(g_A) + \underbrace{P_B}_{=(1-P_A)} W^J(g_B) - \frac{1}{2}\left[(C_A^J)^2 + (C_B^J)^2\right]$$ w.r.t.

here ideology plays no role.

$$\text{s.t.} \quad P_A = \frac{1}{2} + \psi\left\{ \sum_J \alpha^J\left[W^J(g_A) - W^J(g_B)\right] + h\sum_J O^J\alpha^J(C_A^J - C_B^J) \right\}$$

FOC w.r.t. C_A^J: $\psi \cdot h \cdot O^J \alpha^J \left[W^J(g_A) - W^J(g_B)\right] - C_A^J$

$$\Rightarrow C_A^J = \max\left\{0,\ \psi h O^J\alpha^J\left[W^J(g_A) - W^J(g_B)\right]\right\}$$
$$C_B^J = \max\left\{0,\ \psi h O^J\alpha^J\left[W^J(g_B) - W^J(g_A)\right]\right\}.$$
as $C_A^J \geq 0$, $C_B^J \geq 0$.

So. each organized group only contributes to who gives the group highest utility

因为非0项至多有一个. 因此必须一个为0. → 只contribute 1个人.

Stage 1: announcing g_A. g_B 会consider C_A^J. C_B^J.

→ By symmetry $\longrightarrow$ convergence to same equilibrium policy.

$$P_A = \frac{1}{2} + \psi \cdot \left\{ \sum_J \alpha^J\left[W^J(g_A) - W^J(g_B)\right] + h \cdot \sum_J O^J\alpha^J \left[\max\left\{0,\ \psi h\left[W^J(g_A) - W^J(g_B)\right]\right\} - \max\left\{0,\ \psi h\left[W^J(g_B) - W^J(g_A)\right]\right\}\right] \right\}$$

$$= \frac{1}{2} + \sum_J \alpha^J \cdot \left[\psi + O^J(h\psi)^2\right] \cdot \left[W^J(g_A) - W^J(g_B)\right]$$ taking g_B as given

$$P_B = \frac{1}{2} + \quad - - - \quad \left[W^J(g_B) - W^J(g_A)\right]$$ taking g_A as given.

Pr

Each candidate solves $\max_{g} \sum_{J} \alpha^J [\psi + (h\psi)^2 O^J] \cdot W^J(g)$.

If $O^J = 1$ for all J, or $O^J = 0$ for all J $\rightarrow$ eq 3 utilitarian optimum 口 g^*.

$\rightarrow$ Departures arise when only subset of groups organized.

at Eq. no contributions are paid $\rightarrow$ organized groups extract all rents.

For Public finance.

$$\max_{g} \sum_{J} \alpha^J [\psi + (h\psi)^2 O^J] \cdot W^J(g)$$

$$= \max_{g} \sum_{J} \alpha^J [\psi + (h\psi)^2 O^J] \cdot \left[(\bar{g} - g) \frac{y^J}{y} + H(g) \right]$$

FOC w.r.t. g $\rightarrow$ $\sum_J \alpha^J [\psi + (h\psi)^2 O^J] H(g) = \sum_J \alpha^J [\psi + (h\psi)^2 O^J] \cdot \frac{y^J}{y}$

$g^{lobby} = H_g^{-1}\left(\frac{\hat{y}}{y}\right)$. $\hat{y} = \dfrac{\sum_J \alpha^J (1 + h^2 \psi O^J) y^J}{\sum_J \alpha^J (1 + h^2 \psi O^J)}$

When all groups are organized $\hat{y} = \frac{\sum \alpha^J y^J}{\sum \alpha^J} = y$. $g^{LOB} = g^* = H_g^{-1}(1)$

Otherwise, organized groups $\rightarrow$ higher weights $\rightarrow$ 偏向 organized groups 利益

E.g. Only rich organized $\rightarrow$ $\hat{y} > y$. $\rightarrow$ $g^{LOB} < g^*$. $\rightarrow$ smaller than g^*

$\rightarrow$ spending is smaller if R's stake in policy is larger: y^R 相对 y 更高.

$\rightarrow$ more effective is campaign spending in swaying the vote (higher h).

In Prob Voting Model, income inequality $\rightarrow$ smaller gov.

Candidates only want to win $\rightarrow$ bias towards lobbies help them.

$\rightarrow$ Organized Groups have more influence: 贸易

$\rightarrow$ largest stake in policy 更易 organized $\rightarrow$ rich & poor vs middle 更好 organize.

Strong organizations can influence general policy than candidates' bias.

$\boxed{\text{Less symmetric} \longrightarrow \text{increase policy distance}}$

Partisan Politicians.

So far. Candidates driven by exoge rent R. of public office.

but → policy outcomes directly motivate them.

Public Finance. income continuously distributed. set rent $R=0$

$\boxed{\text{Motivate to win is to implement desired policy}}$

For voter with income y^i. policy preference $W^i(g)$.

2 Exoge candidate $P = L, R$ (Citizen - Candidate Model)

P's income y^P : $y^L < y^m < y^R$. left wing & right wing.

Assume : announce. g_P. → elections → Implement winner's policy

$$P_L = \begin{cases} 0 & W^m(g_L) < W^m(g_R) \\ \frac{1}{2} & W^m(g_L) = W^m(g_R) \\ 1. & W^m(g_L) > W^m(g_R) \end{cases}$$

[3] median-voter model.

When setting policy : L max utility $\mathbb{E}[W^L(g)] = P_L W^L(g_L) + (1-P_L) W^L(g_R)$

given R's platform g_R. while R solves symmetric

For L. given policy $g_R < g^m$.

① Centrifugal 离心 force. rise utility (conditional on winning) by $g_L \uparrow$ toward own bliss point.

② Centripetal 向心. increase chance of winning by $g_L \downarrow$ to median voter's preference.

Optimal : decrease g^L just enough from bliss point to raise P_L to unity but not any further

Same for R → $g_L^* = g^m = g_R$ [3] Equilibrium.

Assume Candidate chose preelectoral announcement. $P \in \{L, R\}$ will

$g_P = g^P = H_g^{-1}\left(\frac{y^P}{y}\right)$. , no other point have credibility.

Monotonicity of g in y^i →. median voter pivotal.

L wins if $W^m(g_L) > W^m(g_R)$ ... vice versa.

→ R-wing gover choose lower spending & taxes than L-wing gov.

We can interpret shifts in power as reflecting shifting positions of candidates / or of the electorate 可以政权解释为候选人/选民之偏变ee.

As candidates & pivotal voters have concave $H(g)$, all have long-run preference for stable policy in middle than 两极政策.

Repeated elections → parties can coordinate on self-enforcing cooperative equilibrium with middle-ground policies

endogenous candidates → centrist politicians may arise.

Enfranchisement matters for policies adopted $\xleftarrow{\text{可由}}$ Downsian electoral competition 解释.

Citizen-Candidate Models → Identity of elected politician matters

Q: We assume exog. candidates what if endog ?

- Small partisan effect.
- gender representation
- Disadvantaged minorities representation

May arise centrist politicians → middle of policy spectrum identities

Assume: citizen can enter as a candidate at cost ε

Citizens vote for candidate who max expected utility given other citizens' votes

→ over half wins → 平票 flip coin. elected policy g_P. nobody runs $\bar{g}$ (defaults)

Stage 3. No commitment → elected citizen with income y^P set policy $g_P = H_g^{-1}\left(\frac{y^P}{y}\right)$ to max own utility

Stage 2. Election, given politicians' expected choice & voters monotonic preferences over policy. voters have monotonic pref over candidates

Stage 1. Decide enter or not, foreseeing voters' behavior at S2.

Enter if expected utility > ε given other citizens' decisions

Multiple Eq: 1-candidate, 2-candidates, multiple-candidates

2 conditions for pure-strategy Eq: i must willing to run given J running

Entry-proofness: no other candidate than i & J are willing to enter.

Case 1: 1-candidate

median candidate m enters, y^m win → set policy g^m.

$\boxed{\text{Condorcet winner}}$ no other find worthwhile to incur entry cost

no other with y^m enter, as only incur entry cost without influencing policy

when m runs: { uncontested candidate / in an election with ≥ 2 candidates. 2选一.

Condition for Eq: $W^m(g^m) - W^m(\bar{g}) \geq \varepsilon$, → $\bar{g}$ 要 far enough from g^m

Confirms: { Centripetal pull endogenous candidate toward middle / predicted policy outcome = median-voter eq.

Case 2: 2-candidate, R & L.

y^R must find it worth to run given L is running, vice versa.

→ each candidate must stand some chance of winning → $W^m(g_L) = W^m(g_R)$

$$\tfrac{1}{2}[W^R(g_R) - W^R(g_L)] \geq \varepsilon \ ; \ \tfrac{1}{2}[W^L(g_L) - W^L(g_R)] \geq \varepsilon$$

y^P apply policy $g_P = H_g^{-1}\left(\frac{y^P}{y}\right)$.

Can be fulfilled if g_L, g_R on both sides of y^m, and that policy pref are different enough s.t. expected utility gain outweight cost.

If $y^L < y^i < y^R$ enters (i), he only captures y^m given all other voters stick to equilibrium voting strategies → i won't enter.

Majoritarian (赢者通吃) representation or electoral quorums (选举人数) → barriers distort pref

Adv: facilitate gov formation given fractionalization

Disadv: preference distortion

minimum threshold required for a party to enter parliament.

P9

Preferences also manipulated in illicit ways:

① Vote buying — 'pork barrel politics' gov spending for local projects to bring back political support

② 'Gerrymandering' — redistrict constituencies for electoral advantages
重划选区，以获得选民支持优势.

③ 'logrolling': voters cooperate, 'trade' support,
vote for projects dislike to get support for own projects
→ excess spending.

War as bargaining failure

2 States A, B preference interval $X=[0,1]$
Fix: A prefer close to 1, B prefer close to 0.
Potential outcomes $x \in X$, utility functions $u_A(x)$, $u_B(1-x)$. C^1. $u'>0$, $u''\leq 0$ risk neutral & risk averse
$u_i(0)=0$, $u_i(1)=1$ $i=A,B$
In case of war, A prevails with $p\in[0,1]$, winner grabs everything
B prevails $1-p\in[0,1]$
Cost of war $C_A, C_B > 0$.

$E[u_A] = p\,u_A(1) + (1-p)\,u_A(0) - C_A = p - C_A$

$E[u_B] = (1-p)\,u_B(1) + p\,u_B(0) - C_B = 1-p-C_B$

$\therefore \exists\, X^0 \subset X$ s.t. $\forall x \in X^0$. $u_A(x) > p - C_A$, $u_B(1-x) > 1-p-C_B$

Risk-neutral $u_A(x)=x$ $u_B(1-x)=1-x$

Both states strictly prefer any peaceful agreement in interval $(p-C_A, p+C_B)$ to fighting.

Commitment problem: incentives to renege on peace deals

① preemptive war & offensive advantage: $\underset{\substack{\uparrow \\ \text{first strike winning prob}}}{p_f} > p > p_s$

② A's winning prob ↑. lack of credible commitment.

③ objects bargain about can be source of military power

Powell: commitment problems & informational problems are main source of bargaining failure.

indivisibilities or risk-acceptance can be reduced to commitment problems

War — costly lottery.. all players better off agreeing to equiv. costless lottery to avoid fighting if commitment is possible.

Political Bias. Conflict $C_A, C_B > 0$. but who decide on war are not who bear the cost.

Jackson, Matthew. Morelli 2007: interaction between {domestic pol / econ incentive} to determine war.

Findings. Unbiased countries - acceptable transfers to avoid
biased countries — strong enough bias, cannot avoid

Simultaneous game: country i, j {war, no war}. W_j — wealth of j.

Tech of war → $P_j(W_j, W_i) = P_{ji}$. prob that j wins. $P_{ij} = P(i \text{ wins})$

$P_{ij} = 1 - P_{ji}$. Cost for j: $C \times W_j$. (independent of success)

Benefit: $G \times W_i$ upon j winning.

Assume $C + G \leq 1$

j's wealth losing — $W_j(1 - C - G)$. → positive wealth level (even lose war).

j's wealth winning — $W_j(1-C) + GW_i$.

a_j is fraction of W_j controlled by pivotal agent in j's decision

a_j': fraction of spoils 战利品 of war for pivotal agent.

If no transfers. pivotal agent in j. goes to war iff:

$$(1-C)a_j W_j - (1-P_{ji})a_j G W_j + P_{ji} a_j' G W_i > a_j W_j.$$

Political bias of j: $B_j = \dfrac{a_j'}{a_j}$. share of benefit / share of cost

$B_j \to 1$: similar. $B_j > 1$ → country leader has positive bias

Unbiased leader: $B_j = 1$.

$(1-C)a_j w_j - (1-P_{ji}) a_j G w_j + P_{ji} a_j' G w_i > a_j w_j$.

$\Rightarrow \quad P_{ji} B_j w_i > \left[\frac{C}{G} + (1-P_{ji})\right] w_j$.

Biased leader with larger potential gains when $B_j > 1$

Incentives for war: $B_j \uparrow$. $G \uparrow$. $C \downarrow$.

depend on $\frac{C}{G}$ not on absolute values

depend on B_j. not absolute values of a_j', a_j.

effects of wealth are ambiguous & depend on tech of war.

Eg 1. proportional prob of winning $P_{ji} = \frac{w_j}{w_i + w_j}$

$\Rightarrow \quad \frac{(B_j - 1) G w_i}{w_i + w_j} > C$

politically unbiased countries never goes to war

$B_j > 1$. then tendency of war $\uparrow$ as $w_i \uparrow$

$\downarrow$ as $w_j \uparrow$.

Eg 2. Fixed prob $P_{ji} = \frac{1}{2}$.

$\Rightarrow \quad B_j \cdot \frac{w_i}{w_j} > 1 + \frac{2C}{G}$.

unbiased country could want war. only if wealth relatively low.

Eg 3. Higher wealth wins $P_{ji} = \begin{cases} 1 & w_j > w_i \\ 0.5 & w_j = w_i \\ 0 & w_j < w_i \end{cases}$

j goes to war iff. $w_j > w_i$ and $B_j G w_i > C w_j$.

Scene: no transfer. one would go war. one not. transfer. avoid a war.

commit to peace | conditional transfer.

Assume: transfer from i to j. j get a_j'. i loses a_i. for decision maker

We want to find transfer. avoid a war. i pays j t_{ij}.

j wants war, i not: $\begin{cases} P_{ji} > \frac{1 + \frac{C}{G}}{1 + B_j w_{ij}} \\ 1 - P_{ji} > \frac{1 + \frac{C}{G}}{1 + B_i w_{ji}} \text{ doesn't hold} \end{cases}$ $w_{ij} = \frac{w_i}{w_j}$, $w_{ji} = \frac{w_j}{w_i}$, If transfer $t_{ij} > 0$ made $\longrightarrow$ avoid war, then

$$(1-C)a_j w_j - G a_j w_j + P_{ji} G(a_j w_j + a_j' w_i) \le a_j w_j + a_j' t_{ij}.$$

$$\Rightarrow P_{ji} G(w_j + B_j w_i) \le (C+G) w_j + B_j t_{ij}.$$

For i willing to pay to avoid.

$$(1-P_{ji}) G(w_i + B_i w_j) \le (C+G) w_i - t_{ij}.$$

Let $t_i^P(B_i)$ max transfer i is willing to pay to avoid war.

$$t_i^P(B_i) = (C+G) w_i - (1-P_{ji}) G(w_i + B_i w_j)$$

$$\Rightarrow P_{ji} G(w_j + B_j w_i) \le (C+G) w_j + B_j (C+G) w_i - B_j (1-P_{ji}) G(w_i + B_i w_j)$$

Combine with $P_{ji} > \dfrac{1+\frac{C}{G}}{1+B_j w_{ij}}$.

$$\Rightarrow \underbrace{P_{ji}(1+B_j w_{ij}) - 1 > \frac{C}{G}}_{j \text{ want war without transfer}} > \underbrace{\frac{(1-P_{ji})(B_i B_j - 1)}{1+B_j w_{ij}}}_{i \text{ wants to avoid war by transfer}}$$

range of $\dfrac{\text{costs}}{\text{gains}}$, $\dfrac{C}{G}$ where transfer can avoid war, when.

$B_i \uparrow$, $P_{ji} \uparrow$, and $\dfrac{w_i}{w_j} \uparrow$ (P_{ji} fixed).

Unbiased conditions $B_i = B_j = 1$ never goes to war if can transfer, to each other, & receiver commit no war. RHS=0. willing to buy off.

If. Cannot commit no war. $\rightarrow$ after transfer. war is not of interest

① make target poorer, less appealing
② make challenger richer. more to lose.
③ increase prob challenger will win.

countervailing effect. if prob not increased much $\rightarrow$ may avoid war.

Case of no-commitment is strict subset of commitment case

Difference. Commitment: j compares value of no war (wealth plus transfer) to what they would gain from war without transfer

No commitment: j compares value of no war to what they would gain from war after transfers have been made.

Democracies rarely go to war — 2 unbiased countries can find transfers to avoid war.

War as rent-seeking: Take conflict as given, focus on how many resources devoted to "appropriative activities" in equilibrium.

→ explain "intensity" or duration of conflict.

Assume 2 risk-neutral players: fight to appropriate a prise R

Each face a time constraint: $f + l = 1$, f – fighting. l – labor

Payoff: $\pi_i = P_i(f_i^*, f_j^*)R + w_i(1 - f_i^*)$ * – eq level.

$\pi_j = (1 - P_i(f_i^*, f_j^*))R + w_j(1 - f_j^*)$ P_i = prob of i winning

w = wage

Contest success function: $P_i = \dfrac{\rho_i f_i^*}{\rho_i f_i^* + \rho_j f_j^*}$. ρ_i is fighting tech for i.

Take FOC w.r.t f.

$$\begin{cases} \dfrac{\partial \pi_i}{\partial f_i} = R \cdot \dfrac{\rho_i(\rho_i f_i + \rho_j f_j) - \rho_i(\rho_i f_i)}{(\rho_i f_i + \rho_j f_j)^2} - w_i = 0 = R\dfrac{\rho_i \rho_j f_j}{(\rho_i f_i + \rho_j f_j)^2} - w_i = 0 \\ \dfrac{\partial \pi_j}{\partial f_j} = R\dfrac{\rho_i \rho_j f_i}{(\rho_i f_i + \rho_j f_j)^2} - w_j = 0 \end{cases}$$

$$\Rightarrow f_j = f_i \cdot \frac{w_i}{w_j}.$$

⇒ Nash Equilibrium

$$\begin{cases} f_i^* = \dfrac{\rho_i \rho_j w_j R}{(\rho_j w_i + \rho_i w_j)^2} \\ f_j^* = \dfrac{\rho_i \rho_j w_i R}{(\rho_j w_i + \rho_i w_j)^2} \end{cases} \Rightarrow P_i = \frac{\rho_i f_i^*}{\rho_i f_i^* + \rho_j f_j^*} = \frac{\rho_i w_j}{\rho_i w_j + \rho_j w_i}$$

$$\Rightarrow \begin{cases} \pi_i = R\dfrac{\rho_i^2 w_j^2}{(\rho_j w_i + \rho_i w_j)^2} + w_i \\ \pi_j = R\dfrac{\rho_j^2 w_i^2}{(\rho_j w_i + \rho_i w_j)^2} + w_j \end{cases} \longrightarrow \text{Waste of war} = R \cdot \frac{2\rho_i \rho_j w_i w_j}{(\rho_j w_i + \rho_i w_j)^2}$$

掠夺,侵占

More appropriation when prize R is larger, and when opportunity cost of fighting w are small

When $w_i = w_j$. $\Rightarrow$ both have same fighting efforts $f_i^* = f_j^*$ $\Rightarrow$ Success of war fully determined by tech $p_i = \frac{\beta_i}{\beta_i + \beta_j}$

Hirshleifer's 'Paradox of Power'

Put $\beta_i = \beta_j$. $w_i < w_j$. then $f_i^* > f_j^*$, $P_i = \frac{w_j}{w_i + w_j} > 0.5$. the poorer player fights harder and thus has better chances of winning.

Ethnic polarization: $\text{Polarization} = 1 - \sum_{i=1}^{N} \left(\frac{\frac{1}{2} - \pi_i}{\frac{1}{2}} \right)^2 \pi_i$.

π_i is proportion of people who belong to ethnic group i, N is number of groups. $\sum_i \pi_i = 1$.

This index captures how far the distribution is from $(\frac{1}{2}, 0, \cdots, 0, \frac{1}{2})$. bipolar, which is highest level of polarization, only 2 groups of similar size

Esteban & Ray. 2008 AER Salience of Ethnic Conflict

Class conflict (Rich vs. Poor) has been less salient than ethnic conflict (Hindu vs. Muslim)

Contest for public goods. $\rightarrow$ control more budget for public goods for own's interest

For pure public goods, population sizes of groups don't matter

Assume. no alliance no conflict: { class alliance $\rightarrow$ Class budget conflict. / Ethnic alliance $\rightarrow$ ethnic budget conflict.

Result: most parameter values the conditions of conflicts against class conflict & for ethnic conflict

$\rightarrow$. It's the complementarity between larger Financial Resources (rich) and lower opportunity cost of time (poor) that makes ethnic alliances ~~more successful than class alliances~~

Ethnic polarization associated with conflict. – Causal link? – Different.

General Eq of conflict.

labor-intensive appropriative activities in canonical models of trade.

→ predict on effect of price shocks in K or L-intensive sectors on incentives for conflicts, and how conflict affects economic structure.

2×2. sector 1 & 2. $\tilde{K}$ & L. price. r & w. 1 - K intensive than 2.

endowments $\bar{K}$ & $\bar{L}$. prices internationally determined. $p_1 \equiv p$. for 1. $p_2 = 1$. (numeraire)

Another appropriation sector that uses only labor L_A. with $A(L_A)$ $A(0) \geq 0$. $A' > 0$

amounted appropriated is appropriation $A(\bar{L}) \leq 1$. $A'' < 0$

$$A(L_A)(pq_1 + q_2) = A(L_A)[r\bar{K} + w(\bar{L} - L_A)]$$

a_{ij} is the amount of input j used to produce 1 unit of output i at minimum cost

Given p. $\bar{K}$, $\bar{L}$. eq determines r, w. q_1. q_2 and use of factors K_1. K_2. L_1. L_2.

Zero Profit condition: $r a_{1K} + w a_{1L} = p$ Factor clearing $q_1 a_{1K} + q_2 a_{2K} = \bar{K}$

$r a_{2K} + w a_{2L} = 1$. $q_1 a_{1L} + q_2 a_{2L} = \bar{L} - L_A$

No arbitrage condition (same return) in appropriation & production

$$\frac{A(L_A)}{L_A}[r\bar{K} + w(\bar{L} - L_A)] = [1 - A(L_A)]w.$$

Prop. Existence of appropriation sector makes owners of K & L worse off.

Gross rental prices unchanged but net rental prices: $\begin{cases} [1 - A(L_A)]r < r. \\ [1 - A(L_A)]w < w. \end{cases}$

Prop: Appropriation sector $\begin{cases} \text{increase production of K-intensive good} \\ \text{reduces production of } \cancel{L}\text{-intensive good} \end{cases}$

$$\begin{cases} q_1 = \dfrac{a_{2L}\bar{K} - a_{2K}(\bar{L} - L_A)}{a_{1K}a_{2L} - a_{1L}a_{2K}} \\ q_2 = \dfrac{a_{1L}(\bar{L} - L_A) - a_{1L}\bar{K}}{a_{1K}a_{2L} - a_{1L}a_{2K}}. \end{cases}$$

$L_A \uparrow \to q_1 \uparrow$, $q_2 \downarrow$, when $\dfrac{a_{2K}}{a_{2L}} < \dfrac{a_{1K}}{a_{1L}}$.

Application of Rybczynski's Theorem.

Stolper - Samuelson: $p\uparrow \to r\uparrow . w\downarrow$. $\frac{dr}{dp}>0$. $\frac{dw}{dp}<0$. $\left(\begin{array}{l}\text{set } p_1=p\\ p_2=1\end{array}\right)$.

$$r=\frac{p\cdot a_{2L}-a_{1L}}{a_{1K}a_{2L}-a_{1L}a_{2K}}$$

$$w=\frac{a_{1K}-pa_{2K}}{a_{1K}a_{2L}-a_{1L}a_{2K}}$$

$\to$ $\frac{dr}{dp}=\frac{a_{2L}}{a_{1K}a_{2L}-a_{1L}a_{2K}}>0$

$\frac{dw}{dp}=-\frac{a_{2K}}{a_{1K}a_{2L}-a_{1L}a_{2K}}<0$ as $\frac{a_{1K}}{a_{1L}}>\frac{a_{2K}}{a_{2L}}$.

<u>Prop</u>: $p\uparrow \to$ increase conflict, i.e. $\frac{dL_A}{dp}>0$.

$$A(L_A)=\frac{L_A}{(\frac{r}{w})\bar{K}+\bar{L}} \to \frac{dL_A}{dp}=-\Big[A'-\underbrace{\frac{1}{(\frac{r}{w})\bar{K}+\bar{L}}}_{<0}\Big]^{-1}\Big[\frac{\bar{K}L_A}{[(\frac{r}{w})\bar{K}+\bar{L}]^2}\frac{d(r/w)}{dp}\Big]>0$$

<u>Intuition</u>: price of K-intensive good $\uparrow$ expand the sector. labor sector $\downarrow$.

then releases more labor per unit of K than previously $\to$ absorb initial

factor prices $\longrightarrow$ lower wages & opportunity cost of appropriation. $\to$ more conflict

<u>Prop</u>. Neutral Tech progress $\xrightarrow{\text{in K-sector}}$ increase in conflict.

Consider this neutral tech progress in K-sector. θ. $\to$ $(1+\theta)$ more productive

then Z-profit Condition $\to$ $ra_{1K}+wa_{1L}=(1+\theta)p$ $\to$ same effect as price increase in K-good.

So. give subsidy to L reduces level of conflict.

So some departure from laissez faire (放任) is beneficial.

Intuition: subsidizing productive labor $\to$ increase opportunity cost of appropriation

$\to$ shift labor towards productive sectors of the economy $\to$ second best

(distortionary policy can be optimal given an initial distortion)

<u>Trust & Conflict</u>. Mutual Distrust $\longrightarrow$ arms races & conflict.

Security dilemma. multiple eq. $\to$ without trust, lead to the bad one

Mistaken signals $\longrightarrow$ trigger conflict | "Conflict Snowballs"

P.17

Bilateral conflict escalates ⟶ aggressive actions become uninformative

⟶ a group to experiment with cooperation ⟶ if group not bat

⟶ conflict to an end

Rohner. Theory 2013 RES. Rational Choice theory of trust, trade & war.

war erodes inter-ethnic trust ⟶ reduce trade opportunities and opportunity cost of future war ↓ ⟶ recurrent war.

Distrust may be 'unwarranted' without being irrational.

Culprit: imperfect information ; learning trap : information cascades

Bad luck may result in permanent war trap.

Sr. business relations are key to preserve stable peace.

Peace-keeping forces may secure peace but fail to restore trade and economic cooperation.

Policy aimed at restoring trade & trust can be promising.