ECON 460: Graduate International Trade
Problem Set #1
Spring 2015, Professor Treb Allen
Due: Tuesday April 14 at the beginning of class

1. *Dixit-Stiglitz Preferences.* Suppose that a consumer has wealth W, consumes from a set of differentiated varieties $\omega \in \Omega$, and solves the following CES maximization problem:

$$\max_{\{q(\omega)\}} U = \left(\int_{\Omega} q(\omega)^{\frac{\sigma-1}{\sigma}} d\omega\right)^{\frac{\sigma}{\sigma-1}} \text{ s.t. } \int_{\Omega} p(\omega)\, q(\omega) \leq W, \tag{1}$$

where $\sigma > 0$, $q(\omega)$ is the quantity consumed of variety ω and $p(\omega)$ the price of variety ω.

   (a) Find a price index P such that in equilibrium $U = \frac{W}{P}$.
   (b) Derive the optimal $q(\omega)$ as a function of W, P and $p(\omega)$.
   (c) Show that σ is the elasticity of substitution, i.e. for any $\omega, \omega' \in \Omega$, $\sigma = \frac{d\ln\left(\frac{q(\omega)}{q(\omega')}\right)}{d\ln\left(\frac{\partial U/\partial q(\omega')}{\partial U/\partial q(\omega)}\right)}$.
   (d) What happens as $\sigma \to \infty$? $\sigma \to 1$? $\sigma \to 0$?

2. *Monopolistic competition.* Now consider the problem of a firm producing one of the differentiated varieties ω where consumer preferences are as in equation (1).

   (a) State the profit maximizing condition of the firm.
   (b) Derive the optimal price and quantity the firm charges.
   (c) How does the markup of the price over marginal cost vary with the elasticity of substitution? What is the intuition?
   (d) How does the relationship between profits and revenue vary with the elasticity of substitution? What is the intuition?

3. *The Frechet distribution.* For all $n \in \{1, ..., N\}$, suppose that the random variable $z_n \geq 0$ is distributed according to the Frechet distribution, i.e.:

$$\Pr\{z_n \leq z\} = \exp\left(-T_n z^{-\theta}\right),$$

where $T_i > 0$ and $\theta > 0$ are known parameters. Define the random variable $p_n = \frac{c_n}{z_n}$. Calculate:

$$\pi_{in} \equiv \Pr\left\{p_n \leq \min_{k \neq n} p_k\right\}$$

4. *The Pareto distribution.* Suppose that the random variable $z_n \in [b_n, \infty)$ is distributed according to the Pareto distribution, i.e.:

$$\Pr\{z_n \leq z\} = 1 - \left(\frac{b_n}{z}\right)^{\theta},$$

where $b_n > 0$ and $z_n > 0$ are known parameters. What is the distribution of z_n conditional on being greater than $c_n > b_n$?

5. *Isomorphisms.* Define X_{ij} to be the value of trade flows from i to j. Consider the following generalized gravity equation:

$$X_{ij} = K_{ij} \gamma_i \delta_j, \tag{2}$$

where K_{ij} is a bilateral trade friction, γ_i is an origin fixed effect, and δ_j is a destination fixed effect.

   (a) For each of the following trade models, show how equilibrium trade flows can be expressed as equation (2). That is, write down the mapping between the generalize gravity equation and model fundamentals.

i. Armington model (Anderson '79).
ii. Monopolistic competition with free entry (Krugman '80).
iii. Perfect competition with heterogeneous technologies (Eaton and Kortum '02).
iv. Heterogeneous firms (with Pareto distribution) (Melitz '03 / Chaney '08).

(b) Suppose we only observe trade flows in the data. Can we empirically distinguish between the above models? If not, what other data would you need to observe in order to distinguish between the models?