Macroeconomics A; EI056

Short problems

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1 Growth rate decomposition

1.1 The production function

Question: Consider the production function that we saw in class. Output Y_t is produced using capital K_t, labor L_t, and a general productivity A_t:

$$Y_t = A_t \left(K_t\right)^{1-\alpha} \left(L_t\right)^{\alpha}$$

Write an approximation of this relation around a point where $Y_t = \bar{Y}$, $K_t = \bar{K}$, $L_t = \bar{L}$, and $A_t = \bar{A}$. Specifically, show that:

$$\frac{dY_t}{\bar{Y}} = \frac{dA_t}{\bar{A}} + (1-\alpha)\frac{dK_t}{\bar{K}} + \alpha\frac{dL_t}{\bar{L}}$$

Answer: Recall that the general formula for taking an approximation of a function $Y_t = f\left(K_t, L_t\right)$ is:

$$dY_t = \frac{\partial f}{\partial K_t} dK_t + \frac{\partial f}{\partial L_t} dL_t$$

where $dY_t = Y_t - \bar{Y}$, and similarly for dK_t and dL_t, and all derivatives are evaluated at $Y_t = \bar{Y}$, $K_t = \bar{K}$, $L_t = \bar{L}$, and $A_t = \bar{A}$.

In our case, this implies:

$$\begin{aligned}
dY_t &= \left(\bar{K}\right)^{1-\alpha}\left(\bar{L}\right)^{\alpha} dA_t + (1-\alpha)\bar{A}\left(\bar{K}\right)^{-\alpha}\left(\bar{L}\right)^{\alpha} dK_t + \alpha\bar{A}\left(\bar{K}\right)^{1-\alpha}\left(\bar{L}\right)^{\alpha-1} dL_t \\
dY_t &= \bar{A}\left(\bar{K}\right)^{1-\alpha}\left(\bar{L}\right)^{\alpha}\frac{dA_t}{\bar{A}} + (1-\alpha)\bar{A}\left(\bar{K}\right)^{1-\alpha}\left(\bar{L}\right)^{\alpha}\frac{dK_t}{\bar{K}} + \alpha\bar{A}\left(\bar{K}\right)^{1-\alpha}\left(\bar{L}\right)^{\alpha}\frac{dL_t}{\bar{L}} \\
dY_t &= \bar{A}\left(\bar{K}\right)^{1-\alpha}\left(\bar{L}\right)^{\alpha}\left[\frac{dA_t}{\bar{A}} + (1-\alpha)\frac{dK_t}{\bar{K}} + \alpha\frac{dL_t}{\bar{L}}\right] \\
\frac{dY_t}{\bar{Y}} &= \frac{dA_t}{\bar{A}} + (1-\alpha)\frac{dK_t}{\bar{K}} + \alpha\frac{dL_t}{\bar{L}}
\end{aligned}$$

1.2 Expressing in growth rates

Question: Rearrange the expression to show that in terms of growth rates we get:

$$g\left(Y_t\right) = g\left(A_t\right) + (1-\alpha)\, g\left(K_t\right) + \alpha g\left(L_t\right)$$

where $g(Y_t)$ is the growth rate of Y between period $t-1$ and period t.

Answer: All we have to do is take the values at $t-1$ to be the ones around which the approximation is taken: $\bar{Y} = Y_{t-1}$, $\bar{K} = K_{t-1}$, $\bar{L} = L_{t-1}$, and $\bar{A} = A_{t-1}$. Notice then that $dY_t = Y_t - Y_{t-1}$, as similarly for the other variables.:

$$\frac{Y_t - Y_{t-1}}{Y_{t-1}} = \frac{A_t - A_{t-1}}{A_{t-1}} + (1-\alpha)\frac{K_t - K_{t-1}}{K_{t-1}} + \alpha\frac{L_t - L_{t-1}}{L_{t-1}}$$

Now recall the definition of a growth rate: $g(Y_t) = \frac{Y_t - Y_{t-1}}{Y_{t-1}}$, so we get:

$$g(Y_t) = g(A_t) + (1-\alpha)\, g(K_t) + \alpha g(L_t)$$

1.3 A more direct approach

Question: In this particular case, could we have done the computation more rapidly?
Hint: recall that the growth rate is also the difference between the logs:$g(Y_t) = \ln(Y_t) - \ln(Y_{t-1})$

Answer: The Cobb-Douglas production function is a product of level, and is thus linear in logs:

$$\ln(Y_t) = \ln\left(A_t (K_t)^{1-\alpha} (L_t)^{\alpha}\right)$$
$$\ln(Y_t) = \ln(A_t) + (1-\alpha)\ln(K_t) + \alpha\ln(L_t)$$

Now, write the same expression for period $t-1$:

$$\ln(Y_{t-1}) = \ln(A_{t-1}) + (1-\alpha)\ln(K_{t-1}) + \alpha\ln(L_{t-1})$$

and then take the difference between the period t expression and the period $t-1$ expression:

$$\ln(Y_t) - \ln(Y_{t-1}) = \ln(A_t) - \ln(A_{t-1}) + (1-\alpha)\left[\ln(K_t) - \ln(K_{t-1})\right] + \alpha\left[\ln(L_t) - \ln(L_{t-1})\right]$$
$$g(Y_t) = g(A_t) + (1-\alpha)\, g(K_t) + \alpha g(L_t)$$

2 Evolution of the Phillips curve

2.1 Getting the data

Question: This question requires you to dowload some data and do some computations.
We use the Fred database: https://fred.stlouisfed.org/ Use the search field to look for

- Unemployment rate: you should pick from the list and get to https://fred.stlouisfed.org/series/UNRATE.
- Consumer price index: you should pick from the list and get to https://fred.stlouisfed.org/series/CPIAUCSL.

For both, use the tools to transform the data as you need. Specifically, use "edit graph" on the right to choose an annual frequency, and for the consumer price transform it into percentage change from a year ago.

Use the date ranges to pick data starting in 1950.

Then use the download option to get the data in an excel format (or one of the other formats).

Answer: Following the steps above, you should have the numbers in the excel file provided on the moodle page.

2.2 Plotting the data

Question: In excel (or whatever software you prefer), do a scatter plot with inflation on the vertical axis, and the unemployment rate on the horizontal axis.

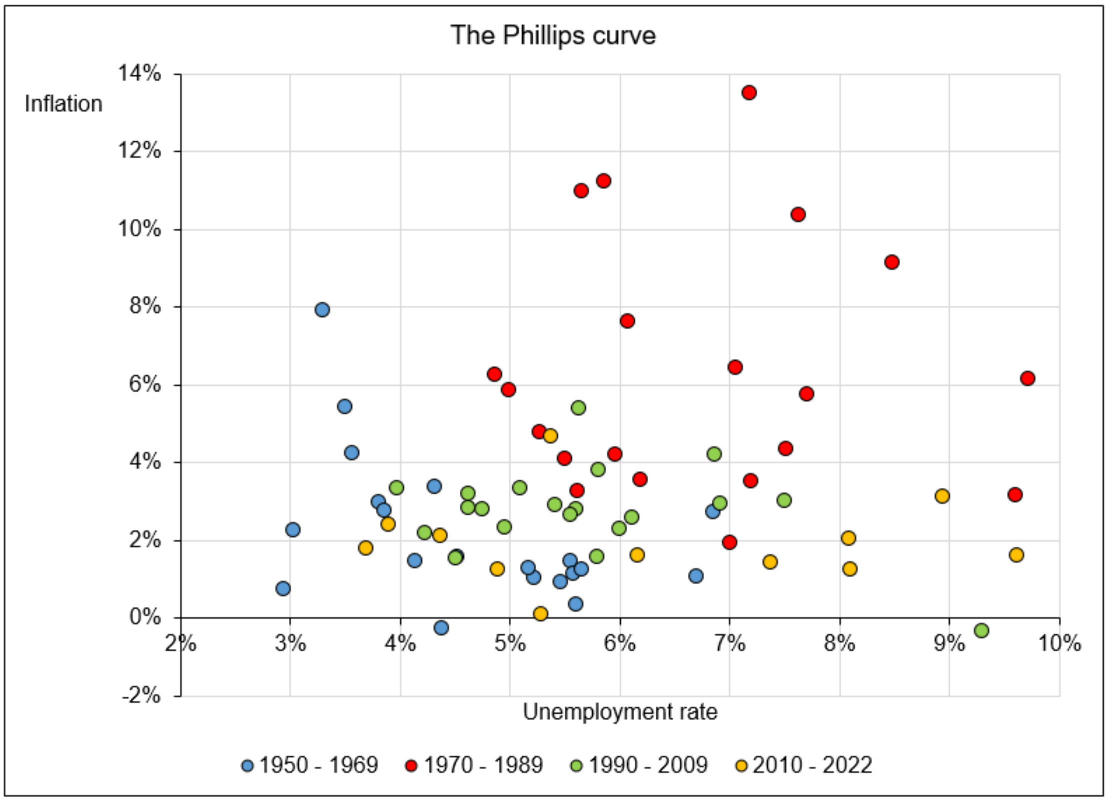
Distinguish between various periods by having them appears in dots of different colors. Specifically, consider a) 1950-1969, b) 1970-1989, c) 1990-2009, d) 2010-2022.

Answer: The figure is below (also in the excel file provided on the moodle page).

We see that initially the relation between the unemployment rate and inflation is negative, at low values of inflation (the blue dots are relatively low).

Starting in the 1970's (red dots) the points move up and to the right. We therefore get both high inflation and high unemployment.

Since then points have moved back down. This corresponds to the period where central banks became more focused on inflation.



2.3 Simple econometrics

Question: We now look at how the sensitivity of inflation to the unemployment rate has evolved.

To do so, we regress the inflation on a constant and the unemployment rate. You can do that in an econometric software, but as we are at the beginning of the semester, you can also do it simply on excel with the function "intercept" and "slope".

How did the relation evolve over the periods a) 1950-1969, b) 1970-1989, c) 1990-2009, d) 2010-2022?

Answer: The results are given in the table below.

Initially inflation has a high intercept, but is quite reactive to the unemployment rate, so on average it is low. The sensitivity to the unemployment rate can tempt policy makers to exploit an apparent inflation - unemployment trade-off.

Things change in the 1970's: the intercept is higher, and – more importantly – the sensitivity to unemployment goes down. One thus cannot get low unemployment as a gain for accepting more inflation.

Sample	Constant	Coefficient on the unemployment rate
1950 - 1969	5.72%	-0.7564
1970 - 1989	6.35%	-0.0039
1990 - 2009	4.39%	-0.2835
2010 - 2022	4.19%	-0.2823

The sensitivity comes back since 1990, but with a low intercept. This is a period where policy makers know there is a sensitivity, but do not try to exploit it.

In the year since the financial crisis, the results are similar as before. Note that this is driven by the recent inflation surge (if you run the regression until 2020 the intercept is lower, and the sensitivity to the unemployment rate is very limited).