Macroeconomics A; EI060

Short problems

Cédric Tille

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1 Optimal exchange rate choice in a floating regime

Question: The output gap reflects unexpected exchange rate movements:

$$x_t = \theta\left(\Delta e_t - \Delta e_t^{\text{expected}}\right)$$

The loss function reflects the volatility of the output gap, around a reference point $\overline{x}$, the volatility of the exchange rate movements, and a fixed cost if the country deviates from the peg.

$$L = \frac{1}{2}\left\{\phi\left(\theta\left(\Delta e_t - \Delta e_t^{\text{expected}}\right) - \overline{x}\right)^2 + \left(\Delta e_t\right)^2\right\} + c_{\Delta e_t > 0}$$

Show that if the country deviates from the peg, it sets:

$$\Delta e_t = \frac{\phi\theta}{1+\phi\theta^2}\left(\theta\Delta e_t^{\text{expected}} + \overline{x}\right)$$

Conditional on the country opting for a floating exchange rate, show that in equilibrium $\Delta e_t = \phi\theta\overline{x}$.

2 Welfare

Question: Show that the loss function if the bank abandons the peg is:

$$L^{\text{float}} = \frac{\phi}{2\left(1+\phi\theta^2\right)}\left(\overline{x} + \theta\Delta e_t^{\text{expected}}\right)^2 + c$$

Show that the loss function of the peg is maintained is:

$$L^{\text{peg}} = \frac{\phi}{2}\left(\overline{x} + \theta\Delta e_t^{\text{expected}}\right)^2$$

3 Threshold

Question: Show that the country chooses to abandon the peg if:

$$c < c^{\text{critical}} = \frac{(\phi\theta)^2}{2(1+\phi\theta^2)}\left(\overline{x} + \theta \Delta e_t^{\text{expected}}\right)^2$$

Show that $c = c^{\text{critical}}$ translates into a threshold for the exchange rate expectations:

$$\Delta \overline{e}_t^{\text{expected}} = \frac{\sqrt{2(1+\phi\theta^2)\, c}}{\phi(\theta)^2} - \frac{\overline{x}}{\theta}$$

Show that there is also another value of $\Delta \overline{e}_t^{\text{expected}}$ which is unambiguously negative

4 Multiple equilibria

Question: Show that multiple equilibria are not feasible if $\Delta \overline{e}_t^{\text{expected}} < 0$ or $\Delta \overline{e}_t^{\text{expected}} > \phi\theta\overline{x}$. Show that multiple equilibria are possible if:

$$\overline{x} < \frac{\sqrt{2(1+\phi\theta^2)\, c}}{\phi\theta} < \left(1+\phi\theta^2\right)\overline{x}$$