Macroeconomics A; EI056

Short problems

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1 Debt dynamics

1.1 Flow constraints

Question: In nominal terms, the government spends G_t to purchase goods and services, and pays an interest rate i_t on its debt B_t. This is finance by taxes T_t or new debt, $B_{t+1} - B_t$:

$$G_t + i_t B_t = T_t + B_{t+1} - B_t$$

Nominal GDP Y_t grows at a rate $g_t = Y_t/Y_{t-1} - 1$. The primary deficit is $D_t^{\text{prim}} = G_t - T_t$.

Show that the flow constraint in terms of variables scaled by GDP (for example $d_t^{\text{prim}} = D_t^{\text{prim}}/Y_t$) is:

$$b_{t+1} - b_t = \frac{d_t^{\text{prim}}}{1 + g_{t+1}} + \frac{i_t - g_{t+1}}{1 + g_{t+1}} b_t$$

1.2 Steady state

Question: Consider that all variables are constant. Is a primary surplus, $d^{\text{prim}} < 0$, needed?

1.3 Intertemporal constraint

Question: Consider that i_t and g_t are constant. Show that:

$$b_t = \frac{1}{1+i} \sum_{s=0}^{\infty} \left(\frac{1+g}{1+i} \right)^s \left(-d_{t+s}^{\text{prim}} \right) + \lim_{k \to \infty} \left(\frac{1+g}{1+i} \right)^k b_{t+k+1}$$

Do we have the usual transversality condition, as in last week?

1.4 Alternative discount

Question: Show that we can rewrite the flow budget constraint as:

$$b_{t+1} - b_t = \frac{d_t^{\text{prim}}}{1 + g_{t+1}} + \frac{i_t - m_t}{1 + g_{t+1}} b_t + \frac{m_t - g_{t+1}}{1 + g_{t+1}} b_t$$

where m_t is the private sector budget discount factor, that we can think of as the rate of return on private capital. We assume that this rate of return is higher than the GDP growth rate.

Assuming that i_t, g_t and m_t are constant, show that we get

$$b_t = \frac{1}{1+m}\sum_{s=0}^{\infty}\left(\frac{1+g}{1+m}\right)^s\left[-d_{t+s}^{\text{prim}} + (m-i)\,b_{t+s}\right] + \lim_{k\to\infty}\left(\frac{1+g}{1+m}\right)^k b_{t+k+1}$$

Do we have the usual transversality condition, as in last week? Is there a new source of income for the government.

2 Political economy of deficit

2.1 Preferences, and final choice

Question: Consider the model of heterogeneous preferences seen in class. We have two periods. In each there is an endowment W. Government spending is used to purchases two types of goods, M and N. The government's budget constraints in the two periods are (B denotes debt):

$$\begin{aligned} M_1 + N_1 &= W + B \\ M_2 + N_2 &= W - B \end{aligned}$$

Individuals are different depending on their preferences for the two types of goods the government can purchase. Specifically, the utility of individual i is:

$$V_i = E\sum_{t=1}^{2}\left[\alpha_i ln\left(M_t\right) + \left(1-\alpha_i\right) ln\left(N_t\right)\right]$$

Individuals differ according to their preference for good M which is captured by the weight $\alpha_i \in [0,1]$.

Show that in the second period:

$$M_2 = \alpha_{med,2}\left(W - B\right) \qquad ; \qquad N_2 = \left(1-\alpha_{med,2}\right)\left(W-B\right)$$

2.2 Initial choice

Question: In the first period, the median voter maximizes:

$$\begin{aligned} & E\sum_{t=1}^{2}\left[\alpha_{med,1} ln\left(M_t\right) + \left(1-\alpha_{med,1}\right) ln\left(N_t\right)\right] \\ =& \alpha_{med,1} ln\left(M_1\right) + \left(1-\alpha_{med,1}\right) ln\left(N_1\right) + E\left[\alpha_{med,1} ln\left(M_2\right) + \left(1-\alpha_{med,1}\right) ln\left(N_2\right)\right] \end{aligned}$$

Show that this utility is:

$$\begin{aligned} & \alpha_{med,1} ln\left(M_1\right) + \left(1-\alpha_{med,1}\right) ln\left(W+B-M_1\right) + ln\left(W-B\right) \\ & + \alpha_{med,1} E ln\left(\alpha_{med,2}\right) + \left(1-\alpha_{med,1}\right) E ln\left(1-\alpha_{med,2}\right) \end{aligned}$$

Show that the optimal choice for M_1 is:

$$M_1 = \alpha_{med,1}\left(W+B\right) \qquad ; \qquad N_1 = \left(1-\alpha_{med,1}\right)\left(W+B\right)$$

2.3 Intertemporal choice

Question: Using the results, show that the utility is:

$$ln\left(W+B\right)+ln\left(W-B\right)+\alpha_{med,1}ln\left(\alpha_{med,1}\right)+\left(1-\alpha_{med,1}\right)ln\left(1-\alpha_{med,1}\right)$$
$$+\alpha_{med,1}Eln\left(\alpha_{med,2}\right)+\left(1-\alpha_{med,1}\right)Eln\left(1-\alpha_{med,2}\right)$$

What is the optimal debt level B?