Macroeconomics A, EI056

Class 6

The Real Business Cycle (RBC) model

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What you will get from today class

- How to measure business cycles: the **filtering approach** (Stata program on the Moodle page).
- Fluctuations in a general equilibrium model (*Real Business Cycles*) with **endogenous labor** supply and exogenous random **shocks**.
  - Setup of the model, solution under uncertainty (Matlab program on the Moodle page).
  - Correct way of doing optimization under uncertainty.
  - Impact of productivity shock.
- **Extensions** of the baseline framework.
  - Making the measure of "technology" more realistic.
  - Impact of demand (government spending) shocks (extra slides).

A question to start

Improvements in productivity make workers more efficient, and should thus stimulate labor supply. The RBC does not capture this as it implies that labor increases only temporarily, followed by a decrease.

Do you agree? Why or why not?

MEASURING BUSINESS CYCLES

The filtering approach

- Macroeconomic time series y_t (such as the log of GDP) have a **trend** y_t^{Trend} and a **cycle** around it $y_t^{Cycle} = y_t - y_t^{Trend}$.
- **Linear** trend: $y_t^{Trend} - y_{t-1}^{Trend}$ is constant $(= y_{t+1}^{Trend} - y_t^{Trend})$.
- **Hodrick-Prescott filter** (HP) computes the trend to minimize:

$$\sum_t \left\{ \underbrace{\left[y_t - y_t^{Trend} \right]^2}_{\text{Data close to trend}} + \lambda \underbrace{\left[\left(y_{t+1}^{Trend} - y_t^{Trend} \right) - \left(y_t^{Trend} - y_{t-1}^{Trend} \right) \right]^2}_{\text{Trend smooth across time}} \right\}$$

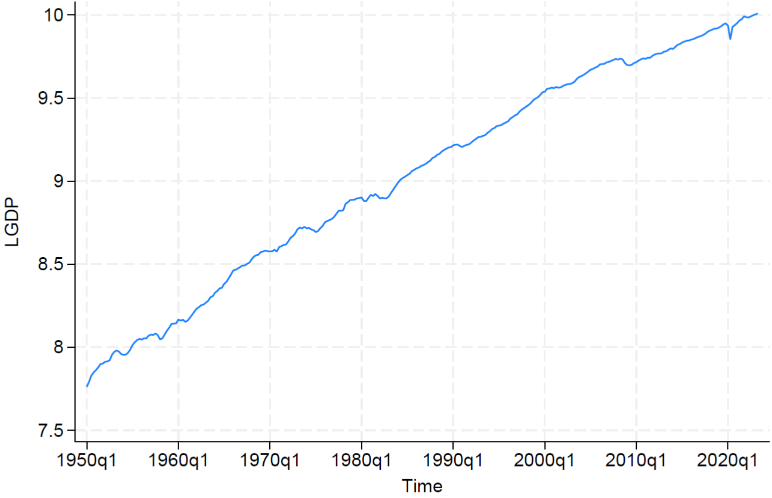
- Trade-off between keeping the trend close to the data and having a smooth trend. If $\lambda = 0$ the trend is the data. If $\lambda \to \infty$: the trend is linear.
- With quarterly data usual value is $\lambda = 1600$. Trend that follows low frequency movements in the data (of periodicity above 8 years).

Properties of trends and cycles

- GDP shows **long-lasting "waves"** of much lower frequency (i.e. last longer) that what we think of as the business cycle.
- Linear trend: the waves go into the cyclical component.
    - Unrealistic feature of long business cycles.
- HP filter: the wave go into the trend, gives a shorter business cycle.
    - The HP cycle however include high-frequency noise.
- Bypass filter (Baxter and King) also removes very high frequency variations. Cycle is fluctuations with periodicity between 1.5 and 8 years.
- Filters in standard econometric packages (Stata "do file" on the moodle page).
- Caveat: filters give imprecise end-of-sample estimates, can introduce spurious movements.

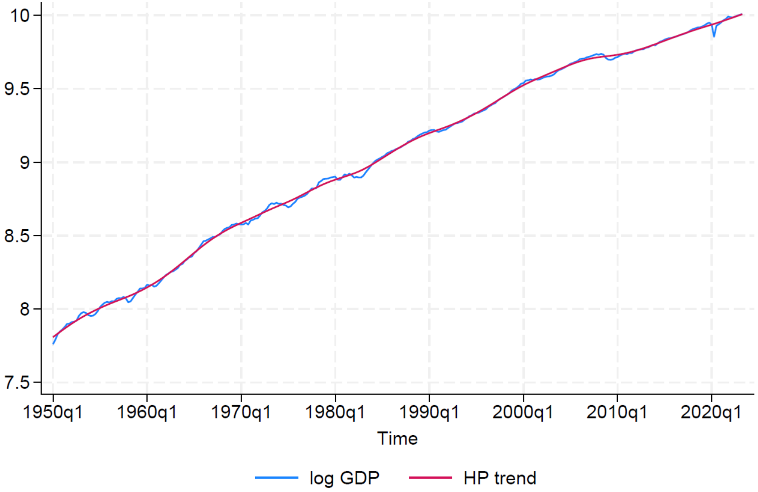
Raw data

- Log level of US GDP since 1950.



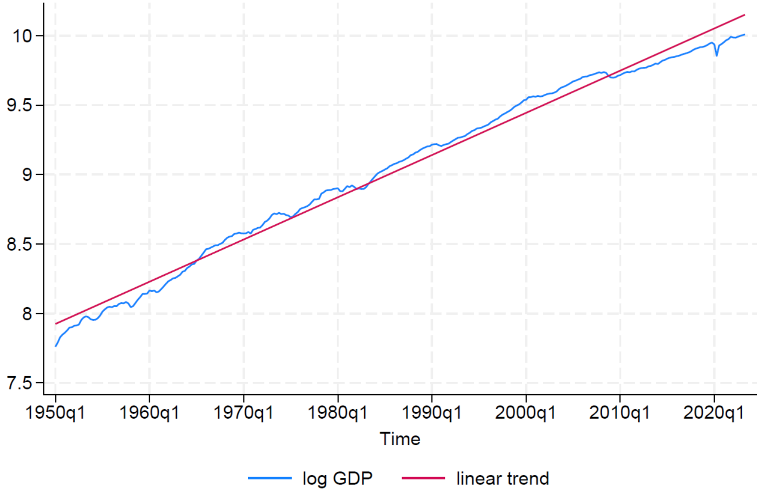
HP trend

- The trend tracks the long waves.



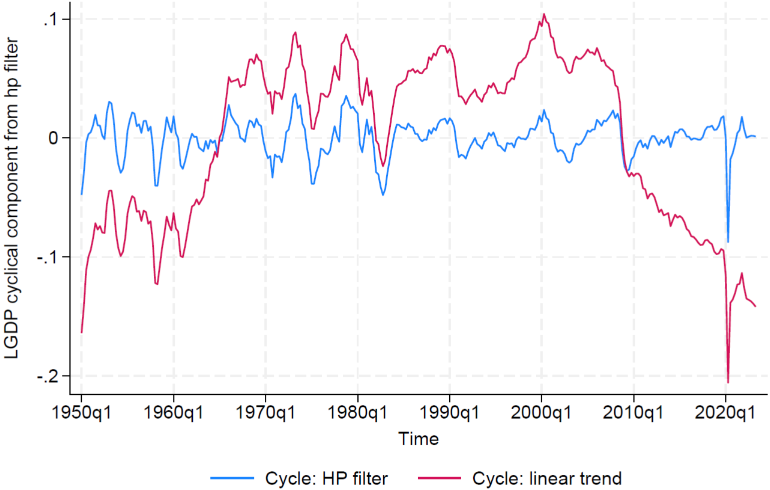
Linear trend

- Sustained gaps of data from trend.



Cycles

- HP cycle is stationary, linear cycle is not.



Stylized facts of the business cycle

- Consumption is less volatile than output, **investment** is much more volatile than output.
- **Labor** is as volatile as output. Overall hours worked are driven by **employment**, not hours per worker.
- Productivity is not observed, but inferred from the production function (in terms of growth rates), **Solow** residuals:

$$g(Y_t) = \alpha g(K_t) + (1-\alpha)(g(A_t) + g(L_t))$$

$$SR_t = (1-\alpha) g(A_t) = g(Y_t) - [\alpha g(K_t) + (1-\alpha) g(L_t)]$$

- The Solow residuals are **volatile** and highly **correlated with output**.
  - Careful in interpreting them as true productivity (more below).

RBC MODEL : INTRODUCTION

The general approach

- Can a model with **no frictions or inefficiencies** explain business cycles?
- If yes, the economy responds efficiently to (unpleasant) business cycles.
  - Build the model from the ground up (households and firms **optimize**) to avoid the Lucas critique.
- Ramsey model, augmented in two main ways:
  - Volatility: **unexpected shocks** affect the economy (productivity and government spending).
  - Endogenous variations in **labor** input.

Technology and shocks

- Discrete time model, Cobb-Douglas production function.

$$Y_t = (K_t)^{\alpha} (A_t L_t)^{1-\alpha} \quad ; \quad 0 < \alpha < 1$$

- Output used for private consumption C, government consumption G, and investment, I. Capital depreciates at a rate δ.

$$Y_t = C_t + G_t + I_t \quad ; \quad K_{t+1} = (1-\delta) K_t + I_t$$

- Productivity A moves around a level $\bar{A}$, with persistence:

$$\ln(A_t) - \ln(\bar{A}) = \hat{A}_t = \rho_A \hat{A}_{t-1} + \varepsilon_{A,t}$$

  - $\varepsilon_{A,t}$ is a shock of expected value zero, and $-1 < \rho_A < 1$ (usually $0 < \rho_A < 1$). Appendix with trends in productivity and population.
- Similar process for government spending G:

$$\ln(G_t) - \ln(\bar{G}) = \hat{G}_t = \rho_G \hat{G}_{t-1} + \varepsilon_{G,t}$$

Optimization by the firm

- Maximize profits: output minus wage bill and rental cost of capital (including depreciation):

$$\Pi_t = (K_t)^\alpha (A_t L_t)^{1-\alpha} - w_t L_t - [(1 + r_t) - (1 - \delta)] K_t$$

  - Firm buys capital by borrowing at cost $1 + r_t$, resells the remaining $1 - \delta$ capital (compared to last week: $r_t^{today} = r_t^{last\ week} - \delta$).

- Marginal product equal to marginal cost. **Labor demand**:

$$\partial \Pi_t / \partial L_t = 0 \Rightarrow w_t = (1 - \alpha) \left(\frac{K_t}{A_t L_t} \right)^\alpha A_t$$

- **Demand for capital**:

$$\partial \Pi_t / \partial K_t = 0 \Rightarrow r_t = \alpha \left(\frac{K_t}{A_t L_t} \right)^{\alpha - 1} - \delta$$

RBC MODEL : HOUSEHOLD

Utility and budget constraint

- Representative agent (appendix considers population growth).
- Utility from consumption C and labor L (leisure is $1 - L$). Time preference with discount factor ρ.
- **Expected** utility: E_t denotes expectation at time t (log utility of consumption for simplicity):

$$U_t = E_t \sum_{s=0}^{\infty} \frac{1}{(1+\rho)^s} \left[\ln\left(C_{t+s}\right) + b \ln\left(1 - L_{t+s}\right)\right]$$

- Budget constraint: consumption + investment = wage + capital income - taxes (equal to government spending in a balanced budget):

$$\begin{aligned} C_t + I_t &= w_t L_t + \left[\left(1+r_t\right) - (1-\delta)\right] K_t - G_t \\ C_t + K_{t+1} &= w_t L_t + \left(1+r_t\right) K_t - G_t \end{aligned}$$

States of nature

- Should we write the Lagrangian and take the derivatives with respect to C_t and C_{t+1} or $E_t C_{t+1}$?
  - Not quite: there is **not one** C_{t+1} but many different C_{t+1}, one for each possible **state of nature** at $t+1$.
  - Optimizing with respect to "future consumption" does make sense. W.r.t "consumption in a state of nature" does.
- States of nature at time $t+s$, indexed by x_{t+s}. The **probability** of x_{t+s} happening is $\pi(x_{t+s})$.
- Expected value: **sum across the states**, weighted by the probabilities ($C(x_{t+s})$ is consumption at time $t+s$ if state x_{t+s} is realized).

$$E_t C_{t+s} = \sum_{x_{t+s}} \pi(x_{t+s}) \cdot C(x_{t+s})$$

Re-writing the utility

- Expected utility is written as:

$$
\begin{aligned}
& E_t \sum_{s=0}^{\infty} \frac{1}{(1+\rho)^s} \left[\ln\left(C_{t+s}\right) + b \ln\left(1 - L_{t+s}\right)\right] \\
= & \sum_{x_{t+s}} \pi\left(x_{t+s}\right) \left(\sum_{s=0}^{\infty} \frac{1}{(1+\rho)^s} \left[\ln\left(C\left(x_{t+s}\right)\right) + b \ln\left(1 - L\left(x_{t+s}\right)\right)\right] \right) \\
= & \underbrace{\sum_{s=0}^{\infty} \frac{1}{(1+\rho)^s}}_{\text{some for all states}} \underbrace{\sum_{x_{t+s}} \pi\left(x_{t+s}\right) \left[\ln\left(C\left(x_{t+s}\right)\right) + b \ln\left(1 - L\left(x_{t+s}\right)\right)\right]}_{\text{specific by state of nature}}
\end{aligned}
$$

Budget constraints

- One budget constraint for each state of nature:

$$
\begin{aligned}
& C\left(x_{t+s}\right)+K\left(x_{t+s+1}\right) \\
= & w\left(x_{t+s}\right) L\left(x_{t+s}\right)+\left(1+r\left(x_{t+s}\right)\right) K\left(x_{t+s}\right)-G\left(x_{t+s}\right)
\end{aligned}
$$

- The Lagrangian is then:

$$
\begin{aligned}
\mathcal{L} = & \sum_{s=0}^{\infty} \sum_{x_{t+s}} \frac{\pi\left(x_{t+s}\right)}{(1+\rho)^{s}}\left[\ln C\left(x_{t+s}\right)+b \ln \left(1-L\left(x_{t+s}\right)\right)\right] \\
& -\sum_{s=0}^{\infty} \sum_{x_{t+s}}\left\{\frac{\pi\left(x_{t+s}\right) \varphi\left(x_{t+s}\right)}{(1+\rho)^{s}}\left[\begin{array}{c}
C\left(x_{t+s}\right) \\
+K\left(x_{t+s+1}\right)+G\left(x_{t+s}\right) \\
-w\left(x_{t+s}\right) L\left(x_{t+s}\right) \\
-\left(1+r\left(x_{t+s}\right)\right) K\left(x_{t+s}\right)
\end{array}\right]\right\}
\end{aligned}
$$

- $\varphi\left(x_{t+s}\right)$: multiplier on the constraint for state x_{t+s} at time $t+s$.

Labor supply

- In **period** t, the state of nature of period t is known. Index variables just with time.
- First order conditions with respect to consumption and labor:

$$\frac{1}{C_t} = \varphi_t \qquad ; \qquad \frac{b}{1 - L_t} = \varphi_t w_t$$

- Combine to get the **labor supply**: marginal utility of leisure = wage * marginal utility of consumption:

$$\frac{b}{1 - L_t} = \frac{1}{C_t} w_t$$

- The link between wage and labor is conditional on consumption.

Euler condition

- First order condition for consumption in a **specific state** at $t+1$:

$$\frac{1}{C\left(x_{t+1}\right)}=\varphi\left(x_{t+1}\right)$$

- Capital for $t+1$ is chosen in t through investment. **Capital will be in place in all possible states** of nature.
- Trade off between current value of resources and expected future value across states (adjusted by the interest rate and impatience):

$$\varphi_t=\frac{1}{1+\rho}\sum_{x_{t+1}}\pi\left(x_{t+1}\right)\varphi\left(x_{t+1}\right)\left(1+r\left(x_{t+1}\right)\right)=E_t\left(\varphi_{t+1}\frac{1+r_{t+1}}{1+\rho}\right)$$

- **Euler condition with the expected** marginal utility of future consumption and interest rate (notice: no $E_t\left(C_{t+1}\right)$ per se):

$$\frac{1}{C_t}=E_t\left(\frac{1}{C_{t+1}}\frac{1+r_{t+1}}{1+\rho}\right)$$

Intertemporal view of labor supply

- Static relation between consumption, labor and the real wage:

$$C_t = w_t \left(1 - L_t\right) / b$$

- Higher wage today does not necessarily raises labor supply. Consumption can move in an offsetting way.
- Euler condition links the wage and labor dynamics (labor supply is driven by **intertemporal substitution**). Abstracting from uncertainty:

$$\frac{b}{1-L_t} / \left(\frac{b}{1-L_{t+1}}\right) = \frac{w_t \varphi_t}{w_{t+1}\varphi_{t+1}} \Rightarrow \frac{1-L_{t+1}}{1-L_t} = \frac{w_t}{w_{t+1}} \frac{1+r_{t+1}}{1+\rho}$$

- Current **labor increases** when **wages are temporarily high** or when the interest rate is high (future discounted wage looks low).

PUTTING IT TOGETHER

3 key relations

- **State variables** (at time t): capital K_t, government spending G_t, productivity, A_t. **Control variables**: consumption, C_t, future capital, K_{t+1} (that is investment, given K_t), labor, L_t.
- 3 relations: Euler, labor-consumption trade-off, capital dynamics:

$$
\begin{aligned}
\frac{1}{C_t} &= E_t \left[\frac{1}{C_{t+1}} \frac{1 + \alpha \left(\frac{K_{t+1}}{A_{t+1} L_{t+1}} \right)^{\alpha - 1} - \delta}{1 + \rho} \right] \\
\frac{b}{1 - L_t} C_t &= w_t = (1 - \alpha) \left(\frac{K_t}{A_t L_t} \right)^{\alpha} A_t \\
K_{t+1} &= (1 - \delta) K_t + I_t \\
&= (K_t)^{\alpha} (A_t L_t)^{1 - \alpha} + (1 - \delta) K_t - G_t - C_t
\end{aligned}
$$

- Solution with log-linearization around a steady state.

Deviations from steady state

- Steady state real interest rate offsets discount: $\bar{r} = \rho$. Gives the capital-labor ratio.
- Log-deviations (percent) around the steady state, $\hat{Y}_t = \ln Y_t - \ln \bar{Y} = \left(Y_t - \bar{Y} \right) / \bar{Y}$.
- Taylor approximation of three key relations (this takes some steps).
  - Solution using undetermined coefficients, or Blanchard-Kahn method.

$$\begin{aligned}
\hat{C}_t &= a_{CK}\hat{K}_t + a_{CA}\hat{A}_t + a_{CG}\hat{G}_t \\
\hat{L}_t &= a_{LK}\hat{K}_t + a_{LA}\hat{A}_t + a_{LG}\hat{G}_t \\
\hat{K}_{t+1} &= b_{KK}\hat{K}_t + b_{KA}\hat{A}_t + b_{KG}\hat{G}_t
\end{aligned}$$

Calibration

- **Calibrate** the model. Pick values for the main parameters and selected steady state variables:

$$\alpha = 1/3 \quad ; \quad \delta = 2.5\% \quad ; \quad \rho_A = \rho_G = 0.9$$

$$\bar{r} = 1.5\% \quad ; \quad \bar{L} = 1/3 \quad ; \quad \bar{G}/\bar{Y} = 0.2$$

- Gives the relations between the state and control variables:

$$\begin{aligned}
\hat{C}_t &= 0.61 \cdot \hat{K}_t + 0.37 \cdot \hat{A}_t - 0.12 \cdot \hat{G}_t \\
\hat{L}_t &= -0.33 \cdot \hat{K}_t + 0.35 \cdot \hat{A}_t + 0.15 \cdot \hat{G}_t \\
\hat{K}_{t+1} &= 0.95 \cdot \hat{K}_t + 0.08 \cdot \hat{A}_t - 0.004 \cdot \hat{G}_t
\end{aligned}$$

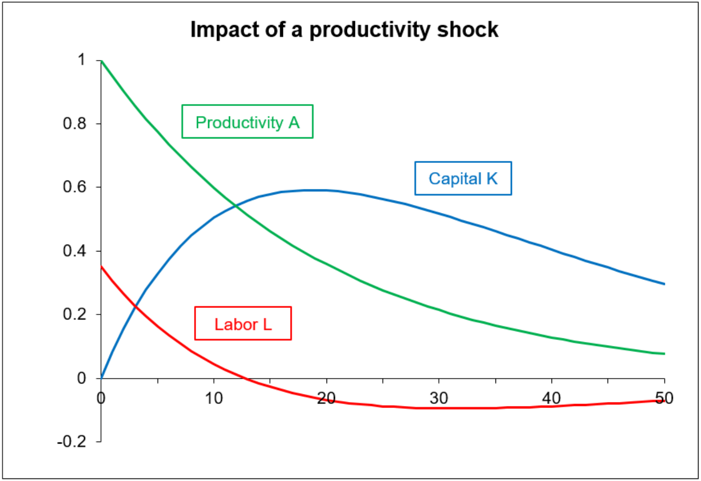
- **Impulse responses** of variables following shocks.

A productivity increase

- Productivity goes up, and gradually converges back to zero.
- **Capital** increases over several periods, and then gradually moves back to the steady path.
- **Labor** temporarily increases (real wage temporarily high).
- **Investment** and **consumption** increase, with investment increasing a lot on impact.
- Higher real wages and higher real interest rate (temporarily).

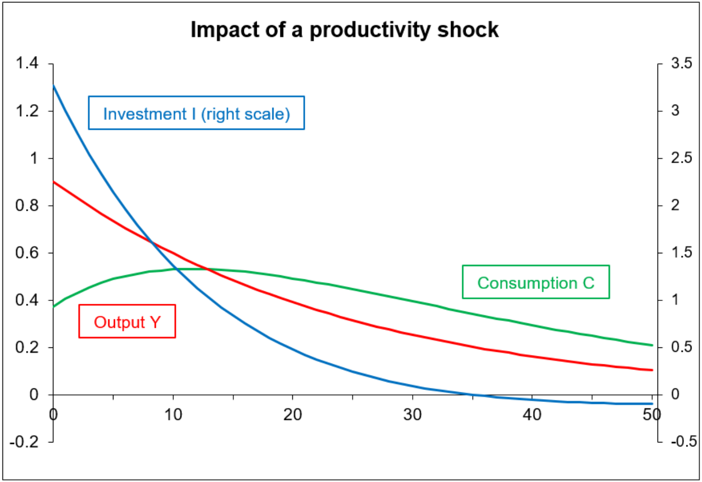
Productivity shock and factors

- Higher productivity raises labor (temporarily, with later offset) and capital (with a delay).



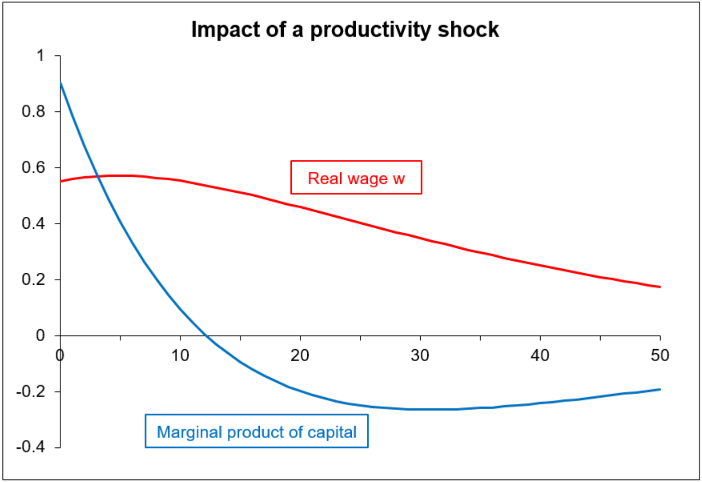
Composition of GDP

- Initially higher investment drives GDP, then consumption becomes the driver.



Factor prices

- Large, but short-lived, increase in real interest rate, more persistent (but still temporary) increase in real wage.



FIT AND EXTENSIONS

Bringing the model to the data

- Use estimated variance of productivity shocks and calibrated model to generate variances and correlations of the various variables.
- Output is nearly as volatile as in the data.
  - Consumption is less volatile than output, and investment is more volatile (3 times).
- There are problems:
  - Labor less volatile than output in the model, but empirically equally volatile.
  - High correlation between labor and output-labor ratio in the model, not in the data.
- One shock (productivity) brings us a long way towards the empirical pattern of the business cycle.
  - But requires unrealistically high degree of intertemporal substitution (households willing to shift labor a lot across time).
- Little **amplification**: we need volatile shocks to get output volatility.
  - Volatile and persistent Solow residuals (frequent decreases in productivity).

Making the model more realistic

- **Extensive margin** in labor input (variation in the share of people working a set amount) rather than **intensive margin** (variation in the number of hours worked by each person).
  - Aggregate labor supply more sensitive to wages than the true individual labor supply (see technical exercises).
- Vary **intensity of utilization** of inputs. Capital can be worked more (intensity z_t), at a cost of higher depreciation:

$$Y_t = F\left(z_t K_t\right)^{1-\alpha}\left(A_t L_t\right)^{\alpha} \quad ; \quad K_{t+1} = \left(1-\delta\left(z_t\right)\right)K_t + \left(Y_t - C_t\right)$$

  - First-order condition: $(1-\alpha)\left(K_t\right)^{1-\alpha}\left(A_t L_t\right)^{\alpha} = \left(z_t\right)^{\alpha}\delta_z\left(z_t\right)K_t$
  - Higher productivity $\rightarrow$ higher utilization, looks like extra productivity (affects "naive" Solow residuals).
  - Output responds more to productivity shocks.
  - True Solow residuals less volatile (70% less) and more realistic (likelihood of quarterly regress cut by 4).

Current developments (Kehoe, Midrigan and Pastorino)

- RBC imposes discipline. Parameters from the structural aspects (e.g. utility function) have to be estimated from sources other than business cycle data.
- Extensions: limited price adjustment, financial markets with frictions (more on this next week).
- Current research with **heterogeneous firms** instead of an aggregate one.
  - Firms have different productivity levels. Shocks that increase the presence of low productivity firms can lower aggregate productivity.
  - Firms have individual (idiosyncratic) productivity shocks. Variance of productivity shocks across firms (zero on average) can have aggregate effects.

Bottom line

- Taken literally, the view that productivity shocks are the cause of efficient business cycles is debatable.
- Many extensions to make the model more realistic.
- Key contribution: emphasis on dynamic stochastic general equilibrium (**DSGE**) models, with optimizing behavior (avoids the Lucas critique).
- Modelling approach used in models with nominal frictions (such as price stickiness) used to analyze monetary policy.

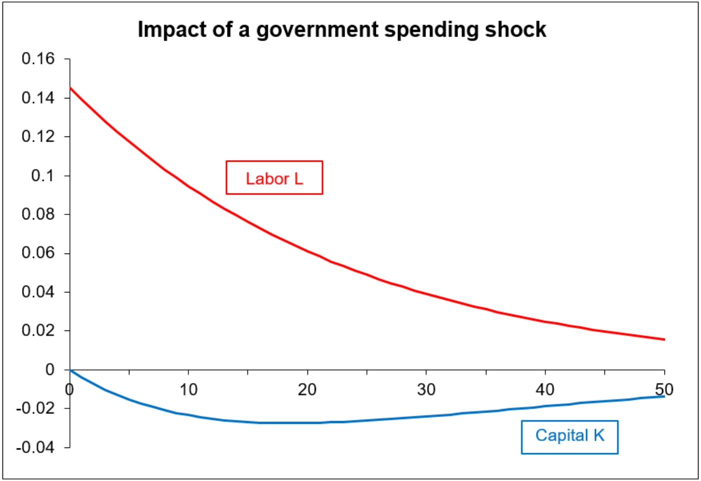
EXTRA : GOVERNMENT SPENGING SHOCK

Effect of increase in spending

- Reduces investment and the capital stock: **crowding out** because the interest rate is higher.
- Labor increases: household is **poorer** because of taxes and works more to get income (wealth effect).
- Output increases but only thanks to the government spending (lower consumption and investment).
- Lower wage as labor supply is increased, and higher interest rate higher.

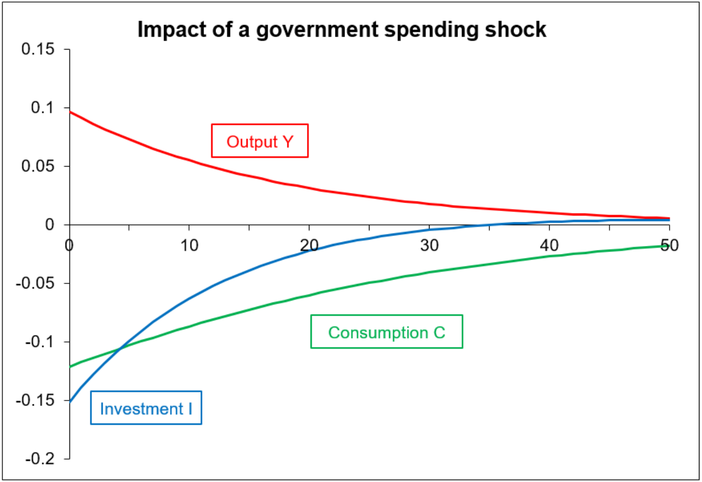
Production and factors

- Reduction in capital and increase in labor (no offset later through negative labor).



Composition of GDP

- Higher output, but lower private consumption and investment.



Factor prices

- Higher real interest rate and lower wage.