Macroeconomics A; EI060

Short problems

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1 Exchange rate with flexible prices

Question: In class, we showed that the nominal exchange rate can be written as a function of the current real exchange rate gap and the stream of current and future money supplies:

$$e_t - \bar{q} = \frac{1-\phi\delta}{1+\eta\psi\delta}(q_t - \bar{q}) + \frac{1}{1+\eta}\sum_{s=t}^{\infty}\left(\frac{\eta}{1+\eta}\right)^{s-t} m_s$$

Consider that the money supply is growing at a rate μ, so $m_s = m_t + \mu(s-t)$.

When prices are flexible, the real exchange rate is always at its long-run value, as nominal variables have no impact on real variables. Show that:

$$e_t^{flex} = \bar{q} + m_t + \mu\eta$$

It is useful to recall that:

$$\sum_{s=t}^{\infty}\left[\left(\frac{\eta}{1+\eta}\right)^{s-t}(s-t)\right] = \eta(1+\eta)$$

Answer: Take the expression for the exchange rate, and use $q_t = \bar{q}$:

$$\begin{aligned}
e_t^{flex} - \bar{q} &= \frac{1}{1+\eta}\sum_{s=t}^{\infty}\left(\frac{\eta}{1+\eta}\right)^{s-t} m_s \\
e_t^{flex} - \bar{q} &= \frac{1}{1+\eta}\sum_{s=t}^{\infty}\left(\frac{\eta}{1+\eta}\right)^{s-t} [m_t + \mu(s-t)] \\
e_t^{flex} - \bar{q} &= m_t\frac{1}{1+\eta}\sum_{s=t}^{\infty}\left(\frac{\eta}{1+\eta}\right)^{s-t} + \mu\frac{1}{1+\eta}\sum_{s=t}^{\infty}\left(\frac{\eta}{1+\eta}\right)^{s-t}(s-t) \\
e_t^{flex} - \bar{q} &= m_t\frac{1}{1+\eta}\frac{1}{1-\frac{\eta}{1+\eta}} + \mu\frac{1}{1+\eta}\eta(1+\eta) \\
e_t^{flex} &= \bar{q} + m_t + \mu\eta
\end{aligned}$$

2 Nominal exchange rate: sticky vs. flexible prices

Question: Consider that until time 0 we are in a steady state with money growing at a rate μ. At time 0 the central bank announces, unexpectedly, that the money growth rate will now be $\mu' > \mu$.

The flexible exchange rate at time 0 just before and just after the announcement are:

$$\begin{aligned} e_0^{flex} &= \bar{q} + m_0 + \eta\mu \\ \left(e_0^{flex}\right)' &= \bar{q} + m_0 + \eta\mu' \end{aligned}$$

Before the announcement the real exchange rate is $\bar{q}$, and the nominal exchange rate is equal to its flexible price value e_0^{flex}. The price level is then $p_0 = e_0^{flex} - \bar{q}$ (we set the foreign price level at zero).

The price level is sticky and does not vary after the announcement. Show that the real exchange rate is then:

$$q_0 - \bar{q} = e_0 - e_0^{flex}$$

Using this result, show that:

$$e_0 - e_0^{flex} = \frac{1+\eta\psi\delta}{\phi\delta+\eta\psi\delta}\left[\left(e_0^{flex}\right)' - e_0^{flex}\right]$$

Answer: The real exchange rate after the announcement is:

$$\begin{aligned} q_0 &= e_0 - p_0 \\ q_0 &= e_0 - e_0^{flex} + \bar{q} \\ q_0 - \bar{q} &= e_0 - e_0^{flex} \end{aligned}$$

Next, we use the expression of the nominal exchange rate under sticky prices:

$$\begin{aligned} e_0 - \bar{q} &= \frac{1-\phi\delta}{1+\eta\psi\delta}\left(q_0 - \bar{q}\right) + \frac{1}{1+\eta}\sum_{s=0}^{\infty}\left(\frac{\eta}{1+\eta}\right)^s m_s \\ e_0 - \bar{q} &= \frac{1-\phi\delta}{1+\eta\psi\delta}\left(q_0 - \bar{q}\right) + \frac{1}{1+\eta}\sum_{s=t}^{\infty}\left(\frac{\eta}{1+\eta}\right)^{s-t}\left[m_0 + s\mu'\right] \\ e_0 &= \bar{q} + \frac{1-\phi\delta}{1+\eta\psi\delta}\left(q_0 - \bar{q}\right) + m_0 + \eta\mu' \\ e_0 &= \bar{q} + m_0 + \eta\mu' + \frac{1-\phi\delta}{1+\eta\psi\delta}\left(e_0 - e_0^{flex}\right) \end{aligned}$$

Using the flexible exchange rate just after the announcement:

$$\begin{aligned} e_0 &= \left(e_0^{flex}\right)' + \frac{1-\phi\delta}{1+\eta\psi\delta}\left(e_0 - e_0^{flex}\right) \\ \left[1 - \frac{1-\phi\delta}{1+\eta\psi\delta}\right] e_0 &= \left(e_0^{flex}\right)' - \frac{1-\phi\delta}{1+\eta\psi\delta} e_0^{flex} \\ \frac{\eta\psi\delta+\phi\delta}{1+\eta\psi\delta} e_0 &= \left(e_0^{flex}\right)' - \frac{1-\phi\delta}{1+\eta\psi\delta} e_0^{flex} \end{aligned}$$

$$
\begin{aligned}
\frac{\eta\psi\delta+\phi\delta}{1+\eta\psi\delta}\left(e_0-e_0^{flex}\right)+\frac{\eta\psi\delta+\phi\delta}{1+\eta\psi\delta}e_0^{flex} &= \left(e_0^{flex}\right)'-\frac{1-\phi\delta}{1+\eta\psi\delta}e_0^{flex}\\
\frac{\eta\psi\delta+\phi\delta}{1+\eta\psi\delta}\left(e_0-e_0^{flex}\right) &= \left(e_0^{flex}\right)'-\left(\frac{1-\phi\delta}{1+\eta\psi\delta}+\frac{\eta\psi\delta+\phi\delta}{1+\eta\psi\delta}\right)e_0^{flex}\\
\frac{\eta\psi\delta+\phi\delta}{1+\eta\psi\delta}\left(e_0-e_0^{flex}\right) &= \left(e_0^{flex}\right)'-e_0^{flex}\\
e_0-e_0^{flex} &= \frac{1+\eta\psi\delta}{\phi\delta+\eta\psi\delta}\left[\left(e_0^{flex}\right)'-e_0^{flex}\right]
\end{aligned}
$$

If $\phi\delta<1$, a shock that pushes $\left(e_0^{flex}\right)'$ above e_0^{flex} pushes e_0 even more above e_0^{flex}.

3 Exchange rate dynamics

Question: Recall the following difference between the exchange rate and the level that prevails under flexible prices:

$$
e_t-\left(e_t^{flex}\right)' = \frac{1-\phi\delta}{1+\eta\psi\delta}\left(q_t-\bar{q}\right)
$$

Show that:

$$
e_0-\left(e_0^{flex}\right)' = \frac{1-\phi\delta}{1+\eta\psi\delta}\left(q_0-\bar{q}\right)
$$

Using these results, and the autoregressive dynamics for the real exchange rate seen in class, show that:

$$
e_t-\left(e_t^{flex}\right)' = (1-\psi\delta)^t\left(e_0-\left(e_0^{flex}\right)'\right)
$$

as well as:

$$
e_0-\left(e_0^{flex}\right)' = \frac{1-\phi\delta}{\phi\delta+\eta\psi\delta}\eta\left(\mu'-\mu\right)
$$

Answer: The starting point comes from the general exchange rate relation:

$$
\begin{aligned}
e_t-\bar{q} &= \frac{1-\phi\delta}{1+\eta\psi\delta}\left(q_t-\bar{q}\right)+\frac{1}{1+\eta}\sum_{s=t}^{\infty}\left(\frac{\eta}{1+\eta}\right)^{s-t}m_s\\
e_t &= \frac{1-\phi\delta}{1+\eta\psi\delta}\left(q_t-\bar{q}\right)+\left(e_t^{flex}\right)'
\end{aligned}
$$

Recall that:

$$
\begin{aligned}
q_0-\bar{q} &= e_0-e_0^{flex}\\
q_0-\bar{q} &= \left(e_0-\left(e_0^{flex}\right)'\right)+\left(\left(e_0^{flex}\right)'-e_0^{flex}\right)\\
q_0-\bar{q} &= \left(e_0-\left(e_0^{flex}\right)'\right)+\frac{\phi\delta+\eta\psi\delta}{1+\eta\psi\delta}\left(q_0-\bar{q}\right)\\
\left(1-\frac{\phi\delta+\eta\psi\delta}{1+\eta\psi\delta}\right)\left(q_0-\bar{q}\right) &= \left(e_0-\left(e_0^{flex}\right)'\right)\\
\frac{1-\phi\delta}{1+\eta\psi\delta}\left(q_0-\bar{q}\right) &= e_0-\left(e_0^{flex}\right)'
\end{aligned}
$$

Recall the real exchange rate dynamics we derived in class:

$$q_t - \bar{q} = (1 - \psi\delta)^t (q_0 - \bar{q})$$

Using this autoregressive relation, we get:

$$\begin{aligned}
e_t - \left(e_t^{flex}\right)' &= \frac{1 - \phi\delta}{1 + \eta\psi\delta} (q_t - \bar{q}) \\
e_t - \left(e_t^{flex}\right)' &= (1 - \psi\delta)^t \frac{1 - \phi\delta}{1 + \eta\psi\delta} (q_0 - \bar{q}) \\
e_t - \left(e_t^{flex}\right)' &= (1 - \psi\delta)^t \left(e_0 - \left(e_0^{flex}\right)'\right)
\end{aligned}$$

The initial exchange rate gap is:

$$\begin{aligned}
e_0 - \left(e_0^{flex}\right)' &= \left(e_0 - e_0^{flex}\right) - \left[\left(e_0^{flex}\right)' - e_0^{flex}\right] \\
e_0 - \left(e_0^{flex}\right)' &= \frac{1 + \eta\psi\delta}{\phi\delta + \eta\psi\delta} \left[\left(e_0^{flex}\right)' - e_0^{flex}\right] - \left[\left(e_0^{flex}\right)' - e_0^{flex}\right] \\
e_0 - \left(e_0^{flex}\right)' &= \frac{1 - \phi\delta}{\phi\delta + \eta\psi\delta} \left[\left(e_0^{flex}\right)' - e_0^{flex}\right] \\
e_0 - \left(e_0^{flex}\right)' &= \frac{1 - \phi\delta}{\phi\delta + \eta\psi\delta} \left[(\bar{q} + m_0 + \eta\mu') - (\bar{q} + m_0 + \eta\mu)\right] \\
e_0 - \left(e_0^{flex}\right)' &= \frac{1 - \phi\delta}{\phi\delta + \eta\psi\delta} \eta (\mu' - \mu)
\end{aligned}$$

If $\phi\delta < 1$, the exchange rate depreciates by more than it would under flexible prices: $e_0 > \left(e_0^{flex}\right)'$.

4 Interest rates

Question: Recall the autoregressive relation of the nominal exchange rate gap:

$$e_t - \left(e_t^{flex}\right)' = (1 - \psi\delta)^t \left(e_0 - \left(e_0^{flex}\right)'\right)$$

Show that:

$$e_{t+1} - e_t = \left(e_{t+1}^{flex}\right)' - \left(e_t^{flex}\right)' - \psi\delta (1 - \psi\delta)^t \left(e_0 - \left(e_0^{flex}\right)'\right)$$

Using the interest parity, show that:

$$i_{t+1} - (i^* + \mu) = (\mu' - \mu) - \psi\delta (1 - \psi\delta)^t \left(e_0 - \left(e_0^{flex}\right)'\right)$$

and:

$$i_1 - (i^* + \mu) = \phi\delta \frac{1 + \eta\psi\delta}{\phi\delta + \eta\psi\delta} (\mu' - \mu)$$

Answer: Take the autoregressive relation in two subsequent periods:

$$
\begin{aligned}
\left[e_{t+1}-\left(e_{t+1}^{flex}\right)'\right]-\left[e_t-\left(e_t^{flex}\right)'\right] &= \left[(1-\psi\delta)^{t+1}-(1-\psi\delta)^t\right]\left(e_0-\left(e_0^{flex}\right)'\right) \\
\left[e_{t+1}-\left(e_{t+1}^{flex}\right)'\right]-\left[e_t-\left(e_t^{flex}\right)'\right] &= -\psi\delta\,(1-\psi\delta)^t\left(e_0-\left(e_0^{flex}\right)'\right) \\
e_{t+1}-e_t &= \left(e_{t+1}^{flex}\right)'-\left(e_t^{flex}\right)'-\psi\delta\,(1-\psi\delta)^t\left(e_0-\left(e_0^{flex}\right)'\right)
\end{aligned}
$$

We use this in the interest parity:

$$
\begin{aligned}
i_{t+1} &= i^*+e_{t+1}-e_t \\
i_{t+1} &= i^*+\left(e_{t+1}^{flex}\right)'-\left(e_t^{flex}\right)'-\psi\delta\,(1-\psi\delta)^t\left(e_0-\left(e_0^{flex}\right)'\right) \\
i_{t+1} &= i^*+m_{t+1}-m_t-\psi\delta\,(1-\psi\delta)^t\left(e_0-\left(e_0^{flex}\right)'\right) \\
i_{t+1} &= i^*+\mu'-\psi\delta\,(1-\psi\delta)^t\left(e_0-\left(e_0^{flex}\right)'\right) \\
i_{t+1}-(i^*+\mu) &= (\mu'-\mu)-\psi\delta\,(1-\psi\delta)^t\left(e_0-\left(e_0^{flex}\right)'\right)
\end{aligned}
$$

We evaluate this relation at $t=0$:

$$
\begin{aligned}
i_1-(i^*+\mu) &= (\mu'-\mu)-\psi\delta\left(e_0-\left(e_0^{flex}\right)'\right) \\
i_1-(i^*+\mu) &= (\mu'-\mu)-\psi\delta\frac{1-\phi\delta}{\phi\delta+\eta\psi\delta}\eta\,(\mu'-\mu) \\
i_1-(i^*+\mu) &= \phi\delta\frac{1+\eta\psi\delta}{\phi\delta+\eta\psi\delta}\,(\mu'-\mu)
\end{aligned}
$$

A shock depreciating the currency ($\mu'>\mu$) thus clearly increases the nominal interest rate, by contrast to a shock on the money level.