PS 1 due 30 September 2025 – Solutions

Version: May 19, 2025

1 RBC model without Household labour

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Q1. *Show that the first order conditions associated with this problem are:* There is nothing to add here.

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Q2. *Show that the first order conditions of the above maximization problem are as follows:* There is nothing to add here.

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Q3. *Some these equations are as follows; you must fill in the remainder.* The equations are

$$
\begin{aligned}
&\frac{C_t}{C_{t-1}} = \beta \left(R_t + 1 - \delta\right) \\
&Y_t = A_t K_{t-1}^{\alpha} L_t^{1-\alpha} \\
&W_t = \frac{1-\gamma}{\gamma} \frac{C_t}{(1-L_t)} \\
&I_t = Y_t - C_t \\
&K_t = I_t + (1-\delta) K_{t-1} \\
&W_t = (1-\alpha) \frac{Y_t}{L_t} \\
&R_t = \alpha \frac{Y_t}{K_t} \\
&\log\left(A_t\right) = \rho \log\left(A_{t-1}\right) + e_t
\end{aligned}
$$

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Q4. *Calculate the steady-state values of the other model variables and check that your values are the same as those found in Table 2 below.* There is nothing to add here.

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Q5. *Using the attached skeleton of a Dynare .mod file for this model, simulate it with the parameters indicated in Table 1 and comment on your results.* $Note$ *that for obscure technical reasons, Dynare will not solve the model unless you* $lead$ *the variables* C *and* R *of the Euler equation by one period each (try it* *without so doing to see this).* The model I used to produce the results below is as follows:

```
(J:\MyCourseDSGEs2025\HW\PS1\RBC_4PS2_Q5.mod)

% Basic RBC Model 1 for PS1, Q5

var Y, C, I, K, L, W, R, A;
varexo e;
parameters
alpha ${\alpha}$ (long_name= 'Capital share')
beta ${\beta}$ (long_name= 'Discount factor')
delta ${\delta}$ (long_name= 'Depreciation rate')
gamma ${\gamma}$ (long_name= 'Preferences parameter')
rho ${\rho}$ (long_name= 'TFP persistence')
;
% Initial calibration
alpha = 0.35;
beta  = 0.97;
delta = 0.06;
gamma = 0.40;
rho   = 0.95;

model;
(C(+1)/C) = beta*(R(+1)+(1-delta)); % Euler Equation <== Note the lead on C and R
Y = A*(K(-1)^alpha)*(L^(1-alpha)); % Production Function
W=((1-gamma)/gamma)*C/(1-L); % Labour Supply Function
I = Y-C; % Resource Constraint
K = I+(1-delta)*K(-1);
W = (1-alpha)*Y/L;
R = alpha*Y/K;
log(A) = rho*log(A(-1))+ e;
end;

% Steady-state values
initval;
Y = 0.75;
C = 0.57;
L = 0.36;
K = 2.87;
I = 0.17;
W = (1-alpha)*Y/L;
R = alpha*Y/K;
A = 1;
e = 0;
end;

resid;
steady;
check;

shocks;
var e; stderr 0.01;
end;

stoch_simul(Tex,irf=40);

write_latex_parameter_table;
write_latex_dynamic_model;
write_latex_definitions;
collect_latex_files;
if system(['pdflatex -halt-on-error -interaction=batchmode ' M_.fname '_TeX_binder.tex'])
    error('TeX-File did not compile.')
end
```

This produces the nice model description below:

$$\frac{C_{t+1}}{C_t} = \beta\ (R_{t+1} + 1 - \delta)$$

$$Y_t = A_t\, {K_{t-1}}^{\alpha}\, {L_t}^{1-\alpha}$$

$$W_t = \frac{C_t\, \frac{1-\gamma}{\gamma}}{1 - L_t}$$

$$I_t = Y_t - C_t$$

$$K_t = I_t + (1 - \delta)\ K_{t-1}$$

$$W_t = \frac{Y_t\ (1 - \alpha)}{L_t}$$

$$R_t = \frac{Y_t\, \alpha}{K_t}$$

$$log\,(A_t) = \rho\, log\,(A_{t-1}) + e_t$$

with

Table 4: Parameter Values

Parameter	Value	Description
α	0.350	Capital share
β	0.970	Discount factor
δ	0.060	Depreciation rate
γ	0.400	Preferences parameter
ρ	0.950	TFP persistence

and the IRFs below:

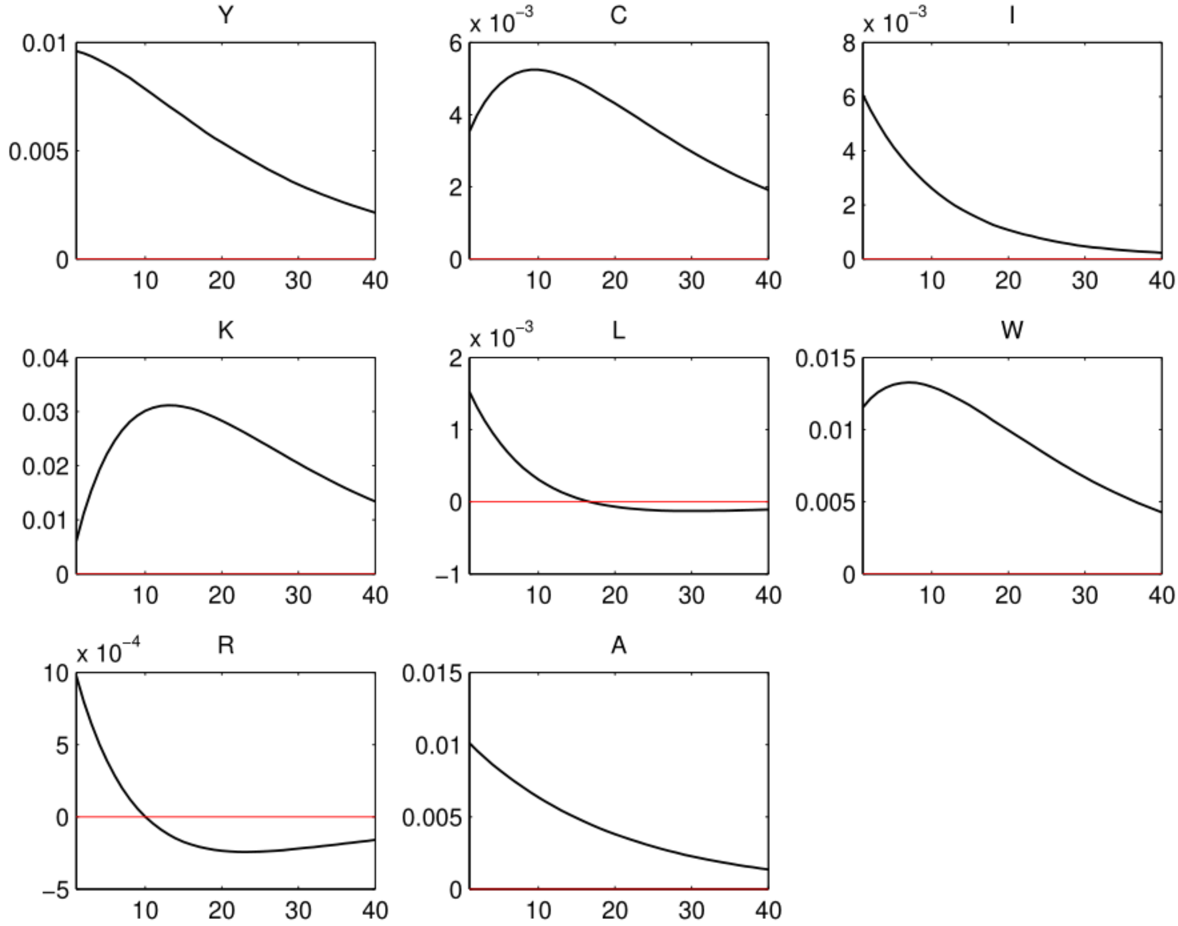

Figure 1: Impulse response functions (orthogonalized shock to e).

The above figure shows the effects of this shock on the variables of the model over 40 quarters/10 years after the shock occurs. We assume that initially there is an increase in the FFP by 1pc, but with a relatively high persistence (degree of autocorrelation) assumed for the TFP shock process. It can be observed that the level of production Y increases on impact, deviating positively from its steady-state value. Subsequently, the positive deviation begins to decrease, but showing significant persistence over time. In fact, after 20 periods have elapsed, the level of production is still 0.5pc higher than its steady-state value. Note for the factors of production that the level of production increases initially without changing the amount of capital (K) and with a slight increase in labour usage (L). This is because the shock involves an instantaneous increase in TFP, which is transferred directly to production.

Second, consumption (C) also increases significantly relative to its steady-state value, but to a lesser extent than for production. Subsequently, this deviation continues increasing until it reaches a maximum after 10 periods, to decrease later: the response of consumption in the face of this shock is thus bell-shaped. This consumption behavior is explained by the behavior of both the production level and investment (I). Investment increases instantaneously as a result of the productivity shock, and then decreases rapidly towards its steady-state value.

For its part, the capital stock of the economy (K) also shows a bell-shaped impulse-response function. Thus, initially the increase in investment also causes an increase of capital stock (net investment is positive). However, as investment decreases, the capital stock reaches a maximum from which it begins to descend, but always remains above its steady-state value.

The effect on employment (L) is very limited. In fact, employment initially increases, although by a very small percentage (approximately 0.2pc) and subsequently decreases, even reaching values slightly below its steady-state level. Finally, regarding factor prices, the wage rate (W) increases on impact, until reaching a maximum, from which it decreases, but remains always above its steady-state value, reflecting the positive effect of the aggregate productivity shock on the marginal productivity of labour. For its part, the (net) real interest rate (R) experiences a slight positive variation initially, given the increase in the marginal productivity of capital, but subsequently decreases very slightly below its steady-state value as a consequence of the capital accumulation process generated by the change in investment.
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2 RBC model with Household labour

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Q6. *Show that the first order conditions associated with the firm's problem are:* There is nothing to add here.

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Q7. *Show that the first order conditions of the above maximization problem are as follows:* There is nothing to add here.

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Q8. *Some these equations are as follows; you must fill in the remainder.* The equations are:

$$\frac{(1-\gamma)}{1-L_{m,t}-L_{h,t}} = \frac{\gamma\omega C_{m,t}^{\eta-1}}{\left[\omega C_{m,t}^{\eta}+(1-\omega)C_{h,t}^{\eta}\right]} W_t$$

$$\beta\frac{\gamma\omega C_{m,t+1}^{\eta-1}}{\left[\omega C_{m,t+1}^{\eta}+(1-\omega)C_{h,t+1}^{\eta}\right]}\left(R_{t+1}+1-\delta\right) = \frac{\gamma\omega C_{m,t}^{\eta-1}}{\left[\omega C_{m,t}^{\eta}+(1-\omega)C_{h,t}^{\eta}\right]}$$

$$\frac{(1-\gamma)}{1-L_{m,t}-L_{h,t}} = \frac{\gamma(1-\omega)C_{h,t}^{\eta-1}}{\left[\omega C_{m,t}^{\eta}+(1-\omega)C_{h,t}^{\eta}\right]}\theta B_t L_{h,t}^{\theta-1}$$

$$Y_t = C_t + I_t$$

$$Y_t = A_t K_t^{\alpha} L_{m,t}^{1-\alpha} C_{h,t} = B_t L_{h,t}^{\theta}$$

$$K_t = (1-\delta)K_{t-1} + I_t$$

$$I_t = Y_t - C_{m,t}$$

$$W_t = (1-\alpha)\frac{Y_t}{L_{m,t}}$$

$$R_t = \alpha\frac{Y_t}{K_t}$$

$$Ctot_t = \omega C_{m,t}^{\eta} + (1-\omega)C_{h,t}^{\eta}$$

$$\ln A_t = (1-\rho_A)\ln\bar{A} + \rho_A \ln A_{t-1} + \varepsilon_t^A$$

$$\ln B_t = (1-\rho_B)\ln\bar{B} + \rho_B \ln B_{t-1} + \varepsilon_t^B$$

==

Q9. *Calculate the steady-state values of the other model variables and check that your values are the same as those found in Table 2 below.* There is nothing to add here.

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Q10. *Using the attached skeleton of a Dynare .mod file for this model, simulate it with the parameters indicated in Table 1 and comment on your results.* The model I used to produce the results below is shown below.

```
(J:\MyCourseDSGEs2025\HW\PS1\RBC_4PS2_Q10.mod)
% Basic RBC Model with Household Work for PS1, Q10

var Y, Cm, Ch, I, K, Lm, Lh, W, R, A, B, Ctot, Ltot; % Ch, Lh = home activities; Cm, Lm = market activities
varexo e, u;
parameters
alpha ${\alpha}$ (long_name= 'Capital share')
beta ${\beta}$ (long_name= 'Discount factor')
delta ${\delta}$ (long_name= 'Depreciation rate')
gamma ${\gamma}$ (long_name= 'Preferences parameter')
omega ${\omega}$ (long_name= 'Rel wt mkt vs home')
eta ${\eta}$ (long_name= 'Goods substitution parameter')
theta ${\theta}$ (long_name= 'Home production parameter')
rho1 ${\rho1}$ (long_name= 'TFP persistence')
rho2 ${\rho2}$ (long_name= 'Home production persistence')
;
% Initial calibration
alpha = 0.35;
beta  = 0.97;
delta = 0.06;
gamma = 0.60;
omega = 0.45;
eta   = 0.80;
theta = 0.8;
rho1  = 0.95;
rho2  = 0.95;

model;
gamma*omega*(Cm^(eta-1))/(omega*Cm^eta+(1-omega)*Ch^eta)=(1-gamma)/(W*(1-Lm-Lh));
gamma*(1-omega)*(Ch^(eta-1))/(omega*Cm^eta+(1-omega)*Ch^eta)=(1-gamma)/(theta*B*Lh^(theta-1)*(1-Lm-Lh));
((Cm^(eta-1))/(omega*Cm^eta+(1-omega)*Ch^eta))/((Cm(+1)^(eta-1))/(omega*Cm(+1)^eta+(1-omega)*Ch(+1)^eta))=beta*(R(+1)+1-delta);
Y = A*(K(-1)^alpha)*(Lm^(1-alpha));
Ch = B*Lh^theta;
K = (Y-Cm)+(1-delta)*K(-1);
I = Y-Cm;
W = (1-alpha)*Y/Lm;
R = alpha*Y/K;
Ctot = (omega*Cm^eta+(1-omega)*Ch^eta)^(1/eta);
Ltot = Lm + Lh;
log(A) = rho1*log(A(-1))+e;
log(B) = rho2*log(B(-1))+u;
end;

% Steady-state values
initval;
Y  = 1;
Cm = 0.750;
Ch = 0.2;
Lm = 0.3;
Lh = 0.1;
K = 3.5;
I = 0.25;
W = (1-alpha)*Y/Lm;
R = alpha*Y/K;
A = 1;
B = 1;
e = 0;
u = 0;
end;

resid;
steady;
check;

shocks;
var e; stderr 0.01;
var u; stderr 0.01;
end;

stoch_simul(Tex,irf=40);

%----------------------------------------------------------------
% generate LaTeX output
% use only if you have LaTex installed
%----------------------------------------------------------------

write_latex_dynamic_model;
write_latex_parameter_table;
write_latex_definitions;
write_latex_prior_table;
collect_latex_files;
if system(['pdflatex -halt-on-error -interaction=batchmode ' M_.fname '_TeX_binder.tex'])
    error('TeX-File did not compile.')
```

```
end;
```

This produces the nice model description below:

$$\frac{\gamma\,\omega\,Cm_t^{\eta-1}}{\omega\,Cm_t^{\eta}+(1-\omega)\ Ch_t^{\eta}}=\frac{1-\gamma}{W_t\ (1-Lm_t-Lh_t)} \tag{1}$$

$$\frac{\gamma\ (1-\omega)\ Ch_t^{\eta-1}}{\omega\,Cm_t^{\eta}+(1-\omega)\ Ch_t^{\eta}}=\frac{1-\gamma}{(1-Lm_t-Lh_t)\ \theta\,B_t\,Lh_t^{\theta-1}} \tag{2}$$

$$\frac{\frac{Cm_t^{\eta-1}}{\omega\,Cm_t^{\eta}+(1-\omega)\,Ch_t^{\eta}}}{\frac{Cm_{t+1}^{\eta-1}}{\omega\,Cm_{t+1}^{\eta}+(1-\omega)\,Ch_{t+1}^{\eta}}}=\beta\ (1+R_{t+1}-\delta) \tag{3}$$

$$Y_t=A_t\,K_{t-1}^{\alpha}\,Lm_t^{1-\alpha} \tag{4}$$

$$Ch_t=B_t\,Lh_t^{\theta} \tag{5}$$

$$K_t=Y_t-Cm_t+K_{t-1}\ (1-\delta) \tag{6}$$

$$I_t=Y_t-Cm_t \tag{7}$$

$$W_t=\frac{Y_t\ (1-\alpha)}{Lm_t} \tag{8}$$

$$R_t=\frac{Y_t\,\alpha}{K_t} \tag{9}$$

$$Ctot_t=\left(\omega\,Cm_t^{\eta}+(1-\omega)\ Ch_t^{\eta}\right)^{\frac{1}{\eta}} \tag{10}$$

$$Ltot_t=Lm_t+Lh_t \tag{11}$$

$$\log\left(A_t\right)=\rho 1\ \log\left(A_{t-1}\right)+e_t \tag{12}$$

$$\log\left(B_t\right)=\rho 2\ \log\left(B_{t-1}\right)+u_t \tag{13}$$

with

Table 4: Parameter Values

Parameter	Value	Description
α	0.350	Capital share
β	0.970	Discount factor
δ	0.060	Depreciation rate
γ	0.600	Preferences parameter
ω	0.450	Rel wt mkt vs home
η	0.800	Goods substitution parameter
θ	0.800	Home production parameter
$\rho 1$	0.950	TFP persistence
$\rho 2$	0.950	Home production persistence

The second model contains two shocks, one each to productivity in the market goods sector and the household goods sector.

As there was only a shock to the aggregate goods sector in the model of Part 1, it is possible to compare the results only for this shock.

The figure below shows such a comparison for aggregate output and consumption, and for the wage rate (which are all three comparable in the two models).

Also shown is the response of Labour devoted to household chores in the model of Part 2, as compared to that of total Labour in the model of Part 1.

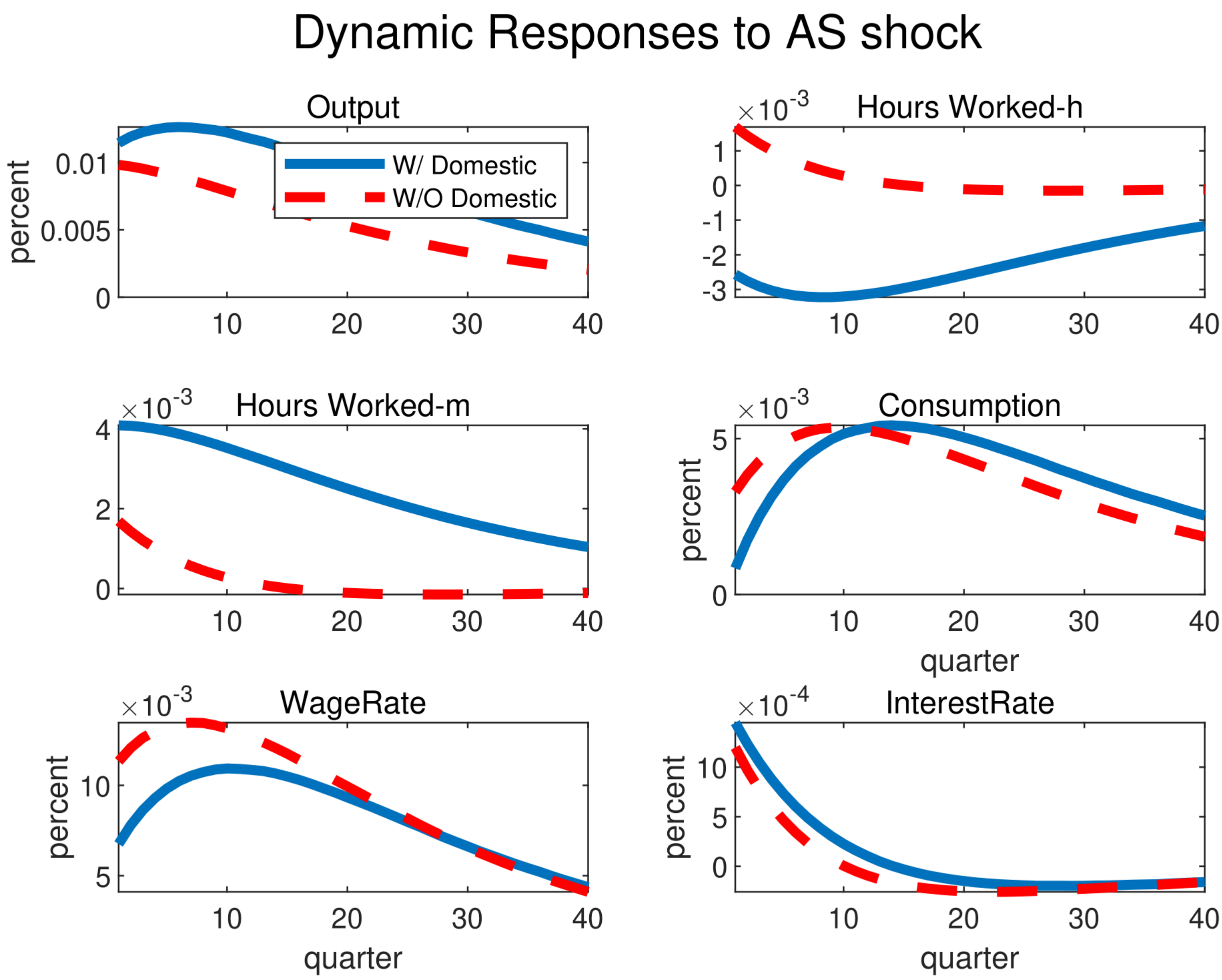
The code used to produce the figure below is:

```
(J:\MyCourseDSGEs2025\HW\PS1\run_IRF_comp_PS2.m)
% NB: This version includes response of Lh to prod shock (compared to L in orig model)

dynare RBC_4PS2_Q10
oo_EHL=oo_;
dynare RBC_4PS2_Q5 noclearall
oo_SGU=oo_;

hh=figure('Name','Dynamic Responses to AS shock');
subplot(3,2,1)
plot(1:options_.irf,oo_EHL.irfs.Y_e,'-',1:options_.irf,oo_SGU.irfs.Y_e,'r--','LineWidth',3)
xlim([1 options_.irf]);
ylabel('percent')
title('Output')
ll=legend('W/ Domestic','W/O Domestic');
set(ll,'Location','NorthEast');
subplot(3,2,2)
plot(1:options_.irf,oo_EHL.irfs.Lh_e,'-',1:options_.irf,oo_SGU.irfs.L_e,'r--','LineWidth',3)
xlim([1 options_.irf]);
title('Hours Worked-h')
subplot(3,2,3)
plot(1:options_.irf,oo_EHL.irfs.Lm_e,'-',1:options_.irf,oo_SGU.irfs.L_e,'r--','LineWidth',3)
xlim([1 options_.irf]);
title('Hours Worked-m')
subplot(3,2,4)
plot(1:options_.irf,oo_EHL.irfs.Ctot_e,'-',1:options_.irf,oo_SGU.irfs.C_e,'r--','LineWidth',3)
xlim([1 options_.irf]);
xlabel('quarter')
ylabel('percent')
title('Consumption')
```

```
subplot(3,2,5)
plot(1:options_.irf,oo_EHL.irfs.W_e,'-',1:options_.irf,oo_SGU.irfs.W_e,'r--','LineWidth',3)
xlim([1 options_.irf]);
xlabel('quarter')
ylabel('percent')
title('WageRate')
subplot(3,2,6)
plot(1:options_.irf,oo_EHL.irfs.R_e,'-',1:options_.irf,oo_SGU.irfs.R_e,'r--','LineWidth',3)
xlim([1 options_.irf]);
xlabel('quarter')
ylabel('percent')
title('InterestRate')
set(findall(hh,'-property','FontWeight'),'FontWeight','normal')
```



Now the impact of the TFP shock on output (blue) is greater than what would be obtained without a household goods sector (red). This is a consequence of the consideration of the sector of domestic goods.

The key element is in the behaviour of employment, derived from agents' time decisions. Now the hours worked in the market (Lm) increase significantly, given that the productivity shock causes an increase in the profitability of work. In this case there is a substitution effect of hours of market work (Lm) for hours of work in the home (Lh), which decrease. Thus, the agent can increase the number of hours worked in the market without reducing leisure hours.

In summary, the inclusion of the household goods sector increases the effects of an aggregate TFP shock, given that the agent can, in the face of such a shock, increase the number of hours worked in the market, reducing the number of hours dedicated to household chores. The amplification of the effects of the shock depends of course on the degree of substitution between both types of working hours or, equivalently, the degree of substitution between market goods and household goods.