Chapter 5
REAL-BUSINESS-CYCLE THEORY

5.1 Introduction: Some Facts about Economic Fluctuations

Modern economies undergo significant short-run variations in aggregate output and employment. At some times, output and employment are falling and unemployment is rising; at others, output and employment are rising rapidly and unemployment is falling. For example, the U.S. economy underwent a severe contraction in 2007–2009. From the fourth quarter of 2007 to the second quarter of 2009, real GDP fell 3.8 percent, the fraction of the adult population employed fell by 3.1 percentage points, and the unemployment rate rose from 4.8 to 9.3 percent. In contrast, over the previous 5 years (that is, from the fourth quarter of 2002 to the fourth quarter of 2007), real GDP rose at an average annual rate of 2.9 percent, the fraction of the adult population employed rose by 0.3 percentage points, and the unemployment rate fell from 5.9 to 4.8 percent.

Understanding the causes of aggregate fluctuations is a central goal of macroeconomics. This chapter and the two that follow present the leading theories concerning the sources and nature of macroeconomic fluctuations. Before we turn to the theories, this section presents a brief overview of some major facts about short-run fluctuations. For concreteness, and because of the central role of the U.S. experience in shaping macroeconomic thought, the focus is on the United States.

A first important fact about fluctuations is that they do not exhibit any simple regular or cyclical pattern. Figure 5.1 plots seasonally adjusted real GDP per person since 1947, and Table 5.1 summarizes the behavior of real GDP in the eleven postwar recessions.[1] The figure and table show that output declines vary considerably in size and spacing. The falls in real GDP range from 0.3 percent in 2000–2001 to 3.8 percent in the recent recession.

[1] The formal dating of recessions for the United States is not based solely on the behavior of real GDP. Instead, recessions are identified judgmentally by the National Bureau of Economic Research (NBER) on the basis of various indicators. For that reason, the dates of the official NBER peaks and troughs differ somewhat from the dates shown in Table 5.1.

TABLE 5.1 Recessions in the United States since World War II

Year and quarter of peak in real GDP	Number of quarters until trough in real GDP	Change in real GDP, peak to trough
1948:4	2	−1.7%
1953:2	3	−2.6
1957:3	2	−3.7
1960:1	3	−1.6
1970:3	1	−1.1
1973:4	5	−3.2
1980:1	2	−2.2
1981:3	2	−2.9
1990:2	3	−1.4
2000:4	1	−0.3
2008:2	4	−3.8

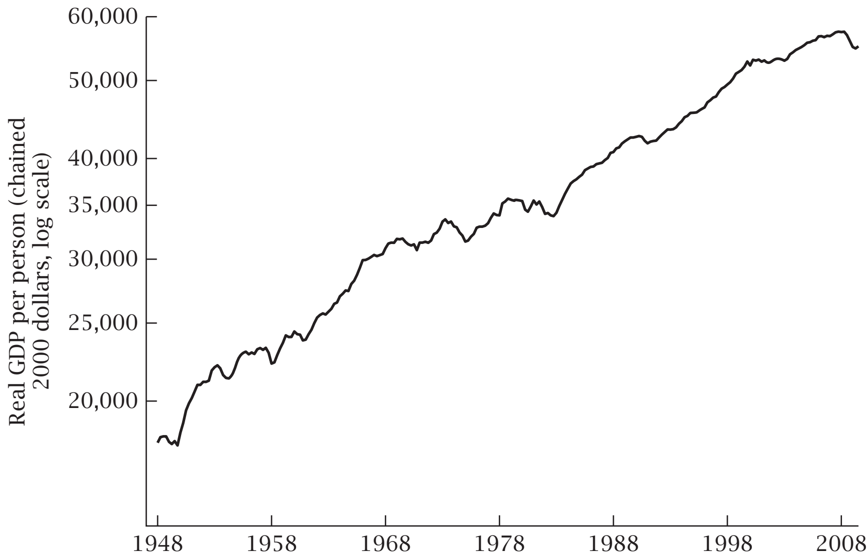

FIGURE 5.1 U.S. real GDP per person, 1947:1–2009:3

The times between the end of one recession and the beginning of the next range from 4 quarters in 1980–1981 to almost 10 years in 1991–2000. The patterns of the output declines also vary greatly. In the 1980 recession, over 90 percent of the overall decline of 2.2 percent took place in a single quarter; in the 1960 recession, output fell for a quarter, then rose slightly, and then fell again; and in the 1957–1958 and 1981–1982 recessions, output fell sharply for two consecutive quarters.

Because output movements are not regular, the prevailing view is that the economy is perturbed by disturbances of various types and sizes at more or less random intervals, and that those disturbances then propagate through

TABLE 5.2 Behavior of the components of output in recessions

Component of GDP	Average share in GDP	Average share in fall in GDP in recessions relative to normal growth
Consumption		
Durables	8.9%	14.6%
Nondurables	20.6	9.7
Services	35.2	10.9
Investment		
Residential	4.7	10.5
Fixed nonresidential	10.7	21.0
Inventories	0.6	44.8
Net exports	−1.0	−12.7
Government purchases	20.2	1.3

the economy. Where the major macroeconomic schools of thought differ is in their hypotheses concerning these shocks and propagation mechanisms.[2]

A second important fact is that fluctuations are distributed very unevenly over the components of output. Table 5.2 shows both the average shares of each of the components in total output and their average shares in the declines in output (relative to its normal growth) in recessions. As the table shows, even though inventory investment on average accounts for only a trivial fraction of GDP, its fluctuations account for close to half of the shortfall in growth relative to normal in recessions: inventory accumulation is on average large and positive at peaks, and large and negative at troughs. Consumer purchases of durable goods, residential investment (that is, housing), and fixed nonresidential investment (that is, business investment other than inventories) also account for disproportionate shares of output fluctuations. Consumer purchases of nondurables and services, government purchases, and net exports are relatively stable.[3] Although there is some variation across recessions, the general pattern shown in Table 5.2 holds in most. And the same components that decline disproportionately when aggregate output is falling also rise disproportionately when output is growing at above-normal rates.

A third set of facts involves asymmetries in output movements. There are no large asymmetries between rises and falls in output; that is, output growth is distributed roughly symmetrically around its mean. There does, however, appear to be asymmetry of a second type: output seems to be

[2] There is an important exception to the claim that fluctuations are irregular: there are large seasonal fluctuations that are similar in many ways to conventional business-cycle fluctuations. See Barsky and Miron (1989) and Miron (1996).

[3] The entries for net exports indicate that they are on average negative over the postwar period, and that they typically grow—that is, become less negative—during recessions.

characterized by relatively long periods when it is slightly above its usual path, interrupted by brief periods when it is relatively far below.[4]

A fourth set of facts concerns changes in the magnitude of fluctuations over time. One can think of the macroeconomic history of the United States since the late 1800s as consisting of four broad periods: the period before the Great Depression; the Depression and World War II; the period from the end of World War II to about the mid-1980s; and the mid-1980s to the present. Although our data for the first period are highly imperfect, it appears that fluctuations before the Depression were only moderately larger than in the period from World War II to the mid-1980s. Output movements in the era before the Depression appear slightly larger, and slightly less persistent, than in the period following World War II; but there was no sharp change in the character of fluctuations. Since such features of the economy as the sectoral composition of output and role of government were very different in the two eras, this suggests either that the character of fluctuations is determined by forces that changed much less over time, or that there was a set of changes to the economy that had roughly offsetting effects on overall fluctuations.[5]

The remaining two periods are the extremes. The collapse of the economy in the Depression and the rebound of the 1930s and World War II dwarf any fluctuations before or since. Real GDP in the United States fell by 27 percent between 1929 and 1933, with estimated unemployment reaching 25 percent in 1933. Over the next 11 years, real GDP rose at an average annual rate of 10 percent; as a result, unemployment in 1944 was 1.2 percent. Finally, real GDP declined by 13 percent between 1944 and 1947, and unemployment rose to 3.9 percent.

In contrast, the period following the recovery from the 1981–1982 recession was one of unprecedented macroeconomic stability (McConnell and Perez-Quiros, 2000). Indeed, this period has come to be known as the "Great Moderation." From 1982 to 2007, the United States underwent only two mild recessions, separated by the longest expansion on record.

The crisis that began in 2007 represents a sharp change from the economic stability of recent decades. But one severe recession is not enough to bring average volatility since the mid-1980s even close to its average in the early postwar decades. And it is obviously too soon to know whether the recent events represent the end of the Great Moderation or a one-time aberration.

Finally, Table 5.3 summarizes the behavior of some important macroeconomic variables during recessions. Not surprisingly, employment falls and

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[4] More precisely, periods of extremely low growth quickly followed by extremely high growth are much more common than periods exhibiting the reverse pattern. See, for example, Sichel (1993).

[5] For more on fluctuations before the Great Depression, see C. Romer (1986, 1989, 1999) and Davis (2004).

TABLE 5.3 Behavior of some important macroeconomic variables in recessions

Variable	Average change in recessions	Number of recessions in which variable falls
Real GDP*	−4.1%	11/11
Employment*	−3.1%	11/11
Unemployment rate (percentage points)	+1.8	0/11
Average weekly hours, production workers, manufacturing	−2.3%	11/11
Output per hour, nonfarm business*	−1.7%	10/11
Inflation (GDP deflator; percentage points)	−0.3	5/11
Real compensation per hour, nonfarm business*	−0.5%	7/11
Nominal interest rate on 3-month Treasury bills (percentage points)	−1.6	10/11
Ex post real interest rate on 3-month Treasury bills (percentage points)	−1.4	9/11
Real money stock (M-2/GDP deflator)*†	−0.5%	3/8

*Change in recessions is computed relative to the variable's average growth over the full postwar period, 1947:1–2009:3.

†Available only beginning in 1959.

unemployment rises during recessions. The table shows that, in addition, the length of the average workweek falls. The declines in employment and the declines in hours in the economy as a whole (though not in the manufacturing sector) are generally small relative to the falls in output. Thus productivity—output per worker-hour—almost always declines during recessions. The conjunction of the declines in productivity and hours implies that the movements in the unemployment rate are smaller than the movements in output. The relationship between changes in output and the unemployment rate is known as *Okun's law*. As originally formulated by Okun (1962), the "law" stated that a shortfall in GDP of 3 percent relative to normal growth produces a 1 percentage-point rise in the unemployment rate; a more accurate description of the current relationship is 2 to 1.

The remaining lines of Table 5.3 summarize the behavior of various price and financial variables. Inflation shows no clear pattern. The real wage, at least as measured in aggregate data, tends to fall slightly in recessions. Nominal and real interest rates generally decline, while the real money stock shows no clear pattern.

5.2 An Overview of Business-Cycle Research

It is natural to begin our study of aggregate fluctuations by asking whether they can be understood using a Walrasian model—that is, a competitive model without any externalities, asymmetric information, missing markets, or other imperfections. If they can, then the analysis of fluctuations may not require any fundamental departure from conventional microeconomic analysis.

As emphasized in Chapter 2, the Ramsey model is the natural Walrasian baseline model of the aggregate economy: the model excludes not only market imperfections, but also all issues raised by heterogeneity among households. This chapter is therefore devoted to extending a variant of the Ramsey model to incorporate aggregate fluctuations. This requires modifying the model in two ways. First, there must be a source of disturbances: without shocks, a Ramsey economy converges to a balanced growth path and then grows smoothly. The initial extensions of the Ramsey model to include fluctuations emphasized shocks to the economy's technology—that is, changes in the production function from period to period.[6] Subsequent work in this area also emphasizes changes in government purchases.[7] Both types of shocks represent real—as opposed to monetary, or nominal—disturbances: technology shocks change the amount that is produced from a given quantity of inputs, and government-purchases shocks change the quantity of goods available to the private economy for a given level of production. For this reason, the models are known as *real-business-cycle* (or *RBC*) models.

The second change that is needed to the Ramsey model is to allow for variations in employment. In all the models we have seen, labor supply is exogenous and either constant or growing smoothly. Real-business-cycle theory focuses on the question of whether a Walrasian model provides a good description of the main features of observed fluctuations. Models in this literature therefore allow for changes in employment by making households' utility depend not just on their consumption but also on the amount they work; employment is then determined by the intersection of labor supply and labor demand.

Although a purely Walrasian model is the natural starting point for studying macroeconomic fluctuations, we will see that the real-business-cycle models of this chapter do a poor job of explaining actual fluctuations. Thus we will need to move beyond them. At the same time, however, what these models are trying to accomplish remains the ultimate goal of business-cycle research: building a general-equilibrium model from microeconomic foundations and a specification of the underlying shocks that explains, both qualitatively and quantitatively, the main features of macroeconomic fluctuations. Thus the models of this chapter do not just allow us to explore how far we can get in understanding fluctuations with purely Walrasian models; they also illustrate the type of analysis that is the goal of business-cycle

[6] The seminal papers include Kydland and Prescott (1982); Long and Plosser (1983); Prescott (1986); and Black (1982).

[7] See Aiyagari, Christiano, and Eichenbaum (1992), Baxter and King (1993), and Christiano and Eichenbaum (1992).

research. Fully specified general-equilibrium models of fluctuations are known as *dynamic stochastic general-equilibrium* (or DSGE) models. When they are quantitative and use additional evidence to choose parameter values and properties of the shocks, they are *calibrated* DSGE models.

As we will discuss in Section 5.9, one way that the RBC models of this chapter appear to fail involves the effects of monetary disturbances: there is strong evidence that contrary to the predictions of the models, such disturbances have important real effects. As a result, there is broad (though not universal) agreement that nominal imperfections or rigidities are important to macroeconomic fluctuations. Chapters 6 and 7 therefore build on the analysis of this chapter by introducing nominal rigidities into business-cycle models.

Chapter 6 drops almost all the complexities of the models of this chapter to focus on nominal rigidity alone. It begins with simple models where nominal rigidity is specified exogenously, and then moves on to consider the microeconomic foundations of nominal rigidity in simple static models. Chapter 6 illustrates an important feature of research on business cycles: although the ultimate goal is a calibrated DSGE model rich enough to match the main features of fluctuations, not all business-cycle research is done using such models. If our goal is to understand a particular issue relevant to fluctuations, we often learn more from studying much simpler models.

Chapter 7 begins to put nominal rigidity into DSGE models of fluctuations. We will see, however, that—not surprisingly—business-cycle research is still short of its ultimate goal. Much of the chapter therefore focuses on the "dynamic" part of "dynamic stochastic general-equilibrium," analyzing dynamic models of price adjustment. The concluding sections discuss some of the elements of leading models and some main outstanding challenges.

5.3 A Baseline Real-Business-Cycle Model

We now turn to a specific real-business-cycle model. The assumptions and functional forms are similar to those used in most such models. The model is a discrete-time variation of the Ramsey model of Chapter 2. Because our goal is to describe the quantitative behavior of the economy, we will assume specific functional forms for the production and utility functions.

The economy consists of a large number of identical, price-taking firms and a large number of identical, price-taking households. As in the Ramsey model, households are infinitely lived. The inputs to production are again capital (K), labor (L), and "technology" (A). The production function is Cobb–Douglas; thus output in period t is

$$Y_t = K_t^{\alpha}(A_t L_t)^{1-\alpha}, \qquad 0 < \alpha < 1. \tag{5.1}$$

Output is divided among consumption (C), investment (I), and government purchases (G). Fraction δ of capital depreciates each period. Thus the capital stock in period $t+1$ is

$$\begin{aligned} K_{t+1} &= K_t + I_t - \delta K_t \\ &= K_t + Y_t - C_t - G_t - \delta K_t. \end{aligned} \tag{5.2}$$

The government's purchases are financed by lump-sum taxes that are assumed to equal the purchases each period.[8]

Labor and capital are paid their marginal products. Thus the real wage and the real interest rate in period t are

$$\begin{aligned} w_t &= (1-\alpha)K_t^{\alpha}(A_tL_t)^{-\alpha}A_t \\ &= (1-\alpha)\left(\frac{K_t}{A_tL_t}\right)^{\alpha}A_t, \end{aligned} \tag{5.3}$$

$$r_t = \alpha\left(\frac{A_tL_t}{K_t}\right)^{1-\alpha} - \delta. \tag{5.4}$$

The representative household maximizes the expected value of

$$U = \sum_{t=0}^{\infty} e^{-\rho t}u(c_t, 1-\ell_t)\frac{N_t}{H}. \tag{5.5}$$

$u(\bullet)$ is the instantaneous utility function of the representative member of the household, and ρ is the discount rate.[9] N_t is population and H is the number of households; thus N_t/H is the number of members of the household. Population grows exogenously at rate n:

$$\ln N_t = \overline{N} + nt, \qquad n < \rho. \tag{5.6}$$

Thus the level of N_t is given by $N_t = e^{\overline{N}+nt}$.

The instantaneous utility function, $u(\bullet)$, has two arguments. The first is consumption per member of the household, c. The second is leisure per member, which is the difference between the time endowment per member (normalized to 1 for simplicity) and the amount each member works, ℓ.

[8] As in the Ramsey model, the choice between debt and tax finance in fact has no impact on outcomes in this model. Thus the assumption of tax finance is made just for expositional convenience. Section 12.2 describes why the form of finance is irrelevant in models like this one.

[9] The usual way to express discounting in a discrete-time model is as $1/(1+\rho)^t$ rather than as $e^{-\rho t}$. But because of the log-linear structure of this model, the exponential formulation is more natural here. There is no important difference between the two approaches, however. Specifically, if we define $\rho' = e^{\rho} - 1$, then $e^{-\rho t} = 1/(1+\rho')^t$. The log-linear structure of the model is also the reason behind the exponential formulations for population growth and for trend growth of technology and government purchases (see equations [5.6], [5.8], and [5.10]).

Since all households are the same, $c = C/N$ and $\ell = L/N$. For simplicity, $u(\bullet)$ is log-linear in the two arguments:

$$u_t = \ln c_t + b \ln(1 - \ell_t), \qquad b > 0. \tag{5.7}$$

The final assumptions of the model concern the behavior of the two driving variables, technology and government purchases. Consider technology first. To capture trend growth, the model assumes that in the absence of any shocks, $\ln A_t$ would be $\overline{A} + gt$, where g is the rate of technological progress. But technology is also subject to random disturbances. Thus,

$$\ln A_t = \overline{A} + gt + \tilde{A}_t, \tag{5.8}$$

where $\tilde{A}$ reflects departures from trend. $\tilde{A}$ is assumed to follow a *first-order autoregressive process*. That is,

$$\tilde{A}_t = \rho_A \tilde{A}_{t-1} + \varepsilon_{A,t}, \qquad -1 < \rho_A < 1, \tag{5.9}$$

where the $\varepsilon_{A,t}$'s are *white-noise* disturbances—a series of mean-zero shocks that are uncorrelated with one another. Equation (5.9) states that the random component of $\ln A_t$, $\tilde{A}_t$, equals fraction ρ_A of the previous period's value plus a random term. If ρ_A is positive, this means that the effects of a shock to technology disappear gradually over time.

We make similar assumptions about government purchases. The trend growth rate of per capita government purchases equals the trend growth rate of technology; if this were not the case, over time government purchases would become arbitrarily large or arbitrarily small relative to the economy. Thus,

$$\ln G_t = \overline{G} + (n + g)t + \tilde{G}_t, \tag{5.10}$$

$$\tilde{G}_t = \rho_G \tilde{G}_{t-1} + \varepsilon_{G,t}, \qquad -1 < \rho_G < 1, \tag{5.11}$$

where the ε_G's are white-noise disturbances that are uncorrelated with the ε_A's. This completes the description of the model.

5.4 Household Behavior

The two most important differences between this model and the Ramsey model are the inclusion of leisure in the utility function and the introduction of randomness in technology and government purchases. Before we analyze the model's general properties, this section discusses the implications of these features for households' behavior.

Intertemporal Substitution in Labor Supply

To see what the utility function implies for labor supply, consider first the case where the household lives only for one period and has no initial wealth. In addition, assume for simplicity that the household has only one member.

In this case, the household's objective function is just $\ln c + b\ln(1-\ell)$, and its budget constraint is $c = w\ell$.

The Lagrangian for the household's maximization problem is

$$\mathcal{L} = \ln c + b\ln(1-\ell) + \lambda(w\ell - c). \tag{5.12}$$

The first-order conditions for c and ℓ, respectively, are

$$\frac{1}{c} - \lambda = 0, \tag{5.13}$$

$$-\frac{b}{1-\ell} + \lambda w = 0. \tag{5.14}$$

Since the budget constraint requires $c = w\ell$, (5.13) implies $\lambda = 1/(w\ell)$. Substituting this into (5.14) yields

$$-\frac{b}{1-\ell} + \frac{1}{\ell} = 0. \tag{5.15}$$

The wage does not enter (5.15). Thus labor supply (the value of ℓ that satisfies [5.15]) is independent of the wage. Intuitively, because utility is logarithmic in consumption and the household has no initial wealth, the income and substitution effects of a change in the wage offset each other.

The fact that the level of the wage does not affect labor supply in the static case does not mean that variations in the wage do not affect labor supply when the household's horizon is more than one period. This can be seen most easily when the household lives for two periods. Continue to assume that it has no initial wealth and that it has only one member; in addition, assume that there is no uncertainty about the interest rate or the second-period wage.

The household's lifetime budget constraint is now

$$c_1 + \frac{1}{1+r}c_2 = w_1\ell_1 + \frac{1}{1+r}w_2\ell_2, \tag{5.16}$$

where r is the real interest rate. The Lagrangian is

$$\begin{aligned} \mathcal{L} = {} & \ln c_1 + b\ln(1-\ell_1) + e^{-\rho}[\ln c_2 + b\ln(1-\ell_2)] \\ & + \lambda\left[w_1\ell_1 + \frac{1}{1+r}w_2\ell_2 - c_1 - \frac{1}{1+r}c_2\right]. \end{aligned} \tag{5.17}$$

The household's choice variables are c_1, c_2, ℓ_1, and ℓ_2. Only the first-order conditions for ℓ_1 and ℓ_2 are needed, however, to show the effect of the relative wage in the two periods on relative labor supply. These conditions are

$$\frac{b}{1-\ell_1} = \lambda w_1, \tag{5.18}$$

$$\frac{e^{-\rho}b}{1-\ell_2} = \frac{1}{1+r}\lambda w_2. \tag{5.19}$$

To see the implications of (5.18)–(5.19), divide both sides of (5.18) by w_1 and both sides of (5.19) by $w_2/(1+r)$, and equate the two resulting expressions for λ. This yields

$$\frac{e^{-\rho}b}{1-\ell_2}\frac{1+r}{w_2} = \frac{b}{1-\ell_1}\frac{1}{w_1}, \tag{5.20}$$

or

$$\frac{1-\ell_1}{1-\ell_2} = \frac{1}{e^{-\rho}(1+r)}\frac{w_2}{w_1}. \tag{5.21}$$

Equation (5.21) implies that relative labor supply in the two periods responds to the relative wage. If, for example, w_1 rises relative to w_2, the household decreases first-period leisure relative to second-period leisure; that is, it increases first-period labor supply relative to second-period supply. Because of the logarithmic functional form, the elasticity of substitution between leisure in the two periods is 1.

Equation (5.21) also implies that a rise in r raises first-period labor supply relative to second-period supply. Intuitively, a rise in r increases the attractiveness of working today and saving relative to working tomorrow. As we will see, this effect of the interest rate on labor supply is crucial to employment fluctuations in real-business-cycle models. These responses of labor supply to the relative wage and the interest rate are known as *intertemporal substitution* in labor supply (Lucas and Rapping, 1969).

Household Optimization under Uncertainty

The second way that the household's optimization problem differs from its problem in the Ramsey model is that it faces uncertainty about rates of return and future wages. Because of this uncertainty, the household does not choose deterministic paths for consumption and labor supply. Instead, its choices of c and ℓ at any date potentially depend on all the shocks to technology and government purchases up to that date. This makes a complete description of the household's behavior quite complicated. Fortunately, we can describe key features of its behavior without fully solving its optimization problem. Recall that in the Ramsey model, we were able to derive an equation relating present consumption to the interest rate and consumption a short time later (the Euler equation) before imposing the budget constraint and determining the level of consumption. With uncertainty, the analogous equation relates consumption in the current period to *expectations* concerning interest rates and consumption in the next period. We will derive this equation using the informal approach we used in equations (2.22)–(2.23) to derive the Euler equation.[10]

Consider the household in period t. Suppose it reduces current consumption per member by a small amount Δc and then uses the resulting greater wealth to increase consumption per member in the next period above what it otherwise would have been. If the household is behaving optimally, a marginal change of this type must leave expected utility unchanged.

Equations (5.5) and (5.7) imply that the marginal utility of consumption per member in period t, c_t, is $e^{-\rho t}(N_t/H)(1/c_t)$. Thus the utility cost of this change is $e^{-\rho t}(N_t/H)(\Delta c/c_t)$. Since the household has e^n times as many members in period $t+1$ as in period t, the increase in consumption per member in period $t+1$, c_{t+1}, is $e^{-n}(1+r_{t+1})\Delta c$. The marginal utility of period-t+1 consumption per member is $e^{-\rho(t+1)}(N_{t+1}/H)(1/c_{t+1})$. Thus the expected utility benefit as of period t is $E_t[e^{-\rho(t+1)}(N_{t+1}/H)e^{-n}(1+r_{t+1})/c_{t+1}]\Delta c$, where E_t denotes expectations conditional on what the household knows in period t (that is, conditional on the history of the economy up through period t). Equating the costs and expected benefits implies

$$e^{-\rho t}\frac{N_t}{H}\frac{\Delta c}{c_t} = E_t\left[e^{-\rho(t+1)}\frac{N_{t+1}}{H}e^{-n}\frac{1}{c_{t+1}}(1+r_{t+1})\right]\Delta c. \tag{5.22}$$

Since $e^{-\rho(t+1)}(N_{t+1}/H)e^{-n}$ is not uncertain and since $N_{t+1} = N_t e^n$, this condition simplifies to

$$\frac{1}{c_t} = e^{-\rho}E_t\left[\frac{1}{c_{t+1}}(1+r_{t+1})\right]. \tag{5.23}$$

This is the analogue of equation (2.20) in the Ramsey model.

Note that the expression on the right-hand side of (5.23) is *not* the same as $e^{-\rho}E_t[1/c_{t+1}]E_t[1+r_{t+1}]$. That is, the tradeoff between present and future consumption depends not just on the expectations of future marginal utility and of the rate of return, but also on their interaction. Specifically, the expectation of the product of two variables equals the product of their expectations plus their covariance. Thus (5.23) implies

$$\frac{1}{c_t} = e^{-\rho}\left\{E_t\left[\frac{1}{c_{t+1}}\right]E_t[1+r_{t+1}] + \mathrm{Cov}\left(\frac{1}{c_{t+1}}, 1+r_{t+1}\right)\right\}, \tag{5.24}$$

where $\mathrm{Cov}(1/c_{t+1}, 1+r_{t+1})$ denotes the covariance of $1/c_{t+1}$ and $1+r_{t+1}$. Suppose, for example, that when r_{t+1} is high, c_{t+1} is also high. In this case, $\mathrm{Cov}(1/c_{t+1}, 1+r_{t+1})$ is negative; that is, the return to saving is high in the times when the marginal utility of consumption is low. This makes saving less attractive than it is if $1/c_{t+1}$ and r_{t+1} are uncorrelated, and thus tends to raise current consumption.

Chapter 8 discusses the impact of uncertainty on optimal consumption further.

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[10] The household's problem can be analyzed more formally using *dynamic programming* (see Section 10.4 or Ljungqvist and Sargent, 2004). This also yields (5.23) below.

The Tradeoff between Consumption and Labor Supply

The household chooses not only consumption at each date, but also labor supply. Thus a second first-order condition for the household's optimization problem relates its current consumption and labor supply. Specifically, imagine the household increasing its labor supply per member in period t by a small amount $\Delta\ell$ and using the resulting income to increase its consumption in that period. Again if the household is behaving optimally, a marginal change of this type must leave expected utility unchanged.

From equations (5.5) and (5.7), the marginal disutility of labor supply in period t is $e^{-\rho t}(N_t/H)[b/(1-\ell_t)]$. Thus the change has a utility cost of $e^{-\rho t}(N_t/H)[b/(1-\ell_t)]\,\Delta\ell$. And since the change raises consumption per member by $w_t\,\Delta\ell$, it has a utility benefit of $e^{-\rho t}(N_t/H)(1/c_t)w_t\,\Delta\ell$. Equating the cost and benefit gives us

$$e^{-\rho t}\frac{N_t}{H}\frac{b}{1-\ell_t}\,\Delta\ell = e^{-\rho t}\frac{N_t}{H}\frac{1}{c_t}w_t\,\Delta\ell, \tag{5.25}$$

or

$$\frac{c_t}{1-\ell_t} = \frac{w_t}{b}. \tag{5.26}$$

Equation (5.26) relates current leisure and consumption, given the wage. Because it involves current variables, which are known, uncertainty does not enter. Equations (5.23) and (5.26) are the key equations describing households' behavior.

5.5 A Special Case of the Model

Simplifying Assumptions

The model of Section 5.3 cannot be solved analytically. The basic problem is that it contains a mixture of ingredients that are linear—such as depreciation and the division of output into consumption, investment, and government purchases—and ones that are log-linear—such as the production function and preferences. In this section, we therefore investigate a simplified version of the model.

Specifically, we make two changes to the model: we eliminate government, and we assume 100 percent depreciation each period.[11] Thus

[11] With these changes, the model corresponds to a one-sector version of Long and Plosser's (1983) real-business-cycle model. McCallum (1989) investigates this model. In addition, except for the assumption of $\delta = 1$, the model corresponds to the basic case considered by Prescott (1986). It is straightforward to assume that a constant fraction of output is purchased by the government instead of eliminating government altogether.