PS3 Solutions

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Solution (a).

- **Unit (Firm) Fixed Effects:** Each firm i has intercept α_i. When the number of firms N is large but the time period T is fixed, we have to estimate almost $N-T$ parameters. Such problem can lead to inconsistency in the estimation of the common parameters, becaus ethe estimation error in α_i doesn't vanish as $N \to \infty$.
- **Time Fixed Effects:** In contrast, time fixed effects add only T dummies, the number of parameters associated with time dummies is fixed. Hence, their estimation does not create an incidental parameters problem.

In short, while unit fixed effects can cause IPP when T is relative small to N, the addition of time fixed effects does not because the number of time dummies remains fixed and is asymptotically negligible.

Table 1: Fixed Eects Model

	(1) FE b/se
log of employment	0.737***
	(0.063)
log of deflated capital	0.096**
	(0.044)
log of deflated R&D	0.144***
	(0.028)
Observations	2971

```
use GMdata.dta, clear
xtset index yr
xtreg ldsal lemp ldnpt ldrnd i.yr, fe robust
```

Solution (b).
Starting from the original equation for firm i at time t:

$$\Delta ldsal_{it} = \beta_1 \Delta lemp_{it} + \beta_2 \Delta ldnpt_{it} + \beta_3 \Delta ldrnd_{it} + (f_t - f_{t-1}) + \Delta u_{it}$$

Since α_i does not vary over time, it drops out in the differencing.
The time effects appear as differences $f_t - f_{t-1}$. Thus, in the first-differenced equation the levels of the year dummies disappear, but their differences remain.

Solution (c).
As $d357_{it}$ equals 1 for firms in industry 357 and is constant over time, then its effect is absorbed by the firm fixed effect α_i. In the fixed effects (within) estimator, the coefficient on $d357$ is not separately identified. n the first-differenced model, any time-invariant variable will vanish $\Delta d357_{it} = 0 \forall t$.
Including a time-invariant dummy does not worsen the IPP since it adds only one parameter that is either absorbed (or eliminated in first differences).

Solution (d).
As we define $\ddot{y}_{it} = y_{it} - \overline{y}_{it}$ and $\ddot{X}_{it} - \overline{X}_{it}$, we have:

$$\hat{\beta}_{FE-W} = \left(\sum_{i,t} \ddot{X}'_{it}\ddot{X}_{it}\right) \sum_{i,t} \ddot{X}_{it}\ddot{y}_{it}$$

$$\hat{\beta}_{RE} = \left(X'\Omega^{-1}X\right)^{-1} X'\Omega^{-1}y$$

```
use "GMdata_balanced.dta", clear
xtset index yr
gen d357 = (sic3 == 357)

xtreg ldsal lemp ldnpt ldrnd i.yr##i.d357, fe robust
eststo fe_w
esttab fe_w using d1.tex, replace label booktabs title("FE-W Estimator")
    ///
    cells("b(fmt(3)) se(fmt(3))")

xtreg ldsal lemp ldnpt ldrnd i.yr##i.d357, re robust
eststo RE
esttab RE using d2.tex, replace label booktabs title("RE Estimator") ///
    cells("b(fmt(3)) se(fmt(3))")
```

Solution (e).
The Hausman test statistics is:

$$H = \left(\hat{\beta}_{FE} - \hat{\beta}_{RE}\right)' \left[A\mathbb{V}[\hat{\beta}_{FE}] - A\mathbb{V}[\hat{\beta}_{RE}]\right]^{-1} \left(\hat{\beta}_{FE} - \hat{\beta}_{RE}\right)$$

```
hausman fe_w re, sigmamore
```

Table 2: Fixed Effects Model

	(1) log of deflated sales b/se
log of employment	0.650***
	(0.031)
log of deflated capital	0.186***
	(0.025)
log of deflated R&D	0.098***
	(0.019)
year == 73, 78, 83, 88=73	0.000
	(.)
year == 73, 78, 83, 88=78	0.058***
	(0.016)
year == 73, 78, 83, 88=83	-0.012
	(0.020)
year == 73, 78, 83, 88=88	0.251***
	(0.020)
d357=0	0.000
	(.)
d357=1	0.000
	(.)
year == 73, 78, 83, 88=73 × d357=0	0.000
	(.)
year == 73, 78, 83, 88=73 × d357=1	0.000
	(.)
year == 73, 78, 83, 88=78 × d357=0	0.000
	(.)
year == 73, 78, 83, 88=78 × d357=1	1.128***
	(0.073)
year == 73, 78, 83, 88=83 × d357=0	0.000
	(.)
year == 73, 78, 83, 88=83 × d357=1	2.482***
	(0.074)
year == 73, 78, 83, 88=88 × d357=0	0.000
	(.)
year == 73, 78, 83, 88=88 × d357=1	3.433***
	(0.074)
Constant	3.774***
	(0.100)
Observations	856

Table 3: Random Effects Model

	(1) log of deflated sales b/se
log of employment	0.582***
	(0.026)
log of deflated capital	0.340***
	(0.019)
log of deflated R&D	0.067***
	(0.016)
year == 73, 78, 83, 88=73	0.000
	(.)
year == 73, 78, 83, 88=78	0.031*
	(0.017)
year == 73, 78, 83, 88=83	-0.080***
	(0.019)
year == 73, 78, 83, 88=88	0.197***
	(0.019)
d357=0	0.000
	(.)
d357=1	-3.304***
	(0.110)
year == 73, 78, 83, 88=73 × d357=0	0.000
	(.)
year == 73, 78, 83, 88=73 × d357=1	0.000
	(.)
year == 73, 78, 83, 88=78 × d357=0	0.000
	(.)
year == 73, 78, 83, 88=78 × d357=1	1.145***
	(0.076)
year == 73, 78, 83, 88=83 × d357=0	0.000
	(.)
year == 73, 78, 83, 88=83 × d357=1	2.500***
	(0.076)
year == 73, 78, 83, 88=88 × d357=0	0.000
	(.)
year == 73, 78, 83, 88=88 × d357=1	3.439***
	(0.077)
Constant	3.312***
	(0.066)
Observations	856