Geneva Graduate Institute (IHEID)

"Topics in Econometrics" (EI139), Fall 2025

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Problem Set 2

Due: Sunday, 7 December, 23:59

- Prepare concise answers.
- State clearly any additional assumptions, if needed.
- You are encouraged to collaborate in groups but the final write-up should be individual.
- Submit your solutions, along with any code (if applicable), in a **single pdf file** through **Moodle**. If you choose to write your solutions by hand, please make sure your scanned answers are legible.
- Grading scale:

5.5	default grade
6	absolutely no mistakes and particularly appealing write-up (clear and concise answers, decent formatting, etc.)
5	more than a few mistakes, or single mistake and particularly long, wordy answers
4	numerous mistakes, or clear lack of effort (e.g. parts not solved or not really attempted)
1	no submission by due date

Problem 1

Consider the dynamic panel data model

$$y_{it} = \alpha_i + \rho y_{it-1} + u_{it} \; , \quad u_{it} \sim iidN(0,1) \; , \quad i = 1:n \; , \quad t = 1:T \; .$$

with the correlated random effects distribution

$$\alpha_i | (y_{i0}, \phi) \sim N(\phi y_{i0}, 1) \; .$$

(a) What is the incidental parameter problem in this model and how does it manifest itself?

(b) Integrate out α_i using the correlated random effects distribution. Derive

$$p(y_{i1}, \ldots, y_{iT} | y_{i0}, \phi, \rho)$$

(c) Can (ϕ, ρ) be consistently (fixed T and $n \longrightarrow \infty$) estimated based on a likelihood constructed from $p(y_{i1}, \ldots, y_{iT} | y_{i0}, \phi, \rho)$? Explain.

(d) How would you estimate α_i, $i = 1 : n$?

Problem 2

Consider the state-space model

$$y_t = \lambda s_t + u_t \ , \quad s_t = \phi s_{t-1} + \epsilon_t \ .$$

For now assume that

$$u_t \sim iidN(0,1) \ , \quad \epsilon_t \sim iidN(0,1) \ , \quad u_t \perp \epsilon_t \quad .$$

(a) Derive the autocovariance function for y_t.

(b) Are the coefficients of the state-space model identified?

(c) Find an observationally equivalent ARMA representation for the state-space model. Express the ARMA parameters as functions of (λ, ϕ).

(d) Write the following code:

- A script that given numerical values for the parameters (λ, ϕ) (make sure that $0 < \phi < 1$) simulates a sample of size $T = 100$ and store $Y_{1:T}$ and $S_{1:T}$.
- A procedure that implements the Kalman filter. The inputs of that procedure should be $Y_{1:T}, \lambda, \phi$ and the output should be $p(y_t|Y_{1:t-1})$, $\mathbb{E}[s_t|Y_{1:t-1}]$, and $\mathbb{V}[s_t|Y_{1:t}]$.
- A script that calls the Kalman filter procedure and creates two plots: (i) the true states, the filtered states $\mathbb{E}[s_t|Y_{1:t-1}]$ and the error bands $\mathbb{E}[s_t|Y_{1:t-1}] \pm 1.96\sqrt{\mathbb{V}[s_t|Y_{1:t-1}]}$; (ii) the sequence of likelihood increments $p(y_t|Y_{1:t-1})$.

(e) Run the code to generate data and then generate two versions of the two plots: (i) using the same parameters that you used to generate the data; (ii) using different parameters. Discuss your findings.

(f) Write a script that plots the likelihood function $\ln p(Y|\lambda, \phi)$ using the sample generated in (v). Fix λ at its "true" value (the one you chose previously to generate the data) and let $\phi \in \mathcal{P}$, where $\mathcal{P}$ is a grid that ranges from 0 to 1. The plot should indicate the true value of ϕ through a vertical line. The script should also generate the estimator $\text{argmax}_{\phi \in \mathcal{P}} \ \ln p(Y|\phi)$. In order to evaluate the log likelihood function, your script will call the Kalman filter procedure that you wrote previously.

(g) Now generate two additional samples of size $T = 50$ and $T = 500$, and plot the log likelihood function for all three samples (for each T use the same scale for the y-axis of the plots if you generate separate plots, or overlay the three log likelihood functions in a single plot).

(h) Replace the grid search with a gradient-based numerical optimization routine and compare the new estimates to the grid-search based estimates.

(i) Now suppose that u_t and ϵ_t are jointly normally distributed and have a correlation ρ. Rederive the autocovariance function for y_t. Are the coefficients (λ, ϕ, ρ) identified? Find an observationally equivalent ARMA representation.

(j) We derived the Kalman filter iterations under the assumption that the errors in the measurement equation and the state-transition equation are independent. Generalize the Kalman filter iterations for the above state-space model to allow for a non-zero correlation ρ between u_t and ϵ_t.