Demystifying DSGE Models

2. Adding Bells and Whistles – The Calvo Fairy or Price and Wage Stickiness

Outline

I. Under the Hood – Nuts and Bolts of a DSGE Model
II. Adding Bells and Whistles – The NK DSGE in all its Glory
III. Case Study – The Canonical Smets-Wouters DSGE
IV. Bringing DSGE Models to the Data – Beyond Calibration and Simulation
V. Extensions to the NK DSGE: The Financial and Housing Sectors
VI. Smörgåsbord – Unemployment and Environment
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Demystifying DSGE Models

Summary: The Basic RBC

- Last week we looked at a simple RBC model:
- Utility function [CRRA]:

$$U(C_t) - V(L_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{L_t^{1+\varphi}}{1+\varphi}$$

- Production function [Cobb-Douglas]:

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}$$

- Law of motion of capital:

$$K_{t+1} = (1-\delta)K_t + I_t$$

- TFP process:

$$\log A_t = \rho \log A_{t-1} + \epsilon_t$$

- Optimising, model became:

$$C_t^{\sigma} L_t^{\varphi} = \frac{W_t}{P_t}$$ (Labour supply)

$$\left(\frac{E_t C_{t+1}}{C_t}\right)^{\sigma} = \beta\left[(1-\delta) + E_t\left(\frac{R_{t+1}}{P_{t+1}}\right)\right]$$ (Euler Equation)

$$K_{t+1} = (1-\delta)K_t + I_t$$ (Law of motion of capital)

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}$$ (Production function)

$$K_t = \alpha MC_t \frac{Y_t}{R_t}$$ (Demand for capital)

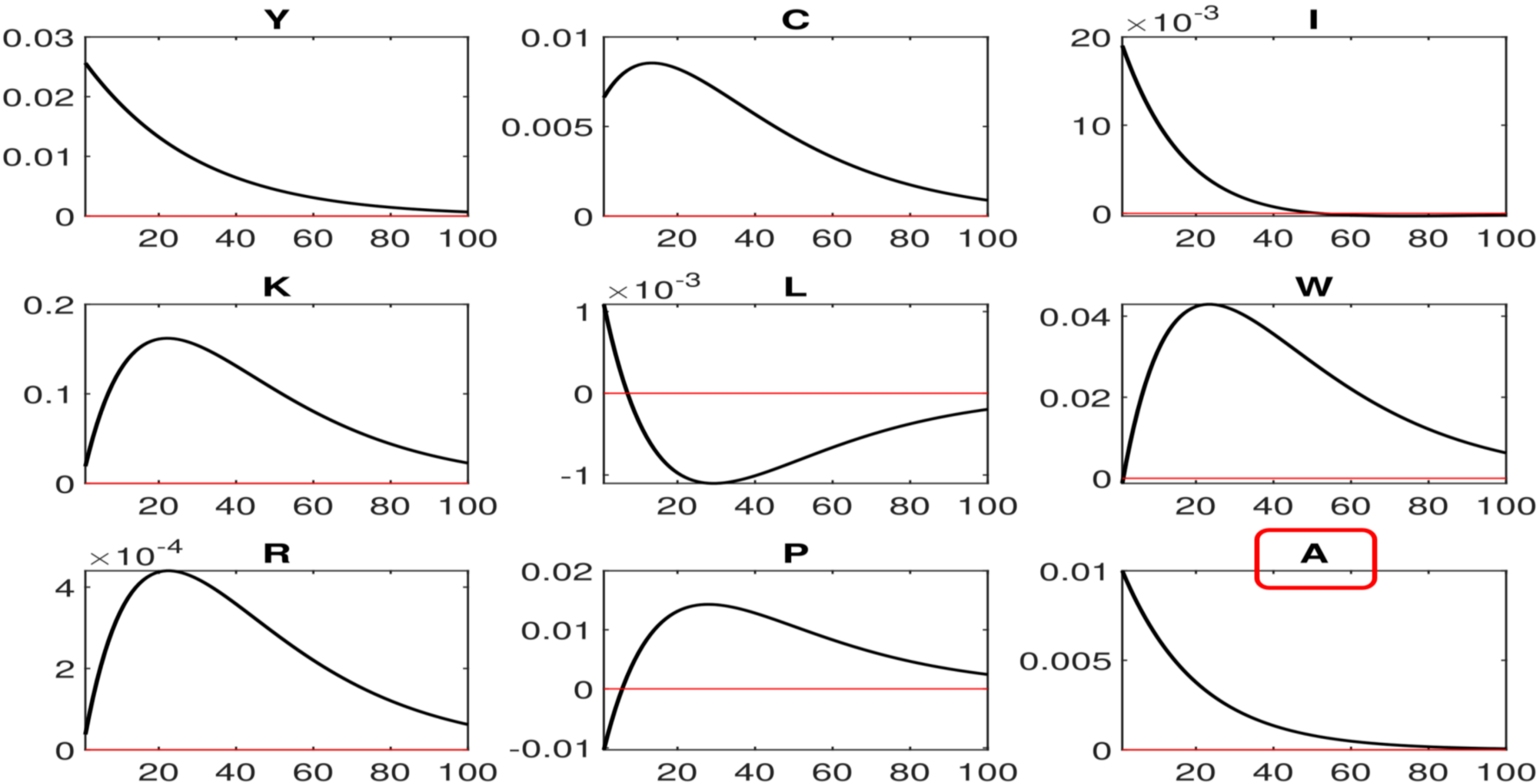
$$L_t = (1-\alpha) MC_t \frac{Y_t}{W_t}$$ (Demand for labour)

$$P_t = \frac{1}{A_t}\left(\frac{W_t}{1-\alpha}\right)^{1-\alpha}\left(\frac{R_t}{\alpha}\right)^{\alpha}$$ (Price level)

$$I_t = Y_t - C_t$$ (Equilibrium condition)

$$\log A_t = \rho_A \log A_{t-1} + \epsilon_t$$ (Productivity shock)

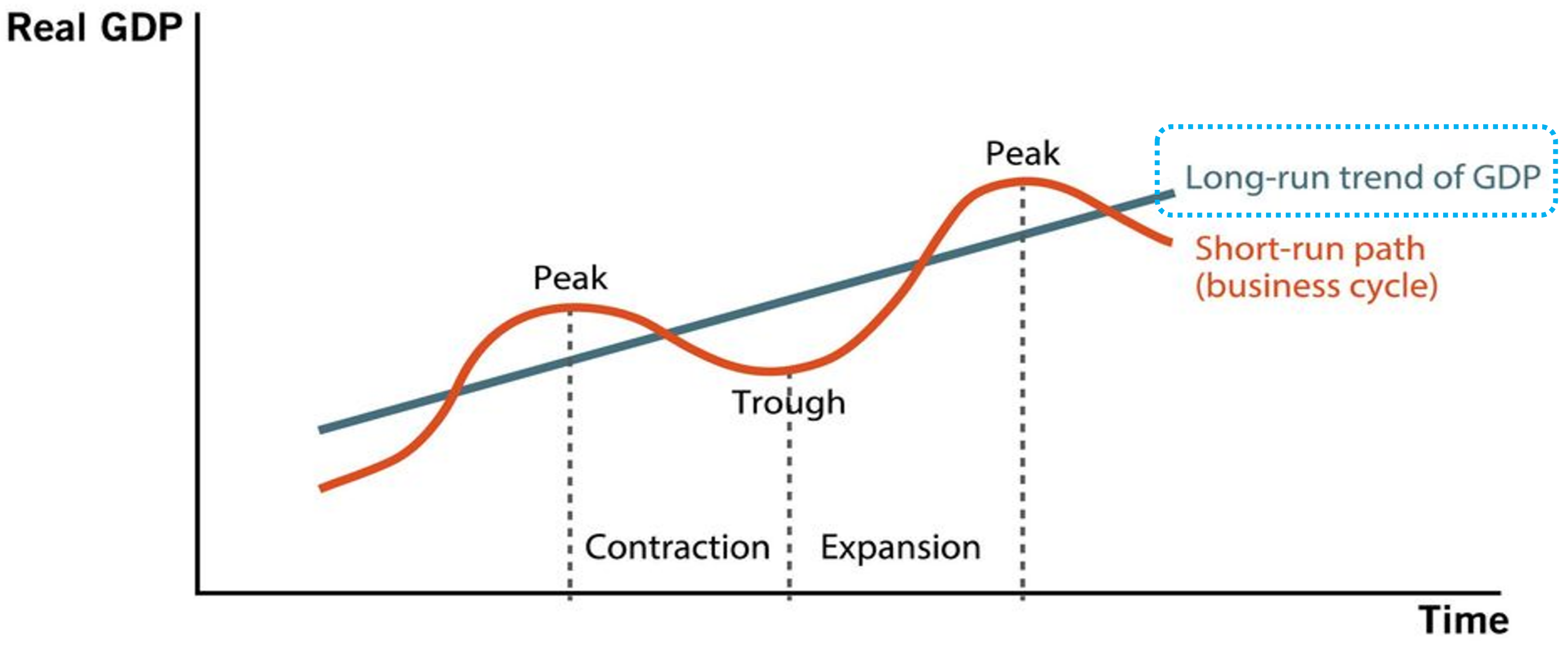
- We then simulated the behaviour of this economy, tracing out the way in which the macro variables responded to a shock in TFP



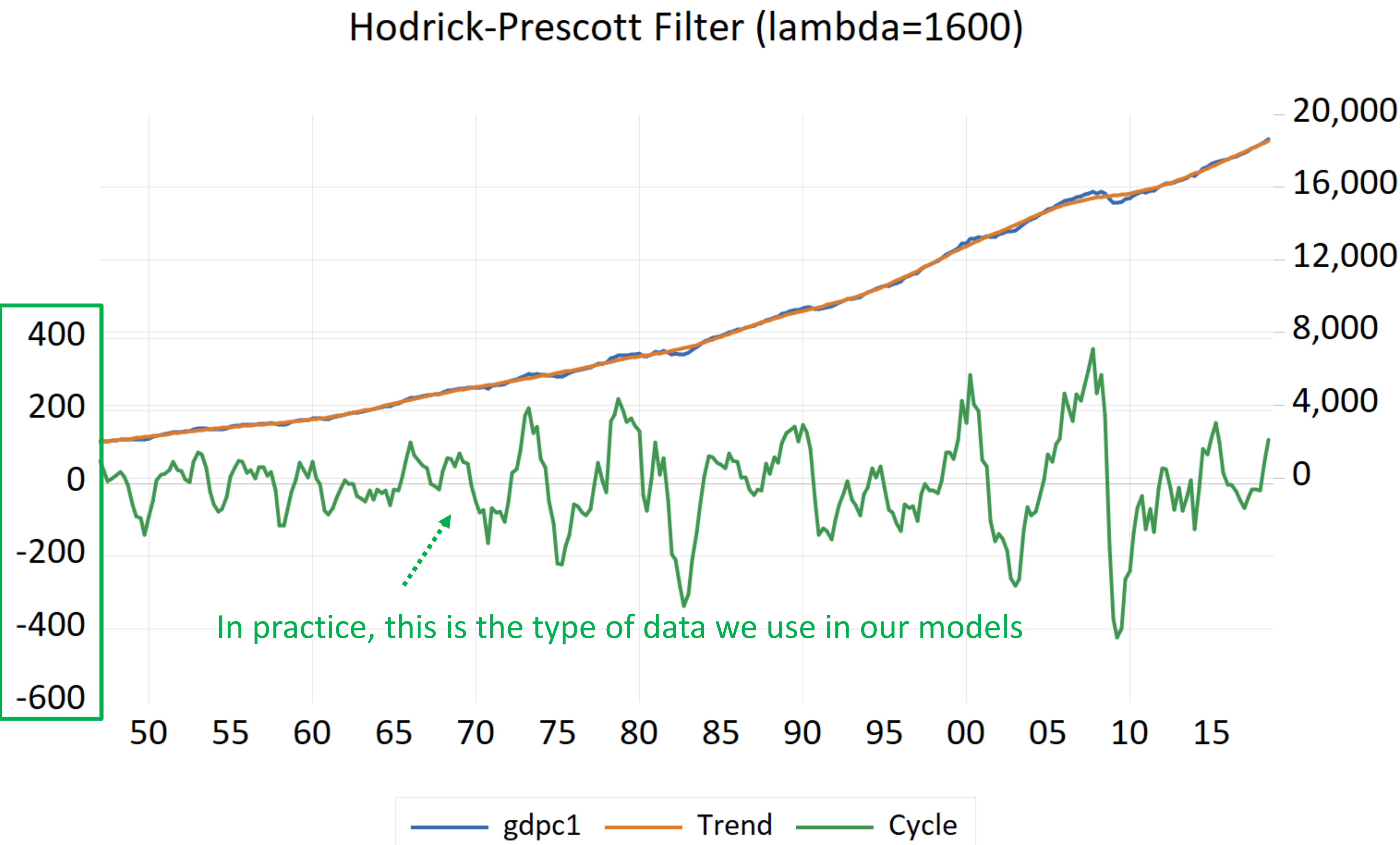
- We might well have stopped there
- But we were ambitious!
- Rather than just tracing out the behaviour of highly trending macro variables, we wanted to look at the fluctuations of the economy around these trends
- This led us to look at data like the **green** line in the slide after next

- "***Long-run trend of GDP***" = "Balanced Growth Path", aka "Steady State"

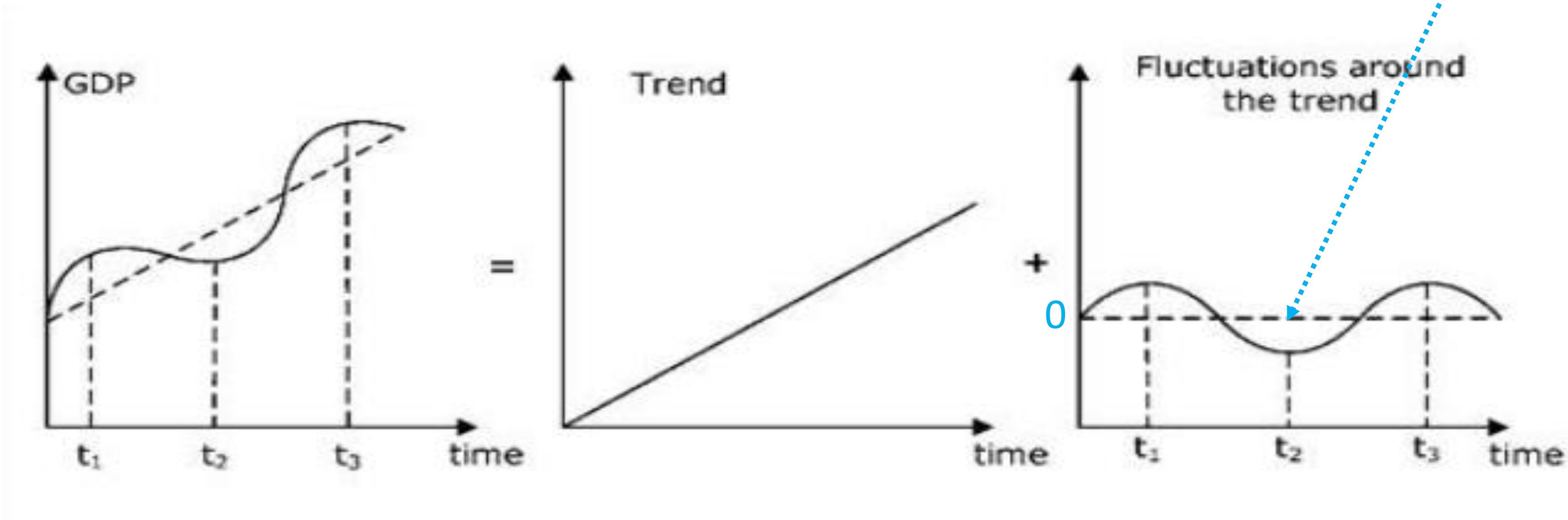
The Business Cycle



This shows ***real GDP per capita*** for the US over the period 1947Q1 – 2018Q3 (blue), the HP filter trend (brown) and the difference between the two (the "cycle") in green



- And we noted that ***on average*** [ie, "steady state"], these fluctuations had a value of 0



- To put our model into a form based on these ***fluctuations***, we ***log-linearised*** [but never forget that this is just an ***approximation*** to an underlying ***nonlinear*** model!]

- Result of doing so was ***simple linear equations*** in the ***transformed*** variables (ie, log deviations from steady state)
- ***Example***: production function

$$y_t = \boxed{a_t} + \alpha \boxed{k_t} + (1 - \alpha) \boxed{l_t}$$

- This states that ***dlogs*** of output are a simple ***weighted average*** of
  - ***dlogs*** of ***capital*** services k (assumed identical to stock)
  - ***dlogs*** of ***labour*** hours l
  - modified by a ***TFP*** productivity process a

- ***Summarising our log-linearised model:***

$\sigma c_t + \varphi l_t = w_t - p_t$ (Labour supply)

$\frac{\sigma}{\beta}\left(c_{t+1} - c_t\right) = \frac{R_{ss}}{P_{ss}}\left(r_{t+1} - p_{t+1}\right)$ (Euler equation)

$k_{t+1} = (1-\delta)\, k_t + \delta i_t$ (Law of motion of capital)

$y_t = a_t + \alpha k_t + (1-\alpha)\, l_t$ (Production function)

$k_t = y_t - (r_t - p_t)$ (Demand for capital)

$l_t = y_t - (w_t - p_t)$ (Demand for labour)

$p_t = (1-\alpha)\, w_t + \alpha r_t - a_t$ (Price level)

$y_t = \frac{C_{ss}}{Y_{ss}} c_t + \frac{I_{ss}}{Y_{ss}} i_t$ (Equilibrium condition)

$a_t = \rho_A a_{t-1} + e_t$ (Productivity shock)

The New-Keynesian Model

- ***NK model*** introduces into basic RBC more realistic specifications:
  - ***imperfect*** competition ⇨ *Today*
  - ***frictions*** in ***prices*** and in ***wages*** ⇨ *Today*
  - ***habit*** formation in consumption ⇨ *Today*
  - ***non-Ricardian*** agents ⇨ *Today*
  - ***adjustment costs*** in investment ⇨ *Today*
  - ***capacity*** (non-)utilisation ***costs*** ⇨ *Today*
  - ***government*** sector (***monetary*** and ***fiscal***) ⇨ *Today if time*
  - ***financial*** sector ⇨ *Week 6*
  - ***housing*** sector ⇨ *Week 6*
  - ***unemployment*** ⇨ *Week 7*
  - ***environment*** ⇨ *Week 7*
  - ***foreign*** trade ⇨ *Week 8*

Imperfect Competition and Price Stickiness

- Price and wage “***frictions***”
- Published evidence about frequency of changes in prices and wages indicates that:
  – Prices and wages are ***temporarily rigid***
  – Prices and wages are ***adjusted [“reset”]*** on average two or three times a year

Thus, In any given quarter, some prices and wages will be reset, but most will have been set some time in the past

  – Prices and wages are ***not*** reset simultaneously
  – Changes in prices of ***tradable*** goods are ***more frequent*** than those of ***non-tradable*** goods
- Hence incorporating price and wage “***stickiness***” into model makes eminent sense

- How to deal with this wage and price stickiness?
- ***"Calvo fairy"*** (or "Calvo devil" as she is sometimes called)
- Calvo fairy visits a certain ***proportion*** of (wholesale) ***firms*** and a (different) proportion of ***households*** in ***each period***
- and ***permits*** those firms or households to ***change*** their prices or wages to their ***utility-maximisation*** levels
- ***Remaining*** firms and households are obliged to follow a ***"Calvo rule"*** for prices and wages

- "Calvo rule" can have many forms:
  - Keeping ***same*** price or wage level as in previous period
  - ***Updating*** prices or wages using ***steady state*** gross inflation rate ("***indexation***")
  - Updating prices or wages using ***previous period's*** gross inflation rate
  - Something else entirely
- Incorporating this in model is straightforward:
- Since ***nothing changes*** as regards optimising behaviour of ***households***, we may simply ***copy over*** utility maximising conditions of basic RBC model

- ➔ ***repeat*** Household's ***labour supply*** function as

σ is the inverse of the intertemporal rate of substitution

$$C_{j,t}^{\sigma} L_{j,t}^{\varphi} = \frac{W_t}{P_t}$$

φ is the inverse of the Frisch elasticity of labour supply ➔ more convex is disutility function for leisure (higher is φ), more ***in***elastic is labour supply ➔ higher real wage

- and ***Euler Equation*** as

$$\left(\frac{E_t C_{j,t+1}}{C_{j,t}}\right)^{\sigma} = \beta\left[(1-\delta) + E_t\left(\frac{R_{t+1}}{P_{t+1}}\right)\right]$$

- and ***law of motion of capital (capital supply)*** as

$$K_{j,t+1} = (1-\delta)K_{j,t} + I_{j,t}$$

- Now add ***imperfect competition*** to NK model
  ➔now have both ***wholesale*** and ***retail*** sector
- ***Retail*** sector is ***lazy***: it just ***aggregates intermediate*** goods produced by ***wholesale*** sector (using ***Dixit-Stiglitz*** aggregator – there are others!)
- ➔ problem of representative ***retail*** firm: maximise its ***profit function*** taking into account ***input costs*** of the "j" varieties of intermediate goods:

$$\max_{Y_{j,t}} \left[P_t Y_t - \int_0^1 P_{j,t} Y_{j,t} dj \right]$$

eg, aggregate "sales" of a retail shop

- ***Dixit-Stiglitz*** aggregator (CES) ➔

$$\max_{Y_{j,t}} P_t \underbrace{\left(\int_0^1 Y_{j,t}^{\frac{\psi-1}{\psi}} dj \right)^{\frac{\psi}{\psi-1}}}_{\mathbf{Y_t}} - P_{j,t} \int_0^1 Y_{j,t} dj$$

- FOCs (using ***Chain Rule*** for differentiation)

$$\frac{\psi}{\psi-1} P_t \left(\int_0^1 Y_{j,t}^{\frac{\psi-1}{\psi}} dj \right)^{\frac{\psi}{\psi-1}-1} \frac{\psi-1}{\psi} Y_{j,t}^{\frac{\psi-1}{\psi}-1} - P_{j,t} = 0$$

- ➔ ***pricing rule for retail goods***

$$P_t = \left[\int_0^1 P_{j,t}^{1-\psi} dj \right]^{\frac{1}{1-\psi}}$$

➔ elasticity of substitution between different varieties is ψ. When variety j is 1% more expensive than average variety, then demand for this particular good will be ψ% lower

- ***Wholesale*** firms ***not lazy*** → ***work*** to produce, then sell their ***differentiated products*** to retail firms
- Wholesale firm solves its problem in ***two stages***
- ***First***, takes prices of ***factors of production*** (return on capital and wages) as given and determines amounts of capital and labour which will minimise ***total production cost***

$$\boxed{\min_{L_{j,t},K_{j,t}}}\left[W_t L_{j,t} + R_t K_{j,t}\right]$$

- ***subject to*** use of a given ***technology*** (typically ***assumed to be Cobb-Douglas***) and law of motion of productivity (typically ***autoregressive***)

- ➔ (copying from first week's RBC model):

$$Y_{j,t} = A_t K_{j,t}^{\alpha} L_{j,t}^{1-\alpha}$$

$$\log A_t = \rho_A \log A_{t-1} + \epsilon_t$$

- Using Lagrangian and solving FOCs ➔ ***demand*** of wholesale firm ***for labour and capital*** (as first week):

$$L_{j,t} = (1-\alpha) MC_{j,t} \frac{Y_{j,t}}{W_t}$$

$$K_{j,t} = \alpha MC_{j,t} \frac{Y_{j,t}}{R_t}$$

- ***Production technology*** here ***same*** as RBC ➔ ***re-use*** first week's RBC equation for $MC_{j,t}$

$$MC_{j,t} = \frac{1}{A_t}\left(\frac{W_t}{1-\alpha}\right)^{1-\alpha}\left(\frac{R_t}{\alpha}\right)^{\alpha}$$

- "***Price stickiness***" distinguishes NK from RBC and will require a ***mark-up*** over perfectly-flexible-price version of MC defined above [ie, $P \neq MC$!!]
- ➔ "***Calvo fairy***" and ***second stage*** of wholesale firm's problem

- ➔ objective of wholesale firm permitted by Calvo fairy to ***optimally reset price (P*)*** of its good is

$$\max_{P^*_{j,t}} E_t \sum_{i=0}^{\infty} (\beta\theta)^i \left(P^*_{j,t} Y_{j,t+i} - TC_{j,t+i} \right)$$

- Maximisation ➔FOCs which ➔ ***optimal price level for price-adjusting firm*** in each period

$$P^*_{j,t} = \left(\frac{\psi}{\psi - 1} \right) E_t \sum_{i=0}^{\infty} (\beta\theta)^i MC_{j,t+i}$$

θ = ***share of firms*** following Calvo rule for ***prices***

ψ = ***elasticity of substitution*** between wholesale ***goods***

- → ***aggregate price level*** each period

share able to reset price = 1-θ

share using Calvo Rule = θ

$$P_t^{1-\psi} = \int_0^{\theta} P_{t-1,j}^{1-\psi} dj + \int_{\theta}^{1} P_{t,j}^{*\,1-\psi} dj$$

Assumed Calvo Rule: could be ***different***

- →

$$P_t = \left[\theta P_{t-1}^{1-\psi} + (1-\theta) P_t^{*\,1-\psi}\right]^{\frac{1}{1-\psi}}$$

- Add ***equilibrium condition*** $Y_t = C_t + I_t$ ➔ model is complete: ***copy*** much from first week's RBC:

$$C_t^{\sigma} L_t^{\varphi} = \frac{W_t}{P_t}$$ (Labour supply)

$$\left(\frac{E_t C_{t+1}}{C_t}\right)^{\sigma} = \beta\left[(1-\delta) + E_t\left(\frac{R_{t+1}}{P_{t+1}}\right)\right]$$ (Euler Equation)

$$K_{t+1} = (1-\delta)K_t + I_t$$ (Law of motion of capital)

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}$$ (Production function)

$$K_t = \alpha MC_t \frac{Y_t}{R_t}$$ (Demand for capital)

$$L_t = (1-\alpha) MC_t \frac{Y_t}{W_t}$$ (Demand for labour)

- ***Completing*** initial NK model:

$$MC_t = \frac{1}{A_t}\left(\frac{W_t}{1-\alpha}\right)^{1-\alpha}\left(\frac{R_t}{\alpha}\right)^{\alpha}$$ (Marginal cost)

$$P_t^* = \left(\frac{\psi}{\psi-1}\right) E_t \sum_{i=0}^{\infty} (\beta\theta)^i MC_{t+i}$$ (Optimal price level)

$$P_t = \left[\theta P_{t-1}^{1-\psi} + (1-\theta) {P_t^*}^{1-\psi}\right]^{\frac{1}{1-\psi}}$$ (General price level)

$$\pi_t = \frac{P_t}{P_{t-1}}$$ (Gross inflation rate)

$$I_t = Y_t - C_t$$ (Equilibrium condition)

$$\log A_t = \rho_A \log A_{t-1} + \epsilon_t$$ (Productivity shock)

- Equation for ***optimal price level [aka, reset price]*** for price-adjusting firm in each period

Mark-up over MC

$$P^{*}_{j,t} = \boxed{\left(\frac{\psi}{\psi - 1}\right)} E_t \sum_{i=0}^{\boxed{\infty}} (\beta\theta)^i MC_{j,t+i}$$

- ***Formidable*** equation ➔ ***infinite sum*** of marginal costs
- ➔***impossible*** to write this (non-linear) model directly in Dynare (without some ***tricks*** …)
- Luckily, we have just such a ***trick***:

- A trick to get rid of ***infinite sums***
- Suppose we have
- $\Sigma_{i=0}^{\infty}(\lambda^i A_{t+i}) = A_t + \lambda A_{t+1} + \lambda^2 A_{t+2} + \lambda^3 A_{t+3} + \lambda^4 A_{t+4} + \ldots$
- Call this (infinite) sum B_t
- Then $B_{t+1} = A_{t+1} + \lambda A_{t+2} + \lambda^2 A_{t+3} + \lambda^3 A_{t+4} + \lambda^4 A_{t+5} + \ldots$
- ➔ $\lambda B_{t+1} = \lambda A_{t+1} + \lambda^2 A_{t+2} + \lambda^3 A_{t+3} + \lambda^4 A_{t+4} + \ldots$
- So
- $B_t - \lambda B_{t+1} = A_t$
- **Neat!!**

$$\boxed{B_t = \Sigma_{i=0}^{\infty}\left(\lambda^i A_{t+i}\right)}$$

- ***Optimal*** price under a ***Calvo*** approach (abstracting from "j") is

$$P_t^* = \boxed{\left(\frac{\psi}{\psi - 1}\right)} E_t \sum_{i=0}^{\infty} (\boxed{\beta\theta})^i \boxed{MC}_{t+i}$$

- Recall, $B_t = \Sigma_{i=0}^{\infty} \left(\lambda^i A_{t+i}\right)$
- In B, set $A = [\psi/(\psi-1)]MC$ and $\lambda = \beta\theta$, ➔ $P_t^* = B_t$
- ***But*** $B_t - \lambda B_{t+1} = A_t$ ➔ $P^*_t - (\beta\theta)P^*_{t+1} = A_t$
- Hence $P^*_t = \beta\theta P^*_{t+1} + \underbrace{[\psi/(\psi-1)]}_{\text{Mark-up over marginal cost}}MC_t$ ← Neat!!

Mark-up **over marginal cost**

- We may now write down new non-linear ***NK model*** directly in **Dynare**
- **predetermined_variables K**;
- **model;**

$$C^{\sigma}_{j,t} L^{\varphi}_{j,t} = \frac{W_t}{P_t}$$

$$\left(\frac{E_t C_{j,t+1}}{C_{j,t}}\right)^{\sigma} = \beta\left[(1-\delta) + E_t\left(\frac{R_{t+1}}{P_{t+1}}\right)\right]$$

- C^sigma*L^phi = W/P;
- (C(+1)/C)^sigma = beta*((1-delta)+R(+1)/P(+1));
- K(+1) = (1-delta)*K+I;

$$A_t K^{\alpha}_{j,t} L^{1-\alpha}_{j,t}$$

- Y = A*(K^alpha)*(L^(1-alpha));
- K = alpha*MC*Y/R;
- L = (1-alpha)*MC*Y/W;

P ≠ MC !!

$$MC_t = \frac{1}{A_t}\left(\frac{W_t}{1-\alpha}\right)^{1-\alpha}\left(\frac{R_t}{\alpha}\right)^{\alpha}$$

- MC = (1/A)*(W/(1-alpha))^(1-alpha)*(R/alpha)^alpha;
- I = Y - C;
- Pstar = beta*theta*Pstar(+1) + (psi/(psi-1))*MC;
- P = (theta*(P(-1)^(1-psi)) + (1-theta)*((Pstar^(1-psi))^(1/(1-psi))));
- pi = P/P(-1);
- log(A) = rhoA*log(A(-1)) + e;
- **end;**
- ***Note new equations in orange box, including definition of (gross) inflation rate***

- We may now proceed to ***simulate*** model
- But since model is ***nonlinear*** [we have not log-linearised here] **Dynare** needs ***steady-state*** values to check for fulfilment of infamous ***Blanchard-Kahn*** conditions
- If no ***analytical*** steady-state solution exists, we may ask **Dynare** to attempt to solve model itself for steady-state values
- But this requires that we provide **Dynare** with ***initial guesses*** at those steady-state values
- And if they are not very good …

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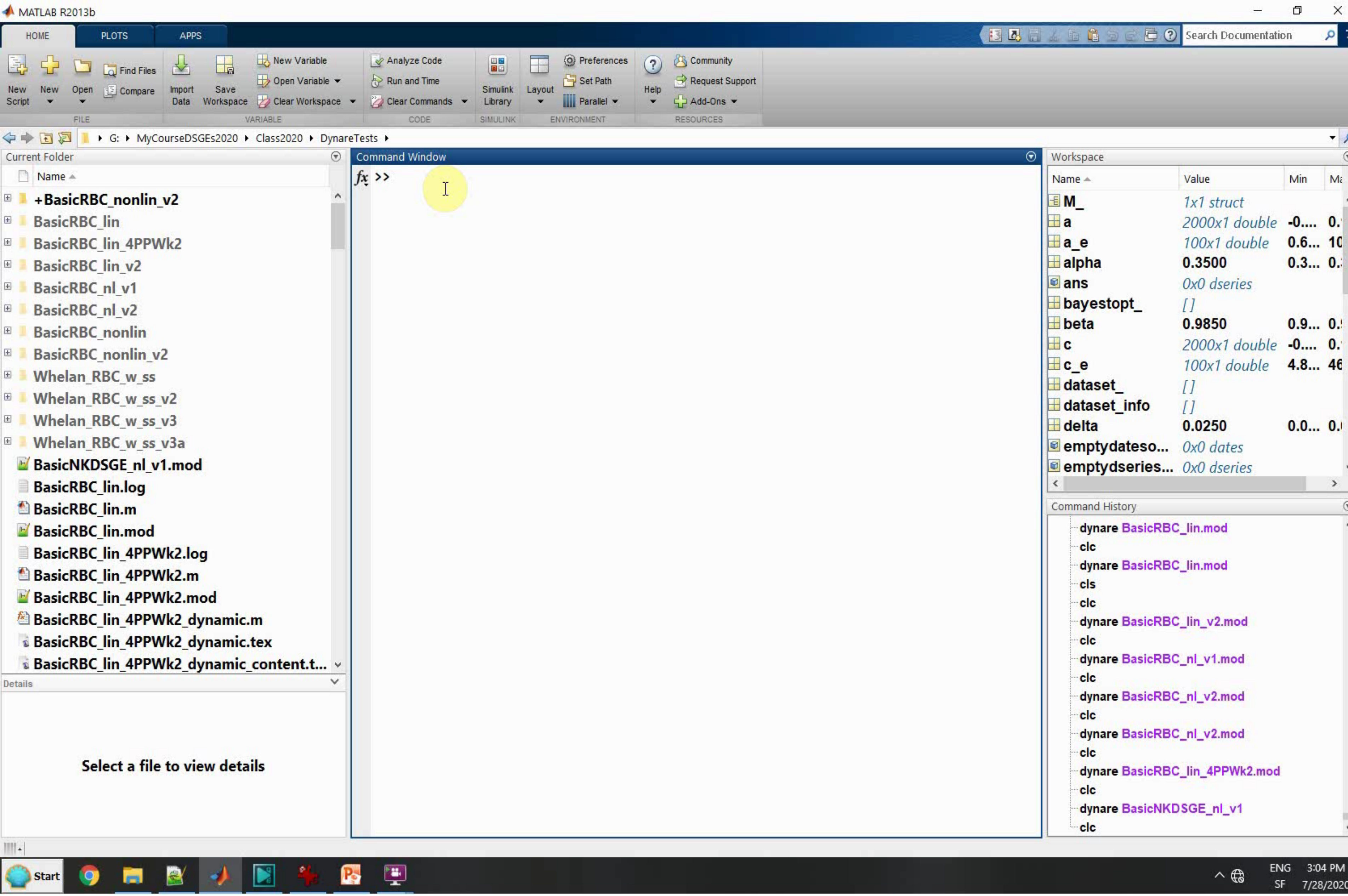

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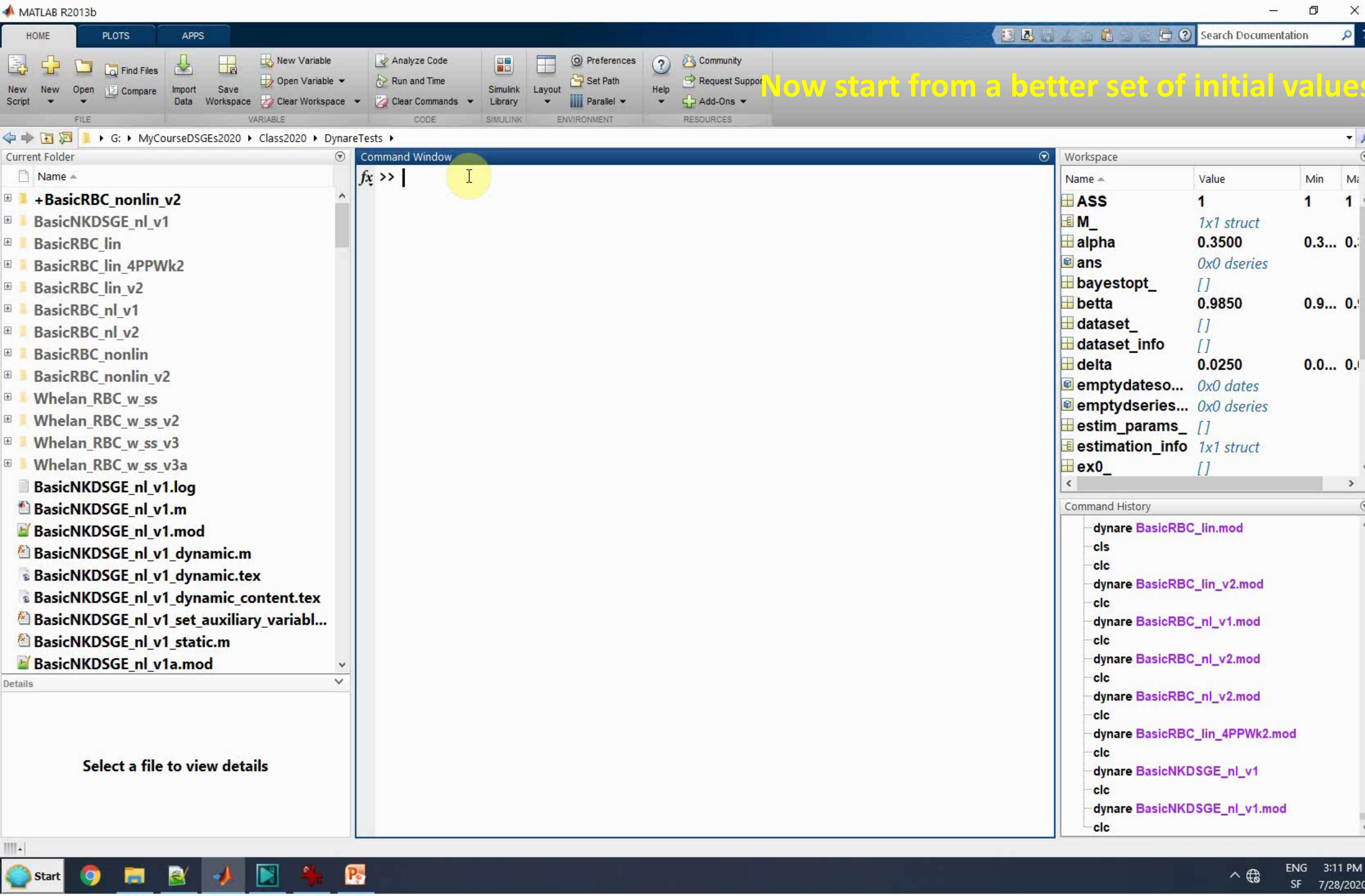
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- As just seen, starting from a ***better set*** of initial values ➔following IRFs

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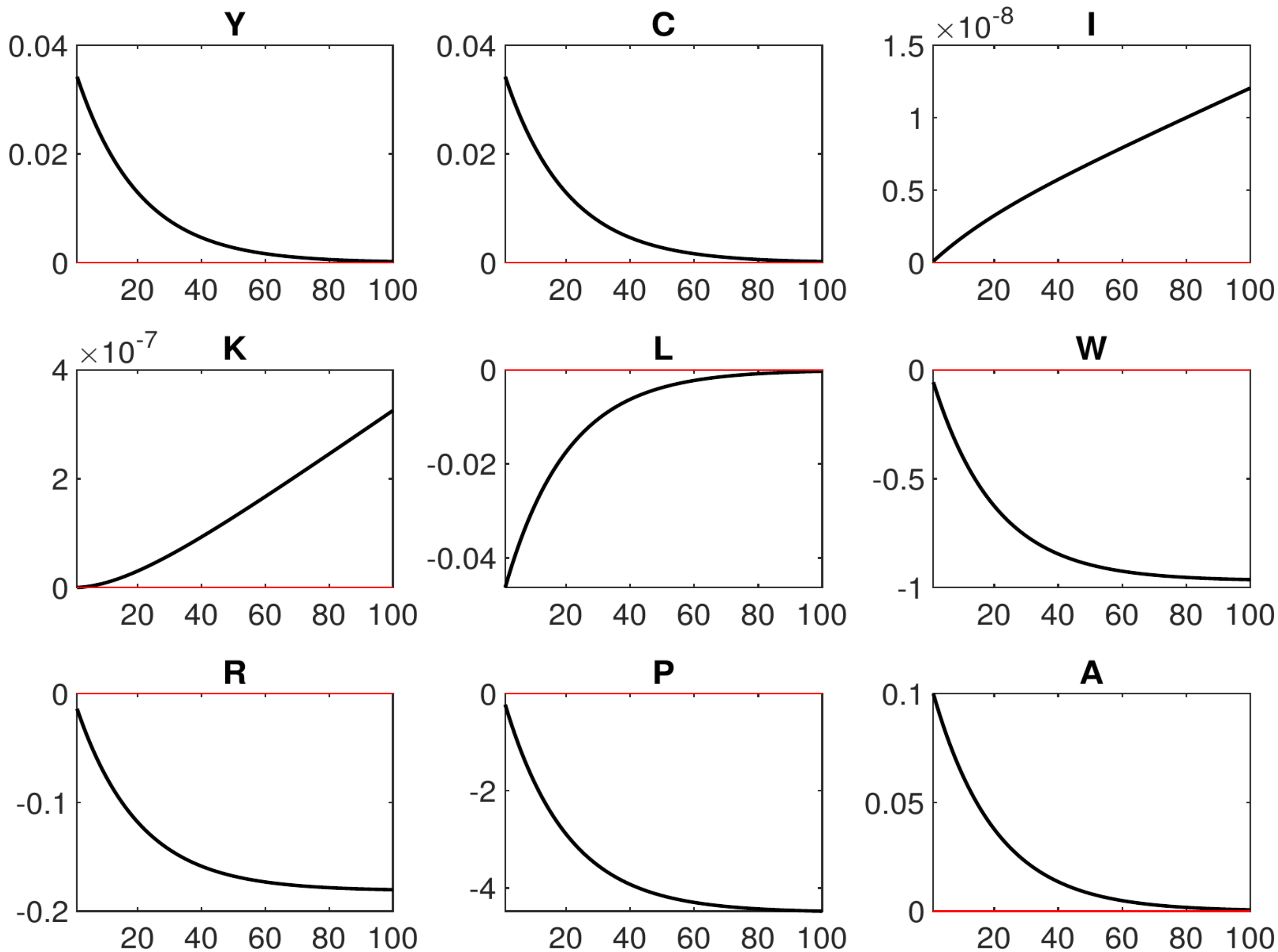

Figure 1: Impulse response functions (orthogonalized shock to e).

Note: these are ***levels***, not log-differences (ie, percentage changes) so shapes are quite different from those found previously

- ***If*** an ***analytical*** solution exists, then ***instead*** of setting Initial Values, we may write down ***steady-state model*** directly
- Fortunately for this model, *Costa* does solution for us *[see pp. 75-79 for details]*
- Results are ***identical***

```
// Steady-state values
steady_state_model;
A = 1; % ==> log(A)=0 ==> no shock until subsequently imposed
P = 1; % Normalisation from Walras' Law (k goods ==> k-1 independent equations in equilibrium ==> may set aggregate price level to 1)
R = P*((1/betta)-(1-delta));
MC = ((psi-1)/psi)*(1-betta*theta)*P;
W = (1-alpha)*(MC^(1/(1-alpha)))*((alpha/R)^(alpha/(1-alpha)));
Y =((R/(R-delta*alpha*MC))^(sigma/(sigma+phi)))*(((W/P)*((W/((1-alpha)*MC))^phi))^(1/(sigma+phi)));
I = delta*alpha*MC*Y/R;
C = (1/(Y^(phi/sigma)))*(((W/P)*(W/((1-alpha)*MC))^phi)^(1/sigma));
L = (1-alpha)*MC*Y/W;
K = alpha*MC*Y/R;
Pstar = P;
pi = 1;
end;
```

- In many cases, especially with more complex models, either ***analytical*** solution is ***unavailable***, or it is ***impossible*** to find initial values which satisfy infamous ***BK conditions***
- In these cases, ***only*** solution is to resort to ***approximating*** non-linear model via ***log-linearisation*** technique
- Although it is not necessary for this simple model, it will be very useful to have done so when model becomes more complex
- Let us begin:

- For ***eight*** equations, there is ***no change*** from first week:
  - labour supply
  - capital supply (law of motion of capital)
  - Euler equation
  - production function
  - demand for capital
  - demand for labour
  - equilibrium condition
  - law of motion of productivity shock
- all ***remain identical*** since ***unaffected*** by price frictions

- ➔ most of ***linearised*** model just ***copied*** over from first week:

$$\sigma c_t + \varphi l_t = w_t - p_t$$ (Labour supply)

$$\frac{\sigma}{\beta}\left(c_{t+1} - c_t\right) = \frac{R_{ss}}{P_{ss}}\left(r_{t+1} - p_{t+1}\right)$$ (Euler Equation)

$$k_{t+1} = (1-\delta)\, k_t + \delta i_t$$ (Law of motion of capital)

$$y_t = a_t + \alpha k_t + (1-\alpha)\, l_t$$ (Production function)

$$k_t = \boxed{mc_t} + y_t - r_t$$ (Demand for capital)

First week we used ***p*** because ***p = mc*** in RBC

$$l_t = \boxed{mc_t} + y_t - w_t$$ (Demand for labour)

$$y_t = \frac{C^*}{Y^*} c_t + \frac{I^*}{Y^*} i_t$$ (Equilibrium condition)

$$a_t = \rho_A a_{t-1} + e_t$$ (Productivity shock)

- Now for something ***New***: Log-linearising Marginal Cost equation

$$MC_t = \frac{1}{A_t}\left(\frac{W_t}{1-\alpha}\right)^{1-\alpha}\left(\frac{R_t}{\alpha}\right)^{\alpha}$$

- New, but yet again using trick

recall that *lower*-case ➔ dlog of *Upper*-case

$$Z = \zeta W^b X^c Y^d \Rightarrow z = bw + cx + dy$$

- ➔ $$mc_t = (1-\alpha)\,w_t + \alpha r_t - a_t$$

Here, $\zeta = (1-\alpha)^{-(1-\alpha)}\alpha^{-\alpha}$

- Also ***New*** : log-linearising ***Phillips Curve*** *[see Costa pp. 80-82 for details]*

$$P_t^* = \left(\frac{\psi}{\psi-1}\right) E_t \sum_{i=0}^{\infty} (\beta\theta)^i MC_{t+i}$$

$$P_t = \left[\theta P_{t-1}^{1-\psi} + (1-\theta) P_t^{*\,1-\psi}\right]^{\frac{1}{1-\psi}}$$

$$\boxed{\pi_t} = \frac{P_t}{P_{t-1}}$$

Gross inflation rate !!

Simple forward-looking Phillips Curve

- ➔ $\boxed{pi_t} = \beta pi_{t+1} + \frac{(1-\beta\theta)(1-\theta)}{\theta}(\underbrace{mc_t - p_t}_{\text{Mark-up over marginal cost}})$

Net inflation rate

- ***Dynare*** **simulation model log-linearised equations :**

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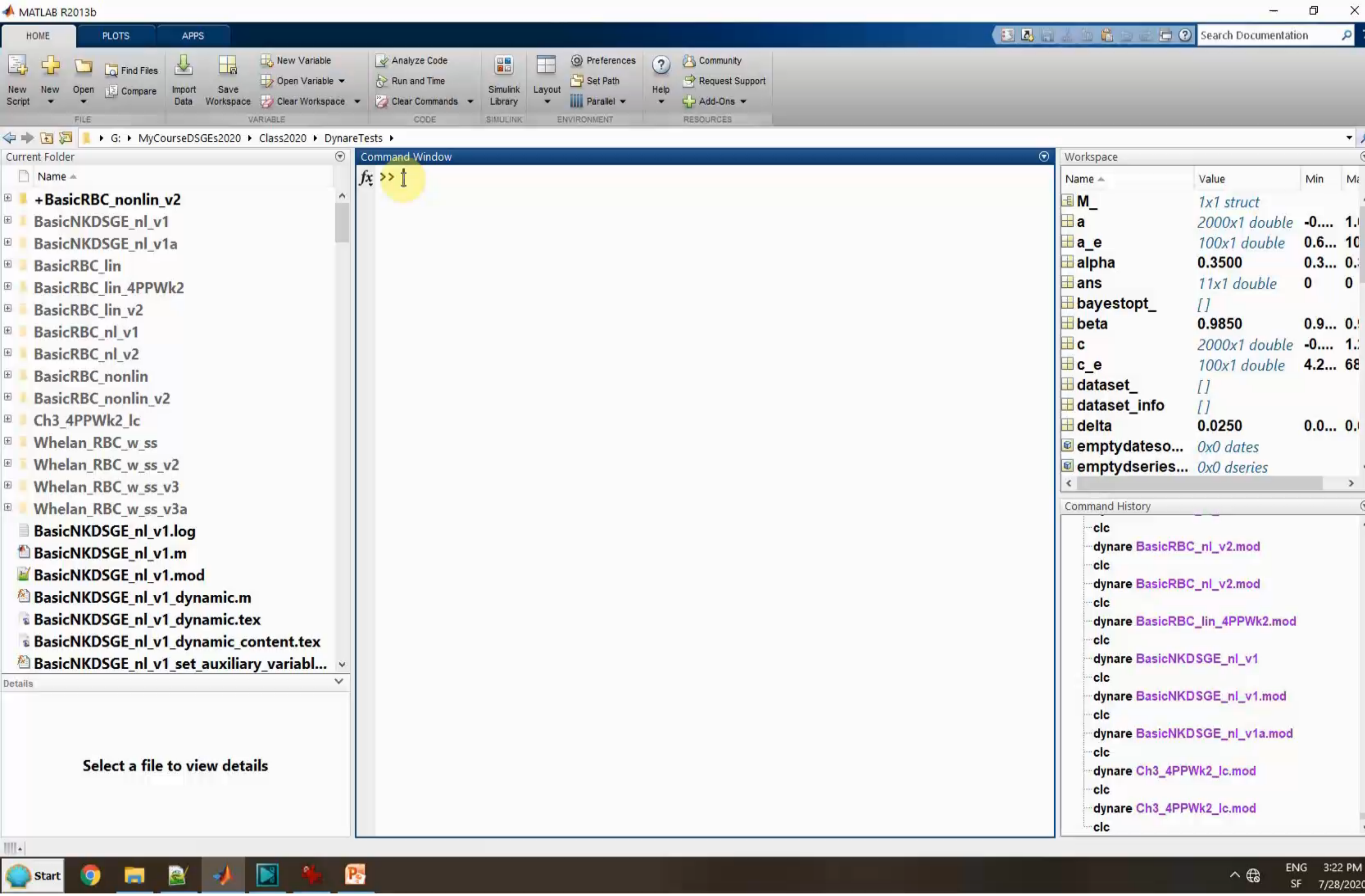
```
//1-Labour Supply
sigma*c + phi*l = w - p;
//2-Euler Equation
(sigma/beta)*(c(+1)-c)=(Rss/Pss)*(r(+1)-p(+1));
//3-Law of motion of Capital
k(+1) = (1-delta)*k + delta*i;
//4-Production Function
y = a + alpha*k + (1-alpha)*l;
//5-Demand for Capital
k = y - (r - p);
//6-Demand for Labour
l = y - (w - p);
//7-Marginal Cost
mc = (1-alpha)*w + alpha*r - a;
//8-Phillips Curve
pi = beta*pi(+1)+((1-theta)*(1-beta*theta)/theta)*(mc-p);
//9-         Inflation Rate
pi = p - p(-1);
//10-Goods Market Equilibrium Condition
Yss*y = Css*c + Iss*i;
//11-Productivity Shock
a = rhoa*a(-1) + e;
```

New !!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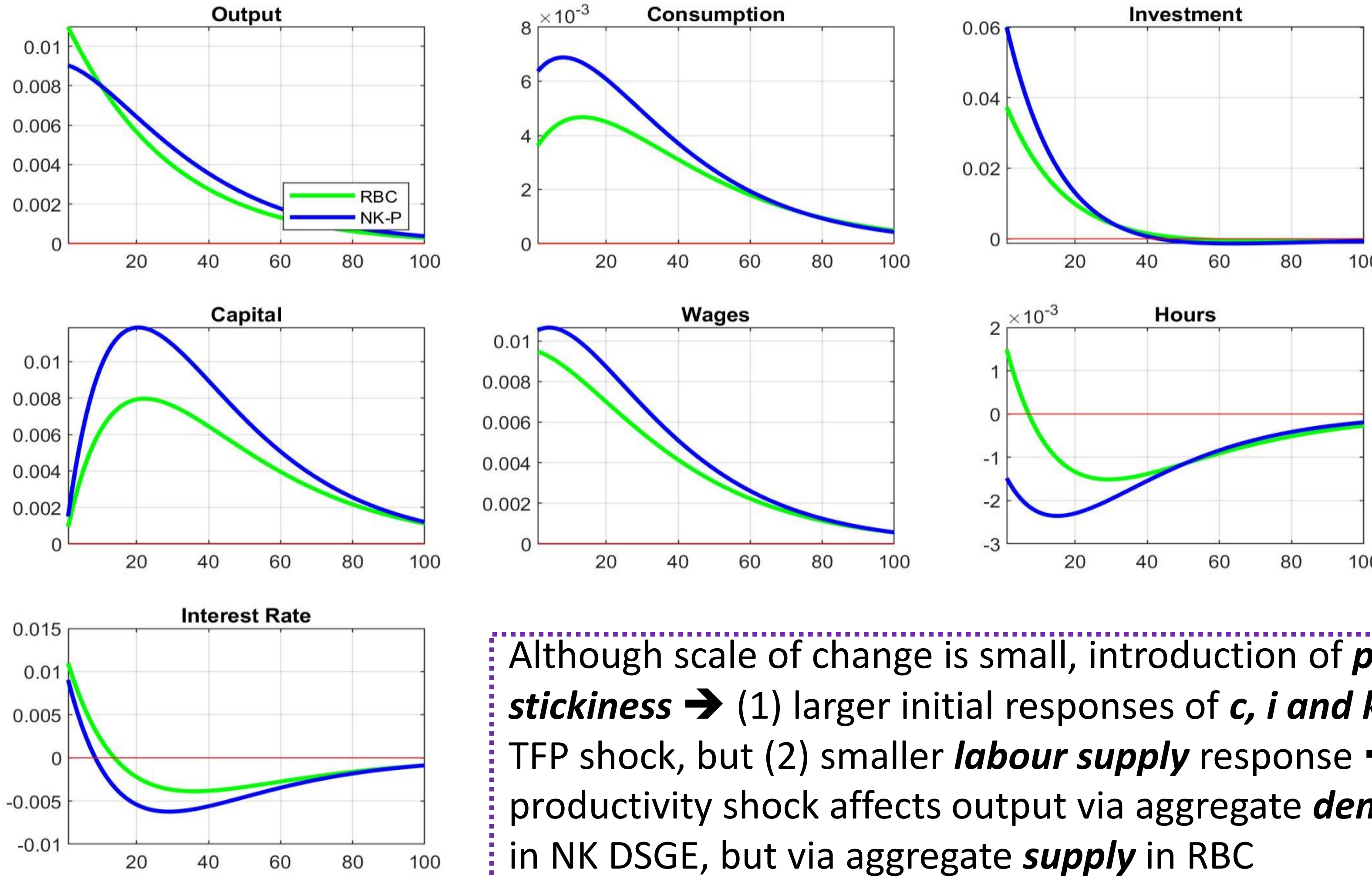
• Comparing IRFs of NK DSGE and RBC:



Although scale of change is small, introduction of ***price stickiness*** ➔ (1) larger initial responses of ***c, i and k*** to TFP shock, but (2) smaller ***labour supply*** response ➔ productivity shock affects output via aggregate ***demand*** in NK DSGE, but via aggregate ***supply*** in RBC

Wage Stickiness

- ***Recall:*** Monopolistic competition → an aggregating retail firm which uses ***Dixit-Stiglitz*** aggregator
- Here ***[NEW]*** assume that households supply their ***differentiated labour*** to a representative union which ***aggregates that labour*** ($L_{j,t}$) into a single labour input (L_t) via ***Dixit-Stiglitz*** aggregator
- →

$$L_t = \left(\int_0^1 L_{j,t}^{\frac{\psi_W - 1}{\psi_W}} \, dj \right)^{\frac{\psi_W}{\psi_W - 1}}$$

- ***Labour aggregating firm*** maximises its profit function

$$\max_{L_{j,t}} \left[W_t L_t - \int_0^1 W_{j,t} L_{j,t} dj \right]$$

- Solving for FOCs and doing a bit of algebra *[see Costa pp. 94-97 for details]* ➔ demand function for ***differentiated labour***

$$L_{j,t} = L_t \left(\frac{W_t}{W_{j,t}} \right)^{\psi_w}$$

- A bit more algebra ➔ ***aggregate wage level***

$$W_t = \left(\int_0^1 W_{j,t}^{1-\psi_w} \, dj \right)^{\frac{1}{1-\psi_w}}$$

- But now, Calvo fairy *[she's back again!]* introduces ***wage stickiness*** into this economy
- ➔ aggregate wage level composed of ***two components***
  - wages for households permitted by ***Calvo fairy*** to select an "optimal" wage
  - wages for those having to follow a ***Calvo Rule***

- Specifically, each household is given
  - probability $1-\theta_W$ of optimally defining its wage
  - probability θ_W of "Calvo Rule" in determining its wage in next period (wage "stickiness")
- Typically, ***Calvo Rule*** implies one of three possibilities
  - maintain previous period's wage
  - update wage using steady-state gross inflation rate (π_{ss})
  - update wage using previous period's gross inflation rate (π_{t-1})
- Here ***assume*** wage remains at previous period's level when applying Calvo Rule

- ***RBC***: Household Lagrangian:

$$\mathscr{L} = E_t \sum_{t=0}^{\infty} \beta^t \left\{ \left[\frac{C_{j,t}^{1-\sigma}}{1-\sigma} - \frac{L_{j,t}^{1+\varphi}}{1+\varphi} \right] \right.$$
$$\left. -\lambda_{j,t} \left[P_t C_{j,t} + \boxed{P_t K_{j,t+1} - P_t(1-\delta) K_{j,t}} - W_t L_{j,t} - R_t K_{j,t} \right] \right\}$$

This is just $P_t I_t$

- But this ***does not take into account*** that only a proportion θ_W of households may now set their wage optimally
- Omitting parts of Lagrangian not related to Labour and Wages, ➔ Household problem becomes …

$$\max_{W^*_{j,t}} E_t \sum_{i=0}^{\infty} (\beta\theta_w)^i \left\{ -\frac{L^{1+\varphi}_{j,t+i}}{1+\varphi} - \lambda_{t+i}\left[-W^*_{j,t} L_{j,t+i} \right] \right\}$$

- Maximisation ➔ FOCs ➔ ***optimal wage*** level for wage-adjusting household in each period

$$W^*_{j,t} = \left(\frac{\psi_w}{\psi_w - 1} \right) E_t \sum_{i=0}^{\infty} (\beta\theta_W)^i \, C^{\sigma}_{j,t+i} L^{\varphi}_{j,t+i} P_{t+i}$$

- All households which Calvo fairy allows to define their optimal wages choose ***same wages W**** (as they are subject to ***same constraints***)
- ➔ ***overall nominal wage*** level each period

$$W_t^{1-\psi_w} = \int_0^{\theta_w} W_{t-1,j}^{1-\psi_w}\, dj + \int_{\theta_w}^{1} W_{t,j}^{*\,1-\psi_w}\, dj$$

- ➔ complete wage model *(does this look familiar?)*

$$W_t = \left[\theta_w W_{t-1}^{1-\psi_W} + (1-\theta_w)\, W_t^{*\,1-\psi_w}\right]^{\frac{1}{1-\psi_w}}$$

So complete model becomes:

$$W^{*}_{j,t} = \left(\frac{\psi_w}{\psi_w - 1}\right) E_t \sum_{i=0}^{\infty} (\beta\theta_W)^i \, C^{\sigma}_{j,t+i} L^{\varphi}_{j,t+i} P_{t+i} \text{ (Definition of optimal wages)}$$

$$W_t^{1-\psi_w} = \int_{j=0}^{\theta_w} W_{t-1}^{1-\psi_w} dj + \int_{j=\theta_w}^{1} W_t^{*1-\psi_w} dj \text{ (Level of aggregate wages)}$$

$$\pi_{W,t} = W_t / W_{t-1} \text{ (Gross wage inflation rate)}$$

$$\left(\frac{E_t C_{t+1}}{C_t}\right)^{\sigma} = \beta\left[(1-\delta) + \left(\frac{E_t R_{t+1}}{E_t P_{t+1}}\right)\right] \text{ (Euler equation)}$$

As before:

$$K_{t+1} = (1-\delta)K_t + I_t$$ (Law of motion of capital)

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}$$ (Production function)

$$K_t = \alpha MC_t \frac{Y_t}{R_t}$$ (Demand for capital)

$$L_t = (1-\alpha) MC_t \frac{Y_t}{W_t}$$ (Demand for labour)

$$MC_t = \frac{1}{A_t}\left(\frac{W_t}{1-\alpha}\right)^{1-\alpha}\left(\frac{R_t}{\alpha}\right)^{\alpha}$$ (Marginal cost)

And again, as before:

$$P_t^* = \left(\frac{\psi}{\psi-1}\right) E_t \sum_{i=0}^{\infty} (\beta\theta)^i MC_{t+i}$$ (Optimal price level)

$$P_t = \left[\theta P_{t-1}^{1-\psi} + (1-\theta) P_t^{*} - \psi\right]^{\frac{1}{1-\psi}}$$ (General price level)

$$\pi_t = \frac{P_t}{P_{t-1}}$$ (Gross inflation rate)

$$Y_t = C_t + I_t$$ (Equilibrium condition)

$$\log A_t = (1-\rho_A)\log A_{ss} + \rho_A \log A_{t-1} + \epsilon_t$$ (Productivity shock)

- Using ***same infinite sum trick*** as before, we may transform

$$W^*_{j,t} = \left(\frac{\psi_w}{\psi_w - 1}\right) E_t \sum_{i=0}^{\infty} (\beta\theta_W)^i \, C^{\sigma}_{j,t+i} L^{\varphi}_{j,t+i} P_{t+i}$$

- into

$$W^*_t = \beta\theta W^*_{t+1} + [\underbrace{\psi_w/(\psi_w-1)}_{\text{Wage Mark-up over marginal cost}}] \, C_t^{\sigma} L_t^{\varphi} P_t$$

- And proceed with new non-linear model
- However, it will be easier later if we continue to ***log-linearise*** this model

- ***Only difference*** between **Dynare** models with and without wage stickiness is ***first two equations***, which replace single labour supply equation

```
//1-Phillips Curve for Wages
piw = beta*piw(+1)+((1-thetaw)*(1-beta*thetaw)/thetaw)*(sigma*c+phi*l-(w-p));
//2-          Wage Inflation Rate
piw = w - w(-1);
```

- ***"Phillips curve for wages"*** is result of log-linearising equation for optimal wage of households allowed by Calvo fairy to set wages optimally
- ***"Wage Inflation Rate"*** is simply a ***definition***

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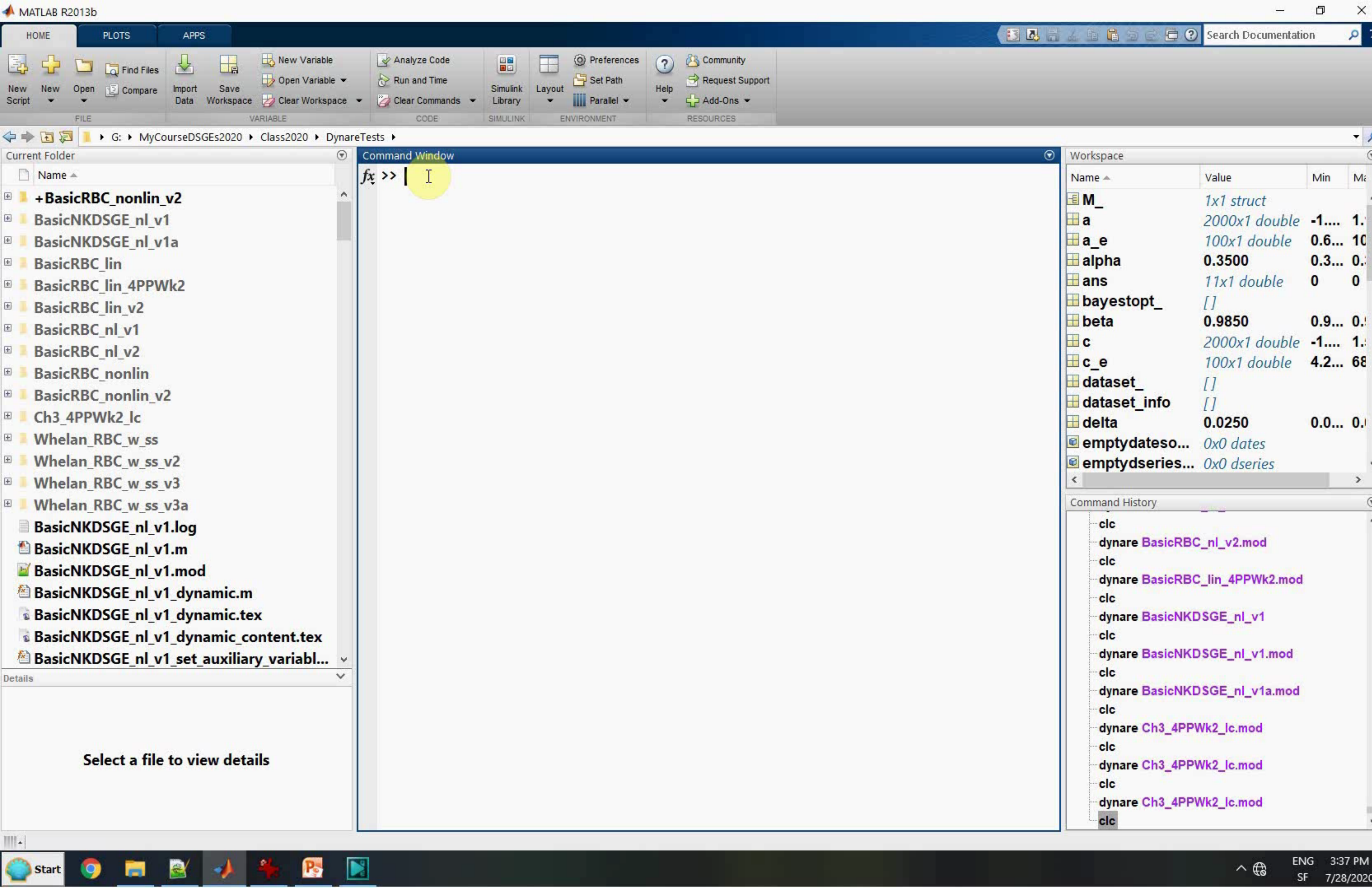
- → ***Model***

```
//1-Phillips Curve for Wages
piw = beta*piw(+1)+((1-thetaw)*(1-beta*thetaw)/thetaw)*(sigma*c+phi*l-(w-p));
//2-          Wage Inflation Rate
piw = w - w(-1);
//3-Euler Equation
(sigma/beta)*(c(+1)-c)=(Rss/Pss)*(r(+1)-p(+1));
//4-Law of Motion of Capital
k(+1) = (1-delta)*k + delta*i;
//5-Production Function
y = a + alpha*k + (1-alpha)*l;
//6-Demand for Capital
k = y - (r - p);
//7-Demand for Labour
l = y - (w - p);
//8-Marginal Cost
mc = ((1-alpha)*w + alpha*r - a);
//9-Phillips Curve
pi = beta*pi(+1)+((1-theta)*(1-beta*theta)/theta)*(mc-p);
//10          Inflation Rate
pi = p - p(-1);
//11-Goods Market Equilibrium Condition
Yss*y = Css*c + Iss*i;
//12-Productivity Shock
a = rhoa*a(-1) + e;
```

New !!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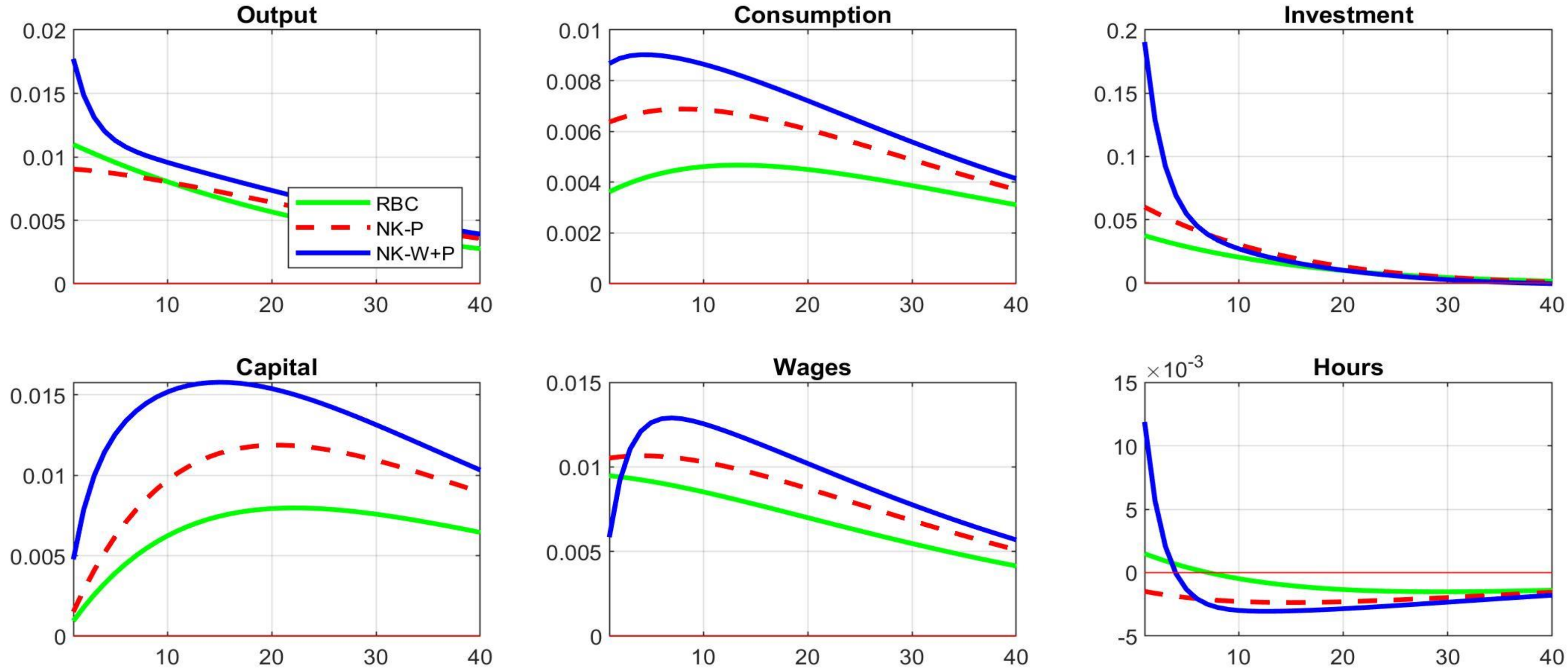
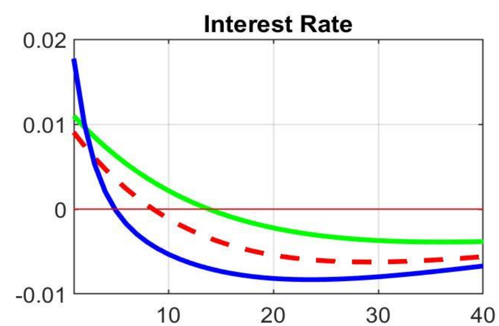
• Comparing IRFs of NK DSGE and RBC:





These graphs show that as ***frictions*** are incorporated, c, i and k become more ***persistent*** ➔ output also. Further, here presence of ***wage stickiness*** ➔ ***labour supply*** initially increases more than in RBC and NK-P models to compensate for initially slower wage rise

- ***NK model*** introduces into basic RBC more realistic specifications:
  - ***imperfect*** competition ☑
  - ***frictions*** in ***prices*** and in ***wages*** ☑
  - ***habit*** formation in consumption ☑
  - ***non-Ricardian*** agents
  - ***adjustment costs*** in investment
  - ***capacity*** (non-)utilisation ***costs***
  - ***government*** sector (***monetary*** and ***fiscal***)
  - ***financial*** sector ⇨ *Week 5*
  - ***housing*** sector ⇨ *Week 5*
  - ***unemployment*** ⇨ *Week 6*
  - ***environment*** ⇨ *Week 6*
  - ***foreign*** trade ⇨ *Week 7*

Non-Ricardian Agents and Habit Formation

- Now enter realm of ***heterogeneous agents*** (“HANK”)
- ***“Ricardian” households:***
  - forward looking
  - base spending not only on current income, but also on ***expected future*** income (“Ricardian equivalence”)
- RBC: all households assumed to have ***free access to financial markets***
- for carrying forward present income to future (via ***saving***) and
- for bringing future income to present (via ***borrowing***)
- ➔ “Complete markets and Arrow-Debreu securities” assumption

- But many agents face ***liquidity constraints:***
  - willing to go into ***debt*** to increase present consumption levels
  - but ***do not have access to credit***
- → cannot maximise their intertemporal utility
- → ***"Non-Ricardian"*** or ***"rule-of-thumb"*** or ***"hand-to-mouth"*** or "***LAMP – Limited Asset Market Participation***" agents
- Many empirical studies both macro- and micro- → a ***significant proportion*** of a country's population is subject to liquidity constraints

- ***NK***: allow ***some*** households to be ***non-Ricardian***
- Liquidity restrictions affecting non-Ricardian households ➔ ***no*** intertemporal maximisation
- ➔ ***consumption*** of non-Ricardian agents ***equals income*** in each period (***assumption*** – ∃ others)
- ➔ Budget constraint of ***Ricardian*** households is

$$P_t\left(C_{R,t}+I_t\right)=W_tL_{R,t}+R_tK_t$$

- But that of ***non-Ricardian*** households will be

$$P_tC_{NR,t}=W_tL_{NR,t}$$

- because ***no saving, no investment*** (➔ no R_tK_t hence no I_t) for non-Ricardians

- Assume ***fraction*** ω_R of households has ***access to financial markets*** (and therefore act as ***Ricardian*** agents)
- but remaining fraction (1 - ω_R) do ***not*** have access to these markets
- ➔ simply consume all current available household income ("***hand-to-mouth***" or "***LAMP***" agents)
- Since only two types of agents ➔ "***TANK***" model
- [If >2 types of agent ➔ "***HANK***" model – "heterogeneous agent NK" model]

- Next, introduce ***habit formation*** into model
- ***RBC***: utility of households depends only on ***current*** consumption and not on ***previous*** consumption
- But empirically, behaviour of households follows a fairly ***regular pattern***
- If household is used to a certain level of consumption and suddenly a ***shock*** [eg, Covid] alters its income, it does not immediately change its pattern of consumption, but uses its ***savings*** to ***mitigate*** this alteration
- ➔ "***habit formation***"

- ***NK***: utility function ***intertemporally additively separable***
- ***New***: maximise utility in terms of ***additional utility*** of consumption today (C_t) over that from "habitual" level (defined as $\varphi_c C_{t-1}$)

➔ ***quasi-difference*** in consumption: $U(C_t - \varphi_c C_{t-1})$

  – where $1 > \varphi_c > 0$ = ***persistence coefficient*** of habit formation (degree to which preferences are ***non-***separable over time)

- If ***persistence coefficient*** φ_c is large (***near 1***) then there will be ***very little additional utility*** (since quasi-difference $C_t - \varphi_c C_{t-1}$ is then close to 0)

- Whereas if persistence coefficient φ_c is small (***near 0***) ➔ ***very little habit formation***, then additional utility will be very substantial (since $C_t - \varphi_c C_{t-1}$ is then close to C_t)
- Why bother with "***habit formation***"?
- Fuhrer (2000) demonstrates ability of habit-formation model to produce ***empirically-observed hump-shaped impulse response*** of consumption to transitory but persistent income shock (which model without habit formation ***fails to do*** adequately)

- Two types of habit formation: ***external and internal***
- ***External*** consumption habits do ***not*** depend on agents' past decisions wrt ***individual*** consumption
- ***instead*** depend on economy's ***aggregate*** consumption ("keeping up with the Joneses")
- ➔ $U(C_{j,t} - \varphi_c \boxed{C_{t-1}})$
- ***Internal*** habit formation: individual's habits determined in terms of ***own past*** consumption
- ➔ $U(C_{j,t} - \varphi_c \boxed{C_{j,t-1}})$
- ***If*** $C_{j,t-1} \approx C_{t-1}$ ➔ no practical difference

Remember, we use *real per capita* consumption in our model

- Now introduce (***internal***) ***habit formation*** to NK model ➔ utility function now includes also (***own***) past consumption
- ➔ ***Ricardian*** households ($j = R$) maximise utility function similar to RBC :

additional utility

$$\max_{C_{R,t},K_{t+1}} E_t \sum_{t=0}^{\infty} \beta^t \left[\frac{\left(\boxed{C_{R,t} - \phi_c C_{R,t-1}}\right)^{1-\sigma}}{1-\sigma} - \frac{L_{R,t}^{1+\varphi}}{1+\varphi} \right]$$

- with budget constraint

$$P_t \left(C_{R,t} + \boxed{I_t} \right) = W_t L_{R,t} + \boxed{R_t K_t}$$

- Lagrangian is set up and maximised as last week:

$$\mathscr{L} = E_t \sum_{t=0}^{\infty} \beta^t \left\{ \left[\frac{\left(\boxed{C_{R,t} - \phi_c C_{R,t-1}}\right)^{1-\sigma}}{1-\sigma} - \frac{L_{R,t}^{1+\varphi}}{1+\varphi} \right] - \lambda_{R,t} \left[P_t C_{R,t} + \boxed{P_t K_{t+1} - P_t(1-\delta)K_t} - W_t L_{R,t} - R_t K_t \right] \right\}$$

$P_t I_t$ recall

- NB: ***only Ricardians save*** ➔ *K* has no "*R*" subscript
- Because of presence of habit formation, FOCs are now rather more complex than previously, where we had the simple formulation

$$\frac{\partial \mathscr{L}}{\partial C_{j,t}} = C_{j,t}^{-\sigma} - \lambda_{j,t} P_t = 0$$

Lagrange Multiplier = "shadow price" of consumption

- Now we have

See derivation pp.124-125 of Costa

$$\frac{\partial \mathscr{L}}{\partial C_{R,t}} = \left(\frac{1}{1-\sigma}\right)\frac{\partial\left[\left(C_{R,t}-\phi_c C_{R,t-1}\right)^{1-\sigma}\right]}{\partial C_{R,t}}$$

$$-\lambda_{R,t}P_t + \left(\frac{1}{1-\sigma}\right)\beta\frac{\partial\left[\left(E_t C_{R,t+1}-\phi_t C_{R,t}\right)^{1-\sigma}\right]}{\partial C_{R,t}} = 0$$

- which can be simplified to

$$\left(C_{R,t}-\phi_c C_{R,t-1}\right)^{-\sigma} - \lambda_{R,t}P_t - \phi_c\beta\left(E_t C_{R,t+1}-\phi_c C_{R,t}\right)^{-\sigma} = 0$$

- Rewriting this ➔ ***Lagrange Multiplier*** explicitly

$$\lambda_{R,t} = \frac{\left(C_{R,t}-\phi_c C_{R,t-1}\right)^{-\sigma}}{P_t} - \phi_c\beta\frac{\left(E_t C_{R,t+1}-\phi_c C_{R,t}\right)^{-\sigma}}{P_t}$$

- So much for Ricardians
- For ***non-Ricardians***, budget constraint is ***simpler*** (K and I drop out, remember):

$$P_t C_{NR,t} = W_t L_{NR,t}$$

- However, Utility function for (hand-to-mouth) ***non-Ricardians*** (*j* = *NR*) identical to that for Ricardians
- ➔ ***Lagrange Multiplier for non-Ricardians***

$$\lambda_{NR,t} = \frac{\left(C_{NR,t} - \phi_c C_{NR,t-1}\right)^{-\sigma}}{P_t} - \phi_c \beta \frac{\left(E_t C_{NR,t+1} - \phi_c C_{NR,t}\right)^{-\sigma}}{P_t}$$

- ***Aggregating*** consumption and labour as

$$C_t = \omega_R C_{R,t} + (1 - \omega_R) C_{NR,t}$$

$$L_t = \omega_R L_{R,t} + (1 - \omega_R) L_{NR,t}$$

- ➔ everything needed since ***all other equations remain identical***
- Log-linearisation ➔ **Dynare** model shown in Appendix
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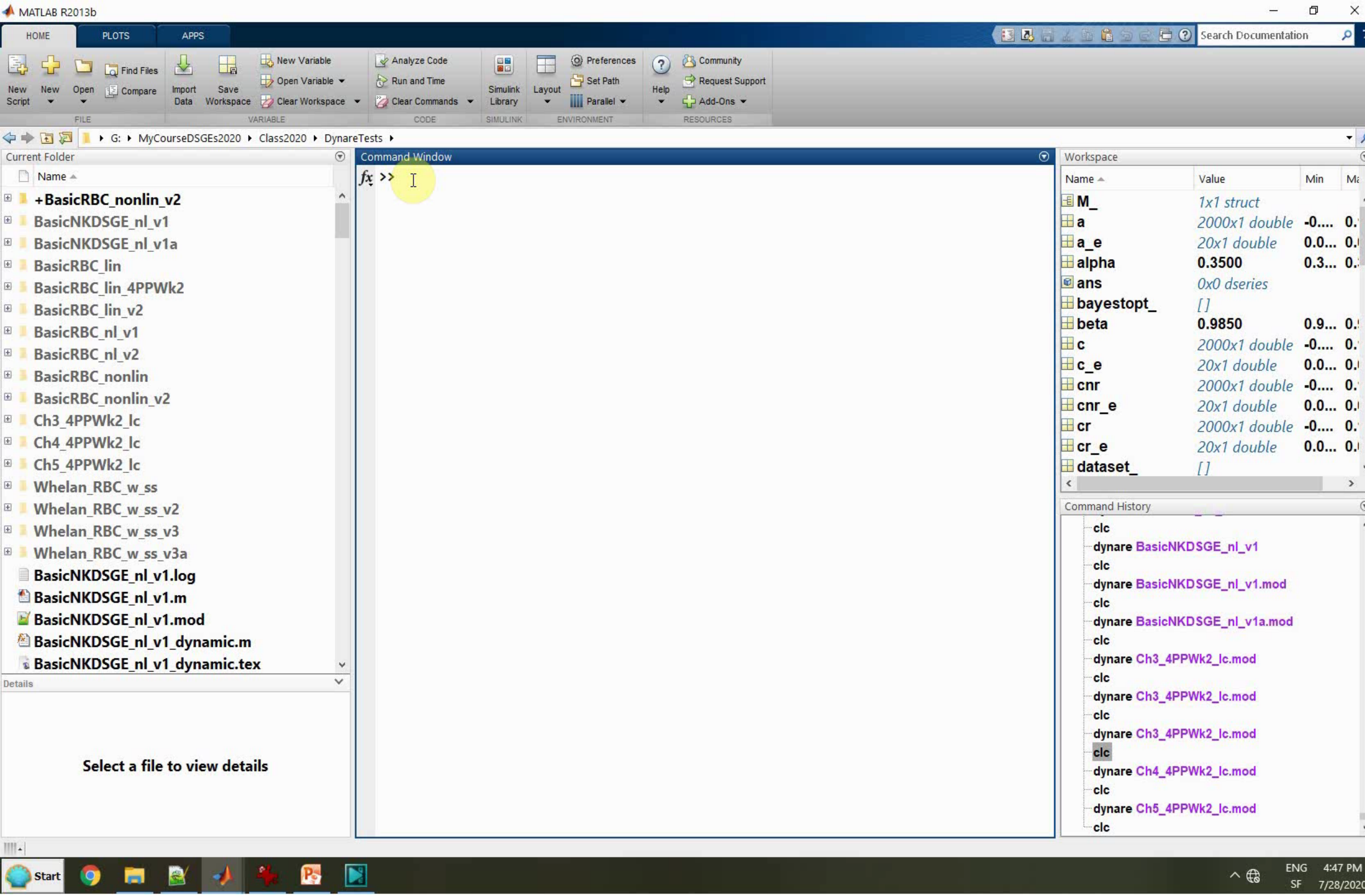
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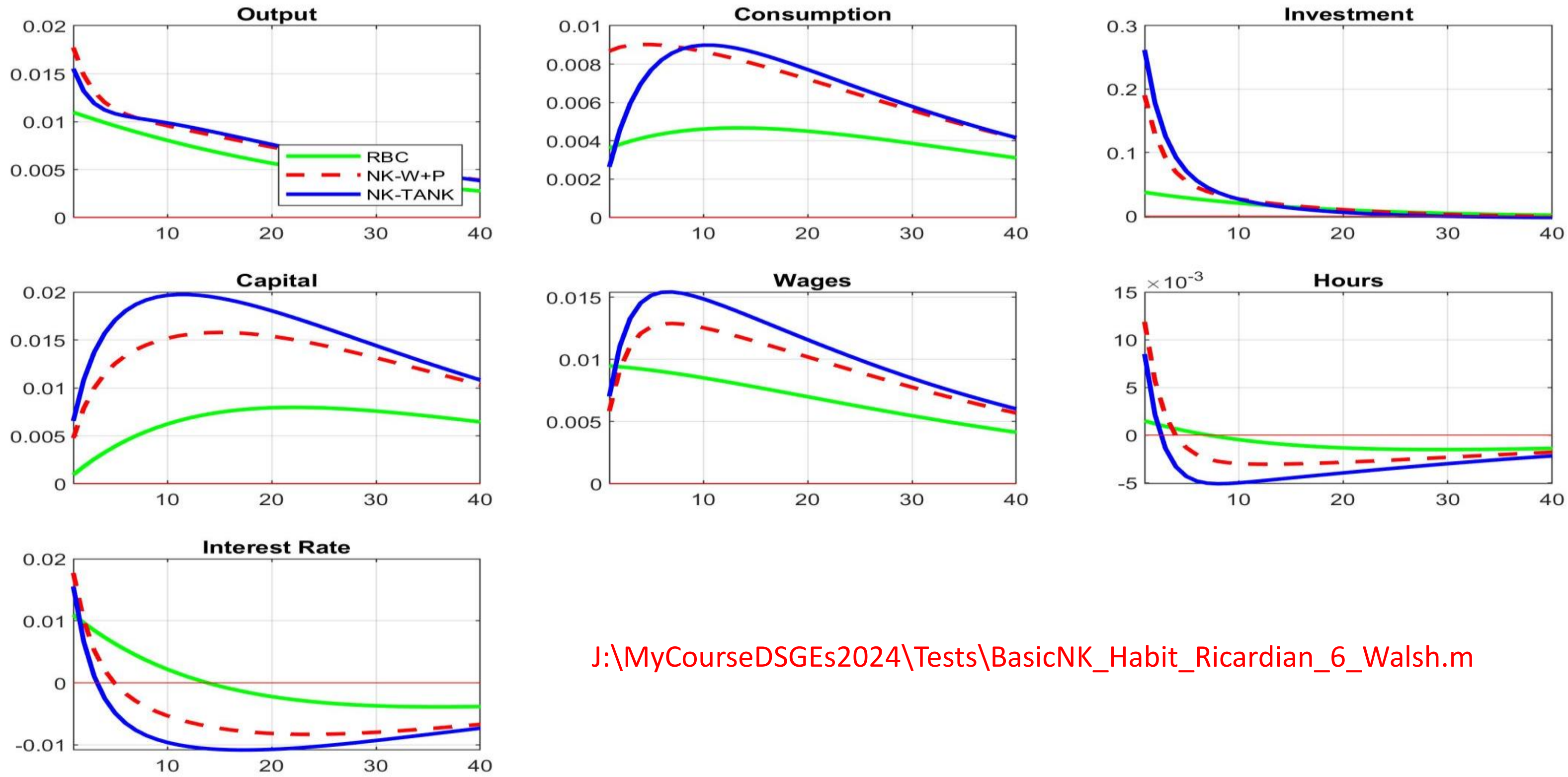
- Comparing results of this model with previous NK (which included imperfect competition and wage and price stickiness) ➔ ***very little impact*** of ***TANK*** and ***HP*** assumptions except on consumption



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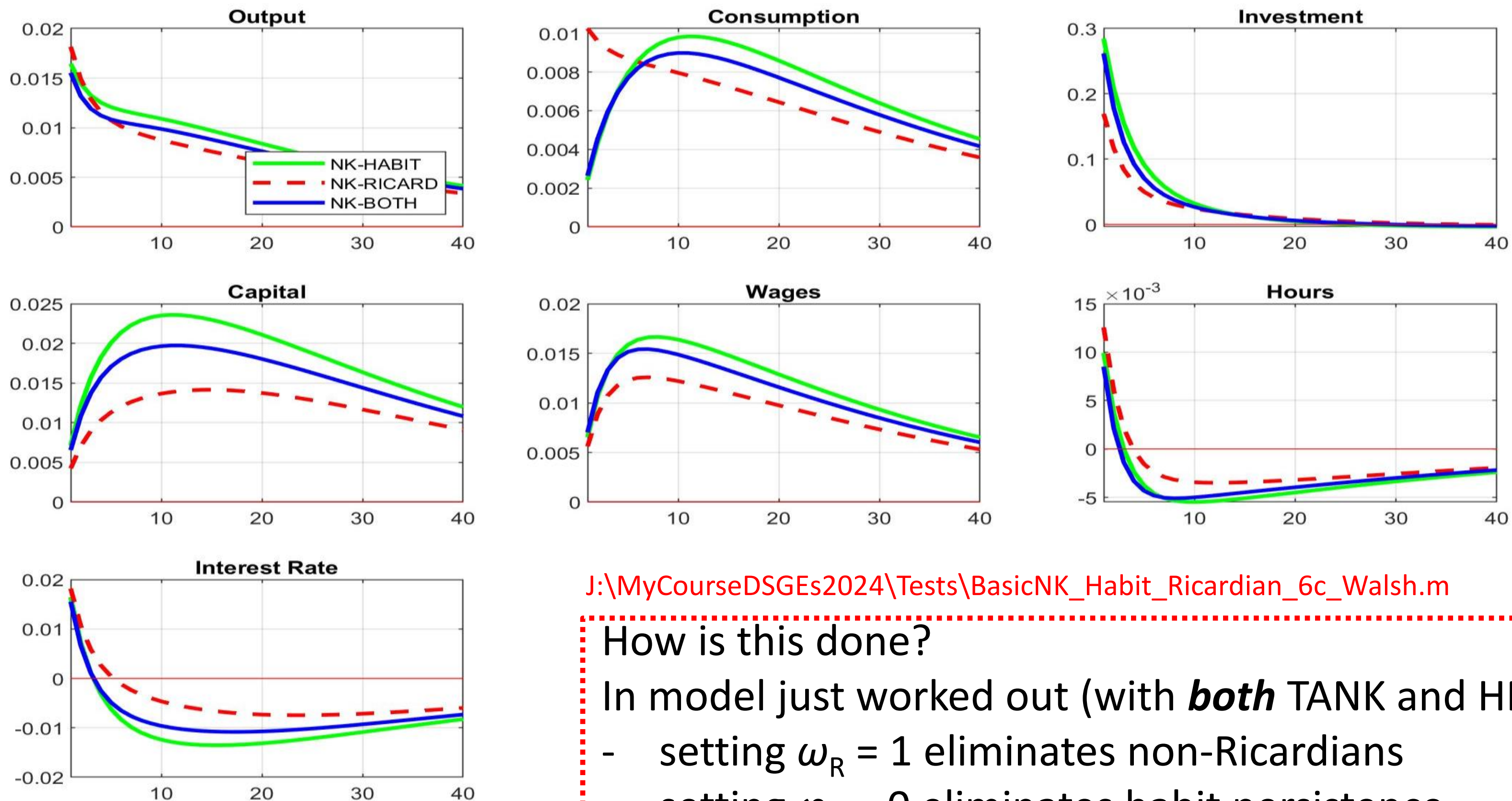
- Separating out ***TANK*** and ***HP*** assumptions shows that impact of inclusion of ***non-Ricardians*** dominates that of ***habit*** persistence



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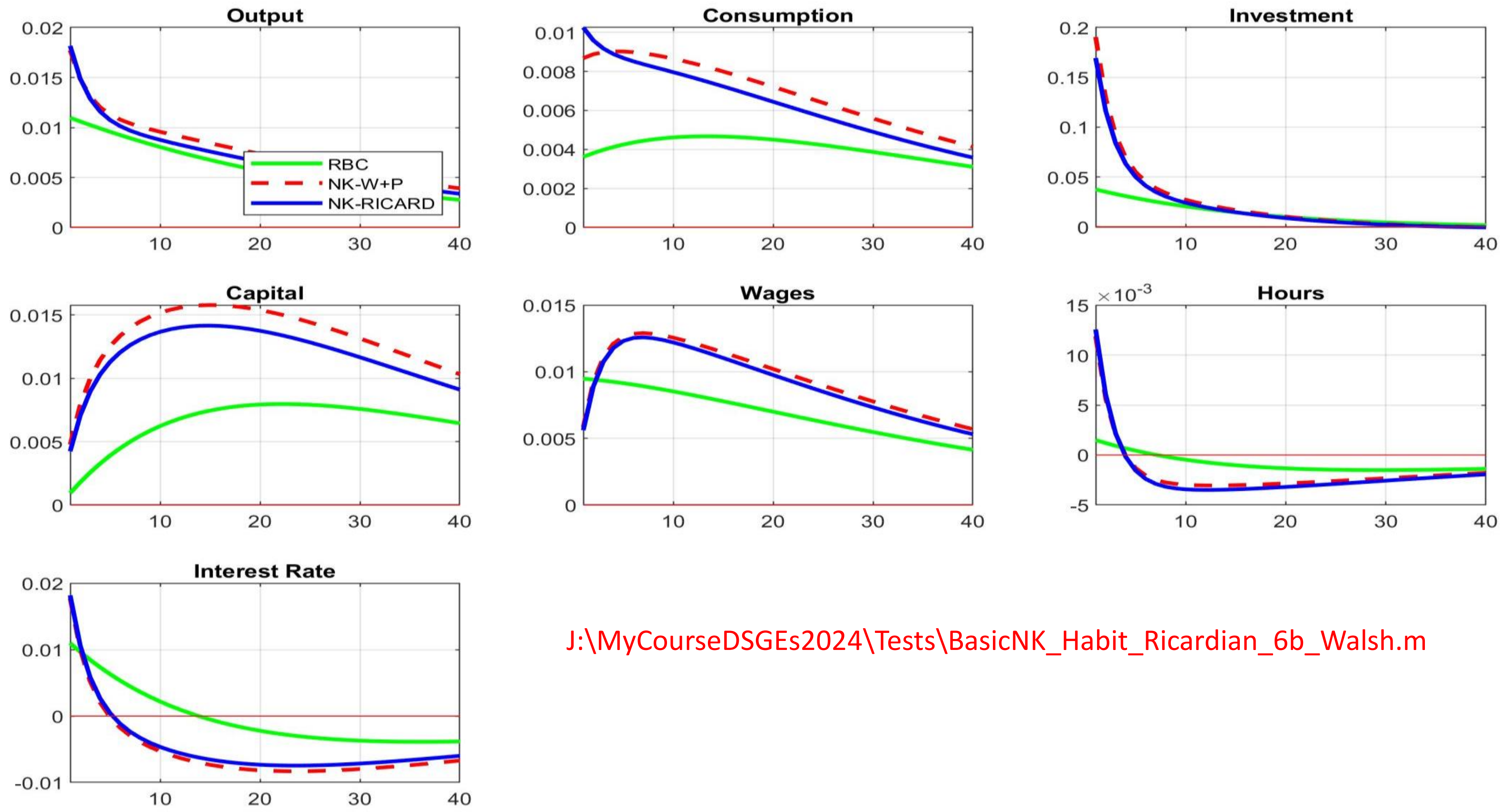
How is this done?
In model just worked out (with ***both*** TANK and HP)
- setting $\omega_R = 1$ eliminates non-Ricardians
- setting $\varphi_C = 0$ eliminates habit persistence

- Compared with difference between ***RBC*** and ***NK-W&P***, inclusion of ***non-Ricardians*** is quite minor, except (as before) for consumption



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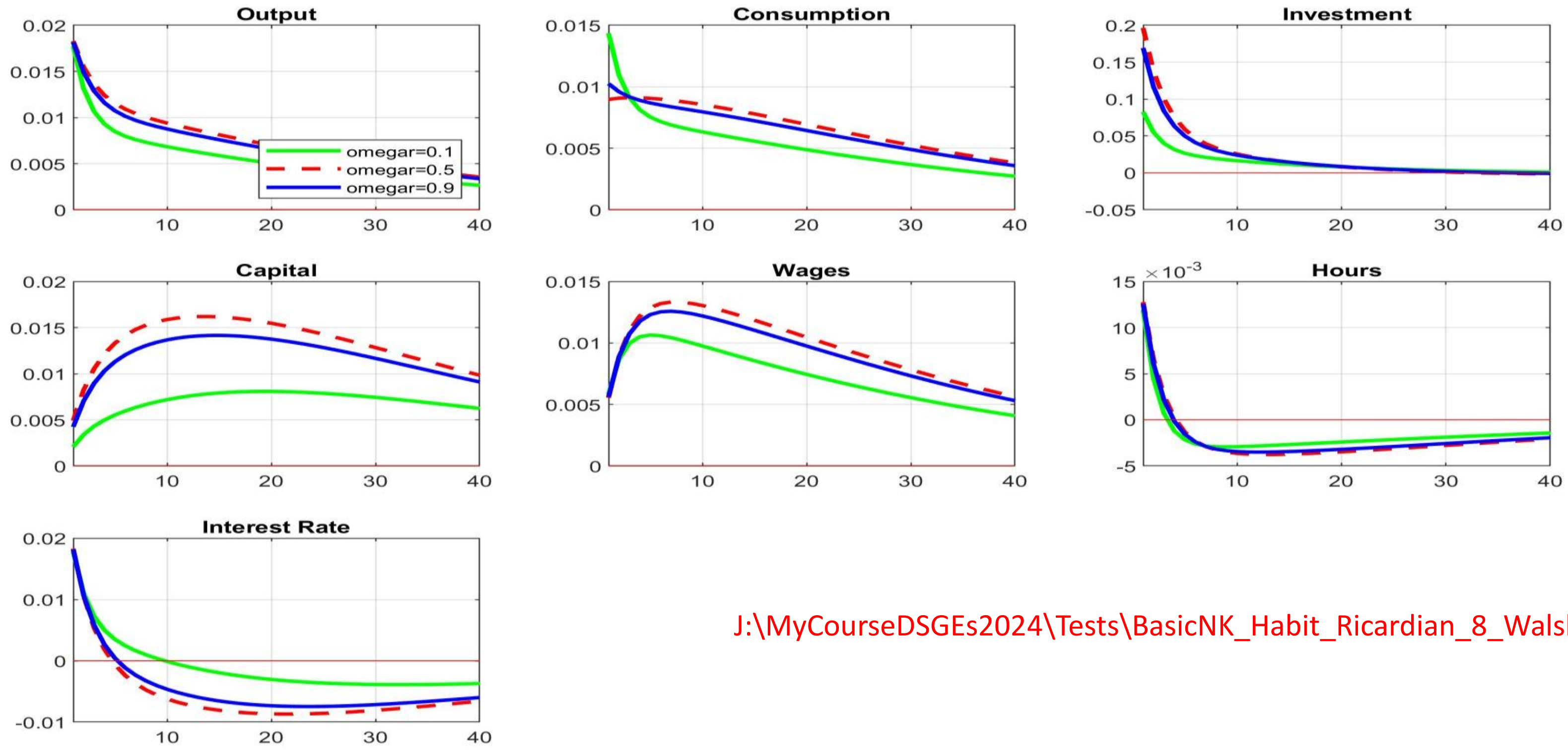
- However, as share of ***non-Ricardians*** increases (ω_R decreases), supply of ***capital*** is compromised as NRs do not have access to ***financial markets*** ➔ lower ***i*** and subsequently lower ***c*** and ***y***



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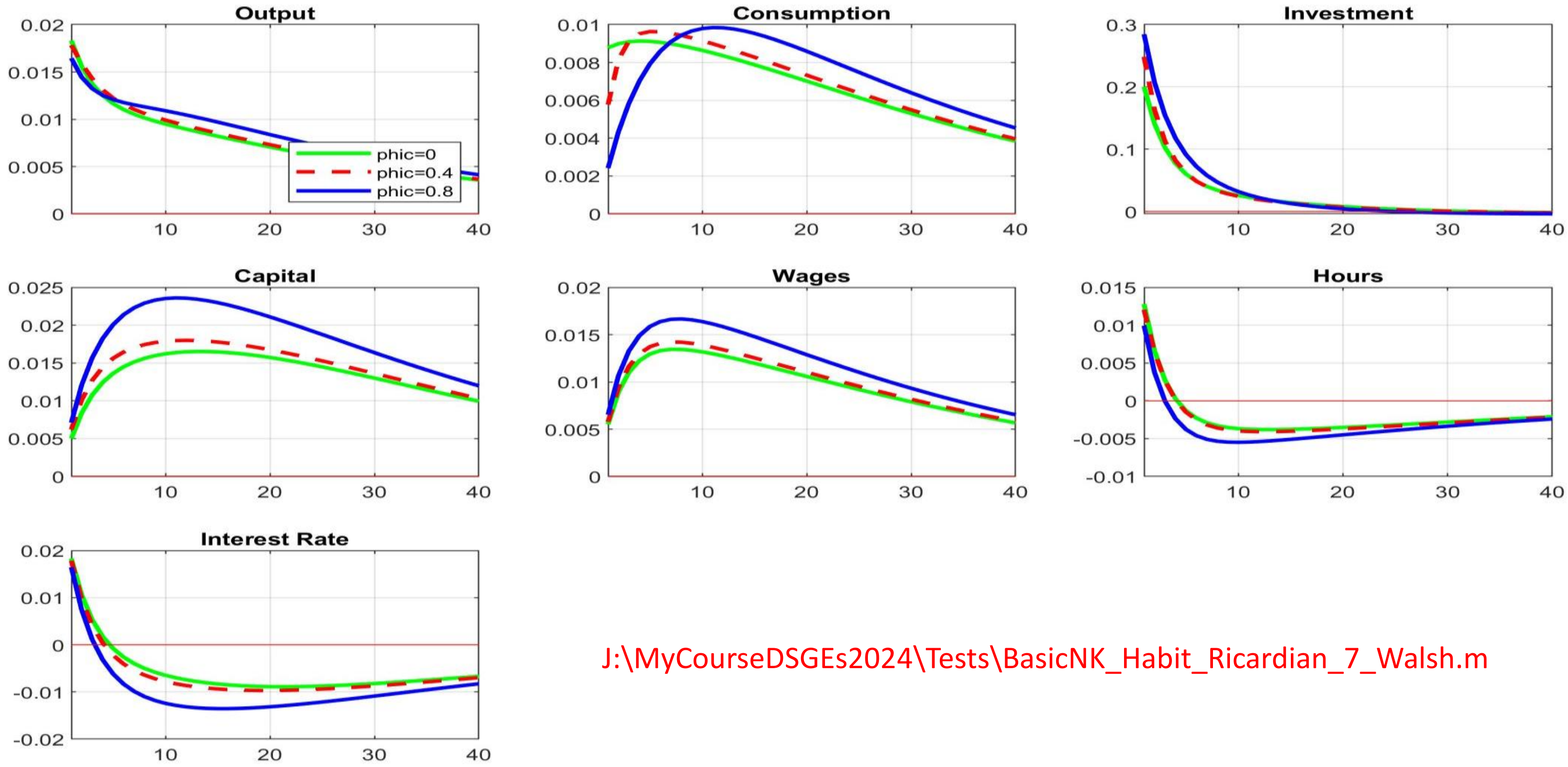
- ***Degree*** of ***habit*** persistence is evidently important for ***consumption***, but also alters ***income effect*** of productivity rise ➔ sharper fall in labour supply, and sharper rises in wages and capital supply



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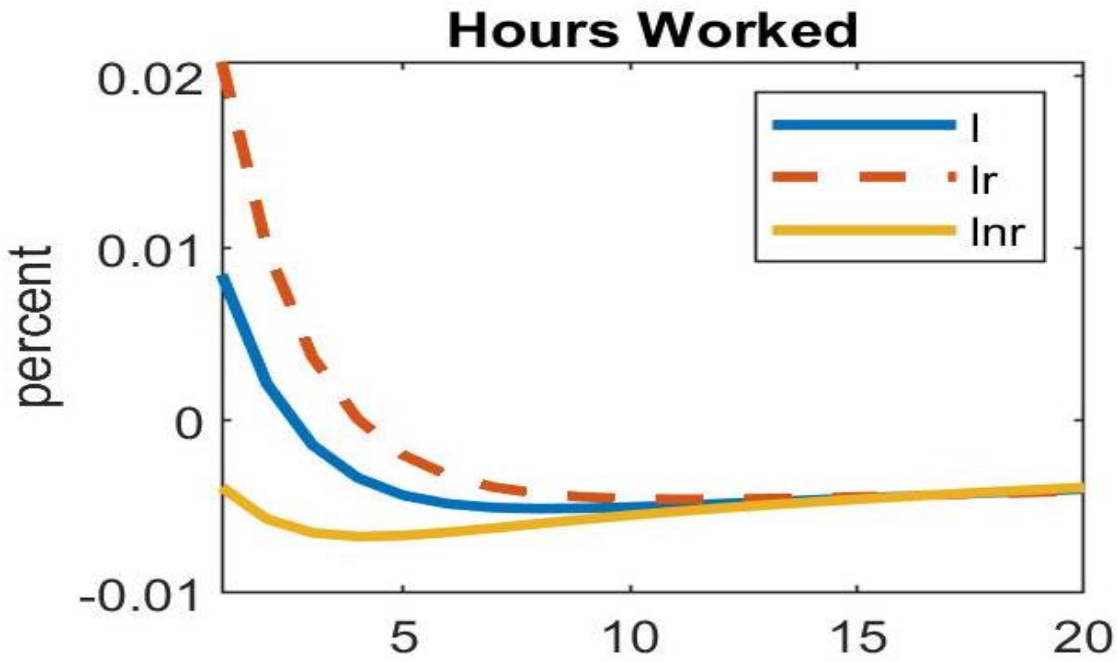
- Behaviour of ***two types of labour***



- *Non-Ricardian* ("hand-to-mouth") households vary their supply of labour ***much less*** than do "optimisers" (*Ricardians*) in response to TFP shock

Adjustment Costs in Investment and Capacity (non-)Utilisation Costs

- **RBC**: capital stock changes ***smoothly*** from period to period ➔ no constraint on investment process
- But capital stock in ***real world*** = factories, warehouses, machinery, equipment, etc
- ***Cannot*** be instantaneously built and installed
- ➔ capital investment for individual firm "***lumpy***"
- Difficult to model
- Could introduce ***Calvo fairy*** here to deal with this!
- But instead …

- Include an ***adjustment cost*** in investment process ➔ changes "law of motion of capital"
- So instead of I_t, we rather have

$$K_{t+1} = (1-\delta)K_t + I_t \boxed{\left[1 - \frac{\chi}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right]}$$

- This strange (quadratic) formulation ➔ adjustment cost = 0 when objective is simply to maintain a ***constant*** level of capital ($I_t = I_{t-1}$ ➔ "steady state")
- What about ***capacity utilisation***?

- **RBC**: installed capital is always used to ***full capacity*** (utilisation rate = 100%)
- But ***empirical*** evidence ➔ capacity utilisation related to firm's position in business cycle
- Firm might have built a plant, but because of weakening economy, it is functioning < maximum capacity
- ➔ opportunity cost

- Let U_t represent rate of ***capacity utilisation*** ➔ Cobb-Douglas production function amended to

$$Y_{j,t} = A_t \left(\boxed{U_t} K_{j,t}\right)^{\alpha} L_{j,t}^{1-\alpha}$$

This changes capital from a ***stock*** concept to a ***flow*** concept

- ➔ changes also to Lagrangian associated with Ricardian households' utility function and to ***budget constraint***

"Effective" flow of services from capital stock

$$P_t\left(C_{R,t} + I_t\right) = W_t L_{R,t} + R_t \boxed{U_t K_t} - P_t K_t \boxed{\left[\Psi_1\left(U_t - 1\right) + \frac{\Psi_2}{2}\left(U_t - 1\right)^2\right]}$$

Adjustment cost function – SW quadratic

- Amended Lagrangian for ***Ricardians***

$$\mathscr{L} = E_t \sum_{t=0}^{\infty} \beta^t \left\{ \left[\frac{\left(C_{R,t} - \phi_c C_{R,t-1}\right)^{1-\sigma}}{1-\sigma} - \frac{L_{R,t}^{1+\varphi}}{1+\varphi} \right] \right.$$

$$-\lambda_{R,t} \left[P_t \left(C_{R,t} + I_t \right) - W_t L_{R,t} - R_t U_t K_t + P_t K_t \boxed{\left[\Psi_1 \left(U_t - 1 \right) + \frac{\Psi_2}{2} \left(U_t - 1 \right)^2 \right]} \right]$$

$$\left. -Q_t \left[K_{t+1} - (1-\delta) K_t - I_t \boxed{\left[1 - \frac{\chi}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 \right]} \right] \right\}$$

- Q_t is "shadow price" [ie, marginal value] of capital, aka ***Tobin's Q*** [recall, ≡ ratio of firm market value to replacement value of firm's assets]

- FOCs ➔ ***Tobin's Q*** [shadow price of capital]

See Costa pp. 157-160 for details

$$Q_t = \beta E_t \left\{ (1-\delta) Q_{t+1} + \lambda_{R,t+1} R_{t+1} U_{t+1} - \lambda_{R,t+1} P_{t+1} \left[\Psi_1 (U_{t+1} - 1) + \frac{\Psi_2}{2} (U_{t+1} - 1)^2 \right] \right\}$$

- FOCs also ➔ equation for ***demand for investment*** assets

$$\lambda_{R,t} P_t - Q_t \left[1 - \frac{\chi}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 - \chi \left(\frac{I_t}{I_{t-1}} \right) \left(\frac{I_t}{I_{t-1}} - 1 \right) \right] = \chi \beta E_t \left[Q_{t+1} \left(\frac{I_{t+1}}{I_t} \right)^2 \left(\frac{I_{t+1}}{I_t} - 1 \right) \right]$$

- and for ***installed capacity*** $\frac{R_t}{P_t} = \Psi_1 + \Psi_2 (U_{t-1})$

Remember, R here is price of capital, ***not*** a policy rate

Only change from previous log-linearised model is in ***investment-related*** equations; rest ***identical***

➔ see details in Appendix

```
//4-Tobin's Q
(Qss/beta)*q = (1-delta)*Qss*q(+1)+LAMBDARss*Rss*Uss
*(lambdar(+1)+r(+1)+u(+1))-LAMBDARss*Pss*Psi1*Uss*u(+1);
//5-Demand for Installed Capacity
(Rss/Pss)*(r-p) = Psi2*Uss*u;
//6-Investment Demand
LAMBDARss*Pss*(lambdar+p)-Qss*q+chi*Qss*(i-i(-1))=chi*beta*Qss*(i(+1)-i);
//13-Production Function
y = a + alpha*(u+k(-1)) + (1-alpha)*l;
//14-Demand for Capital
u + k(-1) = y - (r - p);
```

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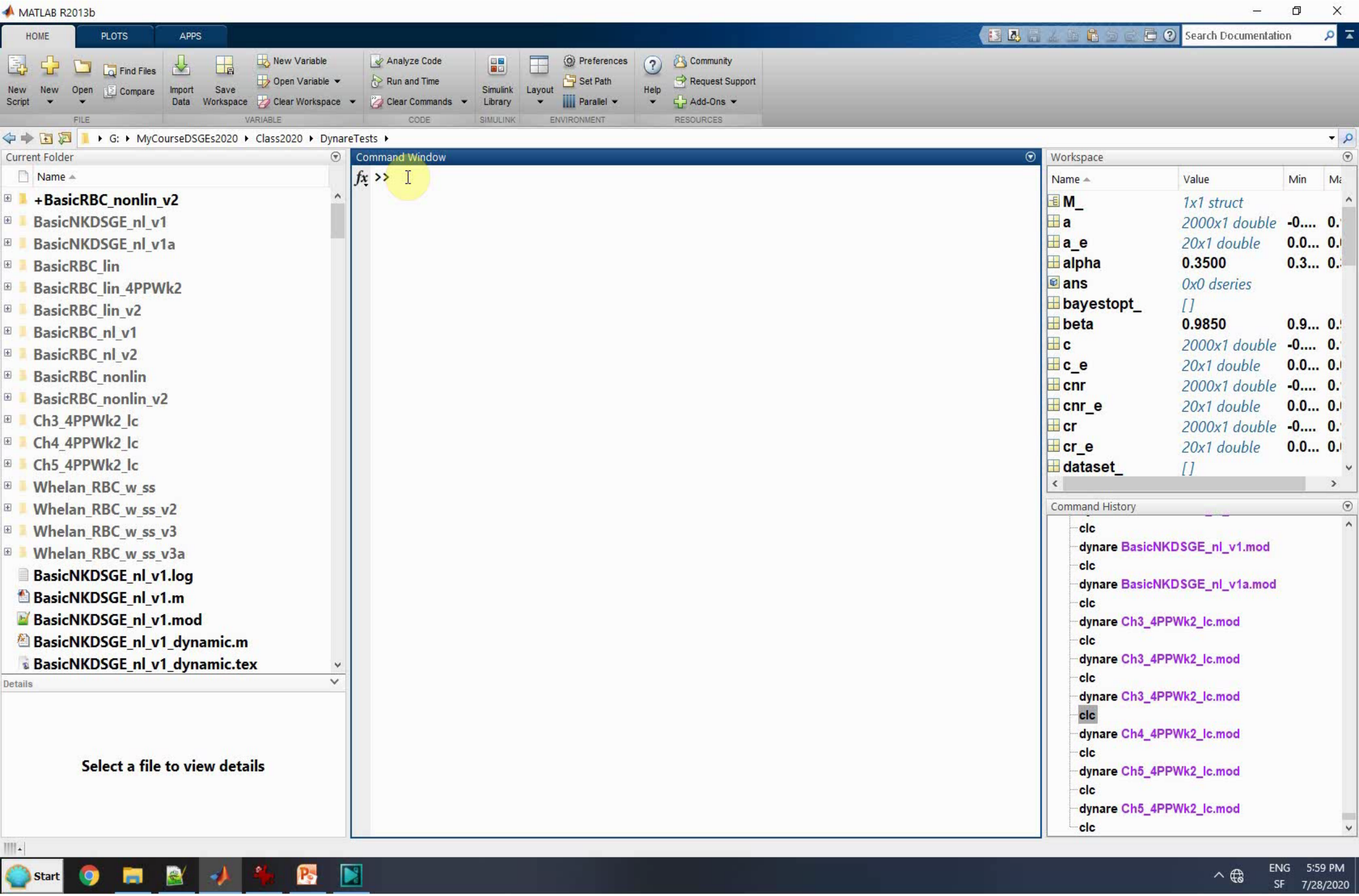
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BasicRBC_nonlin
BasicRBC_nonlin_v2
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Ch5_4PPWk2_lc
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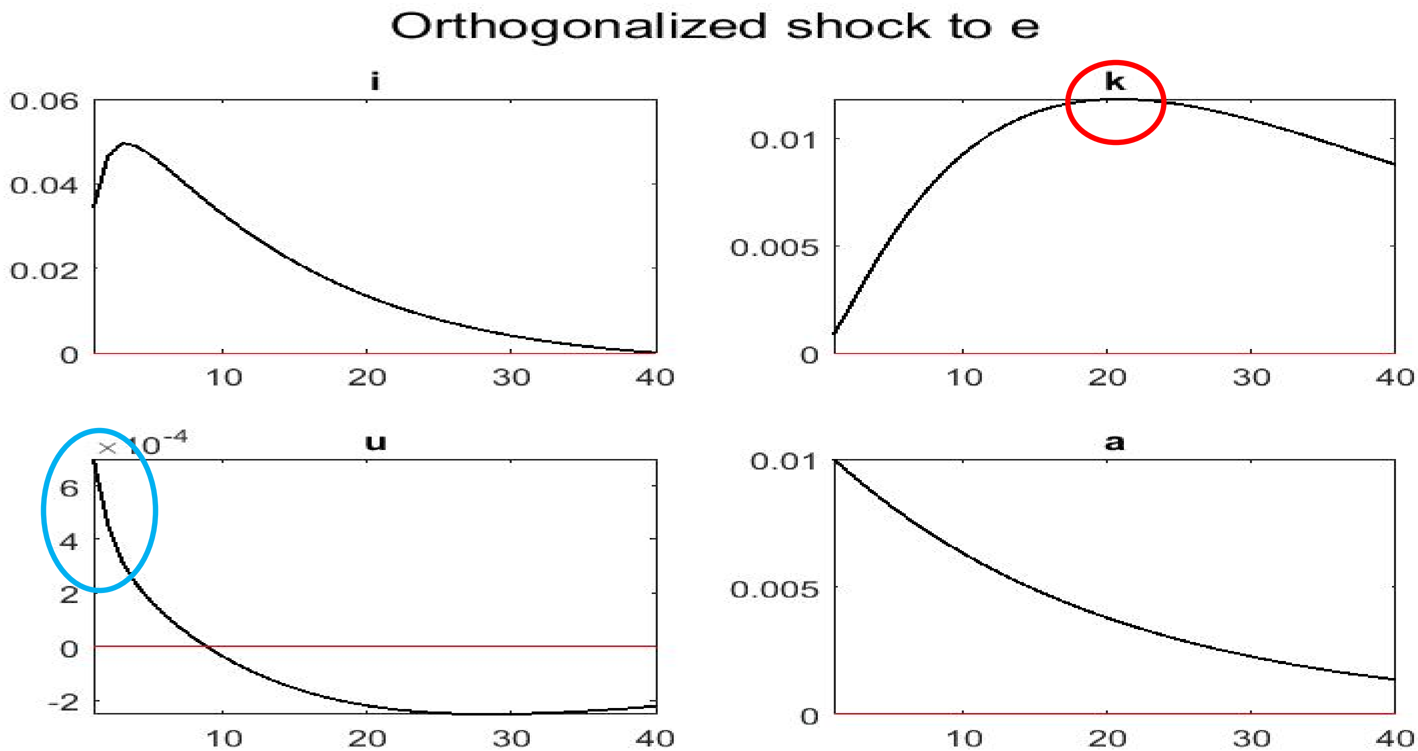
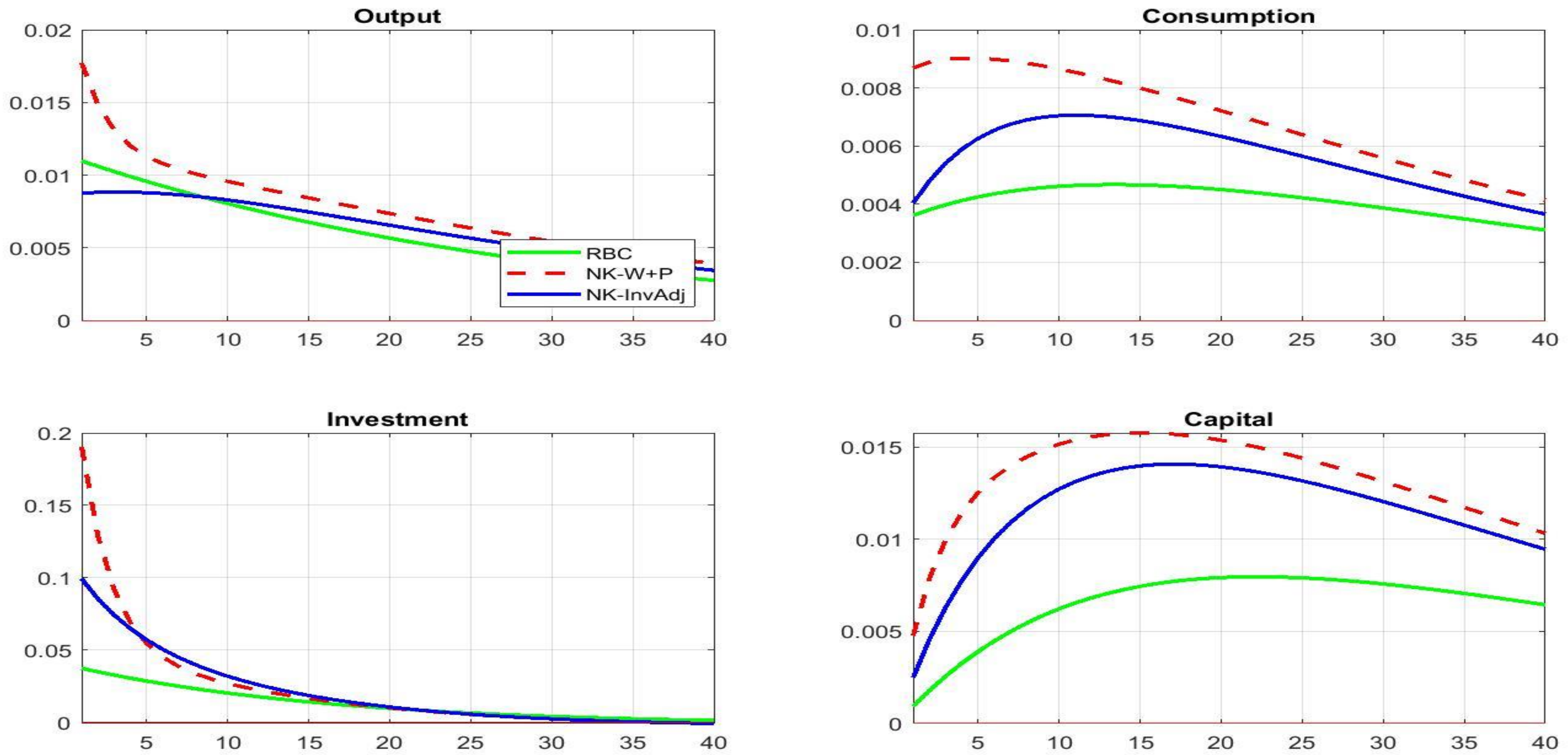
- Investment ***adjustment costs*** introduced here ➔ supply of ***capital*** stock ***responds*** only ***slowly*** to higher demand caused by productivity increase [it takes 5 years = 20 quarters to reach peak]



- ➔ any short-run ***output*** growth is made possible by an ***increase in utilisation*** of installed capacity ***u*** until capital supply responds

- Compared with basic NK model (which includes price and wage stickiness) we see an ***initial sharp downward impact*** of investment and capacity utilisation rigidities on both investment and capital



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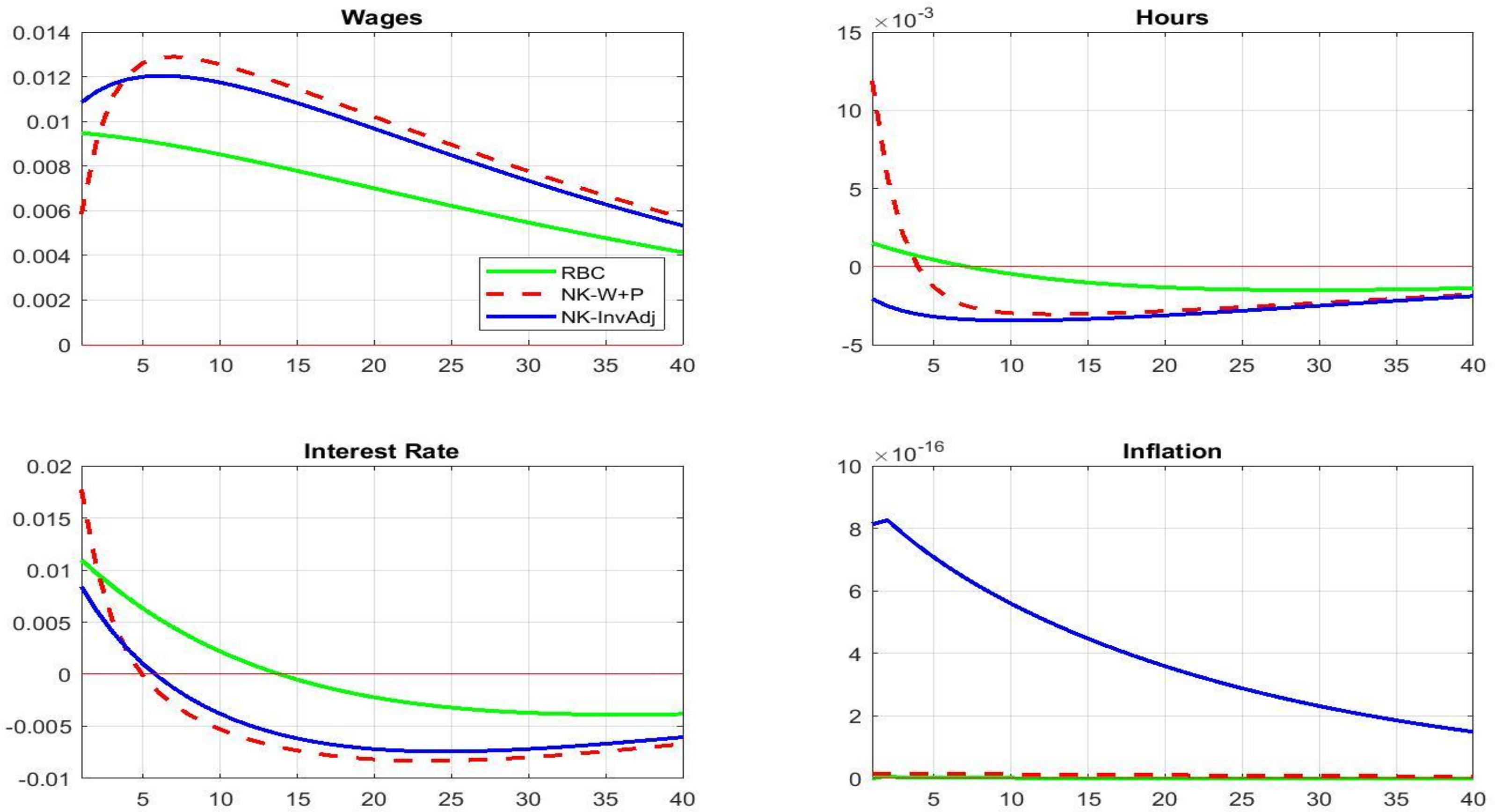
- ***Wage*** rates and ***Hours*** worked also show a ***significant impact*** of addition of these new rigidities: “***smoother***” behaviour than in NK-W+P



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Government sector (monetary and fiscal)

- **RBC**: no government sector
- But in practice ∃:
- ***Monetary*** authority (Central Bank) focuses on ***stabilising prices*** (using Taylor rule in DSGE models)
- ***Fiscal*** authority (Treasury) decides on ***scale and composition of government spending***, among
  - supplying public goods and services to private agents
  - income transfers
  - public capital investment
- But expenditure must be paid for! ➔
  - ***taxation***
  - issuing public ***debt***
  - issuing ***currency***
  - differing costs and benefits to society depending on source

Fiscal Authority

- Taxes affect ***both*** Ricardian and non-Ricardian ***household*** decisions
- Assume following:
  - tax on acquisition of consumer goods and investment assets, τ_c
  - tax on income from labour, τ_l
  - tax on income from capital, τ_k [usually, $\tau_k < \tau_l$]
- Government ***also*** makes ***lump-sum transfer*** of net income ($TRANS_t$) to households, observing the ***proportion*** of Ricardian and non-Ricardian agents

- ***Ricardian*** households ***save*** by acquiring (1-period) **bonds**, discounted at basic rate of interest, ***issued by government*** in each period (B/R^B)

R^B is ***gross*** interest rate

- ➔Ricardian problem

$$\max_{C_{R,t},K^P_{t+1},U_t,I^P_t,B_{t+1}} E_t \sum_{t=0}^{\infty} \beta^t \left[\frac{\left(C_{R,t} - \phi_c C_{R,t-1}\right)^{1-\sigma}}{1-\sigma} - \frac{L_{R,t}^{1+\varphi}}{1+\varphi} \right]$$

- subject to

\+ because tax ***adds to*** cost of consumption and investment goods (VAT)

Net cost of bonds purchased this quarter, and maturing *next* quarter

$$P_t\left(1+\tau_t^c\right)\left(C_{R,t}+I_t^P\right)+\boxed{\frac{B_{t+1}}{R_t^B}}=W_t L_{R,t}\left(1-\tau_t^l\right)+R_t U_t K_t^P\left(1-\tau_t^k\right)$$
$$-P_t K_t^P\left[\Psi_1\left(U_t-1\right)+\frac{\Psi_2}{2}\left(U_t-1\right)^2\right]+\boxed{B_t}+\omega_R P_t TRANS_t$$

Funds received from bonds maturing *this* quarter

- Law of motion of ***private*** capital

$$K_{t+1}^{P} = (1-\delta)K_t^P + I_t^P \left[1 - \frac{\chi}{2}\left(\frac{I_t^P}{I_{t-1}^P} - 1\right)^2\right]$$

- ➔ Lagrangian for Ricardian households

$$\mathcal{L} = E_t \sum_{t=0}^{\infty} \beta^t \Bigg\{ \left[\frac{\left(C_{R,t} - \phi_c C_{R,t-1}\right)^{1-\sigma}}{1-\sigma} - \frac{L_{R,t}^{1+\varphi}}{1+\varphi} \right]$$

$$-\lambda_{R,t} \left\{ \left[P_t\left(1+\boxed{\tau_t^c}\right)\left(C_{R,t} + I_t^P\right) + \frac{B_{t+1}}{R_t^B} - W_t L_{R,t}\left(1-\boxed{\tau_t^l}\right) \right.\right.$$

$$-R_t U_t K_t^P \left(1-\boxed{\tau_t^k}\right) + P_t K_t^P \left[\Psi_1 (U_t - 1) + \frac{\Psi_2}{2}(U_t - 1)^2\right]$$

$$\left.\left. -B_t - \omega_R P_t \boxed{TRANS_t} \right] \right\}$$

$$-Q_t \left[K_{t+1}^P - (1-\delta)K_t^P - I_t^P \left[1 - \frac{\chi}{2}\left(\frac{I_t^P}{I_{t-1}^P} - 1\right)^2\right]\right] \Bigg\}$$

- Solving FOCs → For derivation, see Costa pp. 193-199

1. Lagrange multiplier for Ricardian households:

$$\lambda_{R,t} = \frac{\left(C_{R,t} - \phi_c C_{R,t-1}\right)^{-\sigma}}{P_t \boxed{(1+\tau_t^c)}} - \phi_c \beta \frac{\left(E_t C_{R,t+1} - \phi_c C_{R,t}\right)^{-\sigma}}{P_t \boxed{(1+\tau_t^c)}}$$

2. Tobin's Q:

$$Q_t = \beta E_t \left\{ (1-\delta) Q_{t+1} + \lambda_{R,t+1} R_{t+1} U_{t+1} \boxed{\left(1 - \tau_{t+1}^k\right)} - \lambda_{R,t+1} P_{t+1} \left[\Psi_1 \left(U_{t+1} - 1\right) + \frac{\Psi_2}{2} \left(U_{t+1} - 1\right)^2 \right] \right\}$$

3. Equation for demand for installed capacity:

$$\frac{R_t}{P_t} = \frac{1}{\boxed{\left(1 - \tau_t^k\right)}} \left[\Psi_1 + \Psi_2 \left(U_{t-1}\right)\right]$$

4. Equation for (private) investment demand:

$$\lambda_{R,t} P_t (1 + \tau_t^c) - Q_t \left[1 - \frac{\chi}{2} \left(\frac{I_t^P}{I_{t-1}^P} - 1 \right)^2 - \chi \left(\frac{I_t^P}{I_{t-1}^P} \right) \left(\frac{I_t^P}{I_{t-1}^P} - 1 \right) \right]$$

$$= \chi \beta E_t \left[Q_{t+1} \left(\frac{I_{t+1}^P}{I_t^P} \right)^2 \left(\frac{I_{t+1}^P}{I_t^P} - 1 \right) \right]$$

5. Euler equation for public ***bonds***:

$$\frac{\lambda_{R,t}}{R_t^B} = \beta E_t \lambda_{R,t+1}$$

- ***Non-Ricardian*** households
- As before, but now ***taking into account taxes***:

$$\max_{C_{NR,t}} E_t \sum_{t=0}^{\infty} \beta^t \left[\frac{\left(C_{NR,t} - \phi_c C_{NR,t-1}\right)^{1-\sigma}}{1-\sigma} - \frac{L_{NR,t}^{1+\varphi}}{1+\varphi} \right]$$

- subject to

$$P_t \boxed{(1+\tau_t^c)} C_{NR,t} = W_t L_{NR,t} \boxed{\left(1-\tau_t^l\right)} + (1-\omega_R) P_t \boxed{TRANS_t}$$

- Solving FOCs ➔ Lagrange multiplier for non-Ricardian households

$$\lambda_{NR,t} = \frac{\left(C_{NR,t} - \phi_c C_{NR,t-1}\right)^{-\sigma}}{P_t \boxed{(1+\tau_t^c)}} - \phi_c \beta \frac{\left(E_t C_{NR,t+1} - \phi_c C_{NR,t}\right)^{-\sigma}}{P_t \boxed{(1+\tau_t^c)}}$$

- Wages determined as in previous models, but now taking into account taxes and transfers
- → optimal wage, where "j" represents either Ricardian (R) or non-Ricardian (NR) households, since ***there is no qualitative difference*** in labour which they provide

$$W_{j,t}^{*} = \left(\frac{\psi_W}{\psi_W - 1} \right) E_t \sum_{i=0}^{\infty} (\beta \theta_W)^i \left[\frac{L_{j,t+i}^{\psi}}{\lambda_{j,t+i} \left(1 - \tau_t^l \right)} \right]$$

- thus ***aggregate wage level*** is (as before)

$$W_t = \left[\theta_w W_{t-1}^{1-\psi_W} + (1 - \theta_w) W_t^{*1-\psi_w} \right]^{\frac{1}{1-\psi_W}}$$

Firms

- ***Public capital*** (superscript "G") now also in production function for intermediate goods

$$Y_{j,t} = A_t K^{P}_{j,t} \alpha_1 L^{\alpha_2}_{j,t} K^{\boxed{G}}_{j,t} \alpha_3$$

- ➔ marginal cost

$$MC_{j,t} = \frac{1}{A_t K^{G}_{j,t} \alpha_3} \left(\frac{W_t}{\alpha_2} \right)^{\alpha_2} \left(\frac{R_t}{\alpha_1} \right)^{\alpha_1}$$

- ***Does not affect*** optimal pricing decision, which remains

$$P^{*}_{j,t} = \left(\frac{\psi}{\psi - 1} \right) E_t \sum_{i=0}^{\infty} (\beta\theta)^i MC_{j,t+i}$$

Government

Net income from bonds sold this quarter, but maturing next quarter

- No ***seignorage*** ➔government's ***budget constraint***

$$\boxed{\frac{B_{t+1}}{R_t^B}} - \boxed{B_t} + T_t = P_t G_t + P_t I_t^G + P_t TRANS_t$$

Cost of repaying bonds maturing this quarter

- and ***tax revenue*** is

$$T_t = \tau_t^c P_t \left(C_t + I_t^P \right) + \tau_t^l W_t L_t + \tau_t^k \left(R_t - \delta \right) K_t^P$$

- Also assume that each fiscal instrument evolves following a ***"fiscal policy rule"***
- Many forms possible
- Assume a ***common*** rule of the form

$$\frac{Z_t}{Z_{ss}} = \left(\frac{Z_{t-1}}{Z_{ss}}\right)^{\gamma_Z} \left(\frac{B_t}{Y_{t-1}P_{t-1}} \quad \frac{Y_{ss}P_{ss}}{B_{ss}}\right)^{(1-\gamma_Z)\phi_Z} S_t^Z$$

- where Z represents each of the fiscal policy instruments in turn (τ^c, τ^l, τ^k)
- Why this form?
- Take logs ➔ LHS: $\log(Z_t) - \log(Z_{SS})$ … *looks familiar?*

- This rule also ensures that ***as issued debt rises*** (B_t increases) ***so too do tax rates*** to maintain balance
- A "fiscal shock" S^Z is attached to each rule

$$\log S_t^Z = \rho_Z \log S_{t-1}^Z + \varepsilon_{Z,t}$$

- Law of motion of ***public capital***

$$K_{t+1}^{\boxed{G}} = (1 - \delta_G)\, K_t^G + I_t^G$$

- So much for ***Treasury***; what about ***Central Bank***?
- Central Bank follows a standard ***Taylor Rule*** with a twofold objective: price stability and economic growth $$\frac{R_t^B}{R_{SS}^B} = \left(\frac{R_{t-1}^B}{R_{SS}^B}\right)^{\gamma_R} \left[\left(\frac{\pi_t}{\pi_{ss}}\right)^{\gamma_\pi} \left(\frac{Y_t}{Y_{ss}}\right)^{\gamma_Y}\right]^{(1-\gamma_R)} S_t^m$$
  - where γ_Y and γ_π = sensitivities of basic interest rate in relation to output and inflation rate
  - γ_R = interest rate smoothing parameter
  - R_B = ***policy rate*** (= interest rate on government bonds)
- Again, this form of Taylor Rule is used because taking logs ➔ LHS: $\log(R_t^B) - \log(R_{SS}^B)$

- Here too, I have included a monetary shock, S^m which evolves according to

$$\log S_t^m = \rho_m \log S_{t-1}^m + \varepsilon_{m,t}$$

- Other equations may be ***carried over*** from previous model
- Main ***changes*** are those shown on following slides

$$P_t\left(1+\tau_t^c\right)\left(C_{R,t}+I_t^P\right)+\frac{B_{t+1}}{R_t^B}=W_tL_{R,t}\left(1-\tau_t^l\right)+R_tU_tK_t^P\left(1-\tau_t^k\right)$$

$$-P_tK_t^P\left[\Psi_1\left(U_t-1\right)+\frac{\Psi_2}{2}\left(U_t-1\right)^2\right]+B_t+\omega_RP_tTRANS_t \qquad \text{(Ricardian budget)}$$

$$\frac{\lambda_Rt}{R_t^s}=\beta E_t\lambda_{R,t+1} \qquad \text{(Public bond Euler)}$$

$$\frac{B_{t-1}}{R_t^B}-B_t+T_t=P_tG_t+P_tI_t^G+P_tTRANS_t \qquad \text{(Gov't budget)}$$

$$T_t=\tau_t^cP_t\left(C_t+I_t^P\right)+\tau_t^lW_tL_t+\tau_t^k\left(R_t-\delta\right)K_t^P \qquad \text{(Gov't revenue)}$$

$$\frac{Z_t}{Z_{ss}}=\left(\frac{Z_{t-1}}{Z_{ss}}\right)^{\gamma_Z}\left(\frac{B_t}{Y_{t-1}P_{t-1}}\quad\frac{Y_{ss}P_{ss}}{B_{ss}}\right)^{(1-\gamma_Z)\phi_Z}S_t^Z \qquad \text{(Fiscal policy rule)}$$

$$K_{t+1}^G=\left(1-\delta_G\right)K_t^G+I_t^G \qquad \text{(Public capital)}$$

$$\frac{R_t^B}{R_{Bs}^B}=\left(\frac{R_{t-1}^B-1}{R_{ss}^B}\right)^{\gamma_R}\left[\left(\frac{\pi_t}{\pi_{ss}}\right)^{\gamma_\pi}\left(\frac{Y_t}{Y_{ss}}\right)^{\gamma_s}\right]^{(1-\gamma_R)}S_t^m \qquad \text{(Taylor Rule)}$$

$$Y_t=C_t+I_t^P+I_t^G+G_t \qquad \text{(Equilibrium)}$$

$$\log A_t=\rho_A\log A_{t-1}+\varepsilon_t \qquad \text{(Productivity shock)}$$

$$\log S_t^Z=\rho_Z\log S_{t-1}^Z+\varepsilon_{Z,t} \qquad \text{(Fiscal shock)}$$

$$\log S_t^m=\rho_m\log S_{t-1}^m+\varepsilon_{m,t} \qquad \text{(Monetary shock)}$$

I have also added several "shocks" to model, so that it is possible to observe ***effects*** of ***fiscal*** and ***monetary policy*** [only ***structural*** shock is – as previously – to ***productivity***]

Changing this from 0 permits us to see how altering VAT affects economy – note ***negative*** sign

```
//33 - Income Transfer Shock
STRANS = rhoTRANS*STRANS(-1) + e_TRANS;
//34 - Consumption Tax Shock (Negative => Reduction in Tax Rate!)
Stau_c = rhotau_c*Stau_c(-1) - e_tau_c;
//35 - Labour Income Tax Shock (Negative => Reduction in Tax Rate!)
Stau_l = rhotau_l*Stau_l(-1) - e_tau_l;
//36 - Capital Income Tax Shock (Negative => Reduction in Tax Rate!)
Stau_k = rhotau_k*Stau_k(-1) - e_tau_k;
//37- Monetary Shock (Negative => Reduction in Interest Rate!)
Sm = rhom*Sm(-1)-e_m;
```

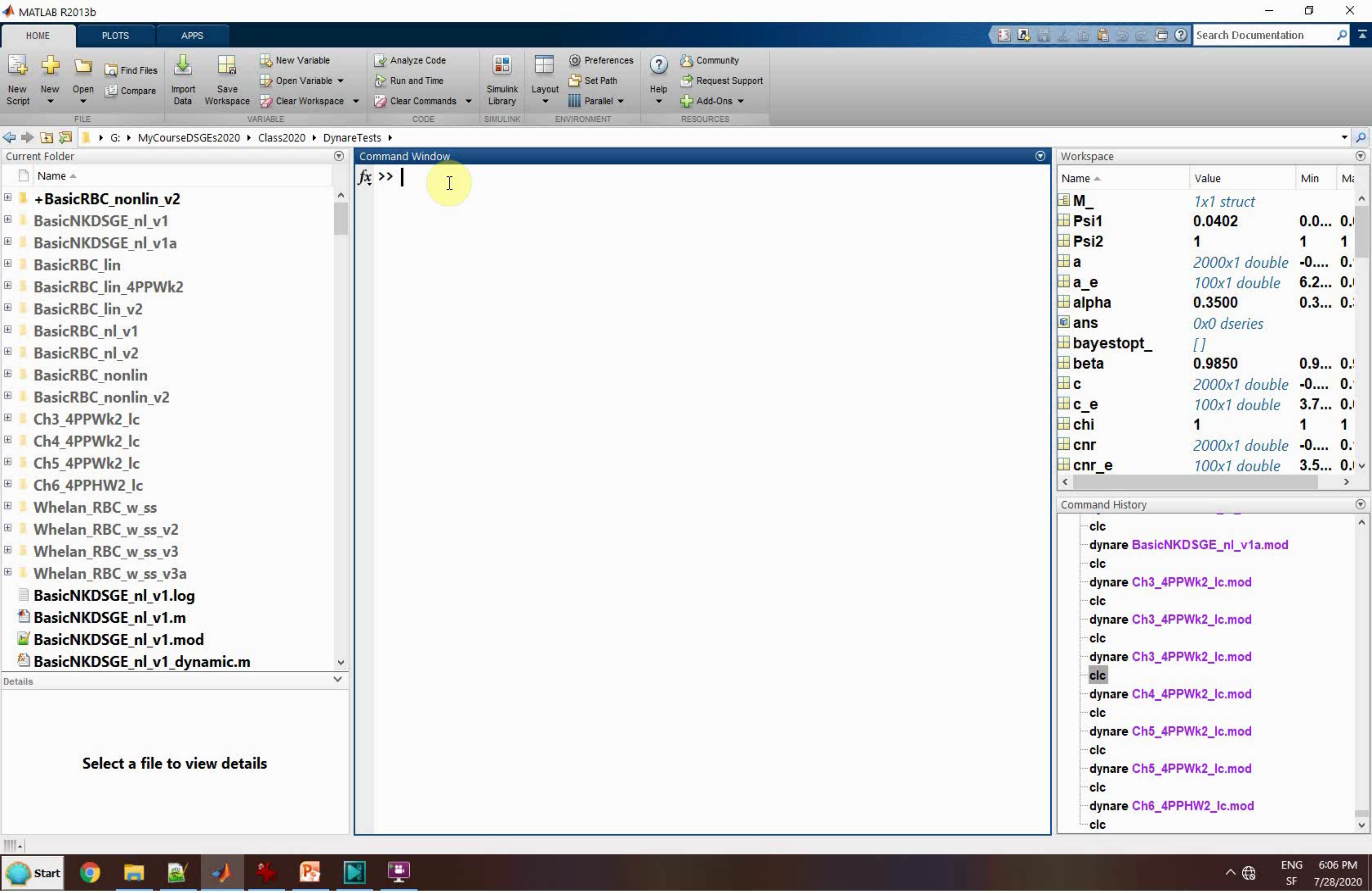
- Model uses following parameters

must sum to unity

α_1	0.300	private capital share
α_2	0.650	labour share
α_3	0.050	public capital share
β	0.985	discount factor
ρ_a	0.900	autocorrelation technology shock
σ	2.000	CRRA coefficient
ϕ	1.500	Frisch elasticity
ψ	8.000	elasticity of substitution - intermediate goods
ψ_w	21.000	elasticity of substitution - labour
δ	0.025	depreciation rate - private
δ_g	0.025	depreciation rate - public
θ	0.750	Calvo parameter - prices
θ_w	0.750	Calvo parameter - wages

- Model uses following parameters

γ_r	0.800	Taylor Rule - interest rate sensitivity
γ_p	1.500	Taylor Rule - inflation sensitivity
γ_y	0.500	Taylor Rule - output sensitivity
τ_c	0.160	Tax rate on consumption SS
τ_l	0.170	Tax rate on labour SS
τ_k	0.080	Tax rate on capital SS
ϕ_c	0.800	Habit persistence
ω_r	0.500	Proportion of Ricardians
ψ_1	0.047	Capacity utilisation parameter 1
ψ_2	1.000	Capacity utilisation parameter 2
χ	1.000	Investment adjustment cost parameter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PLOTS
APPS
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Compare
Import Data
Save Workspace
New Variable
Open Variable
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Analyze Code
Run and Time
Clear Commands
Simulink Library
Layout
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Set Path
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Community
Request Support
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BasicRBC_nl_v1
BasicRBC_nl_v2
BasicRBC_nonlin
BasicRBC_nonlin_v2
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Ch4_4PPWk2_lc
Ch5_4PPWk2_lc
Ch6_4PPHW2_lc
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Whelan_RBC_w_ss_v2
Whelan_RBC_w_ss_v3
Whelan_RBC_w_ss_v3a
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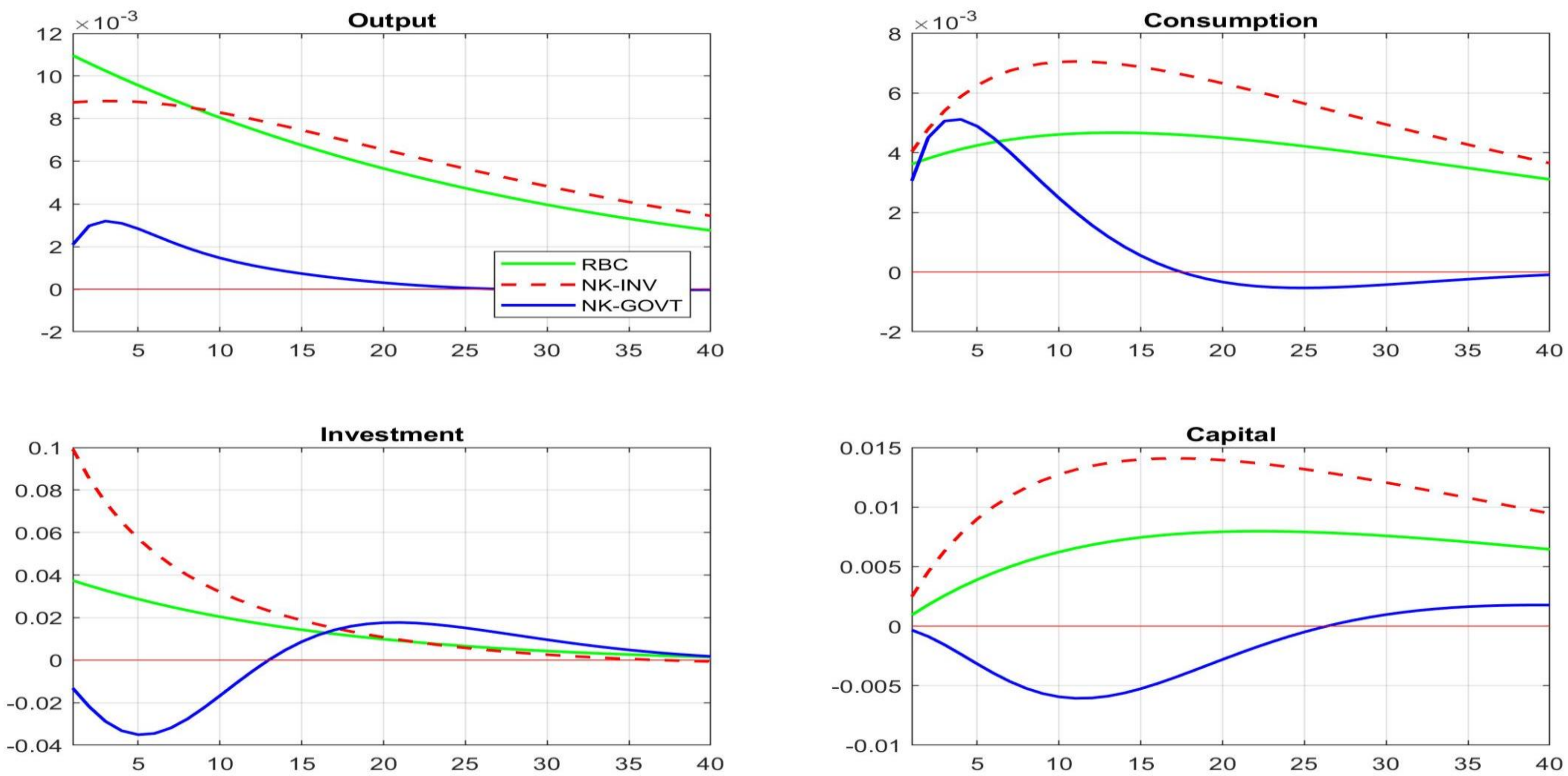
- Compare effects of ***productivity shock*** to that of previous (INV) model J:\MyCourseDSGEs2024\Tests\ch7_run_IRF_comparison1.m



- ➔ inclusion of taxes has a ***sharp effect*** on consumption and investment ➔ income and capital

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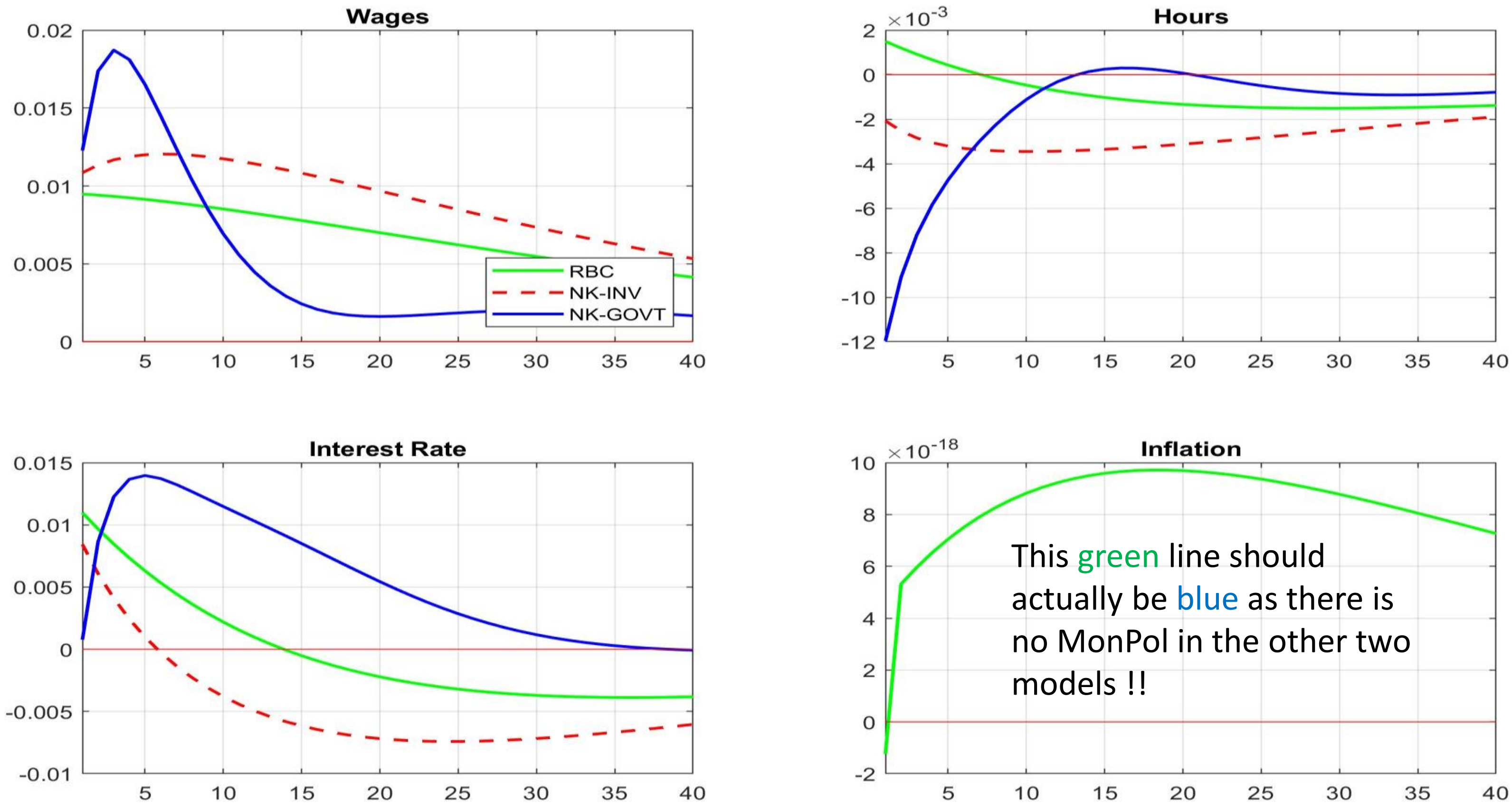


- Evidently monetary policy also weighs sharply on interest rate

Appendix 1: Bounded Rationality

- In a very important new paper, economists from the central banks of Canada and the Netherlands introduce methodology for dealing with the awkwardness of “rational expectations” (RE):
- Hommes *et al* (2023), “Behavioral learning equilibria in New Keynesian models”, *Quantitative Economics*, 14, 1401–1445.
- The RE paradigm [which we have been using] is a model-consistent approach where, by construction, agents’ ***expectations*** are ***on average*** confirmed by the ***realisations*** of the model

- Agents in RE models are ***assumed*** to know a large number of state variables, shocks, and parameters to form their expectations
- But in medium- and large-scale DSGE models, such assumptions lead to ***information sets*** supposed to be used by agents which are ***implausibly large***
- Moreover, to ***match*** the ***persistence*** of macroeconomic variables, RE models typically need to be augmented by highly ***persistent exogenous shocks*** or other sources of persistence, such as consumption ***habits*** [which we have used] and ***indexation*** in prices and wages [ditto]

- Hommes *et al* therefore introduce a form of "***bounded rationality***" in which agents are ***not*** perfectly rational (in the above sense) in their expectations, but instead base them on more ***limited*** information sets
- There are many ways to construct such "boundedly rational" expectations
- Among the simplest (and therefore most appealing for everyday use by real human beings) ways is to use a ***parsimonious*** forecasting scheme, such as simple AR(1), AR(2) or VAR(1) models

- Hommes *et al* use what they call a “Behavioral Learning Equilibrium (BLE)” approach
- Along a BLE, agents ***forecast*** the states of the economy by simple, but optimal, univariate AR(1) rules
- The AR(1) rules are optimal in the sense that it is ***assumed*** that the ***mean*** and the ***first-order autocorrelation*** of all forecasts coincide with the ***actual*** mean and the first-order autocorrelation of the ***realisations***

- In the linearized DSGE framework, RE and BLE are both linear equilibrium models but they satisfy different fixed-point conditions:
- In RE, agents are ***assumed*** to have ***perfect*** structural knowledge of the model
- By contrast, in BLE, agents do ***not*** know the ***cross-correlations*** among the variables and do ***not*** observe the ***shocks***
- but instead use parsimonious univariate AR(1) rules and are ***assumed*** to know the ***correct mean*** and ***first-order autocorrelation*** coefficients

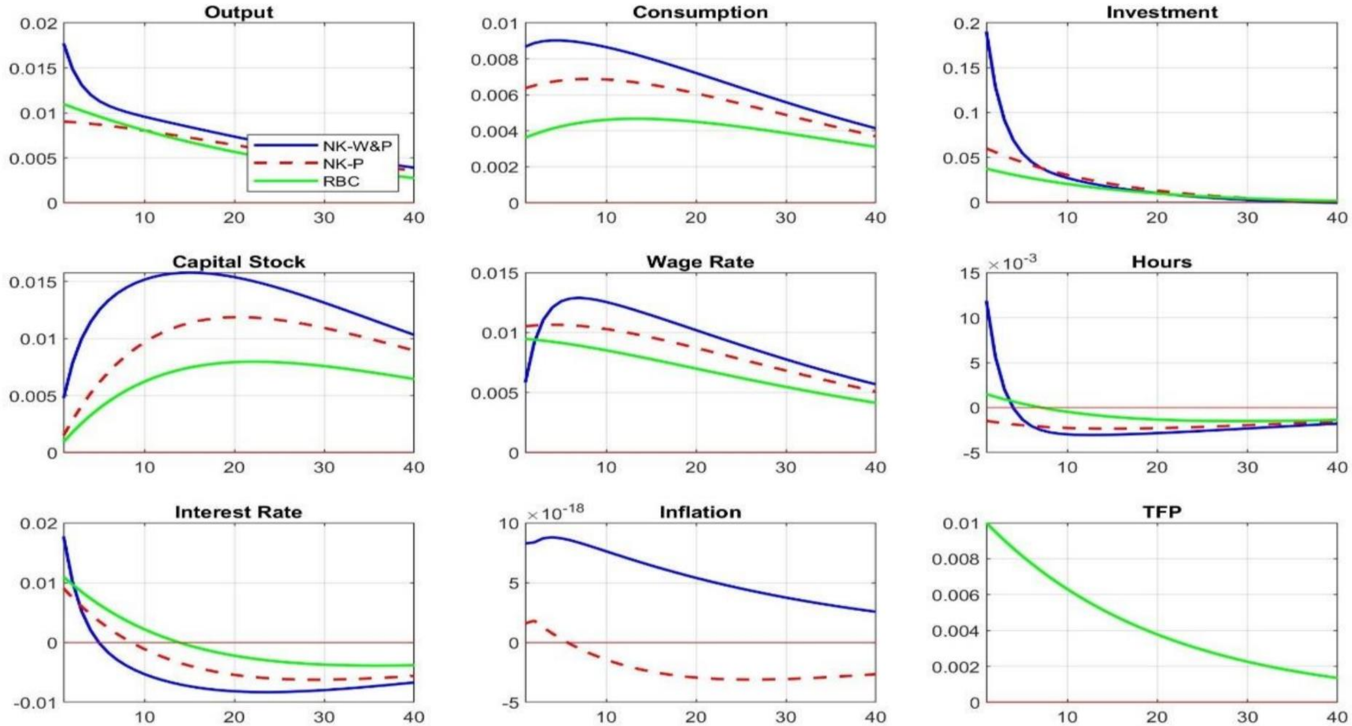
- Hommes *et al* show that in the ***Smets-Wouters*** model which we study next week, most structural parameters have similar values when estimated under RE or BLE assumptions
- But a few have quite ***different*** values, and these are sufficient to make the model behave much better in terms of ***persistence-tracking***
- In addition, ***model fit*** (as measured by the log-likelihood at the mode) is ***much better*** for the BLE model
- Further, when they conduct a pseudo out-of-sample ***forecasting*** exercise, the BLE model beats the RE model for virtually all variables up to 8 quarters ahead

- What of the ***learning*** aspect in BLE?
- Under BLE, agents ***learn*** the two parameters of each AR(1) rule consistent with the ***observable sample*** averages and first-order autocorrelations of the model's state variables
- This is a ***one-time event*** given the data set
- However, it is possible to go beyond this:
- A natural ***learning process*** of a BLE model would be for agents to use a univariate AR(1) rule for every variable and ***update*** their beliefs about the mean and persistence in every period as ***new*** observations become available

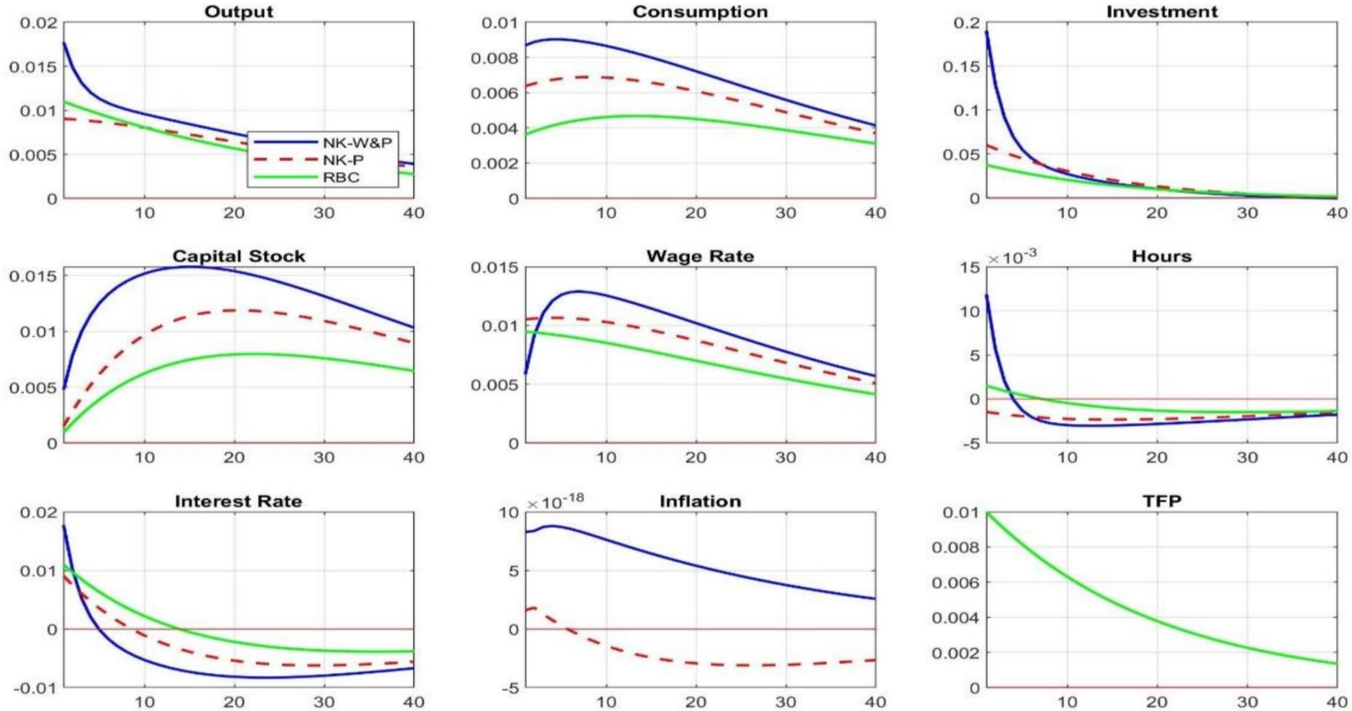
- This process is known as "***sample autocorrelation*** (SAC) ***learning***"
- Hommes *et al* show that when ***SAC learning*** is used for estimating the parameters of the Smet-Wouters model, the results are ***even better*** than for BLE, in terms both of ***model fit*** and ***forecasting***, while changing only little the estimated parameters
- On the other hand, replacing RE with simple AR(1) beliefs (RE vs. BLE) improves the model fit ***more*** than introducing ***time variation*** in AR(1) beliefs (BLE vs. SAC)

Appendix 2: More Details

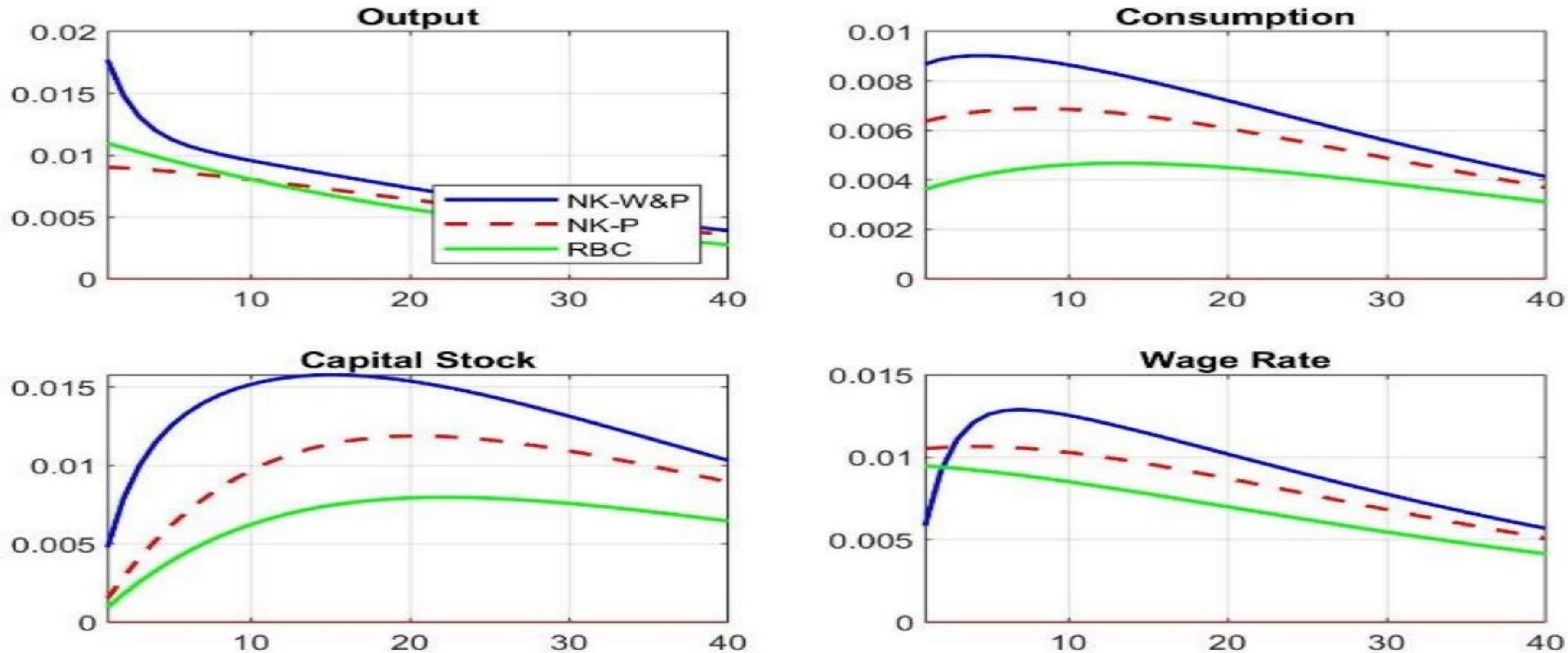
Price and Wage Stickiness



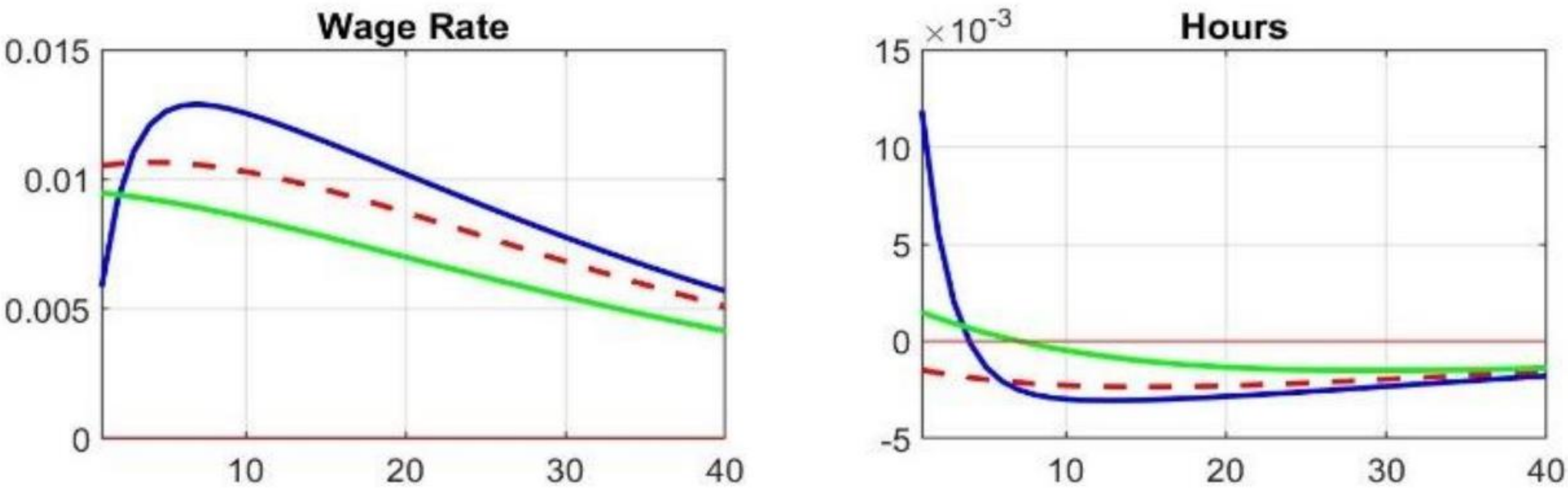
- Productivity shock (***identical*** in all three cases – hence only single colour [green] shown for TFP)
  - first impacts ***investment***, which in all three models rises (relative to SS),
  - ➔ rise (although much smaller – observe carefully the scales!) in ***output*** and ***consumption*** through aggregate spending effect
  - and also (with a lag) spurs growth of capital



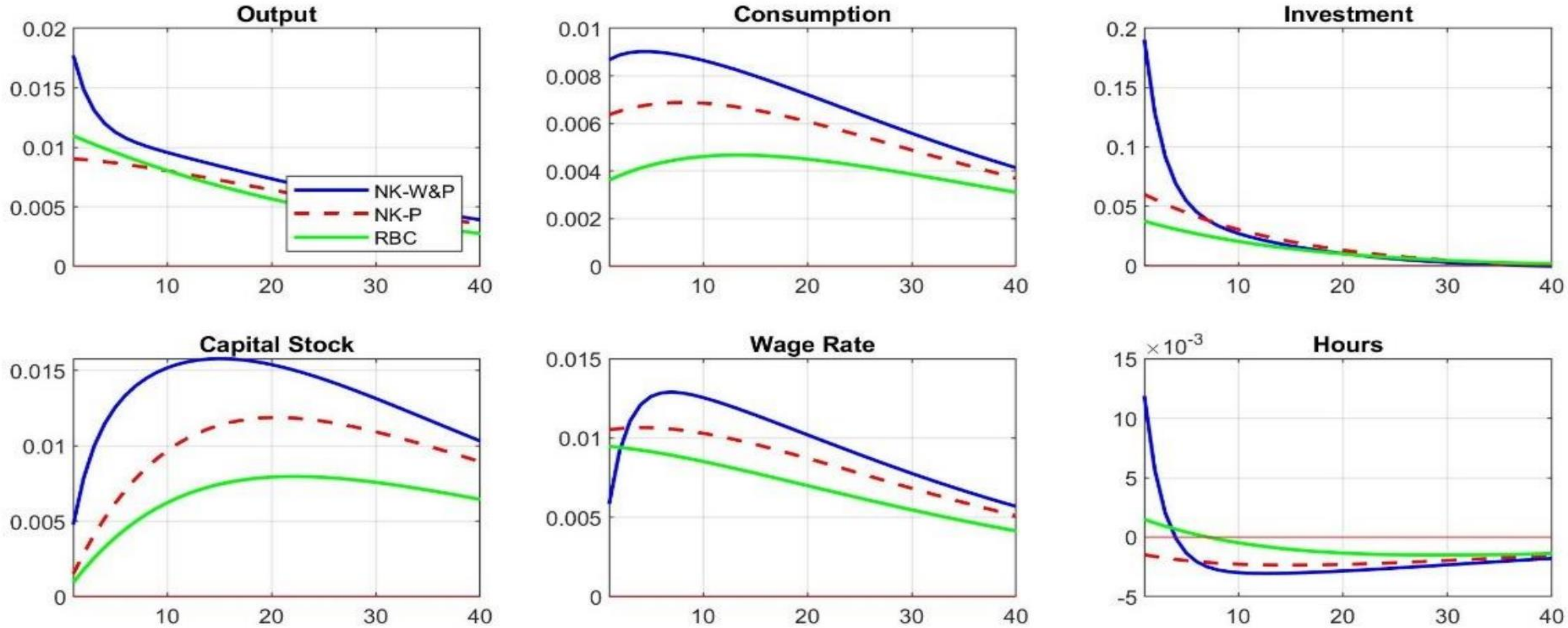
- In these models, there is not yet a government sector, so no Taylor Rule is in operation, and interest rate shown is not a ***policy*** rate, but ***return on capital***, which initially rises with improvement in productivity, before falling back as that shock dies away
- There is no ***inflation*** variable in our RBC model, so there is no green line in that graph, but in any case, values are essentially rounding errors (again, look at scale)



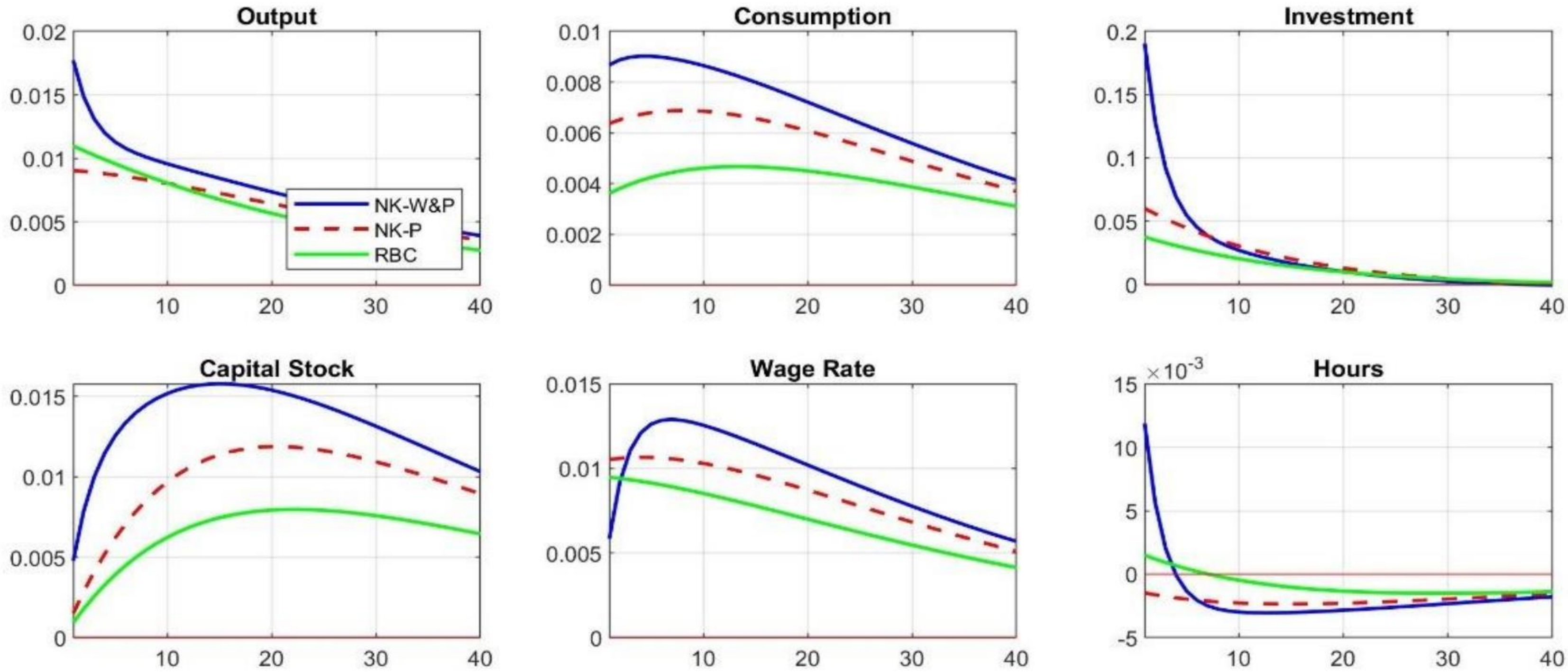
- Next, consider ***differences*** among three models: First, as frictions are incorporated, aggregate variables (***output***, ***consumption*** and ***capital***) become more ***persistent*** (ie, their tails stay higher for longer)
- Hence, RBC (green) line invariably is ***flatter*** and returns more rapidly to zero (ie, its steady state value, as these are log-differences vis-à-vis steady state) than is case for either NK model



- Now, ***hours worked*** and ***wage rate***: In NK-P (dotted red) model, where prices are sticky, but not wages, initial increase in level of wages causes a ***substitution*** of ***leisure*** for work, and consequently dotted red (NK-P) line for ***hours*** is both ***below*** green (RBC) line, and even ***negative***
- By contrast, in NK-W&P (blue line) model, ***stickiness*** of wages causes ***wage rate*** to adjust more ***slowly*** to improved marginal productivity of labour induced by TFP shock, thus ***limiting*** (relative to NK-P and RBC models) increase in household ***income***, and also inducing an actual initial (small – see scale) ***increase*** in ***hours worked*** (blue line), although this rapidly dies out as (lagged) wage rate increase subsequently filters into household income

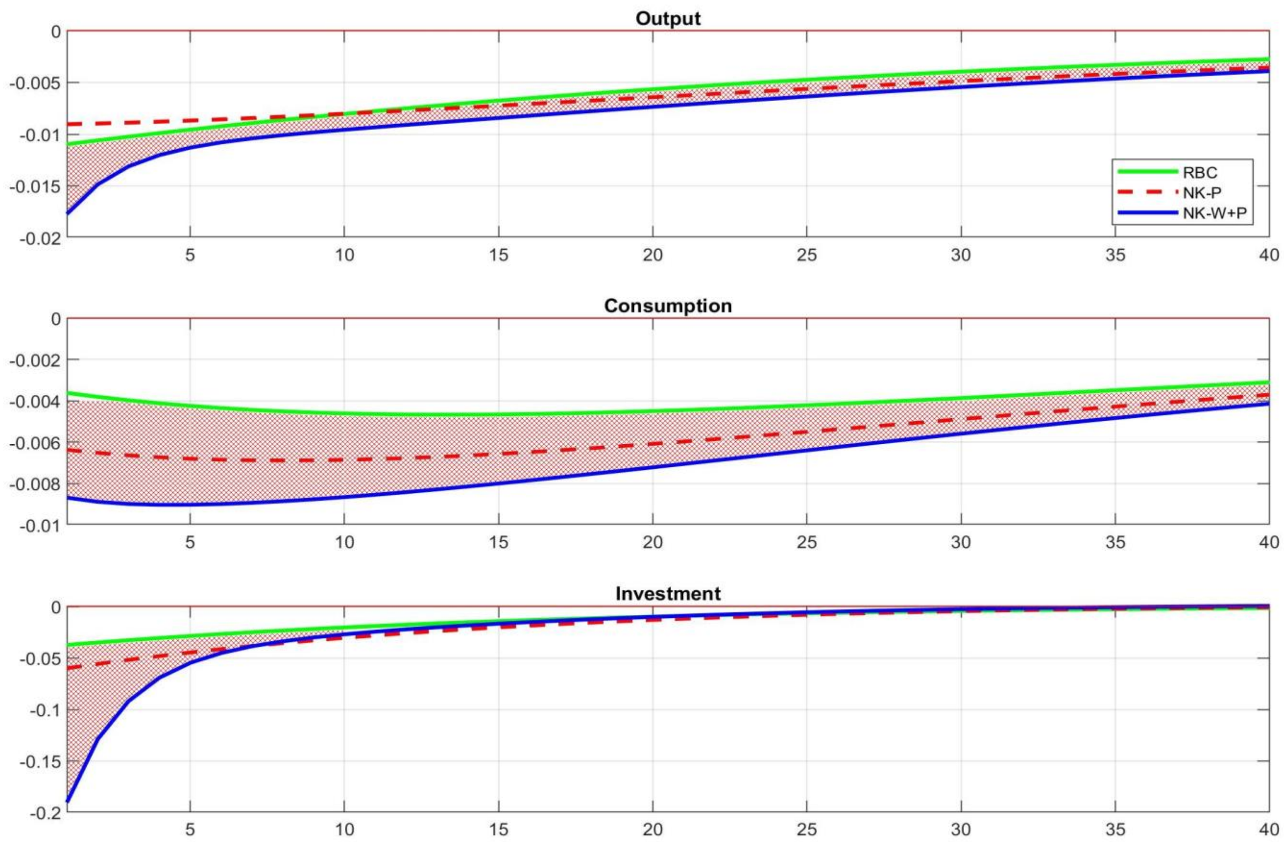


- But why is ***initial impact*** on ***output*** so much higher in NK-W&P (blue line) model versus RBC (green) and NK-P (dotted red)?
- Answer comes from a consideration of ***costs*** involved in producing this output: they are sum of ***capital rental*** and ***labour rental*** costs
- And in NK-W&P model, increase in cost of labour rental (ie, ***wage rate***) initially does not rise as much as it "should" because of ***wage stickiness***
- Hence, investment initially rises much ***more*** in NK-W&P (blue line) model than in the others, causing both ***output*** (***simultaneously***) and ***capital*** (with a ***lag***) also to rise much more, comparatively



- Finally, why is initial impact on output ***smaller*** (although only marginally) in NK-P (dotted red) model than in RBC (green) model?
- Answer was essentially given in a previous slide: ***hours worked*** in NK-P model initially turn ***negative***, and fall below those in RBC model
- Whereas ***investment*** effect is only marginally higher in NK-P (dotted red) model than in RBC (green) model
- Result: ***output*** produced by these smaller working hours also falls short

- What would happen if you were employed by your **central bank** and mistakenly used an ***RBC*** (green) instead of an ***NK DSGE*** with wage and price ***frictions*** (blue) to analyse a negative productivity shock such as recently seen?
- You would be ***fired***, because you would ***underestimate*** the shock's effects and thus propose a ***weaker*** countercyclical policy than necessary:



Non-Ricardian Agents and Habit Formation

New equations related to heterogeneous agents, j ϵ {R, NR}

```
//1-Lagrangian for Ricardian Households
lambdar = (sigma/((1-phic)*(1-phic*beta)))*(phic*beta*(cr(+1)-phic*cr)
-(cr-phic*cr(-1)))-p;
//2-Phillips Curve for Wages for Ricardian Households
piw = beta*piw(+1)+((1-thetaw)*(1-beta*thetaw)/thetaw)*(lr-lambdar-w);
//3-Gross Wage Inflation Rate
piw = w - w(-1);
//4-Euler Equation
(Pss/beta)*(lambdar+p-lambdar(+1))=(1-delta)*Pss*p(+1) + Rss*r(+1);
```

See derivation pp.136-139 of Costa

```
//5-Law of Motion of Capital
k = (1-delta)*k(-1) + delta*i;
//6-Lagrangian for Non-Ricardian Households
lambdanr = (sigma/((1-phic)*(1-phic*beta)))*(phic*beta*(cnr(+1)-phic*cnr)
-(cnr-phic*cnr(-1)))-p;
//7-Phillips Curve for Wages for Non-Ricardian Households
piw = beta*piw(+1)+((1-thetaw)*(1-beta*thetaw)/thetaw)*(lnr-lambdanr-w);
//8-Budget Constraint for for Non-Ricardian Households
p+cnr=w+lnr;
//9-Aggregate Consumption
Css*c = omegar*CRss*cr + (1-omegar)*CNRss*cnr;
//10-Aggregate Labour Supply
Lss*l = omegar*LRss*lr + (1-omegar)*LNRss*lnr;
```

```
//11-Production Function
y = a + alpha*k + (1-alpha)*l;
//12-Demand for Capital
k = y - (r - p);
//13-Demand for Labour
l = y - (w - p);
//14-Marginal Cost
mc = ((1-alpha)*w + alpha*r - a);
//15-Phillips Curve
pi = beta*pi(+1)+((1-theta)*(1-beta*theta)/theta)*(mc-p);
//16-Gross Inflation Rate
pi = p - p(-1);
//17-Goods Market Equilibrium Condition
Yss*y = Css*c + Iss*i;
//18-Productivity Shock
a = rhoa*a(-1) + e;
```

11-18 are just copied over from previous model

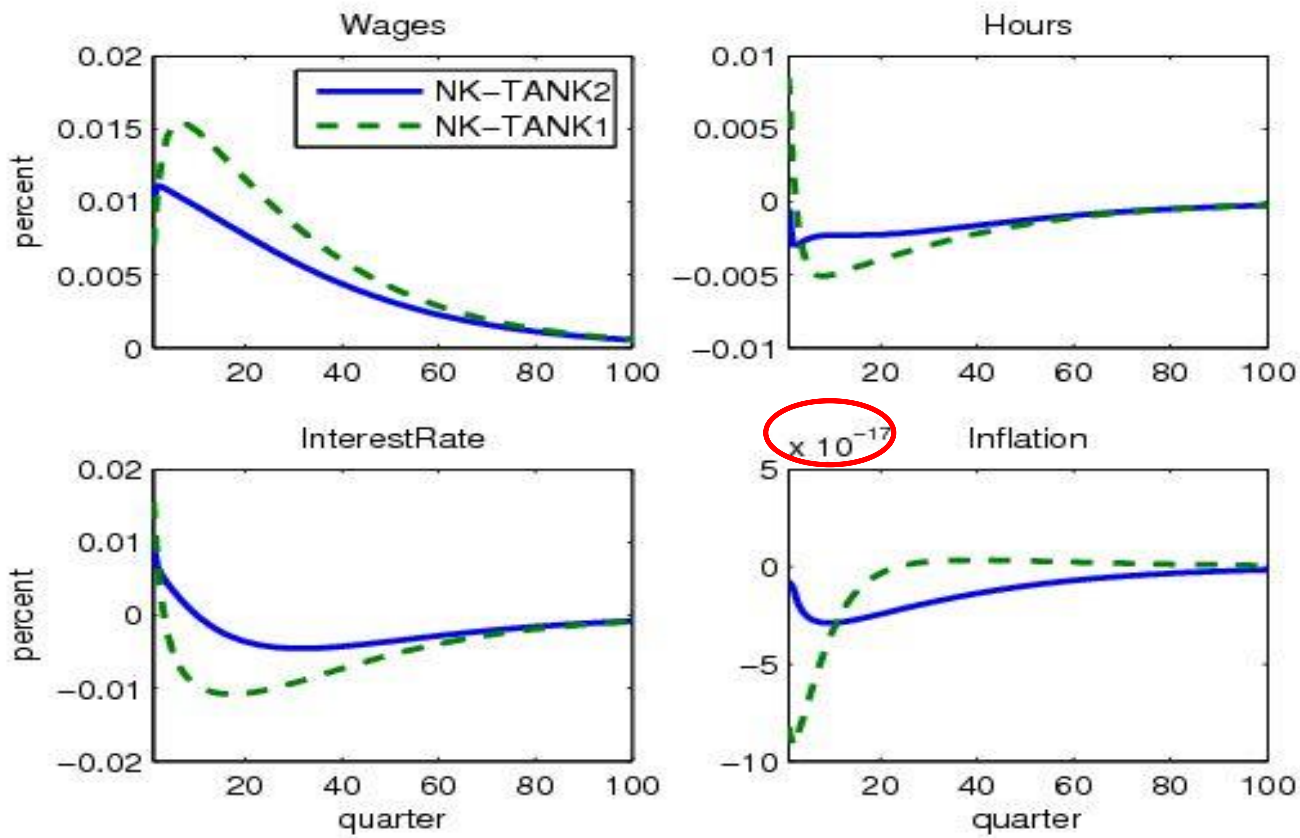
- The results cited were derived from the following

Parameter		Calibrated value
σ	Relative risk aversion coefficient	2
ϕ	Marginal disutility with respect to labor supply	1.5
α	Elasticity of output with respect to capital	0.35
β	Discount factor	0.985
δ	Depreciation rate	0.025
ρ_A	Autoregressive parameter of productivity	0.95
σ_A	Standard deviation of productivity	0.01
θ	Price stickiness parameter	0.75
ψ	Elasticity of substitution between intermediate goods	8
θ_W	Wage stickiness parameter	0.75
ψ_W	Elasticity of substitution between differentiated labour	21
φ_C	Habit persistence	0.8
ω_R	Participation of Ricardian agents in consumption and labor	0.5

- What if we reduce (or increase) degree of price and wage ***stickiness***?

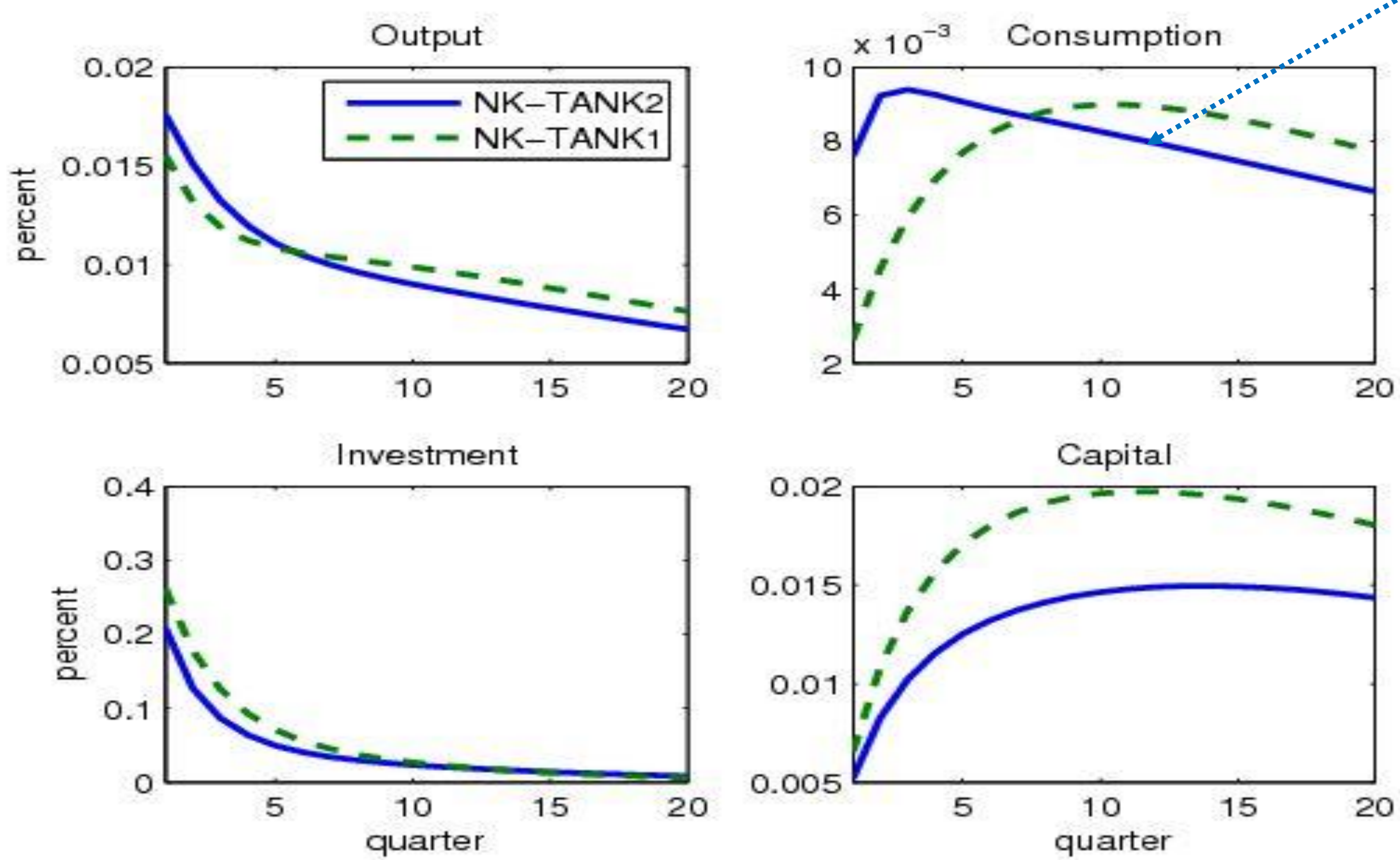
- Fraction of firms and households obliged by Calvo fairy to follow a Calvo Rule was set at three-quarters; what if this is reduced to one-third?

Since a **greater share** can already optimally adjust, they respond less to a shock



- Habit persistence parameter was set at four-fifths; what if this is reduced to one-third?

This means that the additional utility of consumption is anyway much greater, so it responds less to a shock



Adjustment Costs in Investment and Capacity (non-)Utilisation Costs

• ➔ Model

$$\lambda_{R,t} P_t - Q_t \left\{1 - \frac{\chi}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 - \chi\left(\frac{I_t}{I_{t-1}}\right)\left(\frac{I_t}{I_{t-1}} - 1\right)\right\}$$
$$= \chi\beta E_t Q_{t+1}\left(\frac{E_t I_{t+1}}{I_t}\right)^2\left(\frac{E_t I_{t+1}}{I_t} - 1\right) \qquad \text{(Demand for investment assets)}$$

$$K_{t+1} = (1-\delta)K_t + I_t\left[1 - \frac{\chi}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right] \qquad \text{(Law of motion of capital)}$$

$$Q_t = \beta E_t\left\{(1-\delta)Q_{t+1} + \lambda_{R,t+1} R_{t+1} U_{t+1}\right.$$
$$\left. -\lambda_{R,t+1} P_{t+1}\left[\Psi_1\left(U_{t+1} - 1\right) + \frac{\Psi_2}{2}\left(U_{t+1} - 1\right)^2\right]\right\} \qquad \text{(Tobin's Q)}$$

$$\frac{R_t}{P_t} = \Psi_1 + \Psi_2\left(U_t - 1\right) \qquad \text{(Demand for installed capacity)}$$

- ***Remainder*** of model ***familiar***:

$W_{j,t}* = \left(\frac{\psi_W}{\psi_W - 1}\right) E_t \sum_{i=0}^{\infty} (\beta\theta_W)^i \frac{L_{NR,j,t+i}{}^{\psi}}{\lambda_{NR,t+i}}$ (NR optimal wage)

$P_t C_{NR,t} = W_t L_{NR,t}$ (NR budget)

$C_t = \omega_R C_{R,t} + (1 - \omega_R) C_{NR,t}$ (Aggregate consumption)

$L_t = \omega_R L_{R,t} + (1 - \omega_R) L_{NR,t}$ (Aggregate Labour)

$Y_t = A_t \left[U_t K_{j,t}\right]^{\alpha} L_t^{1-\alpha}$ (Production function)

$U_t K_t = \alpha MC_t \frac{Y_t}{R_t}$ (Demand for capital)

New

$L_t = (1-\alpha) MC_t \frac{Y_t}{W_t}$ (Demand for labour)

$MC_t = \frac{1}{A_t}\left(\frac{W_t}{1-\alpha}\right)^{1-\alpha}\left(\frac{R_t}{\alpha}\right)^{\alpha}$ (Marginal cost)

$P_t^* = \left(\frac{\psi}{\psi - 1}\right) E_t \sum_{i=0}^{\infty} (\beta\theta)^i MC_{t+i}$ (Optimal price level)

$P_t = \left[\theta P_{t-1}^{1-\psi} + (1-\theta) P_t^*\right]^i MC_{t+i}$ (General price level)

$\pi_t = P_t / P_{t-1}$ (Equilibrium)

$\log A_t = \rho_A \log A_{t-1} + \epsilon_t$ (Productivity shock)

Government sector (monetary and fiscal)

- Comparing effects of various ***tax rates*** (on consumption, labour and capital)

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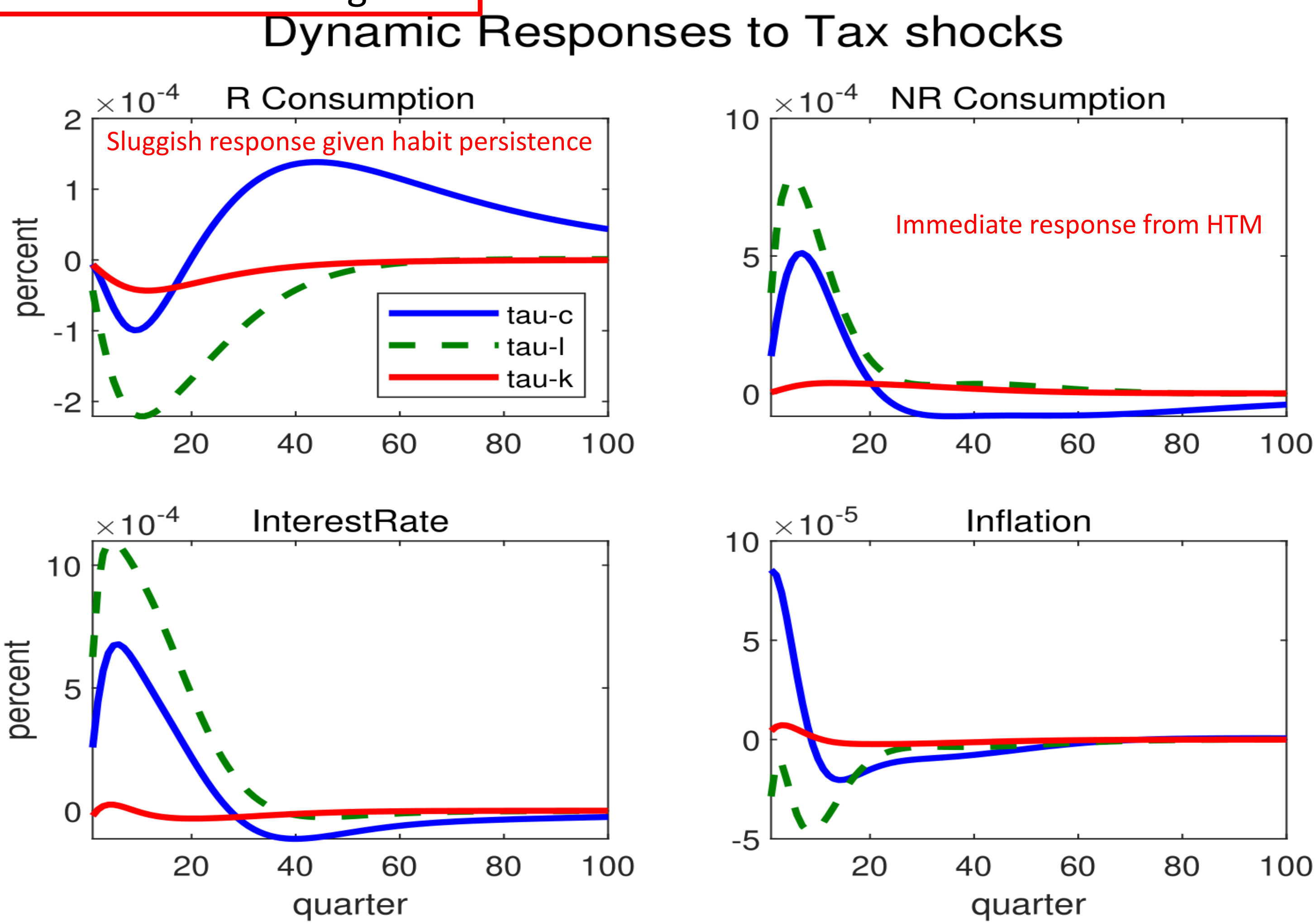
Remember, these shocks are negative!!



- Comparing effects of different Taylor Rule sensitivities to ***interest rate***

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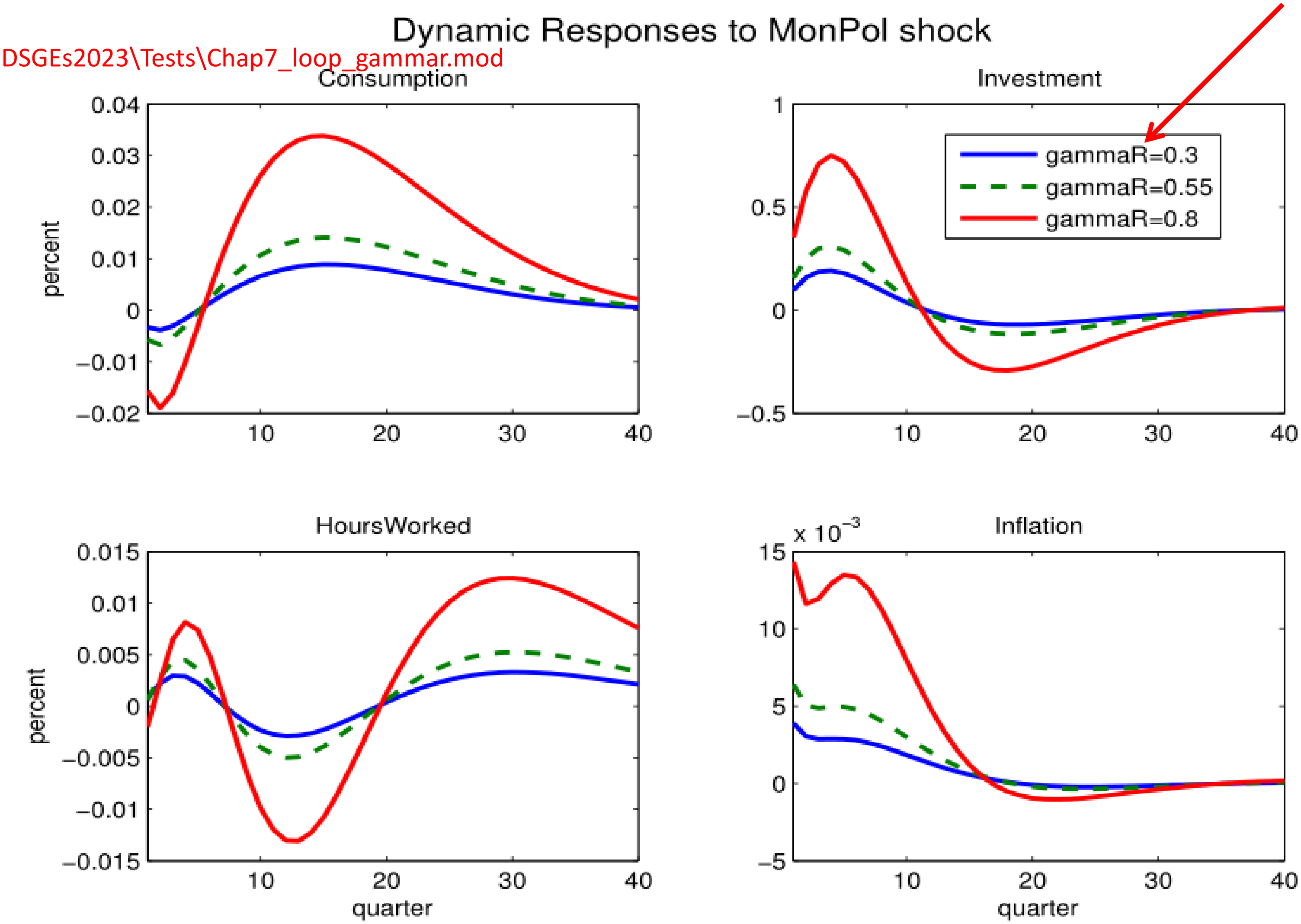

- Comparing effects of different Taylor Rule sensitivities to ***output gap***

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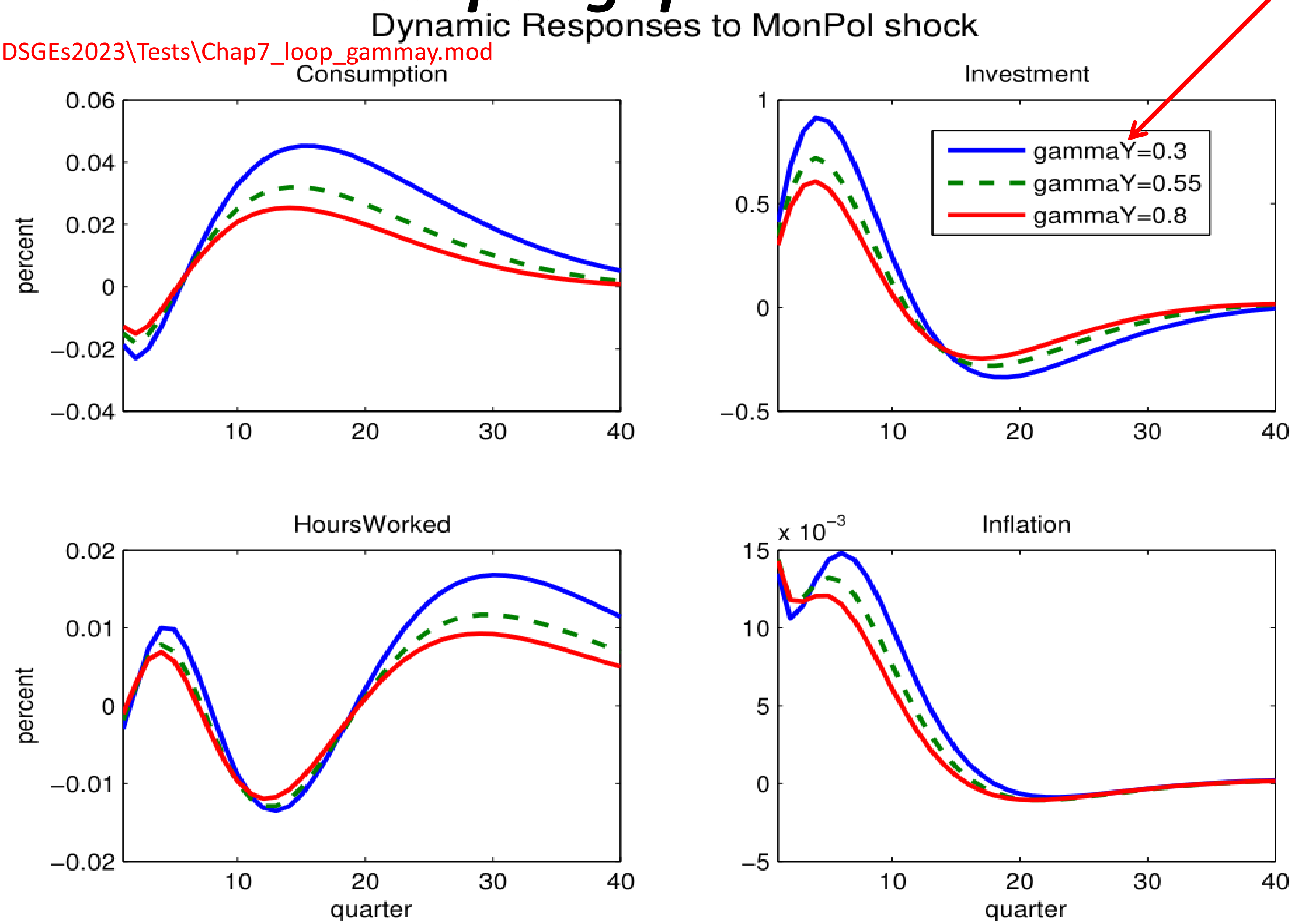

- Comparing effects of different Taylor Rule sensitivities to ***inflation rate***

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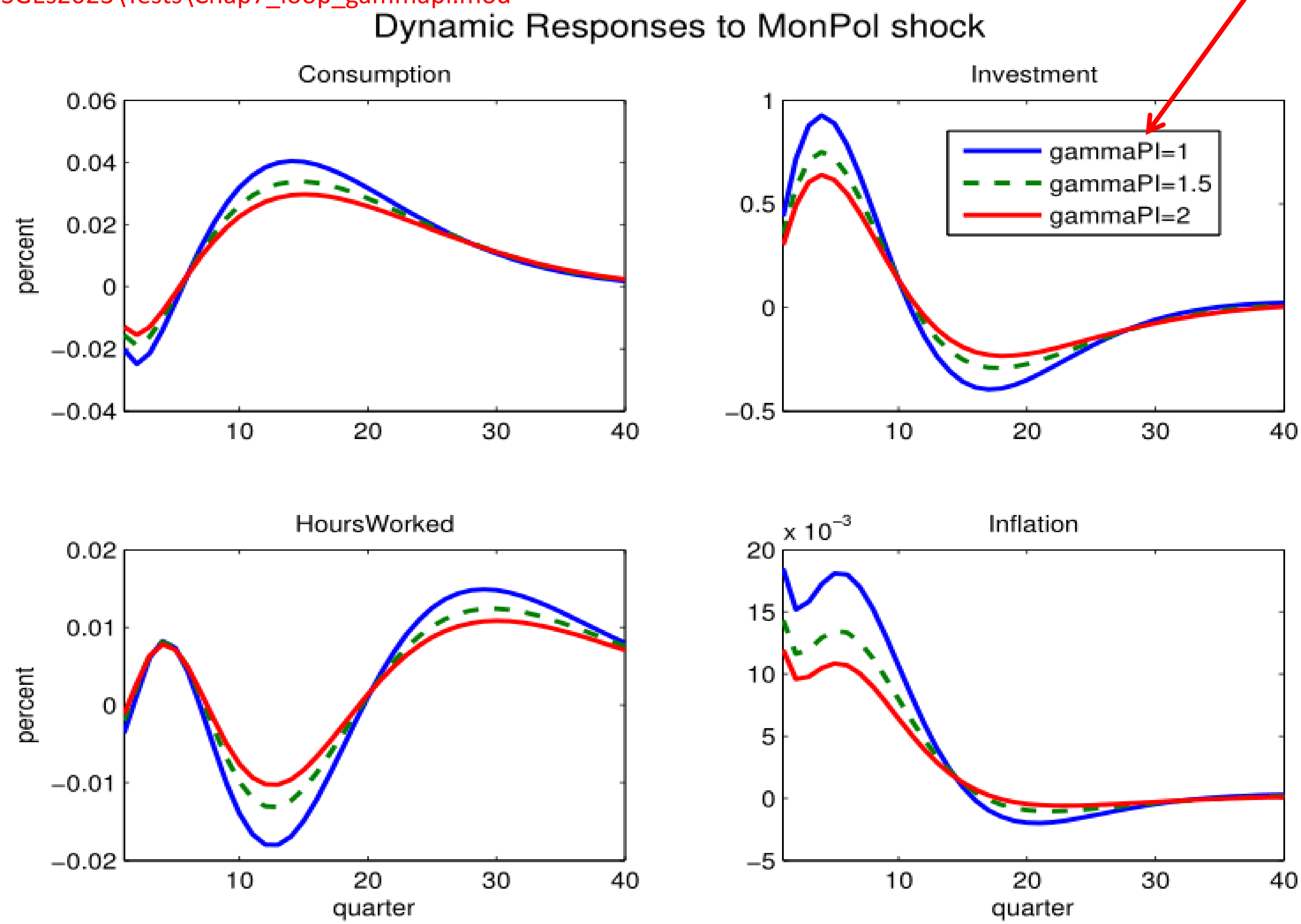

- Comparing effects of different tax rates - ***Labour***

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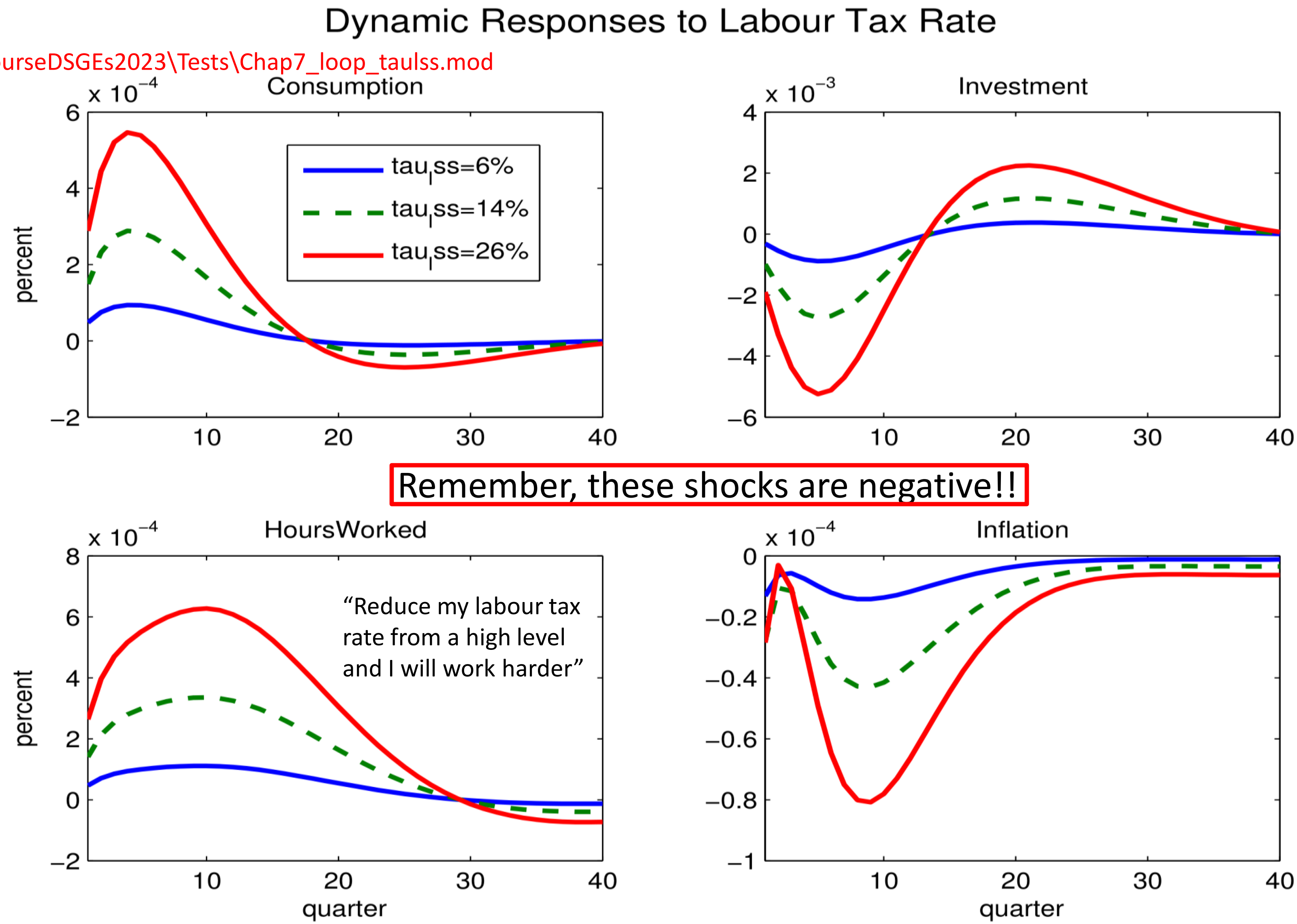
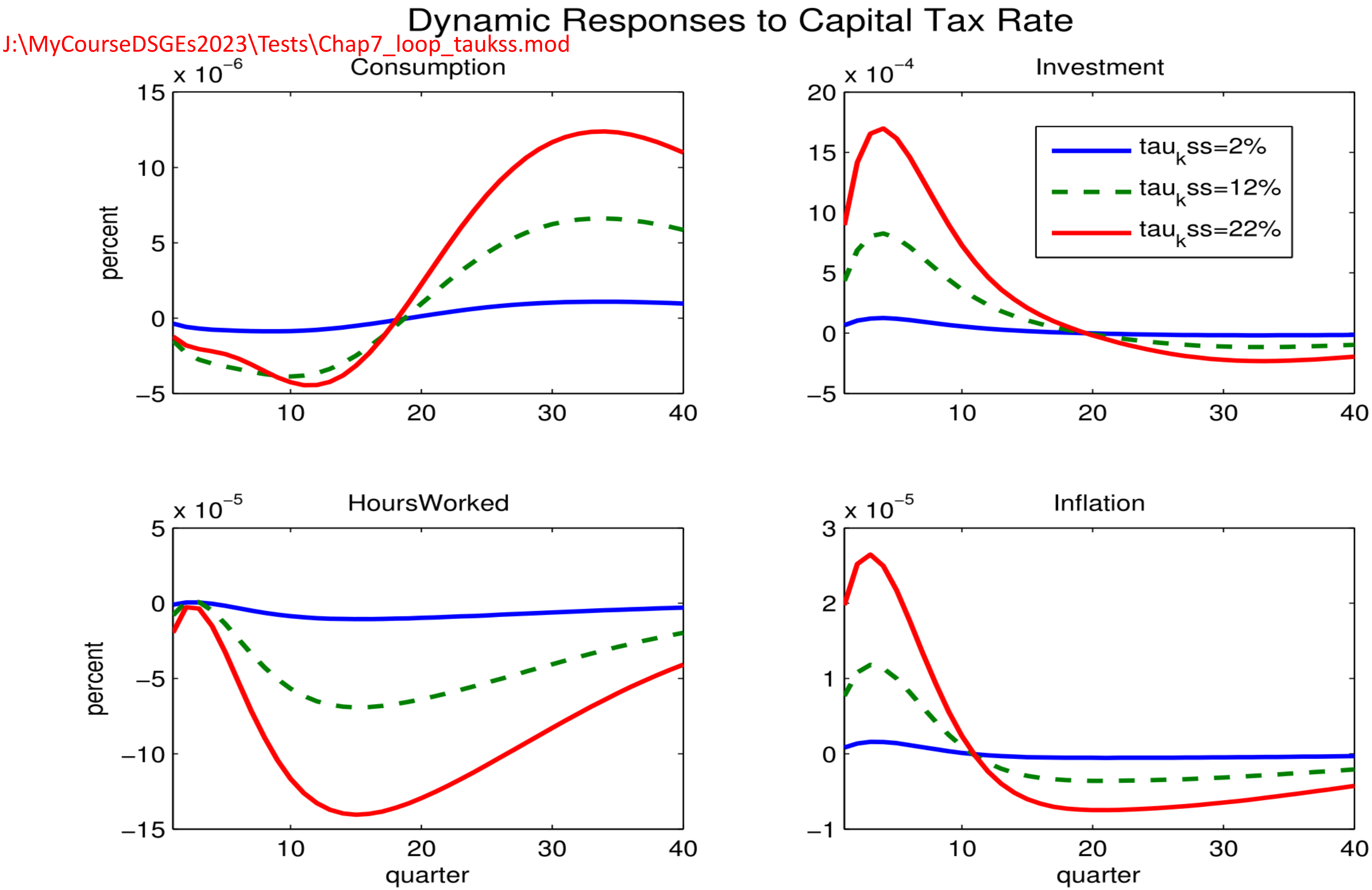


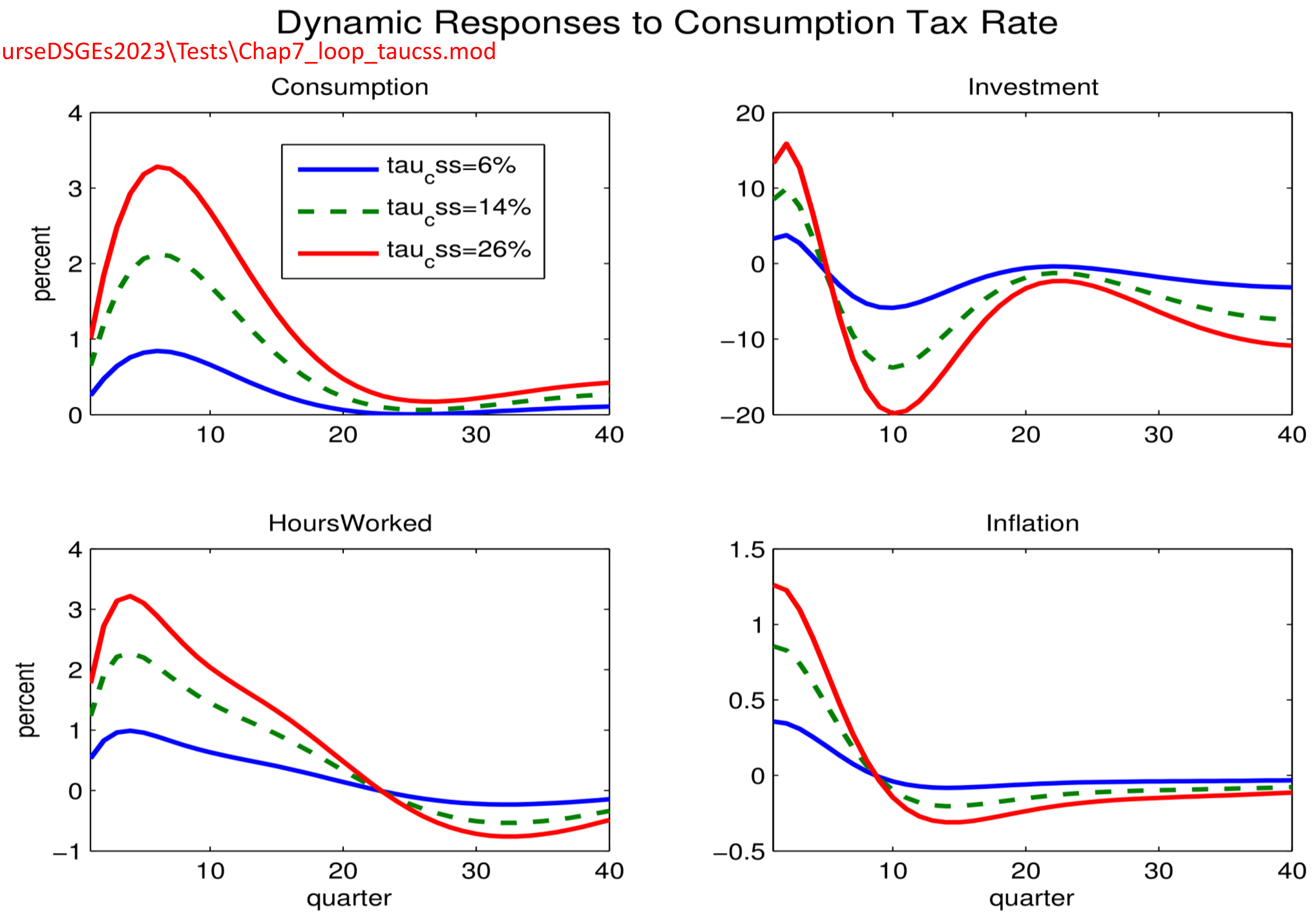
Remember, these shocks are negative!!

"Reduce my labour tax rate from a high level and I will work harder"

- Comparing effects of different tax rates - ***Capital***



- Comparing effects of different tax rates - ***Consumption***



- Comparing effects of different ***Transfer*** rates

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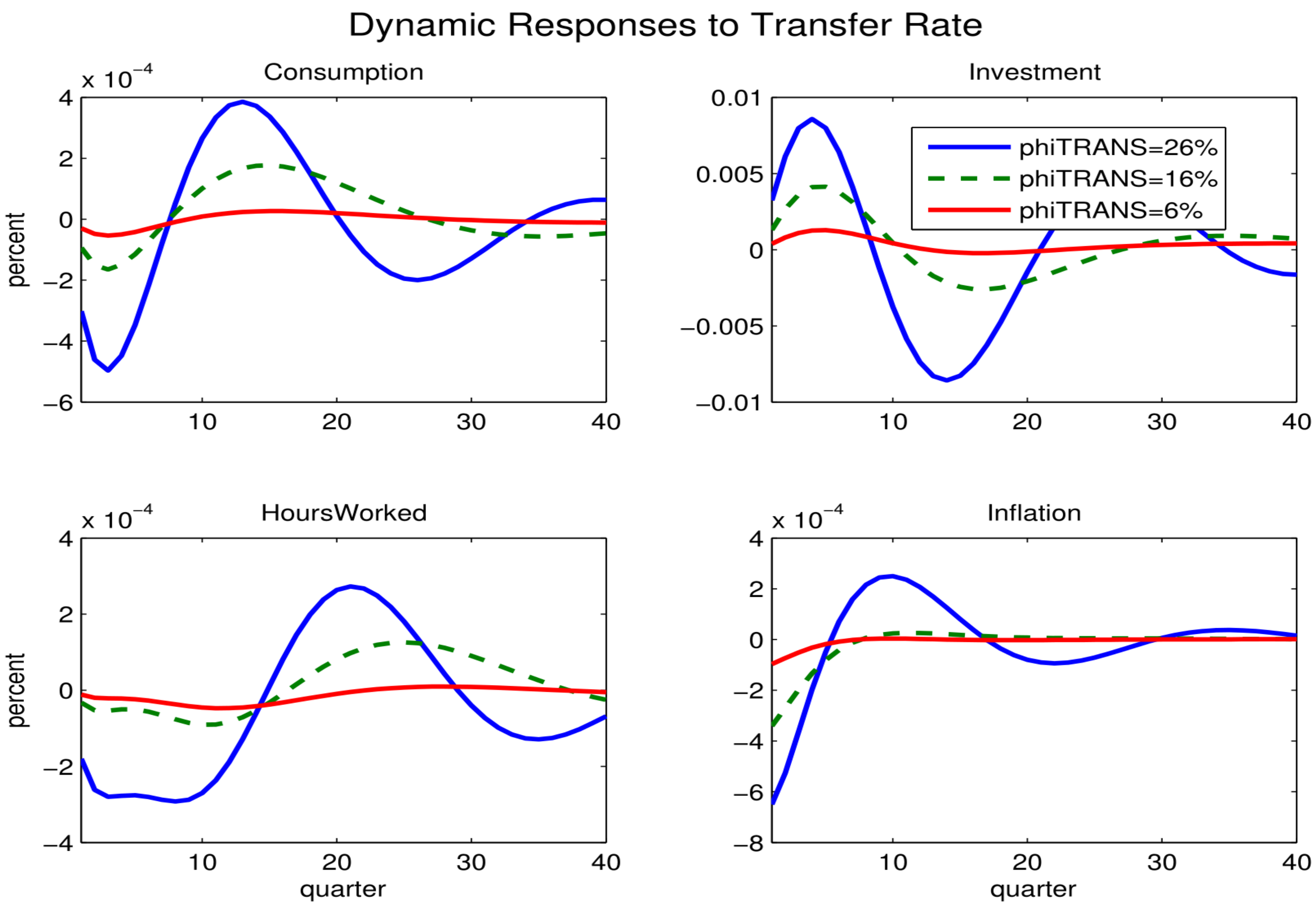
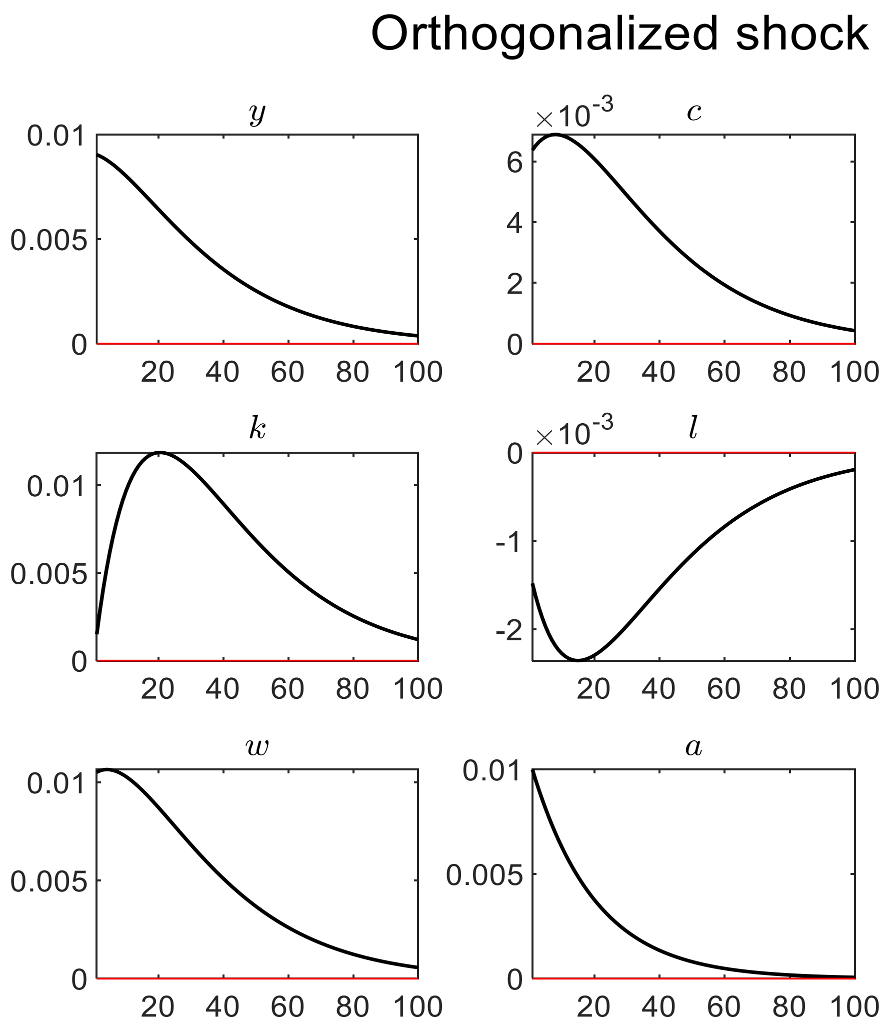


Unused slides

- This produces the following IRFs:

Remember, these are deviations relative to SS values, not levels, caused by corresponding shock



But ***also*** for ***leisure*** (income effect) ➔ fall in ***labour supply***, slowing return of ***wages*** to SS level

As in RBC, a (positive) shock to technology ➔ increases in ***marginal productivity*** of ***factors of production*** ➔ increases in their ***prices*** (***wages*** and ***return on capital***) ➔ increases in household ***income.*** Higher income ➔ increases in ***demand*** for ***consumption*** and ***investment*** goods

- Lagrangian is set up and maximised as first week:

$$\mathscr{L} = E_t \sum_{t=0}^{\infty} \beta^t \left\{ \left[\frac{\left(\boxed{C_{R,t} - \phi_c C_{R,t-1}}\right)^{1-\sigma}}{1-\sigma} - \frac{L_{R,t}^{1+\varphi}}{1+\varphi} \right] - \lambda_{R,t} \left[P_t C_{R,t} + \boxed{P_t K_{t+1} - P_t (1-\delta) K_t} - W_t L_{R,t} - R_t K_t \right] \right\}$$

$\boxed{P_t I_t \text{ recall}}$

- NB: ***only Ricardians save*** ➔ *K* has no "*R*" subscript
- FOCs ➔ ***[NEW] Lagrange Multiplier for Ricardians***

$$\lambda_{R,t} = \frac{\left(C_{R,t} - \phi_c C_{R,t-1}\right)^{-\sigma}}{P_t} - \phi_c \beta \frac{\left(E_t C_{R,t+1} - \phi_c C_{R,t}\right)^{-\sigma}}{P_t}$$

See derivation pp.136-139 of Costa

Lagrange Multiplier = "shadow price" of consumption

- ➔ ***Euler Equation for Ricardians***

$$\lambda_{R,t} P_t = \beta E_t \lambda_{R,t+1} \left[(1-\delta) E_t P_{t+1} + E_t R_{t+1} \right]$$

Euler Equation = ***intertemporal*** restriction on consumption