Financial Integration and Crises

Gianluca Benigno

Graduate Institute Geneva

Lecture 1

Course Information

• Lecturer: Prof. Gianluca Benigno

email: gianluca.benigno@unil.ch homepage: https://sites.google.com/view/gianlucabenigno substack: https://gianlucabenigno.substack.com/

- Office Hours: by appointment
- References:
 - Foundations of International Macroeconomics, Obstfeld and Rogoff, MIT press.
 - Open Economy Macroeconomics,, Martin Uribe and Stephanie Schmitt-Grohe, Princeton University Press.

Introduction

- The Macroeconomic of Crisis:
 - Currency Crisis
 - Sovereign Debt Crisis
 - Financial/Banking Crisis
 - Bubbles

Introduction

Episodes

- Emerging market episodes: Latin America Debt Crisis of 1980s; Asian Crisis 1997-1998.
- Currency Crisis: Collapse of Bretton Woods 1971, ERM crisis 1992.
- ► Global Financial Crisis: mainly advanced economies 2007-2008;
- European Sovereign Debt Crises 2011-2012.
- Covid Crisis: 2020-2021

Introduction

- Questions
 - ▶ What are the factors that have caused a crisis event?
 - ► How do we prevent a crisis from happening?

Course map

- Focus on benchmark economy and key elements of analysis;
- Currency Crisis Models;
- Suddent Stop;
- Deleveraging and Liquidity Traps;
- Sovereign Debt Crisis
- Covid-Crisis

- The global financial crisis (GFC) refers to the period of extreme stress in global financial markets and banking systems between mid 2007 and early 2009.
- During the GFC, a downturn in the US housing market was a catalyst for a financial crisis that spread from the United States to the rest of the world through linkages in the global financial system.
- Many banks around the world incurred large losses and relied on government support to avoid bankruptcy.

Excessive risk taking

- Macro background: Economic growth was strong and stable, and rates of inflation, unemployment and interest were relatively low. In this environment, house prices grew strongly.
- Expectations that house prices would continue to rise led households, in the United States especially, to borrow imprudently to purchase and build houses. (similar patterns arose in Iceland, Ireland, Spain and some countries in Eastern Europe).
- Risky mortgage loans: A large share of such risky borrowing was done by investors seeking to make short-term profits by 'flipping' houses and by 'subprime' borrowers

Excessive risk taking (Banks)

- Competition increased between individual lenders to extend ever-larger amounts of housing loans that, because of the good economic environment, seemed to be very profitable at the time.
- Many lenders providing housing loans did not closely assess borrowers' abilities to make loan repayments. Loans packaged into 'mortgage-backed securities' (MBS).
- Investors who purchased MBS products mistakenly thought that they were buying a very low risk asset: even if some mortgage loans in the package were not repaid, it was assumed that most loans would continue to be repaid.
- These investors included large US banks, as well as foreign banks from Europe and other economies that sought higher returns than could be achieved in their local markets.

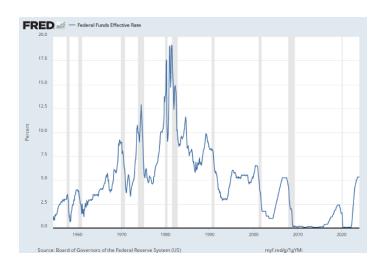
Increased borrowing by banks and investors

- Leverage: banks and other investors in the United States and abroad borrowed increasing amounts to expand their lending and purchase MBS products. Borrowing money to purchase an asset (known as an increase in leverage) magnifies potential profits but also magnifies potential losses.
- Maturity Mismatch: banks and some investors increasingly borrowed money for very short periods, including overnight, to purchase assets that could not be sold quickly.

Regulation and policy errors

- Regulation: insufficient regulation of the institutions that created and sold the complex and opaque MBS to investors.
- As the crisis unfolded, many central banks and governments did not fully
 recognise the extent to which bad loans had been extended during the boom
 and the many ways in which mortgage losses were spreading through the
 financial system.

Global Financial Crisis: Monetary Policy



Global Financial Crisis: Unfolding

US house prices fell, borrowers missed repayments

 Loan repayments were particularly sensitive to house prices in the United States because the proportion of US households (both owner-occupiers and investors) with large debts had risen a lot during the boom and was higher than in other countries.

Global Financial Crisis: Unfolding

Stresses in the financial system

- First emerged clearly around mid 2007. As borrowers miss payments, investors became less willing to purchase MBS products and were actively trying to sell their holdings. As a result, MBS prices declined, which reduced the value of MBS and thus the net worth of MBS investors.
- In turn, investors who had purchased MBS with short-term loans found it much more difficult to roll over these loans, which further exacerbated MBS selling and declines in MBS prices.

Global Financial Crisis: Unfolding

Failure of financial firms, panic in financial markets

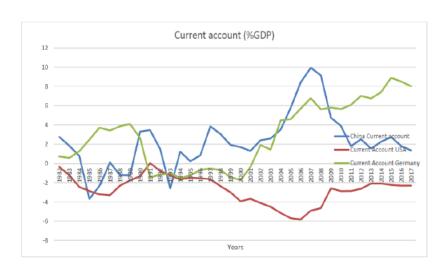
- Failure of Bears Stearns and Lehman Brothers. Investors began pulling their
 money out of banks and investment funds around the world as they did not
 know who might be next to fail and how exposed each institution was to
 subprime and other distressed loans.
- Dysfunctionality of financial markets: everyone tried to sell at the same time
 and many institutions wanting new financing could not obtain it. Businesses
 also became much less willing to invest and households less willing to spend
 as confidence collapsed. As a result, the United States and some other
 economies fell into their deepest recessions since the Great Depression (back
 then).

Global Financial Crisis and Global Imbalances

Macro background

- Key aspect of the world economy pre-GFC was the prresence of large external imbalances associated with declining real interest rates.
- Obstfeld and Rogoff (2009) draw link between global external imbalances and global financial crisis

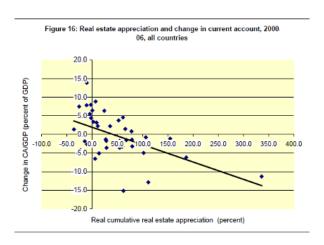
Global Imbalances



Global Imbalances and Real Interest Rates



External Imbalances and House Price Appreciation



Global Financial Crisis and Global Imbalances

Are global imbalances responsible for the global financial crisis?

- Obstfeld and Rogoff argue that the global imbalances of the 2000s and the global financial crisis are closely related.
- In the U.S., the interaction among the Fed's monetary stance, global real
 interest rates, credit market distortions, and financial innovation created the
 toxic mix of conditions making the U.S. the epicentre of the global financial
 crisis.
- Outside the U.S., exchange rate and other economic policies followed by emerging markets such as China contributed to the United States' ability to borrow cheaply abroad and thereby finance its unsustainable housing bubble.

Global Financial Crisis and Global Imbalances

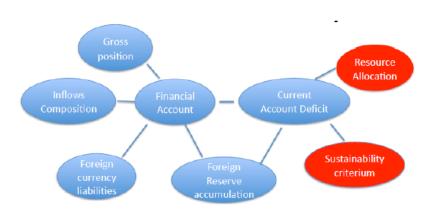
Different perspectives on the nature of global imbalances

- Outcome of integration among economies at different level of domestic development.
- Structure of the international monetary system (Bretton Woods 2).
- Global Saving glut.

External Imbalances Theory

- External Imbalances have often being a symptom of a problem (especially in EMEs)
- To what extent external imbalances are sustainable?
- Is there a criterium for assessing sustainability of external imbalances?

Current Account

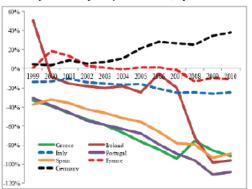


Current Account

- Why do we care?
- Federal Reserve paper (Freund, 2000) studies CA adjustment in industrialized countries.
- A typical current account reversal begins when the current account deficit is about 5 percent of GDP, and that it is associated with slowing income growth and a 10-20 percent real depreciation.
- Real export growth, declining investment, and an eventual levelling off in both the net international investment position and the budget deficit-GDP ratio are also likely to be part of the adjustment.

Net Foreign Assets Evolution (selected countries)

Figure 1. Net foreign asset positions 1999-2010, in percent of GDP



Source: IFS data

- BOP accounts record all the economic transactions in a given time interval between residents of a country and the rest of the world.
- Three types of transactions:
 - Transactions that involve goods and services: current account;
 - ► Transactions that involve financial assets: financial account;
 - Transactions that result in transfer of wealth: capital account (these are normally non-market activities, and often refer to transactions of nonfinancial or intangible assets - small in US data)
- Every international transaction automatically enters the balance of payments twice: once as a credit and once as a debit.

 Current account balance, financial account balance and capital account balance sums up to zero.

$$CA + FA + KA = 0 = BOP$$

- Transactions that involves a payment to foreigners (e.g. importing goods or assets): debit (-)
- Transactions that involves a receipt from foreigners (e.g. exporting goods or assets): credit (+)

- Current Account:
 - Balance on Goods (exports and imports)
 - Service and Income
- Financial Account:
 - Direct Investment
 - Foreign Direct Investment
 - Portfolio Investment
 - Financial Derivatives
 - Changes in Reserve

- Official Reserve Transaction: purchase or sales of official reserve assets by central banks.
- Official Reserve Assets: foreign assets held by central banks.

$$FA = FA^{nonFX} + \Delta FX$$

• To obtain the total change in official reserves:

$$\Delta FX = FA^{nonFX} + CA + KA$$

• If a country is running a current account surplus, and $FA^{nonFX} = KA = 0$ - the country is accumulating foreign reserves

- Modern approach for understanding current account: intertermporal approach to the current account
- Key features:
 - intertemporal optimization
 - flexible price
 - ▶ real model
 - simple international asset market structure

Motivation:

- When do countries run a current account surplus or deficit?
- Are the current account imbalances sustainable?
- How do external imbalances adjust?

Overview on national accounting

$$Y = C + I + G + EX - IM$$

- Y: GDP
- C: Consumption; I: Investment; G: public spending; EX: Exports of goods and services; IM: Imports

$$GNI = Y + rB = C + I + G + \underbrace{EX - IM + rB}_{CA}$$

• Current account : CA = EX - IM + rB with B =net foreign assets (NFA); rB =factor payments on NFA

- Overview on national accounting
- Current account deficit (resp. suplus) means the country is borrowing from (resp. lending to) the rest of the world.
- Current account is a flow. The stock of net external debt = net foreign assets (*NFA* = *B*) = cumulative sum of *CA*

$$B_{t+1} - B_t = CA_t$$

• Important to note here that we abstract from valuation effects of gross foreign assets/liabilities. Traditional approach to *CA*.

Intertemporal Approach to the Current Account

- We consider a two period small open economy model in the spirit of I. Fisher.
 - small open economy model: the rate at which the economy can borrow and lend is given
 - two period economy;
 - representative household ⇒ no distinction between aggregate and individual variables;
 - Endowment economy: agents receive every period an exogenous amount of perishable good Y_t.
- Consumers' preferences:

$$U = u(C_1) + \beta u(C_2)$$

where $\beta \in [0,1]$ is the discount factor. C is consumption good and u is the period utility function with u'>0 and u''<0



Intertemporal approach to the current account

- Asset market structure: B risk-free asset. B is real bond that provides return in terms of consumption good.
- Economy "starts" and "ends" without debt or assets $B_1=0$ (assumption) and $B_3=0$

$$t = 1: B_2 - B_1 = Y_1 + rB_1 - C_1$$

 $t = 2: B_3 - B_2 = Y_2 + rB_2 - C_2$

 By combining period budget constraint, we obtain the intertemporal Budget constraint:

$$C_1 + \frac{C_2}{1+r} = Y_1 + \frac{Y_2}{1+r}$$

with $\frac{1}{1+r}$ is the market discount factor to future consumption (i.e. is the relative price of consumption in period 2 in terms of consumption in period 1). Y is the exogenous endowment of perishable good.

Intertemporal approach to the current account

Optimization problem

$$\max_{C_1,C_2} U = u(C_1) + \beta u(C_2)$$

subject to the intertemporal budget constraint:

$$C_1 + \frac{C_2}{1+r} = Y_1 + \frac{Y_2}{1+r}$$

• First order condition (Euler equation):

$$\frac{\beta u'(C_2)}{u'(C_1)} = \frac{1}{1+r}$$

The marginal rate of intertemporal substitution should be equal to the intertemporal relative price.

- Previous condition determines the path of consumption over time, not the consumption function
- 2 Consumption function is obtained by combining the Intertermpoal budget constraint with the Euler equation.



Intertemporal approach to the current account

 Current Account is the change in net foreign asset position between two periods:

$$CA_t = \underbrace{B_{t+1} - B_t}_{-FA} = \underbrace{Y_t}_{net \ foreign \ income} - C_t$$

 B_{t+1} is defined as the economy's net stock of loans to foreigners at the end of any date t.

- no government consumption
 - no investment
 - no transfers

Intertemporal approach to the current account

- Current Account in our two-period economy.
 - ▶ Period 1 and 2:

$$CA_1 = B_2 - B_1 = Y_1 + rB_1 - C_1$$

 $CA_2 = B_3 - B_2 = Y_2 + rB_2 - C_2$

- Suppose we start with $B_1 = 0$.
- ▶ We have that $B_3 \le 0$ since there is no chance of being repaid after period $2 \Rightarrow$

$$-(Y_1 - C_1) = Y_2 + r(Y_1 - C_1) - C_2$$

This implies that

$$-CA_1 = CA_2$$

Closed economy

• In a closed economy, or in autarky (ie when there is no international borrowing and lending) we would have:

$$t = 1: Y_1 = C_1$$

 $t = 2: Y_2 = C_2$

Recall that *Y* is the exogenous endowment of perishable good.

 In an open economy consumers can choose C₁ and C₂ to maximise their utility, provided that the budget constraint is satisfied. (note that the autarky allocations do satisfy this constraint - so opening to international asset markets can only make consumers better off)

Open Economy

Optimal consumption choice:

$$\frac{\beta u'(C_2)}{u'(C_1)} = \frac{1}{1+r}$$

Consumption profile depends on the first order condition.

- if $\beta < \frac{1}{1+r} \Rightarrow C_1 > C_2$;
- if $\beta > \frac{1}{1+r} \Rightarrow C_1 < C_2$;
- if $\beta = \frac{1}{1+r} \Rightarrow C_1 = C_2$;

Equilibrium consumption level:

use the first order condition and the budget constraint to solve for equilibrium consumption in each period.

In the simplest case in which $\beta = \frac{1}{1+r}$ then:

$$C = \frac{(1+r)Y_1 + Y_2}{2+r}$$

Introduction

Why do countries run current account surpluses/deficits?

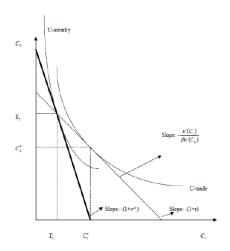
- As in international trade, comparative advantage principle: the country tends to import those commodities whose autarky price are high compared to world prices.
- What are the autarky prices in this context?
 - Determination of autarky relative price: the relative price that would prevail in a closed economy.

$$C_1 = Y_1 \text{ and } C_2 = Y_2$$

$$\Rightarrow \frac{\beta u'(Y_2)}{u'(Y_1)} = \frac{1}{1 + r^a}$$

- Changes in the endowments are the only factor responsible for determining r^a.
- if $r^a < r \Rightarrow CA_1 > 0$; (net lender)
- if $r^a > r \Rightarrow CA_1 < 0$; (net borrower)

Graphical analysis



Autarky interest rate

Changes in output and the autarky interest rate.

-if current endowment increase, $\uparrow Y_1 \Rightarrow$ marginal utility of consumption-1 decreases \Rightarrow relative price of C_1 falls, $r^a \downarrow$

intuition: $\uparrow Y_1 \Rightarrow$ incentive to save \Rightarrow since in autarky this is not possible $\Rightarrow r^a \downarrow$ to reduce incentive to save.

-if future endowment increase, $\uparrow Y_2 \Rightarrow$ marginal utility of consumption-2 decreases and $r^a \uparrow$

intuition: $\uparrow Y_2 \Rightarrow$ incentive to borrow \Rightarrow since in autarky this is not possible $\Rightarrow r^a \uparrow$

Autarky and Market interest rates: an example

• Consider the following example: if $\beta = \frac{1}{1+r}$, then

$$\frac{\beta u'(Y_2)}{u'(Y_1)} = \frac{1}{1+r^a}$$
$$\Rightarrow \frac{u'(Y_2)}{u'(Y_1)} = \frac{1+r}{1+r^a}$$

- Start from $Y_1 = Y_2$ so that the initial equilibrium is the autarky case with no gains from intertemporal trade.
 - If $\uparrow Y_1 \Rightarrow r^a \downarrow$ so that $r^a < r$ and $CA_1 > 0$
 - If $\uparrow Y_2 \Rightarrow r^a \uparrow$ so that $r^a > r$ and $CA_1 < 0$

Global Equilibrium

Determination of world interest rate

- Consider now the situation of two large countries.
- Two-country, endowment economy.
- "Home" and "Foreign" (*) economies have the same structure as before.
 - Foreign economy preferences

$$U^* = u(C_1^*) + \beta^* u(C_2^*)$$

where $\beta^* \in [0,1]$ is the foreign discount factor. C^* is the foreign consumption good and u is the period utility function with u'>0 and u''<0

Foreign economy, intertemporal budget constraint

$$C_1^* + \frac{C_2^*}{1+r} = Y_1^* + \frac{Y_2^*}{1+r}$$

Global Equilibrium

Determination of world interest rate

- Each country solve same problem as before taken as given the world interest rate and determining the consumption (saving) function.
- Equilibrium in the global output market requires:

$$Y_t + Y_t^* = C_t + C_t^* \ t = 1,2$$

or equivalently

$$S_t + S_t^* = 0$$

In autarky

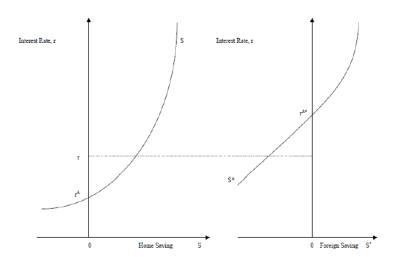
$$S_t = 0 \text{ and } S_t^* = 0$$

And this condition will determine r^a and r^{a^*}

• In an open economy the equilibrium r is determined by condition $S_t + S_t^* = 0$, where saving is an increasing function of interest rates.



Global Equilibrium



Imbalances and Adjustment

Global imbalances: Failure of traditional models

• Log-utility example

$$u(C) = \log C, u(C^*) = \log C^*$$

 Characterize the U.S. economy with higher endowment but slow growth relative to China (*).

$$Y_1 > Y_1^*$$
 with $\Delta Y < \Delta Y^*$

with
$$\Delta Y = Y_2 - Y_1$$
 and $\Delta Y^* = Y_2^* - Y_1^*$.

Imbalances and Adjustment

Global imbalances: Failure of traditional models

Determination of autarky rates

$$\frac{\beta u'(Y_2)}{u'(Y_1)} = \frac{1}{1+r^a} \Rightarrow \frac{\beta Y_1}{Y_2} = \frac{1}{1+r^a}$$
$$\frac{\beta u'(Y_2^*)}{u'(Y_1^*)} = \frac{\beta Y_1^*}{Y_2^*} = \frac{1}{1+r^{a*}}$$

· Comparision of the autarky interest rates

$$\frac{1+r^{a*}}{1+r^a} = \frac{Y_1}{Y_2} \frac{Y_2^*}{Y_1^*}$$

We have that

$$\frac{1+r^{a*}}{1+r^a} > 1 \Leftrightarrow \frac{Y_2^* - Y_1^*}{Y_1^*} - \frac{Y_2 - Y_1}{Y_1} > 0$$

World interest rate is

$$r^a < r < r^{a*}$$

with

$$CA^* < 0, CA > 0$$



Global Imbalances

Global imbalances: Failure of traditional models

- Traditional model: integration would imply current account deficit in the developing country and higher interest rate relative to autarky for the advanced country.
- China: Fast growing country. Very high productivity growth with large investment rate .But savings growing at an even faster pace. Chinese savings puzzle. China runs current account surpluses.
- Interest rates have been trending lower and US economy has run persistent deficits in the external position.

Imbalances

Fall in US Long-Term Interest Rate



Imbalances and Adjustment

Theories of global imbalances: a quick summary

- Different theories:
 - Heterogeneity in Financial market development (Caballero, Fahri and Gourinchas (2008).
 - Precautionary savings and idiosyncratic risk in emerging markets (Ranciere and Jeanne (2006), Jeanne and Caroll (2009))
 - Precautionary savings and differences in financial markets development (financial markets incompleteness): Mendoza, Quadrini and Rios-Rull (2009).

Valuation effects and the Exorbitant Privilege: an example

- Suppose now that the country trades bonds denominated in domestic currency ($B_{H,t}$) but also trades bonds denominated in foreign currency ($B_{F,t}$)
- The budget constraint of the economy becomes:

$$C_t + B_{H,t} + S_t B_{F,t} = B_{H,t-1}(1+i_t) + S_t B_{F,t-1}(1+i_t^*) + Y_t$$

• Countries can have assets and liabilities denominated in different currencies. That is, consider the case where $B_t = A_t - L_t$ where A_t are the assets and L_t the liabilities:

$$C_t + A_{H,t} - L_{H,t} + S_t(A_{F,t} - L_{F,t})$$

= $(A_{H,t-1} - L_{H,t-1})(1 + i_t) + S_t(A_{F,t-1} - L_{F,t-1})(1 + i_t^*) + Y_t$

or

$$\begin{split} \Delta A_{H,t} - \Delta L_{H,t} + S_t (\Delta A_{F,t} - \Delta L_{F,t}) \\ = (A_{H,t-1} - L_{H,t-1})i_t + S_t (A_{F,t-1} - L_{F,t-1})i_t^* + (Y_t - C_t) \end{split}$$

Eg. US current situation

- Most of US assets are in foreign currency so and most liabilities are in dollars
- Assume for simplicity $A_{H,t} = 0$ and $L_{F,t} = 0$

$$CA_t = S_t \Delta A_{F,t} - \Delta L_{H,t} = S_t A_{F,t-1} i_t^* - L_{H,t-1} i_t + \underbrace{(Y_t - C_t)}_{TB}$$

$$CA_t = S_t A_{F,t-1} i_t^* - L_{H,t-1} i_t + TB_t[S_t]$$

where
$$TB_t[S_t] = Y_t - C_t$$

 So a depreciation would not only improve US trade balance (Marshall-Lerner condition) but it would also increase the revenue from their assets relative to the cost of their liabilities.

Some economists argue that the US can maintain a negative trade balance because

- The observed depreciation of the dollar reduces the payments of US liabilities and increase the dollar value of its foreign assets (valuation effect).
- A small depreciation can have a big impact on the CA_t because apart from the improvement in the trade balance this would also generate this positive valuation effect in US balance sheet.
- Because the US is considered a safe heaven, US interest rates tend to pay a negative premium (or trade at a discount). $-i_t^* i_t$ exorbitant privilege.

Infinite Horizon Economy

- Same framework: one good, SOE with constant world interest rate *r*
- Infinite horizon.
- Utility of the representative agent:

$$U = \sum_{s=t}^{\infty} \beta^{s-t} u(C_s)$$

Budget constraint

$$C_t + I_t + G_t + B_{t+1} = Y_t + (1+r)B_t$$

• In production economy

$$Y_t = A_s F(K_s)$$

Infinite Horizon Economy

• Optimization problem:

$$\max_{\{C_{t}, B_{t+1}, K_{t+1}\}_{t=s}^{\infty}} U = \sum_{s=t}^{\infty} \beta^{s-t} u(C_{s})$$

• subject to:

$$C_t + I_t + G_t + B_{t+1} = Y_t + (1+r)B_t$$

 $Y_t = A_s F(K_s)$
 $I_t = K_{t+1} - K_t$

Trade Balance

$$TB_t \equiv Y_t - C_t - I_t - G_t$$

Current account:

$$CA_t \equiv B_{t+1} - B_t$$
$$= TB_t + rB_t$$

• Iterating forward the B.C., imposing the No-Ponzi condition $\lim_{T\to\infty} \frac{B_{t+T+1}}{(1+r)^T} = 0$, we arrive at the intertemporal budget constraint:

$$\sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} C_s = (1+r)B_t + \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} (Y_s - G_s - I_s)$$

 Intertemporal budget constraint: the PV of consumption expenditures must equal the PV of net income.

$$-(1+r)B_{t} = \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} (Y_{s} - G_{s} - C_{s} - I_{s}) = \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} TB_{s}$$

 As before, a negative net foreign asset position at period t must be balanced by future trade surpluses [note: if interest rates could vary, adjustment could be achieved through changes in the intertemporal price of debt; or similarly when values of foreign assets/liabilities is changing over time]

Growth Rate of Output

$$Y_{t+1} = Y_t(1+g)$$

• Constant debt to GDP [=sustainable] implies that B_t grows at rate g

$$B_{t+1} = B_t(1+g) \Rightarrow gB_t = B_{t+1} - B_t = TB_t + rB_t \Rightarrow \frac{TB_t}{Y_t} = \frac{-(r-g)B_t}{Y_t}$$

• Example: in 2009, in the US, Net foreign assets $\frac{B_t}{Y_t} \simeq -30\%$; $r \simeq 3.5\%$ (Gourinchas and Rey (2010)). With low expected growth rate, g = 1%, US should run trade surplus of 0.8% to stabilize their external debt. Trade balance is enough if g = 3.5%.

- Sustainable external debt to GDP ratio
- To keep debt levels constant, the country needs to generate trade surplus/GDP = (r-g) Debt/GDP
- If r>g, country has to run a trade surplus in order to control the Debt/GDP ratio
- 2 If r < g then country can run a trade deficit (of a certain size) without seeing Debt/GDP ratio increase
- Note: shifts in interest rates and growth rates have big impact on external debt dynamics
 - If r increases with Debt/GDP and higher r leads to lower g then countries can rapidly find their external position deteriorating.

Some examples for current account determination

- Consider the case in which $\beta = (1+r)^{-1}$
 - ▶ this implies that $C_s = C_t \ \forall s \ge t$ (From Euler equation)

$$u'(C_t) = \beta(1+r)u'(C_{t+1})$$

Rewrite the intertemporal budget constraint

$$\sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} C_s = \frac{C_t}{1 - \frac{1}{1+r}} = (1+r)B_t + \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} (Y_s - G_s - I_s)$$

$$C_t = \frac{r}{1+r} \left[(1+r)B_t + \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} (Y_s - G_s - I_s) \right]$$

$$C_t = \frac{r}{1+r} W_t$$

- where $W_t \equiv (1+r)B_t + \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} (Y_s G_s I_s)$
- Consumption is equal to annuity value of wealth (amount that you can consume and leave wealth unchanged) $[W_{t+1} = (1+r)(W_t C_t)]$



Some examples for current account determination

• Define now the permanent value of a generic variable \tilde{X}_t as

$$\tilde{X}_t \equiv \frac{r}{1+r} \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} X_s$$

so that now we can re-express current account as

•

$$CA_t = (Y_t - \tilde{Y}_t) - (G_t - \tilde{G}_t) - (I_t - \tilde{I}_t)$$

• Consumption smoothing is motive behind current account fluctuations.

Some examples for current account determination

- Consider the case in which $\beta \neq (1+r)^{-1}$ with CES utility function;
 - ▶ From Euler equation we now have

$$C_{s+1} = \beta^{\sigma} (1+r)^{\sigma} C_s \quad \forall s \ge t$$

Rewrite the intertemporal budget constraint

$$\sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} C_s = \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} \beta^{\sigma(s-t)} (1+r)^{\sigma(s-t)} C_t = W_t$$

$$\frac{C_t}{1-\beta^{\sigma} (1+r)^{\sigma-1}} = W_t$$

- with $\beta^{\sigma} (1+r)^{\sigma-1} < 1$
- We can express now the consumption function as

$$C_t = \left(1 - \frac{\beta^{\sigma}(1+r)^{\sigma}}{1+r}\right)W_t = \frac{r}{1+r}W_t + \frac{1 - \beta^{\sigma}(1+r)^{\sigma}}{1+r}W_t$$



Some examples for current account determination

• Rewrite consumption function:

$$C_t = \left(1 - \frac{\beta^{\sigma} (1+r)^{\sigma}}{1+r}\right) W_t = \frac{r}{1+r} W_t + \frac{1 - \beta^{\sigma} (1+r)^{\sigma}}{1+r} W_t$$

- $\frac{1-\beta^{\sigma}(1+r)^{\sigma}}{1+r}$ is a tilting factor in consumption behavior.
- If $\beta > (1+r)^{-1}$ consumption grows forever; if $\beta < (1+r)^{-1}$ shrinks forever.
- We can re-express current account as

$$CA_t = (Y_t - \tilde{Y}_t) - (G_t - \tilde{G}_t) - (I_t - \tilde{I}_t) - \frac{1 - \beta^{\sigma} (1 + r)^{\sigma}}{1 + r} W_t$$

Note on investment in deterministic setting

• First order conditions from optimization problem

$$u'(C_t) = \beta(1+r)u'(C_{t+1})$$

$$A_{s+1}F'(K_{s+1}) = r$$

- We have that investment decision depends only on world interest rate.
 - small open economy
 - homogenous good
 - no capital market imperfection

Stochastic setting

- Rational expectations: mathematical conditional expectations based on accurate model and all the information about current economic variables (i.e. rational forecast error is uncorrelated with the information on which the forecast was conditioned). We denote with E_t the rational expectation operator
- Optimization problem:

$$\max_{\{C_{t},B_{t+1}\}_{t=s}^{\infty}} U = E_{t} \sum_{s=t}^{\infty} \beta^{s-t} u(C_{s})$$

• subject to:

$$C_t + G_t + B_{t+1} = Y_t + (1+r)B_t$$

From which we get the new Euler equation

$$u'(C_t) = \beta(1+r)E_tu'(C_{t+1})$$



Stochastic setting

• Special case: $\beta = (1+r)^{-1}$ implies

$$u'(C_t) = E_t u'(C_{t+1})$$

 Certainty equivalence: what matters for agents decisions is the conditional mean of future stochastic variables. We need also to assume quadratic utility function

$$u(C) = aC - \frac{b}{2}C^2$$

With both assumptions we obtain

$$C_t = E_t C_{t+s} \, \forall s$$

• Consumption follows a random walk

Stochastic setting

· We get something similar to what we had before

$$C_t = \frac{r}{1+r}W_t$$

with

$$W_t \equiv (1+r)B_t + E_t \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} (Y_s - G_s)$$

and the same current account equation

$$CA_t = Z_t - \tilde{Z}_t$$

with
$$Z_t = Y_t - G_t$$

Stochastic setting

- With only saving decisions, current account is procyclical.
- Now we study economy with investment decisions.
- Questions:
 - How do productivity shocks affect the current account?
 - How can we obtain a countercyclical current account?

Stochastic setting with investment

• Optimization problem:

$$\max_{\{C_{t},B_{t+1},K_{t+1}\}_{t=s}^{\infty}} U = E_{t} \sum_{s=t}^{\infty} \beta^{s-t} u(C_{s})$$

• subject to:

$$C_t + I_t + B_{t+1} = Y_t + (1+r)B_t$$

 $I_t = K_{t+1} - K_t$
 $Y_t = A_t F(K_t)$

From which we get the following first order conditions:

$$u'(C_t) = \beta(1+r)E_tu'(C_{t+1})$$

$$u'(C_t) = \beta E_t \left[u'(C_{t+1})(1+A_{t+1}F'(K_{t+1})) \right]$$

Investment under uncertainty

• If we impose $\beta = (1+r)^{-1}$ we obtain

$$1 + r = Cov_t \left(A_{r+1} F'(K_{t+1}), \frac{u'(C_{t+1})}{u'(C_t)} \right) + E_t \left[1 + A_{t+1} F'(K_{t+1}) \right]$$

• From which we get the investment rule under uncertainty

$$r = Cov_t \left(A_{r+1} F'(K_{t+1}), \frac{u'(C_{t+1})}{u'(C_t)} \right) + E_t \left[A_{r+1} F'(K_{t+1}) \right]$$

- How does the covariance term move? Answer depends on who own the capital stock. In our case capital stock is owned by domestic household.
- If $A_{r+1} \uparrow$, (capital is more productive), C_{t+1} will be higher and $u'(C_{t+1}) \downarrow$ so this means that the covariance is negative.
- Negative covariance implies

$$E_t\left[A_{r+1}F'(K_{t+1})\right] > r$$

and K_{t+1} is lower than in the deterministic case (i.e. $E_t[A_{r+1}F'(K_{t+1})] = r$)

Investment under uncertainty

• If we impose $\beta = (1+r)^{-1}$ we obtain

$$1 + r = Cov_t\left(A_{r+1}F'(K_{t+1}), \frac{u'(C_{t+1})}{u'(C_t)}\right) + E_t\left[1 + A_{r+1}F'(K_{t+1})\right]$$

• From which we get the investment rule under uncertainty

$$r = Cov_t \left(A_{r+1} F'(K_{t+1}), \frac{u'(C_{t+1})}{u'(C_t)} \right) + E_t \left[A_{r+1} F'(K_{t+1}) \right]$$

- How does the covariance term move? Answer depends on who own the capital stock. In our case capital stock is owned by domestic household.
- If $A_{r+1} \uparrow$, (capital is more productive), C_{t+1} will be higher and $u'(C_{t+1}) \downarrow$ so this means that the covariance is negative.
- Negative covariance implies

$$E_t\left[A_{r+1}F'(K_{t+1})\right] > r$$

and K_{t+1} is lower than in the deterministic case (i.e. $E_t[A_{r+1}F'(K_{t+1})] = r$)

Investment under uncertainty

· Let's focus now on the certainty equivalent case

$$E_t\left[A_{r+1}F'(K_{t+1})\right] = r$$

and assume a mean reverting process for the technology shock

$$A_{t+1} - \bar{A} = \rho \left(A_t - \bar{A} \right) + \varepsilon_{t+1}$$

- What are the effects of an increase in ε_t ?
 - Investment increases;
 - Consumption decreases, saving increases.
- What happens to the current account? Remember that now current account is CA = S I

Current Account and Investment

· Let's focus now on the certainty equivalent case

$$E_t\left[A_{r+1}F'(K_{t+1})\right]=r$$

and assume a mean reverting process for the technology shock

$$A_{t+1} - \bar{A} = \rho \left(A_t - \bar{A} \right) + \varepsilon_{t+1}$$

- What are the effects of an increase in ε_t ?
 - Investment increases;
 - Consumption decreases, saving increases.
- What happens to the current account? Remember that now current account is CA = S I

Current Account and Investment

- Consider two cases:
 - No persistence in the shock $(\rho = 0)$; $\varepsilon_t \uparrow \Longrightarrow A_t \uparrow \Longrightarrow Y_t \uparrow$. In this case we have that $E_t A_{t+1} = \bar{A}$ so that investment does not change, while saving will increase because there is a temporary increase in output. Current account is procyclical
 - ▶ In the case in which the shock is permanent ($\rho = 1$): $\varepsilon_t \uparrow \Longrightarrow A_t \uparrow$ and now $E_t A_{t+1} = A_t$. Investment rises. Savings decreases as investment takes one period to adjust. Current account is countercyclical.