Macroeconomics A; EI056

Technical appendix: Tax smoothing and political economy of public deficits

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1 Intertemporal tax smoothing

Consider that government spending G_t, GDP Y_t and the interest rate r are exogenous. The interest rate is also constant.

Raising taxes leads to frictions and a cost that is convex in the ratio of taxes to GDP:

$$\frac{C_t}{Y_t} = f\left(\frac{T_t}{Y_t}\right)$$

where $f(0) = 0$, $f'(0) = 0$ and $f'' > 0$.

The flow budget constraint of the government is:

$$B_1 = (1+r)\,B_0 + G_0 - T_0$$

where B is debt. If we iterate forward and assume that $\lim_{T\to\infty} (1+r)^{-T} B_T = 0$ we get the intertemporal budget constraint:

$$\begin{aligned}
B_1 &= (1+r)\,B_0 + G_0 - T_0 \\
B_0 &= \frac{T_0 - G_0}{1+r} + \frac{1}{1+r} B_1 \\
B_0 &= \frac{T_0 - G_0}{1+r} + \frac{T_1 - G_1}{(1+r)^2} + \frac{1}{(1+r)^2} B_2 \\
B_0 &= \sum_{t=0}^{\infty} \frac{1}{(1+r)^{t+1}} (T_t - G_t) + \lim_{T\to\infty} \frac{1}{(1+r)^T} B_T \\
\sum_{t=0}^{\infty} \frac{1}{(1+r)^{t+1}} T_t &= B_0 + \sum_{t=0}^{\infty} \frac{1}{(1+r)^{t+1}} G_t
\end{aligned}$$

The government minimizes the net present value of the cost of raising taxes subject to the intertemporal budget constraint. The Lagrangian is:

$$\mathcal{L} = \sum_{t=0}^{\infty} \frac{1}{(1+r)^{t+1}} Y_t f\left(\frac{T_t}{Y_t}\right) - \mu \left[\sum_{t=0}^{\infty} \frac{1}{(1+r)^{t+1}} T_t - B_0 - \sum_{t=0}^{\infty} \frac{1}{(1+r)^{t+1}} G_t\right]$$

The first-order condition is:

$$\begin{aligned}
0 &= \frac{\partial \mathcal{L}}{\partial T_t} \\
0 &= \frac{1}{(1+r)^{t+1}} f'\left(\frac{T_t}{Y_t}\right) - \mu \frac{1}{(1+r)^{t+1}} \\
f'\left(\frac{T_t}{Y_t}\right) &= \mu
\end{aligned}$$

therefore taxes are set so that the ratio of taxes to GDP $\tau = T/Y$ is constant.

The intertemporal budget constraint then implies:

$$\begin{aligned}
\sum_{t=0}^{\infty} \frac{1}{(1+r)^{t+1}} T_t &= B_0 + \sum_{t=0}^{\infty} \frac{1}{(1+r)^{t+1}} G_t \\
\tau Y_t \frac{1}{1+r} \sum_{s=0}^{\infty} \frac{1}{(1+r)^{t}} &= B_0 + \sum_{t=0}^{\infty} \frac{1}{(1+r)^{t+1}} G_t \\
\tau Y_t \frac{1}{1+r} \frac{1}{1-\frac{1}{1+r}} &= B_0 + \sum_{t=0}^{\infty} \frac{1}{(1+r)^{t+1}} G_t \\
\tau Y_t &= r\left[B_0 + \sum_{t=0}^{\infty} \frac{1}{(1+r)^{t+1}} G_t\right] \\
\tau Y_t &= r B_0 + G^{perm}
\end{aligned}$$

Where G^{perm} is the permanent-equivalent level of government spending:

$$\begin{aligned}
\sum_{t=0}^{\infty} \frac{1}{(1+r)^{t+1}} G^{perm} &= \sum_{t=0}^{\infty} \frac{1}{(1+r)^{t+1}} G_t \\
G^{perm} \sum_{t=0}^{\infty} \frac{1}{(1+r)^{t+1}} &= \sum_{t=0}^{\infty} \frac{1}{(1+r)^{t+1}} G_t \\
G^{perm} \frac{1}{1+r} \frac{1}{1-\frac{1}{1+r}} &= \sum_{t=0}^{\infty} \frac{1}{(1+r)^{t+1}} G_t \\
G^{perm} &= r \sum_{t=0}^{\infty} \frac{1}{(1+r)^{t+1}} G_t
\end{aligned}$$

The flow budget constraint then implies that debt moves in responses to deviations of government spending from the permanent level:

$$B_{t+1} = (1+r) B_t + G_t - T_t$$

$$
\begin{aligned}
B_{t+1} &= (1+r)\,B_t + G_t - rD_0 - G^{perm} \\
B_{t+1} - B_t &= r\,(B_t - B_0) + (G_t - G^{perm})
\end{aligned}
$$

2 A political economy view of deficits

2.1 General framework

Consider a two period model, setting the real interest rate to zero for brevity. In each period there is an endowment W. Government spending is used to purchases two types of goods, M and N. The government's budget constraints in the two periods are (B denotes debt):

$$
\begin{aligned}
M_1 + N_1 &= W + B \\
M_2 + N_2 &= W - B
\end{aligned}
$$

Individuals are different depending on their preferences for the two types of goods the government can purchase. Specifically, the utility of individual i is:

$$
V_i = E\sum_{t=1}^{2}\left[\alpha_i U\left(M_t\right) + \left(1-\alpha_i\right)V\left(N_t\right)\right]
$$

where U and V are standard concave utility functions. Individuals differ according to their preference for good M which is captured by the weight $\alpha_i \in [0,1]$.

In each period the composition of government spending is done by the median voter, which is the individual whose α_i is the median of the distribution of weights across agents.

2.2 Corner preferences

Consider that agents have corner preferences. Some care only about good M, so that $\alpha_i = \alpha_{High} = 1$ and others care only about good N, so that $\alpha_i = \alpha_{Low} = 0$.

If the agents that care only about good M are more numerous in period 2, then the median voter is one of them and the government only purchases goods M. This happens with probability π. Otherwise, the government only purchases goods N.

We then turn to period 1. Consider first the case where the median voter is an individual that cares only about good M. The intertemporal utility is:

$$
\begin{aligned}
& E\sum_{t=1}^{2}\left[\alpha_{High} U\left(M_t\right) + \left(1-\alpha_{High}\right)V\left(N_t\right)\right] \\
=\; & E\sum_{t=1}^{2} U\left(M_t\right) \\
=\; & U\left(M_1\right) + EU\left(M_2\right) \\
=\; & U\left(W+B\right) + \pi U\left(W-B\right) + \left(1-\pi\right)U\left(0\right)
\end{aligned}
$$

The firs-order condition with respect to debt B is:

$$\begin{aligned} U'(W+B) &= \pi U'(W-B) \\ \frac{U'(W+B)}{U'(W-B)} &= \pi < 1 \end{aligned}$$

As U is concave, U' is a decreasing function. Therefore $W+B > W-B$, that is $B > 0$.

Consider now the case where the median voter is an individual that cares only about good N. The intertemporal utility is:

$$\begin{aligned} & E\sum_{t=1}^{2}\left[\alpha_{Low}U(M_t) + (1-\alpha_{Low})V(N_t)\right] \\ = & E\sum_{t=1}^{2}\left[V(N_t)\right] \\ = & V(N_1) + EV(N_2) \\ = & V(W+B) + \pi V(0) + (1-\pi)V(W-B) \end{aligned}$$

The firs-order condition with respect to debt B is:

$$\begin{aligned} V'(W+B) &= (1-\pi)V'(W-B) \\ \frac{V'(W+B)}{V'(W-B)} &= 1-\pi < 1 \end{aligned}$$

As V is concave, this again implies that $B > 0$.

2.3 Standard preferences

We now look at the situation where agents care about both goods, so α_i is between 0 and 1. For simplicity we set both U and V to be CRRA utilities with a relative risk aversion of θ.

In the second period, the median voter maximizes:

$$\alpha_{med,2}\frac{(M_2)^{1-\theta}}{1-\theta} + (1-\alpha_{med,2})\frac{(W-B-M_2)^{1-\theta}}{1-\theta}$$

The optimal composition of government spending follows from the first-order condition with respect to M_2:

$$\begin{aligned} \alpha_{med,2}(M_2)^{-\theta} &= (1-\alpha_{med,2})(W-B-M_2)^{-\theta} \\ \left(\frac{M_2}{W-B-M_2}\right)^{-\theta} &= \frac{1-\alpha_{med,2}}{\alpha_{med,2}} \\ \frac{M_2}{W-B-M_2} &= \left(\frac{\alpha_{med,2}}{1-\alpha_{med,2}}\right)^{\frac{1}{\theta}} \\ M_2 &= \left(\frac{\alpha_{med,2}}{1-\alpha_{med,2}}\right)^{\frac{1}{\theta}}(W-B-M_2) \end{aligned}$$

$$
\begin{aligned}
M_2 &= \frac{\left(\frac{\alpha_{med,2}}{1-\alpha_{med,2}}\right)^{\frac{1}{\theta}}}{1+\left(\frac{\alpha_{med,2}}{1-\alpha_{med,2}}\right)^{\frac{1}{\theta}}}(W-B) \\
M_2 &= \frac{(\alpha_{med,2})^{\frac{1}{\theta}}}{(1-\alpha_{med,2})^{\frac{1}{\theta}}+(\alpha_{med,2})^{\frac{1}{\theta}}}(W-B)
\end{aligned}
$$

The consumption of the other good is:

$$
N_2 = W - B - M_2 = \frac{(1-\alpha_{med,2})^{\frac{1}{\theta}}}{(1-\alpha_{med,2})^{\frac{1}{\theta}}+(\alpha_{med,2})^{\frac{1}{\theta}}}(W-B)
$$

In the first-period, government spending is set by the median voter with preference $\alpha_{med,1}$. She maximizes:

$$
\begin{aligned}
& E\sum_{t=1}^{2}\left[\alpha_{med,1}\frac{(M_t)^{1-\theta}}{1-\theta}+(1-\alpha_{med,1})\frac{(N_t)^{1-\theta}}{1-\theta}\right] \\
= \; & \alpha_{med,1}\frac{(M_1)^{1-\theta}}{1-\theta}+(1-\alpha_{med,1})\frac{(W+B-M_1)^{1-\theta}}{1-\theta} \\
& +E\left[\alpha_{med,1}\frac{(M_2)^{1-\theta}}{1-\theta}+(1-\alpha_{med,1})\frac{(N_2)^{1-\theta}}{1-\theta}\right] \\
= \; & \alpha_{med,1}\frac{(M_1)^{1-\theta}}{1-\theta}+(1-\alpha_{med,1})\frac{(W+B-M_1)^{1-\theta}}{1-\theta} \\
& +\alpha_{med,1}\frac{1}{1-\theta}E\left(\frac{(\alpha_{med,2})^{\frac{1}{\theta}}}{(1-\alpha_{med,2})^{\frac{1}{\theta}}+(\alpha_{med,2})^{\frac{1}{\theta}}}(W-B)\right)^{1-\theta} \\
& +(1-\alpha_{med,1})\frac{1}{1-\theta}E\left(\frac{(1-\alpha_{med,2})^{\frac{1}{\theta}}}{(1-\alpha_{med,2})^{\frac{1}{\theta}}+(\alpha_{med,2})^{\frac{1}{\theta}}}(W-B)\right)^{1-\theta} \\
= \; & \alpha_{med,1}\frac{(M_1)^{1-\theta}}{1-\theta}+(1-\alpha_{med,1})\frac{(W+B-M_1)^{1-\theta}}{1-\theta}+\frac{(W-B)^{1-\theta}}{1-\theta}\Omega_2
\end{aligned}
$$

where:

$$
\Omega_2 = \alpha_{med,1}E\left(\frac{(\alpha_{med,2})^{\frac{1}{\theta}}}{(1-\alpha_{med,2})^{\frac{1}{\theta}}+(\alpha_{med,2})^{\frac{1}{\theta}}}\right)^{1-\theta}+(1-\alpha_{med,1})E\left(\frac{(1-\alpha_{med,2})^{\frac{1}{\theta}}}{(1-\alpha_{med,2})^{\frac{1}{\theta}}+(\alpha_{med,2})^{\frac{1}{\theta}}}\right)^{1-\theta}
$$

The first-order condition with respect to M_1 implies:

$$
\begin{aligned}
\alpha_{med,1}(M_1)^{-\theta} &= (1-\alpha_{med,1})(W+B-M_1)^{-\theta} \\
M_1 &= \frac{(\alpha_{med,1})^{\frac{1}{\theta}}}{(1-\alpha_{med,1})^{\frac{1}{\theta}}+(\alpha_{med,1})^{\frac{1}{\theta}}}(W+B)
\end{aligned}
$$

The expected utility is then written as:

$$\frac{(W+B)^{1-\theta}}{1-\theta}\Omega_1 + \frac{(W-B)^{1-\theta}}{1-\theta}\Omega_2$$

where:

$$\Omega_1 = \alpha_{med,1}\left(\frac{(\alpha_{med,1})^{\frac{1}{\theta}}}{(1-\alpha_{med,1})^{\frac{1}{\theta}} + (\alpha_{med,1})^{\frac{1}{\theta}}}\right)^{1-\theta} + (1-\alpha_{med,1})\left(\frac{(1-\alpha_{med,1})^{\frac{1}{\theta}}}{(1-\alpha_{med,1})^{\frac{1}{\theta}} + (\alpha_{med,1})^{\frac{1}{\theta}}}\right)^{1-\theta}$$

The first-order condition with respect to debt B is:

$$\begin{aligned}(W+B)^{-\theta}\,\Omega_1 &= (W-B)^{-\theta}\,\Omega_2\\ \frac{W+B}{W-B} &= \left(\frac{\Omega_1}{\Omega_2}\right)^{1/\theta}\end{aligned}$$

We have a deficit in the first period ($B > 0$) if $\Omega_1 > \Omega_2$. We can show numerically[1] that this is the case if $\theta < 1$. There is no deficit and debt if we have a log utility ($\theta = 1$) or there is no heterogeneity in preferences.

3 Delayed reforms

Consider that there are two agents, 1 and 2. The economy would benefit from a reform that will increase the income of agent 1 by R_1 and the income of agent 2 by R_2. While R_1 is known to everybody, R_2 is random and uniformly distributed on an interval $[R_{Low}, R_{High}]$. Only agent 2 know the actual value of R_2.

Undertaking the reform requires a cost T. This cost as to be split between the two agents in the amounts T_1 and T_2. We assume that $R_1 > T$ so that agent 1 could afford to pay for the whole cost. Undertaking the reform is then efficient.

Both agents have to agree to undertake the reform. Agent 1 proposes a tax split T_1 and T_2 to agent 2. If agent 2 accepts the reform is done, and nothing happens otherwise. Agent 2 accepts the reform if $R_2 \geq T_2$. This is for sure the case if $T_2 \leq R_{Low}$, and will never be the case if $T_2 > R_{High}$. If T_2 is between R_{Low} and R_{High} probability of acceptance is:

$$P_{accept} = \frac{R_{High} - T_2}{R_{High} - R_{Low}}$$

The expected payoff of agent 1 is:

$$\begin{aligned}U &= P_{accept}\,(R_1 - T_1)\\ U &= \frac{R_{High} - T_2}{R_{High} - R_{Low}}\,(R_1 - T + T_2)\end{aligned}$$

[1]The simplest way is to set two groups with α_{High} and α_{Low} symmetric around 0.5 (so $\alpha_{Low} = 1 - \alpha_{High}$) and set the probability that α_{High} is the median voter in period 2 to one-half.

The first and second derivatives with respect to T_2 are:

$$\begin{aligned} U' &= \frac{R_{High} - R_1 + T - 2T_2}{R_{High} - R_{Low}} \\ U'' &= \frac{-2}{R_{High} - R_{Low}} < 0 \end{aligned}$$

If the parameters are such that $U' < 0$ when $T_2 = R_{Low}$, agent 1 sets $T_2 = R_{Low}$ to ensure acceptance. Setting it lower will make acceptance impossible, and setting it higher would reduce the expected utility as U is concave.

If $U' > 0$ when $T_2 = R_{Low}$ we have an interior solution where T_2 is set so that $U' = 0$:

$$T_2 = \frac{R_{High} - R_1 + T}{2}$$

The probability of acceptance is:

$$P_{accept} = \frac{R_{High} - T_2}{R_{High} - R_{Low}} = \frac{1}{2} \frac{R_{High} + R_1 - T}{R_{High} - R_{Low}}$$

This is between 0 and 1 so reform may fail.