Exercise 1

(a) Write down the maximization problem

$$\mathcal{L} = U(C_1^T, C_1^N) + \beta(U(C_2^T, C_2^N)) -$$

$$\lambda_1 (C_1^T + p_1 C_1^N - Y_1^T - p_1 Y_1^N + B_2) -$$

$$\beta \lambda_2 (C_2^T + p_2 C_2^N - Y_2^T - p_2 Y_2^N - (1+r) B_2)$$

$$\frac{\partial \mathcal{L}}{\partial C_1^T} = 0 \Longrightarrow U_{C^T}(C_1^T, C_1^N) = \lambda_1$$

$$\frac{\partial \mathcal{L}}{\partial C_2^T} = 0 \Longrightarrow U_{C^T}(C_2^T, C_2^N) = \lambda_2$$

$$\frac{\partial \mathcal{L}}{\partial C_1^N} = 0 \Longrightarrow U_{C^N}(C_1^T, C_1^N) = p_1 \lambda_1$$

$$\frac{\partial \mathcal{L}}{\partial C_2^N} = 0 \Longrightarrow U_{C^N}(C_2^T, C_2^N) = p_2 \lambda_2$$

$$\frac{\partial \mathcal{L}}{\partial B_2} = 0 \Longrightarrow \lambda_1 = \lambda_2 \beta (1+r)$$

Combine expressions:

$$p_1 = \frac{U_{C^N}(C_1^T, C_1^N)}{U_{C^T}(C_1^T, C_1^N)} \qquad p_2 = \frac{U_{C^N}(C_2^T, C_2^N)}{U_{C^T}(C_2^T, C_2^N)}$$

$$U_{C^T}(C_1^T, C_1^N) = \beta(1+r) U_{C^T}(C_2^T, C_2^N)$$

$$U_{C^N}(C_1^T, C_1^N) = \frac{p_1}{p_2} \beta(1+r) U_{C^N}(C_2^T, C_2^N)$$

U'_{C^N,1} = p1/p2 * beta *(1+r) * U'_{C^N,2}

(b) profit maximization by firms:

$$\pi_t^T = a_t^T L_t^T - w_t L_t^T$$

$$\pi_t^N = p_t a_t^N L_t^N - w_t L_t^N$$

Note: wages for the economy are equal

$$\frac{\partial \pi_t^T}{\partial L_t^T} = 0 \Rightarrow w_t = a_t^T$$

$$\frac{\partial \pi_t^N}{\partial L_t^N} = 0 \Rightarrow w_t = p_t a_t^N$$

$$\boxed{p^{-\alpha} = Q}$$

$$p_t = \frac{a_t^T}{a_t^N} \quad \text{(general solution)}$$

$$\boxed{\text{RER} \quad Q_t = \frac{1}{p_t}}$$

$$p_1 = \frac{a_1^T}{a_1^N} \qquad p_2 = \frac{a_2^T}{a_2^N}$$

we have that $a_1^N = a_2^N = a_1^T = a_2^T = 1$:

(c) $\underline{p_1 = 1}$ $\qquad$ $\underline{p_2 = 1}$

Let's go back to the Euler equations

Since ~~that~~ we have log utility:

FOCs:

$$p_1 = \frac{c_1^T}{c_1^N} \Rightarrow c_1^T = c_1^N$$

$$p_2 = \frac{c_2^T}{c_2^N} \Rightarrow c_2^T = c_2^N$$

Also, we have that

Euler eqns: $\dfrac{C_2^T}{C_1^T} = \beta(1+r) = 1$

$$\frac{C_2^N}{C_1^N} = \frac{p_1}{p_2}(\beta(1+r)) = 1$$

therefore: $C_2^T = C_1^T$

$$C_2^N = C_1^N$$

We also know that $C_t^N = Y_t^N$

we can write the budget constraint as follows:

$$C_1^T = Y_1^T - B_2 \qquad (\text{since } p_1 C_1 = p_1 Y_1)$$

$$C_2^T = Y_2^T + (1+r)B_2$$

Combining, we get the CA

$$Y_1^T - B_2 = Y_2^T + (1+r)B_2 \qquad (\text{since } C_1^T = C_2^T$$

$$B_2 = \frac{Y_1^T - Y_2^T}{2+r}$$

$$= \frac{a_1^T C_1^T - a_2^T C_2^T}{2+r} = \frac{C_1^T - C_2^T}{2+r} \qquad (1)$$

To solve for ~~[illegible]~~, labor,

we know that

$$C_t^T = C_t - C_t^N$$

we also know that

$$C_t^N = Y_t^N = a_t^N L_t^N$$

Since $C_1^N = C_2^N$ and $a_1^N = a_2^N = 1$

we know that $C_t^N = C^N$

if C is fixed, so is C^T

therefore

$$\underline{B_2 = 0}$$ (see !)

this implies

$$C_1^T = Y_1^T$$

also from the FOCs

$$C_t^T = C_t^N \Rightarrow$$

$$Y_t^T = Y_t^N \Rightarrow C^T = C^N$$

L^T = 1/2 L $\quad L^T = 1/2 L \qquad L^N = 1/2 L$ L^N = 1/2 L therefore

$$C_1^T = C_1^N = C_2^T = C_2^N = Y_1^T = Y_1^N = Y_2^T = Y_2^T = Y_2^N = \tfrac{1}{2} L$$

(d) We know the CA is

$$B_2 = \frac{Y_1^T - Y_2^T}{2+r} = \frac{C_1^T - C_2^T}{2+r}$$

since $C_1^N = C_2^N \implies L_1^N = L_2^N$

and $L_2 = k L_1$

$$C_1^T = L_1 - C_1^N$$

$$C_2^T = k L_1 - C_1^N$$

$$B_2 = \frac{L_1 - k L_1}{2+r} = \frac{(1-k) L_1}{2+r}$$