Macroeconomics A; EI056

Short problems

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1 IS-LM model under perfect information

Question: The economy is described by three relations, that we put in simplified form abstracting from terms that are not central.

The first relation is the IS curve that summarizes the equilibrium of the market for goods where output y is inversely related to the interest rate i, and subject to a goods demand shock u:

$$y = -\alpha i + u \tag{1}$$

The second relation is the demand for money. It links the quantity of money m used by private agents to output y, the interest rate i and a money demand shock v:

$$m = y - ci + v \tag{2}$$

The final relation links the money m used by private agents to the quantity of money issued by the central bank b, the interest rate i and a shock ω to the transmission between central bank money and the quantity used by agents:

$$m = b + hi + \omega \tag{3}$$

You can think of all three relations as written in terms of deviation from a steady state. The shock are of expected value zero and uncorrelated: $Eu = Ev = E\omega = 0$ and $E(uv) = E(u\omega) = E(v\omega) = 0$.

Show that the solution is:

$$\begin{aligned} y &= \frac{1}{\alpha + c + h}\left[\alpha b + \alpha(\omega - v) + (c + h)u\right] \\ i &= \frac{1}{\alpha + c + h}\left[-b - (\omega - v) + u\right] \\ m &= \frac{1}{\alpha + c + h}\left[(\alpha + c)b + h(v + u) + (\alpha + c)\omega\right] \end{aligned}$$

Consider that the central bank can set b after observing the shocks. What is the monetary policy that stabilizes output (i.e. reaches $y = 0$)?

Answer: Because of (3), the central bank does not directly set m which is now an endogenous variable.

We first combine (2) and (3) to get the interest rate as a function of the central bank money:

$$\begin{aligned}
y - ci + v &= b + hi + \omega \\
-i\left(c + h\right) &= b - y + \omega - v \\
i &= \frac{1}{c+h} y - \frac{b + \omega - v}{c+h}
\end{aligned}$$

We next put this into (1) to solve for output:

$$\begin{aligned}
y &= -\alpha i + u \\
y &= -\frac{\alpha}{c+h} y + \alpha \frac{b + \omega - v}{c+h} + u \\
y \frac{\alpha + c + h}{c+h} &= \alpha \frac{b + \omega - v}{c+h} + u \\
y &= \frac{1}{\alpha + c + h} \left[\alpha b + \alpha\left(\omega - v\right) + \left(c+h\right) u\right]
\end{aligned}$$

The interest rate is then:

$$\begin{aligned}
i &= \frac{1}{c+h} y - \frac{b + \omega - v}{c+h} \\
i &= \frac{1}{c+h} \frac{\alpha}{\alpha + c + h} b + \frac{1}{c+h} \frac{\alpha}{\alpha + c + h} \left(\omega - v\right) + \frac{1}{\alpha + c + h} u - \frac{b + \omega - v}{c+h} \\
i &= \frac{1}{c+h} \left(\frac{\alpha}{\alpha + c + h} - 1\right) b + \frac{1}{c+h} \left(\frac{\alpha}{\alpha + c + h} - 1\right) \left(\omega - v\right) + \frac{1}{\alpha + c + h} u \\
i &= \frac{1}{\alpha + c + h} \left[-b - \left(\omega - v\right) + u\right]
\end{aligned}$$

Finally, (2) or (3) given the money quantity:

$$\begin{aligned}
m &= b + hi + \omega \\
m &= b + \frac{h}{\alpha + c + h} \left[-b - \left(\omega - v\right) + u\right] + \omega \\
m &= \frac{\alpha + c}{\alpha + c + h} b + \frac{h}{\alpha + c + h} \left[v + u\right] + \frac{\alpha + c}{\alpha + c + h} \omega \\
m &= \frac{1}{\alpha + c + h} \left[\left(\alpha + c\right) b + h\left(v + u\right) + \left(\alpha + c\right) \omega\right]
\end{aligned}$$

If the central bank wants to stabilize output, it sets $y = 0$, hence:

$$\begin{aligned}
0 &= \alpha b + \alpha\left(\omega - v\right) + \left(c+h\right) u \\
-\alpha b &= \alpha\left(\omega - v\right) + \left(c+h\right) u \\
b &= -\omega + v - \left(\frac{c+h}{\alpha}\right) u
\end{aligned}$$

The central bank tightens monetary policy ($b < 0$) when the transmission to the larger money aggregate gets stronger ($\omega > 0$), when there is a decrease in money demand ($v < 0$) or when there is an expansionary demand shock ($u > 0$).

2 Policy under imperfect information

Question: Now consider that the central bank cannot observe which shock is realized. All it knows are the variances, that is $E\left(u^2\right)$, $E\left(v^2\right)$ and $E\left(\omega^2\right)$.

The central bank has to set a rule ex-ante. It has two options:

1. Stabilizes the money quantity that it controls, that is set $b = 0$ no matter what. This is monetary targeting.

2. Stabilizes the interest rate, that is set $i = 0$ no matter what. This is interest rate targeting.

Show that output volatility, $E\left(y^2\right)$ is:

$$E\left(y^2\right) = \begin{cases} \left(\frac{1}{\alpha+c+h}\right)^2 \left[\alpha^2\left(E\left(\omega^2\right) + E\left(v^2\right)\right) + (c+h)^2 E\left(u^2\right)\right] & \text{under monetary targeting} \\ E\left(u^2\right) & \text{under interest rate targeting} \end{cases}$$

What is the best policy? explain the intuition

Answer: Under monetary targeting, (3) implies that:

$$m = hi + \omega$$

Our solution of the previous points gives all three endogenous variables:

$$\begin{aligned} y &= \frac{1}{\alpha+c+h}\left[\alpha(\omega - v) + (c+h)u\right] \\ i &= \frac{1}{\alpha+c+h} - (\omega - v) + u \\ m &= \frac{1}{\alpha+c+h}\left[h(v+u) + (\alpha+c)\omega\right] \end{aligned}$$

The variance of output is then (recall that $E(uv) = E(u\omega) = E(v\omega) = 0$):

$$\begin{aligned} E\left(y^2\right) &= \left(\frac{1}{\alpha+c+h}\right)^2 E\left[\alpha\omega - \alpha v + (c+h)u\right]^2 \\ E\left(y^2\right) &= \left(\frac{1}{\alpha+c+h}\right)^2 E\left[\alpha^2\omega^2 + \alpha^2 v^2 + (c+h)^2 u^2\right] \\ &\quad + \left(\frac{1}{\alpha+c+h}\right)^2 E\left[-2\alpha^2\omega v + 2\alpha(c+h)\omega u - 2\alpha(c+h)uv\right] \\ E\left(y^2\right) &= \left(\frac{1}{\alpha+c+h}\right)^2 \left[\alpha^2 E\left(\omega^2\right) + \alpha^2 E\left(v^2\right) + (c+h)^2 E\left(u^2\right)\right] \\ &\quad + \left(\frac{1}{\alpha+c+h}\right)^2 \left[-2\alpha^2 E(\omega v) + 2\alpha(c+h)E(\omega u) - 2\alpha(c+h)E(uv)\right] \\ E\left(y^2\right) &= \left(\frac{1}{\alpha+c+h}\right)^2 \left[\alpha^2\left(E\left(\omega^2\right) + E\left(v^2\right)\right) + (c+h)^2 E\left(u^2\right)\right] \end{aligned}$$

Under interest rate targeting (1) implies:

$$y = u$$

The variance is then simply:

$$E\left(y^2\right) = E\left(u^2\right)$$

The best policy depends on the source of the shock. Interest rate targeting is better if:

$$\begin{aligned}
E\left(u^2\right) &< \left(\frac{1}{\alpha+c+h}\right)^2\left[\alpha^2\left(E\left(\omega^2\right)+E\left(v^2\right)\right)+(c+h)^2 E\left(u^2\right)\right] \\
\left(1-\left(\frac{c+h}{\alpha+c+h}\right)^2\right) E\left(u^2\right) &< \left(\frac{\alpha}{\alpha+c+h}\right)^2\left(E\left(\omega^2\right)+E\left(v^2\right)\right) \\
\left((\alpha+c+h)^2-(c+h)^2\right) E\left(u^2\right) &< \alpha^2\left(E\left(\omega^2\right)+E\left(v^2\right)\right) \\
\left(\alpha^2+(c+h)^2+2\alpha(c+h)-(c+h)^2\right) E\left(u^2\right) &< \alpha^2\left(E\left(\omega^2\right)+E\left(v^2\right)\right) \\
E\left(u^2\right) &< \frac{1}{1+2\frac{c+h}{\alpha}}\left(E\left(\omega^2\right)+E\left(v^2\right)\right)
\end{aligned}$$

Interest rate targeting is preferable if shocks from the real side (market for goods) are less volatile that shocks from the monetary side, i.e. $E\left(u^2\right)$ is low relative to $E\left(\omega^2\right)+E\left(v^2\right)$. This is because under interest rate targeting, output is fully insulated from money shocks. The cost of this insulation is that output is fully exposed to real shocks. Under monetary targeting, output is partially protected from real shocks, but at the cost of being exposed to money shocks. Therefore, if real shocks are moderate, the full exposure to them is a price worth paying to gain the insulation, under interest rate targeting, from money shocks.

The choice is also affected by the impact of interest rate on the monetary side $(c+h)$ relative to the one on the real side (α). If the interest rate has a relatively strong effect on the demand for good (that is $(c+h)/\alpha$ is low), the coefficient in from of $E\left(\omega^2\right)+E\left(v^2\right)$ is high. In other words, interest rate targeting is best even in real shocks are high: the threshold that $E\left(u^2\right)$ must reach for monetary targeting to make sense is higher. Intuitively, when interest rate movements have a strong effect on output, limiting them is particularly important.