Macroeconomics A
Problem Set 1

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1 The AK Model

Consider the production function $Y = AK + BL$, where A and B are positive constants.

a. Is this production function neoclassical? Which of the neoclassical conditions does it satisfy and which one does it not?

b. Write output per person as a function of capital per person. What is the marginal product of k? What is the average product of k?

In what follows, we assume that population grows at the constant rate n and that capital depreciates at the constant rate δ.

c. Write down the fundamental equation of the Solow model.

d. Under what conditions does this model have a steady state with no growth of per capita capital, and under what conditions does the model display endogenous growth?

e. In the case of endogenous growth, how does the growth rate of the capital stock behave over time (that is, does it increase or decrease)? What about the growth rates of output and consumption per capita?

f. If $s = 0.4, A = 1, B = 2, \delta = 0.08$, and $n = 0.02$, what is the long-run growth rate of this economy? What if $B = 5$? Explain the differences.

2 Government in the Solow Model

We introduce government spending in the basic Solow model. Consider the model without technical change and suppose that

$$Y(t) = C(t) + I(t) + G(t)$$

with $G(t)$ denoting government spending at time t. Imagine that government spending is given by $G(t) = \sigma Y(t)$.

a. Discuss how the relationship between income and consumption should be changed. Isit reasonable to assume that $c(t) = cY(t)$, where $c = 1 - s$ from the lectures?

b. Suppose that government spending partly comes out of private consumption, so that $C(t) = (c - \lambda\sigma)Y(t)$, where $\lambda \in [0, 1]$. What is the effect of higher government spending (in the form of higher σ) on the equilibrium of the Solow model?

c. Now suppose that a fraction ϕ of $G(t)$ is invested in the capital stock, so that total investment at time t is given by

$$I(t) = (1 - c - (1 - \lambda)\sigma + \phi\sigma)Y(t).$$

d. Show that if ϕ is sufficiently high, the steady-state level of capital-labor ratio will increase as a result of higher government spending (corresponding to higher σ). Is this reasonable? How would you alternatively introduce public investments in this model?

3 Pollution and the Environmental Kuznets Curve

Consider a version of the standard Solow model: output is

$$Y_t = K_t^{\alpha} L^{1-\alpha}$$

and labor grows at rate n, $L_{t+1} = (1 + n)L_t$. Suppose that each unit of output produced generates Ω_t units of pollution, and that due to exogenous technological progress in pollution abatement, Ω_t decreases at rate g_a, i.e.

$$\Omega_{t+1}/\Omega_t = 1/(1 + g_a).$$

In addition, suppose that there is an "abatement technology" that allows resources to be diverted into pollution reduction. Specifically, if θ represents the share of output used in pollution reduction, then net pollution emissions E_t is given by

$$E_t = a(\theta)\Omega_t Y_t$$

where $a(\theta)$ is assumed to be a positive, decreasing function. For simplicity assume that θ is constant and exogenous.

Let y_t denote net output available for consumption and investment, divided by population:

$$y_t = \frac{(1 - \theta)Y}{L}$$

and similarly let k and e be capital and emissions per capita. We then obtain the following model:

$$\begin{aligned} y_t &= (1-\theta)k_t^{\alpha} \\ \Delta k_{t+1} &= s(1-\theta)k_t^{\alpha} - (\delta + n)k_t \\ e_t &= a(\theta)\Omega_t k_t^{\alpha} \end{aligned}$$

where δ is the depreciation rate of capital.

a. What happens to emissions E in the steady state?

b. It is often claimed that the time path of pollution within economies follows a so-called "Environmental Kuznets Curve", with pollution initially rising and then eventually falling (i.e. it traces out an inverted U-shape). Consider an economy where in the long term the growth rate of E is negative. Under what conditions will this economy feature an Environmental Kuznets Curve? (Note: for this exercise you can approximate the growth rate of a product of two variables as the sum of the growth rates of the two variables. This relationship holds exactly when setting the model in continuous time.)

c. How does a temporary increase in abatement effort θ affect the time path of pollution? Explain intuitively.