

Trade under Imperfect Competition

YUAN ZI¹

¹Graduate Institute of International Studies (yuan.zi@graduateinstitute.ch)

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Introduction

- A large (and growing) part of world trade (1) occurs between very similar countries and (2) takes place intra-industry
- Neoclassical theories unable to explain these facts
- Need for new theories with
 - ① Product differentiation (which also implies...)
 - ② Imperfect competition
 - ③ Increasing returns

The Krugman (1980) model

Basic Ingredients

- CES preferences (love of variety)
- Monopolistic competition
- Increasing returns to scale (IRS) due to fixed production cost
- Trade cost (iceberg)
- Note that IRS is equivalent to $AC > MC$, which implies imperfect competition

The Krugman (1980) Model

Predictions

- Trade Lindt for Ferrero story
- Two identical countries, from autarky to trade:
 - GDP does not change
 - Wages do not change
 - Both enjoy double varieties → gains from trade!

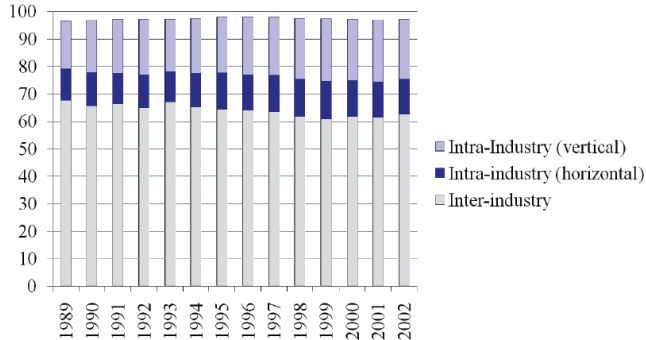
This Lecture

- Intra-industry trade: definition and general data patterns (Grubel and Lloyd, 1975; Brühlhart, 2009)
- Early empirical test of the Krugman model (Head and Ries, 1999)
- Model details are provided in the Appendix

Intra-industry Trade

Inter- or intra-industry trade?

Decomposition of trade (% total)



Source : Fontagné L., Freudenberg M., Gaulier G. (2006). Definitions : Intra-industry trade is identified as simultaneous exports and imports within the same industry. Distinction of vertical and horizontal relies on price differences.

Inter- or intra-industry trade?

Top-10 country pairs (% of bilateral trade, 2000)

Top total IIT shares (per cent)		
Germany	France	86.20
Netherlands	Belgium and Luxembourg	85.01
France	Belgium and Luxembourg	80.42
France	United Kingdom	77.08
Germany	Switzerland	76.99
Germany	Belgium and Luxembourg	76.83
Austria	Germany	76.63
France	Spain	76.55
Germany	Netherlands	76.01
Canada	United States	73.55

Source : Fontagné, Freudenberg & Gaulier (2006)

Intra-industry trade

- How to quantify intra-industry trade?
- Grubel and Lloyd (1975) propose the following index:

$$GL_{ijkt} = 1 - \frac{|X_{ijkt} - M_{ijkt}|}{X_{ijkt} + M_{ijkt}}$$

- Measures for given countries i and j , the proportion of non-overlapping trade flows in total bilateral trade of good k
- Ranges from 0 (perfect inter-industry trade) to 1 (perfect intra-industry trade)
- Issue: what is a “good”?

Intra-industry trade

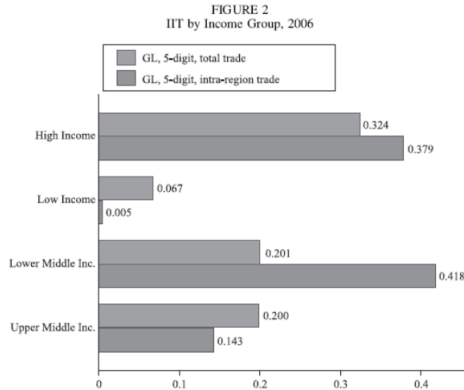
- Brulhart (2009)
- UN-COMTRADE, 1962-2005
- SITC Rev. 1 / 5 digit
- 1,161 products, up to 214 countries

Intra-industry trade

Richer countries do more IIT

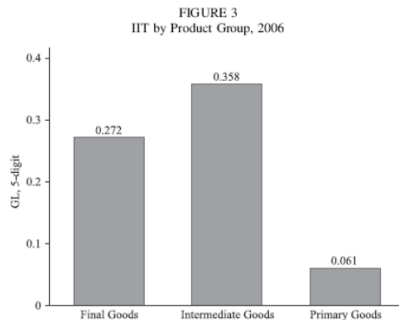
IIT between low income countries is almost inexistent

IIT between middle income countries is large: probably due to processing trade within vertically fragmented industries.



Intra-industry trade

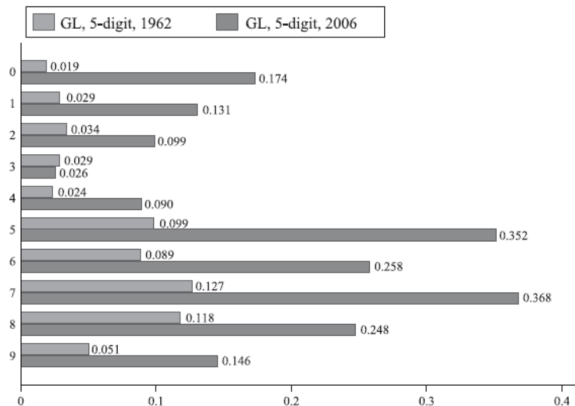
IIT concerns mainly intermediate goods (processing trade)
and a priori sophisticated
and differentiated
goods.



Intra-industry trade

IIT concerns mainly sophisticated and differentiated goods.

FIGURE 6
Global IIT by SITC 1-Digit Sector, 1962 and 2006



Notes:

'wide coverage' dataset; SITC 1-digit sectors: 0 – Food and Live Animals, 1 – Beverages and Tobacco, 2 – Crude Materials Excluding Fuels, 3 – Mineral Fuels Etc., 4 – Animal & Vegetable Oils & Fats, 5 – Chemicals, 6 – Basic Manufactures, 7 – Machines & Transport Equipment, 8 – Misc. Manufactures, 9 – Goods Not Classified by Kind.

Intra-industry trade

- IIT is larger between **richer** countries

TABLE 4
Cross-Country Determinants of IIT, 1965, 1990 and 2006
(Dependent variable = log transformed GL index, estimation by OLS)

	1965				1990				2006			
	<i>All Sectors</i>	<i>Primary</i>	<i>Intermed.</i>	<i>Final</i>	<i>All Sectors</i>	<i>Primary</i>	<i>Intermed.</i>	<i>Final</i>	<i>All Sectors</i>	<i>Primary</i>	<i>Intermed.</i>	<i>Final</i>
<i>log mean per-cap. GDP</i>	1.753*** (0.09)	1.322*** (0.11)	1.944*** (0.11)	1.854*** (0.12)	2.193*** (0.09)	1.855*** (0.10)	2.378*** (0.10)	2.045*** (0.10)	1.617*** (0.08)	1.534*** (0.10)	1.918*** (0.08)	1.513*** (0.08)
<i>log diff per-cap. GDP</i>	-0.0811 (0.08)	0.018 (0.09)	-0.133 (0.11)	-0.210** (0.09)	0.0890 (0.08)	0.00854 (0.08)	0.140* (0.08)	-0.132 (0.09)	0.0444 (0.07)	-0.097 (0.09)	0.189*** (0.07)	-0.0668 (0.07)
<i>log distance</i>	-1.464*** (0.10)	-1.092*** (0.11)	-1.231*** (0.11)	-1.754*** (0.11)	-1.163*** (0.10)	-1.019*** (0.10)	-1.021*** (0.11)	-1.285*** (0.11)	-0.700*** (0.09)	-1.161*** (0.11)	-0.622*** (0.09)	-0.923*** (0.09)
<i>contiguity</i>	1.330*** (0.47)	1.827*** (0.50)	1.464*** (0.51)	0.890* (0.53)	1.486*** (0.48)	1.801*** (0.50)	1.812*** (0.51)	0.969* (0.52)	1.571*** (0.41)	1.672*** (0.53)	2.006*** (0.45)	1.327*** (0.44)
<i>constant</i>	-9.555*** (1.23)	-10.500*** (1.35)	-13.500*** (1.35)	-7.902*** (1.43)	-14.730*** (1.26)	-15.180*** (1.34)	-17.591*** (1.36)	-12.263*** (1.40)	-12.570*** (1.12)	-10.361*** (1.44)	-16.150*** (1.21)	-9.665*** (1.20)
Observations	1,196	1,090	1,101	1,069	1,411	1,340	1,373	1,354	1,375	1,354	1,374	1,373
R ²	0.41	0.27	0.37	0.39	0.41	0.32	0.39	0.36	0.33	0.28	0.34	0.31

***, ** and * indicate statistical significance at the 1 per cent, 5 per cent and 10 per cent levels, respectively. Numbers in parentheses are standard errors.

Intra-industry trade

- IIT is (not really) larger between **more similar** countries

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Intra-industry trade

- IIT is larger between **closer** countries

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Empirical Evidence

Head and Ries (1999)

Overview

- Head and Ries (1999), "Rationalization Effects of Tariff Reductions", *Journal of International Economics*
- Focuses on model selection rather than directly testing the Krugman model
- Confirms economies of scale; does not address gains from trade (increased variety)
- Among the first papers to use firm-level regressions

Head and Ries (1999)

Model Comparison

Table 1
Predicted effects of tariffs on output per plant (q) and the number of plants (n)

Main Assumptions of Model (Authors)	Canadian Tariffs			US Tariffs		
	Fixed n, n^* Δq	Free Entry Δq	Δn	Fixed n, n^* Δq	Free Entry Δq	Δn
Segmented-markets Cournot (Venables, 1985)	+	+	+	—	—	—
Unified-markets Cournot (Horstmann and Markusen, 1986)	NA	0	+	NA	+	—
Monopolistic competition (Helpman and Krugman, 1985)	+	0	+	—	0	—
Tariff-limit pricing (Cox and Harris, 1985 Muller and Rawana, 1990)	—	—	+	NA	NA	NA

Head and Ries (1999)

Data and Specification

- Exploiting the US-Canada Free Trade Agreement (1988)
- 230 Canadian manufacturing industries, 1981 and 1994
- They use the following specification:

$$\ln y_{it} = \alpha_i + \beta_t + \gamma_t \tau_{it} + \gamma_t \tau_{it} + \epsilon_{it} \quad (1)$$

where y_{it} is either output per establishment (q_{it}) or the number of establishments (n_{it}), α_i are industry fixed effects, β_t are year effects, and τ_{it} are industry tariffs.

Head and Ries (1999)

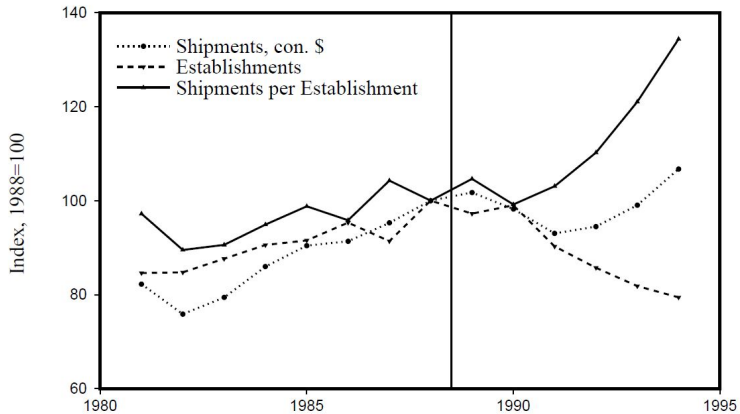


Fig. 1. Scale of Canadian manufacturing, 1981–1994.

Head and Ries (1999)

Table 3
Effects of tariffs on log output per plant ($\ln q$)

	Sample:				
	All	Imp. Com.	IC + Free	IC + Fixed	All
Canadian Tariff	1.134 ^a (0.368)	1.247 ^a (0.411)	0.279 (0.455)	3.824 ^a (0.925)	4.928 ^a (1.135)
U.S. Tariff	-1.638 ^a (0.596)	-2.227 ^a (0.716)	-0.937 (0.828)	-5.632 ^a (1.403)	-6.371 ^a (2.078)
Cdn. Tariff × Turnover					-17.952 ^a (5.489)
U.S. Tariff × Turnover					20.131 ^c (10.289)
1994	0.179 ^a (0.020)	0.172 ^a (0.022)	0.117 ^a (0.025)	0.301 ^a (0.040)	0.186 ^a (0.021)
R^2 (within)	0.175	0.173	0.129	0.338	0.191
Root MSE	0.149	0.152	0.149	0.154	0.149
No. of Obs.	1828	1628	1183	445	1693

Note: Fixed industry year effects are not reported except for 1994 which approximates the percent change from 1988. Standard errors in parentheses. ^a, ^b, ^c indicate significance in a two-tail test at the 1, 5 and 10 percent levels.

- Results on output are consistent with the Krugman model

Head and Ries (1999)

Table 4
Effects of tariffs on log # of plants ($\ln n$)

	Sample: All	Imp. Com.	IC + Free	IC + Fixed	All
Canadian Tariff	1.352 ^a (0.264)	1.629 ^a (0.286)	1.957 ^a (0.305)	-0.384 (0.719)	-2.015 ^b (0.783)
U.S. Tariff	1.218 ^a (0.428)	0.953 ^c (0.499)	1.143 ^b (0.554)	1.781 (1.090)	2.579 ^c (1.433)
Cdn. Tariff × Turnover					14.634 ^a (3.786)
U.S. Tariff × Turnover					-2.195 (7.097)
1994	-0.111 ^a (0.014)	-0.099 ^a (0.015)	-0.087 ^a (0.017)	-0.14 ^a (0.031)	-0.142 ^a (0.014)
R ² (within)	0.438	0.436	0.506	0.290	0.498
Root MSE	0.107	0.106	0.100	0.119	0.103
No. of Obs.	1828	1628	1183	445	1693

Note: Fixed industry and year effects are not reported except for 1994 which approximates the percent change from 1988. Standard errors in parentheses. ^a, ^b, ^c indicate significance in a two-tail test at the 1, 5 and 10 percent levels.

- But strange results on the number of plants

Conclusion

- Problems of the Krugman model: homogeneous firms, factors are immobile across countries
- Next: economic geography and heterogeneous firms to relax these assumptions

Appendix: The Krugman model

The Krugman (1980) model: ingredients

- Increasing returns to scale (IRS) due to fixed production cost
- Monopolistic competition
- Trade cost (iceberg)
- CES preferences
- Note that IRS is equivalent to $AC > MC$, which implies imperfect competition

The Krugman (1980) model: demand side

- One factor, one sector which include a potentially infinite number of varieties of differentiated products ($\omega \in \Omega$)
- **CES demand:** $\max_{q(\omega)} U = \max_{q(\omega)} \left(\int_{\Omega} q(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}}$
- $\sigma > 1$ is the elasticity of substitution between varieties
- **Budget constraint:** $\int_{\Omega} p(\omega) q(\omega) d\omega = wL$
(no K, no profit in equilibrium)
- Yields **demand function:** $q(\omega) = \left(\frac{p(\omega)}{P} \right)^{-\sigma} \frac{wL}{P}$
where $P = \left(\int_{\Omega} p(\omega)^{1-\sigma} d\omega \right)^{\frac{1}{1-\sigma}}$ [► Proof](#)

The Krugman (1980) model: price index and utility

- Interpretation of P : ideal price index
- Can show that with $P = \left(\int_{\Omega} p(\omega)^{1-\sigma} d\omega \right)^{\frac{1}{1-\sigma}}$, $U = \frac{wL}{P}$
- Implies that the utility of the real income is the same regardless of the general level of prices
- We can show that there is preference for diversity: utility increases with the number of varieties
- For a given wL , P varies inversely with utility. P is the price of a unit of utility
- Trade raises utility through an increase in product diversity

The Krugman (1980) model: supply side

- Each firm is a monopolist on a given variety (why?)
- **Fixed cost.** Cost function is $l(q(\omega)) = F + \frac{q(\omega)}{\varphi}$
- **Optimal price.** $p = \frac{\sigma}{\sigma-1} \frac{w}{\varphi}$ (constant mark-up) [▶ Proof](#)
- **Profit.** $\pi(\omega) \equiv p(\omega)q(\omega) - w(F + \frac{q(\omega)}{\varphi}) = w(\frac{q(\omega)}{\varphi(\sigma-1)} - F)$
- **Free entry.** $\pi = 0 \Rightarrow q(\omega) = (\sigma - 1)\varphi F$
(all firms produce the same quantity at the same price)
- **Number of firms.** Solve $L = n(F + \frac{q}{\varphi}) \Rightarrow n = \frac{L}{\sigma F}$
(interpretation?)

The Krugman (1980) model: supply side

- Recall optimal price. $p = \frac{\sigma}{\sigma-1} \frac{w}{\varphi}$ (constant mark-up)
- Plugging into P we can show that $P = \frac{\sigma}{\sigma-1} \frac{w}{\varphi} n^{\frac{1}{1-\sigma}} = \frac{\sigma}{\sigma-1} \frac{w}{\varphi} \left(\frac{L}{\sigma F}\right)^{\frac{1}{1-\sigma}}$
- Lower price index, and therefore higher welfare, in larger economies
- What happens under trade? (so far, this is Dixit-Stiglitz 77)

The Krugman (1980) model: trade

- Assume two countries, identical in everything but their size (L, L^*)
- Assume **iceberg trade costs**, $\tau > 1$
- **Price on the foreign market** is $p^X = \tau \frac{\sigma}{\sigma-1} \frac{w}{\psi} = \tau p$
- Note that domestic price is the same in both markets (why?). Also, there is complete tariff pass-through

The Krugman (1980) model: trade

- Total production. $q = q^D + \tau q^X$
- Total profit. $\pi = pq - w(F + \frac{q}{\psi}) = \frac{wq}{\psi(\sigma-1)} - wF$
- Free entry. $\pi = 0 \Rightarrow q = (\sigma - 1)\psi F$
- Number of firms. n such that $n(F + \frac{q}{\psi}) = L \Rightarrow n = \frac{L}{\sigma F}$
- Comparative static of equilibrium q and n , and the intuition?
- No change in price, no change in output per firm, no change in number of firms. Why?

The Krugman (1980) model: trade and welfare

- Autarky:

$$P = pn^{\frac{1}{1-\sigma}} \quad \text{and} \quad P^* = p^* n^{\frac{1}{1-\sigma}}$$

- Open to trade:

$$P = (p^{1-\sigma} n + (\tau p^*)^{1-\sigma} n^*)^{\frac{1}{1-\sigma}} \quad \text{and} \quad P^* = ((\tau p)^{1-\sigma} n + (p^*)^{1-\sigma} n^*)^{\frac{1}{1-\sigma}}$$

- If trade is costless and countries are symmetric:

$$P = p^* = (2np^{1-\sigma})^{\frac{1}{1-\sigma}} < (np^{1-\sigma})^{\frac{1}{1-\sigma}},$$

as $\sigma > 1$

→ Welfare gains due to diversity

Appendix: derivation of the demand function (1/2)

- Lagrangian: $L = (\int_{\Omega} q(\omega)^{\frac{\sigma-1}{\sigma}} d\omega)^{\frac{\sigma}{\sigma-1}} - \mu(\int_{\Omega} p(\omega)q(\omega) - wL)$

- First order conditions:

$$\frac{\partial L}{\partial q(\omega)} = q(\omega)^{\frac{-1}{\sigma}} (\int_{\Omega} q(\omega)^{\frac{\sigma-1}{\sigma}} d\omega)^{1/(\sigma-1)} - \mu p(\omega) = 0$$

$$\Leftrightarrow q(\omega)^{\frac{-1}{\sigma}} U^{\frac{1}{\sigma}} = \mu p(\omega)$$

$$\Leftrightarrow p(\omega)q(\omega) = U\mu^{-\sigma}p(\omega)^{1-\sigma}$$

- Now take the ratio of demands for two distinct varieties:

$$\frac{q(\omega)}{q(\omega')} = \left(\frac{p(\omega)}{p(\omega')}\right)^{-\sigma} \quad (\text{A})$$

→ relative consumption is a function of relative price.

- σ is the (constant) elasticity of substitution $\frac{\frac{\partial[q(\omega)/q(\omega')]}{q(\omega)/q(\omega')}}{\frac{\partial[p(\omega)/p(\omega')]}{p(\omega)/p(\omega')}}}$

Appendix: derivation of the demand function (2/2)

$$\text{Now } \int_{\Omega} q(\omega)p(\omega)d\omega = \int_{\Omega} p(\omega)q(\omega')\left(\frac{p(\omega)}{p(\omega')}\right)^{-\sigma}d\omega = q(\omega')p(\omega')^{\sigma} \int_{\Omega} p(\omega)^{1-\sigma}d\omega$$

$$\text{This equals } wL \text{ (budget constraint) which leads: } q(\omega') = \frac{wL}{p(\omega')^{\sigma} \int_{\Omega} p(\omega)^{1-\sigma}d\omega} \quad (\text{B})$$

Plugging (B) in (A) and rearranging:

$$q(\omega) = \frac{p(\omega)^{-\sigma} wL}{\int_{\Omega} p(\omega)^{1-\sigma}d\omega}$$

Now, defining the price index:

$$P = \left(\int_{\Omega} p(\omega)^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$$

We finally get:

$$q(\omega) = \frac{p(\omega)^{-\sigma}}{P^{-\sigma}} \frac{wL}{P}$$

→ A percentage rise in $p(\omega)$ reduces demand by σ percent: σ is the price elasticity of demand. [▶ Back](#)

Appendix: derivation of the optimal price

- Start from the firms' profit function: $\pi(\omega) = p(\omega)q(\omega) - w(f + \frac{q(\omega)}{\varphi})$
- Maximize w.r. to price given the demand function from the previous slide (the price index is taken as given by the firm as this is monopolistic competition)
- FOC: $\frac{\partial \pi(\omega)}{\partial p(\omega)} = P^{\sigma-1} wL[(1 - \sigma)p^{-\sigma} + \frac{w}{\varphi}\sigma p^{-\sigma-1}] = 0$
- Rearranging we get: $p = \frac{\sigma}{\sigma-1} \frac{w}{\varphi}$: constant mark-up \times marginal cost

Appendix: price indexes in the two-country model

- The price index is now: $P = (\sum_{\Omega \in H} p(\omega)^{1-\sigma} + \sum_{\Omega \in F} (\tau p^*(\omega))^{1-\sigma})^{\frac{1}{1-\sigma}}$
- In the symmetric equilibrium, $p(\omega) = p \forall \omega \in H$ and $p^*(\omega) = p^* \forall \omega \in F$, so

$$P = (np^{1-\sigma} + n^*(\tau p^*)^{1-\sigma})^{\frac{1}{1-\sigma}} \text{ and } P^* = (n^*(p^*)^{1-\sigma} + n(\tau p)^{1-\sigma})^{\frac{1}{1-\sigma}}$$

- With zero transportation cost ($\tau = 1$), the two indexes are equal to $P = P^* = (2n)^{\frac{1}{1-\sigma}} p$. Note that both indices are lower than under autarky ($P = n^{\frac{1}{1-\sigma}} p$)
- Trade is welfare improving because of preference for diversity

Appendix: wages in the Krugman model

- L is exogenous but w is endogenous
- In order to give the wage level we need the last equation: the goods market equilibrium. Due to Walras law it's equivalent to look at the domestic market, the foreign market or at trade balance

- We use trade balance:

$$X = \lambda \times L \times L^* \times \left(\frac{\tau w^*}{P^*}\right)^{1-\sigma} \times w^* = \lambda \times L \times L^* \times \left(\frac{\tau w}{P}\right)^{1-\sigma} \times w = X^*$$

- Which implies $\frac{w}{w^*} = \left(\frac{P}{P^*}\right)^{\frac{1-\sigma}{\sigma}}$ with $\left(\frac{P}{P^*}\right)^{1-\sigma} = \frac{np^{1-\sigma} + n^*(\tau p^*)^{1-\sigma}}{n(\tau p)^{1-\sigma} + n^*(p^*)^{1-\sigma}}$
- So $\frac{w}{w^*} = \left(\frac{Lw^{1-\sigma} + L^*(\tau w^*)^{1-\sigma}}{L(\tau w)^{1-\sigma} + L^*(w^*)^{1-\sigma}}\right)^{\frac{1}{\sigma}}$

Appendix: specialization (Helpman and Krugman, 1985)

- All manufacturing firms produce the same quantity q that they sell at the same price p :

$$q = q^D + \tau q^X = \mu \left(\frac{p}{P} \right)^{-\sigma} \frac{wL}{P} + \tau \mu \left(\frac{\tau p}{P^*} \right)^{-\sigma} \frac{w^* L^*}{P^*}$$

- Replace price indexes by their open economy expressions:

$$q = \mu \frac{p^{-\sigma}}{n p^{1-\sigma} + n^* (\tau p^*)^{1-\sigma}} wL + \tau \mu \frac{(\tau p)^{1-\sigma}}{n (\tau p)^{1-\sigma} + n^* (p^*)^{1-\sigma}} w^* L^*$$

- Perfect labor mobility across sectors + trade in homogenous goods: same wage. Assume $\varphi = \sigma/(\sigma - 1)$ so that $w = p = 1$. The production of differentiated goods for each variety is:

$$q = \mu \left(\frac{L}{n + n^* \tau^{1-\sigma}} + \frac{\tau^{1-\sigma} L^*}{n \tau^{1-\sigma} + n^*} \right) \quad q^* = \mu \left(\frac{\tau^{1-\sigma} L}{n + n^* \tau^{1-\sigma}} + \frac{L^*}{n \tau^{1-\sigma} + n^*} \right)$$

- Since $q = q^*$ we have $\frac{L}{n + n^* \tau^{1-\sigma}} + \frac{\tau^{1-\sigma} L^*}{n \tau^{1-\sigma} + n^*} = \frac{\tau^{1-\sigma} L}{n + n^* \tau^{1-\sigma}} + \frac{L^*}{n \tau^{1-\sigma} + n^*}$

$$\text{Or } n(1 - \frac{L}{L^*} \tau^{1-\sigma}) = n^* (\frac{L}{L^*} - \tau^{1-\sigma})$$

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