Macroeconomics B, EI060

Class 9

New open economy macroeconomics

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What you will get from today class

- Micro-founded model with nominal rigidities (Obstfeld and Rogoff 10.1).
  - Keynesian model (Mundell-Fleming), with optimizing behavior.
  - Encompasses both the short run (set prices) and the long run (flexible prices).
  - Allows for welfare analysis.
- Extensions.
  - Different elasticities of substitution across countries and within countries.
  - ERPT Partial transmission of exchange rates to import prices.
  - Different dimensions driving the current account.
  - Welfare: beggar-thy-neighbor, or beggar-thyself.
  - Overshooting.

A question to start

M → Y NOT NECESSARILY
LIQUIDITY TRAP, ASSET BUBBLES
NORMAL TIMES: C ↗ UNEMPL ↘

Starting from a situation where output is too low, a monetary expansion that stimulates economic activity is always beneficial.

COVID
BOTTLENECKS
⇒ $\hat{\Pi}$ ↗

L.R ANTICIPATE M ↗

Do you agree? Why or why not?

P ⇒ REAL BENEFIT
! INFLATION

! TOT

TWO COUNTRIES OPTIMIZING MODEL

Optimization and frictions

- "Now open economy macroeconomics".
- Go beyond Mundell-Flemming by introducting optimization, while keeping frictions.
  - Optimal intertemporal choice by households and firms, robust to Lucas critique.
  - Welfare analysis using the household's utility.
- Maintain frictions of the Keynesian model.
  - Prices don't adjust immediately, so monetary policy has real effects.
  - Imperfect competition implies that output is too low (suboptimal use of resources, similar to unemployment), so policy has welfare effects.

Structure

$n \to 0$: SMOE

- Two countries, Home (size n) and Foreign (size $1-n$).
- Each inhabited by a household who consumes a range of brands that are imperfect substitutes (n Home brands, $1-n$ Foreign brands).
- Each brand is produced by a unique firm, which has monopoly power (imperfect competition).
- Some changes compared to OR book (for practicality).
  - 2 levels of consumption basket: Home - Foreign baskets, and brands within each basket. OR has one level.
  - W Household provides labor to firms. Cost of effort, and linear technology. OR by contrast have each different households, each producing a brand.
- We look at the effect of monetary shocks.

Consumption structure

- Home household's basket:

$$C = \left[(n)^{\frac{1}{\lambda}} \left[C(h) \right]^{\frac{\lambda-1}{\lambda}} + (1-n)^{\frac{1}{\lambda}} \left[C(f) \right]^{\frac{\lambda-1}{\lambda}} \right]^{\frac{\lambda}{\lambda-1}}$$

$$C(h) = \left[\left(\frac{1}{n} \right)^{\frac{1}{\theta}} \int_0^n \left[C(z,h) \right]^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}}$$

BRANDS

$$C(f) = \left[\left(\frac{1}{1-n} \right)^{\frac{1}{\theta}} \int_n^1 \left[C(z,f) \right]^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}}$$

$\lambda \leq \theta$

- $\lambda > 0$: elasticity of substitution between Home goods and Foreign goods baskets.
- $\theta > 1$: elasticity of substitution between brands. In the book $\theta = \lambda$.

Consumption demand (1)

- Static allocation of consumption: minimize expenditure, subject to target level of basket.

$$
\begin{aligned}
PC &= P(h)\,C(h) + P(f)\,C(f) \\
&= \int_0^n P(z,h)\,C(z,h)\,dz + \int_n^1 P(z,f)\,C(z,f)\,dz
\end{aligned}
$$

- Consumption of the Home or Foreign goods basket reflects relative prices, time the elasticity of substitution, and overall demand:

$$
C(h) = n\left[\frac{P(h)}{P}\right]^{-\lambda} C \qquad ; \qquad C(f) = (1-n)\left[\frac{P(f)}{P}\right]^{-\lambda} C
$$

- The price index is:

$$
P = \left[n\,[P(h)]^{1-\lambda} + (1-n)\,[P(f)]^{1-\lambda}\right]^{\frac{1}{1-\lambda}}
$$

Consumption demand (2)

- Similar allocation at the level of brands:

$$C(z,h) = \frac{1}{n}\left[\frac{P(z,h)}{P(h)}\right]^{-\theta} C(h) = \left[\frac{P(z,h)}{P(h)}\right]^{-\theta}\left[\frac{P(h)}{P}\right]^{-\lambda} C$$

$$C(z,f) = \frac{1}{1-n}\left[\frac{P(z,f)}{P(f)}\right]^{-\theta} C(f) = \left[\frac{P(z,f)}{P(f)}\right]^{-\theta}\left[\frac{P(f)}{P}\right]^{-\lambda} C$$

- The price indices are:

$$P(h) = \left[\frac{1}{n}\int_0^n [P(z,h)]^{1-\theta} dz\right]^{\frac{1}{1-\theta}}$$

$$P(f) = \left[\frac{1}{1-n}\int_0^n [P(z,f)]^{1-\theta} dz\right]^{\frac{1}{1-\theta}}$$

Foreign variables

- Similar choice in the Foreign country.
  - The Foreign Household also has weights n on Home goods and $1-n$ on Foreign goods, so baskets are similar.
  - Foreign variables, and prices in Foreign currency, denoted by $*$.

$$
\begin{aligned}
C^*(z,h) &= \frac{1}{n}\left[\frac{P^*(z,h)}{P^*(h)}\right]^{-\theta} C^*(h) = \left[\frac{P^*(z,h)}{P^*(h)}\right]^{-\theta}\left[\frac{P^*(h)}{P^*}\right]^{-\lambda} C^* \\
C^*(z,f) &= \frac{1}{1-n}\left[\frac{P^*(z,f)}{P^*(f)}\right]^{-\theta} C^*(f) = \left[\frac{P^*(z,f)}{P^*(f)}\right]^{-\theta}\left[\frac{P^*(f)}{P^*}\right]^{-\lambda} C^*
\end{aligned}
$$

Utility and budget

- Utility of consumption, real balance (to get a money demand, small weight χ), and disutility of labor.

$$U_t = \sum_{s=t}^{\infty} \beta^{s-t} \left[\ln(C_s) + \chi \ln\left(\frac{M_s}{P_s}\right) - \frac{\kappa_s}{2}(L_s)^2 \right]$$

- Purchases domestic money, a bond paying off in Home currency (could have Foreign currency). Income from wages, profits of domestic firms (lump sum), net of tax:

$$B_{t+1} + M_t + P_t C_t = (1 + i_t) B_t + M_{t-1} + W_t L_t + \Pi_t - T_t$$

- Government rebates money creation through a transfer (in appendix, we look at government spending). Bonds are between Home and Foreign households (no government debt):

$$0 = (M_t - M_{t-1}) + T_t$$

Intertemporal optimization

- Maximize utility subject to the budget constraints of each period. Condition with respect to consumptions, bond, labor, and money:

LAGRANGE MULT

$$\lambda_t = \frac{1}{P_t C_t} \quad ; \quad \lambda_{t+1} = \frac{1}{P_{t+1} C_{t+1}}$$

$$\lambda_t = \beta (1 + i_{t+1}) \lambda_{t+1} \quad ; \quad \lambda_t W_t = \kappa_t L_t$$

$$\lambda_t - \beta \lambda_{t+1} = \chi \left(\frac{M_t}{P_t} \right)^{-1} \frac{1}{P_t}$$

- Combining gives the Euler conditions, the money demand, and the labor supply:

$$C_{t+1} = \beta (1 + i_{t+1}) \frac{P_t}{P_{t+1}} C_t$$

$$\frac{M_t}{P_t} = \chi C_t \frac{1 + i_{t+1}}{i_{t+1}}$$

$$\kappa_t L_t C_t = \frac{W_t}{P_t}$$

Foreign optimization

- Similar steps for the Foreign household.

UIP

$1+i^*_t = (1+i_t)\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}$

$$
\begin{aligned}
C^*_{t+1} &= \beta\left(1+i_{t+1}\right)\frac{\mathcal{E}_t P^*_t}{\mathcal{E}_{t+1} P^*_{t+1}} C^*_t \\
\frac{M^*_t}{P^*_t} &= \chi C^*_t \frac{\left(1+i_{t+1}\right)\mathcal{E}_t}{\left(1+i_{t+1}\right)\mathcal{E}_t - \mathcal{E}_{t+1}} \\
\kappa^*_t L^*_t C^*_t &= \frac{W^*_t}{P^*_t}
\end{aligned}
$$

- $\mathcal{E}$: exchange rate in terms of units of Home currency per unit of Foreign currency (an increase is a depreciation of the Home currency)

Firm

- Each brand z is produced by a firm using a linear technology in labor: $Y(z,h) = L(z,h)$.
- Demand faced by the firm computed from allocation of consumption by Home and Foreign Households:

REL PRICE

$$Y(z,h) = nC(z,h) + (1-n)C^*(z,h)$$

- Profits (note that the firm's prices $P(z,h)$ and $P^*(z,h)$ matter with elasticity θ):

$$\begin{aligned}\Pi(z,h) = {} & (P(z,h) - W)\left[\frac{P(z,h)}{P(h)}\right]^{-\theta}\left[\frac{P(h)}{P}\right]^{-\lambda} nC \\ & + (\mathcal{E}P^*(z,h) - W)\left[\frac{P^*(z,h)}{P^*(h)}\right]^{-\theta}\left[\frac{P^*(h)}{P^*}\right]^{-\lambda}(1-n)C^*\end{aligned}$$

Optimal price

- If the firm can set the price, it chooses a markup over marginal cost, and the law of one price:

$$P(z,h) = \mathcal{E} P^{*}(z,h) = \frac{\theta}{\theta - 1} W$$

- Similarly for the Foreign firm:

$$\frac{P(f)}{\mathcal{E}} = P^{*}(f) = \frac{\theta}{\theta - 1} W^{*}$$

- Note that the elasticity θ drives the markup (not λ).

Aggregate output

- All firms within a country make the same choice, so for instance $P(z,h) = P(h)$.
- Home and Foreign outputs are:

$$
\begin{aligned}
Y &= n\left[\frac{P(h)}{P}\right]^{-\lambda} C + (1-n)\left[\frac{P^{*}(h)}{P^{*}}\right]^{-\lambda} C^{*} \\
Y^{*} &= n\left[\frac{P(f)}{P}\right]^{-\lambda} C + (1-n)\left[\frac{P_{t}^{*}(f)}{P^{*}}\right]^{-\lambda} C^{*}
\end{aligned}
$$

- Note that the elasticity λ enters.

Current account

- Bonds are in zero net supply: $nB_t + (1-n) B_t^* = 0$. Home current account:

SALES = Π + WL

$$\begin{aligned} B_{t+1} + P_t C_t &= (1+i_t) B_t + P_t(h) \, n \left[\frac{P_t(h)}{P_t}\right]^{-\lambda} C_t \\ &\quad + \mathcal{E}_t P_t^*(h) (1-n) \left[\frac{P_t^*(h)}{P_t^*}\right]^{-\lambda} C_t^* \end{aligned}$$

- Foreign current account:

$$\begin{aligned} -\frac{n}{1-n} \frac{B_{t+1}}{\mathcal{E}_t} + P_t^* C_t^* &= -\frac{n}{1-n} (1+i_t) \frac{B_t}{\mathcal{E}_t} + n \frac{P_t(f)}{\mathcal{E}_t} \left[\frac{P_t(f)}{P_t}\right]^{-\lambda} C_t \\ &\quad + (1-n) P_t^*(f) \left[\frac{P_t^*(f)}{P_t^*}\right]^{-\lambda} C_t^* \end{aligned}$$

LINEARIZED SOLUTION

Symmetric steady state

- Take linear approximations around a steady state where all countries are identical, with no bond holdings, $B_0 = B_0^* = 0$.
- The interest rate offsets the discount: $1 = \beta(1 + i_0)$.
- Optimal pricing, the labor supply, and the current account gives output and consumption:

$$C_0 = Y_0 = \left(\frac{\theta - 1}{\theta \kappa_0}\right)^{\frac{1}{2}} < \left(\frac{1}{\kappa_0}\right)^{\frac{1}{2}}$$

- The monopolistic distortion ($\theta < \infty$) implies that output is inefficiently low.

↳ PERFECT COMP

- Prices are given by the money demand, and the exchange rate reflects the relative monetary stances:

$$\mathcal{E}_0 = \frac{M_0}{M_0^*}$$

Shocks, and two periods adjustment

- Use log-linear deviations around this steady state: $x_t = \ln X_t - \ln X_0 = (X_t - X_0)/X_0$.
- The economy is initially at the symmetric steady state.
- In period t permanent monetary shocks occur, $\overline{m}$ and $\overline{m}^*$ (fiscal and productivity shocks presented in the appendix). CERTAINTY EQUIL
- Prices cannot adjust in period t (the short run), and are set in the currency of the firm. → S.R. X OPTIMAL PRICE X
  - Prices of domestic goods don't changes, prices of imports change with the exchange rate: $p(h) = p^*(f) = 0$, $p^*(h) = -e$, and $p(f) = e$.

FULL ER PASS-THROUGH

  - A depreciation ($e > 0$) raises the competitiveness of Home goods. PPP holds: $p - p^* = e$. $P(h) \neq \frac{\theta - 1}{\theta} W$
- Prices fully adjust at period $t + 1$. From then on the economy is in a long run steady state, with variables denoted by upper bars.
  - Differ from the original steady state if bond holdings have changed: $\overline{b} = \overline{B}/(P_0 C_0) \neq 0$.
- Convenient to express results in worldwide averages, $x^W = nx + (1-n)x^*$, and cross-country differences (which we focus on), $x - x^*$.

Long run solution

- Solution conditional on the bonds accumulated in the short run, $\bar{b}$.
- If Home accumulated bonds in the short run $(\bar{b} > 0)$, the Home households is better off in the long run:
  - Consumes more.
  - Works less.
  - Benefits from higher terms-of-trade (or lower price competitively to tilt world demand towards Foreign goods).

$$P = \frac{\theta - 1}{\theta} W$$

$$\begin{aligned}
\bar{p}(h) - \bar{p}^*(f) - \bar{e} &= \frac{1}{2\lambda}\frac{1-\beta}{\beta}\frac{\bar{b}}{1-n} \\
\bar{c} - \bar{c}^* &= \frac{1+\lambda}{2\lambda}\frac{1-\beta}{\beta}\frac{\bar{b}}{1-n} \\
\bar{y} - \bar{y}^* &= -\frac{1}{2}\frac{1-\beta}{\beta}\frac{\bar{b}}{1-n}
\end{aligned}$$

Exchange rate dynamics

- Euler conditions in terms of cross-country difference:

$$(\bar{c} - \bar{c}^*) = (c - c^*) + (p - p^* - e) - (\bar{p} - \bar{p}^* - \bar{e})$$

- Purchasing power parity holds in the short and long run: $\bar{p} - \bar{p}^* = \bar{e}$, $p - p^* = e$. The real interest rate is then the same in both countries, and the consumption difference is constant: $(\bar{c} - \bar{c}^*) = (c - c^*)$.
- Money market equilibrium in the short and long run:

UIP in $\frac{m^*}{p^*}$

$$\begin{aligned} \bar{m} - \bar{m}^* - (p - p^*) &= (c - c^*) + \frac{\beta}{1-\beta}(e - \bar{e}) \\ \bar{m} - \bar{m}^* - (\bar{p} - \bar{p}^*) &= (\bar{c} - \bar{c}^*) \end{aligned}$$

- Euler implies that there is no overshooting (with our parameters):

$$\begin{aligned} \bar{m} - \bar{m}^* - \bar{e} &= \bar{m} - \bar{m}^* - e - \frac{\beta}{1-\beta}(e - \bar{e}) \\ e - \bar{e} &= -\frac{\beta}{1-\beta}(e - \bar{e}) \\ e &= \bar{e} \end{aligned}$$

Short run solution: the MM line

- Short run money demand gives a relation between relative consumption and the exchange rate.

PPP

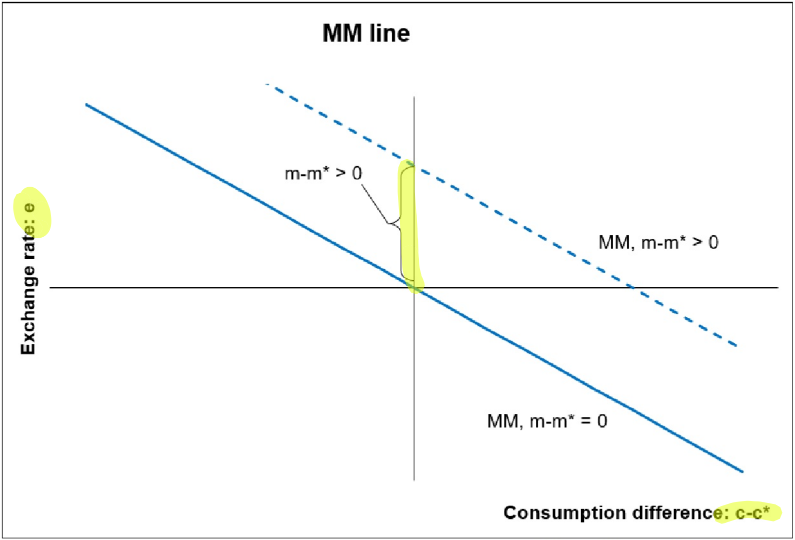
$$(c - c^*) = (\overline{m} - \overline{m}^*) - (p - p^*) = (\overline{m} - \overline{m}^*) - e$$

$$\Rightarrow e = (\overline{m} - \overline{m}^*) - (c - c^*)$$

- Changes in the money supply shift this relation.
- Higher money supply has to be matched with a higher nominal consumption, either through higher consumption or higher prices (i.e. a depreciated home currency).

MM line

- Negative relation between $c - c^*$ and e.



Short run solution: the GG line

- The current account and output demand in the short run imply (using $b = 0$): NOMINAL

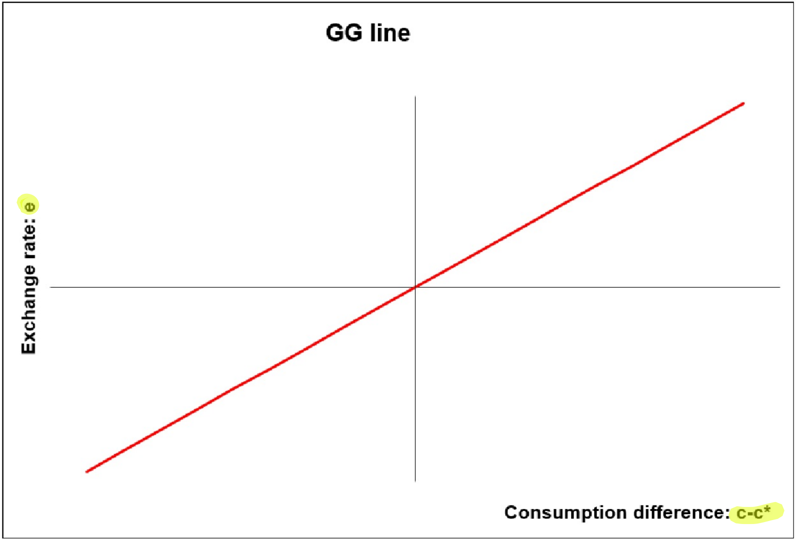
$$\frac{\overline{b}}{1-n} + (c - c^*) = e(\lambda - 1)$$

- Depreciation increases output (as REAL $(y - y^*) = \lambda e$), and worsens the terms of trade (as $(p(h) - p^*(f)) - e = -e$).
  - First effect dominates, so revenue increases by $e(\lambda - 1)$.
  - Additional revenue can finance consumption and savings.
- Long run solution and EULER constant consumption difference, we get a relation between consumption and the exchange rate:

$$e = \left[1 + \frac{\beta}{1-\beta}\frac{2\lambda}{1+\lambda}\right]\frac{1}{\lambda - 1}(c - c^*)$$

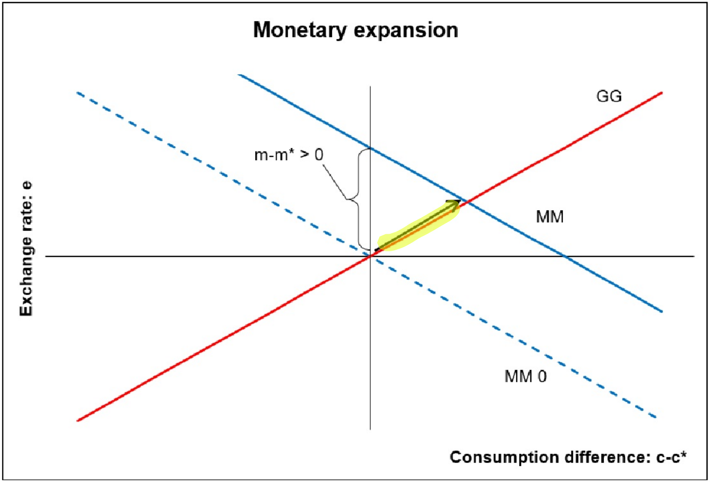
GG line

- Positive relation between $c - c^*$ and e.



Monetary expansion

- An expansion in the Home country, $\overline{m} - \overline{m}^* > 0$, depreciates the exchange rate and raises relative consumption.



Analytical solution

- Combining the two lines, we solve for the exchange rate, consumption and the current account:

IF $\lambda = 1$

$\Rightarrow C - C^* = 0$

$$e = \frac{2\lambda + (1+\lambda)\frac{1-\beta}{\beta}}{2\lambda + \lambda(1+\lambda)\frac{1-\beta}{\beta}} \left(\overline{m} - \overline{m}^*\right)$$

$$c - c^* = \frac{\left(\lambda^2 - 1\right)\frac{1-\beta}{\beta}}{2\lambda + \lambda(1+\lambda)\frac{1-\beta}{\beta}} \left(\overline{m} - \overline{m}^*\right)$$

$$\frac{\overline{b}}{1-n} = \frac{2}{2 + (1+\lambda)\frac{1-\beta}{\beta}} (\lambda - 1) \left(\overline{m} - \overline{m}^*\right)$$

- A monetary expansion raises consumption, depreciates the currency, and leads to a current account surplus provided $\lambda > 1$.

RICHER EXCHANGE RATE EFFECT

Incomplete pass-through

- Import prices reflect the exchange rate only to an extent s. We considered $s = 1$. If $s = 0$ import prices are not affected by the exchange rate. 100%
- Incomplete pass-through of the exchange rate implies that PPP does not hold in the short run:

$s < 1$

$$p - p^* = se$$

- The real interest rate then differs between the two countries. A depreciation of the Home currency reduces its real interest rate, relative to the Foreign country:

$$
\begin{aligned}
\pi^H - \quad & [i - (\bar{p} - p)] - \overbrace{[(i + e - \bar{e}) - (\bar{p}^* - p^*)]}^{\pi^F} \\
= \quad & -(\bar{p} - p) - (e - \bar{e}) + (\bar{p}^* - p^*) \\
= \quad & -(\bar{p} - \bar{p}^* - \bar{e}) + (p - p^* - e) \\
= \quad & -(1 - s)e
\end{aligned}
$$

ln(C); σ = 1

- Consider a more general utility for consumption: $\frac{(C)^{1-\sigma}}{1-\sigma}$. We can show that the current account is given by:

DYN ELAST STATIC

ln(C)

$C_H^{\eta} C_F^{1-\eta}$

$$\frac{\bar{b}}{1-n} = \frac{\sigma(\lambda-1)+(1+\lambda)}{\sigma(\lambda-1)+(1+\lambda)\frac{1}{\beta}}\left[\frac{\sigma-1}{\sigma}(1-s)+(\lambda-1)s\right]e$$

→ σ = λ = 1

- With full pass-through ($s=1$), driven by the elasticity λ.
  - If $\lambda>1$ a depreciation leads to a large shift of demand towards home goods ($y-y^*=\lambda e$).
  - Enough to offset the worsening of the home terms of trade, Home can afford more consumption, smooths through savings.
- Without pass-through ($s=0$) no demand switching ($y-y^*=0$). Current account reflects the intertemporal elasticity σ.

100 €

↓

> 100 CHF

  - Depreciation lowers the Home real interest rate, shifts Home consumption towards the short run.
  - Boosts Home exports earnings. This dominates if consumption is not sensitive to the real interest rate ($\sigma>1$), hence a current account surplus.

Welfare analysis

- Expand the utility of the household:

$$u_t = c - \frac{\theta - 1}{\theta} y + \frac{\beta}{1-\beta}\left[\bar{c} - \frac{\theta-1}{\theta}\bar{y}\right]$$

- Consumption raises welfare, output (effort) lowers it.
  - Smaller weight of effort due to monopolistic distortion $\frac{\theta-1}{\theta}$. < 1
  - Output is too low. Increase both output and consumption leads to a larger consumption utility gain (we get closer to the competitive efficient level).
- A monetary expansion raises global welfare when $\theta < \infty$:

$$u_t^W = \frac{1}{\theta\sigma}\bar{m}^W$$

Welfare difference

- Cross country welfare difference, using the current account relations:

$$u_t - u_t^* = (1 - s)e + \frac{\lambda - \theta}{\lambda\theta}\left[(y - y^*) + \frac{\beta}{1 - \beta}(\bar{y} - \bar{y}^*)\right]$$

- With limited pass-through ($s < 1$), depreciation raises Home welfare.
  - Higher earnings on exports, with limited reduction of export quantity (beggar-thy-neighbor). $U > U^*$
- If $\lambda < \theta$, Home suffers from an output expansion (beggar-thyself).
  - Selling the additional output requires a lower price of Home goods.
  - Cost from worsening of the terms of trade high, extra consumption does not offset the cost of effort.
- No difference if $\lambda = \theta$. Everyone benefits from the global gain.

Overshooting

$\varepsilon = 1$

- If we consider a richer utility of real balances, $\frac{(M/P)^{1-\varepsilon}}{1-\varepsilon}$, we can get overshooting. The Euler, and short and long run money demands are:

$$
\begin{aligned}
(\overline{c} - \overline{c}^*) &= (c - c^*) - (1 - s) e \\
\varepsilon (\overline{m} - \overline{m}^*) - \varepsilon s e &= (c - c^*) + \frac{\beta}{1-\beta} (e - \overline{e}) \\
\varepsilon (\overline{m} - \overline{m}^*) - \varepsilon \overline{e} &= (\overline{c} - \overline{c}^*)
\end{aligned}
$$

- There is overshooting ($e - \overline{e} > 0$) if pass-through is limited ($s < 1$) and the utility of money is very concave ($\varepsilon > 1$):

$$
e - \overline{e} = \frac{1-\beta}{\beta + (1-\beta)\varepsilon} (\varepsilon - 1)(1 - s) e
$$

- The lower real interest in Home brings relative consumption forward $(c - c^*) > (\overline{c} - \overline{c}^*)$ which raises relative money demand.
- If $\varepsilon > 1$ money demand does not increase much, so a difference is needed in nominal returns, i.e. overshooting.

Main takeaways

- Model combining optimization with price rigidities.
  - Consistent short- and long-run solution.
  - Effects in both countries, can be of different sizes.
- Shocks can have long-run effects if they lead to changes in bond holdings.
  - New steady state is not necessarily the initial symmetric one ("unit root" feature).
- Pass-through of exchange rate to import prices matters.
  - Deviation from PPP, exchange rate overshooting.
  - Different channels for the current account.
- Welfare analysis using the household's utility.
  - Higher Home output does not necessarily raise Home welfare (relative to Foreign).