Intermediate Microeconomics

Imperfect Competition II: Oligopoly and Strategic Behavior

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Outline

- Oligopoly (寡头): few sellers, between monopoly and perfect competition
  - Cournot model (古诺)
  - Bertrand model (homogeneous/heterogeneous) (伯川德: 同质/异质)
  - Cartel and collusion (卡特尔与合谋)
  - Stackelberg model (斯塔克伯格)
- Preliminaries in game theory
  - Solution concepts: Nash equilibrium (纳什均衡)
  - Static model
  - Dynamic model

Cournot model (古诺模型)

- Assume the inverse demand of a market is $p = 10 - Q$.
- Two sellers (duopoly: 双寡头). Marginal cost of each seller is 2.
- Each seller chooses to produce a **quantity** that maximizes his/her **own** profit.
  - Seller 1 produces q_1; seller 2 produces q_2. Total quantity is $Q = q_1 + q_2$.
- Inverse demand becomes: $p = 10 - (q_1 + q_2)$.
  - Under monopoly:

$$\max_{Q}(10 - Q)Q - 2Q \Rightarrow Q^m = 4, p^m = 6$$

  - Under perfect competition:

$$p = MC \Rightarrow 10 - Q = 2 \Rightarrow Q^{FB} = 8, p^{FB} = 2$$

Best Responses (最佳回应)

- Seller 1: given the rival's choice q_2, seller 1's profit is $\pi_1(q_1, q_2) = pq_1 - 2q_1$. Seller 1 chooses q_1 that is the best response of q_2

$$\max_{q_1} [10 - q_1 - q_2] q_1 - 2q_1$$

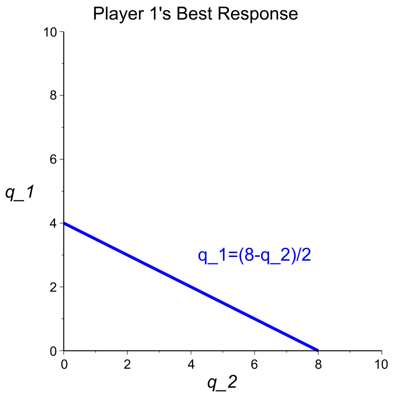
$$\Rightarrow q_1^{BR}(q_2) = \frac{8 - q_2}{2}$$

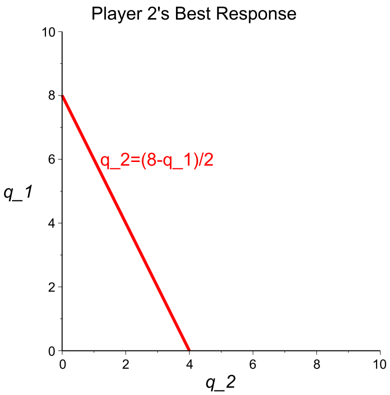
- Seller 2: given the rival's choice q_1, seller 2's profit is $\pi_2(q_1, q_2) = pq_2 - 2q_2$. Seller 2 chooses q_2 that is the best response of q_1

$$\max_{q_2} [10 - q_1 - q_2] q_2 - 2q_2$$

$$\Rightarrow q_2^{BR}(q_1) = \frac{8 - q_1}{2}$$

Best Responses and Rationality

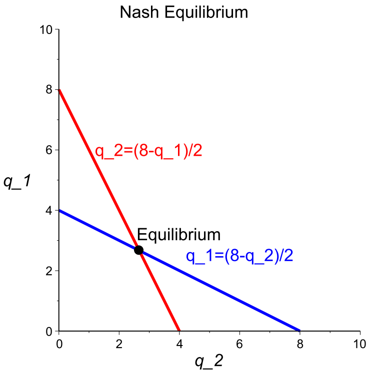




- Both sellers know that they are rational. They both known that each seller will play a best response to the rival's choice.
- They will predict that: the rational strategies will be played where the two best-responses hold simultaneously.

Nash Equilibrium (纳什均衡)

- Rational outcome: the best-responses of two sellers hold simultaneously.



- At the cross-over point, neither party has an incentive to deviate:

$$\begin{cases} q_1 = (8 - q_2)/2 \\ q_2 = (8 - q_1)/2 \end{cases} \Rightarrow \begin{cases} q_1^* = 8/3 \\ q_2^* = 8/3 \end{cases}$$

Monopoly $\leftarrow$ Oligopoly $\rightarrow$ Perfect Competition

- For n sellers ($MC = 2$ for each), each seller chooses q_i, given the other sellers' choices fixed.
- Market demand: $p = 10 - Q = 10 - (q_1 + q_2 + \cdots + q_n)$
- For seller 1: $\max_{q_1}(10 - q_1 - \cdots - q_n)q_1 - 2q_1 \Rightarrow$ $-q_1 + 10 - (q_1 + \cdots + q_n) - 2 = 0$
- Similarly for seller 2, 3, ...
- For seller n: $\max_{q_n}(10 - q_1 - \cdots - q_n)q_n - 2q_n \Rightarrow$ $-q_n + 10 - (q_1 + \cdots + q_n) - 2 = 0$
- The best responses of the n sellers hold simultaneously: $-Q + 10n - nQ - 2n = 0 \Rightarrow Q = \frac{8n}{n+1}$.
  - monopoly: $n = 1 \Rightarrow Q = 4$
  - perfect competition: $n \rightarrow \infty \Rightarrow Q = 8$.

Solution Concept: Nash Equilibrium

- Different from monopoly/perfect competition: under oligopoly, the payoff of each seller depends
  - not only on one's own choice
  - but also on the choice(s) of opponent(s)
- Given others' choices, each seller plays a best response.
- Rational players are aware of that all players will play best responses.
  - Given q_2, player 1 chooses $q_1(q_2)$; given $q_1(q_2)$, player 2 chooses $q_2(q_1(q_2))$; given $q_2(q_1(q_2))$, player 1 chooses $q_1(q_2(q_1(q_2)))$; ...
  - All the above best responses will be **commonly** inferred.
- (Pure-Strategy: 纯策略) Nash equilibrium (NE): Each player plays a best response. At Nash equilibrium, no player has an incentive to deviate.
  - Neither seller has an incentive to deviate from the Nash outcome $(q_1^*, q_2^*) = (8/3, 8/3)$.

Example: Prisoner's Dilemma (囚徒困境)

- Two suspects (player 1 and player 2) are caught. Each is put in a separate room under interrogation. Each of them can choose from
  1. Mum (silence)
  2. Fink (confess)
- If both choose Mum, then each get 2 years in jail (the payoff of each is -2); If both choose fink, then each get 4 years in jail; If one chooses mum while the other chooses fink, then the one who chooses fink gets 1 year but the one who chooses mum gets 5.
- The rule above is **common knowledge**. They make decisions simultaneously.

- Unlike the Cournot model, the strategies taken by each player is finite here: either mum or fink.
- Hence there are 2×2 possible outcomes: in order to represent each outcome, we write player 1's two strategies as two rows; player 2's two strategies as two columns.
- For each outcome, write player 1's payoff (a number) on the left, and player 2's payoff on the right, i.e., $(1\text{'s payoff}, 2\text{'s payoff})$
- The Normal-Form representation of such finite game is

		Player 2	
		Mum	Fink
Player 1	Mum	-2, -2	-5, -1
	Fink	-1, -5	-4, -4

- Next, find the Nash equilibrium.

- Each player plays a best response, given the choice of the other player.
- Underlining each player's best response at the corresponding payoff. For example:
- For player 1:
  - Given player 2's action mum, the best response is fink; Given player 2's action fink, the best response is fink.
- For player 2:
  - Given player 1's action mum, the best response is fink; Given player 1's action fink, the best response is fink.

		Player 2	
		Mum	Fink
Player 1	Mum	-2, -2	-5, <u>-1</u>
	Fink	<u>-1</u>, -5	<u>-4</u>, <u>-4</u>

- The Nash equilibrium is (fink, fink). The two best-responses coincide into the second row/column.

Example

- Player 1 chooses from Upper, Middle and Down. Player chooses Left, Center and Right:

		Player 2		
		Left	Center	Right
	Upper	7, 7	4, 2	1, 8
Player 1	Middle	2, 4	5, 5	2, 3
	Down	8, 1	3, 2	0, 0

- Solve the Nash equilibrium by finding the "cross-over points" of best responses: underlining each player's best responses:

		Player 2		
		Left	Center	Right
	Upper	7, 7	4, 2	1, $\underline{8}$
Player 1	Middle	2, 4	$\underline{5}$, $\underline{5}$	$\underline{2}$, 3
	Down	$\underline{8}$, 1	3, $\underline{2}$	0, 0

- The unique Nash equilibrium is (Middle, Center).

Example of Multiple Nash Equilibria (多重均衡)

- Battle of Sexes: Chris prefers Opera over Football; Alex prefers Football over Opera. Both prefer attending to the same program to attending to different programs.
- Write the game in the matrix form:

		Alex: Opera	Alex: Football
Chris	Opera	2, 1	0, 0
	Football	0, 0	1, 2

- Solve the Nash equilibrium by finding the "cross-over points" of best responses: underlining each player's best response:

		Alex: Opera	Alex: Football
Chris	Opera	$\underline{2}$, $\underline{1}$	0, 0
	Football	0, 0	$\underline{1}$, $\underline{2}$

- Two NE: (Opera, Opera) and (Football, Football)

Example of No Equilibrium (均衡不存在)

- The Rock-paper-scissor game:

			Player 2	
		Rock	Paper	Scissor
	Rock	0, 0	-1, 1	1, -1
Player 1	Paper	1, -1	0, 0	-1, 1
	Scissor	-1, 1	1, -1	0, 0

- Underlining the best response of each player

			Player 2	
		Rock	Paper	Scissor
	Rock	0, 0	-1, $\underline{1}$	$\underline{1}$, -1
Player 1	Paper	$\underline{1}$, -1	0, 0	-1, $\underline{1}$
	Scissor	-1, $\underline{1}$	$\underline{1}$, -1	0, 0

- We cannot find a combination of strategies where each player has no incentive to deviate given the choice of the other: No Nash equilibrium.

Other Examples

$(\underline{-\infty}, -4000)$ $(-1000, \underline{1000})$

$(\underline{-\infty}, -5000)$ $(\underline{0}, \underline{0})$

- You are caught by a robber, who threatens that if you don't tell him the password of your bankcard (with \$1000 cash), he will kill you. Being killed means your payoff is $-\infty$. If the robber kills someone, he faces a punishment that is valued by -\$5000. Should you tell him the password?
- You and your roommate is considering to buy an air conditioner. The air conditioner brings about benefits \$2000 for each person. Buying the air conditioner costs \$3000. If you buy it jointly, you split the cost. Will the air conditional be bought?

Bertrand Model (伯川德): Homogeneous Products

- Two sellers, 1 and 2. There are N consumers in the market.
- Each consumer buys one product, either from seller 1 or seller 2.
- Both sellers offer prices (p_1, p_2) simultaneously. Consumers will buy from the seller who charges a lower price.
- The demand of each seller:

$$q_1 = \begin{cases} N & \text{if } p_1 < p_2 \\ \frac{N}{2} & \text{if } p_1 = p_2 \\ 0 & \text{if } p_1 > p_2 \end{cases}, \quad q_2 = \begin{cases} N & \text{if } p_2 < p_1 \\ \frac{N}{2} & \text{if } p_2 = p_1 \\ 0 & \text{if } p_2 > p_1 \end{cases}$$

- The marginal cost of each seller is assumed to be $c = 0$.

Best Responses and Nash Equilibrium

- Starting from a relatively high price of p_2, consider seller 1's strategies:
  1. If $p_1 > p_2 \Rightarrow q_1 = 0$, the profit of seller 1 is $p_1 q_1 = 0$.
  2. If $p_1 = p_2 \Rightarrow q_1 = N/2$, the profit is $p_1 q_1 = p_2 N/2$.
  3. If $p_1 = p_2 - \epsilon < p_2 \Rightarrow q_1 = N$, the profit is $p_1 q_1 = (p_2 - \epsilon)N$
- Therefore, seller 1's best response is to charge p_1 that is "slightly" below p_2.
- Seller 2 will think in the same way: charging p_2 that is "slightly below p_1.
- The Bertrand-Nash equilibrium: $p_1^* = p_2^* = c = 0$.
- Bertrand Paradox: Only two sellers in the market (oligopoly), but price is equal to the marginal cost (like perfect competition).

Is Collusion (合谋) Stable?

- Recall the prisoner's dilemma:

		Player 2: Mum	Player 2: Fink
Player 1	Mum	-2, -2	-5, -1
	Fink	-1, -5	-4, -4

  The **cooperative outcome** (mum, mum) is better than **Nash outcome** (fink, fink), if the two players collude successfully.
- Consider the following: before the trial, the two players meet and they agree that choosing (mum, mum) is better than (fink, fink). So they claim that each is going to choose "mum" in court.
- Player 1 will think: if player 2 keeps his promise by choosing mum, and if I break the promise by choosing fink instead of mum, I will get -1 which is better than -2.
- Player 2 will think in the same way. And they will realize what the other player is going to do.
- Both players will deviate from the cooperative outcome (which is an incredible threat) to the Nash outcome.

Collusion: Cartel (卡特尔)

- In many countries, one of the objectives of anti-trust policies is to promote competition. Policy makers hold that the collusion of oligopolists leads to Cartel that softens competition.
- Is Cartel stable?
- Recall the Cournot example: $p = 10 - q_1 - q_2$. The Nash outcome is $(q_1^*, q_2^*) = (8/3, 8/3)$. The profit of each is $\pi_1^* = \pi_2^* = (10 - q_1^* - q_2^*) - 2q_i^* = 64/9 \approx 7.11$.
- Consider the following cooperative plan (Cartel): The two sellers form a monopoly and produces the monopolistic quantity. Then each gets 1/2 of the monopolistic profit.
- The Cartel solves

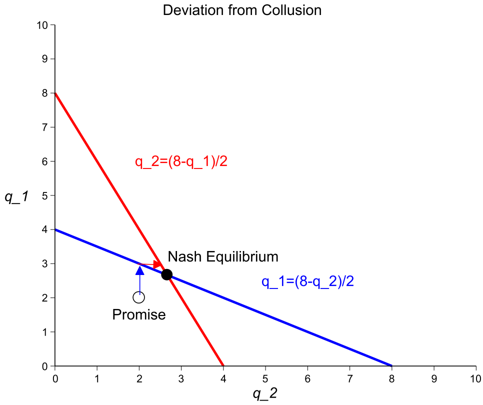
$$\max_{Q}(10 - Q)Q - 2Q \Rightarrow Q^m = 4,\ \pi^m = 16$$

Each seller gets half of the profit $\frac{1}{2}\pi^m = 8 > 7.11 = \pi_i^*$.

- Under Cartel, each seller produces half of the output: $\frac{Q^m}{2} = 2$. Earning $\frac{\pi^m}{2} = 8$, which is more profitable than the Cournot outcome.
- Is there any way to make seller 1 better-off?
- If seller 2 keeps his/her promise, by producing $q_2 = 2$, consider seller 1's best response:

$$\max_{q_1}(10 - 2 - q_1)q_1 - 2q_1 \Rightarrow q_1^d = 3 \Rightarrow \pi_1^d = 9 > 8 = \frac{1}{2}\pi_1^m.$$

  The deviation profit is higher than the collusive profit.
- Therefore, seller 1 will deviate by producing $q_1^d = 3$.
- Seller 2 knows that, and will play a best response to $q_1 = 3$ ...
- Each seller will play the best response to the rival's deviations ... play the Nash equilibrium strategies: $(q_1^*, q_2^*) = (8/3, 8/3)$.



- Sellers have incentives to deviate from the collusive output (incredible promise).
- They will play the Nash equilibrium strategies.

Dynamic Games

- In the above examples, we assume that players move simultaneously.
- In some other circumstances, players move sequentially.
- Example:
  There are 5 rational pirates: A, B, C, D and E. They find 100 gold coins. They must decide how to distribute them. Starting from A, each proposes a distribution of coins. The pirates, including the proposer, then vote on whether to accept this distribution. If the proposed allocation is approved by a majority or a tie vote ($\geq 50\%$), then it happens. Otherwise, the proposer is thrown overboard and dies, and the next makes a new proposal to begin the system again.

Sequential Rationality and Backward Induction (逆向归纳)

A standard procedure to predict a sequential-move-game is to use the backward induction, i.e., starting from the last mover:

- Last step: If D dies, then E gets all 100.
- The last second step: from the view of D, a tie vote (D votes for himself) is sufficient. D gives 100 to himself and 0 to E.
- From the view of C: C knows that if C dies, E gets nothing. So C will give 1 coin for E, 0 to D, and 99 for himself, and E will agree.
- From the view of B: B needs two votes. If B dies, D will get nothing. So B should give D 1 coin, 99 for himself, and nothing to C and E.
- From the view of A: A needs three votes. If A dies, C and E get nothing. So A give C and E each 1 coin, 98 for himself, and nothing to B and D.

So A proposes (98, 0, 1, 0, 1) for (A, B, C, D, E) and A, C and E will agree. The game ends at the first round.

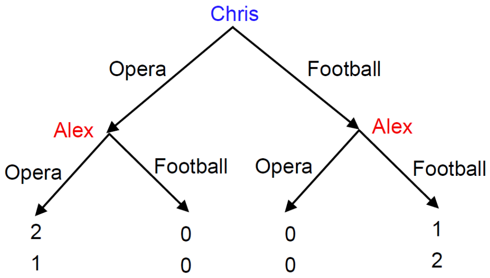
Solution Concept: Subgame Perfect Nash Equilibrium

- The rational outcome for a sequential game, is called subgame perfect Nash equilibrium (子博弈完美). SPNE survives from backward induction.
- To facilitate the analysis, sometimes we use a "game tree" to implement backward induction to find the SPNE.
- Recall the "battle of sexes."

		Alex	
		Opera	Football
Chris	Opera	2, 1	0, 0
	Football	0, 0	1, 2

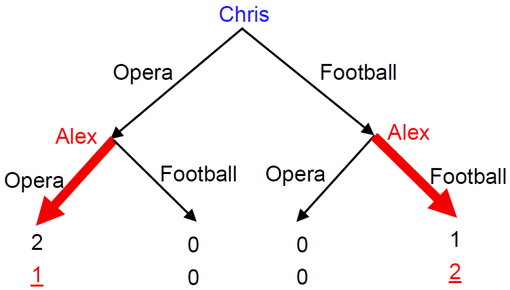
  We have shown that there are two equilibria, if they move simultaneously.
- What about Chris moves first?

- If Chris moves first, and then Alex moves after Chris's action, we can plot the game tree as the following:



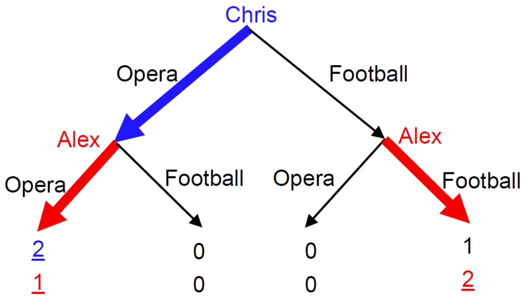
  - Use arrows to represent the strategies and the corresponding orders.
  - For the payoffs, the payoff of the first-mover (Chris), is written at the top; the payoff of the second-mover (Alex), is written at the bottom.

- Using backward induction, find Alex's (second-mover) best responses.
- Underlining Alex's best responses:
  1. Given Chris chooses Opera → Alex prefers Opera over Football.
  2. Given Chris chooses Football → Alex prefers Football over Opera.



- The **bold red arrow** represents Alex's "whole plan" of play.

- Alex's "whole plan" can be commonly inferred by Chris.
- Chris needs to compare the two possible outcomes that will be subsequently played by Alex, and plays a best response to the possible outcomes (i.e., choose one of them to maximize her own payoff)



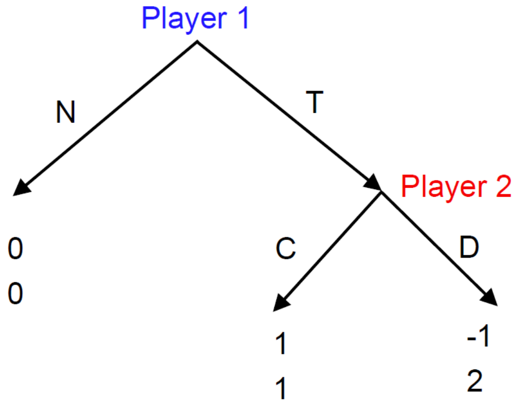
- Then, "Chris chooses Opera, and then Alex chooses Opera" is a subgame perfect Nash equilibrium.

Example: Trust Game

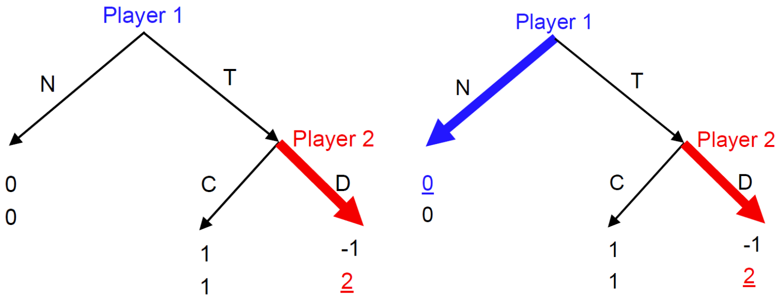
Player 1 first chooses whether to ask for the services of player 2.

1. Player 1 can trust player 2 (T) or not trust him (N), the latter choice given both players a payoff of 0.
2. If player 1 trusts player 2, then player 2 can choose to cooperate (C), which represents offering player 1 some fair level of service, or defect (D), by which he basically cheats player 1 with an inferior, less costly to provide service. Assume that if player 2 cooperates then both players get a payoff of 1, while if player 2 chooses to defect then player 1 gets a payoff of -1 and player 2 gets a payoff of 2.

Game Tree of the Trust Game



Using Backward Induction



At stage 2, player 2 will choose D. Back to stage 1, player 1 compares the payoff from N and T→D, and clearly, N is better than T→D. The SPNE is "player 1 chooses N" and the game ends.

First-Mover Advantage (先行者优势)

- From the example of "battle of sexes," comparing the simultaneous-move setting and sequential-move setting, we can see that there exists a "first-mover advantage."
- However, in some other cases, the first-mover has a disadvantage.
  - Example: for the "Rock-Paper-Scissor" game, the first mover will lose definitely.
  - For the Bertrand game (with homogeneous products), the seller who charges a price (that is above marginal cost) will earn zero demand/profit.
- If sellers compete by setting quantities (Cournot model), and if seller 1 sets q_1 first, and then seller 2 follows by setting q_2, let's check the SPNE.

Stackelberg Model (斯塔克伯格领导者模型)

- Market demand: $p = 10 - Q = 10 - q_1 - q_2$. The game lasts two stages. At stage 1, seller 1 (leader) produces q_1. Then at stage 2, seller 2 (follower) produces q_2.
- Backward induction: at stage 2, q_1 is already given, then seller 2 solves

$$\max_{q_2}(10 - q_1 - q_2)q_2 - 2q_2 \Rightarrow q_2(q_1) = \frac{8 - q_1}{2}.$$

- At stage 1, seller 1 knows that seller 2 will subsequently respond by setting $q_2 = \frac{8-q_1}{2}$. Hence q_2 in seller 1's profit can be replaced by $q_2 = \frac{8-q_1}{2}$, i.e., seller 1 solves

$$\max_{q_1}(10 - q_1 - q_2(q_1))q_1 - 2q_1 \Rightarrow \max_{q_1}\left(10 - q_1 - \frac{8 - q_1}{2}\right)q_1 - 2q_1$$

- Seller 1 solves

$$\max_{q_1} \left(10 - q_1 - \frac{8 - q_1}{2}\right) q_1 - 2q_1 \Rightarrow q_1^{\text{leader}} = 4.$$

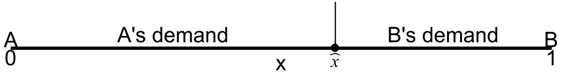
  The profit of seller 1 is $\pi_1^{\text{leader}} = 8 > 7.11$.
- Plug $q_1^L = 4$ into seller 2's best response

$$q_2^{\text{follower}} = \frac{8 - q_1}{2} = 2$$

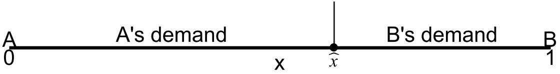
  The profit of seller 2 is
  $\pi_2^{\text{follower}} = (10 - 4 - 2) \cdot 2 - 2 \cdot 2 = 4 < 7.11$
- The first-mover advantage for seller 1.

Bertrand Game with Heterogeneous Products

- In the Bertrand game, we see a "Bertrand paradox" in the sense that each seller earns zero profit, although there are only two sellers.
- In the real world, sellers who compete in prices, are selling differentiated products.
  - Introduce differentiation: horizontal differentiation/Hotelling model/spatial model/linear city model.
- Consider a "linear" beach: $x \in [0, 1]$. One unit of tourists are uniformly distributed along the beach.
- Two ice-cream sellers, A and B, are located at the ends of the beach. A locates at 0 while B locates at 1.
- Each tourist (as a buyer), has the "unit-demand" for ice-cream: each buyer buyers exactly one unit, either from A or from B.



- Both sellers offer prices (p_A, p_B) simultaneously.
- For a buyer located at x, he/she buys one unit either from A or B
  - Utility of buying from A: $V_A - tx - p_A$
  - Utility of buying from B: $V_B - t(1-x) - p_B$
    where x and $1-x$ is the distance between the consumer's location and the location of seller A and B, respectively. t is the per-unit "travel cost."
- Assume $V_A = V_B$. Then for a consumer located at x:
  - if $-tx - p_A > -t(1-x) - p_B$, buy from A.
  - if $-tx - p_A < -t(1-x) - p_B$, buy from B
  - if $-tx - p_A = -t(1-x) - p_B$, indifferent.
- The indifferent margin: $\widehat{x} = \frac{1}{2} + \frac{p_B - p_A}{2t}$.



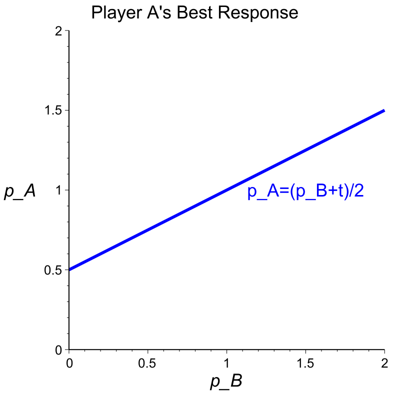
- Seller A's demand: $\Pr(x < \hat{x}) = \int_0^{\hat{x}} \frac{1}{1-0} dx = \frac{1}{2} + \frac{p_B - p_A}{2t}$.
- Seller B's demand: $\Pr(x > \hat{x}) = \int_{\hat{x}}^{1} \frac{1}{1-0} dx = \frac{1}{2} + \frac{p_A - p_B}{2t}$.
- Seller A solves (given p_B)

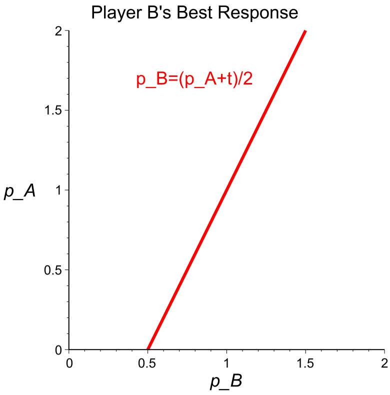
$$\max_{p_A} p_A \left(\frac{1}{2} + \frac{p_B - p_A}{2t} \right) \Rightarrow p_A^{BR}(p_B) = \frac{p_B + t}{2}$$

- Seller B solves (give p_A)

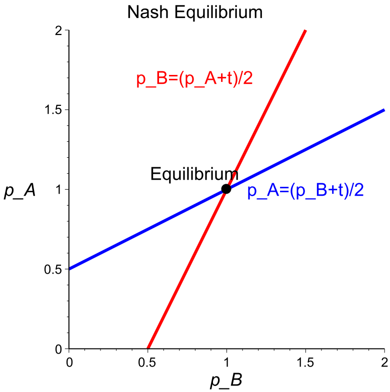
$$\max_{p_B} p_B \left(\frac{1}{2} + \frac{p_A - p_B}{2t} \right) \Rightarrow p_B^{BR}(p_A) = \frac{p_A + t}{2}$$

Best Responses





Nash Equilibrium



$$\begin{cases} p_A^{BR} = \frac{p_B + t}{2} \\ p_B^{BR} = \frac{p_A + t}{2} \end{cases} \Rightarrow (p_A^*, p_B^*) = (t, t)$$

Produce Differentiation and Market Power

- The equilibrium price: $p_A^* = p_B^* = t$. The equilibrium profit: $\pi_A^* = \pi_B^* = \frac{t}{2}$. Both are increasing in t.
- Here, the degree of differentiation is captured by t. A higher t gives rise to a greater level of market power.
- In contrast, if the sellers sell homogeneous products, as is shown by the "Bertrand paradox," the equilibrium price is equal to the marginal cost, i.e., $p_A^* = p_B^* = 0$.

An Application of the Hotelling model

- Still, a unit mass of voters are uniformly distributed over the $x \in [0, 1]$ line.
  - Point 0 represents "left-wing" policies.
  - Point 1 represents "right-wing" policies.
- Two candidates, A and B, each proposes a policy, i.e., each chooses a point in the line $x \in [0, 1]$.
- Each voter will vote one of the two policies that is "closer" to his/her ideal position.
- The candidate with more votes win.
- Nash equilibrium: both candidates propose a policy at $x = 1/2$.
- The "median voter theorem."