L7. The Fundamental Welfare Theorems

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Literature

- MWG (1995), Chapters 10D, 10E

This and the Previous Lecture

Previous lecture: the organization of production and the allocation of the resulting commodities among consumers under *perfect competitive market economies*

$\rightarrow$ positive perspective

Today's lecture: two central results regarding the **optimality** properties of competitive equilibria

$\rightarrow$ normative perspective

Today's Lecture

The two central welfare results:

- If (i) there is a complete set of markets with publicly known prices, and (ii) every agent (firms, consumers) acts perfectly competitively (i.e., as a price taker):

The First Fundamental Welfare Theorem

- then the market outcome is Pareto optimal.

If additionally (iii) household preferences and firm production sets are convex:

The Second Fundamental Welfare Theorem

- then any Pareto optimal outcomes can be achieved as a competitive equilibrium if appropriate lump-sum transfers of wealth are arranged.

Notes on the Two Theorems

The First Fundamental Welfare Theorem

- when markets are complete (i), *any competitive equilibrium* (ii) is necessarily Pareto optimal

- Adam Smith's idea about the "invisible hand" of the market

The Second Fundamental Welfare Theorem

- w. + convexity conditions (iii)

- *all* Pareto optimal outcomes can be implemented through the market mechanism

- a public authority who wants one particular Pareto optimal outcome may do so by appropriately redistributing wealth and then "letting the market work"

Useful Benchmark

If any inefficiencies arise in a market economy, and hence any role for Pareto improving market intervention, we must be able to trace a violation of at least one of the assumptions of the **first fundamental welfare theorem**.

- Various market failures in reality: externalities, market power, info. frictions
- Can all be viewed as different themes developed (deviated) from the benchmark

Based on the **second fundamental welfare theorem**, if any inefficiencies arise in a market economy, one natural starting point of designing optimal policy is to "restore" the perfect competitive environment and let "transfer + market" economy work.

- May not always be feasible, but a useful benchmark to start

- We prove both theorems under a partial equilibrium setting.
- We go through the general equilibrium intuition using the Edgeworth box.
- Check MWG (1995) chapter 15 if you are interested in the general treatment.

Proposition 7.1 (The First Fundamental Welfare Theorem) If the price p^* and allocation $(x_1^*, ..., x_I^*, q_1^*, ..., q_J^*)$ constitute a competitive equilibrium, then this allocation is Pareto optimal.

Proof under the partial equilibrium setting

Because of the quasilinear form of the utility functions, we have unlimited unit-for-unit transfer of utility across consumers through transfers of the numéraire (i.e. one unit of additional numéraire goods' utility gain is the same across consumers).

$u_i = \phi_i(x_i) + m_i$

w_{mi}

The set of utilities that can be attained for the I consumers is given by:

$$\{(u_1, ..., u_I) : \sum_{i=1}^{I} u_i \leq \sum_{i=1}^{I} \phi_i(x_i) + w_m - \sum_{j=1}^{J} c_j(q_j)\}$$

The Pareto optimal allocation must involve quantities $(x_1^*, ..., x_I^*, q_1^*, ..., q_J^*)$ that max the RHS as much as possible → If we "make the cake bigger", there is always a way to achieve mutual benefits. Hence the necessary condition is

$$\max_{(x_1,...,x_I)\geq 0,(q_1,...,q_J)\geq 0} \sum_{i=1}^{I} \phi_i(x_i) + w_m - \sum_{j=1}^{J} c_j(q_j)$$

$$s.t. \sum_{i=1}^{I} x_i - \sum_{j=1}^{J} q_i = 0$$

Lagrange equation:

$$\max_{(x_1,\ldots,x_I)\geq 0,(q_1,\ldots,q_J)\geq 0} \sum_{i=1}^{I} \phi_i(x_i) + w_m - \sum_{j=1}^{J} c_j(q_j) - u\left(\sum_{i=1}^{I} x_i - \sum_{j=1}^{J} q_i\right)$$

FOC:

x^*

q^*

p^*

$$D1 - \quad wrt.\ q). \quad u \leq c_j'(q_j), \text{with equality if } q_j^* > 0 \qquad j = 1, ..., J$$

$$D2 - \quad wrt.\ x). \quad \phi_i'(x_i) \leq u, \text{with equality if } x_i^* > 0 \qquad i = 1, ..., I$$

$$D3 - \quad wrt.\ u). \quad \sum_{i=1}^{I} x_i^* = \sum_{j=1}^{J} q_j^*$$

D1-D3 are exactly parallel to conditions C1-C3, characterizing a competitive equilibrium (Lecture 6), with u replacing p^*!

Q.E.D.

Proposition 7.2 (The Second Fundamental Welfare Theorem of Welfare Economics) For any Pareto optimal levels of utility $(u_1^*, ..., u_I^*)$, there are transfers of the numéraire commodity $(T_1, ..., T_I)$ satisfying $\sum_i T_i = 0$, such that a competitive equilibrium reached for the endowments $(w_{m1} + T_1, ..., w_{mI} + T_I)$ yields precisely the utilities $(u_1^*, ..., u_I^*)$.

Proof under the partial equilibrium setting

Recall from the proof of the first fundamental welfare theorem, we showed that for any Pareto optimal levels of utility $(u_1^*, \dots, u_I^*)$, it must satisfy

$$
\begin{aligned}
&D1. \quad p^* \leq c_j'(q_j), \text{ with equality if } q_j^* = 0 \qquad j = 1, \dots, J \\
&D2. \quad \phi_i'(x_i) \leq p^*, \text{ with equality if } x_i^* > 0 \qquad i = 1, \dots, I \\
&D3. \quad \sum_{i=1}^{I} x_i^* = \sum_{j=1}^{J} q_j^*
\end{aligned}
$$

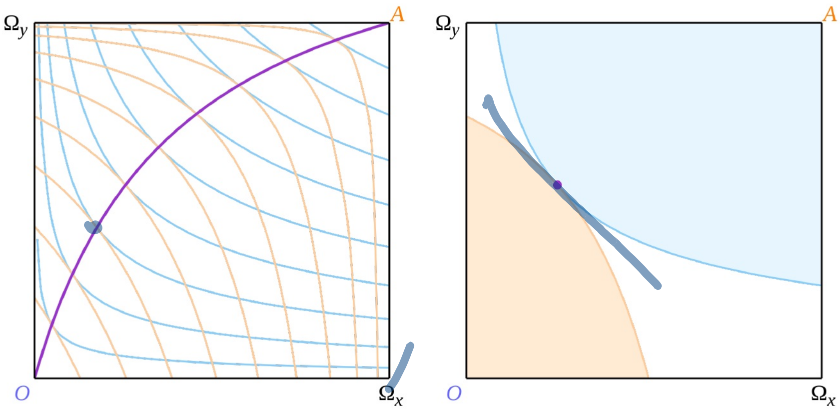
Hence under quasilinear preferences,

$$
\begin{aligned}
\sum_i u_i^* &= \sum_{i=1}^{I} \phi_i(x_i^*) + w_m - \sum_{j=1}^{J} c_j(q_j) \\
&= \sum_{i=1}^{I} (\phi_i(x_i^*) + w_{mi} + T_i - \sum_{j=1}^{J} \theta_{ij} c_j(q_j))
\end{aligned}
$$

By construction the second line constitutes a Pareto optimal utility under transfer.

Edgeworth Box

First Fundamental Welfare Theorem



Edgeworth Box

Second Fundamental Welfare Theorem