Midterm Exam

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EI037 Microeconomics

This is a closed book exam. You need to solve this exam alone and independently. Your answers should be legible, clear, and concise. In order to get full credit you have to give complete answers, including how answers are derived. Partial answers will lead to partial credit. Wrong additional statements (i.e., guessing) might reduce the given credit.

You have 2 hours to complete this exam. Each sub-question is worth 0.5 points, adding up to a total of 6 points. Allocate your time wisely across questions. Good luck!

1. Weak Axiom and Consumer Choice

Let the consumer demand $x(p, w)$ be single valued, homogeneous of degree zero, and satisfies Walras' law. We learned the following definition of the **weak axiom of revealed preference (WA)** in the context of the Walrasian demand function:

> The demand function $x(p, w)$ satisfies the WA, if for any two price-wealth situations (p, w) and (p', w'), we always have that $p \cdot x(p', w') \leq w$ and $x(p', w') \neq x(p, w) \Rightarrow p' \cdot x(p, w) > w'$.

1.a. Explain in words the intuition behind the definition of the WA provided above.

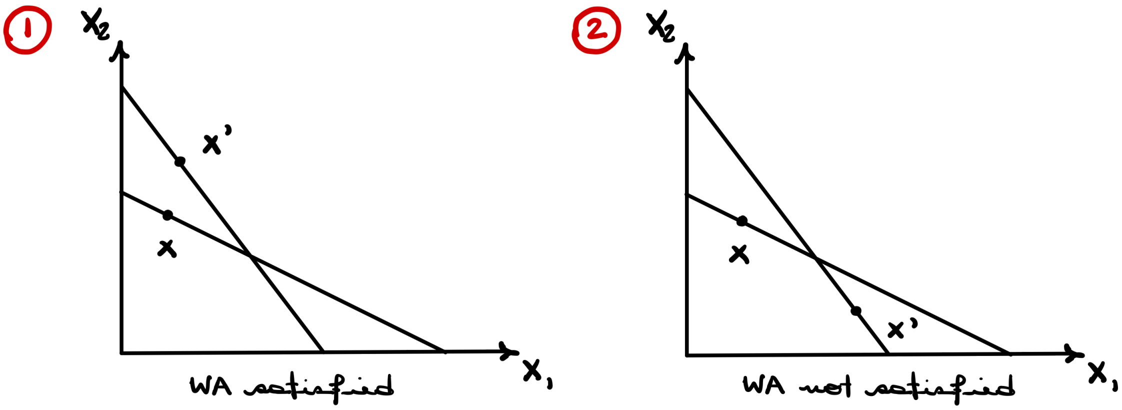
Answer: *The weak axiom of revealed preference (WA) states that if a consumption bundle x is ever chosen when another consumption bundle x' is available, then the bundle x is revealed preferred to bundle x' and it must be the case that whenever bundle x' is chosen, bundle x is not affordable - i.e., too expensive - to the consumer.*

1.b. Provide one graphical example when the WA is violated, and one example when the WA is not violated.

Answer: *Graphs 1 and 2 below represent situations in which the WA is satisfied and not satisfied, respectively. For clarity, in both graphs the bundle x represents the individual's observed consumption choice under a price-wealth situation (p, w), while the bundle x' represents the individual's observed consumption choice under a price-wealth situation (p', w').*

Graph 1: in price-wealth situation (p', w'), the consumer chooses bundle x', although bundle x was also affordable to them. Conversely, in price-wealth situation (p, w), the consumer could not have afforded bundle x', and chooses bundle x instead - that is, bundle x' is outside of the consumer's budget set for (p, w). The WA is therefore satisfied.

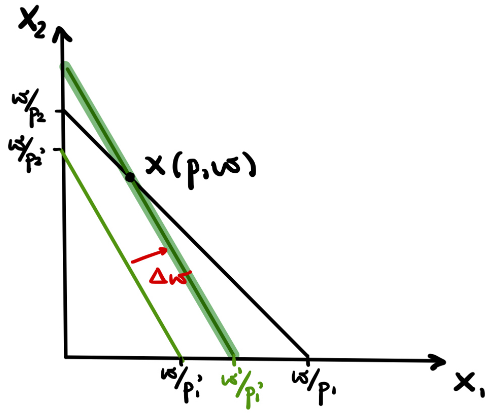
Graph 2: in price-wealth situation (p, w), the consumer could have chosen either bundle x or x', as both were affordable to them. But the consumer chose x, which implies that x is revealed *preferred over x'. In price-wealth situation (p', w'), both consumption bundles x or x' were affordable, but the consumer chose x' and not x. The WA is therefore violated.*



1.c. What is a compensated price change (i.e., Slutsky compensation)? Explain graphically.

Answer: *A compensated price change describes how consumer demand changes after a change in prices for which the consumer's wealth is adjusted such that they are still able to afford their original consumption bundle under the new price situation. It therefore isolate the effect of a price change to be only that of a change in relative prices, while keeping the consumer's purchasing power the same.*

Graphically, we have



Here, we observe that $x(p, w)$ is the consumer's initial consumption bundle under a price-wealth situation (p, w) and we assume that the prices of both goods 1 and 2 increase to p'_1 and p'_2. A compensated price change therefore adjusts the consumer's wealth level by Δw, shifting the consumer's budget line to the right such that they can still afford the original consumption bundle $x(p, w)$ under a new price-wealth situation (p', w').

1.d. Provide a graphical example such that $x(p, w)$ satisfies the WA and the compensated law of demand, but **NOT** the *uncompensated* law of demand.

[Hint: *In case you don't remember the definition*]

We say that the **compensated law of demand (CLD)** holds if for any compensated price change from an initial price-wealth situation (p, w) to a new price-wealth pair $(p', w') = (p', p' \cdot x(p, w))$ we have

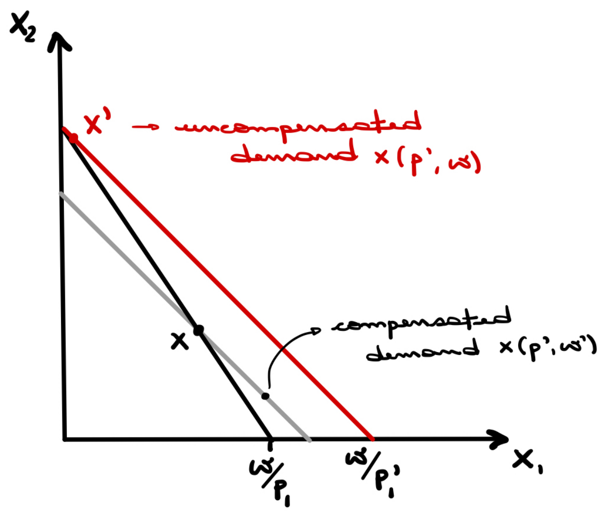
$$(p' - p) \cdot [x(p', w') - x(p, w)] \leq 0$$

with strict inequality whenever $x(p, w) \neq x(p', w')$.

Answer: *Under the definition of the* ***uncompensated*** *demand function, we should have for any two consumption bundles $x \neq x'$ that*

$$(p' - p) \cdot [x(p', w) - x(p, w)] \leq 0$$

where the consumer wealth w remains unchanged (uncompensated) before and after a change in prices from p to p'. The graph below represents a situation in which the WA is satisfied, but ***not*** *the uncompensated law of demand.*



The consumption bundle x' is chosen such that is it just marginally more expensive than the consumer's original budget. Based on this example, we verify that

$$\begin{aligned}(p' - p) \cdot (x' - x) &= p'x' - p'x - px' + px \\ &= w - p'x - px' + w\end{aligned}$$

where $px' = w$ and $p'x < w$, such that $(p' - p)(x' - x) > 0$. We therefore verify that the uncompensated law of demand does not hold, even if the WA is satisfied.

2. Classical Demand Theory

Consider the utility function

$$u(x) = x_1^{\frac{1}{3}} x_2^{\frac{2}{3}}$$

2.a. Find the Walrasian demand function for goods 1 and 2 as a function of prices and wealth.

Answer: *We begin by solving for the consumer's utility maximization problem, in order to obtain their optimal demand of each good. We have*

$$\max_{x_1, x_2 \geq 0} \quad x_1^{\frac{1}{3}} x_2^{\frac{2}{3}} \quad s.t.\ p_1 x_1 + p_2 x_2 \leq w$$

The problem's Lagrangian is therefore

$$\mathcal{L} = x_1^{\frac{1}{3}} x_2^{\frac{2}{3}} - \lambda(p_1 x_1 + p_2 x_2 - w)$$

Since the consumer's utility function is of the Cobb-Douglas form, we can directly derive the optimal consumer demand

$$x_1^*(p, w) = \frac{w}{3p_1} \quad and \quad x_2^*(p, w) = \frac{2w}{3p_2}$$

2.b. Find the Hicksian (compensated) demand function $h(p, u)$ for goods 1 and 2.

Answer: *We solve for the consumer's expenditure minimization problem (EMP)*

$$\min_{x_1, x_2 \geq 0} \quad p_1 x_1 + p_2 x_2 \quad s.t. \quad x_1^{\frac{1}{3}} x_2^{\frac{2}{3}} \geq \bar{u}$$

The consumer's Lagrangian is set up as

$$\mathcal{L} = p_1 x_1 + p_2 x_2 - \lambda(x_1^{\frac{1}{3}} x_2^{\frac{2}{3}} - \bar{u})$$

Take the first order derivative w.r.t x_1, x_2,respectively:

$$\begin{array}{l} \frac{\partial \mathcal{L}}{\partial x_1} = p_1 - \lambda \frac{1}{3} x_1^{-\frac{2}{3}} x_2^{\frac{2}{3}} = 0 \\ \frac{\partial \mathcal{L}}{\partial x_2} = p_2 - \lambda \frac{2}{3} x_1^{\frac{1}{3}} x_2^{-\frac{1}{3}} = 0 \end{array} \quad \Rightarrow \quad \frac{p_1}{p_2} = \frac{\frac{1}{3} x_2}{\frac{2}{3} x_1} \quad \Rightarrow \quad x_1 = \frac{\frac{1}{3} p_2 x_2}{\frac{2}{3} p_1}$$

Substituting x_1 into the consumer's utility constraint, we have

$$\left(\frac{\frac{1}{3} p_2 x_2}{\frac{2}{3} p_1} \right)^{\frac{1}{3}} x_2^{\frac{2}{3}} = \bar{u} \quad \Rightarrow \quad h_1^*(p, u) = \bar{u} \left(\frac{\frac{1}{3} p_2}{\frac{2}{3} p_1} \right)^{\frac{2}{3}}$$

$$and \quad h_2^*(p, u) = \bar{u} \left(\frac{\frac{2}{3} p_1}{\frac{1}{3} p_2} \right)^{\frac{1}{3}}$$

2.c. Find the expenditure function $e(p, u)$, and verify that $h(p, u) = \nabla_p e(p, u)$.

Answer: *The consumer's expenditure function, $e(p,u)$ is given by*

$$
\begin{aligned}
e(p,u) &= p_1 h_1(p,u) + p_2 h_2(p,u) \\
&= p_1 \bar{u} \left(\frac{\frac{1}{3}p_2}{\frac{2}{3}p_1} \right)^{\frac{2}{3}} + p_2 \bar{u} \left(\frac{\frac{2}{3}p_1}{\frac{1}{3}p_2} \right)^{\frac{1}{3}} \;=\; \bar{u} \left(\frac{1}{2}\right)^{\frac{2}{3}} p_1^{\frac{1}{3}} p_2^{\frac{2}{3}} \;+\; \bar{u} \left(\frac{2}{1}\right)^{\frac{1}{3}} p_1^{\frac{1}{3}} p_2^{\frac{2}{3}} \\
&= \bar{u}\; p_1^{\frac{1}{3}} p_2^{\frac{2}{3}} \left[\left(\frac{1}{2}\right)^{\frac{2}{3}} + \left(\frac{2}{1}\right)^{\frac{1}{3}} \right] \;\Rightarrow\; e(p,u) = \; \bar{u}\,(3p_1)^{\frac{1}{3}} \left(\frac{3p_2}{2} \right)^{\frac{2}{3}}
\end{aligned}
$$

We are asked to verify that $h(p,u) = \nabla_p e(p,u)$. Shephard's lemma allows us to derive the Hicksian demand functions for goods 1 and 2 from the consumer's expenditure function, as follows

$$
h_l(p,u) = \frac{\partial e(p,u)}{\partial p_l}
$$

We then have that

$$
\begin{aligned}
\frac{\partial e(p,u)}{\partial p_1} &= \frac{1}{3} \bar{u} \frac{p_1^{-\frac{2}{3}}}{\frac{1}{3}^{\frac{1}{3}}} \left(\frac{p_2}{\frac{2}{3}} \right)^{\frac{2}{3}} \;=\; \bar{u}\,(3p_1)^{-\frac{2}{3}} \left(\frac{p_2}{\frac{2}{3}} \right)^{\frac{2}{3}} \\
&= \bar{u} \left(\frac{p_2}{2p_1} \right)^{\frac{2}{3}}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial e(p,u)}{\partial p_2} &= \frac{2}{3} \bar{u} (3p_1)^{\frac{1}{3}} \frac{p_2^{-\frac{1}{3}}}{\frac{2}{3}^{\frac{2}{3}}} \;=\; \bar{u} (3p_1)^{\frac{1}{3}} \left(\frac{p_2}{\frac{2}{3}} \right)^{-\frac{1}{3}} \\
&= \bar{u} \left(\frac{2p_1}{p_2} \right)^{\frac{1}{3}}
\end{aligned}
$$

2.d. Find the indirect utility function $v(p,w)$, and verify Roy's identity.

Answer: *To obtain the consumer's indirect utility function $v(p,w)$, we plug the optimal Marshallian demands we found in item (a) into the consumer's utility function $u(x_1,x_2)$*

$$
\begin{aligned}
v(p,w) = u(x_1^*, x_2^*) &= \left(\frac{w}{3p_1} \right)^{\frac{1}{3}} \left(\frac{2w}{3p_2} \right)^{\frac{2}{3}} \\
&= w \left(\frac{1}{3p_1} \right)^{\frac{1}{3}} \left(\frac{2}{3p_2} \right)^{\frac{2}{3}}
\end{aligned}
$$

To verify Roy's identity, we first derive each of it's elements separately

$$
\begin{aligned}
\frac{\partial v(p,w)}{\partial p_1} &= -w \left(\frac{1}{3p_1} \right)^{\frac{4}{3}} \left(\frac{2}{3p_2} \right)^{\frac{2}{3}} \\
\frac{\partial v(p,w)}{\partial p_2} &= -w \left(\frac{1}{3p_1} \right)^{\frac{1}{3}} \left(\frac{2}{3p_2} \right)^{\frac{5}{3}}
\end{aligned}
$$

$$\frac{\partial v(p,w)}{\partial w} = \left(\frac{1}{3p_1}\right)^{\frac{1}{3}} \left(\frac{2}{3p_2}\right)^{\frac{2}{3}}$$

Then

$$-\frac{\frac{\partial v(p,w)}{\partial p_1}}{\frac{\partial v(p,w)}{\partial w}} = -\frac{-w\left(\frac{1}{3p_1}\right)^{\frac{4}{3}}\left(\frac{2}{3p_2}\right)^{\frac{2}{3}}}{\left(\frac{1}{3p_1}\right)^{\frac{1}{3}}\left(\frac{2}{3p_2}\right)^{\frac{2}{3}}} = \frac{w}{3p_1} = x_1^*(p,w)$$

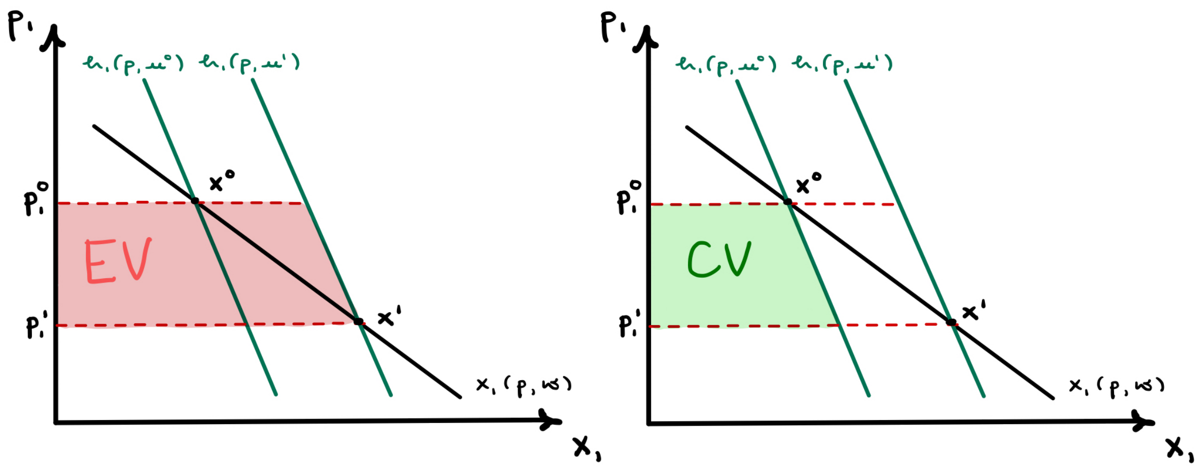
$$-\frac{\frac{\partial v(p,w)}{\partial p_2}}{\frac{\partial v(p,w)}{\partial w}} = -\frac{-w\left(\frac{1}{3p_1}\right)^{\frac{1}{3}}\left(\frac{2}{3p_2}\right)^{\frac{2}{3}}}{\left(\frac{1}{3p_1}\right)^{\frac{1}{3}}\left(\frac{2}{3p_2}\right)^{\frac{5}{3}}} = \frac{2w}{3p_2} = x_2^*(p,w)$$

3. Welfare Economics

3.a. Consider an economy with two goods, 1 and 2. Consider a price change from the initial price vector p^0 to a new price vector $p^1 \leq p^0$ in which only the price of good 1 changes. Show that $CV \leq EV$ if good 1 is a normal good.

Answer: *We observe a change in prices from p^0 to p^1 ($p^0 \geq p^1$), where only the price of good 1 changes in the price vector. If good 1 is a normal good, we know that the slope of the consumer's Hicksian demand function for good 1 will be steeper than that of the Marshallian demand for the same good.*

Graphically, we have

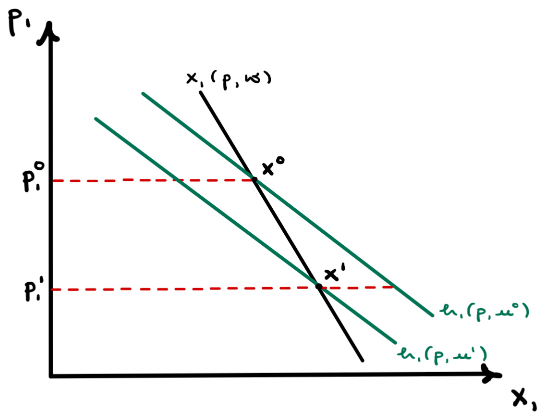


We know that the equivalent variation (EV) can be calculated as the area under the Hicksian demand function for the "new" utility level, $h_1(p,u^1)$, while the compensating variation (CV) will be the area under the Hicksian demand function for the "old" utility, $h_1(p,u^0)$. We know that when prices decrease, both EV and CV are positive. From the figure above, we can therefore *conclude that $CV \leq EV$ for the normal good 1.*

3.b. Consider an economy with two goods, 1 and 2. Consider a price change from the initial price vector p^0 to a new price vector $p^1 \leq p^0$ in which only the price of good 1 changes. Show that $CV \geq EV$ if good 1 is an inferior good.

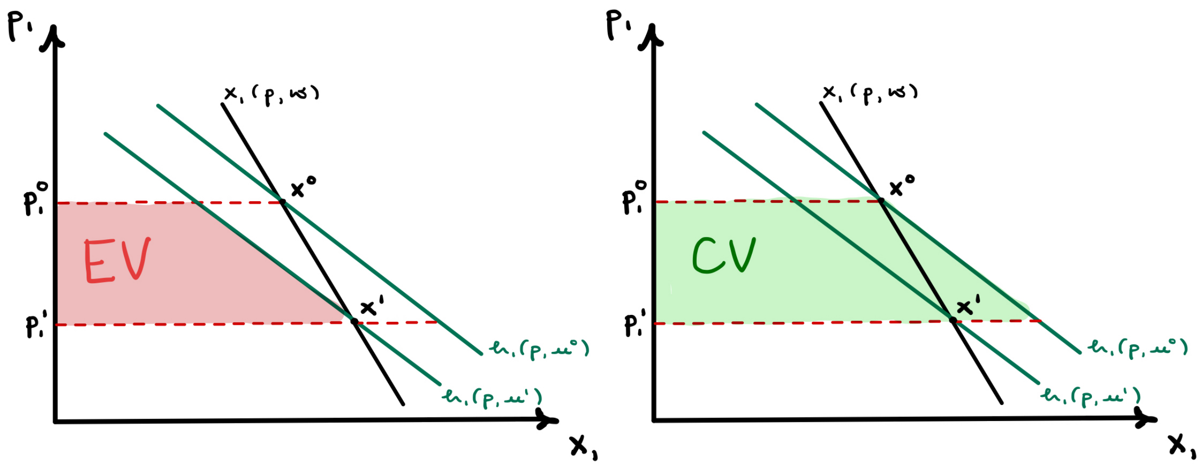
Answer: *We observe a change in prices from p^0 to p^1 ($p^0 \geq p^1$), where only the price of good 1 changes in the price vector. If good 1 is an inferior good, we know that the slope of the consumer's Hicksian demand function for good 1 will be flatter than that of the Marshallian demand for the same good.*

Graphically, we have



We know that the equivalent variation (EV) can be calculated as the area under the Hicksian demand function for the "new" utility level, $h_1(p, u^1)$, while the compensating variation (CV) will be the area under the Hicksian demand function for the "old" utility, $h_1(p, u^0)$.

Graphically, we have



We know that when prices decrease, both EV and CV are positive. From the figure above, we can therefore conclude that $CV \geq EV$ for the inferior good 1.

3.c. Patrick's utility function is $u(x_1, x_2) = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$, where good 1 is food and good 2 is housing. Patrick gets a monthly salary of \$3000. The price of good 1 and the price of good 2 are $p_1 = 2$ and $p_2 = 2$. Patrick's boss is thinking of sending him to another town where the price of food is the same, but the price of housing is 8. The boss offers no raise in pay. Patrick, who understands compensating and equivalent variations perfectly, complains bitterly. He says that although he doesn't mind moving for its own sake and the new town is just as pleasant as the old, having to move is as bad as a cut in pay of \$A. He also says he wouldn't mind moving if - when he moved - he got a raise of \$B. What are A and B equal to?

Answer: *\$A represents Patrick's equivalent variation (EV), while \$B represents their compensating variation (CV). We know that EV and CV are given by*

$$\begin{aligned} EV &= e(p^0, u^1) - e(p^0, u^0) \quad \textit{or} \quad v(p^0, w + EV) = v(p^1, w) = u^1 \\ CV &= e(p^1, u^1) - e(p^1, u^0) \quad \textit{or} \quad v(p^0, w) = v(p^1, w - CV) = u^0 \end{aligned}$$

We begin by solving Patrick's utility maximization problem, in order to obtain their optimal demand of each good. We have

$$\max_{x_1, x_2 \geq 0} \quad x_1^{\frac{1}{2}} x_2^{\frac{1}{2}} \quad s.t. \ p_1 x_1 + p_2 x_2 \leq w$$

The problem's Lagrangian is therefore

$$\mathcal{L} = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}} - \lambda(p_1 x_1 + p_2 x_2 - w)$$

Since the consumer's utility function is of the Cobb-Douglas form, we know that Patrick's optimal demand for goods 1 and 2 will be, respectively

$$x_1^* = \frac{w}{2p_1} \quad \textit{and} \ x_2^* = \frac{w}{2p_2}$$

We can derive Patrick's indirect utility function, $v(p, w)$, from their optimal demand functions

$$v(p, w) = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}} = \left(\frac{w}{2p_1}\right)^{\frac{1}{2}} \left(\frac{w}{2p_2}\right)^{\frac{1}{2}} = \frac{w}{2(p_1 p_2)^{\frac{1}{2}}}$$

We are given that $w = 3000$, $p_1^0 = p_2^0 = 2$, $p_1^1 = 2$ and $p_2^1 = 8$. We can then calculate Patrick's EV and CV as follows

EV: $v(p^0, w + EV) = v(p^1, w) = u^1$

$$\frac{(w + EV)}{2(p_1^0 p_2^0)^{\frac{1}{2}}} = \frac{w}{2(p_1^1 p_2^1)^{\frac{1}{2}}} \quad \Rightarrow \quad \frac{(3000 + EV)}{2 \times (2 \times 2)^{\frac{1}{2}}} = \frac{3000}{2 \times (2 \times 8)^{\frac{1}{2}}} \quad \Rightarrow \quad \frac{3000 + EV}{2} = \frac{3000}{4}$$

$$\Rightarrow \quad (3000 + EV) = 3000 \times \left(\frac{2}{4}\right) \quad \Rightarrow \quad EV = -1500$$

CV: $v(p^0, w) = v(p^1, w - CV) = u^0$

$$\frac{(w)}{2(p_1^0 p_2^0)^{\frac{1}{2}}} = \frac{w - CV}{2(p_1^1 p_2^1)^{\frac{1}{2}}} \quad \Rightarrow \quad \frac{3000}{2 \times (2 \times 2)^{\frac{1}{2}}} = \frac{3000 - CV}{2 \times (2 \times 8)^{\frac{1}{2}}} \quad \Rightarrow \quad \frac{3000}{2} = \frac{3000 - CV}{4}$$

$$\Rightarrow \quad 3000 \times \left(\frac{4}{2}\right) = 3000 - CV \quad \Rightarrow \quad CV = -3000$$

Hence for Patrick, having to move is as bad as a cut in pay of \$1500. He wouldn't mind moving if he got a raise of \$3000. In other words, A equals to 1500 and B equals to 3000.

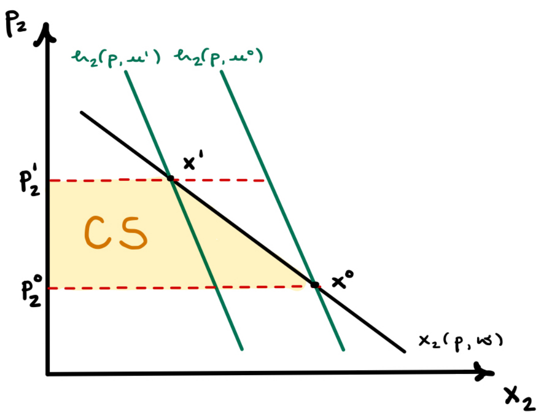
3.d. Patrick's boss does not understand the concepts of compensating and equivalent variations, but he understand the concepts of Marshallian demand and consumer surplus. So he offers offer to raise Patrick's salary by \$C if Patrick moves. What is C equal to?

Answer: *Not understanding the concepts of compensating and equivalent variation, Patrick's boss will offer a raise of \$C to Patrick's salary that corresponds to the change in consumer surplus suffered by Patrick on the occasion of his move.*

We know that the consumer surplus (CS) can be calculated as the area under the Marshallian demand function for good 2, $x_2(p, w)$. We must therefore calculate Patrick's optimal demand of good 2 under both old and new price situations.

$$x_2^*(p^0, w) = \frac{w}{2p_2^0} = \frac{3000}{2 \times 2} = 750 \quad \textit{and} \quad x_2^*(p^1, w) = \frac{w}{2p_2^1} = \frac{3000}{2 \times 8} = 187.5$$

Graphically we have



The consumer surplus will therefore be

$$CS = \frac{(750 + 187.5) \times (8 - 2)}{2} = 2812.5$$

Hence Patrick's boss will offer Patrick a raise of \$2812.5. In other words, C is equal to 2812.5.