7/3/25

REVIEW SESSION (PS 2)

Part 3

Aggregation of Preference

Single crossing condition (SCC)

$q > q'$ & $\alpha'' > \alpha'$

or if $q < q'$ & $\alpha'' < \alpha'$

then $W(q; \alpha^i) \geq W(q'; \alpha')$

Graphically, they can only cross once

Q1) $$W^i(g) = \underbrace{(y - g)\frac{y^i}{y}}_{\text{Linear}} + \underbrace{H(g)}_{\text{Concave}}$$

Preferences should fulfil SCC.

The quasi linear structure of policy preference implies

Concavity Concavity shows single peaked which show SCC holds.

- SCC is satisfied if $g_i > g$ and $y_i' > y_0'$,

the SCC requires

$$W(g_i, y_i') \geq W(g_0, g_i')$$

$$\Downarrow$$

$$W'(g_i, y_0') \geq W'(g_0, y_0')$$

(Slide 19 ↑ α_i implies poor & vice versa in problem set)

$$(y-g_1)\frac{y^i}{y} + H(g_1) \geq (y-g_0)\frac{y^i}{y} + H(g_0)$$

$$H(g_1) - H(g_0) \geq (y-g_0)\frac{y^i}{y} - (y-g_1)\frac{y^i}{y}$$

$$H(g_1) - H(g_0) \geq (\cancel{y} - g_0 - \cancel{y} + g_1)\frac{y_1^i}{y}$$

$$H(g_1) - H(g_0) \geq (g_1 - g_0)\frac{y^i}{y}$$

$$\left[\frac{H(g_1) - H(g_0)}{g_1 - g_0} \geq \frac{y_1^i}{y} > \frac{y_0^i}{y}\right. \qquad (y_1^i > y_0^i)$$

$$\boxed{W^i(g_1, y_1^i) > W^i(g_0, y_0^i)}$$

If $g_2 < g_0$ & $y_2^i < y_0^i$

$$W^i(g_2, y_2^i) \geq W^i(g_0, y_0^i) \Rightarrow W^i(g_2, g_0^i) > W^i(g_0, y_0^i)$$

Q2) a) $w^i = c^i + \alpha^i \ln g$ (population normalised to 1)

$y^i = 1 \ \forall i$

$c^i \leq y - T = 1 - T$ (Budget constraint)

a) g^* of a social planner

$$\tilde{W}(T, g, \alpha^i) = 1 - T + \alpha^i \ln g$$

$$\int_{\alpha_i} T\, dF(\alpha^i) = \int_{\alpha_i} g\, dF(\alpha^i) \Leftrightarrow T = g$$

Total taxes collected = Total govt (public good provision) spending

Find optimal τ that optimises / maximises the citizens policy preference

$$\max_{\tau} \int_{\alpha_i} [1 - \tau + \alpha^i \ln \tau] \, dF(\alpha_i)$$

$$\int_{\alpha_i} [1 - \tau + \alpha^i \ln \tau] \, dF(\alpha_i)$$

$$\underbrace{\int_{\alpha_i} 1 \, dF(\alpha_i)}_{=1} - \int_{\alpha_i} \tau \, dF(\alpha^i) + \int \alpha^i \ln \tau \, dF(\alpha^i)$$

(cont. dependent on α)

$$= 1 - \tau + \ln \tau \int_{\alpha_i} \alpha^i \, dF(\alpha_i)$$

$E[\alpha^i] = \alpha$

$E[X] = \int x f(x) \, dx$

$$\Rightarrow 1 - \tau + \alpha \ln \tau$$

FOC $\quad -1 + \frac{\alpha}{\tau} = 0$

$$g^* = \tau^* = \alpha$$

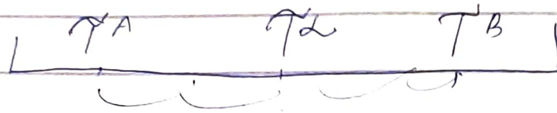
b) $P = A, B$. Platforms τ^A & τ^B

- Exogeneous rent R.
- τ_A, τ_B (votes %)
- $P_P = \text{Prob}(\pi_P \geq \frac{1}{2})$, $P_P \cdot R$

Assume $\alpha^i = \alpha$ (homogenous voters)

$P = A$

$$P_1 = \begin{cases} 0 & \text{if } W(\tau^A,\alpha) < W(\tau^B,\alpha) \\ 1/2 & \text{if } W(\tau^A,\alpha) = W(\tau^B,\alpha) \\ 1 & \text{if } W(\tau^A,\alpha) > W(\tau^B,\alpha) \end{cases}$$



$$\tau^A = \tau^B = \tau^* = \alpha.$$

Each person wins with prob (1/2)

c) $\alpha' \neq \alpha$ (from before) $\forall i$, $\underbrace{W(\tau, \alpha')}_{\text{single peaked}} = \overbrace{1 - \tau}^{\text{linear}} + \underbrace{\alpha' \ln \tau}_{\text{concave}}$

equilibrium if maximised

FOC $\quad -1 + \frac{\alpha'}{\tau} = 0 \Rightarrow \tau^*(\alpha') = \alpha'$

$$\tau^A = \tau^B = \tau^M(\alpha^m) = \alpha^m$$

$$P_{A,B} = \frac{1}{2}$$

d) $\alpha m > \alpha \Rightarrow$ overspending
$\alpha m < \alpha \Rightarrow$ under provision
$\alpha m = \alpha$