Macroeconomics A; EI060

Short problems

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Class of May 7, 2025

1 Expected labor

Question: The Home and Foreign output relations are given by:

$$Y = \frac{1}{2}\left(\frac{PC}{P_H} + \frac{P^*C^*}{P_H^*}\right) = \frac{\mu}{2}\left(\frac{1}{P_H} + \frac{\mu^*}{\mu P_H^*}\right) = \frac{PC}{2}\left(\frac{1}{P_H} + \frac{1}{\mathcal{E} P_H^*}\right)$$

The price indices are:

$$\begin{aligned} P &= 2\left[P_H\right]^{0.5}\left[P_F\right]^{0.5} \\ P^* &= 2\left[P_H^*\right]^{0.5}\left[P_F^*\right]^{0.5} \end{aligned}$$

The various prices are:

$$\begin{aligned} P_H &= \frac{\theta\kappa}{\theta-1} E\left(\frac{\mu}{Z}\right) \\ P_F^* &= \frac{\theta\kappa}{\theta-1} E\left(\frac{\mu^*}{Z^*}\right) \\ P_F &= (\mathcal{E})^{\gamma^*}\frac{\theta\kappa}{\theta-1} E\left(\frac{\mu^*}{Z^*}(\mathcal{E})^{1-\gamma^*}\right) \\ P_H^* &= (\mathcal{E})^{-\gamma}\frac{\theta\kappa}{\theta-1} E\left(\frac{\mu}{Z}(\mathcal{E})^{\gamma-1}\right) \end{aligned}$$

The exchange rate is $\mathcal{E} = \mu/\mu^*$. The technology is $Y = Zl$.

Show that the expected labor is:

$$E(l) = \frac{\theta-1}{\theta\kappa}$$

2 Expected log consumption

Question: We can show that when prices are flexible (take this as given):

$$C^{\text{flex}} = \frac{\theta-1}{\theta 2\kappa}(Z)^{0.5}(Z^*)^{0.5}$$

Show that consumption under sticky prices is:

$$C = \frac{(\mu)^{1-\frac{\gamma^*}{2}} (\mu^*)^{\frac{\gamma^*}{2}}}{\left[E\left(\frac{\mu}{Z}\right)\right]^{0.5} \left[E\left(\frac{\mu^*}{Z^*} (\mathcal{E})^{1-\gamma^*}\right)\right]^{0.5}} \frac{\theta-1}{2\theta\kappa}$$

Show that the gap between the expected log consumption and its value under flexible prices is:

$$\begin{aligned} E(\ln C) - E\left(\ln C^{\text{flex}}\right) &= \left(1-\frac{\gamma^*}{2}\right)\sum_k \pi_k \ln \mu_k + \frac{\gamma^*}{2}\sum_k \pi_k \ln \mu_k^* \\ &\quad -\frac{1}{2}\sum_k \pi_k \ln Z_k - \frac{1}{2}\sum_k \pi_k \ln Z_k^* \\ &\quad -\frac{1}{2}\ln\left[\sum_k \pi_k \frac{\mu_k}{Z_k}\right] - \frac{1}{2}\ln\left[\sum_k \pi_k \frac{(\mu_k)^{1-\gamma^*}(\mu_k^*)^{\gamma^*}}{Z_k^*}\right] \end{aligned}$$

where k is an index of state of nature, and π_k denotes the probability of the state.

3 Optimal Home policy

Question: Show that the value of a specific state μ_k that maximizes $E(\ln C) - E\left(\ln C^{\text{flex}}\right)$ is:

$$0 = \left(1-\frac{\gamma^*}{2}\right) - \frac{1}{2}\frac{\frac{\mu_k}{Z_k}}{\sum_k \pi_k \frac{\mu_k}{Z_k}} - \frac{1-\gamma^*}{2}\frac{\frac{(\mu_k)^{1-\gamma^*}(\mu_k^*)^{\gamma^*}}{Z_k^*}}{\sum_k \pi_k \frac{(\mu_k)^{1-\gamma^*}(\mu_k^*)^{\gamma^*}}{Z_k^*}}$$

We take a log linear approximation around $\mu_k = \mu_k^* = Z_k = Z_k^* = \mu_0 = Z_0 = 1$. For instance:

$$\begin{aligned} \frac{\mu_k}{Z_k} &= \frac{\mu_0}{Z_0} + \frac{1}{Z_0}(\mu_k - \mu_0) - \frac{\mu_0}{(Z_0)^2}(Z_k - Z_0) \\ \frac{\mu_k}{Z_k} &= \frac{\mu_0}{Z_0}\left[1 + \frac{\mu_k - \mu_0}{\mu_0} - \frac{Z_k - Z_0}{Z_0}\right] \\ \frac{\mu_k}{Z_k} &= \frac{\mu_0}{Z_0}\left[1 + \left[\ln(\mu_k) - \ln(\mu_0)\right] - \left[\ln(Z_k) - \ln(Z_0)\right]\right] \\ \frac{\mu_k}{Z_k} &= \frac{\mu_0}{Z_0}\left[1 + \ln(\mu_k) - \ln(Z_k)\right] \end{aligned}$$

Note that to a first order, $E[\ln(\mu_k)] = E[\ln(Z_k)] = 0$. Show that with this approximation:

$$\left[1 + (1-\gamma^*)^2\right]\ln(\mu_k) = \ln(Z_k) + (1-\gamma^*)\ln(Z_k^*) - \gamma^*(1-\gamma^*)\ln(\mu_k^*)$$

4 Joint optimal rule

Question: Following similar steps, we can show (take this as given):

$$\left[1 + (1-\gamma)^2\right]\ln(\mu_k^*) = (1-\gamma)\ln(Z_k) + \ln(Z_k^*) - \gamma(1-\gamma)\ln(\mu_k)$$

Assuming a symmetric situation ($\gamma = \gamma^*$) show that:

$$\begin{aligned} \ln(\mu_k) + \ln(\mu_k^*) &= \ln(Z_k) + \ln(Z_k^*) \\ \ln(\mu_k) - \ln(\mu_k^*) &= \frac{\gamma}{1 + (1-2\gamma)(1-\gamma)}\left[\ln(Z_k) - \ln(Z_k^*)\right] \end{aligned}$$

hence:

$$
\begin{aligned}
\ln\left(\mu_k\right) &= \frac{1-\gamma\left(1-\gamma\right)}{1-\gamma\left(1-\gamma\right)+\left(1-\gamma\right)^2}\ln\left(Z_k\right)+\frac{\left(1-\gamma\right)^2}{1-\gamma\left(1-\gamma\right)+\left(1-\gamma\right)^2}\ln\left(Z_k^*\right) \\
\ln\left(\mu_k\right) &= \frac{1-\gamma\left(1-\gamma\right)}{1-\gamma\left(1-\gamma\right)+\left(1-\gamma\right)^2}\ln\left(Z_k^*\right)+\frac{\left(1-\gamma\right)^2}{1-\gamma\left(1-\gamma\right)+\left(1-\gamma\right)^2}\ln\left(Z_k\right)
\end{aligned}
$$