ECON 39: Undergraduate International Trade
Heckscher-Ohlin Example Questions
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1. Consider a single country (the U.S.) that produces two goods (footballs and soccer balls) using two factors (capital and labor) that are perfectly mobile across sectors. In this example, we will derive the autarkic equilibrium for the following parameters.

$$Q_{US}^{FB} = \left(L_{US}^{FB}\right)^{\frac{1}{2}} \left(K_{US}^{FB}\right)^{\frac{1}{2}}$$
$$Q_{US}^{SB} = \left(L_{US}^{SB}\right)^{\frac{1}{3}} \left(K_{US}^{SB}\right)^{\frac{2}{3}}$$
$$L_{US} = 12$$
$$K_{US} = 12$$

What is also nice about this example is that up until the last step (where we pin down the autarkic equilibrium relative price), the method will be identical for how you would solve for the free trade equilibrium.

(a) In autarky, will the U.S. produce both goods? Why or why not?
Yes, since the preferences are such that the representative agent wants to consume equal quantities of both goods, we know that both goods will be produced.

(b) Using the fact that profits are maximized in each sector, write the equilibrium wage and rental rate in each sector.
We have:

$$w_{US}^{FB} = p_{US}^{FB} \times \frac{\partial Q_{US}^{FB}}{\partial L} = \frac{1}{2} p_{US}^{FB} \left(\frac{L_{US}^{FB}}{K_{US}^{FB}}\right)^{-\frac{1}{2}}$$
$$r_{US}^{FB} = p_{US}^{FB} \times \frac{\partial Q_{US}^{FB}}{\partial K} = \frac{1}{2} p_{US}^{FB} \left(\frac{L_{US}^{FB}}{K_{US}^{FB}}\right)^{\frac{1}{2}}$$
$$w_{US}^{SB} = p_{US}^{SB} \times \frac{\partial Q_{US}^{SB}}{\partial L} = \frac{1}{3} p_{US}^{SB} \left(\frac{L_{US}^{SB}}{K_{US}^{SB}}\right)^{-\frac{2}{3}}$$
$$r_{US}^{SB} = p_{US}^{SB} \times \frac{\partial Q_{US}^{SB}}{\partial K} = \frac{2}{3} p_{US}^{SB} \left(\frac{L_{US}^{SB}}{K_{US}^{SB}}\right)^{\frac{1}{3}}$$

(c) Using the fact that workers and capital owners maximize their own income, write the equilibrium U.S. wage and rental rate as a function of the labor to capital ratio in the football sector AND as a function of the labor to capital ratio in the soccer ball sector.

$$w_{US} = \frac{1}{2} p_{US}^{FB} \left(\frac{L_{US}^{FB}}{K_{US}^{FB}}\right)^{-\frac{1}{2}} = \frac{1}{3} p_{US}^{SB} \left(\frac{L_{US}^{SB}}{K_{US}^{SB}}\right)^{-\frac{2}{3}}$$
$$r_{US} = \frac{1}{2} p_{US}^{FB} \left(\frac{L_{US}^{FB}}{K_{US}^{FB}}\right)^{\frac{1}{2}} = \frac{2}{3} p_{US}^{SB} \left(\frac{L_{US}^{SB}}{K_{US}^{SB}}\right)^{\frac{1}{3}}$$

(d) Using the fact that the total amount of labor and capital employed has to equal the endowment of labor and capital, write down an expression relating the aggregate labor to capital ratio as a function of the labor to capital ratio in both sectors:
In general, we have:

$$\frac{L_{US}}{K_{US}} = \frac{L_{US}^{FB}}{K_{US}} + \frac{L_{US}^{SB}}{K_{US}} = \frac{L_{US}^{FB}}{K_{US}^{FB}} \frac{K_{US}^{FB}}{K_{US}} + \frac{L_{US}^{SB}}{K_{US}^{SB}} \frac{K_{US}^{SB}}{K_{US}}$$

With the particular endowments used above we have:

$$1 = \frac{L_{US}^{FB}}{K_{US}^{FB}} \frac{K_{US}^{FB}}{K_{US}} + \frac{L_{US}^{SB}}{K_{US}^{SB}} \frac{K_{US}^{SB}}{K_{US}}$$

(e) Define $l_{US}^{FB} \equiv \frac{L_{US}^{FB}}{K_{US}^{FB}}$ and $l_{US}^{SB} \equiv \frac{L_{US}^{SB}}{K_{US}^{SB}}$ to be the labor to capital ratios in both sectors. Normalize the price of soccer balls in the U.S. to one, i.e. $p_{US}^{SB} = 1$. Finally, define $\lambda \equiv \frac{K_{US}^{FB}}{K_{US}}$ to be the fraction of capital allocated to the production of footballs. Solve for the equilibrium l_{US}^{FB}, l_{US}^{SB}, and λ as a function of the relative price of footballs p_{US}^{FB}.

To do so, let us first restate the equilibrium conditions from the last two questions using these normalizations:

$$\frac{1}{2}p_{US}^{FB}\left(l_{US}^{FB}\right)^{-\frac{1}{2}} = \frac{1}{3}\left(l_{US}^{SB}\right)^{-\frac{2}{3}} \tag{1}$$

$$\frac{1}{2}p_{US}^{FB}\left(l_{US}^{FB}\right)^{\frac{1}{2}} = \frac{2}{3}\left(l_{US}^{SB}\right)^{\frac{1}{3}} \tag{2}$$

$$1 = \lambda l_{US}^{FB} + (1-\lambda)\, l_{US}^{SB} \tag{3}$$

Note that we have three equations and three unknowns. Hence, we should be able to solve for each of the unknowns. To do so, let us divide equation (2) by equation (1):

$$\frac{\frac{1}{2}p_{US}^{FB}\left(l_{US}^{FB}\right)^{\frac{1}{2}}}{\frac{1}{2}p_{US}^{FB}\left(l_{US}^{FB}\right)^{-\frac{1}{2}}} = \frac{\frac{2}{3}\left(l_{US}^{SB}\right)^{\frac{1}{3}}}{\frac{1}{3}\left(l_{US}^{SB}\right)^{-\frac{2}{3}}} \iff$$

$$l_{US}^{FB} = 2 l_{US}^{SB} \tag{4}$$

Substituting equation (4) back into equation (1) yields:

$$\begin{aligned}
\frac{1}{2}p_{US}^{FB}\left(l_{US}^{FB}\right)^{-\frac{1}{2}} &= \frac{1}{3}\left(l_{US}^{SB}\right)^{-\frac{2}{3}} \iff \\
\frac{1}{2}p_{US}^{FB}\left(2l_{US}^{SB}\right)^{-\frac{1}{2}} &= \frac{1}{3}\left(l_{US}^{SB}\right)^{-\frac{2}{3}} \iff \\
l_{US}^{SB} &= \left(\frac{3}{2^{\frac{3}{2}}}p_{US}^{FB}\right)^{-6}
\end{aligned} \tag{5}$$

Substituting equation (5) into equation (4) then yields:

$$l_{US}^{FB} = 2\left(\frac{3}{2^{\frac{3}{2}}}p_{US}^{FB}\right)^{-6} \tag{6}$$

Finally, substituting equation (4) into equation (3) yields:

$$\begin{aligned}
1 &= \lambda l_{US}^{FB} + (1-\lambda)\, l_{US}^{SB} \iff \\
1 &= \lambda 2 l_{US}^{SB} + (1-\lambda)\, l_{US}^{SB} \iff \\
1 &= (1+\lambda)\, l_{US}^{SB} \iff \\
\lambda &= \frac{1}{l_{US}^{SB}} - 1 \iff \\
\lambda &= \left(\frac{3}{2^{\frac{3}{2}}}p_{US}^{FB}\right)^{6} - 1,
\end{aligned} \tag{7}$$

where the last line used equation (5). Equations (5), (6), and (7) are your answers.

(f) Given the previous answer, what is the equilibrium wage and rental rate as a function of the price? Are equilibrium wages increasing in the relative price of footballs? What about equilibrium rental rates? Why or why not?

From the answer to question (3), we know that:

$$w_{US} = \frac{1}{2}p_{US}^{FB}\left(l_{US}^{FB}\right)^{-\frac{1}{2}}$$

Substituting in equation (5) yields:

$$w_{US} = \frac{1}{2} p_{US}^{FB} \left(2\left(\frac{3}{2^{\frac{3}{2}}} p_{US}^{FB}\right)^{-6}\right)^{-\frac{1}{2}} \iff$$
$$w_{US} = \left(p_{US}^{FB}\right)^4 \frac{3^3}{2^6}$$

Similarly, from the answer to question (3), we know that:

$$r_{US} = \frac{1}{2} p_{US}^{FB} \left(l_{US}^{FB}\right)^{\frac{1}{2}}$$

Substituting in equation (5) yields:

$$r_{US} = \frac{1}{2} p_{US}^{FB} \left(2\left(\frac{3}{2^{\frac{3}{2}}} p_{US}^{FB}\right)^{-6}\right)^{\frac{1}{2}} \iff$$
$$r_{US} = \frac{1}{2^{\frac{1}{2}}} \left(\frac{3}{2^{\frac{3}{2}}}\right)^{-3} \left(p_{US}^{FB}\right)^{-2}$$

Equilibrium wages are increasing in p_{US}^{FB} while equilibrium rental rates are decreasing in p_{US}^{FB}. This is because the production of footballs is labor intensive while the production of soccer balls is capital intensive.

(g) Write the equilibrium production of soccer balls and footballs as a function of the equilibrium price. As the relative price for footballs increases, how does the equilibrium production of footballs and soccer balls respond?

We first have to write the quantity produced as a function of the labor to capital ratio in soccer ball production and the fraction of capital allocated to the production of footballs:

$$Q_{US}^{SB} = \left(L_{US}^{SB}\right)^{\frac{1}{3}} \left(K_{US}^{SB}\right)^{\frac{2}{3}} \iff$$
$$Q_{US}^{SB} = K_{US}^{SB} \left(l_{US}^{SB}\right)^{\frac{1}{3}} \iff$$
$$Q_{US}^{SB} = K_{US} (1-\lambda) \left(l_{US}^{SB}\right)^{\frac{1}{3}}$$

Since we know that $K_{US} = 12$ and we have functions for λ (in equation (7)) and l_{US}^{SB} (in equation (5)), we have:

$$Q_{US}^{SB} = K_{US} (1-\lambda) \left(l_{US}^{SB}\right)^{\frac{1}{3}} \iff$$
$$Q_{US}^{SB} = 12 \times \left(2 - \left(\frac{3}{2^{\frac{3}{2}}} p_{US}^{FB}\right)^{6}\right) \times 2^{\frac{1}{3}} \left(\frac{3}{2^{\frac{3}{2}}} p_{US}^{FB}\right)^{-2} \tag{8}$$

We can do a similar thing for the production of footballs using equations (6) and (7):

$$Q_{US}^{FB} = \left(L_{US}^{FB}\right)^{\frac{1}{2}} \left(K_{US}^{FB}\right)^{\frac{1}{2}} \iff$$
$$Q_{US}^{FB} = K_{US}^{FB} \left(l_{US}^{FB}\right)^{\frac{1}{2}}$$
$$Q_{US}^{FB} = K_{US} \lambda \left(l_{US}^{FB}\right)^{\frac{1}{2}} \iff$$
$$Q_{US}^{FB} = 12 \times \left(\left(\frac{3}{2^{\frac{3}{2}}} p_{US}^{FB}\right)^{6} - 1\right) \left(2^{\frac{1}{2}} \left(\frac{3}{2^{\frac{3}{2}}} p_{US}^{FB}\right)^{-3}\right) \tag{9}$$

It is a little difficult to tell, but as p_{US}^{FB} increases, Q_{US}^{SB} decreases and Q_{US}^{FB} increases. As in the specific factor model, this is because (loosely speaking) it is more profitable for the country as a whole to focus its production on the good which it can sell at the higher price.

(h) Note that thus far we have said nothing about preferences! All that preferences will end up doing is pinning down the equilibrium autarkic relative price. To see this, use the fact that the representative consumer maximizes its utility to write down an expression for the equilibrium price p_{US}^{FB} solely as a function of exogenous variables.

From utility maximization we know that it must be the case that the representative agent consumes and equal amount of both goods, i.e.:

$$C_{US}^{FB} = C_{US}^{SB}$$

Since in autarky, production of each good equals the consumption of each good, we then have:

$$Q_{US}^{FB} = Q_{US}^{SB}$$

From the last question, however, we have expressions for the quantity of footballs and soccer balls produced. By setting equations (8) and (9) equal we have:

$$12 \times \left(2 - \left(\frac{3}{2^{\frac{3}{2}}} p_{US}^{FB}\right)^6\right) \times 2^{\frac{1}{3}} \left(\frac{3}{2^{\frac{3}{2}}} p_{US}^{FB}\right)^{-2} = 12 \times \left(\left(\frac{3}{2^{\frac{3}{2}}} p_{US}^{FB}\right)^6 - 1\right) \left(2^{\frac{1}{2}} \left(\frac{3}{2^{\frac{3}{2}}} p_{US}^{FB}\right)^{-3}\right)$$

This equation could be solved (by a computer, not by hand!) for p_{US}^{FB}.

2. Consider a world composed of many different countries. We will focus on the economy of Lilliput, an island much too small to affect world prices. Suppose that Lilliput is endowed with 16 units of labor and 16 units of capital. Suppose that Lilliputians can produce either footballs or soccer balls. The production function of footballs is given by $Q^{FB} = \left(L^{FB}\right)^{\frac{1}{2}} \left(K^{FB}\right)^{\frac{1}{2}}$, where Q^{FB} is the quantity of footballs produced, L^{FB} is the labor allocated to the production of footballs, and K^{FB} is the capital allocated to the production of soccer balls. Conversely, the production function of soccer balls is given by $Q^{SB} = \left(L^{SB}\right)^{\frac{2}{3}} \left(K^{SB}\right)^{\frac{1}{3}}$, where Q^{SB} is the quantity of footballs produced, L^{SB} is the labor allocated to the production of footballs, and K^{SB} is the capital allocated to the production of soccer balls. Let the world relative price of footballs to soccer balls be denoted by p, which Lilliputians take as exogenous (given their island is so small).

(a) Assume that Lilliputians produce both footballs and soccer balls. Use the profit maximization conditions to derive the equilibrium labor to capital ratios in both the football and soccer balls sectors as function of prices.

- *The profit maximizing equations for labor are:*

$$w = p^{FB} MPL^{FB} = p^{SB} MPL^{SB} \iff$$
$$p = \frac{MPL^{SB}}{MPL^{FB}} = \frac{\frac{2}{3}\left(l^{SB}\right)^{-\frac{1}{3}}}{\frac{1}{2}\left(l^{FB}\right)^{-\frac{1}{2}}}$$

The profit maximizing equations for capital are:

$$r = p^{FB} MPK^{FB} = p^{SB} MPK^{SB} \iff$$
$$p = \frac{MPK^{SB}}{MPK^{FB}} = \frac{\frac{1}{3}\left(l^{SB}\right)^{\frac{2}{3}}}{\frac{1}{2}\left(l^{FB}\right)^{\frac{1}{2}}}$$

where $l^{SB} \equiv \frac{L^{SB}}{K^{SB}}$ and $l^{FB} \equiv \frac{L^{FB}}{K^{FB}}$. We can then solve the following equations simultaneously to get the equilibrium labor to capital ratios as a function of prices in both sectors:

$$p = \frac{\frac{2}{3}\left(l^{SB}\right)^{-\frac{1}{3}}}{\frac{1}{2}\left(l^{FB}\right)^{-\frac{1}{2}}}$$

$$p = \frac{\frac{1}{3}\left(l^{SB}\right)^{\frac{2}{3}}}{\frac{1}{2}\left(l^{FB}\right)^{\frac{1}{2}}}$$

To do so, I first cancel out prices:

$$\frac{\frac{2}{3}\left(l^{SB}\right)^{-\frac{1}{3}}}{\frac{1}{2}\left(l^{FB}\right)^{-\frac{1}{2}}} = \frac{\frac{1}{3}\left(l^{SB}\right)^{\frac{2}{3}}}{\frac{1}{2}\left(l^{FB}\right)^{\frac{1}{2}}} \iff$$
$$2l^{FB} = l^{SB}$$

and then I substitute that back into either expression:

$$p = \frac{\frac{1}{3}\left(l^{SB}\right)^{\frac{2}{3}}}{\frac{1}{2}\left(l^{FB}\right)^{\frac{1}{2}}} \iff$$
$$p = \frac{\frac{1}{3}\left(2l^{FB}\right)^{\frac{2}{3}}}{\frac{1}{2}\left(l^{FB}\right)^{\frac{1}{2}}} \iff$$
$$p = \frac{2}{3}2^{\frac{2}{3}}\left(l^{FB}\right)^{\frac{2}{3}-\frac{1}{2}} \iff$$
$$l^{FB} = p^6\left(\frac{2}{3}2^{\frac{2}{3}}\right)^{-6} \tag{10}$$

and:

$$l^{SB} = p^6 2\left(\frac{2}{3}2^{\frac{2}{3}}\right)^{-6} \tag{11}$$

(b) Combine your answer in part (a) with the market clearing condition to derive the equilibrium fraction of capital allocated to the production of footballs as a function of world prices.

- *The market clearing condition is:*

$$\frac{L}{K} = \frac{L^{FB}}{K^{FB}} \times \frac{K^{FB}}{K} + \frac{L^{SB}}{K^{SB}}\left(1 - \frac{K^{FB}}{K}\right) \iff$$
$$\frac{L}{K} = l^{FB}\lambda + l^{SB}(1-\lambda),$$

where $\lambda \equiv \frac{K^{FB}}{K}$. *We can solve this equation for* λ *as follows:*

$$\frac{L}{K} = l^{FB}\lambda + l^{SB}(1-\lambda) \iff$$
$$\frac{L}{K} = \left(l^{FB} - l^{SB}\right)\lambda + l^{SB} \iff$$
$$\lambda = \frac{\frac{L}{K} - l^{SB}}{l^{FB} - l^{SB}}$$

Substituting in equations (10) and (11) yields:

$$\lambda = \frac{p^6 2\left(\frac{2}{3}2^{\frac{2}{3}}\right)^{-6} - \frac{L}{K}}{p^6\left(\frac{2}{3}2^{\frac{2}{3}}\right)^{-6}} \iff$$
$$\lambda = 2 - \frac{L}{K}p^{-6}\left(\frac{2}{3}2^{\frac{2}{3}}\right)^{6} \tag{12}$$

(c) What is the range of world relative prices such that the Lilliputians will produce both footballs and soccer balls?

- *Note that the Lilliputians produce both footballs and soccer balls as long as* $\lambda \in (0,1)$. *Since* $\frac{L}{K} = 1$ *we can then solve:*

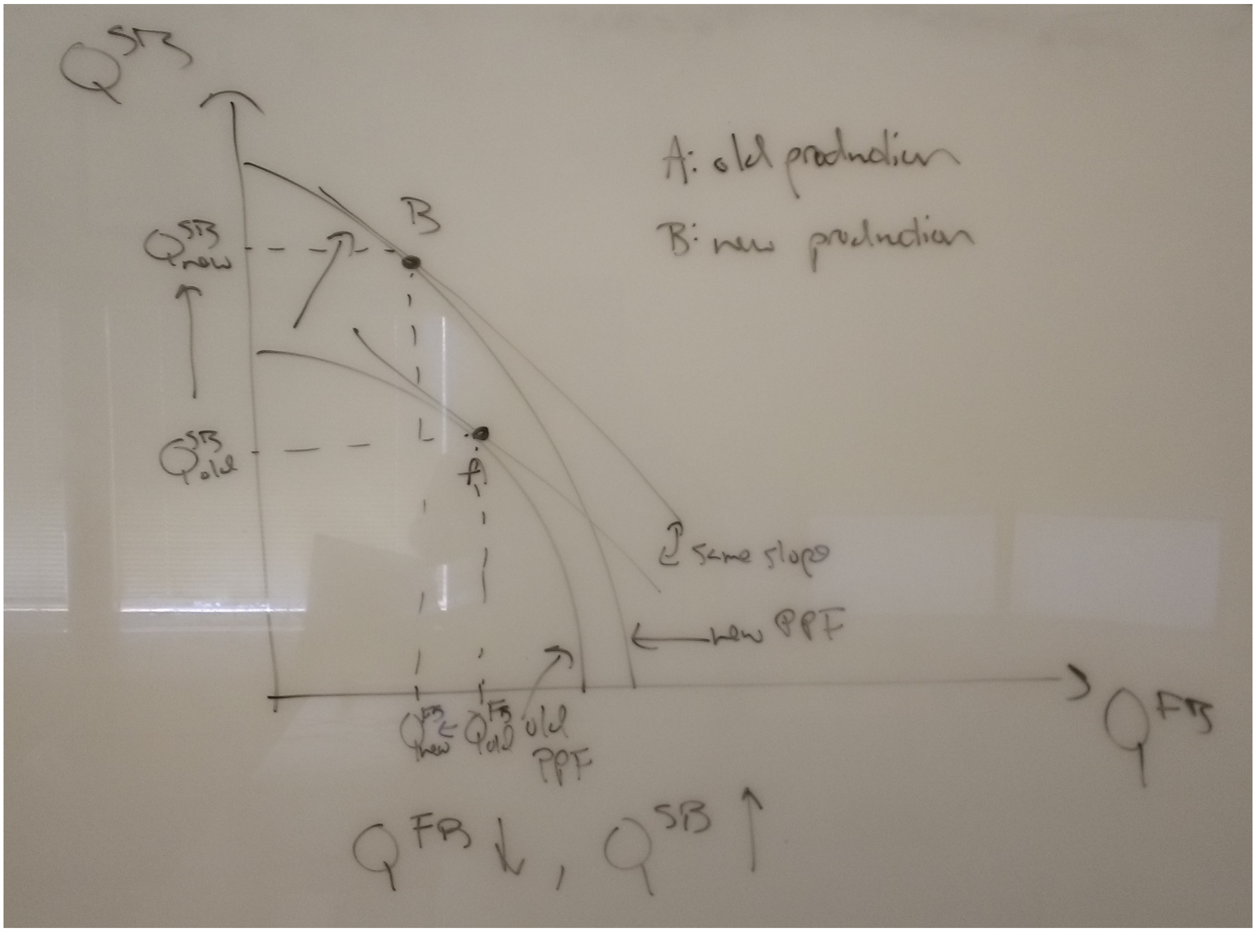
$$
\begin{aligned}
2 - \frac{L}{K} p^{-6} \left(\frac{2}{3} 2^{\frac{2}{3}} \right)^6 < 1 \iff \\
1 < p^{-6} \left(\frac{2}{3} 2^{\frac{2}{3}} \right)^6 \iff \\
p^6 < \left(\frac{2}{3} 2^{\frac{2}{3}} \right)^6 \iff \\
p < \frac{2}{3} 2^{\frac{2}{3}}
\end{aligned}
$$

Similarly:

$$
\begin{aligned}
2 - p^{-6} \left(\frac{2}{3} 2^{\frac{2}{3}} \right)^6 > 0 \iff \\
2 > p^{-6} \left(\frac{2}{3} 2^{\frac{2}{3}} \right)^6 \iff \\
p^6 > \frac{1}{2} \left(\frac{2}{3} 2^{\frac{2}{3}} \right)^6 \iff \\
p > 2^{-6} \times \frac{2}{3} \times 2^{\frac{2}{3}}
\end{aligned}
$$

(d) Suppose a strange man washes ashore on Lilliput, increasing the island's labor endowment. Use a figure to show how this changes the production of footballs and soccer balls. What is the economic intuition?

- *See the attached figure:*



Intuitively, because soccer balls are labor intensive, an increase in the labor endowment will cause a shift in production toward soccer balls. (This is the Rybczynski Theorem).