# Macroeconomics A; EI056

# Short problems

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## 1 Consumption allocation

### 1.1 Optimal choice of a specific variety

**Question**: The consumer wants to reach the highest value of the consumption basket $C$ which is a geometric average of consumption of various varieties indexed by $j$ (distributed over the unit interval $j \in [0, 1]$). Consumption of variety $j$ is denoted by $C_j$. The basket is:

$$C = \left[ \int_0^1 [C_j]^{\frac{\theta-1}{\theta}} \, dj \right]^{\frac{\theta}{\theta-1}}$$

The consumer minimizes the total spending $\int_0^1 P_j C_j dj$ subject to the basket being at a target value $C$. Show that the first-order condition with respect to consumption of various varieties indexed by $j$ is (is the multiplier on the constraint):

$$P_j = \lambda \left[ \int_0^1 [C_j]^{\frac{\theta-1}{\theta}} \, dj \right]^{\frac{1}{\theta-1}} [C_j]^{-\frac{1}{\theta}}$$

### 1.2 Total spending

**Question**: We denote total spending by $PC = \int_0^1 P_j C_j dj$. From the result above, show that: $P = \lambda$.

### 1.3 Demand for a specific variety

**Question**: Using the results so far show that the demand for $C_j$ is:

$$C_j = \left[ \frac{P_j}{P} \right]^{-\theta} C$$

### 1.4 Price index

**Question**: Using the results and the definition of the consumption basket, show that::

$$P = \left[ \int_0^1 [P_j]^{1-\theta} \, dj \right]^{\frac{1}{1-\theta}}$$

## 2 Flexible price allocation

### 2.1 Real marginal cost

**Question**: Consider the New Keynesian model with all prices flexible. The labor supply and the Euler conditions are:

$$\begin{aligned} \frac{W_t}{P_t} (C_t)^{-\sigma} &= \chi (N_t)^{\eta} \\ \frac{1}{P_t} (C_t)^{-\sigma} &= \beta (1 + i_t) E_t \left[ \frac{1}{P_{t+1}} (C_{t+1})^{-\sigma} \right] \end{aligned}$$

The firm producing variety $j$ uses the following technology (N_{j,t} is the labor it uses, $Z_t$ is a productivity parameter):

$$Y_{j,t} = Z_t (N_{j,t})^a$$

The firm pays the wage $W_t$. Show that the real marginal cost is:

$$\Psi_{j,t} = \frac{1}{a} \frac{W_t}{P_t} (Z_t)^{-\frac{1}{a}} (Y_{j,t})^{\frac{1-a}{a}}$$

### 2.2 Optimal price

**Question**: The firm producing variety $j$ knows that it faces the following demand curve (output of any variety goes solely to consumption:

$$Y_{j,t} = \left[ \frac{P_{j,t}}{P_t} \right]^{-\theta} C_t$$

Show that it sets the following real price:

$$\frac{P_{j,t}}{P_t} = \frac{\theta}{\theta - 1} \Psi_t$$

### 2.3 Labor market clearing

**Question**: As all firms set their prices, they choose the same price ($P_{j,t} = P_t$) and produce the same quantity ($Y_{j,t} = Y_t = Y_t$).

Using the optimal price and the labor supply, show that:

$$Y_t = \left[ \frac{a}{\chi} \frac{\theta - 1}{\theta} \right]^{\frac{1}{1+\eta+(\sigma-1)a}} (Z_t)^{\frac{1+\eta}{1+\eta+(\sigma-1)a}}$$

which in terms of log deviations from the steady-state (deviations denoted by lower case letters) is:

$$y_t = \frac{1 + \eta}{1 + \eta + (\sigma - 1) a} z_t$$

### 2.4 Output dynamics

**Question**: Show that taking a log approximation of the Euler condition we get:

$$y_t = E_t y_{t+1} - \frac{1}{\sigma} r_t$$

where the real interest rate is (upper bar denotes steady state values):

$$r_t = \frac{i_t - \bar{r}}{1 + \bar{r}} - (E_t p_{t+1} - p_t)$$