Mathematical Modelling for Starling Murmuration

COP290 Design Practices

April-May 2018

1 Introduction

A murmuration of starlings is an amazing sight - a swooping mass of thousands of birds whirling in the sky above. Accurate modeling of the amazing flocking behavior of starlings has not been possible until now, for they are devilish in their complexity.

The aim of the project is to model and simulate this fascinating phenomenon. We will computationally simulate the phenomenon by modelling each bird as an independent agent communicating and cooperating with other neighbouring agents. The task, however, is not to worry too much about each birds individual behavior but to dream up a set of rules that govern them all. So in this case the behavior of each individual is based on its cruise speed, the position and movement direction of its neighbors, and for the first time other newly identified factors such as each birds attraction to the roost and the simplified physics of aerodynamic flight, which includes banking while turning.

Our objective will be to measure from a realistic simulation the average energy spend by each bird, the angular momentum and the force that each bird has to withstand in a typical flight ritual.

2 Mathematical Model

1. The uncanny co-ordination of these flocks is controlled by several complex real life factors. At individual level, the rues guiding this are relatively simple.
   The flocking behaviour of starlings at the individual level is generally controlled by 3 rules :-

   (a) **Cohesion :-** The birds tend to move towards the center of mass of the other birds in its near vicinity. This steering towards the average position of the local flock mates is termed as the Cohesion. There is an attraction among the birds that are far off from each other upto a certain distance.

   (b) **Alignment :-** Movement just towards the center of mass would lead to the non-alignment of the directions in which the different birds are independently moving.

Alignment, i.e., steering towards the average velocity of neighbours is an important factor deciding the direction of movement of the individual entities.

(c) **Separation :-** The birds cannot be allowed to come close to each other indefinitely. They have to be separated if they come closer than they are expected to. Separation means to avoid crowding neighbours (or to avoid collision). It is short range repulsion.

There are other factors like Obstacles, Predators, Destination etc but the above 3 are the basic rules.

2. **Characteristics of a single bird**
There are some characteristics which would be required to describe the state of each bird independently. Theses characteristics tend to change with time.

   (a) **Position :-** We can define position of a bird by using a co-ordinate system. It will have x, y and z components. The position of a bird changes with time depending upon its velocity.
   (b) **Speed :-** Speed of a bird is the rate with which it is travelling. It is basically the magnitude of velocity.
   (c) **Direction vector :-** It tells the direction in which the bird is travelling. It is given by the velocity vector of the bird divided by the magnitude of its speed.
   (d) **Mass :-** It is the mass of the bird. It does not change with time.

   With the above 4 characteristics, a bird can be completely specified for our model.

3. **Flocking behaviour :-**

   Now, given all the characteristics of a bird at time t, we have to find its characteristics at t + dt. We can determine these characteristics as follows:-

   (a) **Position :-** Position at time t + dt can be determined by using position and velocity at time t.

$$x_{t+dt} = x_t + vx_t * dt$$
$$y_{t+dt} = y_t + vy_t * dt$$
$$z_{t+dt} = z_t + zy_t * dt$$

   (b) **Mass :-** Mass will not change with time.
   (c) **Neighbours :-** As all the birds are moving, the neighbours may change. We will find the new neighbours by calculating the distance between all the birds(by using the position of the birds) and then comparing it with the neighbour radius r.

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2} < r$$

(d) **Velocity :-**

We will first see how the direction of velocity varies. It depends upon the 3 basic rules :-

i. **Cohesion :-** It says that a bird will steer towards the average position of its neighbours. So, firstly , we will calculate the Centre of Mass (COM) of the neighbours. Let it be (xc,yc,zc). So the bird having position p(x,y,z) will move towards (xc-x,yc-y,zc-z). This will be the new steering direction.

ii. **Alignment :-** According to this rule, a bird will steer towards the average velocity of neighbours. If the average velocity of neighbours is in the direction (x,y,z) then the bird will move towards this direction.

iii. **Separation :-** This rule says that the birds have to avoid collision with its neighbours. For this we have to consider a collision radius rc such that for a bird if there is any neighbour in this radius then the bird has to go away from that neighbour. So, if there are n neighbours in the collision radius, then the bird will steer away from the centre of mass(xc,yc,zc) of these neighbours. It means that the bird with position(x,y,z) will move in the direction (x-xc,y-yc,z-zc).

Now each of the above rules can return different moving directions for the bird. So, we have to assign weights to each rule. Let the weight of cohesion rule, alignment rule and separation rule be Fc, Fa, and Fs respectively. Fc is directly proportional to the distance of the bird from the centre of mass of its neighbours while Fs is inversely proportional to it.

$$Fa + Fc + Fs = 1$$
$$V_f = Fc * V_c + Fa * V_a + Fs * V_s$$

Here Vc, Va and Vs are moving directions due to cohesion, alignment and separation respectively. Vf is the final moving direction.
The speed (magnitude of the velocity) varies between a certain range. Its value is maximum when the bird is either very far from its neighbours or it is very close to them(in the collision sphere).

Hence, we have determined how the various characteristics of a bird will change with time.

4. **Boundaries :-**
We have assumed that the birds will be confined within a certain volume (a cube). So whenever, a bird reaches crosses a boundary , it will emerge from the opposite boundary with same velocity.

3 Energy and Force calculations

1. **Average energy spent each bird :-** Two methods can be used to calculate average energy spent by each bird.

(a) We can calculate the average energy of the entire flock at a time t, by using the velocity at time t.

$$E_{avg} = (m_1(v_1)^2 + m_2(v_2)^2 + ... + m_n(v_n)^2)/2n$$

(b) We can integrate the energy of a bird until time T and then take average over it.

$$E_{avg} = \int_0^T m(v^2)dt/T$$

2. **Average angular momentum of a bird :-** Integrate angular momentum(about origin) of the bird untill time T and then take average over it.

$$\vec{L}_{avg} = \int_0^T m(\vec{v} * \vec{r})dt/T$$

3. **Average force experienced by a bird :-** Since average force has to be calculated, we can simply write it as

$$\vec{F}_{avg} = \int_0^T m(\frac{dv}{dt})dt/T = \int_0^T m(dv)/T = m(v_T - v_0)/T$$